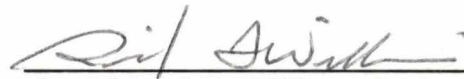


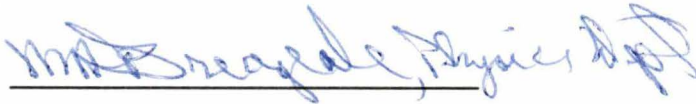
To the Graduate Council:

I am submitting herewith a thesis written by Alexy Kolesnikov entitled "Response of a Three-Component Geophone to Non-Vertically Incident Seismic Waves". I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with major in Geology.

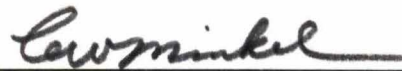


Dr. Richard T. Williams, Major Professor

We have read this thesis
and recommend its acceptance:



Accepted for the Council:



Associate Vice Chancellor
and the Dean of The Graduate School

**RESPONSE OF A THREE-COMPONENT GEOPHONE
TO NON-VERTICALLY INCIDENT SEISMIC WAVES**

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Alexy Yu. Kolesnikov

May, 1996

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ABSTRACT

Geophones are commonly planted on the surface or at shallow depth to record the ground motion produced by various types of seismic waves propagating through the earth's interior. It is generally assumed that the surficial material is heavy rock, and that the instrument is massless by comparison, in which case the instrument passively follows the displacement of the ground caused by the seismic wave. However, it is not always possible to attach the instrument on solid rock, and not all geophones and seismometers are light. Indeed, when a seismometer is installed on a pier, the mass of the pier must be counted as part of the instrument. Depending on the mass and size of the geophone and local soil conditions, the recorded motion may differ considerably from that of the ground without the perturbing instrument mass. This problem is important for various types of seismic sensors used in different fields of seismology, namely marine seismology, earthquake seismology, and exploration geophysics.

In this study I consider the dynamic response of a three-component geophone subjected to obliquely incident seismic waves. My theoretical considerations are based on the equations of the linear theory of elasticity. The theory shows that the motion of a geophone in contact with the ground differs from that of the ground without the instrument, with the difference attributed to the rigid body translations and rotations of the geophone and existence of a secondary wave field produced by the instrument interaction with an incident seismic wave.

It is shown that a particular geophone-soil combination exhibits several

characteristic frequencies at which the recorded seismic signals can be considerably amplified or attenuated. These frequencies are shown to depend on the geophone mass, size and the soil conditions of the installation site, with the amount of amplification depending on all of the above parameters and the angle at which an incident seismic wave approaches the ground surface. The inertial effects of the geophone response are analyzed for the center of mass arbitrarily positioned inside the instrument case. The offset of the center of mass from the vertical axis of the geophone is shown to introduce a coupling between different geophone components and is associated with the increase of the signal amplifications, resulting from the additional displacements recorded due to the rotations of the instrument base.

The validity of the theory was tested using the seismic reflection data recorded by three component geophones used in exploration geophysics. The experimental results prove the validity of the theory, in that it is capable of predicting sharp amplifications of the seismic signals recorded by different components of a seismic recorder, at specific frequency ranges. It therefore provides a theoretical basis for developing frequency filters, suitable for a particular installation site, to remove the amplification effects resulting from the coupling phenomenon from the seismic data, and obtain an improved estimate of the ground motion.

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LIST OF SYMBOLS

Symbol	Description
$\mathbf{U}$	vector of total displacement carried by an incident seismic wave
U	magnitude of the total displacement carried by an incident seismic wave
B	point denoting the center of the geophone base
G	point denoting the center of the geophone mass
ω	angular frequency of an incident seismic wave
f	frequency of an incident wave in Hz
t	time variable
$\mathbf{n}$	unit vector perpendicular to the front of an incident seismic wave
$u_x^{free}, u_y^{free}, u_z^{free}$	components of the surface displacements produced by an incident seismic wave in the XYZ coordinate system whose origin is at G
j_x, j_y, j_z	angles defining the orientation of an incident seismic wave with respect to a geophone
u_{xB}, u_{yB}, u_{zB}	components of the total displacement at the center of the geophone base
B_x, B_y, B_z	coordinates of point B in the XYZ coordinate system

u_{xG}, u_{yG}, u_{zG}	components of the total displacement at the center of the geophone mass
$\theta_x, \theta_y, \theta_z$	rotational displacements of the geophone base
M	geophone mass
P_x, P_y, P_z	components of the forces acting on the geophone as a result of its interaction with an incident seismic wave
C_x, C_y, C_z	couples acting on the geophone as a result of its interaction with an incident seismic wave
I_x, I_y, I_z	moments of inertia of the geophone around the respective coordinate axes
$\mathbf{u}_B$	vector of total displacement at point B
$\mathbf{u}_B^{\text{free}}$	vector of free field displacement at point B
$\mathbf{u}_B^{\text{scattered}}$	vector of scattered displacement at point B
r_0	geophone base radius
h	geophone height
λ, μ	Lamé elastic constants
ρ	density of the soil
σ	ratio of the shear velocity to the compressional velocity
V_p	velocity of a compressional wave in the soil
V_s	velocity of a shear wave in the soil
ν	Poisson's ratio
f_{1x}, f_{2x}	compliance functions for the geophone translations along the

	X axis
f_{1y}, f_{2y}	compliance functions for the geophone translations along the
	Y axis
f_{1z}, f_{2z}	compliance functions for the geophone translations along the Z
	axis
f_{1r}, f_{2r}	compliance functions for the geophone rotations around the
	horizontal axes
f_{1tor}, f_{2tor}	compliance functions for the geophone rotations around the
	vertical axis
K, L, E, P	coefficients used for evaluating the total vertical displacement
	of an axially symmetrical cylinder
T, N, S, R	coefficients used for evaluating the total horizontal
	displacement of an axially symmetrical cylinder
$\{a_{ij}\}, \{b_i\}, \{x_i\}$	coefficients defining the system of equations used for
	evaluating the response of a cylinder whose center of mass is
	offset from the vertical axis
$\Delta_1, \Delta_2, \Delta_3, \Delta$	determinants of the matrixes used for evaluating the response
	of a cylinder whose center of mass is offset from the vertical
	axis
A, D, C	coefficients used for evaluating moments of inertia of a
	cylinder whose center of mass is offset from the vertical axis
π	3.141592654

1. INTRODUCTION

It is well-known that the actual displacements of a geophone can differ from those of the ground without the instrument (Washburn and Willey, 1941, Bycroft, 1978, Hoover and O'Brien, 1980), complicating estimation of the true ground motion. This distortion has been shown on the basis of experimental and theoretical investigations to be a resonant phenomenon strongly dependent on both the size and mass of the geophone, and on the local properties of the soil. The physics of the process has been attributed to scattering of the incident seismic waves from the base of the instrument, which moves as a rigid body relative to the ground contact area, and therefore produces a local distortion in the seismic wave field. To evaluate the total displacement field recorded by a geophone it is necessary to consider a combination of three factors contributing to the recorded signal. These are the input motion produced by an incident seismic wave, the secondary or scattered displacement field produced by interaction between a geophone and the incident seismic wave, and the additional displacements recorded by a geophone due to its rigid body motion.

The free field displacements induced at the surface in the absence of the geophone are well-known, and are generally assumed to coincide with those recorded by a geophone. These displacements have been evaluated for all types of incident seismic waves and a variety of seismic sources (Aki and Richards, 1980, vol. 1). The solution to the problem of the estimation of the secondary displacements produced by the geophone-incident wave interaction is less well known, although a considerable body of literature

exists on this matter. To obtain the displacements, consideration is given to the response of a structure resting on the surface of an elastic half-space and subjected to the action of a periodic force. In the mathematical model of the geophone-soil coupling, the geophone is rigid right cylinder or circular plate attached to the surface of an elastic half-space, and the seismic wave is a periodic force acting on the surface. The boundary conditions are those of continuous normal stress and displacement under the foundation, and zero stresses on the free surface. The solutions of the Lamé equations governing the propagation of elastic waves resulting from the action of a body force are then expressible in terms of integral equations (Miller and Pursey, 1954, Arnold *et al.*, 1955, Bycroft, 1956, Luco and Westman, 1971) which are solved by approximation methods, for the different excitation modes.

The third step of the theoretical analysis must include the determination of the total instrument response by solving equations of its rigid body motion resulting from the interaction with both incident and scattered wave fields. This problem has not previously received a lot of attention, and only a limited body of literature exists on the subject. Simple recommendations concerning the conditions of instrument placement aimed to reduce the influence of the coupling effect were given by Bycroft (1978) and Hoover and O'Brien (1980), based on theoretical studies of single component geophones subjected to horizontally or vertically incident seismic waves only. Bycroft reported the results theoretically calculated for two particular installation sites, computed for two sets of the instrument and soil parameters, while Hoover and O'Brien documented theoretical estimates for a variety of instrument and soil parameters, together with experimental

results obtained using single component geophones. Their theoretical study was restricted to the case of vertical incidence. Their results indicated that the resonant frequency of the plant is strongly influenced by the geophone mass and size, and by consolidation of the soil. They showed that the difference between the motion of the ground in contact with a geophone and the motion of the free surface, increases with increasing geophone mass or decreasing geophone base radius. The amplification of the free surface motion caused by the presence of the perturbing geophone appeared to be higher for more consolidated soils, with the resonant frequency increasing considerably with an increase in soil consolidation. An important limitation of the above studies, which prevents them from providing a theoretical basis for a data processing algorithm to minimize the distortion, is that the geophones were assumed to possess two planes of symmetry. This assumption eliminates the angle of the wave incidence from consideration. The center of mass may be offset some distance from its vertical axis due to manufacture or design of the geophone case. More likely perhaps, the symmetry of a multiple-spike geophone, for example the Mark Products TDC-II land case, may be spoiled if one or two spikes are loose in the soil. In this case all six degrees of freedom are coupled through the equations of the instrument motion, which is especially important for three-component instruments due to the coupling between different geophone components. The angle at which an incident seismic wave approaches the surface defines the amplitude of the displacement amplification, and the input motion exciting one of the geophone components is instantaneously recorded by the other two. As analysis of the response of a structure to strong earthquakes (Duke *et al.*, 1971) and studies of the propagation directions of direct and scattered seismic waves

recorded by a three-component seismic array (Wagner and Owens, 1993) emphasized the importance of considering the influence of the seismic waves incident at angles other than vertical or horizontal. The development of a more complete theory must therefore include consideration of the recording instrument interaction with an obliquely incident seismic wave, for a geophone possessing an arbitrary set of defining parameters.

The prevailing approach has been to improve the instrument-ground coupling using intuitive methods. Thus, in earthquake seismology the instrument is normally mounted on a massive concrete pier, which is cemented to bedrock. While an extensive body of literature concerning the influence of local geology on earthquake seismograms has emerged in the past years (Wagner and Owens, 1993, Beresnev *et al.*, 1995, Kato *et al.*, 1995), the response of the pier to the incident seismic waves as a function of its mass and size remains not well known, and no experimental work exists on this subject. The increasing sophistication of modern seismic recording instruments aimed at providing greater bandwidth and fidelity of the seismic data is undermined by the lack of knowledge concerning their coupling to the earth.

The geophone-soil coupling has been a subject of several experimental studies. Washburn and Willey (1941), showed the coupling to be a resonant phenomenon capable of distorting both amplitude and phase of the higher frequencies of the seismic signal. They demonstrated the strong dependence of the process on the size and mass of the instrument as well as on the properties of the soil. In that study, amplification of a seismic signal increased with the resonant frequency shifting to the lower values, with an increase in the recorder mass or decrease in its base radius. Krohn (1984) conducted a set of

laboratory and field experiments to investigate the coupling phenomenon for vertical and horizontal geophones. She discovered that the coupling resonant frequency is relatively insensitive to changes in mass and size of a vertical geophone, although it is dependent on the firmness of the soil. For horizontal geophones she found that good coupling to the soil is crucial, because rocking of horizontal geophones may occur in shear-wave studies, resulting in distortion of the recorded signal at low frequencies. Hoover and O'Brien (1980) conducted a few experiments to validate their theoretical findings, and their experimental results coincide with those theoretically predicted. However, no experimental data are available concerning the coupling behavior of three-component geophones used in exploration geophysics.

The goal of my work is to develop a general approach, suitable for every installation site, to reduce the amplification effects resulting from the coupling phenomenon from the seismic data, and obtain an improved estimate of the ground motion. The theoretical investigation within this project includes the determination of the total displacement field resulting from the interaction of an obliquely incident seismic wave with a geophone. The theoretical considerations are based on the theory of linear elasticity. To obtain the displacements I solve the equation system governing the motion of a geophone with the center of mass arbitrarily positioned inside the instrument case. The experimental investigation is concerned with testing the theory by applying it to the analysis of seismic data recorded with three-component geophones during a high resolution reflection seismic profiling experiments.

2. THEORY

The theoretical investigation includes the analysis of the distortion produced by interaction between the geophone and a non-vertically incident elastic wave. The total displacement field in the presence of the geophone is determined as a function of the geophone mass and size, the elastic properties of the half-space, and the angle of incidence for the wave.

The geophone is modeled as an infinitely rigid cylinder of mass M , base radius r_0 , and height h , resting on the surface of an isotropic elastic half-space. The physical properties of the half-space are defined by the Lamé elastic constants λ and μ and density ρ . Stresses under the instrument are assumed to correspond to those for static loading. Normal and tangential surface stresses away from the contact area are assumed to be zero, which corresponds to the boundary condition on the free surface.

A monochromatic seismic wave is incident from below. Its orientation with respect to a geophone is defined by the angles between the unit vector normal to the wave front and the coordinate axes. The source of an incident wave is assumed to be remote so that its influence is not considered. The length of an incident wave is assumed to be large compared to the size of the geophone, and the horizontal and vertical displacements of the instrument. A Cartesian coordinate system XYZ is introduced to describe the wave and the instrument motions (Figure 1). Point B corresponds to the center of the instrument base, point G corresponds to the center of the instrument mass, and directions of the XYZ coordinate axes are defined by the orientation of the three geophone components.

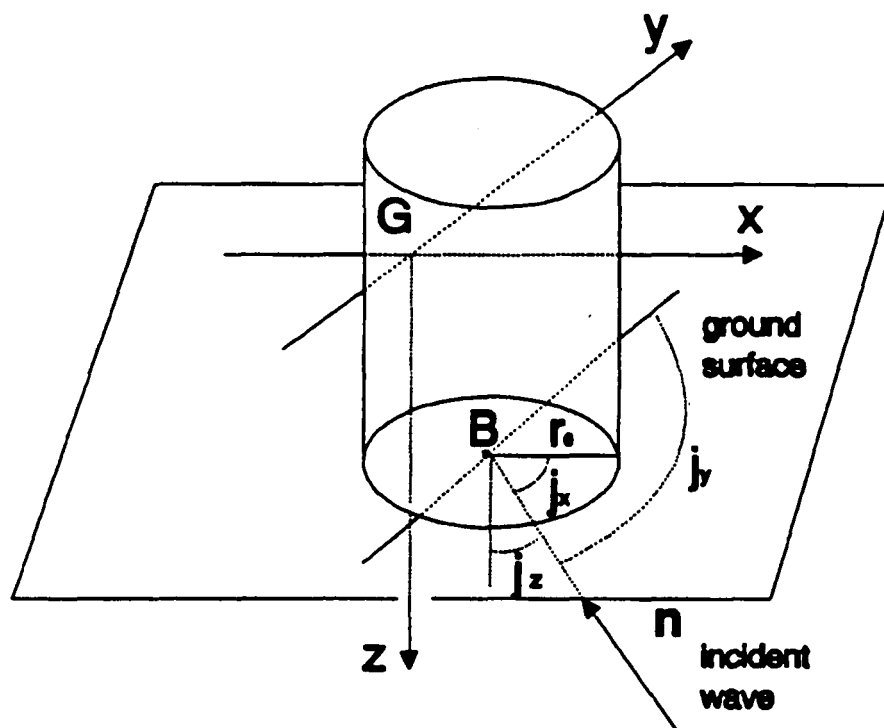


Figure 1. Description of the coordinate system defining the orientation of an incident seismic wave with respect to the seismic recording instrument.

It is assumed that an incident wave carries a displacement of a total magnitude U and the total displacement field produced by a wave can be written in the form

$$\mathbf{U}^{free}(\mathbf{x}, t) = U\mathbf{n}(\mathbf{x}) e^{i\omega t} \quad (1)$$

where $\mathbf{n}$ is a unit vector normal to the wave front, ω is a frequency of an incident wave, t is the time variable and

$$i = \sqrt{-1}$$

The components of the corresponding surface displacements along the axes of the XYZ coordinate system are then found as (Figure 1)

$$u_x^{free} = U \cos j_x e^{i\omega t} \quad (2)$$

$$u_y^{free} = U \cos j_y e^{i\omega t} \quad (3)$$

$$u_z^{free} = -U \cos j_z e^{i\omega t} \quad (4)$$

where

$$U = \sqrt{u_x^{free2} + u_y^{free2} + u_z^{free2}} \quad (5)$$

and j_x, j_y, j_z are the angles between the unit vector $\mathbf{n}$ and the corresponding coordinate axis.

These displacements are those produced by a seismic wave incident upon the

ground surface in the absence of the perturbing instrument mass. They are further referred to as "free-field displacements", and are usually assumed to coincide with the recorded seismic signal.

The motion of the geophone in contact with the ground, however, differs from that of the ground without the instrument. Two different but mutually dependent processes, namely the scattering of the incident seismic wave from the instrument base, and the rigid body motion of the geophone contribute to the distortion of the recorded signal. Therefore the recorded displacement is a combination of the free-field displacements, and additional displacements produced by the scattered wave field, which is further distorted by the rigid body rotations and translations of the recording part of the instrument located in point G. The latter introduces a coupling between the horizontal and vertical components of the geophone.

Let the translational displacements of the center of the instrument mass G in the XYZ coordinate system be denoted u_{xG} , u_{yG} , u_{zG} ; the translational displacements of the center of the instrument base B in the same coordinate system be denoted u_{xB} , u_{yB} , u_{zB} ; and the rotational displacements of the instrument base around the axes of the XYZ coordinate system be denoted θ_x , θ_y , θ_z respectively. Denoting the coordinates of the center of the instrument base B in the XYZ coordinate system as B_x , B_y , B_z and noting that displacements of any point of the cylinder are a combination of translations of its center of mass and rotations around the axes of the XYZ coordinate system we obtain

$$u_{xB} = u_{xG} + B_z \theta_y - B_y \theta_z \quad (6)$$

$$u_{yB} = u_{yG} + B_x \theta_z - B_z \theta_x \quad (7)$$

$$u_{zB} = u_{zG} + B_y \theta_x - B_x \theta_y \quad (8)$$

The interaction of an incident wave with a geophone makes the geophone move as a rigid body, thus inducing three forces P_x , P_y , P_z and three couples C_x , C_y , C_z upon the contact surface area, each corresponding to one of the translational or rotational degrees of freedom. The Euler's equations governing the rigid body displacements of the cylinder are in the form

$$M \frac{d^2 u_{xG}}{dt^2} = P_x \quad (9)$$

$$M \frac{d^2 u_{yG}}{dt^2} = P_y \quad (10)$$

$$M \frac{d^2 u_{zG}}{dt^2} = P_z \quad (11)$$

$$I_x \frac{d^2 \theta_x}{dt^2} = C_x + B_y P_z - B_z P_y \quad (12)$$

$$I_y \frac{d^2 \theta_y}{dt^2} = C_y + B_z P_x - B_x P_z \quad (13)$$

$$I_z \frac{d^2\theta}{dt^2} = C_z + B_x P_y - B_y P_x \quad (14)$$

where I_x , I_y , I_z are moments of inertia of the cylinder about the axes of the XYZ coordinate system.

Each of the induced translational and rotational perturbation modes produces a secondary "scattered" wave field radiating away from the instrument, and thus the displacements of the center of the geophone base are a combination of free-field displacements and those produced by a scattered wave field:

$$\mathbf{u}_B = \mathbf{u}_B^{\text{free field}} + \mathbf{u}_B^{\text{scattered}} \quad (15)$$

The linearity of Lamé equations governing the propagation of the scattered wave field through an elastic half-space, allows each of the modes to be considered independently of the others, with the total scattered displacement field being the sum of the displacement fields produced by each mode (Hsieh, 1962). The modes are coupled by the equations of the instrument motion. The secondary displacement fields of the contact surface area produced by the modes have been shown (Miller and Pursey, 1954; Luco and Westman, 1971) to be the real part of

$$\frac{Pe^{i\omega t}}{\mu r_0} (f_1 + if_2) \quad (16)$$

for the translational modes, and the real part of

$$\frac{Ce^{i\omega t}}{\mu r_0}(f_1 + if_2) \quad (17)$$

for the rotational modes. Here f_1 , f_2 are the complex functions of the frequency of an incident wave and properties of the elastic half-space. They are different for every one of the translational and rotational modes of excitation, and their numerical values have been computed for a broad range of defining parameters (Bycroft, 1978, Luco and Westman, 1971, see Figures 2 to 5). Function f_1 represents the static stiffness of the geophone-soil system, and function f_2 accounts for the energy loss associated with the secondary wave field produced at the geophone base and propagating away from a geophone (Bycroft 1957). As the above functions vary for each of the excitation modes I use the following notation:

$f_1 = f_{1x}; f_2 = f_{2x}$ - for the translation along X axis

$f_1 = f_{1y}; f_2 = f_{2y}$ - for the translation along Y axis

$f_1 = f_{1z}; f_2 = f_{2z}$ - for the translation along Z axis

$f_1 = f_{1r}; f_2 = f_{2r}$ - for the rotations about X and Y axes

$f_1 = f_{1tor}; f_2 = f_{2tor}$ - for the rotations about Z axis

The time dependence of the displacement components is represented by the factor $e^{i\omega t}$, hence taking a time derivative, cancelling $e^{i\omega t}$ term, and using (6)-(8), (15) equations (9)-(14) can be rewritten as

$$\omega^2 M(u_{xB}^{free} + u_{xB}^{scattered} + B_y \theta_z - B_z \theta_y) = P_x \quad (18)$$

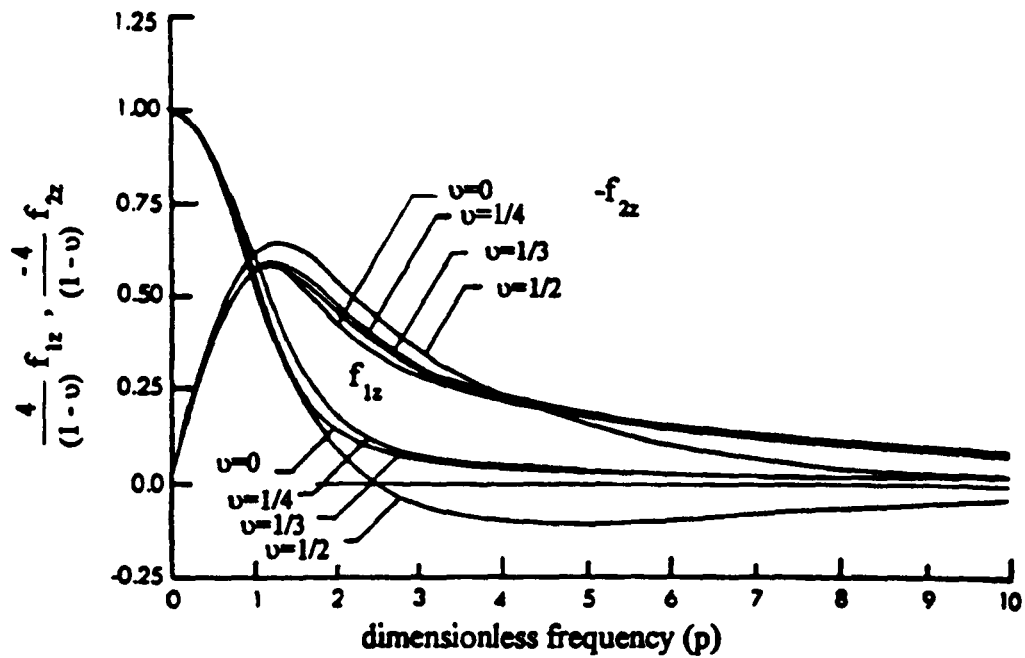


Figure 2. Compliance functions for vertical translation versus dimensionless frequency (p) calculated for different values of the Poisson's ratio ν . Adapted from Bycroft (1978).

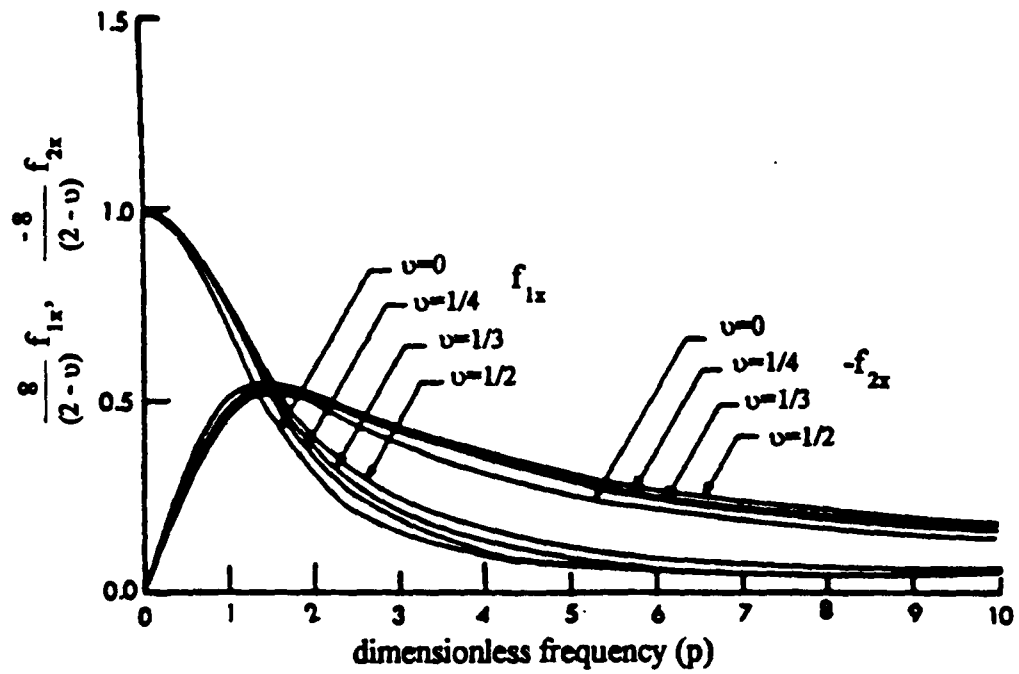


Figure 3. Compliance functions for horizontal translation versus dimensionless frequency (p) calculated for different values of the Poisson's ratio ν . Adapted from Bycroft (1978).

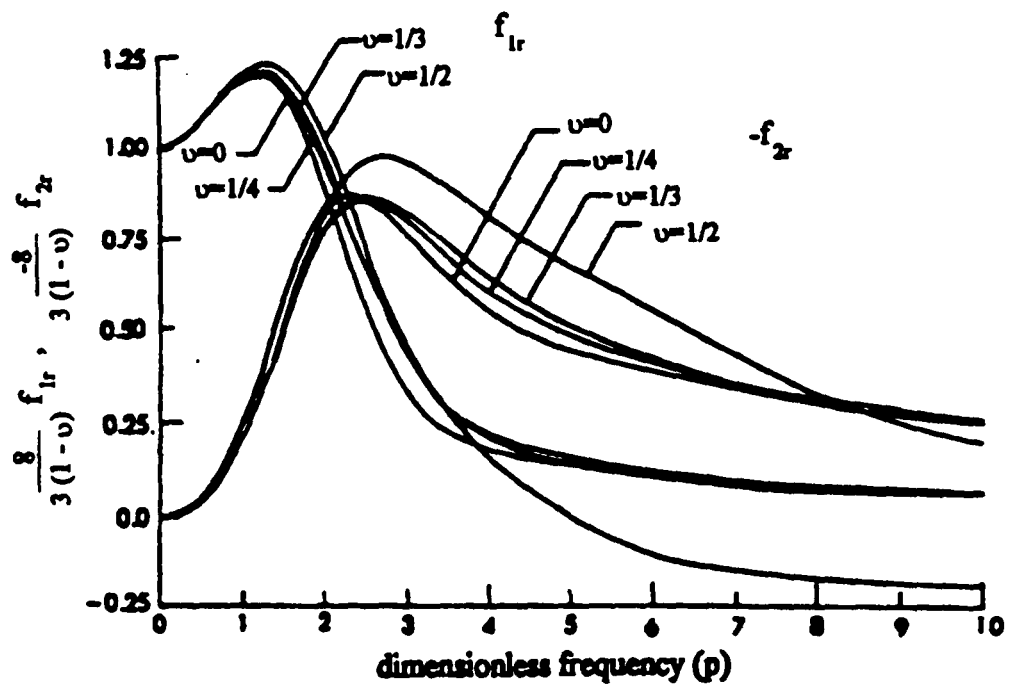


Figure 4. Compliance functions for rotations about the horizontal cylinder axis versus dimensionless frequency (p) calculated for different values of the Poisson's ratio ν . Adapted from Bycroft (1978).

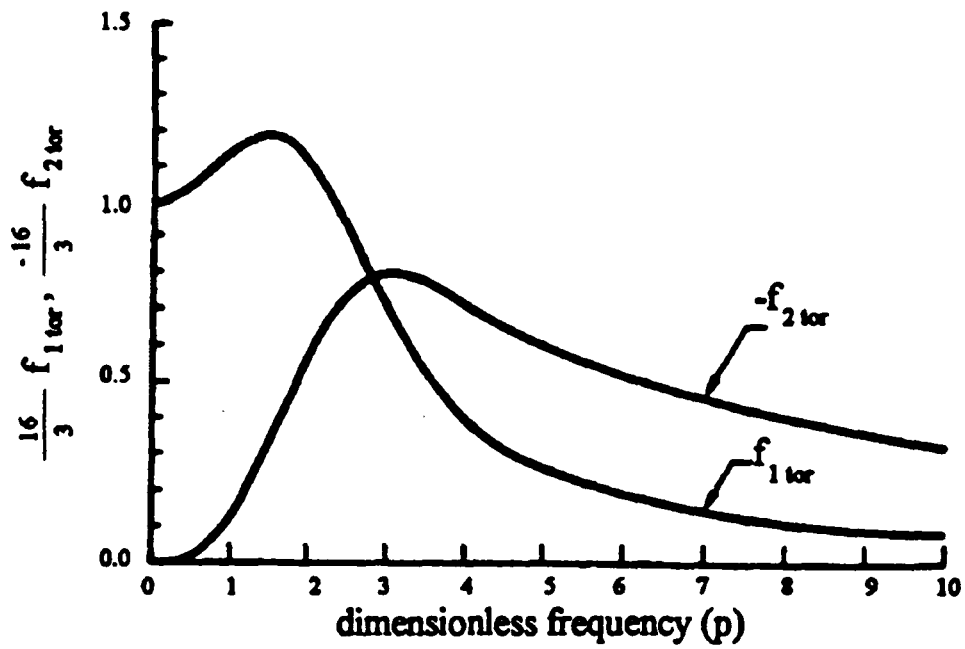


Figure 5. Compliance functions for rotations about the vertical cylinder axis versus dimensionless frequency (p). Function behavior does not depend on the value of the Poisson's ratio ν . Adapted from Luco and Westman (1971).

$$\omega^2 M(u_{yB}^{free} + u_{yB}^{scattered} + B_z \theta_x - B_x \theta_z) = P_y \quad (19)$$

$$\omega^2 M(u_{zB}^{free} + u_{zB}^{scattered} + B_x \theta_y - B_y \theta_x) = P_z \quad (20)$$

$$-\omega^2 I_x \theta_x = -C_x + B_z P_y - B_y P_z \quad (21)$$

$$-\omega^2 I_y \theta_y = -C_y + B_x P_z - B_z P_x \quad (22)$$

$$-\omega^2 I_z \theta_z = -C_z + B_y P_x - B_x P_y \quad (23)$$

Axially Symmetric Geophone.

I first consider the case when geophone is modeled as a cylinder possessing two planes of symmetry, so that B_x and B_y are both equal to zero. Substituting $B_x = B_y = 0$ into (18)-(23) I obtain

$$\omega^2 M(u_{xB}^{free} + u_{xB}^{scattered}) - \omega^2 M B_z \theta_y = P_x \quad (24)$$

$$\omega^2 M(u_{yB}^{free} + u_{yB}^{scattered}) + \omega^2 M B_z \theta_x = P_y \quad (25)$$

$$\omega^2 M(u_{zB}^{free} + u_{zB}^{scattered}) = P_z \quad (26)$$

$$-\omega^2 I_x \theta_x = -C_x + B_z P_y \quad (27)$$

$$-\omega^2 I_y \theta_y = -C_y - B_z P_x \quad (28)$$

$$-\omega^2 I_z \theta_z = -C_z \quad (29)$$

From viewing (24)-(29) it is seen that in this case θ_z is zero (there is no torsion around the vertical axis of the geophone), and vertical and horizontal components of the instrument displacements are not coupled through the equations of motion and can be considered independently. The theoretical basis for a solution of both of the above problems was first provided by Bycroft (1978) who considered a dynamic response of a rigid structure attached to the circular plate rested on the surface of an elastic half-space for vertical or horizontal excitation modes. As the results reported in his paper are limited to those evaluated for two particular installations, I present the details of his solution for the case under consideration in the Appendix and calculate the geophone response as it depends on specific defining parameters. This solution is then compared to the results obtained for a geophone with arbitrarily positioned center of mass.

To calculate the solution I use the soil elastic constants chosen to match those used in the experimental part of my investigation. Free field displacement amplifications for vertical excitation calculated for density $\rho=1690 \text{ kg/m}^3$, compressional wave velocity of $V_p=500 \text{ m/s}$ and various values of the geophone parameters are shown in Figures 6, 7. Its dependence on the variation in the instrument mass is shown in Figure 6 for the values

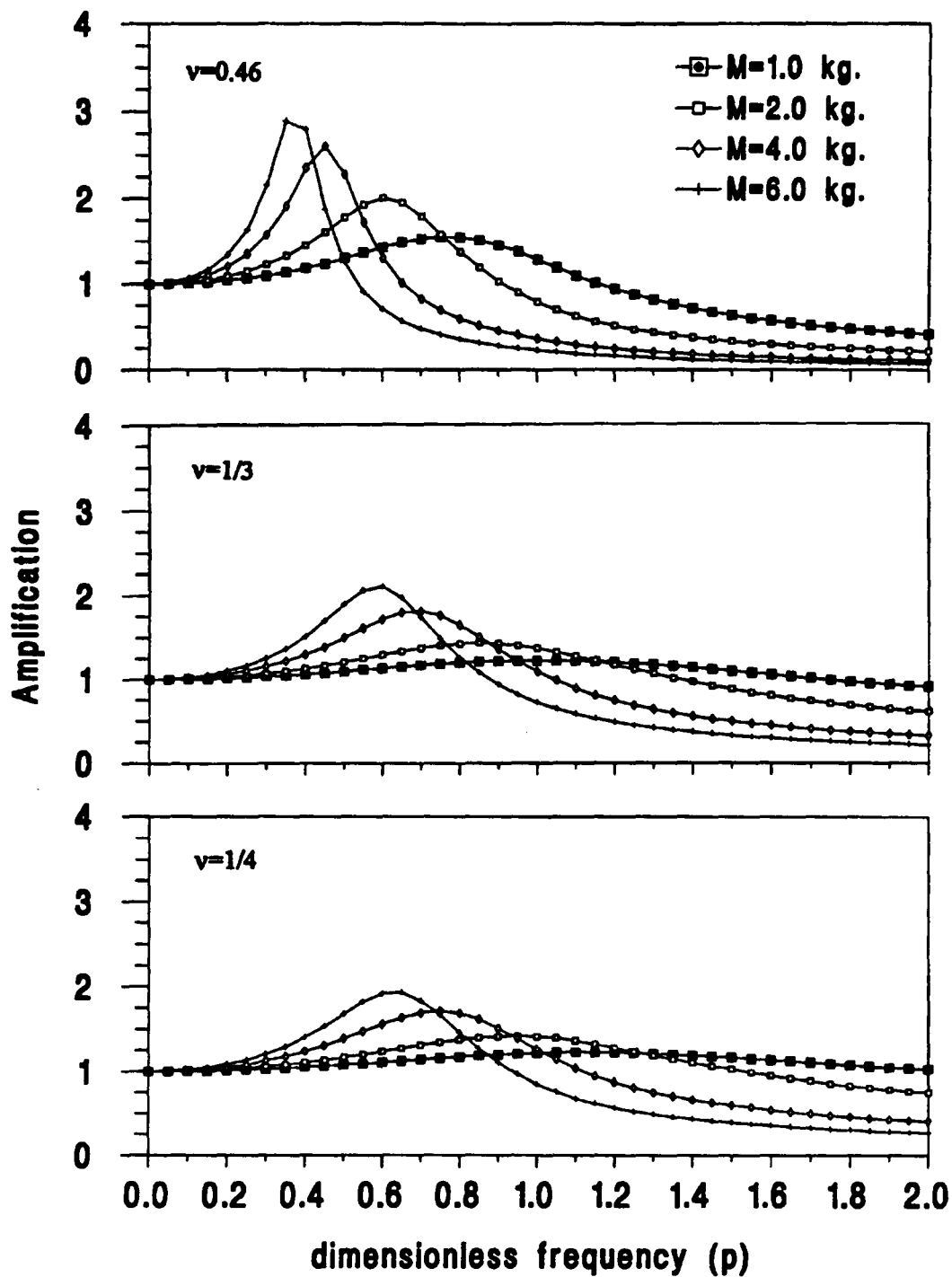


Figure 6. Vertical displacement amplifications as functions of dimensionless frequency (p), calculated for different values of the instrument mass M and Poisson's ratio v . The geophone base radius r_0 is 0.1 m.

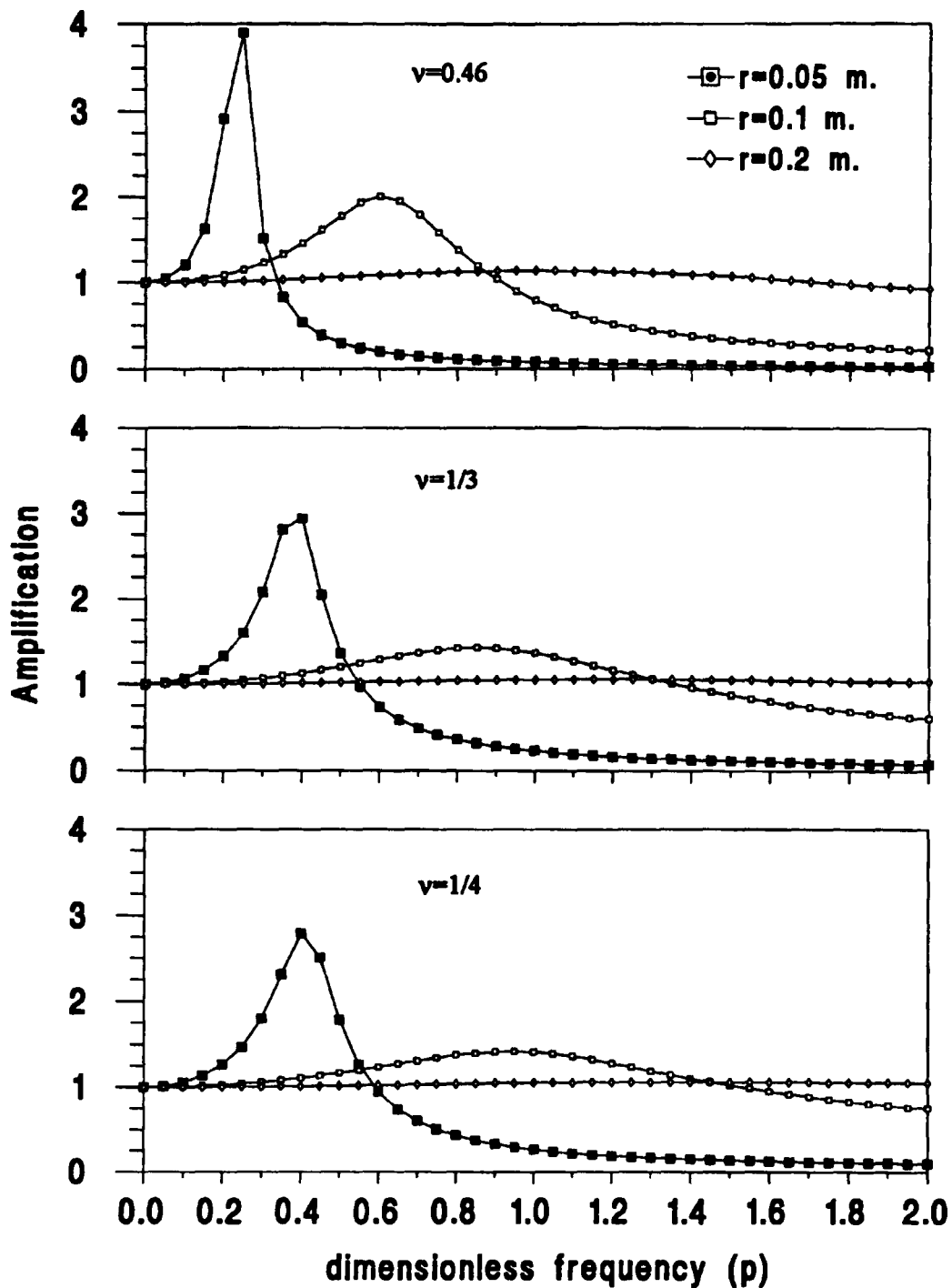


Figure 7. Vertical displacement amplifications as functions of dimensionless frequency (p), calculated for different values of the instrument base radius r_0 and Poisson's ratio v . The geophone mass M is 2 kg.

of the Poisson's ratio of $1/4$, $1/3$, and 0.46 , and its dependence on the variation in the instrument base radius is shown in Figure 7 for the values of the Poisson ratio of $1/4$, $1/3$, and 0.46 . The amplifications for $\nu=0.46$ were computed from the compliance functions obtained by Luco (1971) for $\nu=1/2$, $1/3$, $1/4$, and 0.0 using Lagrange polynomial interpolation.

Free field displacement amplification for the horizontal excitation calculated for density of $\rho=1690 \text{ kg/m}^3$, compressional wave velocity of $V_p=500 \text{ m/s}$ and various values of the instrument parameters and soil constants is shown in Figures 8, 9, 10. Its dependence on the instrument mass is shown in Figure 8 for the values of Poisson's ratio of $\nu=1/4$, and $1/3$, its dependence on the size of the instrument base radius is shown in Figure 9 for the values of Poisson's ratio of $\nu=1/4$, and $1/3$, and its dependence on the height of the instrument's center of mass B_z is shown in Figure 10 for the values of Poisson's ratio of $\nu=1/4$, and $1/3$.

Geophone with arbitrarily positioned center of mass.

By assuming a geophone possessing two planes of symmetry it was possible to considerably simplify the theoretical assessments needed to estimate the ground motion amplification, eliminating the coupling between different geophone components from the consideration. If the center of the instrument mass is offset by some distance from the vertical axis of the geophone, then all six equations of the instrument motion (18)-(23)

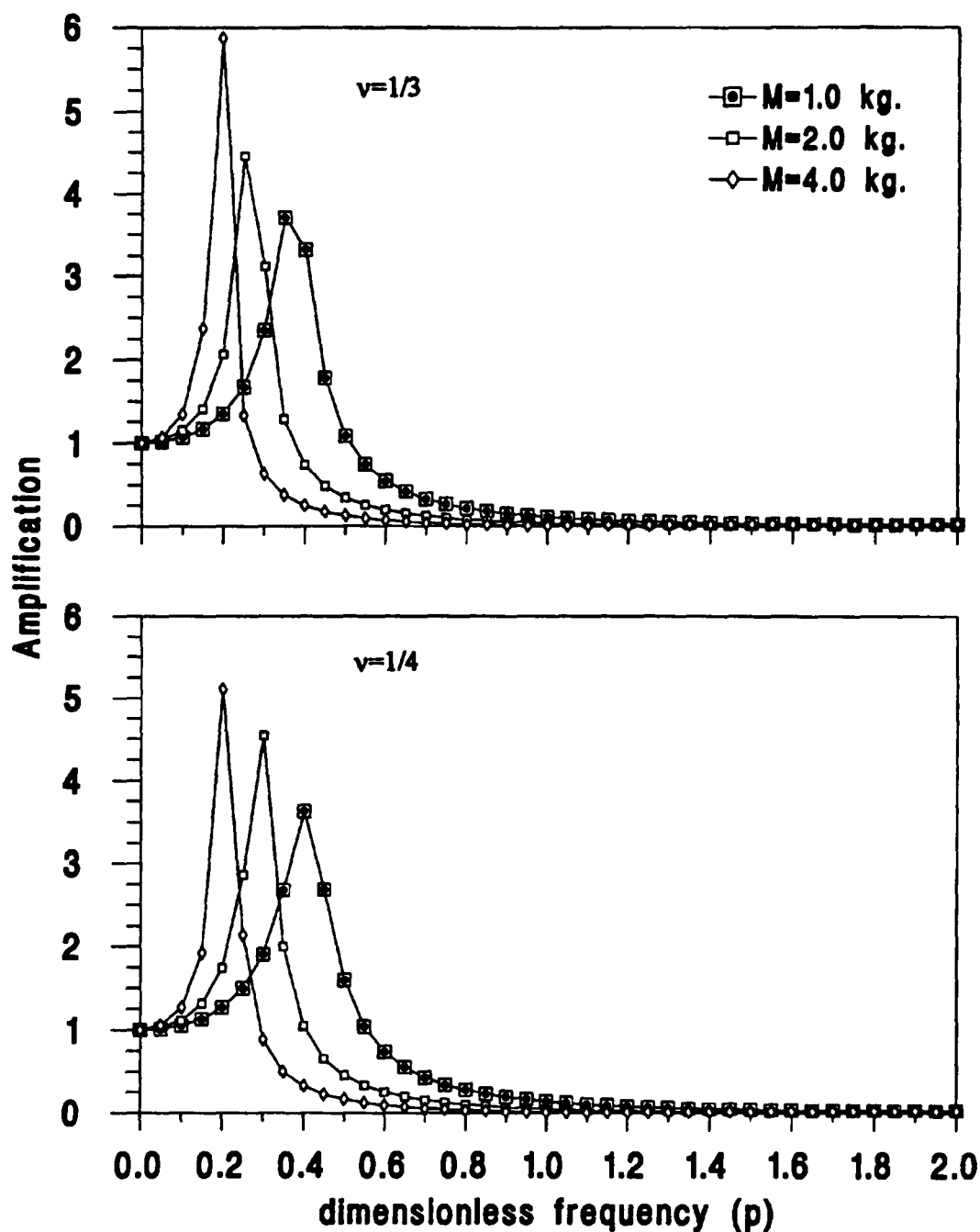


Figure 8. Horizontal displacement amplifications as functions of dimensionless frequency (p), calculated for different values of the instrument mass M and Poisson's ratio ν . The geophone base radius r_0 is 0.1 m., and the elevation of the center of mass above the free-surface B_z is 0.1 m.

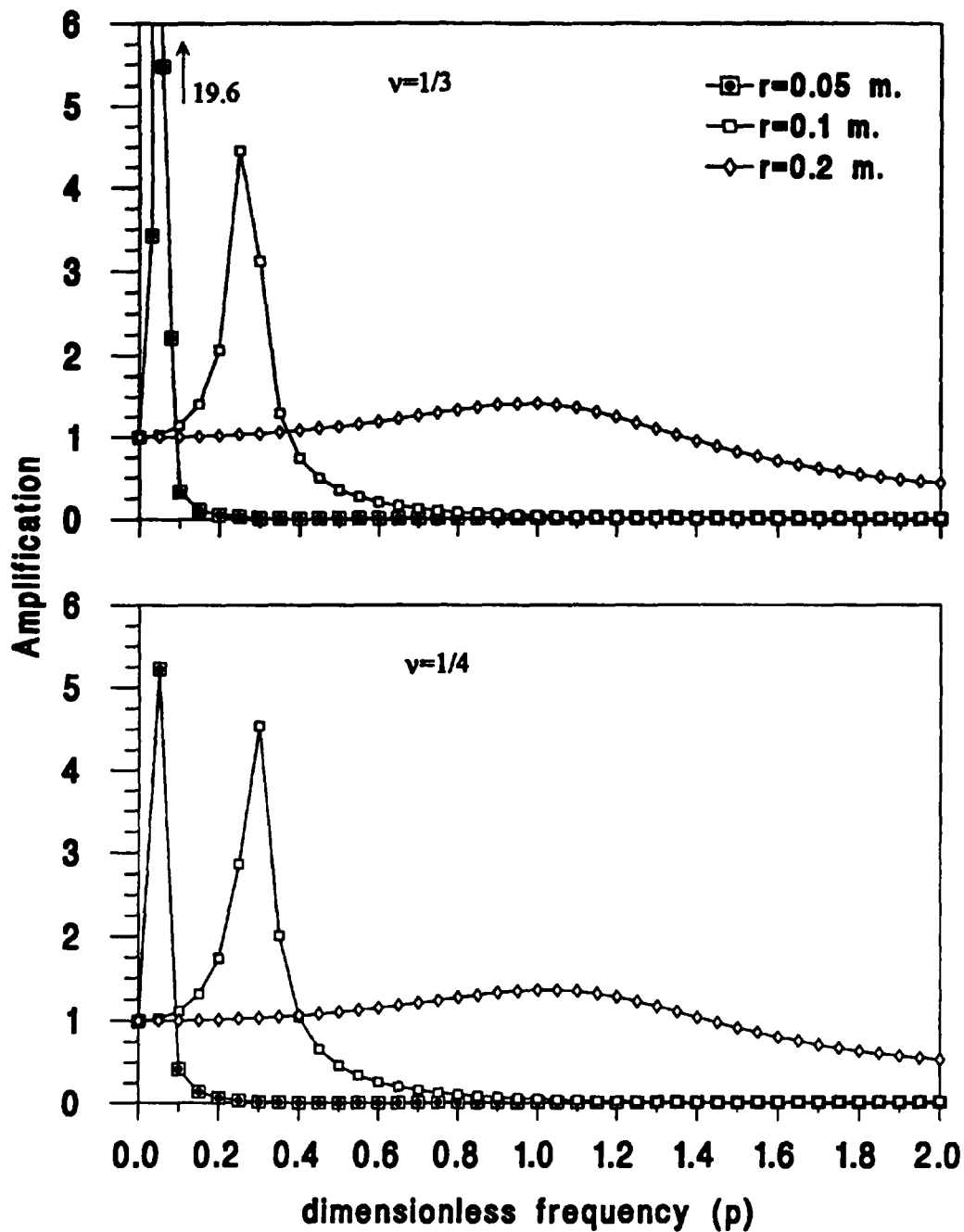


Figure 9. Horizontal displacement amplifications as functions of dimensionless frequency (p) calculated for different values of the instrument base radius r_0 and Poisson's ratio ν . The geophone mass M is 2 kg., and the elevation of the center of mass above the free-surface B_z is 0.1 m.

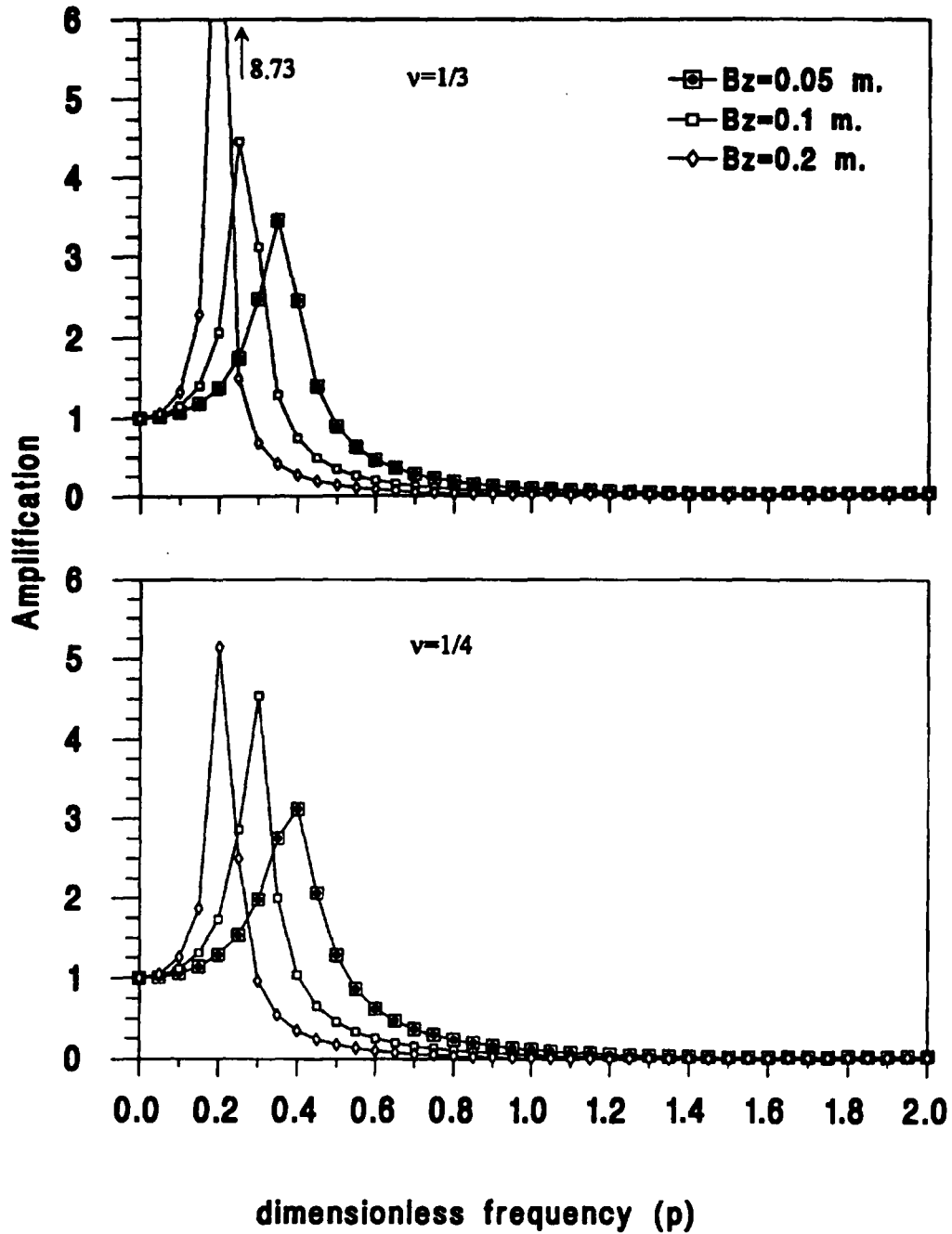


Figure 10. Horizontal displacement amplifications as functions of dimensionless frequency (p) calculated for different values of the elevation of the center of mass above the free-surface B_z and Poisson's ratio ν . The geophone mass M is 2 kg., and the geophone base radius r_0 is 0.1 m.

must be solved together, introducing coupling between translational components of the recorded displacements. Solution of the equation system (18)-(23) is the main goal of my theoretical investigation.

Using (16), (17) I can rewrite the equations (18)-(23) as

$$\begin{aligned} & \omega^2 M(u_{xB}^{free} + \frac{P_x}{\mu r_0}(f_{1x} + if_{2x}) + B_y \frac{C_y}{\mu r_0^3}(f_{1tor} + if_{2tor}) - \\ & - B_z \frac{C_y}{\mu r_0^3}(f_{1r} + if_{2r})) = P_x \end{aligned} \quad (30)$$

$$\begin{aligned} & \omega^2 M(u_{yB}^{free} + \frac{P_y}{\mu r_0}(f_{1y} + if_{2y}) + B_z \frac{C_x}{\mu r_0^3}(f_{1r} + if_{2r}) - \\ & - B_x \frac{C_z}{\mu r_0^3}(f_{1tor} + if_{2tor})) = P_y \end{aligned} \quad (31)$$

$$\begin{aligned} & \omega^2 M(u_{zB}^{free} + \frac{P_z}{\mu r_0}(f_{1z} + if_{2z}) + B_x \frac{C_y}{\mu r_0^3}(f_{1r} + if_{2r}) - \\ & - B_y \frac{C_x}{\mu r_0^3}(f_{1r} + if_{2r})) = P_z \end{aligned} \quad (32)$$

$$-\omega^2 M I_x \frac{C_x}{\mu r_0^3} (f_{1r} + i f_{2r}) = -C_x + B_z P_y - B_y P_z \quad (33)$$

$$-\omega^2 M I_y \frac{C_y}{\mu r_0^3} (f_{1r} + i f_{2r}) = -C_y + B_x P_z - B_z P_x \quad (34)$$

$$-\omega^2 M I_z \frac{C_z}{\mu r_0^3} (f_{1tor} + i f_{2tor}) = -C_z + B_y P_x - B_x P_y \quad (35)$$

(30)- (35) is a linear system of algebraic equations written for unknowns $P_x, P_y, P_z,$

$C_x, C_y, C_z.$

Denoting

$$a_{11} = 1 - \frac{\omega^2 M}{\mu r_0^3} (f_{1x} + i f_{2x}); \quad a_{15} = \frac{\omega^2 M B_z}{\mu r_0^3} (f_{1r} + i f_{2r}) \quad (36)$$

$$a_{16} = -\frac{\omega^2 M B_y}{\mu r_0^3} (f_{1tor} + i f_{2tor}); \quad a_{22} = 1 - \frac{\omega^2 M}{\mu r_0^3} (f_{1y} + i f_{2y}) \quad (37)$$

$$a_{24} = -\frac{\omega^2 M B_z}{\mu r_0^3} (f_{1r} + i f_{2r}); \quad a_{26} = \frac{\omega^2 M B_x}{\mu r_0^3} (f_{1tor} + i f_{2tor}) \quad (38)$$

$$a_{33}=1-\frac{\omega^2 M}{\mu r_0}(f_{1z}+if_{2z}); \quad a_{34}=\frac{\omega^2 MB_y}{\mu r_0^3}(f_{1r}+if_{2r}) \quad (39)$$

$$a_{35}=-\frac{\omega^2 MB_x}{\mu r_0^3}(f_{1r}+if_{2r}); \quad a_{44}=1-\frac{\omega^2 MI_x}{\mu r_0^3}(f_{1r}+if_{2r}) \quad (40)$$

$$a_{55}=1-\frac{\omega^2 MI_y}{\mu r_0^3}(f_{1r}+if_{2r}); \quad a_{66}=1-\frac{\omega^2 MI_z}{\mu r_0^3}(f_{1tor}+if_{2tor}) \quad (41)$$

$$a_{42}=-B_z; \quad a_{43}=B_y; \quad a_{51}=B_z; \quad (42)$$

$$a_{53}=-B_x; \quad a_{61}=-B_y; \quad a_{62}=B_x \quad (43)$$

$$a_{12}=a_{13}=a_{14}=a_{21}=a_{23}=a_{25}=a_{31}=a_{32}=a_{36}=0 \quad (44)$$

$$a_{41}=a_{45}=a_{46}=a_{52}=a_{54}=a_{56}=a_{63}=a_{64}=a_{65}=0 \quad (45)$$

$$b_1=\omega^2 Mu_{xB}^{free}, \quad b_2=\omega^2 Mu_{yB}^{free}, \quad b_3=\omega^2 Mu_{zB}^{free} \quad (46)$$

$$b_4=b_5=b_6=0 \quad (47)$$

$$P_x=x_1; P_y=x_2; P_z=x_3; C_x=x_4; C_y=x_5; C_z=x_6 \quad (48)$$

it is possible to rewrite (30)-(35) using the summation notation as

$$\{a_{nm}\}\{x_m\}=\{b_n\} \quad n,m=1,2,3,4,5,6 \quad (49)$$

In order to find the displacements at the point G it is necessary to evaluate x_1, x_2, x_3 .

Solving (49)

$$x_1=\frac{\Delta_1}{\Delta}; \quad x_2=\frac{\Delta_2}{\Delta}; \quad x_3=\frac{\Delta_3}{\Delta} \quad (50)$$

where Δ is the determinant of the matrix $\{a_{nm}\}$ and Δ_n is obtained by substituting

$$a_{nm}=b_m; \quad n,m=1,2,3,4,5,6 \quad (51)$$

into Δ .

The evaluation of (51) requires specification of I_x, I_y, I_z which are calculated from

$$I_x=\frac{1}{3\pi h r_0^2} \int_A^D (h((B_y+C)^3-(B_y-C)^3)+2C(B_z^3-(B_z-h)^3))dx \quad (52)$$

$$I_y=\frac{2}{\pi h r_0^2} \int_A^D (C(\frac{1}{3}B_z^3-(B_z-h)^3)+hx^2)dx \quad (53)$$

$$I_z=\frac{M}{\pi r_0^2} \int_A^D (2x^2C+\frac{(B_y+C)^3-(B_y-C)^3}{3})dx \quad (54)$$

where

$$A=B_x-\sqrt{r_0^2-B_y^2}; \quad D=B_x+\sqrt{r_0^2-B_y^2} \quad (55)$$

and

$$C=\sqrt{r_0^2-(x-B_x)^2}+B_y \quad (56)$$

by numerical integration.

With (52)-(54) evaluated for a variety of the instrument parameters, the values and magnitudes of P_x , P_y , P_z are then available from (50). The results of computation obtained by evaluating equations (36)-(50) using MATHCAD software are summarized in Figures 11 to 13. Figure 11 shows the amplification trends calculated for different values of the instrument mass. Figure 12 shows amplification dependence on the instrument base radius. Figure 13 shows amplification dependence on the position of the center of mass inside the instrument case. All amplifications were calculated for the following initial soil and recorder parameters: density $\rho=1690 \text{ kg/m}^3$, Poisson's ratio $\nu=1/3$, compressional wave velocity $V_p=500 \text{ m/s}$, $B_y=B_z=0.01 \text{ m}$., recorder height $h=0.1 \text{ m}$. The angles defining the orientation of the incident wave were arbitrarily set as $j_x=45^\circ$, $j_y=60^\circ$ with the vertical free field displacement calculated using (5). As both horizontal components show the same amplification trends I only present results obtained for the X component.

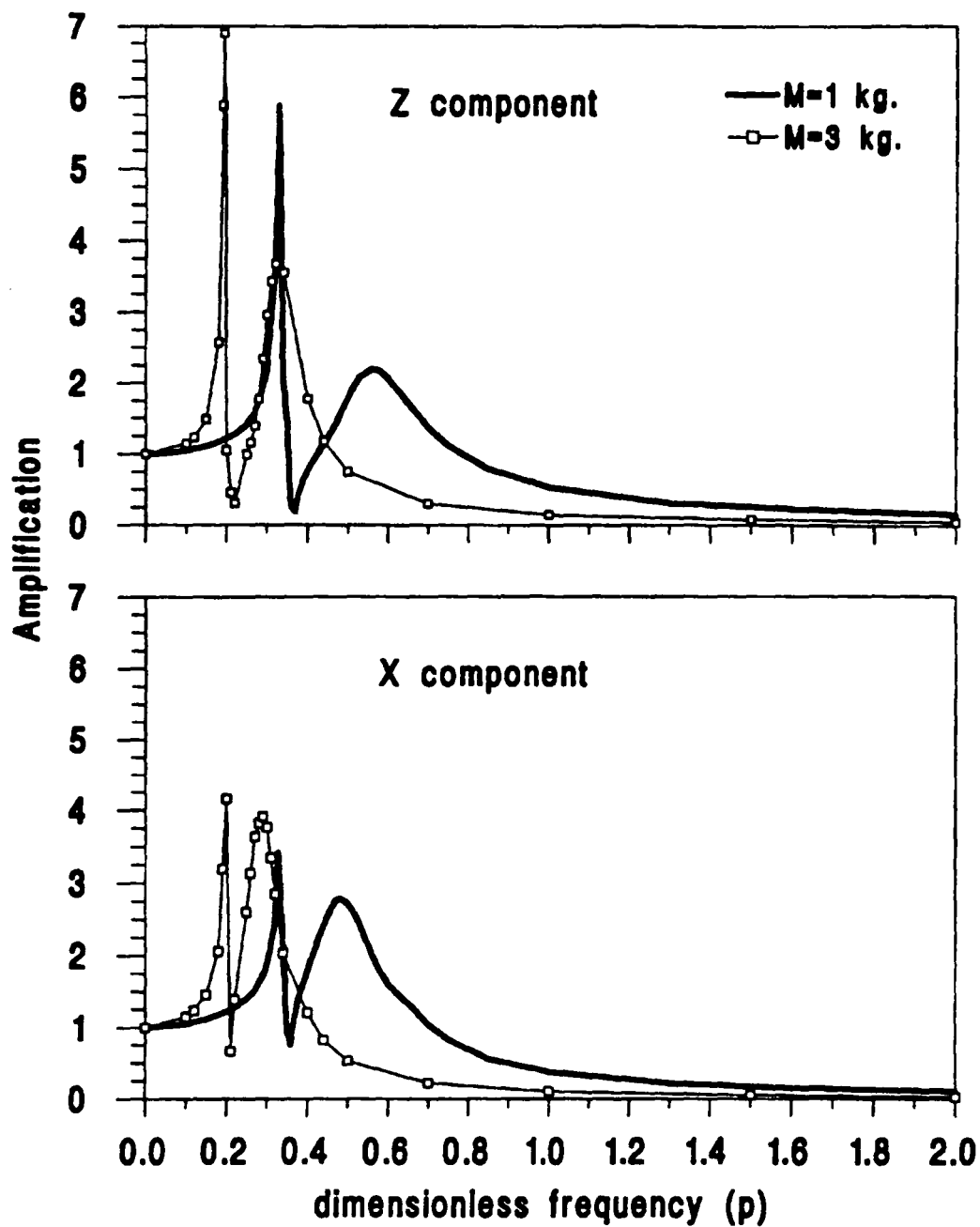


Figure 11. Displacement amplifications as functions of dimensionless frequency (p) calculated for different values of the instrument mass M . The geophone base radius r_0 is 0.05 m., and the offset of the center of mass from the vertical axis of the geophone B_x is 0.01 m.

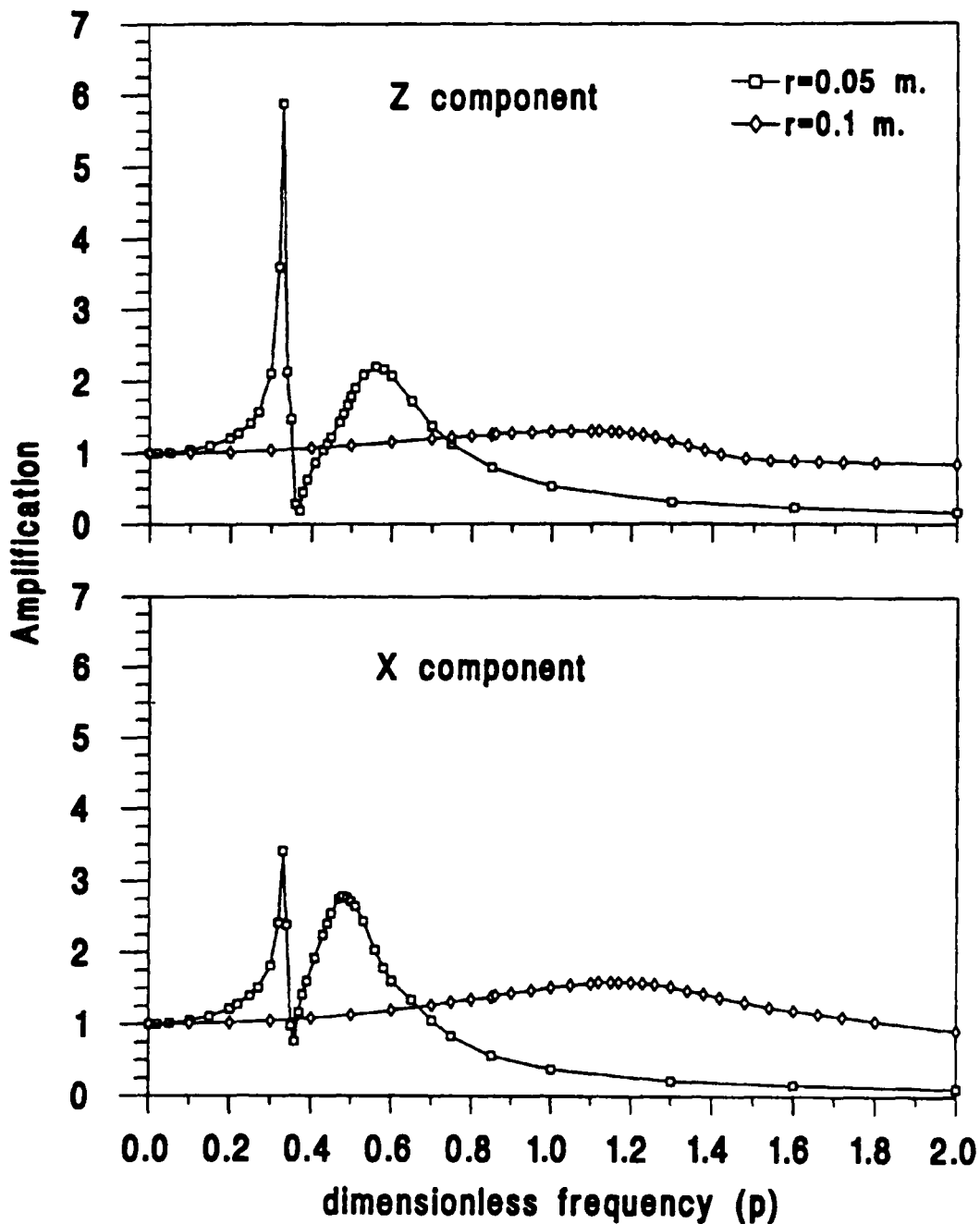


Figure 12. Displacement amplifications as functions of dimensionless frequency (p) calculated for different values of the instrument base radius r_0 . The geophone mass M is 1 kg., and the offset of the center of mass from the vertical axis of the geophone B_x is 0.01 m.

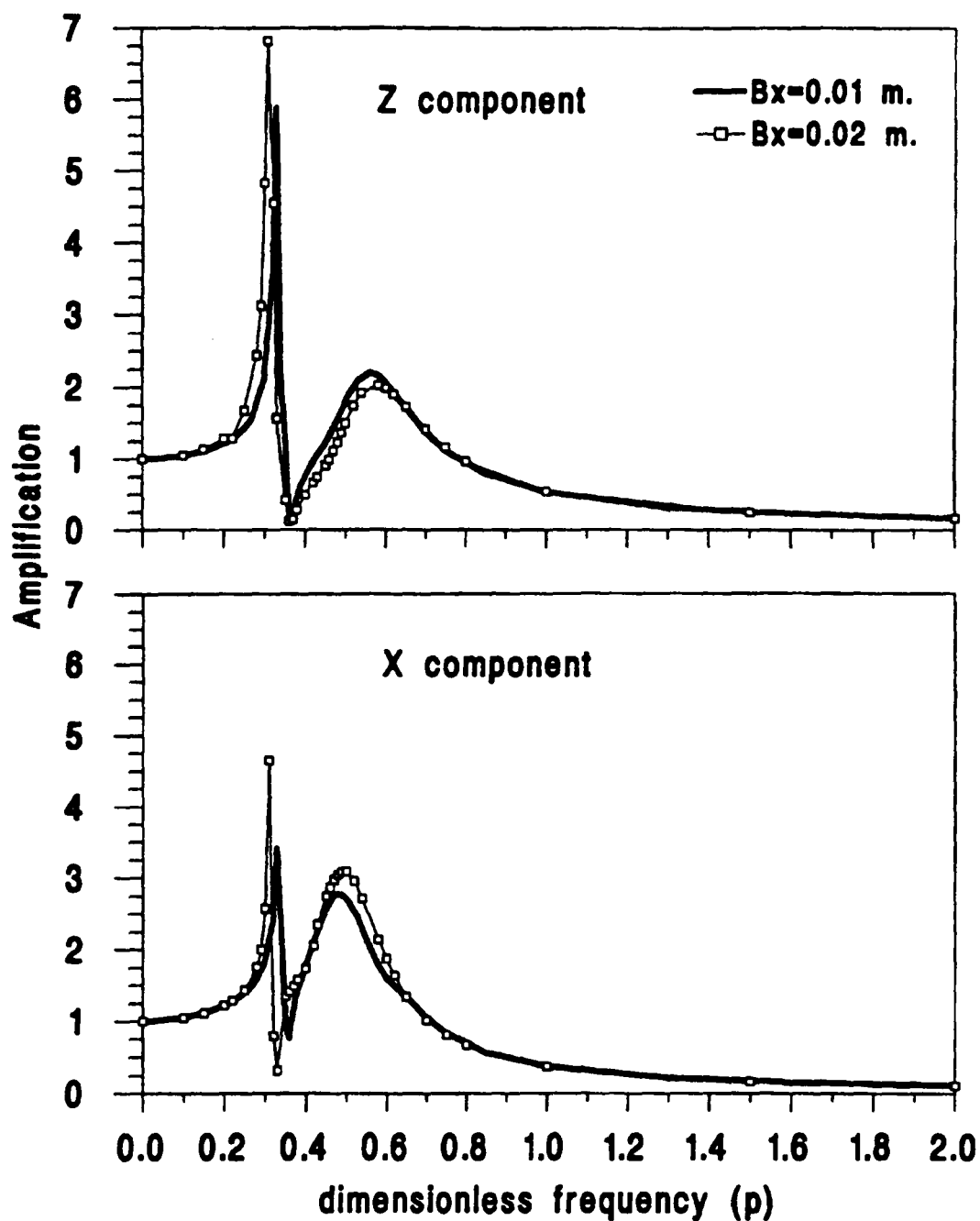


Figure 13. Displacement amplifications as functions of dimensionless frequency (p) calculated for different values of the offset of the center of mass from the vertical axis of the geophone B_x . The geophone mass M is 1 kg., and the geophone base radius r_0 is 0.05 m.

3. THEORETICAL RESULTS AND DISCUSSION.

The theory developed here shows that a recorded seismic signal may be significantly distorted as a result of coupling. The free-field displacement amplification resulting from the presence of the instrument mass is a complex function of the instrument mass, size, and properties of an elastic half-space.

Figures 6 to 13 indicate that the basic trends in amplification are dependent on the instrument parameters and properties of the soil. Amplification is close to unity at low frequencies and the measured displacement corresponds to that of the free-surface. Amplification increases as the frequency of an incident wave increases, reaching its peak at a resonant frequency characteristic of the geophone-soil system. An increase in the instrument mass lowers the resonant frequency and increases amplification. Similar effects result from a decrease in the instrument base radius, or an increase in the elevation of the center of mass above the ground surface. Comparing the results obtained for different soil elastic constants, I conclude that amplification increases and the resonant frequency shifts to the lower values as we progress from consolidated rock ($\nu=1/4$) to sand ($\nu=0.46$). Behavior of the geophone-soil system is therefore analogous to that of a simple harmonic oscillator. Its spring is represented by soil and its mass is represented by a geophone. Low soil consolidation (soft spring) or large geophone mass result in a large displacement recorded by a geophone near the resonant frequency. The results obtained here are consistent with the experimental and theoretical results of Washburn and Willey (1941) and Hoover and O'Brien (1980).

It follows from equations (6)-(8), that the total displacement recorded by each of the geophone components is a product of subtraction of the rotational displacement of the instrument base from the translational displacement of the center of the geophone base. Both of these displacements are amplified by coupling. When a geophone possesses an axial symmetry, rotational displacements recorded by its horizontal components are small and do not contribute significantly to the total displacement. For an unbalanced geophone, whose center of mass is shifted from its vertical axis, rotational displacements become large. Their amplification at a resonant frequency may cause the recorded signal to be attenuated rather than amplified at a specific frequency range. This trend appears in Figures 11 to 13. Note that as the base radius of a geophone increases, diminishing the rotational displacement component, the amplification curve is similar to the case of the axially symmetric geophone (Figure 12). Overall, amplifications for the case of an axially symmetric geophone are lower than when the geophone center of mass is offset from its central axis. This can be attributed to the coupling between different instrument components in the latter case, and the fact that the offset of the center of mass causes extensive rotations of the geophone contributing additional displacements to the recorded signal. The results presented in Figure 13 show that as the offset distance increases so do the displacement amplifications for both horizontal and vertical components.

If a seismic recorder is modeled as an axially symmetric cylinder the amplifications do not depend on the angle of incidence, and a seismic signal exciting one of the recorder components does not trigger any response from the other two. When the geophone center of mass is offset some distance from the vertical axis, then the displacement amplification

becomes dependent on the angle at which an incident seismic wave approaches the surface. To investigate the amplification trends as a function of the angle of incidence, I set $j_y=90^\circ$, making angle j_x an independent variable defining the orientation of an incident wave. The incident wave in this case carries displacements only in the XZ plane of the XYZ coordinate system. Hence, because of the small displacement recorded by the Y horizontal component due to the instrument motion, the amplification of that displacement is equal to infinity. From viewing Figure 14, I note that amplification of the horizontal displacements increases as j_x goes to 90 degrees (vertical incidence), and eventually becomes equal to infinity when a small horizontal displacement is recorded due to the instrument motion for the wave carrying displacements only in the vertical direction. The amplification of the vertical displacement increases with the decrease in j_x as the additional vertical displacement components produced by the scattered wave field and the geophone motion begin contributing significantly to the recorded signal. Likewise, the amplification of the vertical displacement is equal to infinity for the horizontally incident wave carrying displacements only in the X direction. This is consistent with the observations from real seismic data, in that seismic arrivals are usually recorded by all of the geophone components, with the difference in their relative amplitudes being attributed to the variation in the angle of wave incidence (Wagner and Owens, 1993). Allowing for asymmetry of the seismic recording instrument provides a more realistic physical representation, and should be used for obtaining theoretical estimates of the amplification behavior characteristic for a particular site.

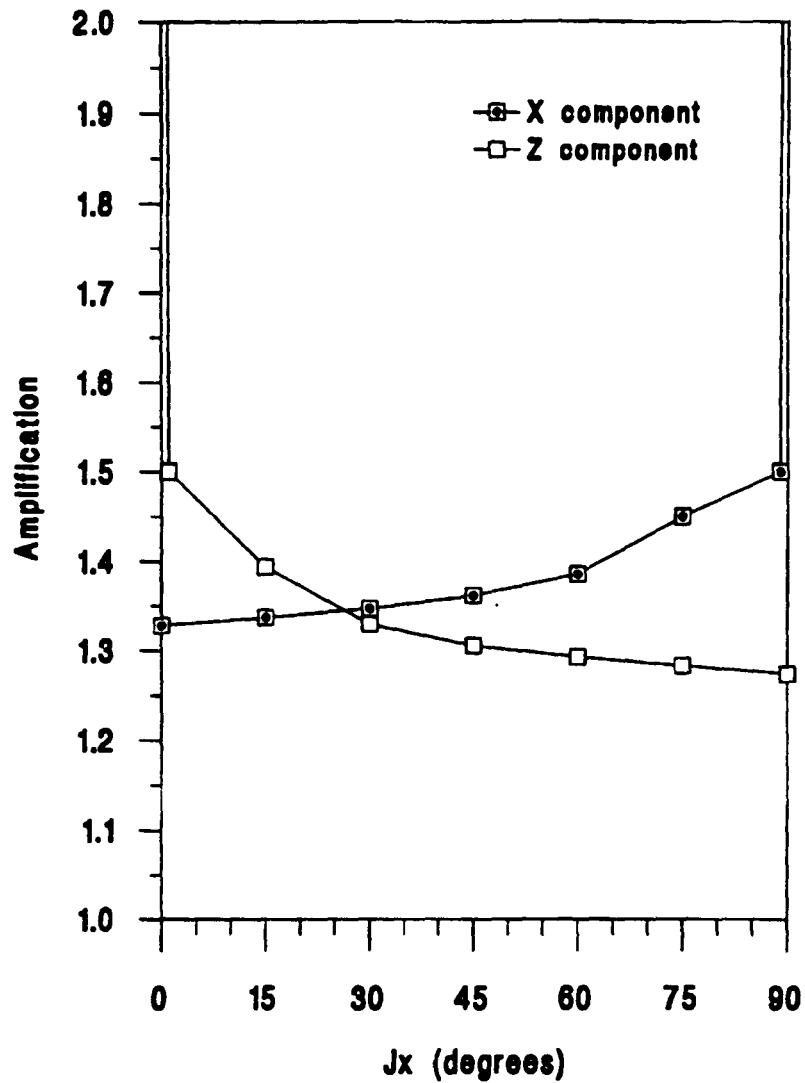


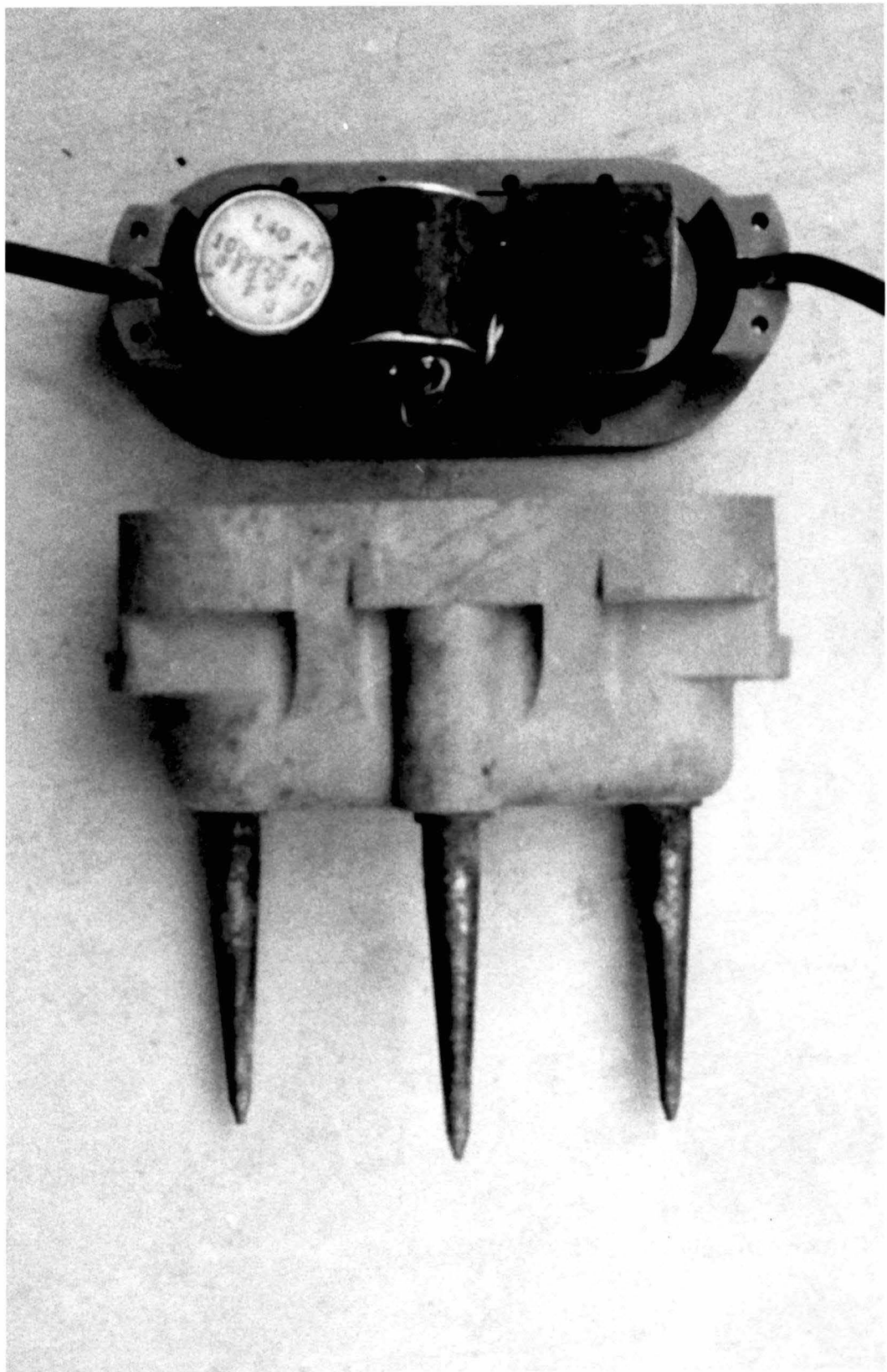
Figure 14. Horizontal and vertical amplifications versus angle of incidence j_x . The geophone mass M is 1 kg., the offset of the center of mass from the vertical axis of the geophone B_x is 0.01 m., and the instrument base radius r_0 is 0.05 m.

4. EXPERIMENT.

The objective of this portion of the study is to test the theory using real seismic data. As shown above there are several characteristic frequencies at which the seismic signals can be considerably amplified or attenuated due to the coupling phenomenon. These frequencies are defined by the instrument's mass, size and the soil conditions of the installation site. The purpose is to determine these frequencies theoretically for a particular seismic recorder, and then compare the predictions to the details of the recorded seismic signal. The seismic data used in this study were collected during two high resolution seismic reflection surveys conducted at different locations.

The first experiment was conducted at Oak Ridge National Laboratory. The details of the experiment together with the obtained results are documented Jacobs *et al.*, (1992) and Goldman (1994). Seismic reflection data were acquired using twenty four 3-component, 100 Hz geophones evenly spaced along a profile at 1.0 m. intervals, with the radial component of each geophone parallel to the direction of the profile. The photograph of a typical geophone used in the experiment is presented in Figure 15. The seismic source was a 21mm. (8 gauge) shotgun fired into the ground at 0.5 m intervals with the first shot point located 0.5 m. to the left of the first geophone. The record produced by each shot consisted of 72 seismic traces, 24 for each of the three components, vertical, radial, transverse. Each trace was corrected (Goldman, 1994) for spherical divergence and attenuation. The parameters of a sample geophone were measured as follows: mass, 1.5 kg., base radius, 0.04 m., the coordinates of the center of

Figure 15. Typical three-component geophone used to acquire the analyzed seismic data. Shown is a Mark Products TDC-II land case.



the instrument's base were measured in assumption that the center of the coordinate system used in the present study corresponds to the instrument's center of mass, which is consistent with the theoretical developments of the previous section, they were measured at $B_x = 0.013$ m., $B_y = 0.003$ m., $B_z = 0.022$ m., and the geophone height was 0.072 m. As reported by Goldman (1994) the seismic velocity of a compressional wave propagating through the soil was 350 m/s. The characteristic soil density of the site at the time of an experiment was 1350 kg/m^3 (Spalding *et al.*, 1992). No shear or surface waves were observed in the data. This indicates that the soil conditions at the site are not favorable for the development of these wave types, and suggests a low value of the shear modulus. I therefore used the Poisson's ratio of 0.46, the value for unconsolidated sand, for my theoretical calculations.

In my study I analyzed the seismic signals recorded by four geophones at different positions along the profile. I considered seismic traces recorded by each of these geophones for different shots, positioned far enough from the geophones to ensure that the records were not affected by an air wave resulting from the shot. Analyzed geophones were labeled so that the geophone number represents its offset from the second shot. True-amplitude plots of data recorded by a typical geophone in the Oak Ridge experiment is shown in Figure 16.

The second data set was obtained during a high resolution seismic reflection survey conducted in San Francisco Bay area by the USGS in collaboration with the University of Tennessee, Knoxville. The purpose of the study was to image the local near-surface faulting. In this case seismic reflection data were acquired using thirty-two three-

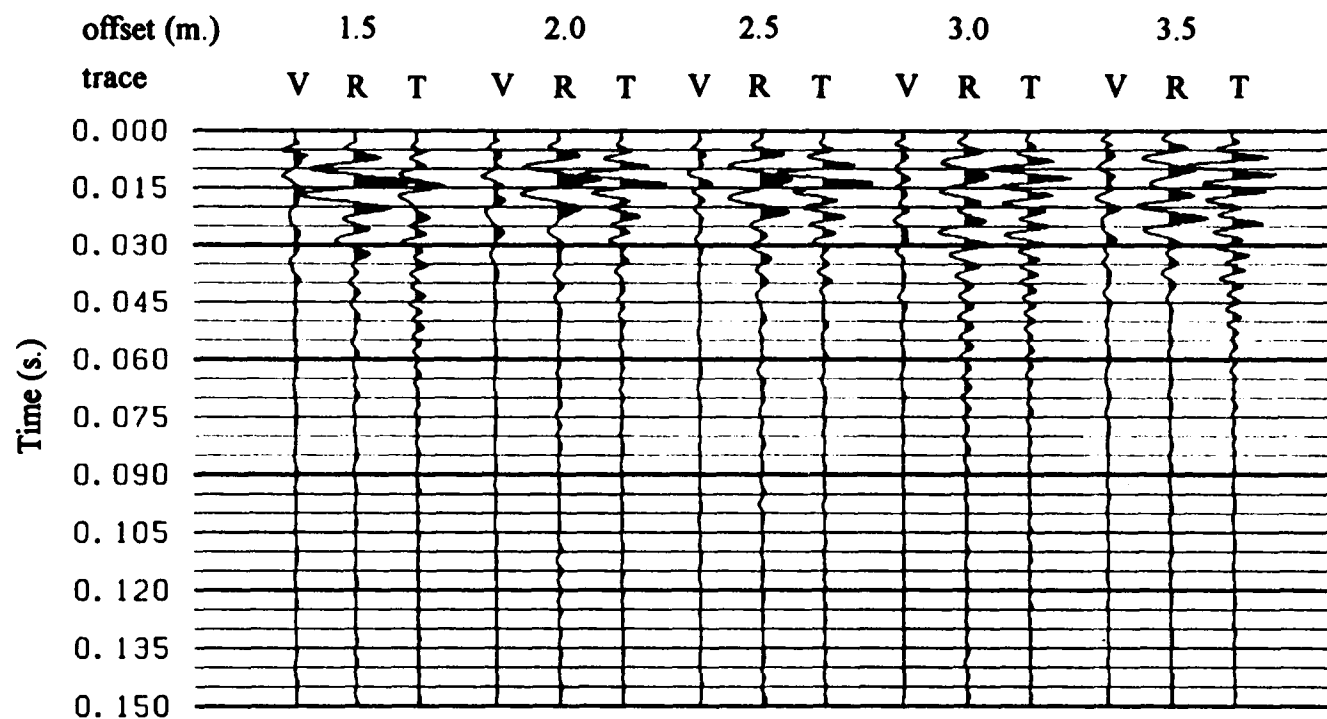


Figure 16. True-amplitude plots of data recorded by a typical geophone in the Oak Ridge experiment. Traces were corrected for spherical divergence, attenuation, and time delays associated with the geophone offset. V- vertical component, R- radial component, T- transverse component.

component 100 Hz geophones evenly spaced along the profile line at 1.0 m. intervals. The seismic source was a 21 mm. (8 gauge) shotgun fired into the ground at 4.0 m. intervals. A data set produced by each shot consisted of 96 seismic traces. Each of the recorded traces was corrected for spherical divergence, and time delays associated with the geophone spacing. The geophones used in this study were the same as the ones used during the Oak Ridge experiment. An analysis of direct arrivals provided the local compressional wave velocity of the weathered subsurface layer at 450 m/s. The analysis of the shear wave data collected at the same site provided the local shear wave velocity at 225 m/s. The value of the Poisson's ratio is therefore $1/3$. The characteristic soil density of the site was assumed to be 1690 kg/m^3 .

I analyzed the seismic signals recorded by three geophones at different positions from shots selected on the basis of considerations outlined above. The selected geophones were labeled so that the geophone number corresponds to its offset from the first shot rounded to the closest integer. True-amplitude plots of data recorded by a typical geophone in the California experiment are shown in Figure 17.

The recorded signal consisted of multiple arrivals with a wide range of frequencies, amplitudes, apparent velocities, and angles of incidence. Figures 18 and 19 show theoretically calculated amplification ratios of horizontal and vertical displacements of a geophone, using different angles of incidence. In calculating these responses I used equations (30)-(35) evaluated for two different values of Poisson's ratio. Figure 18 presents the geophone response for the Poisson's ratio of $1/3$, while the geophone response for that of 0.46 is shown in Figure 19. Note that the horizontal scale for the

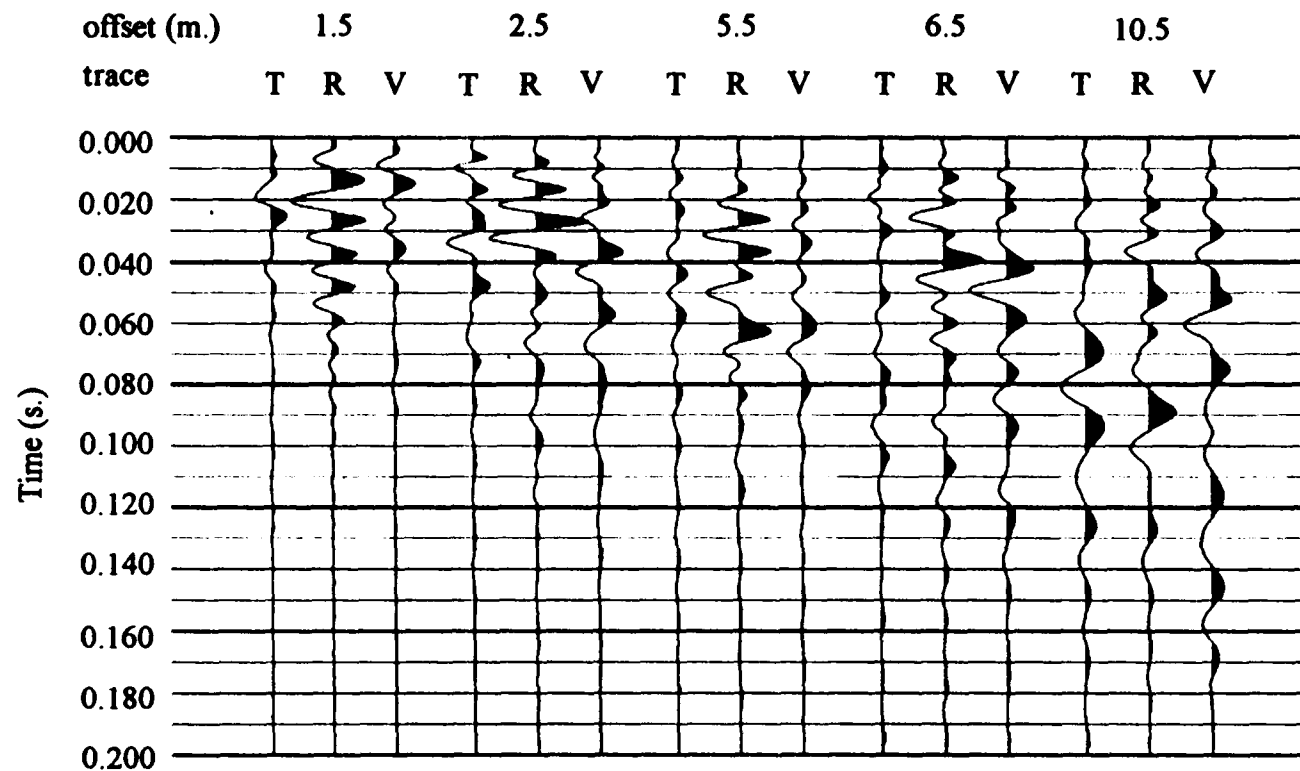


Figure 17. True-amplitude plots of data recorded by a typical geophone in the California experiment. Traces were corrected for spherical divergence and time delays associated with the geophone offset. V- vertical component, R- radial component, T- transverse component.

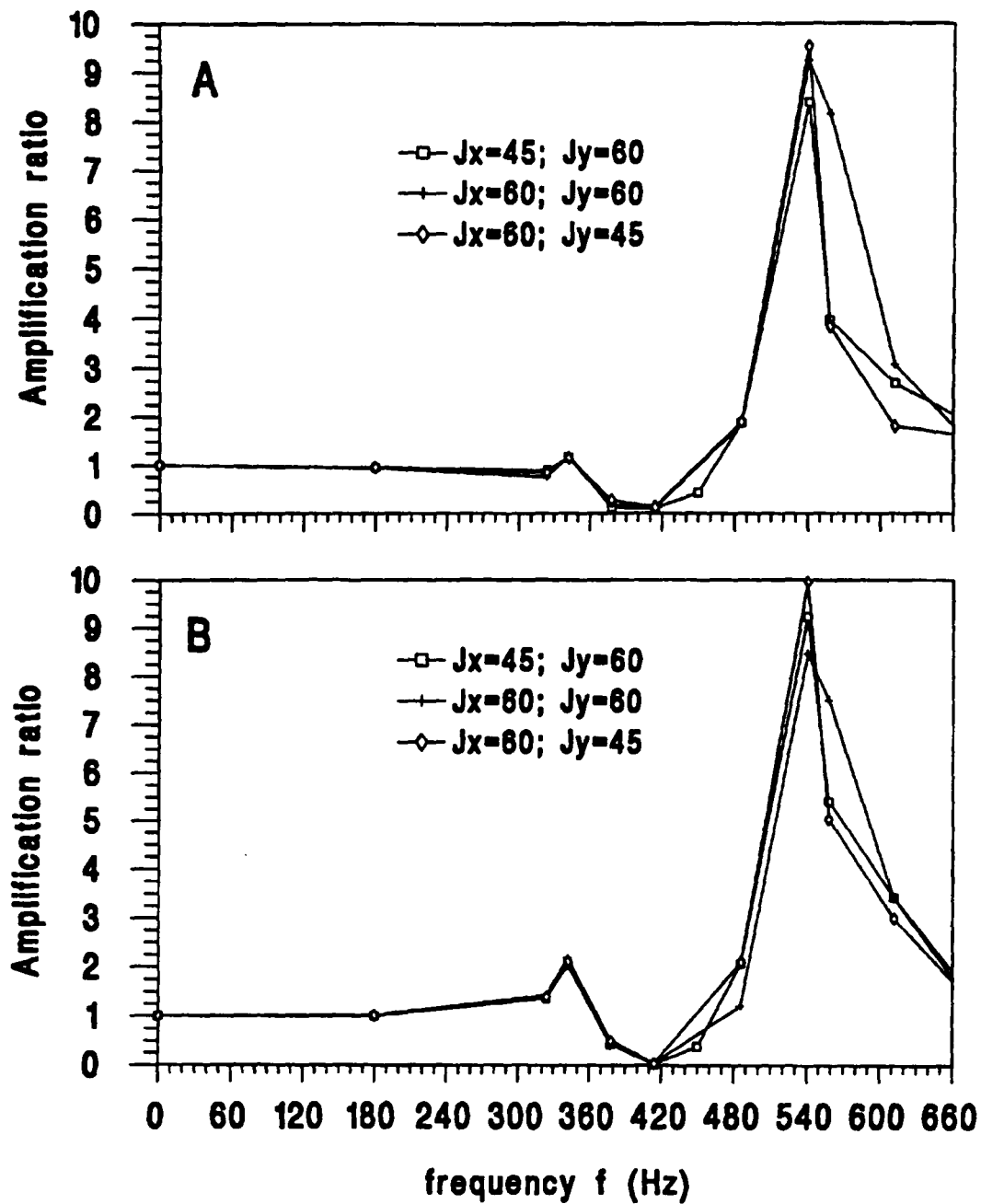


Figure 18. Amplification ratios as functions of frequency (f) calculated for different angles of incidence. Poisson's ratio ν is $1/3$.
 (A) Z component / Y component; (B) X component / Y component.

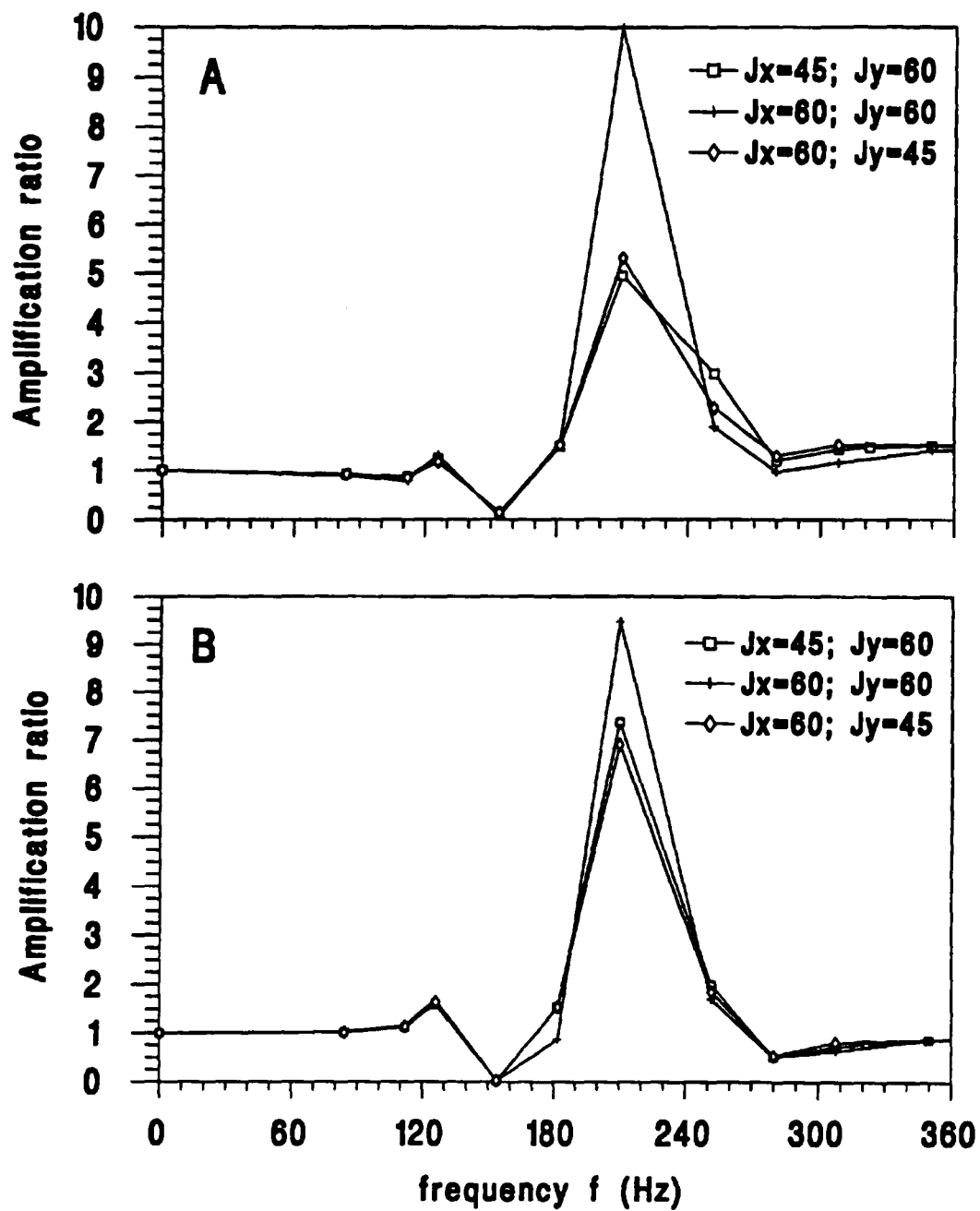


Figure 19. Amplification ratios as functions of frequency (f) calculated for different angles of incidence. Poisson's ratio ν is 0.46.
 (A) Z component / Y component; (B) X component / Y component.

plots is now represented by the frequency (f) in Hz which is computed from the previously used dimensionless frequency (p) as

$$f = \frac{pV_p}{2\pi r_0} \quad (57)$$

It can be seen that although the resonant frequencies are different for different values of Poisson's ratio, the physical behavior of the response curves does not change. While the amplitude ratio strongly depends on the angle at which an incident wave approaches the surface, the resonant frequency of amplification does not, and is the same for all angles of incidence. The resonant frequency of an amplification curve is solely defined by the parameters of the soil-geophone system, and is independent of the incident wave itself. All of the plots show four characteristic frequencies, and associated with them frequency subdomains, each depicting a specific behavior. The signal recorded by one component becomes progressively more dominant over the other as indicated by the slope of their ratio curve. In the present case the frequencies of special interest are 125 Hz, 155 Hz, 220 Hz, 280 Hz for the Poisson's ratio of 0.46 and 345 Hz, 415 Hz, 550 Hz for the Poisson's ratio of 1/3. Every arrival within a seismic trace recorded by the geophone with the measured characteristics is therefore expected to be amplified or attenuated in one of these frequency ranges. This approach allows analysis of the frequency trends shown by a seismic trace as a whole, without singling out separate arrivals, which would have required a detailed knowledge of their origin.

I used multiple-taper spectral analysis to resolve the seismic signals in the

frequency domain (Lees and Park, 1995). The advantage of this approach over a single-taper or autoregressive analyses is that it uses a specially designed orthonormal sequences of tapers to minimize spectral leakage and reduce the variance of the overall spectrum estimate. This leads to a clearer appearance of the dominant signal components in the frequency domain. In my analysis I used five 3-Pi prolate Slepian tapers in combination with the *hires* spectrum estimator whose characteristics are outlined by Lees and Park (1995). The spectral analysis was performed on each of the seismic traces, and the resultant spectra for the transverse and vertical components were divided by that of the radial component to partially eliminate the influence of the seismic source, and produce the data sets comparable in representation to the theoretically obtained ones. The data obtained during the Oak Ridge experiment were taken using 0.5 ms sampling interval and a trace of 150 ms duration. The data obtained during the California experiment were taken using 0.25 ms sampling interval and a trace of 200 ms duration. The useful signal frequency was in the range 60-360 Hz for both data sets. The results of the spectral analysis are presented in Figures 20 to 26. Figures 20 to 23 show the spectral ratios obtained using the seismic data acquired during the Oak Ridge experiment, and Figures 24 to 26 show the spectral ratios obtained using the data acquired during the California experiment.

Oak Ridge data. The spectral analysis indicates that the theory successfully predicted the basic trends in the signal behavior, and the sequence of their appearance in the frequency domain for all considered geophones. All plots show four frequency subdomains, each consistent with the results of the theoretical consideration of the

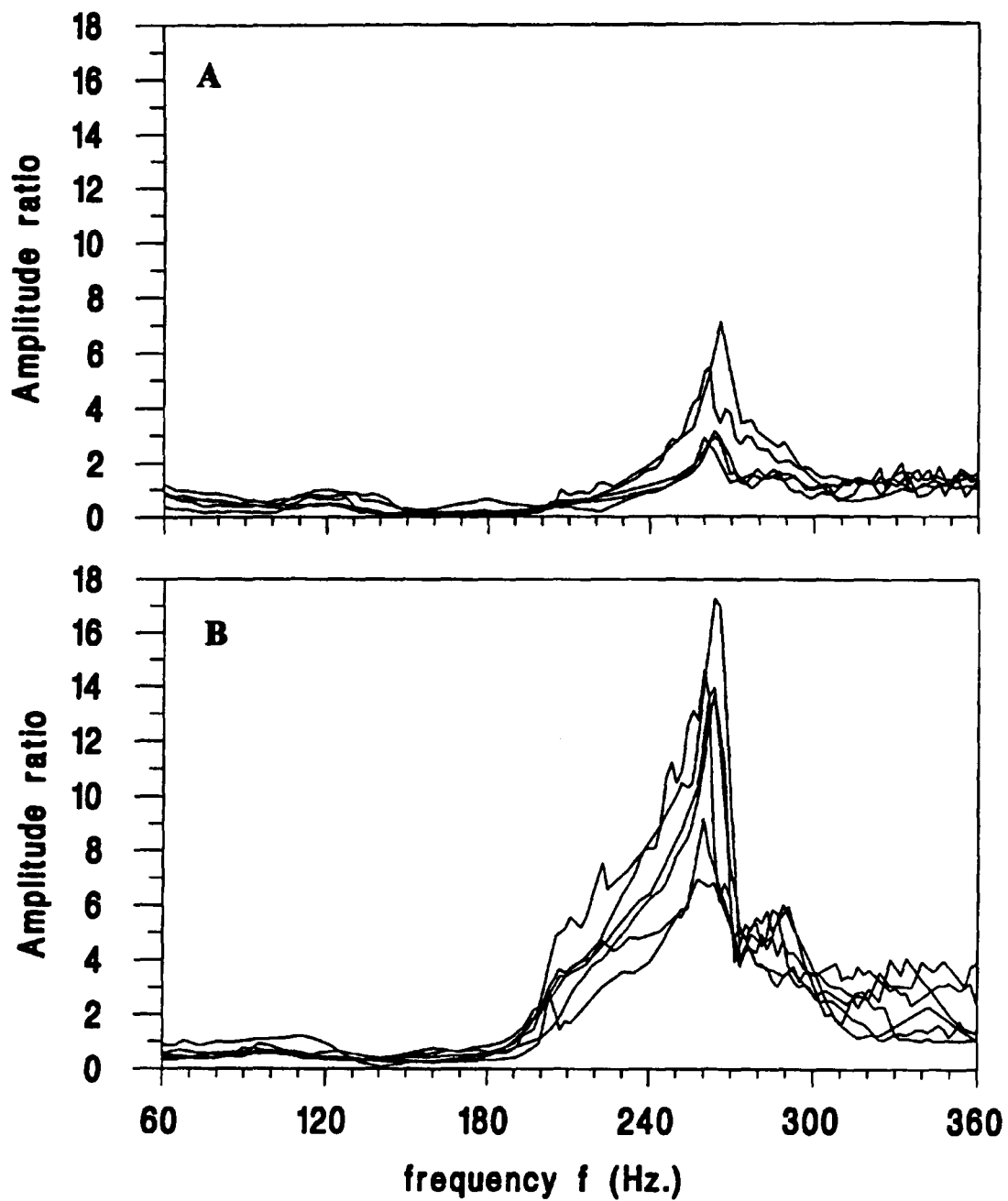


Figure 20. Amplitude ratios of the signals recorded by geophone 4 in the Oak Ridge seismic reflection experiment. (A) Vertical / Radial; (B) Transverse / Radial.

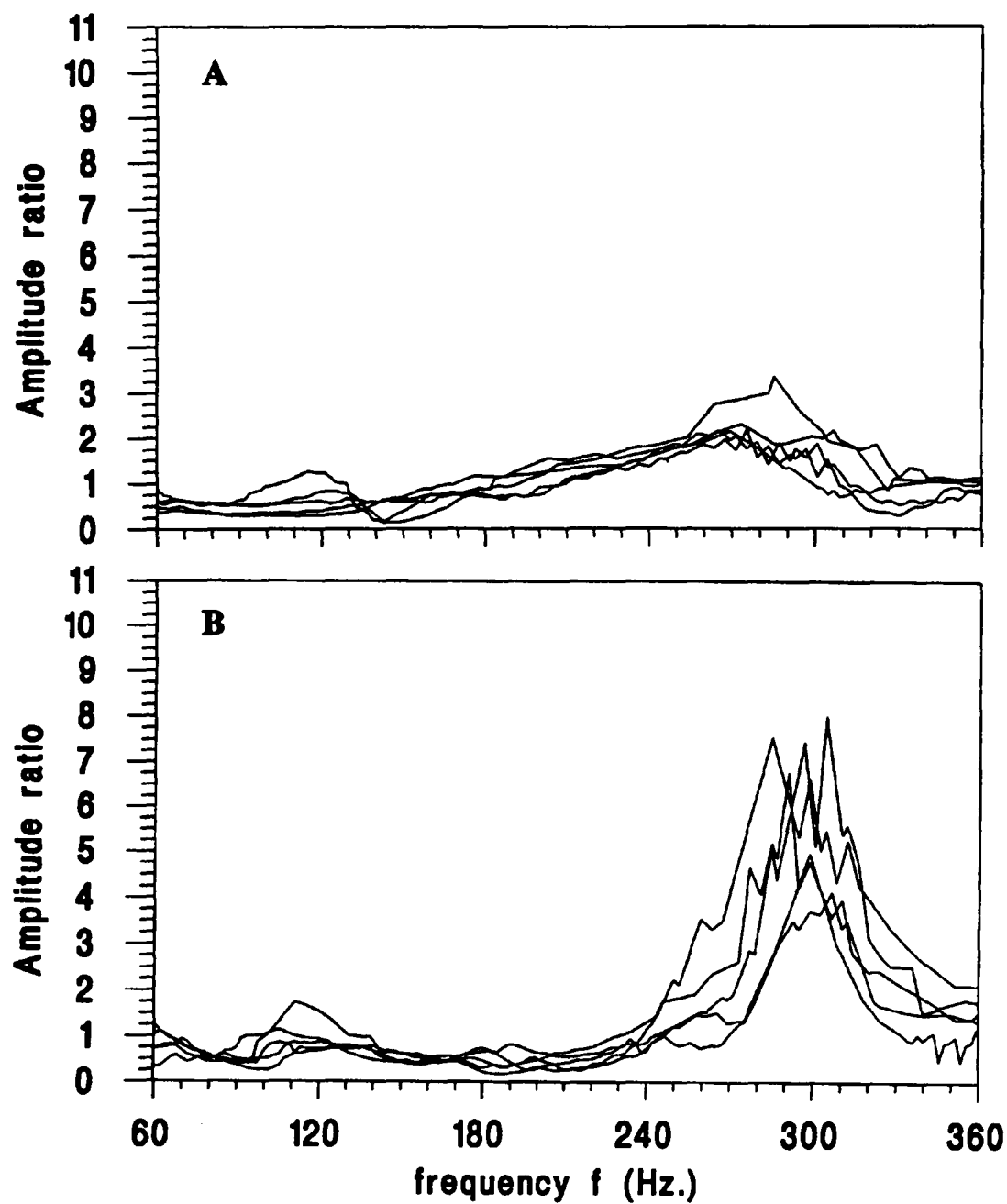


Figure 21. Amplitude ratios of the signals recorded by geophone 5 in the Oak Ridge seismic reflection experiment. (A) Vertical / Radial; (B) Transverse / Radial.

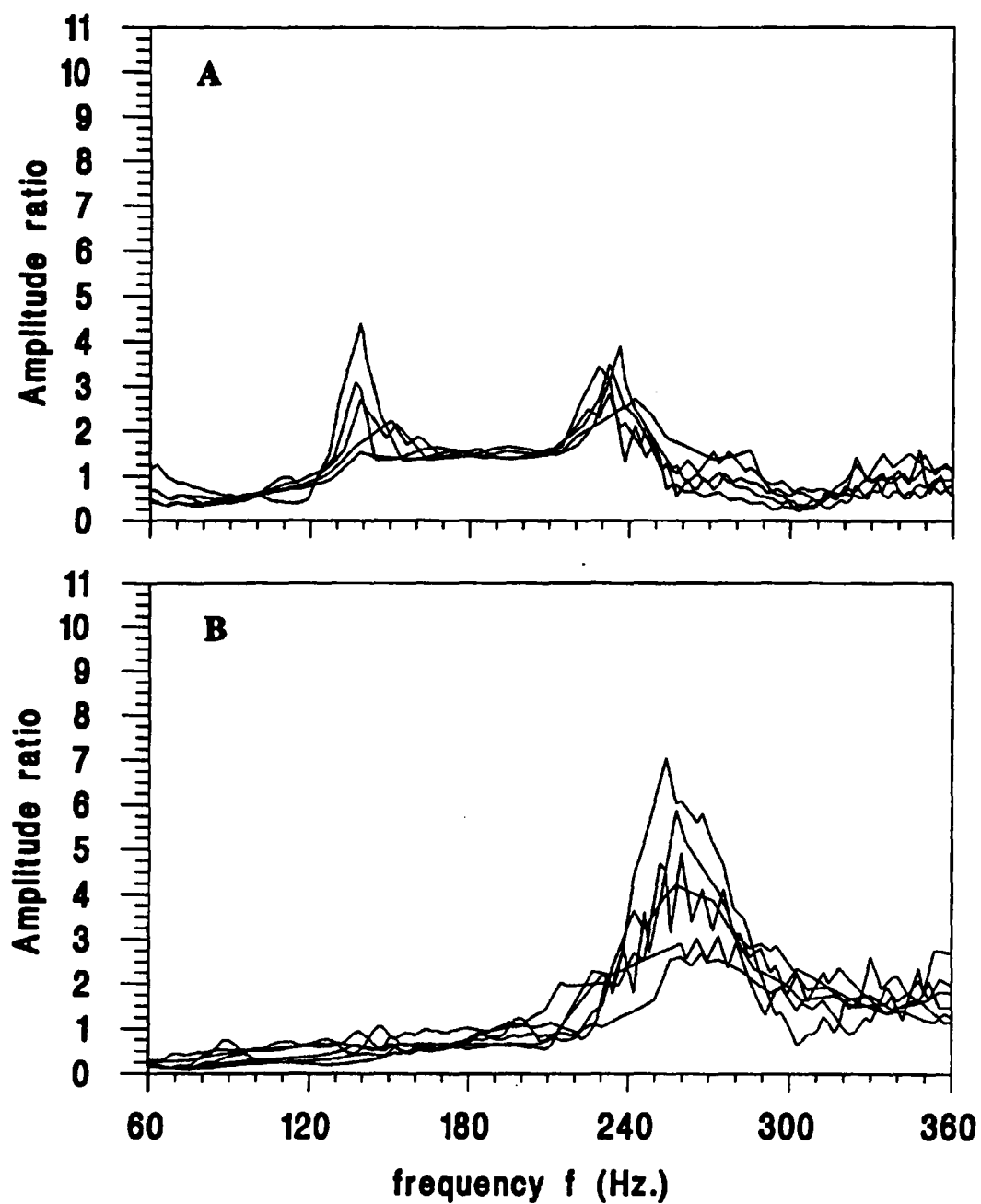


Figure 22. Amplitude ratios of the signals recorded by geophone 6 in the Oak Ridge seismic reflection experiment. (A) Vertical / Radial; (B) Transverse / Radial.

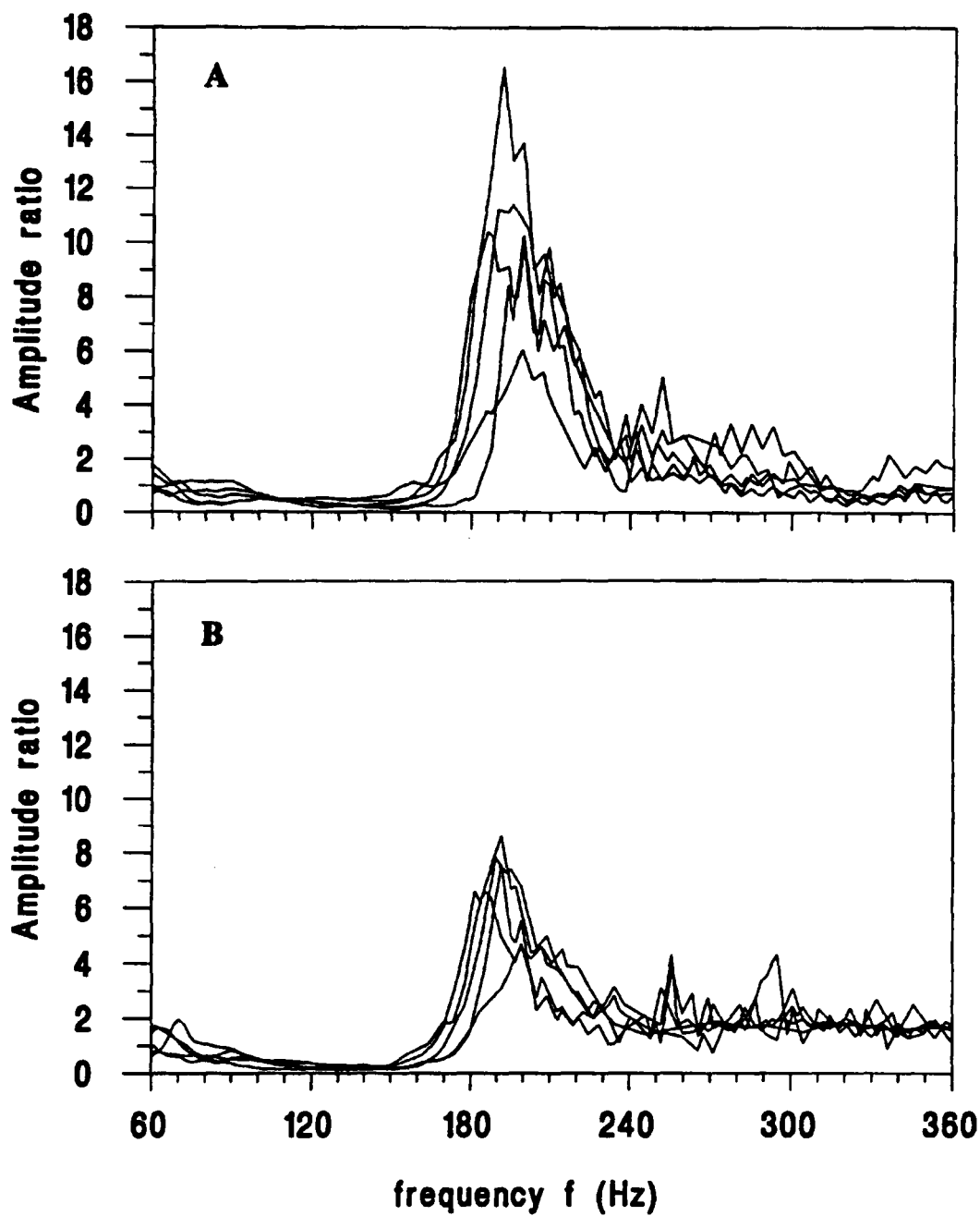


Figure 23. Amplitude ratios of the signals recorded by geophone 9 in the Oak Ridge seismic reflection experiment. (A) Vertical / Radial; (B) Transverse / Radial.

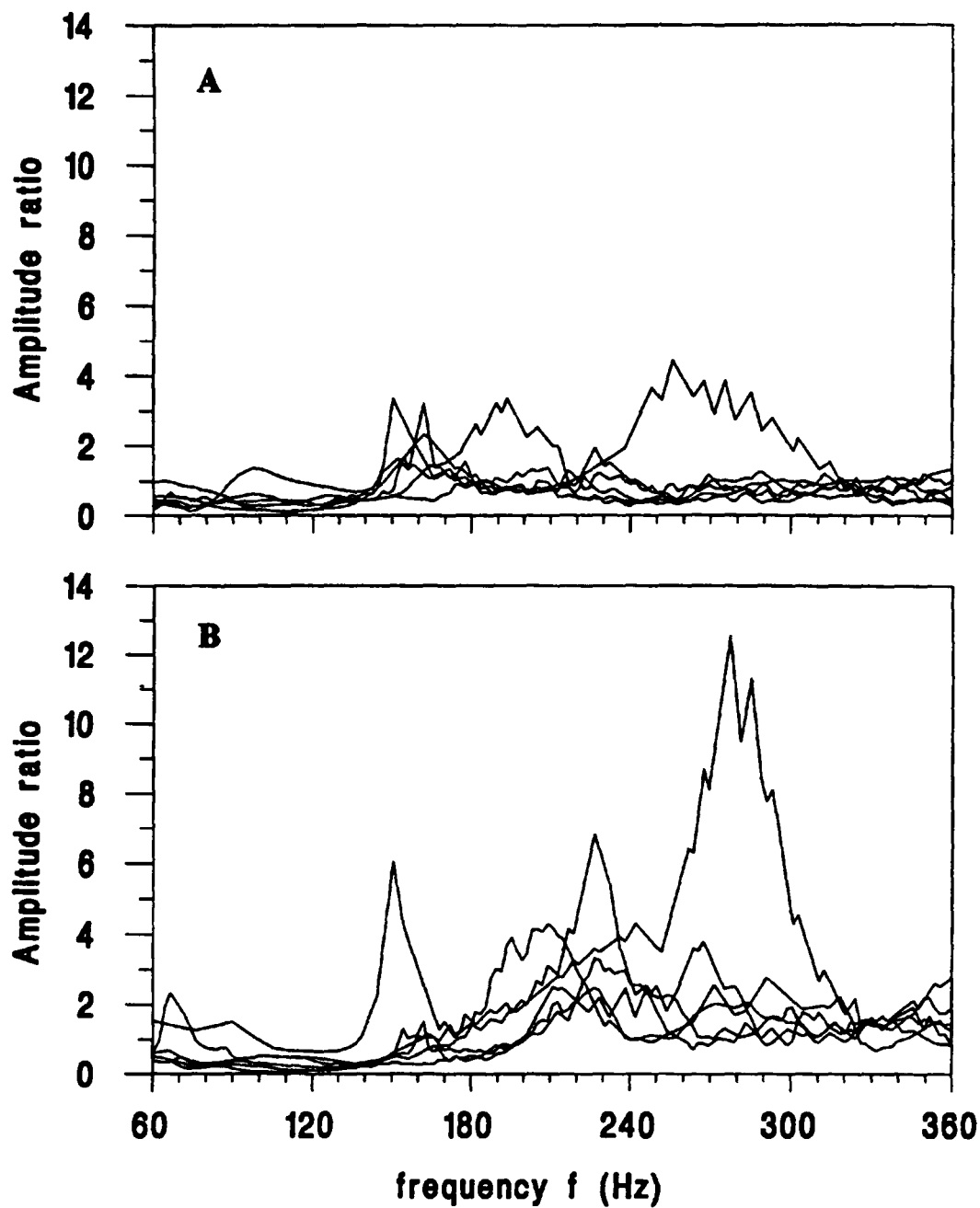


Figure 24. Amplitude ratios of the signals recorded by geophone 2 in the California seismic reflection experiment. (A) Vertical / Radial; (B) Transverse / Radial.

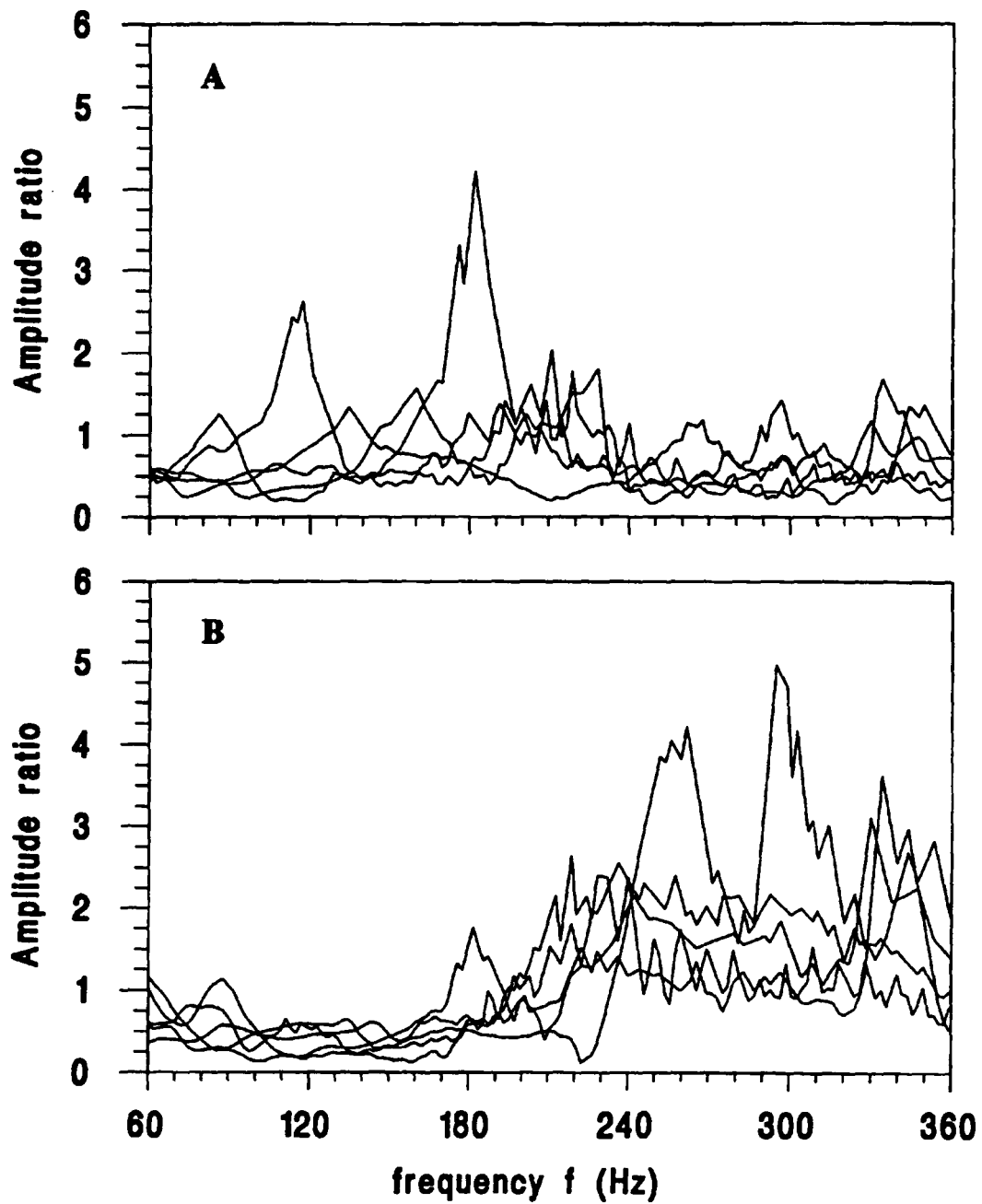


Figure 25. Amplitude ratios of the signals recorded by geophone 3 in the California seismic reflection experiment. (A) Vertical / Radial; (B) Transverse / Radial.

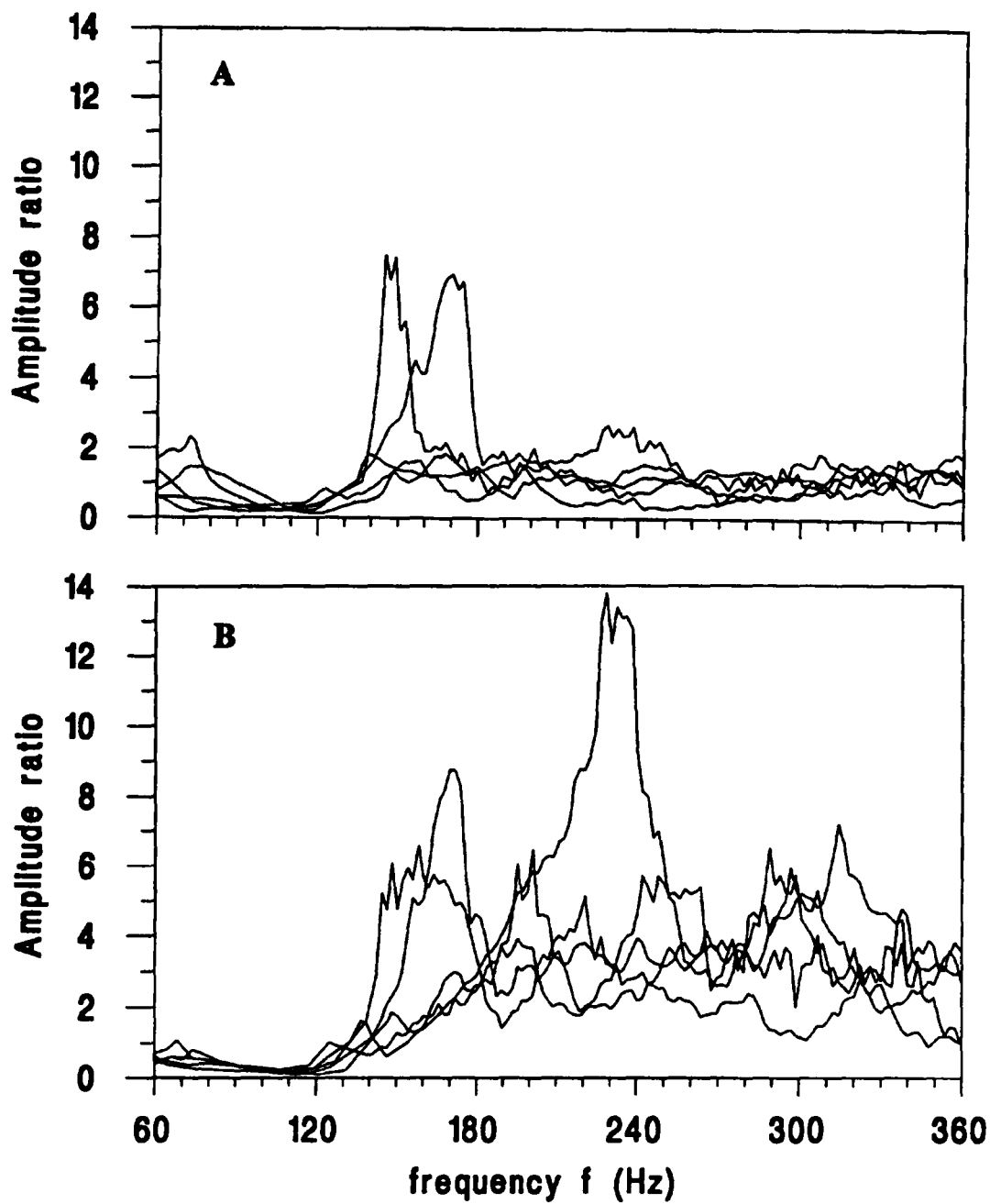


Figure 26. Amplitude ratios of the signals recorded by geophone 6 in the California seismic reflection experiment. (A) Vertical / Radial; (B) Transverse / Radial.

respective components. Each geophone has several resonant frequencies, remaining the same for all seismic signals recorded by a geophone, consistent with the theoretical estimates shown in Figures 18 and 19. In the first subdomain the signals recorded on the radial component dominate those recorded on the transverse and vertical components, with the corresponding ratios decreasing with increasing frequency. This pattern governs the frequency range of 60-100 Hz for geophones 4, 5, and 6 and the frequency range of 60-70 Hz for geophone 9. In the second subdomain the signal recorded on the transverse and vertical components becomes progressively more significant, in most cases eventually exceeding the signal recorded by radial component at the frequency of its maximum amplification. This behavior is characteristic for the frequency range of 100-140 Hz for geophones 4, 5, and 6 and the frequency range of 70-100 Hz for geophone 9. The pattern of the third subdomain repeats that observed in the first with the frequency range of 140-190 Hz for geophones 4, 6; 140-200 Hz for the transverse component of geophone 5, and 100-160 Hz. for geophone 9. The last subdomain shows a sharp increase of the signal recorded by the radial component relative to the other two components.

These results suggest that coupling effects are responsible for the strong appearance of the signals actually observed in the traces recorded by transverse and vertical geophone components, which cannot be explained by the experimental parameters alone. The low shear modulus, absence of any significant subsurface features, and nearly constant compressional velocity of the subsurface layer (Goldman, 1994) all suggest that compressional waves produced at a shotpoint travel nearly horizontally along the array. This would primarily excite the radial geophone component, coinciding with the direction

of the profile line (Aki and Richards, vol. 1, 1980). Consequently this would induce little or no response on the other two components. But the signal amplifications due to the coupling phenomenon result in a strong response to these waves on all three instrument components.

The spectral analysis of the California data do not indicate any consistent trends in the spectral ratio. The higher degree of soil consolidation therefore keeps the influence of the coupling effects above the upper limit of the signal in the frequency domain, which is also predicted by the theoretical estimates (see Figure 18), but may become important for the seismic experiments conducted using the higher frequency range.

The experimental results illustrate that the recorded seismic signal may be significantly distorted due to the interaction of a seismic recorder with an incident seismic wave. It emphasizes the importance of careful consideration of this phenomenon when it is expected to affect a useful frequency range. The ultimate solution to this problem would be the creation of a frequency filter suited to a particular geophone-soil system. The large number of geophones employed in a survey makes this task unreasonably time consuming, but test shots prior to the actual experiment can help to determine which geophones may be subjected to a particularly strong coupling influence. The traces recorded by these geophones may be eliminated from an overall consideration, or the problem of high signal amplifications may be solved in some other appropriate way, for example by increasing the geophone base area. In the particular case of the considered data, geophones 4, and 9 employed in the Oak Ridge seismic experiment provide a good example, as they show higher amplifications compared to geophones 5 and 6.

While the trends in the obtained curves and their sequential order are predicted by the theory, the characteristic frequencies defining their boundaries differ from those calculated theoretically, with the difference ranging from 10 Hz to 80 Hz. Several factors possibly contribute to this inconsistency. Variations in the soil conditions for different geophones, together with variations in the geophone parameters, especially in the position of the center of the instrument's mass within the geophone case, were shown to cause a significant difference in the signal amplification. As noted above, the seismic signals analyzed in the present study consist of a large number of arrivals evidencing a wide range of frequencies, apparent velocities and angles of incidence. The total amplification amplitudes and frequencies thus become a combination of single amplification amplitudes and frequencies of each arrival, which creates effects unaccounted by the theory under consideration. When more precise corrections of the recorded seismic signal for the amplification effects of the coupling phenomenon are required, the exact measurements of the instrument parameters and the soil conditions of the installation site, together with the desirable knowledge concerning the origin of each of the arrivals appearing on the seismic record, necessary to estimate their angle of incidence and frequency range, are needed to obtain stricter theoretical predictions.

The results of analyzing real seismic data in all its complexity demonstrate the validity of the developed theory, which predicts sharp amplifications of the seismic signals recorded by different components of a seismic recorder, at the specific frequency ranges. It provides the theoretical base for developing the frequency filters suited for a particular installation site in order to remove the amplification effects resulting from the coupling

phenomenon from the data, and recover the true ground motion.

5. CONCLUSIONS

Theoretical and experimental investigations within this study indicate that a recorded seismic signal may be significantly amplified or attenuated as a result of coupling between the soil and geophone. The amplitude and frequency of the maximum amplification were shown to be a complex functions of the instrument mass, size, and soil properties. Massive geophones with small bases demonstrate low resonant frequencies and high amplifications. Resonant frequency is dependent upon soil consolidation and decreases with an decrease in consolidation.

The offset of a geophone's center of mass from its vertical axis introduces crosstalk between different components. The seismic signal exciting one of the instrument components is simultaneously recorded by the other two, with the relative observed amplitudes being defined by the angle at which the incident seismic wave approaches the surface. This effect was shown to be responsible for the sharp amplifications of the seismic signals recorded by different geophone components observed in the analyzed seismic reflection data at specific frequencies. The excessive rotations of the geophone base and their amplification at a resonant frequency were shown to result in the attenuation of the recorded signal.

The theoretical considerations within this study were based on the equations of the linear theory of elasticity. Although the physical processes responsible for the amplification effects of coupling are clearly defined by this approach, the consideration of non-linear effects associated with the seismic wave propagation in the Earth would

provide a better estimates of the resonant frequencies and amplification magnitudes.

Based on the results of this study the following measures can be taken to minimize the distortion of the recorded seismic signal due to the coupling phenomenon:

1. The instrument center of mass should be brought as close to the instrument base as possible, and positioned on the instrument's central axis. This can be achieved by manufacturing the geophones with heavier spikes to balance the mass distribution between the spikes which are buried into the ground and the geophone case itself.
2. The instrument mass should be minimized.
3. The small geophone base was shown to introduce high amplifications and significantly distort the recorded signal. As geophone manufacturing is generally aimed at producing compact geophones, the geophone height should be reduced making its base larger by comparison in order to minimize the potential distortion.
4. It is important to pay attention to an excessive amplification of the horizontal displacements for the seismic waves carrying displacements in the vertical direction, and amplifications of the vertical displacements for the seismic waves carrying displacements on the horizontal direction.
5. When conditions of a field experiment do not allow for a precise measurement of the parameters of every seismic recorder, and local soil conditions as is in the case of a seismic reflection survey, spectral analysis of several test shots would provide information about the amplification effects. The geophones producing high amplifications can then be more thoroughly planted to reduce the coupling effects, or the seismic traces recorded by these instruments can be eliminated.

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APPENDIX

APPENDIX

Instrument Response to the Vertical Ground Motion.

Using (16), (26) is

$$\omega^2 M(u_{zB}^{free} + \frac{P_z}{\mu r_0}(f_{1z} + if_{2z})) = P_z \quad (A1)$$

and therefore

$$P_z = \frac{\omega^2 M u_{zB}^{free}}{1 - \frac{\omega^2 M}{\mu r_0} f_{1z} + i \frac{\omega^2 M}{\mu r_0} f_{2z}} = \frac{\omega^2 M u_{zB}^{free}}{K + iL} \quad (A2)$$

where

$$K = 1 - \frac{\omega^2 M}{\mu r_0} f_{1z} \quad (A3)$$

and

$$L = \frac{\omega^2 M}{\mu r_0} f_{2z} \quad (A4)$$

Rewriting P_z as

$$P_z = i \frac{\omega^2 M u_{zB}^{free} L}{K^2 + L^2} - \frac{\omega^2 M u_{zB}^{free} K}{K^2 + L^2} \quad (A5)$$

it is possible to obtain

$$P_z = \frac{\omega^2 M u_{zB}^{free}}{\sqrt{\left(1 - \frac{\omega^2 M}{\mu r_0} f_{1z}\right)^2 + \frac{\omega^4 M^2}{\mu^2 r_0^2} f_{2z}^2}} \quad (A6)$$

Introducing the dimensionless "plant" parameter E as

$$E = \frac{M}{\pi \rho r_0^3} \quad (A7)$$

dimensionless frequency p as

$$p = \frac{\omega r_0}{V_c} \quad (A8)$$

and expressing the ratio between a compressional velocity in the Earth V_p and a shear velocity in the Earth V_s as

$$\sigma = \frac{V_s}{V_p} = \sqrt{\frac{1-2\nu}{2(1-\nu)}} \quad (A9)$$

where ν is the Poisson's ratio, (A6) becomes

$$P_z = \frac{\omega^2 M u_{zB}^{free}}{\sqrt{\left(1 - \frac{p^2 E \pi}{\sigma^2} f_{1z}\right)^2 + \frac{p^4 E^2 \pi^2}{\sigma^4} f_{2z}^2}} \quad (A10)$$

and using (A1) it is possible to calculate the vertical displacement at the center of the geophone mass as

$$u_{zG} = ABS\left(\frac{P_z}{\omega^2 M}\right) = \frac{u_{zB}^{free}}{\sqrt{\left(1 - \frac{P^2 E \pi}{\sigma^2} f_{1z}\right)^2 + \frac{P^4 E^2 \pi^2}{\sigma^4} f_{2z}^2}} \quad (A11)$$

Denoting the amplification of the recorded seismic signal as

$$Amplification = \frac{u_{zG}}{u_{zB}^{free}} \quad (A12)$$

the expression for the amplification is in the form

$$Amplification = \frac{1}{\sqrt{\left(1 - \frac{P^2 E \pi}{\sigma^2} f_{1z}\right)^2 + \frac{P^4 E^2 \pi^2}{\sigma^4} f_{2z}^2}} \quad (A13)$$

which is consistent with the findings of Hsieh (1961) and Bycroft (1978).

Instrument Response to the Horizontal Ground Motion

From viewing (24)-(29) it can be noted that X and Y horizontal components are not coupled and can be considered independently. For the case under consideration the geophone possesses two planes of symmetry and amplification does not depend on the

free field motion and consequently on the geophone orientation. The amplification function is therefore the same for both horizontal components and this study is restricted to the instrument's Y component.

Equations (24)-(29) show that the translation along the Y axis is coupled with the rotation about the X axis and equations (25), (27) must be solved together in order to evaluate the displacement recorded by the instrument's Y component. Using (16) and (17), (25) and (27) can be rewritten as

$$M\omega^2(u_{yB}^{free} + \frac{P_y}{\mu r_0^3}(f_{1y} + if_{2y})) + \frac{B_z C_x}{\mu r_0^3}(f_{1r} + if_{2r})) = P_y \quad (A14)$$

$$-\frac{\omega^2 I_x C_x}{\mu r_0^3}(f_{1r} + if_{2r}) = -C_x + B_z P_y \quad (A15)$$

calculating C_x as a function of P_y from (A15) and substituting into (A14)

$$P_y = \frac{M\omega^2 u_{yB}^{free} (1 - \frac{\omega^2 I_x}{\mu r_0^3}(f_{1r} + if_{2r}))}{(1 - \frac{M\omega^2}{\mu r_0^3}(f_{1y} + if_{2y}))(1 - \frac{\omega^2 I_x}{\mu r_0^3}(f_{1r} + if_{2r})) - \frac{M\omega^2 B_z}{\mu r_0^3}(f_{1r} + if_{2r})} \quad (A16)$$

For convenience P_y can be rewritten as

$$P_y = i \frac{RN - ST}{T^2 + N^2} - \frac{TR + SN}{T^2 + N^2} \quad (\text{A17})$$

where

$$T = 1 - \frac{\omega^2 I_x}{\mu r_0^3} f_{1r} - \frac{M\omega^2}{\mu r_0} f_{1y} + \frac{M\omega^4 I_x}{\mu^2 r_0^4} f_{1y} f_{1r} - \frac{M\omega^4 I_x}{\mu^2 r_0^4} f_{2y} f_{2r} - \frac{M\omega^2 B_z}{\mu r_0^3} f_{1r} \quad (\text{A18})$$

$$R = M\omega^2 u_{yb}^{free} \left(1 - \frac{\omega^2 I_x}{\mu r_0^3} f_{1r} \right) \quad (\text{A19})$$

$$N = -\frac{\omega^2 I_x}{\mu r_0^3} f_{2r} - \frac{M\omega^2}{\mu r_0} f_{2y} + \frac{M\omega^4 I_x}{\mu^2 r_0^4} (f_{1y} f_{2r} + f_{2y} f_{1r}) - \frac{M\omega^2 B_z}{\mu r_0^3} f_{1r} \quad (\text{A20})$$

$$S = -\frac{M\omega^4 u_{yB}^{free} I_x}{\mu r_0^3} f_{2r} \quad (\text{A21})$$

Noticing that P_y is a complex valued function we evaluate its magnitude as

$$|P_y|^2 = \frac{R^2 + S^2}{T^2 + N^2} \quad (\text{A22})$$

and therefore using (A18)-(A21)

$$|P_y| = M\omega^2 u_{yB}^{free} \frac{\sqrt{\left(1 - \frac{\omega^2 I_x^2}{\mu r_0^3} f_{1r}\right)^2 + \frac{\omega^2 I_x^2}{\mu^2 r_0^6} f_{2r}^2}}{\sqrt{T^2 + N^2}} \quad (A23)$$

and from (A14) the horizontal displacement at the center of the geophone mass in the direction of the Y coordinate axis is

$$|u_{yG}| = ABS\left(\frac{P_y}{M\omega^2}\right) = \frac{u_{yB}^{free} \sqrt{\left(1 - \frac{\omega^2 I_x^2}{Mr_0^3} f_{1r}\right)^2 + \frac{\omega^4 I_x^2}{\mu^2 r_0^6} f_{2r}^2}}{\sqrt{T^2 + N^2}} \quad (A24)$$

where instrument moment of inertia about X axis of the XYZ coordinate system is found as (Robson, 1967)

$$I_x = M\left(\frac{r_0^2}{4} + \frac{h^2}{12}\right) \quad (A25)$$

and for the case under consideration, the instrument height is

$$h = 2B_z \quad (A26)$$

Denoting the amplification of the recorded signal as

$$Amplification = \frac{u_{yG}}{u_{yB}^{free}} \quad (A27)$$

the expression for the amplification is available from (A24).

VITA

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