

To the Graduate Council:

I am submitting herewith a dissertation written by William Stevens entitled "Experimental Investigation of Cylinder Back-Pressuring Effects on Isolator Shock Train Dynamics." I have examined the final paper copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Aerospace Engineering.

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Trevor M. Moeller, Major Professor

We have read this dissertation  
and recommend its acceptance:

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John D. Schmisseeur

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Phillip A. Kreth

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(Original signatures are on file with official student records.)



# Experimental Investigation of Cylinder Back-Pressuring Effects on Isolator Shock Train Dynamics

A Dissertation Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

William Stevens

December 2025

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*To Brianna, Hayden, Hanna, and Hope.*

*I love you with all of my heart.*

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*So we fix our eyes not on what is seen, but on what is unseen. For what is seen is temporary, but what is unseen is eternal.*

2 Corinthians 4:18 (NIV<sup>®</sup>)

# Abstract

The operational viability of dual-mode scramjet (DMSJ) engines across a wide Mach range depends on the isolator's ability to condition the flow for combustion while maintaining sufficient margin against inlet unstart. Experimental simulations of isolator back-pressure rise due to combustion often rely on methods that decouple the shock train response from the back-pressuring device itself. While useful for simulating gross pressure rise due to combustion heat release, these approaches fail to fully capture the coupled nature of three-dimensional, shock interaction effects that would be generated by a fuel injection flow blockage. The work presented here aims to address this knowledge gap by quantitatively assessing the formation process, propagation characteristics, and inherent unsteadiness of a shock train using a three-dimensional back-pressuring device.

Experimental testing of a shock train was conducted in the Mach 2.2 UTSI direct-connect isolator facility. Back-pressure was generated using a floor-mounted cylindrical rod inserted perpendicular to the flow at the exit plane of the isolator. The choice of a cylindrical rod was motivated by its known shock-feature similarity to a flush-wall fuel injector. Floor surface pressure measurements and optical diagnostic techniques including high-speed background-oriented schlieren and fast-response pressure sensitive paint were performed, and quantitative time-histories of shock features were extracted from the optical imagery using a custom tracking algorithm. Shock train unsteadiness and identification of the underlying propagation mechanism was analyzed using results from zero-crossing frequency estimates and cross-correlation of the tracked shock signals.

This work reveals the strong coupling between shock train behavior and three-dimensional back-pressuring, evidenced by distinct shock train propagation phases within the isolator that originate from cylinder-induced boundary layer separation events. The intentionally simple method of using a floor-mounted cylinder as a back-pressuring device was designed to integrate multiple, and previously isolated, research areas. Specifically, the cylindrical-rod representation of flow blockage caused by flush-wall fuel injection, the unsteady physics of cylinder-induced shock boundary layer interactions, and the complex sensitivity of shock train propagation to back-pressure effects all converge in this work, providing crucial insights into shock train stability and control strategies for the future development of robust DMSJ propulsion systems.



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# Nomenclature

## List of Symbols

|           |                                          |
|-----------|------------------------------------------|
| $\delta$  | Boundary-layer thickness                 |
| $\tau_0$  | Cross-correlation time delay             |
| $\theta$  | Boundary-layer momentum thickness        |
| $BS_1$    | Axial position of the cylinder bow shock |
| $f$       | Frequency                                |
| $G_{xx}$  | Power spectral density                   |
| $H$       | Isolator test section duct height        |
| $h$       | Cylinder height                          |
| $J$       | Jet momentum flux ratio                  |
| $K_i$     | High-speed pressure transducer $i$       |
| $L_{sep}$ | Characteristic length scale              |
| $P_i$     | Static pressure at port $i$              |
| $R_{xy}$  | Cross-correlation of signal $x$ and $y$  |
| $Re$      | Dimensionless Reynolds number            |

|            |                                       |
|------------|---------------------------------------|
| $S_i$      | Axial position of normal shock $i$    |
| $S_{1f}$   | Axial position of shock 1 foot        |
| $St$       | Dimensionless Strouhal number         |
| $U_\infty$ | Freestream velocity, axial component  |
| $x$        | Axial position from nozzle exit plane |
| $y$        | Vertical position from tunnel floor   |
| $z$        | Lateral position from tunnel sidewall |

### **List of Abbreviations**

|      |                                         |
|------|-----------------------------------------|
| BOS  | Background-Oriented Schlieren           |
| ISSI | Innovative Scientific Solutions Inc.    |
| PLS  | Planar Laser Scattering                 |
| PSP  | Pressure-Sensitive Paint                |
| ROI  | Region Of Interest                      |
| SBLI | Shock-Boundary-Layer Interaction        |
| UTSI | University of Tennessee Space Institute |
| ZCF  | Zero-crossing frequency                 |

# Chapter 1

## Introduction

### 1.1 Motivation

The prospect of reusable hypersonic vehicles for both civilian and military applications hinges on the ability of the propulsion system to produce the necessary thrust to support both takeoff at sea-level and Mach 5+ cruise. The “combined-cycle” engine provides a potential solution to this challenge by integrating low-speed and high-speed propulsion devices onto a single airframe. The choice of high-speed propulsion system is driven primarily by thrust efficiency requirements at the desired cruise speed. One such system first proposed by Curran and Stull [1], called the dual-mode supersonic combustion ramjet or dual-mode scramjet (DMSJ), offers multiple modes of operation depending on the flight condition and is capable of generating the necessary thrust for powered flight from Mach  $\sim 3$  to 8 [2].

The isolator is a critical component of the DMSJ and can significantly impact the performance and operability of the system. Its purpose is to generate the necessary static pressure rise required for combustion and to isolate downstream shock wave boundary layer interactions from propagating upstream and disgorging from the inlet in a event known as inlet unstart. Preventing inlet unstart is critical to maintaining safe vehicle operation as this event is known to cause significant loss of engine thrust [2], large pressure gradients [3], and transient structural loads that exceed steady-state levels by a factor of 10 [2].

The isolator shock train has been the focus of numerous studies in recent decades. The formation of a shock train is the result of complex shock-boundary layer interactions arising from mismatched upstream and downstream pressure conditions imposed on wall-bounded supersonic flow [4]. The structure, location, and dynamic behavior of the shock system has been shown to be sensitive to several factors such as upstream Mach number [5, 6], the ratio of boundary layer scale to duct geometry scale [7–10], and downstream pressure rise or “back-pressure” [11–14]. Furthermore, the shock train has been shown to exhibit significant streamwise fluctuations even when bulk inflow properties and downstream back-pressure are held constant [15–18].

Isolator back-pressure in a DMSJ is primarily a result of thermal blockage due to combustion heat release and physical blockage from fuel injection. Extensive work has been performed over the years investigating the structure and dynamic response of the shock train to these combustor back-pressure effects [3, 10–14, 19, 20]. In general, most experimental investigations of combustor back-pressure fall into one of two categories: with or without fuel injection. Experimental investigations with fuel injection [21–24], while being the most representative of actual DMSJ operation, often require complex test configurations, proper safety precautions for handling corrosive fuels, and adequate hardware cooling. Therefore, simpler approaches are often employed which utilize physical flow choking or blockage devices in cold- or heated-flow wind tunnels.

The most common back-pressuring approaches include decoupled techniques that utilize variable area valves [10, 12, 20] or rotating flow blockage devices [25, 26] several duct heights downstream of the isolator exit or coupled techniques that place rotating flaps [14, 27] or mass flow plugs [28] at the isolator exit. The key factor distinguishing decoupled and coupled back-pressuring techniques is the location of the back-pressuring mechanism in relation to the isolator exit. With the decoupled technique, the back-pressuring device is located several duct heights downstream of the isolator exit; often within or at the end of the wind-tunnel diffuser section where the flow is almost certainly fully subsonic. This would indicate that the blockage effects induced by the device are perceived by the incoming flow well upstream

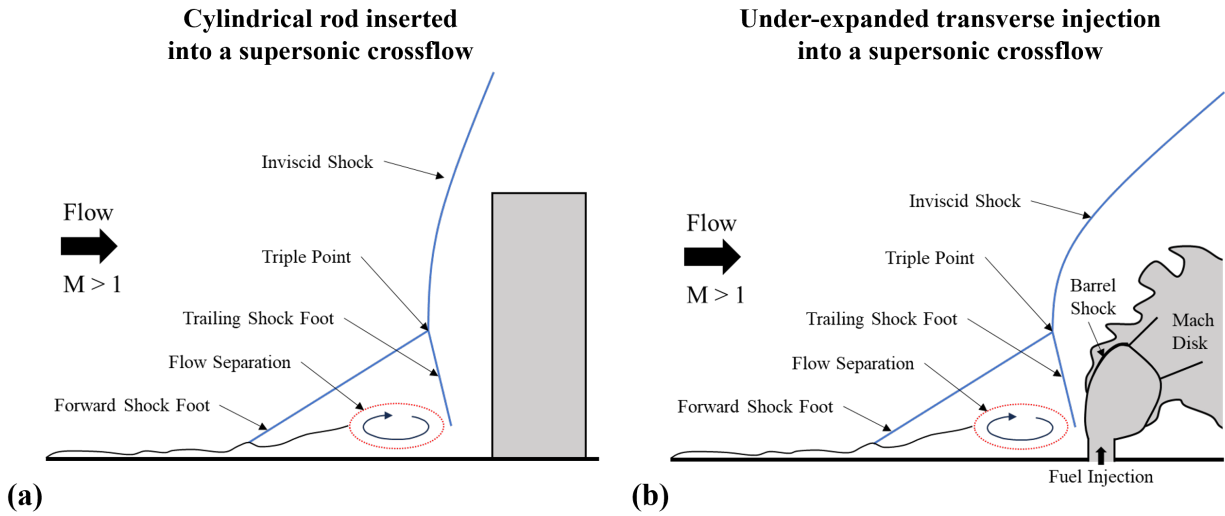
of the device location, and the resulting shock structure takes the form of an isolator pseudo-shock, as Crocco [29] called it. While this approach is suitable for simulating gross thermal blockage back-pressure caused by combustor heat release, it decouples the interaction of the shock train formation and dynamic response from the physical flow disturbance caused by the back-pressuring device.

The coupled back-pressuring approach is more representative of the physical blockage effects caused by fuel injection in a DMSJ system. One of the main challenges associated with supersonic combustion is the need for efficient combustor operation over very short residence times ( $\sim 1$  ms) [24]. This has led to a variety of fuel injection approaches including sidewall injection ports and in-stream injection devices such as ramps and struts. These fuel injection devices are often placed at the combustor entrance, where the flow is still supersonic, to ensure adequate mixing and complete combustion occur downstream. This results in a coupled back-pressure scenario where oblique or normal shock waves form in response to the supersonic flow obstruction. The impingement of these additional shocks on the duct sidewalls has been shown to cause sudden boundary layer separation [30] and rapid upstream propagation of the shock train [14]. The majority of prior experimental investigations have simulated this coupled back-pressure phenomena with a rotating flap placed at the isolator exit [14, 27, 31, 32]. This method, while valid, is two-dimensional in nature and does not fully capture the effects of a three-dimensional flow obstruction that would be generated by a flush-wall fuel injection port. A detailed understanding of the shock train structure and dynamics in response to three-dimensional blockage effects remains a critical gap that must be filled to facilitate the transition of predictive isolator shock train models to real-world systems.

The motivation for investigating a cylinder-induced shock train in this work stems from the extensive landscape of studies on transverse jet interaction with supersonic flow. It is well documented that a transverse jet injected into a supersonic flow produces a highly three-dimensional shock structure [33, 34]. Attempts to model the structure, penetration height, and mixing length of the injected fuel column have resulted in a common simplifying assumption first proposed by Hsia [35]: under similar approach conditions, the structure and



extent of shock-induced flow disturbances caused by a transverse jet can be represented by an equivalent blunt-nosed slender body [35, 36]. In fact, Heiser [2] specifically noted that shock-induced flow disturbances generated by transverse jet fuel injection act as if “a cylindrical rod were inserted into the freestream from the wall at a right angle.” A comparison of these two scenarios is shown in Figure 1.1. Thus, the culminating research question that motivated this study has two parts: If a cylindrical rod mimics the three-dimensional blockage effects of a flush-wall fuel injector, first, can it be used to generate a representative shock train? And second, how does the resulting shock train response compare to those generated by traditional back-pressuring methods?



**Figure 1.1:** Schematics of shock features generated by (a) a cylindrical rod inserted normal to a supersonic crossflow and (b) an under-expanded jet injection normal to a supersonic crossflow (adapted from [37])

## 1.2 Background

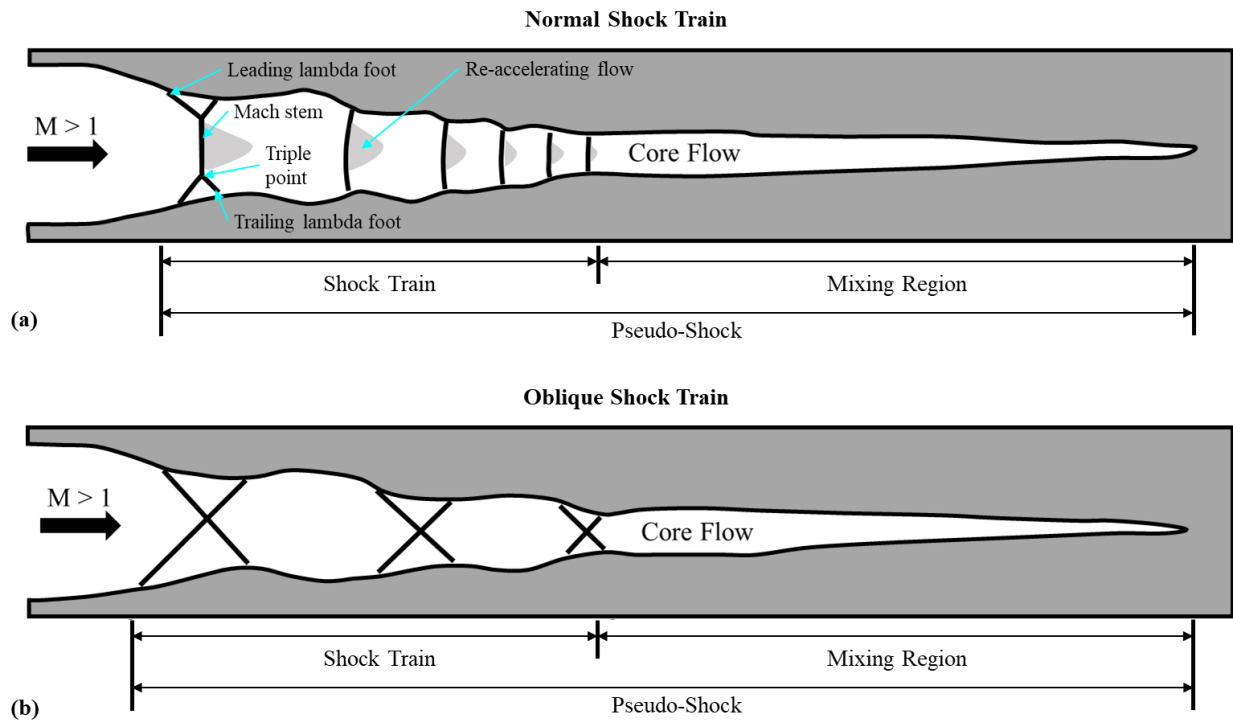
### 1.2.1 The Isolator Shock Train

Scramjet operation relies on compression and deceleration of the incoming airflow to provide suitable conditions for combustion downstream. This process is accomplished by vehicle

forebody compression waves, inlet area contraction, and static pressure rise in the isolator. The isolator shock train is a dynamic flow mechanism that allows the incoming supersonic flow to adjust to the back-pressure transients through a series of normal or oblique shock waves. Depending on the back-pressure conditions, the shock train will translate axially or change in length to accommodate the necessary pressure rise [2]. The isolator must be long enough to contain the shock train and keep disturbances from forcing it upstream into the inlet throat.

Shock trains in constant area ducts are highly dynamic flow phenomena. Shock train structure and shock train dynamics are affected by a variety of factors including the incoming Mach number, boundary layer state, duct cross-sectional area, and back-pressure [38]. Two types of shock train structures are shown in Figure 1.2. Normal shock trains, as shown in Figure 1.2 (a), are most commonly characterized by a bifurcated leading normal shock and subsequent normal shocks which may or may not be bifurcated. Interaction of the strong leading normal shock with the wall boundary layer causes local separation and bifurcates the shock at the triple point, creating leading and trailing shock “feet”. Immediately downstream, the boundary layer reattaches but rapidly thickens, restricting the core flow. This restriction accelerates the subsonic flow behind the normal shock back up to supersonic speed. Oblique shock trains, as shown in Figure 1.2 (b), are most commonly characterized by a series of left and right running oblique shock waves that cross near the centerline. Previous studies [5, 7, 39, 40] on the topic highlight incoming Mach number as the most influential parameter on the type of shock train structure. According to Hunt [10], normal shock trains generally exist at Mach numbers below 1.7 and oblique shock trains generally exist at Mach numbers above 2.2, with a transition region in between that is also dependent on the approach boundary layer thickness to duct half-height ratio, referred to as the confinement ratio. The length of the shock train has been shown to increase with increasing confinement ratio [6, 41], increasing back pressure [12], and increasing incoming Mach number [5].

Although shock trains are typically represented as a two-dimensional or quasi-one-dimensional structures, they are in reality complex highly three-dimensional structures driven by complex shock boundary layer interactions and corner flow effects [9, 20, 42–46].



**Figure 1.2:** Schematic of shock train types redrawn from Matsuo [4] and Hunt [10]: (a) normal shock train; (b) oblique shock train

Previous studies have shown the formation of a hybrid oblique-normal shock front in which the oblique shocks emanate from corner flow separation occurring approximately one duct height upstream of the bifurcated shock Mach stem [44, 45]. Handa [9] suggested the separation of the thinner, low-momentum boundary layer in the corners was a result of intersecting bifurcating shock waves emanating from perpendicularly adjacent walls. Morgan [46] noted the importance of fully resolving wall and corner effects in simulations to accurately capture an unsteady shock train, since two-dimensional simulations typically indicate steady behavior. Detailed studies were carried out by Hunt [10] and Edelman [43] using a combination of high-speed schlieren, stereoscopic PIV, and oil-flow visualization. Flow features extracted from the different optical techniques were combined to present a three-dimensional conceptualization of the shock train. Most notably, the leading shock was found to exhibit a conical hour-glass shape [10] due to large separated regions growing out radially from the corners [43].

Shock trains in constant area ducts exhibit two distinct types of motion: inherent unsteadiness around a mean position and bulk upstream or downstream translation coupled with enlarging or diminishing shock train length. The unsteady behavior of shock trains is known to exist even when bulk inflow and outflow conditions are relatively constant [4, 15–18, 40, 47, 48]. In early experimental work by Ikui [15], the amplitude of shock train oscillation was found to increase with Mach number and approached the scale of 1 duct height as the Mach number approached 3. Yamane [16], noted that shock train unsteadiness was found to have two main components: a high frequency component caused by downstream pressure perturbations propagating upstream and a low frequency component caused by shock oscillation propagating downstream. Analysis by Hunt [10] found the frequency response of the first four shocks in a shock train to be independent of back pressure. Specifically, the first shock exhibited broadband behavior with the majority of energy existing at frequencies less than 200 Hz. While the subsequent trailing shocks were found to exhibit preferred modes occurring around 500 to 3,000 Hz. Lindstrom [49] used multiple-line-of-sight tunable diode laser absorption spectroscopy (TDLAS) to study the shock train oscillation spectra and found similar dominant frequencies occurring around 100 Hz. Later work by

Hunt [18] sought to understand the source of this unsteadiness and discussed different scaling approaches. The authors proposed a frequency scaling argument based on the shock train length and approach velocity, as these were argued to capture the majority of fluid processes that contribute to the unsteadiness. However, it was noted that insufficient experimental studies existed to validate this scaling argument. Detailed cross-spectral analysis of high-speed schlieren shock tracking and wall pressure measurements provided valuable insight on the most influential mechanisms driving shock train unsteadiness: 1) acoustic waves generated by separation bubble instabilities, 2) vortices created by the separation bubble shear layer, and 3) acoustic waves propagating upstream from the diffuser [18].

Bulk shock train motion in response to varying inflow and outflow conditions has been studied extensively [8, 10, 14, 19, 25, 31, 40, 50–58]. Fievet [8] numerically investigated shock train response to varying inflow confinement ratios and found shock train displacement to move upstream with increasing inflow confinement ratio. Additionally, a small perturbation in boundary-layer momentum thickness (0.6% of the channel height) was found to reduce the normal portion of the lambda shock and increase the number of weaker bifurcating shock, leading to an increase in the overall shock train length by as much as two orders of magnitude. Gnani [53] noted hysteresis in the response of the shock train position depending on whether back-pressure was increasing or decreasing. The upstream movement due to increasing back-pressure was nearly double that of the downstream movement for an equivalent decrease in back-pressure. In addition, the shock train motion appeared to track with the sinusoidal back-pressure forcing. However, a phase shift was present between the leading shock position and the min/max back-pressure conditions, which was found to be dependent on the direction of travel. Similar shock train motion characteristics were noticed by Cai and Gillespie [40, 51]. Gillespie [51] numerically simulated the effect of back-pressure disturbances on a shock train and found a linear relationship between the shock train leading edge and back-pressure, after an initial lag, during step forcing. During sinusoidal back-pressure forcing, increasing frequency was found to have reduced effect on shock train motion, while lower frequency forcing caused very large oscillations. Su [58] noted similar shock train response to back-pressure frequency and found back-pressure amplitude to have a greater effect on the shock

train dynamics than forcing frequency. Hunt [10] experimentally investigated the structure and unsteadiness characteristics of a shock train in a constant-area duct as a function of varying back-pressure. Under the conditions tested, the shock train location moved upstream linearly with increasing back-pressure, and the shock train length and corresponding pressure rise were found to be independent of back-pressure condition.

Other studies have investigated the coupled effect of upstream disturbances, such as impinging background waves, with downstream back-pressure oscillations [52, 54–56, 59]. Broadly, these studies were conducted to mimic upstream flow disturbances that are expected in a real scramjet system flying at various attitudes and velocities. Tan [59] experimentally investigated the behavior of shock trains in a hypersonic inlet/isolator model with complex background waves. Close coupling was found between the upstream background waves and the shock train, which switched pointing direction from upward to downward repeatedly during upstream propagation. Additionally, the upstream background waves were found to increase the sustainable back-pressure of the shock train and decrease its length due to the pre-compression effect of the background shock. Hou [54] experimentally investigated the forced oscillation of a shock train in an isolator with background waves and characterized the shock train oscillations into two categories. Type I oscillation, which is a result of back-pressure perturbations and increases with increasing back-pressure amplitude or decreasing back-pressure frequency. The other is a Type II oscillation, which produces a larger oscillation range than Type I and is a result of upstream background waves. Type II oscillation was also found to be minimally effected by back-pressure amplitude or frequency.

### **1.2.2 Back-Pressuring Methods**

Experimental simulations of scramjet isolator back-pressure—arising from fuel injection and combustion heat release—typically employ one of two strategies: live-fire combustion tests in sophisticated facilities, or less complex back-pressure tests conducted in standard cold-flow supersonic wind tunnels. In cold-flow tests, simulated back-pressure is conventionally generated using a mechanical choking device, such as a valve, flow plug, or throttling

mechanism. Other approaches more focused on the inlet unstart process have used floor-mounted flaps or cylindrical pins. It is this author’s opinion that the landscape of current back-pressuring approaches fundamentally fall into two categories: (1) decoupled and (2) coupled methods. The key factor distinguishing these two methods lies in the physical location of the mechanism relative to the isolator exit, which critically determines the local Mach number where the back-pressure is first generated.

Early work by Waltrup and Billig [12, 13] studied shock train behavior over a range of Mach numbers from 1.53 to 2.72. Back-pressure was generated using butterfly valve situated in the diffuser duct nearly 1.5 duct heights downstream from the isolator exit. At each throttle level once the shock train was stabilized, the static pressure profile along was duct was acquired. The resulting pressure rise profiles for all of the tested conditions were found to have similar shapes independent of axial location. Accounting the for the incoming Mach number, boundary layer state ( $\theta$ ,  $Re_\theta$ ), and the duct geometry, the authors were able to collapse the pressure rise into a single quadratic expression; leading to the widely-used Waltrup-Billig correlation. This correlation was later updated to capture rectangular duct effects and validated with experimental data from direct-connect combustor tests [13]. Carroll performed similar experiments at Mach 1.6 and 2.45 using a variable-area diffuser throat to apply back-pressure [6]. The approach was intentionally designed to minimize the adverse pressure gradient effects of the closing the diffuser throat. At Mach 1.6, a normal shock train was observed with local boundary layer separation occurring only at the leading shock, whereas the downstream shocks were non-bifurcated indicating no separation. Hunt [10] and Edelman [19, 20, 43] have used a similar approach to that of Waltrup and Billig for conducting shock train research, with a butterfly valve positioned 13 duct heights downstream of the isolator exit. In [10], Hunt observed that the shock train responded in a nearly linear fashion to back-pressure levels ranging from 2.09 to 3.43. Xiong [50] used a rotating throttling plate located 3.8 duct heights downstream of the isolator exit. Huang [60] used a axially-translating flow plug located 9.8 duct heights downstream of the isolator exit. Cheng [61] used a rotating elliptical shaft located nearly 2.8 duct heights downstream of the isolator exit. The common characteristic shared between all of these approaches

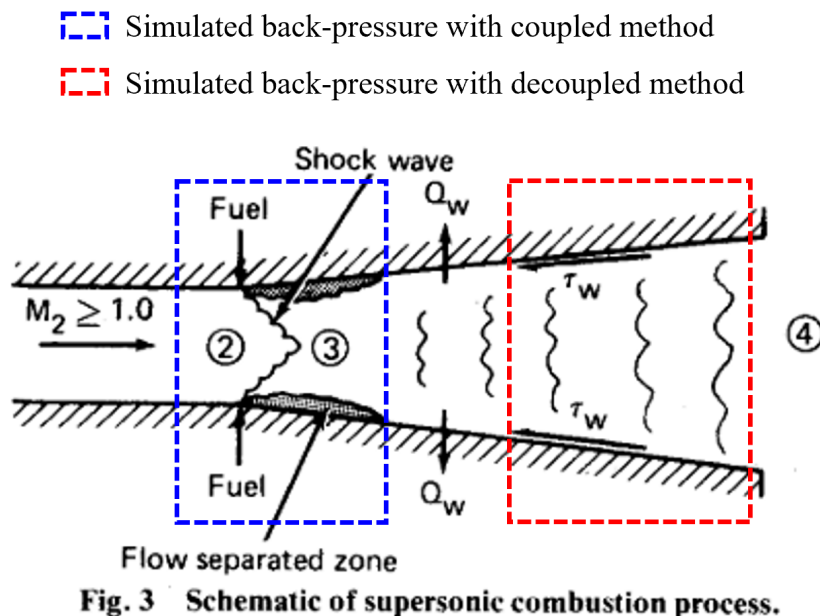
is the multiple duct heights separating the back-pressuring device from the isolator exit. Additionally, the devices are often located within or at the end of the wind-tunnel diffuser section where the flow is almost certainly fully-subsonic.

The most common coupled back-pressuring approach involves the use of 2D movable ramps [14, 31, 52, 54, 55] at the isolator exit plane. Wagner [14] observed multiple shock propagation speeds during ramp deflection. Within the isolator, an initial upstream propagation of 35 m/s was observed followed by a deceleration to approximately 20 m/s at the isolator entrance, near the inlet throat. High-speed schlieren of the isolator exit revealed that initial propagation of the shock system was a direct result of boundary layer separation at the impingement point of the ramp compression shock on the isolator ceiling. It was also suggested that concurrent sidewall separation had occurred due to the impingement of three-dimensional glancing shock interactions. Similar phases of rapid upstream shock propagation were observed in ramp back-pressuring studies by Li [55], Xu [52], and Hou [54], who noted the strong association of this behavior with ramp-induced boundary layer separation on the ceiling.

The ramp back-pressuring method, while valid, is two-dimensional in nature and does not fully capture the effects of a three-dimensional flow obstruction that would be generated by fuel injection in a supersonic combustor. The one implementation of using a three-dimensional back-pressuring device that can be found in the open literature is the early work conducted by Wieting [3]. He used a cylindrical rod inserted downstream of a fuel injection strut to initiate unstart in a Mach 5.3 inlet. The primary objective of this work was to quantify the unsteady pressure loads associated with an unstarted inlet condition. The cylindrical rod injection created a flow area blockage of up to 60%, and the resulting unstart event was observed to be highly transient with two distinct propagation speeds related to the expelling of the normal shock from the inlet and the rate of boundary layer separation propagating upstream. Specifically, the normal shock generated by the cylinder was believed to cause a rapid separation of the boundary layer in the inlet that propagated upstream at a rate of 10 m/s.



The difference in the back-pressure rise that is simulated by the decoupled and coupled methods is best summarized by the annotated schematic from Waltrup [62] shown in Figure 1.3 showing the supersonic combustion process. The fuel injection location is shown at the downstream end of the constant area isolator where the incoming flow is still supersonic. The shock waves generated by the injected fuel stream are shown impinging on the downstream walls leading to localized separation. If the combustor area expansion does not provide sufficient pressure relief, then the separated region is forced to travel upstream; creating a back-pressure disturbance that drives the shock train propagation. In contrast, the pressure rise caused the chemical heat release in the expanding section of the combustor drives the Mach number towards a sonic condition, and any transient disturbances are propagated uniformly upstream.

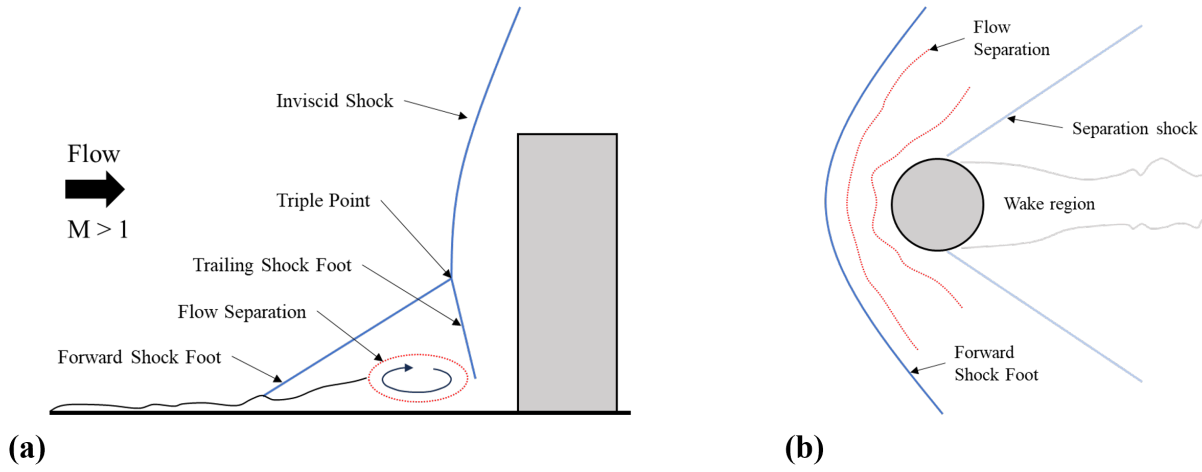


**Figure 1.3:** Annotated schematic of the supersonic combustion process from Waltrup [62].

### 1.2.3 Cylinder-Induced Shock Boundary Layer Interactions

The complex shock-boundary-layer interactions (SBLIs) surrounding a vertical cylinder mounted normal to a supersonic flow has been an active area of research in recent years

[63–68]. The fundamental shock structures associated with this type of interaction are shown in Figure 1.4. An adverse pressure gradient resulting from the inviscid cylinder bow shock causes a localized separation region to form at the base of the cylinder. As a result, a series of compression waves form and coalesce into an oblique shock wave, called the forward shock foot, that merges with the normal inviscid shock. To maintain continuity with the freestream pressure rise caused by passing through the normal shock, a trailing shock foot forms at the base, resulting in a lambda-shock ( $\lambda$ ) structure. The intersection of these shock features forms what is known as the triple point [69].



**Figure 1.4:** Schematic of three-dimensional cylinder SBLI shock features. (a) Side view; (b) Top View.

The key parameters used to characterize the cylinder SBLI are the triple point height, measured vertically from the floor, and the maximum shock separation distance, which is typically defined as the distance from the cylinder face to the upstream point of the forward shock foot. These features have been shown to scale linearly for cylinder normalized cylinder heights ( $h/d$ ) less than 1.13 [70]. For larger cylinder heights, the triple point height and separation distance have been shown to remain constant, reaching what is commonly referred to as a semi-infinite condition [71]. A comprehensive scaling study performed by Lindörfer et al. [67] looking at the effects of varying the ratio of cylinder diameter to boundary-layer thickness on SBLI structures and their corresponding dynamics. In general, the authors found the cylinder diameter and boundary layer thickness to be adequate scaling parameters

when the incoming boundary layer is less than the cylinder height. When the incoming boundary layer is greater than the cylinder height, a power law was found to provide the best fit.

The unsteady dynamics associated with cylinder SBLIs are often characterized in terms of the non-dimensional Strouhal number based on the interaction length scale ( $St_L = fL/U$ ), where  $U$  represents the freestream velocity and  $L$  represents a characteristic length scale equivalent to the maximum shock separation distance. Experimental evidence has shown the unsteadiness to be a result of either high frequency fluctuations in the upstream boundary layer, low frequency fluctuations of the downstream separation bubble, or some combination of the two [72]. Previous studies have shown these low frequency fluctuations, or “shock-breathing” fluctuations, to occur at Strouhal numbers ranging from approximately 0.01 – 0.2 [18, 72–75]. Combs et al. [76] developed a custom shock tracking algorithm for extracting time-histories of cylinder-induced interactions from high-speed schlieren images collected at 100 kHz. The time-histories of the upstream influence shock, forward lambda shock, and trailing lambda shock from interaction with an incoming Mach 1.81 flow were compared with high-speed wall pressure measurements. Comparison of the two showed excellent agreement; validating the novel image-based tracking technique. In a follow-on study, Combs et al. [66] used high-speed schlieren imagery to track the positions of the upstream influence shock, forward lambda shock, and trailing lambda shock of a cylinder-induced SBLI in Mach 1.78 flow. The results revealed the presence of low-frequency fluctuations that occur in the Strouhal number range of 0.04 to 0.08. Furthermore, it was suggested that instabilities arising from the separated flow region were the cause of the observed unsteady interactions.

### 1.3 Scope of the Current Work

The scope of this work is first, to demonstrate the ability to generate a representative shock train using a floor-mounted cylindrical rod, and second, to comprehensively characterize its structure, quasi-static response, and unsteadiness. Experimental testing was performed in the UTSI direct-connect isolator facility with a square duct cross-section measuring 15.24

cm tall and streamwise length of 13 duct heights. Flow entering the isolator test section is representative of nominal Mach 2.2 flow at a Reynolds number per unit length of 41 million per meter. Back-pressure was generated by inserting a floor-mounted cylindrical rod normal to the flow at the exit plane of the isolator. Floor surface pressure measurements and multi-plane optical imagery including background-oriented schlieren, fast-response pressure sensitive paint, and planar laser CO<sub>2</sub> Rayleigh scattering were acquired simultaneously to support the following research objectives.

1. Demonstrate the ability to generate a representative isolator shock train with a cylinder back-pressuring device.
2. Investigate the shock train formation process during cylinder insertion.
3. Characterize the shock train structure, dynamic response, and overall pressure rise as a function of cylinder height.
4. Characterize the inherent unsteadiness of the shock train and attempt to identify the cause.

The results of this work directly address an identified knowledge gap by providing a comprehensive understanding of the back-pressure effects of a three-dimensional flow obstruction on shock train structure and dynamics. Furthermore, because the cylindrical rod mimics the flow disturbance of transverse fuel injection into a supersonic flow, this methodology offers a potential pathway to streamline future scaling studies on shock train response to fuel equivalence ratio levels representative of a DMSJ engine.

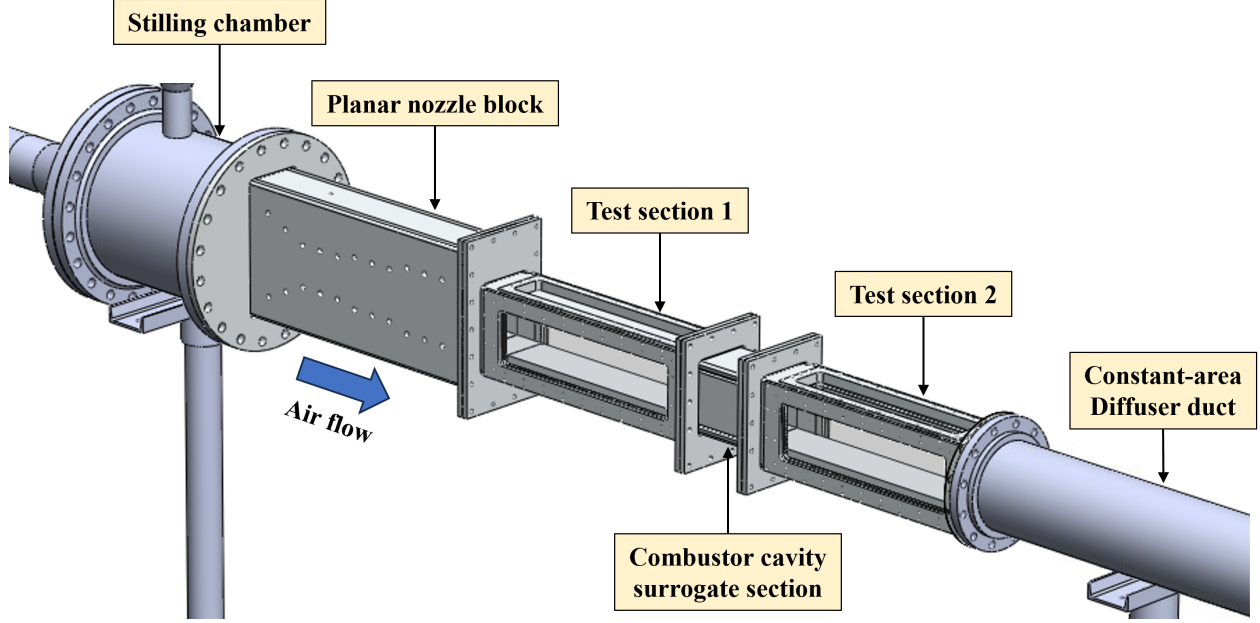
# Chapter 2

## Experimental Setup

### 2.1 UTSI Direct-Connect Isolator Facility

Experiments for this work were conducted in the Direct-Connect Isolator Facility at the University of Tennessee Space Institute (UTSI) , shown in Figure 2.1. The modular design, removable floor segments, and large optical viewing areas make this facility ideal for non-combusting, direct-connect isolator research. The facility is equipped with a planar nozzle and features a test section 15.24 cm wide by 15.24 cm tall by 203.2 cm long. Facility air is supplied by high pressure air tanks with 23.6 m<sup>3</sup> of storage capacity. Several minutes of run time can be achieved with Reynolds numbers ranging from 26-60 million per meter and a nominal Mach number of 2.2 exiting the nozzle.

As shown in Figure 2.1, the isolator test section is divided into test section 1 and test section 2 by a combustor cavity section. This section provides additional capability for studying the impact of different floor and ceiling cavity designs on isolator dynamics. High-pressure air can also be injected as a surrogate for combustor fuel-injectors. For this work, no cavity model was used, and the duct area remained constant throughout the entire test section. Test section 1 and 2 sidewalls and ceiling panels are made from 25.4 mm thick borosilicate glass allowing for substantial optical access throughout. Test section 1 has floor-to-ceiling optical access on both sidewalls starting at the nozzle exit plane and extending



**Figure 2.1:** Schematic of the UTSI Direct-Connect Isolator Facility

71.12 cm in the axial direction. Test section 2 has a single sidewall window with floor-to-ceiling optical access that extends 66.04 cm in the axial direction. The other sidewall in test section 2 is solid aluminum (not shown in Figure 2.1).

## 2.2 Floor-Mounted Cylinder

A 25.4 mm OD stainless steel cylinder positioned at the downstream end of test section 2 was used to generate the necessary back-pressure conditions for studying the shock-train. The cylinder was attached to a single-axis BiSlide<sup>®</sup> slider assembly driven by a VMX<sup>™</sup> stepper motor controller from Velmex. The slider has a linear step size of 0.00635 mm/step and was driven at a maximum speed of 6000 steps/sec (38.1 mm/sec) for all of the data presented in this work.

The cylinder and slider plate assembly were mounted to the tunnel floor with custom 3d-printed bushing inserted in a compression housing. Limit switches on the slider plate guide rail were used to control the starting home position and max cylinder height in the test section. The number of steps required from the motor for each cylinder height was

calibrated pre-test based on the staring position and the linear step size reported in the operation manual. Physical inspection and measurements were performed at each specified cylinder height to ensure correct positioning. Before each run, a user-defined motion profile was loaded into the controller via an RS-232 serial interface. During tunnel operation, a remote trigger was sent to activate the cylinder motion.

## **2.3 Instrumentation**

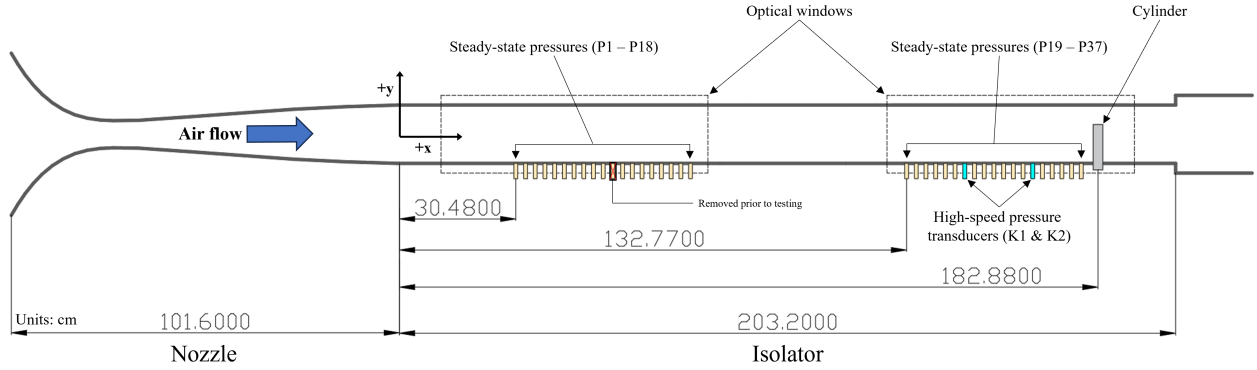
### **2.3.1 Steady-State Wall Pressure Measurements**

Steady-state pressure was measured along the test section floor centerline to capture the undisturbed (no back-pressure) isolator pressure and help characterize the time-averaged shock train structure. Measurements were made with a 64-port miniature electronic scanning pressure module (ESP-64D) from Measurement Specialties, Inc. The module consists of an array of silicon piezo-resistive pressure sensors, one for each pressure channel. The channel outputs are then electronically multiplexed before being sent to a remote analog-to-digital (A/D) converter. The data is acquired on the computer as pressure differential. Absolute pressure of each channel is calculated post-acquisition by adding the local atmospheric pressure from the facility's calibrated Honeywell Precision barometer (SN 44845).

Each pressure output was acquired at 10 Hz using the time stamp from a standard laptop real-time clock. The reported static accuracy of the system is  $\pm 0.05\%$  Full-Scale. Pressure taps were placed starting at 30.48 cm from the nozzle exit plane with a 25.4 mm spacing between, as shown in Figure 2.2. Each pressure tap was connected to the ESP module using equal-length segments of 0.86 mm ID flexible urethane tubing.

### **2.3.2 High-Frequency Wall Pressure Measurements**

High-frequency wall pressure measurements were made using Kulite<sup>®</sup> high temperature, ultra-miniature, pressure transducers (XCE-062-25A). Two Kulites, K1 and K2, (as shown in Figure 2.2) were installed in test section 2 at the upstream and downstream extents of the



**Figure 2.2:** Facility Hardware and Instrumentation Layout (side view)

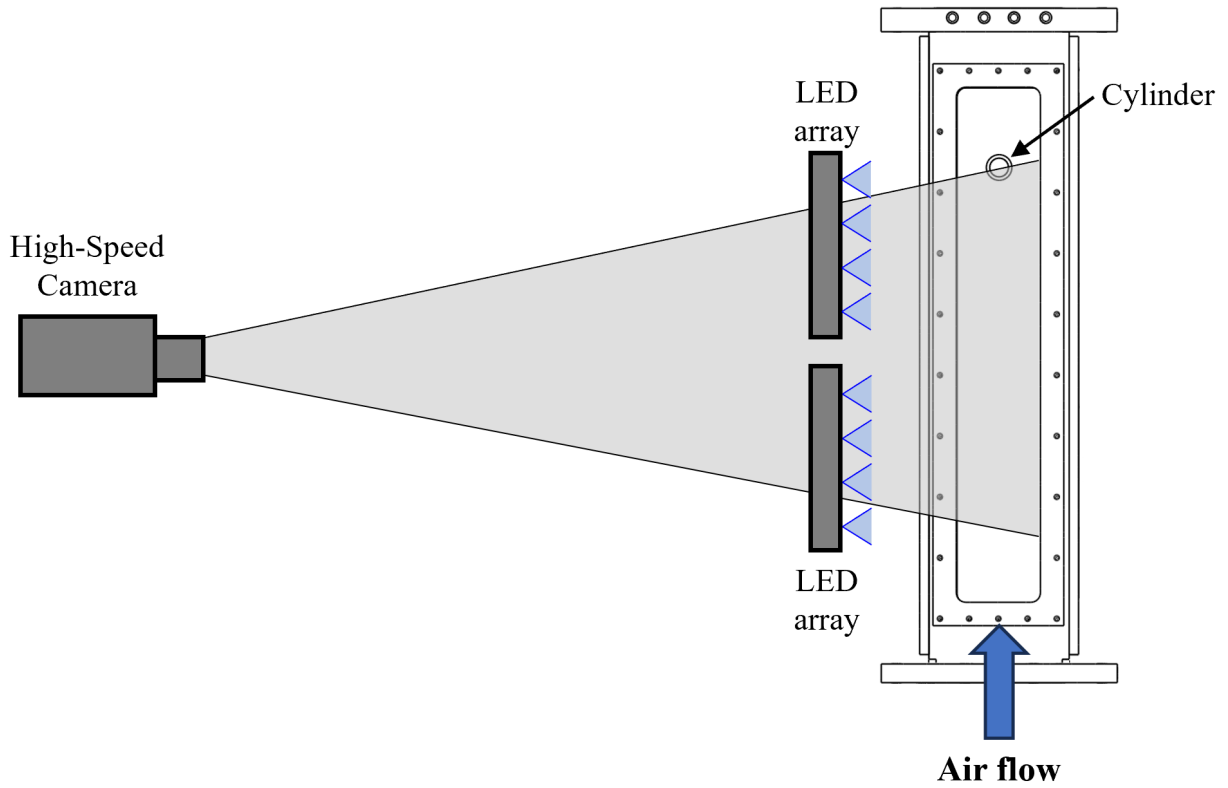
pressure sensitive paint region, respectively. The XCE-062-25A model has a max absolute pressure range of 172.4 kPa (25 psi), covering the full static pressure range experienced in test section 2 during back-pressuring. Data were oversampled at 20 kHz and sent through a KSC-2 power signal conditioner. A total gain factor of 50 was applied to the signal (8 pre-filter and 6.25 post-filter). A 6-pole low-pass filter with a cutoff frequency of 10 kHz was used for anti-aliasing. The filter has a pass-band amplitude response nearly identical to a 6-pole Butterworth filter with sharper roll-off characteristics.

## 2.4 Optical Flow Diagnostics

### 2.4.1 Background-Oriented Schlieren

The restricted optical access from the solid aluminum sidewall in test section 2 drove the need for a background-oriented schlieren (BOS) technique to view the isolator shock train and quantitatively assess dynamic characteristics of certain shock train features. The setup, shown in Figure 2.3, consisted of two LaVision blue LED arrays (VZ17) with a nominal wavelength of 445 nm and a Phantom v2012 high-speed camera with a 135 mm focal length Nikon lens. BOS images were acquired at 10 kHz with an exposure time of 40  $\mu$ s. The camera field of view covers 152.4 mm (i.e.  $1.0H$ ) in the y-direction and 533.3 mm (i.e.  $3.5H$ ) in the x-direction with a resolution of 2.4 px/mm.





**Figure 2.3:** Schematic diagram of the background-oriented schlieren arrangement in test section 2 (top view)

As shown in Figure 2.4, a sinusoidal fringe background pattern (Eq. 2.1) was used in this work; following the recommendation outlined by Vinnichenko *et al.* [77] for imaging thin structures like boundary layers or shock waves. The image intensity,  $I$ , at each  $(x, y)$  pixel coordinate is defined by a 2D cosine wave with fixed wavelength,  $\lambda$ . For a grayscale image the intensity in Eq. 2.1 must be rescaled from 0 to 1 (Eq. 2.2). A zoomed-in region from the high-speed camera in Figure 2.4 (c) shows the resulting pattern has a wavelength of 6 pixels (6-15 pixels is recommended in [77]). The background was printed on an adhesive-backed vinyl decal using a large format plotter printer and applied directly to the solid aluminum sidewall inside test section 2.

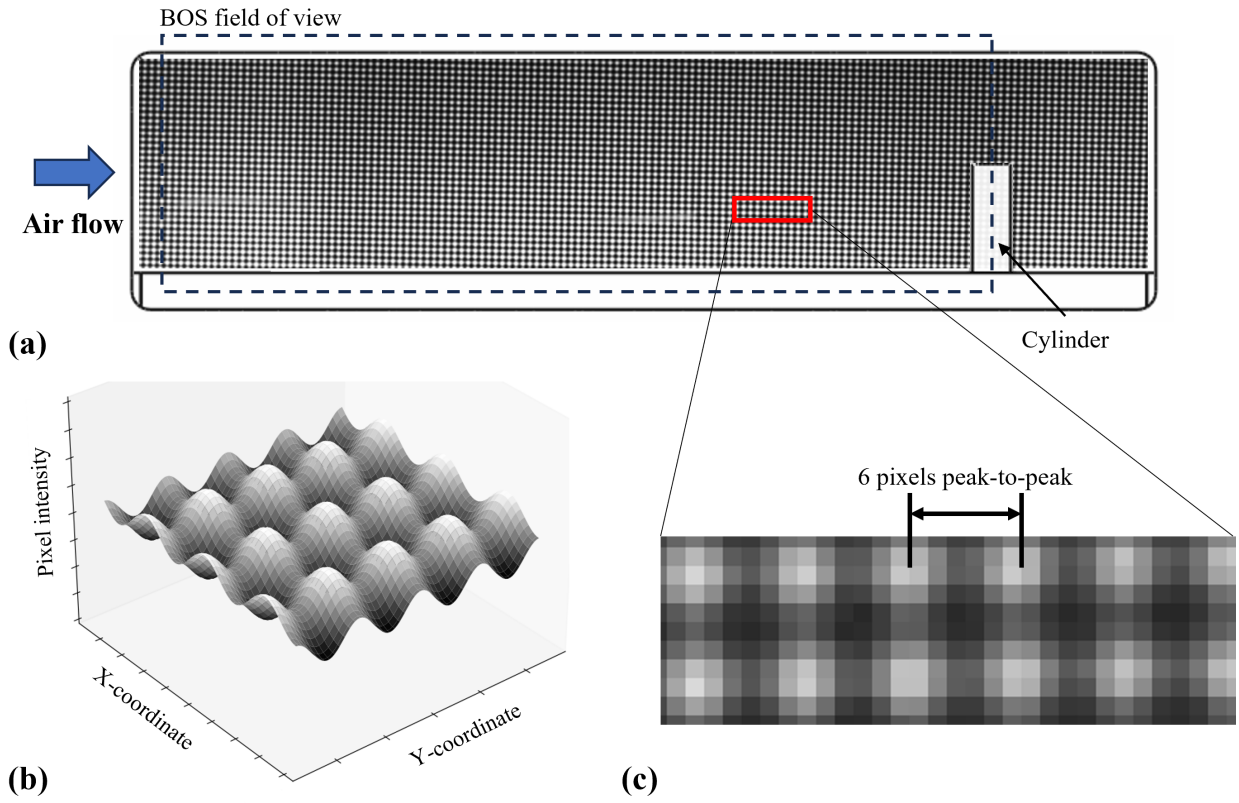
$$I(x, y) = \cos\left(\frac{2 \cdot \pi \cdot x}{\lambda}\right) + \cos\left(\frac{2 \cdot \pi \cdot y}{\lambda}\right) \quad (2.1)$$

$$I_{grayscale}(x, y) = 255 \cdot [0.25 \cdot I(x, y) + 0.5] \quad (2.2)$$

### 2.4.2 Pressure-Sensitive Paint

Fast-response Pressure-Sensitive Paint (PSP) was used to supplement the unsteady STLE behavior found in the high-speed BOS dataset and provide an overhead view showing the 3-dimensional structure of the STLE. Polymer ceramic paint from Innovative Scientific Solutions Inc. (ISSI) was used for this work. ISSI's fast-response paint consists of two parts: (1) an oxygen-permeable binder as the base coat and (2) a UV-excited luminescent top coat. The intensity of the fluorescence is correlated with the local oxygen concentration and can be correlated with local static pressure through an off-line calibration process. The base coat and top coat were applied in a two-step process using a gravity-fed spray gun.

The setup, shown in Figure 2.5, consisted of two water-cooled LED arrays with a nominal wavelength of 400 nm and a Photron Mini AX-200 high-speed camera with a 85 mm focal length Nikon lens. Two lens filters were used to improve the imaging results: (1) a 450 nm long-pass filter to eliminate the excitation wavelength and (2) a 610 nm long-pass filter to



**Figure 2.4:** Schematic of: (a) BOS background pattern in test section 2 (side view); (b) example of 3D-mapped pixel intensity; (c) background imaged by camera

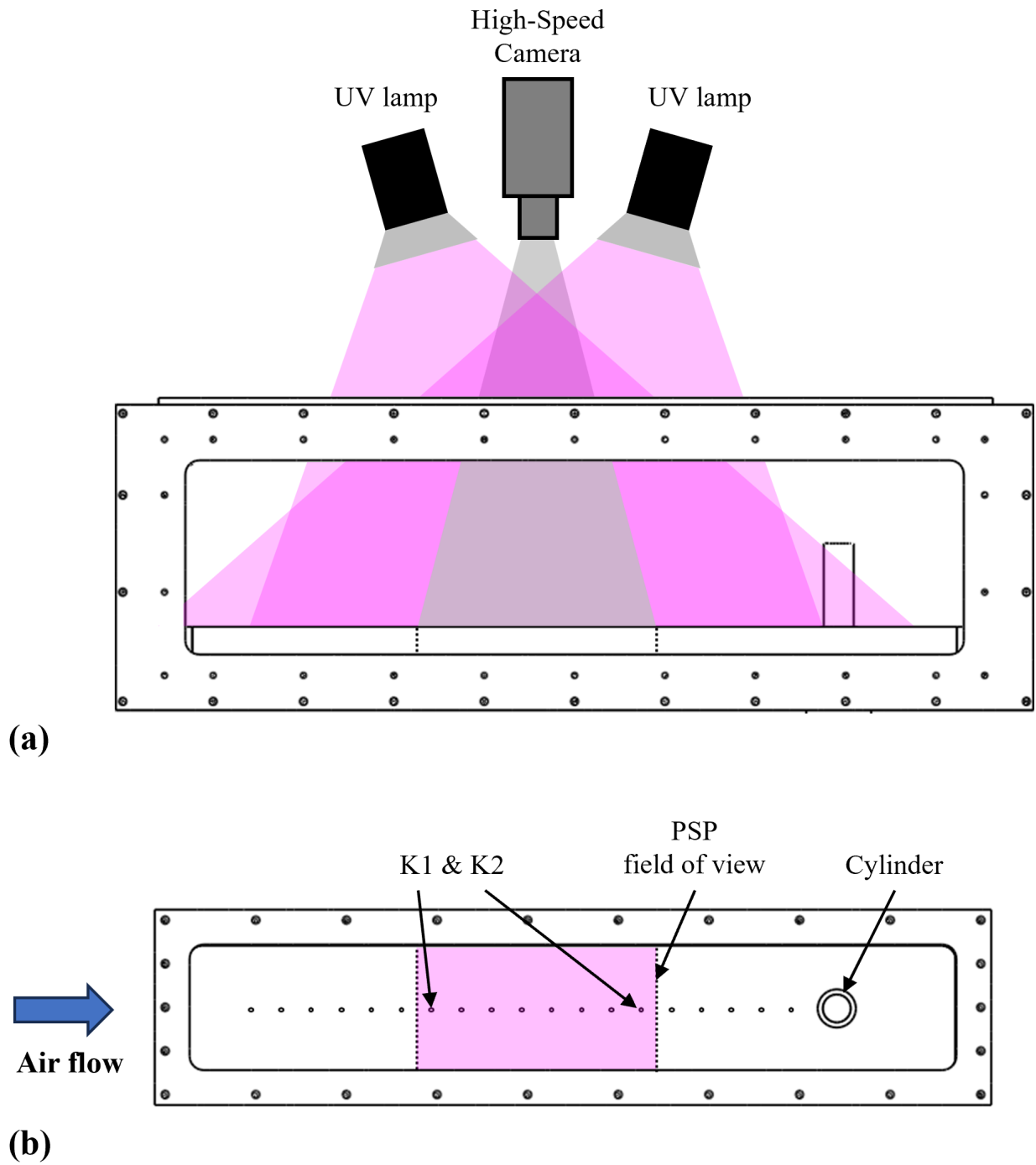
further isolate the emitted fluorescent signal. The camera field of view covers 152.4 mm (i.e.  $1.0H$ ) in the z-direction and 254 mm (i.e.  $1.7H$ ) in the x-direction with a resolution of 4.1 px/mm.

#### 2.4.2.1 Paint Calibration

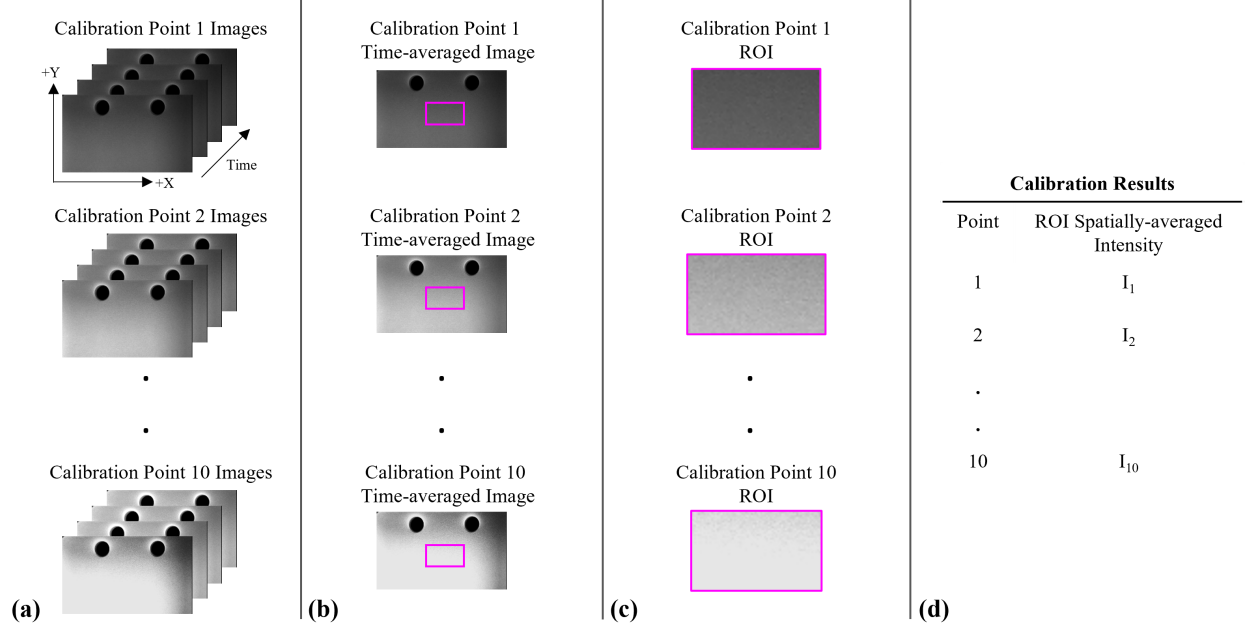
Steady-state calibration of the PSP was performed post-test using a small-scale vacuum chamber rig. A coupon-sized sample of 6061 aluminum was painted with the PSP to replicate the tested configuration as closely as possible. The vacuum chamber was initially open to atmosphere and a series of images were acquired at 10 kHz with the same camera and lighting hardware from main experiment. Next, a small pump pulled vacuum on the chamber, reducing the pressure, and another series of images were acquired. This process was repeated until the minimum vacuum capability of the calibration rig was achieved. From there, the pressure was increased in multiple steps until the chamber returned to atmospheric conditions.

A schematic diagram of the calibration process is shown in Figure 2.6. Starting on the left and working right, the image sequences from each calibration point were time-averaged. Next, a user-defined Region Of Interest (ROI) window was used to crop the time-averaged image. Finally, the spatially-averaged intensity was calculated and stored as a calibration curve point. This process was repeated for various ROI window sizes, resulting in a distribution of calibration points at each pressure condition. The results of the calibration process are shown in Figure 2.7. The pressures and average intensities have been normalized by their respective values at a user-selected reference condition. For this work, the local atmospheric pressure ( $P_{ref}=99.5$  kPa) was chosen as the reference condition.

A modified Stern–Volmer equation suitable to aerodynamic applications [78] was used to determine the calibration curve and is given in Eq. 2.3. The Stern–Volmer coefficients,  $A$ ,  $B$ , and  $C$  were determined using a least-squares polynomial fit of the average static calibration points at each pressure condition. The calibration coefficients and the fit coefficient of determination ( $R^2$  value) are reported in Table 2.1. The distribution of calibration points at each pressure condition was used to estimate a maximum curve fit uncertainty of  $\pm 10\%$ .



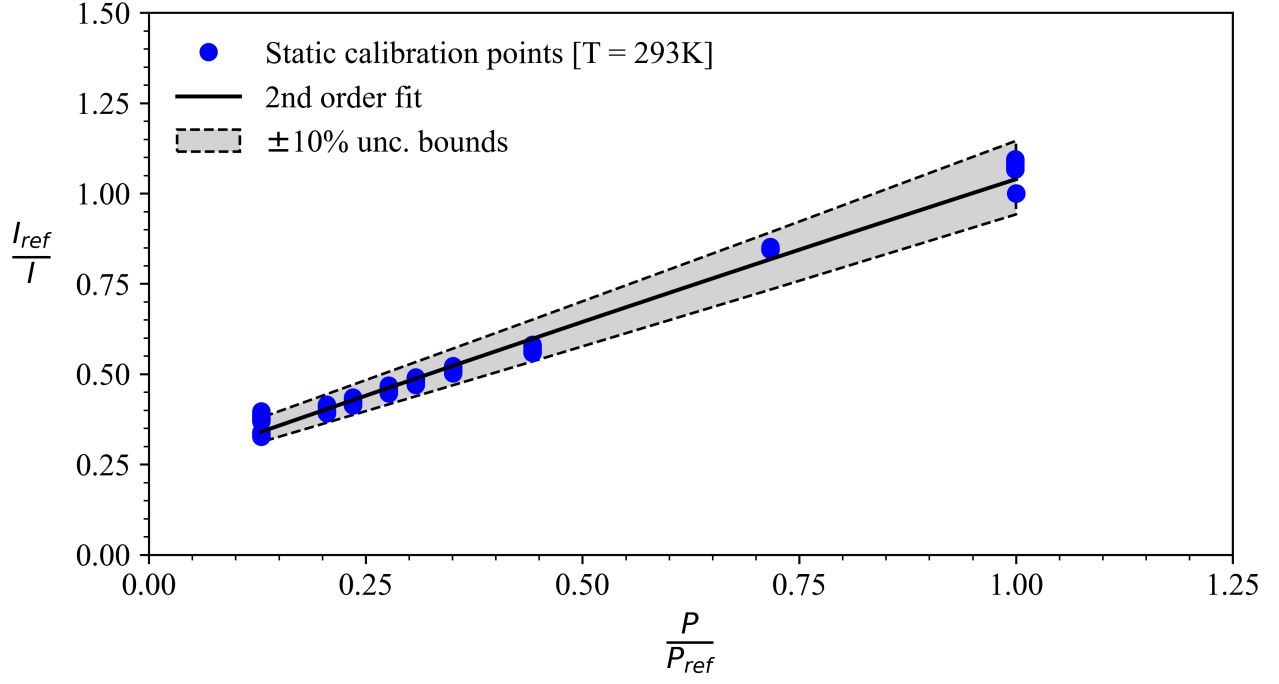
**Figure 2.5:** Schematic diagram of the pressure-sensitive paint arrangement in test section 2. (a) Side view; (b) Top view



**Figure 2.6:** PSP Calibration Process (left to right). (a) Images are acquired at 10 kHz at each calibration condition; (b) Images are time-averaged at each calibration condition; (c) A Region of Interest (ROI) is selected from the time-averaged image; (d) Spatially-averaged intensity from ROI is stored for each calibration condition.

$$\left(\frac{I_{ref}}{I}\right) = A \cdot \left(\frac{P}{P_{ref}}\right)^2 + B \cdot \left(\frac{P}{P_{ref}}\right) + C \quad (2.3)$$

It should be noted that the calibrated pressure range did not cover the max pressure in test section 2 during testing. At high back-pressure (i.e. large cylinder flow area blockage) conditions, pressures downstream of the shock train rose above atmospheric pressure, and an insufficient amount of in-situ calibration data was available to extend the offline calibration data to the higher pressures. Additionally, surface temperature, which is known to influence the proportion of emitted fluorescence, was not controllable during calibration and did not match the low wall temperatures experienced in test section 2. For these reasons, steady-state pressure levels from the PSP data were not considered for shock train pressure characterization, and only the spectral content of the data transients were considered for further analysis.



**Figure 2.7:** PSP Calibration Results

**Table 2.1:** PSP Calibration Curve Fit Results

| A       | B      | C      | $R^2$  | $2\sigma$ uncertainty |
|---------|--------|--------|--------|-----------------------|
| -0.0352 | 0.8432 | 0.2316 | 0.9938 | $\pm 10\%$            |

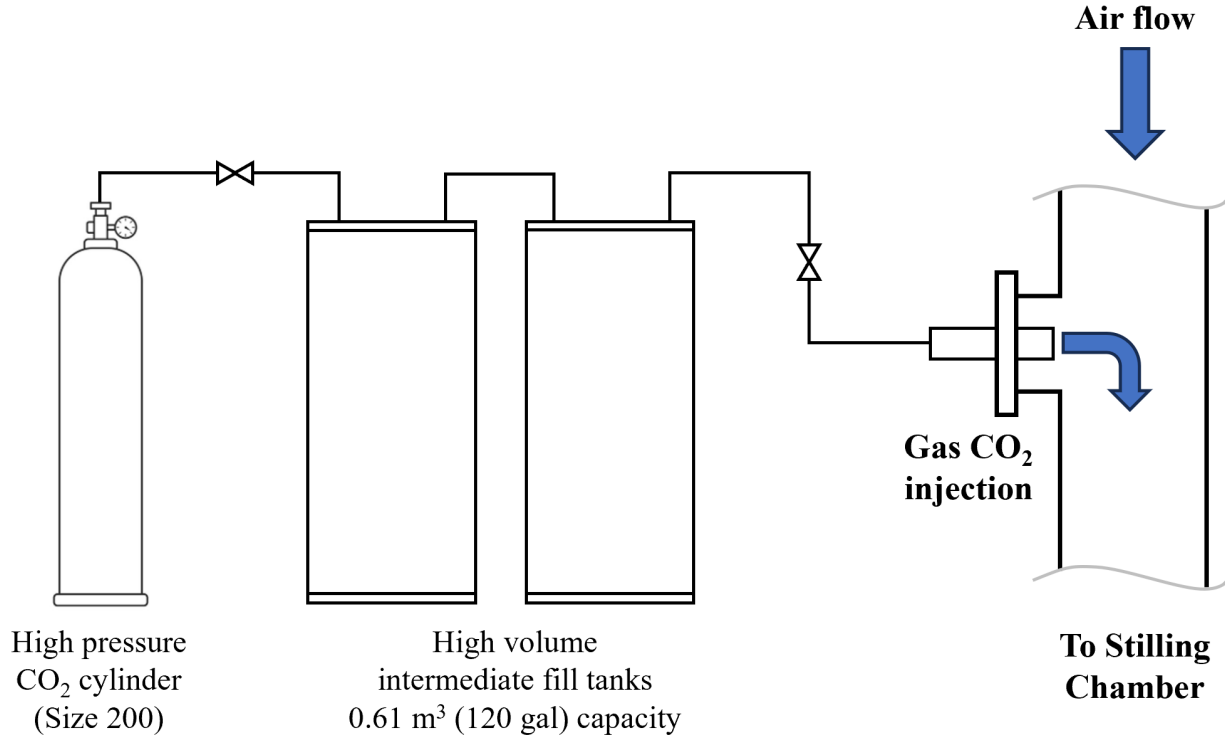
### 2.4.3 Planar Laser Scattering

Planar laser CO<sub>2</sub> Rayleigh scattering (PLS) was performed near the end of the experimental campaign to provide a qualitative view of the core flow and regions of separated flow in test section 2 during back-pressuring. PLS was selected for its relative ease of implementation. CO<sub>2</sub> was selected as the seeding agent due to its relatively low cost and inert nature. While planar laser scattering has been used extensively in the past for optical flow diagnostics at supersonic Mach numbers [79, 80], to the author's knowledge, this work successfully implements CO<sub>2</sub> seeded PLS imaging at Mach numbers below what has been reported in the available literature.

Rayleigh scattering occurs when light is scattered by particles with mean radii much smaller than the incident light ( $\sim 1/10$  the wavelength) [81]. Rayleigh scattering with CO<sub>2</sub> was achieved by generating a light-scattering fog composed of solid, nanoscale CO<sub>2</sub> crystals on the order of 1-10 nm in diameter [80, 82, 83]. The CO<sub>2</sub> fog is generated by a deposition (gas to solid) phase-transition of gaseous CO<sub>2</sub> injected upstream of the tunnel stilling chamber. The low-temperature and low-pressure environment downstream of the converging-diverging nozzle created suitable conditions for CO<sub>2</sub> deposition to occur.

The PLS seeding and optical arrangement are shown in Figures 2.8 and 2.9, respectively. Seeding was performed well upstream of the stilling chamber to ensure adequate mixing of the CO<sub>2</sub> and main tunnel air streams. As shown in Figure 2.8, the CO<sub>2</sub> was supplied by a standard, size 200, high pressure cylinder from Airgas, Inc. The intermediate fill tanks were charged by opening the pressure regulator on the high pressure cylinder. Once filled, the valves were closed to isolate the charged intermediate fill tanks. During air-on operations, a trigger was sent to open the pneumatic solenoid valve downstream of the intermediate fill tanks - releasing the gaseous CO<sub>2</sub> into the main air supply through a hollow tube injection wand. The limited volume of CO<sub>2</sub> gas in this arrangement was sufficient to provide a few seconds of CO<sub>2</sub> fog in test section 2 during air-on operations. The fill tank pressure and piping diameter used in this work allowed for average CO<sub>2</sub> flow rates between 2-4% of the main air supply.



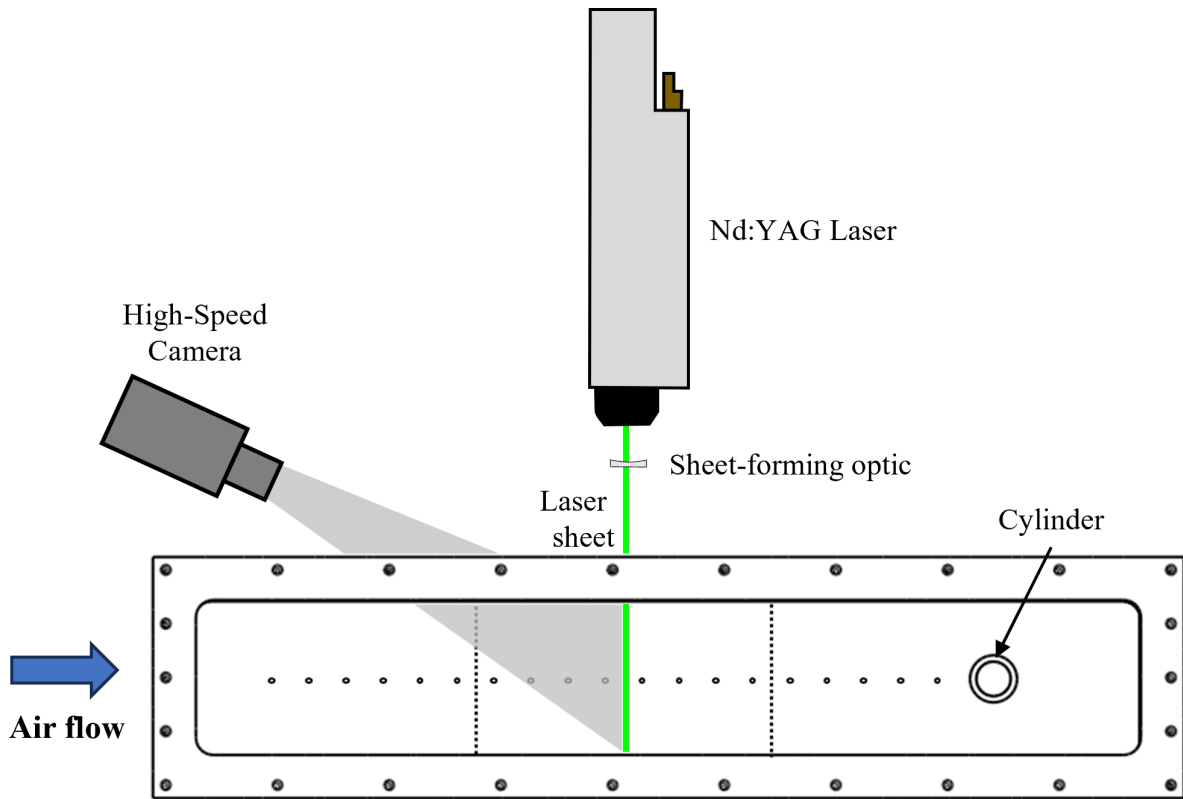


**Figure 2.8:** Schematic diagram of CO<sub>2</sub> seeding arrangement

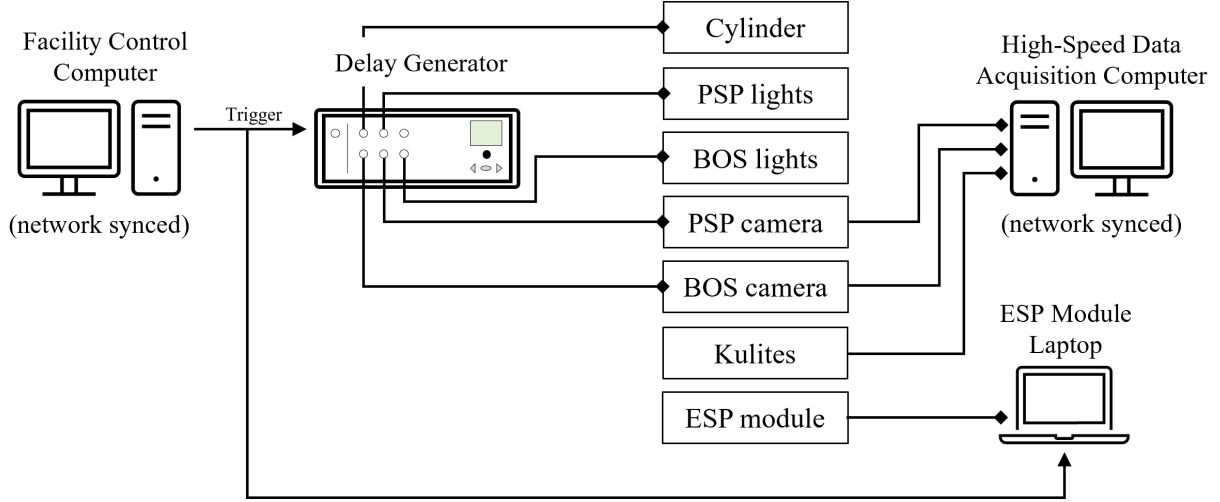
The optical arrangement, shown in Figure 2.9, used a Quantel Q-smart 850, Nd:YAG, pulsed 532 nm laser with a maximum output energy of 430 mJ/pulse and a Photron Mini AX-200 high-speed camera pointing downstream at an angle to capture the full cross-section illuminated by the laser sheet. The delay between the flashlamp and Q-switch was adjusted to 175  $\mu$ s and generated an energy output of 2.88-3.34 mJ/pulse. PLS images were acquired at the maximum laser pulsing frequency of 10 Hz.

## 2.5 Time Synchronization of Data Sources

Time synchronization of the cameras, lighting, and instrumentation was accomplished using a common trigger-out reference time and known time delays. As shown in Figure 2.10, a common trigger-out was sent from the facility control computer to a Berkeley Nucleonics Model 577 Digital Delay/Pulse Generator where the internal delay settings of each channel

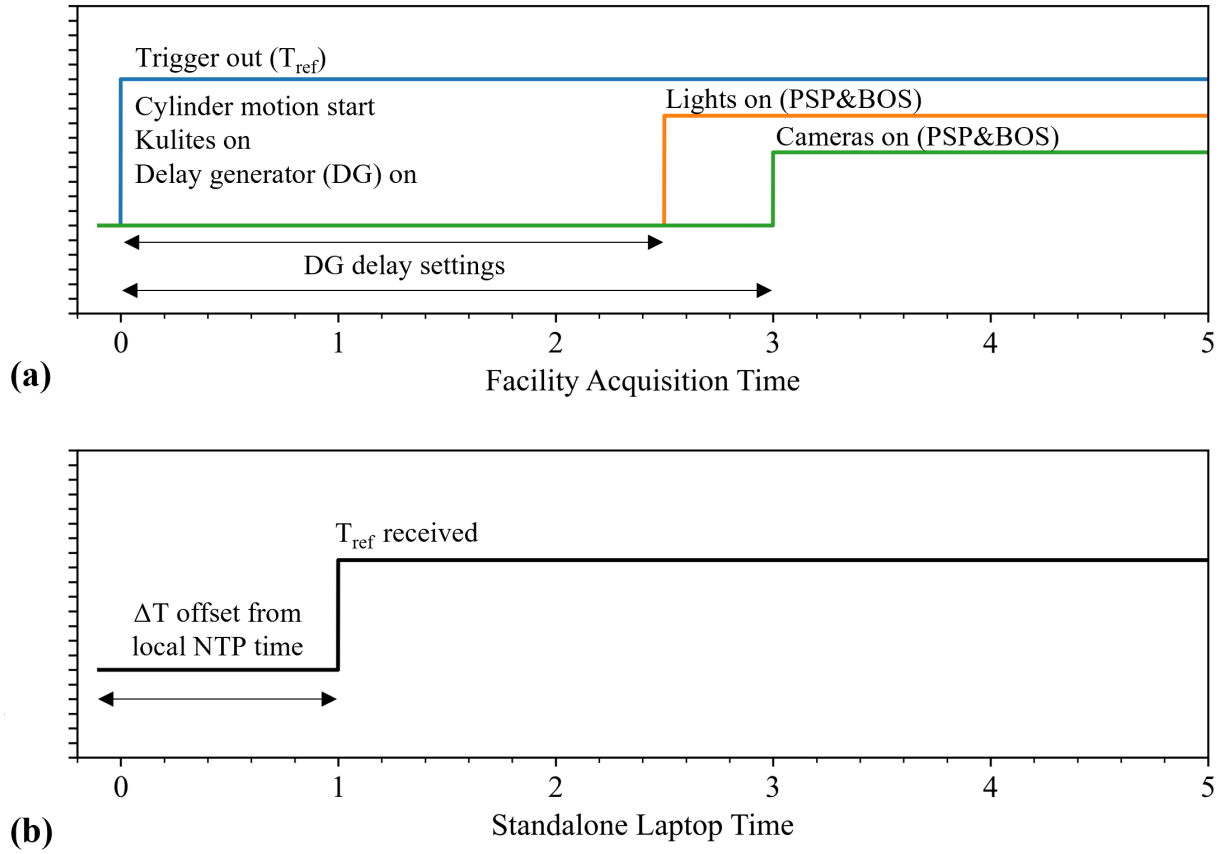


**Figure 2.9:** Schematic diagram of the planar laser Rayleigh scattering arrangement in test section 2 (top view)



**Figure 2.10:** Schematic diagram of the data acquisition arrangement

were stored. The internal pulse rate of the system was set to 10 kHz for a majority of the experimental campaign. The high-speed data from the BOS camera, PSP camera, and Kulites were acquired on a facility acquisition computer, which was time NTP-synchronized with the facility control computer. Due to software restrictions, a standalone laptop was required to initialize and acquire data from the ESP module. The common trigger-out reference was also sent to the standalone laptop, and time alignment was performed using the system's real-time clock (RTC). The trigger time history for a typical experimental run is shown in Figure 2.11.



**Figure 2.11:** Notional trigger events in: (a) facility acquisition computer and (b) standalone laptop time domains

# Chapter 3

## Analysis Methodology

The governing physics, models, assumptions, and processing techniques used to analyze the experimental data are described in this section. First, a simulated jet-to-freestream momentum flux ratio is computed based on the cylinder height. Next, isolator pseudo-shock models are presented based on classical and contemporary quasi-1D flow assumptions. These models are used to predict overall pressure rise and overall shock train length downstream from the point of incipient flow separation. The remaining sub-sections detail image processing techniques and image feature tracking algorithms used in extracting time-dependent data from the BOS and PSP imagery.

### 3.1 Simulated Jet-to-Freestream Momentum Flux

Circular flush-wall fuel injectors are often the first choice when designing high-speed propulsion systems due to their mechanical simplicity and scalability [24]. At a fundamental level, the circular flush-wall fuel injector acts as a transverse jet penetrating a supersonic cross-flow. Numerous studies have been conducted looking at the flow features generated by jet injection in supersonic cross-flow [84, 85]. Additional studies have been conducted to develop models of the jet penetration depth into a supersonic cross-flow [86], which has a direct impact on fuel mixing and combustion performance. The general consensus from

these studies points to a direct correlation between the jet penetration depth and the jet-to-freestream momentum flux ratio, which is a ratio of the dynamic pressures as given in Eq. 3.1 [84, 85].

$$J = \frac{(\rho u^2)_{jet}}{(\rho u^2)_{\infty}} = \frac{(\gamma p M^2)_{jet}}{(\gamma p M^2)_{\infty}} \quad (3.1)$$

Gruber *et al* [87] performed a mixing and penetration study of some basic injector geometries in a Mach 2.0 cross-flow and proposed the following correlation for jet penetration depth,

$$\left( \frac{y}{d_{eff} \cdot J} \right) = 1.23 \left( \frac{x}{d_{eff} \cdot J} \right)^{0.344} \quad (3.2)$$

where  $y$  is the transverse jet penetration depth normalized by the effective jet diameter,  $d_{eff}$ ,  $x/d_{eff}$  is the normalized axial distance from the leading edge of the jet injection site, and  $J$  is the jet-to-freestream momentum flux ratio.

Similarly, Rothstein and Wantuck [88] studied hydrogen jet injection in a Mach 1.5 flow and proposed the correlation given in Eq. 3.3.

$$\frac{y}{d_{eff}} = 2.173 \cdot J^{0.276} \left( \frac{x}{d_{eff}} \right)^{0.281} \quad (3.3)$$

For this work, it was assumed that the height of the cylinder blockage acts as the effective jet penetration depth. It was also assumed that the cylinder diameter represents the axial distance at which the jet reaches its maximum penetration depth (i.e.  $x/d_{eff} = 1$ ). From there, the jet-to-freestream momentum flux ratio simulated by the cylinder height was directly calculated. The results for several cylinder heights based on Eqs. 3.2 and 3.3 are given in Table 3.1.

**Table 3.1:** Simulated jet momentum flux ratio based on normalized cylinder height

| Cylinder height ( $\frac{h}{d}$ ) | $J$ (Mach 2.0) <sup>a</sup> | $J$ (Mach 1.5) <sup>b</sup> |
|-----------------------------------|-----------------------------|-----------------------------|
| 1.00                              | 0.73                        | 0.06                        |
| 2.00                              | 2.07                        | 0.74                        |
| 2.55                              | 2.99                        | 1.79                        |
| 3.00                              | 3.81                        | 3.22                        |
| 4.00                              | 5.86                        | 9.12                        |
| 5.00                              | 8.20                        | 20.48                       |

<sup>a</sup> Gruber *et al* [87]<sup>b</sup> Rothstein & Wantuck [88]

## 3.2 Isolator Pseudo-Shock Models

The isolator pseudo-shock models evaluated in this work fall into two general categories - classical and contemporary. The classical models used for comparison in this work include McLafferty’s [89] flux-conserved model for pressure recovery through a normal shock and the empirically-derived Waltrup-Billig [90] correlation for rectangular isolators. Comparisons are also drawn with the contemporary Smart [91] flux-conserved model. This model derives the governing equations for a differential fluid element of separated flow in a duct with variable area, friction, and heat transfer. The resulting equations can be integrated axially to provide estimates of the flow state throughout the length of the shock train. An excellent review on the predictive performance of these models and other pseudo-shock models is provided by Edelman *et al.* [20].

As noted in [20], each of the current models available today have limitations. Empirically-derived models are restricted to a relatively narrow set of flow conditions and are not applicable beyond the model calibration limits. Flux-conserved models, while more widely applicable, effectively collapse the upstream flow state into a uniform quasi-1D profile and do not accurately model three-dimensional flow distortion of the incoming flow. Despite these limitations, the models still offer a valuable verification source for experimental comparison - particularly when implementing a novel approach for cylinder-induced shock train generation.

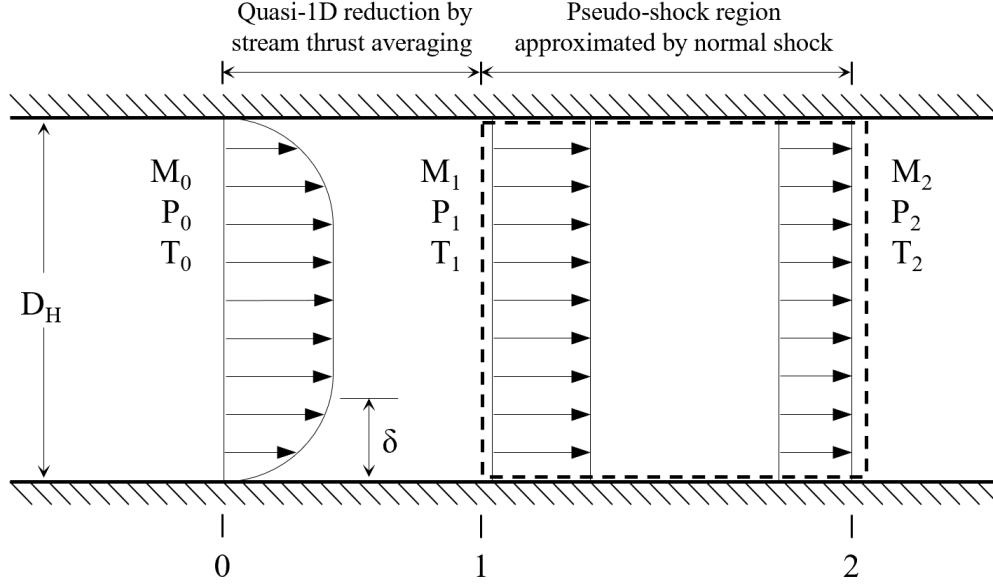
### 3.2.1 Classical Models

The McLafferty model, first proposed in 1953 [89], provides a theoretical estimate of the rise in static pressure across a normal shock in a duct with an initial boundary layer. A schematic of the model is shown in Figure 3.1. The incoming flow (denoted as station 0) consists of a supersonic core at Mach number,  $M_0$ , and pressure,  $P_0$ . A turbulent boundary layer with a  $1/7$  velocity profile is assumed - defining the incoming boundary layer thickness,  $\delta$ , displacement thickness,  $\delta^*$ , and momentum thickness,  $\theta$ . The flow at station 0 is reduced down to a quasi-1D uniform profile at station 1 just upstream of the pseudo-shock interaction region with use of a flux-conserved or stream thrust averaging approach, which conserves mass flux, momentum flux, and energy flux over the fictitious control volume between stations 0 and 1. Upon entering the pseudo-shock interaction region at station 1, the conservation equations are integrated across the control volume between stations 1 and 2 to calculate the downstream subsonic Mach number,  $M_2$  and maximum back-pressure,  $P_2$ . The model also assumes a perfect gas with  $\gamma = 1.4$  and zero wall-skin friction through the shock region. Results of the predicted predicted pressure rise across a normal shock as a function of incoming boundary layer displacement thickness is shown in Figure 3.2.

The Waltrup-Billig [13, 90] correlation for rectangular isolators is an empirical model developed for use with incoming Mach numbers,  $M_0$ , in the range of  $1.5 - 2.7$ . The experimental setup used in [90] allowed a range of back-pressures and partial- to full-strength pseudo-shocks to be investigated. The correlation is given in Eq. 3.4 where  $M_0$ ,  $P_0$ ,  $\theta$ , and  $Re_\theta$  are the Mach number, pressure, momentum thickness, and momentum thickness Reynolds number at the point just upstream of the start of the shock train, respectively. The value  $x$  is the axial location starting just upstream of the shock train, and the value  $P(x)$  is the predicted static pressure profile.

$$\frac{x(M_0^2 - 1)Re_\theta^{1/4}}{D^{1/2}\theta^{1/2}} = 50 \left( \frac{P(x)}{P_0} - 1 \right) + 170 \left( \frac{P(x)}{P_0} - 1 \right)^2 \quad (3.4)$$





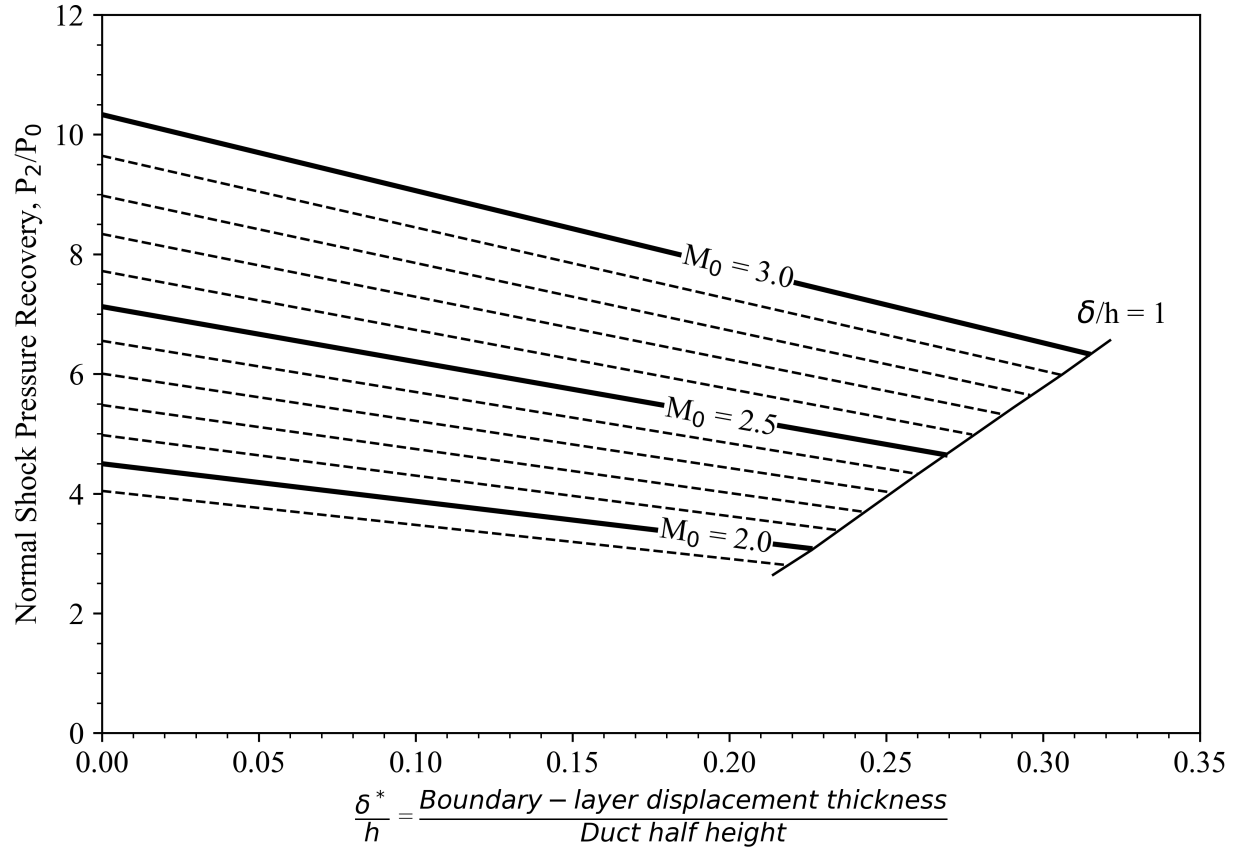
**Figure 3.1:** Schematic of McLafferty's [89] flow model for deriving the theoretical pressure recovery through a normal shock in a duct with an initial boundary layer.

### 3.2.2 Contemporary Models

The Smart [91] flux-conserved model shown in Figure 3.3 assumes the flow through a variable area duct has a supersonic core that is constricted by a fully separated turbulent boundary layer with no wall friction ( $C_f = 0$ ). Along with the standard differential equations for mass, momentum, and energy, Smart [91] proposed two additional equations that are needed to solve for the change in core flow area along the control volume. The equations for the change in Mach number and the change in core flow area ratio ( $A_c/A$ ) are given in Eq. 3.6 and 3.7, respectively, where  $p$  is the pressure,  $C_f$  is the skin friction coefficient just upstream of the the shock train,  $D_H$  is the duct hydraulic diameter, and  $T_t$  is the total temperature.

$$\frac{d\rho}{dx/D_H} = \rho \left( \frac{1}{P} \frac{dP}{dx/D_H} - \frac{1}{T} \frac{dT}{dx/D_H} \right) \quad (3.5)$$

$$\frac{dM^2}{M^2} = - \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \left[ \left( \frac{2}{\gamma M^2} \frac{A}{A_c} \right) \frac{dp}{p} + \left( \frac{A}{A_c} \right) \frac{4C_f dx}{D_H} + \frac{dT_t}{T_t} \right] \quad (3.6)$$



**Figure 3.2:** Static pressure rise across a normal shock,  $P_2/P_0$  predicted by McLafferty model [89] as a function of incoming boundary layer displacement thickness,  $\delta^*$ .

$$\begin{aligned} \frac{d(A_c/A)}{A_c/A} = & \left[ \frac{1 - M^2\{1 - \gamma(1 - A_c/A)\}}{\gamma M^2 A_c/A} \right] \frac{dp}{p} + \left( \frac{1 + (\gamma - 1)M^2}{2A_c/A} \right) 4C_f \frac{dx}{D_H} \\ & + \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \frac{dT_t}{T_t} \end{aligned} \quad (3.7)$$

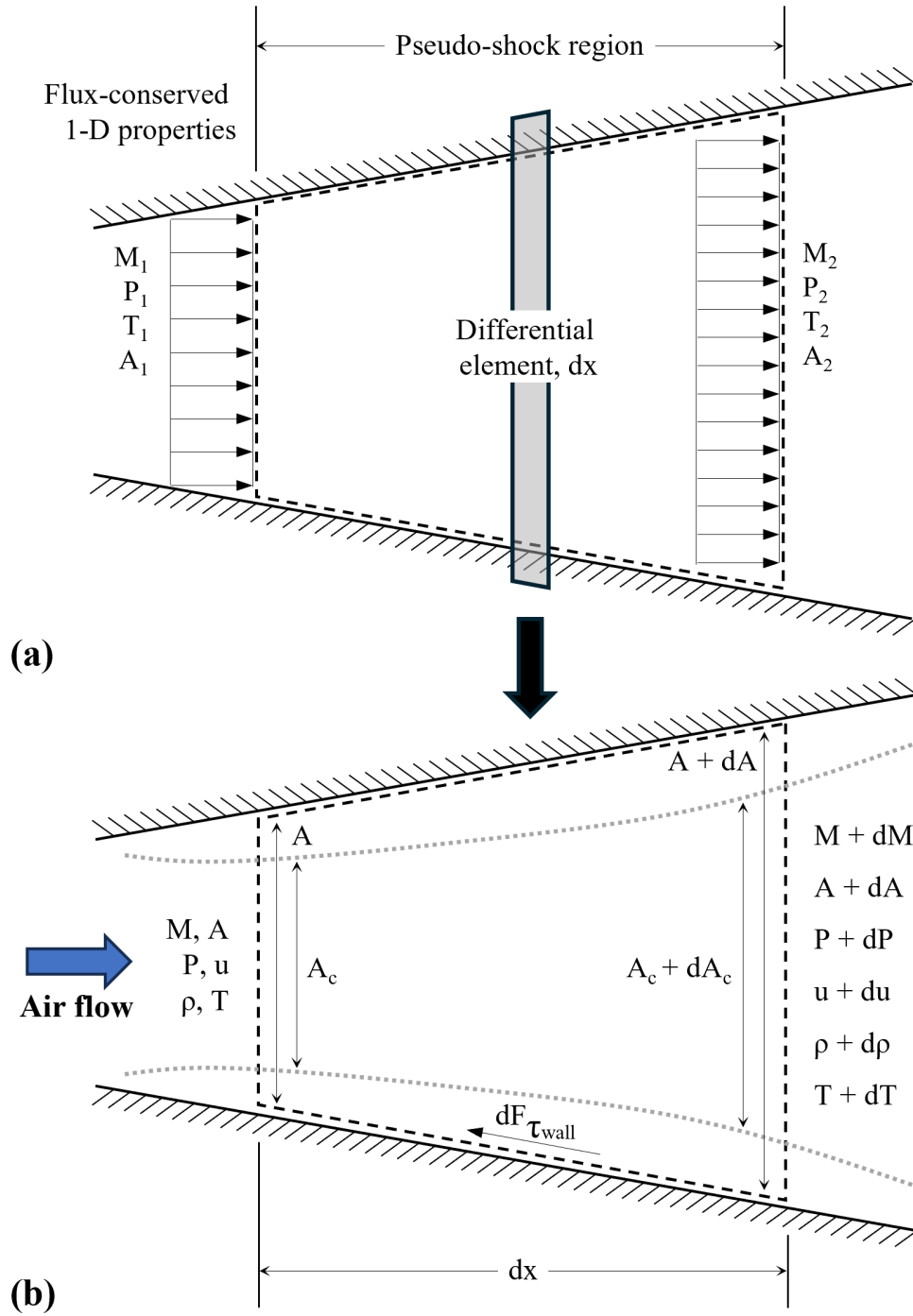
The set of ordinary differential equations in the Smart model is closed by the inclusion of a pressure diffusion model first proposed by Ortwerth [92] as given in Eq. 3.8. This model assumes the rate of pressure rise along a duct with separated flow is proportional to the empirical constant,  $k$ , the skin friction coefficient just upstream of the shock train,  $C_{f0}$ , and the local dynamic pressure,  $\rho V^2/2$ ; and that it is inversely proportional to the duct hydraulic diameter,  $D_H$ .

$$\frac{dp}{dx} = \frac{k}{D_H} C_{f0} \left( \frac{\rho V^2}{2} \right) \quad (3.8)$$

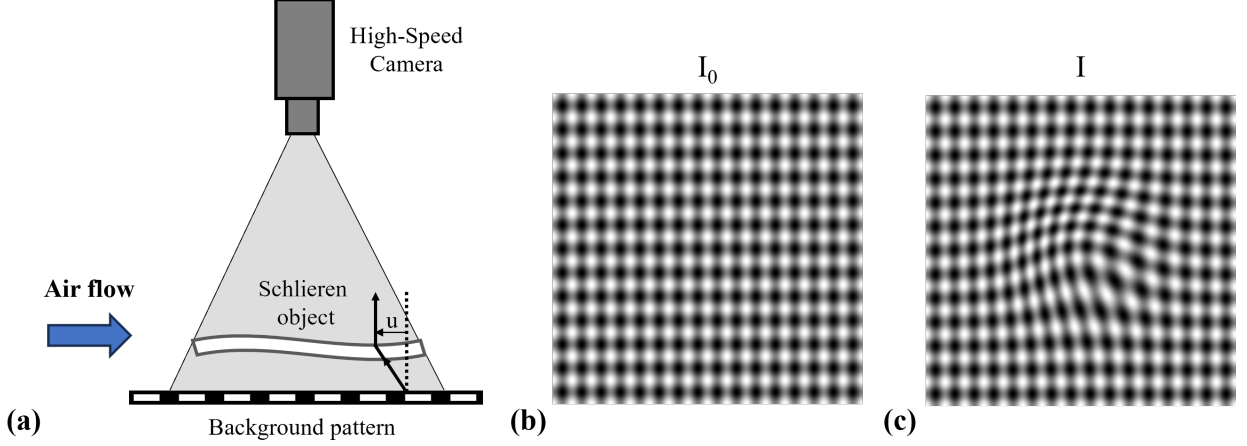
## 3.3 Image Processing

### 3.3.1 Background-Oriented Schlieren

The background-oriented schlieren technique is used for visualizing refractive index gradients caused by inhomogeneities in compressible flows [93]. Its relatively simple setup and scalability to large fields of view made the BOS technique an ideal approach for this work. The basic principle behind the BOS technique is shown in Figure 3.4. The intentional selection of a 2D sinusoidal fringe pattern allows the background to act as a 2D spatial carrier wave that is phase modulated or distorted by refractive index gradients in the flow. The steps for extracting the 2D phase shift vector from an undistorted and distorted image pair are discussed next.



**Figure 3.3:** Schematic of Smart's [91] flow model (neglecting heat transfer to/from the system). (a) Control volume with flux-conserved 1-D flow properties entering the pseudo-shock region; (b) Differential element with reduced core flow area,  $A_c$ , from the separated flow assumption.



**Figure 3.4:** BOS imaging configuration. (a) High-speed camera pointing at a background wall pattern; (b) Undistorted image seen by the camera,  $I_0$ ; (c) Distorted image seen by the camera,  $I$ . The presence of the schlieren object creates a refractive index gradient - distorting the background pattern seen by the camera. The fixed spatial periodicity of the background allows this distortion to be treated as a phase modulated version of the reference background.

The 2D wave equation for the background fringe pattern, as described in Chapter 2, is restated in Eq. 3.9 for convenience. The reference image,  $I_0$ , is expressed as a 2D periodic waveform defined by a fixed wavelength,  $\lambda$ , and spatial image coordinates,  $(x, y)$ .

$$I_0(x, y) = \cos\left(\frac{2 \cdot \pi \cdot x}{\lambda}\right) + \cos\left(\frac{2 \cdot \pi \cdot y}{\lambda}\right) \quad (3.9)$$

Since the distorted image,  $I$ , is considered to be a phase shifted version of the reference image [77], it can be expressed as

$$I(x, y) = \cos\left(\frac{2 \cdot \pi \cdot x}{\lambda} + \Delta\varphi_x(x, y)\right) + \cos\left(\frac{2 \cdot \pi \cdot y}{\lambda} + \Delta\varphi_y(x, y)\right) \quad (3.10)$$

where  $\Delta\varphi_x(x, y)$  and  $\Delta\varphi_y(x, y)$  are the fringe phase shifts in the  $x$  and  $y$  directions, respectively.

Following the approach given in [94], the objective of the schlieren reconstruction problem is to solve for the displacement vector,  $\vec{u}(x, y)$ , that maps the reference image,  $I_0$ , into the distorted image,  $I$ , as expressed in Eq. 3.11. For compactness, the spatial vector  $\vec{r} = (x, y)$  is used in Eq. 3.11.

$$I(\vec{r}) = I_0(\vec{r} - \vec{u}(\vec{r})) \quad (3.11)$$

### 3.3.1.1 Synthetic Reconstruction with Fourier Transform Profilometry

The complex Fourier transform of a general function,  $f$ , in two dimensions is given in Eq. 3.12 as

$$f(\vec{r}) = \sum_{v_1=-\infty}^{\infty} \sum_{v_2=-\infty}^{\infty} a_{v_1 v_2} e^{i(v_1 \vec{k}_1 + v_2 \vec{k}_2) \cdot \vec{r}} \quad (3.12)$$

where  $\vec{k}_1$  and  $\vec{k}_2$  form a set of orthogonal basis vectors in the complex Fourier domain (k-space), and  $v_1$  and  $v_2$  are the corresponding scaling factors, respectively. These basis vectors are orthogonal by design and correspond to the linearly independent carrier wave vectors of the BOS background pattern.

Applying Eq. 3.12 to the reference and distorted image pair yields Eq. 3.13 and Eq. 3.14, respectively.

$$I_0(\vec{r}) = \sum_{v_1=-\infty}^{\infty} \sum_{v_2=-\infty}^{\infty} a_{v_1 v_2} e^{i(v_1 \vec{k}_1 + v_2 \vec{k}_2) \cdot \vec{r}} \quad (3.13)$$

$$I(\vec{r}) = \sum_{v_1=-\infty}^{\infty} \sum_{v_2=-\infty}^{\infty} a_{v_1 v_2} e^{i(v_1 \vec{k}_1 + v_2 \vec{k}_2) \cdot (\vec{r} - \vec{u}(\vec{r}))} \quad (3.14)$$

Under certain conditions, the distorted signal will have  $\vec{k}_1$  and  $\vec{k}_2$  basis vectors in proximity to, but slightly spread out from the reference image basis vectors [94]. From there, filtering is performed in the Fourier (k-space) domain in the region around each principal component axis. Following the recommendation in [77], truncated circular sector filters are applied in this work and will be described in more detail in the following sub-section.

Following the derivation in [94], Fourier domain filtering results in finite complex functions in each principal component direction as given below.

In the  $\vec{k}_1$  direction:

$$\{g_0(\vec{r})\}_1 = a_{v_1} e^{iv_1 \vec{k}_1 \cdot \vec{r}} \quad (3.15)$$

$$\{g(\vec{r})\}_1 = a_{v_1} e^{iv_1 \vec{k}_1 \cdot (\vec{r} - \vec{u}(\vec{r}))} \quad (3.16)$$

In the  $\vec{k}_2$  direction:

$$\{g_0(\vec{r})\}_2 = a_{v_2} e^{iv_2 \vec{k}_2 \cdot \vec{r}} \quad (3.17)$$

$$\{g(\vec{r})\}_2 = a_{v_2} e^{iv_2 \vec{k}_2 \cdot (\vec{r} - \vec{u}(\vec{r}))} \quad (3.18)$$

The phase shifts in each of the carrier wave directions,  $\Delta\varphi_1(\vec{r})$  and  $\Delta\varphi_2(\vec{r})$ , are computed using Eq. 3.19 where \* denotes the complex conjugate.

$$\Delta\varphi(\vec{r}) \equiv \text{Im}(\ln(gg_0^*)) \quad (3.19)$$

From there, the displacement vector,  $\vec{u}(\vec{r})$ , can be solved directly using the following two equations at each pixel location.

$$\Delta\varphi_1(\vec{r}) = -\vec{k}_1 \cdot \vec{u}(\vec{r}) \quad (3.20)$$

$$\Delta\varphi_2(\vec{r}) = -\vec{k}_2 \cdot \vec{u}(\vec{r}) \quad (3.21)$$

### 3.3.1.2 Application to Experimental Imagery

In practice, application of the Fourier Transform Profilometry method is straightforward; thanks to modern image processing tools. An overview of the processing steps is shown in Figure 3.5. A reference image acquired with no flow disturbances present (Fig. 3.5 (a)) is transformed into the Fourier domain, or k-space domain, with a simple 2D FFT (Fig. 3.5 (c)). Because the background pattern is periodic, the carrier wave peaks ( $\vec{k}_1$  and  $\vec{k}_2$ ) appear

symmetrically (in the  $k_x$  and  $k_y$  directions) about the zero-frequency. Recalling from Chapter 2, the background pattern used in this work resulted in a waveform peak-to-peak spacing of 6 pixels in the horizontal and vertical directions - corresponding to spatial frequencies,  $f_x$  and  $f_y$  of  $\approx 0.17 \text{ pix}^{-1}$ . Similar results are shown for the distorted image acquired during wind-on operation with shock disturbances present (Fig. 3.5 (b)). While not much can difference can be seen in the reference and distorted raw images, the k-space projection of the distorted image (Fig. 3.5 (d)) shows the same dominant carrier peaks but with additional “smearing” or modulation around the edges due to the distorted signal.

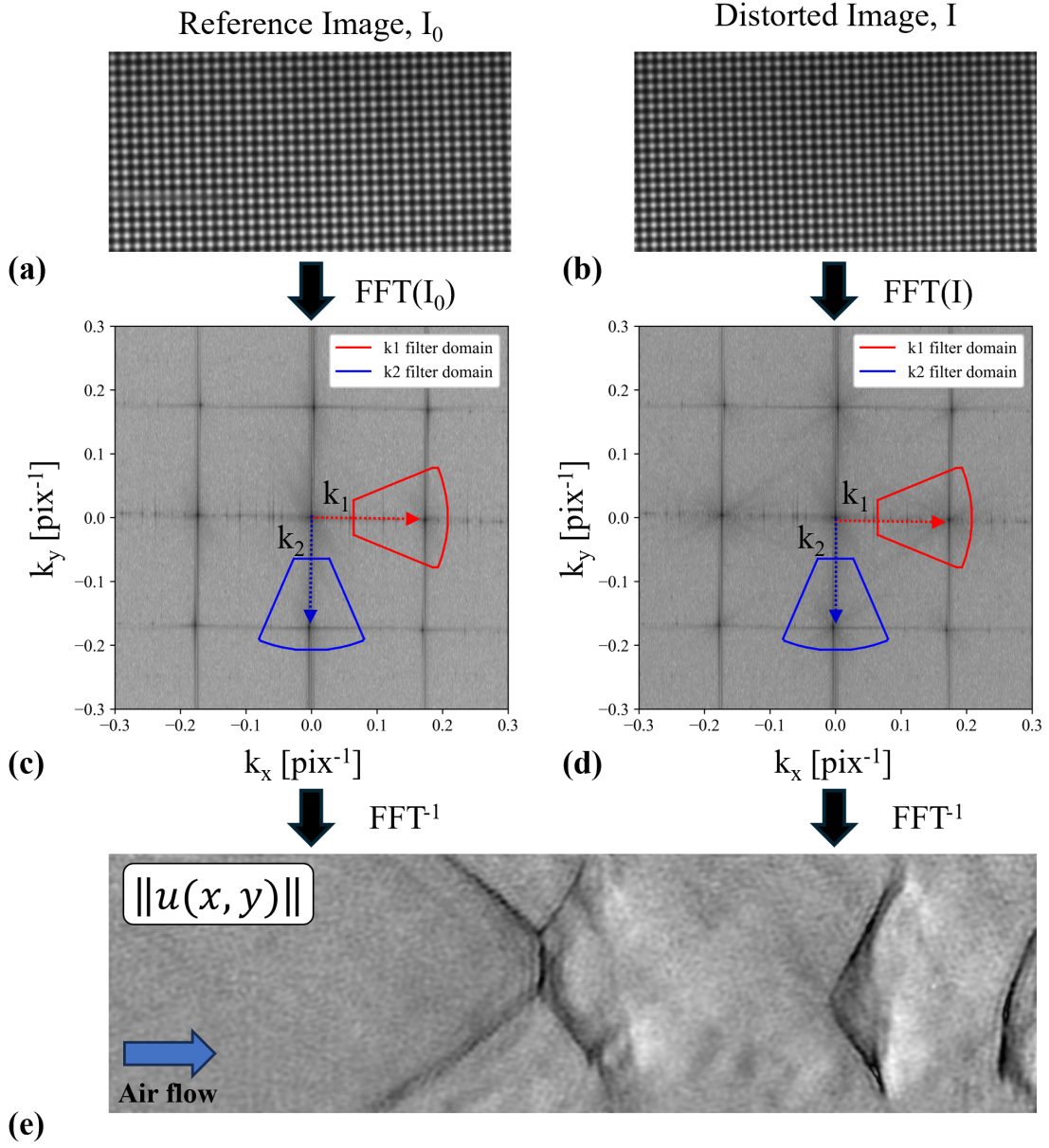
Once the carrier wave peaks were identified, truncated circular sector filters were designed to isolate the signal by applying the filter in the Fourier domain. Following the approach in [77], the filters were designed to have a central sector angle of 0.8 radians and minimum and maximum truncation radii corresponding to  $0.4f_x$  and  $1.2f_x$ , respectively. The resulting filter bounds are shown as the red and blue lines in (Fig. 3.5 (c)). A binary mask was created from each filter region and applied in the Fourier domain. The inverse 2D FFT was then taken to provide image-space reconstructions of the k-space filtered signals. This process was carried out for the reference and distorted signals separately.

From there, the displacement vector field was calculated using Eqs. 3.19 - 3.21. The magnitude of the displacement was calculated and is shown as a grayscale image in Figure 3.5 (e). The process described above was repeated for each distorted image frame acquired during wind-on operation to provide a time-dependent reconstruction of the shock train.

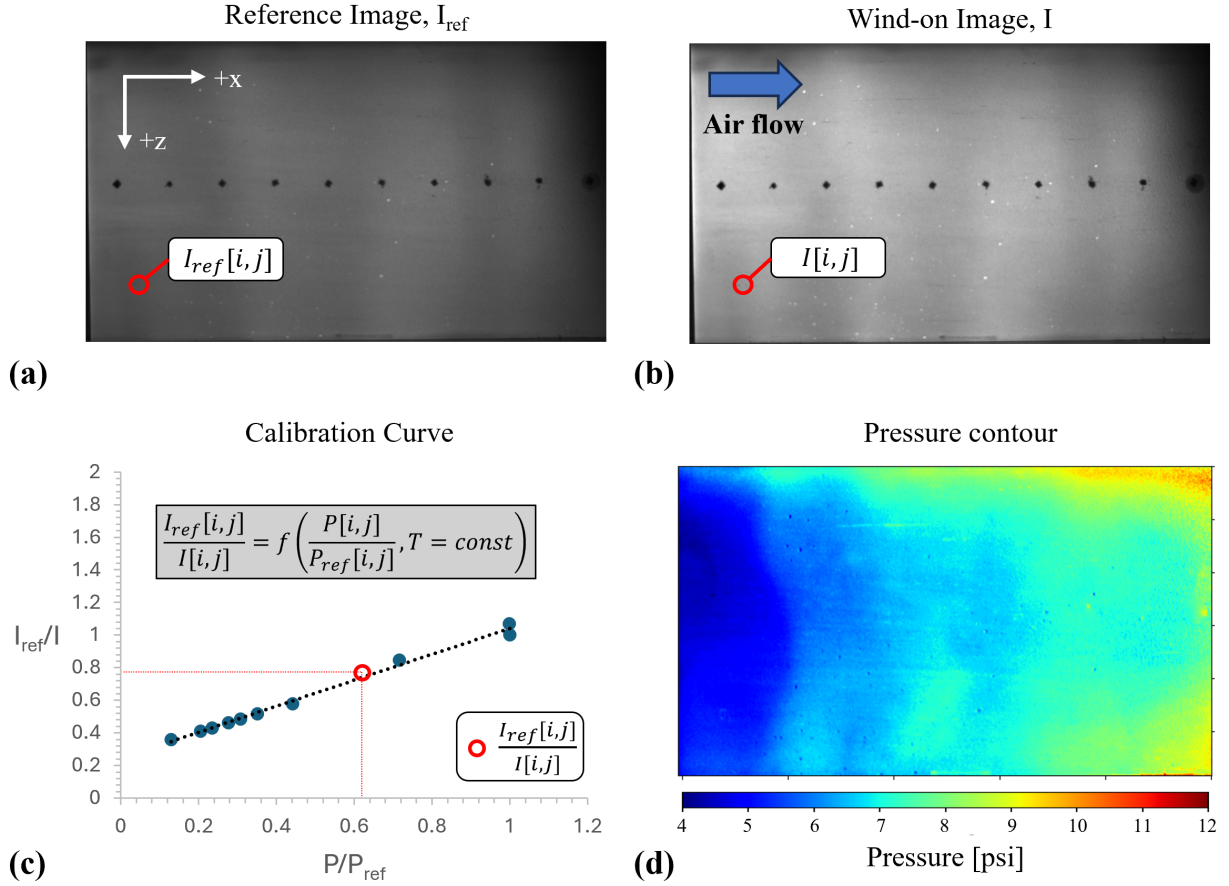
### 3.3.2 Pressure-Sensitive Paint

The processing steps for the PSP imagery are shown in Figure 3.6. A reference image is acquired with the PSP lights operating during wind-off conditions (Fig. 3.6 (a)). The reference pressure is taken from the upstream kulite, K1, and is assumed to remain spatially and temporally constant during acquisition of the reference images. A wind-on image is shown in Figure 3.6 (b). The calibration curve fit is used to map the intensity ratio,  $I_{ref}/I$ , to a unique pressure ratio,  $P/P_{ref}$ , for each pixel in the image (Fig. 3.6 (c)). Finally, the





**Figure 3.5:** Application of Fourier Transform Profilometry to BOS images. (a) Reference image of the sinusoidal fringe background; (b) Distorted image during wind-on operation with shock disturbances present; (c) Reference image in the Fourier domain ( $k$ -space). Carrier peaks  $\vec{k}_1$  and  $\vec{k}_2$  correspond to the linearly independent periodic background waves; (d) Distorted image in the Fourier domain ( $k$ -space). Carrier peaks appear as “smeared” or modulated by the distortion signal. Truncated circular sector filters (red and blue regions in (c) and (d)) are used to isolate regions around each carrier peak; (e) Reconstructed displacement field  $\vec{u}(\vec{r})$  revealing shock disturbances in the flow ( $\|\vec{u}\|$  shown here).



**Figure 3.6:** PSP Image Processing Methodology. (a) Reference image acquired during wind-off period (shown as 16-bit grayscale); (b) Wind-on image (shown as 16-bit grayscale); (c) Notional intensity ratio,  $I_{ref}/I$  of pixel  $[i, j]$  mapped onto calibration curve. This step is performed for each pixel coordinate in the image; (d) Processed PSP contour.

reference pressure is used to convert the pressure ratio into a physical pressure contour as shown in Figure 3.6 (d).

As discussed in Chapter 2, the limited pressure range and lack of surface temperature control during the off-line calibration meant that the PSP steady-state pressure could not be considered accurate with sufficient confidence. It is expected that a large bias offset is present in the steady-state pressure data due to the unknown surface temperature sensitivity of the fast response PSP [95, 96]. For this reason, steady-state PSP pressures are not presented in this report. However, much insight can be gained from analyzing the dynamic behavior of

certain shock train features appearing in the PSP contours. The results of this analysis will be discussed later.

### 3.3.3 Planar Laser Scattering

Due to the viewing angle of the PLS camera with respect to the laser sheet, a perspective transformation of the PLS imagery was performed to reconstruct a view normal to the laser sheet - effectively mapping the skewed camera coordinates into the tunnel flow coordinate system. The process is described in Figure 3.7. A LaVision calibration plate (106-10) was used for performing the perspective correction. The plate has a dot center-to-center distance of 10 mm and a dot diameter of 2.2 mm. The optical setup resulted in a pixel resolution of 4.86 px/mm .

The location of each corner target point in camera coordinates is given as

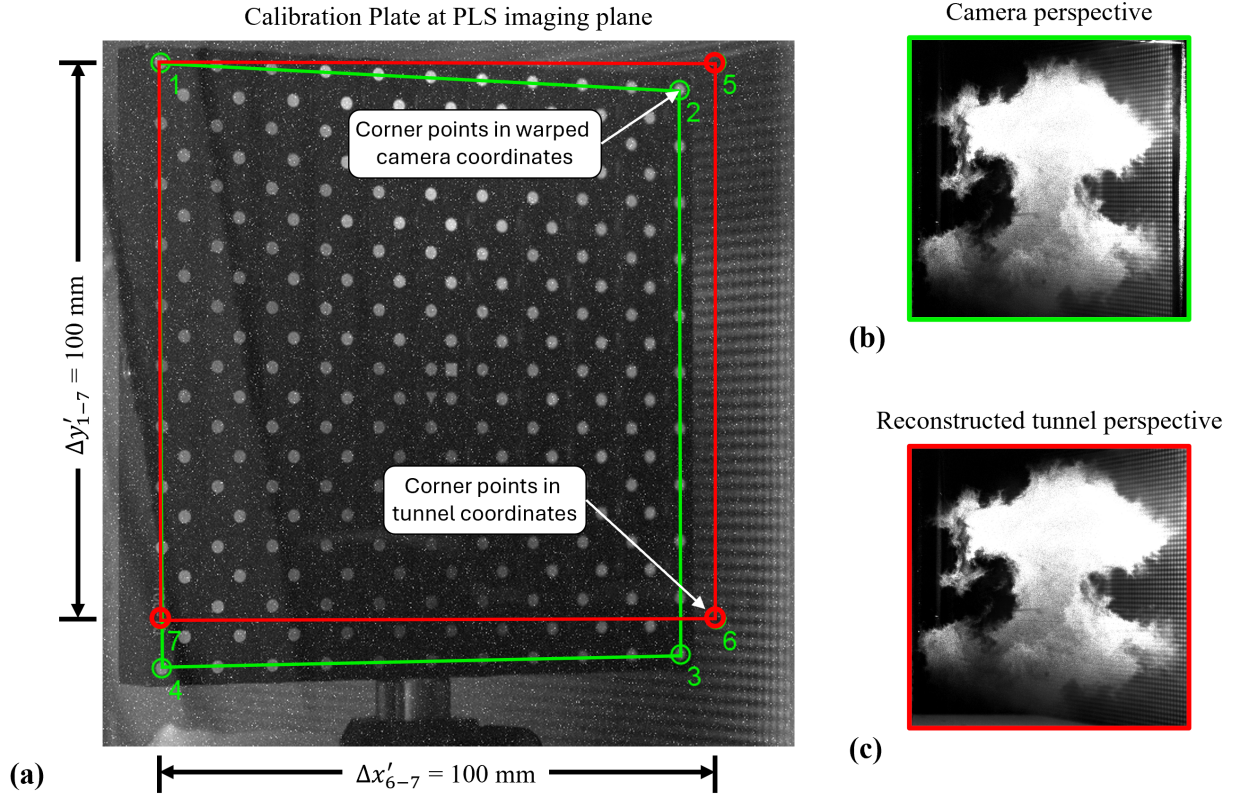
$$(x_i, y_i)_{i=1,2,3,4} \quad (3.22)$$

where  $i$  is the numerical designator for each corner point, as shown in Figure 3.7. The expected locations of the corner points in a tunnel coordinate system are given as

$$(x'_j, y'_j)_{j=1,5,6,7} \quad (3.23)$$

where points 5, 6, and 7 are the expected corner points in a dewarped image plane normal to the laser sheet. These points are estimated using the known location of point 1 and the fixed dot spacing of the calibration plate. A 3x3 transformation matrix between the two coordinate systems was then found by feeding the  $(x_i, y_i)$  and  $(x_j, y_j)$  corner points into the `getPerspectiveTransform` method within the OpenCV image processing library. The transformation matrix,  $M$ , was then used to reconstruct the PLS images in a tunnel frame of reference by Eq. 3.24. The resulting dewarped image is shown in Figure 3.7 (c).

$$I(x', y') = I \left( \frac{M_{11}x + M_{12}y + M_{13}}{M_{31}x + M_{32}y + M_{33}}, \frac{M_{21}x + M_{22}y + M_{23}}{M_{31}x + M_{32}y + M_{33}} \right) \quad (3.24)$$



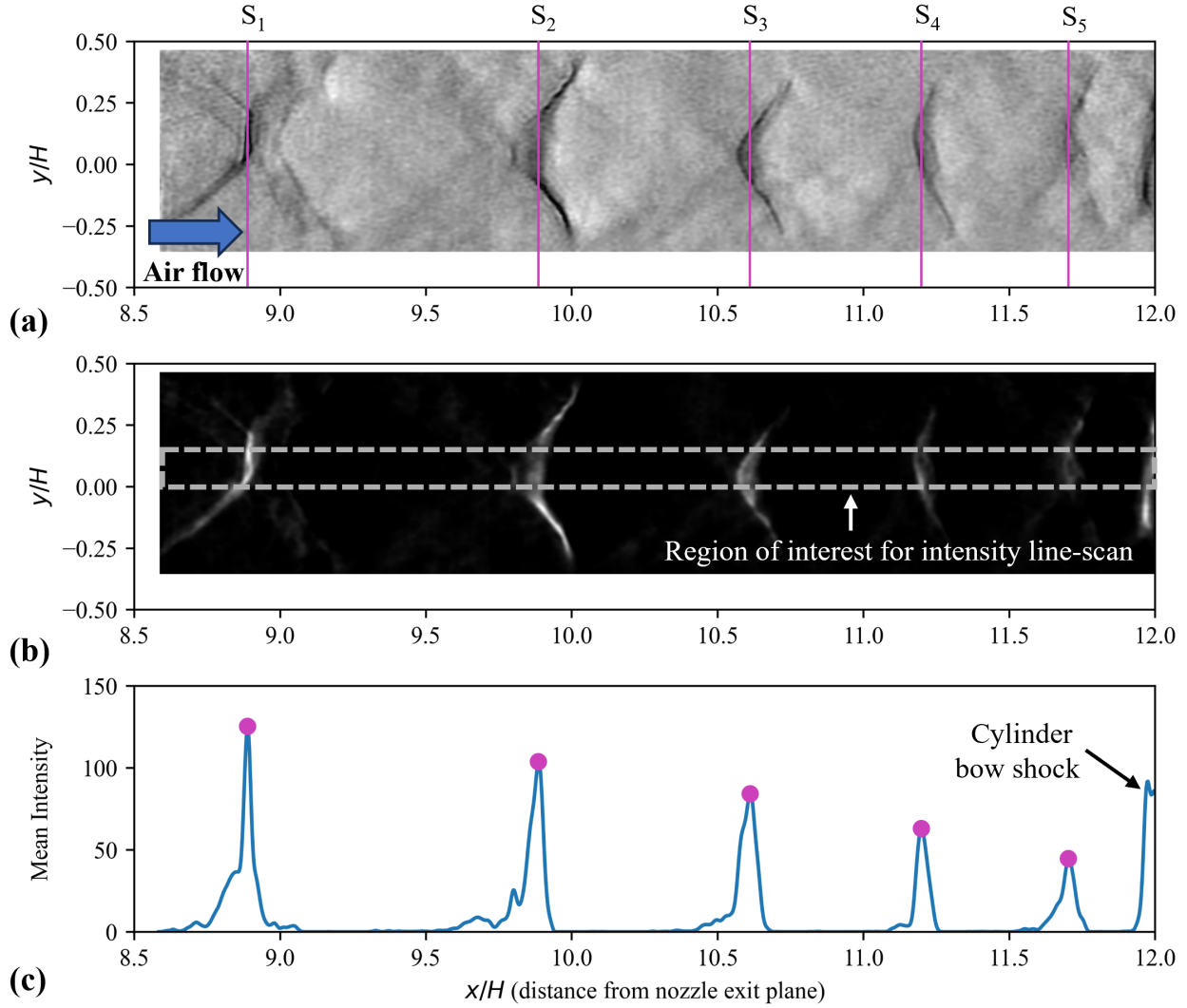
**Figure 3.7:** PLS image perspective transformation. (a) Calibration plate from camera perspective. The green points are the corner target points found in camera coordinates. The red points are where the corner target points would appear if the camera were normal to the laser sheet; (b) PLS image from camera perspective; (c) Reconstructed PLS image in tunnel coordinates.

### 3.4 Image-Based Shock Tracking

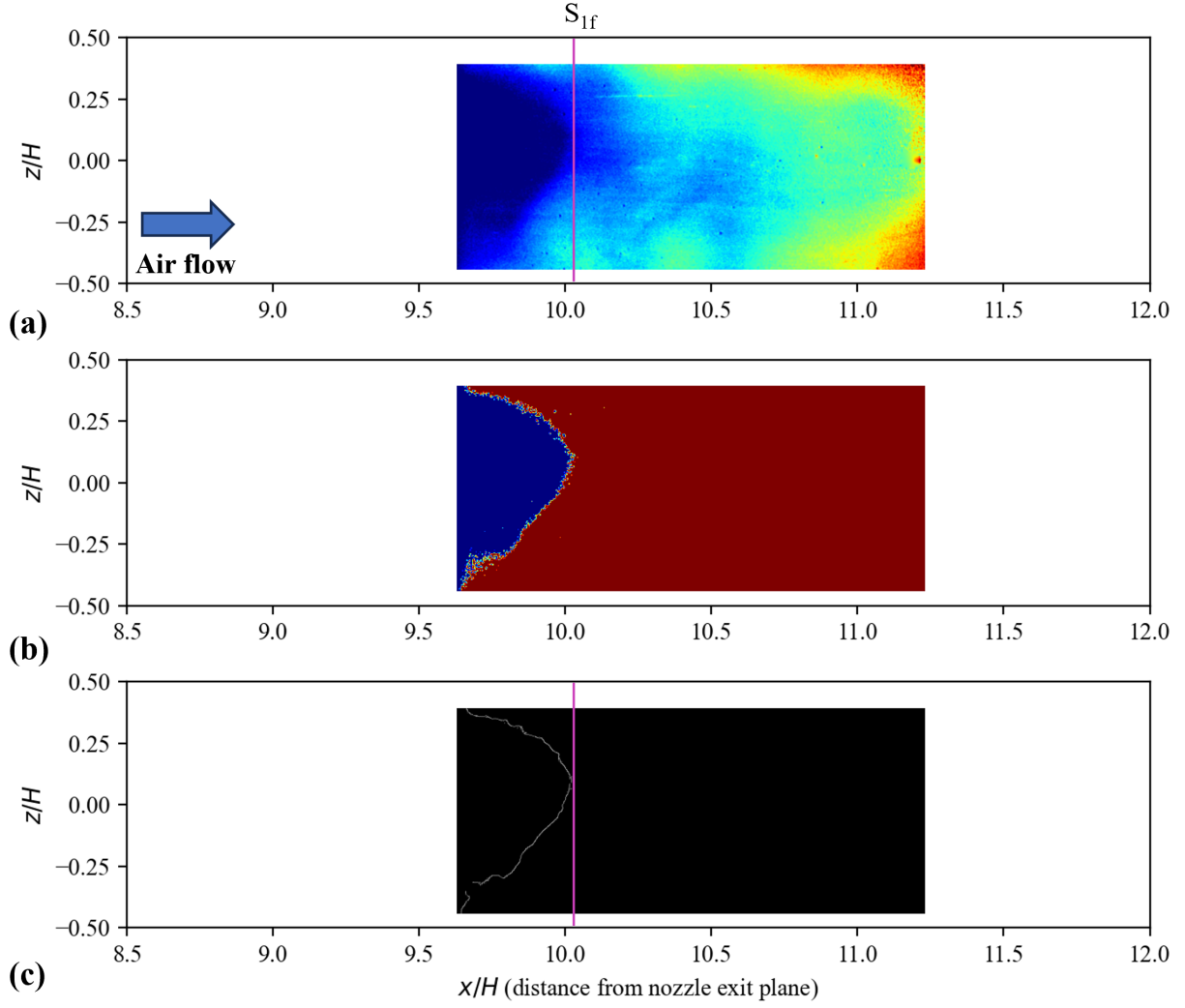
The detection of multiple shock features was performed using time-aligned imagery from the high-speed BOS and PSP techniques. The detection and tracking process using the BOS imagery is shown in Figure 3.8. Starting with the grayscale BOS image (a), a mean intensity of the background flow was determined and then subtracted from (a). Intensity-based thresholding was then applied to the mean subtracted image to enhance the shock features, as shown in (b). From (b), image cropping was performed to isolate a centerline region of interest approximately 100 pixels tall. A line-scan of the average pixel intensity at each axial location was then performed on the cropped region. At this point, the shock

features could easily be distinguished by their corresponding peaks in (c). In this sample image, five shocks were separately tracked and are indicated by labels  $S_1 - S_5$ . The axial location of each shock in the shock train was recorded with a timestamp to later correlate with the cylinder height during the run. This process was repeated for each frame recorded during the test run.

The overhead orientation of the PSP setup allowed for tracking of the leading bifurcated normal shock foot, as shown in Figure 3.9. In (a), a sample PSP image is shown capturing the time when the leading shock foot is directly over the PSP panel, indicated by the label  $S_{1f}$ . RGB-color based thresholding was then applied to isolate the initial pressure rise from the shock leading edge. The result of this step is shown in (b). In the final step, a Canney edge detection algorithm was applied; resulting in the binary image shown in (c). The shock foot leading edge was identified based on the axial location of the most downstream edge in (c). This process was repeated for each frame recorded during the test run.



**Figure 3.8:** BOS shock tracking process. (a) Sample BOS image with tracked shock features  $S_1 - S_5$ ; (b) Mean-Subtracted BOS image with threshold enhancing; (c) Line-scan of region of interest average intensity. Shock features  $S_1 - S_5$  were identified based on the axial location of each peak in (c).



**Figure 3.9:** PSP shock tracking process. (a) Sample PSP image with tracked leading shock foot,  $S_{1f}$ ; (b) RGB-threshold image isolating the shock foot front; (c) Binary Canny edge detection image of (b).  $S_{1f}$  was identified based on the axial location of the most downstream edge.

# Chapter 4

## Characterization of the Quasi-Static Shock Train

### 4.1 Baseline Flow Conditions

Flow conditions are considered baseline when the tunnel is running at nominal conditions without the presence of the cylinder back-pressure. This will also be referred to as a “clean” tunnel flow throughout the remainder of this document. Flow properties along the tunnel test sections were computationally characterized by Stokes [97] and were leveraged as a part of this work. The flow conditions and boundary layer properties for the clean tunnel configuration at the most upstream and most downstream shock train leading edge locations are reported in Table 4.1. For shock locations in between, the data was linearly interpolated. The pressure ratio, denoted as  $P_2/P_0$  to be consistent with isolator pseudo-shock naming convention, corresponds to the baseline pressure rise starting from the most upstream location  $P_{37}/P_4$  and from the most downstream location  $P_{37}/P_{30}$ . For this work, the “isolator exit” was defined to be just upstream of the cylinder leading edge at port 37.



**Table 4.1:** Baseline flow conditions and boundary layer properties at the most upstream and most downstream shock train leading edge locations

| Tunnel axial location ( $x$ )         | $2.5H$             | $10.7H$            | Units             |
|---------------------------------------|--------------------|--------------------|-------------------|
| Mach number                           | 2.15               | 2.06               | —                 |
| Pressure ratio ( $P_2/P_0$ )          | 1.23 <sup>a</sup>  | 1.04 <sup>b</sup>  | —                 |
| Velocity ( $U_\infty$ )               | 519.31             | 507.64             | m/s               |
| Temperature                           | 138.77             | 144.73             | K                 |
| Density                               | 0.68               | 0.76               | kg/m <sup>3</sup> |
| Boundary-layer thickness ( $\delta$ ) | 12.07              | 15.56              | mm                |
| Momentum thickness ( $\theta$ )       | 1.25               | 2.34               | mm                |
| Confinement ratio ( $2\delta/H$ )     | 0.16               | 0.21               | —                 |
| Reynolds number ( $Re_\theta$ )       | $1.26 \times 10^4$ | $2.31 \times 10^4$ | —                 |

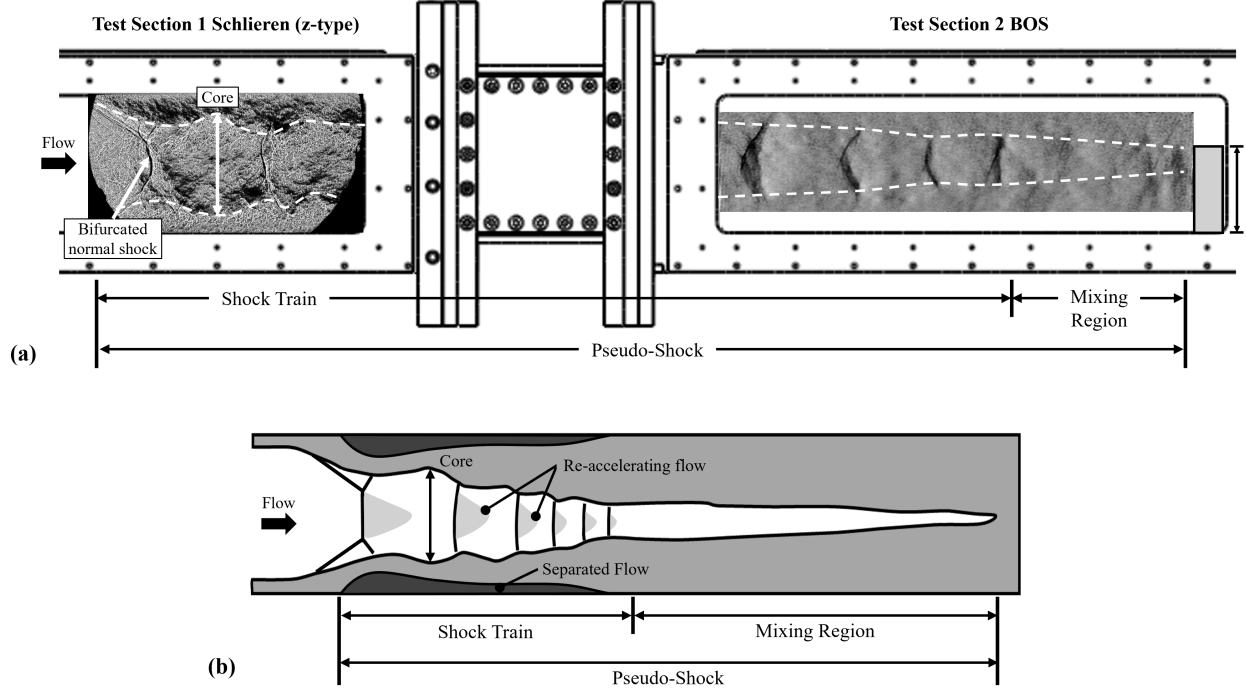
<sup>a</sup> ( $P_{37}/P_4$ )

<sup>b</sup> ( $P_{37}/P_{30}$ )

## 4.2 Shock Train Structure

The use of the term shock train in this work is consistent with Matsuo’s [4] definition: a series of repeated shocks in line that are visible by optical observations. A pseudo-shock, as Crocco [29] called it, is the full compression system that is comprised of the shock train and a shockless subsonic/supersonic mixing region downstream of the shock train. A substantial amount of duct length, anywhere from 6 - 15 hydraulic diameters [89], is required for a complete pseudo-shock to develop.

Figure 4.1 shows a comparison of the pseudo-shock generated using the cylinder back-pressuring method and the commonly used pseudo-shock representation from Matsuo [4]. A full pseudo-shock system is visible when the cylinder height is increased beyond  $4.86\delta$ , as shown in (a). Simultaneous images from z-type schlieren in test section 1 and BOS in test section 2 are overlaid on the tunnel geometry. At this level of back-pressure, the shock train leading edge has pushed upstream into the first test section, where the rapidly thickening boundary layer is more evident. Test section 2 contains the trailing shocks that continue to diminish in height as the boundary layer grows. The mixing region can be seen



**Figure 4.1:** Comparison of experimentally simulated pseudo-shock to Matsuo's [4] representation. (a) Simultaneous z-type schlieren and BOS images at high back-pressure conditions ( $h_{cylinder} > 4.86\delta$ ) overlaid on tunnel geometry; (b) Pseudo-shock schematic from Matsuo [4].

downstream of the last shock. In this region, the supersonic core has rapidly diminished and the average Mach number is most likely subsonic. This is further supported by the shock-free BOS results and the disappearance of the cylinder bow shock. The close comparison of the experimentally generated pseudo-shock provides confidence that the cylinder-induced approach is generating a proper pseudo-shock system. However, the lack of optical access and instrumentation availability in the coupling block between the test sections drove this work to focus entirely on the shock train characteristics and dynamics while in test section 2.

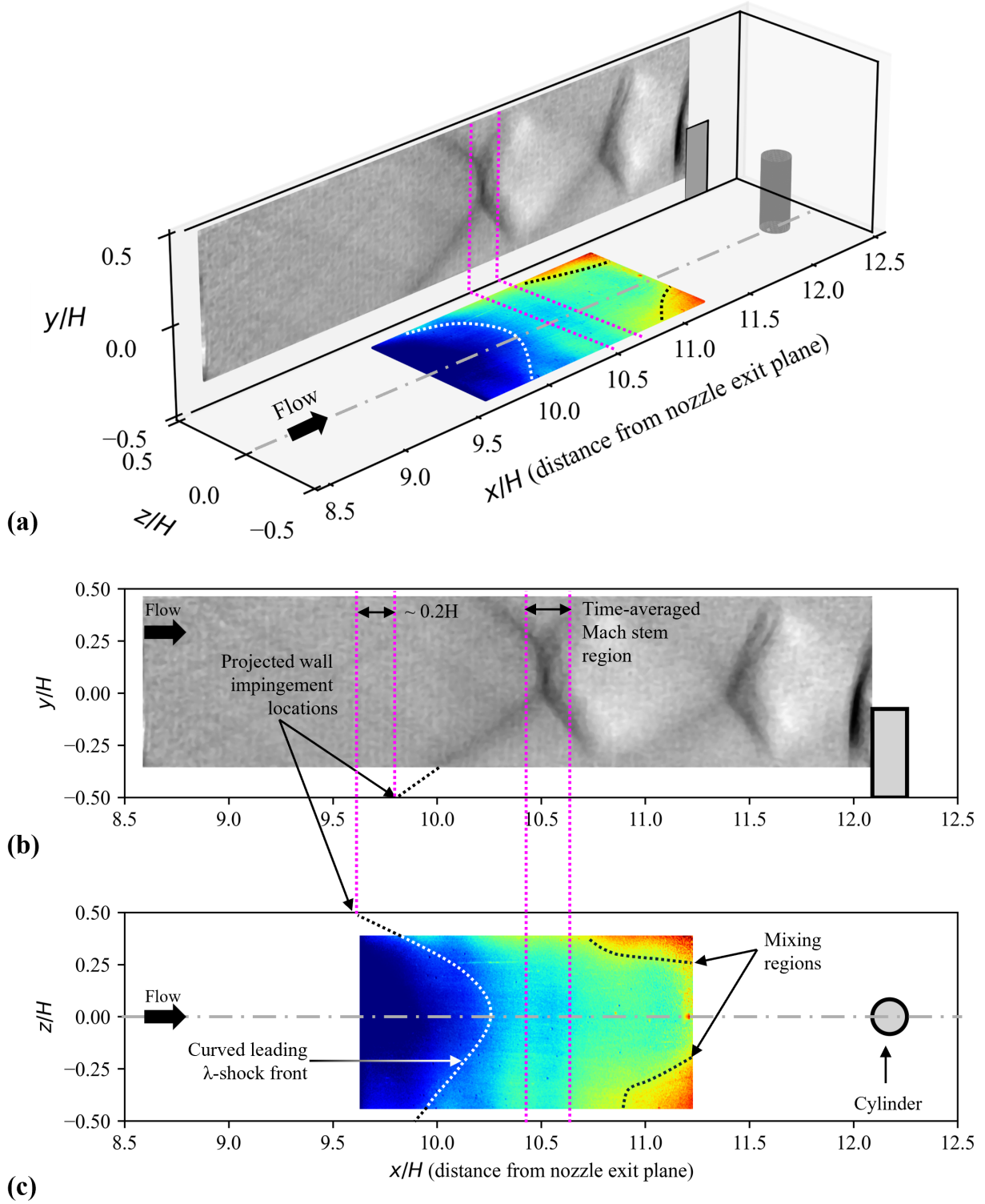
The three-dimensional structure of the shock train was evaluated by viewing the orthogonal BOS, PSP, and PLS imaging planes simultaneously. Time-averaged BOS and PSP images at a cylinder height of  $h = 4.13\delta$  are shown in Figure 4.2. Evaluation of the BOS image (b) reveals a bifurcated normal shock centered around  $x = 10.5H$  with a second

normal shock downstream around  $x = 11.5H$ . The inviscid cylinder shock at  $x = 12H$  is not considered to be part of the shock train. The curvature of the leading shock foot can be seen in the overhead PSP (c) view. This curvature is indicative of a three-dimensional conical shock structure resulting from upstream corner flow separation [10, 43]. The leading shock foot locations were linearly extrapolated to the wall for both the PSP and BOS images. A comparison of the two locations shows an axial difference of  $\sim 0.2H$  and highlights the limitation of using a path-integrated technique, like schlieren, for determining the shock train leading edge. At the downstream end of the PSP panel, the subsonic/supersonic mixing region can be seen starting to develop.

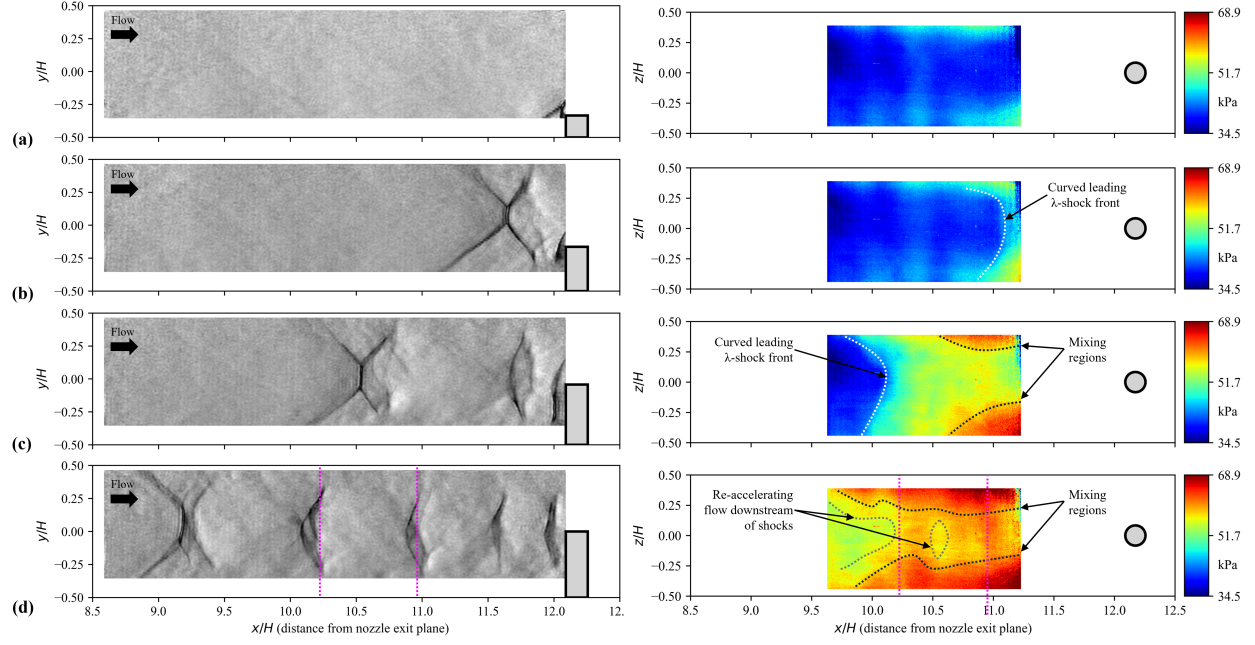
A series of instantaneous BOS and PSP images at increasing cylinder heights is shown in Figure 4.3. The shock train leading edge can be seen propagating upstream as the cylinder height increases (starting at (a) and going to (d)). In (d) the shock train leading edge is upstream of the PSP section. A larger mixing region developing from the sidewalls can be seen as well as regions of pressure peaks and valleys - indicating potential pockets of re-accelerating flow behind the normal shocks.

Core flow profiles upstream and downstream of the shock train leading edge are shown in Figures 4.4– 4.7, respectively. The white region in the PLS image is the cloud of dry ice nanoparticles illuminated by the laser sheet. The existence of solid  $\text{CO}_2$  particles illuminated by the laser sheet indicates supersonic flow with static pressure and temperatures at sufficiently low conditions for the  $\text{CO}_2$  to remain in a solid state. The static pressure and temperature rise in the subsonic flow regions cause the solid  $\text{CO}_2$  particles to sublime; resulting in dark regions of the image.

In Figures 4.4 and 4.5, the shock train leading edge is downstream of the PLS imaging plane. Large regions emanating from both sidewalls can be seen protruding into the core flow up to a significant portion of the test section’s half-span ( $\sim 70\%$ ). These regions are indicative of sidewall-tied streamwise vortices originating from the upstream 2-D converging-diverging nozzle as noted by Andrews [79]. At the centerline ( $z/H = 0$ ), the ceiling and floor boundary layer heights upstream of the shock train leading edge appear to be relatively



**Figure 4.2:** Time-averaged BOS and PSP images at cylinder height  $h = 4.13\delta$ . (a) Isometric view mapping BOS and PSP images to the sidewall and floor, respectively; (b) Time-averaged BOS image (side view); (c) Time-averaged PSP image (top view); Flow is left to right in (b) and (c).



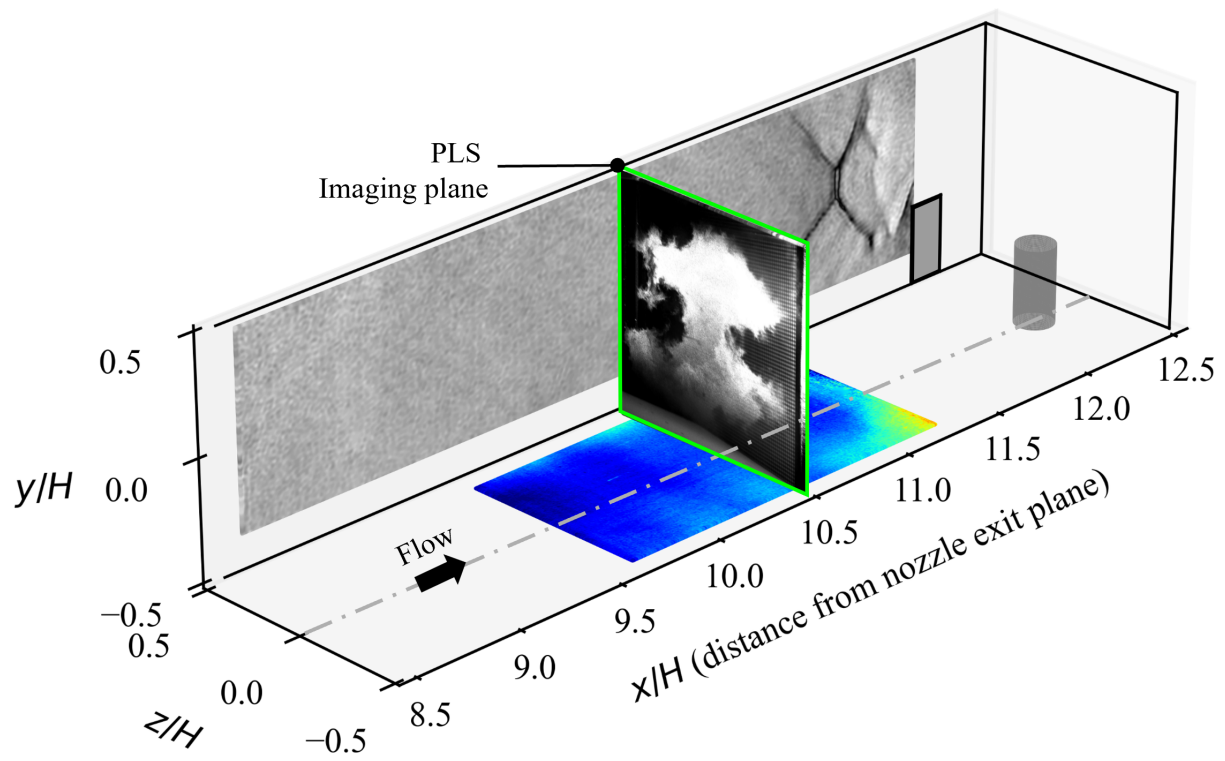
**Figure 4.3:** Instantaneous BOS (left column) and PSP (right column) images of shock train features at increasing cylinder heights. (a)  $h = 1.62\delta$ ; (b)  $h = 3.24\delta$ ; (c)  $h = 4.37\delta$ ; (d)  $h = 4.86\delta$

equal; further supporting the claim that the asymmetrical shift of the shock train towards the ceiling is an effect of the downstream asymmetric cylinder-induced shock separation.

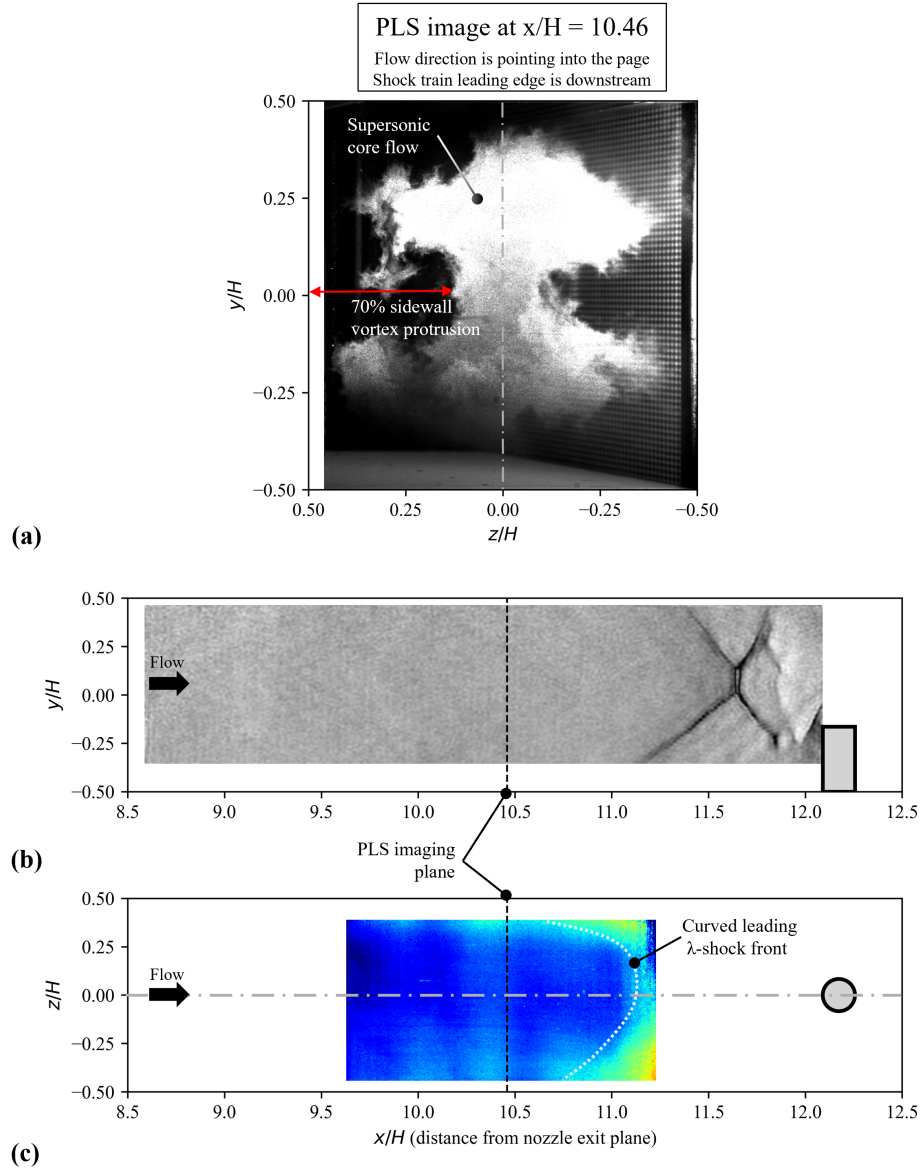
In Figures 4.6 and 4.5, the shock train leading edge is upstream of the PLS imaging plane. The result is a substantial reduction in the core flow area - as indicated by the smaller illuminated  $\text{CO}_2$  particle cloud. This reduced area is indicative of the re-accelerating core flow regions that are known to exist downstream of each normal shock in the shock train. The PSP further supports this claim - showing small regions of pressure drops between regions of higher pressure.

### 4.3 Shock Train Formation Process

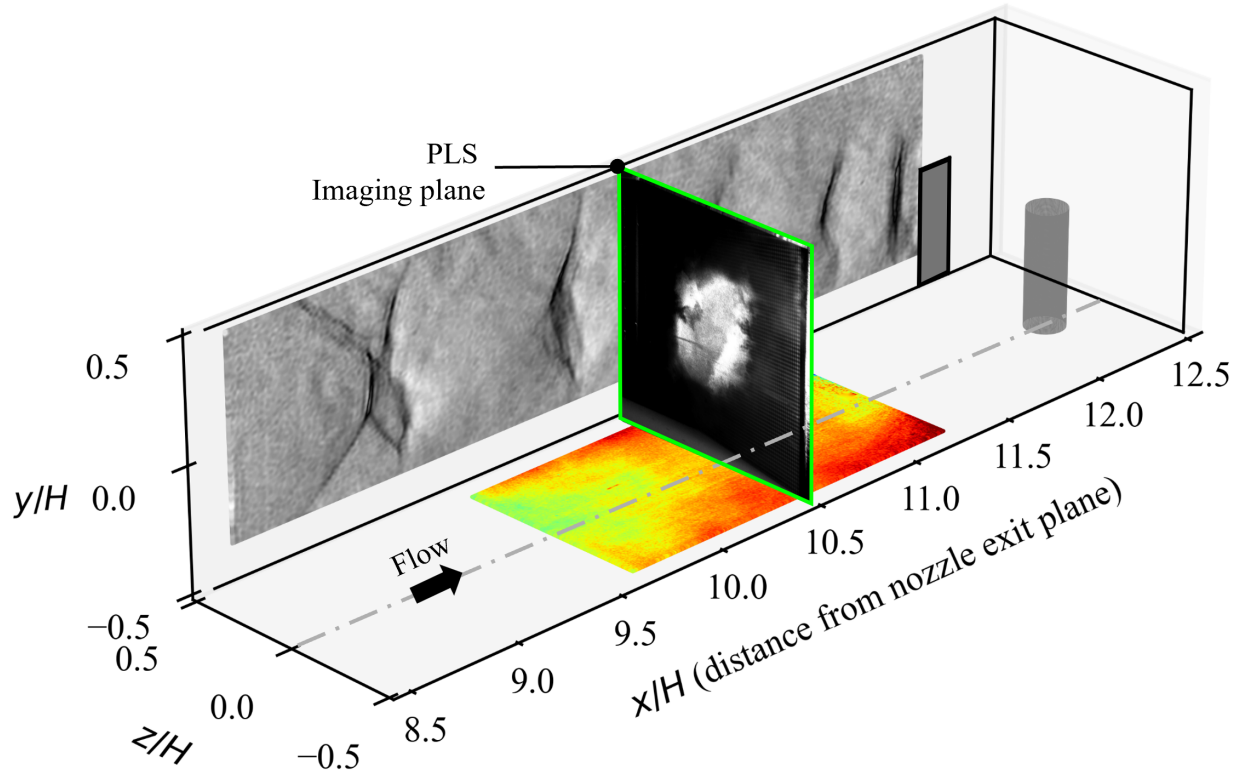
A series of BOS images acquired at increasing cylinder heights is shown in Figure 4.8. In the left-side column, the flow is left to right and dimensions are normalized by the duct height,  $H$ . The vertical dimension represents the floor-to-ceiling extent of the duct. The white areas



**Figure 4.4:** Isometric view of BOS, PSP, and PLS images at cylinder height  $h = 3.24\delta$ .

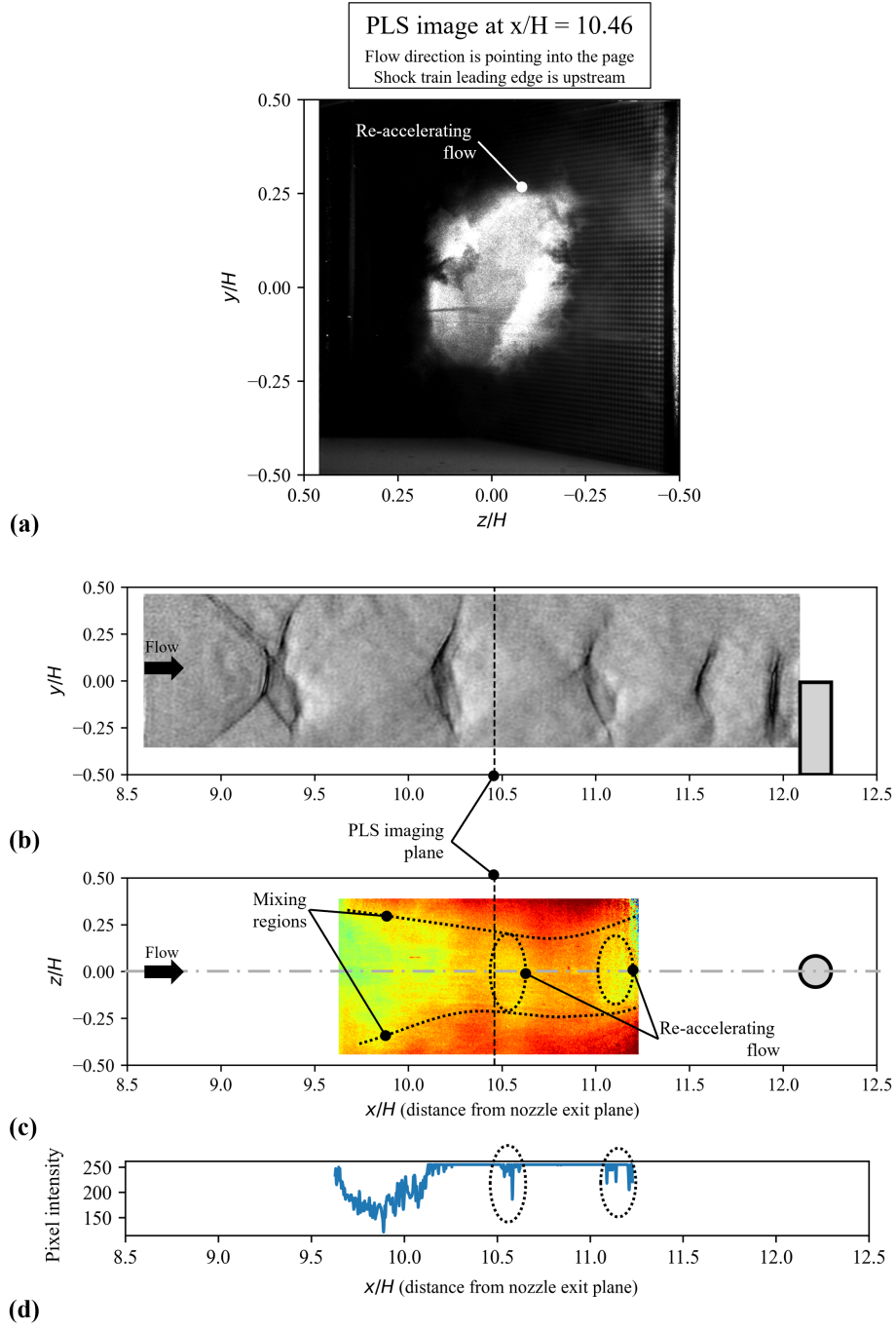


**Figure 4.5:** Three image view of (a) PLS plane; (b) BOS; and (c) PSP at cylinder height  $h = 3.24\delta$ . At this cylinder height, the shock train leading edge is downstream of the PLS imaging plane. PLS image has been un-distorted so that the view is aligned with the flow direction - normal to the laser sheet.



**Figure 4.6:** Isometric view of BOS, PSP, and PLS images at cylinder height  $h = 4.86\delta$ .



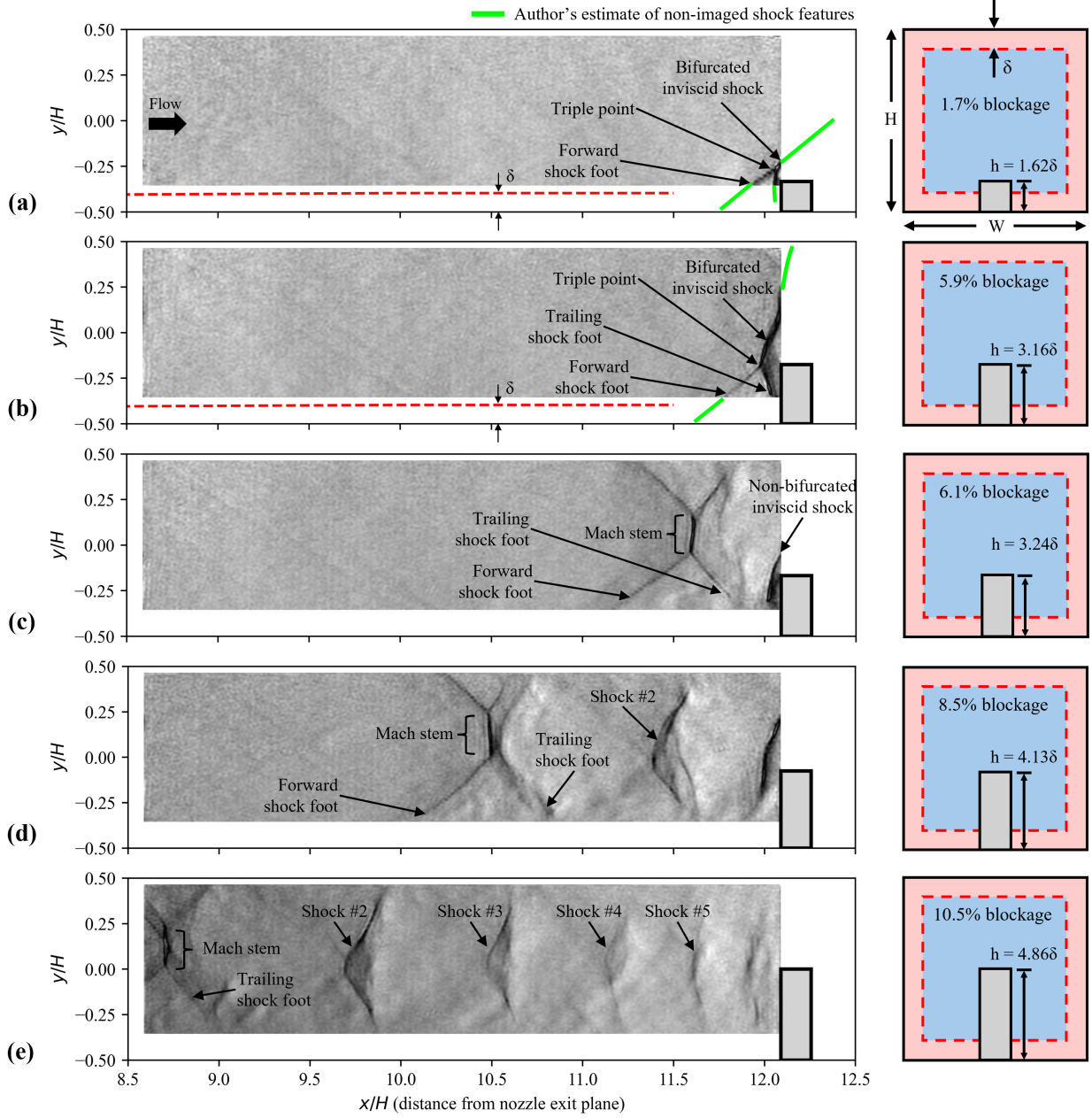


**Figure 4.7:** Three image view of (a) PLS plane; (b) BOS; and (c) PSP at cylinder height  $h = 4.86\delta$ . The centerline PSP pixel intensity is shown in (d) to highlight the re-accelerating flow regions. At this cylinder height, the shock train leading edge is upstream of the PLS imaging plane. PLS image has been un-distorted so that the view is aligned with the flow direction - normal to the laser sheet.

are where no BOS imagery was acquired or where image cropping was performed to remove saturated pixels from the reflective aluminum top and bottom walls. Greater reflection issues occurred on the bottom wall ( $y/H < -0.35$ ) due to the placement of the LEDs. The right-side column shows a simplified schematic of a spanwise (y-z) plane (flow into the page) at the cylinder centerline for each cylinder height. Core flow blockage area is reported as a percent of the total core flow area and is calculated using the estimated boundary layer thickness from [97],  $\delta$ , of the undisturbed upstream flow field and the projected frontal area of the cylinder. This approach assumes equal boundary layer profiles on the top, bottom, and side walls.

In Figure 4.8 (a) the cylinder begins to extend beyond the estimated floor boundary layer - generating an adverse pressure gradient strong enough to cause localized separation and lead to the formation of an upstream oblique compression wave, or forward shock foot. The forward shock foot intersects with the inviscid cylinder bow shock at the triple point - creating what's often called a bifurcated shock root. Dolling [71] and Price [98] noted this structure is expected to occur when the cylinder height is roughly equivalent to the triple point height.

As the cylinder height is further increased in (b), leading shock foot standoff distance increases. Beyond a height of approximately 1 cylinder diameter, the standoff distance plateaus at a maximum value of 3 cylinder diameters. This is consistent the maximum normalized distance of 2.65 noted by Westkaemper [70]. The forward shock foot extends farther upstream, and the free end of the shock downstream of the triple point begins to turn more normal to the flow - extending toward the top wall. Further increasing the cylinder height causes the top wall boundary layer to separate from the upper bow shock impingement. This separation gives rise to a single bifurcated normal shock, as shown in (c). In addition, the inviscid cylinder shock no longer exhibits a bifurcated root due to the increased boundary layer thickness downstream of the leading bifurcated shock. Additional increase in back-pressure produces the beginning of a shock train of increasing length and shock count, as shown in (d) and (e). The cylinder-induced boundary layer separation region gives rise to asymmetric boundary layer profiles on the ceiling and floor - with the

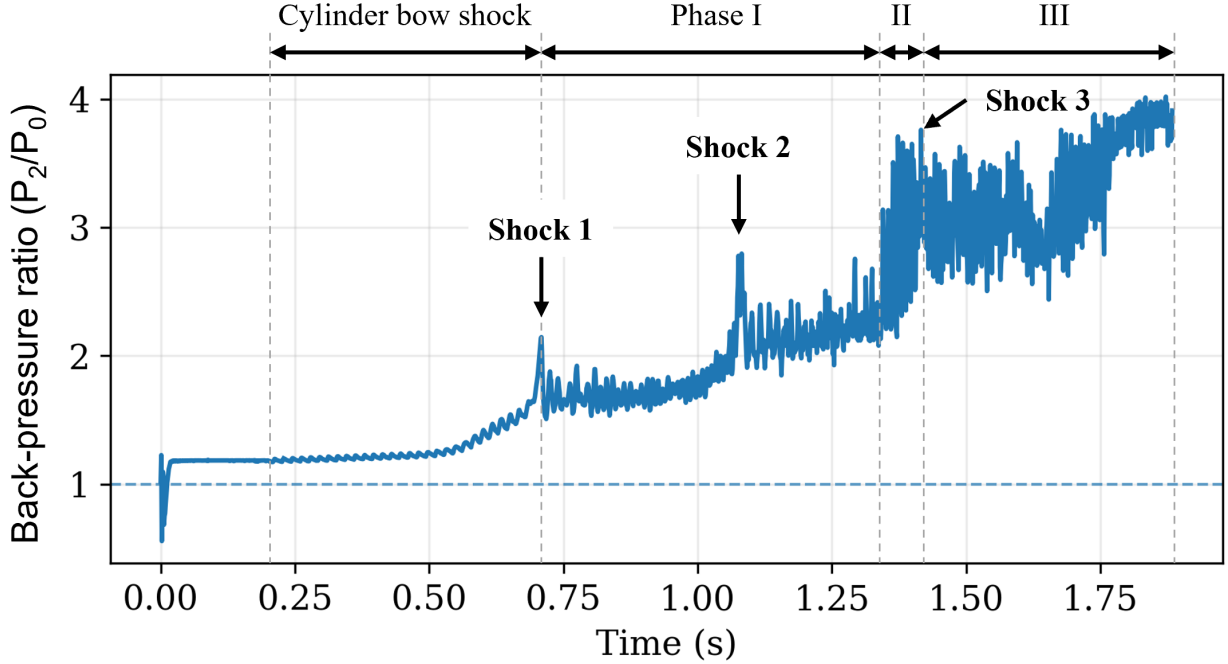


**Figure 4.8:** BOS images of shock features at increasing cylinder heights. (a)  $h = 1.62\delta$ ; (b)  $h = 3.16\delta$ ; (c)  $h = 3.24\delta$ ; (d)  $h = 4.13\delta$ ; (e)  $h = 4.86\delta$ . Boundary layer thickness,  $\delta$ , of the undisturbed upstream flow field from a prior CFD study [97] is overlaid as the red dashed line.

thicker floor boundary layer causing longer forward and trailing shock feet and shifting the shock train centerline towards the ceiling. The boundary layer rapidly thickens after each subsequent shock and causes a restriction of the core flow area. This restriction causes some of the post-shock subsonic flow to re-accelerate and gives rise to additional normal shocks. The reduced vertical height of the shocks 3, 4, and 5 and the reduced intensity in the BOS image is a clear indication of reduced shock strength from a reduction in local Mach number caused by thickening of the subsonic mixing region.

The shock train formation results presented here offer a clear distinction from those seen in other studies, which is directly attributed to the coupled cylinder-induced back-pressuring method. The three-dimensional blockage generated by the cylinder uniquely interacts with the duct walls, leading to distinct boundary layer separation events and the formation of additional shocks within the shock train structure. Furthermore, the substantial length of the facility isolator ( $\sim 13H$ ) provides a unique opportunity to fully visualize the complete phases of shock train formation before the facility nozzle unstarts.

Analysis of results from the computational study performed by Stokes [97] is provided here to provide further insight on the shock train formation process. Due to differences in boundary layer state between simulation and experiment, the results are not compared for shock train position similarity at corresponding cylinder heights. Instead, the results are used primarily to form insights on significant separation events driving the shock train upstream propagation. A time history of the back-pressure ratio during cylinder insertion is shown in Figure 4.9. The simulation is initialized with a cylinder height of  $1.62\delta$ . Cylinder motion begins at 0.20 s and follows a constant speed of 38.1 mm/s. The initial pressure rise is caused by the increasing width and height of the cylinder bow shock. The formation of the first shock,  $S_1$ , begins the start of the first propagation phase, called Phase I. Phase I contains the formation of shocks  $S_1$  and  $S_2$  and exhibits a gradual pressure rise overall. Some time after the formation of shock  $S_2$ , a rapid pressure rise occurs over a period of  $\sim 0.1$  s along with the formation of shock  $S_3$ . This event is designated as Phase II behavior and indicates significant upstream propagation of the shock train. Phase III is characterized by



**Figure 4.9:** Back-pressure ratio time history during cylinder insertion from computational study by Stokes [97]

the formation of shock  $S_3$ , significant back-pressure ratio unsteadiness, and a more gradual pressure rise.

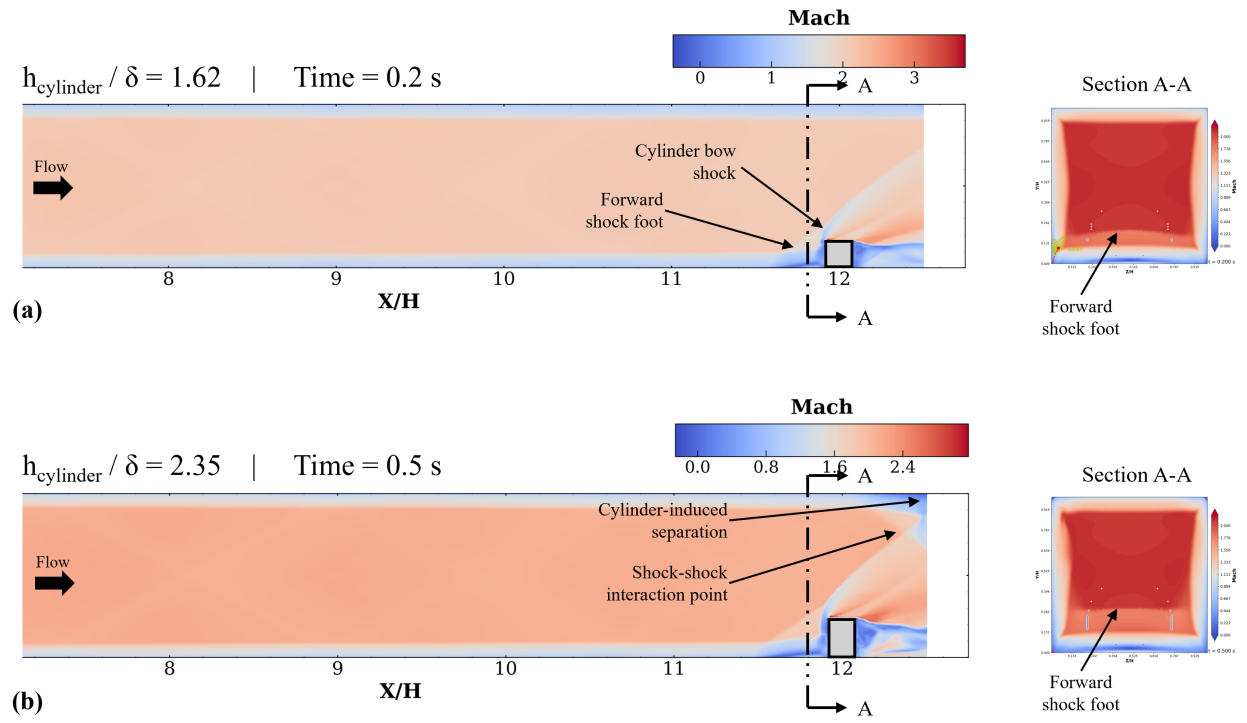
Additional insight was obtained on the driving mechanism for each phase by evaluating the Mach number contour results from [97] shown in Figures 4.10 – 4.13. Each figure contains a streamwise (x-y plane) and spanwise (y-z plane) view of key features that are exhibited in the different phases. The cylinder bow shock phase shown in Figure 4.10 shows the initial time (a) and a time just before the onset of shock  $S_1$  formation. In (b) the cylinder bow shock can be seen impinging on the top wall causing a localized separation region to form. This in turn generates an upstream compression shock that intersects with the top of the bow shock. Additional thickening of the top wall separation region causes the intersection point to propagate upstream until it is disgorged upstream of the cylinder plane forming the first shock  $S_1$ . During Phase I (Figure 4.11), the upstream propagation of shock  $S_1$  is accompanied by separated boundary layer regions in each of the four corners, as shown by the spanwise view (Section A-A). The top, bottom, and sidewall boundary layers appear to

remain attached just upstream of the cylinder face. Further increasing the cylinder height drives the formation of shock  $S_2$ . The spanwise view in (b) shows swelling of the bottom and sidewall boundary layers with recirculating regions that begin to appear - indicating the onset of separation. In Phase II (Figure 4.12), the top, bottom, and sidewall boundary layers separate just upstream of the cylinder, as shown in the spanwise view. The passage of shock  $S_3$  can also be seen as it propagates upstream. This phenomena correlates with the substantial increase in back-pressure ratio displayed in Figure 4.9 and is believed to be the root cause of the rapid upstream propagation of the shock train. The rapid thickening of the separated region restricts the core flow area causing a further reduction in Mach number and a corresponding increase in static pressure, which in combination drive the rapid upstream propagation. The remaining back-pressure rise displayed in Phase III (Figure 4.13) is believed to be primarily a result of increased blockage area from the cylinder and enhanced mixing of the subsonic boundary region with the diminishing low-supersonic core flow region. This behavior is similar to what would be expected from a decoupled back-pressuring approach.

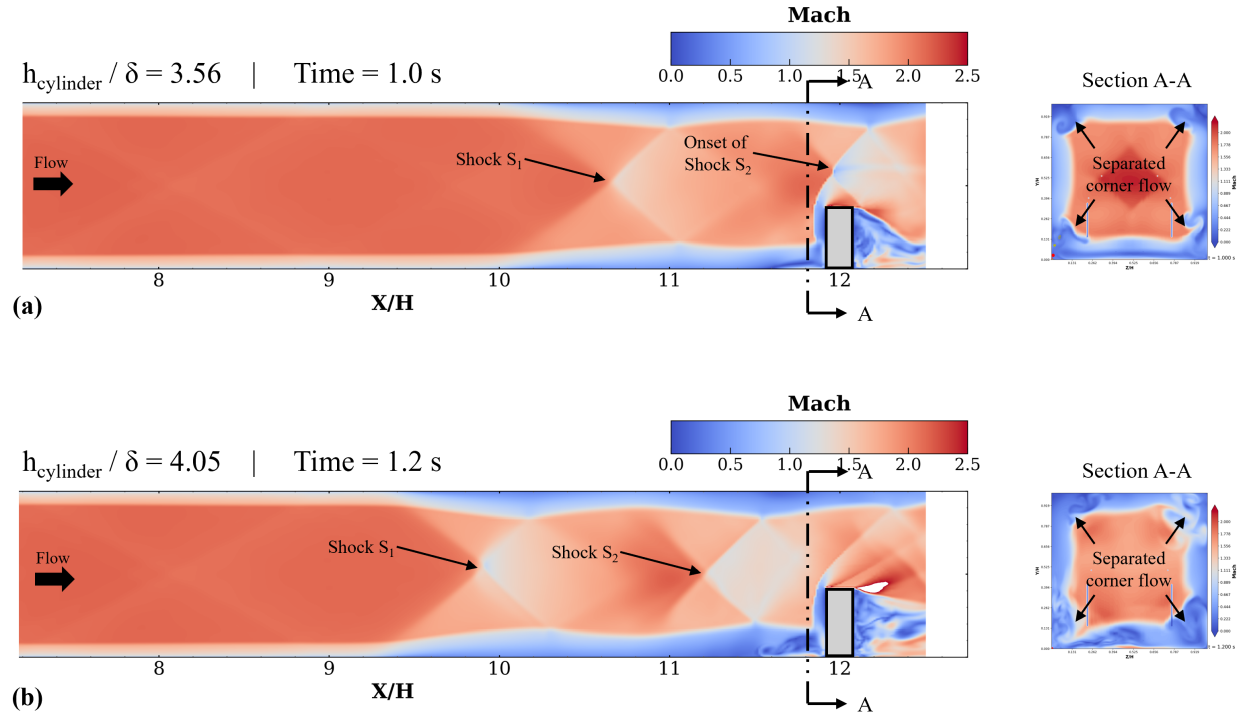
## 4.4 Response to Cylinder Back-Pressuring

The quasi-static motion of the shock train response to cylinder back-pressuring is assessed in this section. Here, the term quasi-static motion refers to the large time-scale response of the shock train motion driven by the cylinder forcing function and is used as a way to differentiate from the small time-scale unsteadiness, which is addressed in the next chapter. Image-based detection was performed on the BOS imagery and was used to track each shock in the shock train as a function of time. The cylinder height as a function of time was also known, since the cylinder was inserted into the flow at a constant vertical speed of 38.1 mm/s. This allowed for the tracked shock motion to be directly correlated with a specific cylinder height.

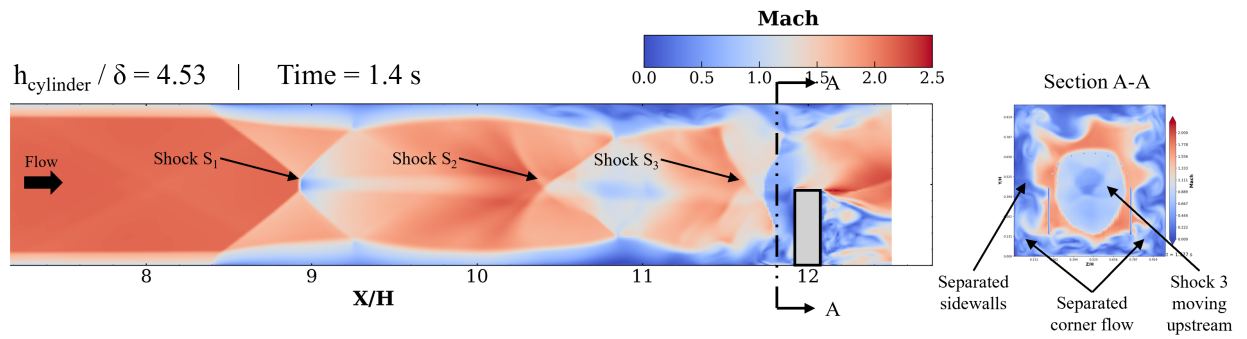
Time histories of a cylinder insertion run and a cylinder extraction run are shown in Figure 4.14 (a) and (b), respectively. In (a), the cylinder began at a height of  $1.62\delta$ . BOS acquisition began at 2.5 s and cylinder motion began at 2.7 s. Shock tracking results are



**Figure 4.10:** Time-accurate Mach number contours of the cylinder bow shock phase at (a) 0.2 s and (b) 0.5 s; from computational study by Stokes [97]

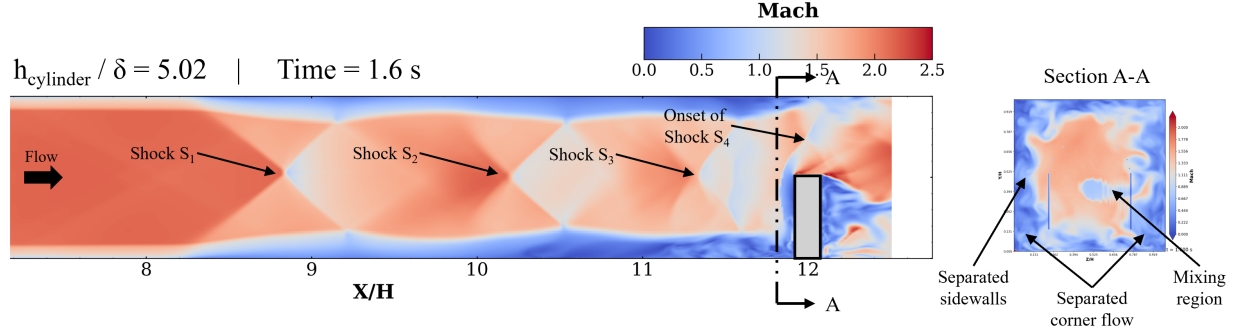


**Figure 4.11:** Time-accurate Mach number contours of Phase I at (a) 1.0 s and (b) 1.2 s; from computational study by Stokes [97]



**Figure 4.12:** Time-accurate Mach number contour of Phase II at 1.4 s; from computational study by Stokes [97]





**Figure 4.13:** Time-accurate Mach number contour of Phase III at 1.6 s; from computational study by Stokes [97]

shown for shocks  $S_1 - S_4$  starting from the time of first formation. The shock positions were tracked until the cylinder reached the max setpoint height of  $4.86\delta$ . Similar results are shown in (b) for a cylinder extraction run. For this run, the cylinder began at a height of  $4.86\delta$  and was extracted out of the flow until the height reached the minimum setpoint of  $1.62\delta$ . BOS acquisition was used to track shocks  $S_1 - S_4$  during the run until the shocks were no longer visible.

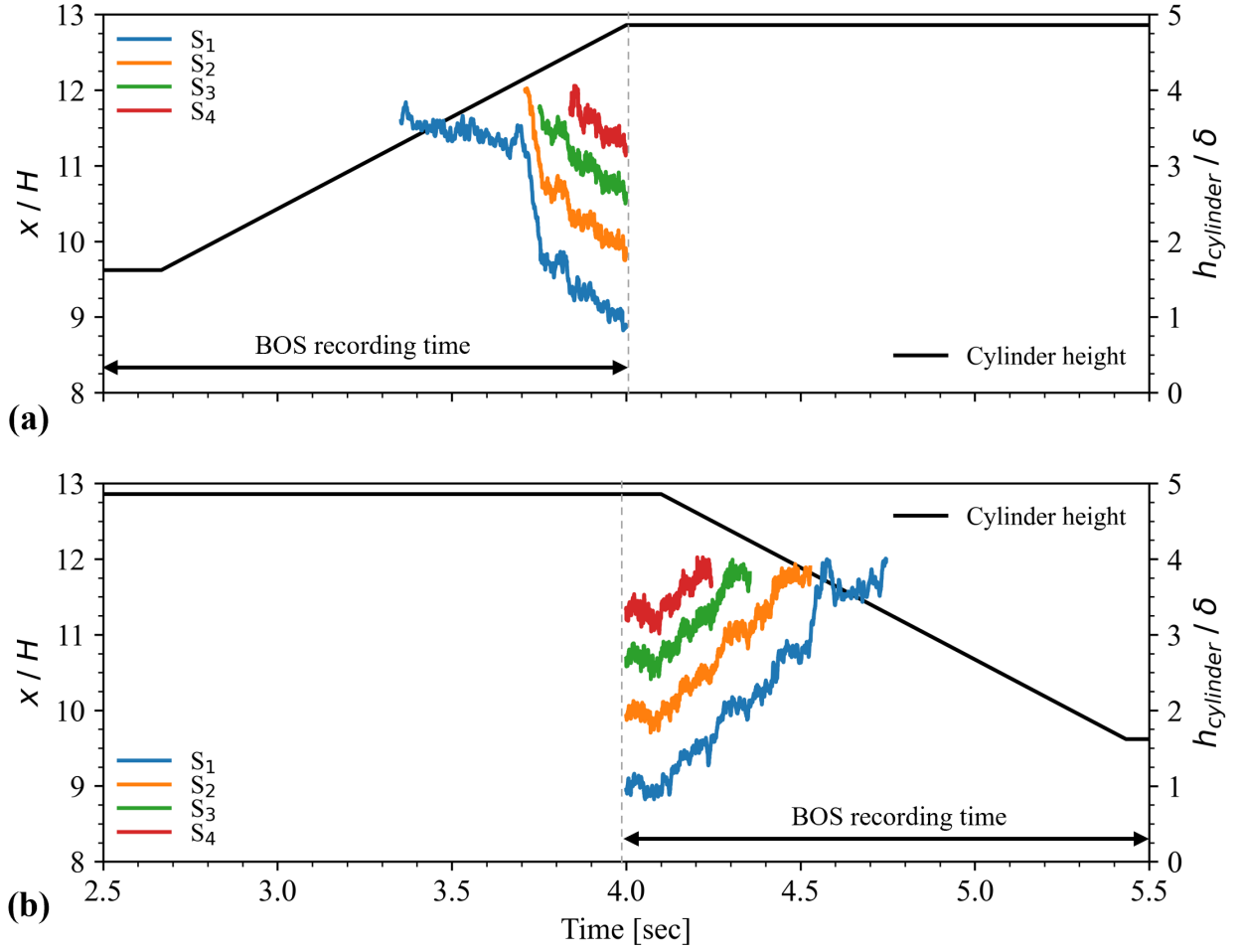
Figure 4.15 presents the same run data, but plotted as a function of cylinder height rather than time. This alternative presentation allowed for a better correlation of shock features across different runs at matching cylinder positions. The most noticeable trend across both data sets is the multi-phase response of the shocks during cylinder motion. The proposal introduced in the previous section of a three-phase shock train response is evaluated here by applying phase labels based on visual inspection of the average shock propagation slope. In (a), the first bifurcated normal shock,  $S_1$  appears at  $x = 12H$  when the cylinder height reaches  $\sim 2.9\delta$ . It continues to propagate upstream at a nearly constant rate until  $h_{cylinder} = 4.13\delta$ . This is consistent with Phase I behavior. The second shock,  $S_2$ , appears at the onset of rapid upstream motion; propagating at nearly the same rate as  $S_1$ . The rapid upstream motion occurs over a period of 0.1 s and is initiated at the formation of  $S_2$ . This fits the criteria defined previously for Phase II behavior. For  $h_{cylinder} > 4.28\delta$ , the shock train enters Phase III behavior indicated by the formation of shocks  $S_3$  and  $S_4$  form and a nearly constant propagation rate shared across all four shocks. In addition, the constant

separation distances between subsequent shocks in Phase II suggests rigid-body motion of the shock train. Edelman [19] noted this type of rigid-body-like response when investigating the forced response of a shock train to 1.5 Hz periodic back-pressuring in a constant area duct at Mach 2.

In (b), the cylinder height starts at  $4.86\delta$  and is retracted from the flow; reducing the back-pressure over time. At maximum height, the shock train begins in Phase III and remains in Phase III as the cylinder height is reduced. Shocks  $S_3$  and  $S_4$  disappear at approximately the same cylinder height as they appeared in (a). However, Phase III in (b) does not end with the disappearance of shocks  $S_3$  and  $S_4$ . Instead, shock  $S_2$  continues to exhibit Phase III behavior until  $\sim 3.82\delta$  - well below the Phase II / Phase III transition point in (a). At the onset of Phase II, shock  $S_2$  disappears and shock  $S_1$  exhibits a rapid downstream propagation before entering Phase I.

Further analysis, shown in Figure 4.16, highlights the path-dependent behavior of shock  $S_1$  - often referred to as shock train hysteresis [60, 99, 100]. The difference in shock  $S_1$  location at the Phase II / Phase III transition point of each run equates to an apparent shift in cylinder height of  $\sim 0.4\delta$ . For each Phase, a least-squares linear regression was performed, and the calculated slopes were used to estimate a shock propagation speed for each shock. The results are tabulated in Table 4.2. and represent the average speed for each Phase. A max propagation speed of  $0.01 U_\infty$  (5.1 m/s) occurred during Phase II, and it was found that the estimated speed during each Phase was independent of increasing or decreasing back-pressure effects from the cylinder. Additionally, as shown in Figure 4.17, each shock in the shock train followed nearly the same Phase propagation speed - further supporting the rigid-body assumption of Edelman [19].

The observation of multiple propagation speed phases aligns with the findings of Wagner [14] and Wieting [3]. Wagner calculated an isolator shock train speed of  $0.035 U_\infty$  using a ramp positioned at the isolator exit for back-pressuring and concluded that upstream propagation was initiated by boundary layer separation due to the ramp compression shock impinging on the ceiling. This separation mechanism is relevant because the boundary layer is known to exhibit a dual-solution (bistable) state [101], meaning reattachment occurs only

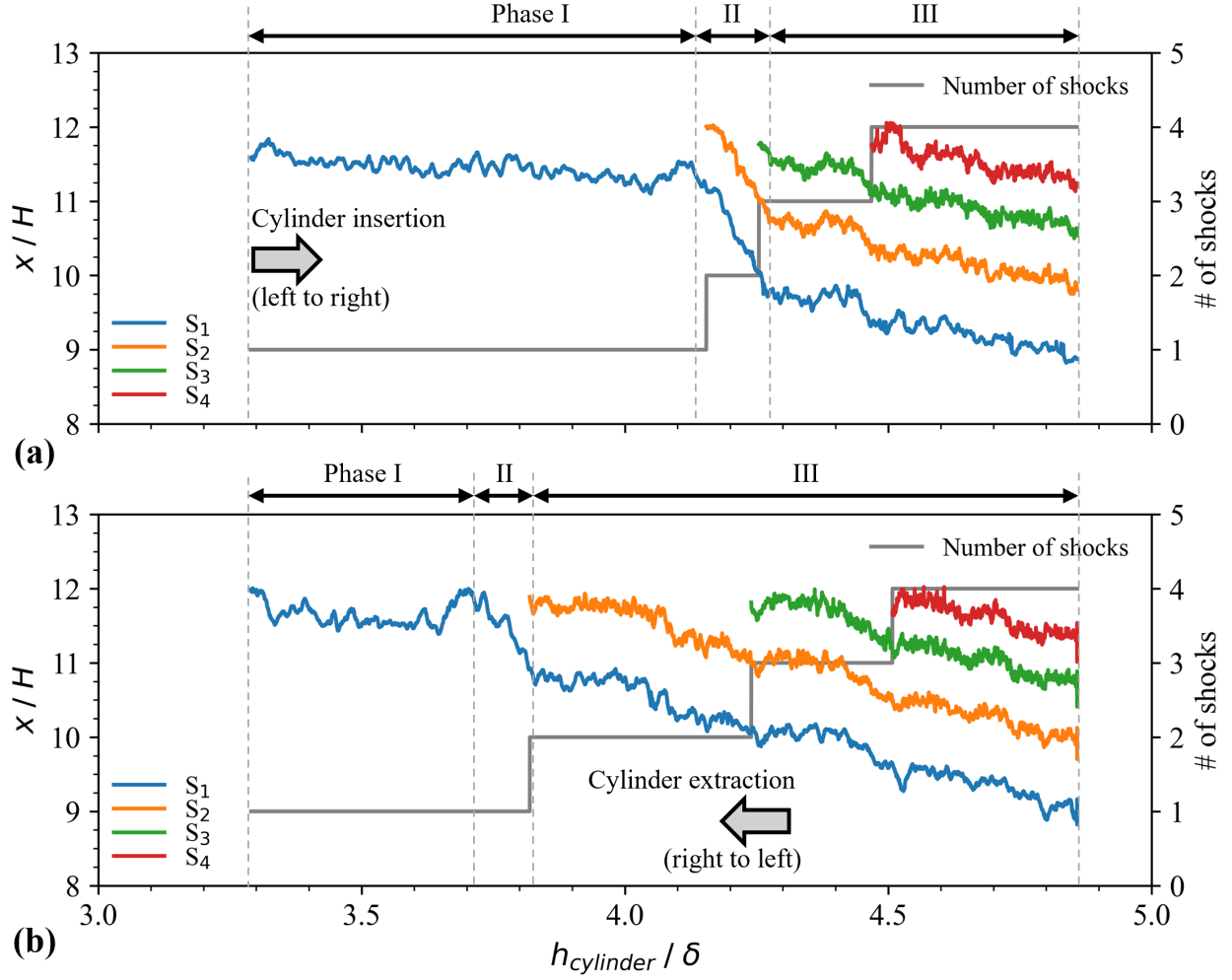


**Figure 4.14:** Shock tracking and cylinder height time histories for (a) a cylinder insertion run and (b) a cylinder extraction run.

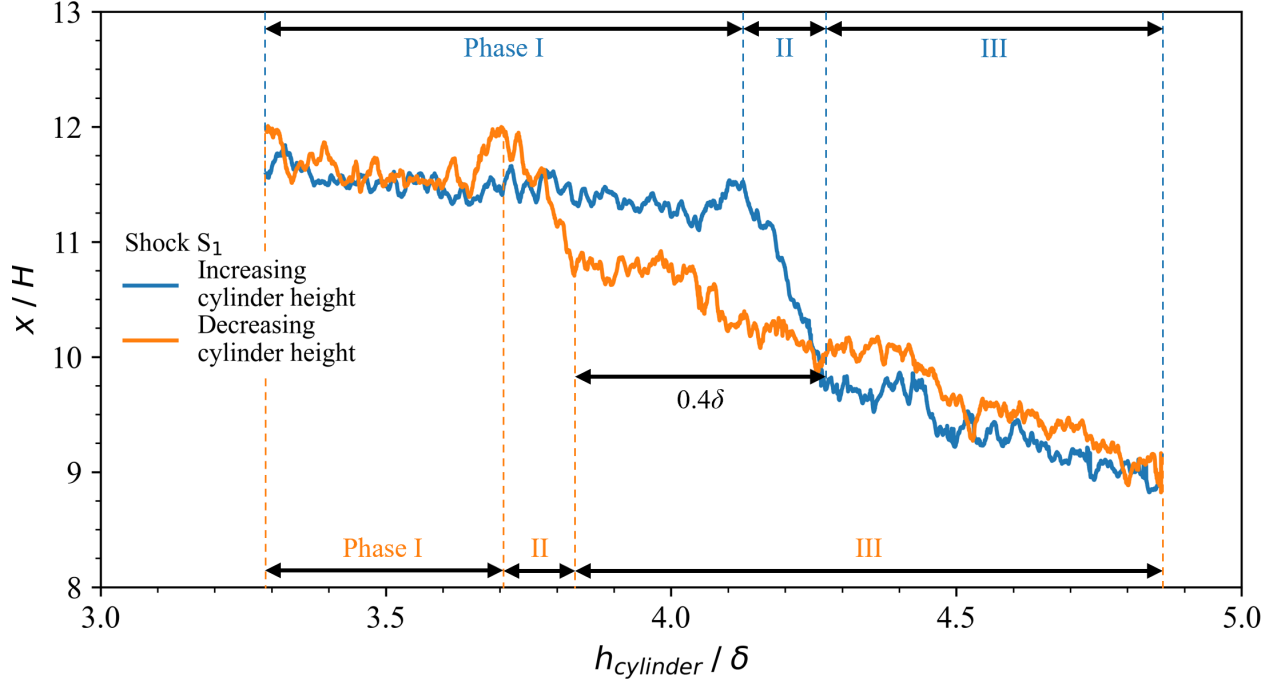
**Table 4.2:** Average speed during each shock train propagation phase.

| Propagation Phase | Average Speed*                | Units |
|-------------------|-------------------------------|-------|
| I                 | $3 \times 10^{-4} U_{\infty}$ | m/s   |
| II                | $1 \times 10^{-2} U_{\infty}$ | m/s   |
| III               | $1 \times 10^{-3} U_{\infty}$ | m/s   |

\*  $U_{\infty} = 507.64$  m/s



**Figure 4.15:** Shock tracking results as function of cylinder height for (a) a cylinder insertion run and (b) a cylinder extraction run. The cumulative sum of shocks in the shock train during each run is shown as the solid gray line.

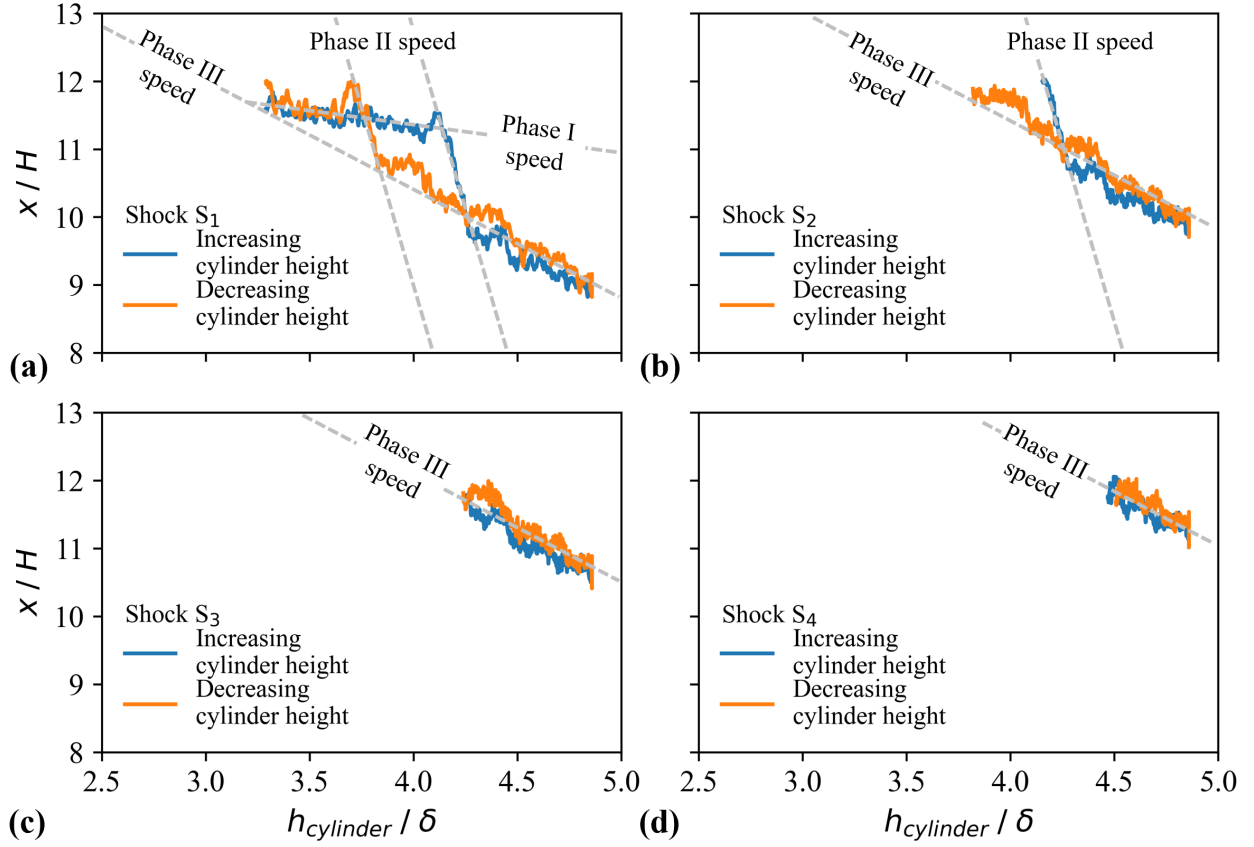


**Figure 4.16:** Leading shock,  $S_1$ , hysteresis behavior.

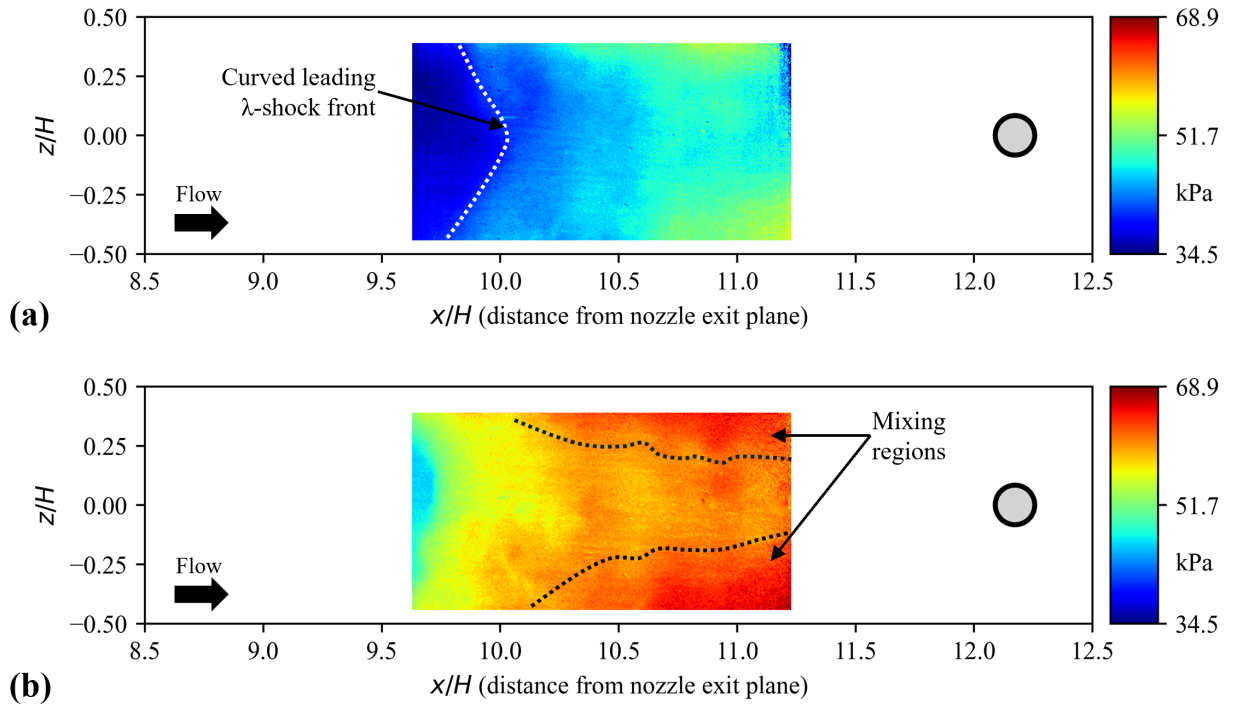
after a significant reduction in the adverse pressure gradient. Wieting observed an upstream propagation speed of 10 m/s to occur during unstart of a Mach 5.3 inlet, which was believed to be dictated by the upstream rate of boundary layer separation.

The PSP results shown in Figure 4.18 provide additional evidence supporting this dual-solution concept. Panels (a) and (b) compare images from the upward and downward cylinder sweeps, respectively, at an identical cylinder height. Panel (a) shows the upstream shock train leading edge with a largely supersonic core flow area downstream, whereas Panel (b) reveals a large mixing region of low momentum sidewall flow and a distinctly different core flow static pressure. This substantial difference in shock location and static pressure suggests that the downstream boundary layer remains separated during the cylinder extraction, indicating the existence of a dual-mode boundary layer state solution.

The shock train exhibits a distinct multi-phase response during cylinder back-pressuring, a behavior that was successfully verified using both the current experimental data set and the computational results from Stokes [97]. Analysis of the computational work revealed that this response is directly caused by distinct boundary layer separation events—triggered



**Figure 4.17:** Individual responses of (a) Shock  $S_1$ ; (b) Shock  $S_2$ ; (c) Shock  $S_3$ ; and (d) Shock  $S_4$  with constant propagation speed lines overlaid.



**Figure 4.18:** Comparison of PSP floor pressure during (a) an increasing cylinder height run and (b) a decreasing cylinder height run at an equivalent cylinder height of  $4.2\delta$ . Top view looking down.

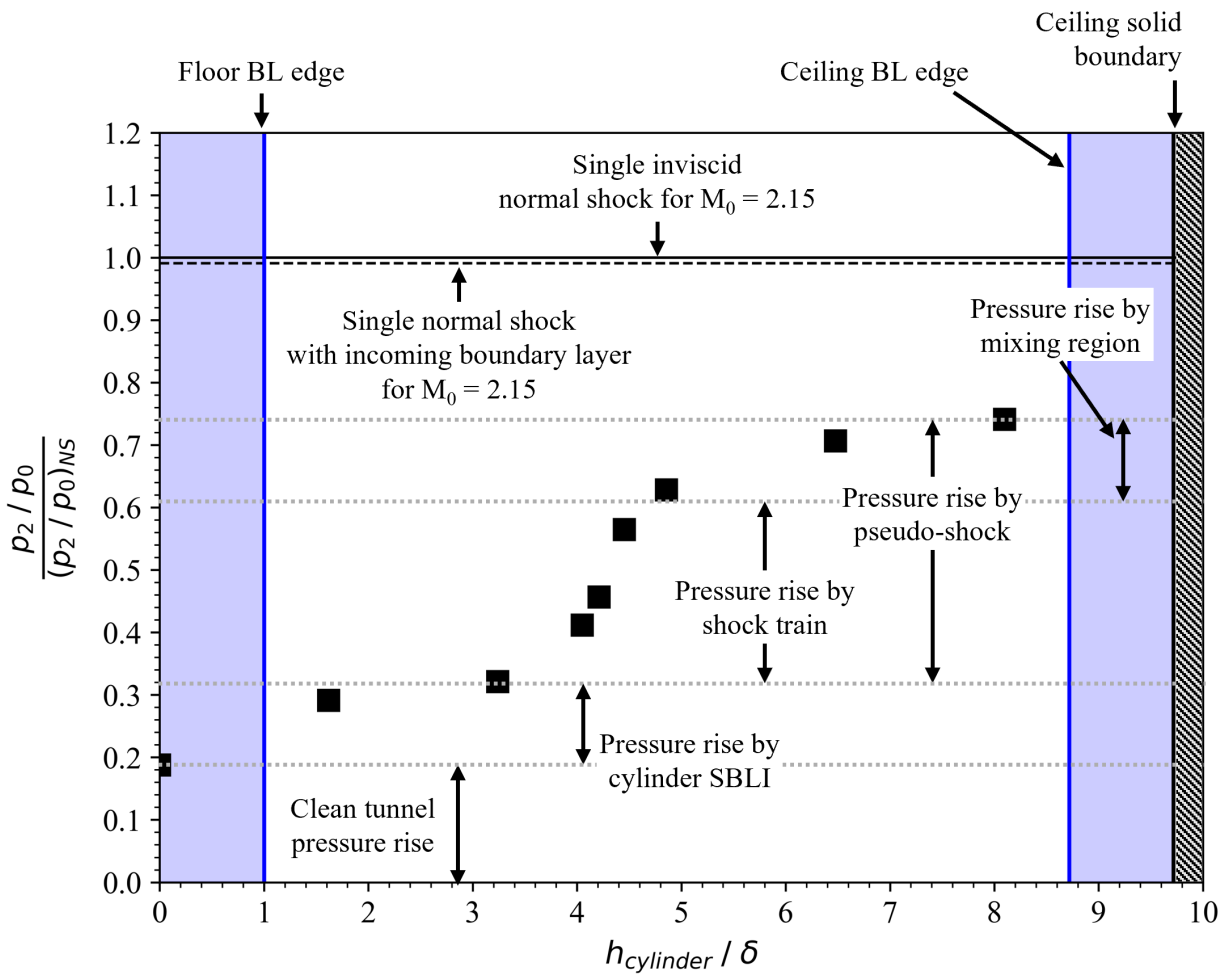
by the impingement of the three-dimensional cylinder bow shock—which lead to separate shock train propagation phases. The similarity in propagation behavior between this work and the computational model provides sufficient evidence to conclude that similar separation events are the fundamental cause of the unique multi-phase dynamics observed here.

## 4.5 Shock Train Pressure Rise and Overall Length

The calculated pressure rise across the shock train is presented in Figure 4.19 as a function cylinder height normalized by the local boundary layer thickness just upstream of the cylinder face during clean tunnel operation,  $h_{cylinder}/\delta$ . Consistent with other studies on this topic, the pressure rise produced by the shock train was defined as the ratio of the downstream pressure just before the cylinder face,  $p_2$  to the pressure just upstream of the shock train leading edge,  $p_0$ . This ratio was then normalized by the theoretical static pressure rise across a single inviscid normal shock  $(p_2/p_0)_{NS}$  at the freestream Mach number just upstream of the shock train leading edge,  $M_0$ . For this work, pressure port P37 ( $x = 11.71H$ ) was used as the downstream pressure. The upstream port location varied depending on the shock train leading edge location. Freestream conditions at any axial location in the tunnel were interpolated from the baseline flow conditions calculated by Stokes [97].

As shown in Figure 4.19, the pressure rise due to increasing cylinder height spans several different flow states. At lower cylinder heights,  $h_{cylinder} < 4\delta$ , only the cylinder lambda shock was present; accounting for nearly 24% of the total pressure rise generated over the baseline tunnel pressure ratio. For  $4\delta \leq h_{cylinder} < 4.5\delta$ , the back-pressure is sufficiently high enough to generate a shock train in test section 2. This flow state accounts for nearly 55% of the total pressure rise. The sharp pressure rise is a result of the normal shock train growing in both axial length and number of shocks. For  $h_{cylinder} > 4.5\delta$ , the flow state is indicative of a pseudo-shock system, where the back-pressure continues to rise gradually in the subsonic/supersonic mixing region downstream of the shock train and reaches maximum of 74% of a normal shock pressure rise.

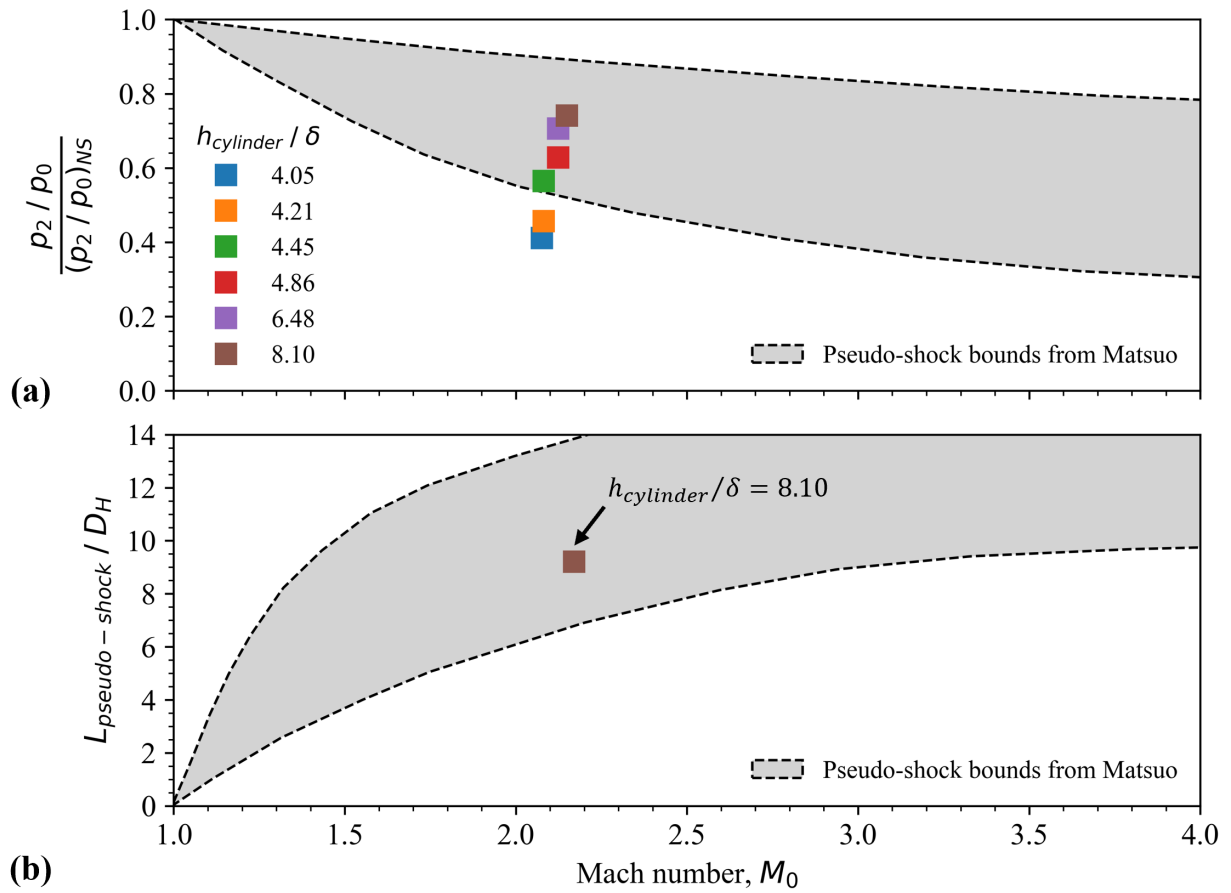




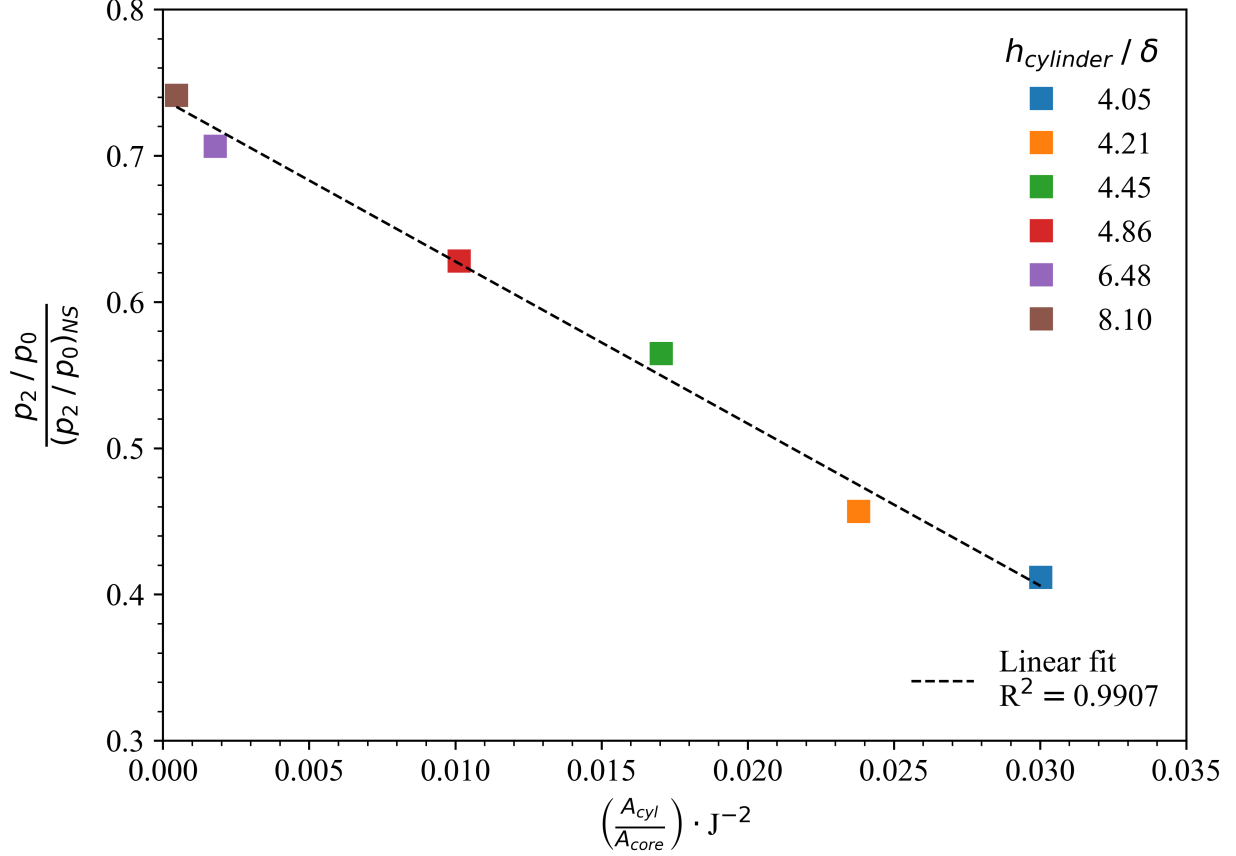
**Figure 4.19:** Measured static pressure rise normalized by the pressure rise from a single normal shock as a function of cylinder height. Bounds are shown for a single inviscid normal shock and a single normal shock with incoming boundary layer (McLafferty [89]) as the solid and dashed black lines, respectively. Boundary layer estimates are based on Stokes’ work [97].

A combination of the BOS imagery and the static pressure taps on the floor were useful in determining whether the flow state was a full pseudo-shock or a shock train by itself. Matsuo [4] compiled data from several studies and generated empirical bounds for the pressure rise and length of a typical pseudo-shock based on the upstream freestream Mach number,  $M_0$ . The experimentally derived pressure rise and pseudo-shock length from this work are presented in Figure 4.20. In (a) the pressure rise from varying cylinder heights are plotted versus upstream freestream Mach number. Both the pressure rise and upstream Mach number increase with increasing cylinder height. This is to be expected since the increased back-pressure forces the shock train leading edge upstream where freestream Mach number is greater. Cylinder heights less than  $\sim 4.5\delta$  fell outside the bounds of a typical pseudo-shock and were deemed to consist of only a normal shock train system. In (b) the pseudo-shock length was determined from the schlieren imagery. Due to the lack of optical access in between test sections 1 and 2, only the length from the most upstream pseudo-shock condition could be evaluated. The length at this condition was estimated to be  $\sim 9.2H$  - consistent with the empirical bounds. It is common for the length to be non-dimensionalized by the duct hydraulic diameter,  $D_H$ , as shown in (b). For this work, since the tunnel has a rectangular cross-section with an aspect ratio of 1, the duct height and hydraulic diameter are used here interchangeably.

Here, a method for linearizing the shock train/pseudo-shock pressure rise is proposed based on the simulated jet-to-freestream momentum flux ratios reported in the previous chapter. For back-pressures producing at least one bifurcated normal shock, a change of variables was performed to convert the normalized cylinder height into a function of jet momentum flux ratio scaled by the cylinder blockage ratio. The non-linearity present in the pressure rise was effectively collapsed to a linear trend when plotted against the inverse square of the simulated jet momentum flux ratio,  $J^{-2}$ . The results of this approach are shown in Figure 4.21. A linear fit of the data, given in Eq. 4.1, was performed; resulting in a coefficient of determination of  $R^2 = 0.9907$ . The general trend shows a linear increase in static pressure rise across the shock train/pseudo-shock system for increasing jet momentum



**Figure 4.20:** Comparison of measured (a) static pressure rise normalized by the pressure rise from a single normal shock; and (b) Non-dimensional pseudo-shock length as a function of freestream Mach number,  $M_0$ , upstream of the shock train leading edge. Bounded regions in gray represent empirical bounds determined by Matsuo [4].



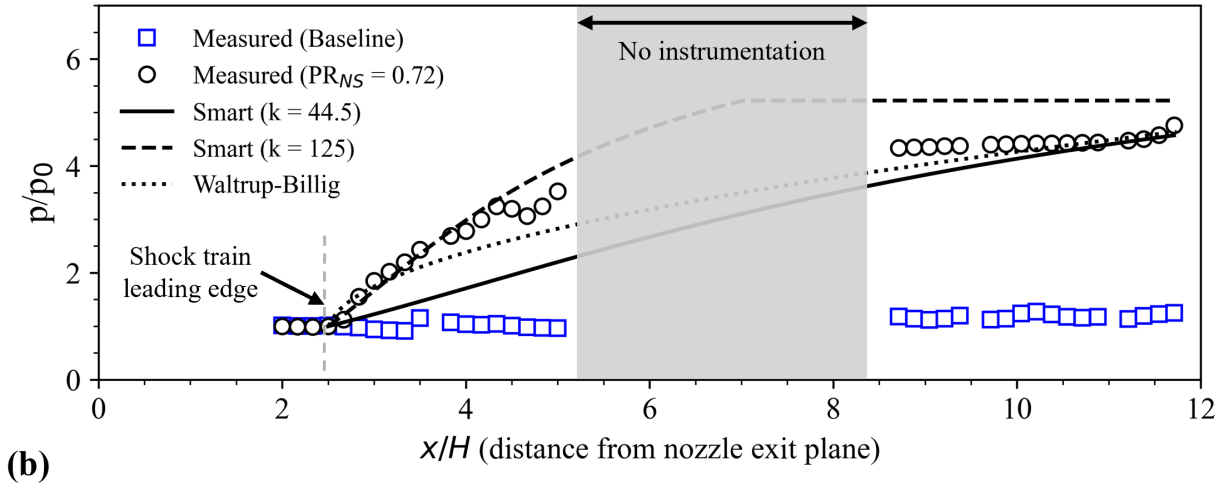
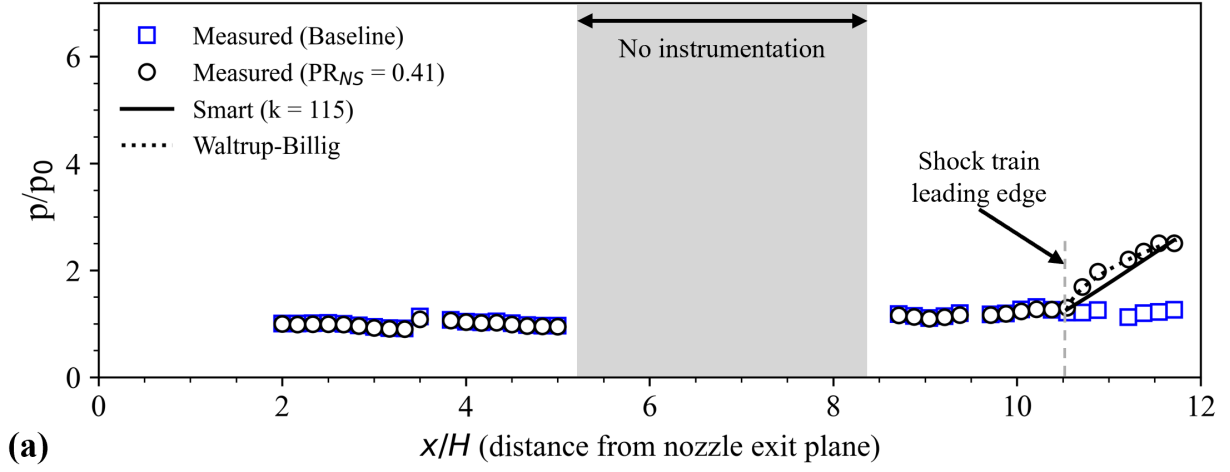
**Figure 4.21:** Measured static pressure rise normalized by the pressure rise from a single normal shock as a function of cylinder blockage ratio,  $A_{cyl}/A_{core}$ , and simulated jet momentum flux ratio,  $J$ . Momentum flux ratio estimates were based on the Rothstein and Wantuck [88] correlation, as defined in Table 3.1.

flux ratios. Although this model does not directly account for the effects of the cylinder-induced boundary layer separation phases, the necessary use of a non-linear change of variable to achieve a linear trend provides compelling indirect evidence of their influence.

$$\frac{p_2/p_0}{(p_2/p_0)_{NS}} = -11.08 \left[ \left( \frac{A_{cyl}}{A_{core}} \right) \cdot J^{-2} \right] + 0.74 \quad (4.1)$$

A comparison of the static pressure pressure profile measured along the tunnel floor centerline to the well-established Waltrup-Billig [13, 90] and Smart [91] isolator model predictions are shown in Figure 4.22. Results from the most downstream and most upstream shock leading edge positions are plotted as black circles in (a) and (b), respectively. For

perspective, the baseline pressure profiles during clean tunnel operation are plotted as blue squares. In (a), both models show good agreement with the overall pressure rise. The Waltrup-Billig correlation, which utilizes only the upstream flow conditions, does an excellent job of capturing the curvature throughout the pressure rise. The Smart model predicts the overall pressure rise, but fails to capture the initial high-gradient region at the shock leading edge. This is likely a result of the condensed shock train region, which only consists of a single bifurcated normal shock, and the absence of a monolithic, separated boundary layer assumed by the pressure gradient closure model. In (b), the high-back pressure forces the shock train upstream to  $x = 2.5H$  - resulting in a fully developed pseudo-shock. Again, the predicted overall pressure rise from the Waltrup-Billig model shows excellent agreement. However, it is limited in its monotonic pressure rise assumption, which does not properly account for the complex fluid dynamics downstream of the initial pressure rise. The Smart model is shown in (b) with two different empirical constants for the Orwerth pressure gradient closure model ( $k = 44.5$  and  $k = 125$ ). As noted in [20], tuning of  $k$  can be used to bound particular features of the predicted pressure rise. For  $k = 125$ , the pressure gradient closure model is heavily weighted, allowing the initial high-gradient region at the shock train leading edge to be captured and supporting the assumption of a fully separated boundary layer in the upstream region. Downstream of the  $x = 4.5H$  axial station, the model diverges from the measure data, indicating reattachment of the local boundary layer in the experimental data. Setting  $k = 44.5$  essentially extends separated flow region assumed by the pressure diffusion model and causes the pressure to rise more gradually; eventually matching the total pressure rise at the end of the duct. The choice of  $k$  varies widely across the literature. Orwerth [92] and Smart [91] reported  $k$  values of 44.5 and 50.0, respectively, and Edelman [20] found that  $k$  values of 15 and 35 provided adequate bounds for low-fidelity design considerations.



**Figure 4.22:** Measured pressure profile comparison with Smart and Waltrup-Billig pressure rise models at cylinder heights of (a)  $h = 4.05\delta$  and (b)  $h = 8.10\delta$

# Chapter 5

## Characterization of the Shock Train Unsteadiness

### 5.1 Unsteadiness of Tracked Shock Features

Characterization of the shock train unsteadiness was evaluated by analyzing the image-based shock tracking results during constant back-pressure conditions (i.e. a fixed cylinder height). For each back-pressure condition, the test procedure was to (1) bring the tunnel on-condition with no cylinder, (2) inject the cylinder at a rate of 38.1 mm/sec, (3) hold the cylinder at a fixed height while acquiring simultaneous BOS and PSP imagery of the shock train motion. Image-based shock tracking of each BOS and PSP frame was performed post-test and used to support additional investigation.

The results from a test run with cylinder injection up to  $4.13\delta$  are shown in Figure 5.1. Time histories of the (a) BOS shock tracking results, (b) PSP shock tracking results, (c)  $K_1$  pressure transducer response, and (d)  $K_2$  pressure transducer response are shown during cylinder injection (Time  $< 1.8$  [sec]) and while the cylinder height is fixed (Time  $> 1.8$  [sec]). The upstream propagation of the shocks can be seen by following the pressure transducer traces in (c) and (d). As the cylinder is injected, the transducer closest to the cylinder ( $K_2$  in (d)) experiences an instantaneous pressure rise from the leading shock foot,  $S_{1f}$ , crossing,

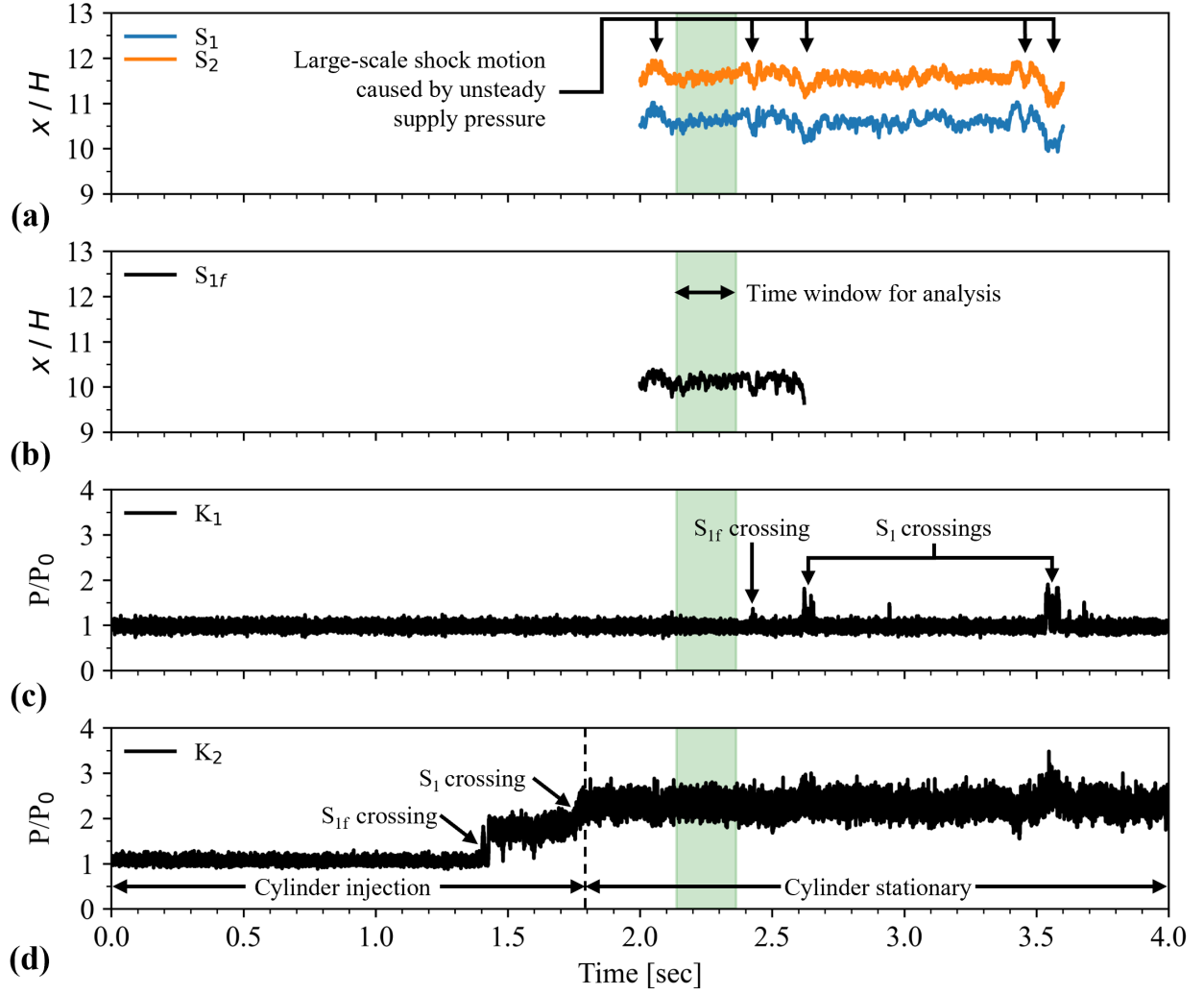
followed by another pressure rise from the mach stem crossing,  $S_1$ . The upstream pressure transducer,  $K1$ , does not experience any pressure rise, as the leading shock foot remains downstream at this cylinder height.

The tracking results in (a) and (b) reveal both large and small-scale motions during the stationary cylinder period. The large-scale motions were found to be a result of fluctuations in the stilling chamber supply pressure caused by an unpredictable lag response of the main control valve. The smaller-scale motions in the tracking results represent the phenomena of interest, the inherent shock train unsteadiness. A windowed-region of the data, represented by the green box in Figure 5.1, was extracted during a period of constant supply pressure to use for subsequent analysis.

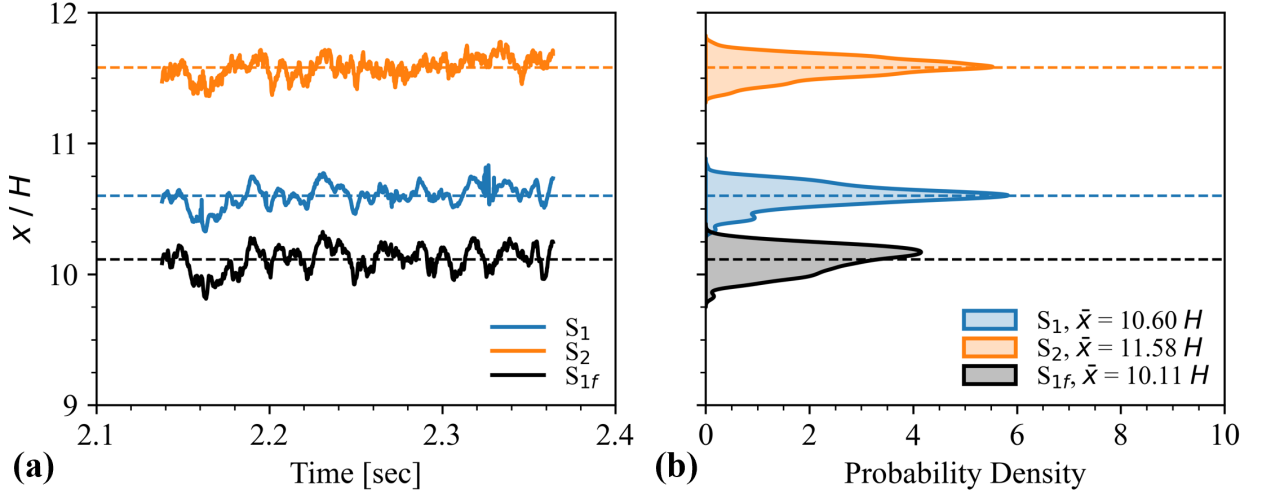
The results of the clipped shock tracking results along with probability density histograms are shown in Figure 5.2 (a) and (b), respectively. A summary of the key statistics is given in Table 5.1. In general, there is a near-synchronous motion between all the shock features, and the distances between subsequent shocks remains nearly constant - further supporting the claim of a rigid-body like response. The  $\sim 1H$  distance between shock features  $S_1$  and  $S_2$  is consistent with other studies [10]. Additionally, the tracked shock features exhibit a stochastic behavior with some noted periodicity. Leading shock features,  $S_1$  and  $S_{1f}$ , are dominated by high amplitude, lower frequency excursions from the mean, while the second shock,  $S_2$ , exhibits higher frequency, small scale motions. The increased variance of the leading shock foot,  $S_{1f}$ , in comparison to the leading shock Mach stem,  $S_1$ , suggests a greater sensitivity near the boundary layer region than the core flow region. This behavior is attributed to the high-degree of sensitivity of the leading shock foot to inflow disturbances, such as upstream boundary layer properties [7, 8] and Mach number [5, 18]. Downstream of the leading shock, the boundary layer has rapidly thickened; making it less susceptible to upstream disturbances and effectively acting as a low-frequency damping mechanism for the second shock.

Similar results for a cylinder height of  $4.86\delta$  are shown in Figures 5.3 and 5.4. At this condition, a four-shock shock train was generated, and the leading shock was at the upstream extent of the BOS field of view. This limited tracking of the leading shock features to just the





**Figure 5.1:** Time histories of (a) BOS shock tracking results; (b) PSP shock tracking results; (c)  $K_1$  pressure transducer response; and (d)  $K_2$  pressure transducer response during a cylinder injection up to  $h_{cylinder} = 4.13\delta$ .

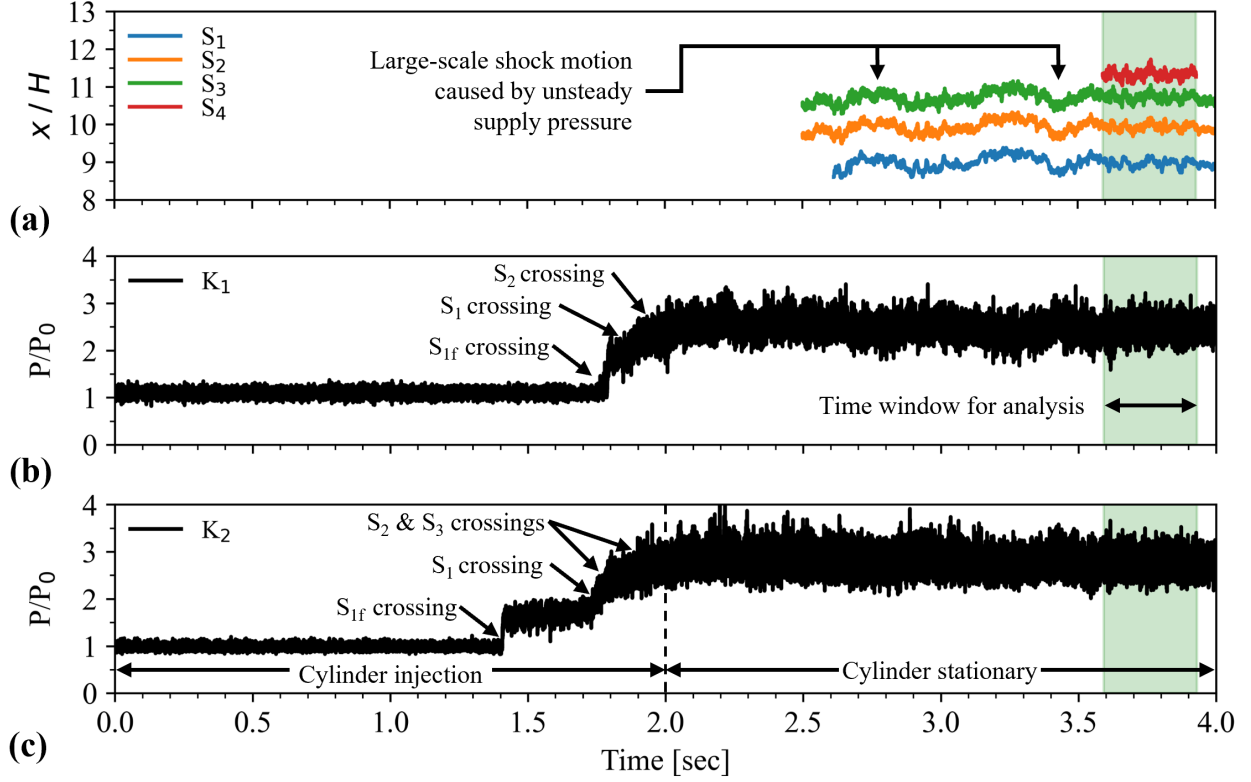


**Figure 5.2:** Shock tracking results at a fixed cylinder height,  $h_{cylinder} = 4.13\delta$ . (a) Time history of shocks  $S_1$  and  $S_2$  from BOS imagery and leading shock foot,  $S_{1f}$ , from PSP imagery; (b) Probability density histograms of tracking results. Sample means are drawn as dashed lines.

**Table 5.1:** Statistics of tracked shock features at  $h_{cylinder} = 4.13\delta$

| Statistic          | Tracked Shock Feature |          |          |
|--------------------|-----------------------|----------|----------|
|                    | $S_{1f}$              | $S_1$    | $S_2$    |
| Mean               | $10.11H$              | $10.60H$ | $11.58H$ |
| Standard Deviation | $0.10H$               | $0.08H$  | $0.08H$  |
| Skewness           | -0.47                 | -0.75    | -0.34    |
| Kurtosis*          | -0.43                 | 0.41     | -0.16    |

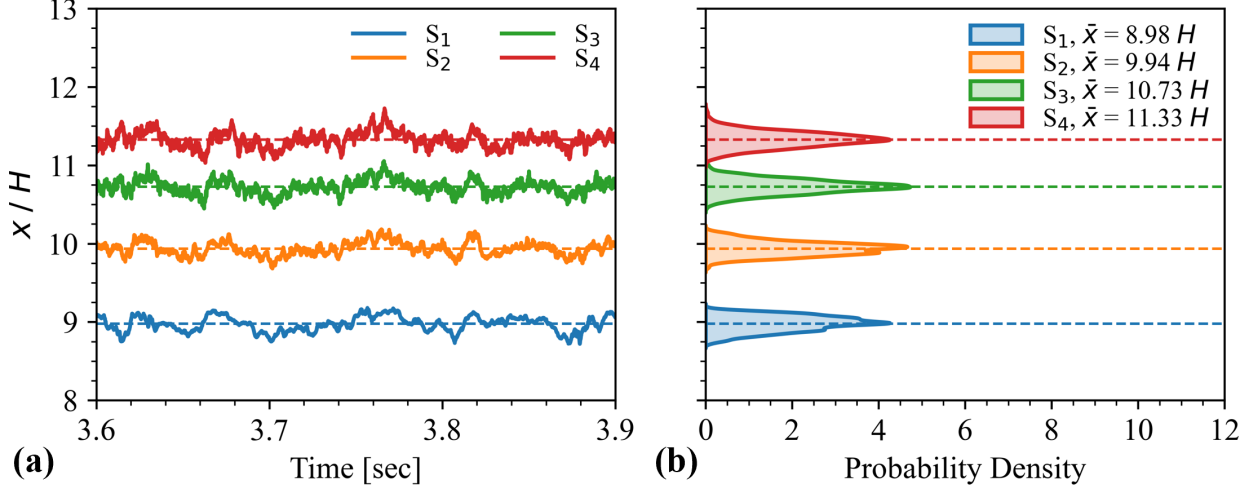
\* Normalized so a normal distribution = 0



**Figure 5.3:** Time history of (a) BOS shock tracking results; (b)  $K_1$  pressure transducer response; and (c)  $K_2$  pressure transducer response during a cylinder injection up to  $h_{cylinder} = 4.86\delta$ .

Mach stem,  $S_1$ . While shock features were visible in the PSP view at this condition, there was not enough dynamic range sensitivity to conduct image-based tracking with confidence. Similar to the  $4.13\delta$  condition, the cylinder was injected at a rate of 38.1 mm/sec. The shocks can be seen crossing the pressure transducers as the cylinder is injected ((b) and (c)). In (a) the four tracked shock features,  $S_1 - S_4$ , are shown. Image-based tracking of the fourth shock feature,  $S_4$ , proved to be difficult during portions of the run - most likely due to uneven lighting and reduced sensitivity in the downstream region of test section 2. For this reason, the data was windowed for further analysis based on two criteria: (1) time when the supply pressure was relatively constant and (2) time when tracking of  $S_4$  was successful.

Results of the windowed shock tracking results for  $4.86\delta$  are shown in Figure 5.4. The shocks exhibit a rigid-body like response, with time-synchronized motion and constant spacing between subsequent shocks. The time-averaged spacing between shocks decreases



**Figure 5.4:** Shock tracking results at a fixed cylinder height,  $h_{cylinder} = 4.86\delta$ . (a) Time history of shocks  $S_1$ ,  $S_2$ ,  $S_3$ , and  $S_4$  from BOS imagery; (b) Probability density histograms of tracking results. Sample means are drawn as dashed lines.

along the length of the shock train by  $\sim 0.2H$  each time. Additionally, the spacing between shocks  $S_1$  and  $S_2$  remains constant at  $\sim 1H$  for both cylinder height conditions - indicating that the shock spacing is independent of back-pressure. Statistics of the tracked shock features are given in Table 5.2.

For this work, a simple zero-crossing frequency analysis was performed to determine the dominant frequency component of each tracked shock feature. The zero-crossing frequency represents a time-averaged frequency by estimating the number of crossings of the signal

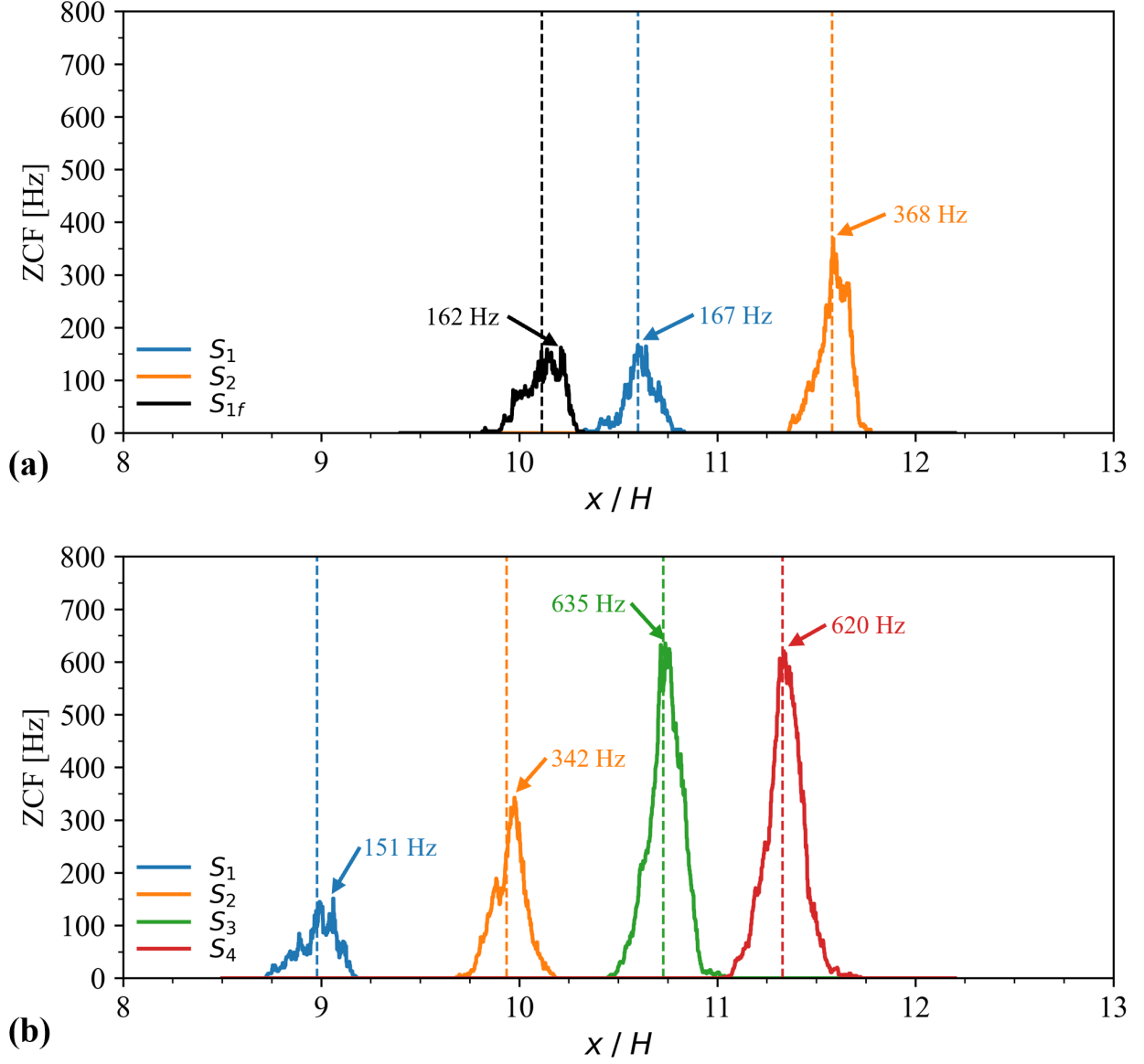
**Table 5.2:** Statistics of tracked shock features at  $h_{cylinder} = 4.86\delta$

| Statistic          | Tracked Shock Feature |         |          |          |
|--------------------|-----------------------|---------|----------|----------|
|                    | $S_1$                 | $S_2$   | $S_3$    | $S_4$    |
| Mean               | $8.98H$               | $9.94H$ | $10.73H$ | $11.33H$ |
| Standard Deviation | $0.08H$               | $0.08H$ | $0.09H$  | $0.1H$   |
| Skewness           | -0.31                 | 0.13    | 0.03     | 0.13     |
| Kurtosis*          | -0.56                 | -0.27   | 0.03     | 0.25     |

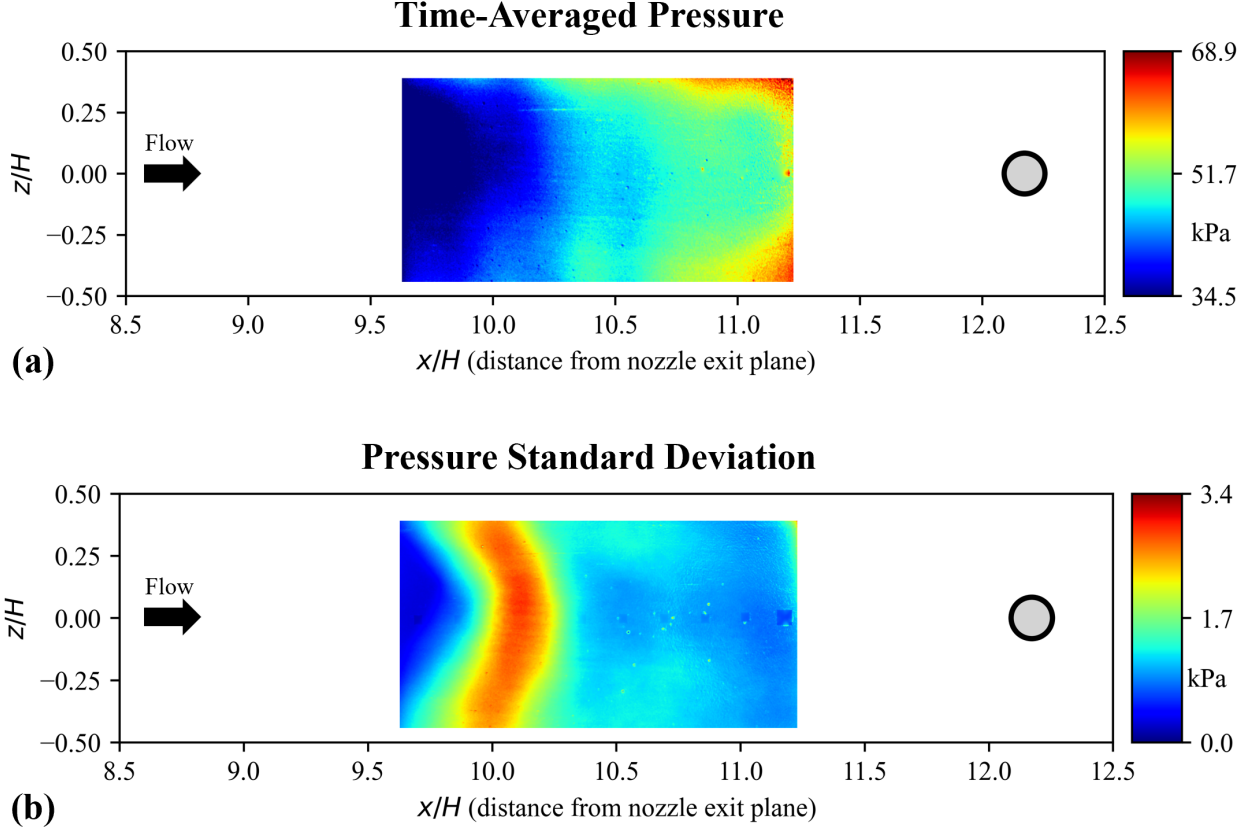
\* Normalized so a normal distribution = 0

per unit time [102], and was useful for determining each shock feature’s dominant frequency component. The zero-crossing frequency (ZCF) results for the  $4.13\delta$  and  $4.86\delta$  cylinder height conditions are shown in Figure 5.5 (a) and (b), respectively. The curves represent the average rate of signal crossings multiplied by the sampling frequency, calculated at each  $x/H$ . The dominant frequency is found by locating the peak of each distribution. In (a), the leading shock features,  $S_{1f}$  and  $S_1$ , exhibit the nearly equivalent zero-crossing frequencies at  $\sim 160$  Hz, while the second shock moves at  $\sim 2.3$  times that rate. In (b), increased back-pressure has forced shocks  $S_1$  and  $S_2$  upstream, but they maintain nearly the same zero-crossing frequencies. The dominant frequency of shock  $S_3$  is 635 Hz,  $\sim 1.8$  times the rate of  $S_2$ . From  $S_1$  to  $S_3$ , this equates to a linear frequency increase of  $\sim 2x$  Hz per  $1H$  of downstream shock separation distance. This trend, however, does not hold for downstream shock  $S_4$ . The  $S_4$  maximum crossing frequency appears to reach a plateau around 620 Hz - remaining nearly constant to  $S_3$ .

A spectral analysis of the global PSP results was also performed to analyze the shock foot unsteadiness at a fixed cylinder height of  $4.13\delta$ . First, the pixel time-averages and pixel standard deviations from 800 images (0.08 s) are presented in Figure 5.6. The mean pressure profile in (a) is similar to what has been discussed in the previous chapter. In (b), the shock foot unsteadiness is clearly visible by the red banded region. Additionally, downstream of the shock foot, the sidewalls appear to exhibit increased variance compared to the centerline flow; indicating the existence of overhead compression shocks emanating from the sidewall and converging towards the Mach stem region. The image spectral analysis was performed by applying a fast Fourier Transform to each pixel’s time history using the `welch` function in the Scipy signal processing library. Per Welch’s method [103], windowing of each pixel’s time history was performed using a Hann window consisting 512 samples and a 50% overlap. This resulted in a frequency resolution of 19.53 Hz. The resulting power spectral density  $G_{xx}$  was then normalized by each pixel’s variance. Finally, the normalized spectral density was integrated over frequency bands corresponding to the zero-crossing frequency results presented previously. Frequency bands of  $[0 - 250)$  Hz,  $[250 - 500)$  Hz, and  $[500 - 750]$  Hz were selected and the results are shown in Figure 5.7 (a), (b), and (c), respectively. As



**Figure 5.5:** Zero-crossing frequency results of tracked shock features at (a)  $h_{cylinder} = 4.13\delta$  and (b)  $h_{cylinder} = 4.86\delta$ .

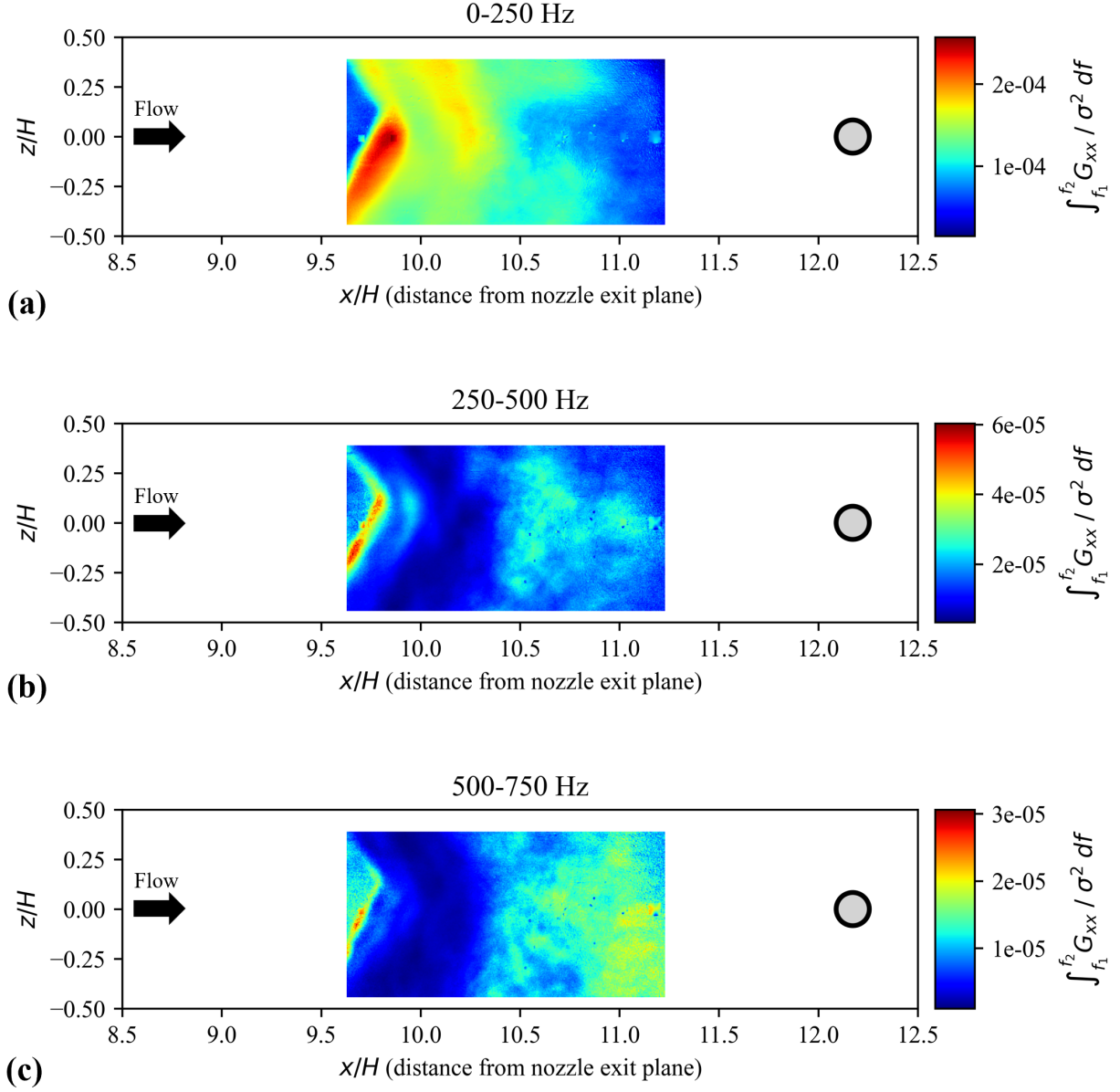


**Figure 5.6:** (a) Time-averaged pressure and (b) standard deviation of fast-response PSP results at a fixed cylinder height,  $h_{cylinder} = 4.13\delta$ .

expected, the majority of the power is contained in the  $[0 - 250)$  Hz frequency band around the shock foot location ( $x = 9.7H - 10.3H$ ). This is consistent with the calculated crossing frequency of 162 Hz. Higher frequency content is exhibited in the trailing shock foot region of influence ( $x = 10.5H - 11.3H$ ) but at a significantly lower power than the dominant unsteadiness in the  $[0 - 250)$  Hz frequency band. This is believed to be a result of localized boundary layer separation that is known to exist downstream of the first shock [43].

The zero-crossing frequency analysis yielded two key insights regarding shock train unsteadiness:

**(1) Frequency Independence from Back-Pressure:** The dominant crossing frequency of each shock appears to be independent of the cylinder-induced back-pressure. Specifically, shocks  $S_1$  and  $S_2$  maintain their preferred crossing frequencies regardless of their axial



**Figure 5.7:** PSP spectral heat maps for the (a) 0 – 250 Hz, (b) 250 – 500 Hz, and (c) 500 – 750 Hz frequency bands at a fixed cylinder height,  $h_{cylinder} = 4.13\delta$ .



location in the tunnel, a frequency independence similar to that noted by Hunt [10].

**(2) Frequency Dependence on Local Boundary Layer State:** The dominant crossing frequency of each shock can be directly attributed to the three propagation phases described previously, which are defined by the local boundary layer state.

- **Upstream of  $S_1$ :** The flow remains attached and undisturbed, providing a constant upstream condition for  $S_1$ .
- **Between  $S_1$  and  $S_2$ :** Local separation occurs only in the four corners, while the main ceiling, floor, and sidewall boundary layers remain attached.
- **Between  $S_2$  and  $S_3$ :** Boundary layer instability from the separated corner flow has caused the remaining ceiling, floor, and sidewall boundary layers to locally separate.
- **Downstream of  $S_3$ :** The boundary layer remains locally separated around the duct perimeter and the supersonic core flow area is significantly diminished.

This suggests that the dominant crossing frequency of each shock in the shock train is fundamentally dependent on the local upstream and downstream boundary layer states. The near-identical crossing frequencies of shocks  $S_3$  and  $S_4$  are therefore explained by the fact that both shocks exist in a region defined by similar upstream and downstream boundary layer states.

## 5.2 Identification of the Unsteadiness Propagation Mechanism

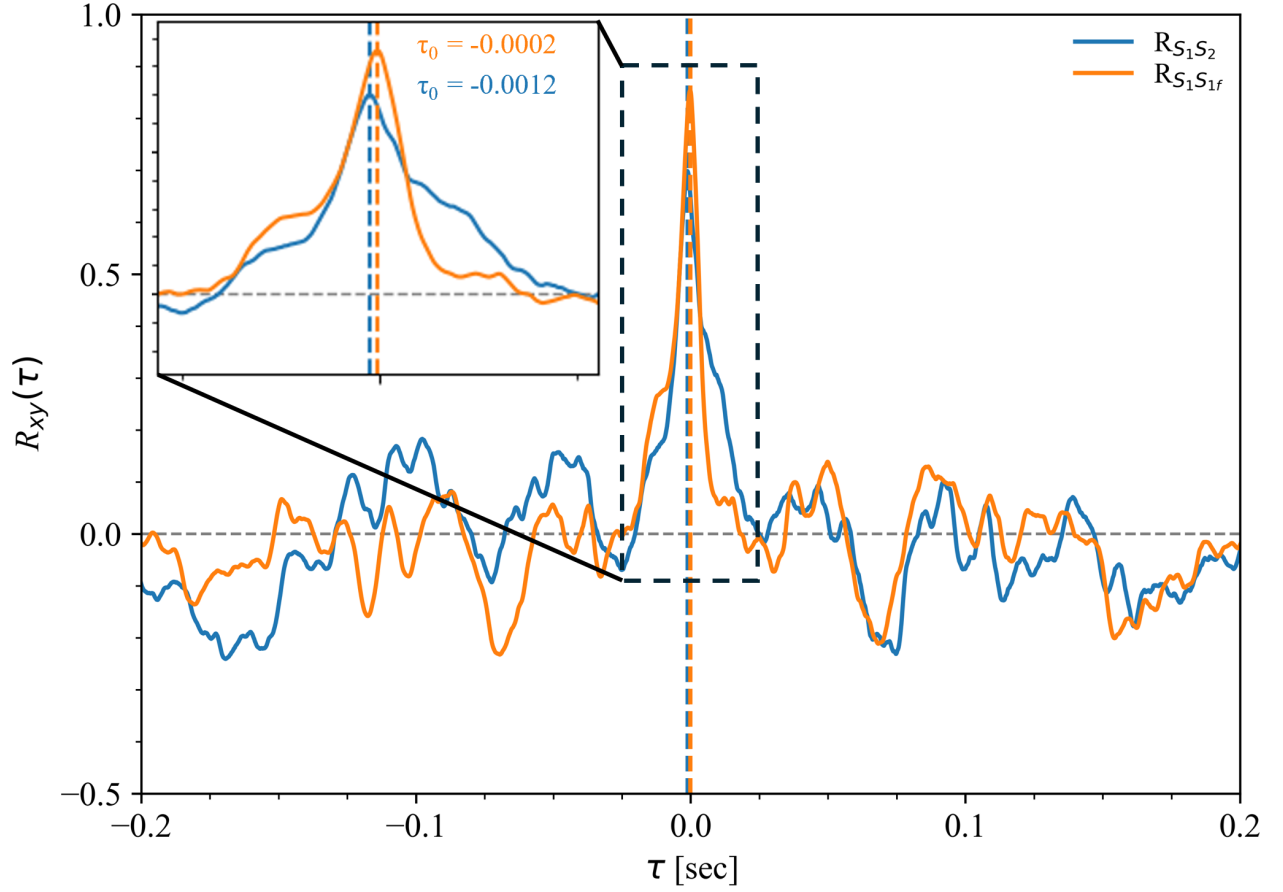
A cross-correlation analysis was performed to examine the temporal order of shock feature motion and identify the mechanism driving the unsteadiness. The cross-correlation provides a way to determine the temporal correlation between two signals based on their joint probability density function - independent of the frequency domain. The cross-correlation between two signals  $x$  and  $y$ ,  $R_{xy}(\tau)$  is computed as a function of the delay variable,  $\tau$ , which

is used to temporally shift the second signal relative to the first. The estimated time delay between the two signals,  $\tau_0$  is taken as the location of maximum correlation. A negative time delay ( $\tau_0 < 0$ ) indicates that signal  $x$  leads signal  $y$ . A positive time delay ( $\tau > 0$ ) indicates that signal  $x$  lags signal  $y$ .

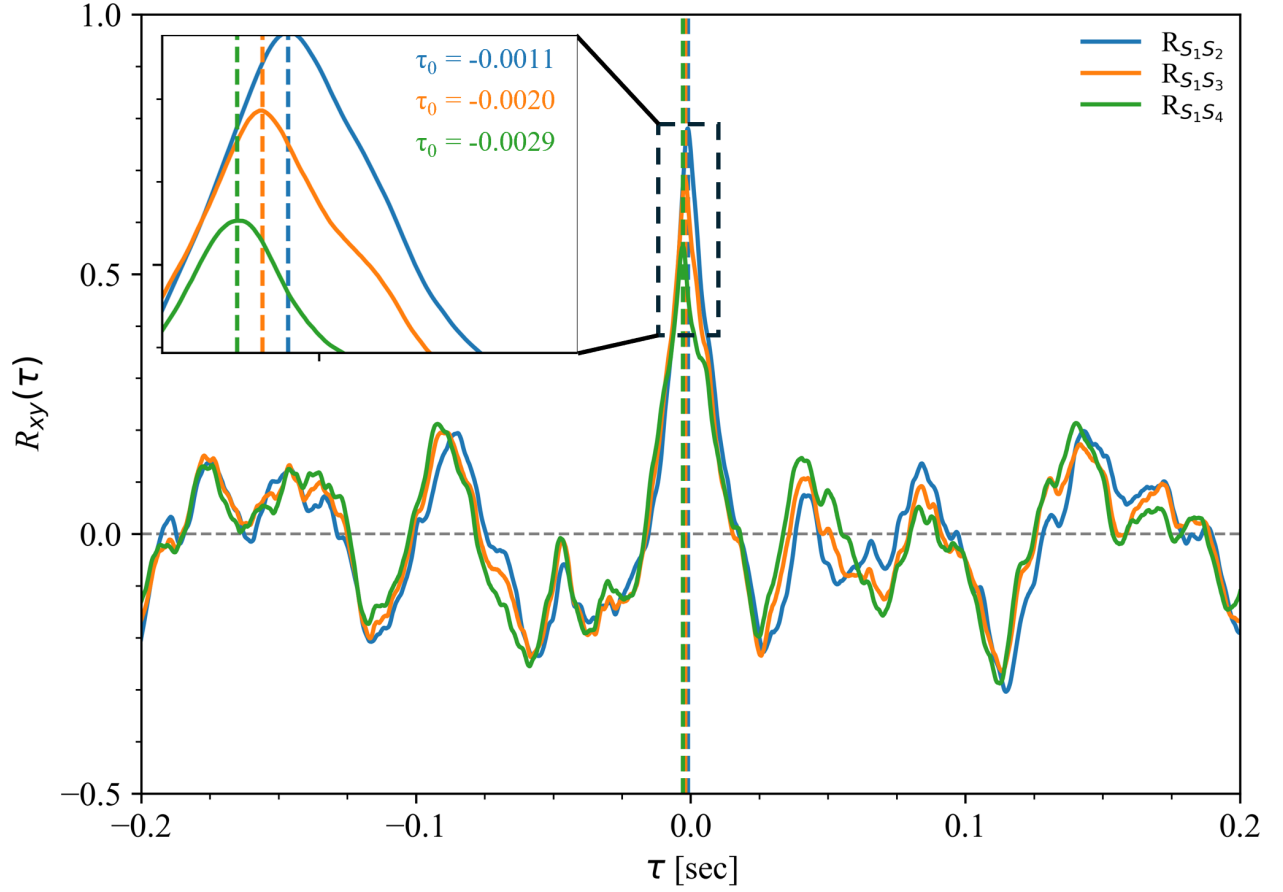
Results of the cross-correlation for tracked shock features at  $h_{cylinder} = 4.13\delta$  are shown in Figure 5.8. Each curve,  $R_{xy}(\tau)$ , represents the cross-correlation of the leading shock Mach stem,  $S_1$ , with the other tracked features,  $S_2$  and  $S_{1f}$ . The near zero time delay associated with the leading shock features  $R_{S_1 S_{1f}}$  indicates approximately synchronous motion of the Mach stem and shock foot. While the time delay associated with the first two Mach stems,  $R_{S_1 S_2}$ , is negative meaning the leading shock Mach stem fluctuates before the the second shock Mach stem. Similar results are shown in Figure 5.9 for a  $4.86\delta$  cylinder height. In this case, only the tracked Mach stem locations were compared. The time delay of all the downstream shocks are negative compared to the leading shock and grow larger with increasing streamwise location. This indicates that the leading shock fluctuates first, and is followed by subsequent motion of the downstream shocks.

The estimated time delays,  $\tau_0$  are plotted in Figure 5.10 as a function of mean shock location,  $x/H$ . For each cylinder height condition, a curve fit was applied to the data to calculate a convective propagation velocity. A zero time delay was added for the  $\tau_{S_1 S_1}$  data point in each plot to allow for a more accurate linear fit. The magnitude of the slope indicates the speed of the disturbance, and the negative sign of the slope indicates a downstream direction of travel. A strong linear trend between the time delay and the normalized axial location can be seen for both cylinder heights, and the resulting slope indicates a downstream convecting perturbation traveling at a speed of  $0.25 U_\infty$  (127 m/s).

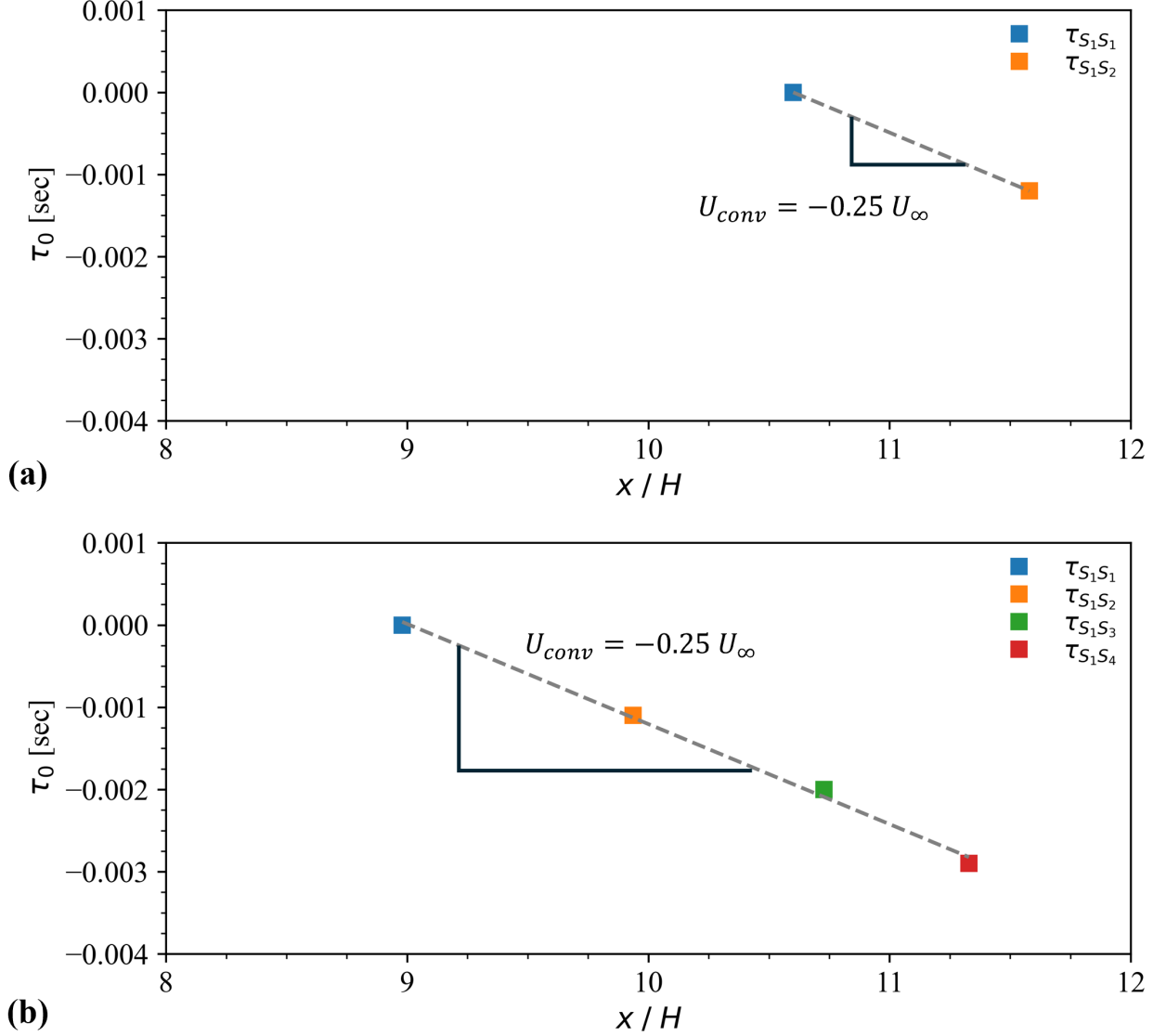
To better understand the disturbance propagation behavior downstream of the last shock in the shock train, a cross-correlation was performed on cylinder bow shock,  $BS_1$ , and Mach stem  $S_2$  tracking results. Limited cylinder bow shock imagery was available for tracking at the  $4.86\delta$  cylinder height - restricting analysis to only the  $4.13\delta$  cylinder height. The results are shown in Figure 5.11. The max correlation between the cylinder bow shock,  $BS_1$ , and



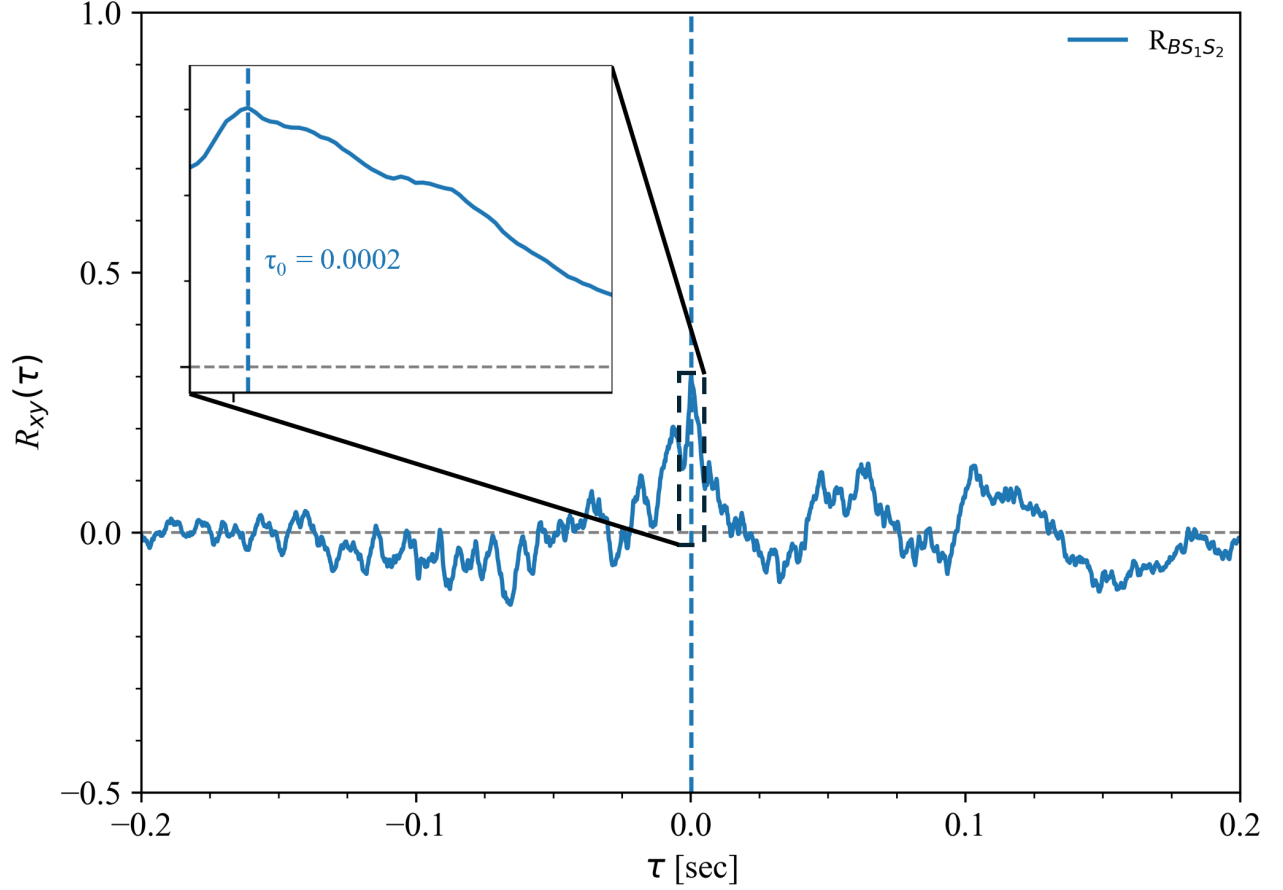
**Figure 5.8:** Cross-correlation results,  $R_{xy}(\tau)$ , of tracked shock features at a fixed cylinder height,  $h_{cylinder} = 4.13\delta$ . The estimated time delay,  $\tau_0$ , between signal  $x$  and signal  $y$  occurs at the location of maximum correlation. A negative time delay ( $\tau_0 < 0$ ) indicates signal  $x$  leads signal  $y$ .



**Figure 5.9:** Cross-correlation results,  $R_{xy}(\tau)$ , of tracked shock features at a fixed cylinder height,  $h_{cylinder} = 4.86\delta$ . The estimated time delay,  $\tau_0$ , between signal  $x$  and signal  $y$  occurs at the location of maximum correlation. A negative time delay ( $\tau_0 < 0$ ) indicates signal  $x$  leads signal  $y$ .



**Figure 5.10:** Convective propagation speed of shock train unsteadiness at fixed cylinder heights (a)  $h_{cylinder} = 4.13\delta$  and (b)  $h_{cylinder} = 4.86\delta$ . Time delays,  $\tau_0$ , are extracted from the cross-correlation results. Shock locations,  $x/H$ , are the shock tracking sample mean locations. A negative slope,  $U_{conv} < 0$ , indicates downstream propagation of the disturbance.



**Figure 5.11:** Cross-correlation result,  $R_{xy}(\tau)$ , of tracked cylinder bow shock,  $BS_1$ , and tracked shock  $S_2$  at a fixed cylinder height,  $h_{cylinder} = 4.13\delta$ .

the second shock Mach stem,  $S_2$ , occurs at a positive time delay - indicating that the mach stem fluctuates slightly before the cylinder bow shock.

Finally, a scaling of the unsteady shock characteristics was performed using the dimensionless Strouhal number. The Strouhal number represents a ratio of the oscillation frequency to the characteristic frequency of the freestream flow and is given as

$$St = \frac{fL_{sep}}{U_\infty} \quad (5.1)$$

where  $L_{sep}$  is the characteristic length and  $U_\infty$  is the freestream velocity. In cylinder shock boundary layer interaction studies, the characteristic length is often taken to be the axial distance between the upstream cylinder face and the leading edge of the cylinder

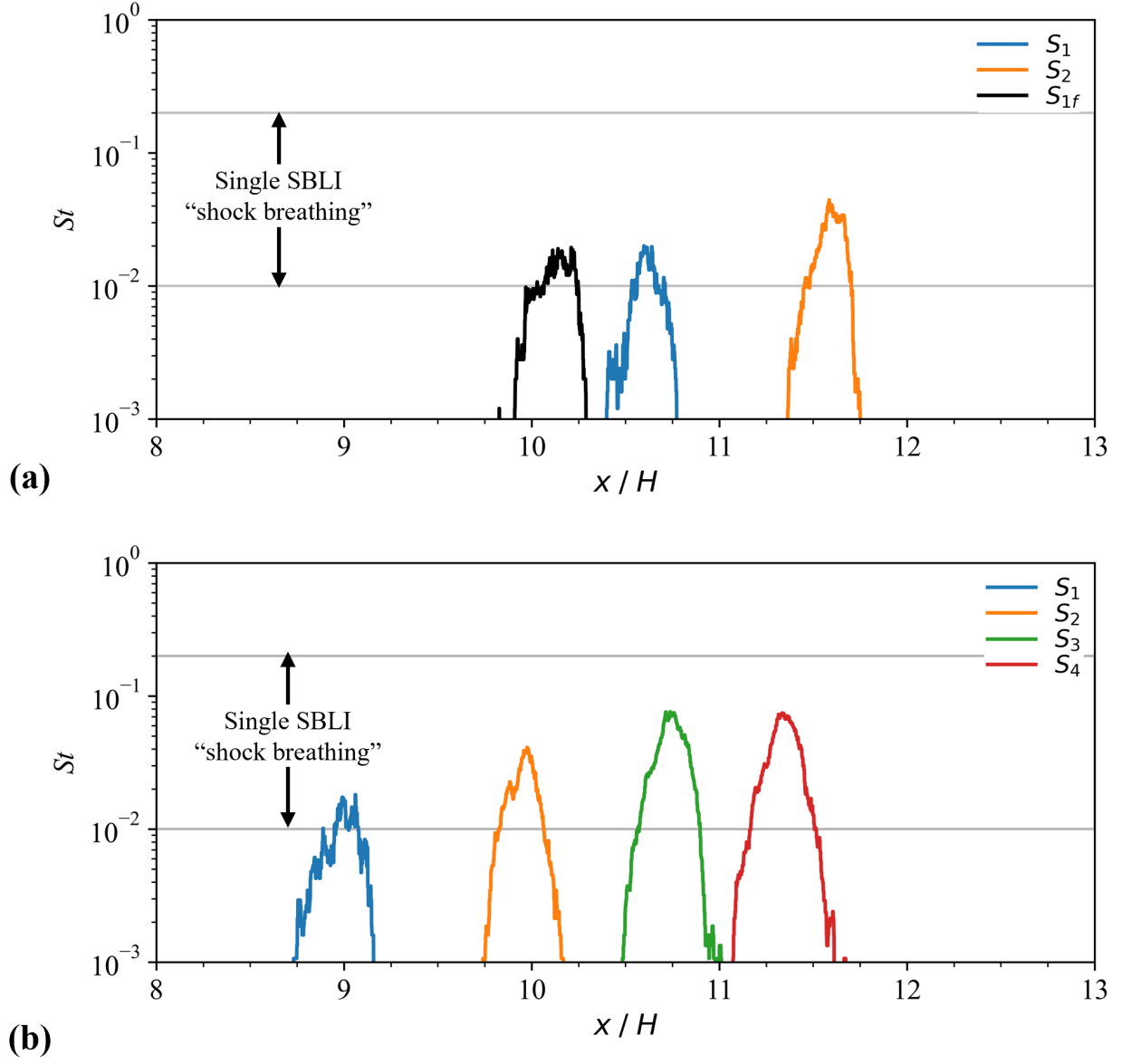
separation shock foot. For this work, the separation length was estimated by extrapolating the separation shock foot from the BOS imagery down to the floor. The approach assumed a constant shock angle from the triple point down to the floor. The resulting parameters used for the scaling were  $L_{sep} = 0.061$  m and  $U_{\infty} = 507.64$  m/s.

The zero-crossing frequency results were scaled to an equivalent Strouhal number, and the results are shown in Figure 5.12. Previous studies have shown Strouhal numbers ranging from approximately 0.01 – 0.2 to be associated with “shock breathing” as a result of cylinder-induced shock boundary layer interaction instabilities [18, 73–75]. Detailed analysis performed by Combs et al. [66] found these low-frequency fluctuations to occur at a Strouhal number range of 0.04 to 0.08 for a semi-infinite cylinder inserted at a Mach 1.78 approach condition. As shown in (a) and (b), the shock train unsteadiness quantified in this work ranges from 0.02 to 0.08; consistent with the low frequency “shock-breathing” fluctuations noted in other studies. Shock features  $S_{1f}$  and  $S_1$  are captured at the lower bound of this range, suggesting that the cylinder-induced fluctuations contribute in some part to the observed shock unsteadiness. Furthermore, the frequency plateau exhibited between  $S_3$  and  $S_4$  indicates that the cylinder-induced fluctuation sets an upper limit on the maximum zero-crossing frequency of the Mach stem near the cylinder blockage region.

The cylinder back-pressuring method used in this work creates a unique coupling between the fluctuating characteristics of a single cylinder-induced SBLI and the inherent unsteadiness of a shock train. The results presented in this section suggest that the shock train unsteadiness is driven by downstream cylinder induced perturbations. Based on these findings, the following hypothesis is proposed regarding the disturbance propagation mechanism: Instabilities generated by the cylinder-induced separation bubble travel upstream through the boundary layer, and their effect is modulated by the local boundary layer state both upstream and downstream of each shock. When a perturbation reaches the leading shock, it prompts an axial shift to correct for the instantaneous change in boundary layer conditions. This response triggers a cascading displacement among the downstream shocks as they sequentially adjust to the new upstream flow state. Motion of the trailing shock in the shock train causes the cylinder bow shock to adjust to the upstream disturbance.

This adjustment, in turn, directly generates new instability at the impingement point of the cylinder-induced separation bubble, thus closing the feedback loop.





**Figure 5.12:** Zero-crossing Strouhal numbers of tracked shock features at fixed cylinder heights (a)  $h_{cylinder} = 4.13\delta$  and (b)  $h_{cylinder} = 4.86\delta$ .

# Chapter 6

## Conclusions

### 6.1 Summary and Conclusions

The characteristics of a shock train generated by cylinder-induced back-pressuring were investigated in the UTSI direct-connect isolator facility. The motivation for using a floor-mounted cylinder stemmed from the structural similarity that has been observed between the three-dimensional shock patterns generated by a transverse jet and those produced by a solid-body cylinder in supersonic flow. This specific experimental approach offered a novel, low-cost means of simulating the three-dimensional blockage effects that are characteristic of a flush-wall fuel injector in a high-speed propulsion flowpath.

The objectives of this study were to:

1. Demonstrate the ability to generate a representative isolator shock train with a cylinder back-pressuring device.
2. Investigate the shock train formation process during cylinder insertion.
3. Characterize the shock train structure, dynamic response, and overall pressure rise as a function of cylinder height.
4. Characterize the inherent unsteadiness of the shock train and identify the cause.

The primary data collection methods used in this study included simultaneous steady-state and high-speed pressure measurements at discrete axial locations along the floor centerline, high-speed background-oriented schlieren imaging and fast-response pressure sensitive paint from the side and overhead viewing angles, respectively, and high-resolution core flow profile images using planar laser CO<sub>2</sub> Rayleigh scattering. A 25.4 mm OD stainless steel cylinder positioned at the downstream end of test section 2 was inserted into the flow at various heights generating back-pressure, and producing a time-phased formation of shock features ranging from a single bifurcated cylinder bow shock to a full pseudo-shock system. The static pressure taps along the floor centerline were used to characterize the quasi-static pressure rise of the shock train and validate the predictive capability of commonly used isolator pseudo-shock models. The time-resolved BOS and PSP images were acquired at 10 kHz and allowed for image-based tracking of certain shock features. A number of shock features were tracked at each sampling interval and used to characterize the streamwise propagation of the shock system in response to changing back-pressure conditions and to support frequency analysis of the inherent shock train unsteadiness.

The downstream cylinder back-pressuring approach successfully generated multiple shock structures, providing a unique opportunity to visualize the full formation process of an isolator pseudo-shock, beginning with a normal shock train. The shock train observed in the BOS imagery displayed a classic normal shock train structure: a leading bifurcated shock followed downstream by non-bifurcated shocks of diminishing height. Additionally, overhead PSP imagery revealed a curved leading shock foot that extended upstream of the BOS-viewed Mach stem by approximately one duct height. This three-dimensional structure was further confirmed by the spanwise PLS imagery, which captured a circular core flow region downstream of the shock train leading edge. The convergence of features across the PSP and PLS visualizations provided conclusive evidence of the three-dimensional, bulb-shaped structure of these re-accelerating supersonic flow regions.

A notable discovery that is attributed to the unique back-pressuring method employed in this work was evidence of distinct shock train propagation phases. Prior to shock train formation, the cylinder blockage generates a classic inviscid bow shock with a bifurcated

shock foot. Increasing the cylinder height creates additional spanwise blockage and causes the vertical extension of the bow shock to impinge on the duct ceiling. This creates a strong enough adverse pressure gradient to generate a separation bubble and produce upstream propagation of the first normal shock. This process repeats as the cylinder height is further increased causing the sequential formation of additional shocks. Based on shock tracking results, the shock train propagation during cylinder back-pressuring was shown to follow three distinct phases summarized below:

- **Phase I propagation:** At a cylinder height of approximately  $2.9\delta$ , separation of the corner flow boundary layer occurs and causes an upstream propagation of 0.5 duct heights at an average speed of approximately  $3 \times 10^{-4} U_\infty$ .
- **Phase II propagation:** At a cylinder height of approximately  $4.13\delta$ , separation of the floor, ceiling, and sidewall boundary layers occurs causing a rapid upstream propagation of 1.5 duct heights at an average speed of approximately  $1 \times 10^{-2} U_\infty$ .
- **Phase III propagation:** At cylinder heights greater than  $4.28\delta$ , upstream propagation continues due to increased back-pressure that is primarily driven by the gross blockage effect of the cylinder and the reduced core flow Mach number at the tail of the shock train. Propagation occurs at an average speed of approximately  $1 \times 10^{-3} U_\infty$ .

In Phase II and III, the shock train behaved like a rigid-body with synchronous motion of all shocks and constant separation distances between adjacent shocks. Additionally, there was a significant amount of path-dependent hysteresis behavior between the insertion and extraction runs; leading to an apparent shift in cylinder height of approximately  $0.4\delta$ . During cylinder extraction, the shock train exhibited extended Phase III behavior over a period equivalent to a cylinder height difference of  $0.73\delta$  from the cylinder insertion run.

The overall pressure rise and length of the cylinder-induced shock system fell within empirical predictions. For cylinder heights  $< 4\delta$ , the single cylinder SBLI accounted for nearly 24% of the overall pressure rise. For cylinder heights between  $4\delta - 4.5\delta$ , the back-pressure was sufficiently high enough to generate a normal shock train which accounted for

nearly 55% of the overall pressure rise. Beyond  $4.5\delta$ , a pseudo-shock system is believed to exist based on visual inspection of the schlieren imagery. At the maximum cylinder height tested, the pseudo-shock reached a length of 8.1 duct heights and produced an overall pressure rise of 74% of that compared to a normal shock at the same approach Mach number. By incorporating a second order effect of jet momentum flux ratio,  $J$ , the non-linear pressure rise observed during cylinder back-pressuring was found to be effectively collapsed into a single linear trend by applying a non-linear change of variables.

Experimental streamwise pressure profiles were compared with the well-established Waltrup-Billig empirical and Smart flux-conserved models. In general, both models demonstrated the ability to capture the overall pressure rise of the shock system. The Waltrup-Billig correlation proved most useful in identifying the initial pressure gradient produced by the shock train leading edge and predicting the downstream pressure rise. However, its monotonic pressure rise assumption makes it ill-suited for predicting the pressure profile shape downstream of the initial pressure rise for long pseudo-shock systems. The Smart model approximates the flow as quasi-1D with separation initiating just downstream of the shock train leading edge and utilizes the Ortwerth pressure diffusion model with an empirical constant  $k$ . The value of  $k$  determines the magnitude of the initial pressure rise caused by the separated region and was found to range between 44.5 and 125 for comparisons with this work.

Feature tracking results from high-speed BOS and PSP imagery showed the inherent unsteadiness of the shock train about its time-averaged location despite having constant upstream and downstream boundary conditions. The zero-crossing frequency of the first four shocks in the shock train were assessed for cylinder heights up to  $4.86\delta$ . The maximum crossing frequency of the leading shock occurred at approximately 160 Hz. The frequency increased for each downstream shock at a rate of approximately 2x Hz per duct height. The third and fourth downstream shocks plateaued at a maximum crossing frequency of approximately 620 Hz. This trend was found independent of cylinder back-pressure. Furthermore, the near-identical crossing frequencies of the third and fourth shocks suggest

that the dominant crossing frequency of each shock in the shock train is fundamentally dependent on the local upstream and downstream boundary layer states.

Cross-correlation of the shock positions was used to assess the time delay between signals and determine the temporal order of the shock train fluctuations. The upstream shock foot and Mach stem features of the leading shock were found to fluctuate first with nearly zero lag, indicating a rigi-body response. The perturbation caused by the leading shock fluctuation was found to convect downstream in a linear fashion at a speed of  $0.25 U_\infty$ .

The dimensionless Strouhal number was used to scale the shock train unsteadiness results in a manner consistent with traditional cylinder-induced shock/boundary layer interaction studies. In general, Strouhal numbers based on the zero-crossing frequency analysis were found to range from  $0.02 - 0.08$ . This places the observed shock train unsteadiness within the approximate bounds of “shock-breathing” fluctuations commonly observed in cylinder-induced single SBLI studies, and the frequency plateau exhibited by the third and fourth shocks suggests that the cylinder-induced fluctuation sets an upper limit on the maximum zero-crossing frequency of these oscillations.

The cylinder back-pressuring method used in this work creates a unique coupling between the fluctuating characteristics of a single cylinder-induced SBLI and the inherent unsteadiness of a shock train. The observed findings in this work have led to the following hypothesis regarding the unsteadiness propagation mechanism:

- Cylinder-induced “shock-breathing” fluctuations emanating from separation bubble instabilities travel upstream through the boundary layer and are modulated by the local boundary layer state both upstream and downstream of each shock.
- Upon arrival of the perturbation, the leading shock shifts axially to correct for the instantaneous change in boundary layer condition; triggering a cascading displacement of the downstream shocks as they sequentially adjust to the new upstream flow state.
- Motion of the shock train tail causes the cylinder bow shock to adjust to the upstream disturbance, which, in turn, directly generates new instability at the impingement point of the cylinder-induced separation bubble, thus closing the feedback loop.

This work has successfully demonstrated the tightly-coupled nature of shock train behavior with a three-dimensional back-pressuring method, achieved through the insertion of a cylindrical rod at the isolator exit. The primary findings that distinguish this study from previous back-pressuring research are:

1. The observation of distinct shock train propagation phases within the isolator, identified to be a direct result of cylinder-induced boundary layer separation events.
2. The direct influence of cylinder-induced fluctuations on the inherent unsteadiness characteristics of the shock train.

The intentionally simple method of using a floor-mounted cylinder as a back-pressuring device was designed to integrate multiple, and previously isolated, research areas. Specifically, the cylindrical-rod representation of flow blockage caused by flush-wall fuel injection, the unsteady physics of cylinder-induced shock boundary layer interactions, and the complex sensitivity of shock train propagation to back-pressure effects all converge in this work, providing crucial insights for the future development of robust dual-mode scramjet propulsion systems.

## 6.2 Recommendations for Future Work

The exploratory nature of this study brought about many lessons-learned and led to the following recommendations for future work.

**Cylinder sensitivity study:** The decision to use a 25.4 mm OD cylindrical rod mounted on the floor centerline was based primarily on availability and schedule. Future studies investigating various cylinder diameters, cylinder mounting configurations, and number of cylinders would be valuable for determining the sensitivity of these results to different three-dimensional blockage shapes. Complimentary to that work could be an investigation of different cylinder injection rates. The relatively slow injection speed used in this study is

not representative of the actual time scale of pulsed fuel injection. The extent of back-pressuring hysteresis observed in this work could change depending on the rate of cylinder motion.

**Validation study of a cylindrical rod as a surrogate for fuel injection:** The idea of using of a solid cylindrical body for representing the blockage effects of a transverse jet in supersonic flow is seen throughout the literature, however, very little information exists validating this experimentally. A valuable study would be an experimental validation of this idea using a suite of modern optical diagnostic approaches. The results of such a study could lay the groundwork for scaling the cylinder back-pressuring configuration to meet a certain fuel equivalence ratio target.

**Identification of optimal sensor placement for shock train detection:** Using data from the current work, time-based correlations of PSP response with BOS tracking results could be used to determine regions of interest for sensor placement. The results of such an analysis would provide valuable insights on the optimal location and quantity of sensors that are needed to track and predict the location of the shock train. Future work with a larger region of PSP would help validate the accuracy these preliminary findings.



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# Appendix

# Appendix A

## Selected Scripts

### A.1 BOS Image Processing | Python Code

```
###  
  
import numpy as np  
from scipy import fft  
import cv2  
  
###  
  
# DEFINE FUNCTIONS  
  
# 2D FFT  
def calculate_2dft(im):  
    ft = fft.ifftshift(im)  
    ft = fft.fft2(ft)  
    return fft.fftshift(ft)
```

```

# 2D INVERSE FFT
def calculate_2dift(im):
    ift = np.fft.ifftshift(im)
    ift = np.fft.ifft2(ift)
    ift = np.fft.fftshift(ift)
    return ift

#%%

# LOOP THRU EACH WIND-ON IMAGE IN THE STACK
for im_index in np.arange(im_start, im_stop, step=skip_every, dtype=int):

    # LOAD WIND-OFF (UNDISTORTED) REFERENCE IMAGE
    imref = cv2.imread(imref_files[im_index], -1) # load wind-off image
    im_rows, im_cols = imref.shape

    # LOAD WIND-ON DISTORTED IMAGE
    im2 = cv2.imread(im2_files[im_index], -1) # load wind-on image

    # PERFORM 2D FFT ON UNDISTORTED IMAGE
    im0_fft = calculate_2dft(imref)
    freq_rows = fft.fftfreq(im_rows)
    freq_cols = fft.fftfreq(im_cols)

    # CREATE SQUARE GRID OF FFT FREQUENCIES
    freq0_x, freq0_y = np.meshgrid(fft.fftshift(freq_cols), fft.fftshift(freq_rows))

```



```

# PERFORM 2D FFT ON DISTORTED IMAGE
im_fft = calculate_2dft(im2)
freq_rows = fft.fftfreq(im_rows)
freq_cols = fft.fftfreq(im_cols)

# CREATE SQUARE GRID OF FFT FREQUENCIES
freq_x, freq_y = np.meshgrid(fft.fftshift(freq_cols), fft.fftshift(freq_rows))

# CREATE CIRCULAR ARC MASKS FOR FILTERING IN FOURIER DOMAIN
c1_arc_mask = np.zeros_like(freq0_x, dtype="complex")
c2_arc_mask = np.zeros_like(freq0_x, dtype="complex")

# DEFINE BASIS VECTORS REPRESENTING BACKGROUND WAVE PATTERN
k1 = np.array([[0.173523, -0.00337]])
k2 = np.array([[-0.00399, -0.17290]])

# DEFINE RADIAL BOUNDS OF FILTER
outer_rad = 1.2*np.linalg.norm(k1)
inner_rad = 0.4*np.linalg.norm(k1)

outer_cond = np.sqrt(freq0_x**2 + freq0_y**2) <= outer_rad
inner_cond = np.sqrt(freq0_x**2 + freq0_y**2) >= inner_rad

rad_cond = inner_cond & outer_cond

# DEFINE ANGULAR BOUNDS OF FILTER
c1_max_angle = 0.8 # radians from positive x-axis
c1_min_angle = -0.8 # radians from positive x-axis

```

```

c2_max_angle = c1_max_angle + (np.pi/2)
c2_min_angle = c1_min_angle + (np.pi/2)

max_cond = np.arctan2(freq0_y, freq0_x) < c1_max_angle
min_cond = np.arctan2(freq0_y, freq0_x) > c1_min_angle
angle_cond = max_cond & min_cond

# C1 MASK AROUND K1
c1_arc_mask[rad_cond & angle_cond] = 1

max_cond = np.arctan2(freq0_y, freq0_x) < c2_max_angle
min_cond = np.arctan2(freq0_y, freq0_x) > c2_min_angle
angle_cond = max_cond & min_cond

# C2 MASK AROUND K2
c2_arc_mask[rad_cond & angle_cond] = 1

# APPLY C1 AND C2 MASKS TO UNDISTORTED IMAGE
# Extract carrier waves (c1 and c2 of ref image)
c1 = im0_fft * c1_arc_mask
c2 = im0_fft * c2_arc_mask

im0_c1 = calculate_2dift(c1)
im0_c2 = calculate_2dift(c2)

```

```

# APPLY C1 AND C2 MASKS TO DISTORTED IMAGE
# Extract distorted carrier waves (m1 and m2 of distorted image)
m1 = im_fft * c1_arc_mask
m2 = im_fft * c2_arc_mask

im_m1 = calculate_2dift(m1)
im_m2 = calculate_2dift(m2)

# Calculate phase shift in each carrier wave direction
phi1 = np.imag(np.log(np.conj(im0_c1)*im_m1)) # (im_rows, im_cols)
phi2 = np.imag(np.log(np.conj(im0_c2)*im_m2))

k1_inv = 1/k1
k2_inv = 1/k2

# CALCULATE DISPLACEMENT VECTOR, U
Ux = np.zeros_like(phi1) # (im_rows, im_cols)
Uy = np.zeros_like(phi2)

for i in range(im_rows):
    for j in range(im_cols):
        phi_vec = np.array([[phi1[i,j]], [phi2[i,j]]])
        Ux[i,j] = np.dot(k1_inv, phi_vec)[0][0]
        Uy[i,j] = np.dot(k2_inv, phi_vec)[0][0]

# CALCULATE MAGNITUDE OF U FOR PROCESSED BOS IMAGE
U_mag = np.sqrt(Ux**2 + Uy**2)

```

# Vita

William Stevens graduated from David Lipscomb High School (now Lipscomb Academy) in May 2010. He completed his Bachelor of Science in Mechanical Engineering at Tennessee Technological University in May 2014. He earned his Master of Science in Mechanical Engineering from the University of Tennessee Space Institute in May 2016. Upon graduation, he began working as a wind tunnel test analyst for the 716th Test Squadron at Arnold Engineering Development Complex in August 2016. In January 2017, he enrolled at the University of Tennessee Space Institute as a part-time PhD student. He joined the Air Force Research Laboratory in August 2022 working for the High-Speed Systems Division, where he currently supports conceptual vehicle design and propulsion system integration.