To the Graduate Council:

I am submitting herewith a thesis written by Nancy J. Hamilton entitled "Machine Recognition of Void Fraction for Two-Phase Flow Measurements." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

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MACHINE RECOGNITION OF VOID FRACTION FOR TWO-PHASE FLOW MEASUREMENTS

A Thesis

Presented for the

Master of Science

Degree

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ABSTRACT

An investigation was made of various histogram shapes as a means of determining the void fraction of two-phase flow of steam-water mixtures. Machine recognition techniques based on curve fitting procedures were implemented as a means of computing the void fraction. The third and fourth central moments associated with each histogram were used as the key elements in the void fraction identification. This method of void fraction prediction was found to be easily implementable, as the information necessary for the construction of the histograms could be computed and stored during the execution of the experiment. The calculations for the void fraction identification could then be completed when processing time was available after the conclusion of the experiment.

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INTRODUCTION

Machine recognition of void fraction represents an approach to the problem of discrimination among two-phase flow signals. The creation of a histogram is one method whereby the amplitude of a two-phase flow signal can be graphically illustrated. Observation of such histograms for a wide range of flow conditions is a procedure that may be easily implemented whereby the histograms may be categorized with respect to their moments.

The objective of this thesis is the investigation of the use of moments to classify various void fractions of a two-phase flow and how machine recognition techniques may be used to distinguish among the moments corresponding to the different histograms. There are several stages to the general problem being considered, as shown in Figure 1.1. The preprocessing stage represents any conditioning of the original signal such as filtering and analog-to-digital conversion. Feature extraction is a process whereby a set of discriminatory features of the signal are determined. In this case, the features are the central moments calculated from the histograms. Classification of a set of features comprises the final step in the machine recognition process. In this stage, a decision is made as to the nature of the signal based on the extracted set of features.

In general, a finite number of classes are defined such that the classifier assigns the signal to one of the classes. For a

Figure 1.1. Pattern Recognition System.

two-phase flow, the various flow regimes could represent the different classes. Thus, in order to determine the void fraction for a given signal, the signal would first have to be classified as a particular flow regime. A second classification would have to be done to determine the void fraction. This thesis presents a more direct method for the determination of a void fraction.

The classification technique employed is a direct implementation of curve fitting, or the representation of tabular data with a mathematically expressible function. Thus the task of determining a void fraction for a set of sample data is reduced to the evaluation of a scalar quantity. Appropriate restrictions are considered in using a curve fitting approach since the tabular data extends over a wide range.

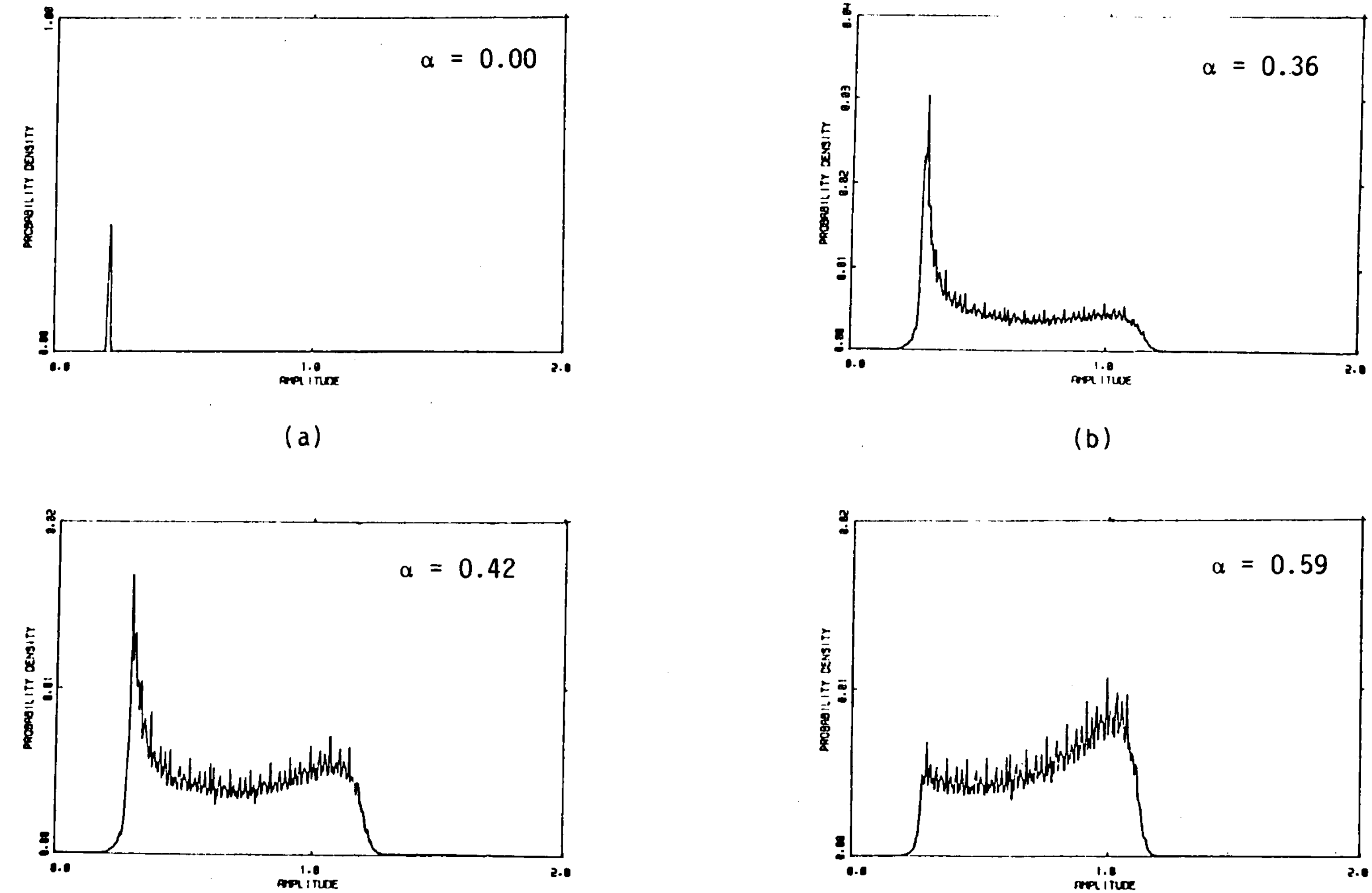
Motivation

It is always of interest to the designer or operator of a system to know as accurately as possible the status of the system or the conditions under which it is operating. This knowledge helps to control and optimize the performance of the system. At the very least, some type of warning should be given when it is operating below minimum standards of performance or entering early stages of failure. Generally, some type of sensor will be attached to the system to output a waveform which is indicative of the performance of the system. In order to interpret this waveform, specific parameters must be chosen for the system which will relate to the information desired by the operator.

These features, in order to be useful in determining a system's status, must be defined and extracted from the sensor output to provide meaningful information. The machine recognition of void fraction, which in this case is the fraction of the channel occupied by the steam phase of a steam-water flow, is one case in particular where the features can be calculated directly from the sensor output.

During the simulation of the refill and reflood following a loss of coolant accident, the ability to determine void fraction is required in order to predict what is occurring within a reactor vessel as coolant is injected into the cooling chamber. Since it is not humanly possible to directly observe the cooling process, a method is required whereby void fraction can be determined from an output waveform generated by the fluid flow.

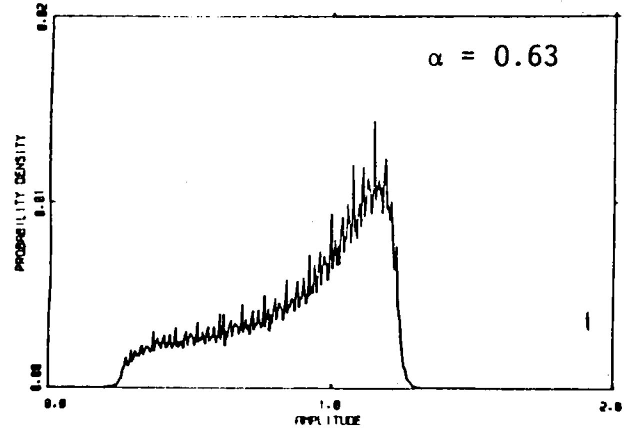
One approach to the calculation of void fraction for a two-phase flow is an analysis of the histograms computed from the output waveform. In this case, the waveform is an impedance signal generated by a two-phase flow and distinctive histograms can be constructed from the signal for various void fraction values, as shown in Figure 1.2. Each of these histograms has a unique shape which corresponds to a particular void fraction range. Histograms represented by a tall spike are indicative of void fractions whose values are very close to either zero or one, representing all water or all steam, respectively. Histograms which are positively skewed are representative of void fractions in the range of 2 to 40%, indicating more water in the fluid flow than steam. Histograms which are relatively flat indicate an almost equal concentration of steam and water and



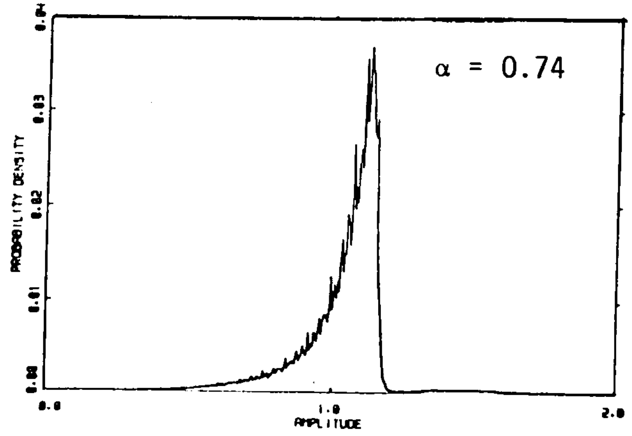
(a) (b)

(c) (d)

Figure 1.2. Histograms Representing Steam Void Fractions of (a) 0%, (b) 36%, (c) 42%, (d) 59%, (e) 63%, (f) 74%, (g) 86% and (h) 100%.



(e)



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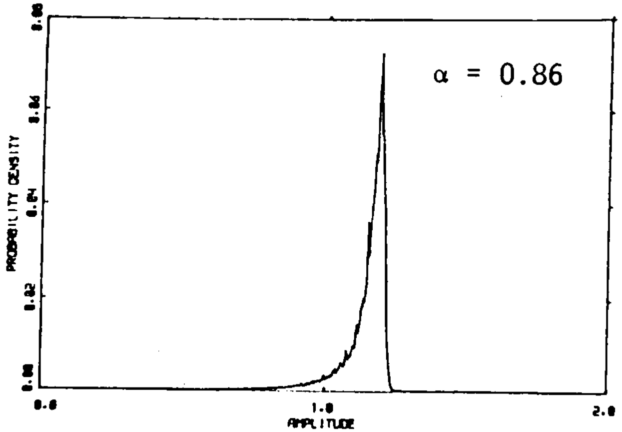
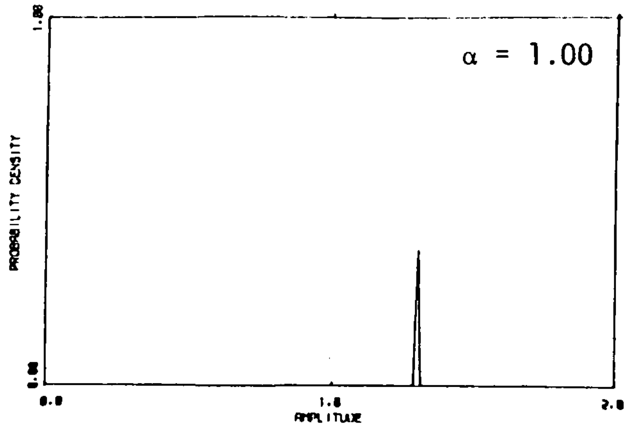


(g)



(h)

Figure 1.2. (continued)

characterize void fractions in the range of 40 to 60%. Impedance signals representing void fractions of 60 to 98% produce histograms which have a marked negative skewness.

The shape of each histogram can be described by moments, thus reducing the information present in the impedance signal to a set of moments computed from the histograms. This set of features extracted from the data remains well-behaved and exhibits characteristics which are significant in the discriminating process, as will be discussed in more detail in Chapter 2. Other feature sets which could possibly be used include power spectral density and level crossings. The interpretation of a system's performance may be difficult to obtain in terms of the power spectral density, while an examination of level crossings for the extraction of information for a two-phase flow has been done previously.

Literature Background

A wide variety of techniques have been proposed for void fraction measurements, reflecting the great importance of this parameter. A knowledge of the void fraction is important in estimating the behavior of a nuclear reactor system; since the content of the absorbing phase, water, will affect the neutron economy. In order to develop techniques for determining void fraction, it is often helpful to analyze the system and classify the flow into a number of "flow patterns" or "flow regimes." This analysis and classification helps in obtaining a qualitative understanding of the flow and, it is hoped, will also lead to a better prediction method for void fraction. Typical classifications of two-phase flow regimes are discussed by Hewitt and Hall-Taylor [1].

The number of contributors in the area of void fraction identification is extremely large. Since Hewitt and Lovegrove [2] have provided an extensive survey of experimental methods in two-phase flow studies, the following discussion will list only some individuals and their endeavors. The most widely used technique for the measurement of void fraction is measurement of the absorption of a beam of gamma rays in the flow. This method was used in determining the void fraction of the steam-water mixture in the test loop. The test loop provided the data used in formulating the void fraction recognition approach developed in this thesis. Mitchell and Gonzalez [3] investigated the second most widely used class of methods for determining void fraction. Specifically, those methods involved the measurement of the conductance and capacitance of a flow.

While neither histograms nor moments have been previously used to directly determine the void fraction of a flow, they have been used to discriminate among flow regimes. Kim, Gonzalez and Leavell [4] examined the possibility of using moments to categorize two-phase flow data. Jones and Zuber [5] used the probability density of the instantaneous void fraction as a means of defining the flow pattern for the three dominant patterns of bubbly, slug and annular flows. Moments have been used previously by Dudani [6] and Hu [7] for pattern recognition purposes.

Tou and Gonzalez [8] outlined the basic concepts of pattern recognition as shown in Figure 1.1 (page 2). While the technique utilized in this thesis is not specifically a pattern recognition technique; the basic concepts of taking a signal, performing preprocessing and feature selection functions upon it, and then classifying the signal according to some predetermined criteria will still apply for the machine recognition of void fraction.

Chapter Outline

A brief outline of the topics of this thesis is as follows. Chapter 2 discusses the general theoretical considerations for the determination of the classification features represented by the central moments calculated from the histograms. The classification approach is also presented. Chapter 3 describes the test equipment used to generate the data for the experiments. The recognition results are discussed, along with the results of the testing and evaluation of the recognition approach. Concluding remarks are given in Chapter 4.

CHAPTER 2

RECOGNITION APPROACH

The machine recognition of void fraction is a process which involves extracting specific features from a signal and classifying these features according to some predetermined criteria. In this case, the feature set consisted of the central moments calculated from the histograms. The classification criteria included the values of the first, third and fourth central moments extracted from the histograms. Since the method used in this thesis for classifying void fractions had not been previously examined, a training phase had to be employed using known void fractions to produce a set of void fraction curves and equations for the classification stage of the machine recognition of void fraction.

Histograms

The distinctive histograms in Chapter 1 were created with data used in the experiments for the recognition of void fraction. Figure 2.1 illustrates a representative graph of a general histogram. To construct a histogram, the range of measurement is divided into equal intervals, selecting the interval length c to be a simple one in terms of the original units of measurement. The x-axis is then divided into intervals of length c, called class intervals. The boundaries of the class interval are placed at $(x_i - c/2)$ and $(x_i + c/2)$, x_i being the mid-point value for that class, or central value. To create the histogram, each class is represented by a rectangle whose base is proportional to the width of the class interval

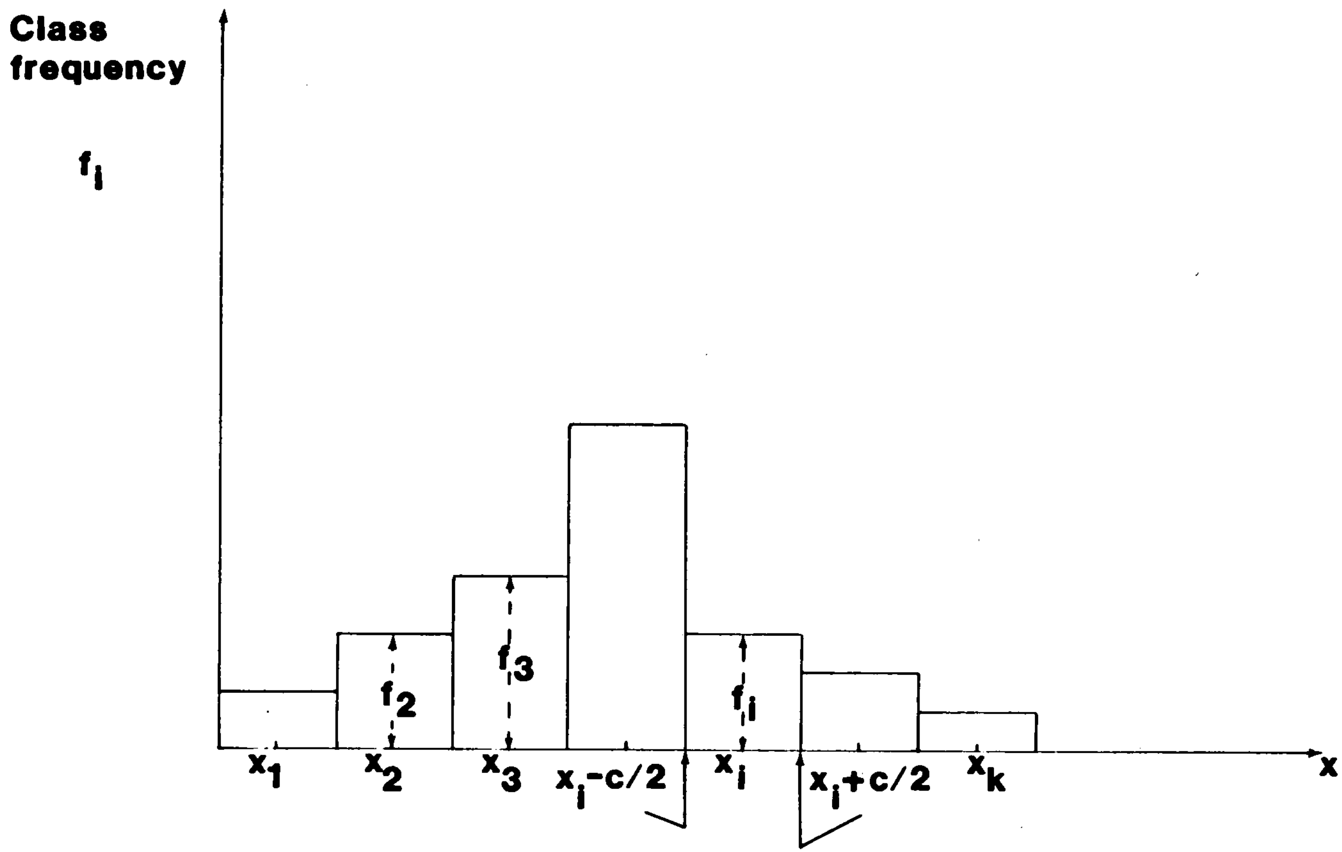


Figure 2.1. General Histogram.

and whose height is proportional to the class frequency, or the number of observations falling into the i^{th} class interval. The figure of juxtaposed rectangles furnishes a rough approximation to the true probability density function.

In order to construct a histogram, the measurements being grouped must be represented by digital quantities. If the measurements are given in analog form, they must be converted to digital representations. During the conversion process, the digital elements of the signal are assigned to particular levels within the analog-to-digital (A/D) converter. These levels can be used as the class intervals when constructing the histogram. The class frequency for a particular A/D level can be determined if a count of the total number of times a digital element is assigned to a specific level is kept.

Moments

Probability density functions and histograms may be described completely by the values of their moments. For the machine recognition of void fraction, the first, third and fourth central moments were chosen as the main classification features because they can determine the mean, skewness and kurtosis of a probability density function. These three geometrical properties can be used to discriminate among the histograms shown in Figure 1.2 (page 5).

The arithmetic mean is the first moment or statistical expectation of x. The mean of a discrete function over a finite interval is defined by

$$\bar{x} = \sum_{x} x \, f(x), \tag{2.1}$$

where f(x) is the class frequency corresponding to the variable x. The arithmetic mean of a histogram is the point on the x-axis which corresponds to the [x,y] coordinate pair representing the centroid of the area enclosed by the histogram. It is so fundamental in theory and in practice that it is customary, once it has been determined, to take it as a new origin and refer all higher moments to this origin. These moments are then called central moments, because the mean is subtracted from each measurement before it is raised to a power.

The second moment about the mean is of special importance because it tells something about the dispersion of the measurements. The dispersion or variance of the measurements is defined by

$$u_2 = \sum_x (x - \bar{x})^2 f(x), \qquad (2.2)$$

where $\bar{x}$ is the mean of the histogram as computed from Equation 2.1. While the second moment is not used for the machine recognition of void fraction, it can be used to indicate whether the values of x are spread densely or widely on either side of the mean.

Many continuous distributions can be represented by a characteristic bell-shaped curve. This distribution is said to be symmetric since it can be folded along an axis so that the two sides coincide. A distribution that lacks symmetry with respect to a vertical axis is said to be skewed. Figure 2.2(a) illustrates a distribution that is said to be skewed to the right, since it has a long right tail and a much shorter left tail. Distributions with these two characteristics are said to positively skewed. In Figure 2.2(b) the distribution is symmetric, while in Figure 2.2(c) it is skewed to the left, or is said to have a negative skewness. The histograms shown in Figure 1.2 (page 5) are variations of the three basic configurations shown in Figure 2.2.

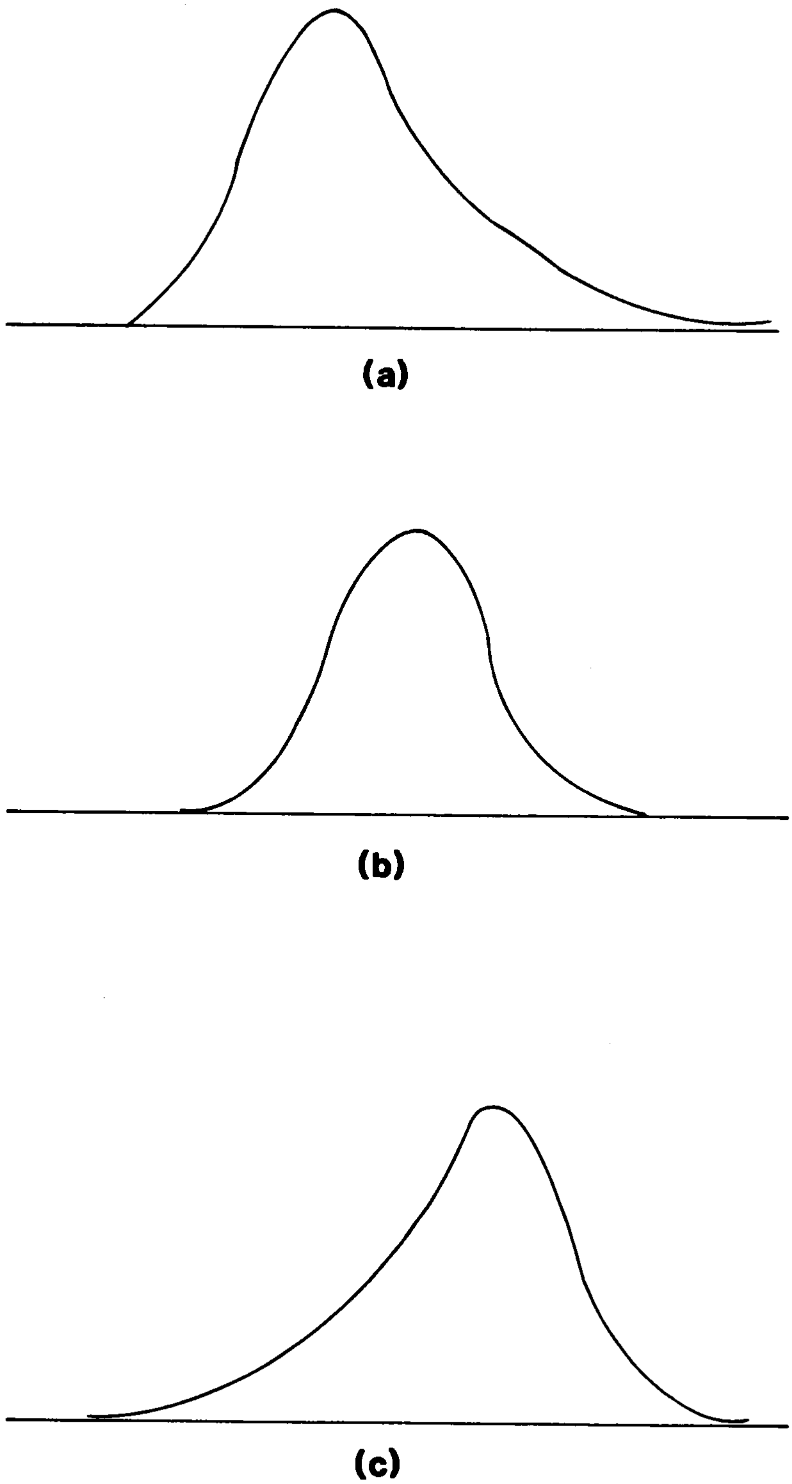


Figure 2.2. Skewness of Data.

The third moment is a measure of the skewness of a distribution and is calculated by the following equation,

$$u_3 = \sum_x (x - \bar{x})^3 f(x). \tag{2.3}$$

If the distribution is symmetric, u_3 is equal to zero. If the long tail of the distribution is on the side of the positive values of $(x - \bar{x})$, the cubes of the positive values of $(x - \bar{x})$ will outweigh the cubes of the negative values, resulting in u_3 being positive. This situation identifies the distribution as having a positive skewness. In the same manner, if the long tail of the curve is on the side of the negative values of $(x - \bar{x})$, the third moment will be negative and the distribution will have a negative skewness.

The fourth moment is also of importance when describing distributions. It is a measure of the degree of flattening or kurtosis for a distribution. The fourth moment is estimated by

$$u_4 = \sum_x (x - \bar{x})^4 f(x). \tag{2.4}$$

Two distributions may have the same mean, the same standard deviation, the same skewness, and yet may differ in that the curve of one may be more flattened at the center than that of the other, as shown in Figure 2.3.

Since moments of order higher than four have no simple geometrical interpretation, only the first, third and fourth moments were chosen for this experiment. The assumption that the three moment values are sufficient to discriminate among the various histogram shapes is verified by the recognition results presented in Chapter 3.

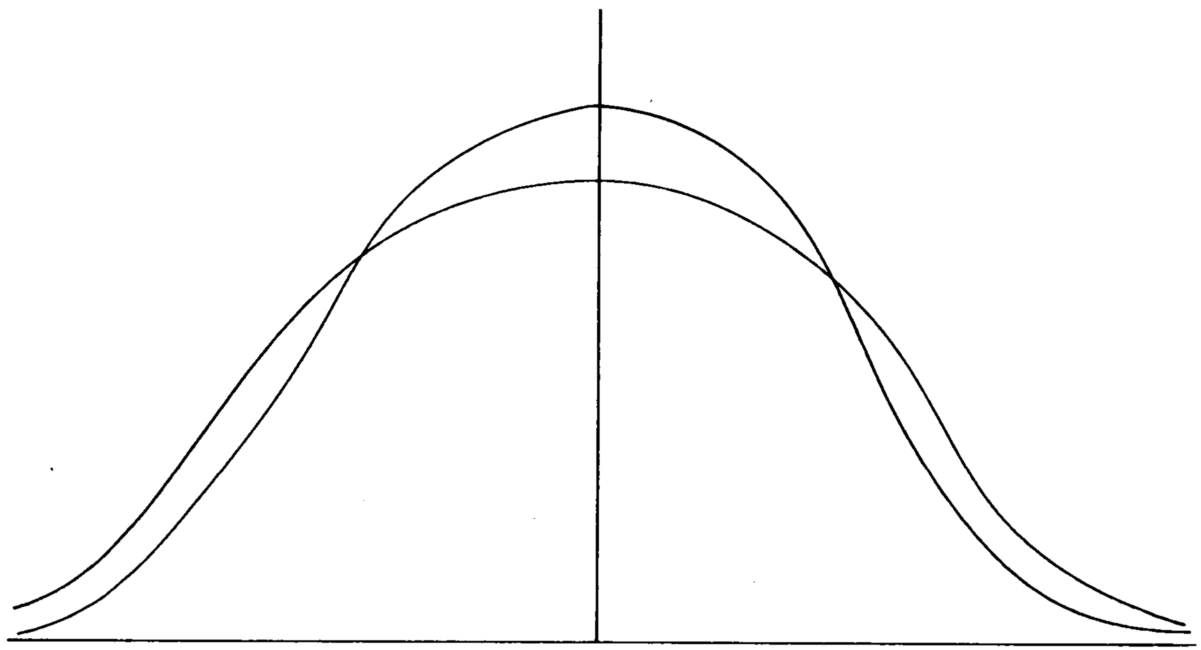


Figure 2.3. Kurtosis of Data .

Classification Approach

The classification approach for the machine recognition of void fraction is outlined by the flowcharts in Figures 2.4 and 2.5. Figure 2.4 outlines the procedure for the training phase of the experiment described in Chapter 3. It uses the principles developed in this chapter and the curve fitting techniques in Appendix A to formulate the equations and graphs necessary for the calculation of a signal's void fraction. The classification phase of the experiment is presented in flowchart form in Figure 2.5. It uses information derived from the training phase of the experiment to aid in void fraction identification.

The first three steps for either the training or classification phase of the experiment are the same. The analog signal is converted to a digital signal by an A/D converter and a histogram is created from the digitized signal. Three moments are calculated from the histogram. The first moment, or mean, indicates the central tendency of the histogram. The third moment gives an indication of the direction in which the histogram is skewed, and the fourth moment gives an indication of the flatness, or kurtosis of the histogram.

During the training phase of the experiment, the signals which were examined had known void fractions associated with them. It was with these void fraction values that a recognition approach for void fraction identification was possible. The object of the training phase was to determine some type of relationship between a two-phase flow signal and its void fraction. From an observation of the histograms representative of the known void fractions, it was noted that for each void fraction value, a distinctive histogram was formed.

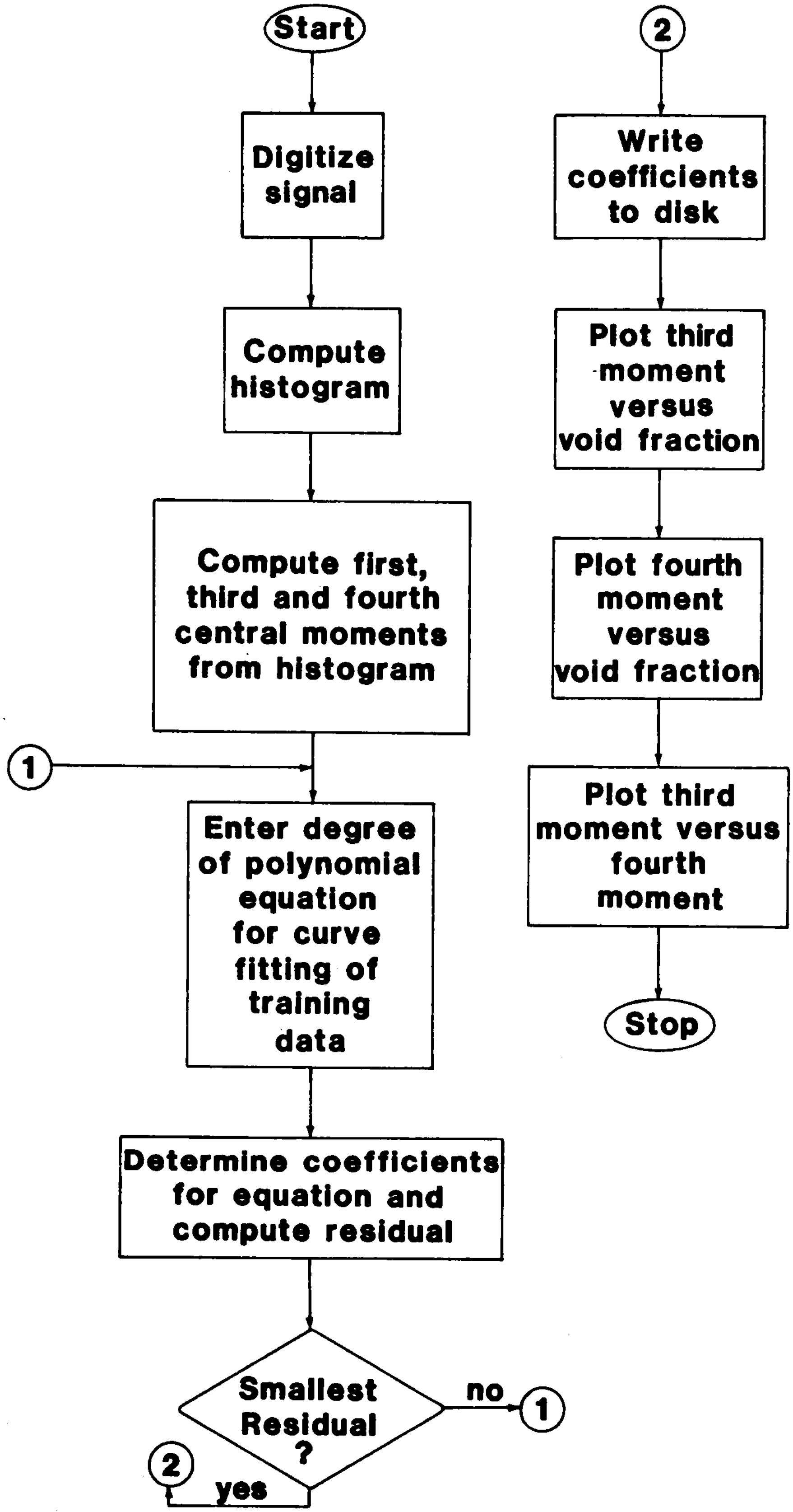


Figure 2.4. Training Phase.

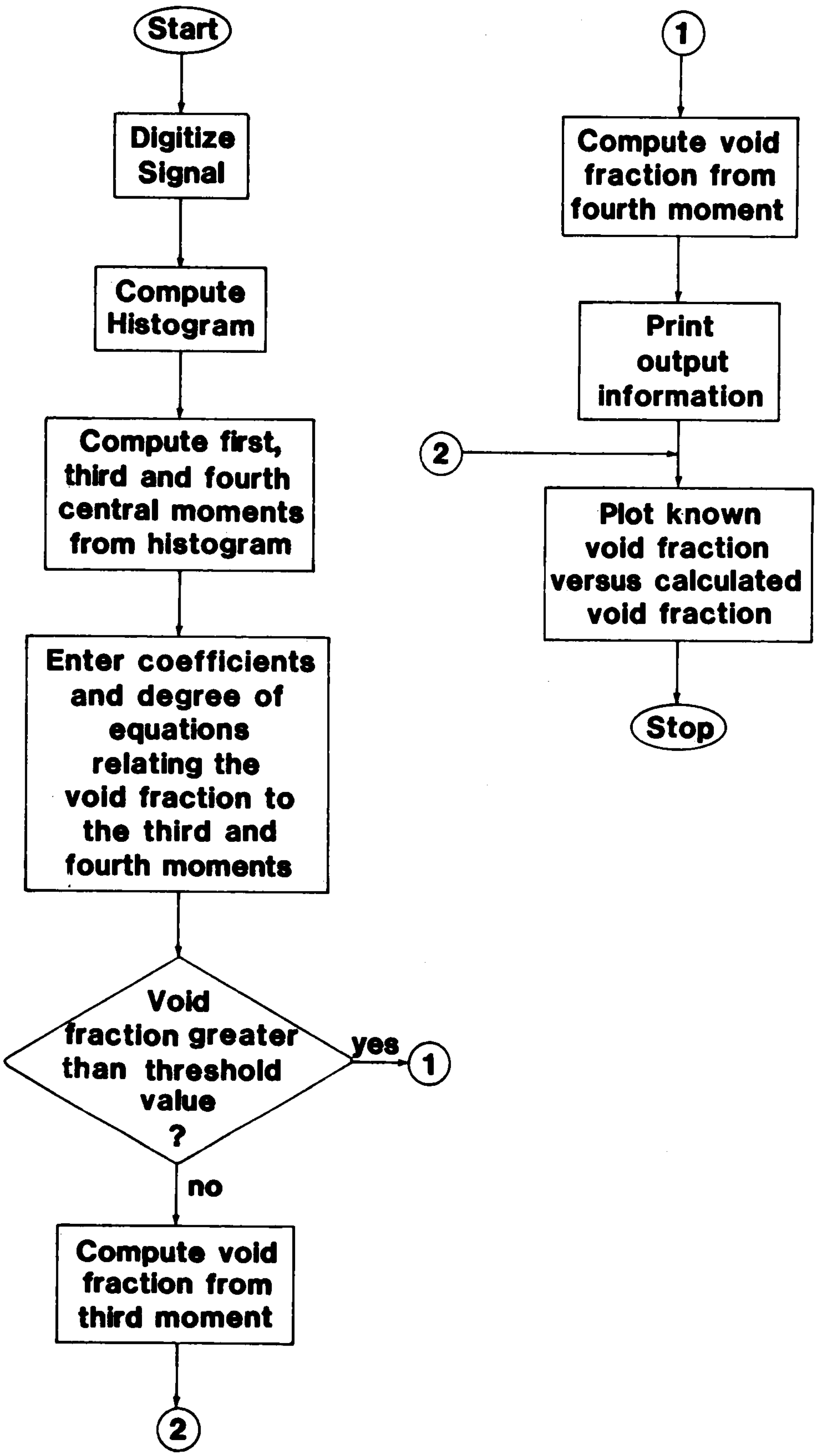


Figure 2.5. Classification Phase .

The different histogram shapes fell into three main categories. As shown in Figure 1.2 (page 5), they were either symmetrical, positively skewed or negatively skewed. Since the third moment is a measure of skewness, it was decided to examine the third moment of each histogram. Symmetrical histograms had a third moment of zero. Histograms that were positively skewed had positive third moments and those which were negatively skewed had negative third moments. Plotting the third moments from the histograms against their known void fractions, a definite pattern emerged. Plots of the fourth moments versus void fraction also displayed a distinctive pattern. Examples of the third and fourth moments plotted versus void fraction are presented in Chapter 3 for void fractions ranging in value from 0 to 100%.

Viewing plots of the third and fourth moments versus void fraction, it could be seen that portions of some curves would result in better void fraction estimates than others. Figure 2.6 shows a typical plot of the third moment versus void fraction. It is noted that for void fractions which are greater than 80%, discrimination among void fraction values is difficult. Hence, the curves were divided and equations were formulated for individual sections of each curve. The end products of the training phase were the coefficients for two equations relating the void fraction to either a third or a fourth moment value.

The coefficients were determined by curve fitting techniques using the method of least squares. The approximating function was a polynomial of a single variable. The equation of condition had the form

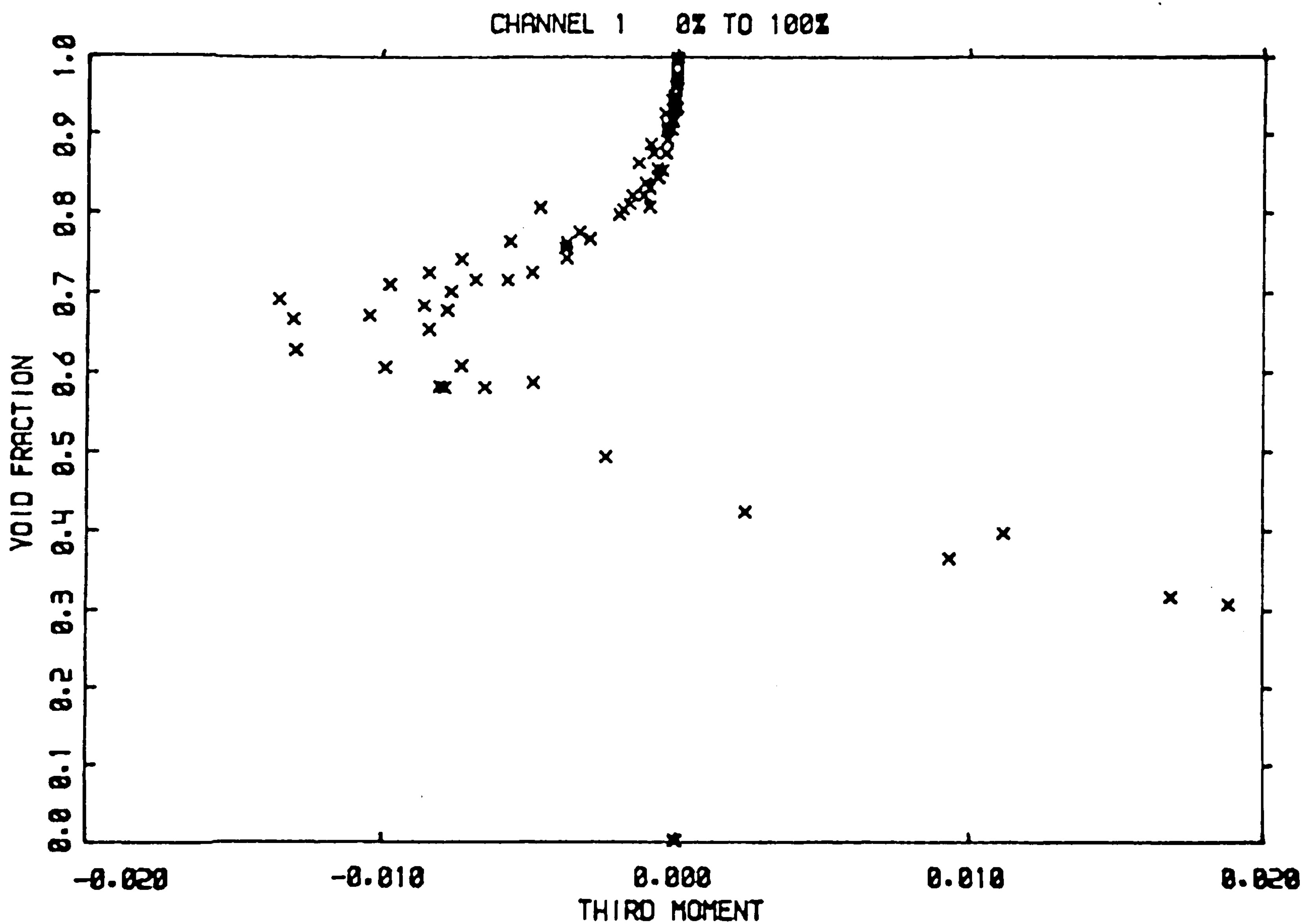


Figure 2.6. Typical Plot of Third Moment Versus Void Fraction.

$$y = a_1 + a_2x + a_3x^2 + a_4x^3 + \ldots + a_kx^{k-1}. \tag{2.5}$$

Equations of order one through seven were tested, with the equations having the smallest residuals being chosen as the equations of condition for the recognition approach. No equations of order greater than seven were tested, as they did not result in single-valued functions of x for the data points contained in several data sets.

The coefficients and the degree of the equations to which they belong served as input for the classification phase of the experiment. Depending upon the value of the fourth moment calculated from the signal, and whether or not it indicated that the void fraction was less than or greater than a threshold value, the void fraction was computed from either the third or the fourth moment. The third moment was used for classifying void fractions in the range of 30 to 70%, with the fourth moment being used to compute void fractions in the range of 70 to 100%. The data for the experiment contained no void fractions of less than 30%, if one ignores the all water or 0% void fractions; and hence, no identification scheme was attempted for void fractions in the range of 0 to 30%.

Several other approaches to the problem of void fraction identification were considered in addition to the recognition approach discussed in this thesis. One approach consisted of first classifying the various two-phase flows into pattern classes based on their flow regimes. The pattern classes were then divided into void fraction ranges. This method was not chosen for void fraction identification as the classifier could not place two-phase flow samples into the correct pattern classes.

Other approaches included the use of moment invariants and lognormal probability density functions to discriminate among various void fractions. However, neither of these approaches could be implemented.

CHAPTER 3

INSTRUMENTATION AND EXPERIMENTAL RESULTS

Background

The data used in the experiments presented in this thesis were obtained from the Advanced Instrumentation for Reflood Studies (AIRS) Test Stand, which is used for testing two-phase flow instruments under development at Oak Ridge National Laboratory (ORNL) for the AIRS program [9]. The most commonly tested instrument configuration is an instrumented guide tube upon which various types of impedance probes can be mounted.

The main portion of the AIRS Test Stand is shown in Figure 3.1. It consists of vertically mounted piping, a portion of which serves as the test section. The test section contains a bundle assembly of instrumented and uninstrumented rods. The array of rods has flow passages similar to the flow passages of the reflood test facilities in which the instrumented rods are to be used. The void fractions and velocities of the two-phase steam-water mixtures flowing vertically upward through the test bundle are controlled and measured as accurately as possible with conventional instrumentation.

Saturated steam is throttled to a specific pressure before entering the test stand mixing chamber. Demineralized water is injected into the steam as it passes through a mixing chamber. The mixing chamber is upstream of the test section. The steam-water mixture void fractions and velocities are maintained at specified test conditions by controlling the steam flow and water injection rates.

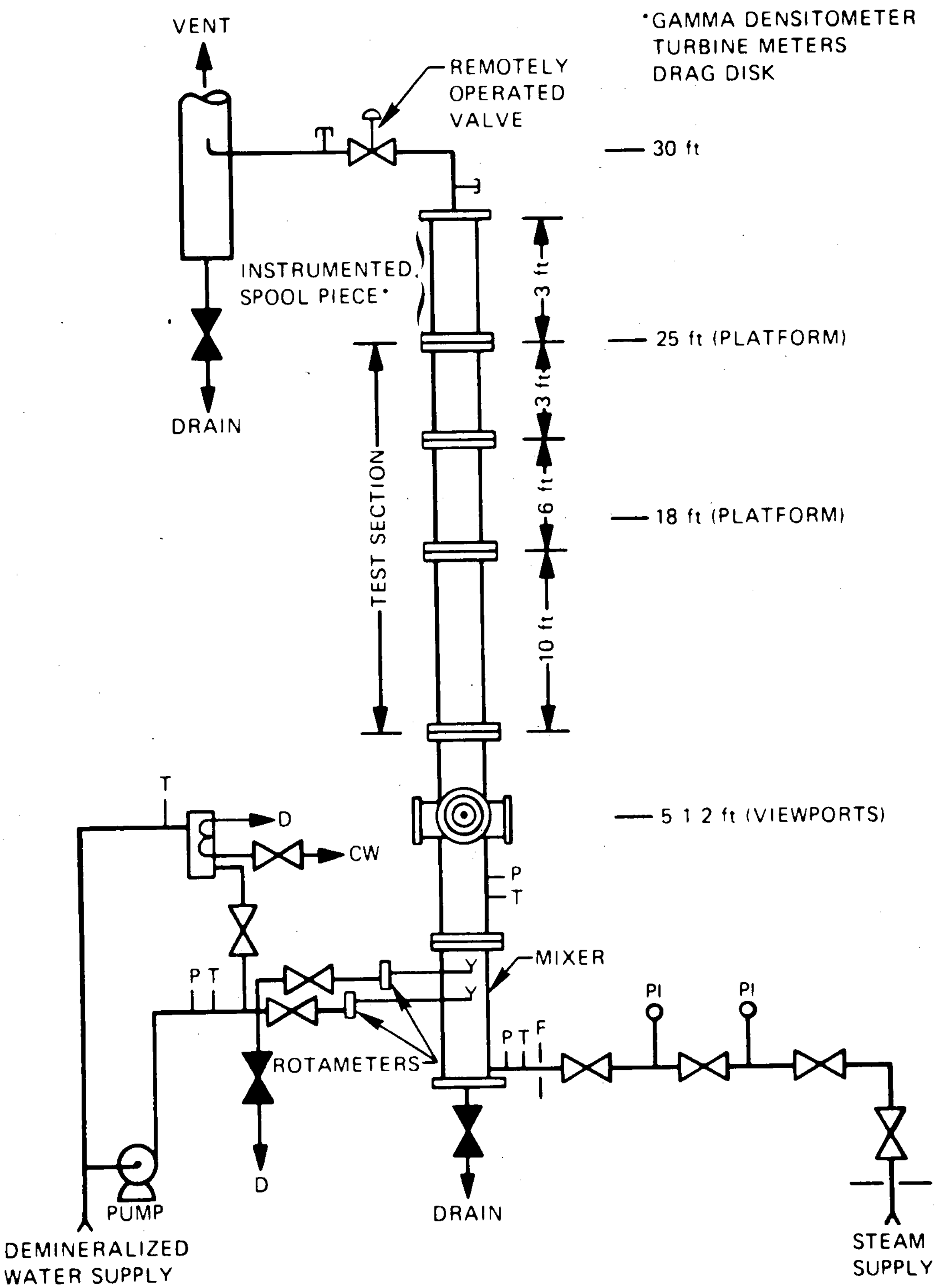


Figure 3.1. AIRS Steam-Water Test Stand.

Steam and water mixtures leaving the test bundle flow through an instrumented spool piece containing a drop disk, thermocouple, pressure transducer, three-beam gamma densitometer and turbine meter. Downstream of the instrumented spool is a throttling valve for flow control. The steam-water mixtures are then exhausted into a large pipe which extends through the roof of the building. Condensate from that line is routed to a building drain.

Test conditions that are possible in the AIRS Test Stand are vertical upflow, maximum temperature, maximum pressure, maximum steam-flow rate, maximum demineralized water injection rate and steam-water void fractions of 0 to 100%. The void fractions are measured with the gamma densitometer while other conditions are evaluated by the electronics system.

Electronics System

The AIRS Electronics System is one component of a total system used for measuring loss-of-coolant and reflood flow rates in non-nuclear pressurized water reactor test facilities. The function of the electronics system is to determine the impedance of a probe located in the two-phase flow by generating an analog signal proportional to the impedance. The probe was developed by the Oak Ridge National Laboratory and can take various forms. Several configurations of sensor probes under development are shown in Figure 3.2.

A probe assembly generally consists of two impedance probes. Analog signals proportional to the magnitude of the impedance of each probe are used to determine the velocity of the two-phase flow. The value of the void fraction, which is a function of the steam-to-water

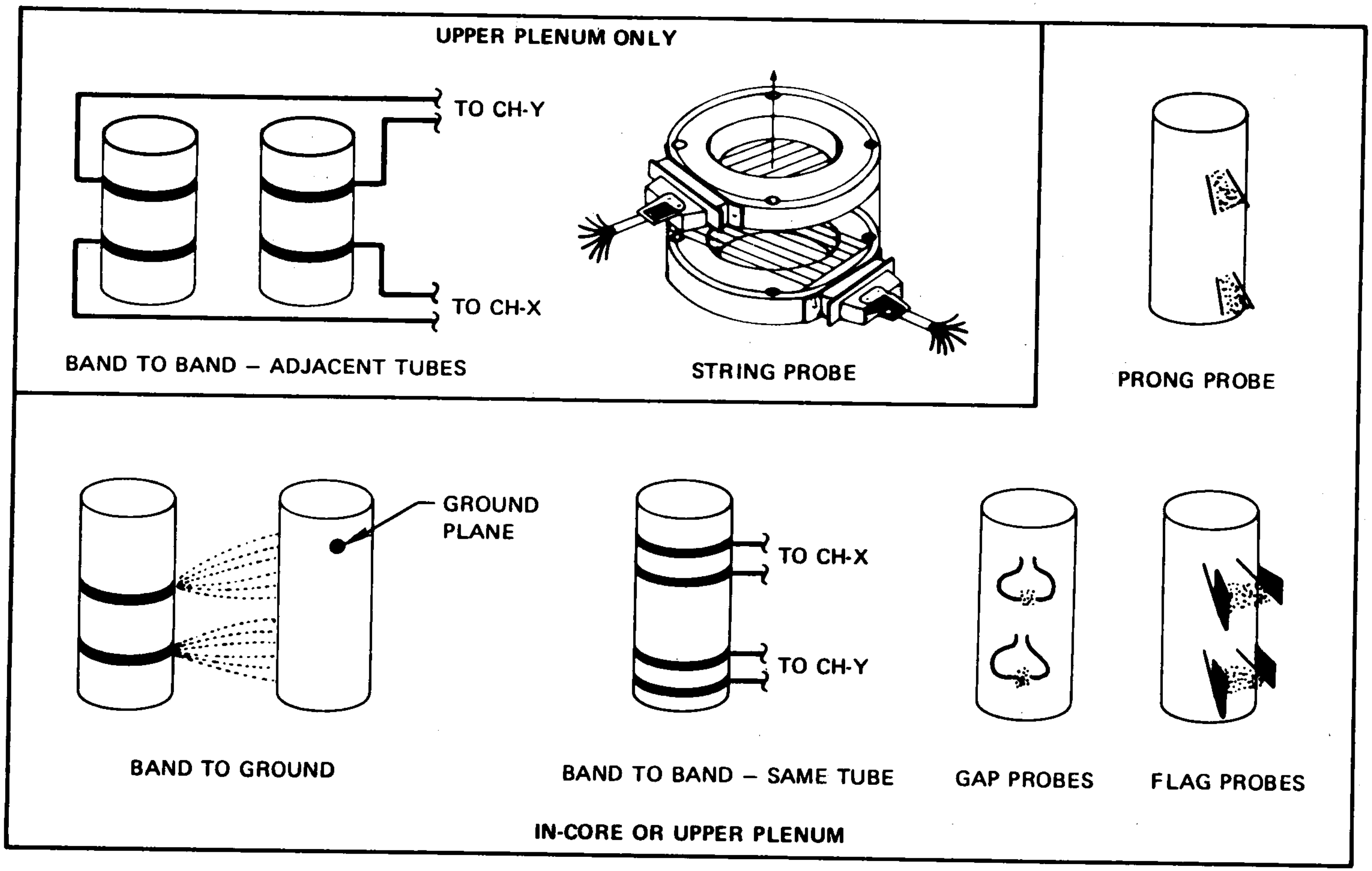


Figure 3.2. A Variety of Sensor Configurations.

ratio of the two-phase flow, depends on both the magnitude and the phase of the probe impedance.

The probe used for this experiment was a flag probe. For void fraction measurements, the flag probe has been shown to have the most reproducible results. The flag probe type and flag probe sensors are shown in Figures 3.3 and 3.4, respectively. These probes are generally located remote from the electronic instrumentation; and by cross-correlation of the electrical output of the two sensors, the velocity and direction of the fluid can be inferred.

The output voltage proportional to the magnitude of the impedance is proportional to the inverse of the capacitance of the signal. The sensitivity of the system to changes in probe capacitance is proportional to the inverse of the probe capacitance squared. Therefore, the AIRS Electronics System is most sensitive to small changes in probe impedance at very high values of void fraction.

Background for Recognition Experiments

As illustration of some of the principles of the machine recognition of void fraction, results are presented showing the formulation and testing of the recognition approach. The data for the experiments was obtained from the AIRS Test Stand. The analog signal was recorded on tape and later digitized with a PDP 11/20 minicomputer. The resolution of the conversion was ten bits. A Rockland Model 852 Dual Hi/Lo Filter was used to filter the signals prior to analog-to-digital conversion. A general description of each signal is presented with the discussion of the corresponding experiment later in this chapter.

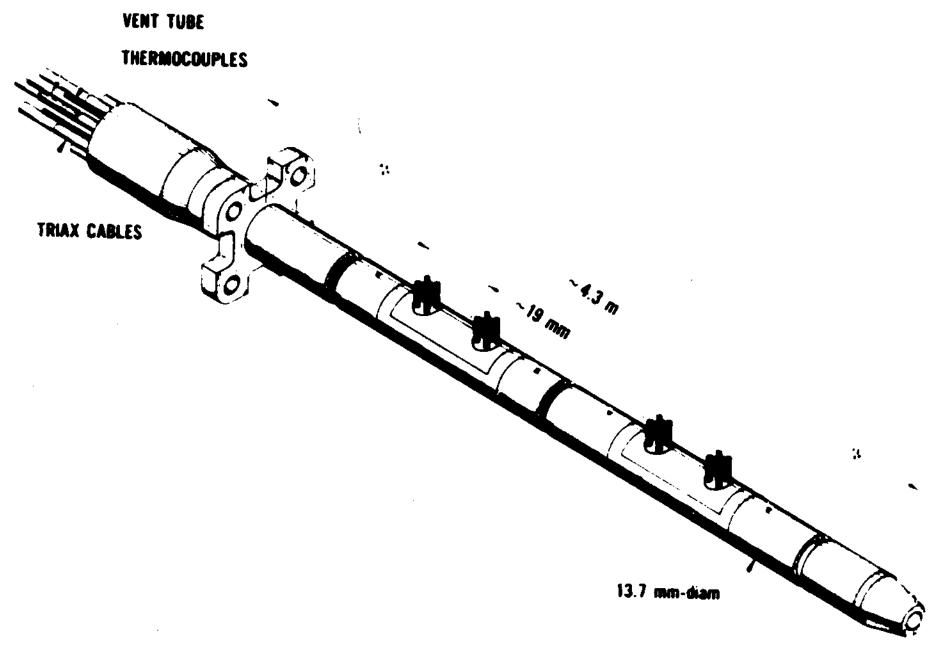


Figure 3.3. In-Core Guide Tube Impedance Measurement Assembly Flag Probe Type.

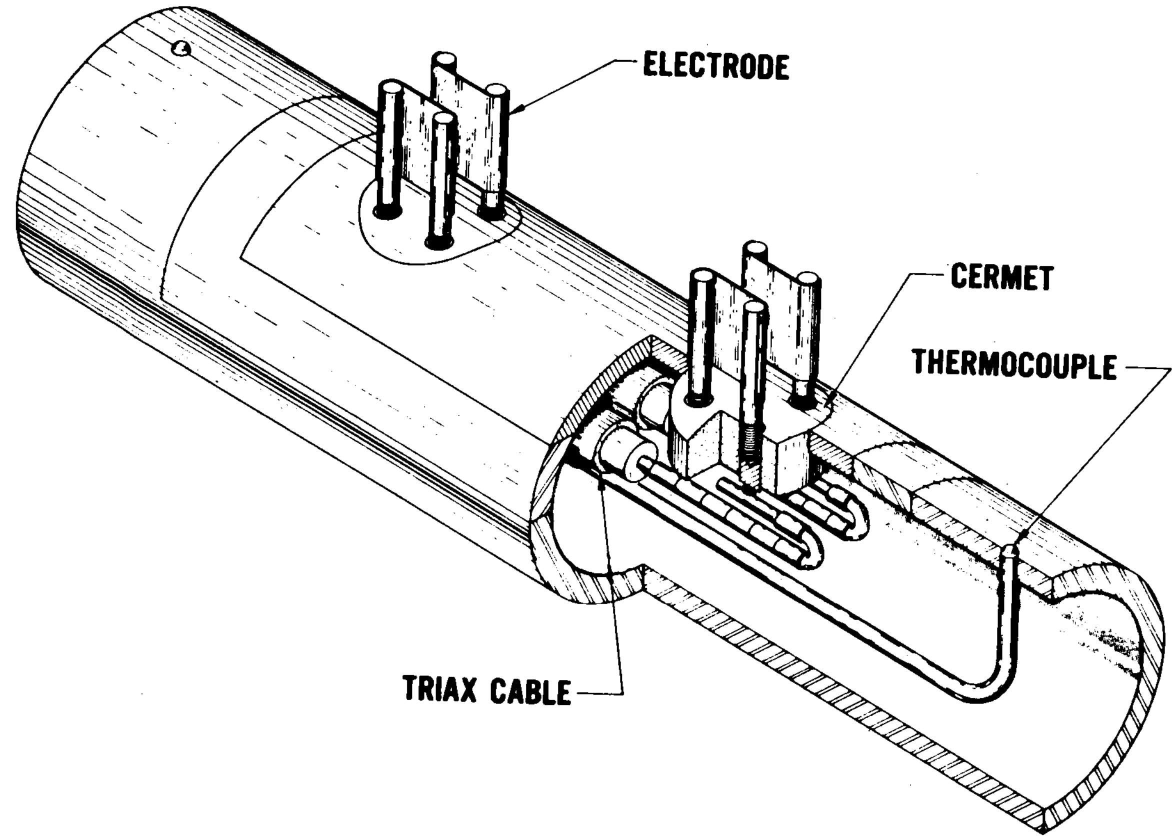


Figure 3.4. In-Core Guide Tube Impedance Measurement Assembly Flag Probe Sensor.

The first experiment describes the results of the recognition approach. The recognition approach was used to evaluate the steam content of a two-phase steam-water flow. Other experiments were concerned with testing and evaluating the recognition approach. One experiment tested the approach with data obtained from three prongs of the flag probe adjacent to the flag probe prong being used for recognition purposes. Another experiment tested the approach with data acquired from different probe configurations, and a third experiment examined the ability of the approach to formulate an accurate void fraction estimate for small segments of data.

Recognition Results

The data for this experiment was obtained from the AIRS Test Stand. The impedance signal which served as the output of the system was digitized with 10,000 samples being taken each second. Two hundred and fifty-six seconds of data were digitized resulting in 2,560,000 points being used to compose a histogram. This high sampling rate was chosen in order to fill consecutive bins in the A/D converter during the digitization of the signal. Sampling rates of less than 10,000 samples per second skipped several A/D bins when the signal jumped sharply from one level to another. The original impedance signal was proportional to a voltage range of 0 to 10 volts and was later attenuated by a factor of 7. After attenuation, 0 volts represented the all water case while 1.414 volts was indicative of all steam.

As the signal was being digitized, a count of the number of digital elements assigned to each particular A/D bin was kept. With this information, a histogram was created for each individual signal. Ninety-four signals were examined, resulting in 94 distinct histograms. The gamma densitometer located at the top of the AIRS Test Stand recorded a void fraction for each signal, thus associating each histogram with a known void fraction. Examples of the histograms along with their void fraction values were shown in Figure 1.2 (page 5). As indicated earlier, no void fractions of less than 30% were contained in the data used in the formulation of the recognition approach. One exception to this was the case of all water or 0% void fraction.

The third and fourth moments were computed from the various histograms representing the 94 runs. Plots of the third moment versus void fraction and the fourth moment versus void fraction are shown in Figures 3.5 and 3.6, respectively. From an observation of the two plots, void fractions in the range of 30 to 70% were seen to be more easily distinguishable if the third moment was used as a means of void fraction identification. For void fractions between 70 and 100%, the fourth moment was observed to be the better descriptor. By observing Figure 3.7, a plot of the fourth moment versus the third moment, this phenomenon can be seen more clearly. The point where the curve breaks sharply upward represents a void fraction value of approximately 70%. Points located to the left of this mark represent void fractions in the range of 70 to 100%, with the fourth moments associated with 100% void fractions lying in the range between 6 X 10^{-9} and 4 X 10^{-8} volts. Void fractions between 30 and 70% had fourth moments ranging in value from

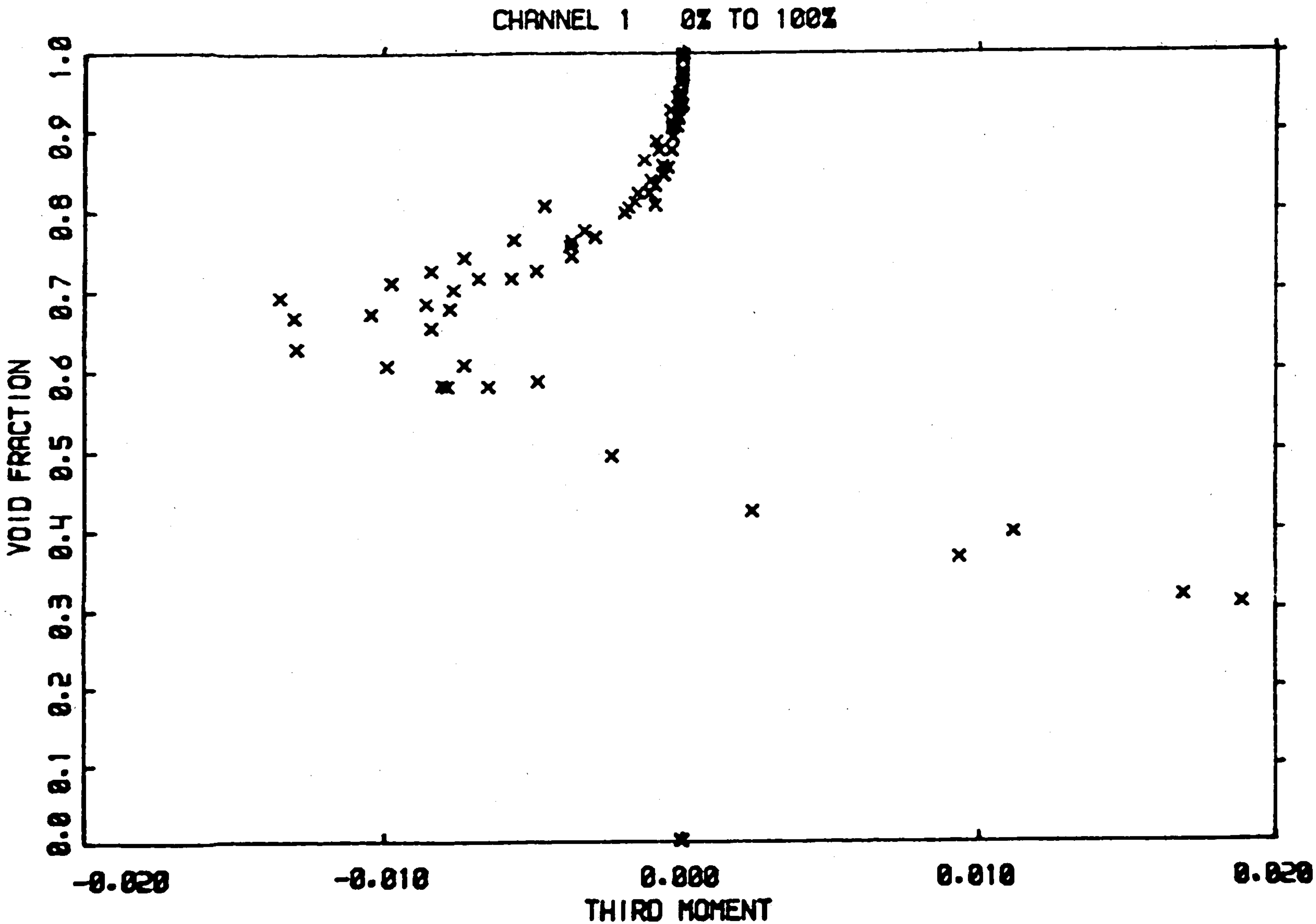


Figure 3.5. Third Moment Versus Void Fraction.

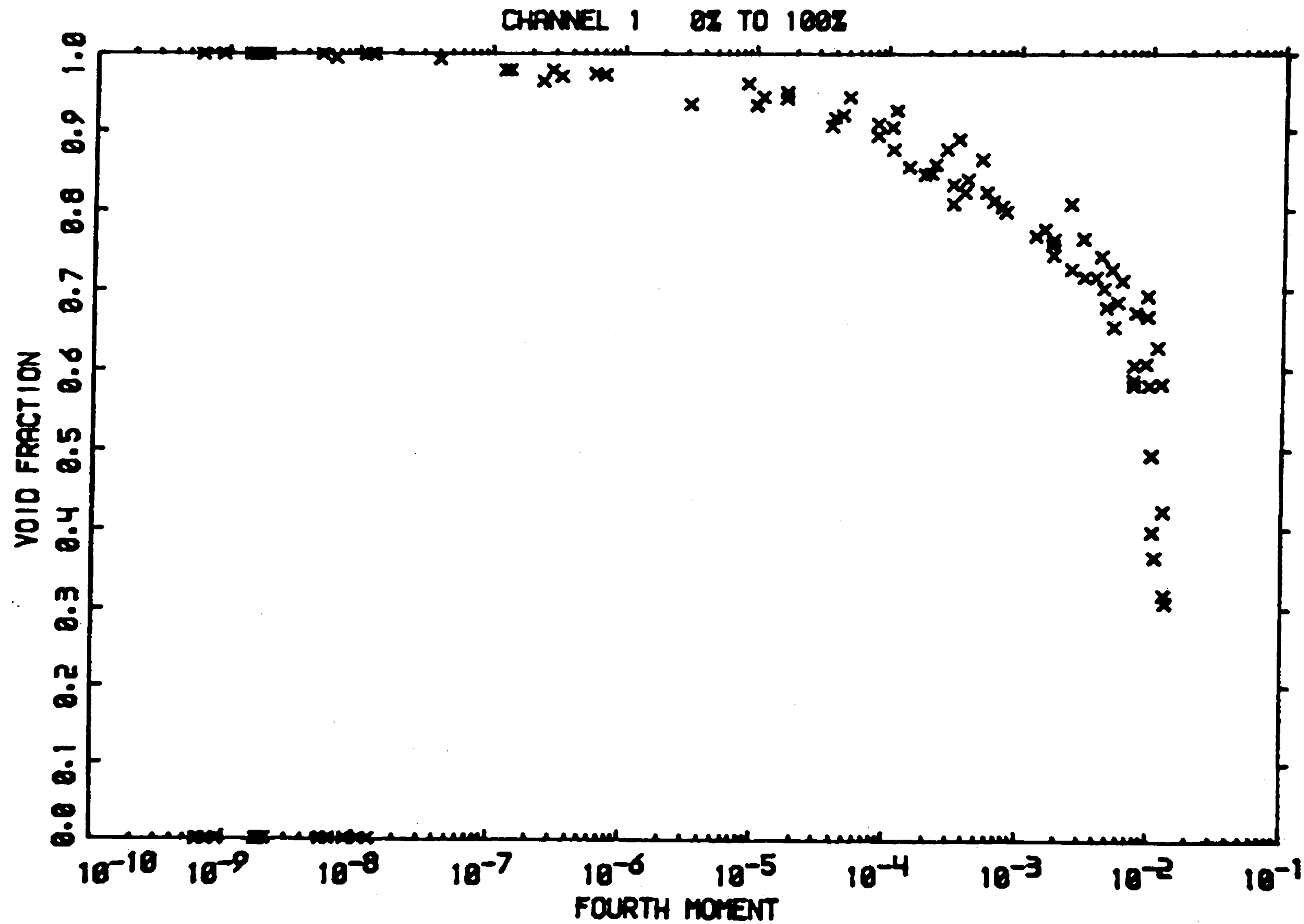


Figure 3.6. Fourth Moment Versus Void Fraction.

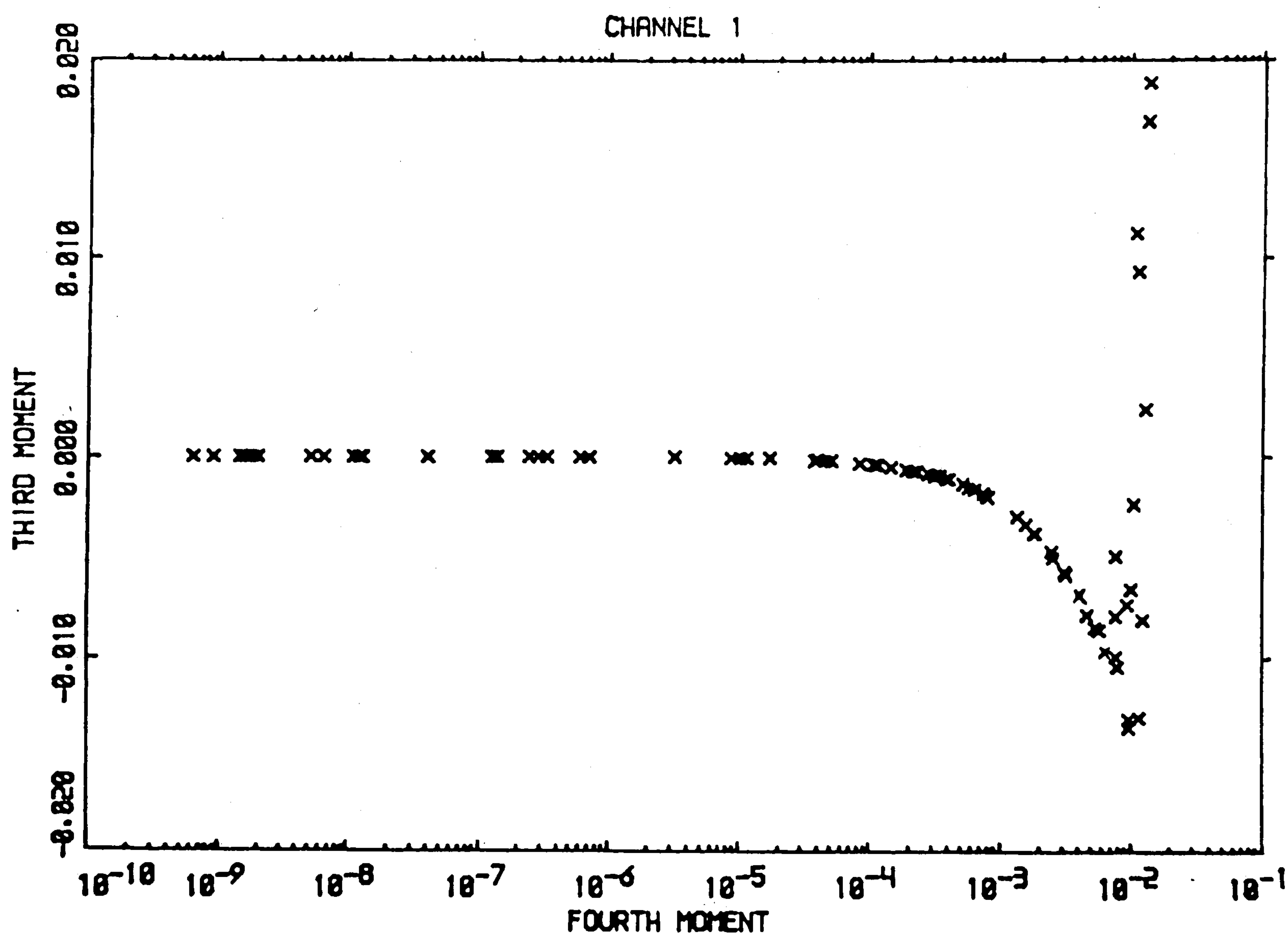


Figure 3.7. Fourth Moment Versus Third Moment.

approximately 4 X 10^{-2} to 2 X 10^{-1} volts, not allowing much of a range for discrimination between the various void fraction values using the fourth moment. As can be seen from the graph, the third moment is a much better descriptor for void fractions in that range.

Using the classification approach described in Chapter 2 and the curve fitting techniques outlined in Appendix A, a curve was fitted through the third moment values for void fractions between 30 and 70%. The best fit or curve which resulted in the smallest residual or sum of the squares of the errors between the known void fraction values and the fitting curve is shown in Figure 3.8. The best fit of a curve for the fourth moment values associated with void fractions of 70 to 100% is shown in Figure 3.9.

Half of the original 94 runs were randomly chosen to create a second recognition approach based on only 47 points. The resulting void fraction curves and their residuals are shown in Figures 3.10 and 3.11, respectively. Figure 3.10 is a plot of the third moment versus void fraction and Figure 3.11 is a plot of the fourth moment versus void fraction.

The equations representing the best curve fit between the third and fourth moments and their associated void fraction values comprise the essence of the recognition approaches presented. The approaches were tested for accuracy with the experiments outlined below.

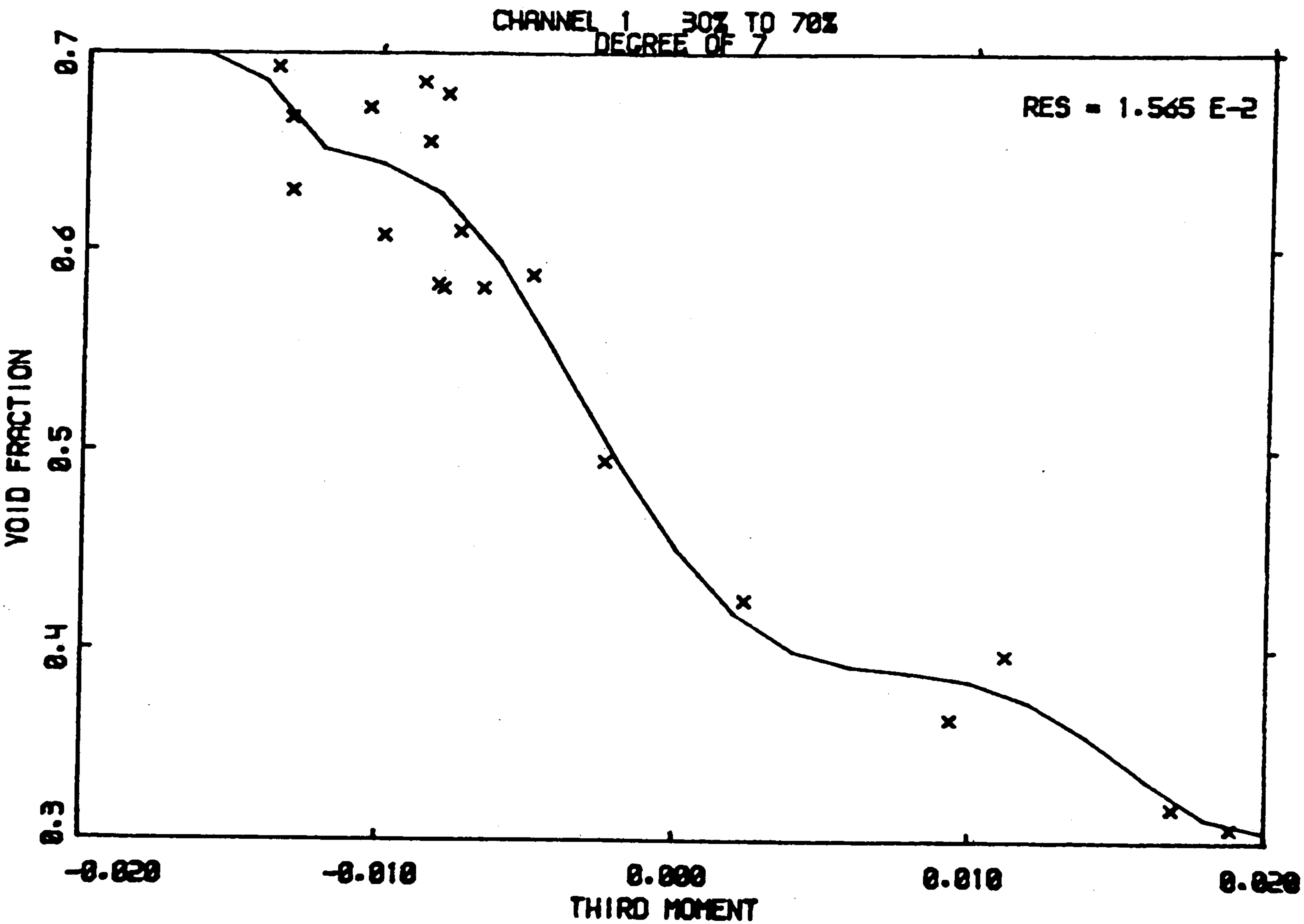


Figure 3.8. Curve Fit for Void Fractions Between 30 and 70% (94 Points).

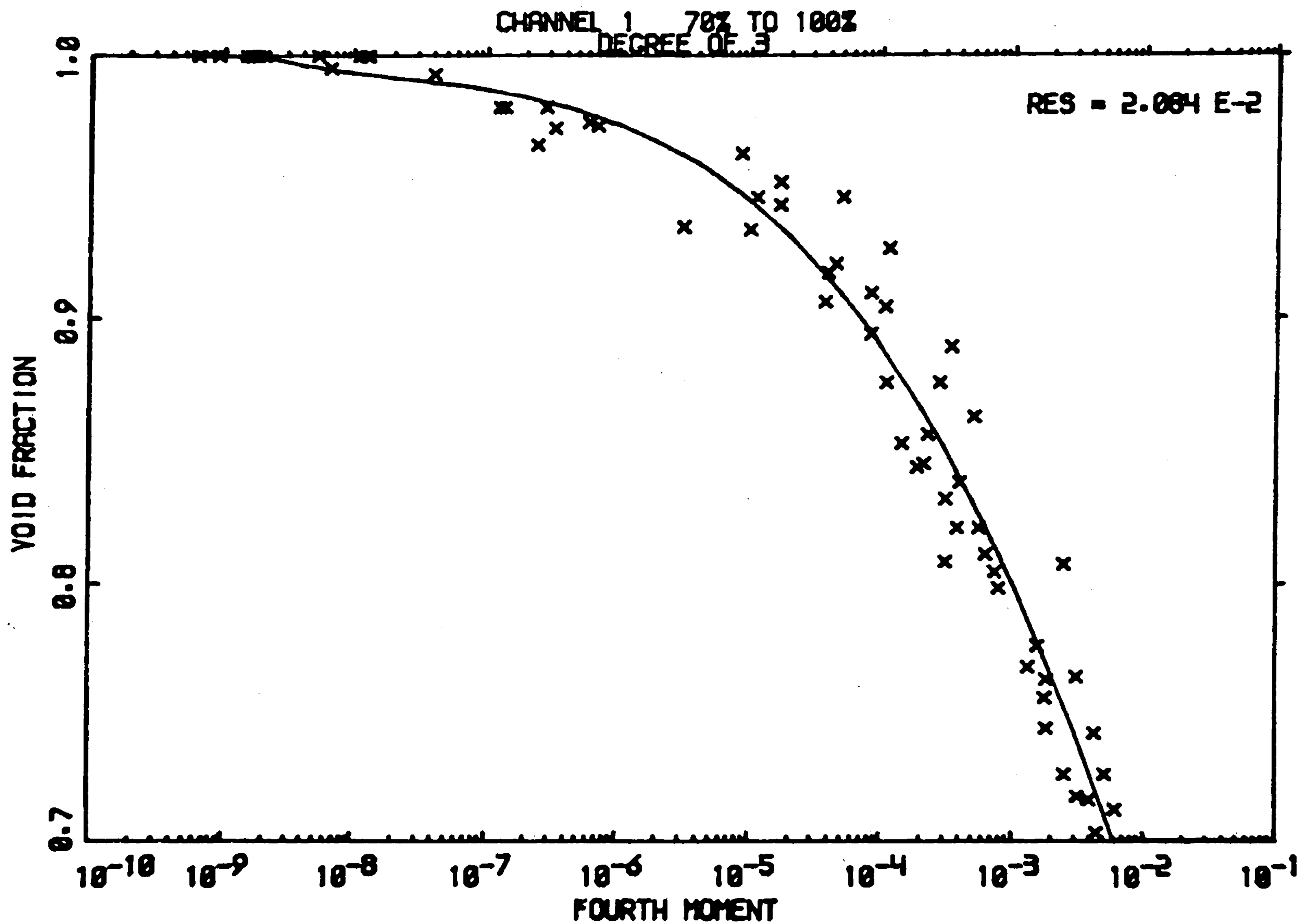


Figure 3.9. Curve Fit for Void Fractions Between 70 and 100% (94 Points).

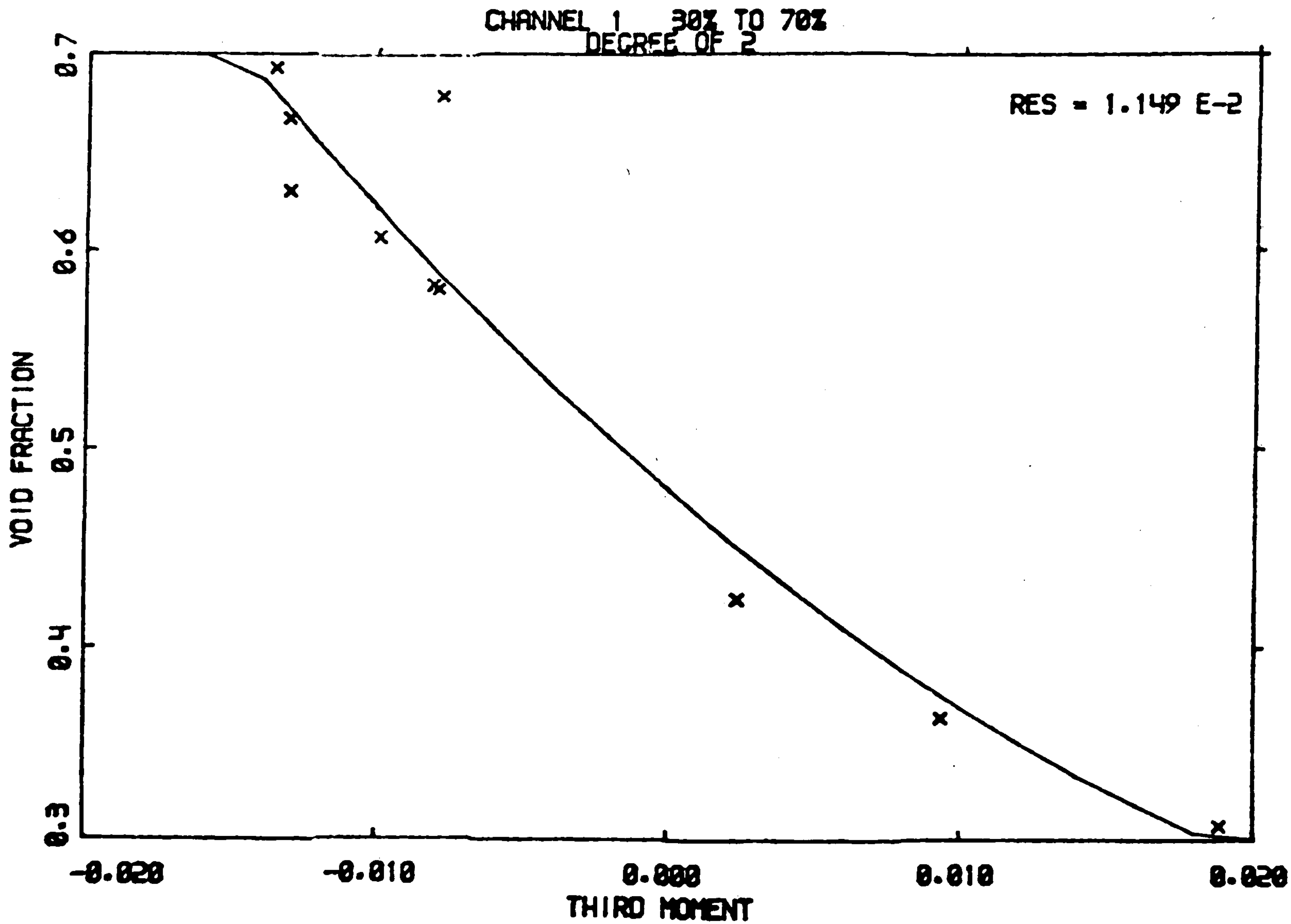


Figure 3.10. Curve Fit for Void Fractions Between 30 and 70% (47 Points).

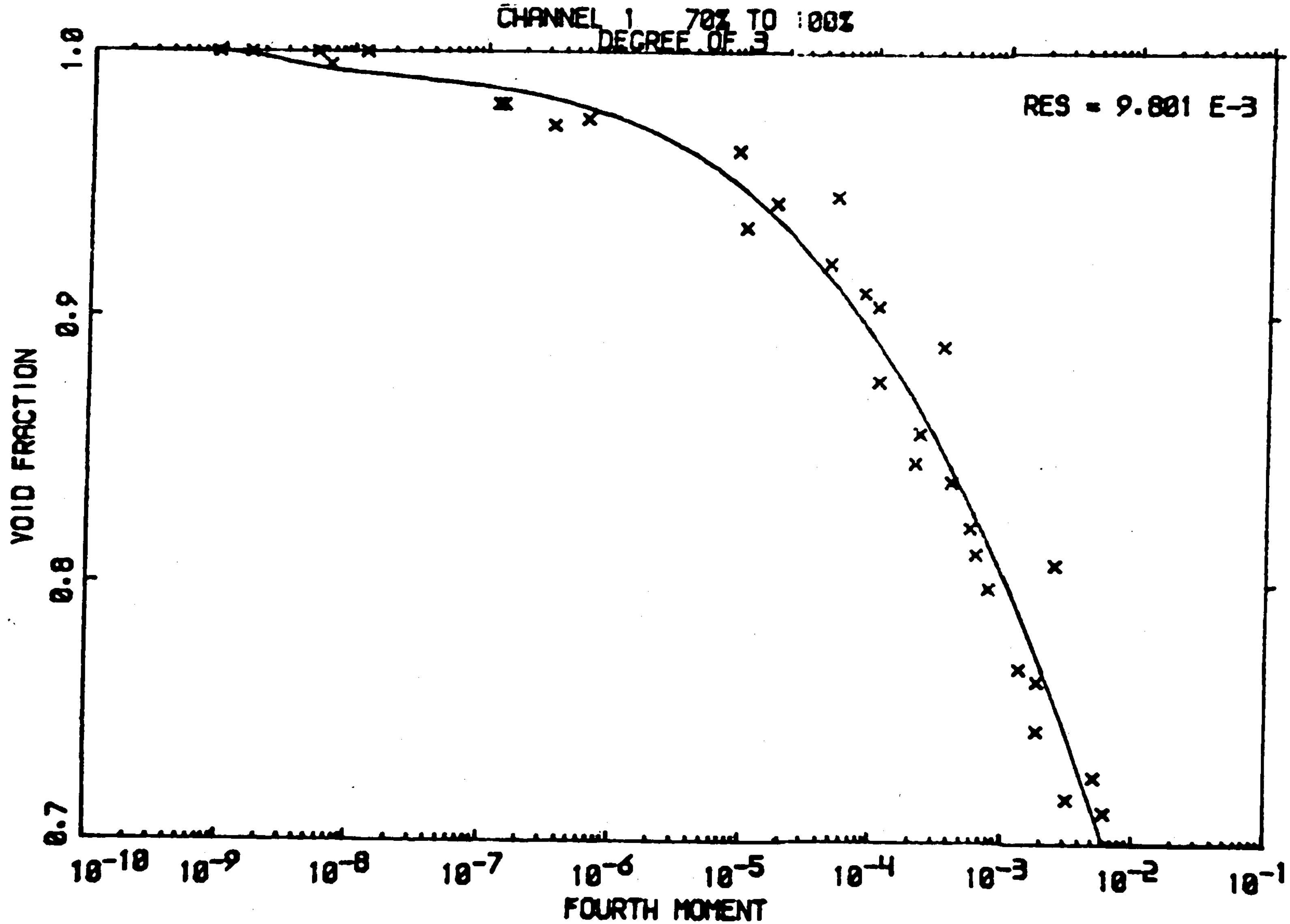


Figure 3.11. Curve Fit for Void Fractions Between 70 and 100% (47 Points).

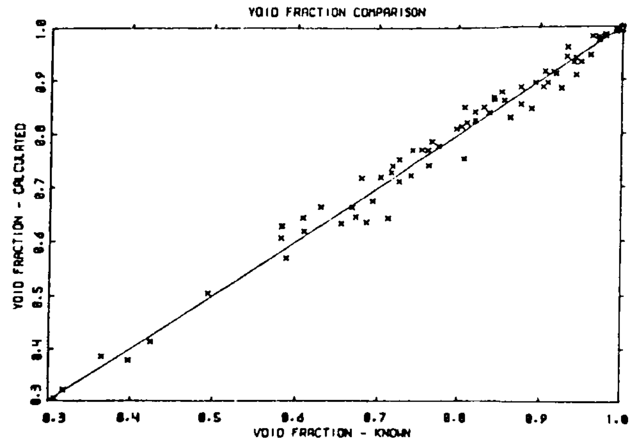
Testing and Evaluation

The flag probe used for recognition purposes consisted of four prongs. Each prong corresponded to one of four channels on the tape recorder. The channel used for the recognition results was channel one. As the flow moved vertically upward, it encountered channels one, two, three and four, in sequence. This experiment consisted of using the data obtained from channels two, three and four of the tape recorder and the recognition equations already developed to obtain void fraction estimates for various test runs. Table 3.1 gives a numerical indication of how the void fractions from these three channels compared with the recognition results obtained with 94 points. The average residual, defined as the difference between

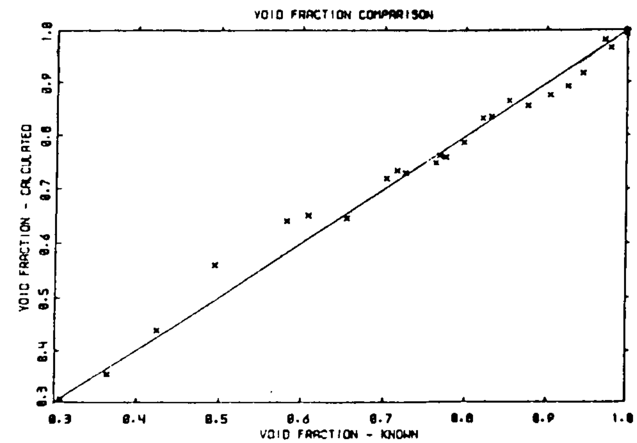
Table 3.1. Recognition Results with 94 Points.

Channel	Average Residual	Average Percent Deviation
1	0.01645	2.265
2	0.01690	2.518
3	0.02649	3.901
4	0.03071	5.226

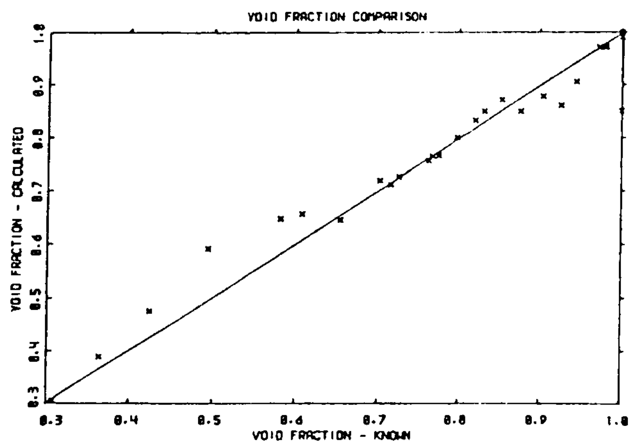
the known and calculated void fraction value, is shown, along with the average percent deviation from the original void fraction value. The graphs shown in Figure 3.12 relate the known and calculated void fractions. If there were a perfect match between the two values, the point representing the two values would lie on the line which indicates a



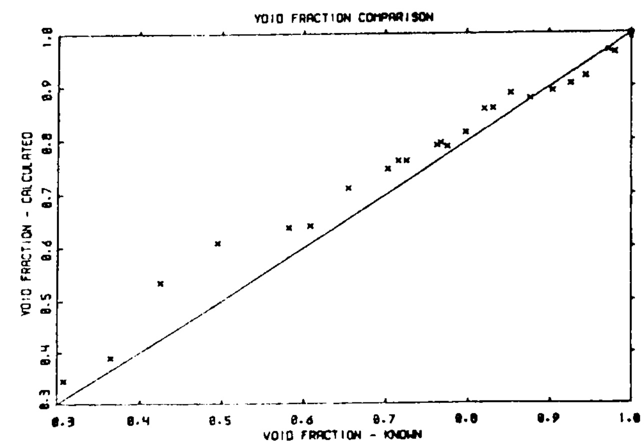
(a) Channel One



(b) Channel Two



(c) Channel Three



(d) Channel Four

Figure 3.12. Known Void Fraction Versus Calculated Void Fraction (94 Points).

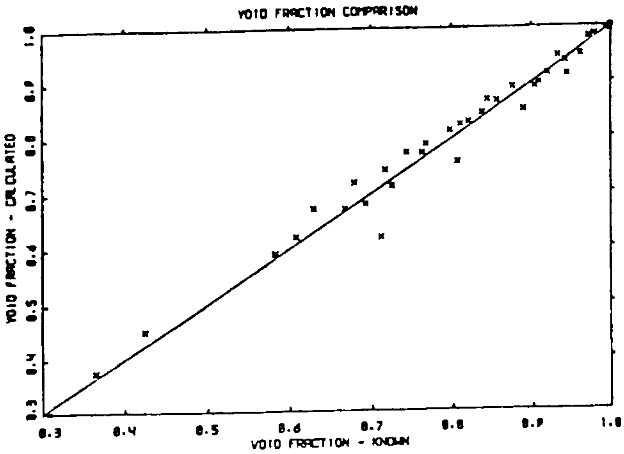
perfect match between the known and calculated void fraction. Part (a) of Figure 3.12 depicts the 94 points used for the initial recognition approach. Parts (b), (c) and (d) of the graph represent testing the initial approach with data from channels two, three and four of the tape recorder, respectively.

Table 3.2 gives an indication of how well the 47 point recognition approach identified void fractions. Figure 3.13 graphically shows the same relationships as did Figure 3.12. Part (a) again represents the void fraction values used to formulate the 47 point recognition approach. Part (b) represents the 47 void fraction values taken from channel one but not used in the recognition process. Parts (c), (d) and (e) represent void fraction values taken from channels two, three and four, respectively.

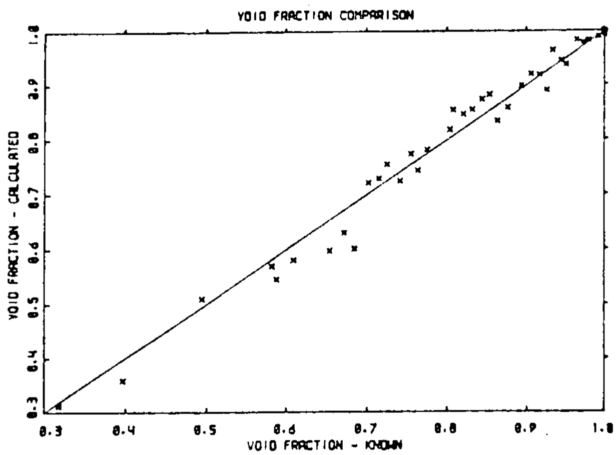
As shown in Figure 3.2 (page 27), there are several different configurations for impedance probes. The probe used in this thesis

Table 3.2. Recognition Results with 47 Points.

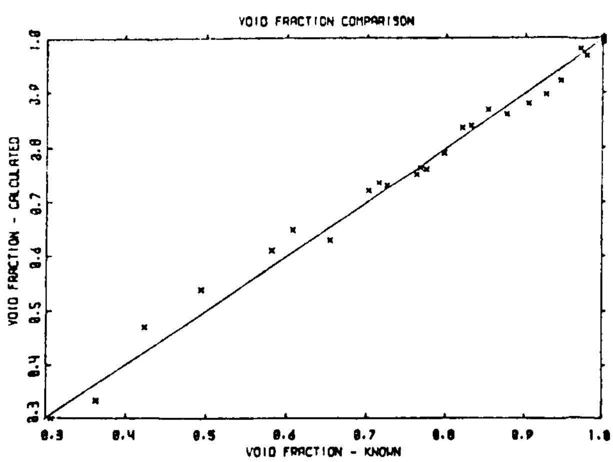
Channel	Average Residual	Average Percent Deviation
1	0.01652	2.277
1	0.01963	2.764
2	0.01758	2.807
3	0.02637	3.931
4	0.02744	4.502



(a) Channel One

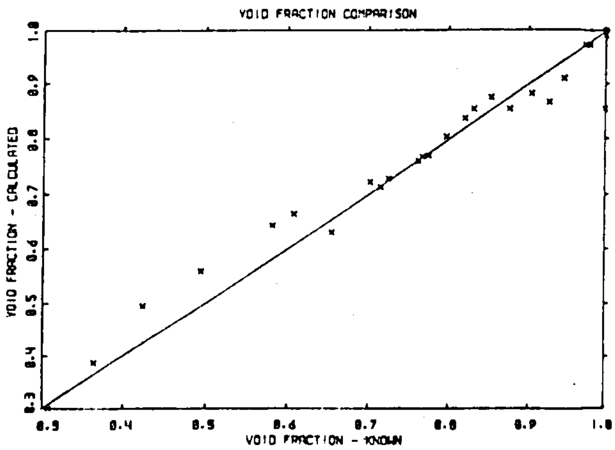


(b) Channel One

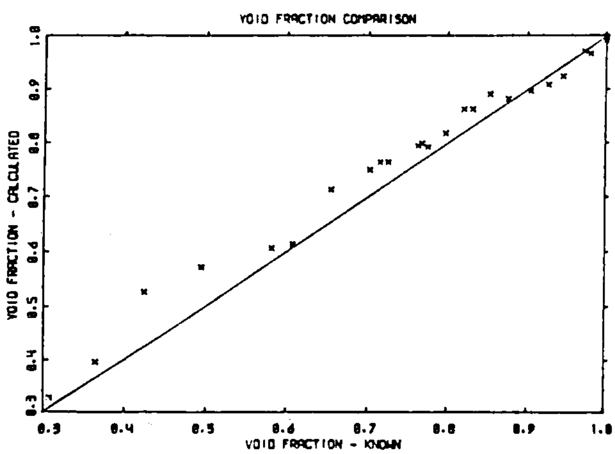


(c) Channel Two

Figure 3.13. Known Void Fraction Versus Calculated Void Fraction (47 Points).



(d) Channel Three



(e) Channel Four

Figure 3.13. (continued)

for the recognition approach was a flag probe. Table 3.3 shows void fraction classification results for data taken with a flag probe, but during a different experiment. Table 3.4 shows classification results for a string probe mounted in the AIRS Test Stand. Only signals with high void fractions were tested against the recognition approach. For signals with low void fractions, the probe became saturated and reliable data could not be obtained.

Figure 3.14 and Figure 3.15 show histograms created from small segments of data. The data for this experiment was composed of sample points acquired at the rate of 1000 samples per second. Part (a) of each figure displays a histogram composed of 256 points, or approximately one quarter of a second of data. The histogram in part (b) contains 512 points or approximately one half second of data. The histograms in parts (c) and (d) exhibit one second and two seconds worth of data, respectively. Table 3.5 and Table 3.6 show the classification results for Figure 3.14 representing a void fraction of 42% and Figure 3.15, whose histograms represent a void fraction of 72%. To a certain extent Table 3.5 and Table 3.6 show that as the size of the sample increases, the accuracy of the void fraction estimate also increases.

Table 3.3. Flag Probe Classification Results.

Average Residual	Average Percent Deviation
0.03475	3.936

Table 3.4. String Probe Classification Results.

Average Residual	Average Percent Deviation
0.04962	5.083

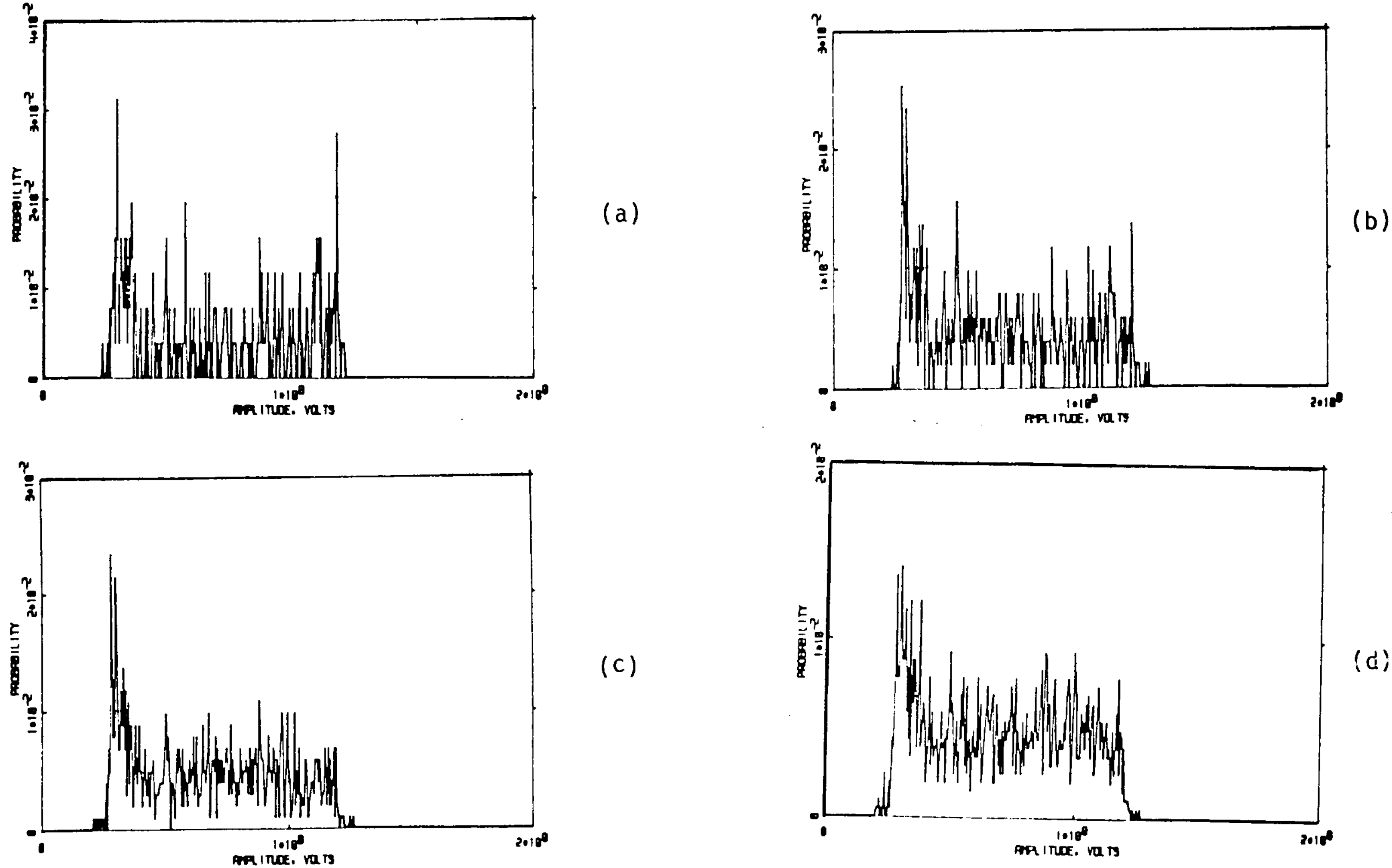


Figure 3.14. Histograms Representing Void Fraction of 42% for (a) 1/4, (b) 1/2, (c) 1 and (d) 2 Seconds of Data.

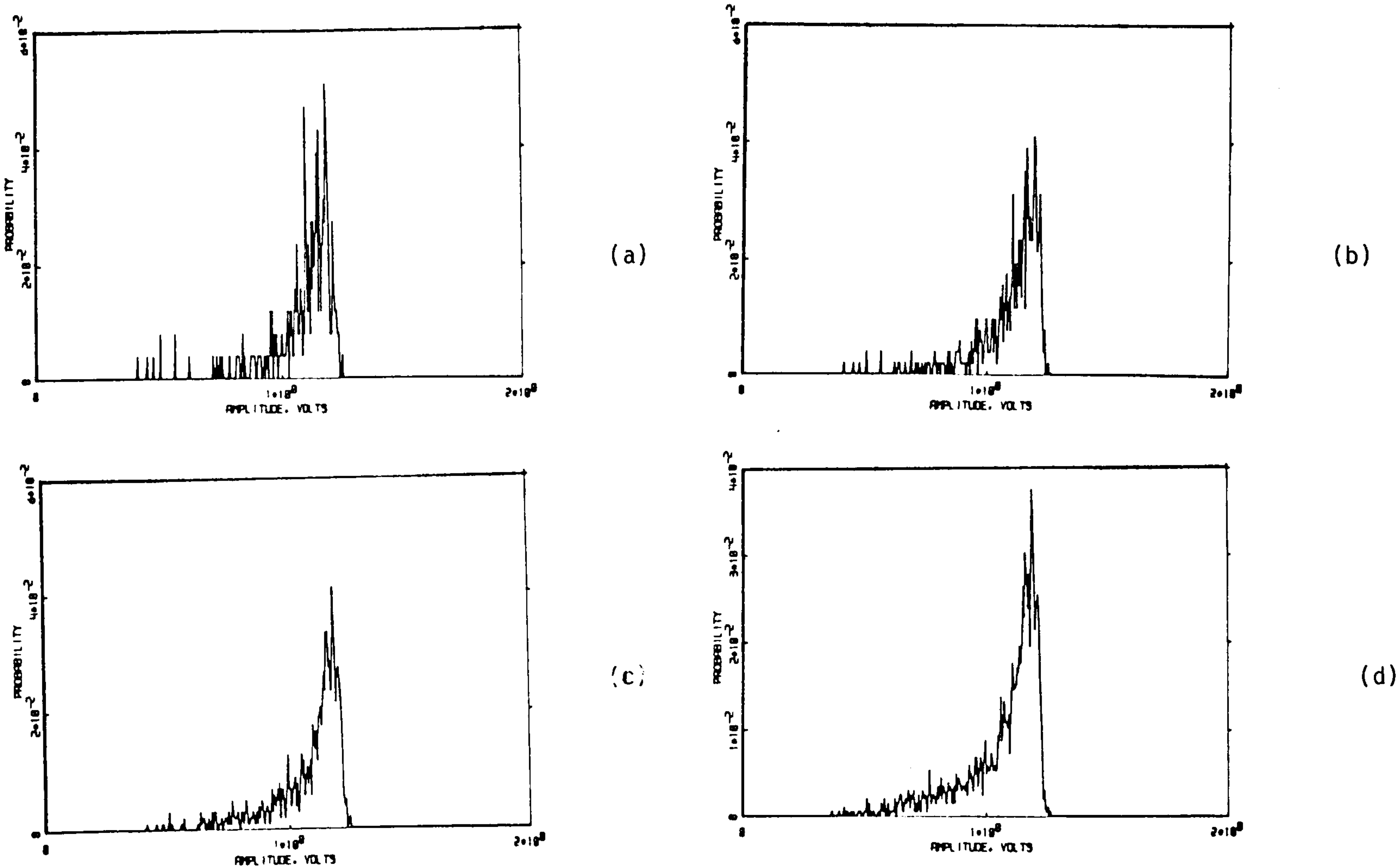


Figure 3.15. Histograms Representing Void Fraction of 72% for (a) 1/4, (b) 1/2, (c) 1 and (d) 2 Seconds of Data .

Table 3.5. Segments of Data with 42% Void Fraction.

Time (sec)	Average Residual	Average Percent Deviation
1/4	0.02111	3.376
1/2	0.02016	3.192
1	0.02248	3.382
2	0.01736	2.676

Table 3.6. Segments of Data with 72% Void Fraction.

Time (sec)	Average Residual	Average Percent Deviation
1/4	0.02277	2.946
1/2	0.02638	3.973
1	0.02092	3.053
2	0.01389	1.977

CHAPTER 4

CONCLUDING REMARKS

A histogram represents an easily obtainable measure of the amplitude of the digitized elements representative of a signal. When comparing a group of signals, a comparison of the histograms representing the various signals can be used to distinguish among the different signals. In particular, the shapes of the various histograms give an indication of the amplitude of the components comprising the signal. By computing the first, third and fourth moments of each histogram, a numerical measure can be obtained whereby each histogram can be characterized by its central tendency, skewness and flatness.

The property of the signals used in this experiment which enabled the use of histograms and their moments was the fact that the histograms of the two-phase flow signals were either positively skewed, symmetrical or negatively skewed. While this was true for two-phase flows composed of steam and water, the histogram shapes for two-phase flows composed of air and water could not be as easily categorized. For example, a histogram representing a slug flow for an air-water mixture has two very distinct peaks, one at either end of the voltage range.

In order for the recognition approach to be useful in the identification of void fraction, the signals being examined must extend over the same voltage range. This is most apparent in the calculation of the fourth moment. The width of the void fraction range is inversely proportional to the magnitude of the fourth moment of a histogram representing an impedance signal. Hence, as the signal is extended over an increasingly wider range, the value of its fourth moment will become increasingly smaller. This will allow finer distinctions to be made among histograms having almost the same shape.

The major difficulty in using the method described in this thesis for void fraction identification lies in the impedance probes and the response of these probes to two-phase flow mixtures. Often the probes become coated or saturated by the two-phase flow, rendering inaccurate the output signal of the impedance probe. The benefits of this method arise from the ease of implementation and the relevance of the information interpreting the steam content of a two-phase steam-water flow.

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APPENDICES

APPENDIX A

CURVE FITTING

The term curve fitting refers to the process of finding a mathematical formula for approximating a set of data values or for approximating another mathematical formula [10]. The method of least squares is used for fitting both data and functions. The criterion of best fit of the least squares method is that the sum of the squares of the errors be a minimum. When expressed mathematically, the quantity is called a residual. The primary advantage of the method of least squares is that methods of calculus can be employed to find the minimum sum of squares of errors. This is done in the following manner.

Suppose that a set of values y_i which depend on certain other variables $x_1, x_2, \ldots, x_n$ are to be represented by some function such as

$$u = g(x_1, x_2, \ldots, x_n). \tag{A.1}$$

This equation will also contain certain parameters a_j that must be determined. Therefore, Equation A.1 can be rewritten as follows,

$$u = g(a_1, a_2, \ldots, a_k, x_1, x_2, \ldots, x_n). \tag{A.2}$$

This equation is called the equation of condition. Let S be the sum of the squares of all errors

$$S = \sum_{i=1}^{N} (y_i - u_i)^2 \tag{A.3}$$

where N is the total number of cases. Substituting Equation A.2, evaluated for the i^{th} set of independent variables into Equation A.3

yields

$$S = \sum_{i=1}^{N} (y_i - g(a_1,a_2,\ldots,a_k,x_{1i},x_{2i},\ldots,x_{ni}))^2. \qquad (A.4)$$

A necessary condition that a minimum for the function S exists is that the partial derivatives with respect to each of the parameters $a_1,a_2,\ldots,a_k$ be zero. The equations

$$\begin{aligned} \frac{\delta S}{\delta a_1} &= 0 \\ \frac{\delta S}{\delta a_2} &= 0 \\ \vdots\quad & \quad\vdots \\ \frac{\delta S}{\delta a_k} &= 0 \end{aligned} \qquad (A.5)$$

are satisfied only when S is minimum. Equation A.5 gives k equations in k unknowns.

The following examples of equations of condition, although non-linear functions of the independent variables, are linear functions of the parameters.

$$\begin{aligned} y &= a_1 + a_2x_1 + a_3x_2 + a_4x_1x_2 \\ y &= a_1 + a_2x + a_3x^2 + a_4x^3 \\ y &= a_1 \sin x + a_2. \end{aligned} \qquad (A.6)$$

These equations can be reduced to the general form

$$y = a_1 + a_2x_1 + a_3x_2 + a_4x_3 + \ldots \qquad (A.7)$$

by making suitable substitutions such as $x_3 = x_1x_2$, $x_2 = x^2$, $x_3 = x^3$ and $x_1 = \sin x$. The function S then becomes

$$S = \sum_{i=1}^{N} (y_i - a_1 - a_2x_{1i} - \ldots - a_{n+1}x_{ni})^2. \tag{A.8}$$

By taking partial derivatives of Equation A.8, the following equations can be formed

$$\frac{\delta S}{\delta a_1} = -2 \sum_{i=1}^{N} (y_i - a_1 - a_2x_{1i} - \ldots -a_{n+1}x_{ni}) = 0$$

$$\frac{\delta S}{\delta a_2} = -2 \sum_{i=1}^{N} x_{1i}(y_i - a_1 - a_2x_{1i} - \ldots - a_{n+1}x_{ni}) = 0$$

$$\frac{\delta S}{\delta a_3} = -2 \sum_{i=1}^{N} x_{2i}(y_i - a_1 - a_2x_{1i} - \ldots - a_{n+1}x_{ni}) = 0 \tag{A.9}$$

$$\vdots$$

$$\frac{\delta S}{\delta a_{n+1}} = -2 \sum_{i=1}^{N} x_{ni}(y_i - a_1 - a_2x_{1i} - \ldots - a_{n+1}x_{ni}) = 0$$

Equations A.9 can be simplified to the following form, where the limits i=1 to i=N have been omitted from summations and subscripts on the variables for simplicity in writing.

$$a_1N + a_2 \sum x_1 + a_3 \sum x_2 + \ldots + a_{n+1} \sum x_n = \sum y$$

$$a_1 \sum x_1 + a_2 \sum x_1^2 + a_3 \sum x_1x_2 + \ldots + a_{n+1} \sum x_1x_n = \sum x_1y$$

$$a_1 \sum x_2 + a_2 \sum x_1x_2 + a_3 \sum x_2^2 + \ldots + a_{n+1} \sum x_2x_n = \sum x_2y$$

$$\vdots \tag{A.10}$$

$$a_1 \sum x_n + a_2 \sum x_1x_n + a_3 \sum x_2x_n + \ldots + a_{n+1} \sum x_n^2 = \sum x_ny.$$

Equations A.10 represent a set of n+1 simultaneous linear equations in n+1 unknowns $a_1, a_2, \ldots, a_{n+1}$. In order to solve the simultaneous equations, write the coefficients into the matrix

$$[X] \begin{bmatrix} N & \sum x_1 & \sum x_2 & \sum x_3 & \ldots \sum x_n \\ \sum x_1 & \sum x_1^2 & \sum x_1 x_2 & \sum x_1 x_3 & \ldots \sum x_1 x_n \\ \sum x_2 & \sum x_1 x_2 & \sum x_2{}^2 & \sum x_2 x_3 & \ldots \sum x_2 x_n \\ \vdots & & & & \vdots \\ \sum x_n & \sum x_1 x_n & \sum x_2 x_n & \sum x_3 x_n & \ldots \sum x_n^2 \end{bmatrix} . \tag{A.11}$$

Define a (n+1) X 1 column matrix called [A] and another (n+1) X 1 matrix called [Y] as follows:

$$[A] = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{n+1} \end{bmatrix} \quad [Y] = \begin{bmatrix} \sum y \\ \sum x_1 y \\ \sum x_2 y \\ \vdots \\ \sum x_n y \end{bmatrix} . \tag{A.12}$$

Because of the special way in which matrix multiplication is defined, Equation A.7 could be rewritten as

$$[X]\,[A] = [Y]. \tag{A.13}$$

Thus, for example, the first term in [Y], the term $\sum y$, can be obtained by a term-by-term multiplication of the first row of [X], containing the terms N, $\sum x_1$, ..., $\sum x_n$, by the first, and only, column of [A], containing the terms $a_1, a_2, \ldots, a_{n+1}$, and adding the partial products. This is exactly the same as writing the first equation in A.10

$$a_1 N + a_2 \sum x_1 + a_3 \sum x_2 + \ldots + a_{n+1} \sum x_n = \sum y. \qquad \text{(A.14)}$$

Since division is not defined for matrices, the form [X] [A] = [Y] can be solved by multiplying by the inverse of [X].

$$\begin{aligned} &[X]\,[A] = [Y] \\ &[X]^{-1}\,[X]\,[A] = [X]^{-1}\,[Y] \qquad \text{(A.15)} \\ &[A] = [X]^{-1}\,[Y]. \end{aligned}$$

Once the values for [A] have been determined, the value of y in Equation A.7, the general form, can be evaluated. This value can then be compared with the observed value of y. When, for different equations of condition, the sum of the squares of the errors between the observed value and computed values is a minimum, the method of least squares fulfills the criterion of best fit.

As an actual numerical example, consider the values listed in Table A.1 and fit the data with a straight line.

Table A.1. Experimental Data.

x	y
0	1.10
1	1.47
2	2.08
3	2.53
4	3.06

The equation of condition would be

$$y = a_0 + a_1 x. \tag{A.16}$$

The sum of the squares of the errors is as follows:

$$S = \sum_{i=1}^{5} (y_i - a_0 - a_1 x_i)^2. \tag{A.17}$$

Taking the partial derivatives of Equation A.17, the following equations can be formed.

$$\begin{aligned} \frac{\partial S}{\partial a_0} &= -2 \sum_{i=1}^{5} (y_i - a_0 - a_1 x_i) = 0 \\ \frac{\partial S}{\partial a_1} &= -2\, x_i \sum_{i=1}^{5} (y_i - a_0 - a_1 x_i) = 0. \end{aligned} \tag{A.18}$$

Equations A.18 can be simplified to the following form.

$$\begin{aligned} 5a_0 + a_1 \sum x &= \sum y \\ a_0 \sum x + a_1 \sum x^2 &= \sum xy. \end{aligned} \tag{A.19}$$

Evaluating the summations for the data in Table A.1, the equations become

$$\begin{aligned} 5a_0 + 10a_1 &= 10.24 \\ 10a_0 + 30a_1 &= 25.46. \end{aligned} \tag{A.20}$$

To solve the equations in A.20, place the coefficients of the unknowns in a 2 X 2 matrix, [X], and the unknowns and summations for y in 2 X 1 matrices, [A] and [Y], respectively.

$$[X] = \begin{bmatrix} 5 & 10 \\ 10 & 30 \end{bmatrix} \qquad [A] = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \qquad [Y] = \begin{bmatrix} 10.24 \\ 25.46 \end{bmatrix}. \tag{A.21}$$

The values in [A] can be determined from Equation A.15.

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 30 & -10 \\ -10 & 5 \end{bmatrix} \begin{bmatrix} 10.24 \\ 25.46 \end{bmatrix}. \tag{A.22}$$

Solution of Equation A.22 yields $a_0 = 1.052$ and $a_1 = 0.498$. The sum of the squares of the errors is 0.009544, and this is the smallest value that can be obtained.

APPENDIX B

PROGRAM DESCRIPTIONS

Four basic programs were used in this thesis for the machine recognition of void fraction. One program samples the data and creates a histogram. A second program calculates the first through the fourth central moments of the histogram. Once the moments have been calculated, a third program makes use of curve fitting techniques to determine the coefficients for an equation relating the void fraction to the values of the third and fourth moments. Classification is performed with a fourth program. Each program is described in detail in the following pages.

There are several programs used in conjunction with the four main programs. Some are concerned with the creation of input and output files while others aid in the sampling of the data. One subroutine used during the curve fitting process inverts matrices. The plotting subroutines are part of a system subroutine package containing the elements necessary for the creation of a plot. The programs were written for use on a PDP 11/20 or PDP 11/34 minicomputer. They could be used on a different type of computer with slight modifications.

B.1. Program HGRMD.FOR

This program samples one channel of an analog-to-digital converter whose voltage range is 0 to 5 volts. By keeping a count of the total number of digitized elements and the number of elements falling within the various A/D voltage bins, the frequency for any particular bin can be determined. This is done by dividing the number of elements falling within a particular voltage bin by the total number of digitized elements. If the voltage range and frequency of all of the bins are plotted, a histogram will result. The output of this program is a plot of the histogram and a file containing run identification information and the frequency of each voltage bin of the A/D converter. The run identification information consists of a run number, channel number, water flow rate, pattern class number and void fraction, and is present in all of the input and output files used for the machine recognition of void fraction. HGRMD.FOR was originally written by J. A. Mullens and has been revised for void fraction identification purposes. The subroutines associated with HGRMD.FOR, which aid in the sampling process, were also written by J. A. Mullens and are not listed in this thesis.

B.2. Program MOMENT.FOR

This program is used to compute the first through the fourth central moments of a histogram. The first moment is the mean, and once it has been determined, it is taken as a new origin and all higher moments are referred to this origin. The input to the program is a file which contains, in addition to the run identification information, the frequency of each voltage bin in the A/D converter. The output of the program is a file which contains the run identification information and the values of the first four central moments for specified histograms. In order to calculate moments of order higher than four, the array containing the central moment values must be expanded, as well as the index function of several DO loops. It is assumed that the input and output files contain the same number of records. Thus, the location of information for a specific run number is the same in either the input or output file. For example, if information pertaining to run number 74 is found in record number 8 of the input file, or histogram file, the corresponding moments for run number 74 are located in record number 8 of the output file.

B.3. Program PLTMOM.FOR

The purpose of this program is to determine a relationship between a set of numerical values and the void fraction of a two-phase flow. The numerical values are the third and fourth moments computed from a histogram representative of a two-phase flow. Associated with each histogram is a known void fraction. Making use of curve fitting techniques using the method of least squares, a relationship can be found between the third and fourth moments and the void fractions associated with a set of histograms. The final result of this program is a set of coefficients for a polynomial equation of a single variable expressing the void fraction as a function of either the third or the fourth moment. In order to determine coefficients for the equation of a curve representing the best fit of a line through a set of data points, the program has been designed to test equations of various degrees. It was found that since some data sets contained only a few points, equations of order greater than seven often resulted in void fractions which were not unique functions of a single moment value. Thus, the program is structured only for equations of order one through seven. Three graphs are produced by this program. The first graph is a plot of the third moment versus void fraction. The second graph plots the fourth moment versus void fraction, and the third graph is a plot of the fourth moment versus the third moment.

B.4. Program CLASS.FOR

This program is used to compute a void fraction for a two-phase flow. The equations necessary to calculate the void fraction were determined in PLTMOM.FOR, and the coefficients of these equations serve as input to this program. Also used as input are the third and fourth moments of two-phase flow signals for which void fractions are to be determined. Depending upon the value of the fourth moment, one of two equations is used to compute the void fraction. In order to determine which equation to use, the one expressing the void fraction as a function of the third moment or the one expressing the void fraction as a function of the fourth moment, the fourth moment is compared to a single value determined during the training phase of the machine recognition of void fraction. The equation using the third moment is for classifying void fractions in the range of 30 to 70%. The equation using the fourth moment classifies void fractions in the range of 70 to 100%. This program was not designed to compute void fractions of less than 30%. The output of this program is a listing of known and calculated void fractions, run identification information consisting of a run number and channel number, the difference between the two void fraction values and the percent deviation between the calculated void fraction and the fitting curve.

APPENDIX C

PROGRAM LISTINGS

```
      PROGRAM HGRMD
C
C***********************************************************************
C
C     THIS PROGRAM SAMPLES CHANNEL ONE OF THE AR11 ANALOG-
C     TO-DIGITAL CONVERTER (ANALOG VOLTAGE RANGE IS ZERO
C     TO FIVE VOLTS).  FROM THE SAMPLED VALUES, A HISTOGRAM
C     IS CREATED.  THE OUTPUT OF THE PROGRAM IS A FILE
C     CONTAINING RUN IDENTIFICATION INFORMATION AND THE
C     RELATIVE FREQUENCY FOR EACH LEVEL USED IN THE A/D
C     CONVERTER.  THE RUN IDENTIFICATION INFORMATION
C     CONSISTS OF A RUN NUMBER, CHANNEL NUMBER, WATER FLOW
C     RATE, PATTERN CLASS NUMBER AND VOID FRACTION VALUE.
C
C     THE PROGRAM LINKS WITH AR11CK, AD1 (MODIFIED FOR
C     ZERO TO FIVE VOLT SAMPLING), HG1, HGRM1, OUTPUT
C     AND MODPLT (CONTAINING SEVERAL SYSTEM SUBROUTINES).
C     AR11CK, AD1, HG1 AND HGRM1 WERE WRITTEN BY
C     J. A. MULLENS AND ARE NOT LISTED IN THE PROGRAM LISTINGS.
C
C     AR11CK DETERMINES WHICH BITS TO SET FOR THE AR11 CLOCK
C     STATUS REGISTER AND PRESET REGISTER TO OBTAIN THE
C     DESIRED SAMPLING RATE.
C
C     AD1 PERFORMS A/D SAMPLING FOR CHANNEL ONE OF THE AR11
C     AND STORES THE SAMPLES IN CORE MEMORY.
C
C     HG1 OBTAINS THE DATA FROM THE SAMPLER AND PLACES IT IN
C     A BIN CORRESPONDING TO A PARTICULAR VOLTAGE VALUE.
C     THE RESULTING HISTOGRAM IS DETERMINED FROM AN
C     ANALYSIS OF THE BINS.
C
C     HGRM1 CONTROLS THE OPERATION OF HG1, WHICH IS  A FORTRAN
C     CONTROLLED SUBROUTINE.
C
C     LINKS WITH:  AD1, AR11CK, HG1, HGRM1, OUTPUT AND MODPLT.
C
C             OUTPUT FILE:   LBLK = 826 WORDS
C                            NBLK = VARIABLE
C
C     WRITTEN:  J. A.  MULLENS, 02-OCT-79 (FROM AN EARLIER VERSION)
C     REVISED:   N. J.  HAMILTON, 07-NOV-79
C
C***********************************************************************
C
      COMMON IRUN,VOID,LBLK,NBLK
      DIMENSION IDATA(2048),RNPTS(410),X(410),XDAT(4),YDAT(4)
      DIMENSION RJUNK(1024)
      LOGICAL*1 LABEL(72),FILNAM(14)
      INTEGER*4 NPTS(1024),IZERO
      EQUIVALENCE (RNPTS(1),NPTS(1)),(RJUNK(1),IDATA(1))
      DATA NBIN,NREC/1024,100/
      MBIN = 410
C
      LBLK=1024
```

```
      CALL INITL(3)
      CALL ERASE
C
C     ENTER SAMPLING INFORMATION
C
      TYPE 1010
 1010 FORMAT (1H0,'ENTER NBLK OF 1024 SAMPLES (AN EVEN NUMBER):  ',$)
      ACCEPT 2011, NBLK
 2011 FORMAT (I10)
 2010 FORMAT(Q,8I10)
C
      TYPE 1020
 1020 FORMAT (1H0,'ENTER SAMPLING RATE:  ',$)
      ACCEPT 2061, SR
 2061 FORMAT (F12.6)
      CALL ARTICK(SR,SRSEL,IBASE,ITICK,IER)
C
C     ENTER RUN IDENTIFICATION INFORMATION
C
  103 TYPE 1030
 1030 FORMAT(1H0,'MCGILL RUN #, CHANNEL # : '$)
      ACCEPT 2024,IRUN,ICHN
 2024 FORMAT(2I10)
C
  104 TYPE 1035
 1035 FORMAT(1H0,'GAMMA DENSITOMETER VOID FRACTION: '$)
      ACCEPT 2060,NC,VOID
 2060 FORMAT (Q,F12.6)
      IF(VOID.LT.0.OR.VOID.GT.1.0E0) GO TO 104
C
  124 TYPE 1040
 1040 FORMAT(1H0,'ENTER PATTERN CLASS NUMBER: '$)
      ACCEPT 2010, NC, IPC
      IF (NC.EQ.0) GO TO 124
C
  125 TYPE 1043
 1043 FORMAT (1H0,'ENTER WATER FLOW RATE:  ',$)
      ACCEPT 2010, NC, IWFR
      IF (NC.EQ.0) GO TO 125
C
      TYPE 1044
 1044 FORMAT (1H0,'ENTER TITLE:  ',$)
      ACCEPT 1046, NT, (LABEL(I),I=1,NT)
 1046 FORMAT (Q,72A1)
C
C     OPEN OUTPUT FILE
C
      CALL OUTPUT (FILNAM,ILBLK,INBLK,IREC,1)
C
      READ (1'1) IDUMMY
  105 TYPE 1050
 1050 FORMAT(1H0,'ENTER RECORD TO WRITE: '$)
      ACCEPT 2012,NC,IREC
 2012 FORMAT(Q,I10)
      IF(NC.EQ.0) GO TO 105
```

```
C
C
C       ZERO OUT INTEGER*4 HISTOGRAM ARRAY NPTS
C
  110   K=JICVT(0,IZERO)
        DO 10 I=1,NBIN
           K=JMOV(IZERO,NPTS(I))
   10   CONTINUE
C
C       GET THE HISTOGRAM
C
        CALL HGRM1(LBLK,NBLK,IDATA,IBASE,ITICK,NPTS)
        DO 30 I=1,MBIN
           RNPTS(I)=AJFLT(NPTS(I))
   30   CONTINUE
C
C       CREATE X-AXIS VALUE
C
        DELX = 5.0 / 1024.0
        DO 25 I = 1,MBIN
        X(I) = DELX * FLOAT(I-1)
   25   CONTINUE
C
C       GET THE NUMBER OF SAMPLES (SUM) AND CALCULATE THE
C       VOLTAGE VALUE OF EACH BIN
C
        SUM=0.0
        DO 40 I=1,MBIN
           SUM=SUM+RNPTS(I)
   40   CONTINUE
C
        TYPE 8005, SUM
 8005   FORMAT(1H0,'SUMMATION = ',E16.7)
C
C       NORMALIZE HISTOGRAM TO GET APD, FIND FIRST AND LAST
C       NON-ZERO BINS
C
        MINBIN=0
        DO 45 I=1,MBIN
           RNPTS(I)=RNPTS(I)/SUM
           IF(MINBIN.EQ.0.AND.RNPTS(I).NE.0.0E0) MINBIN=I
           IF(RNPTS(I).NE.0.0E0) MAXBIN=I
   45   CONTINUE
C
C       WRITE RECORD TO DISK
C
  300   WRITE(1'IREC) IRUN, ICHN, IWFR, IPC, VOID, RNPTS
C
C       PRINT RESULTS
C
        TYPE 4045, MINBIN,MAXBIN
 4045   FORMAT(1H0,'MINIMUM AND MAXIMUM BINS :  ',2I8)
C
C       PLOT HISTOGRAM
C
```

```
      CALL WAITS
      CALL ERASE
      CALL SCALE(RNPTS,MBIN,YDAT,0)
      XDAT(1) = 2.0
      XDAT(2) = 0.0
      CALL SCALE(X,0,XDAT,0)
      CALL PLOTS(X,XDAT,0,-1,MBIN,RNPTS,YDAT,0,0,0)
      CALL LABELS('AMPLITUDE, VOLTS',16,'PROBABILITY',11)
      CALL TITLE ('TWO PHASE FLOW APD',18,1)
      CALL TITLE(LABEL,NT,0)
      CALL WAITS
C
      STOP 'PROGRAM FINISHED'
      END
```

```
      PROGRAM MOMENT
C
C     MOMENT CALCULATES THE FIRST THROUGH THE FOURTH
C     CENTRAL MOMENTS OF A HISTOGRAM.
C
C     THE INPUT TO THE PROGRAM IS A FILE CONTAINING RUN
C     IDENTIFICATION INFORMATION ALONG WITH THE RELATIVE
C     FREQUENCY OF EACH LEVEL USED IN THE A/D CONVERTER.
C
C     THE OUTPUT FILE CONTAINS THE RUN IDENTIFICATION INFORMATION
C     ALONG WITH FOUR MOMENT VALUES.
C
C     THE RUN IDENTIFICATION INFORMATION CONSISTS OF A RUN NUMBER,
C     CHANNEL NUMBER, WATER FLOW RATE, PATTERN CLASS
C     NUMBER AND VOID FRACTION VALUE.
C
C               INPUT FILE LENGTH:    826 WORDS
C               OUTPUT FILE LENGTH:    14 WORDS
C
C     LINKS WITH:   INPUT, OUTPUT
C
C     WRITTEN:   N J HAMILTON
C     DATE:      03-AUG-79
C
      DIMENSION RNPTS(410),DX(411),CENMOM(4)
      LOGICAL*1 FILNAM(14)
C
C     OPEN INPUT AND OUTPUT FILES
C
      CALL INPUT (FILNAM,LBLK,NBLK,IREC,1)
C
      CALL OUTPUT (FILNAM,ILBLK,INBLK,JREC,2)
C
      READ (2'1) IDUMMY
C
C     ENTER RUN IDENTIFICATION INFORMATION
C
  200 TYPE 1030
 1030 FORMAT (1H0,'ENTER RUN #, CHANNEL #:   ',$)
      ACCEPT 2030, IRUN, ICHN
 2030 FORMAT (2I10)
C
C     CORRECT FILE OPENED?
C
      DO 100 I = 1,NBLK
      READ (1'IREC) IR,ICH
      IF (IR.EQ.IRUN.AND.ICH.EQ.ICHN) GO TO 120
  100 CONTINUE
C
  120 IF (I.GT.NBLK) STOP  'NOT IN SPECIFIED FILE'
      IREC = I
      TYPE 23, IREC
   23 FORMAT (1H0,'RECORD = ',I10)
      JREC = IREC
C
```

```
C        ZERO ARRAY FOR MOMENTS CALCULATION
C
         DO 130 I = 1,4
         CENMOM(I) = 0.0
  130    CONTINUE
C
C        READ VALUES FROM DISK
C
         READ (1'IREC) IIR, IIC, IWFR, IPC, VF, RNPTS
C
C        CALCULATE MEAN OF DATA
C
         RMEAN = 0.0
         DELX = 5.0 / 1024.0
         DX(1) = DELX / 2.0
         DO 150 I = 1,410
         RMEAN = RMEAN + DX(I) * RNPTS(I)
         DX(I+1) = DX(I) + DELX
  150    CONTINUE
         CENMOM(1) = RMEAN
         TYPE 27
   27    FORMAT (1H0)
C
C        SUBTRACT MEAN FROM POINTS
C
         DO 155 I = 1,410
         DX(I) = DX(I) - RMEAN
  155    CONTINUE
C
C        CALCULATE CENTRAL MOMENTS
C
         SUM = 0.0
         DO 160 I = 2,4
         DO 170 J = 1,410
         SUM = SUM + (RNPTS(J) * DX(J)**I)
  170    CONTINUE
         CENMOM(I) = SUM
         SUM = 0.0
  160    CONTINUE
C
C        TYPE MOMENT VALUES ON CRT SCREEN
C
         DO 29 J = 1,4
         TYPE 30, J, CENMOM(J)
   29    CONTINUE
   30    FORMAT (1H ,'CENMOM',I5,3X,' = ',E16.7)
C
C        WRITE RESULTS TO DISK
C
         WRITE (2'JREC) IIR, IIC, IWFR, IPC, VF, CENMOM
C
         IREC = 1
         GO TO 200
         STOP
         END
```

```
      PROGRAM PLTMOM
C
C     PLTMOM PLOTS THE THIRD AND FOURTH MOMENTS
C     VERSUS VOID FRACTION.  THE METHOD USED FOR
C     PLOTTING PURPOSES IS CURVE FITTING, USING THE
C     METHOD OF LEAST SQUARES.  THE EQUATION USED
C     FOR CURVE FITTING IS A POLYNOMIAL EQUATION OF
C     ORDER ONE TO SEVEN, DEPENDING UPON THE
C     USER.  FOR EACH EQUATION TESTED, A RESIDUAL,
C     OR SUMMATION OF LEAST SQUARES IS OBTAINIED.
C     THE EQUATION USED FOR TEST PURPOSES IS
C     THE ONE WHICH RESULTS IN THE SMALLEST RESIDUAL
C     VALUE.
C
C     THE INPUT TO THE PROGRAM IS A FILE CONTAINING
C     MOMENT VALUES.  THE NUMBER OF RECORDS IS VARIABLE.
C     EACH RECORD HAD A LENGTH OF FOURTEEN WORDS.
C
C     THE OUTPUT CONSISTS OF THREE GRAPHS AND
C     AN OUTPUT FILE.  THE FIRST GRAPH IS A PLOT
C     OF THE THIRD MOMENT VERSUS VOID FRACTION.  THE
C     SECOND GRAPH PLOTS THE FOURTH MOMENT VERSUS VOID
C     FRACTION.  THE THIRD GRAPH IS A PLOT OF THE
C     THIRD MOMENT VERSUS THE FOURTH MOMENT.  THE
C     OUTPUT FILE CONSISTS OF THE COEFFICIENTS OF THE
C     EQUATIONS OF CONDITION TESTED BY THE PROGRAM.
C     THEY CONTAIN SIXTEEN RECORDS, WITH EACH RECORD
C     HAVING A LENGTH OF TWO WORDS.
C
C     LINK WITH:  INPUT,OUTPUT,DEGREE,FITINV,MODPLT
C
      LOGICAL*1 FILNAM(14),LABEL(24),LAB(11)
      DIMENSION CENMOM(4),YM3(100),YM4(100),X(100),XDAT(4),YDAT(4)
      DIMENSION ZX(21),ZX4(91),ZY3(21),ZY4(91)
      DATA LABEL/'C','H','A','N','N','E','L',' ',' ',' ',' ',' ',' ',
     6 ' ',' ',' ',' ',' ',' ',' ',' ',' ',' ',' '/
      DATA LAB/'D','E','G','R','E','E',' ','O','F',' ',' '/
C
C     ENTER THE NUMBER OF POINTS FOR PLOTTING AND THE
C     VOID FRACTION RANGE OF THESE POINTS
C
      TYPE 505
  505 FORMAT (1H0,'ENTER NUMBER OF POINTS TO PLOT:  ',$)
      ACCEPT 600, NPTS
  600 FORMAT (I10)
      TYPE 507
  507 FORMAT (1H0,'ENTER VOID FRACTION RANGE:  ',$)
      ACCEPT 508, NC, (LABEL(I),I=13,24)
  508 FORMAT (Q,17A1)
C
C     OPEN INPUT FILE
C
      CALL INPUT(FILNAM,LBLK,NBLK,IREC,1)
C
C     ENTER RUN NUMBERS FOR POINTS TO PLOT
```

```
C
      TYPE 39
   39 FORMAT (1H )
  500 DO 27 J = 1,NPTS
  800 TYPE 1030
 1030 FORMAT (1H ,'RUN # :   ',$)
      ACCEPT 2030, IRUN
 2030 FORMAT (2I10)
      ICHN = 1
C
C     OPEN CORRECT FILE?
C
      DO 100 II = 1,NBLK
      READ (1'IREC) IR, ICH
      IF (IR.EQ.IRUN.AND.ICH.EQ.ICHN) GO TO 120
  100 CONTINUE
  120 IF (II.GT.NBLK) GO TO 700
      IREC = II
C
C     READ MOMENT VALUES AND RUN IDENTIFICATION INFO
C
      READ (1'IREC) IIR, IIC, IWFR, IPC, VF, CENMOM
      YM3(J) = CENMOM(3)
      YM4(J) = CENMOM(4)
      X(J) = VF
      IREC = 1
      GO TO 27
  700 CALL CLOSE (1)
C
C     INCORRECT FILE, TRY AGAIN
C
      TYPE 34
   34 FORMAT (1H0,'NOT IN FILE - RE-ENTER FILE NAME AND SIZE')
      CALL INPUT (FILNAM,LBLK,NBLK,IREC,1)
      GO TO 800
   27 CONTINUE
      CALL CLOSE (1)
C
C     CALL SUBROUTINE TO DETERMINE THE VALUES OF THE
C     COEFFICIENTS NEEDED FOR CURVE FITTING
C
  501 CALL DEGREE (X,YM3,YM4,NPTS,ZX,ZX4,ZY3,ZY4,SUM3,SUM4,I3,I4)
C
C     PLOT THIRD MOMENT VERSUS VOID FRACTION
C
      TYPE 30
   30 FORMAT (1H0,'PAUSE - CARRIAGE RETURN TO PLOT')
  750 CALL INITL(3)
      CALL ITOCHR(IIC,LABEL(9),NO)
      CALL WAITS
      CALL ERASE
      XDAT(1) = 0.7
      XDAT(2) = 0.3
      XDAT(3) = 1. / (XDAT(1) - XDAT(2))
      XDAT(4) = 0.1
```

```
      YDAT(1) = 0.020
      YDAT(2) = -0.020
      CALL SCALE (YM3,0,YDAT,0)
      CALL PLOTS (YM3,YDAT,-4,-1,NPTS,X,XDAT,-2,-4,1)
      CALL SPLOTS(ZY3,YDAT,-4,-1,21,ZX,XDAT,-2,0,1)
C     CALL DOTZ (XDAT,0.3,ILOC1,0,1)
C     CALL DOTZ (XDAT,0.7,ILOC2,0,1)
C     CALL DOTIT (ILOC1,72,700,0)
C     CALL DOTIT (ILOC2,72,700,0)
      CALL VALPUT ('RES = ',6,SUM3,2,1)
      CALL LABELS ('THIRD MOMENT',12,'VOID FRACTION',13)
      CALL TITLE (LABEL,24,1)
      CALL ITOCHR (I3,LAB(11),NO)
      CALL TITLE (LAB,11,0)
      CALL WAITS
      CALL ERASE
C
C     PLOT FOURTH MOMENT VERSUS VOID FRACTION
C
      XDAT(1) = 1.0
      XDAT(2) = 0.7
      XDAT(3) = 1. / (XDAT(1) - XDAT(2))
      XDAT(4) = 0.1
      YDAT(1) = ALOG10(0.1)
      YDAT(2) = ALOG10(1.0E-10)
      CALL SCALE (YM4,0,YDAT,1)
      CALL PLOTS (YM4,YDAT,1,-1,NPTS,X,XDAT,-2,-4,1)
      CALL SPLOTS (ZY4,YDAT,1,-1,91,ZX4,XDAT,-2,0,1)
C     CALL DOTIT (ILOC2,72,700,0)
      CALL VALPUT ('RES = ',6,SUM4,2,1)
      CALL LABELS ('FOURTH MOMENT',13,'VOID FRACTION',13)
      CALL TITLE(LABEL,24,1)
      CALL ITOCHR (I4,LAB(11),NO)
      CALL TITLE (LAB,11,0)
      CALL WAITS
      CALL ERASE
C
C     PLOT THIRD MOMENT VERSUS FOURTH MOMENT
C
      YDAT(1) = 0.020
      YDAT(2) = -0.020
      CALL SCALE (YM3,0,YDAT,0)
      XDAT(1) = ALOG10(0.1)
      XDAT(2) = ALOG10(1.0E-10)
      CALL SCALE(YM4,0,XDAT,1)
```

```
      CALL PLOTS (YM4,XDAT,1,-1,NPTS,YM3,YDAT,-4,-4,1)
      CALL LABELS ('FOURTH MOMENT',13,'THIRD MOMENT',12)
      CALL TITLE (LABEL,9,1)
      CALL WAITS
      CALL ERASE
      TYPE 2060
 2060 FORMAT (1H0,'ANOTHER DEGREE? 0-Y, 1-N:   ',$)
      ACCEPT 2030, IANS
      IF (IANS.EQ.0) GO TO 501
C

       STOP 'PROGRAM FINISHED'
       END
```

```
      SUBROUTINE DEGREE(XA,YM3,YM4,NPTS,ZX,ZX4,ZY3,ZY4,SUM3,SUM4,I3,I4)
C
C     SUBROUTINE DEGREE DETERMINES THE VALUES OF THE
C     COEFFICIENTS NEEDED FOR CURVE FITTING FOR UP TO
C     A SEVENTH DEGREE EQUATION.  FOR THE DETERMINATION
C     OF THE COEFFICIENTS FOR THE PLOT OF THE FOURTH
C     MOMENT VERSUS VOID FRACTION, THE LOGARITHM
C     OF THE FOURTH MOMENT VALUE MUST FIRST BE TAKEN.
C
      LOGICAL*1 FILNAM(14)
      REAL MAT3,MAT4
      DIMENSION XA(100),YM3(100),YM4(100),MAT3(8,8),MAT4(8,8)
      DIMENSION RHOLD(8,8),Y3HOL(8),Y4HOL(8),ZX(21),ZX4(91)
      DIMENSION ZY3(21),ZY4(91),Y(15),X(15),A(8),B(8)
C
C     ZERO ARRAYS
C
      DO 50 I = 1,15
      X(I) = 0.0
      Y(I) = 0.0
   50 CONTINUE
C
      DO 60 I = 1,8
      A(I) = 0.0
      B(I) = 0.0
      Y3HOL(I) = 0.0
      Y4HOL(I) = 0.0
   60 CONTINUE
C
C     COMPUTE VALUES FOR MATRIX
C
      X(1) = NPTS
      DO 100 I = 1,NPTS
      XVAL = YM3(I)
      DO 100 J = 2,15
      K = J - 1
      X(J) = X(J) + XVAL**K
  100 CONTINUE
C
C     CONSTRUCT MATRIX FOR THIRD MOMENT FIT
C
      K = 0
      DO 400 I = 1,8
      DO 500 J = 1,8
      L = K + J
      MAT3(I,J) = X(L)
  500 CONTINUE
      K = K + 1
  400 CONTINUE
C
C     COMPUTE Y SUMMATIONS FOR THIRD MOMENT FIT
C
      TYPE 250
  250 FORMAT (1H0,'ENTER ORDER FOR THIRD MOMENT FIT:  ',$)
      ACCEPT 255, IFIT3
```

```
  255   FORMAT (I10)
C
        I3 = IFIT3
        IFIT3 = IFIT3 + 1
        DO 200 I = 1,NPTS
        XVAL = YM3(I)
        Y3VAL = XA(I)
        Y3HOL(1) = Y3HOL(1) + Y3VAL
        DO 200 J = 2,IFIT3
        K = J - 1
        Y3HOL(J) = Y3HOL(J) + Y3VAL * XVAL**K
  200   CONTINUE
C
C       COMPUTE VALUES FOR MARTIX FOR FOURTH ORDER FIT
C
        Y(1) = NPTS
        DO 1100 I = 1,NPTS
        YVAL = ALOG10(YM4(I))
        DO 1100 J = 2,15
        K = J - 1
        Y(J) = Y(J) + YVAL**K
 1100   CONTINUE
C
C       CONSTRUCT MATRIX FOR FOURTH MOMENT FIT
C
        K = 0
        DO 405 I = 1,8
        DO 410 J = 1,8
        L = K + J
        MAT4(I,J) = Y(L)
  410   CONTINUE
        K = K + 1
  405   CONTINUE
C
C       COMPUTE Y SUMMATIONS FOR FOURTH MOMENT FIT
C
        TYPE 350
  350   FORMAT (1H0,'ENTER DEGREE FOR FOURTH MOMENT FIT:   ',$)
        ACCEPT 255, IFIT4
C
        I4 = IFIT4
        IFIT4 = IFIT4 + 1
        DO 300 I = 1,NPTS
        XVAL = ALOG10(YM4(I))
        Y4VAL = XA(I)
        Y4HOL(1) = Y4HOL(1) + Y4VAL
        DO 300 J = 2, IFIT4
        K = J - 1
        Y4HOL(J) = Y4HOL(J) + Y4VAL * XVAL**K
  300   CONTINUE
C
C       GET MATRIX FOR THIRD MOMENT EVALUATION
C
        DO 600 I = 1,IFIT3
        DO 600 J = 1,IFIT3
```

```
      RHOLD(I,J) = MAT3(I,J)
  600 CONTINUE
C
C     PRINT MATRIX VALUES FOR INVERTED MATRIX
C
      CALL FITINV (RHOLD,IFIT3)
      WRITE (6,24)
   24 FORMAT (1H0)
      DO 610 I = 1,IFIT3
      WRITE (6,620) (RHOLD(I,J),J=1,IFIT3)
  620 FORMAT (1H ,'M3 =',8E16.7)
  610 CONTINUE
C
C     COMPUTE COEFFICIENTS FOR THIRD MOMENT EQUATION
C
      DO 700 I = 1,IFIT3
      T = Y3HOL(I)
      DO 700 J = 1,IFIT3
      A(J) = A(J) + RHOLD(J,I) * T
  700 CONTINUE
C
C     PRINT COEFFICIENT VALUES FOR THIRD MOMENT EQUATION
C
      WRITE (6,24)
      DO 800 I = 1,IFIT3
      WRITE (6,801) I, A(I)
  801 FORMAT (1H ,'A',I1,' = ',E16.7)
  800 CONTINUE
      CALL OUTPUT (FILNAM,LBLK,NBLK,IREC,1)
C
C     GET MATRIX FOR FOURTH MOMENT EVALUATION
C
      DO 900 I = 1,IFIT4
      DO 900 J = 1,IFIT4
      RHOLD(I,J) = MAT4(I,J)
  900 CONTINUE
C
C     PRINT MATRIX VALUES FOR INVERTED MATRIX
C
      CALL FITINV (RHOLD,IFIT4)
      WRITE (6,24)
      DO 910 I = 1,IFIT4
      WRITE (6,915) (RHOLD(I,J),J=1,IFIT4)
  915 FORMAT (1H ,'M4 =',8E16.7)
  910 CONTINUE
C
C     COMPUTE COEFFICIENTS FOR FOURTH MOMENT EQUATION
C
      DO 950 I = 1,IFIT4
      F = Y4HOL(I)
      DO 950 J = 1,IFIT4
      B(J) = B(J) + RHOLD(J,I) * F
  950 CONTINUE
      WRITE (6,24)
C
```

```
C         PRINT COEFFICIENT VALUES FOR FOURTH MOMENT EQUATION
C
          DO 975 I = 1,IFIT4
          WRITE (6,985) I, B(I)
  985     FORMAT (1H ,'B',I1,' = ',E16.7)
  975     CONTINUE
C
C         WRITE COEFFICIENTS TO DISK
C
          DO 108 I = 1,8
          K = I
          J = K + 8
          WRITE (1'K) A(I)
          WRITE (1'J) B(I)
  108     CONTINUE
          CALL CLOSE (1)
C
C         DETERMINE ARRAY OF POINTS FOR PLOTTING THE
C         THIRD MOMENT VERSUS VOID FRACTION
C
          RINC = -0.020
          DO 1000 I = 1,21
          ZY3(I) = RINC
          VAL = A(1)
          DO 1101 J = 2,IFIT3
          K = J - 1
          VAL = VAL + A(J) * RINC**K
 1101     CONTINUE
          ZX(I) = VAL
          RINC = RINC + 0.002
 1000     CONTINUE
C
C         DETERMINE ARRAY OF POINTS FOR PLOTTING THE FOURTH
C         MOMENT VERSUS VOID FRACTION
C
          M = 1
          N = -10
          DO 1050 I = 1,91
          RNUM = M * 10.**N
          RINC = ALOG10(RNUM)
          ZY4(I) = RNUM
          VAL = B(1)
          DO 1200 J = 2,IFIT4
          K = J - 1
          VAL = VAL + B(J) * RINC**K
 1200     CONTINUE
          ZX4(I) = VAL
          M = M + 1
          IF (M.NE.10) GO TO 1050
          M = 1
          N = N + 1
 1050     CONTINUE
C
C         COMPUTE RESIDUAL FOR THIRD MOMENT CURVE
C
```

```
      SUM3 = 0.0
      SUM4 = 0.0
      DO 1300 I = 1,NPTS
      RINC = YM3(I)
      VAL = A(1)
      DO 1400 J = 2,IFIT3
      K = J - 1
      VAL = VAL + A(J) * RINC**K
 1400 CONTINUE
      SUM3 = SUM3 + (XA(I) - VAL)**2
C
C     COMPUTE RESIDUAL FOR FOURTH MOMENT CURVE
C
      R4INC = ALOG10(YM4(I))
      VAL = B(1)
      DO 1500 J = 2,IFIT4
      K = J - 1
      VAL = VAL + B(J) * R4INC**K
 1500 CONTINUE
      SUM4 = SUM4 + (XA(I) - VAL)**2
 1300 CONTINUE
C
C     PRINT RESIDUALS ON LINE PRINTER
C
      WRITE (6,1600) SUM3 , SUM4
 1600 FORMAT (1H0,'SUM3 = ',E16.7,'  SUM4 = ',E16.7)
C
      RETURN
      END
```

```
      SUBROUTINE FITINV(CINV,NDIM)
C
C     FITINV DETERMINES THE INVERSE OF A MATRIX.  THE MATRIX
C     TO BE INVERTED IS ONE OF THE PARAMETERS PASSED DURING
C     THE SUBROUTINE CALL.
C
      DIMENSION CINV(8,8),B(9),C(9),IP(9),IQ(9)
C
C     DETERMINATION OF PIVOT ELEMENT
C
      DO 1 K = 1,NDIM
      PIVOT = 0.0
      DO 100 I = K,NDIM
      DO 100 J = K,NDIM
      IF (ABS(CINV(I,J)) - ABS (PIVOT)) 100,100,101
  101 PIVOT = CINV(I,J)
      IP(K) = I
      IQ(K) = J
  100 CONTINUE
C
C     CHECK FOR SINGULAR MATRIX
C
      IF (PIVOT) 102,900,102
  102 IPK = IP(K)
      IQK = IQ(K)
C
C     EXCHANGE OF THE PIVOTAL ROW WITH THE KTH ROW
C
      IF (IPK-K) 200,299,200
  200 DO 201 J = 1,NDIM
      Z = CINV(IPK,J)
      CINV(IPK,J) = CINV(K,J)
      CINV(K,J) = Z
  201 CONTINUE
C
C     EXCHANGE OF THE PIVOTAL COLUMN WITH THE KTH COLUMN
C
  299 IF (IQK-K) 300,399,300
  300 DO 301 I = 1,NDIM
      Z = CINV(I,IQK)
      CINV(I,IQK) = CINV(I,K)
      CINV(I,K) = Z
  301 CONTINUE
C
C     JORDAN ELIMINATION STEP
C
  399 DO 400 J = 1,NDIM
      IF (J-K) 403,402,403
  402 B(J) = 1./PIVOT
      C(J) = 1.
      GO TO 404
  403 B(J) = -CINV(K,J) / PIVOT
      C(J) = CINV(J,K)
  404 CINV(K,J) = 0.0
  400 CINV(J,K) = 0.0
```

```
C
      DO 405 I = 1,NDIM
      DO 405 J = 1,NDIM
  405 CINV(I,J) = CINV(I,J) + C(I) * B(J)
    1 CONTINUE
C
C     REORDERING THE MATRIX
C
      K = NDIM
      DO 500 KDUM = 1,NDIM
      IPK = IP(K)
      IQK = IQ(K)
      IF (IPK - K) 501,502,501
  501 DO 503 I = 1,NDIM
      Z = CINV(I,IPK)
      CINV(I,IPK) = CINV(I,K)
  503 CINV(I,K) = Z
  502 IF (IQK-K) 504,500,504
  504 DO 506 J = 1,NDIM
      Z = CINV(IQK,J)
      CINV(IQK,J) = CINV(K,J)
  506 CINV(K,J) = Z
  500 K = K - 1
C
      RETURN
  900 TYPE 850
  850 FORMAT (1H0,'THE MATRIX IS SINGULAR, DOES NOT HAVE AN INVERSE')
      RETURN
      END
```

```
      PROGRAM CLASS
C
C     THIS PROGRAM WILL COMPUTE A VOID FRACTION VALUE GIVEN
C     A PAIR OF MOMENT VALUES.  THE MOMENT VALUES ARE THE
C     THIRD AND FOURTH MOMENTS CALCULATED FROM THE
C     FREQUENCY HISTOGRAM OF A SIGNAL WHOSE VOID FRACTION
C     IS TO BE DETERMINED.
C
C     THE INPUT REQUIRED FOR THIS PROGRAM CONSISTS OF
C     THREE PARTS.  THE FIRST PART REQUIRES TWO VALUES
C     TO BE ENTERED, REPRESENTING THE DEGREES OF THE
C     POLYNOMIAL EQUATION TO BE USED FOR PLOTTING THE
C     VOID FRACTION.  THE SECOND PART OF THE
C     INPUT CONSISTS OF TWO FILES -- ONE CONTAINS
C     THE COEFFICIENTS FOR PLOTTING THE THIRD MOMENTS AND THE
C     OTHER FOR PLOTTING THE FOURTH MOMENTS.  THE THIRD
C     SECTOR OF THE INPUT CONSISTS OF RUN IDENTIFICATION
C     INFORMATION.  THE PROGRAM ASKS FOR SPECIFIC RUN AND
C     CHANNEL NUMBERS.  IF THE DESIRED RUN IS FOUND IN THE OPEN
C     INPUT FILE, A VOID FRACTION IS COMPUTED FOR THAT
C     RUN.  IF THE DESIRED RUN IS NOT FOUND IN THE INPUT FILE,
C     THE PROGRAM IS FINISHED AND THE OUTPUT OF THE PROGRAM
C     IS DISPLAYED.
C
C     THE OUTPUT OF THE PROGRAM CONSISTS OF A GRAPH AND
C     A PRINTOUT OF THE VOID FRACTION VALUES COMPUTED
C     BY THE PROGRAM.  OTHER INFORMATION PERTAINING TO
C     THE VOID FRACTION COMPUTATION IS ALSO GIVEN, SUCH
C     AS THE RUN AND CHANNEL NUMBER, AND THE RESIDUAL AND
C     PERCENT DEVIATION BETWEEN THE KNOWN AND CALCULATED
C     VOID FRACTION.
C
C     COEFFICIENT INPUT FILE:   NBLK = 16
C                               LBLK =  2
C
C     MOMENT INPUT FILE:   NBLK = VARIABLE
C                          LBLK = 14
C
C     LINKS WITH:   INPUT, PLTERR, MODPLT
C
      LOGICAL*1 FILNAM(14)
      DIMENSION A(8),RBILL(100),B(8),CENMOM(4),V(100),R(100)
C
C     ENTER DEGREES FOR THIRD AND FOURTH MOMENT
C     POLYNOMIAL EQUATIONS
C
      TYPE 1000
 1000 FORMAT (1H0,'ENTER DEGREES FOR PLOTS:   ',$)
      ACCEPT 2000, I3, I4
 2000 FORMAT (2I10)
C
C     ENTER FILES CONTAINING EQUATION COEFFICIENTS
C
      CALL INPUT (FILNAM,LBLK,NBLK,IREC,1)
      DO 100 I = 1,8
```

```
      J = I
      READ (1'J) A(I)
100   CONTINUE
      CALL CLOSE (1)
      SRES = 0.0
      RNUM = 0.0
      P = 0.0
      N = 0
      CALL INPUT (FILNAM,LBLK,NBLK,IREC,1)
      DO 150 J = 9,16
      K = J
      I = J - 8
      READ (1'K) B(I)
150   CONTINUE
      CALL CLOSE (1)
C
C     OPEN FILES FOR VOID FRACTION IDENTIFICATION
C
      CALL INPUT (FILNAM,LBLK,NBLK,IREC,1)
C
C     PRINT HEADINGS FOR OUTPUT
C
      WRITE (6,900)
900   FORMAT (1H0,21X,'VOID',11X,'CALCULATED',9X,'RUN',9X,'CHANNEL',
     1 9X,'RESIDUAL',10X,'PERCENT')
      WRITE (6,902)
902   FORMAT (1H ,19X,'FRACTION',12X,'VOID',57X,'DEVIATION')
C
C     ENTER RUN IDENTIFICATION INFORMATION
C
286   TYPE 3000
3000  FORMAT (1H ,'ENTER RUN # AND CH #:   ',$)
      ACCEPT 2000, IRR, ICC
      DO 200 I = 1,NBLK
      READ (1'IREC) IR, IC
      IF (IR.EQ.IRR.AND.IC.EQ.ICC) GO TO 300
200   CONTINUE
300   IF (I.EQ.NBLK+1) GO TO 700
      IREC = I
C
C     READ MOMENT VALUES FOR PROGRAM
C
      READ (1'IREC) IR, IC, IWFR, IPC, VF, CENMOM
C
C     DECIDE IF VOID FRACTION LESS THAN OR GREATER
C     THAN SEVENTY PERCENT
C
      IF (CENMOM(4).LT.0.52E-2) GO TO 400
C
C     COMPUTE VOID FRACTION FROM THIRD MOMENT VALUE
C
      VOID = A(1)
      DO 350 I = 2,13+1
      K = I - 1
      VOID = VOID + A(I) * CENMOM(3)**K
```

```
  350   CONTINUE
        IF (VOID.LT.0.3) VOID = 0.3
        IF (VOID.GT.0.7) VOID = 0.7
        GO TO 500
C
C       COMPUTE VOID FRACTION FROM FOURTH MOMENT VALUE
C
  400   VOID = B(1)
        DO 450 I = 2,14+1
        K = I - 1
        VOID = VOID + B(I) * (ALOG10(CENMOM(4)))**K
  450   CONTINUE
        IF (VOID.LT.0.7) VOID = 0.7
        IF (VOID.GT.1.0) VOID = 1.0
C
C       COMPUTE THE DIFFERENCE AND PERCENT DEVIATION
C       BETWEEN KNOWN AND CALCULATED VOID FRACTION VALUES
C
  500   RES = VF - VOID
        SRES = SRES + ABS(RES)
        RNUM = RNUM + 1.0
        N = N + 1
        V(N) = VF
        R(N) = RES
        RBILL(N) = VOID
        PERDEV = (ABS(RES)/VF) * 100.0
        P = P + PERDEV
C
C       WRITE RESULTS TO PRINTER
C
        WRITE (6,600) VF, VOID, IR, IC, RES, PERDEV
  600   FORMAT (1H0,14X,E13.4,6X,E13.4,7X,I5,8X,I5,8X,E13.4,8X,F7.3)
        IREC = 1
        GO TO 286
C
C       COMPUTE AVERAGE RESIDUAL AND PERCENT DEVIATION
C       FOR ALL VOID FRACTIONS CALCULATED DURING THE
C       PROGRAM RUN.
C
  700   ARES = SRES / RNUM
        AP = P / RNUM
        WRITE (6,800) ARES, AP
  800   FORMAT (/,/,1H0,17X,'AVERAGE RESIDUAL = ',E13.4,5X,
     2  'AVERAGE PERCENT DEVIATION = ',F7.3)
C
C       PLOT KNOWN VOID FRACTION VERSUS CALCULATED
C       VOID FRACTION
C
        CALL PLTERR (N,V,RBILL)
        STOP
        END
```

```
      SUBROUTINE PLTERR(N,V,RBILL)
C
C     SUBROUTINE PLTERR PLOTS THE KNOWN VOID FRACTION VERSUS
C     THE CALCULATED VOID FRACTION.  THE ERROR BETWEEN THE
C     TWO VALUES CAN BE SEEN BY HOW FAR THE PLOTTED POINTS
C     DEVIATE FROM THE FORTY-FIVE DEGREE LINE INDICATIVE OF
C     A PERFECT MATCH BETWEEN THE TWO VALUES.
C
      DIMENSION RBILL(100),XDAT(4),X(2),Y(2),V(100)
C
      CALL INITL(3)
      CALL WAITS
      CALL ERASE
C
C     SCALE AXES
C
      XDAT(1) = 1.0
      XDAT(2) = 0.3
      XDAT(4) = 0.1
      XDAT(3) = 1.0 / (XDAT(1) - XDAT(2))
C
C     PLOT POINTS AND LABEL GRAPH
C
      CALL PLOTS (V,XDAT,-2,-1,N,RBILL,XDAT,-2,-4,1)
      X(1) = 0.3
      X(2) = 1.0
      CALL SPLOTS (X,XDAT,-2,-1,2,X,XDAT,-2,0,0)
      CALL LABELS ('VOID FRACTION - KNOWN',21,'',0)
      CALL LABELS ('',0,'VOID FRACTION - CALCULATED',26)
      CALL TITLE ('VOID FRACTION COMPARISON',24,1)
      CALL WAITS
      RETURN
      END
```

```
      SUBROUTINE INPUT (FILNAM,LBLK,NBLK,IREC,NUM)
C
C     SUBROUTINE INPUT IS USED DURING THE EXECUTION OF A
C     PROGRAM TO OPEN AN INPUT FILE.  IT PASSES THE FILE NAME
C     AND THE SIZE OF THE FILE BACK TO THE MAIN PROGRAM.
C
      LOGICAL*1 FILNAM(14)
C
C     ACCEPT NUMBER AND LENGTH OF RECORDS
C
      TYPE 1000
 1000 FORMAT (1H0,'ENTER INPUT NBLK, LBLK:   ',$)
      ACCEPT 2000, NBLK, LBLK
 2000 FORMAT (2I10)
C
C     ACCEPT INPUT FILE NAME
C
      TYPE 1010
 1010 FORMAT (1H0,'ENTER INPUT FILE:   ',$)
      ACCEPT 2010, NC, (FILNAM(I),I=1,NC)
 2010 FORMAT (Q,14A1)
C
      CALL ASSIGN (NUM,FILNAM,NC)
      DEFINE FILE NUM(NBLK,LBLK,U,IREC)
      IREC = 1
      RETURN
      END
```

```
      SUBROUTINE OUTPUT (FILNAM,LBLK,NBLK,IREC,NUM)
C
C     SUBROUTINE OUTPUT IS USED DURING THE EXECUTION OF A
C     PROGRAM TO OPEN AN OUTPUT FILE.  IT PASSES THE FILE NAME
C     AND THE SIZE OF THE FILE BACK TO THE MAIN PROGRAM.
C
      LOGICAL*1 FILNAM(14)
C
C     ACCEPT NUMBER AND LENGTH OF RECORDS
C
      TYPE 1000
 1000 FORMAT (1H0,'ENTER OUTPUT NBLK, LBLK:  ',$)
      ACCEPT 2000, NBLK, LBLK
 2000 FORMAT (2I10)
C
C     ACCEPT OUTPUT FILE NAME
C
      TYPE 1010
 1010 FORMAT (1H0,'ENTER OUTPUT FILE:  ',$)
      ACCEPT 2010, NC, (FILNAM(I),I=1,NC)
 2010 FORMAT (Q,14A1)
C
      CALL ASSIGN (NUM,FILNAM,NC)
      DEFINE FILE NUM(NBLK,LBLK,U,IREC)
      IREC = 1
      RETURN
      END
```

```
      PROGRAM INITIAL
C
C     THIS PROGRAM CREATES AND INITIALIZES A FILE.
C     TO INITIALIZE THE FILE, IT PLACES A
C     MINUS ONE IN THE FIRST WORD OF EACH RECORD.
C
      LOGICAL*1 FILNAM(14)
C
C     ENTER NUMBER OF RECORDS AND LENGTH OF FILE
C
      TYPE 1010
 1010 FORMAT (1H0,'ENTER NUMBER OF RECORDS:   ',$)
      ACCEPT 2010,NREC
 2010 FORMAT (I10)
      TYPE 1030
 1030 FORMAT (1H0,'ENTER LENGTH OF BLOCK:   ',$)
      ACCEPT 2010, LBLK
C
C     ACCEPT FILE NAME
C
      TYPE 1020
 1020 FORMAT (1H0,'ENTER FILE NAME:   ',$)
      ACCEPT 2020, NC, (FILNAM(I),I=1,NC)
 2020 FORMAT (Q,14A1)
C
      CALL ASSIGN (1,FILNAM,NC)
      DEFINE FILE 1(NREC,LBLK,U,IREC)
      IREC = 1
      NUM = -1
C
C     INITIALIZE FIRST WORD OF EACH RECORD
C
      DO 100 I = 1,NREC
      WRITE (1'IREC) NUM
  100 CONTINUE
      STOP
      END
```

VITA

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