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COMPUTER-AIDED REDUCTION OF PLASMA DATA

A Thesis

Presented for the

Master of Engineering

Degree

The University of Tennessee, Knoxville

Saeid Shariati

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ABSTRACT

Poor signal-to-noise ratios and the nonlinear characteristics of the data obtained by most plasma diagnostic equipment makes computer-aided data handling a desirable feature in plasma laboratories. The Lecroy 3500-SA32 signal analyzer is used as the data handling system in this work. The performance of the Lecroy 3500-SA32 signal analyzer, used for recording and reducing plasma data under the noisy environment of the laboratory, is reported. The data characteristics and software programs are discussed for three types of plasma diagnostic equipment: (1) Langmuir Probes; (2) Charge Exchange Neutral Energy Analyzers; (3) Retarding Potential Energy Analyzers. Three computer programs in FORTRAN 80 are included which obtain an iterated best fit of experimental data to the corresponding analytical expressions for each case. Plasma parameters such as ion kinetic temperature, electron kinetic temperature, electron number density, plasma potential, etc. are available on a real-time basis.

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INTRODUCTION

There is a growing need for more systematic data handling in plasma laboratories. Poor signal-to-noise ratios and the nonlinear characteristics of the data obtained by most plasma diagnostic equipment makes data handling and reduction without a computer slow, subjective, manpower intensive, and (usually) unavailable on a real-time basis. By using properly programmed computers, significantly more and more precise information may be obtained about the plasma than is possible with conventional methods of data handling.

The UTK Plasma Science Laboratory is equipped with two types of plasma diagnostic equipment which can benefit from computer-assisted data reduction: (1) Langmuir probes; and (2) retarding potential energy analyzers. Computer-based data reduction techniques are applied to data generated by these diagnostic instruments. A new data acquisition and handling system will enable the user to do a real-time analysis of data by using the interactive features of the Lecroy 3500 system (Appendix C).

Three computer programs in FORTRAN 80 are included which obtain an iterated best fit of experimental data based on the least squares method for three types of diagnostic instruments: (1) Langmuir probes; (2) retarding potential energy analyzers (computer program was originally written by Loretta R. Ellis); and (3) charge exchange neutral energy analyzers. The third program, written for the charge exchange neutral energy analyzer, uses a set of pseudo data, and will

not be used in the UTK Plasma Science Laboratory for the purpose of real-time analysis of plasma data in the immediate future. These computer programs can be run on the Lecroy 3500 computer (Appendix A).

The analog data from Langmuir probes and/or retarding potential analyzers are fed into a signal conditioning system (to make the curves as smooth as possible) and digitized by a Lecroy TK8837 transient recorder (Appendix C). The digitized data are stored in the histogram memory of the Lecroy 3500 system, which can be read by the corresponding computer program on a real-time basis or can be stored on a floppy disk for later use. An isolation transformer, ISAFIL (Appendix C) is used to protect the computer internal circuitry against power line noise.

CHAPTER I

LANGMUIR PROBE

A. THEORY

The Langmuir probe is one of the oldest and most widely used diagnostic tools in plasma laboratories, largely because of its mechanical and electrical simplicity. A schematic drawing of a cylindrical Langmuir probe is shown in Figure 1.1. When the voltage on a Langmuir probe is changed, a characteristic current-voltage curve, similar to that shown in Figure 1.2, is traced out. As the potential increases, the current to the probe decreases slightly in the region A to B. In this region, the probe will collect only positive ions, and the nearly constant current is equal to the random ion current. This random current is determined by the rate at which the ions reach the probe as a result of their random motions in the plasma, and is given by

$$I_{ni} = An_i e\sqrt{\frac{KT_i}{2\pi m_i}} \tag{1.1}$$

where n_i is the concentration of positive ions in the plasma, T_i is the ion kinetic temperature, and A the probe area.

As the potential of the probe is increased, a few of the most energetic and fastest moving electrons penetrate the positive ion sheath and reach the probe, with a resulting decrease in the net

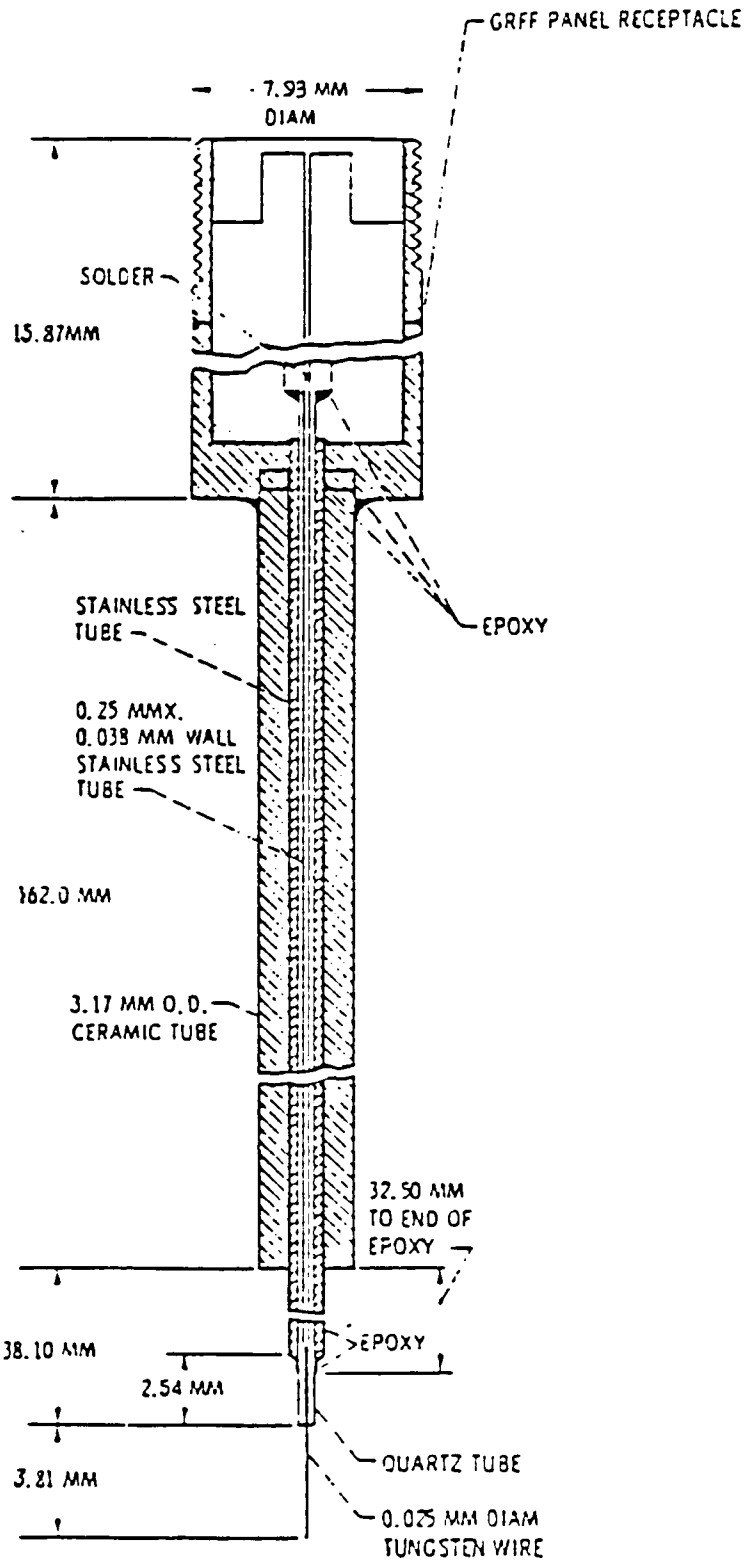


Figure 1.1. Schematic drawing of cylindrical Langmuir probe.

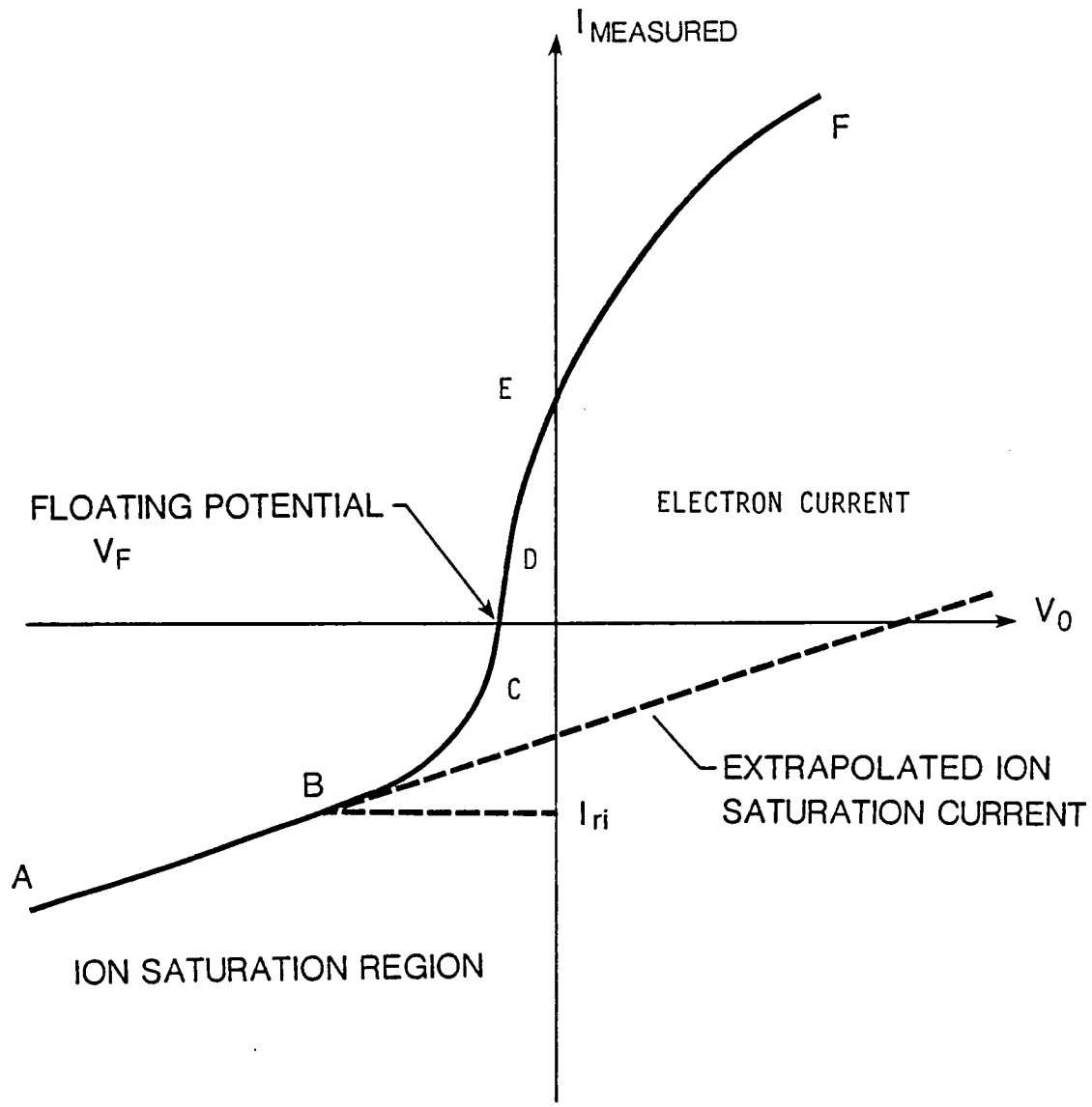


Figure 1.2. Current-voltage characteristic of a Langmuir probe.

probe current. A potential, point C, will be reached for which the net current is zero. This is the floating potential, V_F, of the plasma. The electron current exceeds the positive ion current for all probe potentials more positive than point C. At point E the probe is at the plasma potential, V_p, and receives the entire random current due to both electrons and positive ions from the plasma. The region EF is an accelerating field region for electrons, while positive ions are being reflected. The region EF is called the electron saturation region, in which the entire distribution of electrons is collected by the probe. The current in the region EF increases slightly with the applied voltage, due to the increase in sheath area. For cylindrical probes, used in the UTK Plasma Science Laboratory, the region EF is not flat, i.e., of constant current as the voltage is increased. Consider the region CE in Figure 1.2; the number of electrons reaching the probe exceeds the number of positive ions, so that the probe may be regarded as being surrounded by a sheath in which electrons predominate. Only electrons with sufficient energy to overcome the repulsion of the sheath will be able to penetrate it and reach the probe. If V is the potential difference across the sheath, between the bulk of the plasma and the probe surface, the required energy is just eV. Assuming the Boltzmann distribution law to apply, then

$$n_o = n_e \exp\left(\frac{eV}{KT_e}\right) \tag{1.2}$$

where n_e is the electron density in the bulk of the plasma, n_o is the density at the probe surface, and T_e is the electron temperature.

The electron current at any stage is proportional to the electron density at the surface, hence from Eq. (1.2)

$$I_e = I_{re} \exp(\frac{eV}{KT_e}) \tag{1.3}$$

where I_{re} is the electron saturation current. I_{re} is the random electron current at the point E (plasma potential). At this point the potential across the sheath has been decreased to zero by the external potential applied to the probe. Hence, at any point along the curve

$$V = V_0 - V_p \tag{1.4}$$

where V_0 is the applied probe potential, V_p is the plasma potential, and V is the potential difference across the sheath. Combining Eq. (1.3) and Eq. (1.4) we have

$$I_e = I_{re} \exp\left(\frac{e(V_0 - V_p)}{KT_e}\right) \tag{1.5}$$

Taking the logarithms of both sides of Eq. (1.5) results in

$$\ell n\ I_e = \ell n\ I_{re} + \frac{eV_0}{KT_e} - \frac{eV_p}{KT_e}$$

$$\ell n\ I_e = (\ell n\ I_{re} - \frac{eV_p}{KT_e}) + \frac{eV_0}{KT_e}$$

$$\ell n\ I_e = \text{constant} + \frac{eV_0}{KT_e} \tag{1.6}$$

The random electron current for a probe of area A square meters is given by:

$$I_{re} = An_e e\sqrt{\frac{KT_e}{2\pi m_e}} \tag{1.7}$$

where n_e is the electron density of the plasma [1,2].

B. DATA CHARACTERISTICS AND REDUCTION

In analyzing Langmuir probe data, one must first plot Eq. (1.6) on a semilog paper with the voltage on the linear (horizontal) scale and the electron current I_e on the logarithmic (vertical) scale. It follows that a plot of the experimental values of $\ell n\ I_e$ against the applied potential V_0 should be a straight line with a slope e/KT_e. Hence, the electron temperature T_e in the plasma can be found. It must be noted that the measured probe current after point E (Figure 1.2) is equal to the difference between the electron current I_e and the random ion current I_{ri}. The actual electron currents, plotted on the logarithmic scale, are then obtained from the difference between the extrapolated ion saturation current (to the right of region AB) and the net current. Hence, for an arbitrary point like D the actual electron current is (Figure 1.2):

$$I_e(V) = I_{measured}(V) - I_{ri}(V)\ . \tag{1.8}$$

Any departure of the plot from linearity implies that a Maxwell-

Boltzmann distribution does not exist for the electrons. In such a case, an electron kinetic temperature cannot be defined.

Once the electron temperature is known, the electron density n_e can be obtained by means of Eq. (1.7). The random electron current I_{re} is the current corresponding to the plasma potential V_p (point E on Figure 1.2). This point is difficult to locate on the linear-linear scale; however, it can be located on a semilog plot of current as a function of voltage as the knee of the curve, where the current departs from the original straight line. The potential corresponding to this break in the curve is the plasma potential V_p.

Assuming that the electron and ion density in the plasma is almost equal, then Eq. (1.1) can be used to obtain an estimate of the ion kinetic temperature T_i. Since for cylindrical Langmuir probes the ion saturation current region is not flat, I_{ri} is taken to be the current corresponding to the knee of the I-V characteristic at point B, as shown in Figure 1.2.

C. COMPUTER PROGRAM

Experimental data from the Langmuir probe cannot be reduced on-line in any simple way. A computer program in FORTRAN 80 was written which uses the data reduction procedure discussed in the previous section to find the values of plasma parameters, including floating potential, plasma potential, electron number density, ion kinetic temperature, and electron kinetic temperature. The computer

program is run on a Lecroy 3500 small computer (for detailed information on the Lecroy 3500 system, refer to Appendix C).

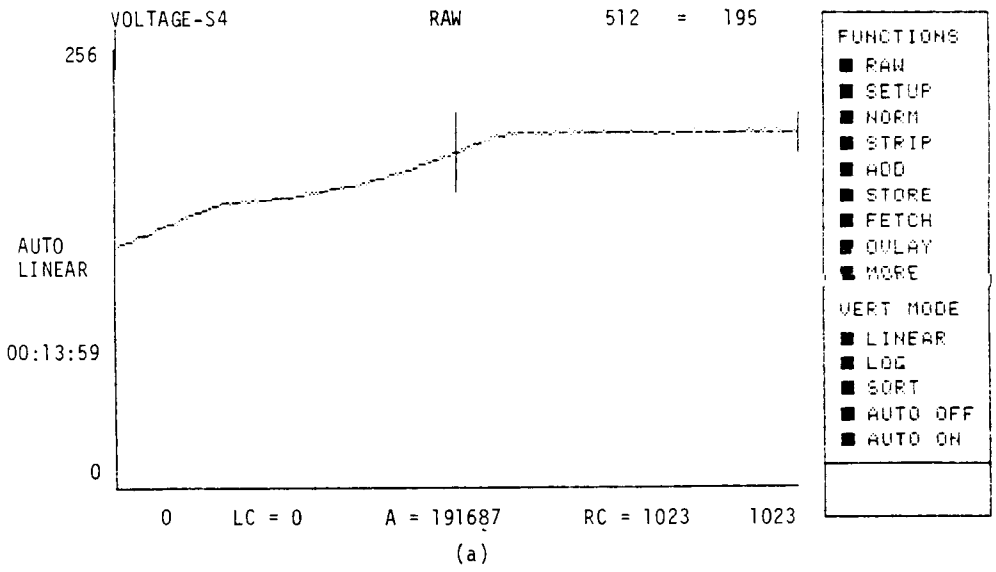
Since it is not possible to fit a single curve to the experimental data, the light pen provided by the Lecroy system is used to divide the current-voltage characteristic shown in Figure 1.2) into three segments: (1) AB-ion saturation region; (2) BE-electron current region; (3) EF-electron saturation region. The last two segments are found from the semilog plot of I_e vs. voltage. A straight line is then fitted to each segment according to the least squares method (Appendix B) (BE and EF appear as straight lines on the semilog plot) to find the corresponding unknown constants, from which the plasma parameters are derived [4,5]. To be able to use the light pen, negative plots are drawn on the screen. The mass of the ion and the probe dimensions are supplied by the user through the questions asked by the computer. This computer program, its operation, and its application to data analysis is discussed in Appendix A.

Since the Langmuir probe curve takes about twenty-five seconds to trace, one k-byte of the histogram memory for each axis and a sampling frequency of f_s = 30 Hz (generated by an external clock) are utilized to trace the whole plot. First and second k-byte of the histogram memory are used to store the probe voltage and current, respectively. Two TR-8837 transient records are used to record the probe voltage and current simultaneously.

To reduce the effect of noise, every forty consecutive points along each axis (voltage and current) are averaged and taken as a single point. As a result, the number of data points is the total number of data points divided by forty, twenty-five in this case.

Real-time plots of voltage and current for two different runs (AQD1 and AQD4) are shown in Figures 1.3 and 1.4. The horizontal axis in each of these figures represents the channels or memory locations that each data point is stored in. The horizontal axis in these figures can also be thought of as the time axis. Since it takes $1/f_s$ seconds to fill up each channel, the maximum number (1023) times $1/f_s$ is the total time required to trace the Langmuir probe plot. The vertical axis value times two is the actual value of the signal recorded by the transient recorder. This value is then properly scaled inside the program to represent its corresponding real value.

Figures 1.5 and 1.6 show the computer plots (the current-voltage characteristic and the semilog plot of the electron current) for two experimental runs, AQD1 and AQD4, respectively. In these plots, a cross is used to represent the actual data point (crosses are printed beside the data point), and the continuous line represents the fitted curve. The tabulated values of the probe current and voltage for each run is shown in Tables I and II.



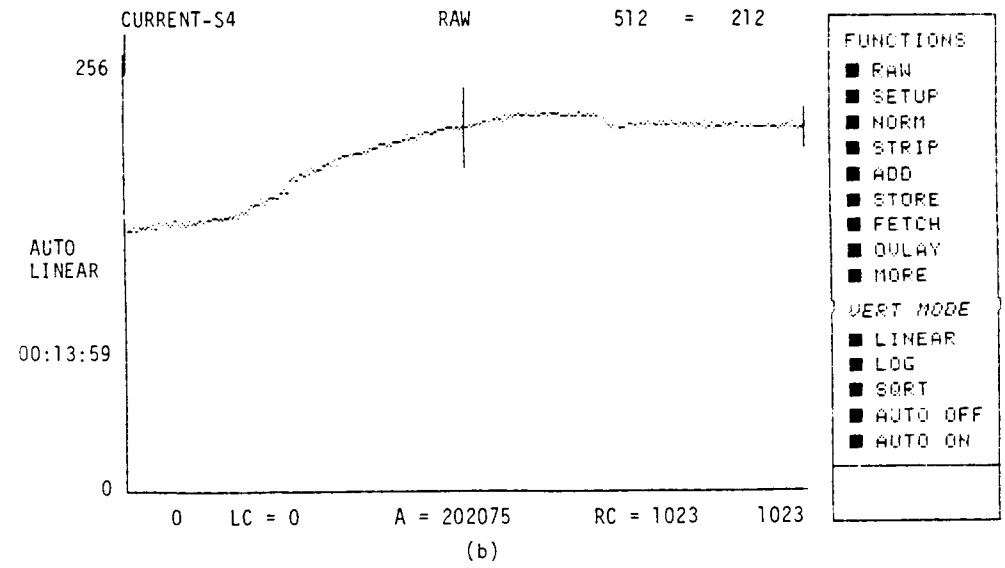


Figure 1.3. (a) Real-time probe voltage plot (run #AQD1); (b) real-time probe current plot (run #AQD1).

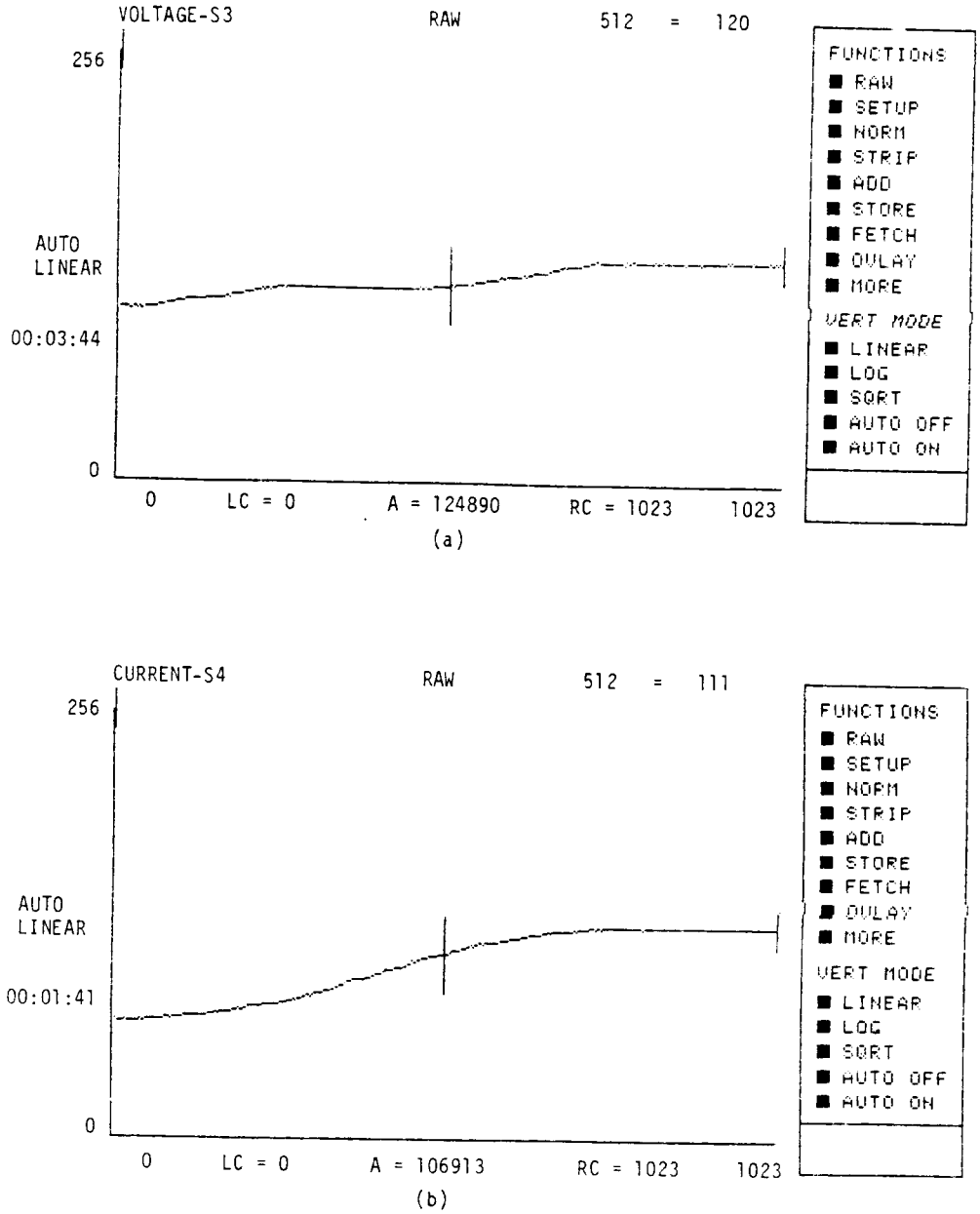
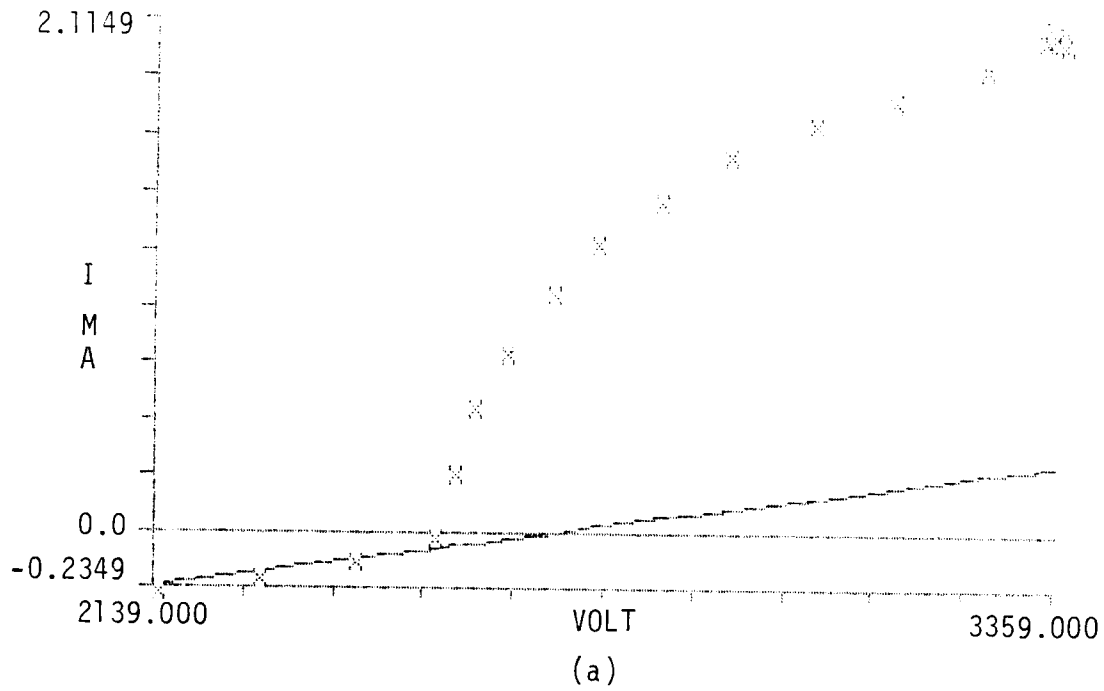
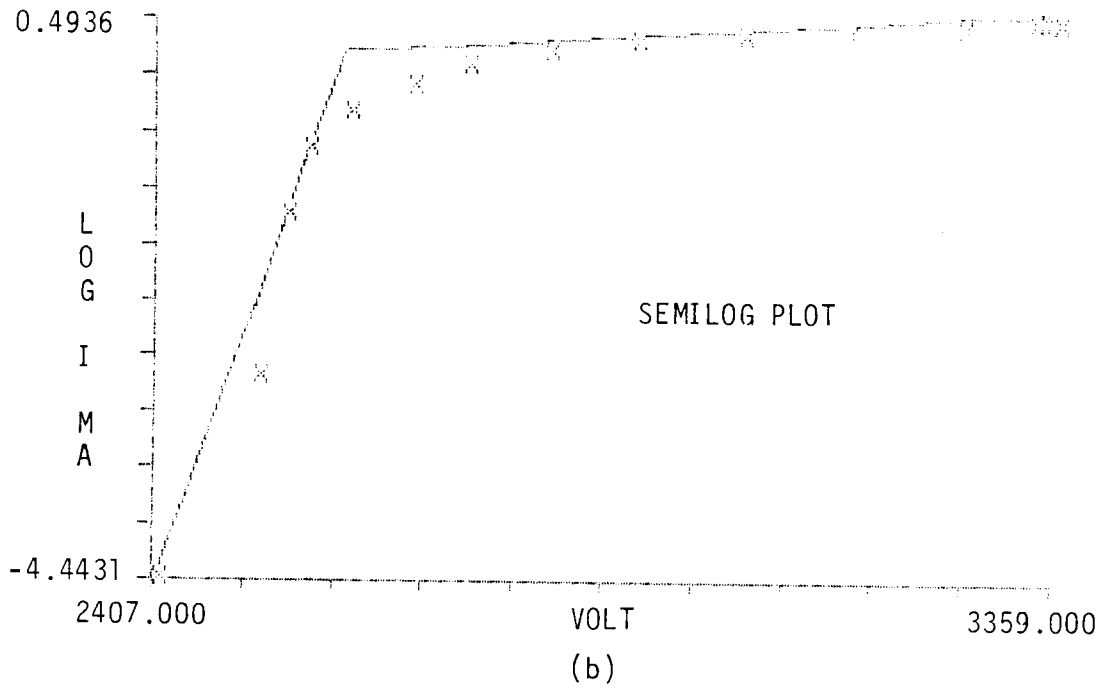


Figure 1.4. (a) Real-time probe voltage plot (run #AQD4); (b) real-time probe current plot (run #AQD4).

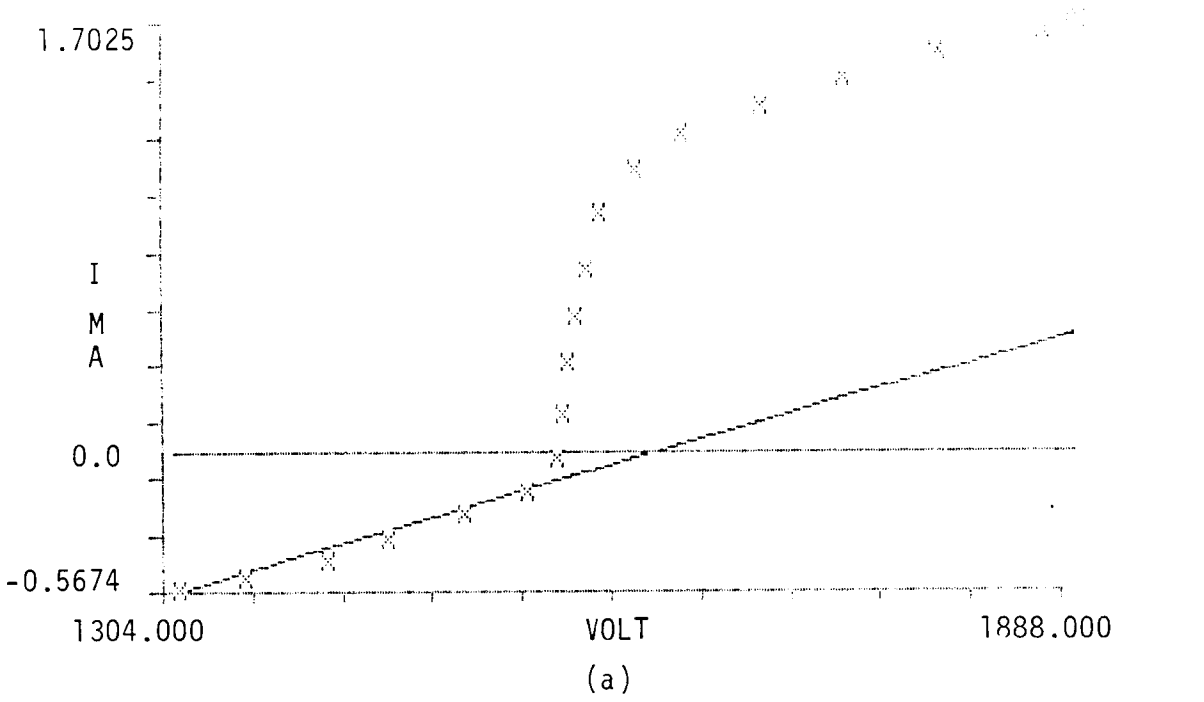


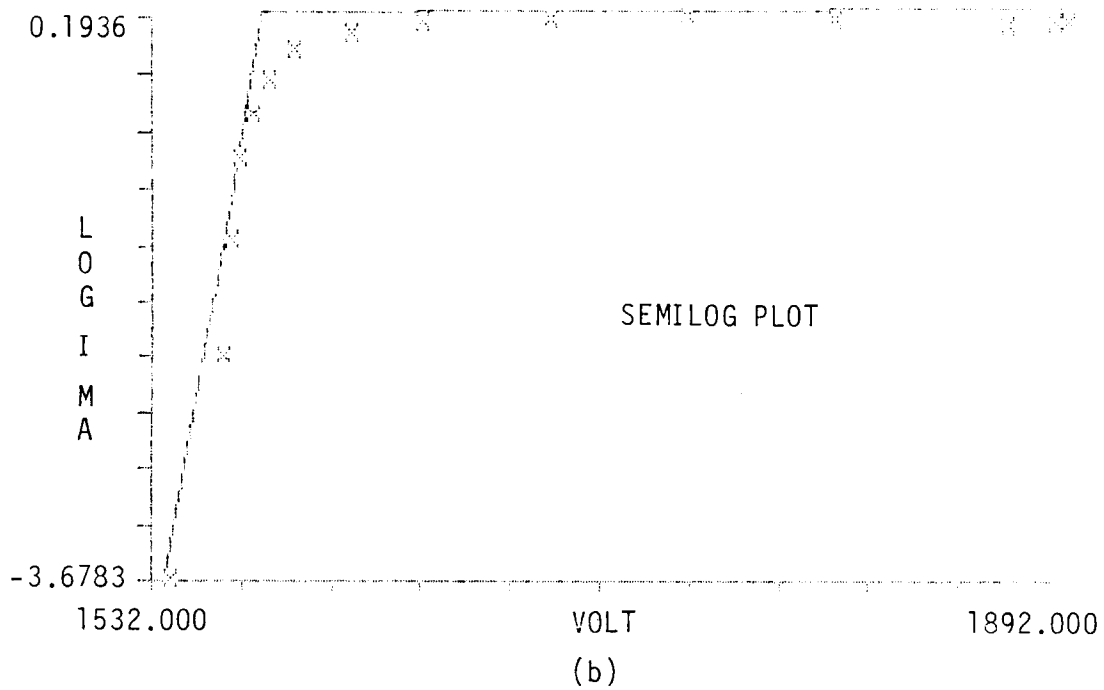


TI = 31.46 EV
TE = 42.39 EV
NE = .1279E+16/M3

VF = 2507.75 V
VP = 2609.76 V

Figure 1.5. (a) Current-voltage probe characteristic with ion saturation line; (b) log of electron current vs. probe voltage (run #AQD1) mass of the ion = 4 amu; probe radius = .015 cm; probe length = .6 cm.





TI = 22.82 EV VF = 1557.62 V
TE = 10.12 EV VP = 1577.41 V
NE = .2652E+16/M3

Figure 1.6. (a) Current-voltage probe characteristic with ion saturation line (run #AQD4); (b) log of electron current vs. probe voltage (run #AQD4) mass of the ion = 4 amu; probe radius = .015 cm; probe length = .6 cm.

TABLE I

EXAMPLE OF INPUT TO LANGMUIR PROBE PROGRAM (RUN #AQD1)

V	I
2141.00000	-.21900
2277.00024	-.16200
2410.00000	-.09300
2515.00000	.01500
2544.00000	.27900
2569.50024	.55100
2611.50000	.78300
2677.50000	1.03800
2736.50000	1.24300
2822.50000	1.42200
2915.00000	1.60800
3028.50000	1.73200
3138.50000	1.83100
3261.00000	1.95900
3338.50000	2.07900
3345.50000	2.12900
3353.50000	2.10300
3353.50000	2.10800
3351.00000	2.11800
3348.00000	2.13100
3348.50000	2.11300
3359.50024	2.08200
3353.50000	2.10200
3363.50000	2.06700

TABLE II

EXAMPLE OF INPUT TO LANGMUIR PROBE PROGRAM (RUN #AQD4)

V	I
1311.50000	-.51400
1355.50000	-.46800
1408.00000	-.40300
1446.50000	-.31900
1497.00000	-.22000
1537.50000	-.12800
1558.50012	.01000
1562.00000	.18000
1565.00000	.39100
1571.00012	.58600
1577.50000	.77700
1587.00000	.99100
1610.00000	1.15900
1639.50000	1.30300
1691.00012	1.42300
1745.00000	1.52600
1804.50012	1.62500
1874.00000	1.70400
1892.50000	1.74600
1895.49988	1.75100
1897.99988	1.74800
1892.00012	1.75600
1897.00012	1.75500
1897.50000	1.75000

CHAPTER II

RETARDING POTENTIAL ENERGY ANALYZER

A. THEORY

Retarding potential electrostatic analyzers are widely used in plasma physics to determine the distribution of energy or momentum in a flux of charged particles [1,3]. Such analyzers use electrostatic forces to measure the energy distribution function of either ions or electrons. The type of retarding potential energy analyzer used in the UTK Plasma Science Laboratory is shown in Figure 2.1. A plasma exists on the right, and both ions and electrons escape from this plasma and pass through a small hole in a grounded plate which is located in front of the analyzer opening. After the particles enter the analyzer, they impinge on grid 3, the repeller grid, which may be biased to repel all charges of either positive or negative sign. The remaining particles then approach grid 2, the retarding grid, which is biased to repel all particles, the energy of which in the direction of the analyzer axis is below the value corresponding to the potential on this grid. The particles with momentum above this value pass through grid 2, and then pass through the negatively biased grid 1, the suppressor grid, the function of which is to prevent secondary electrons from leaving the collector. Varying the retarding potential V on grid 2 from zero to a value much higher than that corresponding to the particle kinetic temperature produces a

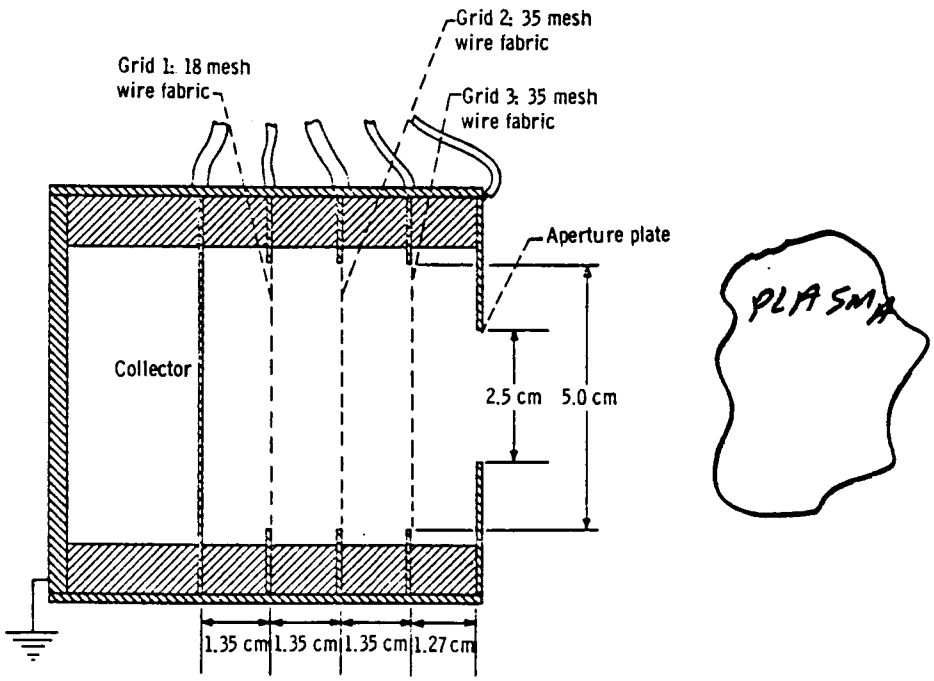


Figure 2.1. Schematic drawing of gridded electrostatic analyzer used in experimental investigation.

retarding potential curve of V as a function of collector current. This curve represents the integrated distribution of particle energy along the collector axis. Since the analyzer has been assumed to be one dimensional, it will provide information only about the momentum along the analyzer axis, and not about the total energy of the particles. Analytical expressions are derived in the following analysis, which relate the integrated distribution of axial energy observed by the collector to the effective escape cone angle, the floating potential and the kinetic temperature of the particles as they are incident on the analyzer. Four different integrated energy spectra are considered in the following derivation: (1) steady state; (2) pulsed; (3) delta function; and (4) square-wave function [1,3].

a. Integrated Steady-State Spectra from Arbitrary Cone in Velocity Space

It is assumed in this discussion that the particles reaching the analyzer are isotropic within a cone in velocity space of half-angle θ_0, as shown in Figure 2.2, and obey the Maxwellian distribution of velocities along a radius in velocity space. Analytical expressions are derived based on the assumption that the magnetic field lines, electric field, and analyzer axis are parallel. The source of this plasma is assumed to float at some potential V_1 with respect to ground of the analyzer, so that an electric field may exist between the plasma and the analyzer. This floating potential is not that encountered in Langmuir probe theory.

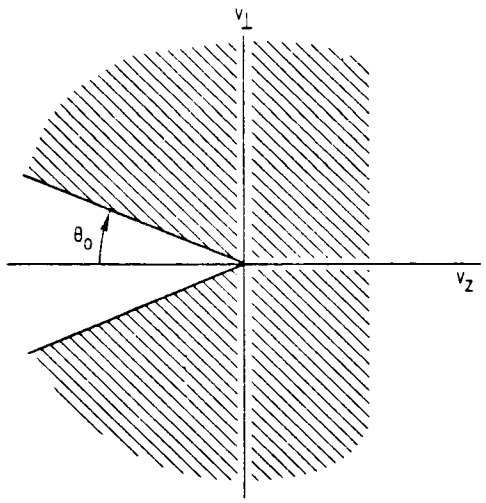


Figure 2.2. Schematic drawing of velocity space with particles confined to escape cone of half-angle θ_0.

The plasma at the analyzer, under the stated assumptions, will have a distribution function in velocity space given by [3]:

$$dn(v,\theta,\phi) = \frac{n_0}{\pi^{3/2}} \frac{v^2}{v_0^3} \exp(-\frac{v^2}{v_0^2}) \sin\theta \, dv \, d\theta \, d\phi$$

$$0 < \theta < \theta_0 \qquad (2.1)$$

where v is particle speed, v_0 is the most probable speed, n_0 is the particle number density at the entrance to the analyzer, θ is the angle with respect to the z-axis (or magnetic field line), and ϕ is the rotational angle in a plane normal to the z-axis. The most probable speed v_0 is given in terms of the kinetic temperature $KT = eV_0$ by

$$v_0 = \sqrt{\frac{2KT}{m}} = \sqrt{\frac{2eV_0}{m}} \, . \qquad (2.2)$$

The speed v may be written in terms of the velocity v_z along the analyzer axis using Figure 2.3

$$v_z = v \cos \theta \ , \tag{2.3a}$$

$$v = v_z \sec \theta \ . \tag{2.3b}$$

Changing the variable of integration from v to v_z in Eq. (2.1), and integrating over the azimuthal angle $0 \leq \phi \leq 2\pi$, we have

$$dn(v_z,\theta) = \frac{2n_0}{\pi^{1/2}} \frac{v_z^2}{v_0^3} \exp(-\frac{v_z^2 \sec^2 \theta}{v_0^2}) \sec^3\theta \sin\theta \, dv_z \, d\theta \tag{2.4}$$

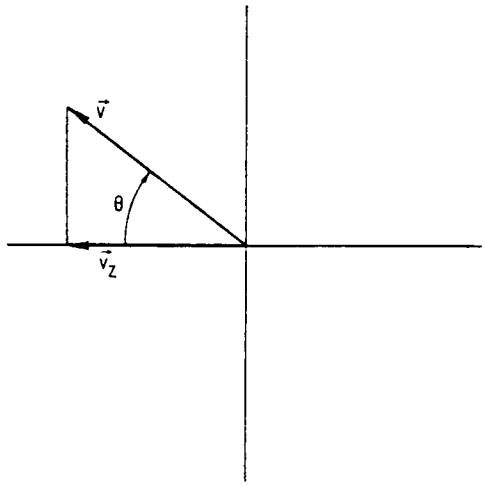


Figure 2.3. Geometric relation of velocities.

The differential current density in amperes per square meter of particles impinging on the analyzer is

$$dI(v_z,\theta) = ev\cos\theta\, dn(v,\theta) = ev_z\, dn(v_z,\theta)\ . \tag{2.5}$$

Substituting Eq. (2.4) into Eq. (2.5) results in

$$dI(v_z,\theta) = \frac{2n_0 e}{\pi^{1/2}}\frac{v_z^3}{v_0^3}\exp\left(-\frac{v_z^2\sec^2\theta}{v_0^2}\right)\sec^2\theta\tan\theta\, dv_z\, d\theta \tag{2.6}$$

Integrating this velocity distribution over the cone in velocity space which contains the particles $0 \le \theta < \theta_0$ yields

$$\begin{aligned} dI(v_z) &= \frac{2n_0 e}{v_0^3\pi^{1/2}}\int_0^{\theta_0} v_z^3\exp\left(-\frac{v_z^2\sec^2\theta}{v_0^2}\right)\sec^2\theta\tan\theta\, d\theta\, dv_z \\ &= e\frac{n_0 v_z}{\pi^{1/2}v_0}\left[\exp\left(-\frac{v_z^2}{v_0^2}\right) - \exp\left(-\frac{v_z^2\sec^2\theta_0}{v_0^2}\right)\right]dv_z \end{aligned} \tag{2.7}$$

Integrating Eq. (2.7) over all $v_z > v_{zmin}$ results in the current flowing to the collector. v_{zmin} is given by

$$v_{zmin} = \pm\sqrt{\frac{2|eV_1|}{m}} \tag{2.8}$$

where V_1 is the plasma potential.

The total ion current density collected by the electrode is given by integrating Eq. (2.7) over the range $v_{z0} \le v_z \le \infty$.

$$I = \frac{en_0}{2\pi^{1/2}v_0}\int_{v_{z0}}^{\infty} 2v_z\left[\exp\left(-\frac{v_z^2}{v_0^2}\right) - \exp\left(-\frac{v_z^2\sec^2\theta_0}{v_0^2}\right)\right]dv_z \tag{2.9}$$

where

$$v_{z0} = \sqrt{\frac{2e}{m}(V - V_1)} \tag{2.10}$$

Performing the integration and using Eqs. (2.10) and (2.2), the collector current as a function of retarding potential V is

$$I = \frac{n_0 \, ev_0}{2\pi^{1/2}} \left[\exp\left(- \frac{V - V_1}{V_0} \right) - \cos^2\theta_0 \exp\left(- \frac{(V - V_1)\sec^2 \theta_0}{V_0} \right) \right]$$

$$V \geq V_1 \tag{2.11}$$

in amperes per square meter. The maximum possible current available to the analyzer occurs when $V = V_1$, the plasma potential, because all particles emitted by the source are collected. This current is given by

$$I_1 = \frac{n_0 \, ev_0}{2\pi^{1/2}} \sin^2\theta_0 \, . \tag{2.12}$$

Using the mean thermal velocity $\bar{v}$ of the particles and assuming an isotropic distribution over a hemisphere in velocity space, Eq. (2.12) reduces to

$$I_1 = \frac{n_0 \, e\bar{v}}{4} \qquad \theta_0 = \pi/2 \, . \tag{2.13}$$

Normalizing Eq. (2.11) by using Eq. (2.13), and adding the zero offset of the curve I_0 we have

$$I = I_1 + I_0 \qquad V \leq V_1 \tag{2.14}$$

$$I = I_0 + I_1 \exp\left(-\frac{(V-V_1)}{V_0}\right)\left(\frac{1 - \cos^2\theta_0 \exp\left[-\frac{(V-V_1)\tan^2\theta_0}{V_0}\right]}{\sin^2\theta_0}\right) V \geq V_1 \tag{2.15}$$

Equation (2.15) is the desired expression for the retarding potential curve. It is a function of the plasma potential V_1, the kinetic temperature of the plasma V_0, and the effective escape cone angle in velocity space θ_0 [1,3].

b. Integrated Pulsed Spectra from Arbitrary Cone in Velocity Space [5]

In the following derivation, it is assumed that the arrival rates of particles, averaged over time, are proportional to the number of particles within each velocity increment dv. The particles are emitted from the plasma in repetitive bursts. The assumption of a Maxwellian distribution and "isotropy" in velocity space are retained along with the other assumptions of the previous case to simplify the analysis. From Eqs. (2.1) and (2.3b) the distribution function of particles at the analyzer is

$$dn(v_z,\theta,\phi) = \frac{n_0}{\pi^{3/2}} \frac{v_z^2}{v_0^3} \exp\left(-\frac{v_z^2 \sec^2\theta}{v_0^2}\right)\sec^2\theta \sin\theta \, dv_z \, d\theta \, d\phi$$

$$0 \leq \theta \leq \theta_0 \tag{2.16}$$

where the total number of particles reaching the analyzer per pulse is $[N_0(1 - \cos \theta_0)]/2$. N_0 cannot be related directly to a spatial number density at the analyzer because the particles with different velocities will be spread in space. If there are W bursts of particles per second, the collected current in amperes after the integration over ϕ will be

$$dI(v_z,\theta) = \frac{2eN_0W}{\pi^{1/2}} \frac{v_z^2}{v_0^3} \exp(- \frac{v_z^2 \sec^2\theta_0}{v_0^2}) \sec^2\theta \tan\theta \, dv_z \, d\theta$$

$$0 \leq \theta \leq \theta_0 . \qquad (2.17)$$

The current collected when the analyzer is set to repel all particles within the escape cone θ_0 and with a z-momentum less than that corresponding to v_{z0} is given by

$$I = \frac{2eN_0W}{v_0^3 \pi^{1/2}} \int_{v_{z0}}^{\infty} \int_0^{\theta_0} v_z^2 \exp(- \frac{v_z^2 \sec^2\theta}{v_0^2}) \sec^2\theta \tan\theta \, dv_z \, d\theta . \qquad (2.18)$$

Performing the integration over the angle θ, we have

$$I = \frac{eN_0W}{v_0\pi^{1/2}} \int_{v_{z0}}^{\infty} \left[\exp(- \frac{v_z^2}{v_0^2}) - \exp(- \frac{v_z^2 \sec^2\theta_0}{v_0^2}) \right] dv_z \qquad (2.19)$$

Performing the integration over v_z and using Eqs. (2.2) and (2.9) results in

$$I = \frac{eN_0W}{2}\left[\operatorname{erfc}\left(\frac{V-V_1}{V_0}\right)^{1/2} - \cos\theta_0 \operatorname{erfc}\left(\frac{(V-V_1)\sec^2\theta_0}{V_0}\right)^{1/2}\right] \quad V \geq V_1 \tag{2.20}$$

The maximum possible current to the analyzer occurs when $V = V_1$, the plasma potential, and is given by

$$I_1 = \frac{eN_0W}{2}(1 - \cos\theta_0) \ . \tag{2.21}$$

Using Eq. (2.21) to normalize Eq. (2.20), and adding the zero offset I_0, gives the shape of the retarding potential curve:

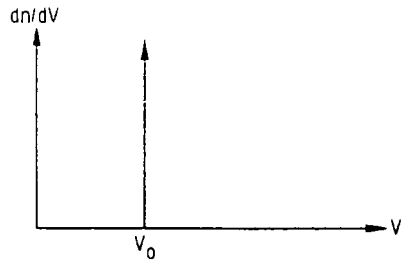
$$I = I_0 + I_1 \qquad V < V_1$$

$$I = I_0 + I_1\left[1 - \frac{\operatorname{erf}\left(\frac{V-V_1}{V_0}\right)^{1/2} - \cos\theta_0 \operatorname{erf}\left(\frac{(V-V_1)}{V_0}\sec^2\theta_0\right)^{1/2}}{1 - \cos\theta_0}\right]$$

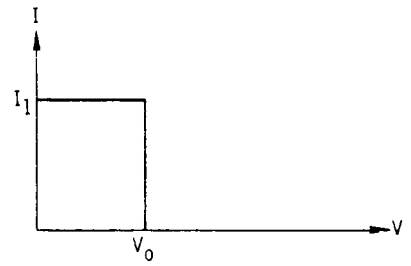
$$V \geq V_1 \tag{2.22}$$

c. Other Integrated Energy Spectra [5]

The two cases considered in (a) and (b) are, in a way, extremes. They give the maximum and minimum arrival rates (and hence V_0) consistent with locally Maxwellian distributions. It is often of interest to compare these two cases with the opposite limit of a perfectly ordered distribution. Two such ordered distributions were included in the computer program. The first is the delta-function energy distribution shown in Figure 2.4a. This corresponds to a monoenergetic



(a) Delta-function energy distribution



(b) Integrated delta-function distribution

Figure 2.4. Schematic drawing of delta-function energy spectra.

beam of particles. The integrated energy distribution shown in Figure 2.4b would be a square wave of the form

$$I = I_1 \qquad V < V_0 \tag{2.23}$$

and

$$I = 0 \qquad V \geq V_0 \ . \tag{2.24}$$

A second simple distribution function is the square-wave distribution, in which all the particles reaching the analyzer are evenly distributed in energy between $V_1 \leq V \leq V_2$, as shown in Figure 2.5a. Such a particle distribution will yield the integrated distribution shown in Figure 2.5b, whose analytical form is [1,3]:

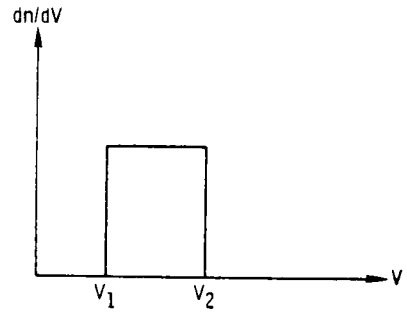
$$I = I_1 \qquad V < V_1 \tag{2.25}$$

$$I = I_1\left(\frac{V_2-V}{V_2-V_1}\right) \qquad V_1 \leq V \leq V_2 \tag{2.26}$$

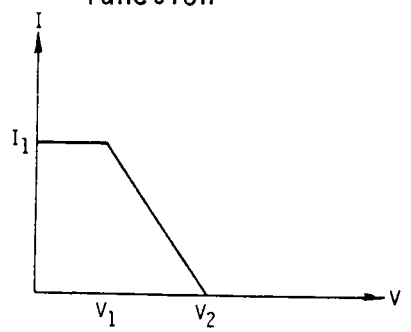
B. COMPUTER PROGRAM

Experimental data obeying the equations (2.15) and (2.22) cannot be reduced in any simple way, since the curves are not, in general, simple exponentials. In the special case of an isotropic, steady-state distribution over the entire hemisphere in velocity space, Eq. (2.15) reduces to

$$I = I_0 + I_1 \exp\left(-\frac{V-V_1}{V_0}\right) \qquad \theta_0 = \pi/2, \qquad V \geq V_1 \tag{2.28}$$



(a) Square-wave distribution function



(b) Integrated square-wave distribution

Figure 2.5. Schematic drawing of square-wave energy spectra.

which is an exponential distribution. The kinetic temperature of the plasma in question may then be determined by plotting the data on a semilogarithmic graph and measuring the slope of the resulting straight line. When $\theta_0 \neq \pi/2$, however, the integrated energy spectra is no longer an exponential curve.

A computer program in FORTRAN 80 was written which provides an iterated best fit of the four integrated energy spectra discussed previously, to a set of experimental data. The computer program is run on a Lecroy 3500 small computer (Appendix C) (the computer program was originally written by Loretta R. Ellis). The integrated distribution functions used in the computer program are Eqs. (2.15) and (2.22). For the delta-function distribution, from Eqs. (2.23) and (2.24)

$$I = I_0 + I_1 \qquad V < V_0 \tag{2.29}$$

$$I = I_0 \qquad V \geq V_0 \, . \tag{2.30}$$

Finally, for the square-wave distribution, from Eqs. (2.25) to (2.27)

$$I = I_0 + I_1 \qquad V < V_1 \tag{2.31}$$

$$I = I_0 + I_1\left(\frac{V_2 - V}{V_2 - V_1}\right) \qquad V_1 \leq V \leq V_2 \tag{2.32}$$

$$I = I_0 \qquad V > V_2 \, . \tag{2.33}$$

In the preceding equations, I_0 is the zero offset of the curve on the y-axis.

The computer program takes as input the retarding potential curves in the form of a table of values of V and its corresponding collector current I. Input is read directly from the histogram memory. One k-byte of memory is used to store the data for each axis (voltage is stored in the first k-byte). Two TR8837 transient recorders with a sampling frequency of f_s = 18 Hz, generated by an external clock, are used simultaneously to sample the data (voltage and current). To reduce the effect of noise, every thirty consecutive points are then averaged and taken as a single point.

Initial values for the constants I_0, I_1, V_0, and V_1 are supplied by the user through the questions asked by the computer. These initial values are used as approximate starting values. The last channel or memory location, which indicates the end of the plot, will also be supplied by the user. The last channel number can be obtained by moving the marker to the desired end of the real-time current plot and reading the corresponding channel number off the screen.

Through an iterative process, the program converges to best-fitting values of the five parameters I_0, I_1, V_0, V_1, and θ_0, appropriate to the distribution function in question.

The least-squares method was used for each curve fit [4,5], To use this method conveniently, the equations should be linear in the unknown constants. Equations (2.15) and (2.22) are not of this

form; therefore, new expressions were derived which were linear in corrections for approximate values of the constants (Appendix B).

The computer output illustrated in Tables III and IV consists of the raw data for V and I in the first two columns, and the best-fitting curve points for the four energy distribution in the next four columns. At the bottom of each column the best-fitting values of I_0, I_1, V_0, V_1, and θ_0 for that distribution are printed out. The mean square error of the best-fitting curve is also printed out.

Real time plots of voltage and current for two runs, AQE1 and AQE3 are shown in Figures 2.6 and 2.7. Figures 2.8 through 2.11 show the computer output plots for run #AQE1, and Figures 2.12 through 2.15 show the computer output plots for run #AQE3. In these plots, a cross indicates the actual data point (each cross is printed beside the data point) and the straight line is the fitted curve to the data. This computer program, its operation, and its application to data analysis are discussed in Appendix A.

TABLE III

EXAMPLE OF OUTPUT FROM COMPUTER PROGRAM (RUN #AQE1)

	V	I	Steady	Sq. Wave	Delta	Pulsed
1	94.667	600.000	618.305	607.202	548.123	620.328
2	150.667	590.340	601.647	594.341	548.123	601.951
3	229.333	579.473	578.189	576.274	548.123	576.690
4	301.333	564.621	557.012	559.739	548.123	554.425
5	366.667	546.267	538.269	544.734	548.123	535.076
6	474.667	526.706	508.660	519.931	548.123	505.117
7	577.333	502.677	482.429	496.353	548.123	479.081
8	668.000	474.904	460.981	475.530	369.142	458.074
9	780.000	447.494	436.791	449.808	369.142	434.627
10	880.000	422.862	417.333	426.842	369.142	415.921
11	964.000	399.919	402.500	407.551	369.142	401.721
12	1072.000	379.271	385.355	382.748	369.142	385.325
13	1188.000	355.846	369.189	356.107	369.142	369.889
14	1293.333	338.700	356.354	331.916	369.142	357.571
15	1417.333	329.161	343.270	325.672	369.142	344.927
16	1538.667	322.520	332.349	325.672	369.142	334.208
17	1632.000	326.504	325.069	325.672	369.142	326.869
	I0 =	300.000	275.000	325.672	369.142	274.000
	I1 =	600.000	437.259	531.406	178.981	442.117
	V0 =	300.000	556.720	1156.942	668.000	658.296
	V1 =	100.000	-371.259	-993.359	0.000	-253.267
	THETA =	40.000	14.768	0.000	0.000	10.202
	ERROR =		2.01793	.75729	7.69199	2.31624

Note: V is in volts and I is in amperes x 10^{-8}.

TABLE IV

EXAMPLE OF OUTPUT FROM COMPUTER PROGRAM (RUN #AQE3)

	V	I	Steady	Sq. Wave	Delta	Pulsed
1	90.667	170.000	169.513	178.791	159.585	183.857
2	165.333	172.099	169.513	174.635	159.585	178.758
3	269.333	171.822	169.513	168.846	159.585	171.178
4	364.000	167.941	169.513	163.577	159.585	164.120
5	497.333	163.506	165.386	156.156	159.585	154.311
6	626.667	154.121	155.579	148.958	159.585	145.190
7	760.000	143.785	143.847	141.537	159.585	136.391
8	898.667	133.410	132.050	133.819	92.319	127.927
9	1017.333	124.144	122.962	127.214	92.319	121.313
10	1150.667	115.432	114.075	119.793	92.319	114.515
11	1289.333	107.393	106.289	112.075	92.319	108.171
12	1433.333	101.018	99.614	104.060	92.319	102.303
13	1578.667	94.326	94.124	95.971	92.319	97.091
14	1718.667	89.732	89.823	88.179	92.319	92.635
15	1856.000	85.416	86.381	80.535	92.319	88.836
16	2002.667	82.129	83.399	77.990	92.319	85.207
17	2176.000	79.555	80.619	77.990	92.319	81.549
18	2301.333	78.189	79.012	77.990	92.319	79.165
19	2445.333	75.041	77.500	77.990	92.319	76.781
20	2608.000	75.476	76.141	77.990	92.319	74.330
	I0 =	75.000	71.200	77.990	92.319	62.700
	I1 =	170.000	98.313	163.219	67.266	129.824
	V0 =	1600.000	669.283	1466.254	898.667	843.870
	V1 =	200.000	379.070	-1030.786	0.000	-91.789
	THETA =	40.000	57.537	0.000	0.000	3.975
	ERROR =		.82769	2.35541	9.68843	3.10728

Note: V is in volts and I is in amperes x 10^{-8}.

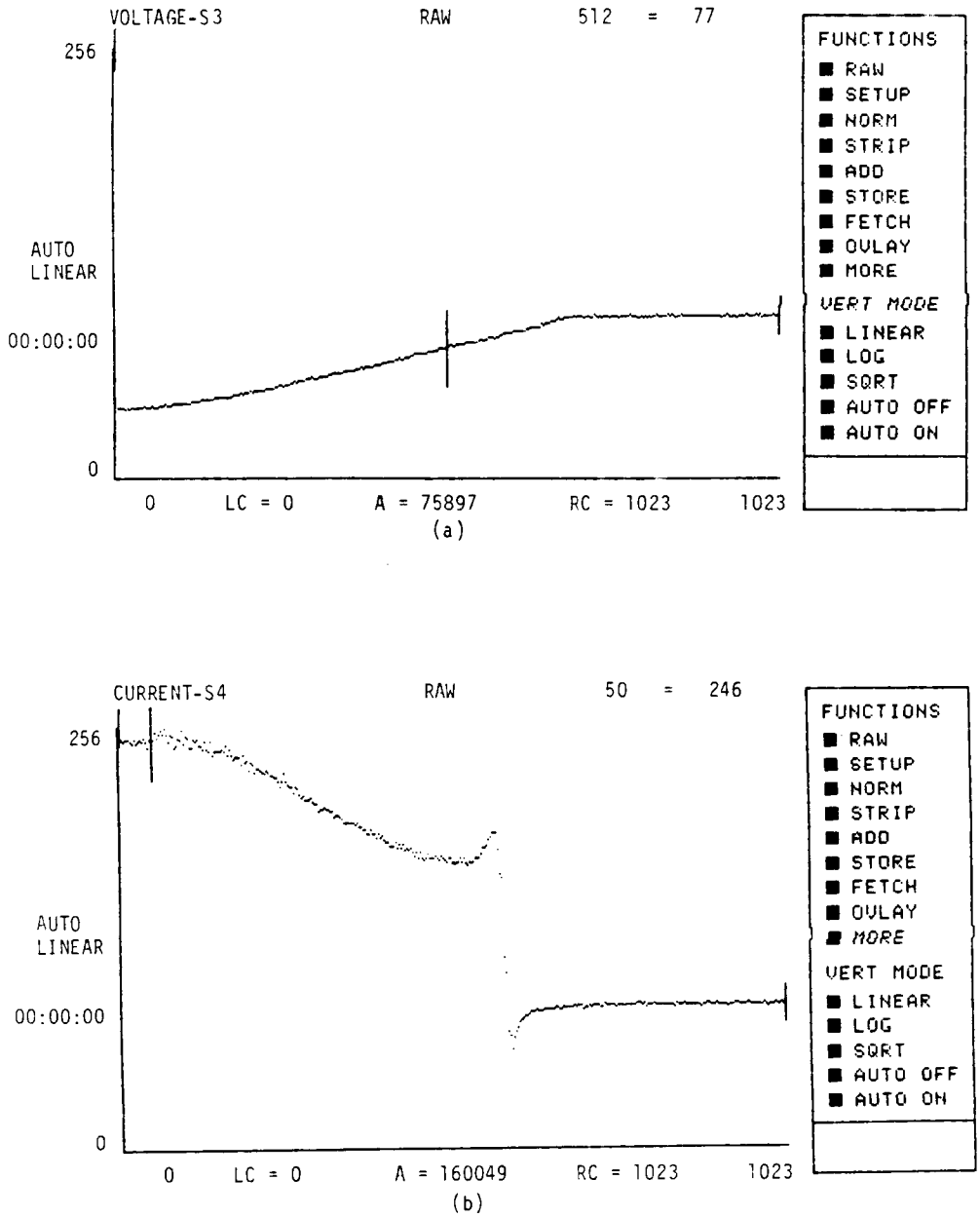


Figure 2.6. (a) Real-time plot of the voltage (run #AQE1); (b) real-time plot of the current (run #AQE1).

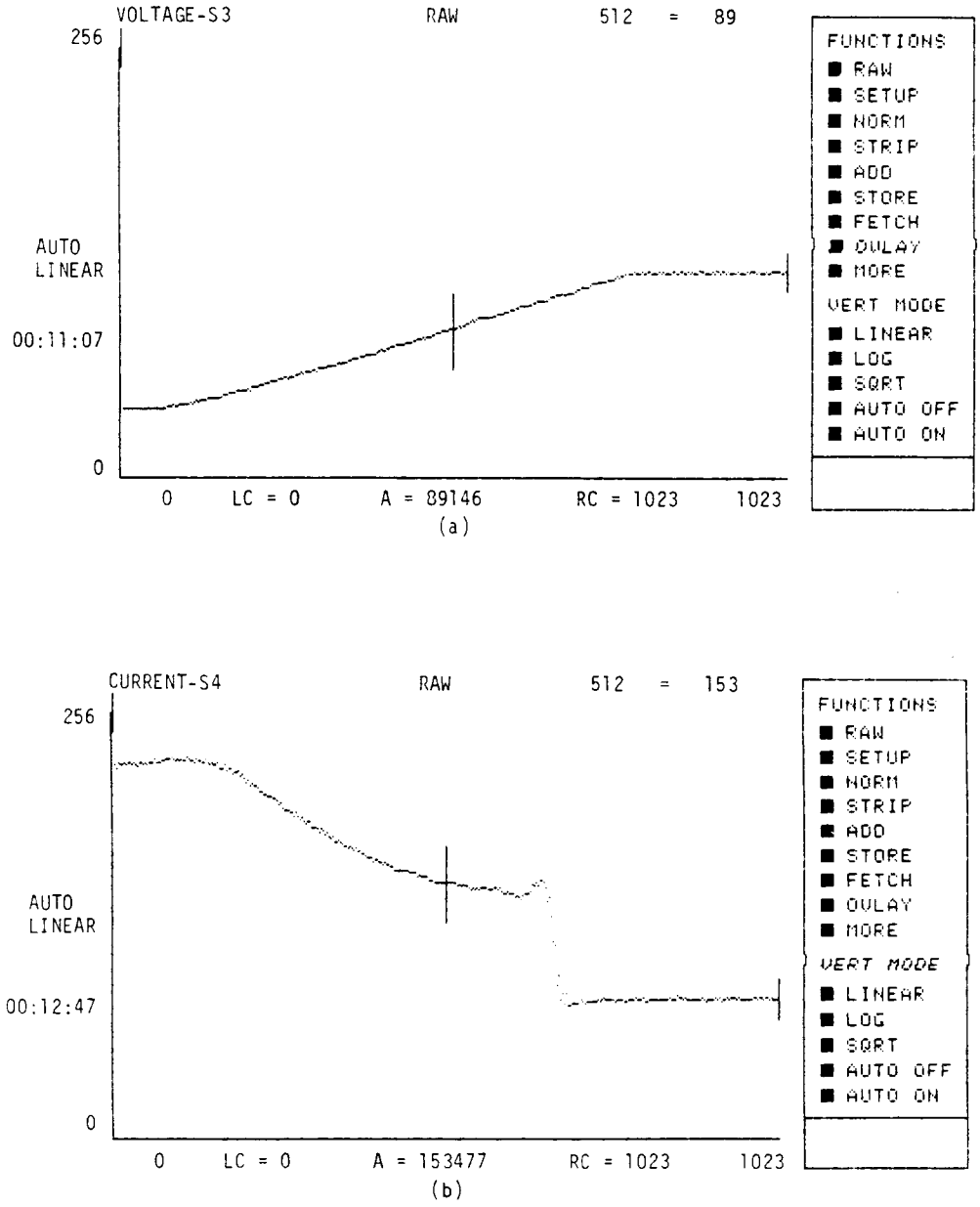


Figure 2.7. (a) Real-time plot of voltage (run #AQE3);
(b) real-time plot of current (run #AQE3).

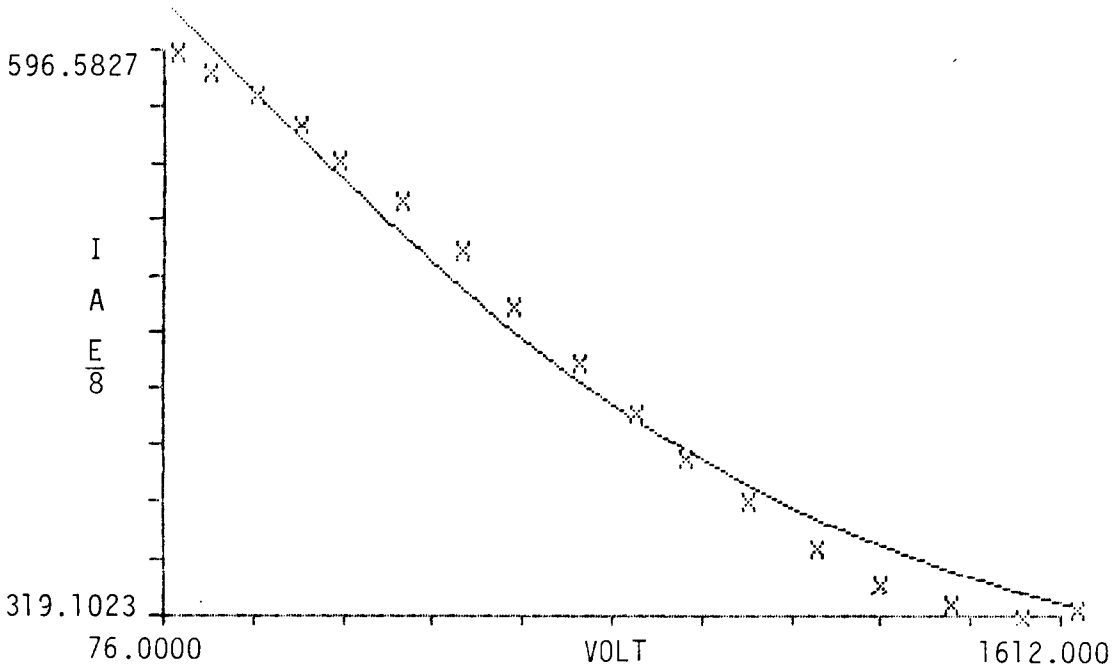


Figure 2.8. Steady (run #AQE1).

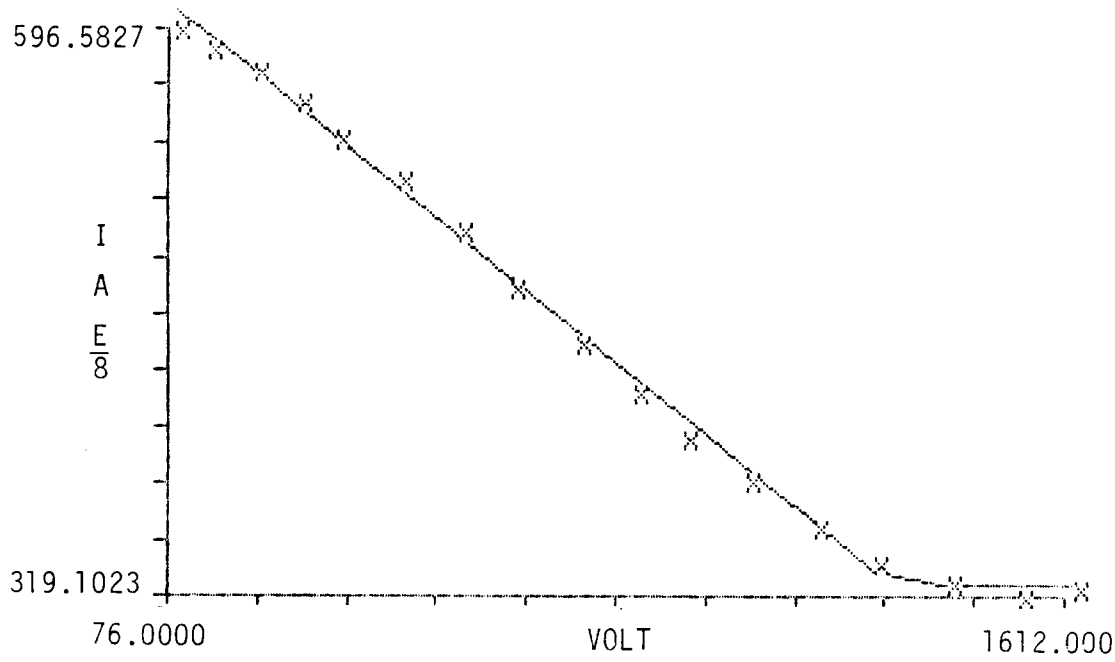


Figure 2.9. Square-wave (run #AQE1).

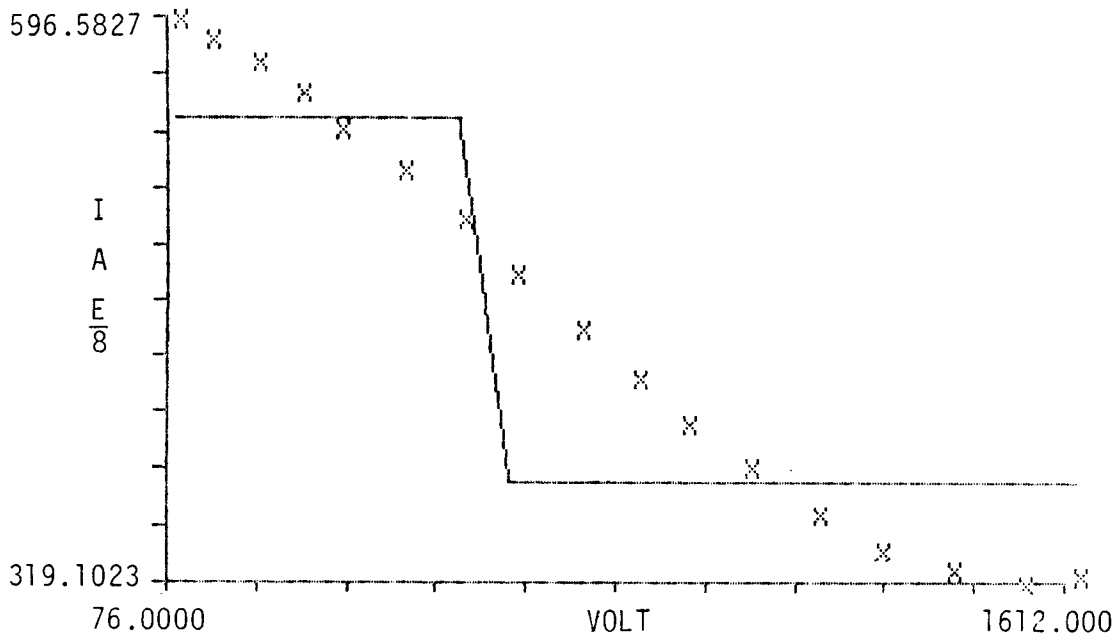


Figure 2.10. Delta (run #AQE1).

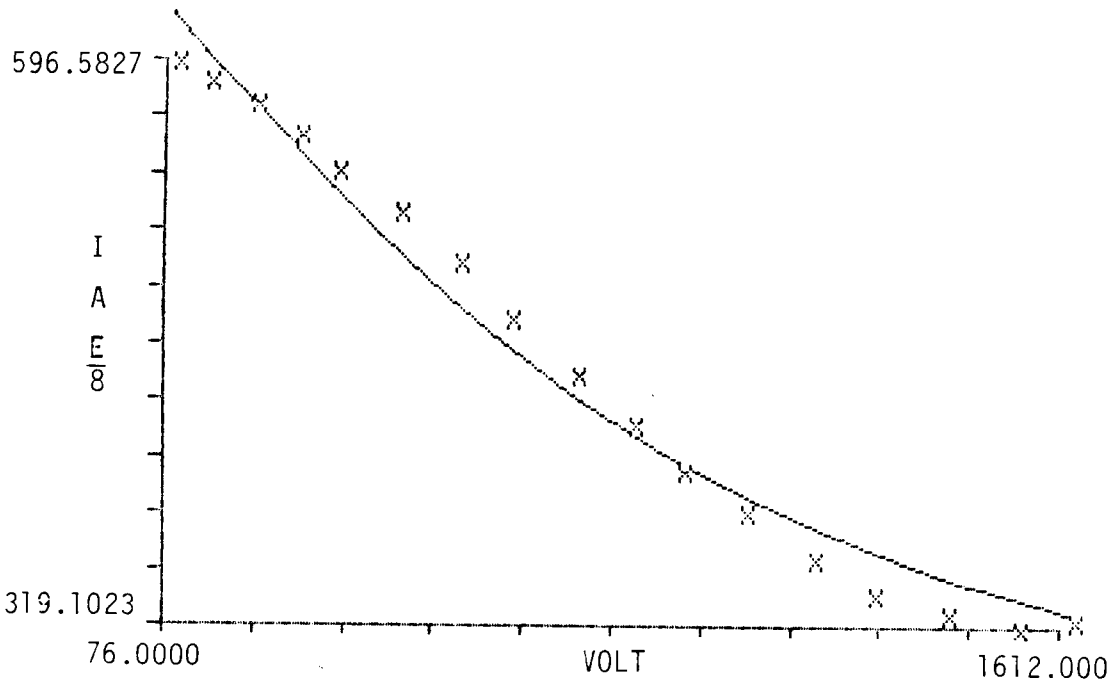


Figure 2.11. Pulsed (run #AQE1).

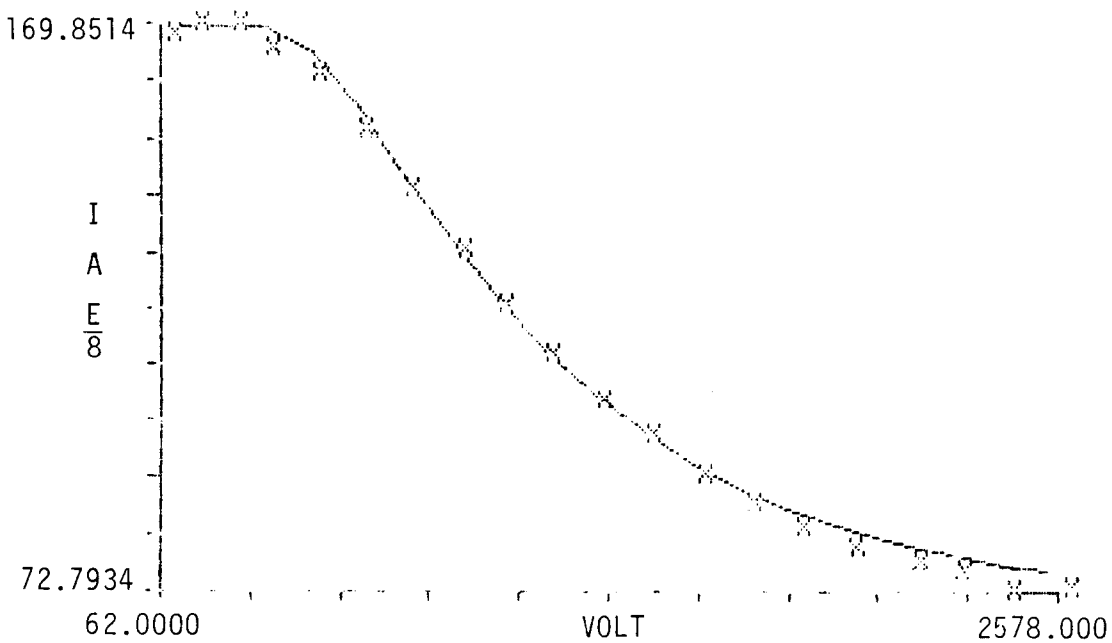


Figure 2.12. Steady (run #AQE3).

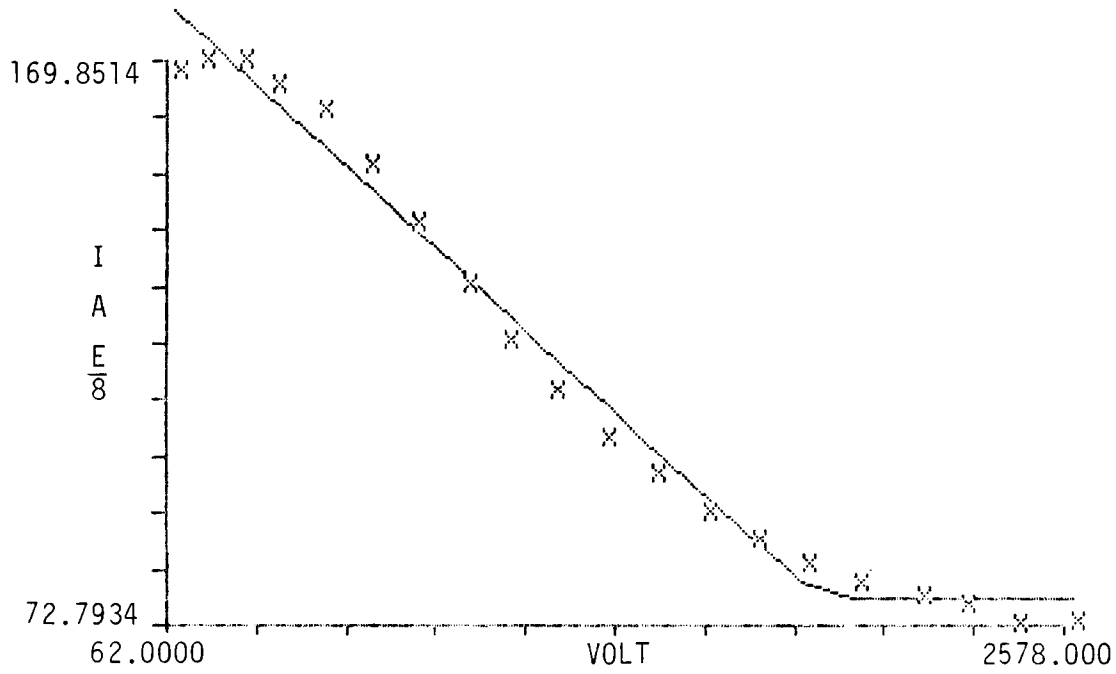


Figure 2.13. Square-wave (run #AQE3).

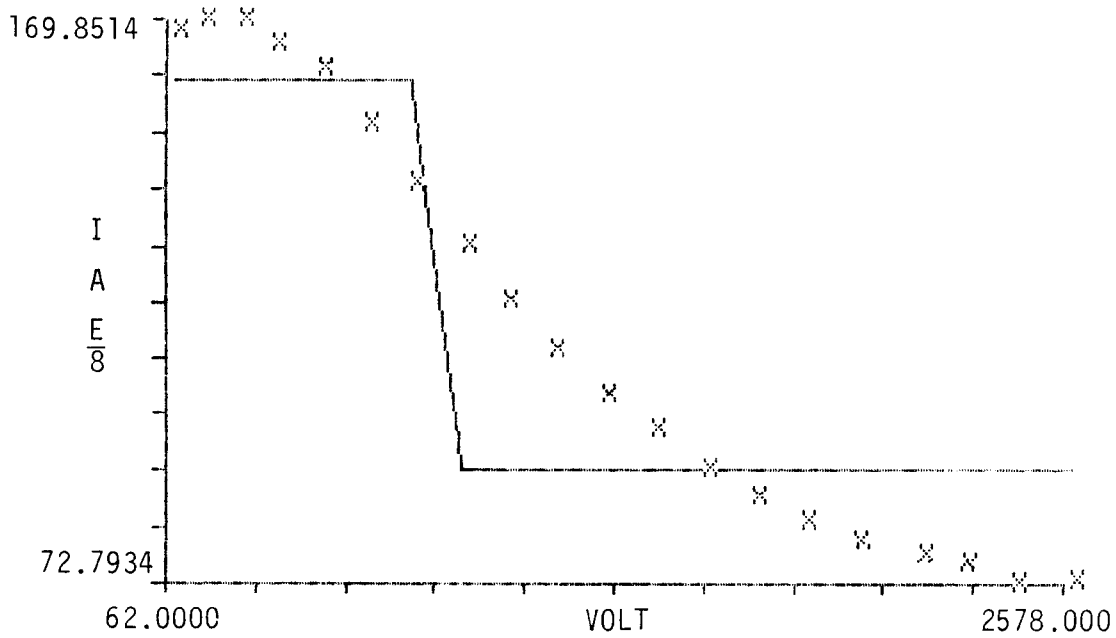


Figure 2.14. Delta (run #AQE3).

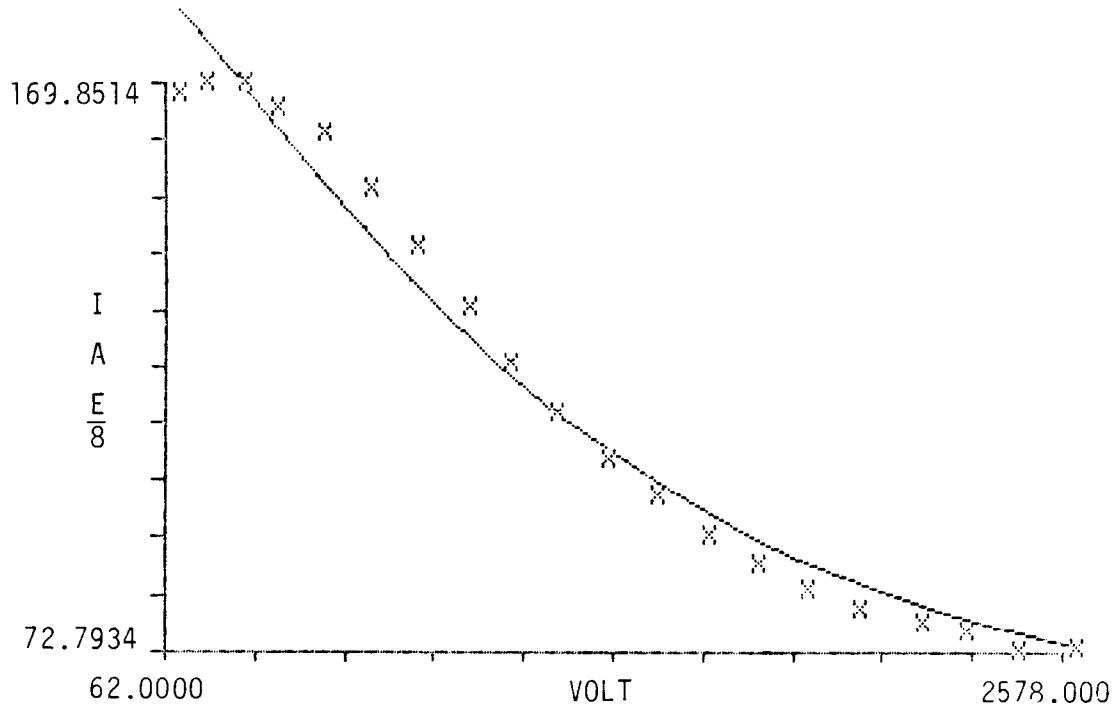


Figure 2.15. Pulsed (run #AQE3).

CHAPTER III

CHARGE EXCHANGE NEUTRAL ENERGY ANALYZER

A. THEORY

The curved plate ion energy analyzer is the essential component in charge exchange neutral energy analyzers. The ion energy distribution of a hot ($T' > 100$ eV) plasma can be measured with a charge exchange neutral energy analyzer. The principle of operation is based on charge exchange reactions on background neutral gas within the plasma, which produces a flux of energetic neutral atoms. The energies of these neutral atoms reflect the energy distribution functions of the original ions. Let us consider a deuterium plasma, and find the distribution functions of the fast deuterium ions D_F^+. These ions can interact with either molecular or atomic neutral deuterium according to the following charge exchange reactions:

$$D_F^+ + D_2 \rightarrow D_2^+ + D_F^0 \tag{3.1a}$$

$$D_F^+ + D \rightarrow D^+ + D_F^0 \tag{3.1b}$$

The electron associated with the neutral atomic or molecular deuterium is charge-exchanged to a fast deuterium ion. The low energy ionized atomic or molecular deuterium and the energetic fast neutral deuterium atom are the products of the charge exchange reactions. The fast

deuterium atom energy and momentum is essentially identical with that of the fast deuterium neutrals. The fast deuterium atoms are no longer confined by the magnetic field and leave the plasma. In the neutral detector some of the fast neutrals pass through a series of columnating slits and are re-ionized according to the following reaction by (usually) nitrogen gas:

$$D_F^0 + N_2 \rightarrow N_2^- + D_F^+ \ . \tag{3.2}$$

The resulting deuterium ions are then directed into a set of 90° electrostatic deflector plates. The energy of the ions which pass through the deflector plates is determined according to the following equation:

$$V = \frac{R}{d} V_a \tag{3.3}$$

where V_a is the plate's potential and R/d is the ratio of the mean plate radius to the separation of the two curved plates. By changing the voltage applied between the deflector plates one may sweep out the energy distribution function of the re-ionized particles. The ions are passed through a second series of slits and impinge on a photo-multiplier detector which counts individual particles. A schematic drawing of the charge exchange neutral detector is shown in Figure 3.1.

To obtain the original ion distribution function, one must correct for the charge exchange process of atomic deuterium ions on

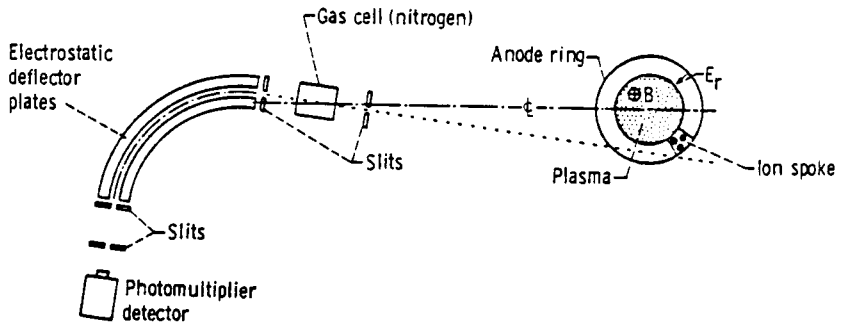


Figure 3.1. Schematic drawing of charge exchange neutral energy analyzer.

atomic or molecular deuterium according to Eqs. (3.1), the re-ionization in the nitrogen gas cell, and the energy resolution of the electrostatic analyzer. The flux I(V) of individual particles reaching the photomultiplier is given by:

$$I(V) = C_0 n_i n_o \sigma_{CX}(V) v_i f(V) K(V) G_a(V) \tag{3.4}$$

where V is the ion energy in electron volts, C_0 is a geometrical constant, f(V) is the desired ion energy distribution, K(V) is the re-ionization probability of atomic deuterium in a nitrogen gas cell, and $G_a(V)$ is a correction for the energy resolution of the analyzer. $G_a(V)$ can be derived as follows:

$$\text{Energy Resolution} = \frac{dV}{V} = \text{constant} \tag{3.5}$$

$$G_a(V) = dV = \text{constant} \times V \tag{3.6}$$

$$\therefore G_a(V) = K_1 V \ . \tag{3.7}$$

The factor $n_i n_o \sigma_{CX}(V) v_i$ in Eq. (3.4) is the number of charge exchange reactions per cubic centimeter per second, n_i and n_o are the ion and neutral number identity, respectively, and v_i is the deuterium ion velocity and is related to the ion energy as follows:

$$eV = 1/2\ m_i v_i^{\,2} \tag{3.8}$$

$$\therefore v_i = K_2 V^{1/2} \ . \tag{3.9}$$

The correction factor K(V) for the nitrogen gas cell is related to the ion energy according to [1]:

$$K(V) = K_3 V^{0.56} \ . \tag{3.10}$$

Using Eqs. (3.7), (3.9), and (3.10), Eq. (3.4) can be written as

$$I(V) = C_0 n_i n_o \sigma_{CX}(V)(K_2 V^{.5}) f(V)(K_3 V^{.56})(K_1 V) \tag{3.11}$$

$$I(V) = \underbrace{C_0 n_i n_o K_2 K_3 K_1}_{Q}\ \sigma_{CX}(V) f(V)(V^{2.06}) \tag{3.12}$$

$$I(V) = Q \sigma_{CX}(V) f(V) V^{2.06} \tag{3.13}$$

and finally

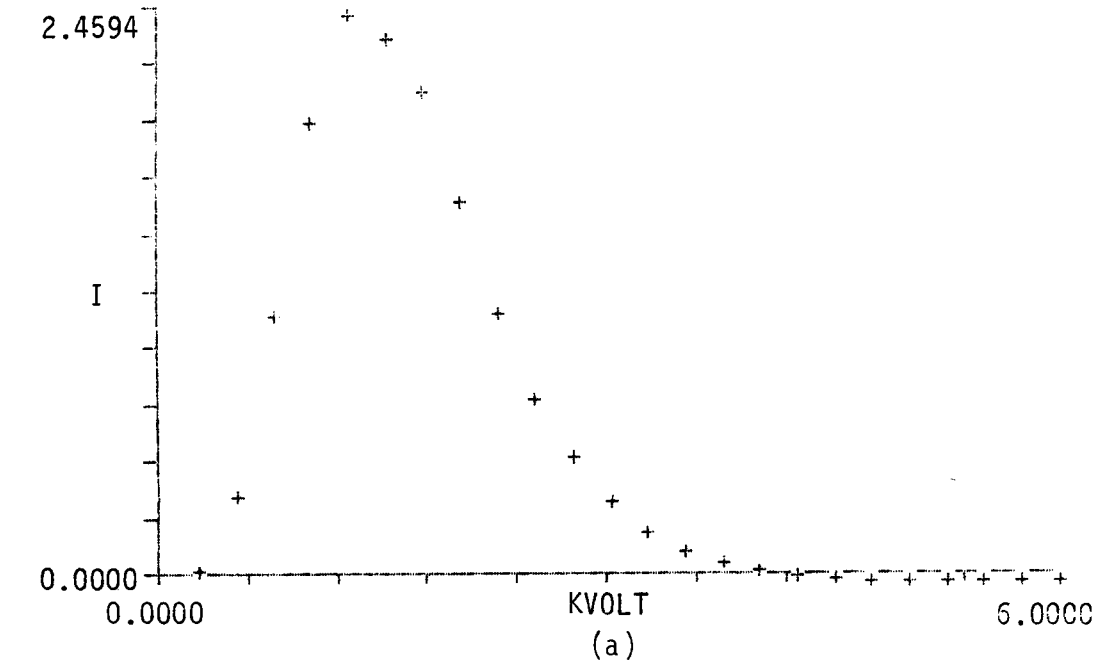
$$f(V) = \frac{C_1 I(V)}{V^{2.06}\sigma_{CX}(V)} \tag{3.14}$$

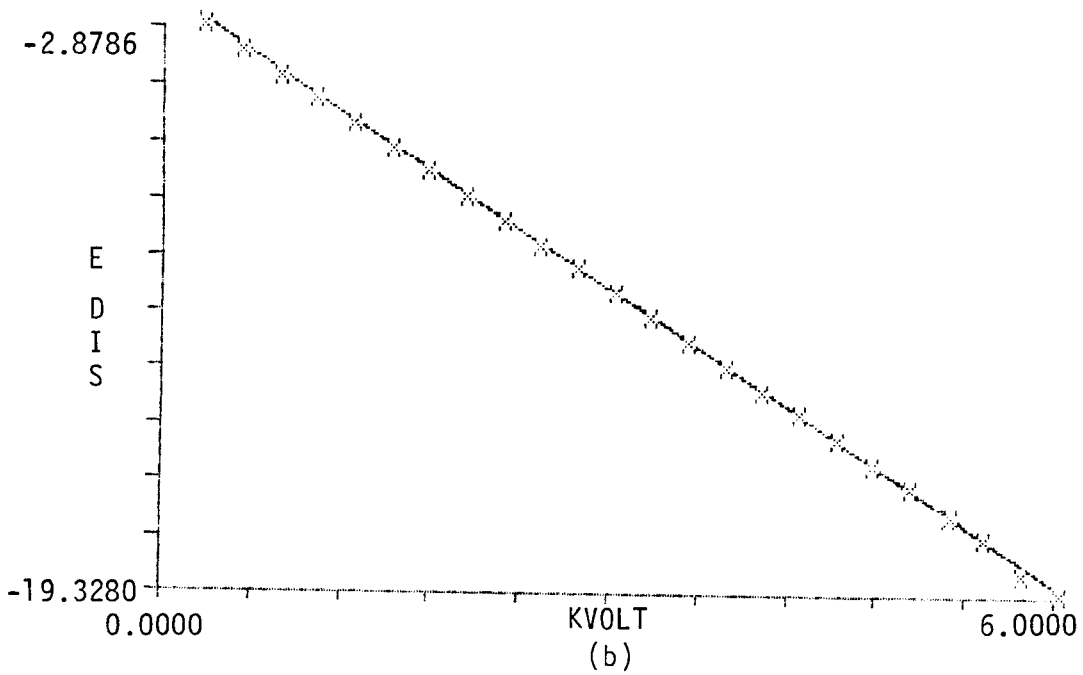
where C_1 is a constant for the apparatus [1].

B. COMPUTER PROGRAM

A computer program in FORTRAN 80 was written which finds the ion kinetic temperature of a plasma. The computer program is run on the Lecroy 3500 small computer (Appendix C), and takes as input the current measured by the detector at the output of the curved plate analyzer, and the voltage applied across the analyzer plates. The ion energy distribution function obtained from Eq. (3.14) (divided by $V^{.5}$) was used as input to the computer program,which obtains a Maxwellian distribution with a least-squares best fit to the experimentally determined ion energy distribution function f(v). For a Maxwellian distribution, the logarithm of $(f(v)/v^{.5})$ as a function of voltage is a straight line. The slope of this line gives the ion kinetic temperature. The computer program finds the best-fitted straight line according to least-squares method to $\log[f(v)/v^{.5}]$ as a function of v [4,5],

The computer program is programmed for both molecular and atomic deuterium. Figure 3.2 shows the computer output plots for the pseudo data shown in Table IV, since the actual instrument is not functioning at this time. This computer program, its operation, and its application to data analysis are discussed in Appendix A.





TI = .3505578 KEV

Figure 3.2. (a) Maxwellian distribution of ions; current vs. voltage. (b) Logarithm of ion energy distribution [$dN/dE(v \cdot^5)$] vs. voltage.

TABLE V

EXAMPLE OF INPUT TO CHARGE EXCHANGE
NEUTRAL ENERGY ANALYZER
(PSEUDO DATA)

V	I
.75000	1.16400
1.00000	2.00000
1.25000	2.46000
1.50000	2.36000
1.75000	2.13000
2.00000	1.66290
2.25000	1.17910
2.50000	.80390
2.75000	.54940
3.00000	.35660
3.25000	.22590
3.50000	.14050
3.75000	.08960
4.00000	.05350
4.25000	.03180
4.50000	.01870
4.75000	.01040
5.00000	.00607
5.25000	.00326
5.50000	.00186
5.75000	.00076
6.00000	.00050
.25000	.05200
.50000	.37980

REFERENCES

REFERENCES

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APPENDIXES

APPENDIX A

COMPUTER PROGRAMS

I. LANGMUIR PROBE PROGRAM

The main program LANPROB (Figure A.1) reads the input values of voltage and current directly from the histogram memory of the Lecroy 3500. The mass of the ion, and the length and radius of the Langmuir probe are then entered into the program. The limits for each segment are calculated based on the light pen approximation. Two hits are required to identify the limits (start and the end). The actual electron current I_e is computed in this part, and then is passed to subroutine PLOT2 to be plotted against the applied probe voltage on a semilog scale. The plasma potential and floating potential are calculated using the light pen approximation. The electron kinetic temperature is found based on a least squares fit to the electron current region on a semilog plot (the electron kinetic temperature is the inverse of the slope of this straight line). Electron number density, ion number density, and ion kinetic temperature are calculated using the analytical expressions (1.1) and (1.7).

Subroutine PLOT1 (Figure A.2) generates a negative plot of current-voltage characteristics. A light pen is used on this plot to identify the limits of the ion saturation region (AB in Figure 1.2, p. 5). PLOT1 also generates a negative plot of the current-voltage characteristic with the ion saturation line (extrapolated) superimposed. The computer plots are shown in Figures 1.4a and 1.5a (pp. 13 and 14). The light pen is then used on the latest plot to

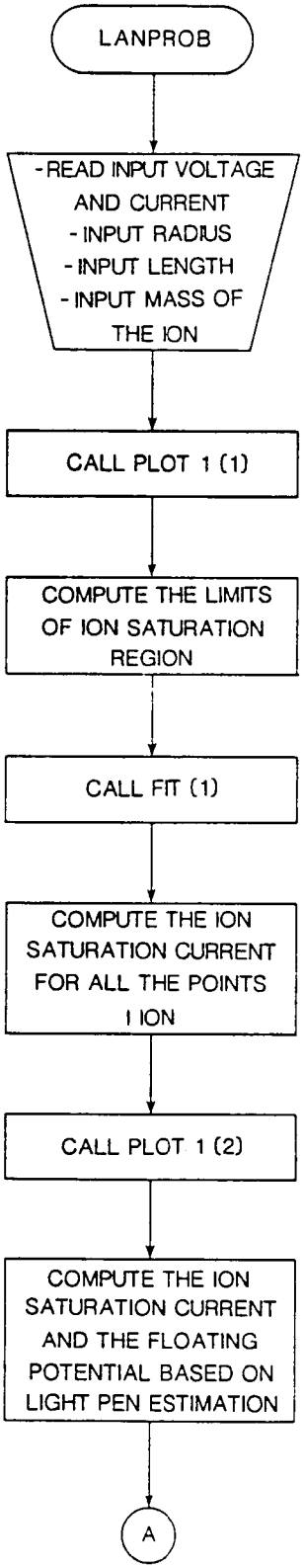

Figure A.1. Flowchart for subroutine LANPROB.

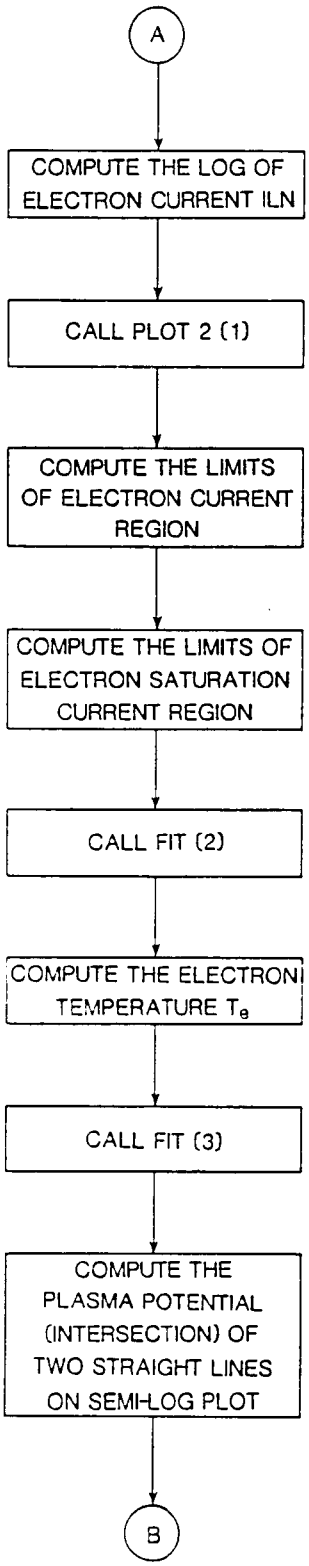

Figure A.1 (continued)

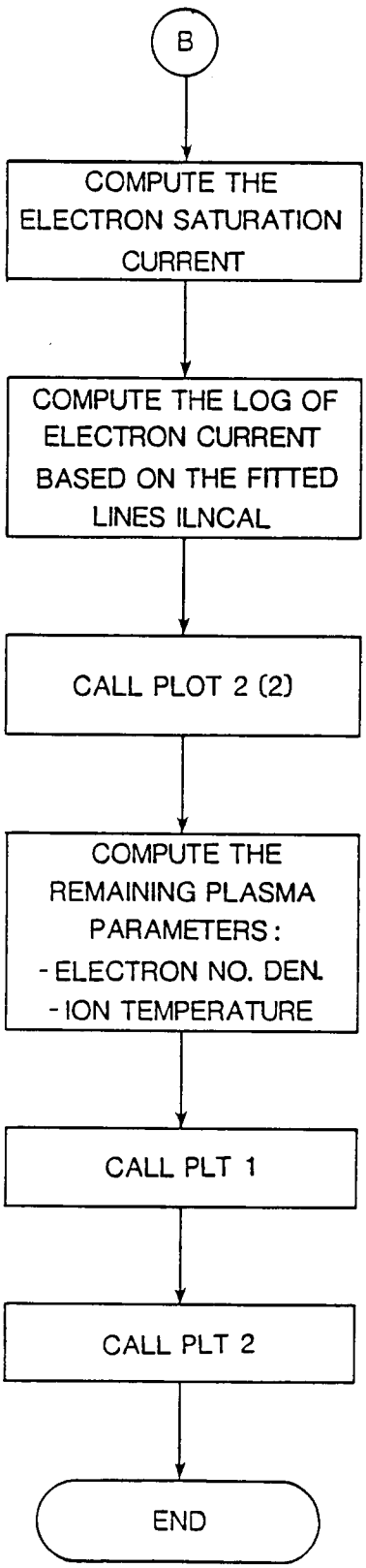

Figure A.1 (continued)

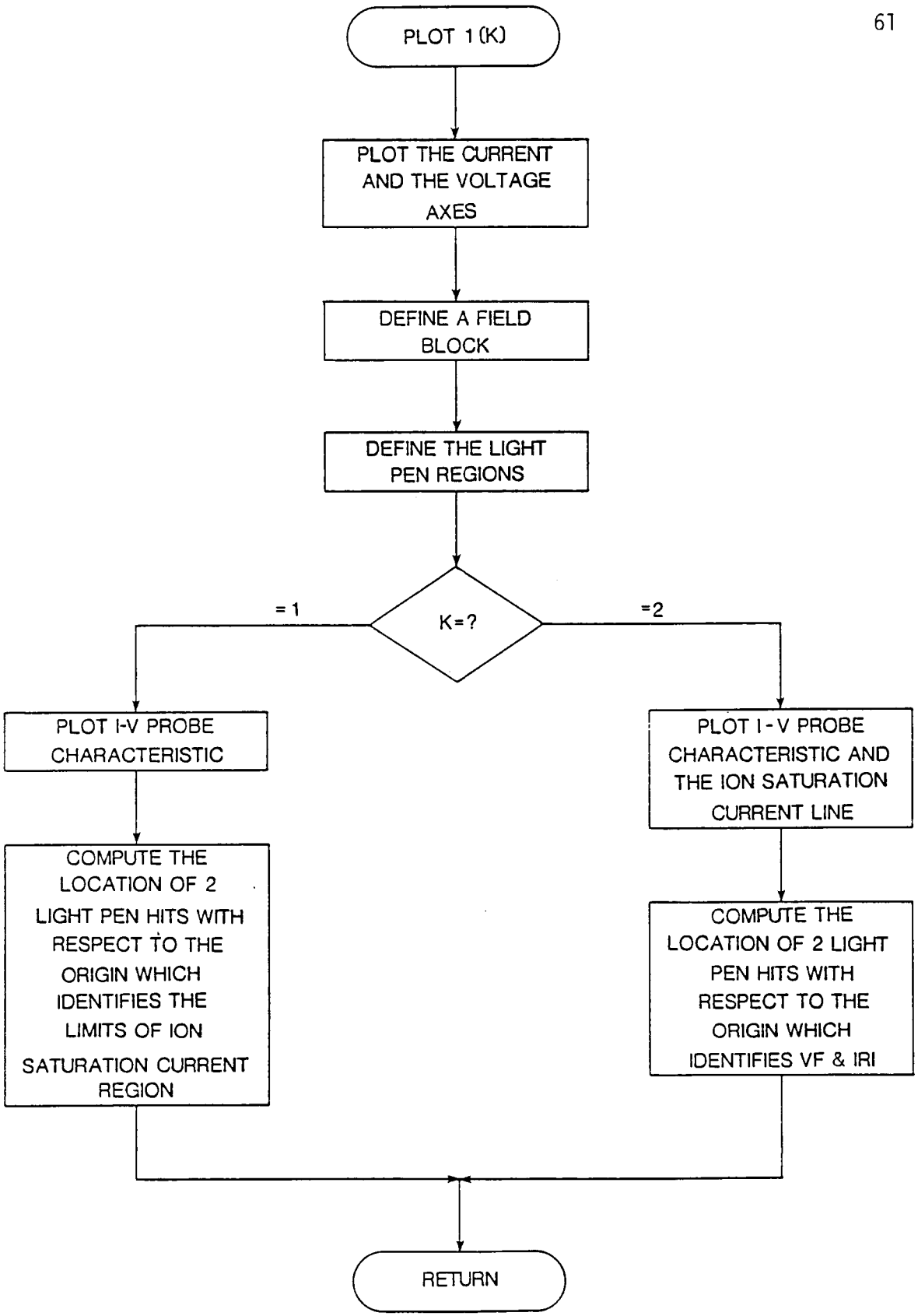

Figure A.2. Flowchart for subroutine PLOT1.

estimate the light pen regions corresponding to the ion saturation current and the floating potential of the plasma, respectively. This information is then passed to the main program LANPROB to calculate ion saturation current and the floating potential. The equation for the ion saturation line is obtained through subroutine FIT(1).

Subroutine PLOT2 (Figure A.3) generates a negative semilog plot of electron current I_e as a function of applied probe potential V_0. The light pen is used on this plot to identify the limits of the electron current region and the electron saturation region. Each of these regions appears as a straight line on a semilog plot. The computer outputs for this plot are shown in Figures 1.4b and 1.5b (pp. 13 and 14).

Subroutine FIT (Figure A.4) finds the normal equations according to the least squares method and computes the constants describing the fitted straight line (slope and Y-intercept) for each segment.

Subroutines PLT1 (Figure A.5) and PLT2 (Figure A.6) display the current-voltage characteristic (Figures 1.4a and 1.5a, pp. 13 and 14) and semilog plot of electron current as a function of voltage (Figures 1.4b and 1.5b), respectively. These plots are then screen-dumped on the Lecroy 3931A printer.

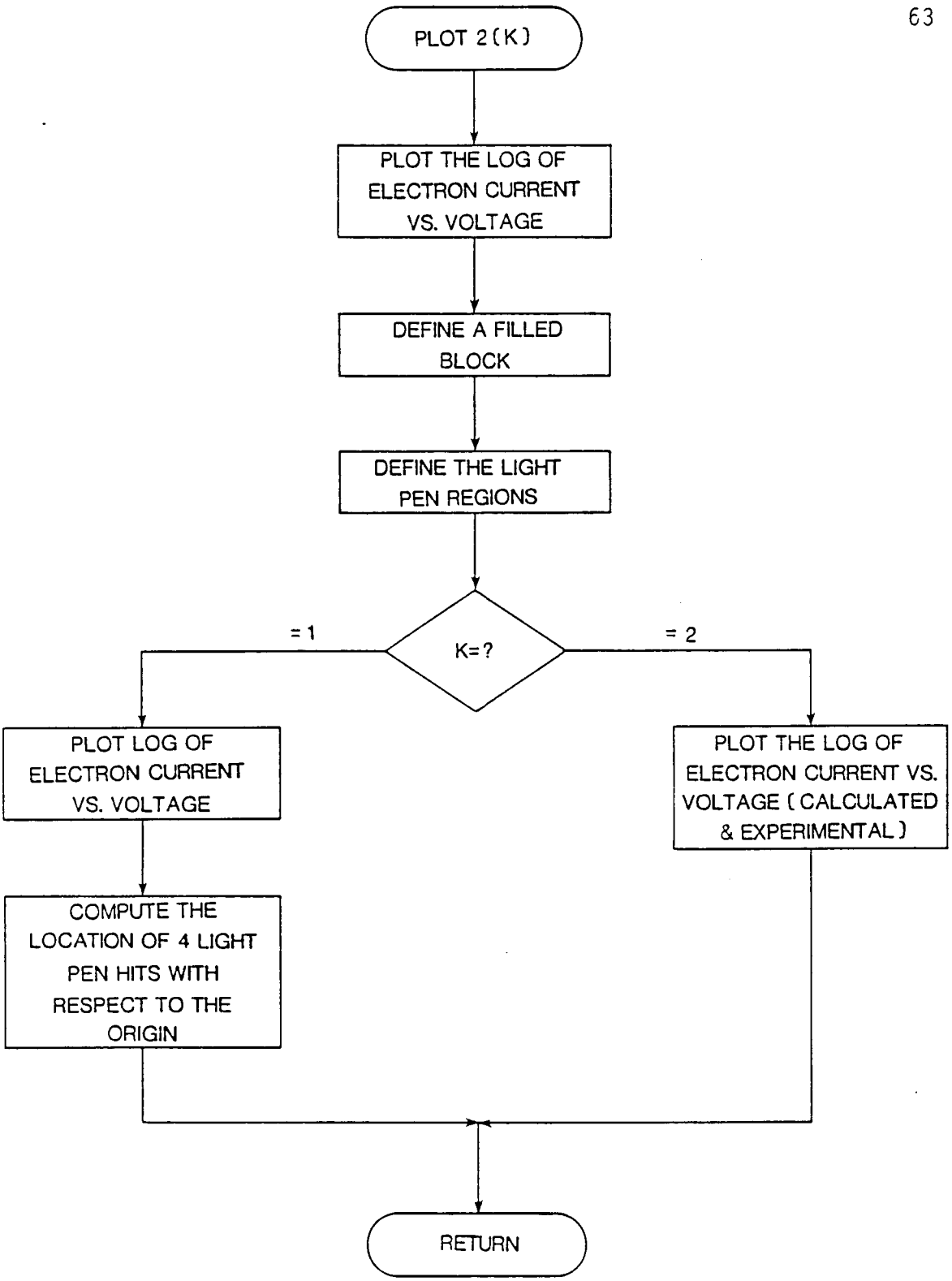

Figure A.3. Flowchart for subroutine PLOT2.

Note: First two hits identifies the limits of electron current region. Next two hits identify the limits of electron saturation current region.

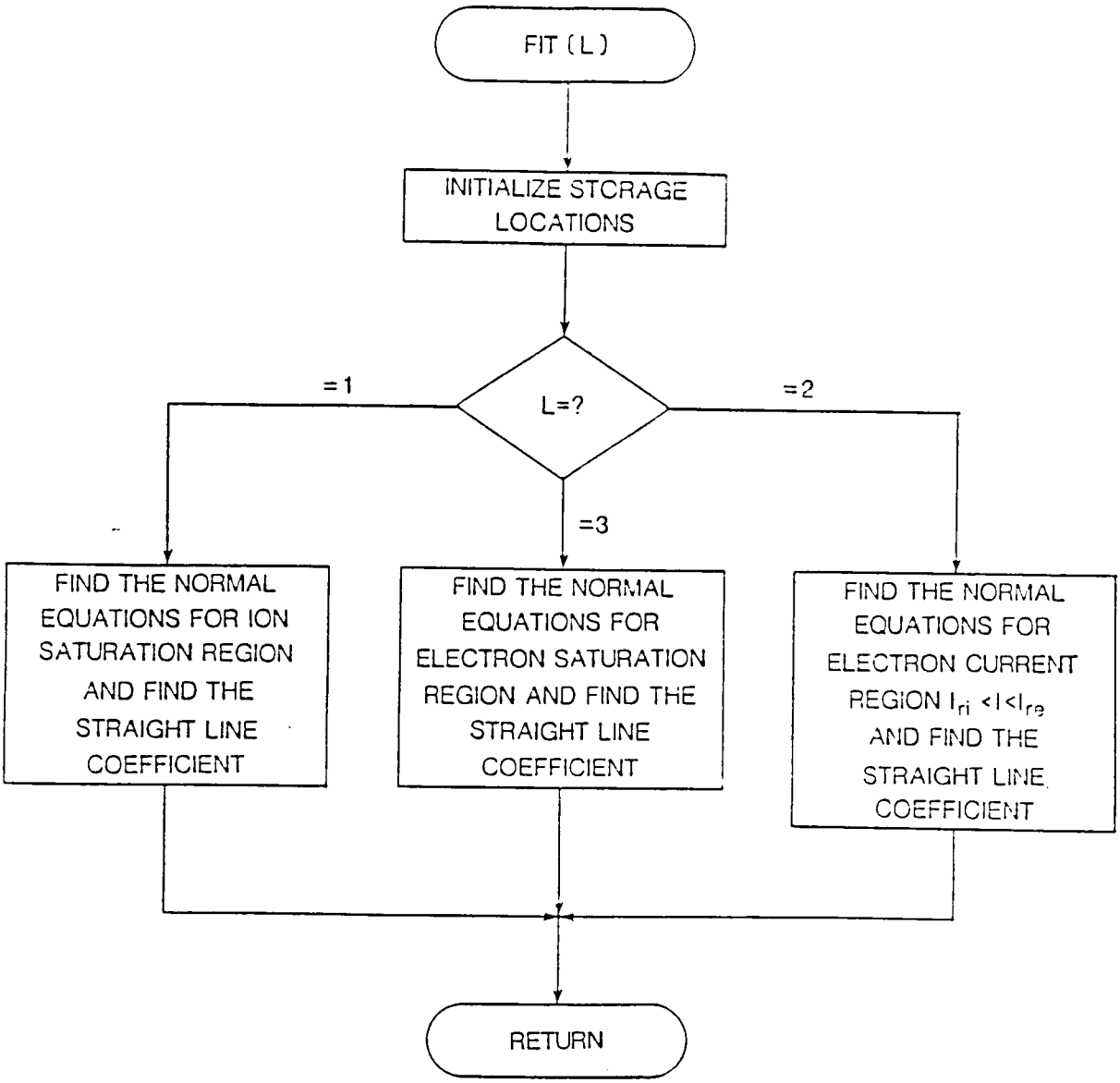

Figure A.4. Flowchart for subroutine FIT.

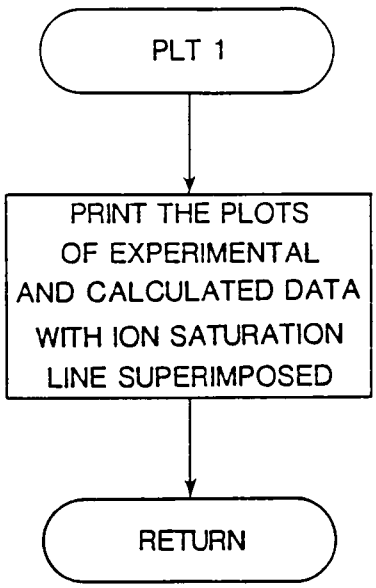

Figure A.5. Flowchart for subroutine PLT1.

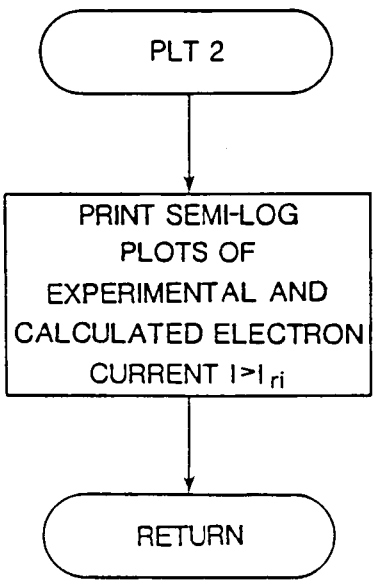

Figure A.6. Flowchart for subroutine PLT2.

```
      PROGRAM LANPROB
      DIMENSION IELEC(70),ARRAY(1000)
      REAL I,IION,IRI,IRE,ILN,IELEC,ILNCAL,NE
      COMMON/ALL/N,N1,M1,V(70),I(70),IION(70),ILN(70),ILNCAL(70)
     1,A,B,M,VCHAN(4),VEST(4)
C     N IS THE NO. OF DATA POINTS
C     N1 IS THE NO. OF DATA POINTS BELOW THE ION SAT. CURRENT
C     M IS THE NO. OF DATA POINTS USED FOR EACH FIT
C     THE STRAIGHT LINE FORMULA IS: I=A+B*V AND LN(IELEC)=A+B*V
C     V(J)   VOLTAGE INPUT
C     I(J)   CURRENT INPUT
C     IION(J)  CALCULATED ION SAT. CURRENT FOR ALL THE POINTS
C     ILN(J)  LOG OF ELECTRON CURRENT
C     IELEC(J)  CALCULATED ELECTRON CURRENT
C     ILNCAL(J)  CALCULATED LOG OF ELECTRON CURRENT
C     VEST(4)  ESTIMATED POINTS USING THE LIGHT PEN
C     VCHAN(4) CHANGE IN VOLTAGE FROM THE LIGHT PEN TO THE
C     FIRST POINT OF THE PLOT
C
C     VFFLOATING POTENTIAL
C     VPPLASMA POTENTIAL
C     TIION TEMPERATURE
C     TEELECTRON TEMPERATURE
C     NEELECTRON NUMBER DENSITY
C     PLOT1   PLOT2 ARE NEGATIVE PLOTS TO BE USED
C     BY THE LIGHT PEN
C     PLT1  PLT2 ARE POSITIVE PLOTS FOR THE OUTPUT
      N=24
      CALL HMFRD (ARRAY,20,1000)
      II=1
      K=40
1     SUMAVG=0.0
      DO 2 J=II,K
2     SUMAVG=SUMAVG+ARRAY(J)
      AVG=SUMAVG/40.0
      KK=K/40
      V(KK)=(AVG-39.0)*2.0*10.0
      II=II+40
      K=K+40
      IF (K.LE.1000) GO TO 1
      CALL HMFRD(ARRAY,1044,1000)
```

```
      II=1
      K=40
3     SUMAVG=0.0
      DO 4 J=II,K
4     SUMAVG=SUMAVG+ARRAY(J)
      AVG=SUMAVG/40.0
      KK=K/40
      I(KK)=((AVG-83.0)*2.0)/50.0
      II=II+40
      K=K+40
      IF (K.LE.1000) GO TO 3
      WRITE (5,5)
5     FORMAT(15H RADIUS IN CM ?)
      READ (5,6) R
6     FORMAT (F7.6)
      WRITE (5,7)
7     FORMAT (15H LENGTH IN CM ?)
      READ (5,9) RL
9     FORMAT (F7.5)
      WRITE (5,12)
12    FORMAT (25H MASS OF THE ION IN AMU ?)
      READ (5,14) RMI
14    FORMAT (F6.2)
      R=R/100.
      RL=RL/100.
      RMI=RMI*1.67E-27
      AREA=6.283185*R*RL
C     PLOT I VS V CHARACTERISTIC (NEGATIVE PLOT)
      CALL PLOT1(1)
C     CALCULATE THE BEGINNING OF THE ION SAT. REGION
      VEST(1)=V(1)+VCHAN(1)
C     CALCULATE THE END OF THE ION SAT. REGION
      VEST(2)=V(1)+VCHAN(2)
C     FIT A STRAIGHT LINE BETWEEN VEST(1)  VEST(2)
      CALL FIT(1)
      DO 15 J=1,N
15    IION(J)=A+B*V(J)
C     PLOT ION SAT. CURRENT (IION)  PROBE CURRENT I VS V
C     NEGATIVE PLOT
      CALL PLOT1(2)
C     CALCULATE THE FLOATING POTENTIAL USING THE LIGHT PEN
```

```
      VF=V(1)+VCHAN(2)
      VRI=V(1)+VCHAN(1)
C     CALCULATE THE ION SATURATION CURRENT
      IRI=A+B*VRI
      N1=1
C     CALCULATE THE ELECTRON CURRENT (IELEC)  THE LOG OF
C     ELECTRON CURRENT (ILN)  FIND NO. OF DATA POINTS
C     BELOW ION SAT. CURRENT (N1)
      DO 20 J=1,N
      IF (V(J).LE.VRI) GO TO 17
      IELEC(J)=I(J)-IION(J)
      ILN(J)=ALOG(IELEC(J))
      GO TO 20
17    N1=N1+1
20    CONTINUE
C     PLOT LOG OF ELECTRON CURRENT (ILN) VS VOLTAGE
C     NEGATIVE PLOT
      CALL PLOT2(1)
C     CALCULATE THE START  THE END OF ELECTRON CURRENT
C     REGION ON SEMI LOG PLOT (VEST(1)  VEST(2)).ALSO
C     CALCULATE THE START  THE END OF ELECTRON SATURATION
C     CURRENT REGION ON SEMI LOG PLOT (VEST(3)  VEST(4))
C              USING THE LIGHT PEN ESTIMATION
      DO 22 JJ=1,4
22    VEST(JJ)=VCHAN(JJ)+V(N1)
C     FIT A STRAIGHT LINE TO ELECTRON CURRENT REGION
      CALL FIT(2)
C     CALCULATE THE ELECTRON TEMPERATURE
      TE=1/B
      SA=A
      SB=B
C     FIT A STRAIGHT LINE TO ELECTRON SATURATION REGION
      CALL FIT(3)
C     CALCULATE THE PLASMA POTENTIAL (INTERCEPT OF FIT(2)
C     FIT(3) STRAIGHT LINES)
      VP=(A-SA)/(SB-B)
      IRE=EXP(SA+SB*VP)
      M1=0
C     CALCULATE THE LOG OF ELECTRON CURRENT (ILNCAL) BASED
C     ON FITTED LINE (FIT(2)  FIT(3)).ALSO FIND NO. OF DATA
C     POINTS BETWEEN ION SAT. CURRENT  THE BEGINNING OF
```

```
C     ELECTRON CURRENT REGION (M1)
      DO 30 J=N1,N
      IF (V(J).LT.VEST(1)) GO TO 27
      IF(V(J).GE.VP) GO TO 25
      ILNCAL(J)=SA+SB*V(J)
      GO TO 30
25    ILNCAL(J)=A+B*V(J)
      GO TO 30
27    M1=M1+1
30    CONTINUE
      N1=N1+M1
      CALL PLOT2(2)
C     CALCULATE ELECTRON NO. DENSITY (NE)
      NE=IRE/(2.6756E-11*AREA*SQRT(TE))
C     CALCULATE THE ION TEMPERATURE (TI)
      PTI=1.5339E37*((IRI/1000.)**2*RMI)
      TI=(1.0E18/((AREA**2)*(NE**2)))*PTI
      WRITE (2,120)
      WRITE (2,115) (V(J),I(J),J=1,N)
      IFLAG=0
      CALL PLT1
50    WRITE (2,60) TI,TE,NE,VF,VP
60    FORMAT(5H TI= ,F13.2,3H EV/5H TE= ,F13.2,3H EV/5H NE=
     1,E13.4,4H /M3/5H VF= ,F13.2,3H V /5H VP= ,F13.2,2H V)
115   FORMAT (4X,F10.5,10X,F10.5)
120   FORMAT (10X,3H V ,15X,3H I )
      IF (IFLAG.EQ.1) GO TO 140
      IFLAG=1
      CALL PLT2
      GO TO 50
140   END
C
C
C
      SUBROUTINE PLOT1(K)
C     K=1 NEGATIVE PLOTT OF I VS V TO BE USED BY THE LIGHT PEN
C     K=2 NEGATIVE PLOT OF I VS V AND IION VS V
      DIMENSION X(50),Y(50)
      REAL I,IION,IRI,ILN,IELEC,ILNCAL
      COMMON/ALL/N,N1,M1,V(70),I(70),IION(70),ILN(70),ILNCAL(70)
     1,A,B,M,VCHAN(4),VEST(4)
```

```
      DO 10 J=1,N
10    X(J)=I(J)
      S=3.8
      SY=S
      CALL SCALE (X,N,1,S,XMIN,DX)
      YMIN=XMIN
      DY=DX
      CALL PLINIT
      CALL AXIS (82,51,'I MA',-4,S,90.0,XMIN,DX)
      DO 20 J=1,N
20    X(J)=V(J)
      S=4.0
      CALL SCALE (X,N,1,S,XMIN,DX)
C     ROUND OFF THE XMIN=MIN. VOLTAGE  DX=VOLT/INCH
      IXMIN=XMIN
      XMIN=IXMIN
      IDX=DX
      DX=IDX
      CALL AXIS (82,51,'VOLT',4,S,0.0,XMIN,DX)
C     CALCULATE THE DISTANCE BETWEEN THE FIRST DATA POINT AND
C     THE ORIGIN ON THE PLOT (IN PIXELS)
      OFFSET=(V(1)-XMIN)*(82.0/DX)
      IY=51.0*SY+51.0
      IX=82.0+OFFSET
      IXSIZE=S*82.0-OFFSET
      IYSIZE=SY*51.0
      CALL BLOCK(IX,IY,IXSIZE,IYSIZE,-1)
      IXSIZE=IXSIZE/50
      CALL ZPEN
C     DEFINE 50 LIGHT PEN REGION STARTING FROM THE FIRST
C     DATA POINT ON THE PLOT
      DO 22 NUM=1,50
      CALL DEFPEN(IX,IY,IXSIZE,IYSIZE,NUM)
22    IX=IX+IXSIZE
C     PLOT THE ZERO CURRENT LINE
      IY=(0.0-YMIN)*(51.0/DY)+51.0
      CALL PLOT (82,IY,5)
      IX=S*82.0+82.0
      CALL PLOT(IX,IY,4)
C     PLACE THE PEN AT THE ORIGIN  PLOT THE ION SAT. LINE
      CALL PLOT(82,51,5)
```

```
      IF (K.NE.2) GO TO 27
      DO 25 J=1,N
      IX=(V(J)-XMIN)*(82.0/DX)+82.0
      IY=(IION(J)-YMIN)*(51.0/DY)+51.0
25    CALL PLOT (IX,IY,4)
C     PLOT THE I VS V PROBE CHARACTERISTIC
27    CALL PLOT (82,51,5)
      DO 30 J=1,N
      IX=(V(J)-XMIN)*(82.0/DX)+82.0
      IY=(I(J)-YMIN)*(51.0/DY)+51.0
30    CALL PLOT (IX,IY,4)
      DEL=(V(N)-V(1))/50.0
      GO TO (65,80) ,K
C     FIND THE LIGHT PEN REGION MOST RECENTLY HIT AND
C     THE VOLTAGE CHANGE WITH RESPECT TO THE FIRST
C     POINT (VCHAN)
65    DO 75 JJ=1,2
      WRITE (5,70) JJ
70    FORMAT (I3)
72    CALL SLEEP(240)
      CALL LPEN(NUM)
      IF (NUM.EQ.0) GO TO 72
      RNUM=NUM
75    VCHAN(JJ)=(RNUM-1.0)*DEL
      GO TO 95
80    WRITE (5,85)
85    FORMAT (36H ESTIMATE THE ION SATURATION CURRENT)
87    CALL SLEEP(240)
      CALL LPEN(NUM)
      IF(NUM.EQ.0) GO TO 87
      RNUM=NUM
      VCHAN(1)=(RNUM-1.0)*DEL
      WRITE (5,90)
90    FORMAT (28H FIND THE FLOATING POTENTIAL)
92    CALL SLEEP(240)
      CALL LPEN(NUM)
      IF (NUM.EQ.0) GO TO 92
      RNUM=NUM
      VCHAN(2)=(RNUM-1.0)*DEL
95    CALL PLTCLS
      RETURN
```

```
      END
C
C
C
      SUBROUTINE FIT(L)
C     FITS A STRAIGHT LINE BASED ON LEAST SQUARE METHOD
C     A IS THE Y INTERCEPT
C     B IS THE SLOPE
C     L=1 FIT FOR ION SATURATION REGION
C     L=2 FIT FOR ELECTRON CURRENT REGION (SEMI LOG PLOT)
C     L=3 FIT FOR ELECTRON SATURATION REGION (SEMI LOG PLOT)
      DIMENSION SI(70),RES(70),SV(70)
      REAL I,IION,IRI,ILN,IELEC,ILNCAL,MINERR
      COMMON/ALL/N,N1,M1,V(70),I(70),IION(70),ILN(70),ILNCAL(70)
     1,A,B,M,VCHAN(4),VEST(4)
      SUMV2=0.0
      SUMV=0.0
      SUMSI=0.0
      SUMVSI=0.0
      M=0
      GO TO (10,30,50),L
10    DO 15 J=1,N
      IF (V(J).LT.VEST(1).OR.V(J).GE.VEST(2)) GO TO 15
      M=M+1
      SI(M)=I(J)
      SV(M)=V(J)
15    CONTINUE
      GO TO 60
30    DO 35 J=N1,N
      IF (V(J).LT.VEST(1).OR.V(J).GT.VEST(2)) GO TO 35
      M=M+1
      SI(M)=ILN(J)
      SV(M)=V(J)
35    CONTINUE
      GO TO 60
50    DO 55 J=N1,N
      IF (V(J).LT.VEST(3).OR.V(J).GT.VEST(4)) GO TO 55
      M=M+1
      SI(M)=ILN(J)
      SV(M)=V(J)
55    CONTINUE
```

```
C     CALCULATE THE COEFFICIENTC OF THE NORMAL EQUATIONS
60    DO 70 J=1,M
      SUMV=SUMV+SV(J)
      SUMSI=SUMSI+SI(J)
      SUMV2=SUMV2+SV(J)*SV(J)
70    SUMVSI=SUMVSI+SV(J)*SI(J)
      RNOM=M
      DENOM=(RNOM*SUMV2)-(SUMV*SUMV)
C     CALCULATE THE Y INTERCEPT (A)  THE SLOPE (B) OF THE
C     FITTED STRAIGHT LINE
      A=(SUMV2*SUMSI-SUMV*SUMVSI)/DENOM
      B=(RNOM*SUMVSI-SUMV*SUMSI)/DENOM
80    RETURN
      END
C
C
C
      SUBROUTINE PLOT2(K)
C     K=1   WILL PLOT THE LOG OF ELECTRON CURRENT VS VOLTAGE
C     K=2   WILL PLOT THE FITTED LINES ALSO
      DIMENSION X(50)
      REAL I,IION,IRI,ILN,IELEC,ILNCAL
      COMMON/ALL/N,N1,M1,V(70),I(70),IION(70),ILN(70),ILNCAL(70)
     1,A,B,M,VCHAN(4),VEST(4)
      DO 10 J=N1,N
      MM=J-N1+1
10    X(MM)=ILN(J)
      S=3.8
      SY=S
      NOM=N-N1+1
      CALL SCALE  (X,NOM,1,S,XMIN,DX)
      YMIN=XMIN
      DY=DX
      CALL PLINIT
      CALL AXIS (82,51,'LOG I MA',-8,S,90.0,XMIN,DX)
      DO 20 J=N1,N
      MM=J-N1+1
20    X(MM)=V(J)
      S=4.0
      CALL SCALE (X,NOM,1,S,XMIN,DX)
      IXMIN=XMIN
```

```
      XMIN=IXMIN
      IDX=DX
      DX=IDX
      CALL AXIS(82,51,'VOLT',4,S,0.0,XMIN,DX)
      OFFSET=(V(N1)-XMIN)*(82.0/DX)
      IY=51.0*SY+51.0
      IX=82.0+OFFSET
      IXSIZE=S*82.0-OFFSET
      IYSIZE=SY*51.0
      CALL BLOCK(IX,IY,IXSIZE,IYSIZE,-1)
      IXSIZE=IXSIZE/50
      CALL ZPEN
      DO 21 NUM=1,50
      CALL DEFPEN(IX,IY,IXSIZE,IYSIZE,NUM)
21    IX=IX+IXSIZE
      CALL PLOT(82,51,5)
      IF (K.EQ.1) GO TO 23
      DO 22 J=N1,N
      IX=(V(J)-XMIN)*(82.0/DX)+82.0
      IY=(ILNCAL(J)-YMIN)*(51.0/DY)+51.0
22    CALL PLOT(IX,IY,4)
23    CALL PLOT (82,51,5)
      DO 25 J=N1,N
      IX=(V(J)-XMIN)*(82.0/DX)+82.0
      IY=(ILN(J)-YMIN)*(51.0/DY)+51.0
25    CALL PLOT (IX,IY,4)
      IF(K.EQ.2) GO TO 50
      DEL=(V(N)-V(N1))/50.0
      DO 40 JJ=1,4
      WRITE (5,35) JJ
35    FORMAT (4I3)
37    CALL SLEEP(240)
      CALL LPEN(NUM)
      IF (NUM.EQ.0) GO TO 37
      RNUM=NUM
40    VCHAN(JJ)=(RNUM-1.0)*DEL
      GO TO 60
50    CALL SLEEP(600)
60    CALL PLTCLS
      RETURN
      END
```

```
C
C
C
      SUBROUTINE PLT1
C     POSITIVE PLOT OF I VS V PROBE CHARACTERISTIC  IION
      DIMENSION X(100),Y(100)
      REAL I,IION,IRI,ILN,IELEC,ILNCAL
      COMMON/ALL/N,N1,M1,V(70),I(70),IION(70),ILN(70),ILNCAL(70)
     1,A,B,M,VCHAN(4),VEST(4)
      DO 10 J=1,N
      Y(J)=IION(J)
10    X(J)=I(J)
      S=4.0
      CALL SCALE (X,N,1,S,XMIN,DX)
      YMIN=XMIN
      DY=DX
      CALL PLINIT
      CALL AXIS (82,25,'I MA',-4,S,90.0,XMIN,DX)
      DO 20 J=1,N
20    X(J)=V(J)
      S=4.0
      CALL SCALE (X,N,1,S,XMIN,DX)
      IXMIN=XMIN
      XMIN=IXMIN
      IDX=DX
      DX=IDX
      CALL AXIS (82,25,'VOLT',4,S,0.0,XMIN,DX)
      CALL PLOT(82,25,-3)
      CALL LINE(X,Y,N,1,XMIN,DX,YMIN,DY)
      DO 25 J=1,N
25    Y(J)=0.0
      CALL LINE(X,Y,N,1,XMIN,DX,YMIN,DY)
      IY=(0.0-YMIN)*(51.0/DY)
      CALL WRTSYM(-30,IY,'0.0',3,0.0)
      DO 30 J=1,N
      IX=(V(J)-XMIN)*(82.0/DX)
      IY=(I(J)-YMIN)*(51.0/DY)
30    CALL WRTSYM(IX,IY,'X',1,0.0)
      CALL SCRDMP
      CALL PLTCLS
      RETURN
```

```
      END
C
C
C
      SUBROUTINE PLT2
C     POSITIVE SEMILOG PLOT OF ELECTRON CURRENT (ILN ILNCAL)C
RON CURRENT (ILN ILNCAL)
      DIMENSION X(100),Y(100)
      REAL I,IION,IRI,ILN,IELEC,ILNCAL
      COMMON/ALL/N,N1,M1,V(70),I(70),IION(70),ILN(70),ILNCAL(70)
     1,A,B,M,VCHAN(4),VEST(4)
      DO 10 J=N1,N
      MM=J-N1+1
      Y(MM)=ILNCAL(J)
10    X(MM)=ILN(J)
      S=4.0
      NOM=N-N1+1
      CALL SCALE(X,NOM,1,S,XMIN,DX)
      YMIN=XMIN
      DY=DX
      CALL PLINIT
      CALL AXIS (82,25,'LOG I MA',-8,S,90.0,XMIN,DX)
      DO 20 J=N1,N
      MM=J-N1+1
20    X(MM)=V(J)
      S=4.0
      CALL SCALE(X,NOM,1,S,XMIN,DX)
      IXMIN=XMIN
      XMIN=IXMIN
      IDX=DX
      DX=IDX
      CALL AXIS(82,25,'VOLT',4,S,0.0,XMIN,DX)
      CALL PLOT(82,25,-3)
      CALL LINE(X,Y,NOM,1,XMIN,DX,YMIN,DY)
      DO 30 J=N1,N
      IX=(V(J)-XMIN)*(82.0/DX)
      IY=(ILN(J)-YMIN)*(51.0/DY)
30    CALL WRTSYM(IX,IY,'X',1,0.0)
      CALL WRTSYM(164,102,'SEMILOG PLOT',13,0.0)
      CALL SCRDMP
      CALL PLTCLS
      RETURN
      END
```

II. RETARDING POTENTIAL ENERGY ANALYZER PROGRAM

The main program RPA (Figure A.7) reads the input data, supplies this information to the subprograms, stores the results until all the calculations are completed for a given set of data, then prints the results.

Subroutine MODE (Figure A.8) finds the best-fitting values for the constants I_0, I_1, V_0, V_1, θ_0 for the steady-state and for the pulsed-state expressions. Subroutines STEADY (Figure A.9), PULSED (Figure A.10), STDY1 (Figure A.11), and PLSD1 (Figure A.12) supply the derivatives and residuals for those expressions for the least squares evaluation of the corrections needed (step (1) and (2) following). Subroutine CALC (Figure A.13) provides the necessary calculations for step (3). The iteration cycle consists of three steps:

(1) Maximum of 3 iterations to find V_0 and V_1, keeping I_0, I_1, and θ_0 fixed until the corrections are small enough (correction/quantity) < .001.

(2) Maximum of 3 iterations to find I_1 and θ_0, keeping I_0, V_0, and V_1 fixed until (a) corrections are small enough; (b) $|\Delta\theta_0| > 0.5$; (c) $(\theta_0 + \Delta\theta_0)$ falls outside the range $(0, \pi/2)$.

(3) Maximum of 50 iterations to find I_0, keeping I_1, V_0, V_1, and θ_0 fixed.

The values of the constants are updated after each iteration. On completion of step (3), the values are compared with those supplied at the beginning of the cycle. If they do not agree within

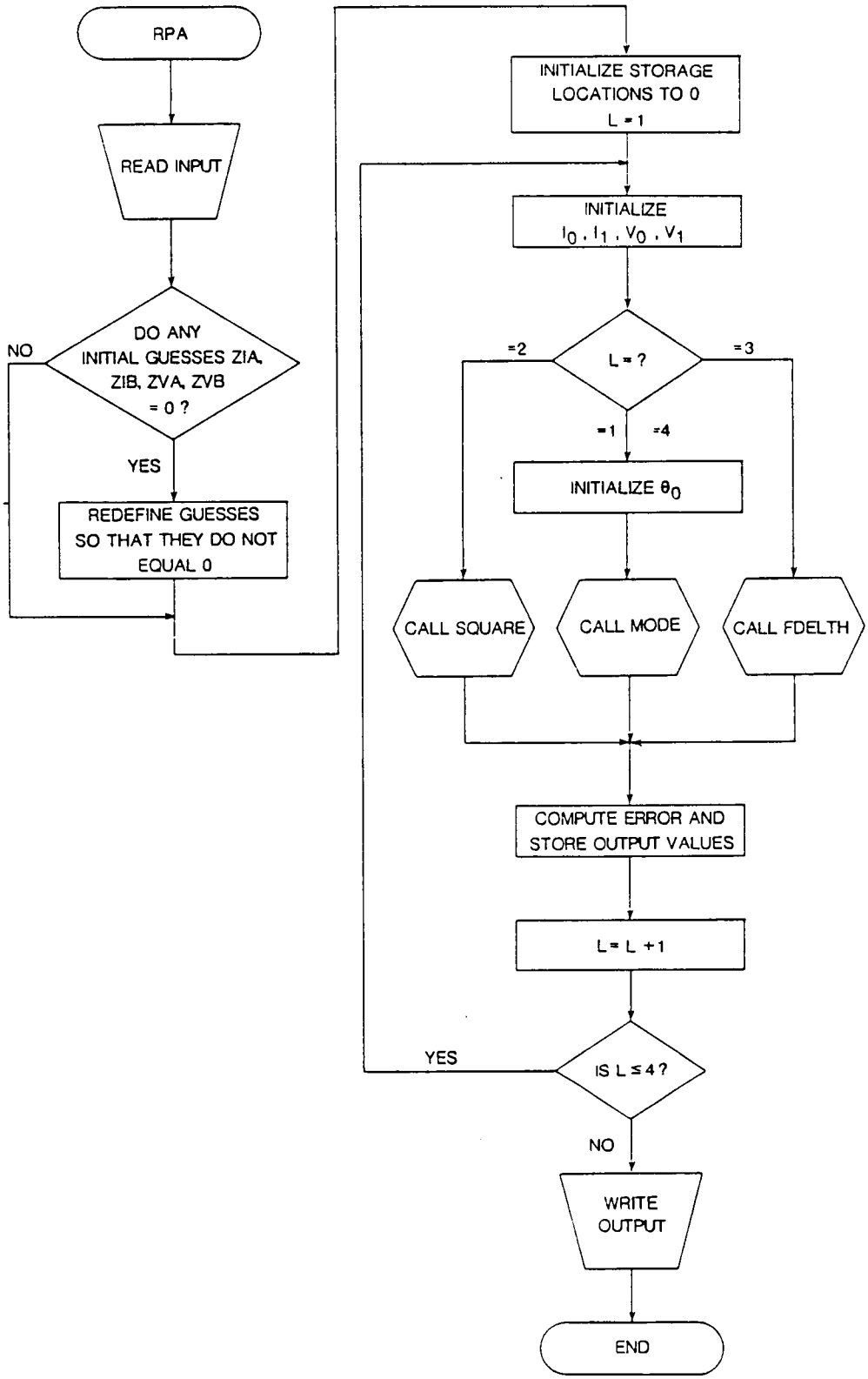

Figure A.7. Flowchart for main program RPA.

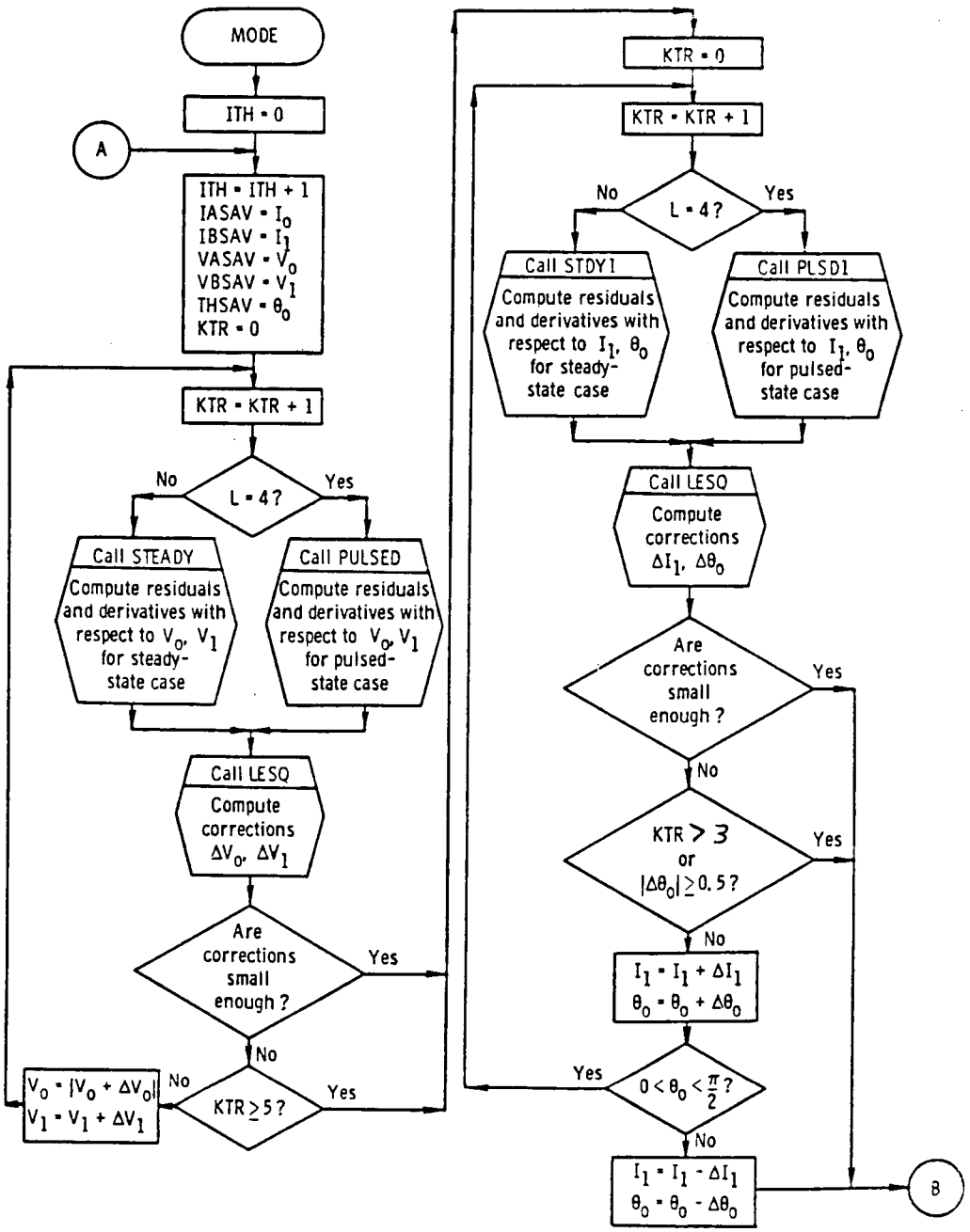

Figure A.8. Flowchart for subroutine MODE. Taken from [3].

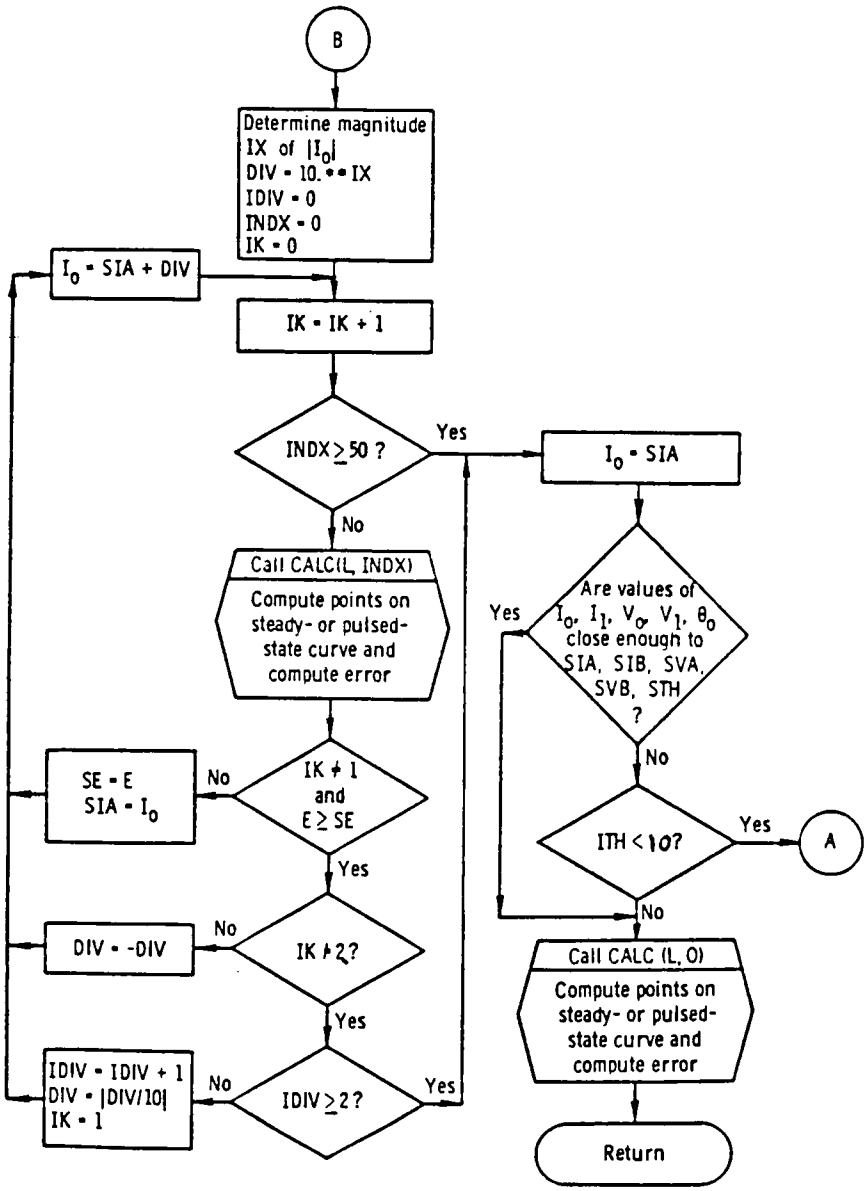

Figure A.8 (continued)

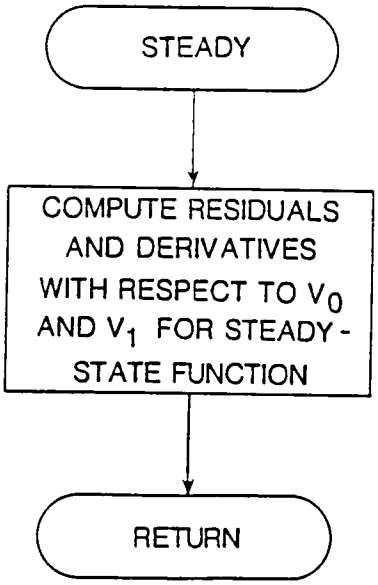

Figure A.9. Flowchart for subroutine STEADY.

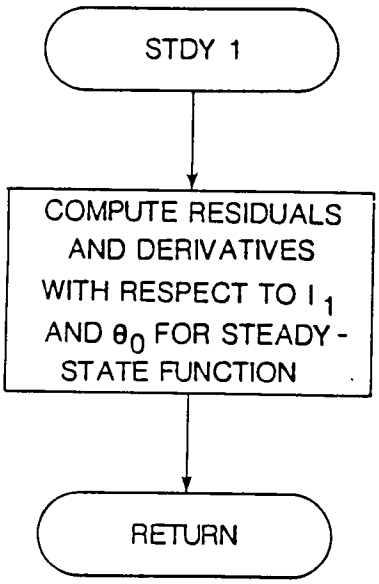

Figure A.10. Flowchart for subroutine STDY1.

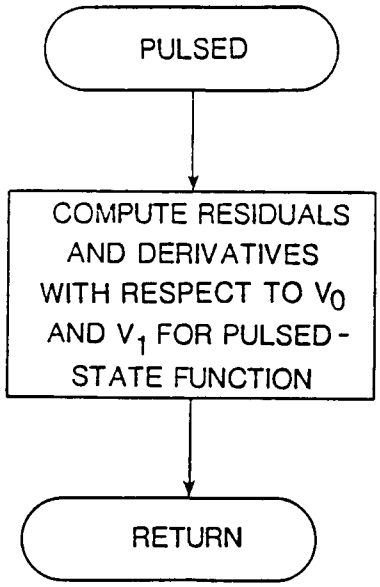

Figure A.11. Flowchart for subroutine PULSED.

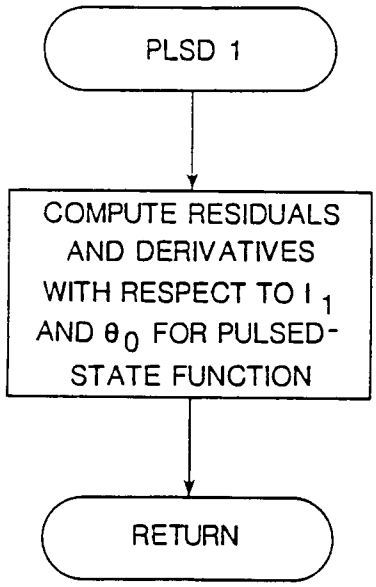

Figure A.12. Flowchart for subroutine PLSD1.

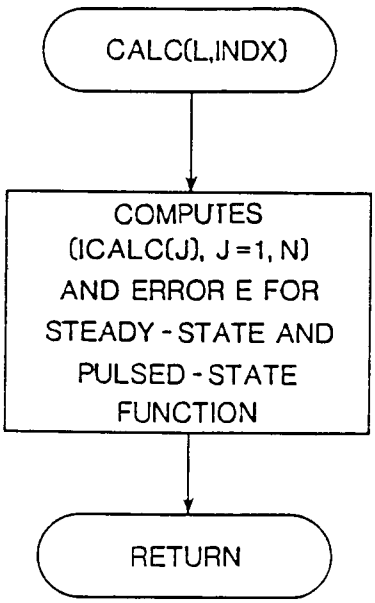

Figure A.13. Flowchart for subroutine CALC.

the limits specified, the preceding three steps are repeated for a maximum of 10 cycles or until the values agree to three significant digits. The values at the end of a cycle become starting values at the beginning of the next cycle.

Function ERF (Figure A.14) finds the value of the error function required for the pulsed mode curve fit.

Subroutine SQUARE (Figure A.15) finds the best fitting values for the constants I_0, I_1, V_0, and V_1 for the square energy distribution function. The method consists of two parts:

(1) Maximum of five iterations to find V_0 and V_1, keeping I_0 and I_1 fixed until the corrections are small enough.

(2) Maximum of five iterations to find I_0 and I_1, keeping V_0 and V_1 fixed, until the corrections are small enough.

The values of the constants are now compared with those from the beginning of the cycle. If they do not agree within the limits specified, the preceding two steps are repeated for a maximum of 50 cycles or until the values agree to three significant digits. The values at the end of one cycle become starting values at the beginning of the next cycle.

Subroutine FDELTA (Figure A.16) finds the best-fitting values for the constants I_0, I_1, and V_0 for the delta function energy distribution. Values of I_0 and I_1 are found as V_0 takes successive values of V from the input curve. When the mean square error between the computed curve and the input curve reaches a minimum, the iteration stops.

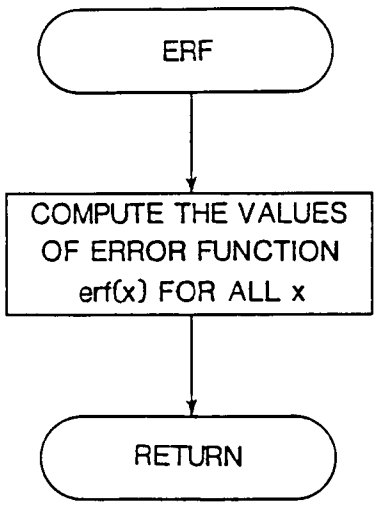

Figure A.14. Flowchart for function ERF.

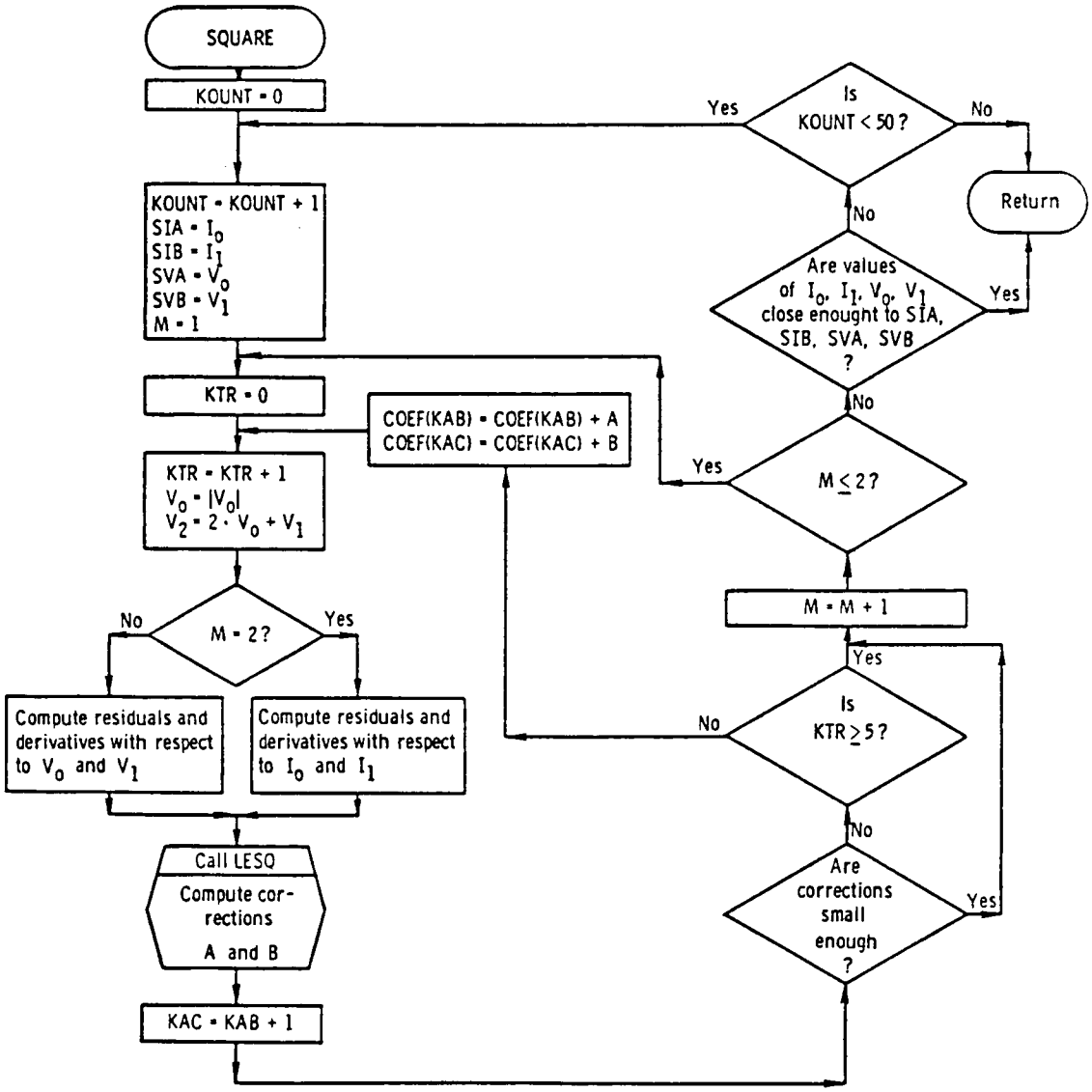

Figure A.15. Flowchart for subroutine SQUARE. Taken from [3].

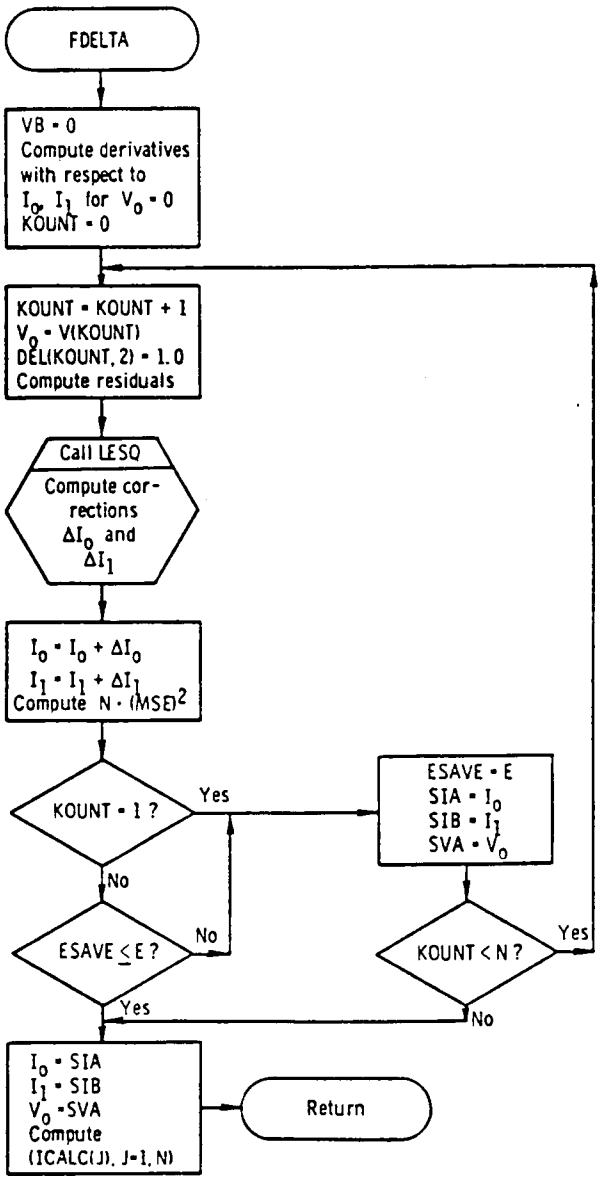

Figure A.16. Flowchart for subroutine FDELTA. Taken from [5].

```
      PROGRAM RPA
      DIMENSION ARRAY(1000)
      DIMENSION COEF(5),Z(10,5),ZX(5),CO(5)
      REAL ISCALE
      REAL I,ICALC,IA,IB,IASAV,IBSAV
      COMMON/ALL2/ZI(100,5)
      COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
      COMMON /ALL1/IA,IB,VA,VB,TH,A,B,E,M,KOUNT,KTR
      EQUIVALENCE (COEF(1),IA),(ZX(1),ZIA),(ZX(2),ZIB)
     1 ,(ZX(3),ZVA),(ZX(4),ZVB),(ZX(5),ZTH)
       EQUIVALENCE (CO(1),IASAV),(CO(2),IBSAV),(CO(3),VASAV),
     1 (CO(4),VBSAV),(CO(5),THSAV)
      DATA ZTH,ZZTH/40.,0.69813/
C
C     RUN IS THE RUN NUMBER
C     (V,I) ARE THE (X,Y)COORDINATESVOLTAGE VS CURRENTC

C     NONE OF THE INITIAL GUESSES ZIA,ZIB,ZVA,ZVB SHOULD BE ZERO
C     N= NO. OF POINTS (V,I)
C     V(J) VOLTAGE INPUT  I(J) CURRENT INPUTC
C     ZI(J,L) CALCULATED CURRENT USING FINAL VALUES OF IA,IB,
C     VA,VB,TH.
C     INITIAL VALUES OF IA,IB,VA,VB,TH (INITIAL GUESSES) ARE
C     STORED IN ZX(5)
C     FINAL VALUES OF IA,IB,VA,VB,TH ARE STORED IN Z(K,L) FOR
C     DIFFERENT MODES :
C     L=1,4          L=1STEADYSTATE MODEC      L=1,4
C                    L=2SQUAREWAVE FUNCTIONC
C                    L=3DELTA FUNCTION
C                    L=4PULSED MODE
C
C     K=1,10         K=1IAI0
C                    K=2IBI1
C                    K=3VAV0
C                    K=4VBV1
C                    K=5THTHETA
C                    K=6NO. OF ITERATIONS
C                    K=7MEAN SQUARE ERROR
C
      WRITE(5,200)
200   FORMAT(15H LAST CHANNEL ?)
      READ(5,210) CH
```

```
210     FORMAT(F5.1)

        WRITE(5,220)
220     FORMAT(16H OFFSET CURRENT?)
        READ(5,230) ZIA
230     FORMAT(F7.2)

        WRITE(5,240)
240     FORMAT(14H MAX. CURRENT?)
        READ(5,250) ZIB
250     FORMAT(F7.2)

        WRITE(5,260)
260     FORMAT(13H PLASMA POT.?)
        READ(5,270) ZVB
270     FORMAT(F7.2)

        WRITE(5,280)
280     FORMAT(15H KINETIC TEMP.?)
        READ(5,290) ZVA
290     FORMAT(F7.2)

        N=CH/30.0
        LCH=CH-20.0
        CALL HMFRD(ARRAY,50,LCH)
        II=1
        K=30
1       SUMAVG=0.0
        DO 3 J=II,K
3       SUMAVG=SUMAVG+ARRAY(J)
        AVG=SUMAVG/30.0
        KK=K/30
        V(KK)=(AVG-39.0)*2.0*20.0
        II=II+30
        K=K+30
        IF(K.LE.LCH) GO TO 1
        CALL HMFRD(ARRAY,1074,LCH)
        S=0.0
        DO 4 J=1,30
4       S=S+ARRAY(J)
        AVG=S/30.0
```

```
      ISCALE=ZIB/(AVG-83.0)
      II=1
      K=30
5     SUMAVG=0.0
      DO 7 J=II,K
7     SUMAVG=SUMAVG+ARRAY(J)
      AVG=SUMAVG/30.0
      KK=K/30
      I(KK)=(AVG-83.0)*ISCALE
      II=II+30
      K=K+30
      IF(K.LE.LCH) GO TO 5
      IF (ZIA.EQ.0.0) ZIA=I(N)*0.001
      IF (ZIA.EQ.0.0) ZIA=0.001
      IF (ZIB.EQ.0.0) ZIB=I(1)
      IF (ZVA.EQ.0.0) ZVA=V(2)
      IF (ZVB.EQ.0.0) ZVB=V(2)
      FLN=N
      DO 20 J=1,5
      DO 20 K=1,10
20    Z(K,J)=0.0
      DO 80 L=1,4
      IA=ZIA
      IB=ZIB
      VA=ZVA
      VB=ZVB
      GO TO (30,40,50,30) ,L
30    TH=ZZTH
      CALL MODE(L)
      Z(5,L)=57.2957*TH
      GO TO 60
40    CALL SQUARE
      GO TO 60
50    CALL FDELTA
60    S=0.0
      DO 70 J=1,N
      ZI(J,L)=ICALC(J)
70    S=S+(I(J)-ICALC(J))**2
      Z(1,L)=IA
      Z(2,L)=IB
      Z(3,L)=VA
```

```
      Z(4,L)=VB
      Z(6,L)=KOUNT
80    Z(7,L)=100./I(1)*SQRT(S/FLN)
      WRITE (2,120)
      WRITE (2,130) (J,V(J),I(J),(ZI(J,L),L=1,4),J=1,N)
      WRITE (2,135)
      WRITE (2,140) (ZX(J),(Z(J,L),L=1,4),J=1,5)
      WRITE (2,150) (Z(7,L),L=1,4)
      CALL PLT
C
C
120   FORMAT (10X,1HV,10X,1HI,8X,6HSTEADY,
     1 5X,7HSQWAVE,5X,5HDELTA,6X,6HPULSED/)
130   FORMAT (I4,6F11.3)
135   FORMAT (//17X,7HINITIAL,5X,6HSTEADY,5X,7HSQWAVE,

     1 6X,5HDELTA,6X,7HPULSED /)
140   FORMAT (6X,8H I0=    ,5F11.3/6X,8H I1=    ,5F11.3/
     1 6X,8H V0=    ,5F11.3/6X,8H V1=    ,5F11.3/6X,
     1 8HTHETA=  ,5F11.3)
150   FORMAT (6X,8HERROR=  ,11X,4F11.5)
180   END
C
C
C
      SUBROUTINE MODE (L)
      DIMENSION CO(10),COEF(5)
      REAL I,ICALC,IA,IB,IASAV,IBSAV
      COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
      COMMON /ALL1/IA,IB,VA,VB,TH,A,B,E,M,ITH,KTR
      EQUIVALENCE (COEF(1),IA)
      EQUIVALENCE (CO(1),IASAV),(CO(2),IBSAV),(CO(3),VASAV),
     1 (CO(4),VBSAV),(CO(5),THSAV)
      DATA CONV/.001/
      ITH=0
10    ITH=ITH+1
      IASAV=IA
      IBSAV=IB
      VASAV=VA
      VBSAV=VB
      THSAV=TH
      KTR=0
```

```
20      KTR=KTR+1
        IF (L.EQ.4) GO TO 30
        CALL STEADY
        GO TO 40
30      CALL PULSED
40      CALL LESQ
        IF ((ABS(A/VA).LE.CONV).AND.(ABS(B/VB).LE.CONV)) GO TO 50
        IF (KTR.GE.3) GO TO 50
        VA=ABS(VA+A)
        VB=VB+BB
        GO TO 20
50      KTR=0
60      KTR=KTR+1
        IF (L.EQ.4) GO TO 70
        CALL STDY1
        GO TO 80
70      CALL PLSD1
80      CALL LESQ
        IF ((ABS(A/IB).LE.CONV).AND.(ABS(B/TH).LE.CONV))GO TO 90
        IF (KTR.GE.3.OR.ABS(B).GT.0.5) GO TO 90
        IB=IB+A
        TH=TH+B
        IF (TH.GT.0.0.AND.TH.LE.1.57079) GO TO 60
        IB=IB-A
        TH=TH-B
90      X=ABS(IA)
        IX=0
100     IF(X.LT.10.0) GO TO 110
        IX=IX+1
        X=X/10.0
        GO TO 100
110     IF (X.GE.1.0) GO TO 120
        IX=IX-1
        X=X*10.0
        GO TO 110
120     DIV=10.0**IX
        IDIV=0
        INDX=0
        IK=0
130     IK=IK+1
        IF (INDX.GE.50) GO TO 170
```

```
      CALL CALC (L,INDX)
      IF (IK.NE.1.AND.E.GE.SE) GO TO 140
      SE=E
      SIA=IA
      GO TO 160
140   IF (IK.NE.2) GO TO 150
      DIV=-DIV
      GO TO 160
150   IF (IDIV.GE.2) GO TO 170
      IDIV=IDIV+1
      DIV=ABS(DIV/10.0)
      IK=1
160   IA=SIA+DIV
      GO TO 130
170   IA=SIA
      IN=0
      DO 180 J=1,5
180   IF (ABS((COEF(J)-CO(J))/CO(J)).LE.CONV) IN=IN+1
      IF (IN.NE.5.AND.ITH.LT.12) GO TO 10
      CALL CALC (L,0)
      RETURN
      END
C
C
C
      SUBROUTINE STEADY
      REAL I,ICALC,IA,IB
      COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
      COMMON /ALL1/IA,IB,VA,VB,ANG,A,B,E,M,KOUNT,KTR
C
C     DERIVATIVES WITH RESPECT TO VA AND VB
C
10    IF (KTR.GT.1) GO TO 20
      SA=SIN(ANG)**2
      CA=COS(ANG)**2
      TA=SA/CA
      ZA=IB/SA
20    ZB=ZA/VA
      DO 40 J=1,N
      IF(V(J).LE.VB) GO TO 30
      X=(V(J)-VB)/VA
```

```
      EX=EXP(-X)
      EXT=EXP(-X*TA)
      TXY=ZB*EX*(1.0-EXT)
      ICALC(J)=IA+ZA*EX*(1.0-CA*EXT)
      DEL(J,1)=TXY*X
      DEL(J,2)=TXY
      GO TO 40
30    ICALC(J)=IA+IB
      DEL(J,1)=0.0
      DEL(J,2)=0.0
40    DEL(J,3)=I(J)-ICALC(J)
      RETURN
      END
C
C
C
      SUBROUTINE STDY1
      DIMENSION Y(100),TX(100)
      REAL I,ICALC,IA,IB
      COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
      COMMON /ALL1/IA,IB,VA,VB,ANG,A,B,E,M,KOUNT,KTR
C
C     DERIVATIVES WITH RESPECT TO IB AND TH
C
      IF (KTR.GT.1) GO TO 70
      DO 60 J=1,N
      IF (V(J).LE.VB) GO TO 50
      Y(J)=(V(J)-VB)/VA
      TX(J)=EXP(-Y(J))
      GO TO 60
50    DEL(J,1)=1.0
      DEL(J,2)=0.0
60    CONTINUE
70    CA=COS(ANG)
      CB=CA**2
      SA=SIN(ANG)
      SB=SA*SA
      SC=SA*SB
      CA=0.5*CA*SC
      SA=SB/CB
      DO 90 J=1,N
```

```
      IF (V(J).LE.VB) GO TO 80
      TXY=EXP(-Y(J)*SA)
      DEL(J,1)=TX(J)*(1.0-CB*TXY)/SB
      DEL(J,2)=(TX(J)*((CB+Y(J)*SB)*TXY-CB)/CA)*IB
80    ICALC(J)=IA+IB*DEL(J,1)
90    DEL(J,3)=I(J)-ICALC(J)
      RETURN
      END
C
C
C
      FUNCTION ERF(Y)
C     FINDS THE VALUE OF THE ERROR FUNCTION
      IF (Y.GE.2.0) GO TO 40
      AI=0.0
      BI=0.0
      DO 20 I=0,5
      Z=1.0
      F=1.0
      IF (I.EQ.0) GO TO 17
      K=2*I
      DO 5 J=1,K
5     F=F*J
      K=K+1
      DO 10 L=1,K
10    Z=Z*L
17    AI=AI+Y**(4*I+1)/((4*I+1)*F)
      BI=BI+Y**(4*I+3)/((4*I+3)*Z)
20    CONTINUE
      ERF=(2/SQRT(3.1415927))*(AI-BI)
      GO TO 50
40    ERF=0.999
50    RETURN
      END
C
C
C
      SUBROUTINE PULSED
      REAL I,ICALC,IA,IB
      DIMENSION Y(100),TX(100)
      COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
```

```
      COMMON /ALL1/IA,IB,VA,VB,ANG,A,B,E,M,KOUNT,KTR
C
C     DERIVATIVES WITH RESPECT TO VA AND VB
C
10    IF (KTR.GT.1) GO TO 20
      CA=COS(ANG)
      CB=1.0-CA
      CA2=CA**2
      ZB=IB/CB/1.772454
20    ZA=ZB/VA
      DO 40 J=1,N
      IF (V(J).LE.VB) GO TO 30
      X2=(V(J)-VB)/VA
      X=SQRT(X2)
      TXY=ZA*(EXP(-X2)-EXP(-X2/CA2))
      ICALC(J)=IA+IB*(1.0-(ERF(X)-CA*ERF(X/CA))/CB)
      DEL(J,1)=X*TXY
      DEL(J,2)=TXY/X
      GO TO 40
30    ICALC(J)=IA+IB
      DEL(J,1)=0.0
      DEL(J,2)=0.0
40    DEL(J,3)=I(J)-ICALC(J)
      RETURN
      END
C
C
C
      SUBROUTINE PLSD1
      REAL I,ICALC,IA,IB
      DIMENSION Y(100),TX(100)
      COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
      COMMON /ALL1/IA,IB,VA,VB,ANG,A,B,E,M,KOUNT,KTR
C
C     DERIVATIVES WITH RESPECT TO IB AND TH
C
      IF (KTR.GT.1) GO  TO 70
      DO 60 J=1,N
      IF (V(J).LE.VB) GO TO 50
      Y(J)=SQRT((V(J)-VB)/VA)
      TX(J)=ERF(Y(J))
```

```
      GO TO 60
50    DEL(J,1)=1.0
      DEL(J,2)=0.0
60    CONTINUE
70    SA=SIN(ANG)
      CA=COS(ANG)
      CB=1.0-CA
      CC=IB*SA/CB**2
      CD=1.1283792*CB/CA
      DO 90 J=1,N
      IF (V(J).LE.VB) GO TO 80
      X=Y(J)
      XX=ERF(X/CA)
      DEL(J,1)=1.0-(TX(J)-CA*XX)/CB
      DEL(J,2)=CC*(CD*EXP(-(X/CA)**2)*X-XX+TX(J))
80    ICALC(J)=IA+IB*DEL(J,1)
90    DEL(J,3)=I(J)-ICALC(J)
      RETURN
      END
C
C
C
      SUBROUTINE CALC(L,INDX)
      REAL I,ICALC,IA,IB,IASAV,IBSAV
      DIMENSION TX(100)
      COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
      COMMON /ALL1/IA,IB,VA,VB,ANG,A,B,E,M,KOUNT,KTR
      E=0.0
      INDX=INDX+1
      IF (INDX.GT.1) GO TO 50
      MM=1+L/3
      GO TO (10,30),MM
10    SA=SIN(ANG)**2
      CA=COS(ANG)**2
      TA=SA/CA
      DO 20 J=1,N
      IF (V(J).LE.VB) GO TO 20
      X=(V(J)-VB)/VA
      TX(J)=EXP(-X)/SA*(1.0-CA*EXP(-X*TA))
20    CONTINUE
      GO TO 50
```

```
30      CA=COS(ANG)
        CB=1.0-CA
        DO 40 J=1,N
        IF (V(J).LE.VB) GO TO 40
        X=SQRT((V(J)-VB)/VA)
        TX(J)=1.0-(ERF(X)-CA*ERF(X/CA))/CB
40      CONTINUE
50      DO 70 J=1,N
        IF (V(J).LE.VB) GO TO 60
        ICALC(J)=IA+IB*TX(J)
        GO TO 70
60      ICALC(J)=IA+IB
70      E=E+(I(J)-ICALC(J))**2
        RETURN
        END
C
C
C
        SUBROUTINE LESQ
        REAL I,ICALC,IA,IB
        DIMENSION SUM(5,6)
        COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
        COMMON /ALL1/IA,IB,VA,VB,ANG,A,B,E,M,KOUNT,KTR
        DO 20 K=1,2
        DO 20 J=K,3
        SUM(K,J)=0.0
        DO 10 L=1,N
10      SUM(K,J)=SUM(K,J)+DEL(L,K)*DEL(L,J)
20      SUM(J,K)=SUM(K,J)
        A=0.0
        B=0.0
        DENOM=SUM(1,1)*SUM(2,2)-SUM(1,2)**2
        IF (DENOM.EQ.0.0) GO TO 30
        A=(SUM(1,3)*SUM(2,2)-SUM(1,2)*SUM(2,3))/DENOM
        B=(SUM(1,1)*SUM(2,3)-SUM(1,3)*SUM(2,1))/DENOM
30      RETURN
        END
C
C
C
        SUBROUTINE SQUARE
```

```
      REAL I,ICALC,IA,IB
      DIMENSION COEF(5)
      COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
      COMMON /ALL1/IA,IB,VA,VB,ANG,A,B,E,M,KOUNT,KTR
      EQUIVALENCE (IA,COEF(1))
      DATA CONV/0.001/
      KOUNT=0
10    KOUNT=KOUNT+1
      SIA=IA
      SIB=IB
      SVA=VA
      SVB=VB
      DO 120 M=1,2
      KTR=0
20    KTR=KTR+1
      VA=ABS(VA)
      VC=2.0*VA+VB
      IF (M.EQ.2) GO TO 70
C
C     DERIVATIVES WITH RESPECT TO VA AND VB
C
      ALPHA=IB/VA/2.0
      KAB=3
      DO 60 J=1,N
      IF (V(J).LE.VB) GO TO 30
      IF (V(J).GT.VC) GO TO 40
      DEL(J,2)=ALPHA
      DEL(J,1)=DEL(J,2)*(V(J)-VB)/VA
      ICALC(J)=IA+DEL(J,2)*(VC-V(J))
      GO TO 60
30    ICALC(J)=IA+IB
      GO TO 50
40    ICALC(J)=IA
50    DEL(J,1)=0.0
      DEL(J,2)=0.0
60    DEL(J,3)=I(J)-ICALC(J)
      GO TO 110
C
C     DERIVATIVES WITH RESPECT TO IA AND IB
C
70    KAB=1
```

```
      DO 100 J=1,N
      DEL(J,1)=1.0
      IF (V(J).LE.VB) GO TO 80
      IF (V(J).GT.VC) GO TO 90
      DEL(J,2)=(VC-V(J))/VA/2.0
      ICALC(J)=IA+IB*DEL(J,2)
      GO TO 100
80    ICALC(J)=IA+IB
      DEL(J,2)=1.0
      GO TO 100
90    ICALC(J)=IA
      DEL(J,2)=0.0
100   DEL(J,3)=I(J)-ICALC(J)
110   CALL LESQ
      KAC=KAB+1
      IF ((ABS(A/COEF(KAB)).LE.CONV).AND.(ABS(B/COEF(KAC)).LE.
     1 CONV)) GO TO 120
      IF (KTR.GE.5) GO TO 120
      COEF(KAB)=COEF(KAB)+A
      COEF(KAC)=COEF(KAC)+B
      GO TO 20
120   CONTINUE
      IF ((ABS((SIA-IA)/SIA).LE.CONV).AND.(ABS((SIB-IB)/SIB)
     1 .LE.CONV).AND.(ABS((SVA-VA)/SVA).LE.CONV).AND.(ABS((SVB
     1 -VB)/SVB).LE.CONV)) GO TO 130
      IF (KOUNT.LT.50) GO TO 10
130   RETURN
      END
C
C
C
      SUBROUTINE FDELTA
      REAL I,ICALC,IA,IB
      COMMON /ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
      COMMON /ALL1/IA,IB,VA,VB,ANG,A,B,E,M,KOUNT,KTR
      VB=0.0
      DO 10 J=1,N
      DEL(J,2)=0.0
10    DEL(J,1)=1.0
      KOUNT=0
20    KOUNT=KOUNT+1
```

```
      VA=V(KOUNT)
      DEL(KOUNT,2)=1.0
      E=0.0
      DO 30 J=1,N
      ICALC(J)=IA+IB*DEL(J,2)
30    DEL(J,3)=I(J)-ICALC(J)
      CALL LESQ
      IA=IA+A
      IB=IB+B
      DO 40 J=1,N
      ICALC(J)=IA+IB*DEL(JJ,2)
40    E=E+(I(J)-ICALC(J))**2
      IF (KOUNT.EQ.1) GO TO 50
      IF (ESAVE.LE.E) GO TO 60
50    ESAVE=E
      SIA=IA
      SIB=IB
      SVA=VA
      IF (KOUNT.LT.N) GO TO 20
60    IA=SIA
      IB=SIB
      VA=SVA
      DO 70 J=1,N
      ICALC(J)=IA
70    IF (V(J).LT.VA) ICALC(J)=IA+IB
      RETURN
      END
C
C
C
      SUBROUTINE PLT
      DIMENSION X(100),Y(100)
      REAL I,ICALC,IA,IB,IASAV,IBSAV
      COMMON/ALL/N,V(100),I(100),ICALC(100),DEL(100,5)
      COMMON/ALL2/ZI(100,5)
      DO 80 L=1,4
      DO 10 J=1,N
      Y(J)=ZI(J,L)
10    X(J)=I(J)
      S=4.0
      CALL SCALE (X,N,1,S,XMIN,DX)
```

```
      YMIN=XMIN
      DY=DX
      CALL PLINIT
      CALL AXIS (82,25,'I A E-8',-7,S,90.0,XMIN,DX)
      DO 20 J=1,N
20    X(J)=V(J)
      S=4.0
      CALL SCALE (X,N,1,S,XMIN,DX)
      IXMIN=XMIN
      XMIN=IXMIN
      IDX=DX
      DX=IDX
      CALL AXIS (82,25,'VOLT',4,S,0.0,XMIN,DX)
      CALL PLOT(82,25,-3)
      CALL LINE (X,Y,N,1,XMIN,DX,YMIN,DY)
      DO 30 J=1,N
      IX=(V(J)-XMIN)*(82.0/DX)
      IY=(I(J)-YMIN)*(51.0/DY)
30    CALL WRTSYM(IX,IY,'X',1,0.0)
      CALL SCRDMP
      GO TO (40,50,60,70),L
40    WRITE (2,45)
45    FORMAT (40X,7H STEADY)
      GO TO 80
50    WRITE (2,55)
55    FORMAT (40X,12H SQUAREWAVE)55     FORMAT (40X,12H SQUAREWAVE)
      GO TO 80
60    WRITE (2,65)
65    FORMAT (40X,6H DELTA)
      GO TO 80
70    WRITE (2,75)
75    FORMAT (40X,7H PULSED)
80    CONTINUE
      CALL PLTCLS
      RETURN
      END
```

Subroutine LESQ (Figure A.17) finds the normal equations, which are two equations in two unknowns. It then uses these equations to solve for the unknowns, the unknowns being the corrections.

Subroutines STEADY, PULSED, STDY1, PLSD1, and CALC supply the necessary computations (derivatives, residuals, etc.) for subroutine MODE.

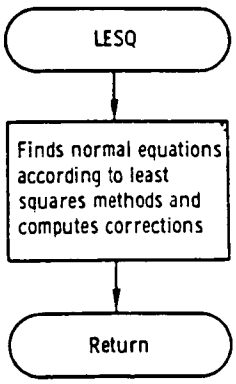

Figure A.17. Flowchart for subroutine LESQ. Taken from [5].

```
      PROGRAM CX
      DIMENSION E(50)
      REAL I
      COMMON/ALL/N,A,B,V(50),ELN(50),VSIG(30),SIG(30),ECAL(50)
      COMMON/ALL1/I(50)
      CALL OPEN (6,'CXDAT1  DAT',0)
      READ (6,60) (VSIG(J),J=1,18)
      READ (6,60) (SIG(J),J=1,18)
      READ (6,80) N
      READ (6,70) (V(J),J=1,N)
      READ (6,70) (I(J),J=1,N)
      DO 10 J=1,N
      ARG=V(J)
      CALL CRSSEC(ARG,SIGCAL)
      E(J)=I(J)/(V(J)**2.56*SIGCAL)
10    ELN(J)=ALOG(E(J))
      CALL LESQ
      TI=-(1/B)
      DO 20 J=1,N
20    ECAL(J)=A+B*V(J)
      WRITE (2,100)
      WRITE (2,90) (V(J),I(J),J=1,N)
      CALL PLT1
      CALL PLT
      WRITE (2,40) TI
40    FORMAT (4H TI=,F10.7,4H KEV)
60    FORMAT (6F8.3)
70    FORMAT (5F10.7)
80    FORMAT (I2)
90    FORMAT (4X,F10.5,10X,F10.5)
100   FORMAT (10X,3H V ,15X,3H I )
      END
C
C
C
      SUBROUTINE CRSSEC(ARG,SIGCAL)
      COMMON/ALL/N,A,B,V(50),ELN(50),VSIG(30),SIG(30),ECAL(50)
      DO 10 L=1,18
      IF (ARG.GT.VSIG(L)) GO TO 10
      IF (ARG.EQ.VSIG(L)) GO TO 20
      IF (ARG.EQ.VSIG(L-1)) GO TO 30
```

```
      GO TO 40
10    CONTINUE
20    SIGCAL=SIG(L)
      GO TO 50
30    SIGCAL=SIG(L-1)
      GO TO 50
40    CHAN=ABS((SIG(L)-SIG(L-1))/(VSIG(L)-VSIG(L-1)))
      SIGCAL=(ARG-VSIG(L-1))*CHAN+SIG(L-1)
50    RETURN
      END
C
C
C
      SUBROUTINE LESQ
      COMMON/ALL/N,A,B,V(50),ELN(50),VSIG(30),SIG(30),ECAL(50)
      SUMV2=0.0
      SUMV=0.0
      SUME=0.0
      SUMEV=0.0
      DO 10 J=1,N
      SUMV=SUMV+V(J)
      SUME=SUME+ELN(J)
      SUMV2=SUMV2+V(J)*V(J)
10    SUMEV=SUMEV+V(J)*ELN(J)
      RNOM=N
      DENOM=(RNOM*SUMV2)-(SUMV*SUMV)
      A=(SUMV2*SUME-SUMV*SUMEV)/DENOM
      B=(RNOM*SUMEV-SUMV*SUME)/DENOM
      RETURN
      END
C
C
C
      SUBROUTINE PLT
      DIMENSION X(50),Y(50)
      COMMON/ALL/N,A,B,V(50),ELN(50),VSIG(30),SIG(30),ECAL(50)
      M=N
      DO 10 J=1,N
      Y(J)=ECAL(J)
10    X(J)=ELN(J)
      S=4.0
```

```
      CALL SCALE(X,M,1,S,XMIN,DX)
      YMIN=XMIN
      DY=DX
      CALL PLINIT
      CALL AXIS (82,25,'E DIS',-5,S,90.,XMIN,DX)
      X(1)=0.
      M=N+1
      DO 20 J=1,N
20    X(J+1)=V(J)
      S=4.
      CALL SCALE(X,M,1,S,XMIN,DX)
      CALL AXIS(82,25,'KVOLT',5,S,0.0,XMIN,DX)
      CALL PLOT(82,25,-3)
      DO 25 J=1,N
25    X(J)=V(J)
      CALL LINE(X,Y,N,1,XMIN,DX,YMIN,DY)
      DO 30 J=1,N
      IX=(V(J)-XMIN)*(82.0/DX)
      IY=(ELN(J)-YMIN)*(51.0/DY)
30    CALL WRTSYM(IX,IY,'X',1,0.0)
      CALL SCRDMP
      CALL PLINIT
      CALL PLTCLS
      RETURN
      END
C
C
C
      SUBROUTINE PLT1
      REAL I
      DIMENSION X(50),Y(50)
      COMMON/ALL/N,A,B,V(50),ELN(50),VSIG(30),SIG(30),ECAL(50)
      COMMON/ALL1/I(50)
      M=N
      DO 10 J=1,N
      Y(J)=I(J)
10    X(J)=I(J)
      S=4.
      CALL SCALE(X,M,1,S,XMIN,DX)
      YMIN=XMIN
      DY=DX
```

```
      CALL PLINIT
      CALL AXIS(82,25,'I',-1,S,90.,XMIN,DX)
      M=N+1
      X(1)=0.
      DO 20 J=1,N
20    X(J+1)=V(J)
      CALL SCALE(X,M,1,S,XMIN,DX)
      CALL AXIS(82,25,'KVOLT',5,S,0.,XMIN,DX)
      CALL  PLOT(82,25,-3)
      DO 30 J=1,N
      IX=(V(J)-XMIN)*(82./DX)
      IY=(I(J)-YMIN)*(51./DY)
30    CALL WRTSYM(IX,IY,'+',1,0.0)
      CALL SCRDMP
      RETURN
      END
```

III. CHARGE EXCHANGE NEUTRAL ENERGY ANALYZER PROGRAM

The main program CX (Figure A.18) reads the input values of voltage and current from a data file written by the user. It then computes the ion energy distribution $f(v)/v^{.5}$ using Eq. 3.14, and finds the logarithm of $f(v)/v^{.5}$. Finally, it calculates the ion kinetic temperature of the plasma, based on the least squares fit to the semilog plot of $f(v)/v^{.5}$ as a function of v.

Subroutine CRSSEC (Figure A.19) computes the values of the cross section $\sigma_{CX}(V)$, using linear interpolation. This subroutine uses the values of σ_{CX}, known as a function of ion energy, between 60 eV and 6 KeV, to find the cross section σ_{CX} for any ion kinetic temperature. The cross section is computed for both atomic and molecular deuterium.

Subroutine LESQ (Figure A.20) finds the normal equations according to least squares method (Appendix B) for the fitted straight line to the semilog plot of the ion energy distribution. It then computes the coefficients of the straight line (slope and Y intercept).

Subroutines PLT (Figure A.21) and PLT1 (Figure A.22) display the semilog plot of the ion energy distribution and current as a function of voltage, respectively. These plots are screen dumped on the Lecroy 3931 printer.

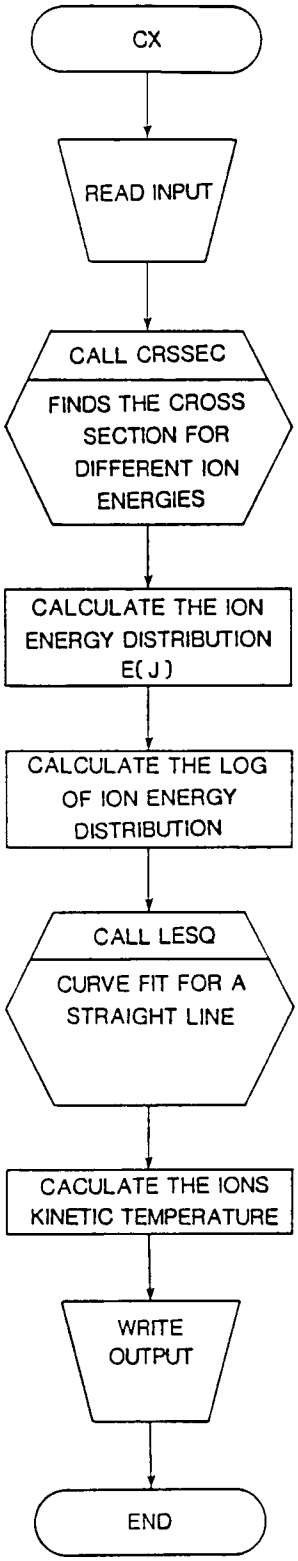

Figure A.18. Flowchart for the main program CX.

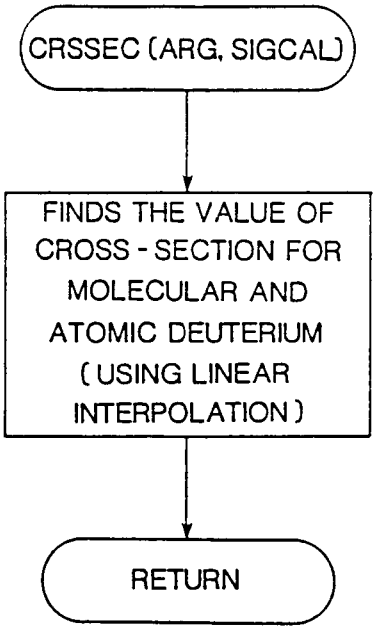

Figure A.19. Flowchart for subroutine CRSSEC.

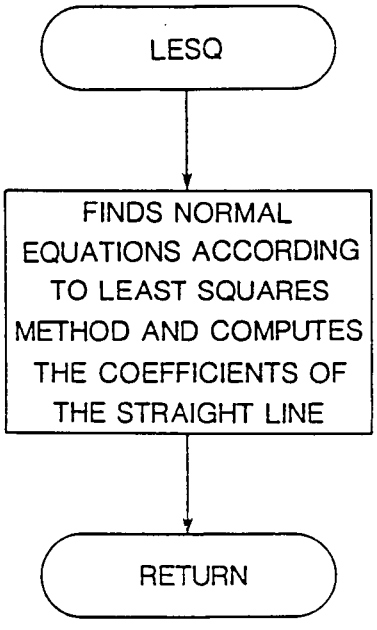

Figure A.20. Flowchart for subroutine LESQ.

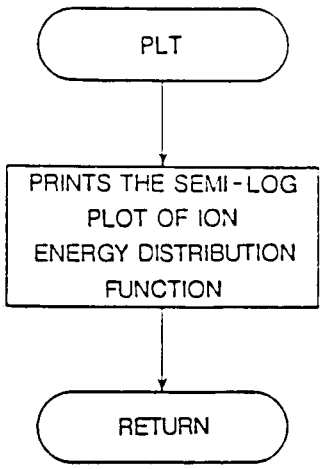

Figure A.21. Flowchart for subroutine PLT.

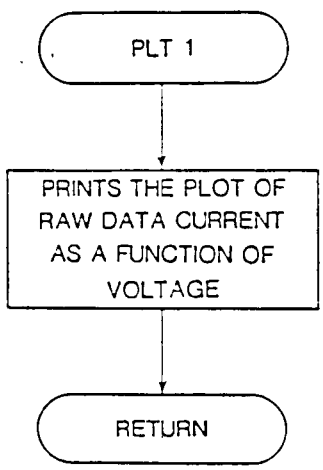

Figure A.22. Flowchart for subroutine PLT1.

APPENDIX B

METHODS OF CURVE FITTING

I. INTRODUCTION

A phenomenological correlation, or empirical formula, is one whose form is inferred from the results of experiment or observation and for which any constant factors are determined from experimental or observational data. For example, it is known that the speed of a ship varies with the horsepower according to the formula

$$P = a + b\,V^3 \,. \tag{B.1}$$

The constants to be determined in this formula are a and b, and for the purpose of determining them we should take several sets of readings of the speed and the corresponding horsepower. These sets of simultaneous values of V and P would, when substituted in the given formula, give several equations in the two unknowns a and b. The next step would be to find the best values for a and b from the several equations. For the solution of this part of the problem three methods are available: the graphical method or method of selected parts, the method of averages, and the method of least squares. The method of averages assumes that the best representative curve is that for which the algebraic sum of the residuals is zero. The method of averages is the shortest and easiest method for finding the constants in an empirical formula, but the residual equations can be grouped in several ways, and each different grouping will give different values for the unknown constants. There is no way to determine in advance just what grouping will give the best result.

The graphic method can be used whenever the given formula can be plotted as a straight line either directly or after a suitable transformation. Results of this method depend on one's vision and the graph paper used. This method, in general, is not recommended except for obtaining approximate values of the constants.

The method of least squares gives only one formula and that is always the best possible one. Only the least squares method (linear and nonlinear) used in computer programs is illustrated in this appendix.

II. THE METHOD OF LEAST SQUARES

This method says that the best representative curve is that for which the sum of the squares of the residuals is a minimum. Since the squares of the residuals are positive quantities, the requirement that their sum will be as small as possible insures that the numerical values of the residuals will be small, and this means that in the case of a series of plotted points the best representative curve will pass as closely as possible to all the points.

Let us consider the formula

$$y = a + bx + cx^2 \tag{B.2}$$

and find the values of a, b, and c which will make the graph of (B.2) pass as near as possible to each of the n pairs of observed values (x_1,y_1), (x_2,y_2) . . . (x_n,y_n). Substituting in (B.2) each of the n pairs of values in turn, we get the following residual equations:

$$
\begin{aligned}
v_1 &= a + bx_1 + cx_1^2 - y_1 , \\
v_2 &= a + bx_2 + cx_2^2 - y_2 , \\
&\vdots \\
v_n &= a + bx_n + cx_n^2 - y_n .
\end{aligned}
\tag{B.3}
$$

The principle of least squares says that the best values of the unknown constants a, b, and c are those which make the sum of the squares of the residuals a minimum, or

$$\sum v^2 = v_1^2 + v_2^2 + \ldots + v_n^2$$

must be a minimum. Hence

$$
\begin{aligned}
\sum(a + bx + cx^2 - y)^2 &= (a + bx_1 + cx_1^2 - y_1)^2 \\
&\quad + (a + bx_2 + cx_2^2 - y_2)^2 + \ldots + (a + bx_n + cx_n^2 - y_n)^2 \\
&= f(a, b, c)
\end{aligned}
$$

is to be a minimum.

The condition that f(a, b, c) be a maximum or a minimum is that its partial derivatives with respect to a, b, and c shall each be zero. After taking the derivatives and dividing through by 2, we get the following three normal equations:

$$(a + bx_1 + cx_1^2 - y_1) + (a + bx_2 + cx_2^2 - y_2)$$
$$+ \ldots + (a + bx_n + cx_n^2 - y_n) = 0$$

$$(x_1(a + bx_1 + cx_1^2 - y_1) + x_2(a + bx_2 + cx_2^2 - y_2)$$
$$+ \ldots + x_n(a + bx_n + cx_n^2 - y_n) = 0 \qquad \text{(B.4)}$$

$$x_1^2(a + bx_1 + cx_1^2 - y_1) + x_2^2(a + bx_2 + cx_2^2 - y_2)$$
$$+ \ldots + x_n^2(a + bx_n + cx_n^2 - y_n) = 0 \ .$$

The normal equations are solved by the ordinary methods of algebra for simultaneous equations of the first degree in two or more unknowns. The number of normal equations is always the same as the number of unknown constants to be determined, whereas the number of residual equations is equal to the number of observations. The number of observations must always be greater than the number of undetermined constants if the method of least squares is to be of any benefit in the solution.

The least squares rule can be applied to any formula which can be reduced to a form linear in the constants.

III. NONLINEAR FORMULAS

The relation between the pressure p and temperature t of saturated steam can be expressed by

$$p = a(10)^{bt/(c+t)}$$

where a, b, c, are unknown constants. These constants do not enter the formula linearly, and no transformation of the formula will give a linear relation among them. Therefore, they cannot be determined by the methods of least squares.

Consider a formula involving two variables, x and y, and three undetermined constants, a, b, c:

$$y = f(x, a, b, c) \ . \tag{B.5}$$

Let a_0, b_0, c_0 be approximate values of a, b, c, obtained from a graph, and let α, β, γ be the corrections on a_0, b_0, c_0, respectively. Hence

$$\begin{aligned} a &= a_0 + \alpha \\ b &= b_0 + \beta \\ c &= c_0 + \gamma \end{aligned} \tag{B.6}$$

Then

$$y' = f(x, a_0, b_0, c_0) \tag{B.7}$$

will be a function whose graph approximates the graph (B.5). The values of this approximating function corresponding to x_1, x_2, . . . x_n will be

$$
\begin{aligned}
y_1' &= f(x_1, a_0, b_0, c_0) \\
y_2' &= f(x_2, a_0, b_0, c_0) \\
&\cdot \\
&\cdot \\
&\cdot \\
y_n' &= f(x_n, a_0, b_0, c_0)
\end{aligned}
\tag{B.8}
$$

If (B.5) is the most probable function and its graph is the best representative curve, the residuals will be

$$
\begin{aligned}
v_1 &= f(x_1, a, b, c) - y_1 \\
v_2 &= f(x_2, a, b, c) - y_2 \\
&\cdot \\
&\cdot \\
&\cdot \\
v_n &= f(x_n, a, b, c) - y_n \ .
\end{aligned}
\tag{B.9}
$$

Substituting in (B.9) the values of a, b, c as given by (B.6), we have for the first residual

$$v_1 = f(x_1, a_0 + \alpha, b_0 + \beta, c_0 + \gamma) - y_1$$

or

$$v_1 + y_1 = f(x_1, a_0 + \alpha, b_0 + \beta, c_0 + \gamma) \tag{B.10}$$

Expanding (B.10) by Taylor's Theorem for a function of several variables, we have

$$v_1 + y_1 = f(x_1, a_0, b_0, c_0) + \alpha\left(\frac{\partial f_1}{\partial a}\right)_0 + \beta\left(\frac{\partial f_1}{\partial b}\right)_0 + \gamma\left(\frac{\partial f_1}{\partial c}\right)_0 + \ . \ . \ . \tag{B.11}$$

where $\left(\frac{\partial f_1}{\partial a}\right)_0$ means

$$\left(\frac{\partial f}{\partial a}\right)_{\substack{x=x_1 \\ a=a_0 \\ b=b_0 \\ c=c_0}}, \text{ etc.}$$

Since $y_1' = f(x_1, a_0, b_0, c_0)$, (B.11) becomes

$$v_1 + y_1 = y_1' + \alpha\left(\frac{\partial f_1}{\partial a}\right)_0 + \beta\left(\frac{\partial f_1}{\partial b}\right)_0 + \gamma\left(\frac{\partial f_1}{\partial c}\right)_0 + \ldots$$

or

$$v_1 = \alpha\left(\frac{\partial f_1}{\partial a}\right)_0 + \beta\left(\frac{\partial f_1}{\partial b}\right)_0 + \gamma\left(\frac{\partial f_1}{\partial c}\right)_0 + y_1' - y_1 .$$

Now let

$$r_1 = y_1' - y_1,\ r_2 = y_2' - y_2,\ \ldots,\ r_n = y_n' - y_n .$$

Then the residuals become

$$\begin{aligned} v_1 &= \alpha\left(\frac{\partial f_1}{\partial a}\right)_0 + \beta\left(\frac{\partial f_1}{\partial b}\right)_0 + \gamma\left(\frac{\partial f_1}{\partial c}\right)_0 + r_1 \\ v_2 &= \alpha\left(\frac{\partial f_2}{\partial a}\right)_0 + \beta\left(\frac{\partial f_2}{\partial b}\right)_0 + \gamma\left(\frac{\partial f_2}{\partial c}\right)_0 + r_2 \\ &\vdots \\ v_n &= \alpha\left(\frac{\partial f_n}{\partial a}\right)_0 + \beta\left(\frac{\partial f_n}{\partial b}\right)_0 + \gamma\left(\frac{\partial f_n}{\partial c}\right)_0 + r_n . \end{aligned} \qquad \text{(B.12)}$$

These equations are linear in the corrections α, β, γ, and the method of least squares can be applied from this point on.

APPENDIX C

TECHNICAL DATA

I. LECROY 3500SA32

(Lecroy Technical Data)

1. Acquisition

Inputs (TR8837 transient recorders):

Number	2
Sensitivity	512 mV peak-to-peak maximum
Offset	±256 mV adjustable via front panel potentiometer
Impedance Impedance	50 ohms ±1%, 12 PF
Frequency response	>16 MHz (1 dB) for full scale signal

Trigger:

Voltage levels	TTL: positive edge sensitive; 0 to +5 volts, 10 ns min, 50 ohms

Acquisition Buffer: 8K points may be subdivided into as many as 16 segments

Memory:

ADC Dynamic Range: 8-bits

Aperture Uncertainty: ±15 picoseconds, maximum

Dynamic Accuracy: $\geq$ 7.0 effective bits for 16 MHz full scale sine wave

Linearity: Better than ±.4% of full scale

Sampling Period: 31.25 nanoseconds to 2.0 microseconds per sample point. Selected by keypad entry.

External Clock:

Overall Repetition Rate: Depends on sample record length. Typical rate of >700 Hz for 1000 point record length.

2. Fast Averaging

Processor: Bit-slice processor; 68-bit wide instruction; 167 nanosecond cycle time; 1K x 48-bit micro-program storage memory.

Averaging: 8192 (8K) x 24-bit (16,777,215 counts) standard. Static RAM.

Memory: Up to 64K x 24 bits optional in 8K increments.

3. Signal Processing

Display and Archiving

Processors:

- CPU - Intel 8085A
- Arithmetic - AM9511; 2 MHz; 32-bit floating point calculations
- Video - Intel 8085A with private 8K x 8-bit EPROM and 8K x 8-bit RAM.

Built-In Processing Routines	Addition/ Subtraction	Any acquired waveforms may be added or subtracted from each other, with the resultant curve stored in memory.
	Normalize/ Overlay	Two curves may be overlayed on the screen for comparison. Offset and scale factors can be automatically calculated and applied to one of the curves for proper scaling.
	Expansion and Regions of Interest	Up to 256 separate areas of the waveform can be selected for detailed, expanded display or storage.
	Other Mathematical Processing	Curves can be displayed in Linear, Log, or Square Root with single light pen commands. X-axis (time) smoothing; 3, 5, or 9 points. X-axis calibration. Peak search and report.
	Fast Fourier Transform	Performs 512-point Fast Fourier Transform on selected portion of waveform. FFT and raw curve can be displayed simultaneously.
Auto Analysis	A series of up to 600 keystroke or light pen commands can be "memorized" for automatic data collection and manipulations.	

Archiving — Displayed curves can be stored in RAM memory or on floppy disks with simple light pen commands.

Processing and Display Memories

Computer Memory: 64K x 8-bit dynamic RAM.

Display Memory: 128K x 1-bit dynamic RAM. dedicated.

Scratchpad Memory: 1K x 8-bit battery-backed-up CMOS Static RAM. Used for storage of acquisition setup parameters.

4. Output

Serial Input/Output: Conforms to EIA-RS-232-C. Baud rates from 75 to 19,200.

Composite Video: Back Panel BNC connector (RS-170) outputs to external video monitor.

Printer: Model 3931A Graphics Printer; 9 x 11 fanfold paper; 90 characters/second. Provides "screen dump" graphic output of CRT screen.

Additional Features

CP/M Computer — System operates as a full feature microcomputer with CP/M based dual floppy disk system. Supports BASIC or FORTRAN software.

Terminal — System can operate as a computer terminal via the RS-232-C Serial Data Port.

System Dimensions, Weight, Temperature and Power Requirements

System Console: 15.85 inches (40.2 cm) high x 17.38 inches (44.1 cm) wide x 32.57 inches (82.5 cm) deep. Depth includes keyboard. 24.77 inches (62.9 cm) is depth of mainframe only. Rack mounting by means of optional rack-mounting shelf.

Dual Drive Floppy Disk	5.25 inches (13.3 cm) high x 17.6 inches (44.7 cm) wide x 21 inches (53.4 cm) deep. Rack mounting by optional shelf brackets.
Weight:	Console 80 pounds (36 Kg) Disk Drive 48 pounds (21.6 Kg).
Operating Temperature Range:	Complete system 15-40 degrees C (60-104 degrees F). Relative Humidity: 10%-95%.
Power Requirements:	115/230 Volts ac, 50/60 Hz. 600 watts including dual drive floppy disk.

II. ISAFIL

(Control Concept Corporation)

ISAFIL is a combination of the newest in magnetic technology plus active tracking filters designed to provide an isolation and attenuation of common and normal mode noise. ISAFIL works for sinusoidal or random transient noise, and attenuates IEEE 587-80 category A and B transients.

Whatever the origin of the transients, they can be classified in two simple categories:

1. Common Mode: This is the disturbances and noise between one or more power line conductors and the earthen ground.

2. Normal or Transverse Mode: This is the noise potential between conductors.

ISAFIL incorporates a shielded isolation transformer, which is superior in common mode noise attenuation, and an Islatrol, the active tracking filter offering a normal mode transient attenuation.

Instrument specifications are:

- Two-terminal input and two-terminal output, all floating. Fifth terminal, super shield, attaches to earth ground.
- Attenuation of noise voltages between power lines and ground to less than one part in 20 million (146 dB).
- Attenuation of noise voltages across the power line to less than one part in 500 (54 dB).
- Greater than 50 dB attenuation for IEEE 587-80.
- Through-capacitance (input to output with shield grounded), less than .0005 PF.
- Active filtering squelches either spike or oscillatory noise voltages across input terminals within 10 nanoseconds.
- Power 1 KVA
- Input Voltage 120 V
- Output Voltage 120 V

VITA

The author was born in Tehran, Iran, on December 21, 1957. He attended primary school from August 1963 to June 1969 and high school from August 1969 to June 1975.

He started his university work in August 1975 at the Arya-Mehr University of Technology in the Physics Department, and in September 1978 he came to the United States to continue his education.

He has been attending The University of Tennessee, Knoxville in the Electrical Engineering Department since April 1979. He received the degree of Bachelor of Science in Electrical Engineering in March 1981.

He began his studies for the Master of Engineering degree in the Electrical Engineering Department with a teaching assistantship in April 1981.

He was an instructor of physics and computer science at Knoxville College from September 1982 to June 1983. He received a research assistantship (funded by the Office of Naval Research) in the UTK Plasma Science Laboratory in September 1983, where he did his thesis research.