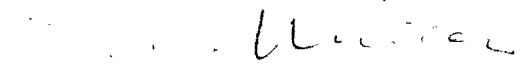


May 28, 1955

To the Graduate Council:

I am submitting herewith a thesis written by Carol G. Doss entitled "Certain Equivalence Relations in Transformation Semigroups." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Arts, with a major in Mathematics.


Major Professor

We have read this thesis
and recommend its acceptance:




Accepted for the Council:


Dean of the Graduate School

CERTAIN EQUIVALENCE RELATIONS IN TRANSFORMATION SEMIGROUPS

A THESIS

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Carol G. Doss

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CERTAIN EQUIVALENCE RELATIONS IN TRANSFORMATION SEMIGROUPS

0. Introduction

The general object of this thesis is to study certain equivalence relations defined on a semigroup, in particular, to study certain equivalence relations defined on semigroups of single-valued transformations. We are interested in semigroups of transformations partly because every semigroup has as homomorphic image a semigroup of transformations (and hence a subsemigroup of a transformation semigroup of degree n for some n). This is a well-known fact, analogous to the Cayley Theorem on abstract groups, but we shall give a brief proof in Section 1. Section 1 is devoted to definitions and basic concepts. In Section 2 we prove some theorems concerning certain equivalence relations defined on a transformation semigroup of degree n . In Section 3 we present some results concerning the transformation semigroup T_3 of degree 3.

As an appendix, we have listed all subsemigroups of T_3 and the minimal generating sets of each such subsemigroup. The regular subsemigroups of T_3 are marked by an asterisk and the pseudo-inverses of each element of T_3 are listed.

1. Definitions and basic concepts

A semigroup is a system consisting of a non-empty set closed under a single-valued binary associative operation which we call multiplication. The cardinal number of the set is called the order of the semigroup. For union, intersection,

inclusion, proper inclusion, and the empty set we use $\cup$, $\cap$, $\subseteq$, $\subset$, and $\emptyset$, respectively. If S is a semigroup and $\emptyset \subset S_1 \subseteq S$ and S_1 is closed under the semigroup operation, we say that S_1 is a subsemigroup of S ; if S_1 is a group we call S_1 a subgroup of S . We call S_1 a proper subsemigroup of S if $S_1 \subset S$. We assume that the reader is acquainted with the algebra of sets. A non-empty subset of the set of elements of the semigroup S will be called a complex. If M_1 and M_2 are complexes in S then the product $M_1 M_2$ is defined to be the set of all products $m_1 m_2$ where $m_1 \in M_1$ and $m_2 \in M_2$. The relations $M_1 \cdot M_2 M_3 = M_1 M_2 \cdot M_3$, $M_1 \cdot (M_2 \cup M_3) = M_1 M_2 \cup M_1 M_3$, $(M_1 \cup M_2) \cdot M_3 = M_1 M_3 \cup M_2 M_3$, $M_1 \cdot (M_2 \cap M_3) \subseteq M_1 M_2 \cap M_1 M_3$, $(M_1 \cap M_2) \cdot M_3 \subseteq M_1 M_3 \cap M_2 M_3$ are well known¹ and we shall use them constantly.

A particularly important class of subsets of a semigroup comprises ideals, left, right, and two-sided. A non-empty subset L of a semigroup S is called a left-ideal in S if $SL \subseteq L$, and a non-empty subset R of S is called a right ideal if $RS \subseteq R$. If a subset S is both a left ideal and a right ideal we call it a two-sided ideal. Ideals of any of these three kinds are called proper ideals if they are properly contained in S . It is obvious that any left, right, or two-sided ideal of S is a subsemigroup of S , and that

¹See, for example, P. Dubreil, Algèbre (Paris: Gauthier-Villars, 1946), p. 31.

if M is any complex in S then the complexes SM , MS , and SMS are, respectively, left, right, and two-sided ideals of S , as are the complexes $SM \cup M$, $M \cup MS$, and $M \cup SM \cup MS \cup SMS$, respectively. In particular when M is a single element a we call the complexes $Sa \cup a$, $a \cup aS$, $a \cup Sa \cup aS \cup SaS$, the principal left, right, and two-sided ideals of S , respectively, generated by a . If M is a left [right, two-sided] ideal of S , and M is contained in a subsemigroup S_1 of S , then M is a left [right, two-sided] ideal of S_1 , and is said to be a left [right, two-sided] subideal of S_1 . However, a complex $M \subseteq S_1$ may be an ideal of the semigroup S_1 without being a subideal of S_1 , i.e., without being an ideal of S .

We shall consider certain equivalence relations, introduced by Green,² that are definable in an arbitrary semigroup. Elements a and b of a semigroup S are said to be $\mathcal{L}$ -equivalent (written $a \mathcal{L} b$) in S if they generate the same principal left ideal of S , and to be $\mathcal{R}$ -equivalent (written $a \mathcal{R} b$) in S if they generate the same principal right ideal of S . We say that elements a and b of S are $\mathcal{D}$ -equivalent (written $a \mathcal{D} b$) in S if there exists an element $z \in S$ such that $a \mathcal{L} z$ and $z \mathcal{R} b$ in S . It is almost obvious that the relations $\mathcal{L}$ and $\mathcal{R}$ have the properties of reflexivity, symmetry, and transitivity, and that $\mathcal{D}$ is reflexive and transitive. It is not so obvious that the relation $\mathcal{D}$ is symmetric; a proof may be found in Green's paper.³ Finally, a and b are $\mathcal{J}$ -equivalent in S if

²J. A. Green, Annals of Mathematics 54 (1951), pp. 163-172.

³Green, op. cit., pp. 164-165.

they are both $\mathcal{L}$ - and $\mathcal{R}$ -equivalent in S . It is easily seen that if elements a and b are $\mathcal{J}$ -equivalent in S they are $\mathcal{L}$ - and $\mathcal{R}$ -equivalent in S ; and if they are $\mathcal{L}$ -equivalent or $\mathcal{R}$ -equivalent in S then they are $\mathcal{D}$ -equivalent in S .

We are interested primarily in semigroups of transformations, i.e., in semigroups of single-valued mappings of a set A into itself. The set A is called the domain of the mapping φ and the set $A\varphi$ of images $a\varphi$ (a subset of A) is called the range of the mapping. The mapping that leaves every element of A fixed is called the identity mapping and a mapping that maps all elements of A into a single element is called a constant mapping. Transformations φ and ψ of a domain A are equal provided that $a\varphi = a\psi$ for every $a \in A$. The product of φ and ψ (in that order) is defined to be the mapping $\varphi\psi$ such that $a(\varphi\psi) = (a\varphi)\psi$ for every $a \in A$. It is well known⁴ that this multiplication of mappings is associative and that, therefore, under this operation, the class of all transformations of the set A is a semigroup. If the cardinal number of A is n we call this semigroup the transformation semigroup of degree n and we denote it by T_n . If A is a finite set we find it convenient to represent it by the set

⁴Birkhoff and MacLane, A Survey of Modern Algebra (New York: MacMillan, 1953), p. 120.

$(1, 2, \dots, n)$ of positive integers and we represent a transformation φ of A by $\varphi = \begin{pmatrix} 1 & 2 & \dots & n \\ a_1 & a_2 & \dots & a_n \end{pmatrix}$.

This representation of a transformation is similar to that commonly used for a permutation of A ; however, a transformation of A need not be one-to-one as in the case of a permutation. Although it is convenient to represent the elements of A by positive integers, we shall call these elements letters in order to emphasize the fact that A is an abstract set in which no relations or operations are defined.

To prove that every semigroup has a homomorph among the transformation semigroups, we may proceed as follows. With each $a \in S$ we associate the transformation (right translation) φ_a defined by $x\varphi_a = xa$ for all $x \in S$. Then $x\varphi_{ab} = x \cdot ab = xa \cdot b = (x\varphi_a) \cdot b = (x\varphi_a)\varphi_b = x \cdot \varphi_a\varphi_b$ for all $x \in S$. Hence if $a \rightarrow \varphi_a$ and $b \rightarrow \varphi_b$ then $ab \rightarrow \varphi_a\varphi_b$, i.e., the correspondence $a \rightarrow \varphi_a$ is a homomorphism of S onto the semigroup of all right translations of S , which is in turn a subsemigroup of the semigroup of all transformations of S .

The homomorphism need not be an isomorphism, as one may see by considering any semigroup S in which, for some distinct elements a and b , $xa = xb$ for all $x \in S$ (e.g., the semigroup S_c discussed below).

An element a of a semigroup S is called a regular element (or an inversive element) of S if $axa = a$ for some $x \in S$. If every element of S is regular then S is

said to be a regular semigroup⁵ (or an inverse semigroup⁶). If $a \in S$ and $axa = a$ and $xax = x$ for some $x \in S$ then x is called a pseudo-inverse of a . Any regular element has at least one pseudo-inverse; indeed, if $axa = a$ then the element $y = xax$ is a pseudo-inverse of a . The most familiar semigroup in which every regular element has a unique pseudo-inverse is a group; other examples exist, however.

A subset M of a semigroup S is said to generate S if every element of S is a product of positive integral powers of elements of M . If S is generated by a single element then S is called a cyclic semigroup. If at least one element of a generating set M of S is a product of powers of other elements of M , then M is said to be a dependent generating set of S ; otherwise, it is called an independent generating set of S . Given a generating set M with cardinal number m , we say that M is a minimal generating set of S if S has no generating set with cardinal number less than m . It is clear that any finite minimal generating set is independent; the converse, however, does not hold even in finite groups.

⁵J. Von Neumann, Proc. Nat. Acad. Sci., 22 (1936), p. 708. Von Neumann's definition is stated for rings, but since the notion is purely multiplicative it may be applied at once to semigroups.

⁶Gabriel Thierrin, Comptes Rendus Acad. Sci., 232 (1951), p. 376.

2. Certain equivalence relations in T_n

Lemma 1. Let S be a semigroup and let $a, b \in S$. Then $a \mathcal{L} b$ if and only if either $a = b$ or else $Sa = Sb$, $a \in Sa$, and $b \in Sb$.

Proof. Suppose $a \mathcal{L} b$, i.e., $Sa \cup a = Sb \cup b$. If $a \neq b$ then $a \in Sb$ and $b \in Sa$, whence $a = s_1 b$ and $b = s_2 a$ for some $s_1, s_2 \in S$. Hence if $s \in S$ then $sa = ss_1 b \in Sb$, whence $Sa \subseteq Sb$, and $sb = ss_2 a \in Sa$, whence $Sb \subseteq Sa$. Therefore $Sa = Sb$ and $a \in Sa$ and $b \in Sb$.

Conversely, if $a = b$ then $a \mathcal{L} b$ is immediate; hence suppose $Sa = Sb$, $a \in Sa$, and $b \in Sb$. Then $Sa \cup a = Sa$ and $Sb \cup b = Sb$, whence $Sa \cup a = Sb \cup b$, i.e., $a \mathcal{L} b$.

Corollary. If $a \notin Sa$ then a is $\mathcal{L}$ -equivalent in S to no other element of S .

By left-right duality we obtain the following lemma and corollary.

Lemma 2. Let S be a semigroup and let $a, b \in S$. Then $a \mathcal{R} b$ if and only if either $a = b$ or else $aS = bS$, $a \in aS$, and $b \in bS$.

Corollary. If $a \notin aS$ then a is $\mathcal{R}$ -equivalent in S to no other element of S .

Lemma 3. Let S be a subsemigroup of a semigroup T , and let $a \mathcal{L} b$ [$a \mathcal{R} b$, $a \mathcal{D} b$] in S . Then $a \mathcal{L} b$ [$a \mathcal{R} b$, $a \mathcal{D} b$] in T .

Proof. By Lemma 1, if $a \mathcal{L} b$ in S then either (1) $a = b$ or (2) $Sa = Sb$, $a \in Sa$, and $b \in Sb$. If $a = b$

it is immediate that $a \mathcal{L} b$ in T ; hence suppose (2) holds. Let $t \in T$. Then from $a \in S_a$ and $S_a = S_b$ we obtain $ta \in tS_a = tS_b \subseteq T_b$, whence $T_a \subseteq T_b$; and from $b \in S_b$ and $S_a = S_b$ we have $tb \in tS_b = tS_a \subseteq T_a$, whence $T_b \subseteq T_a$. Therefore $T_a = T_b$, and we have shown that $a \mathcal{L} b$ in S implies $a \mathcal{L} b$ in T . A similar proof shows that $a \mathcal{R} b$ in S implies $a \mathcal{R} b$ in T . Finally, suppose $a \mathcal{D} b$ in S ; then there is an element $z \in S$ such that $a \mathcal{L} z$ in S and $z \mathcal{R} b$ in S . It follows at once from what we have just proved that $a \mathcal{L} z$ in T and $z \mathcal{R} b$ in T , whence $a \mathcal{D} b$ in T .

The converse of Lemma 3 is not necessarily true, as is shown by the following example, which is a subsemigroup of T_3 . Let us denote this subsemigroup, which we shall use in other counter examples to follow, by S_c .

		B_{03}	B_{04}	C_{01}	C_{02}
S_c :	B_{03}	C_{01}	C_{02}	C_{01}	C_{02}
	B_{04}	C_{01}	C_{02}	C_{01}	C_{02}
	C_{01}	C_{01}	C_{02}	C_{01}	C_{02}
	C_{02}	C_{01}	C_{02}	C_{01}	C_{02}

From the multiplication table of T_3 it is easily verified that B_{03} and B_{04} are both $\mathcal{L}$ - and $\mathcal{R}$ -equivalent in T_3 , but they are not even $\mathcal{D}$ -equivalent in the subsemigroup shown above.

For any $t \in T_n$, let the range of t be denoted by R_t . For any set M , let the cardinal number of M be denoted by

M. We state without proof the following obvious lemmas and corollary.

Lemma 4. If $t, t' \in T_n$ then $R_{tt'} \subseteq R_{t'}$.

Lemma 5. If $t, t' \in T_n$ then $\bar{R}_{tt'} \subseteq \bar{R}_t$.

Corollary. If $t, t' \in T_n$ then $\bar{R}_{tt'} \subseteq \min [\bar{R}_t, \bar{R}_{t'}]$.

Theorem 1. If two elements of a subsemigroup S of T_n are $\mathcal{L}$ -equivalent in S then they have the same range.

Proof. Let $a, b \in S$, $a \mathcal{L} b$ in S . Then

$Sa \cup a = Sb \cup b$. If $a = b$, the result is immediate. If $a \neq b$ then, for some $s \in S$, $a = sb$. By Lemma 4, $R_a = R_{sb} \subseteq R_b$. Similarly, $R_b \subseteq R_a$. Therefore $R_a = R_b$.

Theorem 1 fails if $\mathcal{R}$ is substituted for $\mathcal{L}$, even if $S = T_n$. From the multiplication table of S_c (given under Lemma 3) it is easily seen that C_{01} and C_{02} are $\mathcal{R}$ -equivalent in S_c , but they do not have the same range. If $S = T_n$, in particular T_3 , we see from the multiplication table of T_3 that $B_{01} \mathcal{R} B_{07}$, but they do not have the same range.

Theorem 2. If two elements of T_n have the same range then they are $\mathcal{L}$ -equivalent in T_n .

Proof. Let $a, b \in T_n$, $R_a = R_b$. Since T_n contains an identity element, $a \in T_na$ and $b \in T_nb$. Hence $a \mathcal{L} b$ if and only if $T_na = T_nb$. To prove that $T_na \subseteq T_nb$, we let ta be an arbitrary element of T_na and proceed to define a transformation $t' \in T_n$ such that $ta = t'b$. For each $i = 1, \dots, n$, let the images of i under ta , t' , and b

be c_i , x_i , and b_i , respectively. Since $R_{ta} \subseteq R_a \subseteq R_b$, each c_i occurs at least once among the b_i . For each i , select one such occurrence, say $b_j = c_i$, and define x_i to be j . Then the image of i under $t'b$ is c_i . With this definition of t' , $ta = t'b$ and we have, when such a t' has been found for each $t \in T_n$, $T_na \subseteq T_nb$. Similarly, $T_nb \subseteq T_na$. Hence $T_na = T_nb$, whence $a \mathcal{L} b$.

Theorem 2 fails if $\mathcal{R}$ is substituted for $\mathcal{L}$. The elements B_{01} and B_{03} of T_3 have the same range, but from the multiplication table of T_3 we see that they are not $\mathcal{R}$ -equivalent.

If two elements of a subsemigroup S of T_n have the same range they are not necessarily $\mathcal{L}$ -equivalent in S . Consider the elements B_{03} and B_{04} of S_c . From the multiplication table of S_c we see that B_{03} and B_{04} are not $\mathcal{L}$ -equivalent.

Corollary. Two elements of T_n are $\mathcal{L}$ -equivalent in T_n if and only if they have the same range.

We remark that, in the special case $T = T_n$, the part of Lemma 3 that refers to $\mathcal{L}$ -equivalence is an immediate consequence of Theorems 1 and 2.

Lemma 6. If two elements of a subsemigroup S of T_n are $\mathcal{D}$ -equivalent in S then they have the same rank.

Proof. Let $a, b \in S$, $a \mathcal{D} b$ in S . Then there is an element $z \in S$ such that $a \mathcal{L} z$ in S and $z \mathcal{R} b$ in S . By Theorem 1, $R_a = R_z$. If $z = b$ then $R_z = R_b$ and we

actually have $R_a = R_b$. If $z \neq b$ then, by Lemma 2, $z = bs$ for some $s \in S$. But, by Lemma 5, $\bar{R}_z = \bar{R}_{bs} \leq \bar{R}_b$. Similarly, by Lemma 2, $b = zs'$ for some $s' \in S$ and so, by Lemma 5, $\bar{R}_b = \bar{R}_{zs'} \leq \bar{R}_z$. Therefore $\bar{R}_z = \bar{R}_b$, and from $R_a = R_z$ it follows that $\bar{R}_a = \bar{R}_b$. The converse of Lemma 6 is not necessarily true. Again consider the elements B_{03} and B_{04} of S_c . From the multiplication table of S_c we see that B_{03} is $\mathcal{L}$ -equivalent to no element of S_c except itself, but B_{03} is not $\mathcal{R}$ -equivalent to B_{04} . Hence, there is no element $z \in S_c$ such that $B_{03} \mathcal{L} z$ and $z \mathcal{R} B_{04}$. However, the converse of Lemma 6 does hold (and is our Lemma 8) in the special case $S = T_n$.

Corollary. If two elements of a subsemigroup S of T_n are either $\mathcal{L}$ -equivalent or $\mathcal{R}$ -equivalent in S then they have the same rank.

Definition. Transformations $a = \begin{pmatrix} 1 & 2 & \dots & n \\ a_1 & a_2 & \dots & a_n \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 2 & \dots & n \\ b_1 & b_2 & \dots & b_n \end{pmatrix}$ are said to be similar provided that

$a_i = a_j$ if and only if $b_i = b_j$.

Lemma 7. If z and b are elements of T_n having the same rank, and if $z = bx$ for some $x \in T_n$, then z and b are similar.

Proof. The hypothesis $z = bx$ may be written

$$\begin{pmatrix} 1 & 2 & \dots & n \\ z_1 & z_2 & \dots & z_n \end{pmatrix} = \begin{pmatrix} 1 & 2 & \dots & n \\ b_1 & b_2 & \dots & b_n \end{pmatrix} \begin{pmatrix} 1 & 2 & \dots & n \\ x_1 & x_2 & \dots & x_n \end{pmatrix}$$

Let $r = \overline{\overline{R}}_z = \overline{\overline{R}}_b$. If $r = 1$, the conclusion is immediate.

We shall assume henceforth that $r > 1$.

We show first that if $z_i = z_j$ then $b_i = b_j$. Suppose, to the contrary, that for some i and j ($1 \leq i < j \leq n$) $z_i = z_j$ but $b_i \neq b_j$. Without loss of generality, we may re-label the letters $1, 2, \dots, n$ (i.e., we may re-arrange the columns of the transformation symbols) so that $i = 1$ and $j = 2$. Let $m = z_1 = z_2$; then the number of distinct letters other than m occurring in the sequence $z_3, z_4, \dots, z_n$ is $r - 1$. Call these distinct letters $p_1, p_2, \dots, p_{r-1}$. We continue our re-labeling so as to make $z_{i+2} = p_i$ ($i = 1, 2, \dots, r-1$). This accomplished, z is represented by the symbol

$$\begin{pmatrix} 1 & 2 & 3 & 4 & \dots & r+1 & r+2 & \dots & n \\ m & m & p_1 & p_2 & \dots & p_{r-1} & c_1 & \dots & c_{n-r-1} \end{pmatrix},$$

in which the set $R_z = [m, p_1, p_2, \dots, p_{r-1}]$ consists of r distinct letters and $c_i \in R_z$ ($i = 1, 2, \dots, n - r - 1$).

By our supposition, $b_1 \neq b_2$; let $u = b_1$ and $v = b_2$. Then the equality $z = bx$ now reads

$$\begin{pmatrix} 1 & 2 & 3 & \dots & r+1 & r+2 & \dots & n \\ m & m & p_1 & \dots & p_{r-1} & c_1 & \dots & c_{n-r-1} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ u & v & b_3 & \dots & b_n \end{pmatrix} \begin{pmatrix} 1 & \dots & u & \dots & v & \dots & n \\ x_1 & \dots & m & \dots & m & \dots & x_n \end{pmatrix},$$

in which $u \neq v$. By Lemma 4, $R_z \subseteq R_x$. Hence each p_i ($i = 1, \dots, r - 1$) occurs at least once among the x_j ($j = 1, \dots, n$; $u \neq j \neq v$). In fact, since p_i is the image of $i + 2$ under z , $p_i = x_j$ where $j = b_{i+2}$ ($i = 1, \dots, r - 1$)

and $u \neq j \neq v$. Therefore $b_3, b_4, \dots, b_{r+1}$ is a sequence of $r - 1$ distinct letters and none of them is either u or v . Hence R_b contains the $r + 1$ distinct letters $u, v, b_3, \dots, b_{r+1}$, contrary to the hypothesis $\overline{R}_b = \overline{R}_z = r$.

To show that $b_i = b_j$ implies $z_i = z_j$, the hypothesis $\overline{R}_z = \overline{R}_b$ is not needed. We need only observe that if i and j have the same image under b then they have the same image under bx , whence, by the hypothesis $z = bx$, they have the same image under z .

We conclude that z and b are similar.

Theorem 3. A necessary and sufficient condition that two elements of T_n be $\mathcal{R}$ -equivalent in T_n is that they be similar.

Proof of necessity. Let $z, b \in T_n, z \mathcal{R} b$ in T_n .

If $z = b$ the conclusion is immediate. If $z \neq b$ then, by Lemma 2, $z = bx$ for some $x \in T_n$. And, by the corollary to Lemma 6, z and b have the same rank. Hence, by Lemma 7, z and b are similar.

Proof of sufficiency: Let z and b be similar elements of T_n . First, we shall determine an element $x \in T_n$ such that $z = bx$. If a letter m occurs in the range of b , say as b_i , we assign as the image of m under x the letter z_i . If m occurs more than once as a b -image, say as x_i and as x_j , then by hypothesis $z_i = z_j$; hence the image of m under x is uniquely determined. If a letter m does not occur in the range of b , we assign an arbitrary letter as the image of m under x . Now, for any i , the image of

i under bx is z_i ; hence $z = bx$. In the same way we may determine an element $y \in T_n$ such that $b = zy$. But then, for any $t \in T_n$, $zt = bxt \in bT_n$, whence $zT_n \subseteq bT_n$; and $bt = zyt \in zT_n$, whence $bT_n \subseteq zT_n$. Therefore $zT_n = bT_n$. And, since T_n contains the identity transformation, $z \in zT_n$ and $b \in bT_n$. Thus the conditions of Lemma 2 are fulfilled, and we conclude that $z \mathcal{R} b$ in T_n .

Lemma 8. If a and b are elements of T_n having the same rank r then $a \mathcal{D} b$ in T_n .

Proof. By hypothesis, there is a one-to-one correspondence φ between the range of a and the range of b . Let us label the elements h_α of the range of a and the elements k_α of the range of b so that $h_\alpha = k_\alpha$ ($\alpha = 1, \dots, r$). We define a mapping z as follows. For any i ($1 \leq i \leq n$), if $b_i = k_\alpha$ define z_i to be h_α . From this definition it is immediate that $R_z \subseteq R_a$. Furthermore, to any $h_\alpha \in R_a$ there corresponds under φ a letter $k_\alpha \in R_b$. But k_α is the image under b of at least one letter i , and we have defined z_i to be h_α . Hence $h_\alpha \in R_z$. Therefore $R_a \subseteq R_z$. We now have $R_a = R_z$, whence, by the corollary to Theorem 2, $a \mathcal{L} z$ in T_n .

To prove that $z \mathcal{R} b$ we show first that $z_i = z_j$ implies $b_i = b_j$. This follows at once from the definition of z , for if $b_i = k_\alpha$ and $b_j = k_\beta$ and $b_i \neq b_j$ then $\alpha \neq \beta$ and so $z_i = h_\alpha \neq h_\beta = z_j$. Conversely, if $b_i = b_j = k_\alpha$ then $z_i = h_\alpha = z_j$. Hence $z_i = z_j$ if and only if $b_i = b_j$,

i.e., z and b are similar. By Theorem 3, then

$z \mathcal{Q} b$ in T_n .

From the definition of $\mathcal{D}$ -equivalence it follows that $a \mathcal{D} b$ in T_n .

From Lemmas 6 and 8 we have at once:

Theorem 4. Two elements of T_n are $\mathcal{D}$ -equivalent in T_n if and only if they have the same rank.

Theorem 5. The transformation semigroup of degree n is a regular semigroup.

Proof. For any $a \in T_n$ we must show that there exists an $x \in T_n$ such that $axa = a$, i.e.,

$$\begin{pmatrix} 1 & 2 & \dots & n \\ a_1 & a_2 & \dots & a_n \end{pmatrix} \begin{pmatrix} 1 & 2 & \dots & n \\ x_1 & x_2 & \dots & x_n \end{pmatrix} \begin{pmatrix} 1 & 2 & \dots & n \\ a_1 & a_2 & \dots & a_n \end{pmatrix} = \begin{pmatrix} 1 & 2 & \dots & n \\ a_1 & a_2 & \dots & a_n \end{pmatrix}$$

Let us define x thus: (1) if $j = a_i$ for some i , let x_j be any i such that $a_i = j$, (2) if $j \neq a_i$ for all i ($i = 1, \dots, n$) let x_j be any letter. Now, for any i , the image of i under axa is a_i . Hence $axa = a$.

Theorem 6. A regular element a of T_n has the same rank as any pseudo-inverse of a .

Proof. Let $a, x \in T_n$, $axa = a$, $xax = x$. Since $axa = a$, $\bar{\bar{R}}_a = \bar{\bar{R}}_{a.xa}$ and by Lemma 4 $\bar{\bar{R}}_{a.xa} \leq \bar{\bar{R}}_{xa}$. By Lemma 5 $\bar{\bar{R}}_{xa} \leq \bar{\bar{R}}_x$. Now $\bar{\bar{R}}_x = \bar{\bar{R}}_{x.ax}$ and by Lemma 4 and Lemma 5, applied in that order, we have $\bar{\bar{R}}_{x.ax} \leq \bar{\bar{R}}_{ax} \leq \bar{\bar{R}}_a$. Hence, $\bar{\bar{R}}_a = \bar{\bar{R}}_{a.xa} \leq \bar{\bar{R}}_{xa} \leq \bar{\bar{R}}_x = \bar{\bar{R}}_{x.ax} \leq \bar{\bar{R}}_{ax} \leq \bar{\bar{R}}_a$. Therefore, $\bar{\bar{R}}_a = \bar{\bar{R}}_x$.

If $axa = a$ for some a and x in T_n this does not imply that $\bar{\bar{R}}_a = \bar{\bar{R}}_x$. Examples: (For notation, see appendix)

- (1) If $a = B_{03}$ and $x = A_{04}$ we have $axa = a$, but $\bar{R}_a \neq \bar{R}_x$.
 (2) If $a = B_{09}$ and $x = A_{02}$ we have $axa = a$, but $\bar{R}_a \neq \bar{R}_x$.

3. The transformation semigroup of degree three

We have occasion frequently to think of the transformation semigroup of degree three, T_3 , as subdivided into five disjoint sets, α , β_1 , β_2 , β_3 and ϑ , where α is the set $\{A_i\}$ ($i = 01, \dots, 06$) of all mappings of the set $\{a, b, c\}$ onto itself; β_1 is the set $\{B_i\}$ ($i = 01, \dots, 06$) of all mappings of the set $\{a, b, c\}$ into the set $\{a, b\}$; β_2 is the set $\{B_i\}$ ($i = 07, \dots, 12$) of all mappings of the set $\{a, b, c\}$ into the set $\{a, c\}$; β_3 is the set $\{B_i\}$ ($i = 12, \dots, 18$) of all mappings of the set $\{a, b, c\}$ into the set $\{b, c\}$; and ϑ is the set $\{C_i\}$ ($i = 01, 02, 03$) of all mappings of the set $\{a, b, c\}$ into a single letter, a or b or c .

We say that a letter x in a set A remains fixed under a transformation φ of A if $x\varphi = x$.

By an examination of the mappings that belong to each of the five disjoint sets, α , β_1 , β_2 , β_3 , and ϑ , we see that there is only one mapping, the identity mapping A_{01} , that leaves every letter in the set $\{a, b, c\}$ fixed. We also see that each of the sets, β_1 , β_2 , and β_3 contains exactly two mappings that leave no letters fixed, two that leave one letter fixed and two that leave two letters fixed. Each mapping in ϑ leaves only one letter fixed.

Let us consider a subdivision of each β_i ($i = 1, 2, 3$) into three disjoint sets of two elements each, two mappings belonging to the same set if and only if they leave the same number of letters fixed. For example, in β_1 , B_{01} and B_{02} belong to the same set since each mapping leaves two letters fixed, B_{03} and B_{04} belong to the same set since each mapping leaves one letter fixed, and B_{05} and B_{06} belong to the same set since each mapping leaves no letters fixed. Now $B_{01}B_{02} = B_{01}$, $B_{02}B_{01} = B_{02}$, $B_{01}^2 = B_{01}$, and $B_{02}^2 = B_{02}$. Thus, in β_1 , the product of two mappings, each of which leaves two letters fixed, is a mapping leaving two letters fixed. And $B_{01}B_{03} = B_{02}B_{03} = C_{01}$, $B_{03}B_{01} = B_{03}B_{02} = B_{03}$, $B_{01}B_{04} = B_{02}B_{04} = C_{02}$, $B_{04}B_{01} = B_{04}B_{02} = B_{04}$. Thus the product of a mapping in β_1 leaving two letters fixed by a mapping in β_1 leaving one letter fixed is a mapping (either in β_1 or in $\mathcal{O}$) leaving one letter fixed. These examples are instances of the following general statements, which may be verified by reference to the multiplication table of T_3 :

(1) a product of two mappings in the same β -class is either in that β -class or is one of two elements of $\mathcal{O}$; (2) the number of letters left fixed by a product of mappings belonging to the same β -class is determined by the numbers of letters left fixed by the given mappings, and is independent of the order in which the mappings are multiplied. Furthermore, if at least one factor in a product of mappings is in $\mathcal{O}$ then the product is in $\mathcal{O}$. If to a β -class we adjoin

the appropriate pair of elements of $\mathcal{V}$ (adjoin c_{01} and c_{02} to β_1 , c_{01} and c_{03} to β_2 , c_{02} and c_{03} to β_3) we obtain a set of eight elements which is a subsemigroup of T_3 . Each such semigroup of order 8 may be partitioned into three subsets, the set 0 consisting of two elements leaving no letter fixed, the set 1 of four elements leaving one letter fixed, and the set 2 of two elements leaving two letters fixed. For example, in the semigroup $\{\beta_1, c_{01}, c_{02}\}$ the set 0 consists of B_{05} and B_{06} , the set 1 consists of B_{03} , B_{04} , c_{01} , c_{02} , and the set 2 consists of B_{01} and B_{02} . From the foregoing remarks and the multiplication table of T_3 it follows that, for each such semigroup of order 8, the correspondence that associates with each element of the semigroup the set (0, 1, or 2) containing it is a homomorphism of the semigroup onto the commutative semigroup of order 3 whose multiplication table is

	0	1	2	
0	2	1	0	
1	1	1	1	.
2	0	1	2	

The three semigroups of order 8 are in fact isomorphic to one another, corresponding elements appearing on the same line in the following array.

B ₀₁	B ₀₇	B ₁₃
B ₀₂	B ₀₈	B ₁₄
B ₀₃	B ₀₉	B ₁₅
B ₀₄	B ₁₀	B ₁₇
B ₀₅	B ₁₁	B ₁₆
B ₀₆	B ₁₂	B ₁₈
C ₀₁	C ₀₁	C ₀₂
C ₀₂	C ₀₃	C ₀₃

Lemma 9. In each β -class there is at least one subset S of order 3 that generates a subsemigroup of T_3 containing that β -class.

Proof. Let us consider β_1 . We need only to show that we can find one set S of three elements in β_1 such that every element in β_1 is either an element of S or a product of powers of elements of S . From the multiplication table for T_3 we see that $B_{05}^2 = B_{01}$, $B_{06}^2 = B_{02}$ and $B_{03}B_{05} = B_{04}$. Hence, every element in β_1 is either an element of the set $S = \{B_{03}, B_{05}, B_{06}\}$ or is the product of elements in S . Therefore, S generates a subsemigroup containing β_1 . Similarly for β_2 and β_3 .

Lemma 10. Any set S containing exactly five elements of a β -class β_1 generates a subsemigroup of T_3 that contains β_1 .

Proof. Again, consider β_1 . It is sufficient to show that the element of β_1 that is not in S is a product of powers of the other five elements of β_1 .

Suppose $B_{01} \notin S$. Since there are only six elements in β_1 , obviously $B_{05} \in S$. Now, $B_{05}^2 = B_{01}$. Hence, S generates a subsemigroup containing B_{01} and therefore containing β_1 . Similarly, we may show that any $B_i \in \beta_1$ is the product of powers of the other five elements in β_1 : $B_{06}^2 = B_{02}$, $B_{04} \cdot B_{05} = B_{03}$, $B_{03} \cdot B_{05} = B_{04}$, $B_{01} \cdot B_{06} = B_{05}$ and $B_{02} \cdot B_{05} = B_{06}$. Therefore, S generates a subsemigroup of T_3 that contains β_1 . Similarly for β_2 and β_3 .

Theorem 7: Any subsemigroup of T_3 generated by a set S of six elements that contains either four, five, or six elements of one β -class can be generated by a set of five or fewer elements, provided that S contains no A_i .

The proof of this theorem will consist of three cases: (1) S contains six elements of one β -class, (2) S contains five elements of one β -class, and (3) S contains four elements of one β -class. Only in the last case shall we need to consider subcases. We prove the theorem for the β -class β_1 ; similar proofs hold for the other β -classes.

Case 1: $S = \beta_1$. Let S' be the subsemigroup of T_3 generated by S ($= \beta_1$). By Lemma 9, there is a subset R (of order 3) of S that generates a subsemigroup R' of T_3 containing S . Since $S \subseteq R'$, $S' \subseteq R'$. But, since $R \subseteq S$, $R' \subseteq S'$. Therefore $R' = S'$, i.e., the set R of three elements generates the same subsemigroup as S .

Case 2: S contains exactly five elements of β_1 .

Let S_1 be the set of five elements in β_1 and let x be the sixth element in S . By Lemma 10 we know that S_1

generates a subsemigroup containing β_1 . Hence, as in Case 1, S_1 and β_1 generate the same subsemigroup. But, by Case 1 there is a set R of three elements of β_1 that generates the same subsemigroup as β_1 . Therefore, the sets S_1 and R generate the same subsemigroup of T_3 , whence $S_1 \cup x$ and $R \cup x$ generate the same subsemigroup of T_3 . But $S = S_1 \cup x$. Hence $R \cup x$ generates the same subsemigroup as S ; and since $R \cup x$ contains at most four elements, the theorem is proved in Case 2.

Case 3: S contains exactly four elements of β_1 .

Subcase 1: S contains two mappings (B_{03}, B_{04}) in β_1 that leave one letter fixed and two mappings (B_{01}, B_{02}) that leave two letters fixed. It is easily seen from the multiplication table for T_3 that the set $\beta_1 \cap S$ is independent and does not generate a subsemigroup containing β_1 .

Let us consider a fifth element x in S ; it must be either a B_i in β_2 or β_3 , or a C_i in $\mathcal{O}$. If x is B_{07}, B_{09}, B_{10} , or B_{11} then, since $B_{07}B_{04} = B_{11}B_{03} = B_{05}$ and $B_{09}B_{04} = B_{10}B_{03} = B_{06}$, $(\beta_1 \cap S) \cup x$ generates a subsemigroup S'_1 containing five elements of β_1 . But then, by Lemma 10, $\beta_1 \subseteq S'_1$. Since $(\beta_1 \cap S) \cup x \subseteq \beta_1 \cup x$, S'_1 is contained in the subsemigroup generated by $\beta_1 \cup x$. But, since $\beta_1 \subseteq S'_1$ and $x \in S'_1$, the subsemigroup generated by $\beta_1 \cup x$ is contained in S'_1 . Hence S'_1 is exactly the subsemigroup generated by $\beta_1 \cup x$. But now, as in Case 1, there is a set R of three elements of β_1 which generates the same subsemigroup

R' that β_1 generates, whence $R \cup x$ generates the same subsemigroup as $\beta_1 \cup x$. Therefore $R \cup x$ generates S'_1 , whence, if y is the sixth element of S , $R \cup x \cup y$ and $(\beta_1 \cap S) \cup x \cup y$ generate the same subsemigroup of T_3 . But $S = (\beta_1 \cap S) \cup x \cup y$, and $R \cup x \cup y$ is a set of at most five elements. Thus the theorem is proved in Subcase 1 provided S contains at least one of the four elements B_{07} , B_{09} , B_{10} , B_{11} .

If S contains either B_{08} or B_{12} then S is dependent since $B_{08}B_{01} = B_{03}$ and $B_{12}B_{01} = B_{04}$.

If S contains no element of β_2 but contains at least one element of β_3 , the foregoing argument may be repeated with the substitution of B_{13} , B_{15} , B_{17} , B_{16} , B_{14} , B_{18} for B_{07} , B_{09} , B_{10} , B_{11} , B_{08} , B_{12} , respectively.

Finally, if S contains no element of β_2 or β_3 then it must contain two elements of $\mathcal{V}$, and so must contain either C_{01} or C_{02} . But then S is dependent, for $B_{01}B_{03} = C_{01}$ and $B_{01}B_{04} = C_{02}$. This completes the proof for Subcase 1.

Subcase 2: S contains two mappings (B_{03}, B_{04}) in β_1 that leave one element fixed and two mappings (B_{05}, B_{06}) in β_1 that leave no elements fixed. Since $0 \cdot 0 = 2$, $S_1 (= \beta_1 \cap S)$ generates a subsemigroup S'_1 containing five elements of β_1 , and hence, by Lemma 10, containing β_1 . As in Case 1, there is a set R of three elements of β_1 that generates S'_1 . Therefore, letting x and y be the other two elements of

S , $S_1 \cup x \cup y$ and $R \cup x \cup y$ generate the same subsemigroup of T_3 . But $S = S_1 \cup x \cup y$ and $R \cup x \cup y$ is a set of five elements. Thus the theorem is proved in Subcase 2.

Subcase 3: S contains two mappings (B_{05}, B_{06}) in β_1 that leave no elements fixed and two mappings (B_{01}, B_{02}) in β_1 that leave two elements fixed.

Since, $B_{05}^2 = B_{01}$, S is a dependent generating set.

Subcase 4: S contains one mapping $B_i \in \beta_1$ which leaves no elements fixed, both mappings that leave one element fixed, and one B_j that leaves two elements fixed.

Since $0 \cdot 0 = 2$, B_i^2 is a mapping that leaves two elements fixed. If $B_i^2 = B_j$, then S is dependent. If B_i^2 is not an element of S then S generates a subsemigroup containing five elements in β_1 , and by Subcase 2 the subsemigroup generated by S can be generated by a set of five or fewer elements.

Subcase 5: S contains one mapping B_i in β_1 that leaves no elements fixed, one mapping that leaves one element fixed, and both mappings that leave two elements fixed.

Since $0 \cdot 0 = 2$, B_i^2 is a mapping that leaves two elements fixed. But S contains both mappings in β_1 that leave two elements fixed. Hence, S is a dependent set.

Subcase 6: S contains two mappings (B_{05}, B_{06}) that leave no elements fixed, one mapping that leaves one element fixed, and one (either B_{01} or B_{02}) that leaves two elements fixed.

Since $B_{05}^2 = B_{01}$ and $B_{06}^2 = B_{02}$, S is a dependent set.

q.e.d.

We have been unable to prove a theorem similar to Theorem 7 for which the generating set of six elements contains fewer than four elements of a β -class except by referring to the multiplication table of T_3 for a large number of products. However, by actual computation of the subsemigroups of T_3 generated by sets of n ($n = 1, \dots, 6$) elements we have found that every subsemigroup of T_3 generated by a set of six elements can be generated by a set of five elements or fewer, if A_{01} is not one of the six.

Theorem 8. If every subsemigroup of T_3 generated by a set of six elements can be generated by a set of five elements or fewer, then every subsemigroup of T_3 can be generated by a set of five elements or fewer.

Proof. We shall use induction on the number n of elements in a generating set.

Case 1. $n = 7$. Let S be a set of seven elements and S' the subsemigroup of T_3 generated by S . Let S_1 be a subset of S containing exactly six elements of S , and S'_1 the subsemigroup generated by S_1 . We may write $S = S_1 \cup x$, where x is the element of S not in S_1 . Now, since S'_1 is generated by a set of six elements, there is, by hypothesis, a set S_2 of five elements or fewer that generates S'_1 . Therefore, the sets S_1 and S_2 generate the same subsemigroup

whence $S_1 \cup x$ and $S_2 \cup x$ generate the same subsemigroup of T_3 . But $S = S_1 \cup x$. Hence $S_2 \cup x$ generates the same subsemigroup as S . Now $S_2 \cup x$ contains at most six elements, whence, by hypothesis, there is a set of five elements or fewer that generates the same subsemigroup of T_3 as $S_2 \cup x$. Thus the theorem is proved in Case 1.

Case 2: Assume that every subsemigroup generated by a set of n elements ($7 < n < 27$) can be generated by a set of five elements or fewer and consider a subsemigroup S' generated by a set S of $n + 1$ elements. Let S_1 be a set of n elements such that $S_1 \subset S$, and S_1' the subsemigroup generated by S_1 . Let x be the element of S not in S_1 . By the assumption there is a set S_2 of five elements or fewer that generates the same subsemigroup of T_3 as S_1' . Hence, $S_1 \cup x$ and $S_2 \cup x$ generate the same subsemigroup of T_3 . But $S = S_1 \cup x$. Therefore, $S_2 \cup x$ generates the same subsemigroup as S . Since S_2 contains five elements or fewer, $S_2 \cup x$ contains at most six elements. But then, by hypothesis, there is a set of five elements or fewer that generates the same subsemigroup of T_3 as $S_2 \cup x$. Therefore, the subsemigroup generated by S can be generated by a set of five elements or fewer.

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APPENDIX

APPENDIX

In this appendix are listed the multiplication table of the transformation semigroup T_3 of degree 3, and what the author believes to be all subsemigroups of T_3 and the minimal generating sets of each such subsemigroup. The multiplication table of T_3 was taken from a multiplication table of the transformation semigroup T_4 of degree 4 in a thesis on "Endomorphisms and Translations of Semigroups" by E. E. Posey, multigraphed at the University of Tennessee in August 1949. We use the same notation for transformations as that used by Posey. Two mistakes found in the multiplication table of T_4 given by Posey have been corrected in this thesis. The regular subsemigroups of T_3 are marked by an asterisk and the pseudo-inverses of each element of T_3 are listed. Also the L -, R -, I -, and D -structure of T_3 is given.

It would be futile to expect tabulations of this sort to be entirely free from errors. The author has checked his work repeatedly, and has rectified several mistakes, but others have doubtless escaped his notice. He will appreciate receiving corrections from users of the tabulations.

We have omitted from our list the subsemigroups of T_3 obtainable trivially by adjoining to a subsemigroup S the identity transformation A_{01} .

ELEMENTS OF THE TRANSFORMATION SEMIGROUP OF DEGREE THREE

(Mappings of the set (1, 2, 3) into itself)

A ₀₁ : 1 2 3	B ₀₂ : 1 2 1	B ₀₉ : 1 3 1	B ₁₆ : 2 3 2
A ₀₂ : 1 3 2	B ₀₃ : 1 1 2	B ₁₀ : 3 1 3	B ₁₇ : 2 3 3
A ₀₃ : 2 1 3	B ₀₄ : 2 2 1	B ₁₁ : 3 1 1	B ₁₈ : 3 3 2
A ₀₄ : 2 3 1	B ₀₅ : 2 1 1	B ₁₂ : 3 3 1	C ₀₁ : 1 1 1
A ₀₅ : 3 1 2	B ₀₆ : 2 1 2	B ₁₃ : 3 2 3	C ₀₂ : 2 2 2
A ₀₆ : 3 2 1	B ₀₇ : 1 3 3	B ₁₄ : 2 2 3	C ₀₃ : 3 3 3
B ₀₁ : 1 2 2	B ₀₈ : 1 1 3	B ₁₅ : 3 2 2	

PSEUDO-INVERSES OF EACH ELEMENT OF T_3

A ₀₁ : A ₀₁	B ₀₄ : B ₁₀ B ₁₁ B ₁₃ B ₁₅	B ₁₃ : B ₀₂ B ₀₄ B ₁₃ B ₁₄
A ₀₂ : A ₀₂	B ₀₅ : B ₀₅ B ₀₆ B ₁₀ B ₁₁	B ₁₄ : B ₀₈ B ₁₀ B ₁₃ B ₁₄
A ₀₃ : A ₀₃	B ₀₆ : B ₀₅ B ₀₆ B ₁₆ B ₁₇	B ₁₅ : B ₀₂ B ₀₄ B ₀₉ B ₁₂
A ₀₄ : A ₀₅	B ₀₇ : B ₀₁ B ₀₃ B ₀₇ B ₀₈	B ₁₆ : B ₀₃ B ₀₆ B ₁₆ B ₁₈
A ₀₅ : A ₀₄	B ₀₈ : B ₀₇ B ₀₈ B ₁₄ B ₁₇	B ₁₇ : B ₀₃ B ₀₆ B ₀₈ B ₁₀
A ₀₆ : A ₀₆	B ₀₉ : B ₀₁ B ₀₃ B ₁₅ B ₁₈	B ₁₈ : B ₀₉ B ₁₂ B ₁₆ B ₁₈
B ₀₁ : B ₀₁ B ₀₂ B ₀₇ B ₀₉	B ₁₀ : B ₀₄ B ₀₅ B ₁₄ B ₁₇	C ₀₁ : C ₀₁ C ₀₂ C ₀₃
B ₀₂ : B ₀₁ B ₀₂ B ₁₃ B ₁₅	B ₁₁ : B ₀₄ B ₀₅ B ₁₁ B ₁₂	C ₀₂ : C ₀₁ C ₀₂ C ₀₃
B ₀₃ : B ₀₇ B ₀₉ B ₁₆ B ₁₇	B ₁₂ : B ₁₁ B ₁₂ B ₁₅ B ₁₈	C ₀₃ : C ₀₁ C ₀₂ C ₀₃

THE L -, R -, I -, AND D -STRUCTURE OF T_3

	L_1	L_2	L_3	L_4	L_5	L_6	L_7
R_1	A_1 $(i=01, \dots, 06)$						
R_2		B_{01}, B_{05}	B_{07}, B_{11}	B_{15}, B_{17}			
R_3		B_{02}, B_{06}	B_{09}, B_{10}	B_{13}, B_{16}			
R_4		B_{03}, B_{04}	B_{08}, B_{12}	B_{14}, B_{18}			
R_5					C_{01}	C_{02}	C_{03}

THE TRANSFORMATION SEMIGROUP OF DEGREE THREE

	A01	A02	A03	A04	A05	A06	B01	B02	B03	B04	B05	B06	B07	B08
A01	A01	A02	A03	A04	A05	A06	B01	B02	B03	B04	B05	B06	B07	B08
A02	A02	A01	A04	A03	A06	A05	B01	B03	B02	B06	B05	B04	B07	B09
A03	A03	A05	A01	A06	A02	A04	B06	B05	B03	B04	B02	B01	B10	B08
A04	A04	A06	A02	A05	A01	A03	B04	B05	B02	B06	B03	B01	B12	B09
A05	A05	A03	A06	A01	A04	A02	B06	B03	B05	B01	B02	B04	B10	B11
A06	A06	A04	A05	A02	A03	A01	B04	B02	B05	B01	B03	B06	B12	B11
B01	B01	B07	B05	B17	B11	B15	B01	B01	C01	C02	B05	B05	B07	C01
B02	B02	B09	B06	B16	B10	B13	B02	B02	C01	C02	B06	B06	B09	C01
B03	B03	B08	B04	B14	B12	B18	B03	B03	C01	C02	B04	B04	B08	C01
B04	B04	B12	B03	B18	B08	B14	B04	B04	C01	C02	B03	B03	B12	C01
B05	B05	B11	B01	B15	B07	B17	B05	B05	C01	C02	B01	B01	B11	C01
B06	B06	B10	B02	B13	B09	B16	B06	B06	C01	C02	B02	B02	B10	C01
B07	B07	B01	B17	B05	B15	B11	B01	C01	B01	B05	B05	C02	B07	B07
B08	B08	B03	B14	B04	B18	B12	B03	C01	B03	B04	B04	C02	B08	B08
B09	B09	B02	B16	B06	B13	B10	B02	C01	B02	B06	B06	C02	B09	B09
B10	B10	B06	B13	B02	B16	B09	B06	C01	B06	B02	B02	C02	B10	B10
B11	B11	B05	B15	B01	B17	B07	B05	C01	B05	B01	B01	C02	B11	B11
B12	B12	B04	B18	B03	B14	B08	B04	C01	B04	B03	B03	C02	B12	B12
B13	B13	B16	B10	B09	B06	B02	C02	B02	B06	B02	C01	B06	C03	B10
B14	B14	B18	B08	B12	B03	B04	C02	B04	B03	B04	C01	B03	C03	B08
B15	B15	B17	B11	B07	B05	B01	C02	B01	B05	B01	C01	B05	C03	B11
B16	B16	B13	B09	B10	B02	B06	C02	B06	B02	B06	C01	B02	C03	B09
B17	B17	B15	B07	B11	B01	B05	C02	B05	B01	B05	C01	B01	C03	B07
B18	B18	B14	B12	B08	B04	B03	C02	B03	B04	B03	C01	B04	C03	B12

THE TRANSFORMATION SEMIGROUP OF DEGREE THREE (continued)

	A01	A02	A03	A04	A05	A06	B01	B02	B03	B04	B05	B06	B07	B08
C01	C01	C01	C02	C02	C03	C03	C01	C01	C01	C02	C02	C02	C01	C01
C02	C02	C03	C01	C03	C01	C02	C02	C02	C01	C02	C01	C01	C03	C01
C03	C03	C02	C03	C01	C02	C01	C02	C01	C02	C01	C01	C02	C03	C03

THE TRANSFORMATION SEMIGROUP OF DEGREE THREE (continued)

	B09	B10	B11	B12	B13	B14	B15	B16	B17	B18	C01	C02	C03
A01	B09	B10	B11	B12	B13	B14	B15	B16	B17	B18	C01	C02	C03
A02	B08	B12	B11	B10	B18	B16	B15	B14	B17	B13	C01	C02	C03
A03	B11	B07	B09	B12	B17	B14	B16	B15	B13	B18	C01	C02	C03
A04	B11	B07	B08	B10	B17	B16	B14	B15	B18	B13	C01	C02	C03
A05	B08	B12	B09	B07	B18	B15	B16	B14	B13	B17	C01	C02	C03
A06	B09	B10	B08	B07	B13	B15	B14	B16	B18	B17	C01	C02	C03
B01	B07	B11	B11	C03	B15	C02	B15	B17	B17	C03	C01	C02	C03
B02	B09	B10	B10	C03	B13	C02	B13	B16	B16	C03	C01	C02	C03
B03	B08	B12	B12	C03	B18	C02	B18	B14	B14	C03	C01	C02	C03
B04	B12	B08	B08	C03	B14	C02	B14	B18	B18	C03	C01	C02	C03
B05	B11	B07	B07	C03	B17	C02	B17	B15	B15	C03	C01	C02	C03
B06	B10	B09	B09	C03	B16	C02	B16	B13	B13	C03	C01	C02	C03
B07	C01	C03	B11	B11	C03	B17	B15	C02	B17	B15	C01	C02	C03
B08	C01	C03	B12	B12	C03	B14	B18	C02	B14	B18	C01	C02	C03
B09	C01	C03	B10	B10	C03	B16	B13	C02	B16	B13	C01	C02	C03
B10	C01	C03	B09	B09	C03	B13	B16	C02	B13	B16	C01	C02	C03
B11	C01	C03	B07	B07	C03	B15	B17	C02	B15	B17	C01	C02	C03
B12	C01	C03	B08	B08	C03	B18	B14	C02	B18	B14	C01	C02	C03
B13	B09	B10	C01	B09	B13	B13	C02	B16	C03	B16	C01	C02	C03
B14	B12	B08	C01	B12	B14	B14	C02	B18	C03	B18	C01	C02	C03
B15	B07	B11	C01	B07	B15	B15	C02	B17	C03	B17	C01	C02	C03
B16	B10	B09	C01	B10	B16	B16	C02	B13	C03	B13	C01	C02	C03
B17	B11	B07	C01	B11	B17	B17	C02	B15	C03	B15	C01	C02	C03
B18	B08	B12	C01	B08	B18	B18	C02	B14	C03	B14	C01	C02	C03

THE TRANSFORMATION SEMIGROUP OF DEGREE THREE (continued)

	B ₀₉	B ₁₀	B ₁₁	B ₁₂	B ₁₃	B ₁₄	B ₁₅	B ₁₆	B ₁₇	B ₁₈	C ₀₁	C ₀₂	C ₀₃
C ₀₁	C ₀₁	C ₀₃	C ₀₃	C ₀₃	C ₀₃	C ₀₂	C ₀₃	C ₀₂	C ₀₂	C ₀₃	C ₀₁	C ₀₂	C ₀₃
C ₀₂	C ₀₃	C ₀₁	C ₀₁	C ₀₃	C ₀₂	C ₀₂	C ₀₂	C ₀₃	C ₀₃	C ₀₃	C ₀₁	C ₀₂	C ₀₃
C ₀₃	C ₀₁	C ₀₃	C ₀₁	C ₀₁	C ₀₃	C ₀₃	C ₀₂	C ₀₂	C ₀₃	C ₀₂	C ₀₁	C ₀₂	C ₀₃

NOTE

This table is extracted from Posey's thesis (see introduction to Appendix) except that the products $A_{02}B_{02} = B_{02}$ and $A_{06}B_{13} = B_{17}$ have been corrected to read $A_{02}B_{02} = B_{03}$ and $A_{06}B_{13} = B_{13}$.

SUBSEMIGROUPS OF T_3 GENERATED BY ONE ELEMENT

* A_{01}	(A_{01})	$B_{09} C_{01}$	(B_{09})
* $A_{01} A_{02}$	(A_{02})	$B_{10} C_{03}$	(B_{10})
* $A_{01} A_{03}$	(A_{03})	* $B_{07} B_{11}$	(B_{11})
* $A_{01} A_{04} A_{05}$	$(A_{04}) (A_{05})$	* $B_{08} B_{12}$	(B_{12})
* $A_{01} A_{06}$	(A_{06})	* B_{13}	(B_{13})
* B_{01}	(B_{01})	* B_{14}	(B_{14})
* B_{02}	(B_{02})	$B_{15} C_{02}$	(B_{15})
$B_{03} C_{01}$	(B_{03})	* $B_{13} B_{16}$	(B_{16})
$B_{04} C_{02}$	(B_{04})	$B_{17} C_{03}$	(B_{17})
* $B_{01} B_{05}$	(B_{05})	* $B_{14} B_{18}$	(B_{18})
* $B_{02} B_{06}$	(B_{06})	* C_{01}	(C_{01})
* B_{07}	(B_{07})	* C_{02}	(C_{02})
* B_{08}	(B_{08})	* C_{03}	(C_{03})

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTSOrder 2

* B ₀₁ B ₀₂	* B ₀₈ B ₁₄
* B ₀₁ B ₀₇	* B ₀₈ C ₀₁
* B ₀₁ C ₀₁	* B ₀₈ C ₀₃
* B ₀₁ C ₀₂	* B ₁₃ B ₁₄
* B ₀₂ B ₁₃	* B ₁₃ C ₀₂
* B ₀₂ C ₀₁	* B ₁₃ C ₀₃
* B ₀₂ C ₀₂	* B ₁₄ C ₀₂
* B ₀₂ C ₀₃	* B ₁₄ C ₀₃
* B ₀₇ B ₀₈	* C ₀₁ C ₀₂
* B ₀₇ C ₀₁	* C ₀₁ C ₀₃
* B ₀₇ C ₀₃	* C ₀₂ C ₀₃

Order 3

* A ₀₁ A ₀₂ C ₀₁	(A ₀₂ C ₀₁)	B ₀₁ B ₀₄ C ₀₂	(B ₀₁ B ₀₄)
* A ₀₁ A ₀₃ C ₀₃	(A ₀₃ C ₀₃)	* B ₀₁ B ₁₄ C ₀₂	(B ₀₁ B ₁₄)
* A ₀₁ A ₀₆ C ₀₂	(A ₀₆ C ₀₂)	B ₀₁ B ₁₄ C ₀₂	(B ₀₁ B ₁₅)
B ₀₁ B ₀₃ C ₀₁	(B ₀₁ B ₀₃)	* B ₀₁ C ₀₂ C ₀₃	(B ₀₁ C ₀₃)

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTS (continued)Order 3 (continued)

$B_{02} B_{03} C_{01}$	$(B_{02} B_{03})$	$B_{08} B_{10} C_{03}$	$(B_{08} B_{10})$
$B_{02} B_{04} C_{02}$	$(B_{02} B_{04})$	* $B_{08} C_{01} C_{02}$	$(B_{08} C_{02})$
* $B_{02} B_{08} C_{01}$	$(B_{02} B_{08})$	$B_{09} C_{01} C_{03}$	$(B_{09} C_{03})$
$B_{02} B_{09} C_{01}$	$(B_{02} B_{09})$	$B_{10} B_{13} C_{03}$	$(B_{10} B_{13})$
$B_{03} B_{08} C_{01}$	$(B_{03} B_{08})$	$B_{10} C_{01} C_{03}$	$(B_{10} C_{01})$
$B_{03} C_{01} C_{02}$	$(B_{03} C_{02})$	$B_{13} B_{15} C_{02}$	$(B_{13} B_{15})$
$B_{04} B_{14} C_{02}$	$(B_{04} B_{14})$	$B_{13} B_{17} C_{03}$	$(B_{13} B_{17})$
$B_{04} C_{01} C_{02}$	$(B_{04} C_{01})$	* $B_{13} C_{01} C_{03}$	$(B_{13} C_{01})$
$B_{07} B_{09} C_{01}$	$(B_{07} B_{09})$	$B_{14} B_{15} C_{02}$	$(B_{14} B_{15})$
$B_{07} B_{10} C_{03}$	$(B_{07} B_{10})$	$B_{14} B_{17} C_{03}$	$(B_{14} B_{17})$
* $B_{07} B_{13} C_{03}$	$(B_{07} B_{13})$	* $B_{14} C_{01} C_{02}$	$(B_{14} C_{01})$
$B_{07} B_{17} C_{03}$	$(B_{07} B_{17})$	$B_{15} C_{02} C_{03}$	$(B_{15} C_{03})$
* $B_{07} C_{02} C_{03}$	$(B_{07} C_{02})$	$B_{17} C_{02} C_{03}$	$(B_{17} C_{02})$
$B_{08} B_{09} C_{01}$	$(B_{08} B_{09})$		

Order 4

* $A_{01} A_{02} B_{01} B_{07}$	$(A_{02} B_{01}) (A_{02} B_{07})$
* $A_{01} A_{02} C_{02} C_{03}$	$(A_{02} C_{02}) (A_{02} C_{03})$
* $A_{01} A_{03} B_{08} B_{14}$	$(A_{03} B_{08}) (A_{03} B_{14})$
* $A_{01} A_{03} C_{01} C_{02}$	$(A_{03} C_{01}) (A_{03} C_{02})$
* $A_{01} A_{06} B_{02} B_{13}$	$(A_{06} B_{02}) (A_{06} B_{13})$

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTS (continued)Order 4 (continued)

* A ₀₁ A ₀₆ C ₀₁ C ₀₃	(A ₀₆ C ₀₁) (A ₀₆ C ₀₃)
* B ₀₁ B ₀₂ B ₀₅ B ₀₆	(B ₀₁ B ₀₆) (B ₀₂ B ₀₅) (B ₀₅ B ₀₆)
B ₀₁ B ₀₃ B ₀₈ C ₀₁	(B ₀₁ B ₀₈)
* B ₀₁ B ₀₅ B ₀₇ B ₁₁	(B ₀₅ B ₀₇) (B ₀₅ B ₁₁) (B ₀₁ B ₁₁)
B ₀₁ B ₁₃ B ₁₅ C ₀₂	(B ₀₁ B ₁₃)
B ₀₁ B ₁₇ C ₀₂ C ₀₃	(B ₀₁ B ₁₇)
* B ₀₁ B ₀₅ C ₀₁ C ₀₂	(B ₀₅ C ₀₁) (B ₀₅ C ₀₂)
B ₀₂ B ₀₄ B ₁₄ C ₀₂	(B ₀₂ B ₁₄)
* B ₀₂ B ₀₆ B ₁₃ B ₁₆	(B ₀₆ B ₁₃) (B ₀₂ B ₁₃) (B ₀₆ B ₁₆)
* B ₀₂ B ₀₆ C ₀₁ C ₀₂	(B ₀₆ C ₀₁) (B ₀₆ C ₀₂)
B ₀₂ B ₀₇ B ₀₉ C ₀₁	(B ₀₂ B ₀₇)
B ₀₂ B ₁₀ C ₀₁ C ₀₃	(B ₀₂ B ₁₀)
B ₀₃ B ₀₄ C ₀₁ C ₀₂	(B ₀₃ B ₀₄)
B ₀₃ B ₁₄ C ₀₁ C ₀₂	(B ₀₃ B ₁₄)
B ₀₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ C ₀₃)
B ₀₄ B ₀₈ C ₀₁ C ₀₂	(B ₀₄ B ₀₈)
B ₀₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₄ C ₀₃)
* B ₀₇ B ₀₈ B ₁₁ B ₁₂	(B ₀₇ B ₁₂) (B ₀₈ B ₁₁) (B ₁₁ B ₁₂)
* B ₀₇ B ₁₁ C ₀₁ C ₀₃	(B ₁₁ C ₀₁) (B ₁₁ C ₀₃)
B ₀₇ B ₁₄ B ₁₇ C ₀₃	(B ₀₇ B ₁₄)
B ₀₇ B ₁₅ C ₀₂ C ₀₃	(B ₀₇ B ₁₅)
B ₀₈ B ₁₀ B ₁₃ C ₀₃	(B ₀₈ B ₁₃)

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTS (continued)Order 4 (continued)

* B ₀₈ B ₁₂ B ₁₄ B ₁₈	(B ₀₈ B ₁₈) (B ₁₂ B ₁₄) (B ₁₂ B ₁₈)
* B ₀₈ B ₁₂ C ₀₁ C ₀₃	(B ₁₂ C ₀₁) (B ₁₂ C ₀₃)
B ₀₉ B ₁₀ C ₀₁ C ₀₃	(B ₀₉ B ₁₀)
B ₀₉ B ₁₃ C ₀₁ C ₀₃	(B ₀₉ B ₁₃)
B ₀₉ C ₀₁ C ₀₂ C ₀₃	(B ₀₉ C ₀₂)
B ₁₀ C ₀₁ C ₀₂ C ₀₃	(B ₁₀ C ₀₂)
* B ₁₃ B ₁₄ B ₁₆ B ₁₈	(B ₁₃ B ₁₈) (B ₁₄ B ₁₆) (B ₁₆ B ₁₈)
* B ₁₃ B ₁₆ C ₀₂ C ₀₃	(B ₁₆ C ₀₂) (B ₁₆ C ₀₃)
* B ₁₄ B ₁₈ C ₀₂ C ₀₃	(B ₁₈ C ₀₂) (B ₁₈ C ₀₃)
B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₁₅ B ₁₇)
B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₁₅ C ₀₁)
B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₁₇ C ₀₁)

Order 5

* B ₀₁ B ₀₂ B ₀₇ B ₀₉ C ₀₁	(B ₀₁ B ₀₉)
* B ₀₁ B ₀₂ B ₁₃ B ₁₅ C ₀₂	(B ₀₂ B ₁₅)
* B ₀₁ B ₀₃ B ₀₇ B ₀₈ C ₀₁	(B ₀₃ B ₀₇)
* B ₀₁ B ₀₄ B ₁₄ B ₁₅ C ₀₂	(B ₁₄ B ₁₅)
* B ₀₁ B ₀₅ B ₁₄ C ₀₁ C ₀₂	(B ₀₅ B ₁₄)
* B ₀₁ B ₀₅ C ₀₁ C ₀₂ C ₀₃	(B ₀₅ C ₀₃)
* B ₀₁ B ₁₄ B ₁₈ C ₀₂ C ₀₃	(B ₀₁ B ₁₈)
* B ₀₂ B ₀₃ B ₀₈ B ₀₉ C ₀₁	(B ₀₃ B ₀₉)

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTS (continued)Order 5 (continued)

* B ₀₂ B ₀₄ B ₁₃ B ₁₄ C ₀₂	(B ₀₄ B ₁₃)
* B ₀₂ B ₀₆ B ₀₈ C ₀₁ C ₀₂	(B ₀₆ B ₀₈)
* B ₀₂ B ₀₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₆ C ₀₃)
* B ₀₂ B ₀₈ B ₁₂ C ₀₁ C ₀₃	(B ₀₂ B ₁₂)
* B ₀₂ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₈)
* B ₀₇ B ₀₈ B ₁₄ B ₁₇ C ₀₃	(B ₀₈ B ₁₇)
* B ₀₇ B ₁₀ B ₁₃ B ₁₇ C ₀₃	(B ₁₀ B ₁₇)
* B ₀₇ B ₁₁ B ₁₃ C ₀₁ C ₀₃	(B ₁₁ B ₁₃)
* B ₀₇ B ₁₃ B ₁₆ C ₀₂ C ₀₃	(B ₀₇ B ₁₆)
* B ₀₇ B ₁₁ C ₀₁ C ₀₂ C ₀₃	(B ₁₁ C ₀₂)
* B ₀₈ B ₁₀ B ₁₃ B ₁₄ C ₀₃	(B ₁₀ B ₁₄)
* B ₀₈ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₁₂ C ₀₂)
* B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₁₆ C ₀₁)
* B ₁₄ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₁₈ C ₀₁)

Order 6

* A ₀₁ A ₀₂ A ₀₃ A ₀₄ A ₀₅ A ₀₆	(A ₀₂ A ₀₃) (A ₀₂ A ₀₄) (A ₀₂ A ₀₅) (A ₀₂ A ₀₆) (A ₀₃ A ₀₄) (A ₀₃ A ₀₅) (A ₀₃ A ₀₆) (A ₀₄ A ₀₆) (A ₀₅ A ₀₆)
* A ₀₁ A ₀₂ B ₀₁ B ₀₅ B ₀₇ B ₁₁	(A ₀₂ B ₀₅) (A ₀₂ B ₁₁)

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTS (continued)Order 6 (continued)

* A ₀₁ A ₀₂ B ₁₃ B ₁₄ B ₁₆ B ₁₈	(A ₀₂ B ₁₃) (A ₀₂ B ₁₄) (A ₀₂ B ₁₆) (A ₀₂ B ₁₈)
A ₀₁ A ₀₂ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(A ₀₂ B ₁₅) (A ₀₂ B ₁₇)
* A ₀₁ A ₀₃ B ₀₁ B ₀₂ B ₀₅ B ₀₆	(A ₀₃ B ₀₁) (A ₀₃ B ₀₂) (A ₀₃ B ₀₅) (A ₀₃ B ₀₆)
A ₀₁ A ₀₃ B ₀₃ B ₀₄ C ₀₁ C ₀₂	(A ₀₃ B ₀₃) (A ₀₃ B ₀₄)
* A ₀₁ A ₀₃ B ₀₈ B ₁₂ B ₁₄ B ₁₈	(A ₀₃ B ₁₂) (A ₀₃ B ₁₈)
* A ₀₁ A ₀₄ A ₀₅ C ₀₁ C ₀₂ C ₀₃	(A ₀₄ C ₀₁) (A ₀₄ C ₀₂) (A ₀₄ C ₀₃) (A ₀₅ C ₀₁) (A ₀₅ C ₀₂) (A ₀₅ C ₀₃)
* A ₀₁ A ₀₆ B ₀₂ B ₀₆ B ₁₃ B ₁₆	(A ₀₆ B ₀₆) (A ₀₆ B ₁₆)
* A ₀₁ A ₀₆ B ₀₇ B ₀₈ B ₁₁ B ₁₂	(A ₀₆ B ₀₇) (A ₀₆ B ₀₈) (A ₀₆ B ₁₁) (A ₀₆ B ₁₂)
A ₀₁ A ₀₆ B ₀₉ B ₁₀ C ₀₁ C ₀₃	(A ₀₆ B ₀₉) (A ₀₆ B ₁₀)
B ₀₁ B ₀₃ B ₀₄ B ₀₅ C ₀₁ C ₀₂	(B ₀₃ B ₀₅) (B ₀₄ B ₀₅)
B ₀₂ B ₀₃ B ₀₄ B ₀₆ C ₀₁ C ₀₂	(B ₀₃ B ₀₆) (B ₀₄ B ₀₆)
* B ₀₂ B ₀₆ B ₀₈ B ₁₂ C ₀₁ C ₀₃	(B ₀₆ B ₁₂)
B ₀₇ B ₀₉ B ₁₀ B ₁₁ C ₀₁ C ₀₃	(B ₀₉ B ₁₁) (B ₁₀ B ₁₁)
B ₀₈ B ₀₉ B ₁₀ B ₁₂ C ₀₁ C ₀₃	(B ₀₉ B ₁₂) (B ₁₀ B ₁₂)
B ₁₃ B ₁₅ B ₁₆ B ₁₇ C ₀₂ C ₀₃	(B ₁₅ B ₁₆) (B ₁₆ B ₁₇)
B ₁₄ B ₁₅ B ₁₇ B ₁₈ C ₀₂ C ₀₃	(B ₁₅ B ₁₈) (B ₁₇ B ₁₈)

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTS (continued)Order 7

- * A₀₁ A₀₂ B₀₂ B₀₃ (A₀₂ B₀₂) (A₀₂ B₀₃) (A₀₂ B₀₈) (A₀₂ B₀₉)
 B₀₈ B₀₉ C₀₁
- * A₀₁ A₀₃ B₀₇ B₁₀ (A₀₃ B₀₇) (A₀₃ B₁₀) (A₀₃ B₁₃) (A₀₃ B₁₇)
 B₁₃ B₁₇ C₀₃
- * A₀₁ A₀₆ B₀₁ B₀₄ (A₀₆ B₀₁) (A₀₆ B₀₄) (A₀₆ B₁₄) (A₀₆ B₁₅)
 B₁₄ B₁₅ C₀₂
- B₀₁ B₀₃ B₀₄ B₀₅ (B₀₅ B₀₈)
 B₀₈ C₀₁ C₀₂
- * B₀₁ B₀₃ B₁₄ B₁₇ (B₀₃ B₁₇)
 C₀₁ C₀₂ C₀₃
- * B₀₁ B₀₅ B₁₄ B₁₈ (B₀₅ B₁₈)
 C₀₁ C₀₂ C₀₃
- B₀₁ B₀₅ B₁₅ B₁₇ (B₀₅ B₁₅) (B₀₅ B₁₇)
 C₀₁ C₀₂ C₀₃
- B₀₁ B₁₃ B₁₅ B₁₆ (B₀₁ B₁₆)
 B₁₇ C₀₂ C₀₃
- B₀₂ B₀₃ B₀₄ B₀₆ (B₀₆ B₁₄)
 B₁₄ C₀₁ C₀₂
- * B₀₂ B₀₄ B₀₈ B₁₀ (B₀₄ B₁₀)
 C₀₁ C₀₂ C₀₃
- B₀₂ B₀₆ B₀₉ B₁₀ (B₀₆ B₀₉) (B₀₆ B₁₀)
 C₀₁ C₀₂ C₀₃

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTS (continued)Order 7 (continued)

B ₀₂ B ₀₇ B ₀₉ B ₁₀	(B ₀₂ B ₁₁)
B ₁₁ C ₀₁ C ₀₃	
B ₀₃ B ₀₄ B ₀₈ B ₁₂	(B ₀₃ B ₁₂) (B ₀₄ B ₁₂)
C ₀₁ C ₀₂ C ₀₃	
B ₀₃ B ₀₄ B ₁₄ B ₁₈	(B ₀₄ B ₁₈) (B ₀₃ B ₁₈)
C ₀₁ C ₀₂ C ₀₃	
* B ₀₇ B ₀₉ B ₁₃ B ₁₅	(B ₀₉ B ₁₅)
C ₀₁ C ₀₂ C ₀₃	
* B ₀₇ B ₁₁ B ₁₃ B ₁₆	(B ₁₁ B ₁₅) (B ₁₁ B ₁₇)
C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₁₁ B ₁₅ B ₁₇	(B ₁₁ B ₁₅) (B ₁₁ B ₁₅)
C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₁₄ B ₁₅ B ₁₇	(B ₀₇ B ₁₈)
B ₁₈ C ₀₂ C ₀₃	
B ₀₈ B ₀₉ B ₁₀ B ₁₂	(B ₁₂ B ₁₃)
B ₁₃ C ₀₁ C ₀₃	
B ₀₉ B ₁₀ B ₁₃ B ₁₆	(B ₀₉ B ₁₆) (B ₁₀ B ₁₆)
C ₀₁ C ₀₂ C ₀₃	

Order 8

B ₀₁ B ₀₃ B ₀₄ B ₀₈ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₂)
B ₀₁ B ₀₅ B ₁₃ B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₅ B ₁₃)

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTS (continued)Order 8 (continued)

B ₀₂ B ₀₆ B ₀₇ B ₀₉ B ₁₀ C ₀₁ C ₀₂ C ₀₃	(B ₀₆ B ₀₇)
B ₀₇ B ₁₁ B ₁₄ B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₁₁ B ₁₄)
B ₀₇ B ₁₁ B ₁₅ B ₁₇ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₁₁ B ₁₈)
B ₀₈ B ₀₉ B ₁₀ B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₈ B ₁₆)

Order 9

B ₀₁ B ₀₃ B ₀₄ B ₀₅ B ₀₈ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₅ B ₁₂)
B ₀₁ B ₀₅ B ₁₃ B ₁₅ B ₁₆ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₅ B ₁₆)
B ₀₂ B ₀₃ B ₀₄ B ₀₆ B ₁₄ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₆ B ₁₈)
B ₀₂ B ₀₆ B ₀₇ B ₀₉ B ₁₀ B ₁₁ C ₀₁ C ₀₂ C ₀₃	(B ₀₆ B ₁₁)
B ₀₈ B ₀₉ B ₁₀ B ₁₂ B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₁₂ B ₁₆)

Order 11

* B ₀₁ B ₀₂ B ₀₅ B ₀₆ B ₀₇ B ₀₉ B ₁₀ B ₁₁ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₀) (B ₀₅ B ₀₉) (B ₀₅ B ₁₀)
* B ₀₁ B ₀₂ B ₀₅ B ₀₆ B ₁₃ B ₁₅ B ₁₆ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₇) (B ₀₆ B ₁₅) (B ₀₆ B ₁₇)
* B ₀₁ B ₀₃ B ₀₄ B ₀₅ B ₀₇ B ₀₈ B ₁₁ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₁₁) (B ₀₄ B ₀₇) (B ₀₄ B ₁₁)
* B ₀₁ B ₀₃ B ₀₄ B ₀₅ B ₁₄ B ₁₅ B ₁₇ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₁₅) (B ₀₄ B ₁₇)

SUBSEMIGROUPS OF T_3 GENERATED BY TWO ELEMENTS (continued)Order 11 (continued)

- * $B_{02} B_{03} B_{04} B_{06} B_{08} B_{09}$ $(B_{03} B_{10}) (B_{04} B_{09})$
 $B_{10} B_{12} C_{01} C_{02} C_{03}$
- * $B_{02} B_{03} B_{04} B_{06} B_{13} B_{14}$ $(B_{03} B_{13}) (B_{03} B_{16}) (B_{04} B_{16})$
 $B_{16} B_{18} C_{01} C_{02} C_{03}$
- * $B_{07} B_{08} B_{11} B_{12} B_{14} B_{15}$ $(B_{08} B_{15}) (B_{12} B_{15}) (B_{12} B_{17})$
 $B_{17} B_{18} C_{01} C_{02} C_{03}$
- * $B_{07} B_{09} B_{10} B_{11} B_{13} B_{15}$ $(B_{09} B_{17}) (B_{10} B_{15})$
 $B_{16} B_{17} C_{01} C_{02} C_{03}$
- * $B_{08} B_{09} B_{10} B_{12} B_{13} B_{14}$ $(B_{09} B_{14}) (B_{09} B_{18}) (B_{10} B_{18})$
 $B_{16} B_{18} C_{01} C_{02} C_{03}$

Order 13

- * $A_{01} A_{02} B_{02} B_{03} B_{04} B_{06} B_{08}$ $(A_{02} B_{04}) (A_{02} B_{06})$
 $B_{09} B_{10} B_{12} C_{01} C_{02} C_{03}$ $(A_{02} B_{10}) (A_{02} B_{12})$
- * $A_{01} A_{03} B_{07} B_{09} B_{10} B_{11} B_{13}$ $(A_{03} B_{09}) (A_{03} B_{11})$
 $B_{15} B_{16} B_{17} C_{01} C_{02} C_{03}$ $(A_{03} B_{15}) (A_{03} B_{16})$
- * $A_{01} A_{06} B_{01} B_{03} B_{04} B_{05} B_{14}$ $(A_{06} B_{03}) (A_{06} B_{05})$
 $B_{15} B_{17} B_{18} C_{01} C_{02} C_{03}$ $(A_{06} B_{17}) (A_{06} B_{18})$

Order 24

- * $A_{01} A_{04} A_{05} B_i C_j$ $(A_{04} B_i), i = 01, \dots, 18$
 $(i = 01, \dots, 18; j = 01, 02, 03)$ $(A_{05} B_i), i = 01, \dots, 18$

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTSOrder 3

* B ₀₁ B ₀₂ C ₀₁	* B ₀₇ B ₀₈ C ₀₃
* B ₀₁ B ₀₂ C ₀₂	* B ₀₇ C ₀₁ C ₀₃
* B ₀₁ B ₀₇ C ₀₁	* B ₀₈ B ₁₄ C ₀₃
* B ₀₁ C ₀₁ C ₀₂	* B ₀₈ C ₀₁ C ₀₃
* B ₀₂ B ₁₃ C ₀₂	* B ₁₃ B ₁₄ C ₀₂
* B ₀₂ B ₁₃ C ₀₃	* B ₁₃ B ₁₄ C ₀₃
* B ₀₂ C ₀₁ C ₀₂	* B ₁₃ C ₀₂ C ₀₃
* B ₀₂ C ₀₁ C ₀₃	* B ₁₄ C ₀₂ C ₀₃
* B ₀₂ C ₀₂ C ₀₃	* C ₀₁ C ₀₂ C ₀₃
* B ₀₇ B ₀₈ C ₀₁	

Order 4

B ₀₁ B ₀₂ B ₀₃ C ₀₁	(B ₀₁ B ₀₂ B ₀₃)
B ₀₁ B ₀₂ B ₀₄ C ₀₂	(B ₀₁ B ₀₂ B ₀₄)
* B ₀₁ B ₀₂ C ₀₂ C ₀₃	(B ₀₁ B ₀₂ C ₀₃)
B ₀₁ B ₀₃ C ₀₁ C ₀₂	(B ₀₁ B ₀₃ C ₀₂)
B ₀₁ B ₀₄ B ₁₄ C ₀₂	(B ₀₁ B ₀₄ B ₁₄)
B ₀₁ B ₀₄ C ₀₁ C ₀₂	(B ₀₁ B ₀₄ C ₀₁)
* B ₀₁ B ₀₇ C ₀₂ C ₀₃	(B ₀₁ B ₀₇ C ₀₂) (B ₀₁ B ₀₇ C ₀₃)
B ₀₁ B ₁₄ B ₁₅ C ₀₂	(B ₀₁ B ₁₄ B ₁₅)
* B ₀₁ B ₁₄ C ₀₁ C ₀₂	(B ₀₁ B ₁₄ C ₀₁)
* B ₀₁ B ₁₄ C ₀₂ C ₀₃	(B ₀₁ B ₁₄ C ₀₃)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 4 (continued)

B ₀₁ B ₁₅ C ₀₂ C ₀₃	(B ₀₁ B ₁₅ C ₀₃)
* B ₀₁ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ C ₀₁ C ₀₃)
B ₀₂ B ₀₃ B ₀₈ C ₀₁	(B ₀₂ B ₀₃ B ₀₈)
B ₀₂ B ₀₃ C ₀₁ C ₀₂	(B ₀₂ B ₀₃ C ₀₂)
B ₀₂ B ₀₄ C ₀₁ C ₀₂	(B ₀₂ B ₀₄ C ₀₁)
B ₀₂ B ₀₈ B ₀₉ C ₀₁	(B ₀₂ B ₀₈ B ₀₉)
* B ₀₂ B ₀₈ C ₀₁ C ₀₂	(B ₀₂ B ₀₈ C ₀₂)
* B ₀₂ B ₀₈ C ₀₁ C ₀₃	(B ₀₂ B ₀₈ C ₀₃)
B ₀₂ B ₀₉ C ₀₁ C ₀₃	(B ₀₂ B ₀₉ C ₀₃)
* B ₀₂ B ₁₃ C ₀₁ C ₀₃	(B ₀₂ B ₁₃ C ₀₁)
B ₀₃ B ₀₈ C ₀₁ C ₀₂	(B ₀₃ B ₀₈ C ₀₂)
B ₀₄ B ₁₄ C ₀₁ C ₀₂	(B ₀₄ B ₁₄ C ₀₁)
B ₀₇ B ₀₈ B ₀₉ C ₀₁	(B ₀₇ B ₀₈ B ₀₉)
B ₀₇ B ₀₈ B ₁₀ C ₀₃	(B ₀₇ B ₀₈ B ₁₀)
B ₀₇ B ₀₉ C ₀₁ C ₀₃	(B ₀₇ B ₀₉ C ₀₃)
B ₀₇ B ₁₀ B ₁₃ C ₀₃	(B ₀₇ B ₁₀ B ₁₃)
B ₀₇ B ₁₀ C ₀₁ C ₀₃	(B ₀₇ B ₁₀ C ₀₁)
B ₀₇ B ₁₃ B ₁₇ C ₀₃	(B ₀₇ B ₁₃ B ₁₇)
* B ₀₇ B ₁₃ C ₀₁ C ₀₃	(B ₀₇ B ₁₃ C ₀₁)
* B ₀₇ B ₁₃ C ₀₂ C ₀₃	(B ₀₇ B ₁₃ C ₀₂)
B ₀₇ B ₁₇ C ₀₂ C ₀₃	(B ₀₇ B ₁₇ C ₀₂)
* B ₀₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ C ₀₁ C ₀₂)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 4 (continued)

B ₀₈ B ₀₉ C ₀₁ C ₀₃	(B ₀₈ B ₀₉ C ₀₃)
B ₀₈ B ₁₀ C ₀₁ C ₀₃	(B ₀₈ B ₁₀ C ₀₁)
* B ₀₈ B ₁₄ C ₀₁ C ₀₂	(B ₀₈ B ₁₄ C ₀₁) (B ₀₈ B ₁₄ C ₀₂)
* B ₀₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₈ C ₀₂ C ₀₃)
B ₁₀ B ₁₃ C ₀₁ C ₀₃	(B ₁₀ B ₁₃ C ₀₁)
B ₁₃ B ₁₄ B ₁₅ C ₀₂	(B ₁₃ B ₁₄ B ₁₅)
B ₁₃ B ₁₄ B ₁₇ C ₀₃	(B ₁₃ B ₁₄ B ₁₇)
B ₁₃ B ₁₅ C ₀₂ C ₀₃	(B ₁₃ B ₁₅ C ₀₃)
B ₁₃ B ₁₇ C ₀₂ C ₀₃	(B ₁₃ B ₁₇ C ₀₂)
* B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₁₃ C ₀₁ C ₀₂)
B ₁₄ B ₁₅ C ₀₂ C ₀₃	(B ₁₄ B ₁₅ C ₀₃)
B ₁₄ B ₁₇ C ₀₂ C ₀₃	(B ₁₄ B ₁₇ C ₀₂)
* B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₁₄ C ₀₁ C ₀₃)

Order 5

* A ₀₁ A ₀₂ B ₀₁ B ₀₇ C ₀₁	(A ₀₂ B ₀₁ C ₀₁) (A ₀₂ B ₀₇ C ₀₁)
* A ₀₁ A ₀₂ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ C ₀₁ C ₀₂)
* A ₀₁ A ₀₃ B ₀₈ B ₁₄ C ₀₃	(A ₀₃ B ₀₈ C ₀₃) (A ₀₃ B ₁₄ C ₀₃)
* A ₀₁ A ₀₃ C ₀₁ C ₀₂ C ₀₃	(A ₀₃ C ₀₁ C ₀₃) (A ₀₃ C ₀₂ C ₀₃)
* A ₀₁ A ₀₆ C ₀₁ C ₀₂ C ₀₃	(A ₀₆ C ₀₁ C ₀₂) (A ₀₆ C ₀₂ C ₀₃)
B ₀₁ B ₀₂ B ₀₃ B ₀₈ C ₀₁	(B ₀₁ B ₀₂ B ₀₈)
B ₀₁ B ₀₂ B ₀₄ B ₁₄ C ₀₂	(B ₀₁ B ₀₂ B ₁₄)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 5 (continued)

B ₀₁ B ₀₃ B ₀₄ C ₀₁ C ₀₂	(B ₀₁ B ₀₃ B ₀₄)
B ₀₁ B ₀₃ B ₁₄ C ₀₁ C ₀₂	(B ₀₁ B ₀₃ B ₁₄)
B ₀₁ B ₀₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₃ C ₀₃)
B ₀₁ B ₀₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₄ C ₀₃)
B ₀₁ B ₀₇ B ₁₅ C ₀₂ C ₀₃	(B ₀₁ B ₀₇ B ₁₅)
B ₀₁ B ₀₇ B ₁₇ C ₀₂ C ₀₃	(B ₀₁ B ₀₇ B ₁₇)
B ₀₁ B ₀₃ B ₀₈ C ₀₁ C ₀₂	(B ₀₁ B ₀₈ C ₀₂)
B ₀₁ B ₁₃ B ₁₄ B ₁₅ C ₀₂	(B ₀₁ B ₁₃ B ₁₄)
B ₀₁ B ₁₃ B ₁₅ C ₀₂ C ₀₃	(B ₀₁ B ₁₃ C ₀₃)
B ₀₁ B ₁₄ B ₁₇ C ₀₂ C ₀₃	(B ₀₁ B ₁₄ B ₁₇)
B ₀₁ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₀₁ B ₁₅ B ₁₇)
B ₀₁ B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₅ C ₀₁)
B ₀₁ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₇ C ₀₁)
B ₀₂ B ₀₃ B ₀₄ C ₀₁ C ₀₂	(B ₀₂ B ₀₃ B ₀₄)
B ₀₂ B ₀₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₃ C ₀₃)
B ₀₂ B ₀₄ B ₀₈ C ₀₁ C ₀₂	(B ₀₂ B ₀₄ B ₀₈)
B ₀₂ B ₀₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₄ C ₀₃)
B ₀₂ B ₀₇ B ₀₈ B ₀₉ C ₀₁	(B ₀₂ B ₀₇ B ₀₈)
B ₀₂ B ₀₇ B ₀₉ C ₀₁ C ₀₃	(B ₀₂ B ₀₇ C ₀₃)
B ₀₂ B ₀₈ B ₁₀ C ₀₁ C ₀₃	(B ₀₂ B ₀₈ B ₁₀)
B ₀₂ B ₀₉ B ₁₀ C ₀₁ C ₀₃	(B ₀₂ B ₀₉ B ₁₀)
B ₀₂ B ₀₉ B ₁₃ C ₀₁ C ₀₃	(B ₀₂ B ₀₉ B ₁₃)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 5 (continued)

B ₀₂ B ₀₉ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₉ C ₀₂)
B ₀₂ B ₁₀ B ₁₃ C ₀₁ C ₀₃	(B ₀₂ B ₁₀ B ₁₃)
B ₀₂ B ₁₀ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₀ C ₀₂)
B ₀₂ B ₀₄ B ₁₄ C ₀₁ C ₀₂	(B ₀₂ B ₁₄ C ₀₁)
B ₀₃ B ₀₄ B ₀₈ C ₀₁ C ₀₂	(B ₀₃ B ₀₄ B ₀₈)
B ₀₃ B ₀₄ B ₁₄ C ₀₁ C ₀₂	(B ₀₃ B ₀₄ B ₁₄)
B ₀₃ B ₀₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₀₄ C ₀₃)
B ₀₃ B ₀₈ B ₁₄ C ₀₁ C ₀₂	(B ₀₃ B ₀₈ B ₁₄)
B ₀₃ B ₀₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₀₈ C ₀₃)
B ₀₃ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₁₄ C ₀₃)
B ₀₄ B ₀₈ B ₁₄ C ₀₁ C ₀₂	(B ₀₄ B ₀₈ B ₁₄)
B ₀₄ B ₀₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₄ B ₀₈ C ₀₃)
B ₀₄ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₄ B ₁₄ C ₀₃)
B ₀₇ B ₀₈ B ₁₀ B ₁₃ C ₀₃	(B ₀₇ B ₀₈ B ₁₃)
* B ₀₇ B ₀₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₀₈ C ₀₂)
B ₀₇ B ₀₉ B ₁₀ C ₀₁ C ₀₃	(B ₀₇ B ₀₉ B ₁₀)
B ₀₇ B ₀₉ B ₁₃ C ₀₁ C ₀₃	(B ₀₇ B ₀₉ B ₁₃)
B ₀₇ B ₀₉ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₀₉ C ₀₂)
B ₀₇ B ₁₀ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₀ C ₀₂)
B ₀₇ B ₁₃ B ₁₄ B ₁₇ C ₀₃	(B ₀₇ B ₁₃ B ₁₄)
B ₀₇ B ₁₃ B ₁₅ C ₀₂ C ₀₃	(B ₀₇ B ₁₃ B ₁₅)
B ₀₇ B ₁₄ B ₁₇ C ₀₂ C ₀₃	(B ₀₇ B ₁₄ C ₀₂)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 5 (continued)

B ₀₇ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₀₇ B ₁₅ B ₁₇)
B ₀₇ B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₅ C ₀₁)
* B ₀₇ B ₁₄ B ₁₆ C ₀₂ C ₀₃	(B ₀₇ B ₁₆ C ₀₂) (B ₀₇ B ₁₆ C ₀₃)
B ₀₇ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₇ C ₀₁)
B ₀₈ B ₀₉ B ₁₀ C ₀₁ C ₀₃	(B ₀₈ B ₀₉ B ₁₀)
B ₀₈ B ₀₉ C ₀₁ C ₀₂ C ₀₃	(B ₀₈ B ₀₉ C ₀₂)
B ₀₈ B ₁₀ C ₀₁ C ₀₂ C ₀₃	(B ₀₈ B ₁₀ C ₀₂)
B ₀₈ B ₁₀ B ₁₃ C ₀₁ C ₀₃	(B ₀₈ B ₁₃ C ₀₁)
B ₀₉ B ₁₀ B ₁₃ C ₀₁ C ₀₃	(B ₀₉ B ₁₀ B ₁₃)
B ₀₉ B ₁₀ C ₀₁ C ₀₂ C ₀₃	(B ₀₉ B ₁₀ C ₀₂)
B ₀₉ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₉ B ₁₃ C ₀₂)
B ₁₀ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₁₀ B ₁₃ C ₀₂)
* B ₁₃ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₁₃ B ₁₄ C ₀₁)
B ₁₃ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₁₃ B ₁₅ B ₁₇)
B ₁₃ B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₁₃ B ₁₅ C ₀₁)
B ₁₃ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₁₃ B ₁₇ C ₀₁)
B ₁₄ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₁₄ B ₁₅ B ₁₇)
B ₁₄ B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₁₄ B ₁₅ C ₀₁)
B ₁₄ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₁₄ B ₁₇ C ₀₁)
B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₁₅ B ₁₇ C ₀₁)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 6

* A ₀₁ A ₀₂ B ₀₁ B ₀₇ C ₀₂ C ₀₃	(A ₀₂ B ₀₁ C ₀₂) (A ₀₂ B ₀₁ C ₀₃) (A ₀₂ B ₀₇ C ₀₂) (A ₀₂ B ₀₇ C ₀₃)
A ₀₁ A ₀₃ B ₀₈ B ₁₄ C ₀₁ C ₀₂	(A ₀₃ B ₀₈ C ₀₁) (A ₀₃ B ₀₈ C ₀₂) (A ₀₃ B ₁₄ C ₀₂) (A ₀₃ B ₁₄ C ₀₁)
B ₀₁ B ₀₃ B ₀₄ B ₀₈ C ₀₁ C ₀₂	(B ₀₁ B ₀₄ B ₀₈)
* B ₀₁ B ₀₂ B ₀₅ B ₀₆ C ₀₁ C ₀₂	(B ₀₁ B ₀₆ C ₀₁) (B ₀₁ B ₀₆ C ₀₂) (B ₀₂ B ₀₅ C ₀₁) (B ₀₂ B ₀₅ C ₀₂) (B ₀₅ B ₀₆ C ₀₁) (B ₀₅ B ₀₆ C ₀₂)
B ₀₁ B ₀₇ B ₁₃ B ₁₅ C ₀₂ C ₀₃	(B ₀₁ B ₀₇ B ₁₃)
B ₀₁ B ₀₇ B ₁₄ B ₁₇ C ₀₂ C ₀₃	(B ₀₁ B ₀₇ B ₁₄)
B ₀₁ B ₀₃ B ₀₈ B ₁₄ C ₀₁ C ₀₂	(B ₀₁ B ₀₈ B ₁₄)
B ₀₁ B ₀₃ B ₀₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₈ C ₀₃)
B ₀₁ B ₁₃ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₀₁ B ₁₃ B ₁₇)
B ₀₁ B ₁₃ B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₃ C ₀₁)
* B ₀₁ B ₁₄ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₈ C ₀₁)
* B ₀₁ B ₀₂ B ₁₃ B ₁₅ C ₀₂ C ₀₃	(B ₀₂ B ₁₅ C ₀₃)
* B ₀₁ B ₀₅ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₅ B ₁₄ C ₀₃)
B ₀₂ B ₀₃ B ₀₄ B ₁₄ C ₀₁ C ₀₂	(B ₀₂ B ₀₃ B ₁₄)
B ₀₂ B ₀₄ B ₀₈ B ₁₄ C ₀₁ C ₀₂	(B ₀₂ B ₀₈ B ₁₄)
B ₀₂ B ₀₄ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₄ C ₀₃)
* B ₀₂ B ₀₆ B ₀₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₆ B ₀₈ C ₀₃)
B ₀₂ B ₀₇ B ₀₉ B ₁₀ C ₀₁ C ₀₃	(B ₀₂ B ₀₇ B ₁₀)
B ₀₇ B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₆ C ₀₁)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 6 (continued)

B ₀₂ B ₀₇ B ₀₉ B ₁₃ C ₀₁ C ₀₃	(B ₀₂ B ₀₇ B ₁₃)
B ₀₂ B ₀₇ B ₀₉ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₇ C ₀₂)
B ₀₂ B ₀₈ B ₁₀ B ₁₃ C ₀₁ C ₀₃	(B ₀₂ B ₀₈ B ₁₃)
B ₀₂ B ₀₈ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₂ C ₀₂)
* B ₀₇ B ₀₈ B ₁₁ B ₁₂ C ₀₁ C ₀₃	(B ₀₇ B ₁₂ C ₀₁) (B ₀₇ B ₁₂ C ₀₃) (B ₀₈ B ₁₁ C ₀₁) (B ₀₈ B ₁₁ C ₀₃) (B ₁₁ B ₁₂ C ₀₁) (B ₁₁ B ₁₂ C ₀₃)
* B ₀₇ B ₁₁ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₁₁ B ₁₃ C ₀₂)
B ₀₇ B ₁₄ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₀₇ B ₁₄ B ₁₅)
* B ₀₇ B ₁₄ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₆ C ₀₁)
B ₀₇ B ₁₄ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₄ C ₀₁)
B ₀₈ B ₀₉ B ₁₀ B ₁₃ C ₀₁ C ₀₃	(B ₀₈ B ₀₉ B ₁₃)
B ₀₈ B ₁₀ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₈ B ₁₃ C ₀₂)
* B ₁₃ B ₁₄ B ₁₆ B ₁₈ C ₀₂ C ₀₃	(B ₁₃ B ₁₈ C ₀₂) (B ₁₃ B ₁₈ C ₀₃) (B ₁₆ B ₁₈ C ₀₃) (B ₁₄ B ₁₆ C ₀₂) (B ₁₄ B ₁₆ C ₀₃) (B ₁₆ B ₁₈ C ₀₂)
* B ₁₃ B ₁₄ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₁₃ B ₁₈ C ₀₁)

Order 7

A ₀₁ A ₀₂ B ₁₅ B ₁₇	(A ₀₂ B ₁₅ C ₀₁) (A ₀₂ B ₁₇ C ₀₁)
C ₀₁ C ₀₂ C ₀₃	

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 7 (continued)

A01 A03 B03 B04 C01 C02 C03	(A03 B03 C03) (A03 B04 C03)
* B01 B02 B03 B07 B08 B09 C01	(B01 B03 B09) (B01 B08 B09) (B02 B03 B07) (B03 B07 B09)
* B01 B02 B04 B13 B14 B15 C02	(B01 B04 B13) (B02 B04 B15) (B02 B14 B15) (B04 B13 B15)
* B01 B02 B05 B06 C01 C02 C03	(B01 B06 C03) (B02 B05 C03) (B05 B06 C03)
* B01 B02 B07 B09 C01 C02 C03	(B01 B09 C02) (B01 B09 C03)
* B01 B02 B13 B15 C01 C02 C03	(B02 B15 C01)
B01 B03 B04 B05 B14 C01 C02	(B03 B05 B14) (B04 B05 B14)
B01 B03 B04 B05 C01 C02 C03	(B03 B05 C03) (B04 B05 C03)
* B01 B03 B07 B08 C01 C02 C03	(B03 B07 C02) (B03 B07 C03)
* B01 B04 B14 B15 C01 C02 C03	(B04 B15 C01) (B04 B15 C03)
* B01 B05 B07 B11 C01 C02 C03	(B01 B11 C01) (B01 B11 C02) (B01 B11 C03) (B05 B07 C01) (B05 B07 C02) (B05 B07 C03) (B05 B11 C01) (B05 B11 C02) (B05 B11 C03)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 7 (continued)

B ₀₁ B ₁₄ B ₁₅ B ₁₇ B ₁₈ C ₀₂ C ₀₃	(B ₀₁ B ₁₄) (B ₀₁ B ₁₅ B ₁₈) (B ₀₁ B ₁₇ B ₁₈)
B ₀₂ B ₀₃ B ₀₄ B ₀₆ B ₀₈ C ₀₁ C ₀₂	(B ₀₃ B ₀₆ B ₀₈) (B ₀₄ B ₀₆ B ₀₈)
B ₀₂ B ₀₃ B ₀₄ B ₀₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₀₆ C ₀₃) (B ₀₄ B ₀₆ C ₀₃)
B ₀₂ B ₀₃ B ₀₈ B ₀₉ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₀₉ C ₀₂) (B ₀₃ B ₀₉ C ₀₃)
* B ₀₂ B ₀₄ B ₁₃ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₄ B ₁₃ C ₀₁) (B ₀₄ B ₁₃ C ₀₃)
* B ₀₂ B ₀₆ B ₀₈ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₆ B ₀₈ B ₁₂) (B ₀₆ B ₁₂ C ₀₂)
* B ₀₂ B ₀₆ B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₆ C ₀₁) (B ₀₂ B ₁₆ C ₀₂) (B ₀₂ B ₁₆ C ₀₃) (B ₀₆ B ₁₃ C ₀₁) (B ₀₆ B ₁₃ C ₀₂) (B ₀₆ B ₁₃ C ₀₃) (B ₀₆ B ₁₆ C ₀₁) (B ₀₆ B ₁₆ C ₀₂) (B ₀₆ B ₁₆ C ₀₃)
B ₀₂ B ₀₈ B ₀₉ B ₁₀ B ₁₂ C ₀₁ C ₀₃	(B ₀₂ B ₀₉ B ₁₂) (B ₀₂ B ₁₀ B ₁₂)
* B ₀₇ B ₀₈ B ₁₀ B ₁₃ B ₁₄ B ₁₇ C ₀₃	(B ₀₇ B ₁₀ B ₁₄) (B ₀₈ B ₁₀ B ₁₇) (B ₀₈ B ₁₃ B ₁₇) (B ₁₀ B ₁₄ B ₁₇)
* B ₀₇ B ₀₈ B ₁₁ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₂ C ₀₂) (B ₀₈ B ₁₁ C ₀₂) (B ₁₁ B ₁₂ C ₀₂)
* B ₀₇ B ₀₈ B ₁₄ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₈ B ₁₇ C ₀₁) (B ₀₈ B ₁₇ C ₀₂)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 7 (continued)

B ₀₇ B ₀₉ B ₁₀ B ₁₁	(B ₀₉ B ₁₁ B ₁₃) (B ₁₀ B ₁₁ B ₁₃)
B ₁₃ C ₀₁ C ₀₃	
B ₀₇ B ₀₉ B ₁₀ B ₁₁	(B ₀₉ B ₁₁ C ₀₂) (B ₁₀ B ₁₁ C ₀₂)
C ₀₁ C ₀₂ C ₀₃	
* B ₀₇ B ₁₀ B ₁₃ B ₁₇	(B ₁₀ B ₁₇ C ₀₁) (B ₁₀ B ₁₇ C ₀₂)
C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₁₃ B ₁₅ B ₁₆	(B ₀₇ B ₁₅ B ₁₆) (B ₀₇ B ₁₆ B ₁₇)
B ₁₇ C ₀₂ C ₀₃	
B ₀₈ B ₀₉ B ₁₀ B ₁₂	(B ₀₉ B ₁₂ C ₀₂) (B ₁₀ B ₁₂ C ₀₂)
C ₀₁ C ₀₂ C ₀₃	
* B ₀₈ B ₁₀ B ₁₃ B ₁₄	(B ₁₀ B ₁₄ C ₀₁) (B ₁₀ B ₁₄ C ₀₂)
C ₀₁ C ₀₂ C ₀₃	
* B ₀₈ B ₁₂ B ₁₄ B ₁₈	(B ₀₈ B ₁₈ C ₀₁) (B ₀₈ B ₁₈ C ₀₂) (B ₀₈ B ₁₈ C ₀₃)
C ₀₁ C ₀₂ C ₀₃	(B ₁₂ B ₁₄ C ₀₁) (B ₁₂ B ₁₄ C ₀₂) (B ₁₂ B ₁₄ C ₀₃)
	(B ₁₂ B ₁₈ C ₀₁) (B ₁₂ B ₁₈ C ₀₂) (B ₁₂ B ₁₈ C ₀₃)
* B ₁₃ B ₁₄ B ₁₆ B ₁₈	(B ₁₄ B ₁₆ C ₀₁) (B ₁₆ B ₁₈ C ₀₁)
C ₀₁ C ₀₂ C ₀₃	
B ₁₃ B ₁₅ B ₁₆ B ₁₇	(B ₁₅ B ₁₆ C ₀₁) (B ₁₆ B ₁₇ C ₀₁)
C ₀₁ C ₀₂ C ₀₃	
B ₁₄ B ₁₅ B ₁₇ B ₁₈	(B ₁₅ B ₁₈ C ₀₁) (B ₁₇ B ₁₈ C ₀₁)
C ₀₁ C ₀₂ C ₀₃	

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 8

A ₀₁ A ₀₂ B ₀₁ B ₀₇ B ₁₅	(A ₀₂ B ₀₁ B ₁₅) (A ₀₂ B ₀₁ B ₁₇)
B ₁₇ C ₀₂ C ₀₃	(A ₀₂ B ₀₇ B ₁₅) (A ₀₂ B ₀₇ B ₁₇)
* A ₀₁ A ₀₂ B ₁₃ B ₁₄ B ₁₆	(A ₀₂ B ₁₃ C ₀₂) (A ₀₂ B ₁₃ C ₀₃)
B ₁₈ C ₀₂ C ₀₃	(A ₀₂ B ₁₄ C ₀₂) (A ₀₂ B ₁₄ C ₀₃)
	(A ₀₂ B ₁₆ C ₀₃) (A ₀₂ B ₁₈ C ₀₃)
	(A ₀₂ B ₁₆ C ₀₂) (A ₀₂ B ₁₈ C ₀₂)
* A ₀₁ A ₀₃ B ₀₁ B ₀₂ B ₀₅	(A ₀₃ B ₀₁ C ₀₁) (A ₀₃ B ₀₁ C ₀₂)
B ₀₆ C ₀₁ C ₀₂	(A ₀₃ B ₀₂ C ₀₁) (A ₀₃ B ₀₂ C ₀₂)
	(A ₀₃ B ₀₅ C ₀₁) (A ₀₃ B ₀₅ C ₀₂)
	(A ₀₃ B ₀₆ C ₀₁) (A ₀₃ B ₀₆ C ₀₂)
A ₀₁ A ₀₃ B ₀₃ B ₀₄ B ₀₈	(A ₀₃ B ₀₃ B ₀₈) (A ₀₃ B ₀₃ B ₁₄)
B ₁₄ C ₀₁ C ₀₂	(A ₀₃ B ₀₄ B ₀₈) (A ₀₃ B ₀₄ B ₁₄)
A ₀₁ A ₀₆ B ₀₂ B ₀₉ B ₁₀	(A ₀₆ B ₀₂ B ₀₉) (A ₀₆ B ₀₂ B ₁₀)
B ₁₃ C ₀₁ C ₀₃	(A ₀₆ B ₀₉ B ₁₃) (A ₀₆ B ₁₀ B ₁₃)
* A ₀₁ A ₀₆ B ₀₇ B ₀₈ B ₁₁	(A ₀₆ B ₀₇ C ₀₁) (A ₀₆ B ₀₇ C ₀₃)
B ₁₂ C ₀₁ C ₀₃	(A ₀₆ B ₀₈ C ₀₁) (A ₀₆ B ₀₈ C ₀₃)
	(A ₀₆ B ₁₁ C ₀₁) (A ₀₆ B ₁₁ C ₀₃)
	(A ₀₆ B ₁₂ C ₀₁) (A ₀₆ B ₁₂ C ₀₃)
B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₅	(B ₀₁ B ₀₃ B ₀₆) (B ₀₁ B ₀₄ B ₀₆)
B ₀₆ C ₀₁ C ₀₂	(B ₀₂ B ₀₃ B ₀₅) (B ₀₂ B ₀₄ B ₀₅)
	(B ₀₃ B ₀₅ B ₀₆) (B ₀₄ B ₀₅ B ₀₆)
B ₀₁ B ₀₃ B ₀₄ B ₀₅ B ₀₈	(B ₀₅ B ₀₈ C ₀₃)
C ₀₁ C ₀₂ C ₀₃	

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 8 (continued)

B ₀₁ B ₀₃ B ₀₄ B ₀₅ B ₀₈	(B ₀₅ B ₀₈ B ₁₄)
B ₁₄ C ₀₁ C ₀₂	
B ₀₁ B ₀₃ B ₀₄ B ₁₄ B ₁₈	(B ₀₁ B ₀₃ B ₁₈) (B ₀₁ B ₀₄ B ₁₈)
C ₀₁ C ₀₂ C ₀₃	
B ₀₁ B ₀₅ B ₁₄ B ₁₅ B ₁₇	(B ₀₅ B ₁₄ B ₁₅) (B ₀₅ B ₁₄ B ₁₇)
C ₀₁ C ₀₂ C ₀₃	
B ₀₁ B ₀₇ B ₁₃ B ₁₅ B ₁₆	(B ₀₁ B ₀₇ B ₁₆)
B ₁₇ C ₀₂ C ₀₃	
B ₀₁ B ₀₇ B ₁₄ B ₁₅ B ₁₇	(B ₀₁ B ₀₇ B ₁₈)
B ₁₈ C ₀₂ C ₀₃	
B ₀₁ B ₁₃ B ₁₅ B ₁₆ B ₁₇	(B ₀₁ B ₁₆ C ₀₁)
C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₃ B ₀₄ B ₀₆ B ₀₈	(B ₀₆ B ₀₈ B ₁₄)
B ₁₄ C ₀₁ C ₀₂	
B ₀₂ B ₀₃ B ₀₄ B ₀₆ B ₁₄	(B ₀₆ B ₁₄ C ₀₃)
C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₃ B ₀₄ B ₀₈ B ₁₂	(B ₀₂ B ₀₃ B ₁₂) (B ₀₂ B ₀₄ B ₁₂)
C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₃ B ₀₄ B ₁₄ B ₁₈	(B ₀₂ B ₀₃ B ₁₈) (B ₀₂ B ₀₄ B ₁₈)
C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₄ B ₁₈)
B ₀₂ B ₀₆ B ₀₈ B ₀₉ B ₁₀	(B ₀₆ B ₀₈ B ₀₉) (B ₀₆ B ₀₈ B ₁₀)
C ₀₁ C ₀₂ C ₀₃	

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 8 (continued)

B ₀₂ B ₀₇ B ₀₉ B ₁₀ B ₁₁ B ₁₃ C ₀₁ C ₀₃	(B ₀₂ B ₁₁ B ₁₃)
B ₀₂ B ₀₇ B ₀₉ B ₁₀ B ₁₁ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₁ C ₀₂)
B ₀₂ B ₀₈ B ₀₉ B ₁₀ B ₁₂ B ₁₃ C ₀₁ C ₀₃	(B ₀₂ B ₁₂ B ₁₃)
B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₁ B ₁₂ C ₀₁ C ₀₃	(B ₀₇ B ₀₉ B ₁₂) (B ₀₇ B ₁₀ B ₁₂) (B ₀₈ B ₀₉ B ₁₁) (B ₀₈ B ₁₀ B ₁₁) (B ₀₉ B ₁₁ B ₁₂) (B ₁₀ B ₁₁ B ₁₂)
B ₀₇ B ₀₉ B ₁₀ B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₀₉ B ₁₆) (B ₀₇ B ₁₀ B ₁₆)
B ₀₇ B ₁₁ B ₁₃ B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₁₁ B ₁₃ B ₁₅) (B ₁₁ B ₁₃ B ₁₇)
B ₀₇ B ₁₄ B ₁₅ B ₁₇ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₈ C ₀₁)
B ₀₈ B ₀₉ B ₁₀ B ₁₂ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₁₂ B ₁₃ C ₀₂)
B ₁₃ B ₁₄ B ₁₅ B ₁₆ B ₁₇ B ₁₈ C ₀₂ C ₀₃	(B ₁₃ B ₁₅ B ₁₈) (B ₁₃ B ₁₇ B ₁₈) (B ₁₄ B ₁₅ B ₁₆) (B ₁₄ B ₁₆ B ₁₇) (B ₁₅ B ₁₆ B ₁₈) (B ₁₆ B ₁₇ B ₁₈)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 9

* A ₀₁ A ₀₂ A ₀₃ A ₀₄ A ₀₅	(A ₀₂ A ₀₃ C ₀₁) (A ₀₂ A ₀₃ C ₀₂)
A ₀₆ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ A ₀₃ C ₀₃) (A ₀₂ A ₀₄ C ₀₁)
	(A ₀₂ A ₀₄ C ₀₂) (A ₀₂ A ₀₄ C ₀₃)
	(A ₀₂ A ₀₅ C ₀₁) (A ₀₂ A ₀₅ C ₀₂)
	(A ₀₂ A ₀₅ C ₀₃) (A ₀₂ A ₀₆ C ₀₁)
	(A ₀₂ A ₀₆ C ₀₂) (A ₀₂ A ₀₆ C ₀₃)
	(A ₀₃ A ₀₄ C ₀₁) (A ₀₃ A ₀₄ C ₀₂)
	(A ₀₃ A ₀₄ C ₀₃) (A ₀₃ A ₀₅ C ₀₁)
	(A ₀₃ A ₀₅ C ₀₂) (A ₀₃ A ₀₅ C ₀₃)
	(A ₀₃ A ₀₆ C ₀₁) (A ₀₃ A ₀₆ C ₀₂)
	(A ₀₃ A ₀₆ C ₀₃) (A ₀₄ A ₀₆ C ₀₁)
	(A ₀₄ A ₀₆ C ₀₂) (A ₀₄ A ₀₆ C ₀₃)
	(A ₀₅ A ₀₆ C ₀₁) (A ₀₅ A ₀₆ C ₀₂)
	(A ₀₅ A ₀₆ C ₀₃)
* A ₀₁ A ₀₂ B ₀₁ B ₀₂ B ₀₃	(A ₀₂ B ₀₁ B ₀₂) (A ₀₂ B ₀₁ B ₀₃)
B ₀₇ B ₀₈ B ₀₉ C ₀₁	(A ₀₂ B ₀₁ B ₀₈) (A ₀₂ B ₀₁ B ₀₉)
	(A ₀₂ B ₀₂ B ₀₇) (A ₀₂ B ₀₃ B ₀₇)
	(A ₀₂ B ₀₇ B ₀₈) (A ₀₂ B ₀₇ B ₀₉)
* A ₀₁ A ₀₂ B ₀₂ B ₀₃ B ₀₈	(A ₀₂ B ₀₂ C ₀₂) (A ₀₂ B ₀₂ C ₀₃)
B ₀₉ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₀₃ C ₀₂) (A ₀₂ B ₀₃ C ₀₃)
	(A ₀₂ B ₀₈ C ₀₂) (A ₀₂ B ₀₈ C ₀₃)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 9 (continued)

* A ₀₁ A ₀₂ B ₀₁ B ₀₅ B ₀₇ B ₁₁ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₀₅ C ₀₁) (A ₀₂ B ₀₅ C ₀₂) (A ₀₂ B ₀₅ C ₀₃) (A ₀₂ B ₁₁ C ₀₁) (A ₀₂ B ₁₁ C ₀₂) (A ₀₂ B ₁₁ C ₀₃)
* A ₀₁ A ₀₂ B ₁₃ B ₁₄ B ₁₆ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₁₃ C ₀₁) (A ₀₂ B ₁₄ C ₀₁) (A ₀₂ B ₁₆ C ₀₁) (A ₀₂ B ₁₈ C ₀₁)
* A ₀₁ A ₀₃ B ₀₁ B ₀₂ B ₀₅ B ₀₆ C ₀₁ C ₀₂ C ₀₃	(A ₀₃ B ₀₁ C ₀₃) (A ₀₃ B ₀₂ C ₀₃) (A ₀₃ B ₀₅ C ₀₃) (A ₀₃ B ₀₆ C ₀₃)
* A ₀₁ A ₀₃ B ₀₇ B ₀₈ B ₁₀ B ₁₃ B ₁₄ B ₁₇ C ₀₃	(A ₀₃ B ₀₇ B ₀₈) (A ₀₃ B ₀₇ B ₁₄) (A ₀₃ B ₀₈ B ₁₀) (A ₀₃ B ₀₈ B ₁₃) (A ₀₃ B ₀₈ B ₁₇) (A ₀₃ B ₁₀ B ₁₄) (A ₀₃ B ₁₃ B ₁₄) (A ₀₃ B ₁₄ B ₁₇)
* A ₀₁ A ₀₃ B ₀₇ B ₁₀ B ₁₃ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(A ₀₃ B ₀₇ C ₀₁) (A ₀₃ B ₀₇ C ₀₂) (A ₀₃ B ₁₀ C ₀₁) (A ₀₃ B ₁₀ C ₀₂) (A ₀₃ B ₁₃ C ₀₁) (A ₀₃ B ₁₃ C ₀₂) (A ₀₃ B ₁₇ C ₀₁) (A ₀₃ B ₁₇ C ₀₂)
* A ₀₁ A ₀₃ B ₀₈ B ₁₂ B ₁₄ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₃ B ₁₂ C ₀₁) (A ₀₃ B ₁₂ C ₀₂) (A ₀₃ B ₁₂ C ₀₃) (A ₀₃ B ₁₈ C ₀₁) (A ₀₃ B ₁₈ C ₀₂) (A ₀₃ B ₁₈ C ₀₃)
* A ₀₁ A ₀₆ B ₀₁ B ₀₂ B ₀₄ B ₁₃ B ₁₄ B ₁₅ C ₀₂	(A ₀₆ B ₀₁ B ₀₂) (A ₀₆ B ₀₁ B ₁₃)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 9 (continued)

* A ₀₁ A ₀₆ B ₀₁ B ₀₄ B ₁₄	(A ₀₆ B ₀₁ C ₀₃) (A ₀₆ B ₀₁ C ₀₁)
B ₁₅ C ₀₁ C ₀₂ C ₀₃	(A ₀₆ B ₀₄ C ₀₁) (A ₀₆ B ₀₄ C ₀₃)
	(A ₀₆ B ₁₄ C ₀₁) (A ₀₆ B ₁₄ C ₀₃)
	(A ₀₆ B ₁₅ C ₀₁) (A ₀₆ B ₁₅ C ₀₃)
* A ₀₁ A ₀₆ B ₀₂ B ₀₆ B ₁₃	(A ₀₆ B ₀₆ C ₀₁) (A ₀₆ B ₀₆ C ₀₂)
B ₁₆ C ₀₁ C ₀₂ C ₀₃	(A ₀₆ B ₀₆ C ₀₃)
* A ₀₁ A ₀₆ B ₀₇ B ₀₈ B ₁₁	(A ₀₆ B ₀₇ C ₀₂) (A ₀₆ B ₀₈ C ₀₂)
B ₁₂ C ₀₁ C ₀₂ C ₀₃	(A ₀₆ B ₁₁ C ₀₂) (A ₀₆ B ₁₂ C ₀₂)
B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₅	(B ₀₁ B ₀₆ B ₀₈) (B ₀₂ B ₀₅ B ₀₈)
B ₀₆ B ₀₈ C ₀₁ C ₀₂	(B ₀₅ B ₀₆ B ₀₈)
B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₅	(B ₀₁ B ₀₆ B ₁₄) (B ₀₂ B ₀₅ B ₁₄)
B ₀₆ B ₁₄ C ₀₁ C ₀₂	(B ₀₅ B ₀₆ B ₁₄)
B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₈	(B ₀₁ B ₀₂ B ₁₂)
B ₁₂ C ₀₁ C ₀₂ C ₀₃	
B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₁₄	(B ₀₁ B ₀₂ B ₁₈)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	
B ₀₁ B ₀₂ B ₀₇ B ₀₉ B ₁₃	(B ₀₁ B ₀₉ B ₁₃) (B ₀₁ B ₀₉ B ₁₅)
B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₇ B ₁₅) (B ₀₂ B ₀₉ B ₁₅)
B ₀₁ B ₀₃ B ₀₄ B ₀₅ B ₁₄	(B ₀₃ B ₀₅ B ₁₈) (B ₀₄ B ₀₅ B ₁₈)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	
* B ₀₁ B ₀₃ B ₀₇ B ₀₈ B ₁₄	(B ₀₁ B ₀₈ B ₁₇) (B ₀₃ B ₀₇ B ₁₄)
B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₀₇ B ₁₇) (B ₀₃ B ₀₈ B ₁₇)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 9 (continued)

B ₀₁ B ₀₅ B ₀₇ B ₁₁ B ₁₄ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₅ B ₀₇ B ₁₄) (B ₀₅ B ₁₁ B ₁₄)
B ₀₁ B ₀₅ B ₀₇ B ₁₁ B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₁ B ₁₅) (B ₀₁ B ₁₁ B ₁₇) (B ₀₅ B ₀₇ B ₁₅) (B ₀₅ B ₀₇ B ₁₇) (B ₀₅ B ₁₁ B ₁₅) (B ₀₅ B ₁₁ B ₁₇)
B ₀₁ B ₀₅ B ₁₃ B ₁₄ B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₅ B ₁₃ B ₁₄)
B ₀₁ B ₀₅ B ₁₄ B ₁₅ B ₁₇ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₅ B ₁₅ B ₁₈) (B ₀₅ B ₁₇ B ₁₈)
B ₀₁ B ₁₃ B ₁₄ B ₁₅ B ₁₆ B ₁₇ B ₁₈ C ₀₂ C ₀₃	(B ₀₁ B ₁₃ B ₁₈) (B ₀₁ B ₁₄ B ₁₆) (B ₀₁ B ₁₆ B ₁₈)
B ₀₂ B ₀₃ B ₀₄ B ₀₆ B ₀₈ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₀₆ B ₁₂) (B ₀₄ B ₀₆ B ₁₂)
* B ₀₂ B ₀₄ B ₀₈ B ₁₀ B ₁₃ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₀ B ₁₄) (B ₀₄ B ₀₈ B ₁₃) (B ₀₄ B ₁₀ B ₁₃) (B ₀₄ B ₁₀ B ₁₄)
B ₀₂ B ₀₆ B ₀₇ B ₀₈ B ₀₉ B ₁₀ C ₀₁ C ₀₂ C ₀₃	(B ₀₆ B ₀₇ B ₀₈)
B ₀₂ B ₀₆ B ₀₈ B ₀₉ B ₁₀ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₆ B ₀₉ B ₁₂) (B ₀₆ B ₁₀ B ₁₂)
B ₀₂ B ₀₆ B ₀₉ B ₁₀ B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₉ B ₁₆) (B ₀₂ B ₁₀ B ₁₆) (B ₀₆ B ₀₉ B ₁₃) (B ₀₆ B ₀₉ B ₁₆) (B ₀₆ B ₁₀ B ₁₃) (B ₀₆ B ₁₀ B ₁₆)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 9 (continued)

B ₀₂ B ₀₇ B ₀₈ B ₀₉ B ₁₀	(B ₀₂ B ₀₇ B ₁₂) (B ₀₂ B ₀₈ B ₁₁)
B ₁₁ B ₁₂ C ₀₁ C ₀₃	(B ₀₂ B ₁₁ B ₁₂)
B ₀₃ B ₀₄ B ₀₈ B ₁₂ B ₁₄	(B ₀₃ B ₀₈ B ₁₈) (B ₀₃ B ₁₂ B ₁₄)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₁₂ B ₁₈) (B ₀₄ B ₀₈ B ₁₈)
	(B ₀₄ B ₁₂ B ₁₄) (B ₀₄ B ₁₂ B ₁₈)
B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₁	(B ₀₇ B ₁₂ B ₁₃) (B ₀₈ B ₁₁ B ₁₃)
B ₁₂ B ₁₃ C ₀₁ C ₀₃	(B ₁₁ B ₁₂ B ₁₃)
B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₃	(B ₀₇ B ₀₈ B ₁₆)
B ₁₆ C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₀₉ B ₁₀ B ₁₁ B ₁₃	(B ₀₉ B ₁₁ B ₁₆) (B ₁₀ B ₁₁ B ₁₆)
B ₁₆ C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₁₀ B ₁₁ B ₁₅ B ₁₇	(B ₀₇ B ₁₁ B ₁₈)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₁₁ B ₁₃ B ₁₄ B ₁₅	(B ₁₁ B ₁₃ B ₁₄)
B ₁₇ C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₁₁ B ₁₃ B ₁₅ B ₁₆	(B ₁₁ B ₁₅ B ₁₆) (B ₁₁ B ₁₆ B ₁₇)
B ₁₇ C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₁₁ B ₁₄ B ₁₅ B ₁₇	(B ₁₁ B ₁₄ B ₁₈)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 10

A ₀₁ A ₀₂ B ₁₃ B ₁₄ B ₁₅ B ₁₆ B ₁₇ B ₁₈ C ₀₂ C ₀₃	(A ₀₂ B ₁₃ B ₁₅) (A ₀₂ B ₁₃ B ₁₇) (A ₀₂ B ₁₅ B ₁₆) (A ₀₂ B ₁₅ B ₁₈) (A ₀₂ B ₁₄ B ₁₅) (A ₀₂ B ₁₄ B ₁₇) (A ₀₂ B ₁₆ B ₁₇) (A ₀₂ B ₁₇ B ₁₈)
A ₀₁ A ₀₃ B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₅ B ₀₆ C ₀₁ C ₀₂	(A ₀₃ B ₀₁ B ₀₃) (A ₀₃ B ₀₁ B ₀₄) (A ₀₃ B ₀₃ B ₀₅) (A ₀₃ B ₀₃ B ₀₆) (A ₀₃ B ₀₂ B ₀₃) (A ₀₃ B ₀₂ B ₀₄) (A ₀₃ B ₀₄ B ₀₅) (A ₀₃ B ₀₄ B ₀₆)
* A ₀₁ A ₀₆ B ₀₁ B ₀₂ B ₀₄ B ₁₃ B ₁₄ B ₁₅ C ₀₁ C ₀₂	(A ₀₆ B ₀₂ B ₀₄) (A ₀₆ B ₀₂ B ₁₄) (A ₀₆ B ₀₂ B ₁₅) (A ₀₆ B ₀₄ B ₁₃)
A ₀₁ A ₀₆ B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₁ B ₁₂ C ₀₁ C ₀₃	(A ₀₆ B ₀₉ B ₁₁) (A ₀₆ B ₀₉ B ₁₂) (A ₀₆ B ₁₀ B ₁₁) (A ₀₆ B ₁₀ B ₁₂) (A ₀₆ B ₀₇ B ₀₉) (A ₀₆ B ₀₇ B ₁₀) (A ₀₆ B ₀₈ B ₀₉) (A ₀₆ B ₀₈ B ₁₀)
B ₀₁ B ₀₃ B ₀₄ B ₀₈ B ₁₂ B ₁₄ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₈ B ₁₈) (B ₀₁ B ₁₂ B ₁₄) (B ₀₁ B ₁₂ B ₁₈)
B ₀₁ B ₀₅ B ₀₇ B ₁₁ B ₁₃ B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₁ B ₁₃) (B ₀₅ B ₀₇ B ₁₃) (B ₀₅ B ₁₁ B ₁₃)
B ₀₁ B ₀₅ B ₀₇ B ₁₁ B ₁₄ B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₁ B ₁₄) (B ₀₅ B ₀₇ B ₁₄) (B ₀₅ B ₁₁ B ₁₄)
B ₀₂ B ₀₃ B ₀₄ B ₀₈ B ₁₂ B ₁₄ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₈ B ₁₈) (B ₀₂ B ₁₂ B ₁₄) (B ₀₂ B ₁₂ B ₁₈)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 10 (continued)

$B_{02} B_{06} B_{07} B_{09} B_{10} B_{13}$	$(B_{02} B_{07} B_{16}) (B_{06} B_{07} B_{13})$
$B_{16} C_{01} C_{02} C_{03}$	$(B_{06} B_{07} B_{16})$
$B_{02} B_{06} B_{08} B_{09} B_{10} B_{13}$	$(B_{02} B_{08} B_{16}) (B_{06} B_{08} B_{13})$
$B_{16} C_{01} C_{02} C_{03}$	$(B_{06} B_{08} B_{16})$

Order 11

$A_{01} A_{02} B_{01} B_{05} B_{07} B_{11}$	$(A_{02} B_{05} B_{15}) (A_{02} B_{05} B_{17})$
$B_{15} B_{17} C_{01} C_{02} C_{03}$	$(A_{02} B_{11} B_{15}) (A_{02} B_{11} B_{17})$
$A_{01} A_{03} B_{03} B_{04} B_{08} B_{12}$	$(A_{03} B_{03} B_{12}) (A_{03} B_{03} B_{18})$
$B_{14} B_{18} C_{01} C_{02} C_{03}$	$(A_{03} B_{04} B_{12}) (A_{03} B_{04} B_{18})$
$A_{01} A_{06} B_{02} B_{06} B_{09} B_{10}$	$(A_{06} B_{06} B_{09}) (A_{06} B_{06} B_{10})$
$B_{13} B_{16} C_{01} C_{02} C_{03}$	
$B_{01} B_{03} B_{04} B_{05} B_{08} B_{12}$	$(B_{05} B_{12} B_{14}) (B_{05} B_{08} B_{18})$
$B_{14} B_{18} C_{01} C_{02} C_{03}$	$(B_{05} B_{12} B_{18})$
$B_{01} B_{05} B_{13} B_{14} B_{15} B_{16}$	$(B_{05} B_{13} B_{18}) (B_{05} B_{14} B_{16})$
$B_{17} B_{18} C_{01} C_{02} C_{03}$	$(B_{05} B_{16} B_{18})$
$B_{02} B_{03} B_{04} B_{06} B_{08} B_{12}$	$(B_{06} B_{08} B_{18}) (B_{06} B_{12} B_{14})$
$B_{14} B_{18} C_{01} C_{02} C_{03}$	$(B_{06} B_{12} B_{18})$
$B_{02} B_{06} B_{07} B_{08} B_{09} B_{10}$	$(B_{06} B_{07} B_{12}) (B_{06} B_{08} B_{11})$
$B_{11} B_{12} C_{01} C_{02} C_{03}$	$(B_{06} B_{11} B_{12})$
$B_{02} B_{06} B_{08} B_{09} B_{10} B_{12}$	$(B_{02} B_{12} B_{16}) (B_{06} B_{12} B_{13})$
$B_{13} B_{16} C_{01} C_{02} C_{03}$	$(B_{06} B_{12} B_{16})$

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 11 (continued)

B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₁ B ₁₂	(B ₀₇ B ₁₂ B ₁₆) (B ₀₈ B ₁₁ B ₁₆)
B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₁₁ B ₁₂ B ₁₆)
B ₀₇ B ₁₁ B ₁₃ B ₁₄ B ₁₅ B ₁₆	(B ₁₁ B ₁₃ B ₁₈) (B ₁₁ B ₁₄ B ₁₆)
B ₁₇ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₁₁ B ₁₆ B ₁₈)

Order 12

A ₀₁ A ₀₂ B ₀₁ B ₀₇ B ₁₃	(A ₀₂ B ₀₁ B ₁₃) (A ₀₂ B ₀₁ B ₁₄)
B ₁₄ B ₁₅ B ₁₆ B ₁₇	(A ₀₂ B ₀₁ B ₁₆) (A ₀₂ B ₀₁ B ₁₈)
B ₁₈ C ₀₂ C ₀₃	(A ₀₂ B ₀₇ B ₁₃) (A ₀₂ B ₀₇ B ₁₄)
	(A ₀₂ B ₀₇ B ₁₆) (A ₀₂ B ₀₇ B ₁₈)
A ₀₁ A ₀₃ B ₀₁ B ₀₂ B ₀₃	(A ₀₃ B ₀₁ B ₀₈) (A ₀₃ B ₀₁ B ₁₄)
B ₀₄ B ₀₅ B ₀₆ B ₀₈	(A ₀₃ B ₀₂ B ₀₈) (A ₀₃ B ₀₂ B ₁₄)
B ₁₄ C ₀₁ C ₀₂	(A ₀₃ B ₀₅ B ₀₈) (A ₀₃ B ₀₅ B ₁₄)
	(A ₀₃ B ₀₆ B ₀₈) (A ₀₃ B ₀₆ B ₁₄)
A ₀₁ A ₀₆ B ₀₂ B ₀₇ B ₀₈	(A ₀₆ B ₀₂ B ₀₇) (A ₀₆ B ₀₂ B ₀₈)
B ₀₉ B ₁₀ B ₁₁ B ₁₂	(A ₀₆ B ₀₂ B ₁₁) (A ₀₆ B ₀₂ B ₁₂)
B ₁₃ C ₀₁ C ₀₃	(A ₀₆ B ₀₂ B ₁₂) (A ₀₆ B ₀₇ B ₁₃)
	(A ₀₆ B ₀₈ B ₁₃) (A ₀₆ B ₁₁ B ₁₃)
	(A ₀₆ B ₁₂ B ₁₃)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 15

A ₀₁ A ₀₂ B ₀₁ B ₀₅ B ₀₇ B ₁₁	(A ₀₂ B ₀₅ B ₁₃) (A ₀₂ B ₀₅ B ₁₄)
B ₁₃ B ₁₄ B ₁₅ B ₁₆ B ₁₇	(A ₀₂ B ₀₅ B ₁₆) (A ₀₂ B ₀₅ B ₁₈)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₁₁ B ₁₃) (A ₀₂ B ₁₁ B ₁₄)
	(A ₀₂ B ₁₁ B ₁₆) (A ₀₂ B ₁₁ B ₁₈)
A ₀₁ A ₀₃ B ₀₁ B ₀₂ B ₀₃ B ₀₄	(A ₀₃ B ₀₁ B ₁₂) (A ₀₃ B ₀₁ B ₁₈)
B ₀₅ B ₀₆ B ₀₈ B ₁₂ B ₁₄	(A ₀₃ B ₀₂ B ₁₂) (A ₀₃ B ₀₂ B ₁₈)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₃ B ₀₅ B ₁₂) (A ₀₃ B ₀₅ B ₁₈)
	(A ₀₃ B ₀₆ B ₁₂) (A ₀₃ B ₀₆ B ₁₈)
A ₀₁ A ₀₆ B ₀₂ B ₀₆ B ₀₇ B ₀₈	(A ₀₆ B ₀₆ B ₀₇) (A ₀₆ B ₀₆ B ₀₈)
B ₀₉ B ₁₀ B ₁₁ B ₁₂ B ₁₃	(A ₀₆ B ₀₆ B ₁₁) (A ₀₆ B ₀₆ B ₁₂)
B ₁₆ C ₀₁ C ₀₂ C ₀₃	
* B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₅ B ₀₆	(B ₀₁ B ₀₃ B ₁₀) (B ₀₁ B ₀₄ B ₀₉)
B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₁	(B ₀₁ B ₀₄ B ₁₀) (B ₀₁ B ₀₈ B ₁₀)
B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₉ B ₁₂) (B ₀₁ B ₁₀ B ₁₂)
	(B ₀₂ B ₀₃ B ₁₁) (B ₀₂ B ₀₄ B ₀₇)
	(B ₀₂ B ₀₄ B ₁₁) (B ₀₃ B ₀₅ B ₀₉)
	(B ₀₃ B ₀₅ B ₁₀) (B ₀₃ B ₀₆ B ₀₇)
	(B ₀₃ B ₀₆ B ₁₁) (B ₀₃ B ₀₇ B ₁₀)
	(B ₀₃ B ₀₉ B ₁₁) (B ₀₃ B ₁₀ B ₁₁)
	(B ₀₄ B ₀₅ B ₀₉) (B ₀₄ B ₀₅ B ₁₀)
	(B ₀₄ B ₀₆ B ₀₇) (B ₀₄ B ₀₆ B ₁₁)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 15 (continued)

	(B ₀₄ B ₀₇ B ₀₉) (B ₀₄ B ₀₇ B ₁₀)
	(B ₀₄ B ₀₉ B ₁₁) (B ₀₄ B ₁₀ B ₁₁)
	(B ₀₅ B ₀₈ B ₀₉) (B ₀₅ B ₀₈ B ₁₀)
	(B ₀₅ B ₀₉ B ₁₂) (B ₀₅ B ₁₀ B ₁₂)
* B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₅ B ₀₆	(B ₀₁ B ₀₃ B ₁₃) (B ₀₁ B ₀₃ B ₁₆)
B ₁₃ B ₁₄ B ₁₅ B ₁₆ B ₁₇	(B ₀₂ B ₀₃ B ₁₅) (B ₀₂ B ₀₃ B ₁₇)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₄ B ₁₇) (B ₀₂ B ₁₄ B ₁₇)
	(B ₀₂ B ₁₅ B ₁₈) (B ₀₂ B ₁₇ B ₁₈)
	(B ₀₃ B ₀₅ B ₁₃) (B ₀₃ B ₀₅ B ₁₆)
	(B ₀₃ B ₀₆ B ₁₅) (B ₀₃ B ₀₆ B ₁₇)
	(B ₀₃ B ₁₃ B ₁₅) (B ₀₃ B ₁₃ B ₁₇)
	(B ₀₃ B ₁₅ B ₁₆) (B ₀₃ B ₁₆ B ₁₇)
	(B ₀₄ B ₀₅ B ₁₃) (B ₀₄ B ₀₅ B ₁₆)
	(B ₀₄ B ₀₆ B ₁₅) (B ₀₄ B ₀₆ B ₁₇)
	(B ₀₄ B ₁₃ B ₁₇) (B ₀₄ B ₁₅ B ₁₆)
	(B ₀₄ B ₁₆ B ₁₇) (B ₀₆ B ₁₄ B ₁₅)
	(B ₀₆ B ₁₄ B ₁₇) (B ₀₆ B ₁₅ B ₁₈)
	(B ₀₆ B ₁₇ B ₁₈)
* B ₀₁ B ₀₂ B ₀₅ B ₀₆ B ₀₇ B ₀₉	(B ₀₁ B ₀₉ B ₁₆) (B ₀₁ B ₀₉ B ₁₇)
B ₁₀ B ₁₁ B ₁₃ B ₁₅ B ₁₆	(B ₀₁ B ₁₀ B ₁₃) (B ₀₁ B ₁₀ B ₁₅)
B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₀ B ₁₆) (B ₀₁ B ₁₀ B ₁₇)
	(B ₀₂ B ₀₇ B ₁₇) (B ₀₂ B ₀₉ B ₁₇)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 15 (continued)

	(B ₀₂ B ₁₀ B ₁₅) (B ₀₂ B ₁₀ B ₁₇)
	(B ₀₂ B ₁₁ B ₁₅) (B ₀₂ B ₁₁ B ₁₇)
	(B ₀₅ B ₀₉ B ₁₃) (B ₀₅ B ₀₉ B ₁₅)
	(B ₀₅ B ₀₉ B ₁₆) (B ₀₅ B ₀₉ B ₁₇)
	(B ₀₅ B ₁₀ B ₁₃) (B ₀₅ B ₁₀ B ₁₅)
	(B ₀₅ B ₁₀ B ₁₆) (B ₀₅ B ₁₀ B ₁₇)
	(B ₀₆ B ₀₇ B ₁₅) (B ₀₆ B ₀₇ B ₁₇)
	(B ₀₆ B ₀₉ B ₁₅) (B ₀₆ B ₀₉ B ₁₇)
	(B ₀₆ B ₁₀ B ₁₅) (B ₀₆ B ₁₀ B ₁₇)
	(B ₀₆ B ₁₁ B ₁₅) (B ₀₆ B ₁₁ B ₁₇)
* B ₀₁ B ₀₃ B ₀₄ B ₀₅ B ₀₇ B ₀₈	(B ₀₁ B ₀₈ B ₁₅) (B ₀₁ B ₁₂ B ₁₅)
B ₁₁ B ₁₂ B ₁₄ B ₁₅ B ₁₇	(B ₀₁ B ₁₂ B ₁₇) (B ₀₃ B ₀₇ B ₁₅)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₀₇ B ₁₈) (B ₀₃ B ₀₈ B ₁₅)
	(B ₀₃ B ₁₁ B ₁₄) (B ₀₃ B ₁₁ B ₁₅)
	(B ₀₃ B ₁₁ B ₁₇) (B ₀₃ B ₁₁ B ₁₈)
	(B ₀₃ B ₁₂ B ₁₅) (B ₀₃ B ₁₂ B ₁₇)
	(B ₀₄ B ₀₇ B ₁₄) (B ₀₄ B ₀₇ B ₁₅)
	(B ₀₄ B ₀₇ B ₁₇) (B ₀₄ B ₀₇ B ₁₈)
	(B ₀₄ B ₀₈ B ₁₅) (B ₀₄ B ₀₈ B ₁₇)
	(B ₀₄ B ₁₁ B ₁₄) (B ₀₄ B ₁₁ B ₁₅)
	(B ₀₄ B ₁₁ B ₁₇) (B ₀₄ B ₁₁ B ₁₈)
	(B ₀₄ B ₁₂ B ₁₅) (B ₀₄ B ₁₂ B ₁₇)
	(B ₀₅ B ₀₈ B ₁₅) (B ₀₅ B ₀₈ B ₁₇)
	(B ₀₅ B ₁₂ B ₁₅) (B ₀₅ B ₁₂ B ₁₇)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 15 (continued)

* B ₀₂ B ₀₃ B ₀₄ B ₀₆ B ₀₈ B ₀₉	(B ₀₂ B ₀₉ B ₁₄) (B ₀₂ B ₀₉ B ₁₈)
B ₁₀ B ₁₂ B ₁₃ B ₁₄ B ₁₆	(B ₀₂ B ₁₀ B ₁₈) (B ₀₃ B ₀₈ B ₁₃)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₃ B ₀₈ B ₁₆) (B ₀₃ B ₀₉ B ₁₄)
	(B ₀₃ B ₀₉ B ₁₆) (B ₀₃ B ₀₉ B ₁₈)
	(B ₀₃ B ₁₀ B ₁₃) (B ₀₃ B ₁₀ B ₁₄)
	(B ₀₃ B ₁₀ B ₁₆) (B ₀₃ B ₁₀ B ₁₈)
	(B ₀₃ B ₁₂ B ₁₃) (B ₀₃ B ₁₂ B ₁₆)
	(B ₀₄ B ₀₈ B ₁₆) (B ₀₄ B ₀₉ B ₁₃)
	(B ₀₄ B ₀₉ B ₁₄) (B ₀₄ B ₀₉ B ₁₆)
	(B ₀₄ B ₀₉ B ₁₈) (B ₀₄ B ₁₀ B ₁₆)
	(B ₀₄ B ₁₀ B ₁₈) (B ₀₄ B ₁₂ B ₁₃)
	(B ₀₄ B ₁₂ B ₁₆) (B ₀₆ B ₀₉ B ₁₄)
	(B ₀₆ B ₀₉ B ₁₈) (B ₀₆ B ₁₀ B ₁₄)
	(B ₀₆ B ₁₀ B ₁₈)
* B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₁ B ₁₂	(B ₀₇ B ₀₉ B ₁₄) (B ₀₇ B ₀₉ B ₁₈)
B ₁₃ B ₁₄ B ₁₅ B ₁₆ B ₁₇	(B ₀₇ B ₁₀ B ₁₈) (B ₀₈ B ₀₉ B ₁₅)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₈ B ₀₉ B ₁₇) (B ₀₈ B ₁₀ B ₁₅)
	(B ₀₈ B ₁₃ B ₁₅) (B ₀₈ B ₁₅ B ₁₆)
	(B ₀₈ B ₁₆ B ₁₇) (B ₀₉ B ₁₁ B ₁₄)
	(B ₀₉ B ₁₁ B ₁₈) (B ₀₉ B ₁₂ B ₁₅)
	(B ₀₉ B ₁₂ B ₁₇) (B ₀₉ B ₁₄ B ₁₅)
	(B ₀₉ B ₁₄ B ₁₇) (B ₀₉ B ₁₅ B ₁₈)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 15 (continued)

$(B_{09} B_{17} B_{18}) (B_{10} B_{11} B_{14})$
 $(B_{10} B_{11} B_{18}) (B_{10} B_{12} B_{15})$
 $(B_{10} B_{12} B_{17}) (B_{10} B_{14} B_{15})$
 $(B_{10} B_{15} B_{18}) (B_{10} B_{17} B_{18})$
 $(B_{12} B_{13} B_{15}) (B_{12} B_{13} B_{17})$
 $(B_{12} B_{15} B_{16}) (B_{12} B_{16} B_{17})$

Order 17

* A ₀₁ A ₀₂ B ₀₁ B ₀₂ B ₀₃	(A ₀₂ B ₀₁ B ₀₄) (A ₀₂ B ₀₁ B ₀₆)
B ₀₄ B ₀₅ B ₀₆ B ₀₇	(A ₀₂ B ₀₁ B ₁₀) (A ₀₂ B ₀₁ B ₁₂)
B ₀₈ B ₀₉ B ₁₀ B ₁₁	(A ₀₂ B ₀₂ B ₀₅) (A ₀₂ B ₀₂ B ₁₁)
B ₁₂ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₀₃ B ₀₅) (A ₀₂ B ₀₃ B ₁₁)
	(A ₀₂ B ₀₄ B ₀₅) (A ₀₂ B ₀₄ B ₀₇)
	(A ₀₂ B ₀₄ B ₁₁) (A ₀₂ B ₀₅ B ₀₆)
	(A ₀₂ B ₀₅ B ₀₈) (A ₀₂ B ₀₅ B ₀₉)
	(A ₀₂ B ₀₅ B ₁₀) (A ₀₂ B ₀₅ B ₁₂)
	(A ₀₂ B ₀₉ B ₁₁) (A ₀₂ B ₀₆ B ₁₁)
	(A ₀₂ B ₀₇ B ₁₀) (A ₀₂ B ₀₈ B ₁₁)
	(A ₀₂ B ₀₉ B ₁₁) (A ₀₂ B ₁₀ B ₁₁)
	(A ₀₂ B ₁₁ B ₁₂)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 17 (continued)

* A ₀₁ A ₀₂ B ₀₂ B ₀₃ B ₀₄	(A ₀₂ B ₀₂ B ₁₃) (A ₀₂ B ₀₂ B ₁₄)
B ₀₆ B ₀₈ B ₀₉ B ₁₀	(A ₀₂ B ₀₂ B ₁₆) (A ₀₂ B ₀₂ B ₁₈)
B ₁₂ B ₁₃ B ₁₄ B ₁₆	(A ₀₂ B ₀₃ B ₁₃) (A ₀₂ B ₀₃ B ₁₄)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₀₃ B ₁₆) (A ₀₂ B ₀₃ B ₁₈)
	(A ₀₂ B ₀₄ B ₁₃) (A ₀₂ B ₀₄ B ₁₄)
	(A ₀₂ B ₀₄ B ₁₆) (A ₀₂ B ₀₄ B ₁₈)
	(A ₀₂ B ₀₆ B ₁₃) (A ₀₂ B ₀₆ B ₁₄)
	(A ₀₂ B ₀₆ B ₁₆) (A ₀₂ B ₀₆ B ₁₈)
	(A ₀₂ B ₀₈ B ₁₃) (A ₀₂ B ₀₈ B ₁₄)
	(A ₀₂ B ₀₈ B ₁₆) (A ₀₂ B ₀₈ B ₁₈)
	(A ₀₂ B ₀₉ B ₁₃) (A ₀₂ B ₀₉ B ₁₄)
	(A ₀₂ B ₀₉ B ₁₆) (A ₀₂ B ₀₉ B ₁₈)
	(A ₀₂ B ₁₀ B ₁₃) (A ₀₂ B ₁₀ B ₁₄)
	(A ₀₂ B ₁₀ B ₁₆) (A ₀₂ B ₁₀ B ₁₈)
	(A ₀₂ B ₁₂ B ₁₃) (A ₀₂ B ₁₂ B ₁₄)
	(A ₀₂ B ₁₂ B ₁₆) (A ₀₂ B ₁₂ B ₁₈)
* A ₀₁ A ₀₃ B ₀₁ B ₀₂ B ₀₅	(A ₀₃ B ₀₁ B ₀₇) (A ₀₃ B ₀₁ B ₀₉)
B ₀₆ B ₀₇ B ₀₉ B ₁₀	(A ₀₃ B ₀₁ B ₁₀) (A ₀₃ B ₀₁ B ₁₁)
B ₁₁ B ₁₃ B ₁₅ B ₁₆	(A ₀₃ B ₀₁ B ₁₃) (A ₀₃ B ₀₁ B ₁₅)
B ₁₇ C ₀₁ C ₀₂ C ₀₃	(A ₀₃ B ₀₁ B ₁₆) (A ₀₃ B ₀₁ B ₁₇)
	(A ₀₃ B ₀₂ B ₀₇) (A ₀₃ B ₀₂ B ₀₉)
	(A ₀₃ B ₀₂ B ₁₀) (A ₀₃ B ₀₂ B ₁₁)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 17 (continued)

	(A ₀₃ B ₀₂ B ₁₃) (A ₀₃ B ₀₂ B ₁₅)
	(A ₀₃ B ₀₂ B ₁₆) (A ₀₃ B ₀₂ B ₁₇)
	(A ₀₃ B ₀₅ B ₀₇) (A ₀₃ B ₀₅ B ₀₉)
	(A ₀₃ B ₀₅ B ₁₀) (A ₀₃ B ₀₅ B ₁₁)
	(A ₀₃ B ₀₅ B ₁₃) (A ₀₃ B ₀₅ B ₁₅)
	(A ₀₃ B ₀₅ B ₁₆) (A ₀₃ B ₀₅ B ₁₇)
	(A ₀₃ B ₀₆ B ₀₇) (A ₀₃ B ₀₆ B ₀₉)
	(A ₀₃ B ₀₆ B ₁₀) (A ₀₃ B ₀₆ B ₁₁)
	(A ₀₃ B ₀₆ B ₁₃) (A ₀₃ B ₀₆ B ₁₅)
	(A ₀₃ B ₀₆ B ₁₆) (A ₀₃ B ₀₆ B ₁₇)
* A ₀₁ A ₀₃ B ₀₇ B ₀₈ B ₀₉	(A ₀₃ B ₀₇ B ₁₂) (A ₀₃ B ₀₇ B ₁₈)
B ₁₀ B ₁₁ B ₁₂ B ₁₃	(A ₀₃ B ₀₈ B ₀₉) (A ₀₃ B ₀₈ B ₁₁)
B ₁₄ B ₁₅ B ₁₆ B ₁₇	(A ₀₃ B ₀₈ B ₁₅) (A ₀₃ B ₀₈ B ₁₆)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₃ B ₀₉ B ₁₂) (A ₀₃ B ₀₉ B ₁₄)
	(A ₀₃ B ₀₉ B ₁₈) (A ₀₃ B ₁₀ B ₁₂)
	(A ₀₃ B ₁₀ B ₁₈) (A ₀₃ B ₁₁ B ₁₂)
	(A ₀₃ B ₁₁ B ₁₄) (A ₀₃ B ₁₁ B ₁₈)
	(A ₀₃ B ₁₂ B ₁₃) (A ₀₃ B ₁₂ B ₁₅)
	(A ₀₃ B ₁₂ B ₁₆) (A ₀₃ B ₁₂ B ₁₇)
	(A ₀₃ B ₁₃ B ₁₈) (A ₀₃ B ₁₄ B ₁₅)
	(A ₀₃ B ₁₄ B ₁₆) (A ₀₃ B ₁₅ B ₁₈)
	(A ₀₃ B ₁₆ B ₁₈) (A ₀₃ B ₁₇ B ₁₈)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 17 (continued)

* A ₀₁ A ₀₆ B ₀₁ B ₀₃ B ₀₄	(A ₀₆ B ₀₁ B ₀₇) (A ₀₆ B ₀₁ B ₀₈)
B ₀₅ B ₀₇ B ₀₈ B ₁₁	(A ₀₆ B ₀₁ B ₁₁) (A ₀₆ B ₀₁ B ₁₂)
B ₁₂ B ₁₄ B ₁₅ B ₁₇	(A ₀₆ B ₀₃ B ₀₇) (A ₀₆ B ₀₃ B ₀₈)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₆ B ₀₃ B ₁₁) (A ₀₆ B ₀₃ B ₁₂)
	(A ₀₆ B ₀₄ B ₀₇) (A ₀₆ B ₀₄ B ₀₈)
	(A ₀₆ B ₀₄ B ₁₁) (A ₀₆ B ₀₄ B ₁₂)
	(A ₀₆ B ₀₅ B ₀₇) (A ₀₆ B ₀₅ B ₀₈)
	(A ₀₆ B ₀₅ B ₁₁) (A ₀₆ B ₀₅ B ₁₂)
	(A ₀₆ B ₀₇ B ₁₄) (A ₀₆ B ₀₇ B ₁₅)
	(A ₀₆ B ₀₇ B ₁₇) (A ₀₆ B ₀₇ B ₁₈)
	(A ₀₆ B ₀₈ B ₁₄) (A ₀₆ B ₀₈ B ₁₅)
	(A ₀₆ B ₀₈ B ₁₇) (A ₀₆ B ₀₈ B ₁₈)
	(A ₀₆ B ₁₁ B ₁₄) (A ₀₆ B ₁₁ B ₁₅)
	(A ₀₆ B ₁₁ B ₁₇) (A ₀₆ B ₁₁ B ₁₈)
	(A ₀₆ B ₁₂ B ₁₄) (A ₀₆ B ₁₂ B ₁₅)
	(A ₀₆ B ₁₂ B ₁₇) (A ₀₆ B ₁₂ B ₁₈)

Order 21

* B ₁ C _j (i = 01, ..., 18;	(B ₀₁ B ₀₈ B ₁₃) (B ₀₁ B ₀₈ B ₁₆)
j = 01, 02, 03)	(B ₀₁ B ₀₉ B ₁₄) (B ₀₁ B ₀₉ B ₁₈)
	(B ₀₁ B ₁₀ B ₁₄) (B ₀₁ B ₁₀ B ₁₈)
	(B ₀₁ B ₁₂ B ₁₃) (B ₀₁ B ₁₂ B ₁₆)

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 21 (continued)

$(B_{02} B_{07} B_{14}) (B_{02} B_{07} B_{18})$
 $(B_{02} B_{08} B_{15}) (B_{02} B_{08} B_{17})$
 $(B_{02} B_{11} B_{14}) (B_{02} B_{11} B_{18})$
 $(B_{02} B_{12} B_{15}) (B_{02} B_{12} B_{17})$
 $(B_{03} B_{07} B_{13}) (B_{03} B_{07} B_{16})$
 $(B_{03} B_{09} B_{15}) (B_{03} B_{09} B_{17})$
 $(B_{03} B_{10} B_{15}) (B_{03} B_{10} B_{17})$
 $(B_{03} B_{11} B_{13}) (B_{03} B_{11} B_{16})$
 $(B_{04} B_{07} B_{13}) (B_{04} B_{07} B_{16})$
 $(B_{04} B_{09} B_{15}) (B_{04} B_{09} B_{17})$
 $(B_{04} B_{10} B_{15}) (B_{04} B_{10} B_{17})$
 $(B_{04} B_{11} B_{13}) (B_{04} B_{11} B_{16})$
 $(B_{05} B_{08} B_{13}) (B_{05} B_{08} B_{16})$
 $(B_{05} B_{09} B_{14}) (B_{05} B_{09} B_{18})$
 $(B_{05} B_{10} B_{14}) (B_{05} B_{10} B_{18})$
 $(B_{05} B_{12} B_{13}) (B_{05} B_{12} B_{16})$
 $(B_{06} B_{07} B_{14}) (B_{06} B_{07} B_{18})$
 $(B_{06} B_{08} B_{15}) (B_{06} B_{08} B_{17})$
 $(B_{06} B_{11} B_{14}) (B_{06} B_{11} B_{18})$
 $(B_{06} B_{12} B_{15}) (B_{06} B_{12} B_{17})$

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 23

* $A_{01} A_{02} B_i C_j$ ($i = 01, \dots, 18;$ $j = 01, 02, 03$)	$(A_{02} B_{02} B_{15}) (A_{02} B_{02} B_{17})$ $(A_{02} B_{03} B_{15}) (A_{02} B_{03} B_{17})$ $(A_{02} B_{04} B_{15}) (A_{02} B_{04} B_{17})$ $(A_{02} B_{06} B_{15}) (A_{02} B_{06} B_{17})$ $(A_{02} B_{08} B_{15}) (A_{02} B_{08} B_{17})$ $(A_{02} B_{09} B_{15}) (A_{02} B_{09} B_{17})$ $(A_{02} B_{10} B_{15}) (A_{02} B_{10} B_{17})$ $(A_{02} B_{12} B_{15}) (A_{02} B_{12} B_{17})$
* $A_{01} A_{03} B_i C_j$ ($i = 01, \dots, 18;$ $j = 01, 02, 03$)	$(A_{03} B_{03} B_{07}) (A_{03} B_{03} B_{09})$ $(A_{03} B_{03} B_{10}) (A_{03} B_{03} B_{11})$ $(A_{03} B_{03} B_{13}) (A_{03} B_{03} B_{15})$ $(A_{03} B_{03} B_{16}) (A_{03} B_{03} B_{17})$ $(A_{03} B_{04} B_{07}) (A_{03} B_{04} B_{09})$ $(A_{03} B_{04} B_{10}) (A_{03} B_{04} B_{11})$ $(A_{03} B_{04} B_{13}) (A_{03} B_{04} B_{15})$ $(A_{03} B_{04} B_{16}) (A_{03} B_{04} B_{17})$
* $A_{01} A_{06} B_i C_j$ ($i = 01, \dots, 18;$ $j = 01, 02, 03$)	$(A_{06} B_{01} B_{09}) (A_{06} B_{01} B_{10})$ $(A_{06} B_{03} B_{09}) (A_{06} B_{03} B_{10})$ $(A_{06} B_{04} B_{09}) (A_{06} B_{04} B_{10})$ $(A_{06} B_{05} B_{09}) (A_{06} B_{05} B_{10})$ $(A_{06} B_{07} B_{16}) (A_{06} B_{08} B_{16})$ $(A_{06} B_{09} B_{14}) (A_{06} B_{09} B_{15})$

SUBSEMIGROUPS OF T_3 GENERATED BY THREE ELEMENTS (continued)Order 23 (continued) $(A_{06} B_{09} B_{16}) (A_{06} B_{09} B_{17})$ $(A_{06} B_{09} B_{18}) (A_{06} B_{10} B_{14})$ $(A_{06} B_{10} B_{15}) (A_{06} B_{10} B_{16})$ $(A_{06} B_{10} B_{17}) (A_{06} B_{10} B_{18})$ $(A_{06} B_{11} B_{16}) (A_{06} B_{12} B_{16})$ Order 27 $A_i B_j C_k$ $(i = 01, \dots, 06;$ $j = 01, \dots, 18$ $k = 01, 02, 03)$ $(A_{02} A_{03} B_i), i = 01, \dots, 18$ $(A_{02} A_{04} B_i), i = 01, \dots, 18$ $(A_{02} A_{05} B_i), i = 01, \dots, 18$ $(A_{02} A_{06} B_i), i = 01, \dots, 18$ $(A_{03} A_{04} B_i), i = 01, \dots, 18$ $(A_{03} A_{05} B_i), i = 01, \dots, 18$ $(A_{03} A_{06} B_i), i = 01, \dots, 18$ $(A_{04} A_{06} B_i), i = 01, \dots, 18$

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTSOrder 4

- * B₀₁ B₀₂ C₀₁ C₀₂
- * B₀₂ B₁₃ C₀₂ C₀₃
- * B₀₂ C₀₁ C₀₂ C₀₃
- * B₀₇ B₀₈ C₀₁ C₀₃
- * B₁₃ B₁₄ C₀₂ C₀₃

Order 5

- | | |
|---|---|
| B ₀₁ B ₀₂ B ₀₃ C ₀₁ C ₀₂ | (B ₀₁ B ₀₂ B ₀₃ C ₀₂) |
| B ₀₁ B ₀₂ B ₀₄ C ₀₁ C ₀₂ | (B ₀₁ B ₀₂ B ₀₄ C ₀₂) |
| * B ₀₁ B ₀₂ C ₀₁ C ₀₂ C ₀₃ | (B ₀₁ B ₀₂ C ₀₁ C ₀₃) |
| B ₀₁ B ₀₄ B ₁₄ C ₀₁ C ₀₂ | (B ₀₁ B ₀₄ B ₁₄ C ₀₁) |
| * B ₀₁ B ₀₇ C ₀₁ C ₀₂ C ₀₃ | (B ₀₁ B ₀₇ C ₀₁ C ₀₂) (B ₀₁ B ₀₇ C ₀₁ C ₀₃) |
| B ₀₁ B ₁₄ B ₁₅ C ₀₂ C ₀₃ | (B ₀₁ B ₁₄ B ₁₅ C ₀₃) |
| * B ₀₁ B ₁₄ C ₀₁ C ₀₂ C ₀₃ | (B ₀₁ B ₁₄ C ₀₁ C ₀₃) |
| B ₀₂ B ₀₃ B ₀₈ C ₀₁ C ₀₂ | (B ₀₂ B ₀₃ B ₀₈ C ₀₂) |
| * B ₀₂ B ₀₆ C ₀₁ C ₀₂ C ₀₃ | (B ₀₂ B ₀₆ C ₀₁ C ₀₃) (B ₀₂ B ₀₆ C ₀₂ C ₀₃) |
| B ₀₂ B ₀₈ B ₀₉ C ₀₁ C ₀₃ | (B ₀₂ B ₀₈ B ₀₉ C ₀₃) |
| * B ₀₂ B ₁₃ C ₀₁ C ₀₂ C ₀₃ | (B ₀₂ B ₁₃ C ₀₁ C ₀₂) |
| * B ₀₂ B ₀₈ C ₀₁ C ₀₂ C ₀₃ | (B ₀₂ B ₀₈ C ₀₂ C ₀₃) |
| B ₀₇ B ₀₈ B ₀₉ C ₀₁ C ₀₃ | (B ₀₇ B ₀₈ B ₀₉ C ₀₃) |
| B ₀₇ B ₀₈ B ₁₀ C ₀₁ C ₀₃ | (B ₀₇ B ₀₈ B ₁₀ C ₀₁) |
| B ₀₇ B ₁₀ B ₁₃ C ₀₁ C ₀₃ | (B ₀₇ B ₁₀ B ₁₃ C ₀₁) |

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 5 (continued)

B07 B13 B17 C02 C03	(B07 B13 B17 C02)
* B07 B13 C01 C02 C03	(B07 B13 C01 C02)
B08 B10 B13 C01 C03	(B08 B10 B13 C01)
* B08 B14 C01 C02 C03	(B08 B14 C01 C03) (B08 B14 C02 C03)
B13 B14 B15 C02 C03	(B13 B14 B15 C03)
B13 B14 B17 C02 C03	(B13 B14 B17 C02)

Order 6

B01 B02 B03 B04 C01 C02	(B01 B02 B03 B04)
B01 B02 B03 B08 C01 C02	(B01 B02 B08 C02)
B01 B02 B03 C01 C02 C03	(B01 B02 B03 C03)
B01 B02 B04 B14 C01 C02	(B01 B02 B14 C01)
B01 B02 B04 C01 C02 C03	(B01 B02 B04 C03)
* B01 B02 B05 C01 C02 C03	(B01 B02 B05 C03)
B01 B03 B04 B14 C01 C02	(B01 B03 B04 B14)
B01 B03 B04 C01 C02 C03	(B01 B03 B04 C03)
B01 B03 B08 C01 C02 C03	(B01 B03 C02 C03)
B01 B03 B14 C01 C02 C03	(B01 B03 B14 C03)
B01 B04 B14 C01 C02 C03	(B01 B04 B14 C03)
* B01 B05 B14 C01 C02 C03	(B01 B05 B14 C03)
B01 B07 B15 B17 C02 C03	(B01 B07 B15 B17)
B01 B07 B15 C01 C02 C03	(B01 B07 B15 C01)
B01 B07 B17 C01 C02 C03	(B01 B07 B17 C01)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 6 (continued)

B01 B13 B14 B15 C02 C03	(B01 B13 B14 C03)
B01 B14 B15 B17 C02 C03	(B01 B14 B15 B17)
B01 B14 B15 C01 C02 C03	(B01 B14 B15 C01)
B01 B14 B17 C01 C02 C03	(B01 B14 B17 C01)
B01 B15 B17 C01 C02 C03	(B01 B15 B17 C01)
B02 B03 B04 C01 C02 C03	(B02 B03 B04 C03)
B02 B03 B08 C01 C02 C03	(B02 B03 B08 C03)
B02 B04 B08 C01 C02 C03	(B02 B04 B08 C03)
B02 B07 B08 B09 C01 C03	(B02 B07 B08 C03)
B02 B07 B09 C01 C02 C03	(B02 B07 C02 C03)
B02 B08 B09 C01 C02 C03	(B02 B08 B09 C02)
B02 B08 B09 B10 C01 C03	(B02 B08 B10 C02)
B02 B08 B10 C01 C02 C03	(B02 B08 B09 B10)
B02 B09 B10 C01 C02 C03	(B02 B09 B10 C03) (B02 B09 B10 C02)
B02 B09 B10 B13 C01 C03	(B02 B09 B10 B13)
B02 B09 B13 C01 C02 C03	(B02 B09 B13 C02)
B02 B10 B13 C01 C02 C03	(B02 B10 B13 C02)
B03 B04 B08 B14 C01 C02	(B03 B04 B08 B14)
B03 B04 B08 C01 C02 C03	(B03 B04 B08 C03)
B03 B04 B14 C01 C02 C03	(B03 B04 B14 C03)
B03 B08 B14 C01 C02 C03	(B03 B08 B14 C03)
B04 B08 B14 C01 C02 C03	(B04 B08 B14 C03)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 6 (continued)

B07 B08 B09 B10 C01 C03	(B07 B08 B09 B10)
B07 B08 B09 C01 C02 C03	(B07 B08 B09 C02)
B07 B08 B10 B13 C01 C03	(B07 B08 B13 C01)
B07 B08 B10 C01 C02 C03	(B07 B08 B10 C02)
B07 B09 B10 B13 C01 C03	(B07 B09 B10 B13)
B07 B09 B10 C01 C02 C03	(B07 B09 B10 C02)
B07 B09 B13 C01 C02 C03	(B07 B09 B13 C02)
B07 B10 B13 C01 C02 C03	(B07 B10 B13 C02)
B07 B13 B14 B17 C02 C03	(B07 B13 B14 C02)
B07 B13 B15 B17 C02 C03	(B07 B13 B15 B17)
B07 B13 B15 C01 C02 C03	(B07 B13 B15 C01)
B07 B13 B17 C01 C02 C03	(B07 B13 B17 C01)
B07 B15 B17 C01 C02 C03	(B07 B15 B17 C01)
B08 B09 B10 C01 C02 C03	(B08 B09 B10 C02)
B09 B10 B13 C01 C02 C03	(B09 B10 B13 C02)
B13 B14 B15 B17 C02 C03	(B13 B14 B15 B17)
B13 B14 B15 C01 C02 C03	(B13 B14 B15 C01)
B13 B14 B17 C01 C02 C03	(B13 B14 B17 C01)
B14 B15 B17 C01 C02 C03	(B14 B15 B17 C01)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 7

* A ₀₁ A ₀₂ B ₀₁ B ₀₇ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₀₁ C ₀₁ C ₀₂)
	(A ₀₂ B ₀₁ C ₀₁ C ₀₃)
	(A ₀₂ B ₀₇ C ₀₁ C ₀₂)
	(A ₀₂ B ₀₇ C ₀₁ C ₀₃)
* A ₀₁ A ₀₃ B ₀₈ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(A ₀₃ B ₀₈ C ₀₁ C ₀₃)
	(A ₀₃ B ₀₈ C ₀₂ C ₀₃)
	(A ₀₃ B ₁₄ C ₀₂ C ₀₃)
	(A ₀₃ B ₁₄ C ₀₁ C ₀₃)
* A ₀₁ A ₀₆ B ₀₂ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(A ₀₆ B ₀₂ C ₀₁ C ₀₂)
	(A ₀₆ B ₀₂ C ₀₂ C ₀₃)
B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₈ C ₀₁ C ₀₂	(B ₀₁ B ₀₂ B ₀₄ B ₀₈)
B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₁₄ C ₀₁ C ₀₂	(B ₀₁ B ₀₂ B ₀₃ B ₁₄)
B ₀₁ B ₀₂ B ₀₃ B ₀₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₂ B ₀₈ C ₀₃)
B ₀₁ B ₀₂ B ₀₄ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₂ B ₁₄ C ₀₃)
B ₀₁ B ₀₃ B ₀₄ B ₀₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₄ B ₀₈ C ₀₃)
B ₀₁ B ₀₃ B ₀₄ B ₀₈ B ₁₄ C ₀₁ C ₀₂	(B ₀₁ B ₀₄ B ₀₈ B ₁₄)
B ₀₁ B ₀₃ B ₀₈ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₈ B ₁₄ C ₀₃)
B ₀₁ B ₀₇ B ₁₃ B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₇ B ₁₃ C ₀₁)
B ₀₁ B ₀₇ B ₁₄ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₀₁ B ₀₇ B ₁₄ B ₁₅)
B ₀₁ B ₀₇ B ₁₄ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₇ B ₁₄ C ₀₁)
B ₀₁ B ₁₃ B ₁₄ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₀₁ B ₁₃ B ₁₄ B ₁₇)
B ₀₁ B ₁₃ B ₁₄ B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₃ B ₁₄ C ₀₁)
B ₀₁ B ₁₃ B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₃ B ₁₇ C ₀₁)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 7 (continued)

B ₀₂ B ₀₃ B ₀₄ B ₀₈ B ₁₄ C ₀₁ C ₀₂	(B ₀₂ B ₀₃ B ₀₈ B ₁₄)
B ₀₂ B ₀₃ B ₀₄ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₃ B ₁₄ C ₀₃)
B ₀₂ B ₀₄ B ₀₈ B ₁₄ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₈ B ₁₄ B ₀₃)
B ₀₂ B ₀₇ B ₀₈ B ₀₉ B ₁₀ C ₀₁ C ₀₃	(B ₀₂ B ₀₇ B ₀₈ B ₁₀)
B ₀₂ B ₀₇ B ₀₈ B ₀₉ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₇ B ₀₈ C ₀₂)
B ₀₂ B ₀₇ B ₀₉ B ₁₀ B ₁₃ C ₀₁ C ₀₃	(B ₀₂ B ₀₇ B ₁₀ B ₁₃)
B ₀₂ B ₀₇ B ₀₉ B ₁₀ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₇ B ₁₀ C ₀₂)
B ₀₂ B ₀₇ B ₀₉ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₇ B ₁₃ C ₀₂)
B ₀₂ B ₀₈ B ₀₉ B ₁₀ B ₁₃ C ₀₁ C ₀₃	(B ₀₂ B ₀₈ B ₀₉ B ₁₃)
B ₀₂ B ₀₈ B ₁₀ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₈ B ₁₃ C ₀₂)
B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₃ C ₀₁ C ₀₃	(B ₀₇ B ₀₈ B ₀₉ B ₁₃)
B ₀₇ B ₀₈ B ₁₀ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₀₈ B ₁₃ C ₀₂)
B ₀₇ B ₁₃ B ₁₄ B ₁₅ B ₁₇ C ₀₂ C ₀₃	(B ₀₇ B ₁₃ B ₁₄ B ₁₅)
B ₀₇ B ₁₃ B ₁₄ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₃ B ₁₄ C ₀₁)
B ₀₈ B ₀₉ B ₁₀ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₈ B ₀₉ B ₁₃ C ₀₂)

Order 8

B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₈	(B ₀₁ B ₀₂ B ₀₈ B ₁₄)
B ₁₄ C ₀₁ C ₀₂	
B ₀₁ B ₀₃ B ₀₄ B ₀₅ B ₁₄	(B ₀₃ B ₀₅ B ₁₄ C ₀₃) (B ₀₄ B ₀₅ B ₁₄ C ₀₃)
C ₀₁ C ₀₂ C ₀₃	
B ₀₁ B ₀₇ B ₁₃ B ₁₄ B ₁₅	(B ₀₁ B ₀₇ B ₁₃ B ₁₄)
B ₁₇ C ₀₂ C ₀₃	

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 8 (continued)

B ₀₁ B ₀₇ B ₁₃ B ₁₅ B ₁₆	(B ₀₁ B ₀₇ B ₁₃ B ₁₆)
B ₁₇ C ₀₂ C ₀₃	
B ₀₁ B ₁₄ B ₁₅ B ₁₇ B ₁₈	(B ₀₁ B ₁₅ B ₁₈ C ₀₁)
C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₃ B ₀₄ B ₀₆ B ₀₈	(B ₀₃ B ₀₆ B ₀₈ C ₀₃) (B ₀₄ B ₀₆ B ₀₈ C ₀₃)
C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₇ B ₀₈ B ₀₉ B ₁₀	(B ₀₂ B ₀₇ B ₀₈ B ₁₃)
B ₁₃ C ₀₁ C ₀₃	
B ₀₂ B ₀₈ B ₀₉ B ₁₀ B ₁₂	(B ₀₂ B ₀₉ B ₁₂ C ₀₂) (B ₀₂ B ₁₀ B ₁₂ C ₀₂)
C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₀₉ B ₁₀ B ₁₁ B ₁₃	(B ₀₉ B ₁₁ B ₁₃ C ₀₂) (B ₁₀ B ₁₁ B ₁₃ C ₀₂)
C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₁₃ B ₁₅ B ₁₆ B ₁₇	(B ₀₇ B ₁₅ B ₁₆ C ₀₁) (B ₀₇ B ₁₆ B ₁₇ C ₀₁)
C ₀₁ C ₀₂ C ₀₃	

Order 9

A ₀₁ A ₀₂ B ₀₁ B ₀₇ B ₁₅	(A ₀₂ B ₀₁ B ₁₅ C ₀₁) (A ₀₂ B ₀₁ B ₁₇ C ₀₁)
B ₁₇ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₀₇ B ₁₅ C ₀₁) (A ₀₂ B ₀₇ B ₁₇ C ₀₁)
* A ₀₁ A ₀₂ B ₁₃ B ₁₄ B ₁₆	(A ₀₂ B ₁₃ C ₀₁ C ₀₂) (A ₀₂ B ₁₃ C ₀₁ C ₀₃)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₁₄ C ₀₁ C ₀₂) (A ₀₂ B ₁₄ C ₀₁ C ₀₃)
	(A ₀₂ B ₁₆ C ₀₁ C ₀₃) (A ₀₂ B ₁₆ C ₀₁ C ₀₂)
	(A ₀₂ B ₁₈ C ₀₁ C ₀₃) (A ₀₂ B ₁₈ C ₀₁ C ₀₂)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 9 (continued)

* A01 A03 B01 B02 B05 B06 C01 C02 C03	(A03 B01 C01 C03) (A03 B01 C02 C03) (A03 B02 C01 C03) (A03 B02 C02 C03) (A03 B05 C01 C03) (A03 B05 C02 C03) (A03 B06 C01 C03) (A03 B06 C02 C03)
A01 A03 B03 B04 B08 B14 C01 C02 C03	(A03 B03 B08 C03) (A03 B03 B14 C03) (A03 B04 B08 C03) (A03 B04 B14 C03)
A01 A06 B02 B09 B10 B13 C01 C02 C03	(A06 B02 B09 C02) (A06 B02 B10 C02) (A06 B09 B13 C02) (A06 B10 B13 C02)
* A01 A06 B07 B08 B11 B12 C01 C02 C03	(A06 B07 C01 C02) (A06 B07 C02 C03) (A06 B08 C01 C02) (A06 B08 C02 C03) (A06 B11 C01 C02) (A06 B11 C02 C03) (A06 B12 C01 C02) (A06 B12 C02 C03)
B01 B02 B03 B04 B05 B06 C01 C02 C03	(B01 B03 B06 C03) (B01 B04 B06 C03) (B02 B03 B05 C03) (B02 B04 B05 C03) (B03 B05 B06 C03) (B04 B05 B06 C03)
* B01 B02 B03 B07 B08 B09 C01 C02 C03	(B01 B03 B09 C02) (B01 B03 B09 C03) (B01 B08 B09 C02) (B01 B08 B09 C03) (B02 B03 B07 C02) (B02 B03 B07 C03) (B03 B07 B09 C02) (B03 B07 B09 C03)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 9 (continued)

* B ₀₁ B ₀₂ B ₀₄ B ₁₃ B ₁₄	(B ₀₁ B ₀₄ B ₁₃ C ₀₁) (B ₀₁ B ₀₄ B ₁₃ C ₀₃)
B ₁₅ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₄ B ₁₅ C ₀₁) (B ₀₂ B ₀₄ B ₁₅ C ₀₃)
	(B ₀₂ B ₁₄ B ₁₅ C ₀₁) (B ₀₂ B ₁₄ B ₁₅ C ₀₃)
	(B ₀₄ B ₁₃ B ₁₅ C ₀₁) (B ₀₄ B ₁₃ B ₁₅ C ₀₃)
B ₀₁ B ₀₃ B ₀₄ B ₀₅ B ₀₈	(B ₀₅ B ₀₈ B ₁₄ C ₀₃)
B ₁₄ C ₀₁ C ₀₂ C ₀₃	
B ₀₁ B ₀₇ B ₁₃ B ₁₅ B ₁₆	(B ₀₁ B ₀₇ B ₁₆ C ₀₁)
B ₁₇ C ₀₁ C ₀₂ C ₀₃	
B ₀₁ B ₀₇ B ₁₄ B ₁₅ B ₁₇	(B ₀₁ B ₀₇ B ₁₈ C ₀₁)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₃ B ₀₄ B ₀₆ B ₀₈	(B ₀₆ B ₀₈ B ₁₄ C ₀₃)
B ₁₄ C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₇ B ₀₈ B ₀₉ B ₁₀	(B ₀₂ B ₀₇ B ₀₉ B ₁₂) (B ₀₂ B ₀₇ B ₁₀ B ₁₂)
B ₁₁ B ₁₂ C ₀₁ C ₀₃	(B ₀₂ B ₀₈ B ₀₉ B ₁₁) (B ₀₂ B ₀₉ B ₁₁ B ₁₂)
	(B ₀₂ B ₀₈ B ₁₀ B ₁₁) (B ₀₂ B ₁₀ B ₁₁ B ₁₂)
B ₀₂ B ₀₇ B ₀₉ B ₁₀ B ₁₁	(B ₀₂ B ₁₁ B ₁₃ C ₀₂)
B ₁₃ C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₇ B ₀₉ B ₁₀ B ₁₁	(B ₀₂ B ₁₁ B ₁₃ C ₀₂)
B ₁₃ C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₈ B ₀₉ B ₁₀ B ₁₂	(B ₀₂ B ₁₂ B ₁₃ C ₀₂)
B ₁₃ C ₀₁ C ₀₂ C ₀₃	

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 9 (continued)

B07 B08 B09 B10 B11	(B07 B09 B12 C02) (B07 B10 B12 C02)
B12 C01 C02 C03	(B08 B09 B11 C02) (B08 B10 B11 C02)
	(B09 B11 B12 C02) (B10 B11 B12 C02)
* B07 B08 B10 B13 B14	(B07 B10 B14 C01) (B07 B10 B14 C02)
B17 C01 C02 C03	(B08 B10 B17 C01) (B08 B10 B17 C02)
	(B08 B13 B17 C01) (B08 B13 B17 C02)
	(B10 B14 B17 C01) (B10 B14 B17 C02)
B13 B14 B15 B16 B17	(B13 B15 B18 C01) (B13 B17 B18 C01)
B18 C01 C02 C03	(B14 B15 B16 C01) (B14 B16 B17 C01)
	(B15 B16 B18 C01) (B16 B17 B18 C01)

Order 10

B01 B02 B03 B04	(B01 B06 B08 C03) (B02 B05 B08 C03)
B05 B06 B08	(B05 B06 B08 C03)
C01 C02 C03	
B01 B02 B03 B04	(B01 B06 B08 B14) (B02 B05 B08 B14)
B05 B06 B08	(B05 B06 B08 B14)
B14 C01 C02	
B01 B02 B03 B04	(B01 B03 B06 B12) (B01 B04 B06 B12)
B05 B06 B12	(B02 B03 B05 B12) (B02 B04 B05 B12)
C01 C02 C03	(B03 B05 B06 B12) (B04 B05 B06 B12)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 10 (continued)

B ₀₁ B ₀₂ B ₀₃ B ₀₄	(B ₀₁ B ₀₆ B ₁₄ C ₀₃)
B ₀₅ B ₀₆ B ₁₄	
C ₀₁ C ₀₂ C ₀₃	
B ₀₁ B ₀₂ B ₀₃ B ₀₄	(B ₀₁ B ₀₃ B ₀₆ B ₁₈)
B ₀₅ B ₀₆ B ₁₈	
C ₀₁ C ₀₂ C ₀₃	
B ₀₁ B ₀₅ B ₀₇ B ₁₁	(B ₀₁ B ₁₁ B ₁₅ B ₁₆) (B ₀₁ B ₁₁ B ₁₆ B ₁₇)
B ₁₅ B ₁₆ B ₁₇	(B ₀₅ B ₀₇ B ₁₅ B ₁₆) (B ₀₅ B ₀₇ B ₁₆ B ₁₇)
C ₀₁ C ₀₂ C ₀₃	(B ₀₅ B ₁₁ B ₁₅ B ₁₆) (B ₀₅ B ₁₁ B ₁₆ B ₁₇)
B ₀₁ B ₀₇ B ₁₃ B ₁₄	(B ₀₁ B ₀₇ B ₁₄ B ₁₆) (B ₀₁ B ₀₇ B ₁₆ B ₁₈)
B ₁₅ B ₁₆ B ₁₇	
B ₁₈ C ₀₂ C ₀₃	
B ₀₁ B ₁₃ B ₁₄ B ₁₅	(B ₀₁ B ₁₃ B ₁₈ C ₀₁)
B ₁₆ B ₁₇ B ₁₈	
C ₀₁ C ₀₂ C ₀₃	
B ₀₂ B ₀₇ B ₀₈ B ₀₉	(B ₀₂ B ₀₇ B ₁₂ B ₁₃) (B ₀₂ B ₀₈ B ₁₁ B ₁₃)
B ₁₀ B ₁₁ B ₁₂	(B ₀₂ B ₁₁ B ₁₂ B ₁₃)
B ₁₃ C ₀₁ C ₀₃	
B ₀₂ B ₀₇ B ₀₈ B ₀₉	(B ₀₂ B ₀₇ B ₁₂ C ₀₂) (B ₀₂ B ₀₈ B ₁₁ C ₀₂)
B ₁₀ B ₁₁ B ₁₂	(B ₀₂ B ₁₁ B ₁₂ C ₀₂)
C ₀₁ C ₀₂ C ₀₃	

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 10 (continued)

B ₀₇ B ₀₈ B ₀₉ B ₁₀	(B ₀₇ B ₁₂ B ₁₃ C ₀₂) (B ₀₈ B ₁₁ B ₁₃ C ₀₂)
B ₁₁ B ₁₂ B ₁₃	(B ₁₁ B ₁₂ B ₁₃ C ₀₂)
C ₀₁ C ₀₂ C ₀₃	
B ₀₇ B ₁₃ B ₁₄ B ₁₅	(B ₀₇ B ₁₃ B ₁₈ C ₀₁) (B ₀₇ B ₁₄ B ₁₆ C ₀₁)
B ₁₆ B ₁₇ B ₁₈	(B ₀₇ B ₁₆ B ₁₈ C ₀₁)
C ₀₁ C ₀₂ C ₀₃	

Order 11

* A ₀₁ A ₀₂ B ₀₁ B ₀₂ B ₀₃ B ₀₇	(A ₀₂ B ₀₁ B ₀₂ C ₀₂) (A ₀₂ B ₀₁ B ₀₂ C ₀₃)
B ₀₈ B ₀₉ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₀₁ B ₀₃ C ₀₂) (A ₀₂ B ₀₁ B ₀₃ C ₀₃)
	(A ₀₂ B ₀₁ B ₀₈ C ₀₂) (A ₀₂ B ₀₁ B ₀₈ C ₀₃)
	(A ₀₂ B ₀₁ B ₀₉ C ₀₂) (A ₀₂ B ₀₁ B ₀₉ C ₀₃)
	(A ₀₂ B ₀₂ B ₀₇ C ₀₂) (A ₀₂ B ₀₂ B ₀₇ C ₀₃)
	(A ₀₂ B ₀₃ B ₀₇ C ₀₂) (A ₀₂ B ₀₃ B ₀₇ C ₀₃)
	(A ₀₂ B ₀₇ B ₀₈ C ₀₂) (A ₀₂ B ₀₇ B ₀₈ C ₀₃)
	(A ₀₂ B ₀₇ B ₀₉ C ₀₂) (A ₀₂ B ₀₇ B ₀₉ C ₀₃)
A ₀₁ A ₀₂ B ₁₃ B ₁₄ B ₁₅ B ₁₆	(A ₀₂ B ₁₃ B ₁₅ C ₀₁) (A ₀₂ B ₁₃ B ₁₇ C ₀₁)
B ₁₇ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₁₄ B ₁₅ C ₀₁) (A ₀₂ B ₁₄ B ₁₇ C ₀₁)
	(A ₀₂ B ₁₅ B ₁₆ C ₀₁) (A ₀₂ B ₁₅ B ₁₈ C ₀₁)
	(A ₀₂ B ₁₆ B ₁₇ C ₀₁) (A ₀₂ B ₁₇ B ₁₈ C ₀₁)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 11 (continued)

A01 A03 B01 B02 B03 B04 B05 B06 C01 C02 C03	(A03 B01 B03 C03) (A03 B01 B04 C03) (A03 B02 B03 C03) (A03 B02 B04 C03) (A03 B03 B05 C03) (A03 B03 B06 C03) (A03 B04 B05 C03) (A03 B04 B06 C03)
* A01 A03 B07 B08 B10 B13 B14 B17 C01 C02 C03	(A03 A07 B08 C01) (A03 B07 B08 C02) (A03 B07 B14 C01) (A03 B07 B14 C02) (A03 B08 B10 C01) (A03 B08 B10 C02) (A03 B08 B13 C01) (A03 B08 B13 C02) (A03 B08 B17 C01) (A03 B08 B17 C02) (A03 B10 B14 C01) (A03 B10 B14 C02) (A03 B13 B14 C01) (A03 B13 B14 C02) (A03 B14 B17 C01) (A03 B14 B17 C02)
* A01 A06 B01 B02 B04 B13 B14 B15 C01 C02 C03	(A06 B01 B06 C03) (A06 B01 B13 C03)
A01 A06 B07 B08 B09 B10 B11 B12 C01 C02 C03	(A06 B09 B11 C02) (A06 B09 B12 C02) (A06 B10 B11 C02) (A06 B10 B12 C02) (A06 B07 B09 C02) (A06 B07 B10 C02) (A06 B08 B09 C02) (A06 B08 B10 C02)
B01 B02 B03 B04 B05 B06 B08 B12 C01 C02 C03	(B01 B06 B08 B12) (B02 B05 B08 B12) (B05 B06 B08 B12)
B01 B02 B03 B04 B05 B06 B12 B14 C01 C02 C03	(B01 B06 B14 C03) (B02 B05 B14 C03) (B05 B06 B14 C03)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 11 (continued)

B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₅ B ₀₆ B ₁₄ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₆ B ₁₄ B ₁₈) (B ₀₂ B ₀₅ B ₁₄ B ₁₈) (B ₀₅ B ₀₆ B ₁₄ B ₁₈)
B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₈ B ₁₂ B ₁₄ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₂ B ₀₈ B ₁₈) (B ₀₁ B ₀₂ B ₁₂ B ₁₄) (B ₀₁ B ₀₂ B ₁₂ B ₁₈)
B ₀₁ B ₀₅ B ₀₇ B ₁₁ B ₁₃ B ₁₄ B ₁₅ B ₁₇ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₁₁ B ₁₃ B ₁₄) (B ₀₅ B ₀₇ B ₁₃ B ₁₄) (B ₀₅ B ₁₁ B ₁₃ B ₁₄)
B ₀₁ B ₀₇ B ₁₃ B ₁₄ B ₁₅ B ₁₆ B ₁₇ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₁ B ₀₇ B ₁₄ B ₁₆) (B ₀₁ B ₀₇ B ₁₃ B ₁₈) (B ₀₁ B ₀₇ B ₁₆ B ₁₈)
B ₀₂ B ₀₆ B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₁ B ₁₂ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₆ B ₀₇ B ₁₂) (B ₀₂ B ₀₆ B ₀₈ B ₁₁) (B ₀₂ B ₀₆ B ₁₁ B ₁₂) (B ₀₆ B ₀₇ B ₀₈ B ₁₁) (B ₀₆ B ₀₇ B ₀₈ B ₁₂) (B ₀₆ B ₀₇ B ₀₉ B ₁₂) (B ₀₆ B ₀₉ B ₁₁ B ₁₂) (B ₀₆ B ₀₇ B ₁₀ B ₁₂) (B ₀₆ B ₀₈ B ₀₉ B ₁₁) (B ₀₆ B ₀₈ B ₀₉ B ₁₂)
B ₀₂ B ₀₆ B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₇ B ₀₈ B ₁₆) (B ₀₆ B ₀₇ B ₀₈ B ₁₃) (B ₀₆ B ₀₇ B ₀₈ B ₁₆)
B ₀₂ B ₀₆ B ₀₇ B ₀₉ B ₁₀ B ₁₁ B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₁ B ₁₃ B ₁₆) (B ₀₂ B ₀₆ B ₀₇ B ₁₃) (B ₀₂ B ₀₆ B ₀₇ B ₁₆) (B ₀₂ B ₀₆ B ₁₁ B ₁₃) (B ₀₂ B ₀₆ B ₁₁ B ₁₆)
B ₀₂ B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₁ B ₁₂ B ₁₃ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₇ B ₁₂ B ₁₃) (B ₀₂ B ₀₈ B ₁₁ B ₁₃) (B ₀₂ B ₁₁ B ₁₂ B ₁₃)
B ₀₇ B ₀₈ B ₀₉ B ₁₀ B ₁₁ B ₁₂ B ₁₃ B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₇ B ₁₀ B ₁₂ B ₁₆) (B ₀₈ B ₀₉ B ₁₁ B ₁₆) (B ₀₈ B ₁₀ B ₁₁ B ₁₆) (B ₀₉ B ₁₁ B ₁₂ B ₁₆) (B ₁₀ B ₁₁ B ₁₂ B ₁₆)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 11 (continued)

B ₀₇ B ₁₁ B ₁₃ B ₁₄ B ₁₅ B ₁₆	(B ₁₁ B ₁₃ B ₁₄ B ₁₆) (B ₁₁ B ₁₄ B ₁₆ B ₁₈)
B ₁₇ B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₁₁ B ₁₄ B ₁₅ B ₁₆) (B ₁₁ B ₁₅ B ₁₆ B ₁₈)
	(B ₁₁ B ₁₄ B ₁₆ B ₁₇) (B ₁₁ B ₁₆ B ₁₇ B ₁₈)

Order 13

A ₀₁ A ₀₂ B ₀₁ B ₀₇ B ₁₃	(A ₀₂ B ₀₁ B ₁₃ C ₀₁) (A ₀₂ B ₀₁ B ₁₄ C ₀₁)
B ₁₄ B ₁₅ B ₁₆ B ₁₇	(A ₀₂ B ₀₁ B ₁₆ C ₀₁) (A ₀₂ B ₀₁ B ₁₈ C ₀₁)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(A ₀₂ B ₀₇ B ₁₃ C ₀₁) (A ₀₂ B ₀₇ B ₁₄ C ₀₁)
	(A ₀₂ B ₀₇ B ₁₆ C ₀₁) (A ₀₂ B ₀₇ B ₁₈ C ₀₁)
A ₀₁ A ₀₃ B ₀₁ B ₀₂ B ₀₃	(A ₀₃ B ₀₁ B ₀₈ C ₀₃) (A ₀₃ B ₀₁ B ₁₄ C ₀₃)
B ₀₄ B ₀₅ B ₀₆ B ₀₈	(A ₀₃ B ₀₂ B ₀₈ C ₀₃) (A ₀₃ B ₀₂ B ₁₄ C ₀₃)
B ₁₄ C ₀₁ C ₀₂ C ₀₃	(A ₀₃ B ₀₅ B ₀₈ C ₀₃) (A ₀₃ B ₀₅ B ₁₄ C ₀₃)
	(A ₀₃ B ₀₆ B ₀₈ C ₀₃) (A ₀₃ B ₀₆ B ₁₄ C ₀₃)
A ₀₁ A ₀₆ B ₀₂ B ₀₇ B ₀₈	(A ₀₆ B ₀₂ B ₀₇ C ₀₂) (A ₀₆ B ₀₂ B ₀₈ C ₀₂)
B ₀₉ B ₁₀ B ₁₁ B ₁₂	(A ₀₆ B ₀₂ B ₁₁ C ₀₂) (A ₀₆ B ₀₂ B ₁₂ C ₀₂)
B ₁₃ C ₀₁ C ₀₂ C ₀₃	(A ₀₆ B ₀₇ B ₁₃ C ₀₂) (A ₀₆ B ₀₈ B ₁₃ C ₀₂)
	(A ₀₆ B ₁₁ B ₁₃ C ₀₂) (A ₀₆ B ₁₂ B ₁₃ C ₀₂)
B ₀₁ B ₀₂ B ₀₃ B ₀₄ B ₀₅	(B ₀₁ B ₀₆ B ₀₈ B ₁₈) (B ₀₁ B ₀₆ B ₁₂ B ₁₄)
B ₀₆ B ₀₈ B ₁₂ B ₁₄	(B ₀₁ B ₀₆ B ₁₂ B ₁₈) (B ₀₂ B ₀₅ B ₁₂ B ₁₄)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₀₅ B ₀₈ B ₁₈) (B ₀₂ B ₀₅ B ₁₂ B ₁₈)
	(B ₀₅ B ₀₆ B ₀₈ B ₁₈) (B ₀₅ B ₀₆ B ₁₂ B ₁₄)
	(B ₀₅ B ₀₆ B ₁₂ B ₁₈) (B ₀₂ B ₀₆ B ₁₂ B ₁₄)
	(B ₀₂ B ₀₆ B ₁₂ B ₁₈)

SUBSEMIGROUPS OF T_3 GENERATED BY FOUR ELEMENTS (continued)Order 13 (continued)

B ₀₁ B ₀₅ B ₀₇ B ₁₁ B ₁₃	(B ₀₁ B ₁₁ B ₁₃ B ₁₈) (B ₀₁ B ₁₁ B ₁₄ B ₁₆)
B ₁₄ B ₁₅ B ₁₆ B ₁₇	(B ₀₁ B ₁₁ B ₁₆ B ₁₈) (B ₀₅ B ₁₁ B ₁₃ B ₁₈)
B ₁₈ C ₀₁ C ₀₂ C ₀₃	(B ₀₅ B ₁₁ B ₁₄ B ₁₆) (B ₀₅ B ₁₁ B ₁₆ B ₁₈)
	(B ₀₅ B ₀₇ B ₁₃ B ₁₈) (B ₀₅ B ₀₇ B ₁₄ B ₁₆)
	(B ₀₅ B ₀₇ B ₁₆ B ₁₈) (B ₀₅ B ₁₁ B ₁₃ B ₁₈)
	(B ₀₅ B ₁₁ B ₁₄ B ₁₆) (B ₀₅ B ₁₁ B ₁₆ B ₁₈)
B ₀₂ B ₀₆ B ₀₇ B ₀₈ B ₀₉	(B ₀₂ B ₀₈ B ₁₁ B ₁₆) (B ₀₂ B ₀₇ B ₁₂ B ₁₆)
B ₁₀ B ₁₁ B ₁₂ B ₁₃	(B ₀₆ B ₀₇ B ₁₂ B ₁₃) (B ₀₆ B ₀₇ B ₁₂ B ₁₆)
B ₁₆ C ₀₁ C ₀₂ C ₀₃	(B ₀₂ B ₁₁ B ₁₂ B ₁₆) (B ₀₆ B ₁₁ B ₁₂ B ₁₃)
	(B ₀₆ B ₀₈ B ₁₁ B ₁₃) (B ₀₆ B ₁₁ B ₁₂ B ₁₃)
	(B ₀₆ B ₀₈ B ₁₁ B ₁₆) (B ₀₂ B ₀₇ B ₁₂ B ₁₆)
	(B ₀₂ B ₁₁ B ₁₂ B ₁₆)

SUBSEMIGROUPS OF T_3 GENERATED BY FIVE ELEMENTSOrder 7

B01 B02 B03 B04 C01 C02 C03	(B01 B02 B03 B04 C03)
B01 B03 B04 B14 C01 C02 C03	(B01 B03 B04 B14 C03)
B01 B07 B15 B17 C01 C02 C03	(B01 B07 B15 B17 C01)
B01 B14 B15 B17 C01 C02 C03	(B01 B14 B15 B17 C01)
B02 B08 B09 B10 C01 C02 C03	(B02 B08 B09 B10 C02)
B02 B09 B10 B13 C01 C02 C03	(B02 B09 B10 B13 C02)
B03 B04 B08 B14 C01 C02 C03	(B03 B04 B08 B14 C03)
B07 B08 B09 B10 C01 C02 C03	(B07 B08 B09 B10 C02)
B07 B08 B10 B13 C01 C02 C03	(B07 B08 B13 C01 C02)
B07 B09 B10 B13 C01 C02 C03	(B07 B09 B10 B13 C02)
B07 B13 B14 B17 C01 C02 C03	(B07 B13 B14 C01 C02)
B07 B13 B15 B17 C01 C02 C03	(B07 B13 B15 B17 C01)
B13 B14 B15 B17 C01 C02 C03	(B13 B14 B15 B17 C01)

Order 8

B01 B02 B03 B04 B08 C01 C02 C03	(B01 B02 B04 B08 C03)
B01 B02 B03 B04 B14 C01 C02 C03	(B01 B02 B03 B14 C03)
B01 B03 B04 B08 B14 C01 C02 C03	(B01 B04 B08 B14 C03)
B01 B07 B14 B15 B17 C01 C02 C03	(B01 B07 B14 B15 C01)
B01 B13 B14 B15 B17 C01 C02 C03	(B01 B13 B14 B17 C01)
B02 B03 B04 B08 B14 C01 C02 C03	(B02 B03 B08 B14 C03)
B02 B07 B08 B09 B10 C01 C02 C03	(B02 B07 B08 B10 C02)

SUBSEMIGROUPS OF T_3 GENERATED BY FIVE ELEMENTS (continued)Order 8 (continued)

B02 B07 B09 B10 B13 C01 C02 C03	(B02 B07 B10 B13 C02)
B02 B08 B09 B10 B13 C01 C02 C03	(B02 B08 B09 B13 C02)
B07 B08 B09 B10 B13 C01 C02 C03	(B07 B08 B09 B13 C02)
B07 B13 B14 B15 B17 C01 C02 C03	(B07 B13 B14 B15 C01)

Order 9

B01 B02 B03 B04 B08 B14 C01 C02 C03	(B01 B02 B08 B14 C03)
B01 B07 B13 B14 B15 B17 C01 C02 C03	(B01 B07 B13 B14 C01)
B02 B07 B08 B09 B10 B13 C01 C02 C03	(B02 B07 B08 B13 C02)

Order 10

B02 B07 B08 B09	(B02 B07 B09 B12 C02)	(B02 B07 B10 B12 C02)
B10 B11 B12	(B02 B08 B09 B11 C02)	(B02 B09 B11 B12 C02)
C01 C02 C03	(B02 B08 B10 B11 C02)	(B02 B10 B11 B12 C02)

Order 11

B01 B02 B03 B04 B05 B06	(B01 B06 B08 B14 C03)
B08 B14 C01 C02 C03	(B02 B05 B08 B14 C03)
	(B05 B06 B08 B14 C03)
B02 B07 B08 B09 B10 B11	(B02 B07 B12 B13 C02)
B12 B13 C01 C02 C03	(B02 B08 B11 B13 C02)
	(B02 B11 B12 B13 C02)