

To the Graduate Council:

I am submitting herewith a thesis written by Purnachandra Rao Surapaneni entitled "Numerical Boundary Integral Analysis of Heat Transfer From Circular Tubes in a Semi-Infinite Medium." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.

*Rao V. Arimilli*

Rao V. Arimilli, Major Professor

We have read this thesis  
and recommend its acceptance:

*[Signature]*  
*V. Arimilli*

*James A. Eul*

Accepted for the Council:

*C. W. Mink*

The Graduate School

NUMERICAL BOUNDARY INTEGRAL ANALYSIS OF HEAT TRANSFER  
FROM CIRCULAR TUBES IN A SEMI-INFINITE MEDIUM

A Thesis  
Presented for the  
Master of Science  
Degree  
The University of Tennessee, Knoxville

Purnachandra Rao Surapaneni

March 1985

Dedicated to  
Lord Sri Venkateswara Swamy

## ACKNOWLEDGEMENTS

The author wishes to express his sincere gratitude to Dr. Rao V. Arimilli, major professor and advisor, for his invaluable advice, inspiration, guidance, and help during the period of the author's graduate study and untiring efforts in reviewing the manuscript of this thesis. The author is also indebted to Dr. Masood Parang, his second advisor, for personal and professional advice, encouragement and careful review of this work. The author wishes to sincerely thank Dr. J. A. Euler and Dr. J. R. Parsons for serving on the thesis committee.

The author like to thank Dr. Donald R. Pitts, Head of the Department, and the Department of Mechanical and Aerospace Engineering for the partial financial support in the form of graduate assistantship and for providing the funds for the computer work associated with this thesis.

The help of my friend and fellow graduate student Mr. Peter E. Raad in dealing with certain computer-related problems is greatly appreciated.

I express my deepest gratitude to my grand parents, Haripurushottam and Lalitamba, my parents, Venkata Kuchela Rao and Annapurna Devi, my uncle and aunty, Subba Rao and Komalatha Devi, and my brother, Gangadhara Rao, for their continuous encouragement, love and support.

My deepest admiration and love to my fiancée, Sailaja, for her spiritual support and understanding. She makes all of this worthwhile.

## ABSTRACT

A two-dimensional analysis is presented for the steady heat transfer from a set of parallel isothermal tubes of arbitrary diameters and positions located in a semi-infinite medium. The plane surface is exposed to uniform convective environment. The analysis uses a special form of boundary integral equations, developed specially for circular interior boundaries, for the solution of two-dimensional Laplace's equation. Three computer programs are developed for the solution of the stated problem in its general form, the problem with two tubes, and the problem with two tubes with symmetry conditions. Criteria used for the selection of segment length, plane convective surface length, and the location of the axes are discussed. For illustration purposes, several problems of interest are solved. The results are presented and discussed. Also, the temperature distribution on the plane convective surface is presented.

# TABLE OF CONTENTS

| CHAPTER                                                                                                                     | PAGE |
|-----------------------------------------------------------------------------------------------------------------------------|------|
| I. INTRODUCTION .....                                                                                                       | 1    |
| Review of Pertinent Literature .....                                                                                        | 2    |
| Outline of the Work to be Presented .....                                                                                   | 5    |
| II. FORMULATION OF INTEGRAL EQUATIONS .....                                                                                 | 8    |
| Levels of Approximations .....                                                                                              | 16   |
| III. BOUNDARY CONDITIONS AND NONDIMENSIONALIZATION .....                                                                    | 19   |
| IV. THE TWO-TUBE PROBLEM .....                                                                                              | 28   |
| Symmetry Considerations .....                                                                                               | 36   |
| V. THE GENERAL PROBLEM .....                                                                                                | 40   |
| VI. CRITERIA USED FOR THE SELECTION OF LOCATION OF<br>THE COORDINATE AXES, $\bar{\xi}_{\max}$ , AND $\Delta\bar{\xi}$ ..... | 47   |
| The Location of the Coordinate Axes .....                                                                                   | 47   |
| Selection of $\bar{\xi}_{\max}$ from the Results of<br>of Two-Tube Problem .....                                            | 47   |
| Selection of $\Delta\bar{\xi}$ from Results of<br>Two-Tube Problem .....                                                    | 56   |
| VII. VALIDATION OF COMPUTER PROGRAMS .....                                                                                  | 62   |
| Comparison with an Exact Solution .....                                                                                     | 64   |
| VIII. RESULTS AND DISCUSSION .....                                                                                          | 68   |
| Single-Tube Problem .....                                                                                                   | 68   |
| Two-Tube Problem .....                                                                                                      | 71   |
| Special Cases of General Problem .....                                                                                      | 83   |
| Horizontal Array of Tubes(M=1,2,...8) .....                                                                                 | 83   |
| A Special Four-Tube Problem .....                                                                                           | 88   |
| Problem of Different Arrangements of Six Tubes .....                                                                        | 92   |
| Problem of Variable Diameter and Temperature .....                                                                          | 95   |
| IX. CONCLUSIONS .....                                                                                                       | 100  |
| LIST OF REFERENCES .....                                                                                                    | 102  |

| CHAPTER          | PAGE |
|------------------|------|
| APPENDIXES ..... | 105  |
| Appendix A ..... | 106  |
| Appendix B ..... | 110  |
| Appendix C ..... | 120  |
| Appendix D ..... | 130  |
| Appendix E ..... | 144  |
| VITA .....       | 146  |

# LIST OF TABLES

| TABLE                                                                                                                                                                                                                                                                                                                                                     | PAGE |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 1. Results for One Two-Tube Problem to Illustrate the Effect of $\bar{\epsilon}_{\max}$ on the Accuracy of the Solution (BI=0.5; $\Delta\bar{\epsilon}=0.4$ ; $\bar{D}=6.0$ ; $2\bar{s}=10.0$ ) .....                                                                                                                                                     | 49   |
| 2. Results for a Second Two-Tube Problem to Illustrate the Effect of $\bar{\epsilon}_{\max}$ on the Accuracy of the Solution (BI=1.0; $\Delta\bar{\epsilon}=1.0$ ; $\bar{D}=25.0$ ; $2\bar{s}=10.0$ ) .....                                                                                                                                               | 49   |
| 3. Results for One Two-Tube Problem Showing the Influence of $\Delta\bar{\epsilon}$ on the Accuracy of the Solution (BI=0.5; $\bar{D}=18.0$ ; $2\bar{s}=8.0$ ; $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}} = 96.0$ ) .....                                                                                                                | 57   |
| 4. Results of Second Two-Tube Problem Showing the Influence of $\Delta\bar{\epsilon}$ on the Accuracy of the Solution (BI=1.0; $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}} = 164.0$ ; $\bar{D}=30.0$ ; $2\bar{s}=14.0$ ) .....                                                                                                            | 57   |
| 5. Results of the Problem of Figure 10 Showing the Effect of $\bar{\epsilon}_{\max}$ Criteria (Developed from the Two-Tube Problem) on the Accuracy of the Solution (BI=0.1; $\bar{D}=10.0$ ; $X T_{\max}=6.0$ ; $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}} = 87.5$ ) .....                                                              | 59   |
| 6. Results for the Problem of Figure 11 Showing the Effect of $\Delta\bar{\epsilon}$ Criteria (Developed from the Two-Tube Problem) on the Accuracy of the Solution (BI=0.3; $\bar{D}=30.0$ ; $X T_{\max}=7.5$ ; $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}}$ ) .....                                                                     | 60   |
| 7. Two-Tube Results of Problem Shown in Figure 4 Showing the Symmetry in the Solution ( $\bar{q}_1=\bar{q}_2$ , $\bar{q}_1=\bar{q}_2$ , $\bar{q}_1=-\bar{q}_2$ , $\phi_{k+5}=\phi_{k-6}$ ) (BI = 0.5; $2\bar{s}=10.0$ ; $\Delta\bar{\epsilon}=\bar{D}/16$ ; $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}}$ ) .....                          | 63   |
| 8. Two-Tube Results of Problem Shown in Figure 4 Illustrated to Compare the Solutions of TWOSYMM and TWOTUBE Programs (BI=0.5; $2\bar{s}=10.0$ ; $\Delta\bar{\epsilon}=\bar{D}/16$ ; $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}}$ ) .....                                                                                                 | 63   |
| 9. GENTUBE Results for the Problem in Figure 12 Showing the Symmetry in the Solution ( $\bar{q}_1=\bar{q}_2$ , $\bar{q}_3=\bar{q}_4$ , $\bar{q}_5=\bar{q}_6$ ) (BI=0.5; $\Delta\bar{\epsilon}=1.5$ ; $\phi_r=212$ F; $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}} = 186.0$ ; A(I)=1.0; $\phi_{k+8}=0.05338$ ; $\phi_{k-9}=0.05317$ ) ..... | 65   |

10. Comparison of the Results of Present Method with the Results Obtained from the Exact Solution of a Single Tube Problem ( $Bi=10$ ;  $X_{T_{max}}=10.0$ ;  $a=1.0$ ;  $\bar{\epsilon}_{max} = 11.3 * \sqrt{X_{T_{max}} * D}$  ) ..... 67
11. Results for  $\bar{q}_0$ ,  $\bar{q}_1$ , and  $\bar{q}_2$  for Various Tubes in Configurations Shown in Fig. 30a and Fig. 30b ..... 93
12. GENTUBE Results for  $\bar{q}_0$ ,  $\bar{q}_1$ , and  $\bar{q}_2$  for Various Tubes in the Problem of Figure 33. Results Indicate Solution Symmetry ( $\bar{q}_1=\bar{q}_2$ ,  $\bar{q}_3=\bar{q}_4$ ,  $\bar{q}_5=\bar{q}_6$ ,  $\bar{q}_7=\bar{q}_8$ , etc.). (Reference Tube no. : 7; Reference Tube Temp. =425F; Reference Tube Radius = 3.0) ..... 98

# LIST OF FIGURES

| FIGURE                                                                                                                                                                                                                                   | PAGE |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 1. The Schematic of Two-Dimensional Circular Tubes in a Semi-Infinite Medium. The Dimensional Form .....                                                                                                                                 | 9    |
| 2. Two-Dimensional Region with Circular Tubes: Geometry and Notation(from Reference 23) .....                                                                                                                                            | 10   |
| 3. The Schematic of Two-Dimensional Circular Tubes in a Semi-Infinite Medium. The Dimensional Form .....                                                                                                                                 | 20   |
| 4. The Schematic of Two Tubes in a Semi-Infinite Medium .....                                                                                                                                                                            | 29   |
| 5. A Sketch of the Two-Dimensional Problem and Boundary Conditions in Nondimensional Form .....                                                                                                                                          | 41   |
| 6. Temperature Distribution on the Plane Convective Surface Using $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * D}$ Criterion for Two Tubes in a Semi-Infinite Medium. TWOTUBE Results.(TWOSYMM Yielded Identical Results) .....    | 52   |
| 7. Temperature Distribution on the Plane Convective Surface Using $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * D}$ Criterion for Two Tubes in a Semi-Infinite Medium. TWOSYMM Results .....                                        | 53   |
| 8. Temperature Distribution as a Function of $\bar{\xi}$ and $\bar{\epsilon}_{\max}$ (Range of $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * D}$ to $20 \bar{D}$ ) for Two Tubes in a Semi-Infinite Medium. TWOSYMM Results ...     | 54   |
| 9. Temperature Distribution as a Function of $\bar{\xi}$ and $\bar{\epsilon}_{\max}$ (Range of $\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * D}$ to $20 \bar{D}$ ) for Two Tubes at a Shallow Depth in a Semi-Infinite Medium ..... | 55   |
| 10. Sketch of the Problem Considered to Illustrate the Effect of $\bar{\epsilon}_{\max}$ Criterion on the Accuracy of the GENTUBE Solution .....                                                                                         | 58   |
| 11. Sketch of the Problem Considered to Illustrate the Effect of $\Delta \bar{\xi}$ Criterion on the Accuracy of the GENTUBE Solution .....                                                                                              | 58   |
| 12. Sketch of the Problem Considered to Validate the GENTUBE Computer Program .....                                                                                                                                                      | 65   |
| 13. Variation of $\bar{q}_0$ with Nondimensional Depth at Different Biot Numbers for a Single Tube in a Semi-Infinite Medium .....                                                                                                       | 69   |

|     |                                                                                                                                                  |    |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 14. | Variation of $\bar{q}_1$ with Nondimensional Depth at Different Biot Numbers for a Single Tube in a Semi-Infinite Medium .....                   | 70 |
| 15. | Variation of Temperature Along the Plane Convective Surface at Different Tube Depths for a Single Tube in a Semi-Infinite Medium .....           | 72 |
| 16. | Variation of Temperature Along the Plane Convective Surface at Different Biot Numbers for a Single Tube in a Semi-Infinite Medium .....          | 73 |
| 17. | Variation of Total Value of $\bar{q}_0$ with Nondimensional Depth at Different Biot Numbers for Two Tubes .....                                  | 74 |
| 18. | Variation of Total Value of $\bar{q}_1$ with Nondimensional Depth at Different Biot Numbers for Two Tubes .....                                  | 75 |
| 19. | Variation of value of $\bar{q}_2^1$ with Nondimensional Depth at Different Biot Numbers for Two Tubes ( $\bar{q}_2^1 = -\bar{q}_2^2$ ) .....     | 76 |
| 20. | Variation of Total Value of $\bar{q}_0$ with Nondimensional Depth for Different Tube Spacings of Two Tubes .....                                 | 78 |
| 21. | Variation of Total Value of $\bar{q}_1$ with Nondimensional Depth for Different Tube Spacings of Two Tubes .....                                 | 79 |
| 22. | Variation of Value of $\bar{q}_2$ with Nondimensional depth at Different Tube Spacings for Two Tubes (Note: $\bar{q}_2^1 = -\bar{q}_2^2$ ) ..... | 80 |
| 23. | Variation of Temperature Along the Plane Convective Surface for Different Tube Depths for Two Tubes .....                                        | 81 |
| 24. | Variation of Temperature Along the Plane Convective Surface at Different Biot Numbers for Two Tubes .....                                        | 82 |
| 25. | Variation of Normalized Total Value of $\bar{q}_0$ with Nondimensional Depth for M Tubes Shown in Figures 31-a ( $Bi=0.5, 2\bar{s}=5.0$ ) .....  | 84 |
| 26. | Variation of Total Value of $\bar{q}_1$ with Nondimensional Depth for M Tubes Shown in Figure 31-a ( $Bi=0.5, 2\bar{s}=5.0$ ) .....              | 85 |
| 27. | Variation of Total Value of Positive $\bar{q}_2$ with Nondimensional Depth for M Tubes Shown in Figure 31-a ( $Bi=0.5, 2\bar{s}=5.0$ ) .....     | 86 |

| Figure                                                                                                                                                                                                                                                                                              | Page |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 28. Variation of Normalized Total Value of $\bar{q}_0$ with Nondimensional Depth for M Tubes Shown in Figures 31-a ( $Bi=0.05, 2\bar{s}=5.0$ ) .....                                                                                                                                                | 89   |
| 29. Variation of Total Value of $\bar{q}_1$ with Nondimensional Depth for M Tubes Shown in Figure 31-a ( $Bi=0.05, 2\bar{s}=5.0$ ) .....                                                                                                                                                            | 90   |
| 30. Sketch of the Problems Considered (for Comparative Evaluation of $\bar{q}_0, \bar{q}_1$ , and $\bar{q}_2$ for Two Tube Arrangements) are Shown in (a) with $2\bar{s}=7.5$ , and $a_1=a_2=a_3=a_4=a_{\max}=A$ and in (b) with $2\bar{s}=5$ , $a_7=a_8=a_{\max}$ , and $a_5=a_6=a_{\max}/3$ ..... | 91   |
| 31. Sketch of Problem Considered for Comparative Evaluation of $\bar{q}_0$ for Two Tube Arrangements Shown in (a) and (b) ( $Bi=0.5, 2\bar{s}=5.0$ ) .....                                                                                                                                          | 94   |
| 32. Variation of Total $\bar{q}_0$ with Nondimensional Depth for Configurations (a) and (b) Shown in Figure 31 ( $Bi=0.5, 2\bar{s}=5.0$ ) .....                                                                                                                                                     | 96   |
| 33. Sketch of a Symmetrical Problem Involving Tubes at Arbitrary Isothermal Temperatures .....                                                                                                                                                                                                      | 97   |
| A-1. Sketch for the Evaluation of Logarithmic Integrals .....                                                                                                                                                                                                                                       | 107  |

# LIST OF SYMBOLS

$a$  = outer radius of a tube

$A$  = radius of the reference tube

$b_{\alpha i} = a_{\alpha} / \gamma_{\alpha i}$

$b_{\alpha \beta} = \varepsilon_{\alpha} / \gamma_{\alpha \beta}$

$Bi = ha/k$

$c_j$  =  $j$  th segment of discretized boundary

$D$  = normal distance of center of a tube from convective plane surface

$g(X,Y)$  = harmonic function defined in the text

$g_0^{\beta}(X), g_1^{\beta}(X), g_2^{\beta}(X)$  = Special kernel functions defined in equation (6);  
same as reference[23]

$h$  = convective heat transfer coefficient

$k$  = coefficient of thermal conductivity

$K$  and  $L$  - denote order of the series expansion in Equations

$M$  = number of tubes in the semi-infinite medium

$n$  = unit normal

$q$  = normal derivative of temperature,  $\frac{\partial \phi}{\partial \eta}$

$Q_0^{\alpha}$  = dimensional heat transfer at tube surface per unit length  
of the tube

$\bar{q}_0^{\alpha} = \frac{\partial \phi}{\partial \eta}$  at hole  $\alpha$  = dimensionless heat transfer  

$$= \frac{-Q_0^{\alpha} A}{2\pi a_{\alpha} k \phi_r}$$

$T$  = temperature

$T_a$  = ambient temperature at the plane convecting surface

= Far-field temperature in the semi-infinite medium.

$T_t$  = tube temperature

$X, Y$  - position vectors of points on boundary

$XT_{\max}$  - horizontal distance of the farthest located tube  
used for criterion

### Greek Symbols

$\alpha, \beta$  - tube identification numbers; used as subscripts and superscripts

$\gamma$  = lengths defined in Figure 2

$\xi, \eta$  - coordinate axes

$\xi^\beta$  = position vector of center of tube

$\theta$  = azimuthal angle

$\phi$  = temperature difference =  $T - T_a$

$\Phi$  = dimensionless temperature defined in the text

$\phi_r$  = surface temperature of the reference tube

$\psi$  - angles defined in Figure 2

### Subscripts

0 - denotes zeroth-order term of the series expansion at tube surface

1, 2 - denote coefficients of sine and cosine terms, respectively, of the series expansion at the tube surface

## CHAPTER I

### INTRODUCTION

There are many problems of engineering interest which require the solution of Laplace's equation in the regions with interior circular boundaries. Some of the well known examples are flow of an inviscid incompressible fluid around circular cylinders, heat conduction in dies or moulds with cylindrical heating or cooling lines, buried heat exchanger tubes of ground-coupled heat pumps, and elastic torsion in a bar with circular holes. To be specific, the use of buried utility lines for mass or energy transport is commonly encountered in various applications such as heat transfer from ground-coupled heat exchanger tubes[1,2], buried electric power cables[3], and buried pipe lines for the transportation of fluids involving thermal effects. For example, the buried heat exchangers of ground-coupled heat pump systems use ground advantageously as a superior heat source or sink relative to ambient air.

The use of integral methods for solving these problems have been a subject of interest for many investigators for several years. The fundamental method used in the potential theory for solving the integral equation associated with Laplace's equation is the use of Green's function[4]. In earlier times, the application of this method was confined to those problems involving simple geometry and boundary conditions although the method itself is quite general. In recent times, this method is modified and this modified version was found to

be less complex and more practical when compared to the previous version[5-7].

### Review of Pertinent Literature

The use of boundary integral equation techniques has become popular in recent years. Russian mathematicians such as Mikhlin[8] made these techniques popular in Western Europe. The boundary integral techniques became popular with engineers by the continued work of Jaswon[5], Symm[7], and many others[8,9]. These authors first applied integral equations for the solution of problems governed by Laplace's equation.

Considerable amount of research on boundary integral equation methods was conducted in Southampton University and several important conferences on boundary element methods were held there. The researchers at the Southampton University, where the words "Boundary Elements" were first used, came up with the first book in 1978 entitled "The Boundary Element Method for Engineers"[9]. The proceedings of those conferences were edited by Brebbia, C. A. (Ed)[10,11] and are now used as standard references.

Boundary element method(BEM) has been used successfully to solve problems in potential theory, unsteady conduction heat transfer, and some fluid flow problems. Thus, the method can be used to solve boundary value problems of engineering interest. One example of recent application has been for the solution of time-dependent problems. Chang, et al. [12] solved the problem in time domain using piecewise constant values for the space and time variables. Wrobel and

Brebbia[13] have used the approach in [12] to solve problems involving irregular boundaries. BEM was also used for solving the fluid flow problems governed by the Navier-Stokes equations[10,11,13].

Cruse[14] applied the BEM to problems in Elasto-dynamics, Elasto-statics, and Fracture mechanics. Some useful expressions, for use in BEM, for strains and stresses in three dimensional bodies was published by Bui[15] in 1978.

The 3rd international conference on boundary element method was held in California during July, 1981. From the proceedings of this conference, one can observe the significant progress made in the application of BEM to many fields of engineering.

One of the primary topics of discussion by several authors in this conference was numerical integration. Some practical ideas have been put forward. A number of papers dealt with potential flow problems in this conference. The solution of time dependent diffusion problem by the integration over time as well as in space was also addressed[16].

Recent developments in numerical integration techniques has made this method a more useful tool now known as the boundary integral equation(BIE) method [10,11,13,17].

The BIE method was applied to a variety of steady conduction heat transfer problems by Khader and Hanna[18,19]. This method has also been extended for the solution of the time dependent problems[12,20-22]. The established numerical procedures by the finite-difference method as well as the finite-element method would require discretization of the entire interior and the boundary of the

region, and would involve the solution of very large systems of algebraic equations especially for two- and three-dimensional problems. Thus, these two methods generate solutions at all internal nodes whether such information is needed or not. From an engineering point of view, in many problems, only the solution on the boundary is of interest. Boundary integral equation method is well suited for such problems.

In addition to the boundary values in some problems, interior values at specific positions may be of interest. In such cases, once a problem is solved for boundary values using the BIE method, the solution required at any interior point can be calculated from the boundary values explicitly[18].

The conventional boundary integral equation methods which solve problems of this type do not take explicit advantage of the circular nature of the internal boundaries. Therefore, they usually require fine discretization near the hole boundaries, which in turn increases the number of simultaneous equations needed to generate a reasonably accurate solution. Such fine discretization could lead to conditioning problems especially when the region is very large relative to the size of the holes.

In a recent paper, Barone and Caulk[23] derived a system of special boundary integral equations for solving Laplace's equation in two-dimensional regions with circular holes. In their paper, the integrals around the circular boundaries were evaluated analytically, thereby eliminated the need for fine discretization normally required on such boundaries and the associated numerical conditioning problems.

They applied the method to solve an optimization problem[24]. Arimilli and Parang[25] have used the method and analyzed the problem of two parallel tubes each at the same constant temperature and buried under the ground with convective boundary condition at the surface of the ground.

#### Outline of the Work to be Presented

In the present thesis, a general two-dimensional problem of steady conduction heat transfer from a set of parallel tubes of arbitrary diameters, positions, and temperatures, located in a semi-infinite medium exposed to convective environment is formulated by using the method developed by Barone and Caulk[23]. This problem is analyzed, solved, and results are presented and discussed. A special case of the general problem involving two parallel tubes of same diameter and temperature in a semi-infinite medium are also analyzed and results are discussed. Criteria for selecting the parameters(segment length,etc.) of the numerical formulation are developed. Each of the tubes is considered to be at a uniform surface temperature in the above problems. The plane surface is assumed to be exposed to a specified uniform convective environment. The general nature of this analysis makes it applicable to a wide variety of problems. For example, this analysis can be applied to the design and optimization of heat exchangers of ground-coupled heat pump systems or to determine their performance characteristics under various operating conditions. Another application of this analysis is that of the calculation of thermal effects in a field of utility lines for power

and fluid transport. The criteria for the selection of maximum length used to represent the infinite region( $\xi_{\max}$ ) and segment length( $\Delta\xi$ ) is developed and discussed.

The BIE method, based on Green's second identity, formulates the temperature at any point in terms of the boundary values of the temperatures and their normal derivatives over the domain of interest. This identity when applied to the discretized boundaries of the domain usually leads to a set of linear algebraic equations involving these variables. Furthermore, in this analysis a special procedure, developed by Barone and Caulk[23], is employed for interior regions with circular boundaries where its temperature and its normal derivative of temperature are analytically integrated. This procedure eliminates the need for discretization on the circular boundaries, and thus avoids the numerical conditioning problems associated with the fine discretization normally required on such boundaries.

In this approach, boundary quantities on the holes are expanded in circular harmonics with unknown coefficients. These unknown coefficients represent normal derivative of temperature on the circular holes. The governing equations are first nondimensionalized. The nondimensional equations are applied to the problem in its general form, i.e., arbitrary locations, diameters, and temperatures. The integrals over the outer boundary are discretized and analytically integrated over each segment.

In the present thesis, zeroth- and first-order terms of the expansion are included and as such the results presented represent the first-order approximation. A computer program named GENTUBE is

developed to solve for the unknown expansion coefficients on the circular boundaries and the temperature distribution at the convective plane surface for this general problem.

As a special case of the general problem, the nondimensional equations are reduced for the problem of two circular tubes of equal diameters and temperatures located at the same depth from the plane convective surface in a semi-infinite medium. A computer program called TWOTUBE is developed for this two-tube problem without invoking the symmetry conditions present in this problem. Separately, the equations of the two-tube problem are further simplified by making use of the symmetry conditions present in the problem. A computer program called TWOSYMM is developed in order to solve these simplified equations for the unknown expansion coefficients and the temperature distributions. This TWOSYMM program involves half as many unknowns as the TWOTUBE program and, therefore, requires substantially less computational time than the TWOTUBE program. The TWOSYMM program can be validated by comparing the results with those from TWOTUBE program for symmetric problems. Further, the results of these two programs can be used to validate the GENTUBE program for special two-tube problem.

For illustration purposes, several problems of interest are solved. The results obtained for various tube arrangements are presented in a form suitable for use by designers of thermal systems.

Even though only a selected number of problems of special interest are solved, the GENTUBE program itself is quite general and can be used for any tube configuration having the same type of boundary conditions used in this formulation.

## CHAPTER II

### FORMULATION OF INTEGRAL EQUATIONS

Let  $R$  be a two-dimensional semi-infinite region of the physical problem of interest as shown in Figure 1. A sketch of the geometry and notation is shown in Figure 2. Consider a steady-state conduction heat transfer problem in a medium of constant thermal conductivity. The governing equation for this steady conduction heat transfer problem is

$$\nabla^2 \phi = 0 \quad (\text{Laplace's Equation})$$

where  $\phi$  is a harmonic function representing temperature in  $R$  which satisfies boundary conditions on  $\partial R$  and  $\partial C^\alpha$ .

The function  $\phi$  is said to be harmonic within the region  $R$ , if the function  $\phi(\xi, \eta)$  and its first derivatives are single-valued and continuous within and on the boundaries of region  $R$ , if second derivatives exist, and if the function  $\phi$  satisfies the Laplace's equation

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} = 0$$

The problem can be formulated in terms of the boundary integral equation known as the Green's second identity which is written as

$$\lambda \phi(Y) + \int_{\text{Boundary}} \left( \phi \frac{\partial g}{\partial n} - g \frac{\partial \phi}{\partial n} \right) ds = 0 \quad (1)$$

where  $Y$  is a position vector of points on boundary,  $\phi(Y)$  is the

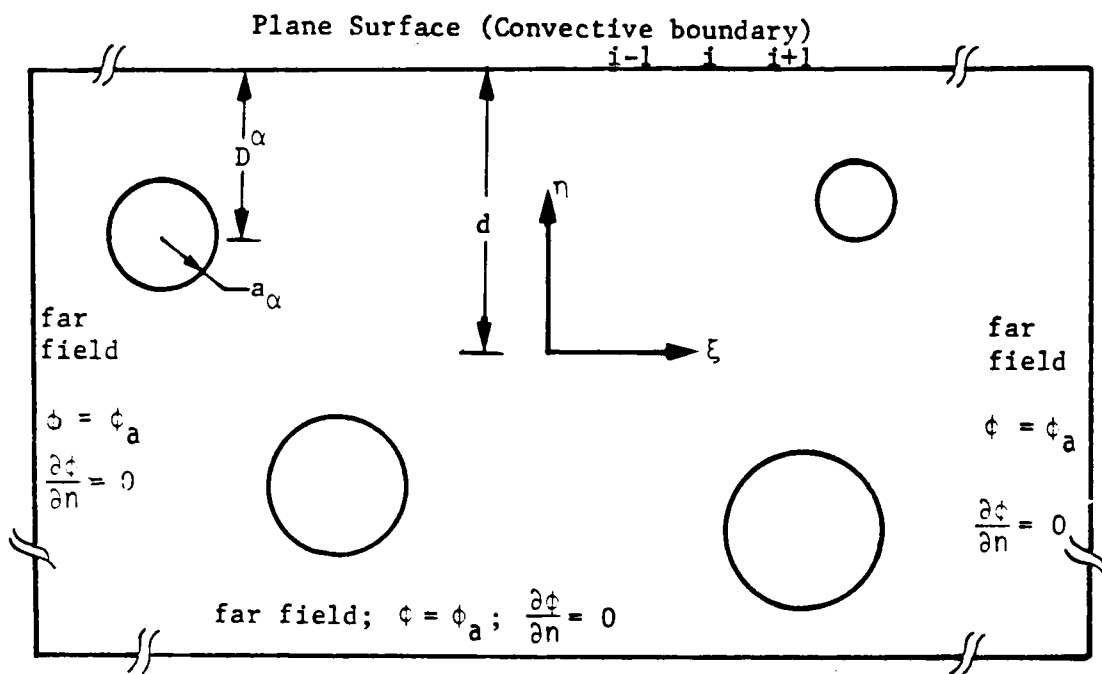


Figure 1. The Schematic of two-dimensional circular tubes in a semi-infinite medium. The dimensional form.

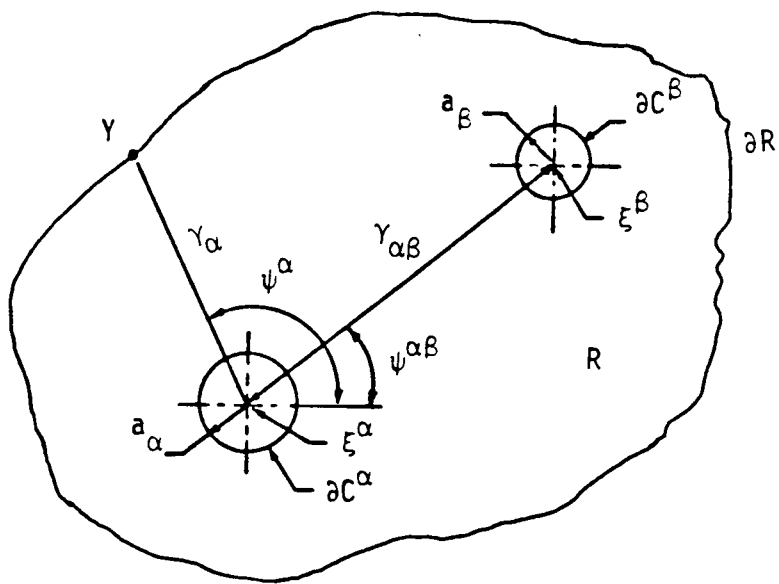


Figure 2. Two-dimensional region with circular tubes:  
geometry and notation (from reference 23)

temperature at  $y$ ,  $g(X,Y)$  is a second harmonic function over the region of interest and is defined as

$$g(x,y) = -\frac{1}{2\pi} \log |x - y|$$

and

$$\lambda = \begin{cases} 1 & , y \in R \\ \frac{1}{2} & , y \in \{\partial R, \partial C^\alpha\} \\ 0 & , y \notin \{R, \partial R, \partial C^\alpha\} \end{cases}$$

The harmonic function  $\phi$  in equation (1) satisfies boundary conditions on the circular boundary ( $\partial C^\alpha$ ) and the outer boundary of the region ( $\partial R$ ). The integral term in this equation (1) can be written as two terms, one for integration over external boundary and the other for summation of the integrals over each of the internal boundaries. The constant  $\lambda$  in equation (1) becomes 1 if  $Y$  belongs to the entire region but here the interest is only in internal and external boundaries where  $\lambda$  is  $1/2$ .

For the problem shown in Figure 1, this equation (1) can be written as

$$\frac{1}{2} \phi(y) + \int_{\partial R} \left( \phi \frac{\partial g}{\partial n} - g \frac{\partial \phi}{\partial n} \right) ds + \sum_{\alpha=1}^M \int_{\partial C^\alpha} \left( \phi^\alpha \frac{\partial g}{\partial n} - g q^\alpha \right) ds = 0 \quad (2)$$

where  $\phi^\alpha$  and  $q^\alpha$  are values of  $\phi$  and its outward normal derivative on the hole  $\partial C^\alpha$ .

The integral over the external boundary can be discretized by dividing the boundary into N segments. The integrals over the interior boundaries can be evaluated analytically following the approach of Barone and Caulk[23]. For each point Y on the boundary, the discretization yields one algebraic equation in terms of all of the boundary values of temperature and their normal derivatives. Thus N equations can be written for the N points on the boundary.

Barone and Caulk[23] take advantage of the circular geometry of the tube. On each of the circular interior boundaries they represent  $\phi^\alpha$  and  $q^\alpha$  by a finite sum of circular harmonics on  $\partial C^\alpha$  in the following form:

$$\phi^\alpha = \phi_0^\alpha + \sum_{n=1}^K (\phi_{1n}^\alpha \sin n \theta^\alpha + \phi_{2n}^\alpha \cos n \theta^\alpha) \quad (3-a)$$

$$q^\alpha = q_0^\alpha + \sum_{n=1}^L (q_{1n}^\alpha \sin n \theta^\alpha + q_{2n}^\alpha \cos n \theta^\alpha) \quad (3-b)$$

$$\alpha = 1, 2, \dots, M$$

Here  $\phi_0^\alpha$  and  $q_0^\alpha$  are constants representing the zeroth-order terms. K and L represent the order of approximation, and  $\theta^\alpha$  is the polar angle centered at  $\xi^\alpha$ , measured relative to the X-axis. Here M denotes the number of interior tubes. In general, the order of approximation need not be the same for all the holes.

Substituting equations(3) into equation(2) and performing the integration for  $y \in \{\partial R, \partial C^\beta\}$ , the equation(2) reduces to[23]

$$G(Y) + \int_{\partial R} \left( \phi \frac{\partial g}{\partial n} - g \frac{\partial \phi}{\partial n} \right) ds - \sum_{\alpha=1}^M \left[ \sum_{n=1}^K \frac{1}{2} b_{\alpha}^n (\phi_{1n}^{\alpha} \sin n \psi^{\alpha} + \phi_{2n}^{\alpha} \cos n \psi^{\alpha}) - a_{\alpha} \{ q_0^{\alpha} \log \gamma_{\alpha} - \sum_{n=1}^K \frac{1}{2n} b_{\alpha}^n (q_{1n}^{\alpha} \sin n \psi^{\alpha} + q_{2n}^{\alpha} \cos n \psi^{\alpha}) \} \right] = 0 \quad (4)$$

where (see Figure 2)

$$b_{\alpha} = \frac{a_{\alpha}}{\gamma_{\alpha}}, \quad \gamma_{\alpha} = |Y - \xi^{\alpha}|, \quad \psi^{\alpha} = \theta^{\alpha}(Y),$$

$$G(Y) = \begin{cases} \phi(Y) & , Y \in R \\ \frac{1}{2} \phi(Y) & , Y \in \{\partial R, \partial C^{\beta}\} \end{cases}$$

For isothermal conditions on each circular tube,  $\phi^{\alpha} = \phi_0^{\alpha}$ . Now equation (4) reduces to

$$G(Y) + \int_{\partial R} \left( \phi \frac{\partial g}{\partial n} - g \frac{\partial \phi}{\partial n} \right) ds + \sum_{\alpha=1}^M a_{\alpha} \{ q_0^{\alpha} \log \gamma_{\alpha} - \sum_{n=1}^L \frac{1}{2n} b_{\alpha}^n (q_{1n}^{\alpha} \sin n \psi^{\alpha} + q_{2n}^{\alpha} \cos n \psi^{\alpha}) \} = 0 \quad (5)$$

The integral term over the external boundary ( $\partial R$ ) in equation (5) contains  $N$  unknown values of  $\phi_j$  (or  $q_j$ ), where  $\phi_j$  is temperature approximating the segment  $j$  and  $j+1$  on plane convective surface, for  $N$  discretized segments. Similarly, the summation term over the holes ( $\partial C^{\alpha}$ ) in equation (5) has  $(M+2LM)$  number of unknown values ( $q_0^{\alpha}$ ,  $q_{1n}^{\alpha}$ , and  $q_{2n}^{\alpha}$ ) for  $M$  number of tubes and  $L$  number of order

of approximations. Thus, for first-order approximation, equation (5) will have  $(N+3M)$  number of unknowns for  $N$  number of discretized segments on the external boundary and  $M$  number of tubes. However, for  $N$  discrete segments on external boundary, equation (5) yields only  $N$  number of equations. The additional  $3M$  equations required to complete the equation set can be generated by considering different set of integral equations with the following choice of Kernel functions.

$$g_0^\beta(x) = g(x, \xi^\beta)$$

$$g_{1n}^\beta(x) = \frac{\sin n\theta^\beta}{|x - \xi^\beta|^n} \quad (6)$$

$$g_{2n}^\beta(x) = \frac{\cos n\theta^\beta}{|x - \xi^\beta|^n}$$

With the substitution of these special functions, the equation(2) now gives

$$\int_{\partial R} \left( \phi \frac{\partial g_0^\beta}{\partial n} - g_0^\beta \frac{\partial \phi}{\partial n} \right) ds + \sum_{\alpha=1}^M \int_{\partial C^\alpha} \left( \phi^\alpha \frac{\partial g_0^\beta}{\partial n} - g_0^\alpha q^\alpha \right) ds = 0 \quad (7)$$

$$\int_{\partial R} \left( \phi \frac{\partial g_{\lambda n}^\beta}{\partial n} - g_{\lambda n}^\beta \frac{\partial \phi}{\partial n} \right) ds + \sum_{\alpha=1}^M \int_{\partial C^\alpha} \left( \phi^\alpha \frac{\partial g_{\lambda n}^\beta}{\partial n} - g_{\lambda n}^\beta q^\alpha \right) ds = 0 \quad (8)$$

Equation (8) represents two equations corresponding to the cases of  $\lambda = 1$  and  $\lambda = 2$ .

In general, equation (7) involves M number of equations for M number of tubes. Equation (8) involves ML number of equations for each value of  $\lambda$ , where L is the order of approximation and M is the number of tubes. Thus, equations (7) and (8) together represent  $M(2L+1)$  number of equations. Substitution of equations (3b), (6) and the condition  $\phi^\alpha = \phi_0^\alpha$  into (7) and (8) yield (see [23] for details) the following two equations.

$$\int_{\partial R} \left( \phi \frac{\partial g_0}{\partial n} - g_0^\beta \frac{\partial \phi}{\partial n} \right) ds + \phi_0^\beta + q_0^\beta a_\beta \log a_\beta + \sum_{\alpha=1}^M a_\alpha \{ q_0^\alpha \log \gamma_{\alpha\beta} - \sum_{n=1}^L \frac{1}{2n} b_{\alpha\beta}^n (q_{1n}^\alpha \sin n\psi^{\alpha\beta} + q_{2n}^\alpha \cos n\psi^{\alpha\beta}) \} = 0 \quad (9)$$

$$\int_{\partial R} \left( \phi \frac{\partial g_{\lambda n}}{\partial n} - g_{\lambda n}^\beta \frac{\partial \phi}{\partial n} \right) ds - \pi a_\beta^{1-n} q_{\lambda n}^\beta - \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M \int_{\partial C^\alpha} g_{\lambda n}^\beta q^\alpha ds = 0 \quad (10)$$

$$\beta = 1, 2, \dots, M, \quad \lambda = 1, 2, \quad n = 1, 2, \dots, L$$

where

$$b_{\alpha\beta}^n = a_\alpha / \gamma_{\alpha\beta}, \quad \gamma_{\alpha\beta} = |\xi^\alpha - \xi^\beta|, \quad \psi^{\alpha\beta} = \theta^\alpha(\xi^\beta).$$

Equation (10) vanishes if only the zeroth-order approximation is considered.

### Levels of Approximations

In general, the normal heat flux on each tube boundary can be represented by any finite number of harmonics in equation (3-b) and it is reasonable to expect the accuracy of the solution to improve as this number is increased. The zeroth-order representation is  $q^\alpha = q_0^\alpha$ .

In this case, equations (5) and (9) reduce to

$$\frac{1}{2} \phi_i(Y) + \int_{\partial R} \left( \phi \frac{\partial g}{\partial n} - g \frac{\partial \phi}{\partial n} \right) ds + \sum_{\alpha=1}^M a_\alpha \{q_0^\alpha \log \gamma_\alpha\} = 0, \quad y \in \partial R \quad (11)$$

$$\int_{\partial R} \left( \phi \frac{\partial g_0^\beta}{\partial n} - g_0^\beta \frac{\partial \phi}{\partial n} \right) ds + \phi_0^\beta + q_0^\alpha a_\beta \log a_\beta + \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M a_\alpha \{q_0^\alpha \log \gamma_{\alpha\beta}\} = 0 \quad (12)$$

The equations (11) and (12) are referred to as the zeroth-order equations and their solution the zeroth-order solution.

But as discussed in [23], the accuracy of the results may not be reasonable. To improve the accuracy, first-order equations are considered in this thesis. For the first-order approximations ( $L=1$ ), equation (3-b) can be written as

$$q^\alpha = q_0^\alpha + q_1^\alpha \sin \theta^\alpha + q_2^\alpha \cos \theta^\alpha$$

With the substitution of this, equations (5), (9) and (10) reduce to

$$\begin{aligned} \frac{1}{2} \phi_i(Y) + \int_{\partial R} \left( \phi \frac{\partial g}{\partial n} - g \frac{\partial \phi}{\partial n} \right) ds + \sum_{\alpha=1}^M a_\alpha \{q_0^\alpha \log \gamma_\alpha - \\ - \frac{1}{2} b_\alpha (q_1^\alpha \sin \psi^\alpha + q_2^\alpha \cos \psi^\alpha)\} = 0, \quad y \in \partial R \end{aligned} \quad (13)$$

$$\int_{\partial R} \left( \phi \frac{\partial g_0^\beta}{\partial n} - g_0^\beta \frac{\partial \phi}{\partial n} \right) ds + \phi_0^\beta + q_0^\alpha a_\beta \log a_\beta + \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M a_\alpha \{ q_0^\alpha \log \gamma_{\alpha\beta} - \frac{1}{2} b_{\alpha\beta} (q_1^\alpha \sin \psi^{\alpha\beta} + q_2^\alpha \cos \psi^{\alpha\beta}) \} = 0 \quad (14)$$

$$\int_{\partial R} \left( \phi \frac{\partial g_1^\beta}{\partial n} - g_1^\beta \frac{\partial \phi}{\partial n} \right) ds - \pi q_1^\beta + \pi \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M \{ 2b_{\alpha\beta} q_0^\alpha \sin \psi^{\alpha\beta} + b_{\alpha\beta}^2 (q_2^\alpha \sin 2\psi^{\alpha\beta} - q_1^\alpha \cos 2\psi^{\alpha\beta}) \} = 0 \quad (15)$$

$$\int_{\partial R} \left( \phi \frac{\partial g_2^\beta}{\partial n} - g_2^\beta \frac{\partial \phi}{\partial n} \right) ds - \pi q_2^\beta + \pi \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M \{ 2b_{\alpha\beta} q_0^\alpha \cos \psi^{\alpha\beta} + b_{\alpha\beta}^2 (q_1^\alpha \sin 2\psi^{\alpha\beta} + q_2^\alpha \cos 2\psi^{\alpha\beta}) \} = 0 \quad (16)$$

Equation (13) is a result of using  $g(X,Y)$  as the Kernel function. It represents  $N$  algebraic equations corresponding to the  $N$  points on the external boundary. Equations (14), (15), and (16) are the result of using  $g_0^\beta(X)$ ,  $g_1^\beta(X)$ , and  $g_2^\beta(X)$  as the Kernel functions, respectively. For  $M$  tubes in the region, equations (14), (15), and (16) each represent  $M$  equations.

Thus, the total number of resulting algebraic equations is  $(N+3M)$  for the formulation involving the first-order approximation.

### CHAPTER III

#### BOUNDARY CONDITIONS AND NONDIMENSIONALIZATION

It should be pointed out, before proceeding with the nondimensionalization, that the  $\phi$  appearing in the various forms of the boundary integral equations(1 through 37) should be interpreted as a temperature difference. In other words,  $\phi$  represents temperature above some fixed reference temperature.

Consider a general steady-state conduction heat transfer problem in a semi-infinite medium as shown in Figure 3.

In the absence of tubes, in this general problem, the semi-infinite medium is considered to be in thermal equilibrium with the ambient medium. In other words, the temperature within the medium is uniform and is equal to  $T_a$ , the temperature of the ambient medium. Thus, under steady-state conditions, the heat transfer through the medium is zero. But when isothermal tubes at temperatures different from  $T_a$  are present in the medium, under steady-state conditions, the far-field temperature within the medium would remain at  $T_a$  and heat transfer at the plane surface would be by convection.

The governing equation for this problem is

$$\nabla^2 \phi = 0 \quad (17)$$

where  $\phi = T - T_a$

The boundary conditions for this problem are the following:

At far field,  $\phi = 0$

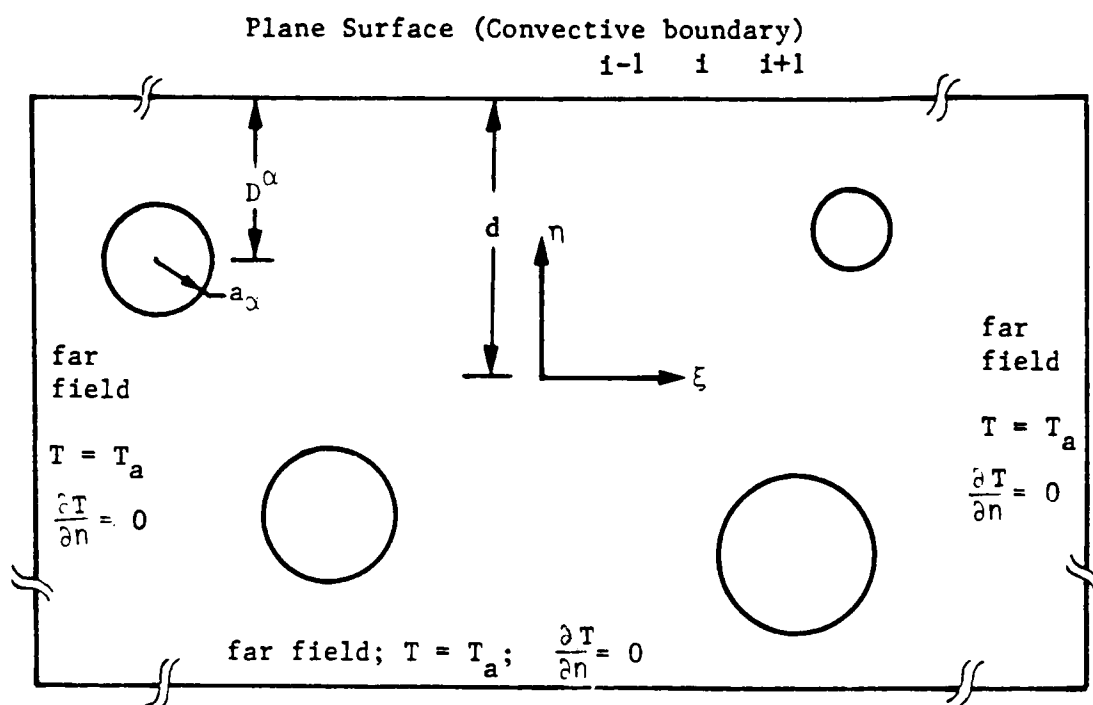


Figure 3. The Schematic of two-dimensional circular tubes in a semi-infinite medium. The dimensional form.

On the  $\alpha$  th tube surface,  $\phi = \phi_t^\alpha = T_t^\alpha - T_a$

At the plane surface, heat transfer is by convection and is given by

$$\left. \frac{\partial \phi}{\partial \eta} \right|_{\eta=d} = - \frac{h}{k} \phi \Big|_{\eta=d}$$

Let  $q$  denote  $\frac{\partial \phi}{\partial n}$

The dimensional form of the four governing equations (13), (14), (15), and (16) representing first-order approximation of the problem in Figure 3 can now be written in the discretized form as

$$\begin{aligned} \frac{1}{2} \phi_i + \sum_{j=1}^N [\phi_j \int_{C_j} \frac{\partial g_i}{\partial n} ds - q_j \int_{C_j} g_i ds] \\ + \sum_{\alpha=1}^M a_\alpha [q_0^\alpha \log \gamma_{\alpha i} - \frac{1}{2} b_{\alpha i} (q_1^\alpha \sin \psi^{\alpha i} + q_2^\alpha \cos \psi^{\alpha i})] = 0 \quad (18) \end{aligned}$$

$$\begin{aligned} \sum_{j=1}^N \{ \phi_j \int_{C_j} \frac{\partial g_0^\beta}{\partial n} ds - q_j \int_{C_j} g_0^\beta ds \} + \phi_0^\beta + q_0^\beta a_\beta \log a_\beta \\ + \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M a_\alpha [q_0^\alpha \log \gamma_{\alpha \beta} - \frac{1}{2} b_{\alpha \beta} (q_1^\alpha \sin \psi^{\alpha \beta} + q_2^\alpha \cos \psi^{\alpha \beta})] = 0 \quad (19) \end{aligned}$$

$$\sum_{j=1}^N \{ \phi_j \int_{C_j} \frac{\partial g_1^\beta}{\partial n} ds - q_j \int_{C_j} g_1^\beta ds \} - \pi q_1^\beta +$$

$$\pi \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M [2b_{\alpha\beta} q_0^\alpha \sin \psi^{\alpha\beta} + b_{\alpha\beta}^2 (q_2^\alpha \sin 2\psi^{\alpha\beta} - q_1^\alpha \cos 2\psi^{\alpha\beta})] = 0 \quad (20)$$

$$\sum_{j=1}^N \{ \phi_j \int_{C_j} \frac{\partial g_2^\beta}{\partial n} - q_j \int_{C_j} g_2^\beta ds \} - \pi q_2$$

$$+ \pi \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M [2b_{\alpha\beta} q_0^\alpha \cos \psi^{\alpha\beta} + b_{\alpha\beta}^2 (q_1^\alpha \sin 2\psi^{\alpha\beta} + q_2^\alpha \cos 2\psi^{\alpha\beta})] = 0 \quad (21)$$

where the parameters are as defined in the nomenclature.

Defining the nondimensional temperature,

$$\phi(\bar{\xi}, \bar{\eta}) \equiv \frac{\phi(\xi, \eta)}{\phi_r} \equiv \frac{T(\xi, \eta) - T_a}{T_r - T_a}$$

where  $T_a$  and  $T_r$  are constants representing the far-field temperature, and the temperature of the reference tube, respectively.

Nondimensionalizing  $\eta$  and  $d$  with the reference tube radius  $A$ ,

$$\bar{\eta} = \frac{\eta}{A}$$

$$\bar{d} = \frac{d}{A}$$

The nondimensional form of governing equation becomes,

$$\nabla^2 \phi = 0$$

The nondimensional boundary conditions are:

At far field,  $\phi = 0$

On the  $\alpha$  th tube surface,

$$\phi^\alpha = \frac{\phi_t^\alpha}{\phi_r} \equiv \frac{T_t^\alpha - T_a}{T_r - T_a}$$

At plane surface,

$$\frac{\partial \phi}{\partial \bar{\eta}} \Big|_{\bar{\eta}=\bar{d}} = -\frac{hA}{\kappa} \phi \Big|_{\bar{\eta}=\bar{d}}$$

Defining Biot number,  $Bi = hA/k$  and using  $\bar{q}$  to denote normal derivative,

$$\bar{q} \equiv \frac{\partial \phi}{\partial \bar{\eta}}$$

the boundary condition at the plane surface can now be written as

$$\bar{q} \Big|_{\bar{\eta}=\bar{d}} = -Bi * \phi \Big|_{\bar{\eta}=\bar{d}}$$

As can be seen from nondimensional boundary conditions, the nondimensional temperature at far-field goes to zero. The other length and geometric parameters can now be nondimensionalized with the

reference tube radius A. However, if two or more tubes have the same highest temperature, then the tube with the largest radius is taken as the reference tube in this formulation.

$$\bar{s} = \frac{s}{A}$$

$$\bar{\gamma}_{\alpha i} = \frac{\gamma_{\alpha i}}{A}$$

$$\bar{b}_{\alpha i} = \frac{\bar{a}_{\alpha}}{\bar{\gamma}_{\alpha i}}$$

$$\bar{\xi} = \frac{\xi}{A}$$

$$d\bar{s} = \frac{ds}{A}$$

The governing equations (18) through (21) can now be nondimensionalized using the above dimensionless parameters. But first the nondimensionalization of the typical terms appearing in those equations is shown below:

$$\phi_i = \phi_r \cdot \Phi_i$$

$$\phi_j = \phi_r \cdot \Phi_j$$

$$\int_{C_j} \frac{\partial g_i}{\partial n} ds = 0 \quad \text{as } g_i \neq f(n) \text{ for the problem under consideration}$$

$$q_j = \frac{\partial \phi_j}{\partial n} = \phi_r \cdot \frac{\partial \Phi_j}{\partial n}$$

$$= \frac{\phi_r}{A} \cdot \frac{\partial \Phi_j}{\partial \bar{n}}$$

$$= \frac{\phi_r}{A} (-Bi \Phi_{\bar{n}=\bar{D}_{\alpha}})$$

$$ds = d(\xi_j - \xi_i) = A \cdot d(\bar{\xi}_j - \bar{\xi}_i) = A \cdot d\bar{s}$$

$$a_{\alpha} = A \bar{a}_{\alpha}$$

$$\begin{aligned} g_i &= -\frac{1}{2\pi} \log(\xi_j - \xi_i) \\ &= -\frac{1}{2\pi} \{ \log(\bar{\xi}_j - \bar{\xi}_i) + \log A \} \end{aligned}$$

$$\begin{aligned} q_0^{\alpha} &= \frac{\partial \phi_0^{\alpha}}{\partial n} \\ &= \frac{\phi_r}{A} \cdot \frac{\partial \phi_0^{\alpha}}{\partial n} \\ &= \frac{\phi_r}{A} \cdot \bar{q}_0^{\alpha} \quad (\text{Define } \frac{\partial \bar{\phi}_0^{\alpha}}{\partial n} = \bar{q}_0^{\alpha}) \end{aligned}$$

$$q_1^{\alpha} = \frac{\phi_r}{A} \bar{q}_1^{\alpha}$$

$$q_2^{\alpha} = \frac{\phi_r}{A} \bar{q}_2^{\alpha}$$

$$\gamma_{\alpha i} = A \cdot \gamma_{\alpha i}$$

$$b_{\alpha i} = \frac{a_{\alpha}}{\gamma_{\alpha i}} = \frac{\bar{a}_{\alpha}}{\bar{\gamma}_{\alpha i}}$$

$$\sin \psi^{\alpha i} = \frac{\bar{D}_{\alpha}}{\bar{\gamma}_{\alpha i}}$$

$$\cos \psi^{\alpha i} = \frac{(\bar{\xi}_i - \bar{s}_{\alpha})}{\bar{\gamma}_{\alpha i}}$$

$$g_0^{\beta} = g(x, \varepsilon^{\beta}) = -\frac{1}{2\pi} \log |\xi_j - \xi^{\beta}| = -\frac{1}{2\pi} \{ \log |\bar{\xi}_j - \bar{\xi}^{\beta}| + \log A \}$$

$$g_1^{\beta}(\xi) = \frac{\sin \theta}{|\xi - \xi^{\beta}|} = \frac{\sin \theta}{A |\bar{\xi}_j - \bar{\xi}^{\beta}|} = \frac{\bar{D}_{\beta}}{A \{ |\bar{\xi}_j - \bar{\xi}^{\beta}| \}^2}$$

$$g_2^{\beta}(\xi) = \frac{\cos \theta}{|\xi - \xi^{\beta}|} = \frac{\cos \theta}{A |\bar{\xi}_j - \bar{\xi}^{\beta}|} = \frac{\bar{\xi}_j - \bar{s}_{\beta}}{\{ |\bar{\xi}_j - \bar{\xi}^{\beta}| \}^2}$$

Substituting these terms into equations (18) through (21), dividing each equation by  $\phi_r$ , and writing the known terms on right hand side of each of the equation will result in the following dimensionless equations.

$$\begin{aligned} \frac{1}{2} \phi_i - \frac{Bi}{2\pi} \sum_{j=1}^N \phi_j \int_{\bar{\xi}=\bar{\xi}_j}^{\bar{\xi}_{j+1}} \{ \log |\bar{\xi}_j - \bar{\xi}_i| + \log A \} d|\bar{\xi}_j - \bar{\xi}_i| \\ + \sum_{\alpha=1}^M [\bar{a}_{\alpha} \{ \log \bar{\gamma}_{\alpha i} + \log A \} \bar{q}_0^{\alpha} - \frac{1}{2} \frac{\bar{a}_{\alpha}^2}{\bar{\gamma}_{\alpha i}} \sin \psi^{\alpha i} \bar{q}_1^{\alpha} \\ - \frac{1}{2} \frac{\bar{a}_{\alpha}^2}{\bar{\gamma}_{\alpha i}} \cos \psi^{\alpha i} \bar{q}_2^{\alpha}] = 0.0 \end{aligned} \quad (22)$$

where

$$\bar{\gamma}_{\alpha i} = \sqrt{(\bar{\xi}_i - \bar{s}_{\alpha})^2 + \bar{D}_{\alpha}^2}$$

$\bar{D}_{\alpha}$  = Depth of  $\alpha^{th}$  tube from plane convective surface

$\bar{s}_{\alpha}$  = Distance from  $\alpha$ -axis to the center of  $\alpha^{th}$  tube parallel to the plane convective surface

$$\begin{aligned} - \frac{Bi}{2} \sum_{j=1}^N \phi_j \int_{\bar{\xi}=\bar{\xi}_j}^{\bar{\xi}_{j+1}} \{ \log |\bar{\xi}_j - \bar{\xi}^{\beta}| + \log A \} d(\bar{\xi}_j - \bar{\xi}^{\beta}) \\ + \bar{a}_{\beta} \log \{ \bar{a}_{\beta} + \log A \} \bar{q}_0^{\beta} + \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M [\bar{a}_{\alpha} (\log \bar{\gamma}_{\alpha \beta} + \log A) \bar{q}_0^{\alpha} \\ - \frac{1}{2} \frac{\bar{a}_{\alpha}^2}{\bar{\gamma}_{\alpha \beta}} \sin \psi^{\alpha \beta} \bar{q}_1^{\alpha} - \frac{1}{2} \frac{\bar{a}_{\alpha}^2}{\bar{\gamma}_{\alpha \beta}} \cos \psi^{\alpha \beta} \bar{q}_2^{\alpha}] = -\epsilon_0^{\beta} \end{aligned} \quad (23)$$

where

$$\bar{\gamma}_{\alpha \beta} = \sqrt{(\bar{D}_{\beta} - \bar{D}_{\alpha})^2 + (\bar{s}_{\alpha} - \bar{s}_{\beta})^2}$$

$$\begin{aligned}
B_i \quad \sum_{j=1}^N \{ \phi_j \int_{\bar{\xi}=\bar{\xi}_j}^{\bar{\xi}_{j+1}} \frac{\sin \theta}{A |\bar{\xi}_j - \bar{\xi}^\beta|} d(\bar{\xi}_j - \bar{\xi}^\beta) - \frac{\pi}{A} \bar{q}_1^\beta \\
+ \pi \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M \left[ \frac{2}{A} \frac{\bar{a}_\alpha}{\bar{\gamma}_{\alpha\beta}} \sin \psi^{\alpha\beta} \bar{q}_0^\alpha + \right. \\
\left. + \frac{\bar{a}_{\alpha\beta}^2}{A \cdot \bar{\gamma}_{\alpha\beta}^2} \sin 2\psi^{\alpha\beta} \cdot \bar{q}_2^\alpha - \frac{\bar{a}_{\alpha\beta}^2}{A \cdot \bar{\gamma}_{\alpha\beta}^2} \cos 2\psi^{\alpha\beta} \bar{q}_1^\alpha \right] = 0 \quad (24)
\end{aligned}$$

$$\begin{aligned}
B_i \quad \sum_{j=1}^N \{ \phi_j \int_{\bar{\xi}=\bar{\xi}_j}^{\bar{\xi}_{j+1}} \frac{\cos \theta}{A |\bar{\xi}_j - \bar{\xi}^\beta|} d(\bar{\xi}_j - \bar{\xi}^\beta) - \frac{\pi}{A} \bar{q}_2^\beta \\
+ \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M \left[ \frac{2}{A} \frac{\bar{a}_\alpha}{\bar{\gamma}_{\alpha\beta}} \cos \psi^{\alpha\beta} \bar{q}_0^\alpha + \right. \\
\left. + \frac{\bar{a}_\alpha^2}{A \cdot \bar{\gamma}_{\alpha\beta}^2} \sin 2\psi^{\alpha\beta} \bar{q}_1^\alpha + \frac{\bar{a}_\alpha^2}{A \cdot \bar{\gamma}_{\alpha\beta}^2} \cos 2\psi^{\alpha\beta} \bar{q}_2^\alpha \right] = 0 \quad (25)
\end{aligned}$$

The equations (22) through (25) represent the dimensionless form of equations (18) through (21), respectively, and the integrals over the discretized segments will be evaluated analytically in the next chapter.

## CHAPTER IV

### THE TWO-TUBE PROBLEM

In this chapter, the first-order equations (22-25) are reduced for the case of two circular tubes of same diameter and uniform temperature located at the same depth from a plane convective surface in a semi-infinite medium. The first-order circular harmonic expansion for the normal derivatives is used.

The external boundary is divided into N discrete segments, and the integrals of the Kernel functions will be evaluated analytically over the boundary segments. Jaswon and Symm[5,7] have developed a method for integration of logarithmic functions on a straight line. The details of that method are discussed in Appendix A.

The sketch for this two-tube case is shown in Figure (4).

$$\sin \psi^{1i} = \frac{\bar{D}}{\sqrt{\bar{D}^2 + (\bar{\xi}_i - \bar{s})^2}} \quad ; \quad \cos \psi^{1i} = \frac{(\bar{\xi}_i - \bar{s})}{\sqrt{\bar{D}^2 + (\bar{\xi}_i - \bar{s})^2}}$$

$$\sin \psi^{2i} = \frac{\bar{D}}{\sqrt{\bar{D}^2 + (\bar{\xi}_i + \bar{s})^2}} \quad ; \quad \cos \psi^{2i} = \frac{(\bar{\xi}_i + \bar{s})}{\sqrt{\bar{D}^2 + (\bar{\xi}_i + \bar{s})^2}}$$

$$\bar{\gamma}_{1i} = \sqrt{\bar{D}^2 + (\bar{\xi}_i - \bar{s})^2} \quad ; \quad \bar{\gamma}_{2i} = \sqrt{\bar{D}^2 + (\bar{\xi}_i + \bar{s})^2}$$

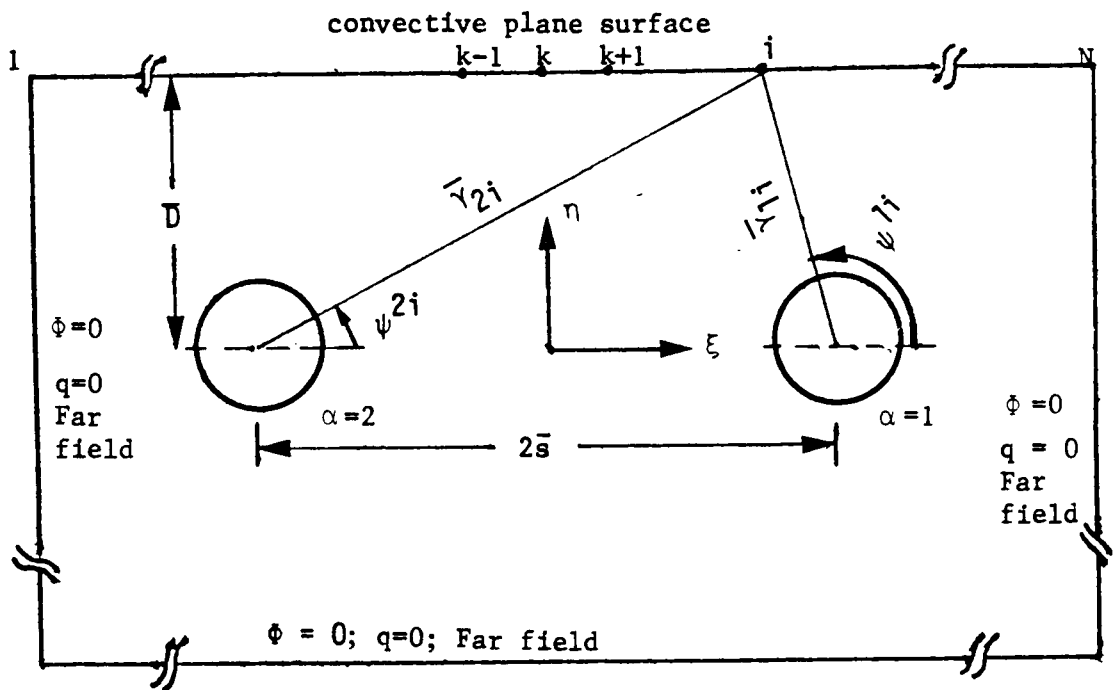


Figure 4. The schematic of two tubes in a semi-infinite medium.

Since diameter is same for both of the tubes, the nondimensional radius of tubes becomes equal to 1. With the substitution of these values, equation (22) becomes

$$\begin{aligned}
 \frac{1}{2} \phi_i - \frac{Bi}{2\pi} \sum_{j=1}^N \phi_j [T_{ij}] + \{\log \bar{\gamma}_{1j} + \log A\} \bar{q}_0^1 \\
 + \{\log \bar{\gamma}_{2i} + \log A\} \bar{q}_0^2 - \frac{1}{2} \frac{\bar{D}}{\bar{\gamma}_{1i}^2} \bar{q}_1^1 - \frac{1}{2} \frac{\bar{D}}{\bar{\gamma}_{2i}^2} \bar{q}_2^2 \\
 - \frac{1}{2} \frac{(\bar{\epsilon}_i - \bar{s})}{\bar{\gamma}_{1i}^2} \bar{q}_2^1 - \frac{1}{2} \frac{(\bar{\epsilon}_i + \bar{s})}{\bar{\gamma}_{2i}^2} \bar{q}_2^2 = 0.0 \quad (26)
 \end{aligned}$$

where

$$T_{ij} = u_{ij} \quad \text{for } i < j$$

$$T_{ij} = -u_{ij} \quad \text{for } i > j \quad \text{and } i \neq j + 1$$

$$T_{ij} = |\bar{\epsilon}_{j+1} - \bar{\epsilon}_i| \{[\log |\bar{\epsilon}_{j+1} - \bar{\epsilon}_i| - 1] + \log A\} \quad \text{for } i = j$$

$$T_{ij} = |\bar{\epsilon}_j - \bar{\epsilon}_i| \{[\log |\bar{\epsilon}_j - \bar{\epsilon}_i| - 1] + \log A\} \quad \text{for } i = j + 1$$

and

$$\begin{aligned}
 u_{ij} = [|\bar{\epsilon}_{j+1} - \bar{\epsilon}_i| \{[\log |\bar{\epsilon}_{j+1} - \bar{\epsilon}_i| - 1] + |\bar{\epsilon}_{j+1} - \bar{\epsilon}_i| \log A\} \\
 - [|\bar{\epsilon}_j - \bar{\epsilon}_i| \{[\log |\bar{\epsilon}_j - \bar{\epsilon}_i| - 1] + |\bar{\epsilon}_j - \bar{\epsilon}_i| \log A\}]
 \end{aligned}$$

$$\bar{\gamma}_{12} = 2\bar{s} = \bar{\gamma}_{21}$$

$$\sin \psi^{12} = \sin 180^\circ = 0$$

$$\sin \psi^{21} = \sin 0^\circ = 0$$

$$\cos \psi^{12} = \cos 180^\circ = -1$$

$$\cos \psi^{21} = \cos 0^\circ = +1$$

Performing the same type of integration as done for equation (22), the equation (23) can be written as two equations, one when  $i$  is at the center of the first tube ( $\beta = 1$  case) and the other when  $i$  is at the center of the second tube ( $\beta = 2$  case).

For  $\beta = 1$

$$\begin{aligned}
 & - \frac{Bi}{2} \sum_{j=1}^N \phi_j \left[ \left\{ \frac{(\bar{\xi}_j - \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{\xi}_j)^2 - (\bar{\xi}_{j+1} - \bar{s})^2}{2(\bar{\xi}_{j+1} - \bar{\xi}_j)} \right\} \right. \\
 & \quad \left[ \log \sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2} - \log \sqrt{(\bar{\xi}_{j+2} - \bar{s})^2 + \bar{D}^2} \right] \\
 & \quad + (\bar{\xi}_{j+1} - \bar{\xi}_j) (\log \sqrt{(\bar{\xi}_{j+1} - \bar{s})^2 + \bar{D}^2} - 1) + \sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2} \\
 & \quad \cdot \cos^{-1} \left\{ \frac{(\bar{\xi}_j - \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{s})^2 + 2\bar{D}^2 - (\bar{\xi}_{j+1} - \bar{\xi}_j)^2}{2\sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2} \sqrt{(\bar{\xi}_{j+1} - \bar{s})^2 + \bar{D}^2}} \right\} \\
 & \quad \left. \left\{ \sin \left[ \cos^{-1} \left\{ \frac{(\bar{\xi}_j - \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{\xi}_j)^2 - (\bar{\xi}_{j+1} - \bar{s})^2}{2\sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2} (\bar{\xi}_{j+1} - \bar{\xi}_j)} \right\} \right] \right\} \right. \\
 & \quad \left. + \log A[(\bar{\xi}_{j+1} - \bar{\xi}_j)] \right] + (\log 2\bar{s} + \log A) \bar{q}_0^2 \\
 & \quad - \frac{1}{2} \frac{1}{2\bar{s}} \bar{q}_2^2 = - \Phi_0^1 \quad (27a)
 \end{aligned}$$

For  $\beta = 2$

$$\begin{aligned}
& - \frac{Bi}{2} \sum_{j=1}^N \Phi_j \left[ \left\{ \frac{(\bar{\xi}_j + \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{\xi}_j)^2 - (\bar{\xi}_{j+1} + \bar{s})^2}{2(\bar{\xi}_{j+1} - \bar{\xi}_j)} \right\} \right. \\
& \quad \left[ \log \sqrt{(\bar{\xi}_j + \bar{s})^2 + \bar{D}^2} - \log \sqrt{(\bar{\xi}_{j+2} + \bar{s})^2 + \bar{D}^2} \right] \\
& \quad + (\bar{\xi}_{j+1} - \bar{\xi}_j) (\log \sqrt{(\bar{\xi}_{j+1} + \bar{s})^2 + \bar{D}^2} - 1) + \sqrt{(\bar{\xi}_j + \bar{s})^2 + \bar{D}^2} \\
& \quad \cdot \cos^{-1} \left\{ \frac{(\bar{\xi}_j + \bar{s})^2 + (\bar{\xi}_{j+1} + \bar{s})^2 + 2\bar{D}^2 - (\bar{\xi}_{j+1} - \bar{\xi}_j)^2}{2\sqrt{(\bar{\xi}_j + \bar{s})^2 + \bar{D}^2} \sqrt{(\bar{\xi}_{j+1} + \bar{s})^2 + \bar{D}^2}} \right\} \\
& \quad \left. \left\{ \sin \left[ \cos^{-1} \left\{ \frac{(\bar{\xi}_j + \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{\xi}_j)^2 - (\bar{\xi}_{j+1} + \bar{s})^2}{2\sqrt{(\bar{\xi}_j + \bar{s})^2 + \bar{D}^2} (\bar{\xi}_{j+1} - \bar{\xi}_j)} \right\} \right] \right\} \right. \\
& \quad \left. + \log A[(\bar{\xi}_{j+1} - \bar{\xi}_j)] \right] + (\log 2\bar{s} + \log A) \bar{q}_0^1 \\
& \quad - \frac{1}{2} \frac{1}{2\bar{s}} \bar{q}_2^1 = - \Phi_0^2 \quad (27b)
\end{aligned}$$

Similar to equation (23), equation (24) has two equations, one for  $i$  at the center of each tube.

Equation (24) now becomes, for  $\beta = 1$

$$\begin{aligned}
 & \int_{\bar{\xi}=\bar{\xi}_j}^{\bar{\xi}_{j+1}} \frac{\sin \theta}{A|\bar{\xi}_j - \bar{\xi}^\beta|} d(\bar{\xi}_j - \bar{\xi}^\beta) \\
 &= \int_{\bar{\xi}=\bar{\xi}_j}^{\bar{\xi}_{j+1}} \frac{\bar{D}}{(\bar{\xi}_j - s)^2 + \bar{D}^2} \frac{1}{A\sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2}} d(\bar{\xi}_j - \bar{s}) \\
 &= \frac{1}{A} [-\sin^{-1} \left\{ \frac{\bar{D}}{\sqrt{(\bar{\xi}_{j+1} - \bar{s})^2 + \bar{D}^2}} \right\} + \sin^{-1} \left\{ \frac{\bar{D}}{\sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2}} \right\}]
 \end{aligned}$$

and for  $\beta = 2$ ,

$(\bar{\xi} - \bar{s})$  terms in the above equation become  $(\bar{\xi} + \bar{s})$

Performing the necessary integration,

For  $\beta = 1$ ,

$$\begin{aligned}
 & -\frac{Bi}{A} \sum_{j=1}^N \phi_j \left[ \sin^{-1} \left\{ \frac{\bar{D}}{(\bar{\xi}_{j+1} - \bar{s})^2 + \bar{D}^2} \right\} - \sin^{-1} \left\{ \frac{\bar{D}}{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2} \right\} \right] \\
 & -\frac{\pi}{A} \bar{q}_1^1 - \frac{\pi}{A} \frac{1}{4s^2} \bar{q}_1^2 = 0 \quad (28a)
 \end{aligned}$$

For  $\beta = 2$ ,

$$\begin{aligned}
 & -\frac{Bi}{A} \sum_{j=1}^N \phi_j \left[ \sin^{-1} \left\{ \frac{\bar{D}}{(\bar{\xi}_{j+1} + \bar{s})^2 + \bar{D}^2} \right\} - \sin^{-1} \left\{ \frac{\bar{D}}{(\bar{\xi}_j + \bar{s})^2 + \bar{D}^2} \right\} \right] \\
 & -\frac{\pi}{A} \bar{q}_1^2 - \frac{\pi}{A} \frac{1}{4s^2} \bar{q}_1^1 = 0 \quad (28b)
 \end{aligned}$$

(A programming note: Some facts need to be considered in equation (28) while evaluating the unknown values. Because of the way it was taken to compute the summation terms, the sign of this term should be changed to get the correct values if  $\bar{\xi}_j$  is less than  $\bar{s}$ . It should also be noted that there may be a special case of  $\bar{\xi}_j$  less than  $s$  and  $\bar{\xi}_{j+1}$  greater than  $\bar{s}$ , in which case  $2 \sin(1.)$  should be subtracted from the second term before multiplying by  $Bi/A$ . All this has been taken care of in the computer program developed for this two-tube problem.)

Equation (25) has 2 equations for two-tube problem, one for  $i$  at the center of the each tube.

For  $\beta = 1$ ,

$$\begin{aligned} \int_{\bar{\xi}=\bar{\xi}_j}^{\bar{\xi}_{j+1}} \frac{\cos \theta}{A|\bar{\xi}_j - \bar{\xi}_\beta|} d(\bar{\xi}_j - \bar{\xi}_\beta) &= \\ &= \int_{\bar{\xi}=\bar{\xi}_j}^{\bar{\xi}_j} \frac{(\bar{\xi}_j - \bar{s})}{\sqrt{(\bar{\xi}_j - \bar{s})^2 + D^2}} \frac{1}{A\sqrt{(\bar{\xi}_j - \bar{s})^2 + D^2}} d(\bar{\xi}_j - \bar{s}) \\ &= \frac{1}{2A} [\ln\{(\bar{\xi}_j - \bar{s})^2 + D^2\}]_{\bar{\xi}_j}^{\bar{\xi}_{j+1}} \end{aligned}$$

For  $\beta = 2$

$$(\bar{\xi}_j - \bar{s})^2 \text{ terms become } (\bar{\xi}_j + \bar{s})^2$$

Equation (25) is now written as

For  $\beta = 1$

$$\frac{Bi}{2A} \sum_{j=1}^N \phi_j [\log\{(\bar{\xi}_{j+1} - \bar{s})^2 + D^2\} - \log\{(\bar{\xi}_j - \bar{s})^2 + D^2\}]$$

$$-\frac{\pi}{A} \frac{1}{q_2} + \frac{\pi}{As} \frac{1}{q_0^2} + \frac{\pi}{A \cdot 4s^2} \frac{1}{q_2^2} = 0 \quad (29a)$$

and for  $\beta = 2$

$$\frac{Bi}{2A} \sum_{j=1}^N \phi_j [\log \{(\bar{\xi}_{j+1} + \bar{s})^2 + \bar{D}^2\} - \log \{(\bar{\xi}_j + \bar{s})^2 + \bar{D}^2\}]$$

$$- \frac{\pi}{A} \bar{q}_2^2 + \frac{\pi}{As} \bar{q}_0^1 + \frac{\pi}{A \cdot 4s^2} \bar{q}_2^1 = 0 \quad (29b)$$

Thus (N+6) algebraic equations for (N+6) unknown values have been developed. These (N+6) unknowns are  $\bar{q}_0^1$ ,  $\bar{q}_1^1$ ,  $\bar{q}_2^1$ ,  $\bar{q}_0^2$ ,  $\bar{q}_1^2$ , and  $\bar{q}_2^2$  on tube surface, and  $\phi_1$ ,  $\phi_2$  ...  $\phi_N$  on the convective plane surface. The above algebraic equations are solved numerically. Thus, the temperature or its normal derivative at the boundaries alone are determined and determination of the variables at interior points is avoided in this boundary integral technique.

A computer program has been developed for this problem of two-tubes to evaluate these unknown values. The criteria used to choose the values of  $\bar{\xi}_{max}$  (the maximum value used in the computer program to represent the distance to the far field on the plane surface) and  $\Delta \bar{\xi}$  (segment length) is discussed in chapter VI of this thesis.

### Symmetry Considerations

As two tubes are located at the same depth, and are having the same temperature and diameter, some symmetrical conditions can be deduced. From the Figure (4), it can be seen that  $\bar{q}_0^1 = \bar{q}_0^2$ ,  $\bar{q}_1^1 = -\bar{q}_1^2$ , and  $\bar{q}_2^1 = -\bar{q}_2^2$ . With the substitution of these symmetry conditions, the six equations (27a-28b) reduce to three equations, thus reducing the number of equations to be solved from (N+6) to (N+3). A symmetry condition also exists on the plane convective surface. This symmetry condition can be explained in the following manner. Suppose K is a nodal point which is located at the same distance from each tube on the plane convective surface. This means K-th node is at the center of the plane convective surface considered for evaluation purposes since same length of the convective surface is considered on either side of the tubes because of the tube symmetry. Because of these symmetrical conditions, the temperature between the nodes K and (K+1) should be equal to the temperature between the nodes K and (K-1). Similarly, the temperature between the nodes (K+1) and (K+2) should be equal to the temperature between the nodes (K-1) and (K-2), and so on. This symmetry condition further reduces the number of equations from (N+3) to (N/2+3). For the convenience, N is selected as even number.

The equations (26-29) which represent seven equations now reduce to four equations for the two tube problem when symmetry is taken into account. The first of these equations representing equation (26) now becomes equation (30).

$$\begin{aligned}
& -\frac{1}{2} \phi_i - \frac{Bi}{2\pi} \sum_{j=1}^N \phi_j [\tau_{ij}] + (\log \bar{\gamma}_{1i} + \log \bar{\gamma}_{2i} + 2 \log A) \bar{q}_0^1 \\
& - \left\{ \frac{\bar{D}}{2} \left( \frac{1}{\bar{\gamma}_{1i}} + \frac{1}{\bar{\gamma}_{2i}} \right) \right\} \bar{q}_1^1 - \frac{1}{2} \left\{ \left( \frac{\bar{\xi}_i - \bar{s}}{\bar{\gamma}_{1i}^2} - \frac{(\bar{\xi}_i + \bar{s})}{\bar{\gamma}_{2i}^2} \right) \right\} \bar{q}_2^1 \\
& = 0.0
\end{aligned} \tag{30}$$

where  $\tau_{ij}$  is as defined following equation (26).

Overall, equation (30) represents  $N$  equations corresponding to the  $N$  points on the plane boundary. These equations are reduced to  $N/2$  equations with the use of the symmetry conditions.

The second equation representing equation (27) when symmetry is taken into account becomes equation (31):

$$\begin{aligned}
& -\frac{Bi}{2} \sum_{j=1}^N \phi_j \left[ \left\{ \frac{(\bar{\xi}_j - \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{\xi}_j)^2 - (\bar{\xi}_{j+1} - \bar{s})^2}{2(\bar{\xi}_{j+1} - \bar{\xi}_j)} \right\} \right. \\
& \quad \left[ \log \sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2} - \log \sqrt{(\bar{\xi}_{j+1} - \bar{s})^2 + \bar{D}^2} \right] \\
& \quad + (\bar{\xi}_{j+1} - \bar{\xi}_j) (\log \sqrt{(\bar{\xi}_{j+1} - \bar{s})^2 + \bar{D}^2} - 1) + \sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2} \\
& \quad \cdot \cos^{-1} \left\{ \frac{(\bar{\xi}_j - \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{s})^2 + 2\bar{D}^2 - (\bar{\xi}_{j+1} - \bar{\xi}_j)^2}{2\sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2} \sqrt{(\bar{\xi}_{j+1} - \bar{s})^2 + \bar{D}^2}} \right\}
\end{aligned}$$

$$\begin{aligned}
& \left\{ \sin \left[ \cos^{-1} \left\{ \frac{(\bar{\xi}_j - \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{\xi}_j)^2 - (\bar{\xi}_{j+1} - \bar{s})^2}{2\sqrt{(\bar{\xi}_j - \bar{s})^2 + D^2(\bar{\xi}_{j+1} - \bar{\xi}_j)}} \right\} \right] \right\} + \\
& \left\{ \frac{(\bar{\xi}_j + \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{\xi}_j)^2 - (\bar{\xi}_{j+1} + \bar{s})^2}{2(\bar{\xi}_{j+1} - \bar{\xi}_j)} \right\} \\
& [\log \sqrt{(\bar{\xi}_j + \bar{s})^2 + D^2} - \log \sqrt{(\bar{\xi}_{j+2} + \bar{s})^2 + D^2}] \\
& + (\bar{\xi}_{j+1} - \bar{\xi}_j) (\log \sqrt{(\bar{\xi}_{j+1} + \bar{s})^2 + D^2} - 1) + \sqrt{(\bar{\xi}_j + \bar{s})^2 + D^2} \\
& \cdot \cos^{-1} \left\{ \frac{(\bar{\xi}_j + \bar{s})^2 + (\bar{\xi}_{j+1} + \bar{s})^2 + 2D^2 - (\bar{\xi}_{j+1} - \bar{\xi}_j)^2}{2\sqrt{(\bar{\xi}_j + \bar{s})^2 + D^2}\sqrt{(\bar{\xi}_{j+1} + \bar{s})^2 + D^2}} \right\} \\
& \left\{ \sin \left[ \cos^{-1} \left\{ \frac{(\bar{\xi}_j + \bar{s})^2 + (\bar{\xi}_{j+1} - \bar{\xi}_j)^2 - (\bar{\xi}_{j+1} + \bar{s})^2}{2\sqrt{(\bar{\xi}_j + \bar{s})^2 + D^2}(\bar{\xi}_{j+1} - \bar{\xi}_j)} \right\} \right] \right\} \\
& + 2 \log A[(\bar{\xi}_{j+1} - \bar{\xi}_j)] + (\log 2\bar{s} + \log A) \bar{q}_0^1 \\
& - \frac{1}{2} \frac{1}{2\bar{s}} \bar{q}_2^1 = - \phi_0^1 \tag{31}
\end{aligned}$$

The third equation representing equation (28) when symmetry is taken into account becomes equation (32):

$$\begin{aligned}
 & -\frac{Bi}{A} \sum_{j=k}^N \phi_j \left[ \sin^{-1} \frac{\bar{D}}{\sqrt{(\bar{\xi}_{j+1} - \bar{s})^2 + \bar{D}^2}} - \sin^{-1} \left\{ \frac{\bar{D}}{\sqrt{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2}} \right\} \right. \\
 & \quad \left. + \sin^{-1} \left\{ \frac{\bar{D}}{\sqrt{(\bar{\xi}_{j+1} + \bar{s})^2 + \bar{D}^2}} \right\} - \sin^{-1} \left\{ \frac{\bar{D}}{\sqrt{(\bar{\xi}_j + \bar{s})^2 + \bar{D}^2}} \right\} \right] \\
 & \quad - \left\{ \frac{\pi}{A} \left( 1 + \frac{1}{4s^2} \right) \bar{q}_1^1 \right\} = 0 \quad (32)
 \end{aligned}$$

The special situations which were discussed earlier following equation (28) also are applicable to equation (32) and need to be considered.

The fourth equation representing equation (29) when symmetry is taken into account becomes equation (33).

$$\begin{aligned}
 & \frac{Bi}{2A} \sum_{j=k}^N \phi_j \left[ \log\{(\bar{\xi}_{j+1} - \bar{s})^2 + \bar{D}^2\} - \log\{(\bar{\xi}_j - \bar{s})^2 + \bar{D}^2\} \right. \\
 & \quad \left. - \log\{(\bar{\xi}_{j+1} + \bar{s})^2 + \bar{D}^2\} - \log\{(\bar{\xi}_j + \bar{s})^2 + \bar{D}^2\} \right] \\
 & \quad + \frac{\pi}{A \cdot \bar{s}} \bar{q}_0^1 - \left\{ \frac{\pi}{A} \left( 1 + \frac{1}{4s^2} \right) \right\} \bar{q}_2^1 = 0 \quad (33)
 \end{aligned}$$

Therefore, the total number of equations to be solved are  $(N/2+3)$  instead of  $(N+6)$ . This represents a considerable reduction of computational time, especially if the distance,  $2\bar{s}$ , between the tubes is large. The reduction in number of equations also improves the accuracy of the solution as the round-off errors are minimized.

## CHAPTER V

### THE GENERAL PROBLEM

In the previous chapter the problem of two tubes of equal diameter and temperature located equidistant from the plane surface is formulated. However, the more general problem of use to practicing engineers may involve arbitrary number of isothermal tubes with arbitrary diameters and locations. The tubes may be at different temperatures. The equations for such a general problem are developed in this chapter. Tubes maintained at different temperatures can be found in buried utility lines which transport fluids or power involving thermal effects. The equations are developed for the analysis of two-dimensional heat transfer from a set of parallel tubes of arbitrary diameters and positions, located in a semi-infinite medium. The surface temperature of each tube is considered to be uniform. Similar to the two-tube problem, the plane surface is considered to be exposed to uniform convective environment. The notation may slightly vary from that of two-tube and symmetry problems because of the complexity involved in this problem. The same nondimensionalization parameters have been used as in the case of two-tube problem.

A sketch of the cross-section of two dimensional circular tubes embedded in a semi-infinite medium in its nondimensional form is presented in Figure 5. As stated earlier, this is a steady-state problem with constant thermal conductivity.

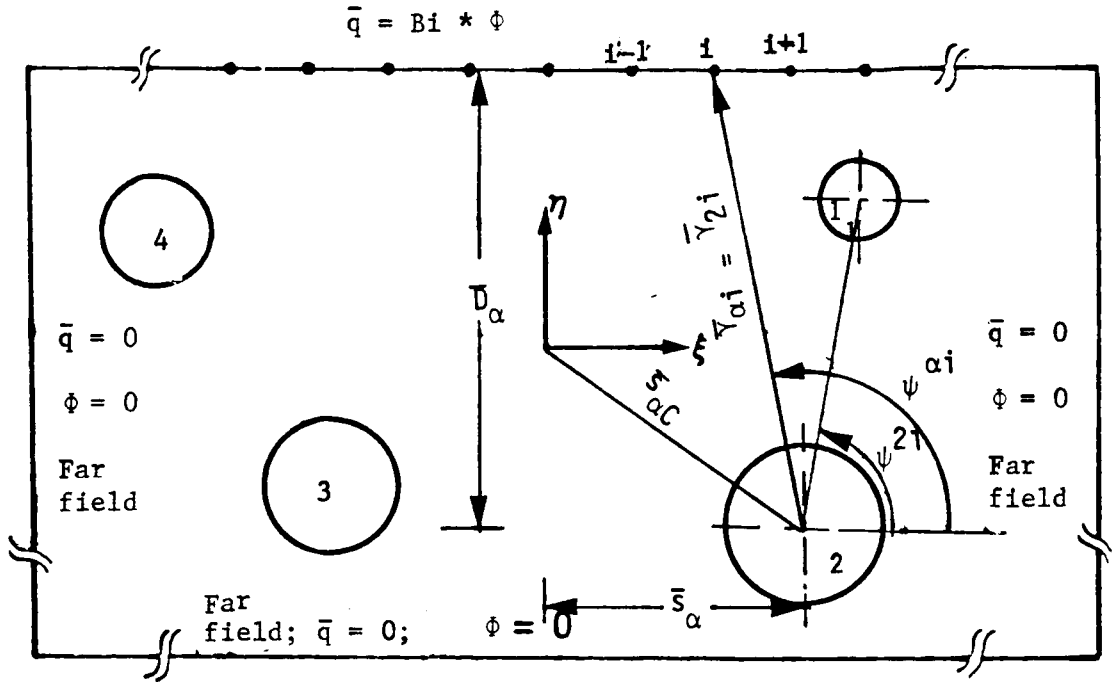


Figure 5. A sketch of the two-dimensional problem and boundary conditions in nondimensional form.

The nondimensional equations (22 - 25) can be used to derive the equations for this problem.

In equation (22) the parameters which need to be evaluated differently from that of two-tube problem are redefined here with the notation shown in Figure 5.

$$\bar{\gamma}_{\alpha i} = \sqrt{(\bar{\xi}_i - \bar{s})^2 + D^2}$$

where  $D$  = depth of  $\alpha$  th tube,

$\bar{s}_\alpha$  = distance from the  $\eta$  -axis to  $\alpha$  tube parallel to the plane surface. If  $s_{\alpha C}$  is specified by user instead of  $s_\alpha$ ,  $s_\alpha$  can be evaluated by

$$\begin{aligned}\bar{s}_\alpha &= \sqrt{\bar{s}_{\alpha c}^2 - (\bar{D}_\alpha - \bar{D}_\beta)^2} \\ \sin \psi^{\alpha i} &= \frac{\bar{D}_\alpha}{\bar{\gamma}_{\alpha i}} \\ \cos \psi^{\alpha i} &= \frac{(\bar{\xi}_i - \bar{s}_\alpha)}{\bar{\gamma}_{\alpha i}}\end{aligned}$$

With these parameters, equation (22) becomes

$$\begin{aligned}\frac{1}{2} \phi_i - \frac{Bi}{2\pi} \sum_{j=1}^N \phi_j [\tau_{ij}] + \sum_{\alpha=1}^N [\bar{a}_\alpha \{ \log \bar{\gamma}_{\alpha i} + \log A \} \bar{q}_1^\alpha \\ - \frac{1}{2} \bar{a}_\alpha^2 \cdot \frac{\bar{D}_\alpha}{\bar{\gamma}_{\alpha i}} \bar{q}_1^\alpha - \frac{1}{2} \bar{a}_\alpha^2 \frac{(\bar{\xi}_i - \bar{s}_\alpha)}{\bar{\gamma}_{\alpha i}} \bar{q}_2^\alpha] = 0.0\end{aligned}\quad (34)$$

As discussed in the two-tube problem, the second term of this equation changes depending on relative position of  $i$  and  $j$ . i.e. whether  $i = j$  or  $i = j+1$  or  $i > j+1$  or  $i < j$ . It changes in the same manner as it did in the problem of two tubes.

This equation (34) yields  $N$  number of equations corresponding to  $N$  discretized segments of the plane wall boundary.

Now some terms in equation (23) are defined here with new notation.

$$\bar{\gamma}_{\alpha\beta} = \sqrt{(\bar{D}_\beta - \bar{D}_\alpha)^2 + (\bar{s}_\alpha - \bar{s}_\beta)^2}$$

where  $\bar{D}_\beta$  and  $\bar{D}_\alpha$  are the depths of  $\beta$  and  $\alpha$  tubes, respectively.  $\beta$  is the tube where  $i$  is centered and,  $\bar{s}_\beta$  is the distance from the  $\eta$ -axis to  $\beta$ th tube parallel to the plane surface.

$$\sin \psi^{\alpha\beta} = \frac{(\overline{D}_\alpha - \overline{D}_\beta)}{\overline{\gamma}_{\alpha\beta}}$$

$$\cos \psi^{\alpha\beta} = \frac{(\overline{s}_\beta - \overline{s}_\alpha)}{\overline{\gamma}_{\alpha\beta}}$$

With this new notation and previously defined terms, equation (23) now becomes for general problem,

$$\begin{aligned} & -\frac{Bi}{2\pi} \sum_{j=1}^N \Phi_j \left[ \frac{(\overline{\xi}_j - \overline{s}_\beta)^2 + (\overline{\xi}_{j+1} - \overline{\xi}_j)^2 - (\overline{\xi}_{j+1} - \overline{s}_\beta)^2}{2(\overline{\xi}_{j+1} - \overline{\xi}_j)} \right. \\ & \quad \left. \{ \log \sqrt{(\overline{\xi}_j - \overline{s}_\beta)^2 + \overline{D}_\beta^2} - \log \sqrt{(\overline{\xi}_{j+1} - \overline{s}_\beta)^2 + \overline{D}_\beta^2} \} \right. \\ & \quad \left. + (\overline{\xi}_{j+1} - \overline{\xi}_j) (\log \sqrt{(\overline{\xi}_{j+1} - \overline{s}_\beta)^2 + \overline{D}_\beta^2} - 1) + \sqrt{(\overline{\xi}_j - \overline{s}_\beta)^2 + \overline{D}_\beta^2} \right. \\ & \quad \left. \cos^{-1} \left\{ \frac{(\overline{\xi}_j - \overline{s}_\beta)^2 + 2\overline{D}_\beta^2 + (\overline{\xi}_{j+1} - \overline{s}_\beta)^2 - (\overline{\xi}_{j+1} - \overline{\xi}_j)^2}{\sqrt{2(\overline{\xi}_j - \overline{s}_\beta)^2 + \overline{D}_\beta^2} \sqrt{(\overline{\xi}_j - \overline{s}_\beta)^2 + \overline{D}_\beta^2}} \right\} \right. \\ & \quad \left. \{ \sin [\cos^{-1} \left\{ \frac{(\overline{\xi}_j - \overline{s}_\beta)^2 + (\overline{\xi}_{j+1} - \overline{\xi}_j)^2 - (\overline{\xi}_{j+1} - \overline{s}_\beta)^2}{2\sqrt{(\overline{\xi}_j - \overline{s}_\beta)^2 + \overline{D}_\beta^2} (\overline{\xi}_{j+1} - \overline{\xi}_j)} \right\}] \} \right. \\ & \quad \left. + (\overline{\xi}_{j+1} - \overline{\xi}_j) \log A \right] + \{ \overline{a}_\beta (\log \overline{a}_\beta + \log A) \} \overline{q}_0^\beta + \\ & \quad + \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M [\{ \overline{a}_\alpha (\log \overline{\gamma}_{\alpha\beta} + \log A) \} \overline{q}_0^\alpha - \frac{1}{2} \frac{\overline{a}_\alpha^2 (\overline{D}_\alpha - \overline{D}_\beta)}{\overline{\gamma}_{\alpha\beta}^2} \overline{q}_1^\alpha \\ & \quad - \frac{1}{2} \frac{\overline{a}_\alpha^2 (\overline{s}_\beta - \overline{s}_\alpha)}{\overline{\gamma}_{\alpha\beta}^2} \overline{q}_2^\alpha ] = -\Phi_0^\beta \end{aligned} \quad (35)$$

Thus, this equation (35) yields M number of equations for M number of tubes.

Similar to equations (22) and (23), some of the terms in equations (24) and (25) are defined here with new notation.

$$\sin 2\psi^{\alpha\beta} = \sin[2 \sin^{-1} \frac{(\overline{D}_\alpha - \overline{D}_\beta)}{\sqrt{(\overline{D}_\beta - \overline{D}_\alpha)^2 + (\overline{s}_\beta - \overline{s}_\alpha)^2}}]$$

$$\cos 2\psi^{\alpha\beta} = \cos[2 \cos^{-1} \{ \frac{(\overline{s}_\beta - \overline{s}_\alpha)}{(\overline{D}_\beta - \overline{D}_\alpha)^2 + (\overline{s}_\beta - \overline{s}_\alpha)^2} \}]$$

With this new notation, equations (24) and (25) now become,

$$\begin{aligned} & - \frac{Bi}{A} \sum_{j=1}^N \Phi_j [\sin^{-1} \{ \frac{\overline{D}_\beta}{\sqrt{(\overline{\xi}_{j+1} - \overline{s}_\beta)^2 + \overline{D}_\beta^2}} \} - \sin^{-1} \{ \frac{\overline{D}_\beta}{\sqrt{(\overline{\xi}_j - \overline{s}_\beta)^2 + \overline{D}_\beta^2}} \}] \\ & - \frac{\pi}{A} \overline{q}_1^\beta + \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M [ \frac{2\pi \overline{a}_\alpha}{A} \cdot \frac{\overline{D}_\alpha - \overline{D}_\beta}{\overline{\gamma}_{\alpha\beta}} \overline{q}_0^\alpha + \\ & \quad \frac{\overline{a}_\alpha^2}{A} \cdot \frac{\pi}{\overline{\gamma}_{\alpha\beta}^2} \{ \sin(2 \sin^{-1} \{ \frac{\overline{D}_\alpha - \overline{D}_\beta}{\overline{\gamma}_{\alpha\beta}} \}) \} \overline{q}_2^\alpha \\ & - \{ \frac{\overline{a}_\alpha^2}{A} \cdot \frac{\pi}{\overline{\gamma}_{\alpha\beta}} \{ \cos(2 \cos^{-1} \{ \frac{(\overline{s}_\beta - \overline{s}_\alpha)}{\overline{\gamma}_{\alpha\beta}} \}) \} \} \overline{q}_1^\alpha ] = 0 \end{aligned} \quad (36)$$

$$\begin{aligned}
& \frac{Bi}{2A} \sum_{j=1}^N \phi_j [\log\{(\bar{\xi}_{j+1} - \bar{s}_\beta)^2 + \bar{D}_\beta^2\} - \log\{(\bar{\xi}_j - \bar{s}_\beta)^2 + \bar{D}_\beta^2\}] - \frac{\pi}{A} \bar{q}_2^\beta \\
& + \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M \left[ \left\{ \frac{2\pi \bar{a}_\alpha}{A} \cdot \frac{1}{\bar{\gamma}_{\alpha\beta}} \frac{(\bar{s}_\beta - \bar{s}_\alpha)}{\bar{\gamma}_{\alpha\beta}} \right\} \bar{q}_0^\alpha \right. \\
& + \left\{ \frac{\bar{a}_\alpha^2}{A} \cdot \frac{\pi}{\bar{\gamma}_{\alpha\beta}^2} \sin\left(2 \sin^{-1} \frac{(\bar{D}_\alpha - \bar{D}_\beta)}{\bar{\gamma}_{\alpha\beta}}\right) \right\} \bar{q}_1^\alpha \\
& + \left\{ \frac{\pi \bar{a}_\alpha^2}{A \bar{\gamma}_{\alpha\beta}^2} \cos\left(2 \cos^{-1} \frac{(\bar{s}_\beta - \bar{s}_\alpha)}{\bar{\gamma}_{\alpha\beta}}\right) \right\} \bar{q}_2^\alpha \right] = 0 \quad (37)
\end{aligned}$$

(A programming note : As discussed in the two-tube problem, here too, two special cases occur while evaluating the first terms of the equations (36) and (37) and they need to be dealt with in a similar manner. The first case when  $\bar{\xi}_j < XT(L)$  and  $\bar{\xi}_{j+1} > XT(L)$  and second case when  $\bar{\xi}_j$  and  $\bar{\xi}_{j+1}$  both less than  $XT(L)$  where  $XT(L)$  is the location of the L number tube parallel to the convective surface.

In addition to the above, another special case occurs while evaluating the coefficients of  $\bar{q}^\alpha$  and  $\bar{q}^\beta$  . i.e. when  $XT(K)$  is less than  $XT(L)$  where K represents  $\beta$  th tube and L represents  $\alpha$  th tube. In this case, the sign of  $\bar{q}_2$  coefficient in equation (36) and the sign of  $\bar{q}_1$  coefficient in equation (37) need to be changed. This situation has been taken care in the computer program that was developed for general problem.)

Equations (36) and (37) yield M number of equations each for M number of tubes. Thus equations (34) - (37) yield (N+3M) number of equations for (N+3M) number of unknown values.

The unknown values, temperatures at different nodes on the convective surface and  $\bar{q}$  s on tube surfaces given the temperatures on tube surfaces can be evaluated numerically by using the computer program that was developed.

The criteria used for the selection of  $\Delta\bar{\xi}$ ,  $\bar{\xi}_{\max}$  and location of **axes** are discussed in the next chapter.

## CHAPTER VI

### CRITERIA USED FOR THE SELECTION OF LOCATION OF THE

#### COORDINATE AXES, $\bar{\xi}_{\max}$ , AND $\Delta\bar{\xi}$

##### The Location of the Coordinate Axes

In the general problem it is desirable to locate the coordinate axes approximately at the center of array of the tubes. In the case of two-tube problem, it has always been placed halfway between the center of the two tubes to take advantage of the symmetry in the problem. However, this may not be exactly possible in the general problem as the tube array may not even have an axis of symmetry. For this reason, a criteria was established in the computer program of general problem to move the axes approximately to the center of the tubes. Such a choice is helpful in the selection of maximum length used ( $\bar{\xi}_{\max}$ ) to represent the infinite region to near minimum value so that the number of equations can be kept small in order to reduce the computational time. The user can prescribe the tube positions relative to an arbitrary location of the coordinate axes. The computer program will move the center of the coordinate axes to the approximate center of the array of tubes.

##### Selection of $\bar{\xi}_{\max}$ from the Results of Two-Tube Problem

In the numerical formulation, the point  $\bar{\xi} \rightarrow \infty$  is represented with  $\bar{\xi} \rightarrow \bar{\xi}_{\max}$  where  $\bar{\xi}_{\max}$  is sufficiently large. Thus, for computational purposes a finite region is chosen to represent the infinite region. As  $\bar{\xi}_{\max}$  is decreased, the computational time will be

decreased when segments of constant length are used to discretize the plane-wall boundary. Thus, the proper selection of minimum value of  $\bar{\xi}_{\max}$  is necessary for significant reduction in the number of equations to be solved and hence the computational time. Numerical experiments were conducted to establish appropriate criteria for the selection of  $\bar{\xi}_{\max}$ . These are discussed below in detail.

Many solutions were obtained for two-tube problem for different values of input parameters, each time varying the value of  $\bar{\xi}_{\max}$  and keeping other parameters constant, and the effect of  $\bar{\xi}_{\max}$  value on the accuracy of the results was determined. Some of these results are shown in Tables 1 and 2. From these values, it can be seen that selection of  $\bar{\xi}_{\max}$  value has a definite effect on the accuracy of the solution. Therefore, it would be desirable to determine the optimum value for  $\bar{\xi}_{\max}$  that would yield a solution within a prior prescribed accuracy of, say, 1 %. It should also be noted that in general as  $\bar{\xi}_{\max}$  value is increased, the number of equations would increase representing the increased number of segments on the convective plane surface. This in turn would require more computational time to solve for the unknown coefficients.

From the results shown in Tables 1 and 2, an attempt was made to develop suitable criterion that can be readily incorporated in the general computer program. As can be seen from these results, the driving parameters in the selection of  $\bar{\xi}_{\max}$  value should be the depth, the distance to the center of the farthest located tube ( $X_{T_{\max}}$ ) and Biot number. Separately, a small computer program, based on least squares method, was developed for selecting the best fit

Table 1 : Results for one two-tube problem to illustrate the effect of  $\bar{\xi}_{\max}$  on the accuracy of the solution.  
(BI = 0.5;  $\Delta\bar{\xi}$  = 0.4;  $\bar{D}$  = 6.0;  $2\bar{s}$  = 10.0)

| $\bar{\xi}_{\max}$ | $\bar{q}_0$ | % accu-<br>racy of<br>$\bar{q}_0$ | $\bar{q}_1$ | % accu-<br>racy of<br>$\bar{q}_1$ | $\bar{q}_2$ | % accu-<br>racy of<br>$\bar{q}_2$ | Temp.<br>at<br>$\bar{\xi}=6.0$ | % accu-<br>racy of<br>Temp. |
|--------------------|-------------|-----------------------------------|-------------|-----------------------------------|-------------|-----------------------------------|--------------------------------|-----------------------------|
| 10                 | 0.1843      | -                                 | 0.1190      | -                                 | -0.0042     | -                                 | 0.4024                         | -                           |
| 18                 | 0.2741      | 32.75                             | 0.0964      | 23.41                             | 0.0308      | 113.67                            | 0.2628                         | 53.16                       |
| 28                 | 0.3010      | 8.94                              | 0.0843      | 14.31                             | 0.0382      | 19.40                             | 0.2331                         | 12.70                       |
| 38                 | 0.3091      | 2.62                              | 0.0802      | 5.16                              | 0.0399      | 4.21                              | 0.2250                         | 3.60                        |
| 54                 | 0.3136      | 1.43                              | 0.0778      | 3.08                              | 0.0407      | 1.97                              | 0.2207                         | 1.95                        |
| 62                 | 0.3146      | 0.32                              | 0.0773      | 0.65                              | 0.0408      | 0.25                              | 0.2197                         | 0.46                        |
| 72                 | 0.3154      | 0.25                              | 0.0768      | 0.65                              | 0.0409      | 0.24                              | 0.2189                         | 0.37                        |
| 90                 | 0.3162      | 0.25                              | 0.0764      | 0.57                              | 0.0410      | 0.26                              | 0.2182                         | 0.32                        |

Table 2 : Results for a second two-tube problem to illustrate the effect  $\bar{\xi}_{\max}$  on the accuracy of the solution.  
(BI = 1.0;  $\Delta\bar{\xi}$  = 1.0;  $D$  = 25.0;  $2s$  = 10.0)

| $\bar{\xi}_{\max}$ | $\bar{q}_0$ | % accu-<br>racy of<br>$\bar{q}_0$ | $\bar{q}_1$ | % accu-<br>racy of<br>$\bar{q}_1$ | $\bar{q}_2$ | % accu-<br>racy of<br>$\bar{q}_2$ | Temp.<br>at<br>$\bar{\xi}=11.0$ | % accu-<br>racy of<br>Temp. |
|--------------------|-------------|-----------------------------------|-------------|-----------------------------------|-------------|-----------------------------------|---------------------------------|-----------------------------|
| 10                 | 0.0762      | --                                | 0.0260      | --                                | 0.0105      | --                                | --                              | --                          |
| 18                 | 0.1189      | 35.9                              | 0.0250      | 4.16                              | 0.0200      | 47.67                             | 0.0622                          | --                          |
| 28                 | 0.1454      | 18.23                             | 0.0221      | 13.26                             | 0.0264      | 24.27                             | 0.0376                          | 65.73                       |
| 38                 | 0.1586      | 0.833                             | 0.0196      | 12.70                             | 0.0296      | 10.82                             | 0.0306                          | 22.55                       |
| 60                 | 0.1705      | 6.98                              | 0.0166      | 18.00                             | 0.0324      | 8.46                              | 0.0259                          | 18.41                       |
| 90                 | 0.1756      | 2.90                              | 0.0151      | 10.08                             | 0.0334      | 3.20                              | 0.0242                          | 7.08                        |
| 110                | 0.1769      | 0.73                              | 0.0146      | 3.42                              | 0.0337      | 0.89                              | 0.0236                          | 2.54                        |
| 126                | 0.1775      | 0.34                              | 0.0144      | 1.39                              | 0.0338      | 0.30                              | 0.0235                          | 0.43                        |
| 150                | 0.1782      | 0.39                              | 0.0142      | 1.41                              | 0.0340      | 0.59                              | 0.0233                          | 0.86                        |
| 180                | 0.1786      | 0.22                              | 0.0140      | 1.08                              | 0.0341      | 0.26                              | 0.0232                          | 0.65                        |

\* percentage difference from the previous value.

equation to select  $\bar{\xi}_{\max}$ . Three types of curve fits representing linear, exponential, and power functions were used in this program. Using this curve fitting computer program with the results in Tables 1 and 2, it was found that power function yielded the best fit of the three functions(see Appendix-E for details).

The results in Tables 1 and 2 yielded the following best-fit relationship in the sense of least squares.

$$\bar{\xi}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}}$$

In the computer program developed for two-tube problem, the starting value of  $\bar{\xi}_{\max}$  has been taken as

$$0.87 * 11.3 * \sqrt{X T_{\max} * \bar{D}}$$

and in the second iteration, it has been increased to

$$11.3 * \sqrt{X T_{\max} * \bar{D}}$$

If the difference between  $\bar{q}_0$  values for these two iterative values of  $\bar{\xi}_{\max}$  is less than 1 %, the value of  $\bar{\xi}_{\max}$  that was selected in the later iteration is considered as satisfactory and that no further adjustments in  $\bar{\xi}_{\max}$  are needed. If not, the value of  $\bar{\xi}_{\max}$  is increased by 12.5 % for each iteration after the second iteration till the difference in the values of present iteration and previous iteration falls below 1 %. In most of the problems, it was observed that the 1 % accuracy criterion is satisfied within two iterations. However, in few problems, where depth of tubes and Biot number are very small, three or more iterations were needed to meet the required

accuracy. Thus, the prescribed accuracy for heat transfer rate is achieved by using the iterative technique to select the value for  $\bar{\epsilon}_{\max}$ .

But it is also observed that the above criterion for  $\bar{\epsilon}_{\max}$ , while highly satisfactory for determining the heat transfer from the tubes, is not satisfactory if the solution of interest is the temperature distribution at the plane convective surface which should satisfy the far-field boundary condition that  $\phi \rightarrow 0$  as  $\bar{\epsilon} \rightarrow \bar{\epsilon}_{\max}$ . This can be seen clearly in Figures 6 and 7. The temperature starts raising at about  $0.75 \bar{\epsilon}_{\max}$ . However, it can be seen from the results shown in Figures 8 and 9 that this temperature distribution is a function of  $\bar{\epsilon}_{\max}$ . It is observed from these Figures that it is necessary to take  $\bar{\epsilon}_{\max}$  values equal to 20 to 25 depths to satisfy the far-field boundary condition.

$$\bar{\epsilon}_{\max} = 23 \bar{D}$$

$$\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}}$$

In summary, equation (a) gives good accuracy for both  $\bar{q}$  and  $\phi$  solutions, where as equation (b) gives good accuracy for  $\bar{q}$  solutions only. However, the CPU time when equation (a) is used is typically 16 times that required when equation (b) is used. Thus, the iteration technique developed earlier can be used much more economically, without any reservation, when the heat transfer from the tubes is only of interest.

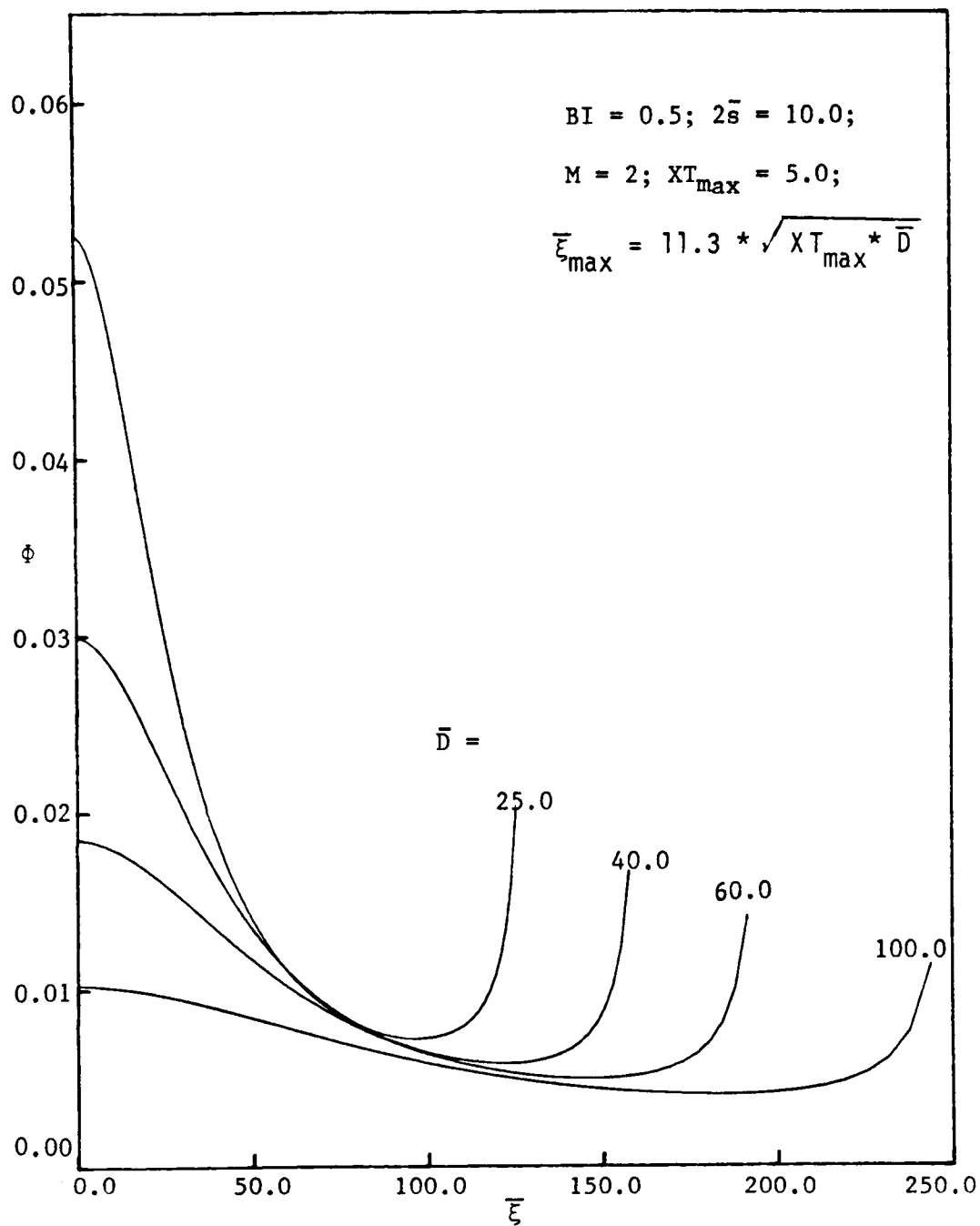


Figure 6. Temperature distribution on the plane convective surface using  $\bar{\xi}_{\max} = 11.3 * \sqrt{XT_{\max} * \bar{D}}$  criterion for two tubes in a semi-infinite medium. TWOTUBE results. (TWOSYMM yielded identical results)

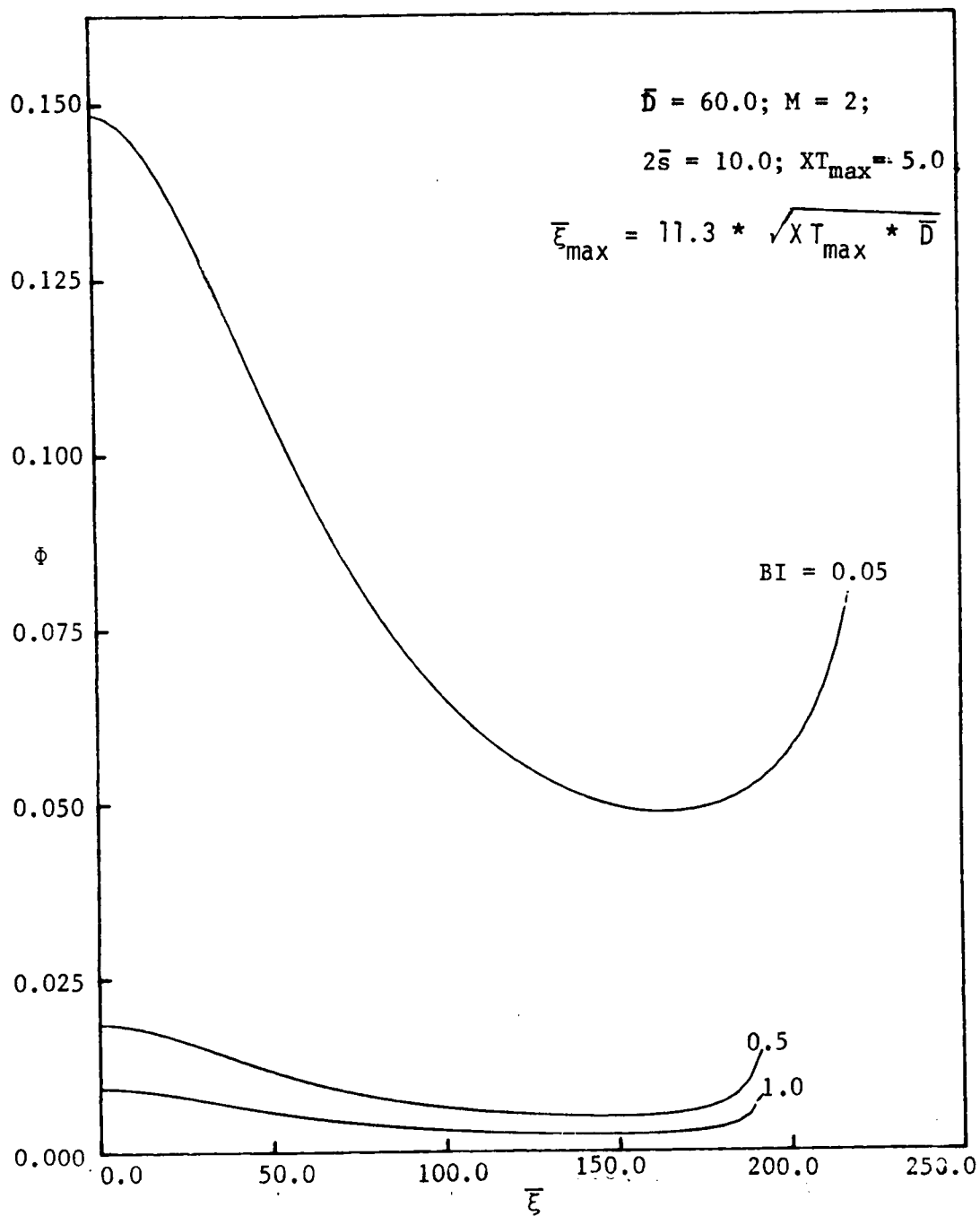


Figure 7. Temperature distribution on the plane convective surface using  $\bar{\xi}_{\max} = 11.3 * \sqrt{XT_{\max} * \bar{D}}$  criterion for two tubes in a semi-infinite medium. TWOSYMM results.

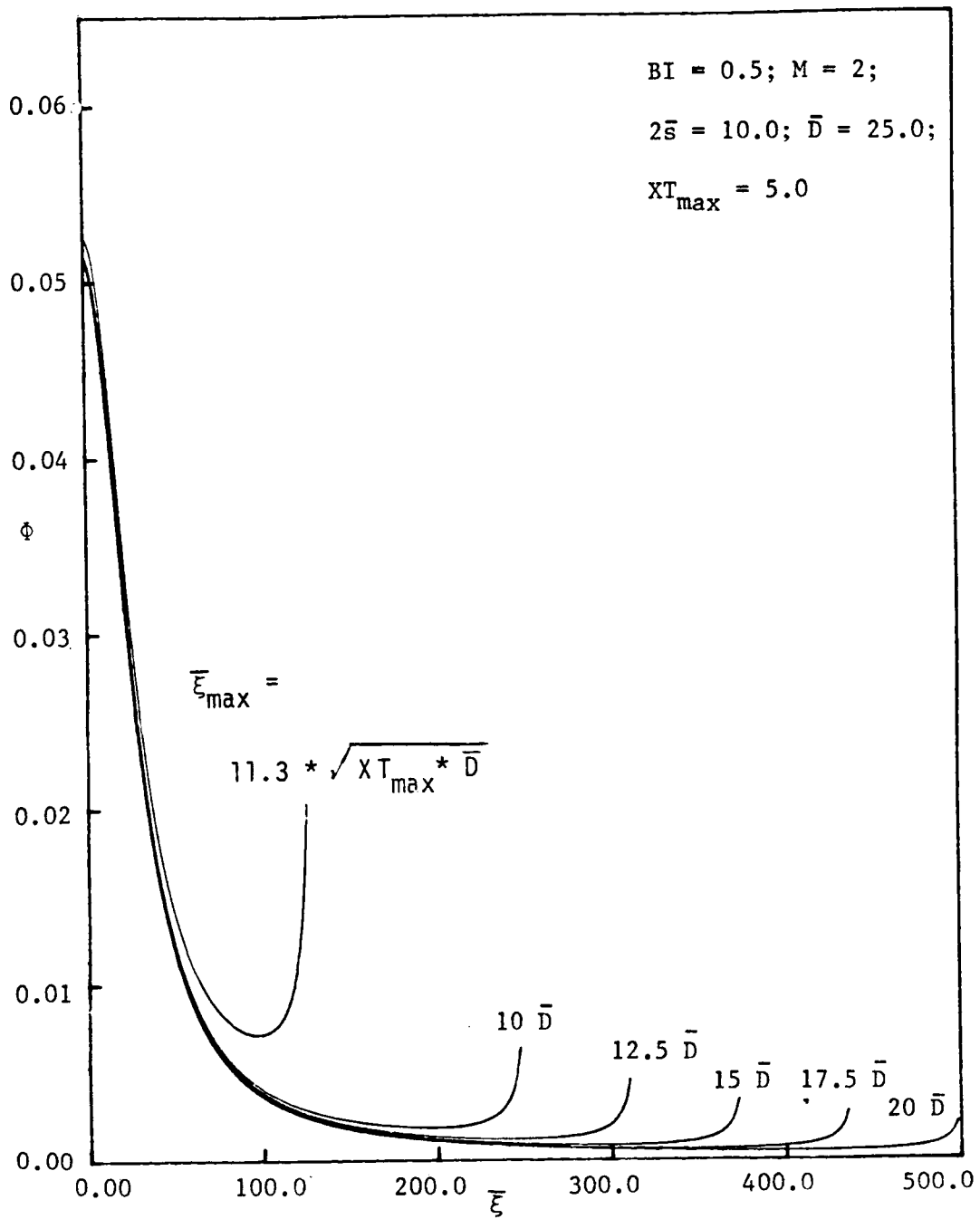


Figure 8. Temperature distribution as a function of  $\bar{\xi}$  and  $\bar{\xi}_{max}$  (range of  $\bar{\xi}_{max}$ :  $11.3 * \sqrt{X T_{max} * \bar{D}}$  to  $20 \bar{D}$ ) for two tubes in a semi-infinite medium. TWOSYMM results.

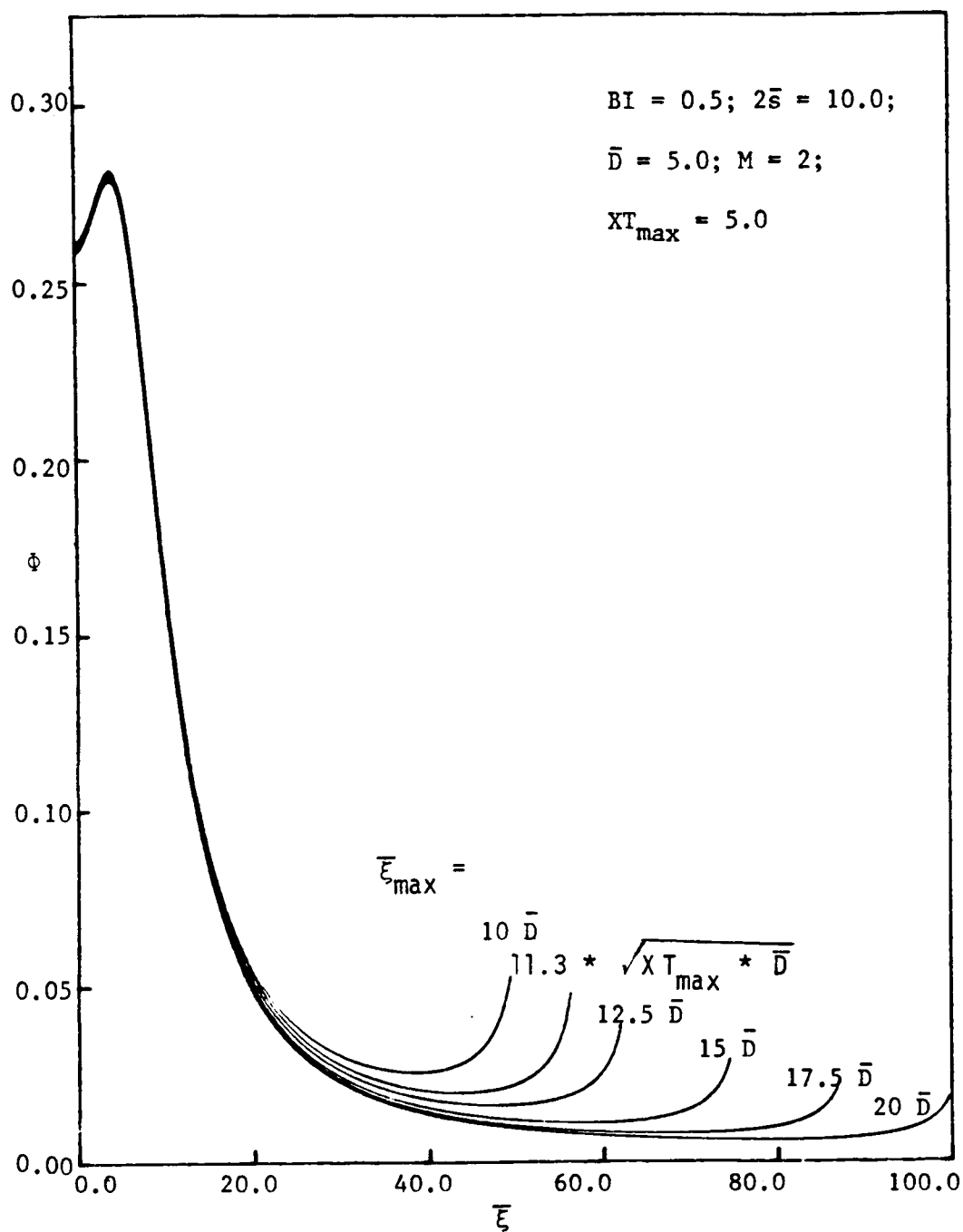


Figure 9. Temperature distribution as a function of  $\bar{\xi}$  and  $\bar{\xi}_{\max}$  ( $\bar{\xi}_{\max}$  range:  $10 \bar{D}$  to  $20 \bar{D}$ ) for two tubes at a shallow depth in a semi-infinite medium.

### Selection of $\Delta \bar{\xi}$ from Results of Two-Tube Problem

With the criteria established for the selection of  $\bar{\xi}_{\max}$ , it is possible to establish a criterion for selection of segment length(  $\Delta \bar{\xi}$  ). From the results shown in Tables 3 and 4, it is clear that  $\Delta \bar{\xi}$  is not a function of either Biot number or  $\bar{\xi}_{\max}$ . Same trends were observed at these two Biot numbers and  $\bar{\xi}_{\max}$ . As can be expected,  $\Delta \bar{\xi}$  was found to be a function of depth of the tubes. From these results at different tube depths for different  $\Delta \bar{\xi}$ , it was concluded that the selection of value of  $\Delta \bar{\xi}$  as one fifteenth of depth of the tube that is nearest to the plane convective boundary.

Having established criteria for the selection of  $\bar{\xi}_{\max}$  and  $\Delta \bar{\xi}$  values in the problem of two tubes, these criteria can now be applied to determine its applicability for two test problems that are representative of the general case. The three tube problem of Figure 10 is chosen as the first test problem and the results are shown in Table 5. In this Table, two values are given for heat transfer rate  $\bar{q}$  s for each tube. The first value is obtained with the first iteration and the second value is obtained with the second iteration. The problem of five tubes shown in Figure 11 is chosen as the second test problem and the results are given in Table 6. At each  $\Delta \bar{\xi}$ , there are five values given for  $\bar{q}$  s. The first values represent the  $\bar{q}$  s from tube 1 of Figure 11 and second value from tube 2 and so on. The values tabulated are for the final iteration which turned out to be the second iteration. From these results, it can be seen that the criteria established for two-tube problem is equally satisfactory for the general problem. Therefore, the same criteria which has been

Table 3 : Results for one two-tube problem showing the influence of  $\Delta\bar{\xi}$  on the accuracy of the solution.

(BI = 0.5;  $\bar{D} = 18.0$ ;  $2s = 8.0$ ;

$$\bar{\xi}_{\max} = 11.3 * \sqrt{X T_{\max}} * \bar{D} = 96.0)$$

| $\Delta\bar{\xi}$ | $\bar{q}_0^1$ | $\bar{q}_1^1$ | $\bar{q}_2^1$ | Surface<br>Temp. at<br>$\bar{\xi} = 40.0$ |
|-------------------|---------------|---------------|---------------|-------------------------------------------|
| 0.5               | 0.1913        | 0.0201        | 0.0451        | 0.0166                                    |
| 0.7               | 0.1913        | 0.0201        | 0.0451        | 0.0166                                    |
| 0.875             | 0.1913        | 0.0201        | 0.0451        | 0.0163                                    |
| 1.0               | 0.1913        | 0.0201        | 0.0451        | 0.0165                                    |
| 2.0               | 0.1913        | 0.0201        | 0.0451        | 0.0165                                    |

Table 4 : Results for a second two-tube problem showing the influence of  $\Delta\bar{\xi}$  on the accuracy of the solution.

(BI = 1.0;  $\bar{\xi}_{\max} = 11.3 * \sqrt{X T_{\max}} * \bar{D} = 164.0$ ;

$\bar{D} = 30.0$ ;  $2s = 14.0$ )

| $\Delta\bar{\xi}$ | $\bar{q}_0^1$ | $\bar{q}_1^1$ | $\bar{q}_2^1$ | Surface<br>temp. at<br>$\bar{\xi} = 30.0$ |
|-------------------|---------------|---------------|---------------|-------------------------------------------|
| 1.0               | 0.1777        | 0.0116        | 0.0240        | 0.0122                                    |
| 2.0               | 0.1777        | 0.0116        | 0.0240        | 0.0120                                    |
| 3.0               | 0.1778        | 0.0116        | 0.0240        | 0.0119                                    |
| 4.0               | 0.1778        | 0.0116        | 0.0240        | --                                        |
| 5.0               | 0.1778        | 0.0116        | 0.0240        | 0.0115                                    |

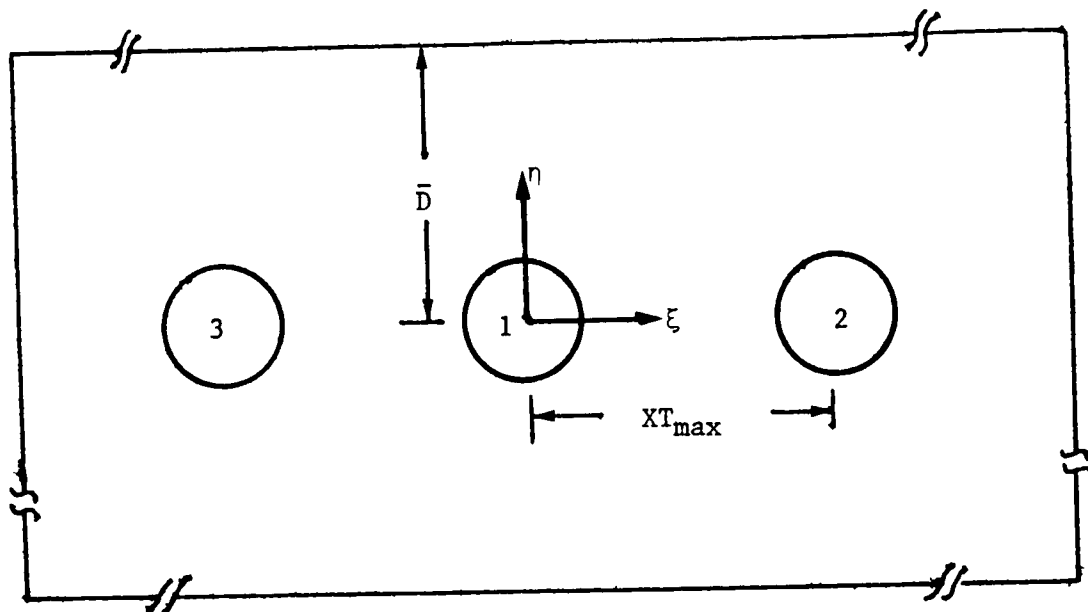


Figure 10. Sketch of the problem considered to illustrate the effect of  $\bar{\xi}_{max}$  criterion on the accuracy of the GENTUBE solution.

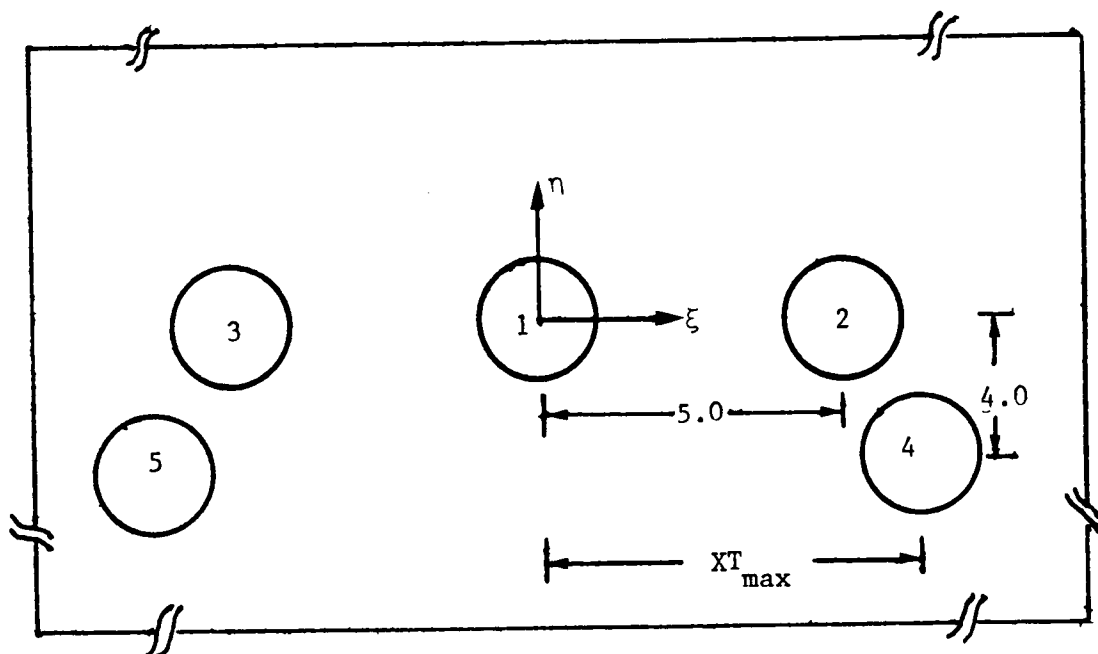


Figure 11. Sketch of the problem considered to illustrate the effect of  $\Delta \bar{\xi}$  on the accuracy of the GENTUBE solution.

Table 5 : Results of the problem of Figure 10 showing the effect of  $\xi_{\max}$  criteria (developed from the two-tube problem) on the accuracy of the solution. (BI = 0.1; D = 10.0;  $XT_{\max} = 6.0$ ;  $\xi_{\max} = 11.3 * \sqrt{XT_{\max} * D} = 87.5$ )

| $\xi_{\max}$ | Tube No. | $\bar{q}_0$ | % $q_0$ difference between iterations | $\bar{q}_1$ | $\bar{q}_2$ |
|--------------|----------|-------------|---------------------------------------|-------------|-------------|
| 87.5         | 1        | 0.1208      | 0.82                                  | 0.0348      | 0.0004      |
|              |          | 0.1218      |                                       | 0.0343      | 0.0004      |
| 87.5         | 2        | 0.1804      | 0.93                                  | 0.0342      | 0.0631      |
|              |          | 0.1821      |                                       | 0.0337      | 0.0637      |
| 87.5         | 3        | 0.1801      | 0.88                                  | 0.0340      | -0.0629     |
|              |          | 0.1817      |                                       | 0.0335      | -0.0635     |

Table 6 : Results for the problem of Figure 11 showing the effect of  $\Delta \bar{\xi}$  criteria (developed from the two-tube problem) on the accuracy of the solution. (BI = 0.3;  $\bar{D} = 30.0$ ;  $X T_{\max} = 7.5$ ;  $\bar{\xi}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}}$ )

| $\Delta \bar{\xi}$ | $\bar{\xi}_{\max}$ | $\bar{q}_0$ | $\bar{q}_1$ | $\bar{q}_2$ | Temp. at $\bar{\xi} = 60.0$ |
|--------------------|--------------------|-------------|-------------|-------------|-----------------------------|
| 1.2                | 169.2              | 0.0584      | 0.0254      | 0.0000      | 0.0212                      |
|                    |                    | 0.0670      | 0.0581      | 0.0129      |                             |
|                    |                    | 0.0670      | 0.0581      | -0.0129     |                             |
|                    |                    | 0.0980      | -0.0210     | 0.0576      |                             |
|                    |                    | 0.0980      | -0.0210     | -0.0576     |                             |
| 1.875              | 168.75             | 0.0584      | 0.0254      | 0.0000      | 0.0211                      |
|                    |                    | 0.0670      | 0.0581      | 0.0129      |                             |
|                    |                    | 0.0670      | 0.0581      | 0.0129      |                             |
|                    |                    | 0.0980      | -0.0210     | 0.0576      |                             |
|                    |                    | 0.0980      | -0.0209     | -0.0575     |                             |
| 2.4                | 170.40             | 0.0584      | 0.0253      | 0.0000      | 0.0209                      |
|                    |                    | 0.0670      | 0.0581      | 0.0129      |                             |
|                    |                    | 0.0670      | 0.0581      | -0.0129     |                             |
|                    |                    | 0.0980      | -0.0210     | 0.0576      |                             |
|                    |                    | 0.0980      | -0.0210     | -0.0576     |                             |
| 3.0                | 171.00             | 0.0584      | 0.0253      | 0.0000      | 0.0207                      |
|                    |                    | 0.0670      | 0.0581      | 0.0129      |                             |
|                    |                    | 0.0670      | 0.0581      | -0.0129     |                             |
|                    |                    | 0.0981      | -0.0210     | 0.0576      |                             |
|                    |                    | 0.0980      | -0.0210     | -0.0576     |                             |

incorporated into the computer program of two-tube problem is also utilized in the computer program of general problem. This iteration technique is especially useful in the general problem where tube locations can be arbitrary. It should be emphasized that the criterion is based on 1 % accuracy on  $\bar{T}$ s. Should the temperature distribution is of the interest in the general problem, it would become necessary much higher values for  $\bar{\epsilon}_{\max}$  as discussed in two-tube case.

With the equations developed, computer programs written, and the criteria for the selection of different values established and incorporated, it is now matter of running different problems and compile the results. To check the accuracy of the results obtained from the numerical schemes developed, the results are compared with the exact solution of a single tube problem for a wide range of depths. These and other results are compiled in the next two chapters.

## CHAPTER VII

### VALIDATION OF COMPUTER PROGRAMS

The validity of computer programs that were developed for two-tube problem (TWOTUBE and TWOSYMM) and general problem (GENTUBE) were checked. There is a plane of symmetry in the two-tube problem since both tubes are located at a vertical depth of  $D$  from the plane convective surface. Because of the symmetrical location of these two tubes, the following conditions should be satisfied:  $\bar{q}_0^1 = \bar{q}_0^2$ ,  $\bar{q}_1^1 = \bar{q}_1^2$ , and  $\bar{q}_2^1 = -\bar{q}_2^2$  and also  $\phi_k = \phi_{k-1}$ ,  $\phi_{k+1} = \phi_{k-2}$  and so on where  $k$  is the center node of the plane surface and  $\phi$  represents the temperature of  $k$  and  $k+1$  segment. It was observed that these conditions have been satisfied by the results obtained by using the TWOTUBE program (Refer Fig 4 and Table 7).

Results are also obtained for this two-tube problem by using the TWOSYMM program which is formulated with the symmetry conditions included in it. These results were compared with the results obtained from the TWOTUBE program for the same input values. It was observed that the results are matching very well (Refer Table 8). Thus, it was assumed that the computer programs developed for two-tube problem are working well and are valid to use.

The GENTUBE program is validated in the following manner. First, the results were obtained for two tubes by keeping the same conditions as was in the two-tube problem and found the results were matching with the results obtained from the TWOTUBE program. Next, various tube configurations were taken with symmetrical conditions between the

Table 7 : Two-tube results of problem shown in Figure 4 showing the symmetry in the solution.

$$\left( \bar{q}_0^1 = \bar{q}_0^2, \bar{q}_1^1 = \bar{q}_1^2, \bar{q}_2^1 = -\bar{q}_2^2, \phi_{k+5} = \phi_{k-6} \right) (BI = 0.5; 2\bar{s} = 10.0; \Delta \bar{\xi} = \bar{D}/16; \bar{\xi}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}} )$$

| $\bar{\xi}_{\max}$ | $\bar{D}$ | $\bar{q}_0^1$ | $\bar{q}_0^2$ | $\bar{q}_1^1$ | $\bar{q}_1^2$ | $\bar{q}_2^1$ | $\bar{q}_2^2$ | $\phi_{k+5}$ | $\phi_{k-6}$ |
|--------------------|-----------|---------------|---------------|---------------|---------------|---------------|---------------|--------------|--------------|
| 138.75             | 30.0      | 0.1657        | 0.1657        | 0.0113        | 0.0113        | 0.0320        | -0.0320       | 0.0389       | 0.0387       |
| 174.00             | 48.0      | 0.1432        | 0.1432        | 0.0064        | 0.0064        | 0.0281        | -0.0281       | 0.0220       | 0.0220       |
| 195.0              | 60.0      | 0.1344        | 0.1344        | 0.0049        | 0.0049        | 0.0264        | -0.0264       | 0.0169       | 0.0168       |

Table 8 : Two-tube results of problem shown in Figure 4 illustrated to compare the solutions of TWOSYMM and TWOTUBE programs. (BI=0.5  $2\bar{s} = 10.0; \Delta \bar{\xi} = \bar{D}/16; \bar{\xi}_{\max} = 11.3 * \sqrt{X T_{\max} * \bar{D}} )$

| $\bar{\xi}_{\max}$ | $\bar{D}$ | $\bar{q}_0^{(2-T)}$ | $\bar{q}_0^{(SYM)}$ | $\bar{q}_1^{(2-T)}$ | $\bar{q}_1^{(SYM)}$ | $\bar{q}_2^{(2-T)}$ | $\bar{q}_2^{(SYM)}$ | $\phi_{k+5}^{(2-T)}$ | $\phi_{k+5}^{(SYM)}$ |
|--------------------|-----------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|----------------------|----------------------|
| 138.75             | 30.0      | 0.1657              | 0.1658              | 0.0113              | 0.0113              | 0.0320              | 0.0320              | 0.0389               | 0.0389               |
| 174.0              | 48.0      | 0.1432              | 0.1432              | 0.0064              | 0.0064              | 0.0281              | 0.0281              | 0.0220               | 0.0220               |
| 195.0              | 60.0      | 0.1344              | 0.1343              | 0.0049              | 0.0049              | 0.0264              | 0.0264              | 0.0169               | 0.0169               |

2-T Indicates value from two-tube computer program.

SYM indicates value from symmetry computer program.

tubes and the results were compared and was found matching well. To show the results, an example problem with the tube arrangement as shown in Figure 12 is taken and the results are given in Table 9 for each tube. The temperature and diameter of all tubes are the same in this problem. The results shown in Table 9 for the problem of Figure 12 is a good indication of the validity of the computer program developed for the general problem.

These computer programs were checked for many other values of input and tube configurations to see the validity of these computer programs even though a limited number of cases are presented in this thesis. The solution accuracy is checked with a known exact solution for a single tube.

#### Comparison with an Exact Solution

In order to establish the accuracy of the present method, consider the model problem of a tube of radius  $a$  that is at a constant surface temperature of  $\phi_t$  and located at a depth  $D$  from the plane convective surface (maintained at  $\phi = 0$ )

The exact solution for this problem in dimensional form is given by

$$q_0 = \frac{\phi_t}{a \cos^{-1} h \left( \frac{D}{a} \right)}$$

Changing this  $q_0$  to nondimensional  $\bar{q}_0$ ,

$$\frac{\phi_r}{a} \bar{q}_0 = \frac{\phi_t}{a \cos^{-1} h \left( \frac{D}{a} \right)}$$

Table 9 : GENTUBE results for the problem in Figure 12 showing the symmetry in the solution. ( $\bar{q}_0^1 = \bar{q}_0^2, \bar{q}_0^3 = \bar{q}_0^4, \bar{q}_0^5 = \bar{q}_0^6$   
 $(BI = 0.5; \Delta \bar{\xi} = 1.5; \phi_r = 212 \text{ F}; A(1) = 1.0; \phi_{k+8} = 0.05338;$   
 $\phi_{k-9} = 0.05317; \bar{\xi}_{\max} = 11.3 * \sqrt{\chi T_{\max} * D})$

| Tube no | XY(I) | YT(I) | $\bar{q}_0$ | $\bar{q}_1$ | $\bar{q}_2$ |
|---------|-------|-------|-------------|-------------|-------------|
| 1       | 7.5   | 0.0   | 0.0690      | 0.0037      | 0.0291      |
| 2       | -7.5  | 0.0   | 0.0690      | 0.0036      | -0.0291     |
| 3       | 5.0   | 12.0  | 0.1160      | 0.0449      | 0.0268      |
| 4       | -5.0  | 12.0  | 0.1160      | 0.0448      | -0.0268     |
| 5       | 4.0   | -12.0 | 0.0682      | -0.0215     | 0.0214      |
| 6       | -4.0  | -12.0 | 0.0683      | -0.0215     | -0.0214     |

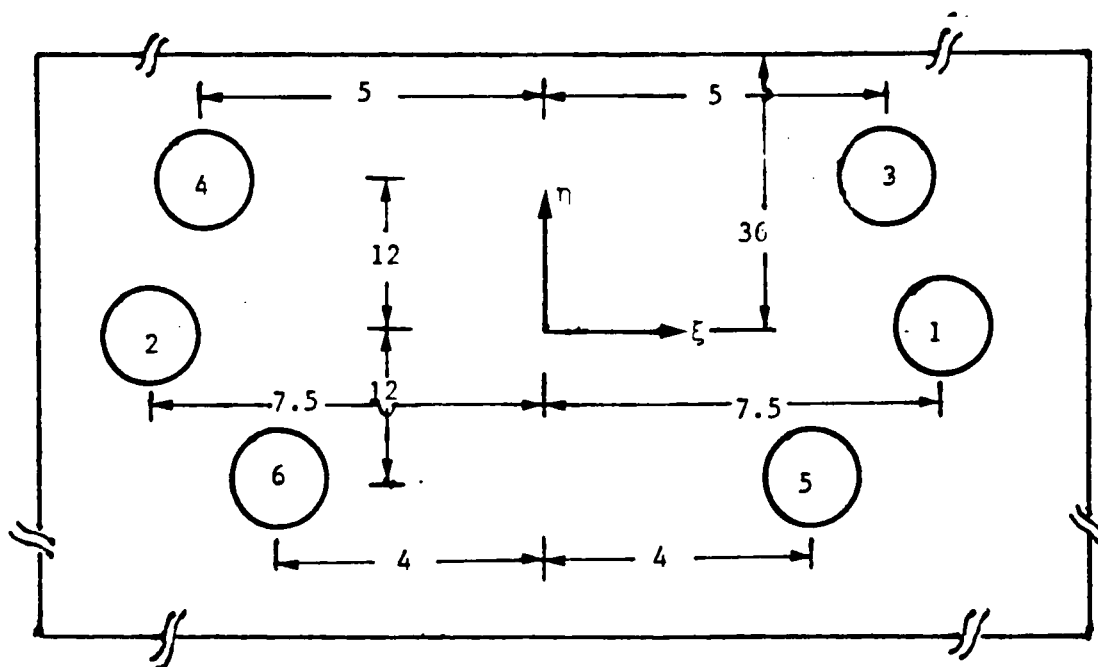


Figure 12. Sketch of the problem considered to validate the GENTUBE computer program.

$$\bar{q}_0 = \frac{\phi_t}{\phi_r \cos^{-1} h \left( \frac{\bar{D}}{A} \right)}$$

For a single tube problem  $\phi_r = \phi_t$ . Therefore,

$$\bar{q}_0 = \frac{1}{\cos^{-1} h \left( \frac{\bar{D}}{A} \right)}$$

The convective plane boundary problem formulated in this thesis can be reduced to the above problem in the limit of  $Bi \rightarrow \infty$ . Using  $Bi = 10^5$ , the solution by the present method with a step size of  $\bar{D}/16$  yielded a zero dimensionless surface temperature everywhere on plane wall. The results of this present method are compared with the results obtained from an exact solution and are given in Table 10. As can be seen from the Table, that the present method results are matched the exact solution within 1% accuracy for upto  $\bar{D}$  equal to 100. In general, the accuracy is found to improve even more with decreasing  $\bar{D}$ .

Table 10 : Comparison of the results of present method with the results obtained from the exact solution of a single tube problem.  
 $(Bi = 10^5; a = 1.0; XT_{max} = 10.0;$   
 $\bar{E}_{max} = 11.3 * \text{SQRT}((XT_{max} * \bar{D}))$

| $\bar{D}$ | $\bar{q}_0$<br>(exact) | $\bar{q}_0$<br>(numerical) | % diff-<br>rence<br>between<br>$\bar{q}_{0, \text{exact}}$<br>and $\bar{q}_{0, \text{numerical}}$ |
|-----------|------------------------|----------------------------|---------------------------------------------------------------------------------------------------|
| 25        | 0.2556                 | 0.2547                     | 0.36                                                                                              |
| 30        | 0.2443                 | 0.2432                     | 0.41                                                                                              |
| 40        | 0.2282                 | 0.2270                     | 0.51                                                                                              |
| 50        | 0.2172                 | 0.2156                     | 0.73                                                                                              |
| 60        | 0.2089                 | 0.2075                     | 0.67                                                                                              |
| 80        | 0.1970                 | 0.1954                     | 0.83                                                                                              |
| 100       | 0.1887                 | 0.1869                     | 1.02                                                                                              |

## CHAPTER VIII

### RESULTS AND DISCUSSION

The methods developed in the previous chapters have been applied to many example problems. The number of tubes in these problems was varied from one tube to as high as eight tubes. However, the computer program can be used for any number of tubes. Problems of variable diameter and variable temperature are also considered here. The results are presented in an appropriate form and are discussed below.

#### Single Tube Problem

The first problem considered and solved is the problem of heat transfer from a single tube located at a depth  $D$  below convective boundary with a radius of  $a$  and temperature of  $T_s$ . Figures 13 and 14 represent the zeroth order( $\bar{q}_{0s}$ ) and first order( $\bar{q}_{1s}$ ) solution of normal derivative of temperature at tube surface for this problem. The second coefficient of expansion  $\bar{q}_{2s}$  is identically zero for this problem due to vertical symmetry in the problem. Here  $\bar{q}_0$  represents the overall nondimensional heat transfer rate from the tube surface. The results for  $\bar{q}_{0s}$  and  $\bar{q}_{1s}$  are plotted as a function of Biot number,  $Bi$ , and normalized depth,  $\bar{D}$ . It is observed that, in general, both  $\bar{q}_0$  and  $\bar{q}_1$  decrease with decreasing Biot number and increasing depth. This result can be justified with the argument that decrease in Biot number means decrease in heat transfer coefficient at the boundary, since thermal conductivity and tube diameter are constants, which means less heat transfer at the convective boundary

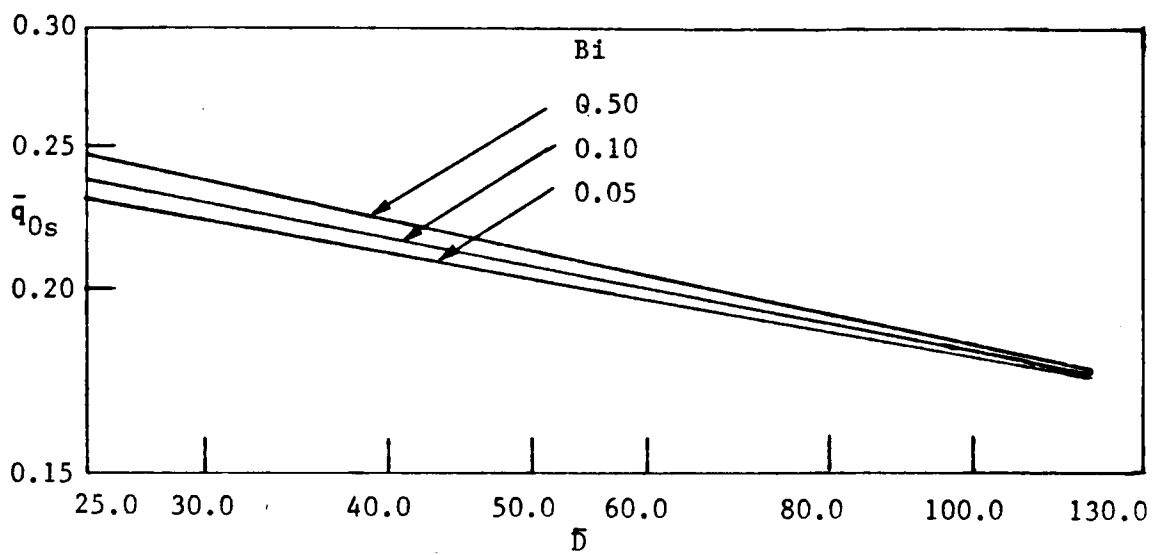


Figure 13. Variation of  $\bar{q}_0$  with nondimensional depth at different Biot numbers for a single tube in a semi-infinite medium.

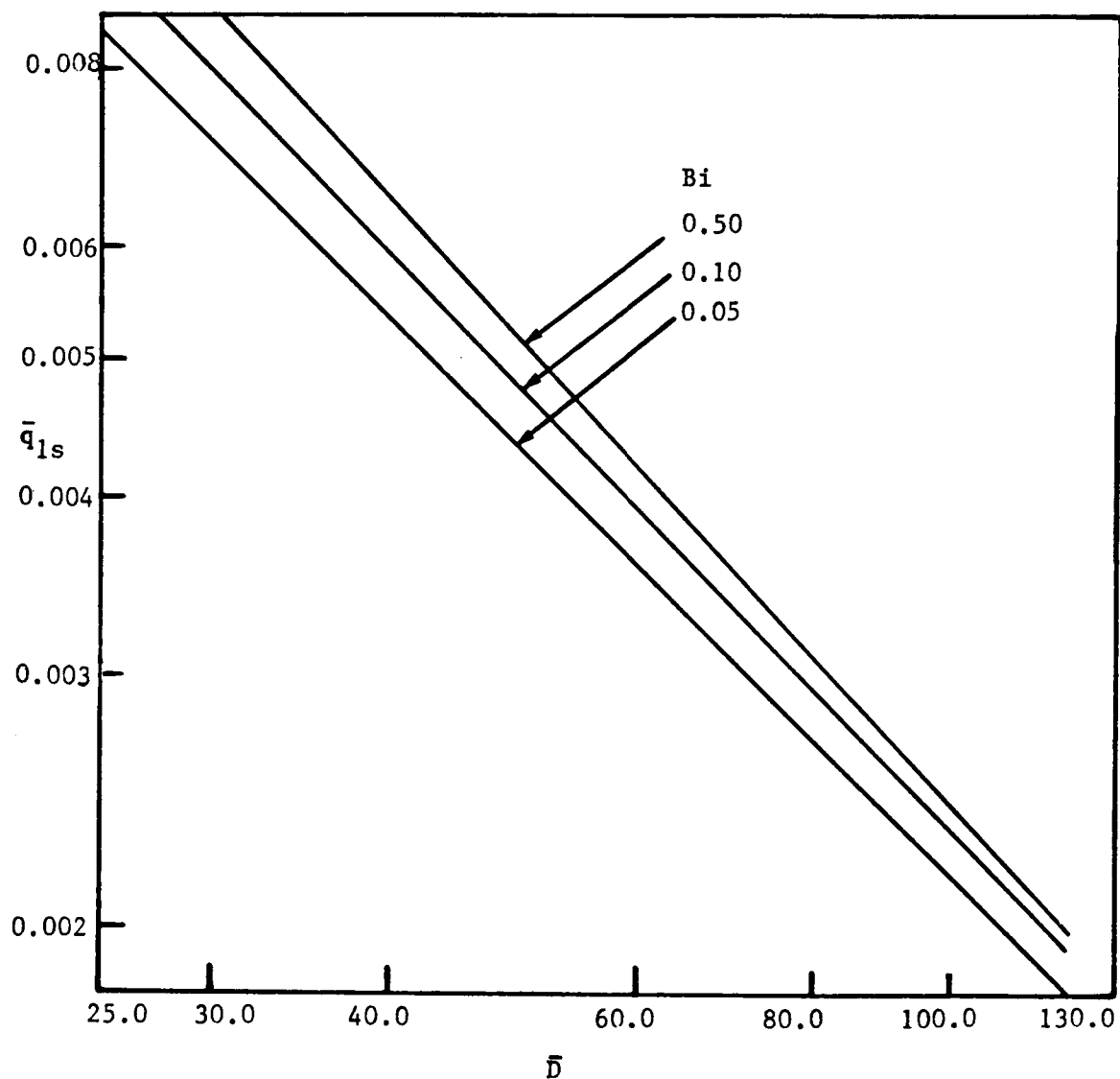


Figure 14. Variation of  $\bar{q}_1$  with nondimensional depth at different Biot numbers for a single tube in a semi-infinite medium.

which in turn results in the decrease of heat transfer from the tube. It is also observed that changes in heat transfer at the convective boundary have progressively smaller effect with increasing depth as expected. The values of the zeroth-order term of the expansion,  $\bar{q}_0$ , are at least 25 times bigger in magnitude than that of the first-order term,  $\bar{q}_1$ .

Figures 15 and 16 represent the temperature distribution at the convective surface for this problem as a function of depth and Biot number, respectively. Here, in both cases, the plane convective surface length ( $\bar{\xi}_{\max}$ ) is taken as 23 depths. As can be seen from the Figures, the nondimensional is  $\phi$  reaching to zero value at  $\bar{\xi} \rightarrow 0.95 \bar{\xi}_{\max}$  and beyond that there is a small temperature rise. This rise is limited to less than 2 % of the peak value. It is to be noted that, the temperature drop is steep. It drops from near peak values at  $\bar{\xi} = 0$  to values less than 10 % of peak value at  $\bar{\xi} = 5\bar{D}$ . As expected, the temperature everywhere on the plane surface decreases with the increasing depth and Biot number.

#### Two-tube Problem

Next, a problem of 2-tubes located at a depth  $\bar{D}$  below convective boundary and spaced at a distance  $2\bar{S}$  as shown in Figure 1 is considered. Both tubes are assumed having the same diameter and temperature. Figures 17, 18, and 19 represent the zeroth-order ( $\bar{q}_0$ ) and first-order ( $\bar{q}_1$  and  $\bar{q}_2$ ) solution of normal derivative of temperature at the tube surface for this problem. These results are plotted as a function of Biot number and depth,  $\bar{D}$ . In Figure 19,  $\bar{q}_2$  is taken

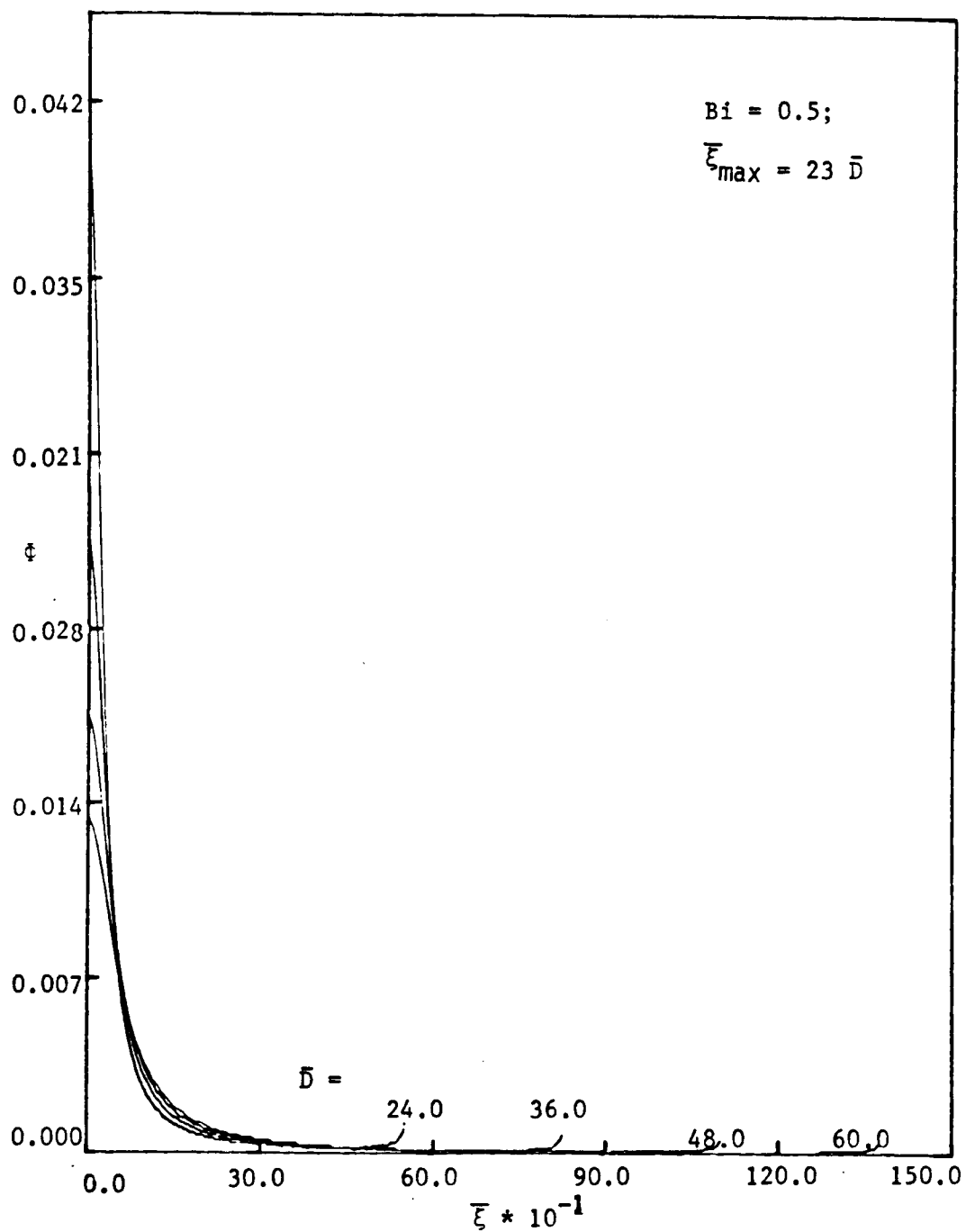


Figure 15. Variation of temperature along the plane convective surface at different tube depths for a single tube in a semi-infinite medium.

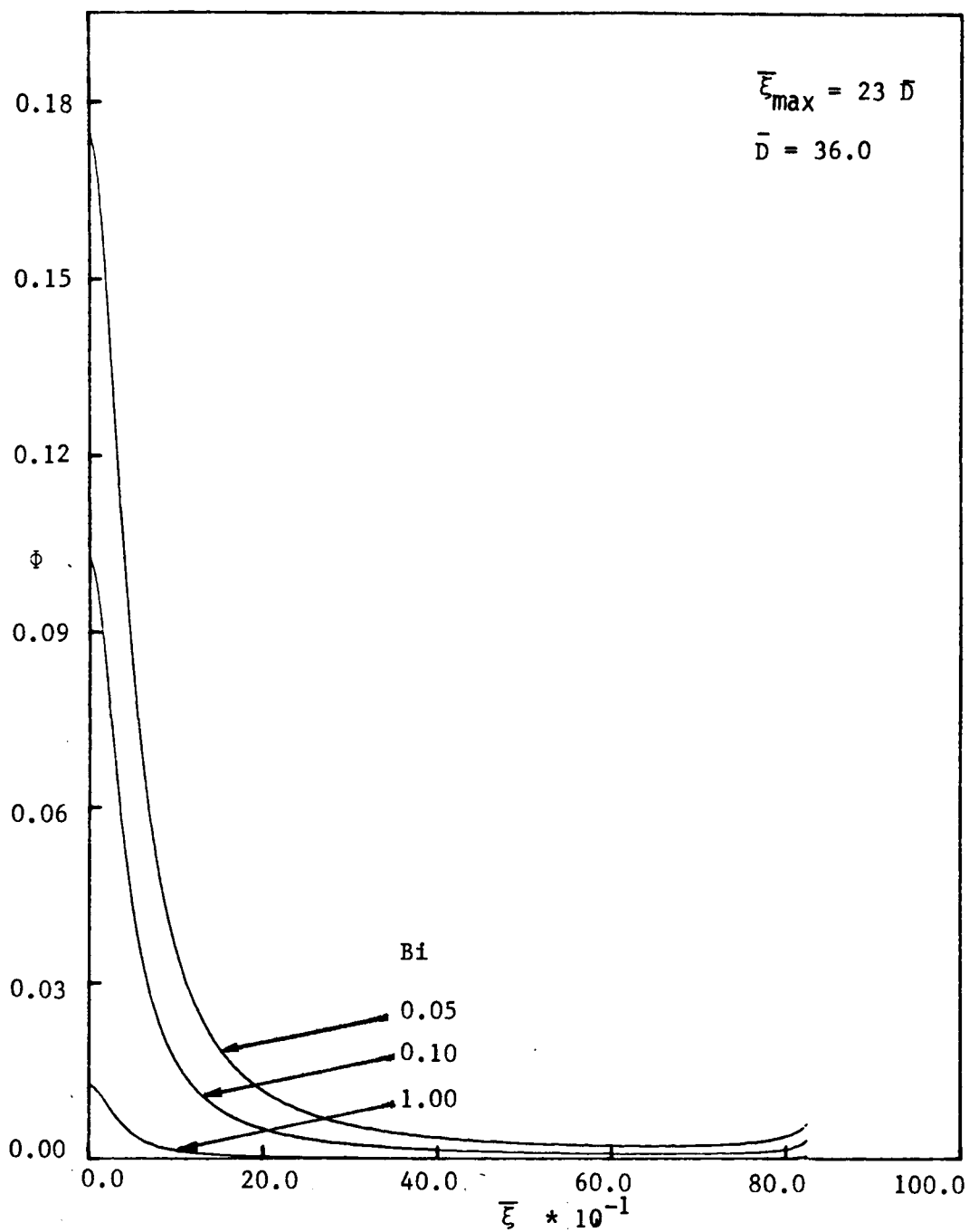


Figure 16. Variation of temperature along the plane convective surface at different Biot numbers for a single tube in a semi-infinite medium.

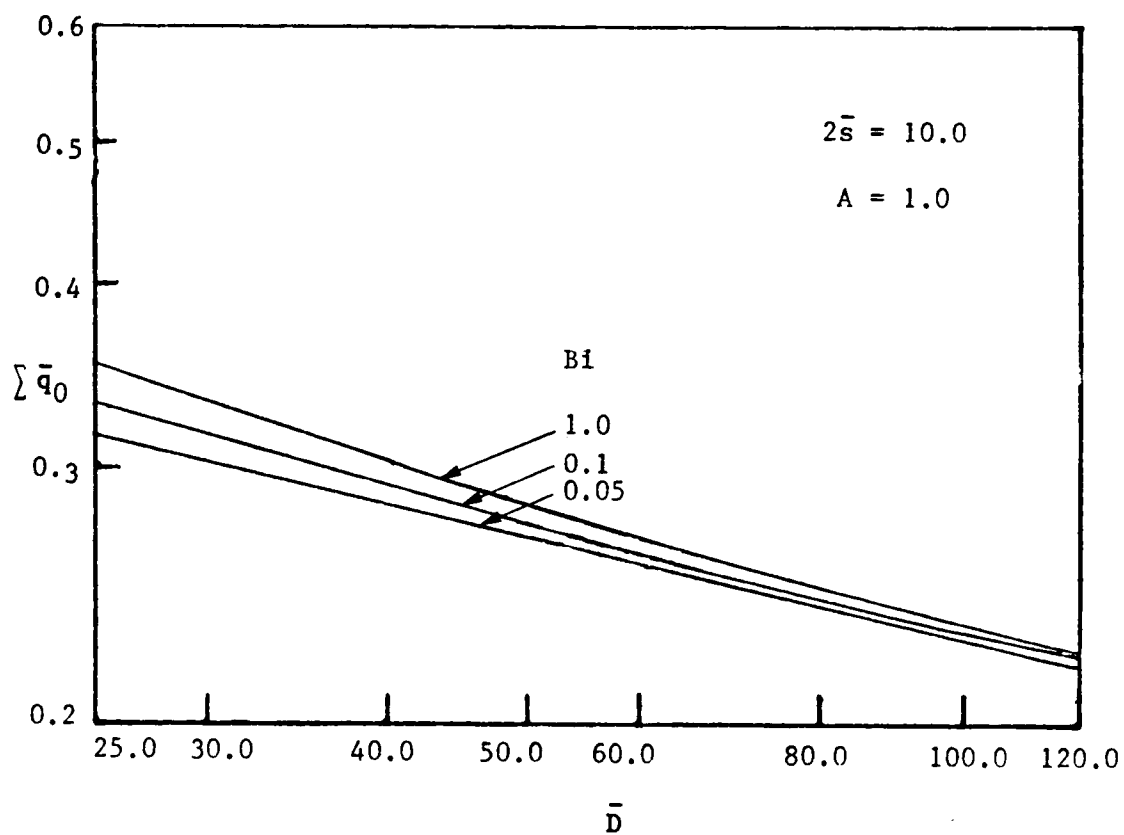


Figure 17. Variation of total value of  $\bar{q}_0$  with nondimensional depth at different Biot numbers for two tubes.

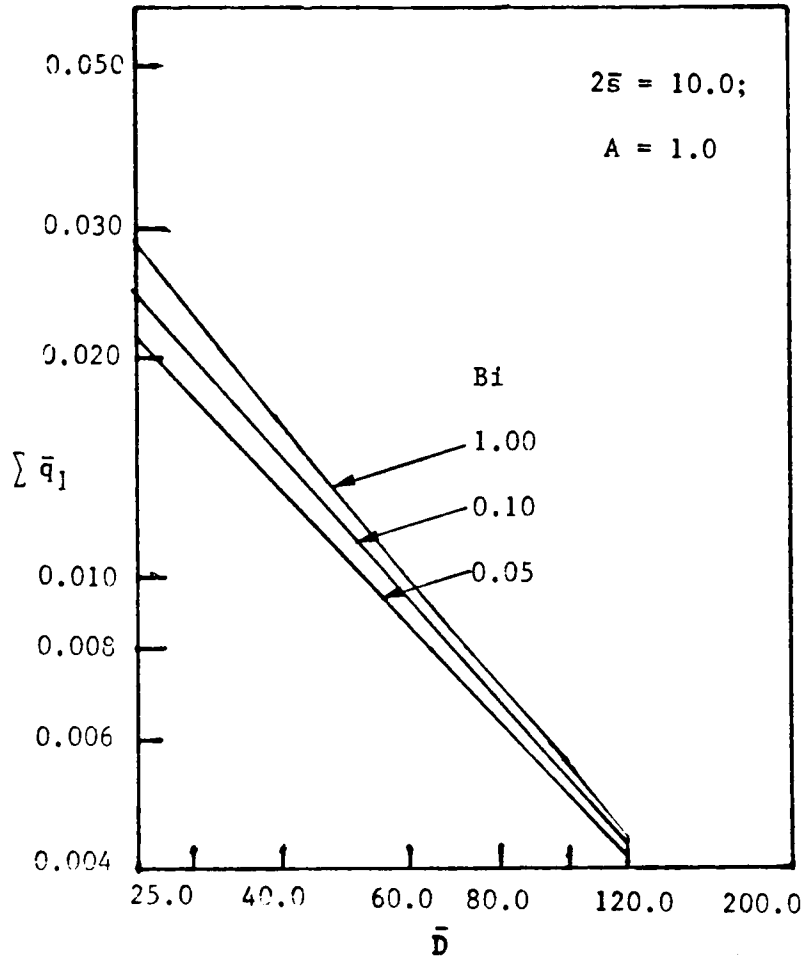


Figure 18. Variation of total value of  $q$  with nondimensional depth at different Biot numbers for two tubes.

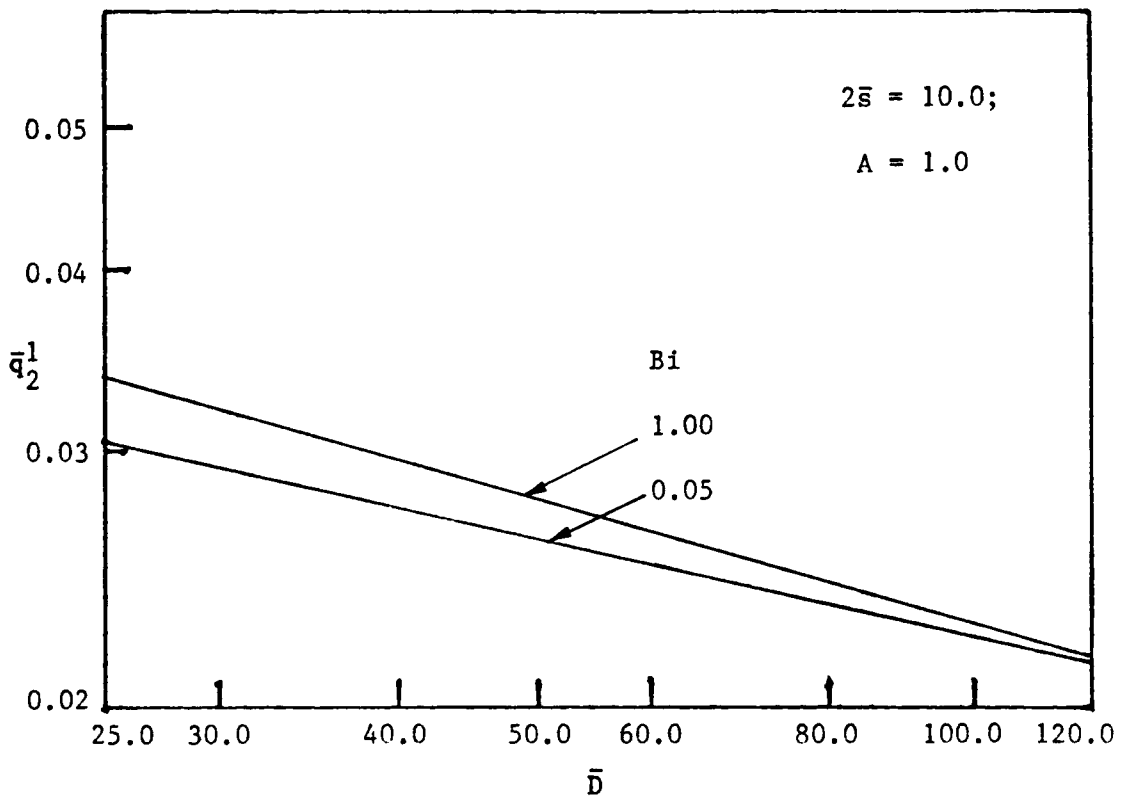


Figure 19. Variation of total value of  $\bar{q}_2^1$  with nondimensional depth at different Biot numbers for two tubes.  
(note :  $\bar{q}_2^1 = -\bar{q}_2^2$  )

instead of  $\sum \bar{q}_2$  since  $\bar{q}_2^1 = -\bar{q}_2^2$ . It can be seen from these Figures that all  $\bar{q}_0$ ,  $\bar{q}_1$  and  $\bar{q}_2$  decrease with decrease in Biot number and increase in depth. This can be justified with the same reasoning as was discussed in the problem of single tube. It is also observed that the values of  $\bar{q}_0$  are at least 10 times bigger than  $\bar{q}_1$  and 5 times bigger than  $\bar{q}_2$ .

The heat transfer results for a constant Biot number are plotted as a function of tube spacing ( $2\bar{S}$ ) and depth,  $\bar{D}$ . Figures 20, 21, and 22 represent the solution for  $\bar{q}_0$ ,  $\bar{q}_1$ , and  $\bar{q}_2$  for this problem. It is observed that both  $\bar{q}_0$  and  $\bar{q}_1$  increase with increasing spacing and decreasing depth but  $\bar{q}_2$  decreases with increasing spacing and increasing depth. Once again, the magnitude of the value of  $\bar{q}_0$  is at least 10 times bigger than  $\bar{q}_1$ .

Figures 23 and 24 represent the temperature distribution on the convective surface as a function of depth and Biot number, respectively, for the problem of two tubes at fixed spacing and constant temperature. For  $\bar{D} > 25$ , the trends of these temperature distributions are similar to those observed and discussed in the single tube problem. These Figures indicate that for the spacing chosen ( $2\bar{S} = 10.0$ ), the peak in the temperature distribution occurs at the origin, and not directly above the center of the tubes ( $\bar{\xi} = \bar{S}$ ) as one might expect. This is perhaps indicative of the fact that the depths considered in this are not sufficiently small. Here too, the temperature falls rapidly from its peak value at  $\bar{\xi} = 0$  to 10 % of the peak as  $\bar{\xi}$  goes to only  $5\bar{D}$ . As observed in the results of the single tube problem, the temperature on the plane surface of the two-tube problem also decreases with increase in Biot number and depth.

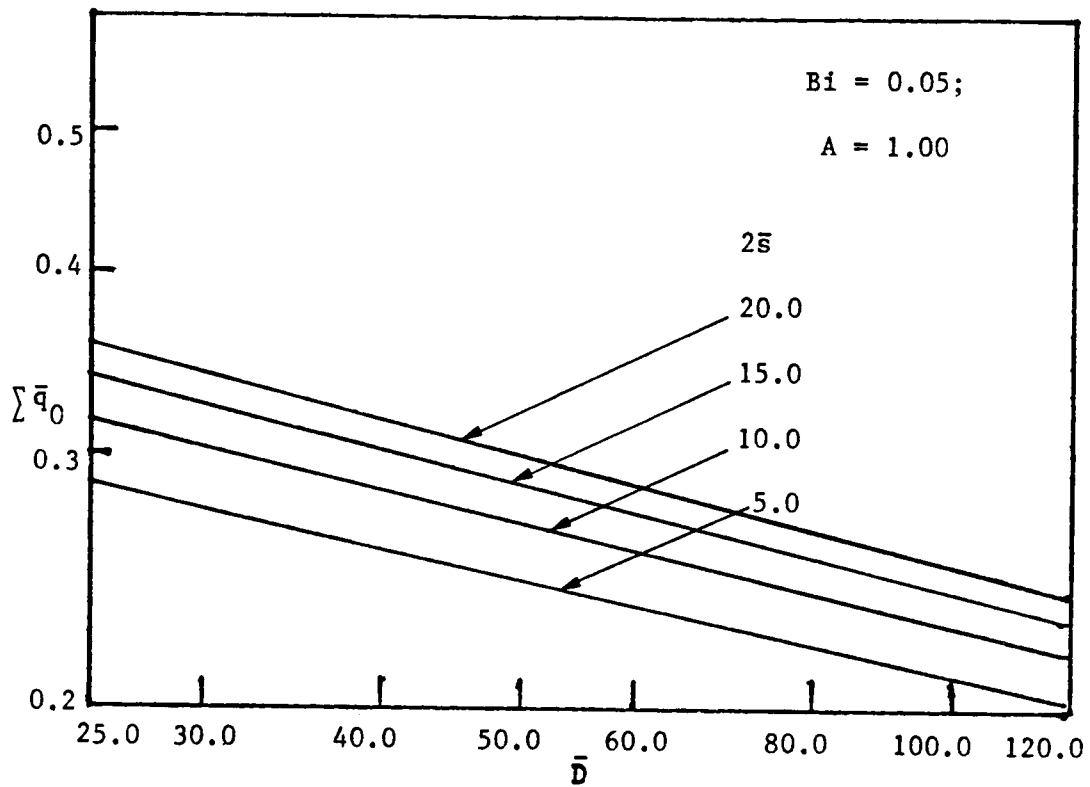


Figure 20. Variation of total value of  $\bar{q}_0$  with nondimensional depth for different tube spacings of two tubes.

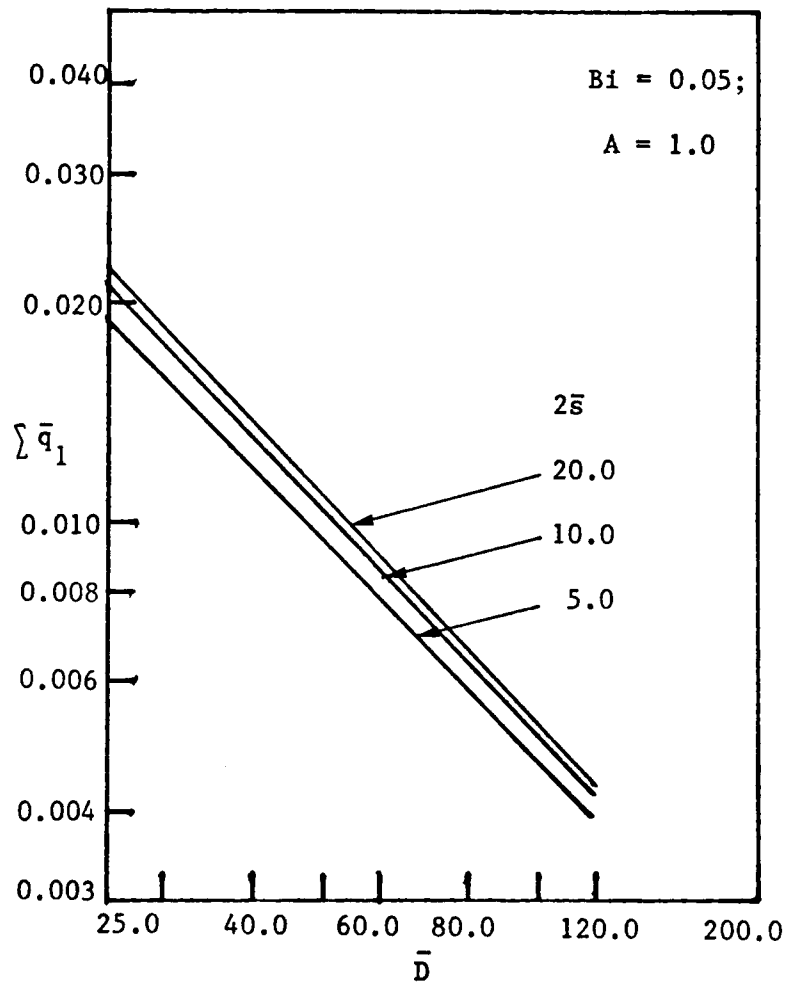


Figure 21. Variation of total value of  $\bar{q}_1$  with nondimensional depth for different tube spacings of two tubes.

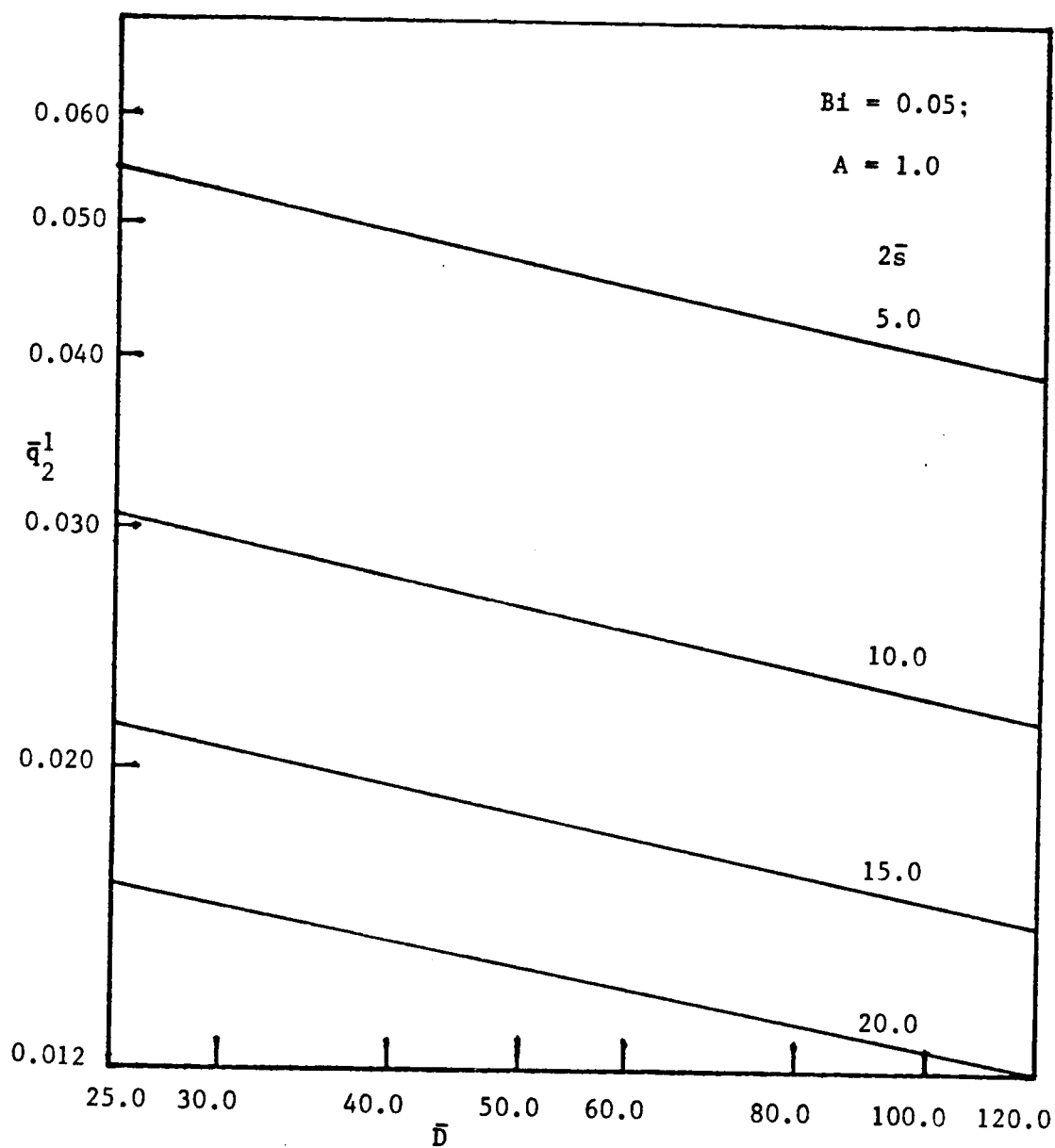


Figure 22. Variation of value of  $\bar{q}_2^1$  with nondimensional depth at different tube spacings for two tubes. ( note :  $\bar{q}_2^1 = -\bar{q}_2^2$  )

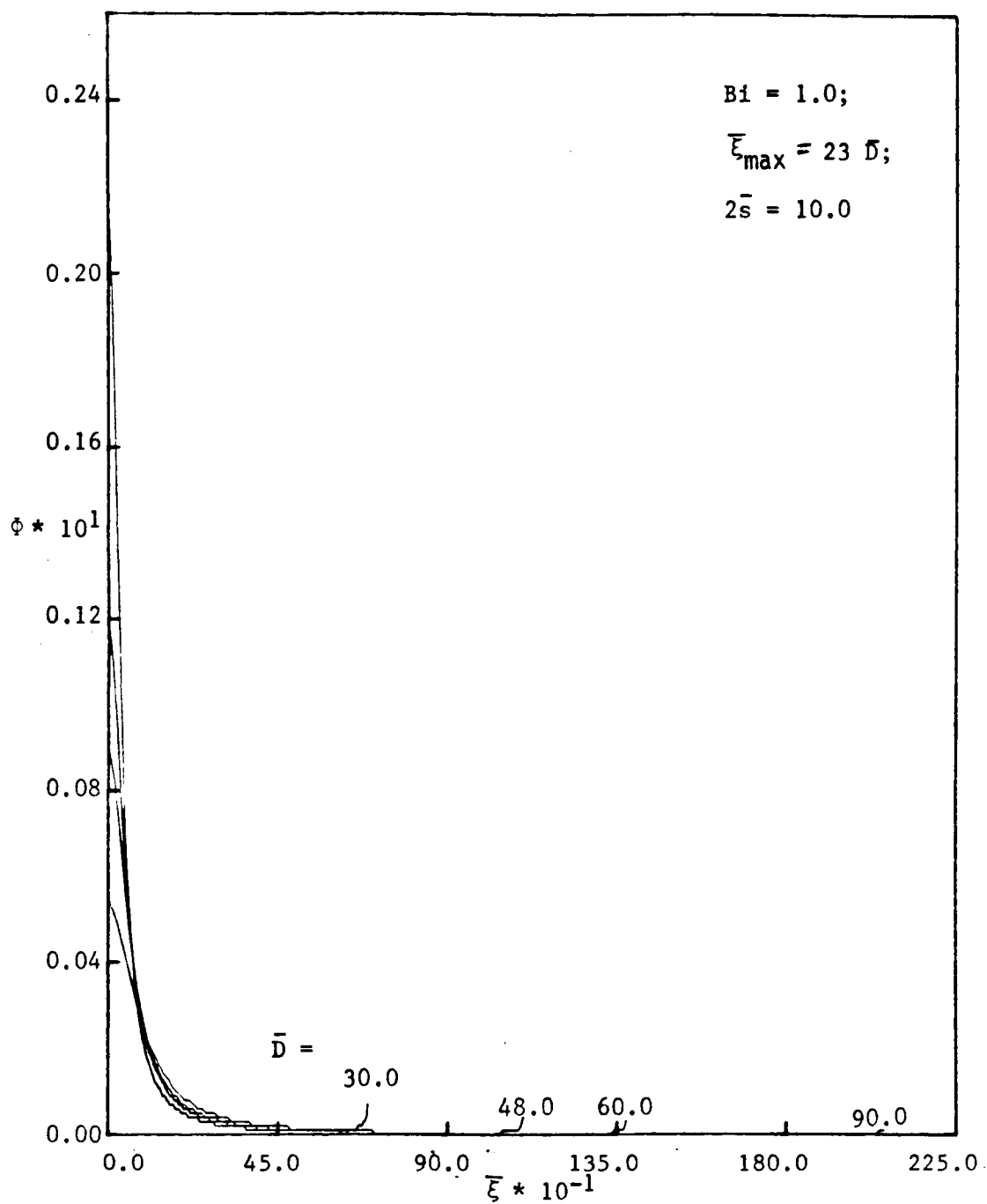


Figure 23. Variation of temperature along the plane convective surface for different tube depths for two tubes.

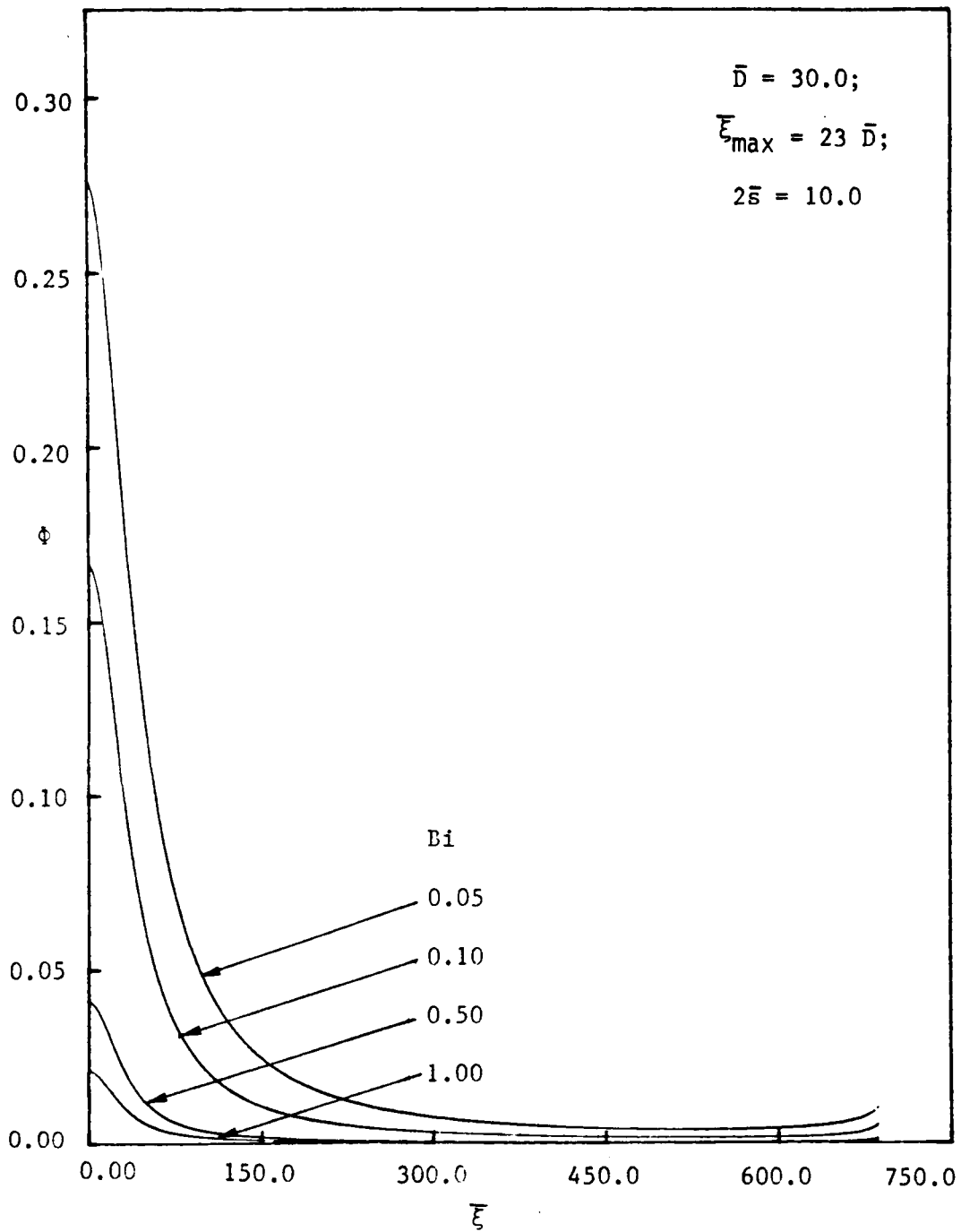


Figure 24. Variation of temperature along the plane convective surface at different Biot numbers for two tubes.

The problem of two tubes that are kept at a shallow depth is also of special interest. The temperature distribution solutions for this case is shown in Figure 9. For the shallow depth chosen ( $\bar{D} = 5$ ), the peak value of temperature on plane convective surface occurs directly above the location of the tubes. This trend is expected as the tubes are very close to the surface. The temperature distributions in Figures 8 and 9 indicate that for  $\bar{D}$  values some where between 5 and 25, the peak in the temperature distribution can be expected to shift from a position directly above the center of the tubes to the plane of symmetry.

#### Special Cases of General Problem

##### Horizontal Array of Tubes ( $M = 1, 2, \dots, 8$ )

Figures 25, 26, and 27 show the results for  $\bar{q}_0$ ,  $\bar{q}_1$ , and  $\bar{q}_2$  for  $M = 1$  through  $M = 8$  cases for the problem of  $M$  horizontal constant-diameter tubes located at a depth of  $\bar{D}$  below the convective boundary. All the tubes are assumed to have identical constant temperature and equal spacing ( $2\bar{x}$ ) as shown in Figure (31a). In all these cases,  $Bi = 0.5$  and  $2\bar{x} = 5.0$  were taken as typical values encountered in buried heat exchanger tubes. Also, the sum of  $\bar{q}_0$  for  $M$  tubes in each case is normalized with respect to  $M$  times the heat transfer  $\bar{q}_0$  from a single tube at same conditions ( $M\bar{q}_{0s}$ ). These normalized values of total  $\bar{q}_0$  are presented as a function of  $\bar{D}$  and are as shown in Figure 25. The results show the same qualitative behaviour as in the case of previous problems i.e. decreasing heat transfer with decreasing depth. However, it is clearly observed that the increase in

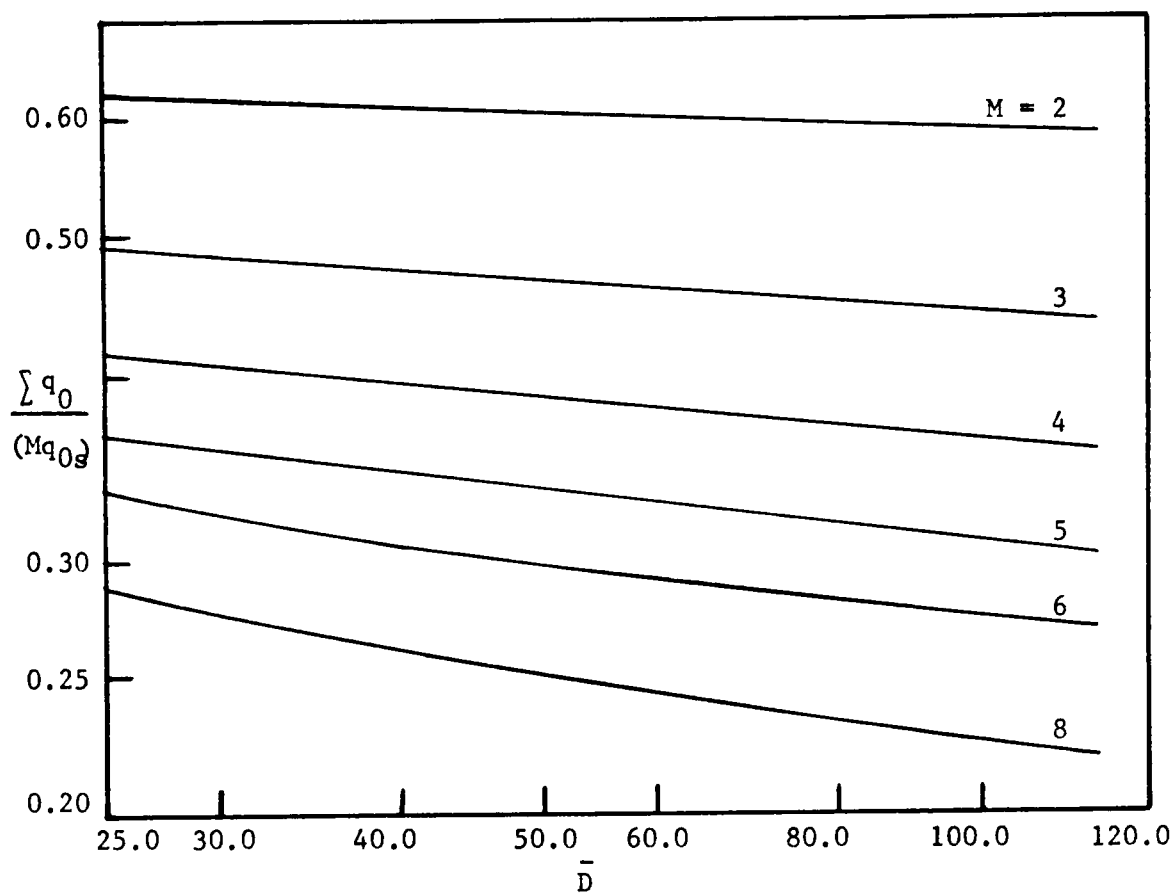


Figure 25. Variation of normalized total value of  $\bar{q}_0$  with nondimensional depth for M tubes shown in Figure 31-a ( $Bi = 0.5, 2\bar{\epsilon} = 5.0$ )

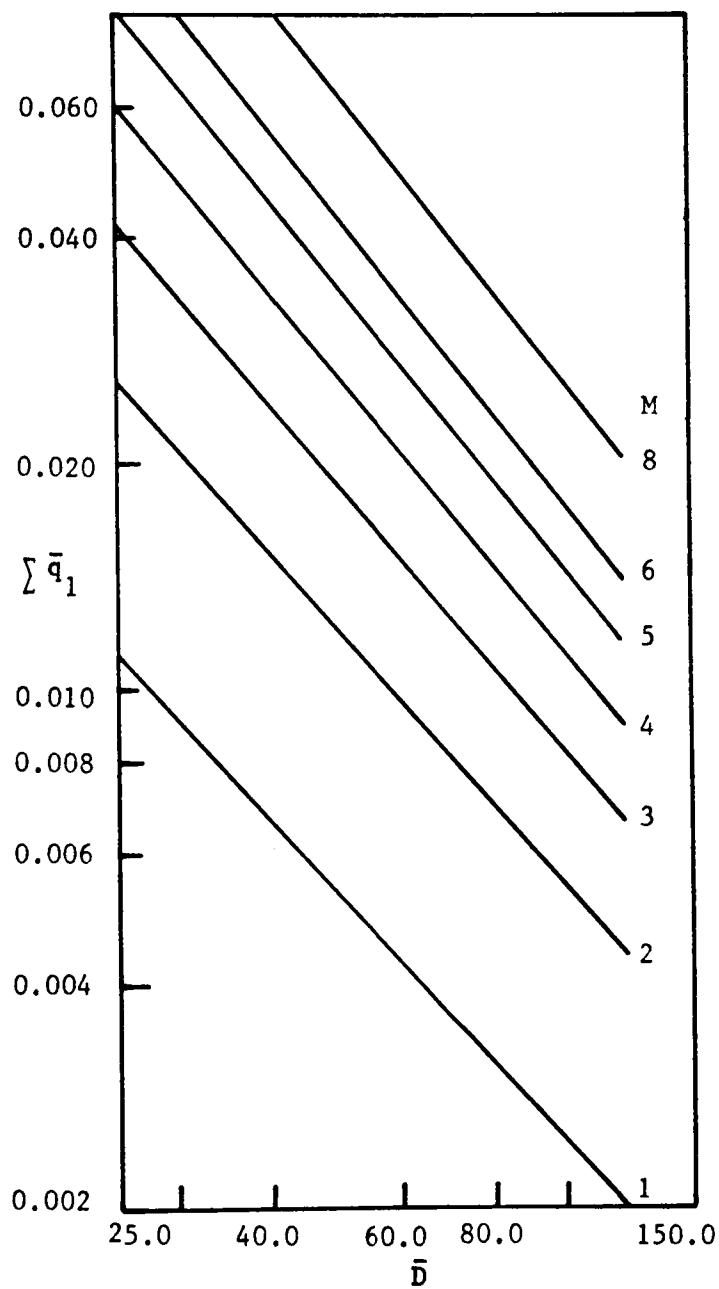


Figure 26. Variation of total value of  $\bar{q}_1$  with nondimensional depth for M tubes shown in Figure 31-a ( $Bi = 0.5$ ,  $2s = 5.0$ )

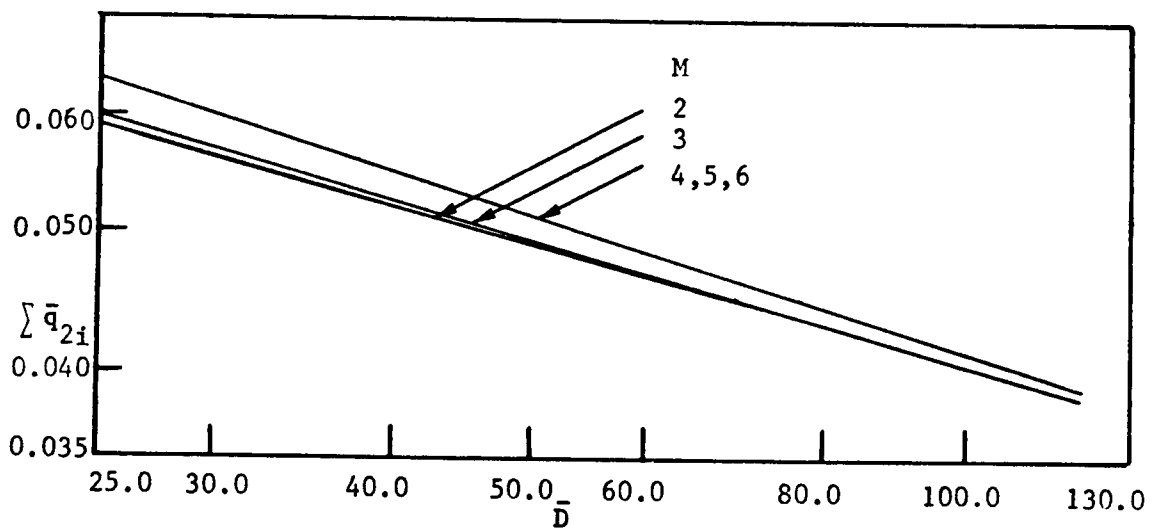


Figure 27. Variation of total value of positive  $\bar{q}_2$  with nondimensional depth for  $M$  tubes shown in Figure 31-a ( $Bi = 0.5$ ,  $2\bar{S} = 5.0$ )

the number of tubes gives a progressively decreasing value for the normalized  $\bar{q}_0$  which is expected since the middle tubes in an array have a significantly lower heat transfer than the tubes located at the outer regions of the array. For example when  $M=2$ , under similar conditions, the presence of a second tube results in a cumulative heat transfer that is only 25 % higher than the heat transfer that would occur when only a single tube is present. The ordinate in Figure 25 may be viewed as the ratio of total heat transfer from a given number of tubes at the specified spacing to the heat transfer from the same number of tubes spaced far apart such that each tube effectively performs as an isolated single tube. So, one should take into account the installation and equipment costs before proceeding with two or more tubes.

In Figure 26, the total values of  $\bar{q}_1$  of  $M$  number of tubes are given as a function of  $\bar{D}$  for arrays of  $M = 1$  through  $M = 8$ . The results show that  $\bar{q}_1$  decreases as  $\bar{D}$  increases and also  $\bar{q}_1$  is significantly smaller than  $\bar{q}_0$ . It is also observed that  $\bar{q}_1$  values are positive for all values of  $M$ .

Figure 27 represents the values of  $\bar{q}_2$  as a function of  $D$ . Due to the symmetry condition, the  $\bar{q}_2$  values are zero for  $M = 1$ , as mentioned earlier. For all configurations with even number of tubes, the symmetry condition gives positive values of  $\bar{q}_2$  for half of the tubes (located on the positive side of the horizontal axis) and negative values with same magnitude for half of the tubes (located on the negative side of the horizontal axis). Likewise for all configurations with odd number of tubes, the symmetry condition gives

$\bar{q}_2 = 0$  for the center tube and positive and negative values with the same magnitude for tubes located on the positive and negative side of the horizontal axis, respectively. For this reason, only the sum of positive values of  $\bar{q}_2$  are presented in Figure 27 for each case. Thus for a total of M tubes, the vertical axis in this Figure 27 represents

$$\sum_{i=1}^{M/2} \bar{q}_{2i} \quad \text{and} \quad \sum_{i=2}^{(M-1)/2} \bar{q}_{2i}$$

for even and odd values of M, respectively. The results show that for large values of M ( $M > 4$ ), the change in the value of  $\bar{q}_{2i}$  becomes insignificant. As expected and discussed before, the total value of  $\bar{q}_{21}$  is seen to decrease with increase in depth,  $\bar{D}$ . The value of  $\bar{q}_2$  is bigger than  $\bar{q}_1$  but significantly lower than  $\bar{q}_0$ .

Similar trends are observed when Biot number is changed to 0.05, and the results are presented in Figures 28 and 29 for  $\bar{q}_0$  and  $\bar{q}_1$ , respectively. It is observed that while the trends of these  $\bar{q}_0$  and  $\bar{q}_1$  are similar to those observed in the previous problem ( $Bi = 0.5$ ), their magnitude turn out to be different.

#### A Special Four-Tube Problem

The next problem was chosen to illustrate the effect of variation of tube diameter on heat transfer rates. The two sub-problems presented in Figures 30a and 30b were considered. Here the total heat transfer rate from a set of four constant diameter tubes as shown in Figure 30a was compared to the total heat transfer rate from a set of four tubes illustrated in Figure 30b. The tube surface

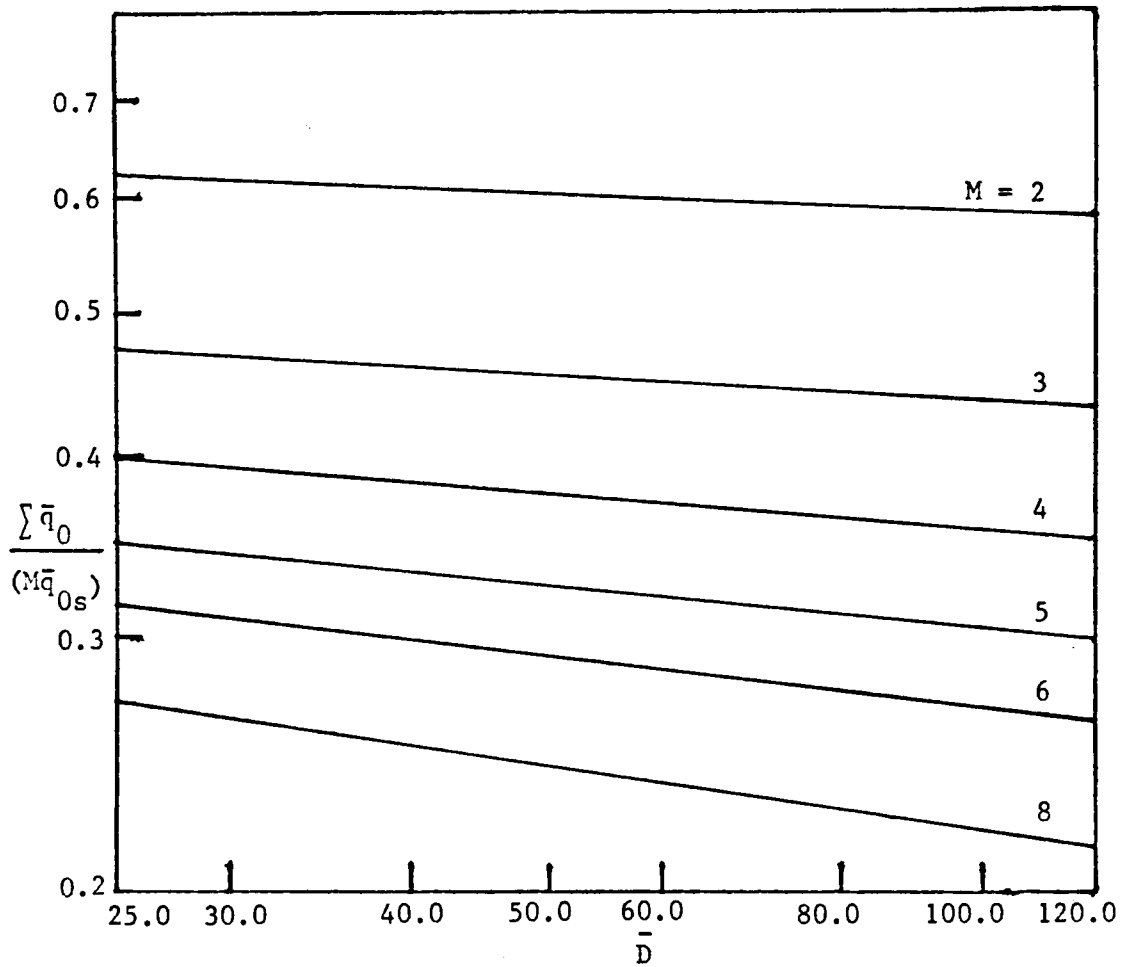


Figure 28. Variation of normalized total value of  $\bar{q}_0$  with nondimensional depth for M tubes shown in Figure 31-a ( $Bi = 0.05, 2\bar{x} = 5.0$ )

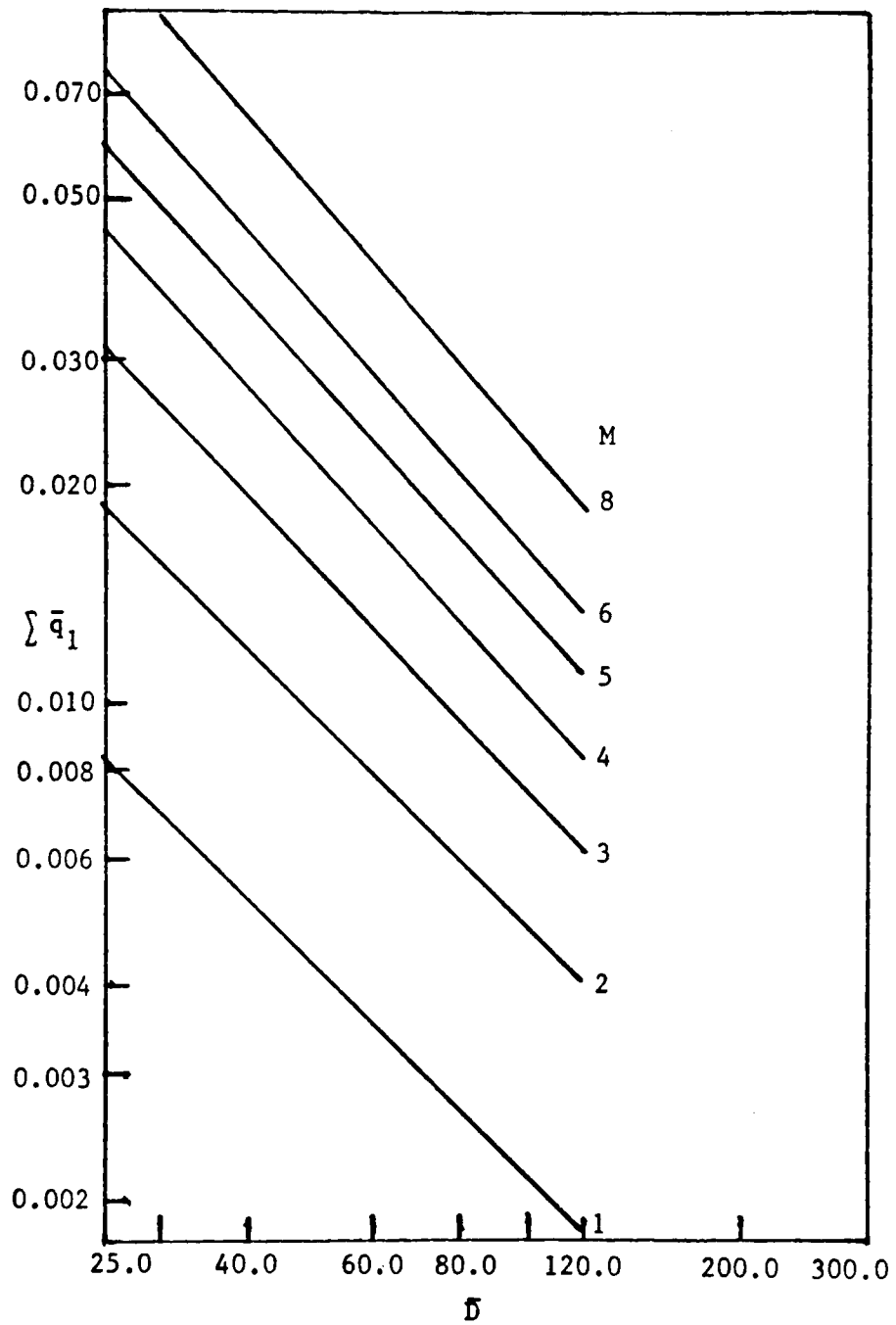
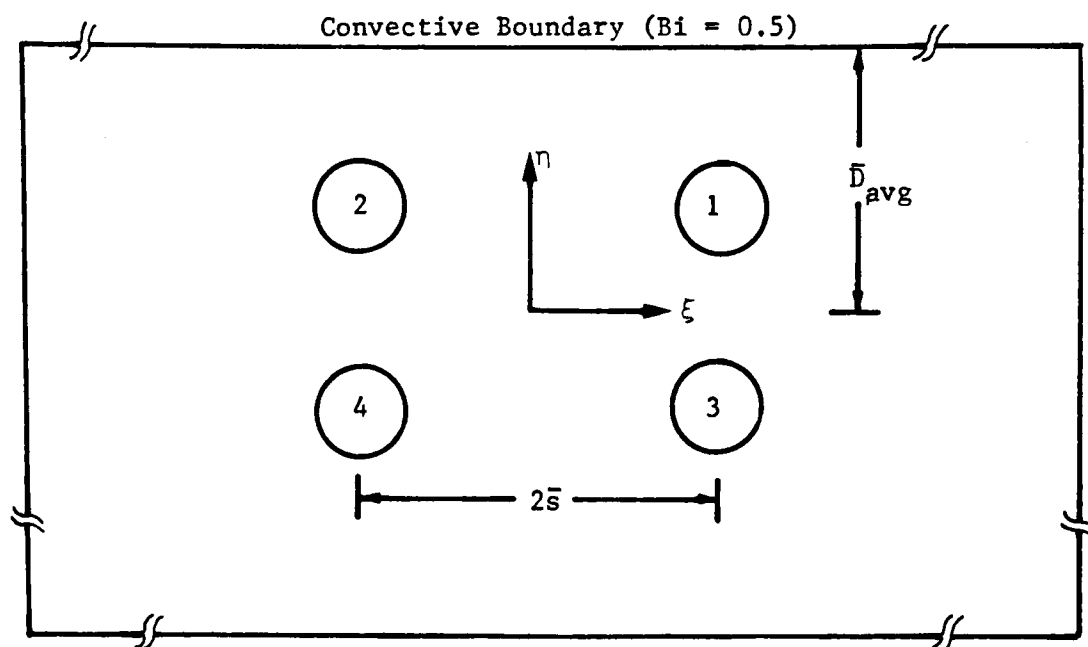
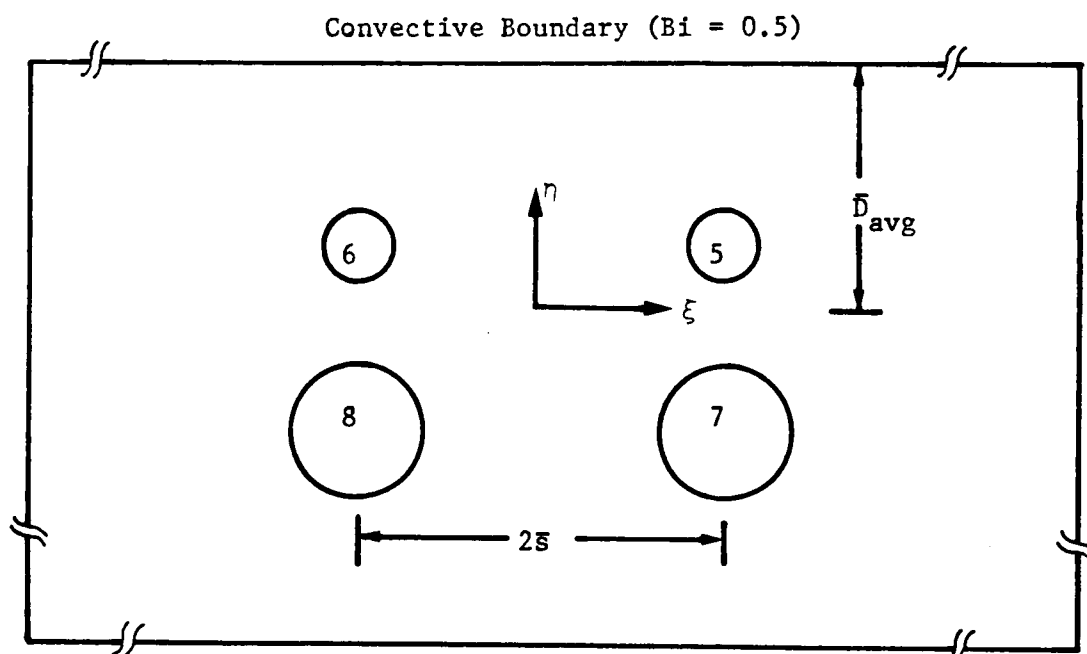


Figure 29. Variation of total value of  $\bar{q}_1$  with nondimensional depth for  $M$  tubes shown in Figure 31-a ( $Bi = 0.05$ ,  $2\bar{s} = 2.5$ )



(a)



(b)

Figure 30. Sketch of the problems considered (for comparative evaluation of  $q_0$ ,  $\bar{q}_1$ , and  $\bar{q}_2$  for two tube arrangements) are shown in (a) with  $2\bar{s}=7.5$ , and  $a_1=a_2=a_3=a_4=a_{\max}=A$  and in (b) with  $2s=5$ ,  $a_7=a_8=a_{\max}$ , and  $a_5=a_6=a_{\max}/3$ .

temperature, spacing, and average depth( $\bar{D}_{avg}$ ) in both cases are assumed equal, but the tube diameters are taken to be different in the two cases as shown in Figures 30a and 30b. As is the case in all the remaining problems, all the length scales were nondimensionalized with respect to the tube radius in Figure 30a(all tubes have the same diameter) and with respect to the maximum tube radius in Figure 30b(all tubes do not have the same diameter) since the temperature is same on all the tube surfaces. The results for  $\bar{q}_0$ ,  $\bar{q}_1$ , and  $\bar{q}_2$  are presented in Table 11 for tubes numbered 1, 3, 5, and 7. The symmetry conditions can be readily used(i.e.  $\bar{q}_0^1 = \bar{q}_0^2$ ,  $\bar{q}_1^1 = \bar{q}_1^2$ , and  $\bar{q}_2^1 = -\bar{q}_2^2$  etc.) to obtain the values for tubes marked 2, 4, 6, and 8. It is observed that the arrangement of Figure (a) gives higher rate of heat transfer(approximately 12.50 % increase in total heat transfer rate). Then, the large diameter tubes and small diameter tubes in Figure 30b were interchanged and heat transfer rates were observed. Arrangement of Figure 30(a) still yields higher rate of heat transfer(approximately 10 % increase in total heat transfer rate).

#### Problem of Different Arrangements of Six Tubes

The next problem was selected to illustrate the effect of arrangement of tube arrays on the heat transfer rates. The problems sketched in Figure 31a and 31b were considered. All tubes were assumed to have the same diameter, surface temperature, and spacing. The average depth in the second case (b) was kept the same as the depth of the single array case (a). The total heat transfer rate of an horizontal array of six tubes(Figure 31a) is compared to that of two

Table 11 : Results for  $\bar{q}_0$ ,  $\bar{q}_1$ , and  $\bar{q}_2$  for various tubes in configurations shown in Fig. 30a and Fig. 30b.

| Tube<br>no. | Depth<br>( $\bar{D}_{\text{avg}}/a_{\text{max}}$ ) | Radius<br>( $a/a_{\text{max}}$ ) | $\bar{q}_0$ | $\bar{q}_1$ | $\bar{q}_2$ |
|-------------|----------------------------------------------------|----------------------------------|-------------|-------------|-------------|
| 1           | 50.0                                               | 1.0                              | 0.1099      | 0.0137      | 0.0291      |
| 3           | 100.0                                              | 1.0                              | 0.0831      | -0.0033     | 0.0223      |
| 5           | 33.33                                              | 0.33                             | 0.2649      | 0.0192      | 0.0353      |
| 7           | 66.67                                              | 1.0                              | 0.0804      | -0.0032     | 0.0313      |

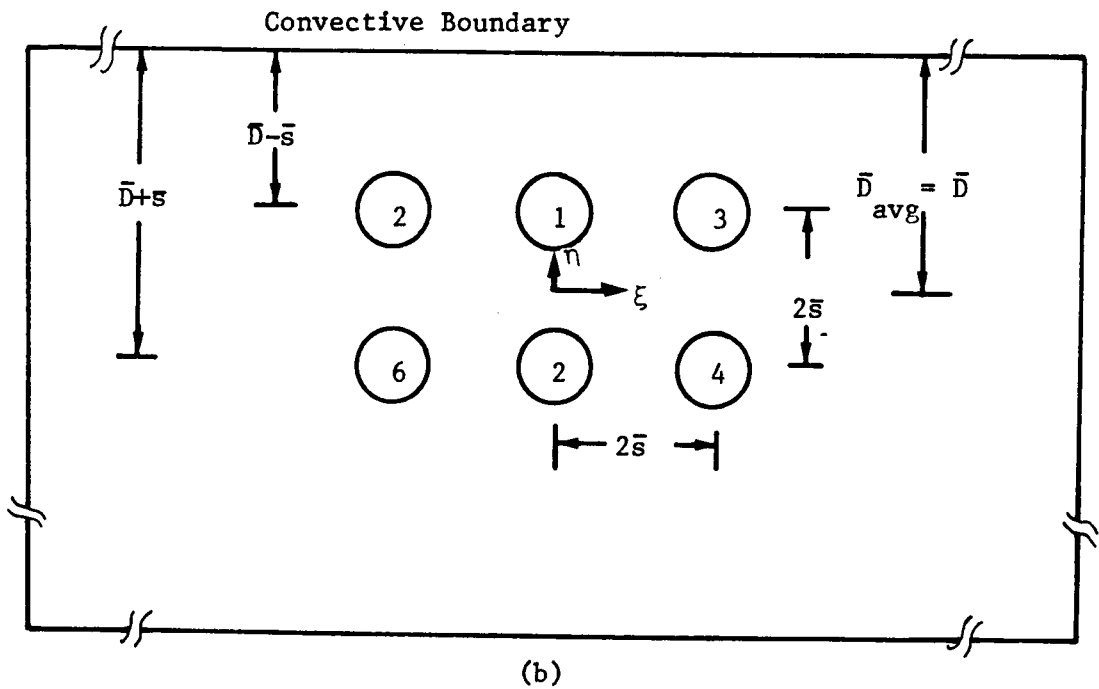
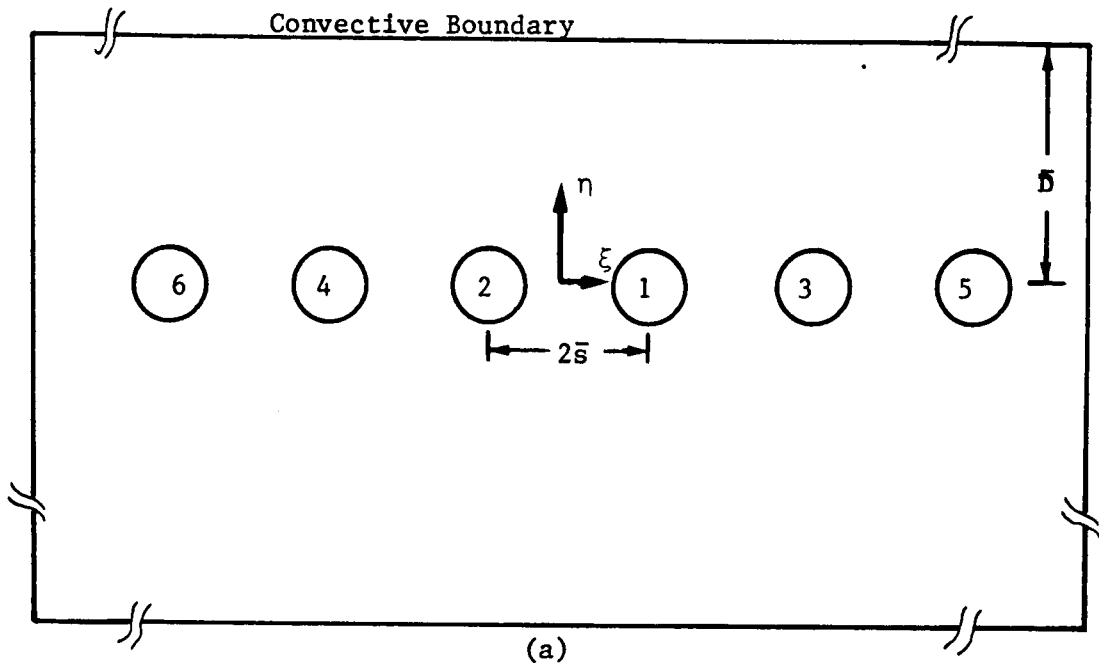


Figure 31. Sketch of problem considered for comparative evaluation of  $\bar{q}$  for two tube arrangements shown in (a) and (b) ( $Bi = 0.5$ ,  $2\bar{s} = 5.0$ )

arrays of three tubes (shown in Figure 31b). The results for total values of  $\bar{q}_0$  are presented as a function of nondimensional depth,  $\bar{D}$  in Figure 32. The results clearly indicate the advantage of single array arrangement at all the depths in maximizing the total heat transfer rate. The increase of heat transfer rate in the arrangement of Figure 31a from that arrangement of Figure 31b is about 14.5 % at a depth of 25 and 12.14 at a depth of 120. It is also understood that increasing depth results in lower heat transfer rates. This trend is observed consistently in all the other problems presented in this thesis.

#### Problem of Variable Diameter and Temperature

The next problem was chosen to illustrate the effect of variation of temperature from one tube to another tube. It should be noted that the temperature on each tube surface is uniform. The tube diameters are also varied. The problem sketched in Figure 33 was considered. The results are tabulated in Table 12. Tubes 1, 2, 5, and 6 give negative  $\bar{q}_0$  values. This may be because of tubes 1 and 2 are very close to 7 and 8 which have the highest temperatures on their surfaces, and 5 and 6 have the lowest surface temperatures and are close to tubes 1, 2, 3, and 4. However, all tubes have positive  $\bar{q}_1$  values, and half tubes have positive  $\bar{q}_2$  values and other half tubes have negative  $\bar{q}_2$  values because of the symmetry conditions between the tubes. Tubes 3, 4, 7, and 8 have the positive  $\bar{q}_0$  values. Tubes 7 and 8 have the highest  $\bar{q}_0$  values as expected since they are having the highest temperature as well as the largest diameter among those tubes. The total heat transfer rate of tubes 7 and 8 is approximately 88 % higher

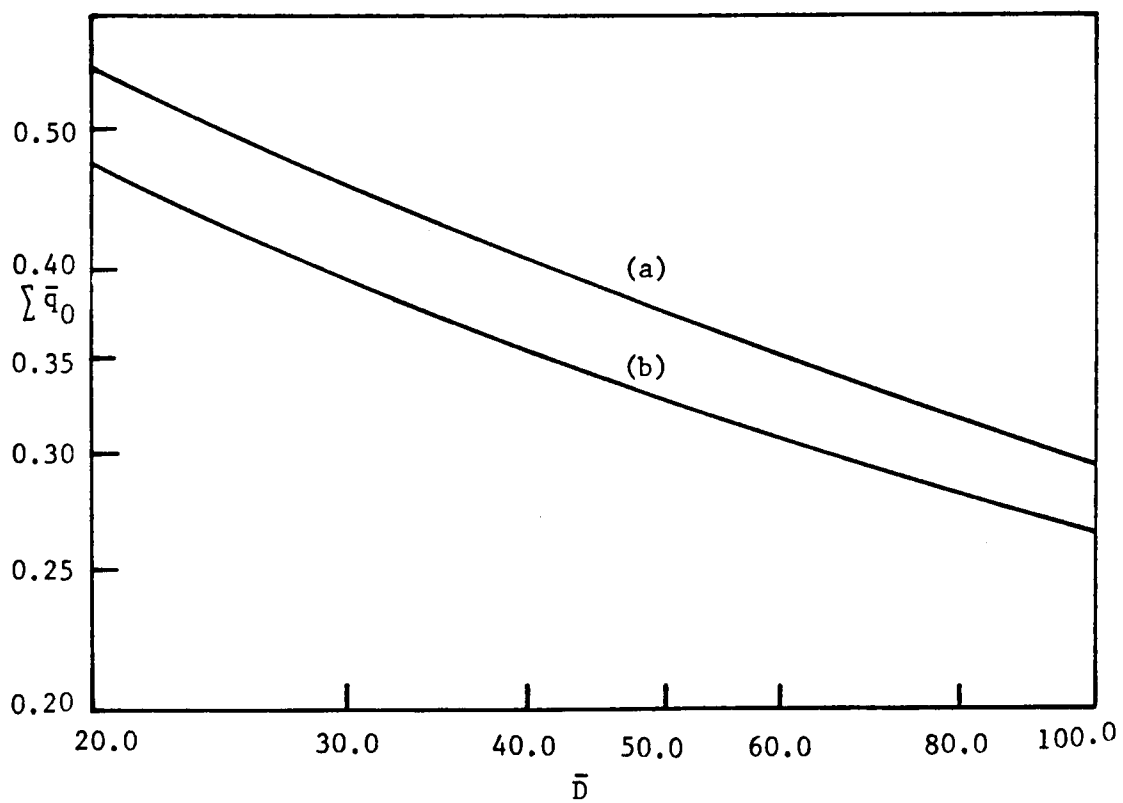


Figure 32. Variation of total  $\bar{q}_0$  with nondimensional depth for configurations (a) and (b) shown in Figure 31. ( $Bi = 0.5$ ,  $2s = 5.0$ )

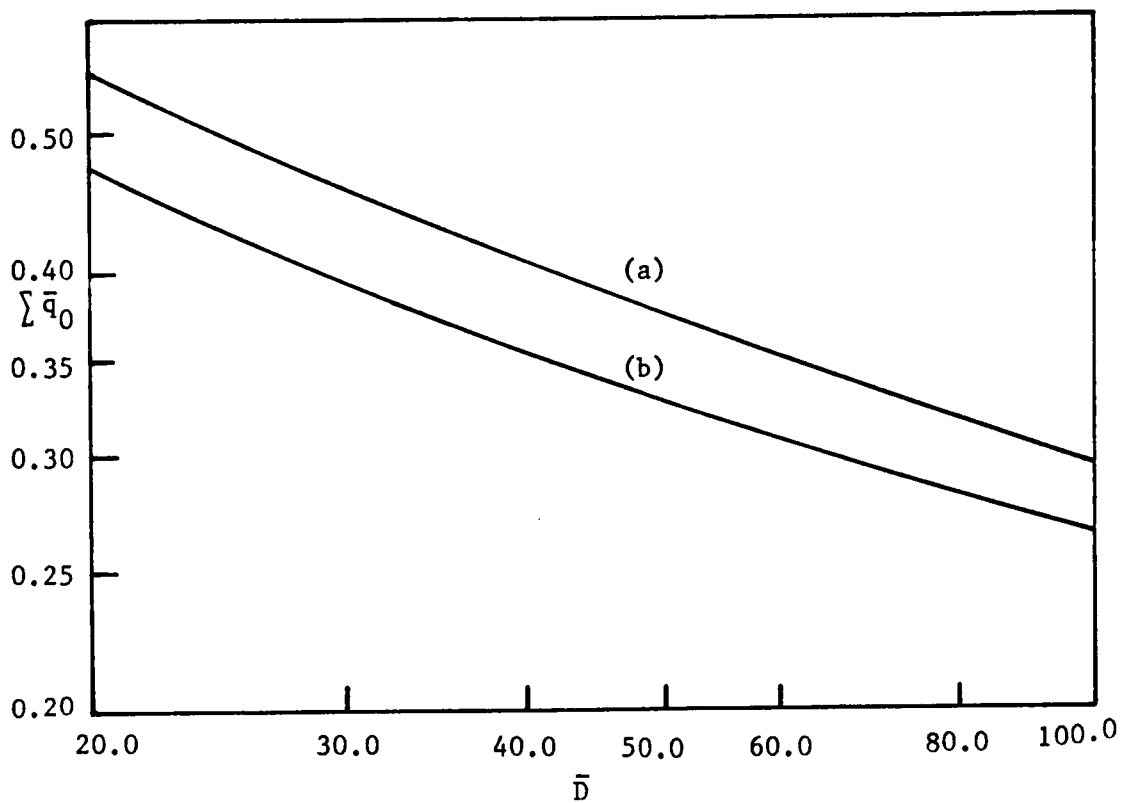


Figure 32. Variation of total  $\bar{q}_0$  with nondimensional depth for configurations (a) and (b) shown in Figure 31. ( $Bi = 0.5$ ,  $2s = 5.0$ )

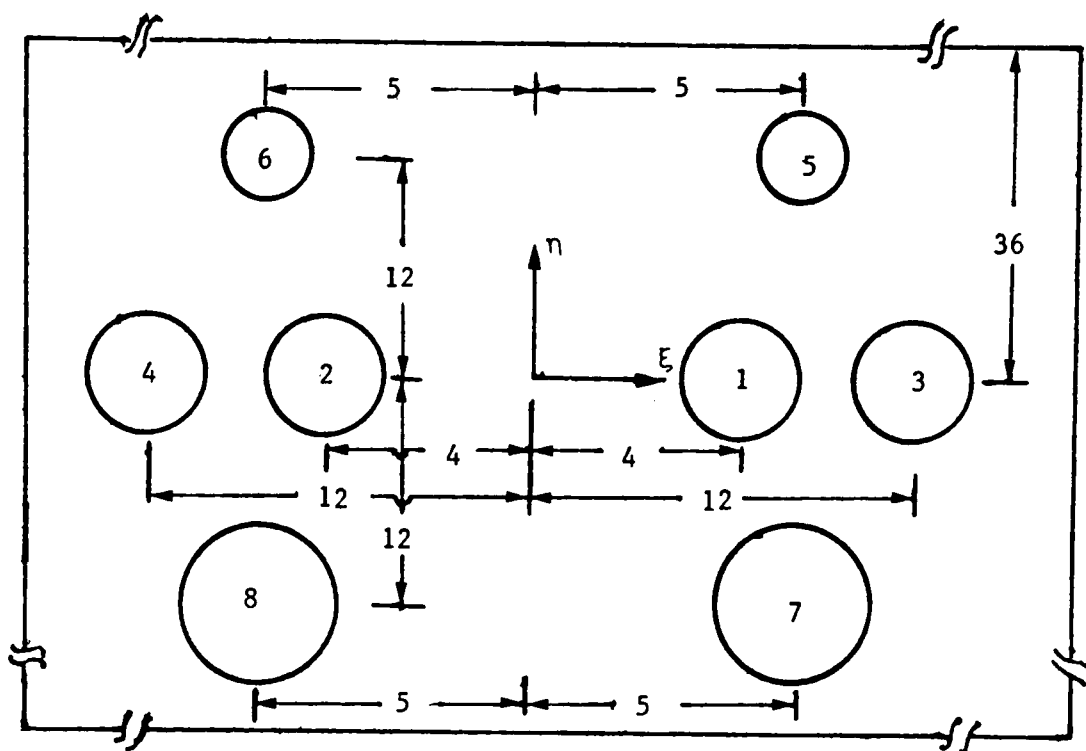


Figure 33. Sketch of a symmetrical problem involving tubes at arbitrary isothermal temperatures.

Table 12 : GENTUBE results for  $\bar{q}_0$ ,  $\bar{q}_1$ , and  $\bar{q}_2$  for various tubes in the problem of Figure 33. Results indicate solution symmetry ( $\bar{q}_0^1 = \bar{q}_0^2$ ,  $\bar{q}_0^3 = \bar{q}_0^4$ ,  $\bar{q}_0^5 = \bar{q}_0^6$ ,  $\bar{q}_0^7 = \bar{q}_0^8$  etc.). (Reference tube no. : 7; Reference tube temp. = 425 F; Reference tube radius = 3.0 )

| Tube No. | Temp. $\Phi_t$ | Radius (A(I)) | $\bar{q}_0$ | $\bar{q}_1$ | $\bar{q}_2$ |
|----------|----------------|---------------|-------------|-------------|-------------|
| 1        | 0.7647         | 0.67          | -0.0742     | 0.1805      | -0.0194     |
| 2        | 0.7647         | 0.67          | -0.0742     | 0.1805      | 0.0196      |
| 3        | 0.7647         | 0.67          | 0.0332      | 0.1171      | 0.0065      |
| 4        | 0.7647         | 0.67          | 0.0332      | 0.1171      | -0.0063     |
| 5        | 0.5765         | 0.50          | -0.0377     | 0.0890      | -0.0151     |
| 6        | 0.5765         | 0.50          | -0.0377     | 0.0890      | 0.0155      |
| 7        | 1.0000         | 1.00          | 0.1909      | 0.0569      | 0.0826      |
| 8        | 1.0000         | 1.00          | 0.1910      | 0.0569      | -0.0823     |

than that total heat transfer rate of tubes 3 and 4. This type of problems usually occur in buried utility lines. The method developed in this thesis can be used effectively to find the heat transfer rate for any tube configuration of arbitrary tube diameters, locations and temperatures.

## CHAPTER IX

### CONCLUSIONS

A two-dimensional analysis of steady heat transfer from a set of parallel isothermal tubes of arbitrary diameters and positions, located in a semi-infinite medium is presented. The plane surface is exposed to uniform convective environment. The boundary integral equations were nondimensionalized, discretized, and the integrals were evaluated analytically. A numerical model based on the boundary integral equation method was developed for the steady problem. Several criteria were developed for the selection of the segment length, and the length of the plane convective surface. The criteria were discussed in detail and are incorporated in the computer programs. Based on numerical model and the criteria, three main computer programs were developed. The first one is for the problem of two tubes, second one is for the problem of two tubes with symmetry conditions incorporated, and the last one is for the general problem of arbitrary number of tubes, tube locations, temperatures and sizes. These programs were used, for demonstration purposes, to solve six problems of interest. Each problem was solved for a wide range of parameters. The zeroth- and first-order heat transfers from the tubes were determined and presented as a function of depth, spacing, and Biot number. Temperature distributions on the plane convective surface were evaluated and presented as a function of depth and Biot number. The effect of selection of convective plane surface length on the solution of the temperature distribution was discussed and the

results were presented in a graphical form. Using a model problem with a known closed-form solution, the accuracy of the numerical model was found to be less than 1 percent.

From the results, it was observed that the numerical boundary integral method gives high accuracy results. The computer program that was developed for general problem is very flexible and can be used for any tube configuration, size and temperature. The computer program that was developed for two-tube problem with symmetrical conditions helps in reducing the computational time by reducing the number of points needed to nearly half of the points needed in two tube computer program without symmetry conditions.

The high accuracy of the results and the efficiency associated with the boundary integral method make the model presented a highly useful tool in solving the class of problems discussed.

## LIST OF REFERENCES

## LIST OF REFERENCES

1. Kalman, M. D., "Earth Heat Exchangers for Ground Source Heat Pumps," M.S. Thesis, Georgia Institute of Technology, 1980.
2. Ahmed, A. E., "Simulation of Simultaneous Heat Moisture Transfer in Soils Heated by Buried Pipes," Ph.D. Dissertation, Ohio State University, 1980.
3. Black, W. Z., Hartley, J. G., Bush, R. A., and Martin, M. A., Jr., "Thermal Stability of Soils Adjacent to Underground Transmission Power Cables," Report EPRI EL-2595, September, 1982.
4. Morse, P. M., and Feshbach, H., "Methods of Theoretical Physics," Part II, McGraw-Hill, New York, 1961.
5. Jaswon, M. A., "Integral Equation Methods in Potential Theory," Proceedings Royal Society, Vol. 275 A, P23, 1963.
6. MacMillan, W. D., "The Theory of Potential," Dover, New York, 1958.
7. Symm, G. T., "Integral Equation Methods in Potential Theory," Proceedings Royal Society, Vol. 275 A, P 33, 1963.
8. Mikhlin, S. G., "Integral Equations", Pergamon Press, New York(1957).
9. Brebbia, C. A., "The Boundary Element Method for Engineers," Pentech Press, London, Halstead, New York,(1978).
10. Brebbia, C. A.,(ED.), "Recent Advances in Boundary Element Methods," Proc. 1st Int. Conference Boundary Element Methods, Southampton University, 1978, Pentech Press, London(1978).
11. Brebbia, C. A.(ED.), "New Developments in Boundary Element Methods, Southampton University, 1980, CML publications, Southampton(1980).
12. Chang, Y. P., Kang, C. S., and Chen, D. J., "The Use of Fundamental Greens Functions for the Solution of Problems of Heat Conduction in Anisotropic Media," International Journal of Heat and Mass Transfer, Vol 16, PP. 1905-1918, 1973.
13. Wrobel, L. C., and Brebbia, C., "The Boundary Element Method for Steady-State and Transient Heat Conduction," 1st Int. Conf. Numerical Methods in Thermal Problems, Swansea, Taylor, C. and Morgan, K.(Eds), Pineridge Press(1979).
14. Cruse, T. A., and Vanduren, W., "Three Dimensional Elastic Stress Analysis of a Fracture Specimen with Edge Crack," Int. J. Fract. Mech., 7, PP. 1-15(1971).

15. Bui, H. D., "Some Remarks about the Formulation of Three-Dimensional Thermoelastoplastic Problems by Integral Equation," Int. J. Solids Structure, No. 14, PP. 935-939(1978)
16. Curran, D. A. S. , B. A. Lewis and M. Cross, Numerical Solution of the Diffusion Equation by a Potential Method, in this book
17. Brebbia, C. A., Telles, J. C. F., and Wrobel, L. C., "Boundary Element Techniques," Springer Verlag, Berlin Heidelberg, New York(1984)
18. Khader, M. S., and Hanna, M. C., "An Iterative Boundary Integral Numerical Solution for General Steady State Heat Conduction Problems," Journal of Heat Transfer, Vol. 103, PP. 26-31, February, 1981.
19. Khader, M. S., and Hanna, M. C., "Surface Integral Numerical Solution for General Steady Heat Conduction in Composite Media," Proc. Seventh International Heat Transfer Conference, Munchen, FRG, Volume 2, PP. 15-20, 1982.
20. Rizzo, F. J. and Shippy, d. J., "A Method of Solution for Certain Problems of Transient Heat Conduction," AIAA Journal, Vol. 8, No. 11, pp. 2004-2009, Nov 1970.
21. Shaw, R. P., "An Integral Equation Approach to Diffusion," International Journal of Heat and Mass Transfer, Vol. 17, PP. 693-699, 1974.
22. Wrobel, L. C. and Brebbia, C. A., "A Formulation of the Boundary Element Method for Axisymetric Transient Heat Conduction," International Journal of Heat and Mass Transfer, Vol. 24, No. 5, PP. 843-850, 1981.
23. Barone, M. R., and Caulk, D. A., "Special Boundary Integral Equations for Approximate Solution of Laplaces Equation in Two-Dimensional Regions with Circular Holes," The Quarterly Journal of Mechanics and Applied Mathematics, Vol. XXXIV, Pt. 3, PP. 265-286, 1981.
24. Barone, M. R., and Caulk, D. A., "Optimal Arrangement of Holes in a Two-Dimensional Heat Conduction by Special Boundary Integral Method," International Journal for Numerical Methods in Engineering, Vol. 18, PP. 675-685, 1982.
25. Arimilli, R. V., and Parang, M., "Numerical Analysis of Heat Transfer in Buried Horizontal Heat Exchanger Tubes," AIChE Symposium Series 225, Vol. 79, PP. 121-129, 1983

## APPENDIXES

## APPENDIX A

# EVALUATION OF LOGARITHMIC INTEGRALS

Consider Figure A-1 to show the method of integration[5,7]

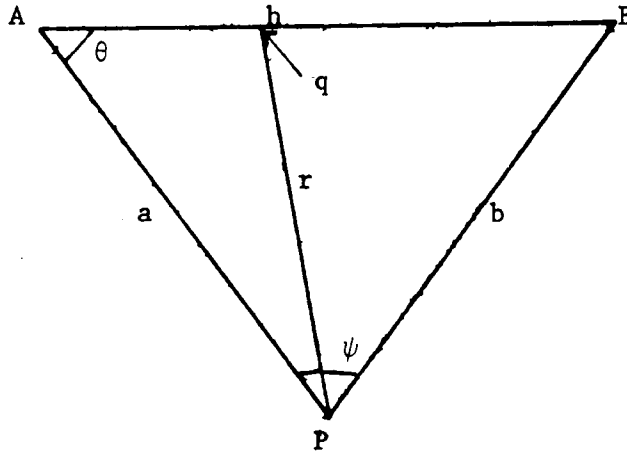


Figure A-1. Sketch for the evaluation of logarithmic integrals.

Consider the interval of integration over the straight line segment AB. Let h be the distance between points A and B,  $\psi$  be the subtending angle for the straight line AB at point P, and a and b represent distances from P to A and p to B, respectively.

$$\psi = \cos^{-1} \left( \frac{a^2 + b^2 - h^2}{2ab} \right) \quad , \quad 0 \leq \psi \leq \Pi$$

$$\theta = \cos^{-1} \left( \frac{a^2 + h^2 - b^2}{2ah} \right) , 0 \leq \theta \leq \pi ,$$

$$T \equiv \int_0^h \log |p - q| \, ds$$

$$= a \cos \theta (\log a - \log b) + h(\log b - 1) + a \psi \sin \theta$$

This is known as straight-line integration formula.

The following special cases need to be considered

(i) When P, A, and B are collinear,  $\theta = 0$  or  $\pi$

$$\sin \theta = 0 \quad \cos \theta = 1 \text{ or } -1$$

Specifically,

when  $\theta = 0$ ,  $a = (h+b)$  and the position of the points

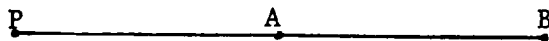
would be as shown below:



$$\text{Thus, } T = a(\log a - 1) - b(\log b - 1)$$

when  $\theta = \pi$ ,  $h = (b-a)$  and the position of the points

would be as shown below:



$$\text{Thus, } T = b(\log b - 1) - a(\log a - 1).$$

(ii) For a case of  $P = A$ , or  $P = B$ ,

$$T = h(\log h - 1)$$

where for  $P = A$ ,  $h = b$ , and for  $P = B$ ,  $h = a$

Using this method, the integral terms of equations (22) and (23) are evaluated.

From the above, it can be seen that the two-tube problem will have the following four special cases:

- when
- (i)  $i < j$ ,
  - (ii)  $i > j$  (but  $i \neq j+1$ ),
  - (iii)  $i = j$ , and
  - (iv)  $i = j+1$

As it is evident from the equations, the cases (i) and (ii) are negative of each other.

## APPENDIX B

# TWOTUBE COMPUTER PROGRAM

```

C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C      TWOTUBE COMPUTER PROGRAM DEVELOPED TO CALCULATE
C      TEMPERATURE DISTRIBUTION ON THE PLANE CONVECTIVE
C      SURFACE AND NORMAL DERIVATIVES OF TEMPERATURES
C      ON TUBE SURFACES FOR A PROBLEM OF TWO TUBES OF
C      SAME DIAMETER AND TEMPERATURE, AND LOCATED AT THE
C      SAME DEPTH FOR A PROBLEM IN A SEMI-INFINITE MEDIUM
C      (WITHOUT ASSUMING SYMMETRY)
C
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C      INPUT USED
C
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C      XI(I) = NON-DIM. LENGTH IN XI-DIR. FROM
C              POINT 1 TO THE POINT WHERE I LIES.
C      A(I,J)= MATRIX OF ORDER J BY I.
C      R1(I) = NON-DIM. DISTANCE FROM THE CENTER OF
C              THE TUBE 1 TO THE POINT WHERE I LIES
C      R2(I) = NON-DIM. DISTANCE FROM THE CENTER OF
C              THE TUBE 2 TO THE POINT WHERE I LIES
C      N      = NUMBER OF DISCRETIZED POINTS(THAT IS
C              FOR (N-1) DISCRETIZED SEGMENTS, THERE
C              WILL BE N NUMBER OF POINTS.
C      A1      = TUBE DIA. WHICH IS USED TO NON-DIM.
C              ALL LENGTH PARAMETERS.
C      S      = NON-DIM. DISTANCE BETWEEN THE TUBE
C              CENTERS ON XI-DIR.
C      D      = NON-DIM. DEPTH OF THE TUBE.
C      BI      = BIOT NUMBER.
C      PHIT    = TUBE SURFACE TEMP.
C      PHIA    = FAR FIELD TEMP.
C      I AND J = J MOVES ALL OVER THE BOUNDARY
C              FOR EACH FIXED I.
C      DELXI   = DISCRETIZED SEGMENT LENGTH.
C
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C      DIMENSION XI(260),A(260,260),R1(260),R2(260)
C      DIMENSION C(260),WKAREA(260)
C      DIMENSION F1(260),F2(260),F3(260),F4(260),F5(260)
C      DATA N,D,S,BI,M/250,25.0,5.0,1.0,2/
C      DATA A12,PHID,PHIA/1.0,274.0,62.0/
C      DO 5 LL=1,4
C      READ (21,*)

```

```

      NSPECI = N
      S = S/A12
      D = D/A12
      PHIT = (PHID-PHIA)/(PHID-PHIA)
      A1 = A12/A12
C     FIND XTMAX (HERE S = XTMAX)
      FAC = 11.3*0.87
      IR = 0
700    IF(IR.EQ.1) FAC =11.3
750    QOOLD = QONEW
      XI(1) = -FAC*(SQRT(D*S))
      N = NSPECI

C
C     FIND SEGMENT LENGTH DELXI
C     EQUAL SEGMENTS HAVE BEEN USED.
C
      DELXI = D/16.
C     DELXI = 1.0
C
C     CALCULATE NO. OF POINTS REQUIRED
      NREQ = (ABS(XI(1))/DELXI)*2
C
C     IF NREQ IS GREATER THAN N,STOP THE
C     PROGRAM AND CHANGE THE DIMENSIONS,
C     OTHERWISE N=NREQ. MAKE SURE N IS
C     ODD NUMBER(SINCE WE ARE DENOTING
C     FIRST POINT AS 1)
C
      IF(NREQ.EQ.N) GOTO 180
      IF(NREQ.LT.N) GOTO 160
      WRITE(23,170)NREQ
170    FORMAT(1H ,'NREQ = ',I6)
      STOP
160    NN = N
190    NN = NN-2
      IF(NN.GT.NREQ) GOTO 190
      IF(NN.EQ.NREQ) GOTO 180
      IF(NN.LT.NREQ) NREQ=NN+2
180    N = NREQ
C
      XI(1) = -(N/2*DELXI)
C     EVALUATE LENGTHS AT EVERY POINT.
      DO 10 I=1,N
      XI(I+1) = XI(I)+DELXI
10    CONTINUE
C
      PI = 3.1415926
      D2 = D*D
      D2S = D/(2*S)
C
C     -----

```

```

C      EQUATION 26
C      -----
C      BOTH I AND J ARE MOVING ON THE BOUNDARY
C      EVALUATE COEFF. OF PHI(J)
      DO 20 I=1,N
      DO 30 J=1,N
      C1 = ABS(XI(J+1)-XI(I))
      B1 = ABS(XI(J)-XI(I))
      IF(C1.EQ.0.0)A(I,J)=-BI/(2.*PI)*B1*(ALOG(B1)-1.)
      IF(B1.EQ.0.0)A(I,J)=-BI/(2.*PI)*C1*(ALOG(C1)-1.)
      IF((C1.EQ.0.0).OR.(B1.EQ.0.0)) GOTO25
      A(I,J) = -BI/(2.*PI)*(C1*(ALOG(C1)-1.)-B1*
1          (ALOG(B1)-1.))
      IF(I.GT.J+1) A(I,J)=-A(I,J)
25      IF(I.EQ.J) A(I,J)=0.5+A(I,J)
30      CONTINUE
20      CONTINUE
C
C      EVALUATION OF COEFF. OF Q0,Q1, AND Q2 FOR 2 TUBES.
C
      DO 40 I=1,N
      R1(I) = SQRT((XI(I)-S)**2+D2)
      R2(I) = SQRT((XI(I)+S)**2+D2)
C      AS LONG AS D NOT EQUAL TO ZERO,WE SHOULD
C      NEVER HAVE R1 OR R2 EQUAL TO ZERO.
      A(I,N+1) = ALOG(R1(I))
      A(I,N+2) = -0.5*(A1*(D/R1(I)**2))
      A(I,N+3) = -0.5*(A1*((XI(I)-S)/R1(I)**2))
      A(I,N+4)=ALOG(R2(I))
      A(I,N+5)=-0.5*(A1*(D/R2(I)**2))
      A(I,N+6)=-0.5*(A1*((XI(I)+S)/R2(I)**2))
40      CONTINUE
C
C      J MOVING ON THE BOUNDARY WHERE I IS AT THE
C      CENTER OF THE TUBE
C      -----
C      GENERATE COEFF. OF EQ. 27
C      -----
C
C      EVALUATE COEFF.PHI(J)
      DO 45 J=1,N
      F1(J) = XI(J+1)-S
      F2(J) = XI(J+1)+S
      F3(J) = XI(J)-S
      F4(J) = XI(J)+S
      F5(J) = XI(J+1)-XI(J)
45      CONTINUE
      DO 50 J=1,N
      Z1=((F3(J)**2+F5(J)**2-F1(J)**2)/(2.*F5(J)))

```

```

1      *(ALOG(SQRT(F3(J)**2+D2))-ALOG(SQRT
1      (F1(J)**2+D2)))+F5(J)*(ALOG(SQRT
1      (F1(J)**2+D2))-1.)
      Z2=R1(J)*ACOS((F3(J)**2+F1(J)**2+2.*D2-
1      F5(J)**2)/(2.*R1(J)*SQRT(F1(J)**2+D2)))*SIN
1      (ACOS((F3(J)**2+F5(J)**2-F1(J)**2)/
1      (2.*R1(J)*F5(J))))
      A(N+1,J)=-BI/(2.*PI)*(Z1+Z2)
      Z3=((F4(J)**2+F5(J)**2-F2(J)**2)/(2.*F5(J)))
1      *(ALOG(SQRT(F4(J)**2+D2))-ALOG(SQRT
1      (F2(J)**2+D2)))+F5(J)*(ALOG(SQRT
1      (F2(J)**2+D2))-1.)
      Z4=R2(J)*ACOS((F4(J)**2+F2(J)**2+2.*D2-
1      F5(J)**2)/(2.*R2(J)*SQRT(F2(J)**2+D2)))*SIN
1      (ACOS((F4(J)**2+F5(J)**2-F2(J)**2)/
1      (2.*R2(J)*F5(J))))
      A(N+2,J) = -BI/(2.*PI)*(Z3+Z4)
50    CONTINUE
C    EVALUATE COEFF. OF Q0,Q1 AND Q2 FOR 2 TUBES.
      A(N+1,N+1)=0.0
      A(N+1,N+2)=0.0
      A(N+1,N+3)=0.0
      A(N+1,N+4)=ALOG(2.*S)
      A(N+1,N+5)=0.0
      A(N+1,N+6)=- (1./(4.*S))
      A(N+2,N+1)=ALOG(2.*S)
      A(N+2,N+2)=0.0
      A(N+2,N+3)=1./(4.*S)
      A(N+2,N+4)=0.0
      A(N+2,N+5)=0.0
      A(N+2,N+6)=0.0

C    -----
C    EQ.28
C    -----
C    EVALUATE COEFF. OF PHI(J)
      TH1=2.*ASIN(1.)
      S1 = -S
      DO 60 J=1,N
      IF(XI(J).LT.S.AND.XI(J+1).GT.S) GOTO 61
      A(N+3,J) = -BI*(ASIN(D/(SQRT(F1(J)*F1(J)+D2)))
1      -ASIN(D/(SQRT(F3(J)*F3(J)+D2))))
      IF(XI(J).LT.S) A(N+3,J) = -A(N+3,J)
      GOTO 62
61    A(N+3,J) = -BI*(ASIN(D/(SQRT(F1(J)*F1(J)+D2)))
1      +ASIN(D/(SQRT(F3(J)*F3(J)+D2)))-TH1)
62    IF(XI(J).LT.S1.AND.XI(J+1).GT.S1) GOTO 65
      A(N+4,J) = -BI*( ASIN(D/(SQRT(F2(J)*F2(J)+D2)))
1      -ASIN(D/(SQRT(F4(J)*F4(J)+D2))))
      IF(XI(J).LT.S1) A(N+4,J) = -A(N+4,J)
      GOTO 60

```

```

65      A(N+4,J) = -BI*( ASIN(D/(SQRT(F2(J)*F2(J)+D2)))
1          +ASIN(D/(SQRT(F4(J)*F4(J)+D2)))-TH1)
60      CONTINUE
C      EVALUATE COEFF. OF Q0,Q1, AND Q2 FOR 2 TUBES.
      A(N+3,N+1)=0.0
      A(N+3,N+2)=-PI
      A(N+3,N+3)=0.0
      A(N+3,N+4)=0.0
      A(N+3,N+5)=-(PI/(4.*S**2))
      A(N+3,N+6)=0.0
      A(N+4,N+1)=0.0
      A(N+4,N+2)=-(PI/(4.*S**2))
      A(N+4,N+3)=0.0
      A(N+4,N+4)=0.0
      A(N+4,N+5)=-PI
      A(N+4,N+6)=0.0
C      -----
C      EQ 29
C      -----
C      EVALUATE COEFF. OF PHI(J)
      DO 70 J=1,N
1      A(N+5,J) = 0.5*BI*(ALOG(F1(J)*F1(J)+D2)
          -ALOG(F3(J)*F3(J)+D2))
1      A(N+6,J)=0.5*BI*(ALOG(F2(J)*F2(J)+D2)
          -ALOG(F4(J)*F4(J)+D2))
70      CONTINUE
C      EVALUATE COEFF.OF Q0,Q1. AND Q2 FOR 2 TUBES.
      A(N+5,N+1)=0.0
      A(N+5,N+2)=0.0
      A(N+5,N+3)=-PI
      A(N+5,N+4)=PI/S
      A(N+5,N+5)=0.0
      A(N+5,N+6)=PI/(4.*S**2)
      A(N+6,N+1)=-(PI/S)
      A(N+6,N+2)=0.0
      A(N+6,N+3)=PI/(4.*S**2)
      A(N+6,N+4)=0.0
      A(N+6,N+5)=0.0
      A(N+6,N+6)=-PI
C      -----
C      EVALUATION OF R.H.S.
C      -----
C      EQ. 26
C      -----
      DO 80 I=1,N
      C(I) = 0.0
80      CONTINUE
C      -----
C      EQ. 27
C      -----

```

```

C      C(N+1) = -1.0
C      C(N+2) = -1.0
C      -----
C      EQ. 28
C      -----
C      C(N+3) = 0.0
C      C(N+4) = 0.0
C      -----
C      EQ. 29
C      -----
C      C(N+5) = 0.0
C      C(N+6) = 0.0
C
C      SOLVE MATRIX
C
C      NN = N+6
C      IDGT IS AN INPUT OPTION.
C      IF IDGT IS GREATER THAN ZERO, THE
C      ELEMENTS OF MATRIX AND R.H.S.
C      VECTOR ARE ASSUMED TO BE CORRECT
C      TO IDGT DECIMAL DIGITS AND ROUTINE
C      PERFORMS AN ACCURACY TEST.
C
C      IF IDGT EQUAL TO ZERO, THE ACCURACY
C      IS BYPASSED.
C
C      IDGT = 4
C      CALL LEQTI1F(A,1,NN,260,C,IDGT,WKAREA,IER)
C      PRINT ALL INPUTS USED ON OUTPUT.
C      WRITE(23,90)
90      FORMAT(1H1//1X,75(1H*)//15X,
1      'INPUT QUANTITIES'//1X,75(1H*)//)
      WRITE(23,520) N,BI,DELXI,PHIT,A1,D,D2S
520     FORMAT(1H , 'N = ',I3,2X,'BIOT NO. = ',F10.3,
1           2X,'SEGMENT LENGTH = ',F10.4//1X,
1           'MAX. TEMP. = ',F10.4,2X,'RADIUS',
1           ' OF MAX. TEMP. TUBE = ',F10.6//1X,
1           'DEPTH OF THE TUBE FROM THE PLANE CONVECTIVE',
1           'SURFACE = ',F10.6,2X,'D/2S = ',F8.3)
C      PRINT OUTPUT
      WRITE(23,110)
110     FORMAT(1H1//1X,75(1H*)//15X,
1           'OUTPUT QUANTITIES'//75(1H*)//)
C      PRINT ERROR MESSAGES FROM MATRIX SOLVER
      IF(IER.EQ.129) WRITE(23,120)
120     FORMAT(1H ,15X,'ERROR MESSAGES'//75(1H*),
1           //'MATRIX IS ALGORITHMICALLY SINGULAR')
      IF(IER.EQ.34) WRITE(23,130)
130     FORMAT(1H ,15X,'ERROR MESSAGES'//75(1H*),
1           '//2X,'ACCURACY TEST FAILED.THE COMPUTED '

```

```

1  , 'SOLUTION MAY BE IN ERROR BY',/2X' MORE THAN CAN'
1  , ' BE ACCOUNTED FOR BY THE UNCERTAINTY OF '
1  , 'THE DATA',/)
  IR = IR+1
  WRITE(5,*)IR
C
C  PRINT HEAT TRANSFER SOLUTION.
C
  QONEW = 0.0
  Q1TOT=0.0
  Q2TOT=0.0
C
C
  WRITE(23,140) C(N+1),C(N+2),C(N+3)
140  FORMAT(/5X,'Q01 = ',F10.6,5X,'Q11 = ',F10.6,
1  5X,'Q21 = ',F10.6,/)
C
  WRITE(23,146) C(N+4),C(N+5),C(N+6)
146  FORMAT(/5X,'Q02 = ',F10.6,5X,'Q12 = ',F10.6,
1  5X,'Q22 = ',F10.6,/)
C
  QONEW = C(N+1)+C(N+4)
  Q1TOT = C(N+2)+C(N+5)
  Q2TOT = C(N+3)+C(N+6)
C
  WRITE(23,555) QONEW,Q1TOT,Q2TOT
555  FORMAT(//2X,'Q0TOTAL = ',F10.6,2X,'Q1TOT = ',F10.6,
1  2X,'Q2TOT = ',F10.6)
  IF(IR.EQ.1) GOTO 700
  IF(IR.EQ.7)GOTO 575
  QDIFF = QONEW-QOOLD
  PERCEN = (QDIFF/QONEW)*100.0
  IF(PERCEN.LT.1.0)GOTO 575
  FAC = 1.125*FAC
  GOTO 750
C
575  WRITE(23,573)IR,PERCEN
573  FORMAT(//,2X,'NO. OF ITERATIONS = ',I3,3X,
1  '% DIFF. = ',F6.3)
C
C  PRINT TEMP. DISTRIBUTION
C
  WRITE(23,841)
841  FORMAT(1H ,3X,'I',10X,'XI',7X,'TEMPERATURE'
1  ,11X,'I',7X,'XI',8X,'TEMPERATURE'
1  ,12X,'I',9X,'XI',8X,'TEMPERATURE'//)
  NN3 = N/3
  NN4 = NN3*3
  NN5 = N-NN4
  IF(NN5.EQ.1) GOTO 871

```

```

      IF(NN5.EQ.2) GOTO 861
      IF(NN5.EQ.0) GOTO 881
871  N2 = NN3
      II1 = 1
      II2 = NN3+2
      II3 = NN3*2+2
      DO 874 I=1,N2
        WRITE(23,875)II1,XI(II1),C(II1),II2,
1          XI(II2),C(II2),II3,
1          XI(II3),C(II3)
875  FORMAT(2X,I3,2X,F12.5,4X,F10.7,
1      11X,I3,2X,F12.5,4X,F10.7,
1      11X,I3,2X,F12.5,4X,F10.7)
      II1 = II1+1
      II2 = II2+1
      II3 = II3+1
874  CONTINUE
      NN6 = N2+1
      WRITE(23,876)NN6,XI(NN6),C(NN6)
876  FORMAT(2X,I3,2X,F12.5,4X,F10.7)
      GOTO 900
861  N2 = NN3
      II1 = 1
      II2 = NN3+2
      II3 = NN3*2+3
      DO 864 I=1,N2
        WRITE(23,865)II1,XI(II1),C(II1),II2,
1          XI(II2),C(II2),II3,
1          XI(II3),C(II3)
865  FORMAT(2X,I3,2X,F12.5,4X,F10.7,
1      11X,I3,2X,F12.5,4X,F10.7,
1      11X,I3,2X,F12.5,4X,F10.7)
      II1 = II1+1
      II2 = II2+1
      II3 = II3+1
864  CONTINUE
      NN6 = N2+1
      N3 = N2+NN6+1
      WRITE(23,866)NN6,XI(NN6),C(NN6),
1      N3,XI(N3),C(N3)
866  FORMAT(2X,I3,2X,F12.5,4X,F10.7,
1      11X,I3,2X,F12.5,4X,F10.7)
      GOTO 900
881  N2 = NN3
      II1 = 1
      II2 = 1+NN3
      II3 = 1+NN3*2
      DO 884 I=1,N2
        WRITE(23,885)II1,XI(II1),C(II1),II2,
1          XI(II2),C(II2),II3,

```

```

1          XI(II3),C(II3)
885  FORMAT(2X,I3,2X,F12.5,4X,F10.7,
1      11X,I3,2X,F12.5,4X,F10.7,
1      11X,I3,2X,F12.5,4X,F10.7)
      II1 = II1+1
      II2 = II2+1
      II3 = II3+1
884  CONTINUE
C 5   CONTINUE
900  STOP
      END

```

## APPENDIX C

# TWOSYMM COMPUTER PROGRAM

```

C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C      TWOSYMM COMPUTER PROGRAM DEVELOPED TO CALCULATE
C      TEMPERATURE DISTRIBUTION ON THE PLANE CONVECTIVE
C      SURFACE AND NORMAL DERIVATIVES OF TEMPERATURES
C      ON TUBE SURFACES FOR A PROBLEM OF TWO TUBES OF
C      SAME DIAMETER AND TEMPERATURE, AND LOCATED AT THE
C      SAME DEPTH FOR A PROBLEM IN A SEMI-INFINITE MEDIUM
C      (WITH INSERTING SYMMETRY CONDITIONS, I.E. Q01=Q02,
C      Q11=Q12, Q21=Q22)
C
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C      INPUT USED
C
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C      XI(I) = NON-DIM. LENGTH IN XI-DIR. FROM
C      POINT 1 TO THE POINT WHERE I LIES.
C      A(I,J)= MATRIX OF ORDER J BY I.
C      R1(I) = NON-DIM. DISTANCE FROM THE CENTER OF
C      THE TUBE 1 TO THE POINT WHERE I LIES
C      R2(I) = NON-DIM. DISTANCE FROM THE CENTER OF
C      THE TUBE 2 TO THE POINT WHERE I LIES
C      N      = NUMBER OF DISCRETIZED POINTS(THAT IS
C      FOR (N-1) DISCRETIZED SEGMENTS, THERE
C      WILL BE N NUMBER OF POINTS.
C      A1     = TUBE DIA. WHICH IS USED TO NON-DIM.
C      ALL LENGTH PARAMETERS.
C      S      = NON-DIM. DISTANCE BETWEEN THE TUBE
C      CENTERS ON XI-DIR.
C      D      = NON-DIM. DEPTH OF THE TUBE.
C      BI     = BIOT NUMBER.
C      PHIT   = TUBE SURFACE TEMP.
C      PHIA   = FAR FIELD TEMP.
C      I AND J = J MOVES ALL OVER THE BOUNDARY
C      FOR EACH FIXED I.
C      DELXI  = DISCRETIZED SEGMENT LENGTH.
C
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C      DIMENSION XI(200),A(110,110),R1(110),R2(110)
C      DIMENSION C(110),WKAREA(110)
C      DIMENSION F1(110),F2(110),F3(110),F4(110),F5(110)
C      DATA N,D,S,BI/190,30.0,5.0,0.5/
C      DATA A12,PHID,PHIA/1.0,274.0,62.0/
C      DO 5 LL=1,4
C      READ (21,*)

```

```

      NSPECI = N
      S = S/A12
      D = D/A12
      PHIT = (PHID-PHIA)/(PHID-PHIA)
      A1 = A12/A12
C      FIND XTMAX (HERE S = XTMAX)
      FAC = 11.3*0.87
      IR = 0
      700 IF(IR.EQ.1) FAC =11.3
      750 QOOLD = QONEW
      XI(1) = -FAC*(SQRT(D*S))
      N = NSPECI

C
C      FIND SEGMENT LENGTH DELXI
C      EQUAL SEGMENTS HAVE BEEN USED.
C
      DELXI = D/16.
C      DELXI = 3.1
C
C      CALCULATE NO. OF POINTS REQUIRED
      NREQ = (ABS(XI(1))/DELXI)*2
C
C      IF NREQ IS GREATER THAN N,STOP THE
C      PROGRAM AND CHANGE THE DIMENSIONS,
C      OTHERWISE N=NREQ. MAKE SURE N IS
C      ODD NUMBER(SINCE WE ARE DENOTING
C      FIRST POINT AS 1)
C
      IF(NREQ.EQ.N) GOTO 180
      IF(NREQ.LT.N) GOTO 160
      WRITE(22,170)NREQ
      170 FORMAT(1H , 'NREQ = ',I6)
      STOP
      160 NN = N
      190 NN = NN-2
      IF(NN.GT.NREQ) GOTO 190
      IF(NN.EQ.NREQ) GOTO 180
      IF(NN.LT.NREQ) NREQ=NN+2
      180 N = NREQ
      XI(1) = -(N/2*DELXI)
      K = N/2+1
C      EVALUATE LENGTHS AT EVERY POINT.
      DO 10 I=1,N
      XI(I+1) = XI(I)+DELXI
      10 CONTINUE
      PI = 3.1415926
      D2 = D*D
      D2S=D/(2.*S)
C      -----
C      EQUATION 26
C      -----

```

```

C      BOTH I AND J ARE MOVING ON THE BOUNDARY
C      EVALUATE COEFF. OF PHI(J)
      KI=0
      DO 20 I=K,N
      KI = KI+1
      KJ = 0
      JL = K
      DO 30 J=K,N
      KJ = KJ+1
      C1 = ABS(XI(J+1)-XI(I))
      B1 = ABS(XI(J)-XI(I))
      IF(C1.EQ.0.0)AIJ1=-BI/(2.*PI)*B1*(ALOG(B1)-1.)
      IF(B1.EQ.0.0)AIJ1=-BI/(2.*PI)*C1*(ALOG(C1)-1.)
      IF((C1.EQ.0.0).OR.(B1.EQ.0.0)) GOTO25
      AIJ1 = -BI/(2.*PI)*(C1*(ALOG(C1)-1.)-B1*
1          (ALOG(B1)-1.))
      IF(I.GT.J+1) AIJ1=-AIJ1
25      IF(I.EQ.J) AIJ1=0.5+AIJ1
      JL = JL+1
      C2 = ABS(XI(JL+1)-XI(I))
      B2 = ABS(XI(JL)-XI(I))
      IF(C2.EQ.0.0)AIJ2=-BI/(2.*PI)*B2*(ALOG(B2)-1.)
      IF(B2.EQ.0.0)AIJ2=-BI/(2.*PI)*C2*(ALOG(C2)-1.)
      IF((C2.EQ.0.0).OR.(B2.EQ.0.0)) GOTO 27
      AIJ2 = -BI/(2.*PI)*(C2*(ALOG(C2)-1.)-B2*
1          (ALOG(B2)-1.))
      IF(I.GT.JL+1)AIJ2=-AIJ2
27      IF(I.EQ.JL)AIJ2=0.5+AIJ2
      A(KI,KJ) = AIJ1+AIJ2
30      CONTINUE
20      CONTINUE
C
C      EVALUATION OF COEFF. OF Q0,Q1, AND Q2.
C
      JL=0.0
      DO 40 I=K,N
      JL=JL+1
      R1(JL) = SQRT((XI(I)-S)**2+D2)
      R2(JL) = SQRT((XI(I)+S)**2+D2)
C      AS LONG AS D NOT EQUAL TO ZERO,WE SHOULD
C      NEVER HAVE R1 OR R2 EQUAL TO ZERO.
      A(JL,K) = ALOG(R1(JL)*R2(JL))
      A(JL,K+1) = -0.5*(A1*(D/R1(JL)**2)+A1*(D/R2(JL)**2))
      A(JL,K+2) = -0.5*(A1*((XI(I)-S)/R1(JL)**2)-
1          A1*((XI(I)+S)/R2(JL)**2))
40      CONTINUE
C      J MOVING ON THE BOUNDARY WHERE I IS AT THE
C      CENTER OF THE TUBE
C      -----
C      GENERATE COEFF. OF EQ. 27
C      -----
C      EVALUATE COEFF.PHI(J)

```

```

JL =0
DO 45 J=K,N
JL=JL+1
F1(JL) = XI(J+1)-S
F2(JL) = XI(J+1)+S
F3(JL) = XI(J)-S
F4(JL) = XI(J)+S
F5(JL) = XI(J+1)-XI(J)
45 CONTINUE
JL=0
DO 50 J=K,N
JL =JL+1
Z1=((F3(JL)**2+F5(JL)**2-F1(JL)**2)/(2.*F5(JL)))
1 *(ALOG(SQRT(F3(JL)**2+D2))-ALOG(SQRT
1 (F1(JL)**2+D2)))+F5(JL)*(ALOG(SQRT
1 (F1(JL)**2+D2))-1.)
Z2=R1(JL)*ACOS((F3(JL)**2+F1(JL)**2+2.*D2-
1 F5(JL)**2)/(2.*R1(JL)*SQRT(F1(JL)**2+D2)))*SIN
1 (ACOS((F3(JL)**2+F5(JL)**2-F1(JL)**2)/
1 (2.*R1(JL)*F5(JL))))
Z3=((F4(JL)**2+F5(JL)**2-F2(JL)**2)/(2.*F5(JL)))
1 *(ALOG(SQRT(F4(JL)**2+D2))-ALOG(SQRT
1 (F2(JL)**2+D2)))+F5(JL)*(ALOG(SQRT
1 (F2(JL)**2+D2))-1.)
Z4=R2(JL)*ACOS((F4(JL)**2+F2(JL)**2+2.*D2-
1 F5(JL)**2)/(2.*R2(JL)*SQRT(F2(JL)**2+D2)))*SIN
1 (ACOS((F4(JL)**2+F5(JL)**2-F2(JL)**2)/
1 (2.*R2(JL)*F5(JL))))
A(K,JL) = -BI/(2.*PI)*(Z1+Z2+Z3+Z4)
50 CONTINUE
C EVALUATE COEFF. OF Q0,Q1 AND Q2
A(K,K) = ALOG(2.*S)
A(K,K+1) = 0.0
A(K,K+2) = 1./(4.*S)
C -----
C EQ.28
C -----
C EVALUATE COEFF. OF PHI(J)
C
C IF J GREATER THAN OR EQUAL TO K BUT
C LESS THAN S, TAKE T5 EQUAL TO NEGATIVE
C OF T5. THIS IS BECAUSE THAT MY ALGEBRA
C SAYS SIN(THETA) = SIN(PI-THETA) BUT
C COMPUTER WHEN TAKING ARCSIN, TAKE IT
C AS THETA.
TH1=2.*ASIN(1.)
JL=0
DO 60 J=K,N
JL=JL+1
IF(XI(J).LT.S.AND.XI(J+1).GT.S) GOTO 55

```

```

      T5 = ASIN(D/(SQRT(F1(JL)*F1(JL)+D2)))
1      -ASIN(D/(SQRT(F3(JL)*F3(JL)+D2)))
      IF(XI(J).LT.S) T5=-T5
      GOTO 58
55     T5 = ASIN(D/(SQRT(F1(JL)*F1(JL)+D2)))
1      +ASIN(D/(SQRT(F3(JL)*F3(JL)+D2)))-TH1
58     T6 = ASIN(D/(SQRT(F2(JL)*F2(JL)+D2)))
1      -ASIN(D/(SQRT(F4(JL)*F4(JL)+D2)))
      A(K+1,JL) = -BI*(T5+T6)
60     CONTINUE
C      EVALUATE COEFF. OF Q0,Q1, AND Q2
      A(K+1,K) = 0.0
      A(K+1,K+1) = -PI*(1.+1./(4.*S**2))
      A(K+1,K+2) = 0.0
C      -----
C      EQ 29
C      -----
C      EVALUATE COEFF. OF PHI(J)
      JL=0
      DO 70 J=K,N
      JL=JL+1
      A(K+2,JL) = 0.5*BI*(ALOG(F1(JL)*F1(JL)+D2)
1      -ALOG(F2(JL)*F2(JL)+D2)-ALOG(F3(JL)*F3(JL)
1      +D2)+ALOG(F4(JL)*F4(JL)+D2))
70     CONTINUE
C      EVALUATE COEFF.OF Q0,Q1. AND Q2
      A(K+2,K) = PI/S
      A(K+2,K+1) = 0.0
      A(K+2,K+2) = -PI*(1.+1./(4.*S**2))
C      -----
C      EVALUATION OF R.H.S.
C      -----
C      EQ. 26
C      -----
      IL=K-1
      DO 80 I=1,IL
      C(I) = 0.0
80     CONTINUE
C      -----
C      EQ. 27
C      -----
      C(IL+1) = -1.0
C      -----
C      EQ. 28
C      -----
      WRITE(5,*)BI
      C(IL+2) = 0.0
      WRITE(5,*)BI
C      -----
C      EQ. 29
C      -----

```

```

      C(IL+3) = 0.0
C
C      SOLVE MATRIX
C
      NN = N/2+3
C      IDGT IS AN INPUT OPTION.
C      IF IDGT IS GREATER THAN ZERO, THE
C      ELEMENTS OF MATRIX AND R.H.S.
C      VECTOR ARE ASSUMED TO BE CORRECT
C      TO IDGT DECIMAL DIGITS AND ROUTINE
C      PERFORMS AN ACCURACY TEST.
C
C      IF IDGT EQUAL TO ZERO, THE ACCURACY
C      IS BYPASSED.
C
      IDGT = 4
      CALL LEQT1F(A,1,NN,110,C,IDGT,WKAREA,IER)
C
C      PRINT ALL INPUTS USED ON OUTPUT.
C
      WRITE(22,90)
90      FORMAT(1H1,/,1X,55(1H*),1X,'INPUT PARAMETERS',1X,55(1H*))
      WRITE(22,100) N,DELXI,S,D,BI,D2S
100     FORMAT(/2X,'N = ',I4,', ',3X,'DELXI = ',F10.4,
1       ', ',3X,'S = ',F10.4,', ',3X,'D = ',
1       F10.4,', ',3X,'BI = ',F10.4,', ',
1       3X,'D/2S = ',F10.6)
C
C      PRINT OUTPUT
C
      WRITE(22,102)
102     FORMAT(/1X,60(1H*),1X,'OUTPUT',1X,60(1H*))
      WRITE(22,106)
106     FORMAT(1H,/,1X,'ACCURACY TEST FAILED.THE COMPUTED '
1       ', 'SOLUTION MAY BE IN ERROR BY',/, ' MORE THAN CAN'
1       ', ' BE ACCOUNTED FOR BY THE UNCERTAINTY OF '
1       ', 'THE DATA',/)
      IR = IR+1
C      PRINT HEAT TRANSFER SOLUTION
      QONEW = 0.0
      M11=N-K+2
      M22=N-K+3
      M33=N-K+4
      WRITE(22,140) C(M11),C(M22),C(M33)
140     FORMAT(5X,'QO = ',F10.6,5X,'Q1 = ',F10.6,
1       5X,'Q2 = ',F10.6,/)
      QONEW = C(M11)
      IF(IR.EQ.1) GOTO 700
      IF(IR.EQ.7)GOTO 575
      QDIFF = QONEW-QOOLD

```

```

        PERCEN = (QDIFF/QONEW)*100.0
        IF(PERCEN.LT.1.0)GOTO 575
C      FAC = (QONEW/QOOLD*4.0-3.0)*FAC
C      FAC = FAC+2.0
        FAC = 1.125*FAC
        GOTO 750
C
    575  WRITE(22,573)IR,PERCEN
    573  FORMAT(/,2X,'NO. OF ITERATIONS = ',I3,3X,
        1      '% DIFF. = ',F6.3)
C
C      PRINT TEMP. DISTRIBUTION IN THREE COLUMNS.
C
        WRITE(22,141)
    141  FORMAT(1H ,3X,'I',7X,'XI',7X,'TEMPERATURE'
        1      ,11X,'I',7X,'XI',8X,'TEMPERATURE'
        1      ,11X,'I',7X,'XI',8X,'TEMPERATURE'//)
        NN2 = N-K+1
        NN3 = NN2/3
        NN4 = 3*NN3+NN2
        NN5 = N-NN4
        IF(NN5.EQ.1) GOTO 171
        IF(NN5.EQ.2) GOTO 161
        IF(NN5.EQ.0) GOTO 181
    171  N2 = K+NN3-1
        II1 = K
        II2 = K+NN3+1
        II3 = K+NN3*2+1
        M44 = II1-K+1
        M55 = II2-K+1
        M66 = II3-K+1
        DO 174 I=K,N2
        WRITE(22,175)M44,XI(II1),C(M44),M55,
        1      XI(II2),C(M55),M66,
        1      XI(II3),C(M66)
    175  FORMAT(2X,I3,2X,F10.5,4X,F10.7,
        1      11X,I3,2X,F10.5,4X,F10.7,
        1      11X,I3,2X,F10.5,4X,F10.7)
        M44 = M44+1
        M55 = M55+1
        M66 = M66+1
        II1 = II1+1
        II2 = II2+1
        II3 = II3+1
    174  CONTINUE
        NN6 = N2+1
        NN7 = NN3+1
        WRITE(22,176)NN7,XI(NN6),C(NN7)
    176  FORMAT(2X,I3,2X,F10.5,4X,F10.7)
        GOTO 200

```

```

161      N2 = K+NN3-1
        II1 = K
        II2 = K+NN3+1
        II3 = K+NN3*2+2
        M44 = II1-K+1
        M55 = II2-K+1
        M66 = II3-K+1
        DO 164 I=K,N2
          WRITE(22,165)M44,XI(II1),C(M44),M55,
1          XI(II2),C(M55),M66,
1          XI(II3),C(M66)
165      FORMAT(2X,I3,2X,F10.5,4X,F10.7,
1          11X,I3,2X,F10.5,4X,F10.7,
1          11X,I3,2X,F10.5,4X,F10.7)
        M44 = M44+1
        M55 = M55+1
        M66 = M66+1
        II1 = II1+1
        II2 = II2+1
        II3 = II3+1
164      CONTINUE
        NN6 = N2+1
        NN7 = NN3+1
        N3 = N2+NN7+1
        NN8 = 2*NN7
        WRITE(22,166)NN7,XI(NN6),C(NN7),
1        NN8,XI(N3),C(NN8)
166      FORMAT(2X,I3,2X,F10.5,4X,F10.7,
1          11X,I3,2X,F10.5,4X,F10.7)
        GOTO 200
181      N2 = K+NN3-1
        II1 = K
        II2 = K+NN3
        II3 = K+NN3*2
        M44 = II1-K+1
        M55 = II2-K+1
        M66 = II3-K+1
        DO 184 I=K,N2
          WRITE(22,185)M44,XI(II1),C(M44),M55,
1          XI(II2),C(M55),M66,
1          XI(II3),C(M66)
185      FORMAT(2X,I3,2X,F10.5,4X,F10.7,
1          11X,I3,2X,F10.5,4X,F10.7,
1          11X,I3,2X,F10.5,4X,F10.7)
        M44 = M44+1
        M55 = M55+1
        M66 = M66+1
        II1 = II1+1
        II2 = II2+1
        II3 = II3+1

```

|     |          |
|-----|----------|
| 184 | CONTINUE |
| 200 | STOP     |
|     | END      |

## APPENDIX D

# GENTUBE COMPUTER PROGRAM

```

C *****
C
C GENTUBE computer program developed to compute
C temperature distribution on the plane convective
C surface and normal derivative of temperature on
C tube surfaces for a general problem of arbitrary
C tube locations, diameters and temperatures in a
C semi-infinite medium.
C
C *****
C
C DIMENSION XI(370),R(370,6),A(370,370)
C DIMENSION C(370),WKAREA(370),RAB(6,6)
C DIMENSION G1(6,6),G2(6,6),XT(6),YT(6)
C DIMENSION F1(6,370),F2(6,370),F3(6,370)
C DIMENSION AA(6),PHIT(6)
C REAL LARGE
C
C MAKE SURE THAT THE DIMENSIONS ARE
C EQUAL TO (N+3*M)
C
C N = NUMBER OF DISCRETIZED POINTS ON THE PLANE
C CONVECTIVE SURFACE
C M = NUMBER OF TUBES.
C Y = DEPTH OF LOCATION OF THE TUBE FROM THE PLANE
C CONVECTIVE SURFACE.
C PHIA = FAR-FIELD TEMPERATURE USED FOR
C NONDIMENSIONALIZATION.
C YT = LOCATION OF THE TUBE WITH RESPECTIVE TO AXIS
C XT = LOCATION OF THE TUBE WITH RESPECTIVE TO AXIS
C AA = TUBE RADIUS
C PHIT = TUBE TEMPERATURE
C
C DATA N,BI,M/352,1.0,6/
C NSPECI = N
C DATA Y,PHIA/24.0,62.0/
C DATA YT(1),XT(1),AA(1),PHIT(1)/0.0,0.0,1.0,274.0/
C DATA YT(2),XT(2),AA(2),PHIT(2)/0.0,6.0,1.0,274.0/
C DATA YT(1),XT(1),AA(1),PHIT(1)/0.0,4.0,1.0,274.0/
C DATA YT(2),XT(2),AA(2),PHIT(2)/0.0,-4.0,1.0,274.0/
C DATA AA(3),YT(3),XT(3),PHIT(3)/1.0,12.0,5.0,274.0/
C DATA AA(4),YT(4),XT(4),PHIT(4)/1.0,12.0,-5.0,274.0/
C DATA AA(5),YT(5),XT(5),PHIT(5)/1.0,-12.0,4.0,274.0/
C DATA AA(6),YT(6),XT(6),PHIT(6)/1.0,-12.0,-4.0,274.0/
C DATA AA(7),YT(7),XT(7),PHIT(7)/3.0,-12.0,5.0,425.0/
C DATA AA(8),YT(8),XT(8),PHIT(8)/3.0,-12.0,-5.0,425.0/
C
C FIND MAX. TEMP. AND RADIOUS OF
C THAT MAX. TEMP. TUBE.

```

```

        PHID = 0.0
C      THIS DO LOOP IS USEFUL IF ALL
C      TEMPERATURES ARE NEGATIVE.
        DO 200 I=1,M
          IF(PHIT(I).LT.PHID)PHID=PHIT(I)
200      CONTINUE
          DO 210 I=1,M
            IF(PHIT(I).LE.PHID) GOTO 210
            AD = AA(I)
            PHID = PHIT(I)
210      CONTINUE
C
C      TO CHECK WHETHER THERE ARE MORE
C      THAN ONE TUBE HAVING MAX. TEMP.
C      IF THERE ARE, WE SHOULD TAKE THE
C      RADIOUS OF THE MAX. RADIOUS TUBE
C      AMONG THOSE TUBES FOR NON-DIM.
C      PURPOSE.
C
        DO 220 I=1,M
          IF(PHID.NE.PHIT(I)) GOTO 220
          IF(AA(I).GT.AD)AD=AA(I)
220      CONTINUE
C
C      NON-DIMENSIONLIZE TEMPS. AND LENGTHS.
        DO 225 I=1,M
          PHIT(I) = (PHIT(I)-PHIA)/(PHID-PHIA)
          AA(I) = AA(I)/AD
          XT(I) = XT(I)/AD
          YT(I) = YT(I)/AD
225      CONTINUE
          Y = Y/AD
C
        DO 12 K=1,M
          DO 14 L=1,M
            IF(K.EQ.L) GOTO 14
            G1(K,L) = (Y-YT(L))-(Y-YT(K))
            G2(K,L) = XT(K)-XT(L)
            RAB(K,L) = SQRT(G1(K,L)**2+G2(K,L)**2)
14          CONTINUE
12        CONTINUE
C
C      CHECK WHETHER ANY TUBE IS INTERSECTING
C      WITH ANY OTHER TUBE.
C
        DO 92 K=1,M
          DO 94 L=1,M
            IF(K.EQ.L) GOTO 94
            XTOT = AA(K)+AA(L)+0.05
            IF(RAB(K,L).GE.XTOT)GOTO 94
            WRITE(22,96)K,XT(K),L,XT(L)
96          FORMAT(1H , 'XT(' I3' ) = ',F7.2,

```

```

1 2X,'XT('I3') = ',F7.2)
94 CONTINUE
92 CONTINUE
DO 97 K=1,M
DO 98 L=1,M
IF(K.EQ.L) GOTO 98
XTOT = AA(K)+AA(L)+0.05
IF(RAB(K,L).GE.XTOT)GOTO 98
STOP
98 CONTINUE
97 CONTINUE
C
C CHECK WHETHER ANY TUBE IS ABOVE THE SURFACE.
C IF IT IS,STOP THE PROGRAM AND CHECK. NO TUBE
C SHOULD BE ABOVE THE SURFACE FOR THIS CASE.
C
DO 110 I=1,M
IF((YT(I)+AA(I)).LT.Y) GOTO 110
WRITE(22,120) I,YT(I)
120 FORMAT(1H ,'YT('I3,') = ',F7.2)
110 CONTINUE
DO 130 I=1,M
IF((YT(I)+AA(I)).LT.Y) GOTO 130
STOP
130 CONTINUE
C TO FIND WHERE THE LOCATION OF AXIS
C SHOULD BE AND ABS(MAX(XT(I)))
C PP = ABS(MAX(XT(I)))
10 SMALL = 0.0
LARGE = 0.0
C NOW FIND LARGE AND SMALL WHERE LARGE
C IS THE LONGEST DISTANCE TUBE ON R.H.S.
C OF ETA(Y) AXIS AND SMALL IS THE LONGEST
C DISTANCE TUBE ON L.H.S. OF ETA AXIS.
DO 20 I = 1,M
IF(XT(I).GT.LARGE) LARGE = XT(I)
IF(XT(I).LT.SMALL) SMALL = XT(I)
20 CONTINUE
C NOW CHECK HOW MANY TUBES ARE L.H.S.
C OF ETA-AXIS. IF 95 PERCENT OF NO. OF
C TUBES OF ONE SIDE ARE GREATER THAN
C THE OTHER SIDE, THEN THE AXIS SHOULD
C BE MOVED TO THE SIDE WHERE THERE ARE
C MORE TUBES UNTIL THIS 95 PERCENT
C CRITERIA IS SATISFIED.
C
C KN=NO. OF TUBES ON L.H.S. OF ETA-AXIS
C KP=NO. OF TUBES ON R.H.S. OF ETA-AXIS
KN = 0
KP = 0
PP = 0.0
XNEG = SMALL

```

```

XPOS = LARGE
DO 30 I=1,M
IF(ABS(XT(I)).GT.PP) PP=ABS(XT(I))
IF(XT(I).LT.0.0) GOTO 40
IF(XT(I).GT.0.0) GOTO 50
IF(XT(I).EQ.0.0) GOTO 30
40  KN = KN+1
    IF(XT(I).GT.XNEG) XNEG=XT(I)
    GOTO 30
50  KP = KP+1
    IF(XT(I).LT.XPOS) XPOS=XT(I)
30  CONTINUE
    KN1 = 0.95*KN
    KP1 = 0.95*KP
    IF(KN1.GT.KP) GOTO 60
    IF(KP1.GT.KN) GOTO 70
    IF((KN1.LE.KP).AND.(KP1.LE.KN))GOTO 100
60  DO 80 I=1,M
    XT(I) = XT(I)-XNEG
80  CONTINUE
    GOTO 10
70  DO 90 I=1,M
    XT(I) = XT(I)-XPOS
90  CONTINUE
    GOTO 10
C
C    TO FIND THE TUBES OF MAX AND MIN DEPTHS
C    FROM THE PLANE CONVECTIVE SURFACE.
C
100  CHANGE = Y-YT(1)
    DO 140 I=1,M
    IF((Y-YT(I)).LT.CHANGE)CHANGE=Y-YT(I)
140  CONTINUE
C
C    SHORT=THE NEAREST TUBE DISTANCE.
C
    SHORT = CHANGE
C    YL = MAX. DEPTH TUBE
    YL = CHANGE
    DO 145 I = 1,M
    IF(Y-YT(I).GT.YL)YL=(Y-YT(I))
145  CONTINUE
C
C    SELECT LENGTH IN SUCH A WAY THAT
C    ABS OF XI(N) AND XI(1) SHOULD BE
C    10 TIMES GREATER THAN ABS(MAX(XT(I)))
C
    IF(M.EQ.1.AND.XT(1).EQ.0.0)PP = 10.0
    XI(1) = -14.*PP
    QONEW=0.0
    FAC = 11.3*0.87
    IR=0

```

```

700  IF(IR.EQ.1)FAC=11.3
750  QOOLD=QONEW
      XI(1) = -FAC*(SQRT(YL*PP))
      N = NSPECI
C
C      DELXI SHOULD BE SELECTED IN SUCH A
C      WAY THAT IT IS 1/10 OF THE DISTANCE
C      OF THE NEAREST TUBE FROM THE GROUND
C      SURFACE.
C      CHANGE = MIN(YT(I))
C
C      DO 150 I=1,M
C      IF((YT(I)-Y).GT.SHORT)SHORT=(YT(I)-Y)
C 150  CONTINUE
C
C      FIND SEGMENT LENGTH DELXI
C      EQUAL SEGMENTS HAVE BEEN USED.
C
C      DELXI = ABS(SHORT)/16.
C      DELXI = 4.0
C
C      CALCULATE NO. OF POINTS REQUIRED
C      NREQ = (ABS(XI(1))/DELXI)*2
C
C      IF NREQ IS GREATER THAN N,STOP THE
C      PROGRAM AND CHANGE THE DIMENSIONS,
C      OTHERWISE N=NREQ. MAKE SURE N IS
C      ODD NUMBER(SINCE WE ARE DENOTING
C      FIRST POINT AS 1)
C
C      IF(NREQ.EQ.N) GOTO 180
C      IF(NREQ.LT.N) GOTO 160
C      WRITE(22,170)NREQ
170  FORMAT(1H , 'NREQ = ',I6)
      STOP
160  NN = N
190  NN = NN-2
      IF(NN.GT.NREQ) GOTO 190
      IF(NN.EQ.NREQ) GOTO 180
      IF(NN.LT.NREQ) NREQ=NN+2
180  N = NREQ
      XI(1) = -(N/2*DELXI)
      D2S = Y/(ABS(XT(2)-XT(1)))
C
C      EVALUATE DISTANCE TO VARIOUS POINTS
C      FROM ETA-AXIS.
C
C      DO 230 I=1,N
C      XI(I+1)=XI(I)+DELXI
230  CONTINUE
      PI = 3.1415926
C
C      EVALUATE COEFFICIENTS OF PHI(J),

```

```

C      Q0, Q1, AND Q2.
C
C      -----
C      EQUATION 26
C      -----
C
C      BOTH I AND J ARE MOVING ON THE
C      BOUNDARY.
C
C      COEFF. OF PHI(J)
C
      T = 2.*PI
      DO 240 I=1,N
      DO 250 J=1,N
      C1 = ABS(XI(J+1)-XI(I))
      B1 = ABS(XI(J)-XI(I))
      IF(C1.EQ.0.0)A(I,J) = -BI/T*
1      (B1*(ALOG(B1)-1.)+B1*ALOG(AD))
      IF(B1.EQ.0.0)A(I,J) = -BI/T*
1      (C1*(ALOG(C1)-1.)+C1*ALOG(AD))
      IF((C1.EQ.0.0).OR.(B1.EQ.0.0)) GOTO 260
      A(I,J) = -BI/T*((C1*(ALOG(C1)-1.)+C1*
1      ALOG(AD))-(B1*(ALOG(B1)-1.)+
1      B1*ALOG(AD)))
      IF(I.GT.J+1) A(I,J) = -A(I,J)
260      IF(I.EQ.J) A(I,J)=0.5+A(I,J)
250      CONTINUE
240      CONTINUE
C
C      COEFF. OF Q0, Q1, AND Q2.
C
      DO 255 I=1,N
      M1 = N+1
      DO 265 L=1,M
      R(I,L) = SQRT((XI(I)-XT(L))**2+(Y-YT(L))**2)
      A(I,M1) = AA(L)*(ALOG(R(I,L))+ALOG(AD))
      A(I,M1+1) = -(0.5*AA(L)**2*(Y-YT(L)))/(R(I,L)**2)
      A(I,M1+2) = -(0.5*AA(L)**2*(XI(I)-XT(L)))/(R(I,L)**2)
      M1 =M1+3
265      CONTINUE
255      CONTINUE
C
C      -----
C      EQUATION 27
C      -----
C
C      COEFF. PHI(J)
C
      DO 270 L=1,M
      R(N+1,L) = SQRT((XI(N+1)-XT(L))**2+(Y-YT(L))**2)
270      CONTINUE
      DO 280 L=1,M

```

```

DO 290 J=1,N
F1(L,J)=(XI(J)-XT(L))
F2(L,J)=(XI(J+1)-XI(J))
F3(L,J)=(XI(J+1)-XT(L))
Z1=((F1(L,J)**2+F2(L,J)**2-F3(L,J)**2)/
1      (2.*F2(L,J)))*(ALOG(R(J,L))-ALOG(R(J+1,L)))
1      +F2(L,J)*(ALOG(R(J+1,L))-1.0)
Z2=R(J,L)*ACOS((R(J,L)**2+R(J+1,L)**2
1      -F2(L,J)**2)/(2.*R(J,L)*R(J+1,L)))
1      *SIN(ACOS((F1(L,J)**2+F2(L,J)**2
1      -F3(L,J)**2)/(2.*R(J,L)*F2(L,J))))
A(N+L,J) = -BI/T*((Z1+Z2)+F2(L,J)*ALOG(AD))
290  CONTINUE
280  CONTINUE
C
C      COEFF. OF Q0, Q1, AND Q2.
C
DO 292 K=1,M
DO 294 L=1,M
IF(K.EQ.L) GOTO 294
G1(K,L) = (Y-YT(L))-(Y-YT(K))
G2(K,L) = XT(K)-XT(L)
RAB(K,L) = SQRT(G1(K,L)**2+G2(K,L)**2)
294  CONTINUE
292  CONTINUE
C
DO 300 K=1,M
M1 = 1
DO 310 L=1,M
IF(K.EQ.L) GOTO 320
A(N+K,N+M1) = AA(L)*(ALOG(RAB(K,L))+ALOG(AD)))
A(N+K,N+M1+1) = -(0.5*AA(L)**2*G1(K,L))/
1      RAB(K,L)**2
A(N+K,N+M1+2) = -(0.5*AA(L)**2*G2(K,L))/
1      RAB(K,L)**2
GOTO 330
320  A(N+K,N+M1) = AA(K)*(ALOG(AD)+ALOG(AA(K)))
A(N+K,N+M1+1) = 0.0
A(N+K,N+M1+2) = 0.0
330  M1 = M1+3
310  CONTINUE
300  CONTINUE
C
C      -----
C      EQUATION 28
C      -----
C      COEFF. OF PHI(J)
C
TH1 = 2.*ASIN(1.)
DO 340 L=1,M
DO 350 J=1,N
IF(XI(J).LT.XT(L).AND.XI(J+1).GT.XT(L)) GOTO 345

```

```

      A(N+M+L,J) = -(BI/AD)*(ASIN((Y-YT(L))/R(J+1,L))
1      -ASIN((Y-YT(L))/R(J,L)))
      IF(XI(J).LT.XT(L)) A(N+M+L,J) = -A(N+M+L,J)
      GOTO 350
345  A(N+M+L,J) = -(BI/AD)*(ASIN((Y-YT(L))/R(J+1,L))
1      +ASIN((Y-YT(L))/R(J,L))-TH1)
350  CONTINUE
340  CONTINUE
C
C      COEFF. OF Q0, Q1, AND Q2.
C
      DO 360 K=1,M
      M1 = 1
      DO 370 L=1,M
      IF(K.EQ.L) GOTO 380
      A(N+M+K,N+M1) = (2.*PI*AA(L)*G1(K,L))/
1      (AD*RAB(K,L)**2)
      A(N+M+K,N+M1+1) = -((AA(L)**2*PI)/(AD*RAB(K,L)**2))
1      *(COS(2.*ACOS(G2(K,L)/RAB(K,L))))
      IF(XT(K).LT.XT(L)) GOTO 385
      A(N+M+K,N+M1+2) = -((AA(L)**2*PI)/(AD*RAB(K,L)**2))
1      *(SIN(2.*ASIN(G1(K,L)/RAB(K,L))))
      GOTO 390
385  A(N+M+K,N+M1+2) = -((AA(L)**2*PI)/(AD*RAB(K,L)**2))
1      *(-(SIN(2.*ASIN(G1(K,L)/RAB(K,L)))))
      GOTO 390
380  A(N+M+K,N+M1) = 0.0
      A(N+M+K,N+M1+1) = -PI/AD
      A(N+M+K,N+M1+2) = 0.0
390  M1 = M1+3
370  CONTINUE
360  CONTINUE
C
C      -----
C      EQUATION 29
C      -----
C
C      COEFF. PHI(J)
C
      KKK = 2*M
      DO 400 L=1,M
      DO 410 J=1,N
      A(N+KKK+L,J) = (BI/2.*AD)*(ALOG(R(J+1,L)**2)
1      -ALOG(R(J,L)**2))
410  CONTINUE
400  CONTINUE
C
C      COEFF. OF Q0, Q1, AND Q2.
C
      DO 420 K=1,M
      M1 = 1
      DO 430 L=1,M
      IF(K.EQ.L) GOTO 440

```

```

      A(N+KKK+K,N+M1) = (2.*PI*AA(L)*G2(K,L))/
1      (AD*RAB(K,L)**2)
      IF(XT(K).LT.XT(L)) GOTO 445
      A(N+KKK+K,N+M1+1) = ((PI*AA(L)**2)/(AD*
1      RAB(K,L)**2))*(SIN(
1      2.*ASIN(G1(K,L)/RAB(K,L))))
      A(N+KKK+K,N+M1+2) = ((PI*AA(L)**2)/(AD*RAB(K,L)**2))
1      *(COS(2.*ACOS(G2(K,L)/RAB(K,L))))
      GOTO 450
445      A(N+KKK+K,N+M1+1) = ((PI*AA(L)**2)/(AD*
1      RAB(K,L)**2))*(-(SIN(
1      2.*ASIN(G1(K,L)/RAB(K,L))))
      A(N+KKK+K,N+M1+2) = ((PI*AA(L)**2)/(AD*RAB(K,L)**2))
1      *(COS(2.*ACOS(G2(K,L)/RAB(K,L))))
      GOTO 450
440      A(N+KKK+K,N+M1) = 0.0
      A(N+KKK+K,N+M1+1) = 0.0
      A(N+KKK+K,N+M1+2) = -PI/AD
450      M1 =M1+3
430      CONTINUE
420      CONTINUE
C      -----
C      EVALUATION OF R.H.S.
C      -----
C      EQUATION 26
C      -----
      DO 460 I=1,N
      C(I) = 0.0
460      CONTINUE
C      -----
C      EQUATION 27
C      -----
      DO 470 L=1,M
      C(N+L) = -PHIT(L)
470      CONTINUE
C      -----
C      EQUATION 28
C      -----
      DO 480 L=1,M
      C(N+M+L) = 0.0
480      CONTINUE
C      -----
C      EQUATION 29
C      -----
      DO 490 L=1,M
      C(N+KKK+L) = 0.0
490      CONTINUE
C
C      SOLVE MATRIX
C
      NN = N+3*M
      IDGT = 4

```

```

        CALL LEQT1F(A,1,NN,370,C,IDGT,WKAREA,IER)
        IF(IER.EQ.34) WRITE(22,500)
500    FORMAT(1H ,/2X,'ACCURACY TEST FAILED'/)
        IF(IER.EQ.129) WRITE(22,510)
510    FORMAT(1H ,2X,'MATRIX IS ALGORITHMICALLY',
1      'SINGULAR')
        WRITE(22,520) N,BI,DELXI,PHID,AD,Y,D2S
520    FORMAT(1H , 'N = ',I3,2X,'BIOT NO. = ',F10.3,
1      2X,'SEGMENT LENGTH = ',F10.4,/,1X,
1      'MAX. TEMP. = ',F10.4,2X,'RADIUS',
1      ' OF MAX. TEMP. TUBE = ',F10.6,/,1X,
1      'LOCATION OF THE AXIS FROM THE PLANE CONVECTIVE',
1      'SURFACE = ',F10.6,2X,'D/2S = ',F8.3)
        WRITE(22,522)
522    FORMAT(/1H ,2X,'I',9X,'AA(I)',7X,
1      'XT(I)',7X,'YT(I)',7X,'PHIT(I)'/)
        DO 523 I=1,M
        WRITE(22,525)I, AA(I),XT(I),YT(I),PHIT(I)
525    FORMAT(/1H ,I3,5X,4F12.6)
523    CONTINUE
        IR = IR+1
        WRITE(5,*)IR
C
C      PRINT HEAT TRANSFER SOLUTION.
C
        QONEW = 0.0
        Q1TOT=0.0
        Q2TOT=0.0
        WRITE(22,530)
530    FORMAT(/2X'TUBE',7X,'QO',10X,'Q1',7X,'Q2')
        M1 = N+1
        M2 = M1+1
        M3 = M1+2
        DO 540 L=1,M
        WRITE(5,*)L,C(M1),C(M2),C(M3)
        WRITE(22,550)L,C(M1),C(M2),C(M3)
550    FORMAT(/1X,I3,5X,3F10.6)
        QONEW = C(M1)+QONEW
        Q1TOT = C(M2)+Q1TOT
        Q2TOT = C(M3)+Q2TOT
        CM1 = C(N+1)
        CM2 = C(N+2)
        CM3 = C(N+3)
        M1 = M1+3
        M2 = M1+1
        M3 = M1+2
540    CONTINUE
        WRITE(22,555) QONEW,Q1TOT,Q2TOT
555    FORMAT(/2X,'QOTOTAL = ',F10.6,2X,'Q1TOT = ',F10.6,
1      2X,'Q2TOT = ',F10.6)
        IF(IR.EQ.1) GOTO 700
C      IF(IR.EQ.7)GOTO 575

```

```

      QDIFF = QONEW-QOOLD
      PERCEN = (QDIFF/QONEW)*100.0
      IF(PERCEN.LT.1.0)GOTO 575
C     FAC = (QONEW/QOOLD*4.0-3.0)*FAC
C     FAC = FAC+2.0
      FAC = 1.125*FAC
      GOTO 750
C
  575  WRITE(22,573)IR,PERCEN
  573  FORMAT(//,2X,'NO. OF ITERATIONS = ',I3,3X,
    1      '% DIFF. = ',F6.3)
C     FOLLOWING 4 LINES FOR 3-TUBE CASE
C     QORAT = CM1/QONEW
C     Q1RAT = CM2/Q1TOT
C     Q2RAT = CM3/Q2TOT
C     WRITE(22,610)QORAT,Q1RAT,Q2RAT
C 610  FORMAT(//2X,'QOCEN/SIGQ0 = ',F10.6,2X,
C     1  'Q1CEN/SIGQ1 = ',F10.6,2X,'Q2CEN/SIGQ2 = ',F10.6//)
C     FOLLOWING 9 LINES FOR 4 TUBES CASE.
C     Q012 = C(N+1)+C(N+4)
C     Q034 = C(N+7)+C(N+10)
C     Q112 = C(N+2)+C(N+5)
C     Q134 = C(N+8)+C(N+11)
C     WRITE(22,615)Q012,Q034,Q112,Q134
C 615  FORMAT(//2X,'Q012 = ',F10.6,2X,
C     1      'Q034 = ',F10.6,2X,
C     1      'Q112 = ',F10.6,2X,
C     1      'Q134 = ',F10.6//)
C
C     PRINT TEMP. DISTRIBUTION
C
C     WRITE(22,560)
C 560  FORMAT(//1H ,3X,'I',8X,'XI',
C     1      11X,'TEMPERATURE'//)
C     DO 570 I=1,N
C     WRITE(22,580)I,XI(I),C(I)
C 580  FORMAT(2X,I3,3X,F13.6,5X,F10.7)
C 570  CONTINUE
      WRITE(22,841)
  841  FORMAT(//1H ,3X,'I',10X,'XI',7X,'TEMPERATURE'
    1      ,11X,'I',7X,'XI',8X,'TEMPERATURE'
    1      ,12X,'I',9X,'XI',8X,'TEMPERATURE'//)
      NN3 = N/3
      NN4 = NN3*3
      NN5 = N-NN4
      IF(NN5.EQ.1) GOTO 871
      IF(NN5.EQ.2) GOTO 861
      IF(NN5.EQ.0) GOTO 881
  871  N2 = NN3
      II1 = 1
      II2 = NN3+2
      II3 = NN3*2+2

```

```

      DO 874 I=1,N2
      WRITE(22,875)II1,XI(II1),C(II1),II2,
1          XI(II2),C(II2),II3,
1          XI(II3),C(II3)
875  FORMAT(2X,I3,2X,F12.5,4X,F10.7,
1          11X,I3,2X,F12.5,4X,F10.7,
1          11X,I3,2X,F12.5,4X,F10.7)
      II1 = II1+1
      II2 = II2+1
      II3 = II3+1
874  CONTINUE
      NN6 = N2+1
      WRITE(22,876)NN6,XI(NN6),C(NN6)
876  FORMAT(2X,I3,2X,F12.5,4X,F10.7)
      GOTO 900
861  N2 = NN3
      II1 = 1
      II2 = NN3+2
      II3 = NN3*2+3
      DO 864 I=1,N2
      WRITE(22,865)II1,XI(II1),C(II1),II2,
1          XI(II2),C(II2),II3,
1          XI(II3),C(II3)
865  FORMAT(2X,I3,2X,F12.5,4X,F10.7,
1          11X,I3,2X,F12.5,4X,F10.7,
1          11X,I3,2X,F12.5,4X,F10.7)
      II1 = II1+1
      II2 = II2+1
      II3 = II3+1
864  CONTINUE
      NN6 = N2+1
      N3 = N2+NN6+1
      WRITE(22,866)NN6,XI(NN6),C(NN6),
1      N3,XI(N3),C(N3)
866  FORMAT(2X,I3,2X,F12.5,4X,F10.7,
1          11X,I3,2X,F12.5,4X,F10.7)
      GOTO 900
881  N2 = NN3
      II1 = 1
      II2 = 1+NN3
      II3 = 1+NN3*2
      DO 884 I=1,N2
      WRITE(22,885)II1,XI(II1),C(II1),II2,
1          XI(II2),C(II2),II3,
1          XI(II3),C(II3)
885  FORMAT(2X,I3,2X,F12.5,4X,F10.7,
1          11X,I3,2X,F12.5,4X,F10.7,
1          11X,I3,2X,F12.5,4X,F10.7)
      II1 = II1+1
      II2 = II2+1
      II3 = II3+1
884  CONTINUE

```

900      STOP  
         END

## APPENDIX E

## LEAST SQUARE FITS

The three equations used for developing criteria to find the value of  $\bar{\epsilon}_{\max}$  are

(1) Straight line  $Y = A \cdot X + B$

(2) Exponential  $Y = A \cdot (\exp(B \cdot X))$  and

(3) Power law  $Y = A \cdot X^B$

where  $Y = \bar{\epsilon}_{\max}$

$$X = X T_{\max}^{*D}$$

and A,B are Constants

A computer program is developed to find these constants A and B for the given set of input, and standard error for these three equations have been found. From a number of computer program runs for different sets of input, it has been observed that Logarithmic equation has low standard error(S.E.).

From the computer program, it has been calculated that the values of A and B are 11.3 and 0.5, respectively.

Therefore, the equation for  $\bar{\epsilon}_{\max}$  is written as

$$\bar{\epsilon}_{\max} = 11.3 * \sqrt{X T_{\max}^{*D}}$$

## VITA

Purnachandra Rao Surapaneni was born on May 16, 1959, in Vijayawada, in the state of Andhra Pradesh, India. He spent most of his childhood in his native place, Maredumaka of Krishna district, in the state of Andhra Pradesh. He attended Andhra Loyola College, Vijayawada, and received his Intermediate(Pre-University Course) diploma in 1975. In September 1976, he entered Siddaganga Institute of Technology, Tumkur, Bangalore University, and received the Bachelor degree in Mechanical Engineering in September 1981. In the fall of 1982, he came to the United States and started graduate work in Mechanical Engineering at the University of Tennessee, Knoxville, from where he received his Master's degree. While at the University of Tennessee, he was a graduate assistant in the department of Mechanical and Aerospace Engineering. He is a coauthor of a paper entitled "Numerical Boundary Integral Analysis of Heat Transfer from Circular Tubes in a Semi-Infinite Medium", which was presented in the 1984 ASME Winter Annual Meeting, New Orleans, Louisiana.

The author is a student member of the American Society of Mechanical Engineers, and the American Institute of Aeronautics and Astronautics. He also assisted in the conduct of the 1983 AIAA Guidance and Control Conference, Gatlinburg, Tennessee. He was the president of the Telugu Cultural Association in 1980-81 when he was a senior student.