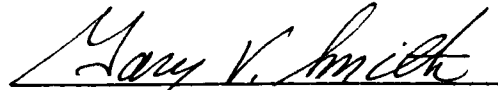


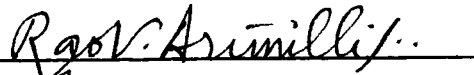
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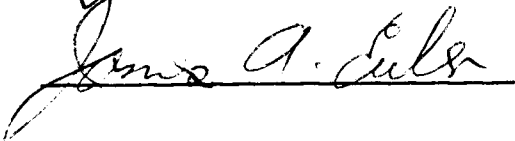
I am submitting herewith a thesis written by Mark W. Wendel entitled "Potential Flow Analysis of the Transition from an External Flow to an Internal Flow of a Receiver/Diffuser." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.



Gary V. Smith, Major Professor

We have read this thesis
and recommend its acceptance:





Accepted for the Council:



Vice Provost
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POTENTIAL FLOW ANALYSIS OF THE
TRANSITION FROM AN EXTERNAL FLOW
TO AN INTERNAL FLOW OF A
RECEIVER/DIFFUSER

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Mark W. Wendel

December 1985

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ABSTRACT

A two-dimensional computational potential flow model has been developed to describe the flow field characteristics of a submerged jet impinging on a receiver/diffuser. This model is a first effort at understanding the transition from the external jet flow to the flow internal to the receiver/diffuser.

The flow field is divided into one upper flow region and one lower flow region which interact across a free-surface boundary. Since the orientation of the free surface dividing these two regions is not known initially, an iterative scheme is used to solve the flow field. As the dividing free surface orientation and associated boundary conditions are changed through iteration, a Cauchy Boundary Integral method is used to solve both flow regions.

The method is verified with the exact solution of flow around a cylinder by shaping a receiver/diffuser to a cylindrical shape with a trailing "tail" section. The receiver/diffuser problem is solved using diffuser half angles of one, three, and five degrees. Comparisons of flow rates and wall pressure distributions are made with published results. Additionally a two-dimensional pressure distribution within the receiver/diffuser is plotted.

It was discovered that the model differed from the receiver/diffuser problem by predicting "induced" velocity-magnitudes in the surrounding fluid as large as the jet velocity, although this large amount of flow did not pass through the receiver/diffuser. However, the pressure distribution trend along the inside of the receiver/diffuser wall matches the trend in the experimental results.

It is expected that the model would be useful in analyzing critical internal aspects of the receiver/diffuser design.

The recommendation is made to continue the analysis by changing the boundary conditions on the upper flow region and fixing the receiver/diffuser back pressure as the independent variable.

TABLE OF CONTENTS

SECTION	PAGE
I. INTRODUCTION	1
Problem Motivation	1
Statement of Problem	1
Review of Literature	6
II. MATHEMATICAL ANALYSIS	8
Method of Solution	8
Boundary Integral Method	12
Solution Algorithm	14
III. COMPUTATIONAL RESULTS	17
Method Verification	17
Receiver/Diffuser Results	21
IV. DISCUSSION OF RESULTS AND RECOMMENDATIONS	27
BIBLIOGRAPHY	35
APPENDIXES	37
A. PROGRAM LISTING	38
B. ADDITIONAL COMPUTATIONAL MODEL INFORMATION	64
VITA	66

LIST OF FIGURES

FIGURE	PAGE
1. Cross-sectional view of a submerged jet impinging on a receiver/diffuser	2
2. Approximated flow field for computation	5
3. Boundary conditions on the flow field approximation	10
4. Computational domain for flow around a cylinder	18
5. Comparison of analytic streamline with the computed streamline in flow around a cylinder	19
6. Comparison of analytic pressure distribution with the computed results in flow around a cylinder	20
7. Dimensions of the receiver/diffuser used in computation ..	22
8. Computed and experimental flow rates through the receiver/diffuser	23
9. Computed wall pressure distribution through a 1° receiver/diffuser	24
10. Computed wall pressure distribution through a 5° receiver/diffuser	25
11. Segment of the two-dimensional pressure field for a 5° receiver/diffuser with a back pressure ratio of .5	26
12. Exact wall pressure distribution for internal potential flow through a diffuser	28
13. Experimental wall pressure distribution through an axisymmetric 5° receiver/diffuser	29
A.1. Computational flow domain with program variables defined .	39

NOMENCLATURE

SYMBOL	PARAMETER	UNITS
CP	coefficient of pressure	--
h	half-height of the entering fluid jet	m
H_L	lower region Bernoulli constant	m^2/s^2
H_U	upper region Bernoulli constant	m^2/s^2
p	pressure	Pa
P	density-adjusted pressure	m^2/s^2
P_{in}	jet density-adjusted pressure	m^2/s^2
P_o	jet density-adjusted stagnation pressure	m^2/s^2
P_b	diffuser density-adjusted back pressure	m^2/s^2
PR	ratio between R/D back pressure and inlet jet stagnation pressure	--
QR	ratio between R/D flow rate and inlet jet flow rate	--
s	coordinate along the dividing streamline	m
u	x-velocity component	m/s
v	y-velocity component	m/s
V	total velocity magnitude	m/s
V_{iL}	velocity of the inlet jet	m/s
V_{iU}	velocity in upper region inlet immediately above the free surface	m/s
x,y	cartesian coordinates	m
z	complex coordinate vector	m

Greek Symbols

θ	diffuser half angle	degrees
ρ	fluid density	kg/m^3
ϕ	velocity potential function	m^2/s
ψ	velocity stream function	m^2/s

I. INTRODUCTION

Problem Motivation

Recent work (ref. 7) in the area of fluid pumping systems utilizing venturi-like reverse-flow diverters (RFD's) has revealed the need for a mathematical model of the interaction between a submerged fluid jet and a receiver/diffuser (R/D). Such a flow (Fig. 1) consists of a fluid jet (possibly emanating from a nozzle) moving through an otherwise undisturbed region of the same fluid and impinging on the inlet of a R/D. If the pressure at the outlet of the R/D is low enough, a secondary flow through the device will be induced consisting of fluid which originates from the jet surroundings. This problem, which requires modeling the transition of an external flow into an internal flow was the original motivation for this thesis work. An additional application of such a model would be predicting the pressure-flow characteristics of jet pipe valves (ref.1).

Statement of Problem

The purpose of this thesis is the development of a model and solution scheme which describes this transitioning (external to internal) flow associated with a R/D and will be a first effort at understanding and predicting pressure and velocity distributions associated with the R/D design. These distributions are critical to the R/D performance.

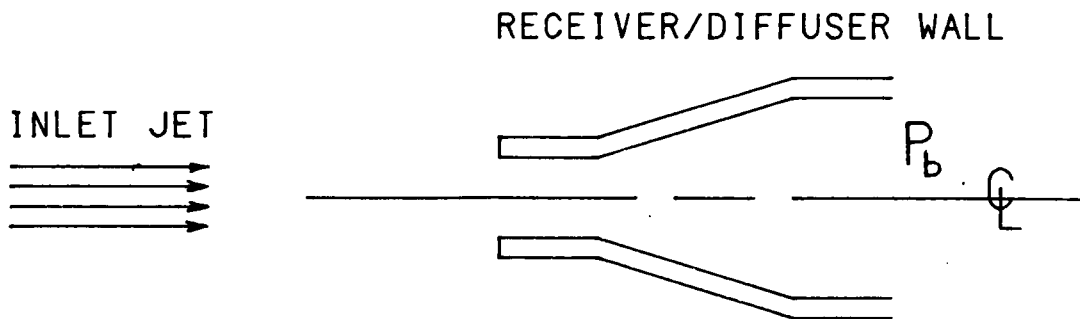


Fig. 1. Cross-sectional view of a submerged jet impinging on a receiver/diffuser.

It has been observed (ref. 7) that for a liquid jet with a given stagnation pressure P_0 , flow through the R/D increases as the back pressure P_b , is lowered. However, for some jet stagnation pressures there exists some critical back pressure below which, the flow rate through the R/D remains essentially unchanged. It is believed that this "saturation effect" is caused by cavitation of the fluid as it encounters low pressures immediately inside the R/D. Using a mathematical model to predict velocity and pressure distributions inside the R/D, it is conceivable that an improved design of the R/D could be achieved which would reduce severe velocity gradients that exist for low P_b . Alleviation of these steep velocity gradients should tend to eliminate or reduce the negative effects of cavitation caused by these low pressures.

It is believed that the phenomenon under consideration is primarily of a potential flow nature. That is, the induction of secondary flow through the R/D and saturation of the output flow is primarily due to the streamwise pressure gradients as opposed to viscous effects. For this reason a potential flow model was anticipated to be useful. The simplest potential flow solution to the problem would be to consider the entire flow domain as a single potential field. That is, at every point in the flow, the sum of the density-adjusted pressure ($P = \frac{2p}{\rho}$) and the square of the velocity magnitude is equal to a single Bernoulli constant, H .

It is assumed that the source of the jet lies within one or two jet heights of the inlet plane so that not much mixing has occurred between the jet and surrounding fluid. It therefore is expected that

an extremely sharp velocity gradient exists between the entering jet and the surrounding fluid. This gradient is expected to be sufficiently sharp such that it can be practically treated as a velocity discontinuity. Physically, for steady state the pressure must be continuous throughout the fluid. These expectations of the flow contradict the idea of a single potential field model since differential patches of fluid next to but on opposite sides of the velocity discontinuity must have the same static pressure, but do not necessarily have the same velocity. This would tend to indicate the existence of two different Bernoulli constants, and therefore it was decided to treat the flow field as two interacting potential flow regions which are separated by a free-surface streamline (Fig. 2). The resulting solution must meet the condition that pressure at any point along that streamline be the same when calculated in either flow region. Since the boundaries of both flow regions are unknown to the extent of the dividing streamline, an iterative solution technique is used to solve the flow regions simultaneously.

The solution of one flow region, whether it be the upper or lower, is accomplished by means of a Cauchy Integral boundary integral method. The boundary integral technique is particularly suited to this problem since throughout the iterative process the flow region geometries change. Moving the boundary location requires considerably less effort than regenerating a portion of a two-dimensional grid which would be required in a finite-difference control-volume technique.

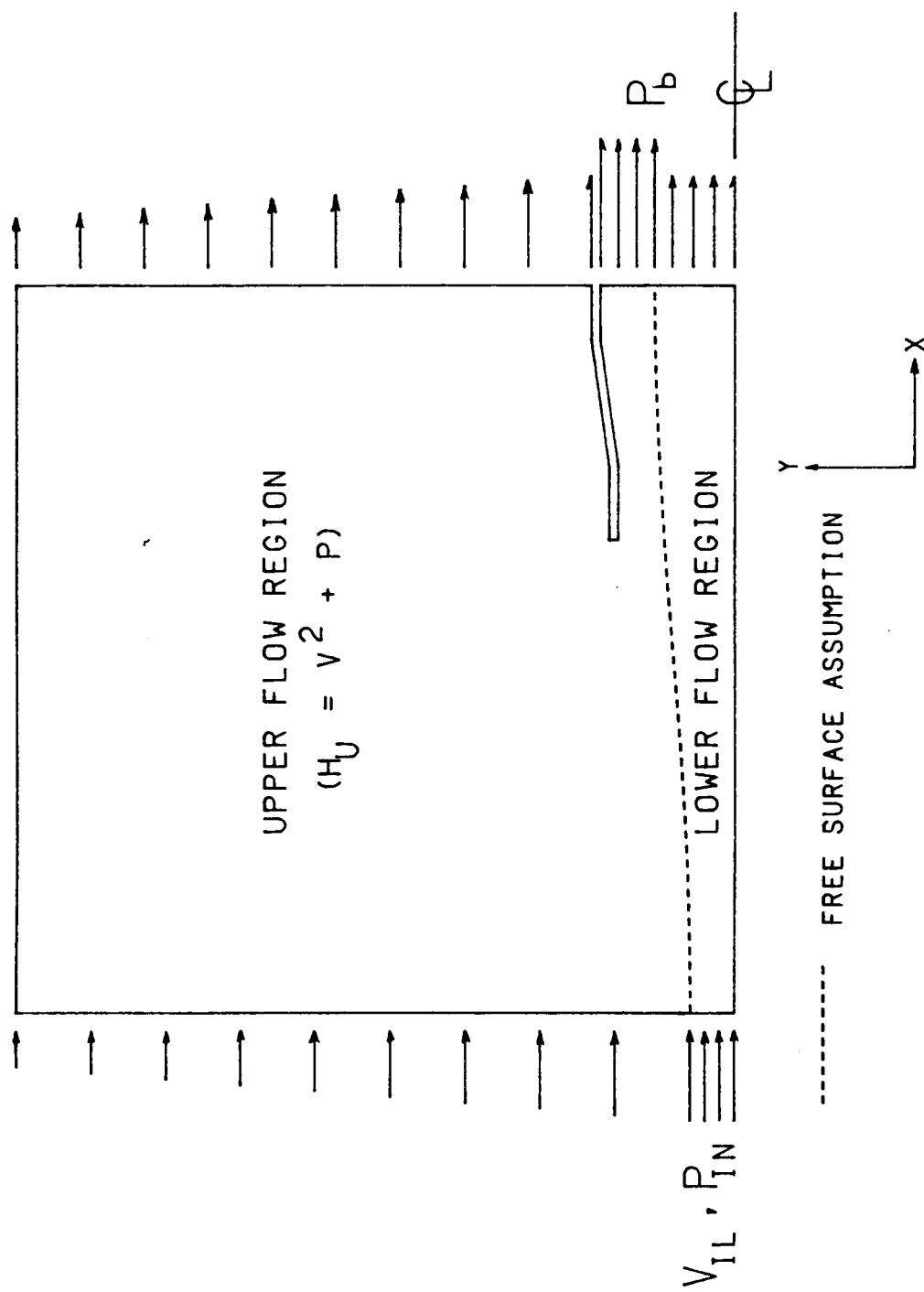


Fig. 2. Approximated flow field for computation.

Review of Literature

Experimental data for the jet and R/D configuration has been taken by Reid (ref. 5) and Smith and Counce (ref. 7); the latter includes discussion of the RFD operation. The exact solution for internal potential flow through a straight-walled diffuser was derived by Smith (ref. 6) using a complex-variable mapping. Söhngen and Bischoff (ref. 8) describe a boundary-integral, free-surface-iterative technique applied specifically to underflow below an inclined sluice gate with two iterative schemes given. An introductive development of the general boundary element method is explained by Hromadka and Guymon (ref. 3). It must be understood that the Cauchy Integral Equation (CIE) which is used in this thesis, is only one type of integral equation to which the boundary integral method may be applied. Ferziger (ref. 2) briefly explains the CIE technique: he indicates that the imaginary part of the complex system of equations that is the result of reducing the CIE is generally a better conditioned system than is the real part. White (ref. 9) explains the technique in greater detail including the reduction procedure from the CIE to the algebraic system of equations.

Interestingly, the boundary integral technique is useful in analyzing flows through porous media which are dominated by viscous forces (extremely low Reynolds numbers). This is because Darcy's empirical relation defining a velocity potential coupled with the incompressible continuity equation yields Laplace's equation in the velocity potential variable, similar to inviscid incompressible flow.

Hunt and Isaacs (ref. 4) specifically apply the CIE to groundwater seepage beneath a dam.

II. MATHEMATICAL ANALYSIS

Method of Solution

The model, which is two-dimensional, consists of two separate, interacting potential flow regions separated by a free-surface streamline. The governing equations for steady potential flow are:

Conservation of Mass:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

or

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

where

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \phi}{\partial y} = - \frac{\partial \psi}{\partial x}$$

Conservation of Momentum:

$$P + V^2 = H = \text{Bernoulli constant}$$

H is constant throughout one flow region.

$$P = \frac{2p}{\rho}$$

The two regions are distinct in that they have different Bernoulli constants; H_L (for the lower region) and H_U (for the upper region). Note that the lower region Bernoulli constant is equal to the density-adjusted stagnation pressure of the inlet jet.

Boundary conditions on the non-free-surface boundaries for both regions are indicated in Fig. 3. A constant stream ($\psi = \text{constant}$) boundary means that no mass convects across that boundary. A constant potential ($\phi = \text{constant}$) boundary means that no mass convects tangentially along that boundary.

The upper region is bounded on the left and right (outside of the R/D) by a constant potential boundary. Notice that such a condition does not imply that the velocity normal to the boundary is constant along that boundary. However, the portion of the left boundary across which the inlet jet flows is specified as a boundary of uniform velocity with a zero tangential component. In this analysis the back pressure range in the R/D will be low enough such that the flow out of the R/D is always greater than the inlet jet flow; hence, the exit plane of the R/D lies in both flow regions. The portion of this plane lying in the upper flow region is treated as a constant velocity boundary with zero a tangential component. The portion lying in the lower region is treated similarly; however, a velocity discontinuity across the free surface streamline is allowed.

At any time during the iterative solution, some assumption for the free surface orientation will exist. During solution of the lower region this free surface boundary is treated as a streamline. During solution of the upper flow region, it is assumed that the potential distribution along the free surface boundary is known.

Measurements which have been taken by Reid (ref. 5) and Smith and Counce (ref. 7) were obtained by varying the back pressure P_b ,

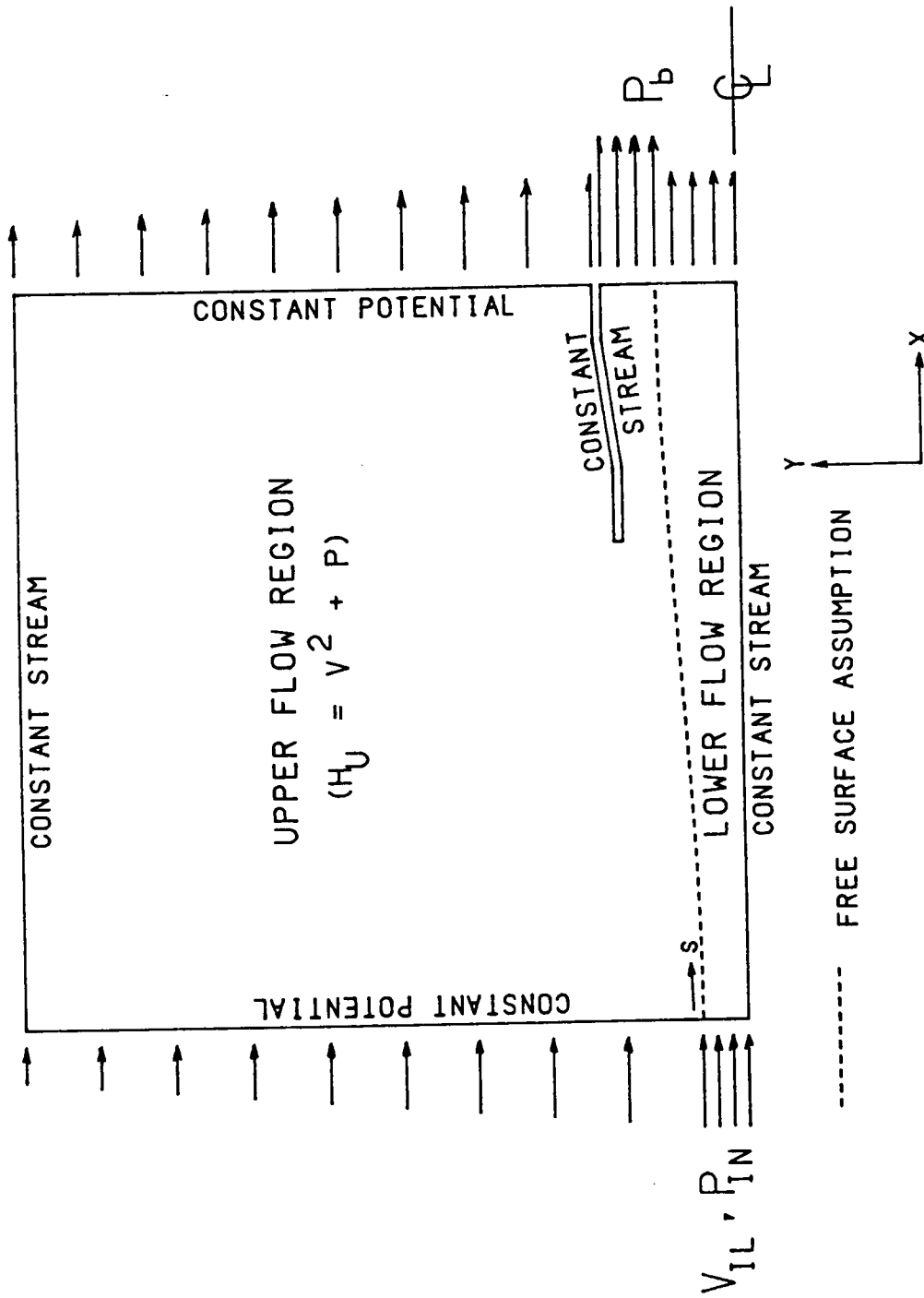


Fig. 3. Boundary conditions on the flow field approximation.

for a fixed inlet jet velocity and pressure, and then measuring the resulting R/D flow rate and associated parameters. Just as in the experiments, the computational model requires that the R/D geometry and inlet jet conditions be known. However, it was decided to choose the upper region Bernoulli constant H_U as the independent variable rather than P_b . Leaving H_U as a dependent variable would necessitate an additional iterative loop nested inside of the streamline-orientation iterations. The free-surface-iterative scheme which was used consists of solving the lower region for a velocity distribution, and hence pressure distribution (from the Bernoulli relation) along the free surface boundary. In order to impose this pressure distribution as a boundary condition on the upper region, H_U must be known to translate the condition in terms of velocity (potential gradient),

$$V(s) = [H_U - P(s)]^{1/2} \quad .$$

If H_U is not known, then some sort of iteration must be used to solve the upper region with the imposed boundary condition. This double-iterative method was anticipated to require significantly greater computational effort than the method using H_U as the independent variable. It was found that this simplifying assumption was a necessary first step in developing a solution scheme for this complex problem.

Boundary Integral Method

The boundary integral method described by White (ref. 9) will be used to solve either the upper or lower flow region with ϕ (or ψ) known within a constant at each point on the region boundary by calculating ψ (or ϕ) at each boundary point.

The method consists of satisfying Cauchy's integral equation by integrating around the boundary of a flow region.

$$f(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z - z_0} dz$$

where

$$f(z) = \phi(z) + i\psi(z) ,$$

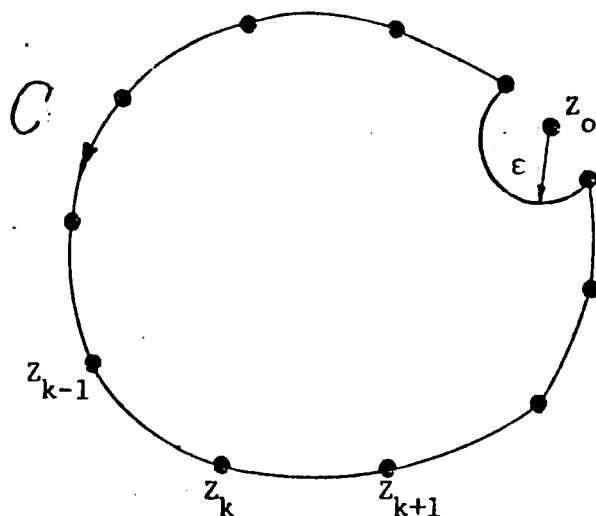
$f(z)$ is analytic, and z_0 lies within the region bounded by C .

The above equation is reduced by taking z_0 as a point on the boundary and then indenting the boundary about z_0 . Thus,

$$\int_C \frac{f(z)}{z - z_0} dz = 0$$

since the integrand is now non-singular within the region bounded by C .

The indentation is made as a circular arc, and the integral on the arc is performed as its radius ϵ approaches zero.



Integration over the remaining portion of the contour is accomplished by discretizing the region boundary into N segments using N discrete boundary points, and letting $f(z)$ vary only linearly between these discrete points,

$$f(z) = f(z_k) + \frac{z - z_k}{z_{k+1} - z_k} f(z_{k+1})$$

After integration is performed, the equation is reduced to an algebraic equation containing N values of $f(z)$. Since $f(z) = \phi(z) + i\psi(z)$, and either ϕ or ψ is known at each point, this algebraic equation contains N unknowns.

$$\sum_{k=1}^N A_k f(z_k) = 0$$

A_k = complex coefficient obtained by integration

The N equations necessary for solution of the unknown ϕ 's and/or ψ 's

can be obtained by writing N similar algebraic equations as each discrete boundary point takes its turn as z_0 .

$$\sum_{k=1}^N A_{jk} f(z_k) = 0 \quad j = 1, 2, \dots, N$$

Since the resulting system is complex, $2N$ equations will exist, both a real and an imaginary system. Either of these may be used to solve for the unknown ϕ 's and/or ψ 's. Important information about the discretization of the boundary and the actual number of equations used can be found in Appendix B.

$$\sum_{k=1}^N [\operatorname{Re}(A_{jk}) + i \operatorname{Im}(A_{jk})][\phi_k + i \psi_k] = 0 \quad j = 1, 2, \dots, N$$

$$\text{Real System: } \sum_{k=1}^N [\operatorname{Re}(A_{jk})\phi_k - \operatorname{Im}(A_{jk})\psi_k] = 0 \quad j = 1, 2, \dots, N$$

$$\text{Imaginary System: } \sum_{k=1}^N [\operatorname{Re}(A_{jk})\psi_k + \operatorname{Im}(A_{jk})\phi_k] = 0 \quad j = 1, 2, \dots, N$$

Solution Algorithm

The following steps make up the iterative free-surface procedure which was used to solve the flow field. Reference to Fig. 3 will be helpful.

1. Specify the R/D geometry, inlet jet conditions, and upper region Bernoulli constant H_0 .

2. Assume some dividing-line orientation as an initial guess for the free-surface streamline which divides the two flow regions.

3. Now that the complete geometry for both flow regions has been assumed, and the inlet jet velocity is known (hence, the stream function is known), solve the lower region for the potential function on the boundary.

4. Differentiate the potential function for the velocity distribution along the dividing-line.

$$V_L(s) = \frac{\partial \phi(s)}{\partial s}$$

5. Calculate the corresponding pressure distribution along the dividing-line.

$$P(s) = H_L - V_L^2(s)$$

6. Calculate the velocity distribution along the dividing-line as viewed from the upper region using the newly-calculated $P(s)$.

$$V_U(s) = [H_U - P(s)]^{1/2}$$

7. Integrate this velocity distribution for the potential distribution along the free-surface assumption.

$$\phi(s) = \int_0^s V_U(s) ds + \phi(0)$$

8. Using $\phi(s)$ as a boundary condition, solve the upper flow region. This solution will include $\psi(s)$.

9. Determine the component of velocity normal to the free-surface assumption by differentiating the stream function.

$$V_{\text{norm}}(s) = \frac{\partial \psi(s)}{\partial s}$$

10. Add $V_{\text{norm}}(s)$ to the tangential velocity (taken as $V(s)$ in step 5) for the Cartesian-referenced velocity components u and v .

11. Since $\frac{v}{u} = \frac{dy}{dx}$ can be interpreted as a corrected slope for the dividing-line orientation, integrate for a new guess at the free-surface orientation.

$$y_{\text{new}}(x) = \int_0^x \frac{v}{u} dx + y(0)$$

12. Under-relax the process to avoid iterative divergence.

$$y_{\text{new}}(x) = y_{\text{old}}(x) + \alpha (y_{\text{new}}(x) - y_{\text{old}}(x))$$

α = under-relaxation factor (less than 1)

13. Return to step (3) with the corrected dividing line and continue the process until the orientation of the free-surface converges to a solution. When convergence is obtained, ϕ and ψ are known at every boundary point in both flow regions: velocity and pressure can then be calculated at any point.

III. COMPUTATIONAL RESULTS

Method Verification

To insure that the algorithm would converge to a correct solution, the computational method was applied to the problem of flow around a cylinder (Fig. 4). In order to keep the boundary conditions the same as the region shown in Fig. 2, it was necessary to include a "tail" of negligible thickness on the cylinder. The cylinder-"tail" configuration now acts as the R/D, and if the Bernoulli constants of the upper and lower regions are equal, then the uniform flow is established.

Since the analytic solution for a streamline in flow around a cylinder is known, the computed solution can be verified. This comparison is made graphically in Fig. 5. The computed results were obtained using $\alpha = .3$ after 25 iterations.

In addition, the computed pressure distribution on the cylinder is compared with the exact solution in Fig. 6. These results indicated that the solution was converging properly. Through attempts to improve accuracy of this solution it was discovered that the system of equations taken from the imaginary part of the approximation to the Cauchy integral equation (see Sect. II) yielded better results than did the real system. This is consistent with Ferziger's findings (ref. 2).

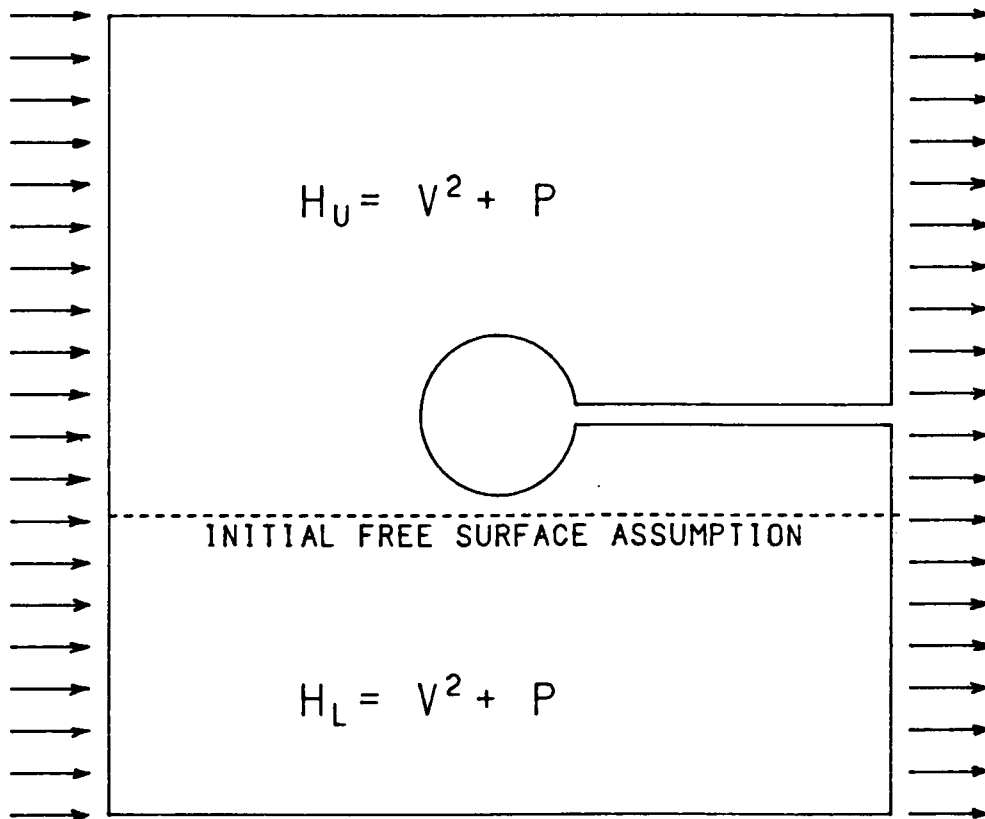


Fig. 4. Computational domain for flow around a cylinder.

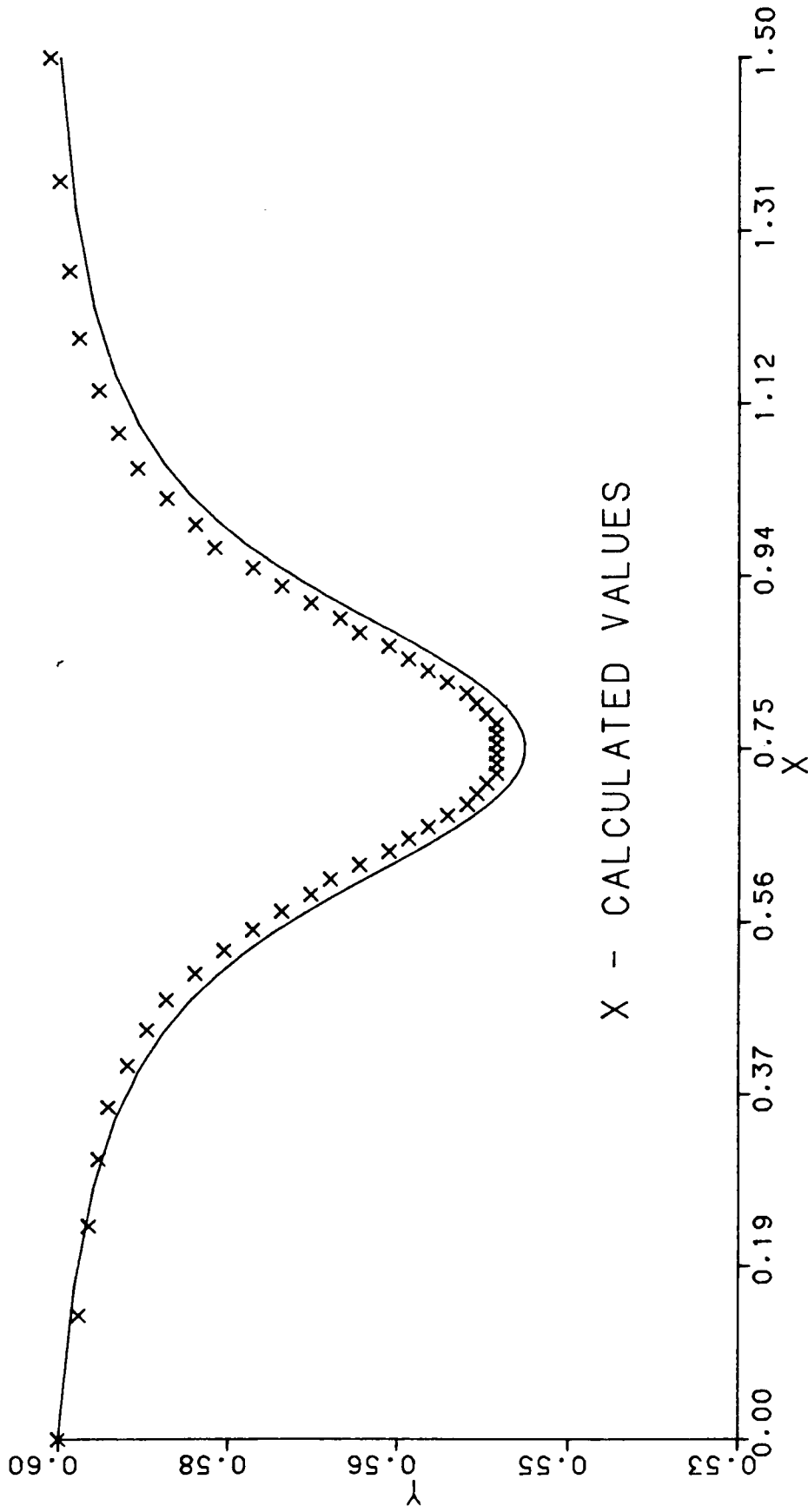


Fig. 5. Comparison of analytic streamline with the computed streamline in flow around a cylinder.

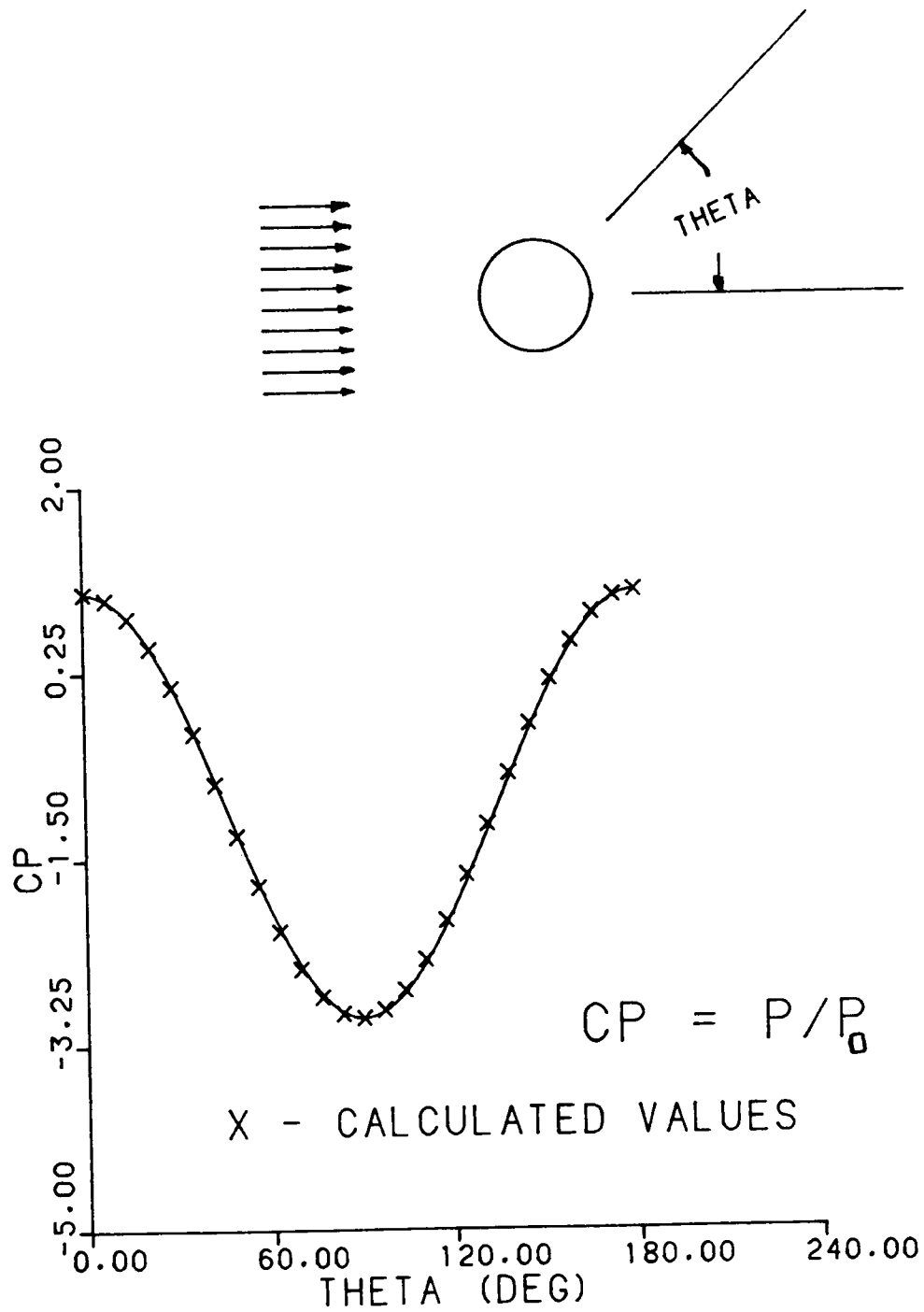


Fig. 6. Comparison of analytic pressure distribution with the computed results in flow around a cylinder.

Receiver/Diffuser Results

Computational results were obtained for a R/D such as that shown in Fig. 7. This straight-walled geometry is similar to the cross-section of Reid's experimental apparatus, although his R/D was axisymmetric as opposed to the two-dimensional model used in this study for computation.

This analysis was carried out for diffuser half angles of one, three, and five degrees. Figure 8 presents the pressure-flow characteristics for each diffuser half angle studied as well as experimental data from Reid (ref. 5). The pressure distributions along the inside part of the R/D wall are presented for diffuser half angles of one and five degrees in Figs. 9 and 10 respectively. In addition a two-dimensional pressure distribution in a critical region of the flow field is shown in Fig. 11(a) for a R/D half angle, of five degrees and a pressure ratio (PR) of .5. The orientation of the rectangular region for which this pressure distribution was determined is shown in Fig. 11(b).

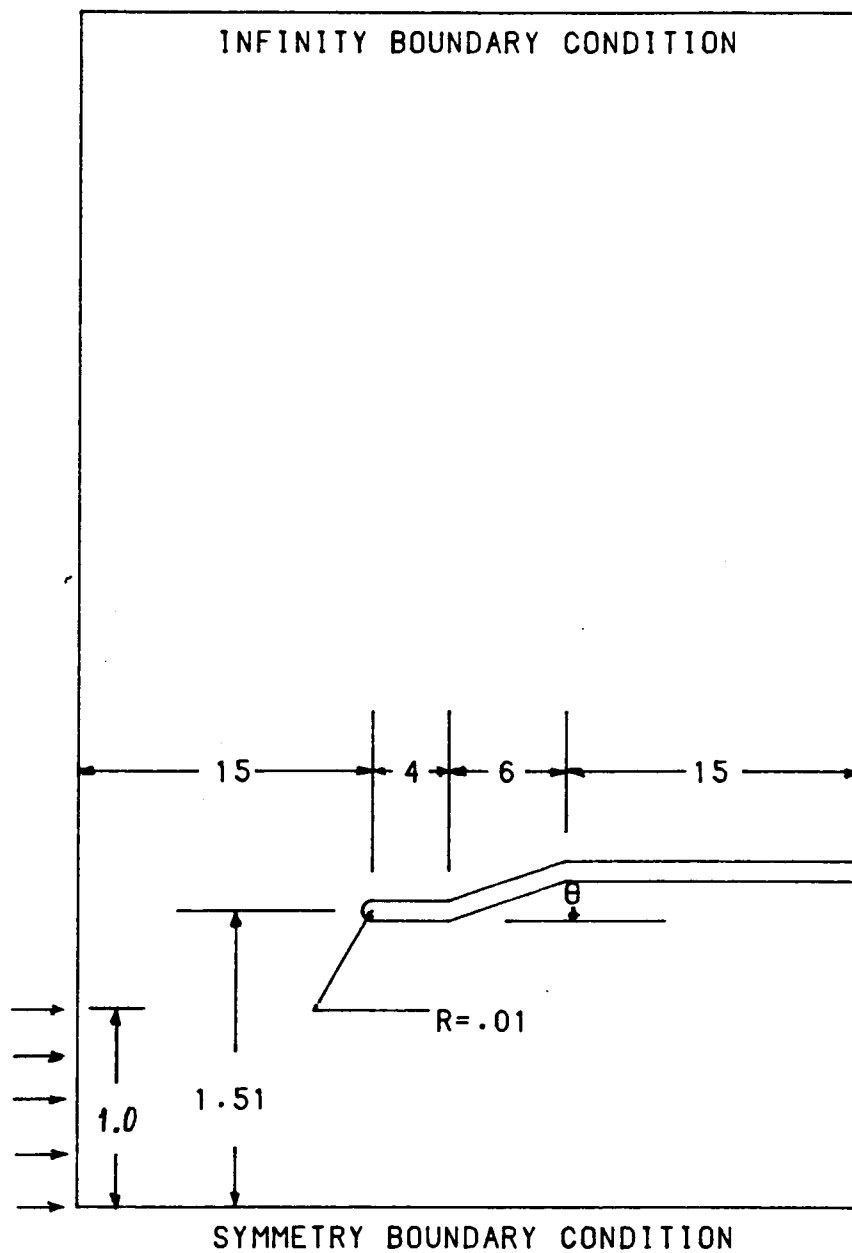


Fig. 7. Dimensions of the receiver/diffuser used in computation.

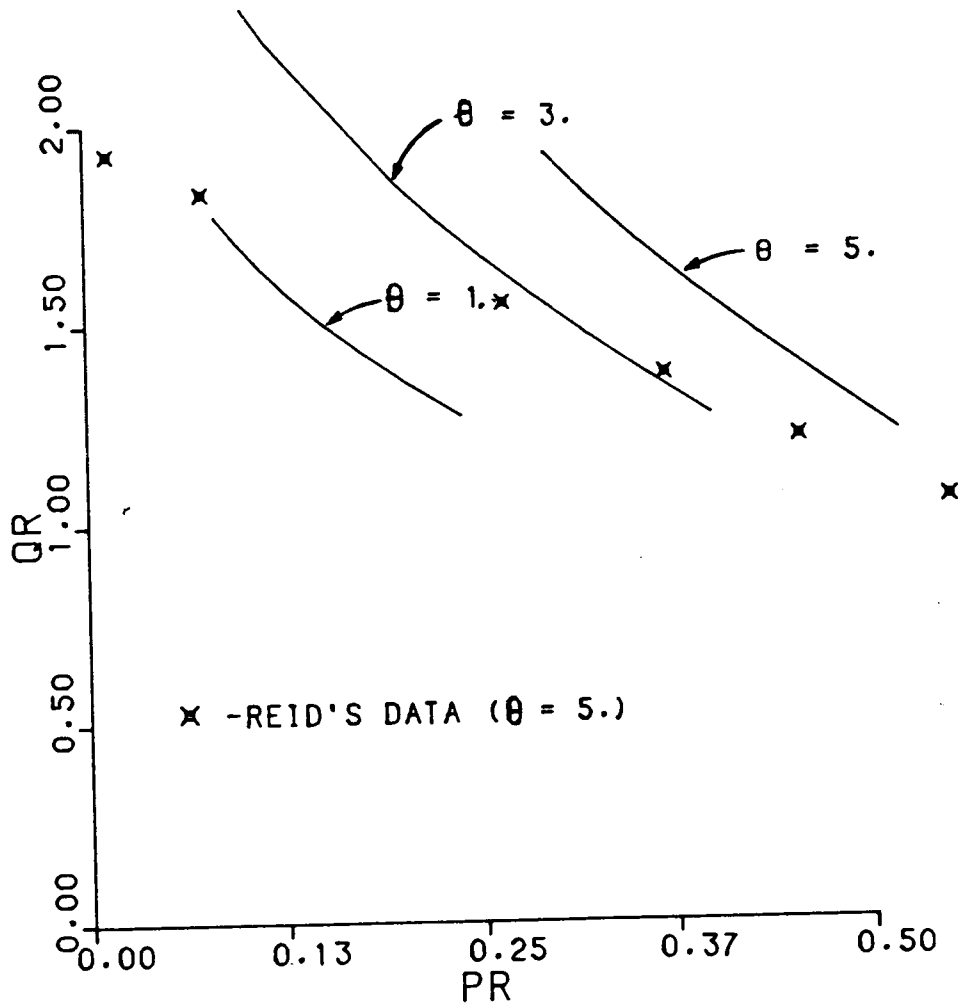


Fig. 8. Computed and experimental flow rates through the receiver/diffuser.

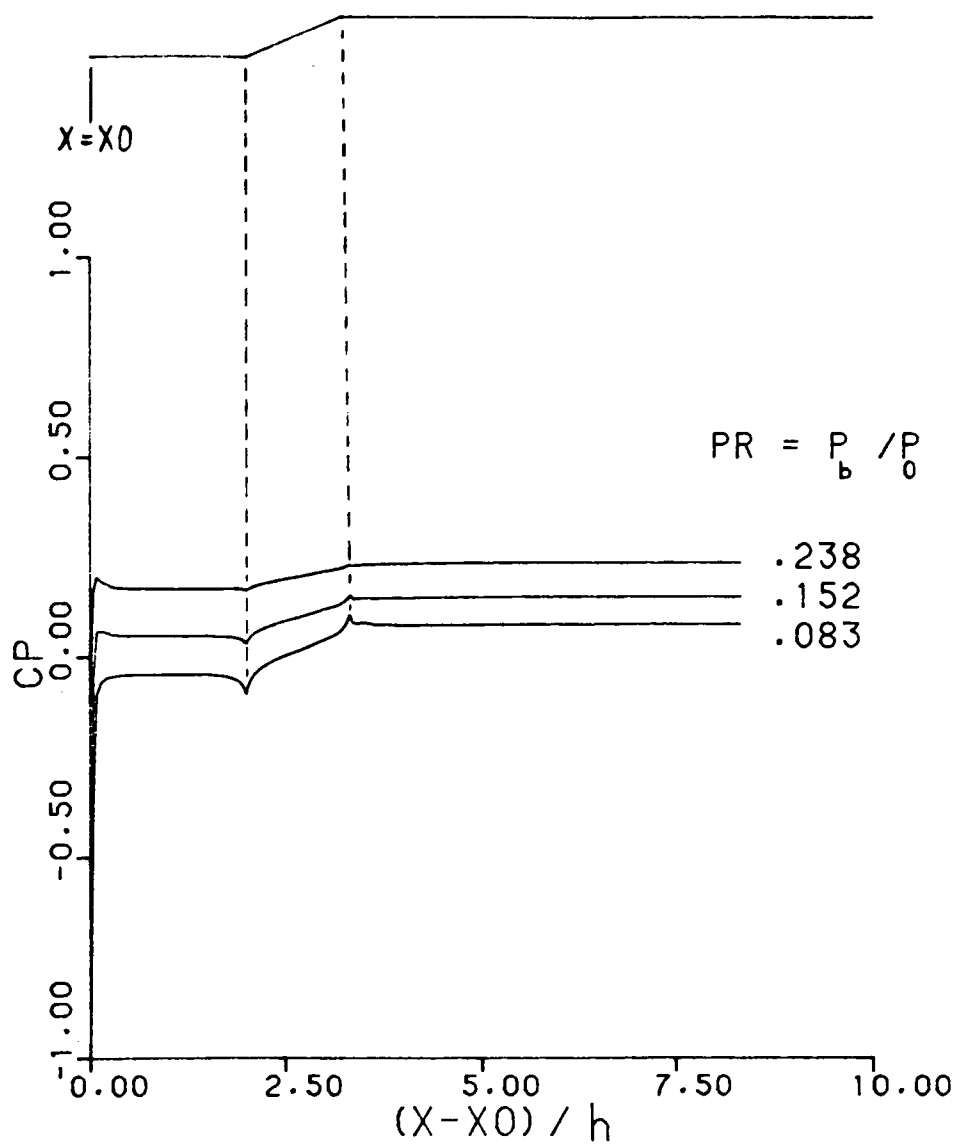


Fig. 9. Computed wall pressure distribution through a 1° receiver/diffuser.

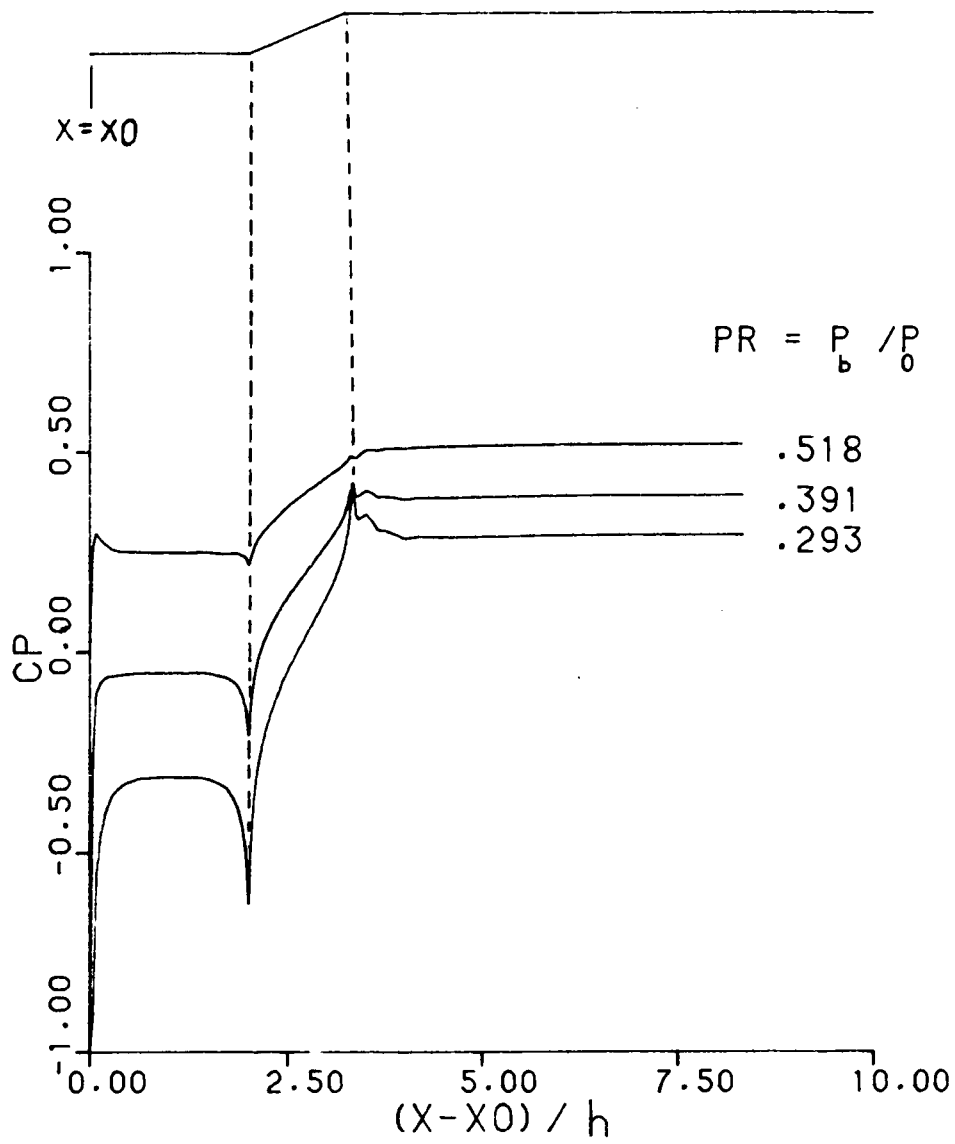
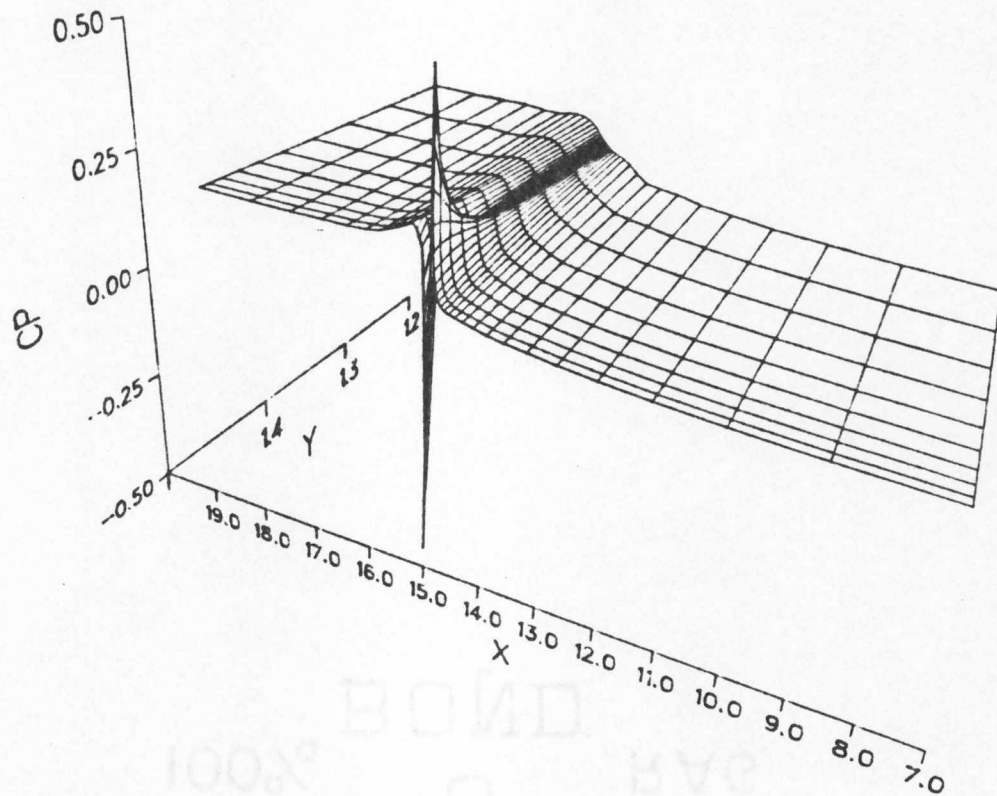
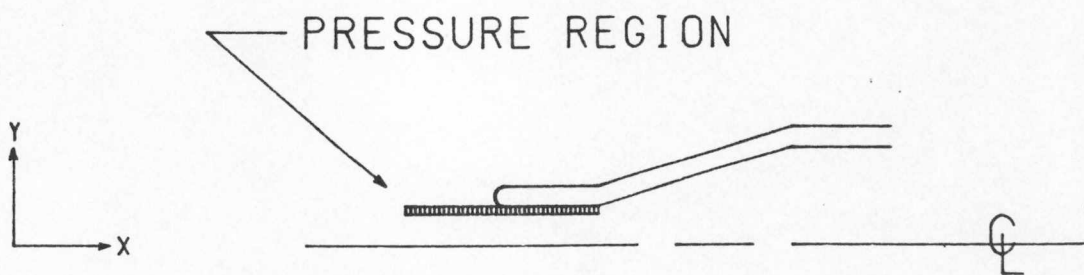


Fig. 10. Computed wall pressure distribution through a 5° receiver/diffuser.



(a)



(b)

Fig. 11. Segment of the two-dimensional pressure field for a 5° receiver/diffuser with a back pressure ratio of .5.

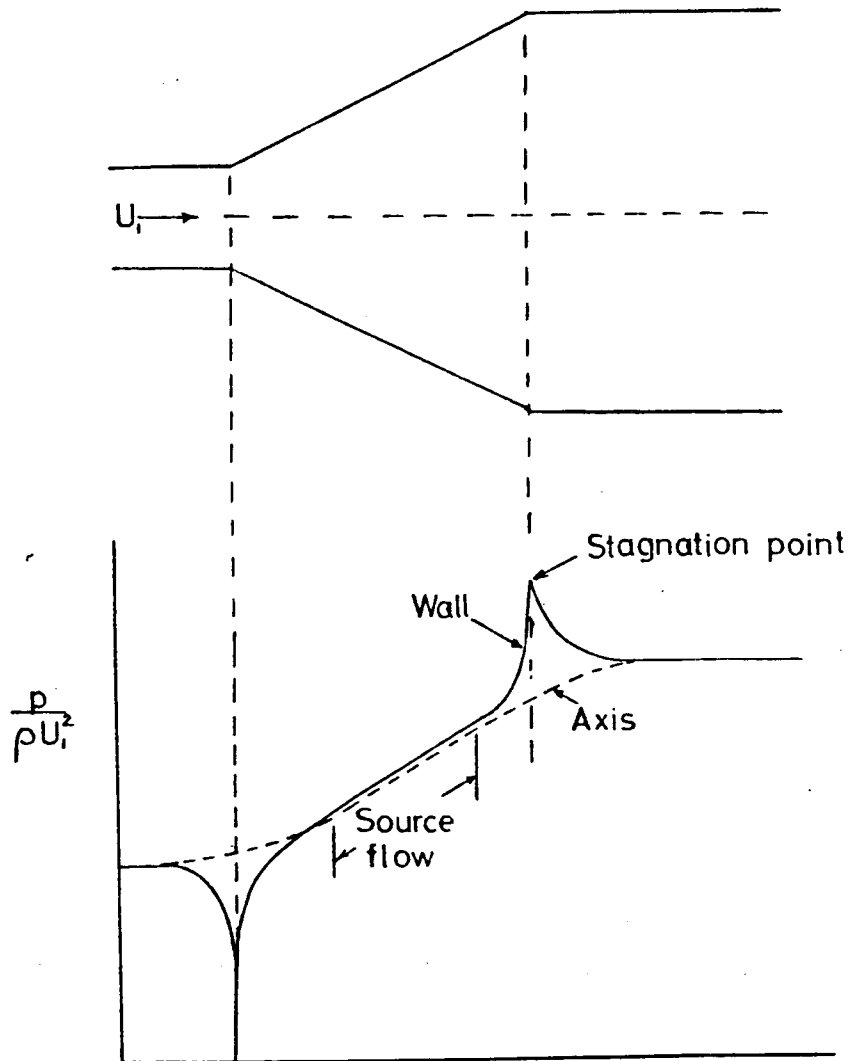
IV. DISCUSSION OF RESULTS AND RECOMMENDATIONS

Confidence in the ability of the technique to obtain a properly converged solution was obtained from the results of the verification problem (Fig. 5, p. 19 and Fig. 6, p. 20). These results were judged close enough to the exact flow solution to indicate that the model was behaving as expected.

The R/D results will be compared with Smith's exact result for two-dimensional potential flow through a straight walled diffuser (ref. 6) (Fig. 12) and with Reid's experimental results (ref. 5) (Fig. 8, p. 23 and Fig. 13, p. 29). Before making these comparisons, the differences between problem definitions in each case should be noted.

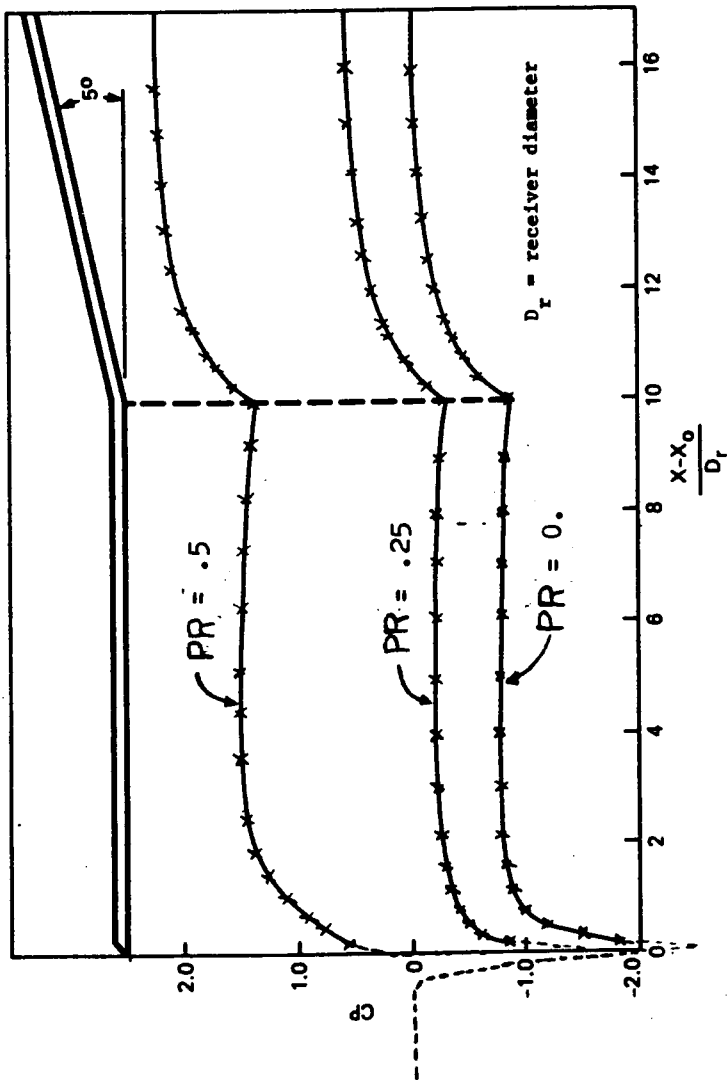
Smith's problem was set up as an entirely internal potential flow and can be used for comparison only inside the R/D where the flow has completed the transition from an external to internal flow. Reid's results are based on an axisymmetric geometry, and, since they are experimental, include viscous effects to some degree. In either case, only qualitative comparisons can be made; hopefully, these comparisons can still lead to a useful assessment of the capabilities of this model.

From Smith's result (ref. 6) (Fig. 12), it can be noted that the pressure distribution along the inside of the diffuser wall varies as would be predicted by substituting the average velocity through the diffuser into Bernoulli's relation. This is true for all points along the wall except those close to the corners. Near the corner where the included angle is greater than 180° , the pressure



Source: Smith, A. J., "Flow in a Diffuser with Straight Walls," Applied Scientific Research, Vol. 31, pp. 21-28, 1975.

Fig. 12. Exact wall pressure distribution for internal potential flow through a diffuser.



Source: Reid, K. N., Static and Dynamic Interaction of a Fluid Jet and a Receiver/Diffuser, Massachusetts Institute of Technology, 1964.

Fig. 13. Experimental wall pressure distribution through an axisymmetric 5° receiver/diffuser.

asymptotically approaches minus infinity, corresponding to a singular point in the flow. At the corner where the included angle is less than 180° , the pressure approaches and reaches the stagnation pressure of the flow.

Similar characteristics can be observed from the computational results. Note that near the corners, where the pressure changes rapidly, the boundary points must be densely packed in order to properly resolve the flow details. It would take an infinite number of boundary points to reproduce the exact solution. The fact that the boundary is computationally represented by discrete points accounts for the differences between the computed pressure distribution and Smith's result. These discrepancies have little bearing on the accuracy of the model since additional resolution at any point in the flow can be obtained by increasing the number of boundary points. This would also, of course, increase the solution computational time.

The noticeable difference between Smith's wall pressure distribution and the computational results shown in Fig. 9 (p. 24) and Fig. 10 (p. 25) is the low pressure at the R/D entrance in the computed results. This difference comes about because of the different problem definitions as mentioned before. Smith's result is for an entirely internal flow and it therefore makes no sense to compare it with computed results that are in, or close to, the external part of the flow.

An additional comparison will be made with Reid's experimental results. Since viscous effects have been neglected in the model,

noticeable differences may exist between the computed results and experimental results (Fig. 8, p. 23 and Fig. 13, p. 29). However, the decision to describe the flow with a potential flow model hinged on the assumption that the design of the R/D could be improved based on a model which neglected viscous effects. This comparison serves to check the validity of the assumption, and hence, the usefulness of the model.

Reid's data (Fig. 13) also shows a measured Bernoulli pressure distribution along the R/D wall. Some pressure loss is noticeable in the straight section due to fluid friction, but this loss is small compared to the changes in the Bernoulli distribution. This result coincides with the distribution predicted by the potential flow models. The point of low pressure at the beginning of the diffusing section is evident, although not as pronounced as in the inviscid models.

Another similarity between Reid's measured pressure distribution and the computed results in Fig. 9 (p. 24) and Fig. 10 (p. 25) is the extremely low pressure at the entrance to the R/D (the dashed line shows extrapolated results). This is true in spite of the difference in the shape of the wall at the entrance. Reid used a sharp edge geometry (Fig. 13) whereas the computational results were for a rounded geometry (Fig. 7, p. 22).

The major difference between the flow field studied in this thesis and Reid's measured flow field is the current flow field appears to be more like a jet submerged in a uniform flow, rather than a submerged jet in an otherwise undisturbed flow. That is,

velocity magnitudes entering the upper region on the left boundary, and exiting above the R/D were computed to be about the same as or larger than the jet velocity magnitude. Thus fluid flowing into the upper region appears more like an imposed fluid jet rather than a flow which is induced by low pressures inside the R/D. This difference is a result of the way in which the computational problem was defined.

One result of this difference is that a stagnation point was computed about midway along the rounded leading edge of the R/D wall. This trend is shown in Fig. 11 (p. 26): the fluid stagnates near the tip of the leading edge and then accelerates around that edge into the R/D. Figure 11 (p. 26) shows that these high and low pressures are greatly localized around the leading edge which indicates that this is a critical section of the R/D design.

Although Reid did not measure the pressure along this leading edge (probably because of the difficulty in physically placing the sensors in such a position), intuitively, if fluid were indeed being induced into the R/D, one would expect the induced flow velocity to be much less than the velocity of the imposed fluid jet and the fluid to accelerate around the leading edge rather than to stagnate. Despite this difference between measured and computed flow fields, the model could still be helpful to the submerged jet-R/D problem. Given a R/D geometry which is a design option, the pressure field trends inside the R/D could be calculated using this model with some assurance (based on the comparisons made) of meaningful results.

Critical aspects of the shape could be resolved by densely packed boundary points.

Turning attention to the flow rate/pressure curves shown in Fig. 8 (p. 23), one clear similarity between Reid's measured flow curve through the axisymmetric R/D and the computed curve through the two-dimensional R/D is that the slopes are about the same. Note that the included diffuser angle does not significantly affect the slope of the computed flow/pressure curve (at least for the three cases shown).

Reid's curve in Fig. 8 (p. 23) was for a 5° diffuser half angle and a diffuser area ratio of 4.0. For the geometries used in the computed results, the dimensions shown in Fig. 7 (p. 22) were held constant while the diffuser angle θ , was varied. This means that the diffuser area ratio was dependent on θ . This method of varying the area ratio and holding the length dimensions fixed was chosen over the method of fixing the area ratio at 4.0 (that value used by Reid) and varying the diffuser length with because in a two-dimensional geometry, the latter method would require a diffusing length of 34 times the receiver height (shown as 1.51 in Fig. 7, p. 22) for a 5° diffuser, 57 times for a 3° diffuser, and 172 times for a 1° diffuser. These great lengths require many more boundary points, resulting in much higher requirements for computer time and storage (more computational effort than was allowed for this investigation). Note that in Reid's axisymmetric R/D, such a long diffuser was not required because the area increases with the square of the duct radius. So, an additional difference in the problem definition with

Reid's experimental results is the diffuser area ratio. For this reason, the flow rates should not be compared quantitatively. However, it is expected that the computed flow rates through the R/D should be higher than the measured values because the computational model neglects viscous losses through the R/D.

One last limitation on the computational model was discovered from the results. If the upper region Bernoulli constant is decreased causing the upper flow to decrease and raising the R/D back pressure, a point is reached where the decreasing upper flow field energy is not enough to balance the rising static pressures at the R/D exit. This means that values for the R/D flow cannot be obtained for back pressures higher than the values plotted in Fig. 8 (p. 23).

For further analysis of the R/D problem, it is recommended that an algorithm in which the back pressure is specified as the independent variable be developed. As mentioned before, it is expected that such an algorithm would require a nested iterative scheme, but such a model might be worth the effort since it should not include the high velocity characteristic of the upper flow region. To insure that the high flow rate is eliminated, it is recommended that the boundary conditions on the upper flow field be changed so that all of the fluid entering the region could only exit the flow domain through the R/D. By mass conservation, the inflow must then be extremely low compared with the outflow through the R/D which has a much smaller flow area.

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BIBLIOGRAPHY

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APPENDIXES

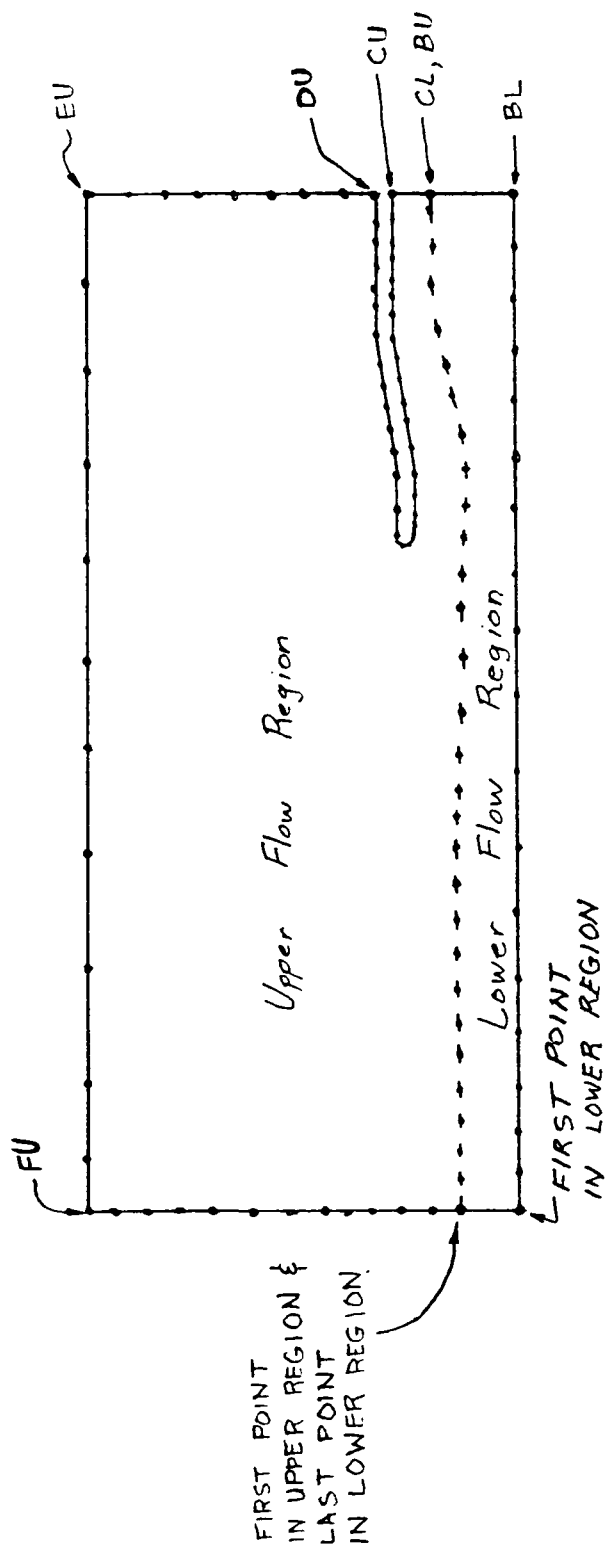
APPENDIX A

PROGRAM LISTING

The FORTRAN listing for the computational model follows. Reference to Fig. A.1 will be helpful in reading the code.

The first program solves for the complex potential at all points along the region boundaries. A sample data set for region coordinates is given.

The second program calculates the pressures on the interior of the upper flow region and writes them out in a format suitable for a plotting routine.



NOTE: ALTHOUGH THE EXACT NUMBER AND LOCATION OF THE
BOUNDARY POINTS USED FOR CALCULATION ARE NOT SHOWN,
THE COORDINATES CAN BE FOUND IN THE INPUT DATA.

A.1. Computational flow domain with program variables defined.

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
 C
 C PROGRAM STREAM IS WRITTEN TO SOLVE BY ITERATION FOR C
 C THE ORIENTATION OF A STREAMLINE BETWEEN TWO REGIONS C
 C OF IDEAL FLUID FLOW (DIFFERENT BERNOULLI CONSTANTS). C
 C THIS IS ACCOMPLISHED BY SATISFYING THE CONDITION C
 C THAT THE PRESSURE ON THE DIVIDING STREAMLINE BE THE C
 C SAME WHEN COMPUTED FROM EITHER SIDE. C
 C

C NOTE: IF MORE THAN 70 POINTS ARE TO BE USED ON THE C
 C DIVIDING STREAMLINE, DIMENSIONS MUST BE C
 C CHANGED IN SUBROUTINE INTG. C
 C

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

C
 C VARIABLE DEFINITIONS: C
 C

C A = MATRIX CONTAINING THE COMPLEX COEFFICIENTS C
 C WHICH ARE DEPENDENT ONLY ON THE BOUNDARY C
 C GEOMETRY C
 C C = COEFFICIENT MATRIX FOR SOLUTION OF C
 C EQUATIONS C
 C CWL = UNDER RELAXATION FACTOR USED ON LINE C
 C ORIENTATION CONVERGENCE C
 C ETA = CONTAINS VECTOR DIFFERENCES BETWEEN C
 C BOUNDARY POINTS C
 C F = CONTAINS COMPLEX POTENTIAL VALUES WHICH C
 C ARE WRITTEN TO TAPE (OR DISK) FOR POST C
 C PROCESSING C
 C H1 = HALF-HEIGHT OF THE ENTERING 2-D JET C
 C H2 = HALF-HEIGHT OF THE FLUID EXITING THE R/D C
 C IN THE LOWER FLOW REGION C
 C HL = BERNOULLI CONSTANT FOR LOWER REGION C
 C HU = BERNOULLI CONSTANT FOR UPPER REGION C
 C HUO = BERNOULLI CONSTANT FOR UPPER REGION FOR C
 C THE FIRST SOLUTION C
 C IA = MOST POINTS ALLOWED ON THE UPPER REGION C
 C BOUNDARY (DIMENSION OF MATRICES) C
 C IBL = POINT NUMBER OF POINT BL C
 C IBU = POINT NUMBER OF POINT BU C
 C ICL = POINT NUMBER OF POINT CL C
 C ICU = POINT NUMBER OF POINT CU C
 C IDU = POINT NUMBER OF POINT DU C
 C IEU = POINT NUMBER OF POINT EU C
 C IFU = POINT NUMBER OF POINT FU C
 C ITMAX = MAXIMUM NUMBER OF ORIENTATION-SOLVE C
 C ITERATIONS ALLOWED C
 C NPTSL = NUMBER OF POINTS APPROXIMATING THE C
 C LOWER REGION BOUNDARY C
 C NPTSU = NUMBER OF POINTS APPROXIMATING THE UPPER C
 C REGION BOUNDARY C
 C PRES = CONTAINS PRESSURES AT BOUNDARY POINTS C
 C PSI = STREAM FUNCTION ALONG THE DIVIDING STREAM C
 C LINE WITH RESPECT TO THE LOWER FLOW REGION C

```

C      PSTAG = STAGNATION PRESSURE OF THE INLET JET      C
C      RHS   = CONTAINS RIGHT HAND SIDE OF SYSTEM OF     C
C              EQUATIONS FOR SOLUTION OF A FLOW REGION    C
C      VEL    = CONTAINS VELOCITY MAGNITUDES AT BOUNDARY  C
C              POINTS                                     C
C      VJETIN = VELOCITY OF THE INLET JET                 C
C      VJETOT = EXIT VELOCITY OF THE LOWER FLOW REGION    C
C      VNORM  = CONTAINS VELOCITY COMPONENT NORMAL TO THE C
C              DIVIDING STREAMLINE GUESS                 C
C      W      = TEMPORARY STORAGE ARRAY USED BOTH FOR     C
C              POTENTIALS AT BOUNDARY POINTS AND FOR SLOPE C
C              OF THE DIVIDING STREAMLINE                 C
C      XL     = LENGTH OF THE COMPUTATIONAL DOMAIN IN THE  C
C              X-DIRECTION                                C
C      Y      = HEIGHT OF THE DIVIDING STREAM LINE        C
C      ZL     = CONTAINS LOWER REGION COORDINATES         C
C      ZU     = CONTAINS UPPER REGION COORDINATES         C
C

```

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

```

```

      IMPLICIT REAL*8 (A-H,P-Z)
      REAL*8 VEL(280),W(280),RHS(280),C(280,280),
1 PRES(280),Y(280),VNORM(280),DATA(10,6)
      COMPLEX*16 A(280,280),ZL(280),ZU(280),F(280),Z(284),
1 ETA(284)
      INTEGER IP(280)
      IA=280
      IAP4=IA+4

```

```

C
C-----SPECIFY GRID PARAMETERS
C

```

```

      NPTSU=280
      NPTSL=120
      NPLM1=NPTSL-1
      NPLM2=NPTSL-2
      NPUM1=NPTSU-1
      IBL=60
      IBU=NPTSL-IBL
      IDU=210
      IEU=235
      IFU=245
      IEUM1=IEU-1
      IFUM1=IFU-1
      IFUP1=IFU+1
      IBUP1=IBU+1
      IBLP1=IBL+1
      IBLM1=IBL-1
      IBUM1=IBU-1
      ICL=IBL+1
      ICLM1=ICL-1
      ICU=IBU+1
      ICLP1=ICL+1
      IEUP1=IEU+1
      IDUP1=IDU+1

```

```

C
C-----CONTROLS ON CONVERGENCE
C
      ITMAX=1
      CWL=.3D0
C
C-----PHYSICAL KNOWNNS
C
      HU0=.94D0**2
      HL=1.D0
      PI=4.D0*DATAN(1.D0)
      PSTAG=HL
      VJETIN=DSQRT(HL)
C
C-----READ IN FIXED REGION COORDINATES
C
      DO 10 I=1,IBL
      READ(5,9999)ZL(I)
10     CONTINUE
      DO 20 I=IBUP1,NPTSU
      READ(5,9999)ZU(I)
20     CONTINUE
C
C-----READ IN INITIAL STREAM LINE GUESS
C
      DO 30 I=ICL,NPTSL
      READ(5,9999)ZL(I)
30     CONTINUE
      NPONTS=1
      DO 430 LL=1,NPONTS
      ILAST=0
      HU=HU0-.1D0*DFLOAT(LL-1)
      DATA(LL,1)=HU
      ICOUNT=0
      WRITE(6,9993)HU
40     CONTINUE
      ICOUNT=ICOUNT+1
      IF(ICOUNT.EQ.ITMAX)ILAST=1
C
C-----COPY  GUESS INTO UPPER GRID
C
      DO 50 I=1,IBU
      ZU(I)=ZL(NPTSL+1-I)
50     CONTINUE
C
C-----CALCULATE ADDITIONAL GEOMETRY INFO
C
      H1=DIMAG(ZL(NPTSL)-ZL(1))
      H2=DIMAG(ZL(ICL)-ZL(IBL))
      XL=DREAL(ZU(IBU)-ZU(1))
      PSI=H1*VJETIN
C

```

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C          LOWER REGION SOLUTION
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C-----GO TO SUBPROGRAM TO CALCULATE GEOMETRICAL COMPLEX
C-----COEFFICIENTS FOR LOWER REGION
C
C          CALL COEFF(NPTSL,IA,IAP4,A,ZL,Z,ETA)
C
C-----CALCULATE RIGHT HAND SIDE AND COEFFICIENT MATRIX FOR
C-----SOLUTION GIVEN THE STREAM FUNCTION TO FIND THE
C-----POTENTIAL
C
C          DO 70 J=2,NPLM1
C          DO 60 K=2,NPTSL
C          IF(K.GT.IBL)RHS(J-1)=RHS(J-1)-DREAL(A(J,K))*PSI
C          IF(K.EQ.NPTSL)GO TO 60
C          C(J-1,K-1)=DIMAG(A(J,K))
60      CONTINUE
70      CONTINUE
C
C-----REDUCE MATRICIES,EXCLUDING EQUATION ICL
C
C          DO 80 J=1,NPLM2
C          C(J,IBLM1)=C(J,IBLM1)+C(J,ICLM1)
80      CONTINUE
C          DO 100 J=1,NPLM2
C          IF(J.LE.ICLM1)M=0
C          IF(J.GT.ICLM1)M=1
C          RHS(J-M)=RHS(J)
C          DO 90 K=1,NPLM2
C          IF(K.LE.ICLM1)N=0
C          IF(K.GT.ICLM1)N=1
C          C(J-M,K-N)=C(J,K)
90      CONTINUE
100     CONTINUE
C          NN=NPTSL-3
C          CALL SOLVE(NN,IA,RHS,IP,C)
C          DO 110 J=2,IBL
C          W(J)=RHS(J-1)
110     CONTINUE
C          DO 120 J=ICLP1,NPLM1
C          W(J)=RHS(J-2)
120     CONTINUE
C          W(ICL)=W(IBL)
C          W(1)=0.DO
C          W(NPTSL)=0.DO

```

```

C
C-----DIFFERENTIATE POTENTIAL FOR VELOCITY ALONG DIVIDING
C-----STREAMLINE
C
      CALL DIFF(NPTSL,IBL,IDU,W,ZL,VEL)
C
C-----CALCULATE PRESSURE ALONG DIVIDING LINE REFERENCING
C-----TO UPPER GRID
C
      PRES(IBU)=VJETIN*VJETIN*(1.DO-H1*H1/(H2*H2))
      PRES(1)=0.DO
      VJETOT=.5DO*(VEL(IBL)+VEL(ICL))
      DO 130 J=2,IBLM1
      PRES(J)=HL-VEL(NPTSL+1-J)*VEL(NPTSL+1-J)
130    CONTINUE
C
C-----SET RHS AND COEFFICIENT MATRICIES TO ZERO
C
      DO 150 I=1,NPTSU
      W(I)=0.DO
      RHS(I)=0.DO
      DO 140 J=1,NPTSU
      A(I,J)=(0.DO,0.DO)
      C(I,J)=0.DO
140    CONTINUE
150    CONTINUE
C
C-----CALCULATE VELOCITY DISTRIBUTION ALONG DIVIDING LINE
C
      DO 160 J=1,IBU
      VEL(J)=DSQRT(HU-PRES(J))
160    CONTINUE
C
C-----INTEGRATE VELOCITY FOR POTENTIAL ALONG DIVIDING LINE
C
      W0=0.DO
      CALL INTG(ZU,IA,VEL,IBU,W0,W)
      WIB=W(IBU)
      IF(ILAST.NE.1)GO TO 180
      DO 170 J=1,IBU
      F(J)=DCMLPX(W(J),0.DO)
170    CONTINUE
180    CONTINUE

```

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                 C
C              UPPER REGION SOLUTION                            C
C                                                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C-----CALCULATE GEOMETRICAL COMPLEX COEFFICIENTS FOR
C-----UPPER REGION
C
C              CALL COEFF(NPTSU,IA,IAP4,A,ZU,Z,ETA)
C
C-----CALCULATE COEFFICIENT MATRIX AND RIGHT HAND SIDE FOR
C-----SOLUTION. NOTE THAT POTENTIAL IS KNOWN ALONG THE BOTTOM.
C
C              DO 200 J=2,NPTSU
C              DO 190 K=2,NPTSU
C              IF(K.GT.IBU)GO TO 191
C              RHS(J)=RHS(J)-DIMAG(A(J,K))*W(K)
C              C(J,K)=DREAL(A(J,K))
C              GO TO 190
191          CONTINUE
C              IF(K.GT.IDU)GO TO 192
C              C(J,ICU)=C(J,ICU)+DREAL(A(J,K))
C              IF(K.EQ.ICU)RHS(J)=RHS(J)-DIMAG(A(J,ICU))*WIB
C              IF(K.EQ.ICU)GO TO 190
C              C(J,K)=DIMAG(A(J,K))
C              GO TO 190
192          CONTINUE
C              IF(K.GT.IEU)GO TO 193
C              C(J,IDU)=C(J,IDU)+DIMAG(A(J,K))
C              IF(K.EQ.IEU)C(J,IFU)=C(J,IFU)+DREAL(A(J,K))
C              IF(K.EQ.IEU)GO TO 190
C              C(J,K)=DREAL(A(J,K))
C              GO TO 190
193          CONTINUE
C              IF(K.GT.IFU)GO TO 194
C              C(J,IFU)=C(J,IFU)+DREAL(A(J,K))
C              IF(K.EQ.IFU)GO TO 190
C              C(J,K)=DIMAG(A(J,K))
C              GO TO 190
194          CONTINUE
C              C(J,K)=DREAL(A(J,K))
190          CONTINUE
200          CONTINUE
C
C-----REDUCE MATRIX BY ELIMATING EQUATION AT IEU
C
C              DO 220 J=2,NPTSU
C              IF(J.LE.IEU)M=1
C              IF(J.GT.IEU)M=2
C              RHS(J-M)=RHS(J)
C              DO 210 K=2,NPTSU
C              IF(K.LE.IEU)N=1

```

```

                IF(K.GT.IEU)N=2
                C(J-M,K-N)=C(J,K)
210          CONTINUE
220          CONTINUE
C
C-----DATA FOR SOLVER
C
                NN=NPTSU-2
                CALL SOLVE(NN,IA,RHS,IP,C)
C
C-----FILL IN STREAM/POTENTIAL MATRIX WITH SOLUTION
C
                QUDIF=RHS(ICU-1)
                QUIN=RHS(IFU-2)
                W(1)=0.DO
                W(IFU)=0.DO
                DO 230 J=2,IEUM1
                IF(J.EQ.ICU)GO TO 230
                W(J)=RHS(J-1)
230          CONTINUE
                DO 240 J=IEUP1,NPTSU
                W(J)=RHS(J-2)
240          CONTINUE
                W(ICU)=WIB
                W(IEU)=W(IDU)
C
C-----DETERMINE THE NEW STREAM LINE FOR COMPARISON WITH
C-----ASSUMED LINE
C
                Y(1)=DIMAG(ZU(1))
C
C-----CALCULATE NORMAL VELOCITY BY DIFFERENTIATING STREAM
C-----FUNCTION
C
                CALL DIFF(NPTSU,IBU,IDU,W,ZU,VNORM)
C
C-----DETERMINE ERROR IN THE VNORM DISTRIBUTION
C
                E=0.DO
                DO 250 J=1,IBUM1
                E=DABS((VNORM(J)+VNORM(J+1))/2.DO)*CDABS(ZU(J+1)-
1          ZU(J))+E
250          CONTINUE
                E=E/DSQRT(HU)
                WRITE(6,9998)ICOUNT,E
                IF(ILAST.NE.1)GO TO 310
C
C-----FILL IN COMPLEX POTENTIAL MATRIX FOR UPPER REGION
C
                DO 260 J=1,IBU
                F(J)=DCMPLX(DREAL(F(J)),W(J))
260          CONTINUE

```

```

DO 270 J=ICU, IDU
F(J)=DCMPLX(W(J), QUDIF)
270 CONTINUE
DO 280 J=IDUP1, IEUM1
F(J)=DCMPLX(W(IDU), W(J))
280 CONTINUE
DO 290 J=IEU, IFU
F(J)=DCMPLX(W(J), QUIN)
290 CONTINUE
DO 300 J=IFUP1, NPTSU
F(J)=DCMPLX(0.D0, W(J))
300 CONTINUE
310 CONTINUE
C
C-----ADD TANGENT AND NORMAL VELOCITIES TO GET X AND Y
C-----COMPONENT VELOCITIES FOR NEW LINE SLOPE= VY/VX
C
DO 330 J=1, IBU
THETA1=DATAN2(DIMAG(ZU(J+1)-ZU(J)), DREAL(ZU(J+1)-
1 ZU(J)))
IF(J.EQ.1)GO TO 320
THETA2=DATAN2(DIMAG(ZU(J)-ZU(J-1)), DREAL(ZU(J)-
1 ZU(J-1)))
THETA=(THETA1+THETA2)/2.D0
320 CONTINUE
IF(J.EQ.IBU)THETA=THETA2
IF(J.EQ.1)THETA=THETA1
C
C-----FILL SLOPE INTO ARRAY W
C
W(J)=(VEL(J)*DSIN(THETA)-VNORM(J)*DCOS(THETA))/
1 (VEL(J)*DCOS(THETA)+VNORM(J)*DSIN(THETA))
330 CONTINUE
C
C-----INTEGRATE SLOPE FOR NEW LINE ORIENTATION
C
YO=DIMAG(ZU(1))
CALL INTG(ZU, IA, W, IBU, YO, Y)
C
C-----WEIGHT NEW LINE WITH THE OLD LINE
C
DO 340 J=2, IBU
Y(J)=DIMAG(ZU(J))+(Y(J)-DIMAG(ZU(J)))*CWL
340 CONTINUE
DO 350 J=2, IBU
ZL(NPTSL-J+1)=DCMPLX(DREAL(ZL(NPTSL-J+1)), Y(J))
350 CONTINUE
IF(ILAST.EQ.1)GO TO 380
C
C-----SET MATRICIES TO ZERO
C
DO 370 I=1, NPTSU
RHS(I)=0.D0

```

```

      W(I)=0.DO
      VEL(I)=0.DO
      PRES(I)=0.DO
      DO 360 J=1,NPTSU
      C(I,J)=0.DO
360    CONTINUE
370    CONTINUE
      GO TO 40
380    CONTINUE
      DO 390 J=ICU,NPTSU
      VEL(J)=VNORM(J)
      PRES(J)=HU-VEL(J)*VEL(J)
390    CONTINUE
      PRATIO=PRES(IBU)/PSTAG
      QIN=VJETIN*H1
      QRATIO=(QIN+QUDIF)/QIN
      DATA(LL,2)=PRATIO
      DATA(LL,3)=QRATIO
      DATA(LL,4)=Y(IBU)
      DATA(LL,5)=E
      WRITE(6,9994)
      DO 400 I=1,NPTSU
      CP=PRES(I)/HL
      WRITE(6,9995)I,ZU(I),VEL(I),CP
400    CONTINUE
      DO 420 I=1,NPTSU
      W(I)=0.DO
      RHS(I)=0.DO
      DO 410 J=1,NPTSU
      A(I,J)=(0.DO,0.DO)
      C(I,J)=0.DO
410    CONTINUE
420    CONTINUE
430    CONTINUE
      WRITE(6,9996)
      DO 440 J=1,NPONTs
      WRITE(6,9997)(DATA(J,I),I=1,5)
440    CONTINUE
C
C-----SAVE UPPER FIELD COMPLEX POTENTIAL FOR INTERIOR
C-----CALCULATIONS
C
      WRITE(8)F,ZU
C
C-----FORMATS
C
9999    FORMAT(2(F20.10))
9998    FORMAT(2X,I3,3X,F10.5)
9997    FORMAT(6(2X,F10.7))
9996    FORMAT(//,6X,' HU',7X,'PRATIO',6X,'QRATIO',7X,
1      'YOUT',8X,'ERROR',/)
9995    FORMAT(1X,I3,2X,F8.4,2X,F8.4,2X,F10.6,2X,F10.6)

```

```

9994  FORMAT(1H1, //4X, '*----- FINAL RESULTS FOR UPPER
      1  REGION -----*',
      1  //1X, 'POINT', 4X, 'X', 10X, 'Y', 6X, 'VELOCITY', 3X,
      1  'PRESSURE', /)
9993  FORMAT(///' SOLUTION FOR UPPER FLOW FIELD ', /,
      1  ' BERNOULLI CONSTANT = ', F10.7, //, 2X, 'ITER.', 4X,
      2  'ERROR', /)
      STOP
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C  SUBROUTINE COEFF CALCULATES THE N**2 COMPLEX          C
C  COEFFICIENTS FOR THE N GRID POINTS ON THE BOUNDARY    C
C  OF FLOW FIELD                                           C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE COEFF(NPTS,IA,IAP4,A,ZZ,Z,ETA)
      IMPLICIT REAL*8 (A-H,P-Z)
      COMPLEX*16 ZZ(IA),Z(IAP4),ETA(IAP4),A(IA,IA)
      PI=4.DO*DATAN(1.DO)
      NPTSP2=NPTS+2
      NPTSP4=NPTS+4
      DO 5 J=3,NPTSP2
      Z(J)=ZZ(J-2)
5      CONTINUE
      Z(2)=Z(NPTS+2)
      Z(1)=Z(NPTS+1)
      Z(NPTS+3)=Z(3)
      Z(NPTS+4)=Z(4)

C
C-----BEGIN ITERATING AROUND CONTOUR
C
      DO 10 J=3,NPTSP2
      DO 20 K=3,NPTSP2
      I1=0
      I2=0
      IF(K.NE.3)GO TO 11
      DO 30 M=1,NPTSP4
      ETA(M)=Z(M)-Z(J)
30      CONTINUE
11      CONTINUE
C
C-----PROCEED TO CALCULATE COEFFICIENT MATRIX
C
      IF(K.NE.J)GO TO 12
      A(J-2,K-2)=CDLOG(-ETA(J+1)/ETA(J-1))-DCMLPX(0.DO,PI)
      GO TO 20
12      CONTINUE
      IF(J.EQ.3.AND.K.EQ.NPTS+2)I1=1
      IF(K.NE.J-1.AND.I1.NE.1)GO TO 13
      A(J-2,K-2)=
1      ETA(J-2)/(ETA(J-2)-ETA(J-1))*CDLOG(ETA(J-1)/ETA(J-2))
      GO TO 20
13      CONTINUE

```

```

      IF(J.EQ.NPTS+2.AND.K.EQ.3)I2=1
      IF(K.NE.J+1.AND.I2.NE.1)GO TO 14
      A(J-2,K-2)=
1    ETA(J+2)/(ETA(J+2)-ETA(J+1))*CDLOG(ETA(J+2)/ETA(J+1))
      GO TO 20
14   CONTINUE
      A(J-2,K-2)=ETA(K+1)/(ETA(K+1)-ETA(K))*CDLOG(ETA(K+1)/
1    ETA(K))+ETA(K-1)/(ETA(K-1)-ETA(K))*CDLOG(ETA(K)/
2    ETA(K-1))
20   CONTINUE
10   CONTINUE
      RETURN
      END

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C    SUBROUTINE DIFF DIFFERENTIATES A POINT BY POINT          C
C    FUNCTION F(Z) WHERE Z IS COMPLEX. DIFFERENTIATION        C
C    IS APPROXIMATED BY A THREE POINT TAYLOR SCHEME.          C
C                                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUBROUTINE DIFF(NPTS,NBOT,ID,W,Z,VEL)
      IMPLICIT REAL*8 (A-H,P-Z)
      REAL*8 W(280),VEL(280)
      COMPLEX*16 Z(280)
      NPTSM1=NPTS-1
      A=CDABS(Z(2)-Z(1))
      B=CDABS(Z(3)-Z(2))
      C=CDABS(Z(NBOT)-Z(NBOT-1))
      D=CDABS(Z(NBOT-1)-Z(NBOT-2))
      E=CDABS(Z(NBOT+1)-Z(NBOT+2))
      F=CDABS(Z(NBOT+2)-Z(NBOT+3))
      G=CDABS(Z(NPTS-1)-Z(NPTS))
      H=CDABS(Z(NPTS-2)-Z(NPTS-1))
      VEL(1)=(W(2)-W(1))/A
      VEL(NBOT)=(W(NBOT)-W(NBOT-1))/C
      IF(NBOT.GE.NPTS)GO TO 10
      VEL(NBOT+1)=(W(NBOT+1)-W(NBOT+2))/E
      VEL(NPTS)=(W(NPTS-1)-W(NPTS))/G
      IF(ID.GE.NPTS)GO TO 10
      VEL(ID)=(W(ID)-W(ID-1))/CDABS(Z(ID)-Z(ID-1))
      VEL(ID+1)=(W(ID+1)-W(ID+2))/CDABS(Z(ID+1)-Z(ID+2))
10   CONTINUE
      DO 20 J=2,NPTSM1
      IF(J.EQ.ID.OR.J.EQ.ID+1)GO TO 20
      IF(J.EQ.NBOT.OR.J.EQ.NBOT+1)GO TO 20
      DSIM=CDABS(Z(J)-Z(J-1))
      DSI=CDABS(Z(J+1)-Z(J))
      R=DSI/DSIM
      VEL(J)=(W(J+1)+(R*R-1.DO)*W(J)-R*R*
1    W(J-1))/R/(1.DO+R)/DSIM
20   CONTINUE

```

```

      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      SUBROUTINE INTG INTEGRATES A DISTRIBUTION OF VARIABLY C
C      SPACED FUNCTION POINTS. THE FUNCTION IS OF TWO C
C      INDEPENDENT VARIABLES C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE INTG(Z,NDIM,F,N,GO,G)
      IMPLICIT REAL*8 (A-H,P-Z)
      COMPLEX*16 Z(NDIM)
      REAL*8 F(70),C(70),S(70),C1(70),C2(70),C3(70),
1      ALPHA(70),B(70),D(70),G(70)
      NM1=N-1

C
C-----CALCULATE S VALUES
C
      S(1)=0.DO
      DO 10 I=2,N
      S(I)=S(I-1)+CDABS(Z(I)-Z(I-1))
10      CONTINUE
      FPO=0.DO
      FPN=0.DO

C
C-----DETERMINE COEFFICIENTS FOR TRI-DIAGONAL MATRIX
C
      DO 20 I=2,NM1
      H=S(I+1)-S(I)
      HM=S(I)-S(I-1)
      ALPHA(I)=3.DO*(F(I+1)*HM-F(I)*(S(I+1)-S(I-1))+
1      F(I-1)*H)/H/HM
20      CONTINUE

C
C-----SOLVE TDM
C
      C1(1)=1.DO
      C3(1)=0.DO
      C2(1)=0.DO
      DO 30 I=2,NM1
      C1(I)=2.DO*(S(I+1)-S(I-1))-(S(I)-S(I-1))*C3(I-1)
      C3(I)=(S(I+1)-S(I))/C1(I)
      C2(I)=(ALPHA(I)-(S(I)-S(I-1))*C2(I-1))/C1(I)
30      CONTINUE
      C1(N)=1.DO
      C2(N)=0.DO
      C(N)=C2(N)
      DO 40 II=1,NM1
      I=NM1-II+1
      C(I)=C2(I)-C3(I)*C(I+1)
      B(I)=(F(I+1)-F(I))/(S(I+1)-S(I))-(S(I+1)-S(I))*
1      (C(I+1)+2.DO*C(I))/3.DO
      D(I)=(C(I+1)-C(I))/3.DO/(S(I+1)-S(I))
40      CONTINUE

```

```

C
C-----INTEGRATE THE APPROXIMATION
C
      G(1)=G0
      DO 50 I=2,N
      DI=F(I-1)*(S(I)-S(I-1))+B(I-1)*(S(I)-S(I-1))**2/2.DO+
1  C(I-1)*(S(I)-S(I-1))**3/3.DO+D(I-1)*(S(I)-S(I-1))**4/
2  4.DO
      G(I)=G(I-1)+DI
50  CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      SUBROUTINE SOLVE SOLVES A FULL N BY N SYSTEM OF      C
C      EQUATIONS.                                           C
C                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUBROUTINE SOLVE(N,NDIM,B,IP,C)
      IMPLICIT REAL*8 (A-H,P-Z)
      REAL*8 B(NDIM),C(NDIM,NDIM)
      INTEGER IP(NDIM)
      IP(N)=1
      DO 6 K=1,N
      IF(K.EQ.N)GO TO 5
      KP1=K+1
      M=K
      DO 1 I=KP1,N
      IF(DABS(C(I,K)).GT.DABS(C(M,K)))M=I
1  CONTINUE
      IP(K)=M
      IF(M.NE.K)IP(N)=-IP(N)
      T=C(M,K)
      C(M,K)=C(K,K)
      C(K,K)=T
      IF(T.EQ.0.DO)GO TO 5
      DO 2 I=KP1,N
      C(I,K)=-C(I,K)/T
2  CONTINUE
      DO 4 J=KP1,N
      T=C(M,J)
      C(M,J)=C(K,J)
      C(K,J)=T
      IF(T.EQ.0.DO)GO TO 4
      DO 3 I=KP1,N
      C(I,J)=C(I,J)+C(I,K)*T
3  CONTINUE
4  CONTINUE
5  IF(C(K,K).EQ.0.DO)IP(N)=0
6  CONTINUE

```

```

C
C-----SOLVE SYSTEM
C
      IF(N.EQ.1)GO TO 9
      NM1=N-1
      DO 7 K=1,NM1
      KP1=K+1
      M=IP(K)
      T=B(M)
      B(M)=B(K)
      B(K)=T
      DO 7 I=KP1,N
      B(I)=B(I)+C(I,K)*T
7      CONTINUE
      DO 8 KB=1,NM1
      KM1=N-KB
      K=KM1+1
      B(K)=B(K)/C(K,K)
      T=-B(K)
      DO 8 I=1,KM1
      B(I)=B(I)+C(I,K)*T
8      CONTINUE
9      B(1)=B(1)/C(1,1)
10     RETURN
      END

```

C
 C-----INPUT COORDINATES FOR BOUNDARY POINTS (INCLUDING
 C-----INITIAL STREAMLINE GUESS) FOR 5 DEGREE R/D PROBLEM
 C

X	Y
0.0000000000	0.0000000000
0.6779661017	0.0000000000
1.3559322034	0.0000000000
2.0338983051	0.0000000000
2.7118644068	0.0000000000
3.3898305085	0.0000000000
4.0677966102	0.0000000000
4.7457627119	0.0000000000
5.4237288136	0.0000000000
6.1016949153	0.0000000000
6.7796610169	0.0000000000
7.4576271186	0.0000000000
8.1355932203	0.0000000000
8.8135593220	0.0000000000
9.4915254237	0.0000000000
10.1694915254	0.0000000000
10.8474576271	0.0000000000
11.5254237288	0.0000000000
12.2033898305	0.0000000000
12.8813559322	0.0000000000
13.5593220339	0.0000000000
14.2372881356	0.0000000000
14.9152542373	0.0000000000
15.5932203390	0.0000000000
16.2711864407	0.0000000000
16.9491525424	0.0000000000
17.6271186441	0.0000000000
18.3050847458	0.0000000000
18.9830508475	0.0000000000
19.6610169492	0.0000000000
20.3389830508	0.0000000000
21.0169491525	0.0000000000
21.6949152542	0.0000000000
22.3728813559	0.0000000000
23.0508474576	0.0000000000
23.7288135593	0.0000000000
24.4067796610	0.0000000000
25.0847457627	0.0000000000
25.7627118644	0.0000000000
26.4406779661	0.0000000000
27.1186440678	0.0000000000
27.7966101695	0.0000000000
28.4745762712	0.0000000000
29.1525423729	0.0000000000
29.8305084746	0.0000000000
30.5084745763	0.0000000000
31.1864406780	0.0000000000
31.8644067797	0.0000000000

32.5423728814	0.0000000000
33.2203389831	0.0000000000
33.8983050847	0.0000000000
34.5762711864	0.0000000000
35.2542372881	0.0000000000
35.9322033898	0.0000000000
36.6101694915	0.0000000000
37.2881355932	0.0000000000
37.9661016949	0.0000000000
38.6440677966	0.0000000000
39.3220338983	0.0000000000
40.0000000000	0.0000000000
40.0000000000	1.8499546541
37.4192193355	1.8499546541
35.2685687817	1.8499546541
33.4763599869	1.8499546541
31.9828526580	1.8499546541
30.7382632171	1.8499546541
29.7011053498	1.8499546541
28.8368071270	1.8499546541
28.1165586080	1.8499546541
27.5163515088	1.8499546541
27.0161789262	1.8499546541
26.5993684406	1.8499546541
26.2520263694	1.8499546541
25.9625746433	1.8499546541
25.7213648716	1.8499546541
25.5203567285	1.8499546541
25.3528499426	1.8499546541
25.2132609543	1.8499546541
25.0969367974	1.8499546541
25.0000000000	1.8499546541
24.9722357603	1.8475255979
24.9389186727	1.8446107304
24.8989381676	1.8411128895
24.8509615614	1.8369154803
24.7933896340	1.8318785893
24.7243033212	1.8258343201
24.6413997457	1.8185811971
24.5419154552	1.8098774495
24.4225343066	1.7994329524
24.2792769282	1.7868995558
24.1073680742	1.7718594799
23.9010774493	1.7538113888
23.6535286995	1.7321536795
23.3564701997	1.7061644284
23.0000000000	1.6749773271
22.6435298003	1.6437902257
22.3464713005	1.6178009746
22.0989225507	1.5961432653
21.8926319258	1.5780951742
21.7207230718	1.5630550983
21.5774656934	1.5505217018
21.4580845448	1.5400772046

21.3586002543	1.5313734570
21.2756966788	1.5241203340
21.2066103660	1.5180760648
21.1490384386	1.5130391738
21.1010618324	1.5088417647
21.0610813273	1.5053439237
21.0277642397	1.5024290562
21.0000000000	1.5000000000
20.8844317294	1.5000000000
20.7457498046	1.5000000000
20.5793314948	1.5000000000
20.3796295232	1.5000000000
20.1399871571	1.5000000000
19.8524163179	1.5000000000
19.5073313109	1.5000000000
19.0932293024	1.5000000000
18.5963068922	1.5000000000
18.0000000000	1.5000000000
17.4036931078	1.5000000000
16.9067706976	1.5000000000
16.4926686891	1.5000000000
16.1475836821	1.5000000000
15.8600128429	1.5000000000
15.6203704768	1.5000000000
15.4206685052	1.5000000000
15.2542501954	1.5000000000
15.1155682706	1.5000000000
15.0000000000	1.5000000000
14.9971826744	1.5004050703
14.9945935918	1.5015874647
14.9924425043	1.5034513927
14.9909036800	1.5058458499
14.9901017856	1.5085768516
14.9901017856	1.5114231484
14.9909036800	1.5141541501
14.9924425043	1.5165486073
14.9945935918	1.5184125353
14.9971826744	1.5195949297
15.0000000000	1.5200000000
15.3000000000	1.5200000000
15.6000000000	1.5200000000
15.9000000000	1.5200000000
16.2000000000	1.5200000000
16.5000000000	1.5200000000
16.8000000000	1.5200000000
17.1000000000	1.5200000000
17.4000000000	1.5200000000
17.7000000000	1.5200000000
18.0000000000	1.5200000000
18.3000000000	1.5200000000
18.6000000000	1.5200000000
18.9000000000	1.5200000000
19.2000000000	1.5200000000
19.5000000000	1.5200000000

19.8000000000	1.5200000000
20.1000000000	1.5200000000
20.4000000000	1.5200000000
20.7000000000	1.5200000000
21.0000000000	1.5200000000
21.1333333333	1.5316651551
21.2666666667	1.5433303103
21.4000000000	1.5549954654
21.5333333333	1.5666606205
21.6666666667	1.5783257757
21.8000000000	1.5899909308
21.9333333333	1.6016560860
22.0666666667	1.6133212411
22.2000000000	1.6249863962
22.3333333333	1.6366515514
22.4666666667	1.6483167065
22.6000000000	1.6599818616
22.7333333333	1.6716470168
22.8666666667	1.6833121719
23.0000000000	1.6949773271
23.1333333333	1.7066424822
23.2666666667	1.7183076373
23.4000000000	1.7299727925
23.5333333333	1.7416379476
23.6666666667	1.7533031027
23.8000000000	1.7649682579
23.9333333333	1.7766334130
24.0666666667	1.7882985681
24.2000000000	1.7999637233
24.3333333333	1.8116288784
24.4666666667	1.8232940336
24.6000000000	1.8349591887
24.7333333333	1.8466243438
24.8666666667	1.8582894990
25.0000000000	1.8699546541
25.7894736842	1.8699546541
26.5789473684	1.8699546541
27.3684210526	1.8699546541
28.1578947368	1.8699546541
28.9473684211	1.8699546541
29.7368421053	1.8699546541
30.5263157895	1.8699546541
31.3157894737	1.8699546541
32.1052631579	1.8699546541
32.8947368421	1.8699546541
33.6842105263	1.8699546541
34.4736842105	1.8699546541
35.2631578947	1.8699546541
36.0526315789	1.8699546541
36.8421052632	1.8699546541
37.6315789474	1.8699546541
38.4210526316	1.8699546541
39.2105263158	1.8699546541
40.0000000000	1.8699546541

40.0000000000	1.8861884719
40.0000000000	1.9072924349
40.0000000000	1.9347275870
40.0000000000	1.9703932846
40.0000000000	2.0167586915
40.0000000000	2.0770337205
40.0000000000	2.1553912581
40.0000000000	2.2572560571
40.0000000000	2.3896802957
40.0000000000	2.5618318060
40.0000000000	2.7856287693
40.0000000000	3.0765648216
40.0000000000	3.4547816896
40.0000000000	3.9464636181
40.0000000000	4.5856501250
40.0000000000	5.4165925840
40.0000000000	6.4968177808
40.0000000000	7.9011105365
40.0000000000	9.7266911190
40.0000000000	12.0999458763
40.0000000000	15.1851770607
40.0000000000	19.1959776004
40.0000000000	24.4100183021
40.0000000000	31.1882712142
40.0000000000	40.0000000000
36.0000000000	40.0000000000
32.0000000000	40.0000000000
28.0000000000	40.0000000000
24.0000000000	40.0000000000
20.0000000000	40.0000000000
16.0000000000	40.0000000000
12.0000000000	40.0000000000
8.0000000000	40.0000000000
4.0000000000	40.0000000000
0.0000000000	40.0000000000
0.0000000000	30.9992882685
0.0000000000	24.0756638597
0.0000000000	18.7497989298
0.0000000000	14.6529797530
0.0000000000	11.5015803862
0.0000000000	9.0774270272
0.0000000000	7.2126936740
0.0000000000	5.7782834024
0.0000000000	4.6748908858
0.0000000000	3.8261274114
0.0000000000	3.1732324312
0.0000000000	2.6710055233
0.0000000000	2.2846771326
0.0000000000	1.9875014474
0.0000000000	1.7589047666
0.0000000000	1.5830611659
0.0000000000	1.4477968577
0.0000000000	1.3437473898
0.0000000000	1.2637093376

0.0000000000	1.2021416052
0.0000000000	1.1547818110
0.0000000000	1.1183512000
0.0000000000	1.0903276532
0.0000000000	1.0687710787
0.0000000000	1.0521890983
0.0000000000	1.0394337287
0.0000000000	1.0296219060
0.0000000000	1.0220743501
0.0000000000	1.0162685378
0.0000000000	1.0118025284
0.0000000000	1.0083671365
0.0000000000	1.0057245274
0.0000000000	1.0036917511
0.0000000000	1.0021280771
0.0000000000	1.0009252509
40.0000000000	1.0000000000
38.1920197291	1.0000000000
36.5211414878	1.0000000000
34.9708203971	1.0000000000
33.5270454644	1.0000000000
32.1778710029	1.0000000000
30.9130482701	1.0000000000
29.7237331765	1.0000000000
28.6022523411	1.0000000000
27.5419143410	1.0000000000
26.5368562827	1.0000000000
25.5819182127	1.0000000000
24.6725396441	1.0000000000
23.8046737802	1.0000000000
22.9747159989	1.0000000000
22.1794439025	1.0000000000
21.4159668026	1.0000000000
20.6816829501	1.0000000000
19.9742431532	1.0000000000
19.2915196942	1.0000000000
18.6315796612	1.0000000000
17.9926619754	1.0000000000
17.3731575252	1.0000000000
16.7715919235	1.0000000000
16.1866104862	1.0000000000
15.6169651018	1.0000000000
15.0615027125	1.0000000000
14.5191551766	1.0000000000
13.9889303156	1.0000000000
13.4699039822	1.0000000000
12.9612130085	1.0000000000
12.4620489164	1.0000000000
11.9716522880	1.0000000000
11.4893077097	1.0000000000
11.0143392144	1.0000000000
10.5461061575	1.0000000000
10.0839994701	1.0000000000
9.6274382415	1.0000000000

9.1758665870	1.0000000000
8.7287507638	1.0000000000
8.2855765023	1.0000000000
7.8458465222	1.0000000000
7.4090782075	1.0000000000
6.9748014162	1.0000000000
6.5425564035	1.0000000000
6.1118918372	1.0000000000
5.6823628886	1.0000000000
5.2535293797	1.0000000000
4.8249539719	1.0000000000
4.3962003783	1.0000000000
3.9668315858	1.0000000000
3.5364080711	1.0000000000
3.1044859941	1.0000000000
2.6706153545	1.0000000000
2.2343380937	1.0000000000
1.7951861261	1.0000000000
1.3526792800	1.0000000000
0.9063231297	1.0000000000
0.4556066975	1.0000000000
0.0000000000	1.0000000000

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C   THIS PROGRAM CALCULATES PRESSURES ON THE INTERIOR      C
C   OF UPPER REGION GIVEN THE COMPLEX POTENTIAL AT ALL    C
C   UPPER REGION BOUNDARY POINTS.                          C
C                                                           C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C   IMPLICIT REAL*8 (A-H,P-Z)
C   REAL*4 TDUM,IDENT(20),X(100),Y(11),P(98,10)
C   COMPLEX*16 ZU(280),ETA(280),A(100,11),ZIN,F(280),
C   1 FP,ETAP,CI,CONE
C   REAL*8 PWALL(50),XWALL(50)
C   DATA IDENT /20*'  '/
C   NPTS=280
C   NWALL=50
C
C-----READ IN COORDINATES OF BOUNDARY AND COMPLEX POTENTIALS
C-----AT THOSE POINTS
C
C   READ(23)F,ZU
C   IWST=130
C   NI=100
C   NIM1=NI-1
C   NIM2=NI-2
C   NJ=11
C   NJM1=NJ-1
C   YWALL=1.5D0
C   H=.94D0**2
C
C-----READ IN COORDINATES OF INTERIOR POINTS
C
C   DO 6 I=1,NI
C   READ(5,99)X(I)
6   CONTINUE
C   DO 7 J=1,NJ
C   READ(5,99)Y(J)
7   CONTINUE
C   DO 5 I=1,NWALL
C   L=NWALL+1-I
C   READ(5,98)XWALL(L),PWALL(L)
5   CONTINUE
C   CONE=DCMPLX(1.D0,0.D0)
C   CI=DCMPLX(0.D0,1.D0)
C   PI=4.D0*DATAN(1.D0)
C
C-----INTEGRATE AROUND THE CONTOUR FOR COMPLEX POTENTIAL AT
C-----EACH INTERIOR POINT
C
C   DO 10 J=1,NJ
C   DO 20 I=1,NI
C   IF(J.EQ.NJ.AND.X(I).GE.XWALL(1))GO TO 36
C   ZIN=CMPLX(X(I),Y(J))

```

```

DO 31 M=1,NPTS
ETA(M)=ZU(M)-ZIN
31 CONTINUE
DO 30 K=1,NPTS
IF(K.EQ.NPTS)GO TO 33
ETAP=ETA(K+1)
FP=F(K+1)
GO TO 34
33 CONTINUE
ETAP=ETA(1)
FP=F(1)
34 CONTINUE
A(I,J)=F(K)*(ETAP/(ETAP-ETA(K))*CDLOG(ETAP/ETA(K))-CONE)+
1 FP*(ETA(K)/(ETA(K)-ETAP)*CDLOG(ETAP/ETA(K))+CONE)+A(I,J)
30 CONTINUE
A(I,J)=-CI*A(I,J)/2.DO/PI
GO TO 20
36 CONTINUE
C
C-----IF 'INTERIOR' POINT ACTUALLY LIES ALONG THE
C-----WALL THEN INTERPOLATE FOR THE COMPLEX POTENTIAL
C
DO 37 M=1,NWALL
X2=XWALL(M)
N2=M
IF(X2.GT.X(I))GO TO 38
N1=N2
X1=X2
37 CONTINUE
WRITE(6,97)
STOP
38 CONTINUE
FACT=(X(I)-X1)/(X2-X1)
A(I,J)=F(IWST+1-N1)+FACT*(F(IWST+1-N2)-F(IWST+1-N1))
20 CONTINUE
10 CONTINUE
C
C-----CALCULATE VELOCITIES AND PRESSURES ON THE INTERIOR
C-----EITHER BY FINITE DIFFERENCE OR INTERPOLATION
C
DO 50 I=2,NIM1
DO 60 J=2,NJ
DYS=Y(J)-Y(J-1)
DXE=X(I)-X(I-1)
DXW=X(I+1)-X(I)
IF(J.EQ.NJ.AND.X(I).GE.XWALL(1))GO TO 51
IF(J.NE.NJ)GO TO 52
V=DREAL(A(I,J)-A(I,J-1))/(Y(J)-Y(J-1))
GO TO 53
52 CONTINUE
DYN=Y(J+1)-Y(J)
V=(A(I,J)*(1.DO/DYS**2-1.DO/DYN**2)+A(I,J+1)/DYN**2-
1 A(I,J-1)/DYS**2)/(1.DO/DYS+1.DO/DYN)
53 CONTINUE

```

```

      U=(A(I,J)*(1.DO/DXE**2-1.DO/DXW**2)+A(I+1,J)/DXW**2-
1  A(I-1,J)/DXE**2)/(1.DO/DXE+1.DO/DXW)
      VT=DSQRT(V*V+U*U)
      P(I-1,J-1)=H-VT*VT
      GO TO 60
51  CONTINUE
      DO 70 M=1,NWALL
      X2=XWALL(M)
      N2=M
      IF(X2.GT.X(I))GO TO 71
      N1=N2
      X1=X2
70  CONTINUE
      WRITE(6,97)
      STOP
71  CONTINUE
      FACT=(X(I)-X1)/(X2-X1)
      P(I-1,J-1)=PWALL(N1)+FACT*(PWALL(N2)-PWALL(N1))
60  CONTINUE
50  CONTINUE
      DO 80 I=1,NIM2
      WRITE(6,96)(P(I,J),J=1,NJM1)
80  CONTINUE
C
C-----WRITE PRESSURE DATA TO TAPE OR DISK FOR
C-----PLOTING PROGRAM
C
      WRITE(8)NIM2,NJM1,IDENT,(X(I),I=2,NIM1),(Y(I),I=2,NJ)
      WRITE(8)TDUM,P
      WRITE(8)P
      WRITE(8)P
      WRITE(8)P
      WRITE(8)P
99  FORMAT(F20.10)
98  FORMAT(5X,F8.4,24X,F10.6)
97  FORMAT(' DATA POINT OUT OF EXPECTED GRID')
96  FORMAT(10F8.4)
      STOP
      END

```

APPENDIX B

ADDITIONAL COMPUTATIONAL MODEL INFORMATION

Discretization

The distribution of the points along the boundary of a region is crucial to keep the total number of points to a minimum and yet sufficiently represent the boundary. Several distributions were used on both the cylinder and R/D problems. It was discovered that using a geometric progression of point spacings while packing points more densely around critical aspects of the geometry yielded the best results.

Reduction of Equations

Although for N discrete boundary points, it is possible to write N equations, for some regions there exist less than N unknowns since both ϕ and ψ can be known at a point. Since the number of equations exceeds the number of unknowns in such a case, the system is overdetermined. This was the case in the application of the method in this thesis. The system was solved by not including the equations written at the points with no unknowns. For the upper region, there exist $N-3$ unknowns and for the lower region, $N-2$.

Convergence of the Solution

The number of iterations required to converge on a streamline orientation ranged between 20 and 30 with under-relaxation factors between .1 and .5. The total computational time on the IBM 3081 in double precision was about 10 minutes.

VITA

Mark W. Wendel was born in Lindsborg, Kansas on June 25, 1961. Subsequently, he grew up and attended elementary schools in Illinois, Nebraska, and Tennessee and eventually graduated from Farragut High School in Knoxville in June 1979. The following September he entered the University of Tennessee at Knoxville, and in June 1984 he received a Bachelor of Science degree in Mechanical Engineering. He immediately entered the University of Tennessee Graduate School funded by a Mechanical Engineering Departmental fellowship. After two quarters of full time graduate school, he accepted a job with Martin Marietta Energy Systems, Inc., in Oak Ridge, Tennessee. He then completed his thesis work and received his Masters degree in Mechanical Engineering in December 1985.

The author is a member of Tau Beta Pi, the American Society of Mechanical Engineers and the American Institute of Aeronautics and Astronautics.