VAPORIZATION OF A PURE LIQUID INTO GAS BUBBLES

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A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN CHEMICAL ENGINEERING

The University of Tennessee

June 1955

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SUMMARY

This investigation, part of a long range research project defined by Peebles (6) for the purpose of calculating Murphree vapor efficiencies, is concerned with the isothermal vaporization of a pure liquid into a gas bubble. The main objective of this phase of the project was to determine whether or not convection currents affect mass transfer within the gas bubble. Accordingly, the following equation was derived on the basis of the assumptions that (1) The bubble is spherical in shape and of constant radius and (2)No convection currents exist in the bubble so that the only mass transfer process is radial diffusion of the vapor from the surface into the bubble. Under these conditions the Murphree efficiency should be

$$E_{mv} = \frac{y_a}{y_e} = 1 - \frac{6}{\pi^2} \sum_{n=1}^{n=\infty} \frac{1}{n^2} e^{-\frac{n^2 \pi^2 D t}{R^2}}$$

where E_{mv} = Murphree vapor efficiency

D = diffusivity constant

t = time of contact between liquid and vapor

R = bubble radius

y_a = average concentration of liquid in bubble

y_e = equilibrium concentration of liquid in bubble

Only a very limited amount of experimental data could be obtained to check the above relation because of experimental difficulties. Accordingly, no conclusions could be drawn.

In order to understand the difficulties encountered, the procedure should be considered. A measured quantity of gas was bubbled through a liquid of known height. The gas-vapor mixture resulting from this procedure was then passed through an adsorption bulb for the purpose of removing the vapor so it could be measured quantitatively. Nitrogen was used as the gas the the liquids, ethyl acetate and water were used. The difficulties encountered occurred in the formation of bubbles and in the quantitative determination of the amount of vapor in the gas. The bubble formation difficulties resulted because of the fact that mercury was placed over the capillary to prevent liquid-bubble contact while the bubble was forming. However, the mercury produced an uneven bubble rate and caused the bubbles to vary in size.

The second difficulty resulted from the fact that an adsorption agent of sufficient activity and capacity could not be found. None at all could be found for the ethyl acetate while silica gel was the only one showing promise for the water vapor. Proposed recommendations for the alleviation of the above difficulties are given in the text of the report.

INTRODUCTION

Definition of Problem

Some of the most important diffusional operations in chemical engineering involve mass transfer between a liquid phase and gas bubbles. One of the most important of these operations is that of distillation for which the equipment usually consists of a vertical circular column in which plates are equally spaced. The flow of vapor and liquid is countercurrent and the plate is equipped with some device such as bubble caps which bubble the gas through the liquid retained on the plate. Diffusional interchange occurs between the rising gas bubbles and the body of the fluid held on the plates.

In the design of contacting columns, the number of theoretical plates required for a given separation are calculated. A theoretical plate is one which fulfills the requirement that the vapor rising from the plate and passing to the plate above is in equilibrium with the liquid leaving the plate and passing to the plate below. In actual practice, such equilibrium is rarely attained. It is thus necessary to convert this calculated number of theoretical plates to the actual number of plates required by means of a plate efficiency. Two main types of efficiency terms are generally used. The first of these is the overall efficiency for the entire column, obtained by dividing the number

of theoretical plates by the number of actual plates found necessary for the required separation. The other efficiency used is the Murphree plate efficiency defined by the following equation:

$$E_{mp} = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} \ 100$$

where y_{n+1} = the composition of vapor rising to the plate

y_n = the composition of the vapor leaving the plate

y_n^* = the composition of vapor in equilibrium with the liquid leaving the plate

E_{mp} = the Murphree plate efficiency

It should be noted that this efficiency may be greater than 100% because of a change in the liquid composition as the liquid flows across the plate. If the liquid composition on a plate remains essentially constant, the efficiency term defined by the above equation is known as the Murphree vapor efficiency, E_{mv}.

At the present time, it is necessary, for the most part, to obtain plate efficiencies experimentally, although some progress has been made in the calculation of Murphree vapor efficiencies from the physical properties of the system. Walter and Sherwood (9) were able to fit the data of their experiments by the empirical equation:

$$E_{mv} = 1 - e^{-m}$$

$$m = \frac{h}{(2.50 + 0.370/HP)\ \mu^{0.68} \omega^{0.33}}$$

where h = the effective fluid depth, inches

H = Henry's law coefficient, (lb moles)/(Ft^3)(atm.)

P = the pressure in atm.

μ = the viscosity in centipoises

w = the slot width in inches

Geddes (4) attempted the calculation of Murphree vapor efficiencies from theoretical considerations only. However, as stated in his paper, "due to incompleteness of existing data certain simplifying assumptions were necessary, and hence there is the need for further refinement. The main objective of the paper will be achieved if it clarifies the interrelation of the variables and elicits the active interest of chemical engineers in more fundamental experimental work."

In accordance with this need for "more fundamental experimental work," Peebles (6) defined a long range research project, of which this paper is a part, for studying mass transfer in liquid-gas bubble systems. This paper is specifically concerned with the isothermal vaporization of a pure liquid into a gas bubble with the primary objective being that of determining whether appreciable convection occurs within the gas bubble. Accordingly, the mathematical analysis of the prob lem is made on the assumption that no convection currents occur in the gas bubble, that

is, that the mass transfer is due only to radial diffusion into the bubble.

In connection with the above statements, it should be noted that the plate efficiency is dependent upon three main factors: (1) The design and construction of the plate and the flow of liquid across it; (2) The inter-diffusion of components between liquid and vapor as the bubbles pass through the liquid layer an the plate; and (3) Those that control the entrainment of fine particles from one plate to that above it (1). It is the second of these factors with which Peebles' long range research project is concerned and of which this paper is a part.

Mathematical Analysis of Problem

In accordance with the origin and objectives of this paper, the following procedure was used. Bubbles of dry gas were passed through a pure liquid of known height, h. The basic assumptions were made that: (1) The bubble is spherical in shape and of constant radius, and (2) No convection currents exist in the bubble so that the only mass transfer process is radial diffusion of the vapor from the surface into the bubble.

Let: y = concentration of liquid in bubble

r = radius of bubble

Q = amount of liquid in bubble at time, t

D = diffusivity constant

Then, from Fick's law which states that the rate of diffusion is proportional to the cross sectional area and to the concentration gradient, the following equation can be obtained:

$$\frac{\partial Q}{\partial t} = D4\pi r^2 \frac{\partial y}{\partial t} \tag{1}$$

where: $y = f(r,t)$

$$Q = \int_o^r 4\pi r^2 y dr \quad \text{or} \quad \frac{\partial Q}{\partial r} = 4\pi r^2 y \tag{2}$$

From equation 1,

$$\frac{\partial Q}{\partial t \partial r} = 4\pi D \left[r^2 \frac{\partial^2 y}{\partial r^2} + 2r \frac{\partial y}{\partial r} \right] \tag{3}$$

From equation 2,

$$\frac{\partial Q}{\partial t \partial r} = 4\pi \left[r^2 \frac{\partial y}{\partial t} + o \right] = 4\pi r^2 \frac{\partial y}{\partial t} \tag{4}$$

From (3) and (4),

$$4\pi r^2 \frac{\partial y}{\partial t} = 4\pi D \left[r^2 \frac{\partial^2 y}{\partial r^2} + 2r \frac{\partial y}{\partial r} \right]$$

or

$$\frac{\partial y}{\partial t} = D \left[\frac{\partial^2 y}{\partial r^2} + \frac{2}{r} \frac{\partial y}{\partial r} \right] \tag{5}$$

Let $V = yr$ or $y = \frac{V}{r}$ (6)

Partial differentiation of (6) gives,

$$\frac{\partial y}{\partial r} = (V)(-1)(r^{-2}) + (r^{-1}) \frac{\partial V}{\partial r} = \frac{1}{r} \frac{\partial V}{\partial r} - \frac{V}{r^2} \tag{7}$$

Partial differentiation of (7) gives,

$$\frac{\partial^2 y}{\partial r^2} = (r^{-1})\frac{\partial^2 V}{\partial r^2} + \frac{\partial V}{\partial r}(-1)(r^{-2}) - (r^{-2})(\frac{\partial V}{\partial r}) + (V)(-2)(r^{-3})$$

$$= \frac{1}{r}\frac{\partial^2 V}{\partial r^2} - \frac{1}{r^2}\frac{\partial V}{\partial r} - \frac{1}{r^2}\frac{\partial V}{\partial r} + \frac{2V}{r^3}$$

$$\frac{\partial^2 y}{\partial r^2} = \frac{1}{r}\frac{\partial^2 V}{\partial r^2} - \frac{2}{r^2}\frac{\partial V}{\partial r} + \frac{2V}{r^3} \qquad (8)$$

Partial differentiation of (6) gives,

$$\frac{\partial y}{\partial t} = \frac{1}{r}\frac{\partial V}{\partial t} \qquad (9)$$

Combination of equations 5, 7, 8, and 9 gives,

$$\frac{1}{r}\frac{\partial V}{\partial t} = D\left[\frac{1}{r}\frac{\partial^2 V}{\partial r^2} - \frac{2}{r^2}\frac{\partial V}{\partial r} + \frac{2V}{r^3} + \frac{2}{r}\left(\frac{1}{r}\frac{\partial V}{\partial r} - \frac{V}{r^2}\right)\right]$$

$$= D\,\frac{1}{r}\frac{\partial^2 V}{\partial r^2}$$

From which

$$\frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial r^2} \qquad (10)$$

To solve equation 10, assume that,

$$V(r,t) = V_s(r) + V_T(r,t) \qquad (11)$$

where $V_s(r)$ = steady state function of r only

$= y_e r$

$V_T(r,t)$ = transient state function of r and t

ye = equilibrium concentration of liquid in bubble

Partial differentiating equation 11,

$$\frac{\partial V}{\partial t} = 0 + \frac{\partial V_T}{\partial t} = \frac{\partial V_T}{\partial t}$$

$$\frac{\partial V}{\partial r} = \frac{\partial V_s}{\partial r} + \frac{\partial V_T}{\partial r} = y_e + \frac{\partial V_T}{\partial r}$$

$$\frac{\partial^2 V}{\partial r^2} = 0 + \frac{\partial^2 V_T}{\partial r^2} = \frac{\partial^2 V_T}{\partial r^2}$$

Therefore, from equation 10,

$$\frac{\partial V_T}{\partial t} = D \frac{\partial^2 V_T}{\partial r^2} \qquad (12)$$

Assume the solution of (12) to be of the form

$$V_T = R(r) \cdot T(t) = RT \qquad (13)$$

$$\frac{\partial V_T}{\partial t} = R\frac{\partial T}{\partial t}$$

$$\frac{\partial V_T}{\partial r} = T\frac{\partial R}{\partial r}$$

$$\frac{\partial^2 V_T}{\partial r^2} = T\frac{\partial^2 R}{\partial r^2}$$

Substituting these values into equation 12,

$$R\frac{\partial T}{\partial t} = DT\frac{\partial^2 R}{\partial r^2} = -K^2$$

or

$$\frac{1}{DT}\frac{\partial T}{\partial t} = \frac{1}{R}\frac{\partial^2 R}{\partial r^2} = -K^2 \tag{14}$$

The constant, $-K^2$, is picked to facilitate the solution of the equation.

Separating equation 13, the following are obtained,

$$\frac{\partial T}{\partial t} + K^2 DT = 0 \tag{15}$$

$$\frac{\partial^2 R}{\partial r^2} + RK^2 = 0 \tag{16}$$

Solving (14) by operator notation,

$$(D^{11} + K^2 D)T = 0$$

$$T = C_1 e^{-K^2 Dt} \tag{17}$$

Solving (15)

$$(D^{11^2} + K^2)R = 0$$

$$D^{11^2} = -K^2$$

$$D^{11} = \pm iK$$

$$R = C_2 \cos Kr + C_3 \sin Kr \tag{18}$$

From equations 13, 17, and 18,

$$V_T(r,t) = C_1 e^{-K^2 Dt}\left[C_2 \cos Kr \quad C_3 \sin Kr\right] \tag{19}$$

The boundary conditions are as follows:

(1) $V_T(0,t) = V(0,t) - 0 = 0$

$V_T(0,t) = 0$

(2) $V_T(r,0) = V(r,0) - y_e r = -y_e r$

$V_T(r,0) = -y_e r$

(3) $V_T(R,0) = V(R,0) - y_e r = y_e r - y_e r = 0$

$V_T(R,0) = 0$

(4) $V_T(r,\infty) = V(r,\infty) - y_e r = y_e r - y_e r = 0$

$V_T(r,\infty) = 0$

From condition 1,

$$V_T(0,t) = 0 = C_1 e^{-K^2 Dt} \left[C_2 \cos 0 + C_3 \sin 0 \right]$$

Therefore, $C_2 = 0$, and

$$V_T(r,t) = B_n e^{-K^2 Dt} \sin Kr$$

where $B_n = C_1 C_3$

From condition 3,

$$V_T(R,0) = 0 = B_n \sin KR$$

From which

$$KR = n\pi \qquad K = \frac{n\pi}{R}$$

$$V_T(r,t) = \sum_{n=1}^{n=\infty} B_n e^{\frac{-n^2\pi^2 Dt}{R^2}} \sin \frac{n\pi r}{R} \qquad (20)$$

From condition 2,

$$V_t(r,0) = -y_e r = \sum_1^\infty B_n \sin \frac{n\pi r}{R}$$

This relation can be expressed as the Fourier sine series of period 2R which represents $-y_e r$ and for which

$$B_n = \frac{2}{R}\int_o^R -y_e r \sin \frac{n\pi r}{R}\, dr$$

$$= -\frac{2y_e}{R}\left[\frac{R^2}{n^2\pi^2}\sin\frac{n\pi r}{R} - \frac{R}{n\pi} r \cos\frac{n\pi r}{R}\right]_o^R$$

$$= \frac{-2y_e}{R}\left[\frac{R^2}{n^2\pi^2}\sin n\pi - \frac{R\cdot R}{n\pi}\cos n\pi\right]$$

For all positive integers,

$$\sin n\pi = 0$$

$$B_n = -\frac{2y_e}{R}\left[-\frac{R\cdot R}{n\pi}\cos n\pi\right]$$

When n is even,

$$B_n = -\frac{2y_e}{R}\left(-\frac{R\cdot R}{n\pi}\right) = \frac{2y_e R}{n\pi}$$

When n is odd,

$$B_n = -\frac{2y_e}{R}\left(-\frac{R\cdot R}{n\pi}\right)(-1) = -\frac{2y_e R}{n\pi}$$

Therefore,

$$B_n = (-1)^n \frac{2y_e R}{n\pi}$$

From this, and equation 20, it follows that

$$V_T(r,t) = \frac{2y_eR}{\pi} \sum_{1}^{\infty} \frac{(-1)^n}{n} e^{-\frac{n^2\pi^2Dt}{R^2}} \sin\frac{n\pi r}{R} \qquad (21)$$

From equations 11 and 21, it follows that

$$yr = y_er + \frac{2y_eR}{\pi} \sum_{1}^{\infty} \frac{(-1)^n}{n} e^{\frac{-n^2\pi^2Dt}{R^2}} \sin\frac{n\pi r}{R}$$

OR

$$y = y_e + \frac{2y_eR}{\pi r} \sum_{1}^{\infty} \frac{(-1)^n}{n} e^{-\frac{n^2\pi^2Dt}{R^2}} \sin\frac{n\pi r}{R} \qquad (22)$$

Let y_a = average concentration of liquid in bubble.

$$y_a = \frac{\int_o^V y\,dV}{V}$$

$$V = \frac{4}{3}\pi r^3$$

$$dV = (\frac{4}{3}\pi)(3r^2dr)$$

$$y_a = \frac{\frac{4}{3}\pi\int_o^R y3r^2dr}{\frac{4}{3}\pi R^3} = \frac{3\int_o^R yr^2dr}{R^3}$$

$$y_a = \frac{3}{R^3}\int_o^R yr^2dr \qquad (23)$$

Substituting equation 22 in 23,

$$y_a = \frac{3}{R_3}\int_o^R \left[y_e + \frac{2y_eR}{\pi r} \sum \frac{(-1)^n}{n} e^{-\frac{n^2\pi^2Dt}{R^2}} \sin\frac{n\pi r}{R} \right] r^2dr$$

$$y_a = \frac{3y_e}{R^3}\int_0^R r^2 dr + \frac{6y_e}{R^2\pi}\int_0^R r\left[\sum_1^\infty \frac{(-1)^n}{n} e^{\frac{-n^2\pi^2 Dt}{R^2}} \sin\frac{n\pi r}{R}\right] dr$$

$$= \frac{3y_e}{R^3}\left[\frac{r^3}{3}\right]_0^R + \frac{6y_e}{R^2\pi}\sum_1^\infty \frac{(-1)^n}{n} e^{\frac{-n^2\pi^2 Dt}{R^2}} \int_0^R r \sin\frac{n\pi r}{R}\, dr$$

$$= \frac{3y_e R^3}{R^3} + \frac{6y_e}{R^2\pi}\sum_1^\infty \frac{(-1)^n}{n} e^{\frac{-n^2\pi^2 Dt}{R^2}} \left[\frac{R^2}{n^2\pi^2}\sin\frac{n\pi r}{R} - \frac{R}{n\pi} r\cos\frac{n\pi r}{R}\right]_0^R$$

$$= y_e + \frac{6y_e}{R^2\pi}\sum_1^\infty \frac{(-1)^n}{n} e^{\frac{-n^2\pi^2 Dt}{R^2}} \left[\frac{R^2}{n^2\pi^2}(\sin n\pi - 0) - \frac{R}{n\pi}(R\cos n\pi - 0)\right]$$

$$= y_e + \frac{6y_e}{R^2\pi}\sum_1^\infty \frac{(-1)^n}{n} e^{\frac{-n^2\pi^2 Dt}{R^2}} \left(-\frac{R^2}{n\pi}\cos n\pi\right)$$

If n is odd,

$$y_a = y_e + \frac{6y_e}{R^2\pi}\sum_1^\infty -\frac{1}{n} e^{\frac{-n^2\pi^2 Dt}{R^2}} \left(\frac{R^2}{n\pi}\right)$$

$$= y_e + \frac{6y_e}{R\pi}\left(-\frac{R^2}{\pi}\right)\sum_1^\infty \frac{1}{n^2} e^{\frac{-n^2\pi^2 Dt}{R^2}}$$

If n is even,

$$y_a = y_e + \frac{6y_e}{R^2\pi} \sum_{1}^{\infty} \frac{1}{n} e^{\frac{-n^2\pi^2 Dt}{R^2}} \left(-\frac{R^2}{n\pi}\right)$$

$$= y_e + \frac{6y_e}{R^2\pi} \left(-\frac{R^2}{\pi}\right) \sum_{1}^{\infty} \frac{1}{n^2} e^{\frac{-n^2\pi^2 Dt}{R^2}}$$

Therefore,

$$y_a = y_e - \frac{6y_e}{\pi^2} \sum_{1}^{\infty} \frac{1}{n^2} e^{\frac{-n^2\pi^2 Dt}{R^2}} \tag{24}$$

By definition,

$$E_{mv} = \frac{y_n - y_{n+1}}{y_e - y_{n+1}}$$

In this case,

$$y_n = y_a$$

$$y_{n+1} = 0$$

So that

$$E_{mv} = \frac{y_a}{y_e}$$

Therefore,

$$E_{mv} = \frac{y_a}{y_e} = 1 - \frac{6}{\pi^2} \sum_{n=1}^{n=\infty} \frac{1}{n^2} e^{\frac{-n^2\pi^2 Dt}{R^2}} \tag{25}$$

This equation, dependent only on the physical properties of the system enables the calculation of the Murphree vapor efficiency if the assumptions made hold true (2, 3, 8).

Calculation of Murphree Vapor Efficiencies by Use of Equation 25

In the above equation, it is seen that E_{mv} depends on the factors D, t, and R. D can be estimated by the Gilliland equation (5),

$$D = 0.0166 \left[\frac{T^{3/2}}{P \left(V_A^{\frac{1}{3}} + V_B^{\frac{1}{3}} \right)^2} \right] \left[\frac{1}{M_A} + \frac{1}{M_B} \right]^{\frac{1}{2}}$$

where T = the absolute temperature, ^{o}K

P = the total pressure, atm.

$V_B = V_A$ = the molecular volumns

$M_B = M_A$ = the molecular weights

D = the diffusivity constant, $Ft^2/hr.$

The time, t, is given by the equation,

$$t = \frac{h}{u}$$

where h = the depth of liquid

u = the velocity

It is possible to calculate u by the method of Peebles and Garber (7). Thus, all of the prerequisites necessary for

the calculation of E_{mv} are at hand. In the following sections of the report, the equipment and procedure used to test the theoretical deductions are described. Then the values of E_{mv} calculated by use of equations 25, 26 and 27 are compared with the experimentally determined values. Finally, the conclusions and recommendations pertaining to the project are given.

DESCRIPTION OF INVESTIGATION

In this investigation, bubbles of nitrogen gas were passed through water and ethyl acetate for the purpose of determining the feasibility of calculating Murphree vapor efficiencies theoretically. The equipment used is shown in Figures 1, 2, and 3, Figure 1 being a flow sheet of the experimental apparatus and Figures 2 and 3 showing the essential construction details of the two columns used. The column in Figure 2 is designed for use with water and other liquids which do not attack plexiglass whereas the column in Figure 3 is designed for use with organic solvents such as the ethyl acetate used in this experiment.

In order to more fully understand the experimental set-up and equipment, reference is made to Figure 1. Gas flows from a hi-pressure cylinder through two reducing valves into the drying bulb at a pressure of about 3.75 psig. The gas then flows through a calibrated capillary tube so that the volumetric flow rate can be determined. After passage through this capillary, the gas flows into a small heat exchanger where the gas is heated to the temperature of the liquid in order to assure isothermal operation. The gas is then bubbled through the vaporization column and the resulting mixture of vapor and gas rises through a gas collector. The purpose of the gas collector is to minimize the effect of the liquid vapor pressure on the results by

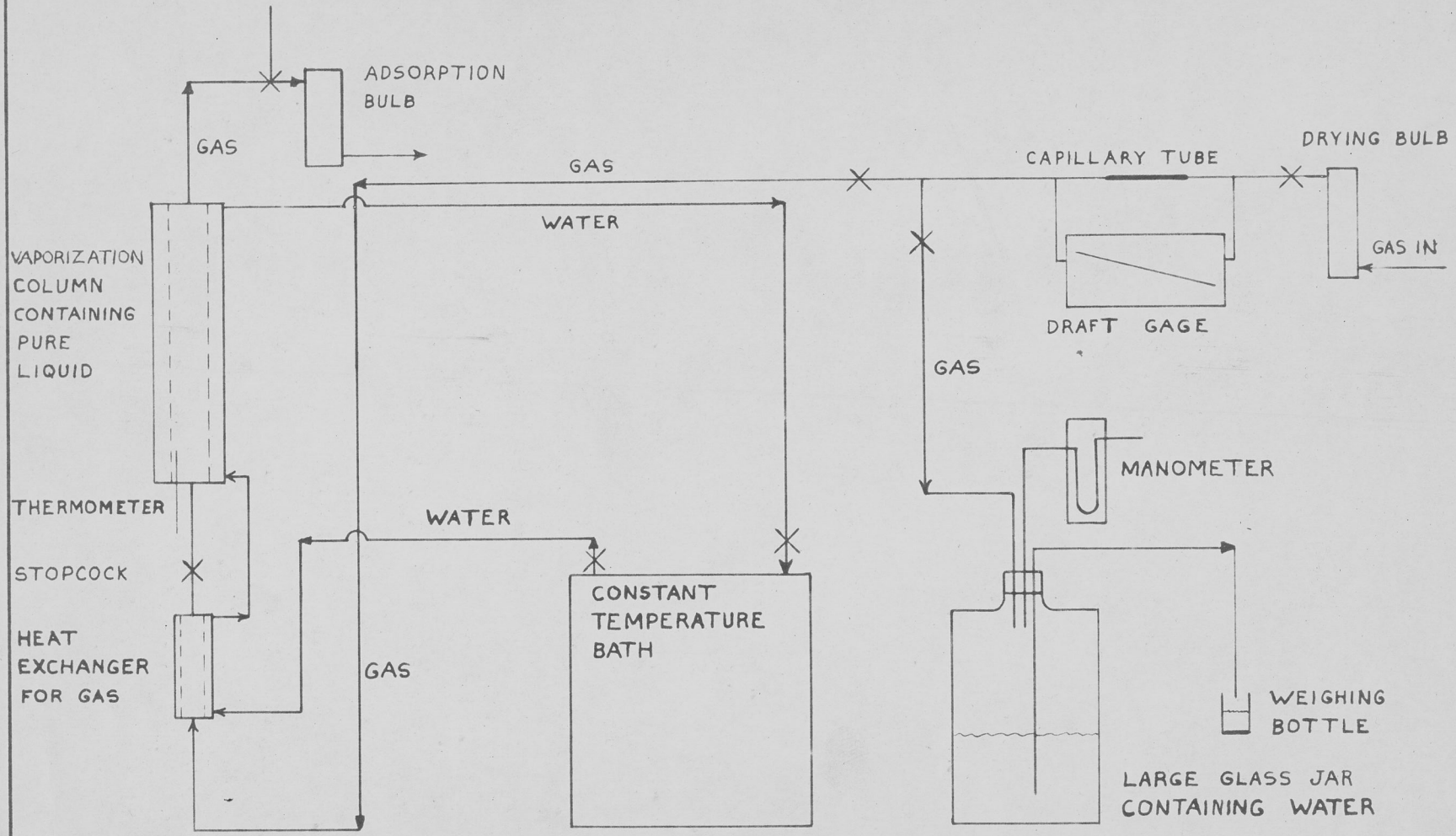


FIG. 1 - APPARATUS FOR STUDYING THE VAPORIZATION OF PURE LIQUIDS INTO GAS BUBBLES

DTM

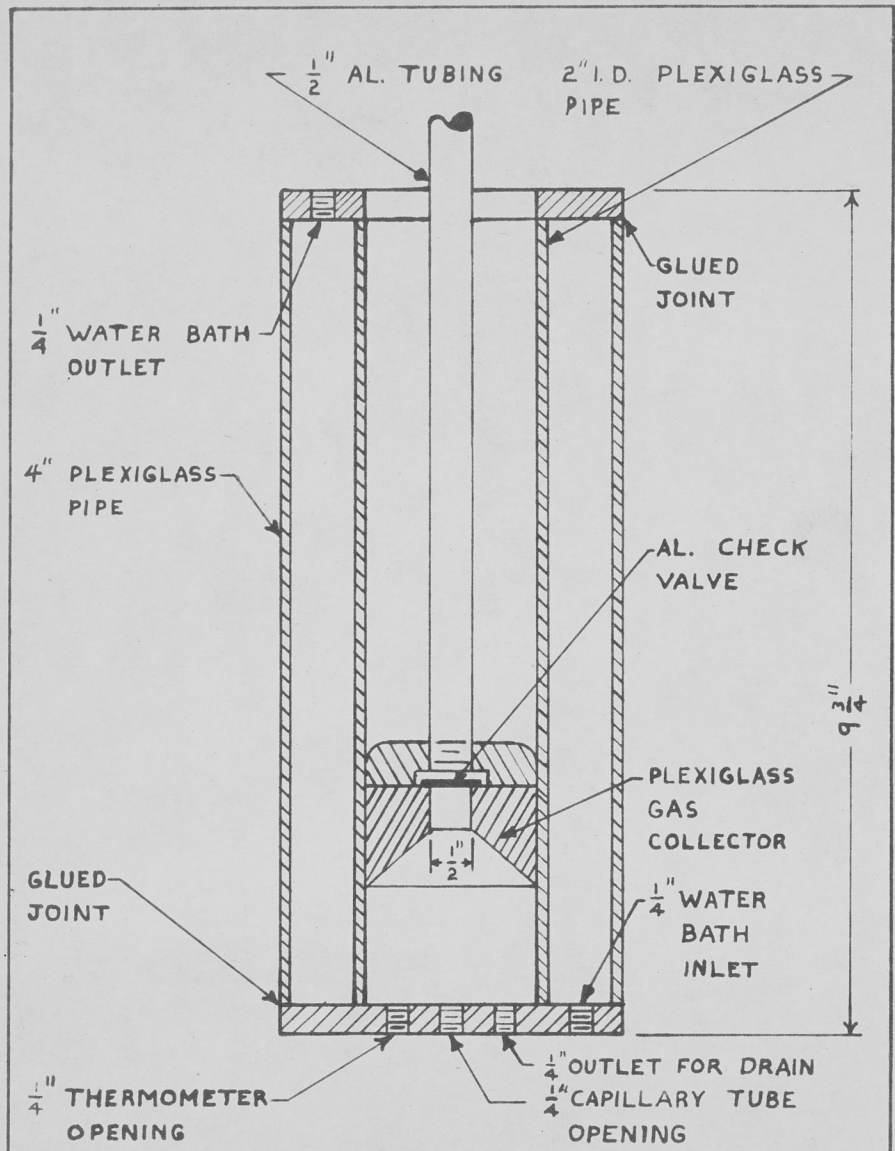


FIG. 2 - CONSTRUCTION DETAILS OF COLUMN I.

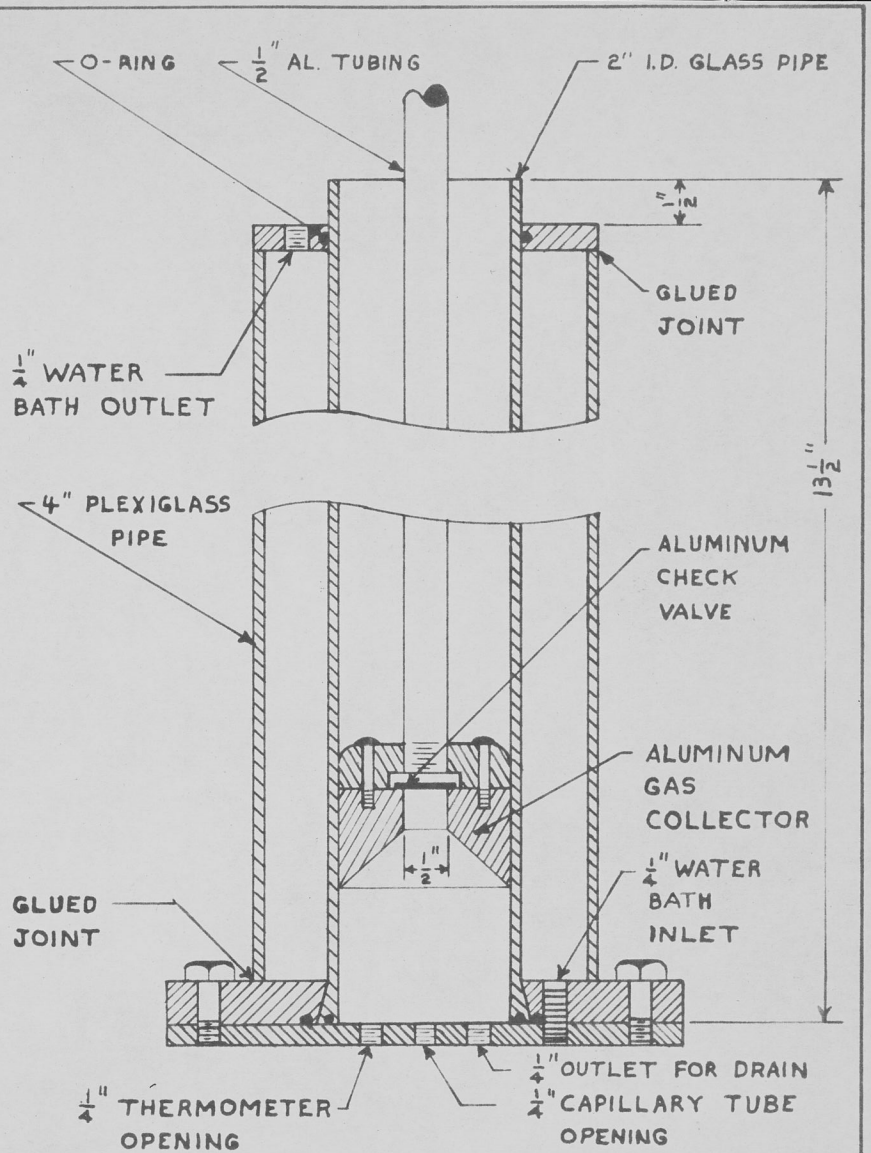


FIG. 3 - CONSTRUCTION DETAILS OF COLUMN II.

DTM

reducing the open space above the liquid surface to a minimum. During a run, the gas collector is placed in the liquid so that the liquid surface is at the top of the cone frustum of the gas collector (see Figures 2 and 3). The gas then passes from the gas collector into an adsorption bottle where the vapor is selectively adsorbed from the gas. Also, water from the constant temperature bath is passed continuously through the system when runs are being made. The experimental data needed for each run are : (1) Time of run, (2) Liquid temperature, (3) Manometer reading for determination of gas flow rate, (4) Number of bubbles/unit time for determination of bubble size, (5) Amount of vapor adsorbed during the time of the run, (6) atmospheric pressure, and (7) Liquid depth. With these values, and with the calibration chart for the capillary tube, it is possible to calculate both the experimental and theoretical Murphree plate efficiencies by the methods shown in the theoretical discussion and the sample calculations (See Appendix D).

The large glass jar of water shown in Figure 1 is for the purpose of calibrating the capillary tube flowmeter. When calibrating, all gas flow is directed into this glass jar. When making a calibration run, the draft gauge is set for a recorded pressure drop across the capillary tube. The gas flows into the jar and displaces the water into a 100 ml. weighed bottle. The weight of water collected in

a specified time is measured. From these data, the water temperature, and handbook values of the density of water, it is possible to calculate the volumetric gas flow rates. One note of caution is necessary. The pressure in the jar as measured by the small oil manometer shown should be the same at the beginning and end of a run.

One of the most important parts of the procedure is that of forming the gas bubble. Dossett and Porter (2) in performing a similar investigation used an unprotected capillary tube to form bubbles. However, this type of bubble formation has serious limitations. First of all, the bubble is in contact with the liquid while it is forming. Secondly, the bubble diameter is limited to a fairly small size. Since in industrial applications, fairly large bubbles are used, some means of obtaining larger bubbles than can be obtained with a capillary is desirable. Third, the liquid runs down the capillary prolonging the time of gas contact with the liquid. In this investigation, an attempt was made to eliminate these difficulties by placing a mercury layer over the capillary. This procedure eliminated the above difficulties for the most part, although the bubble size was still limited to a radius of approximately 0.15-0.20 inches. Unfortunately, new difficulties were encountered. The bubbles formed under the mercury were not released at an even rate. The bubbles formed also

varied greatly in size during a run. For example, a run might begin with a bubble radius of 0.10 inch and end up with a radius of 0.15 inch. One explanation for this action is the fact that a very small movement of the capillary tube up or down from the mercury surface has a pronounced effect on the bubble size. It was also noticed that new mercury worked better than that which had been used for some time. This effect may be the result of a surface contamination of the mercury. The difficulties encountered with this phase of the investigation are partly the reason for the meager amount of data obtained.

Another difficulty encountered was the measurement of the amount of vapor picked up by the gas. Dossett and Porter used activated carbon to pick up both ethyl acetate and acetone vapors from air. In this investigation, an attempt was made to pick up the same vapors with activated carbon with no success. Whether the difficulty was due to a bad supply of activated carbon or to some other phenomena is unknown. It should be noted here that a small error was introduced by the adsorption of the organic vapor in Tygon tubing which was used to carry the vapors from the gas collector to the adsorption bottle. However, even after this tubing had been replaced by rubber tubing so that all the vapor collected reached the activated carbon, very little adsorption took place.

In the second system studied, that of nitrogen gas and water, an attempt was also made to collect the vapor with an adsorption medium. The first one tried was drierite. Two different bottles with the seals unbroken were used. However, even after regenerating the drierite for 2 hours at 400°F, all of the vapor was not picked up by four adsorption bottles placed in series. Since the literature credits good drierite with a high activity and it is known that drierite in the delivered form has a wide variation of quality, it was assumed that this drierite was of poor quality and that much of it was not in the metastable stage. With the failure of drierite, silica gel was used. The silica gel was much more active than the drierite in that the first bottle took up all of the water vapor. The silica gel was regenerated only when the second bottle in the series gained as much as 0.0001 gms. during a run. It was reactivated at 300°F for two hours. Thus, only silica gel showed promise of being satisfactory in adsorbing water vapor.

It should be noted at this point that carbon dioxide was originally scheduled to be used as the gas instead of nitrogen. However, both activated carbon and silica gel picked up the carbon dioxide gas along with the vapor. Thus, one can see some of the difficulties encountered. Recommendations to alleviate these difficulties are given in a following section of the report.

DISCUSSION OF RESULTS

As seen in Table I, the results of this investigation are very meager. The difficulties causing this lack of data have been discussed in the Description of Investigation. The data given for water show a large discrepancy between the values of the Murphree vapor efficiency calculated from experimental data and from the theory given in the investigation. However, the data are insufficient to state whether this discrepancy is due to failure of the theory or to experimental shortcomings. No data are given for ethyl acetate, since no adsorbent was found of sufficient activity and capacity to completely adsorb the vapor from the gas.

TABLE I

EXPERIMENTAL DATA AND RESULTS

Run No.	Time of Run (min.)	Wt. H_2O Collected	Temp. (^oF)	Atm. Pressure (atm.)	Gas Flow Rate ml./min.	No. of Bubbles	Bubble Radius (in.)	Height Liquid (in.)	E_{mv} Exp.	E_{mv} Theoretical
1	50	.0095	86	0.983	10.50	3460	.1305	1.25	58.8	96.78
2	78	.0121	86	0.983	10.65	7800	.1160	1.25	48.1	98.15
3	80	.0160	86	0.983	11.95	5840	.1340	1.25	55.0	96.43
4	60	.0102	86	0.983	11.95	4920	.1290	2.50	57.3	99.83

CONCLUSIONS AND RECOMMENDATIONS

The data obtained by this investigation were too inadequate to draw any conclusions from. The experimental difficulties resulting in this lack of data have been described before. Several recommendations are made to alleviate these difficulties in future work. First of all, it is recommended that the mercury used in the formation of bubbles be kept very clean and not allowed to contact the liquid except at the interface; that is, the liquid should not be allowed to mix excessively with the mercury. Also, extreme care should be taken during a run to make certain that the capillary is not moved. If these suggestions do not alleviate the difficulties encountered in the bubble size and bubble rate, it is recommended that some other means of forming bubbles of sufficiently constant size and rate be sought. The other major difficulty occurred in the analysis of the vapor content of the gas stream. It is suggested that some method more positive than gas adsorption be used. It might be possible to surround a weighed adsorption bulb completely with dry ice. Then, when the vapor passes through, it would condense or freeze out in the bulb. At the end of a run, all stopcocks on the bulb could be closed so that the liquid collected could not escape as it warmed up to room temperature. It is possible that two bulbs would have to be arranged in series in order to allow the

expansion of the gas as it warmed up. From the weight before and after, the amount of vapor could be determined.

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APPENDIX

APPENDIX A

NOMENCLATURE

D	=	Diffusivity constant
E_{mp}	=	Murphree plate efficiency
E_{mv}	=	Murphree vapor efficiency
g	=	Acceleration of gravity
g_c	=	Correction factor
h	=	Height of liquid
$M_A = M_B$	=	Molecular weight of components A and B, respectively
P	=	Atmospheric pressure
Q	=	Amount of liquid in bubble at time, t
R	=	Radius of air bubble
ρ_G	=	Density of gas
ρ_L	=	Density of liquid
γ	=	Surface tension of liquid
t	=	Time of contact
T	=	Temperature
y	=	Concentration of liquid in the gas bubble
y_a	=	Average mole fraction of liquid in bubble
y_e	=	Equilibrium mole fraction of liquid in bubble
μ	=	Viscosity of liquid
u	=	Velocity of bubble through liquid
$V_A = V_B$	=	Molal volume of components A and B, respectively

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APPENDIX B

TABLE II

VALUES OF THE SERIES $\frac{6}{\pi^2} \sum_{n=1}^{n=\infty} \frac{1}{n^2} e^{-n^2 X}$ FOR DIFFERENT VALUES OF X

X	Value	X	Value	X	Value	X	Value
0.00	1.0000	0.48	0.3993	0.96	0.2361	2.20	0.0674
0.02	0.8536	0.50	0.3900	0.98	0.2312	2.30	0.0610
0.04	0.7967	0.52	0.3810	1.00	0.2264	2.40	0.0552
0.06	0.7540	0.54	0.3723	1.05	0.2150	2.50	0.0499
0.08	0.7195	0.56	0.3638	1.10	0.2043	2.60	0.0452
0.10	0.6897	0.58	0.3557	1.15	0.1940	2.70	0.0408
0.12	0.6632	0.60	0.3477	1.20	0.1843	2.80	0.0370
0.14	0.6395	0.62	0.3399	1.25	0.1752	2.90	0.0334
0.16	0.6175	0.64	0.3325	1.30	0.1665	3.00	0.0303
0.18	0.5975	0.66	0.3252	1.35	0.1582	3.20	0.0248
0.20	0.5789	0.68	0.3182	1.40	0.1505	3.40	0.0203
0.22	0.5615	0.70	0.3113	1.45	0.1431	3.60	0.0166
0.24	0.5450	0.72	0.3046	1.50	0.1360	3.80	0.0136
0.26	0.5296	0.74	0.2980	1.55	0.1293	4.00	0.0111
0.28	0.5149	0.76	0.2917	1.60	0.1230	4.50	0.0068
0.30	0.5010	0.78	0.2854	1.65	0.1170	5.00	0.0041
0.32	0.4877	0.80	0.2794	1.70	0.1112	5.50	0.0025
0.34	0.5751	0.82	0.2734	1.75	0.1058	6.00	0.0015
0.36	0.4629	0.84	0.2677	1.80	0.1006	6.50	0.0009
0.38	0.4513	0.86	0.2621	1.85	0.0957	7.00	0.0005
0.40	0.4401	0.88	0.2567	1.90	0.0910	7.50	0.0003
0.42	0.4293	0.90	0.2513	1.95	0.0866	8.00	0.0002
0.44	0.4189	0.92	0.2461	2.00	0.0823	8.50	0.0001
0.46	0.4090	0.94	0.2410	2.10	0.0745		

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APPENDIX C

SUMMARY OF VELOCITY EQUATIONS FOR GAS BUBBLES (7)

	Terminal Velocity	Range of Applicability
Region 1	$U_\infty = \frac{2R^2(\rho_1-\rho_2)g}{9\mu}$	$Re \leq 2$
Region 2	$U_\infty = .33g^{.76}\left(\frac{\rho_1}{\mu}\right)^{.52} R^{1.28}$	$2 \leq Re \leq 4.02G_1^{.214}$
Region 3	$U_\infty = 1.35\left(\frac{\gamma g_c}{\rho_1 R}\right)^{0.50}$	$4.02G_1^{-0.214} \leq Re \leq 3.1G_1^{-.25}$ or $16.32G_1^{0.144} \leq G_2 \leq 5.75$
Region 4	$U_\infty = 1.18\left(\frac{\gamma g g_c}{\rho_1}\right)^{0.25}$	$3.10G_1^{-0.25} \leq Re$ or $5.75 \leq G_2$

where $G_1 = \frac{g\mu^4}{\rho_1\gamma^3 g_c^3}$, dimensionless

$G_2 = \frac{gR^4U^4\rho_1^3}{\gamma^3 g_c^3} = \frac{Re^4 \cdot G_1}{16}$, dimensionless

Re = Reynolds number, $\frac{2R\,U\rho_1}{\mu}$, dimensionless

g = gravitational acceleration 32.17 ft/sec^2

g_c = gravitational conversion factor between force and mass, $32.17\ \frac{(lb.mass)(ft)}{(sec)^2(lb.force)}$

R = equivalent spherical gas bubble radius, ft.

U = particle velocity, relative to fluid, ft/sec

U_∞ = steady state, terminal rising bubble velocity ft/sec

γ = liquid surface tension, lb. force/ft.

μ = liquid viscosity, lb. mass/(ft)(sec)

ρ_1 = liquid density, lb. mass/cu.ft.

ρ_2 = gas density, lb. mass/cu.ft.

APPENDIX D

SAMPLE CALCULATIONS

All calculations shown are for run #1 with a liquid depth of $1\frac{1}{4}$".

I. Calculation of E_{mv} from Experimental Data

Time of run = 50 minutes

Wt. of water vapor collected = 0.0095

Moles of water vapor collected = $\frac{0.0095}{18}$ = 0.000527

Temperature = 30°C

Atmospheric pressure = 0.983

From the calibration data and draft gauge reading, the nitrogen flow rate is 10.50 ml/min or 525 ml. of gas for the entire 50 minutes.

$n = \frac{PV}{RT}$ where n = moles of gas

P = pressure in atm. = 0.983

V = volume in ml. = 525

R = gas constant = 82.057 $\frac{\text{atm-ml}}{\text{mole } ^\circ K}$

T = temperature, $^\circ K$ = 303

$$n = \frac{(0.983)(525)}{(82.06)(303)} = .0208$$

$$\text{Mole fraction of water} = \frac{0.000527}{.0208\ 0.000527} = .0248$$

From the handbook data, the equilibrium mole fraction of water = .0418. Therefore,

$$E_{mv} = \frac{.0248}{.0418} 100 = 58.8\%$$

II. Calculation of E_{mv} from the Theory Developed in the Introduction

$$D = 0.0166 \left[\frac{T^{3/2}}{P(V_A^{\frac{1}{3}} + V_B^{\frac{1}{3}})^2} \right] \left[\left(\frac{1}{M_A} + \frac{1}{M_B} \right)^{\frac{1}{2}} \right]$$

where D = diffusion constant, Ft^2/hr

T = absolute temperature, ^oK, = 303

P = pressure, atm, = 0.983

$V_A = V_B$ = molal volume of water vapor and nitrogen gas = 18.8, 31.2, respectively

$M_A = M_B$ = molecular weight of water and nitrogen gas = 18, 28, respectively

$$D = 0.0166 \left[\frac{(303)^{3/2}}{(0.983)\left[(31.2)^{\frac{1}{3}} + (18.8)^{\frac{1}{3}} \right]^2} \right] \left[\frac{1}{28} + \frac{1}{18} \right]^{\frac{1}{2}}$$

$$= .792 \frac{Ft^2}{hr} = 2.2 \times 10^{-4} \frac{Ft^2}{sec}$$

From the experimental data, the number of bubbles = 3460

$$ml./bubble = \frac{525}{3460} = .1518 \text{ cm}^3 = .00930 \text{ in}^3$$

The volume of a sphere, V, is given by

$$V = \frac{4}{3}\pi r^3, \text{ from which}$$

$$r^3 = \frac{(3)(.00930)}{4\pi}$$

$$r = .1305 \text{ in.} = .01089 \text{ Ft.}$$

Assume the bubble velocity to be in region 3 (See Appendix C). Then,

$$U = 1.35\sqrt{\frac{\gamma g_c}{\rho R}}$$

where U = velocity, Ft/sec

γ = surface tension, lb./Ft., = 4.88×10^{-3}

ρ = density, $\frac{\text{lb}}{\text{Ft}^3}$ = 62.4

R = bubble radius, Ft., = 0.01089 Ft.

g_c = correction factor = $32.2 \frac{\text{lb Ft}}{\text{lb/sec}^2}$

$$U = 1.35\sqrt{\frac{(4.88 \times 10^{-3})(32.2)}{(62.4)(.01089)}} = 0.651 \text{ Ft./sec.}$$

$$R_e = \frac{2 R_b \rho U}{\mu}$$

where R_e = Reynolds number

μ = viscosity

$$R_e = \frac{(2)(.1305/12)(62.4)(.651)}{5.38 \times 10^{-4}} = 1648$$

$$G_1 = \frac{g\,U^4}{\rho\gamma^3 g_c^3}$$

where G_1 = dimensionless

g = gravitational acceleration

$$G_1 = \frac{(32.2)(5.38)\times 10^{-4})^4}{(62.4)(4.88 \times 10^{-3})^3 (32.2)^3} = 11.1 \times 10^{-12}$$

$$3.10\, G_1^{-0.250} = 1700/4.02\, G_1^{-.214} = 907$$

Therefore,

$$4.02 G_1^{-.214} \leq Re \leq 3.10\, G_1^{-.250}$$

and the assumption of Region 3 is correct.

$$t = \frac{h}{u}$$

where t = time of contact between liquid and gas, sec.

h = fluid depth, Ft., = $\frac{1\frac{1}{4}}{12}$ Ft.

$$t = \frac{1.25/12}{0.651} = 0.160 \text{ sec.}$$

$$X = \frac{\pi^2 Dt}{R^2}$$

where X = value in Table II for obtaining value

of $\frac{6}{\pi^2} \sum_1^{\infty} \frac{1}{n^2} e^{-n^2 X}$

$$= \frac{(\pi^2)(2.20 \times 10^{-4})(.160)}{(.01089)^2}$$

$$= 2.94$$

From Table II,

$$\frac{6}{\pi^2} \sum_{1}^{\infty} \frac{1}{n^2} e^{-n^2 X} = .0322$$

$$E_{mv} = 1 - \frac{6}{\pi^2} \sum_{1}^{\infty} \frac{1}{n^2} e^{-n^2 X} = 1 - 0.322$$

$$= .9678 = 96.78\%$$

APPENDIX E

APPROVAL

The foregoing thesis is hereby approved as a creditable study of an engineering subject, carried out and presented in a manner sufficiently satisfactory to warrant its acceptance as a reprequisite to the degree for which it has been submitted. It is to be understood that by this approval the undersigned does not necessarily endorse or approve any statement made, opinions expressed, or conclusions drawn therein, but approves the thesis only for the purpose for which it is submitted.

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Title_____

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