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Date November 30, 1990

OPTIMIZATION OF THE THERMAL DESIGN OF A CRYOGENICALLY-COOLED COMPUTER

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

James C. Anderson
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ABSTRACT

The recent design of large computers having heat dissipation rates in excess of 1 kW demands research in the area of the possibility of cryogenic cooling systems. Direct immersion liquid cooling of the CPU boards can be implemented in order to maintain low board temperatures and avoid damage to the circuit components. In this paper, the performance and cost of three types of cryogenic cooling systems, the "once-through" system, the saturated pool boiling refrigerated sytem, and the subcooled pool boiling refrigerated system, will be analyzed.

In the "once-through" system, the liquid nitrogen, chosen for its liquid temperature range and low cost, is allowed to boil on the CPU boards and is then vented to the surroundings. Such systems are currently in use since many installations requiring the use of large computers have supplies of liquid nitrogen readily available for numerous other applications.

The second cooling system which was investigated is the saturated pool boiling system. In this system, the gaseous nitrogen generated from boiling on the CPU boards is condensed by a condenser placed in the vapor space above the liquid nitrogen pool. The required refrigeration is produced by a cryocooler operating on the Stirling cycle.

The final cooling system which was analyzed is the subcooled pool boiling system. This system is very similar to the saturated system discussed above with the exception that the condenser is completely submerged in the liquid nitrogen, which is subcooled to a temperature below the saturation temperature of 77 K.

Thermal/fluid numerical models used in conjunction with an optimization code, GRG2, were used to predict the performance and cost of the cryogenic cooling systems. The initial cost of the "once-through" system is approximately $41,150. The annual operating cost is approximately $20,540. The initial cost of the saturated pool boiling system is approximately $102,400, while the annual power cost is approximately $16,000. If the liquid nitrogen is subcooled from the saturation temperature of 77 K to 68 K, the uniform annual cost of the cooling system is practically unaffected. However, the critical heat flux for the subcooled system is increased by approximately 40% over the value for the saturated system. This increase in critical heat flux may become significant in preventing thermal failure or degradation of the electronic components if local values of the heat flux exceed approximately 14 W/cm^2. However, due to the extremely high initial cost of the refrigerated systems, the "once-through" cryogenic

cooling system is recommended if the local heat flux on the CPU board remains below 14 W/cm^2.

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LIST OF SYMBOLS

A	Surface area (m^2)
A_c	Cross-sectional area (m^2)
b	Fin parameter $= (L_{fin} + t/2)^{3/2} (h/kA_p)^{1/2}$ for radial fins $= L_{fin}(2h/kt)^{1/2}$ for axial fins
c_p	Specific heat (J/kg-K)
$C_{\Delta p}$	Regenerator pressure drop coefficient
C_{R1}, C_{R2}	Regenerator pressure drop constants
D	Diameter (m)
e	Pipe roughness (m)
f	Darcy friction factor
F_{1-2}	Configuration factor
F_e	Emissivity factor for radiant exchange $= 1/(1/\varepsilon_1 + 1/\varepsilon_2 - 1)$
g	Acceleration due to gravity = 9.81 m/s^2
h	Enthalpy (kJ/kg)
h	Heat transfer coefficient (W/m^2-K)
h_{fg}	Enthalpy of evaporation (kJ/kg)
k	Thermal conductivity (W/m-K)
l	Mesh spacing in regenerator (m)
L	Length (m)
L_p	Amplitude of piston travel in cryocooler (m)
$\dot{m}$	Mass flow rate (kg/s)
n	Number of mesh wire screens in regenerator
Nu_D	Nusselt number
Pr	Prandtl number
P	Pressure (kPa)
P_1 / P_2	Pressure ratio in cryocooler
$\dot{Q}$	Heat transfer rate (W)
q''	Nominal heat flux on CPU board = 0.4 W/cm^2
$\dot{Q}_{board}$	Rate of heat transfer from CPU boards (W)
$\dot{Q}_{condenser}$	Rate of heat transfer in condenser (W)
$\dot{Q}_{leads}$	Rate of heat transfer through the leads (W)
$\dot{Q}_{line}$	Rate of heat transfer through the transfer lines (W)
$\dot{Q}_{ref}$	Refrigeration capacity (W)
$\dot{Q}_{sub}$	Refrigeration required to subcool liquid nitrogen (W)

$\dot{Q}_{vacuum}$	Rate of net radiative exchange across the vacuum insulation layer (W)
$\dot{Q}_{vessel}$	Rate of heat transfer through the CPU vessel (W)
r_1	Inner pipe radius (m)
r_2	Outer pipe radius (m)
r_3	Outer pipe radius including insulation (m)
R	Ideal gas constant (kJ/kg-K)
R''_f	Fouling factor
R_L	Volume fraction of liquid phase
R_{th}	Thermal resistance (K/W)
Ra_D	Rayleigh number based on inner diameter
Re_D	Reynolds number based on inner diameter
t	Fin thickness (m)
T	Temperature (K)
T_c	Temperature of working gas in the compression space of the cryocooler (K)
T_d	Temperature of working gas in the dead space of the cryocooler (K)
T_e	Temperature of working gas in the expansion space of the cryocooler (K)
T_H	Maximum temperature of the working fluid in the Gifford-McMahon cryocooler (K)
T_L	Minimum temperature of the working fluid in the Gifford-McMahon cryocooler (K)
U	Overall heat transfer coefficient (W/m^2 K)
V	Volume (liters)
v	Velocity (m/s)
W_{fan}	Fan power (W)
x	Quality of saturated nitrogen (mass of nitrogen vapor / total mass of nitrogen)
X	Dead volume ratio
X	Lockhart-Martinelli parameter for two-phase flow $= (\Delta p_{liquid} / \Delta p_{vapor})^{1/2}$

GREEK SYMBOLS

α	Thermal diffusivity (m^2/s)
β	Volumetric thermal expansion coefficient (K^{-1})
β	Coefficient of performance of cryocooler
γ	Specific heat ratio $= c_p / c_v$
Δp	Pressure drop (kPa)
Δp_f	Frictional pressure drop (kPa)
Δp_{fan}	Pressure rise across fan (kPa)
ΔT_{LM}	Log mean temperature difference
ε	Emissivity
η_{fan}	Fan efficiency
η_{fin}	Fin efficiency
η_p	Pump efficiency
η_o	Temperature effectiveness of fin $= 1 - (A_{fin} / A) (1 - \eta_{fin})$
θ	Phase angle of piston in cryocooler (radians)
μ	Dynamic viscosity ($N\text{-}s/m^2$)
ν_c	Kinematic viscosity of working gas in cryocooler at T_c (m^2/s)
ν_e	Kinematic viscosity of working gas in cryocooler at T_e (m^2/s)
ν_d	Kinematic viscosity of working gas in cryocooler at T_d (m^2/s)
ρ	Density (kg/m^3)
σ	Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$ $W/m^2\text{-}K^4$
τ	Temperature ratio $= T_c / T_e$
ω	Angular velocity of cryocooler (rad/s)

SUBSCRIPTS

a	Ambient
b	CPU board
c	Cold
cond	Condenser
e	Exit
ext	External
f	Fluid
h	Hot
hx	Heat exchanger
i	Insulation
in	Inlet
int	Internal
l	Liquid
m	Mesh wire in regenerator
p	Pipe
pist	Piston
ref	Refrigerator
reg	Regenerator
s	Surface
sat	Saturated
sub	Subcooled
v	Vapor

CHAPTER I

INTRODUCTION

Motivation for the Research

Cryogenics is the production and use of very cold temperatures. Although the point on the temperature scale where the cryogenic regime begins is not clearly defined, the National Bureau of Standards considers cryogenic temperatures to lie below -150 ^{o} C (123 K). The boiling points of many gases such as helium, hydrogen, neon, nitrogen, oxygen, and air lie below -150 ^{o} C at atmospheric pressure. Oxygen was the first gas to be liquefied. This occurred in the late 1870's. In 1892, a major breakthrough in cryogenic technology occurred when James Dewar developed the vacuum-jacketed vessel for cryogenic fluid storage. In 1895, Carl van Linde became the first man to recognize the industrial applications of gas liquefaction and put these ideas into practice. Linde installed the first air-liquefaction plant in the United States in 1907. The world's first successful rocket flight powered by liquid-oxygen-gasoline propellant was conducted by Dr. Robert H. Goddard in 1926. Around 1942, the world became aware of the military applications of cryogenic technology with the development of the V-2 weapon system, the first large liquid-propellant rocket. NASA used cryogenic fluids extensively in space vehicles during the 1960's. The 1970's have been called the "decade of diversification" for cryogenic engineering (Barron, 1985). Current applications involving cryogenic engineering include rocket propulsion, cryopumping, superconductivity, biological and medical applications, and food processing. Research in the area of cryogenic refrigeration and the use of cryogens (cryogenic refrigerants) to cool large computers is also presently underway.

Computers with relatively small average heat fluxes on their component boards are typically air-cooled. However, air cooling may not be sufficient for computers which must dissipate large amounts of power over a small surface area. Since liquids generally have much higher convective heat transfer coefficients than gases, such as air, they are often used to cool computers with high heat fluxes. Computers employing direct immersion water cooling, a technology introduced by Control Data Corporation in the 1970's, have been designed and marketed by many companies including IBM, Honeywell, and Sperry.

One interesting method of cooling computers involves the use of a cryogenic liquid. A cryogenic coolant allows operation of the computer boards at cryogenic temperatures which provides several significant advantages. Kirschman (1985) has identified several advantages of cryogenically-cooled electronics as: (1) faster switching times of semiconductor devices, (2) increased circuit speed resulting from lower electrical resistances of the interconnecting materials, (3) improved heat removal due to higher thermal conductivity of many circuit materials, (4) reduced power dissipation resulting from lower operating voltage, and (5) reduction of thermally-induced failures of devices and components. The first liquid-nitrogen-cooled supercomputer, the ETA-10, was installed at Florida State University in 1986 (Krane, Parsons, and Bar-Cohen, 1988).

Although cryogenic cooling has many advantages, it also has several disadvantages. The greatest disadvanage of a cryogenic cooling system is the cost. It is anticipated that the initial cost as well as the operating power cost will be high. In a refrigerated system, the refrigerator requires a very large power input in proportion to the useful refrigeration produced. For example, cooling liquid nitrogen to 63 K at an ambient temperature of 300 K results in a coefficient of performance value of only 26.6%. Therefore, the development of a cost-effective, cryogenically-cooled computer will involve extensive thermal design efforts to maximize the utilization of the cryogenic fluid. Other possible problems of cryogenic cooling systems include their complexity and reliability.

Objectives

Based on the preceding discussion, the objectives of this thesis are

(1) The development of thermal/fluid numerical models which will predict the performance of the various alternative cryogenic cooling systems for a computer, and

(2) The manipulation of these thermal/fluid numerical models with an optimization code to minimize the cost of the cryogenic cooling system.

A Brief Review of the Pertinent Literature

Many general references on cryogenic technology and applications exist. The text by Barron (1985) contains useful information on cryogenic fluid storage and transfer systems, cryogenic insulation, and two-phase flow. Walker's two-volume work (1983) is an excellent source of information on cryocoolers and their applications. Cryocoolers are devices which produce refrigeration at temperatures less than 120 K (Walker, 1983). Properties of fluids at cryogenic temperatures were taken for this study from several texts including those by Incropera and DeWitt (1985), Van Wylen and Sonntag (1986), and Barron (1985).

One conceptual design of a cryogenically-cooled computer was proposed by Longsworth and Steyert (1987). In their saturated pool boiling design, shown in Figure 1, liquid nitrogen is used as the coolant and a cryocooler operating on the Gifford-McMahon thermodynamic cycle is employed. The vertically-oriented CPU boards, which are covered with chip packages, are immersed in the liquid nitrogen and the condenser is placed in the vapor space above the liquid pool. As the coolant vapor comes into contact with the condenser, it condenses and falls back into the liquid pool.

Kamerara, Yokouchi, and Niwa (1987) have shown that a saturated pool boiling system, such as the one discussed above, can be easily modified to create a subcooled system by completely filling the CPU vessel with liquid and submerging the condenser in the liquid. A sketch of a typical subcooled pool boiling design is shown in Figure 2. In a submerged condenser system, the liquid coolant can be maintained in a subcooled state and the temperature difference between the board and the fluid can be kept to a minimal value. Both the saturated and subcooled pool boiling designs are based on direct immersion cooling in which there is actual physical contact between the coolant and the components being cooled.

In their paper, Krane, Parsons, and Bar-Cohen (1988) present a preliminary thermal analysis of a candidate cryogenic cooling system. This paper describes the basic configuration of the system, contains information on the possible refrigeration cycles, and presents an overview of the heat transfer modes and correlations involved. The paper indicates that significant advantages are associated with the use of liquid nitrogen as the coolant. Also, it was concluded that the critical heat flux lies between 14.9 W/cm^2 and 23.6 W/cm^2 for nitrogen, depending on the amount of subcooling below 77 K. For liquid nitrogen at atmospheric pressure, the maximum subcooling possible heat

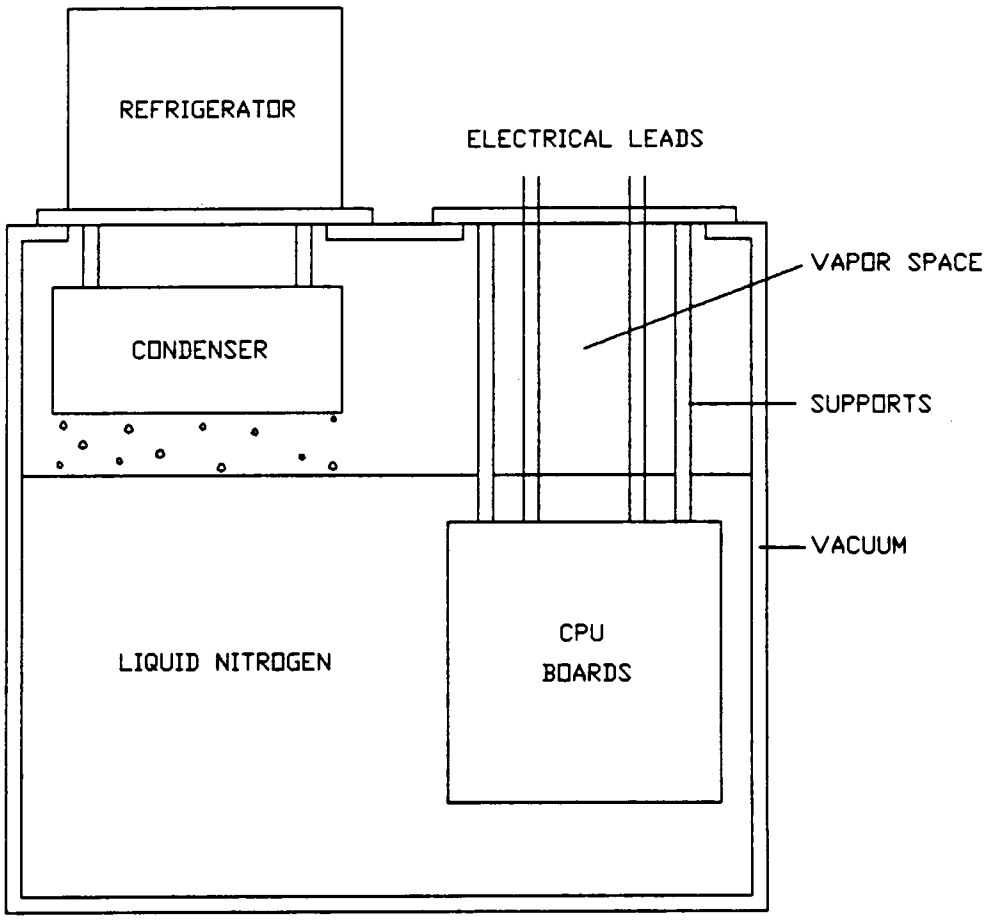

Figure 1. Saturated Pool Boiling Design of Longsworth and Steyert

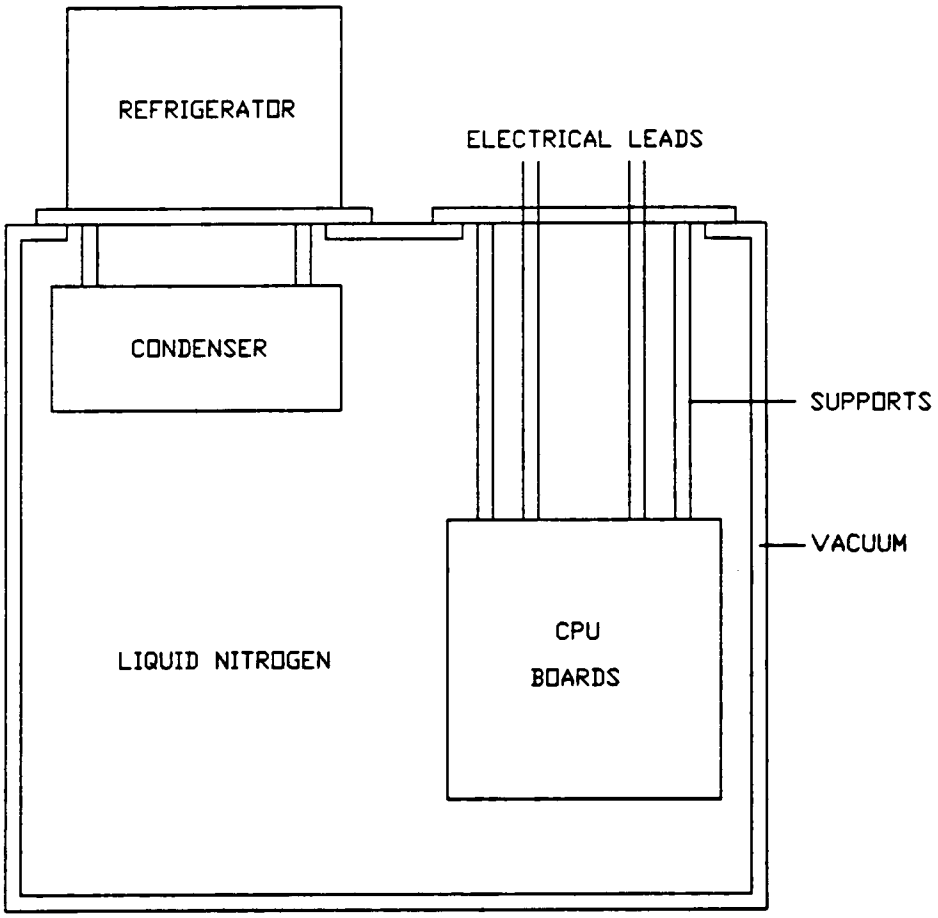

Figure 2. Subcooled Pool Boiling Design of Kamerara, Yokouchi, and Niwa

without freezing is 14.1 K. Therefore, if the board heat flux remains below the critical flux, the heat transfer occurs by natural convection and/or nucleate boiling, which require only a small temperature difference between the board and the fluid. Transition boiling will not occur in a controlled heat flux system such as the one being studied. Therefore, if the heat flux from the CPU board exceeds the critical heat flux, film boiling will result. Film boiling results in high surface temperatures which can lead to destruction or severe degradation of the electronic components and must, therefore, be avoided.

No references could be found in the open literature related to cryogenics and its applications to cooling of computers which specifically satisfy the stated objectives of this work.

CHAPTER II

SPECIFICATION OF THE COOLING SYSTEM CONFIGURATIONS TO BE ANALYZED

Basic Assumptions for the Analyses

The following assumptions concerning the configuration of the cryogenic cooling system are taken from Krane, Parsons, and Bar-Cohen (1988) and Bar-Cohen (1988):

(1) The computer contains one vertically-oriented CPU board which is approximately 50 cm by 50 cm in size.

(2) The nominal electrical power dissipation on the CPU board is 1 kW which corresponds to an average heat flux of 0.4 W/cm^2.

(3) The vessel containing the computer is a vertically-oriented cylinder with an inner diameter of approximately 75 cm and a height of 75 cm.

(4) Cryocoolers having capacities greater than 1 kW are large in size and are characterized by high noise and vibration levels. Since these characteristics are not compatible with a typical computer environment, remote location of the cryocooler is necessary. Thus, it is assumed that the cryocooler for the refrigerated system is located at the maximum possible distance (100 feet, or approximately 30 meters) from the computer.

(5) The cryogenic liquid reservoir may be located a maximum distance of 100 feet, or approximately 30 meters, from the computer.

(6) The computer is assumed to be cooled by direct immersion in liquid nitrogen (the choice of liquid nitrogen as the coolant will be discussed further in Chapter V).

(7) The operating pressure of the CPU vessel is assumed to be very near atmospheric pressure (1 atm).

Definition of a "Once-Through" System

Since many industrial, government, and research facilities which require the use of computers already have reservoirs of liquid nitrogen available for a variety of uses, the use of a "once-through" system may prove to be quite economical. Although an available supply of liquid nitrogen makes the "once-through" system particularly attractive, such a system may also be feasible in situations where liquid nitrogen supplies are not already available. In a "once-through" system, the cryogenic coolant is allowed to boil on the CPU board and the gaseous coolant is then vented to the surroundings. Another alternative to simply venting the gaseous nitrogen to the surroundings would be to employ some type of regenerative cooling in order to reduce the total heat transfer from the surroundings. A schematic of a "once-through" system is shown in Figure 3. The system consists of a vessel which contains the CPU board, a reservoir, or storage vessel, the connecting line, and possibly a pump. Although such a system requires a greater volume of coolant, the "once-through" system does not require a cryocooler as does the refrigerated system. Determining the optimal size of the reservoir may involve a trade-off between volume, which should be as large as possible in order to maximize the time interval between fillings, cost, and heat transfer to the vessel. However, for this study, the reservoir will be assumed to be sufficiently large to supply coolant for a maximum of seven days, assuming continuous operation.

Definition of the Saturated Pool Boiling System with a Cryocooler

The saturated pool boiling design discussed previously will also be analyzed. A schematic of the complete saturated pool boiling refrigerated system is shown in Figure 4. In addition to the CPU vessel, liquid nitrogen transfer line, and reservoir, the system consists of a cryocooler and a helium loop containing a heat exchanger, a condenser, and a circulator, or fan. The additional helium loop is necessary in order to transfer heat from the gaseous coolant and allow condensation. Helium is chosen since it is the most commonly used working fluid in cryocoolers.

Both Stirling and Gifford-McMahon refrigeration cycles will be considered for the cryocooler. The reservoir in the refrigerated system is for emergency and filling purposes only. With the reservoir serving as a backup cooling system, it is assumed for the purposes of this study that the computer should be able to operate in the

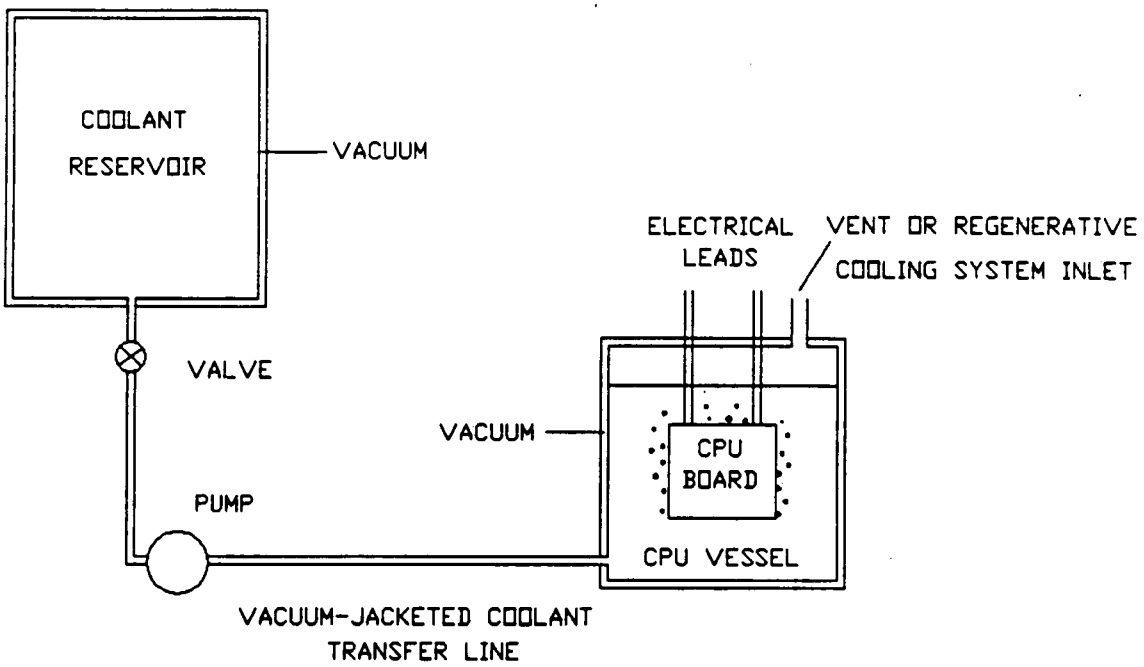

Figure 3. Schematic of a "Once-Through" Cryogenic Cooling System for a Computer

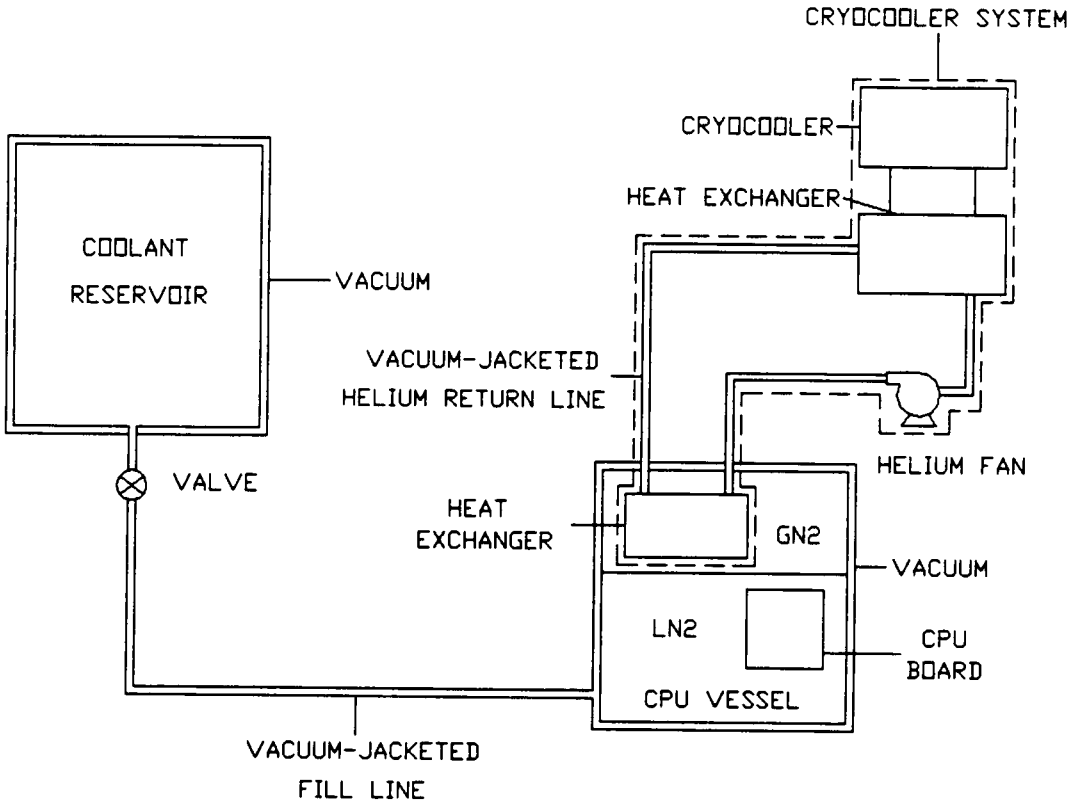

Figure 4. Schematic of a Computer Actively Cooled to Cryogenic Temperatures by a Cryocooler

"once-through" mode for a maximum of twenty-four hours should problems arise with the cryocooler unit. If the cryocooler unit is still non-operational after twenty-four hours, the reservoir will need to be refilled to allow additional time for repair or maintenance.

Definition of the Subcooled Pool Boiling System with a Cryocooler

The subcooled pool boiling design will be identical to the saturated system shown in Figure 4, with two exceptions: (1) the liquid nitrogen will be subcooled a specified amount, and (2) the liquid nitrogen will completely fill the CPU vessel such that the condenser is completely submerged in the subcooled liquid. An additional power cost will be required to subcool the liquid nitrogen from the saturated state, in which it is purchased, to the desired temperature. The major advantage of the subcooled system is its ability to maintain the CPU boards at a lower temperature and therefore reduce the possibilty of thermally-induced failure.

CHAPTER III

MODELING OF A "ONCE-THROUGH" COOLING SYSTEM

Energy Sources

In the cryogenically-cooled computer system, energy dissipated by the electronic components and transferred from the environment must be carried away by a sufficient flowrate of cryogen in order to maintain the appropriate operating temperature. Heat transfer from the environment, which is assumed to be at 300 K, is classified, for the purposes of this study as: (1) heat transfer through the fill line, (2) heat transfer through the walls of the vessel, and (3) heat transfer through the current leads. The CPU board mounts may introduce an additional path for conduction heat transfer; however, it is assumed that this source of heat transfer is insignificant if the board mounts are properly designed.

Heat Transfer Through The Fill Line

The heat transfer through the fill line is calculated according to the type of insulation used. The thermal model allows for vacuum-jacketed insulation or standard insulating materials with known thermal conductivities. Four components of heat transfer are considered in the model. These are: 1) internal convection, 2) conduction through the pipe, 3) heat transfer across the insulation layer, and 4) external convection. Utilizing the electrical resistance analogy for heat transfer (Incropera and DeWitt, 1985), each of these components of heat transfer can be characterized by a thermal resistance. Thermal resistance is defined as the ratio of the driving potential (temperature difference) to the corresponding heat transfer rate. The pipe cross-section and the corresponding thermal network are shown in Figure 5. The rate of heat transfer through the fill line is found by

$$\dot{Q}_{line} = (T_a - T_f) / R_{total} , \qquad (3-1)$$

where :

$$R_{total} = R_{th1} + R_{th2} + R_{th3} + R_{th4}. \qquad (3-2)$$

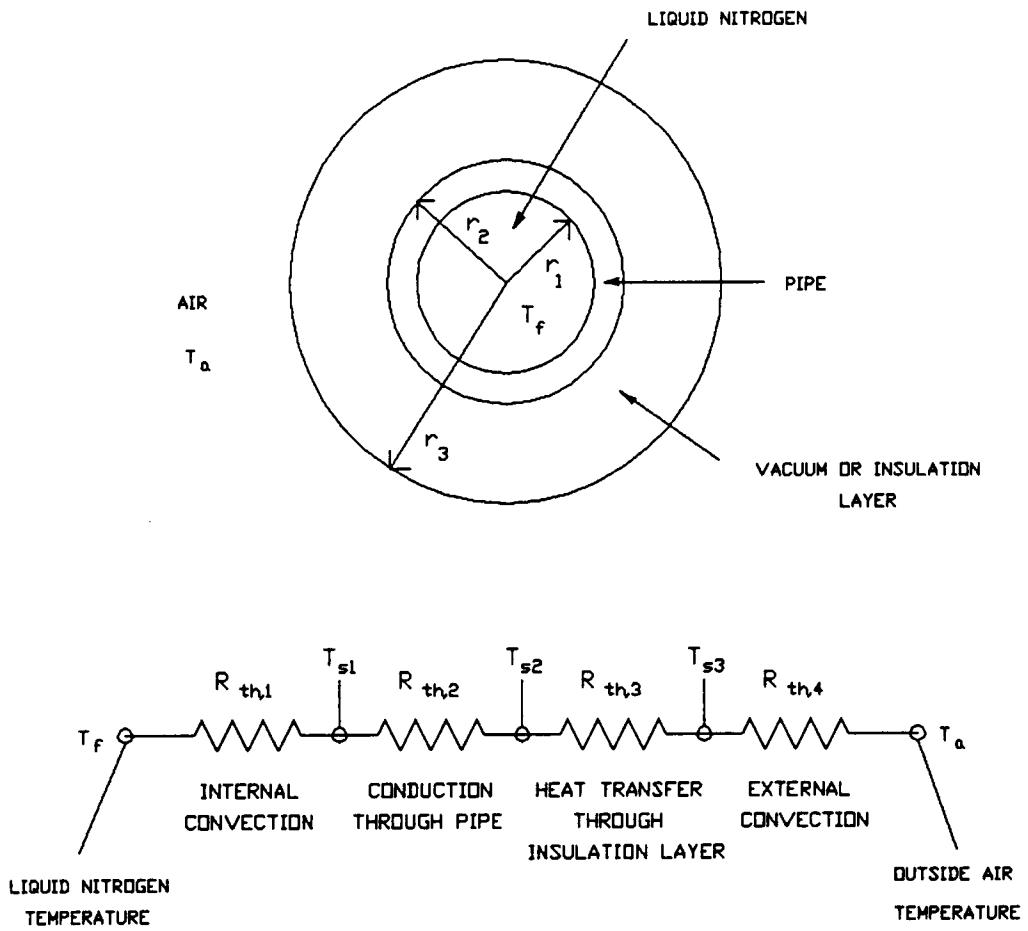

Figure 5. Thermal Network for the Liquid Nitrogen Fill Line

In order to calculate the heat transfer due to internal convection, the internal convection coefficient must first be determined. Correlations for Nusselt numbers from Incropera and DeWitt (1985) were used in the model. A constant value of 4.36 is used for the Nusselt number if the flow is laminar (i.e. Re < 2300) since, according to Barron (1985), the axial heat flux into the pipe can be considered nearly constant along the length of the cryogenic-fluid transfer line. This correlation assumes fully-developed, steady-state flow. A second correlation for Nusselt number as a function of Reynolds and Prandtl number is used if the flow is turbulent with Re > 10000. A correlation was not found which applies to the transition Reynolds number range. Therefore, in order to cover the Reynolds number range between 2300 and 10000, a linear curve was constructed between the endpoints of the two curves. Therefore, the following equations are used in the model:

$$Nu_D = 4.36 \qquad \text{for } Re_D < 2300,\ Pr > 0.6, \qquad (3\text{-}3)$$

$$Nu_D = 4.36 + [(36.5Pr^{1/3} - 4.36) / 7700](Re_D - 2300)$$

$$\text{for } 2300 < Re_D < 10000, \qquad (3\text{-}4)$$

and,

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^{1/3} \qquad \text{for } Re_D > 10000,\ Pr > 0.6. \qquad (3\text{-}5)$$

Once the Nusselt number is known, the internal convection coefficient can be found by

$$h_{int} = Nu_D\, k_f / D. \qquad (3\text{-}6)$$

The thermal resistance due to internal convection is calculated from

$$R_{th1} = 1/(2\,\pi\, r_1\, L\, h_{int}). \qquad (3\text{-}7)$$

The thermal resistance due to conduction through the pipe is calculated as

$$R_{th2} = \ln(r_2 / r_1) / (2 \pi L k_p). \qquad (3-8)$$

The thermal resistance due to external convection can be calculated in a manner similar to that for the internal convection as

$$R_{th3} = 1/(2 \pi r_3 L h_{ext}). \qquad (3-9)$$

Since the external convection is assumed to be free convection in air, a value of 5.0 $W/m^2\text{-}^oF$ is used for the convection heat transfer coefficient as recommended by Incropera and DeWitt (1985).

If standard type insulation, or even multilayer insulation with known thermal conductivity, is used, the thermal resistance of the insulation layer can be calculated as

$$R_{th4} = \ln(r_3/r_2) / (2 \pi L k_i). \qquad (3-10)$$

If long piping runs are present in a cryogenic system, however, vacuum insulated lines are usually employed. The heat transfer to the liquid coolant through the vacuum insulated lines is calculated by assuming it occurs solely through radiation exchange between the hot outer jacket and the cool inner pipe. As shown by Barron (1985), the emissivity factor, F_e, is defined by

$$1/F_e = 1/\varepsilon_1 + A_1/A_2 (1/\varepsilon_2 - 1), \qquad (3-11)$$

and the heat transfer across the vacuum gap is

$$\dot{Q}_{vacuum} = F_e F_{1-2} \sigma A_1 (T_2^4 - T_1^4), \qquad (3-12)$$

where subscript 1 refers to the inner pipe, or enclosed, surface and subscript 2 refers to the outer pipe surface, or the enclosure. However, if the inner pipe is completely

enclosed by the outer pipe, $F_{1-2} = 1$ and the above equation reduces to

$$\dot{Q}_{vacuum} = F_e \sigma A_1 (T_2^4 - T_1^4). \quad (3-13)$$

Consequently,

$$R_{th} = \Delta T / \dot{Q}_{vacuum}. \quad (3-14)$$

Radiation shields between the outer jacket and the pipe are commonly used to reduce the heat transfer through the vacuum insulaton. Since the heat transfer is assumed to occur solely through radiation exchange, this value can be significantly reduced by the insertion of thin, concentric, highly reflective shields. A value of shield emissivity (ε_{shield}) of 0.05, recommended by Siegel and Howell (1981), is used in the analysis. The use of radiation shields requires the following modification of the emissivity factor as described by Siegel and Howell (1981):

$$1/F_e = 1/\varepsilon_1 + A_1/A_2 \, (1 / \varepsilon_2 - 1) + \sum_{i=1}^{n} (A_1/A_{shield,i}) (2 / \varepsilon_{shield,i} - 1), \quad (3-15)$$

where ε_1 is the emissivity of the inner pipe, ε_2 is the emissivity of the outer pipe, and n is the number of radiation shields.

Heat Transfer Through the Walls of the CPU Vessel

The heat transfer through the walls of the CPU vessel is calculated in the same manner as the heat transfer through the fill line. The following components of heat transfer are considered: (1) natural convection over the inner vessel wall, (2) conduction through the vessel wall, (3) heat transfer across the insulation layer, and (4) external convection in air.

Heat Transfer Through the Current Leads

Current leads, which are used to communicate between the CPU and other units of the computer, can be very complicated in design as described by Buyanov (1985). One relatively simple current lead construction consists of tubes and wire bundles formed from materials in common use (Buyanov, 1985). Therefore, for simplicity, the current leads are modeled as a single lead with diameter D_{lead} and length L_{lead}. This assumption should provide reasonable results since most current leads are constructed from tight bundles or braids of wire. The lead is assumed to be copper. The heat transfer through the lead is calculated as

$$\dot{Q}_{lead} = k_{lead} A_{c,lead} \Delta T / L_{lead}. \qquad (3-16)$$

Heat Transfer from the CPU Board

The rate of heat transfer per CPU board is calculated from

$$\dot{Q}_{board} = q'' A_b, \qquad (3-17)$$

where q" is the value of the nominal heat flux on the CPU board and A_b is the area of one side of the CPU board.

Required Mass Flow Rate of Liquid Nitrogen

If the temperature of the liquid nitrogen is known, the total rate of heat transfer to the coolant in a "once-through" system can be determined. If the inlet enthalpy of the cryogen is known and the exit enthalpy is assumed to be the enthalpy of saturated vapor at atmospheric pressure, the required mass flow rate can be determined from the First Law as follows:

$$\dot{m} = \dot{Q}_{total} / (h_e - h_i), \qquad (3-18)$$

where :

$$\dot{Q}_{total} = \dot{Q}_{line} + \dot{Q}_{lead} + \dot{Q}_{board} + \dot{Q}_{vessel}. \qquad (3-19)$$

Pressure Drop in the Fill Line

For a "once-through" system, it must be determined whether or not a pump is necessary in order to transfer the liquid coolant from the reservoir to the CPU vessel. If the total pressure drop in the transfer line is relatively low, it could possibly be overcome simply through an elevation change between the reservoir and the computer. However, such a control scheme would be complicated since the mass flow rate of coolant varies depending on the level of coolant in the reservoir. An alternative solution to supplying a continuous flow of coolant is to allow the level of coolant to fall to a minimum level in the CPU vessel at which time a pump is activated and the CPU vessel is once again filled to its maximum operational level. If the coolant is supplied from the reservoir in a saturated state, vapor will immediately begin to form in the transfer lines due to the heat addition. If the coolant is subcooled, it will remain 100% liquid through a portion of the piping length before vapor formation begins. Thus, both single phase and two-phase pressure drop calculations may be necessary.

Single Phase Pressure Drop

If the cryogen is sufficiently subcooled, the liquid nitrogen will remain in the liquid phase throughout the fill line and the pressure drop may be calculated as a single phase pressure drop. Since the wall heat flux is assumed to be constant, the energy of the liquid increases linearly with distance along the fill line. Therefore, in order to take into account the changing properties of liquid nitrogen, the transfer line is divided into small segments when calculating the pressure drop. In order to calculate the pressure drop, the outlet temperature of the line segment must be known for the evaluation of thermal properties of the coolant. The inlet temperature of the liquid coolant (reservoir temperature) is assumed to be known. First, an outlet temperature is assumed for the first segment. The rate of heat transfer to the line segment ($\dot{Q}_{line}$) is calculated as previously described and an exit enthalpy, h_{e1}, is calculated by applying the First Law and assuming steady flow with negligible changes in kinetic and potential energy to the control volume shown in Figure 6. This gives

$$h_{e1} = \dot{Q}_{line} / \dot{m} + h_{in}. \qquad (3\text{-}20)$$

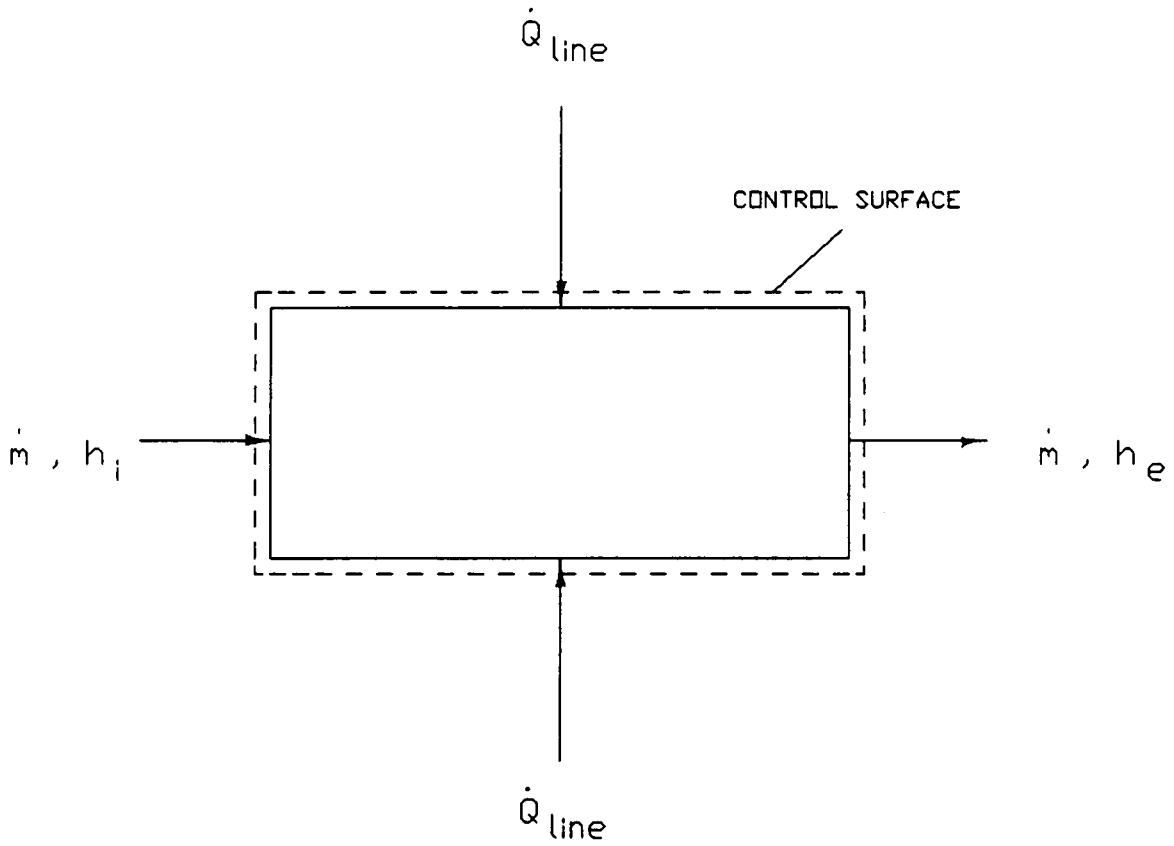

Figure 6. Energy Balance on the Transfer Line Segment

A second value of exit enthalpy, h_{e2}, is determined from an enthalpy curve fit, which will be derived below, for the liquid coolant. The difference between the two enthalpy values is calculated and a new outlet temperature is assumed. The above procedure is repeated until the difference between the two enthalpy values changes signs. When this occurs, the correct outlet temperature has been found and the fluid properties, viscosity and density, can be determined. The error involved in this procedure may be minimized by choosing a sufficiently small value for the temperature increment.

The Darcy friction factor, f, is found from the following set of equations given in White (1986):

$$f = 64 / Re_D \qquad \text{for } Re_D < 2000, \qquad (3-21)$$

$$1/f^{1/2} = 1.74 - 2\log[2e/D + 18.7/Re_D\, f^{1/2}] \qquad \text{for } 2000 < Re_D < 4000, \qquad (3-22)$$

and,

$$1/f^{1/2} = -1.8\log(6.9/Re_D + e/3.7D)^{1.11} \qquad \text{for } Re_D > 4000. \qquad (3-23)$$

When equation 3-22 for the friction factor in transition flow is used, an iterative procedure is required in order to find a value for f. Once the friction factor has been calculated, the segmental pressure drop, ΔP_i, can be calculated from

$$\Delta p_i = \rho g(fL/D)(v^2/2g). \qquad (3-24)$$

Two-Phase Pressure Drop

Two basic models have been developed to predict the pressure drop for two-phase flow. These are the "homogeneous model", in which the two-phase mixture is treated as a pseudofluid, or imaginary liquid/vapor fluid with suitable average properties, and the "separated flow model", in which the two phases are assumed to be artificially

segregated into two streams having different properties and velocities. Lockhart and Martinelli have developed correlations based on the "separated flow" model for two-phase flow in horizontal tubes which predict pressure drops for cryogenic fluids such as nitrogen with good accuracy (Barron,1985).

The Lockhart-Martinelli parameter is defined as

$$X^2 = (\Delta p/\Delta L)_l / (\Delta p/\Delta L)_v, \qquad (3\text{-}25)$$

or,

$$X^2 = [C_L(Re_v)^m \rho_v / C_G(Re_L)^n \rho_l] [(1 - x)/x]^2, \qquad (3\text{-}26)$$

where $(\Delta p/\Delta L)_l$ and $(\Delta p/\Delta L)_v$ are the pressure drops per unit length that would exist if the pipe were completely filled with liquid or vapor alone, respectively. Re_l and Re_v are the liquid and vapor Reynolds numbers, respectively, and x denotes the fluid quality. C_L, C_G, m, and n in Equation 3-26 are constants whose values are dependent on the flow regimes of the individual phases. Values for these constants are given in Table 1.

For diabatic, two-phase flow, the total pressure drop consists of two components-- frictional and momentum. The momentum pressure drop component is due to the acceleration or change in momentum of the fluid resulting from the change in fluid quality.

The frictional pressure drop, Δp_f, may be determined from

$$\Delta p_f = \Delta p_o/(x_e - x_{in}) \int_{x_{in}}^{x_e} (1 - x)^{2-n} \phi_L^2 \, dx, \qquad (3\text{-}27)$$

where :

$$\phi_L = (X^2 + CX + 1)^{1/2} / X. \qquad (3\text{-}28)$$

Equation 3-27 assumes that the axial heat flux is constant, which is frequently the case

Table 1. Two-Phase Pressure Drop Constants

Constant	Laminar	Turbulent	
	Re<2000	3000<Re<50000	Re>50000
m (vapor)	1	0.25	0.20
n (liquid)	1	0.25	0.20
C_G (vapor)	64	0.316	0.184
C_L (liquid)	64	0.316	0.184

Source : Cryogenic Systems, Barron, 1985.

for cryogenic transfer lines (Barron, 1985). C in Equation 3-28 is a constant with values given in Table 1. Δp_o in Equation 3-27 is the pressure drop that would occur if liquid alone were flowing at the total mass flow rate ($\dot{m} = \dot{m}_l + \dot{m}_v$).

The momentum pressure drop component may be determined from the following correlation (Barron, 1985):

$$\Delta p_m = \phi_m (\dot{m}_l + \dot{m}_v)^2 / \rho_l A_p^2, \qquad (3\text{-}29)$$

where :

$$\dot{m}_l = \dot{m} (1 - x), \qquad (3\text{-}30)$$

and,

$$\dot{m}_v = \dot{m} x. \qquad (3\text{-}31)$$

ϕ_m, the momentum pressure drop parameter, is defined by

$$\phi_m = (1 - x_e)^2 / R_{L,e} - (1 - x_{in})^2 / R_{l,in} + [x_e^2 / (1 - R_{L,e}) - x_{in}^2 / (1 - R_{L,in})] (\rho_l / \rho_v). \qquad (3\text{-}32)$$

The quantity R_L, the volume fraction of liquid phase, may be expressed as a function of the Lockhart-Martinelli parameter as follows :

$$R_L = X / (X^2 + CX + 1)^{1/2}. \qquad (3\text{-}33)$$

The total pressure drop for two-phase diabatic flow is simply the sum of the frictional and momentum components given by

$$\Delta p = \Delta p_f + \Delta p_m. \qquad (3\text{-}34)$$

CHAPTER IV

MODELING OF THE REFRIGERATED SYSTEM

The helium loop in the cryocooler of a refrigerated system is a very important segment of the cryocooler since it removes thermal energy from the coolant in the CPU vessel. In a saturated system, heat is transferred from the gaseous nitrogen to the helium resulting in condensation. In a subcooled system, heat is transferred from the liquid nitrogen, thereby maintaining it in a subcooled state. It is undesirable to have two-phase flow in the transfer line of this loop due to the higher pressure drop and pumping problems which may arise. Therefore, the working fluid of the cryocooler should remain in a single phase over the temperature range existing in the loop (approximately 60 to 80 K). Most cryogenic fluids such as nitrogen will have both liquid and gas phase components in this temperature range at a pressure of one atmosphere. Since helium remains 100% vapor in the temperature-pressure range of interest, gaseous helium is commonly used as the working fluid in cryocoolers.

Heat Transfer to the Transfer Lines

The heat transfer to the gaseous helium transfer lines is calculated in a similar manner as that described for the liquid coolant lines. The heat transfers through the CPU vessel, the current leads, and from the CPU boards are calculated in the same manner as described for the "once-through" system.

Heat Transfer in the Condenser

The heat transfer occuring in the condenser can be determined by applying the First Law to the system shown in Figure 7. Assuming steady flow and negligible changes in kinetic and potential energy, the energy balance gives

$$\dot{Q}_{condenser} = \dot{Q}_{vessel} + \dot{Q}_{lead} + \dot{Q}_{board}. \qquad (4-1)$$

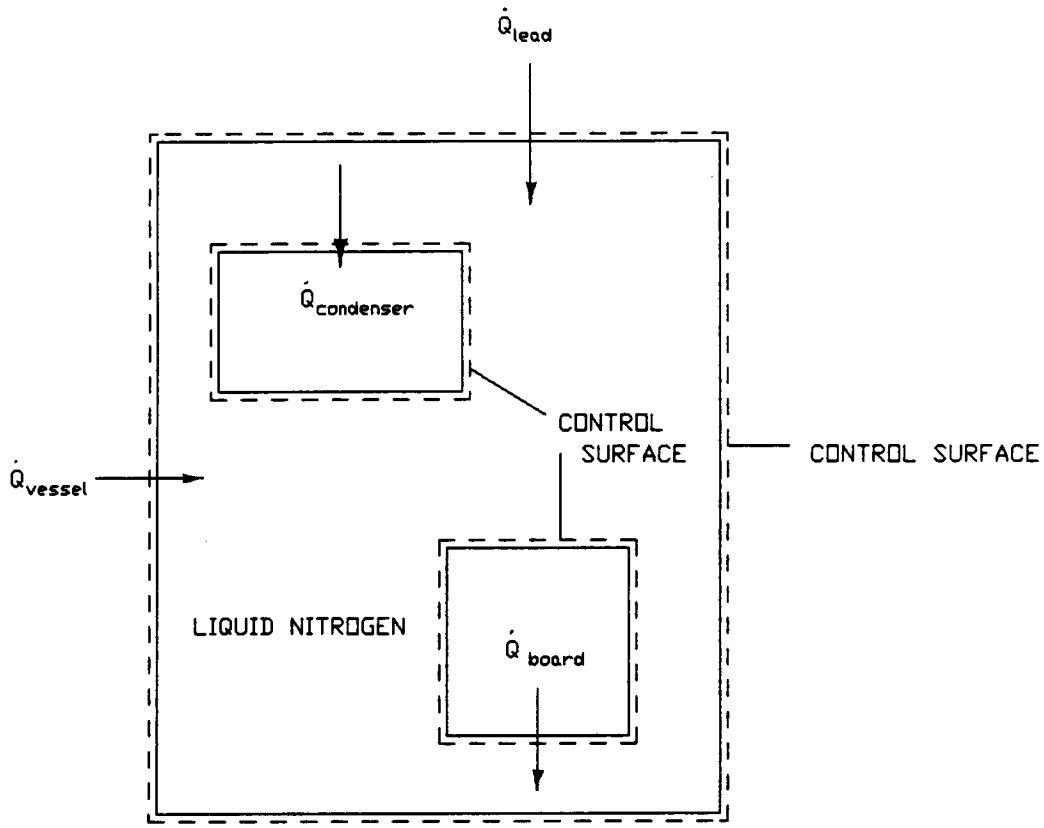

Figure 7. Heat Transfers to the Liquid Coolant

The heat transfer through the liquid coolant transfer lines is not considered since these lines are used only as fill lines in a refrigerated system.

Required Refrigeration Capacity

The required refrigeration capacity can be found by performing an energy balance on the gaseous helium circulation loop. Applying the First Law to the control volume shown in Figure 8, and assuming steady flow and negligible changes in kinetic and potential energy, the refrigerating capacity is given by

$$\dot{Q}_{refrigerator} = \dot{Q}_{condenser} + \dot{Q}_{line} + W_{pump}. \qquad (4-2)$$

Pressure Drop in the Helium Loop

The pressure drop in the gaseous helium transfer lines is calculated in a manner similar to that used to calculate the single phase pressure drop in the liquid coolant lines. However, since helium can be assumed to be a calorically-perfect ideal gas, the change in enthalpy can be calculated as

$$\Delta h = c_p \Delta T. \qquad (4-3)$$

The density and viscosity of gaseous helium are calculated from the following expressions

$$\mu = -2.408678 \times 10^{-12} T^3 + 3.010459 \times 10^{-10} T^2 + 6.483437 \times 10^{-8} T + 2.36983 \times 10^{-6}, \qquad (4-4)$$

and,

$$\rho = -1.731763 \times 10^{-6} T^3 + 5.52761 \times 10^{-4} T^2 - 6.242247 \times 10^{-2} T + 2.937333 \qquad (4-5)$$

which are third order curve fits of data from Barron (1985). These expressions are

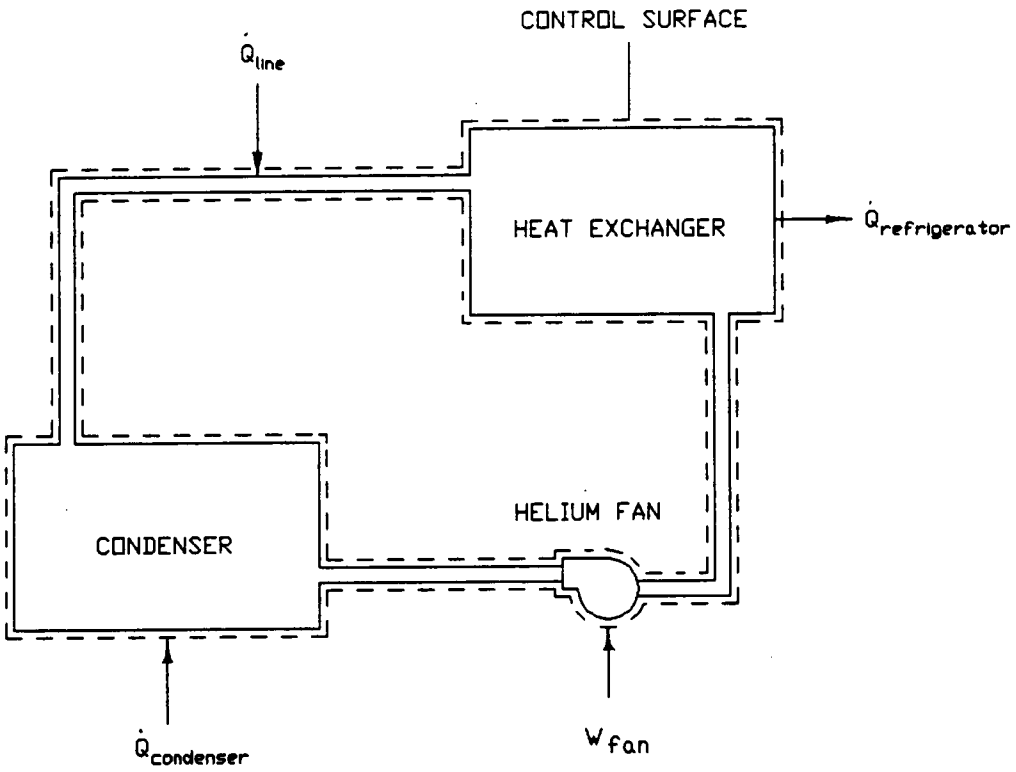

Figure 8. Control Volume for Analysis of the Helium Loop

valid for temperatures between 40 K and 125 K. The data used to obtain these expressions appears in Table B.4 in Appendix B .

Cryocooler Models

The coefficient of performance of the cryocooler (β) is defined as

$$\beta = \text{Refrigeration produced / Work input} = \dot{Q}_{ref} / W_{in}. \qquad (4-6)$$

The Carnot refrigerator, a fictitious device which operates with reversible processes, provides the maximum possible coefficient of performance given by

$$\beta_{carnot} = 1 / (T_H / T_L - 1), \qquad (4-7)$$

where T_H and T_L are the high and low temperatures of the cycle, respectively.

The Stirling Cycle Model

The Stirling cycle device, which was first used as a refrigerator in 1834, is a closed regenerative cycle in which the same fluid (usually gaseous helium) is repeatedly compressed and expanded at different temperature levels. The workspace of the Stirling cycle consists of three regions: the expansion space, which is at the refrigeration temperature, the compression space, which is at ambient temperature, and the dead space, which consists of the internal volumes of the heat exchangers and ducts connecting the other two spaces. As an approximation, the temperature of the dead space is taken to be the average of the expansion and compression space temperatures (Tailor and Narayankhedkar, 1987).

The ideal Stirling cycle consists of two isothermal processes and two constant volume processes. The p-v and T-s diagrams for the ideal Stirling cycle are shown in Figure 9. The four processes are as follows:

Process 1-2: Isothermal compression -- heat transfer from the working fluid

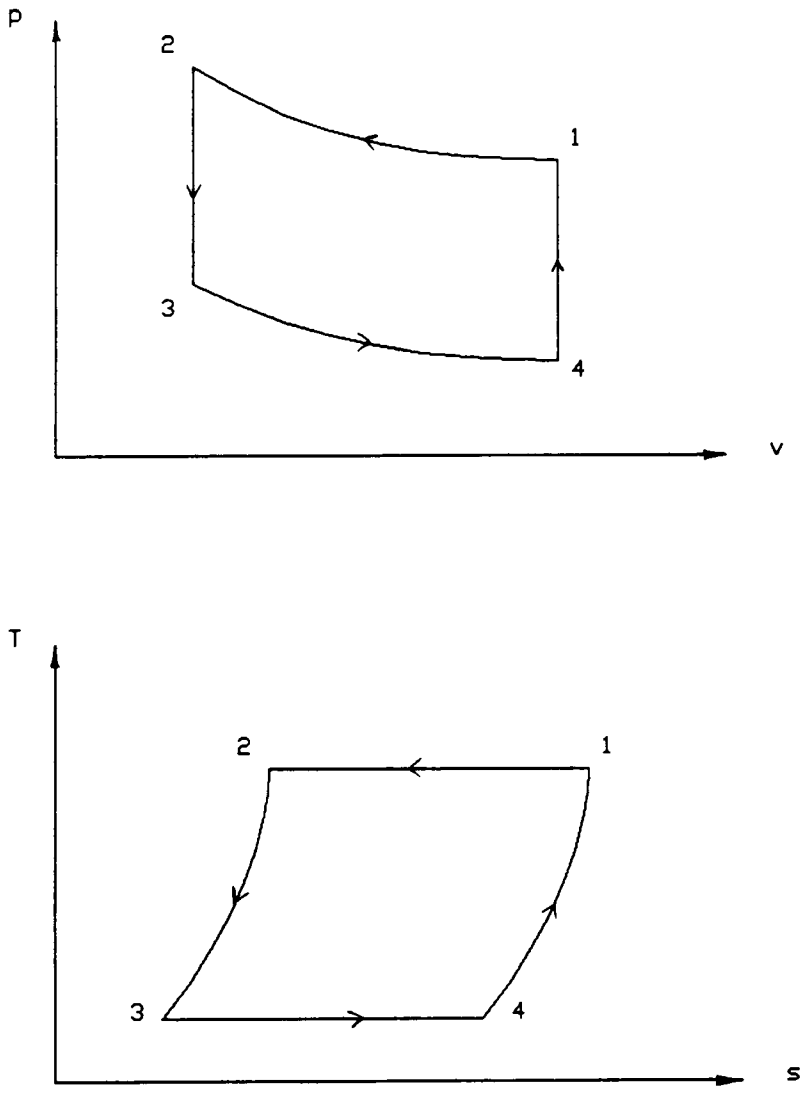

Figure 9. Stirling Cycle p-v and T-s Diagrams

to the external sink at a temperature less than the compression temperature,

Process 2-3: Constant volume -- heat transfer from the working fluid to the regenerative matrix -- gas temperature decreases from ambient (T_a) to refrigeration temperature (T_{ref}),

Process 3-4: Isothermal expansion -- heat transfer to the working fluid from an external source at a temperature greater than the refrigeration temperature (T_{ref}) -- useful refrigeration, $\dot{Q}_{ref}$, is performed during this process,

Process 4-1: Constant volume -- heat transfer to the working fluid from the regenerative matrix --gas returns to ambient temperature (T_a).

The ideal Stirling cycle involves an idealization which is physically impossible to achieve: the reciprocating elements are assumed to move in a discontinuous manner (Walker, 1983). This assumption may be relaxed by assuming harmonic motion of the reciprocating elements. This more realistic case was analyzed by Gustav Schmidt in 1871 (Walker, 1983). This special case of harmonic piston motion along with isothermal compression and expansion has become known as the Schmidt cycle. Schmidt cycle analysis results in the following expression for the coefficient of performance:

$$\beta = T_e / (T_c - T_e). \qquad (4-8)$$

Although the Schmidt cycle is more realistic than the idealized cycle, some assumptions are still not completely valid. Processes 1-2 and 3-4, shown in Figure 9, are more nearly adiabatic than isothermal (Walker, 1983). Departure from isothermal conditions results in a decrease in refrigeration capacity and coefficient of performance. A schematic of the cryocooler which will be modeled appears in Figure 10. The

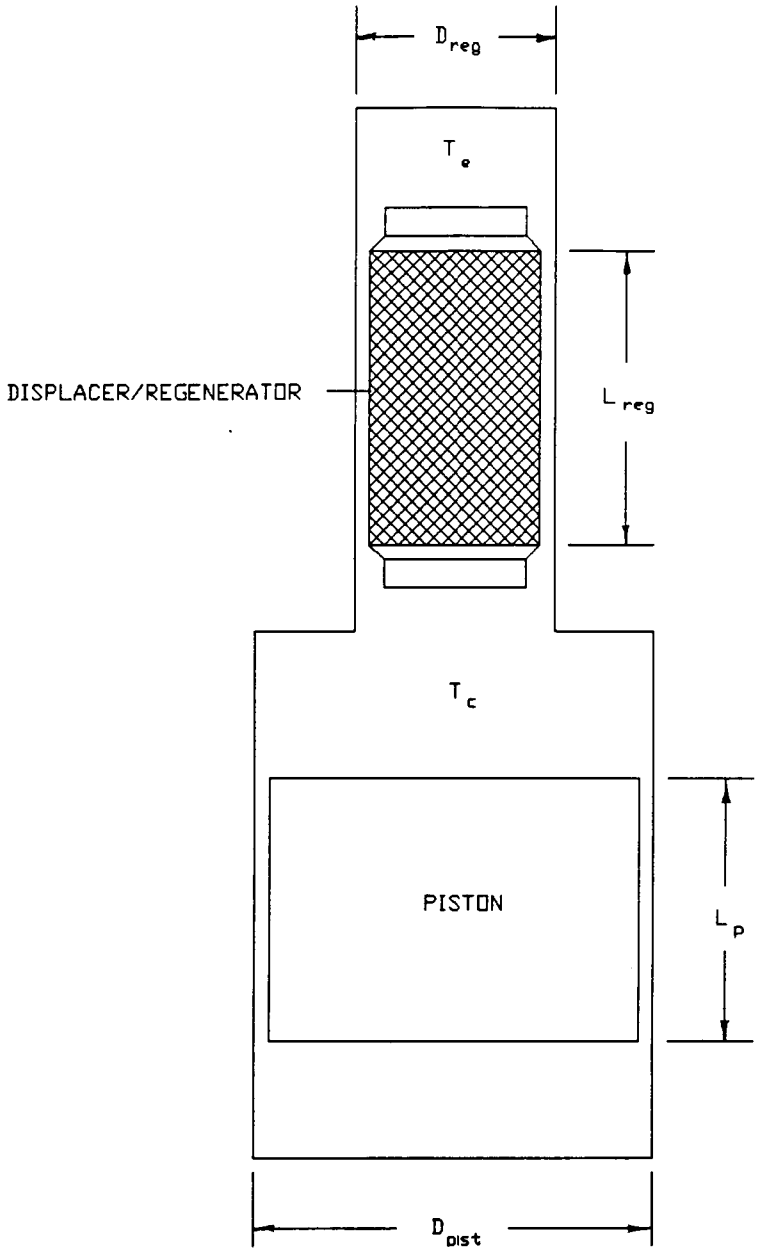

Figure 10. Schematic Representation of a Stirling Cryocooler

cryocooler consists of a piston and displacer/regenerator which operate in a single cylinder. The regenerator is incorporated in the displacer and, therefore, these two components move together. The regenerator can be thought of as a "thermodynamic sponge", which alternately absorbs and releases thermal energy. The regenerator usually consists of a matrix of finely divided metal in the form of wires or strips. The approach used in this work to calculate the coefficient of performance of the Stirling cryocooler in the thermal model is outlined in a paper entitled "Analysis and Performance Prediction of an Electromagnetically-Driven Free Displacer Stirling Cryocooler" by Tailor and Narayankhedkar (1988). The major assumptions of this method are: 1) the workspace consists of three isothermal regions at temperatures T_c, T_e, and T_d, 2) motions of the piston and displacer are sinusoidal and friction free, 3) pressure drops other than the one in the regenerator are negligibly small, and 4) perfect regenerator performance. Perfect regenerator performance implies that the regenerator is 100% effective in transferring heat or the heat transfer to the working fluid in the regenerator due to such things as aerodynamic friction is assumed to be zero. The following equations, which are derived in the aforementioned paper, are used to calculate the coefficient of performance:

$$\lambda = (\nu_d / \nu_c)(1/4\ X)[(1 + \nu_c / \nu_e)^2 \tan^2\theta + (2\ \nu_c / \nu_e)^2]^{1/2}, \quad (4-9)$$

$$B = \tan^{-1}[(\nu_e / \nu_c + 1)\ \tan\theta / 2], \quad (4-10)$$

$$C_{\Delta p} = C_{R1}\ n\ \tau\ \nu_d^2\ A_{c,pist}\ L_p\ \omega \sin\theta / [(\tau + 1)\ R\ T_c\ \nu_c\ A_{c,reg}\ \sigma\ l]$$

$$+C_{R2}\ n\ \tau\ (\nu_d\ A_{c,pist}\ L_p)^2\ \omega^2 \sin^2\theta / [(\tau + 1)\ R\ T_c\ (\nu_c\ A_{c,reg}\ \sigma)^2], \quad (4-11)$$

and,

$$\beta = \lambda \sin B / (1 + \nu_c / \nu_e)[(\lambda \cos B + C_{\Delta p} / 2) \tan\theta - \lambda \sin B], \quad (4-12)$$

where:

$$A_{c,pist} = \pi D_{pist}^{2}/4, \qquad (4-13)$$

$$A_{c,reg} = \pi D_{reg}^{2} / 4 , \qquad (4-14)$$

and,

$$\sigma = [l / (l + D_m)]. \qquad (4-15)$$

The constants C_{R1} and C_{R2} in Equation 4-11 are constants which appear in the correlation for prediction of the flow friction factor for a packed wire screen regenerator matrix (Tailor and Narayankhedkar, 1988).

The kinematic viscosity, ν, is defined as

$$\nu = \mu / \rho. \qquad (4-16)$$

The values of dynamic viscosity and density of gaseous helium are calculated from the following curve fits of data (Barron, 1985) presented in Table B.2 in Appendix B for temperatures between 50 and 300 K:

$$\mu = 2.249898 \times 10^{-13} T^3 - 1.631992 \times 10^{-10} T^2 + 8.801451 \times 10^{-8} T + 2.082927 \times 10^{-6} \qquad (4-17)$$

and,

$$\rho = -1.247751 \times 10^{-7} T^3 + 8.365729 \times 10^{-5} T^2 - 1.905197 \times 10^{-2} T + 1.699258. \qquad (4-18)$$

If the pressure drop in the regenerator is assumed to be zero, the above equation for the coefficient of performance reduces to:

$$\beta = \nu_e / (\nu_c - \nu_e). \qquad (4-19)$$

This is an important result in that the coefficient of performance of a perfect regenerative free displacer Stirling cryocooler is a function of kinematic viscosity ratio rather than the temperature ratio as predicted by the Schmidt cycle analysis. The Schmidt cycle analysis overpredicts the coefficient of performance since it results in a value equal to the Carnot value. For example, if we consider a Stirling cryocooler operating between 63 and 300 K, the Schmidt analysis predicts a coefficient of performance value of 0.2658 (Carnot value), while the method described above results in a value of 0.0773 if helium is used as the working gas.

The Gifford-McMahon Model

The Gifford-McMahon cycle, developed in 1959, is similar to the piston/displacer Stirling engine. However, valves are used to isolate the compressor from the regenerator and displacer units. A schematic of a typical Gifford-McMahon cycle is shown in Figure 11. Due to the fact that not all of the working gas proceeds through the same series of processes, the Gifford-McMahon cycle is very difficult to describe thermodynamically. Due to this complexity, the performance of the cycle will be considered to be the result of several mechanisms which bring energy in or out of the cold region (Ackermann and Gifford, 1971).

The coefficient of performance of an ideal Gifford-McMahon cryocooler can be calculated from the following equation presented in a paper by W.E. Gifford (1965):

$$\beta_{ideal} = R\,(P_1/P_2 - 1) / \{c_p[(P_1/P_2)(T_H / T_L) - 1](P_1/P_2)^{(\gamma-1)/\gamma-1}\}. \quad (4\text{-}20)$$

Equation 4-20 shows that the coefficient of performance is a function of thermodynamic properties of the working gas (i.e. specific heat ratio, gas constant, etc.), the pressure ratio, and the high and low temperatures of the cycle (T_H and T_L, respectively).

The preceding analysis provides an ideal value of the coefficient of performance since it does not take into account any thermal leaks to the cold regions of the machine by conduction, convection, or radiation. Another paper by Ackermann and Gifford entitled "A Heat Balance Analysis of a Gifford-McMahon Cryorefrigerator" (1971) was used in order to modify the ideal coefficient of performance value to include the thermal losses. This paper states that 55% of the ideal refrigeration is lost through the combined effects of pressure drops and void volume degradation and 45% of the remaining

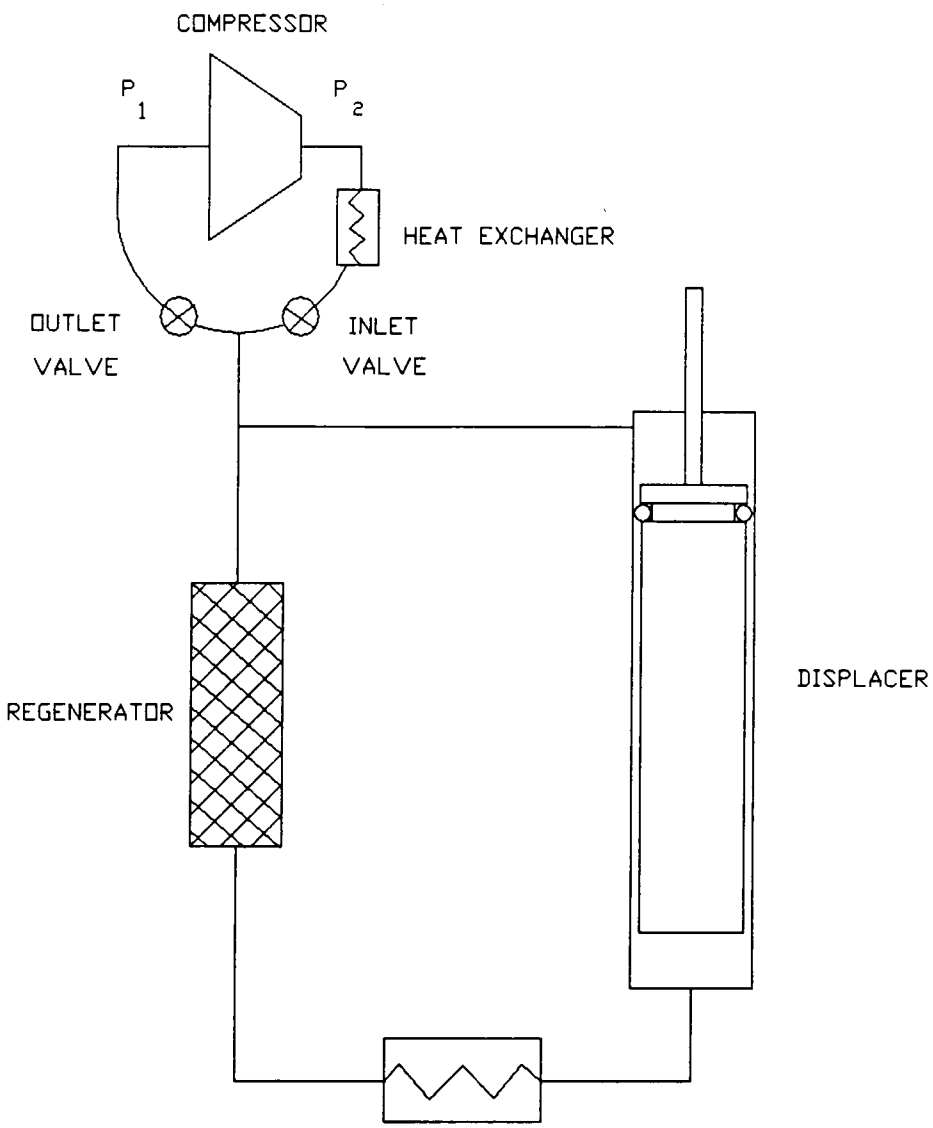

Figure 11. Schematic Representation of a Gifford-McMahon Cryocooler

refrigeration is lost through thermal losses including conduction losses, regenerator losses, and motional heat leaks. Void volume degradation, which accounts for a substantial loss in ideal refrigeration, is due to the presence of void volumes in the cryocooler which result in a reduction of the pressure difference achieved. Motional heat leaks arise from the travel of the piston and the conduction path which is provided through the the clearance between the piston and the housing. Therefore, the actual coefficient of performance is assumed here to be 20.25% of the ideal value

$$\beta_{actual} = 0.2025\ R(P_1/P_2 - 1) / \{c_p[(P_1/P_2)\ (T_H/T_L) - 1](P_1/P_2)^{(\gamma-1)/\gamma-1}\}. \quad (4-21)$$

Heat Exchanger Analysis

Heat Exchanger Analysis for the Saturated Pool Boiling Design

The condenser is assumed to have a temperature distribution as shown in Figure 12. The required mass flow rate of gaseous helium in the condenser, and throughout the helium loop, can be determined as

$$\dot{m}_{He} = \dot{Q}_{condenser} / (c_p\, \Delta T_{condenser}), \quad (4-22)$$

where $\dot{Q}_{condenser}$ is the heat transfer required in the condenser, calculated as previously described. The corresponding log mean temperature difference is given by:

$$\Delta T_{LM} = (T_{c2} - T_{c1})/\ \ln[(T_{sat} - T_{c1}) / (T_{sat} - T_{c2})]. \quad (4-23)$$

The heat exchanger used to transfer thermal energy from the gaseous helium in the loop is assumed to have a temperature distribution as shown in Figure 13. It is assumed that the temperature of the helium in the expansion space of the cryocooler remains relatively constant at the refrigeration temperature (T_{ref}). The amount of heat transfer from the gaseous helium in the loop is found from the following expression:

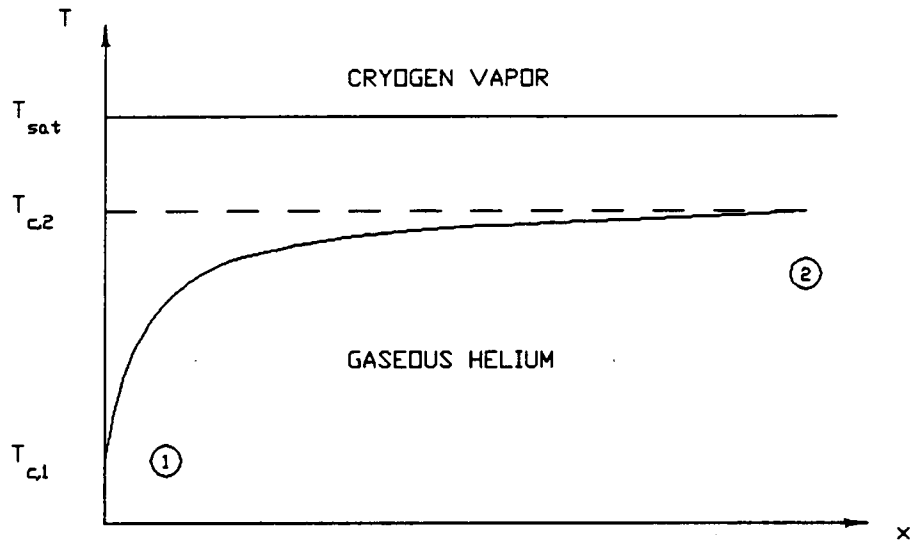

Figure 12. Condenser Temperature Distribution for Saturated Pool Boiling Design

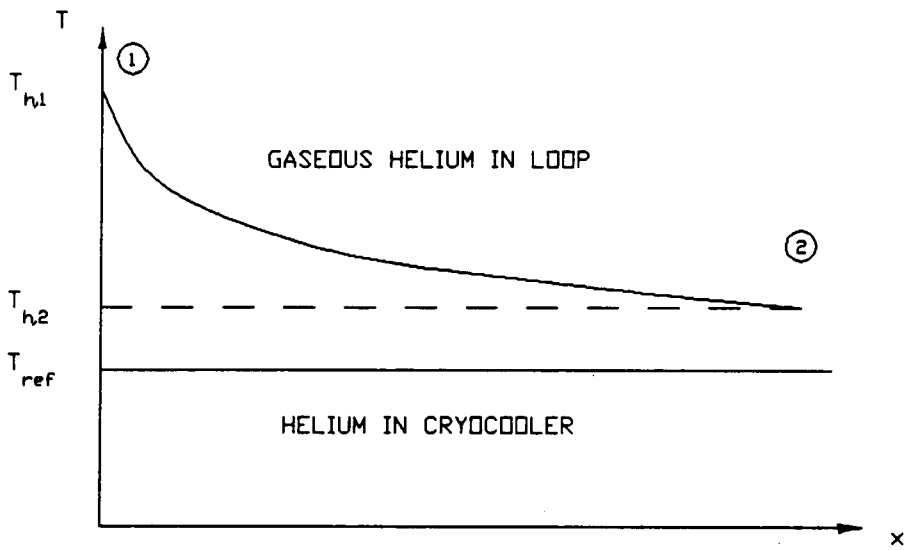

Figure 13. Heat Exchanger Temperature Distribution for Saturated and Subcooled Pool Boiling Designs

$$\dot{Q}_{ref} = \dot{Q}_{hx} = \dot{m}_{He}\, c_p\, (T_{h1} - T_{h2}). \qquad (4\text{-}24)$$

The corresponding log mean temperature difference is given by:

$$\Delta T_{LM} = (T_{h1} - T_{h2}) \,/\, \ln[(T_{h1} - T_{ref}) \,/\, (T_{h2} - T_{ref})]. \qquad (4\text{-}25)$$

The product of the overall heat transfer coefficient (U) and heat exchanger surface area (A) is determined by

$$UA = Q \,/\, \Delta T_{LM}. \qquad (4\text{-}26)$$

The following equation (Incropera and DeWitt, 1985) is used to calculate the length of pipe required for the condenser and heat exchanger:

$$1/UA = 1 \,/\, (hA)_{inner} + R''_{f,inner} \,/\, A_{inner} + \ln(D_o \,/\, D_i) \,/\, 2\pi k L$$
$$+ R''_{f,outer} \,/\, A_{outer} + 1 \,/\, (hA)_{outer}, \qquad (4\text{-}27)$$

where:

$$A = \pi D L. \qquad (4\text{-}28)$$

R''_f , the fouling factor, takes into account such things as fluid impurities which can lead to the deposition of a film on the inner surface of the pipe resulting in an increased resistance to heat transfer. A value of 0.0002 is used for the fouling factor as recommended by Incropera and DeWitt (1985). Equation 4-27 can be solved for the heat exchanger length which gives

$$L = UA \{ 1 \,/\, \pi (hD)_{inner} + R''_{f,inner} \,/\, \pi D_{inner} + \ln(D_o \,/\, D_i)\ 2\pi k$$
$$+ R''_{f,outer} \,/\, \pi D_{outer} + 1 \,/\, \pi (hD)_{outer}. \qquad (4\text{-}29)$$

Heat Exchanger Analysis for the Subcooled Pool Boiling Design

The heat exchanger analysis for the subcooled pool boiling design is identical to the analysis for the saturated pool boiling design with one exception. The condenser is replaced by a heat exchanger that is completely submerged in the liquid nitrogen. The submerged heat exchanger has a temperature distribution shown in Figure 14. The log mean temperature difference in this heat exchanger is

$$\Delta T_{LM} = (T_{c2} - T_{c1}) / \ln[(T_{sub} - T_{c2}) / (T_{sub} - T_{c2})]. \qquad (4-30)$$

Heat Transfer Coefficients for Use in the Heat Exchanger Analysis

If the Nusselt number is known, the heat transfer coefficient can be determined from the following equation:

$$h = Nu_D \, k / D. \qquad (4-31)$$

Correlations for Convective Heat Transfer Coefficients for Flows in Cylindrical Tubes

The convective heat transfer coefficients for the tube flows in both the condenser and heat exchangers in the saturated and subcooled pool boiling designs can be calculated from the following correlations for internal flow:

$$Nu_D = 4.36, \qquad \text{if } Re_D < 2300, \; Pr > 0.6 \qquad (4-32)$$

$$Nu_D = 4.36 + [(36.5Pr^{1/3} - 4.36) / 7700](Re_D - 2300),$$

$$\text{if } 2300 < Re_D < 10000, \; Pr > 0.6, \qquad (4-33)$$

and,

$$Nu_D = 0.023Re_D^{4/5} Pr^{1/3}, \qquad \text{if } Re_D > 10000, \; Pr > 0.6. \qquad (4-34)$$

Correlations for External Convective Heat Transfer Coefficients

Correlations for the Saturated Pool Boiling Design. The convection coefficient over the outside of the tubes in the condenser is calculated from the following

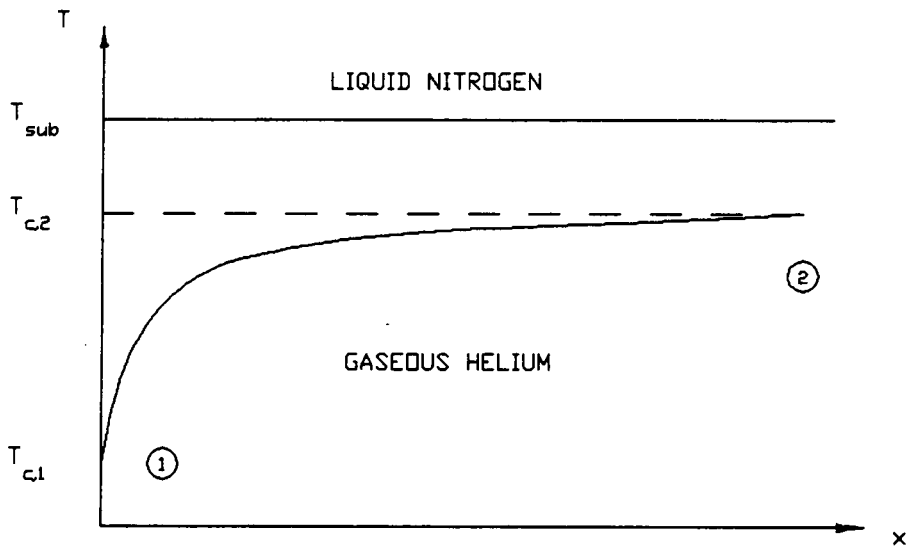

Figure 14. Heat Exchanger Temperature Distribution for Subcooled Pool Boiling Design

correlation for condensation on a vertical tier of N horizontal tubes (Incropera and DeWitt, 1985)

$$h_{D,N} = 0.729 \{[g \rho_l (\rho_l - \rho_v) k_l^3 h'_{fg}] / N \mu_l (T_{sat} - T_s) D\}^{1/4}, \quad (4-35)$$

where:

$$h'_{fg} = h_{fg} + 0.68 c_{p,l} (T_{sat} - T_s). \quad (4-36)$$

All properties are evaluated at the saturation temperature of the liquid cryogen. T_s, the surface temperature of the tube, is taken as the average temperature of the helium in the condenser.

The convection coefficient on the outside of the tubes for the heat exchanger interfacing with the refrigerator is determined from the following correlation for free convection over a horizontal cylinder (Incropera and DeWitt, 1985) given by

$$Nu_D = \{ 0.6 + (0.387 Ra_D^{1/6}) / [1 + (0.559 / Pr)^{9/16}]^{8/27} \}^2, \quad (4-37)$$

where:

$$Ra_D = g \beta (T_s - T_{ref}) D^3 / \nu \alpha. \quad (4-38)$$

Values of viscosity, density, thermal conductivity, and Prandtl number for gaseous helium are obtained from the following linear interpolations between 50 K and 75 K (Barron, 1985):

$$\mu = 7.884 \times 10^{-6} - 7.164 \times 10^{-8} (75 - T_{ref}), \quad (4-39)$$

$$\rho = 0.641 + 1.332 \times 10^{-2} (75 - T_{ref}), \quad (4-40)$$

$$k = 62.4 \times 10^{-3} - 6 \times 10^{-4} (75 - T_{ref}), \quad (4-41)$$

and,

$$Pr = 0.657 + 4.8 \times 10^{-4} (75 - T_{ref}), \qquad (4-42)$$

where T_{ref} is the refrigeration temperature in degrees Kelvin. The data used to obtain these expressions is presented in Tables B-4 and B-5 in Appendix B.

Since helium may be treated as an ideal gas, the value of the thermal expansion coefficient (β) is found from

$$\beta = 1 / T_{ref}. \qquad (4-43)$$

T_s, the surface temperature of the tube, is taken as the average temperature of the helium in the heat exchanger. T_{ref} is the absolute cold temperature produced by the refrigerator.

Correlations for the Subcooled Pool Boiling Design

The convection coefficient on the outside of the tubes for the heat exchanger exposed to the cold space of the cryocooler is calculated in the same manner as described for the saturated pool boiling design. However, the external convection coefficient for the heat exchanger submerged in the liquid nitrogen must now also be calculated as a free convection coefficient. Equation 4-37 can be used to calculate the Nusselt number of liquid nitrogen. The Rayleigh number is calculated as

$$Ra_D = g\beta(T_{sub} - T_s)D^3/\nu\,\alpha, \qquad (4-44)$$

where:

$$\beta = -1/\rho(\Delta\rho / \Delta T)_{p=const}. \qquad (4-45)$$

Values of viscosity, density, thermal conductivity, Prandtl number, and thermal

expansion coefficient for liquid nitrogen are obtained from the following linear interpolations between 65 K and 75 K (Barron, 1985):

$$\mu = -10.5 \times 10^{-6} T_{sub} + 960.5 \times 10^{-6}, \quad (4-46)$$

$$\rho = -4.28 T_{sub} + 1139.1, \quad (4-47)$$

$$k = -1.57 \times 10^{-3} T_{sub} + 260.75 \times 10^{-3}, \quad (4-48)$$

$$Pr = -0.105 T_{sub} + 10.345, \quad (4-49)$$

and,

$$\beta = 7.39 \times 10^{-5} T_{sub} + 5.15 \times 10^{-5}. \quad (4-50)$$

The data used to obtain these expressions is presented in Tables B-2 and B-3 in Appendix B.

Finned Heat Exchanger Analysis

Since the outer free convection coefficients for the heat exchangers are relatively low, the length of pipe required in the heat exchangers becomes large. This, consequently, increases the pressure drop, pump work , and temperature rise across the pump. Since it is desired to keep these quantities to a minimum, the addition of fins to the outside of the tubes will be investigated. Fin addition increases the effective heat transfer area, as well as increasing the thermal resistance and thereby decreasing the pipe length required.

The heat exchanger design is basically a fixed volume design since the available space is restricted. The maximum value of heat transfer per unit length is acheived when the ratio of inner to outer surface area is approximately inversely proportional to the ratio of their respective heat transfer coefficients (Fraas, 1989). For the gaseous helium heat exchanger interfacing with the refrigerator being considered, the internal convection coefficient is approximately 135 W/m^2-K, while the external convection

coefficient is roughly 17 W/m^2 K. Therefore, based on the previously mentioned rule of thumb, the heat exchanger volume would be most efficiently utilized if the outer surface area was approximately 8 times greater than the inner surface area. Fraas (1989) also states that typical fin spacing values range from 6 to 15 fins/in (2.4 - 5.9 fins/cm).

An important parameter which needs to be determined is the fin efficiency, η_f. Fin efficiency is defined as the ratio of the average temperature difference between the bulk free fluid stream and the surface to the temperature difference between the bulk free stream and the base surface (root of fin). Fin efficiency depends on the thermal conductivity of the fin, the heat transfer coefficient, and the fin dimensions (thickness and length).

Once the fin efficiency is known, the temperature effectiveness of the fin, η_o, can be determined from the equation (Incropera and DeWitt, 1985)

$$\eta_o = 1 - (A_{fin} / A)(1 - \eta_f), \qquad (4-51)$$

where A_{fin} is the surface area of the fin and A is the total surface area (fin and base).

If fins are added, Equation 4-27 can be modified to the following form (Incropera and DeWitt, 1985):

$$1/UA = 1 / (hA)_{inner} + R''_{f,inner} / A_{inner} + \ln(D_o / D_i) / 2 \pi k L$$
$$+ R''_{f,outer} / (\eta_o A)_{outer} + 1 / (\eta_o hA)_{outer}. \qquad (4-52)$$

Two types of fins, radial and axial, will be considered.

Radial Fins

A sketch of a typical radial fin is shown in Figure 15. The fins have length L_{fin} and thickness t. The fins are spaced 3t apart; therefore, if two finned tubes are placed close together, a minimum distance of t may still be maintained between adjacent fins. The fin

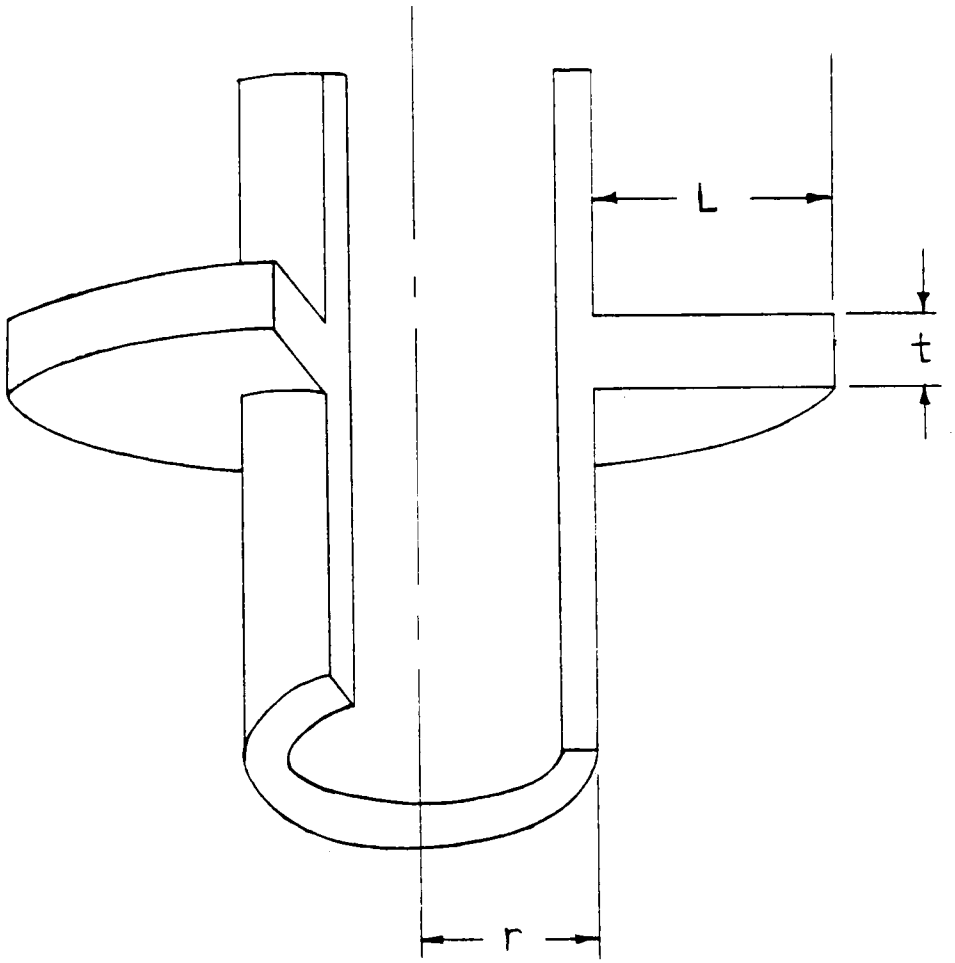

Figure 15. Radial Fin Configuration

efficiency is found from the following polynomial curve fit, obtained from data presented in Table 2 (Incropera and DeWitt,1985):

$$\eta_{fin} = 4.871362 \times 10^{-2}\, b^3 - 9.734929 \times 10^{-2}\, b^2 - 0.3833113\, b + 1.014096. \qquad (4\text{-}53)$$

Equation 4-53 is valid for a fin length approximately equal to the outer pipe diameter. As the fin length increases, the fin efficiency decreases. The fin parameter, b, is defined as

$$b = L_c^{3/2}(h / k A_p)^{1/2}, \qquad (4\text{-}54)$$

where:

$$L_c = L_{fin} + t / 2, \qquad (4\text{-}55)$$

and,

$$A_p = L_c\, t. \qquad (4\text{-}56)$$

The toal number of fins on the heat exchanger piping is determined from

$$N_{fin} = L_{hx} / 4t, \qquad (4\text{-}57)$$

where L_{hx} is the total length of pipe in the heat exchanger.

The base surface area is

$$A_{base} = \pi\, D_{outer}\, (3t)\, N_{fin}. \qquad (4\text{-}58)$$

The fin surface area is

$$A_{fin} = \pi\, (D_{outer} + 2L_{fin})\, t\, N_{fin} + 2\pi\, [(D_{outer} + 2L_{fin})^2 / 4 - D_{outer}^2 / 4]\, N_{fin}. \qquad (4\text{-}59)$$

Table 2. Fin Efficiency of Radial Fins

$L_c^{3/2} (h/kA_p)^{1/2}$	η_{fin}
0	1.00
0.1	0.97
0.2	0.94
0.3	0.90
0.4	0.86
0.5	0.82
1.0	0.55
1.5	0.38
2.0	0.27
2.5	0.20

Source : Heat Exchanger Design, Fraas, 1989.

The total surface area can be expressed as the product of the total pipe length and a constant, c, as follows

$$A_{tot} = A_{base} + A_{fin} = (c)\, L_{hx}, \qquad (4-60)$$

where:

$$c = \pi(D_{outer} - N_{fin}t) + 2L_{fin}N_{fin}. \qquad (4-61)$$

Therefore, the total finned pipe length can be determined from the following relationship:

$$L_{hx} = 1/UA\,[1 / (\pi\, hD)_{outer} + R''_{f,inner} / \pi\, D_{inner} + \ln(D_{outer} / D_{inner}) / 2\,\pi\, k + R''_{f,outer} / (\eta_o\, c) + 1 / (\eta_o\, h_{outer}\, c)]. \qquad (4-62)$$

Axial Fins

A sketch of axial fins is shown in Figure 16. The fins have length L_{fin} and thickness t. Once again, the fins are spaced 3t apart to allow proper meshing. The fin efficiency is found from the following correlation, obtained from data presented in Table 3 (Fraas, 1989):

$$\eta_{fin} = 9.662825 \times 10^{-2}\, b^3 - 0.2780816\, b^2 - 0.1136575\, b + 1.008117. \qquad (4-63)$$

The fin parameter for axial fins is defined as

$$b = L_{fin}\,(2h / k\, t)^{1/2}. \qquad (4-64)$$

The number of axial fins is given by

$$N_{fin} = \pi\, D_{outer} / 4\, t. \qquad (4-65)$$

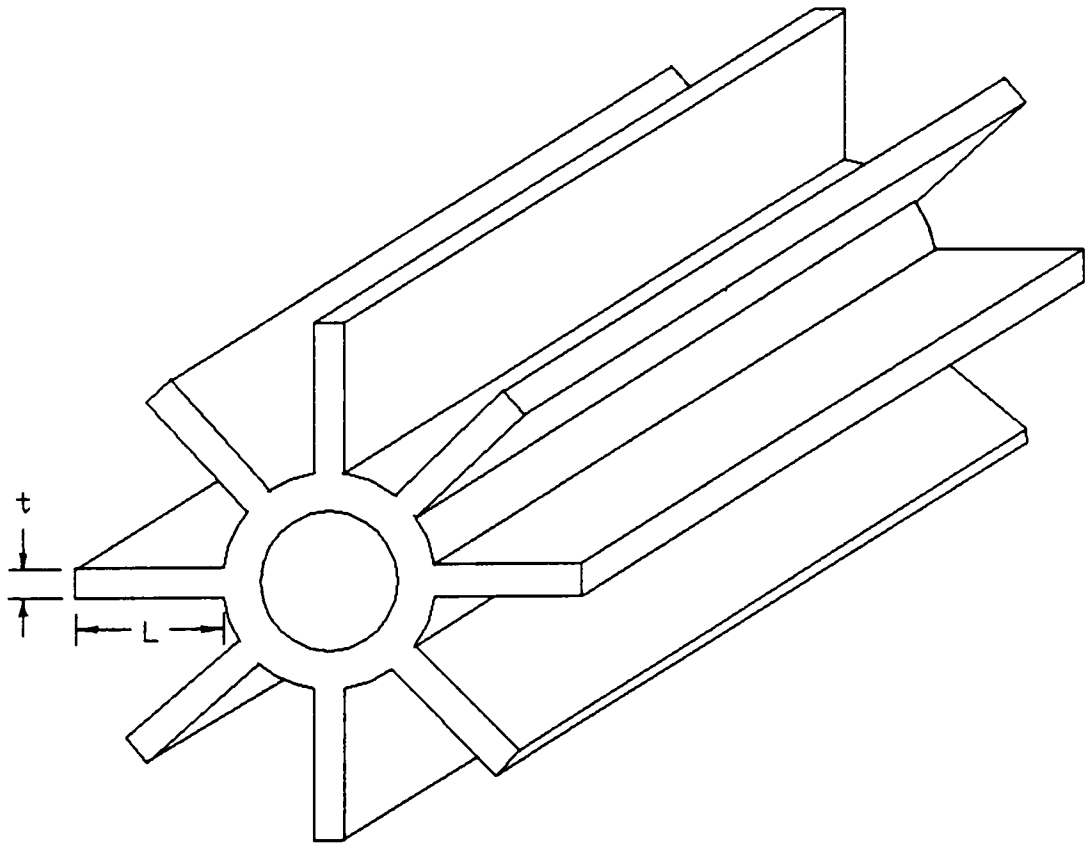

Figure 16. Axial Fin Configuration

Table 3. Fin Efficiency of Axial Fins

$L_{fin}\,(2h/kt)^{1/2}$	η_{fin}
0	1.00
0.2	0.98
0.4	0.94
0.6	0.86
0.8	0.78
1.0	0.70
1.2	0.64
1.4	0.57
1.8	0.47
2.0	0.44

Source : Heat Exchanger Design, Fraas, 1989.

The base surface area is

$$A_{base} = \pi (D_{outer} - N_{fin}\, t)\, L_{fin}. \qquad (4\text{-}66)$$

The fin surface area is

$$A_{fin} = 2L_{fin}\, L_{hx}\, N_{fin}. \qquad (4\text{-}67)$$

The total surface area can be expressed as the product of the total pipe length and a constant, c, as follows

$$A_{tot} = c\, L_{hx}, \qquad (4\text{-}68)$$

where:

$$c = \pi(D_{outer} - N_{fin}t) + 2L_{fin}N_{fin}. \qquad (4\text{-}69)$$

The finned pipe length can be calculated from Equation 4-62.

Analysis of the Gaseous Helium Circulator

The fan work required is given by

$$W_{fan} = \dot{m}_{He}\, \Delta p / \rho\, \eta_{fan}, \qquad (4\text{-}70)$$

where Δp is the total pressure drop occurring in the gaseous helium lines, condenser, and heat exchanger and η_{fan} is the fan efficiency. The temperature rise across the fan is calculated from the relationship

$$\Delta T_{fan} = W_{fan} / \dot{m}_{He}\, c_p. \qquad (4\text{-}71)$$

CHAPTER V

SYSTEM SPECIFICATIONS

Selection of the Coolant

Although an average heat flux of 0.4 W/cm^2 is assumed for the CPU board, the heat flux distribution is typically non-uniform and the actual heat flux value at the chip surface may be as high as 10-20 W/cm^2 (Krane, Parsons, and Bar-Cohen, 1988). Since gases are typically characterized by low convective heat transfer coefficients, a liquid should be employed as the coolant.

Many of the advantages of operating electronic components at cryogenic temperatures are realized between 10 K and 100 K (Krane, Jaeger, Bar-Cohen, and Gaensslen, 1990). Both nitrogen and neon exist in the liquid state within this temperature range. Several other cryogens also exist in the liquid state at these temperatures; however, their use is prohibited due to some undesirable characteristic. Carbon monoxide, CO, is toxic, fluorine, F_2, is corrosive, and hydrogen, H_2, and oxygen, O_2, present fire hazards.

Liquid nitrogen, which can be subcooled to approximately 63 K, boils at 77 K at atmospheric pressure. Liquid nitrogen, obtained by liquefying air, is available in most metropolitan areas for approximately $0.10 per liter (Krane, Parsons, and Bar-Cohen,1988). Neon, which is a liquid between 24.5 K and 44.5 K, boils at 27.09 K at atmospheric pressure. Although gaseous neon can be purchased for as low as $0.05 per liter, liquid neon costs approximately $100 per liter (Krane, Parsons, Bar-Cohen,1988). Based on original cost, liquid nitrogen has a significant economic advantage over liquid neon. The use of neon would also involve increased refrigeration costs due to its lower saturation temperature. A third advantage is that a critical heat flux value nearly twice as large as that of neon can be obtained with liquid nitrogen (Krane, Parsons, and Bar-Cohen,1988). Based on the previous discussion, liquid nitrogen should be used as the working fluid of the cryogenically-cooled computer.

The following curve fits obtained from data in Barron (1985) are used to obtain the enthalpy, viscosity, and density of liquid nitrogen at temperatures between 63 and 77 K:

$$h = -1.33702 \times 10^{-1} T^3 + 2.733992 \times 10^{1} T^2 - 0.1201434 \times 10^{3} T - 222.2402 \times 10^{3}, \quad (5-1)$$

$$\mu = 5.637657 \times 10^{-9} T^3 - 9.241204 \times 10^{-6} T^2 + 35.90146 \times 10^{-6} T + 300.8022 \times 10^{-6}, \quad (5-2)$$

and,

$$\rho = 1.083493 \times 10^{-3} T^3 - 0.2505496 T^2 + 14.82779 T + 658.1284. \quad (5-3)$$

The values used to obtain these correlations appear in Tables B.1 and B.2 in Appendix B.

Design of the Piping System

Since the piping introduces a significant path for heat transfer from the surroundings, the piping system must be carefully designed. Barron (1985) suggests that schedule 5 pipe (the thinnest piping available in stainless steel) or schedule 10 pipe be used for cryogenic applications. Therefore, it is assumed that stainless steel schedule 5 piping will be used for all transfer lines in the cryogenic cooling system. Table 4 contains the standard stainless steel IPS pipe sizes for schedule 5 piping up to a nominal size of 50 mm. The standard wall thickness of schedule 5 piping over this size range is 1.7 mm.

Insulation

Several types of insulation are commonly used for cryogenic applications. These include foam, evacuated powders, vacuum, and multilayer insulations. Common foams used for cryogenic insulation include polyurethane and polystyrene. Foams are characterized by a relatively high heat transfer rate per meter (approximately 20 W/m). Foam insulation also tends to crack upon repeated thermal cycling and loses its insulating value. Evacuated powder insulation consists of a finely divided particulate material such as perlite or SiO_2 packed between the surfaces to be insulated. Multilayer insulation consists of alternating layers of highly refllective material such as aluminum foil or alumnized Mylar and a low conductivity spacer such as fiberglass mat or paper

Table 4. Stainless Steel IPS Pipe Sizes

Nominal size (mm)	Inner Diameter (mm)	Outer Diameter (mm)
15	17.9	21.3
20	23.3	26.7
25	30.0	33.4
32	38.8	42.4
40	44.9	48.3
50	56.9	60.3

Source : Cryogenic Systems, Barron, 1985.

all under a high vacuum (Barron, 1985). Although multilayer insulations provide the greatest resistance to heat transfer from the surroundings, they also have the highest cost per unit volume (Barron, 1985) which makes them impractical for long piping runs. However, vacuum insulations are characterized by a significantly lower cost and therefore are frequently used for cryogenic applications. Therefore, it is assumed that all piping in the cryogenic cooling system is vacuum insulated.

The next problem to be addressed concerns the quantification of the heat transfer rate from the surroundings through the transfer lines. In order to predict the heat transfer rate through the vacuum insulated lines, representative values of heat transfer rates through actual vacuum insulated lines must be obtained and these values must be closely matched by the analytical model by adjusting the insulation thickness and the number of radiation shields. Semiflex^{TM} piping, manufactured by Vacuum Barrier Corporation, was used as the reference in determining the heat transfer rate through the line. Semiflex^{TM} piping consists of two concentric, corrugated copper tubes separated by a spiral Teflon spacer (Vacuum Barrier Corporation, 1989). The space between the tubes is continuously evacuated by a pump. Semiflex^{TM} piping is bendable which simplifies installation and minimizes joints. Heat transfer rates through a 30 meter section of Semiflex piping were obtained from Vacuum Barrier Corporation and are presented in Table 5. These values apply to heat transfer to liquid nitrogen at a temperature of 77 K through the piping from the surroundings at approximately 300 K. The values in Table 5 were used to obtain the following correlation which will be used as a reference in predicting the heat transfer rate through a 30 meter run of piping :

$$\dot{Q}_{line} = 1.246872 \times 10^{-3}\, ID^3 - 0.1437394\ ID^2 + 5.977663\, ID - 33.95257, \qquad (5-4)$$

where $\dot{Q}_{line}$ is in watts and ID is the inner pipe diameter in mm.

For piping having nominal sizes ranging from 15 to 50 mm, the above correlation was used to calculate the steady-state heat transfer rate through a 30 meter section of the transfer line. These values are presented in Table 6. Using the method discussed in Chapter III for calculating the heat transfer rate through the line, a trial and error procedure was used to obtain the combination of insulation thickness and number of radiation shields for each pipe size which would provide values of heat transfer rate

Table 5. Heat Transfer Rates Through 30 Meters of Semiflex^{TM} Piping

Inner Diameter (mm)	Heat Transfer Rate (W)
16	30
32	51
44	57
56	69

Source : Vacuum Barrier Corporation, 1989.

Table 6. Heat Transfer Rates Obtained from Vacuum Barrier Corporation and Calculated by Analytical Model

Nominal Size (mm)	Insulation Thickness (mm)	Heat Transfer Rate	
		Equation 5-4 (W)	Model (W)
15	21.45	34.14	34.19
20	26.85	43.06	43.09
25	27.20	49.68	49.68
32	22.55	54.42	54.44
40	19.55	57.53	57.54
50	22.45	70.50	70.56

comparable to the values obtained from Equation 5-4. By trial and error, it was discovered that the use of one radiation shield in the analytical model provided results which were in the range of values predicted by Equation 5-4. Therefore, assuming one radiation shield, the vacuum insulation thickness was varied until the values predicted by the numerical model were within less than 0.5% of the reference values. Table 6 contains the heat transfer rates through the vacuum insulated piping predicted by the model using the trial and error procedure described above.

According to Vacuum Barrier Corporation (1989), the minimum value of heat transfer rate per meter of piping length achievable is approximately 1 W/m. However, a representative for Macomber Cryogenics (1990) stated that values of heat transfer rate per meter as low as 0.43 W/m can be obtained by using "super insulated" vacuum piping which contains radiation shields. He also stated that bare vacuum insulation containing no radiation shields has a representative heat transfer rate per meter value of approximately 3.8 W/m. The lowest value of heat transfer rate per meter of length for vacuum insulated pipe, 0.31 W/m, was quoted by a reresentative of Cryogenic Services (1990).

Using the vacuum insulation thickness for 15 mm nominal size pipe appearing in Table 6 (21.45 mm), the heat transfer rate per meter of length for 15 mm nominal size pipe containing various numbers of radiation shields was calculated using the numerical model. These values for heat transfer rate per unit length for vacuum insulation containing zero to four radiation shields are presented in Table 7. The values presented in Table 7 are in good agreement with values obtained from cryogenic suppliers (Vacuum Barrier, 1989, Macomber Cryogenics, 1990, and Cryogenic Services, 1990)

Based on the preceding discussion, the actual piping system modeled consists of stainless steel schedule 5 piping with vacuum insulation containing a minimum of zero and a maximum of four radiation shields. A cross-section of the modeled insulated piping system is shown in Figure 17.

Preliminary "Once-Through" System Specifications

In order to quantify parameters associated with the "once-through" system such as heat transfer rates, required liquid nitrogen flow rate, and pressure drop in the transfer line, preliminary calculations were made prior to the optimization studies.

Table 7. Calculated Heat Transfer Rate per Meter Through the Vacuum Insulated Lines as a Function of the Number of Radiation Shields

Number of Radiation Shields	Heat Transfer Rate per Meter (W/m)
0	4.63
1	1.14
2	0.636
3	0.439
4	0.334

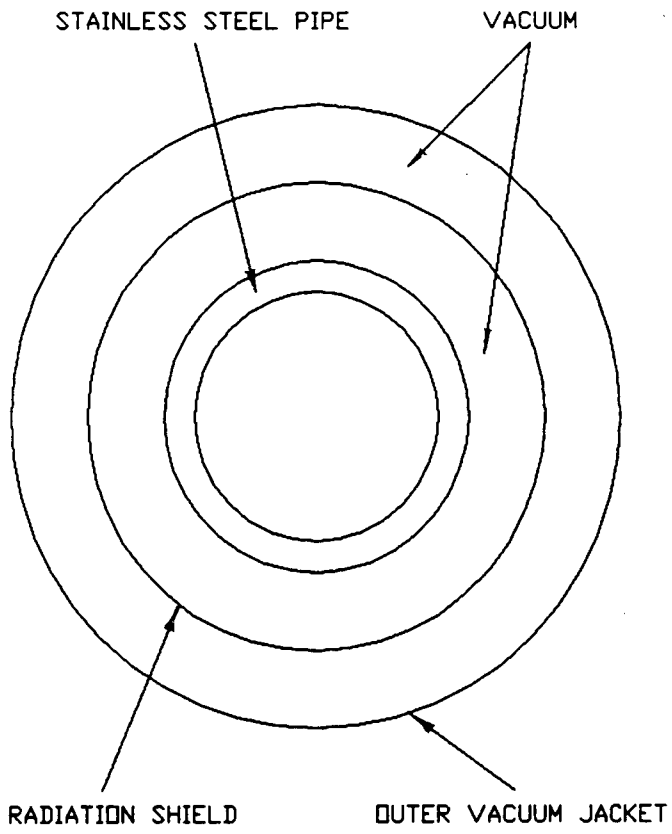

Figure 17. Cross Section of the Modeled Insulated Piping System

Heat Transfer to the Liquid Nitrogen

Heat transfer to the liquid nitrogen occurs through the transfer line, the CPU vessel, the leads, and from the computer boards. Preliminary values for each of these heat transfer rates will be discussed below.

Depending on the pipe size and the number of radiation shields used, the rate of heat transfer through the liquid nitrogen lines ranges from approximately 10 W to 140 W. A typical value of evaporation rate for liquid nitrogen dewars was found to be approximately 1.5% volume per day (Minnesota Valley Engineering, 1989). This boil-off rate is based on a liquid nitrogen temperature of -320 ^{o}F (77 K) and an ambient temperature of 70 ^{o}F (294 K). If the CPU vessel is filled with liquid nitrogen at 77 K and has an inner diameter of 75 cm and a height of 75 cm, the boil-off rate is approximately 5.75 x 10^{-8} m^3/s. Multiplying by the enthalpy of evaporation and the density of liquid nitrogen at 77 K, the steady state heat loss from the CPU vessel is calculated as 9.26 W. Once again, a trial and error procedure was used to determine the vacuum insulation thickness and the number of radiation shields used in the model which will give a heat transfer rate through the CPU vessel comparable with the value of 9.26 W obtained above. The values of insulation thickness and number of radiation shields for the CPU vessel which will be used in the analytical model were determined to be 5.75 cm and 2, respectively.

If the leads are modeled as a single copper lead as previously described, the heat transfer to the liquid nitrogen through the leads by lateral conduction may be calculated from Equation 3-16. Buyanov (1985) discusses a typical current lead which consists of 500 wires of 0.1 mm diameter. Therefore, the modeled lead is assumed to have a diameter of 5 mm. A lead length of 30 cm (approximately 1 ft) is assumed. If the ends of the lead are exposed to temperatures of 300 K (ambient) and 77 K, the heat transfer to the liquid nitrogen through the lead is approximately 7 W.

If the computer has one vertical CPU board with a surface area of 0.25 m^2 having an average heat flux of 4000 W/m^2 (Krane, Parsons, and Bar-Cohen, 1988), the total power dissipation is 1 kW. Based on the above information, a total of approximately 1.1 kW of heat is transferred to the liquid nitrogen in a "once-through" system (1 kW from the CPU board, a maximum of approximately 140 W through the transfer line, approximately 10 W through the CPU vessel, and approximately 7 W through the leads).

Liquid Nitrogen Flow Parameters

Table 8 contains values of mass flow rate, velocity, and Reynolds number for liquid nitrogen flow at 77 K for nominal pipe sizes ranging from 15 mm to 50 mm. The liquid nitrogen flow in the transfer line is laminar with a mass flow rate of approximately 5.5 x 10^{-3} kg/s.

Pressure Drop and Temperature Rise in the Transfer Line

Values of pressure drop and CPU vessel inlet temperature for pipe runs of 30 meters are given in Table 9 for an inlet liquid nitrogen temperature of 63 K (maximum subcooling). As the pipe size increases, the exit temperature of the liquid nitrogen increases due to the increased heat leak through the lines. However, even for the largest pipe size considered (50 mm), almost 6 K of subcooling is maintained at the entrance to the CPU vessel.

If the liquid nitrogen enters the transfer lines as a saturated liquid at 77 K, any heat transfer will result in some vaporization and two-phase flow will exist in the line. Table 10 gives values of the two-phase pressure drop as well as the quality at the inlet of the CPU vessel after 30 meters of travel. In comparing the pressure drop values in Tables 9 and 10, it can be seen that the two-phase pressure drops are a factor of approximately 2.5 to 6.0 greater than the single phase pressure drops; however, the two-phase pressure drop values are still extremely low due to the low flow rates. As the pipe size increases, the quality of the saturated nitrogen at the inlet of the CPU vessel increases as a result of the increased heat transfer through the line. If large fractions of vapor exist in the transfer line, additional equipment such as a phase separator may be required in order to avoid pumping problems. Therefore, it is desired to the keep the liquid nitrogen quality as low as possible.

Preliminary Refrigerated System Specifications

In an attempt to quantify parameters associated with the refrigerated system such as the coefficient of performance of the cryocooler, refrigeration capacity, required work input, heat transfer coefficients in the heat exchangers, piping lengths required in the heat exchangers , and pressure drops in the system, preliminary calculations were performed prior to the optimization studies. In order to perform the preliminary calculations, values for the pipe sizes, refrigeration temperature, outlet temperature of

Table 8. Calculated Liquid Nitrogen Flow Parameters

Nominal Pipe Size (mm)	Mass Flow Rate (kg/s)	Flow Velocity (m/s)	Reynolds Number
15	5.27×10^{-3}	1.16×10^{-4}	10.6
20	5.31×10^{-3}	9.03×10^{-5}	10.7
25	5.35×10^{-3}	7.06×10^{-5}	10.8
32	5.37×10^{-3}	5.48×10^{-5}	10.9
40	5.39×10^{-3}	4.75×10^{-5}	10.9
50	5.45×10^{-3}	3.80×10^{-5}	11.0

Table 9. Calculated Pressure Drop and Temperature Rise in Transfer Lines

Nominal Pipe Size (mm)	Pressure Drop (Pa)	CPU Vessel Inlet Temperature (K)
15	17.84	66.76
20	6.15	67.67
25	2.22	68.35
32	0.78	68.83
40	0.44	69.12
50	0.17	70.40

Table 10. Calculated Two-Phase Pressure Drop and CPU Vessel Inlet Quality Values

Nominal Pipe Size (mm)	Pressure Drop (Pa)	CPU Vessel Inlet Quality (%)
15	109.6	3.26
20	14.46	4.06
25	4.85	4.66
32	1.73	5.09
40	0.99	5.35
50	0.41	6.50

the heat exchanger exposed to the cold space of the cryocooler, and temperature rise across the condenser or submerged heat exchanger must be specified. The following list of assumptions were necessary only for the preliminary calculations presented in this chapter (these parameters along with others will be optimized in the final design):

(1) All pipe sizes, including gaseous helium transfer lines and heat exchanger and condenser piping, are assumed to have a nominal size of 50 mm.

(2) The cold temperature produced in the expansion space of the cryocooler, the refrigeration temperature, is assumed to be 63 K.

(3) The outlet temperature of the heat exchanger in the upper portion of the helium loop is assumed to be 66 K.

(4) The temperature rise across the condenser (saturated pool boiling design) or the submerged heat exchanger (subcooled pool boiling design) is assumed to be 6 K.

Coefficient of Performance of the Stirling Cryocooler

Since the coefficient of performance of the Stirling cycle is dependent on the kinematic viscosity ratio, as shown by Equation 4-12, it is therefore dependent on the working fluid used. Neglecting the pressure drop in the regenerator, and using Equation 4-19, gaseous helium produces a coefficient of performance value of 0.0773. This value is based on a refrigeration temperature of 63 K.

Several parameters must be determined before the coefficient of performance of the Stirling cryocooler can be calculated. These include the dead volume ratio (X), the sinusoidal phase shift between the piston and the displacer (θ), piston diameter (D_{pist}), the amplitude of the piston stroke (L_p), the running frequency of the cryocooler (ω), the mesh distance (l) and the diameter of mesh wire in the regenerator (D_m), the number of mesh punchings in the regenerator (n), and the diameter of the regenerator matrix (D_{reg}). The dead volume ratio in the cryocooler is the total internal volume of the heat exchangers and associated ducts expressed as a fraction of the swept

volume in the expansion space of the cryocooler. The regenerator is assumed to be composed of a series of stacked wire screens. The mesh distance refers to the spacing between the wires in the screen and the number of mesh punchings in the regenerator refers to the actual number of stacked wire screens.

Walker (1983) suggests that a dead volume ratio of between 1 and 2 be used in Stirling cryocoolers. The value of the dead volume ratio should be relatively low in order to maximize the the pressure difference achieved in the cryocooler. The results of Tailor and Narayankhedkar (1988) show that the maximum refrigerating capacity as well as the coefficient of performance of the cryocooler decreases as the dead volume ratio increases. Tailor and Narayankhedkar (1988) also indicate that the maximum refrigerating capacity is acheived when the phase shift between the piston and displacer is 45°.

In a paper by Walker (1983), some technical specifications for a Philips Stirling cryocooler (model PPG 102) are given. At 80 K, the cryocooler has a refrigerating capacity of 1 kW and a coefficient of performance of 11.1%. The running frequency of the cryocooler is 1450 rpm and the working fluid is gaseous helium. The piston has a diameter of 7 cm and a stroke of 5.3 cm. This paper also indicates that well designed Stirling cryocoolers achieve coefficient of performance values that are between 35-45% of the Carnot value. Although these coefficient of performance values will not be used in this study, they will serve as a means of comparison with the value obtained from the analytical model used. A table of geometric factors of wire screens commonly used in thermal regenerators of Stirling cryocoolers, including values of mesh wire diameters and mesh distances, appears in a paper by Miyabe, Takahashi, and Hamaguchi (1982). The values for mesh distance range from 0.044 mm to 2.032 mm and the values for wire diameter range from 0.04 mm to 0.508 mm.

Walker (1983) suggests that the regenerator length should be at least three times the frontal diameter of the flow section in order to maximze the heat transfer occurring in the regenerator. The regenerator diameter is typically smaller than the piston diameter. If a regenerator diameter is assumed, the regenerator length may be approximated as three times the diameter. If the regenerator length and the diameter of mesh wires in the regenerator are known, the number of mesh punchings in the regenerator can be calculated as

$$n = L_{reg} / D_m. \qquad (5-5)$$

A comprehensive list of the input values for the Stirling cycle coefficient of performance calculation are presented in Table 11. Using the values in Table 11, the coefficient of performance of the Stirling cycle, including the regenerator pressure drop, is 0.0533. This corresponds to a Second Law efficiency of 20.05%. The Second Law efficiency of the cryocooler is defined as the ratio of the actual coefficient of performance to the Carnot coefficient of performance. Since this value is considerably lower than the value of 35-45% predicted by Walker (1983), this model is conservative even though it involves several simplifying assumptions including perfect regenerator performance.

Coefficient of Performance of the Gifford-McMahon Cryocooler

Gifford (1965) indicates that the Gifford-McMahon cycle works well at a pressure ratio between 1.5 and 2.5 since maximum values of coefficient of performance are achieved in this range. Helium will be used as the working gas in the cryocooler. Assuming a pressure ratio of 2.0 and values of T_H and T_L as 300 K and 63 K, respectively, the coefficient of performance of the Gifford-McMahon cryocooler, including estimated losses documented by Gifford (1965) due to pressure drops, the presence of void volume, and conduction, is 0.0296. The corresponding Second Law efficiency is 11.14%. This calculated value of the coefficient of performance agrees well with the values of two Gifford-McMahon cryocoolers discussed in Walker's text (1983). The AL01 has a coefficient of performance of 0.0222, corresponding to a Second Law efficiency of 8.35%, when operated at 80 K. The AL02 has a coefficient of performance value of 0.0267, corresponding to a Second Law efficiency of 10.05%, at 80 K.

Flow Parameters

The heat transfer required in the condenser is approximately 1016 W, which is the sum of the heat transfers through the CPU vessel walls, the current leads, and from the CPU boards. Based on this value of heat transfer and the assumed value of 6 K for the condenser temperature rise, the required mass flow rate of gaseous helium is approximately 3.26×10^{-2} kg/s. If 50 mm nominal size piping is used, the velocity of

Table 11. Input Values for the Stirling Cryocooler Coefficient of Performance Calculation

T_c	300 K
T_e	63 K
X	2
θ	45°
D_{pist}	7 cm
L_p	5.3 cm
ω	1450 rpm
l	0.153 mm
D_m	0.101 mm
n	1500
D_{reg}	5 cm

the gaseous helium is approximately 18 m/s with a corresponding Reynolds number of approximately 96200.

Refrigeration Capacity and Required Power Input

Values of the refrigeration capacity, which is equal to the heat transfer in the heat exchanger, and the power input to the refrigerator are given in Table 12 for three types of heat exchangers: an unfinned heat exchanger, a heat exchanger with radial fins, and a heat exchanger with axial fins. For all three types, the piping is assumed to have a nominal size of 50 mm. For both the radial and axial finned heat exchangers, the fins are assumed to have a length of 30 mm and a thickness of 1 mm. The fin length value of 30 mm was chosen as a maximum limit due to the space constraints. The value of fin thickness will be optimized in the final design.

Fan Power

A gaseous helium fan which supplies a flow in the range of 5×10^{-2} kg/s over a pressure drop of approximately 3.5 kPa has a design efficiency of 55% (Barber-Nichols Engineering). Fan efficiency is defined as the ratio of the actual power delivered to the fluid to the power required to drive the fan. The values of fan power for the same conditions and the three types of heat exchangers as described above are given in Table 13. As seen from these results, the use of a radially-finned heat exchanger results in the minimum required amount of fan power.

Heat Transfer Coefficients for the Heat Exchangers and Condenser

Accurate predictions of the heat transfer coefficients for the condenser and the heat exchanger are extremely important since they play an important role in the determination of the condenser and heat exchanger lengths.

Saturated Pool Boiling Design. The internal heat transfer coefficient for the condenser as determined by Equation 4-34 is approximately 200 W/m^2-K. The external heat transfer coefficient for the condenser is due to condensation and may be predicted by Equation 4-35. As expected, gaseous nitrogen condensation provides excellent heat transfer with a heat transfer coefficient of approximately 2100 W/m^2-K. The internal heat transfer coefficient for the heat exchanger is approximately 204 W/m^2-K. The external heat transfer coefficient, calculated from Equation 4-37, is

Table 12. Calculated Values of the Refrigeration Capacity and Refrigerator Power Input

Type of Heat Exchanger	Refrigeration Capacity (W)	Power Input (W)
Unfinned	1959	36751
Radial fins	1604	30104
Axial fins	1633	30637

Table 13. Calculated Values of the Fan Power

Type of Heat Exchanger	Fan Power (W)
Unfinned	617
Radial fins	263
Axial fins	291

due to free convection. As expected, the free convection coefficient is relatively low at approximately 17 W/m^2-K.

Subcooled Pool Boiling Design. The convective heat transfer coefficients between the helium and the inner surface of the pipe for both heat exchangers in the subcooled pool boiling design are approximately 200 W/m^2-K. The external heat transfer coefficient for the submerged heat exchanger is in the range of 300 W/m^2-K for saturated liquid nitrogen and 500 W/m^2-K for liquid nitrogen with maximum subcooling. The convective heat transfer coefficient between the cold helium produced in the expansion space of the cryocooler and the outer surface of the heat exchanger is approximately 15 W/m^2-K.

Piping Lengths

Saturated Pool Boiling Design. For the conditions previously described, a total of approximately 7 meters of piping are required in the condenser. Values of piping length and total surface area for unfinned, radially-finned, and axially-finned heat exchangers are given in Table 14. The fins are assumed to have a length of 30 mm and a thickness of 1 mm. Using the values presented in Table 14, the unfinned heat exchanger has approximately 0.189 m^2 of surface area per meter of length. The radially-finned heat exchanger has approximately 4.49 m^2 of surface area per meter of length, which is almost 23 times as much surface area as the unfinned pipe. The axially-finned heat exchanger has approximately 2.86 m^2 of surface area per meter of length. Based on this information, the radial fins provide the most efficient means of adding surface area and thus decreasing the piping length required.

Subcooled Pool Boiling Design. The length of pipe required in the submerged heat exchanger is approximately 4 meters, assuming the addition of radial fins having a length of 30 mm and a thickness of 1 mm. The heat exchanger interfacing with the refrigerator must have approximately 19 meters of piping, assuming the addition of radial fins with the same dimensions as given above.

Pressure Drop in the Gaseous Helium Transfer Lines

It is desired to keep the pressure drop occurring in the gaseous helium lines to a minimum. This is accomplished by keeping the piping lengths to a minimum. If the

Table 14. Calculated Values of the Piping Length and Surface Area in the Heat Exchanger

Type of Heat Exchanger	Piping Length (m)	Total Surface Area (m^2)
Unfinned	90.03	17.06
Radial	10.96	49.23
Axial	18.01	51.53

pressure drop becomes too high, the fan work required results in a large temperature rise across the fan. If this temperature rise is high enough, the inlet temperature at the condenser is too high to allow condensation of the liquid nitrogen and the coolant system fails.

If a radially-finned heat exchanger consisting of 10.96 m of piping, a condenser with a total piping length of 7 m, and 60 m of transfer line are used, a total pressure drop of 3.1 kPa is produced in the helium loop.

CHAPTER VI

COST ESTIMATES FOR A CRYOGENIC COOLING SYSTEM

Component Cost Estimation

Since the purpose of this study is to minimize the overall cost of the cryogenic cooling system, the initial costs of the individual components must be determined. The costs of all components, including those necessary for both a "once-through" and a refrigerated system, will be discussed.

For a "once-through" system, initial cost estimates for the following components must be determined :

(1) Piping,
(2) Vacuum insulation,
(3) Liquid nitrogen reservoir,
(4) CPU vessel, and
(5) Liquid nitrogen pump.

In addition to the components listed above, the refrigerated system requires cost estimates for the following components :

(6) Cryocooler,
(7) Heat exchangers and/or condenser in the helium loop, and
(8) Gaseous helium fan.

Piping

Prices (in $ / m) for 304 stainless steel schedule 5 piping, ranging from a nominal size of 15 mm in to 50 mm, appear in Table 15 (Hajoca, 1989). These values were used to generate the following expression for pipe cost :

$$\text{Piping cost} = (47881.49\ R^3 - 4932.401\ R^2 + 343.1831\ R + 1.363372)\ L, \qquad (6-1)$$

Table 15. 304 Stainless Steel Schedule 10 Piping Costs

Nominal size (cm)	Cost ($/m)
15	13.62
20	14.99
25	18.77
32	21.39
40	23.20
50	27.07

Source : Hajoca piping manufacturer, 1989.

where R is the radius and L is the pipe length, both in meters.

Vacuum Insulation

Vacuum insulation costs ranging from approximately $250 per meter to $500 per meter were obtained (Vacuum Barrier Corporation, 1989, Macomber Cryogenics, 1990, and Cryogenic Services, 1990). The cost of the vacuum insulation relies heavily on its insulating ability; however, a correlation for vacuum insulation cost as a function of the heat transfer rate through the insulation could not be obtained. In the analytical model, the heat transfer rate through the vacuum insulation is a direct function of the number of radiation shields as shown from the results presented in Chapter V. Therefore, the following "artificial" correlation for vacuum insulation cost as a function of the number of radiation shields was assumed:

$$\text{Vacuum Insulation Cost} = [250 + 250(n/4)]L, \qquad (6-2)$$

where n is the number of radiation shields in the insulation and L is the length of pipe in meters. The number of radiation shields can vary from zero to four.

Dewars

Costs of liquid nitrogen dewars ranging in volume from 160 liters (standard laboratory size dewars) to over 3400 liters (bulk tanks) were obtained from two manufacturers (Minnesota Valley Engineering, 1989, and Taylor-Wharton, 1989). This cost information is presented in Table 16. The information presented in this table was used to obtain the following expression for dewar price as a function of volume :

$$\text{Dewar cost} = 3.105242 \times 10^{-7} V^3 - 3.412452 \times 10^{-3} V^2 + 14.35143 V - 625.181, \qquad (6-3)$$

where V is the volume of the dewar in liters.

Liquid Nitrogen Pump

The cost of a liquid nitrogen pump which will transfer the liquid nitrogen from the reservoir to the CPU vessel is approximately $5000 (Barber-Nichols, 1990). Such a pump is well suited for intermittent operation.

Table 16. Liquid Nitrogen Dewar Prices

Volume (liters)	Price ($)
160	1686
185	1754
230	2559
1893	16420
3407	20940

Sources : Minnesota Valley Engineering and Taylor-Wharton, 1989.

Cryocooler

The cost of the cryocooler can be approximated by a correlation appearing in a paper by Longsworth and Steyert (1987). The following correlation gives the cost of the refrigerator and dewar as a function of the refrigerating capacity, $\dot{Q}_{ref}$, in watts:

$$\text{Cryocooler cost} = 750\ \dot{Q}_{ref}^{0.52}. \qquad (6-4)$$

Equation 6-4 is valid for cryocoolers in the 10 to 1000 W capacity range and in quantities of tens of thousands (Longsworth and Steyert, 1987).

Heat Exchangers

A review of the recent literature on heat exchanger cost estimation proved unsucessful in locating correlations to predict the cost of finned-tube heat exchangers. However, the following correlation (Fraas,1989) can be used to approximate the cost of shell-and-tube heat exchangers:

$$\text{Heat exchanger cost} = 235\ (\text{Area})^{0.665}. \qquad (6-5)$$

Fraas(1989) indicates that the cost of finned-tube heat exchangers with aluminum fins on carbon steel tubes runs about 25% the cost of conventional carbon steel shell-and-tube heat exchangers. Therefore, the following modification of Equation 6-5 may be used to estimate the price of finned-tube heat exchangers:

$$\text{Finned heat exchanger cost} = 58.75\ (\text{Area})^{0.665}. \qquad (6-6)$$

Helium Fan

The approximate cost of a helium fan which will provide approximately 5×10^{-2} kg/s of flow at an inlet temperature of 63 K to 77 K across a pressure drop of approximately 3.45 kPa was obtained from Barber-Nichols (1989). Due to the one-time, nonrecurring engineering costs, a fan meeting these requirements would cost approximately $30,000 to $40,000 to construct. However, a standard cryogenic fan available from Barber-Nichols, model BNHeP-03, which operates at a higher flowrate, would cost approximately $18,000. This fan could be used in conjunction with a frequency invertor to produce the desired flow (Barber-Nichols, 1989).

Additional Costs

Liquid Nitrogen

Liquid nitrogen is fairly inexpensive and is available in most metropolitan areas for approximately $0.10 per liter, if purchased in bulk quantities (Krane, Parsons, Bar-Cohen, 1988).

Power

Power costs were obtained from the Knoxville Utility Board. For the first 2000 kW-hr, the cost is 5.352 cents per kW-hr and 5.941 cents per kW-hr for any additional consumption (Knoxville Utility Board, 1989). These rates are residential rates. Commercial rates, which were used in the analysis, are 1.5 times higher.

Liquid Nitrogen Subcooling Cost

The required refrigeration for subcooling the liquid nitrogen from the saturated state (77 K) to the subcooled temperature, T_{sub}, is

$$Q_{sub} = mc_p(T_{sat} - T_{sub})/\beta, \qquad (6-7)$$

where β is the coefficient of performance of the cryocooler. The cost of producing the refrigeration to subcool the liquid nitrogen can be determined using the electrical power rates given above.

Total "Once-Through" System Cost

The initial cost of a "once-through" system may be approximated as

$$\text{Initial "once-through" system cost} = \text{Piping cost} + \text{Reservoir cost} + \text{CPU vessel cost} + \text{Pump Cost}. \qquad (6-8)$$

The annual cost of a "once-through" system is

Annual "once-through" system cost = Liquid nitrogen cost. (6 - 9)

Maintenance costs for the "once-through" system are practically nonexistent with the exception of the cost involved in refilling the liquid nitrogen reservoir.

Total Refrigerated System Cost

The initial cost of a refrigerated system may be approximated as

Initial refrigerated system cost = Piping cost + Reservoir cost + CPU vessel cost + Cryocooler cost + Condenser cost + Heat exchanger cost + Helium fan cost. (6 - 10)

The annual cost of a saturated pool boiling refrigerated system is

Annual refrigerated system cost = Power cost. (6 - 11)

The annual cost of a subcooled pool boiling refrigerated system is

Annual refrigerated system cost = Power cost
+ Liquid nitrogen subcooling cost. (6 - 12)

The maintenance costs for the refrigerated system are substantially more than for the "once-through" system. The cryocooler, the helium circulator, and the heat exchangers in the helium loop would probably all require regular maintenance and overhauls. Since maintenance costs are not included in the analysis, the total cost of the refrigerated system obtained will be conservative.

CHAPTER VII

OPTIMIZATION OF THE THERMAL DESIGN

The Optimization Code

The designs of the cryogenic cooling systems described in the earlier chapters were optimized by manipulating the thermal model of each system with an optimization code, GRG2. An optimum system is defined here as the minimum annual cost system which satisfies all of the design and performance constraints. GRG2 is a FORTRAN code which solves nonlinear problems by the Generalized Reduced Gradient Method. GRG2 can be used to minimize or maximize an objective function, g(x), subject to given constraints where x is a vector of n real-valued design variables. GRG2 computes first partial derivatives of g with respect to each design variable, x_i, using a method such as the finite forward difference approximation. The user must supply a subroutine GCOMP which computes values of the objective function, g, for a given vector x. The user must also provide data specifying any upper and lower bounds on the variables. These variable bounds, along with other parameters are communicated to GRG2 through a sequential input file, FORT5.DAT, the format of which is specified in the GRG2 User's Guide (Lasdon, 1986). Further information on GRG2 may be found in this User's Guide.

Design Variables for the "Once-Through" System

The thermal design of the "once-through" system is a function of 25 independent variables, 23 of which have fixed values. Only two independent variables, the inner liquid nitrogen pipe radius and the number of radiation shields in the vacuum insulated lines, will be manipulated in the optimization of the "once-through" system. The values of the 23 fixed design variables, such as the ambient and coolant saturation temperatures, piping lengths, vessel size, thermal conductivities and emissivities are presented in Table 17.

Table 17. Design Variables for the "Once-Through" System Having Specified Values

1. Coolant saturation temperature	77 K
2. Initial coolant temperature	77 K
3. Ambient temperature	300 K
4. CPU board surface area	0.25 m^2
5. Number of CPU boards	1
6. Piping length between coolant reservoir and computer	30 m
7. Pipe roughness	4.6×10^{-5} m
8. Pipe thickness	1.7 mm
9. Pipe emissivity	0.2
10. Vessel emissivity	0.2
11. Height of CPU vessel	75 cm
12. Inner radius of CPU vessel	37.5 cm
13. CPU vessel thickness	0.75 cm
14. CPU vessel insulation thickness	5.75 cm
15. Diameter of modeled lead	5 mm
16. Length of modeled lead	30 cm
17. Lead conductivity	427 W/m-K
18. Nominal heat flux from CPU board	4000 W/m^2
19. Pipe thermal conductivity	14 W/m-K
20. Vessel thermal conductivity	14 W/m-K
21. CPU vessel and transfer line external free convection coefficient in air	5 W/m^2-K
22. Number of radiation shields in vacuum-insulated CPU vessel	2
23. Radiation shield emissivity	0.05

Design Variables for the Saturated and Subcooled Refrigerated Systems

The thermal design of the saturated pool boiling refrigerated system is a function of 44 independent variables, 33 of which have fixed values. The values of these 33 variables are listed in Table 18. The first 23 values are identical to those of the corresponding variables for the "once-through" system. The following 11 variables will be manipulated in the saturated pool boiling refrigerated system optimization:

(1) Refrigeration temperature (T_{ref}), the cold temperature produced in the expansion space of the cryocooler,

(2) Dead volume ratio (X),

(3) Diameter of mesh wire in cryocooler regenerator (D_m),

(4) Mesh distance in regenerator (l),

(5) Radius of pipe in gaseous helium loop (r_{He}),

(6) Temperature rise in condenser (ΔT_{cond}),

(7) Inner diameter of condenser piping (D_{cond}),

(8) Inner diameter of heat exchanger piping (D_{hx}),

(9) Heat exchanger piping fin thickness (t),

(10) Inner radius of coolant line (r_c),

(11) Number of radiation shields in the vacuum insulation (n).

The thermal design of the subcooled pool boiling refrigerated system is identical to the design of the saturated pool boiling system with the exception of one additional design variable to be optimized -- the subcooled liquid nitrogen temperature.

Design Constraints on the Refrigerated Systems

The design of the refrigerated system is constrained in several ways. First, the helium outlet temperature from the condenser or submerged heat exchanger cannot exceed 77 K. The second restriction involves the allowable size of the condenser or submerged heat exchanger. As stated previously, in the saturated pool boiling design

Table 18. Design Variables for the Refrigerated Systems Having Specified Values

1. Coolant saturation temperature	77 K
2. Initial coolant temperature	77 K
3. Ambient temperature	300 K
4. CPU board surface area	0.25 m^2
5. Number of CPU boards	1
6. Piping length between coolant reservoir and computer	30 m
7. Pipe roughness	4.6×10^{-5} m
8. Pipe thickness	1.7 mm
9. Pipe emissivity	0.2
10. Vessel emissivity	0.2
11. Height of CPU vessel	75 cm
12. Inner radius of CPU vessel	37.5 cm
13. CPU vessel thickness	0.75 cm
14. CPU vessel insulation thickness	5.75 cm
15. Diameter of modeled lead	5 mm
16. Length of modeled lead	30 cm
17. Lead conductivity	427 W/m-K
18. Nominal heat flux from CPU board	4000 W/m^2
19. Pipe thermal conductivity	14 W/m-K
20. Vessel thermal conductivity	14 W/m-K
21. CPU vessel and transfer line external free convection coefficient in air	5 W/m^2-K
22. Number of radiation shields in vacuum-insulated CPU vessel	2
23. Radiation shield emissivity	0.05
24. Temperature of compression space in cryocooler	300 K
25. Phase angle of piston in cryocooler	45°

Table 18. Design Variables for the Refrigerated Systems Having Specified Values (continued)

26. Piston diameter	7 cm
Regenerator pressure drop coefficients :	
27. C_{R1}	52.15
28. C_{R2}	0.334
29. Amplitude of piston in regenerator	5.3 cm
30. Running frequency of cryocooler	151.84 rad/s
31. Gas constant of working fluid in cryocooler	2.077×10^3
32. Diameter of regenerator matrix	5 cm
33. Distance between cryocooler and computer	30 m

being considered, the condenser is placed in the vapor space above the CPU boards. In the subcooled pool boiling refrigerated system, the submerged heat exchanger must also be placed in the same confined volume. The CPU vessel is rather small, having an inner diameter of only 75 cm and a height of 75 cm. The CPU board is 50 cm high, which leaves a maximum of approximately 20 cm of height for the condenser. It is assumed that the top edge of the board should be at least 5 cm below the liquid surface in the saturated pool boiling system. If the condenser or submerged heat exchanger must fit inside an envelope which has the shape of rectangular box, the maximum dimensions of the envelope are approximately 53 cm in length, 20 cm in height, and 53 cm in width. The tubes in the condenser or submerged heat exchanger must be spaced a minimum of 1/2 OD apart in order to allow space for the radial fins. If the largest nominal size pipe considered (50 mm) is used, six rows of tubing will fit within the 53 cm width. Using the maximum height of 20 cm, only two 53 cm x 53 cm levels of tubing may be implemented. Therefore, the maximum condenser or submerged heat exchanger piping length which can be accomodated in the aforementioned CPU vessel is approximately 6.36 m. Based on this information, there is a possible that the size of the CPU vessel may need to be increased in order to allow space for the volume required by the condenser or submerged heat exchanger.

Limits on Selected Design Variables for the Refrigerated Systems

The dead volume ratio of the Stirling cryocooler will be restricted to values between 1.0 and 2.0, as suggested by Walker (1983). The limits for the diameter of mesh wire and mesh wire distance in the cryocooler regenerator were taken from a table of geometric factors of wire screens commonly used in the regenerator matrix of Stirling cryocoolers (Miyabe, Takahashi, and Hamaguchi, 1982). For standard wire size numbers ranging from 25 to 48, the wire diameter values cover a range of 0.004 mm to 0.508 mm, while the mesh distance values vary from 0.044 mm to 2.032 mm. The heat exchanger fin thickness range was determined from the rule of thumb of 6-15 fins per inch given by Fraas (1989). Assuming the fins are spaced a distance of 3t apart as discussed in Chapter IV, a fin thickness range of approximately 0.5 mm to 1.0 mm is obtained. The piping nominal size can range from 15 mm to 50 mm.

Preliminary analysis of the refrigerated system indicates that the refrigeration temperature produced in the expansion space of the cryocooler cannot exceed

approximately 63 K with a corresponding 10 K temperature rise in the condenser or submerged heat exchanger. The refrigeration temperature can extend beyond 63 K if the temperature rise in the condenser or submerged heat exchanger is decreased by a corresponding amount. For the initial optimization, the refrigeration temperature will be assigned a lower limit of 55 K and an upper limit of 63 K. A refrigeration temperature below 55 K would probably not be practical since the minimum temperature of the liquid nitrogen is approximately 63 K. The temperature rise in the condenser or submerged heat exchanger will be given a minimum value of 3 K and an upper limit of 10 K. Preliminary results indicate that condenser piping lengths in excess of the allowable amount would be required if the temperature difference across the condenser exceeds 10 K. The effects of extending the ranges of the refrigeration temperature and the temperature rise across the condenser or submerged heat exchanger will be studied in more detail after the initial optimization is complete.

System Cost Comparison

Several methods could be used to perform a cost comparison between the "once-through" and refrigerated cooling systems. The method which will be used in this study, suggested by Middendorf (1986), involves the calculation of a uniform equivalent annual cost. The estimated economic life of the computer will be assumed as five years since computer systems are rarely used longer than this as a result of the rapid advancement in computer technology. The equivalent uniform annual cost will be determined assuming that future money is to be discounted at 8%. Straight line depreciation, assuming no salvage value after five years, will be used. By definition, the system having the lowest uniform annual cost is the "optimum system" that should be employed.

CHAPTER VIII

RESULTS AND CONCLUSIONS

Results for the "Once-Through" System

Since the "once-through" system optimization involves manipulation of only two variables, the inner liquid nitrogen transfer line radius and the number of radiation shields in the vacuum insulation, the use of the GRG2 optimization code is not necessary. Table 19 presents the uniform annual cost of the "once-through" system as a function of both the pipe radius and the the number of radiation shields in the insulation. As seen from Table 19, the uniform annual cost of the system increases with increasing pipe size over the range of pipe sizes considered and assuming a constant value for the number of radiation shields. This increase in annual cost is a result of the increased heat transfer rate through the line which leads to a higher liquid nitrogen consumption. Increasing the transfer line size from the optimal value of 15 mm to 50 mm results in a system annual cost increase of approximately 3.0%. The only disadvantage of using smaller piping is the increased pressure drop in the line. However, the two-phase pressure drop occurring in the 15 mm nominal size line is still extremely low at a value of approximately 110 Pa. As seen from Table 19, the lowest uniform annual cost is achieved by using 15 mm nominal size pipe containing one radiation shield. Although the use of additional shields provides a lower heat transfer rate through the line, the increase in insulation cost outweighs the savings in liquid nitrogen consumption. Table 20 contains the cost analysis for the optimal "once-through" system. The uniform annual cost of the optimal "once-through" system is approximately $28800. Table 21 contains the costs of the individual components of the "once-through" system. The largest cost is that of the liquid nitrogen reservoir which comprises approximately 55% of the total initial cost. Significant parameters for the optimal system are presented in Table 22.

Results for the Saturated Pool Boiling Refrigerated System with a Stirling Cryocooler

Use of the GRG2 Optimization Code

GRG2 was used to manipulate the saturated pool boiling refrigerated system

Table 19. "Once-Through" System Cost as a Function of Pipe Size and Number of Radiation Shields

Nominal Pipe Size (mm)	Number of Radiation Shields 0	1	2	3	4
15	30679	28779	28824	29067	29369
20	31466	28981	28942	29153	29438
25	32294	29141	29037	29223	29495
32	33174	29263	29112	29281	29544
40	33702	29338	29159	29317	29575
50	35190	29643	29337	29447	29679

Table 20. Uniform Annual Cost of the Optimal "Once-Through" System

Year	Depreciation ($)	Liquid Nitrogen Cost ($)	Total Annual Cost ($)	Discount Factor	Present Value Annual Cost ($)
1	8230	20542	28772	0.926	26643
2	8230	20542	28772	0.857	24658
3	8230	20542	28772	0.794	22845
4	8230	20542	28772	0.735	21147
5	8230	20542	28772	0.681	19594

Uniform Annual Cost= $28772

Table 21. Initial Cost Breakdown for the "Once-Through" System

Component	Cost ($)
Reservoir	22754
Piping	9628
CPU Vessel	3767
Liquid Nitrogen Pump	5000
Total	41149

Table 22. Description of the Optimal "Once-Through" System

Inlet liquid nitrogen temperature	77 K
Transfer line nominal size	15 mm
Number of radiation shields in vacuum insulation	1
Required mass flow rate of liquid nitrogen	5.27×10^{-3} kg/s
Saturated nitrogen quality at CPU vessel entrance	3.27%
Pressure drop in transfer lines	110 Pa
Reservoir volume	4333 liters
Heat transfer rates :	
Transfer line	34.22 W
CPU vessel	10.12 W
Leads	6.23 W
Total	50.58 W

thermal model to obtain the optimum values of the design variables listed in Table 23. The first ten variables listed in Table 23 were optimized simultaneously. The value for the number of radiation shields in the vacuum insulated lines was optimized alone using the optimum values of the first ten design variables obtained. A sample of the output from GRG2 is provided in Appendix C. The termination message provided by GRG2 states that the Kuhn-Tucker conditions were satisfied. Of the six possible termination messages provided by GRG2, this message implies the highest level of confidence that at least a local minimum in the system cost has been found. Several optimization runs were performed with different sets of initial values for the design variables and identical optimum values to those presented in Table 23 were obtained.

Effects of Selected Variables on the Saturated Pool Boiling Design

The effects of selected design variables on the design of the saturated pool boiling system will be discussed below. All plots were constructed using the optimum values of all other design parameters.

The optimization results indicate that the refrigeration temperature, the temperature produced in the expansion space of the cryocooler, should be as high as possible while still allowing coolant condensation. The refrigeration temperature can be lower than the minmum obtainable liquid nitrogen temperature of approximately 63 K. As mentioned earlier, higher refrigeration temperatures result in larger condenser piping lengths. A plot of condenser piping length vs refrigeration temperature is shown in Figure 18. Using the parameter values given in Table 23, the condenser piping length required is 9.52 m, which exceeds the allowable value. Reducing the refrigeration temperature to 61 K results in a reduction of condenser piping length to 6.4 m. A further reduction in refrigeration temperature to 60 K results in a piping length of 5.50 m, which will fit in the available condenser space. The effect of lowering the refrigeration temperature on the annual power cost of the cooling system is shown in Figure 19. Reducing the refrigeration temperature from the optimum value of 63 K to 60 K results in an increase in uniform annual cost of less than 1% .

A cryocooler dead volume ratio of 1.0 is favored over higher values. However, the system cost is insensitive to the dead volume ratio, since a value of 2.0 results in less than a 0.14% increase in uniform annual cost over the cost corresponding to a dead volume ratio of 1.0. The system cost is also relatively unaffected by the value of mesh wire diameter in the cryocooler regenerator. Although a larger value (0.508 mm) is

Table 23. Optimum Values of the Design Variables for the Saturated Pool Boiling System

Parameter	Optimum Value
1. Refrigeration temperature	63.0 K
2. Dead volume ratio in Stirling cryocooler	1.0
3. Mesh wire diameter	0.508 mm
4. Mesh distance	2.032 mm
5. Inner radius of helium lines	28.45 mm
6. Temperature rise across the condenser	10 K
7. Inner diameter of the condenser piping	56.9 mm
8. Inner diameter of the heat exchanger piping	56.9 mm
9. Fin thickness	1.0 mm
10. Inner radius of the liquid nitrogen transfer lines	8.95 mm
11. Number of radiation shields in the vacuum insulation	2

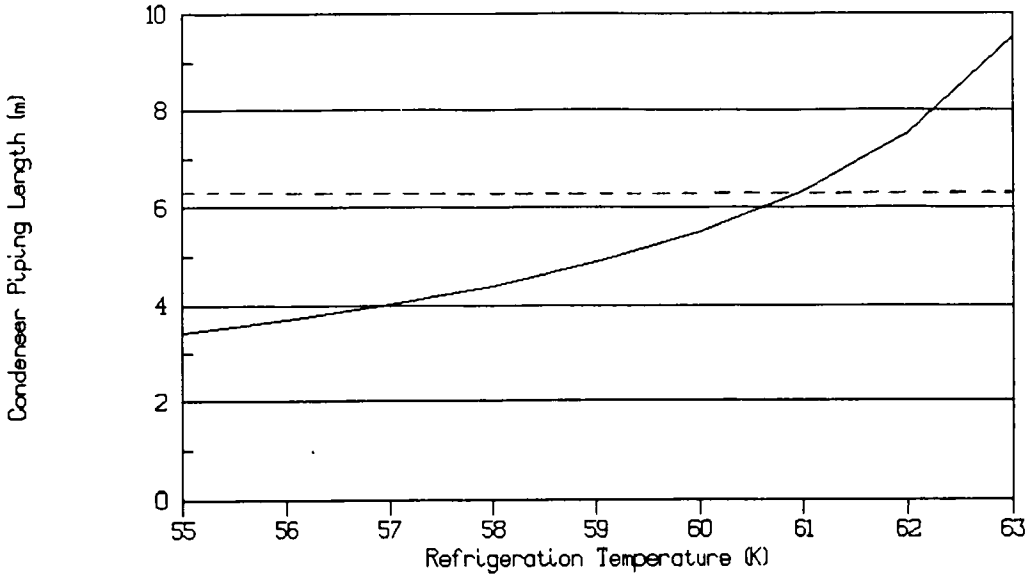

Figure 18. Condenser Piping Length vs Refrigeration Temperature

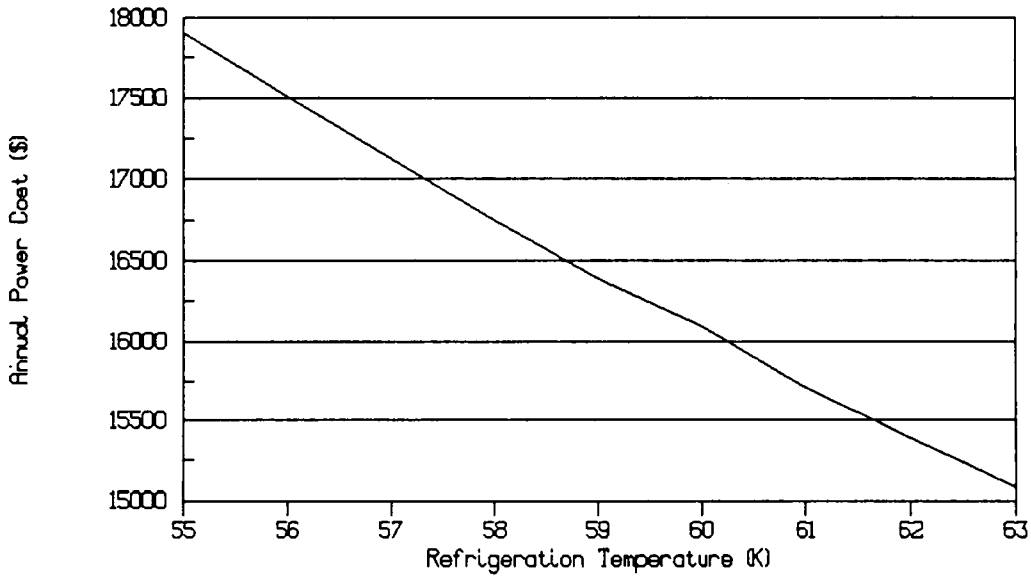

Figure 19. Annual Power Cost vs Refrigeration Temperature

favored, the minimum value of mesh wire diameter (0.04 mm) produces only a 0.79% increase in system cost.

The saturated pool boiling refrigerated system cost is most sensitive to the mesh distance, or wire screen spacing, in the regenerator of the Stirling cryocooler. A plot of mesh distance vs coefficient of performance of the cryocooler is shown in Figure 20. The coefficient of performance of the Stirling cryocooler is a monotonically increasing function of mesh distance and has a maximum value of 6.31% at a mesh distance of 2.032 mm. The COP drops to an extremely low value of 0.17% at a mesh distance of 0.04 mm. This reduction in coefficient of performance results in an increase of power cost by a factor of nearly 37. A plot of mesh distance vs annual power cost is shown in Figure 21. This data supports the fact that the regenerator performance makes a critical contribution to the overall performance of the Stirling cryocooler.

Optimization results indicate that larger nominal size piping in the helium loop (50 mm) is preferred over smaller size piping. Smaller diameter piping produces larger pressure drops which require greater pumping power (larger electrical power costs). In fact, the use of piping of nominal size smaller than 40 mm leads to system failure due the high temperature rise across the pump and a condenser outlet temperature exceeding 77 K. The use of piping smaller than 40 mm also leads to condenser piping lengths which exceed the maximum acceptable value.

The temperature rise across the condenser is a critical parameter in the subcooled pool boiling refrigerated system since its value has a direct effect on the mass flow rate of helium in the loop. As shown in Figure 22, the temperature rise across the condenser can be increased from 10 K to as much as 13 K with a slight decrease in uniform annual cost (approximately 0.7%). However, the corresponding condenser piping length exceeds the maximum allowable value. Lowering the temperature rise across the condenser has both advantages and disadvantages. Although this results in a decrease in condenser size, the heat exchanger size increases since it must handle an increased temperature difference. Figures 23 and 24 show the effect of the condenser temperature rise on the piping lengths in the condenser and heat exchanger, respectively. The use of a smaller condenser temperature rise also requires higher flow rates of helium, as shown in Figure 25, which leads to higher refrigeration capacities, as seen in Figure 26, and increases in work input. Therefore, the condenser temperature rise should be as high as possible without exceeding the condenser size limitations. For the saturated pool boiling system, the optimum value of the condenser temperature rise is approximately 10 K.

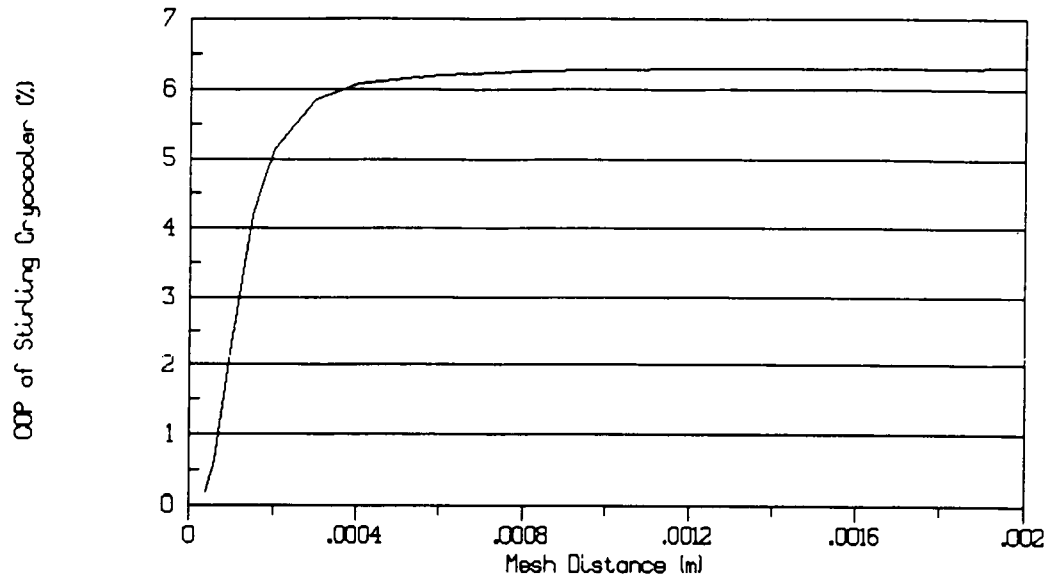

Figure 20. Coefficient of Performance of Stirling Cryocooler vs Mesh Distance in Regenerator

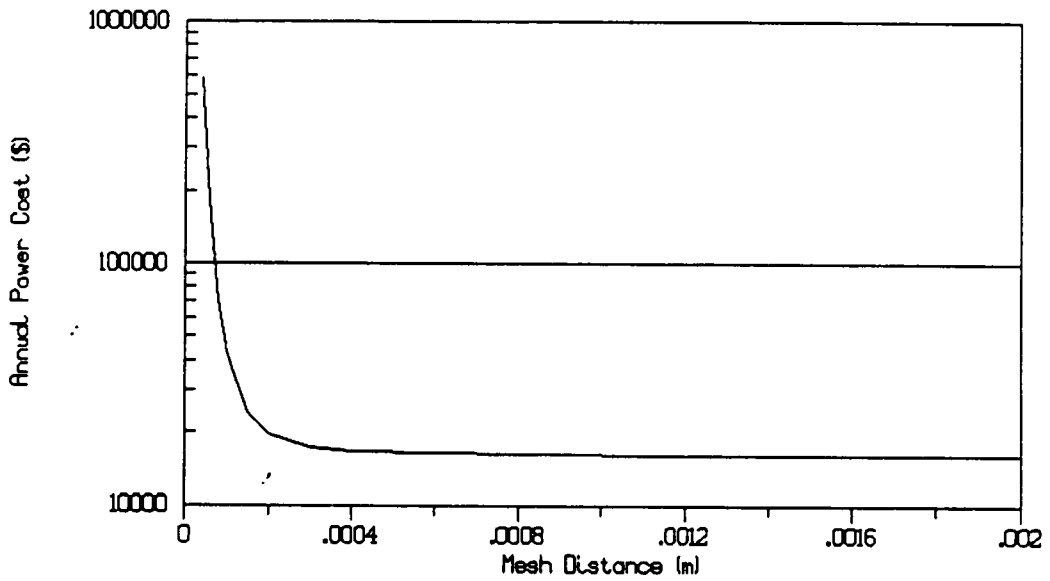

Figure 21. Annual Power Cost vs Mesh Distance in Stirling Cryocooler Regenerator

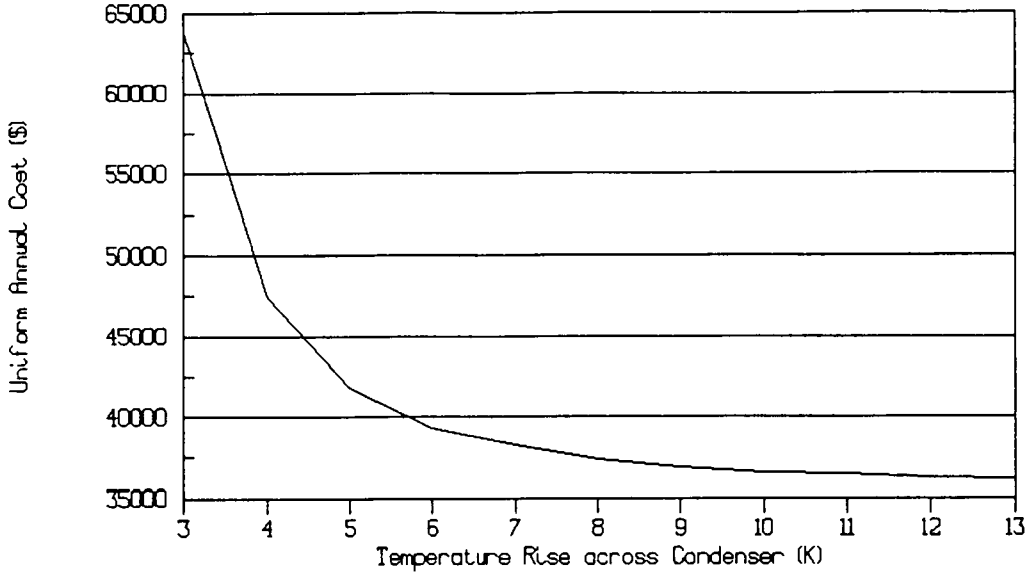

Figure 22. Uniform Annual Cost vs Temperature Rise across Condenser

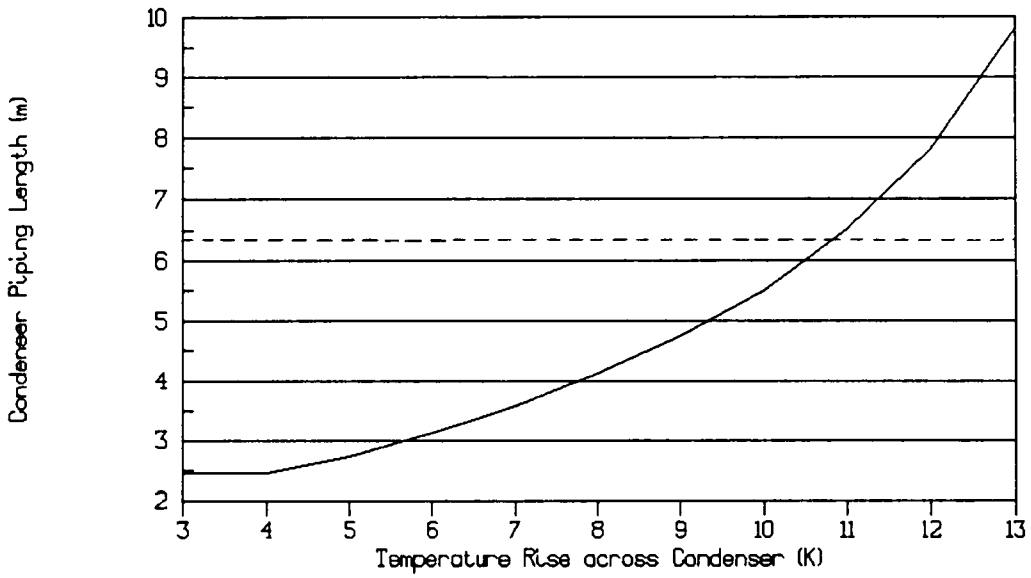

Figure 23. Condenser Piping Length vs Temperature Rise across Condenser

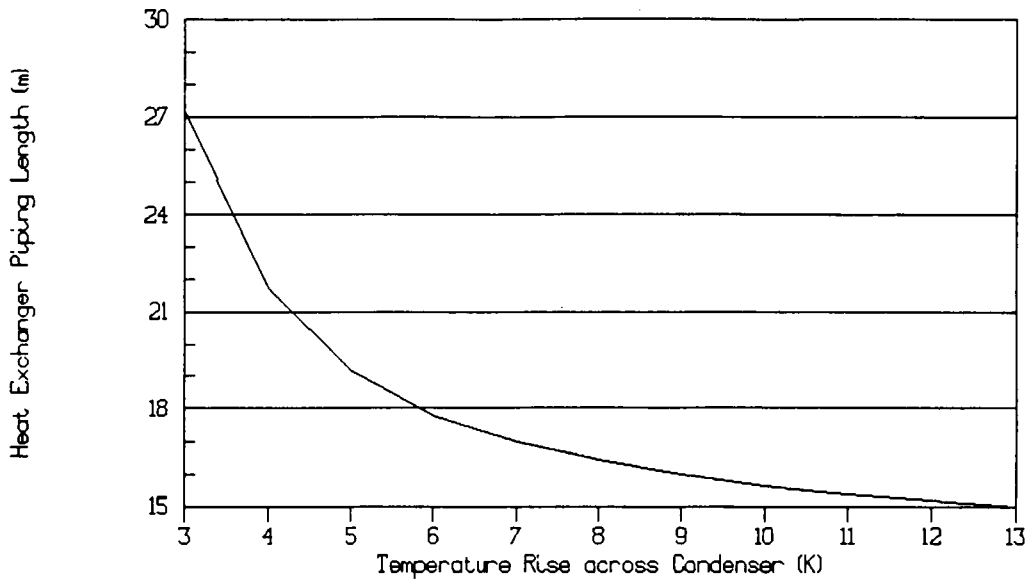

Figure 24. Heat Exchanger Piping Length vs Temperature Rise across Condenser

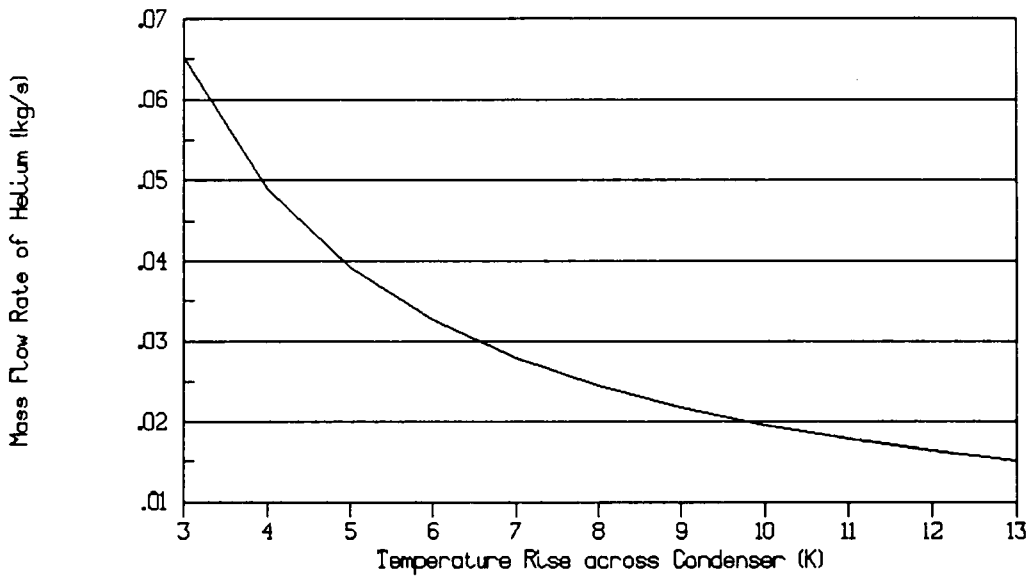

Figure 25. Mass Flow Rate of Helium vs Temperature Rise across Condenser

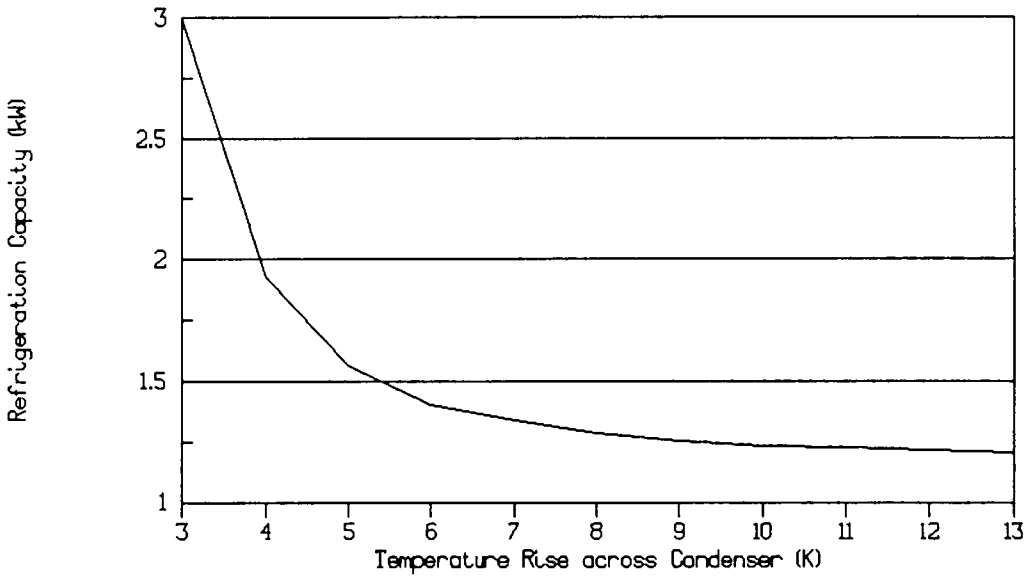

Figure 26. Refrigeration Capacity vs Temperature Rise across Condenser

The optimal fin thickness was determined to be 1 mm, which corresponds to approximately 2.5 fins per cm (6.35 fins per inch). Smaller fin thicknesses would provide more heat transfer area but result in an increased equipment cost, which outweighs the advantages of their increased thermal performance.

Since the line between the reservoir and the computer is used only for filling and emergency purposes such as failure of the cryocooler, its size has no effect on the performance of the saturated pool boiling refrigerated system. Thus, the smallest nominal size pipe (15 mm) is recommended due to its lower cost.

Table 24 contains the cost analysis for the optimal saturated pool boiling refrigerated system. Table 25 provides a breakdown of the initial cost of the system. Significant parameters for the optimal saturated pool boiling refrigerated system are presented in Table 26.

Results for the Subcooled Pool Boiling Refrigerated System with a Stirling Cryocooler

The opimum values of the design variables for the subcooled pool boiling system are listed in Table 27. The optimum subcooled pool boiling system is nearly identical to the optimum saturated pool boiling system except that the liquid nitrogen is subcooled to 68 K. The additional cost involved in subcooling the liquid nitrogen is insignificant (less than $3 per month). If the liquid nitrogen is subcooled to a temperature below 68 K, the submerged heat exchanger analysis predicts an outlet temperature exceeding the subcooled liquid nitrogen temperature, which is impossible. If radially-finned tubes are used in both heat exchangers, the total piping length required in the helium loop is less for the subcooled system than for the saturated system. Also, the size of the reservoir for emergency purposes is approximately 12% smaller for the subcooled system than for the saturated system. The uniform annual cost of the subcooled pool boiling system is approximately $130 less than that of the saturated pool boiling system. However, the major advantage of the subcooled system is the increase in the critical heat flux. By increasing the value of the critical heat flux, the possibility of film boiling, which would result in excessive component temperatures, is reduced. A plot of critical heat flux vs liquid nitrogen subcooled temperature is shown in Figure 27. This plot was constructed using correlations presented in the paper by Krane, Parsons, and Bar-Cohen (1988). Subcooling the liquid nitrogen to 68 K increases the value of the critical heat flux by approximately 40%.

Table 24. Uniform Annual Cost of the Optimal Saturated Pool Boiling System with a Stirling Cryocooler

Year	Depreciation ($)	Power Cost ($)	Total Annual Cost ($)	Discount Factor	Present Annual Cost ($)
1	20485	16041	36526	0.926	33823
2	20845	16041	36526	0.857	31303
3	20845	16041	36526	0.794	29002
4	20845	16041	36526	0.735	26857
5	20845	16041	36526	0.681	24874

Uniform Annual Cost= $36526

Table 25. Initial Cost Breakdown for the Saturated Pool Boiling System

Component	Cost ($)
Reservoir	6338
Piping	46714
Stirling cryocooler and CPU vessel	30352
Condenser	59
Heat exchanger	961
Helium fan	18000
Total	102424

Table 26. Description of the Optimal Saturated Pool Boiling System with a Stirling Cryocooler

Refrigeration capacity	1232 W
Coefficient of performance of the Stirling cryocooler	0.068
Power input to the refrigerator	19535 W
Condenser:	
Heat transfer rate	1016 W
Piping length	5.50 m
Surface area	1.01 m^2
Pressure drop	87 Pa
Heat exchanger:	
Heat transfer rate	1232 W
Piping length	15.62 m
Surface area	66.8 m^2
Pressure drop	245 Pa
Helium flowrate	1.95 $x10^{-2}$ kg/s
Fan power	57 W
Total pressure drop	1.17 kPa

Table 27. Optimum Values of the Design Variables for the Subcooled Pool Boiling System

Parameter	Optimum Value
1. Refrigeration temperature	60.0 K
2. Dead volume ratio in Stirling cryocooler	1.0
3. Mesh wire diameter	0.508 mm
4. Mesh distance	2.032 mm
5. Inner radius of helium lines	28.45 mm
6. Temperature rise across the condenser	10 K
7. Inner diameter of the condenser piping	56.9 mm
8. Inner diameter of the heat exchanger piping	56.9 mm
9. Fin thickness	1.0 mm
10. Inner radius of the liquid nitrogen transfer lines	8.95 mm
11. Number of radiation shields in the vacuum insulation	2
12. Liquid nitrogen subcooled temperature	68 K

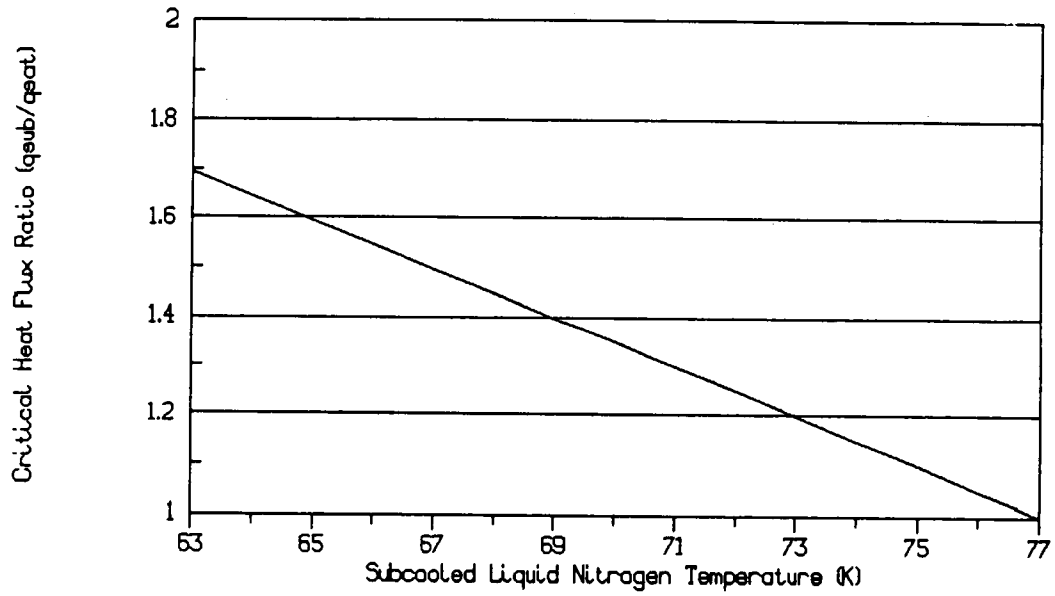

Figure 27. Critical Heat Flux Ratio vs Subcooled Liquid Nitrogen Temperature

Temperature of the CPU Board

The critical heat flux value for saturated liquid nitrogen at atmospheric pressure is approximately 13.95 W/cm^2. Therefore, in order to avoid film boiling, the CPU board temperature must be maintained at a temperature below approximately 86.97 K. Using correlations presented in the paper by Krane, Parsons, and Bar-Cohen (1988), the heat flux at liquid nitrogen boiling incipience is determined to be approximately 0.005 W/cm^2 if the liquid nitrogen is in the saturated state at atmospheric pressure. Therefore, at the nominal heat flux of 0.4 W/cm^2, the heat transfer from the CPU board will occur by nucleate boiling. The temperature of the CPU board at the nominal heat flux is therefore determined to be approximately 79.41 K, which is well below the maximum CPU board temperature limit.

If the liquid nitrogen is subcooled to a temperature of 68 K as in the optimum subcooled pool boiling design, the heat flux at boiling incipience is increased significantly to approximately 0.396 W/cm^2. However, this value is still slightly below the nominal heat flux value of 0.4 W/cm^2 and therefore, nucleate boiling occurs on the CPU board which is at a temperature of approximately 79.41 K.

Conclusions

The optimum refrigerated cryogenic computer cooling systems result in uniform annual costs which are approximately 27% higher than the annual cost of the optimum "once-through" cooling system. Although the daily power cost entailed in operating the refrigerated systems is lower than the daily cost of liquid nitrogen needed for operation of the "once-through" system, the initial cost of the refrigerated systems is over 2.4 times that of the "once-through" system. The largest cost associated with the refrigerated systems is the piping cost, which is in excess of $46,000. The costs of the cryocooler and the helium fan comprise approximately 47.4% of the total system cost. These costs cannot be significantly reduced by design efforts. In order for the uniform annual cost of the refrigerated systems to be equivalent to the annual cost of the "once-through" system, the costs of the piping, cryocooler, and helium fan for the refrigerated systems must all be reduced by a factor of approximately two, which is not deemed possible.

Although maintenance costs were not considered in this analysis, it is

anticipated that the refrigerated systems would involve greater maintenance expense than the "once-through" system. Both the cryocooler and the pump would require periodic overhauls and regular maintenance. The only significant maintenance cost for the "once-through" system is that associated with the refilling of the liquid nitrogen reservoir.

If a refrigerated system was adopted, however, the subcooled pool boiling design should be chosen as opposed to the saturated pool boiling design. Although the uniform annual costs of the two systems are essentially equivalent, the critical heat flux for the subcooled system is approximately 40% higher than that of the saturated system. If local values of the heat flux on the CPU board approach 15 W/cm^2, a subcooled system would be required in order to prevent film boiling. However, when the board heat fluxes are less than 15 W/cm^2, a "once-through" system should be selected, since it is much more economical than the refrigerated systems.

As evident from the preceding results, cryogenic cooling is very expensive and computers having relatively low heat fluxes can be more economically cooled by other methods. However, if the temperature of the electronic components must be maintained in the cryogenic range, systems such as the ones discussed in this paper can be effectively used to provide the necessary cooling.

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APPENDIXES

APPENDIX A

The Numerical Model

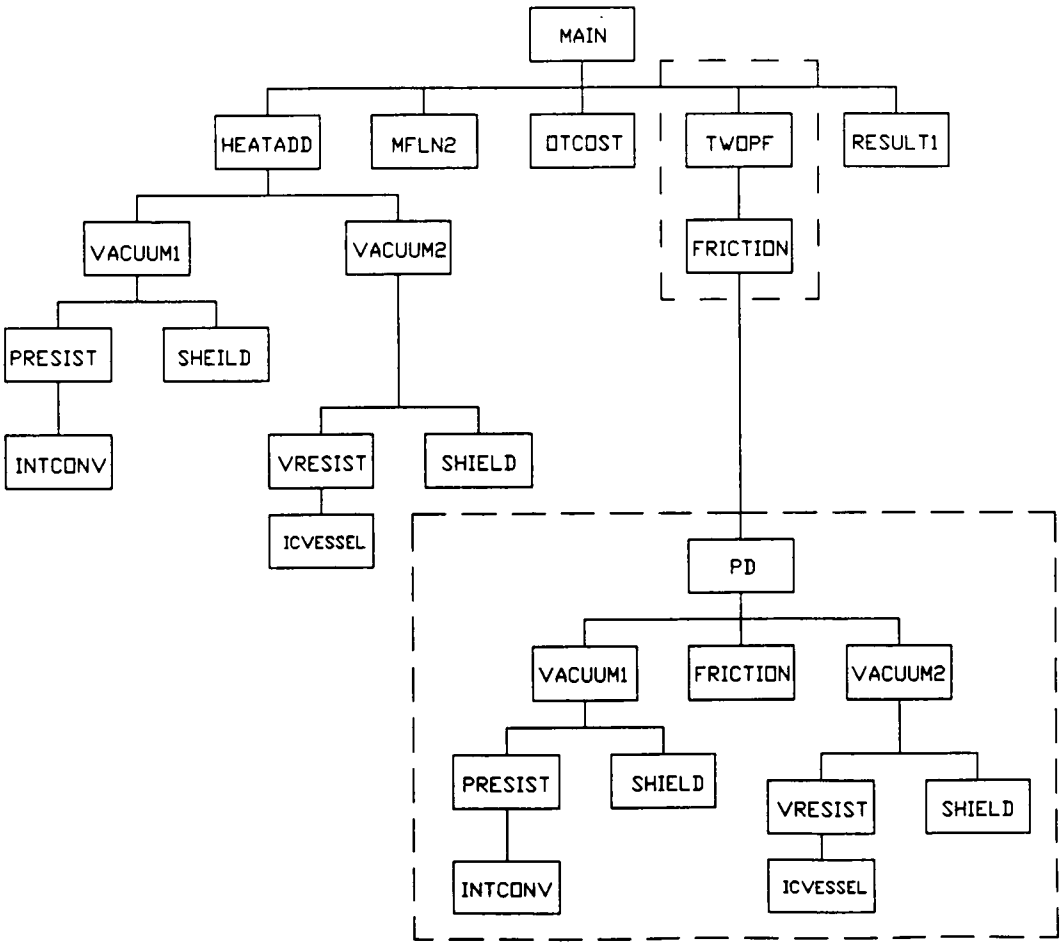

Figure A1. Subroutine Structure of the "Once-Through" System Numerical Model

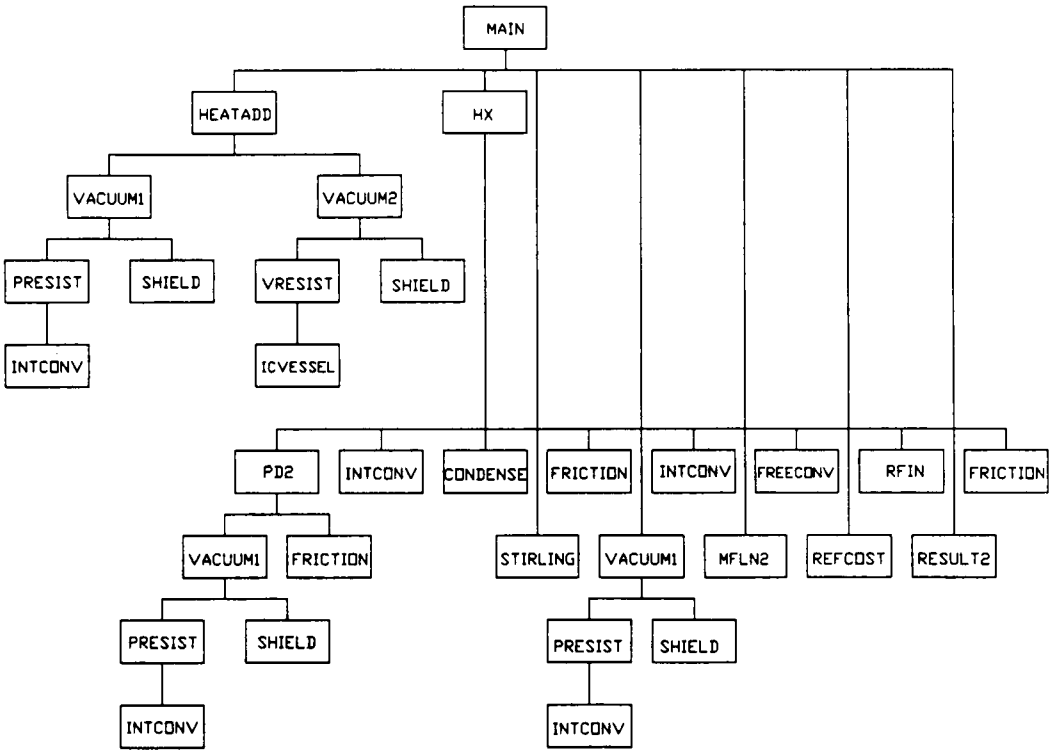

Figure A2. Subroutine Structure of the Refrigerated System Numerical Model

LIST OF VARIABLES

A	Thermal diffusivity
A1L, A2L	Surface areas of pipe
A1V, A2V	Surface areas of CPU vessel
AB	Surface area of CPU board
ABASE	Base surface area
AFIN	Fin surface area
ALPHA	Phase angle of piston in cryocooler
ALR	Surface area-length ratio
AP	Cross-sectional area of piston
AR	Cross-sectional area of regenerator
ATOT	Total surface area
BETA	Thermal expansion coefficient
C	Two-phase pressure drop constant
CCOST	Condenser cost
CDP	Regenerator pressure drop coefficient
CG	Two-phase pressure drop constant
CL	Two-phase pressure drop constant
COP	Coefficient of performance of cryocooler
COST1	Initial cooling system cost
COST2	Annual cooling system cost
CP	Specific heat
CR1, CR2	Stirling pressure drop constants
CRYCOST	Cryocooler cost
CX	Lockhart-Martinelli parameter
D	Diameter
DIC	Inner diameter of condenser piping
DIHX	Inner diameter of heat exchanger piping
DLEAD	Diameter of modeled lead
DM	Diameter of mesh wire in regenerator
DOC	Outer diameter of condenser piping
DOHX	Outer diameter of heat exchanger piping
DP	Pressure drop
DPC	Pressure drop in condenser
DPF	Frictional pressure drop for two-phase flow
DPHX	Pressure drop in heat exchanger
DPIST	Piston diameter
DPM	Momentum pressure drop for two-phase flow
DPO	Pressure drop for two-phase flow if liquid alone were flowing at the total mass flow rate
DREG	Diameter of regenerator matrix
DTCOND	Temperature difference across condenser
DTLMC	Log mean temperature difference of condenser
DTLMHX	Log mean temperature difference of heat exchanger
DX	Quality increment
EP	Emissivity of piping material
ES	Emissivity of shields

EV	Emissivity of vessel material
F	Friction factor
FINEFF	Fin efficiency
FL	Emissivity factor for transfer lines
FLAG	Insulation type indicator
FLUID	Fluid type indicator
FRIC	Frictional two-phase pressure drop integral
FTYPE	Fin type indicator
FV	Emissivity factor for CPU vessel
G	Acceleration of gravity
GAMMA	Specific heat ratio
H	Convection coefficient
HCOND	Condensation heat transfer coefficient
HE	Exit enthalpy
HEPD	Pressure drop in gaseous helium loop
HF	Friction head
HFG	Enthalpy of evaporation
HFREE	Free convection heat transfer coefficient
HI	Inlet enthalpy
HIC	Internal convection coefficient for condenser
HINT	Internal convection coefficient
HIHX	Internal convection coefficient for heat exchanger
HOHX	External convection coefficient for heat exchanger
HP	Pump head
HV	Height of CPU vessel
HXCOST	Heat exchanger cost
K	Thermal conductivity
KF	Thermal conductivity of fluid
KI	Thermal conductivity of insulation
KLEAD	Thermal conductivity of leads
KP	Thermal conductivity of piping material
KV	Thermal conductivity of vessel material
KWHR	Kilowatt hours
L	Length of connecting lines between reservoir and CPU vessel
L2	Length of piping between condenser and cryocooler
LC	Length of piping in condenser
LF	Fin length
LHX	Length of piping in heat exchanger
LLEAD	Length of leads
LN2COST	Liquid nitrogen cost
LNVOL	Liquid nitrogen volume
LP	Amplitude of piston in regenerator
MFR	Mass flow rate of cryogenic fluid
MFRG	Mass flow rate of gas component in two-phase flow
MFRHE	Mass flow rate of gaseous helium
MFRL	Mass flow rate of liquid component in two-phase flow

MID	Intermediate heat loss value
MU	Dynamic viscosity
MUC	Dynamic viscosity of working gas in cryocooler at compression space temperature
MUD	Dynamic viscosity of working gas in cryocooler at dead space temperature
MUE	Dynamic viscosity of working gas in cryocooler at expansion space temperature
MUG	Dynamic viscosity of gas component in two-phase flow
MUL	Dynamic viscosity of liquid component in two-phase flow
N	Number of mesh punchings in regenerator
NB	Number of CPU boards
NF	Number of fins
NSP	Number of radiation shields in vacuum insulated liquid nitrogen lines
NSP2	Number of radiation shields in vacuum insulated gaseous helium lines
NSV	Number of radiation shields in vacuum insulated CPU vessel
NTC	Number of rows of tubes in condenser
NU	Kinematic viscosity
NUN	Nusselt number
NUM	Number of divisions of transfer line
OAR	Opening area ratio
OMEGA	Running frequency of cryocooler
P	Power dissipated on CPU board
PARAM	Fin parameter
PCOST	Piping cost
PDR	Pressure drop in the regenerator
PEFF	Pump efficiency
PHIL	Frictional-pressure-drop parameter
PHIM	Momentum-pressure-drop parameter
PI	π
POWCOST	Power cost
PR	Pressure ratio in the Gifford-McMahon cryocooler
PRESSDROP	Total pressure drop in the transfer lines
PRN	Prandtl number
PRT1	Thermal resistance due to internal convection
PRT2	Thermal resistance of pipe wall
PRT3	Thermal resistance due to external convection
PRT4	Thermal resistance of insulation layer
PRTOT	Total thermal resistance for pipe
PUMPCOST	Pump cost
QCOND	Heat transfer in condenser

QHEL	Heat added through the gaseous helium lines
QHX	Heat transfer in heat exchanger
Q LL	Heat added through the leads
QLOSS	Total heat added to the cryogenic fluid
QLP	Heat added through the transfer lines
QLV	Heat added through the CPU vessel
QNOM	Nominal heat flux on the CPU boards
QREF	Refrigeration capacity
R	Gas constant
R2C	Modified radial fin radius
RAD	Rayleigh number
RADHT	Radiant heat transfer rate
RCOST	Reservoir cost
RE	Reynolds number
REF	Refrigerator type indicator
REG	Reynolds number of gas component in two-phase flow
REL	Reynolds number of liquid component in two-phase flow
REMAX	Maximum Reynolds number in Stirling regenerator
RHL1	Inner radius of piping for helium loop
RHL2	Outer radius of piping for helium loop
RHL3	Outer radius of piping (including insulation) for helium loop
RHO	Density
RHOC	Density of working gas in cryocooler at compression space temperature
RHOD	Density of working gas in cryocooler at dead space temperature
RHOE	Density of working gas in cryocooler at expansion space temperature
RHOG	Density of gas component for two-phase flow
RHOL	Density of liquid component for two-phase flow
RHOV	Density of gaseous nitrogen at saturation temperature
RIC	Inner radius of condenser piping
RIHX	Inner radius of heqat exchanger piping
RL	Volume fraction of liquid
RP	Pipe roughness
RP1	Inner radius of piping for liquid nitrogen transfer lines
RP2	Outer radius of piping for liquid nitrogen transfer lines
RP3	Outer radius of piping (including insulation) for liquid nitrogen transfer lines
RV1	Inner radius of vessel
RV2	Outer radius of vessel
RV3	Outer radius of vessel including insulation

RVOL	Reservoir volume
SAC	Condenser surface area
SAHX	Heat exchanger surface area
SIGMA	Stefan-Boltzmann constant
SL	Mesh distance
SM, SN	Two-phase pressure drop constants
SYSTEM	System indicator (SYSTEM = 1, "once-through" SYSTEM = 2, refrigerated)
TA	Ambient temperature
TB	Temperature on CPU board
TC	Temperature in compression space of cryocooler
TCI	Inlet condenser temperature
TCO	Outlet condenser temperature
TD	Temperature in dead space of cryocooler [TD = (TC+TE)/2]
TE	Temperature in expansion space of cryocooler
TEMPEFF	Temperature effectiveness of fin
TF	Mean temperature of cryogenic fluid
TH	High temperature in Gifford-McMahon cryocooler
THE	Helium temperature
THF	Fin thickness
THXI	Inlet heat exchanger temperature
THXO	Outlet heat exchanger temperature
TI	Initial (inlet) temperature
TINC	Temperature increment
TL	Low temperature in Gifford-McMahon cryocooler
TMIN	Minimum temperature of cryogenic fluid
TO	Outlet (exit) temperature
TR	Temperature ratio (TR=TC/TE)
TREF	Refrigeration temperature
TS	Saturation temperature of cryogen
TV	Temperature of liquid nitrogen in CPU vessel
UAC	UA factor for condenser
UACOST	Uniform annual cost
UAHX	UA factor for heat exchanger
UMAX	Maximum velocity in Stirlingf regenerator
V	Velocity of cryogenic fluid
VC	Kinematic viscosity of working gas in cryocooler at compression space temperature
VCOST	CPU vessel cost
VD	Kinematic viscosity of working gas in cryocooler at dead space temperature
VE	Kinematic viscosity of working gas in cryocooler at expasnsion space temperature
VHE	Gaseous helium velocity
VRT1	Thermal resistance of CPU vessel wall

VRT2	Thermal resistance due to external convection
VRT3	Thermal resistance of insulation layer
VRTOT	Total thermal resistance of vessel
WIN	Work input to cryocooler
WPUMP	Pump work
WTOT	Total work required
X	Dead volume ratio
XE	Exit quality

```
C*****************************************************************
C*                                                               *
C*                          MODEL.FOR                            *
C*                                                               *
C*****************************************************************
C
      REAL*8 AB,ALPHA,CCOST,COP,COST1,COST2,COST3,CP,CR1,CR2,CRYCOST
      REAL*8 DIC,DIHX,DIV,DLEAD,DM,DOC,DOHX,DP,DPC,DPHX,DPIST
      REAL*8 DREG,DTCOND,EP,ES,EV,F,FLAG,FLUID,FTYPE,G,GAMMA,H,HCOND
      REAL*8 HE,HEPD,HF,HFG,HI,HIC,HIHX,HOHX,HV,HXCOST,KF,KI,KLEAD
      REAL*8 KP,KV,L,L2,LC,LF,LH,LHX,LLEAD,LNCOST,LN2COST
      REAL*8 LP,MFR,MFRHE,MID,MUL,MUG,N,NB,NSP,NSV,NTC,NUM,NUM2,OMEGA
      REAL*8 P,PCOST,PDR,PI,PR,PRESSDROP,PUMPCOST,QCOND,QHEL,QHX,QLL
      REAL*8 QLOSS,QLP,QLV,QNOM,QREF,R,RCOST,REF,RHL1,RHL2,RHL3,RHO
      REAL*8 RHOL,RP,RP1,RP2,RP3,RV1,RV2,RV3,RVOL,SAC,SAHX,SIGMA,SL
      REAL*8 SYSTEM,TA,TC,TCI,TCO,TF,TH,THE,THF,THXI,THXO,TI,TI2,TINC
      REAL*8 TO,TREF,TS,UACOST,V,VCOST,WIN,WPUMP,X,XE
C     OPEN DATA FILES
      OPEN(UNIT=10,FILE='MODEL.DAT',STATUS='OLD')
      OPEN(UNIT=100,FILE='REFR.DAT',STATUS='OLD')
      OPEN(UNIT=200,FILE='HX.DAT',STATUS='OLD')
      OPEN(UNIT=300,FILE='RESULT1.DAT',STATUS='NEW')
      OPEN(UNIT=400,FILE='RESULT2.DAT',STATUS='NEW')
C     READ DATA FROM FILES
      READ(10,*) TS,TI,TA
      READ(10,*) AB,NB,L,RP1
      READ(10,*) RP,EP
      READ(10,*) EV,HV,RV1,RV2
      READ(10,*) RV3,DLEAD,LLEAD,KLEAD
      READ(10,*) QNOM,KP,KV,KI
      READ(10,*) H,FLAG,NUM,REF
      READ(10,*) TINC,NSP,NSV,ES
      READ(10,*) FLUID,SYSTEM
      READ(100,*) TC,TREF,X,ALPHA
      READ(100,*) DPIST,CR1,CR2
      READ(100,*) LP,OMEGA,SL,DM
      READ(100,*) R,DREG,TH
      READ(100,*) PR
      READ(200,*) RHL1,L2
      READ(200,*) NUM2,DTCOND,DIC
      READ(200,*) DIHX,NTC
      READ(200,*) FTYPE,THF
C     SET VALUES FOR CONSTANTS
      PI=3.14159265
      SIGMA=5.67E-08
      G=9.81
      RP2=RP1+1.7E-03
      RP3=13501.32*RP1**3-774.3287*RP1**2+14.56183*RP1-0.04752474
      RHL2=RHL1+1.7E-03
      RHL3=13501.32*RHL1**3-774.3287*RHL1**2+14.56183*RHL1
     @ -0.04752474
      DOC=DIC+1.7E-03
      DOHX=DIHX+1.7E-03
```

```
      THXO=TREF+1
      LF=DOHX/2
C     CALCULATE THE POWER DISSIPATED ON THE CPU BOARDS
      P=QNOM*AB*NB
C     PERFORM ANALYSIS ON ONCE-THROUGH SYSTEM
      IF (SYSTEM .EQ. 1) THEN
C     CALL SUBROUTINE HEATADD TO CALCULATE THE HEAT ADDED TO THE
C     LIQUID NITROGEN
      CALL HEATADD(PI,SIGMA,FLAG,TA,L,KP,KV,KI,G,FLUID,H,EP,
     @ ES,EV,HV,RP1,RP2,RP3,RV1,RV2,RV3,NSP,NSV,KLEAD,DLEAD,LLEAD,
     @ TI,MFR,MID,QLV,QLL,QLOSS)
C     CALL SUBROUTINE MFLN2 TO CALCULATE THE REQUIRED MASS FLOW RATE
C     OF LIQUID NITROGEN
      CALL MFLN2(P,TI,QLOSS,MFR)
C     CALL SUBROUTINE OTCOST TO CALCULATE THE COST OF A ONCE-THROUGH
C     SYSTEM
      CALL OTCOST(TI,L,NSP,RP1,MFR,RCOST,PCOST,PUMPCOST,VCOST,
     @ LN2COST,COST1,COST2,UACOST,RVOL)
      IF (TI .EQ. 77) THEN
         QLP=MID
         CALL TWOPF(PI,RP1,RP,L,G,MFR,QLP,NUM,PRESSDROP,XE)
         GOTO 10
      ENDIF
C     CALL SUBROUTINE PD TO CALCULATE THE PRESSURE DROP IN THE
C     LIQUID NITROGEN TRANSFER LINES
      CALL PD(NSP,NSV,ES,L,NUM,FLAG,PI,RP1,RP2,RP3,EP,SIGMA,TA,
     @ KP,KI,H,TI,MFR,G,RV1,RV2,RV3,EV,HV,KV,TINC,FLUID,PRESSDROP,
     @ QLP,QLV,TO)
10    ENDIF
C     PERFORM ANALYSIS ON A REFRIGERATED SYSTEM
      IF (SYSTEM .EQ. 2) THEN
C     CALL SUBROUTINE HEATADD TO CALCULATE THE HEAT ADDED TO THE
C     LIQUID NITROGEN
      CALL HEATADD(PI,SIGMA,FLAG,TA,L,KP,KV,KI,G,FLUID,H,EP,
     @ ES,EV,HV,RP1,RP2,RP3,RV1,RV2,RV3,NSP,NSV,KLEAD,DLEAD,LLEAD,
     @ TI,MFR,MID,QLV,QLL,QLOSS)
C     CALCULATE THE HEAT TRANSFER REQUIRED IN THE CONDENSER
      QCOND=QLOSS+P-MID
C     CALL SUBROUTINE HX
      FLUID=2
      CALL HX(L2,NUM2,FLAG,PI,RHL1,RHL2,RHL3,EP,SIGMA,TA,KP,KI,H,
     @ NSP,L2,NUM2,ES,THXO,G,TINC,TREF,FLUID,RP,QCOND,DTCOND,DIC,DOC,
     @ DIHX,DOHX,NTC,THXI,FTYPE,LF,THF,QHX,SAC,SAHX,WPUMP,MFRHE,HEPD,TCI
     @ ,TCO,HIC,HCOND,LC,DPC,HIHX,HOHX,LHX,DPHX,TI)
C     CALL SUBROUTINE STIRLING OR GM TO GET VALUE OF COP
      IF (REF .EQ. 1) THEN
         CALL STIRLING(X,ALPHA,PI,DPIST,CR1,CR2,TC,TREF,LP,OMEGA,SL,
     @       DM,R,DREG,PDR,COP)
      ENDIF
      IF (REF .EQ. 2) THEN
         CALL GM(R,PR,TH,TREF,COP)
      ENDIF
C     CALCULATE THE TOTAL LENGTH OF GASEOUS HELIUM TRANSFER LINE
      L=2*L2
```

```
C       CALCULATE THE AVERAGE TEMPERATURE OF THE GASEOUS HELIUM
        TF=(THXI+THXO)/2
C       CALCULATE THE HEAT ADDED THROUGH THE GASEOUS HELIUM LINES
        CALL VACUUM1(PI,KP,H,RHL1,RHL2,RHL3,L,EP,SIGMA,TA,TF,NSP,
     @ ES,FLUID,G,MFRHE,MID)
        QHEL=MID
C       CALCULATE THE REFRIGERATING CAPACITY
        QREF=QHX
C       CALCULATE THE REQUIRED WORK INPUT TO THE REFRIGERATOR
        WIN=QREF/COP
C       CALL SUBROUTINE MFLN2 TO CALCULATE THE REQUIRED MASS FLOW RATE
C       OF LIQUID NITROGEN
        CALL MFLN2(P,TI,QLOSS,MFR)
        CALL REFCOST(L,L2,NSP,QREF,WIN,WPUMP,RP1,RHL1,SAC,SAHX,
     @ MFR,TI,COST1,COST2,UACOST,PCOST,RCOST,CRYCOST,LNCOST,CCOST,
     @ HXCOST,PUMPCOST,RVOL,COP,COST3)
        ENDIF
C       PRINT RESULTS
        IF (SYSTEM .EQ. 1) THEN
        CALL RESULT1(RP1,MID,QLV,QLL,QLOSS,MFR,RCOST,TI,PCOST,PUMPCOST
     @ ,VCOST,LN2COST,COST1,COST2,PRESSDROP,TO,XE,UACOST,RVOL)
        ENDIF
        IF (SYSTEM .EQ. 2) THEN
        CALL RESULT2(TREF,QREF,COP,WIN,COST1,COST2,DIC,TCI,
     @ TCO,QCOND,HIC,HCOND,LC,SAC,DPC,DIHX,THXI,THXO,QHX,HIHX,HOHX,
     @ LHX,SAHX,DPHX,RHL1,MFRHE,HEPD,WPUMP,UACOST,PCOST,RCOST,CRYCOST
     @ ,LNCOST,CCOST,HXCOST,PUMPCOST,RVOL,TI,COST3,LF,THF,FTYPE)
        ENDIF
        END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE AXFIN                           *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE FIN EFFICIENCY AND             *
C*     TEMPERATURE EFFECTIVENESS OF AXIAL FINS                       *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE AXFIN(DOHX,LHX,FTYPE,LF,THF,PI,HOHX,TEMPEFF,ALR)
      REAL*8 ABASE,AFIN,ALR,ATOT,DOHX,FINEFF,FTYPE,HOHX,K,LF
      REAL*8 LHX,NF,PARAM,PI,TEMPEFF,THF
      K=9
C     CALCULATE THE NUMBER OF FINS
      NF=INT(PI*DOHX/(4*THF))
C     CALCULATE THE SURFACE AREA OF THE BASE (UNFINNED PORTION)
      ABASE=PI*(DOHX-NF*THF)*LHX
C     CALCULATE THE SURFACE AREA OF THE FINS
      AFIN=2*LF*LHX*NF
C     CALCULATE THE FIN PARAMETER
      PARAM=LF*(2*HOHX/(K*THF))**0.5
C     CALCULATE THE FIN EFFICIENCY
      FINEFF=9.662825E-02*PARAM**3-0.2780816*PARAM**2-0.1136575*
     @ PARAM+1.008117
C     CALCULATE THE TOTAL SURFACE AREA
      ATOT=ABASE+AFIN
C     CALCULATE THE TEMPERATURE EFFECTIVENESS OF THE FINS
      TEMPEFF=1-(AFIN/ATOT)*(1-FINEFF)
C     CALCULATE THE AREA/LENGTH RATIO FOR THE HEAT EXCHANGER
      ALR=ATOT/LHX
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                          SUBROUTINE CONDENSE                      *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE CONVECTION COEFFICIENT         *
C*     FOR CONDENSATION OF LIQUID NITROGEN ON N HORIZONTAL TUBES.    *
C*                                                                   *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE CONDENSE(G,TF,DOC,NTC,HCOND)
      REAL*8 CP,DOC,G,HCOND,HFG,HFGP,K,MUL,NTC,RHOL,RHOV,TF,TSAT
C     PROPERTIES OF LIQUID NITROGEN AT 77 K
      RHOL=807.3
      RHOV=4.604
      K=139.6E-03
      HFG=199.3E03
      MUL=158E-06
      CP=2.051E03
      TSAT=77
      HFGP=HFG+0.68*CP*(TSAT-TF)
C     CALCULATE THE CONVECTION COEFFICIENT FOR CONDENSATION
      HCOND=0.729*((G*RHOL*(RHOL-RHOV)*K**3*HFGP)/(NTC*MUL*(TSAT-TF
     @ )*DOC))**0.25
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE FREECONV                        *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE CONVECTION COEFFICIENT         *
C*     FOR FREE CONVECTION OVER A HORIZONTAL TUBE.                   *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE FREECONV(RAD,PRN,K,D,HFREE)
      REAL*8 D,HFREE,K,NUD,NUD1,NUD2,NUD3,PRN,RAD
C     CALCULATE THE NUSSELT NUMBER
      NUD1=0.387*RAD**(0.16666667)
      NUD2=(0.559/PRN)**(0.5625)
      NUD3=(1+NUD2)**(0.2962963)
      NUD=(0.6+NUD1/NUD3)**2
C     CALCULATE THE CONVECTION COEFFICIENT FOR FREE CONVECTION
      HFREE=NUD*K/D
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE FRICTION                        *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE FRICTION FACTOR IN THE         *
C*     TRANSFER LINES.                                               *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE FRICTION(MFR,PI,RP1,RP,MUL,F)
      REAL*8 F,FG,MFR,MUL,PI,RE,RP,RP1
C     CALCULATE THE REYNOLDS NUMBER IN THE PIPE
      RE=(4*MFR)/(PI*(RP1*2)*MUL)
C     CALCULATE THE FRICTION FACTOR IF THE FLOW IS LAMINAR
      IF (RE .LT. 2000) F=64/RE
      IF (RE .LT. 2000) GOTO 120
C     CALCULATE THE FRICTION FACTOR IF THE FLOW IS FULLY TURBULENT
      IF (RE .GT. 4000) THEN
      F=1/(-1.8*LOG10(6.9/RE+(RP/((RP1*2)*3.7))**1.11))
      ENDIF
      IF (RE .GT. 4000) F=F**2
      IF (RE .GT. 4000) GOTO 120
C     USE AN ITERATIVE TECHNIQUE TO CALCULATE THE FRICTION FACTOR IF
C     THE FLOW IS TRANSITIONAL
      FG=0.02
110   F=(1/(1.74-2*LOG10(2*RP/(RP1*2)+18.7/(RE*FG**0.5))))**2
      IF (ABS(F-FG) .LT. 0.0001) GOTO 120
      FG=F
      GOTO 120
120   RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                          SUBROUTINE GM                            *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE COEFFICIENT OF PERFORMANCE     *
C*     OF A GIFFORD-MCMAHON CRYOCOOLER.                              *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE GM(R,PR,TH,TREF,COP)
      REAL*8 COP,CP,GAMMA,PR,R,TH,TL,TREF
      TL=TREF
      CP=5.20E03
      GAMMA=1.67
C     CALCULATE THE IDEAL COEFFICIENT OF PERFORMANCE
      COP=(R*(PR-1))/(CP*(PR*(TH/TL)-1)*(PR**((GAMMA-1)/GAMMA) -1))
C     CALCULATE THE ACTUAL COEFFICIENT OF PERFORMANCE (INCLUDES LOSSES)
      COP=0.2025*COP
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE HEATADD                         *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE HEAT ADDED TO THE LIQUID       *
C*     NITROGEN                                                      *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE HEATADD(PI,SIGMA,FLAG,TA,L,KP,KV,KI,G,FLUID,H,EP,
     @ ES,EV,HV,RP1,RP2,RP3,RV1,RV2,RV3,NSP,NSV,KLEAD,DLEAD,LLEAD,
     @ TI,MFR,MID,QLV,QLL,QLOSS)
      REAL*8 DLEAD,EP,ES,EV,FLAG,FLUID,G,H,HV,KI,KLEAD
      REAL*8 KP,KV,L,LLEAD,MFR,MID,NSP,NSV,PI,QLL,QLOSS,QLV
      REAL*8 RP1,RP2,RP3,RV1,RV2,RV3,SIGMA,TA,TF,TI
      TF=TI
C     CALCULATE THE HEAT ADDED THROUGH THE LINES AND THE CPU VESSEL
      IF (FLAG .EQ. 0) THEN
         CALL VACUUM1(PI,KP,H,RP1,RP2,RP3,L,EP,SIGMA,TA,TF,NSP,
     @      ES,FLUID,G,MFR,MID)
         CALL VACUUM2(PI,KV,H,RV1,RV2,HV,RV3,EV,SIGMA,TA,TF,NSV,
     @      ES,QLV)
      ENDIF
      IF (FLAG .EQ. 1) THEN
         CALL MLI1(RP1,RP2,RP3,PI,L,KP,KI,H,TA,TF,FLUID,G,MFR,MID)
         CALL MLI2(RV1,RV2,RV3,PI,HV,KV,KI,H,TA,TF,QLV)
      ENDIF
C     CALCULATE THE HEAT ADDED THROUGH THE LEADS (QLL)
      QLL=(KLEAD*(PI*DLEAD**2/4)*(TA-TF))/LLEAD
C     CALCULATE THE TOTAL HEAT ADDED TO THE SYSTEM
      QLOSS=MID+QLV+QLL
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                          SUBROUTINE HX                            *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE SIZES OF THE TWO HEAT          *
C*     EXCHANGERS IN THE HELIUM LOOP. THE PRESSURE DROPS AND         *
C*     TEMPERATURES OF GASEOUS HELIUM ARE ALSO CALCULATED.           *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE HX(L2,NUM2,FLAG,PI,RHL1,RHL2,RHL3,EP,SIGMA,TA,
     @ KP,KI,H,NSP,L2,NUM2,ES,THXO,G,TINC,TREF,FLUID,RP,QCOND,
     @ DTCOND,DIC,DOC,DIHX,DOHX,NTC,THXI,FTYPE,LF,THF,QHX,SAC,SAHX,
     @ WPUMP,MFRHE,HEPD,TCI,TCO,HIC,HCOND,LC,DPC,HIHX,HOHX,LHX,DPHX,
     @ TI)
      REAL*8 A,ALR,BETA,CP,D,DIC,DIHX,DOC,DOHX,DPC,DPHX,DTCOND
      REAL*8 DTLMC,DTLMHX,EP,ES,F,FLAG,FLUID,FTYPE,G,H,HCOND,HEPD
      REAL*8 HF,HFREE,HIC,HINT,HIHX,HOHX,HP,INTER,K,KI,KP
      REAL*8 L,L2,LC,LF,LHX,MFR,MFRHE,MU,NSP,NTC
      REAL*8 NU,NUM2,PEFF,PI,PRESSDROP2,PRN,QCOND,QHX
      REAL*8 RAD,RE,RHO,RHL1,RHL2,RHL3,RIC,RIHX,RP,RP1,SAC,SAHX
      REAL*8 SIGMA,TA,TCI,TCO,TEMP,TEMPEFF,TF,THF,THXI,THXO,TI2
      REAL*8 TI,TINC,TO2,TREF,TV,UAC,UAHX,V,WPUMP
      CP=5.20E03
      WPUMP=0
C     CALCULATE THE REQUIRED MASS FLOW RATE OF GASEOUS HELIUM IN THE
C     CONDENSER
      MFRHE=QCOND/(CP*DTCOND)
130    TI2=THXO
      L=L2/NUM2
C     CALL SUBROUTINE PD2 TO CALCULATE THE INLET TEMPERATURE TO THE
C     CONDENSER AND THE PRESSURE DROP IN THE LINE
      CALL PD2(NSP,ES,L,NUM2,FLAG,PI,RHL1,RHL2,RHL3,
     @ EP,SIGMA,TA,KP,KI,H,TI2,MFRHE,G,TINC,FLUID,HEPD,TO2)
C     CALCULATE THE INLET TEMPERATURE TO THE CONDENSER
      TCI=TO2+WPUMP/(MFRHE*CP)
      IF ((TCI+DTCOND) .GT. 77) THEN
         PRINT*,'INLET TEMPERATURE TO CONDENSER IS TOO HIGH'
      ENDIF
C     CALCULATE THE OUTLET TEMPERATURE OF GASEOUS HELIUM FOR THE
C     CONDENSER
      TCO=TCI+DTCOND
      TI2=TCO
      PRESSDROP2=HEPD
C     CALL SUBROUTINE PD2 TO CALCULATE THE INLET TEMPERATURE OF GASEOUS
C     HELIUM FOR THE HEAT EXCHANGER AND THE PRESSURE DROP IN THE LINE
      CALL PD2(NSP,ES,L,NUM2,FLAG,PI,RHL1,RHL2,RHL3,
     @ EP,SIGMA,TA,KP,KI,H,TI2,MFRHE,G,TINC,FLUID,HEPD,TO2)
      PRESSDROP2=PRESSDROP2+HEPD
      HEPD=PRESSDROP2
      THXI=TO2
      TV=77
C     CALCULATE THE LOG MEAN TEMPERATURE DIFFERENCE FOR THE CONDENSER
      DTLMC=(TCO-TCI)/LOG((TV-TCI)/(TV-TCO))
```

```
C       CALCULATE UA FOR THE CONDENSER
        UAC=QCOND/DTLMC
C       CALCULATE THE LOG MEAN TEMPERATURE DIFFERENCE FOR THE HEAT EXCHANGER
        DTLMHX=(THXI-THXO)/LOG((THXI-TREF)/(THXO-TREF))
C       CALCULATE THE HEAT TRANSFER IN THE HEAT EXCHANGER
        QHX=MFRHE*CP*(THXI-THXO)
C       CALCULATE UA FOR THE HEAT EXCHANGER
        UAHX=QHX/DTLMHX
C       CALCULATE THE MEAN TEMPERATURE OF THE HELIUM IN THE CONDENSER
        TF=(TCI+TCO)/2
        MFR=MFRHE
        RP1=DIC/2
C       CALCULATE THE INTERNAL CONVECTION COEFFICIENT FOR THE CONDENSER
        CALL INTCONV(TF,MFR,PI,RP1,G,FLUID,HINT)
        HIC=HINT
C       CALCULATE THE OUTER CONVECTION COEFFICIENT FOR THE CONDENSER
        IF (TI .EQ. 77) THEN
           CALL CONDENSE(G,TF,DOC,NTC,HCOND)
           ALR=PI*DOC
           GOTO 135
        ENDIF
C       CALCULATE THE LENGTH OF THE HEAT EXCHANGER FOR A SUBCOOLED
C       SYSTEM
C       CALCULATE THE PRANDTL NUMBER OF THE LIQUID NITROGEN AT THE
C       SUBCOOLED TEMPERATURE
        D=DOC
        PRN=-0.105*TI+10.345
C       CALCULATE THE VISCOSITIES AND DENSITY OF LIQUID NITROGEN AT THE
C       SUBCOOLED TEMPERATURE
        MU=-10.5E-06*TI+960.5E-06
        RHO=-4.28*(TI-65)+860.9
        NU=MU/RHO
C       CALCULATE THE THERMAL DIFFUSIVITY OF LIQUID NITROGEN
        A=NU/PRN
C       CALCULATE THE THERMAL EXPANSION COEFFICIENT OF LIQUID NITROGEN
        BETA=7.39E-05*TI+5.15E-05
        IF (TI .LT. TF) TI=TF+1
C       CALCULATE THE RAYLEIGH NUMBER OF LIQUID NITROGEN
        RAD=(G*BETA*(TI-TF)*D**3)/(NU*A)
C       CALCULATE THE THERMAL CONDUCTIVITY OF LIQUID NITROGEN
        K=-1.57E-03*TI+260.75E-03
C       CALCULATE THE OUTER CONVECTION COEFFICIENT FOR THE HEAT EXCHANGER
        CALL FREECONV(RAD,PRN,K,D,HFREE)
        HCOND=HFREE
C       SET THE THERMAL CONDUCTIVITY OF THE PIPE EQUAL TO 9 W/m-K
        K=9
        ALR=PI*DOC
        TEMPEFF=1
C       CALCULATE THE REQUIRED LENGTH OF PIPE IN THE HEAT EXCHANGER
132     LC=UAC*(1/(PI*HIC*DIC)+0.0002/(PI*DIC)+LOG(DOC/DIC)/
     @ (2*PI*K)+0.0002/(TEMPEFF*ALR)+1/(HCOND*TEMPEFF*ALR))
        IF (FTYPE .EQ. 0) GOTO 150
        IF (ABS(INTER-LC) .LT. 0.01) GOTO 137
        IF (FTYPE .EQ. 1) THEN
```

```
         CALL RFIN(DOC,LC,FTYPE,LF,THF,PI,HCOND,TEMPEFF,ALR)
       ENDIF
       IF (FTYPE .EQ. 2) THEN
         CALL AXFIN(DOC,LC,FTYPE,LF,THF,PI,HCOND,TEMPEFF,ALR)
       ENDIF
       INTER=LC
       GOTO 132
135    K=9
C      CALCULATE THE LENGTH OF PIPE REQUIRED IN THE CONDENSER
       LC=UAC*(1/(PI*HIC*DIC)+0.0002/(PI*DIC)+LOG(DOC/DIC)/(2*PI*K)+
     @ 0.0002/(PI*DOC)+1/(PI*HCOND*DOC))
C      CALCULATE THE PROPERTIES OF HELIUM AT THE AVERAGE TEMPERATURE
C      IN THE CONDENSER
137    ALR1=ALR
       MU=7.884E-06-7.164E-08*(75-TF)
       RHO=0.641+1.332E-02*(75-TF)
C      CALCULATE THE VELOCITY OF GASEOUS HELIUM IN THE CONDENSER
       V=4*MFRHE/(RHO*PI*DIC**2)
C      CALCULATE INNER RADIUS OF CONDENSER LINE
       RIC=DIC/2
C      CALL SUBROUTINE FRICTION TO FIND THE FRICTION FACTOR
       CALL FRICTION(MFRHE,PI,RIC,RP,MU,F)
C      CALCULATE THE FRICTION HEAD
       HF=F*LC*V**2/(DIC*2*G)
C      CALCULATE THE PRESSURE DROP IN THE CONDENSER
       DPC=RHO*HF*G
C      CALCULATE THE AVERAGE TEMPERATURE OF THE GASEOUS HELIUM IN
C      THE HEAT EXCHANGER
       TF=(THXI+THXO)/2
       RP1=DIHX/2
C      CALCULATE THE INTERNAL CONVECTION COEFFICIENT FOR THE HEAT
C      EXCHANGER
       CALL INTCONV(TF,MFR,PI,RP1,G,FLUID,HINT)
       HIHX=HINT
C      CALCULATE THE PRANDTL NUMBER OF THE GASEOUS HELIUM AT THE
C      REFRIGERATION TEMPERATURE
       D=DOHX
       PRN=0.657+4.8E-04*(75-TREF)
C      CALCULATE THE VISCOSITIES AND DENSITY OF GASEOUS HELIUM AT THE
C      REFRIGERATION TEMPERATURE
       MU=7.884E-06-7.164E-08*(75-TREF)
       RHO=0.641+1.332E-02*(75-TREF)
       NU=MU/RHO
C      CALCULATE THE THERMAL DIFFUSIVITY OF GASEOUS HELIUM
       A=NU/PRN
C      CALCULATE THE THERMAL EXPANSION COEFFICIENT OF GASEOUS HELIUM
       BETA=1/TREF
C      CALCULATE THE RAYLEIGH NUMBER OF GASEOUS HELIUM
       RAD=(G*BETA*(TF-TREF)*D**3)/(NU*A)
C      CALCULATE THE THERMAL CONDUCTIVITY OF GASEOUS HELIUM
       K=62.4E-03-6E-04*(75-TREF)
C      CALCULATE THE OUTER CONVECTION COEFFICIENT FOR THE HEAT EXCHANGER
       CALL FREECONV(RAD,PRN,K,D,HFREE)
       HOHX=HFREE
```

```
C       SET THE THERMAL CONDUCTIVITY OF THE PIPE EQUAL TO 9 W/m-K
        K=9
        ALR=PI*DOHX
        TEMPEFF=1
C       CALCULATE THE REQUIRED LENGTH OF PIPE IN THE HEAT EXCHANGER
140     LHX=UAHX*(1/(PI*HIHX*DIHX)+0.0002/(PI*DIHX)+LOG(DOHX/DIHX)/
     @ (2*PI*K)+0.0002/(TEMPEFF*ALR)+1/(HOHX*TEMPEFF*ALR))
        IF (FTYPE .EQ. 0) GOTO 150
        IF (ABS(INTER-LHX) .LT. 0.01) GOTO 150
        IF (FTYPE .EQ. 1) THEN
           CALL RFIN(DOHX,LHX,FTYPE,LF,THF,PI,HOHX,TEMPEFF,ALR)
        ENDIF
        IF (FTYPE .EQ. 2) THEN
           CALL AXFIN(DOHX,LHX,FTYPE,LF,THF,PI,HOHX,TEMPEFF,ALR)
        ENDIF
        INTER=LHX
        GOTO 140
C       CALCULATE VISCOSITY AND DENSITY OF GASEOUS HELIUM AT THE AVERAGE
C       TEMPERATURE IN THE HEAT EXCHANGER
150     ALR2=ALR
        MU=7.884E-06-7.164E-08*(75-TF)
        RHO=0.641+1.332E-02*(75-TF)
C       CALCULATE VELOCITY OF GASEOUS HELIUM IN THE HEAT EXCHANGER
        V=4*MFRHE/(RHO*PI*DIHX**2)
C       CALCULATE THE INNER RADIUS OF THE HEAT EXCHANGER LINE
        RIHX=DIHX/2
C       CALL SUBROUTINE FRICTION TO GET THE FRICTION FACTOR
        CALL FRICTION(MFRHE,PI,RIHX,RP,MU,F)
C       CALCULATE THE FRICTION HEAD
        HF=F*LHX*V**2/(DIHX*2*G)
C       CALCULATE THE PRESSURE DROP IN THE HEAT EXCHANGER
        DPHX=RHO*HF*G
C       CALCULATE THE TOTAL PRESSURE DROP
        HEPD=HEPD+DPC+DPHX
        PEFF=0.55
C       CALCULATE THE PUMP HEAD REQUIRED
        HP=HEPD/(RHO*G)
C       CALCULATE THE WORK OF THE PUMP
        WPUMP=MFRHE*G*HP/PEFF
C       ITERATE TO FIND AN ACCURATE VALUE OF PUMP WORK
        IF (ABS(WPUMP-TEMP) .LT. 0.001) GOTO 160
        TEMP=WPUMP
        GOTO 130
160     SAC=ALR1*LC
        SAHX=ALR2*LHX
        RE=4*MFRHE/(PI*2*RHL1*MU)
        RETURN
        END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE ICVESSEL                        *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE THERMAL RESISTANCE DUE TO      *
C*     NATURAL CONVECTION ON THE INTERIOR OF THE CPU VESSEL.         *
C*                                                                   *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE ICVESSEL(G,TA,TF,HV,RV1,VRT3)
      REAL*8 ALPHA,BETA,CP,G,HINT,HV,K,MU,NU,NUL,PI,PR,RAL,RHO,RV1
      REAL*8 TA,TF,VRT3
      PI=3.14159
      RHO=1.083493E-03*TF**3-0.2505496*TF**2+14.82779*TF+658.1284
      MU=5.637657E-09*TF**3-0.9241204E-06*TF**2+35.90146E-06*TF
     @ +300.8022E-06
      BETA=7.39E-05*TF+5.15E-05
      CP=(4.43644E-06*TF**3-9.025098E-04*TF**2+6.437882E-02*TF+0.4
     @ 181636)*1.0E03
      K=(6.916212E-04*TF**3-0.1218814*TF**2+5.284699*TF+140.1524)*
     @ 1.0E-03
      PR=4.793316E-05*TF**3-8.096186E-03*TF**2+0.3239784*TF+3.5086
     @ 39
      NU=MU/RHO
      ALPHA=K/(RHO*CP)
      RAL=(G*BETA*(TA-TF)*HV**3)/(NU*ALPHA)
      NUL=(0.825+((0.387*RAL**0.167)/((1+(0.492/PR)**0.5625)**0.29
     @ 63)))**2
      HINT=NUL*K/HV
      VRT3=1/(2*PI*RV1*HV*HINT)
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE INTCONV                         *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE NUSSELT NUMBER AND THE         *
C*     CORRESPONDING VALUE OF CONVECTION COEFFICIENT FOR INTERNAL    *
C*     FLOW IN A PIPE.                                               *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE INTCONV(TF,MFR,PI,RP1,G,FLUID,HINT)
      REAL*8 CP,FLUID,G,HINT,KF,MFR,MU,NUN,PI,PRN,RE,RP1,SLOPE
      REAL*8 TF,V
C     CALCULATE THE PROPERTIES OF THE FLUID
      IF (FLUID .EQ. 1) THEN
         CP=4.43644E-03*TF**3-9.025098E-01*TF**2+6.437882E01*TF+
     @   0.4181636E03
         MU=5.637657E-09*TF**3-0.9241204E-06*TF**2+35.90146E-06*TF
     @   +300.8022E-06
         KF=5.567163E-07*TF**3-9.294225E-05*TF**2+3.220027E-03*TF+
     @   0.1891409
      ENDIF
      IF (FLUID .EQ. 2) THEN
         CP=5.20E03
         MU=-2.408678E-12*TF**3+3.010459E-10*TF**2+6.483437E-08*TF
     @   +2.36983E-06
         KF=2.529307E-08*TF**3-8.014686E-06*TF**2+1.273839E-03*TF
     @   +9.554924E-04
      ENDIF
C     CALCULATE THE PRANDTL NUMBER
      PRN=CP*MU/KF
C     CALCULATE THE REYNOLDS NUMBER
      RE=4*MFR/(PI*2*RP1*MU)
C     CALCULATE THE NUSSELT NUMBER FOR LAMINAR FLOW
      IF (RE .LT. 2300) THEN
         NUN=4.36
         GOTO 100
      ENDIF
C     CALCULATE THE NUSSELT NUMBER FOR TURBULENT FLOW
      IF (RE .GT. 10000) THEN
         NUN=0.023*RE**0.8*PRN**0.3333
         GOTO 100
      ENDIF
C     CALCULATE THE NUSSELT NUMBER FOR TRANSITIONAL FLOW
      SLOPE=(0.023*10000**0.8*PRN**0.3333-4.36)/(10000-2300)
      NUN=4.36+SLOPE*(RE-2300)
C     CALCULATE THE INTERNAL CONVECTION COEFFICIENT
100   HINT=NUN*KF/(2*RP1)
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                          SUBROUTINE MFLN2                         *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE REQUIRED MASS FLOWRATE OF      *
C*     LIQUID NITROGEN FOR A ONCE THROUGH SYSTEM BASED ON THE        *
C*     HEAT ADDITIONS TO THE SYSTEM                                  *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE MFLN2(P,TI,QLOSS,MFR)
      REAL*8 HE,HI,MFR,P,QLOSS,TF,TI
      TF=TI
C     CALCULATE THE INLET AND EXIT ENTHALPIES OF LIQUID NITROGEN
      HI=-1.133702E-01*TF**3+2.733992E01*TF**2-0.1201434E03*TF-
     @ 223.2402E03
      HE=77.212E03
C     CALCULATE THE REQUIRED MASS FLOW RATE OF CRYOGENIC FLUID
      MFR=(QLOSS+P)/(HE-HI)
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE MLI1                            *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE HEAT ADDED TO THE CRYOGENIC    *
C*     FLUID THROUGH THE TRANSFER LINES IF MULTILAYER INSULATION     *
C*     IS USED.                                                      *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE MLI1(RP1,RP2,RP3,PI,L,KP,KI,H,TA,TF,FLUID,G,MFR,
     @ MID)
      REAL*8 FLUID,G,H,HI,KF,KI,KP,L,MFR,MID,PI,PRT1,PRT2,PRT3,PRT4
      REAL*8 PRTOT,RP1,RP2,RP3,TA,TF
C     CALL PRESIST TO CALCULATE THE THERMAL RESISTANCES ASSOCIATED
C     WITH CONVECTION AND CONDUCTION THROUGH THE PIPE
      CALL PRESIST(RP1,RP2,RP3,TF,PI,L,KP,H,FLUID,G,MFR,PRT1,PRT2,
     @ PRT3)
C     CALCULATE THE THERMAL RESISTANCE OF THE INSULATION LAYER
      PRT4=LOG(RP3/RP2)/(2*PI*L*KI)
C     CALCULATE THE TOTAL THERMAL RESISTANCE
      PRTOT=PRT1+PRT2+PRT3+PRT4
C     CALCULATE THE HEAT ADDED THROUGH THE LINE SEGMENT
      MID=(TA-TF)/PRTOT
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE MLI2                            *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE HEAT ADDED TO THE CRYOGENIC    *
C*     FLUID THROUGH THE CPU VESSEL IF MULTILAYER INSULATION OF      *
C*     SPECIFIED THICKNESS IS USED.                                  *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE MLI2(RV1,RV2,RV3,PI,HV,KV,KI,H,TA,TF,QLV)
      REAL*8 H,HV,KI,KV,PI,QLV,RV1,RV2,RV3,TA,TF,VRT1,VRT2,VRT3
      REAL*8 VRT4,VRTOT
      G=9.81
C     CALL VRESIST TO CALCULATE THE THERMAL RESISTANCES ASSOCIATED
C     WITH CONVECTION AND CONDUCTION THROUGH VESSEL
      CALL VRESIST(G,TA,TF,RV1,RV2,RV3,PI,HV,KV,H,VRT1,
     @ VRT2,VRT3)
C     CALCULATE THE THERMAL RESISTANCE OF THE INSULATION LAYER
      VRT4=LOG(RV3/RV2)/(2*PI*HV*KI)
C     CALCULATE THE TOTAL THERMAL RESISTANCE
      VRTOT=VRT1+VRT2+VRT3+VRT4
C     CALCULATE THE HEAT ADDED THROUGH THE VESSEL
      QLV=(TA-TF)/VRTOT
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE OTCOST                          *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE TOTAL COST OF A ONCE-THROUGH   *
C*     SYSTEM                                                        *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE OTCOST(TI,L,NSP,RP1,MFR,RCOST,PCOST,PUMPCOST,VCOST,
     @ LN2COST,COST1,COST2,UACOST,RVOL)
      REAL*8 COST1,COST2,L,LN2COST,MFR,NSP,PCOST,PUMPCOST,RCOST
      REAL*8 RHOL,RP1,RVOL,TI,TIME1,TIME2,UACOST,VCOST
C     SET TIME1=7  DAYS
      TIME1=604800.0
C     SET TIME2=1 DAY
      TIME2=86400
C     CALCULATE THE DENSITY OF LIQUID NITROGEN AT TI
      RHOL=1.083493E-03*TI**3-0.2505496*TI**2+14.82779*TI
     @ +658.1284
C     CALCULATE THE VOLUME OF THE LIQUID NITROGEN RESERVOIR
      RVOL=1.1*((MFR*TIME1/RHOL)*1000)
C     CALCULATE THE COST OF THE LIQUID NITROGEN RESERVOIR
      RCOST=3.105242E-07*RVOL**3-3.412452E-03*RVOL**2+14.35143*
     @ RVOL-625.181
C     CALCULATE THE COST OF THE PIPING SYSTEM
      PCOST=(75+(NSP/4)*75)*3.2808*L+(47881.49*RP1**3-4932.401
     @ *RP1**2+343.1831*RP1+1.363372)*3.2808*L
C     ENTER THE PUMP COST
      PUMPCOST=5000
C     CALCULATE THE COST OF THE CPU VESSEL
      VCOST=3.105242E-07*331.34**3-3.412452E-03*331.34**2+14.35143*
     @ 331.34-625.181
C     CALCULATE THE COST OF THE LIQUID NITROGEN PER DAY
      LN2COST=MFR*TIME2/RHOL*1000*0.1
C     CALCULATE THE TOTAL INITIAL COST
      COST1=RCOST+PCOST+PUMPCOST+VCOST
C     CALCULATE THE DAILY COST
      COST2=LN2COST
      COST2=COST2*365
      UACOST=(0.926*(COST1/5+COST2)+0.857*(COST1/5+COST2)+0.794*(
     @ COST1/5+COST2)+0.735*(COST1/5+COST2)+0.681*(COST1/5+COST2))
     @ *0.2505
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE PD                              *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE PRESSURE DROP OCCURRING IN     *
C*     THE TRANSFER LINES CONTAINING SUBCOOLED LIQUID NITROGEN.      *
C*     SINCE HEAT IS BEING ADDED TO THE LINES, THE TEMPERATURE       *
C*     AND THEREFORE THE PROPERTIES OF THE FLUID ARE CHANGING.       *
C*     THE TRANSFER LINE IS DIVIDED INTO SMALL SEGMENTS AND THE      *
C*     PRESSURE DROP IS CALCULATED FOR EACH SEGMENT.                 *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE PD(NSP,NSV,ES,L,NUM,FLAG,PI,RP1,RP2,RP3,EP,SIGMA,TA,
     @ KP,KI,H,TI,MFR,G,RV1,RV2,RV3,EV,HV,KV,TINC,FLUID,PRESSDROP
     @ ,QLP,QLV,TO)
      REAL*8 DIFF,DP,EP,ES,EV,F,FLAG,FLUID,G,H,HE1,HE2,HF,HI,HV,KI,KP
      REAL*8 KV,L,MFR,MID,MUL,NSP,NSV,NUM,PI,PRESSDROP,QL,QLP,QLV,RHOL
      REAL*8 RP,RP1,RP2,RP3,RV1,RV2,RV3,SIGMA,TA,TEMP,TF,TI,TINC,TO,V
      INTEGER I
C     INITIALIZE VARIABLES
      QLP=0
      PRESSDROP=0
C     DIVIDE THE PIPE LENGTH INTO SMALL SEGMENTS
      L=L/NUM
      DO 50 I=1,NUM
C      CALCULATE THE ENTHALPY OF THE FLUID AT THE INLET
         HI=-1.133702E-01*TI**3+2.733992E01*TI**2-0.1201434E03*TI-
     @    223.2402E03
C      ASSUME AN OUTLET TEMPERATURE
         TO=TI+TINC
C      CALCULATE THE AVERAGE FLUID TEMPERATURE
30       TF=(TI+TO)/2
C      CALCULATE THE HEAT ADDED TO THE LINE SEGMENT
         IF (FLAG .EQ. 0) THEN
             CALL VACUUM1(PI,KP,H,RP1,RP2,RP3,L,EP,SIGMA,TA,TF,NSP,
     @           ES,FLUID,G,MFR,MID)
         ENDIF
         IF (FLAG .EQ. 1) THEN
            CALL MLI1(RP1,RP2,RP3,PI,L,KP,KI,H,TA,TF,FLUID,G,MFR,
     @          MID)
         ENDIF
C      CALCULATE THE EXIT ENTHALPY BY TWO METHODS
         HE1=MID/MFR+HI
         HE2=-1.133702E-01*TO**3+2.733992E01*TO**2-0.1201434E03*TO-
     @    223.2402E03
         DIFF=HE2-HE1
C      CHECK TO SEE IF THE TWO VALUES OF EXIT ENTHALPY ARE CLOSE
         IF (((DIFF .GT. 0) .AND. (TEMP .LT. 0)) .OR. ((DIFF .LT. 0)
     @    .AND. (TEMP .GT. 0))) THEN
           GOTO 40
         ENDIF
         TEMP=DIFF
         IF (HE2 .GT. HE1) THEN
```

```
            PRINT*,'WARNING: TEMPERATURE INCREMENT TOO LARGE'
          ENDIF
C       ASSUME A NEW EXIT TEMPERATURE
          TO=TO+TINC
          GOTO 30
40        QL=MID
          QLP=QLP+QL
C       CALCULATE THE DYNAMIC VISCOSITY AND DENSITY OF THE FLUID
          MUL=5.637657E-09*TF**3-0.9241204E-06*TF**2+35.90146E-06*
     @    TF+300.8022E-06
          RHOL=1.083493E-03*TF**3-0.2505496*TF**2+14.82779*TF+658.1284
C       CALCULATE THE VELOCITY IN THE LINE
          V=MFR/(RHOL*PI*RP1**2)
C       CALCULATE THE FRICTION FACTOR
          CALL FRICTION(MFR,PI,RP1,RP,MUL,F)
C       CALCULATE THE FRICTION HEAD
          HF=F*L*V**2/(RP1*4*G)
C       CALCULATE THE PRESSURE DROP IN THE LINE SEGMENT
          DP=RHOL*HF*G
C       ADD THE SEGMENT PRESSURE DROP TO THE TOTAL PRESSURE DROP
          PRESSDROP=PRESSDROP+DP
C       SET THE INLET TEMPERATURE OF THE NEW SEGMENT EQUAL TO THE
C       OUTLET TEMPERATURE OF THE PREVIOUS SEGMENT
          TI=TO
50      CONTINUE
C       CALCULATE THE HEAT ADDED THROUGH THE CPU VESSEL
        IF (FLAG .EQ. 0) THEN
          CALL VACUUM2(PI,KV,H,RV1,RV2,HV,RV3,EV,SIGMA,TA,TF,NSV,
     @    ES,QLV)
        ENDIF
        IF (FLAG .EQ. 1) THEN
          CALL MLI2(RV1,RV2,RV3,PI,HV,KV,KI,H,TA,TF,QLV)
        ENDIF
C       SET L EQUAL TO TOTAL LENGTH
        L=L*NUM
        RETURN
        END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE PD2                             *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE PRESSURE DROP OCCURRING        *
C*     IN THE GASEOUS HELIUM LOOP.                                   *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE PD2(NSP,ES,L,NUM2,FLAG,PI,RHL1,RHL2,RHL3,
     @ EP,SIGMA,TA,KP,KI,H,TI2,MFRHE,G,TINC,FLUID,HEPD,TO2)
      REAL*8 CP,DIFF,DP,EP,ES,F,FLAG,FLUID,G,H,HE1,HE2,HEPD,HF,HI
      REAL*8 KP,KI,L,MFRHE,MID,MU,NSP,NUM2,PI,QL,QLP,RHL1
      REAL*8 RHL2,RHL3,RHO,RP,SIGMA,TA,TEMP,TF,TI2,TINC,TO2,VHE
      INTEGER I
C     INITIALIZE VARIABLES
      QLP=0
      HEPD=0
      FLUID=2
      CP=5.20E03
      DO 190 I=1,NUM2
C     CALCULATE THE ENTHALPY OF THE HELIUM AT THE INLET
         HI=CP*TI2
C     ASSUME AN OUTLET TEMPERATURE
         TO2=TI2+TINC
C     CALCULATE THE AVERAGE FLUID TEMPERATURE
170      TF=(TI2+TO2)/2
C     CALCULATE THE HEAT ADDED TO THE LINE SEGMENT
         IF (FLAG .EQ. 0) THEN
            CALL VACUUM1(PI,KP,H,RHL1,RHL2,RHL3,L,EP,SIGMA,TA,TF,
     @         NSP,ES,FLUID,G,MFRHE,MID)
         ENDIF
         IF (FLAG .EQ. 1) THEN
            CALL MLI1(RHL1,RHL2,RHL3,PI,L,KP,KI,H,TA,TF,FLUID,G,
     @         MFRHE,MID)
         ENDIF
C     CALCULATE THE EXIT ENTHALPY BY TWO METHODS
         HE1=MID/MFRHE+HI
         HE2=CP*TF
         DIFF=HE2-HE1
C     CHECK TO SEE IF THE TWO VALUES OF EXIT ENTHALPY ARE CLOSE
         IF (((DIFF .GT. 0) .AND. (TEMP .LT. 0)) .OR. ((DIFF .LT. 0)
     @    .AND. (TEMP .GT. 0))) THEN
            GOTO 180
         ENDIF
         TEMP=DIFF
         IF (HE2 .GT. HE1) THEN
            PRINT*,'WARNING: TEMPERATURE INCREMENT TOO LARGE'
         ENDIF
C     ASSUME A NEW EXIT TEMPERATURE
         TO2=TO2+TINC
         GOTO 170
180      QL=MID
         QLP=QLP+QL
```

```
C       CALCULATE THE DYNAMIC VISCOSITY AND DENSITY OF THE HELIUM
           MU=-2.408678E-12*TF**3+3.010459E-10*TF**2+6.483437E-08*TF
     @     +2.36983E-06
           RHO=-1.731763E-06*TF**3+5.52761E-04*TF**2-6.242247E-02*TF
     @     +2.937333
C       CALCULATE THE VELOCITY IN THE LINE
           VHE=MFRHE/(RHO*PI*RHL1**2)
C       CALCULATE THE FRICTION FACTOR
           CALL FRICTION(MFRHE,PI,RHL1,RP,MU,F)
C       CALCULATE THE FRICTION HEAD
           HF=F*L*VHE**2/(RHL1*4*G)
C       CALCULATE THE PRESSURE DROP IN THE LINE SEGMENT
           DP=RHO*HF*G
C       ADD THE SEGMENT PRESSURE DROP TO THE TOTAL PRESSURE DROP
           HEPD=HEPD+DP
C       SET THE INLET TEMPERATURE OF THE NEW SEGMENT EQUAL TO THE
C       OUTLET TEMPERATURE OF THE PREVIOUS SEGMENT
           TI2=TO2
190     CONTINUE
        RETURN
        END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE PRESIST                         *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE THERMAL RESISTANCES DUE        *
C*     TO INTERNAL CONVECTION, CONDUCTION THROUGH THE PIPE, AND      *
C*     EXTERNAL CONVECTION.                                          *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE PRESIST(RP1,RP2,RP3,TF,PI,L,KP,H,FLUID,G,MFR,PRT1,
     @ PRT2,PRT3)
      REAL*8 FLUID,G,H,HINT,KF,KP,L,MFR,PI,PRT1,PRT2,PRT3,RP1,RP2
      REAL*8 RP3,TF
C     CALL SUBROUTINE INTCONV TO GET VALUE OF INTERNAL CONVECTION
C     COEFFICIENT
      CALL INTCONV(TF,MFR,PI,RP1,G,FLUID,HINT)
C     CALCULATE THE THERMAL RESISTANCE DUE TO INTERNAL CONVECTION
      PRT1=1/(2*PI*RP1*L*HINT)
C     CALCULATE THE THERMAL RESISTANCE OF THE PIPE
      PRT2=LOG(RP2/RP1)/(2*PI*L*KP)
C     CALCULATE THE THERMAL RESISTANCE DUE TO EXTERNAL CONVECTION
      PRT3=1/(2*PI*RP3*L*H)
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE REFCOST                         *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE TOTAL COST OF A REFRIGERATED   *
C*     CRYOGENIC IMMERSION COOLING SYSTEM                            *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE REFCOST(L,L2,NSP,QREF,WIN,WPUMP,RP1,RHL1,SAC,SAHX,
     @ MFR,TI,COST1,COST2,UACOST,PCOST,RCOST,CRYCOST,LNCOST,CCOST,
     @ HXCOST,PUMPCOST,RVOL,COP,COST3)
      REAL*8 CCOST,COP,COST1,COST2,COST3,CP,CRYCOST,HXCOST,KWHR,L
      REAL*8 L2,LNCOST,LNVOL,MASS,MFR,NSP,PCOST,PI,POWCOST,PUMPCOST
      REAL*8 QREF,QSUB,RCOST,RHL1,RHO,RP1,RVOL,SAC,SAHX
      REAL*8 TI,UACOST,WIN,WPUMP,WTOT
C     CALCULATE THE COST OF THE PIPING
      PCOST=(75+(NSP/4)*75)*3.2808*L+(47881.49*RP1**3-4932.401
     @ *RP1**2+343.1831*RP1+1.363372)*L*3.2808
     @ +(75+(NSP/4)*75)*3.2808*2*L2+(47881.49*RHL1**3-
     @ 4932.401*RHL1**2+343.1831*RHL1+1.363372)*2*L2*3.2808
C     CALCULATE THE COST OF THE REFRIGERATOR AND THE CPU VESSEL
      CRYCOST=750*QREF**0.52
C     CALCULATE THE DENSITY OF THE LIQUID NITROGEN IN THE RESERVOIR
      RHO=1.083493E-03*TI**3-0.2505496*TI**2+14.82779*TI+658.1284
C     CALCULATE THE VOLUME OF THE RESERVOIR
      RVOL=MFR*86400/RHO*1000
C     CALCULATE THE COST OF THE RESERVOIR
      RCOST=3.105242E-07*RVOL**3-3.412452E-03*RVOL**2+14.35143*
     @ RVOL-625.181
C     CALCULATE THE TOTAL VOLUME OF LIQUID NITROGEN
      LNVOL=MFR*86400/RHO+(PI*0.75**2/4)*0.75
C     CALCULATE THE COST OF LIQUID NITROGEN
      LNCOST=LNVOL*1000*0.1
C     CALCULATE THE COST OF THE CONDENSER
      CCOST=0.25*235*SAC**0.665
C     CALCULATE THE COST OF THE HEAT EXCHANGER
      HXCOST=0.25*235*SAHX**0.665
C     ENTER THE COST OF THE PUMP
      PUMPCOST=18000
C     CALCULATE THE TOTAL POWER FOR THE REFRIGERATOR AND THE PUMP
      WTOT=(WIN+WPUMP)/1000
C     CALCULATE THE TOTAL kW-HR REQUIRED PER MONTH
      KWHR=WTOT*24*31
C     CALCULATE THE POWER COST FOR THE REFRIGERATOR AND PUMP PER MONTH
      IF (KWHR .LT. 2000) THEN
         POWCOST=1.5*0.05352*KWHR
         GOTO 20
      ENDIF
      KWHR=KWHR-2000
      POWCOST=1.5*0.05352*2000+1.5*0.05941*KWHR
C     CALCULATE THE INITIAL COST OF A REFRIGERATED SYSTEM
20    COST1=PCOST+RCOST+CRYCOST+CCOST+HXCOST+PUMPCOST
      COST2=POWCOST
```

```
      COST2=COST2*12
      IF (TI .EQ. 77) GOTO 25
C     CALCULATE THE COST OF SUBCOOLING THE LIQUID NITROGEN
C     CALCULATE THE DENSITY AT THE SUBCOOLED TEMPERATURE
      RHO=-4.28*(TI-65)+860.9
C     CALCULATE THE SPECIFIC HEAT OF THE LIQUID NITROGEN AT THE
C     SUBCOOLED TEMPERATURE
      CP=2008+3.4*(TI-65)
C     CALCULATE THE MASS OF LIQUID NITROGEN TO SUBCOOL
      MASS=LNVOL*RHO
C     CALCULATE THE REFRIGERATION REQUIRED TO SUBCOOL THE LIQUID
C     NITROGEN TO THE DESIRED TEMPERATURE
      QSUB=(MASS*CP*(77-TI))/COP
C     CALCULATE THE SUBCOOLING COST
      IF ((QSUB/3.6E06) .LT. 2000) THEN
         COST3=1.5*0.05352*(QSUB/3.6E06)
         GOTO 25
      ENDIF
      COST3=1.5*0.05352*2000+1.5*0.05941*(QSUB/3.6E06)
25    COST2=COST2+LNCOST*12+COST3*12
      UACOST=(0.926*(COST1/5+COST2)+0.857*(COST1/5+COST2)+0.794*(
     @ COST1/5+COST2)+0.735*(COST1/5+COST2)+0.681*(COST1/5+COST2))
     @ *0.2505
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                          SUBROUTINE RESULT1                       *
C*                                                                   *
C*     THIS SUBROUTINE PRINTS THE RESULTS FOR A ONCE-THROUGH         *
C*     SYSTEM.                                                       *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE RESULT1(RP1,MID,QLV,QLL,QLOSS,MFR,RCOST,TI,PCOST,
     @ PUMPCOST,VCOST,LN2COST,COST1,COST2,PRESSDROP,TO,XE,UACOST,RVOL)
      REAL*8 COST1,COST2,LN2COST,MFR,MID,PCOST,PUMPCOST,PRESSDROP,QLL
      REAL*8 QLOSS,QLV,RCOST,RP1,RVOL,TI,TO,VCOST,UACOST,XE
      WRITE(300,200)
      WRITE(300,*)
      WRITE(300,*)
      WRITE(300,210) 2000*RP1
      WRITE(300,220) MFR
      WRITE(300,230) RVOL
      IF (TI .EQ. 77) THEN
         WRITE(300,240)
         WRITE(300,250) XE*100
         WRITE(300,260) PRESSDROP*1.4504E-04
         GOTO 190
      ENDIF
      WRITE(300,270) TI
      WRITE(300,280) TO
      WRITE(300,290) PRESSDROP*1.4504E-04
190   WRITE(300,*)
      WRITE(300,*)
      WRITE(300,300)
      WRITE(300,*)
      WRITE(300,310) MID
      WRITE(300,320) QLV
      WRITE(300,330) QLL
      WRITE(300,340) QLOSS
      WRITE(300,*)
      WRITE(300,*)
      WRITE(300,350)
      WRITE(300,*)
      WRITE(300,360) RCOST
      WRITE(300,370) PCOST
      WRITE(300,375) PUMPCOST
      WRITE(300,380) VCOST
      WRITE(300,390) COST1
      WRITE(300,400) COST2
      WRITE(300,410) UACOST
200   FORMAT(5X,'ONCE-THROUGH SYSTEM')
210   FORMAT(5X,'INNER PIPE DIAMETER = ',F6.3,' mm')
220   FORMAT(5X,'REQUIRED MASS FLOWRATE OF LIQUID NITROGEN = '
     @ ,E9.4,' kg/s')
230   FORMAT(5X,'RESERVOIR VOLUME = ',F7.2,' liters')
240   FORMAT(5X,'INLET LIQUID NITROGEN TEMPERATURE IS 77 K')
250   FORMAT(5X,'EXIT QUALITY IS ',F4.2,' %')
```

```
260     FORMAT(5X,'TWO-PHASE PRESSURE DROP IN THE LIQUID NITROGEN LINES='
     @ ,E9.4,' psi')
270     FORMAT(5X,'INLET LIQUID NITROGEN TEMPERATURE = ',F5.2,' K')
280     FORMAT(5X,'THE TEMPERATURE OF LIQUID NITROGEN AFTER 30 m = '
     @ ,F5.2,' K')
290     FORMAT(5X,'SINGLE PHASE PRESSURE DROP IN THE LIQUID NITROGEN
     @ LINES = ',F5.3,' psi')
300     FORMAT(5X,'HEAT ADDITIONS TO THE LIQUID NITROGEN:')
310     FORMAT(5X,'HEAT ADDED THROUGH LIQUID NITROGEN LINES = '
     @ ,F6.2,' W')
320     FORMAT(5X,'HEAT ADDED THROUGH CPU VESSEL = ',F5.2,' W')
330     FORMAT(5X,'HEAT ADDED THROUGH LEADS=',F5.2,' W')
340     FORMAT(5X,'TOTAL HEAT ADDED TO THE LIQUID NITROGEN = '
     @ ,F6.2,' W')
350     FORMAT(5X,'ONCE-THROUGH SYSTEM COST:')
360     FORMAT(5X,'THE COST OF THE RESERVOIR = $ ',F8.2)
370     FORMAT(5X,'THE COST OF THE PIPING = $ ',F8.2)
375     FORMAT(5X,'THE COST OF THE PUMP = $ ',F8.2)
380     FORMAT(5X,'THE COST OF THE CPU VESSEL = $ ',F8.2)
390     FORMAT(5X,'THE INITIAL COST OF A ONCE-THROUGH SYSTEM = $ '
     @ ,F8.2)
400     FORMAT(5X,'THE ANNUAL COST OF A ONCE-THROUGH SYSTEM = $ ',
     @ F8.2)
410     FORMAT(5X,'THE UNIFORM ANNUAL COST = $ ',F8.2)
        RETURN
        END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE RESULT2                         *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE PRINTS THE RESULTS FOR A REFRIGERATED         *
C*     SYSTEM.                                                       *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE RESULT2(TREF,QREF,COP,WIN,COST1,COST2,DIC,TCI,TCO,
     @ QCOND,HIC,HCOND,LC,SAC,DPC,DIHX,THXI,THXO,QHX,HIHX,HOHX,LHX,
     @ SAHX,DPHX,RHL1,MFRHE,HEPD,WPUMP,UACOST,PCOST,RCOST,CRYCOST,
     @ LNCOST,CCOST,HXCOST,PUMPCOST,RVOL,TI,COST3,LF,THF,FTYPE)
      REAL*8 CCOST,COP,COST1,COST2,COST3,CRYCOST,DIC,DIHX,DPC,DPHX
      REAL*8 HCOND,HEPD,HIC,HIHX,HOHX,HXCOST,LC,LF,LHX,LNCOST,MFRHE
      REAL*8 QCOND,QHX,QREF,PCOST,PUMPCOST,RCOST,RHL1,RVOL,SAC,SAHX
      REAL*8 TCI,TCO,THF,THXI,THXO,TI,TREF,UACOST,WIN,WPUMP,FTYPE
      WRITE(400,500)
      WRITE(400,*)
      IF (TI .EQ. 77) THEN
         WRITE(400,505)
         GOTO 490
      ENDIF
      WRITE(400,507)
      WRITE(400,508) TI
490   WRITE(400,*)
      WRITE(400,510)
      WRITE(400,*)
      WRITE(400,520) TREF
      WRITE(400,530) QREF
      WRITE(400,540) COP
      WRITE(400,550) WIN
      WRITE(400,*)
      IF(TI .EQ. 77) THEN
         WRITE(400,560)
         WRITE(400,*)
         GOTO 495
      ENDIF
      WRITE(400,565)
      WRITE(400,*)
      IF (FTYPE .EQ. 1) THEN
          WRITE(400,645) THF*1000,LF*1000
      ENDIF
      IF (FTYPE .EQ. 2) THEN
         WRITE(400,647) THF*1000,LF*1000
      ENDIF
495   WRITE(400,570) 1000*DIC
      WRITE(400,580) TCI
      WRITE(400,590) TCO
      WRITE(400,600) QCOND
      WRITE(400,610) HIC
      WRITE(400,620) HCOND
      WRITE(400,630) LC
      WRITE(400,640) SAC
```

```
      WRITE(400,650) DPC*1.4504E-04
      WRITE(400,*)
      IF (TI .EQ. 77) THEN
         WRITE(400,660)
         WRITE(400,*)
         GOTO 496
      ENDIF
      WRITE(400,665)
      WRITE(400,*)
496   IF (FTYPE .EQ. 1) THEN
         WRITE(400,645) THF*1000,LF*1000
      ENDIF
      IF (FTYPE .EQ. 2) THEN
         WRITE(400,647) THF*1000,LF*1000
      ENDIF
      WRITE(400,670) 1000*DIHX
      WRITE(400,680) THXI
      WRITE(400,690) THXO
      WRITE(400,700) QHX
      WRITE(400,710) HIHX
      WRITE(400,720) HOHX
      WRITE(400,730) LHX
      WRITE(400,740) SAHX
      WRITE(400,750) DPHX*1.4504E-04
      WRITE(400,*)
      WRITE(400,760)
      WRITE(400,*)
      WRITE(400,770) 2000*RHL1
      WRITE(400,780) MFRHE
      WRITE(400,790) HEPD*1.4504E-04
      WRITE(400,800) WPUMP
      WRITE(400,*)
      WRITE(400,810)
      WRITE(400,*)
      WRITE(400,820) COST1
      WRITE(400,830) PCOST
      WRITE(400,840) RCOST
      WRITE(400,845) RVOL/1000
      WRITE(400,850) CRYCOST
      WRITE(400,860) CCOST
      WRITE(400,870) HXCOST
      WRITE(400,880) LNCOST*12
      WRITE(400,890) PUMPCOST
      WRITE(400,900) COST2
      WRITE(400,910) UACOST
      IF (TI .LT. 77) THEN
         WRITE(400,905) COST3*12
      ENDIF
500   FORMAT(5X,'REFRIGERATED SYSTEM')
505   FORMAT(5X,'SATURATED POOL BOILING')
507   FORMAT(5X,'SUBCOOLED POOL BOILING')
508   FORMAT(5X,'SUBCOOLED TEMPERATURE = ',F4.1,' K')
510   FORMAT(5X,'REFRIGERATOR:')
520   FORMAT(5X,'REFRIGERATION TEMPERATURE = ',F4.1,' K')
```

```
530     FORMAT(5X,'REFRIGERATION CAPACITY = ',F7.2,' W')
540     FORMAT(5X,'COEFFICIENT OF PERFORMANCE = ',E9.4)
550     FORMAT(5X,'WORK INPUT TO THE REFRIGERATOR = ',F9.2,' W')
560     FORMAT(5X,'CONDENSER:')
565     FORMAT(5X,'HEAT EXCHANGER I:')
570     FORMAT(5X,'INNER PIPE DIAMETER = ',F6.3,' mm')
580     FORMAT(5X,'INLET TEMPERATURE = ',F5.2,' K')
590     FORMAT(5X,'OUTLET TEMPERATURE = ',F5.2,' K')
600     FORMAT(5X,'HEAT TRANSFER = ',F7.2,' W')
610     FORMAT(5X,'INTERNAL CONVECTION COEFFICIENT = ',F7.2,
     @ ' W/m^2-K')
620     FORMAT(5X,'EXTERNAL CONVECTION COEFFICIENT = ',F7.2,
     @ ' W/m^2-K')
630     FORMAT(5X,'LENGTH OF PIPE = ',F5.2,' m')
640     FORMAT(5X,'SURFACE AREA = ',F6.2,' m^2')
645     FORMAT(5X,'RADIAL FINS: THICKNESS = ',F4.1,' mm, LENGTH = ',
     @ F4.1,' mm')
647     FORMAT(5X,'AXIAL FINS: THICKNESS = ',F4.1,' mm, LENGTH = ',
     @ F4.1,' mm')
650     FORMAT(5X,'PRESSURE DROP = ',E9.4,' psi')
660     FORMAT(5X,'HEAT EXCHANGER:')
665     FORMAT(5X,'HEAT EXCHANGER II:')
670     FORMAT(5X,'INNER PIPE DIAMETER = ',F6.3,' mm')
680     FORMAT(5X,'INLET TEMPERATURE = ',F5.2,' K')
690     FORMAT(5X,'OUTLET TEMPERATURE = ',F5.2,' K')
700     FORMAT(5X,'HEAT TRANSFER = ',F7.2,' W')
710     FORMAT(5X,'INTERNAL CONVECTION COEFFICIENT = ',F6.2,
     @ ' W/m^2-K')
720     FORMAT(5X,'EXTERNAL CONVECTION COEFFICIENT = ',F6.2,
     @ ' W/m^2-K')
730     FORMAT(5X,'LENGTH OF PIPE = ',F6.2,' m')
740     FORMAT(5X,'SURFACE AREA = ',F6.2,' m^2')
750     FORMAT(5X,'PRESSURE DROP = ',E9.4,' psi')
760     FORMAT(5X,'HELIUM LOOP')
770     FORMAT(5X,'INNER PIPE DIAMETER = ',F6.3,' mm')
780     FORMAT(5X,'MASS FLOW RATE OF GASEOUS HELIUM = ',E9.4,
     @ ' kg/s')
790     FORMAT(5X,'TOTAL PRESSURE DROP IN GASEOUS HELIUM LOOP = ',
     @ E9.4,' psi')
800     FORMAT(5X,'PUMP WORK REQUIRED = ',F6.2,' W')
810     FORMAT(5X,'REFRIGERATED SYSTEM COST:')
820     FORMAT(5X,'THE INITIAL COST OF THE REFRIGERATED SYSTEM = $ ',
     @ F9.2)
830     FORMAT(5X,'THE PIPING COST = $ ',F8.2)
840     FORMAT(5X,'THE RESERVOIR COST = $ ',F8.2)
845     FORMAT(5X,'THE RESERVOIR SIZE = ',F7.2,' m^3')
850     FORMAT(5X,'THE CRYOCOOLER COST = $ ',F8.2)
860     FORMAT(5X,'THE CONDENSER COST = $ ',F7.2)
870     FORMAT(5X,'THE HEAT EXCHANGER COST = $ ',F7.2)
880     FORMAT(5X,'THE ANNUAL LIQUID NITROGEN COST = $ ',F6.2)
890     FORMAT(5X,'THE PUMP COST = $ ',F8.2)
900     FORMAT(5X,'THE POWER COST OF THE REFRIGERATED SYSTEM = $ ',
     @ F9.2,' PER YEAR')
905     FORMAT(5X,'THE COST OF SUBCOOLING THE LIQUID NITROGEN = $ ',
```

```
     @ F6.2,' PER YEAR')
910    FORMAT(5X,'THE UNIFORM ANNUAL COST = $ ',F9.2)
       RETURN
       END
```

```
C*****************************************************************
C*                                                               *
C*                        SUBROUTINE RFIN                        *
C*                                                               *
C*     THIS SUBROUTINE CALCULATES THE FIN EFFICIENCY AND         *
C*     TEMPERATURE EFFECTIVENESS OF RADIAL FINS                  *
C*                                                               *
C*****************************************************************
C
      SUBROUTINE RFIN(DOHX,LHX,FTYPE,LF,THF,PI,HOHX,TEMPEFF,ALR)
      REAL*8 ABASE,AFIN,ALR,APF,ATOT,DOHX,FINEFF,FTYPE,HOHX,K,LF
      REAL*8 LFP,LHX,NF,PARAM,PI,R2C,TEMPEFF,THF
      K=9
C     CALCULATE THE NUMBER OF FINS
      NF=INT(LHX/(4*THF))
C     CALCULATE THE SURFACE AREA OF THE BASE (UNFINNED PORTION)
      ABASE=PI*DOHX*3*THF*NF
C     CALCULATE THE SURFACE AREA OF THE FINS
      AFIN=PI*(DOHX+2*LF)*THF*NF+(PI/2)*((DOHX+2*LF)**2-DOHX**2)
     @ *NF
C     CALCULATE PARAMETERS NECESSARY TO DETERMINE THE FIN
C     EFFICIENCY
      R2C=(DOHX+LF)+THF/2
      LFP=LF+THF/2
      APF=LFP*THF
      PARAM=LFP**3/2*(HOHX/(K*APF))**1/2
C     CALCULATE THE FIN EFFICIENCY
      FINEFF=4.871362E-02*PARAM**3-9.734929E-02*PARAM**2-0.3833113*
     @ PARAM+1.014096
C     CALCULATE THE TOTAL SURFACE AREA
      ATOT=ABASE+AFIN
C     CALCULATE THE TEMPERATURE EFFECTIVENESS OF THE FINS
      TEMPEFF=1-(AFIN/ATOT)*(1-FINEFF)
C     CALCULATE THE AREA/LENGTH RATIO FOR THE HEAT EXCHANGER
      ALR=ATOT/LHX
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE SHIELD                          *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE EMISSIVITY FACTOR FOR A        *
C*     GIVEN NUMBER OF SHIELDS WITH EMISSIVITY FACTOR FS             *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE SHIELD(A1L,A2L,RP2,RP3,NSP,PI,L,EP,ES,FL)
      REAL*8 A(20),A1L,A2L,DR,EP,ES,FL,L,NSP,PI,NRP,RP2,RP3,SUM1
      INTEGER I
C     CALCULATE THE SPACING BETWEEN THE SHIELDS
      DR=(RP3-RP2)/(NSP+1)
C     CALCULATE THE SURFACE AREAS OF THE SHIELDS
      NRP=RP2+DR
      DO 80 I=1,NSP
         A(I)=2*PI*NRP*L
         NRP=NRP+DR
80    CONTINUE
C     CALCULATE THE EMISSIVITY FACTOR FOR GIVEN NUMBER OF SHIELDS
      SUM1=0
      DO 90 I=1,NSP
         SUM1=SUM1+(A1L/A(I))*(2/ES-1)
90    CONTINUE
      FL=1/(1/EP+(A1L/A2L)*(1/EP-1)+SUM1)
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE STIRLING                        *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE COEFFICIENT OF PERFORMANCE     *
C*     OF AN ELECTROMAGNETICALLY-DRIVEN FREE DISPLACER STIRLING      *
C*     CRYOCOOLER.                                                   *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE STIRLING(X,ALPHA,PI,DPIST,CR1,CR2,TC,TREF,LP,
     @ OMEGA,SL,DM,R,DREG,PDR,COP)
      REAL*8 ALPHA,AP,AR,ARG,B,CDP,CDP1,CDP2,COP,CR1,CR2,DM,DPIST
      REAL*8 DREG,FRE,LAMBDA,LP,MUC,MUD,MUE,N,OAR,OMEGA,PDR,PI,R
      REAL*8 REMAX,RHOC,RHOD,RHOE,SL,TC,TD,TE,TR,TREF,UMAX,VC,VD
      REAL*8 VE,X
      N=INT((3*DREG)/DM)
      TE=TREF
C     CALCULATE THE TEMPERATURE IN THE DEAD SPACE
      TD=(TC+TE)/2
C     EVALUATE THE DENSITY, DYNAMIC AND KINEMATIC VISCOSITIES OF
C     HELIUM AT THE COMPRESSION SPACE TEMPERATURE (TC)
      RHOC=-1.247751E-07*TC**3+8.365729E-05*TC**2-1.905197E-02*TC
     @ +1.699258
      MUC=2.249898E-13*TC**3-1.631992E-10*TC**2+8.801451E-08*TC
     @ +2.082927E-06
      VC=MUC/RHOC
C     EVALUATE THE DENSITY, DYNAMIC AND KINEMATIC VISCOSITIES OF
C     HELIUM AT THE EXPANSION SPACE TEMPERATURE (TE)
      RHOE=-1.247751E-07*TE**3+8.365729E-05*TE**2-1.905197E-02*TE
     @ +1.699258
      MUE=2.249898E-13*TE**3-1.631992E-10*TE**2+8.801451E-08*TE
     @ +2.082927E-06
      VE=MUE/RHOE
C     EVALUATE THE DENSITY, DYNAMIC AND KINEMATIC VISCOSITIES OF
C     HELIUM AT THE DEAD SPACE TEMPERATURE (TD)
      RHOD=-1.247751E-07*TD**3+8.365729E-05*TD**2-1.905197E-02*TD
     @ +1.699258
      MUD=2.249898E-13*TD**3-1.631992E-10*TD**2+8.801451E-08*TD
     @ +2.082927E-06
      VD=MUD/RHOD
C     CALCULATE VALUES OF LAMBDA AND B
      LAMBDA=(VD/VC)*(1/(4*X))*((1+VC/VE)**2*TAN(ALPHA)**2+
     @ (2*VC/VE)**2)**0.5
      ARG=(VE/VC+1)*TAN(ALPHA)/2
      B=ATAN(ARG)
C     CALCULATE THE CROSS-SECTIONAL AREA OF THE PISTON
      AP=PI*DPIST**2/4
C     CALCULATE THE TEMPERATURE RATIO
      TR=TC/TE
C     CALCULATE THE CROSS-SECTIONAL AREA OF THE REGENERATOR MATRIX
      AR=PI*DREG**2/4
C     CALCULATE THE OPENING AREA RATIO
      OAR=(SL/(SL+DM))**2
```

```
C       CALCULATE THE PRESSURE DROP IN THE REGENERATOR
        UMAX=(OMEGA*VD*LP/(OAR*VC*2))*((1+VC/VE)**2-2*(1+VC/VE)*X*COS
     @ (ALPHA)+X**2)**0.5
        REMAX=SL*UMAX/VD
        FRE=CR1+CR2*REMAX
        PDR=FRE*N*RHOD*(VD/SL)**2*REMAX/2
C       CALCULATE THE REGENERATOR PRESSURE DROP COEFFICIENT
        CDP1=CR1*N*TR*VD**2*AP*LP*OMEGA*SIN(ALPHA)
        CDP1=CDP1/((TR+1)*R*TC*VC*AR*OAR*SL)
        CDP2=CR2*N*TR*(VD*AP*LP)**2*OMEGA**2*SIN(ALPHA)**2
        CDP2=CDP2/((TR+1)*R*TC*(VC*AR*OAR)**2)
        CDP=CDP1+CDP2
C       CALCULATE THE COEFFICIENT OF PERFORMANCE OF THE STIRLING
C       CRYOCOOLER
        COP=(LAMBDA*SIN(B))/((1+VC/VE)*((LAMBDA*COS(B)+CDP/2)*
     @ TAN(ALPHA)-LAMBDA*SIN(B)))
        RETURN
        END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE TWOPF                           *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE TOTAL PESSURE DROP             *
C*     (FRICTIONAL AND MOMENTUM) FOR TWO PHASE FLOW IN THE           *
C*     TRANSFER LINES. THE LOCKHART-MARTINELLI METHOD IS USED.       *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE TWOPF(PI,RP1,RP,L,G,MFR,QLP,NUM,PRESSDROP,XE)
      REAL*8 AP,C,CG,CL,CX1,CX2,DPF,DPM,DPO,DX,F,FRIC,G,HF,HFG,L
      REAL*8 MFR,MFRG,MFRL,MID1,MID2,MUG,MUL,NUM,PHIL1,PHIL2
      REAL*8 PHIM,PI,PRESSDROP,QLP,REG,REL,RHOG,RHOL,RL1,RL2,RP,RP1
      REAL*8 SM,SN,SUM,V,X,XE,XI
C     PROPERTIES OF NITROGEN AT SATURATION TEMPERATURE
      RHOL=807.3
      RHOG=4.614
      MUL=158E-06
      MUG=54.17E-07
      HFG=199.3E03
C     CALCULATE THE CROSS-SECTIONAL AREA OF THE PIPE
      AP=(RP1*2)**2*PI/4
C     CALL SUBROUTINE FRICTION TO CALCULATE THE FRICTION FACTOR
      CALL FRICTION(MFR,PI,RP1,RP,MUL,F)
C     CALCULATE THE VELOCITY
      V=MFR/(RHOL*PI*RP1**2)
C     CALCULATE THE FRICTION HEAD
      HF=F*L*V**2/((RP1*2)*2*G)
C     CALCULATE THE PRESSURE DROP IF LIQUID ALONE WERE FLOWING AT
C     THE TOTAL MASS FLOW RATE
      DPO=RHOL*G*HF
C     CALCULATE THE APPROXIMATE HEAT ADDED PER LINE SEGMENT
      QLP=QLP/NUM
C     INITIALIZE VARIABLES
      FRIC=0
      XI=1.0E-06
C     CREATE A DO LOOP TO CALCULATE THE FRICTIONAL PRESSURE DROP
C     FOR EACH LINE SEGMENT
      DO 70 I=1,NUM
C     CALCULATE THE EXIT QUALITY
         XE=XI+QLP/(MFR*HFG)
C     CALCULATE THE MASS FLOW RATES OF THE LIQUID AND GAS COMPONENTS
         MFRL=MFR*(1-XE)
         MFRG=MFR*XE
C     CALCULATE THE REYNOLDS NUMBER OF THE LIQUID AND GAS COMPONENTS
         REL=4*MFRL/(PI*(RP1*2)*MUL)
         REG=4*MFRG/(PI*(RP1*2)*MUG)
C     OBTAIN VALUES OF CONSTANTS C,CG,CL,SM,AND SN
         IF ((REL .LT. 2000) .AND. (REG .LT. 2000)) C=5.0
         IF ((REL .LT. 2000) .AND. (REG .GT. 2000)) C=12.0
         IF ((REL .GT. 2000) .AND. (REG .LT. 2000)) C=10.0
         IF ((REL .GT. 2000) .AND. (REG .GT. 2000)) C=20.0
         IF (REL .LT. 2000) THEN
```

```
            SN=1.0
            CL=64.0
          ENDIF
          IF ((REL .GT. 2000) .AND. (REL .LT. 50000)) THEN
            SN=0.25
            CL=0.316
          ENDIF
          IF (REL .GT. 50000) THEN
            SN=0.2
            CL=0.184
          ENDIF
          IF (REG .LT. 2000) THEN
            SM=1.0
            CG=64.0
          ENDIF
          IF ((REG .GT. 2000) .AND. (REG .LT. 50000)) THEN
            SM=0.25
            CG=0.316
          ENDIF
          IF (REG .GT. 50000) THEN
            SM=0.2
            CG=0.184
          ENDIF
          DX=(XE-XI)/10
          SUM=0
C       USE NUMERICAL INTEGRATION TECHNIQUE TO CALCULATE INTEGRAL (SUM)
          DO 60 X=XI,XE,DX
            CX1=(CL*REG**SM*RHOG*(1-X)**2/(CG*REL**SN*RHOL*X
     @      **2))**0.5
            PHIL1=((CX1**2+C*CX1+1)**0.5)/CX1
            MID1=(1-X)**(2-SN)
            CX2=(CL*REG**SM*RHOG*(1-X-DX)**2/(CG*REL**SN*RHOL*
     @      (X+DX)**2))**0.5
            PHIL2=((CX2**2+C*CX2+1)**0.5)/CX2
            MID2=(1-X-DX)**(2-SN)
            SUM=SUM+0.5*(MID1*PHIL1**2+MID2*PHIL2**2)*DX
60        CONTINUE
          FRIC=FRIC+SUM
C         SET EXIT QUALITY EQUAL TO INLET QUALITY FOR NEXT SEGMENT
          XI=XE
70      CONTINUE
C       RESET INITIAL QUALITY
        XI=1.0E-06
C       CALCULATE THE FRICTIONAL PRESSURE DROP (DPF)
        DPF=DPO*FRIC/(XE-XI)
C       CALCULATE THE MOMENTUM PRESSURE DROP (DPM) FOR DIABATIC TWO
C       PHASE FLOW
        CX1=(CL*REG**SM*RHOG*(1-XI)**2/(CG*REL**SN*RHOL*XI**2))**0.5
        CX2=(CL*REG**SM*RHOG*(1-XE)**2/(CG*REL**SN*RHOL*XE**2))**0.5
        RL1=CX1/(CX1**2+C*CX1+1)**0.5
        RL2=CX2/(CX2**2+C*CX2+1)**0.5
        PHIM=(1-XE)**2/RL2-(1-XI)**2/RL1+(XE**2/(1-RL2)-XI**2/(1-RL1
     @ ))*(RHOL/RHOG)
        DPM=(PHIM*(MFRL+MFRG)**2)/(RHOL*AP**2)
```

```
C       CALCULATE THE TOTAL PRESSURE DROP FOR TWO PHASE FLOW
        PRESSDROP=DPF+DPM
        RETURN
        END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE VACUUM1                         *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE HEAT ADDED TO THE CRYOGENIC    *
C*     FLUID THROUGH THE TRANSFER LINES IF VACUUM INSULATION IS      *
C*     USED.                                                         *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE VACUUM1(PI,KP,H,RP1,RP2,RP3,L,EP,SIGMA,TA,TF,NSP,
     @ ES,FLUID,G,MFR,MID)
      REAL*8 A1L,A2L,ES,EP,FL,FLUID,G,H,KP,L,MFR,MID,NSP,PI,PRT1
      REAL*8 PRT2,PRT3,PRT4,PRTOT,RADHT,RP1,RP2,RP3,SIGMA,TA,TF
C     CALL PRESIST TO CALCULATE THE THERMAL RESISTANCES ASSOCIATED
C     WITH CONVECTION AND CONDUCTION THROUGH THE PIPE
      CALL PRESIST(RP1,RP2,RP3,TF,PI,L,KP,H,FLUID,G,MFR,PRT1,PRT2,
     @ PRT3)
C     CALCULATE THE SURFACE AREAS OF THE PIPE AND SHELL
      A1L=2*PI*RP2*L
      A2L=2*PI*RP3*L
C     CALCULATE THE EMISSIVITY FACTOR
      FL=1/(1/EP+A1L/A2L*(1/EP-1))
      IF (NSP .GT. 0) THEN
         CALL SHIELD(A1L,A2L,RP2,RP3,NSP,PI,L,EP,ES,FL)
      ENDIF
C     CALCULATE THE HEAT ADDED THROUGH THE LINE SEGMENT DUE TO RADIATION
C     EXCHANGE
      RADHT=FL*SIGMA*A1L*(TA**4-TF**4)
C     CALCULATE THE THERMAL RESISTANCE ASSOCIATED WITH THE VACUUM
C     INSULATION
      PRT4=(TA-TF)/RADHT
C     CALCULATE THE TOTAL THERMAL RESISTANCE
      PRTOT=PRT1+PRT2+PRT3+PRT4
C     CALCULATE THE TOTAL HEAT ADDED THROUGH THE LINE SEGMENT
      MID=(TA-TF)/PRTOT
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                        SUBROUTINE VACUUM2                         *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE HEAT ADDED TO THE CRYOGENIC    *
C*     FLUID THROUGH THE CPU VESSEL IF VACUUM INSULATION OF          *
C*     SPECIFIED THICKNESS IS USED.                                  *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE VACUUM2(PI,KV,H,RV1,RV2,HV,RV3,EV,SIGMA,TA,TF,
     @ NSV,ES,QLV)
      REAL*8 A1V,A2V,EV,ES,FV,G,H,HV,KV,NSV,PI,QLV,QLV1,RV1,RV2
      REAL*8 RV3,SIGMA,TA,TF,VRT1,VRT2,VRT3,VRT4,VRTOT
      G=9.81
C     CALL VRESIST TO CALCULATE THE THERMAL RESISTANCES ASSOCIATED
C     WITH CONVECTION AND CONDUCTION THROUGH THE VESSEL
      CALL VRESIST(G,TA,TF,RV1,RV2,RV3,PI,HV,KV,H,VRT1,
     @ VRT2,VRT3)
C     CALCULATE THE SURFACE AREAS OF THE VESSEL AND SHELL
      A1V=2*PI*RV2*HV
      A2V=2*PI*RV3*HV
C     CALCULATE THE EMISSIVITY FACTOR
      FV=1/(1/EV+A1V/A2V*(1/EV-1))
      IF (NSV .GT. 0) THEN
         CALL SHIELD(A1V,A2V,RV2,RV3,NSV,PI,HV,EV,ES,FV)
      ENDIF
C     CALCULATE THE HEAT ADDED THROUGH THE CPU VESSEL
      QLV1=FV*SIGMA*A1V*(TA**4-TF**4)
C     CALCULATE THE THERMAL RESISTANCE ASSOCIATED WITH THE VACUUM
C     INSULATION
      VRT4=(TA-TF)/QLV1
C     CALCULATE THE TOTAL THERMAL RESISTANCE
      VRTOT=VRT1+VRT2+VRT3+VRT4
C     CALCULATE THE TOTAL HEAT ADDED THROUGH THE CPU VESSEL
      QLV=(TA-TF)/VRTOT
      RETURN
      END
```

```
C*********************************************************************
C*                                                                   *
C*                          SUBROUTINE VRESIST                       *
C*                                                                   *
C*                                                                   *
C*     THIS SUBROUTINE CALCULATES THE THERMAL RESISTANCES DUE TO     *
C*     CONDUCTION THROUGH THE VESSEL WALL AND EXTERNAL CONVECTION.   *
C*                                                                   *
C*                                                                   *
C*********************************************************************
C
      SUBROUTINE VRESIST(G,TA,TF,RV1,RV2,RV3,PI,HV,KV,H,VRT1,
     @ VRT2,VRT3)
      REAL*8 G,H,HV,KV,PI,RV1,RV2,RV3,TA,TF,VRT1,VRT2,VRT3
C     CALCULATE THE THERMAL RESISTANCE OF THE VESSEL
      VRT1=LOG(RV2/RV1)/(2*PI*HV*KV)
C     CALCULATE THE THERMAL RESISTANCE DUE TO EXTERNAL CONVECTION
      VRT2=1/(2*PI*RV3*HV*H)
C     CALCULATE THE THERMAL RESISTANCE DUE TO INTERNAL FREE
C     CONVECTION OVER THE CPU VESSEL
      CALL ICVESSEL(G,TA,TF,HV,RV1,VRT3)
      RETURN
      END
```

APPENDIX B

Property Values

Table B-1. Enthalpy Values of Liquid Nitrogen

T (K)	h_f (kJ/kg)
63.143	-150.348
65	-146.691
70	-136.569
75	-126.287
77.347	-121.433

Source: Cryogenic Systems, Barron, 1985.

Table B-2. Viscosity and Density of Liquid Nitrogen

T (K)	μ (μPa-s)	ρ (kg/m^3)
65	278	860.9
70	220	840.0
75	173	818.1
77.36	158	807.3

Source : Cryogenic Systems, Barron, 1985.

Table B-3. Thermal Conductivity and Prandtl Number of Liquid Nitrogen

T (K)	k (mW/m-K)	Pr
65	158.7	3.52
70	149.9	2.97
75	143.0	2.47
77.36	139.6	2.32

Source : Cryogenic Systems, Barron, 1985.

Table B-4. Viscosity and Density of Gaseous Helium

T (K)	μ (μPa-s)	ρ (kg/m^3)
40	5.275	1.218
50	6.093	0.974
75	7.884	0.641
100	9.470	0.487
125	10.47	0.390
150	12.37	0.325
175	13.69	0.278
200	14.96	0.244
225	16.19	0.217
250	17.40	0.195
275	18.63	0.177
300	19.87	0.163

Source : Cryogenic Systems, Barron, 1985.

Table B-5. Thermal Conductivity and Prandtl Number of Gaseous Helium

T (K)	k (mW/m-K)	Pr
40	40.9	0.627
50	47.4	0.669
75	62.4	0.657
100	73.3	0.672
125	84.4	0.676
150	95.1	0.676
175	105.1	0.675
200	115.4	0.674
225	124.9	0.674
250	133.9	0.675
275	142.5	0.680
300	150.7	0.686

Source : Cryogenic Systems, Barron, 1985.

APPENDIX C

Computer Output

ONCE-THROUGH SYSTEM

INNER PIPE DIAMETER = 17.900 mm
REQUIRED MASS FLOWRATE OF LIQUID NITROGEN = .5270E-02 kg/s
RESERVOIR VOLUME = 4333.48 liters
INLET LIQUID NITROGEN TEMPERATURE IS 77 K
EXIT QUALITY IS 3.26 %
TWO-PHASE PRESSURE DROP IN THE LIQUID NITROGEN LINES
= 0.1590E-01 psi

HEAT ADDITIONS TO THE LIQUID NITROGEN:

HEAT ADDED THROUGH LIQUID NITROGEN LINES = 34.22 W
HEAT ADDED THROUGH CPU VESSEL = 10.12 W
HEAT ADDED THROUGH LEADS= 6.23 W
TOTAL HEAT ADDED TO THE LIQUID NITROGEN = 50.58 W

ONCE-THROUGH SYSTEM COST:

THE COST OF THE RESERVOIR = $ 22753.89
THE COST OF THE PIPING = $ 9628.24
THE COST OF THE PUMP = $ 5000.00
THE COST OF THE CPU VESSEL = $ 3766.68
THE INITIAL COST OF A ONCE-THROUGH SYSTEM = $ 41148.81
THE ANNUAL COST OF A ONCE-THROUGH SYSTEM = $ 20541.84
THE UNIFORM ANNUAL COST = $ 28778.69

REFRIGERATED SYSTEM

SATURATED POOL BOILING

REFRIGERATOR:

REFRIGERATION TEMPERATURE = 60.0 K
REFRIGERATION CAPACITY = 1232.05 W
COEFFICIENT OF PERFORMANCE = .6307E-01
WORK INPUT TO THE REFRIGERATOR = 19534.64 W

CONDENSER:

INNER PIPE DIAMETER = 56.900 mm
INLET TEMPERATURE = 62.34 K
OUTLET TEMPERATURE = 72.34 K
HEAT TRANSFER = 1016.36 W
INTERNAL CONVECTION COEFFICIENT = 134.73 W/m^2-K
EXTERNAL CONVECTION COEFFICIENT = 1846.40 W/m^2-K
LENGTH OF PIPE = 5.50 m
SURFACE AREA = 1.01 m^2
PRESSURE DROP = .1260E-01 psi

HEAT EXCHANGER:

RADIAL FINS: THICKNESS = 1.0 mm, LENGTH = 29.3 mm
INNER PIPE DIAMETER = 56.900 mm
INLET TEMPERATURE = 73.12 K
OUTLET TEMPERATURE = 61.00 K
HEAT TRANSFER = 1232.05 W
INTERNAL CONVECTION COEFFICIENT = 134.67 W/m^2-K
EXTERNAL CONVECTION COEFFICIENT = 16.74 W/m^2-K
LENGTH OF PIPE = 15.62 m
SURFACE AREA = 66.80 m^2
PRESSURE DROP = .3562E-01 psi

HELIUM LOOP

INNER PIPE DIAMETER = 56.900 mm
MASS FLOW RATE OF GASEOUS HELIUM = .1955E-01 kg/s
TOTAL PRESSURE DROP IN GASEOUS HELIUM LOOP = .1742E+00 psi
PUMP WORK REQUIRED = 57.14 W

REFRIGERATED SYSTEM COST:

THE INITIAL COST OF THE REFRIGERATED SYSTEM = $ 102424.36
THE PIPING COST = $ 46714.26
THE RESERVOIR COST = $ 6338.35
THE RESERVOIR SIZE = 0.55 m^3
THE CRYOCOOLER COST = $ 30352.02
THE CONDENSER COST = $ 59.23

```
THE HEAT EXCHANGER COST = $  960.50
THE ANNUAL LIQUID NITROGEN COST = $ 665.62
THE PUMP COST = $ 18000.00
THE POWER COST OF THE REFRIGERATED SYSTEM = $  16041.16
   PER YEAR
THE UNIFORM ANNUAL COST = $  36535.04
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REFRIGERATED SYSTEM

SUBCOOLED POOL BOILING
SUBCOOLED TEMPERATURE = 68.0 K

REFRIGERATOR:

REFRIGERATION TEMPERATURE = 60.0 K
REFRIGERATION CAPACITY = 1232.07 W
COEFFICIENT OF PERFORMANCE = .6307E-01
WORK INPUT TO THE REFRIGERATOR = 19534.91 W

HEAT EXCHANGER I:

RADIAL FINS: THICKNESS = 1.0 mm, LENGTH = 29.3 mm
INNER PIPE DIAMETER = 56.900 mm
INLET TEMPERATURE = 62.34 K
OUTLET TEMPERATURE = 72.34 K
HEAT TRANSFER = 1016.63 W
INTERNAL CONVECTION COEFFICIENT = 134.76 W/m^2-K
EXTERNAL CONVECTION COEFFICIENT = 234.52 W/m^2-K
LENGTH OF PIPE = 5.05 m
SURFACE AREA = 21.60 m^2
PRESSURE DROP = .1159E-01 psi

HEAT EXCHANGER II:

RADIAL FINS: THICKNESS = 1.0 mm, LENGTH = 29.3 mm
INNER PIPE DIAMETER = 56.900 mm
INLET TEMPERATURE = 73.12 K
OUTLET TEMPERATURE = 61.00 K
HEAT TRANSFER = 1232.07 W
INTERNAL CONVECTION COEFFICIENT = 134.70 W/m^2-K
EXTERNAL CONVECTION COEFFICIENT = 16.74 W/m^2-K
LENGTH OF PIPE = 15.63 m
SURFACE AREA = 66.80 m^2
PRESSURE DROP = .3564E-01 psi

HELIUM LOOP

INNER PIPE DIAMETER = 56.900 mm
MASS FLOW RATE OF GASEOUS HELIUM = .1955E-01 kg/s
TOTAL PRESSURE DROP IN GASEOUS HELIUM LOOP = .1732E+00 psi
PUMP WORK REQUIRED = 56.85 W

REFRIGERATED SYSTEM COST:

THE INITIAL COST OF THE REFRIGERATED SYSTEM = $ 102038.12
THE PIPING COST = $ 46714.26
THE RESERVOIR COST = $ 5557.77
THE RESERVOIR SIZE = 0.48 m^3

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THE CRYOCOOLER COST = $ 30352.24
THE CONDENSER COST = $  453.34
THE HEAT EXCHANGER COST = $  960.51
THE ANNUAL LIQUID NITROGEN COST = $ 580.91
THE PUMP COST = $ 18000.00
THE POWER COST OF THE REFRIGERATED SYSTEM = $  15988.08
   PER YEAR
THE UNIFORM ANNUAL COST = $  36404.67
THE COST OF SUBCOOLING THE LIQUID NITROGEN = $  31.64 PER
   YEAR
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REFRIGERATED SYSTEM OPTIMIZATION

NUMBER OF VARIABLES IS    10

NUMBER OF FUNCTIONS IS     2

SPACE RESERVED FOR HESSIAN HAS DIMENSION    10

LIMIT ON BINDING CONSTRAINTS IS     1

ACTUAL LENGTH OF Z ARRAY IS    272
ROWS
 R    1   1 0.000000E-01 7.700000E+01
END
BOUNDS
 R    1   1 5.800000E+01 6.300000E+01
 R    2   2 1.000000E+00 2.000000E+00
 R    3   3 4.000000E-05 5.080000E-04
 R    4   4 4.400000E-05 2.032000E-03
 R    5   5 8.950000E-03 2.845000E-02
 R    6   6 3.000000E+00 1.000000E+01
 R    7   7 1.790000E-02 5.690000E-02
 R    8   8 1.790000E-02 5.690000E-02
 R    9   9 5.000000E-04 1.000000E-03
 R   10  10 8.950000E-03 2.845000E-02
END
INITIAL
TOGETHER
  6.3000000E+01  1.0000000E+00  5.0800000E-04  2.0320000E-03
  2.8450000E-02  1.0000000E+01  5.6900000E-02  5.6900000E-02
  1.0000000E-03  8.9500000E-03
PRINT
IPR     0
END
GO

EPNEWT =  1.0000E-04  EPINIT =  1.0000E-04  EPSTOP =  1.0000E-04
 EPPIV =  1.0000E-03  PH1EPS = 0.00000E-01

NSTOP =    3  ITLIM =   10  LIMSER =     10000

IPR =   0  PN4 =   0  PN5 =   0  PN6 =   0  PER =   0  DUMP =   0

TANGENT VECTORS WILL BE USED FOR INITIAL ESTIMATES OF BASIC VARIABLES

THE FINITE DIFFERENCE PARSH USING FORWARD DIFFERENCES WILL BE USED

OBJECTIVE FUNCTION WILL BE MINIMIZED.

DFP WILL BE USED IF # SUPERBASICCS < = 10
```

OUTPUT OF INITIAL VALUES

REFRIGERATED SYSTEM OPTIMIZATION

SECTION 1 -- FUNCTIONS

FUNCTION NO.	NAME	STATUS	TYPE	INITIAL VALUE	LOWER LIMIT	UPPER LIMIT
1			RNGE	7.5442824E+01	0.0000000E-01	7.7000000E+01
2			OBJ	3.5612581E+04		

SECTION 2 -- VARIABLES

VARIABLE NO.	NAME	STATUS	INITIAL VALUE	LOWER LIMIT	UPPER LIMIT
1		UL	6.3000000E+01	5.8000000E+01	6.3000000E+01
2		LL	1.0000000E+00	1.0000000E+00	2.0000000E+00
3		UL	5.0800000E-04	4.0000000E-05	5.0800000E-04
4		UL	2.0320000E-03	4.4000000E-05	2.0320000E-03
5		UL	2.8450000E-02	8.9500000E-03	2.8450000E-02
6		UL	1.0000000E+01	3.0000000E+00	1.0000000E+01
7		UL	5.6900000E-02	1.7900000E-02	5.6900000E-02
8		UL	5.6900000E-02	1.7900000E-02	5.6900000E-02
9		UL	1.0000000E-03	5.0000000E-04	1.0000000E-03
10		LL	8.9500000E-03	8.9500000E-03	2.8450000E-02

FINAL RESULTS

REFRIGERATED SYSTEM OPTIMIZATION

SECTION 1 -- FUNCTIONS

NO.	NAME	INITIAL VALUE	FINAL VALUE	STATUS	DISTANCE FROM NEAREST BOUND	LAGRANGE MULTIPLIER
1		7.54428E+01	7.54428E+01	FREE	1.557E+00:U	
2		3.56126E+04	3.56126E+04	OBJ		

SECTION 2 -- VARIABLES

NO.	NAME	INITIAL VALUE	FINAL VALUE	STATUS	DISTANCE FROM NEAREST BOUND	REDUCED GRADIENT
1		6.30000E+01	6.30000E+01	NONBASIC	UPPERBND	-2.71632E+02
2		1.00000E+00	1.00000E+00	NONBASIC	LOWERBND	5.81772E+01
3		5.08000E-04	5.08000E-04	NONBASIC	UPPERBND	-3.77573E+04
4		2.03200E-03	2.03200E-03	NONBASIC	UPPERBND	-2.60641E+04
5		2.84500E-02	2.84500E-02	NONBASIC	UPPERBND	-1.15055E+05
6		1.00000E+01	1.00000E+01	NONBASIC	UPPERBND	-4.67110E+02
7		5.69000E-02	5.69000E-02	NONBASIC	UPPERBND	-9.47858E+03
8		5.69000E-02	5.69000E-02	NONBASIC	UPPERBND	-1.32678E+04
9		1.00000E-03	1.00000E-03	NONBASIC	UPPERBND	-5.12663E+04
10		8.95000E-03	8.95000E-03	NONBASIC	LOWERBND	1.45517E+04

VITA

James C. Anderson was born in Knoxville, Tennessee on February 1, 1966. He attended elementary schools in that city and graduated from Powell High School in June 1984. The following September he entered The University of Tennessee, Knoxville, and in May 1988 he received a Bachelor of Science degree in Mechanical Engineeering. In the summer of 1988 he accepted a research assistantship at The University of Tennessee, Knoxville and began study toward a Master's degree. He received the Master's degree with a major in Mechanical Engineering in December 1990.

The author is a member of Tau Beta Pi, Pi Tau Sigma, and the American Society of Mechanical Engineers. Mr. Anderson is currently employed by Martin Marietta Energy Systems in Oak Ridge, Tennessee.