

To the Graduate Council:

I am submitting herewith a thesis written by Bharat S. Shah entitled "Design and Analysis of a Linear Motor Drive System for Machine Tooling." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

Walter L. Green
Walter L. Green, Major Professor

We have read this thesis and
recommend its acceptance:

Marshall Pace

Accepted for the Council:

Edward G. Cook
Vice Chancellor for
Graduate Studies and Research

Thesis
no.
53-1
July 12

DESIGN AND ANALYSIS OF A LINEAR MOTOR
DRIVE SYSTEM FOR MACHINE TOOLING

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Bharat S. Shah

March 1980

1 116284

DEDICATION

This thesis is dedicated to my

parents

Mr. Subodhchandra M. Shah

and

Mrs. Taxila S. Shah.

ACKNOWLEDGMENTS

The author wishes to acknowledge with deepest appreciation to his major professor, Dr. Walter L. Green, for his advice and patience in the preparation of this thesis. Professor Green has never missed a chance to provide his assistance, encouragement, and constructive criticism when needed in preparation of this thesis. Appreciation is also extended to committee members, Dr. Marshall O. Pace and Dr. J. M. Bailey, for their valuable suggestions and review of the thesis. A special thanks is due to Dr. J. M. Bailey for his occasional technical guidance and great friendly attitude.

For their very capable typing and pleasant manner, thanks to Mrs. Denise Smiddy and Miss Cheryl Williams. The author also wants to extend his gratitude to Mr. Jimmie Killingsworth and Mr. Michael J. Frazier for their help in editing the thesis.

The author wishes to express his appreciation to the Electrical Engineering Department for the research assistantship as well as the graduate teaching assistantship which made his graduate studies possible.

Last, but not least, the author wants to extend his appreciation to his parents, brothers and sister for their encouragement, patience, and assistance during the preparation of this work.

ABSTRACT

This thesis presents the modeling and control system design of a linear motor used for controlling a machine tool system.

The mathematical model of the linear motor was developed using mass, resistance, inductance, flux density, back e.m.f., and length of the control coil as its parameters.

The controllers were designed for the linear motor tool control system using position and velocity feedback loops. Two methods of compensation techniques were used during the course of this work to satisfy system requirements. The performance of the system was judged by the surface finished products and the minimum position error signal. The compensation technique used the Bode diagram, the phase diagram, the step response, and the closed loop response of the system. The analytic compensation technique was based on the high value of the velocity error constant and the ratio of pole (zero) to zero (pole) of the compensator ($\alpha \leq 15$). Whereas in the experimental compensation technique approximate values of RC components of the compensation were selected first and later tuned to achieve minimum position error signal. The analytic design was developed after the experimentally obtained compensation was working well.

The final compensation of the linear motor system obtained by these two different techniques was simulated on the computer using the Continuous System Modeling Program. The results achieved by the computer simulation of different techniques were compared for overall system performance. Finally, the experimentally determined compensation was installed on the linear motor control system.

TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
Linear Motor	2
Scope of Thesis	5
II. SYSTEM COMPONENTS	7
Position Control	7
Description of Components	9
Linear Motor	14
Laser Interferometer	17
Frequency-to-Voltage Converters	17
Numerical Control Unit	18
Direct Current Power Amplifier	19
Air Bearing Slide	19
III. ANALYTICAL DEVELOPMENT OF THE SYSTEM	23
Linear Motor	23
Frequency-to-Voltage Converter	31
Direct Current Power Amplifier	34
Numerical Control Unit	34
IV. DESIGN AND ANALYSIS OF THE LINEAR MOTOR CONTROL SYSTEM . . .	37
Fundamental Design Considerations	37
Lag Compensation	40
Lead Compensation	40
Lag-Lead Compensation	41
System Design	41
The Uncompensated System	43

CHAPTER	PAGE
Selection of Compensation	44
Design 1. Analytical Design (Inner Loop Lag Compensator $\alpha \leq 15$), ...	45
Design 2. Experimental Design and Achievement of Analytic Stability	70
V. CONCLUSIONS AND RECOMMENDATIONS	86
LIST OF REFERENCES.	88
APPENDIXES.	90
Appendix A.	91
Appendix B.	93
Appendix C.	97
Appendix D.	99
VITA.	103

LIST OF TABLES

TABLE	PAGE
4.1. Comparison of Results for Design 1 and Design 2	84
B.1. Parameters of the Linear Motor at Different Positions of the Moving Member	95

LIST OF FIGURES

FIGURES	PAGE
1.1. Axial Flux Distribution in the Linear Motor	3
1.2. Radial Flux Distribution in the Linear Motor	4
2.1. Position Control System	8
2.2. Position Control With Inner Loop Velocity Feedback . . .	10
2.3. The Linear Motor Tool Control System	11
2.4. Block Diagram of the Linear Motor Set Up	12
2.5. Complete Block Diagram of the System Components for Simulation	13
2.6. The Linear Motor and Its Components	15
2.7. The Linear Motor Used for Cutting a Metal Piece	16
2.8. The Base has "v" and Rectangular Notches for Air Bearing Slide	21
2.9. Bottom View of the X Axis Slide	22
3.1. Equivalent Circuit of Linear Motor Drive by a Power Amplifier	25
3.2. Block Diagram of the Transfer Function of the Linear Motor Considering Load Disturbance	28
3.3. Block Diagram of the Transfer Function of the Linear Motor	30
3.4. Position and Transfer Function of the F/V Converter in the System	33
3.5. The Transfer Function Block Diagram of the Complete System	36

FIGURE	PAGE
4.1. Block Diagram of the Uncompensated System	42
4.2. The Frequency Response of $G_2(s)H_2(s)$ of Equation 4.1 .	47
4.3. The Frequency Plot of $G_1(s)H_1(s)$ of Equations 4.11 (Uncompensated) and 4.13 (Compensated)	49
4.4. A CSMP Block Diagram of the Compensated System Without Inner Loop Lag Compensation	51
4.5. The CSMP Program of the Step Response of Design 1 Shown in Figure 4.4	52
4.6. The Step Response of Design 1 (No Inner Lag Compensa- tion) Shown in Figure 4.4	53
4.7. The Frequency Plot of $G_2(s)H_2(s)$ of Equations 4.15 (Uncompensated) and 4.20 (Compensated)	57
4.8. Block Diagram of Compensated (With Inner Lag $\alpha \leq 15$) Velocity Loop of Design 1	59
4.9. The Frequency Plot of $G_1(s)H_1(s)$ of Equations 4.24 (Uncompensated) and 4.26 (Compensated)	61
4.10. (a) Root Locus of $G_1(s)H_1(s)$ of Design 1 of Equation 4.26; (b) Enlargement of the Root Locus at the Origin of Figure 4.10 (a)	63
4.11. A CSMP Block Diagram of the Compensated System ($\alpha \leq 15$)	66
4.12. The CSMP Program of the Compensated System ($\alpha \leq 15$) Shown in Figure 4.11 for the Step Response	67
4.13. The Step Response of the Compensated System ($\alpha \leq 15$) of Figure 4.11	68

FIGURE	PAGE
4.14. The Closed Loop Frequency Response of Equation 4.27 . .	69
4.15. Realization of Compensation by RC Component Which Makes the System Stable	72
4.16. A CSMP Block Diagram of the Experimentally Compensated System (Without Change in Gains in the Velocity Loop) .	73
4.17. The Frequency Plot of $G_1(s)H_1(s)$ of Equation 4.31 for Design 2	75
4.18. The CSMP Program of the Step Response of Design 2 Shown in Figure 4.16	77
4.19. The Step Response of Design 2 Shown in Figure 4.16. . .	78
4.20. A CSMP Block Diagram of the Experimentally Compensated System (With Change in Gains in the Velocity Loop) . .	80
4.21. The Frequency Plot of $G_1(s)H_1(s)$ of Equation 4.35 . . .	82
A.1. A Plot of Approximate Current Required for Different Size Linear Motors at Three Different Positions of Moving Coil for 20 lb. Forces	92
B.1. The Magnitude Plot of the Linear Motor Transfer Func- tion at Three Different Positions of the Moving Member.	96
D.1. (a) General Representation of Compensation Using Operational Amplifier and RC Network; (b) Realization of Lag Compensator; (c) Realization of Lead Compens- ation	100

CHAPTER I

INTRODUCTION

In recent years the demand to produce parts having high quality surface finishes and precision contours in the machine tool industry has accelerated. One area in which these precision parts are required is laser fusion research and development. Surface finishes ranging from one to three microinches, peak to valley, and having a contour accuracy of ten to twenty microinches are desired to insure correct focus and reflection of laser beams. A principal method for producing these parts requires a turning machine using a diamond point tool. The turning machine consists of a spindle which rotates the part at a near constant RPM, two slide axes, usually referred to as the x and y axis, and auxiliary hardware for precision control of these axes.

The x and y slides are normally driven by torque motors. Each torque motor is connected to a lead screw-ball nut combination that is used to convert the rotary motion of the torque motor into linear slide motion. There are two common problems encountered with this system. First, the lead screw will not be exactly straight. Thus for each revolution of the lead screw there will be a slight movement perpendicular to the lead screw axis. This slight movement will cause a cyclic error in the surface finish of the part. The second problem is the friction between the lead screw and ball nut. This friction causes a stick-slip effect in the slide motion.

To circumvent these problems, direct drive systems has been considered. One such system is a linear motor which has been developed

so that a smooth and accurate slide motion can be obtained.

The linear motor produces motion parallel to the control axis. This motion is caused by a force established between a strong magnetic field and current carrying conductors. The detailed construction and working principles are discussed in the following paragraphs.

Linear Motor

Linear motion implied motion along a straight line. A force producing linear motion is termed thrust, propulsion force, or traction force.⁶ A machine which produces this propulsion force is called a linear motor. The linear motor also is said to produce a solenoidal action. The principle of the linear motor can be understood easily with analogy to the conventional rotating motor.⁶ The stator is designated as the primary or exciting coil and the rotor as the secondary or controlling coil. The current flowing through the primary coil creates a strong magnetic field along the axis, producing a force which causes the exciting coil to move. The conventional rotating machine exerts torque about the axis or rotation and angular acceleration of the motor, whereas the linear motor exerts force on the translator or rotor and produces linear acceleration. The axial and radial flux distribution of the linear motor are shown in Figure 1.1 and Figure 1.2, respectively. The theory of coil windings is the same as employed in conventional rotary electric machinery. It should be noted that the rotor or secondary cylinder is made of iron and is hollow. The control coil is wound around the secondary cylinder. The stator or the primary cylinder has an exciting coil and serves as a magnetic return path. The radial component of the

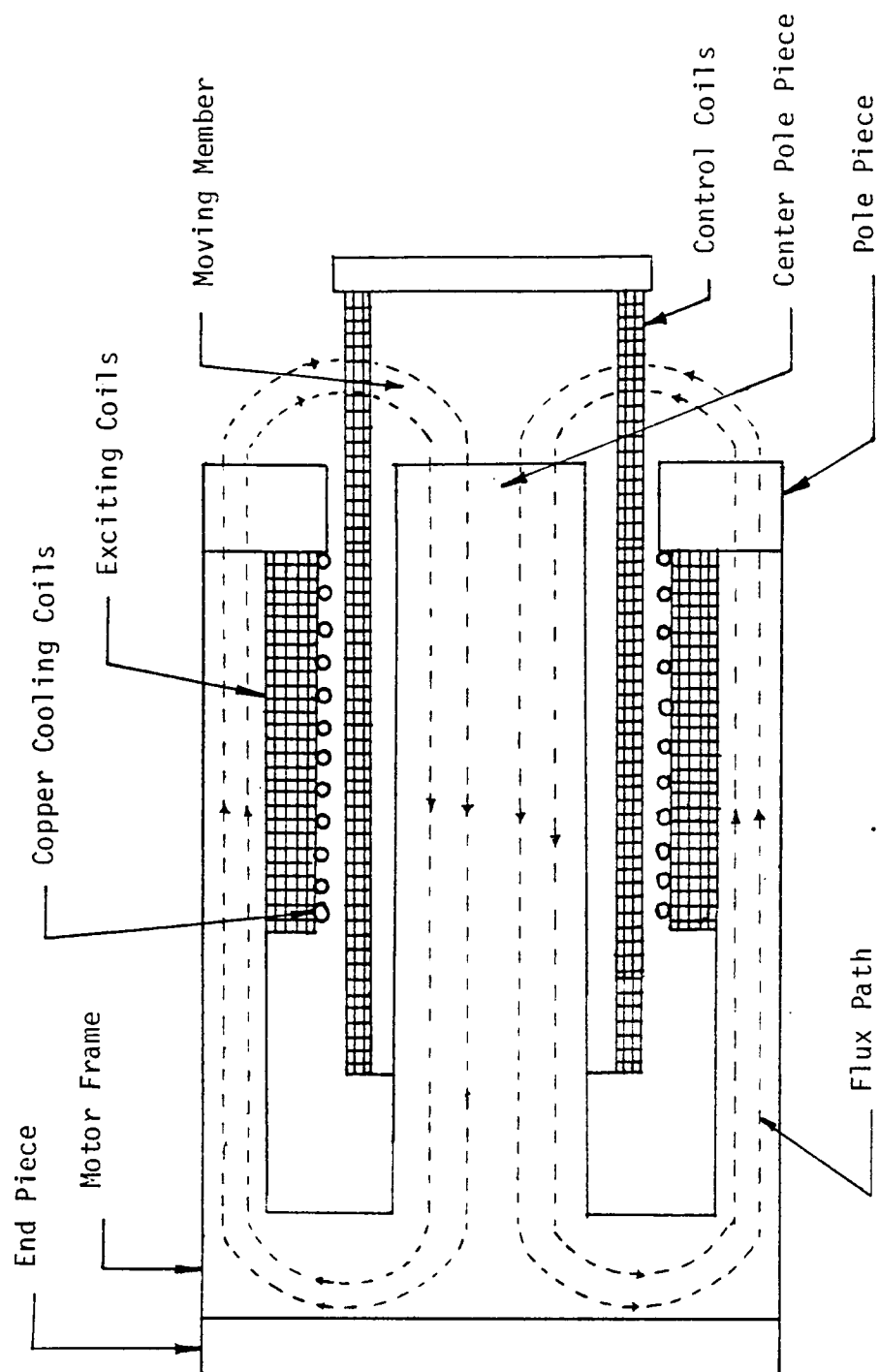


Figure 1.1. Axial Flux Distribution in the Linear Motor.

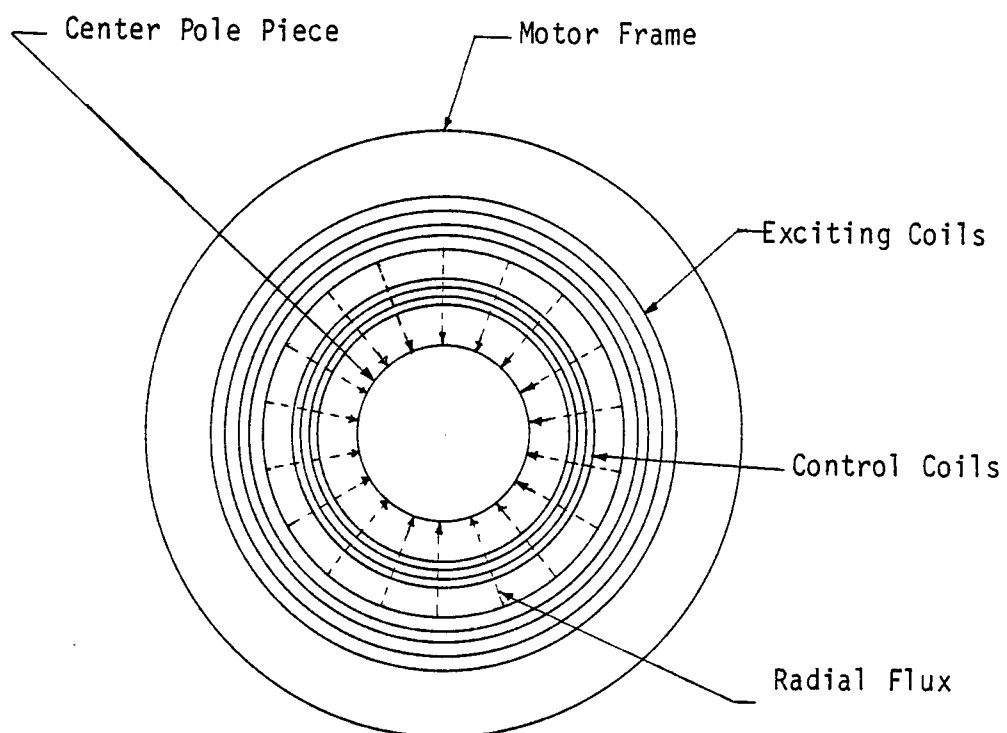


Figure 1.2. Radial Flux Distribution in the Linear Motor.

air gap flux which interacts with the current circulating in the control coil (secondary) to produce an axial force. The direction of an axial force may be found by applying the left hand thumb rule. By changing the direction of the current in the control coil externally, the direction of travel of the moving member or secondary cylinder is changed.⁵

Linear electric motors have broad industrial applications. In most of these applications either the linear "DC" motor, the linear induction motor (LEM) or the linear synchronous motor (LSM)⁶ is used. This thesis presents the analysis and design of a control system utilizing linear "DC" motors for a turning machine having a diamond point tool.

Scope of Thesis

This thesis is devoted to the modeling, simulation, and control of a linear motor drive system. The model will be used to develop a working digital simulation of the total system. Actual test data from an existing prototype hardware system will be used to verify the system simulation.

The advantage of having an accurate simulation model is that control methods can be easily verified without the expense of installing actual hardware on a trial and error basis. In this application, the controlling technique utilizes feedback from the position loop as well as the velocity loop. The controlling technique of a system which is very stiff and insensitive to external disturbances is of great interest. This type of analysis, simulation, and control design will be valuable in developing present as well as future linear motor systems.

The following areas of the linear motor drive system will be considered:

- (1) Development of a complete analytical model of the system using actual system parameters such as mass, flux density, rotor winding resistance, winding inductance and force constants.
- (2) Performance of dynamic tests on the prototype hardware system and modification of the simulation model as necessary for corresponding performance.
- (3) Evaluation of the performance of the system that can be achieved using conventional compensation techniques.

The selection and description of the system components are given in Chapter II. The derivation of the general transfer functions of the basic components are developed in Chapter III. A detailed analysis and design of different compensators is given in Chapter IV. The comparison of the results from the prototype hardware model and simulation using block diagrams of the system are included in Chapter IV. Chapter V contains several suggestions, the conclusion and recommendations for future work.

CHAPTER II

SYSTEM COMPONENTS

An important phase in the design of a control system is the selection of components. In general, this depends on several factors such as economics, individual component requirements, precision, and system performance.

The system components that have been selected for the Linear Machine - Tool Control System include:

- A linear motor,
- A laser interferometer system,
- Frequency-to-voltage converters,
- A Mark Century numerical control unit,
- A direct current power amplifier and
- Compensators.

The criteria used in the selection of the above components will be discussed in this chapter. The development of transfer functions will be presented in Chapter III.

Position Control

In a position control system, it is desirable to have the final position of the output to be in exact correspondence with the reference position. The comparison between the input and the output position yields an error signal which is applied to a controller. The controller develops a correcting signal for the horizontal motion which is delivered to the output shaft to reduce the position error. The block diagram of a basic position control system is shown in Figure 2.1.

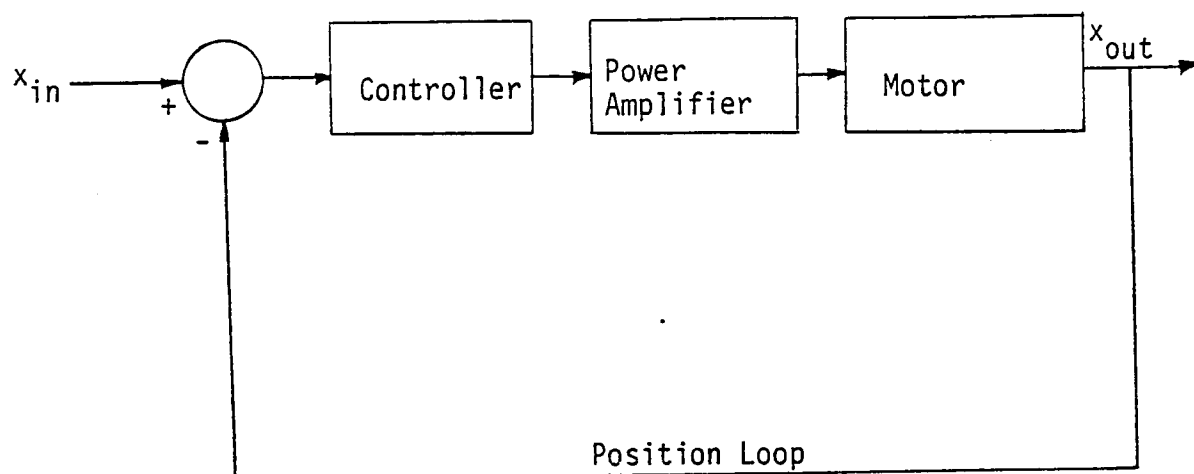


Figure 2.1. Position Control System.

Another configuration which improves the damping of the system is shown in the block diagram of Figure 2.2. In this case, minor loop rate feedback is used to improve the stability of the system by introducing additional damping.² The improvement in the system behavior is discussed at length in Chapter IV.

Description of Components

When the components of the system are selected, it is required that each must satisfy a number of individual specifications involving space limitation, weight, power consumption, voltage level, and others. After the components have been selected, they are coupled together, and their performance as a unit should satisfy system requirements based on sensitivity to disturbance, transient, and steady state specifications.

The linear motor tool control system shown in Figure 2.3 consists of air-bearing slides and an air bearing spindle. The system also includes a linear motor, a laser interferometer system, and a numerical control unit. The simple block diagram of the linear motor configuration is shown in Figure 2.4. Figure 2.5 is a complete block diagram of the system for purposes of simulation. The position command is given in the form of pulses within the numerical control unit. The numerical control unit also receives pulses from the laser interferometer as an output of the system. The laser interferometer detects the position of the linear motor. Hence, the laser interferometer system creates the position feedback loop. The position error signal and the output of the frequency-to-voltage converter are applied through compensation electronics to a direct current power amplifier whose output drives the

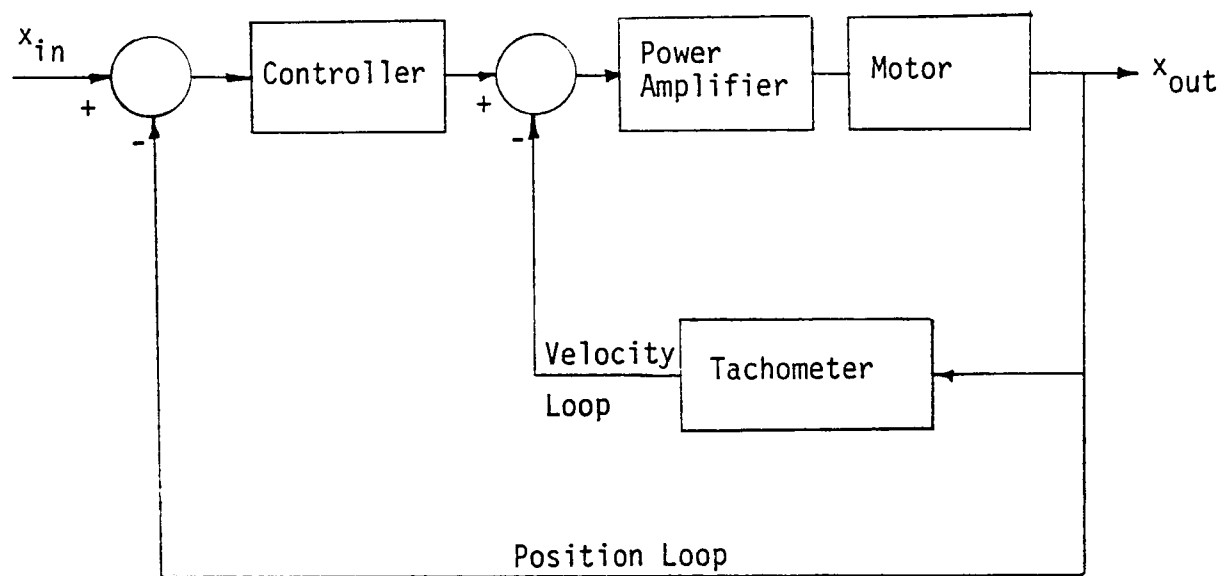


Figure 2.2. Position Control With Inner Loop Velocity Feedback.

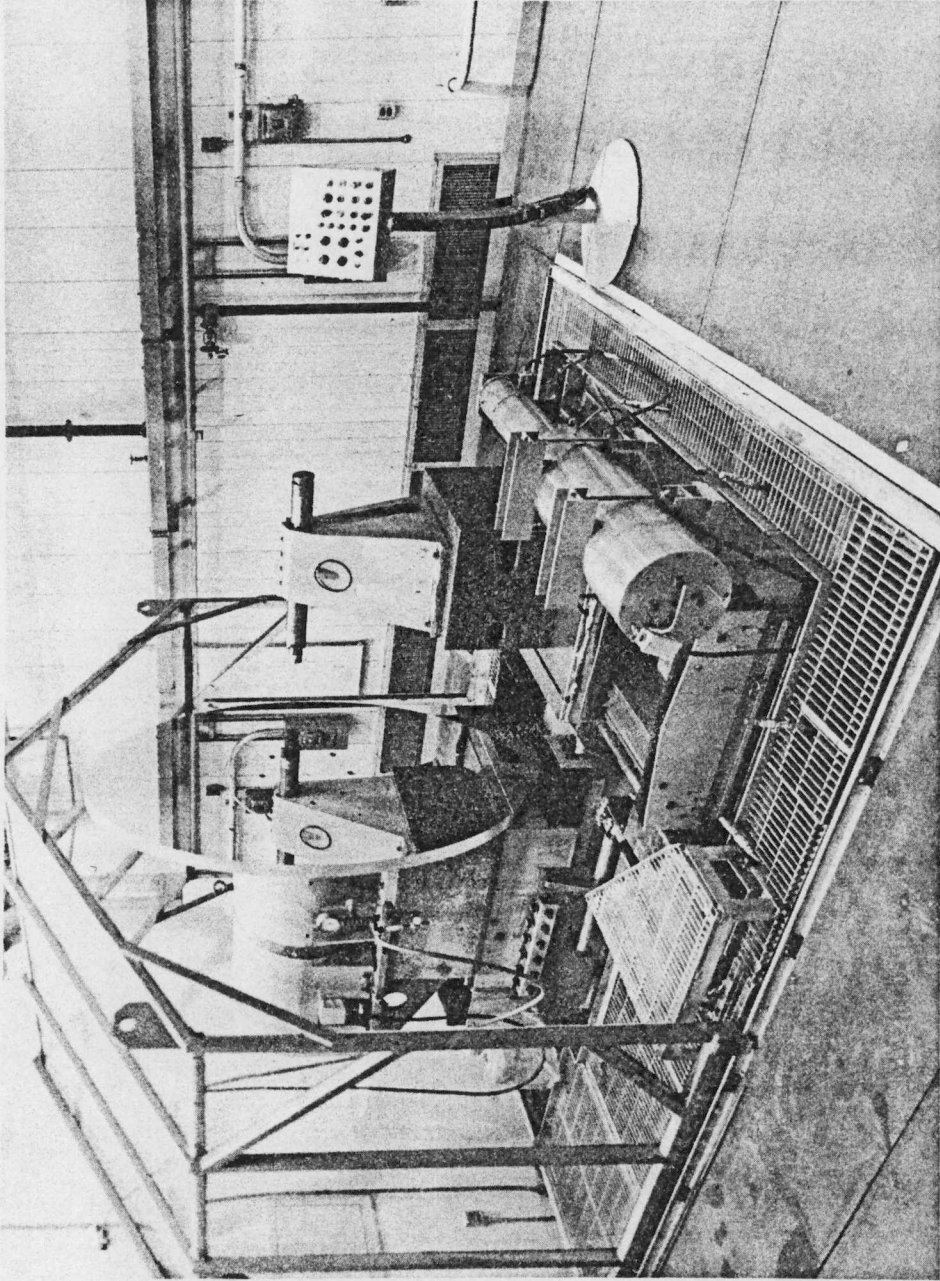


Figure 2.3. The Linear Motor Tool Control System.

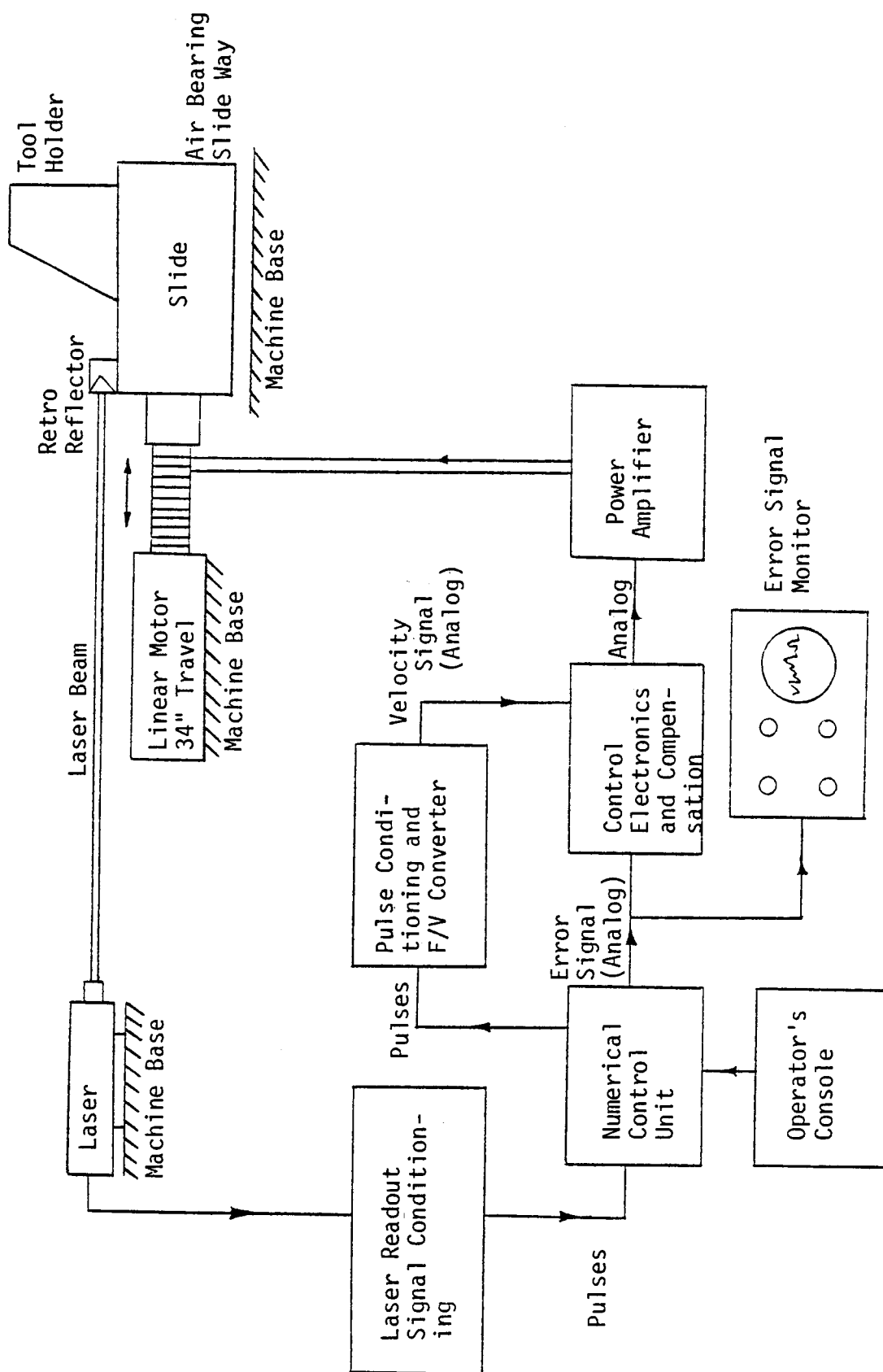


Figure 2.4. Block Diagram of the Linear Motor Set Up.

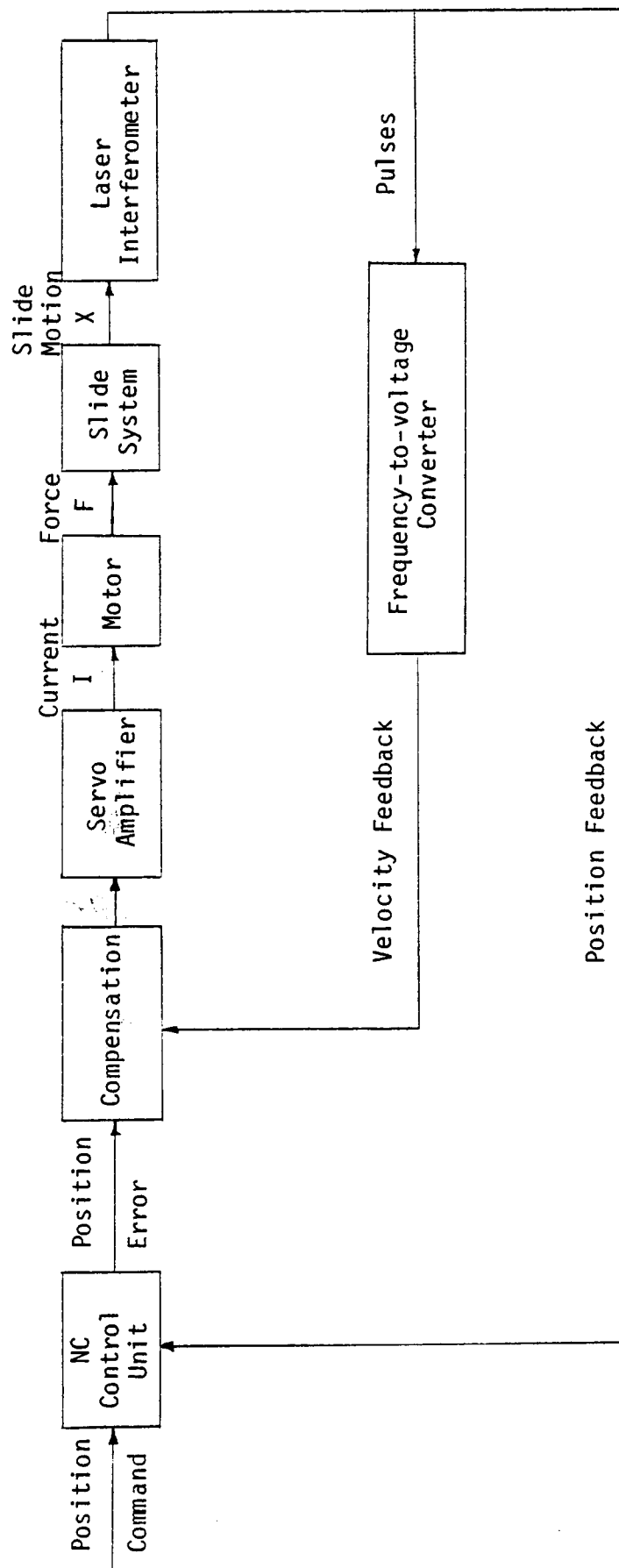


Figure 2.5. Complete Block Diagram of the System Components for Simulation.

linear motor. A detailed description of each of the basic system components is given below.¹

Linear Motor

The linear motor consists mainly of a stator or stationary member and a translator or moving member which surround the central pole piece. A cross section of the linear motor is shown in Figure 2.6. The stator has an exciting coil which generates the direct current magnetic fields. The exciting coil has layers of round wire and has twenty two hundred turns. The maximum current, 11 amperes, is delivered by a 34 volt DC power supply. The copper cooling coils, with circulating water, are provided to cool the stator winding.¹

The control coil, which is wound around the moving member, has two layers of square wire and has seven hundred turns.¹ The supply voltage of the control coil is also direct current. Reversing the polarity of the control coil changes the travel direction of the moving member. The moving member is attached to an air-bearing slide. The operation of the linear motor is analogous to a direct current torque motor except for the travel limit imposed by the moving member stroke. Figure 2.7 shows the moving member, control coil, motor frame, end piece, tool base and mounting for cutting a metal work-piece.

Analysis shows that the axial force is a function of the electric current flowing through the control coil, flux density in the air gap, and length of the control coil. The linear motor is designed to have a nominal force rating of 20.0 pounds with a maximum available force of 50.0 pounds. The force generated is approximately equal to 4.0 times the

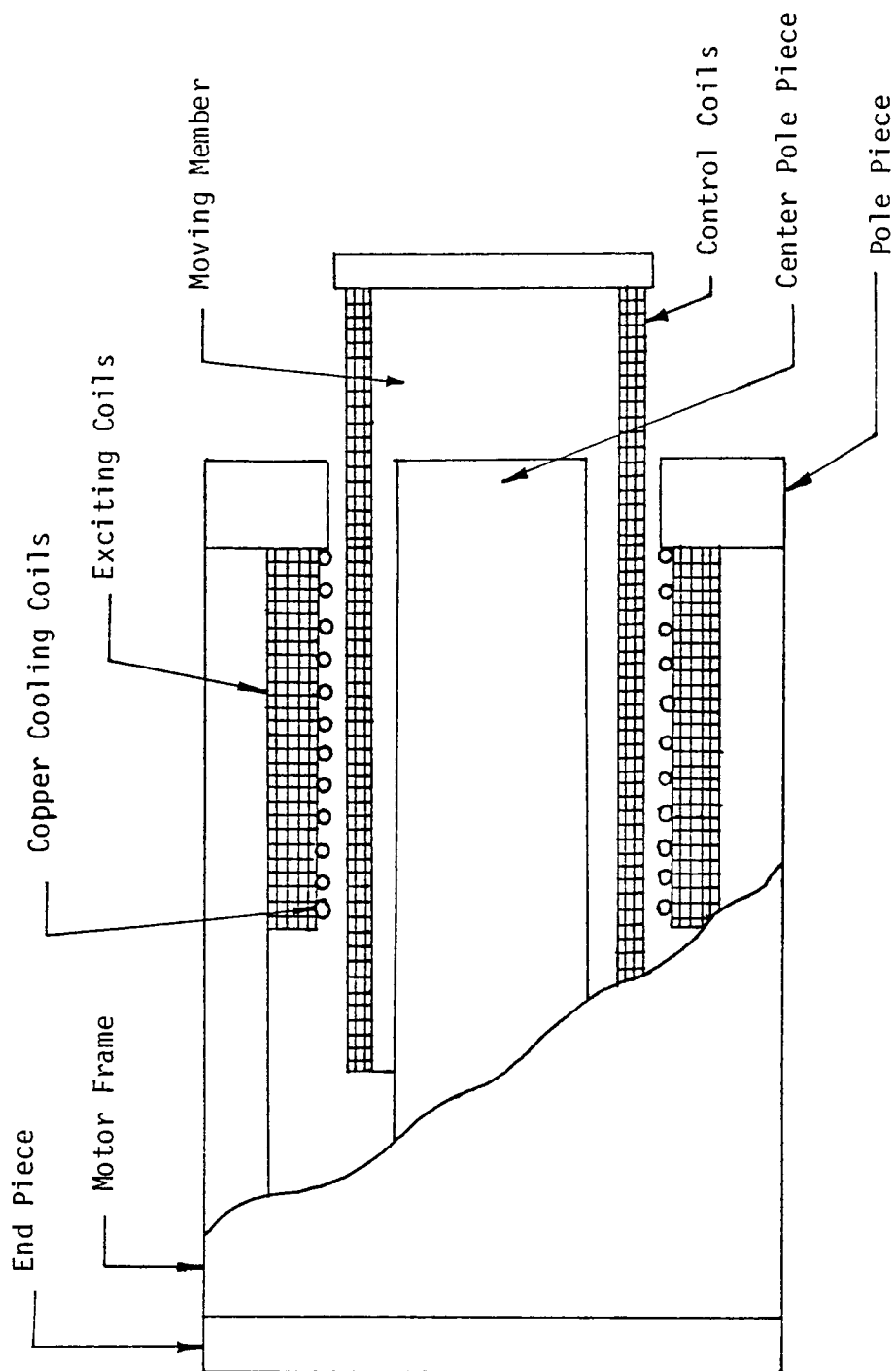
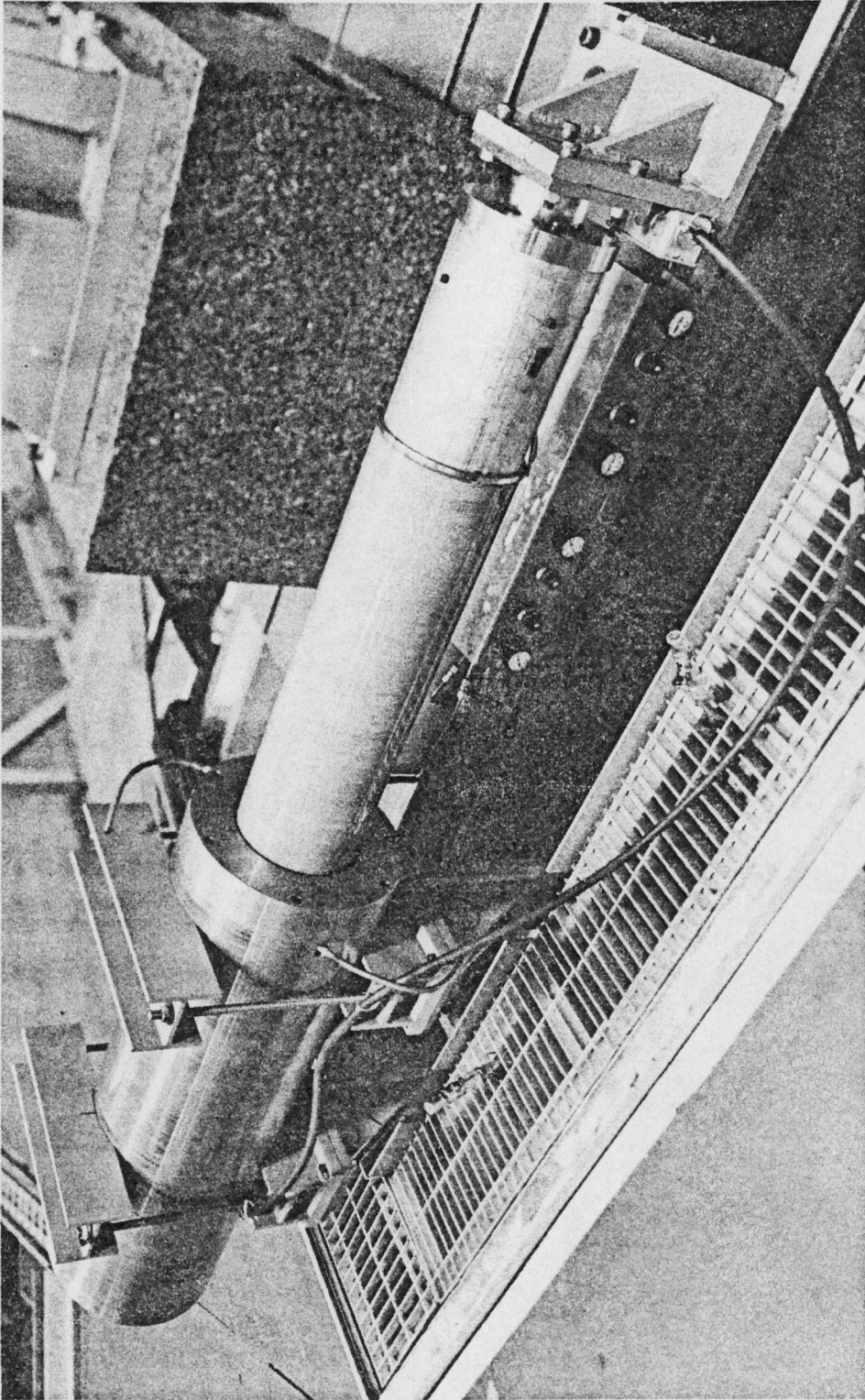


Figure 2.6. The Linear Motor and Its Components.



2.7. The Linear Motor Used For Cutting a Metal Piece.

control coil current in amperes. A plot of approximate current required for different size linear motors at three different positions of moving coil for 20 lb. forces is shown in Appendix A.¹ Control current is responsible for yielding a force to move the moving member both inward and outward.

Laser Interferometer

The machine motion in the horizontal direction is recorded or monitored by a feedback measuring unit, which may be electric, mechanical, optical, electromechanical, or of some other form. For this particular system, Hewlett-Packard laser interferometer produces pulses which are used to determine position. Pulses from the laser interferometer system are also applied to frequency-to-voltage converters (F/V) to determine velocity as explained in the following section. A resolution extender is used to obtain a pulse resolution of approximately 16 millimeters (0.6228 microinch) per pulse.¹ The laser interferometer system has an accuracy of ± 0.5 parts per million and a maximum velocity of 72 inches per minute.

Frequency-to-Voltage Converters

Since the linear motor has no rotating parts or components, it is not feasible to use a tachometer to provide a velocity signal. The information regarding the position of the linear motor obtained through a laser interferometer system is in digital form. The laser pulses are coupled to individual forward and reverse direction frequency-to-voltage (F/V) converters to generate an analog signal. The output of

the frequency-to-voltage converters are located in the feedback path. The full scale input frequency is 10 KiloHertz and the full scale output voltage is 10 volts.

Numerical Control Unit⁸

The Mark Century 7500 Series Numerical Contouring Control continuously directs two motions of a machine tool (normally called the x and y motion) and provides storage for auxiliary machine-functions. To understand how the control performs these functions, it is broken into four major function-areas of the control.

(1) Timing and Reference Signals Area

Signals are present or absent during a particular time period with reference to some established zero time; the timing and reference signals allow the identification of any bit or bits of information.

(2) Computations Area

The Computations area performs almost all of the numerical operations required by the control. It consists of word registers containing binary coded decimal (BCD) data representing the buffer and active position, velocity, departure, and other dimensions for the axes of motion.

(3) Per-Motion Area

The Per-Motion area receives rate and direction signals from the computations area. It generates the necessary signals to run the machine motions in order to execute the programmed commands.

(4) Data Input

The Data Input section of the control accepts, distributes, and stores command information from punched tape or from the manual data keyboard. The data readout may be used to display information contained in any storage area of the control.

In general, the position signals of the linear motor are fed to the numerical control unit to compare it with the input signal. The output of the numerical control unit is in analog form.

Direct Current Power Amplifier

The linear motor must be driven by a power amplifier. The particular amplifier is a 100 watts, Inland, transistorized unit used for small machine tool systems. It is ideally suited for low power industrial drives such as those employed for cutting, drilling, grinding, and milling machines. This versatile transistorized drive system can be controlled either of two ways:

- (1) By a NC control for contouring,
- (2) With switch closures for point-to-point applications.

The output voltage is $\pm 15\text{V.DC}$. The bandwidth of the amplifier is 300 Hertz. The power amplifier contributes a gain of 10 to the system transfer function.

Air Bearing Slide

The linear motor must move a very heavy load. A typical load is 6000 lbs. The base of the linear motor is mounted on hydraulic pads to avoid picking up the building vibrations. The building vibrations

have an adverse effect on part finishes. The base has honed "v" and rectangular notches for linear slide. Figure 2.8 shows the base for both the x and y axes. The x slide carries the diamond tool for cutting a workpiece. The slide rides on the top of the base which is shown in Figure 2.9. The slide has graphite pads in three sections. A nominal 90 psi air pressure applied to the graphite pads causes the slide to be lifted from the base to form a non-contact slide. Hence, large loads can be moved by the relatively small force produced by the linear motor.

The derivation of the transfer function of each component is discussed in Chapter III.

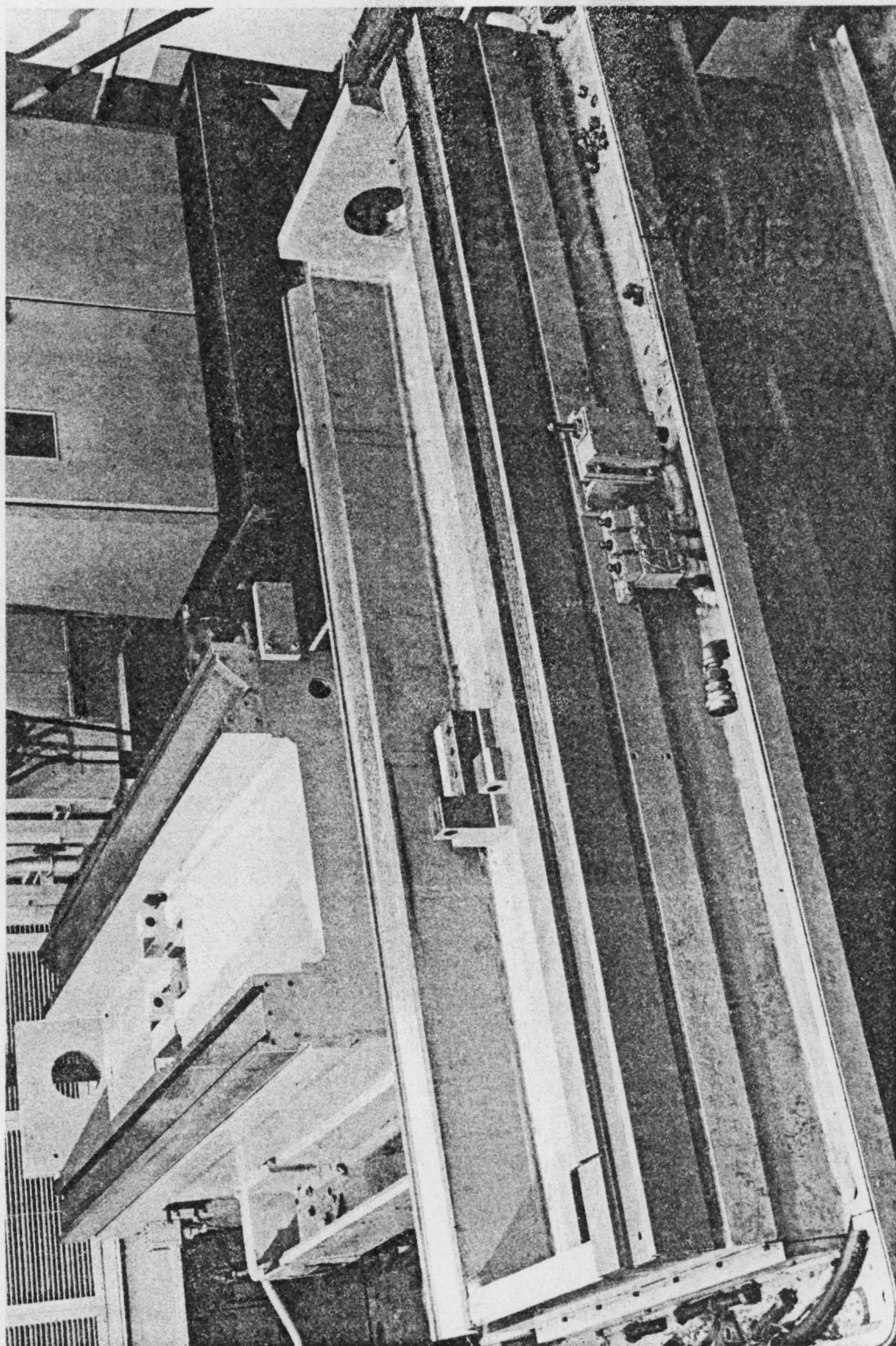


Figure 2.8. The Base has "v" and Rectangular Notches for Air Bearing Slide.

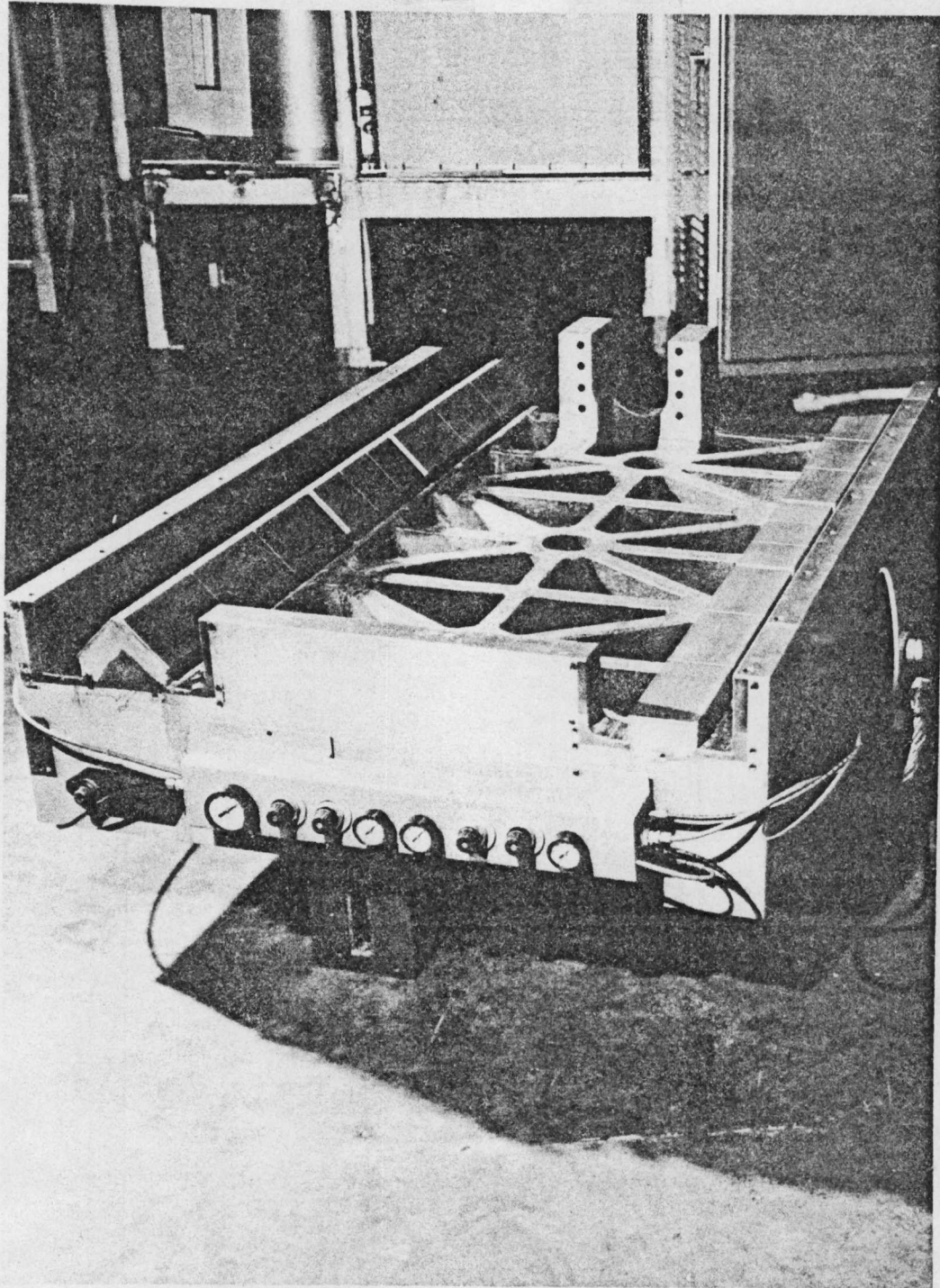


Figure 2.9. Bottom View of the X Axis Slide.

CHAPTER III

ANALYTICAL DEVELOPMENT OF THE SYSTEM

Any linear stationary, continuous control system can be described by a set of linear differential equations with constant coefficients. The differential equations constitute the mathematical model of the physical system, from which the characteristics of the system can be studied. A convenient tool for this study is the Laplace transform.

The transfer function of a system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input with all the initial conditions of the system equal to zero. Since the transfer function is obtained by Laplace transforming the differential equations of the system, it contains the basic information concerning the essential characteristics of the system. Transfer functions play an important role in control theory.

Forming the block diagram from the transfer functions of each component is the first step for studying the system behavior. The derivation of the transfer function of any system component is achieved after performing several tests in order to determine certain parameters. The parameters of the system are then used in deriving an analytical transfer function. Analytical development of the transfer function of a linear motor, a frequency-to-voltage converter, NC unit, and a direct current power amplifier is given in this chapter.

Linear Motor

It is necessary to know the operating principle of the linear motor before deriving its transfer function. The basic principle has

been discussed in Chapters I and II. As stated earlier, the linear motor operation is analogous to a D.C. torque motor, with the exception of the travel limit imposed by the moving member stroke. A complete, straightforward derivation of the transfer function is presented here, along with some comments on important related characteristics.

In order to derive the transfer function of the linear motor, the equivalent circuit of Figure 3.1 is used. The following notations will be used in deriving the transfer function of the linear motor:

k_b = back electromotive force (volt/inch/sec.)

x = motion of the moving members and control coil (inch/sec.)

M = weight to be moved by the moving members (pounds)

R = resistance of the control coil (ohm)

R_s = equivalent resistance of the power source (ohm)

L = inductance of the control coil (henry)

$e(t)$ = voltage applied to the control coil (volts)

$i(t)$ = current flowing through the control coil (ampere).

It is assumed that constant magnetic flux density is maintained in the air gap by the exciting coil.

Kirchoff's law of voltage may be applied to the electric circuit shown in Figure 3.1 which gives:³

$$e(t) = i(t)(R + R_s) + L \frac{di(t)}{dt} + k_b \frac{dx}{dt} . \quad (3.1)$$

The force developed by the linear motor is proportional to the product of flux density, length of the control coil, and current flowing through the control coil.

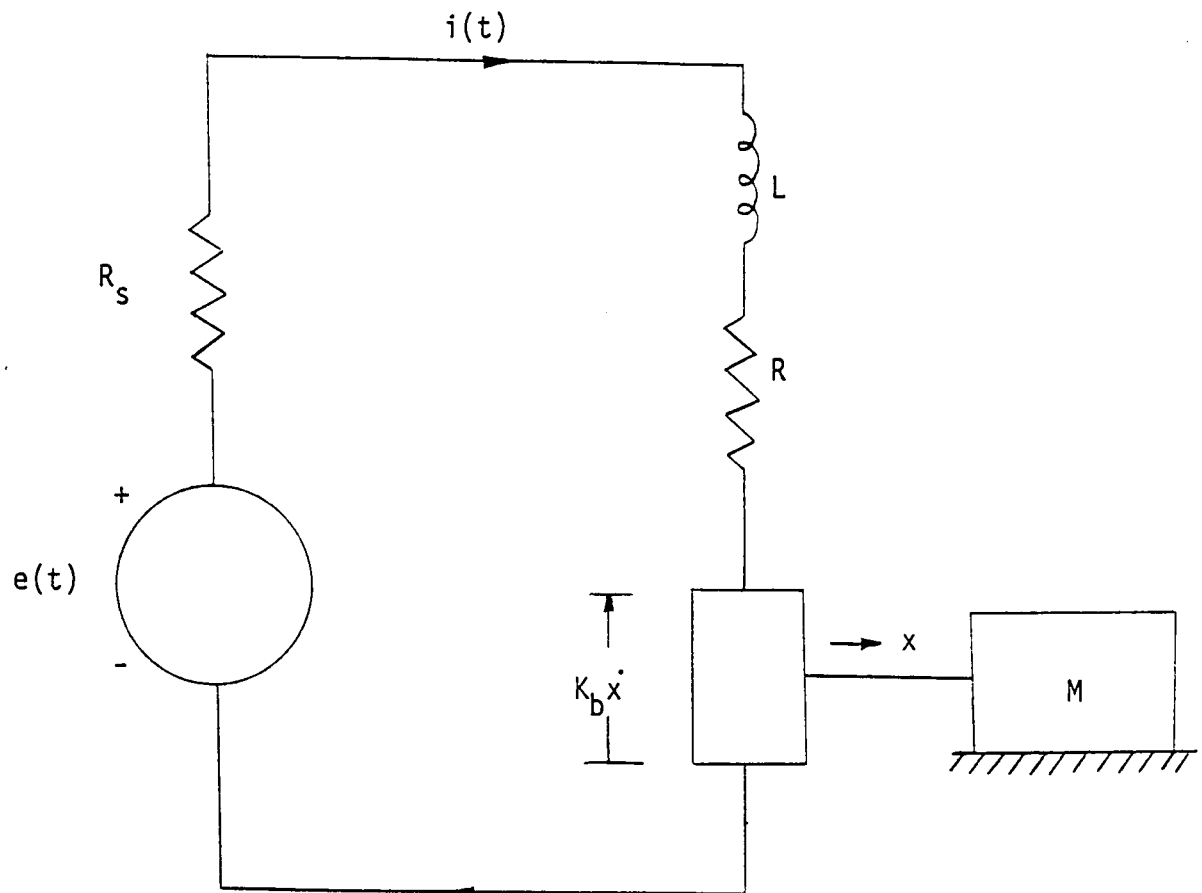


Figure 3.1. Equivalent Circuit of Linear Motor Drive by a Power Amplifier.

$$F_d(t) = BL_1 i(t) . \quad (3.2)$$

where

B = flux density in the air gap (lines/inch²).

L_1 = length of the control coil (inch).

The developed force must be equal to the accelerating force applied to the load, assuming that the load viscous and static friction are negligible.³

$$F_a(t) = M \frac{d^2x}{dt^2} . \quad (3.3)$$

$$F_a(t) = F_d(t) . \quad (3.4)$$

The developed force must overcome the effect of static friction, load viscous friction, and accelerate the load. The load viscous friction and static friction are the primary causes of the disturbance in the linear motor. Simply, it can be represented as

$$F_d(t) = F_a(t) + D(t) . \quad (3.5)$$

where

$D(t)$ = disturbance effect due to static and viscous friction of the load.

The electromotive force (emf) is directly proportional to the horizontal motion of the moving members, as expressed by³

$$EMF = K_b \frac{dx}{dt} . \quad (3.6)$$

To determine the transfer function of the linear motor, the Laplace transform of Equation 3.1, 3.2, 3.3, 3.5, and 3.6 must be found assuming zero initial conditions.

$$E(s) = I(s)(R + R_s) + sI(s)L + sX(s)K_b. \quad (3.7)$$

$$F_d(s) = BL_1 I(s). \quad (3.8)$$

$$F_d(s) = F_a(s) + D(s). \quad (3.9)$$

$$F_a(s) = Ms^2 X(s). \quad (3.10)$$

From Equations 3.4, 3.8 and 3.10, it can be shown that

$$BL_1 I(s) = Ms^2 X(s). \quad (3.11)$$

Defining $sX(s) = V(s)$, Equation 3.11 becomes

$$BL_1 I(s) = MsV(s). \quad (3.12)$$

Solving Equation 3.7 for $I(s)$ gives

$$I(s) = \frac{E(s) - K_b V(s)}{R + R_s + sL} = \frac{E(s) - K_b V(s)}{R_T + sL} \quad (3.13)$$

where

$$R_T = R + R_s.$$

The block diagram of the system using the above equations is shown in Figure 3.2. At this point, the change in position output, $X(s)$ due to the disturbance, $D(s)$ and input reference, $E(s)$ is developed by using Mason's formula as

$$X(s) = \frac{\left[\frac{BL_1 E(s)}{MsR_T(1 + \frac{L}{R_T}s)} - \frac{D(s)}{Ms} \right]}{s \left[1 + \frac{K_b BL_1}{MsR_T(1 + \frac{L}{R_T}s)} \right]}. \quad (3.14)$$

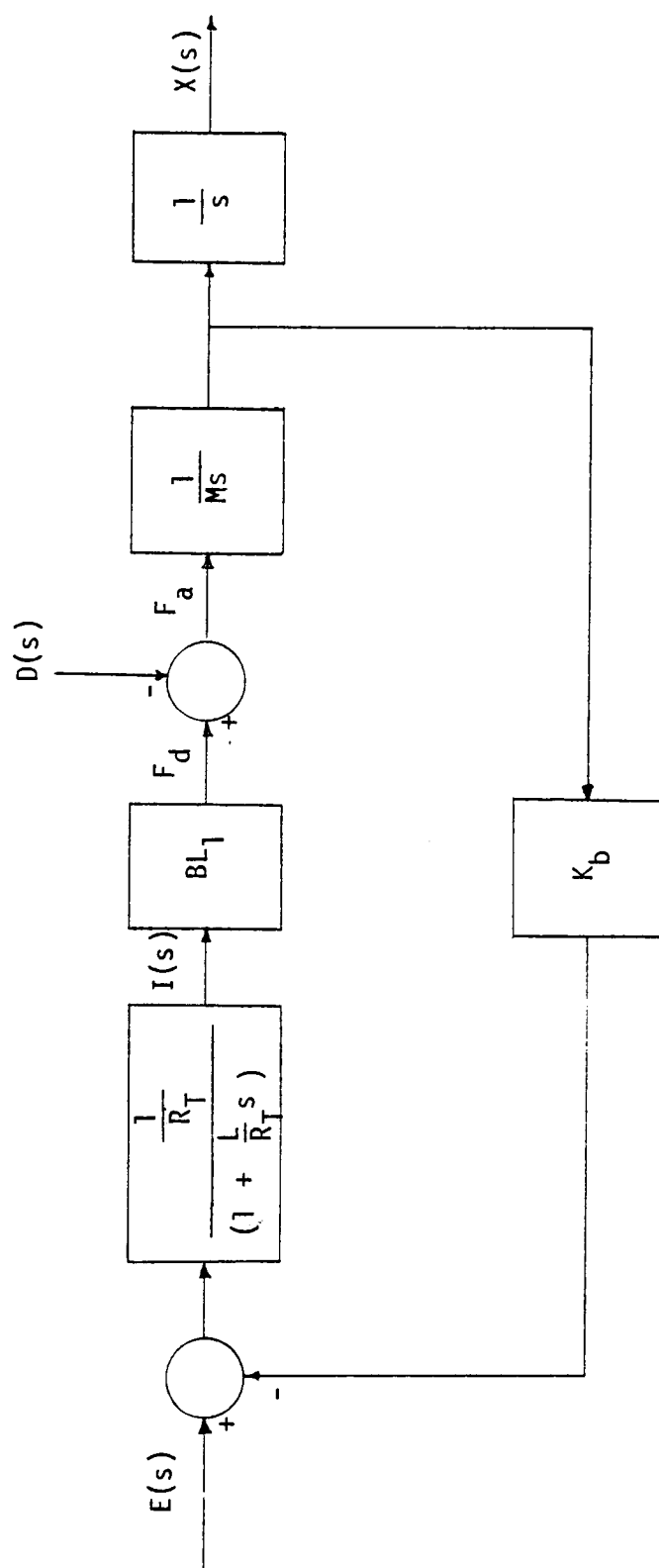


Figure 3.2. Block Diagram of the Transfer Function of the Linear Motor Considering Load Disturbance.

However, it seems beneficial to give a system block diagram without considering the disturbance effect in order to derive the simplified transfer functions of the system as shown in Figure 3.3. Hence the transfer function for $V(s)/E(s)$ is given by

$$\frac{V(s)}{E(s)} = \frac{\frac{BL_1}{Ms(R_T + sL)}}{1 + \frac{BL_1 K_b}{Ms(R_T + sL)}} \quad (3.15)$$

Equation 3.15 can be simplified to

$$\frac{V(s)}{E(s)} = \frac{1/k_b}{s^2 \frac{ML}{BL_1 K_b} + \frac{s MR_T}{BL_1 K_b} + 1} \quad (3.16)$$

It can reasonably be assumed that

$$\frac{MR_T}{BL_1 K_b} \gg \frac{L}{R} \quad .$$

Using this approximation, one can write Equation 3.16 as

$$\frac{V(s)}{E(s)} = \frac{1/k_b}{s^2 \frac{LM}{BL_1 K_b} + s \left(\frac{MR_T}{BL_1 K_b} + \frac{L}{R_T} \right) + 1} \quad (3.17)$$

or

$$\frac{V(s)}{E(s)} = \frac{1/k_b}{\left(1 + \frac{MR_T s}{BL_1 K_b}\right) \left(1 + \frac{L}{R_T} s\right)} \quad (3.18)$$

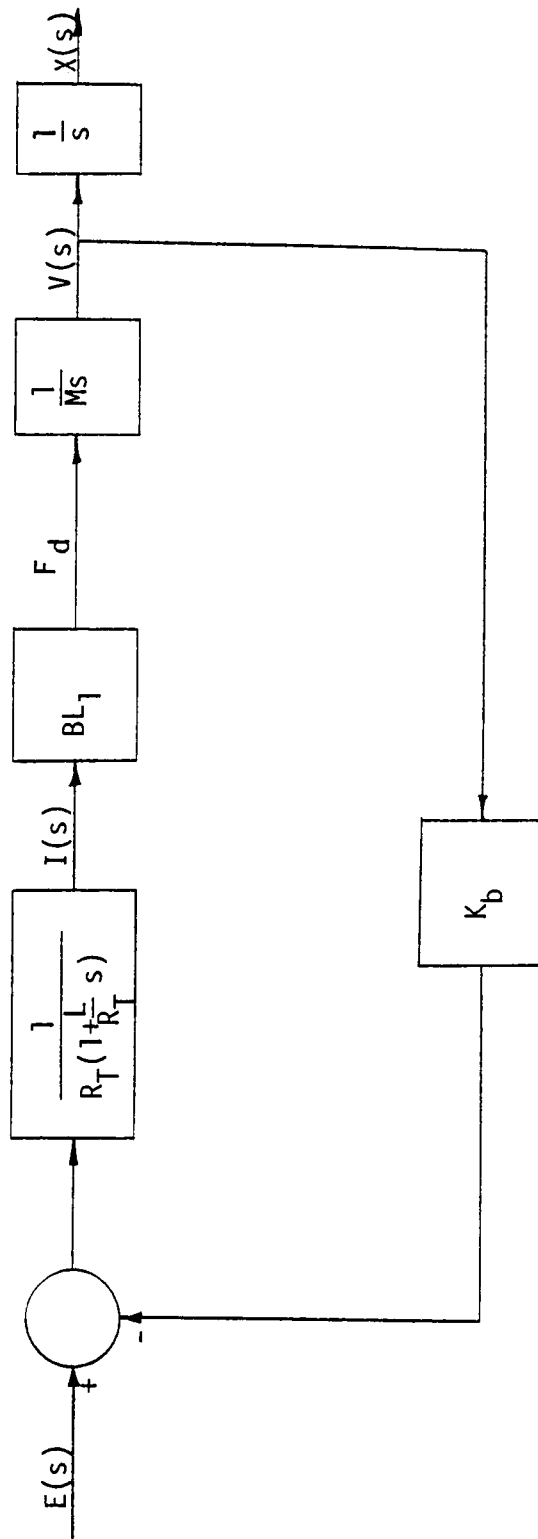


Figure 3.3. Block Diagram of the Transfer Function of the Linear Motor.

Let $\frac{MR_T}{BL_1K_b} = T_m$ and $\frac{L}{R_T} = T_e$

T_m and T_e are called the mechanical time constant and the electrical time constant, respectively. Each time constant has units of seconds.

Using the definitions for T_m and T_e simplifies Equation 3.18 to,

$$\frac{V(s)}{E(s)} = \frac{1/K_b}{(1 + T_ms)(1 + T_es)} \quad (3.19)$$

Hence,

$$\frac{X(s)}{E(s)} = \frac{1/K_b}{s(1 + T_ms)(1 + T_es)} \quad (3.20)$$

Equation 3.20 is the total transfer function of the linear motor without considering the disturbance effect. The simplified transfer function is a tool used for designing compensators and for simulation purposes. The calculation of the mechanical time constant and the electrical time constant can be found in Appendix B where it is shown that $T_m = 9.0$ seconds, $T_e = 0.0027$ seconds, and $K_b = 0.63$ volts/inch/seconds. Inserting the values of T_m , T_e , and K_b in Equation 3.20 gives

$$\frac{X(s)}{E(s)} = \frac{1.5873}{s(1 + 9.0s)(1 + 0.0027s)} \quad (3.21)$$

Frequency-to-Voltage Converter

Since the linear motor does not have rotating components, it is not possible to use a tachometer to provide a velocity signal. Instead the

laser pulses are coupled to frequency-to-voltage (F/V) converters to generate an analog signal. The frequency of the laser pulses is proportional to the slide velocity. A separate converter is used to determine velocity in the positive and negative directions. The output of the frequency-to-voltage converters provide an analog velocity signal and is summed with the output of the numerical control unit to form the signal subsequently applied to the servo amplifier signal.

The block diagram of the frequency-to-voltage converter is shown in Figure 3.4. The input to the frequency-to-voltage converters is the velocity of the linear motion; that is, the input is taken from just before the integration block. The velocity of the linear motor is measured in inches per second. The laser interferometer detects the position of the linear motor by pulses. One pulse corresponds to a movement of the linear motor of 0.6228 micro-inches. The output of the laser interferometer is ten volts if it receives one hundred thousand pulses per second. From this information, the gain of the frequency-to-voltage converter is calculated as follows. If 10 volts = 100,000 pulses/second and 1 pulse = 0.6228 micro-inches, then, 1 inch/second = 160.56 volts.

According to the information provided by the manufacturer, the frequency-to-voltage converter has a first order lag. The time constant of the frequency-to-voltage converter is given by the empirical formula as $T_{f/v} = 10 (0.0022 + C_{ext})$ milliseconds where C_{ext} is an external capacitor expressed in microfarad. In this case, $C_{ext} = 1$ microfarad.

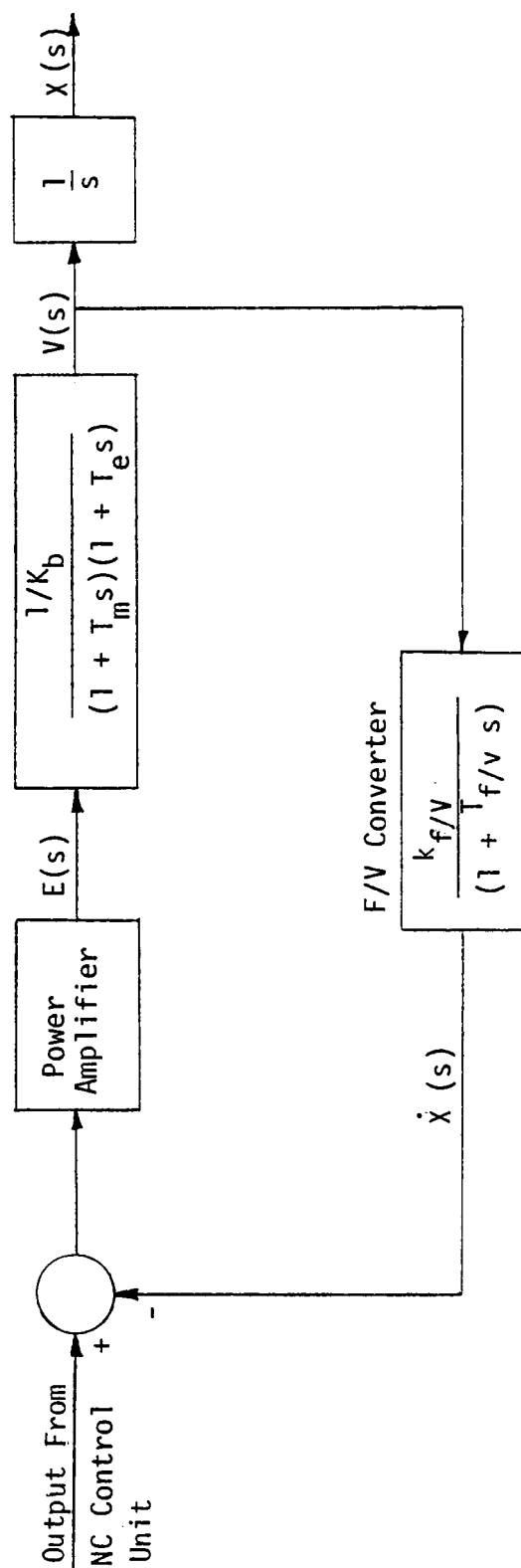


Figure 3.4 . Position and Transfer Function of the F/V Converter in the System.

Thus, $T_{f/v} = 0.01$ seconds. The complete transfer function of the frequency-to-voltage converter is given below and shown in Figure 3.4.

$$\frac{\dot{X}(s)}{V(s)} = \frac{K_{f/v}}{1 + T_{f/v}s} \quad (3.22)$$

Inserting the values of $K_{f/v}$ and $T_{f/v}$ in Equation 3.22 gives

$$\frac{\dot{X}(s)}{V(s)} = \frac{160.56}{1 + 0.01s} \quad (3.23)$$

Actual laboratory measurements have confirmed the correctness of this transfer function.

Direct Current Power Amplifier

The overall gain of the direct current power amplifier is measured as a ratio of the output signal to the input signal. The gain of the power amplifier is 10 volts/volt. As a transfer function, the power amplifier can be treated as a fixed gain of 10 since it has a bandwidth of 300 Hz which is well beyond the system bandwidth.

Numerical Control Unit

As stated earlier, the numerical control unit receives pulses from the laser interferometer and compares these pulses with the input command pulses. The conversion of the digital signal to an analog signal to form the position error takes place within the numerical control unit. The conversion factor is 10^6 volts/inch.

The transfer functions of all components are connected together to form a complete system which is shown in Figure 3.5.

The different compensators and controlling techniques suitable for this system are discussed in Chapter IV.

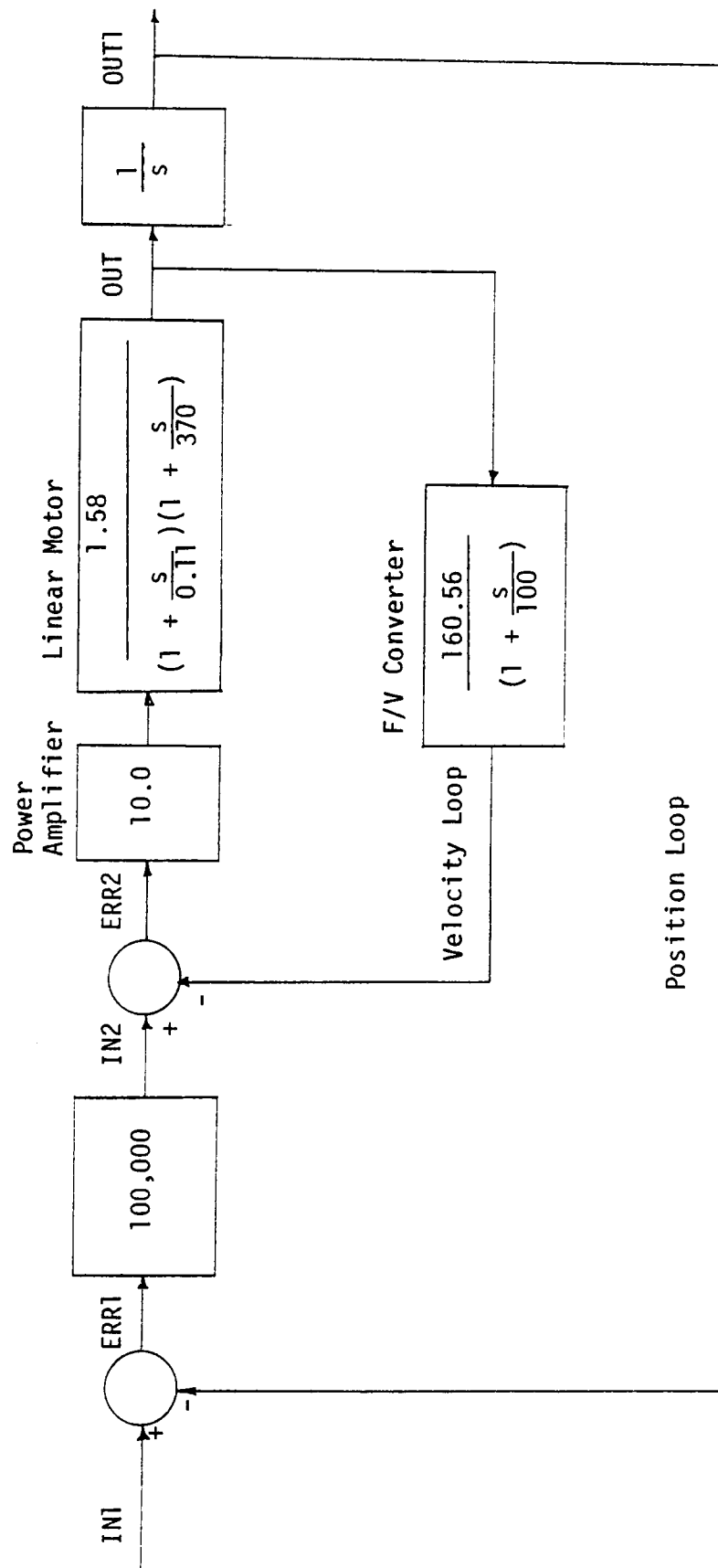


Figure 3.5. The Transfer Function Block Diagram of the Complete System.

CHAPTER IV

DESIGN AND ANALYSIS OF THE LINEAR MOTOR CONTROL SYSTEM

The application of control system design and analysis techniques can be most effectively illustrated by considering the overall analysis of a complete control system. Based on the concepts developed in Chapter III, a complete design and analysis for the linear motor control system will be discussed here. The linear motor system must provide a fast response for various inputs. Bode diagrams, root locus methods, and digital computer simulation are applicable to fast responding system design and are used in this analysis.

The system under study must meet the following specifications:

- (1) A velocity error coefficient, K_v , of 10^6 or greater.
- (2) Step response overshoot less than 40%.
- (3) Settling time less than one second.

Fundamental Design Considerations^{2,7,10}

The transient performance of a system is determined by the location of the poles and zeros of the closed loop transfer function. For a second order system, generalized time response curves based on damping ratio are available. Many higher order systems have a dominant complex pole pair and additional poles located far to the left of the imaginary axis so that the system can be approximated by a second order system. The effects on the system performance, of additional poles and zeros nearer to the dominant poles should be taken into account. However, if a pole is very near to a zero, then both the pole and the zero may be

neglected. The effects of other poles and zeros near the dominant poles are difficult to determine except in the simplest cases. The typical effect of an additional pole near the dominant pole pair is to increase the peak time, and the effect of a zero is to decrease the peak time while increasing the overshoot. Peak time can be increased by shifting zeros to the left and poles to the right. This knowledge about the effects of noncritical poles and zeros on the system performance should be a helpful guide to the designer in the placement of closed loop poles and zeros.¹⁰

The process of introducing additional combinations of poles and zeros into a system to reshape its root locus and Bode diagram, in order to improve system performance, is called compensation or stabilization.

When applying cascade compensation techniques to produce a low steady state error, care must be taken that the location of the compensators' poles and zeros will allow satisfactory transient response. During the tuning of the controller (compensator), the location of poles and zeros should be noted and compared with those of previous cases. The direction in which the poles and zeros should be moved may be predicted for the next adjustment of the controller. A well compensated system will have a good transient response and a gain large enough to meet the steady state error specification. The object of providing compensation is also to prevent any abnormal values of damping factor, overshoot, phase margin, gain margin, rise time and settling time from adversely affecting the overall efficiency of the system.¹⁰ The compensation devices may consist of electrical networks or mechanical equipment, such as levers, springs, and dash pots.

The overall performance of a control system may be judged by its transient response and its steady state error. The steady state error is defined as the difference between the input and the output as time tends to infinity. The steady state error may be found using the final value theorem.⁹

$$e_{ss} = \lim_{s \rightarrow 0} [sE(s)]$$

For a unit ramp input to a type 1 system, it can be readily shown that⁷

$$e_{ss} = \frac{1}{\lim_{s \rightarrow 0} sG(s)} = \frac{1}{K_v}$$

where $G(s)$ is the open loop transfer function of a unity negative feedback system. For the above case

$$K_v = \lim_{s \rightarrow 0} s G(s)$$

K_v is denoted as the velocity error coefficient and has units of $(\text{sec})^{-1}$. This error coefficient is used to determine the steady state error of the output with respect to a ramp input or command. This error is often referred to as the system following error. The steady state error for a ramp input will decrease when the velocity error constant is increased. For a type 1 system, the steady state error is zero for a step input, finite for a ramp input, and infinite for a parabolic input.

It has been determined from practical considerations, such as system disturbances from an interrupted cut, that the K_v for the linear motor system must be much greater than the K_v of a system using a ball nut and lead screw.

Selection of the location for inserting the compensator is largely dependent on the control system. Cascade compensators are generally inserted before the major energy source (power amplifier) in the forward path so that power dissipation is relatively small. Cascade networks used for compensation are usually called lag, lead, and lag-lead compensators.

Lag Compensation

Lag compensation is used to increase the error constant of the system. Well designed lag compensator will not significantly influence the dominant poles of the system as displayed on the root locus. Furthermore, a lag compensator has a pole to the right of a zero and nearer to the origin of the s-plane. This pole-zero location usually causes a slight increase in the step response overshoot of the system for the same root locus gain as without compensation. However, the lag compensator allows the low frequency gain to be increased and hence the steady state error is decreased.

Lead Compensation

The lead compensator accomplishes improvement of the transient response. The effect of lead compensation on the system is to increase

the speed of the transient response by changing the natural frequency of the dominant poles. A larger value of the natural frequency gives a faster transient response.

Lag-Lead Compensation

A lag-lead compensation is basically a lag compensator and a lead compensator in cascade. This form of compensation is used to improve both the transient response and the steady state response. Realization of all three compensators (lag, lead, lag-lead) with resistors, capacitors, and operational amplifiers is discussed in Appendix D.

These design considerations are applied to the present system to meet the required specifications.

System Design

Several controller designs will now be applied to the system shown in Figure 4.1. Before presenting these designs, it is best to determine the performance of the system that can be achieved without additional compensators. The following transfer function notation is frequently used in the process of designing compensations:

$G_1(s)H_1(s)$: Open loop transfer function of the position loop.

$\frac{OUT1(s)}{IN1(s)}$: Closed loop transfer function of the position loop.

$G_2(s)H_2(s)$: Open loop transfer function of the velocity loop.

$\frac{OUT(s)}{IN2(s)}$: Closed loop transfer function of the velocity loop.

The Uncompensated System

The pole-zero location of the open loop transfer function and the closed loop transfer function can be used to give insight into the system's behavior. This behavior generally serves as a guideline for designing compensators. The closed loop response of the uncompensated system can be judged by calculating the closed loop poles and zeros. In the process of determining the closed loop performance, one must first use the open loop transfer function of the velocity loop to determine the inner loop closed loop transfer function. For this case, the velocity open loop transfer function is given by

$$G_2(s)H_2(s) = \frac{10372675}{(s+0.11)(s+370)(s+100)} \quad (4.1)$$

The inner loop closed loop transfer function is expressed by

$$\frac{OUT(s)}{IN2(s)} = \frac{G_2(s)}{1 + G_2(s)H_2(s)} \quad (4.2)$$

Upon simplifying this expression,

$$\frac{OUT(s)}{IN2(s)} = \frac{646 (s+100)}{(s+45.2+j26.3)(s+379.7)} \quad (4.3)$$

The velocity error constant can be found from the open loop transfer function of the outer loop (position loop).

$$G_1(s)H_1(s) = \frac{64603000 (s+100)}{s(s+45.2+j26.3)(s+379.7)} \quad (4.4)$$

$$K_V = \lim_{s \rightarrow 0} s G_1(s)H_1(s) = 622.0 \text{ sec}^{-1}.$$

The velocity error constant of the uncompensated linear motor system is low compared to requirement of 10^6 or higher. The system closed loop transfer function is given by

$$\frac{OUT1(s)}{INT(s)} = \frac{G_1(s)}{1+G_1(s)H_1(s)} \quad (4.5)$$

With appropriate substitution and simplification,

$$\frac{OUT1(s)}{INT(s)} = \frac{64603000 (s+100)}{(s+86.6)(s+590.9)(s-103.7 \pm j340)} \quad (4.6)$$

Equation 4.6 indicates that the closed loop dominant complex poles are located in the right half of the s-plane. This clearly indicates that the uncompensated system is unstable, and in addition has a low value for the velocity error constant.

Selection of Compensation

There are numerous ways to compensate a system with different gains and compensators at different locations. Several compensators were designed and tested on the linear motor control system. The linear motor control system did not respond satisfactorily to all combinations of gains and compensators because the input voltage to the operational amplifier was high enough to saturate it and the position error was high. A compensation format that gave satisfactory results is as follows:

- (1) A lag compensator in the forwards path of the velocity loop.
- (2) A lag-lead compensator in the forward path of the position loop.
- (3) Gain in the velocity feedback path.

The designs presented below are distinguished from one another by the parameter α of the compensator. α as used here is the ratio of zero to pole for a lag compensator and the ratio of the pole to zero for the lead compensator. A value of α smaller than 15 allows compensator construction with reasonable small capacitors in its RC networks. However, depending upon the system performance, values of α greater than 15 are acceptable. In this case, the experimental design has an α value equal to 188 for the lag compensator in the forward path of the velocity loop. The calculation of RC components of the compensator is given in Appendix D. The different designs mainly emphasized are listed below:

Design 1. Analytic design (inner loop lag compensation $\alpha \leq 15$).

Design 2. Experimental design and adjustment of gain to achieve analytic stability (inner loop lag compensation $\alpha = 188$).

Design 1. Analytical Design (Inner Loop Lag Compensator $\alpha \leq 15$)

As discussed earlier, a high value of the velocity error constant must be obtained. To meet the requirement of the velocity error constant, the following criteria were followed while designing the compensation:

- (1) Gain in the velocity feedback path should be as small as possible.
- (2) Gain of the forward path of the velocity loop should be larger than that of the feedback path gain.
- (3) The compensation in the forward path of the velocity loop does not affect the velocity error constant of the system.

First consider the velocity loop of the uncompensated system. The

open loop transfer function is given by Equation 4.1 which is

$$G_2(s)H_2(s) = \frac{10372675.34}{(s+0.11)(s+370)(s+100)} \quad (4.1)$$

The Bode diagram and phase plot of $G_2(s)H_2(s)$ are shown in Figure 4.2. The gain crossover point occurs at frequency $\omega = 144$ rad/sec. The phase margin is only 14° . Additional phase is required in order to decrease the overshoot and to increase the stability of the inner loop. From Figure 4.2, it is noted that reduction in gain can increase the phase margin. The gain crossover point can be changed from $\omega = 144$ rad/sec to $\omega = 0.3$ rad/sec after shifting the Bode plot down by approximately 60 db. This reduction in gain gives a phase of -70° . Thus the new phase margin of the inner loop is 110° . The gain reduction of 60 db can be achieved in the velocity feedback path by changing the gain from unity to 0.001. The object of reducing this gain is to increase the velocity error constant of the system. It is obvious that the system with 0.001 gain in the velocity feedback path has a slow responding step response compared to unity feedback gain for the velocity loop. The open and closed loop transfer functions of the velocity loop, with the changed gain in the velocity feedback path, are given by

$$G_2(s)H_2(s) = \frac{10373}{(s+0.11)(s+370)(s+100)} \quad (4.7)$$

and

$$\frac{OUT(s)}{IN2(s)} = \frac{646 (s+100)}{(s+0.39)(s+99.6)(s+370.1)} \quad (4.8)$$

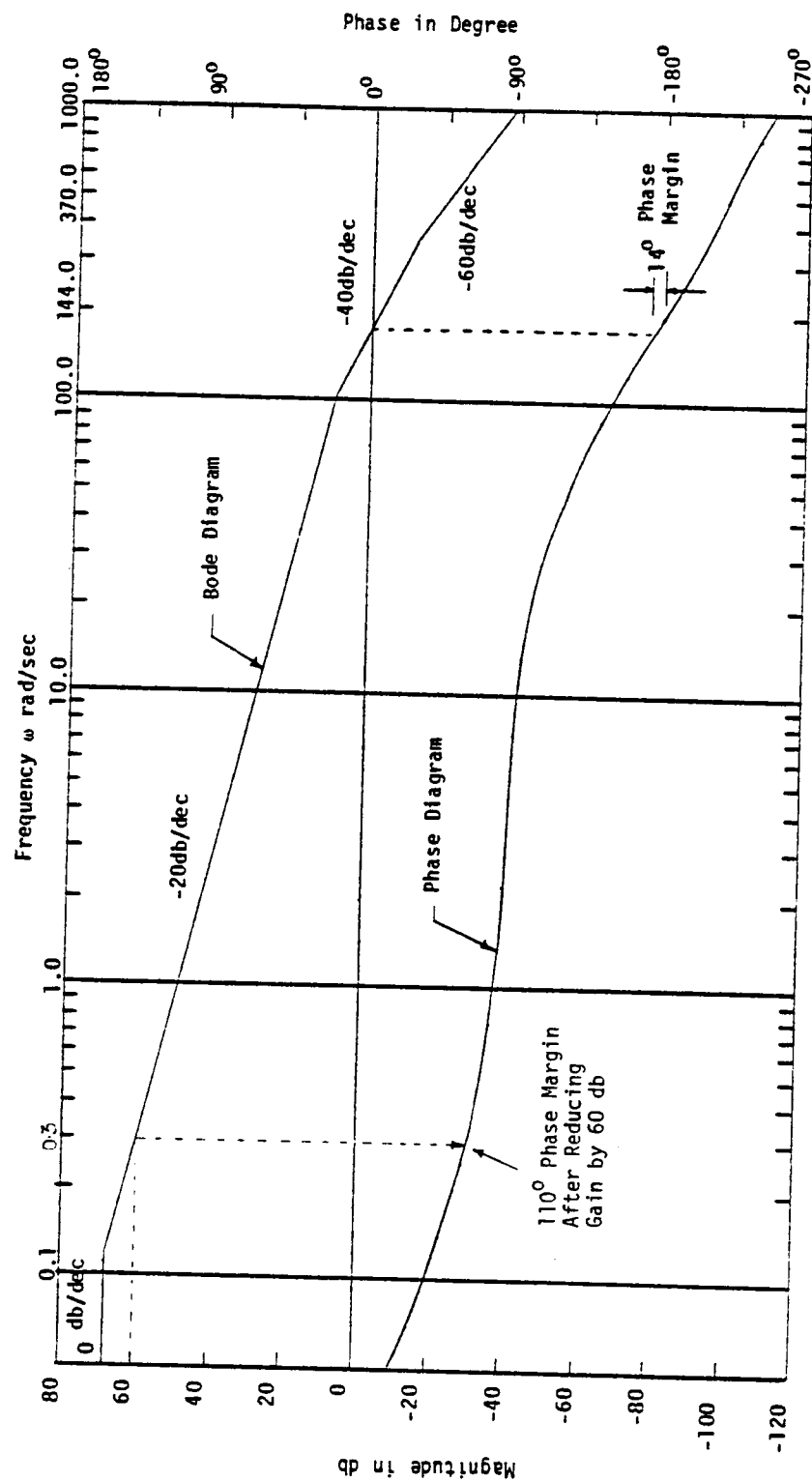


Figure 4.2. The Frequency Response of $G_2(s)H_2(s)$ of Equation 4.1.

From Equation 4.8, the pole at -99.6 is very close to the zero at -100. The resultant effect of the pole at -99.6 and the zero at -100 is negligible. Hence Equation 4.8 reduces to

$$\frac{OUT(s)}{IN2(s)} = \frac{646}{(s+0.39)(s+370.1)} \quad (4.9)$$

The open loop transfer function of the position loop is calculated as

$$G_1(s)H_1(s) = \frac{64600000}{s(s+0.39)(s+370.1)} \quad (4.10)$$

which gives

$$K_V = \lim_{s \rightarrow 0} s G_1(s)H_1(s) = 448327 \text{ sec}^{-1}.$$

A gain increase of 2.23 in the forward path of the position loop is required in order to raise the velocity error constant, K_V , to one million. With this gain increase, the open loop transfer function of the position loop becomes

$$G_1(s)H_1(s) = \frac{144122600}{s(s+0.39)(s+370.1)} \quad (4.11)$$

The Bode diagram and phase plot corresponding to Equation 4.11 are shown in Figure 4.3. The gain crossover occurs at $\omega = 500$ rad/sec with a phase of -230° . Thus, the system is unstable. A lag-lead compensator may be used to stabilize the system and to provide acceptable transient response. In general, there is no fixed procedure available for the design of the lag-lead controller.⁷ Usually a trial-and-error procedure, using the design techniques outlined for the phase lag and the phase lead controller,

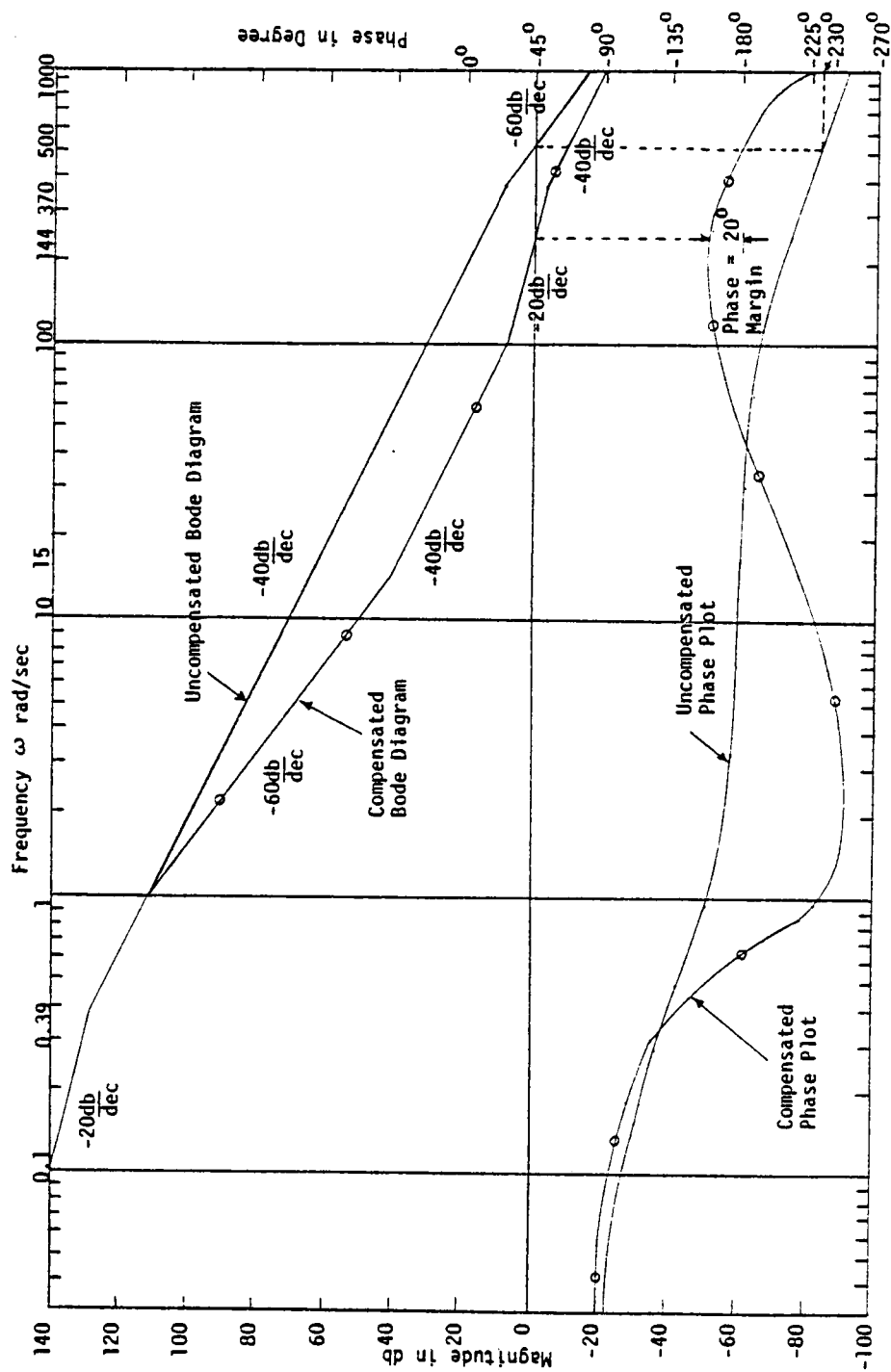


Figure 4.3. The Frequency Plot of $G_1(s)H_1(s)$ of Equations 4.11 (Uncompensated) and 4.13 (Compensated).

provides a satisfactory design arrangement. Using a trial-and-error approach leads to the following lag-lead controller.

$$\begin{aligned} G_c(s) &= \left[\frac{1 + \frac{s}{15}}{1 + \frac{s}{1}} \right] \left[\frac{1 + \frac{s}{100}}{1 + \frac{s}{1000}} \right] . \\ \text{(Lag-Lead)} & \qquad \qquad \text{Phase Lag} \qquad \text{Phase Lead} \end{aligned} \quad (4.12)$$

The lag-lead compensator has a lag α of 15 and a lead α of 10. The compensated open loop transfer function and the closed loop transfer function of the position loop of the system are expressed as

$$G_1(s)H_1(s) = \frac{96081733 (s+15)(s+100)}{s (s+0.39)(s+370.1)(s+1)(s+1000)} \quad (4.13)$$

and

$$\frac{OUT1(s)}{INT(s)} = \frac{96081733 (s+15)(s+100)}{(s+14.9)(s+148.2)(s+51.3+j237.7)(s+1105.7)} . \quad (4.14)$$

The Bode diagram and the phase plot of the compensated system of Equation 4.13 is given in Figure 4.3 where the following is observed. The system is conditionally stable with the gain crossover of 200 rad/sec and a phase of -160° . The low phase margin of 20° for the system predicts high overshoot for the step input.

Computer Simulation. A CSMP¹¹ block diagram of the compensated system is shown in Figure 4.4. The CSMP program and unit step response for the compensated system are shown in Figures 4.5 and 4.6, respectively. Figure 4.6 reveals that the transient response has a peak value of 1.79

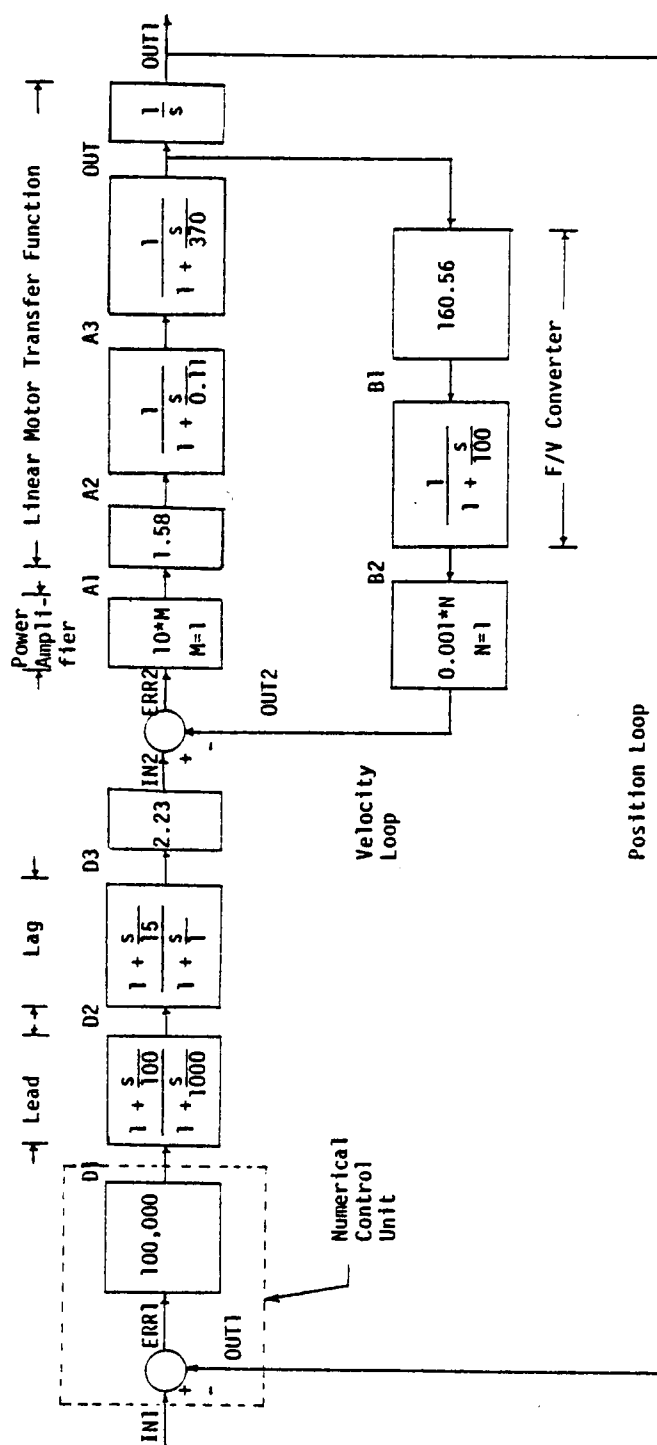


Figure 4.4. A CSMP Block Diagram of the Compensated System Without Inner Loop Lag Compensation.

```

INITIAL
PARAM M=1.0, N=1.0
DYNAMIC
IN1 = STEP(0.0)
ERR1 = IN1-OUT1
D1 = 100000.0*ERR1
D2 = LEDLAG(1.0/100.0,1.0/1000.0,D1)
D3 = LEDLAG(1.0/15.0,1.0/1.0,D2)
IN2 = 2.23*D3
ERR2 = IN2-OUT2
A1 = 10.0*M*ERR2
A2 = 1.58*A1
A3 = REALPL(0.0,1.0/0.11,A2)
OUT = REALPL(0.0,1.0/370.0,A3)
B1 = 160.56*OUT
B2 = REALPL(0.0,1.0/100.0,B1)
OUT2 = 0.001*N*B2
OUT1 = INTGRL(0.0,OUT)
PRTPLT OUT1
TIMER FINTIM = 0.5, OUTDEL = 0.005
LABEL LM-17, GAIN CHANGE VEL-LOOP, LEDLAG OUTER LOOP

```

Figure 4.5. The CSMP Program of the Step Response of Design 1 Shown in Figure 4.4.

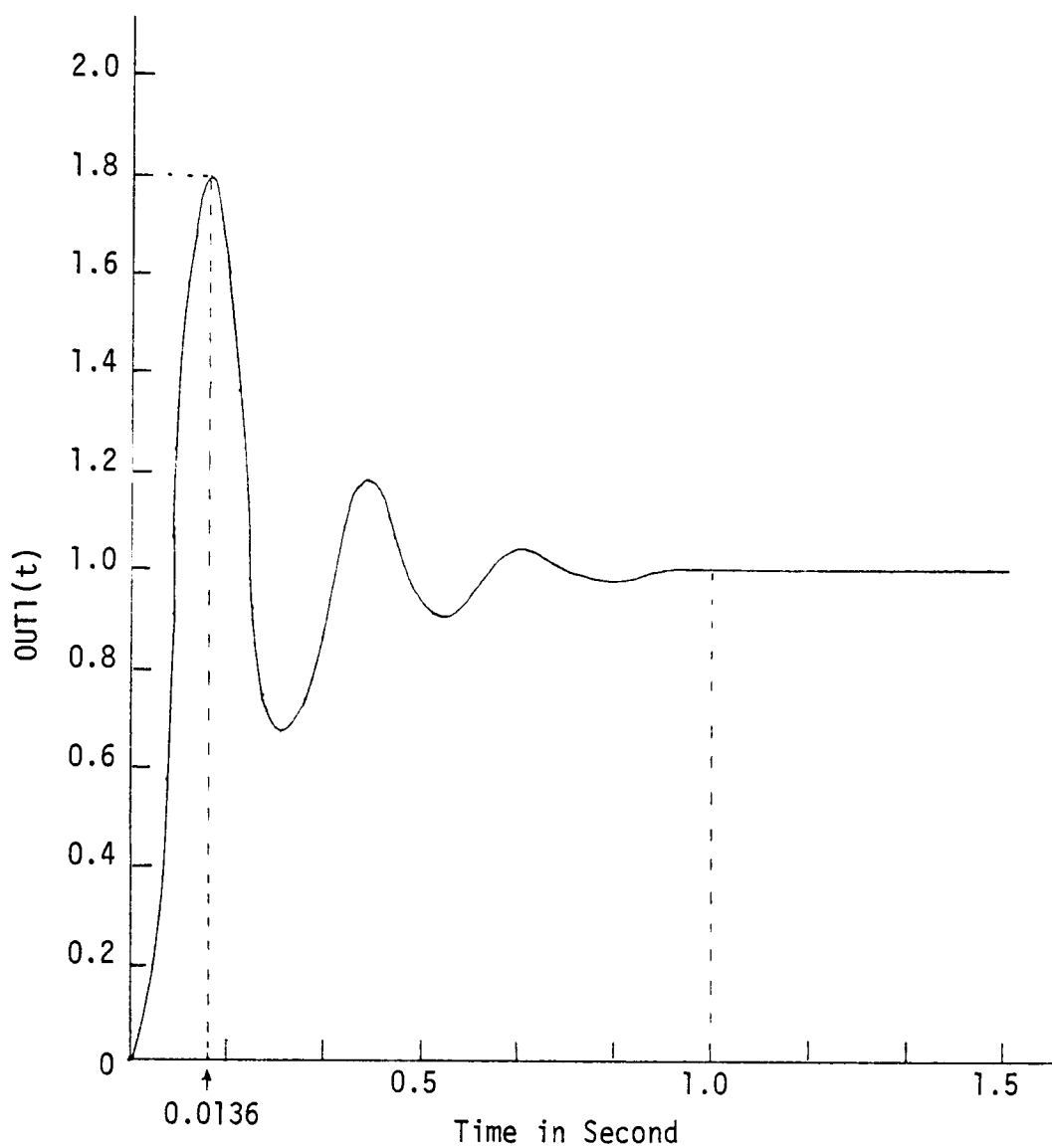


Figure 4.6. The Step Response of Design 1 (No Inner Lag Compensation) Shown in Figure 4.4.

at 0.0136 second. The system has 79% overshoot for the unit step input. The excessive overshoot of the system is due to the following:

- (1) The uncompensated Bode diagram of the position loop shown in Figure 4.3 reveals that the slope at the gain crossover frequency, $\omega = 500$ rad/sec, -60 db/dec. At break frequency, $\omega = 370$ rad/sec, the slope changes to -40 db/dec from -60 db/dec as ω decreases. The slope of -40 db/dec and -60 db/dec give a -230° phase at the gain crossover frequency.
- (2) Figure 4.3 shows that the uncompensated magnitude plot has a single break frequency at $\omega = 0.39$ rad/sec which gives a high gain crossover frequency of $\omega = 500$ rad/sec, and a low phase of -230° .
- (3) Due to the factors described in (1) and (2), a lag-lead compensator could not be designed which, in itself, could eliminate the excessive overshoot.
- (4) The closed loop poles of the compensated system are -14.86, -148.19, $-51.34 \pm j237.73$ and -1105.67. The damping ratio of the dominant complex poles is approximately 0.21 if the required K_V is maintained.

Gain reduction in the velocity feedback path and the addition of lag-lead compensation in the forward path of the position loop is not sufficient to meet specifications as outlined earlier in this chapter. In order to meet specifications, a lag compensator is required in the forward path of the velocity loop. Use of a lag compensator has the

following advantages:

- (1) In the phase lag compensation block, the ratio of the pole to the zero is less than unity. Hence the root locus gain of the velocity loop is less than that of an uncompensated velocity loop. As a result, the break point due to the low frequency pole may be obtained from the uncompensated open loop transfer function of the position loop.
- (2) From Figure 4.2, page 47, it is observed that phase increases as frequency decreases from the gain crossover frequency of 144 rad/sec. Therefore, a lag compensator can be designed in the frequency range of $\omega = 0.1$ rad/sec to $\omega = 10$ rad/sec. The compensator should be designed such that it provides the dominant complex poles for the closed loop transfer function of the velocity loop.
- (3) Due to lag compensation, the closed loop transfer function of the velocity loop has complex poles. As a result, the Bode diagram for the position loop has a double break frequency. A -40 db/dec slope can be achieved at the gain crossover point of the Bode diagram for the uncompensated position loop. A -40 db/dec slope at the gain crossover point gives a better phase margin than a -60 db/dec slope. Considering the above mentioned points, a lag compensator is designed to meet the system specifications.

First consider the velocity loop of the uncompensated system. The open loop transfer function of the uncompensated velocity loop is given

by

$$G_2(s)H_2(s) = \frac{10372675.34}{(s+0.11)(s+370)(s+100)} \quad (4.15)$$

The Bode diagram and phase plot of Equation 4.15 are shown in Figure 4.7. The gain crossover point occurs at frequency $\omega = 144$ rad/sec. The phase margin is only 14° . An additional 56° of phase is introduced in order to decrease the overshoot and to increase the stability of the inner loop. Thus the new phase margin of the inner loop is 70° . As broadly explained earlier, a lag compensation⁷ is required in the forward path of the velocity loop. The compensation is assumed to have the form of

$$\begin{aligned} G_c(s) \\ (\text{Lag}) \end{aligned} = \frac{1 + \frac{T}{\alpha} s}{1 + T s} \quad \alpha \leq 15 \quad (4.16)$$

From Figure 4.7, it is noted that when the phase is -110° (70° phase margin), $\omega_c' = 25$ rad/sec and the gain is 20.4 db. ω_c' is the frequency at which the desired phase margin occurs. For this case,

$$\frac{1}{\alpha} = \frac{-}{10} \left| \frac{20.4}{20} \right| = 0.0955. \quad (4.17)$$

Placing the zero of the lag compensator one decade below the proposed new crossover frequency of ω_c' yields

$$\frac{\alpha}{T} = \frac{\omega_c'}{10} = \frac{25}{10} = 2.5. \quad (4.18)$$

Thus,

$$\begin{aligned} G_c(s) \\ (\text{Lag}) \end{aligned} = \frac{1 + \frac{s}{2.5}}{1 + \frac{s}{0.24}} \quad (4.19)$$

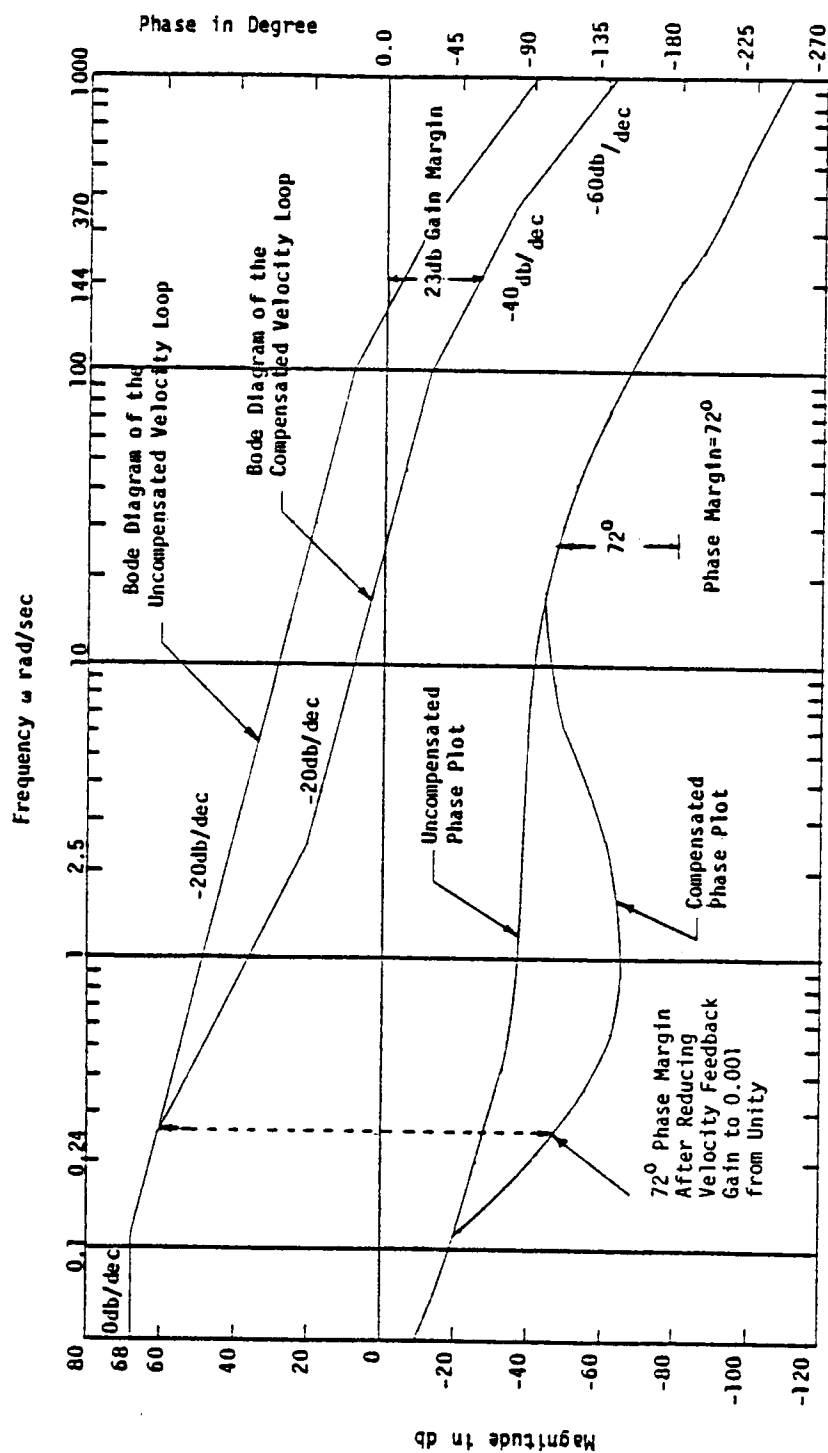


Figure 4.7. The Frequency Plot of $G_2(s)H_2(s)$ of Equations 4.15 (Uncompensated) and 4.20 (Compensated).

The frequency plot including the lag compensator of Equation 4.19 is shown in Figure 4.7. At $\omega = 25$ rad/sec, the phase is -108° and hence the phase margin is 72° . This same phase margin can be achieved at $\omega = 0.26$ rad/sec by reducing the loop gain. The gain crossover point can be changed from $\omega = 25$ rad/sec to $\omega = 0.26$ rad/sec after shifting the Bode plot down by approximately 60 db. The gain reduction of 60 db achieved in the velocity feedback path by changing the gain from unity to 0.001. The object of reducing this gain is to increase the velocity error constant of the system. It is obvious that the system with 0.001 gain in the velocity feedback path has a slow responding step response compared to unity feedback gain for the velocity loop. The velocity feedback gain of 0.001 is used for this design. The block diagram of the compensated velocity loop is depicted in Figure 4.8. The open loop transfer function and the closed loop transfer function of the velocity loop are calculated as

$$G_2(s)H_2(s) = \frac{991.1 (s+2.5)}{(s+0.24)(s+0.11)(s+370)(s+100)} \quad (4.20)$$

and

$$\frac{OUT(s)}{INT(s)} = \frac{61.73 (s+2.5)(s+100)}{(s+0.19+j0.24)(s+99.96)(s+370)} \quad (4.21)$$

In Equation 4.21, the pole at -99.96 is very near to the zero at -100 . Hence Equation 4.21 reduces to

$$\frac{OUT(s)}{INT(s)} = \frac{61.73 (s+2.5)}{(s+0.19+j0.24)(s+370)} \quad (4.22)$$

The open loop transfer function of the position loop becomes

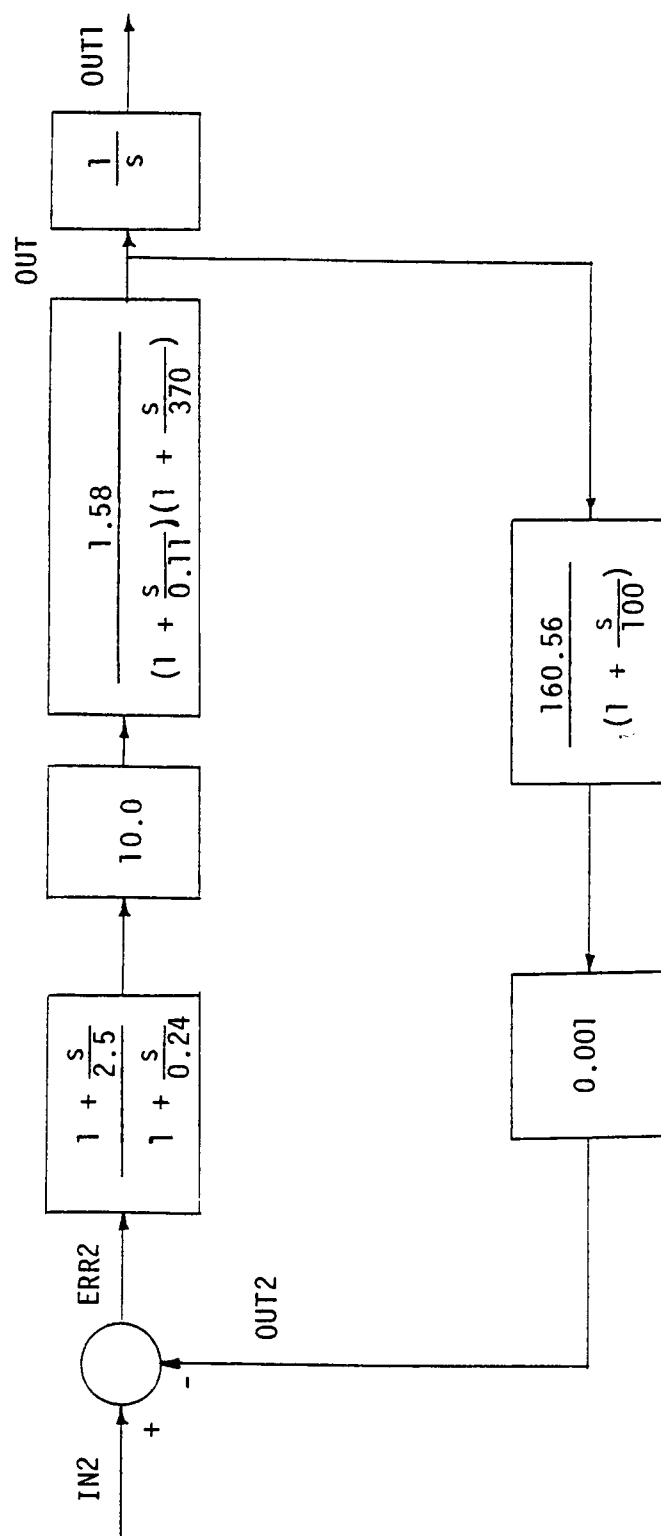


Figure 4.8. Block Diagram of Compensated (With Inner Lag $\alpha \leq 15$) Velocity Loop of Design 1.

$$G_1(s)H_1(s) = \frac{6173000(s+2.5)}{s(s+0.19+j0.24)(s+370)} \quad (4.23)$$

which gives

$$K_V = \lim_{s \rightarrow 0} s G_1(s)H_1(s) = 446000 \text{ sec}^{-1}.$$

A gain increase of 2.3 in the forward path of the position loop is required in order to raise K_V to one million. With this gain increase, the open loop transfer function of the position loop becomes

$$G_1(s)H_1(s) = \frac{13858385(s+2.5)}{s(s+0.19+j0.24)(s+370)} \quad (4.24)$$

The Bode diagram and the phase plot corresponding to Equation 4.24 are shown in Figure 4.9. The gain crossover occurs at $\omega = 184 \text{ rad/sec}$ with a phase of -207° . Thus, the system is unstable. A lag-lead compensator may be used to stabilize the system and to provide acceptable transient response. In general, there is no fixed procedure available for the design of the lag-lead controller.⁷ Usually a trial-and-error procedure, using the design techniques outlined for the phase lag and the phase lead controller, provides a satisfactory design arrangement. Using a trial-and-error approach leads to the following lag-lead controller.

$$G_c(s) = \left[\frac{1 + \frac{s}{30}}{1 + \frac{s}{300}} \right] \left[\frac{1 + \frac{s}{0.15}}{1 + \frac{s}{0.01}} \right] \quad (4.25)$$

(Lag-Lead) Phase Lead Phase Lag

The lag-lead compensator has a lag α of 15 and a lead α of 10.

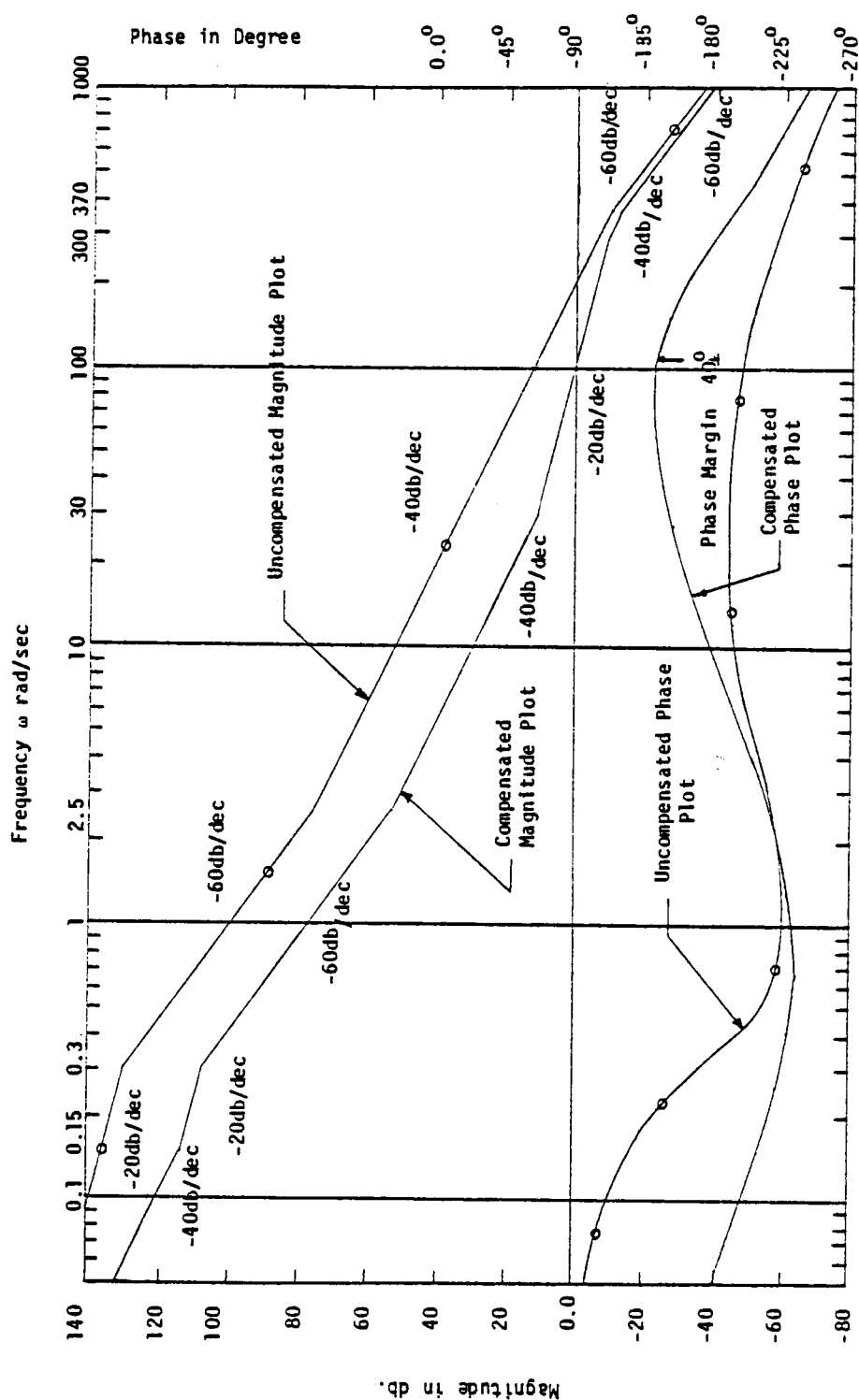


Figure 4.9. The Frequency Plot of $G_1(s)H_1(s)$ of Equations 4.24 (Uncompensated) and 4.26 (Compensated).

The compensated open loop transfer function of the position loop of the system is now given by

$$G_1(s)H_1(s) = \frac{9243542 (s+2.5)(s+30)(s+0.15)}{s(s+0.19+j0.24)(s+370)(s+300)(s+0.01)} \quad (4.26)$$

The phase and magnitude of the compensated system is given in Figure 4.9 where the following is observed. The system is conditionally stable with the gain crossover at 88 rad/sec and the phase margin 40° . The operating range of frequency for the system lies between 9 rad/sec and 300 rad/sec. Simultaneously, the maximum variation of the outer loop gain between the two extreme operating frequencies is 126, that is, 42 db.

The root locus of the open loop transfer function of the position loop is shown in Figures 4.10 (a) and (b). Figure 4.10 (b) is an enlargement of the locus near the origin. There are three crossings of the imaginary axis by the locus. The locus is in the right half of the s-plane whenever the gain is between 5 and 300,000 or higher than 60,000,000. Hence the system is conditionally stable. The closed loop transfer function of the system is given by

$$\frac{OUT1(s)}{IN1(s)} = \frac{9243542 (s+2.5)(s+30)(s+0.15)}{(s+0.15)(s+2.49)(s+60.76)(s+66.5+j72.5)(s+475)} \quad (4.27)$$

The closed loop poles of the system are -0.15, -2.49, -60.76, -66.5 +j72.5, -100, and -475. Due to the complexity of the system, it is difficult to find the effect of poles and zeros on the complex poles. The damping ratio of the complex pole pair is approximately 0.68 and

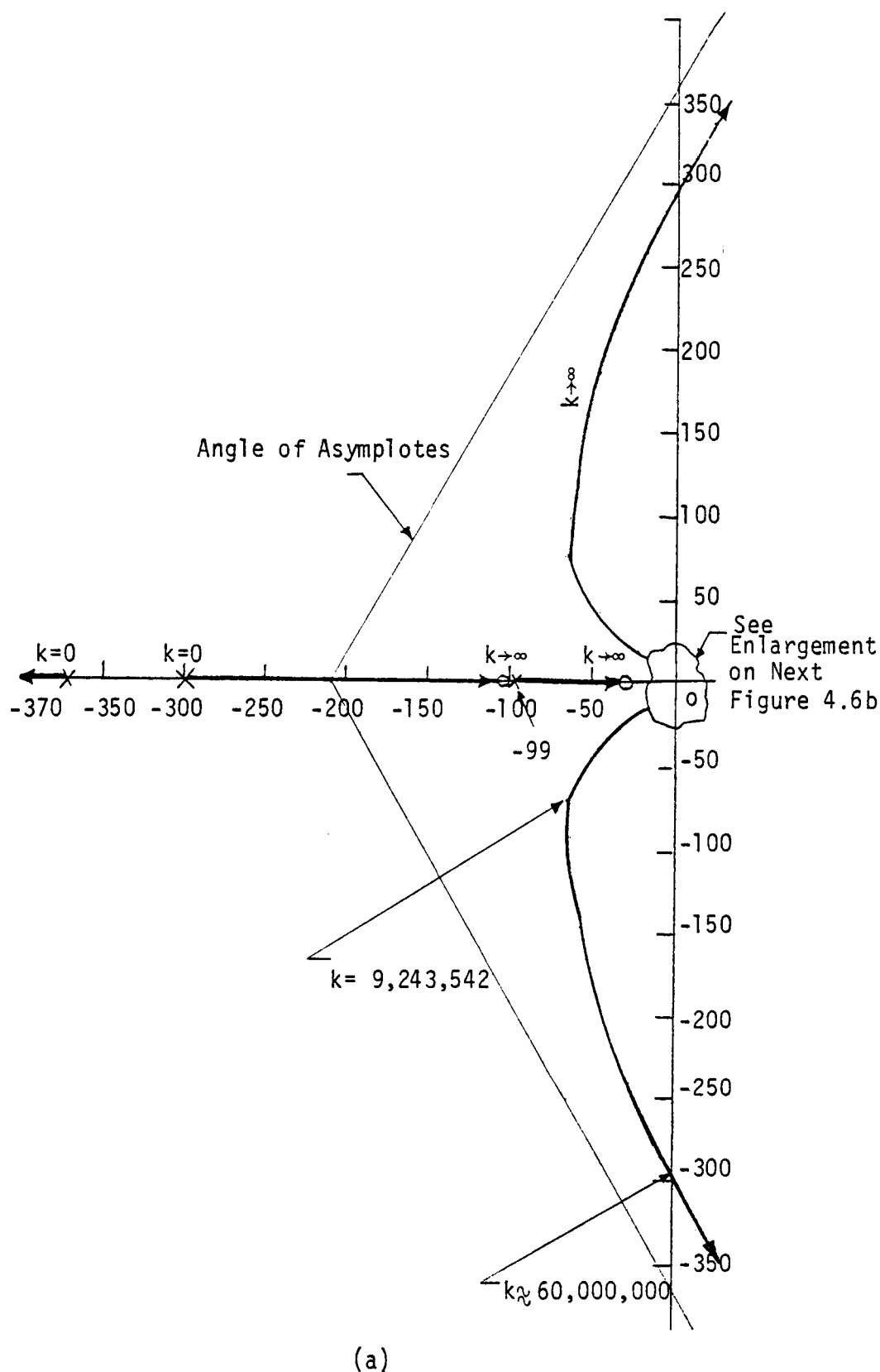
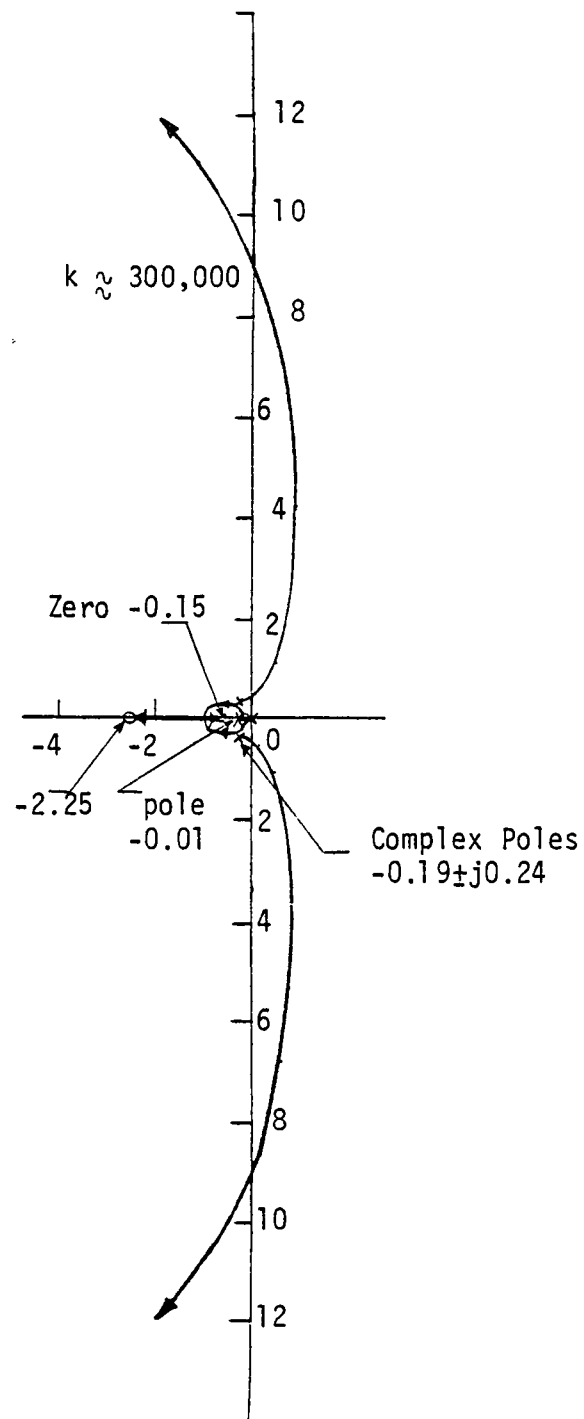


Figure 4.10. (a) Root Locus of $G_1(s)H_1(s)$ of Design 1 of Equation 4.26; (b) Enlargement of the Root Locus at the Origin of Figure 4.10 (a).



(b)

Figure 4.10 (continued)

the natural frequency is 98.4 rad/sec. The rise time of the system can be approximated by

$$t_r \approx \frac{0.8 + 2.5 \zeta}{\omega_n} \quad 0 < \zeta < 1. \quad (4.28)$$

where ζ is the damping ratio and ω_n is the natural frequency of the dominant poles. For this case $t_r = 0.025$ seconds.

Computer Simulation. A CSMP¹¹ block diagram of the compensated system is shown in Figure 4.11. The CSMP program and step response for the compensated linear motor system are shown in Figure 4.12 and 4.13, respectively. The characteristics of the step response are given below:

Rise time	= 0.017 second
First Peak time	= 0.035 second
Settling time	= 0.08 second
Overshoot	= 39.68%

These characteristics indicate that the system's performance will be satisfactory for this design.

The stability of the system may also be checked by the closed loop frequency response shown in Figure 4.14. The peak resonance M_p is 1.5, which is satisfactory. The system bandwidth is 155 rad/sec, that is, 24.67 hertz. However, in general, a steep cutoff characteristic may be accompanied by a large peak resonance, which corresponds to a system with a low stability margin.⁷ In this case, the peak resonance is 1.5. Hence the system has a slow cutoff rate and high stability margin. In the process


```

INITIAL
PARAM M=1.0, N=0.001
DYNAMIC
IN1 = STEP(0.0)
ERR1 = IN1-OUT1
D1 = 100000.0*ERR1
D2 = LEDLAG(1.0/0.15,1.0/0.01,D1)
D3 = LEDLAG(1.0/30.0,1.0/300.0,D2)
IN2 = 2.245*D3
ERR2 = IN2-OUT2
A1 = LEDLAG(1.0/2.5,1.0/0.24,ERR2)
A2 = 10.0*A1
A3 = A2
A4 = 1.58*M*A3
A5 = REALPL(0.0,1.0/0.11,A4)
OUT = REALPL(0.0,1.0/370.0,A5)
B1 = 160.56*OUT
B2 = REALPL(0.0,1.0/100.0,B1)
OUT2 = N*B2
OUT1 = INTGRL(0.0,OUT)
PRTPLT OUT1
TIMER FINTIM = 1.0, OUTDEL = 0.01
LABEL SP-X-D, LAG INNER, LEAD OUTER
END
STOP
ENDJOB

```

Figure 4.12. The CSMP Program of the Compensated System ($\alpha < 15$) Shown in Figure 4.11 for the Step Response.

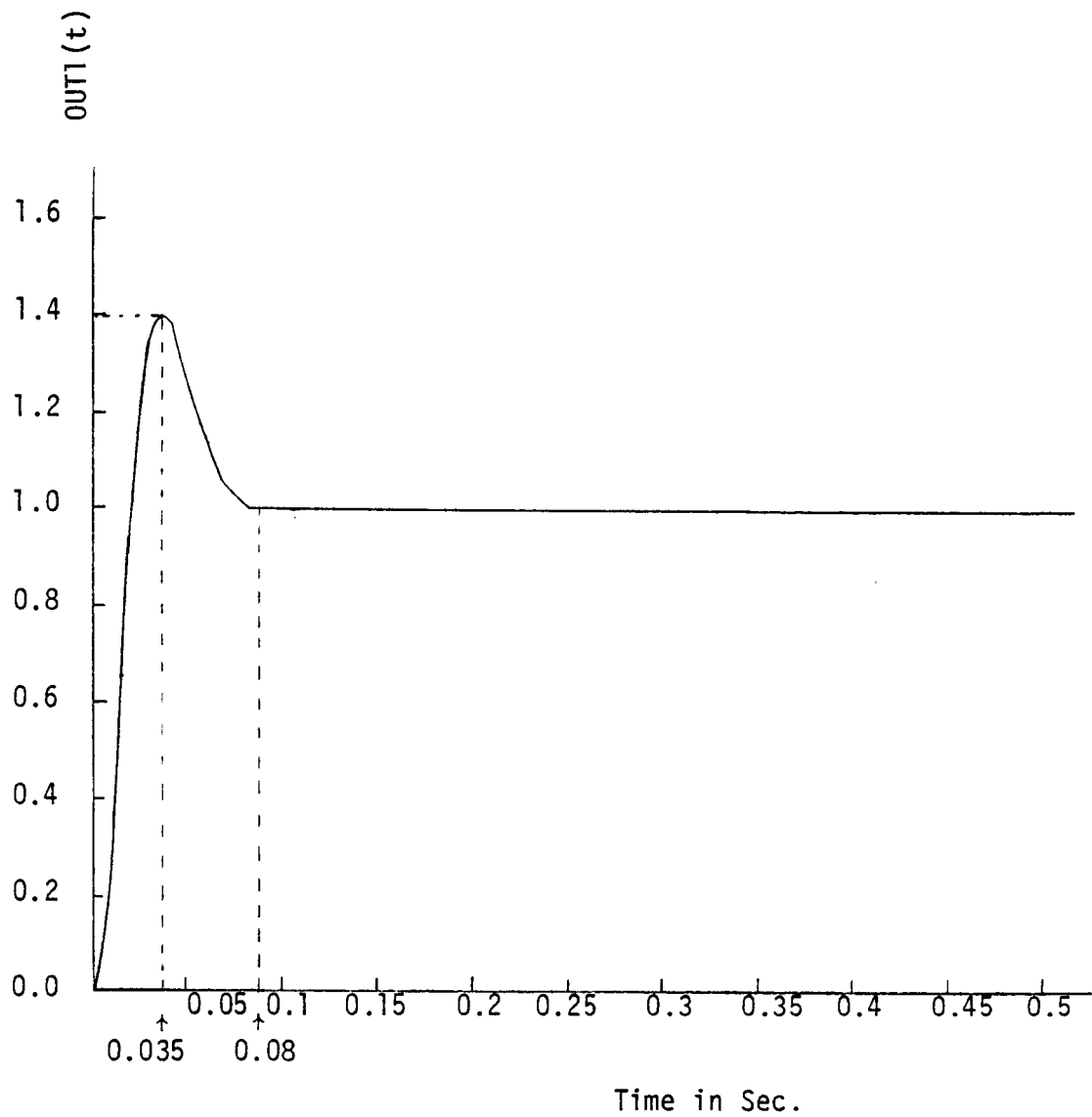


Figure 4.13. The Step Response of the Compensated System ($\alpha \leq 15$) of Figure 4.11.

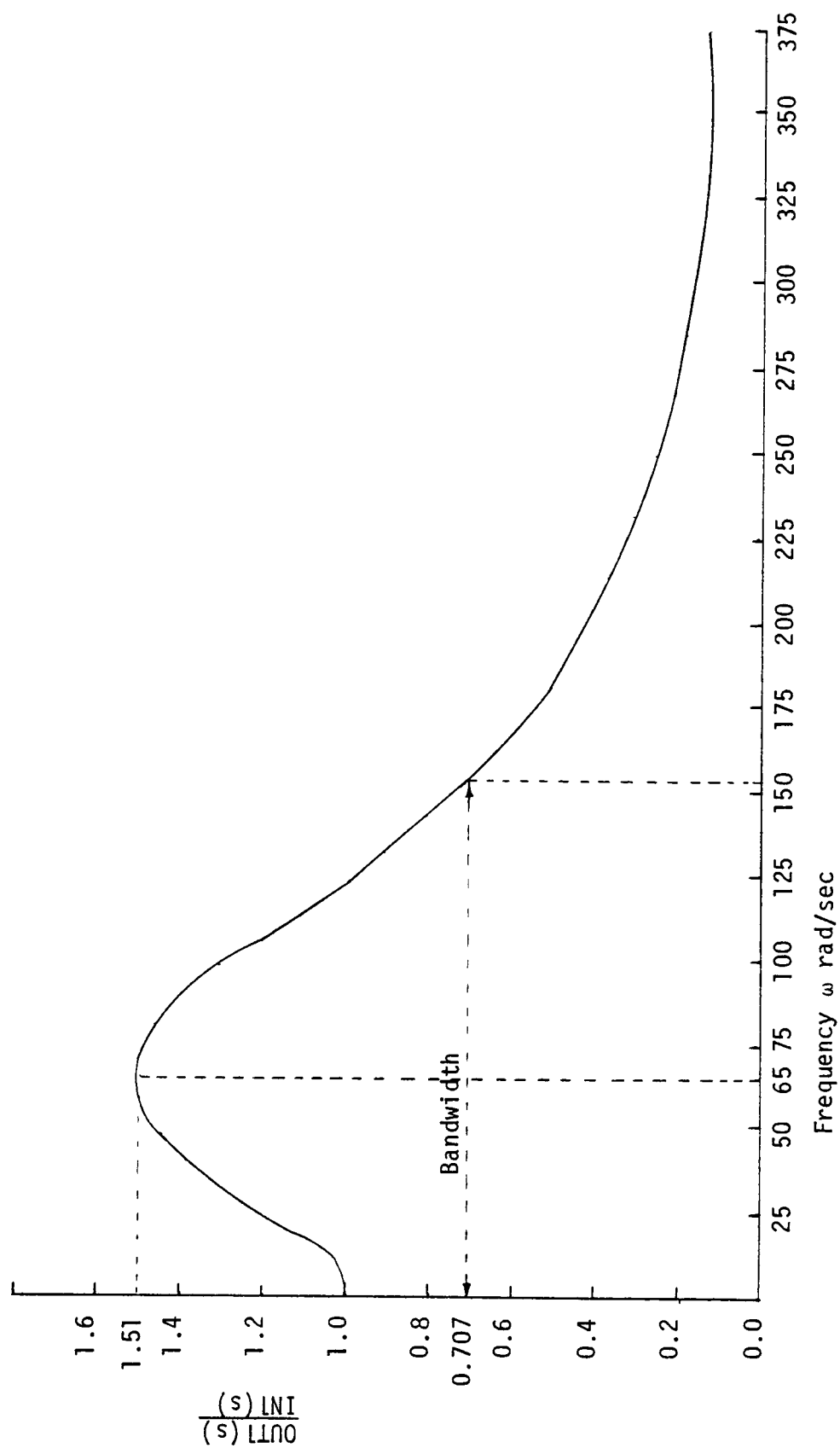


Figure 4.14. The Closed Loop Frequency Response of Equation 4.27.

of designing the compensators, difficulties arise because of the compensation format restrictions. Such restrictions are: α should be less than 15, and the high value of the velocity error constant.

This design can be realized by RC networks and operational amplifiers. Design 1 was developed after Design 2 was installed on the system. The system is operational and cannot be shutdown for the purpose of changing controls. Thus, Design 1 has not been placed on the system.

Design 2. Experimental Design and Achievement of Analytic Stability

Design 1 was developed after the system was operational with the experimental design and controllers. Modification of the resistor and capacitor values were made in the compensator using the change in performance of the system as a guide. Experimental design was achieved by adjusting the RC component values for best response to the velocity inputs and insensitivity to disturbance. Acceptable performance for the expected operating range was obtained in this manner.

The nature of this system, with the two feedback loops, cause several factors to be taken into account before any attempt to modify the system's performance can be made.

- (1) The input voltage level to the operational amplifier should be below the voltage that causes the output of the amplifier to saturate.
- (2) The output of the system is a series of pulses through the laser interferometer; therefore, it is not possible to measure directly the analog signal of the position output.

- (3) The output of the laser interferometer is connected to the numerical control unit internally.
- (4) The command signal to the numerical control unit is a series of pulses and is controlled by the velocity and the displacement of the cutting tool.
- (5) The output of the numerical control unit should be as small as possible to improve the quality of the finished product. Otherwise, the register's overflow indicator on the numerical control unit turns on and the reset procedure must be performed. Hence, the output waveform of the numerical control unit with disturbance input and the overflow indication are the only measures to judge the stability of the system.

Considering the above points, approximate compensation is connected to the linear motor system shown in Figure 4.1, page 42 . The values of resistors, capacitors, and potentiometer gains are adjusted such that the system yields minimum position error signal from the output of the numerical control unit. Also input disturbances (such as those obtained by striking the tool post) must be minimized.

Using the values of RC components shown in Figure 4.15 results in system stabilization experimentally. Conversion of the RC network compensator into a block diagram of the system including the compensators and gain which make the system stable is shown in Figure 4.16.

The experimentally determined compensation should be justified analytically by plotting frequency response diagram and calculating the closed loop poles of the system. The velocity error constant is

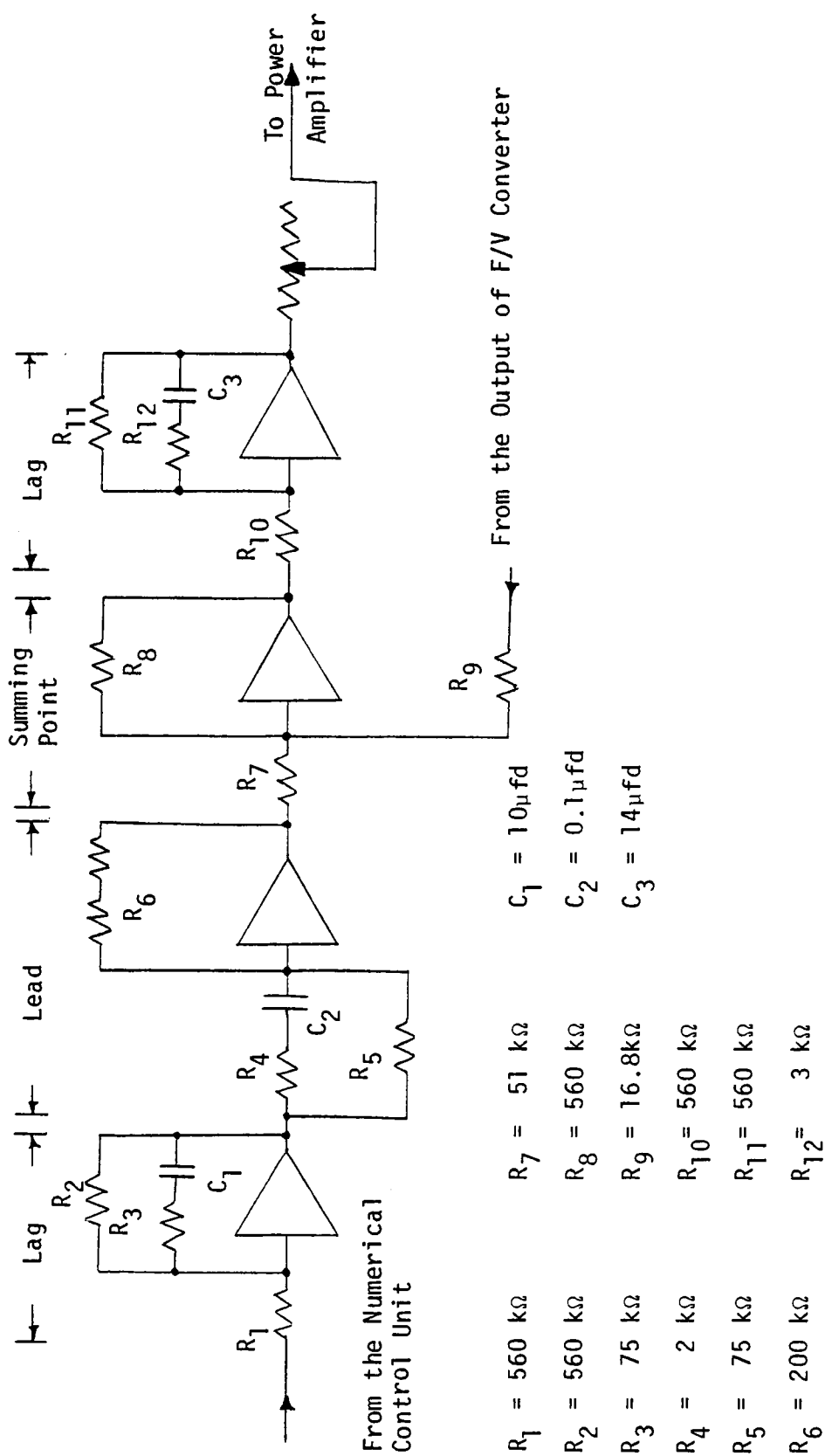


Figure 4.15. Realization of Compensation by RC Component Which Makes the System Stable.

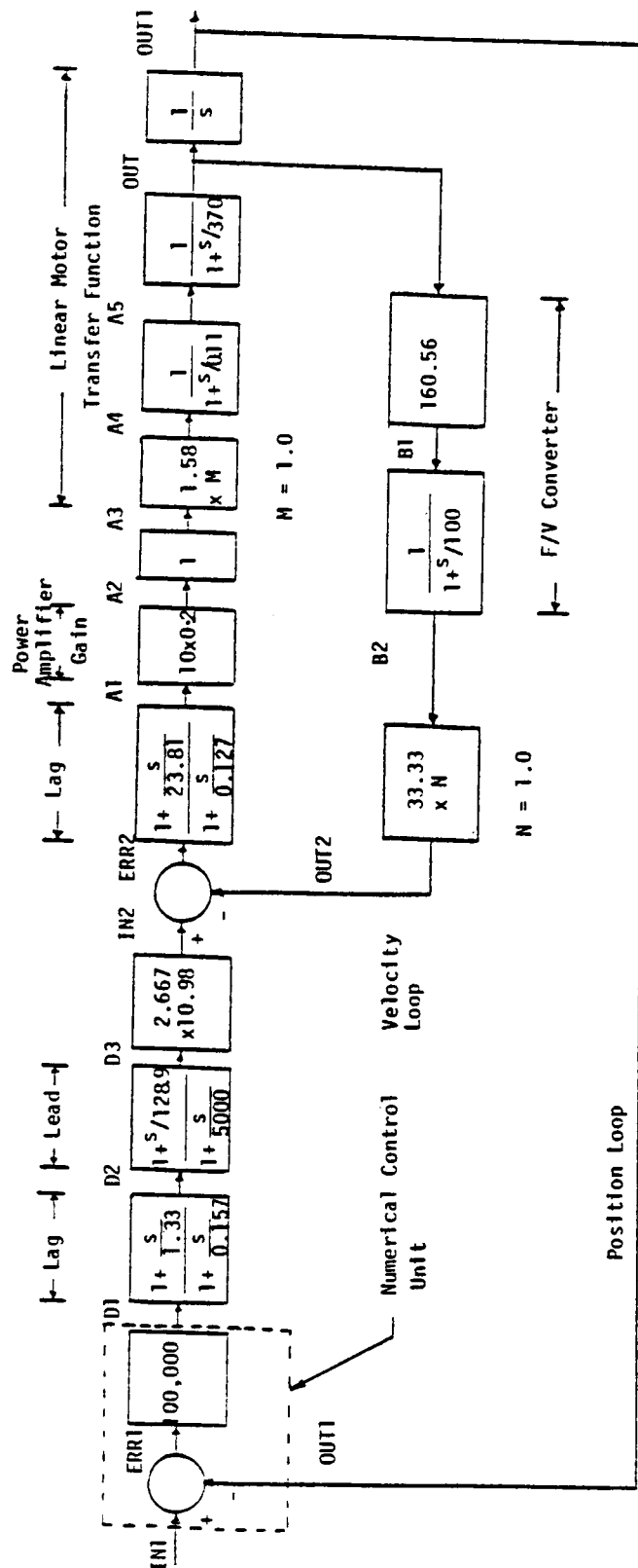


Figure 4.16. A CSMP Diagram of the Experimentally Compensated System (Without Change in Gains in the Velocity Loop).

calculated prior to calculation of the closed loop poles of the system. The open loop and the closed loop transfer functions of the velocity loop are calculated as

$$G_2(s)H_2(s) = \frac{367110 (s+23.8)}{(s+0.11)(s+0.127)(s+100.0)(s+370.0)} \quad \text{and} \quad (4.29)$$

$$\frac{OUT(s)}{IN2(s)} = \frac{0.686 (s+23.8)(s+100.0)}{(s+3.8+j15.7)(s+89.2)(s+373.4)} \quad (4.30)$$

The open loop transfer function of the position loop is given by

$$G_1(s)H_1(s) = \frac{13321003 (s+1.3)(s+130)}{s(s+0.16)(s+5000.0)} \times \frac{OUT(s)}{IN2(s)} \quad (4.31)$$

$$K_V = \lim_{s \rightarrow 0} sG_1(s)H_1(s) = 547 \text{ sec}^{-1}.$$

The velocity error constant is low relative to 10^6 sec^{-1} . The Bode diagram and the phase diagram of the open loop transfer function of the position loop of Equation 4.31 are shown in Figure 4.17. The following results are observed from the frequency plot:

Gain crossover point: $\omega = 33 \text{ rad/sec}$, phase = -197°

Phase crossover point: $\omega = 21 \text{ rad/sec}$, gain = 13 db
 $\omega = 64 \text{ rad/sec}$, gain = 12.33 db

Phase margin = -17° .

The gain crossover point falls where the phase is approximately -17° .

The stability of the system may be checked by the location of the closed loop poles. The closed loop transfer function is given by

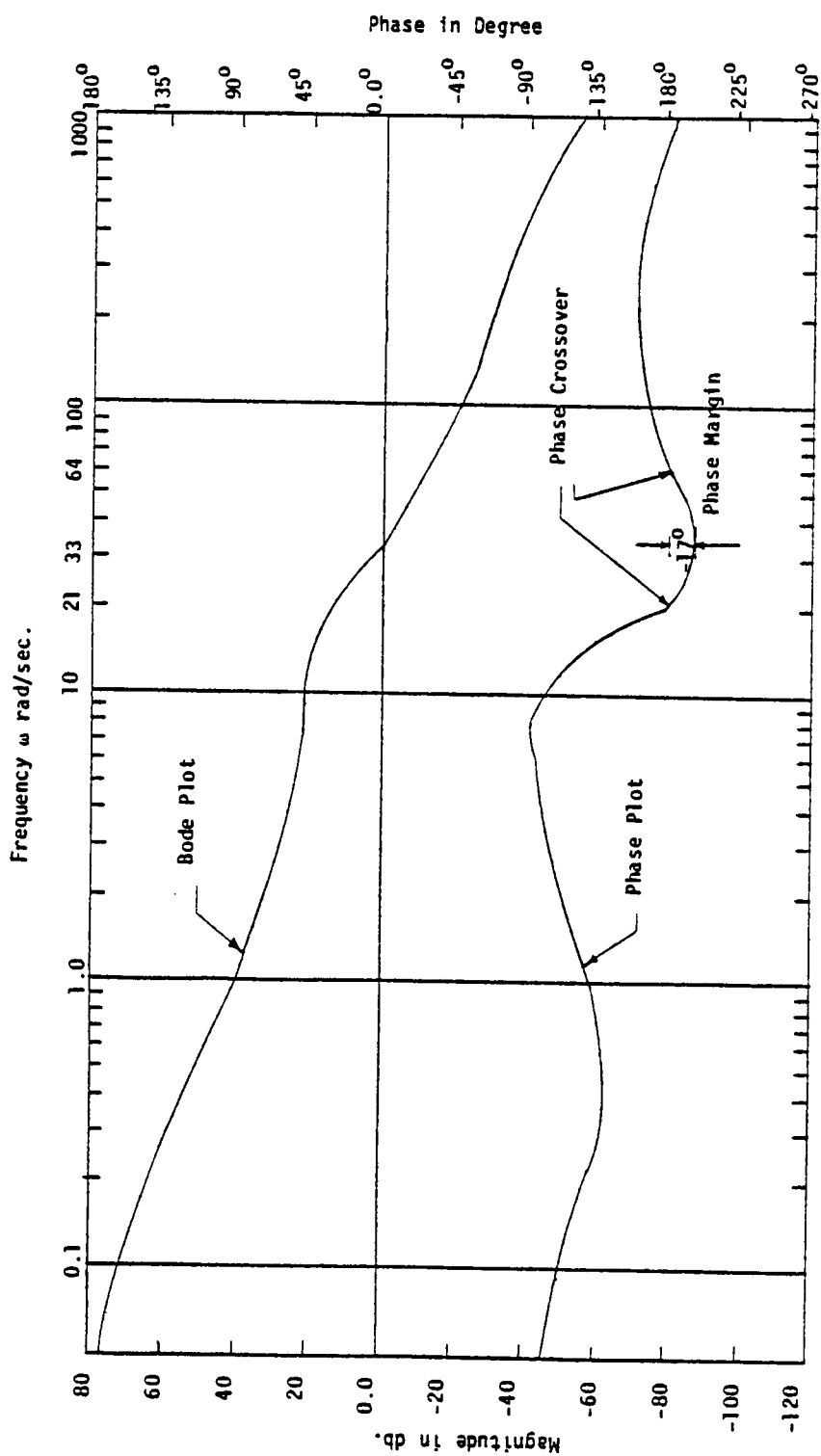


Figure 4.17. The Frequency Plot of $G_1(s)H_1(s)$ of Equation 4.31 for Design 2.

$$\frac{OUT1(s)}{IN1(s)} = \frac{9138208(s+23.8)(s+100)(s+1.33)(s+129.87)}{(s+1.4)(s+15)(s+90)(s+370.2)(s-3\pm j33.4)(s+5000)} \quad (4.32)$$

The location of the closed loop complex pole pair is located in the right half of s-plane. Hence, the system is conditionally stable.

Computer Simulation. The Continuous System Modeling Program (CSMP) for any system's simulation using block diagrams is given in Appendix C. The CSMP program for the step input response of the system of Figure 4.16 is shown in Figure 4.18. The computer plot of the step response of Figure 4.19 also shows that the system is unstable, that is, the magnitude of step response is increasing with time.

Referring to the frequency plot of Figure 4.17 and the closed loop poles, the system was conditionally stable. This design was not stable for the given model that was used. This is not because the model was very inaccurate but rather due to the conditionally stable system. The non-linearities (not included in the model due to the difficulty of determining non-linearity properties of air bearing slides and the day-to-day change in the slide system air pressure) cause the model to be inaccurate enough that the system operates in the unstable region. Changing the gain in the feedback path and the forward path of the velocity loop makes the system stable. It is believed that the system non-linearities caused the effect gain to be changed such that the system operated in a stable region.

```

INITIAL
PARAM M=1.0, N=1.0
DYNAMIC
IN1 = STEP(0.0)
ERR1 = IN1-OUT1
D1 = 100000.0*ERR1
D2 = LEDLAG(1.0/129.87,1.0/5000.0,D1)
D3 = LEDLAG(1.0/1.33,1.0/0.1575,D2)
IN2 = 10.98*2.667*D3
ERR2 = IN2-OUT2
A1 = LEDLAG(1.0/23.81,1.0/0.127,ERR2)
A2 = 10.0*0.2*A1
A3 = A2
A4 = 1.58*M*A3
A5 = REALPL(0.0,1.0/0.11,A4)
OUT = REALPL(0.0,1.0/370.0,A5)
B1 = 160.56*OUT
B2 = REALPL(0.0,1.0/100.0,B1)
OUT2 = 33.33*B2
OUT1 = INTGRL(0.0,OUT)
PRTPLT OUT1
TIMER FINTIM = 0.5, OUTDEL = 0.01
LABEL SP-X-D, LAG INNER. LEAD OUTER
END
STOP
ENDJOB

```

Figure 4.18. The CSMP Program of the Step Response of Design 2 Shown in Figure 4.16.

TIME	OUT1	MINIMUM	OUT1	MAXIMUM
		-1.1952E+01	I	4.0913E+00
0.0000E+00	0.0000E+00		-----+	
2.0000E-02	1.7952E-01		-----+	
4.0000E-02	6.2094E-01		-----+	
6.0000E-02	1.1943E+00		-----+	
8.0000E-02	1.6971E+00		-----+	
1.0000E-01	1.9254E+00		-----+	
1.2000E-01	1.7614E+00		-----+	
1.4000E-01	1.2354E+00		-----+	
1.6000E-01	5.3161E-01		-----+	
1.8000E-01	-6.9600E-02		-----+	
2.0000E-01	-2.9770E-01		-----+	
2.2000E-01	-7.2107E-03		-----+	
2.4000E-01	7.4562E-01		-----+	
2.6000E-01	1.7018E+00		-----+	
2.8000E-01	2.4838E+00		-----+	
3.0000E-01	2.7405E+00		-----+	
3.2000E-01	2.2990E+00		-----+	
3.4000E-01	1.2593E+00		-----+	
3.6000E-01	-1.3419E-02		-----+	
3.8000E-01	-1.0106E+00		-----+	
4.0000E-01	-1.2769E+00		-----+	
4.2000E-01	-6.0922E-01		-----+	
4.4000E-01	8.2638E-01		-----+	
4.6000E-01	2.5188E+00		-----+	
4.8000E-01	3.7852E+00		-----+	
5.0000E-01	4.0377E+00		-----+	

Figure 4.19. The Step Response of Design 2
Shown in Figure 4.16.

Achievement of Analytic Stability. Although Design 2 provides acceptable performance, analytic performance on the CSMP program is unstable, as seen in Figure 4.19. Since the design is workable, changing of the resistance and capacitance values in the compensator is not desirable. However, by changing the values of gain in the forward path and the feedback path of the velocity loop, the system can be made analytically stable.

The value of the velocity error constant must be high enough to reduce the friction in the system. System with numerical control unit, the following error should be minimum to avoid overflow of the registers in the numerical control unit. High gain in the position loop or high velocity error constant provides a stiff system. Design 2 provides a satisfactory velocity error constant to insure experimentally acceptable performance. By changing the gains in the velocity loop in Design 2, the calculated velocity error constant is increased by approximately an order of magnitude. This provides acceptable analytic performance on the CSMP program without any change in the RC values of the compensation.

The value of the velocity error constant for Design 2 was calculated to be 547. After changing the gain in the forward path of the velocity loop from 0.2 to ten, and decreasing the velocity feedback gain from 33.33 to 3.3, the new calculated velocity error constant is found to be 5527. A CSMP block diagram of the system with adjusted gain is shown in Figure 4.20. The open loop and the closed loop transfer functions of the velocity loop are calculated as

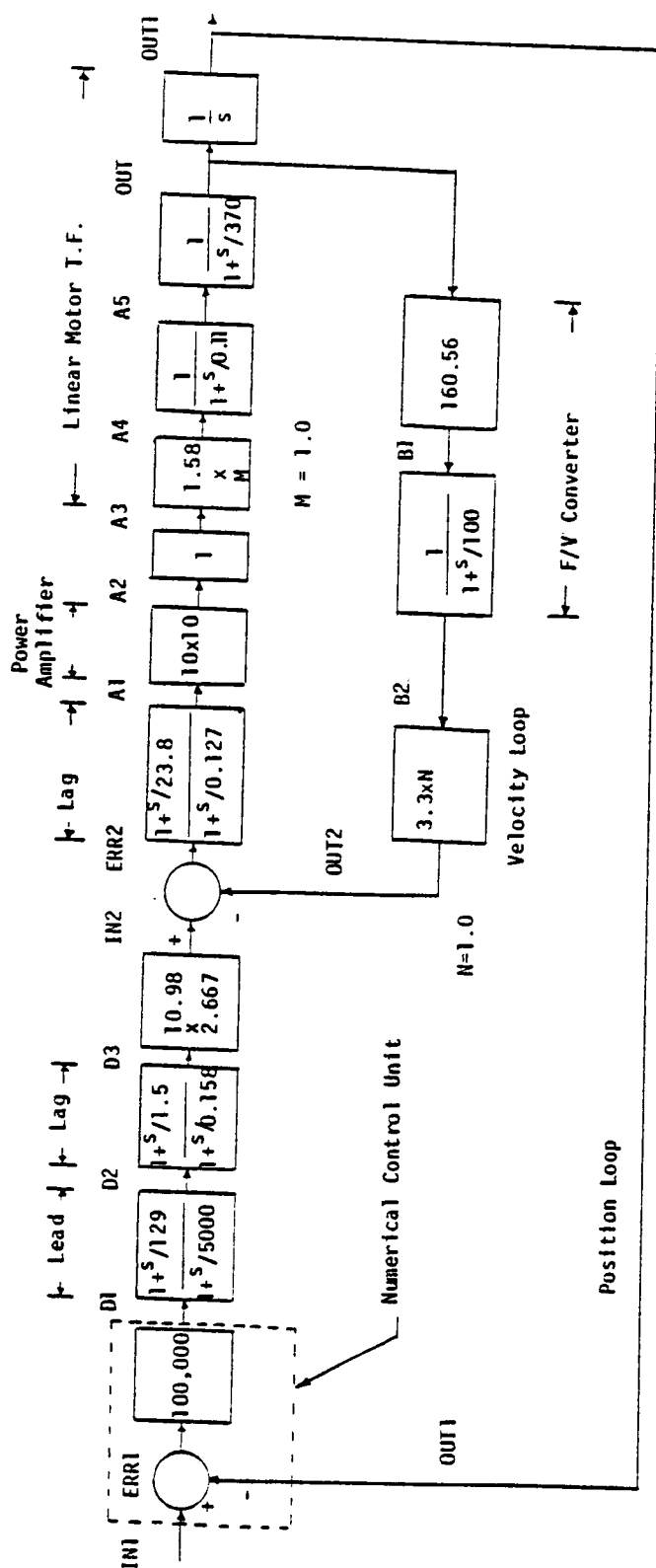


Figure 4.20. A CSMP Block Diagram of the Experimentally Compensated System (With Change in Gains in the Velocity Loop).

$$G_2(s)H_2(s) = \frac{1817378(s+23.8)}{(s+0.11)(s+0.127)(s+100)(s+370)} \cdot \quad (4.33)$$

$$\frac{OUT(s)}{IN2(s)} = \frac{34.3(s+23.8)(s+100)}{(s+42.35)(s+21.2+j46.9)(s+385.5)} \cdot \quad (4.34)$$

The open loop transfer function of the position loop is given by

$$G_1(s)H_1(s) = \frac{11837931(s+129.9)(s+1.5)}{s(s+5000)(s+0.158)} \times \frac{OUT(s)}{IN2(s)} \cdot \quad (4.35)$$

$$K_V = \lim_{s \rightarrow 0} sG_1(s)H_1(s) = 5527 \text{ sec}^{-1}.$$

The frequency plot of the Equation 4.35 is sketched in Figure 4.21. The frequency response gives the following results:

Gain crossover point: $\omega = 229 \text{ rad/sec}$, phase = -161°

Phase margin = 19°

Phase crossover point: $\omega = 900 \text{ rad/sec}$, gain = -26.5db .

A significant feature of the phase diagram is that the system has a positive phase margin until a frequency of 900 rad/sec is reached. The gain crossover frequency must be less than the phase crossover frequency; thus the closed loop stability is also determined by this criterion. In this case, the gain phase crossover frequency is 229 rad/sec and the phase crossover frequency is 900 rad/sec . Thus, the system can operate over a wide range of frequency. The closed loop poles can be a guideline for interpreting the system's behavior.

$$\frac{OUT1(s)}{IN1(s)} = \frac{406041046(s+129.9)(s+1.5)(s+23.8)(s+100)}{(s+15)(s+23)(s+79)(s+45+j225)(s+260)(s+5017)} \cdot \quad (4.36)$$

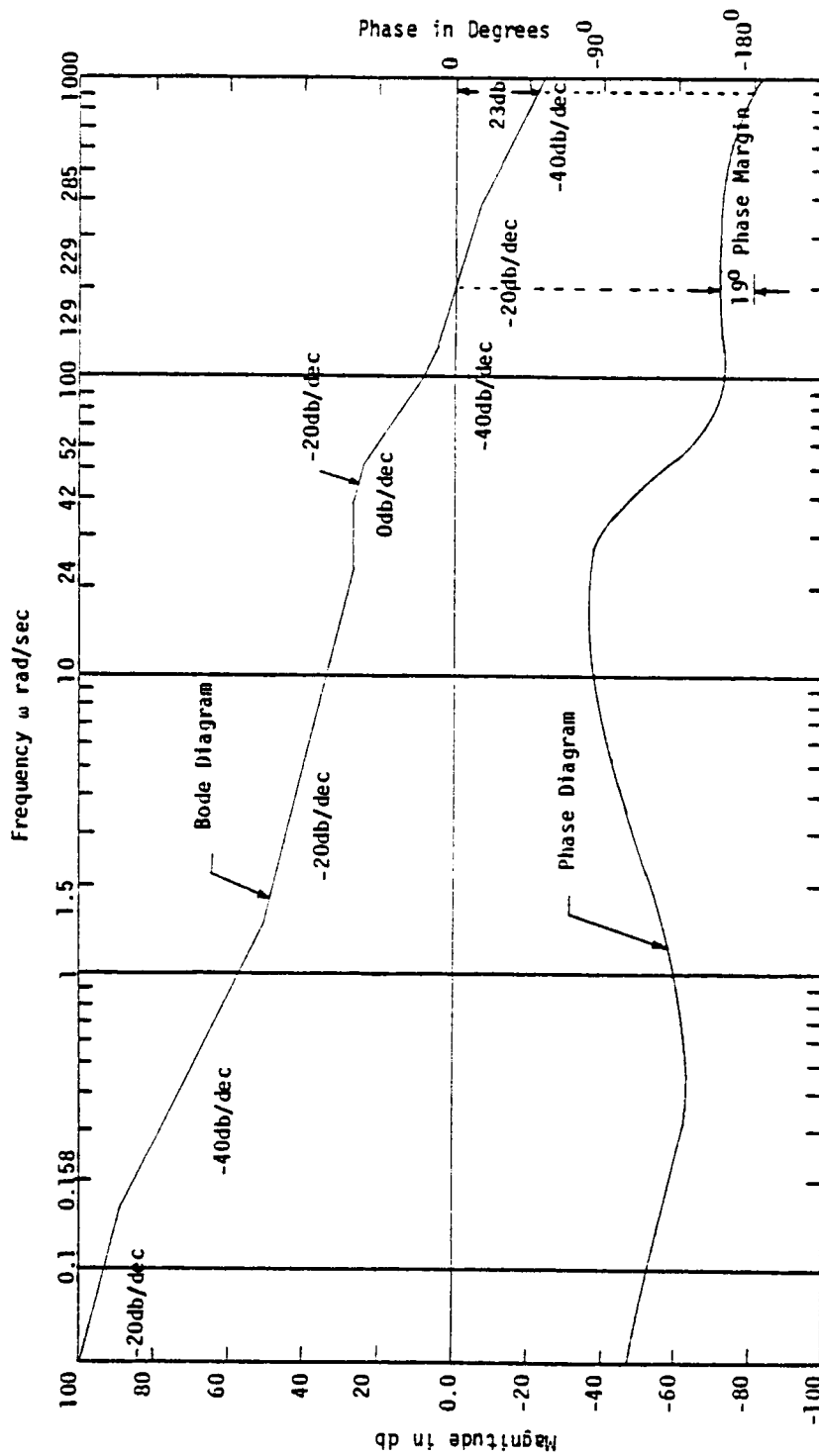


Figure 4.21. The Frequency Plot of $G_1(s)H_1(s)$ of Equation 4.35.

The dominant complex pole pair has a damping ratio of 0.2 and natural frequency of 229 rad/sec. The rise time can be calculated by the empirical formula

$$t_r \approx \frac{0.8 + 2.5 \zeta}{\omega_n} \quad 0 < \zeta < 1. \quad (4.27)$$

where ζ = damping ratio

ω_n = natural frequency rad/sec.

Therefore,

$$t_r \approx \frac{0.8 + 2.5 \times 0.2}{229} = 0.006 \text{ sec.} \quad (4.28)$$

The system therefore responds very fast to step input commands. The step response of the system with experimentally obtained compensation has the following characteristics:

Rise time = 0.006 second

First peak time = 0.015 second

Settling time = 0.12 second

Overshoot = 63%

The comparison of results of Design 1 (with inner loop lag compensator) and Design 2 is tabulated in Table 4.1. The step response of the system with experimentally obtained compensation has 63% and first peak occurs at 0.015 second.

In this chapter, Design 1 is achieved analytically and meets the system specifications. Using the basic design technique, the RC values were changed experimentally until minimum position error was achieved.

Table 4.1 Comparison of Results for Design 1 and Design 2

Type	K_v	Overshoot %	Rise Time Sec	Settling Time Sec	Phase Margin Degree	Resonant Peak m_p	Bandwidth rad/sec	Cut-off Rate
Design 1 $\alpha \leq 15$	10^6	39	0.017	0.08	40°	1.51	144	Moderate
Design 2	547	Unstable	-	-	-17°	3.58	52	Very Fast
Gain Change in the Velocity Loop	5527	63	0.007	0.12	19°	3.08	450	Fast

Using experimentally determined compensation tried on computer, the results were unacceptable, although performance in a practical situation was good. Changing gains, the system was found to work well on the computer. Design 2 was achieved in this manner. Results of Design 1 and Design 2 are tabulated and compared from Table 4.1.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

A dynamic model for the linear motor machine-tool control system has been developed in terms of the electrical time constant, and the mechanical time constant. The linear motor model has been found to resemble the model of a D.C. torque motor. Conventional compensation using the Bode diagram, the phase diagram, and the root locus techniques have been designed which give acceptable performance.

In Design 1 ($\alpha \leq 15$), compensation is designed analytically and meets the system specifications. The settling time for the step response is 0.08 second which is better than the rest of the designs. The system has a bandwidth of 144 rad/sec and moderate cut off rate of the closed loop frequency response. It has 39.1% overshoot.

Using the same basic design, values of resistance and capacitance were varied experimentally until minimum position error was achieved. Using the experimentally determined resistance and capacitance values, the system design was again tried on the computer. This design was not stable for a given model that was used. This is not because the model was very inaccurate but rather due to the conditionally stable system. The computer results were unaccepted (63% overshoot), although in actual practice performance was good.

As the system was operational with Design 2, no more changes were made on the compensators. However, Design 2, which retains the same values of resistor and capacitor but changes the gain in the forward and the feedback paths of the velocity loop, was found to work well on the

computer. After achieving Design 2, no change was implemented on the actual machine because Design 2 cuts high quality surface finished parts. The step response of Design 2 has a 0.006 second rise time. The settling time is 0.12 second. The overshoot is 63% which is higher than that of Design 1. Since system operation is acceptable with Design 2 ($\alpha = 188$), no further modifications were made in the practical system.

Several areas of research not covered in the scope of this work are apparent and could be the subject of future projects to further improve knowledge of the linear motor-tool control systems. Some of these areas are:

- (1) The study of the sensitivity of the linear motor-tool control system to variation of system parameters.
- (2) The determination of the effects of friction and non-linearities on the stability of the system.
- (3) The effect on the stability of the system when making interrupted cuts.

LIST OF REFERENCES

LIST OF REFERENCES

1. Barkman, W. E., A Linear-Induction-Motor Slide Drive, Oak Ridge, Tennessee: The U. S. Department of Energy, Y-2089, October 1977.
2. Green, W. L. Unpublished Class Notes, EE 5240, The University of Tennessee, Knoxville: 1976.
3. Inland Motor Corporation, Systems Analysis, Section 2.
4. Green, C. W., Paul, J. A., Application of D.C. Linear Motor As Short Stroke and Static Actuator, Proceeding IEE, Vol. 116, No. 4 April 1969, pp. 599-604.
5. Green, C. W., Paul, J.A., Assessmant of Flux Distributions, Proceeding IEE Vol. 118, No. 10, October 1972. pp. 1413-1420.
6. Nasar, S. A., Boldea, I., Linear Motion Electric Machines, New York: John Wiley and Sons: 1976.
7. Kuo, B. C., Automatic Control Systems, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., Third Edition, 1975.
8. General Electric, Mark Century 1500 Series Numerical Contouring Control, Waynesboro, Virginia: Numerical Equipment Control Department, 1969.
9. Eveleigh, V. W., Introduction to Control Systems Design, New York: McGraw Hill Book Company, 1972.
10. Gupta, Someshwar. C., Hasdorff, L., Fundamentals of Automatic Control, New York: John Wiley and Sons, Inc., 1970.
11. Green, W. L., Speckhart, F. H., A Guide to Using the Continuous System Modeling Program, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1976.

APPENDIXES

APPENDIX A

This appendix gives plots of approximate current required for different size linear motors at three different positions of moving coil for 20 lb. forces shown in Figure A.1.

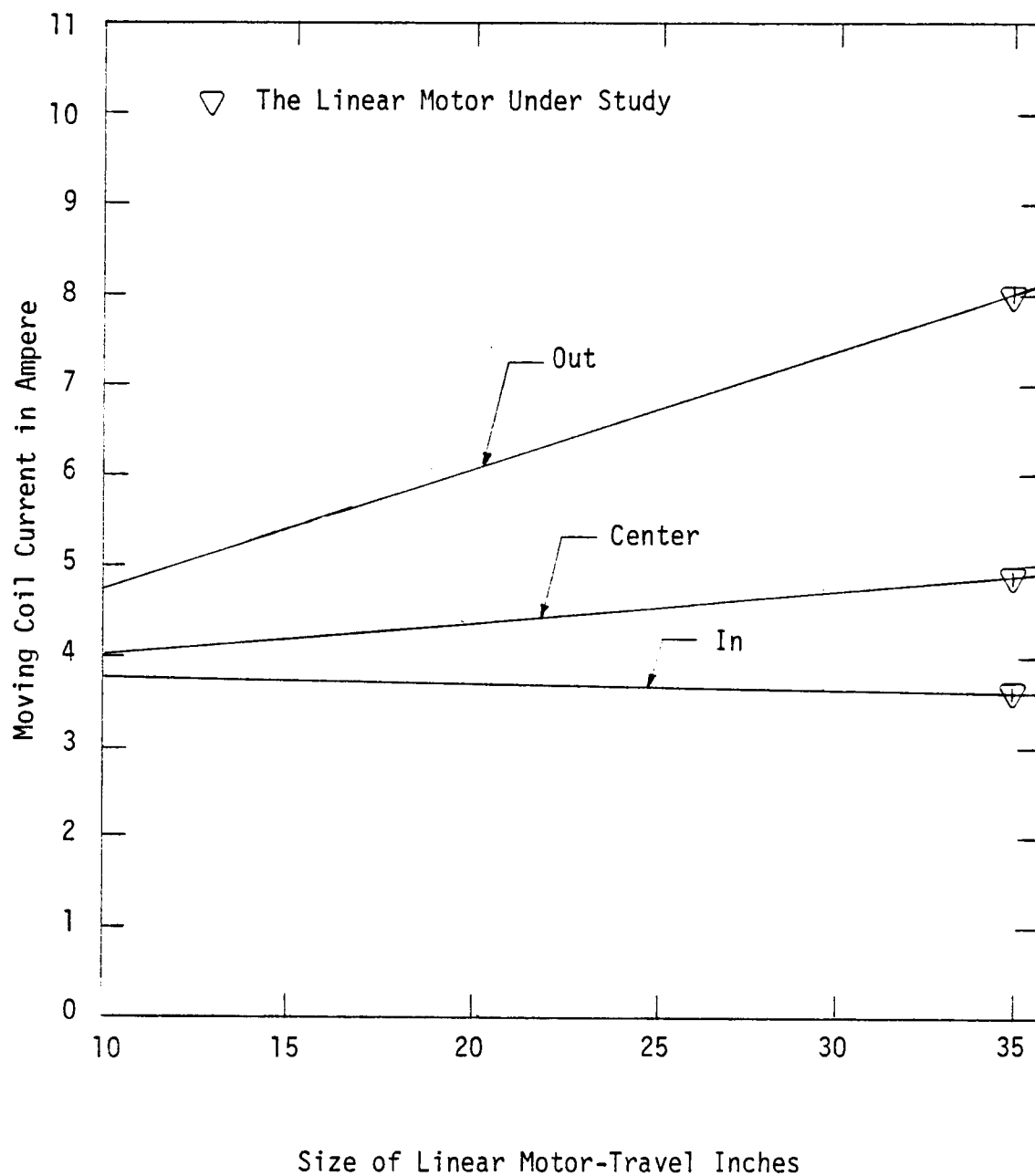


Figure A.1. A Plot of Approximate Current Required for Different Size Linear Motors at Three Different Positions of Moving Coil for 20 lb. Forces.

APPENDIX B

This appendix contains the determination of the electrical time constant, T_e and the mechanical time constant, T_m of the linear motor.

The linear motor develops a force in the horizontal direction due to the interaction between the strong magnetic flux established by exciting coil and the current carrying conductors of the moving member. Force developed and inductance of the linear motor are relative to the position of the moving member. Hence, the parameters of the linear motor are calculated when the moving member is fully extended, in the center and fully retracted. Parameter values of the linear motor obtained at the center position of the moving member are considered for design and simulation purposes. The maximum and minimum flux linkage are obtained when the moving member is fully retracted and fully extended, respectively. From Chapter III, force developed, the mechanical time constant, and the electrical time constant are derived as

$$F = BL_l I$$

$$T_m = \frac{MR_T}{BL_l K_b}$$

$$T_e = \frac{L}{R_T}$$

The values of M , R_T , and K_b are constant irrespective of the position of the moving member. It is measured that mass, $m = 6000$ lb, $R_T = 1.45$ ohm, and $K_b = 0.63$ volts/inch/second. Whereas, the values of L and BL_l are not constant at different position of the moving member.

The bridge circuit is used to measure the inductance, L , of the linear motor at different positions of the moving member. Force developed by the linear motor is calculated analytically at three different positions of the moving member. The values of inductance, force, the mechanical time constant and the electrical time constant are shown in Table B.1. The transfer function of the linear motor is given by Equation 3.20.

$$\frac{X(s)}{E(s)} = \frac{\frac{1}{K_b}}{s(1 + T_m s)(1 + T_e s)}$$

The magnitude plot of the linear motor transfer function at three different positions of the moving member is shown in Figure B.1. It is concluded that consideration of parameters of the linear motor at the center position of the moving member allows the variation of parameters whether it is fully extended or fully retracted.

Table B.1. Parameters of the Linear Motor at Different Positions of the Moving Member

Position of the Moving Member	Inductance L MilliHenry	Force $F = BL_1 I$ lb.	Mechanical Time Constant T_m Second	Electrical Time Constant T_e Second
Fully Extended	5.5	2.5 I	14.4	0.0038
Center	3.9	4 I	9.0	0.0027
Fully Retracted	2.35	5.5 I	6.5	0.0016

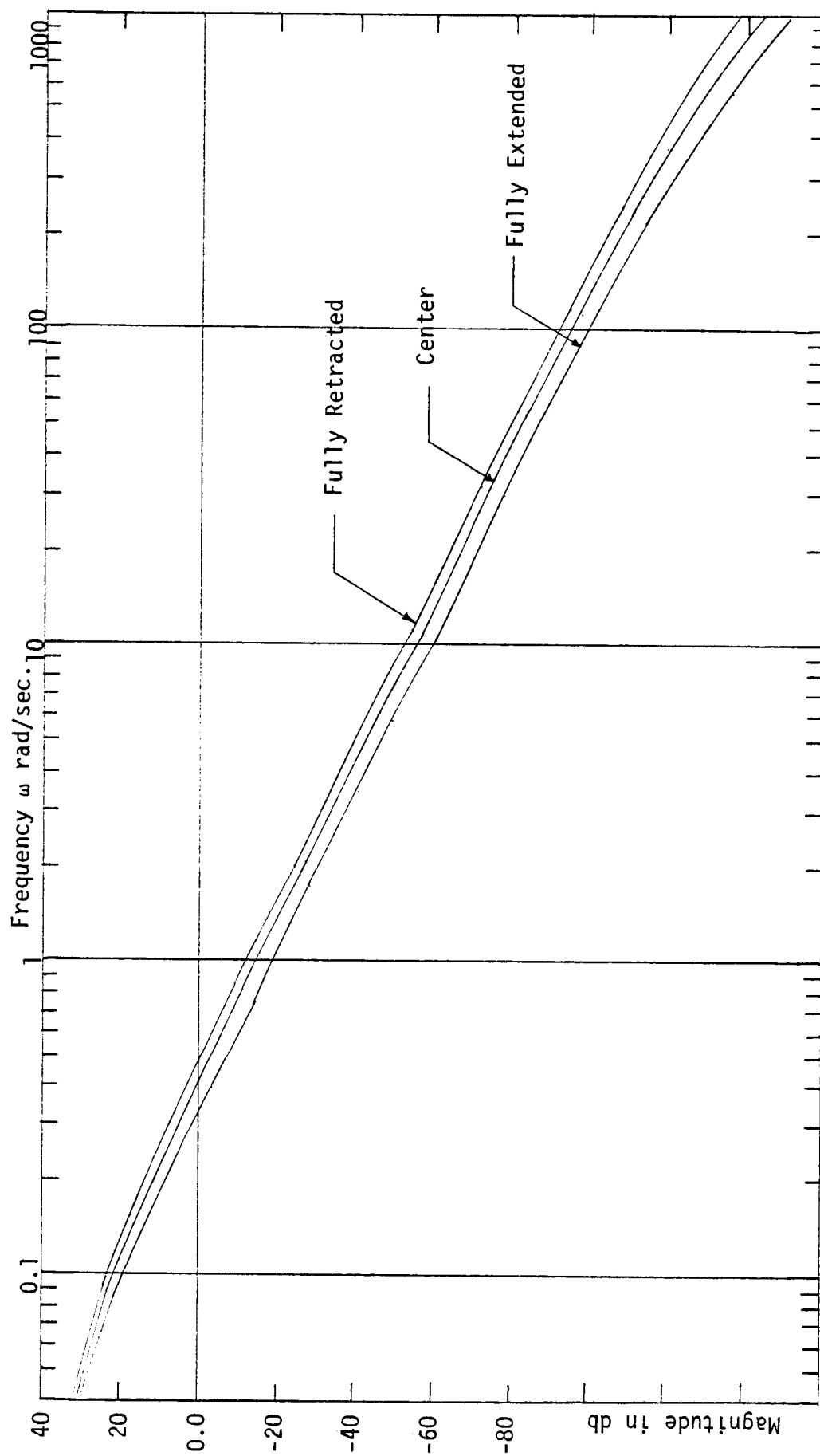
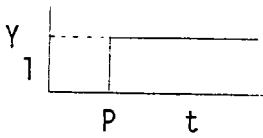


Figure B.1. The Magnitude Plot of the Linear Motor Transfer Function at Three Different Positions of the Moving Member.

APPENDIX C

This appendix gives an overview of the continuous system modeling program (CSMP) which was developed by IBM. It provides an application oriented input language that accepts problems expressed in the form of either an analog block diagram or system of ordinary differential equations. Data input and output are facilitated by means of application oriented control statements. A brief description of the functions is given along with equivalent Laplace transform.

Function	Equivalent Laplace Transform	General Form
$Y = \int_0^t x \, dt + I_c$	$\frac{1}{s}$	Y=INTGRL (IC,x) Y(0) + IC INTEGRATOR
$Y = \frac{dx}{dt}$	s	Y=DERIV (IC,x) x'(t=0)=IC DERIVATIVE
$PY' + Y = x$	$\frac{1}{Ps+1}$	Y=REALPL (IC, P,x) Y(0) = IC 1st ORDER LAG (REAL POLE)
$P_2Y' + Y = P_1x' + x$	$\frac{P_1s + 1}{P_2s + 1}$	Y=LEDLAG(P1,P2,x) P1 and P2 are parameters; initial conditions are not used with function. LAG-LEAD
Y=0 t<P Y=1 t>P	Signal Source 	Y=STEP (P) P=Starting time of the step. STEP FUNCTION

For the control system under analysis, the block diagram is shown in Figure 4.7, page 57. The CSMP statements for this block diagram are given in Figure 4.8, page 59. The output at any stage can be printed and plotted by the use of Executive Control statements. The following are Executive Control statements:

(1) PRTPLT OUT1 (ERR1)

The quantity OUT1 (Final Position Output) will be printed and plotted while quantity ERR1 within parenthesis will only be printed at each step.

(2) TIMER FINTIM = 1.0, OUTDEL = 0.01

This statement causes OUT1 to be printer-plotted every 0.01 seconds until 1.0 seconds is reached. OUTDEL controls the output increment of the independent variable for printer-plots.

APPENDIX D

This appendix contains the realization of lag and lead compensations using RC network and operation amplifier. Figure D.1 (a) shows the general representation of compensation. $E_{in}(s)$ and $E_{out}(s)$ are the input voltage and the output voltage of the operational amplifier, respectively. $Z_f(s)$ and $Z_{in}(s)$ are the feedback impedance and the input impedance of the operation amplifier, respectively. Hence, the input voltage and the output voltage are expressed as

$$\frac{E_{out}(s)}{E_{in}(s)} = - \frac{Z_f(s)}{Z_{in}(s)} \quad (D.1)$$

The negative sign is due to signal inversion of the operational amplifier.

Lag Compensation

A lag compensator is expressed as

$$G_c(s) = \frac{1 + \frac{T}{\alpha} s}{1 + T s} \quad \alpha \leq 15. \quad (D.2)$$

(Lag)

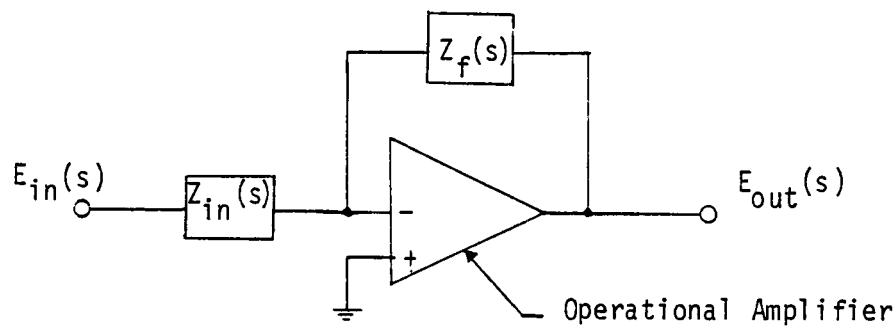
Lag compensation using an RC network is shown in Figure D.1 (b). The feedback impedance, $Z_f(s)$ and the input impedance, $Z_{in}(s)$ are calculated from Figure D.1 (b) as

$$Z_f(s) = \frac{R_2 (1 + R_1 C_1 s)}{1 + (R_1 + R_2) C_1 s} \quad (D.3)$$

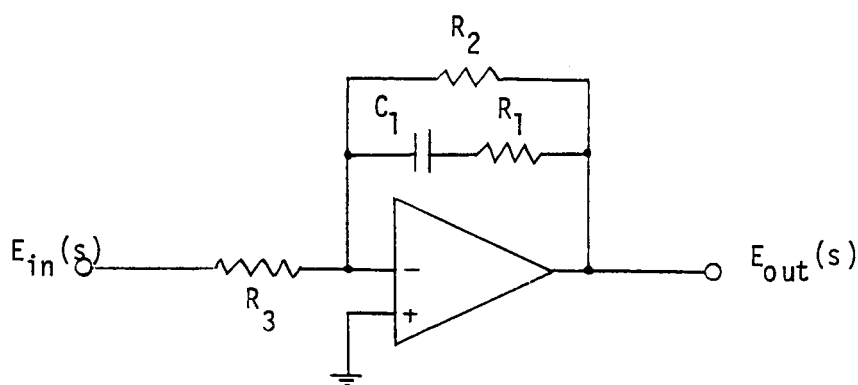
and

$$Z_{in}(s) = R_3 \quad (D.4)$$

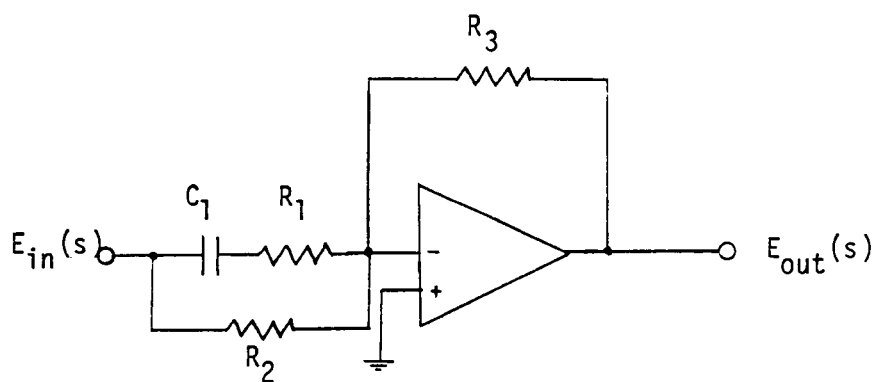
Using Equations D.3 and D.4, $\frac{E_{out}(s)}{E_{in}(s)}$ may be written as



(a)



(b)



(c)

Figure D.1. (a) General Representation of Compensation Using Operational Amplifier and RC Network; (b) Realization of Lag Compensator; (c) Realization of Lead Compensation.

$$\frac{E_{out}(s)}{E_{in}(s)} = \frac{R_2 (1 + R_1 C_1 s)}{R_3 [1 + (R_1 + R_2) C_1 s]} \quad (D.5)$$

By comparing the coefficients of Equation D.2 and Equation D.5, values of resistors and capacitor are calculated for realization of the lag compensator.

Thus,

$$R_2 = R_3$$

$$R_1 C_1 = \frac{T}{\alpha}$$

$$(R_1 + R_2) C_1 = T$$

For simplicity the value of capacitor is chosen first, and then R_1 , R_2 and R_3 are found.

Lead Compensation

A lead compensator is assumed to have the form of

$$\begin{matrix} G_c(s) \\ \text{(Lead)} \end{matrix} = \alpha \frac{1 + \frac{T}{\alpha} s}{1 + T s} \quad \alpha < 1. \quad (D.6)$$

Lead compensation using an RC network is shown in Figure D.1 (c). The feedback impedance, $Z_f(s)$ and the input impedance, $Z_{in}(s)$ are calculated from Figure D.1 (c) as

$$\begin{aligned} Z_f(s) &= R_3 \\ Z_{in}(s) &= \frac{R_2(1 + R_1 C_1 s)}{1 + (R_1 + R_2) C_1 s} \end{aligned} \quad (D.8)$$

Substituting $Z_f(s)$ and $Z_{in}(s)$ in Equation D.1, $\frac{E_{out}(s)}{E_{in}(s)}$ is found.

$$\frac{E_{out}(s)}{E_{in}(s)} = - \frac{R_3 [1 + (R_1 + R_2) C_1 s]}{R_2 (1 + R_1 C_1 s)} . \quad (D.9)$$

By comparing the coefficients of Equation D.9 and Equation D.6, values of resistors and the capacitor are calculated for realization of the lead compensator. This yields;

$$R_2 = R_3$$

$$(R_1 + R_2) C_1 = \frac{T}{\alpha}$$

$$R_1 C_1 = T$$

For convenience the value of capacitor is selected first and then R_1 , R_2 and R_3 are found.

VITA

Bharat Subodhchandra Shah was born in Bombay, India, on February 10, 1953. He attended primary school from June 1959 to May 1965 and high school from June 1965 to April 1969. He graduated from Swastik Shishuvihar High School, Ahmedabad in April 1969.

He began his university work in M. G. Science Institute, Ahmedabad, affiliated with the Gujarat University. He received the degree of Bachelor of Science in Electronics and Control System Engineering from the Gujarat University, Ahmedabad in April 1975. As an undergraduate, summer employment included training at Ahmedabad Electricity Board. He worked as a sales engineer for New India Electric Motor Manufacturing Company for three months. He also worked for Gujarat Electricity Board as a Junior Engineer for two months. He accepted employment at Gujarat State Fertilizer Corporation, Baroda, India in February 1976 and continued there until he left India for further studies.

He entered the Graduate School at The University of Tennessee, Knoxville where he began his graduate studies toward the Master of Science degree in Electrical Engineering with a major in Control Systems. From August 1977 to July 1978, he was a research assistant under the guidance of Professor Dr. Walter L. Green for a project sponsored by the Y-12 Plant of Union Carbide Nuclear Division, Oak Ridge, Tennessee. At present he is a graduate teaching assistant in the Electrical Engineering Department. Currently, he is working towards his second Master of Science degree in Engineering Administration at The University of Tennessee, Knoxville.