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ESTIMATION OF SURFACE TEMPERATURE AND HEAT FLUX USING
EMBEDDED THERMOCOUPLES

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee Space Institute

Jeffrey Daniel King
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Abstract

This thesis investigates several new methods for estimating surface temperature and heat flux based on thermocouples embedded within the interior of a test article. The test apparatus was a Mach 2 axisymmetric rocket nozzle with probes embedded at depths of 1/8" and 1/4" from the inner surface of the nozzle at the throat and exit locations. Three methods for numerically estimating temperature and heat flux at the surface of the nozzle were considered: a one-thermocouple slab-coordinate method, a two-thermocouple slab-coordinate method, and a finite-element analysis (FEA) based on a one-dimensional cylindrical coordinate method. Comparisons between the methods, as well as temperature and heat flux uncertainties caused by the axial location of the thermocouple probes (and resulting radius of curvature of the nozzle materials considerations), are presented. The slab-coordinate two-thermocouple and FEA methods show good agreement in both thermocouple probe axial locations, but the one-thermocouple measurement showed significant uncertainty for the throat location, but offered good agreement at the exit location. Suggestions are made for future improvements in experimentation and analysis methods.

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Chapter 1 – Introduction

The accurate prediction of surface temperature and heat flux conditions has always been a critical capability in the development of new technologies. In many cases, direct surface measurement is impractical due to limitations of size or the nature of the surface. Many high temperature or high heat flux surfaces can not be instrumented directly because no probe would survive the extreme conditions outside the surface. Methods are required to deduce or estimate surface conditions based on measurements that can be taken internally in the material. This thesis examines three such methods, all of which rely on a number of embedded temperature probes to predict both surface temperature and surface heat flux.

Of these three methods, two were developed by Dr. Jay Frankel of the University of Tennessee. Both of these analytical methods are based on a one-dimensional slab (Cartesian) coordinate system. The third method, developed by the author, is based on a simple numerical finite-element analysis (FEA) for a one-dimensional cylindrical coordinate system and was developed to account for the specific geometrical features of the test apparatus. Each method is discussed in detail in Chapter 2.

The test apparatus for the data used in this thesis is a Mach 2 rocket nozzle made from monolithic graphite with a total of four embedded thermocouples. Two of the temperature probes are embedded at the throat plane of the nozzle at depths 1/8" and 1/4" from the surface, and two of the probes are placed near the exit plane of the nozzle at the same depths from the surface. Each test involved a 5-second firing of the rocket engine with a known propellant chemistry. From these tests, the measurements from each of the embedded thermocouples were used in the methods described in Chapter 2 to predict surface conditions. Chapter 2 provides the background information on the experimental setup and information about the embedded probe data.

The analysis processes for each of the methods are somewhat similar. The temperature data measured during each test of the experiment must first be filtered to remove noise. Each analysis method is then applied, resulting in a prediction of surface temperature and surface heat flux. A description of each step of the process is located in Chapter 3. Also included in Chapter 3 is a discussion of the sensitivity analysis performed on each of the methods.

Three representative cases were selected for presentation within this thesis. The cases represent the primary combustion mixtures used to produce various properties within the combustion gases. Each test case was analyzed using all three methods, and the results of all of the analyses are presented in Chapter 4. Trends within the results are discussed in Chapters 5 and 6. Chapter 5 includes trends noted within each analysis method and between analysis methods within one sample case, as well as suggestions for improvements in the results and methodologies used. Chapter 6 offers final conclusions about the experiment and analyses.

Chapter 2 – Background

This chapter offers an explanation of the experiments that produced the data used for analysis and the mathematical models used to perform the analysis. A description of the experimental setup, with the placement of temperature probes within the test section as well as key test parameters such as sampling rates and measurement times, is provided. The mathematical background offers the primary equations used to predict the surface temperature and heat flux based on the probe data. Where derivations are required, they have been simplified to show only the key steps preceding the final results.

Experimental Setup

The primary test section for this experiment is a converging-diverging graphite nozzle through which hot exhaust gases flow. The upstream end of the nozzle is connected to a combustion chamber while the downstream end connects to a conductivity channel. Most test firings of the combustor lasted 5 seconds, though some tests were run for shorter duration to investigate various aspects of the experiment. The nozzle itself is a graphite core with a stainless steel shell. Silica sand is packed between the core and the shell to maintain alignment. Four thermocouples were embedded within the nozzle: two at the plane of the throat and two near the exit plane. This axial placement of the probes is shown in Figure 2-1. The internal diameter of the throat is 2", and the diameter at the exit plane is 3". [1]

At each axial location, one probe is located at a depth of 1/8" from the inner surface of the nozzle while the second probe is 1/4" from the surface. This spacing can be seen in Figure 2-1, which also illustrates the angular spacing between the probes. Both aspects of spacing are important in the analysis performed in this thesis. One assumption used in both the analytical and numerical methods of the analysis in this thesis was that the heat transfer is one dimensional, so the angular spacing of the probes, a physical necessity, introduces a small level of uncertainty in the results. The second key assumption of the analysis was that the probes are uniformly spaced radially. That is, the distance between the probes was equal to the distance from the first probe to the surface. Uniform spacing allows for significant simplification of the equations and is key to the results obtained (see Equations (2-3) and (2-4) for simplifications resulting from this assumption).

Several test firings of the apparatus were performed. Many of these firings were short duration and were used to test components and procedures. Of the full-length firings, 28 were considered a good set for initial application of this analysis. These firings represent the bulk of the propellant chemistry tests where different materials were added into the combustor to achieve certain flow properties through the downstream conductivity channel. All tests include 30 seconds of data: approximately 10 seconds prior to the firing, the 5-second burn time, and a cool-down period for the remainder. During this period of data collection, thermocouple readings were taken at 1 kHz, resulting in 30,000 measurements of temperature for each probe.

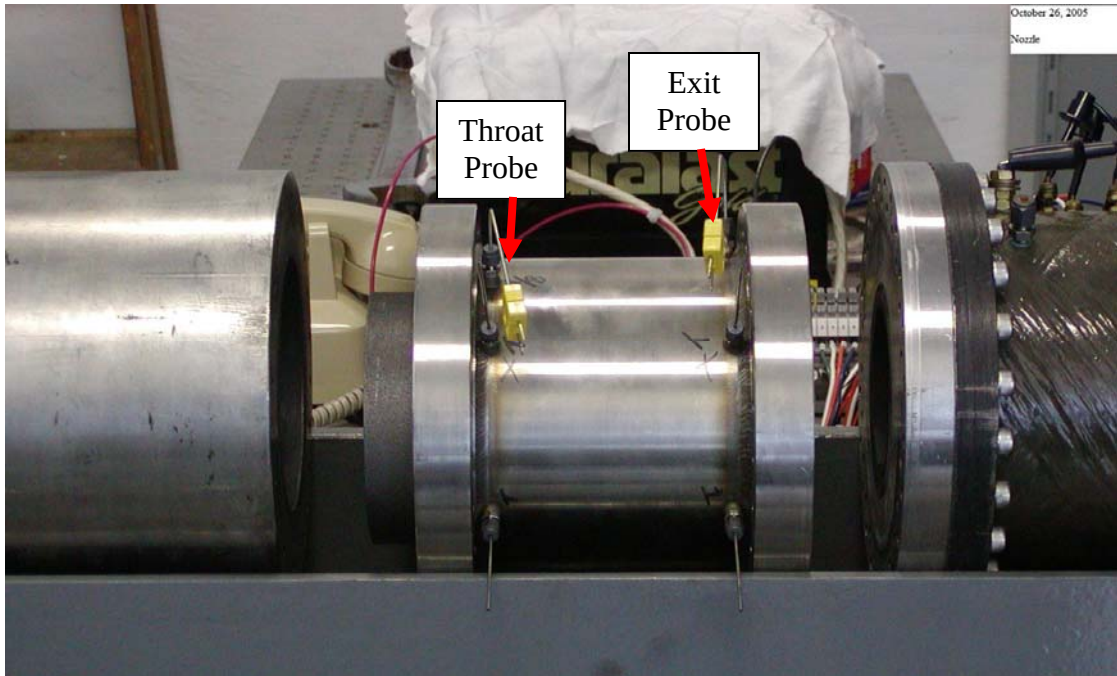


Figure 2-1 – Axial Locations of the Thermocouples

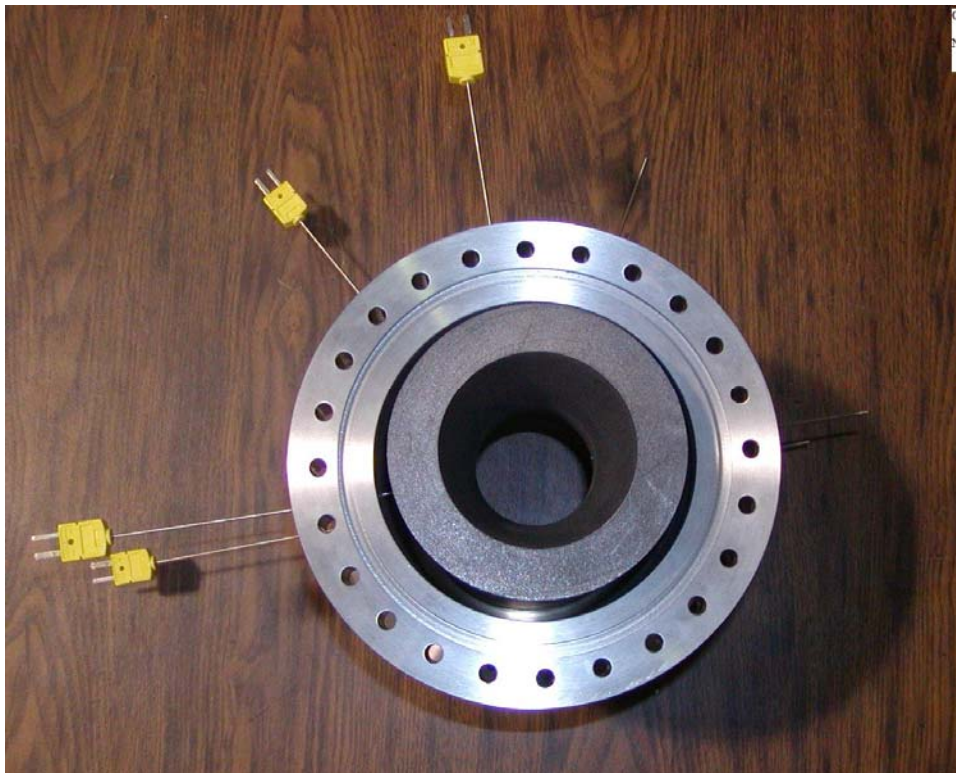


Figure 2-2 – Angular and Radial Probe Installations

1-D Slab Analysis

An estimate of the surface temperature and heat flux in a nozzle can be obtained by applying a one-dimensional semi-infinite slab model developed by Dr. Jay Frankel of the University of Tennessee [2]. Frankel developed relationships in a planar half-space between the temperature at locations within a material and the temperature at the surface. His method uses the heat equation ($\partial T / \partial t = \alpha \partial^2 T / \partial x^2$) together with Taylor series expansions of the temperature and heat flux that involve temperature measurements at the known point or points to estimate the surface conditions. The Taylor series spatial derivatives are converted into forms involving time derivatives by use of the heat equation. Details of this derivation can be found Reference 2. The formulation can be used for either a single probe located at a known distance η from the surface or two probes located at positions η and 2η . As described above, the probes for this experiment were placed 1/8" and 1/4" from the surface. The use of a planar half-space approach with a cylindrical geometry results in deviations of the theoretical results from the true values. It is expected that this error is larger at the throat, where the radius of curvature of the material is smaller, since it more poorly approximates the mathematical assumption than at the exit plane, where there is a larger radius. Both the single-point formulation and the two-point formulation developed by Frankel were used in this analysis. In addition, the single-point formulation was used with the 1/4" probe to estimate the temperature at a depth of 1/8". This estimate was then compared to the recorded data at 1/8" to provide a basis for comparison for this method. Frankel's equation used to estimate surface temperature, based on a single embedded probe, is given as Equation (2-1). The single-probe heat flux equation is given as Equation (2-2) below:

$$T(0,t) = T(\eta,t) + q''(\eta,t) \frac{\eta}{1!k} + \frac{\partial T}{\partial t}(\eta,t) \frac{\eta^2}{2!\alpha} + \frac{\partial q''}{\partial t}(\eta,t) \frac{\eta^3}{3!\alpha k} + O(\eta^4) \quad (2-1)$$

$$q''(0,t) = q''(\eta,t) + \frac{k}{\alpha} \frac{\partial T}{\partial t}(\eta,t) \frac{\eta}{1!} + \frac{1}{\alpha} \frac{\partial q''}{\partial t}(\eta,t) \frac{\eta^2}{2!} + \frac{k}{\alpha^2} \frac{\partial^2 T}{\partial t^2}(\eta,t) \frac{\eta^3}{3!} + O(\eta^4) \quad (2-2)$$

The equations for surface temperature and heat flux based on two embedded thermocouples are derived from the Taylor series expansions around each thermocouple combined to eliminate the terms for heat flux within the temperature equation and the terms for temperature in the heat flux equation. The result is two equations which base a surface condition solely on the same condition at the embedded locations. Equation (2-3) is the two-thermocouple surface temperature equation, while the heat flux is given by Equation (2-4).

$$T(0,t) = -T(2\eta,t) + 2T(\eta,t) + \frac{1}{\alpha} \frac{\partial T}{\partial t}(\eta,t) \eta^2 + O(\eta^4) \quad (2-3)$$

$$q''(0,t) = -q''(2\eta,t) + 2q''(\eta,t) + \frac{1}{\alpha} \frac{\partial q''}{\partial t}(\eta,t) \eta^2 + O(\eta^4) \quad (2-4)$$

Each of these four equations contains a truncation error term on the order of η^4 .

In order to use these approximations, a value of the heat flux and its time derivative must be known at η and 2η as a function of time. Frankel has shown that Equation (2-5), below, expresses heat flux at some location x in the planar half-space as a function of the time derivative of temperature at x [2]. That is, to determine the heat flux at $x = \eta$ and $x = 2\eta$ all that is required is the time derivative of the known temperature data at those locations.

$$q''(x,t) = \sqrt{\frac{\rho c_p k}{\pi}} \int_0^t \left(\frac{\partial T}{\partial u}(x,u) \frac{du}{\sqrt{t-u}} \right) \quad (2-5)$$

The full analysis process, including discussion of the numerical differentiation of discreet, noisy temperature data, required filtering of the data, and application of these equations, will be presented in the next chapter.

Finite Element Analysis

Cylindrical Coordinates

The assumption of slab geometry neglects the cylindrical nature of the nozzle geometry. One alternative for representing temperature and heat flux in the nozzle that includes the geometrical concerns is a finite-element analysis (FEA). For this method, each thermocouple, or node, is considered the center of a small volume element, with heat passing through the surface or being stored in the volume of each element. A third element includes the surface temperature node, though there is no thermocouple at that location. A simple schematic of the problem is shown in Figure 2-3. This approach also offers the advantage that it can be applied to composite materials where the transport properties of each layer are known. The numerical approach used for this method is based on the method described in Reference 3.

Development of the governing equation begins with the first law of thermodynamics expressed at the center node, node 1, as given in Equation (2-6).

$$\dot{Q}_{in} = \dot{Q}_{out} + \dot{E}_{stored} \quad (2-6)$$

The total heat transfer into the element from the surface element must balance the heat transferred out of the element into the outer element plus the total energy stored within the element, in the form of a temperature rise. The total heat transfer can be expressed through Fourier's Law at each surface, and the energy storage is simply the rate of change of internal energy at the node itself. Making these substitutions results in equation (2-7).

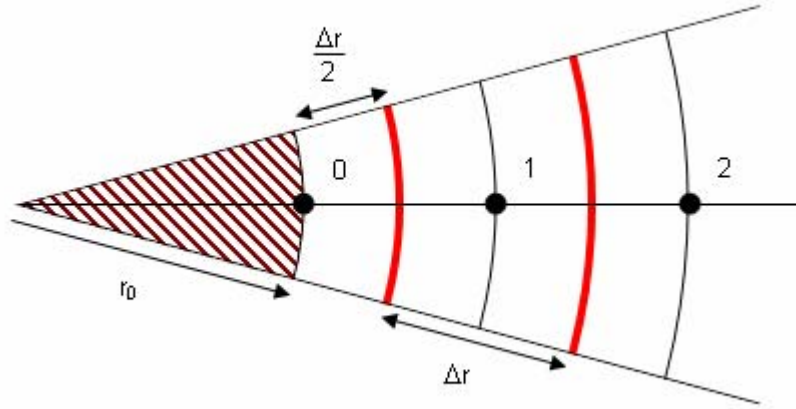


Figure 2-3 – Cylindrical Finite Element Schematic

$$-kA \frac{\partial T}{\partial r} \Big|_{r=r_0+\frac{\Delta r}{2}} = -kA \frac{\partial T}{\partial r} \Big|_{r=r_0+\frac{3\Delta r}{2}} + \rho c_p V \frac{\partial T}{\partial t} \Big|_{r=r_0+\Delta r} \quad (2-7)$$

The area in each heat flux term is the total heat transfer area for each surface and therefore is not the same in each term. Each spatial derivative is approximated as a finite difference between the two points across the boundary. For example, the difference across the boundary at $r_0 + \Delta r/2$ is $T_1 - T_0$. The time derivative is not approximated as a simple finite difference since it was needed for the slab approach and can be calculated with more accuracy. Expanding equation (2-7) based on these assumptions results in equation (2-8).

$$-k \left[2\pi \left(r_0 + \frac{\Delta r}{2} \right) \right] L \frac{T_1 - T_0}{\Delta r} = -k \left[2\pi \left(r_0 + \frac{3\Delta r}{2} \right) \right] L \frac{T_2 - T_1}{\Delta r} + \rho c_p \pi \left[\left(r_0 + \frac{3\Delta r}{2} \right)^2 - \left(r_0 + \frac{\Delta r}{2} \right)^2 \right] L \frac{\partial T}{\partial t} \Big|_{r=r_0+\Delta r} \quad (2-8)$$

Finally, solving for the only unknown temperature, T_0 , reduces the equation to equation (2-9).

$$T_0 = T_1 + \frac{r_0 + \frac{3\Delta r}{2}}{r_0 + \frac{\Delta r}{2}} (T_1 - T_2) + \frac{\Delta r}{\alpha(2r_0 + \Delta r)} \left[\left(r_0 + \frac{3\Delta r}{2} \right)^2 - \left(r_0 + \frac{\Delta r}{2} \right)^2 \right] \frac{\partial T}{\partial t} \quad (2-9)$$

This equation includes the cylindrical geometry and provides a method to calculate the surface temperature using two thermocouples without calculating heat flux at the embedded location.

To calculate the surface heat flux, all that is required is an appropriate application of Fourier's law at the surface. Equation (2-10) is the equation for heat flux at the surface, assuming axisymmetric heat transfer. Where other spatial derivatives were approximated by two-point difference formulas, this equation requires foreknowledge of three temperature values, so a slightly more accurate three-point formula can be used. The equation for the spatial derivative of measured temperature is given in Chapter 3 as Equation (3-5).

$$q'' = -k \frac{\partial T}{\partial r} \bigg|_{r=r_0} \quad (2-10)$$

Half-space Planar Coordinates

Finite element analysis is not limited to cylindrical coordinates. The same process can be applied to the slab coordinates used to derive Frankel's equations. Again, the derivation starts with a simple three-node layout as shown in Figure 2-4.

Applying the first law of thermodynamics at the center node produces equation (2-11), which reduces to equation (2-12) under the conditions of slab geometry.

$$-kA \frac{\partial T}{\partial x} \bigg|_{x=\frac{\Delta x}{2}} = -kA \frac{\partial T}{\partial x} \bigg|_{x=x_0+\frac{3\Delta x}{2}} + \rho c_p V \frac{\partial T}{\partial t} \bigg|_{x=\Delta x} \quad (2-11)$$

$$-k \frac{\partial T}{\partial x} \bigg|_{x=\frac{\Delta x}{2}} = -k \frac{\partial T}{\partial x} \bigg|_{x=x_0+\frac{3\Delta x}{2}} + \rho c_p (\Delta x) \frac{\partial T}{\partial t} \bigg|_{x=\Delta x} \quad (2-12)$$

Again, applying finite difference approximations to the spatial derivatives, but not time derivatives, yields equation (2-13).

$$-k \frac{T_1 - T_0}{\Delta x} = -k \frac{T_2 - T_1}{\Delta x} + \rho c_p (\Delta x) \frac{\partial T}{\partial t} \quad (2-13)$$

Solving the equation as before for the only unknown, T_0 , produces equation (2-14).

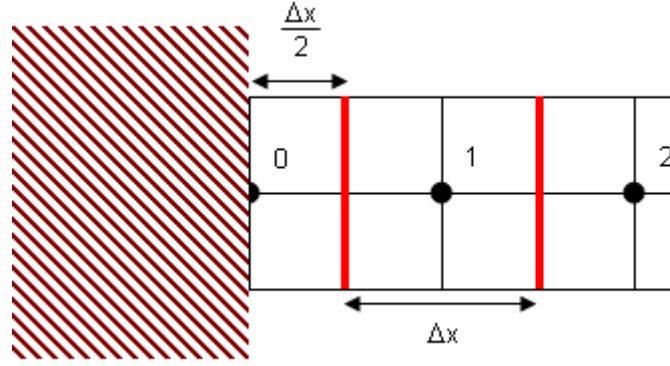


Figure 2-4 – Planar Half-space Finite Element Schematic

$$T_0 = 2T_1 - T_2 + \frac{(\Delta x)^2}{\alpha} \frac{\partial T}{\partial t} \quad (2-14)$$

Frankel's two-thermocouple equation, as given in equation (2-3) is

$$T(0,t) = -T(2\eta,t) + 2T(\eta,t) + \frac{1}{\alpha} \frac{\partial T}{\partial t}(\eta,t)\eta^2 + O(\eta^4) \quad (2-3)$$

Accounting for differing nomenclature, the finite element approach for slab coordinates produces the same equation as Frankel's Taylor series approach. This suggests that the cylindrical finite element analysis, which also includes only three nodes, should provide similar accuracy, despite lower-order truncation error.

Chapter 3 – Analysis Methods

The process used to analyze the test data is presented in this chapter. The first section explains the need for data filtering and how an appropriate filter was applied. The final sections describe the calculations required to apply the equations of Chapter 2. Examples of the calculations are presented to aid the reader in understanding the process, but full results are presented in the next chapter. Discussion of each method individually and in relation to the entire project will be presented in Chapters 5 and 6.

Data Filtering

As mentioned in the Experimental Setup section of Chapter 2, each firing of the combustor resulted in 30,000 data points for each thermocouple. As with any experimental data set, there is some level of noise associated with the measurement. On a representative plot of the temperature data over the entire measurement time, as in Figure 3-1, the noise is hardly noticeable. However, the noise is clearly visible when the scale of the plot is decreased. As can be seen in Figure 3-2, the data over a time range of a tenth of a second clearly illustrates the noise inherent in the data.

Every analysis method presented in Chapter 2 requires an evaluation of the time derivative of this discrete, noisy data. Performing this derivative on the data in Figure 3-1 produces the plot of Figure 3-3. Even in the 10 seconds before the firing, when the temperature can be seen to be nearly constant in Figure 3-1, the time rate of change of the temperature varies between ± 200 °C / s, or roughly 40% of the peak value

To solve the problem of noisy data, an appropriate filter must be applied. The power spectrum of the data, obtained through the use of a Fourier transform of the temperature data, provides information about the bounds required on a filter. Figure 3-4 is the power spectrum of the data presented in Figure 3-1.

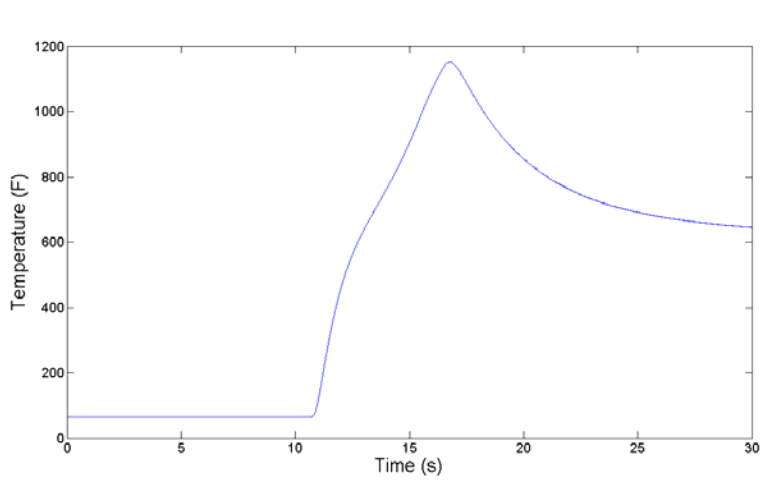


Figure 3-1 – Probe Data at 1/8” from Surface, Full Measurement Time (Run 24, 7 Nov. 2005)

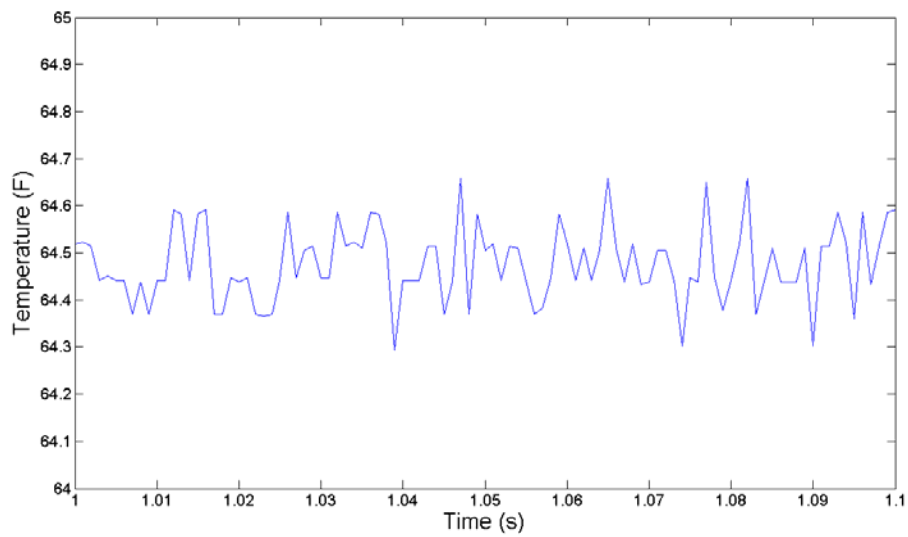


Figure 3-2 – Probe Data at 1/8" from Surface, Small Time Scale (Run 24, 7 Nov. 2005)

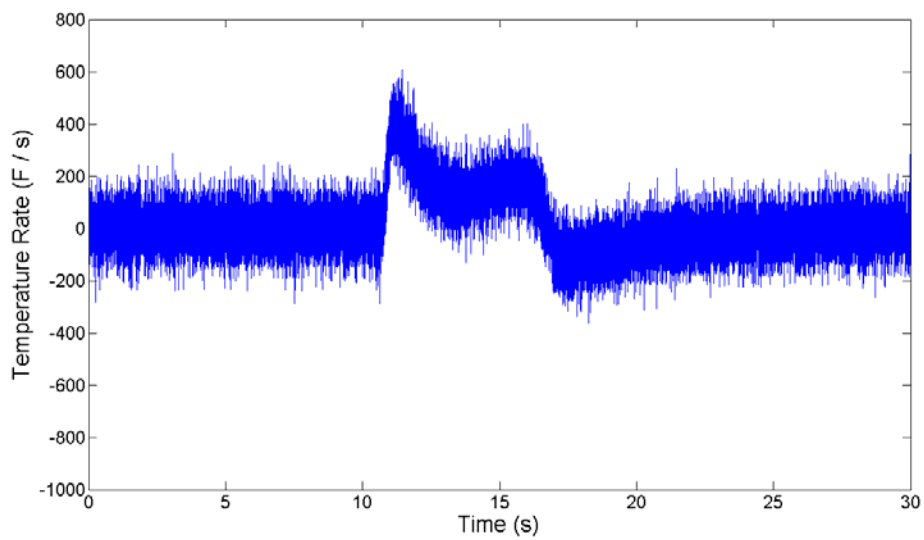


Figure 3-3 – Temperature Rate of Noisy Data at 1/8" from Surface (Run 24, 7 Nov. 2005)

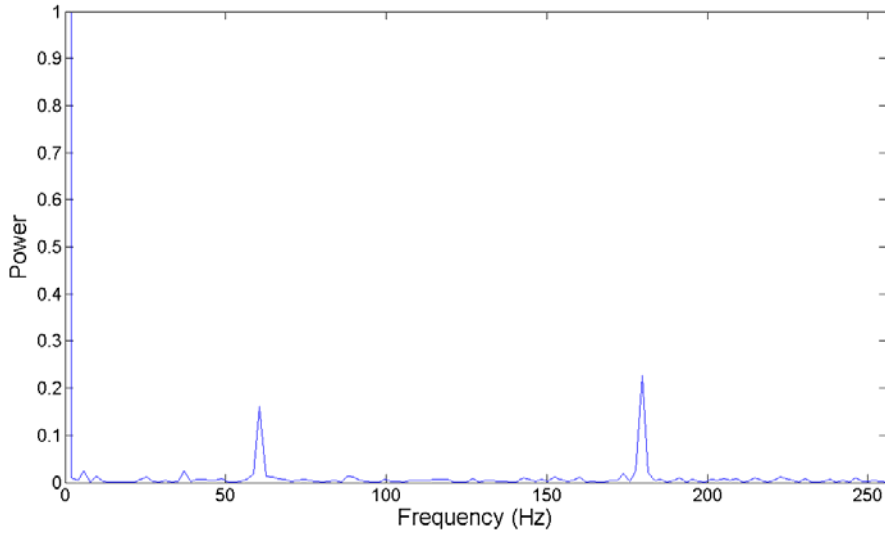


Figure 3-4 – Power Spectrum of Temperature Data at 1/8” from Surface (Run 24, 7 Nov. 2005)

The power spectrum shows a large DC shift with smaller peaks at 60 Hz and 180 Hz. The DC shift is several orders of magnitude larger than either higher frequency peak and is not plotted to full scale in Figure 3-4 in order to illustrate the peaks at higher frequencies. The 60 Hz signal is due to the frequency of the power system used on the experiment. The peak at 180 Hz is a harmonic of that power system, and other, smaller peaks would be seen if the frequency axis were continued past 256 Hz. From the power spectrum, it is clear that the signal is not greatly influenced by frequencies significantly greater than the DC shift. Therefore, a low-pass filter can be applied to the data to remove the high frequency signals without affecting the integrity of the true signal. Frankel suggests a Gaussian low-pass filter for use with his slab analysis [2]. The formula for Frankel’s filter is given as Equation 1. The parameter ω_c is the cutoff angular frequency, or 2π times the cutoff frequency. The cutoff frequency was chosen as 30 Hz in order to balance the removal of high frequency noise while maintaining the lower frequency signal.

$$\bar{T}_{filtered}(t_k) = \frac{\sum_{n=0}^{N-1} T(t_n) e^{-\frac{(t_k - t_n)^2 \omega_c^2}{4}}}{\sum_{n=0}^{N-1} e^{-\frac{(t_k - t_n)^2 \omega_c^2}{4}}} \quad (3-1)$$

Other filters were investigated, such as a combination of a notch filter with a Parks-McClellan differentiator [4]. Preliminary analysis of this combination indicated that it would not provide derivative data sufficiently smooth for use in the equations. The Gaussian filter does reduce the amplitude of all non-zero frequencies in the power

spectrum. However, this reduction is very small for low frequencies and does not significantly affect the data, which is primarily within the DC shift region of the power spectrum.

The Gaussian filter was applied to each data set before any other calculations were made. A representative result of the filter applied to the data is shown in Figure 3-5. The original data is the same as presented in Figure 3-2, while the smoother curve is the result of the filter. A small, low frequency oscillation can still be seen within the data. This small oscillation has little effect on the derivative, however, as can be seen in Figure 3-6. The noise in the derivative is small and will present a problem only if it becomes necessary to take a second derivative in time. Note that Equation (2-2) does require a second time derivative in order to achieve fourth order accuracy. This issue will be addressed later in this chapter when applying Equation (2-4) as a predictor of surface heat flux.

Calculation of $q''(\eta, t)$

A key component of the one-thermocouple equations, as well as the two-thermocouple heat flux equation, is the heat flux at the probe locations. This parameter, which Frankel calls $q''(\eta, t)$ [2], must be evaluated at $x = \eta$ from Equation 2-5.

$$q''(x, t) = \sqrt{\frac{\rho c_p k}{\pi}} \int_{u=0}^t \left(\frac{\partial T}{\partial u}(x, u) \frac{du}{\sqrt{t-u}} \right) \quad (2-5)$$

The first step in evaluating this expression is computing the time derivative of the temperature data. A simple five-point formula [5], given in Equation (3-2), was used for this step.

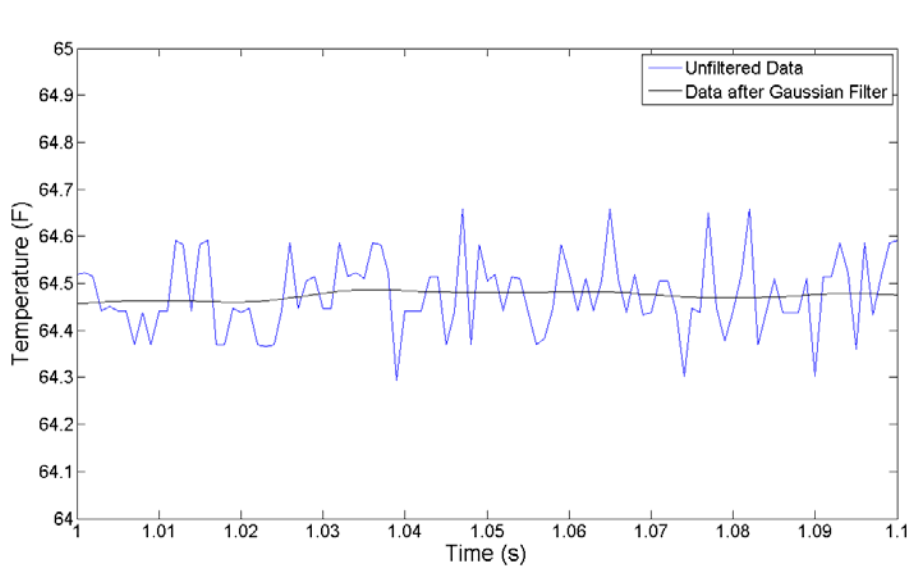


Figure 3-5 – Unfiltered data and filtered data at 1/8" from Surface (Run 24, 7 Nov. 2005)

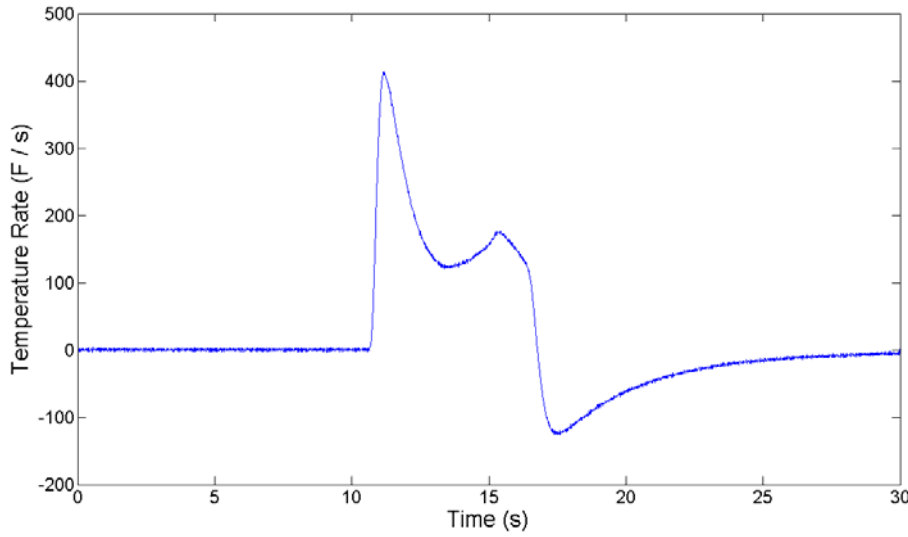


Figure 3-6 – Temperature Rate of Filtered Data at 1/8” from Surface (Run 24, 7 Nov. 2005)

$$f'(t_0) = \frac{1}{12\Delta t} [f(t_0 - 2\Delta t) - 8f(t_0 - \Delta t) + 8f(t_0 + \Delta t) - f(t_0 + 2\Delta t)] + O(\Delta t^4) \quad (3-2)$$

Once the derivative term is evaluated, the integral is calculated. Since the integrand in Equation (2-5) is singular at the limit $u = t$ (due to the square root term in the denominator), the simplest method of evaluating the expression was to use a simple left-hand summation. Using the left-hand term avoids the singularity which exists only at the right end of the summation. Higher-order methods would require an evaluation at that singularity. Based on this method, the integral was rewritten as Equation (3-3) [2].

$$q''(x, u_j) = \sqrt{\frac{\rho c_p k \Delta u}{\pi}} \sum_{i=1}^{j-1} \frac{\frac{\partial T}{\partial u}(x, u_i)}{\sqrt{j-i}} \quad (3-3)$$

The integration (or summation) of the slightly noisy temperature rate data has the effect of smoothing the heat flux curve, as evidenced by Figure 3-7.

Calculation of Surface Temperature and Heat Flux

One Thermocouple Equation

With the quantity q'' now defined, it is possible to move to methods of estimating the surface conditions. The first of these methods requires data from only one thermocouple embedded within the test material [2]. There are several advantages to this method. It

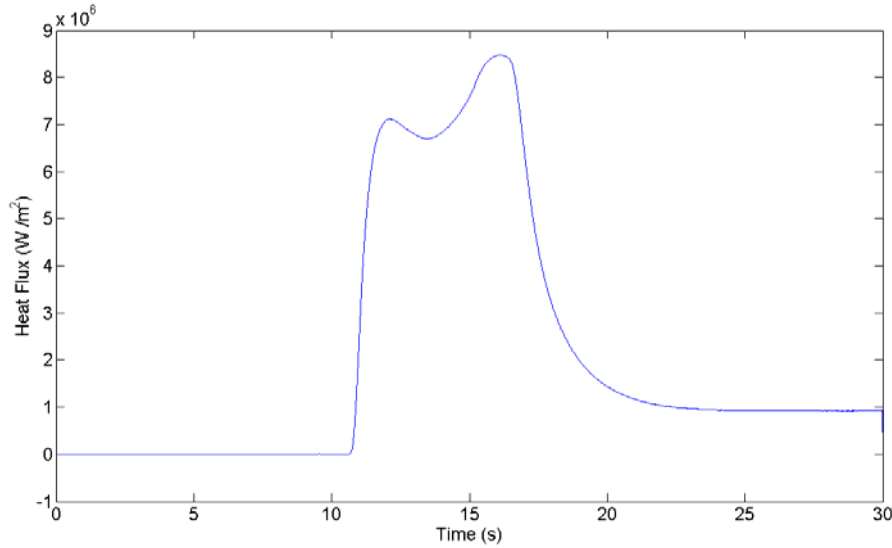


Figure 3-7 – Heat Flux at Embedded Location, 1/8” from Surface (Run 24, 7 Nov. 2005)

requires the minimum amount of data to provide an initial estimate for surface conditions. Applying this method to two adjacent probes, it is possible to compare the extrapolated or predicted temperature of one probe to the data from the other probe, which provides a standard of comparison of the estimate. The primary disadvantage with regard to this experiment is that the analysis formulation is in slab rather than cylindrical coordinates.

A key parameter in all heat transfer is the thermal diffusivity, α . It can be shown that the thermal diffusivity is the only parameter involved in the one thermocouple temperature equation, Equation (2-1). The coefficient on the integral of Equation (2-5) can be rewritten by substituting from the definition of thermal diffusivity, resulting in Equation (3-4).

$$q''(x,t) = \frac{k}{\sqrt{\alpha\pi}} \int_{u=0}^t \left(\frac{\partial T}{\partial u}(x,u) \frac{du}{\sqrt{t-u}} \right) \quad (3-4)$$

The thermal conductivity in the numerator of Equation (3-4) cancels the same term in the denominator of the q'' terms of Equation (2-1), leaving a surface temperature equation dependent on only one parameter, the thermal diffusivity. The surface heat flux equation represented by the expansion in Equation (2-2) still requires both thermal conductivity and diffusivity.

The nature of the nozzle material, graphite, makes both the thermal diffusivity and thermal conductivity difficult to determine. Published values of thermal diffusivity range by as much as a factor of 10 and often depend on orientation of the graphite. Both parameters depend on the temperature of the material, which changes over the course of a test run. The equations presented in this thesis, however, are derived for constant

properties. For the purposes of this analysis, average values of the thermal diffusivity and thermal conductivity were chosen. Thermal conductivity was set at 120 W / (m K) and thermal diffusivity was assumed as $4\text{E-}5 \text{ m}^2 / \text{s}$. These values are based on data collected and evaluated by Mr. Robert Rhodes [6]. His estimation method involved matching the temperature change between the temperature probes to an estimate of the surface temperature, and he manually iterated on an estimate for thermal diffusivity to achieve an optimal match between the temperature data from the probes and the prediction of his method. The thermal diffusivity and conductivity associated with Rhodes' best fit was used in the analysis presented in this thesis.

Since Equation (2-1) is a Taylor expansion in Cartesian coordinates, it applies for any linear change of size η . This allowed the equation to be applied using the data from one embedded probe to predict the temperature at the other probe location, with the assumption that the rectangular geometry would not produce very different results from the measured values at the cylindrical radius of interest. Since there is data from the probe at the second location, the prediction can be compared with the data associated with that location to provide an estimate of the effect of radius of curvature on the slab geometry assumption. The comparison at the throat is shown in Figure 3-8. Performing the same analysis at the exit plane of the nozzle results in the curve shown in Figure 3-9. It is evident that the slab assumption is better suited to the analysis at the larger radius associated with the nozzle exit plane. Further discussion of the error associated with these estimates will be presented in Chapter 5.

Comparisons of the predicted temperatures to actual data are helpful in judging the overall accuracy of the method, but the goal is to predict the surface temperatures. Equation (2-1) can be applied directly to the data from the probe embedded at a location η (1/8") to predict surface conditions. The truncation error associated with Equation (2-1) depends on η^4 , so selecting the probe closer to the surface should produce a smaller truncation error. Figure 3-10 shows the prediction of the surface temperature based on the 1/8" probe.

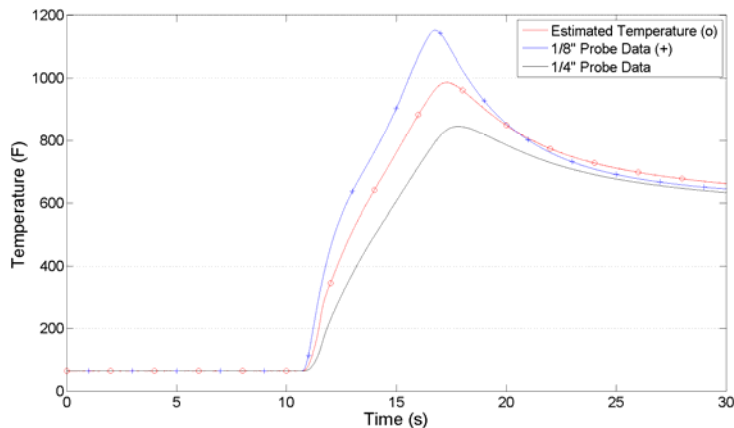


Figure 3-8 – One Thermocouple Estimate of Probe Data at Nozzle Throat (Run 24, 7 Nov. 2005)

It is also possible to predict the surface temperature using the 1/4" probe data, though the truncation error is increased significantly. Figure 3-11 shows this estimate along with the 1/8" probe prediction. It is obvious from Figure 3-11 that the estimate using the 1/4" probe underestimates the surface temperature since it barely exceeds the 1/8" probe data. The use of slab analysis also contributes to the discrepancy in the predicted throat surface temperatures. A further discussion of these results can be found in Chapter 5.

Heat flux can be estimated in the same way as temperature. Instead of comparing the Taylor series prediction to data at the 1/8" probe location, though, the comparison is made between the surface heat flux given by the Taylor series approximation, Equation (2-2), calculated at 1/8", to the values calculated using the integral relation given by Equation (2-5). The curves for the 1/8" location heat flux for the throat and exit planes are given in Figure 3-12 and 3-13, respectively.

These comparisons suggest that the one thermocouple planar half-space equations offer better accuracy at the exit plane, where the radius of the cylinder is larger. The smaller radius at the throat has a much greater impact on the error associated with the assumption of a planar half-space.

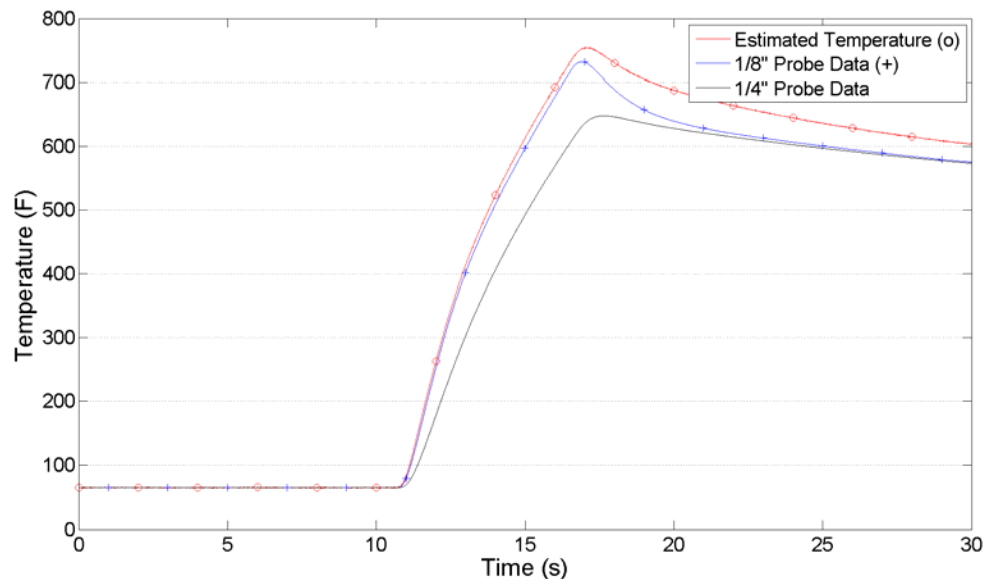


Figure 3-9 – One Thermocouple Estimate of Probe Data at Nozzle Exit (Run 24, 7 Nov. 2005)

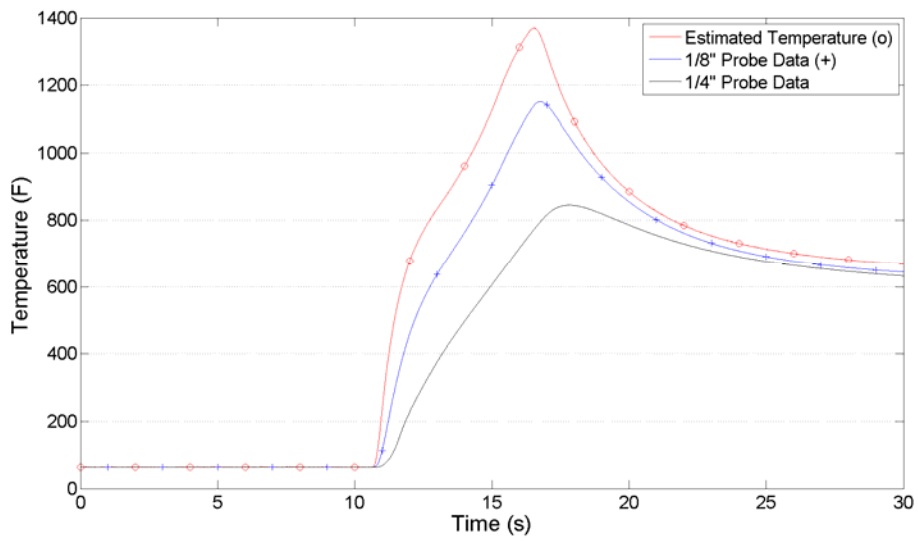


Figure 3-10 – Surface Temperature Estimate Based on 1/8" Probe – Throat (Run 24, 7 Nov. 2005)

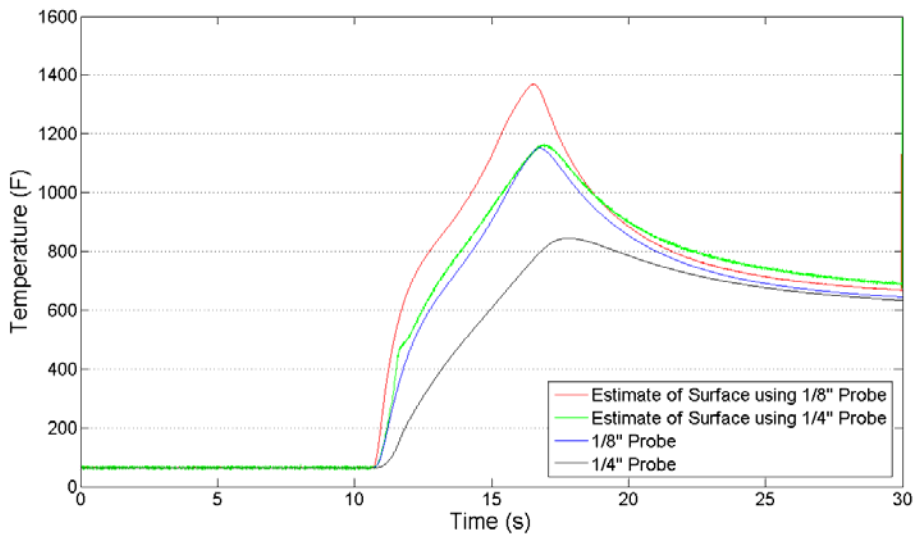


Figure 3-11 – Surface Temperature Prediction Based on Each Probe – Throat (Run 24, 7 Nov. 2005)

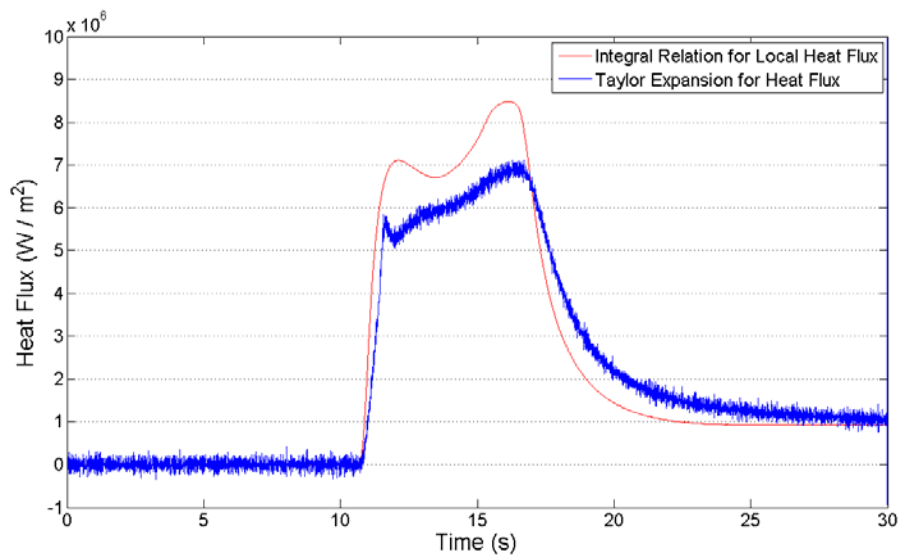


Figure 3-12 – Heat Flux at 1/8" Probe Location – Throat (Run 24, 7 Nov. 2005)

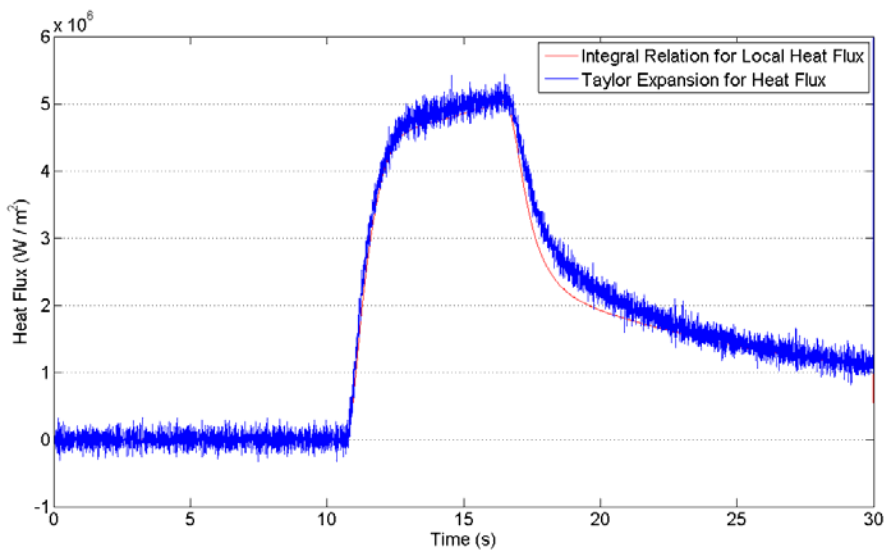


Figure 3-13 – Heat Flux at 1/8" Probe Location – Exit (Run 24, 7 Nov. 2005)

As mentioned in Chapter 2, Equation (2-2) requires the second derivative of temperature with respect to time. Applying the filter to the data results in a smoother rate curve (Figure 3-6 compared to Figure 3-3) but does not remove all of the noise. It is the remaining noise which causes the Taylor expansion heat flux curves of Figures 3-12, 3-13, 3-14, and 3-15 to be jagged. Additional filtering could remove this noise and smooth the heat flux curves. However, the filtering process is computationally inefficient, and the time spent on additional filtering may not result in substantially better results, especially considering the availability of other analysis methods which do not require the noisy second derivative. Further judgments and discussions will be made in the Chapter 5.

Two Thermocouple Equation

The second method for surface temperature and heat flux estimation is based on using data from two equally-spaced embedded thermocouples. As mentioned in Chapter 2, this physical arrangement offers simplifications over the one thermocouple equation and produces an approximation which requires only first-order rate data. Equation (2-3) is used to predict surface temperatures while Equation (2-4) estimates surface heat flux. The surface temperature estimate at the throat for this method is shown in Figure 3-14. The corresponding heat flux is shown in Figure 3-15. Surface temperature and heat flux for the exit plane are given in Figures 3-16 and 3-17, respectively.

Comparisons of these results with the one thermocouple results, associated errors, and possible improvements will be made in Chapter 5.

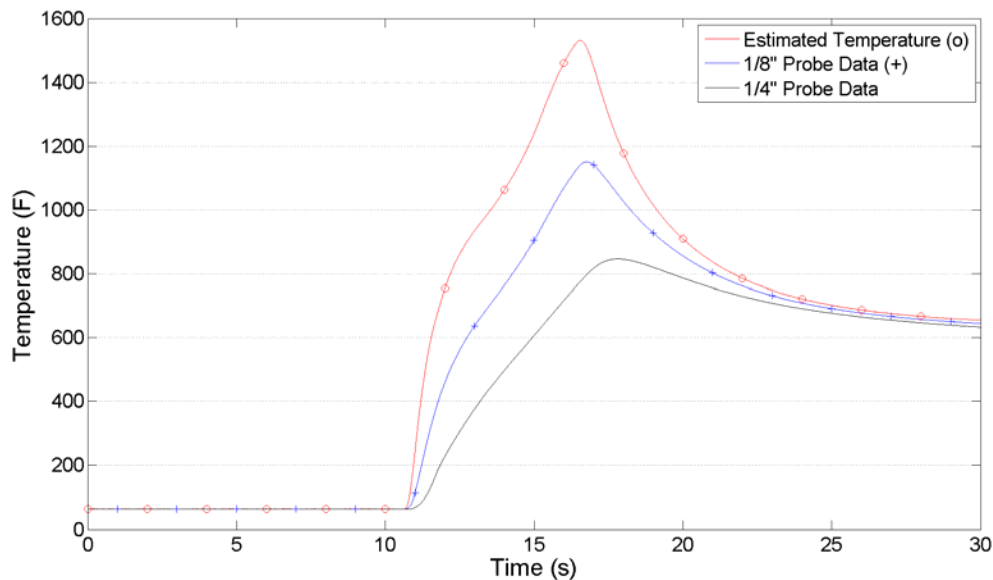


Figure 3-14 – Two Thermocouple Surface Temperature Estimate – Throat (Run 24, 7 Nov. 2005)

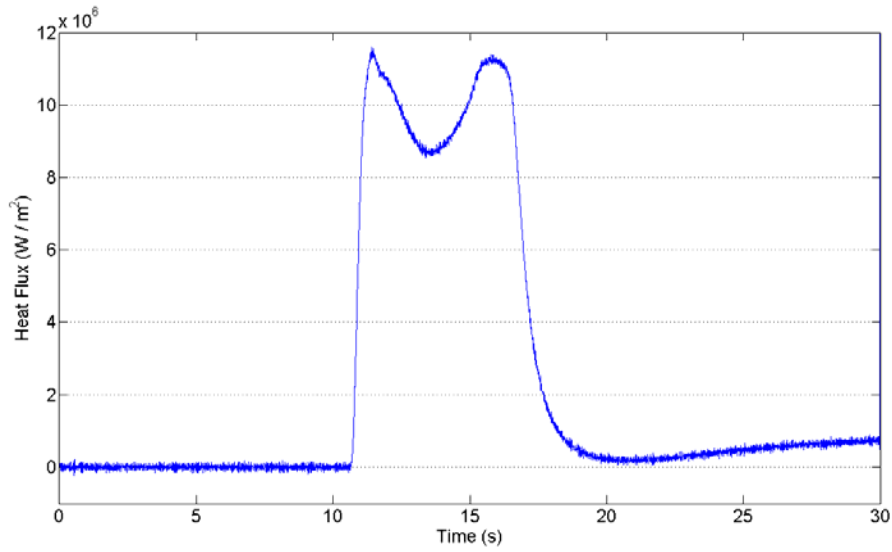


Figure 3-15 – Two Thermocouple Surface Heat Flux Estimate – Throat (Run 24, 7 Nov. 2005)

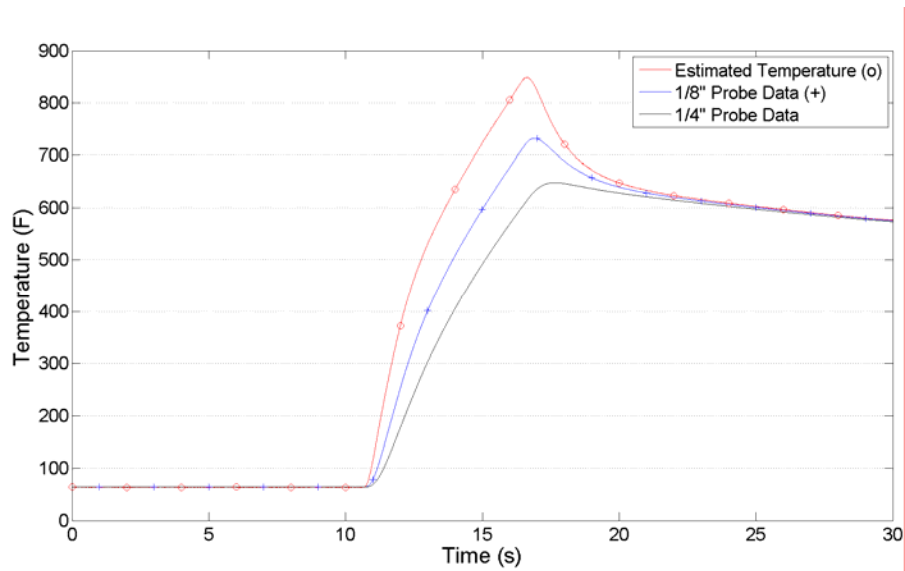


Figure 3-16 – Two Thermocouple Surface Temperature Estimate – Exit (Run 24, 7 Nov. 2005)

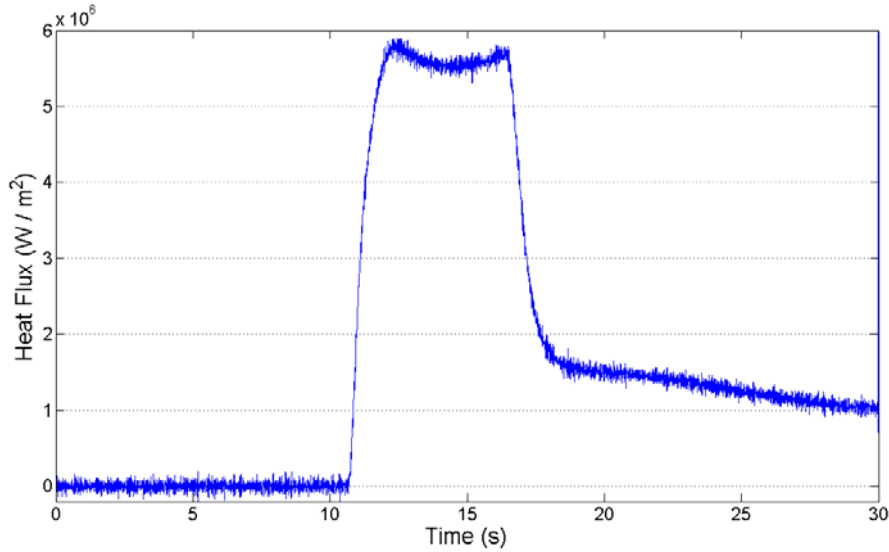


Figure 3-17 – Two Thermocouple Surface Heat Flux Estimate – Throat (Run 24, 7 Nov. 2005)

Cylindrical Finite Element

The final estimation method used for this analysis was the simple three-point finite element approach, Equation (2-9), to predict the temperature at the surface. The surface heat flux is approximated using Equation (2-10) and a three-point derivative formula, Equation (3-5). While not as accurate as the five-point formula used for the time derivatives, the three-point derivative formula [5] requires only information that has already been determined – temperature data at each node.

$$f'(r_0) = \frac{1}{2\Delta r} [-3f(r_0) + 4f(r_0 + \Delta r) - f(r_0 + 2\Delta r)] + O(\Delta r^2) \quad (3-5)$$

Applying this method at the plane of the nozzle throat produced the surface temperature estimate shown in Figure 3-18 and the surface heat flux prediction of Figure 3-19. Surface temperature and heat flux at the exit plane are shown in Figures 3-20 and 3-21, respectively.

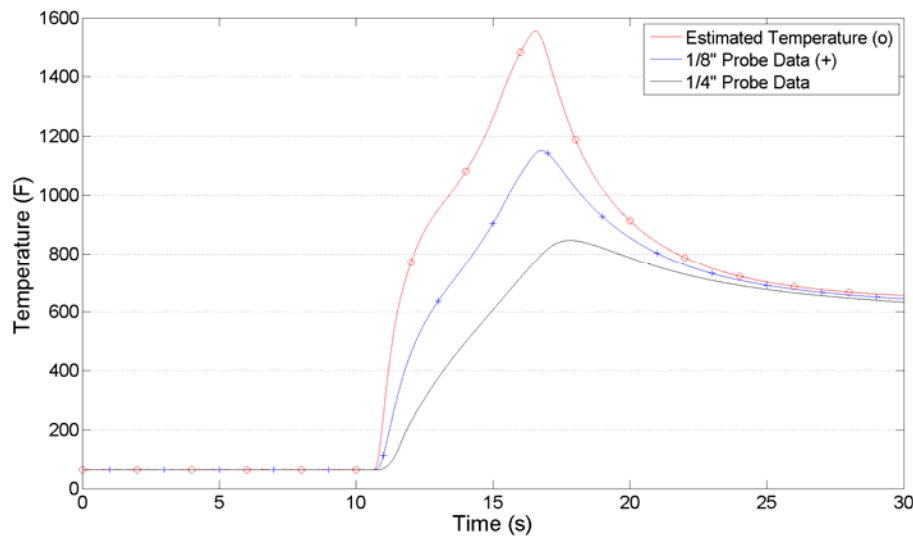


Figure 3-18 – Finite Element Surface Temperature Estimate – Throat (Run 24, 7 Nov. 2005)

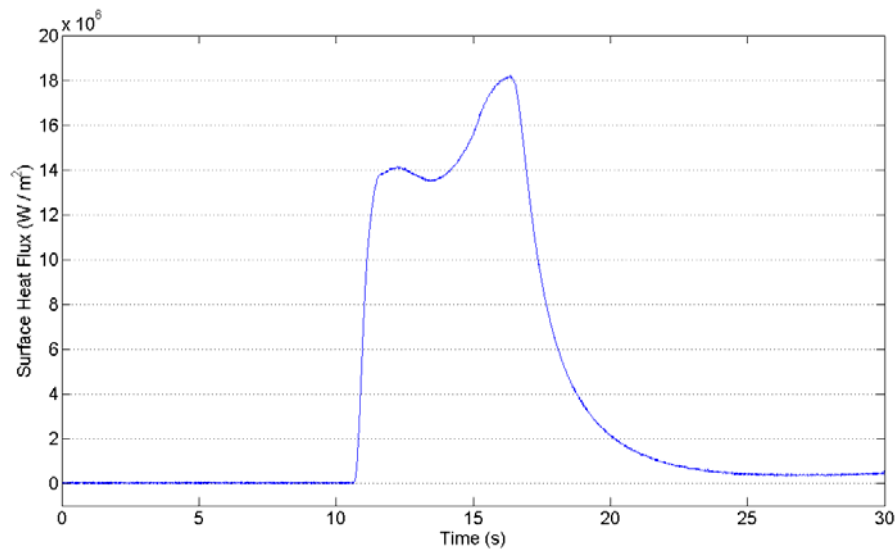


Figure 3-19 – Finite Element Surface Heat Flux Estimate – Throat (Run 24, 7 Nov. 2005)

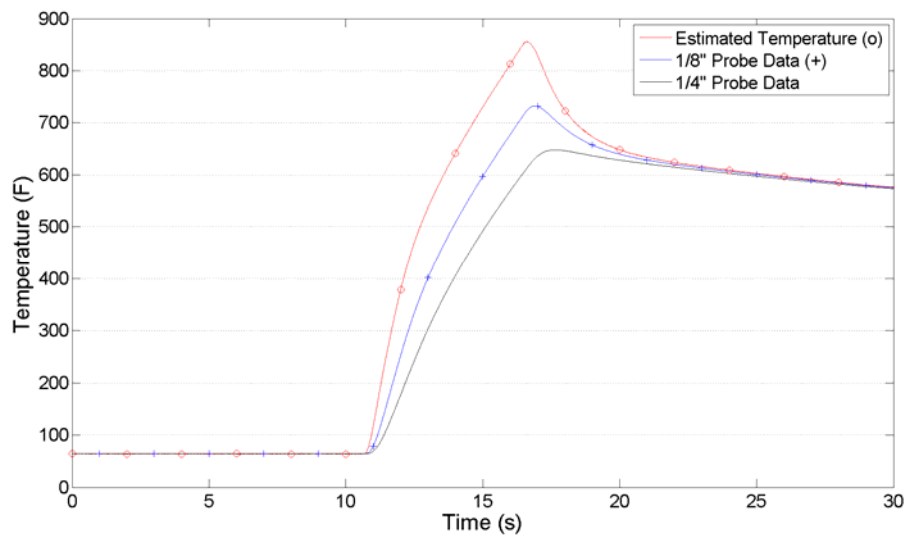


Figure 3-20 – Finite Element Surface Temperature Estimate – Exit (Run 24, 7 Nov. 2005)

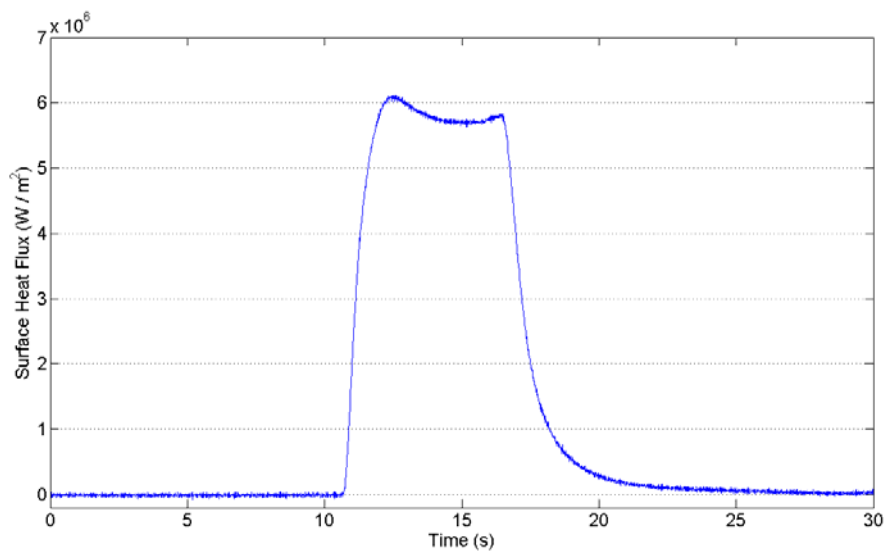


Figure 3-21 – Finite Element Surface Heat Flux Estimate – Exit (Run 24, 7 Nov. 2005)

Sensitivity Analysis

As mentioned earlier in this chapter, published values of the thermal diffusivity of graphite can vary over a wide range. While the value of the thermal diffusivity used in this analysis is consistent with results from other analyses of the same data, it is important to determine the sensitivity of the equations to this parameter. To determine the sensitivity, the value of the thermal diffusivity was changed and the entire analysis program was run for the new value. Two types of changes were made: a logarithmic or order of magnitude change in thermal diffusivity and a linear change. These results are presented in the final section of Chapter 4.

Discussion of all results can be found in Chapter 5.

Chapter 4 – Results

The objective of this chapter is to present representative charts, graphs, and tables from the analysis of data relating to each selected test firing. The data sets are grouped by propellant chemistry, and the results of one run from each group are presented. Discussion of the results will be held until Chapter 5.

Once the analysis process had been defined, it was applied to all of the data sets. As mentioned in Chapter 2, a total of 31 test runs were considered for the primary analysis. Table 4-1 lists these test runs, date of firing and propellant additive percentages. The additive percentages are defined separately because of the nature of the combustion chemistry. The aluminum is considered a fuel additive, so its percentage is defined as the ratio of the mass of aluminum to the fuel mass, which is the sum of the mass of aluminum and the mass of the jet fuel. The potassium is not considered to be a part of the fuel, but rather an additive to the flow as whole, so its percentage is defined as the ratio of the mass of potassium to the total flowing mass, including fuel, oxidizer, and all other additives. The purpose of these additives is to alter the combustion temperature and the electrical conductivity of the gas to make it a plasma.

There are five total propellant combinations, though only 3 occur a significant number of times within the sample set. The combinations containing 2% potassium (K) with 10% and 25% aluminum (Al) were run only twice within the sample set, Runs 34 and 35 and Runs 38 and 39, respectively. In addition, Runs 50 and 51 include only a 2-second burn time, rather than the 5-second burn used for the other tests. From the standpoint of calculating the surface temperature using the methods described in Chapter 2, these differences have no effect on the analysis. They may, however, illustrate strengths and weaknesses among the methods. A full discussion of any such judgments will be presented in Chapter 5.

A representative case was selected for each propellant combination to be presented in detail in this thesis. From the group of 2% K and 0% Al (Group 1), Run 24 was selected. Run 30 was chosen from Group 2 (0% K, 0% Al), and Run 36 was used from Group 3 (2% K, 15% Al). Each of these cases is simply the first run for each propellant combination. For the sensitivity analysis, only Run 24 is presented. The trends evident from this single case apply to each of the representative cases.

Detailed explanations of the methods used to predict surface temperature and heat flux have been presented in Chapters 2 and 3. Chapter 2 explains the mathematical models used for each estimate type, and Chapter 3 follows a sample analysis which was then applied to each data set considered in this chapter. The following figures (4-1 through 4-96), 12 for each axial location of each run plus sensitivity analysis, provide the basis for comparison and discussion in Chapter 5.

Table 4-1 – Test Firing Dates and Chemistry

Run #	Date	K (%)	Al (%)
24*	11-07-05	2	0
25	11-14-05	2	0
26	12-01-05	2	0
27	12-08-05	2	0
28	12-12-05	2	0
29	12-14-05	2	0
30*	01-06-06	0	0
32	01-12-06	0	0
33	01-25-06	0	0
34	01-31-06	2	10
35	02-02-06	2	10
36*	02-15-06	2	15
37	02-24-06	2	15
38	03-17-06	2	25
39	03-30-06	2	25
40	03-30-06	2	15
41	04-19-06	0	0
42	04-20-06	0	0
43	05-16-06	0	0
44	05-26-06	0	0
45	05-31-06	0	0
46	06-12-06	0	0
48	06-28-06	0	0
49	07-07-06	0	0
50	07-13-06	0	0
51	07-13-06	0	0
72	02-27-07	2	0
73	03-02-07	2	0
74	03-12-07	2	0
75	03-16-07	2	15
76	03-19-07	2	15

* Runs analyzed

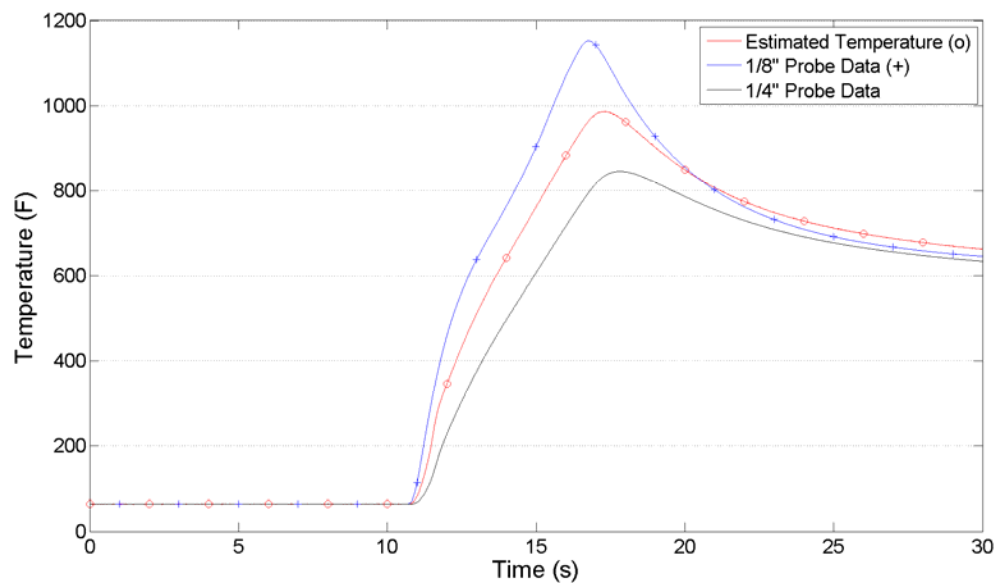


Figure 4-1 – One Thermocouple Estimate – 1/4" Probe Predicting 1/8" Temperature (Throat, Run 24, 7 Nov. 2005)

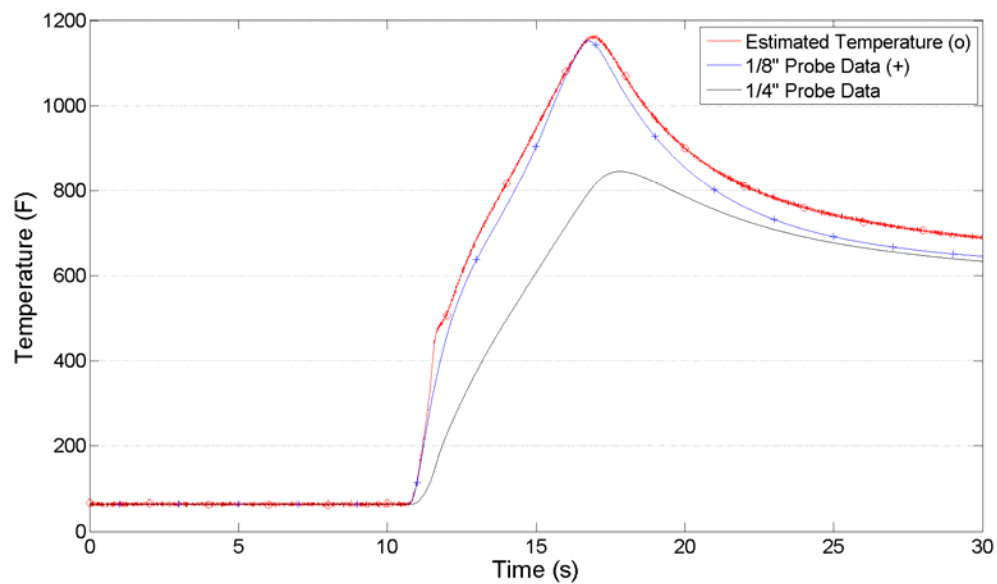


Figure 4-2 – One Thermocouple Estimate – 1/4" Probe Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

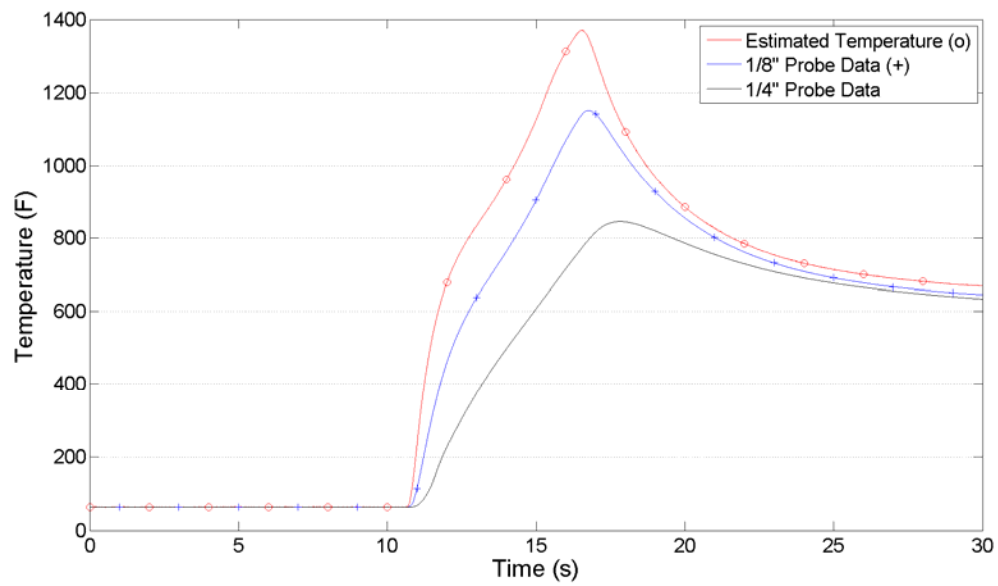


Figure 4-3 – One Thermocouple Estimate – 1/8" Probe Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

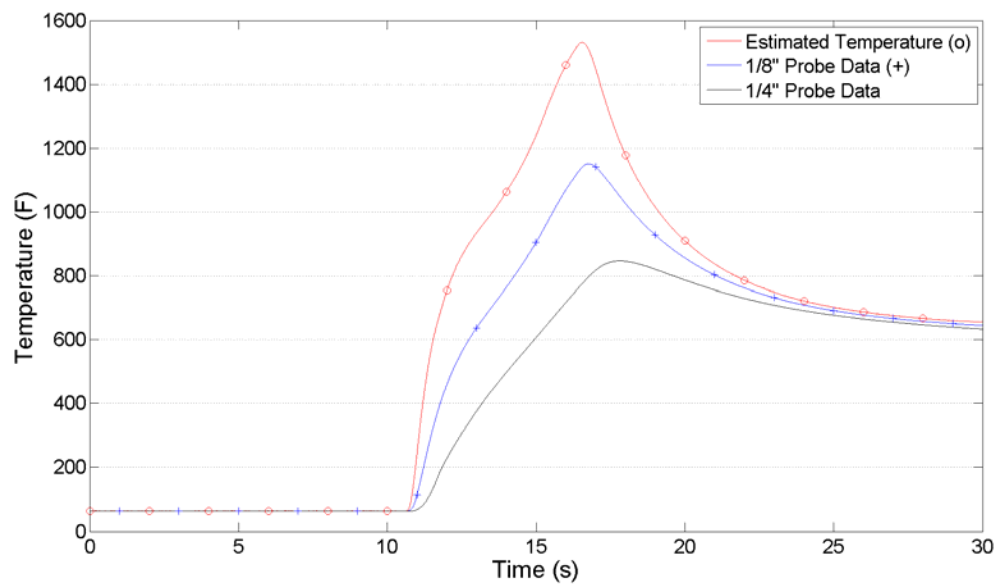


Figure 4-4 – Two Thermocouple Estimate – Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

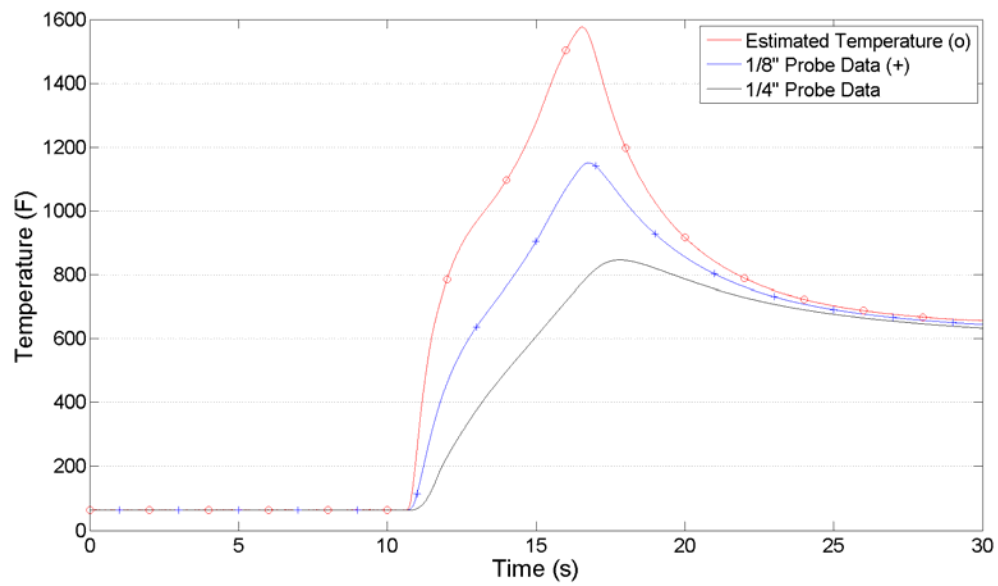


Figure 4-5 – Finite Element Estimate – Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

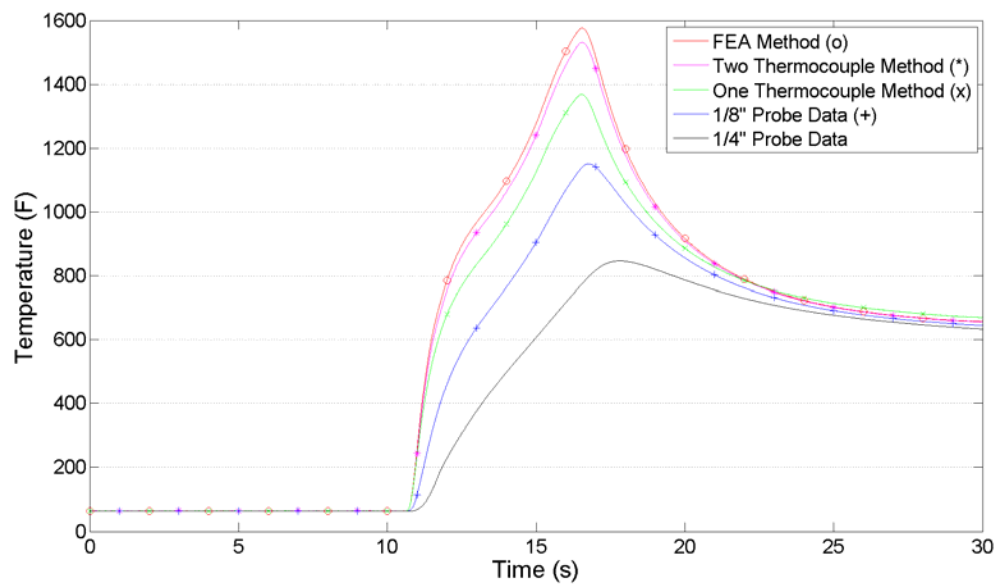


Figure 4-6 – Comparison of Estimate Methods – 1/8" Probe, Two Probe, and FEA Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

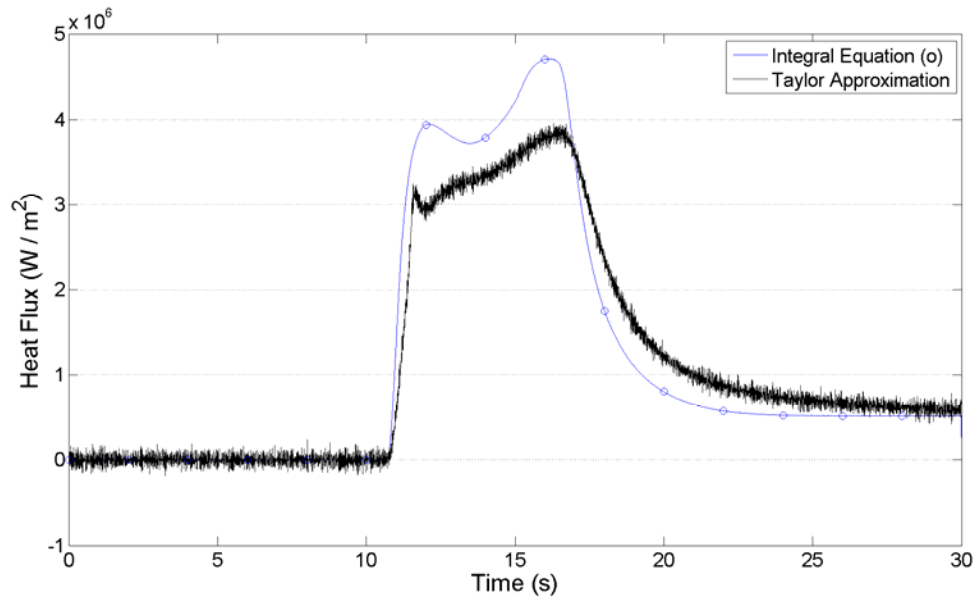


Figure 4-7 – One Thermocouple Estimate – 1/4” Probe Predicting 1/8” Heat Flux (Throat, Run 24, 7 Nov. 2005)

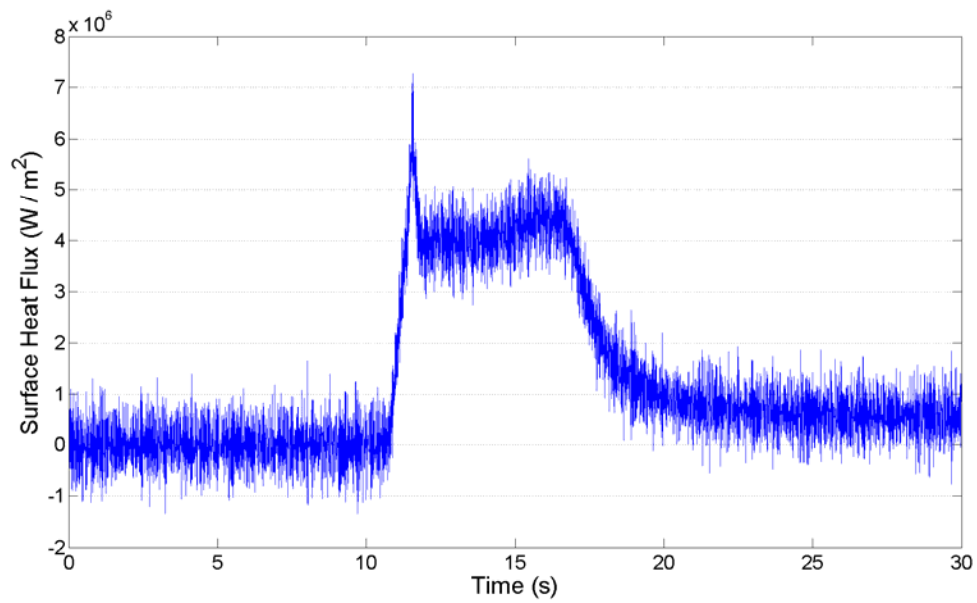


Figure 4-8 – One Thermocouple Estimate – 1/4” Probe Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

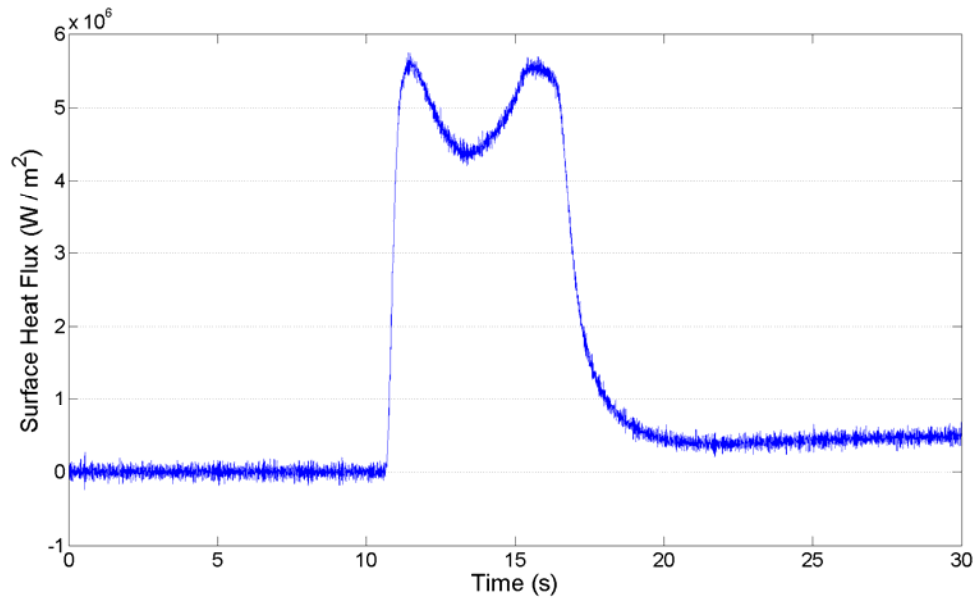


Figure 4-9 – One Thermocouple Estimate – 1/8" Probe Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

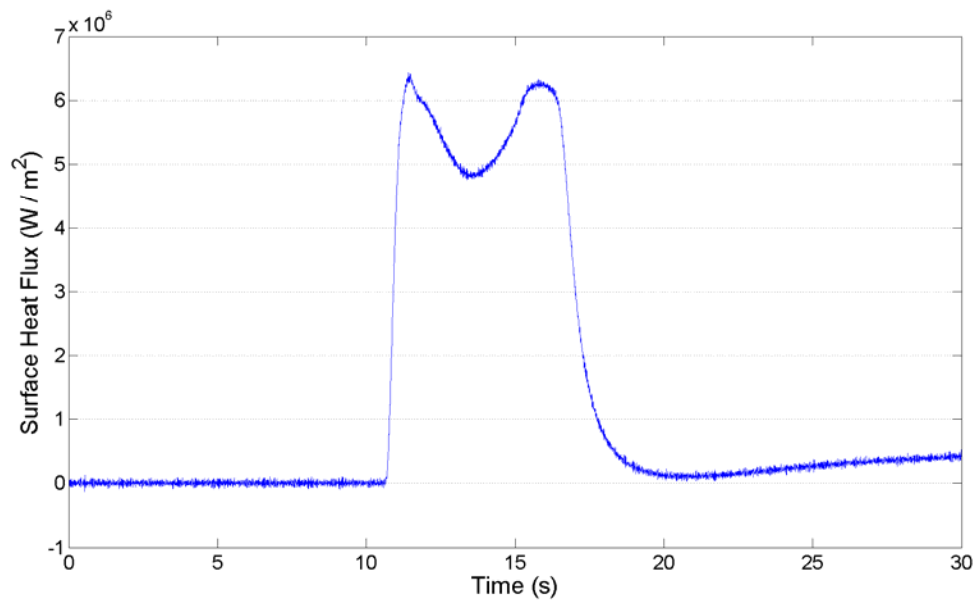


Figure 4-10 – Two Thermocouple Estimate – Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

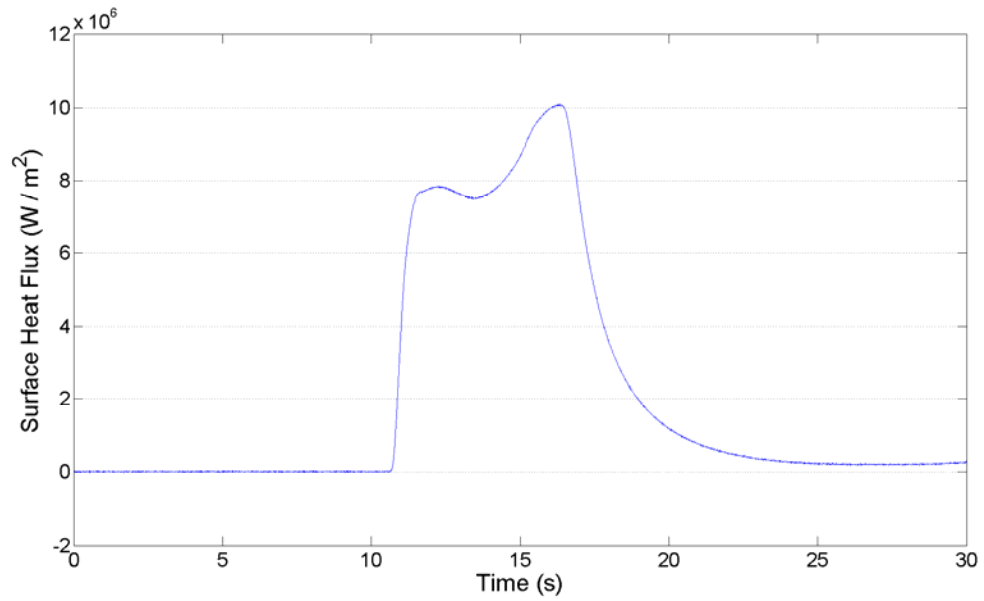


Figure 4-11 – Finite Element Estimate – Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

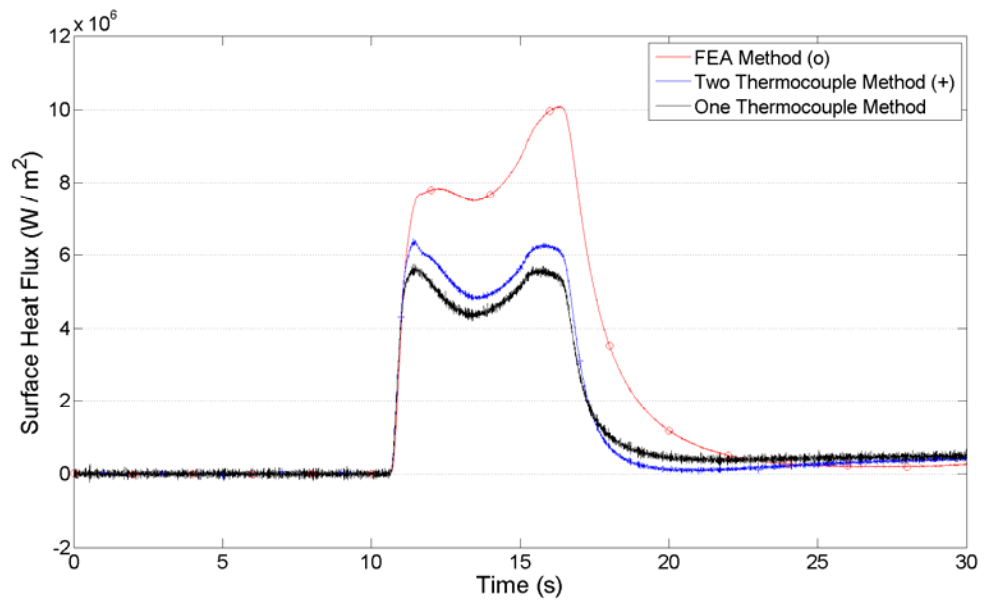


Figure 4-12 – Comparison of Estimate Methods – 1/8” Probe, Two Probe, and FEA Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

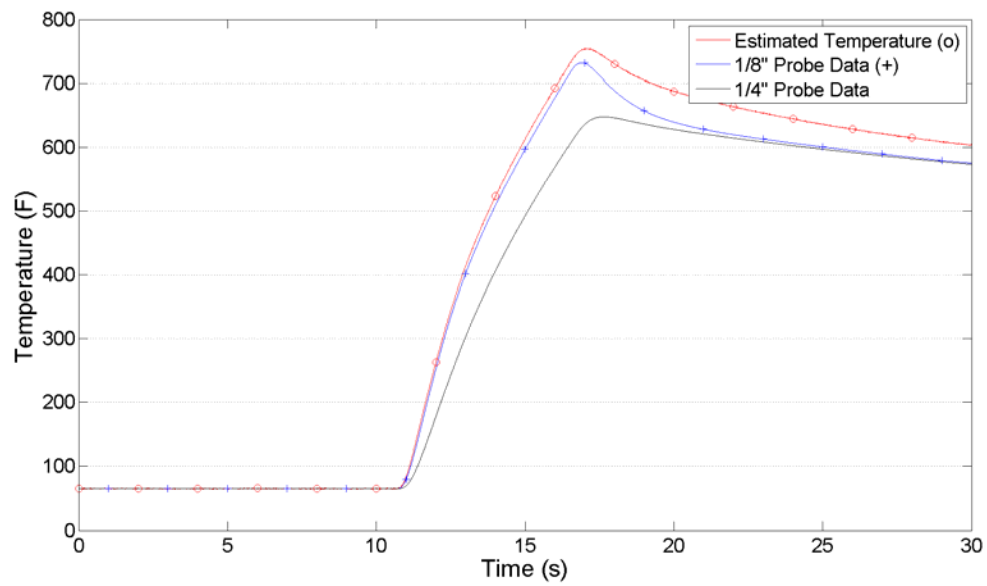


Figure 4-13 – One Thermocouple Estimate – 1/4” Probe Predicting 1/8” Temperature (Exit, Run 24, 7 Nov. 2005)

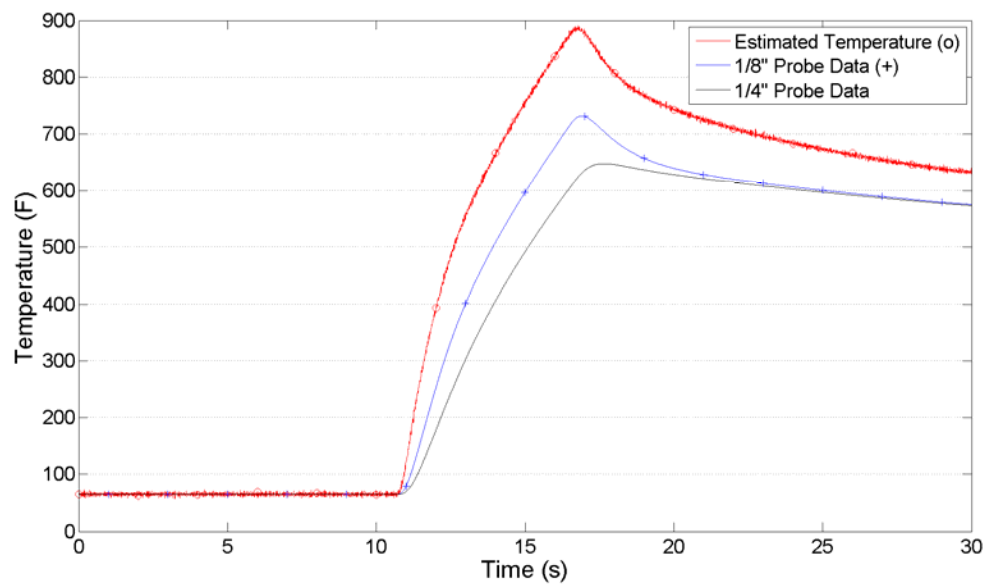


Figure 4-14 – One Thermocouple Estimate – 1/4” Probe Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

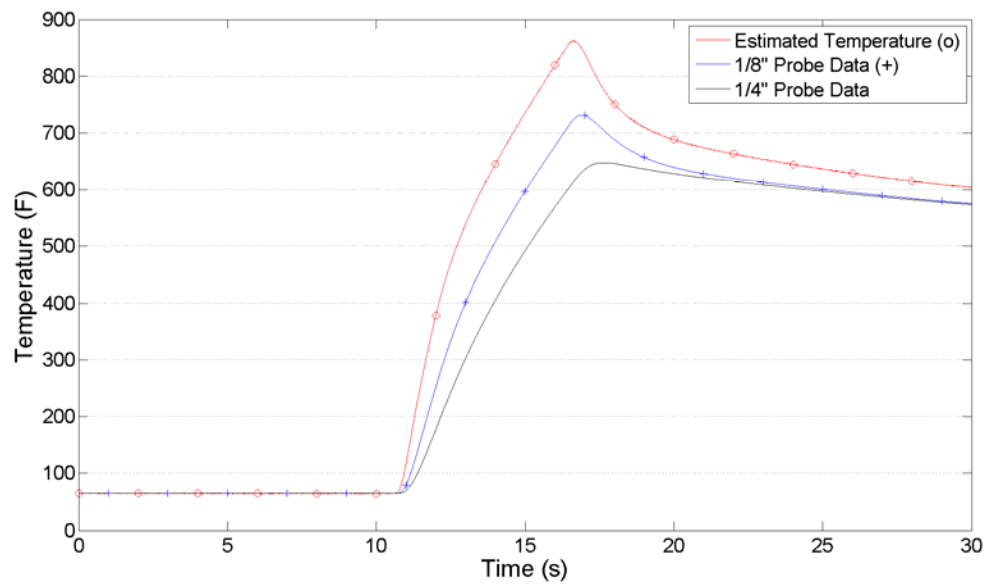


Figure 4-15 – One Thermocouple Estimate – 1/8" Probe Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

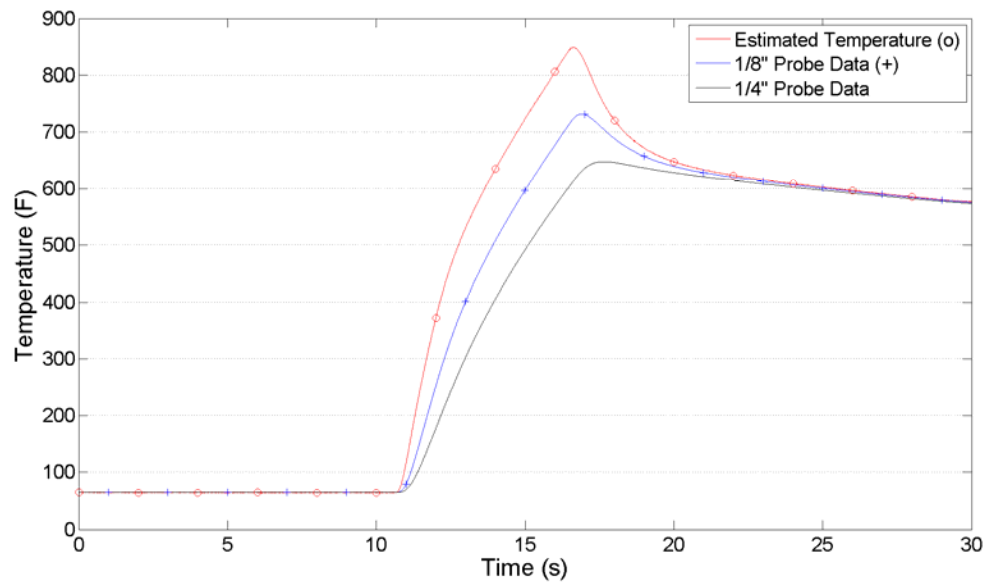


Figure 4-16 – Two Thermocouple Estimate – Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

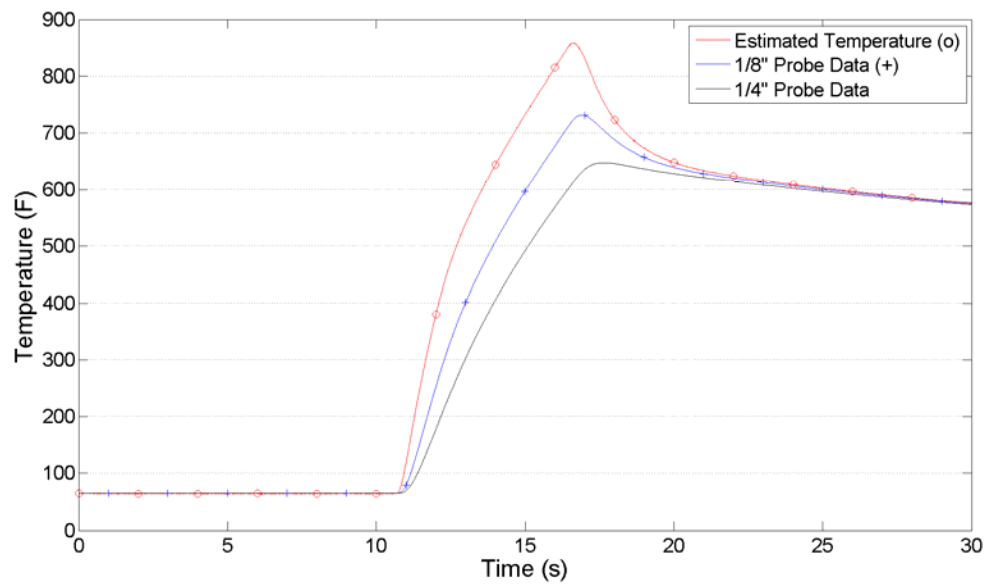


Figure 4-17 – Finite Element Estimate – Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

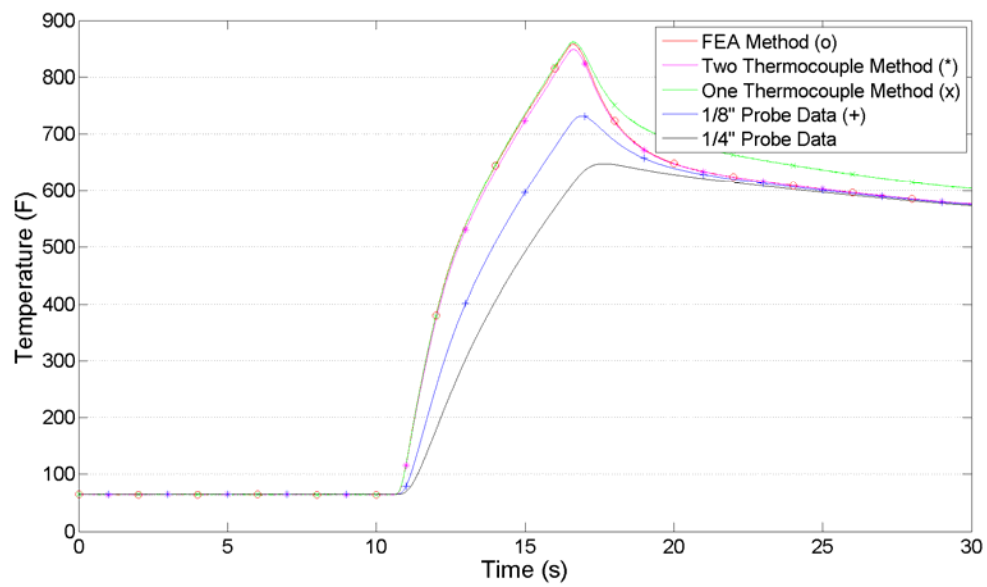


Figure 4-18 – Comparison of Estimate Methods – 1/8" Probe, Two Probe, and FEA Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

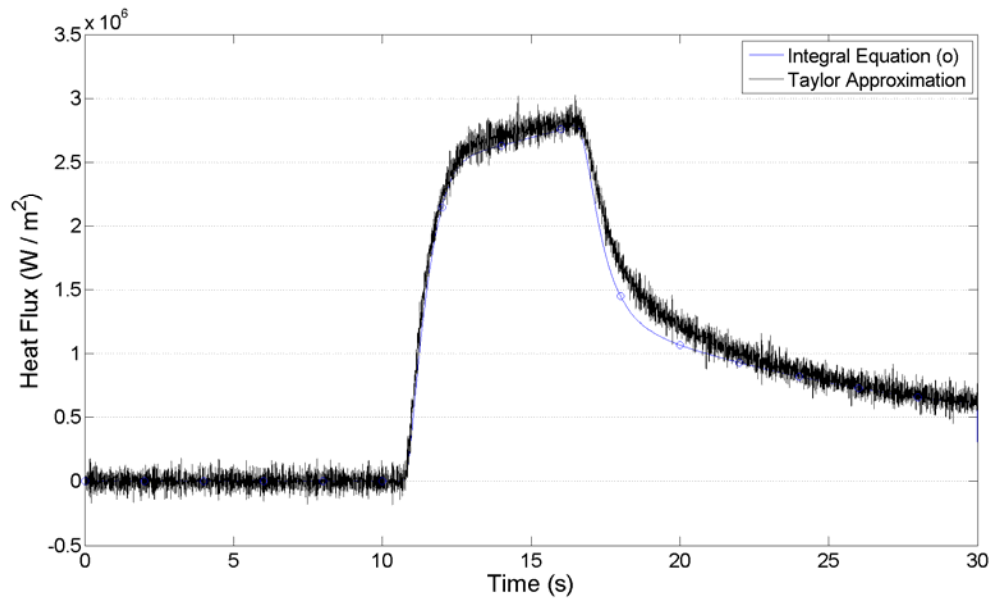


Figure 4-19 – One Thermocouple Estimate – 1/4” Probe Predicting 1/8” Heat Flux (Exit, Run 24, 7 Nov. 2005)

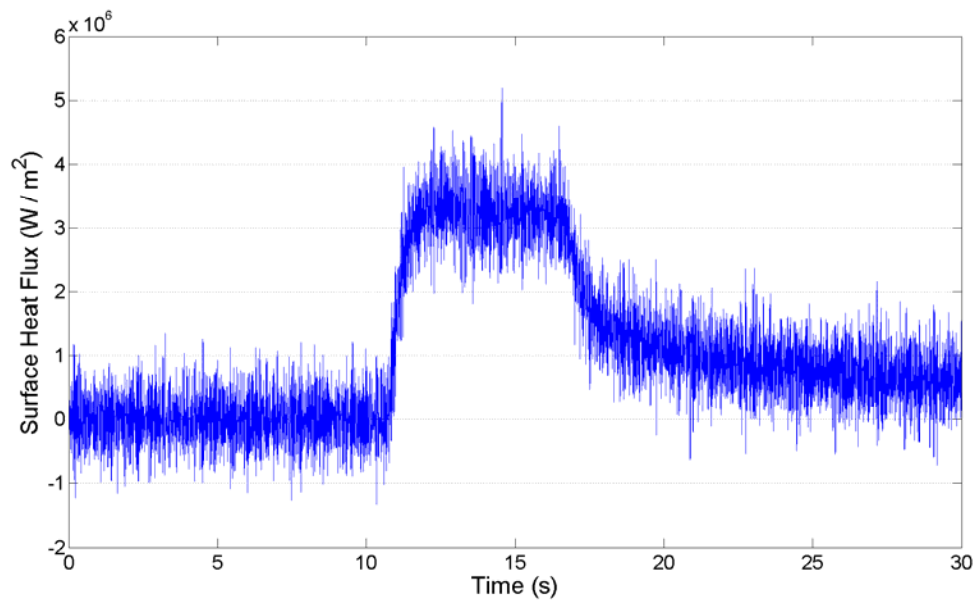


Figure 4-20 – One Thermocouple Estimate – 1/4” Probe Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

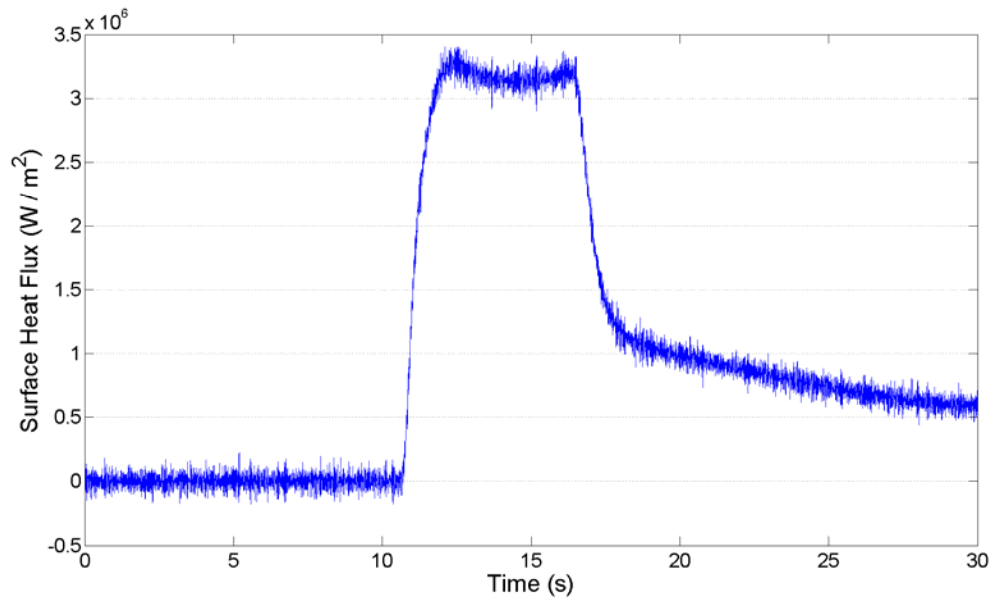


Figure 4-21 – One Thermocouple Estimate – 1/8” Probe Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

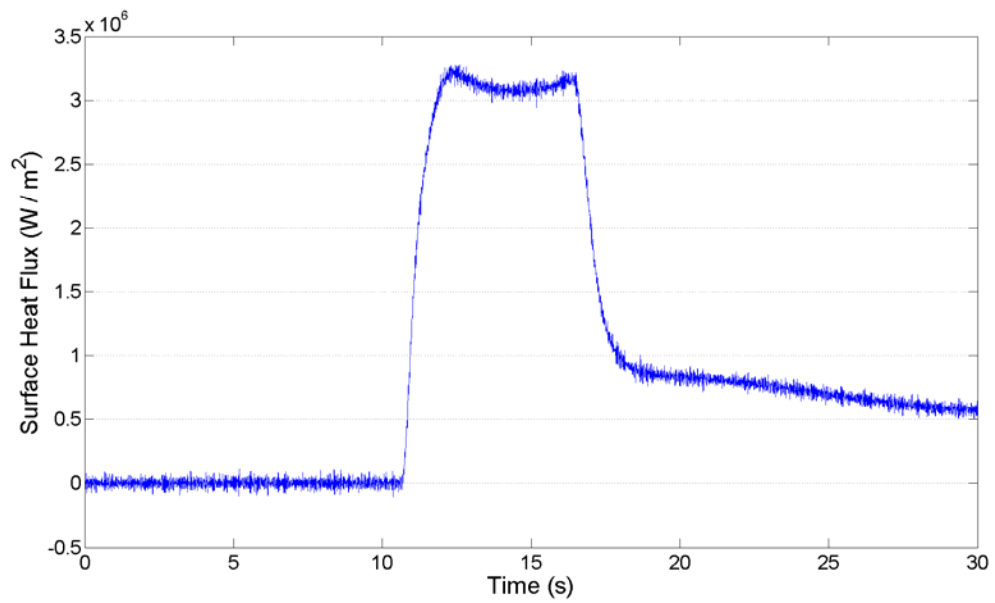


Figure 4-22 – Two Thermocouple Estimate – Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

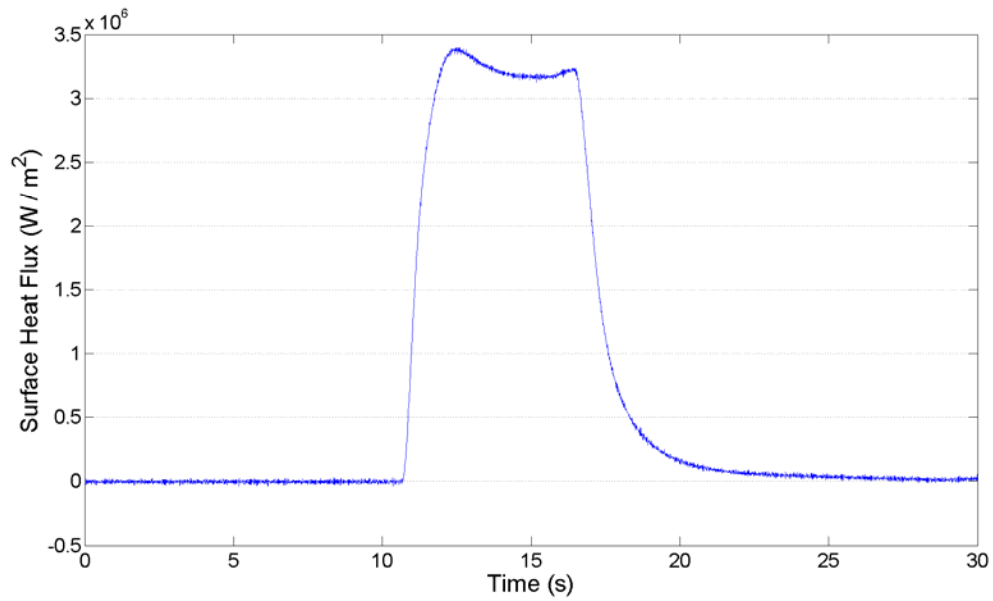


Figure 4-23 – Finite Element Estimate – Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

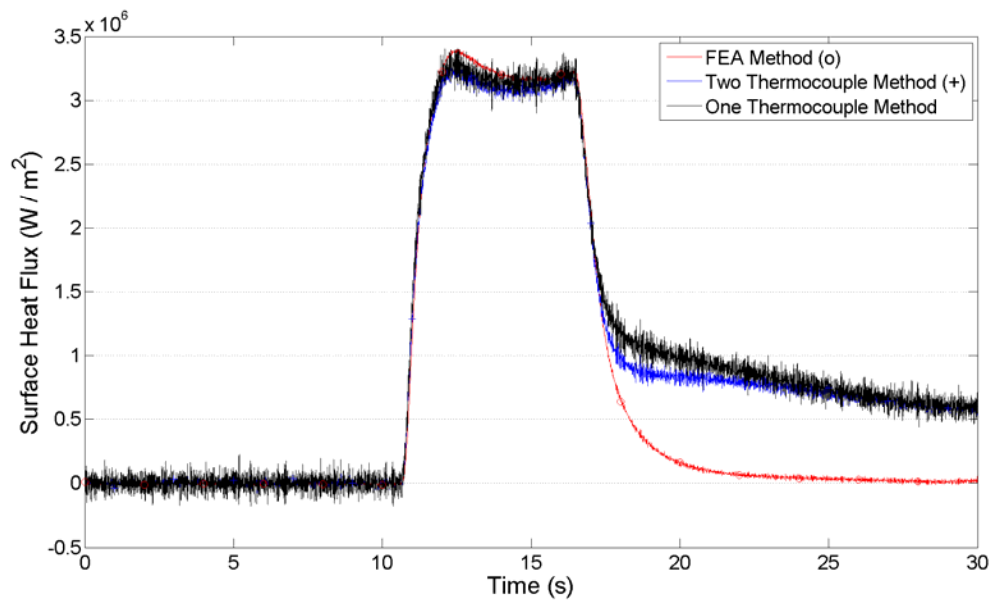


Figure 4-24 – Comparison of Estimate Methods – 1/8” Probe, Two Probe, and FEA Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

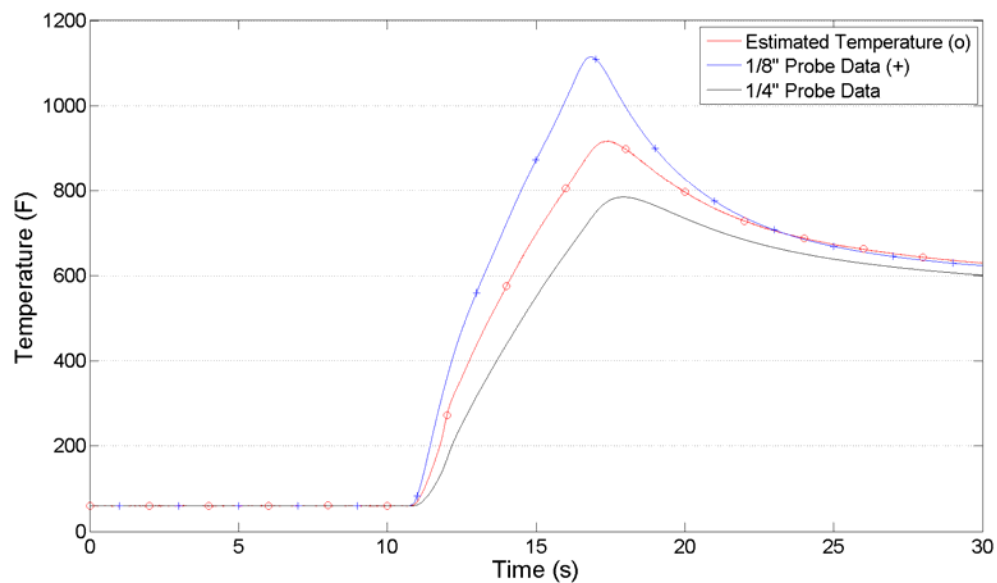


Figure 4-25 – One Thermocouple Estimate – 1/4" Probe Predicting 1/8" Temperature (Throat, Run 30, 6 Jan. 2006)

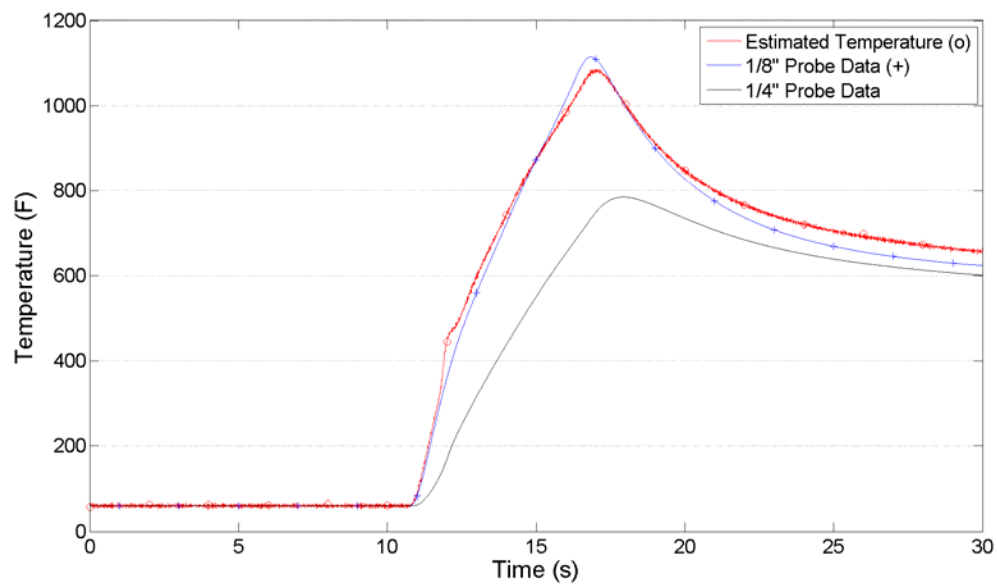


Figure 4-26 – One Thermocouple Estimate – 1/4" Probe Predicting Surface Temperature (Throat, Run 30, 6 Jan. 2006)

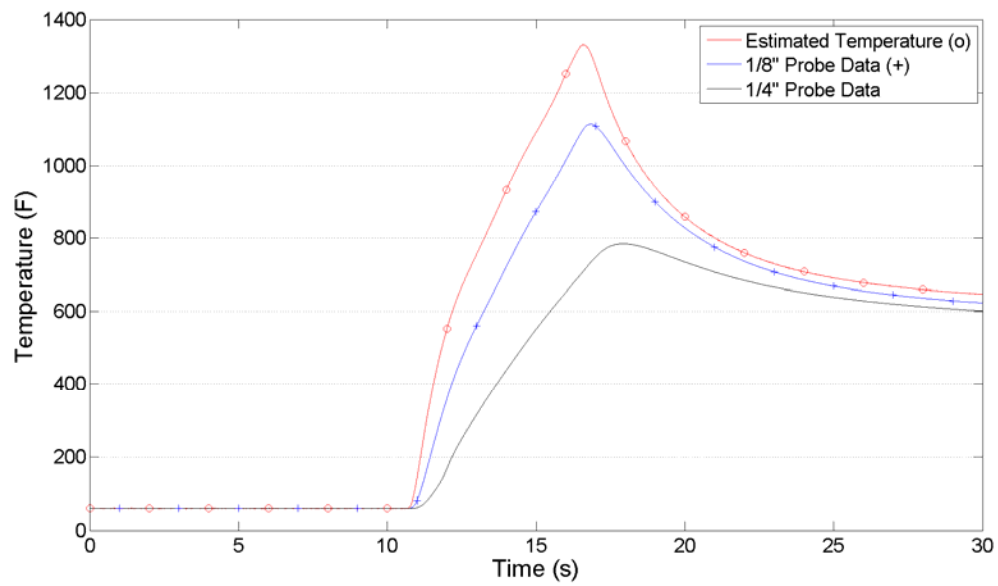


Figure 4-27 – One Thermocouple Estimate – 1/8" Probe Predicting Surface Temperature (Throat, Run 30, 6 Jan. 2006)

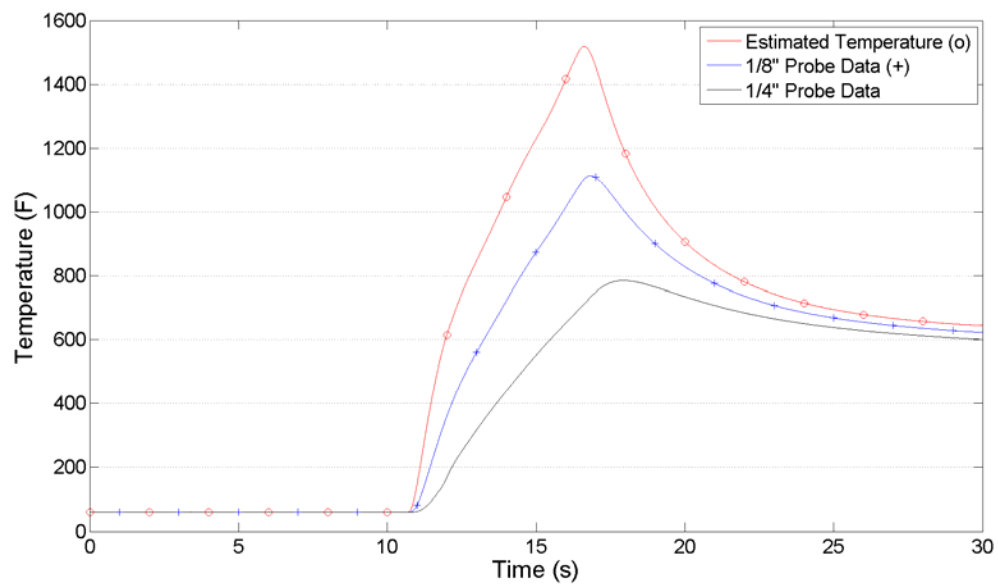


Figure 4-28 – Two Thermocouple Estimate – Predicting Surface Temperature (Throat, Run 30, 6 Jan. 2006)

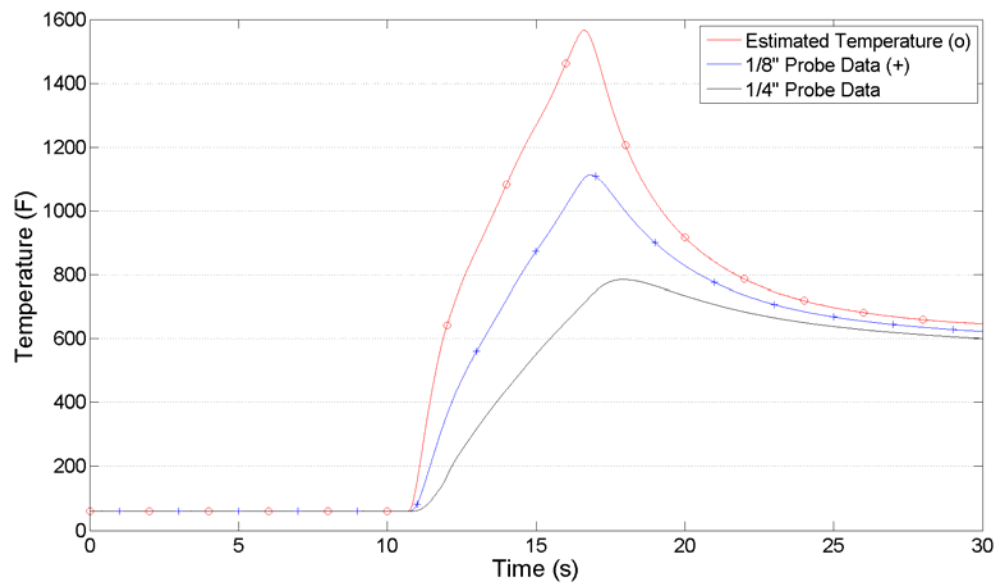


Figure 4-29 – Finite Element Estimate – Predicting Surface Temperature (Throat, Run 30, 6 Jan. 2006)

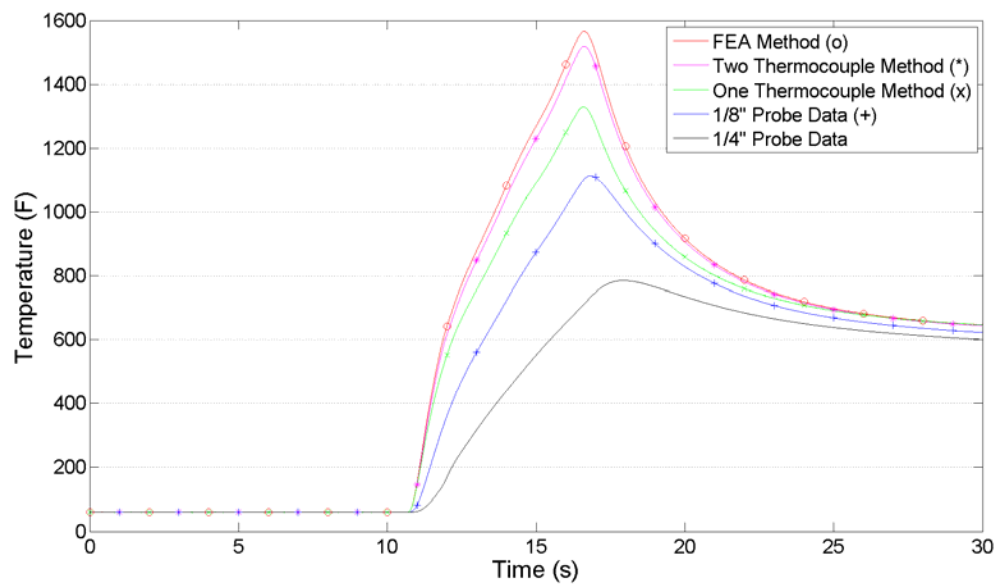


Figure 4-30 – Comparison of Estimate Methods – 1/8" Probe, Two Probe, and FEA Predicting Surface Temperature (Throat, Run 30, 6 Jan. 2006)

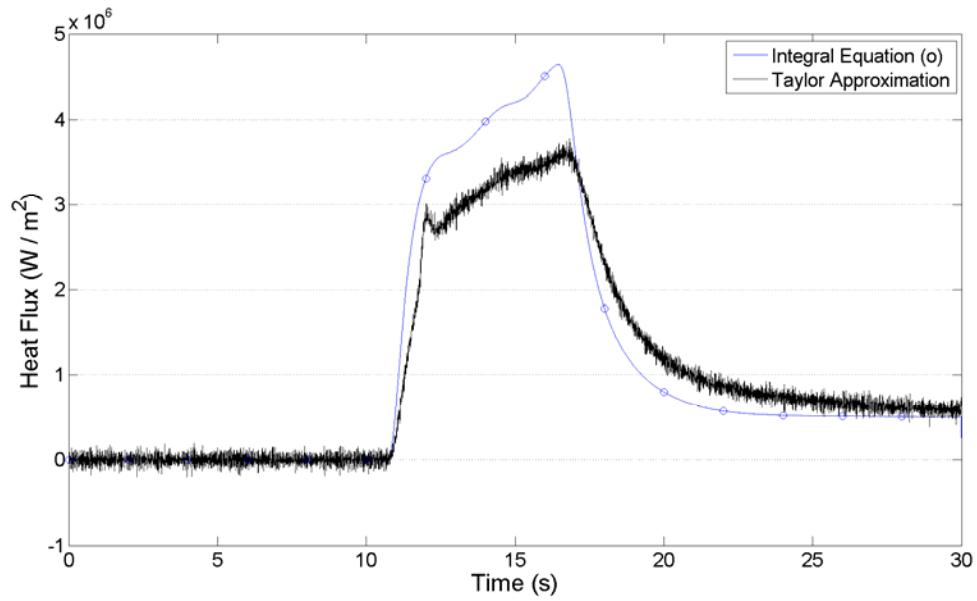


Figure 4-31 – One Thermocouple Estimate – 1/4” Probe Predicting 1/8” Heat Flux (Throat, Run 30, 6 Jan. 2006)

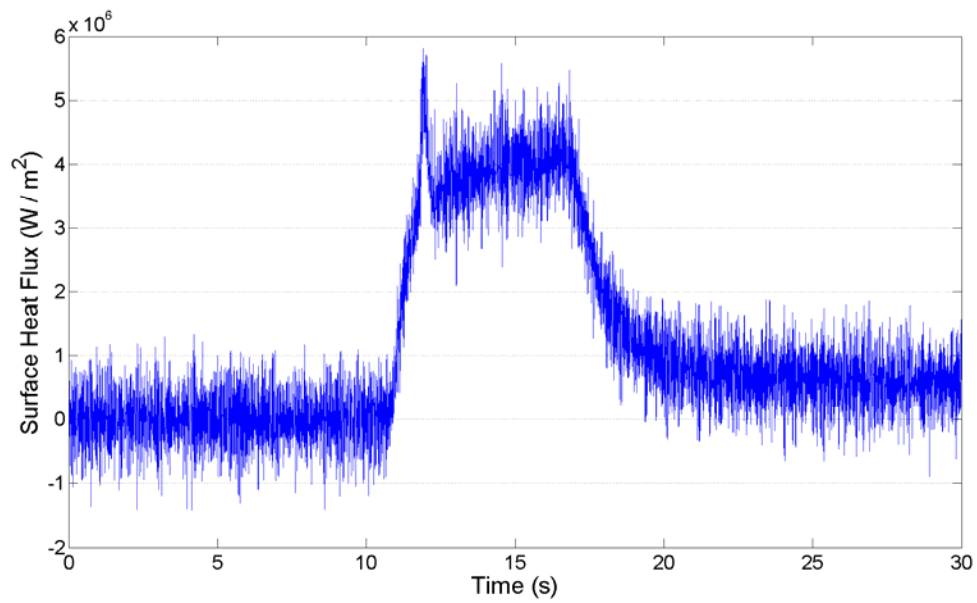


Figure 4-32 – One Thermocouple Estimate – 1/4” Probe Predicting Surface Heat Flux (Throat, Run 30, 6 Jan. 2006)

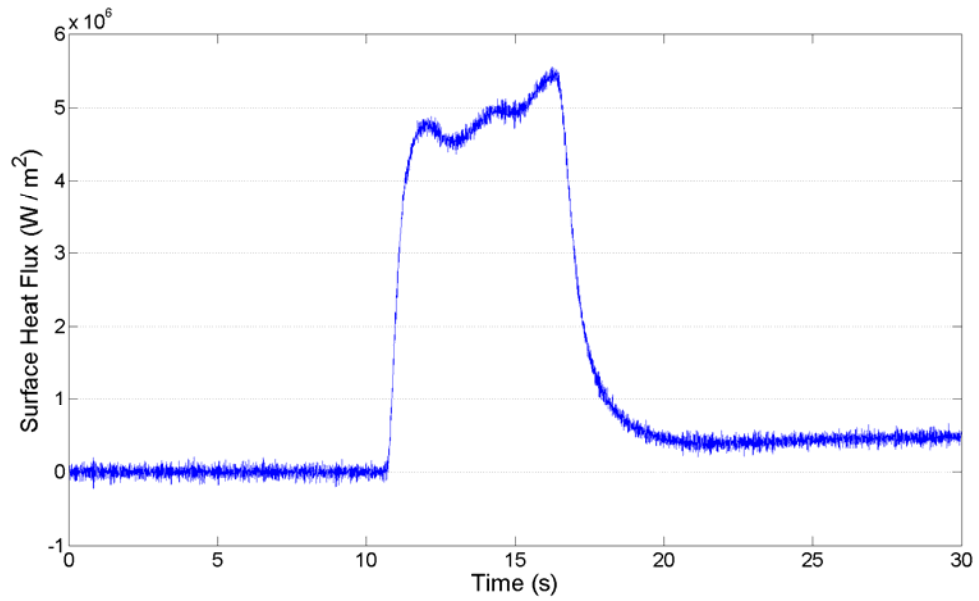


Figure 4-33 – One Thermocouple Estimate – 1/8" Probe Predicting Surface Heat Flux (Throat, Run 30, 6 Jan. 2006)

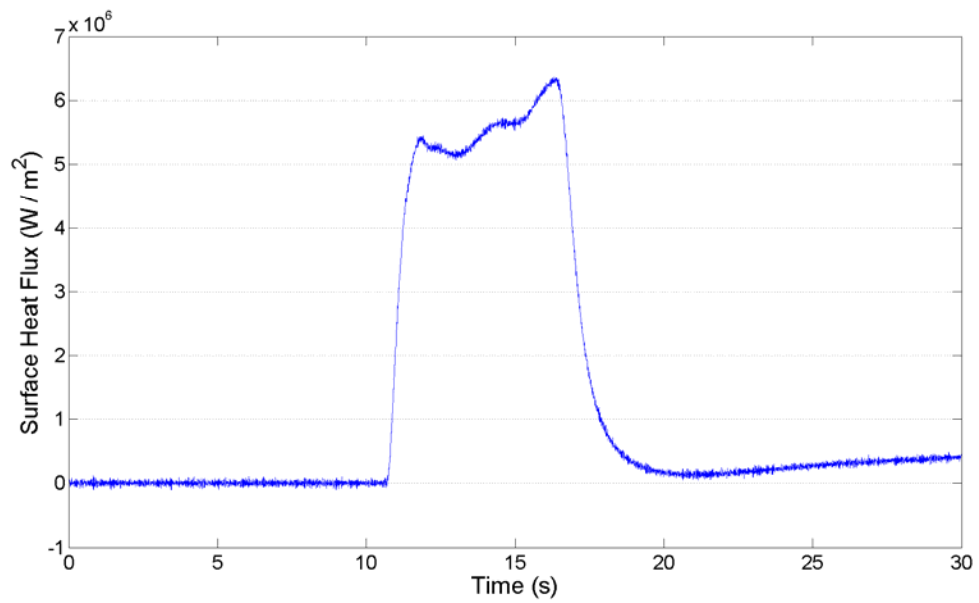


Figure 4-34 – Two Thermocouple Estimate – Predicting Surface Heat Flux (Throat, Run 30, 6 Jan. 2006)

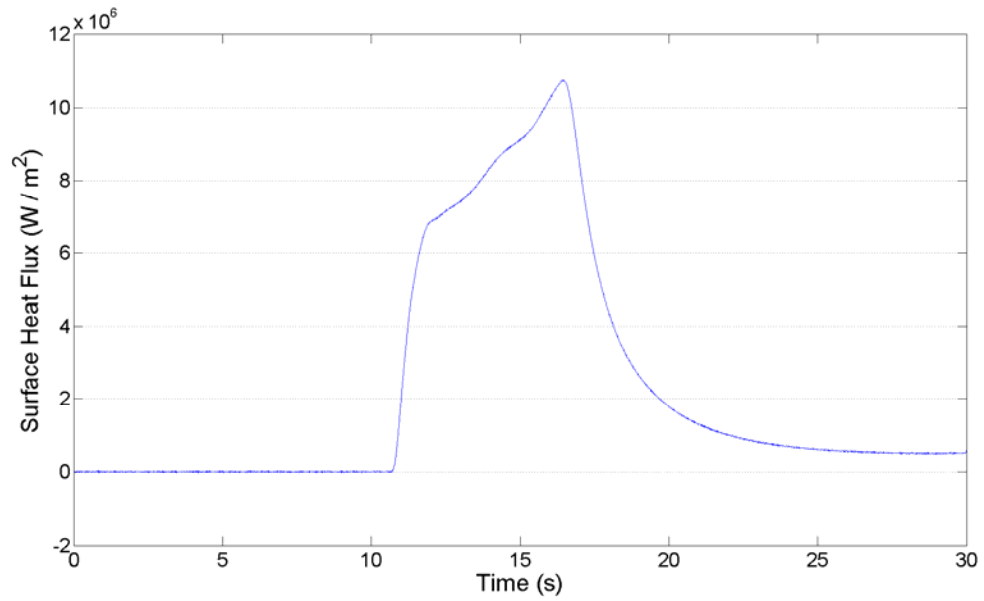


Figure 4-35 – Finite Element Estimate – Predicting Surface Heat Flux (Throat, Run 30, 6 Jan. 2006)

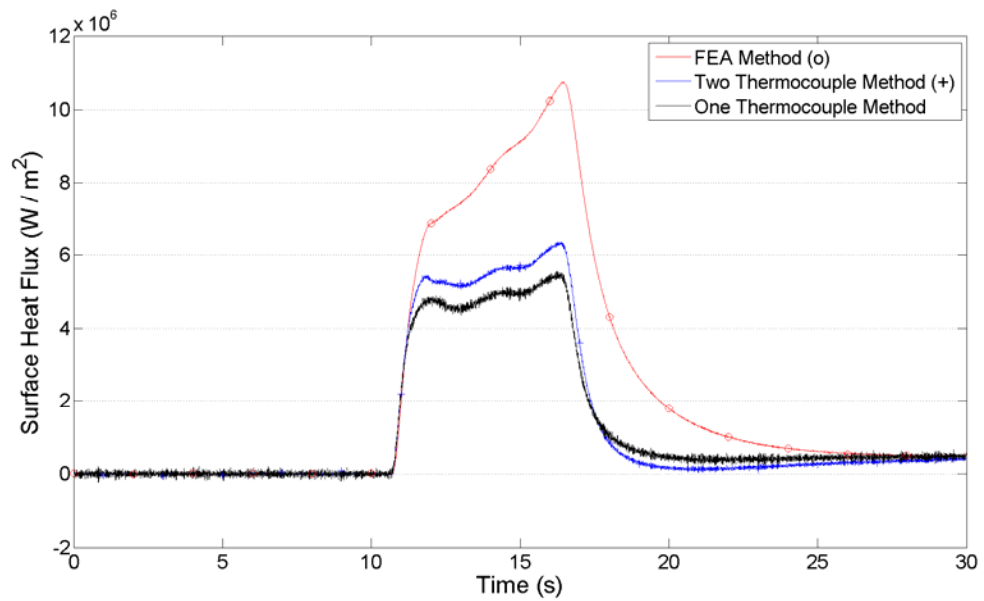


Figure 4-36 – Comparison of Estimate Methods – 1/8” Probe, Two Probe, and FEA Predicting Surface Heat Flux (Throat, Run 30, 6 Jan. 2006)

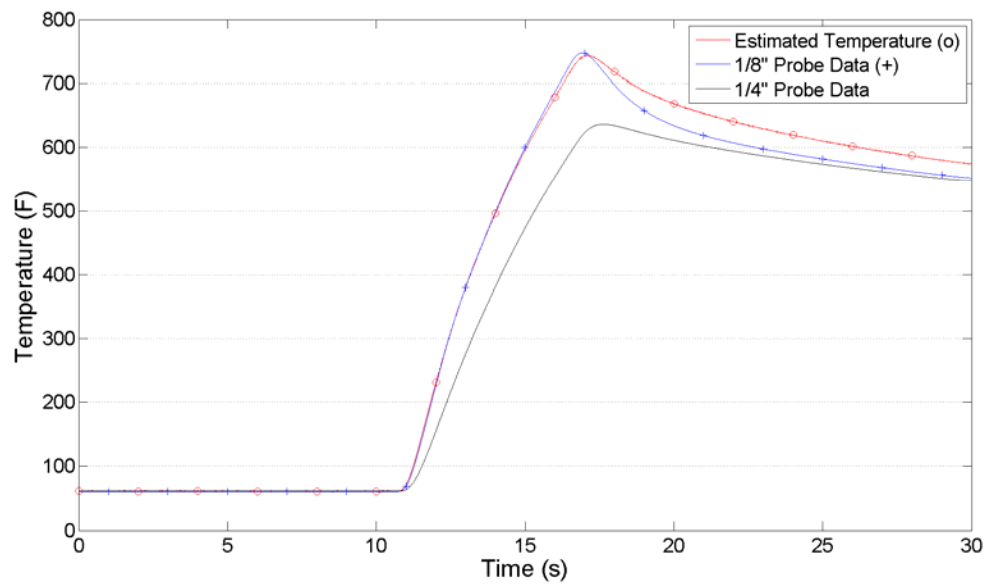


Figure 4-37 – One Thermocouple Estimate – 1/4" Probe Predicting 1/8" Temperature (Exit, Run 30, 6 Jan. 2006)

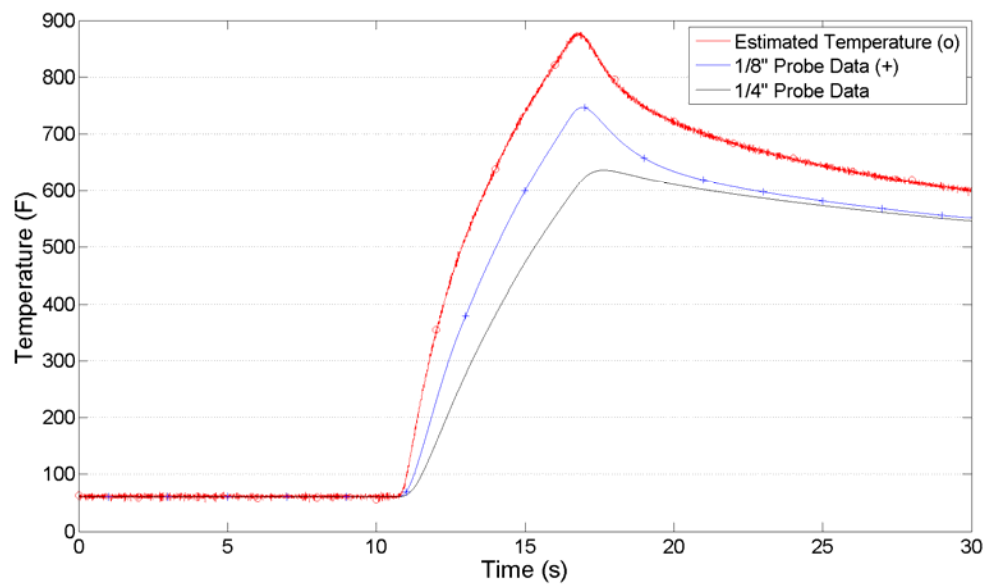


Figure 4-38 – One Thermocouple Estimate – 1/4" Probe Predicting Surface Temperature (Exit, Run 30, 6 Jan. 2006)

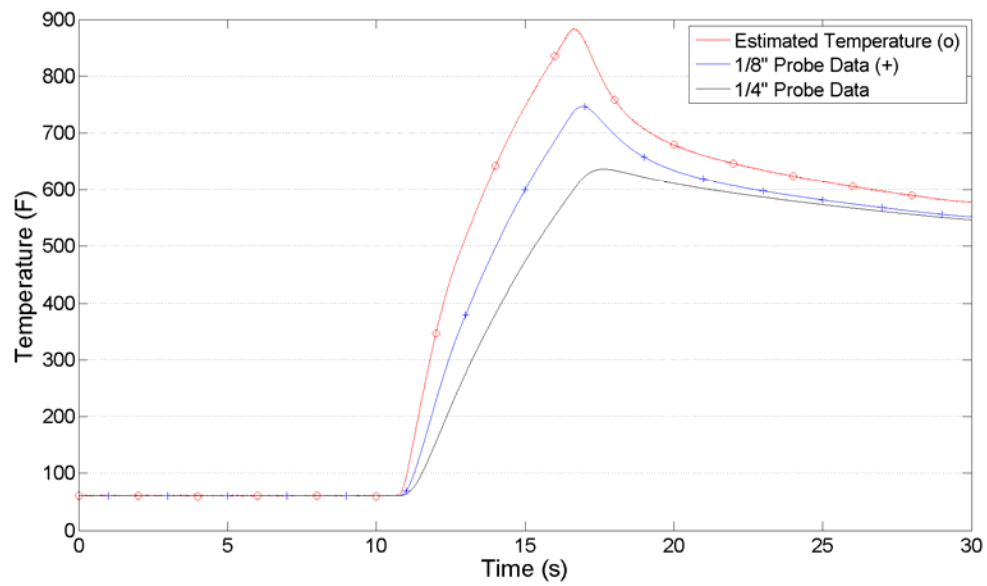


Figure 4-39 – One Thermocouple Estimate – 1/8" Probe Predicting Surface Temperature (Exit, Run 30, 6 Jan. 2006)

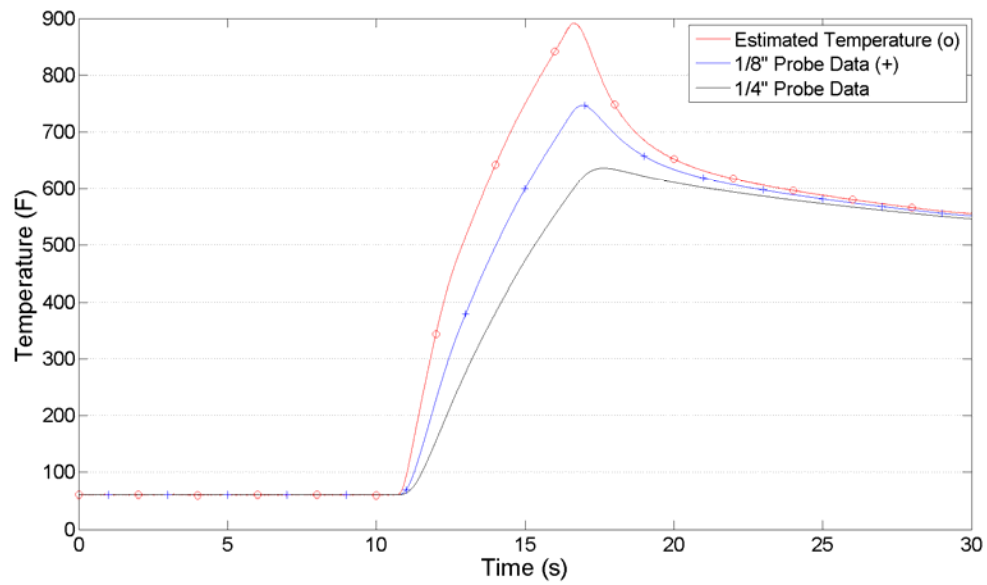


Figure 4-40 – Two Thermocouple Estimate – Predicting Surface Temperature (Exit, Run 30, 6 Jan. 2006)

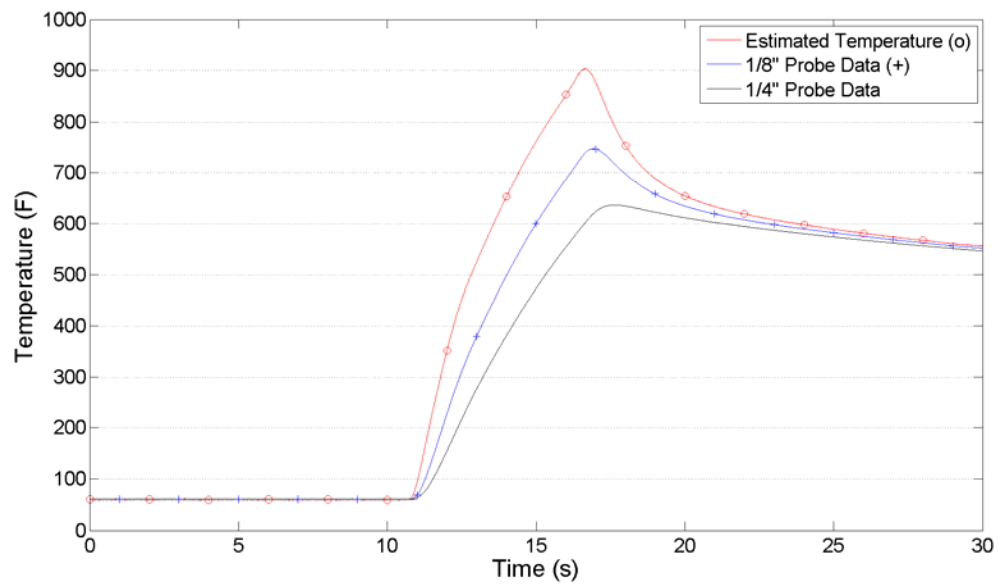


Figure 4-41 – Finite Element Estimate – Predicting Surface Temperature (Exit, Run 30, 6 Jan. 2006)

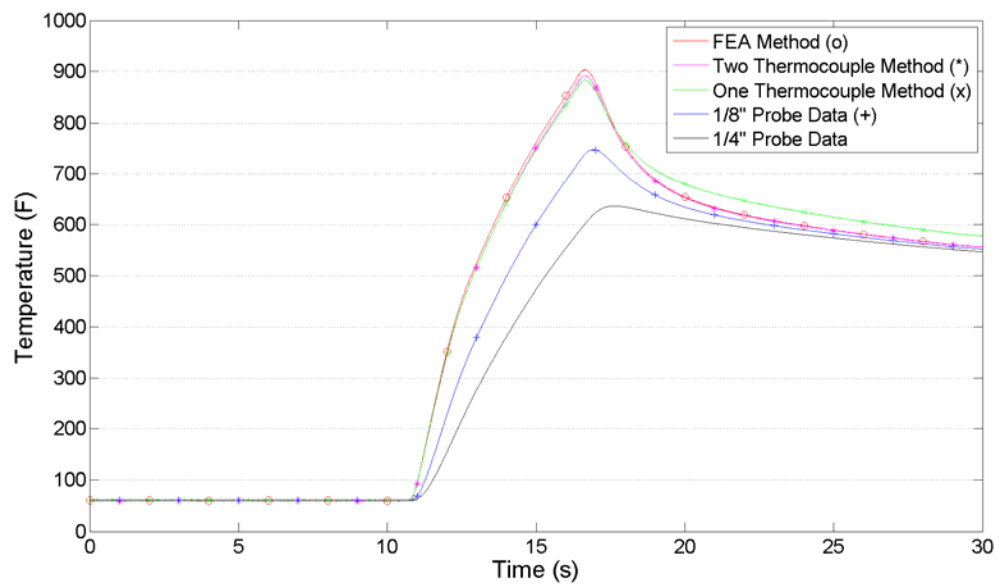


Figure 4-42 – Comparison of Estimate Methods – 1/8" Probe, Two Probe, and FEA Predicting Surface Temperature (Exit, Run 30, 6 Jan. 2006)

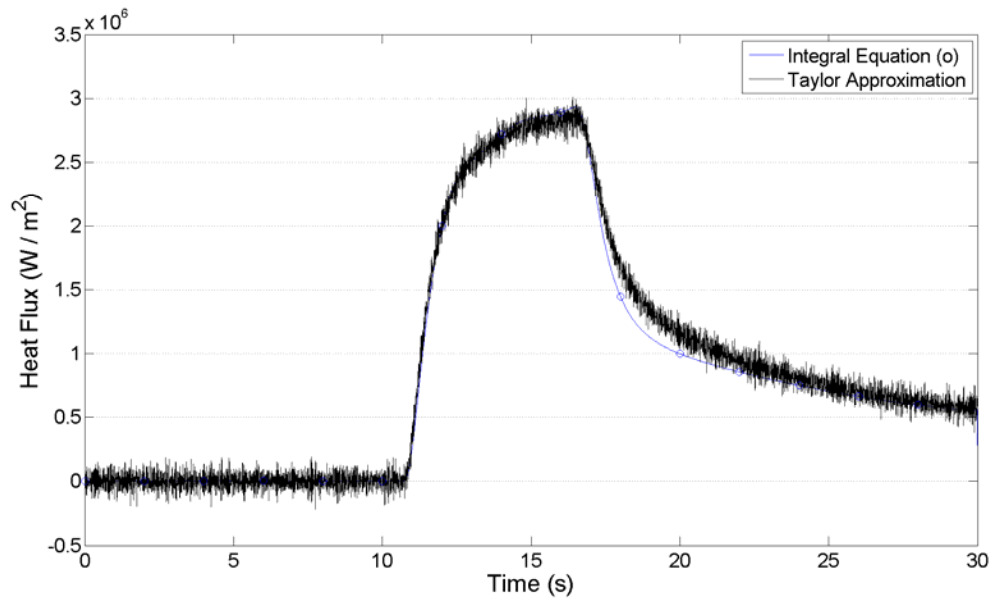


Figure 4-43 – One Thermocouple Estimate – 1/4” Probe Predicting 1/8” Heat Flux (Exit, Run 30, 6 Jan. 2006)

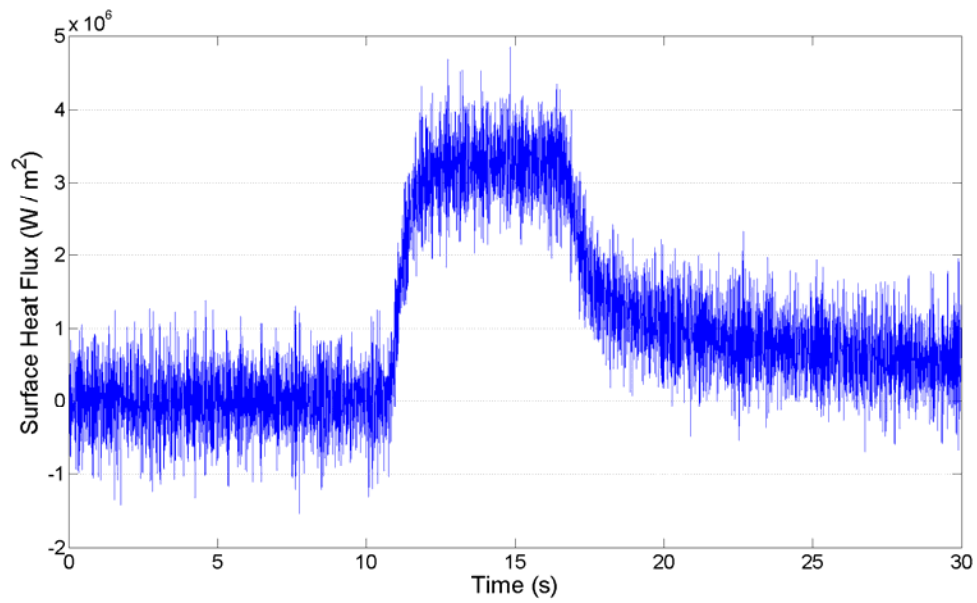


Figure 4-44 – One Thermocouple Estimate – 1/4” Probe Predicting Surface Heat Flux (Exit, Run 30, 6 Jan. 2006)

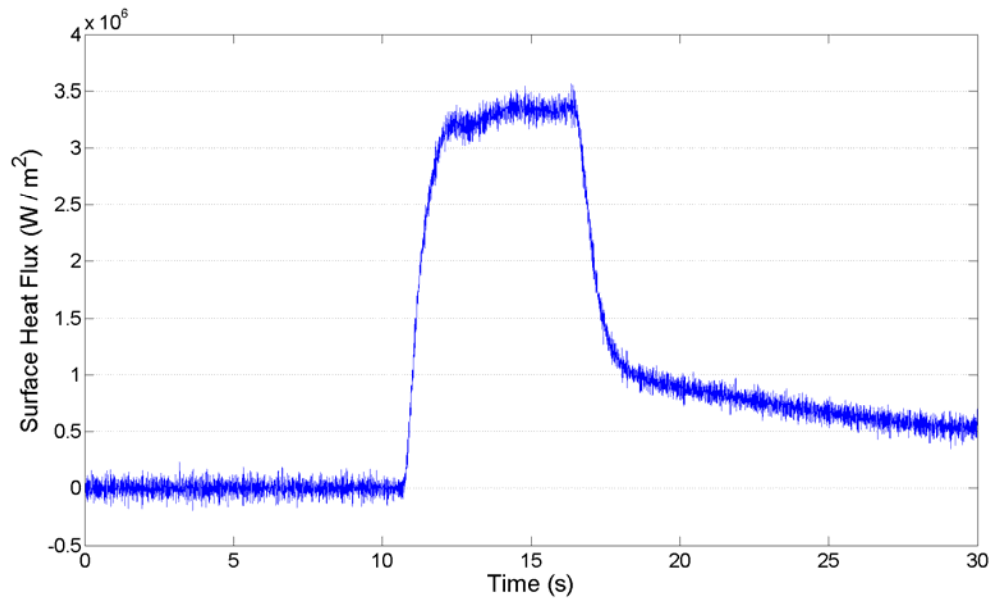


Figure 4-45 – One Thermocouple Estimate – 1/8” Probe Predicting Surface Heat Flux (Exit, Run 30, 6 Jan. 2006)

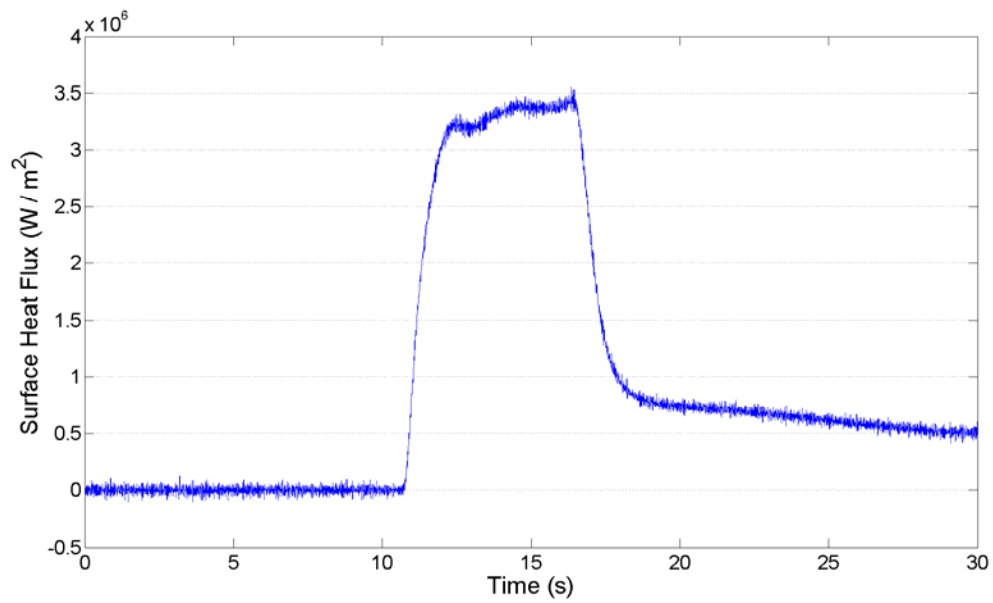


Figure 4-46 – Two Thermocouple Estimate – Predicting Surface Heat Flux (Exit, Run 30, 6 Jan. 2006)

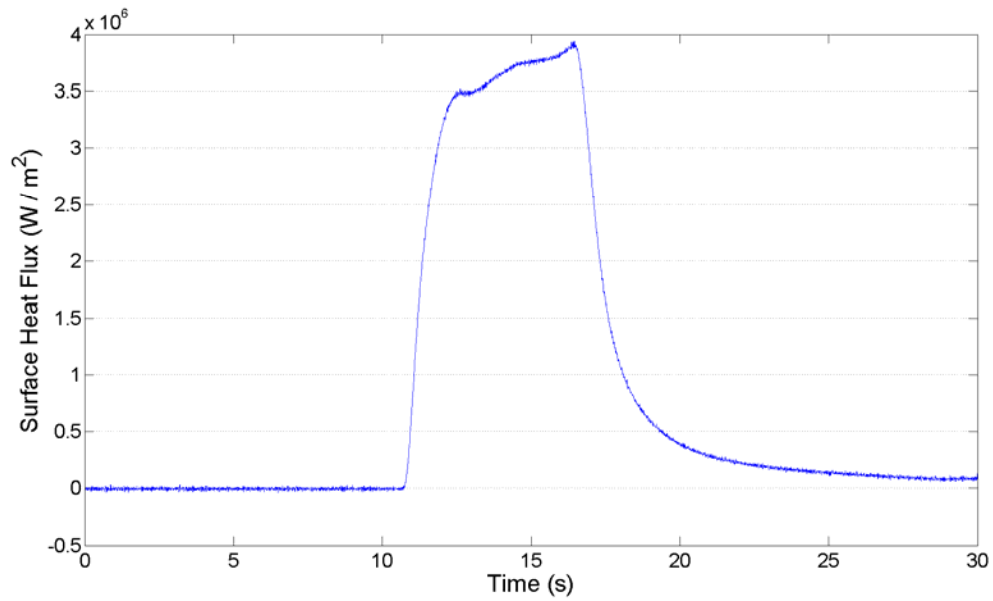


Figure 4-47 – Finite Element Estimate – Predicting Surface Heat Flux (Exit, Run 30, 6 Jan. 2006)

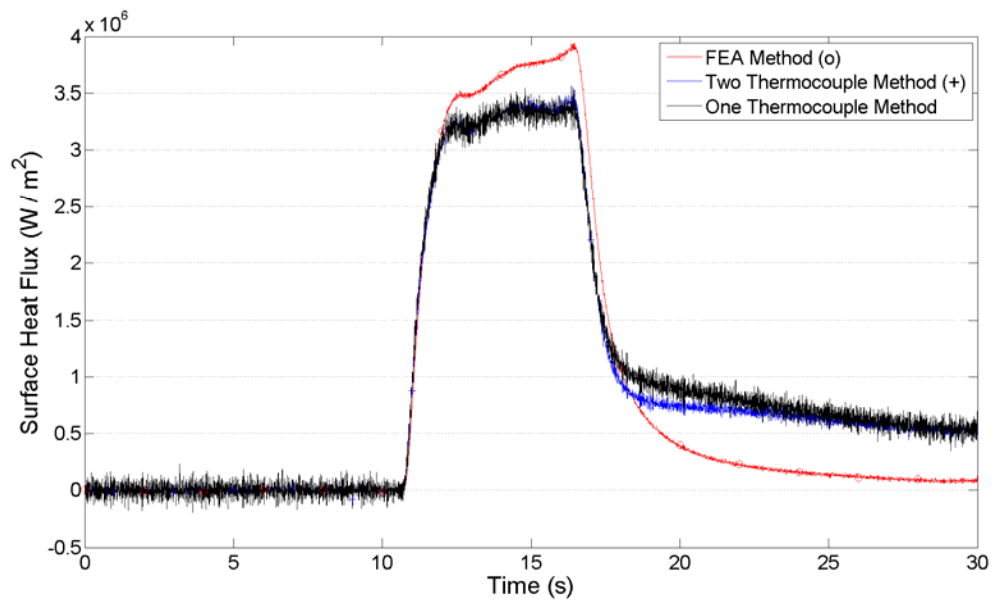


Figure 4-48 – Comparison of Estimate Methods – 1/8” Probe, Two Probe, and FEA Predicting Surface Heat Flux (Exit, Run 30, 6 Jan. 2006)

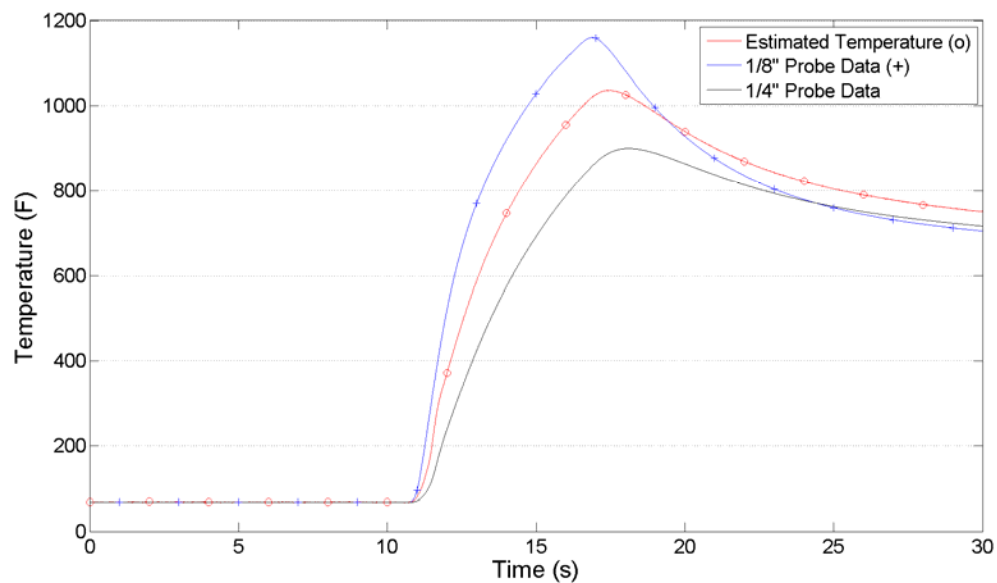


Figure 4-49 – One Thermocouple Estimate – 1/4" Probe Predicting 1/8" Temperature (Throat, Run 36, 15 Feb. 2006)

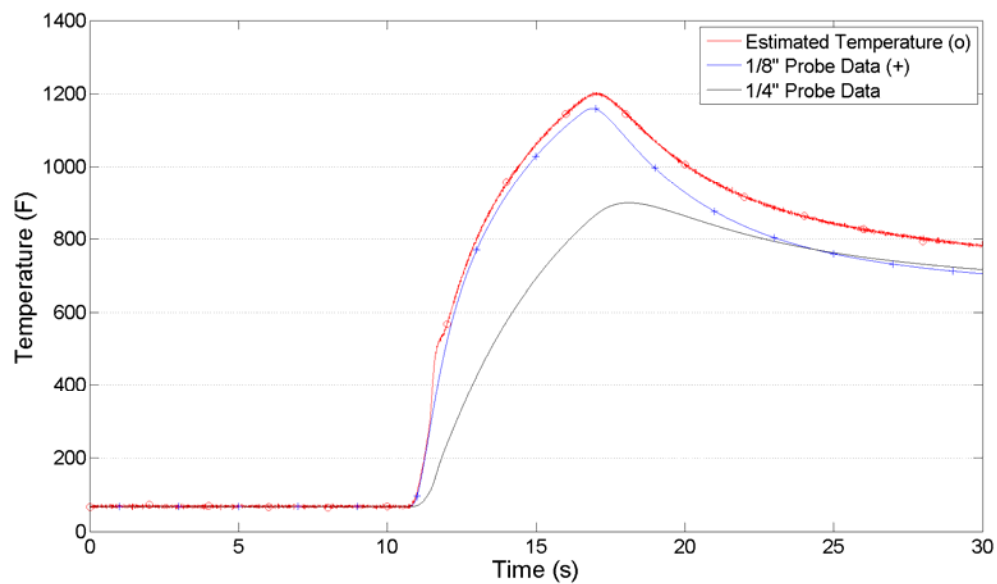


Figure 4-50 – One Thermocouple Estimate – 1/4" Probe Predicting Surface Temperature (Throat, Run 36, 15 Feb. 2006)

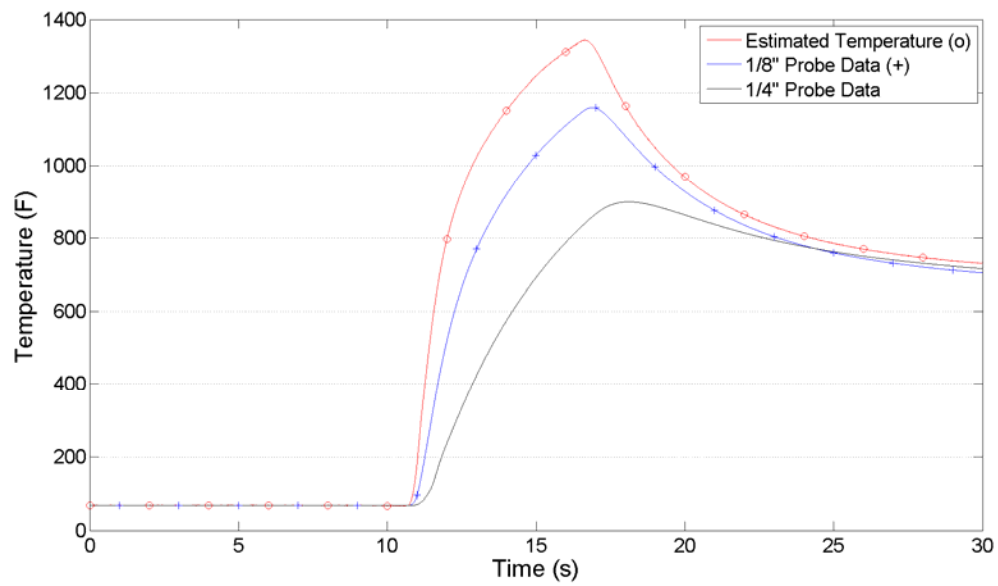


Figure 4-51 – One Thermocouple Estimate – 1/8" Probe Predicting Surface Temperature (Throat, Run 36, 15 Feb. 2006)

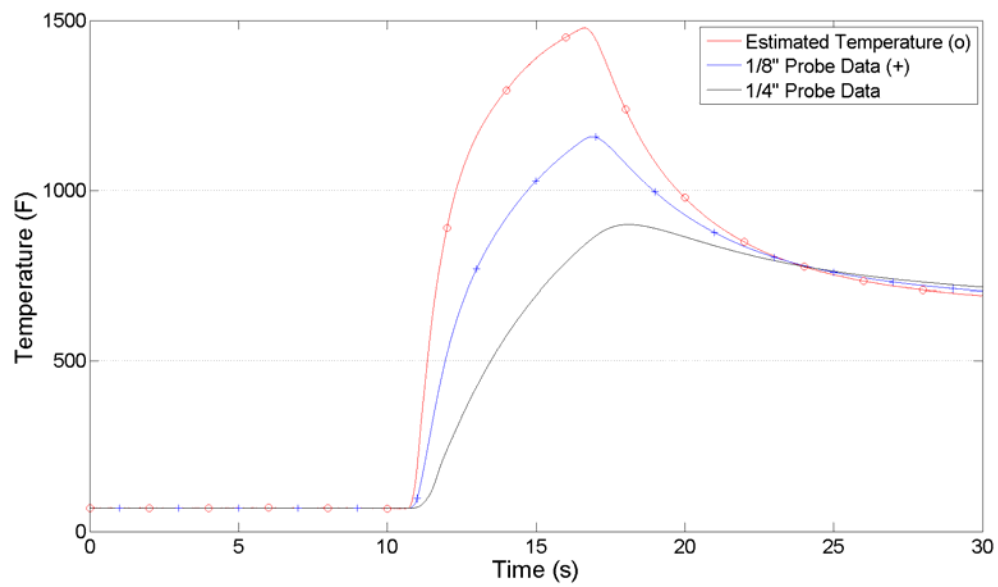


Figure 4-52 – Two Thermocouple Estimate – Predicting Surface Temperature (Throat, Run 36, 15 Feb. 2006)

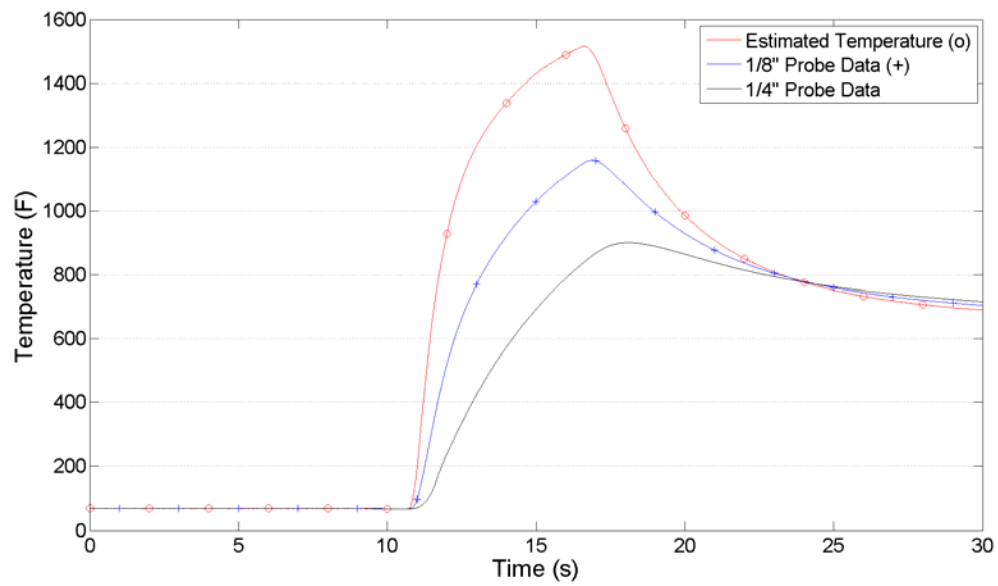


Figure 4-53 – Finite Element Estimate – Predicting Surface Temperature (Throat, Run 36, 15 Feb. 2006)

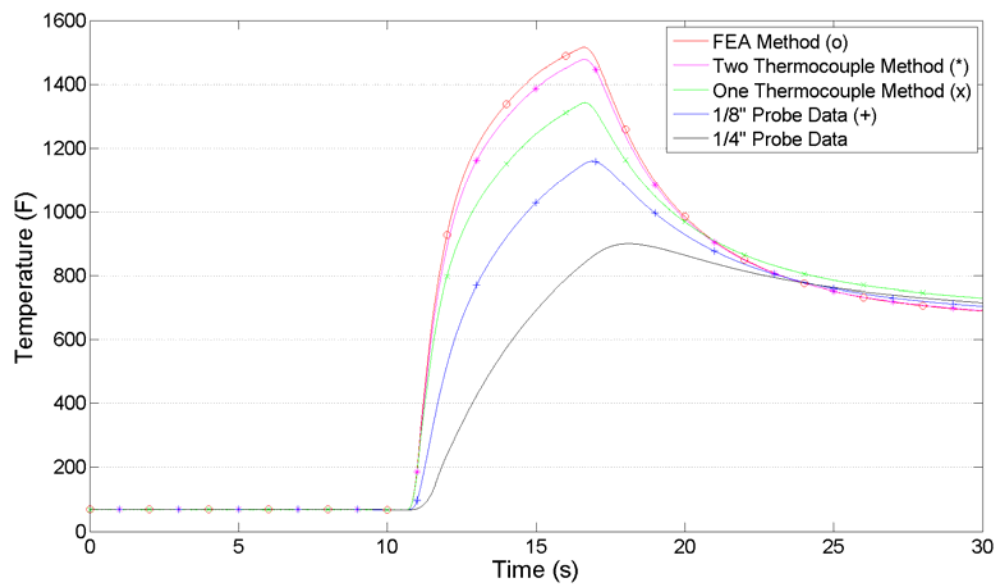


Figure 4-54 – Comparison of Estimate Methods – 1/8" Probe, Two Probe, and FEA Predicting Surface Temperature (Throat, Run 36, 15 Feb. 2006)

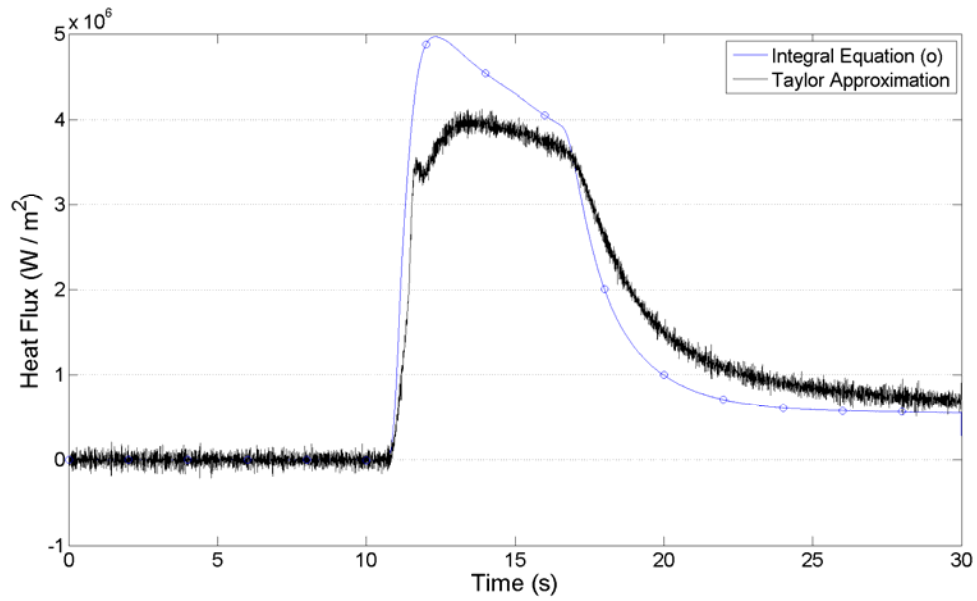


Figure 4-55 – One Thermocouple Estimate – 1/4” Probe Predicting 1/8” Heat Flux (Throat, Run 36, 15 Feb. 2006)

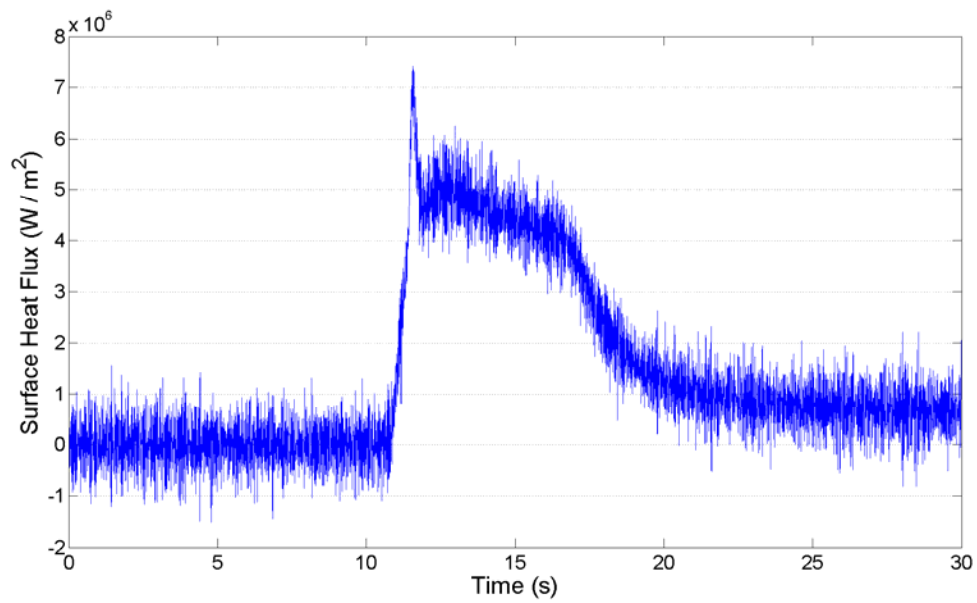


Figure 4-56 – One Thermocouple Estimate – 1/4” Probe Predicting Surface Heat Flux (Throat, Run 36, 15 Feb. 2006)

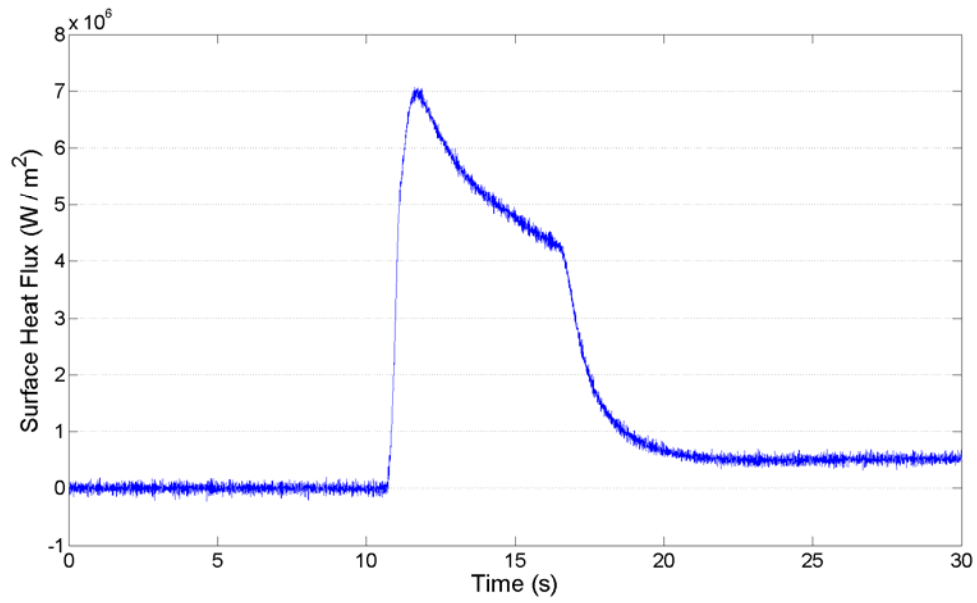


Figure 4-57 – One Thermocouple Estimate – 1/8” Probe Predicting Surface Heat Flux (Throat, Run 36, 15 Feb. 2006)

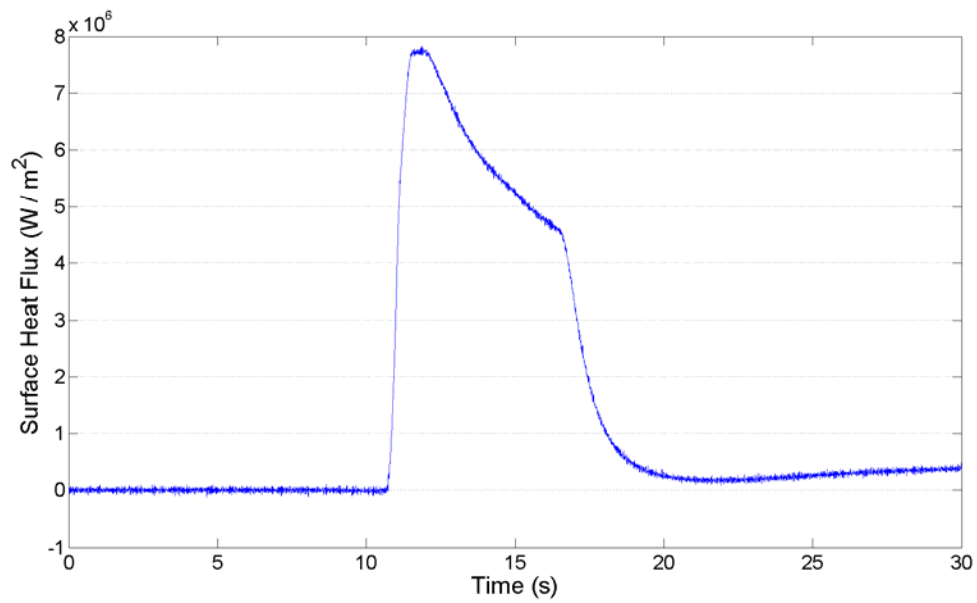


Figure 4-58 – Two Thermocouple Estimate – Predicting Surface Heat Flux (Throat, Run 36, 15 Feb. 2006)

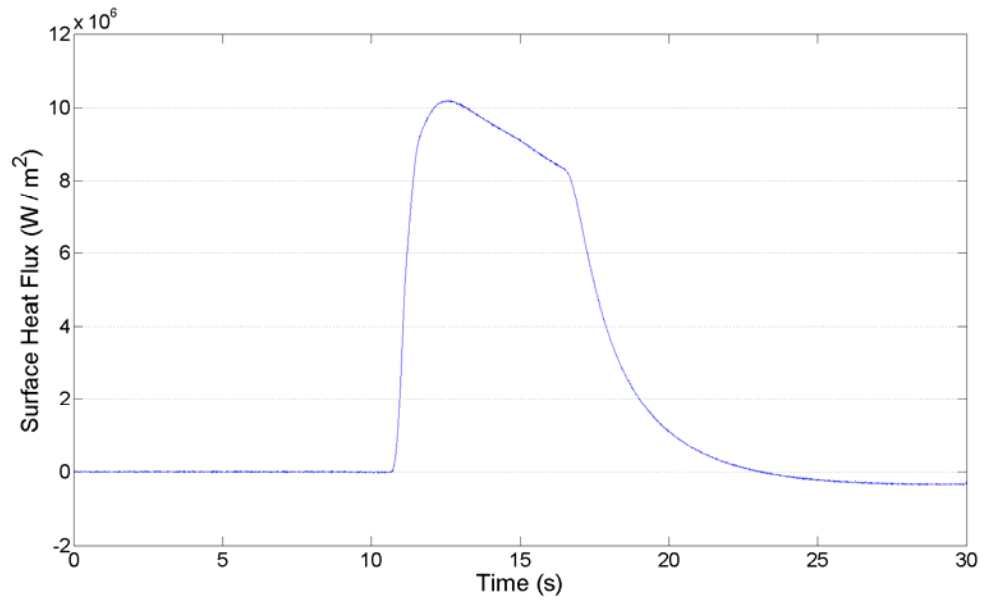


Figure 4-59 – Finite Element Estimate – Predicting Surface Heat Flux (Throat, Run 36, 15 Feb. 2006)

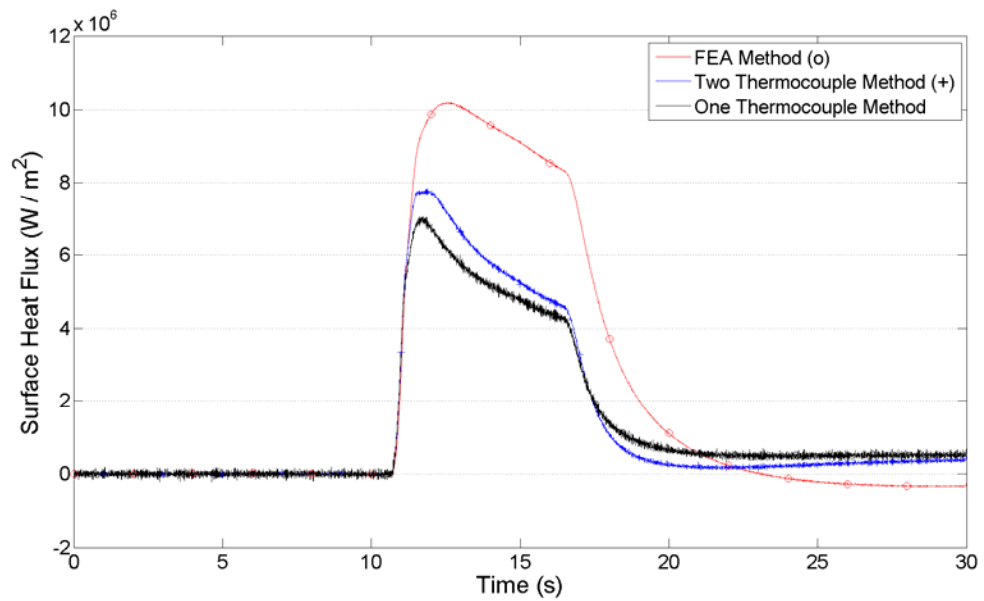


Figure 4-60 – Comparison of Estimate Methods – 1/8” Probe, Two Probe, and FEA Predicting Surface Heat Flux (Throat, Run 36, 15 Feb. 2006)

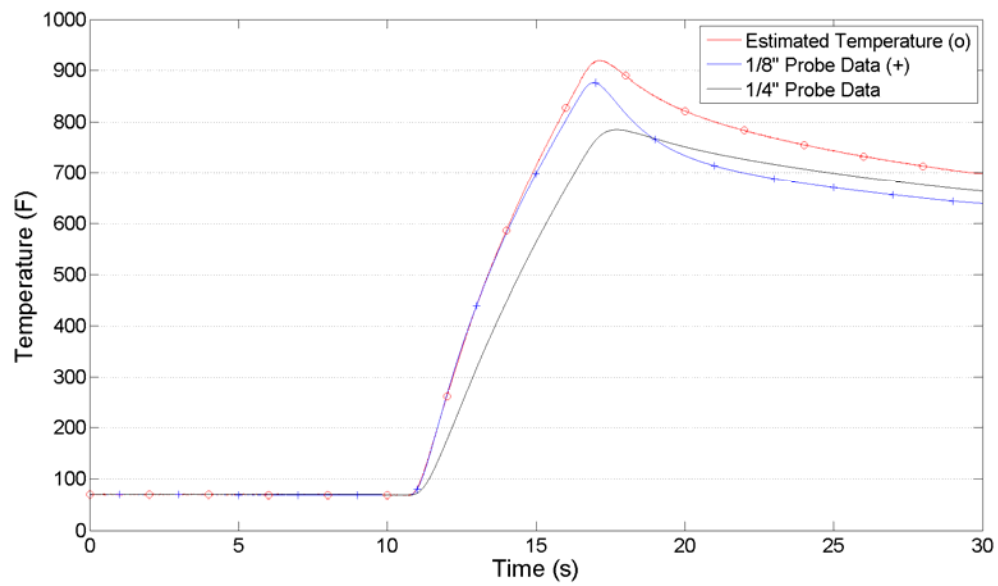


Figure 4-61 – One Thermocouple Estimate – 1/4" Probe Predicting 1/8" Temperature (Exit, Run 36, 15 Feb. 2006)

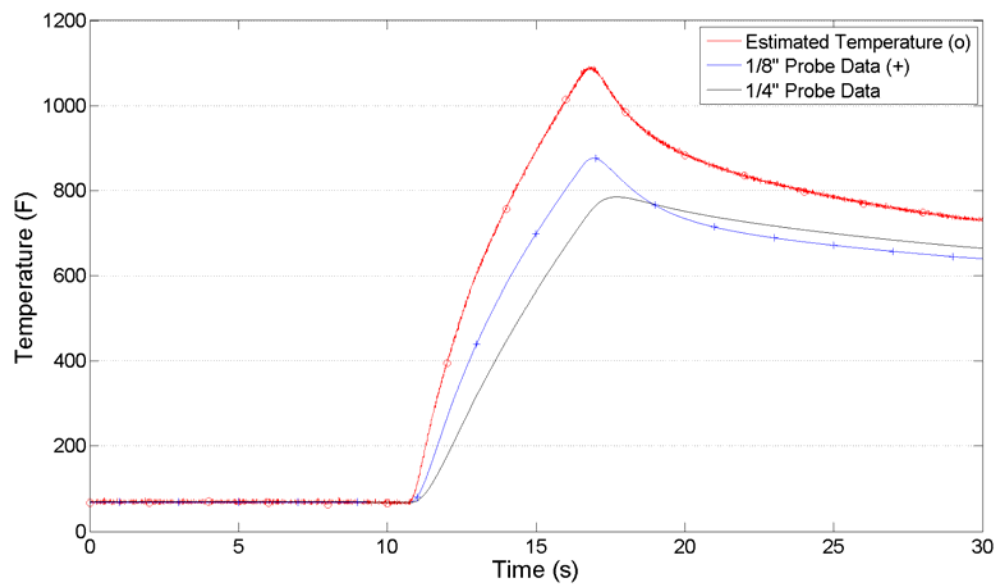


Figure 4-62 – One Thermocouple Estimate – 1/4" Probe Predicting Surface Temperature (Exit, Run 36, 15 Feb. 2006)

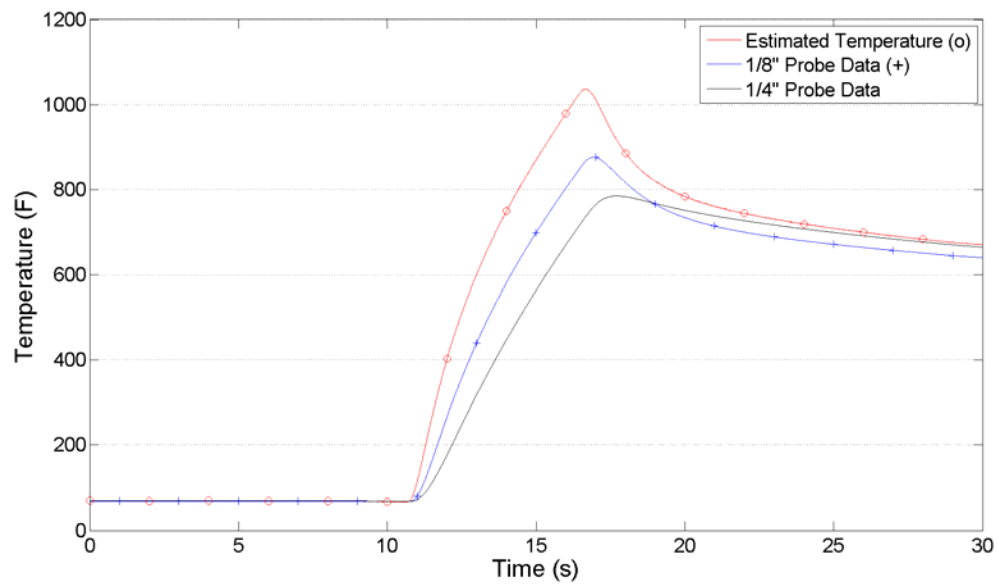


Figure 4-63 – One Thermocouple Estimate – 1/8" Probe Predicting Surface Temperature (Exit, Run 36, 15 Feb. 2006)

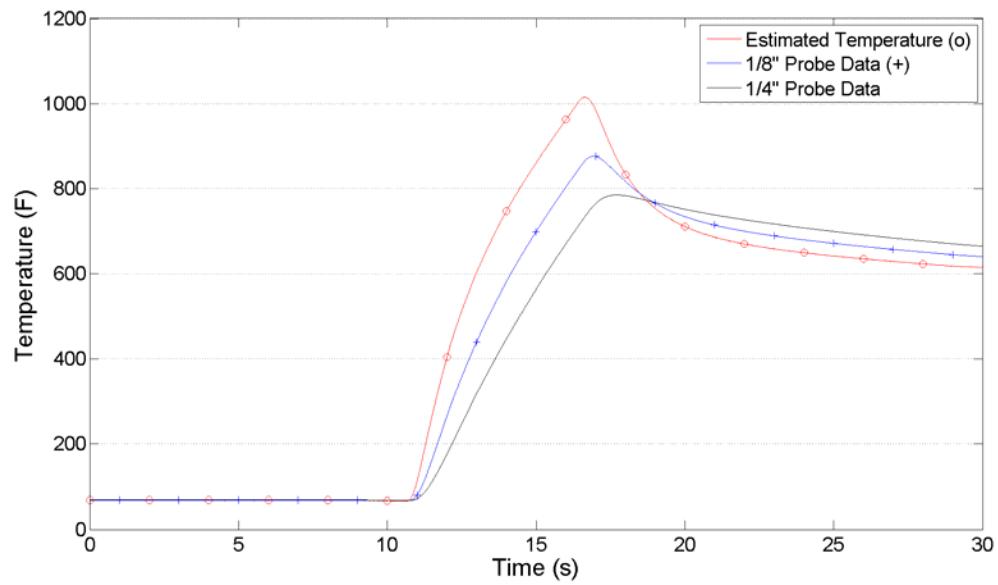


Figure 4-64 – Two Thermocouple Estimate – Predicting Surface Temperature (Exit, Run 36, 15 Feb. 2006)

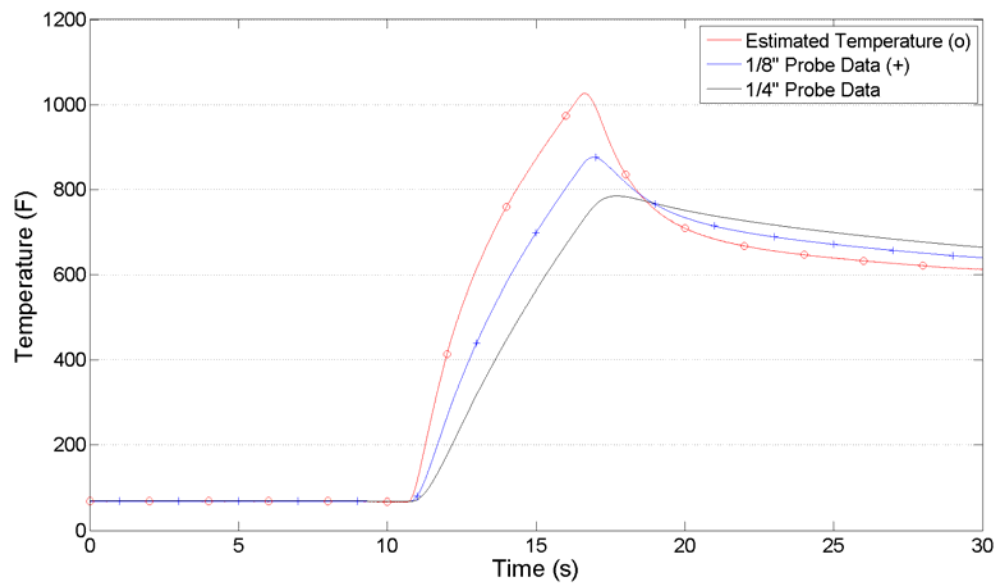


Figure 4-65 – Finite Element Estimate – Predicting Surface Temperature (Exit, Run 36, 15 Feb. 2006)

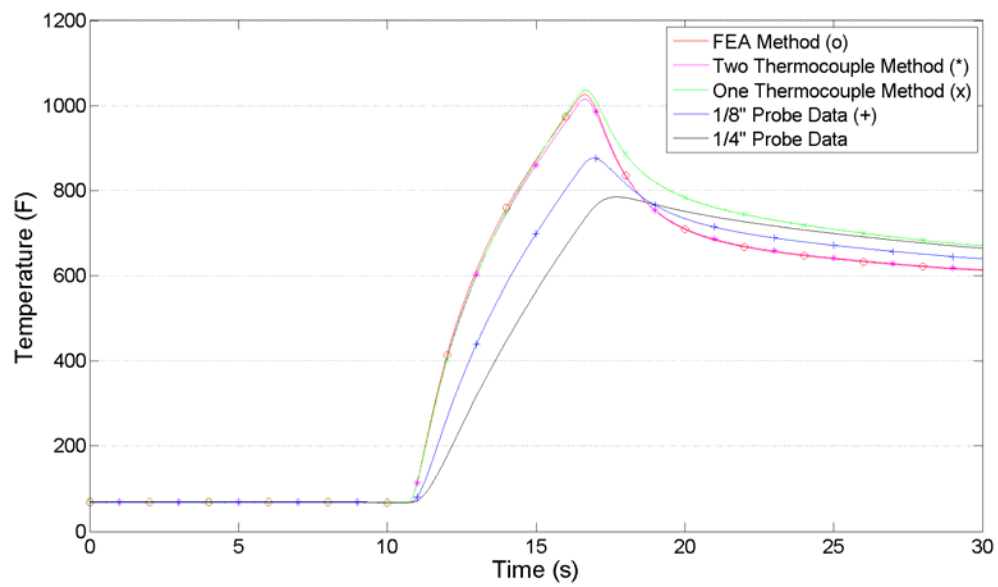


Figure 4-66 – Comparison of Estimate Methods – 1/8" Probe, Two Probe, and FEA Predicting Surface Temperature (Exit, Run 36, 15 Feb. 2006)

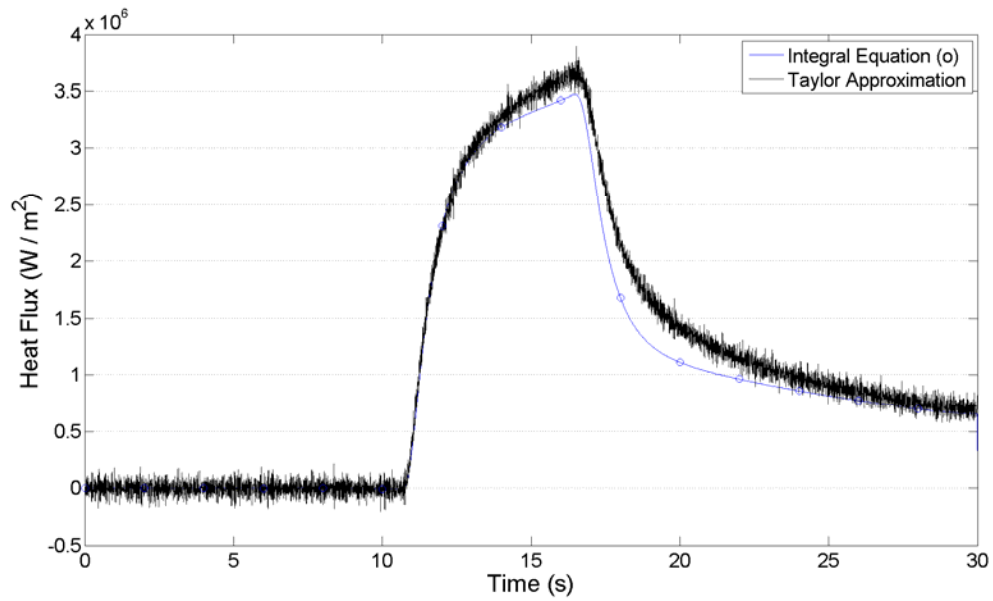


Figure 4-67 – One Thermocouple Estimate – 1/4” Probe Predicting 1/8” Heat Flux (Exit, Run 36, 15 Feb. 2006)

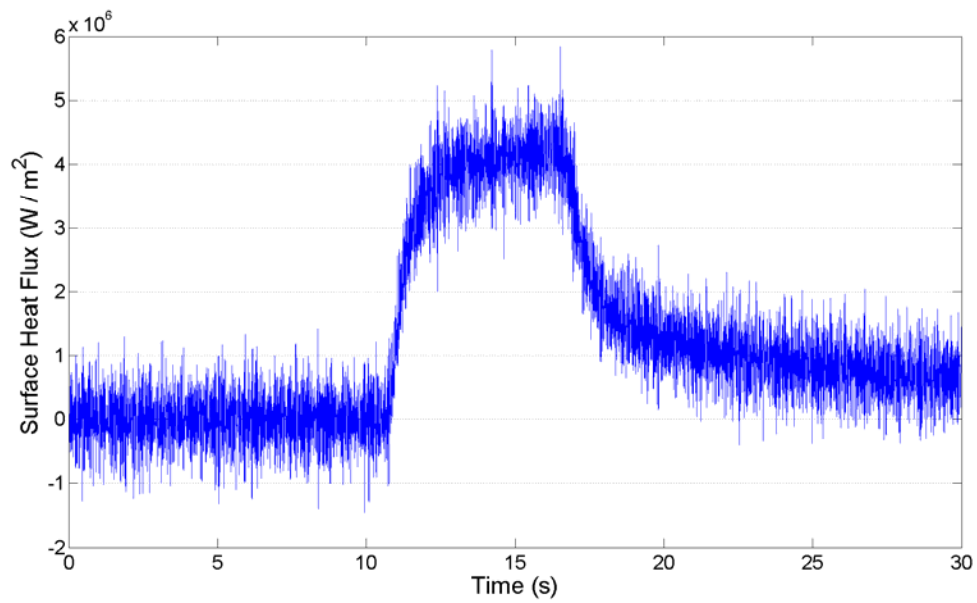


Figure 4-68 – One Thermocouple Estimate – 1/4” Probe Predicting Surface Heat Flux (Exit, Run 36, 15 Feb. 2006)

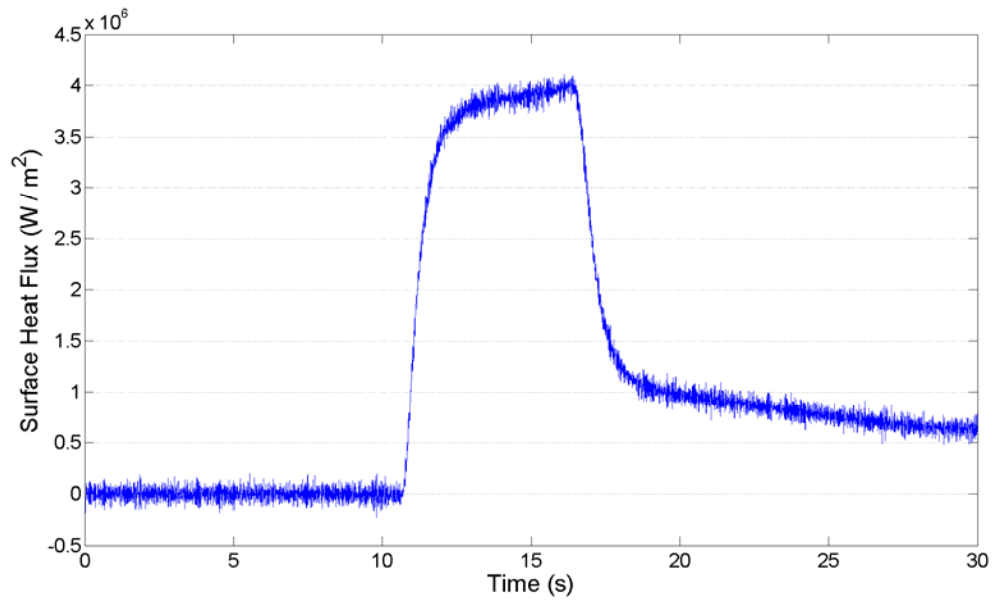


Figure 4-69 – One Thermocouple Estimate – 1/8” Probe Predicting Surface Heat Flux (Exit, Run 36, 15 Feb. 2006)

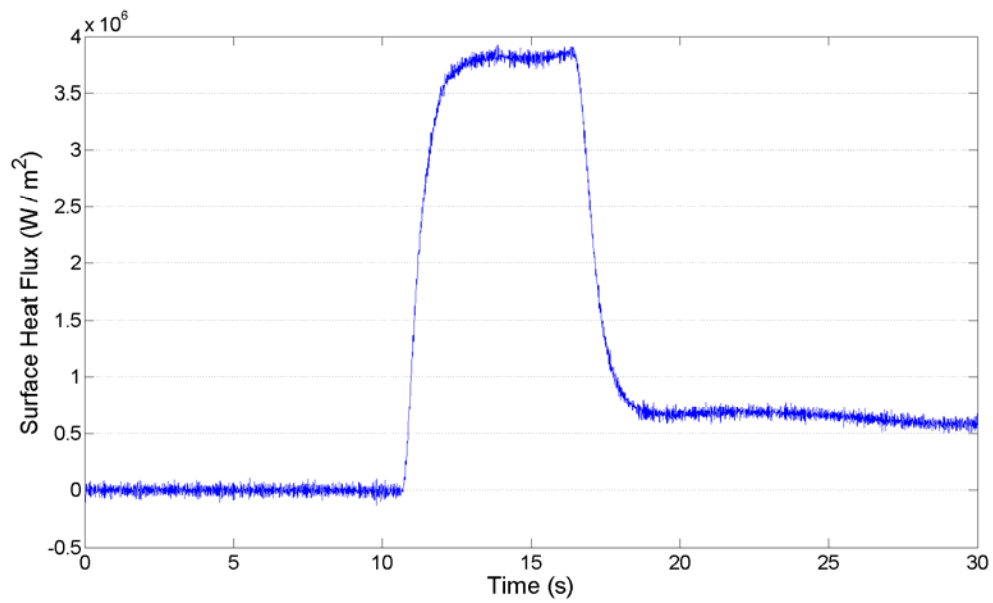


Figure 4-70 – Two Thermocouple Estimate – Predicting Surface Heat Flux (Exit, Run 36, 15 Feb. 2006)

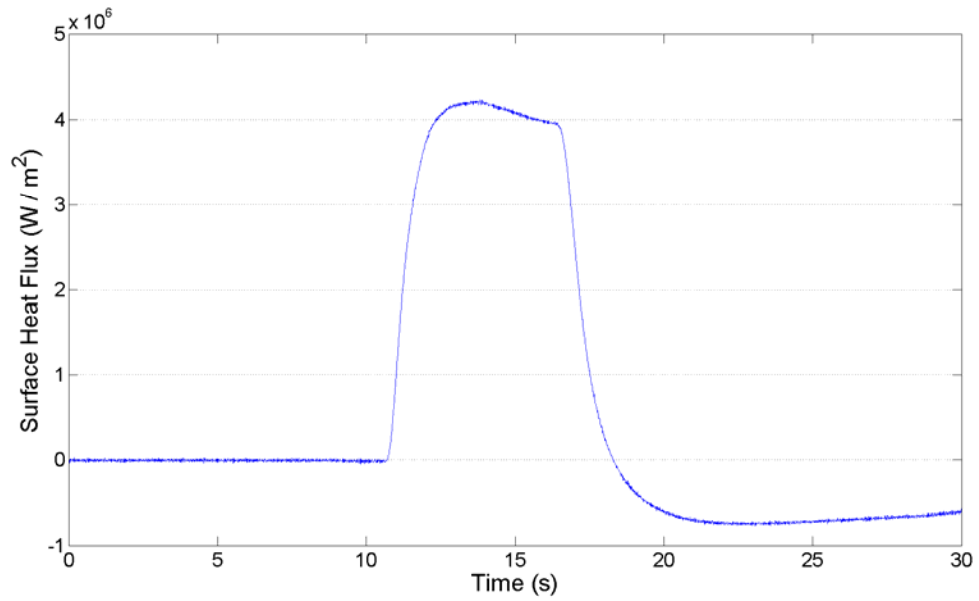


Figure 4-71 – Finite Element Estimate – Predicting Surface Heat Flux (Exit, Run 36, 15 Feb. 2006)

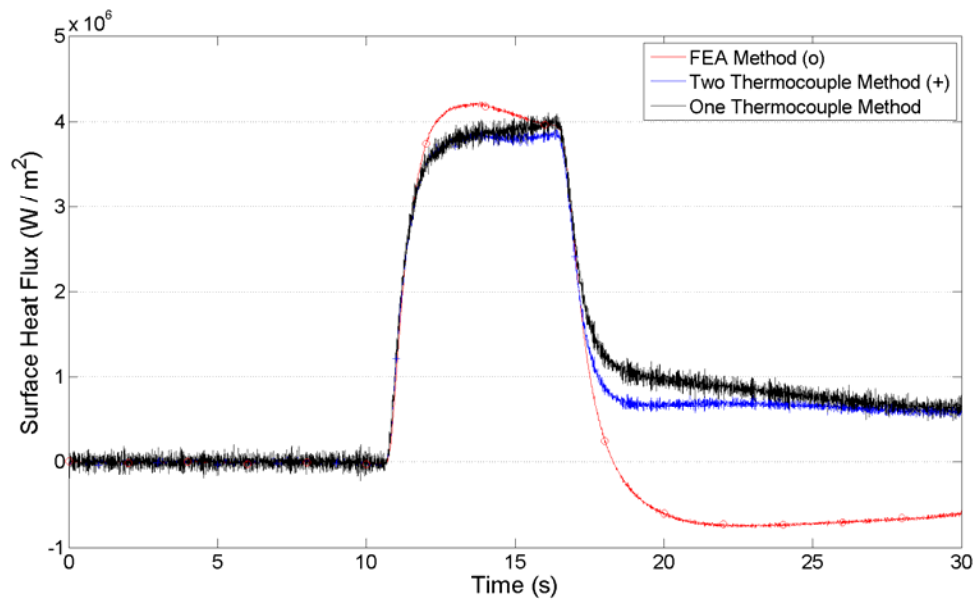


Figure 4-72 – Comparison of Estimate Methods – 1/8” Probe, Two Probe, and FEA Predicting Surface Heat Flux (Exit, Run 36, 15 Feb. 2006)

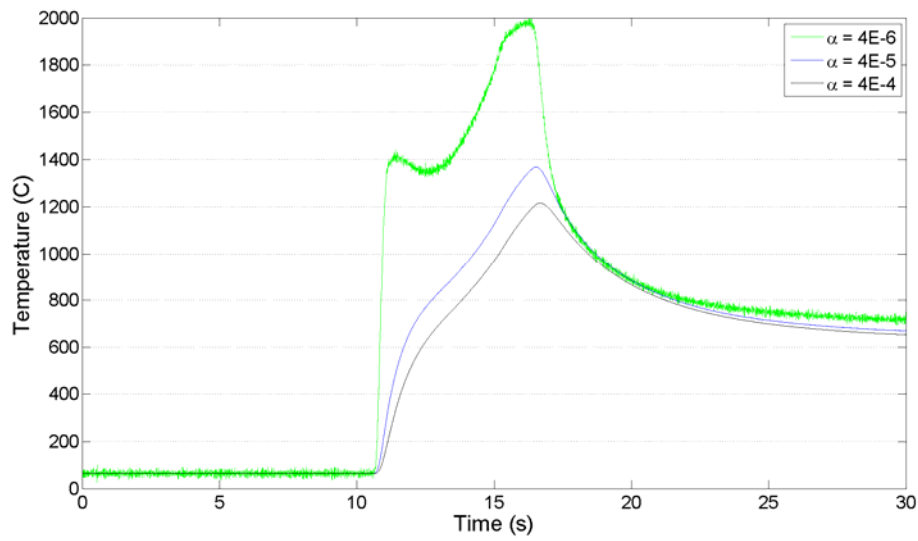


Figure 4-73 – Logarithmic Sensitivity of One Thermocouple Estimate – 1/8” Probe Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

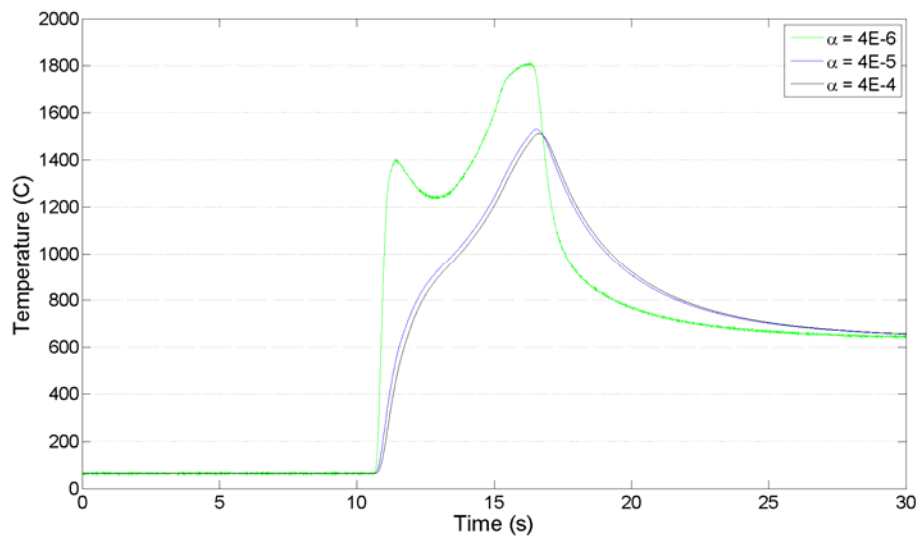


Figure 4-74 – Logarithmic Sensitivity of Two Thermocouple Estimate – Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

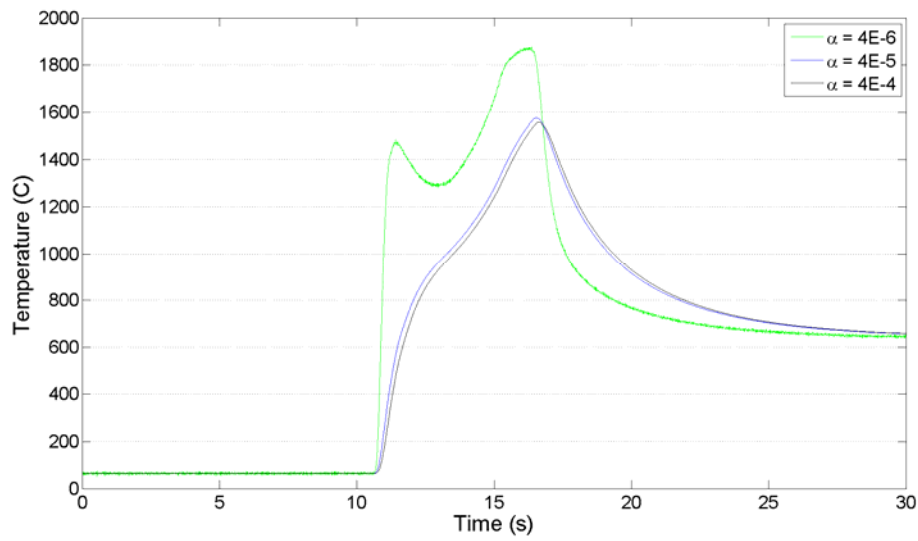


Figure 4-75 – Logarithmic Sensitivity of Finite Element Estimate – Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

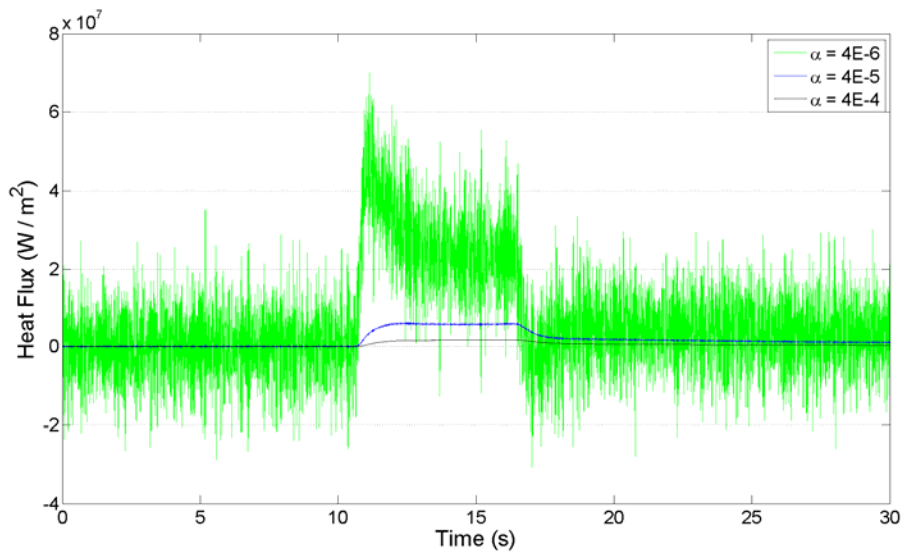


Figure 4-76 – Logarithmic Sensitivity of One Thermocouple Estimate – 1/8” Probe Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

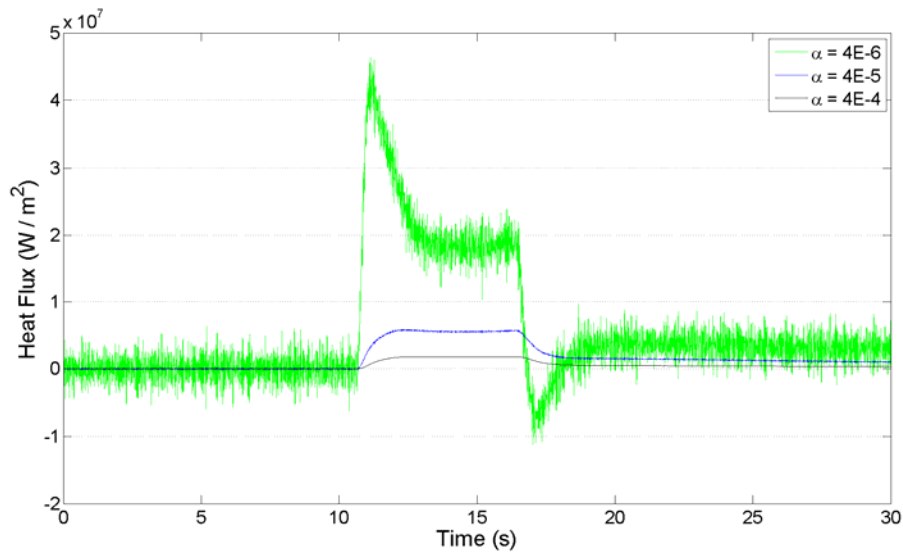


Figure 4-77 – Logarithmic Sensitivity of Two Thermocouple Estimate – Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

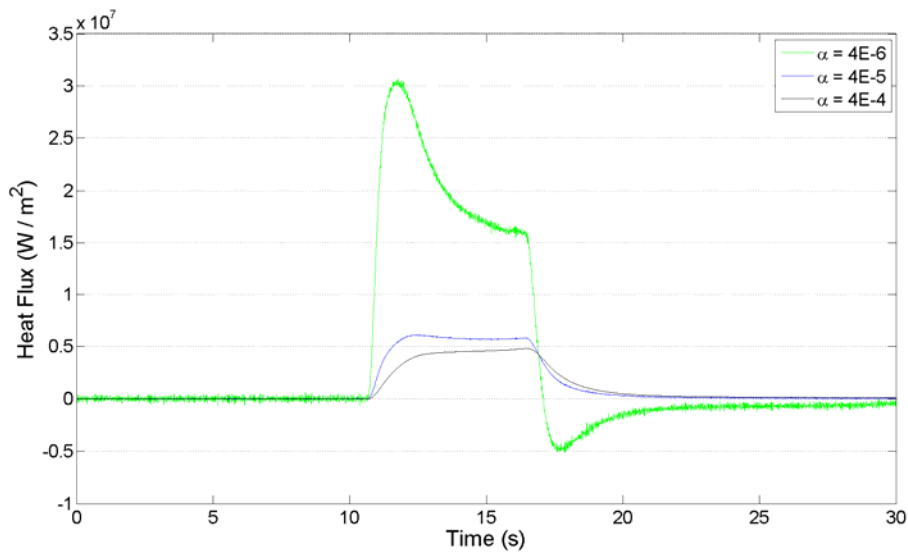


Figure 4-78 – Logarithmic Sensitivity of Finite Element Estimate – Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

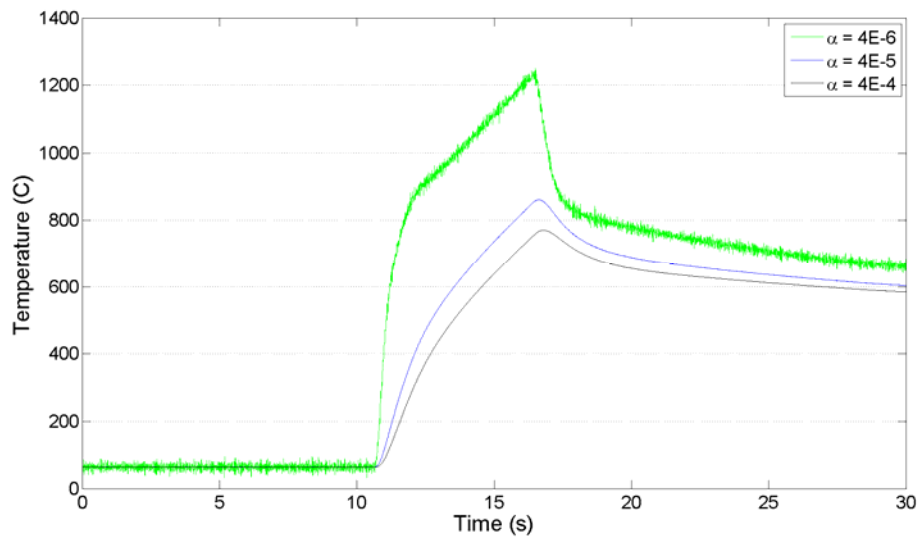


Figure 4-79 – Logarithmic Sensitivity of One Thermocouple Estimate – 1/8” Probe Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

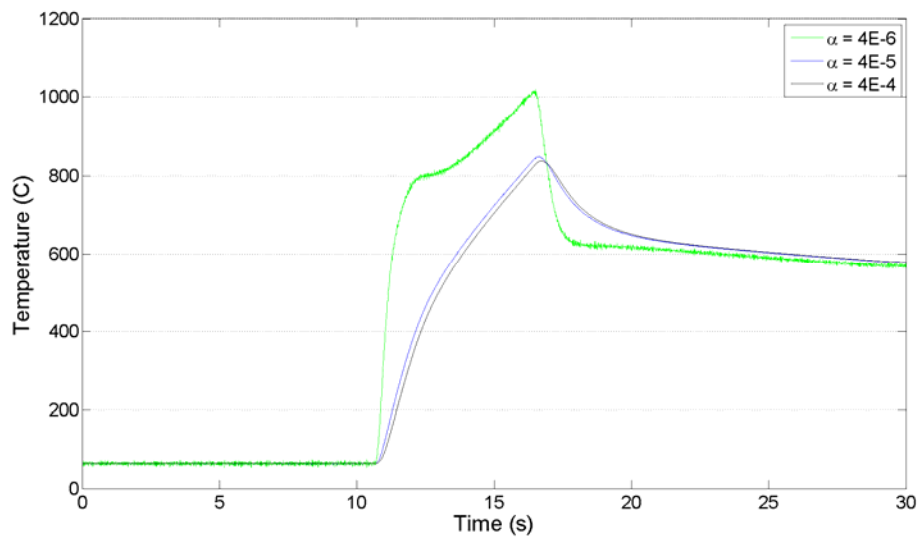


Figure 4-80 – Logarithmic Sensitivity of Two Thermocouple Estimate – Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

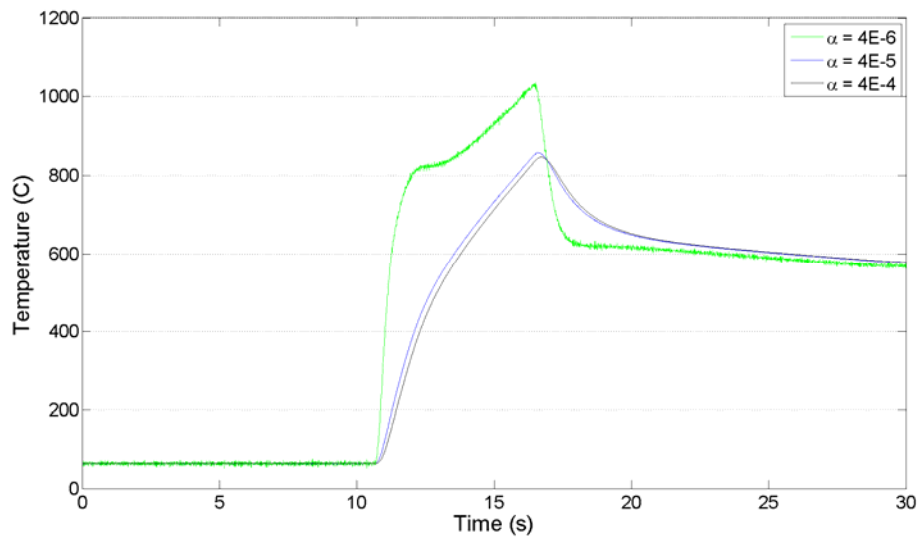


Figure 4-81 – Logarithmic Sensitivity of Finite Element Estimate – Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

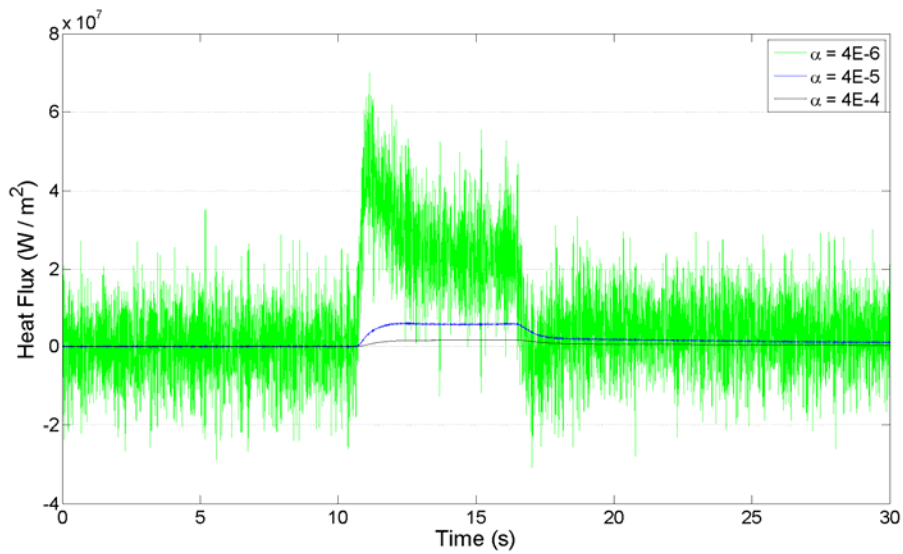


Figure 4-82 – Logarithmic Sensitivity of One Thermocouple Estimate – 1/8” Probe Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

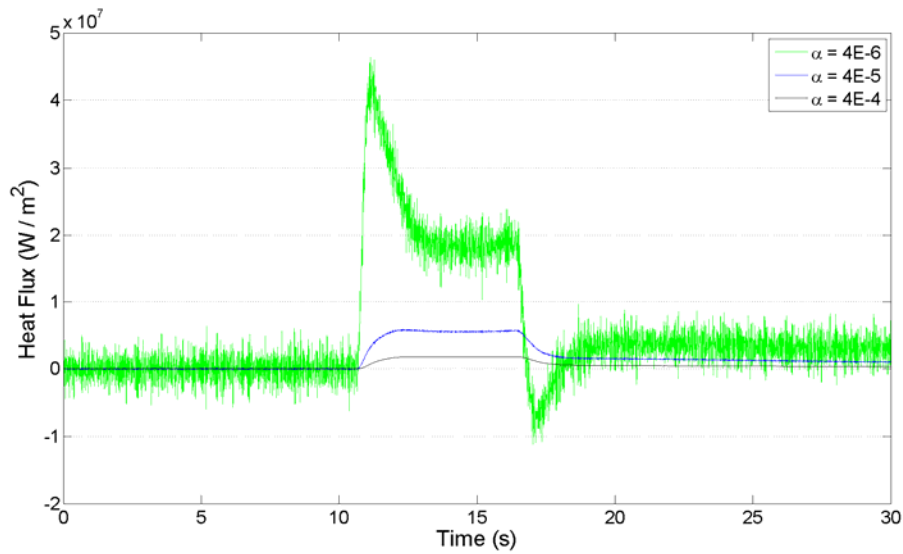


Figure 4-83 – Logarithmic Sensitivity of Two Thermocouple Estimate – Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

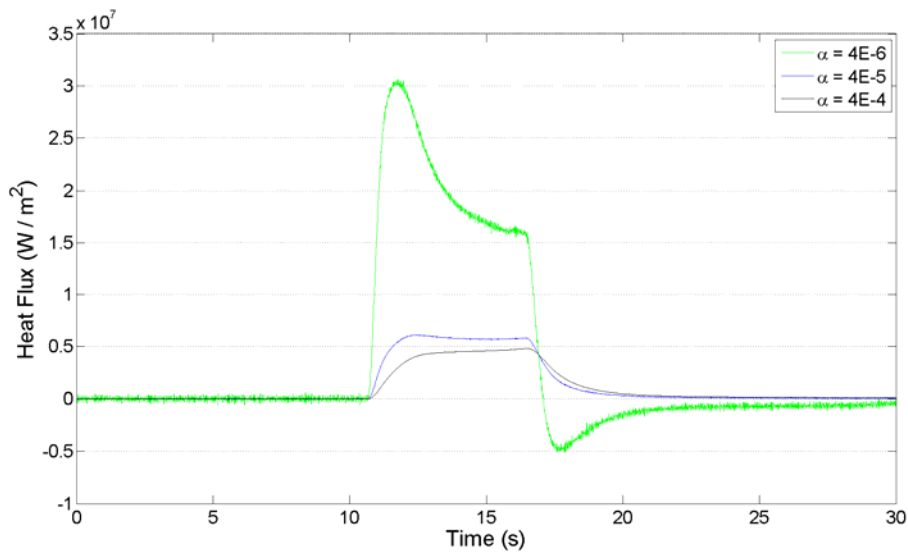


Figure 4-84 – Logarithmic Sensitivity of Finite Element Estimate – Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

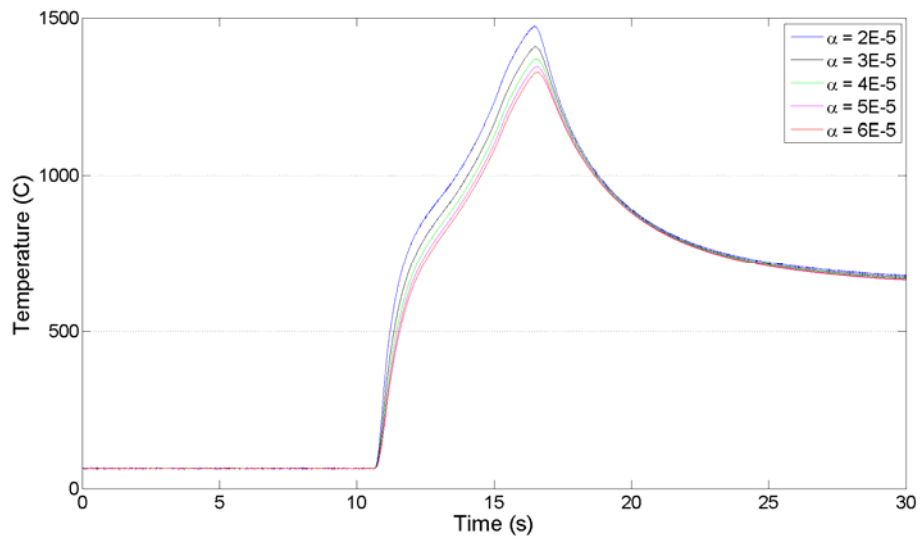


Figure 4-85 – Linear Sensitivity of One Thermocouple Estimate – 1/8” Probe Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

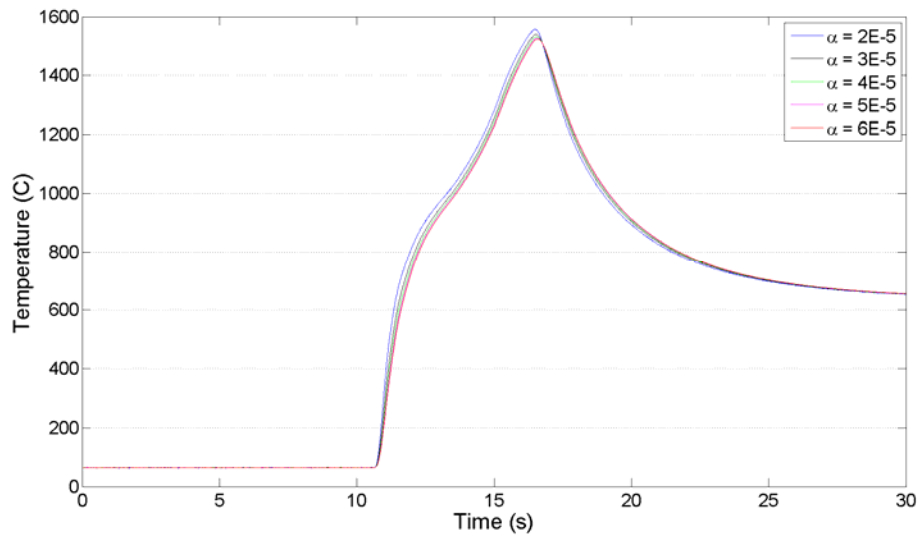


Figure 4-86 – Linear Sensitivity of Two Thermocouple Estimate – Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

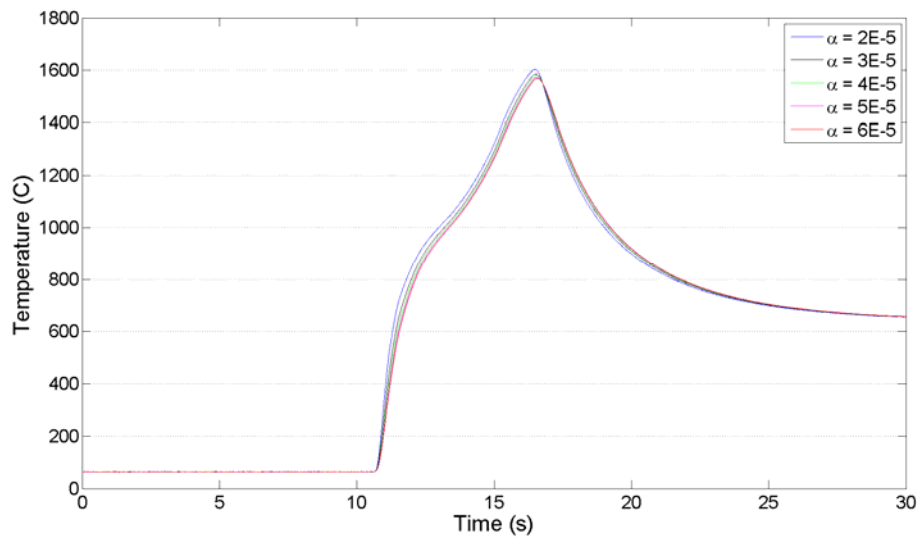


Figure 4-87 – Linear Sensitivity of Finite Element Estimate – Predicting Surface Temperature (Throat, Run 24, 7 Nov. 2005)

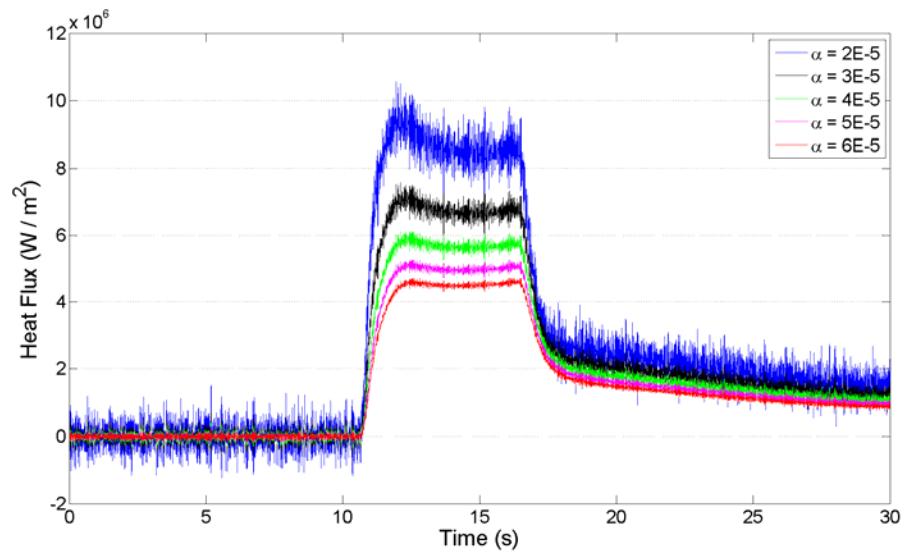


Figure 4-88 – Linear Sensitivity of One Thermocouple Estimate – 1/8” Probe Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

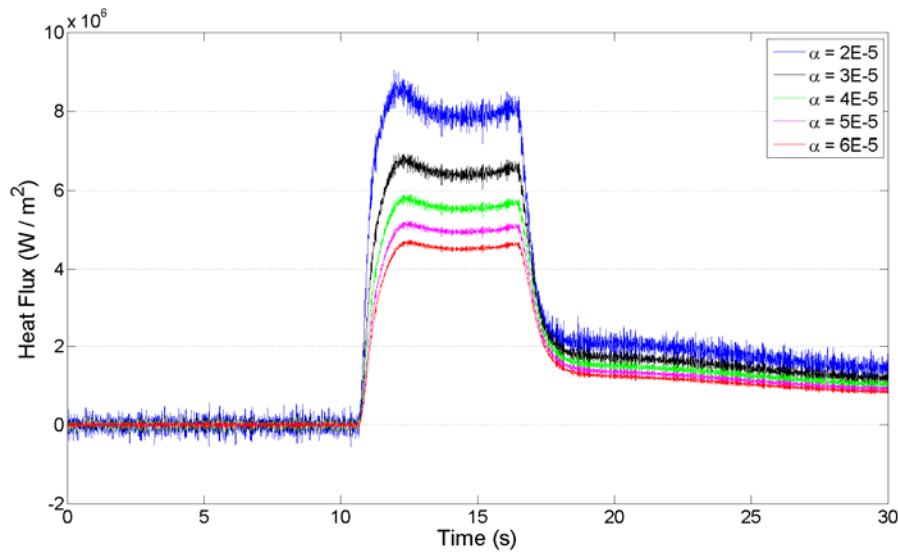


Figure 4-89 – Linear Sensitivity of Two Thermocouple Estimate – Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

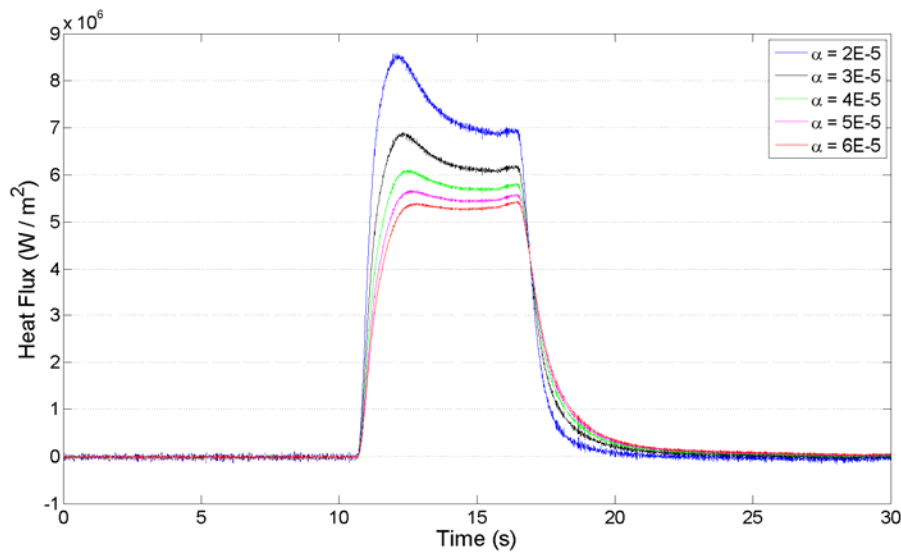


Figure 4-90 – Linear Sensitivity of Finite Element Estimate – Predicting Surface Heat Flux (Throat, Run 24, 7 Nov. 2005)

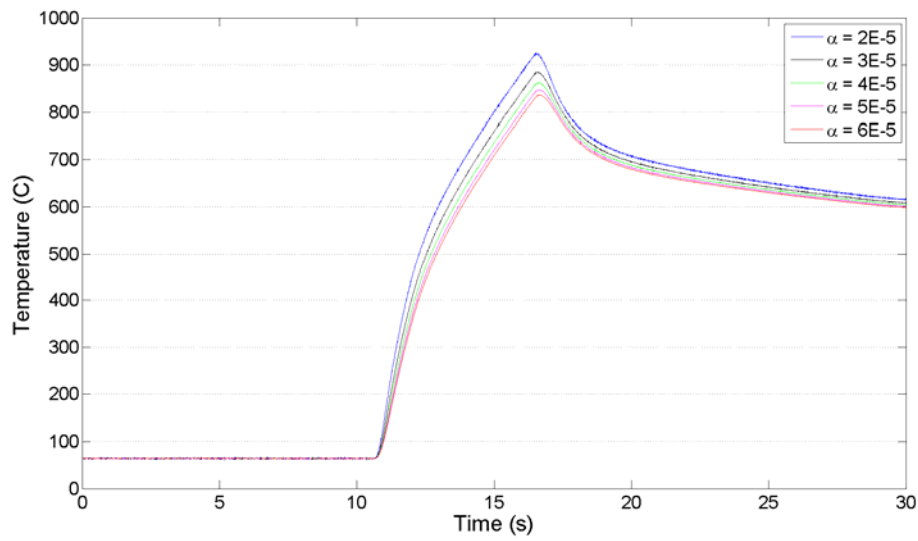


Figure 4-91 – Linear Sensitivity of One Thermocouple Estimate – 1/8” Probe Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

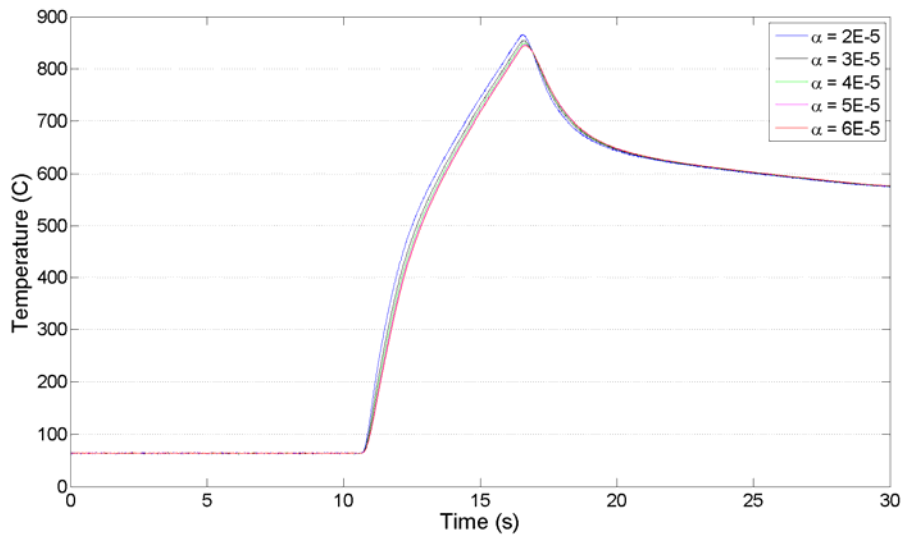


Figure 4-92 – Linear Sensitivity of Two Thermocouple Estimate – Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

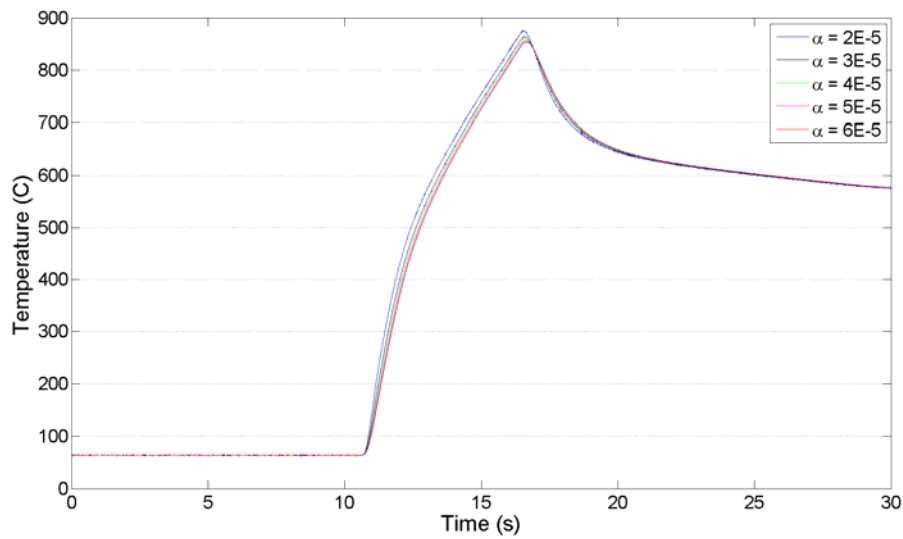


Figure 4-93 – Linear Sensitivity of Finite Element Estimate – Predicting Surface Temperature (Exit, Run 24, 7 Nov. 2005)

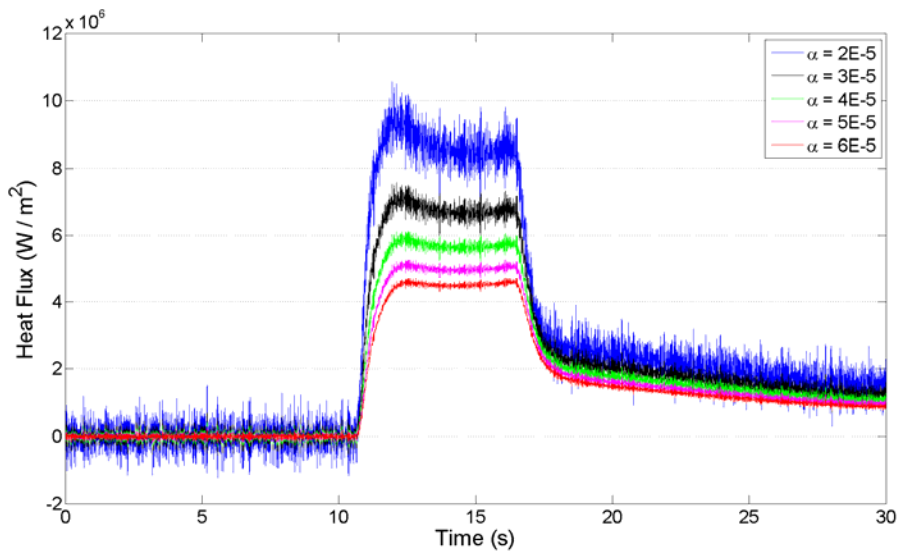


Figure 4-94 – Linear Sensitivity of One Thermocouple Estimate – 1/8” Probe Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

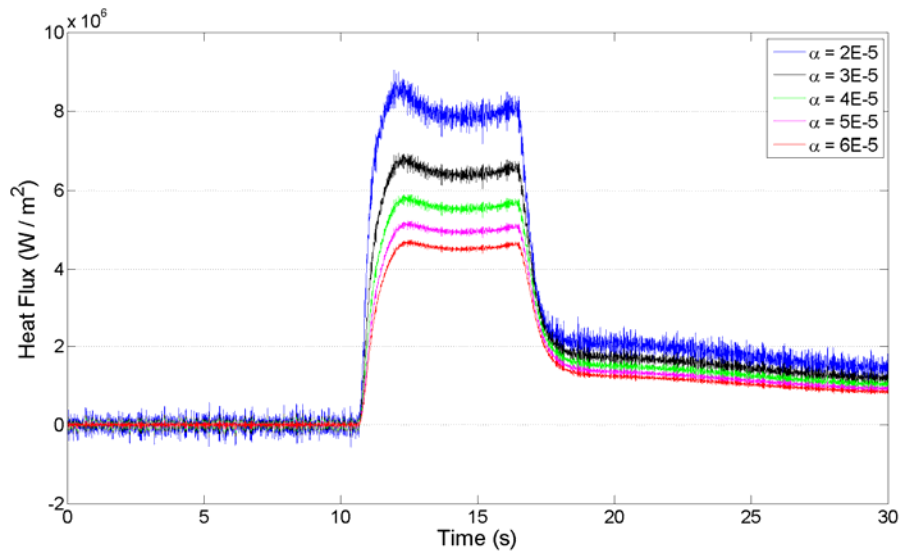


Figure 4-95 – Linear Sensitivity of Two Thermocouple Estimate – Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

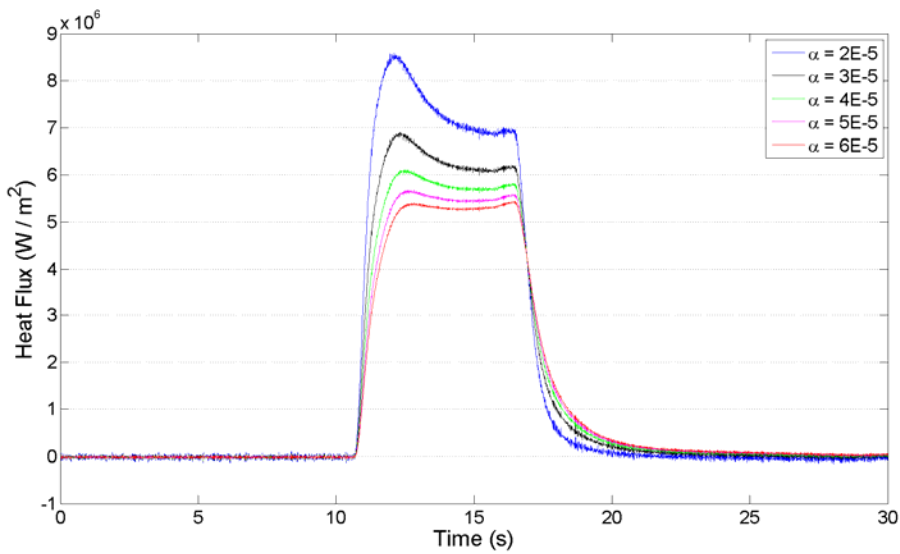


Figure 4-96 – Linear Sensitivity of Finite Element Estimate – Predicting Surface Heat Flux (Exit, Run 24, 7 Nov. 2005)

Chapter 5 – Discussion

This chapter provides descriptions and explanations of the results presented in Chapter 4. Each method of analysis will be considered for accuracy within the present context as well as for applicability to wider problems. The graphs of Chapter 4 will often be referred to as examples of the trends described. A final set of conclusions and recommendations for moving forward with this analysis will follow in Chapter 6.

Embedded Temperature Predictions

It is first important to identify the reason for largest difference between the results of Chapter 4. For this type of analysis, the fuel mixture ratios did not appear as critical as the geometry of the test section. For each run, the maximum 1/8" probe temperature at the throat varied by approximately 10% (1000°C for Run 24 in Figure 4-1, 900°C for Run 30 in Figure 4-25, and 1050°C for Run 36 in Figure 4-49). The same trend is clear at the nozzle exit plane (725°C for Run 24, 750°C for Run 30, and 875°C for Run 36 in Figures 4-13, 4-37, and 4-61, respectively). The larger difference is realized between the nozzle throat temperature and the exit plane temperature. This temperature difference at the axial locations is due to the compressible nature of the flow within the nozzle and the expanding gases caused by the nozzle as well as heat loss while the gases travel down the nozzle.

The nozzle geometry also affects the slab geometry assumption required for the one- and two-thermocouple equations developed by Dr. Frankel [2]. A slab geometry is one where the surface radius tends toward infinity, so the larger surface radius at the nozzle exit (1.5" compared to 1" at the throat) should reduce the error associated with this assumption. That this, in fact, occurred is evident in the graphs of Chapter 4. For Run 24, a comparison can be made between Figure 4-1 and Figure 4-13. Both figures demonstrate the prediction of the temperature at the 1/8" location made by the one-thermocouple equation, Equation (2-1), applied at the 1/4" probe. Figure 4-1 shows this prediction at the nozzle throat; Figure 4-13 shows it at the nozzle exit. In Figure 4-1, the curve giving the prediction is approximately halfway between the curves for the probe data, an underestimate of approximately 180°C. The predicted curve and the measured curve are nearly identical for much of the test in Figure 4-13, departing only approximately 25°C at the peak and then with increasing difference during the cooling period after the firing concluded. This trend is even more evident between Figures 4-25 and 4-37 (Run 30) where the estimate at the exit plane varies by no more than approximately 30°C, or less than 0.5% of the peak temperature. At the throat, the peak temperature predicted by the equation is almost 200°C, or nearly 20%, lower than the measured value. This influence of the nozzle geometry will continue to have a significant effect on the results of these methods, especially in those that use the slab assumption.

Surface Temperature Estimates – One Thermocouple

The next set of graphs in Chapter 4 illustrates the use of the one-thermocouple equation to predict the surface temperature, that is, applying Equation (2-1). Figures 4-2, 4-26, and 4-50 show the application of Equation (2-1) using the 1/4" probe data at the throat, while Figures 4-3, 4-27, and 4-51 use the 1/8" probe data taken at the throat. At the exit, the surface temperature predictions using the 1/4" probe data are shown in Figures 4-14, 4-38, and 4-62, and the predictions using the 1/8" probe data are given in Figures 4-15, 4-39, and 4-63. For each of the graphs where the 1/4" probe was used to predict the surface temperature at the throat (Figures 4-2, 4-26, and 4-50), the predicted value at the surface nearly falls on top of 1/8" probe data. This matches the trend seen in Figures 4-1, 4-25, and 4-49 where the one-thermocouple estimate using the 1/4" probe was significantly lower than the 1/8" probe data it was trying to predict. This suggests that applying the one-thermocouple equation to a geometry with significant curvature and axial conduction will produce an underestimate of the true temperatures. At the exit plane, however, the surface temperature predictions using Equation (2-1) with data from each of the two probes are very similar. The most noticeable differences between Figures 4-14 and 4-15, 4-38 and 4-39, and 4-62 and 4-63 are that the predictions using the deeper probe (the first listed figure of each pair) show a higher level of noise (evident on the graph as a thicker line) and a larger difference during the analysis of the cooling period. It is important to note, however, that the one-thermocouple equation seems to overestimate the temperature during the cooling period. This can also be seen in the estimates of the 1/8" location using the 1/4" probe (Figures 4-1, 4-13, 4-25, 4-37, 4-49, and 4-61). The one-thermocouple equation predicts lower temperatures than measured when applied during a heating process and higher temperatures than measured when applied to a cooling process. This suggests that the prediction trails the data – it does not accurately predict change and may be better suited for steady-state measurements or those with a very slow rate of change.

Figures 4-62 and 4-63 also present an interesting point of comparison for the other methods – the probe data curves cross during the time scale of the test. Since the nozzle is heated from the inner core permeating outwards, the 1/8" probe gets hotter sooner than the 1/4" probe, which is twice as far from the heat of the combustion gases. This is referred to as the penetration time of the heat. Cooling would also be expected to follow a similar pattern, that is, the 1/8" probe would register cooling sooner than the 1/4" probe since the larger radius associated with the deeper probe translates to a greater mass which must cool down. All figures in Chapter 4 which include the probe data show the temperatures decreasing and converging within the analysis time, but only the exit plane probes for Run 36 show distinct separation in the temperatures as the nozzle cools. (The probes at the nozzle throat also show a slight cross and separation, but the occurrence is much more noticeable at the exit plane.) It is also expected that the nozzle surface should cool faster than the 1/8" probe, since it is exposed to the air. However, since a measurement of surface temperature was not taken during testing, this is a hypothesis.

Surface Temperature Estimates – Two Thermocouple

The next method used to estimate the surface temperature used both embedded probes in the same equation, Equation (2-3). Since all measured data were required to calculate the surface temperature, it was not possible to produce comparisons such as those used in the one-thermocouple analysis. A good deal was still determined from the single estimate available for each of the 6 data sets. The figures produced with this analysis are Figure 4-4, Figure 4-16, Figure 4-28, Figure 4-40, Figure 4-52, and Figure 4-64. Figure 4-4 is a clear example of the two-thermocouple estimate following the data. For this set of test data each probe shows a slight inflection within the temperature data, with the inflection being slightly more noticeable on 1/8" probe data. The predicted curve shows a similar inflection with even more emphasis than the 1/8" probe. Another physical aspect that is well-represented with these figures is the penetration time of the heat. As mentioned above, it is expected that the surface must experience a temperature rise before heat can be transferred into the nozzle itself. This results in a slight difference in the time of the peaks of each curve. The difference in peak times is very clear in Figure 4-64. The predicted surface temperature reaches its maximum approximately 0.5s before the first embedded probe, which, in turn, peaks about 0.5s before the next probe. Returning to the case where the probe temperature data crossed, Figure 4-64 includes the data as well as the two-thermocouple prediction of the surface temperature. Figures 4-62 and 4-63 did not show that the surface temperature should be lower than the two probes, but the two-thermocouple estimate shown in Figure 4-64 does include the expected drop in surface temperature. The surface temperature prediction even drops below the 1/8" probe data before the 1/8" probe data drops below the 1/4" probe data, as would be expected with the penetration time. The thermocouple response time also supports this observation.

Since the two-thermocouple equation is also based on a slab coordinate system, some judgment must be made about its accuracy to the cylindrical geometry present in the nozzle. Comparing figures from the same run but different locations (such as Figures 4-28 and 4-40), the predicted surface temperature may demonstrate a larger change from the 1/8" probe than is seen from the 1/4" probe to the 1/8" probe. Without further evidence, there is no way to verify this observation, much less attribute it to the curvature of the nozzle. This issue will be revisited later in the chapter when the temperature estimation methods are compared against one another.

Surface Temperature Estimates – Finite Element

The final temperature estimation method was the cylindrical finite element analysis, and was derived in Chapter 2 (Equation (2-9)). This method also required both sets of data to predict the surface temperature so no inherent comparisons can be made between the predictions and the data. The primary advantage of the FEA, however, is that it does consider the radius of the nozzle and does not require the assumption of a slab geometry. The results of this analysis are shown in Figures 4-5, 4-29, and 4-53 for the throat location and Figures 4-17, 4-41, and 4-65 for the exit location. Similar arguments can be made for the FEA prediction as for the two-thermocouple estimate. The same inflection

predicted in Figure 4-4 can be seen in Figure 4-5, the penetration time is evident on each figure, and the surface temperature prediction crosses the recorded temperature curves in a similar way. The significant advantages to the FEA method are that it includes cylindrical coordinates and can be expanded to include more data, should it become available. The derivation in Chapter 2 took less than a page, including explanations. Deriving a formula to include more data points or layers of dissimilar materials would be only slightly more complex. The form of Equation (2-9) is slightly more complicated than Equation (2-3), which governs the two-thermocouple estimate, but does not require heat flux estimates and heat flux rates like Equation (2-1) does.

A simple standard in analysis is that more data produces better results. Equation (2-1) requires only one input (temperature data for a single location); Equation (2-3) has two inputs (temperature data at two locations); Equation (2-9) requires temperature data at two locations, plus knowledge of the radius of curvature of the test section. While approximations may discount this rule-of-thumb in higher-order analysis, the fact remains that the FEA model is the only explicit representation of surface temperature currently available in cylindrical coordinates.

Comparison of Temperature Estimates

The last temperature figures for each run and location are direct comparisons of the methods on the same set of axes. The one-thermocouple prediction uses the 1/8" probe data since the 1/4" probe estimate was shown to be a significant underestimate for each of the sample cases. It is important to note again the differences between the results at the nozzle throat (Figures 4-6, 4-30, and 4-54) and results at the exit plane (Figures 4-18, 4-42, and 4-66). Based on the three results for the nozzle throat, the one-thermocouple estimate predicts the lowest surface temperature of the three methods while the FEA method predicts the highest. The one-thermocouple estimate can be seen to trail off higher than either of the other two methods during the cooling at the end of the run. The two-thermocouple and FEA predictions show significant agreement considering the slab assumption used for the two-thermocouple equation. The FEA solution does tend slightly toward the extreme (higher during heating, lower during cooling) but with variation from the two-thermocouple results by less than 50°C (3%). It seems the issue discussed with the two-thermocouple solution of a slight increase in the spacing between surface and 1/8" probe compared to 1/8" probe and 1/4" probe is a natural jump caused more by the higher temperatures and having less to do with the curvature of the nozzle. It is also possible that this difference is caused by axial heat transfer, which was assumed negligible, but which may play a significant role for the deeper probe.

At the exit plane, the results match even more closely with approximately 20°C (2%) difference between any two predictions at the maximum temperature. However, the one-thermocouple prediction still tends higher than the other methods during the cooling period. In addition, the one-thermocouple estimate predicts the highest peak temperature for both Run 24 (Figure 4-18) and Run 36 (Figure 4-66). For Run 30 (Figure 4-42), the FEA method predicts the highest peak temperature just as it did for the throat results.

The excellent agreement of the results at the exit plane suggests that the radius of curvature of the nozzle has very little effect at that axial plane. While more studies would need to be conducted to confirm these results, it seems that a ratio of probe spacing to internal radius of less than 10% may be an acceptable first-pass condition for the use of the slab-coordinate methods applied to a cylindrical geometry.

Heat Fluxes

Since temperature is a measured quantity at the probe locations, it is the easiest to compare directly. However, many engineering design processes, computational fluid dynamics (CFD) codes, and other software packages also require a surface heat flux as either a known quantity an input parameter. The one-thermocouple and two-thermocouple methods use a Taylor series approach to calculate surface heat flux in a manner similar to the calculation of surface temperature. The FEA method adapts Fourier's Law to compute the surface heat flux. A discussion and comparison of these methods will illustrate advantages and disadvantages to each method.

Much of what has been said regarding the temperature measurements can be applied to the calculated heat flux values. Each temperature prediction technique corresponds to a method of calculating surface heat flux. The one-thermocouple method uses the Taylor series expansion around a single probe location to develop a relationship for the surface heat flux as given in Equation (2-2). Two Taylor series and two probes are used to generate Equation (2-4) for the two-thermocouple method. The FEA method, Equation (2-10), relies on the calculated surface temperature as well as the measured data from the two probes to predict the surface heat flux.

Embedded Heat Flux Predictions

As with the temperature estimates, the first comparison to be made for heat flux uses the one-thermocouple equation applied to the 1/4" probe to predict the heat flux at the 1/8" location. Rather than comparing the prediction of the Taylor series to measured data, however, the Taylor series estimate must be compared to the integral definition of local heat flux for slab coordinates, Equation (2-5). The integral is computed for use within the equations, so the issue is not with the calculation. The problem with applying the comparison is that both the prediction (the Taylor series) and the exemplar (the integral formulation) assume a slab coordinate system. The most that can be said about the comparison, then, is that the two definitions may not be consistent, but no judgment can be made as to which calculation is more affected by the assumption.

In the comparison of the integral relationship for heat flux at the 1/8" location to the Taylor series prediction using the 1/4" probe data, Figures 4-7, 4-31, and 4-55 represent the throat location, and Figures 4-19, 4-43, and 4-67 represent the exit plane location. From the graphs of the throat heat flux, a pattern similar to the one noted for temperature estimates is evident. The predicted heat flux (Taylor series) is smaller than the integral relationship. In Figure 4-31, the peak of the Taylor series approximation is near 6.5 MW

/ m² (neglecting noise) while the integral relation calculates a maximum heat flux of 8.5 MW / m². This underestimate does not appear significantly in the exit plane heat flux predictions, as the Taylor series approximation often overlaps and obscures the integral relation. These comparisons highlight a key aspect of the heat flux calculations – even with filtered data, the Taylor series approximations are noisy. This is especially true of the one-thermocouple equation since it includes the second temporal derivative term. No additional filtering of the results was done so that noise level could be evaluated as a criterion for comparison. Applying another filter (or applying the same Gaussian filter initially used on the data) would smooth the data for other uses but would remove an important consideration in the analysis.

Surface Heat Flux Estimates – One Thermocouple

The next estimate performed was the prediction of surface heat flux using the one-thermocouple equation and the 1/4” probe (Figures 4-8, 4-20, 4-32, 4-44, 4-56, and 4-68). The most obvious feature of this prediction is the noise. Where the noise level for the 1/8” probe location was 0.25 MW / m² or less, the noise for this calculation is 2 MW / m². A larger scale noise was also evident on the surface temperature predictions of this type, though it was only noticeable on the graphs as a slightly thicker line. This suggests an important trend with the one-thermocouple predictions: the larger the spatial step, the more noise that gets introduced into the estimate. This general rule probably holds for each of the estimation methods, but is only evident in the one-thermocouple equation because the data allow only one step size to be used for the other analysis methods. Such a trend is expected because of the η^4 truncation term.

Before concluding the discussion of the surface heat flux estimates using the 1/4” probe, it is important to compare the results to those produced by the 1/8” probe (Figures 4-9, 4-21, 4-33, 4-45, 4-57, and 4-69). The noise level, while still noticeable in the 1/8” predictions, is much lower than the 1/4” predictions. The factor under consideration, however, is the curve itself. For comparison, the average of the noisy data curve is considered. For Run 24 at the throat (Figures 4-8 and 4-9), the surface heat flux prediction for the 1/4” probe is between 6 and 8 MW / m² during the test firing (time between approximately 12 and 17 seconds). The prediction for the 1/8” probe is between 8 and 10 MW / m². At the throat, the 1/4” prediction is significantly lower than the 1/8” prediction, matching the results of the temperature analysis. Figures 4-44 and 4-45 show the two surface heat flux predictions at the exit plane for Run 30. Here, however, there is significant agreement between the average value of the surface heat flux based on the 1/4” probe and surface heat flux based on the 1/8” probe. Both grow sharply as the test begins, level off around 6 MW / m² for most of the firing, then peak at approximately 7 MW / m². The noise causes the prediction of Figure 4-44 to appear to spike much higher and also affects the scale of the graph, but the average value tracks well with the curve of Figure 4-45.

An interesting feature of the surface heat flux estimates which is first seen in the one-thermocouple 1/8” probe surface predictions is the shape of the heat flux curve. Since

each of these tests was conducted under similar conditions with the major change being only the propellant chemistry, it might be expected that the heat flux would follow a consistent pattern. On the contrary, the heat flux predictions exhibit several different characteristics. The run 24 throat prediction (Figure 4-9) shows an increase in heat flux at the start of the firing, a dip, and then followed by a rise toward the end of the firing before dropping off again after the firing is complete. The surface heat flux is shown to increase with time at the throat of the nozzle during the run 30 firing (Figure 4-33), even showing two minor peaks during the test. The prediction for Run 36 at the throat (Figure 4-58) demonstrates a monotonic drop after the initial peak. The exit surface heat flux for each run (Figures 4-21, 4-45, and 4-69) does not exhibit a significant dip, but remains nearly constant throughout the firing. It is possible that the noise still present in the estimates masks some of the trends noted at the nozzle throat. It may also be that the geometric differences tend to cause a more even heat flux at the larger radius nozzle exit.

Surface Heat Flux Estimates – Two Thermocouple

The two-thermocouple method for predicting surface heat flux are presented in Figures 4-10, 4-22, 4-34, 4-46, 4-58, and 4-70. Where noise was a major factor in the one-thermocouple estimates, it is only slightly noticeable in these graphs. Some fluctuations can be seen, but they appear more as a thickened line than jagged peaks and valleys. The shapes observed in the one-thermocouple estimates are clearer in the two-thermocouple predictions because of the lower noise levels. The exit plane of Run 24 (Figure 4-22) displays a dip similar to, but smaller than, the dip seen in either Figure 4-9 for the one-thermocouple prediction or Figure 4-10 which uses the two-thermocouple method. The rising heat flux with minor peaks and valleys is clear in both the throat and exit estimate for Run 30 (Figures 4-34 and 4-46). For Run 36 the curve of Figure 4-70 remains nearly constant, which does not follow the trend of Figure 4-58 where the heat flux decreases with time. The two-thermocouple prediction relies on the same slab assumption as the one-thermocouple method and so is still influenced by the radius of curvature.

Surface Heat Flux Estimates – Finite Element

The final surface heat flux prediction is based on the cylindrical FEA method and Fourier's law. The noise of this set of results is the smallest of the methods. The method does, however, seem to produce predictions that are of a different nature than either of the other two methods. As a first example, Figure 4-11 shows the surface heat flux estimate for Run 24 at the throat. Where both Figures 4-9 and 4-10 show two nearly equal peaks, Figure 4-11 predicts that the first peak is significantly lower than the second with a very small decrease in heat flux during the time between the peaks. The clear periodic nature seen in Figures 4-33 and 4-34 for the throat prediction for Run 30 is damped in Figure 4-35 where it might go unnoticed were one not looking for it. Finally, the sharp decline in heat flux present in Figure 4-57 and 4-58 for Run 36 at the throat is a smoother and more gradual decrease in Figure 4-59. The three graphs related to the nozzle exit (Figures 4-23, 4-47, and 4-71) also tend to be more rounded, with sharp corners eliminated.

Comparison of Heat Flux Estimates

As with the temperature predictions, the final heat flux graph for each combustor condition is a comparison graph containing the surface heat flux predictions of each of the three methods (the one-thermocouple estimate based on the 1/4" probe was eliminated due to excessive noise within the estimate). Immediately apparent in all six graphs (Figures 4-12, 4-24, 4-36, 4-48, 4-60, and 4-72) is the agreement between the one- and two-thermocouple predictions and the difference in the FEA prediction. For the exit plane estimates, the FEA prediction matches well during the firing but during the cooling period drops well below the other predictions. For the predictions at the throat, the FEA method produces a curve significantly higher than the two slab-coordinate methods. The sharp decrease seen in the exit plane estimates using the FEA method are actually more physically correct. As mentioned above, after combustion the surface should cool faster than the inner probe locations, so heat should begin to flow from the inside toward the surface, or opposite the direction it had been flowing during the firing. Figure 4-72 shows this net outflow of heat while Figures 4-24 and 4-48 show that the heat flux is dropping but has not reached the condition where it has changed direction. It is also possible that the numerical application of Fourier's law causes an overshoot of the actual heat flux. The misapplication of Fourier's law could explain both the much higher heat flux during heating for the throat-location graphs (Figures 4-12, 4-36, and 4-60) as well as the much more extreme cooling flux for the nozzle exit graphs. However, the error associated with Fourier's law would be expected to show the same overestimate during each heating cycle and underestimate during each cooling cycle, rather than just heating for one radius of curvature and cooling for another.

Sensitivity Analysis – Logarithmic

The first set of sensitivity plots in Chapter 4 is those with a logarithmic change in the thermal diffusivity, that is, a change in the order of magnitude. Three values of α were considered: $4\text{E-}6 \text{ m}^2 / \text{s}$, $4\text{E-}5 \text{ m}^2 / \text{s}$, and $4\text{E-}4 \text{ m}^2 / \text{s}$. These reflect the nominal case ($4\text{E-}5 \text{ m}^2 / \text{s}$) and a change of one order of magnitude in each direction. Each estimation method was run for each value of the thermal diffusivity, with results plotted by analysis type.

The first three figures in this section (Figures 4-73, 4-74, and 4-75) contain the predicted surface temperature curves at the throat. Immediately obvious is that the curve for $\alpha = 4\text{E-}6 \text{ m}^2 / \text{s}$ is much higher than the other curves for all three graphs. The peak for the $\alpha = 4\text{E-}6 \text{ m}^2 / \text{s}$ curve is 600°C higher than the nominal curve for the one-thermocouple estimate. For both the two-thermocouple estimate and the FEA prediction, it is nearly 300°C higher. The curve for $\alpha = 4\text{E-}4 \text{ m}^2 / \text{s}$ is lower than the nominal curve on all graphs but is almost 200°C lower on the one-thermocouple graph and is within 20°C for the other two cases. These comparisons immediately suggest that the one-thermocouple method is much more sensitive to thermal diffusivity than either of the other methods.

This is clear from Equation (2-1), as well, since α appears in every term of the approximation. Equations (2-3) and (2-9) depend on α in a single term each.

Not only does the magnitude of the $\alpha = 4\text{E-}6 \text{ m}^2 / \text{s}$ curve change significantly, so, too, does the shape. Where the nominal curve is smooth and monotonic during the firing, the $\alpha = 4\text{E-}6 \text{ m}^2 / \text{s}$ curve shows a small peak near the ignition time, a drop during the early stages of the firing, and finally a larger peak at approximately the time of maximum temperature in the other curves. This pattern can be seen in each of the three figures. Where it was said above that the one-thermocouple analysis trailed the data during cool down, each analysis is anticipating the data. This is a result of the equations, all of which include α in the denominator of any term in which it appears. When the magnitude of α drops significantly, any term with α in the denominator more significantly dominates the equation. Since this term is the temporal derivative, the equation over-predicts changes, causing the extreme reactions.

Surface heat flux for the throat is addressed in the next set of plots (Figure 4-76, 4-77, and 4-78). Again, the $\alpha = 4\text{E-}6 \text{ m}^2 / \text{s}$ prediction is much higher than either other value of α and exhibits higher peaks and valleys. Also noticeable in these figures is the noise amplification that accompanies the decrease in thermal diffusivity, especially in Figures 4-76 and 4-77. This is also due to the nature of the equations, Equation (2-2) and (2-4). Each q'' term includes an implicit dependence on α (from Equation (3-4)) in addition to any explicit dependence within the equation. Heat flux is clearly affected even more strongly by the magnitude of the thermal diffusivity than temperature is.

Figures 4-79, 4-80, and 4-81 show the surface temperature prediction at the exit plane based on each of the three methods. While many of the same trends mentioned for the throat apply to the exit plane, one trend that was present in the throat data is clearer at the exit plane: an almost vertical drop in temperature immediately after the peak for the $\alpha = 4\text{E-}4 \text{ m}^2 / \text{s}$ case. At the throat, this drop was significant but smooth. At the exit plane, the temperature reaches its maximum, and then falls nearly 400°C in a few thousandths of a second before leveling off for the duration of the test. This is more evidence of the general methodology which effectively anticipates changes and causes an over-prediction of the results.

The heat flux curves of Figures 4-82, 4-83, and 4-84 exhibit the same behavior as discussed for Figures 4-76, 4-77, and 4-78. They verify that the 0.5" internal radius of curvature difference is dwarfed by an order-of-magnitude change in thermal diffusivity, as are most other factors. The nature of all of the heat flux curves (shape, magnitude, noise levels) which were important in discussing the effect of each method are masked by the large changes caused by the large thermal diffusivity difference.

Sensitivity Analysis – Linear

The final group of figures in Chapter 4 compares results of a linear change in thermal diffusivity. Five values of α were considered: $2\text{E-}5 \text{ m}^2 / \text{s}$, $3\text{E-}5 \text{ m}^2 / \text{s}$, $4\text{E-}5 \text{ m}^2 / \text{s}$ (the

nominal value), $5\text{E-}5 \text{ m}^2 / \text{s}$, and $6\text{E-}5 \text{ m}^2 / \text{s}$. This type of change could occur physically in a number of ways, the most likely of which are different ambient conditions raising and lowering the steady-state temperature slightly or a shift in properties over time due to thermal fatigue.

Throat surface temperature estimates are shown in Figures 4-85, 4-86, and 4-87. All 5 curves match well, with the one-thermocouple predictions (Figure 4-85) showing the maximum difference of 25°C at the peak. This supports the conclusion that the one-thermocouple estimate is the most sensitive to changes in α , even with linear changes barely noticeable using other methods. These curves also suggest that a small error in α would not significantly affect surface temperature predictions using either the two-thermocouple or the FEA method.

Heat flux, however, requires a more accurate value of α in order to provide a more confident estimate. The spread in the five curves of Figures 4-88, 4-89, and 4-90 suggest that heat flux is affected more directly by even small changes in α . As mentioned in the discussion of logarithmic changes, this is due to the nature of the equations, which require α in every term (implicitly in the case of the FEA method). Another trend seen in the logarithmic sensitivity curves is that heat flux increases for a decreasing value of α . While it stays on the same order of magnitude, the predicted heat flux is seen to almost double, from $10 \text{ MW} / \text{m}^2$ to $18 \text{ MW} / \text{m}^2$ from the nominal case to the $2\text{E-}5 \text{ m}^2 / \text{s}$ case in Figure 4-88. The change is less extreme in the other two figures but exhibits the same pattern.

Figures 4-91 – 4-93 and 4-94 – 4-96 show surface temperature and surface heat flux, respectively, for the exit plane. The spread in the peak temperatures increases to nearly 100°C for the one-thermocouple case but decreases for both the two-thermocouple and FEA cases. Heat flux exhibits the same general change as seen at the throat with no noticeable change in relative difference between the curves.

A similar sensitivity analysis was performed for the radial positions of the embedded probes. This analysis indicated that for a change in radial spacing of $\pm 10\%$, no discernable difference existed in the results. Sensitivity to radius of curvature is inherent to the consideration of the two axial planes of the nozzle.

General Discussion

Two major assumptions govern the mathematical models. The first assumption is that all transport properties of the graphite nozzle are constant. Constant properties are required for simplification of every equation in Chapter 2. The question arises whether this assumption is valid over such a large range of temperatures (as much as 1600°C in several test runs). As seen in the linear sensitivity plots in Chapter 4 (Figures 4-85 – 4-96), small changes in the thermal diffusivity during the entire analysis period have only a small effect. Using an average value of the thermal diffusivity would still allow for

predictions accurate within 15% for the one-thermocouple equation and less than 5% for the two-thermocouple and FEA methods.

The second major assumption inherent to every equation in Chapter 2 is one-dimensional heat transfer. One-dimensional heat transfer simplifies the governing equations as the assumption of constant properties did but is required for another reason as well. A two- or three-dimensional analysis is not practical for this test. Expanding to a two-dimensional model would require several more pairs of thermocouples along the axis of the nozzle. To include angular effects, thermocouples would have to be placed at the test depths (1/8" and 1/4") at locations around the circumference of the nozzle. While future tests can consider this, the existing test apparatus was not built to include that instrumentation. With no test data and no way to gather data at new locations on this test platform, the equations which form the basis for the mathematical model must be simplified to their one-dimensional radial form, as was done in Chapter 2.

Besides the constraints of the experiment in the placement of thermocouples, the thermocouples themselves must be considered as sources of error. As with any measurement devices, thermocouples have a response time, or a time required from change in input to a change in output. For a thermocouple, the response time is affected by the type of thermocouple, including the manufacturing process, and the thermal contact it makes with the test location. For the thermocouples embedded in the nozzle, the manufacturer quotes a response time of 2 seconds in air and 0.1s in water. Since the thermocouples are sealed into place with a graphite paste the response time can be assumed to be approximately closer to that in water than in air. It is estimated that it may take as long as 0.5s to report a change in temperature using the test thermocouple.

These assumptions affect every equation and analysis method in some way – from a mathematical simplification to variation in the data used in the equations. It is the nature of experimentation that such errors exist, but the relatively small differences they make do not change the overall trends seen within the results. Judgments can still be made about advantages and disadvantages of the various methods.

Each method used to determine surface temperature and surface heat flux offers specific benefits and drawbacks to its application. The first method used, the one-thermocouple method in slab coordinates, is the simplest of the methods. It requires only one data set to produce its predictions. Based on the comparison graphs of Chapter 4, the estimate from the one-thermocouple equation is relatively inconsistent with the more complex methods. Because it requires only the one data set, however, the one-thermocouple method is more sensitive to the other parameters. The one-thermocouple curves for the different radii of curvature show the greatest difference of any of the methods. In the sensitivity analysis, the one-thermocouple equation showed the greatest spread for small changes in thermal diffusivity. The one-thermocouple temperature equation is the most computationally inefficient, relying on a calculated integral for the heat flux term, a temperature rate, and the rate of the heat flux. Calculating heat flux for the one-thermocouple method requires the second time derivative of temperature, which

introduces significant noise. Where only one temperature measurement can be taken, only the one-thermocouple equation can be applied and must be used for the prediction of surface conditions, but when more than one embedded temperature reading can be taken, the one-thermocouple method should not be the primary analysis method and should instead be replaced by one of the two-thermocouple methods.

The simplest two-thermocouple prediction technique is the one-dimensional slab coordinate equation based on the Taylor series expansion, Equations (2-3) and (2-4). This method requires a more complicated experiment since the number of thermocouple probes is doubled and the probes must be spaced correctly within the test material. However, this method shows less sensitivity to thermal diffusivity and changes in the radius of curvature than the one-thermocouple method. Computational savings in the method are balanced by the need for filtering of an additional data set, so the methods are comparable in calculation time. However, Equations (2-3) and (2-4) do not require any second derivative terms, so the noise level is reduced significantly, as mentioned in Chapter 5. Both this method and the one-thermocouple method have truncation error terms on the order of η^4 . By the nature of the equation, this indicates a truncation error beginning in the fifth or sixth decimal place, well below the precision of the thermocouples.

The final method, which also requires data from two thermocouples, is the cylindrical FEA. In addition to the two thermocouple measurements, the FEA solution requires knowledge of the radius of curvature. An additional advantage to the FEA method is that it can be expanded to include any number of thermocouples with only minor changes. This could prove useful for experiments that use composite or layered materials rather than solid pieces such as the graphite nozzle. The one-dimensional FEA equation can also be written for two or three dimensions, if data becomes available for a higher-dimension analysis. Equation (2-9) (FEA) is as computationally efficient as Equation (2-3) (slab two-thermocouple), requiring two filtered data sets but no numerical integration. Equation (2-10) (surface heat flux for the FEA method) is much more efficient than either of its counterparts in the slab coordinates, requiring only a numerical derivative. However, Equation (2-10) also produces results which are significantly higher than the other methods. This point of inconsistency requires further investigation beyond the scope of this thesis before any conclusions can be drawn. The approximations used for the spatial derivatives have a truncation error on the order of η^2 , but applying the same approximations in slab coordinates produced Equation (2-14), which is identical to Equation (2-3). The truncation error on the order of η^4 associated with Equation (2-3) suggests that the FEA truncation error may be of the same order. Whatever the truncation error, the noise level for the FEA method is very low.

All three methods considered in this thesis show promise to applications beyond a rocket nozzle. The two-thermocouple equations offer substantial benefits in noise reduction and a decrease in sensitivity to small changes in parameters without much sacrifice in terms of analysis time or effort. The significant benefit to the one-thermocouple equation is that it can be applied to situations where it is possible to place only a single probe. The

FEA equation is a more general two-thermocouple equation and could be applied to a larger class of experiments but may not offer the same small truncation error as the slab-coordinate equation. Each method has an area of analysis where it can be considered the best method. The one-thermocouple equations are best when only one measurement can be taken. The slab-coordinate equations offer known truncation errors for flat surfaces. The FEA solution ensures that radius of curvature is considered in the analysis. Applying all three at once, as was done in this thesis, provides a range in which the surface conditions are expected to fall. While accuracy can only be determined when the actual surface conditions are known, the agreement between the slab two-thermocouple equation and cylindrical FEA equation suggests that both offer very good accuracy for the type of predictions made for this thesis.

Looking forward to future experiments which may employ the methods considered in this thesis, several recommendations become apparent. The first is to use a material with well-known thermal transport properties or employ a method for determining these properties. One method of determining thermal diffusivity and thermal conductivity is to include more thermocouples within the test area and apply the prediction equations between them, as was done for the 1/4" probe predicting the 1/8" temperature in Chapter 4. The reason that method was not applied here was the radius of curvature and the high sensitivity of the one-thermocouple equation to even small changes in α . With at least three probes, however, any of the methods could be applied to the three known temperatures to determine thermal properties.

Another significant benefit for future work in this area is the development of Taylor series-style equations for cylindrical coordinates. The nature of cylindrical coordinates and the heat flux equation makes this equation difficult to produce in an explicit form. This new cylindrical Taylor series equation would answer the question about truncation error in the FEA solution, as well as expand the capabilities of the heat flux prediction.

Chapter 6 – Conclusions

A number of methods have been studied for the estimation of surface temperatures and heat fluxes based on one or two embedded thermocouple probes. Based on the results of the analysis in this thesis, it is expected that the predictions offer higher accuracy and greater consistency when obtained with one of the two-thermocouple methods. The one-thermocouple prediction should be reserved for applications where two thermocouples cannot be properly mounted in the test article. As a general guideline, the two-thermocouple method in slab coordinates should not be considered accurate for cylindrical coordinates when the radial step size between measurements exceeds 10% of the radius of curvature of the test article. For those cases, the axisymmetric FEA model currently offers the only method for computing surface conditions. For temperature predictions, the results from the different methods are in good agreement and suggest an error of no more than 10% for the case where the slab approximation was deemed a good estimate of the curved geometry. Heat flux predictions show a wider range and require further investigation to determine of an estimate of their accuracy. A sensitivity analysis showed that small changes in the thermal diffusivity do not cause large variations in the predicted surface conditions, but that heat flux was more sensitive to change than the temperature estimates. As new methods are developed, the ones suggested in this thesis will certainly be re-evaluated and refined. This is an active area of research and should remain so as engineering problems continue to require the estimates provided by these methods.

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APPENDICES

Appendix A – List of Symbols and Abbreviations

Symbol or Abbreviation	Meaning
A	Area (m^2)
c_p	Specific Heat ($\text{J} / \text{kg K}$)
$\dot{E}$	Energy Storage Rate (W)
FEA	Finite Element Analysis
k	Thermal Conductivity ($\text{W} / \text{m K}$)
L	Length (m)
N	Total number of data points
$\dot{Q}$	Heat Transfer Rate (W)
q''	Heat Flux (W / m^2)
r	Radius (m)
r_0	Radius to Surface (m)
Δr	Spatial Step Size (m)
T	Temperature ($^{\circ}\text{C}$)
t	Time (s)
Δt	Time Step Size (s)
u	Integration Variable
V	Volume (m^3)
x	Distance from Surface (m)
Δx	Spatial Step Size (m)
α	Thermal Diffusivity (m^2 / s)
η	Spatial step size (m)
ρ	Density (kg / m^3)
ω_c	Cutoff Angular Frequency (rad / s)

Appendix B – MATLAB Code Used for Analysis

Program 1: Filtering the measured data

```
function data_filtering

% This program is designed to take data from an input text file, filter
% it
% with the Gaussian filter defined in the function below, and produce a
% new
% text file with the filtered data.
%
% As written, the program expects the input data file without headers
% and
% including five columns of data. The first column is the time at
% which
% each measurement was taken. The other four columns correspond to
% specific probe locations.

% Input Section

% MATLAB reads the column data from the .dat file into rows of the A
% matrix
fid = fopen( 'data/data76.dat' ); % Opens data file
A = fscanf( fid, '%g %g %g %g %g', [ 5 inf ] ); % Reads 5 columns
% of data
fclose( fid ); % Closes data file

A = A'; % Transposes the data into columns

% Assignment Section
%
% Takes input data and assigns it to specific vectors
time = A(:,1);
nozzle_throat_1_8_temp = A(:,2);
nozzle_throat_1_4_temp = A(:,3);
nozzle_exit_1_8_temp = A(:,4);
nozzle_exit_1_4_temp = A(:,5);

% Define Parameters for Filtering
delta_t = 0.001; % Time step for calculations - may be different than
% data
time2 = 0:delta_t:30;
N_max = length( time );
N2_max = length( time2 );
f_cutoff = 30; % [Hz]
omega_cutoff = 2*pi*f_cutoff; % [rad / s]

% Filter each data set from the input
filtered_nozzle_throat_1_8_temp = gauss_filter( nozzle_throat_1_8_temp,
N_max, N2_max, time, time2, omega_cutoff );
filtered_nozzle_throat_1_4_temp = gauss_filter( nozzle_throat_1_4_temp,
N_max, N2_max, time, time2, omega_cutoff );
```

```

filtered_nozzle_exit_1_8_temp = gauss_filter( nozzle_exit_1_8_temp,
N_max, N2_max, time, time2, omega_cutoff );
filtered_nozzle_exit_1_4_temp = gauss_filter( nozzle_exit_1_4_temp,
N_max, N2_max, time, time2, omega_cutoff );

% Output Section
%
% Writes output data to text file in similar format to the input file
fid = fopen( 'data/data76f.dat', 'w' );
for i = 1:1:N2_max
    fprintf( fid, '%16.8g\t%16.8g\t%16.8g\t%16.8g\t%16.8g\r\n',
time2(i), filtered_nozzle_throat_1_8_temp(i),
filtered_nozzle_throat_1_4_temp(i), filtered_nozzle_exit_1_8_temp(i),
filtered_nozzle_exit_1_4_temp(i) );
end
fclose(fid);

```

```

% Filtering subroutine
function b = gauss_filter( a, Na, Nb, timea, timeb, omega_cutoff )

```

```

% Variables:
% b = filtered vector
% a = unfiltered vector
% Na = length of a (number of entries in a)
% Nb = length of b
% timea = vector of measurement times for a
% timeb = vector of measurement times for b
% omega_cutoff = cutoff angular frequency

```

```

% Preallocate for speed
numerator = zeros( Na, 1 );
denominator = zeros( Na, 1 );
b = zeros( Nb, 1 );

```

```

% Filtering loop
for j = 1:1:Nb
    for i = 1:1:Na
        numerator(i) = a(i) * exp( - ( ( timeb(j) - timea(i) )^2 *
omega_cutoff^2 ) / 4 );
        denominator(i) = exp( - ( ( timeb(j) - timea(i) )^2 *
omega_cutoff^2 ) / 4 );
    end
    b(j) = sum( numerator ) / sum( denominator );
end

```

Program 2: Applying the estimation methods

```

function surface_estimation

```

```

% Input Section

```

```

run_number = 24;
run_string = num2str( run_number );

filename1 = strcat( 'data/data', run_string, 'f.dat' ); % Input data
file
filename2 = strcat( 'data/data', run_string, 't.dat' ); % Output data
file
filename3 = strcat( 'data/data', run_string, 'e.dat' ); % Output data
file

% Extension 'f' indicates filtered data which has been output from the
% filtering function
% Extensions 't' and 'e' indicate throat and exit locations,
respectively

fid = fopen( filename1 ); % Opens data file
A = fscanf( fid, '%g %g %g %g %g', [ 5 inf ] ); % Reads 5 columns
of data
fclose( fid ); % Closes data file

A = A'; % Transposes the data into columns

time = A(:,1);
nozzle_throat_1_8_temp = A(:,2);
nozzle_throat_1_4_temp = A(:,3);
nozzle_exit_1_8_temp = A(:,4);
nozzle_exit_1_4_temp = A(:,5);

% Variable Definitions

r_throat = 1. / 39.37; % Measurements converted to meters for
calculations
r_exit = 1.5 / 39.37;
delta_r = 1/8 / 39.37;

delta_t = 0.001; % Time step matches the one used for data filtering
time2 = 0:delta_t:30;
N_max = length( time );
N2_max = length( time2 );

% Thermal transport properties determined independently
k = 120.;
alpha = 4e-5;

% Throat

% Use Slab One and Two Probe Formulas to Determine Surface Conditions
[ T_surface_throat_1_8, q_local_throat_1_8, q_surface_throat_1_8 ] =
one_thermocouple( nozzle_throat_1_8_temp, N2_max, delta_t, alpha, k,
delta_r );

```

```

[ T_at_1_8_throat_1_4, q_local_throat_1_4, q_at_1_8_throat_1_4 ] =
one_thermocouple( nozzle_throat_1_4_temp, N2_max, delta_t, alpha, k,
delta_r );
[ T_surface_throat_1_4, q_local_throat_1_4, q_surface_throat_1_4 ] =
one_thermocouple( nozzle_throat_1_4_temp, N2_max, delta_t, alpha, k, 2
* delta_r );
[ T_surface_two_thermocouple, q_surface_two_thermocouple ] =
two_thermocouple( nozzle_throat_1_8_temp, nozzle_throat_1_4_temp,
q_local_throat_1_8, q_local_throat_1_4, N2_max, delta_t, alpha, k,
delta_r );

% Finite Element Analysis
dTdt_1_8 = dfdt( nozzle_throat_1_8_temp, delta_t, N2_max );
T_surface_throat_FEA = nozzle_throat_1_8_temp + ( r_throat + 3 *
delta_r / 2 ) ./ ( r_throat + delta_r / 2 ) .* ( nozzle_throat_1_8_temp
- nozzle_throat_1_4_temp ) + delta_r / ( alpha * ( 2 * r_throat +
delta_r ) ) * ( ( r_throat + 3 * delta_r / 2 ) ^ 2 - ( r_throat +
delta_r / 2 ) ^ 2 ) * dTdt_1_8;
q_surface_throat_FEA = -k / ( 2 * delta_r ) * ( -3 * T_surface_throat_FEA
+ 4 * nozzle_throat_1_8_temp - nozzle_throat_1_4_temp );

% Exit

% Use Slab One and Two Probe Formulas to Determine Surface Conditions
[ T_surface_exit_1_8, q_local_exit_1_8, q_surface_exit_1_8 ] =
one_thermocouple( nozzle_exit_1_8_temp, N2_max, delta_t, alpha, k,
delta_r );
[ T_at_1_8_exit_1_4, q_local_exit_1_4, q_at_1_8_exit_1_4 ] =
one_thermocouple( nozzle_exit_1_4_temp, N2_max, delta_t, alpha, k,
delta_r );
[ T_surface_exit_1_4, q_local_exit_1_4, q_surface_exit_1_4 ] =
one_thermocouple( nozzle_exit_1_4_temp, N2_max, delta_t, alpha, k, 2 *
delta_r );
[ T_surface_exit_two_thermocouple, q_surface_exit_two_thermocouple ] =
two_thermocouple( nozzle_exit_1_8_temp, nozzle_exit_1_4_temp,
q_local_exit_1_8, q_local_exit_1_4, N2_max, delta_t, alpha, k,
delta_r );

% Finite-Element Analysis
dTdt_exit_1_8 = dfdt( nozzle_exit_1_8_temp, delta_t, N2_max );
T_surface_exit_FEA = nozzle_exit_1_8_temp + ( r_exit + 3 * delta_r / 2
) ./ ( r_exit + delta_r / 2 ) .* ( nozzle_exit_1_8_temp -
nozzle_exit_1_4_temp ) + delta_r / ( alpha * ( 2 * r_exit + delta_r ) )
* ( ( r_exit + 3 * delta_r / 2 ) ^ 2 - ( r_exit + delta_r / 2 ) ^ 2 ) *
dTdt_exit_1_8;
q_surface_exit_FEA = -k / ( 2 * delta_r ) * ( -3 * T_surface_exit_FEA + 4
* nozzle_exit_1_8_temp - nozzle_exit_1_4_temp );

% Throat Plane Variables
%
% nozzle_throat_1_8_temp - filtered probe data
% nozzle_throat_1_4_temp - filtered probe data

```

```

% T_surface_throat_1_8 - one thermocouple prediction of surface
temperature
%           based on 1/8" probe
% q_local_throat_1_8 - Frankel's formula for heat flux at 1/8" probe
%           location
% q_surface_throat_1_8 - one thermocouple prediction of surface heat
flux
%           based on 1/8" probe
% T_at_1_8_throat_1_4 - one thermocouple prediction of 1/8" temperature
%           based on 1/4" probe
% q_local_throat_1_4 - Frankel's formula for heat flux at 1/8" probe
%           location
% q_at_1_8_throat_1_4 - one thermocouple prediction of heat flux at
1/8"
%           based on 1/4" probe
% T_surface_throat_1_4 - one thermocouple prediction of surface
temperature
%           based on 1/4" probe
% q_local_throat_1_4 - Frankel's formula for heat flux at 1/8" probe
%           location
% q_surface_throat_1_4 - one thermocouple prediction of surface heat
flux
%           based on 1/4" probe
% T_surface_two_thermocouple - surface temperature based on Frankel's
%           two-thermocouple equation
% q_surface_two_thermocouple - surface heat flux based on Frankel's
%           two-thermocouple equation
% T_1_8_one_thermocouple - cylindrical coordinate estimate of 1/8"
location
%           based on Cartesian surface temperature estimate
% T_1_4_one_thermocouple - cylindrical coordinate estimate of 1/4"
location
%           based on Cartesian surface temperature estimate
% T_surface_throat_FEA - cylindrical finite element estimate of surface
temperature

% Exit Plane Variables
%
% nozzle_exit_1_8_temp - filtered probe data
% nozzle_exit_1_4_temp - filtered probe data
% T_surface_exit_1_8 - one thermocouple prediction of surface
temperature
%           based on 1/8" probe
% q_local_exit_1_8 - Frankel's formula for heat flux at 1/8" probe
%           location
% q_surface_exit_1_8 - one thermocouple prediction of surface heat flux
%           based on 1/8" probe
% T_at_1_8_exit_1_4 - one thermocouple prediction of 1/8" temperature
%           based on 1/4" probe
% q_local_exit_1_4 - Frankel's formula for heat flux at 1/8" probe
%           location
% q_at_1_8_exit_1_4 - one thermocouple prediction of heat flux at 1/8"
%           based on 1/4" probe
% T_surface_exit_1_4 - one thermocouple prediction of surface
temperature

```



```

% Preallocate for speed
surface_T = zeros( N, 1 );
surface_q = zeros( N, 1 );

% Taylor series evaluations
for i = 1 : 1 : N
    surface_T(i) = T(i) + local_q(i) * eta / k + dTdt(i) * eta ^ 2 / (
2 * alpha ) + dqdt(i) * eta ^ 3 / ( 6 * alpha * k );
    surface_q(i) = local_q(i) + k / alpha * dTdt(i) * eta + 1 / alpha *
dqdt(i) * eta^2 / 2 + k / alpha^2 * d2Tdt2(i) * eta^3 / 6;
end

% Slab coordinate two-thermocouple analysis function
function [ surface_T, surface_q ] = two_thermocouple( T1, T2, q1, q2,
N, delta_t, alpha, k, eta)

% Variables:
%
% surface_T = surface temperature estimate based on this method
% surface_q = surface heat flux estimate based on this method
% T1 = vector of temperatures from 1/8" probe
% T2 = vector of temperatures from 1/4" probe
% q1 = local heat flux at 1/8" location
% q2 = local heat flux at 1/4" location
% N = number of entries of T1, T2, q1, and q2
% delta_t = time step of entries
% alpha = thermal diffusivity
% k = thermal conductivity
% eta = spatial step size

% Temporal derivatives required for formulas
dTdt1 = dfdt ( T1, delta_t, N );
dqdt1 = dfdt( q1, delta_t, N );

% Preallocate for speed
surface_T = zeros( N, 1 );
surface_q = zeros( N, 1 );

% Taylor series evaluations
for i = 1 : 1 : N
    surface_T(i) = -T2(i) + 2 * T1(i) + 1 / alpha * dTdt1(i) * eta^2;
    surface_q(i) = -q2(i) + 2 * q1(i) + 1 / alpha * dqdt1(i) * eta^2;
end

% Numerical derivative
function b = dfdt ( a, delta_t, N )

```

```

% Variables:
%
% b = numerical derivative
% a = original data
% delta_t = time step of entries
% N = number of entries

% Preallocate for speed
b = zeros( N, 1 );

% Five-point forward difference formula
b(1) = 1 / ( 12 * delta_t ) * ( -25 * a( 1 ) + 48 * a( 2 ) - 36 * a( 3 ) + 16 * a( 4 ) - 3 * a( 5 ) );
b(2) = 1 / ( 12 * delta_t ) * ( -25 * a( 2 ) + 48 * a( 3 ) - 36 * a( 4 ) + 16 * a( 5 ) - 3 * a( 6 ) );

% Five-point central difference formula
for i = 3:1:N-2
    b(i) = 1 / ( 12 * delta_t ) * ( a( i-2 ) - 8 * a( i-1 ) + 8 * a( i+1 ) - a( i+2 ) );
end

% Five-point backward difference formula
b(N-1) = 1 / ( 12 * delta_t ) * ( -25 * a( N-1 ) + 48 * a( N-2 ) - 36 * a( N-3 ) + 16 * a( N-4 ) - 3 * a( N-5 ) );
b(N) = 1 / ( 12 * delta_t ) * ( -25 * a( N ) + 48 * a( N-1 ) - 36 * a( N-2 ) + 16 * a( N-3 ) - 3 * a( N-4 ) );

```

Program 3: Producing Output Graphs

```

function analysis_plots

% Input Section

run_number = 24;
run_string = num2str( run_number );
location_string = 'e'; % 't' or 'e' for throat or exit data

filename1 = strcat( 'data/data', run_string, location_string, '.dat' );

close all; % Closes ant figures already open to prevent cross-over data

fid = fopen( filename1 ); % Opens data file
A = fscanf( fid, '%g %g %g %g %g %g %g %g %g %g %g %g %g %g %g %g', [ 16 inf ] ); % Reads 16 columns of data
fclose( fid ); % Closes data file

A = A';

time2 = A(:,1);
nozzle_1_8_temp = A(:,2);

```

```

nozzle_1_4_temp = A(:,3);
T_surface_1_8 = A(:,4);
q_local_1_8 = A(:,5);
q_surface_1_8 = A(:,6);
T_at_1_8_1_4 = A(:,7);
% q_local_1_4 = A(:,8); % - repeat variable
q_at_1_8_from_1_4 = A(:,9);
T_surface_1_4 = A(:,10);
q_local_1_4 = A(:,11);
q_surface_1_4 = A(:,12);
T_surface_two_thermocouple = A(:,13);
q_surface_two_thermocouple = A(:,14);
T_surface_FEA = A(:,15);
q_surface_FEA = A(:,16);

% Plotting section

figure(1)
set(gcf,'PaperPositionMode','auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2(1:29997), T_surface_1_8(1:29997), '-r' );
    hold on
plot( time2, nozzle_1_8_temp, '-b' );
plot( time2, nozzle_1_4_temp, '-k' );
    hold off
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Temperature (C)', 'FontSize', 20 );
    legend( 'Estimated Temperature', '1/8" Probe Data', '1/4" Probe
Data' );
    set( gca, 'YGrid', 'on' );

% Save figure for later access
picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_one_thermocouple_T_surface_1_8_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(2)
set(gcf,'PaperPositionMode','auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2, q_local_1_8, '-b' );
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Local Heat Flux (W / m^2)', 'FontSize', 20 );
    set(gca, 'YGrid', 'on' );

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_q_local_1_8_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

```

```

figure(3)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2(1:29997), q_surface_1_8(1:29997), '-b' );
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Surface Heat Flux (W / m^2)', 'FontSize', 20 );
    set(gca, 'YGrid', 'on')

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_q_surface_1_8_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(4)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2(1:29997), T_at_1_8_1_4(1:29997), '-r' );
    hold on
plot( time2, nozzle_1_8_temp, '-b' );
plot( time2, nozzle_1_4_temp, '-k' );
    hold off
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Temperature (C)', 'FontSize', 20 );
    legend( 'Estimated Temperature', '1/8" Probe Data', '1/4" Probe
Data' );
    set( gca, 'YGrid', 'on' );

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_T_at_1_8_from_1_4_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(5)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2, q_local_1_4, '-b' );
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Local Heat Flux (W / m^2)', 'FontSize', 20 );
    set(gca, 'YGrid', 'on');

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_q_local_1_4_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

```

```

figure(6)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2, q_local_1_8, '-b' );
    hold on;
plot( time2(1:29997), q_at_1_8_from_1_4(1:29997), '-k' );
    hold off;
xlabel( 'Time (s)', 'FontSize', 20 );
ylabel( 'Heat Flux (W / m^2)', 'FontSize', 20 );
set(gca, 'YGrid', 'on');
legend( 'Integral Equation', 'Taylor Approximation' );

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_q_at_1_8_from_1_4_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(7)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2(1:29997), T_surface_1_4(1:29997), '-r' );
    hold on
plot( time2, nozzle_1_8_temp, '-b' );
plot( time2, nozzle_1_4_temp, '-k' );
    hold off
xlabel( 'Time (s)', 'FontSize', 20 );
ylabel( 'Temperature (C)', 'FontSize', 20 );
legend( 'Estimated Temperature', '1/8" Probe Data', '1/4" Probe
Data');
set( gca, 'YGrid', 'on');

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_T_surface_1_4_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(8)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2(1:29997), q_surface_1_4(1:29997), '-b' );
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Surface Heat Flux (W / m^2)', 'FontSize', 20 );
    set(gca, 'YGrid', 'on')

```

```

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_q_surface_1_4_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(9)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2, T_surface_two_thermocouple, '-r' );
    hold on
plot( time2, nozzle_1_8_temp, '-b' );
plot( time2, nozzle_1_4_temp, '-k' );
    hold off
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Temperature (C)', 'FontSize', 20 );
    legend( 'Estimated Temperature', '1/8" Probe Data', '1/4" Probe
Data' );
    set( gca, 'YGrid', 'on' );

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_T_surface_two_thermocouple_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(10)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2(1:29997), q_surface_two_thermocouple(1:29997), '-b' );
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Surface Heat Flux (W / m^2)', 'FontSize', 20 );
    set(gca, 'YGrid', 'on')

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_q_surface_two_thermocouple_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(11)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2, T_surface_FEA, '-r' );
    hold on
plot( time2, nozzle_1_8_temp, '-b' );

```

```

plot( time2, nozzle_1_4_temp, '-k' );
    hold off
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Temperature (C)', 'FontSize', 20 );
    legend( 'Estimated Temperature', '1/8" Probe Data', '1/4" Probe
Data');
    set( gca, 'YGrid', 'on' );

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_T_surface_FEA_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(12)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2, q_surface_FEA, '-b' );
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Surface Heat Flux (W / m^2)', 'FontSize', 20 );
    set(gca, 'YGrid', 'on')

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_q_surface_FEA_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

figure(13)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2, T_surface_FEA, '-r' );
    hold on
plot( time2, T_surface_two_thermocouple, '-m' );
plot( time2(1:29997), T_surface_1_8(1:29997), '-g' );
plot( time2, nozzle_1_8_temp, '-b' );
plot( time2, nozzle_1_4_temp, '-k' );
    hold off
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Temperature (C)', 'FontSize', 20 );
    legend( 'FEA Method', 'Two Thermocouple Method', 'One Thermocouple
Method', '1/8" Probe Data', '1/4" Probe Data');
    set( gca, 'YGrid', 'on' );

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_T_surface_comparison_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

```

```

figure(14)
set(gcf, 'PaperPositionMode', 'auto');
maximize( gcf );
axes( 'FontSize', 16 );
plot( time2, q_surface_FEA, '-r' );
    hold on
plot( time2(1:29997), q_surface_two_thermocouple(1:29997), '-b' );
plot( time2(1:29997), q_surface_1_8(1:29997), '-k' );
    xlabel( 'Time (s)', 'FontSize', 20 );
    ylabel( 'Surface Heat Flux (W / m^2)', 'FontSize', 20 );
    legend( 'FEA Method', 'Two Thermocouple Method', 'One Thermocouple
Method' );
    set(gca, 'YGrid', 'on')

picname = strcat( 'plots/run', run_string, '/run_', run_string,
'_q_surface_comparison_', location_string );
saveas(gcf, picname, 'fig' );
saveas(gcf, picname, 'png' );

```

VITA

Jeffrey Daniel King was born in Memphis, TN on February 8, 1983. He was raised in Memphis and attended Shelby County public schools. He graduated from Germantown High School in nearby Germantown, TN in 2001. From there, he went to Christian Brothers University in Memphis and received a B.S. in Mechanical Engineering in 2005. While at Christian Brothers, he was active in the American Society of Mechanical Engineers and Tau Beta Pi, the Engineering Honor Society. At his graduation ceremony, he was presented with the Outstanding School of Engineering Award and the T. Herbert Darnell Mechanical Engineering Graduate Award. Upon graduation, he attended the University of Tennessee Space Institute in Tullahoma, TN and received his M.S. in Mechanical Engineering.