CIRCLE DIAGRAMS

FOR

SYNCHRONOUS MOTORS

by

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and

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I

INTRODUCTION

It is often desireable to know the characteristics of operation of a synchronous motor over the entire range of operation. In many different problems, where several variables are involved, it is desireable to construct a circle diagram from which all of the characteristics of the particular appliance can be obtained. The advantages of a circle diagram are: (1) that all the data required can be combined in one diagram, (2) that the performance of the appliance can be constructed from certain constants without the necessity of making several series of tests over the entire range of operation, and (3) that the operation of the proposed project can be predicted before it is constructed.

This paper sets out the results the authors obtained in constructing a circle diagram to include all losses of a synchronous motor. The study of circle diagrams for synchronous motors is included in the curricula of most universities, but these studies do not include all of the losses of the motor in the diagram. The common diagram for synchronous motors includes a series of resistance and reactance. The resistance is determined by the actual resistance of the machine, which in most cases is taken as the D.C. resistance. The reactance is the synchronous reactance of the machine. From this it is evident that the mechanical losses, hysteresis losses, eddy current losses, stray current losses, and D.C. field losses are not included in the diagram. The diagram that the authors constructed contains all of these losses except the D.C. field losses. These could easily be included as will be shown later.

II

GENERAL SURVEY OF FIELD

Synchronous motors are used today primarily to correct the low power factors that most shops operate on, due to the induction motor loads or other appliance that may be drawing a lagging current. By operating a synchronous motor with a high field current, it can be made to draw a leading current. Since the total current drawn by the shop or factory is the vector sum of the currents drawn by each piece of machinery, the power factor of the factory may be made to approach unity by using the proper number of synchronous motors operating at a leading power factor. If the manager of the shop had a circle diagram for the synchronous motors he considered installing, he could tell very closely what number of motors to purchase and also at what field current they must be operated.

III

STATEMENT OF PURPOSE

The students mind is usually curious. We had worked with circle diagrams for several different kinds of equipment, and all that we worked with, except that for a synchronous motor, included all the losses incured in operating the piece of equipment. Since there was only one exception, we dicided to determine the difficulties incurred in applying the circle diagram to all of the losses of the synchronous motor. As this could not be accomplished by any theoretical method that we tried to work out, we resorted to the trial and error method. We made a complete test of the synchronous motor and then found the equivalent diagram to fit the experimental results as near as possible. The intention of this paper is to show

how we adopted an equivalent diagram to the experimental results, what the errors are in our assumptions, and the magnitude of these errors.

IV

THEORY OF CIRCLE DIAGRAM

It is possibel to construct a circle diagram for any kind of a circuit that has no moving parts and has a constant reactance. Since a synchronous motor has a rotating armature, it is necessary to resolve the actual circuit of the motor into an equivalent circuit that has no moving parts. This circuit is shown later in the report. The general equations for any circle diagram are as follows.

$$E_s = AE_r + BI_r$$

Where E_s is the voltage at the sending end, and E_r is the voltage at the receiving end, I_r is the current at the receiving end, and A and B are constants determined by the circuit for which the circle diagram is being constructed.

$$I_s = CE_r + DI_r$$

where I_s is the current at the sending end, and the constants C and D determined by the circuit. These two equations are for the receiving circle part of the diagram. The equations for the sending circle can be obtained form them as follows:

transposing $\quad AE_r = E_s - BI_r$

$$CE_r = I_s - DI_r$$

and $\quad ACE_r = CE_s - BCI_r$

$$ACE_r = AI_s - ADI_r$$

subtracting $$I_r = \frac{C}{AD - BC} E_s - \frac{A}{AD - BC} I_s$$

also $$E_r = \frac{D}{AD - BC} E_s - \frac{B}{AD - BC} I_s$$

Let $A = a_1 + ja_2$

and $b = b_1 + jb_2$

Then $\alpha = \tan^{-1} \frac{a_2}{a_1}$

$$\beta = \tan^{-1} \frac{b_2}{b_1}$$

The construction of these angles are shown on the following page in the diagram. In this diagram E_r is the steady vector, E_s is a constant and moves along the arc, B is a constant; therefore I_r must change in angle and magnitude. The angle θ_r is the power factor angle and the X and Y axes are so placed that θ_r is the angle between BI_r and the X axis. The solution for the angles as shown is merely a metter of geometry. If all sieds of the triangle OMN are multiplied by $\frac{Er}{B}$, the values shown in figure two are obtained. This is the actual receiver circle and can be applied to any circuit within the limits set above. The sending circle is obtained in the same manner, but the center for the sending circle will be in the fourth quadrant instead of the second. The angles for the sending circle will be as follows:

$$\Delta = \tan^{-1} \frac{k_2}{k_1}$$

$$\tau = \tan^{-1} \frac{h_2}{h_1}$$

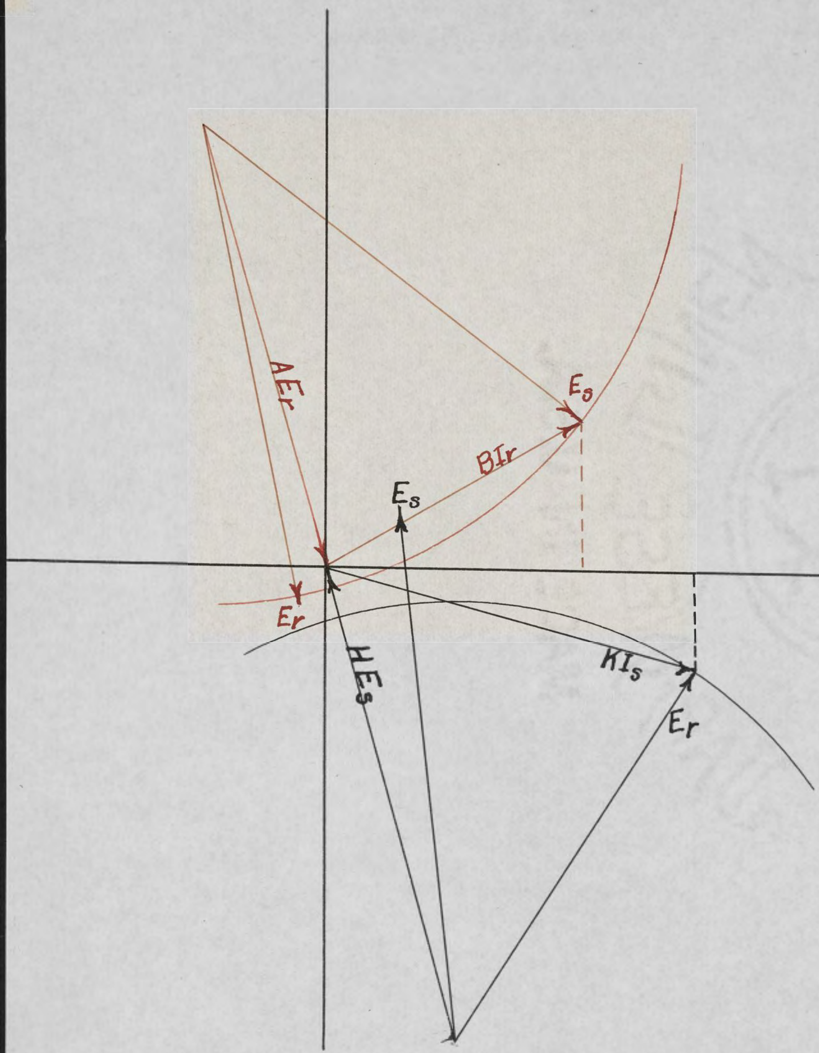

Fig. One

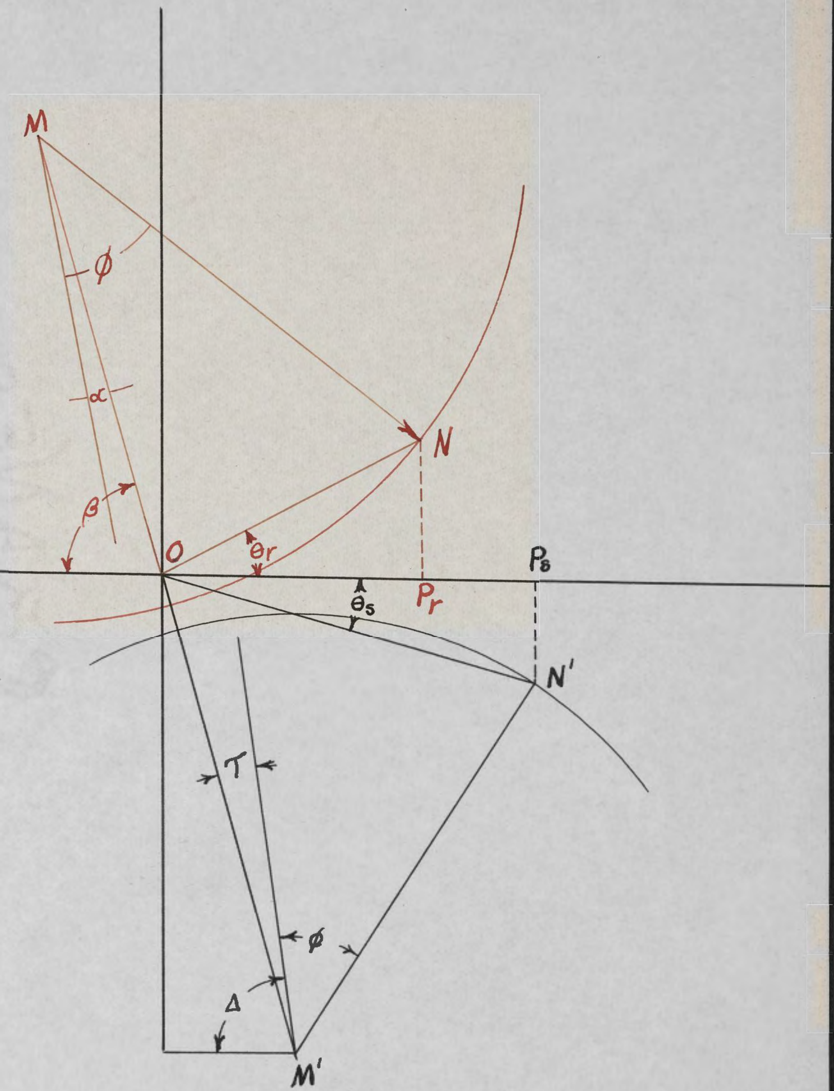

Fig. Two

These formulas are for the general circle diagram and may be applied to any case. The constants must be worked out for any case before the diagram can be applied. The constants are determined by writing an equation for E_s and I_s for the particular circuit and solving for A, B, C, and D. Following are the computations for our circuit.

$E_s = E_r + ZI_r$ from which $A = 1$ and $B = Z$

$I_s = gE_s + I_r$ from which $C = g$ and $D = 1$

$$OM' = \frac{HE_s^2}{K} = \frac{\frac{1}{1 - gZ} E_s^2}{\frac{Z}{1 - gZ}} = \frac{E_s^2}{Z}$$

$$M'N' = \frac{E_s E_r}{K} = \frac{E_s E_r (1 - gZ)}{Z}$$

$$ON' = E_s I_s$$

$$\alpha = \tan^{-1} \frac{a_2}{a_1} = 0$$

$$\beta = \tan^{-1} \frac{b_2}{b_1} = \frac{X_{SA}}{R}$$

And for the receiving circuit

$$E_r = \frac{1}{1 - gZ} E_s - \frac{Z}{1 - gZ} I_s$$

We can see form this that $H = \frac{1}{1 - gZ}$

and $K = \frac{Z}{1 - gZ}$

Then $MO = \frac{E_r^2}{Z}$

$$MN = \frac{E_s E_r}{Z}$$

$$ON = E_r I_r$$

$$\Delta = \tan^{-1} \frac{k_2}{k_1} \quad \text{where } K = \frac{r \neq jx}{1 - g(r \neq jx)}$$

We find that g is so small that the $g(r \neq jx)$ term may be dropped with a maximum error of 9' (minutes) which is too small to effect the acuracy in a 20" times 16" diagram. (This term cannot be dropped in the numerical value of Z)

Then we can say $\Delta = \tan^{-1} \frac{X_{SA}}{R} = \alpha$

and $\tau = \tan^{-1} \frac{h_2}{h1}$ where $H = \frac{1}{1 - g(r \neq jx)}$

so we may assume $\tau = 0$

These assumptions were used in constructing the large circle diagram included in this paper.

V

SYNCHRONOUS MOTORS

Any one not familiar with synchronous motors should make a further study of them else where for a thorough understanding of this paper, but it will help to refresh the mind of the reader to give a short discussion of a synchronous motor. A synchronous motor is a three phase machine in all practical use, although it is possible to use some other number of phases. It must have a rotating field in the stator, and a three phase stator is the most economical. The rotor may have either salient or non-salient pole windings and is excited by direct current. This sets up a field in the rotor that rotates with the rotor. Some auxiliary means must be used to bring a synchronous motor up to speed, but when it is brought to synchronous speed, the field of the rotor stays in step with the rotating field of the stator and holds the rotor at a constant speed no matter what the load is on the motor. This is assuming that the field is great enough to keep the motor in step.

It is the object of this paper to determine all the losses of a synchronous motor, incorporate them in an equivalent electrical diagram, and construct a circle diagram for this equivalent diagram. These losses that must be considered are the copper losses in the armature, mechanical losses, hysteresis losses and eddy current losses. The mechanical losses will be constant and therefore may be represented by a shunt across the sending end of the line. The copper losses and the eddy current losses vary as the square of the current in the stator winding. The hysteresis losses, according to theory, is proportional to the 1.6 power of the stator current for constant field current. We calculated the hysteresis and eddy current losses together in the tests we made and found that sum of the two varied as the 1.8 power of the stator current at the low power factors, but approached the square of the stator current as the power factor approached 100%.

Another loss is the D.C. field loss. This will be constant for each field current, as it is the product of the square of the field current and the resistance of the D.C. field. It would be possible to include this in the shunt resistance the same as the mechanical losses, but as it is not a part of the A.C. losses of the motor we did not include it in our diagram. It can be included in the circle diagram without including it in the equivalent diagram as will be shown later.

VI

DESCRIPTION OF METHOD USED

As was stated above, there is no way to work this problem out in a purely theoretical manner. Therfore it is necessary to go to the trial and error method. Probably the best way to present this is to give the diagram that gave the best results and show what steps were taken in deciding on this diagram.

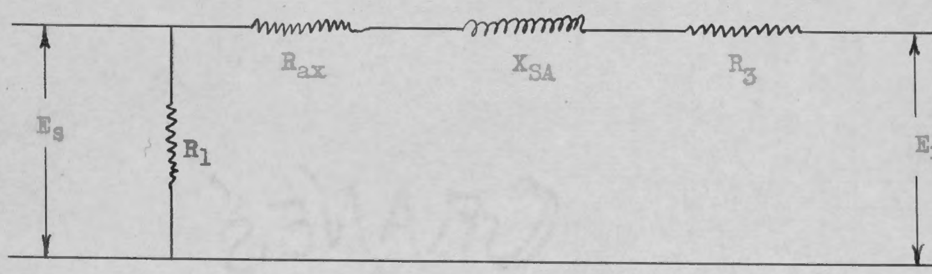

Fig. Three

Al these values are worked up per phase, so to get the total these values must be multiplied by three.

The resistance marked R_1 is shunted across the sending end of the diagram to take care of the mechanical losses in the machine. As the machanical losses are constant at all conditions of operation of the motor, this will give the exact mechanical losses at all times.

The resistance R_{ax} is a resistance determined by the actual resistance of the armature per phase. The purpose of R_{ax} is to represnet the actual I^2R loss in the motor. It seem strange that the actual resistance of the armature could not be used, but we found that we could get more accurate results in this manner. In an actual motor all the current flowing to the motor would flow through the resistance of the armature, but in this equivalent circuit, part of the current flowing to the sending end of the diagram does not flow through R_{ax}. This necessitates a higher value of R_{ax} than the resistance of the motor would give.

Since the applied voltage is constant and R_1 is constant, the current flowing through the R_1 part of the circuit is easily calculated. This value subtracted from the rated value of the motor will give an approximate value of the current flowing in R_{ax} at full load. From this it is possible to calculate a value of R_{ax} to give the same I^2R loss as is obtained with the total current flowing through the phase resistance of the motor. This assumption will produce a slight error in two ways. The first is due to the fact that the total current will not be the same at different field currents, and the second is caused by the power factor changing as the load is decreased at the same field current. In working up the diagram we calculated a value of R_{ax} for each of the field currents as is shown in table "Data for Circle Diagram" taking into account both the difference in the currents and the power factor at full load. The results show that the former of the two sources of error mentioned above will not cause a noticeable error in the final results. The latter will cause a slight error at part load, but this is counter balanced by an error in using a resistance to represnet the sum of the hysteresis and eddy current losses. This error in R_{ax} exagerates the I^2 R loss at the lower loads.

The next thing in the diagram is the impedance, either inductance or capacity. The best method of determining this is the A.I.E.E. method, Any one not familiar with this can find it in any of the higher text books of electrical engineering. This will be different for each field current but will be constant for any field current.

The last resistance R_3 is a resistance to represnet the hysteresis and eddy current loss in the motor. When we assume that these losses can be represented by a resistance in series with the line, we are assuming that they vary as the square of the current. Theoretically the eddy current loss will vary as the square of the motor current but the hysteresis loss will be proportional to the 1.6 power of the current. Assuming that these two losses are equal they would be proportional to the 1.8 power of the current. In these tests we made on the motor we found that this is about the right exponent for power factors below 85%, but as the power factor aproaches unity this exponent is increased to almost two.

In our discussion above of R_{ax} we saw that there was an error that exagerated the losses at the lower loads. The tendency of R_3 is to exagerate the losses at the higher loads. These two errors balance each other very nicely. It was stated above that the hysteresis and eddy current losses approached the R_3 loss very nearly at 100% power factor. If the error in R_{ax} were the same at low and high power factors, it would cause a greater error at unity power factor. But since the error in R_{ax} is caused by the change in power factor it is eliminated at 100% power factor, which is another tendency for the errors in the diagram to balance each other.

There is no provision made for the D.C. field loss in this diagram. If a diagram is desired that will include the D.C. field loss, the center of the receiver circle may be moved to the left by an amount equal to the field loss. It is a simple matter to get the field loss of the machine by a measurement of the field resistance.

DATA FOR MECHANICAL LOSSES

SHORT CIRCUIT

I_f	Wt.	H.P.	Watts	$I_{s.c.}$
0	1.687	0.580	433	0.0
1.2	2.187	0.750	562	11.0
2.1	2.375	0.816	610	19.2
2.6	2.562	0.881	658	23.5
3.0	2.750	0.946	706	26.5
3.2	2.875	0.989	739	28.5
3.5	3.125	1.078	804	31.5
4.0	3.687	1.270	949	35.5
4.5	4.312	1.482	1109	39.5
5.1	5.187	1.783	1332	44.5
5.8	6.125	2.104	1575	49.5
6.6	7.625	2.621	1960	49.0

NO LOAD

I_f	Wt.	H.P.	Watts	E_g	$\frac{E_g}{3}$
0.0	2.187	0.754	564	0	0
1.28	2.187	0.754	564	73	43
2.72	2.312	0.715	594	152	90
3.38	2.437	0.839	626	182	108
4.67	2.562	0.883	660	229	135
6.11	3.000	1.032	771	267	158
6.94	3.187	1.099	820	282	167
8.01	3.437	1.182	886	298	176
9.82	3.687	1.270	948		

MOTOR LOAD TESTS

NO LOAD

I_f	I_a	K.W.	I^2R	Mcl.Loss	H.& E.C.
2.05	20.8	1.34	0.156	0.54	0.644
2.50	16.8	0.66	0.102	0.54	0.018
2.65	14.7	0.67	0.076	0.54	0.054
2.90	.319	0.63	0.070	0.54	0.020
3.10	12.2	0.62	0.055	0.54	0.025
3.65	8.7	0.59	0.027	0.54	0.023
4.05	5.6	0.57	0.010	0.54	0.020
4.20	4.6	0.56	0.006	0.54	0.019
4.50	2.0	0.56	0.000	0.54	0.025
5.10	3.0	0.55	0.000	0.54	0.010
5.35	4.9	0.55	0.009	0154	0.001
5.95	9.2	0.57	0.030	0.54	0.000
6.40	12.2	0.61	0.055	0.54	0.015
6.70	14.4	0.63	0.075	0.54	0.015
6.85	15.8	0.66	0.090	0.54	0.030
7.22	18.4	0.68	0.142	0.54	0.000

ONE THIRD LOAD

I_f	I_a	Input	Output	Total L.	I^2R	Mcl.L.	H.& E.C.
1.50	29	5.150	4.04	1.110	0.310	0.54	
2.12	24	5.060	4.04	1.020	0.207	0.54	0.273
2.68	20	4.930	4.04	0.890	0.147	0.54	0.203
3.20	17	4.850	4.04	0.810	0.108	0.54	0.163
3.58	15	4.793	4.04	0.753	0.083	0.54	0.130
4.25	12	4.740	4.04	0.700	0.056	0.54	0.104
4.70	12	4.729	4.04	0.689	0.056	0.54	0.093
5.30	13	4.728	4.04	0.688	0.063	0.54	0.085
6.00	16	4.760	4.04	0.720	0.095	0.54	0.085
6.40	19	4.808	4.04	0.768	0.131	0.54	0.095
7.00	22	4.844	4.04	0.835	0.179	0.54	0.085
7.80	27	4.910	4.04	0.870	0.269	0.54	0.141

TWO THIRDS LOAD

I_f	I_a	Input	Output	Total L.	I^2R	Mcl.L.	H.& E.C.
2.15	41.5	10.42	8.75	1.67	0.636	0.54	0.494
2,73	29.6	10.10	8,75	1.35	0.580	0.54	0.230
3.38	30.3	10.00	8.75	1.25	0.246	0.54	0.364
4.10	26.5	9.86	8.75	1.11	0.255	0,54	0.315
5.11	25.0	9.82	8.75	1.07	0.227	0.54	0.303
6.60	27.0	9.86	8.75	1.11	0.267	0.54	0.303
7.60	32.5	9.98	8.75	1.23	0.395	0.54	0.295
8.60	37.0	10.06	8.75	1.31	0.510	0.54	0.260
9.10	40.0	10.10	8.75	1.35	0.591	0.54	0.211

Full Load

I_f	I_a	Input	Output	Total L.	I^2R	Mcl.L.	H.&. E.C.
4.41	39.0	14.9	12.98	1.92	0.567	0.54	0.813
5.00	38.5	14.8	12.98	1.82	0.551	0.54	0.729
5.90	37.0	14.6	12.98	1.62	0.518	0.54	0.572
7.80	41.5	14.86	12.98	1.88	0.621	0.54	0.719

RESISTANCE

FIELD

E	I	R
12.8	1.32	9.70
37.0	3.98	9.30
61.0	8.20	7.45

ARMATURE

Phase	E	I	R
1-5	3.67	14.9	0.246
1-9	3.67	14.9	0.246
5-9	3.67	14.9	0.246

X_s (A.I.E.E.)

E_{ac}	I_{ac}	I_f
168	39	7.8

DATA FOR CIRCLE DIAGRAM

I_f	R_{ax}	R_3	R	R_{phase}	Z	X_{SA}	E_r	E_s
4	0.420	0.890	1.312	0.438	2.07	1.94	127	127
5	0.425	0.800	1.222	0.407	1.84	1.70	139	127
6	0.423	0.427	0.849	0.282	1.50	1.47	153	127
7	0.421	0.248	0.670	0.223	1.37	1.32	165	127
8	0.419	0.190	0.612	0.204	1.15	1.10	172	127

I_f	g	(1-gZ)	MN	MO	M'N'	M'O	Delta
4	0.0111	0.977	7800	7800	7610	7770	77.2°
5	0.0111	0.980	9590	10500	9390	8750	76.5
6	0.0111	0.983	12920	15620	12730	10730	79.0
7	0.0111	0.985	15800	19820	15050	11760	80.2
8	0.0111	0.987	18990	25730	18720	14020	79.4

VIII

RESULTS

Below is a table showing the per cent error in the efficiency obtained from the diagram and the actual efficiency of the motor at full and one half load.

FULL LOAD

I_f	Actual Eff.	Eff. from Diagram	Error
4	86.7%	86.7%	0
5	87.5	87.0	-0.5%
6	89.0	98.8	0.8
7	89.3	98.2	-0.1
8	86.4	86.7	0.3

HALF LOAD

I_f	Actual Eff.	Eff. from Diagram	Error
4	87.5%	88.5%	1.0%
5	87.9	88.5	0.6
6	88.1	88.9	0.8
7	87.2	88.2	1.0
8	85.0	84.8	-0.2

IX

CONCLUSIONS

From the above resluts the conclusion may be drawn that it is possible to include all the losses of a synchronous motor into a circle diagram and obtain correct results. This being true it is possible to predict from the design the operation of the synchronous motor before it is constructed. We only made the test on one motor, but it is reasonable to assume that the same type of equivalent diagram would apply to any other synchronous motor.

X

APPROVAL

The foregoing thesis is hereby approved as a creditable study of an engineering subject, carried out and presented in a manner sufficiently satisfactory to warrant its acceptance as a prerequisite to the degree for which it has been submitted. It is to be understood that by this approval the undersigned does not necessarily endorse or approve any statement made, opinions expressed, or conclusions drawn therein, but approves the thesis only for the purpose for which it is submitted.

Name J. G. Tarboux

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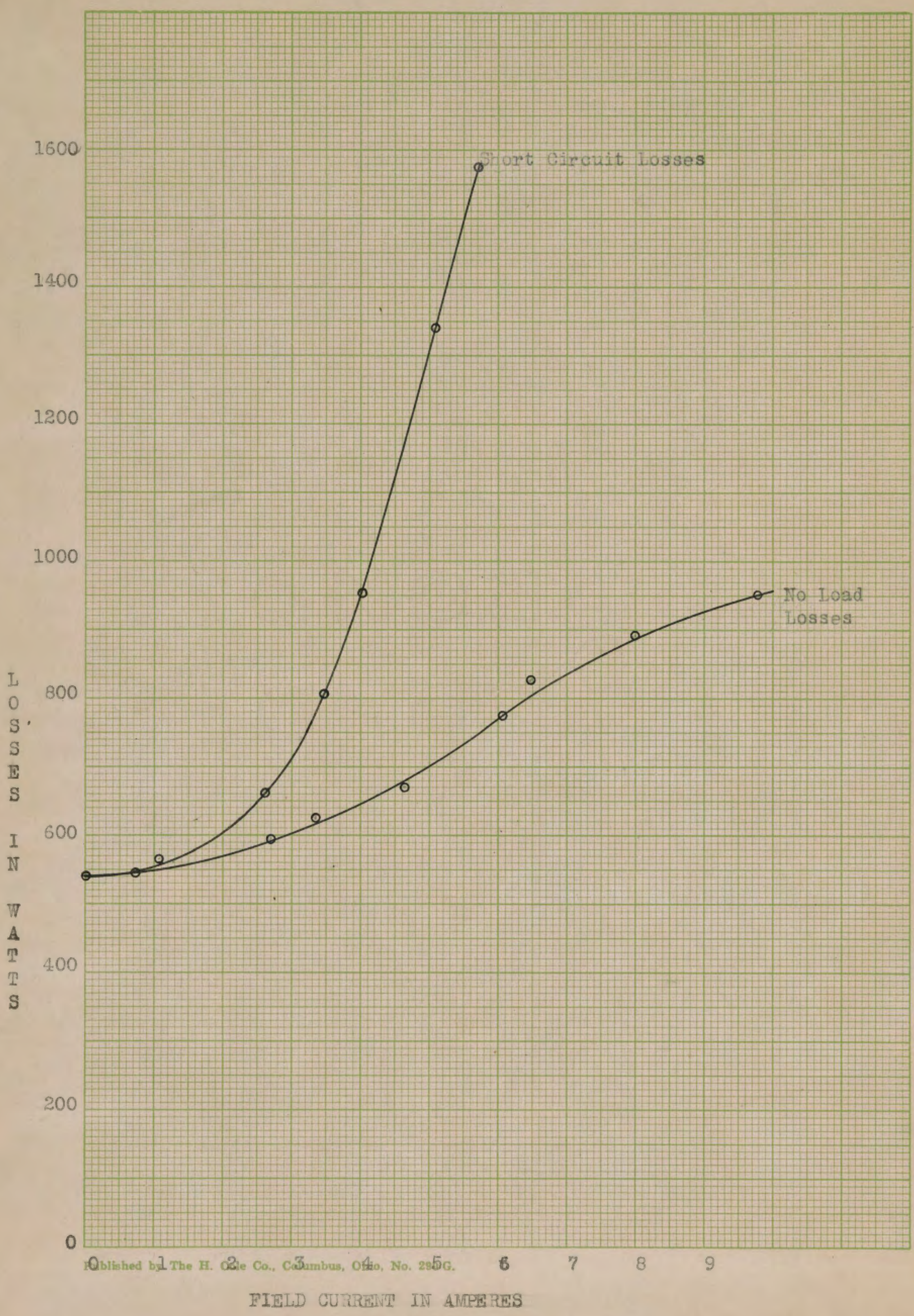
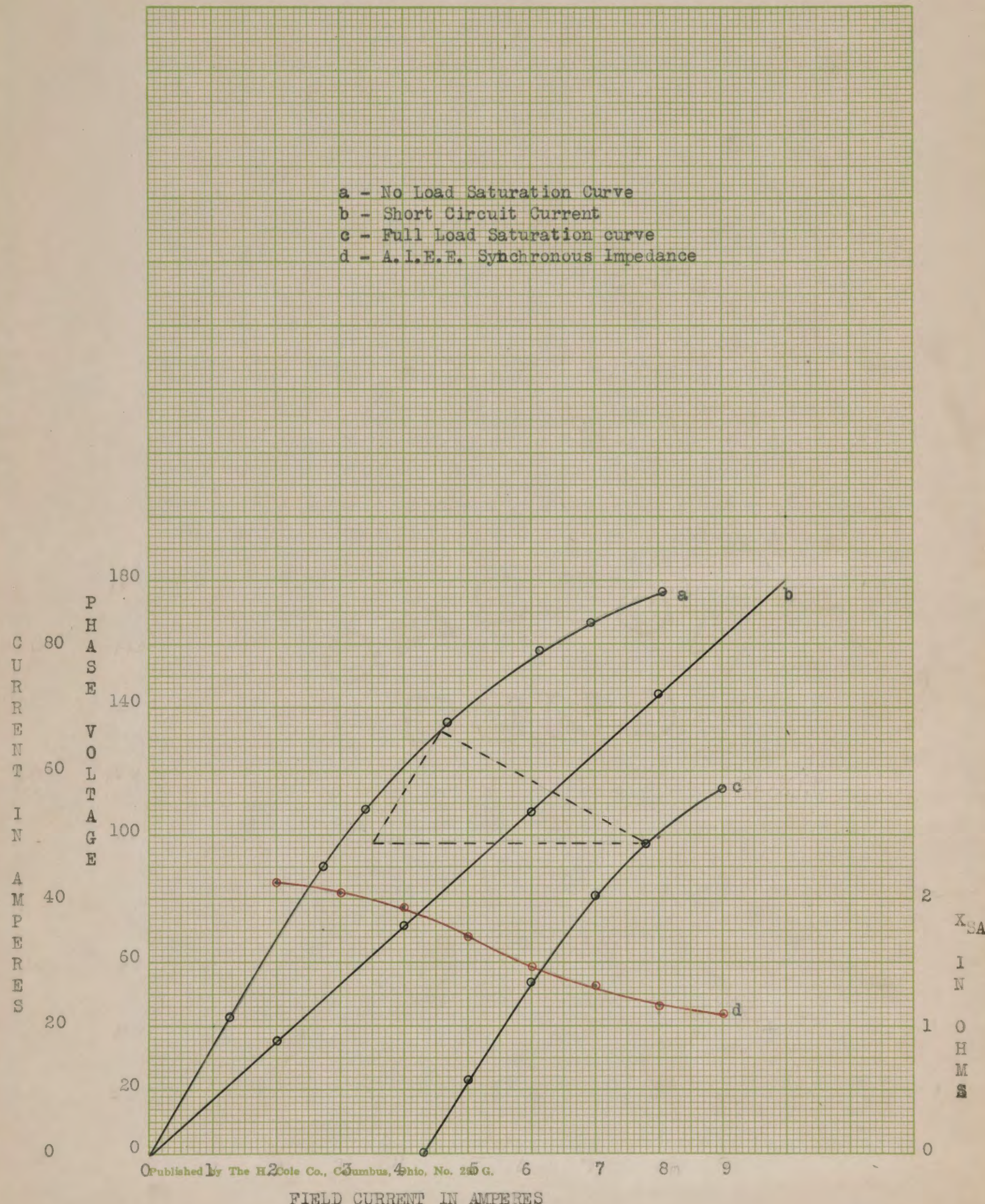
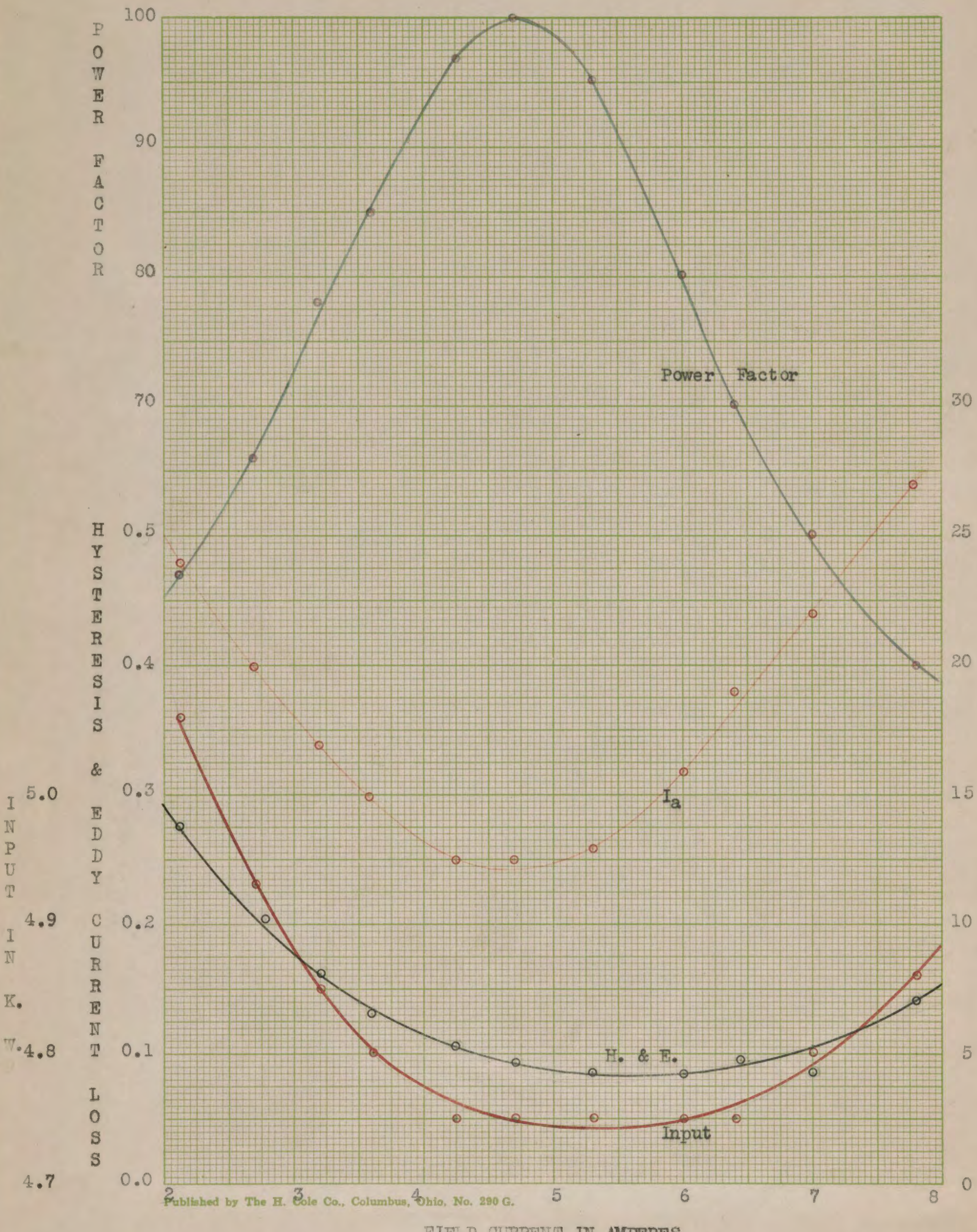

Short Circuit Losses
No Load Losses
1600
1400
1200
1000
800
600
400
200
0
LOSSES IN WATTS
0
1
2
3
4
5
6
7
8
9
FIELD CURRENT IN AMPERES


a - No Load Saturation Curve
b - Short Circuit Current
c - Full Load Saturation curve
d - A.I.E.E. Synchronous Impedance
180
140
100
60
20
0
80
60
40
20
0
PHASE VOLTAGE
CURRENT IN AMPERES
2
1
0
X_{SA} IN OHMS
a
b
c
d
0 1 2 3 4 5 6 7 8 9
FIELD CURRENT IN AMPERES

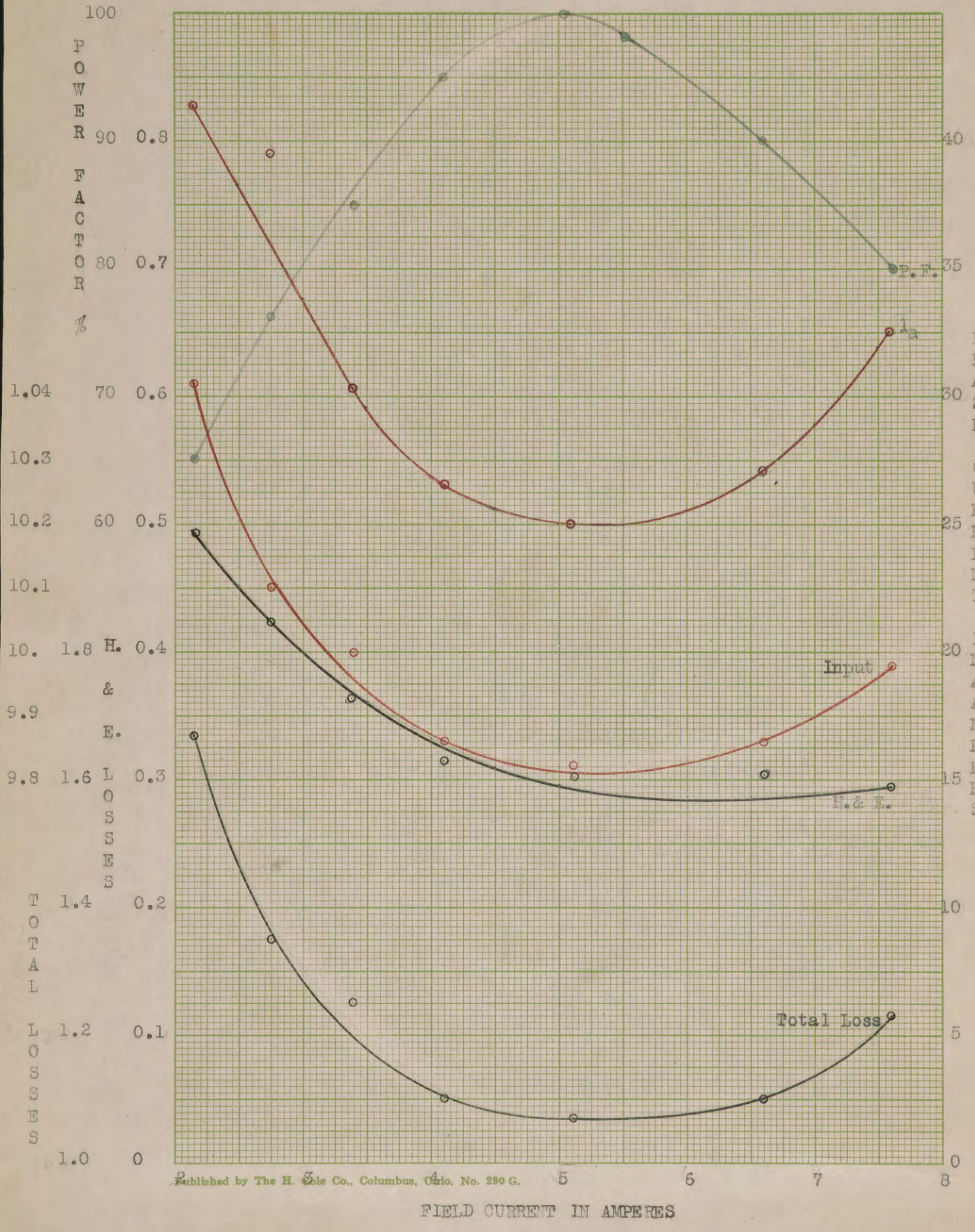
1/3 LOAD

OUTPUT = 4.04 K.W.



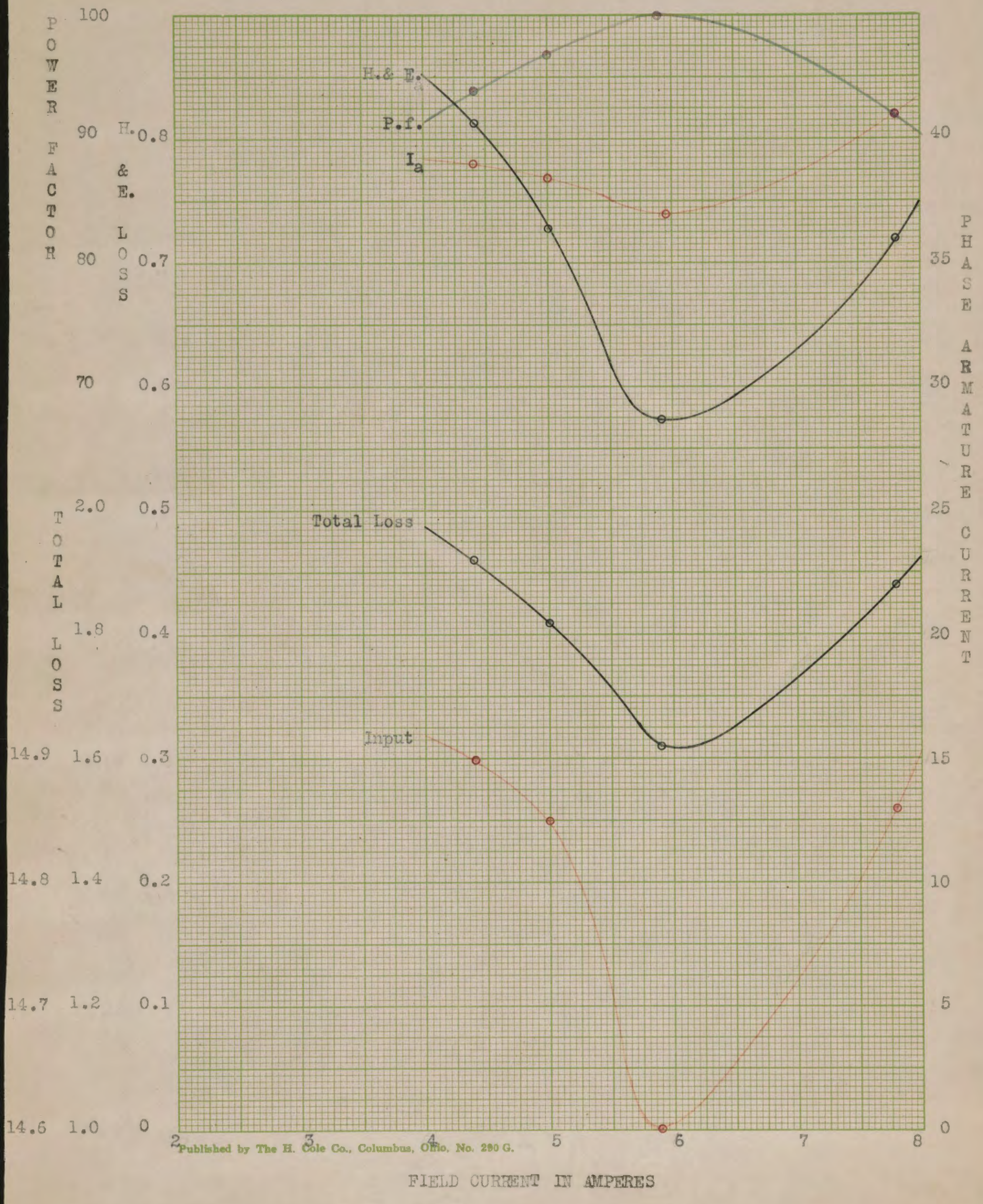
2/3 LOAD

OUTPUT = 8.75 K.W.

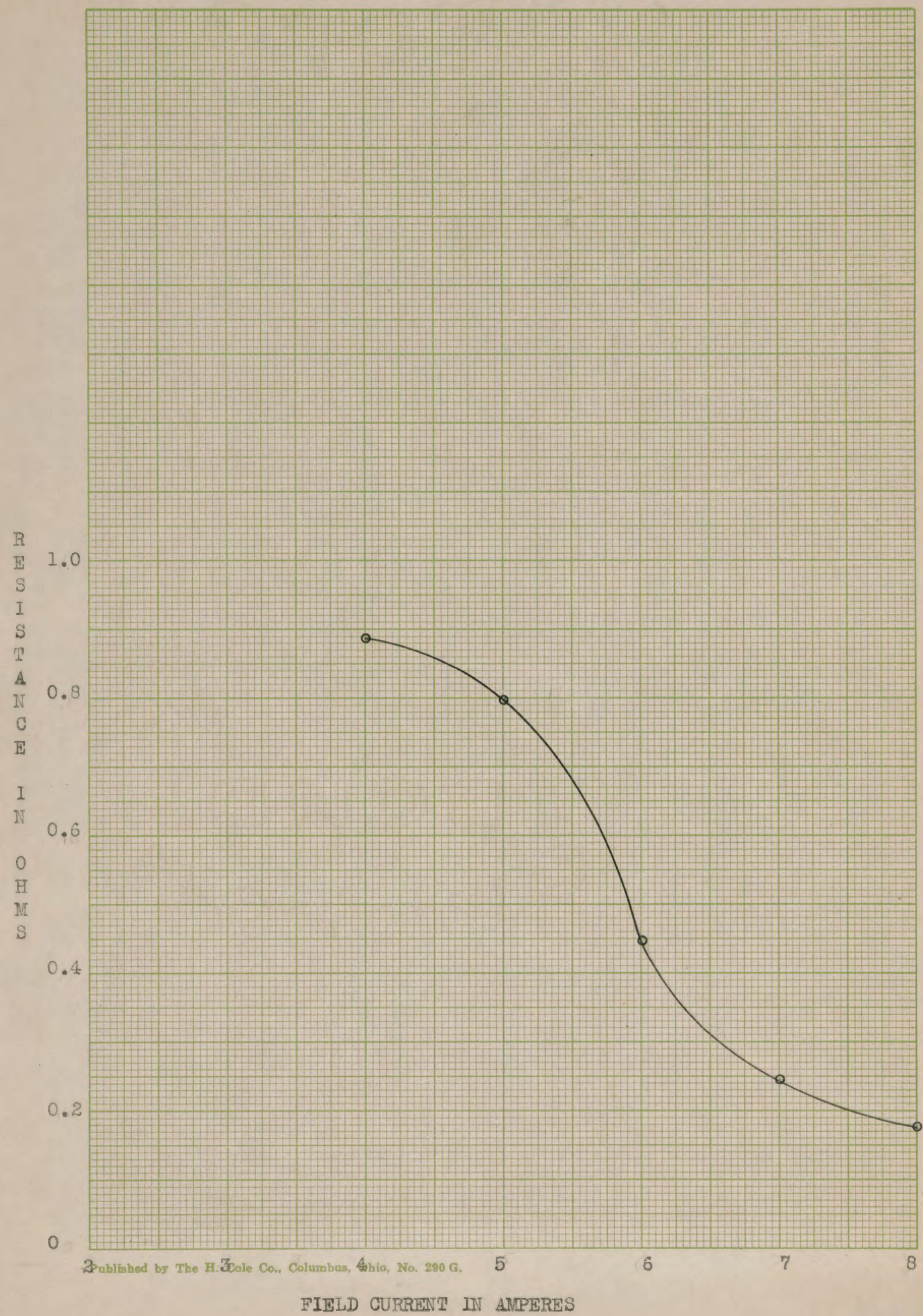


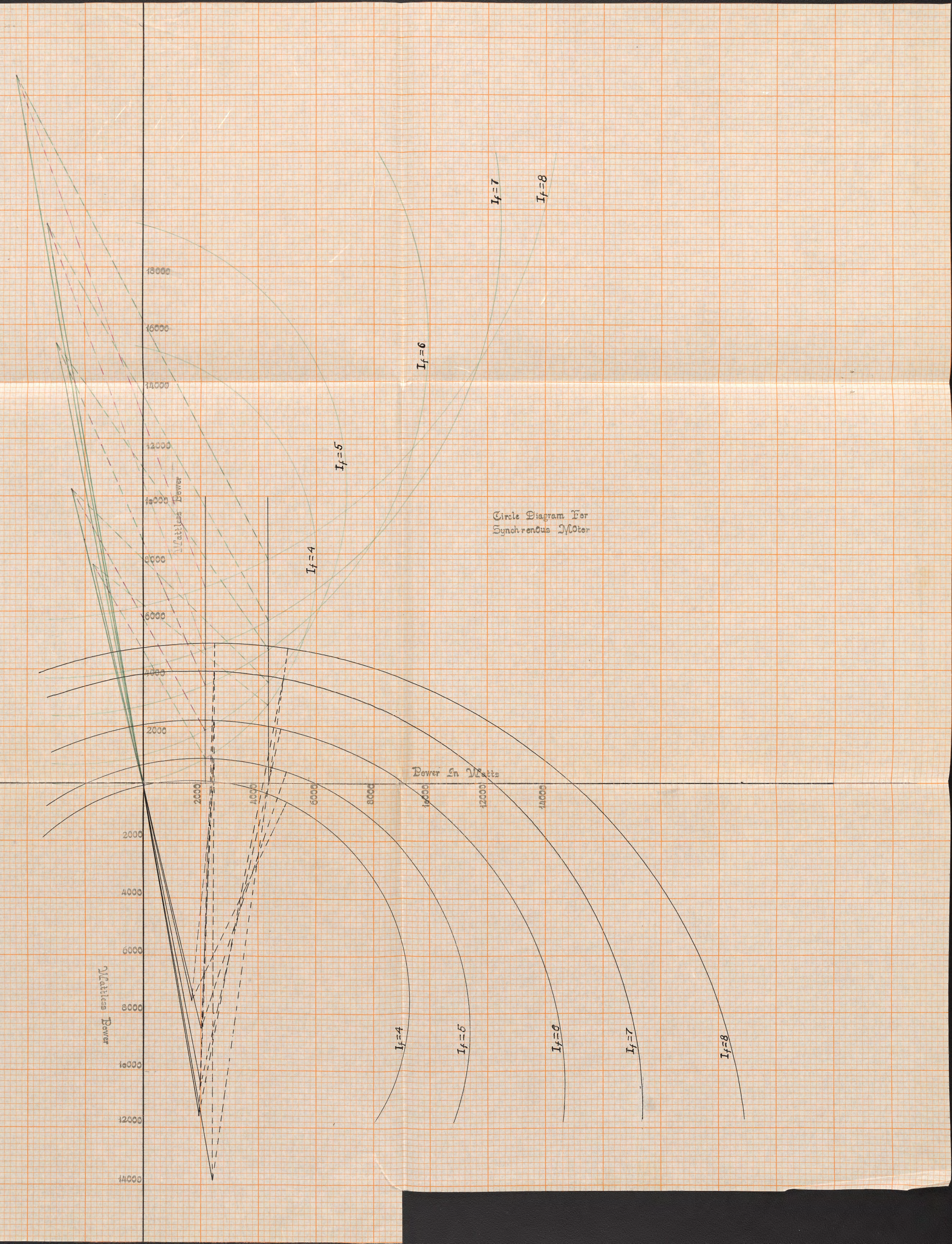
FULL LOAD

OUTPUT = 12.98 K.W.



R₃ - HYSTERESIS AND EDDY CURRENT LOSSES




Circle Diagram For
Synchronous Motor
Power In Watts
Wattless Power
Wattless Power
$I_f=4$
$I_f=5$
$I_f=6$
$I_f=7$
$I_f=8$
$I_f=4$
$I_f=5$
$I_f=6$
$I_f=7$
$I_f=8$
18000
16000
14000
12000
10000
8000
6000
4000
2000
2000
4000
6000
8000
10000
12000
14000
2000
4000
6000
8000
10000
12000
14000