

**STRUCTURAL PROTECTION AND RESONANCE DISRUPTION USING THE
VARIABLE INERTIA ROTATIONAL MECHANISM**

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DEDICATION

To Abbu, Dr. M. A. Rashid Sarkar

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ABSTRACT

Mitigating structural damage and other losses from earthquakes and extreme winds is an ongoing challenge for civil infrastructure. Various passive control strategies and supplemental devices have been developed to control structural vibration. Building off the work on traditional passive control devices, such as tuned mass dampers, viscous dampers and base isolation systems, recent research has focused on the potential of rotational inertial mechanisms. Linear rotational inertial mechanisms, otherwise known as inerters, can transfer translational motion to the rotational motion of a flywheel, consequently creating significant mass amplification effects in the host structure. While inerters can modify a structure's dynamic properties, the modifications these linear devices provide are constant, and resonant conditions still exist. As research advances in this area, investigations have been carried out on nonlinear rotational inertial mechanisms and how the variable inertia these devices produce could prove advantageous for resonance avoidance. This thesis focuses on a nonlinear rotational inertia mechanism known as the variable inertial rotational mechanism (VIRM), in which masses can radially move inside the device's flywheel, continuously changing the moment of inertia of the flywheel and the mass effects the device provides during the response of the structure. This thesis explores the effect of VIRM parameters on the natural frequencies and structural response of single-degree and multi-degree-of-freedom structures, considering different load types and amplitudes. Furthermore, a small-scale VIRM prototype is developed, and its individual components are experimentally characterized to understand their behavior and interaction with the attached structure. Through numerical and experimental investigations, the effectiveness of the VIRMs at mitigating structural response under various excitations is evaluated and compared to a fixed inertia rotational mechanism. The combined numerical and experimental efforts provide valuable insights into the dynamic characteristics of VIRMs, highlighting their potential for adaptive frequency shifting and enhanced vibration control.

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CHAPTER 1 Introduction

1.1 Problem Statement

Civil structures, including buildings, bridges and other critical infrastructure networks are fundamental components of modern civilization. These structures are crucial for transportation, housing, ensuring the safety and well-being of communities, and supporting economic activities. Traditionally designed to withstand static loads, these structures are also vulnerable to dynamic loads from natural and man-made hazards.

Changing weather patterns are escalating the frequency and severity of natural hazards, leading to significant economic and life losses globally. The Center for Research on Epidemiology of Disasters reports a staggering annual loss due to natural hazards: \$45 billion in the United States, \$27 billion in China, and \$24 billion in Japan. In 2017 alone, the United States experienced a record of over \$300 billion in economic losses from weather disasters (W. Zhang & Gruber, 2020). Damage and destruction to human-made structures, particularly civil infrastructures, are significant contributors to economic losses and pose serious threats to public safety.

Wind, earthquakes, and vibrations from machinery and vehicles are major sources of the dynamic loads that can result in negative consequences, including catastrophic structural failures. The collapse of the Tacoma Narrows Bridge in 1940 stands as a stark reminder of the destructive power of wind-induced torsional vibration. More recently, the I-35W bridge collapse in Minneapolis in 2007, tragically resulted in 13 deaths and 145 injuries and highlighted the dangers of fatigue, particularly in structures with inherent structural weaknesses.

Earthquakes present a global challenge, with moderate to high seismic hazard zones found on every continent. The 1999 Chi-Chi earthquake in Taiwan (2,415 deaths, 11,305 injuries, 51,000 buildings damaged, \$10 billion in economic losses) and the 1994 Northridge earthquake in California (\$44 billion in economic losses, 57 fatalities) illustrate the devastating impact of these events (Wu & Lindell, 2004). Despite ongoing research in seismic mitigation, the highly variable nature of earthquakes, soil conditions, and soil-structure interaction continues to pose significant challenges for engineers (Thenhaus et al., 2003).

To mitigate structural damage, traditional approaches have often focused on structural modifications, such as stiffening and strengthening members and materials. However, incorporating structural control mechanisms often proves to be a more economically

advantageous solution. The effectiveness of structural control is evident in real-world applications:

- Taipei 101, Taiwan: During a recent earthquake in April 2024, the Taipei 101 skyscraper, equipped with a 660-metric-ton tuned mass damper, swayed gently despite the 7.4 magnitude quake, safeguarding the structure and its occupants.
- Kyobashi Seiwa Building, Tokyo: In 1989, two active mass drivers were installed on the roof of this 11-story building to mitigate vibrations from strong winds and moderate seismic events (Soong & Spencer, 2002).
- Walnut Creek Bridge, USA: The first full-scale implementation of semi-active control in the United States was on this bridge, showcasing the effectiveness of variable damper technology (Neff Patten et al., 1999).

The examples highlighted above demonstrate the potential of structural control mechanisms in mitigating the impact of dynamic loads. However, the complex nonlinear behavior of structures under extreme loading conditions and the need for cost-effective and adaptable control strategies, demands the need for continued research and development in the structural control field.

1.2 Structural Control Systems

Structural control systems aim to absorb, curtail or dissipate energy from dynamic excitations, such as wind, earthquake, waves, and traffic. When a structure is subjected to these dynamic excitations, the energy in the system appears as kinetic energy or elastic and inelastic deformation energy, which is potentially associated with damage. By incorporating a structural control system, energy in the structure can be more effectively attenuated, and most importantly, the inelastic deformation demand can be reduced. Structural control systems can be primarily categorized as passive, semi-active, and active control systems. Although considerable research has been conducted on all these systems, the complexity, cost-effectiveness, reliability, and the conservative nature of the construction industry has limited the implementation of semi-active and active control systems in the United States (Spencer & Nagarajaiah, 2003).

Passive control devices are often more desirable in the civil engineering industry due to their lower complexity, that they do not have power and sensor requirements, and that they still maintain an ability to modify the structural dynamics and enhance the energy dissipation potential. Passive control devices include base isolation system, tuned mass damper, tuned liquid

dampers, viscous, viscoelastic, metallic and friction dampers (Soong & Dargush, 1997). Base isolation systems are one of the most successful passive control systems and can isolate the structure from ground motion. However, potentially the most popular type of passive control device is the tuned mass damper (TMD). TMDs consist of mass, spring and damper and they are typically tuned to the most critical mode of vibration of the primary structure by selecting their mass and stiffness. TMDs change the dynamic response of the structure near the tuned mode and reduces the response of the structure by transmitting energy to the TMD (Elias & Matsagar, 2017).

TMDs have gained widespread popularity in real-world applications because they utilize linear dynamics to reduce structural response. Examples of structures incorporating TMDs include prominent skyscrapers such as Taipei 101 in Taipei, Taiwan, Citygroup Center in New York, US, Sydney Tower in Sydney, Australia, Shanghai World Financial Center in Shanghai China (Soto, 2012). While TMDs have been widely implemented, the mass of the TMD needs to be relatively large enough to effectively control the response of the structure. For example, the TMDs for the Taipei 101 and the Citygroup Center are 660,000 kg and 370,000 kg (Soto, 2012), with a mass ratio of 1.25% and 2% of the first mode modal mass, respectively (Chung et al., 2013; Kwok & Samali, 1995). However, a key limitation of TMDs is that they are tuned to a specific mode of vibration, and as a result, they can not effectively mitigate the response of other modes, which are often also present during extreme natural hazard events. Similar to a TMD, other linear passive devices are often designed to meet a specific performance objective and may not adapt well to varying properties of the primary structure, damage and changes in the external excitation.

To address these limitations, researchers have explored incorporating nonlinearities into passive control devices. This approach aims to broaden the effective frequency range and improve performance under a wider range of system parameters and external excitations. Nonlinear energy sinks (NES), bistable nonlinear tuned vibration absorbers (BNTVA), magnetic vibration absorbers (MVA), particle impact dampers (PID), and nonlinear viscous dampers (NVD) are a few examples of nonlinear passive control device (Habib & Romeo, 2017; Lo Feudo et al., 2019; Lu et al., 2018). Both NES and BNTVA systems utilize strongly nonlinear stiffness, resulting in a non-constant device natural frequency that eliminates the need for tuning to a specific frequency (Lu et al., 2018). In addition to this, the nonlinear dynamic excitation further enhance the damping capacity of the BNTVA (Dekemele, 2023). These nonlinear devices often

outperform their linear counterparts in terms of broadband frequency response and vibration reduction. However, they are not without their own challenges. For instance, NES and PIDs may not be as effective in highly damped structures. NES performance can also be limited by specific excitation characteristics and structural properties, while PIDs tend to perform better under high-frequency, large-amplitude conditions (Lu et al., 2018). NVDs, on the other hand, can be susceptible to performance variations due to ambient temperature changes and potential damage from prolonged reciprocating motion (R. J. Smith & Willford, 2007). Therefore, while nonlinear passive control devices offer significant advantages, it's crucial to acknowledge that their optimal performance relies on careful consideration of both the external excitation and the characteristics of the structure itself (Lu et al., 2018).

1.3 Variable Inertia Rotational Mechanism

The variable inertia rotational mechanism (VIRM) is an innovative nonlinear structural control mechanism that alters its inertia to adapt to external dynamic loads. This adaptive response allows the VIRM to mitigate a wider range of vibrations and enhance the overall dynamic performance of structures. VIRM can be employed as active, semi-active or passive control devices and are part of a relatively new branch of structural control devices utilizing rotational inertia mechanisms (RIMs). RIMs amplify motion and provide large effective mass to the structure with a relatively small physical mass. These devices utilize ball-screw, rack and pinion, and lead-screw to convert translational motion of the structure to the rotational motion of a small mass designed to generate a large effective mass. For example, a RIM has been manufactured to produce an amplified effective mass that is up to 9000 times the physical mass of the device (Sugimura et al., 2012). Due to its significant mass amplification effect, and the possible economic benefits of a smaller mass device that requires less space than typical alternatives, further consideration of RIMs has gain traction in academia and in industry (Ma et al., 2021).

RIMs can be classified as linear RIMs and nonlinear RIMs. Linear RIMs, also known as inerters, generate a restoring force that is proportional to the relative acceleration between its two terminals and produce a constant effective mass (M. C. Smith, 2002). RIMs have been incorporated with several other structural control applications, such as TMD, toggle braces, a friction pendulum system, external rocking walls for frame structures (De Domenico, Ricciardi, et al., 2020; Hwang et al., 2007; Javidialesaadi & Wierschem, 2018; Thiers-Moggia & Málaga-Chuquitaype, 2020) and can be physically incorporated in between structures, in outrigger

systems and in base isolation systems (L. Li & Liang, 2020; Takehiko et al., 2015). The results of this previous work show that incorporating RIMs can decrease the structure's drift, decrease its acceleration response in some circumstances, and reduce the natural frequency of the structure, as compared to the uncontrolled structure. However, similar to limitations observed in linear passive devices, the effectiveness of the linear inerters can be hindered by their fixed dynamic properties, which leads to resonance frequencies being shifted but not disrupted. This motivates the exploration of nonlinear RIMs, which showcases adaptable behavior of the nonlinear devices to operate in a wider range of frequencies and loading conditions and disrupt the linear dynamics of structures.

Given the limitations of linear inerters, nonlinear RIMs present a growing area of exploration within the field. These mechanisms feature intentional nonlinearities that vary the effective mass produced by the device based on the type of nonlinearity. Nonlinear RIMs can be broadly classified as state-switching mechanisms and functionally varied mechanisms. An example of a state-switching mechanism is a clutch inerter damper (CID), which exhibits abrupt changes in the rotational inertia based on the state of the system. The engagements and disengagements of the CID flywheel result in shifts in the instantaneous natural frequencies, demonstrating the dynamic nature of the state-switching RIMs (Makris & Kampas, 2016; Málaga-Chuquitaype et al., 2019; Talley et al., 2023).

In contrast, functionally varied RIMs generally offer smooth variations in inertance as the rotational inertia varies as the function of the structure's response, such as displacement or velocity of the structure. The variable inertia rotational mechanism (VIRM) falls under this category and modifies the rotational inertia of the flywheel by incorporating moving masses inside the flywheel (Su & Liu, 2010). When the flywheel's rotational velocity changes as a result of the structure's responses, the masses inside move back and forth and continuously shift the natural frequency of the structure.

While the earliest applications of VIRMs in research appeared in 1978, the studies involving these mechanisms have been mostly limited to wind-power applications as an energy storage system or as tunable wave energy converter to improve energy harvesting efficiency (Jauch, 2015; Q. Li et al., 2021; Ullman, 1978; Y. Yang et al., 2021). In recent years, VIRMs have been considered in vehicle suspension, diesel generator vibration suppression and civil structure applications (Sarkar et al., 2024; S. Yang et al., 2019; Yuan et al., 2010). There has been very limited work done on semi-active, active and passive VIRMs. While some numerical and

experimental work on these different types of VIRMs have been conducted, the majority of the research to date has focused primarily on understanding the intrinsic behavior and rotational inertia changes, rather than their effects when integrated with a structure. Additionally, the mathematical model used in the numerical simulations have been linearized (Xu et al., 2015a), which simplifies computations, but fails to capture the additional damping contribution of the VIRMs to the attached structure. In recent years, the effect of VIRM to mitigate the response of the attached structure has been studied, mostly limited to a single-degree-of-freedom structure and considering mostly harmonic loading. However, a key gap in knowledge exists to clearly understand the VIRM's effect on the primary structure's dynamics and response mitigation under different load scenarios. Additionally, the effect of VIRMs on multi-degree-of-freedom structures and under earthquake loads have not been investigated yet. To fully realize the potential of VIRMs, a combined approach involving experimental realization, numerical modeling and comparative analysis is crucial, yet currently underdeveloped.

1.4 Dissertation Overview

The primary objective of this dissertation is to better understand the dynamic characteristics and assess the performance of the VIRM in structures subjected to dynamic excitation. Therefore, this work tests the central hypothesis that the VIRM can be an effective passive control system in civil engineering structures. The hypothesis is validated by analyzing the natural frequency shifts and response attenuation under different load types using mathematical modeling and physical testing. This dissertation focuses on three key areas to evaluate the central hypothesis:

Focus Area 1: Impact of VIRM on the natural frequency of structures. There is uncertainty about how VIRM properties affect natural frequencies when subjected to different load type. This area explores changes in natural frequency using mathematical modeling of VIRM attached on SDOF and MDOF structures.

Focus Area 2: Influence of VIRM on the effectiveness of structural control. This area examines the impact of variable inertia on the performance of structures under impulse, harmonic and other complex loads. Various response measures, such as displacement, interstory displacements and acceleration are monitored to assess the performance of the structure with VIRM. Moreover, the trend in performance will be compared to the trends in natural frequency focused in Focus Area 1 to draw connections between the structure's dynamics and the response.

Focus Area 3: Experimental VIRM realization and numerical comparison. This area focuses on the development of a physical small-scale VIRM model and its representation through numerical modeling. Experimental investigations, including controlled testing of individual components of the VIRM and system-level dynamic analysis, are employed to characterize the VIRM's behavior and interaction with the attached structure. The findings from this investigation helps to refine the mathematical model and provide valuable insights into the dynamic characteristics of VIRMs.

The subsequent chapters of this dissertation are described below.

Chapter 2 provides an overall literature review of work related to this dissertation. This review includes discussion on traditional structural control devices, different RIMs, and details on previous VIRM work.

Chapter 3 details the mathematical model derivation of the VIRM in a SDOF and MDOF structure. The mathematical models of linear RIMs attached to a SDOF and MDOF structures are also presented in this chapter. Additionally, several response measures to estimate natural frequency and performance are also discussed. The response measures include instantaneous natural frequency, average instantaneous natural frequency, overall natural frequency, H2 response measure, median absolute interstory displacement, and median absolute acceleration response.

Chapter 4 investigates the effect of variable inertia on the natural frequency and response of the SDOF structure, directly addressing aspects of both Focus Area 1 and Focus Area 2 in a SDOF structure. A range of system properties of the VIRM, such as damping and stiffness, are examined under different load types and load intensities in this chapter. Comparisons are made between VIRM and linear RIM systems using various natural frequency measures and the H2 response measure.

Chapter 5 extends the investigation to a two DOF structure subjected to white noise loading and seismic ground motion. This chapter addresses the gaps in existing literature by examining frequency shifts produced by VIRM in MDOF structures and the performance of these systems under seismic excitation. This chapter builds upon Focus Area 1 by examining frequency shifts in a MDOF structures, and expands upon Focus Area 2 by evaluating VIRM performance in MDOF structure under complex loads. Incremental dynamic analysis is employed to

investigate different VIRM and linear RIM configurations, highlighting the interplay between VIRM parameters, frequency shifts, and seismic performance.

Chapter 6, addressing Focus Area 3, delves into the dynamic behavior of VIRMs through a combined experimental and numerical approach. This chapter details the development of the physical small-scale VIRM model integrated into a SDOF primary structure. To accurately characterize the VIRM's behavior, a range of experimental methods are employed, including controlled compressive testing of springs and computer-vision-aided free vibration testing for estimating stiffness and damping parameters, respectively. These experimental findings are then used to refine the mathematical model of the VIRM structure, leading to a deeper understanding of its dynamic characteristics and revealing key discrepancies between experimental and numerical observations.

Chapter 7 summarizes the dissertation's contributions, highlighting key conclusions from each chapter and outlining potential future research directions for VIRM technology.

This dissertation provides a comprehensive analysis of VIRMs as passive control systems for civil engineering structures. By combining mathematical modeling, numerical simulations, and experimental validation, this research enhances the understanding of VIRM behavior and their potential for improving structural response under dynamic loading.

CHAPTER 2 Literature review

This chapter presents a comprehensive literature review on structural control systems, with a focus on nonlinear passive control systems. The review covers different types of structural control systems, including active, semi-active, and passive control systems. Particular emphasis is placed on rotational inertia mechanisms a class of passive control systems. The review delves deeper into the variable inertia rotational mechanism (VIRM) which constitutes the nonlinear passive control system that is the central subject of this dissertation. The VIRM section provides an overview of previous studies, identifies key knowledge gaps, and outlines how this research aims to address those gaps.

2.1 Structural control of civil engineering structures

Civil structures can experience significant dynamic forces, including wind and seismic responses in buildings, floor vibrations, and deck and cable vibration in bridges. These dynamic responses can cause severe and prolonged vibrations, which can impact the structural integrity, functionality and human occupants. The level of protection requirement depends on factors such as, ensuring reliable operation, occupant comfort, safeguarding the lives of the occupants and the structure itself. While these responses can be mitigated through structural modifications, it is often more economically advantageous to incorporate structural control devices. The purpose of the structural control devices is to absorb and reflect energy introduced by the dynamic forces. The different types of structural control systems used for this purpose are broadly classified as passive, active, and semi-active control systems (Housner et al., 1997).

2.1.1 Active control systems

Active control systems use sensors to gather information, a controller to analyze it and determine necessary adjustments, and actuators to implement these adjustments, all within a feedback loop to regulate the structure's response to external forces. These systems utilize global response information to provide control forces to counteract the dynamic load coming to the structure. Their ability to use global response information allows them to determine the appropriate control forces over the entire structure, mitigating not only the 1st mode response but also other vibration modes and accommodating different loading conditions (Housner et al., 1997). An example of active control with a controllable mass in a structural system is called active mass driver. In this system, accelerometers or displacement sensors are utilized to collect structural response data and the control computer sends position commands to the control mass, which then moves with the specified dynamic motion to shift the structural dynamics and attenuate the structure's response.

While active control strategies have demonstrated increased performance over passive control, the major drawbacks involve the need for an external power supply, complicated required control algorithm, complex configuration, and overall higher cost (Spencer & Nagarajaiah, 2003). Furthermore, if the controller is incorrectly designed, it can shift the dynamics in a way that can cause the structure to go unstable. Despite active control systems having been extensively studied over the past few decades, their disadvantages have led to limited applications for civil structures (Casciati et al., 2012).

2.1.2 *Passive control systems*

Passive control systems are mechanical devices attached to the structure and impart forces that are developed in response to the motion of the structure. Unlike active control systems, passive control systems do not require external energy to operate and are inherently stable. Hence, passive control devices have become a popular alternative to improve the dynamic response of structures. Some examples of passive vibration control devices include tuned mass dampers (TMDs), tuned liquid dampers, friction dampers, viscous dampers, metallic yielding dampers, rotational inertia devices and more (Housner et al., 1997; Saaed et al., 2015).

Passive control devices can be divided into two categories: linear passive control devices and nonlinear passive control devices. Linear passive devices operate based on linear dynamic principles, where the relationship between the applied force and the resulting response remains linear and time-invariant. While these devices are widely recognized for their inherent simplicity, their fixed response characteristics make them lack the robustness needed to effectively handle varying and unpredictable loads, resulting in potential performance degradation over time (Soong & Dargush, 1997). Another major drawback of such linear passive devices is their inability to adapt to varying performance objectives under different load levels. This limitation can result in suboptimal performance across a range of loading scenarios, particularly in environments where loads are unpredictable and variable (Housner et al., 1997; Soong & Dargush, 1997).

Tuned mass dampers are the most commonly employed linear passive devices and are designed such that their natural vibration frequencies match the fundamental frequency of the primary structure, which allows them to absorb and dissipate vibrational energy (Yamaguchi & Harnpornchai, 1993). Although tuned mass dampers can reduce response of the structure, a significant challenge in the design and implementation of these devices is that they require a relatively large mass to be effective. Furthermore, these devices primarily impact only one mode of vibration, limiting their overall effectiveness in multi-modal structures. Additionally, changes

in structural properties due to extreme events or loading conditions over prolonged period can detune the TMD, further reducing its effectiveness (Rana & Soong, 1998).

In contrast to a linear passive control device, nonlinear passive control devices can have a wide frequency band of vibration attenuation and high robustness (H. Li et al., 2015; Pazooki et al., 2018). Nonlinearities in these devices come in many different forms, such as, cubic stiffness, bistable systems, negative stiffness behavior, etc. These nonlinear characteristics enable these devices to overcome limitations of linear passive devices, allowing them to adapt to varying load levels and conditions. For example, a nonlinear energy sink (NES) utilizes a mass connected through a nonlinear spring to the primary structure. The lack of linear fundamental frequency allows the NES to dissipate energy over a wide frequency range (Vakakis et al., 2008). While these devices offer significant advantages, they also introduce complexities in design and analysis, potentially leading to unpredictable behavior under certain conditions.

This dissertation focuses on a nonlinear passive control device utilizing the variable inertia rotational mechanism (VIRM) which is based on the principle of the rotational inertia mechanism (RIM). An in-depth review of RIM and the VIRM is presented later in Section 2.2 and 2.3, respectively.

2.1.3 *Semi-active control systems*

Semi-active control is an advanced control methodology that combines features of active and passive control devices. In these devices, sensors measure the structural response, and a control algorithm and actuator adjust the properties of the passive devices, such as dampers or stiffness elements, in real-time to respond to changing environmental and operational conditions of the structure. This approach provides adaptability of the active systems while maintaining the stability of the passive systems and ensures low energy requirements (Fenz & Constantinou, 2008; Nagarajaiah, 2009).

Some examples of semi-active devices are a hydraulic damper with a variable orifice, semi-active control variable stiffness device, and magnetorheological, electrorheological, and friction damper with variable damping force (Symans & Constantinou, 1999). The performance of these systems depend on both the active and passive components and these systems can potentially approach the effectiveness of an active control system; however, they can require significant computational resources and processing time, accuracy and reliability of the sensor and actuator systems.

2.2 Rotational inertia mechanisms

To improve performance of traditional passive control devices, an innovative family of component referred to as rotational inertia mechanisms (RIMs) have been developed that can be employed in a passive control device to mitigate vibration. RIMs can transfer translational motion to rotational motion and in doing so create a mass amplification effect. For example, a RIM device currently in service in a building in Japan can produce a mass amplification effect of about 9000 times the physical mass of the device (Sugimura et al., 2012). The conversion from linear to rotational motion can be achieved using a ball-screw or rack-and-pinion in combination with a flywheel, fluid-based mechanisms, or electromagnetic-based mechanisms (De Domenico, Ricciardi, et al., 2020; Gonzalez-Buelga et al., 2015; M. C. Smith, 2002). Two devices illustrating this mechanism are shown in Figure 0-1.

RIMs are categorized into two types: linear RIMs and nonlinear RIMs. Linear RIMs provide constant inertial effects, whereas, nonlinear RIMs can vary their inertial properties in response to the system's response. The majority of the systems studied and used in civil engineering applications consist of linear RIMs. Linear RIMs are typically referred to as inerters and are defined as two terminal mechanical elements that generate force proportional to the relative acceleration between the two terminals. The proportionality constant of an inerter is known as 'inertance', which is equivalent to the effective inertial mass and has the same unit as mass. The relationship between the force and the relative acceleration in the inerter can be expressed as

$$F = b(\ddot{u}_1 - \ddot{u}_2) \quad (2.1)$$

where, F is the equal forces on the two terminals, $\ddot{u}_1$ and $\ddot{u}_2$ are the acceleration of the two terminals and b is the proportionality constant inertance.

Inerter-based passive control devices can be categorized into three groups: energy dissipator, dynamic vibration absorber and vibration isolator (Ma et al., 2021). These devices have been primarily utilized as inerter-based mass dampers and inerter-based dampers in structural control. The different inerter-based devices their schematic models and references are summarized in Table 2-1. These devices can be physically incorporated in between structures, in

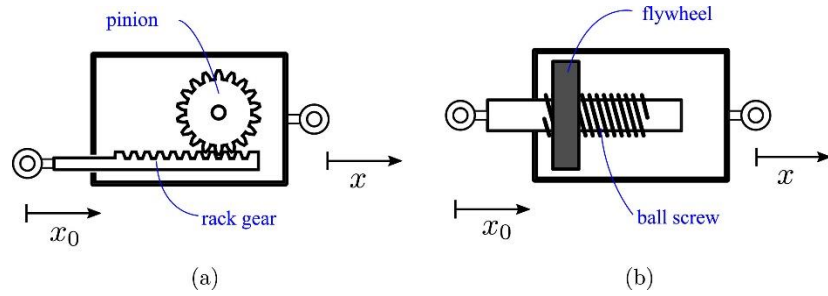
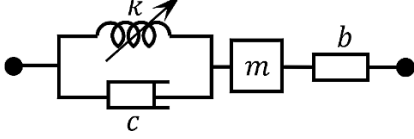
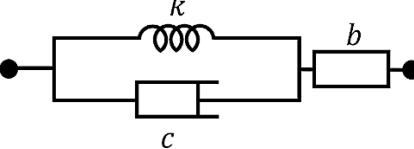
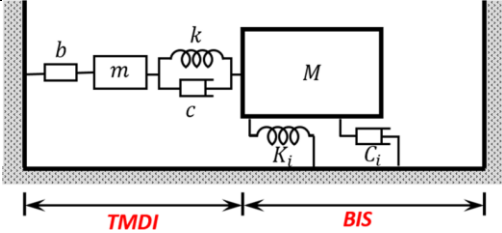


Figure 0-1. Physical rotational inertia mechanism realizations (a) rack and pinion mechanism (b) ball screw mechanism (figure from (Kuhnert et al., 2021))

Table 0-1. Summary of inerter-based devices

Name	Schematic model	Reference
Inerter-based energy dissipators		
Rotational inertia viscous damper (RIVD)		(Hwang et al., 2007)
Tuned viscous mass damper (TVMD)		(Ikago et al., 2012)
Inerter-based dynamic vibration absorber		
Tuned mass damped inerter (TMDI)		(Marian & Giaralis, 2014)
Tuned inerter damper (TID)		(Lazar et al., 2014)
Rotational inertia double tuned mass damper (RIDTMD)		(Garrido et al., 2013)

Table 2-1 Continued

Nonlinear energy sink inertor (NESI)		(Javidialesaadi & Wierschem, 2019a)
Inertor-based vibration isolator		
Friction pendulum inertor system (FPIS)		(Zhao et al., 2019)
Base isolation system TMDI (BIS-TMDI)		(De Domenico & Ricciardi, 2018)

outrigger systems and in base isolation systems (De Domenico, Ricciardi, et al., 2020; L. Li & Liang, 2020; Takehiko et al., 2015), and can be incorporated with other structural control devices to develop inerter-based systems. Hwang et al. proposed an inerter-based energy dissipator named rotational inertia viscous damper (RIVD) which uses a ball screw mechanism to transform inter-story drift into the rotation of a flywheel immersed in viscous fluid (Hwang et al., 2007). The tuned viscous mass damper (TVMD), developed by Ikago et al. combines an inertial mass damper with a spring and is tuned close to the fundamental period of the primary structure to maximize the damping efficiency. The experimental results show that the TVMD can outperform the conventional viscous dampers and IMD, reducing maximum inter-story drift by up to 70% (Ikago et al., 2012).

Inerters have been incorporated with dynamic vibration absorbers such as tuned devices to reduce the physical mass of the secondary structure. The tuned mass damper inerter (TMDI), as proposed by Marian and Giaralis, connects a tuned mass via an inerter to amplify the mass effect and showed significant improvements in seismic and wind-induced vibrations in high-rise buildings (Marian & Giaralis, 2014). The tuned inerter damper (TID), studied by Lazar et al., operates similar to TMDI with an inerter installed in series with spring and damper elements resulting in highly space-efficient design and suitable for various structural applications including beams, cables, and multi-story buildings (Lazar et al., 2014). The RIDTMD, researched by Wang and Lin, introduces a dual tuned mass system connected through rotational inertia mechanisms, broadening the frequency range for vibration mitigation and providing superior damping efficiency across multiple modes of vibration (Wang & Lin, 2019). While the TMDI and TID primarily focus on enhancing performance with reduced mass and space requirements, the RIDTMD can be designed to tackle multi-mode vibrations effectively, making it ideal for complex structural systems subjected to diverse dynamic loads. Additionally, the Nonlinear Energy Sink Inerter (NESI), studied by Qian and Zuo, combines a nonlinear energy sink with an inerter to enhance energy dissipation capabilities, providing substantial reductions in RMS responses and offering effective vibration control across a wide frequency range (Qian & Zuo, 2021). These devices collectively advance the field of structural vibration control by offering tailored solutions that leverage the unique benefits of inerter technology.

Among the inerter-based vibration isolators, friction pendulum inerter system (FPIS) and the base isolation system tuned mass damper inerter (BIS TMDI) are two prominent examples. The FPIS integrates inerters with friction pendulum bearings, combining mass amplification with

robust energy dissipation to improve seismic isolation and reduce structural responses during earthquakes (Zhao et al., 2019). Meanwhile, the BIS-TMDI merges base isolation principles with a tuned mass damped inerter, reducing displacement demands and enhancing structural stability under seismic loads by decreasing the required secondary mass, making it a space-efficient and cost-effective solution (De Domenico & Ricciardi, 2018). These systems exemplify how inerter technology can be effectively employed to improve vibration isolation in civil engineering applications.

Nonlinear rotational inertia mechanisms are related to inerters but the inertance produced by these mechanisms is not constant. Nonlinear rotational inertia mechanisms can be classified into state-switching mechanisms and functionally varied mechanisms. In a state-switching mechanism, the rotational inertia produced by the mechanism abruptly changes based on the state of the system. An example of a nonlinear rotational inertia mechanism with a state-switching nonlinearity is the clutch inerter damper (CID), which includes two flywheels and a passive clutch system, as presented in Figure 0-2 (Makris & Kampas, 2016). With a CID, a flywheel is engaged when the absolute velocity of the system is larger than an interaction velocity; when the acceleration reverses, the CID flywheel is no longer engaged. Consequently, the natural frequencies of the system with a CID will shift as the device engages and disengages with the structure (Javidialesaadi & Wierschem, 2019b; Makris & Kampas, 2016; Makris & Moghimi, 2019; Málaga-Chuquitaype et al., 2019). Due to its unique behavior, CID have demonstrated superior control effectiveness in seismic protection of SDOF structure, multi-story steel structures and rocking structures (Cupp & Wierschem, 2024; Málaga-Chuquitaype et al., 2019; Thiers-Moggia & Málaga-Chuquitaype, 2020; M. Wang & Sun, 2018). With functionally varied rotational inertia mechanisms, the inertance can vary as a function of many different parameters, such as displacement of the structure, rotational velocity of the flywheel etc. Unlike state-switching mechanisms, the inertance in functionally varied rotational inertia mechanisms generally varies, smoothly resulting in constantly shifting natural frequencies.

A type of functionally varied RIM is proposed by using geometrically nonlinear inerters (Moraes et al., 2018) or incorporating a linear inerter into a quasi-zero stiffness vibration isolator (J. Yang et al., 2020), as illustrated in Figure 0-3. The key component of this device is the orientation of the inerter in the configuration, where the linear inerters are arranged obliquely

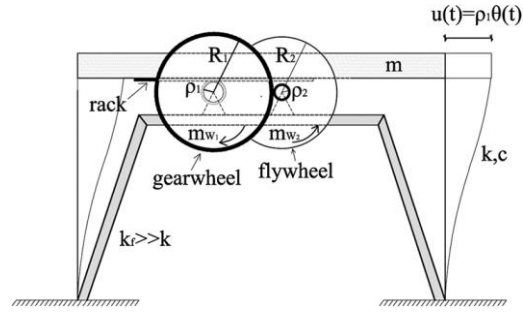


Figure 0-2. Two flywheels and a clutch system forming a clutch inerter damper (Makris & Kampas, 2016)

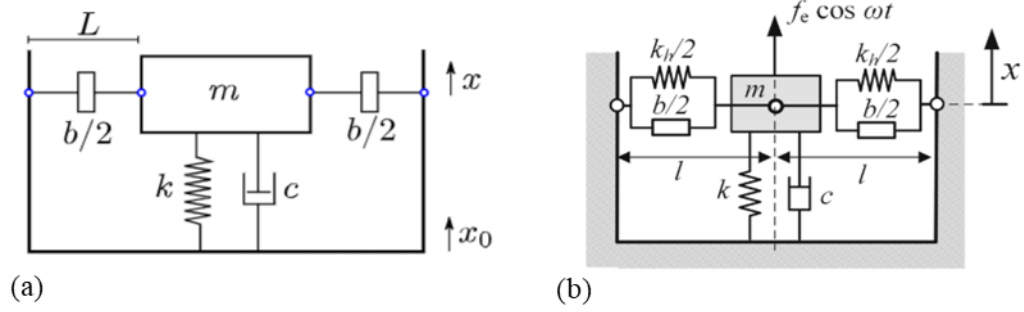


Figure 0-3. (a) Geometrically nonlinear inerter, (b) quasi-zero isolation with inerter attachment. (Moraes et al., 2018; J. Yang et al., 2020)

with respect to the structural mass. This setup creates a geometric nonlinearity because the effective inertia changes with the displacement of the structure, leading to variable inertial forces that enhance vibration isolation over a broader frequency range (Moraes et al., 2018). Through varying the gear ratio in a continuously variable transmission (CVT) or varying the lead screw or ball screw pitch, functionally variable inertance can be generated and the inertance generated in this case is function of the displacement of the structure. In an adaptive tuned mass inerter damper (ATMID), the CVT mechanism works alongside an inerter to retune its natural frequency by adjusting the inertance and damping to match the excitation frequency. This configuration results in an adaptive inertance that enhances the oscillation suppression more than traditional passive devices (Ali Sadeghian et al., 2021).

Additionally, the variable lead mechanism, as discussed by Faraj et al., leverages a ball-screw inerter to mitigate impact loadings by adjusting the thread lead dynamically. When the structure attached with the variable lead mechanisms moves, the variable thread lead changes the inertance and introduces a damping like term, providing a constant decelerating force (Faraj et al., 2019).

This dissertation focuses on a functionally varied nonlinear RIM called the variable inertia rotational mechanism (VIRM) where the flywheel rotational inertia can shift due to moving masses incorporated inside the flywheel. The next section discusses the previous research on VIRM and identifies gaps in the current understanding of passive VIRM.

2.3 Variable Inertia Rotational Mechanism

The variable inertia rotational mechanism (VIRM) features a 'host' flywheel frame with symmetrically spaced discrete masses, guide tracks and springs, as illustrated in Figure 0-4. The design of the VIRM allows the flywheel's rotational inertia to adjust in response to changing operational conditions. As the rotational velocity of the flywheel increases, the centrifugal force drives the masses towards the outer boundary of the guide track. Conversely, the restoring force of the springs make the masses slide back to their original position as the rotational velocity decreases, creating a variable equivalent mass effect. The guide track limits the mass movement along the radial direction and the guide boundary restricts the maximum mass displacement. This unique ability to adjust the flywheel's inertia in real-time has drawn significant interest of researchers in the field of structural dynamics and control systems, particularly in the application energy storage, power system stability and vibration control. While the term 'variable

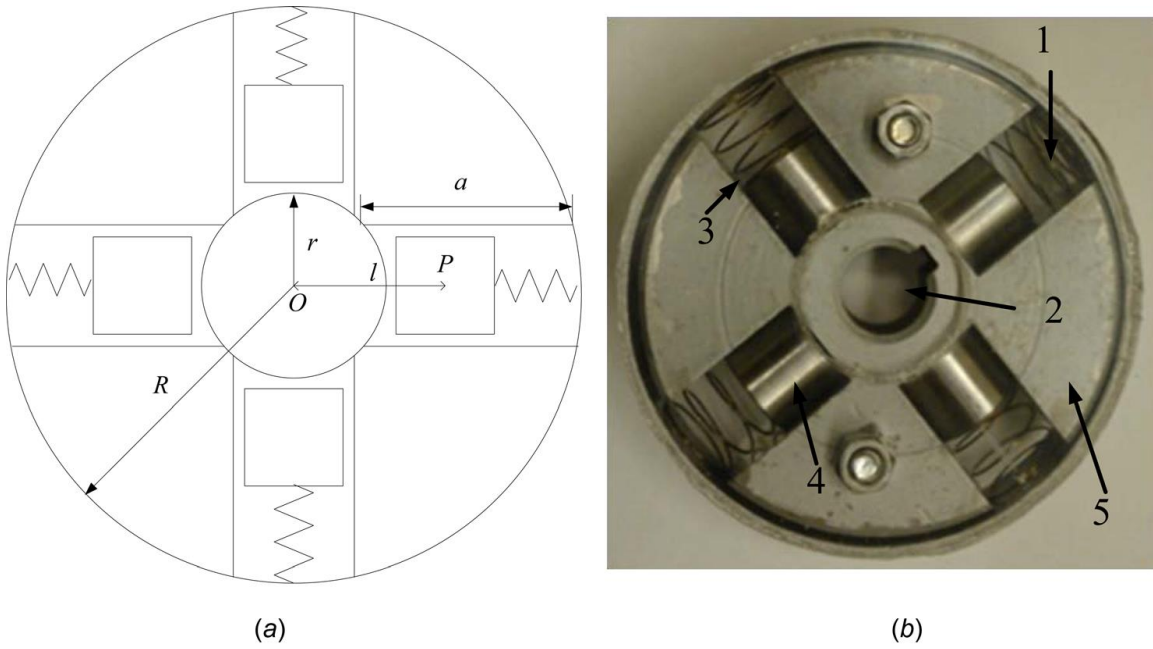


Figure 0-4. VIRM device: (a) schematic diagram and (b) physical model (1- spring, 2- inner hole of the shaft, 3- slot, 4 – slider and 5- frame) (S. Yang et al., 2016a)

inertia flywheel' has been used in some previous literature to describe this mechanism, the more of technically precise term 'variable inertia rotational mechanism' is used in this dissertation, as it falls under the subcategory of rotational inertia mechanisms.

While VIRMs have definite benefits in various applications, the current body of research is very limited, with only a few studies investigating the fundamental characteristics, conceptual design experimentation, and modeling in targeted applications. Among the early explorations of VIRMs, Ullman and Velkoff's work provides a foundational introduction to the concept of VIRM, a simple mathematical model and a potential design. Ullman named this device the variable inertia flywheel (VIF) as a type of passive control device for innovative energy storage solutions. His theoretical work laid the groundwork for the VIF's ability to provide controllable energy storage solutions and vibration control by varying the distribution of mass within a flywheel (Ullman, 1978).

Recent work by Xu and Yang contributed the understanding and application of passive VIRM in vehicle suspension systems, demonstrating its versatility and effectiveness in dynamic environments (Xu et al., 2015a; S. Yang et al., 2016a, 2019). In 2015, Xu et al. proposed a two-terminal mass-based vibration absorber with a variable moment of inertia (VMI) for passive vibration suspension. This system uses a hydraulic mechanism to convert linear motion into rotational motion, adjusting the position of the embedded slider masses in response to the vehicle's vertical oscillation. Their numerical model focused on vehicle dynamics under impulse and harmonic load excitations and the results show improved response time, better road handling and enhanced rider comfort compared to the constant moment of inertia counterpart (Xu et al., 2015a). Yang et al. extended this concept in 2016 by mounting the VMI on a two-terminal hydraulic device and validated its varying force generation due to changing mass effects against a mathematical model. Figure 0-5 (a) presents the experimental rig used for measuring variable inertia force in Yang's study. The results showed good agreement between the experimental data and model predictions with the flywheel effectively generating variable inertia forces in response to excitation inputs (S. Yang et al., 2016a). In a subsequent study in 2019, Yang introduced an adaptive lightweight, high-strength nylon flywheel attached to an inverse screw mechanism. Simulations showed that the adaptive flywheel provides better performance in terms of rider comfort and tire grip compared to the traditional suspension systems, especially under zero and impulse inputs (S. Yang et al., 2019).

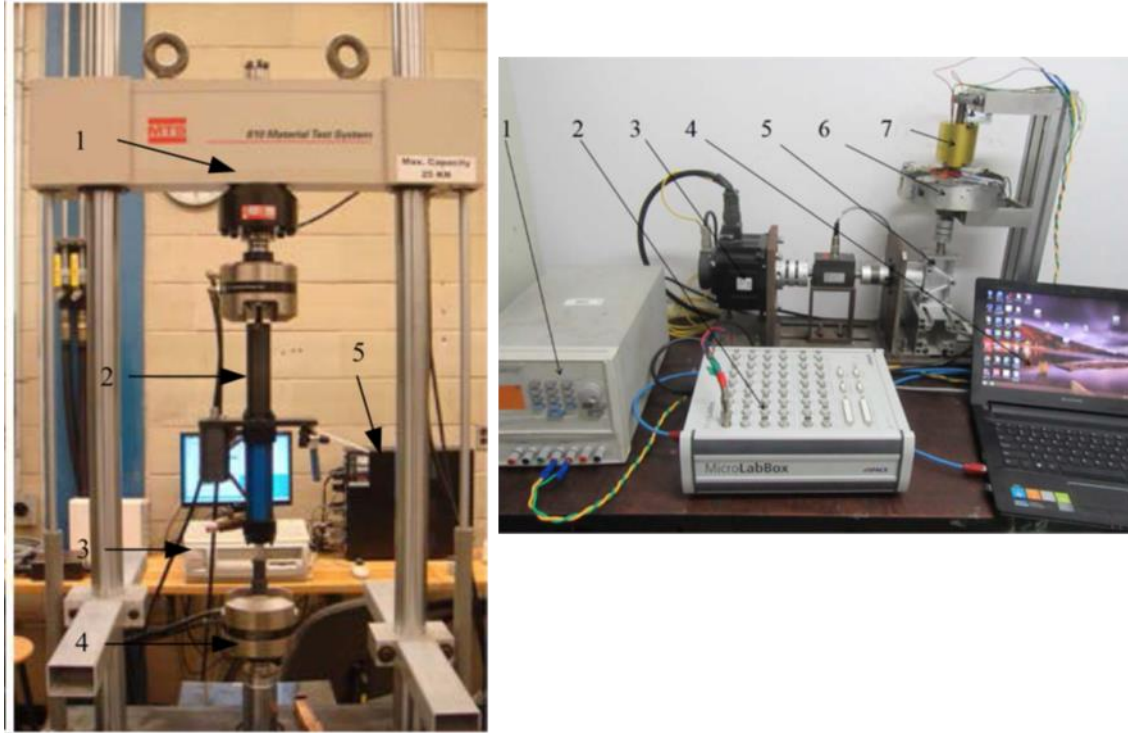


Figure 0-5. Moment of inertia test setups in different studies (a) VIF test rig (Notations: 1-force cell, 2-two-terminal hydraulic system, 3-computer, 4-actuator, and 5-controller) (S. Yang et al., 2016a), (b) MRVIF test rig (Notations: 1- DC power, 2- DAQ, 3- servo motion, 4- computer, 5- bevel gear, 6- MRVIF, 7- conductive slip ring) (Dong et al., 2018)

While the VIRMs employed in systems in the discussion so far have been incorporated as passive control systems, Dong et al. proposed a magneto-rheological VIF (MRVIF) for semi-active vibration control (Dong et al., 2018). Figure 0-5(b) shows the experimental setup of the MRVIF study. The results show that the MRVIF adjusts the moment of inertia via MR dampers, placed in the VIF, in the similar way as the slider masses, and allows semi-active tuning of the flywheel's inertia. Following this innovation, Li et al. integrated semi-active adaptive inerter and damper using the MR technology (W. Li et al., 2021). This work integrates variable damping and inertia, controlled by a fuzzy PID controller, and demonstrated superior performance in ride comfort and suspension effectiveness compared to traditional vehicle MR suspensions.

The flywheel has been employed in rotating machinery as an energy storage solution to release stored kinetic energy to accommodate transient load requirements. Due to its complex structure and maintenance difficulty, it has rarely been used to decrease the speed fluctuations of rotating machineries. In 2010, Yuan et al. focused on integrating a VIF into a diesel generator unit under pulse load (Yuan et al., 2010). In 2019, Zhang et al. advanced the understanding of VIF's application in the diesel generator by proposing a passive VIF to improve the speed stability of diesel generators against sudden loading impacts (Y. Zhang et al., 2020). Both studies highlight the advantages of this application, particularly improving speed stability, reducing sensitivity to loading impacts and enhancing the power quality of diesel generators. Jauch integrated VIFs within the wind turbine rotor. This system employs internal weights within the rotor blades; by shifting these weights closer to the center or the tip of the blades, the system can effectively control the flywheel's inertia. Numerical simulation of this work demonstrated that the controllable inertia addresses the transient wind speed variations and mitigates the power output fluctuations from wind turbines, thereby enhancing the stability of the power system frequency. Unlike traditional methods that employ separate flywheel units, integrating the flywheel within the wind turbine rotor leverages the large radius of modern rotor blades to achieve significant inertia effects (Jauch, 2015). In recent years, VIF's impact on hydraulic power systems have also been investigated, and the results showed that VIF significantly reduce speed fluctuations in hydraulic motor drives, enhances reliability and vibration reduction (Kushwaha et al., 2020; Mahato et al., 2019a).

Additionally, researchers have recognized the potential of harnessing the dynamic inertia of VIFs for efficient energy harvesting. Su and Liu analyzed hybrid power systems incorporating VIF for energy storage and regeneration. Their study demonstrated that VIFs could significantly

enhance the efficiency of hybrid systems, particularly in heavy-vehicle applications with frequent stop-and-go operations. The VIF improved energy recovery performance by adjusting the inertia dynamically to match varying operational conditions, leading to substantial efficiency gains compared to traditional hybrid systems (Su & Liu, 2010). Li et al. and Yang et al. explored VIF in a wave energy harvester by combining with a coaxial mechanical motion rectifier (MMR) to improve power output and efficiency (Q. Li et al., 2021; Y. Yang et al., 2021). Chen and Yang also developed a VIF-based vibration energy harvester for bicycles by incorporating a directional VIF and a coaxial MMR. Laboratory and road tests revealed that the harvester achieved an average output power of 2.11 W, effectively powering onboard devices such as GPS and Wi-Fi modules (P. Chen & Yang, 2023). Table 0-2 showcases various VIRM research milestone.

2.3.1 *Knowledge gaps in VIRM research*

While the aforementioned studies have contributed significantly to the understanding of VIRMs, several key knowledge gaps remain to be addressed.

Modeling complexity and validation: Most research on VIRMs has relied on numerical simulations using simplified equations of motion, often neglecting factors like tangential velocity of masses and complex spring forces, which limits the accuracy of these models (Xu et al., 2015a; S. Yang et al., 2016a; Y. Zhang et al., 2020). While some experimental studies have been conducted to validate certain aspects of VIRM behavior, these have typically focused on moment of inertia investigation of VIRM systems rather than their integration and effect on structures (Dong et al., 2018; S. Yang et al., 2016a). In this dissertation, the complex nonlinearities of the VIRM system are thoroughly characterized using mathematical modeling and verified through comprehensive experimental studies.

Experimental investigation of VIRM-structure interaction: A key gap in the research is the lack of experimental investigation into the dynamic behavior of VIRMs when coupled with other structures. The current experimental setups, such as uniaxial testing machines, have limitations in accurately replicating the real-world boundary conditions encountered in practical applications (P. Chen & Yang, 2023; Xu et al., 2015a; S. Yang et al., 2016a). This lack of experimental investigation leaves the combined VIRM-structure behavior uncertain. In this investigation, the variable inertia rotational mechanism (VIRM) is integrated into a single-degree-of-freedom (SDOF) structural system and comprehensively tested under various loading conditions and amplitudes. The response of the coupled VIRM-structure system is then systematically evaluated.

Table 0-2. VIRM research in various applications

Year	Authors	Contribution	Type of VIRM	Tested	Application
1978	Ullman and H. Velkoff	Introduced fundamental concept, and potential design	Passive	Theoretical	Energy storage
2010	Su and Liu	Improved energy recovery of hybrid power system	Passive	Numerical	Energy harvesting
2015	Jauch	Demonstrated stable power and higher energy storage in wind-power applications	Passive	Theoretical	Energy storage and stability of wind turbine
2015	Xu et al.	Developed hydraulic vibration absorber with VIRM for vehicle suspensions, increased rider comfort	Passive	Experimental	Vibration control
2016	Yang et al.	Used uniaxial testing machine to characterize VIRM in vehicle suspensions, demonstrated variable mass	Passive	Experimental	Vibration control
2018	Dong et al.	Studied semi-active VIRM with MR fluid for adjusting inertia, validated with MRVIF prototype	Semi-active	Numerical and Experimental	Vibration control
2019	Mahato et al.	Mounted VIRM on hydraulic device for energy absorption and stability, focused on semi-active systems	Passive	Numerical	Stability of machinery
2020	Zhang et al.	Proposed VIRM for speed stability in diesel generators, modeled in Simulink	Passive	Numerical	Stability of machinery
2020	Kushwaha et al.	Compared dynamic responses of variable and fixed inertia flywheels in hydraulic systems, validated experimentally	Passive	Numerical and Experimental	Stability of machinery
2021	Q. Li et al.	Improved energy harvesting in wave energy converters by adjusting resonant frequency	Passive	Numerical	Energy harvesting
2021	W. Li et al.	Studied semi-active VIRM with MR fluid, focused on vehicle suspension applications	Semi-active	Numerical and Experimental	Vibration control

Limited applications in structural vibration mitigation: Most of the existing research on VIRMs has focused on applications in energy storage and mechanical systems, such as diesel generators and hydraulic drives (Kushwaha et al., 2020; Y. Zhang et al., 2020). However, there remains a significant gap in understanding the integration and impact of VIRMs in structural systems like buildings and bridges. In this dissertation, VIRMs have been integrated into simplified SDOF and MDOF structures. The simplified lumped mass model analysis of VIRM installed structures can help to identify the key parameters and provide guidance for future research into more complex structural systems.

In summary, the dissertation aims to address the aforementioned knowledge gaps. The findings of this research will contribute to the deeper understanding of VIRMs and their potential for passive control of structures under extreme dynamic loading. The research will focus on developing more accurate models that capture the complex nonlinear dynamics of VIRMs and conducting experimental investigations to validate the VIRM-structure behavior. The outcomes of this dissertation are expected to provide valuable insights and guidelines for future research and practical implementations of VIRMs for enhancing resilience of structures.

CHAPTER 3 Mathematical model and response measures

This chapter provides a detailed derivation of the mathematical model of the variable inertia rotational mechanism (VIRM) installed in single-degree-of-freedom (SDOF) and two-degree-of-freedom (2DOF) primary structures, which are investigated in the subsequent chapters of the dissertation. The analyses conducted in Chapter 4 and 5 utilize these mathematical models. In Chapter 6, the model is refined, and validated through experimental testing. Additionally, this chapter introduces various response measures employed in the later chapters. These include different natural frequency measures, and performance measures using H2 norm analysis and incremental dynamic analysis.

3.1 Mathematical Model

In this dissertation, the VIRM is analyzed in both SDOF and 2DOF structures. This section provides a detailed derivation of the mathematical model for a VIRM attached to a SDOF primary structure. Subsequently, the formulation of different VIRM configurations on a 2DOF structure is presented. To compare the VIRM behavior and effectiveness, the fixed inertia rotational mechanism (FIRM) is also studied in this dissertation. The FIRM is a rotational inertia mechanism (RIM) where the masses are fixed in a position in the flywheel and is examined within both SDOF and 2DOF structures, and the corresponding mathematical model are provided in this section.

The VIRM is attached between the ground and the lumped mass (m_s) of the SDOF system, as shown in Figure 0-1.a. The VIRM is in parallel with a spring with stiffness (k_s) and a viscous damper with damping coefficient (c_s). When the structure is subjected to an external force $P(t)$ and the absolute rotational velocity ($\dot{\theta}$) of the flywheel increases, masses (m_{sd}) inside the flywheel move radially. The rotational velocity of the flywheel is proportionally related to the relative velocity of the structure between the terminals of the VIRM

$$\dot{\theta} = \alpha \dot{u}_s \quad (3.1)$$

where $\dot{u}_s$ is the relative velocity of the structure between the terminals of the VIRM and α is the coefficient governing this relationship. α is equal to $\frac{2\pi}{\rho}$ for the ball screw assembly where ρ is the lead of the ball screw.

The moment of inertia of the VIRM flywheel comprises a constant and a variable component. The constant portion of the moment of inertia is expressed as

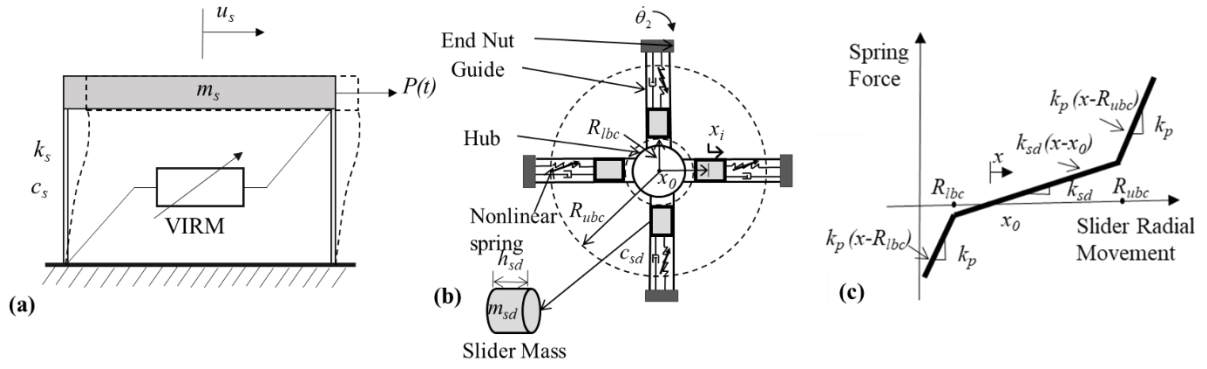


Figure 0-1. (a) SDOF structure with a VIRM attached (b) schematic diagram of VIRM with slider masses at its initial position, (c) spring force-displacement relationship of the nonlinear spring attached to the slider masses of VIRM

$$J_{\text{constant}} = \left(\frac{1}{12} n m_{sd} \left(\frac{3}{4} d_{sd}^2 + h_{sd}^2 \right) + \frac{1}{2} m_c r^2 \right) \quad (3.2)$$

where n , m_{sd} , d_{sd} , h_{sd} , m_c and r are the number of sliders in the VIRM, slider masses, slider diameter, slider height, mass of the flywheel without the sliders, and radius of the flywheel, respectively. The variable portion of the moment of inertia depends on the radial position of the slider masses (x). As the sliders move within the guide tracks of the VIRM flywheel, the moment of inertia changes. The larger the moment of inertia of the VIRM, the more pronounced are the mass effects to the structure. With the exception of the slider masses' mobility during the structural response, the VIRM and FIRM flywheels have identical physical properties. In addition to the constant moment of inertia portion of the FIRM flywheel, the fixed slider position also contributes to the moment of inertia of the FIRM. The fixed slider mass position is denoted using 'y', which is set as either x_0 , the initial position of the masses in the VIRM or R_{ubc} , the upper bound contact point. The moment of inertia of the VIRM and FIRM flywheels can be expressed as

$$\begin{aligned} J_{\text{VIRM}} &= J_{\text{constant}} + n m_{sd} x^2 \\ J_{\text{FIRM}} &= J_{\text{constant}} + n m_{sd} y^2 \end{aligned} \quad (3.3)$$

The nonlinear springs in the VIRM are assumed to have a trilinear elastic force-displacement relationship with a soft central spring (k_{sd}) and two equal but stiffer penalty spring segments (k_p), as illustrated in Figure 0-1c. The relatively stiff penalty spring stiffness is activated when the slider masses approach the center or close to the end of the guide tracks of the flywheel, denoted by R_{lbc} and R_{ubc} , to restrain the movement of the sliders inside the flywheel. Note that, R_{lbc} and R_{ubc} are not the actual minimum and maximum values of the flywheel's radius, rather the initiation position of the penalty springs. This force displacement relationship is similar to the spring behavior as it becomes stiffer when fully compressed.

The equation of motion of the primary structure with VIRM can be derived using Lagrange's equation. The Lagrangian, L , is defined as

$$L = T - V \quad (3.4)$$

where T presents the structure's kinetic energy and V presents the potential energy. The kinetic energy includes the structure's motion, the rotational velocity of the VIRM flywheel, and velocity of the slider masses. It is assumed that the multiple slider masses in the VIRM are moving

synchronously with the same radial motion. The potential energy consists of the energy stored in the structure's springs, and the nonlinear VIRM springs. It is noted that the gravitational effect on the VIRM slider movement is disregarded in this derivation. While gravity does not influence a horizontally positioned flywheel, its effect on a vertically positioned flywheel can potentially have a significant impact on the VIRM's dynamics.

The kinetic energy, potential energy and the virtual work of the non-conservative force, ∂W_{NC} , of the structure is formulated as

$$T = \frac{1}{2} m_s \dot{u}_s^2 + \frac{1}{2} J_{\text{VIRM}} \dot{\theta}^2 + n \frac{1}{2} m_{sd} \dot{x}^2 \quad (3.5)$$

$$V = \frac{1}{2} k_s u_{s_1}^2 + V_{sd}(x) \quad (3.6)$$

$$\partial W_{NC} = (P(t) - c_s \dot{u}_s) \partial u_s - n c_{sd} \dot{x} \partial x \quad (3.7)$$

where $V_{sd}(x)$ in Equation (3.6) is the potential energy of the VIRM trilinear springs and can be expressed as

$$V_{sd} = \left\{ \begin{array}{l} n \frac{1}{2} k_{sd} (R_{lbc} - x_0)^2 + n \frac{1}{2} k_p (x_i - R_{lbc})^2 + n k_{sd} (R_{lbc} - x_0)(x - R_{lbc}); x < R_{lbc} \\ n \frac{1}{2} k_{sd} (x - x_0)^2; R_{lbc} \leq x < R_{ubc} \\ n \frac{1}{2} k_{sd} (R_{ubc} - x_0)^2 + n \frac{1}{2} k_p (x - R_{ubc})^2 + n k_{sd} (R_{ubc} - x_0)(x - R_{ubc}); x \geq R_{ubc} \end{array} \right\} \quad (3.8)$$

The generalized coordinates of the Lagrange's equation for the structure is given as

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = p_i(t), \quad i = 1, 2; \quad (3.9)$$

where q_1 and q_2 are the displacement of the primary structure and the radial displacement of the slider mass. The resulting equation of motion of the VIRM combined with the primary structure can be expressed as

$$\begin{aligned} m_s \ddot{u}_s + J_{VIRM} \alpha^2 \ddot{u}_s + 2nm_{sd} x \dot{x} \alpha^2 \dot{u}_s + nm_{sd} x^2 \alpha^2 \ddot{u}_s + c_s \dot{u}_s + k_s u_s &= P(t) \\ m_{sd} \ddot{x} - m_{sd} x \alpha^2 \dot{u}_s^2 + F_{bsd}(x) + c_{sd} \dot{x} &= 0 \end{aligned} \quad (3.10)$$

where $F_{bsd}(x)$ represents the restoring force from the trilinear springs, defined as

$$F_{bsd} = \begin{cases} k_{sd}(R_{lbc} - x_0) + k_p(x - R_{lbc}), & x < R_{lbc} \\ k_{sd}(x - x_0), & R_{lbc} \leq x < R_{ubc} \\ k_{sd}(R_{ubc} - x_0) + k_p(x - R_{ubc}), & x \geq R_{ubc} \end{cases} \quad (3.11)$$

For comparative analysis of the FIRM structures, the VIRM flywheels are replaced with identical FIRM flywheels. The equations of motion of the FIRM in a SDOF primary structure under external force, $P(t)$ is expressed as

$$m_s \ddot{u}_s + J_{FIRM} \alpha^2 \ddot{u}_s + c_s \dot{u}_s + k_s u_s = P(t) \quad (3.12)$$

Similar to VIRM in a SDOF structure, the Lagrange's equation is taken to derive the equation of motion of a 2DOF structure with VIRM. In this dissertation, two configurations of VIRMs were installed to a 2DOF structure (See Figure 0-2), one configuration - referred to as 2VIRM structure - is where one VIRM is installed between the ground floor and the first floor and another VIRM between first floor and second floor, and another configuration - referred to as 1VIRM structure - is where only one VIRM is installed between the ground and first floor. The first and second story masses, stiffness and viscous damping coefficients are m_{s_1} and m_{s_2} , k_{s_1} , and k_{s_2} , and c_{s_1} and c_{s_2} , respectively and VIRMs on both floors have identical properties with the same notations as the VIRM in SDOF structure.

The potential energy, kinetic energy and the virtual work of the non-conservative forces of the 2DOF structure under ground motion excitation, $(u_g, \dot{u}_g)$ are presented in Equation (3.13).

$$\begin{aligned} T &= \frac{1}{2} m_{s_1} (\dot{u}_{s_1} + \dot{u}_g)^2 + \frac{1}{2} m_{s_2} (\dot{u}_{s_2} + \dot{u}_g)^2 + \frac{1}{2} J_{VIRM_1} \dot{\theta}_1^2 + \frac{1}{2} J_{VIRM_2} \dot{\theta}_2^2 + n_1 \frac{1}{2} m_{sd_1} \dot{x}_1^2 + n_2 \frac{1}{2} m_{sd_2} \dot{x}_2^2 \\ V &= \frac{1}{2} k_{s_1} (u_{s_1} - u_g)^2 + \frac{1}{2} k_{s_2} (u_{s_2} - u_{s_1})^2 + V_{sd_1}(x_1) + V_{sd_2}(x_2) \\ W_{NC} &= (c_{s_2} \dot{u}_{s_2} - (c_{s_1} + c_{s_2}) \dot{u}_{s_1} + c_{s_1} \dot{u}_g) \partial u_{s_1} + (c_{s_2} \dot{u}_{s_1} - c_{s_2} \dot{u}_{s_2} + P(t)) \partial u_{s_2} - n_1 c_{sd_1} \dot{x}_1 \partial x_1 - n_2 c_{sd_2} \dot{x}_2 \partial x_2 \end{aligned} \quad (3.13)$$

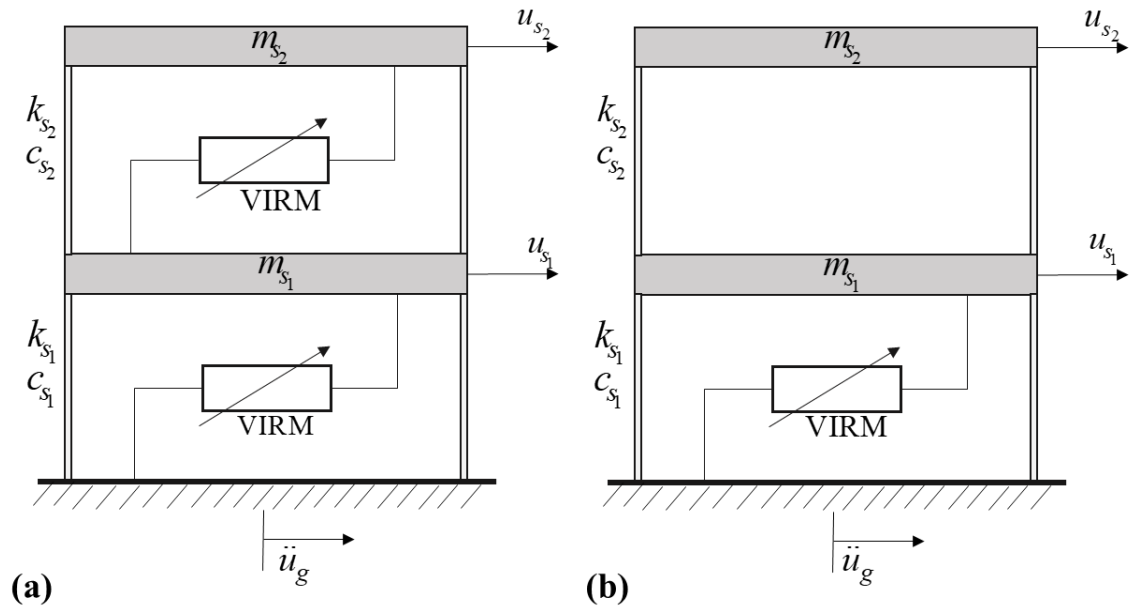


Figure 0-2. (a) 2DOF primary structure with two VIRMs (2VIRM structure) (b) 2DOF primary structure with one VIRM (1VIRM structure)

Here u_g and $\dot{u}_g$ indicate the ground displacement and ground velocity, respectively. Additionally, '1' and '2' subscripts denote the properties of the 1st floor and 2nd floor and the VIRM attachments between the ground and 1st floor and the 1st and the 2nd floor. Due to the presence of two VIRM at both stories in a 2VIRM structure, there are two potential energy components, (V_{sd_1}, V_{sd_2}) , each of them can be identified using Equation (3.8). The equations of motion of the 2VIRM structure can have influence of a combination of V_{sd_1} and V_{sd_2} expressions between the two VIRMs (a total of nine) as the potential energy depends on the radial position of the slider masses.

The equations of motion of the 2VIRM structure and 1VIRM structure subjected to an absolute ground motion acceleration are

$$\begin{aligned}
& m_{s_1} \ddot{u}_{s_1} + J_{VIRM_1} \alpha^2 \ddot{u}_{s_1} - J_{VIRM_2} \alpha^2 \ddot{u}_{s_2} + (2n_1 m_{sd} x_1 \alpha^2 \dot{u}_{s_1}) \dot{x}_1 - (2n_2 m_{sd} x_2 \alpha^2 \dot{u}_{s_2}) \dot{x}_2 \\
& + (c_{s_2} + c_{s_1}) \dot{u}_{s_1} - c_{s_2} \dot{u}_{s_2} + (k_{s_1} + k_{s_2}) u_{s_1} - k_{s_2} u_{s_2} = -m_{s_1} \ddot{u}_g \\
& m_{s_2} \ddot{u}_{s_2} + J_{VIRM_2} \alpha^2 \ddot{u}_{s_2} + 2n_2 m_{sd} x_2 \alpha^2 \dot{u}_{s_2} \dot{x}_2 + k_{s_2} (u_{s_2} - u_{s_1}) - c_{s_2} \dot{u}_{s_1} + c_{s_2} \dot{u}_{s_2} = -m_{s_2} \ddot{u}_g \\
& m_{sd} \ddot{x}_1 - m_{sd} x_1 \alpha^2 \dot{u}_{s_1}^2 + F_{bsd_1} (x_1) + c_{sd} \dot{x}_1 = 0 \\
& m_{sd} \ddot{x}_2 - m_{sd} x_2 \alpha^2 \dot{u}_{s_2}^2 + F_{bsd_2} (x_2) + c_{sd} \dot{x}_2 = 0
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
& m_{s_1} \ddot{u}_{s_1} + J_{VIRM_1} \alpha^2 \ddot{u}_{s_1} + (2n m_{sd} x_1 \alpha^2 \dot{u}_{s_1}) \dot{x}_1 + (c_{s_2} + c_{s_1}) \dot{u}_{s_1} - c_{s_2} \dot{u}_{s_2} + (k_{s_1} + k_{s_2}) u_{s_1} - k_{s_2} u_{s_2} = -m_{s_1} \ddot{u}_g \\
& m_{s_2} \ddot{u}_{s_2} + k_{s_2} (u_{s_2} - u_{s_1}) + c_{s_2} (\dot{u}_{s_2} - \dot{u}_{s_1}) = -m_{s_2} \ddot{u}_g \\
& m_{sd} \ddot{x}_1 - m_{sd} x_1 \alpha^2 \dot{u}_{s_1}^2 + F_{bsd_1} (x_1) + c_{sd} \dot{x}_1 = 0
\end{aligned} \tag{3.15}$$

Additionally, the equations of motion of the two FIRMs in a 2DOF structure and one FIRM in a 2DOF structure under ground motion are shown in Equation (3.15) and (3.16), respectively.

$$\begin{aligned}
& m_{s_1} \ddot{u}_{s_1} + J_{FIRM_1} \alpha^2 (\ddot{u}_{s_1} - \ddot{u}_g) - J_{FIRM_2} \alpha^2 (\ddot{u}_{s_2} - \ddot{u}_{s_1}) + (c_{s_2} + c_{s_1}) \dot{u}_{s_1} - c_{s_2} \dot{u}_{s_2} - c_{s_1} \dot{u}_g \\
& + (k_{s_1} + k_{s_2}) u_{s_1} - k_{s_2} u_{s_2} = -m_{s_1} \ddot{u}_g \\
& m_{s_2} \ddot{u}_{s_2} + J_{FIRM_2} \alpha^2 (\ddot{u}_{s_2} - \ddot{u}_{s_1}) + k_{s_2} (u_{s_2} - u_{s_1}) - c_{s_2} \dot{u}_{s_1} + c_{s_2} \dot{u}_{s_2} = -m_{s_2} \ddot{u}_g
\end{aligned} \tag{3.16}$$

$$\begin{aligned}
& m_{s_1} \ddot{u}_{s_1} + J_{FIRM_1} \alpha^2 \ddot{u}_{s_1} + (c_{s_2} + c_{s_1}) \dot{u}_{s_1} - c_{s_2} \dot{u}_{s_2} + (k_{s_1} + k_{s_2}) u_{s_1} - k_{s_2} u_{s_2} = -m_{s_1} \ddot{u}_g \\
& m_{s_2} \ddot{u}_{s_2} + k_{s_2} (u_{s_2} - u_{s_1}) + c_{s_2} (\dot{u}_{s_2} - \dot{u}_{s_1}) = -m_{s_2} \ddot{u}_g
\end{aligned} \tag{3.17}$$

The equations of motion of the structure with the FIRM is linear in contrast to the structure with the VIRM which exhibit nonlinear equations of motion. The nonlinear components include a coefficient on an acceleration term that grows with increased radial position of the slider, as well as a complicated nonlinear term combining slider velocity, slider position, and primary system velocity. Although several previous studies have developed VIRM EOM, most of them did not account for the tangential velocity component of the sliders (Xu et al., 2015a; J. Yang et al., 2020; Y. Zhang et al., 2020). For example, the kinetic energy generated from the tangential velocity component of the sliders are ignored. This work accounts for forces acting radially and tangentially to the direction of motion. Additionally, unlike most other work in this area, penalty springs are considered to restrain the excessive motion of the slider masses (Y. Wang et al., 2019; Xu et al., 2015a; S. Yang et al., 2016a). Also, the effect of gravity is ignored in this study, as in other studies. Although some of the previous research have considered frictional forces (Dong et al., 2018; C. Li et al., 2012; Y. Zhang et al., 2020), due to the nominal frictional effect it has been ignored in this study.

3.2 Response Measures

In this study, the response of the structure with the VIRM or the FIRM are simulated under different loadings using the MATLAB implicit solver (MathWorks, 2022a) with an output frequency set at 4000 Hz. The loading considered in different chapters of this dissertation are harmonic excitations with different input frequencies, initial displacements, and white noise excitation.

To quantitatively evaluate the response of the dynamic systems studied, several measures are calculated from the system responses. The natural frequency measures, which are described in detail below, include the quantitative measures, the instantaneous frequency (ω_{IF}), the weighted average instantaneous frequency ($\bar{\omega}_w$), and the overall fundamental frequency (ω_0), and a qualitative measure identified using the wavelet spectrum. The ω_{IF} , $\bar{\omega}_w$, and ω_0 are normalized using the natural frequency of the structure without the VIRM or the FIRM, ω_{s_0} , which can be expressed as

$$\Psi_{\omega_{IF}} = \frac{\omega_{IF}}{\omega_{s_0}}, \quad \Psi_{\bar{\omega}_w} = \frac{\bar{\omega}_w}{\omega_{s_0}}, \quad \Psi_{\omega_0} = \frac{\omega_0}{\omega_{s_0}} \quad (3.18)$$

where $\Psi_{\omega_{IF}}$, Ψ_{ω_w} , and Ψ_{ω_0} are the normalized ω_{IF} ratio, normalized $\bar{\omega}_w$ ratio and normalized ω_0 ratio, respectively.

Additionally, the harmonic loading frequency (ω_l) and the response frequency (ω_r) considered in the study are also normalized by the natural frequency of the structure without the VIRM or the FIRM. These are expressed as

$$\Psi_l = \frac{\omega_l}{\omega_{s_0}}, \Psi_r = \frac{\omega_r}{\omega_{s_0}} \quad (3.19)$$

where Ψ_l and Ψ_r are the loading frequency ratio and response frequency ratio, respectively.

To evaluate the performance measures, H_2 norm analog (H_2) is determined using the frequency response function. This measure is quantified for both SDOF and MDOF structures, and the results are presented in 0 and 0. Additionally, incremental dynamic analysis (IDA) is performed to measure the effectiveness of VIRM under seismic excitation. In this dissertation, the maximum absolute story drift (MASD) and the maximum absolute story acceleration (MASA) are the response measures quantified from the IDA and presented in 0.

3.2.1 *Natural frequency measures*

3.2.1.1 *Instantaneous natural frequency*

The instantaneous frequency, ω_{IF} , which is a measure of the preferred vibration frequency of a dynamic system, depends on the structure properties and response. For linear primary structures, the ω_{IF} remains constant across time during the structure's response and is equivalent to the structure's natural frequency. However, in nonlinear systems, the ω_{IF} can fluctuate continuously with the structure's response. Various methods exist to determine the ω_{IF} , as outlined in (Frank Pai, 2010; N. E. Huang et al., 2009; P. Wang & Gao, 2013). While these methods typically assume a linear system dynamic response, the ω_{IF} can still be helpful in characterizing specific dynamic systems along their nonlinear time-history response (Moaveni & Asgarieh, 2012). This study utilizes tangent mass and tangent stiffness matrices to derive instantaneous modal parameters through eigenvalue analysis of the linearized system of the

nonlinear structure. The detailed process of determining the ω_{F} at each time instant is presented below.

In a nonlinear structure, the general force equilibrium equation can be expressed as

$$\mathbf{P}_{\text{ext}} = \mathbf{F}_{\text{int}}(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}) \quad (3.20)$$

where $\mathbf{P}_{\text{ext}}$ and $\mathbf{F}_{\text{int}}$ present the externally applied loads and internal forces, respectively. The internal forces are functions of the system degrees of freedom (DOF) $(\mathbf{u}_0)$ and their derivatives $(\dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0)$. To linearize any nonlinear equilibrium equation, one can perturb the force about a known set of evaluation points, $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, and $\ddot{\mathbf{u}}_0$. A small perturbation in the applied load corresponds to a perturbation in the nodal DOFs. A first-order Taylor series expansion of the internal forces can then be expressed as

$$\mathbf{P}_{\text{ext}} + d\mathbf{P} = \mathbf{F}_{\text{int}}(\mathbf{u}_0, \dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0) + \left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \mathbf{u}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} d\mathbf{u} + \left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \dot{\mathbf{u}}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} d\dot{\mathbf{u}} + \left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \ddot{\mathbf{u}}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} d\ddot{\mathbf{u}} \quad (3.21)$$

As $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, and $\ddot{\mathbf{u}}_0$ satisfy Eq (3.20) and Eq. (3.21), can be reduced to

$$d\mathbf{P} = \left(\left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \mathbf{u}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} \right) d\mathbf{u} + \left(\left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \dot{\mathbf{u}}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} \right) d\dot{\mathbf{u}} + \left(\left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \ddot{\mathbf{u}}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} \right) d\ddot{\mathbf{u}} \quad (3.22)$$

$$d\mathbf{P} = [\mathbf{K}_t(\mathbf{u}_0, \dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0)]d\mathbf{u} + [\mathbf{C}_t(\mathbf{u}_0, \dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0)]d\dot{\mathbf{u}} + [\mathbf{M}_t(\mathbf{u}_0, \dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0)]d\ddot{\mathbf{u}}$$

where $\mathbf{K}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})$, $\mathbf{C}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})$, and $\mathbf{M}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})$ are the tangent stiffness, tangent damping, and tangent mass matrices, respectively. These matrices can be expressed as follows for an n degree of freedom system.

$$\begin{aligned}
[\mathbf{K}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})] &= \frac{\partial \mathbf{F}_{\text{int}}}{\partial \mathbf{u}} = \begin{bmatrix} \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \mathbf{u}_1} & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \mathbf{u}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \mathbf{u}_n} \\ \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \mathbf{u}_1} & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \mathbf{u}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \mathbf{u}_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \mathbf{u}_1} & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \mathbf{u}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \mathbf{u}_n} \end{bmatrix}; [\mathbf{C}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})] = \frac{\partial \mathbf{F}_{\text{int}}}{\partial \dot{\mathbf{u}}} = \begin{bmatrix} \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \dot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \dot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \dot{\mathbf{u}}_n} \\ \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \dot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \dot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \dot{\mathbf{u}}_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \dot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \dot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \dot{\mathbf{u}}_n} \end{bmatrix}; \\
[\mathbf{M}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})] &= \frac{\partial \mathbf{F}_{\text{int}}}{\partial \ddot{\mathbf{u}}} = \begin{bmatrix} \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \ddot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \ddot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \ddot{\mathbf{u}}_n} \\ \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \ddot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \ddot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \ddot{\mathbf{u}}_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \ddot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \ddot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \ddot{\mathbf{u}}_n} \end{bmatrix}
\end{aligned} \tag{3.23}$$

The presence of a VIRM attachment adds an additional degree-of-freedom to the structure. Hence, the eigenvalue analysis presents additional instantaneous frequencies corresponding the VIRM at each time step. For example, the eigenvalue analysis yields two ω_{IF} for VIRM in SDOF structure, three ω_{IF} for the 1VIRM structure and four ω_{IF} for the 2VIRM structures. The ω_{IF} which has substantial engagement with the primary structure's mode shape is considered the ω_{IF} . In the 2DOF structure, the eigenfrequencies associated with the two modes of the primary structure are sorted in descending order and denoted as ω_{IF_1} and ω_{IF_2} , respectively.

3.2.1.2 Weighted average instantaneous frequency

The weighted average instantaneous frequency, $\bar{\omega}_w$, is a unique measure which considers the time-history of the structure's total energy as the weight while averaging the time-history of the ω_{IF} . This measure can help to reflect the overall behavior of the structure and can be calculated as

$$\bar{\omega}_w = \frac{\sum_{i=1}^N W_i \omega_{IF,i}}{\sum_{i=1}^{T_n} W_i} \quad (3.24)$$

where W_i is the total energy and $\omega_{IF,i}$ is the ω_{IF} of the primary structure at time step i , respectively, and N is the total number of time steps considered. For structures with FIRM, since ω_{IF} is constant, the ω_{IF} and $\bar{\omega}_w$ are identical.

3.2.1.3 Overall fundamental frequency

The overall fundamental frequency, ω_0 , helps to evaluate a system's total dynamic response by identifying the response's dominant frequency component. The ω_0 of a linear system remains constant but can shift in nonlinear system due to changes in the loading type and loading amplitude. In this study, the ω_0 is identified using the estimated auto power spectral densities (APSDs) of numerically simulated displacement time-histories of the primary structure. These estimated APSDs are obtained using Welch's averaged, modified periodogram method of spectral estimation (MathWorks, 2022b), with a signal duration of 100 seconds for each analysis. For harmonic and white noise loading, a Hanning window with 10% dataset length and a 50% overlap is used, while for initial displacement condition, an exponential window with no overlap and a decay constant of 0.5 is applied.

Additionally, for both the white noise loading and initial displacement loading conditions, the highest non-zero frequency peak in the APSD function is defined as the ω_0 . However, for a generalized harmonic loading, the highest peak of the auto-spectrum may correspond to the forcing frequency rather than the ω_0 . Therefore, in harmonic loading cases, the ω_0 is defined as the loading frequency that produces the largest peak APSD value, which serves as a pseudo resonance frequency. This measure is evaluated by analyzing the response and APSD across a range of harmonic loading frequencies of the system as the highest peak could instead come from the frequency of the forcing function.

3.2.1.4 Wavelet spectrum

Fourier transform (FT) is a widely used technique to analyze the structural response in the frequency domain, which is utilized to estimate the auto-spectrum and transfer function in this

dissertation. However, to capture the time-dependent frequency characteristics of the VIRM structure, wavelet transform (WT) has also been employed in this dissertation. Both FT and WT decompose signals into linear combinations of basis functions. The key distinction is that WT uses an infinite set of possible wavelet functions (referred to as mother wavelets), while FT uses a single set of basis functions consisting of only sine functions.

Wavelet spectra is produced by the continuous wavelet transform (CWT), which decomposes a function $x(t)$ into the frequency-time domain, defined as

$$W_x(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t) \psi^* \left(\frac{t - \tau}{s} \right) dt \quad (3.25)$$

where s , τ and Ψ^* are the wavelet scale, localized time index and complex conjugate of the normalized mother wavelet, respectively. By evaluating the amplitude of the wavelet coefficient $W_x(\tau, s)$ across a range of wavelet scale and time indices, the wavelet spectrum is obtained, which illustrates the amplitude of the features in the original signal and the variation with time. Depending on the specific mother wavelet used, a relationship may exist between the wavelet scaling factor and the vibration frequency. If the relationship exists, a wavelet spectrum generated with this particular mother wavelet can be used to directly analyze the time-dependent frequency content of a signal. Note that the Morlet wavelet is used as the mother function in this study.

3.2.2 *Performance measures*

3.2.2.1 *H2 norm analog*

The H_2 norm measures the variance amplification and is used to evaluate the effect of the VIRM on the normalized displacement of the primary structure under dynamic loading. The H_2 of the structure is defined by

$$H_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} |H_{zp}(j\omega)|^2 d\omega} \quad (3.26)$$

where $H_{zp}(j\omega)$ is the frequency domain transfer function between the loading and the response.

H_2 norm of a linear time invariant system can be directly computed using the above formulation. However, since the structure with VIRM is nonlinear and lacks an analytical transfer function, an H_2 analog, denoted as $\tilde{H}_2$ is used as the response measure. In the cases of initial displacement and harmonic loading, $\tilde{H}_2$ is calculated by squaring the area of the estimated APSD function under 0 rad/sec and the Nyquist frequency. For white noise case, the estimated transfer function is used instead of the APSD curve to calculate $\tilde{H}_2$. To normalize the $\tilde{H}_2$ response measure (Σ) in the SDOF structure, the $\tilde{H}_2$ of the VIRM equipped structure is normalized by the $\tilde{H}_2$ of the FIRM-equipped structure for initial displacement and white noise case. In harmonic loading scenarios, the peak $\tilde{H}_2$ value from the range of loading frequencies with VIRM is normalized by the peak $\tilde{H}_2$ value with the FIRM. In the 2DOF structure, $\tilde{H}_2$ of each configuration considered is normalized by the $\tilde{H}_2$ of the no-control structure. The normalized $\tilde{H}_2$ response measure (Σ) of the SDOF and 2DOF structure are expressed in Eq. (3.27) and Eq. (3.28).

$$\Sigma = \frac{\tilde{H}_2(\text{VIRM})}{\tilde{H}_2(\text{FIRM (at } x_0)})} \quad (3.27)$$

$$\Sigma = \frac{\tilde{H}_2(\text{RIM configuration})}{\tilde{H}_2(\text{no control})} \quad (3.28)$$

This normalization is performed to quantify the relative comparison of the VIRM's effectiveness in reducing structural response compared to a baseline structure. This approach allows for a consistent evaluation of the VIRM's vibration mitigation capabilities across different loading scenarios. A lower Σ value indicates a greater reduction in structural response achieved by the VIRM compared to the baseline structure.

3.2.2.2 MASD and MASA measures

The impact of the VIRM on structural performance under seismic loads is assessed using incremental dynamic analysis (IDA). This analysis involves conducting numerous nonlinear dynamic analyses of a structural model exposed to a variety of ground motion. Each ground

motion is scaled to multiple intensity measures. Through this process, IDA curves are produced to depict important measures of the resulting structural response in relation to the intensity measures for each ground motion. For this dissertation, the key structural response measures chosen are the maximum absolute interstory displacement (MASD) and the maximum absolute story acceleration (MASA).

The IDA is conducted for a set of 50 ground motions from the FEMA P695 dataset, which includes both far-field and near-field earthquakes (U.S. Department Security & Federal Emergency Agency, 2009). The near-field records are further divided into those with and without pulses, comprising 14 instances each. The ground motions are scaled in two steps, as outlined in Equation (3.29) and (3.30). First, a normalization factor from FEMA P695 is applied to adjust the peak ground velocity of each ground motion to match the median of the dataset. Following this, a scaling factor is used to align the normalized ground motion with the targeted intensity measure for the analyses.

$$\ddot{x}_{g,applied} = \text{normalization factor} * \text{scaling factor} * \ddot{x}_{g,recorded} \quad (3.29)$$

The IDA curves generated in this study use the spectral intensity, S_T , as the intensity measure. In the expression for S_T in Equation (3.30), S_{NRT} represents the median 5% damped spectral acceleration of the normalized record set at the first mode period of the uncontrolled structure. The S_{NRT} values in this work, obtained through linear interpolation from FEMA P695 values based on the uncontrolled structure's first mode period (0.2784 s), are 0.9837 g for near-field earthquakes and 0.7767 g for far-field earthquakes.

$$S_T = S_{NRT} * \text{scaling factor} \quad (3.30)$$

Typically, IDA is conducted until a structure reaches a limit state, such as collapse or yield. However, since the structural models in this study are simplified and do not account for material nonlinearity and collapse, the analysis is conducted up to a predetermined spectral intensity of 2g. To compare and analyze the IDA results, the MASD and MASA measures of the RIM configurations are normalized against the no-control case. Median MASD and median MASA are calculated by aggregating seismic record results at each intensity level for each configuration. Additionally, the median absolute deviation is computed to measure variability, analogous to the standard deviation but using medians.

CHAPTER 4 Investigation of the properties of variable inertia rotational mechanism on the system dynamics and vibration mitigation

This chapter is part of the development of state of the art in variable inertia rotational mechanism. It includes two manuscripts which focuses on the effect of variable inertia on the natural frequency and response of single-degree-of-freedom structures.

The subsection 0, titled ‘Influence of the variable inertia rotational mechanism on natural frequency and structural response’ is a version of a journal manuscript that is under review.

Sarkar A. T., Wierschem, N.E., “Influence of the variable inertia rotational mechanism on natural frequency and structural response,” Manuscript under review as of August 2024

The subsection 4.2, titled ‘Investigation of the impact of slider mass stiffness on the behavior of the variable inertia rotational mechanism for structural vibration mitigation’, is a version of a conference paper presented at the International Modal Analysis Conference 2024.

Sarkar, A.T. and Wierschem, N.E., “Investigation of the impact of slider mass stiffness on the behavior of the variable inertia rotational mechanism for structural vibration mitigation,” 42nd International Modal Analysis Conference (IMAC), Orlando, FL, USA, January 29 – February 1, 2024.

4.1 **Influence of the variable inertia rotational mechanism on natural frequency and structural response**

4.1.1 ***Introduction***

Excessive vibration in civil engineering structures can cause human discomfort, fatigue, damage, and even the collapse of the structure; thus, vibration control is critical in many structures. In recent years, rotational inertial mechanisms, most prominently the inerter, have attracted attention in academia and industry as a potential part of passive vibration control systems. The term ‘inerter’ was proposed by Smith in the context of the force-current analogy between electrical and mechanical networks (M. C. Smith, 2002). The inerter is a two terminal mechanical element that can be realized with various physical means, e.g., the ball-screw assembly, rack-and-pinion, and fluid-based mechanisms (M. C. Smith, 2002, 2008, 2012; Swift et al., 2013). The inerter generates force proportional to the relative acceleration across its two terminals, and the constant of proportionality is known as ‘inertance’, which has the same units as mass. The inerter can produce a mass amplification effect by transforming translational motion into the rotational motion of a flywheel. Hence, the inerter has been applied to a number of novel vibration control systems as well as to reduce the mass of modified conventional vibration control systems. Examples of inerter-based vibration control devices include the tuned-mass-damper-inerter (TMDI), rotational inertia double-tuned mass damper (RIDTMD), rotational inertia viscous damper, inertia mass damper with an electromagnetic damper, friction pendulum inerter system, and inerters combined with an external rocking wall of a frame system (Di Egidio et al., 2021; Giaralis & Petrini, 2017; Z. Huang et al., 2019; Hwang et al., 2007; Jangid, 2022; Javidialesaadi & Wierschem, 2019b; Marian, Laurentiu & Giaralis, Agathoklis, 2017; Nakamura et al., 2014; Z. H. Wang et al., 2019; Zhao et al., 2019).

Most of the previously studied devices featuring rotational inertial components generate constant effective mass, whereas very few provide variable effective mass (W. Li et al., 2021; Mahato et al., 2019a; Xu et al., 2015a; Y. Zhang et al., 2020). Due to their constant effective mass, inerters can be described as linear rotational inertial mechanisms. However, as the research in this field progresses, there is growing interest in understanding the dynamics of nonlinear rotational inertial mechanisms that produce variable inertance. The inertance of such nonlinear rotational inertial mechanisms is not fixed; rather, the inertance can depend on factors related to the device’s response, such as the relative displacement of the device and the rotational velocity of the device’s flywheel. These nonlinear rotational inertial mechanisms can be divided into

categories including functionally varied inertance mechanisms ($b = f(u, \dot{u}, \ddot{u})$, where b is inertance that varies as a smooth function of u , $\dot{u}$, $\ddot{u}$, the displacement, velocity, or the acceleration of the structure) and state-switching mechanisms ($b=b_1$ if condition 1, $b=b_2$ if condition 2, and so on). While this study will consider a functionally varied inertance mechanism, an example of a state switching mechanism is found in the clutch-inerter damper (CID). The CID consists of two flywheels and a passive clutch system, which transforms translational motion into flywheel rotational motion and prevents the kinetic energy in the flywheel from transferring back to the structure (Javidialesaadi & Wierschem, 2019b; Makris & Kampas, 2016; Makris & Moghimi, 2019; Málaga-Chuquitaype et al., 2019; Talley et al., 2023; M. Wang & Sun, 2018).

Functionally varied inertance mechanisms that have attracted much attention in recent years include a geometrically nonlinear inertance mechanism that has been used as a vibration isolator (Moraes et al., 2018; J. Yang et al., 2020). The geometrically nonlinear inertance mechanism features a pair of oblique inerters and the studies' results demonstrated that the geometrically nonlinear inertance mechanism possesses several performance benefits over conventional isolators, such as, improved higher frequency isolation and smaller displacement amplitude and force transmissibility, depending on the frequency range and amplitude of the excitation. Alternatively, other functionally varied inertance mechanisms have been developed by using a controller or a continuously variable transmission to adjust inertance in traditional tuned damper systems. Some examples of these are an adaptive tuned heave plate to mitigate heave motion of floating platforms (Ma et al., 2023), an adaptive tuned viscous inertance damper (Ali Sadeghian et al., 2021), and a tuned mass damper with changeable inertance (Brzeski et al., 2017).

In addition to these, another functionally varied inertance mechanism can be produced by modifying the rotational inertia of a device's flywheel (Dong et al., 2018; Jauch, 2015; Kushwaha et al., 2020; Q. Li et al., 2021; Mahato et al., 2019a; Xu et al., 2015a; Y. Zhang et al., 2020). Nonlinear rotational inertia from the variable moment of inertia of a flywheel can be obtained through flywheel designs that allow the distribution of mass to change during the response of the flywheel; for example, a flywheel with masses that can move within slots radially on the flywheel. These variable inertia rotational mechanisms (VIRM) have the potential to cause significant passive changes to the dynamics of the system they are attached to, which may be exploited as part of a vibration control strategy. However, the majority of VIRM research has been pursued through active or semi-active control methodologies that require input energy and

intricate hardware. This includes work on a variable inertia flywheel (VIF) with energy storage and power control functionality, and VIF with magneto-rheological technology to adjust natural frequency and suppress the torsional vibration of a system (Dong et al., 2018; W. Li et al., 2021; Ullman, 1978; Yuan et al., 2010). In contrast to the above active and semi-active mechanisms, Xu and Yang proposed a VIF for a passive vehicle suspension system and showed improved rider comfort, better road handling and safety and reduced suspension deflection under most circumstances (Xu et al., 2015a; S. Yang et al., 2019). Additionally, researchers have explored installing VIF in various rotary systems such as hydraulic motors, diesel generators, wave energy converters and wind turbine rotors. These studies aimed to enhance stability, increase power absorption bandwidth and enable passive vibration control of the systems (Jauch, 2015; Kushwaha et al., 2020; Q. Li et al., 2021; Mahato et al., 2019a; S. Yang et al., 2016a).

Although research has been conducted on VIRMs to evaluate their effectiveness in reducing the response amplitude of dynamic systems, studies investigating passive VIRMs are rare. Furthermore, the existing limited research on passive VIRMs has predominantly concentrated on their application in rotating machinery systems, while their potential utilization in other types of mechanical or civil engineering structures has been unexplored. The majority of the prior investigations have primarily examined the VIRM's performance under sinusoidal or impulse loading conditions, thus its behavior under random excitation scenarios remain unknown. As a result, there is a notable lack of understanding of the impact of variable rotational inertia on the natural frequency changes in the structure they are utilized in under different loading scenarios. While limited research exists in understanding the effect of constant rotational inertia on a system's natural frequencies (M. Z. Q. Chen et al., 2014), additional investigation is imperative to characterize the effect of variable rotational inertia on the natural frequency and the response of a structure equipped with a VIRM under external excitation. To address these gaps in knowledge, the main objective of this study is to numerically evaluate the effects of a passive VIRM on the natural frequency and response amplitude of a single-degree-of-freedom (SDOF) structure under different types of excitations.

In this study, a passive VIRM is considered in which multiple mass-spring-damper elements are symmetrically positioned on a circular flywheel, which is connected to a ball screw and attached to a SDOF structure. The vibration of the structure with the VIRM drives the ball screw and results in the rotation of the VIRM flywheel. Centrifugal force pushes the masses on the flywheel outward towards the boundary of the baseplate causing a dynamically changing

inertia. In this numerical study, the VIRM is incorporated into a SDOF structure and numerical simulations are carried out given a collection of initial displacement conditions, harmonic loading, and white noise loading to investigate the evolution of the structure's frequencies during the response. The effects of the damper properties of the VIRM on the natural frequency shift, overall fundamental frequency, and the response reduction are also explored. Comparisons in this study are made with a fixed inertia rotational mechanism (FIRM).

The organization of this paper is as follows. In Section 0, the mechanism of the VIRM and a dynamic model of it installed in a SDOF structure are presented. The methodologies for determining relevant response measures, such as the instantaneous frequency, overall fundamental frequency, and an H_2 response measure, are described in Section 4.1.3. The effect of the VIRM on the instantaneous and overall fundamental frequencies are presented in Sections 4.1.4 and 4.1.5, respectively. The effectiveness of the device to reduce the structure's displacement response under different loads is discussed in Section 4.1.6. In Section 4.1.7, the contributions of this study are summarized and conclusions are drawn.

4.1.2 *Variable inertia rotational mechanism*

In this study, the variable inertance of the VIRM is produced by utilizing the flywheel rotational velocity to passively alter the arrangement of masses in the flywheel. A schematic diagram of the VIRM flywheel considered is presented in Figure 0-1(a). This VIRM flywheel consists of a circular plate with four symmetrically spaced mass-spring-damper elements. These masses are constrained such that they can only move radially in the flywheel. When the flywheel is at rest, the springs attached to these masses are at their free length and the masses are located at an initial radial position near the center of the flywheel, x_0 . As the absolute rotational velocity of the flywheel increases, the masses move, in general, radially outward from the center of the flywheel due to the increasing centrifugal force. When the absolute flywheel velocity decreases, the masses will slide back towards the center of the flywheel. The movement of these masses results in the variable moment of inertia of the flywheel. The viscous dampers (c_{sd}) in the VIRM flywheel damp the motion of these masses and dissipate energy in the system. The radial motion of the masses causes a restoring force in the springs that increases with the change in the radial position of the masses. The springs are assumed to have a trilinear elastic force-displacement relationship with a soft central spring k_{sd} and two equal, but much stiffer, penalty spring

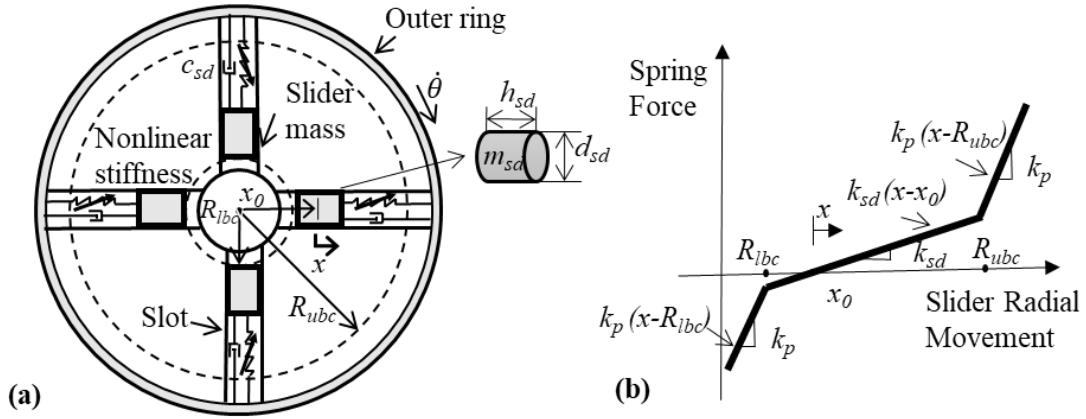


Figure 0-1 (a) Schematic diagram of the VIRM with slider masses shown located at their initial position, (b) Force-displacement relationship of the trilinear spring attached to the slider masses on the flywheel of the VIRM

segments, as illustrated in Figure 0-1(b). The relatively stiff penalty spring stiffness (k_p) in the VIRM is activated when the slider mass approaches the center or the upper bound contact point near the outer ring of the flywheel and works to restrain the movement of the slider mass. The position of the two locations on the radius where the penalty spring stiffness segments are engaged are called the lower bound contact point, R_{lbc} and upper bound contact point, R_{ubc} , respectively. Centrifugal force on the slider masses increases with larger flywheel rotational velocity and increased radial position of the masses; thus, the slider masses are expected to move significantly beyond R_{ubc} with increasing system excitation. However, due to the lower centrifugal forces near the center of the flywheel, the masses are not expected to go significantly beyond R_{lbc} .

Compared to a VIRM, all of the components of a fixed inertia rotational mechanism (FIRM) are kept fixed in the same configuration regardless of the flywheel's rotational velocity. In this study, comparisons of the resulting dynamics and behavior of a system with a VIRM will be made to a system with a FIRM with its masses fixed in their initial position and with a FIRM with masses fixed at the R_{ubc} position.

The relationship between the flywheel angular velocity ($\dot{\theta}$) and the relative velocity ($\dot{u}$) of the structure between the attachment points of the VIRM or FIRM is as follows

$$\dot{\theta} = \alpha \dot{u} \quad (4.1)$$

where α is the coefficient governing this relationship. Utilizing a ball-screw, α is $2\pi/\rho$ where ρ is the lead of the ball-screw.

The total moment of inertia of the VIRM flywheel assembly can be expressed as

$$J_{VIRM} = \frac{1}{12} n m_{sd} \left(\frac{3}{4} d_{sd}^2 + h_{sd}^2 \right) + \frac{1}{2} m_c r^2 + n m_{sd} x^2 \quad (4.2)$$

where n , m_{sd} , d_{sd} , h_{sd} , m_c , and r are the number of slots, slider masses, slider diameter, height, mass of the flywheel without the sliders, and radius of the flywheel, respectively. Note that the total flywheel moment of inertia includes static components that are dependent on the flywheel mass and dimensions and a component that depends on the radial position of the slider masses. In Eq. (4.2), the last term ($n m_{sd} x^2$) is governed by the radial position (x) of the slider masses. The radial position is replaced with x_0 or R_{ubc} to determine the constant moment of inertia

of the two FIRM flywheels. The inertance generated by the constant moment of inertia of the FIRM configuration is $b_y = \left(1/12 * nm_{sd} \left(3/4 * d_{sd}^2 + h_{sd}^2\right) + 1/2 * m_c r^2 + nm_{sd} y^2\right) \alpha^2$ for the FIRM with slider masses fixed at ‘y’ location. This study replaces the ‘y’ location with x_0 or R_{ubc} . The normalized inertance for the two FIRM configurations is expressed as

$$\lambda_{x_0} = \frac{b_{x_0}}{m_s}, \lambda_{R_{\max}} = \frac{b_{R_{\max}}}{m_s} \quad (4.3)$$

Figure 0-2. SDOF primary structure with VIRM attached shows a model with a VIRM attached to a SDOF structure subjected to an external force, $P(t)$. In this figure, m_s represents the mass of the structure, k_s , and c_s are the stiffness and viscous damping coefficients of the structure. When the VIRM is replaced with a FIRM, the structure behaves like a typical spring-mass-damper-inerter (linear) structure and the inertance value provided to the structure is constant.

The equation of motion of the system can be obtained using Lagrange’s equation. The Lagrangian, L , is defined as

$$L = T - V \quad (4.4)$$

where T and V are the system’s kinetic energy and potential energy, respectively. The kinetic energy of the system is calculated considering the motion of the structure, the rotational velocity of the flywheel, and the components of the slider mass velocity. The potential energy of the structure includes the energy stored in all the springs, this includes springs related to the structural stiffness (k_s) and the VIRM stiffness (k_{sd} , k_p). The effect of gravity is ignored here. The kinetic energy, potential energy, and the virtual work of non-conservative forces, ∂W_{NC} , can be formulated as

$$\begin{aligned} T &= \frac{1}{2} m_s \dot{u}^2 + \frac{1}{2} \left(\frac{1}{12} nm_{sd} \left(\frac{3}{4} d_{sd}^2 + h_{sd}^2 \right) + \frac{1}{2} m_c r^2 \right) \alpha^2 \dot{u}^2 + n \frac{1}{2} m_{sd} \dot{x}^2 + n \frac{1}{2} m_{sd} x^2 \alpha^2 \dot{u}^2 \\ V &= \begin{cases} \frac{1}{2} k_s u^2 + n \frac{1}{2} k_{sd} (R_{lbc} - x_0)^2 + n \frac{1}{2} k_p (x - R_{lbc})^2 + nk_{sd} (R_{lbc} - x_0)(x - R_{lbc}), & x < R_{lbc} \\ \frac{1}{2} k_s u^2 + n \frac{1}{2} k_{sd} (x - x_0)^2, & R_{lbc} \leq x < R_{ubc} \\ \frac{1}{2} k_s u^2 + n \frac{1}{2} k_{sd} (R_{ubc} - x_0)^2 + n \frac{1}{2} k_p (x - R_{ubc})^2 + nk_{sd} (R_{ubc} - x_0)(x - R_{ubc}), & x \geq R_{ubc} \end{cases} \\ \partial W_{NC} &= (P(t) - c_s \dot{u}) \partial u - nc_{sd} \dot{x} \partial x \end{aligned} \quad (4.5)$$

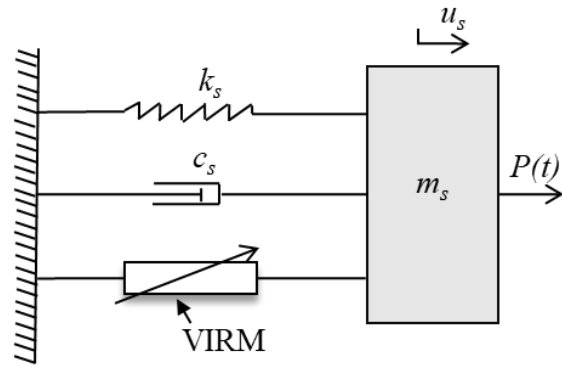


Figure 0-2. SDOF primary structure with VIRM attached

Note that in these equations the multiple slider masses in the VIRM are assumed to be moving synchronously with the same radial motion.

Lagrange's equation for the system in generalized coordinates is given as

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = p_i(t), \quad i = 1, 2; \quad (4.6)$$

where q_1 is the displacement of the primary structure (u) and q_2 is the radial displacement of the slider mass (x). The resulting equations of motion can be expressed as

$$\begin{aligned} m_s \ddot{u} + \left(\frac{1}{12} n m_{sd} \left(\frac{3}{4} d_{sd}^2 + h_{sd}^2 \right) + \frac{1}{2} m_c r^2 \right) \alpha^2 \ddot{u} + 2 n m_{sd} x \dot{x} \alpha^2 \dot{u} + n m_{sd} x^2 \alpha^2 \ddot{u} + c_s \dot{u} + k_s u &= P(t) \\ m_{sd} \ddot{x} - m_{sd} x \alpha^2 \dot{u}^2 + F_{bsd} + c_{sd} \dot{x} &= 0 \end{aligned} \quad (4.7)$$

In the above equations, F_{bsd} represents the restoring force from the trilinear springs in the model. This restoring force varies depending on the radial position of the slider masses and can be defined as

$$\begin{aligned} F_{bsd} &= k_{sd} (R_{lbc} - x_0) + k_p (x - R_{lbc}), \quad x < R_{lbc} \\ F_{bsd} &= k_{sd} (x - x_0), \quad R_{lbc} \leq x < R_{ubc} \\ F_{bsd} &= k_{sd} (R_{ubc} - x_0) + k_p (x - R_{ubc}), \quad x \geq R_{ubc} \end{aligned} \quad (4.8)$$

For the FIRM, the slider mass springs and dampers are removed, and the slider masses are kept fixed with the flywheel frame at either x_0 or R_{ubc} , which results in the constant moment of inertia of the flywheel. The equation of motion of the primary structure with the FIRM is

$$m_s \ddot{u} + \left(\frac{1}{12} n m_{sd} \left(\frac{3}{4} d_{sd}^2 + h_{sd}^2 \right) + \frac{1}{2} m_c r^2 + n m_{sd} y^2 \right) \alpha^2 \ddot{u} + c_s \dot{u} + k_s u = P(t) \quad (4.9)$$

In Eq. (4.9), the fixed position of the slider mass is denoted by y .

Eq. (4.7), Eq. (4.8), and Eq. (4.9) can be written in a normalized form as

$$\begin{aligned} U'' + \left(\frac{1}{12} n \mu_1 \left(\frac{3}{4} d_{sd}^2 + h_{sd}^2 \right) + \frac{1}{2} \mu_2 r^2 \right) \alpha^2 U'' + 2 n \mu_1 \alpha^2 \beta X X' U' + n \mu_1 X^2 \alpha^2 U'' + 2 \xi_s U' + U &= F(t) \\ X'' - X \frac{\alpha^2}{\beta} U'^2 + F_B + 2 \xi_{sd} X' &= 0 \end{aligned} \quad (4.10)$$

$$\begin{aligned} F_B &= R_{lbc} - x_0 + \kappa(X - R_{lbc}), X < R_{lbc} \\ F_B &= X - x_0, R_{lbc} \leq X < R_{ubc} \\ F_B &= R_{ubc} - x_0 + \kappa(X - R_{ubc}), X \geq R_{ubc} \end{aligned} \quad (4.11)$$

$$U'' + \left(\frac{1}{12} n \mu_1 \left(\frac{3}{4} d_{sd}^2 + h_{sd}^2 \right) + \frac{1}{2} \mu_2 r^2 + n \mu_1 Y^2 \right) \alpha^2 U'' + 2 \xi_s U' + U = F(t) \quad (4.12)$$

where

$\mu_1 = m_{sd} / m_s$: slider mass ratio; $\mu_2 = m_c / m_s$: flywheel mass ratio without slider masses;
 $\omega_{s_0} = \sqrt{k_s / m_s}$: natural frequency of the primary structure without VIRM/FIRM;
 $\omega_{sd} = \sqrt{k_{sd} / m_{sd}}$: linearized natural frequency of the the slider mass; $\beta = \omega_{sd} / \omega_{s_0}$: frequency ratio;
 $U = u$: displacement of the primary structure; $U' = \dot{u} / \omega_{s_0}$: normalized velocity of the primary structure;
 $U'' = \ddot{u} / \omega_{s_0}^2$: normalized acceleration of the primary structure;
 $X = x$: displacement of the slider; $X' = \dot{x} / \omega_{sd}$: normalized velocity of the slider;
 $X'' = \ddot{x} / \omega_{sd}^2$: normalized acceleration of the slider; $\xi_s = c_s / 2m_s \omega_{s_0}$: primary structure damping ratio;
 $\xi_{sd} = c_{sd} / 2m_{sd} \omega_{sd}$: linearized damping ratio of the slider mass; $\kappa = k_p / k_{sd}$: stiffness ratio;
 $F(t) = P(t) / k_s$: normalized force amplitude
 F_B : normalized slider stiffness restoring force

In most previous work, the Coriolis force on the slider masses in the VIF was not accounted for (Xu et al., 2015a; J. Yang et al., 2020; Y. Zhang et al., 2020). For example, the kinetic energy generated from the tangential velocity component of the sliders are ignored. This study accounts for forces acting radially and tangentially to the direction of motion. Additionally, unlike most other work in this area, penalty springs that restrain the excessive motion of the slider masses are considered in this study (Dong et al., 2018; Xu et al., 2015a; S. Yang et al., 2016a).

4.1.3 Numerical Analysis and Response Measures

In this study, the response of the structure with the VIRM or the FIRM, presented by the equations of motion in Eq. (4.7) and Eq. (4.9), is simulated under different loadings using the MATLAB implicit solver (MathWorks, 2022a) with an output frequency set at 4000 Hz. The loading considered in different parts of this study are harmonic excitations with different input frequencies, initial displacements, and white noise. The structure and VIRM parameters selected for these numerical simulations are shown in Table 0-1.

Table 0-1. Structure and VIRM parameters

Symbol	Description	Value
n	Slider number	4
μ_1	Slider mass ratio	0.0005
μ_2	Flywheel mass ratio (without sliders)	0.0017
ω_{s_0}	Primary structure natural frequency (without VIRM/FIRM)	2.91 Hz
ω_{sd}	Slider mass linearized natural frequency	42 Hz
β	Frequency ratio	14.64
ξ_s	Primary structure inherent damping ratio	0.02
κ	Stiffness ratio	100
λ_{x_0}	Inertance ratio at initial position, x_0	0.086
$\lambda_{R_{ubc}}$	Inertance ratio at R_{ubc}	0.25
d_{sd}	Slider diameter	0.02 m
h_{sd}	Slider height	0.015 m
r	Radius of the flywheel	0.1 m
x_0	Initial radial position of the slider	0.01 m
α	Proportionality constant between the relative velocity of the structure and the angular velocity of the flywheel	100
R_{ubc}	Radial position of the upper bound contact point of the penalty spring	0.095 m
R_{lbc}	Radial position of the lower bound contact point of the penalty spring	0.005 m

$$\Psi_{\omega_{IF}} = \frac{\omega_{IF}}{\omega_{s_0}}, \quad \Psi_{\bar{\omega}_w} = \frac{\bar{\omega}_w}{\omega_{s_0}}, \quad \Psi_{\omega_0} = \frac{\omega_0}{\omega_{s_0}} \quad (4.13)$$

where $\Psi_{\omega_{IF}}$, $\Psi_{\bar{\omega}_w}$, and Ψ_{ω_0} are the normalized ω_{IF} ratio, normalized $\bar{\omega}_w$ ratio and normalized ω_0 ratio, respectively.

Additionally, the harmonic loading frequency (ω_l) and the response frequency (ω_r) considered in the study are also normalized by the natural frequency of the structure without the VIRM or the FIRM. These are expressed as

$$\Psi_l = \frac{\omega_l}{\omega_{s_0}}, \quad \Psi_r = \frac{\omega_r}{\omega_{s_0}} \quad (4.14)$$

where Ψ_l and Ψ_r are the loading frequency ratio and response frequency ratio, respectively.

4.1.3.1 *Instantaneous Frequency*

The ω_{IF} is a function of the system properties and response and is a measure of the preferred vibration frequency of a dynamic system. The ω_{IF} of a linear primary structure remains constant over all time, regardless of the response of the structure, and is the same as the natural frequency of the structure itself. However, the ω_{IF} of a nonlinear system is not necessarily constant and can vary continuously with the response of the system.

There are numerous potential approaches that could be used to determine the ω_{IF} (Frank Pai, 2010; N. E. Huang et al., 2009; P. Wang & Gao, 2013). Although most of these approaches are based on the assumption that system data corresponds to a linear dynamic response, the ω_{IF} can be used as a tool to characterize specific dynamic systems along their nonlinear time-history response (Moaveni & Asgarieh, 2012). This study uses a method in which instantaneous modal parameters are produced from an eigen analysis using the linearized system properties of the nonlinear structure (i.e., tangent stiffness matrix and tangent mass matrix). Details on the process use to determine ω_{IF} at each time instant are presented below.

In a nonlinear structure, the general force equilibrium equation can be presented by

$$\mathbf{P}_{\text{ext}} = \mathbf{F}_{\text{int}}(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}) \quad (4.15)$$

where $\mathbf{P}_{\text{ext}}$ contains the externally applied loads and $\mathbf{F}_{\text{int}}$ contains the internal forces, which are functions of the system degrees of freedom (DOF) $(\mathbf{u}_0)$ and the derivatives of those DOF $(\dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0)$. Any nonlinear equilibrium equation can be linearized by perturbing the force about a known set of evaluation points, $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, and $\ddot{\mathbf{u}}_0$. A small perturbation in the applied load corresponds to a perturbation in the nodal DOFs, and a first-order Taylor series expansion of the internal forces is expressed as

$$\mathbf{P}_{\text{ext}} + d\mathbf{P} = \mathbf{F}_{\text{int}}(\mathbf{u}_0, \dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0) + \left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \mathbf{u}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} d\mathbf{u} + \left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \dot{\mathbf{u}}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} d\dot{\mathbf{u}} + \left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \ddot{\mathbf{u}}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} d\ddot{\mathbf{u}} \quad (4.16)$$

As $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, and $\ddot{\mathbf{u}}_0$ satisfy Eq. (4.15), Eq. (4.16) can be reduced to

$$d\mathbf{P} = \left(\left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \mathbf{u}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} \right) d\mathbf{u} + \left(\left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \dot{\mathbf{u}}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} \right) d\dot{\mathbf{u}} + \left(\left. \frac{\partial \mathbf{F}_{\text{int}}}{\partial \ddot{\mathbf{u}}} \right|_{\substack{\mathbf{u}=\mathbf{u}_0, \\ \dot{\mathbf{u}}=\dot{\mathbf{u}}_0, \\ \ddot{\mathbf{u}}=\ddot{\mathbf{u}}_0}} \right) d\ddot{\mathbf{u}} \quad (4.17)$$

$$d\mathbf{P} = [\mathbf{K}_t(\mathbf{u}_0, \dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0)]d\mathbf{u} + [\mathbf{C}_t(\mathbf{u}_0, \dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0)]d\dot{\mathbf{u}} + [\mathbf{M}_t(\mathbf{u}_0, \dot{\mathbf{u}}_0, \ddot{\mathbf{u}}_0)]d\ddot{\mathbf{u}}$$

where $\mathbf{K}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})$, $\mathbf{C}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})$, and $\mathbf{M}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})$ are the tangent stiffness, tangent damping, and tangent mass matrices, respectively. These matrices can be expressed as follows for an n degree of freedom system.

$$[\mathbf{K}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})] = \frac{\partial \mathbf{F}_{\text{int}}}{\partial \mathbf{u}} = \begin{bmatrix} \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \mathbf{u}_1} & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \mathbf{u}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \mathbf{u}_n} \\ \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \mathbf{u}_1} & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \mathbf{u}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \mathbf{u}_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \mathbf{u}_1} & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \mathbf{u}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \mathbf{u}_n} \end{bmatrix} \quad (4.18)$$

$$[\mathbf{C}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})] = \frac{\partial \mathbf{F}_{\text{int}}}{\partial \dot{\mathbf{u}}} = \begin{bmatrix} \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \dot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \dot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \dot{\mathbf{u}}_n} \\ \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \dot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \dot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \dot{\mathbf{u}}_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \dot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \dot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \dot{\mathbf{u}}_n} \end{bmatrix} \quad (4.19)$$

$$[\mathbf{M}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})] = \frac{\partial \mathbf{F}_{\text{int}}}{\partial \ddot{\mathbf{u}}} = \begin{bmatrix} \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \ddot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \ddot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \ddot{\mathbf{u}}_n} \\ \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \ddot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \ddot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \ddot{\mathbf{u}}_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \ddot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \ddot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \ddot{\mathbf{u}}_n} \end{bmatrix} \quad (4.20)$$

Accordingly, the tangent stiffness and tangent mass matrices of the structure with the VIRM considered in this study are defined as

$$\mathbf{M}_t = \begin{bmatrix} \frac{\delta F_1}{\delta \ddot{u}} & \frac{\delta F_1}{\delta \ddot{x}} \\ \frac{\delta F_2}{\delta \ddot{u}} & \frac{\delta F_2}{\delta \ddot{x}} \end{bmatrix} = \begin{bmatrix} m_s + n \left(\frac{1}{12} m_{sd} \left(\frac{3}{4} d_{sd}^2 + h_{sd}^2 \right) + \frac{1}{2} m_c r^2 \right) \alpha^2 + n m_{sd} x^2 \alpha^2 & 0 \\ 0 & m_{sd} \end{bmatrix}$$

$$\mathbf{K}_t = \begin{bmatrix} \frac{\delta F_1}{\delta u} & \frac{\delta F_1}{\delta x} \\ \frac{\delta F_2}{\delta u} & \frac{\delta F_2}{\delta x} \end{bmatrix} = \begin{bmatrix} k_s & 2n m_{sd} \dot{x} \alpha^2 \dot{u} + 2n m_{sd} x \alpha^2 \ddot{u} \\ 0 & -m_{sd} \alpha^2 \dot{u}^2 + k_{sd} \end{bmatrix} \quad (4.21)$$

In the above equation for $\mathbf{K}_t$, k_{sd} is replaced by k_p when the penalty spring becomes engaged, i.e., if $x < R_{lbc}$ or $x \geq R_{ubc}$.

Using the above equations, linearized mass and stiffness matrices are produced at each time step in the simulation. Finally, an eigenvalue analysis is performed at each time step with $\mathbf{M}_t$ and $\mathbf{K}_t$ to estimate the ω_{IF} of the system. As the system with a VIRM is a 2DOF system, two natural frequencies will result. The ω_{IF} considered in this measure is defined as the frequency with more substantial engagement of the primary structure in its mode shape.

4.1.3.2 Weighted average instantaneous frequency

While ω_{IF} varies over time, a single measure can be derived from the time history of ω_{IF} that provides information on the overall behavior of the system. One of these measures is $\bar{\omega}_w$, the weighted average value of ω_{IF} . To calculate $\bar{\omega}_w$, the time-history of the total energy of the structure is considered as the weight. The weighted average instantaneous frequency can be calculated as

$$\bar{\omega}_w = \frac{\sum_{i=1}^N W_i \omega_{IF,i}}{\sum_{i=1}^N W_i} \quad (4.22)$$

where W_i and $\omega_{IF,i}$ are the total energy and ω_{IF} of the structure at time step i , respectively, and N is the total number of time steps in the response considered. As ω_{IF} is constant for the structure with the FIRM, $\bar{\omega}_w$ and ω_{IF} have the same value for the structure with the FIRM.

4.1.3.3 Overall fundamental frequency

Similar to $\bar{\omega}_w$, the overall fundamental frequency, ω_0 , can be used to evaluate a system's total dynamic response by identifying the response's lowest dominant frequency component. While ω_0 would not change for a linear system, it can shift due to changes in the loading type and loading amplitude for a nonlinear system.

In this study, estimated auto power spectral densities (APSDs) of resulting numerically simulated primary structure displacement time-histories are utilized to identify ω_0 of the structure. These estimated APSDs are produced using Welch's averaged, modified periodogram method of spectral estimation (MathWorks, 2022b). As the APSD of a response can vary with the

duration of the response considered, the time-history signal duration chosen for each analysis was 100 seconds. For harmonic and white noise loading, a Hanning window with a number of points equal to one tenth of the dataset length is applied and a 50% overlap ratio is utilized. For the initial displacement condition, a 0% overlap ratio is utilized and an exponential window with a length of the same number of points as the dataset and an exponential decay constant of 0.5.

The highest peak at a non-zero frequency in the APSD function for the primary structure's displacement response is defined in this study as ω_0 when considering the white noise loading or an initial displacement. For a generalized harmonic loading, the highest peak of the auto-spectrum would not necessarily yield the actual ω_0 of the system as the highest peak could instead come from the frequency of the forcing function. Consequently, an alternative definition of ω_0 is used in this paper for the harmonic loading case. For harmonic loading, the ω_0 is defined as the loading frequency that results in the largest peak APSD value. This measure can be thought of as a pseudo resonance frequency and is evaluated by considering the response and resulting APSD to harmonic loading over a range of loading frequencies.

4.1.3.4 H_2 norm analog

The H_2 quantifies variance amplification and can be utilized to assess the impact of the VIRM on the normalized primary structure displacement amplitude under dynamic loading. When the structure is subjected to a loading $p(t)$ and its response output is $z(t)$, the H_2 of the structure is defined by

$$H_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} |H_{zp}(j\omega)|^2 d\omega} \quad (4.23)$$

where $H_{zp}(j\omega)$ is the frequency domain transfer function between the loading and response.

For a linear time invariant system with an analytically determined transfer function, the H_2 can be determined using Eq. (4.23). However, the same approach cannot be applied to a nonlinear structure like the structure in this study as an analytical transfer function is not available. Consequently, this study considers an H_2 analog value as a response measure, which is denoted as $\tilde{H}_2$. For the initial displacement and harmonic loading case, $\tilde{H}_2$ is determined by taking the square of the area under the estimated APSD function for the structure's displacement

response between 0 rad/sec and the Nyquist frequency. For the random force excitation case, the estimated transfer function is used to produce $\tilde{H}_2$ instead of the APSD curve.

Normalized values of $\tilde{H}_2$ can then be produced. For the initial displacement and white noise cases, the $\tilde{H}_2$ of the structure with the VIRM is normalized by the $\tilde{H}_2$ of the structure with the FIRM. For the harmonic loading case, the maximum $\tilde{H}_2$ from the analyzed range of loading frequencies with the structure with the VIRM is identified and normalized by the identified maximum $\tilde{H}_2$ for the structure with the FIRM. The normalized $\tilde{H}_2$ response measure (Σ), can be expressed as

$$\Sigma = \frac{\tilde{H}_2(\text{VIRM})}{\tilde{H}_2(\text{FIRM (at } x_0)})} \quad (4.24)$$

4.1.4 *Instantaneous frequency*

To investigate the dynamic effects of the VIRM on the primary structure, this section presents and explores examples of the variation in the $\Psi_{\omega_{IF}}$ of the primary structure with a VIRM subjected to various loading types, loading amplitudes, and loading frequencies and for ranges of slider mass damping. The loads considered in this section are initial conditions, harmonic loading, and white noise. As the structure with the VIRM is nonlinear in nature, the $\Psi_{\omega_{IF}}$ continuously varies over time. To provide points of comparison, the $\Psi_{\omega_{IF}}$ of the structure with the FIRM is provided when the sliders are locked at the initial position (denoted as FIRM (at x_0)) and when the sliders are locked at the R_{ubc} position (denoted as FIRM (at R_{ubc})). Note that the structure with the FIRM (at R_{ubc}) will have a lower $\Psi_{\omega_{IF}}$ than the structure with the FIRM (at x_0) as the slider masses fixed at the R_{ubc} position results in a higher moment of inertia. Figure 0-3(a) shows that for the initial displacement, the structure with the VIRM has a $\Psi_{\omega_{IF}}$ of 0.96 initially, identical to the constant $\Psi_{\omega_{IF}}$ of the structure with the FIRM (at x_0), as expected. As the structure responds to the displacement initial condition, the $\Psi_{\omega_{IF}}$ oscillates between about 0.96 and 0.84

for several cycles, then the amplitude of the changes in the $\Psi_{\omega_{IF}}$ reduce and the $\Psi_{\omega_{IF}}$ remains around 0.96.

The shifts in the $\Psi_{\omega_{IF}}$ observed in Figure 0-3(a) with the VIRM are because of the movement of the slider masses within the VIRM flywheel. The effective force on the slider masses is related to the velocity of the flywheel; thus, oscillations in the response of the structure result in oscillations in the force on and radial displacement response of the slider masses. As the $\Psi_{\omega_{IF}}$ is inversely related to the moment of inertia of the structure, the $\Psi_{\omega_{IF}}$ holds an inverse analog relationship with the slider mass displacement. When the slider masses reach the furthest into the penalty spring region, the VIRM generates a higher moment of inertia than any other orientation of the slider masses and the structure with the VIRM has the lowest observed $\Psi_{\omega_{IF}}$. Note that, while the high stiffness of the penalty spring helps to restrain excessive displacements of the slider mass, the model of the system does not consider a fixed maximum allowable position of the slider masses; thus, the lowest possible $\Psi_{\omega_{IF}}$ of the structure could vary depending on the system's parameters and applied excitation. Additionally, the dynamic response of the structure to this initial displacement dissipates due to damping in the system and the $\Psi_{\omega_{IF}}$ returns to 0.96 with the slider masses coming to rest at their initial positions. Figure 0-3(b-c) also show that the $\Psi_{\omega_{IF}}$ continuously shifts between 0.96 to 0.8 and 0.96 to 0.65 for the harmonic loading case and white noise case, respectively. However, as the harmonic loading and white noise on the structure continues during the entire analysis, the structure does not converge to a constant $\Psi_{\omega_{IF}}$ like the initial displacement case.

In order to investigate the effect of slider damping for different types of loading, the $\Psi_{\omega_{IF}}$ time histories considering undamped (zero slider damping), low damped, and highly damped slider masses are shown in Figure 0-4, Figure 0-5 and Figure 0-6 for the initial displacement, harmonic loading and white noise cases, respectively. The displacement response of the primary structure is also illustrated in the figures for all the damping levels. Note that all structures analyzed, even with the undamped slider masses, have the same 2% inherent damping in the primary structure. Additionally, low damping and high damping scenarios indicate a linearized slider damping ratio, ξ_{sd} , of 0.13 and 20.05, respectively in all the figures mentioned

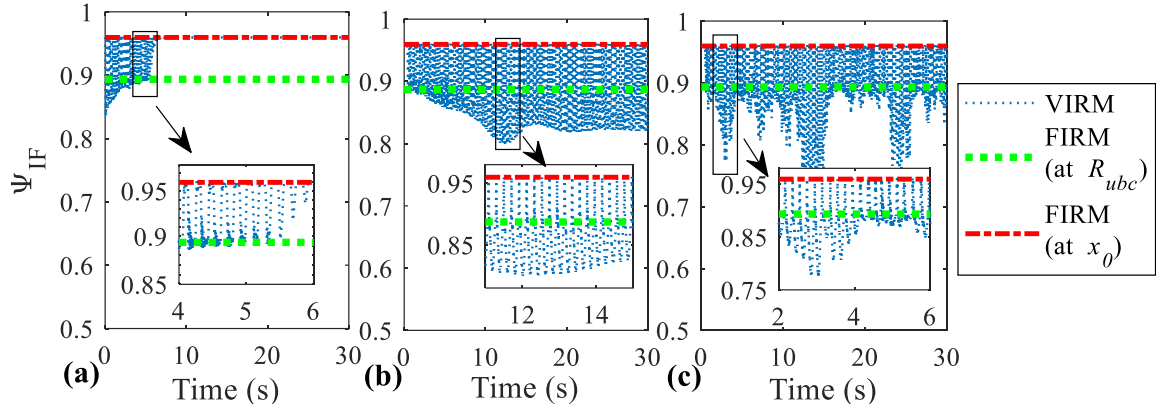


Figure 0-3. Normalized instantaneous frequency, $\Psi_{\omega_{IF}}$, of the primary structure with the VIRM (linearized slider damping ratio, $\xi_{sd}=4$) and the FIRMs for different loading types: (a) initial displacement = 1 m (b) harmonic loading with normalized amplitude of 0.05 m at a loading frequency ratio of 0.85 and (c) white noise with normalized RMS amplitude of 0.3 m/kg

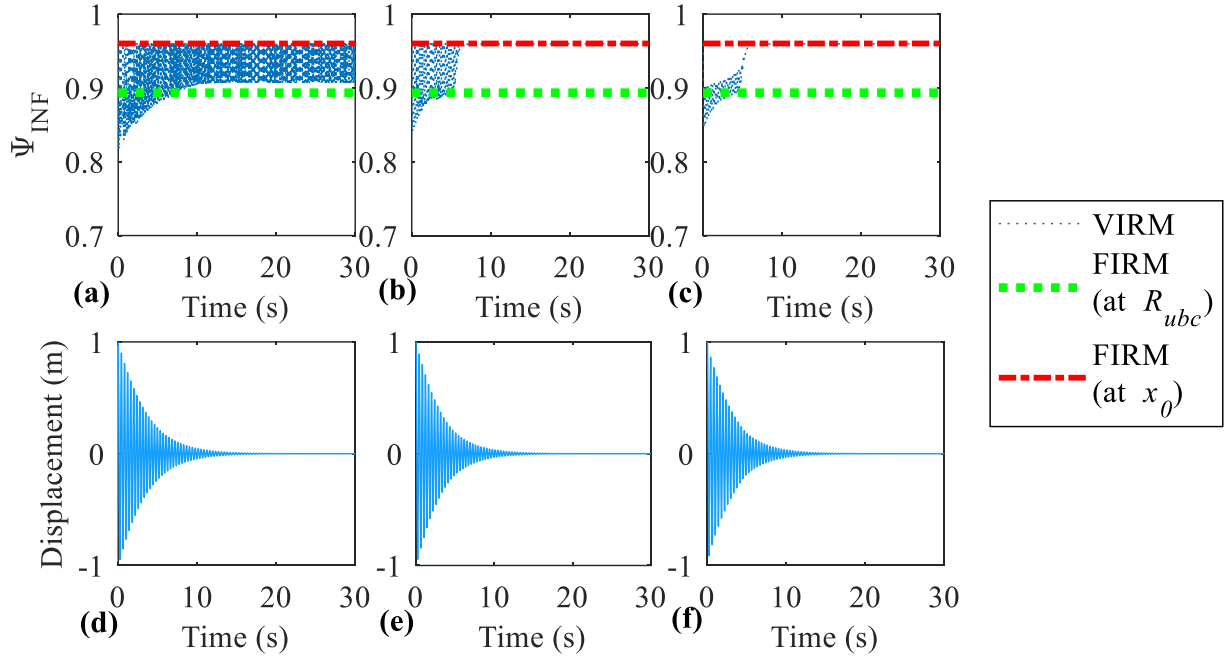


Figure 0-4. Effect of slider damping on the normalized instantaneous frequency, $\Psi_{\omega_{IF}}$, and displacement of the primary structure for an initial displacement=1 m: (a) $\Psi_{\omega_{IF}}$ for zero slider damping ($\xi_{sd}=0$), (b) $\Psi_{\omega_{IF}}$ for low slider damping ($\xi_{sd}=0.13$), (c) $\Psi_{\omega_{IF}}$ for high slider damping ($\xi_{sd}=20.05$), (d) displacement for zero slider damping ($\xi_{sd}=0$), (e) displacement for low slider damping ($\xi_{sd}=0.13$) and (f) displacement for high slider damping ($\xi_{sd}=20.05$)

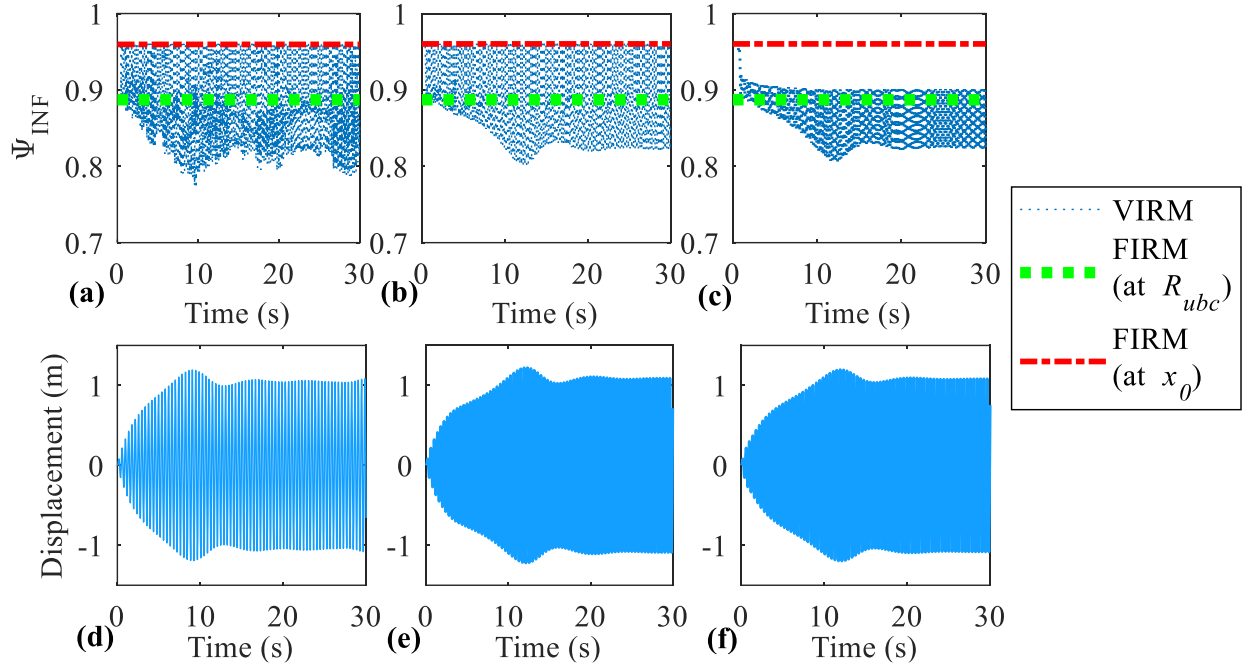


Figure 0-5. Effect of slider damping on the normalized instantaneous frequency, $\Psi_{\omega_{IF}}$, and displacement of the primary structure for harmonic loading with normalized amplitude of 0.05 m at load frequency ratio of 0.86 (zero slider damping) or 0.85 for both slider damping cases: (a) $\Psi_{\omega_{IF}}$ for zero slider damping ($\xi_{sd}=0$), (b) $\Psi_{\omega_{IF}}$ for low slider damping ($\xi_{sd}=0.13$), (c) $\Psi_{\omega_{IF}}$ for high slider damping ($\xi_{sd}=20.05$), (d) displacement for zero slider damping ($\xi_{sd}=0$), (e) displacement for low slider damping ($\xi_{sd}=0.13$) and (f) displacement for high slider damping ($\xi_{sd}=20.05$)

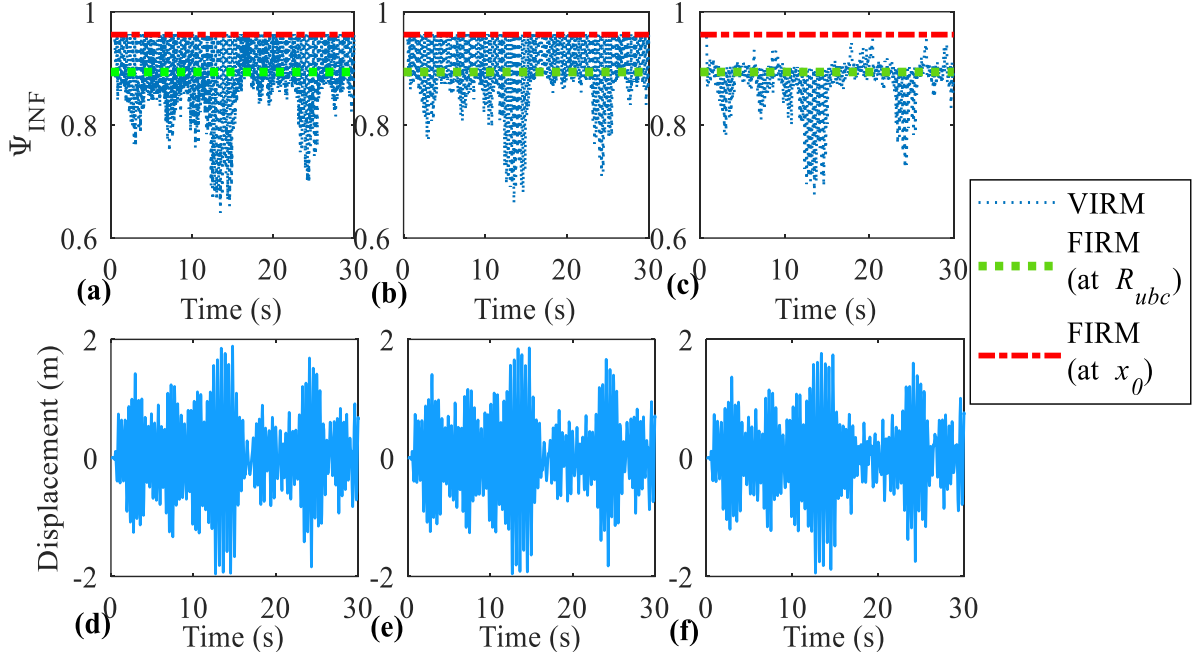


Figure 0-6. Effect of slider damping on the normalized instantaneous frequency, $\Psi_{\omega_{IF}}$, and displacement of the primary structure for white noise with normalized RMS amplitude of 0.3 m/kg: (a) $\Psi_{\omega_{IF}}$ for zero slider damping ($\xi_{sd}=0$), (b) $\Psi_{\omega_{IF}}$ for low slider damping ($\xi_{sd}=0.13$), (c) $\Psi_{\omega_{IF}}$ for high slider damping ($\xi_{sd}=20.05$), (d) displacement for zero slider damping ($\xi_{sd}=0$), (e) displacement for low slider damping ($\xi_{sd}=0.13$) and (f) displacement for high slider damping ($\xi_{sd}=20.05$)

above. Due to the nonlinear nature of the VIRM, the ξ_{sd} is not the same as the damping ratio in general. Rather, the effect of a given value of ξ_{sd} will vary depending on the radial position of the slider masses and relative velocity of the primary structure. Consequently, values of ξ_{sd} greater than 1 do not necessarily mean the slider should behave in an overdamped manner.

It can be observed from Figure 0-4 that the $\Psi_{\omega_{IF}}$ begins to oscillate with a maximum value consistently around 0.96. In the zero slider damping case, the $\Psi_{\omega_{IF}}$ reaches a minimum value of around 0.82 quickly after the initial displacement. As the displacement response of the structure with zero slider damping decays due to inherent structural damping, the minimum value of the $\Psi_{\omega_{IF}}$ increases until about 12 seconds into the response. After this point, the displacement of the structure has mostly attenuated, but sustained oscillations between about 0.96 and 0.91 remain in the $\Psi_{\omega_{IF}}$. These sustained oscillations in $\Psi_{\omega_{IF}}$ are because energy is trapped in the undamped sliders, but cannot transfer back to the primary structure because the primary structure is no longer effectively responding dynamically. When damping is introduced to the slider mass, the fluctuations in the $\Psi_{\omega_{IF}}$ cease after some time and return to their initial values. For instance, in the low damping case, the $\Psi_{\omega_{IF}}$ changes from 0.96 to 0.82 for the first 5 s and smoothly transitions to have a constant $\Psi_{\omega_{IF}}$ of 0.96. When the slider masses are heavily damped, the sliders are restrained from moving rapidly. In this case, the $\Psi_{\omega_{IF}}$ initially keeps changing, but over a narrower frequency range before it finally progresses to a constant 0.96 as the movement of the sliders cease. In this high slider damping case, the $\Psi_{\omega_{IF}}$ of the structure with the VIRM initially oscillates between 0.84 and 0.91, then the $\Psi_{\omega_{IF}}$ shifts between 0.91 and 0.93 at around 5 s, and then steadily converges to 0.96. The displacement response of the primary structure reduces to zero around the same time for the damped cases as the undamped slider case because most of the energy of the structure is dissipated by the structure's inherent damping.

The $\Psi_{\omega_{IF}}$ and the displacement time history of the structure given different slider damping levels under harmonic loading are plotted in Figure 0-5. For all the slider damping levels, the loading frequency was selected such that it results in the largest total response. This figure shows that the $\Psi_{\omega_{IF}}$ oscillates and changes with time for all the slider damping levels.

When the slider masses are undamped, the $\Psi_{\omega_{IF}}$ shifts between a range of 0.96 and 0.78 and includes some abrupt changes in the lower bound envelope of the $\Psi_{\omega_{IF}}$. It is also observed that the $\Psi_{\omega_{IF}}$ oscillates between 0.96 and 0.80 for the low damping level and achieves a much smoother lower bound envelope of $\Psi_{\omega_{IF}}$. Furthermore, the $\Psi_{\omega_{IF}}$ shifts between 0.91 and 0.81 in the highly damped slider mass case after a short amount of time. The overall reduction in the upper value of $\Psi_{\omega_{IF}}$ in the high damped case is because the high damping makes the slider mass less responsive; the slider is not able to quickly return each cycle to the x_0 position and the slider movement ends up restricted to the penalty spring region near the upper bound. This figure also shows that the displacement response and $\Psi_{\omega_{IF}}$ reach mostly steady state behavior after about 25 seconds in both the slider damping cases.

Figure 0-6 shows the $\Psi_{\omega_{IF}}$ and the time-history of the displacement of the structure for different slider damping levels with a white noise loading with a normalized RMS amplitude of 0.3 m/kg. This figure shows that the $\Psi_{\omega_{IF}}$ shifts between 0.96 and 0.65 for the zero slider damping case and the lower bound envelope of the $\Psi_{\omega_{IF}}$ rapidly changes. With the small amount of slider damping that is added to the VIRM in the low slider damping case, the $\Psi_{\omega_{IF}}$ looks much the same, but with a slight increase in the overall lower value of the $\Psi_{\omega_{IF}}$. Even in the high slider damping case, the lower envelope of the $\Psi_{\omega_{IF}}$ changes rapidly due to the continuous but random loading. However, for the high slider damping case, the $\Psi_{\omega_{IF}}$ primarily oscillates between 0.91 and 0.7 and it does not reach the initial $\Psi_{\omega_{IF}}$ value of 0.96 again similar to the harmonic loading case as presented in Figure 0-5(c). Figure 0-6 also shows that the $\Psi_{\omega_{IF}}$ is related to the response amplitude of the primary structure. For example, in all cases, the lowest $\Psi_{\omega_{IF}}$ occurs around 13 seconds, when the primary structure experiences its maximum response.

The influence of the slider damping on the dissipation of energy is illustrated Figure 0-7. This figure presents the energy dissipated by the slider damping, as a percentage of the total energy dissipated, versus ξ_{sd} for different loading types. For this analysis, a unit displacement, a

harmonic loading with a normalized amplitude of 0.05 m at the pseudo resonance frequency, and a white noise of normalized RMS amplitude of 0.3 m/kg are applied to the structure separately. For the initial displacement loading, this figure shows that as the slider damping increases, the dissipated energy percentage increases and reaches a maximum of 16% when ξ_{sd} is 1.34.

Afterwards the dissipation percentage slightly decreases to 14% and stays similar for higher slider damping. A similar trend is also noticed for the harmonic loading case with a maximum energy dissipation of 11% for ξ_{sd} of 0.67 and a close to 5% dissipation for higher slider damping. For the white noise case, there is an initial peak of 10.27% in dissipated energy percentage at ξ_{sd} of 0.80. This is followed by a rapid reduction in dissipation percentage until ξ_{sd} is 1.34, then a long increase in dissipation percentage when ξ_{sd} is higher than 1.34. For example, when ξ_{sd} increases from 6.68 to 13.36, the dissipation percentage increases from 10.78% to 19.78%.

The results in this section have demonstrated that the level of slider damping in a VIRM can result in significant differences in $\Psi_{\omega_{IF}}$ and the percentage of energy dissipated by slider damping; however, the results have also shown that these differences do not always correspond to significant differences in the response of the primary structure. Additionally, these results have shown that the $\Psi_{\omega_{IF}}$ and the percentage of energy dissipated is highly dependent on the loading type.

4.1.5 *Overall fundamental frequency*

different loading types, loading amplitudes, loading frequencies, and for ranges of linearized slider mass damping ratios. The Ψ_{ω_w} is also presented in this section to compare with Ψ_{ω_0} . Moreover, the constant Ψ_{ω_0} and Ψ_{ω_w} of the structure with FIRM for both locked positions are provided to contrast with values of the structure with the VIRM.

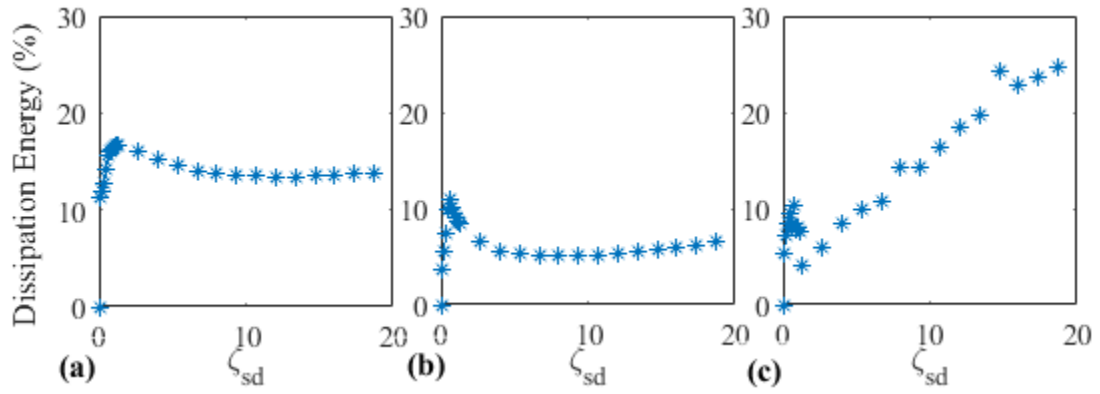


Figure 0-7. Percentage of energy dissipation caused by the slider damping (linearized slider damping ratio, ζ_{sd}) for different loading types: (a) initial displacement =1 m (b) harmonic loading with normalized amplitude of 0.05 m at pseudo resonance frequency (c) white noise of RMS normalized amplitude 0.3 m/kg

Figure 0-8 presents the displacement response APSDs of the primary structure for a linearized slider damping ratio of 4 with an initial displacement condition, harmonic loading, and white noise excitation. For each of the loadings in the figure, APSDs, Ψ_{ω_0} and Ψ_{ω_w} of the structure with the VIRM and the FIRM (at R_{ubc} and at x_0) are shown. The frequency ratio corresponding to the highest peak of the auto-spectrum, Ψ_{ω_0} , indicates the pseudo resonance frequency ratio for the structure with the VIRM. Note that Ψ_{ω_0} is the resonance frequency ratio of the structures with the FIRM. This figure shows that for an initial displacement of 1 m, the structure with the VIRM has a similar, but marginally lower peak APSD displacement amplitude compared to both structures with the FIRM (at R_{ubc} and at x_0). It is also observed that the Ψ_{ω_0} and Ψ_{ω_w} are lower for the structure with the VIRM than structure with the FIRMs. Additionally, Ψ_{ω_w} is consistent with the Ψ_{ω_0} for the structures with the FIRM, as would be expected.

However, there are noticeable differences between Ψ_{ω_0} and Ψ_{ω_w} for the structure with the VIRM, especially for the harmonic loading (Figure 0-8(b)) and white noise case (Figure 0-8(c)). In the harmonic loading case, Figure 0-8(b), the structures are loaded at the forcing frequencies that produced the highest peak APSD. For example, the structure with the VIRM, the FIRM (at R_{ubc}), and the FIRM (at x_0) are oscillated at load frequency ratios 0.85, 0.88 and 0.96, respectively. It is observed that the APSD of the VIRM is noisier than the FIRMs and a super harmonic pseudo resonance frequency can be noticed in the harmonic loading case. This super harmonic pseudo resonance is observed for the structure with the VIRM at a frequency ratio around 2.62 for all loading cases, but it is much more noticeable for the harmonic loading case.

Some of the trends seen in the initial displacement and harmonic loading cases are seen in the results from white noise loading in Figure 0-8(c). One major difference in the results for the white noise case is that in the VIRM APSD curve the Ψ_{ω_0} and the Ψ_{ω_w} are positioned further to the left than for the other load cases and the peak is reduced more relative to the FIRM. Another major difference in these results is that the VIRM APSD curve is more spread out. This increased spread is likely due to the broadband nature of the loading in the white noise case as well as the larger range of $\Psi_{\omega_{IF}}$ covered by the VIRM in this case. Due to this increased spread,

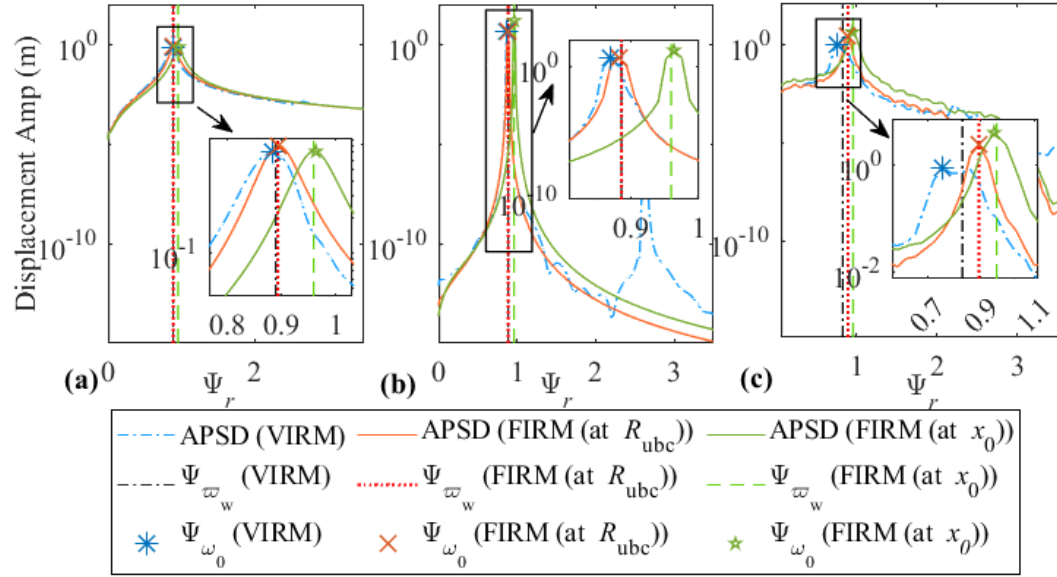


Figure 0-8. Auto spectral density plot and normalized weighted average instantaneous frequency, Ψ_w , of the primary structure with the VIRM or the FIRM for (a) initial displacement = 1 m, (b) normalized harmonic loading of 0.05 m at pseudo resonance/resonance frequency for the structures with the VIRM/FIRM and (c) white noise of normalized RMS amplitude of 0.3 m/kg. Note that the value of normalized overall fundamental frequency, Ψ_0 , for each case is defined as the frequency ratio corresponding to that case's peak point on the auto spectral density plot.

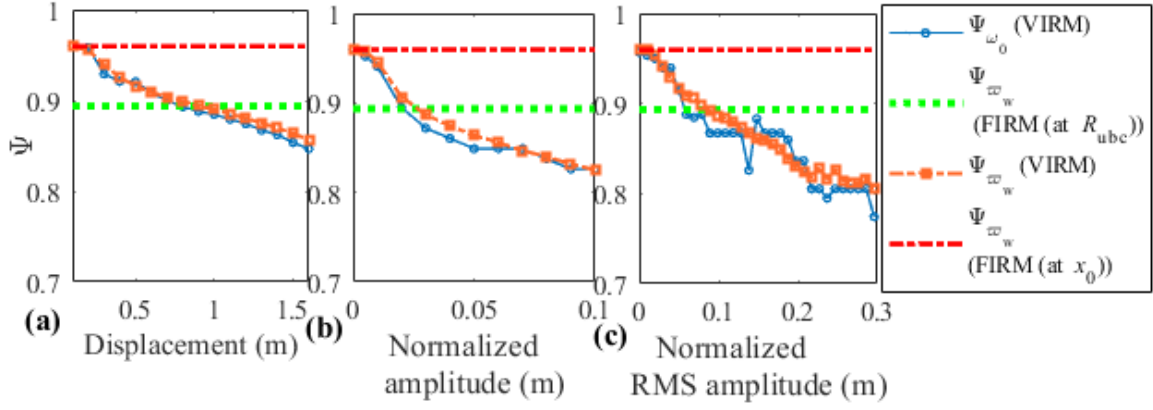


Figure 0-9. Normalized overall fundamental frequency, Ψ_{ω_0} , and normalized weighted average instantaneous frequency, Ψ_{ω_w} , of the primary structure with the VIRM ($\xi_{sd}=4$) and the FIRMs for different loading types and amplitudes: (a) initial displacement condition, (b) structures with VIRM/FIRM excited at pseudo resonance/resonance frequency and (c) white noise excitation

there is expected to be more variability in the resulting identified Ψ_{ω_0} in the white noise loading case. Figure 0-9 can be utilized to contrast Ψ_{ω_0} and Ψ_{ϖ_w} of the structure with the VIRM for different loading types over a range of loading amplitudes. Recall that for the harmonic loading case, Ψ_{ω_0} and Ψ_{ϖ_w} are determined using the forcing frequency that yields the maximum displacement amplitude in the frequency domain. To provide points of comparison, Ψ_{ϖ_w} for the structures with the FIRMs are also presented. As expected, Ψ_{ϖ_w} remains constant in the structures with the FIRM for each loading type as the loading amplitude does not affect the natural frequency of linear structures. However, the added mass effect from the VIRM continuously changes and depends on the loading magnitude and thus the response magnitude. Therefore, Ψ_{ω_0} and Ψ_{ϖ_w} for the VIRM vary for different loading magnitudes. It is observed that Ψ_{ω_0} and Ψ_{ϖ_w} decrease when the loading amplitude increases in nearly all cases. The decreased natural frequency for the higher amplitude load is because the resulting further extension outward of the slider masses of the VIRM adds a higher mass effect to the overall structure, which reduces the fundamental natural frequency of the structure. Although there is a similar trend between the frequency ratio and the loading amplitudes with white noise, Ψ_{ω_0} is more irregular compared to Ψ_{ϖ_w} . This is a result of the broadening of the APSD curves with white noise loading, as seen in Figure 0-9. When the structure is subjected to a comparatively lower loading amplitude, Ψ_{ω_0} and Ψ_{ϖ_w} remained largely unchanged around 0.96, similar to the structure with the FIRM, where the slider masses are fixed at the initial position. In the lower loading amplitude, the slider masses do not significantly move due to the lower rotational velocity of the VIRM. At an initial displacement higher than 1.0 and 0.9, Ψ_{ω_0} and Ψ_{ϖ_w} are below 0.89, which is the resonance frequency of the structure with the FIRM (at R_{ubc}). The reason that Ψ_{ω_0} and Ψ_{ϖ_w} can be lower than the resonance frequency of the structure with the FIRM (at R_{ubc}) is because the slider masses can move into the penalty spring region, causing lower $\Psi_{\omega_{IF}}$. Similarly, Ψ_{ω_0} is lower than 0.89 when the normalized harmonic loading amplitude is higher

than 0.025 m and when the normalized RMS white noise amplitude is higher than 0.08 m/kg. Figure 0-10 presents the influence of linearized slider mass damping ratio ξ_{sd} on the Ψ_{ω_0} and Ψ_{ω_w} of the primary structure under an initial displacement of 0.3 m, a harmonic loading with a normalized amplitude of 0.01 m at pseudo resonance frequencies, and white noise excitation with a normalized RMS amplitude of 0.05 m/kg. In addition, Ψ_{ω_w} of the structures with the FIRM are presented for comparison. It can be observed that there is an overall decreasing trend in the Ψ_{ω_w} for the VIRM with increasing values of ξ_{sd} for all loading types. All deviations from this trend appear minor except that in the harmonic loading case, Ψ_{ω_w} initially increases with increasing ξ_{sd} before it later reduces with increased ξ_{sd} . In contrast, Ψ_{ω_0} of the structure with the VIRM remains almost the same for nearly all ξ_{sd} values for each of the loading types considered. While Ψ_{ω_0} and Ψ_{ω_w} are very similar in nearly all cases, Ψ_{ω_w} is larger than Ψ_{ω_0} in all cases considered for the initial displacement loading. For the harmonic and white noise loading cases, Ψ_{ω_0} becomes larger than Ψ_{ω_w} as ξ_{sd} increases. While $\Psi_{\omega_{IF}}$ for the VIRM clearly varies with the slider damping level, as seen in Figure 0-4, Figure 0-5, and Figure 0-6, the results in Figure 0-9 and Figure 0-10 show that Ψ_{ω_0} and Ψ_{ω_w} are less influenced by the slider damping and more influenced by the loading amplitude changes.

4.1.6 $\tilde{H}_2$ response measure

In this section, $\tilde{H}_2$ and Σ , which are defined in Section 4.1.3.4, are used as measures of the performance of the VIRM related to controlling the response of the primary structure. These measures will be evaluated given different types and amplitudes of loading. Furthermore, variations in these performance measures with changes in ξ_{sd} are considered in this section. As an example of how to interpret a numerical value of the normalized response measure reported in this section, a Σ of 0.6 for a VIRM configuration would indicate a 40% decrease in $\tilde{H}_2$ response measure for the structure with the VIRM relative to the structure with the FIRM at initial position, x_0 .

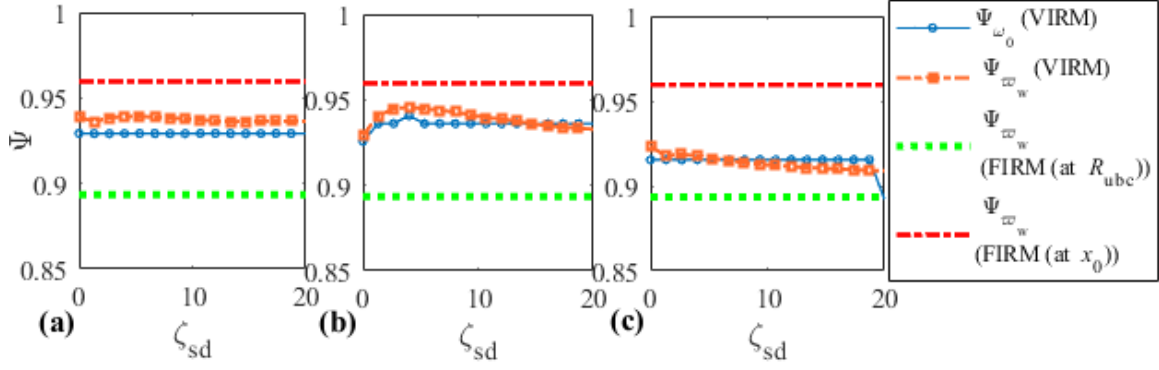


Figure 0-10. Normalized overall fundamental frequency, Ψ_{ω_0} , and normalized weighted average instantaneous frequency, Ψ_{ω_w} , of the primary structure with the VIRM and the FIRMs versus linearized slider damping ratio, ζ_{sd} , for different loading types: (a) initial displacement = 0.3 m, (b) structure with VIRM/FIRM excited at a normalized amplitude of 0.01 m at pseudo resonance/resonance frequency and (c) white noise of normalized RMS amplitude of 0.05 m/kg

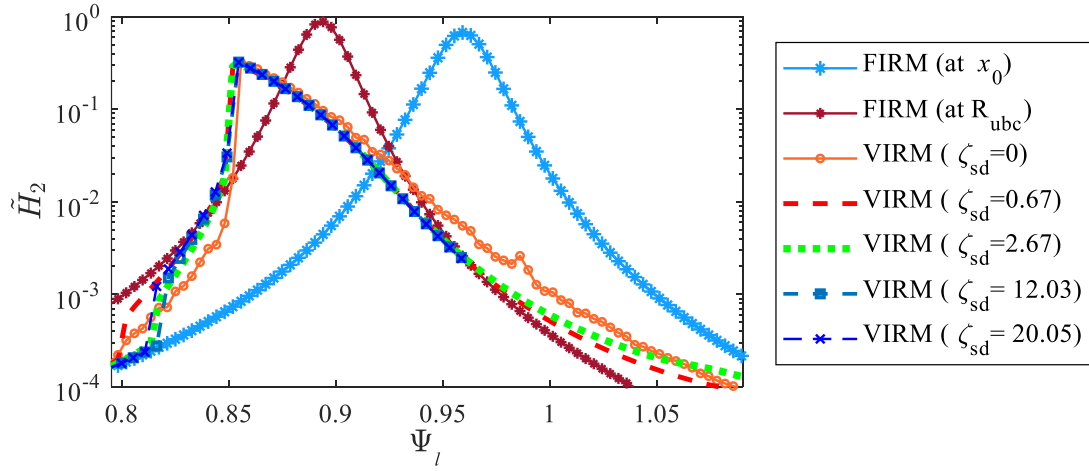


Figure 0-11. $\tilde{H}_2$ of the primary structure versus loading frequency ratio, Ψ_l , with various linearized slider damping ratio, ζ_{sd} , values given a normalized harmonic loading amplitude of 0.05 m

The effect of slider mass damping on the response of the primary structure is presented in Figure 0-11 and Figure 0-12. Figure 0-11 compares the $\tilde{H}_2$ response measure of the structure with the VIRM and the FIRMs for different ξ_{sd} values given harmonic loading with a normalized amplitude of 0.05 m. For each point along a given curve in this figure, the value of that point is the $\tilde{H}_2$ when the harmonic load has a Ψ_l equal to the x-axis value. Figure 0-11 shows that for all ξ_{sd} values considered, the maximum $\tilde{H}_2$ of the structure with the VIRM occurs at a lower Ψ_l value compared to the structure with the FIRMs. This is logical as the $\tilde{H}_2$ of the structure should be greatest when it is excited at the resonance frequency for the structures with the FIRM; however, for the structure with the VIRM, the $\tilde{H}_2$ should be greatest when it is excited at the pseudo-resonance frequency and the additional added mass effects of the VIRM can shift this pseudo-resonance frequency lower than the resonance frequency of the structures with the FIRM. In addition to shifting the location of the peak value, the VIRM reduces the peak $\tilde{H}_2$ for all damping levels and also bends the $\tilde{H}_2$ curve to the lower frequency in a manner usually associated with softening systems (Nayfeh & Mook, 2008). This figure also shows that the peak values of the $\tilde{H}_2$ for the VIRM, and the Ψ_l value that results in this peak value, are all almost the same for the range of ξ_{sd} values considered.

Figure 0-12 presents the normalized $\tilde{H}_2$ response measure (Σ) for the primary structure for different slider damping levels when the structure is subjected to initial displacement conditions, harmonic loading, and white noise. Note that for harmonic loading, each data point on this figure considers a range of Ψ_l values and the Σ of the structure is calculated with the maximum $\tilde{H}_2$ values for the VIRM and the FIRMs. The performance of the structures with the FIRM remains unchanged as the slider masses are locked in position. This figure shows that the structure with the VIRM consistently outperform the FIRMs under initial displacement loading for all the slider mass damping levels considered. Given the initial displacement, Σ drops when a small amount of slider damping is introduced to the VIRM. For instance, Σ decreases to 0.7 at a ξ_{sd} value of 1.34 and further decreases to 0.63 until ξ_{sd} reaches a value of 5.34. Thereafter, ξ_{sd} increases once again as ξ_{sd} further increases. Similarly, the Σ is always lower than 1 for

the harmonic loading and white noise case. However, in these cases Σ is much lower; for example, 0.45 and 0.63 for the harmonic and the white noise cases, respectively, when the VIRM is undamped. As ξ_{sd} increases in the harmonic loading case, Σ decreases to 0.43 when the ξ_{sd} is 1.34 and gradually increases to 0.46 at $\xi_{sd}=5.34$ and remains mostly unchanged in the higher slider damping range. Figure 0-12(c) shows that, for the white noise case, Σ decreases as ξ_{sd} keeps increasing. This decrease in Σ could be attributed to the continued ability of increased slider damping to dissipate more energy in this load case, as observed in Figure 0-7(c).

In order to investigate the effect of loading amplitude on the performance of the structure, $\tilde{H}_2$ are plotted in Figure 0-13 for harmonic loading with different amplitudes across a range of Ψ_l . This figure shows that the maximum $\tilde{H}_2$ occurs at a lower Ψ_l value for the structure with the VIRM compared to the structure with the FIRM at x_0 for the same loading amplitude, which aligns with the previous results in Figure 0-11. Additionally, the structure with the FIRM at R_{ubc} location has a lower Ψ_l compared to the structure at x_0 location and has the maximum $\tilde{H}_2$ value among the three structures. At lower loading amplitudes, for example, for 0.005 m, 0.009 m and 0.02m normalized loading amplitude, the Ψ_l with the maximum $\tilde{H}_2$ is higher than the structure with the FIRM at R_{ubc} . The Ψ_l value with the maximum $\tilde{H}_2$ value of the VIRM is referred to as the pseudo resonance frequency ratio. Near the pseudo resonance frequency ratio, there are multiple stable and unstable steady-state solutions. As the structure approaches the pseudo resonance frequency ratio, the steady-state solution becomes unstable and the structure jumps to a higher stability solution. The jump amplitude of the structure depends on the load amplitude. Additionally, when the loading amplitude increases, the pseudo resonance frequency of the structure with the VIRM decreases.

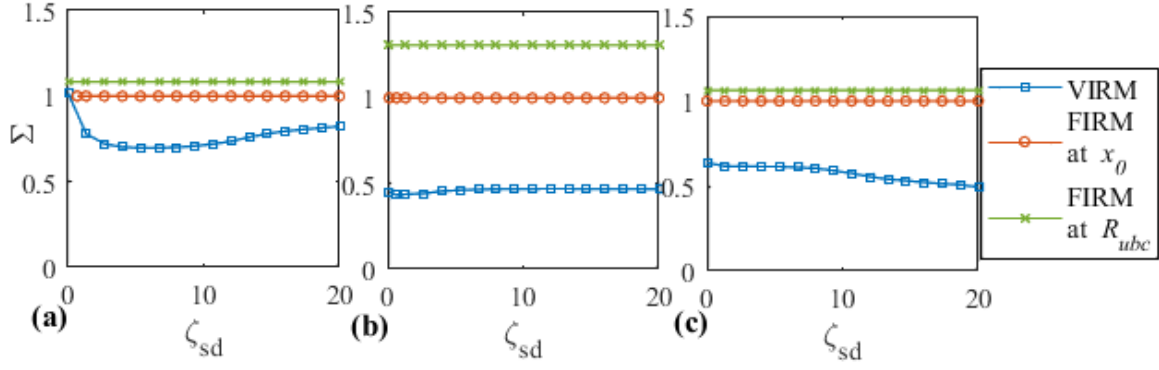


Figure 0-12. Normalized $\tilde{H}_2$ response measure, Σ , of the primary structure versus linearized slider damping ratio, ξ_{sd} , for different loading types: (a) initial displacement of 0.3 m (b) harmonic loading with normalized amplitude of 0.05 m and (c) white noise with normalized RMS amplitude of 0.3 m/kg

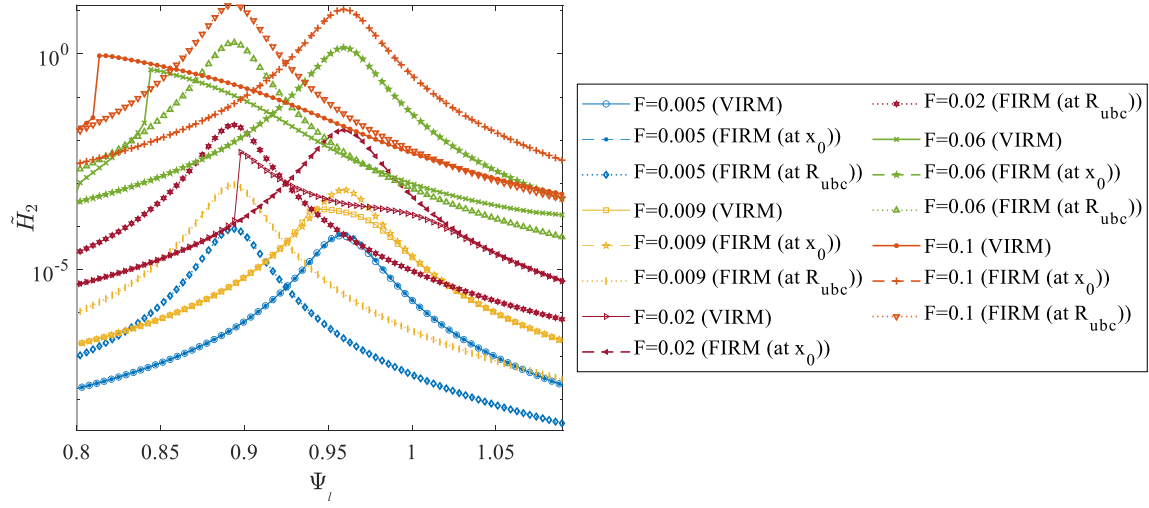


Figure 0-13. $\tilde{H}_2$ of the primary structure with the VIRM ($\xi_{sd}=4$), the FIRM (at x_0) and the FIRM (at R_{abc}) versus loading frequency ratio, Ψ_l , for a range of normalized harmonic loads with different amplitudes

Figure 0-14 can be utilized to investigate the performance of the VIRM at different initial displacements, harmonic loading amplitudes, and white noise levels. In almost every case examined, normalized $\tilde{H}_2$ response measure is less than 1, which indicates that the VIRM is outperforming both the FIRMs. This figure shows that with a small amount of initial displacement, Σ decreases and reaches a minimum of 0.7 when the initial displacement is 0.3 m. However, Σ increases as the structure is excited with a higher initial displacement beyond 0.3 m. The behavior of Σ under different loading amplitudes is very different in the harmonic loading and white noise cases compared to the initial displacement case. Σ for harmonic loading is very low (close to 0.1) for small loading amplitudes and increases with increases in the normalized loading amplitude until a peak value of $\Sigma = 0.57$ is reached, then Σ once again decreases with increased amplitude. For the white noise case, Σ decreases initially to below 0.4 when the normalized RMS amplitude is 0.07 m/kg and increases afterwards until a normalized RMS amplitude of 0.14 m/kg. Finally, Σ decreases again for higher normalized RMS loading beyond 0.14 m/kg.

4.1.7 **Conclusions**

In this paper, an innovative passive nonlinear rotational inertial mechanism called the variable inertia rotational mechanism (VIRM) is described, formulated, and investigated. This paper examines the impact of the VIRM on the natural frequency and the response of a SDOF structure under various types of loads using numerical simulations. The following significant results were observed from this study.

- The incorporation of the VIRM in the primary structure significantly influenced the $\Psi_{\omega_{IF}}$. Although the level of slider damping shifted the $\Psi_{\omega_{IF}}$, the shift in $\Psi_{\omega_{IF}}$ does not always correspond to major changes in the primary structure response. Moreover, $\Psi_{\omega_{IF}}$ is highly dependent on the loading type and loading amplitude.
- Ψ_{ω_w} and Ψ_{ω_0} of the structure with the VIRM reduced significantly when higher loading amplitude is applied to the structure. However, they are less influenced by the slider damping changes.

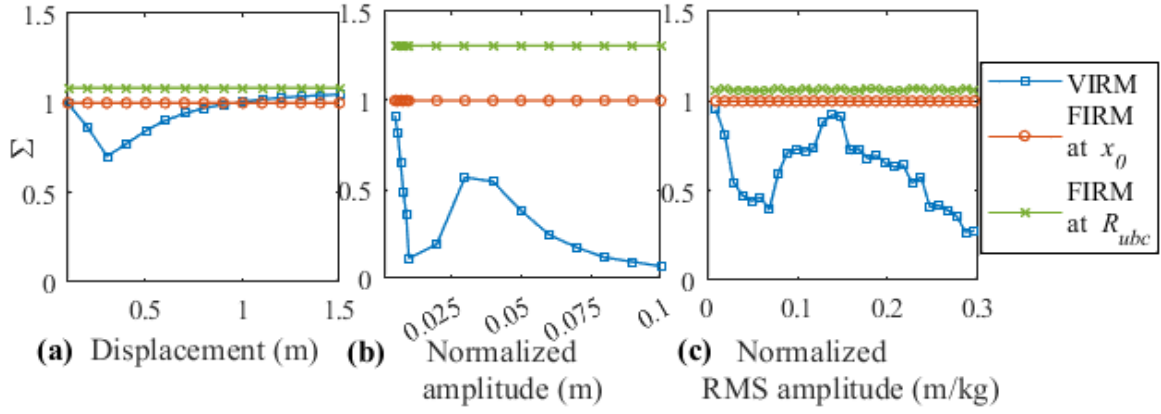


Figure 0-14. Normalized $\tilde{H}_2$ response measure, Σ , of the structure with the VIRM ($\xi_{sd}=4$), the FIRM (at x_0) and the FIRM (at R_{abc}) versus loading amplitude for different loading types: (a) initial displacement (b) harmonic loading and (c) white noise

- The VIRM can notably reduce the pseudo resonance frequency ratio of the structure, and superharmonic pseudo resonances can be observed with the VIRM. Additionally, as the loading amplitude increases, the pseudo resonance frequency ratio and the $\tilde{H}_2$ decrease for the structure with the VIRM relative to the structure with the FIRM at x_0 in all cases considered. The slider mass damping of the VIRM had little to no effect on the shift in the pseudo resonance frequency ratio.
- The performance of the structure with the VIRM, in terms of $\tilde{H}_2$ response measure compared to the FIRMs, changes with the slider damping level considered, but these changes are relatively small for the slider damping levels considered.
- The performance of the VIRM is highly dependent on the type and amplitude of the loading; however, the VIRM outperformed the FIRMs in terms of $\tilde{H}_2$ response measure in almost all the cases considered.

Overall, this paper shows that the VIRM can passively induce significant changes in the dynamics of a primary structure, as evaluated by measures related to the fundamental frequency of the system. Understanding these shifts in fundamental frequency are critical for employing the VIRM as part of a structural control strategy. Although this work with VIRM in a simplified linear model serves as a foundation for future investigation, the nonlinearity introduced by this mechanism significantly impacted the dynamics of the system. Additionally, in most circumstances investigated and with enough input amplitude, the inclusion of the VIRM results in a beneficial reduction in the system response amplitude. Thus, further research on the use of VIRM for passive control of structures, including the presence of intrinsic nonlinearities, under a variety of excitations, including seismic ground motions, is warranted.

4.2 Investigation of the Impact of Slider Mass Stiffness on the Behavior of the Variable Inertia Rotational Mechanism for Structural Vibration Mitigation

4.2.1 Introduction

Rotational inertia mechanisms (RIMs), most commonly the inerters, have been recently developed and the added effective mass these devices can provide has been utilized in a variety of ways to mitigate vibrations. RIMs are two terminal mechanical elements that can be physically realized with different means, such as a ball-screw assembly, rack and pinion assembly, and fluid-based mechanisms (M. C. Smith, 2002, 2012). Linear RIMs produce constant effective

mass. Nonlinear RIMs can produce varying effective mass by utilizing geometric nonlinearities (Moraes et al., 2018; J. Yang et al., 2020), or by modifying its properties as a function of the device's response, such as the relative displacement or rotational velocity (Dong et al., 2018; Mahato et al., 2019b; Xu et al., 2015a; Y. Zhang et al., 2020). One such mechanism that has properties that alter based on the rotational velocity of the device's flywheel is called the variable inertia rotational mechanism (VIRM).

A VIRM can be developed by mounting multiple symmetrically spaced slider masses inside the device's flywheel using springs and guides (see Figure 0-15(b)). A ball-screw assembly connects the flywheel to the structure, causing the flywheel to rotate when there is relative motion between the device's terminals. The flywheel's rotational velocity ($\dot{\theta}$) is proportional to the relative velocity ($\dot{u}$) between the connection terminals of the VIRM. The proportionality constant (α) is $2\pi / \rho$ for a ball-screw, where ρ is the ball screw's lead. The rotation of the flywheel causes the slider masses to move within the guide and larger flywheel rotational velocities result in the slider masses moving further out radially. These movements result in changes in the rotational inertia of the flywheel and, thus, changes in the mass effects the VIRM provides.

The majority of the previous VIRM research has investigated its effectiveness in energy storage, vibration control and stability of attached machinery (Dong et al., 2018; Jauch, 2015; Mahato et al., 2019b; Ullman, 1978). Although previous studies have explored parametric variations in VIRM slider mass stiffness to assess system performance, the slider stiffnesses considered were identical in the assembly. There is a gap in understanding how introducing dissimilarities in the stiffness elements connecting the slider masses in the VIRM flywheel could affect system dynamics and the system's capacity for response mitigation. Additionally, the performance of the structure is typically analyzed using harmonic or impulse loading conditions, thus its behavior under random excitation scenarios remains uncertain. Consequently, critical knowledge gaps exist regarding the impact of dissimilar slider stiffnesses of a VIRM on a system's natural frequency and its ability to mitigate responses when exposed to random vibration.

This work aims to fill these gaps by numerically investigating the natural frequency shifts of a VIRM attached to a single-degree-of-freedom (SDOF) primary system subjected to a random

excitation force (see Figure 0-15(a)). Additionally, this work explores the impact of dissimilar slider stiffness properties on the system behavior and response attenuation performance. The effect of load amplitude changes on the frequency shifts and response reduction are also investigated. Comparisons in this study are made with the behavior and performance of the same primary system with an attached VIRM with sliders with identical stiffness.

The organization of this paper is as follows. In the next section, the mathematical model of a VIRM installed in a single-degree-of-freedom primary system is presented. Additionally, system and VIRM parameters and loading properties are described. The numerical analyses result section describes the simulation results and the last section concludes with a review of the findings of this work.

4.2.2 *Mathematical Model of the System*

In this study, a VIRM is considered in which four slider mass-spring-dampers are symmetrically placed on a circular flywheel, as illustrated in Figure 0-15(b). The springs are modeled to have a trilinear elastic force-displacement relationship, with a soft central stiffness zone, k_{sd} , and two stiffer and equal penalty stiffness zones, k_p (see Figure 0-15(c)). When the slider masses reach a lower bound (R_{lbc}) or upper bound contact point (R_{ubc}) along the guide, the penalty stiffness zone engages and helps to constrain the slider mass movement. The focus of this work is to evaluate the impact of dissimilar slider mass stiffnesses in a VIRM flywheel. In this work, dissimilar slider mass stiffnesses are considered such that two symmetrically spaced slider masses have a central stiffness (k_{sd_1}) and the remaining set are twice as stiff

($k_{sd_2} = 2k_{sd_1}$). Due to the presence of two slider stiffnesses in a VIRM, it is referred to as the dual stiffness VIRM (VIRM-DS). To compare the results of the VIRM-DS, two other VIRM configurations are investigated where all four sliders have identical stiffnesses, with low k_{sd_1} and high k_{sd_2} stiffnesses. These are referred to as low single stiffness VIRM (VIRM-SS (low)) and high single stiffness VIRM (VIRM-SS (high)), respectively. Unless otherwise noted, in each of these configurations, the penalty stiffness values are set at 200 times the soft stiffness:

$k_{p_1} = 200k_{sd_1}, k_{p_2} = 200k_{sd_2}$ for the VIRM-DS and $k_p = 200k_{sd}$ for the VIRM-SS.

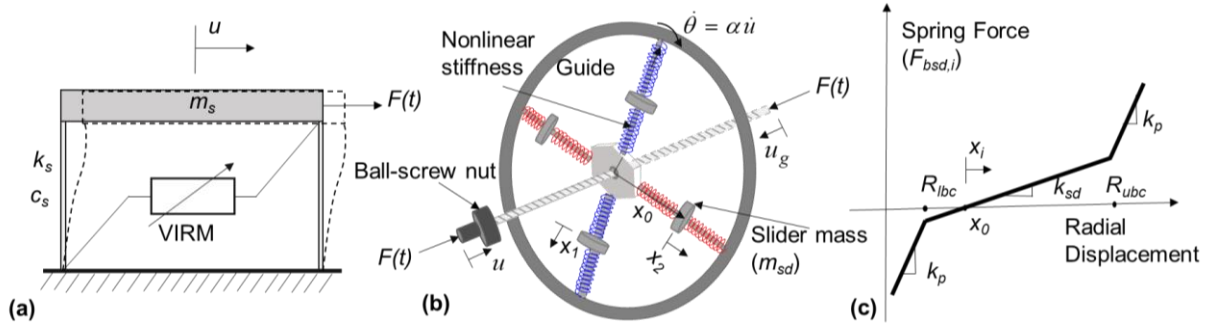


Figure 0-15 Schematic diagram of (a) primary system with VIRM, (b) VIRM-DS flywheel (blue denotes lower stiffness spring, k_{sd_1} and red denotes higher stiffness spring, k_{sd_2}), and (c) trilinear force versus radial position relationship for the slider mass springs

The displacement of the primary structure is u . The presence of two stiffnesses in the VIRM-DS add two additional degrees of freedom (DOF), x_1 and x_2 , to the SDOF primary system compared to one additional DOF for the VIRM-SS, x . The equations of motion of the combined three DOF (VIRM-DS) system and two DOF (VIRM-SS) systems are shown in Eq. (4.25) and (4.26), respectively.

$$\begin{aligned} m_s \ddot{u} + J \alpha^2 \ddot{u} + n m_{sd} \alpha^2 \dot{u} (x_1 \dot{x}_1 + x_2 \dot{x}_2) + \frac{n}{2} m_{sd} \alpha^2 \ddot{u} (x_1^2 + x_2^2) + c_s \dot{u} + k_s u &= F(t) \\ m_{sd} \ddot{x}_1 - m_{sd} x_1 \alpha^2 \dot{u}^2 + F_{bsd,1}(x_1) + c_{sd} \dot{x}_1 &= 0 \\ m_{sd} \ddot{x}_2 - m_{sd} x_2 \alpha^2 \dot{u}^2 + F_{bsd,2}(x_2) + c_{sd} \dot{x}_2 &= 0 \end{aligned} \quad (4.25)$$

$$\begin{aligned} m_s \ddot{u} + J \alpha^2 \ddot{u} + 2 n m_{sd} \alpha^2 \dot{u} x \dot{x} + n m_{sd} \alpha^2 \ddot{u} x^2 + c_s \dot{u} + k_s u &= F(t) \\ m_{sd} \ddot{x} - m_{sd} x \alpha^2 \dot{u}^2 + F_{bsd}(x) + c_{sd} \dot{x} &= 0 \end{aligned} \quad (4.26)$$

where, m_s , c_s , k_s , and $F(t)$ are the primary system's mass, damping, stiffness, and applied load, respectively; the VIRM properties of J , n , m_{sd} , and c_{sd} , are the static rotational inertia, number of sliders, slider mass, and slider damping, respectively. $F_{bsd,1}$, $F_{bsd,2}$ and F_{bsd} represent the restoring force of the VIRM-DS and VIRM-SS trilinear springs of the model, respectively. This trilinear spring relationship is presented by Figure 0-15(c) and can be defined as

$$\begin{aligned} F_{bsd,i} &= k_{sd_i} (R_{lbc} - x_0) + k_{p_i} (x_i - R_{lbc}), x_i < R_{lbc} \\ F_{bsd,i} &= k_{sd_i} (x_i - x_0), R_{lbc} \leq x_i < R_{ubc} \\ F_{bsd,i} &= k_{sd_i} (R_{ubc} - x_0) + k_{p_i} (x_i - R_{ubc}), x_i \geq R_{ubc} \end{aligned} \quad (4.27)$$

In this study, numerical simulations are performed to investigate the response of the primary system with VIRM configurations when the structure is subjected to a band-limited white noise with a frequency range of 1 Hz to 8 Hz. In addition to investigating dissimilar slider stiffness configurations, differences resulting from changes in the overall stiffness of the slider mass springs are investigated in this study. The system and the RIM properties for these numerical simulations are shown in Table 0-2.

Table 0-2. Model parameter properties

Symbol	Description	Value
ω_s	Primary system's natural frequency (without RIM) (Hz)	5.8
n	Number of slider masses	4
k_{sd_1}	Slider soft low stiffness (N/m)	20, 50, 70, 100, 120
k_{sd_2}	Slider soft high stiffness (N/m)	40, 100, 140, 200, 240
m_{sd}	Slider mass (kg)	0.5
c_{sd}	Slider damping coefficient (Ns/m)	5
$\kappa = k_{p_i} / k_{sd_i}$	Slider penalty stiffness ratio	200
J	Flywheel static rotational inertia (kgm ²)	0.0026
ζ_s	Damping ratio of the primary system	2%
x_0	Initial position (m)	0.02
R_{ubc}	Radial position of the upper penalty spring (m)	0.095
R_{lbc}	Radial position of the lower penalty spring (m)	0.005
α	Proportionality constant between the relative velocity of the system and the angular velocity of the flywheel	200

4.2.3 Numerical Analyses Results

This section presents the results of numerical simulations of the primary system with various VIRM configurations, VIRM-DS, VIRM-SS (low), and VIRM-SS (high), and focuses on evaluating the effect of the VIRM slider stiffness on the behavior and response of the attached primary system. Various response measures are computed from the numerical analyses to assess the impact of the VIRM configuration on system behavior. The shift in natural frequencies is examined using wavelet transforms and the instantaneous frequency, which is a measure of the system's preferred vibration frequency. In a linear system, the instantaneous frequency remains constant, while in a nonlinear VIRM system, it can fluctuate with the response of the system. The impact of the VIRMs on vibration attenuation performance is measured using the H_2 norm analog, which quantifies the extent to which the system amplifies or attenuates energy under the applied load condition. To provide a basis of comparison, simulations are also performed with two versions of fixed inertia rotational mechanisms (FIRMs), where the VIRM slider masses are fixed at their initial position, x_0 , or the upper bound contact point, R_{ubc} .

4.2.3.1 Natural Frequency Shift

Figure 0-16 Displacement (a, e) and corresponding wavelets of the primary system connected to different VIRM configurations with low stiffness (a-d ($k_{sd_1}=20$ N/m)) and high stiffness (e-h ($k_{sd_1}=120$ N/m)) for low white noise RMS amplitude of 15 ms-2: (b, f) VIRM-SS (low), (c, g) VIRM-DS, (d, h) VIRM-SS (high)

Figure 0-16 and Figure 0-17 show the displacement of the primary system and the resulting wavelet transform of the VIRM-DS, VIRM-SS (low) and VIRM-SS (high) configurations when the primary structure is subjected to white noise with low and high amplitudes, respectively. In addition to investigating the effect of dissimilar slider stiffness on the response and system behavior, low and high slider stiffnesses are investigated for all the VIRM configurations and the loading amplitudes. Wavelet plots help to visualize the time-dependent frequency components of the corresponding response time series, with dark-shaded regions in the contour plots representing higher amplitudes in system response at the associated time and frequency levels. Additionally, shading in the wavelet subplots are correlated; thus, the darkest shades in these wavelets are indicative of the highest amplitude for all the subplots. The Morlet wavelet is used as the mother wavelet in this study.

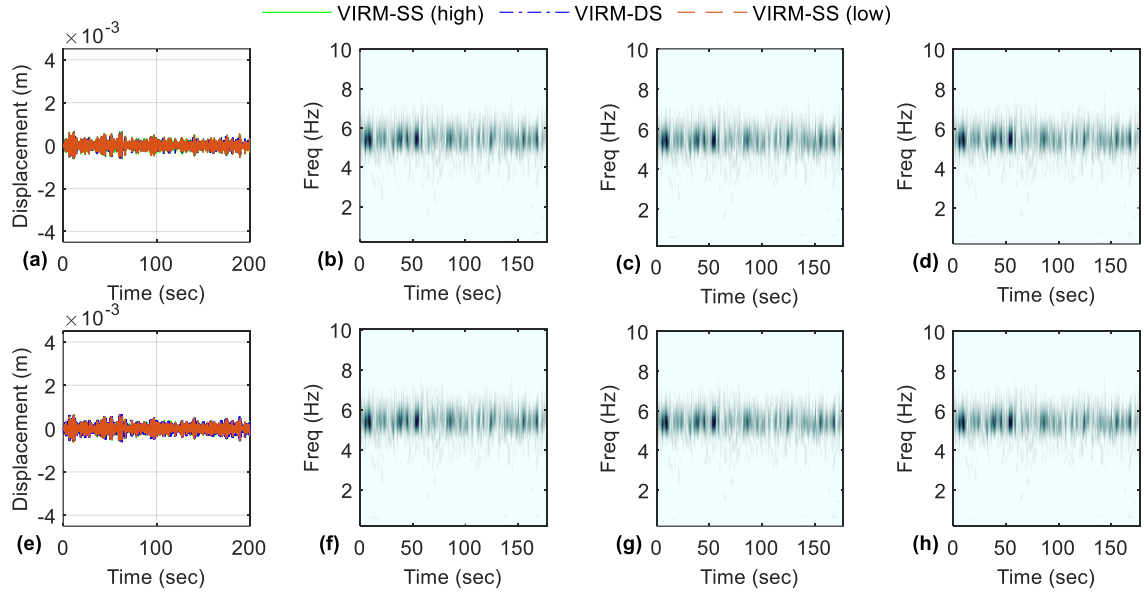


Figure 0-16 Displacement (a, e) and corresponding wavelets of the primary system connected to different VIRM configurations with low stiffness (a-d ($k_{sd_1} = 20$ N/m)) and high stiffness (e-h ($k_{sd_1} = 120$ N/m)) for low white noise RMS amplitude of 15 ms^{-2} : (b, f) VIRM-SS (low), (c, g) VIRM-DS, (d, h) VIRM-SS (high)

Figure 0-16 Displacement (a, e) and corresponding wavelets of the primary system connected to different VIRM configurations with low stiffness (a-d ($k_{sd_1} = 20$ N/m)) and high stiffness (e-h ($k_{sd_1} = 120$ N/m)) for low white noise RMS amplitude of 15 ms-2: (b, f) VIRM-SS (low), (c, g) VIRM-DS, (d, h) VIRM-SS (high)

Figure 0-16(a) and (e) show that at low load amplitude, all the configurations have similar response amplitudes, regardless of the slider stiffnesses or dissimilarities in slider stiffness. It can also be observed from the wavelet results in Figure 0-16(b)-(d), (f)-(h) that there is a similar shading for all the VIRM configurations with the dominant frequencies between 5 Hz and 6 Hz, which indicate that the instantaneous frequency continuously shifts between that range. The limited shift in instantaneous frequency for low load amplitude is because the slider masses do not move significantly due to lower rotational velocity of the VIRM.

The effect of slider stiffness on the system displacement and system behavior with the various VIRM configurations given a white noise with high amplitude is presented in Figure 0-17. Figure 0-17(a) shows that all configurations exhibit comparable responses, similar to the low load amplitude in Figure 0-16(a) and (e). However, the variations are seen in the responses in Figure 0-17(a), which feature systems with overall lower slider spring stiffnesses. The wavelets of the system with VIRM attachments, with $k_{sd_1} = 20$ N/m, presented by Figure 0-17(b-d) show that the frequency content of the system's response varies much more than in the low load amplitude case: between 3 Hz and 6 Hz with the VIRM-SS (low) and between 3.5 Hz and 6 Hz with the VIRM-DS and VIRM -SS (high). The reason behind the lower natural frequency at the high load amplitude, compared to with the low load amplitude, is because at the high load amplitude, the slider masses experience higher centrifugal force, causing them to move further outward radially. It is also notable that the instants of noticeably lower frequency response for the VIRM-SS (low) configuration, at 60, 70, and 125 sec, corresponds to times near where the displacements of the primary system are the highest for the high loading case.

Moreover, the wavelets of Figure 0-17(b-d) show that that the system natural frequencies are relatively higher for the VIRM-DS than VIRM-SS (low) configuration. The higher natural frequency of the system attached with the VIRM-DS could be because two high slider stiffnesses in this configuration inhibits the further outward movement of those slider masses, overall increasing the natural frequency of the system. Figure 0-17 (f-h) present that at the same high

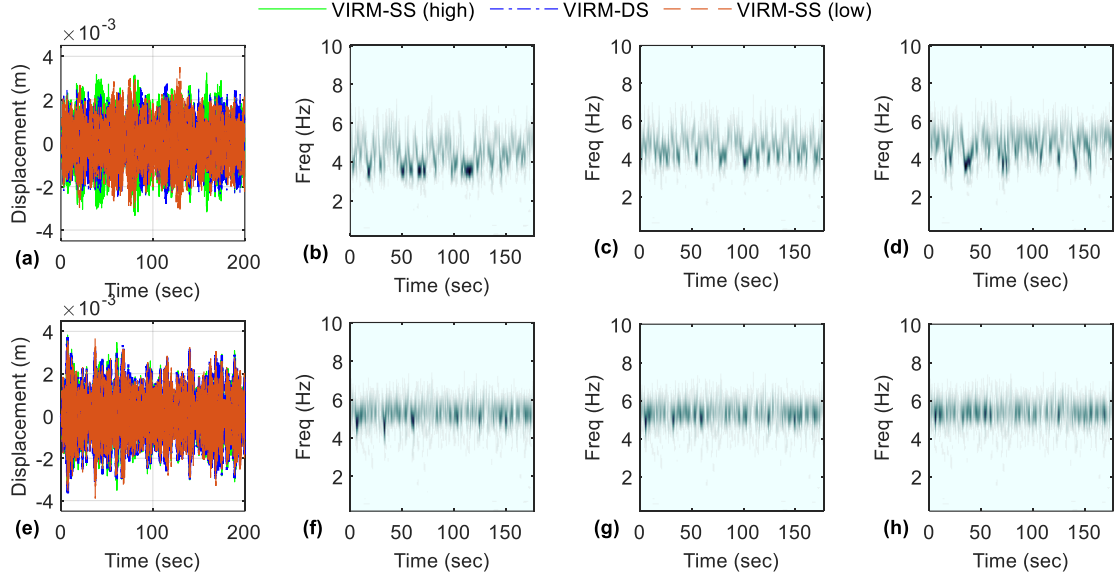


Figure 0-17. Displacement (a, e) and corresponding wavelets of the primary system connected to different VIRM configurations with low stiffness (a-d ($k_{sd_1}=20$ N/m) and high stiffness (e-h ($k_{sd_1}=120$ N/m)) for high white noise RMS amplitude of 116 ms^{-2} : (b, f) VIRM-SS (low), (c, g) VIRM-DS, (d, h) VIRM-SS (high)

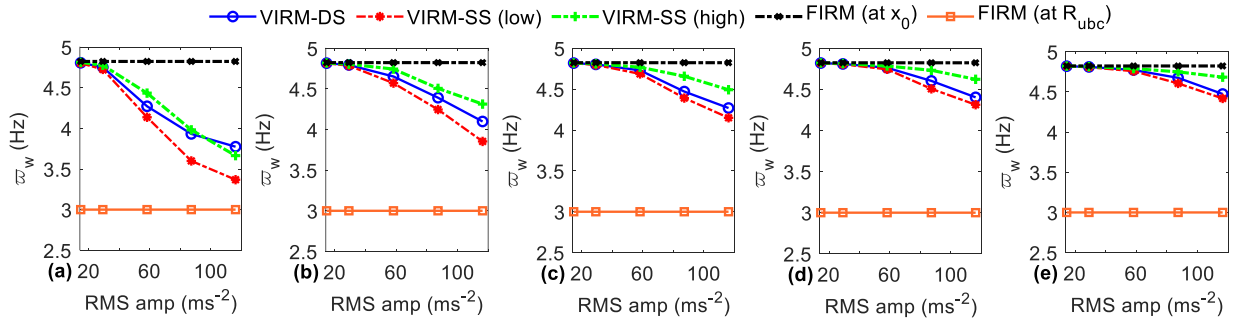


Figure 0-18. Effect on the weighted average instantaneous frequency of load amplitude and slider stiffness, k_{sd_1} : (a) 20 N/m, (b) 50 N/m, (c) 70 N/m, (d) 100 N/m, and (e) 120 N/m

load amplitude and for the slider stiffness of $k_{sd1}=120$ N/m, the overall instantaneous frequency increases and ranges between 4.5 Hz and 6 Hz, similar to the low load amplitude. Furthermore, the increase in the slider stiffness results in a small increase in the displacement of the primary system.

The influence of the load amplitude and the slider stiffness on frequency shifts is investigated with the weighted average instantaneous frequency, ϖ_w , in Figure 0-18. To estimate the instantaneous frequency in this study, the nonlinear equations of motions are linearized by applying the small perturbation method and producing tangent mass and tangent stiffness matrices at each time step in the numerical simulation. Then, an eigenvalue analysis is performed to determine the instantaneous frequency at each time step. The instantaneous frequency of interest is isolated from these results by identifying the eigenvector where the DOF associated with the primary structure has the most significant contribution. Finally, using the total energy time history response as the weighting factor, a weighted average instantaneous frequency is computed.

Figure 0-18 shows that at low load amplitudes VIRM -DS, VIRM-SS (low), and VIRM-SS (high) have the same ϖ_w for all the slider stiffness values, similar to the FIRM with slider masses fixed at the initial position. As the load amplitude increases, ϖ_w decreases for all the VIRM configurations and stiffness values. However, the VIRM-SS (low) have lower ϖ_w compared to VIRM-DS and VIRM-SS (high). Moreover, as the slider stiffness increases, ϖ_w increases for all the configurations, which aligns with the results of the wavelets. The plots suggest that the presence of higher slider stiffness, whether in all of the sliders or in some of them, contribute to a higher weighted average instantaneous frequency as long as the amplitude of response is high enough. It is also observed that the FIRM with slider masses fixed at R_{ubc} have the lowest ϖ_w of all the configurations considered. The higher ϖ_w of the VIRMs than the FIRM (at R_{ubc}) could be because the average slider mass position of the VIRMs was mostly lower than R_{ubc} , adding a lower effective mass than the FIRM (at R_{ubc}).

4.2.3.2 Response Mitigation

System performance can be measured using the H_2 norm analog measure, which is determined by taking the square of the area under the estimated transfer function of the system's

response between the bounded frequency range of 1 Hz to 8 Hz. This frequency range is used to calculate this measure as it corresponds with the frequency range of the band-pass filtered white noise. Figure 0-19 presents the H_2 norm analog of the different VIRM configurations for different load amplitudes and slider stiffnesses. This figure shows that all the VIRM attachments have lower, or approximately the same, H_2 norm analog compared to the FIRM (at x_0). The figure also shows that at low loading amplitudes, the VIRM configurations have a slightly higher H_2 norm analog compared to the FIRM (at R_{ubc}). However, as the load amplitude increases, H_2 norm analog reduces significantly for the VIRM attached systems. Additionally, the VIRM-SS (low) is able to best reduce the response of the system in comparison to the other configurations. This lower response may be because of the larger continuous shifts in frequency and larger shift in overall frequency that are possible with the VIRM-SS (low). This figure also shows that higher slider stiffness, whether in all of the sliders or in some of them, decreases the response mitigation performance of the VIRM. While the results in Figure 0-19 may suggest that lower slider stiffness is always beneficial, this is not always true. For example, if the system with low a stiffness, $k_{sd_1}=20$ N/m, VIRM-DS is investigated at higher load amplitudes, it is observed that the system becomes unstable because the low slider stiffness masses are no longer restrained in the flywheel. This occurs because the effective outward force on the slider masses continuously exceeds the restoring forces working on these masses. In practical terms, this instability would result in a collision of the slider masses at its radial limit or necessitate large flywheel diameters.

4.2.3.2 Penalty stiffness variations

To understand the effect of penalty stiffness on the system behavior and response, the system with VIRM-SS configurations is investigated further and is presented in Figure 0-20. The soft slider stiffnesses of 20 N/m and 70 N/m are considered for this analysis with various penalty stiffness ratios. Figure 0-20(a) shows that for either soft slider stiffness, the ϖ_w increases as the penalty stiffness ratio increases. It is notable that at the softer slider stiffness of 20 N/m and $\kappa=1$, ϖ_w is significantly lower than at the other penalty stiffness ratios, as the penalty stiffness is no longer there to restrain the motion of the slider masses. The changes in response frequency content with different values are seen in the wavelets in Figure 0-20 (c) and (d). While changes in penalty stiffness causes changes in the H_2 norm analog, the change is relatively small, except with softer slider stiffness of 20 N/m and $\kappa=1$, where large changes in H_2 norm analog are also observed.

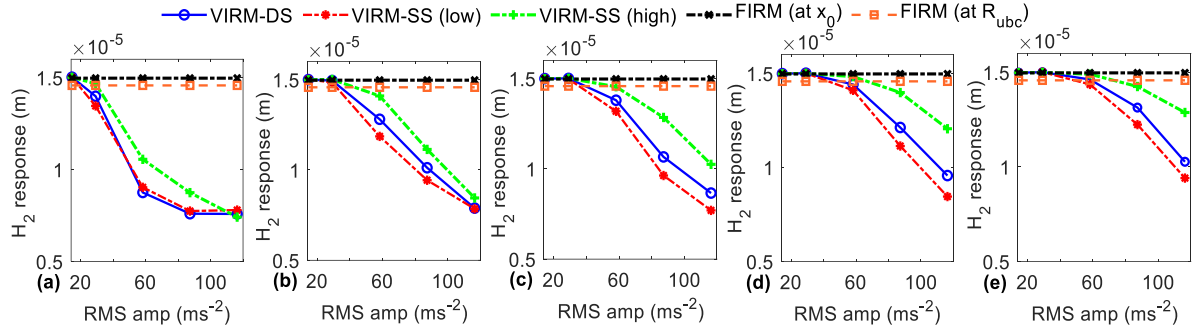


Figure 0-19. Effect on H_2 norm analog of load amplitude and slider stiffness, k_{sd_1} : (a) 20 N/m, (b) 50 N/m, (c) 70 N/m, (d) 100 N/m, and (e) 120 N/m, on the H_2 norm analog

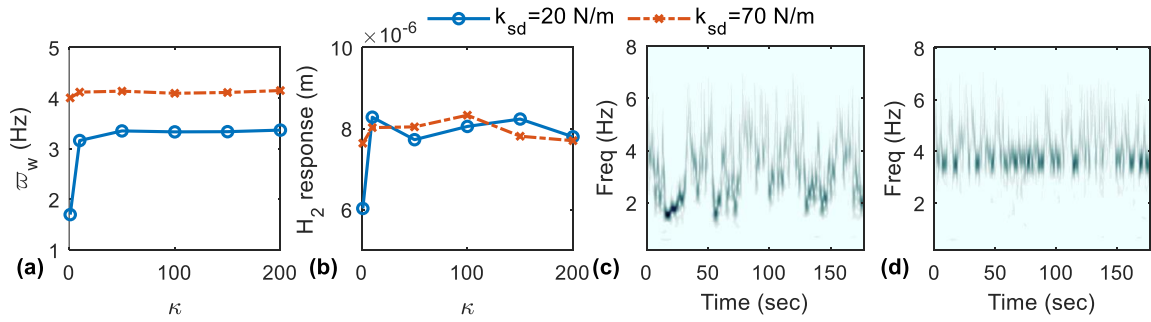


Figure 0-20. Effect of penalty stiffness ratio , κ , for a RMS band-limited white noise of 116 ms^{-2} of the VIRM-SS on the (a) system's weighted average instantaneous frequency, (b) H_2 response; (c) displacement wavelets of the system with VIRM-SS ($k_{sd_1} = 20 \text{ N/m}$, $\kappa = 1$), and (d) displacement wavelets of the system with VIRM-SS ($k_{sd_1} = 20 \text{ N/m}$, $\kappa = 200$)

4.2.4 ***Conclusions***

The variable inertia rotational mechanism (VIRM) numerically investigated in this work includes the dissimilar slider stiffness configuration, which is denoted as dual stiffness VIRM (VIRM-DS) and features a stiffer spring connected to half of its slider masses. The results of the study show that the VIRM-DS generally has a slightly higher to similar weighted average instantaneous frequency than the low single stiffness VIRM (VIRM-SS (low)) across all the stiffnesses considered. However, in conditions of high loading amplitude and low stiffness, the VIRM-SS (low) displays notably lower weighted average instantaneous frequencies compare to the VIRM-DS. Furthermore, the study reveals that the VIRM-DS and different VIRM-SS configurations have similar response mitigation performance with the VIRM-SS (low) generally performing better. Overall, the paper suggests that the slider stiffness plays a significant role in influencing system dynamics and response mitigation. Furthermore, these results suggest that using dissimilar VIRM slider stiffnesses does not have any advantages over using the same slider stiffness. Future work should explore alternative stiffness models and penalty stiffness strategies in depth.

CHAPTER 5 Dynamic analysis and response attenuation of a two-degree-of-freedom structure with a variable inertia rotational mechanism subjected to ground excitation

This paper focuses on the impact of the variable inertia rotational mechanism on a two-degree-of-freedom structure subjected to white noise loading and seismic ground motion. The interplay of VIRM parameters on the frequency shifts and effectiveness of VIRMs for enhancing structural performance and seismic protection is highlighted in this paper. The version of this chapter is published in Engineering Structures.

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<https://doi.org/10.1016/j.engstruct.2024.118586>.

5.1 Introduction

Civil engineering structures can experience adverse effects when subjected to dynamic loads such as seismic excitation and wind. The behavior and response of a structure can be modified by altering the stiffness, strength, and damping of that structure (Kelly, 1993; Kelly et al., 1972; Skinner et al., 1993; Soong & Constantinou, 1994; Soong & Dargush, 1997). However, an alternative passive control device utilizing supplemental rotational inertia has been developed to modify the effective mass of a structure (Arakaki et al., 1999a; Ikago et al., 2012; Javidialesaadi & Wierschem, 2019b; M. C. Smith, 2002; Sugimura et al., 2012). Arakaki et al. first proposed using a rotary damper in the late 1990s, which combines a ball screw assembly and cylindrical mass rotating inside a chamber filled with a viscous fluid, to modify the seismic response of structures (Arakaki et al., 1999a, 1999b). These supplemental rotational inertia devices transform the relative displacement between the terminals to rotational motion of a flywheel. In the force-current analogy, these devices function as a mechanical counterpart of an electrical capacitor. This analogical relationship was initially identified by Smith, who introduced the term ‘inertor’ in 2002 for any mechanical arrangement where the resultant force is proportional to the relative acceleration between its two terminals (M. C. Smith, 2002).

Inerters can be physically realized with mechanical mechanisms such as a ball-screw or rack and pinion assembly with a flywheel, or with fluid-based or electromagnetic-based mechanisms (De Domenico, Ricciardi, et al., 2020; M. C. Smith, 2002, 2008, 2012; Swift et al., 2013). The output force of an inerter is proportional to the relative acceleration between its terminals. The constant of proportionality is called inertance and has the same units as mass. One of the distinguishing features of the inerter is its mass amplification effect. Due to this mass amplification effect, the inerter can produce substantial inertial mass with minimal physical mass.

Upon its conceptual development and initial experimental validation, the inerter has been studied in vehicle mechanics and dynamics to control suspension vibration (M. Chen et al., 2009; Kuznetsov et al., 2011) and has also been proposed for wind and seismic protection of civil structures. In civil structures, these mechanisms are often combined with other control applications, such as tuned mass dampers, toggle braces, external rocking walls, and friction pendulums, to enhance performance (Di Egidio et al., 2021; Giaralis & Petrini, 2017; Hwang et al., 2007; Pietrosanti et al., 2017; Zhao et al., 2019). These systems are examined in applications, such as optimal design and seismic protection of building structures (Giaralis & Taflanidis, 2018; Javidialesaadi & Wierschem, 2018), improving base isolation systems (De Domenico & Ricciardi, 2018; Sugimura et al., 2012), and mitigating wind-induced vibration of tall buildings (Giaralis & Petrini, 2017; Q. Wang et al., 2019). Researchers have also investigated inerter-based devices in multi-degree-of-freedom (MDOF) structures to examine the natural frequency changes (M. Z. Q. Chen et al., 2014) and enhance performance of tuned damper systems (Giaralis & Taflanidis, 2018; Lazar et al., 2014; Marian & Giaralis, 2014). Furthermore, inerter-based dampers, such as multi-tuned damper inerter (De Domenico, Qiao, et al., 2020) and multi-tuned liquid column damper inerter (Q. Wang et al., 2021), have been investigated to control the seismic vibration of adjacent high-rise buildings.

While inerters are characterized by linear behavior and constant resulting inertance, another branch of research explores nonlinear mechanisms with variable inertance (Dong et al., 2018; Moraes et al., 2018; Y. Wang et al., 2019, 2021; Y. Zhang et al., 2020). Variable inertance can be produced in multiple ways including exploiting geometrically nonlinear configurations of otherwise linear inerters (Moraes et al., 2018; J. Yang et al., 2020) or by modifying the rotational inertia of the mechanism's flywheel (Dong et al., 2018; Kushwaha et al., 2020; Q. Li et al., 2021; Mahato et al., 2019b; Xu et al., 2015b; S. Yang et al., 2016b; Y. Zhang et al., 2020). This study considers one such mechanism, called the variable inertia rotational mechanism (VIRM), which employs a flywheel that contains masses that can move within the flywheel and thus produce variations in the flywheel's rotational inertia (Dong et al., 2018; Mahato et al., 2019b; Xu et al., 2015b; S. Yang et al., 2016b; Y. Zhang et al., 2020).

In vehicle systems, VIRMs have demonstrated improvements in rider comfort, better road handling and safety, and decreased suspension deflection under most circumstances (Xu et al., 2015b; S. Yang et al., 2016b). VIRMs have also been studied to increase power absorption bandwidth, increase vibration mitigation, and enhance stability in power systems, such as

hydraulic motors, diesel generators, wave energy converters, and wind turbine rotors (Dong et al., 2018; Jauch, 2015; Kushwaha et al., 2020; Q. Li et al., 2021; Mahato et al., 2019b). However, many of these studies have considered active and semi-active versions of the VIRM, not the passive VIRM (Jauch, 2015; Q. Li et al., 2021; Ullman, 1978).

While applications in other fields have been explored, the investigation of the potential of VIRMs in civil engineering structures is less developed. The existing research has yet to investigate the impact of VIRMs under seismic ground motion or random noise. While limited studies on VIRM indicate effectiveness in reducing the response amplitude of dynamic systems, their influence on instantaneous frequency has not been fully studied. Additionally, the effect of VIRMs on MDOF system dynamics remains unexplored. As a result, there is a notable gap in understanding the impact of variable rotational inertia on the frequency shifts and response of MDOF systems subjected to seismic ground motion and random noise.

To address these gaps in knowledge, this study aims to numerically evaluate the frequency shifts and effectiveness of VIRM in reducing the response of a two-degree-of-freedom (2DOF) structure. This investigation includes scenarios with a structure with single and multiple passive VIRMs that is subjected to seismic ground motion and white noise loading. The study also explores the effect of VIRM inertance ratios and changes in load amplitude on the frequency shifts and response reduction. The computed response quantities for the 2DOF structure with VIRMs will be compared to those of a structure with fixed inertia rotational mechanisms (FIRMs).

This paper is structured as follows. In Section 5.2, the mechanics of the VIRM and mathematical models of a 2DOF structure with one or two VIRMs are described. Section 5.3 introduces the parameters of the structure and the loading considered in this study as well as the response measures that will be used to evaluate the effect of the VIRM. The impact of VIRM on natural frequency shifts is presented in Section 5.4, while Section 5.5 discusses the impact of the VIRM on an H_2 response measure. Section 5.6 presents results from incremental dynamic analyses investigating the effects of VIRM on the response of the structures to various ground motions. The impact of the inertance ratio and loading amplitude are also considered in the results presented in Sections 5.4, 5.5 and 5.6. Finally, the study is summarized and conclusions are presented in Section 5.7.

5.2 Equations of motion of 2DOF structure with variable inertia rotational mechanism

Figure 0-1(a) depicts the 2DOF primary structure that is considered in this numerical study, which has two identical VIRMs attached between the ground and the first floor and between the first and the second floors. In Figure 0-2(b), the two DOF primary structure has one VIRM that is attached between the ground and the first floor of the structure. In both Figure 0-2(a) and Figure 0-2(b), the first and second story masses are m_{s_1} and m_{s_2} , and k_{s_1} , k_{s_2} , c_{s_1} and c_{s_2} are the stiffness and the viscous damping coefficients of the two stories. In this study, two other RIM configurations are investigated in the 2DOF primary structure: one where the two VIRMs are replaced with two identical fixed inertia rotational mechanisms (FIRMs) and another where the single VIRM is replaced with a single FIRM. In comparison to a VIRM, all components of a FIRM maintain a fixed configuration irrespective of the flywheel's rotational velocity.

The VIRM considered in this study is shown in Figure 0-2 and utilizes a ball screw mechanism to transfer the relative translational motion of a story of the structure in to rotational motion of the device's flywheel. In this study, the VIRM features a flywheel hub with four symmetrically spaced guide tracks and four discrete masses on the guide tracks that are connected to the flywheel hub with springs, as illustrated in Figure 0-1. When the flywheel is at rest, the masses are located at an initial radial position, x_0 , near the center of the flywheel. As the flywheel rotational velocity increases, centrifugal force drives the masses to move radially outward, while the springs attached to the flywheel masses make the masses slide back toward their original position with decreases in the rotational velocity. This study assumes that the slider masses in each flywheel are synchronously moving with the same radial motion.

The rotational velocity of the flywheel of the VIRM is proportional to the relative velocity between the two attachment points on the structure. The relationship can be expressed as

$$\begin{aligned}\dot{\theta}_1 &= \alpha_1 (\dot{u}_{s_1} - \dot{u}_g) \\ \dot{\theta}_2 &= \alpha_2 (\dot{u}_{s_2} - \dot{u}_{s_1})\end{aligned}\tag{5.1}$$

where $\dot{\theta}_1$ and $\dot{\theta}_2$ are the rotational velocities of the two VIRM flywheels, and $\dot{u}_{s_1}$, $\dot{u}_{s_2}$, and $\dot{u}_g$ are the absolute velocities of m_{s_1} , m_{s_2} and the ground. α_1 and α_2 are the proportionality coefficients governing this relationship. In this study, $\alpha_1 = \alpha_2 = \alpha$ and $\alpha = \frac{2\pi}{\rho}$ for a ball-screw mechanism where ρ is the lead of the ball-screw.

The mathematical model of the VIRM utilized in this work considers that the springs attached to the flywheel's slider masses have a trilinear elastic force-displacement relationship that has a relatively low stiffness at the center (k_{sd}) between two relatively stiff penalty spring segments (k_p). When the flywheel masses approach the upper bound contact point (R_{ubc}) or the lower bound contact point (R_{lbc}), as shown on Figure 0-1, the penalty spring stiffness segments restrain the movement of the slider masses. The equations defining the restoring force of the trilinear spring (F_{bsd_i}) as a function of the radial displacement of the slider masses of the VIRM in the i^{th} story (x_i) are

$$\begin{aligned} F_{bsd_i} &= k_{sd} (R_{lbc} - x_0) + k_p (x_i - R_{lbc}), \quad x_i < R_{lbc} \\ F_{bsd_i} &= k_{sd} (x_i - x_0), \quad R_{lbc} \leq x_i < R_{ubc} \\ F_{bsd_i} &= k_{sd} (R_{ubc} - x_0) + k_p (x_i - R_{ubc}), \quad x_i \geq R_{ubc} \end{aligned} \quad (5.2)$$

The moment of inertia of the flywheel of the VIRM can be divided into constant and variable parts. The constant portion of the VIRM moment of inertia is

$$J_{\text{constant}} = \left(\frac{1}{12} n m_{sd} \left(\frac{3}{4} d_{sd}^2 + h_{sd}^2 \right) + \frac{1}{2} m_c r^2 \right) \quad (5.3)$$

where n , m_{sd} , d_{sd} , h_{sd} , m_c and r are the number of sliders, slider masses, slider diameter, slider height, mass of the flywheel without the sliders, and radius of the flywheel, respectively. The variable part of the moment of inertia is related to the radial position of the slider masses; when the slider masses move along the guided slot of the VIRM flywheel, the moment of inertia changes. The mass effects provided by the VIRM to the structure increase as the moment of inertia of the VIRM increases. Besides the ability of the slider masses to move during the response of the structure, the VIRM and FIRM flywheels have the same physical properties. The moment of inertia of the flywheel of the VIRM and FIRM flywheels in the i^{th} story can be expressed as

$$\begin{aligned} J_{VIRM_i} &= J_{\text{constant}} + n m_{sd} x_i^2 \\ J_{FIRM_i} &= J_{\text{constant}} + n m_{sd} y^2 \end{aligned} \quad (5.4)$$

The fixed position of the slider mass of the FIRM is denoted by y in Equation (5.4). In this study, y is set as either x_0 , the initial position of the masses in the VIRM, or R_{ubc} , the upper bound contact point.

The equations of motion of the 2DOF primary structure with two VIRM, which is shown in Figure 0-1(a) and hereafter referred to as the 2VIRM structure, can be derived using the Lagrangian method. For this derivation, it is assumed that the two VIRMs have identical properties. The Lagranges' equation of motion in generalized coordinates can be defined as

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = p_i(t), \quad i = 1, 2, 3, 4; \quad (5.5)$$

where q_1, q_2, q_3, q_4 are the displacements relative to the ground of the two masses of the primary structure (u_{s_1} and u_{s_2}) and the radial displacements of the slider masses in the two VIRMs (x_1 and x_2); T and V denote the kinetic energy and potential energy of the dynamic system, respectively.

The kinetic energy of the system can be formulated from the motion of the masses of the structure's floors, rotation of the flywheels, and radial and tangential velocity of the slider masses in the flywheels. The potential energy includes the elastic energy stored in all the springs, including elastic energy stored in the springs related to the structural stiffness (k_{s_1}, k_{s_2}) and the VIRM stiffness (k_{sd}, k_p). The non-conservative forces include the externally applied forces and the damping forces from the viscous dampers.

The equations of motion of the 2VIRM structure subjected to an absolute ground acceleration are

$$\begin{aligned} m_{s_1} \ddot{u}_{s_1} + J_{VIRM_1} \alpha^2 (\ddot{u}_{s_1} - \ddot{u}_g) - J_{VIRM_2} \alpha^2 (\ddot{u}_{s_2} - \ddot{u}_{s_1}) + (2n_1 m_{sd} x_1 \alpha^2 \dot{u}_{s_1}) \dot{x}_1 - (2n_2 m_{sd} x_2 \alpha^2 (\dot{u}_{s_2} - \dot{u}_{s_1})) \dot{x}_2 \\ + (c_{s_2} + c_{s_1}) \dot{u}_{s_1} - c_{s_2} \dot{u}_{s_2} - c_{s_1} \dot{u}_g + (k_{s_1} + k_{s_2}) u_{s_1} - k_{s_2} u_{s_2} - k_{s_1} u_g = -m_{s_1} \ddot{u}_g \\ m_{s_2} \ddot{u}_{s_2} + J_{VIRM_2} \alpha^2 (\ddot{u}_{s_2} - \ddot{u}_{s_1}) + 2n_2 m_{sd} x_2 \alpha^2 (\dot{u}_{s_2} - \dot{u}_{s_1}) \dot{x}_2 + k_{s_2} (u_{s_2} - u_{s_1}) - c_{s_2} \dot{u}_{s_1} + c_{s_2} \dot{u}_{s_2} = -m_{s_2} \ddot{u}_g \\ m_{sd} \ddot{x}_1 - m_{sd} x_1 \alpha^2 (\dot{u}_{s_1} - \dot{u}_g)^2 + F_{bsd_1}(x_1) + c_{sd} \dot{x}_1 = 0 \\ m_{sd} \ddot{x}_2 - m_{sd} x_2 \alpha^2 (\dot{u}_{s_2} - \dot{u}_{s_1})^2 + F_{bsd_2}(x_2) + c_{sd} \dot{x}_2 = 0 \end{aligned} \quad (5.6)$$

The equations of motion for the primary structure with 1 VIRM in its first story, which is shown in Figure 0-1(b) and hereafter referred to as the 1VIRM structure, are shown in Equation (5.7)

$$\begin{aligned}
m_{s_1} \ddot{u}_{s_1} + J_{VIRM_1} \alpha^2 (\ddot{u}_{s_1} - \ddot{u}_g) + (2nm_{sd} x_1 \alpha^2 \dot{u}_{s_1}) \dot{x}_1 + (c_{s_2} + c_{s_1}) \dot{u}_{s_1} - c_{s_2} \dot{u}_{s_2} + (k_{s_1} + k_{s_2}) u_{s_1} - k_{s_2} u_{s_2} &= -m_{s_1} \ddot{u}_g \\
m_{s_2} \ddot{u}_{s_2} + k_{s_2} (u_{s_2} - u_{s_1}) + c_{s_2} (\dot{u}_{s_2} - \dot{u}_{s_1}) &= -m_{s_2} \ddot{u}_g \\
m_{sd} \ddot{x}_1 - m_{sd} x_1 \alpha^2 \dot{u}_{s_1}^2 + F_{bsd_i}(x_1) + c_{sd} \dot{x}_1 &= 0
\end{aligned}
\tag{5.7}$$

In order to evaluate the impact of the VIRM, comparisons will be made to cases where the VIRMs are replaced with FIRMs. The system where identical FIRMs are placed in each story of the primary structure is referred to as the 2FIRM structure and the system where a FIRM is positioned in just the 1st story is referred to as the 1FIRM structure. The equations of motion for the 2FIRM structure and 1FIRM structure are shown in Equation (5.8) and Equation (5.9), respectively.

$$\begin{aligned}
m_{s_1} \ddot{u}_{s_1} + J_{FIRM_1} \alpha^2 (\ddot{u}_{s_1} - \ddot{u}_g) - J_{FIRM_2} \alpha^2 (\ddot{u}_{s_2} - \ddot{u}_{s_1}) + (c_{s_2} + c_{s_1}) \dot{u}_{s_1} - c_{s_2} \dot{u}_{s_2} - c_{s_1} \dot{u}_g \\
+ (k_{s_1} + k_{s_2}) u_{s_1} - k_{s_2} u_{s_2} &= -m_{s_1} \ddot{u}_g \\
m_{s_2} \ddot{u}_{s_2} + J_{FIRM_2} \alpha^2 (\ddot{u}_{s_2} - \ddot{u}_{s_1}) + k_{s_2} (u_{s_2} - u_{s_1}) - c_{s_2} \dot{u}_{s_1} + c_{s_2} \dot{u}_{s_2} &= -m_{s_2} \ddot{u}_g
\end{aligned}
\tag{5.8}$$

$$\begin{aligned}
m_{s_1} \ddot{u}_{s_1} + J_{FIRM_1} \alpha^2 \ddot{u}_{s_1} + (c_{s_2} + c_{s_1}) \dot{u}_{s_1} - c_{s_2} \dot{u}_{s_2} + (k_{s_1} + k_{s_2}) u_{s_1} - k_{s_2} u_{s_2} &= -m_{s_1} \ddot{u}_g \\
m_{s_2} \ddot{u}_{s_2} + k_{s_2} (u_{s_2} - u_{s_1}) + c_{s_2} (\dot{u}_{s_2} - \dot{u}_{s_1}) &= -m_{s_2} \ddot{u}_g
\end{aligned}
\tag{5.9}$$

Additional comparisons will be made with the primary structure without a VIRM or FIRM. This structure will be referred to as the no control structure and the equations of motion are similar to the structures with FIRM, excluding the moment of inertia terms. For convenience, the following ratios are defined and will be utilized throughout this paper.

$\mu_{flywheel} = (n^* m_{sd} + m_c) / m_{s_i}$: flywheel mass ratio; ω_{s_i} : i^{th} mode natural frequency of the primary structure without VIRM/FIRM; $\xi_s = c_s / 2m_s \omega_{s_i}$: primary structure damping ratio;

5.3 System parameters and response measures

This section presents the parameters of the systems and the loading parameters utilized in the analysis of these systems. The range of possible mass effects that are produced by the VIRM given these parameters is also presented. Furthermore, the response measures used in the subsequent section to evaluate system dynamics and performance are introduced.

5.1.1 System and analysis parameters

The results of numerical simulations will be used to compare the dynamic behavior and response of the systems described in the previous section (Equations (5.6)–(5.9)). These numerical simulations are performed using the MATLAB implicit solver (MathWorks, 2022a) with an output frequency set at 4000 Hz. The structure is excited using a band-limited white noise (BLWN) ground acceleration with a frequency range of 1 to 15 Hz, as well as seismic loading. The broadband excitation provided by the BLWN is well-suited for investigating the resulting changes in the structure's dynamic properties. Using the 50 seismic records of the FEMA P695 far-field and near-field earthquake sets (U.S. Department Security & Federal Emergency Agency, 2009), differences in the seismic response of the structural configurations are assessed with a specific focus on median maximum story drift and acceleration.

The system parameters selected for this study are shown in Table 0-1. The resulting natural frequencies of the structure and linearized natural frequencies of the slider mass are between 1-10 Hz. The structural damping coefficients were selected so that the 2DOF structure itself has a 2% inherent damping in each mode and the resulting physical damping coefficients were utilized for the other structural configurations.

Depending on the radial position of the slider masses in the flywheel, the moment of inertia of the flywheels of the VIRM change, as well as the mass effects they provide. The inertance $(b_{2VIRM_1}, b_{2VIRM_2}, b_{1VIRM})$ and inertance ratios $(\beta_{2VIRM_1}, \beta_{2VIRM_2}, \beta_{1VIRM})$ are defined

$$\text{as, } \begin{aligned} b_{2VIRM_1} &= J_{VIRM,1}\alpha^2 + J_{VIRM,2}\alpha^2; b_{2VIRM_2} = J_{VIRM,2}\alpha^2; b_{1VIRM} = J_{VIRM,1}\alpha^2 \\ \beta_{2VIRM_1} &= \frac{b_{2VIRM_1}}{m_{s_1} + m_{s_2}}; \beta_{2VIRM_2} = \frac{b_{2VIRM_2}}{m_{s_1} + m_{s_2}}; \beta_{1VIRM} = \frac{b_{1VIRM}}{m_{s_1} + m_{s_2}}; \end{aligned} \quad (5.10)$$

Figure 0-3 presents the effect of flywheel mass changes and the changes in slider mass position on the inertance ratios of the VIRM in the 2VIRM and 1VIRM structures. The figure shows that the inertance ratio increases as the sliders move out radially and with higher flywheel mass ratios. It is important to note that while the flywheel mass ratio and slider position both impact the inertance ratios, the position of the slider masses has a more significant effect in general. Additionally, it is observed that for the identical parameters considered for both VIRM structures, the maximum inertance ratio of the 2VIRM structure is 5.5, while for the 1VIRM structure, the maximum inertance ratio is 3.

Table 0-1. System parameter values

Symbol	Description	Value
$m_{s_1} = m_{s_2}$	First and second story mass	300 kg
$k_{s_1} = k_{s_2}$	First and second story stiffness	320000 N/m
$\omega_1(\text{no control})$	1 st mode natural frequency (no control)	3.21 Hz
$\omega_2(\text{no control})$	2 nd mode natural frequency (no control)	8.41 Hz
ξ_s	Damping ratio in each mode of the primary system without VIRM/FIRM	2%
μ_{flywheel}	Flywheel mass ratio	0.003, 0.008, 0.015
c_{sd}	Slider damping coefficient	5 Ns/m
k_{sd}	Slider soft stiffness	30 N/m
k_p	Slider penalty stiffness	6000 N/m
d_{sd}	Slider diameter	0.02 m
h_{sd}	Slider height	0.015 m
m_c	Flywheel mass (without sliders)	0.5 kg
R	Flywheel radius	0.1 m
x_0	Initial position	0.02 m
R_{ubc}	Radial position of the upper penalty spring	0.095 m
R_{lbc}	Radial position of the lower penalty spring	0.005 m
α	Proportionality constant between relative structure velocity and flywheel angular velocity	200 (rad/m)

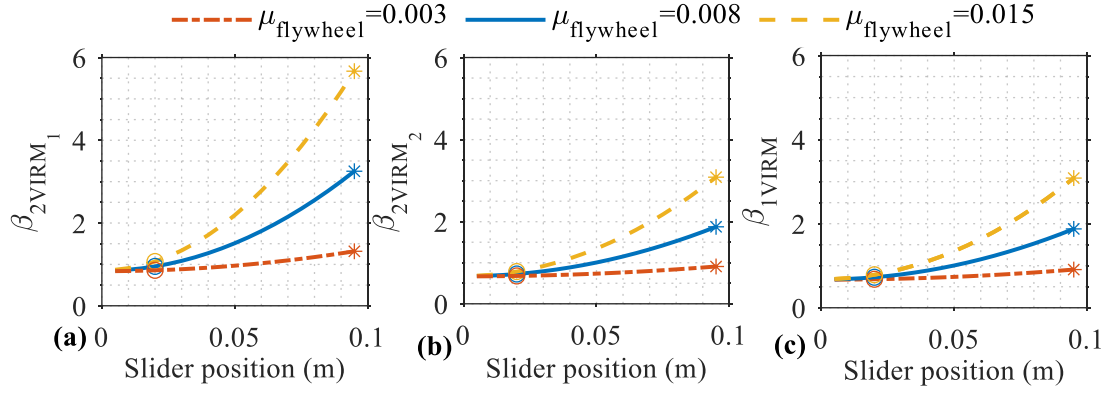


Figure 0-3. Inertance ratios across different flywheel mass ratios and a range of slider mass radial positions (a) β_{2VIRM_1} , (b) β_{2VIRM_2} and (c) β_{1VIRM} . Note that marker legends are Orange:

$\mu_{flywheel} = 0.003$, Blue: $\mu_{flywheel} = 0.008$, Yellow: $\mu_{flywheel} = 0.015$, '*' denotes FIRM (at

R_{ubc}), 'o' denotes FIRM (at x_0)

5.1.2 *Response Measures*

As described in the following sub-sections, various response measures are computed to assess the dynamic behavior and response in this study. The shifts in the natural frequencies are evaluated using wavelet transforms. Other natural frequency measures considered include instantaneous frequency (ω_{IF}) and weighted average instantaneous frequency (ω_w). Peak time-history responses and an estimated H_2 norm analog ($\tilde{H}_2$) are used to investigate structural performance changes with the VIRM. The natural frequency measures and the estimated H_2 norm analog are evaluated considering BLWN loading, while median peak responses from seismic ground motions are determined from a study utilizing incremental dynamic analysis.

5.1.2.1 *Natural Frequency Measures*

5.1.2.1.1 *Wavelet Transform*

The wavelet transform is a tool that can be used to assess vibrational energy distribution in the time and frequency domains. The Morlet wavelet is adopted as the mother wavelet for its simplistic time-domain and frequency-domain analytical forms. As wavelet transforms capture the temporal variability of frequency content, this tool can be effective in examining how the nonlinearity of the VIRM impacts the structure's effective natural frequency over the duration of the analysis time considered.

5.1.2.1.2 *Instantaneous Frequency*

The ω_{IF} represents the preferred vibration frequency of a dynamic system. In a linear system, the ω_{IF} remains constant and matches the system's natural frequency. However, for nonlinear systems, the ω_{IF} continuously changes with the response of the system. In this study, the ω_{IF} is estimated by an eigenvalue analysis of the tangent mass and tangent stiffness matrices at each time step during the response of the system. These matrices for a n-DOF system can be evaluated using Equation (5.11), where $\mathbf{F}_{int}$ contains the internal forces of the nonlinear structure, which is a function of the system DOF (u) and the derivatives of the DOFs ($\dot{u}, \ddot{u}$).

$$\mathbf{K}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}) = \frac{\partial \mathbf{F}_{\text{int}}}{\partial \mathbf{u}} = \begin{bmatrix} \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \mathbf{u}_1} & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \mathbf{u}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \mathbf{u}_n} \\ \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \mathbf{u}_1} & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \mathbf{u}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \mathbf{u}_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \mathbf{u}_1} & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \mathbf{u}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \mathbf{u}_n} \end{bmatrix}; \mathbf{M}_t(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}) = \frac{\partial \mathbf{F}_{\text{int}}}{\partial \ddot{\mathbf{u}}} = \begin{bmatrix} \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \ddot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \ddot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},1}}{\partial \ddot{\mathbf{u}}_n} \\ \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \ddot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \ddot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},2}}{\partial \ddot{\mathbf{u}}_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \ddot{\mathbf{u}}_1} & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \ddot{\mathbf{u}}_2} & \dots & \frac{\partial \mathbf{F}_{\text{int},n}}{\partial \ddot{\mathbf{u}}_n} \end{bmatrix} \quad (5.11)$$

Unlike the addition of the FIRM, the addition of the VIRM adds an additional degree-of-freedom. Hence, the eigenvalue analysis yields three ω_{IF} for the 1VIRM structure and four ω_{IF} for the 2VIRM structure at each time step. The ω_{IF} presented in this study at each time step are the two ω_{IF} that have the most significant engagement of the primary structure, as judged by evaluating their mode shapes, and thus provide information most relevant to the behavior of the primary structure. The resulting two eigen frequencies are then sorted in descending order and are denoted by ω_{IF_1} and ω_{IF_2} , respectively.

5.1.2.1.3 Weighted Average Instantaneous Frequency

As the ω_{IF} changes over time, the weighted average instantaneous frequency, ϖ_w , is derived in order to present information on the overall frequency behavior of the structure. The ϖ_w is determined by averaging the time-history of the ω_{IF} , utilizing a weight factor corresponding to the structure's total energy time-history response, as defined in Equation (5.12). This averaging is performed on both ω_{IF_1} and ω_{IF_2} , with results for the two instantaneous frequencies denoted by ϖ_{w_1} and ϖ_{w_2} , respectively. The ϖ_w can then be normalized using the natural frequencies (ω_1, ω_2) of the no-control structure,

$$\varpi_w = \frac{\sum_{i=1}^n W_i \omega_i}{\sum_{i=1}^n W_i}; \Psi_{\varpi_{w_1}} = \frac{\varpi_{w_1}}{\omega_1}, \Psi_{\varpi_{w_2}} = \frac{\varpi_{w_2}}{\omega_2} \quad (5.12)$$

where W_i and ω_i are the total energy and ω_{IF} of the structure at time step i , respectively, and n is the total number of time steps in the response considered.

5.1.2.2 Performance Measures

5.1.2.2.1 H_2 norm analog

The H_2 norm quantifies system variance amplification; thus, in theory, could be used to assess the impact of the VIRM on the response of the primary structure under dynamic loading. Typically, the H_2 norm is analytically determined for a linear time invariant system using a frequency response function; however, this method is not possible for a structure with a VIRM as it is nonlinear. Alternatively, this study uses a response measure known as the H_2 norm analog, which will be denoted by $\tilde{H}_2$. The $\tilde{H}_2$ is determined by taking the square of the area under the estimated frequency response function of the structure's interstory displacement response due to the ground acceleration over a bounded frequency range of 1-15 Hz, which aligns with the frequency range of the band-pass filtered white noise loading used. The $\tilde{H}_2$ of each configuration considered is normalized by the $\tilde{H}_2$ of the no-control structure, expressed as

$$\Sigma = \frac{\tilde{H}_2(RIM \text{ configuration})}{\tilde{H}_2(no \text{ control})} \quad (5.13)$$

5.1.2.2.2 Incremental Dynamic Analysis Curves

Incremental dynamic analysis (IDA) is performed to quantify the impact of the VIRM on structural performance considering seismic loads. This analysis involves performing multiple nonlinear dynamic analyses of a structural model subjected to a suite of ground motions, with each ground motion scaled to several intensity measures. This process generates IDA curves that represent key measures of the resulting structural response parameterized against the intensity measures for each ground motion. In this study, the maximum absolute interstory displacement and maximum absolute story acceleration are chosen as the key structural response measures.

The IDA is performed using a set of 50 ground motions, sourced from the FEMA P695 dataset, encompassing both far-field and near-field earthquakes (U.S. Department Security & Federal Emergency Agency, 2009). The near-field earthquake records consist of ground motions

with and without pulse, with a total of 14 instances each. The ground motions are scaled in two steps, as outlined in Equation (5.14) and (5.15). Initially, a normalization factor obtained from FEMA P695 (U.S. Department Security & Federal Emergency Agency, 2009) is used to adjust the peak ground velocity of each ground motion to align with the median of the set. Subsequently, a scaling factor is employed to tailor the normalized ground motion to the targeted intensity measure for the specific analyses.

$$\ddot{x}_{g,applied} = \text{normalization factor} * \text{scaling factor} * \ddot{x}_{g,recorded} \quad (5.14)$$

The IDA curves generated in this study use the spectral intensity, S_T , as the intensity measure. In the expression for S_T in Equation (5.15), S_{NRT} represents the median 5% damped spectral acceleration of the normalized record set at the first mode period of the uncontrolled structure.

$$S_T = S_{NRT} * \text{scaling factor} \quad (5.15)$$

Obtained through linear interpolation of values from FEMA P695 using the uncontrolled structure's first mode period (0.2784 s) (U.S. Department Security & Federal Emergency Agency, 2009), the precise S_{NRT} values for near-field and far-field earthquakes are 0.9837 g and 0.7767 g, respectively.

Typically, the IDA is performed until a particular limit state, such as collapse or yield, of a structure is reached. As the structure models employed in this study are simplified and do not account for material nonlinearity and collapse, the analysis is performed up to a predetermined spectral intensity of 2g, rather than a selected limit state.

To capture and compare the overall trends derived from the results of the IDA, the responses of the RIM configurations are normalized against the no control case and the median responses are computed by aggregating the results from seismic records at each intensity level for each configuration. Furthermore, the median absolute deviation is computed as a measure analogous to standard deviation but employing medians.

5.4 Frequency Shifts

In this section, instantaneous frequency (ω_{IF}), weighted average instantaneous frequency (ω_w), and wavelet transforms are used to evaluate the impacts of the different RIM configurations on the structure's natural frequencies when subjected to white noise loading.

These measures for the structures with the VIRM are compared to the measures for the structures with the FIRM with slider masses locked at the initial location, FIRM (at x_0), and with the slider masses locked at R_{ubc} , FIRM (at R_{ubc}).

Figure 0-4 presents the ω_{IF} for the structures with different RIM configurations subject to white noise loading with different amplitudes. This figure shows that the ω_{IF} continuously vary for the structures with VIRM; this is due to the movement of the sliders in the VIRM flywheel during the response of the system. Figure 0-4 also shows that in all the cases, the ω_{IF} of the structures with VIRM are initially at its maximum and identical to the constant ω_{IF} of the structures with FIRM (at x_0), as expected due to the resting position of the slider masses. The range of variation in the ω_{IF} depend on the amplitude of the system loading and response. As the load amplitude increases, a decrease in the overall lower value of the ω_{IF_1} and ω_{IF_2} are observed for the structures with VIRM. For example, the minimum value of ω_{IF_2} for the 2VIRM structure is 4.91 Hz with white noise amplitude of 0.13 ms^{-2} and 2.77 Hz when the load amplitude is 1.03 ms^{-2} . At the low load amplitude of 0.13 ms^{-2} , although ω_{IF} is continuously shifting for both the 1VIRM and 2VIRM structures, the ω_{IF} is closer to the 1FIRM (at x_0) and 2FIRM (at x_0), respectively. Furthermore, lower values of ω_{IF} are consistently observed for the 2VIRM structure, in comparison to the 1VIRM structure, at the two higher load amplitudes, particularly for ω_{IF_2} . This is due to the higher inertance ratio supplied from the attachment of the two VIRM in the structure and aligns with research on inerters in MDOF structures (M. Z. Q. Chen et al., 2014; Wen et al., 2022).

To investigate the effect of different RIM configurations and the load amplitude on the natural frequency shifts and the response of the primary structure, the story drifts and the wavelet transforms of those drifts are plotted in Figure 0-5. Note that, the relative displacement between the 2nd and 1st floors $(u_{s_2} - u_{s_1})$ and between the 1st and ground floors $(u_{s_1} - u_g)$ are defined as the 2nd story drift and 1st story drift, respectively. Figure 0-5 shows the time-histories and wavelet transformations of the 2nd story drift given the white noise loading with RMS amplitude of 1.03

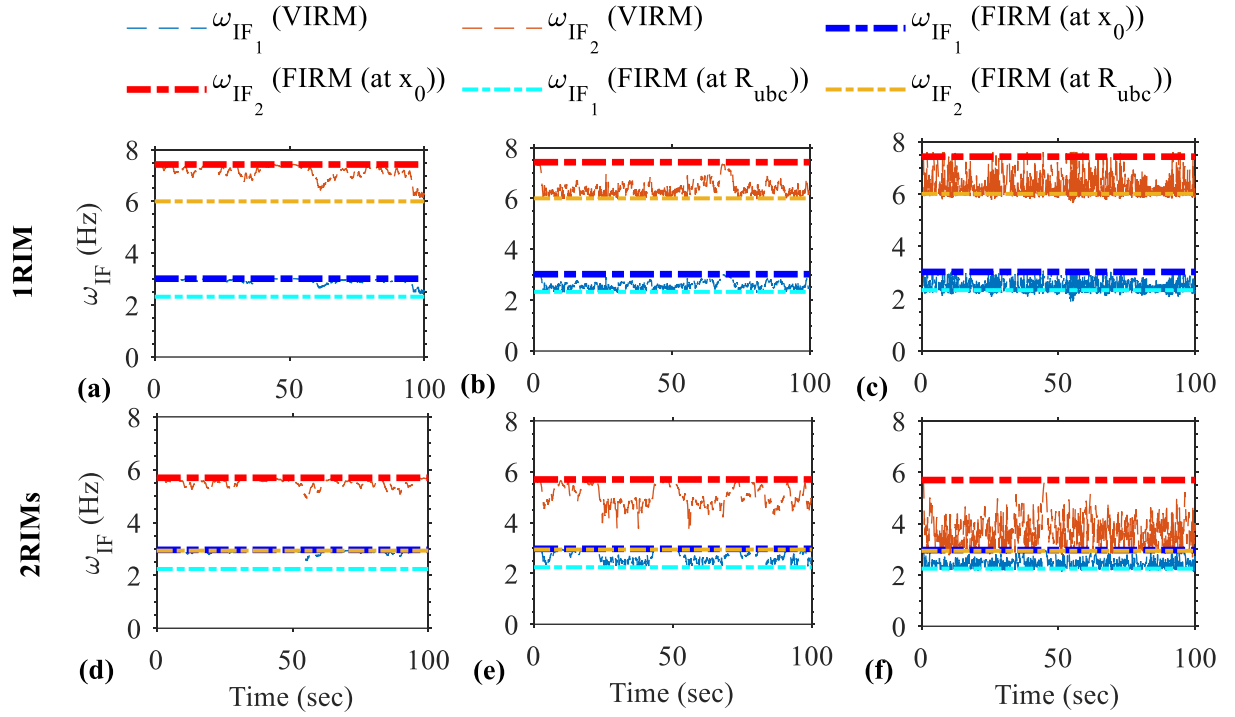


Figure 0-4. Instantaneous frequency of the structure with different RIM configurations (one RIM (a-c) and two RIMs (d-f)) with flywheel mass ratio of 0.008 under RMS white noise amplitudes: (a, d) 0.13 ms⁻², (b, e) 0.30 ms⁻², (c, f) 1.03 ms⁻²

ms^{-2} , which is the same loading considered in Figure 0-4(c), f. It is observed in Figure 0-5(a) that the 2VIRM structure has the lowest 2nd story drift compared to other RIM structures. While Figure 0-5(a) also shows that the 1VIRM structure has an overall lower 2nd story drift time-history amplitude than the 1FIRM structure, the maximum drift amplitudes of both the configurations are more similar than the overall response.

Figure 0-5(b), (c), (d), and e, depict the wavelet transforms of the 2nd story drift of the 2VIRM, 2FIRM, 1VIRM and 1FIRM structures, respectively. In these subplots, the darker shading indicates higher amplitude 2nd story drift response at the time and frequency of the shading. Furthermore, the different wavelet transforms in this figure are plotted such that a direct comparison of relative amplitude can be made when comparing the shading of the different subplots. The wavelet plots show an important 2nd mode response for the 1VIRM structure and a dominant 1st mode response for the 1FIRM and 2FIRM structures. In contrast, the 2VIRM structure shows a small 1st mode response and no noticeable distinct 2nd mode response. The reason for a missing distinct 2nd mode response is that, as seen in Figure 0-4(f), the 2nd mode frequency of the 2VIRM structure has reduced to be, at times, nearly the same as the 1st mode frequency.

The influence of load amplitude on the natural frequency shifts can be investigated using the normalized weighted average instantaneous frequencies $\left(\Psi_{\sigma_{w_1}}, \Psi_{\sigma_{w_2}} \right)$ (see Figure 0-6 and Figure 0-7). These figures present the normalized weighted average instantaneous frequency, as defined in Section 5.1.2.1.3, versus the white noise RMS amplitude for different flywheel mass ratios. Figure 0-6 shows that the first mode normalized weighted average instantaneous frequency $\left(\Psi_{\sigma_{w_1}} \right)$ of the 1VIRM and 2VIRM structures closely match each other for the load amplitudes considered and that $\Psi_{\sigma_{w_1}}$ reduces as the load amplitude increases for both the 1VIRM and 2VIRM structures for all the flywheel mass ratios. As the FIRM structures are linear, their $\Psi_{\sigma_{w_1}}$ does not vary with load amplitude. However, as the flywheel mass ratio increases, $\Psi_{\sigma_{w_1}}$ decreases for both the structures with FIRM and VIRM. Additionally, $\Psi_{\sigma_{w_1}}$ is slightly higher for the 1FIRM structure than the 2FIRM structure for all flywheel mass ratios.

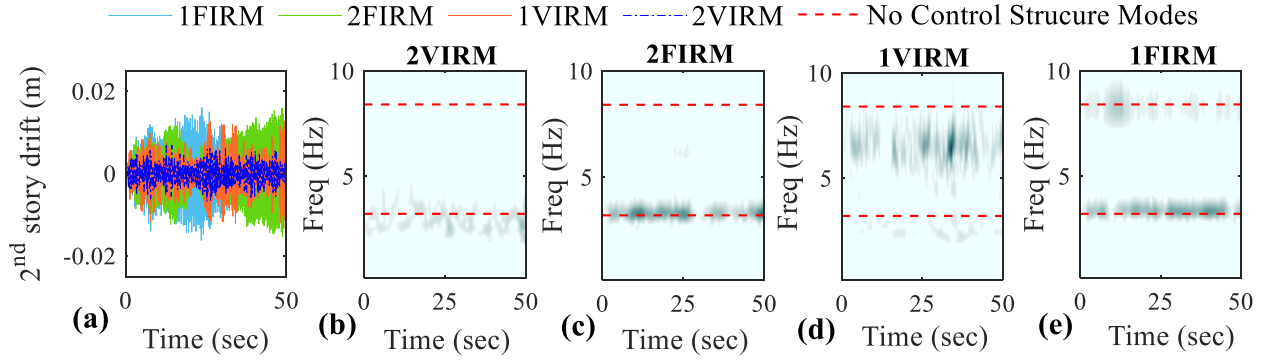


Figure 0-5. 2nd story drift and corresponding wavelet comparison of the structures with RIMs with a flywheel mass ratio 0.008 for a white noise RMS amplitude 1.03 ms⁻²: (a) 2nd story drift, wavelet transform of (b) 2VIRM, (c) 2FIRM, (d) 1VIRM and (e) 1FIRM structures

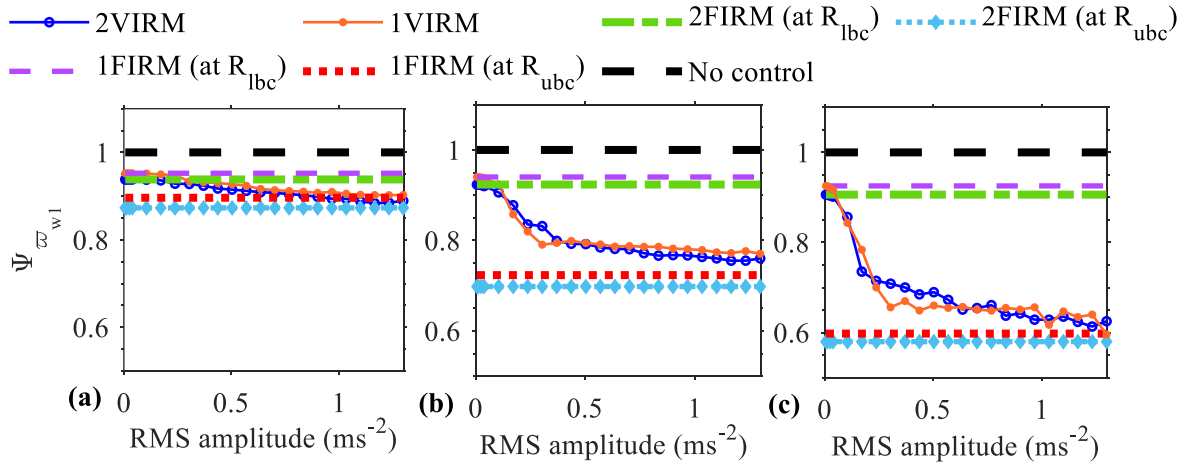


Figure 0-6. Normalized weighted average instantaneous frequency of the first mode, $\Psi_{\sigma_{w1}}$, of various RIM configurations, over a range of white noise amplitudes and for different flywheel mass ratios: (a) 0.003, (b) 0.008, and (c) 0.015

Figure 0-7 shows the second mode normalized weighted average instantaneous frequency $\left(\Psi_{\varpi_{w_2}}\right)$ for different flywheel mass ratios and a range of white noise amplitudes. As with the $\Psi_{\varpi_{w_1}}$ seen in Figure 0-6, the 1FIRM structure has a higher $\Psi_{\varpi_{w_2}}$ than the 2FIRM structure; however, in the case of $\Psi_{\varpi_{w_2}}$, the difference in this value between the 1FIRM and 2FIRM structures is large. Figure 0-7 shows that the 1VIRM structure has a significantly larger $\Psi_{\varpi_{w_2}}$ compared to the 2VIRM structure; however, with the VIRM, the amplitude of $\Psi_{\varpi_{w_2}}$ for both the 1VIRM and 2VIRM structures reduce with higher load amplitude.

The results shown in this section demonstrate that, at a high enough amplitude of system response, a VIRM in the first story or VIRM in both stories can be used to shift both effective natural frequencies of the two-story structure examined. The resulting shifts in the 1st mode are similar for the 1VIRM or 2VIRM structures; however, the shifts in the 2nd mode are much less significant for the 1VIRM structure.

5.5 $\tilde{H}_2$ Response Measure

This section presents the results of the normalized $\tilde{H}_2$ measure, Σ , from the ground motion to story drift frequency response functions to investigate the performance of the VIRM in controlling the interstory displacement of the primary structure. This measure is evaluated for various amplitudes of white noise loading and different flywheel mass ratios.

The effect of flywheel mass ratios and the load amplitudes on the story drifts of the primary structure with different RIM configurations is presented in Figure 0-8. This figure shows that the performance of the structures with the FIRM remains unchanged as the loading amplitude increases, which is expected as the FIRM is linear. Note that, Σ of the structures with the FIRM is determined both when the slider masses are locked at x_0 and locked at R_{ubc} . Σ for the different FIRM configurations is sometimes less than one and sometimes more than one, which indicates that the effect of the FIRM on system performance is not always positive. As the FIRM only produces a constant change in system dynamics, it is believed that the change in Σ for the FIRM structures is mostly a result of the interaction of the specific time history of the finite duration white noise loading and the new shifted system dynamics.

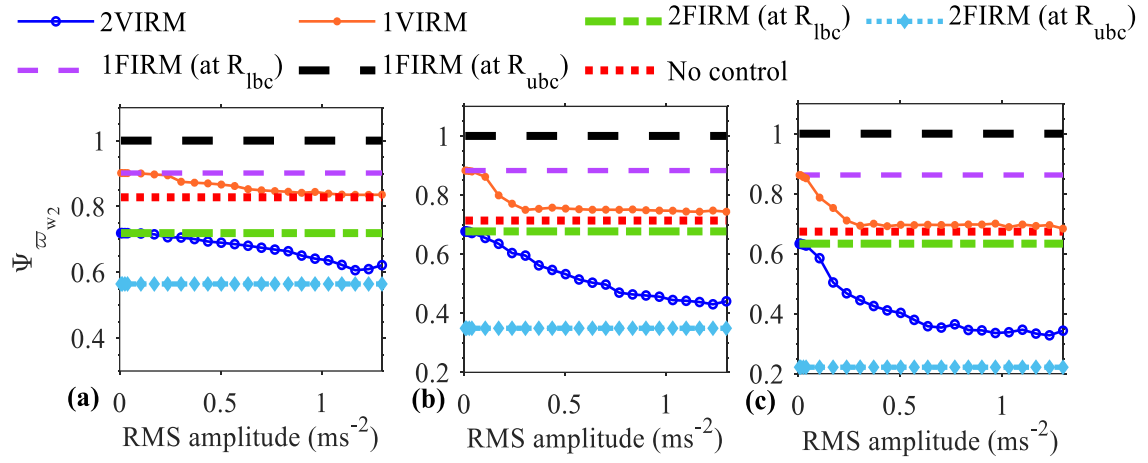


Figure 0-7. Normalized weighted average instantaneous frequency of the second mode, $\Psi_{\omega_{w2}}$, of various RIM configurations, under a range of white noise amplitudes and for different flywheel mass ratios: (a) 0.003, (b) 0.008, and (c) 0.015

Unlike for the structures with the FIRM, it is seen from the results in Figure 0-8 that Σ is always less than 1 for the structures with the VIRM. The Σ performance of the 1VIRM and 2VIRM structures are consistent at low load amplitudes, but increases in amplitude cause their behavior to diverge. At higher load amplitudes, the Σ for the structures with the VIRM reduces and eventually reaches somewhat of a plateau with the 2VIRM structure always having a smaller value of Σ . As flywheel mass ratios increase, a distinct improvement in the performance of the structures with the VIRM is observed, resulting in lower minimum Σ values with higher flywheel mass ratios. Both the 1VIRM and 2VIRM structures have similar performance at controlling the 1st story drift, but the 2VIRM configuration is better at reducing the 2nd story drift compared to the 1VIRM configuration. This is logical as the 1VIRM configuration does not include a VIRM in the second story.

Considering a pair of flywheel mass ratios, the influence of RIMs on the frequency response function (FRF) curves of the ground excitation to the primary structure's 2nd and 1st story drifts are depicted in Figure 0-9. Additionally, the frequency response curve for the no control case is included for reference. The peaks of the FRFs in this figure are utilized to discuss the system natural frequencies, including changes in peak frequency and amplitude, highlighting the overall impact of RIMs on the FRF curves. As the FIRM possesses linear properties, the FRFs for the FIRM structures are characterized by smooth curves. In contrast, the nonlinear behavior of the VIRMs results in non-smooth FRF curves for the VIRM structures.

Figure 0-9, as expected, shows that all of the RIMs lead to reductions in the effective system frequencies seen in the FRFs compared to the no control structure. These reductions in frequencies are more pronounced for the structures with VIRM than the structures with FIRM. The figure also shows that for the 2VIRM structure there appears to be only one dominant effective frequency, the 1st mode frequency. In both story drifts, the 1st mode peak amplitude of the 1VIRM structure is similar to the 2VIRM structure for both flywheel mass ratios considered; however, the 1VIRM structure also has a distinct 2nd mode with a significant peak in the FRF. This high 2nd mode peak amplitude of the 1VIRM structure is the reason for its larger H_2 response measure, as shown in Figure 0-9.

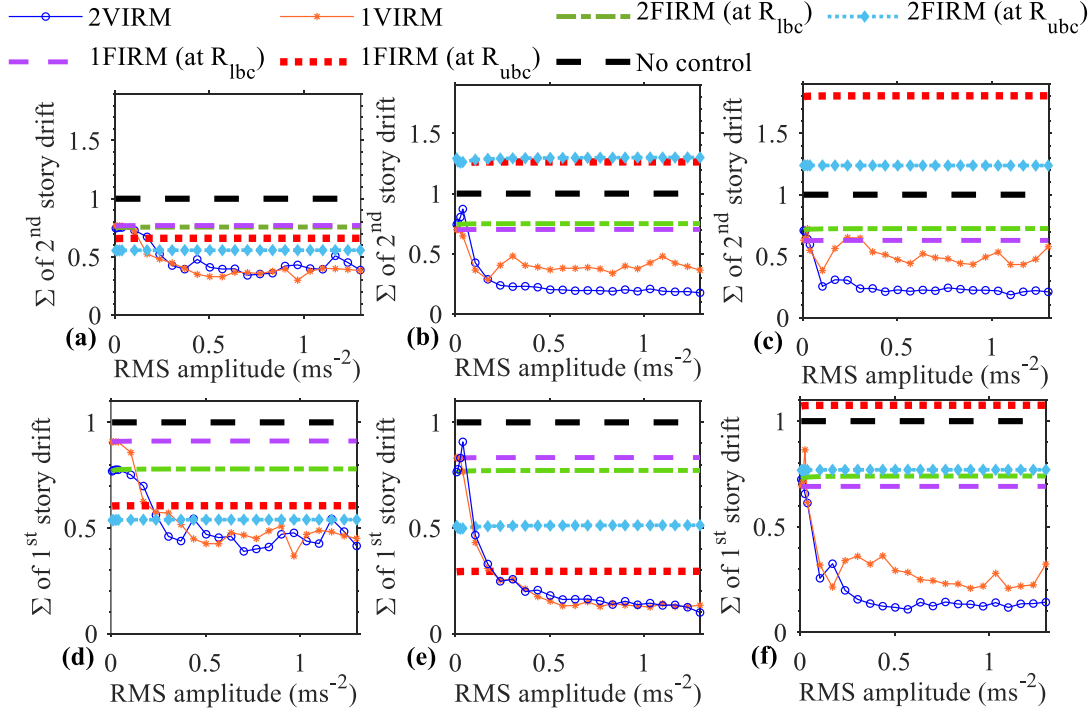


Figure 0-8. Normalized $\tilde{H}_2$, Σ , of the 2nd story drift and 1st story drift of the primary structure with RIMs of different flywheel mass ratios: (a,d) 0.003, (b,e) 0.008, and (c,f) 0.015, over a range of RMS white noise amplitudes.

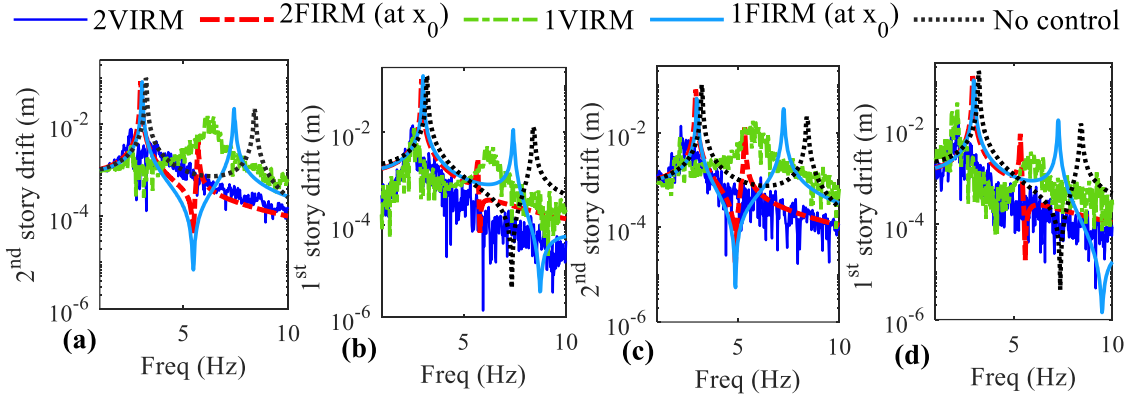


Figure 0-9. Frequency response curves of 2nd story and 1st story drift under a normalized RMS white noise amplitude of 0.30 ms⁻² for different structural configurations with RIM flywheel mass ratios: (a-b) 0.008, and (c-d) 0.015

5.6 Incremental Dynamic Analysis

This section presents the findings of a parametric incremental dynamic analysis (IDA) of the 2DOF structure subjected to the collection of far-field and near-field ground motion set across a range of spectral intensity, S_T , values. Flywheels with various sizes of slider masses are considered in this analysis. In this section, two measures are used to evaluate the response of the system, the maximum absolute story drift (MASD) and maximum absolute story acceleration (MASA). These measures of the RIM configurations are normalized against the no control case and median normalized MASD and MASA values are determined for multiple ground motions.

Figure 0-10 shows the resulting S_T vs MASD curves from the IDAs of the 2DOF structure with different RIM configurations. Each single line represents results for a specific ground motion scaled with increasing spectral intensity. Differences between the lines indicate the differences in response when subjected to different ground motions given each ground motion's unique temporal and frequency characteristics. As the FIRM structures are linear, the MASD increases linearly with increasing scaled amplitude for individual earthquake records. For the 1VIRM and 2VIRM structures, the IDA curves exhibit an initially linear response when the ground motion intensity is low. During these low intensities, the sliders in the VIRM remain very close to the initial positions, resulting in behavior resembling that of the 1FIRM and 2FIRM structures (at x_0). However, as the ground motion intensity increases, the slider masses have more significant radial motion, leading to a nonlinear response that causes the IDA curves for the 1VIRM and 2VIRM structures to deviate from the initial linear segments. As the ground motion intensity increases, the MASD generally exhibits an increasing trend for the VIRM structures, although not necessarily monotonically. The median response for each configuration across all seismic records at each intensity level, as depicted in Figure 0-10, shows the overall trend from the IDA results.

The median normalized MASD IDA curves for RIM configurations with different flywheel mass ratios and the no control case are presented in Figure 0-11. Due to the normalization, x-axis values below one indicate the RIM's effectiveness in mitigating structural response compared to the no control structure. The shaded region of the plots represents the median absolute deviation (MAD) of the normalized response. Notably, the vertical normalized curves for the structures with FIRM signify that the ground motion intensity does not impact the effectiveness of the linear FIRM and the 2FIRM structure has a lower median response than the 1FIRM structure. Figure 0-11 shows that at low spectral intensities, the 2VIRM structure is more

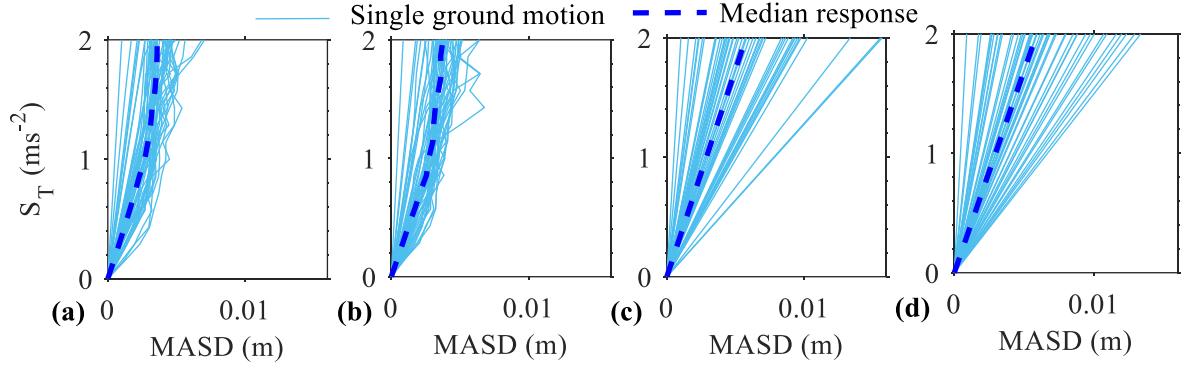


Figure 0-10. IDA curves showing spectral acceleration (S_T) vs maximum absolute story drift (MASD) of the (a) 2VIRM, (b) 1VIRM, (c) 2FIRM, and (d) 1FIRM structures, with a flywheel mass ratio of 0.008

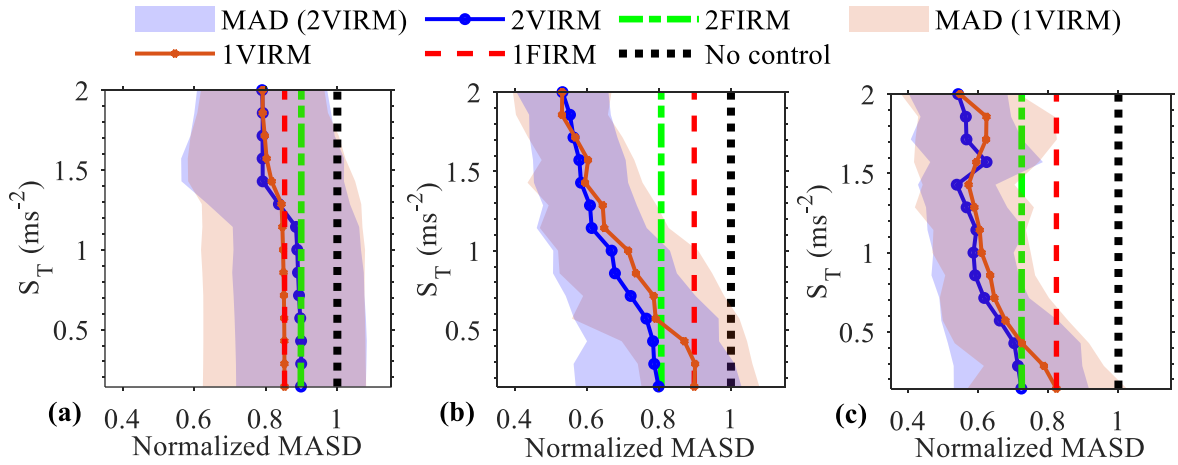


Figure 0-11. S_T vs median normalized MASD IDA curves considering the different RIM configurations and flywheel mass ratios (a) 0.003, (b) 0.008, and (c) 0.015

effective than the 1VIRM structure at reducing the median MASD. However, as the spectral intensity increases, the 2VIRM and 1VIRM structures show similar performance in terms of reducing the maximum absolute story drift. The figure also shows that the increase in flywheel mass ratios reduces the median normalized MASD response of the structures with VIRM and FIRM.

In addition to drift performance, another important parameter to consider is the impact of these devices on the acceleration response of the structures. Figure 0-12 presents the median normalized MASA for the RIM configurations with different flywheel mass ratios and no control structure. Similar trends are observed for the median normalized MASA as the median normalized MASD for the FIRM structures which shows that the 2FIRM structure has lower median response than the 1FIRM structure and as the flywheel mass ratios increase, the median response for both 1FIRM and 2FIRM structures reduce. In the low flywheel mass ratio case, both the 1VIRM and 2VIRM structures were more effective in reducing story acceleration than their counterparts with FIRM at higher intensities. The performance of the 2VIRM structure seen in this figure is also comparable to the normalized MASD performance shown in Figure 0-11.

The most notable difference in the MASA results shown in Figure 0-12 compared to the MASD results shown in Figure 0-11 is the performance of the 1VIRM structure. While the 1VIRM structure exhibits comparable performance to the 2VIRM structure in reducing normalized maximum absolute story drifts, it is significantly less effective in reducing normalized maximum absolute acceleration as the flywheel mass ratio increases. At a flywheel mass ratio of 0.015, there are multiple instances where the normalized median MASA of the 1VIRM structure approaches and occasionally exceeds that of the 1FIRM structure. The poor performance of the 1VIRM structure at higher flywheel mass ratios in terms of acceleration response, compared to other RIM configurations, is likely because the 1VIRM structure was shown to be largely ineffective at reducing the higher mode response.

5.7 Conclusion

This study investigates the use of a variable inertia rotational mechanism (VIRM) for the passive vibration control of a multi-degree-of-freedom structure subjected to base excitation. The study examines the response of a 2DOF structure, in which the VIRM is employed in two configurations: (a) in the 2VIRM structure, one VIRM is employed between the ground and 1st floor and another VIRM is employed between the 1st and the 2nd floor and (b) in the 1VIRM

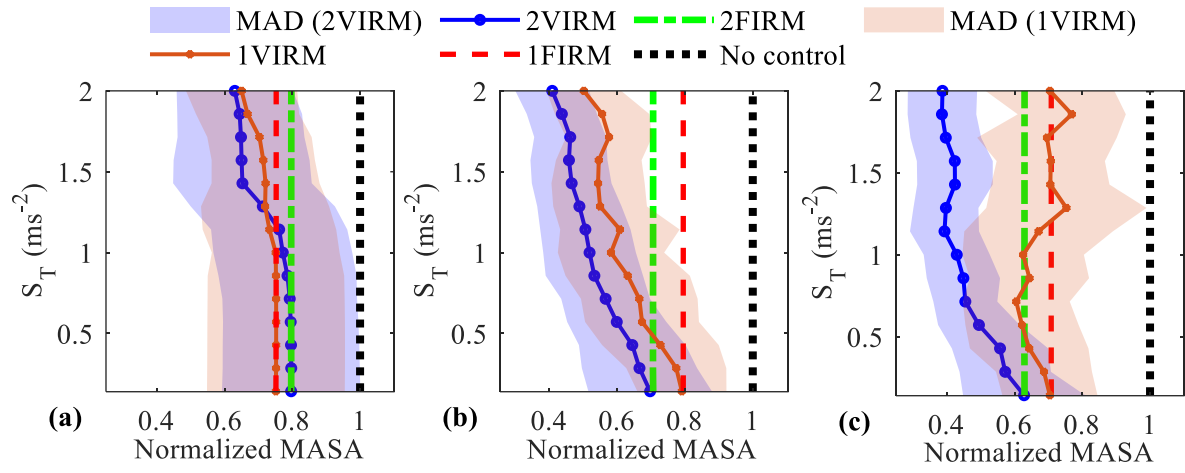


Figure 0-12. S_T vs median normalized MASA IDA curves considering the different RIM configurations and flywheel mass ratios (a) 0.003, (b) 0.008, and (c) 0.015

structure, only one VIRM, positioned between the ground and the 1st floor, is employed. Similar structures with fixed inertia rotational mechanisms (FIRM) are also examined in this study to draw comparisons with the structures with VIRM. The equations of motion of the different structural configurations are formulated and described. The natural frequency and $\tilde{H}_2$ response measures were assembled from the time-history analysis of the structure under white noise base loading. Incremental dynamic analyses were performed to evaluate the maximum absolute story displacement and maximum absolute acceleration response given a suite of seismic ground motions. The following key results were observed from this study.

- The effective natural frequency of the structures with a VIRM depends on the load amplitude and flywheel mass ratios. Higher flywheel mass ratios or load intensities led to decreases in effective natural frequencies for both the 1VIRM and 2VIRM structures.
- The 2VIRM structure was found to be much more effective than the 1VIRM structure at reducing the effective second mode natural frequency.
- The performance of the VIRM in response mitigation is strongly influenced by the intensity of the loading. At low load intensities, VIRMs and FIRMs (slider masses fixed at initial positions) have a similar impact on the responses of the structures. At higher load intensities, improved response reduction is observed for the structures with VIRM, especially the 2VIRM structure.
- Higher flywheel mass ratios contribute to enhanced performance at reducing story drifts for the 2VIRM and 1VIRM structures; however, the 1VIRM structure performance deteriorated at reducing story accelerations with increasing flywheel mass ratios.
- Although a single VIRM can have a large impact on the response of the structure, to effectively mitigate the response at each story and in each mode, VIRMs would be needed at each story.

The exploration of the VIRM's behavior and its impact on frequency shifts and structural response done in this study presents a compelling case for its efficacy in structural control.

Further parametric investigation of the VIRM as a passive control device for structures under diverse loading scenarios and considering more realistic models of structures is imperative for further evaluating the effectiveness of the VIRM.

CHAPTER 6 Experimental Investigation of variable inertia rotational mechanism

6.1 Introduction

Recent advancements in passive control strategies have led to the development of rotational inertia mechanisms (RIMs), which have demonstrated promising results in both research and practical implementation. This mechanism can convert linear motion into the rotational motion through a rack and pinion or ball-screw in combination with a flywheel, fluid-based mechanisms or electromagnetic-based mechanisms (M. C. Smith, 2002, 2008, 2012; Swift et al., 2013). This conversion increases the effective mass of the structure it is installed in, as a result altering its natural frequency and reducing the structural response due to dynamic loads.

The variable inertia rotational mechanism (VIRM) is a nonlinear RIM where the rotational inertia of the flywheel is a function of its rotational velocity. The VIRM flywheel contains multiple symmetric mass-spring-damper systems, with masses that move radially along the flywheel's guide as its absolute velocity changes. The movement of the masses inside the VIRM produces a variable mass effect and can exhibit variable behavior and performance under different load types and amplitudes. Early work by Ullman and Velkoff laid the theoretical foundation for VIRMs in 1978, highlighting their potential in energy storage and harvesting (Ullman, 1978). More recent studies have expanded VIRM's applications to include vibration reduction in vehicle suspensions systems, rotating machineries and civil structures (Sarkar et al., 2024; S. Yang et al., 2019; Yuan et al., 2010). However, the current body of research remains limited, primarily investigating fundamental behavior, conceptual design, and modeling in targeted applications.

The existing research on variable inertia rotational mechanisms (VIRMs) have mostly focused on investigating the variable rotational inertia effect in response to input excitations, often through individual numerical or experimental investigations (Dong et al., 2018; Kushwaha et al., 2020; W. Li et al., 2021; Mahato et al., 2019b; Y. Zhang et al., 2020). Only a limited number of studies have combined both experimental and numerical work, but these have primarily concentrated on the force of the VIRM due to the rotational velocity changes at the two terminals of the flywheel (S. Yang et al., 2016b). Additionally, most existing research have predominantly focused on investigating the intrinsic behavior of VIRM, without considering their interaction and effects when integrated with an attached structural system.

In recent years, VIRMs have been employed in quarter car models, which is a two-degree-of-freedom (2DOF) linear structure. A major limitation of this research is that the equation of motion of the structure with the VIRM is simplified, ignoring the inherent

nonlinearity arising from the tangential velocity of the slider masses (Xu et al., 2015b; S. Yang et al., 2016b). The tangential velocity of the slider masses results in a nonlinear damping effect on the structure, which is an interaction between flywheel velocity, slider mass velocity and slider mass movement. As a result, these studies do not accurately capture the VIRM's impact on structural responses. To address this, an earlier chapter of this dissertation has proposed a more accurate mathematical model that accounts for the nonlinearity associated with the tangential velocity of the slider mass. However, due to lack of experimental investigation, potential unmodeled dynamics may still exist within this model.

Additionally, in the previous chapters of this dissertation and previous literature on the VIRM, the slider stiffness has been considered with a trilinear elastic force-displacement relationship (Dong et al., 2018; Q. Li et al., 2021; Sarkar & Wierschem, 2024) (See Figure 0-1c) or a solely linear force-displacement relationship (Xu et al., 2015a; S. Yang et al., 2019). The trilinear springs include a soft central stiffness and two equal but stiffer penalty springs to better restrain motion when slider masses reach near the center or the outer ring of the flywheel. For the research with only linear stiffness VIRM, the loading amplitude and damping coefficient have been limited to ensure the motion of the slider masses are within set bounds (Dong et al., 2018; W. Li et al., 2021).

This chapter focuses on the experimental realization of the VIRM and its numerical comparison. The primary objective is to develop a small-scale VIRM model and gain insights into the dynamic characteristics of the VIRM and its interaction with a single-degree-of-freedom (SDOF) structure. Controlled experiments are conducted, including both VIRM component-level and primary structure level dynamic analyses, to thoroughly characterize the behavior of the VIRM and its interaction with the attached structure. The experimentally evaluated response and frequency shifts of the VIRM equipped SDOF structure are then compared with the results of the updated mathematical model. This investigation includes a VIRM equipped SDOF structure subjected to random noise ground motion excitation of various amplitudes. The combined experimental and numerical approach can help to identify the shortcomings of the mathematical model and can provide direction towards further refinement of the mathematical model.

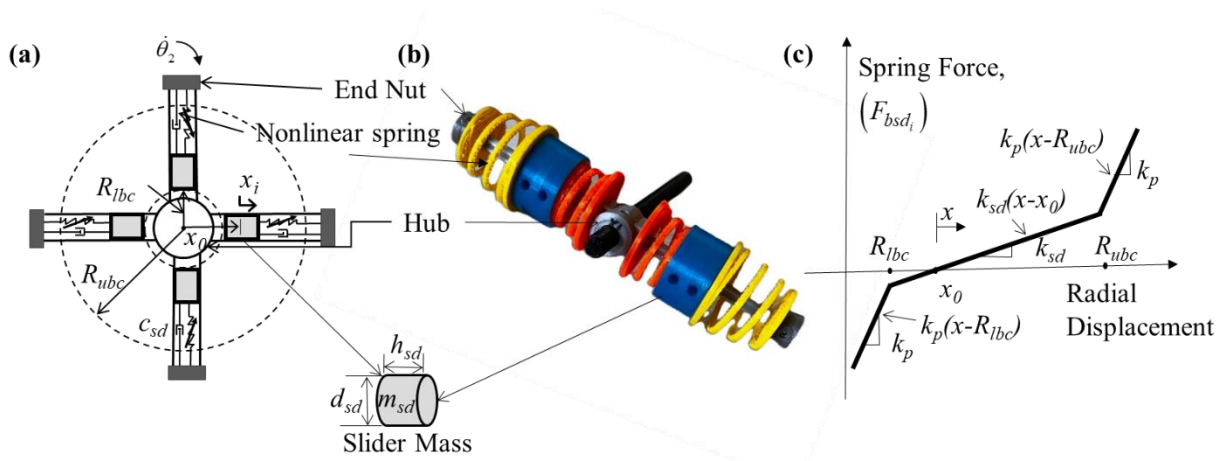


Figure 0-1. (a) Schematic diagram of the VIRM, (b) physical realization of the VIRM, (c) trilinear force displacement relationship of the springs

6.2 Mathematical model of SDOF system with a VIRM

The VIRM is a functionally varied nonlinear rotational inertia mechanism that provides variable mass effects to a structure. The VIRM considered in this work features a passive flywheel with built-in mass-spring-damper elements, where the masses move radially along the flywheel's guide. The relative motion of a rack-and-pinion assembly installed between masses in a system drives the VIRM flywheel to rotate when the structure is vibrated. As the absolute rotational velocity of the flywheel increases, the masses inside the flywheel move outward from the center of the flywheel due to centrifugal force, generating a higher moment of inertia. Likewise, as the flywheel velocity decreases, the masses move back to their initial position near the flywheel's center due to the transient response of the slider mass-spring-damper system. Given dynamic motion of the attached structure, the slider masses will continually change positions, creating a variable mass effect from the VIRM.

The degrees of freedom of the VIRM equipped SDOF primary structure consist of the primary structure's displacement relative to its base (u) and the slider displacement (x). The governing equations of motion of the two-DOF coupled systems, derived using a similar approach to (Sarkar & Wierschem, 2024), are

$$\begin{aligned} m_s \ddot{u} + J(x) \alpha^2 \ddot{u} + 2nm_{sd} x \dot{x} \alpha^2 \dot{u} + c_s \dot{u} + k_s u &= -m_s \ddot{u}_g \\ m_{sd} \ddot{x} - m_{sd} x \alpha^2 \dot{u}^2 + F_{bsd}(x) + c_{sd} \dot{x} &= 0 \end{aligned} \quad (6.1)$$

where, m_s , c_s , k_s , are the mass, damping, and stiffness of the primary system and $\ddot{u}_g$ is the ground acceleration, respectively, while α , n , m_{sd} , c_{sd} denote the moment of inertia, coefficient between the linear and angular motion, number of sliders, slider mass and slider damping. $F_{bsd}(x)$ symbolizes the force exerted by the nonlinear spring of the VIRM and $J(x)$ denote the moment of inertia of the VIRM flywheel, which changes with the radial position of the slider mass.

6.3 Experimental Model

This section provides a detailed description of the experimental setup and methodologies employed in this study. The experimental setup includes the primary structure, VIRM flywheel prototype, rack and pinion gearbox and necessary attachments. Additionally, the instrumentation and data acquisition, including sensors and their placement, is briefly outlined. Finally, the characteristics of the ground motion excitation applied during the experiments are presented.

6.1.1 **Primary Structure**

The primary structure for this experimental study is a custom-designed, one-story structure, as depicted in Figure 0-2. This figure provides an isometric view of the primary structure, illustrating the structure's dimensions, mounting block attachments for the RIMs and the direction of loading applied during tests. The top and base plates have identical dimensions of 416 mm X 152.4 mm X 19.3 mm and the column are 240 mm X 76.2 mm X 3.5 mm, as depicted in Figure 0-2 (b) and (c). The vertical distance between the top and the base plate is 203 mm.

The primary structure components and the mounting blocks for the VIRM are fabricated using fused deposition technique with polylactic acid filament. These 3D printed components are connected using steel fasteners. The columns are orientated such that the structure is flexible in one horizontal direction and much stiffer in the perpendicular horizontal direction and in torsion. Additionally, the structure is designed to remain elastic during testing so that the primary source of nonlinearity arises from the attached VIRM device.

The top and base plate is connected to the VIRM using a set of mounting blocks. A steel plate is rigidly attached to the top plate to provide additional mass to the system. The total moving mass of the system is 5.5 kg, which includes the mass of the top plate, the supplemental steel block, the two column halves, the top mounting block, and associated hardware and excludes the VIRM. The natural frequency of the structure without any added control device was determined to be 2.21 Hz.

6.1.2 **VIRM**

The VIRM used in this study consists of a gearbox housing with a rack and pinion assembly, and a flywheel, as shown in Figure 0-3. The rack connects the two ends of the terminals and moves linearly within the housing, converting linear motion into the rotational motion of the pinion. Neglecting backlash and other nonlinearities, the relative motion of the two terminals and the rotational velocity of the VIRM flywheel are related as follows:

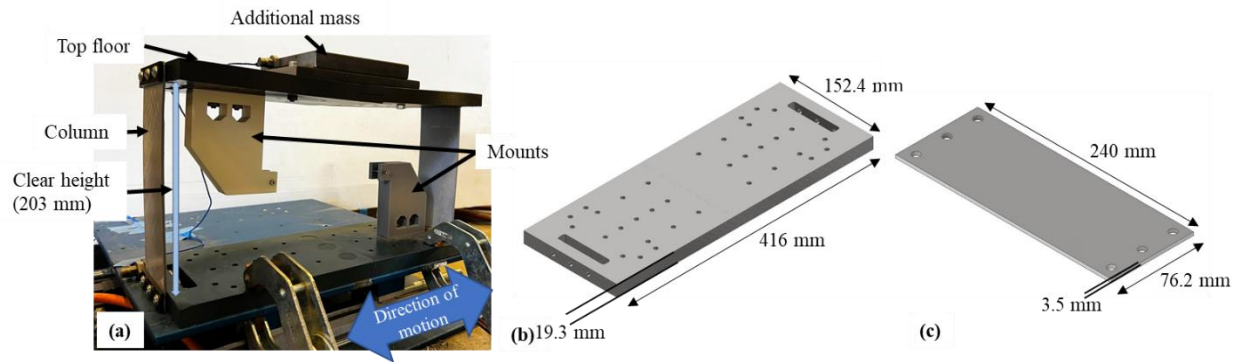


Figure 0-2. (a) Primary structure with mounts attached to the shake table, (b) top plate, (c) column, (dimensions in mm)

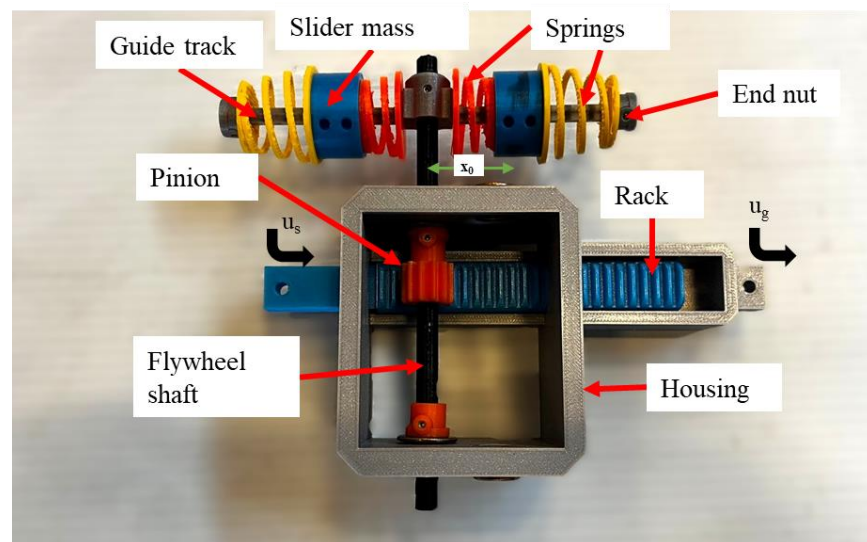


Figure 0-3. VIRM used in this study with annotations identifying different components

$$\dot{\theta} = \alpha \dot{u} \quad (6.2)$$

where $\dot{\theta}$, and $\dot{u}$ represent the rotational velocity of the flywheel and the relative velocity of the structure between the terminals. α for a rack and pinion mechanism is $\frac{1}{r_p}$, where r_p is the radius of the pinion. However, there exist intrinsic friction in the rack and pinion assembly which contributes to additional damping of the VIRM. Assuming a negligible friction in this simplified linear system, the force generated in the two terminals is proportional to the relative acceleration between them and can be expressed as:

$$F = b\ddot{u} \quad (6.3)$$

where, F is the equal forces on the two terminals, $\ddot{u}$ is the relative acceleration of the two terminals and b is the proportionality constant known as inertance. Inertance is equivalent to the effective inertial mass and has the same unit as mass.

The VIRM flywheel incorporates two symmetrically spaced guides with slider mass and spring systems. Each slider mass is only allowed to move radially along the guides within the flywheel. Additionally, the flywheel hub in the center and nuts at the end of the guides restrain the slider masses to movement within the flywheel. Linear bearings are integrated into the slider masses to minimize friction during their radial movement. To ensure smooth motion and prevent impact forces during operation, each guide rack incorporates two springs with a slider mass positioned between them.

The mass of each slider is 0.009 kg, and the inertance of the flywheel without the moving masses is 0.215 kg. When the slider masses are at the initial position (0.0305 m radial distance from flywheel center) and at the radial position with the springs fully compressed near the edge of the guide (0.05 m radial distance from flywheel center), the total inertance produced is 0.5124 kg and 1.015 kg. The resulting mass amplification ratio, calculated as the peak inertance value divided by mass of only the flywheel and attached axles of the VIRM, is 29. In this study, two other RIM configurations are considered where the masses inside the VIRM flywheel are fixed at their initial position or alternatively near the edge of the guide. As the inertance produced with these RIMs are constant, they are referred to as fixed inertia rotational mechanism (FIRM).

6.1.3 *3D printed springs*

The springs attached to the slider masses are one of the most important components of the VIRM as their load deformation relationship will play a large role in dictating the nonlinear response of the system. The preliminary studies of a VIRM design at a similar scale include commercially available springs. While expected shifts in system apparent frequency with changes in loading amplitude were observed, changes in device inertia was less than desired due to limitations on the spring's travel and larger than desired stiffness (Sarkar et al., 2023). Applying fused deposition modeling (FDM) 3D printing to these mechanical springs increases possibilities for the VIRM by enabling the production of custom high-travel and low-stiffness springs with the appropriate geometry.

While FDM 3D printed springs have compelling potential, there is limited research on the mechanical properties and performance of the springs (Enea & Moon, 2022; He et al., 2019). Additionally, design, customization, and deformational control are complex in nature. In this study, the mechanical properties of the 3D-printed springs are investigated using controlled mechanical experiments. Polyethylene terephthalate glycol (PETG) thermoplastic polyester filament, is used to produce two sets of conical springs with different dimensions. The conical shape is specifically chosen to provide a variable spring rate, meaning the stiffness of the spring changes as it is compressed. These springs are printed with a custom support system that is cut away after printing. Each guide of the VIRM consists of two springs and a slider mass, one spring is relatively shorter (Spring A) and near the hub of the flywheel, and the longer spring (Spring B) is between the slider mass and the VIRM end nut. The two springs on both sides of the slider masses are designed to avoid any impact force on the slider masses when the sliders move back and forth during the VIRM flywheel rotation. The springs near the flywheel hub are relatively stiffer compared to the spring positioned closer to the outer circumference of the flywheel. This design choice promotes increased radial displacement of the slider masses away from the center of the flywheel when subjected to higher rotational velocity, therefore generating a higher moment of inertia. Additionally, one end of Spring A and one end of Spring B are fixed with the flywheel hub and the end nut, respectively, causing only compression in the springs. The properties of the springs (Spring A and Spring B) are described in Table 0-1.

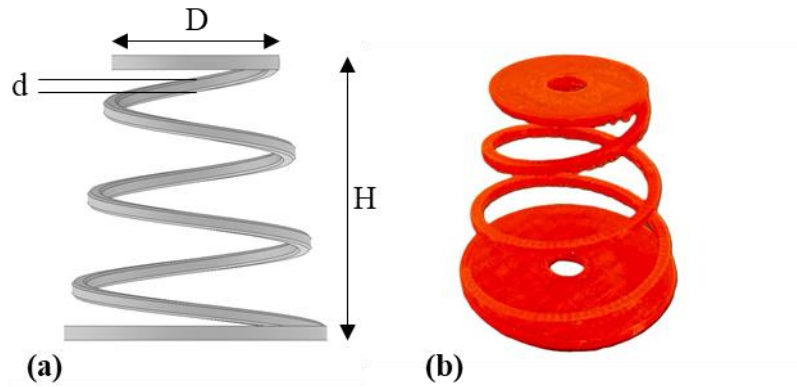


Figure 0-4. Conical spring: (a) side view specifying the geometric parameters defined in Table 0-1 (b) isometric projection of Spring A

Table 0-1. Geometrical parameters of the VIRM conical spring specimens

	Wire thickness, d (mm)	Spring diameter on smaller cross section, D (mm)	Number of windings	Taper percentage (%)	Height, H (mm)
Spring A	1.8	25.4	3	10%	38.1
Spring B	1.8	25.4	2	8%	12.7

6.1.4 ***Instrumentation and Data Acquisition***

The primary structure is attached to the shake table, and PCB 352C33 piezoelectric accelerometers are used to measure the accelerations of the structure's top mass and the shake table in the direction of loading. Data was collected with a sampling frequency of 1652 Hz. A low-pass filter with a cutoff frequency of 15 Hz is applied to the acceleration data to attenuate the high frequency noise. The cutoff frequency is selected such that it is significantly higher than the primary structure's natural frequency. The experimental setup is shown in Figure 0-5

6.1.5 ***Description of ground motions***

The primary structure is mounted on a Quanser Shake Table II, a single-axis shake table with dimensions 0.46 m X 0.46 m and maximum travel distance of ± 0.0762 m. An electric motor combined with a ball-screw actuator controls the displacement of the table in a single-axis. Various types of ground motions are used for testing for this experimental investigation. The intensity of the ground motions is adjusted individually to result in significant movement of the slider masses without causing damage to the experimental setup of the physical model. A band-limited random noise loading with a frequency range of 1 Hz to 25 Hz with 117s duration is employed to identify the first mode frequency, stiffness and damping ratio of the primary structure. To conduct the numerical analyses, this same random noise is scaled to different intensities, allowing for a direct comparison between experimental and simulated responses across various excitation levels.

All ground motions are applied as a horizontal base movement in the flexible direction of the structure. The recorded experimental ground motion, capturing the actual shake table movement, is used directly as the input for the numerical model.

6.4 **Experimental System Identification**

This section details the experimental identification of the dynamic properties characterizing both the primary structure and the VIRM. Robust system identification is a prerequisite for comparing with a numerical models which ensures accurate representation of the dynamic response of structures. The study employs a two-pronged approach, utilizing both frequency and time domain methods to extract system parameters from experimental data. The details of frequency domain identification using band-limited random noise is presented in detail in this section. Additionally, test protocols for estimating VIRM system parameters, such as

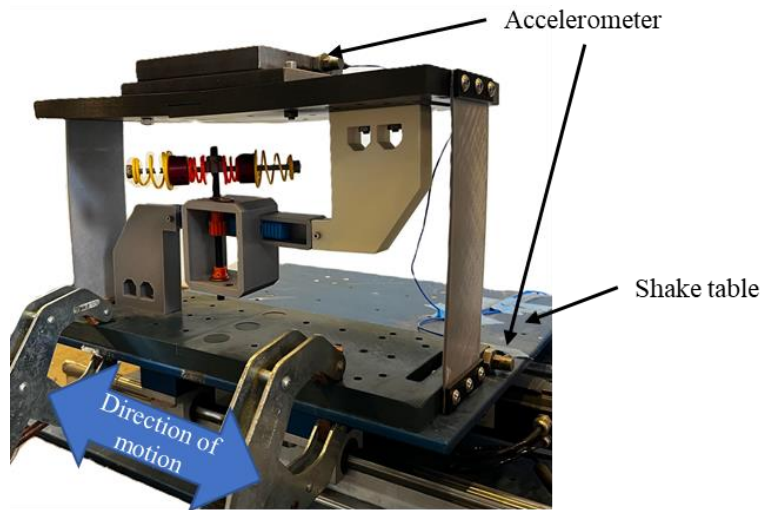


Figure 0-5. Experimental test setup on the shake table platform with notations indicating instrumentation

spring stiffness and damping ratio, are also presented here. The identified parameters are subsequently used to update the mathematical model presented in Section 6.2. The effectiveness of this model updating process is then evaluated in Section 6.5.

6.1.6 **VIRM identification**

This subsection focuses on identifying the stiffness and damping behavior of the VIRM slider masses. Stiffness is determined by analyzing the relationship between the restoring force and displacement of the slider springs. Damping ratios are extracted from the time-history response of slider movement induced by an initial displacement. The following subsections detail both identification techniques.

6.1.6.1 *Stiffness identification*

The stiffness of the 3D printed conical springs within the VIRM flywheel are determined with the aid of displacement-controlled compression testing. The experimental setup, depicted in Figure 0-6, includes a GeoJac digital load actuator (5000 N capacity, 38.1 mm stroke) to compress spring specimen. A load cell placed in line with the actuator measures the applied force, while an LVDT, mounted above the load frame, records displacement. Both Spring A and Spring B are tested under identical conditions: 10% strain rate and a maximum strain of 40%. Data acquisition is performed using a load cell sampling frequency of 1652 Hz and an LVDT sampling rate of 1 Hz. To address the differences in the sampling frequencies, the LVDT data is resampled to match the load cell frequency. Due to the quasi-static nature of the test, the resampling of the LVDT data is not expected to impact the identification results.

Figure 0-7 depicts the force displacement relation of both springs, plotted together considering their respective positions within the VIRM flywheel guide tracks. A low-pass infinite impulse response filter is applied to the raw force displacement data to mitigate the high frequency noise. Note that the displacements in this figure have been shifted such that these displacements reflect the slider position motion relative to the center of the flywheel. The MATLAB curve fitting toolbox is utilized to fit the low pass filtered data to a fifth-degree polynomial model with bisquare weights robustness to minimize the influence of outliers to the fit data. This fitted analytical model is the restoring force of the combined springs and can be expressed as

$$F_{bsd}(x) = p_1x^5 + p_2x^4 + p_3x^3 + p_4x^2 + p_5x + p_6 \quad (6.4)$$

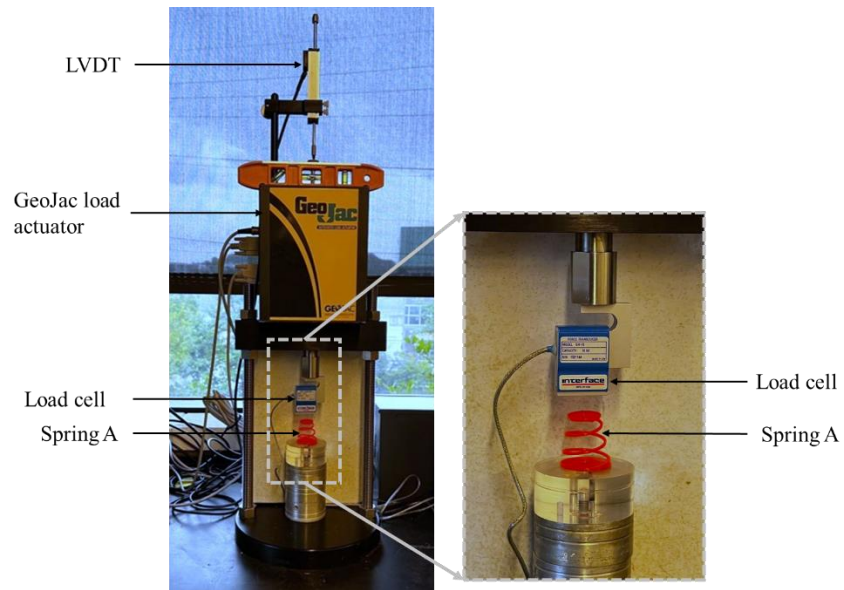


Figure 0-6. Experimental setup of the conical spring specimen on the GeoJac load actuator

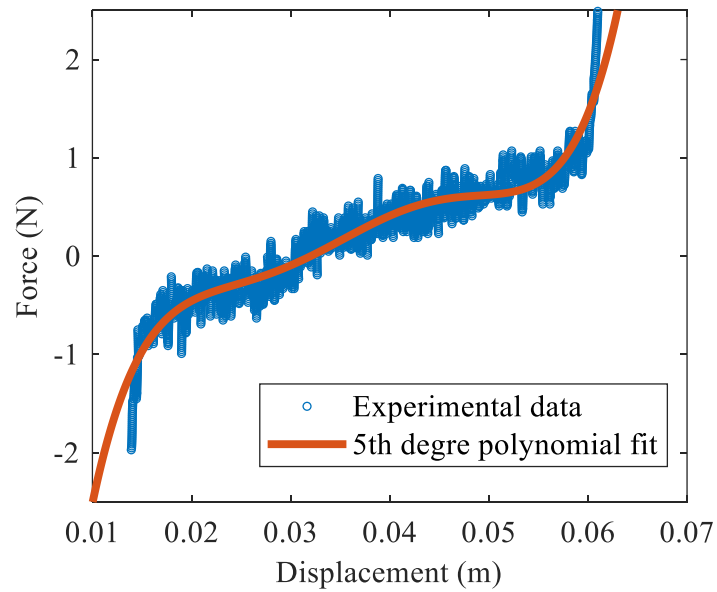


Figure 0-7. Force displacement relationship of the two springs showing the experimental result and the fifth-degree polynomial fit of the experimental data. Note that the displacement is adjusted to reflect the springs radial position on the flywheel guide.

where x is the displacement of the slider mass along the guide track and p_1, p_2, p_3, p_4, p_5 and p_6 are the coefficients with values 1.41E8, -2.5E7, 1.7E6, -5.5E4, 904.8 and -6.5, respectively. This restoring force analytical model is later utilized in the equations of motion of the VIRM slider mass to update the mathematical model.

6.4.1.2 Damping identification

This study employs computer vision technology to determine the damping ratio of the VIRM sliders. Figure 0-8a shows the experimental setup, which consists of the GoPro camera (240 fps), a checkerboard adhesive label attached to the slider mass and the VIRM flywheel itself. The GoPro camera captures the slider's displacement under an initial displacement condition. The checkerboard pattern on the slider mass enables precise position tracking when a video of the motion is processed using the MATLAB computer vision toolbox (The Mathworks Inc., R2024a). The pattern also facilitates the calibration of camera pixels to displacement units. In the processing of each frame, the checkerboard is detected, the centroid of the checkerboard is computed, and the displacement of the centroid is then output relative to the position at the initial frame. Figure 0-8b shows an example of the resulting displacement time history of the slider mass. The damping ratio of the VIRM sliders is then extracted using the logarithmic decrement method, which quantifies the decay rate of free damped vibrations. In this method, the damping ratio is calculated based on the ratio of peak values from cycles during the free vibration response.

The time-history of the slider mass displacement under free vibration measured with computer-vision is presented in Figure 0-8b. The two successive response amplitudes are identified from this time-history and applied to the logarithmic decrement method, as shown in Equation (6.5), to extract the damping ratio of the VIRM sliders.

$$\log \text{ decrement, } \delta = \frac{1}{n} \log \left(\frac{P_i}{P_{i+n}} \right); \text{ damping ratio, } \xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (6.5)$$

where, P_i , P_{i+n} and n are the response amplitude at time 'i', response amplitude after n subsequent cycles and n is the number of cycles between P_i and P_{i+n} . As different geometry springs exist in an individual guide of the VIRM flywheel, the damping ratio also varies as the slider mass engages with the specific spring. When the slider moves outward or inward from its initial position, thereby engaging with Spring A or Spring B, it experiences a damping ratio of 0.17 and 0.13, respectively.

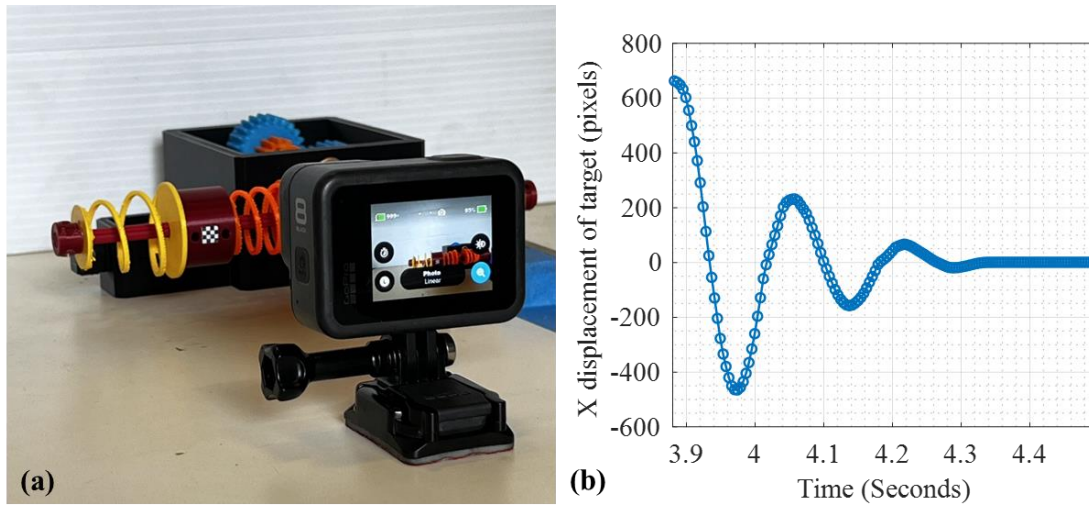


Figure 0-8. (a) Experimental setup of the slider damping ratio measurement using computer vision technology, and (b) measured time-history of slider mass displacement under initial displacement condition using computer vision technology

6.1.7 **Primary structure identification**

The study uses the system response to band-limited random noise (1-25 Hz) to estimate the structure's natural frequency and damping ratio for the different primary system configurations considered. Three structural configurations are investigated in this study: primary structure with no RIM (no control structure), primary structure with FIRM (at R_{ube}), and primary structure with VIRM. The MATLAB signal processing toolbox and the system identification toolbox are utilized to identify the primary structure properties. First, the `tffestimate` function in MATLAB estimates the numerical frequency response function (FRF) between the absolute accelerations at the top and the base plate (The Mathworks Inc., R2023aa). Next, a SDOF analytical dynamic model is fitted to the numerical FRF using the system identification toolbox in MATLAB (The Mathworks Inc., R2023ab). Subsequently, this fitted analytical SDOF model is utilized to extract information about the natural frequency, stiffness and damping ratio of the experimental configuration. The estimated natural frequency and damping ratio of the no control structure is identified as 2.2 Hz and 0.9%, respectively. This natural frequency and the damping ratio are used to compute the stiffness and damping coefficient for the mathematical model. As the stiffness of the primary structure remains the same in the FIRM and VIRM attached structure, the stiffness coefficient identified in the primary structure is utilized for the FIRM and VIRM structures. As there is added damping coming from the gear friction in the RIM structures, the fitted analytical SDOF model is again utilized on the experimental configuration of the FIRM structure to estimate the damping coefficient. This damping coefficient is utilized in both the FIRM and VIRM structures' mathematical model.

6.5 **Experimental Studies**

In this section, frequency response curves of the ground excitation to the structure's top plate absolute acceleration are presented using both experimental investigation and mathematical modeling. In addition to displaying the different structural configurations, frequency response curves of various loading amplitudes are also presented for the VIRM and FIRM structures.

Figure 0-9 presents a comparison between the experimental and numerical results of different structural configurations under random noise excitation with an RMS amplitude of 0.53 ms^{-2} , focusing on the frequency response curves. While the SDOF structure is modeled and designed to behave linearly, the presence of friction from joints introduces nonlinearity to the structure. As a results, the frequency response curves appear non-smooth. Additionally, the rack and pinion gearbox introduces further nonlinearities in the FIRM and VIRM structures. The

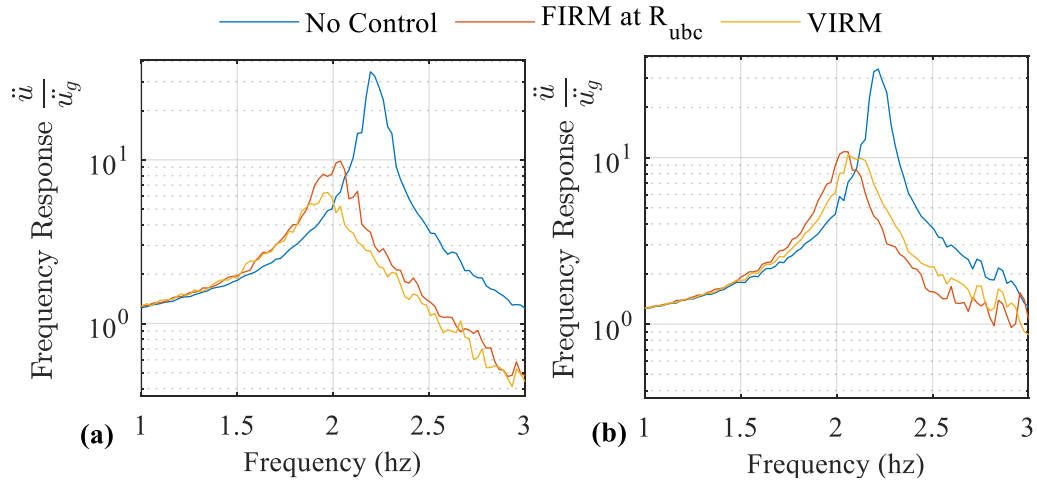


Figure 0-9. Frequency response function of different structural configurations using (a) experimental analysis and (b) numerical analysis

figure shows that for both the experimental and numerical analyses the structure with no control exhibits the highest response amplitude and natural frequency. In contrast, the FIRM structure demonstrates a reduced response amplitude and a lower natural frequency compared to the no control case, which is attributed to the constant added mass effect of the FIRM flywheel. Both experimental and numerical results indicate a consistent trend of response reduction and natural frequency shift, which is also accurately captured in the mathematical model of these structures.

Furthermore, the experimental response of the VIRM structure shows an even greater reduction in response amplitude and natural frequency compared to the other configurations. The superior performance of the VIRM is expected due to the instantaneous frequency shifts enabled by the continuous slider mass movement. However, the numerical analyses on the VIRM do not present the similar trend of reduction in either response or natural frequency. Although the response amplitude of the VIRM structure mathematical model is lower than the FIRM structure, its natural frequency is higher. The anomaly in the experimental and numerical investigation and probable cause are further presented in Figure 0-11. As the mathematical model of the VIRM does not directly model friction, the results are not accurately capturing the observed frequency shift.

Figure 0-10 and Figure 0-11 illustrate a comparative analysis of the frequency response curves between the experimental and mathematical models across varying load amplitudes for the FIRM and VIRM structure, respectively. Figure 0-10 shows an overall consistent alignment between the experimental and mathematical results, with the exception of the low load amplitude case. At this low load level, while the natural frequencies are well-aligned, the response amplitude is noticeably lower in the experimental results compared to the numerical results. This discrepancy is likely due to the fact that, at low loads, the FIRM structure's behavior is significantly influenced by frictional forces. This discrepancy is likely attributed to the more pronounced influence of frictional forces within the rack and pinion gear at these lower loads. While the numerical results at all load levels show a prominent peak in the frequency response near 2.03 Hz, which aligns with the linear nature of the FIRM structure, the experimental results show a slight shift in natural frequency as the load amplitude increases, potentially due to the inherent nonlinearities of the gearbox.

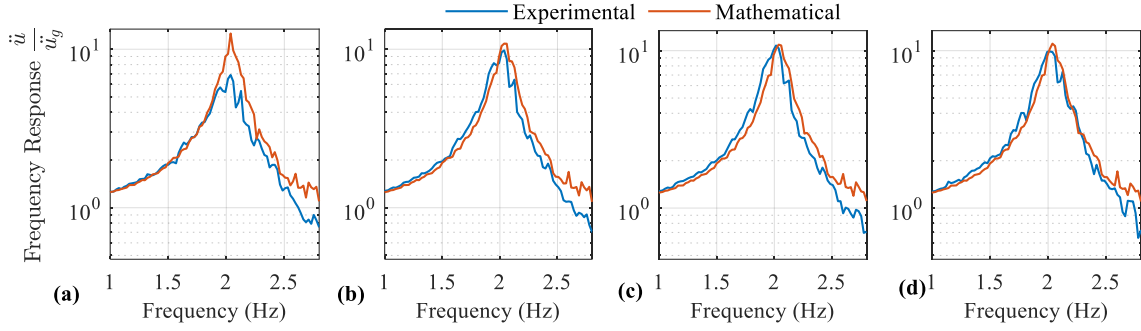


Figure 0-10. Frequency response curves of the ground acceleration to FIRM structure top plates absolute acceleration for various load levels of random noise with RMS: (a) 0.33 ms^{-2} , (b) 0.53 ms^{-2} , (c) 0.73 ms^{-2} , and (d) 0.92 ms^{-2}

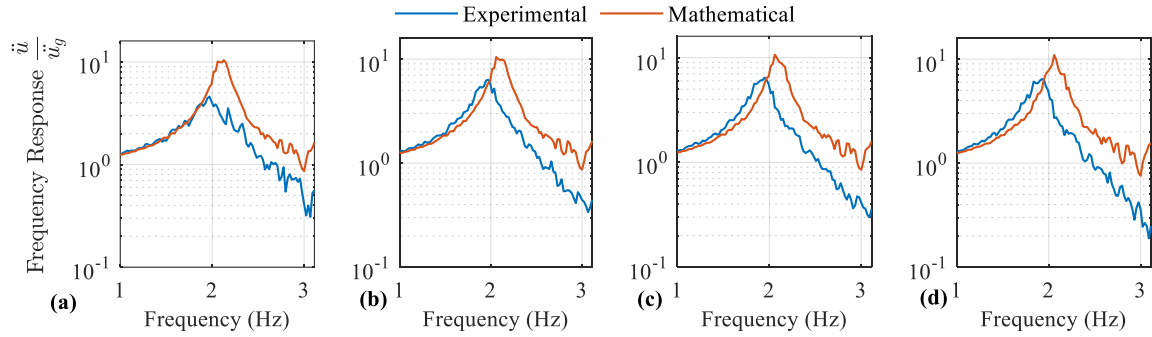


Figure 0-11. Frequency response curves of the ground acceleration to VIRM structure top plates absolute acceleration for various load levels of random noise with RMS: (a) 0.33 ms^{-2} , (b) 0.53 ms^{-2} , (c) 0.73 ms^{-2} , and (d) 0.92 ms^{-2}

Figure 0-11 reveals notable discrepancies between the experimental and mathematical results. The mathematical model consistently predicts a higher overall response amplitude and natural frequency compared to the experimental observations. This could be due to the potential underestimation of damping or other nonlinear phenomenon associated with resonant disruption. At lower load amplitudes, the discrepancies are more amplified, likely due to the dominant friction forces in the experimental setup, which is not adequately modeled in the numerical simulations. This highlights a limitation of the current mathematical model, especially for low load conditions and warrants a need for model refinement, such as incorporating detailed frictional and backlash effects, to enhance the model accuracy.

Furthermore, the experimental results show a clear trend of decreasing natural frequency with increasing load amplitude, a characteristic absent in the numerical results. The shift in the physical system is attributed to the outward movement of the slider masses, resulting in higher inertance and a consequent decrease in natural frequency. The numerical model, however, exhibit less pronounced slider mass movement, suggesting that further refinement of the slider stiffness and damping characteristics might be necessary. Additionally, the observed asynchronous movement of the slider masses in the experimental setup introduces further complexity. The asynchronous behavior, not accounted for in the mathematical model, would lead to complex dynamics behavior.

6.6 Conclusion

This chapter presented the experimental realization of a variable inertia rotational mechanism integrated into a single-degree of freedom structure and compared its experimental response with the results from the updated mathematical model. Additional experimental and numerical comparison are also investigated for a fixed inertia rotational mechanism (FIRM) controlled structure and an uncontrolled structure. After the device was designed and fabricated, this work involved three main stages: system identification using experimental testing, updating the mathematical model using the behavior identified in the first stage, and conducting numerical analyses to compare simulated responses with experimental results.

The experimental investigation confirmed the superior performance of the VIRM in mitigating structural response compared to both uncontrolled and FIRM controlled structures. This advantage stems from the VIRM ability to adjust its inertance dynamically due to the continuous movement of the slider masses, leading to instantaneous shift in natural frequency.

The experimental realization study confirms this continuous movement of the slider masses throughout the response of the structure.

While the developed mathematical models effectively captured the general trends and frequency response behaviors of the FIRM and VIRM structures, some discrepancies between the numerical and experimental results are observed. Specifically:

- The mathematical model overestimated the response amplitude and natural frequencies, particularly for the VIRM structure. The discrepancy is likely due to the unmodeled dynamics associated with nonlinear characteristics, particularly friction within the rack and pinion gearbox, resonance disruption phenomenon and asynchronous slider mass movement.
- The experimental results highlight the significant influence of friction at low load amplitudes for the VIRM and FIRM structures, a factor not fully accounted for in the current mathematical models. The discrepancy underscores the need for friction modeling, especially for low load vibrations.

This chapter emphasizes the importance of considering nonlinearities in the modeling of structural systems equipped with VIRM and FIRM and provides a foundation for future research aimed at refining these models to better capture the complex dynamic interactions present in nonlinear passive control systems. Future work should also consider different types of loading, such as harmonic and seismic, to evaluate the models' robustness and accuracy under a broader range of real-world conditions.

CHAPTER 7 Conclusion

7.1 Summary and conclusions

Civil structures face significant risks from dynamic loads caused by natural and man-made hazards, which can lead to catastrophic failures, economic losses and threats to public safety. The pressing need to protect structures and ensure their resilience has driven extensive research into structural control systems. While semi-active and active control systems have shown promise in protecting structures, cost, complex technology and power reliance have limited their use in civil structures. Linear passive control devices, such as the tuned mass damper (TMD), are increasingly utilized. Although they are effective in reducing structural vibration, the substantial mass and space requirement, and inability to mitigate vibrations outside their tuned frequency pose limitations on their use. However, rotational inertia devices have the potential to be highly effective in passive structural control devices by providing large mass effects to the dynamics of the system.

These increased mass effects have led to the development a new generation of passive control devices. This dissertation expands this newer generation by exploring the incorporation of intentional nonlinearity. This is achieved by developing a device whose effective mass adapts dynamically in response to the structure's motion. The focus of this research is the variable inertia rotational mechanism, and its potential as a practical and effective means of passively protecting structures from dynamic loads caused by natural and man-made hazards. In order to evaluate its potential, the VIRM's effect on structure dynamics, the parametric observations and evaluating the performance of VIRM under different load types, and intensities are achieved in this dissertation using mathematical modeling and shake table testing.

A comprehensive literature review regarding structural control systems, rotational inertia mechanisms (RIMs), and VIRMs and the gaps in literature are discussed in Chapter 2. The derivation behind the mathematical model, and response measures used in this dissertation are presented in Chapter 3. Finally, to meet the goals of the dissertation, the different focus areas outlined in the introduction are investigated and contributions of the specific focus areas are presented in Chapters 4, 5 and 6.

In an attempt to understand the effect of VIRMs on structural behavior, the mathematical model of the VIRM attached to a primary structure was developed and investigated. The different response measures to the understand the fundamental frequency behavior and performance metrics were developed. This assessment was performed numerically with VIRM installed in a simplified linear model to completely understand how the intrinsic nonlinearity of the VIRM

affects the structure. Additionally, effect of various VIRM parameters, different load types and amplitudes were applied to the structure to see its behavior and how it performed under different load types and amplitudes. The VIRM parameters investigated was a range of slider damping and slider stiffness, with single and dual slider stiffness present in a single VIRM across various stiffness levels. The structure behavior was identified using different natural frequency measures, which showed the continuous natural frequency shift and overall lowering of the effective natural frequency compared to a no control case. The results also showed that while the level of slider damping can shift the natural frequency, this shift does not always result in significant changes in the primary structure's response. Additionally, the investigation suggested that dual slider stiffness in VIRM does not offer any advantages over uniform slider stiffness. The findings in this study underscores that the effectiveness of VIRM is highly depended on the amplitude of external loads. For example, under higher loading amplitudes, the structure's response and amplitude decreased and this reduction is less sensitive to the changes in slider damping.

The H_2 norm is a common response measurement to evaluate the performance of a structural control system, however, VIRM being nonlinear, an analytical transfer function solution was not possible. In this work, an approximate H_2 response measure was utilized to measure the performance metric. Under low load intensities, the performance of VIRM is comparable to a linear RIM, named fixed inertia rotational mechanisms (FIRMs), as the load intensities increase VIRM exhibit superior performance in reducing structural response. This performance enhancement is attributed to the VIRM's ability to dynamically adjust the rotational inertia, which results in continuous shifts of the structure's effective frequency. This adaptive frequency not only leads to superior vibration mitigation but also helps to protect structures by preventing sustained resonance conditions.

While the VIRM's dynamic frequency shifting prevents sustained resonance, it is important to acknowledge that certain loading scenarios may introduce brief periods of intensified interaction between the structure and the excitation. These interactions are consequences of the altered frequency and would not occur otherwise. Despite these interactions, the amplitude of the structural response remains bounded as resonant conditions are not maintained. This highlights a crucial finding: while VIRMs show immense promise for vibration mitigation and resonance avoidance, their effectiveness is inherently load dependent. The investigation into different loading scenarios revealed a clear indication: VIRMs excel under harmonic and white noise excitation compared to impulse excitation scenarios. The VIRM's performance in these scenarios

is superior due to its capacity to produce continuous frequency shifting of the structure and prevent it from settling into sustained resonance. However, during impulse excitation, the structure experiences a set excitation energy and the VIRM's frequency shifting cannot disrupt the flow of energy to the structure from the excitation. Furthermore, the degree to which a VIRM can alter the structure's natural frequency appears to be a key factor in its effectiveness, with larger shifts generally yielding better performance.

Overall, the findings show that VIRM can passively induce significant changes in the primary structure's dynamics and show promise for employing as a structural control strategy. A detailed analysis of these findings is presented in Chapter 4. This work laid the groundwork for future research on the use of VIRM in multi-degree-of-freedom structures, as well as the exploration of alternative, more realistic stiffness models in the VIRM and its performance under the realistic motions of civil structures, such as seismic ground motions.

The expanding scope to include various effective mass range of VIRMs in MDOF structures and diverse loadings provides a more comprehensive understanding of VIRM's capacities and limitations. This part of the work focused on two distinct VIRM configurations in a two-degree-of-freedom (2DOF) structure, presented in Chapter 5. In one configuration, termed as the 2VIRM structure, one VIRM is positioned between the ground and the first floor and another placed between the first and the second floor. In the other configuration termed as 1VIRM structure, a single VIRM is positioned between the ground and the first floor. For comparative purposes, the study also examines the 2DOF structure where VIRMs are replaced with FIRMs, named as 2FIRM structure and 1FIRM structure for the two VIRM and one VIRM configurations, respectively. In addition to estimating the response measures previously investigated for the SDOF structure, an incremental dynamic analysis was performed to evaluate the maximum absolute interstory displacement and maximum absolute acceleration responses under a suite of seismic ground motions.

Similar to VIRMs in a SDOF structure, the performance of VIRM in mitigating structural response is shown to be strongly influenced by the intensity of the load. Moreover, the study finds that the 2VIRM structure is significantly more effective than the 1VIRM structure at reducing the effective second mode natural frequency and corresponding vibration. This highlights the potential for employing multiple VIRMs in a structure to achieve better control over its dynamic response, particularly for multiple mode vibration.

The incremental dynamic analysis results show that the higher flywheel mass ratios contribute to better performance in reducing interstory displacements both the 2VIRM and 1VIRM structures. However, an interesting observation is that the performance of the 1VIRM structure deteriorates in terms of reducing story accelerations as the flywheel mass ratios increase. This suggests that while increasing the mass ratios enhance certain aspects of VIRM's performance, it may also introduce complexities in other performance measures if the VIRMs are not uniformly distributed between all the floor levels. This investigation points to the potential need for a distributed application of VIRM in multi-story structures to achieve comprehensive vibration control.

The last focus area of this dissertation investigates the physical realization of the VIRM and compare the numerical and experimental results. While limited work exists on combined numerical and experimental passive VIRM, the focus has been limited to understanding the VIRM's inherent dynamics and not how that corresponds to VIRM attached structure's response. This research aims to bridge the gap by physically realizing a VIRM model, and conduct experimental and numerical analyses to investigate the structure's response. This combined study is essential to compare the experimental results with the mathematical model, particularly under different excitation amplitudes of white noise scenarios. The work in this last part is structured around three primary objectives: system identification, updating the mathematical model and comparing the updated model against experimental data.

The experimental data gathered through white noise excitation testing offers valuable insights into the unmodeled dynamics, ensuring the need for further update of the mathematical model and its robustness needs to be ensured across various configurations and loading scenarios. This combined experimental and numerical work, conducted under varying amplitudes of white noise excitation, revealed key discrepancies between the physical VIRM's behavior and prediction of the established mathematical model. Notably, the model could not accurately capture the movement of the slider masses, one of the crucial aspect governing the system's overall response. These findings underscore the need for further model refinement, incorporating more sophisticated representation of friction and slider dynamics, to enhance the accuracy of the numerical simulations for VIRM integrated structures.

The results of this dissertation highlights VIRM's potential to ensure structural response reduction, providing a novel approach to passive vibration control in civil engineering. The dissertation bridges key knowledge gaps by demonstrating effective passive modifications of

structural dynamics and substantial reduction in dynamic responses. Therefore, further exploration of the use of VIRMs for protecting civil structures is highly recommended.

7.2 Future Work

The promising results of this work highlight the need for continued exploration and development of the VIRM as a part of a structural control strategy. These future directions aim to address the current limitations, explore new applications and combinations of structural control strategies and refine the existing models in a further complex structure. In this section, some potential future directions are described.

7.2.1 *Nonlinear Structural Analysis with VIRM*

The current mathematical model for the Variable Inertia Rotational Mechanism (VIRM) has been developed and tested on linear structures. Future research should extend this to nonlinear structures, incorporating both material and geometric nonlinearity. This would provide a more detailed and realistic model of how VIRM performs under realistic conditions, particularly under severe dynamic loads. Understanding the interaction of VIRM with a nonlinear primary structure will enhance the accuracy of simulations and predictions. Additionally, while the current study primarily focuses on ground acceleration loads, it is crucial to investigate VIRM's performance under different types of dynamic loads, such as wind and impulse loads. Wind loads can induce significant lateral forces, while impulse loads can simulate blast impacts or sudden forces. These investigations will broaden the applicability of VIRM in diverse structural scenarios and provide insights into its behavior under various loading conditions.

7.2.2 *Integration of VIRM with Other Structural Control Strategies:*

While VIRM shows significant promise as a standalone vibration control mechanism, its performance can be further enhanced by integrating it with other structural control strategies. In multi-degree-of-freedom (MDOF) structures, it was observed that a uniform distribution of VIRM generated a single dominant frequency close to the first mode frequency. Integrating a Tuned Mass Damper (TMD) tuned to this dominant frequency could provide substantial benefits. TMDs are effective in reducing vibrations at specific frequencies, and their combination with VIRM could create a combined passive control system that leverages the strengths of both mechanisms. This combined approach can provide more comprehensive vibration control across multiple modes and frequencies.

7.2.3 *Design Optimization and Performance Evaluation of VIRM:*

There is currently no comprehensive study on the design optimization of VIRM. Future research should focus on determining the optimal design parameters of VIRM to maximize its performance. This includes the mass, stiffness, and damping properties of the VIRM components, as well as their arrangement within the structure. Optimization techniques can be employed to identify the best configurations that enhance the VIRM's effectiveness in reducing structural vibrations. Performance evaluation should be conducted under different loading conditions to verify the optimized design. This will involve both numerical simulations and experimental validations to ensure that the optimized VIRM can effectively improve the structural response under real-world conditions. Such studies will provide valuable guidelines for designing VIRMs for various applications in civil engineering.

7.2.4 *Comparative Analysis with Traditional Control Devices*

VIRMs offer versatile implementation, capable of integration into various structural systems. For instance, they can be installed into base isolation system to modify the base isolation frequency, or within bracings or outrigger systems to modify lateral drifts of a structure. While this study primarily focused on benchmarking VIRM performance against linear inerters and uncontrolled scenarios, a comprehensive understanding of their efficacy necessitates comparisons with a wider range of traditional control devices. Future research should prioritize comparative analyses encompassing viscous dampers, metallic yielding dampers, and other devices with similar installation strategies as the VIRM. This broader perspective will illuminate the relative strengths and weaknesses of VIRMs, particularly concerning performance, practicality, and adaptability to specific structural demands. Furthermore, by directly contrasting VIRM behavior with established technologies, this research can provide invaluable insights for engineers navigating the complexities of real-world structural design and retrofitting projects.

7.2.5 *Economic Analysis and Feasibility Study*

A detailed economic analysis is essential to evaluate the cost-effectiveness of VIRM in comparison to traditional vibration control mechanisms. This analysis should consider not only the initial design and installation costs but also the long-term maintenance and operational expenses. By assessing factors such as material costs, maintenance, and potential longevity, a clearer picture of VIRM's market viability could be established. Future research could also

explore the potential for scaling VIRM technology for different types and sizes of structures, ensuring that it is both practical and economically feasible for widespread adoption.

In summary, it is recommended that future investigations focus on extending VIRM's application to nonlinear structures, optimizing its design parameters, and exploring its integration with other control strategies. These efforts will enhance the understanding of VIRM's capabilities and pave the way for its broader adoption in civil engineering, contributing to the development of more resilient and adaptive structures.

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VITA

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