

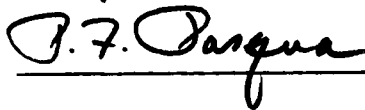
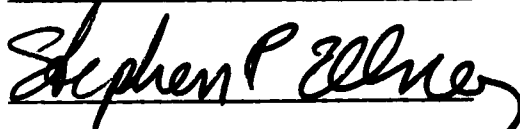
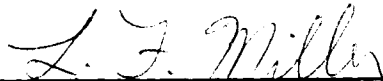
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THE FUNCTIONAL INTERPOLATION METHOD FOR REACTOR
CORE PHYSICS CALCULATIONS

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Meraj Rahimi

June 1986

This work is dedicated toward the path of Allah. May Allah help me to use this knowledge to strengthen my belief in Him and apply this infinitesimal knowledge toward understanding his breathtaking creations.

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ABSTRACT

A novel methodology, the Functional Interpolation Method (FIM), has been developed. The FIM allows the calculation of control rod worth values and power distribution by means of a variational technique which utilizes a special set of trial functions.

This new methodology has been developed for a general multirodded region reactor using the one-group neutron diffusion model.

The FIM algorithm was validated by comparing its predictions to exact analytical solutions of a one-dimensional model of the Fast Flux Test Facility (FFTF) reactor. The outcome of this validation procedure was that given two exact reactor neutronics solutions at two specified reactor core configurations, the FIM methodology provides practically exact results for any intermediate reactor configuration.

Hence, the technique developed in this thesis allows the calculation of control rod worths and power distribution along a reactor fuel cycle, given the eigenvalues and flux profiles for two arbitrary picked points in the cycle.

In view of the algebraic simplicity of the algorithm, the FIM is ideally suitable for on-line computer calculations and hence for the development of reactivity and power distribution monitor systems.

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CHAPTER I

INTRODUCTION

A reactivity and power distribution monitor [1] (RPDM) is needed to provide early detection of abnormal conditions within a reactor core. To this end one must quantify discrepancies between calculated and measured control rod worths and power distributions as functions of isotopic compositions (burn-up). This model calibration approach to a RPDM system was first used to study the reactivity balance during start-up and operation of the High Flux Isotopes Reactor (HFIR) [2,3,4] at Oak Ridge National Laboratory (ORNL). Similar surveillance procedures for anomaly detection have been developed for Boiling Water Reactors (BWR) [5], Pressurized Water Reactors (PWR) [6] and Fast Breeder Reactors (FBR) [7,8].

Because of increasing emphasis on the part of the utilities on the economic operation of nuclear power plants as well as on the maintenance of the integrity of reactor components, there is an interest in using on-line computer systems for monitoring the distribution of core power and reactor performance in general. To implement the on-line monitoring of reactivity and power distribution, the core performance must be predicted at frequent intervals to detect anomalies and update integral performance parameters such as control rod worths and isotopic fuel compositions. In view of the scope of 3-D neutronic calculations and even allowing for substantial improvements in the present computer state-of-the-art, one does not envision the

usage of sophisticated 3-D calculations in economically feasible RPDM systems. There are though several alternate calculational methods which have been proposed such as Coarse-Mesh Nodal Technique and three-dimensional-based Generalized Perturbation Theory (GPT) [1]. Although the nodal method will indeed be even more economical than direct 3-D calculations, it is not clear at this point in time if these techniques will be appropriate for frequent on-line calculations.

On the other hand, despite the powerful features exhibited by the GPT algorithm, this must rely on extensive off-line, 3-D calculations of the core neutronics along the evaluation of the fuel cycle, which would increase the on-line computer memory requirements.

A third alternate methodology has been developed at ORNL Engineering Physics and Mathematics Division by Diffilippo [9]. This method is based on analytical solutions to judiciously simplified neutron balance equations. The role of these simplified calculations is a description of the reactor power distribution in terms of a parameter set that can be determined by fitting the simplified core flux distribution to off-line multidimensional-multigroup calculations of power profiles for various "benchmarking" operating conditions.

Motivated by the crucial role played by accurate on-line computer algorithms in the development of RPDM systems, this thesis presents the development of a new calculational algorithm called the Functional Interpolation Method (FIM). The goal of this method is to effectively implement sophisticated "off-line" core neutronics calculations into simpler yet accurate on-line computer algorithms. The FIM is developed along the following steps:

1. Off-line "exact" (analytical) calculations of the core flux distributions and control rod worth values are performed for two selected core configurations along the fuel cycle.

2. The core neutronics is formulated in terms of a Stationary Functional (Variational Principle), written for the forward and adjoint fluxes and the reactivity eigenvalue $\lambda = 1/K$.

3. Given the pairs of forward and adjoint fluxes (ψ_1, ψ_1^+) and (ψ_2, ψ_2^+) , associated with the respective selected core configurations, one constructs trial functions, ϕ and ϕ^+ , in terms of the forward and the adjoint fluxes:

$$\phi = q \psi_1 + (1 - q)\psi_2 \quad (1-1)$$

$$\phi^+ = \gamma \psi_1^+ + (1 - \gamma)\psi_2^+ \quad (1-2)$$

$$-\infty < q < \infty ; \quad -\infty < \gamma < \infty \quad (1-3)$$

where q and γ are parameters to be determined.

4. The trial functions ϕ and ϕ^+ are inserted into the Variational Principle and the integrals over phase space are performed to reduce the stationary functional to a function of the parameters q , γ , and the eigenvalue λ .

5. The stationarity condition yields three coupled algebraic equations for the determination of the homotopy parameters q , γ and the reactivity eigenvalue λ .

6. The forward and adjoint fluxes are reconstructed by the use of Eqs. (1-1) and (1-2), respectively.

To validate the FIM algorithm, this method will be illustrated using a one-group diffusion treatment of a one-dimensional representation of the Fast Flux Test Facility (FFTF).

The next chapter will provide the basic logic involved in the Variational Principle Technique and Chapters III and IV will illustrate the utilization of this technique to develop the FIM approach for a two-dimensional reactor core. Chapter V will prove the validity of the FIM technique and finally Chapters VI and VII will draw the conclusions and the necessary recommendations.

CHAPTER II

THE VARIATIONAL PRINCIPLE TECHNIQUE

The Variational Principle Technique is a device which utilizes prior physical information on a particular system under consideration to obtain approximate analytical solution describing the actual behavior of the system.

In this chapter we review some features of this technique to facilitate the understanding of the FIM methodology.

2.1. Calculus of Variations

The calculus of variations is concerned with changes in functionals. A functional is a correspondence between a function in some class and the set of real numbers, e.g.,

$$\phi(y) = \int_a^b f(x,y,y')dx , \quad (2-1)$$

$$\phi(y) = \text{MAX}|y(x)| \quad a < x < b. \quad (2-2)$$

The function f is a known function of its arguments. Once $y(x)$ is fully specified, the functional $\phi(y)$, which is a real number, can be calculated.

Having the above argument in mind, we now turn our attention to a property called stationarity property. A function $g(x)$ is stationary at $x = \bar{x}$ when

$$\left. \frac{dg(x)}{dx} \right|_{x=\bar{x}} = 0 , \quad (2-3)$$

$$\lim_{\varepsilon \rightarrow 0} \frac{g(\bar{x} + \varepsilon) - g(\bar{x})}{\varepsilon} = 0 . \quad (2-4)$$

Applying the stationarity property to the functional defined by Eq. (2-1):

$$\lim_{\varepsilon \rightarrow 0} \frac{\phi(y + \varepsilon \eta) - \phi(y)}{\varepsilon} \quad (2-5)$$

where

$$\eta = 0 \text{ @ } x = a, x = b . \quad (2-6)$$

The functions which satisfy the above functional are called extremals. In order to show the conditions which are satisfied by extremals one can consider the following equation:

$$\phi(\varepsilon) = \int_a^b f(x, y + \varepsilon \eta, y' + \varepsilon \eta') . \quad (2-7)$$

Applying the stationarity property,

$$\left. \frac{d\phi(\varepsilon)}{d\varepsilon} \right|_{\varepsilon=0} = \int_a^b dx \left(\eta \frac{\partial f}{\partial y} + \eta' \frac{\partial f}{\partial y'} \right) = 0 . \quad (2-8)$$

Integrating the second term on the right side of Eq. (2-8) will yield

$$\int_a^b \frac{\partial f}{\partial y'} \frac{d\eta}{dx} dx = \left(\eta \frac{\partial f}{\partial y'} \right)_a^b - \int_a^b \eta \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) dx . \quad (2-9)$$

The first term on the right side of Eq. (2-9) is set to zero because of the boundary conditions defined in Eq. (2-6). Therefore, Eq. (2-9) becomes

$$\int_a^b \frac{\partial f}{\partial y'} \frac{d\eta}{dx} dx = - \int_a^b \eta \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) dx . \quad (2-10)$$

Inserting Eq. (2-10) into Eq. (2-8) will result in

$$\left. \frac{d\phi(\epsilon)}{d\epsilon} \right|_{\epsilon=0} = \int_a^b dx \left(\eta \frac{\partial f}{\partial y} - \eta \frac{d}{dx} \frac{\partial f}{\partial y'} \right) = 0 , \quad (2-11)$$

$$\left. \frac{d\phi(\epsilon)}{d\epsilon} \right|_{\epsilon=0} = \int_a^b dx \eta \left(\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} \right) = 0 , \quad (2-12)$$

$$\int_a^b \eta(x) m(x) dx = 0 ,$$

where

$$m(x) = \frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} . \quad (2-13)$$

If $\eta(x)$ vanishes at a and b only, therefore for all other functions of $\eta(x)$, $m(x)$ is equal to zero:

$$m(x) = \frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} = 0 . \quad (2-14)$$

The above relation is called the Euler-Lagrangian Equation for the extremals of the variational problem and the function in (2-7) is called the Lagrangian.

It should be noticed that the variations in the Lagrangian were calculated using a set of boundary conditions (Eq. (2-6)) which are satisfied by the functional. Any other constraints or arbitrary boundary conditions can be met by modifying the Lagrangian. This modification will involve calculations of some terms called Lagrange Multipliers which are obtained by applying the stationarity constraints on the Lagrangian.

2.2. Lagrange Multipliers

Variations can be taken subject to constraints which are handled using Lagrange Multipliers. Considering the problem of making the functional J stationary subject to the condition that a given functional K has a prescribed value K_1 , i.e.,

$$J = \int_a^b F(x, y + \epsilon \eta, y' + \epsilon \eta') dx , \quad (2-15)$$

$$K = \int_a^b G(x, y + \epsilon \eta, y' + \epsilon \eta') dx . \quad (2-16)$$

For an admissible function $y(x)$ one can form the following functional:

$$J^* = \int_a^b dx [F(x, y + \epsilon \eta, y' + \epsilon \eta') + \lambda G(x, y + \epsilon \eta, y' + \epsilon \eta')] \quad (2-17)$$

where λ is the Lagrange Multiplier. Applying the stationarity property, which was discussed in the previous section, to (2-17):

$$\frac{dJ^*}{d\varepsilon} \equiv \delta J^* = \int_a^b dx \left(\eta \frac{\partial F}{\partial y} + \frac{\partial \eta}{\partial x} \frac{\partial F}{\partial y'} + \lambda \left(\eta \frac{\partial G}{\partial y} + \frac{\partial \lambda}{\partial x} \frac{\partial G}{\partial y'} \right) \right) = 0 . \quad (2-18)$$

Now applying integration by parts to the second and fourth term of the right side of the above equation:

$$\begin{aligned} \delta J^* = \int_a^b dx \left(\eta \frac{\partial F}{\partial y} + \lambda \eta \frac{\partial G}{\partial y} \right) &+ \left[\eta \frac{\partial F}{\partial y'} \right]_a^b + \lambda \left[\eta \frac{\partial G}{\partial y'} \right]_a^b \\ &- \int_a^b dx \left(\eta \frac{\partial}{\partial x} \frac{\partial F}{\partial y'} + \lambda \eta \frac{\partial}{\partial x} \frac{\partial G}{\partial y'} \right) = 0 . \end{aligned} \quad (2-19)$$

Rewriting the above equation,

$$\delta J^* = \int_a^b dx \left(\eta \left(\frac{\partial F}{\partial y} + \lambda \frac{\partial G}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} + \lambda \frac{\partial G}{\partial y'} \right) \right) \right) = 0 . \quad (2-20)$$

Since η can be any arbitrary zero or nonzero number, the following relation has to apply:

$$\frac{\partial F}{\partial y} + \lambda \frac{\partial G}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} + \lambda \frac{\partial G}{\partial y'} \right) = 0 . \quad (2-21)$$

λ is obtained by introducing into (2-16) the extremals $\bar{y}(\lambda, x)$, which is obtained from the above equation, and setting $K = K_1$ (prescribed value at the K-functional).

The point of the above exercise was to show a Lagrangian such as (2-15), constrained by any arbitrary condition or functional such as (2-16), can satisfy the stationarity property by inclusion of a term called Lagrange Multiplier such as λ .

The following chapters will show the specific application of the variational techniques in calculating the fluxes and the eigenvalues in reactor physics.

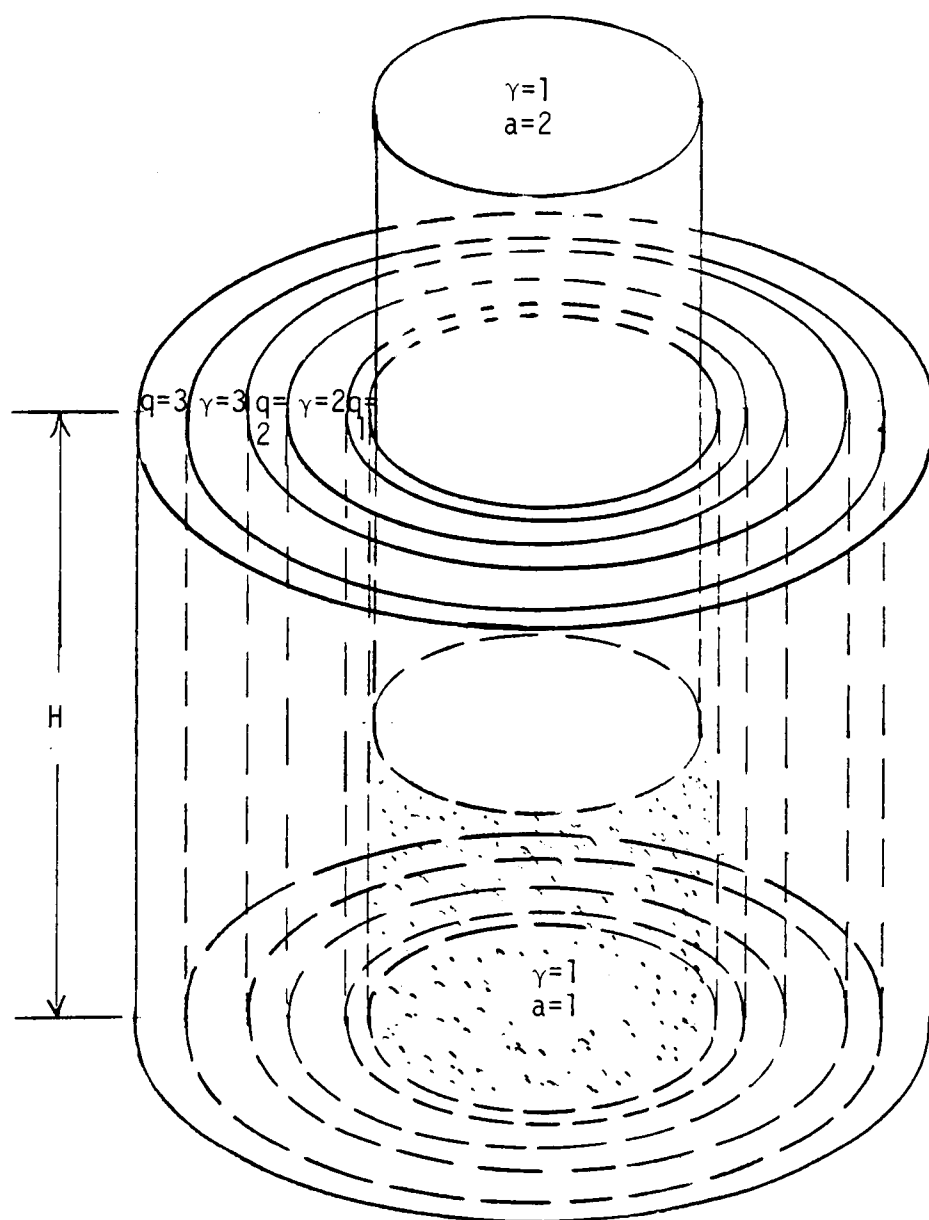
CHAPTER III

A VARIATIONAL PRINCIPLE FOR A MULTIREGION REACTOR WITH PARTIALLY INSERTED RING OF CONTROL RODS

In this chapter a Lagrangian for a multiregion reactor with partially inserted control rod will be developed. One group diffusion equation is used in conjunction with the tools which were developed in the previous chapter. The first section of this chapter will explain the geometry of the model along with the constraints or boundary conditions which are applied on the Lagrangian. In the second part a Lagrangian, which satisfies the stationarity property and the constraints introduced in the first part, will be developed. The level of details was limited to showing the input and output because of the logic explained in the previous chapter and lengthy calculations involved in these types of calculations.

3.1. The Geometry Configuration

A cylindrical reactor as depicted in Figure 3-1 with core height H was considered. In the radial direction there are several core and blanket regions with interdispersed annular regions containing control rods. Rodless regions are assumed homogeneous in both the radial and axial directions. Control rod regions are divided along the axial direction into two subregions, the lower part containing sodium and the upper part filled with the control rod material. A ring of neutron detectors is placed at $r = r_0$ and $z = z_0$.



$q = 1, \dots, Q$ = radial fuel region
 $\gamma = 1, \dots, G$ = radial control rod region
 $a = 1$ = axial sodium region
 $a = 2$ = axial control rod region

Figure 3-1. A multiregion cylindrical reactor core model.

Rodless regions will be identified by the subindex q ($q=1, \dots, Q$). Control rod regions carry two subindexes: the γ subindex ($\gamma=1, \dots, G$) which describes the radial location, and the subindex a ($a=1,2$), describing the sodium and control material subregions, respectively.

Using the neutron diffusion approximation, a set of coupled equations for the rodless region and control rod regions can be written as follows:

$$[D_q \nabla^2 + \frac{F_q}{K} - \Sigma_q] \phi_q = 0 \quad (3-1)$$

where

$$q = 1, \dots, Q,$$

$$K = \text{reactivity eigenvalue},$$

$$F_q = \nu \Sigma_{fq},$$

$$\nu = \text{average number of neutrons born per fission reaction},$$

$$\Sigma_{fq} = \text{macroscopic fission cross section (cm}^{-1}\text{)},$$

$$\Sigma_q = \text{macroscopic absorption cross section (cm}^{-1}\text{)},$$

$$D_q = \text{diffusion coefficient (cm)},$$

and for the control rod regions the diffusion equation will take the following form:

$$[D_{\gamma a} \nabla^2 - \Sigma_{\gamma a}] \phi_{\gamma a} = 0 \quad (3-2)$$

where

$$a = 1,2,$$

$$\Sigma_{\gamma 1} = \text{macroscopic absorption cross section for sodium (cm}^{-1}\text{)},$$

$\Sigma_{\gamma 2}$ = macroscopic absorption cross section for the control material (cm^{-1}),

$D_{\gamma 1}$ = diffusion coefficient for sodium (cm),

$D_{\gamma 2}$ = diffusion coefficient for the control material (cm).

The following boundary conditions apply:

$$\phi_Q(r_Q, Z) = 0, \quad (3-3)$$

$$\text{Flux continuity across all boundaries,} \quad (3-4)$$

$$\phi_q(r, Z = 0) = \phi_q(r, Z = H) = 0. \quad (3-5)$$

For adjoining rodless regions the following boundary condition is applied:

$$D_q \left(\frac{\partial \phi_q}{\partial r} \right)_{r_q} = D_{q+1} \left(\frac{\partial \phi_{q+1}}{\partial r} \right)_{r_q}. \quad (3-6)$$

At the interface between core and control regions the following boundary conditions are applicable:

$$D_{\gamma 1} \left(\frac{\partial \phi_{\gamma 1}}{\partial r} \right)_{r_\gamma} = D_q \left(\frac{\partial \phi_q}{\partial r} \right)_{r_\gamma} \quad 0 < Z < Z_q, \quad (3-7)$$

$$D_{\gamma 2} \left(\frac{\partial \phi_{\gamma 2}}{\partial r} \right)_{r_\gamma} = D_q \left(\frac{\partial \phi_q}{\partial r} \right)_{r_\gamma} \quad Z_q \leq Z < H. \quad (3-8)$$

For each rodless-region the following constraints will apply:

$$D_{\gamma 1} \left(\frac{\partial \phi_{\gamma 1}}{\partial Z} \right)_{Z_\gamma} = D_{\gamma 2} \left(\frac{\partial \phi_{\gamma 2}}{\partial Z} \right)_{Z_\gamma} \quad r_\gamma < r < r_{\gamma+1}, \quad (3-9)$$

and

$$\text{Flux finite at } r = 0 . \quad (3-10)$$

3.2. The Variational Principle

In this section a functional which is stationary with respect to the trial functions satisfying only the boundary conditions (3-3), (3-4), (3-5), and (3-10) was developed.

In developing this functional, one should remember that a prior experience would reduce the number of trials needed to obtain the proper functional. Nevertheless, the general rule for obtaining a functional which is stationary is to multiply the diffusion equation by the adjoint flux and integrating over the phase space. The details of these operations are not shown here, but instead the implementation of the stationarity property is shown. We then write the following functional:

$$\begin{aligned} \mathcal{E} = & \sum_{q=1}^Q \int_q r dr \int_0^H dz W_q (D_q \nabla^2 + \frac{F_q}{K} - \Sigma_q) \phi_q \\ & + \sum_{q=1}^Q \left[\int_q r dr \int_0^{Z_q} dz W_{q1} (D_{q1} \nabla^2 - \Sigma_{q1}) \phi_{q1} + \int_q r dr \int_{Z_q}^H dz W_{q2} (D_{q2} \nabla^2 - \Sigma_{q2}) \phi_{q2} \right] \\ & - \sum_{q=1}^Q r_q \int_0^H dz \left[W_q \left(D_q \left(\frac{\partial \phi_q}{\partial r} \right)_{r_q} - D_{q+1} \left(\frac{\partial \phi_{q+1}}{\partial r} \right)_{r_q} \right) + D_q \left(\frac{\partial W_q}{\partial r} \right) r_q (\phi_{q+1} - \phi_q) r_q \right] \\ & - \sum_{q=1}^{Q-1} r_q \int_0^{Z_q} dz \left[W_{q1} \left(D_{q1} \left(\frac{\partial \phi_{q1}}{\partial r} \right)_{r_q} - D_q \left(\frac{\partial \phi_q}{\partial r} \right)_{r_q} \right) + D_{q1} \left(\frac{\partial W_{q1}}{\partial r} \right) (\phi_q - \phi_{q1}) r_q \right] \\ & - \sum_{q=1}^{Q-1} r_q \int_{Z_q}^H dz \left[W_{q2} \left(D_{q2} \left(\frac{\partial \phi_{q2}}{\partial r} \right)_{r_q} - D_q \left(\frac{\partial \phi_q}{\partial r} \right)_{r_q} \right) + D_{q2} \left(\frac{\partial W_{q2}}{\partial r} \right) (\phi_q - \phi_{q2}) r_q \right] \\ & - \sum_{q=1}^{Q-1} \int_q r dr \left[W_{q1} \left(D_{q1} \left(\frac{\partial \phi_{q1}}{\partial z} \right)_{Z_q} - D_{q2} \left(\frac{\partial \phi_{q2}}{\partial z} \right)_{Z_q} \right) + D_{q1} \left(\frac{\partial W_{q1}}{\partial z} \right)_{Z_q} (\phi_{q2} - \phi_{q1}) Z_q \right] \end{aligned} \quad (3-11)$$

where the quantities W_q , W_{q1} , and W_{q2} are the Lagrange Multipliers and the remaining are the conventional parameters.

To prove the stationarity property of the functional L (i.e., $\delta L = 0$, for arbitrary variations of the trial functions), one can take variations on both sides of Eq. (3-11) and utilizing the following integration by parts.

$$\begin{aligned}
 \sum_{q=1}^Q \int_q r dr \int_0^H dz D_q W_q \nabla^2 \delta \phi_q &= \sum_{q=1}^Q \int_0^H dz D_q \left\{ \left(r_q W_q(r_q) \left(\frac{\partial \delta \phi_q}{\partial r} \right) r_q \right. \right. \\
 &\quad \left. \left. - r_{q-1} W_q(r_{q-1}) \left(\frac{\partial \delta \phi_q}{\partial r} \right) r_{q-1} \right) \right. \\
 &\quad \left. + \left(r_{q-1} \delta \phi_q(r_{q-1}) \left(\frac{\partial W_q}{\partial r} \right) r_{q-1} \right. \right. \\
 &\quad \left. \left. - r_q \delta \phi_q(r_q) \left(\frac{\partial W_q}{\partial r} \right) r_q \right) \right. \\
 &\quad \left. + \int_q r dr \delta \phi_q \nabla^2 W_q \right\}, \tag{3-12}
 \end{aligned}$$

$$\begin{aligned}
 \sum_{q=1}^Q \int_q r dr \int_0^{Z_q} dz W_{q1} \nabla^2 \delta \phi_{q1} &= \sum_{q=1}^{Q-1} D_{q1} \left\{ \int_q r dr \left(W_{q1}(Z_q) \left(\frac{\partial \delta \phi_{q1}}{\partial Z} \right) Z_q - \delta \phi_{q1}(Z_q) \left(\frac{\partial W_{q1}}{\partial Z} \right) Z_q \right) \right. \\
 &\quad \left. + \int_0^{Z_q} dz \left(\left(r_q W_{q1}(r_q) \left(\frac{\partial \delta \phi_{q1}}{\partial r} \right) r_q \right. \right. \right. \\
 &\quad \left. \left. - r_{q-1} W_{q1}(r_{q-1}) \left(\frac{\partial \delta \phi_{q1}}{\partial r} \right) r_{q-1} \right) \right.
 \end{aligned}$$

$$\begin{aligned}
& + (r_{q-1} \delta \phi_{q1}(r_{q-1}) (\frac{\partial W_{q1}}{\partial r})_{r_{q-1}} \\
& - r_q \delta \phi_{q1}(r_q) (\frac{\partial W_{q1}}{\partial r})_{r_q}) \\
& + \int_q r dr \int_0^{Z_q} dZ \delta \phi_{q1} \nabla^2 W_{q1} \Bigg] \quad (3-13)
\end{aligned}$$

$$\begin{aligned}
\sum_{q=1}^Q \int_q r dr \int_{Z_q}^H dZ D_{q2} W_{q2} \nabla^2 \delta \phi_{q2} &= \sum_{q=1}^{Q-1} D_{q2} \left(\int_q r dr \left(-W_{q2}(Z_q) (\frac{\partial \delta \phi_{q2}}{\partial Z})_{Z_q} \right. \right. \\
& + \delta \phi_{q2}(Z_q) (\frac{\partial W_{q2}}{\partial Z})_{Z_q} \Bigg) \\
& + \int_{Z_q}^H dZ \left((r_q W_{q2}(r_q) (\frac{\partial \delta \phi_{q2}}{\partial r})_{r_q} \right. \\
& - r_{q-1} W_{q2}(r_{q-1}) (\frac{\partial \delta \phi_{q2}}{\partial r})_{r_{q-1}}) \\
& + (r_{q-1} \delta \phi_{q2}(r_{q-1}) (\frac{\partial W_{q2}}{\partial r})_{r_{q-1}} \\
& - r_q \delta \phi_{q1}(r_q) (\frac{\partial W_{q2}}{\partial r})_{r_q}) \Bigg) \\
& + \int_q r dr \int_{Z_q}^H dZ \delta \phi_{q2} \nabla^2 W_{q2} \Bigg] \quad (3-14)
\end{aligned}$$

The boundary conditions (3-3), (3-4), (3-5), and (3-10) were used in the derivation of the above equations. By imposing the following boundary conditions on the Lagrange Multipliers:

$$W_q(r,0) = W_q(r,H) = 0 , \quad (3-15)$$

$$W_{q1}(r,0) = 0 ; W_{q2}(r,H) = 0 , \quad (3-16)$$

one obtains in this manner the following result:

$$\delta L = \delta L_1 + \delta L_2 + S_0 + S_1 + S_2 \quad (3-17)$$

where

$$\begin{aligned} \delta L_1 = & \sum_{q=1}^Q \int_q r dr \int_0^H \delta W_q \left(D_q \nabla^2 + \frac{F_q}{K} - \Sigma_q \right) \phi_q \\ & + \sum_{q=1}^Q \int_q r dr \int_0^{Z_q} \delta W_{q1} \left(D_{q1} \nabla^2 - \Sigma_{q1} \right) \phi_{q1} \\ & + \sum_{q=1}^Q \int_q r dr \int_{Z_q}^H \delta W_{q2} \left(D_{q2} \nabla^2 - \Sigma_{q2} \right) \phi_{q2} , \end{aligned} \quad (3-18)$$

$$\begin{aligned} \delta L_2 = & \sum_{q=1}^Q \int_q r dr \int_0^H dZ \delta \phi_q \left((D_q \nabla^2 + \frac{F_q}{K} - \Sigma_q) W_q \right) \\ & + \sum_{q=1}^Q \int_q r dr \int_0^{Z_q} dZ \delta \phi_{q1} \left((D_{q1} \nabla^2 - \Sigma_{q1}) W_{q1} \right) \\ & + \sum_{q=1}^Q \int_q r dr \int_{Z_q}^H dZ \delta \phi_{q2} \left((D_{q2} \nabla^2 - \Sigma_{q2}) W_{q2} \right) , \end{aligned} \quad (3-19)$$

$$\begin{aligned} S_0 = & \sum_{q=1}^{Q-1} \int_q r dr \int_0^H dZ \left(- r_q \left(D_q \frac{\partial \phi_q}{\partial r} - D_{q+1} \frac{\partial \phi_{q+1}}{\partial r} \right) r_q \delta W_q \right. \\ & \left. - r_q D_q (\phi_{q+1} - \phi_q) r_q \left(\frac{\partial \delta W_q}{\partial r} \right) r_q \right) \end{aligned}$$

$$\begin{aligned}
& + r_q \left(D_{q+1} \left(\frac{\partial W_{q+1}}{\partial r} \right) r_q - D_q \left(\frac{\partial W_q}{\partial r} \right) r_q \right) \delta \phi_{q+1}(r_q) \\
& + r_q (W_{q+1} - W_q) r_q D_{q+1} \left(\frac{\partial \delta W_{q+1}}{\partial r} \right) r_q \Bigg] , \quad (3-20)
\end{aligned}$$

$$\begin{aligned}
S_1 = \sum_{q=1}^{Q-1} \int_q r dr \int_0^{Z_q} dZ \Bigg(& - r_q \left(D_q \frac{\partial \phi_{q1}}{\partial r} - D_{q+1} \frac{\partial \phi_{q+1}}{\partial r} \right) r_q \delta W_{q1} \\
& - r_q D_q (\phi_{q1+1} - \phi_{q1}) r_q \left(\frac{\partial \delta W_q}{\partial r} \right) r_q \\
& + r_q \left(D_{q+1} \left(\frac{\partial W_{q1+1}}{\partial r} \right) r_q - D_q \left(\frac{\partial W_{q1}}{\partial r} \right) r_q \right) \delta \phi_{q1+1}(r_q) \\
& + r_q (W_{q1+1} - W_{q1}) r_q D_{q+1} \left(\frac{\partial \delta \phi_{q1+1}}{\partial r} \right) r_q \Bigg] , \quad (3-21)
\end{aligned}$$

$$\begin{aligned}
S_2 = \sum_{q=1}^{Q-1} \int_q r dr \int_{Z_q}^H dZ \Bigg(& - r_q \left(D_q \frac{\partial \phi_{q2}}{\partial r} - D_{q+1} \left(\frac{\partial \phi_{q2+1}}{\partial r} \right) r_q \right) \delta W_{q2} \\
& - r_q D_q (\phi_{q2+1} - \phi_{q2}) r_q \left(\frac{\partial \delta W_{q2}}{\partial r} \right) r_q \\
& + r_q \left(D_{q+1} \left(\frac{\partial W_{q2+1}}{\partial r} \right) r_q - D_q \left(\frac{\partial W_{q2}}{\partial r} \right) r_q \right) \delta \phi_{q2+1}(r_q) \\
& + r_q (W_{q2+1} - W_{q2}) r_q D_{q+1} \left(\frac{\partial \delta \phi_{q2+1}}{\partial r} \right) r_q \Bigg] . \quad (3-22)
\end{aligned}$$

Inspection of Eqs. (3-18) through (3-22) reveals that for arbitrary variations $\delta \phi_q$, $\delta \phi_{q1}$, $\delta \phi_{q2}$, δW_q , δW_{q1} , and δW_{q2} one obtains

$\delta L = 0$, provided that the diffusion equations (3-1) and (3-2) are satisfied together with the boundary conditions (3-3), (3-4), (3-5), (3-6), (3-7), (3-8), (3-9), and (3-10) by both the fluxes ϕ_q , ϕ_{q1} , and ϕ_{q2} as well as the Lagrange Multipliers W_q , W_{q1} , and W_{q2} .

One should notice that the Lagrange Multipliers are the forward fluxes because of the self-adjointness of the one-group neutron diffusion operator. If this operator were to be nonself-adjoint, the Lagrange Multipliers would be defined as adjoint fluxes.

CHAPTER IV

DEVELOPMENT OF THE FIM FOR A TWO-REGION REACTOR WITH PARTIALLY INSERTED RING OF CONTROL RODS

In the previous chapter a Lagrangian for a multiregion reactor core model was developed. As it was shown, the Lagrangian was stationary ($\delta L = 0$) with respect to changes of the flux. A functional for a two-region reactor core model with a partially inserted control rod utilizing the Lagrangian obtained in the previous chapter will be developed. Thereafter, this functional or Lagrangian for the two-region core will be used with the aid of a set of trial fluxes to derive the Functional Interpolation Method (FIM) algorithm.

4.1. Lagrangian Development

A cylindrical reactor core with a partially inserted control rod, as depicted in Figure 4-1, is considered. The void, after the withdrawal of the control rod for Z_1 amount, is filled up with liquid sodium. The radius of the control rod and the outer radius of the fuel region are r_1 and r_2 , respectively.

In order to make the general variational principle Lagrangian (3-11) specific to Figure 4-1, the following equivalences and definitions should be set.

$$W_{11} = \phi_{1S} = \text{neutron flux in the sodium region,}$$

$$W_{21} = \phi_{1C} = \text{neutron flux in the control rod region,}$$

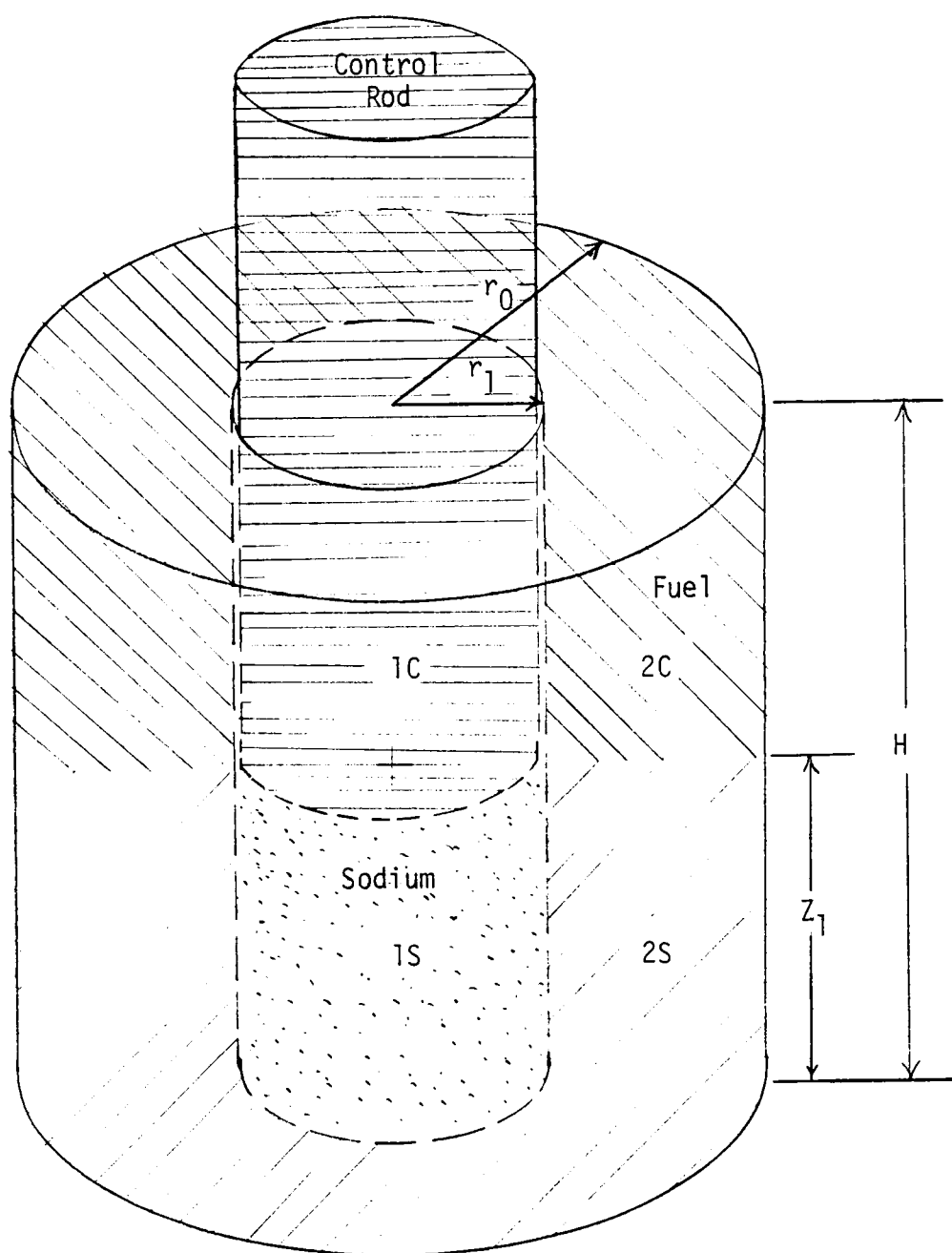


Figure 4-1. A two-region cylindrical reactor core with partially inserted control rod model.

$W_2 = \phi_{2S} (0 \leq Z \leq Z_1)$ = neutron flux in the core region adjoining the sodium zone,

$W_2 = \phi_{2C} (Z_1 \leq Z \leq H)$ = neutron flux in the core region adjoining the control rod zone,

$D_{11} = D_C$ = diffusion coefficient for the control rod region,

$D_{21} = D_S$ = diffusion coefficient for the sodium region,

$D_{12} = D_{22} = D_2$ = diffusion coefficient for the core region,

$\Sigma_{11} = \Sigma_C$ = macroscopic capture cross section for the control region,

$\Sigma_{21} = \Sigma_S$ = macroscopic capture cross section for the sodium region.

Replacing the generic parameter in Eq. (3-11) with the parameters defined in the above for a two-region reactor core, one obtains the following Lagrangian, which satisfies the stationarity property for a two-region reactor core with partially inserted control rod (Figure 4-1).

$$\begin{aligned} \mathcal{L}/2\pi = & \int_0^{r_1} r dr \int_0^{Z_1} \phi_{1S} (D_S \nabla^2 \phi_{1S} - \Sigma_S \phi_{1S}) \\ & + \int_0^{r_1} r dr \int_{Z_1}^H dZ \phi_{1C} (D_C \nabla^2 \phi_{1C} - \Sigma_C \phi_{1C}) \\ & + \int_{r_1}^{r_2} r dr \int_0^{Z_1} dZ \phi_{2S} [D_2 \nabla^2 \phi_{2S} + (F/K - \Sigma_2) \phi_{2S}] \\ & + \int_{r_1}^{r_2} r dr \int_{Z_1}^H dZ \phi_{2C} [D_2 \nabla^2 \phi_{2C} + (F/K - \Sigma_2) \phi_{2C}] \end{aligned}$$

$$\begin{aligned}
& - r_1 \int_0^{Z_1} dZ \left[\phi_{1S}(r_1) \left[D_S \left(\frac{\partial \phi_{1S}}{\partial r} \right) r_1 - D_2 \left(\frac{\partial \phi_{2S}}{\partial r} \right) r_1 \right] \right. \\
& - r_1 \int_{Z_1}^H dZ \left[\phi_{1C}(r_1) \left[D_C \left(\frac{\partial \phi_{1C}}{\partial r} \right) r_1 - D_2 \left(\frac{\partial \phi_{2C}}{\partial r} \right) r_1 \right] \right] \\
& - \int_0^{r_1} r dr \left[\phi_{1S} \left[D_S \left(\frac{\partial \phi_{1S}}{\partial Z} \right) - D_C \left(\frac{\partial \phi_{1C}}{\partial Z} \right) \right] \right]_{Z_1} . \tag{4-1}
\end{aligned}$$

Most of the above symbols were defined. The remaining ones possess their usual computational meaning. The above Lagrangian, as it was mentioned previously, is stationary with respect to changes in flux. The next step is to introduce a set of trial functions in the above Lagrangian and using the Variational Technique, an equation for static eigenvalue will be obtained.

4.2. Implementation of the Variational Technique

In order to obtain the static eigenvalue, a set of trial fluxes needs to be defined. This set can be defined as follows.

$$\phi_{1S}(\vec{r}) = q\psi_{1SA}(\vec{r}) + (1 - q)\psi_{1SB}(\vec{r}) \quad 0 < r < r_1 \quad 0 < Z < Z_1 \tag{4-2}$$

$$\phi_{1C}(\vec{r}) = q\psi_{1CA}(\vec{r}) + (1 - q)\psi_{1CB}(\vec{r}) \quad 0 < r < r_1 \quad 0 < Z < H \tag{4-3}$$

$$\phi_{2S}(\vec{r}) = q\psi_{2SA}(\vec{r}) + (1 - q)\psi_{2SB}(\vec{r}) \quad r_1 < r < r_2 \quad 0 < Z < Z_1 \tag{4-4}$$

$$\phi_{2C}(\vec{r}) = q\psi_{2CA}(\vec{r}) + (1 - q)\psi_{2CB}(\vec{r}) \quad r_1 < r < r_2 \quad Z_1 < Z < H \tag{4-5}$$

where ψ_{1SA} , ψ_{1CA} , ψ_{2SA} , ψ_{2CA} , ψ_{1SB} , ψ_{1CB} , ψ_{2SB} , and ψ_{2CB} are exact, off-line calculations of the flux corresponding to two material

compositions of the reactor core. ϕ_{1S} , ϕ_{1C} , ϕ_{2S} , and ϕ_{2C} are the wanted flux distribution for an arbitrary material make-up. The Variational Principle then searches for the value of the parameter q , which minimizes the Raleigh-Ritz quotient for the static reactivity eigenvalue.

In order to solve for parameter q , the fluxes in the Lagrangian defined by Eq. (4-1) will be replaced by the fluxes defined in Eqs. (4-2) through (4-5).

$$\begin{aligned}
 \mathcal{L}/2\pi = & \int_0^{r_1} r dr \int_0^{Z_1} dz [q\psi_{1SA} + (1-q)\psi_{1SB}] [D_S \nabla^2 (q\psi_{1SA} + (1-q)\psi_{1SB}) \\
 & - \Sigma_S (q\psi_{1SA} + (1-q)\psi_{1SB})] \\
 & + \int_0^{r_1} r dr \int_{Z_1}^H dz [q\psi_{1CA} + (1-q)\psi_{1CB}] [D_C \nabla^2 (q\psi_{1CA} + (1-q)\psi_{1CB}) \\
 & - \Sigma_C (q\psi_{1CA} + (1-q)\psi_{1CB})] \\
 & + \int_{r_1}^{r_2} r dr \int_0^{Z_1} dz [q\psi_{2SA} + (1-q)\psi_{2SB}] [D_2 \nabla^2 (q\psi_{2SA} + (1-q)\psi_{2SB}) \\
 & + (F/K - \Sigma_2)(q\psi_{2SA} + (1-q)\psi_{2SB})] \\
 & + \int_{r_1}^{r_2} r dr \int_{Z_1}^H dz [q\psi_{2CA} + (1-q)\psi_{2CB}] [D_2 \nabla^2 (q\psi_{2CA} + (1-q)\psi_{2CB}) \\
 & + (F/K - \Sigma_2)(q\psi_{2CA} + (1-q)\psi_{2CB})]
 \end{aligned}$$

$$\begin{aligned}
& - r_1 \int_0^{Z_1} dZ [q\psi_{1SA}(r_1) + (1-q)\psi_{1SB}(r_1)] \{ D_S \left[\frac{\partial}{\partial r} (q\psi_{1SA} + (1-q)\psi_{1SB}) \right]_{r_1} \\
& \quad - D_2 \left[\frac{\partial}{\partial r} (q\psi_{2SA} + (1-q)\psi_{2SB}) \right]_{r_1} \} \\
& - r_1 \int_{Z_1}^H dZ [q\psi_{1CA}(r_1) + (1-q)\psi_{1CB}(r_1)] \{ D_C \left[\frac{\partial}{\partial r} (q\psi_{1CA} + (1-q)\psi_{1CB}) \right]_{r_1} \\
& \quad - D_2 \left[\frac{\partial}{\partial r} (q\psi_{2CA} + (1-q)\psi_{2CB}) \right]_{r_1} \} \\
& - \int_0^{r_1} r dr [q\psi_{1SA} + (1-q)\psi_{1SB}]_{Z_1} \{ D_S \left[\frac{\partial}{\partial Z} (q\psi_{1SA} + (1-q)\psi_{1SB}) \right]_{Z_1} \} \\
& + \int_0^{r_1} r dr [q\psi_{1SA} + (1-q)\psi_{1SB}]_{Z_1} \{ D_C \left[\frac{\partial}{\partial Z} (q\psi_{1CA} + (1-q)\psi_{1CB}) \right]_{Z_1} \} .
\end{aligned} \tag{4-6}$$

The above equation, as it was mentioned, needs to be solved for q . Appendix A shows the operation along with the defined terms. Because of the generic nature of this algorithm, the trial fluxes will remain defined in terms of general exact fluxes and their exact fluxes will not be defined further. Appendix A also shows all of the defined terms which involve the integration of the exact fluxes. These fluxes will be obtained upon the determination of a specific core geometry and composition.

After solving the Lagrangian defined by Eq. (4-6) using the Variational Principle Technique, as it is shown in Appendix A, a solution for the static reactivity eigenvalue is obtained.

$$H_1 K^3 + H_2 K + H_3 = 0 . \quad (4-7)$$

Upon the determination of K using Eq. (4-7), the following equation is used to solve for parameter q:

$$q = - \frac{KB_1 + B_2}{2KA_1 + 2A_2} . \quad (4-8)$$

Once the parameter q is known, Eqs. (4-2) through (4-5) are used to calculate the power distribution, given the exact fluxes on the right side of the equation for two forms of the core composition are known. The obtained flux is the desired flux for the specific core composition which has been calculated through an interpolation method.

At this point it is appropriate to reiterate the above Functional Interpolation Method. Given the neutron distribution in a reactor core for the beginning of the fuel cycle and the end of the fuel cycle, the fluxes for the core during the life of the fuel can be obtained on-line using the Functional Interpolation Method. This algorithm requires calculation of the 52 integrals defined in Appendix A. After the calculation of the integrals, the static eigenvalue can be obtained. After the determination of static reactivity eigenvalue, the parameter q can be calculated. Finally, the parameter q is used to obtain the desired flux. One should again be reminded that the requirement for this method is provision of two exact analytical or numerical solutions of the fluxes for the core composition at the

beginning and the end of the fuel cycle. These solutions can be obtained off-line using the existing computer codes. Once these solutions are obtained, the FIM can be used on-line to provide the intermediate solution of the power distribution of the reactor core composition which changes during reactor operation.

In order to verify the FIM, the next chapter provides the calculation of the reactivity eigenvalue and the flux for a reactor core utilizing an analytical method. Then, the flux and the eigenvalue were obtained for the same core composition and geometry using the FIM. The results from both methods were compared. The next chapter provides the detailed operations and additional parameter which were chosen for the validity of the FIM.

CHAPTER V

VALIDATION OF THE FIM

In order to verify the Functional Interpolation Method described in the previous chapter, a reactor core in the form of a one-dimensional slab geometry was chosen. The reason for selection of the one-dimensional geometry was the ability to solve the core model using an analytical method. Since the exact solution of power distribution and reactivity eigenvalue for a reactor core described in a higher dimension of geometry requires the utilization of an existing computer code and there was a budgeting problem, a one-dimensional slab geometry, which allows the solution for the flux and the reactivity eigenvalue to be obtained using hand calculations, was selected. The Fast Flux Test Facility (FFTF) core characteristics were chosen for the one-dimensional core model descriptions in this analysis. Appendix B contains the descriptions and characteristics of the FFTF reactor core. The pertinent information was obtained from Cabell, 1980. The required cross sections were acquired from Mann, 1982. This information is also included in Appendix B. Having all the necessary data, we can now start solving for the flux and reactivity eigenvalue of the reactor core model which is described in the following section.

5.1. Analytical Solution

Considering a reactor core depicted in Figure 5-1, one can write the following equations describing the neutron population using the one-group neutron diffusion theory.

$$(D_1 \frac{d^2}{dx^2} - \Sigma_1) \phi_1(x) = 0 , \quad (5-1)$$

$$(D_2 \frac{d^2}{dx^2} + F/K - \Sigma_2) \phi_2(x) = 0 \quad (5-2)$$

where

D_1 = neutron diffusion coefficient in the control rod region (cm),

Σ_1 = macroscopic cross section for the control rod region (cm^{-1}),

$\phi_1(x)$ = neutron flux in the control rod region ($\text{neutron/cm}^2\text{-sec}$),

D_2 = neutron diffusion coefficient in the fuel region (cm),

Σ_2 = macroscopic cross section for the fuel region (cm^{-1}),

$F = v\Sigma_f$ = fission source for the fuel region,

v = average number of neutrons generated per fission reaction,

Σ_f = macroscopic fission cross section for the fuel region (cm^{-1}),

$\phi_2(x)$ = neutron flux in the fuel region ($\text{neutron/cm}^2\text{-sec}$).

The following boundary conditions are applied to Eqs. (5-1) and (5-2)

$$\left. \frac{d\phi_1}{dx} \right|_{x=0} = 0 , \quad (5-3)$$

Control
Rod
Region

Fuel Region

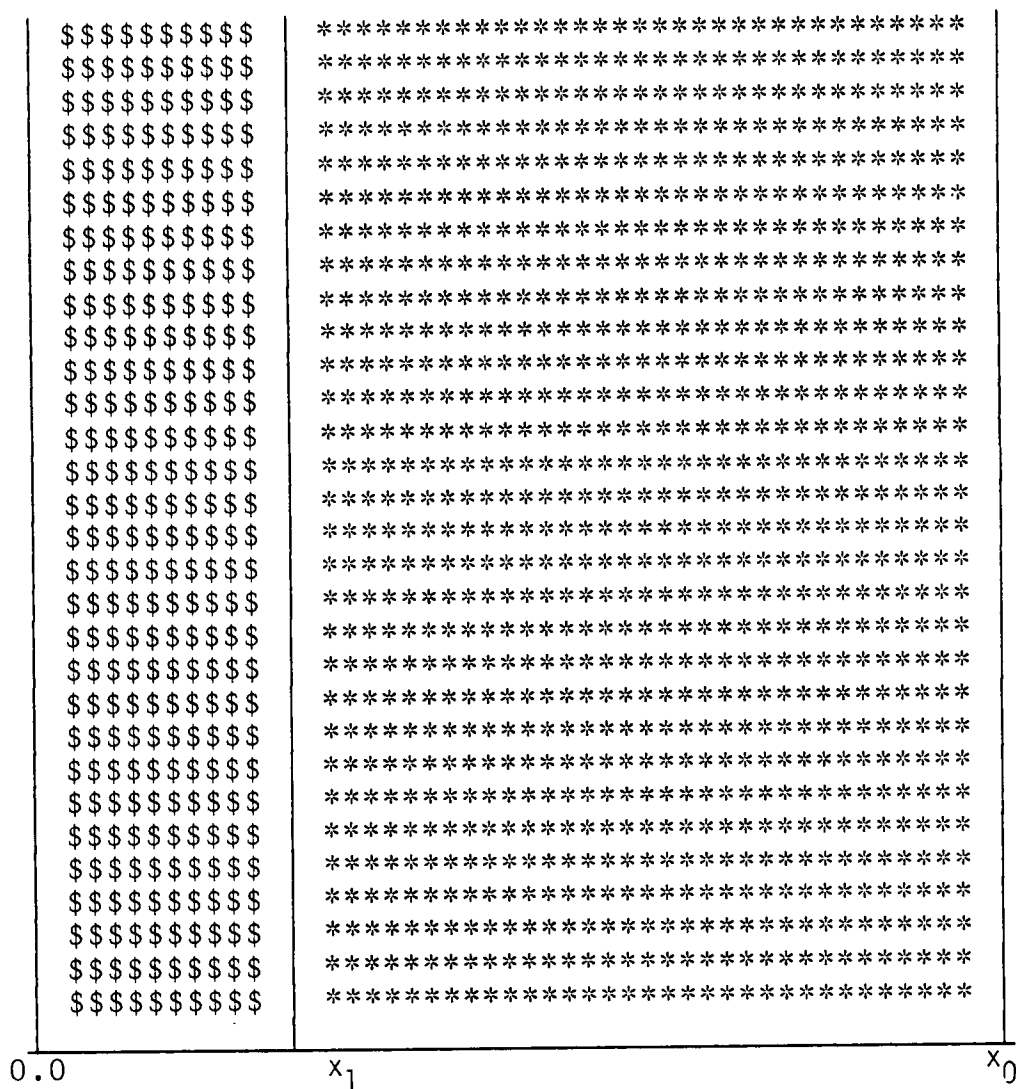


Figure 5-1. One-dimensional representation of the FFTF reactor core.

$$\phi_2(x_0) = 0 , \quad (5-4)$$

$$\phi_1(x_1) = \phi_2(x_1) , \quad (5-5)$$

$$D_1 \left. \frac{d\phi_1}{dx} \right|_{x_1} = D_2 \left. \frac{d\phi_2}{dx} \right|_{x_1} , \quad (5-6)$$

where

$$x_1 = 6.3200 \text{ cm} , \quad (5-7)$$

$$x_0 = \frac{\tilde{a}}{2} \quad (5-8)$$

$$\tilde{a} = \text{extrapolated boundary of the fuel region} = 76.56 \text{ cm.} \quad (5-9)$$

The control rod region thickness and the fuel region length were obtained from the FFTF core parameter table in Appendix B. In addition the following data for the diffusion coefficients and cross sections and other parameters were obtained from the FFTF design. All of the pertinent data and their sources are explained in Appenix B.

$$D_1 = 3.4470 \text{ cm} , \quad (5-10)$$

$$D_2 = 1.1015 \text{ cm} , \quad (5-11)$$

$$\Sigma_1 = 0.0026 \text{ cm} , \quad (5-12)$$

$$\Sigma_2 = 0.0067 \text{ cm} , \quad (5-13)$$

$$F = 0.0082 \text{ cm}^{-1} . \quad (5-14)$$

After obtaining the necessary values one can solve for the flux. Equations (5-1) and (5-2) can be written as follows:

$$\frac{d^2 \phi_1(x)}{dx^2} - \frac{\phi_1(x)}{L^2} = 0 , \quad (5-15)$$

$$\frac{d^2 \phi_2(x)}{dx^2} + b^2 \phi_2(x) = 0 , \quad (5-16)$$

where

$$b^2 = \frac{1}{D_2} (F/K - \Sigma_2) , \quad (5-17)$$

$$L^2 = \frac{D_1}{\Sigma_1} . \quad (5-18)$$

The general solution for Eq. (5-15) is:

$$\phi_1(x) = A_1 e^{\frac{x}{L}} + A_2 e^{-\frac{x}{L}} . \quad (5-19)$$

Applying boundary conditions (5-3) to Eq. (5-19) will result in the solution to one of the above coefficients.

$$\left. \frac{d\phi_1}{dx} \right|_{x=0} = \frac{A_1}{L} - \frac{A_2}{L} = 0 , \quad (5-20)$$

$$A_1 = A_2 . \quad (5-21)$$

Substituting Eq. (5-21) into (5-19):

$$\phi_1(x) = A_1(e^{\frac{x}{L}} + e^{-\frac{x}{L}}) , \quad (5-22)$$

$$\phi_1(x) = 2A_1 \cosh \frac{x}{L} , \quad (5-23)$$

$$\phi_1(x) = A \cosh \frac{x}{L} . \quad (5-24)$$

The general solution for Eq. (5-16) is:

$$\phi_2(x) = C_1 \cos bx + C_2 \sin bx . \quad (5-25)$$

If the boundary condition defined by Eq. (5-4) is applied to Eq. (5-25), a relation for the coefficients in Eq. (5-25) can be obtained:

$$C_1 = - \frac{\sin bx_0}{\cos bx_0} C_2 . \quad (5-26)$$

Inserting Eq. (5-26) into Eq. (5-25) will yield the following:

$$\phi_2(x) = C_2(- \frac{\sin bx_0}{\cos bx_0} \cos bx + \sin bx) . \quad (5-27)$$

Utilizing a trigonometric relationship will result in the following:

$$\phi_2(x) = \frac{C_2}{\cos bx_0} \sin b(x - x_0) . \quad (5-28)$$

At this point there are two solutions described by Eqs. (5-24) and (5-28) for both regions in Figure 5-1. These solutions are not complete until a value for A and C_2 are obtained. In order to obtain

such values the boundary conditions defined by Eqs. (5-5) and (5-6) are applied to both Eqs. (5-24) and (5-28).

$$A \cosh\left(\frac{x_1}{L}\right) = \frac{C_2}{\cos bx_0} \sin b(x_1 - x_0) , \quad (5-29)$$

$$D_1 \frac{A}{L} \sinh \frac{x_1}{L} = \frac{D_2 C_2 b}{\cos bx_0} \cos b(x_1 - x_0) . \quad (5-30)$$

Rewriting the above equations:

$$\frac{A}{C_2} = \frac{\sin b(x_1 - x_0)}{\cos bx_0 \cosh\left(\frac{x_1}{L}\right)} , \quad (5-31)$$

$$\frac{A}{C_2} = \frac{D_2 b L \cos b(x_1 - x_0)}{D_1 \sinh\left(\frac{x_1}{L}\right) \cos bx_0} . \quad (5-32)$$

Equating the right side of Eqs. (5-31) and (5-32) and eliminating any common term from both sides:

$$\frac{\sin b(x - x_0)}{\cosh\left(\frac{x_1}{L}\right)} = \frac{D_2 L}{D_1} \frac{b \cos b(x_1 - x_0)}{\sinh\left(\frac{x_1}{L}\right)} . \quad (5-33)$$

Rewriting the above equation:

$$\frac{D_1 \sin b(x - x_0)}{D_2 L \cosh\left(\frac{x_1}{L}\right)} = b \cot b(x_1 - x_0) . \quad (5-34)$$

Notice that the static eigenvalue is contained within the parameter b . Therefore, Eq. (5-34) has to be solved for b . Obtaining the root of Eq. (5-34) is not a trivial task. The equation can be conformed into the shape and the form that is solvable. Table 5-1 shows the roots of an equation defined as the following:

$$\alpha \cot \alpha = -C . \quad (5-35)$$

This table was obtained from Ref. 14. At this point an attempt should be made to conform Eq. (5-34) into Eq. (5-35).

$$\frac{D_1}{D_2 L} \tanh\left(\frac{x_1}{L}\right) = -b \cot b(x_0 - x_1) . \quad (5-36)$$

If both sides of the above equation are multiplied by $x_0 - x_1$, the following equation can be obtained:

$$-\frac{D_1}{D_2 L} (x_0 - x_1) \tanh\left(\frac{x_1}{L}\right) = b(x_0 - x_1) \cot b(x_0 - x_1) . \quad (5-37)$$

Equation (5-37) is in the form described by Eq. (5-35), with the following defined terms:

$$\alpha = b(x_0 - x_1) , \quad (5-38)$$

$$C = \frac{D_1}{D_2 L} (x_0 - x_1) \tanh\left(\frac{x_1}{L}\right) . \quad (5-39)$$

Equation (5-38), which contains the reactivity eigenvalue, is the unknown term. Equation (5-39), which contains the parameters with

Table 5-1. The Roots of a Transcendental Equation

The first six roots, α_n , of

$$\alpha \cot \alpha + C = 0.$$

C	α_1	α_2	α_3	α_4	α_5	α_6
-1.0	0	4.4934	7.7253	10.9041	14.0662	17.2208
-0.995	0.1224	4.4945	7.7259	10.9046	14.0666	17.2210
-0.99	0.1730	4.4956	7.7265	10.9050	14.0669	17.2213
-0.98	0.2445	4.4979	7.7278	10.9060	14.0676	17.2219
-0.97	0.2991	4.5001	7.7291	10.9069	14.0683	17.2225
-0.96	0.3450	4.5023	7.7304	10.9078	14.0690	17.2231
-0.95	0.3854	4.5045	7.7317	10.9087	14.0697	17.2237
-0.94	0.4217	4.5068	7.7330	10.9096	14.0705	17.2242
-0.93	0.4551	4.5090	7.7343	10.9105	14.0712	17.2248
-0.92	0.4860	4.5112	7.7356	10.9115	14.0719	17.2254
-0.91	0.5150	4.5134	7.7369	10.9124	14.0726	17.2260
-0.90	0.5423	4.5157	7.7382	10.9133	14.0733	17.2266
-0.85	0.6609	4.5268	7.7447	10.9179	14.0769	17.2295
-0.8	0.7593	4.5379	7.7511	10.9225	14.0804	17.2324
-0.7	0.9208	4.5601	7.7641	10.9316	14.0875	17.2382
-0.6	1.0528	4.5822	7.7770	10.9408	14.0946	17.2440
-0.5	1.1656	4.6042	7.7899	10.9499	14.1017	17.2498
-0.4	1.2644	4.6261	7.8028	10.9591	14.1088	17.2556
-0.3	1.3525	4.6479	7.8156	10.9682	14.1159	17.2614
-0.2	1.4320	4.6696	7.8284	10.9774	14.1230	17.2672
-0.1	1.5044	4.6911	7.8412	10.9865	14.1301	17.2730
0	1.5708	4.7124	7.8540	10.9956	14.1372	17.2788
0.1	1.6320	4.7335	7.8667	11.0047	14.1443	17.2845
0.2	1.6887	4.7544	7.8794	11.0137	14.1513	17.2903
0.3	1.7414	4.7751	7.8920	11.0228	14.1584	17.2961
0.4	1.7906	4.7956	7.9046	11.0318	14.1654	17.3019
0.5	1.8366	4.8158	7.9171	11.0409	14.1724	17.3076
0.6	1.8798	4.8358	7.9295	11.0498	14.1795	17.3134
0.7	1.9203	4.8556	7.9419	11.0588	14.1865	17.3192
0.8	1.9586	4.8751	7.9542	11.0677	14.1935	17.3249
0.9	1.9947	4.8943	7.9665	11.0767	14.2005	17.3306
1.0	2.0288	4.9132	7.9787	11.0856	14.2075	17.3364
1.5	2.1746	5.0037	8.0385	11.1296	14.2421	17.3649
2.0	2.2889	5.0870	8.0962	11.1727	14.2764	17.3932
3.0	2.4557	5.2329	8.2045	11.2560	14.3434	17.4490
4.0	2.5704	5.3540	8.3029	11.3349	14.4080	17.5034
5.0	2.6537	5.4544	8.3914	11.4086	14.4699	17.5562
6.0	2.7165	5.5378	8.4703	11.4773	14.5288	17.6072
7.0	2.7654	5.6078	8.5406	11.5408	14.5847	17.6562
8.0	2.8044	5.6669	8.6031	11.5994	14.6374	17.7032
9.0	2.8363	5.7172	8.6587	11.6532	14.6870	17.7481
10.0	2.8628	5.7606	8.7083	11.7027	14.7335	17.7908
15.0	2.9476	5.9080	8.8898	11.8959	14.9251	17.9742
20.0	2.9930	5.9921	9.0019	12.0250	15.0625	18.1136
30.0	3.0406	6.0831	9.1294	12.1807	15.2380	18.3018
40.0	3.0651	6.1311	9.1987	12.2688	15.3417	18.4180
50.0	3.0801	6.1606	9.2420	12.3247	15.4090	18.4953
60.0	3.0901	6.1805	9.2715	12.3632	15.4559	18.5497
80.0	3.1028	6.2058	9.3089	12.4124	15.5164	18.6209
100.0	3.1105	6.2211	9.3317	12.4426	15.5537	18.6650
∞	3.1416	6.2832	9.4248	12.5664	15.7080	18.8496

[†]The roots of this equation are all real if $C > -1$. These negative values of C arise in connection with the sphere.

Source: Reference 14.

the specified values in the previous pages, is the known term.

Therefore, substituting the values specified by Eqs. (5-7) through (5-14) into Eq. (5-39), one will obtain the following value for parameter C:

$$C = 0.4700 . \quad (5-40)$$

Since the value of parameter C is not one of the exact values listed in Table 5-1, an interpolation should be performed. In order to avoid an interpolation for the sake of exactness, the dimension of the control rod region was changed and a value for parameter C was obtained.

$$x_1 = 6.8092 , \quad (5-41)$$

$$C = 0.5000 . \quad (5-42)$$

Using Table 5-1, the value for parameter α corresponding to the value for C can be obtained. Then using formula (5-38) the value for b and finally using formula (5-17) the value for the static eigenvalue is derived.

$$\alpha = 1.8366 , \quad (5-43)$$

$$b = 0.0584 \quad (5-44)$$

$$K = 0.7846 . \quad (5-45)$$

Having the value for the static reactivity eigenvalue, the neutron flux profile in both regions can be obtained. Equations (5-24)

and (5-28) describe such profile. But, as it was mentioned, the two constants, C_2 and A , needed to be determined. Furthermore, it was shown that these constants can be found by applying the boundary conditions to the equations for the flux profile. As a result of this operation Eqs. (5-31) and (5-32) were obtained. At this point we were drifted to go to find a value for the static eigenvalue. Going back to Eq. (5-31) and substituting the equation in the flux profile defined by Eq. (5-28), one will obtain the following:

$$\phi_2(x) = A \frac{\cosh(\frac{x_1}{L}) \sin b(x - x_0)}{\sin b(x_1 - x_0)} . \quad (5-46)$$

Parameter A in the above equation and in Eq. (5-24) is only a scaling factor. This scaling factor can be arbitrarily chosen by normalizing the flux profile.

$$\phi_1(0) = A \cosh \frac{0}{L} = 1 , \quad (5-47)$$

$$A = 1 . \quad (5-48)$$

As a result, the static reactivity eigenvalue and the flux profile for a reactor core as depicted in Figure 5-1 are described by the following equations:

$$K = \frac{F}{D_2 \left(\frac{\alpha}{x_0 - x_1} \right)^2 + \Sigma_2} , \quad (5-49)$$

$$\phi_1(x) = \cosh \frac{x}{L}, \quad 0 \leq x \leq x_1 \quad (5-50)$$

$$\phi_2(x) = \frac{\cosh(\frac{x_1}{L})}{\sin b(x_1 - x_0)} \sin b(x - x_0) \quad x_1 \leq x \leq x_0 \quad (5-51)$$

where

$$\alpha \cot \alpha = -C \quad (5-52)$$

$$C = \frac{D_1}{D_2 L} (x_0 - x_1) \tanh \left(\frac{x_1}{L} \right). \quad (5-53)$$

Figure 5-2 describes the flux profile defined by Eqs. (5-50) and (5-51) graphically. The profile was obtained by using the values given for the parameters at the beginning and a control rod region length of 6.80919 centimeters. The value for corresponding static eigenvalue is given by Eq. (5-45).

Figure 5-2 depicts the flux profile of the reactor core for the beginning of the fuel cycle. As mentioned earlier, the analytical flux profile for two reactor core compositions, beginning of the fuel cycle and end of the fuel cycle are needed for the FIM approach.

The changes in a reactor core composition during operation occur as a result of a nuclear chain reaction. The products produced as a result of this chain reaction change the core composition and consequently the reactor core criticality. This change of the reactor core composition can be expressed in terms of the changes in the neutron diffusion coefficient in the fuel region.

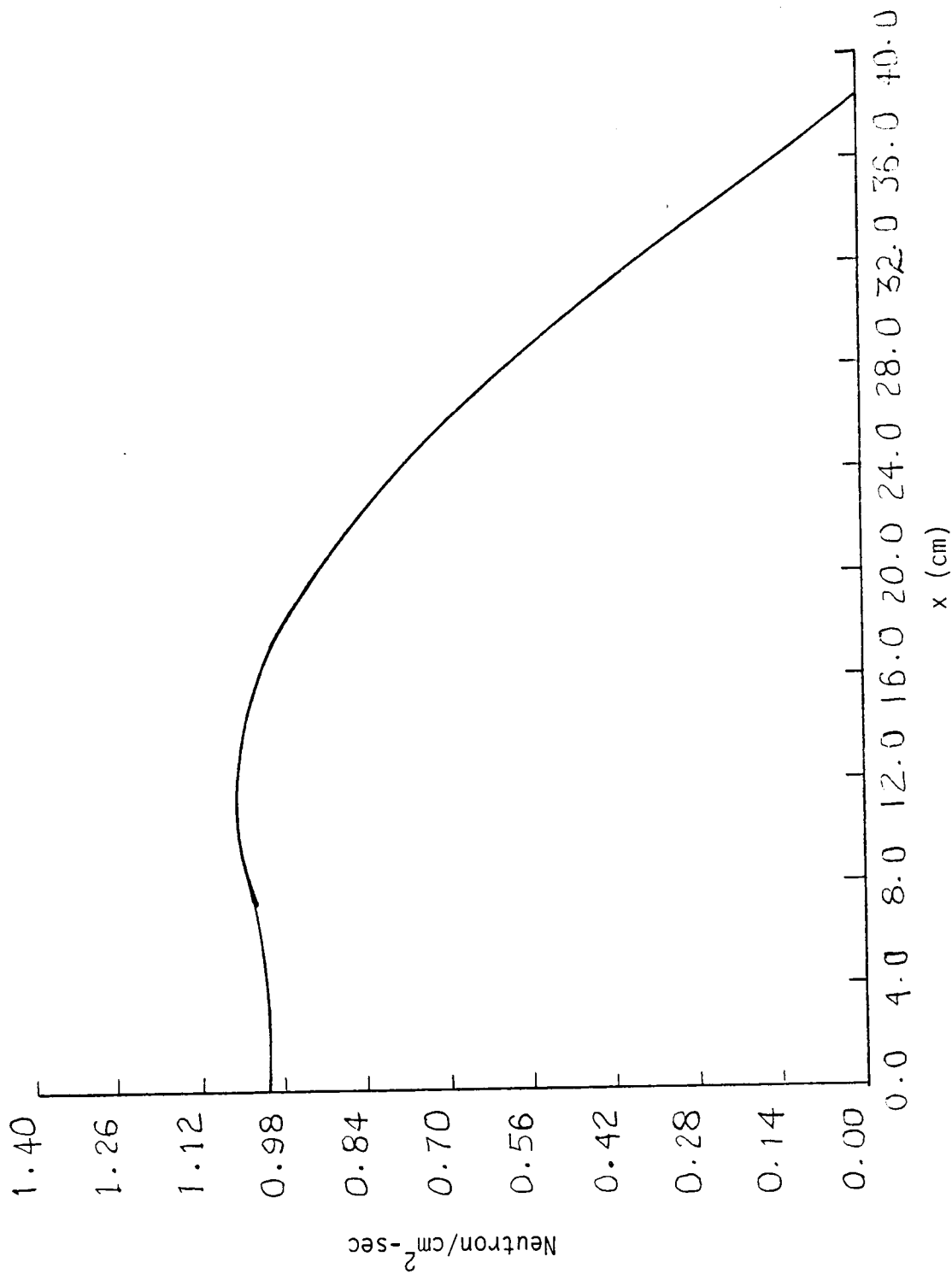


Figure 5-2. Flux profile for the half-way burned-up fuel of a one-dimensional slab reactor core using the analytical approach.

The neutron diffusion coefficient, which is the average distance a neutron travels before its first collision, decreases as a result of the fuel burn-up. This change for the reactor core model shown in Figure 5-1 is obtained by choosing a value for parameter C and obtaining the corresponding value for α using Table 5-1. Then, the following formula is used to obtain the new diffusion coefficient.

$$D_2 = \frac{D_1}{CL} (x_0 - x_1) \tanh \left(\frac{x_1}{L} \right). \quad (5-54)$$

All the other values on the right side of the above equation are assumed to be unchanged. Table 5-2 shows the different values for D_2 and some other parameters such as reactivity eigenvalue corresponding to different reactor compositions.

Table 5-2. The Reactor Parameter Changes as a Function of the Fuel Burn-Up

C	D_2 (cm)	K	b (cm ²)
0.4000	1.3769	0.7349	0.0569
0.5000	1.1015	0.7846	0.0584
0.6000	0.9179	0.8221	0.0597

Another type of analysis which can be used for benchmarking the Functional Interpolation Method is a rod calibration calculation. The dimension of the control rod region in Figure 5-1 was varied and using the following equation the rod calibration was performed:

$$\Delta \rho = \frac{1}{K'} - \frac{1}{K}, \quad (5-55)$$

where

$\Delta\rho$ = the negative reactivity associated with the amount of control rod region present in the core (\$),

K' = the core multiplication factor without the presence of any control (dimensionless),

K = the core multiplication factor for different amount of control region (dimensionless).

Table 5-3 shows the parameters which were calculated in order to perform a rod calibration. Notice that the value of x_1 which corresponds to the length of the control rod region is varied for this analysis. The results are also shown in Figure 5-3 graphically.

At this point we have all the necessary information to show the validity of the FIM. The later calculations, which includes the calculation of the control rod worth, will be used for double benchmarking in order to alleviate any uncertainty about the method. The next section provides the approach to the FIM utilizing the analytical solutions which were obtained in this section.

5.2. The FIM Approach

As mentioned earlier, the Functional Interpolation Method is based on the neutron flux profiles for two core compositions. In this section the neutron flux profiles which were obtained for the reactor core model in Figure 5-1 are used to show the FIM technique. The approach for the FIM, which was shown in Chapter IV for a two-dimensional reactor core model, starts with two sets of equations

Table 5-3. Rod Calibration for a One-Dimensional Reactor Core

D_2	x_1	C	α	K	ρ
1.1015	0	0	1.5708	0.9585	-
1.1015	1.1415	0.1000	1.6320	0.9290	-0.0332
1.1015	2.3623	0.2000	1.6887	0.8977	-0.0707
1.1015	3.6866	0.3000	1.7414	0.8640	-0.1142
1.1015	5.1490	0.4000	1.7906	0.8268	-0.1662
1.1015	6.8092	0.5000	1.8366	0.7846	-0.2313
1.1015	8.7846	0.6000	1.8798	0.7338	-0.3194
1.1015	11.3121	0.7000	1.9203	0.6659	-0.4585

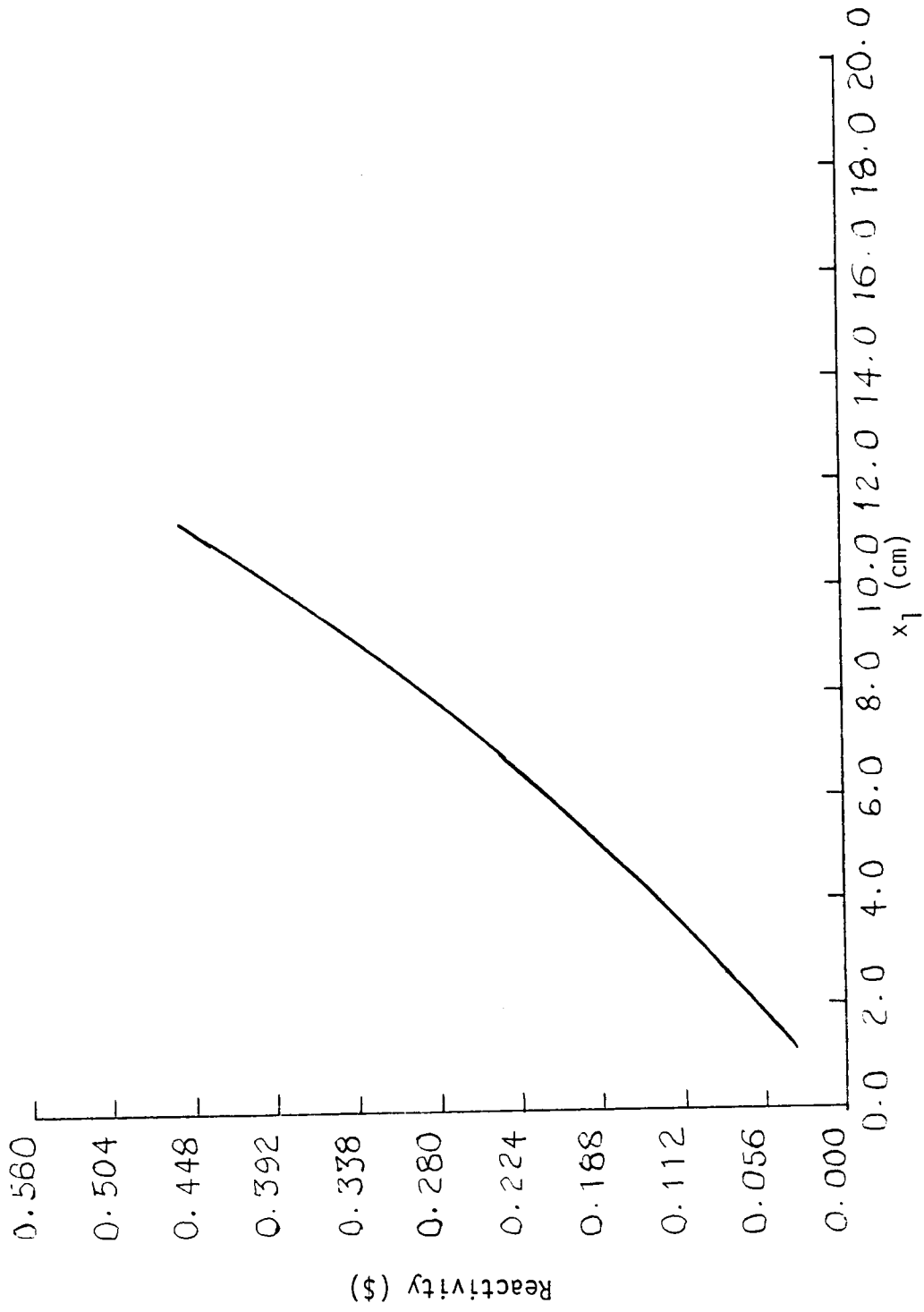


Figure 5-3. Control rod worth for a one-dimensional reactor core.

describing the true neutron flux profile for two types of reactor core compositions. Equations (5-50) and (5-51) can be written for the beginning and end of the fuel cycle of the reactor core model (Figure 5-1).

$$\psi_{1A} = \cosh \left(\frac{x}{L} \right), \quad 0 < x < x_1, \quad (5-56)$$

$$\psi_{2A} = N_A \sin[B_0(x - x_0)] \quad x_1 < x < x_0, \quad (5-57)$$

$$\psi_{1B} = \cosh \left(\frac{x}{L} \right) \quad 0 < x < x_1, \quad (5-58)$$

$$\psi_{2B} = N_B \sin[B_1(x - x_0)] \quad x_1 < x < x_0, \quad (5-59)$$

where

$$N_A = \frac{\cosh \left(\frac{x_1}{L} \right)}{\sin[B_0(x_0 - x_1)]},$$

$$N_B = \frac{\cosh \left(\frac{x_1}{L} \right)}{\sin[B_1(x_0 - x_1)]},$$

$$L = \sqrt{\frac{D_1}{\Sigma_1}},$$

$$x_0 = \frac{\tilde{a}}{2},$$

$$D_1 = 3.4470 \text{ cm},$$

$$\Sigma_1 = 0.0026 \text{ cm}^{-1},$$

$$\tilde{a} = 76.5600 \text{ cm}^{-1},$$

$$B_0 = \frac{1}{D_{2,0}} \left(\frac{F}{K_0} - \Sigma_2 \right) ,$$

$$F = 0.0082 ,$$

$$\Sigma_2 = 0.0067 \text{ cm}^{-1} ,$$

$$D_{2,0} = 1.3769 ,$$

$$K_0 = 0.7349 ,$$

$$B_1 = \frac{1}{D_{2,1}} \left(\frac{F}{K_1} - \Sigma_2 \right)$$

$$D_{2,1} = 0.9179 ,$$

$$K_1 = 0.8221 .$$

Having the solution defined by Eqs. (5-56) through (5-59), we now define a set of trial fluxes.

$$\phi_1 = q \psi_{1A} + (1 - q) \psi_{1B} , \quad (5-60)$$

$$\phi_2 = q \psi_{2A} + (1 - q) \psi_{2B} . \quad (5-61)$$

The above expressions are similar to the trial fluxes in Chapter IV. Remember again, the Variational Principle is utilized to search for the value of parameter q , which minimizes the above equations for the static eigenvalue.

In order to use the Variational Principle technique, as before, a functional called Lagrangian needs to be written for the one-dimensional core geometry shown in Figure 5-1. This Lagrangian, as explained, has to be stationary with respect to changes in the trial

fluxes. The following Lagrangian was obtained in regard to the stationarity property which resulted in inclusion of some Lagrange Multipliers.

$$\begin{aligned}
 2L = & \int_0^{x_1} dx \phi_1 \left[D_1 \frac{d^2 \phi_1}{dx^2} - \Sigma_1 \phi_1 \right] + \int_{x_1}^{x_0} dx \phi_2 \left[D_2 \frac{d^2 \phi_2}{dx^2} - \Sigma_2 \phi_2 + \frac{F}{K} \phi_2 \right] \\
 & + \left[D_2 \phi_1(x_1) \left(\frac{d\phi_2}{dx} \right)_{x_1} - D_1 \phi_2(x_1) \left(\frac{d\phi_1}{dx} \right)_{x_1} \right] . \quad (5-62)
 \end{aligned}$$

The fluxes in the above equation are the trial fluxes described by Eqs. (5-60) and (5-61) which are composed of the two exact fluxes. Using these trial fluxes, solutions to the terms in the above Lagrangian can be obtained.

$$\frac{d^2 \phi_1}{dx^2} = \frac{\Sigma_1}{D_1} \cosh \left(\frac{x}{L} \right) , \quad (5-63)$$

$$\frac{d^2 \phi_2}{dx^2} = -q B_0^2 \phi_{2A} - (1 - q) B_1^2 \phi_{2B} , \quad (5-64)$$

$$\phi_2(x_1) = \phi_1(x_1) = \cosh \left(\frac{x_1}{L} \right) , \quad (5-65)$$

$$\frac{d\phi_2}{dx} = q B_0 N_A \cos B_0(x - x_0) + (1 - q) N_B B_0 \cos B_1(x - x_0) . \quad (5-66)$$

Equations (5-63) through (5-66) are inserted into the Lagrangian, and the terms are multiplied, cancelled, and factored. The following equation shows the Lagrangian after these operations.

$$\begin{aligned}
2L = & q^2 \left(\frac{F}{K} - L_{OF} \right) I_{AA} + [q(2 - q)(L_{1F} - \frac{F}{K}) + (\frac{F}{K} - L_{1F})] I_{BB} \\
& + q(1 - q) \left(\frac{2F}{K} - L_{1F} - L_{OF} \right) I_{AB} + qD_2(S_{0A} - S_{1B}) \\
& + D_2S_{1B} - D_1T_1, \quad (5-67)
\end{aligned}$$

where

$$L_{OF} = D_2 B_0^2 + \Sigma_2, \quad (5-68)$$

$$L_{1F} = D_2 B_1^2 + \Sigma_2, \quad (5-69)$$

$$S_{0A} = N_A B_0 \cos B_0(x_1 - x_0) \cosh \left(\frac{x_1}{L} \right), \quad (5-70)$$

$$S_{1B} = N_B B_1 \cos B_1(x_1 - x_0) \cosh \left(\frac{x_1}{L} \right), \quad (5-71)$$

$$T_1 = \frac{1}{L} \sinh \left(\frac{x_1}{L} \right) \cosh \left(\frac{x_1}{L} \right), \quad (5-72)$$

$$I_{AA} = \int_{x_1}^{x_0} dx \phi_{2A}^2 = \frac{1}{2} N_A^2 \left[x_0 - x_1 + \frac{\sin 2B_0(x_1 - x_0)}{2B_0} \right], \quad (5-73)$$

$$\begin{aligned}
I_{AB} = \int_{x_1}^{x_0} dx \phi_{2A} \phi_{2B} = \frac{1}{2} N_A N_B & \left[\frac{\sin[(B_0 + B_1)(x_1 - x_0)]}{B_0 + B_1} \right. \\
& \left. - \frac{\sin[(B_0 - B_1)(x_1 - x_0)]}{B_0 - B_1} \right], \quad (5-74)
\end{aligned}$$

$$I_{BB} = \int_{x_1}^{x_0} dx \phi_{2B}^2 = \frac{1}{2} N_B^2 \left[x_0 - x_1 + \frac{\sin 2B_1(x_1 - x_0)}{2B_1} \right]. \quad (5-75)$$

One should remember that the goal at this point is to solve for the eigenvalue or multiplication factor K and the parameter q . For this,

K has to be separated from the rest of the parameters on the right side of Eq. (5-67). Therefore, the terms are multiplied by each other and parameters q^2 and q are factored from the right side of Eq. (5-67). The implementation of these operations will result in the following equations.

$$\frac{2L}{FI_{BB}} = (Q_2 - \frac{G_2}{K})q^2 + (Q_1 - \frac{G_1}{K})q + \frac{1}{K} - Q_0, \quad (5-76)$$

where

$$Q_2 = (\frac{1}{K_1} + \frac{1}{K_0}) \frac{I_{AB}}{I_{BB}} - (\frac{1}{K_0} \frac{I_{AA}}{I_{BB}} + \frac{1}{K_1}), \quad (5-77)$$

$$Q_1 = \frac{2}{K_1} - (\frac{1}{K_1} + \frac{1}{K_0}) \frac{I_{AB}}{I_{BB}} + \frac{D_2}{F} (\frac{S_{0A} - S_{1B}}{I_{BB}}), \quad (5-78)$$

$$Q_0 = \frac{D_1 T_1}{F I_{BB}} - \frac{D_2 S_{1B}}{F I_{BB}} + \frac{1}{K_1}, \quad (5-79)$$

$$G_1 = 2 - \frac{2I_{AB}}{I_{BB}}, \quad (5-80)$$

$$G_2 = \frac{2I_{AB}}{I_{BB}} - \frac{I_{AA}}{I_{BB}} - 1, \quad (5-81)$$

$$\frac{1}{K_0} = \frac{L_{OF}}{F}, \quad (5-82)$$

$$\frac{1}{K_1} = \frac{L_{1F}}{F}. \quad (5-83)$$

Since we are seeking for values to K and q and there is only one equation [Eq. (5-76)], we use the stationarity property at this point.

Setting the inverse eigenvalue equal to λ and equating Eq. (5-76) to zero, the following equation is obtained.

$$\lambda = \frac{Q_2^2 + Q_1 q - Q_0}{G_2 q^2 + G_1 q - 1} . \quad (5-84)$$

The second equation is obtained by taking the variation in the Lagrangian with respect to parameter q and setting the result equal to zero. Remember again this is the property of a Lagrangian which is stationary.

$$\frac{\partial L}{\partial q} = 0 , \quad (5-85)$$

$$\therefore q = - \frac{Q_1 - G_1 \lambda}{2(Q_2 - G_2 \lambda)} . \quad (5-86)$$

Solving Eqs. (5-84) and (5-86) simultaneously will result in a second order equation for λ .

$$\lambda^2 + b\lambda + c = 0 , \quad (5-87)$$

where

$$b = - \frac{[2Q_1 G_1 + 4(Q_2 + Q_0 G_2)]}{G_1^2 + 4G_2} ,$$

$$c = \frac{Q_1^2 + 4Q_0 Q_2}{G_1^2 + 4G_2} . \quad (5-88)$$

Solving the second order linear equation [Eq. (5-78)] will result in two solutions for λ . The larger positive value is the value that corresponds to the multiplication factor. After finding a value for K , Eq. (5-86) is used to obtain the parameter q and finally Eqs. (5-60) and (5-61) are utilized to obtain the complete flux profile within the reactor core.

Figure 5-4 shows the flux profile for the reactor model in Figure 5-1 (p. 31) using the Functional Interpolation Method. This flux profile is representative of neutron distribution in the core at a point during the fuel cycle. FFTF parameters were also used for the core characteristics.

In addition, a control rod worth calculation simulating a measurement was performed using the FIM approach. Table 5-4 and Figure 5-5 depict the obtained results. As in the previous case, Formula (5-55) was used for different lengths of the control rod region.

At this point it is appropriate to make a comparison between the results obtained by employing the FIM approach and the solutions as a result of an analytical approach. The next section provides the validation of the FIM technique.

5.3. Comparison of the Analytical and the FIM Results

The last two sections provided the neutron distribution and the control rod worth for a one-dimensional reactor core model using the analytical and the FIM approaches. As explained earlier, the intention of this exercise was to validate the FIM technique.

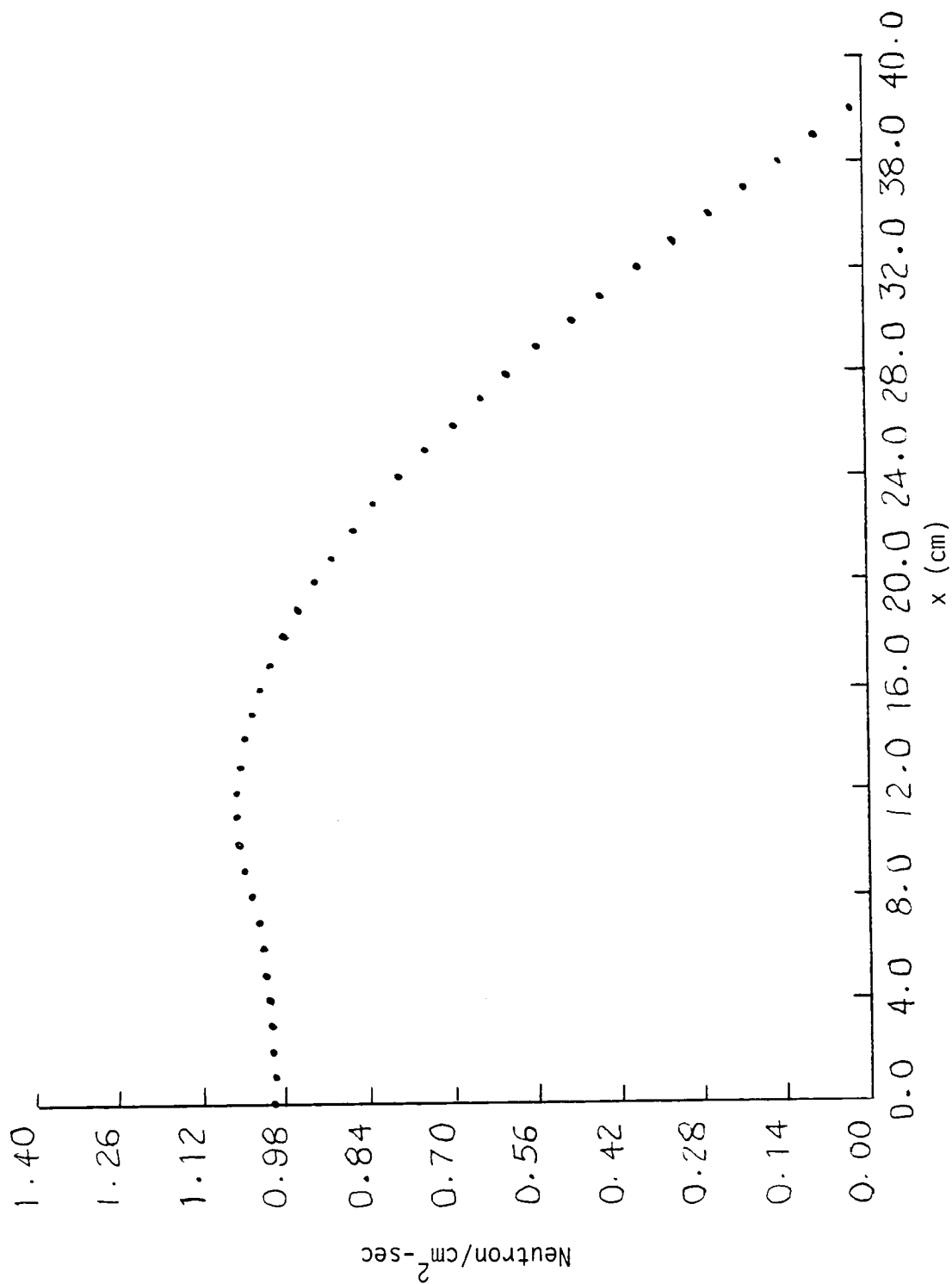


Figure 5-4. Flux profile for the half-way burned-up fuel of a one-dimensional reactor core using the FIM.

Table 5-4. Rod Calibration for a One-Dimensional Reactor Core Using the FIM

D_2	X_1	K	ρ
1.1015	0	0.9583	
1.1015	1	0.9325	-0.0289
1.1015	2	0.9069	-0.0592
1.1015	3	0.8814	-0.0911
1.1015	4	0.8560	-0.1247
1.1015	5	0.8306	-0.1604
1.1015	6	0.8052	-0.1984
1.1015	7	0.7797	-0.2390
1.1015	8	0.7541	-0.2826
1.1015	9	0.7283	-0.3296
1.1015	10	0.7023	-0.3805
1.1015	11	0.6760	-0.4358
1.1015	12	0.6494	-0.4963

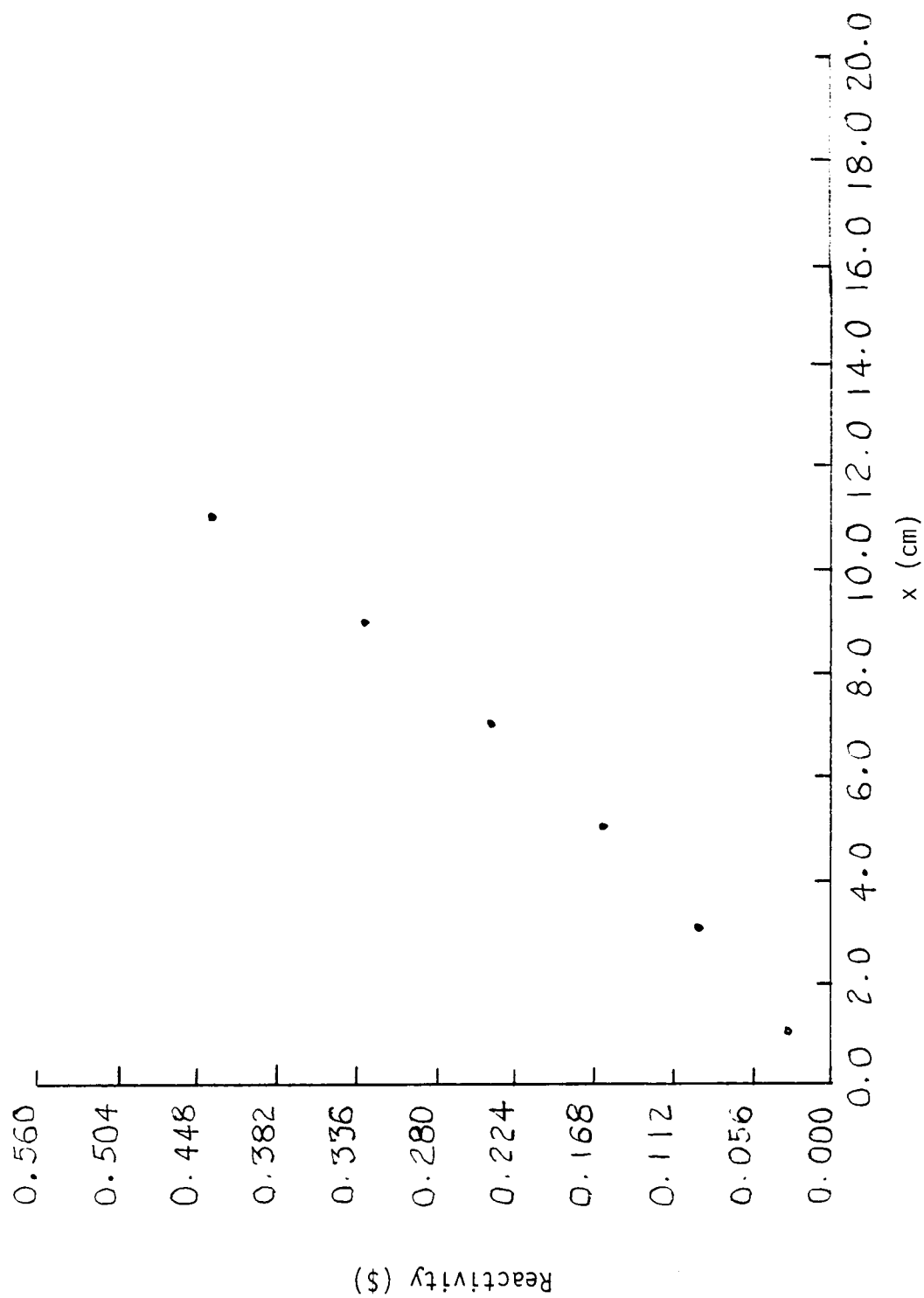


Figure 5-5. Control rod worth for a one-dimensional reactor core using the FIM.

Figures 5-2 through 5-5 (pp. 40,45,53,55) depict the obtained results using both techniques. If the flux profiles in Figures 5-2 and 5-4 are superimposed on each other, Figure 5-6 can be obtained. The same type of analogy can be used for the control rod worth (Figure 5-7). Figures 5-6 and 5-7 reveal a very accurate prediction of the flux profile and the control rod worth by the FIM technique.

In order to perform such a comparison at all phases of the fuel depletion, a simple computer program was written to perform comparisons at the beginning, midway, and the end of the fuel cycle between the analytical flux and the FIM flux. Appendix C shows the pertinent subroutine along with the results of the execution. The percentage of error in all of the cases was negligible. The validity of the FIM technique was also double checked by observing the value of q in Eqs. (5-60) and (5-61). When the analysis is performed for the beginning of the fuel cycle using the FIM technique, the value of q should be equal to one in order for the analytical solution at the end of the fuel cycle to be eliminated and the final solution be equal to the analytical solution at point A. The same type of logic but vice versa should be true for obtaining the flux profile using the FIM technique for the end of the fuel cycle. The latter case calls for the value of q equal to zero. This way the solution at point A is eliminated and the flux turns out to be equal to the analytical flux at the end of the fuel cycle.

Indeed, the value of q was close to one and zero for the above cases, respectively. This last observation assured the accuracy and the acceptability of the FIM technique.

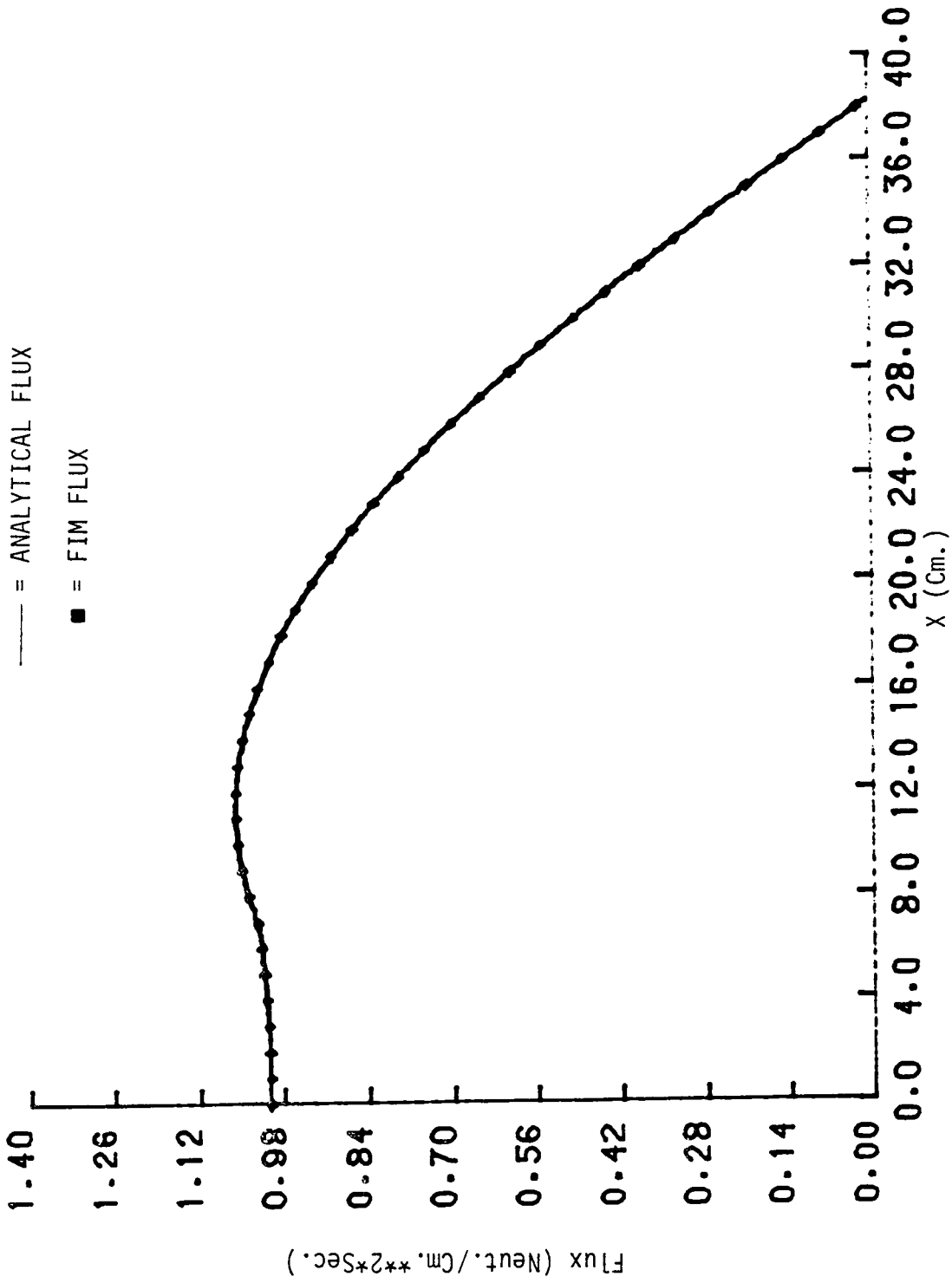


Figure 5-6. Comparison of the fluxes at midway during the fuel cycle.

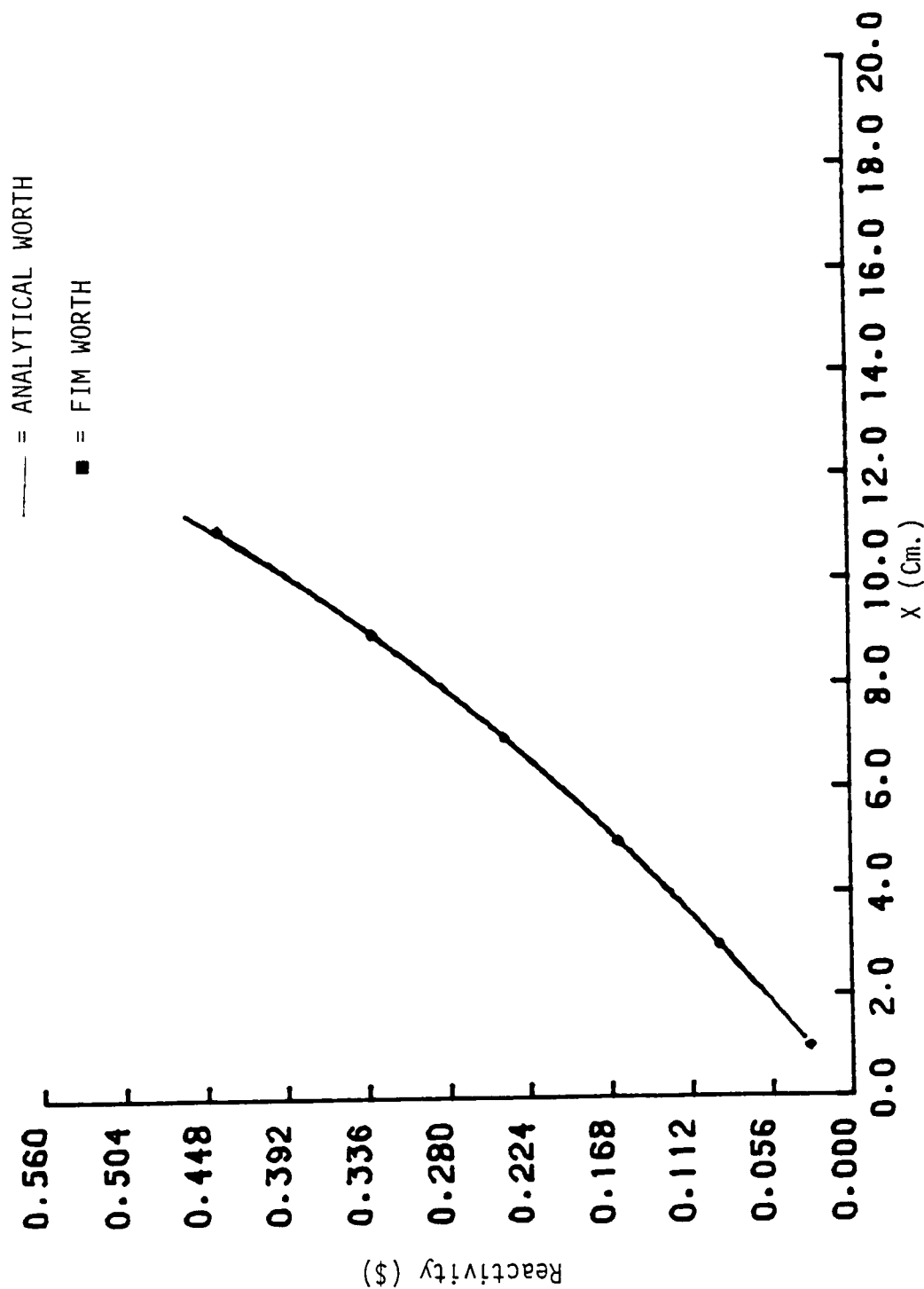


Figure 5-7. Comparison of the control rod worth.

CHAPTER VI

CONCLUSIONS

On-line implementation of a reactivity and power distribution monitor (RPDM) system requires development of fast and reliable computational algorithms to calculate the neutron flux and reactivity eigenvalues. In addition to the reliability and the accuracy conditions set forth on such algorithms, economics of implementing and operating such a system is also an important and parallel constraint, which is not satisfied in most of the algorithms developed in the past. Consequently, there is a need for an algorithm that can predict the neutron flux and the reactivity eigenvalue accurately and on-line during a reactor operation.

The Functional Interpolation Method offers the ability to calculate the power distribution and the reactivity eigenvalue within the perimeter of the above constraints. Chapter IV demonstrated the FIM development with the use of the Variational Principle Technique for a two-dimensional reactor core model. This algorithm can be used to obtain the power distribution and the reactivity eigenvalue upon the determination of the two analytical flux profiles.

Furthermore, Chapter V proved validity of the FIM approach by predicting the neutron population and the multiplication factor for a one-dimensional reactor core model during different core compositions. The results revealed an excellent agreement between the analytical and the FIM predictions. This agreement was demonstrated for both the

neutron flux and the worth of neutron absorber region. In all of the cases the percentage of error did not exceed 2×10^{-3} . This type of results is indicative of an excellent prediction by the FIM approach.

In closing, it should be stressed that the present methodology should not be considered as a new alternate technique for reactor calculations, but rather as a novel method for quick and precise computer on-line calculations for nuclear reactor surveillance.

CHAPTER VII

RECOMMENDATIONS

The Functional Interpolation Method has been formulated for the simultaneous determination of power distributions and reactivity eigenvalues by using two off-line, detailed and accurate solutions for two different reactor core compositions. This thesis has proved the validity of this approach by applying the method to a one-dimensional reactor core model and, furthermore, deriving a general formula for the flux profile and the reactivity eigenvalue by applying the FIM to a two-dimensional reactor core model. There is, however, a need for more work in order to use this algorithm as a part of the RPDM systems for the purposes of surveillance. The following outlines the recommended actions.

1. To perform a detailed numerical neutronics calculation for the reactor model in Chapter IV.
2. To verify the FIM for the two-dimensional geometry by utilizing the above results.
3. To benchmark the FIM two-dimensional results for various fuel compositions during the fuel burn-up cycle.
4. To develop the FIM for a three-dimensional reactor core model.
5. To repeat steps one through three for the three-dimensional reactor core model.

The above actions will insure validation and verification of the FIM approach in a complete and comprehensive way. Furthermore, it will show the trade-offs between time and accuracy of the FIM two-dimensional vs. the FIM three-dimensional.

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APPENDIXES

APPENDIX A

In this appendix the Lagrangian derived in Chapter IV for a generic two-dimensional cylindrical reactor core will be analyzed. Utilizing Variational Principle Technique, an equation will be derived for the static reactivity eigenvalue from the Lagrangian.

Considering the Lagrangian defined by Eq. (4-6), and numbering each of the integrals on the right side of the equation:

$$\begin{aligned} \mathcal{L}/2\pi = & \int_0^{r_1} r dr \int_0^{Z_1} dz [q \psi_{1SA} + (1 - q) \psi_{1SB}] [D_S \nabla^2 (q \psi_{1SA} + (1 - q) \psi_{1SB}) \\ & - \Sigma_S (q \psi_{1SA} + (1 - q) \psi_{1SB})] \quad (1) \end{aligned}$$

$$\begin{aligned} + & \int_0^{r_1} r dr \int_{Z_1}^H dz [q \psi_{1CA} + (1 - q) \psi_{1CB}] [D_C \nabla^2 (q \psi_{1CA} + (1 - q) \psi_{1CB}) \\ & - \Sigma_C (q \psi_{1CA} + (1 - q) \psi_{1CB})] \quad (2) \end{aligned}$$

$$\begin{aligned} + & \int_{r_1}^{r_2} r dr \int_0^{Z_1} dz [q \psi_{2SA} + (1 - q) \psi_{2SB}] [D_2 \nabla^2 (q \psi_{2SA} + (1 - q) \psi_{2SB}) \\ & + (\frac{F}{K} - \Sigma_2) (q \psi_{2SA} + (1 - q) \psi_{2SB})] \quad (3) \end{aligned}$$

$$\begin{aligned} + & \int_{r_1}^{r_2} r dr \int_{Z_1}^H dz [q \psi_{2CA} + (1 - q) \psi_{2CB}] [D_2 \nabla^2 (q \psi_{2CA} + (1 - q) \psi_{2CB}) \\ & + (\frac{F}{K} - \Sigma_2) (q \psi_{2CA} + (1 - q) \psi_{2CB})] \quad (4) \end{aligned}$$

$$\begin{aligned}
& - r_1 \int_0^{Z_1} dz [q \psi_{1SA}(r_1) + (1-q)\psi_{1SB}(r_1)] \{ D_S \left[\frac{\partial}{\partial r} (q \psi_{1SA} + (1-q)\psi_{1SB}) \right] r_1 \\
& \quad - D_2 \left[\frac{\partial}{\partial r} (q \psi_{2SA} + (1-q)\psi_{2SB}) \right] r_1 \} \quad (5)
\end{aligned}$$

$$\begin{aligned}
& - r_1 \int_{Z_1}^H dz [q \psi_{1CA}(r_1) + (1-q)\psi_{1CB}(r_1)] \{ D_C \left[\frac{\partial}{\partial r} (q \psi_{1CA} + (1-q)\psi_{1CB}) \right] r_1 \\
& \quad - D_2 \left[\frac{\partial}{\partial r} (q \psi_{2CA} + (1-q)\psi_{2CB}) \right] r_1 \} \quad (6)
\end{aligned}$$

$$- \int_0^{r_1} r dr [q \psi_{1SA} + (1-q)\psi_{1SB}]_{Z_1} \{ D_S \left[\frac{\partial}{\partial Z} (q \psi_{1SA} + (1-q)\psi_{1SB}) \right]_{Z_1} \} \quad (7)$$

$$+ \int_0^{r_1} r dr [q \psi_{1SA} + (1-q)\psi_{1SB}]_{Z_1} \{ D_C \left[\frac{\partial}{\partial Z} (q \psi_{1CA} + (1-q)\psi_{1CB}) \right]_{Z_1} \} \quad (8)(A-1)$$

The above Lagrangian, which is identified by Formula (A-1), is broken into eight subformulas. Each of these subformulas will be solved separately.

Starting with subformula (1), one can rewrite (1) as follows:

$$\begin{aligned}
(1) &= \int_0^{r_1} r dr \int_0^{Z_1} dz [q \psi_{1SA} + (1-q)\psi_{1SB}] \\
&\quad [D_S q \psi'_{1SA} + D_S (1-q)\psi'_{1SB} - \Sigma_S \psi_{1SA} - \Sigma_S (1-q)\psi_{1SB}] \quad (A-2)
\end{aligned}$$

where

$$\psi'_{1SA} = \nabla^2 \psi_{1SA} , \quad (A-3)$$

$$\psi'_{1SB} = \nabla^2 \psi_{1SB} . \quad (A-4)$$

Multiplying the terms inside the brackets by each other and defining the following terms:

$$I_1 = \int_0^{r_1} r dr \int_0^{Z_1} dz \psi_{1SA} \psi'_{1SA} , \quad (A-5)$$

$$I_2 = \int_0^{r_1} r dr \int_0^{Z_1} dz \psi_{1SA} \psi'_{1SB} , \quad (A-6)$$

$$I_3 = \int_0^{r_1} r dr \int_0^{Z_1} dz \psi_{1SA}^2 , \quad (A-7)$$

$$I_4 = \int_0^{r_1} r dr \int_0^{Z_1} \psi_{1SA} \psi_{1SB} , \quad (A-8)$$

$$I_5 = \int_0^{r_1} r dr \int_0^{Z_1} \psi'_{1SA} \psi_{1SB} , \quad (A-9)$$

$$I_6 = \int_0^{r_1} r dr \int_0^{r_1} \psi'_{1SB} \psi_{1SB} , \quad (A-10)$$

$$I_7 = \int_0^{r_1} r dr \int_0^{r_1} \psi_{1SB}^2 , \quad (A-11)$$

then

$$\begin{aligned}
 (1) = & D_S q^2 I_1 + D_S q(1 - q)I_2 - \Sigma_S q^2 I_3 - 2\Sigma_S q(1 - q)I_4 \\
 & + D_S q(1 - q)I_5 + D_S(1 - q)^2 I_6 - \Sigma_S(1 - q)^2 I_7 .
 \end{aligned} \tag{A-12}$$

Multiplying the terms on the right side of the equation further and factoring q^2 and q , one will obtain:

$$\begin{aligned}
 (1) = & q^2(D_S I_1 - D_S I_2 - S I_3 + 2 S I_4 - D_S I_5 + D_S I_6 - S I_7) \\
 & + (D_S I_6 - S I_2) .
 \end{aligned} \tag{A-13}$$

The same type of analogy is applied to subformula (2).

$$\begin{aligned}
 (2) = & D_C q^2 I_8 + D_C q(1 - q)I_9 - C q^2 I_{10} - 2 C q(1 - q)I_{11} \\
 & + D_C q(1 - q)I_{12} + D_C(1 - q)^2 I_{13} - C(1 - q)^2 I_{14} ,
 \end{aligned} \tag{A-14}$$

where

$$I_8 = \int_0^{r_1} r dr \int_{Z_1}^H dz \psi_{1CA} \psi'_{1CA} , \tag{A-15}$$

$$I_9 = \int_0^{r_1} r dr \int_{Z_1}^H dz \psi_{1CA} \psi'_{1CB} \tag{A-16}$$

$$I_{10} = \int_0^{r_1} r dr \int_{Z_1}^H dz \psi_{1CA}^2 , \tag{A-17}$$

$$I_{11} = \int_0^{r_1} r dr \int_{Z_1}^H dz \psi_{1CA} \psi_{1CB} , \tag{A-18}$$

$$I_{12} = \int_0^{r_1} r dr \int_{Z_1}^H dz \psi'_{1CA} \psi_{1CB} , \quad (A-19)$$

$$I_{13} = \int_0^{r_1} r dr \int_0^{r_1} \psi'_{1CB} \psi_{1CB} , \quad (A-20)$$

$$I_{14} = \int_0^{r_1} r dr \int_0^{r_1} \psi_{1CB}^2 , \quad (A-21)$$

$$\psi'_{1CA} = \nabla^2 \psi_{1CA} , \quad (A-22)$$

$$\psi'_{1CB} = \nabla^2 \psi_{1CB} . \quad (A-23)$$

As before, factoring the parameters q^2 and q in Eq. (A-14):

$$\begin{aligned} (2) = & q^2 (D_C I_8 - D_C I_9 - \Sigma_C I_{10} = 2\Sigma_C I_{11} - D_C I_{12} + D_C I_{13} - \Sigma_C I_{14}) \\ & + q (D_C I_9 - 2\Sigma_C I_{11} + D_C I_{12} - 2\Sigma_C I_{13} + 2\Sigma_C I_{14}) \\ & + (D_C I_{13} - \Sigma_C I_9) . \end{aligned} \quad (A-24)$$

In solving the subformula (3) there are two parameters, q and K , which are needed to be considered. Since the objective is to solve for both unknowns using the "two equations, two unknowns" method, therefore, in some of the subformulas such as (3), q^2 , q , and $1/K$ will be factored on the right side of these equations.

Rewriting subformula (3):

$$\begin{aligned}
 (3) = & \int_{r_1}^{r_2} r dr \int_0^{Z_1} dz [q \psi_{2SA} + (1 - q) \psi_{2SB}] \\
 & [D_2 q \psi'_{2SA} + D_2 (1 - q) \psi'_{2SB} + q \left(\frac{F}{K} - \Sigma_2 \right) \psi_{2SA} \\
 & + (1 - q) \left(\frac{F}{K} - \Sigma_2 \right) \psi_{2SB}] , \tag{A-25}
 \end{aligned}$$

where

$$\psi'_{2SA} = \nabla^2 \psi_{2SA} , \tag{A-26}$$

$$\psi'_{2SB} = \nabla^2 \psi_{2SB} . \tag{A-27}$$

Solving the above subformula for q and $1/K$ will result in the following equalities:

$$\begin{aligned}
 (3) = & q^2 (D_2 I_{15} - D_2 I_{16} - \Sigma_2 I_{17} - D_2 I_{19} + D_2 I_{20} + 2\Sigma_2 I_{18} - \Sigma_2 I_{21}) \\
 & + q (D_2 I_{16} + D_2 I_{19} - 2D_2 I_{20} - 2\Sigma_2 I_{18} + 2\Sigma_2 I_{21}) \\
 & + (D_2 I_{20} - \Sigma_2 I_{21}) \\
 & + \frac{1}{K} (Fq^2 I_{17} + 2Fq I_{18} - 2Fq^2 I_{18} + FI_{21} + FI_{21}q^2 - 2Fq I_{21}) , \tag{A-28}
 \end{aligned}$$

where

$$I_{15} = \int_{r_1}^{r_2} r dr \int_0^{Z_1} dz \psi_{2SA} \psi'_{2SA} , \tag{A-29}$$

$$I_{16} = \int_{r_1}^{r_2} r dr \int_0^{Z_1} dz \psi_{2SA} \psi'_{2SB} , \tag{A-30}$$

$$I_{17} = \int_{r_1}^{r_2} r dr \int_0^{Z_1} dz \psi_{2SA}^2, \quad (A-31)$$

$$I_{18} = \int_{r_1}^{r_2} r dr \int_0^{Z_1} dz \psi_{2SA} \psi_{2SB}, \quad (A-32)$$

$$I_{19} = \int_{r_1}^{r_2} r dr \int_0^{Z_1} dz \psi_{2SB} \psi_{2SB}', \quad (A-33)$$

$$I_{21} = \int_{r_1}^{r_2} r dr \int_0^{Z_1} dz \psi_{2SB}^2. \quad (A-34)$$

The subformulas (4), (5), (6), (7), and (8) can be solved in the same fashion.

$$\begin{aligned} (4) = & q^2 (D_2 I_{22} - D_2 I_{23} - \Sigma_2 I_{24} + 2\Sigma_2 I_{25} - D_2 I_{26} - D_2 I_{27} - \Sigma_2 I_{28}) \\ & + q (D_2 I_{23} - 2\Sigma_2 I_{25} + D_2 I_{26} - 2D_2 I_{27} + 2\Sigma_2 I_{28}) \\ & + (D_2 I_{27} - \Sigma_2 I_{28}) \\ & + \frac{1}{K} (Fq^2 I_{24} + 2FI_{25}q - 2FI_{25}q^2 + FI_{28} + Fq^2 I_{28} - 2Fq I_{28}) \end{aligned} \quad (A-35)$$

$$\begin{aligned} (5) = & q^2 [-r_1 D_S I_{29} + r_1 D_S I_{30} + r_1 D_2 I_{31} - r_1 D_2 I_{32} + r_1 D_S I_{33} \\ & - r_1 D_S I_{34} - r_1 D_2 I_{35} + r_1 D_2 I_{36}] \\ & + q [-r_1 D_S I_{30} + r_1 D_2 I_{32} - r_1 D_S I_{33} + 2r_1 D_S I_{34} + r_1 D_2 I_{35} \\ & + 2r_1 D_2 I_{36}] + [-r_1 D_S I_{34} + r_1 D_2 I_{36}] \end{aligned} \quad (A-36)$$

$$\begin{aligned}
(6) = & q^2 [-r_1 D_S I_{37} + r_1 D_S I_{38} + r_1 D_2 I_{39} - r_1 D_2 I_{40} \\
& + r_1 D_S I_{41} - r_1 D_S I_{42} - r_1 D_2 I_{43} + r_1 D_2 I_{44}] \\
& + q [-r_1 D_S I_{38} + r_1 D_2 I_{40} - r_1 D_S I_{41} + 2r_1 D_S I_{42} + r_1 I_{43} \\
& + 2r_1 D I_{44}] + [-r_1 D_S I_{44} + r_1 D_2 I_{44}] , \tag{A-37}
\end{aligned}$$

$$\begin{aligned}
(7) = & q^2 -D_S I_{45} + D_S I_{46} + D_S I_{47} - D_S I_{48} \\
& + q -D_S I_{46} - D_S I_{47} + 2D_S I_{48} - D_S I_{48} , \tag{A-38}
\end{aligned}$$

$$\begin{aligned}
(8) = & q^2 [D_C I_{49} - D_C I_{50} - D_C I_{51} + D_C I_{52}] \\
& + q [D_C I_{50} + D_C I_{51} - 2D_C I_{52}] + D_C I_{52} , \tag{A-39}
\end{aligned}$$

where

$$\psi'_{2CA} = \nabla \psi_{2CA} , \tag{A-40}$$

$$\psi'_{2CB} = \nabla \psi_{2CB} , \tag{A-41}$$

$$I_{22} = \int_{r_1}^{r_2} r dr \int_{Z_1}^H dz \psi_{2CA} \psi'_{2CA} , \tag{A-42}$$

$$I_{23} = \int_{r_1}^{r_2} r dr \int_{Z_1}^H dz \psi_{2CA} \psi'_{2CB} , \tag{A-43}$$

$$I_{24} = \int_{r_1}^{r_2} r dr \int_{Z_1}^H dz \psi_{2CA}^2 , \tag{A-44}$$

$$I_{25} = \int_{r_1}^{r_2} r dr \int_{Z_1}^H dz \psi_{2CA} \psi_{2CB} , \quad (A-45)$$

$$I_{26} = \int_{r_1}^{r_2} r dr \int_{Z_1}^H dz \psi_{2CB} \psi'_{2CA} , \quad (A-46)$$

$$I_{27} = \int_{r_1}^{r_2} r dr \int_{Z_1}^H dz \psi_{2CB} \psi'_{2CB} , \quad (A-47)$$

$$I_{28} = \int_{r_1}^{r_2} r dr \int_{Z_1}^H dz \psi_{2CB}^2 , \quad (A-48)$$

$$I_{29} = \int_0^{Z_1} dz (\psi_{1SA} \frac{\partial}{\partial r} \psi_{1SA}) r_1 \quad (A-49)$$

$$I_{30} = \int_0^{Z_1} dz (\psi_{1SA} \frac{\partial}{\partial r} \psi_{1SB}) r_1 , \quad (A-50)$$

$$I_{31} = \int_0^{Z_1} dz (\psi_{1SA} \frac{\partial}{\partial r} \psi_{2SB}) r_1 , \quad (A-51)$$

$$I_{32} = \int_0^{Z_1} dz (\psi_{1SA} \frac{\partial}{\partial r} \psi_{2SB}) r_1 , \quad (A-52)$$

$$I_{33} = \int_0^{Z_1} dz (\psi_{1SB} \frac{\partial}{\partial r} \psi_{1SA}) r_1 , \quad (A-53)$$

$$I_{34} = \int_0^{Z_1} dz (\psi_{1SB} \frac{\partial}{\partial r} \psi_{1SB}) r_1 , \quad (A-54)$$

$$I_{35} = \int_0^{Z_1} dz (\psi_{1SB} \frac{\partial}{\partial r} \psi_{2SA}) r_1 , \quad (A-55)$$

$$I_{36} = \int_0^{Z_1} dz (\psi_{1SB} \frac{\partial}{\partial r} \psi_{1SB}) r_1 , \quad (A-56)$$

$$I_{37} = \int_{Z_1}^H dz (\psi_{1CA} \frac{\partial}{\partial r} \psi_{1CA}) r_1 , \quad (A-57)$$

$$I_{38} = \int_{Z_1}^H dz (\psi_{1CA} \frac{\partial}{\partial r} \psi_{1CB}) r_1 , \quad (A-58)$$

$$I_{39} = \int_{Z_1}^H dz (\psi_{1CA} \frac{\partial}{\partial r} \psi_{2CA}) r_1 , \quad (A-59)$$

$$I_{40} = \int_{Z_1}^H dz (\psi_{1CA} \frac{\partial}{\partial r} \psi_{2CB}) r_1 , \quad (A-60)$$

$$I_{41} = \int_{Z_1}^H dz (\psi_{1CB} \frac{\partial}{\partial r} \psi_{1CA}) r_1 , \quad (A-70)$$

$$I_{42} = \int_{Z_1}^H dz (\psi_{1CB} \frac{\partial}{\partial r} \psi_{1CB}) r_1 , \quad (A-71)$$

$$I_{43} = \int_{Z_1}^H dz (\psi_{1CB} \frac{\partial}{\partial r} \psi_{2CA}) r_1 , \quad (A-72)$$

$$I_{44} = \int_{Z_1}^H dz (\psi_{1CB} \frac{\partial}{\partial r} \psi_{2CB}) r_1 , \quad (A-73)$$

$$I_{45} = \int_0^{r_1} r dr (\psi_{1SA} \frac{\partial}{\partial Z} \psi_{1SA}) Z_1 , \quad (A-74)$$

$$I_{46} = \int_0^{r_1} r dr (\psi_{1SA} \frac{\partial}{\partial Z} \psi_{1SB}) Z_1 , \quad (A-75)$$

$$I_{47} = \int_0^{r_1} r dr (\psi_{1SB} \frac{\partial}{\partial Z} \psi_{1SA}) Z_1 , \quad (A-76)$$

$$I_{48} = \int_0^{r_1} r dr (\psi_{1SB} \frac{\partial}{\partial Z} \psi_{1SB}) Z_1 , \quad (A-77)$$

$$I_{49} = \int_0^{r_1} r dr (\psi_{1SA} \frac{\partial}{\partial Z} \psi_{1CA}) Z_1 , \quad (A-78)$$

$$I_{50} = \int_0^{r_1} r dr (\psi_{1SA} \frac{\partial}{\partial Z} \psi_{1CB}) Z_1 , \quad (A-79)$$

$$I_{51} = \int_0^{r_1} r dr (\psi_{1SB} \frac{\partial}{\partial Z} \psi_{1CA}) Z_1 , \quad (A-80)$$

$$I_{52} = \int_0^{r_1} r dr (\psi_{1SB} \frac{\partial}{\partial Z} \psi_{1CB}) Z_1 . \quad (A-81)$$

At this point, the subformulas (1) through (8) are substituted in the Lagrangian defined by Eq. (A-1), and the parameters q^2 , q , and $1/K$ are factored.

$$\mathcal{L}/2\pi = A_1 q^2 + B_1 q + C_1 + (A_2 q^2 + B_2 q + C_2) 1/K , \quad (A-82)$$

where

$$\begin{aligned}
 A_1 = & [D_S I_1 - D_S I_2 - \Sigma_S I_3 + 2\Sigma_S I_4 - D_S I_5 + D_S I_6 - \Sigma_S I_7 \\
 & + D_C I_8 - D_C I_9 - \Sigma_C I_{10} + 2\Sigma_C I_{11} - D_C I_{12} + D_C I_{13} - \Sigma_C I_{14} \\
 & + D_2 I_{15} - D_2 I_{16} - \Sigma_2 I_{17} - D_2 I_{19} + D_2 I_{20} + 2I_{18}\Sigma_2 - \Sigma_2 I_{21} \\
 & + D_2 I_{22} - D_2 I_{23} - \Sigma_2 I_{24} + 2I_{25}\Sigma_2 - D_2 I_{26} + D_2 I_{27} - \Sigma_2 I_{28} \\
 & - r_1 D_S I_{29} + r_1 D_S I_{30} + r_1 D_2 I_{31} - r_1 D_2 I_{32} + r_1 D_S I_{33} \\
 & - r_1 D_S I_{34} - r_1 D_2 I_{35} + r_1 D_2 I_{36} \\
 & - r_1 D_S I_{37} + r_1 D_S I_{38} + r_1 D_2 I_{39} - r_1 D_2 I_{40} \\
 & - r_1 D_S I_{41} - r_1 D_S I_{42} - r_1 D_2 I_{43} + r_1 D_2 I_{44} \\
 & - D_S I_{45} + D_S I_{46} + D_S I_{47} - D_S I_{48} \\
 & - D_C I_{49} - D_C I_{50} - D_C I_{51} + D_C I_{52}] , \tag{A-83}
 \end{aligned}$$

$$\begin{aligned}
 B_1 = & [D_S I_2 - 2\Sigma_S I_4 + D_S I_5 - 2D_S I_6 + 2\Sigma_S I_7 \\
 & + D_C I_9 - 2\Sigma_C I_{11} + D_C I_{12} - 2\Sigma_C I_{13} + 2\Sigma_C I_{14} \\
 & + D_2 I_{16} + D_2 I_{19} - 2D_2 I_{20} - 2\Sigma_2 I_{18} + 2\Sigma_2 I_{21} \\
 & + D_2 I_{23} - 2\Sigma_2 I_{25} + D_2 I_{26} - 2D_2 I_{27} + 2\Sigma_2 I_{28} \\
 & - r_1 D_S I_{30} + r_1 D_2 I_{32} - r_1 D_S I_{33} + 2r_1 D_S I_{34}
 \end{aligned}$$

$$\begin{aligned}
& + r_1 D_2 I_{35} + 2r_1 D_2 I_{36} \\
& - r_1 D_S I_{38} + r_1 D_2 I_{40} - r_1 D_S I_{41} + 2r_1 D_S I_{42} \\
& + r_1 D_2 I_{43} + 2r_1 D_2 I_{44} \\
& - D_S I_{46} - D_S I_{47} + 2D_S I_{48} \\
& + D_C I_5 + D_C I_{51} - 2D_C I_{52}] , \tag{A-84}
\end{aligned}$$

$$\begin{aligned}
C_1 = & [D_S I_6 - \Sigma_2 I_2 + D_C I_{13} - \Sigma_C I_9 + D_2 I_{20} - \Sigma_2 I_{21} \\
& + D_2 I_{27} - \Sigma_2 I_{28} - r_1 D_S I_{34} + r_1 D_2 I_{36} \\
& - r_1 D_S I_{44} + r_1 D_2 I_{44} - D_S I_{48} + D_C I_{52}] , \tag{A-85}
\end{aligned}$$

$$A_2 = (FI_{17} - 2FI_{18} + FI_{21} + FI_{24} - 2FI_{25} + FI_{28}) , \tag{A-86}$$

$$B_2 = (2FI_{18} - 2FI_{21} + 2FI_{25} - 2FI_{28}) , \tag{A-87}$$

$$C_2 = FI_{21} + FI_{28} . \tag{A-88}$$

Setting Eq. (A-82) equal to zero and solving for the static reactivity eigenvalue:

$$K = \frac{A_2 q^2 + B_2 q + C_2}{A_1 q^2 + B_1 q + C_1} . \tag{A-89}$$

Another equation can be obtained by taking the variations of the Lagrangian with respect to changes in the parameter q and setting the resultant expression equal to zero.

$$\frac{\partial L}{\partial q} = 0 , \quad (\text{A-90})$$

$$\frac{\partial L}{\partial q} = 2A_1q + B_1 + 2A_2q/K + B_2/K = 0 , \quad (\text{A-91})$$

$$q = - \frac{KB_1 + B_2}{2KA_1 + 2A_2} . \quad (\text{A-92})$$

Equations (A-89) and (A-92) can be solved simultaneously using "two equations, two unknowns" to obtain a solution for parameter q . After the substitution of Eq. (A-92) into Eq. (A-89), a third order equation is obtained as follows:

$$H_1K^3 + H_2K + H_3 = 0 , \quad (\text{A-93})$$

where

$$H_1 = A_1B_1^2 - 2B_1^2A_1 + 4C_1A_1^2 , \quad (\text{A-94})$$

$$H_2 = -2A_1B_2B_1 - 2A_2B_1^2 + 8C_1A_1A_2 - A_2B_1^2 + 2B_2 , \quad (\text{A-95})$$

$$H_3 = 2B_2^2A_2 - 4C_2A_2^2 - A_2B_2^2 . \quad (\text{A-96})$$

There are three possible sets of solutions associated with the cubic equation defined by (A-93). These sets are differentiated by the discriminant:

$$D = \left(\frac{3H_2 - H_1^2}{9} \right)^3 + \left(\frac{9H_1H_2 - 27H_3 - 2H_1^3}{54} \right)^2 . \quad (\text{A-97})$$

Upon the calculation of the above discriminant, a specific set of solutions can be obtained.

There are three different possible values that a discriminant can have: greater than zero, zero, and less than zero. When the discriminant is greater than zero, two of the solutions possess an imaginary part. Since a static reactivity eigenvalue is a positive real number, the solutions corresponding to the discriminant greater than zero are eliminated.

If the discriminant is equal zero, the following solutions are obtained:

$$K_1 = 2 \sqrt[3]{\frac{9H_1H_2 - 27H_3 - 2H_1^3}{54}} - H_1/3 , \quad (\text{A-98})$$

$$K_2 = - \sqrt[3]{\frac{9H_1H_2 - 27H_3 - 2H_1^3}{54}} - H_1/3 , \quad (\text{A-99})$$

$$K_3 = K_2 . \quad (\text{A-100})$$

As can be seen, there are three roots, one of which is repeated. The static reactivity eigenvalue is the smallest positive of any of the above two values.

The roots of Eq. (A-93) will be different if the discriminant value is less than zero. In this case the following equations should

be used in obtaining the three roots:

$$K_1 = 2 \sqrt{\frac{H_1^2 - 3H_2}{9}} \cos\left(\frac{\theta}{3}\right) - \frac{H_1}{3}, \quad (\text{A-101})$$

$$K_2 = 2 \sqrt{\frac{H_1^2 - 3H_2}{9}} \cos\left(\frac{\theta}{3} = \frac{120 \times \text{PI}}{180}\right) - \frac{H_1}{3}, \quad (\text{A-102})$$

$$K_3 = 2 \sqrt{\frac{H_1^2 - 3H_2}{9}} \cos\left(\frac{\theta}{3} + \frac{240 \times \text{PI}}{180}\right) - \frac{H_1}{3} \quad (\text{A-103})$$

where

$$= -\arctan \frac{-R}{\sqrt{1 + \frac{R^2}{Q^3 - R^2}}} + \frac{\text{PI}}{2}, \quad (\text{A-104})$$

$$R = \frac{9H_1H_2 - 27H_3 - 2H_1^3}{54}, \quad (\text{A-105})$$

$$Q = \frac{3H_2 - H_1^2}{9}, \quad (\text{A-106})$$

$$\text{PI} = 3.14159. \quad (\text{A-107})$$

Again, the static reactivity eigenvalue would be equal to the smallest positive value determined by Eqs. (A-101) through (A-103).

APPENDIX B

The Fast Flux Testing Facility (FFTF) reactor core model was used for verification of the Functional Interpolation Method. In this appendix the FFTF is briefly described and the pertinent information for the core is given. Furthermore, the cross sections needed for the analysis are also presented.

The FFTF is a 400-MW, sodium-cooled, low-pressure, high-temperature, fast neutron flux, nuclear fission reactor plant designed and constructed for irradiation testing of breeder reactor fuels and materials. Since the pertinent information used in this research are the core parameters, we turn our attention to the core descriptions and the information utilized in the analysis rather than the other parts of the FFTF.

Figure B-1 shows the detailed description of the FFTF core and Figure B-2 depicts the listing of the legend for Figure B-1. The FFTF core generates up to 7×10^{15} m/cm²-s of neutron flux at a power level of 400 MW. The core consists of 199 core assemblies. The design includes 74 positions for $\text{PuO}_2\text{-UO}_2$ driver fuel assemblies, which generate the nuclear flux, nine boron carbide control rod absorber assemblies, and eight positions that may be used for independently instrumented test of fuel specimens or reactor core materials. Figure B-3 shows the core for a typical arrangement. Even though the geometry used for benchmarking FIM is a slab geometry, nevertheless the dimensions and materials used are equivalent. Table B-1 provides the additional data needed for the analysis.

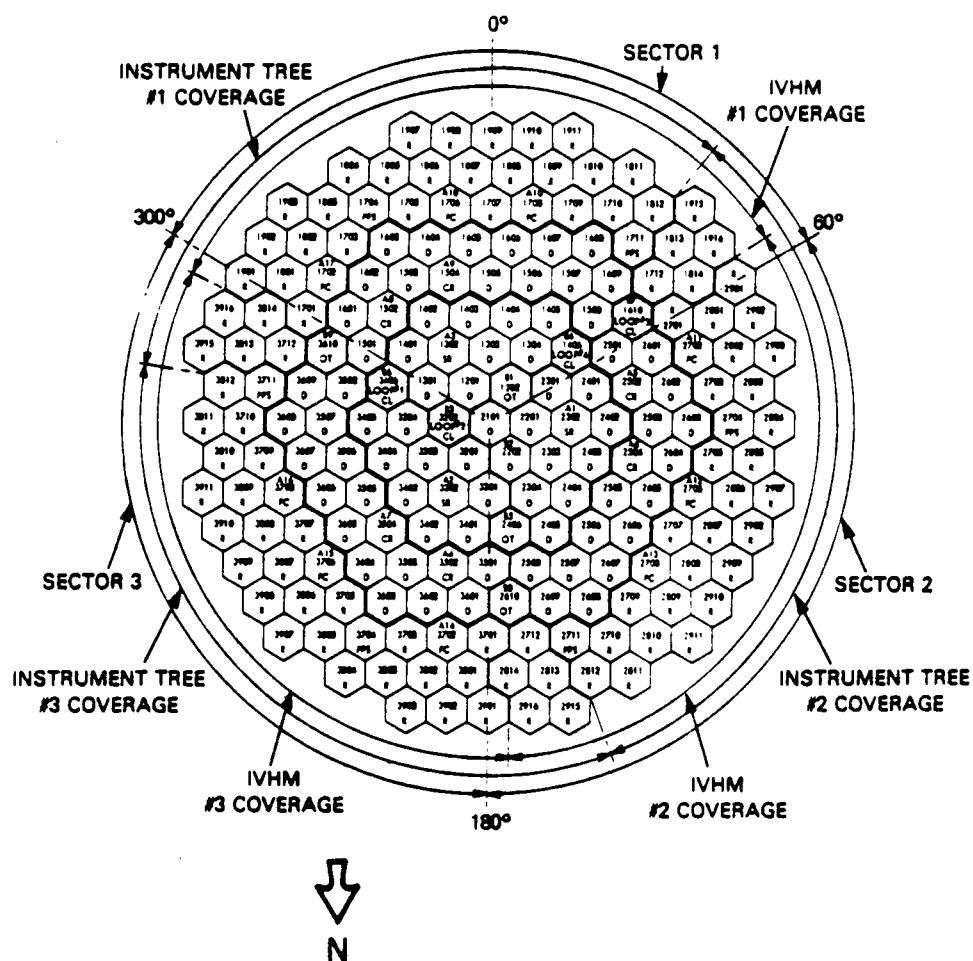
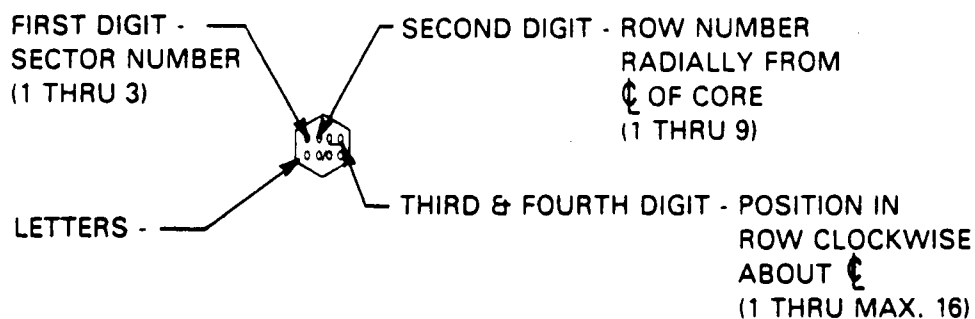


Figure B-1. Detailed map of FFTF core.

Source: Reference 11.



ASSEMBLY FUNCTION

CL	CLOSED LOOPS-4
OT	OPEN TEST ASSEMBLIES-4
D	DRIVER { 29 INNER (ROWS 1 THRU 4)
	POSITIONS { 45 OUTER (ROWS 5 & 6)
SR	SAFETY RODS-3
CR	CONTROL RODS-6
PC	POTENTIAL FIXED SHIM OR MOVABLE PERIPHERAL CONTROL RODS-9
PFS	POTENTIAL FIXED SHIM LOCATIONS-6
R	REFLECTORS-93 TO 108 (ROWS 7-9)

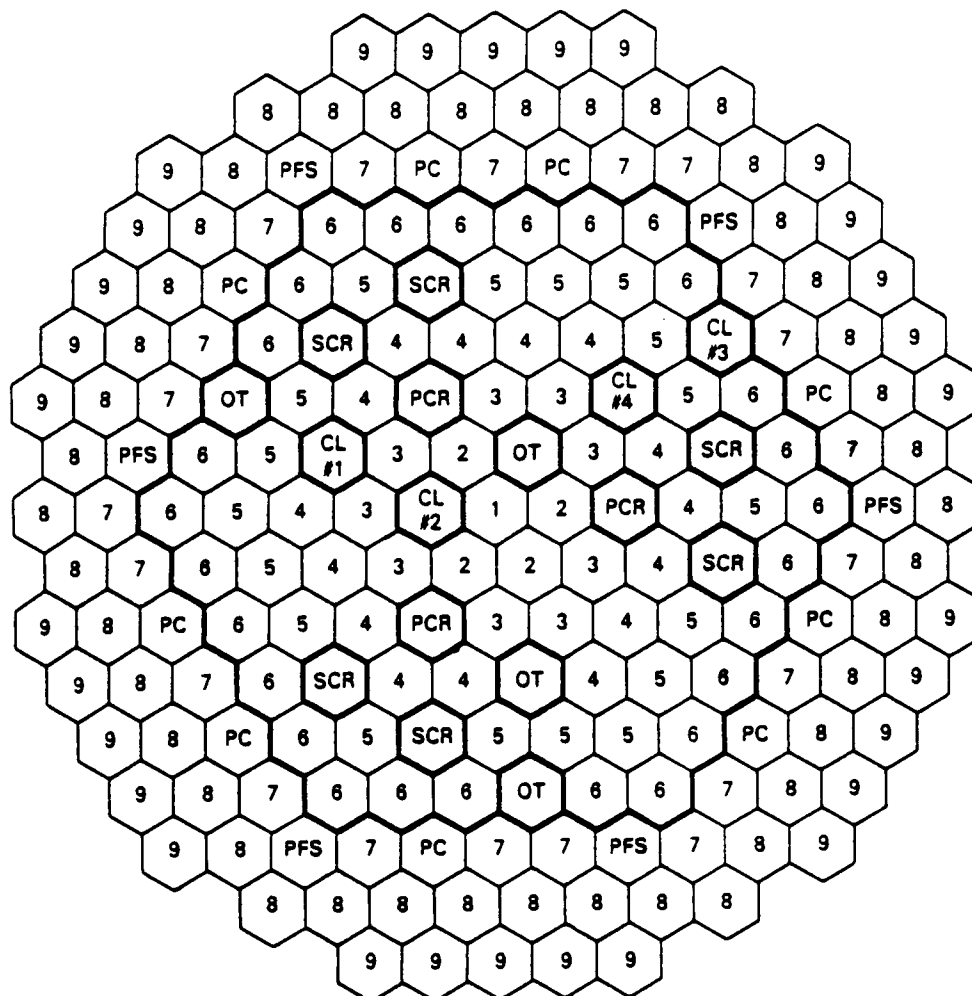
REACTOR VESSEL HEAD PENETRATIONS (REF)

A1-A3 SAFETY RODS-3
 A4-A9 CONTROL RODS-6
 A10-A18 POTENTIAL FIXED SHIM OR MOVABLE PERIPHERAL CONTROL RODS-9

B-1 OT
 B-2 PROXIMITY TEST PLUG
 B-3 LOOP 2 CL
 B-4 LOOP 4 CL
 B-5 OT
 B-6 LOOP 1 CL
 B-7 LOOP 3 CL
 B-8 OT
 B-9 OT

Figure B-2. Legend for detailed map of FFTF core.

Source: Reference 11.



1-9: ROW NUMBER FOR CORE ASSEMBLY. ROWS 1-8 ARE THE FUELED ZONE; ROWS 7-9 ARE THE REFLECTOR ZONE.

CL - CLOSED LOOP POSITION.

SCR - SECONDARY CONTROL ROD ABSORBER POSITION. SIX ASSEMBLIES WERE INSTALLED INITIALLY.

OT - OPEN TEST ASSEMBLY POSITION.

PC - POSITIONS IN WHICH CONTROL RODS MAY BE INSTALLED.

PFS - PERIPHERAL FIXED SHIM ABSORBERS.

PCR - PRIMARY CONTROL RODS. THREE RODS WERE INSTALLED INITIALLY.

Figure B-3. Core map showing typical arrangement.

Source: Reference 11.

Table B-1. FFTF Design Parameters

REACTOR CONTAINMENT VESSEL

Material	Carbon steel
Diameter	135 ft 0 in.
Overall Height	186 ft 8 in.
Depth Below Operating Floor	77 ft 10-1/2 in.
Wall Thickness Above Grade	1 to 1-3/8 in.
Internal Design Pressure	10 psig

REACTOR VESSEL

Material	304 stainless steel
Internal Diameter	243 in. (about 20 ft)
Height	517 in. (about 43 ft)
Wall Thickness	2-3/8 to 2-3/4 in.
Vessel Liner Internal Diameter	19 ft 10-1/2 in.

REACTOR GUARD VESSEL

Material	304 stainless steel
Wall Thickness	1 to 2-1/2 in.
Annulus Between Reactor Vessel and Reactor Guard Vessel	8-in.

REACTOR VESSEL HEAD

Material	Low carbon alloy steel
Diameter	25 ft
Thickness	about 22 in. (plus shielding plates)
Weight	214 tons

GENERAL

Power, Excluding Closed Loops	400 MW
Closed Loops Capability	4 at 2.3 MW each

NUCLEAR CONTROLS

Type of Absorber	B ₄ C Pellets
Boron-10 per Assembly	1.16 to 1.28 kg
Pins per Assembly	61
Pin Cladding - OD	0.474 in.
- Thickness	0.051 in.
Pellet Diameter	0.362 in.
Duct - Wall Thickness	0.120 in.
- Length	12 ft
- Material	316 SS 20% CW
Cladding Material	316 SS 20% CW
Wire Wrap Material	316 SS 17% CW

Table B-1 (continued)

HEAT REMOVAL - MAIN HEAT TRANSPORT SYSTEM

Circuits	3
Rating per Circuit	133 MW
DHX Modules	12
Rating per DHX Module	33 MW
Sodium Flow Rate	43,500 gal/min
Reactor Vessel	680°F nominal
- Inlet Temperature	980°F nominal
- Outlet Temperature	133 psig nominal
- Inlet Pressure	

CORE CONFIGURATION

Core Positions	- Rows 1-6 (Fueled Zone)	91
	- Rows 7-9 (Reflector Zone)	108
	- Total Positions	199
Core Assembly	- Shape	Hexagonal
	- Dimension Across Load Pads	4.715 in.
	- Length (for Most)	12 ft
Active Core	- Fueled Height	36 in.
	- Equivalent Diameter	47.2 in.
	- Volume	1034 liters
Positions for Driver Fuel Assemblies		74
Primary Control Rods (Boron Carbide)		3
Secondary Control Rods (Boron Carbide)		6
Fixed Peripheral Absorber Assemblies		0-15
Positions for 40-ft Independently Instrumented Test Assemblies		8

CORE PHYSICS

Total Fissile Mass	563 kg
Neutron Flux (Peak)	7×10^{15} n/cm ² -s
0.1 MeV	60 to 65%
Conversion Ratio	0.43
Fuel Cycle, Nominal	100 Full Power Days
Average Discharge Burnup	45 MWd/kg
Limiting Peak Burnup	80 MWd/kg
Power Density	0.39 MW/liter
Doppler Coefficient	-0.005
Delayed Neutron Fraction	0.003

DRIVER FUEL

Fuel Assembly Power	3 to 7 MW
Assembly Width at Load Pads	4.715 in.
Assembly Length	12 ft
Fuel Pin Linear Average Power	7.3 kW/ft
Driver Fuel Assembly Average Coolant Flow	18.7×10^4 lb/hr
Initial Core Average Coolant Velocity	21 ft/s
Fuel Type	PuO ₂ -UO ₂
Pu % (of Pu and U) - Rows 1-4	about 22.4%

Table B-1 (continued)

Pu % (of Pu and U) - Rows 5-6	about 27.4%
Pu Fissile Content (^{239}Pu + ^{241}Pu)	88 wt%
Uranium Type	Natural
Duct Wall Thickness	0.120 in.
Duct Material	316 SS 20% CW
Fuel Pins per Fuel Assembly	217
Fuel Pin Cladding - OD	0.23 in.
- Thickness	0.015 in.
- Material	316 SS 20% CW
Fuel Pin Wire Wrap - Diameter	0.056 in.
- Material	316 SS 17% CW
Fuel Pellets per Pin	about 150
Fuel Pin - Length	about 94 in.
- Fueled Length	36 in.

Source: Reference 11.

After obtaining the physical dimensions, the data for cross sections of the FFTF core were obtained from Mann, 1982.¹² This report contains Fast Test Reactor (FTR) set 500 which is a multigroup cross section set for FTR analysis. FTR set 500 is a 53-neutron group, 20-photon group, cross section based on ENDF/B-V cross sections and neutron spectra typical of the FFTF core. The specifications of FTR set 500 and the processing method can be found in Mann, 1982.¹² The obtained microscopic cross sections are tabulated in Table B-2. This table contains all of the scattering and absorption cross section components. This table was used to obtain the macroscopic cross sections needed in Chapter V.

Table B-2. FFTF Spectral-Averaged Cross Sections

Isotope	Cross Sections (barns)									
	Σ_{total}	Σ_{f}	Σ_{a}	Σ_{sc}	Σ_{p}	Σ_{c}	Σ_{r}	Σ_{v}	Σ_{g}	Σ_{h}
^{10}B	5.17	26.06	2.73	2.56	--	--	--	--	--	--
^{11}B	4.02	4.57	4.07	4.57	6	6	6	6	6	6
^{12}C	3.87	4.36	3.87	4.36	--	--	--	--	--	--
^{16}O	3.70	3.83	3.70	3.83	--	--	--	--	--	--
^{23}Na	4.58	6.69	4.47	6.67	6	6	6	6	6	6
^{55}Mn	5.29	6.32	5.13	6.28	0.00004	6	6	6	6	6
^{59}Co	11.24	32.52	10.88	31.92	0.00005	6	6	6	6	6
^{56}Fe	5.14	7.11	5.07	7.07	0.00002	6	6	6	6	6
^{57}Fe	9.20	12.5	9.10	12.4	6	6	6	6	6	6
^{90}Zr	8.10	10.03	7.87	9.95	0.00053	6	6	6	6	6
^{235}U	11.19	22.18	0.22	18.07	0.0011	0.00001	6	6	6	6
^{238}U	11.66	26.68	10.70	18.30	0.0023	0.00003	0.00002	6	6	6
^{239}Pu	10.96	24.96	8.17	10.45	0.00048	6	6	6	6	6
^{240}Pu	11.04	60.07	9.26	16.75	0.00023	6	6	6	6	6
^{241}Pu	12.22	29.00	0.42	10.44	0.00028	0.00003	6	6	6	6
^{241}Am	11.62	25.12	8.94	11.05	0.00019	6	6	6	6	6

Isotope	Cross Sections (barns)									
	Σ_{total}	Σ_{f}	Σ_{a}	Σ_{sc}	Σ_{p}	Σ_{c}	Σ_{r}	Σ_{v}	Σ_{g}	Σ_{h}
^{10}B	2.44	23.49	--	--	--	0.0044	0.0002	--	--	--
^{11}B	6	6	--	--	--	--	--	--	--	--
^{12}C	6	6	--	--	--	--	--	--	--	--
^{16}O	0.0011	0.00001	--	--	--	--	--	--	--	--
^{23}Na	0.00010	6	--	--	--	--	--	--	--	--
^{55}Mn	0.00003	6	6	6	6	6	6	6	6	6
^{59}Co	0.00002	6	6	6	6	6	6	6	6	6
^{56}Fe	0.00004	6	6	6	6	6	6	6	6	6
^{57}Fe	0.00061	6	0.00002	6	6	6	6	6	6	6
^{90}Zr	--	--	--	--	--	--	--	--	--	--
^{235}U	--	--	--	--	--	--	--	--	--	--
^{238}U	--	--	--	--	--	--	--	--	--	--
^{239}Pu	--	--	--	--	--	--	--	--	--	--
^{240}Pu	--	--	--	--	--	--	--	--	--	--
^{241}Pu	--	--	--	--	--	--	--	--	--	--
^{241}Am	--	--	--	--	--	--	--	--	--	--

*less than 10b.

*2101 is the central assembly (midplane); 2009 is a reflector assembly in the last row (midplane). All multigroup cross sections come from 2101 flux weighting.

--not on ENDF/B-V or not processed.

Source: Reference 12.

APPENDIX C

```

10 / =====
20 / = PROGRAMMER: MERAJ RAHIMI =
30 / = = =
40 / = SUBJECT: MATHEMATICAL ALGORITHM =
50 / = =
60 / =====
70 /
80 /
90 DATA 1.1015, .7845812, 1.3768747, .7349419, .9179164, .8220558
100 A=76.56
110 X1=6.80918
120 X0=A/2
130 D1=3.447
140 CASE=0
150 CASE=CASE+1
160 READ D2, TEIGHEN
170 SIGMA1=.0026
180 SIGMA2=.0067
190 F=.0082
200 PI=3.141592
210 L=SQR(D1/SIGMA1)
220 B0=.056897
    B1=.05973
240 /
250 /
260 DEF FNCOSH(X)=(EXP(X)+EXP(-X))/2
270 DEF FNSINH(X)=(EXP(X)-EXP(-X))/2
280 /
290 /
300 NA=FNCOSH(X1/L)/SIN(B0*(X1-X0))
310 NB=FNCOSH(X1/L)/SIN(B1*(X1-X0))
320 /
330 /
340 IAA=(NA^2/2)*(X0-X1+SIN(2*B0*(X1-X0))/(2*B0))
350 IBB=(NB^2/2)*(X0-X1+SIN(2*B1*(X1-X0))/(2*B1))
360 IAB=-(NA*NB/2)*(SIN((B0-B1)*(X1-X0))/(B0-B1)-SIN((B0+B1)*(X1-X0))/(B0+B1))
370 /
380 /
390 LOF=D2*B0^2+SIGMA2
400 L1F=D2*B1^2+SIGMA2
410 SOA=NA*B0*COS(B0*(X1-X0))*FNCOSH(X1/L)
420 S1B=NB*B1*COS(B1*(X1-X0))*FNCOSH(X1/L)
430 T1=(FNSINH(X1/L)*FNCOSH(X1/L))/L
440 /
450 /
460 KO=F/LOF
470 K1=F/L1F
480 Q2=(1/K1+1/KO)*IAB/IBB-(IAA/(KO*IBB)+1/K1)
490 Q1=2/K1-(1/K1+1/KO)*IAB/IBB+D2*(SOA-S1B)/(F*IBB)
500 G2=2*IAB/IBB-IAA/IBB-1
510 G1=2-2*IAB/IBB
520 Q0=D1*T1/(F*IBB)-D2*S1B/(F*IBB)+1/K1
530 /
540 /
550 B=-(2*Q1*G1+4*(Q2+Q0*G2))/(4*G2+G1^2)
560 C=(Q1^2+4*Q0*Q2)/(G1^2+4*G2)

```

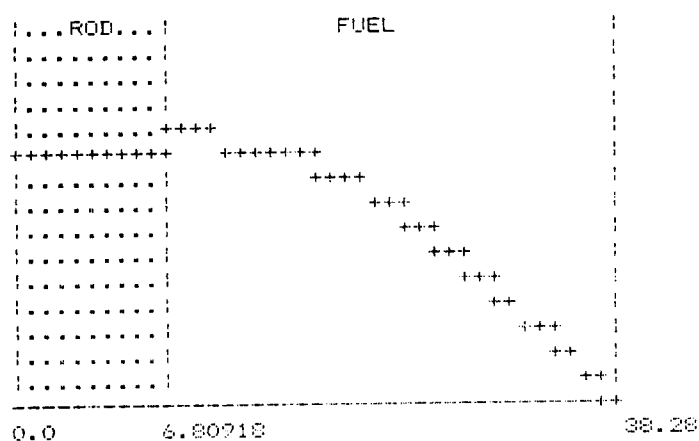

<<<<<<<<<<<<<<< CASE 1 >>>>>>>>>>>>>>>>>>

WITH THE DIFFUSION COEFFICIENT OF THE FUEL REGION= 1.1015
THE STATIC EIGENVALUE IS:

EIGENVALUE= .784582 TRUE EIGENVALUE= .7845812 %ERROR= 1.06358E-04

V. FLUX= +++++

T. FLUX= ****



DO YOU WANT TO DO THE NEXT CASE (Y/N)?

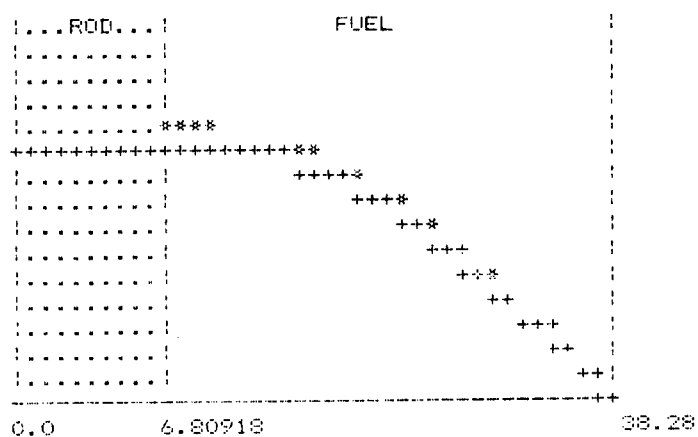
<<<<<<<<<<<<<<< CASE 2 >>>>>>>>>>>>>>>>>>>

WITH THE DIFFUSION COEFFICIENT OF THE FUEL REGION= 1.374875
THE STATIC EIGENVALUE IS:

EIGENVALUE= .7349493 TRUE EIGENVALUE= .7349419 %ERROR= 1.005654E-03

V. FLUX= +++++

T. FLUX= ****



DO YOU WANT TO DO THE NEXT CASE (Y/N)?

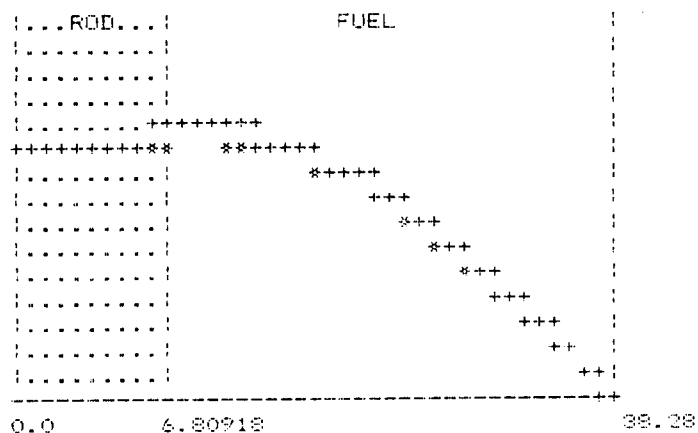
<<<<<<<<<<<<<<<<< CASE 3 >>>>>>>>>>>>>>>>>>>

WITH THE DIFFUSION COEFFICIENT OF THE FUEL REGION= .9179164
THE STATIC EIGENVALUE IS:

EIGENVALUE= .8220661 TRUE EIGENVALUE= .8220558 %ERROR= 1.254368E-03

V. FLUX= +++++

T. FLUX= ****



DO YOU WANT TO DO THE NEXT CASE (Y/N)?

VITA

Meraj Rahimi was born in Tehran, Iran, on March 28, 1959. He attended elementary schools in that city and was graduated from Sina High School in June 1976. He entered English Language School in Charlotte, North Carolina, in February 1977. After obtaining a degree in English, he was accepted at the Greenville Technical College in June 1977. He transferred to The University of Tennessee, Knoxville, in September 1978 and was awarded a Bachelor of Science degree in Nuclear Engineering in June 1981.

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The author is a member of the American Nuclear Society. He is currently employed by a consulting firm in Rockville, Maryland.