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USING THE μ-INTERACTION MEASURE IN THE STABILITY ANALYSIS OF DECENTRALIZED CONTROL STRUCTURES

A Thesis
Presented for the
Master of Science
Degree
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Nick D. Hutson
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Dedicated to my ever-supporting family.

The author wishes to acknowledge, with sincere gratitude, the guidance and suggestions given by his major professor, Dr. Tse-Wei Wang. The author also wishes to thank the members of his research committee, Dr. Charlie Moore and Dr. Duane Bruns. The author is also grateful to The Department of Chemical Engineering at The University of Tennessee for the financial support that made this research possible.

ABSTRACT

This document details the results of using the μ-Interaction Measure as an analysis tool when systematically designing decentralized controllers for multivariable systems. The μ-Interaction Measure indicates the stability of the full-block closed-loop system by providing an upper bound on the magnitude of the system. Several examples found in literature were used to demonstrate this analysis technique. The diagonal controllers were designed using the Internal Model Control (IMC) method. A study was also done including mathematical model uncertainty. The results from the study indicate that although the μ-Interaction Measure may be somewhat conservative, it provides an effective means to guarantee stability of the closed-loop full-block system.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

The occurance and importance of multivariable control has increased rapidly in the chemical processing industries in recent years. Because of this, a flurry of academic research has been done in the design of multivariable control systems. A multivariable plant would ideally be controlled by a single multivariable controller in which each of the controlled inputs is a function of all the measured outputs. Despite the amount of academic research, very few multivariable controllers are applied in industry. Many feel that modern multivariable control theory has promised much but delivered little (Morari, 1986). Multivariable controllers, as opposed to decentralized controllers (with individual input-output pairing), tend to be mathematically complex and computer intensive. Because of this, multivariable controllers tend to have excessive engineering manpower requirements and a high degree of operator nonacceptance. With these thusfar unsolved problems associated with multivariable controllers, the chemical processing industries have avoided the "modern" multivariable controllers in favor of simpler decentralized control systems. With all the knowledge on single-input-single-output (SISO) designs available, it is very attractive to industrial control systems designers to convert a multivariable problem into a set of independent single-input-single-output systems. If the input-output variables are paired correctly, the multivariable problem can be reduced to several SISO loops under most circumstances. What is being sacrificed is decreased performance of the overall multiple SISO system in comparison to an effectively designed true multivariable controller. The industrial attractiveness of this method is that not only does the system have much simpler controller structure, but the tuning and operation can be left to relatively untrained personnel, based on their experience with classical SISO loops. On the other hand, the operation and tuning of a modern multivariable system almost always requires the presence of a specialist and the aid of a computer.

"Decentralized" control, in this sense, is defined as the use of a diagonal controller, designed based on a diagonal plant transfer function matrix, to effectively control the full-block plant with respect to satisfying a set of performance and stability criteria. The simplicity of this method of controller design must, however, be weighed against the effect of "ignoring" the interactions among the loops, which can lead to performance deterioration and even instability (Grosdidier and Morari, 1987). A real shortcoming of this design approach is that the sum of the performance and stability properties of the multiple SISO loops may not be equal to the final performance and stability properties when implemented on the full multivariable system (Doyle and Stein, 1981). With respect to stability, a legitimate question to ask, therefore, would be "how large can the diagonal controller gains be before the closed-loop system (with the full plant transfer function matrix incorporated) becomes unstable when the controller design is based on a diagonal plant transfer function matrix?". Is there a unified analytical rule to calculate such a "maximum" allowable gain? How does this upper bound change when there is model uncertainty associated with the nominal model used for the design of the diagonal controller?

One purpose of any interaction measure (IM) is to indicate under what conditions the stability of the closed-loop decentralized system guarantees the stability of the original closed-loop system. A new dynamic IM for systems under feedback with diagonal controllers has been developed by Grosdidier and Morari (1987). This measure, the μ-Interaction Measure, will not only predict the stability of the closed-loop full-block system with a controller whose design is based on the diagonal plant dynamics, but will also measure the lost performance caused by the diagonalization of the plant matrix. Computation of μ, the structured singular value of a matrix, is a subject under current research by the systems community. However, the computation of the μ-IM, as used in the sense of Grosdidier and Morari (1987), for a 2 x 2 system has been specified, as shown in that reference.

1.2 The Problem Statement

Given the above considerations, the problem statement can be stated as follows:

Design a diagonal proportional-integral-derivative (PID) controller, based on the diagonal elements of a plant transfer function, using the Internal Model Control (IMC) design procedure for a set of multivariable plant systems. Use the μ-Interaction Measure as a stability analysis tool for the full-block closed-loop system under decentralized control. And futher, investigate the design algorithm for the incorporation of plant model uncertainity into the stability analysis using the μ-Interaction Measure index.

1.3 Literature Survey

Systems engineers have been struggling with the problem of multivariable control for quite some time. Decoupling was the earliest method suggested for multivariable control (Boksenbom and Hood, 1949). When confronted with strong variable interactions, a designer introduces into the system special new elements called *decouplers*. The purpose of decouplers is to cancel the interaction effects between the loops and thus render single-input-single-output noninteracting control loops (Stephanopoulas, 1984). One important factor that determines the ultimate success is that the plant has to be known exactly in order for the decoupling to take place exactly and the restriction that the plant dynamics are invertible. However, even after successful decoupling, one is left with a set of multiple SISO loops, and the comment made earlier concerning the sum of effects not equal to the whole effect still applies. Further, if the open-loop system is ill conditioned (Morari and Doyle, 1986), then an inverse based decoupling controller can give rise to serious performance problems in the presence of model uncertainty (Morari, 1986).

While the knowledge of the use of decouplers has increased, in many cases these kinds of controllers simply may not be needed. If the off diagonal elements of an nxn plant transfer function, $\mathbf{G}(s)$, are "small" relative to the diagonal elements, then the plant can be approximated by a block-diagonal plant, $\tilde{\mathbf{G}}(s)$. If the plants, $\mathbf{G}(s)$ and $\tilde{\mathbf{G}}(s)$, are sufficiently "close" then a diagonal controller, $\mathbf{C}(s)$, can be designed, based on $\tilde{\mathbf{G}}(s)$, by using the μ-IM analysis approach to yield a nominally stable feedback loop around $\tilde{\mathbf{G}}(s)$, but will also imply the stability of the feedback loop around $\mathbf{G}(s)$ as well. The purpose of any interaction measure (IM) would be to indicate under what conditions does the nominal stability of the diagonal closed loop system also imply the stability of the full system.

In defining the concept of an interaction measure, it is helpful to first define exactly the term *interaction.* In exploring various interaction measures, Jensen, *et al.* (1986) found that "the various authors have not used a consistent definition of *interaction* and that the proposed dynamic interaction measures usually do not estimate the actual interaction in the *closed-loop* system." By expanding on MacFarlane's (1972) definition, Jensen, *et al.* defines the term *interaction* as follows:

> *Interactions in a closed-loop multivariable system are determined by the transmittances influencing the way in which a reference input, $r_i(s)$, or a disturbance input, $\xi_i(s)$, effects the set of outputs $\{y_j(s) : j \neq i\}$, or alternatively the tranmittances influencing the way in which an output $y_i(s)$ is affected by the set of reference inputs $\{r_j(s) : j \neq i\}$ or disturbance inputs $\{\xi_j(s)\}$.*

Historically, several measures of interaction have been proposed. Jensen, *et al.* (1986), in a review of suggested interaction measures, found that most have the following shortcomings:

- The analysis, using most IM's, is applied to systems with one loop open (and (n-1) loops closed) rather than on fully closed-loop systems.

- They measure the *parallel* ($r_i \rightarrow y_i$ via other loops) transmittance (with one loop open) rather than the true *interaction* ($r_j \rightarrow y_i$ where $j = 1, 2, 3, \ldots, m; j \neq i$, m = number of inputs/outputs) transmittance.

One of the first proposed measures of interaction was the Rijnsdorp (1965) Interaction Quotient. Rijnsdorp considered only the 2 x 2 case, emphasizing operation of a distillation column. For 2 x 2 systems, the Rijnsdorp interaction quotient (IQ) is defined as

$$\kappa(s) = \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)g_{22}(s)} \tag{1.1}$$

where $g_{ij}(s)$ is an element in the plant transfer function matrix, $\mathbf{G}(s)$. The Rijnsdorp IQ is not, however, a measure of interaction, but rather a measure of the relative significance of parallel transmittance to direct transmittance. While this measure does have significance in pre-design analysis, Rijnsdorp's IQ is *not* a measure of *closed-loop* interaction.

The relative gain array (RGA), proposed by Bristol (1966), is one of the first measures of interaction introduced. It is still applied quite frequently and is the topic of many textbooks. The RGA is very attractive in that it is calculated using only the steady state gains and provides direct guidelines for selecting input/output variable pairings when it is applicable. It also predicts when the interactions will be significant. However, it has been shown that the RGA frequently fails to show when a process has significant interaction problems and may give misleading information about steady state interactions. Futhermore, pairing on negative RGA's is not always reliable because it may produce either an unstable system or a system that lacks integrity (will go unstable with some loops on manual, Grosdidier, *et al.*, 1985). There is, also, rarely any indication that the RGA approach has been unsuccessful, that is it results in an unstable system, until the closed-loop system is simulated or operated.

The inadequacies of the RGA led to other IM's which are extentions of the RGA. Witcher and McAvoy (1977) and Bristol (1978) suggested a dynamic version of the RGA called the relative dynamic gain array (RDGA). While the RDGA does not give a measure of the true closed-loop interaction, it does work for some applications, only however, if there is at least one small interaction. Some experts have questioned the theoretical basis of this measure (Grosdidier and Morari, 1986; Jensen, *et al.*, 1986) and it has not been widely accepted.

Tung and Edgar (1977, 1981) used both state space and frequency domain approaches in their interpretation of a relative dynamic gain array. Their work, however, does not seem to be mathematically rigorous (Jensen, *et al.*, 1986) and has not been widely accepted.

Gagnepain and Seborg (1979) used open-loop step responses as the basis of input/output variable pairing. They expanded on Witcher and McAvoy's idea of dynamic potential by using integrals of open-loop step responses to get an average dynamic gain array (ADGA). The ADGA has, however, not had the success of the RDGA and therefore is rarely applied. Jaaksoo (1979) presented an exact time domain equivalent of the RDGA but this too has its drawbacks.

A major problem with the measures or indices discussed thusfar is that they do not give a measure of closed-loop interaction. None take into consideration the structure of the control system, type of controller, or the associated controller gains. Thus the result is that the measures discussed are more a warning of potential difficulties than true measures of closed-loop interaction. Suchanti and Fournier (1973) suggested an interaction measure for each loop and while their procedure did indeed provide a measure of closed-loop interaction, its calculation is mathematically difficult.

The μ-Interaction Measure (Grosdidier and Morari, 1987) does take into consideration the structure of the controller, the closed-loop system interaction, the controller gains, and is mathematically convenient. For these reasons, the μ-IM analysis procedure seems to be a very realistic measure of interaction, superior to those previously proposed.

1.4 Matrix Variable Notation

In this document, all variables represented by **bold face letters** designate **matrices** or **vectors** unless otherwise specified.

CHAPTER 2

BACKGROUND

2.1 Theory and Mathematics of the μ-Interaction Measure

Control engineers are increasingly faced with the problem of designing controllers for multivariable plants (Figure 2.1). Multivariable controllers are often avoided in industry in favor of less complex "decentralized" controllers. By decentralized control, in this sense, it is meant that the controller, $\mathbf{C}(s)$, for a plant, $\mathbf{G}(s)$, is block-diagonal (a square matrix with Jordan-Canonical form) (Figure 2.2). For the equation

$$u_i(s) = C_i(s)(y_i(s) - r_i(s)), \tag{2.1}$$

it is designated that $\mathbf{u}$ and $\mathbf{y}$ are input and output vectors, respectively. With any decentralized design approach, it is further assumed that the plant is square with equal numbers of inputs and outputs, a prerequisite for using the μ-IM approach, and $\mathbf{r}$ is the reference vector. It is also assumed that $\mathbf{u}$, $\mathbf{y}$, and $\mathbf{r}$ have been partitioned in the same manner, $\mathbf{u} = (u_1, u_2, \ldots, u_n)^T$, $\mathbf{y} = (y_1, y_2, \ldots, y_n)^T$, $\mathbf{r} = (r_1, r_2, \ldots, r_n)^T$, as the controller (Grosdidier and Morari, 1987) in the case of the block diagonal configuration.

In the development of the μ-Interaction Measure, it would be helpful to first define what is meant by an *interaction measure.* Grosdidier and Morari (1986)

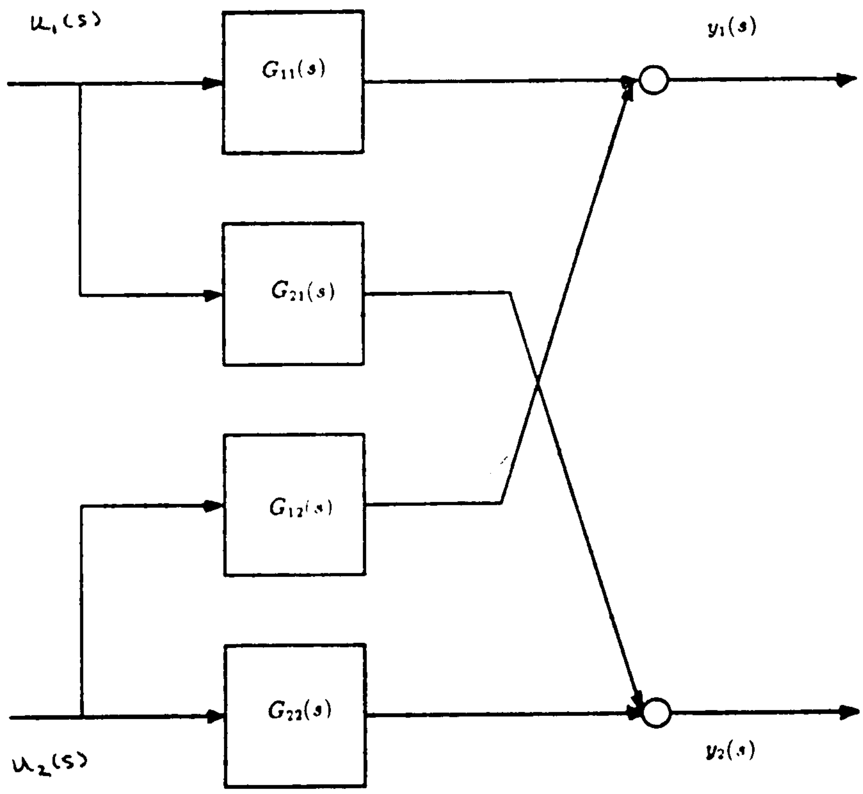


Figure 2.1: Schematic of a Block 2 x 2 Multivariable Plant; $\{u_i\}$ = reference inputs; $\{y_i\}$ = measured outputs; and G_{ij} = the plant transfer function relating the j^{th} input to the i^{th} output.

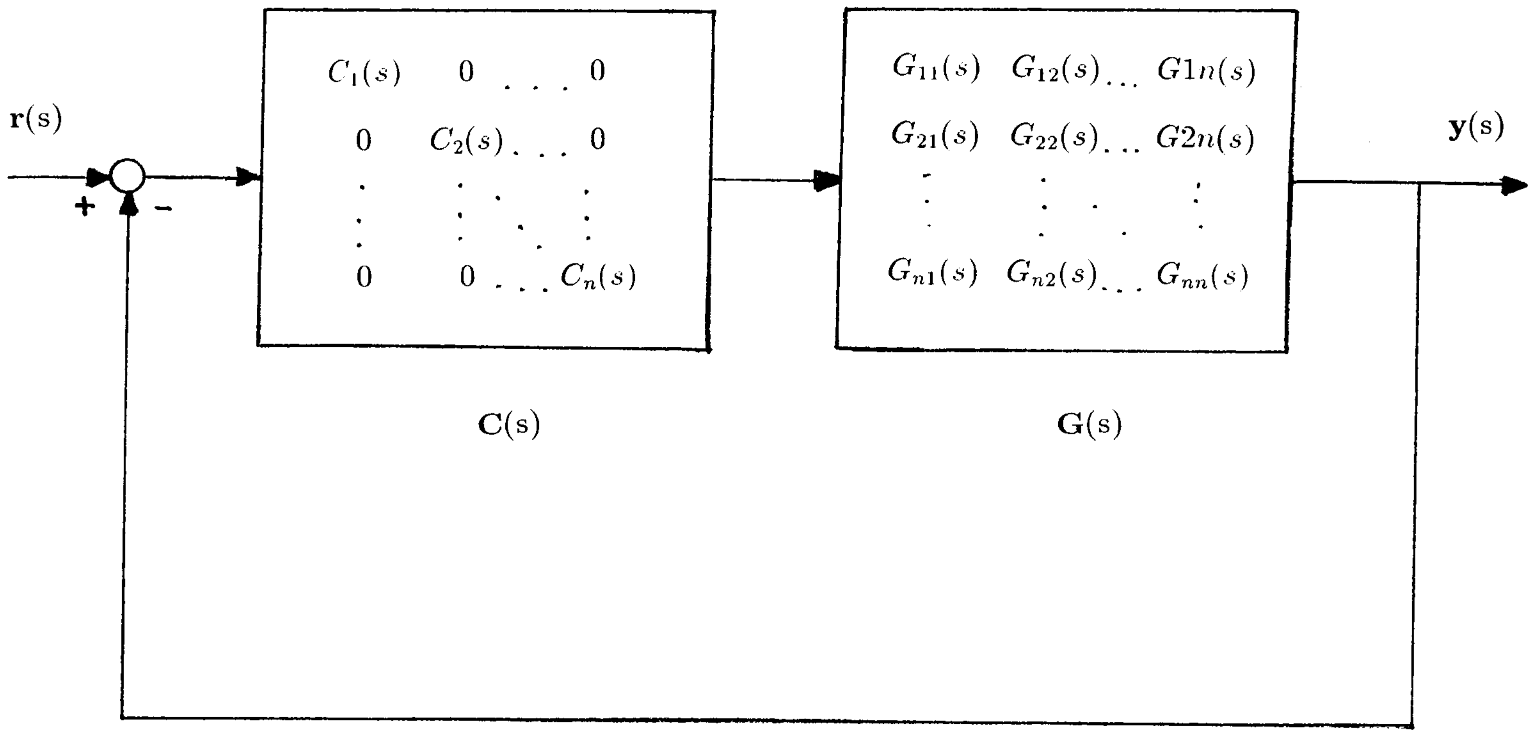


Figure 2.2: General Decentralized Control Structure; $\mathbf{y}$ = system output vector; $\mathbf{r}$ = reference input vector; $\mathbf{C}(s)$ = block diagonal controller; and $\mathbf{G}(s)$ = full-block plant transfer function matrix.

define an interaction measure as follows in terms of Figure 2.3.

A controller

$$\mathbf{C}(s) = diag(C_1(s), C_2(s), \ldots, C_n(s)) \tag{2.2}$$

is to be designed based on the nominal diagonal system

$$\tilde{\mathbf{G}}(s) = diag(G_{11}(s), G_{22}, \ldots, G_{nn}(s)) \tag{2.3}$$

so that the block-diagonal closed-loop system

$$\tilde{\mathbf{H}}(s) = \tilde{\mathbf{G}}(s)\mathbf{C}(s)(\mathbf{I} + \tilde{\mathbf{G}}(s)\mathbf{C}(s))^{-1} \tag{2.4}$$

is stable ($\delta = 0$ in Figure 2.3). An IM expresses the constraints one must follow when choosing the block-diagonal closed-loop structure, $\tilde{\mathbf{H}}(s)$, to guarantee that the full closed-loop system

$$\mathbf{H}(s) = \mathbf{G}(s)\mathbf{C}(s)(\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1} \tag{2.5}$$

is also stable ($\delta = 1$ in Figure 2.3).

Let $\mathbf{E}(s)$ denote the "relative error" obtained from the approximation of the full, original system, $\mathbf{G}(s)$, with the diagonal system, $\tilde{\mathbf{G}}(s)$, and is defined as

$$\mathbf{E}(s) = (\mathbf{G}(s) - \tilde{\mathbf{G}}(s))\tilde{\mathbf{G}}^{-1}(s) \tag{2.6}$$

Since all of the frequently used IM's (relative gain, Rijnsdorp's IQ, Rosenbrock's (1974) Direct Nyquist Array (DNA)) are either implicitly or explicitly defined by interpreting interactions as multiplicative uncertainity, a unified treatment

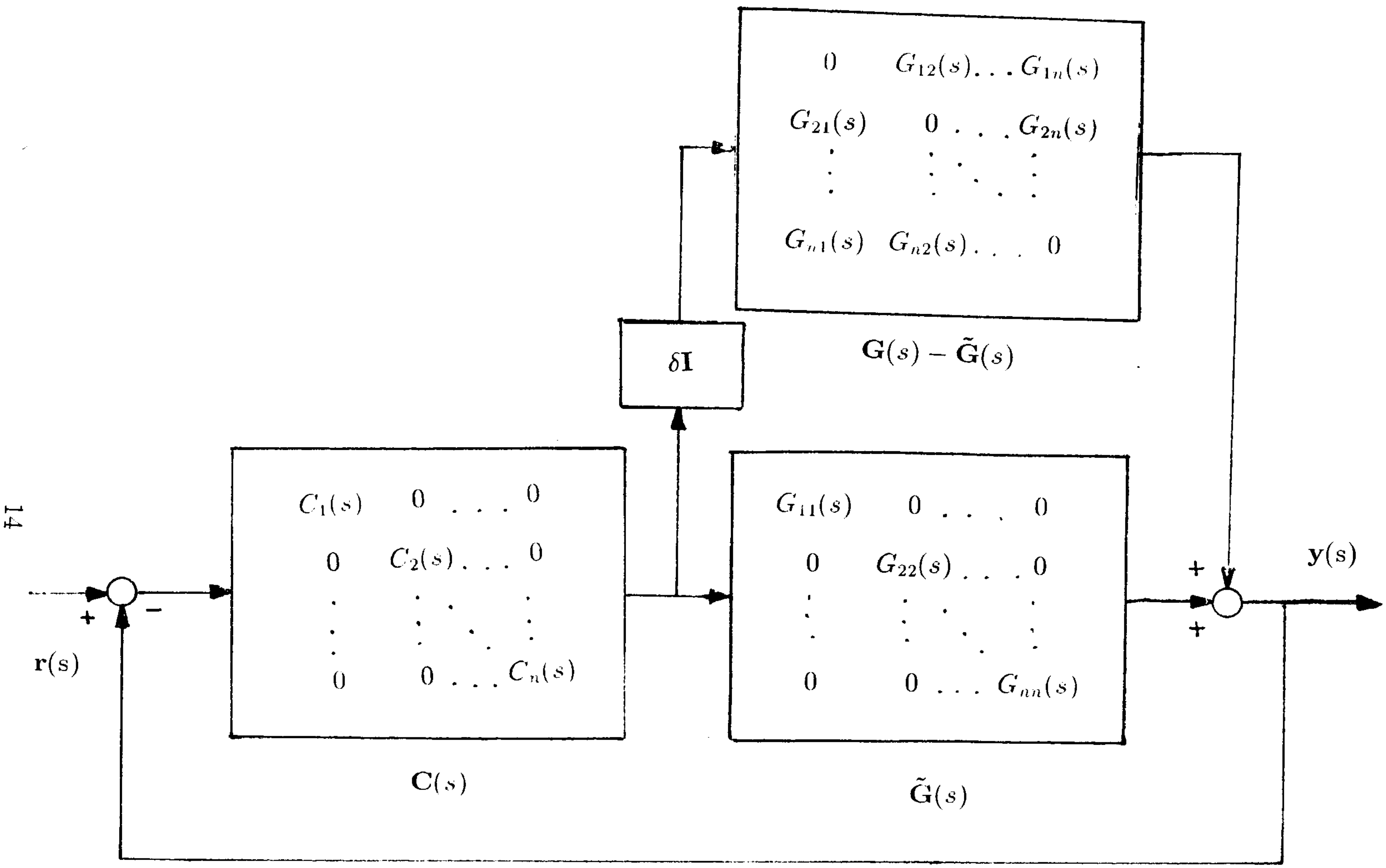


Figure 2.3: Block Diagram Representation of Interactions as Additive Dynamics.

has been given of all currently available measures of interaction (Grosdidier and Morari, 1986).

In the development of the μ-Interaction Measure, Grosdidier and Morari (1986) use the following Nyquist stability criteria. $N(k, g(s))$ denotes the net number of clockwise encirclements of the point $(k, 0)$ on the complex plane by the image of the Nyquist D contour under $g(s)$. Applying the multivariable Nyquist criteria (Postlethwaite and MacFarlane, 1979) to the control system in Figure 2.2, Grosdidier and Morari developed the following theorem: (Theorem 1, 1987)

> *Assume that* $\mathbf{G}(s)$ *and* $\tilde{\mathbf{G}}(s)$ *have the same number of right half plane (RHP) poles and that* $\tilde{\mathbf{H}}(s)$ *is stable. Then the closed-loop system,* $\mathbf{H}(s)$ *is stable if and only if*
>
> $$N(0, det(\mathbf{I} + \mathbf{E}(s)\tilde{\mathbf{H}}(s))) = 0. \tag{2.7}$$

This theorem provides the foundation for further development of constraint specifications on $\tilde{\mathbf{H}}(s)$. This theorem has certains drawbacks, however. First, one should note that the poles of $\tilde{\mathbf{G}}(s)$ are always a subset of the poles of $\mathbf{G}(s)$. Therefore the assumption in the just stated theorem that the number of RHP poles are the same is equivalent to assumption that the location and multiplicity of the RHP poles are the same (Grosdidier and Morari, 1986). Generically the assumption is never satisfied except trivially by stable systems and by systems with integrators.

Grosdidier and Morari then rearranged Figure 2.3, for the case of $\delta = 0$, into an alternate form (Figure 2.4) which yielded the following interpretations:

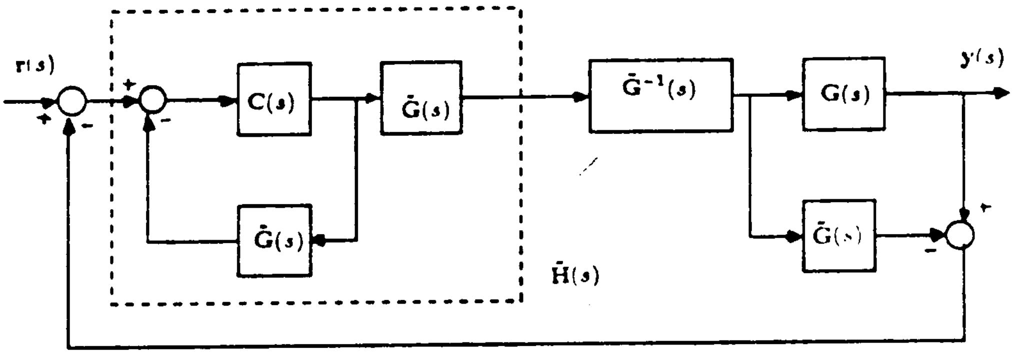


Figure 2.4: Rearranged Classical Control Structure.

Equation 2.7 serves as a condition for $\tilde{\mathbf{H}}(s)$ to remain stable under the multiplicative perturbation, $\mathbf{E}(s)$. Ideally one would want to choose $\tilde{\mathbf{H}}(s) = \mathbf{I}$, *ie.* perfect control. If $\mathbf{G}(s) = \tilde{\mathbf{G}}(s)$ then Equation 2.7 is trivally satisfied. However, if $\mathbf{G}(s) \neq \tilde{\mathbf{G}}(s)$, then $\tilde{\mathbf{H}}(s)$ must be selected so that Equation 2.7 is satisfied. From eqn 2.7 one can see that if $\mathbf{E}(s)$ is "large", then $\tilde{\mathbf{H}}(s)$ must be made "small" to avoid encirclements and thus system instability. As will be shown later, a small $\tilde{\mathbf{H}}(s)$ implies small controller gains and thus a sluggish performance. Qualitatively, one can see the importance and rationale of establishing a bound on $\tilde{\mathbf{H}}(s)$ as imposed by $\mathbf{E}(s)$.

Further development led to the following theorem: (Theorem 2, Grosdidier and Morari, 1986)

Assume that $\mathbf{G}(s)$ and $\tilde{\mathbf{G}}(s)$ have the same RHP poles and that $\tilde{\mathbf{H}}(s)$ is stable. Then the closed-loop system $\mathbf{H}(s)$ is stable if

$$\rho(\mathbf{E}(j\omega)\tilde{\mathbf{H}}(j\omega)) < 1 \qquad \forall \qquad \omega \tag{2.8}$$

where $\rho(\mathbf{A})$ is the spectral radius of $\mathbf{A}$.

This bound turns out to be too restrictive; so using the fact that the spectral radius of a matrix is bounded above by any induced norm of that matrix, one can arrive at the following theorem: (Theorem 2, Grosdidier and Morari, 1987)

Assume $\mathbf{G}(s)$ and $\tilde{\mathbf{H}}(s)$ are stable. Then the closed-loop system, $\mathbf{H}(s)$, is stable if

$$\|\tilde{\mathbf{H}}(j\omega)\| < \|\mathbf{E}(j\omega)\|^{-1} \qquad \forall \qquad \omega \tag{2.9}$$

where $\|\mathbf{A}\|$ denotes any induced norm of $\mathbf{A}$.

$||\mathbf{E}(j\omega)||^{-1}$, by the previous definition, is an interaction measure. $||\mathbf{E}(j\omega)||^{-1}$ is an appealing IM because it can be plotted on a frequency-magnitude diagram to provide an upper bound on the magnitude of $||\tilde{\mathbf{H}}(s)||$. This makes the use of graphical implementation very convenient. The problem with using $||\mathbf{E}(j\omega)||$ as a measure of interaction is that it is too conservative because it does not take into account the block diagonal structure of $\tilde{\mathbf{H}}(s)$ or the directionality of the ignored off-diagonal elements of the plant transfer function.

Grosdidier and Morari (1987) then developed a less conservative measure of interaction defined by the following theorem: (Theorem 4, Grosdidier and Morari, 1987)

Assume that $\mathbf{G}(s)$ and $\tilde{\mathbf{H}}(s)$ are stable. Then the closed-loop system. $\mathbf{H}(s)$ is stable if

$$\sigma_{max}(\tilde{\mathbf{H}}(j\omega)) < \mu^{-1}(\mathbf{E}(j\omega)) \qquad \forall \qquad \omega \tag{2.10}$$

where $\sigma_{max}(\mathbf{A})$ represents the maximum singular value of $\mathbf{A}$ and μ is the structured singular value (SSV) proposed by Doyle (1982) for the analysis of feedback systems with structured uncertainties.

This IM is less conservative because the computation of $\mu(\mathbf{E}(j\omega))$ is based on the block-diagonal structure of $\tilde{\mathbf{H}}(s)$, therefore, taking into consideration the directionality of the elements of $\mathbf{E}(s)$.

A reliable algorithm for the computation of μ has not been formulated as yet, but is an active area of research. There is, however, a reliable means of computation of the structured singular value of $\mathbf{E}(j\omega)$ for 2x2 systems with $\mathbf{E}$ as defined in Equation 2.6. For 2x2 systems, the most commonly used measures of interaction have been the Rijnsdorp (1965) Interaction Measure defined as

$$\kappa(s) = \frac{G_{12}(s)G_{21}(s)}{G_{11}(s)G_{22}(s)} \tag{2.11}$$

and the Relative Gain (Bristol, 1966) defined as

$$\lambda = \frac{1}{1-\kappa(0)} \tag{2.12}$$

and for 2x2 systems, Grosdidier and Morari (1987) show that

$$\mu(\mathbf{E}(j\omega)) = [|\kappa(j\omega)|]^{1/2}. \tag{2.13}$$

In general, $\mu(\mathbf{A})$ has an upper and lower bound as shown below (Doyle, 1981)

$$\rho(\mathbf{A}(j\omega)) \leq \mu(\mathbf{A}(j\omega)) \leq \sigma_{max}(\mathbf{A}(j\omega)) \qquad \forall \qquad \omega \tag{2.14}$$

Where $\rho(\mathbf{A})$ is defined as the spectral norm of $\mathbf{A}$. The aforementioned theorem can be then rephrased directly for a 2x2 system as: (Corollary 4.1, Grosdidier and Morari, 1987)

> *Assume that* $\mathbf{G}(s)$ *and* $\tilde{\mathbf{H}}(s)$ *are stable. Then the closed-loop system,* $\mathbf{H}(s)$*, is stable if*
>
> $$|\tilde{h}_i(j\omega)|_{i=1,2} < |\kappa(j\omega)|^{-1/2} = \mu^{-1}(\mathbf{E}(j\omega)) \qquad \forall \qquad \omega \tag{2.15}$$
>
> where $|\tilde{h}_i(j\omega)|_{i=1,2}$ are indices of the diagonal closed-loop sytem matrix, $\tilde{\mathbf{H}}(j\omega)$

2.2 Use of Internal Model Control (IMC) Design Procedure

With the increasing capabilities of modern computers, virtually any conceivable control theory can be implemented fast and in an efficient manner. While many types of controllers have been developed the most widely used is still the PID type. SISO PID controllers have been used for many years in the chemical processing industries and most operators and control engineers are very comfortable with their operation and tuning. The use of the PID controller is well documented in the literature. Simplicity, as well as, optimality is important in controller tuning. The three parameters of the PID controller (K_c, τ_I, and τ_D) do not translate very effectively into the performance and robustness characteristics desired by the control system designer. The availability of systematic rules which relate model parameters and/or experimental data to controller parameter settings would simplify the design procedure. These empirical "tuning rules" are well documented with the Ziegler-Nichols (1942) rules being possibly the best known. Subsequent fine tuning is usually required to meet the designed criteria.

Rivera, *et al.* (1986), attempt to present a "clearer and more logical" approach to the design of PID controllers. They attempt to formulate a method, based on the Internal Model Control (IMC) (Garcia and Morari, 1982) approach, which is easy to use and simple to understand, while having a sound theoretical basis. The Internal Model Control (IMC) structure (Figure 2.5) provides a suitable framework for satisfying PID controller design objectives. IMC applies to SISO systems, as well as MIMO systems. Using the IMC design procedure, controller complexity depends exclusively on two factors: the complexity of the model and the desired system performance traits. One of the most desire-

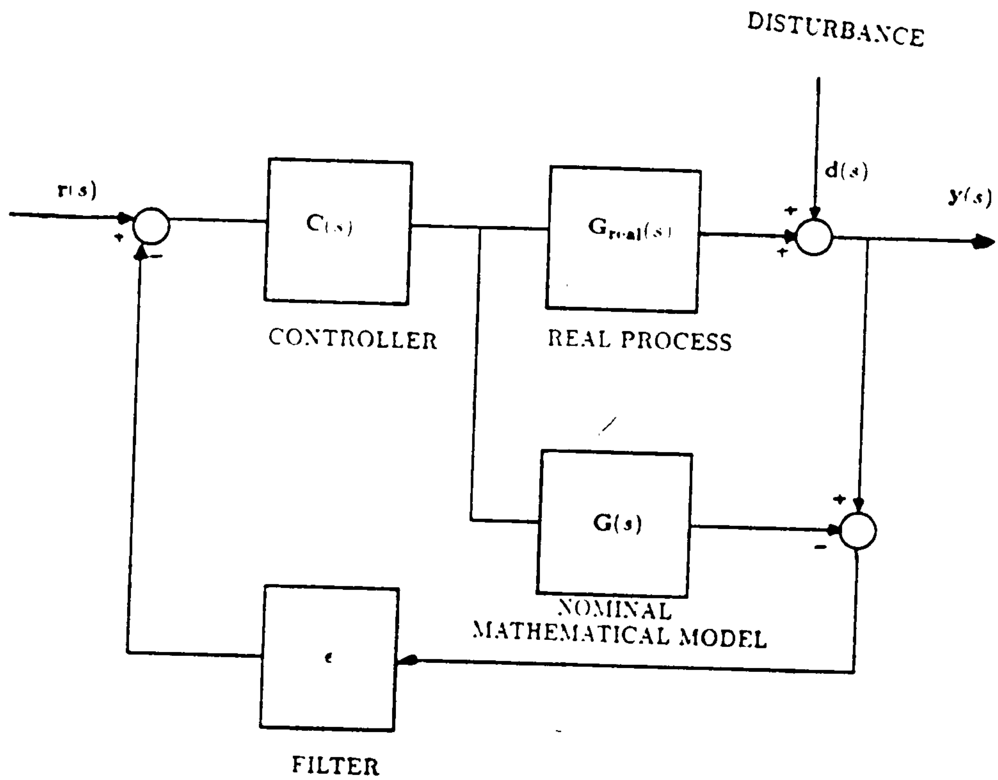


Figure 2.5: The IMC Structure. $\mathbf{y}(s)$ are process outputs; $\mathbf{r}(s)$ are reference inputs.

able features of the IMC structure is that because of the filter, it can handle plant/model mismatch and thus one has a direct means of controlling the robustness of the system. For simple models common to chemical process control, the procedure leads quite often to the PID-type controllers. Rivera, *et al.* (1986) provide a systematic way of using the IMC principles in the design of PID controllers. These PID-type controllers are very attractive because they have only one tuning parameter, ϵ, the closed-loop time constant residing in the filter in the feedback loop. Rivera, *et al.* provide a table from which, by knowing the plant transfer function form, each of the parameters of the controller (K_c, τ_I, and τ_D) can be specified. The only tuning parameter, ϵ, is then adjusted to meet optimally obtainable performance criteria such as integral of the squared error (ISE), integral of the absolute error (IAE), *etc.*.

In using the IMC-PID controller design procedure in conjunction with the μ-Interaction Measure stability analysis, a diagonal PID controller, $\mathbf{C}(s)$, is designed first to meet the requirement that each of the transfer functions, $\tilde{h}_i(s)(i = 1, n)$, is stable and has its magnitude, $|\tilde{h}_i(j\omega)|(i = 1, n)$, bounded by $\mu^{-1}(\mathbf{E}(j\omega))$. If the magnitude of any transfer function, $|\tilde{h}_i(j\omega)|(i = 1, n)$, lies above the $\mu^{-1}(\mathbf{E}(j\omega))$ curve, the system, $\mathbf{H}(s)$, will be unstable and the tuning parameter, ϵ, will be adjusted accordingly until the plot of $|\tilde{h}_i(j\omega)|_{i=1,n}$ lies just below the plot of $\mu^{-1}(\mathbf{E}(j\omega))$. It is further noted that the lower the $\tilde{h}_i(j\omega)$ curve the more sluggish the closed loop system is. Therefore, $\tilde{h}_i$ should be designed so that the plot of $\tilde{h}_i(j\omega)$ vs frequency, ω, lies just below that of $\mu^{-1}(\mathbf{E}(j\omega))$.

2.3 Model Uncertainity and the Concept of *Robustness*

Robustness is the ability of a controller to maintain stability of the closed-loop system in the face of model uncertainty due to unmodeled system dynamics, parameter variations, or perturbation to the system. In the study of physical processes a nominal mathematical model of that system, is developed to best simulate the process. The mathematical model is used as the basis of the control system design. Errors always exist in this nominal mathematical representation. The accuracy of the nominal model description may be limited by inaccuracies in measuring the process outputs or inputs, ignored model dynamics (to achieve simplicity), or linearization of a nonlinear process. An important part of model formulation is the development of bounds on the expected deviation of the physical process dynamics from the nominal description provided by the mathematical model (Birdwell, 1986).

In the past several decades, many procedures have been proposed for the design of linear feedback compensators for systems with model uncertainties. An "optimal" controller, *ie.* H_2- or $H_\infty-$, can be obtained by using the Linear Quadratic Gaussian (LQG) technique (Athans, 1971). Other techniques, like the multivariable frequency domain approaches proposed by Rosenbrock (1974) and MacFarlane (1980) have at times given favorable results (Morari, 1987).

However, when using these techniques one must make certain assumptions about the physical process and its environment. The process is almost always assumed to be linear. Also the model-, disturbance-, and noise parameters must be known exactly. These assumptions rarely hold, and thus controllers designed on the basis of these procedures are often inadequate in the real world of nonlinearities and uncertainty.

Because of the unreliability of recent technologies in the real world, these developments have had almost no effect on the way industry controls its processes. Usually controllers are still tuned by experience or by rule-of-thumb tuning charts. There has been, however, a recent trend toward research using the effects of model uncertainty on system performance and robustness in the design of controllers for feedback systems. In this research, the incorporation of model uncertainty into the μ-IM analysis of a diagonal controller is also investigated.

CHAPTER 3

PROBLEM FORMULATION

3.1 Systems Used

Three example systems from the literature were used in this research effort to test the feasibility of this approach. All of the systems are 2x2 cases and all are distillation models, very common unit operation systems in the chemical processing industry. These "real" systems were used, instead of dreaming up various test plant transfer functions, so that unrealistic problems would be avoided. The systems used are as follows:

1. A methanol-water distillation column by Wood and Berry (1973).

2. A distillation column by Vinante and Luyben (1972).

3. A methanol-ethanol distillation column by Weischedel and McAvoy (1980).

3.1.1 Wood and Berry Column

The first test system used is the Wood and Berry column. This methanol-water distillation column is considered to be easily controlled. A schematic diagram of the column is shown in Figure 3.1 and the plant transfer function matrix is

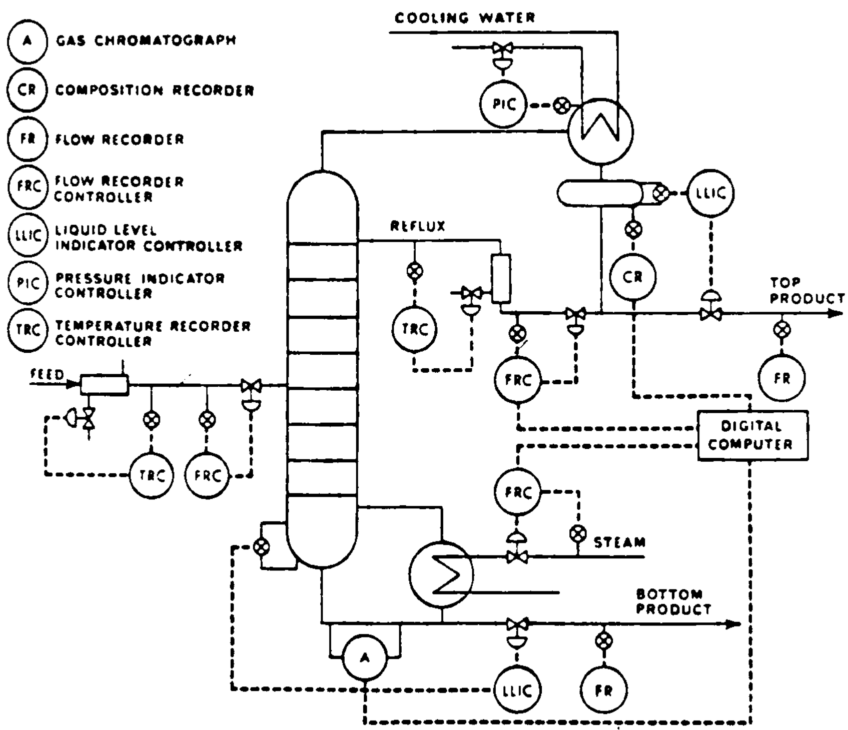


Figure 3.1: Schematic Diagram of the Wood and Berry Column (1973).

as follows:

$$\mathbf{G(s)} = \begin{bmatrix} \frac{12.8e^{-1.0s}}{(16.7s+1)} & \frac{-18.9e^{-3.0s}}{(21.0s+1)} \\ \frac{6.6e^{-7.0s}}{(10.9s+1)} & \frac{-19.4e^{-3.0s}}{(14.4s+1)} \end{bmatrix}. \tag{3.1}$$

In the study reported by Wood and Berry (1973), two control schemes were attempted and then compared to conventional two point control. The results of that study in which Wood and Berry introduce a 15% disturbance to the feed of their system are shown in Figure 3.2 and Figure 3.3. Wood and Berry concluded that while both the ratio- and noninteracting control schemes provide significant improvements over the conventional scheme, the noninteracting scheme (Fig 3.4) is preferred because of better performance characteristics. This scheme, however, requires that the process model be known exactly in order to devolop the inverse-based compensator. This is, in the real world, rarely possible.

3.1.2 Vinante and Luyben Column

The second test system is the Vinante and Luyben column. It has the following plant transfer function matrix:

$$\mathbf{G(s)} = \begin{bmatrix} \frac{-2.2e^{-1.0s}}{(7.0s+1)} & \frac{1.3e^{-0.3s}}{(7.0s+1)} \\ \frac{-2.8e^{-1.8s}}{(9.5s+1)} & \frac{4.3e^{-0.35s}}{(9.2s+1)} \end{bmatrix}. \tag{3.2}$$

In 1986 Luyben used this system to illustrate a newly devised simple method for tuning SISO controllers in multivariable systems. The method, called the "biggest log modulus tuning" (BLT), was applied to the system developed by Vinante and Luyben and the results to a setpoint change in X_1 are shown in

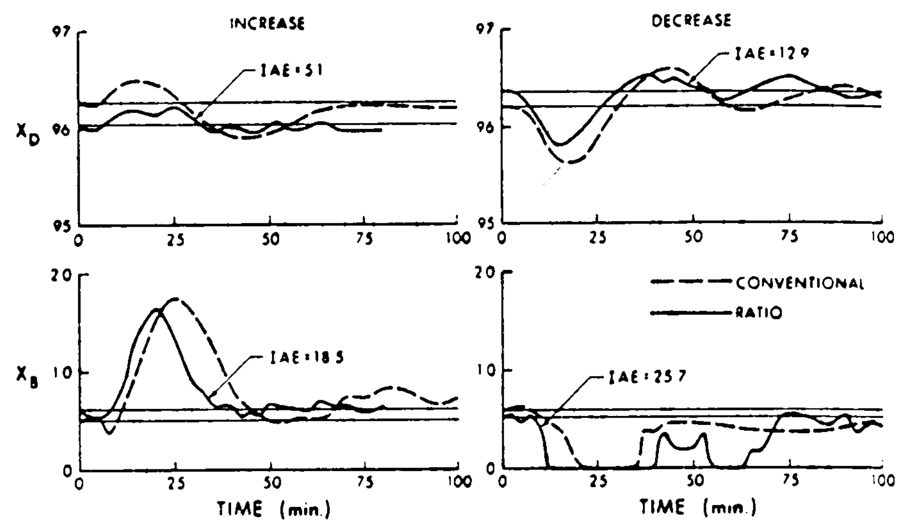


Figure 3.2: Comparison of Performance Under Ratio and Conventional Control for a 15% Feed Disturbance in the Wood and Berry Column.

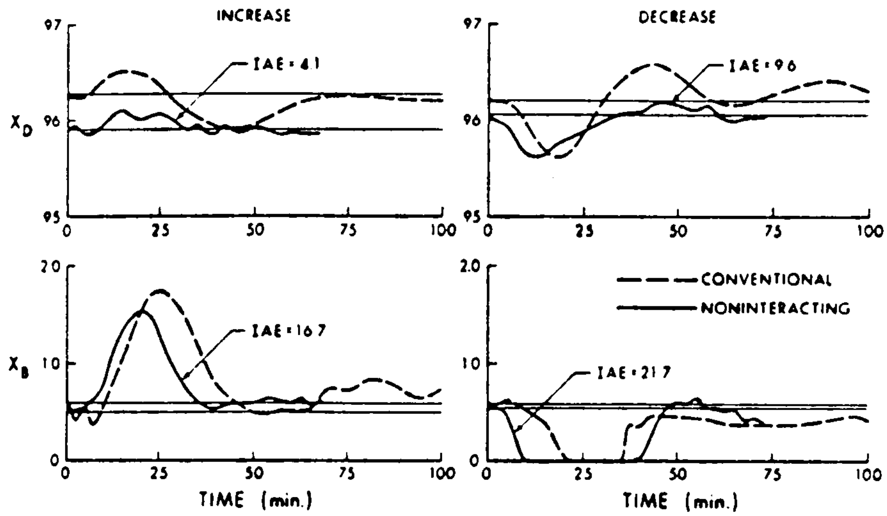


Figure 3.3: Comparison of Performance Under Noninteracting and Conventional Control for a 15% Feed Disturbance in the Wood and Berry Column.

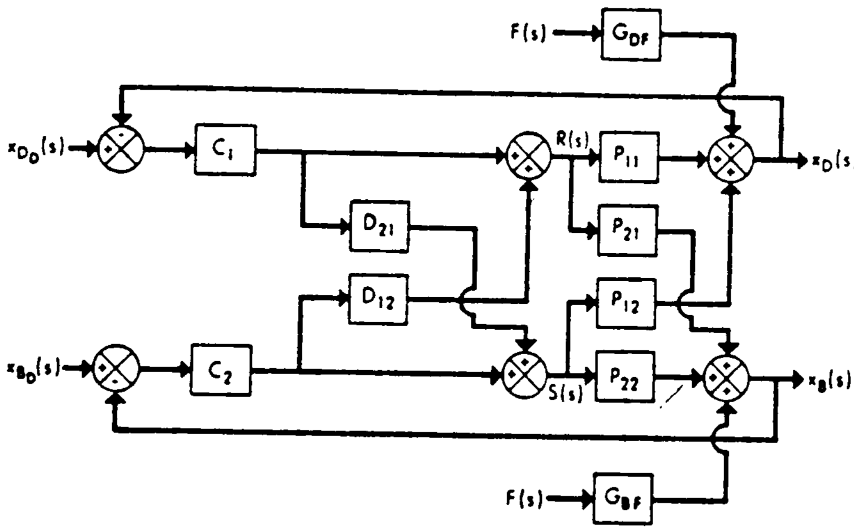


Figure 3.4: Simplified Block of the Noninteracting Column Control System.

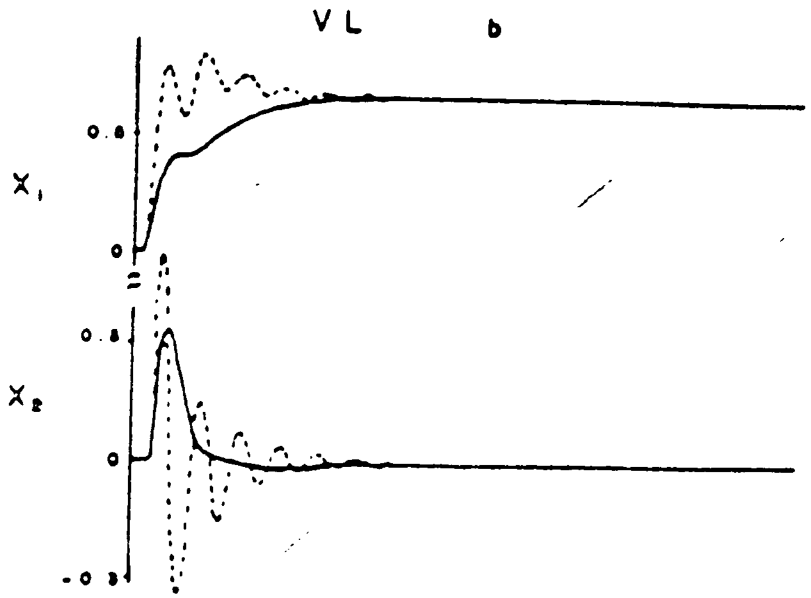


Figure 3.5: X_1 setpoint response for the Vinante and Luyben column. Solid lines are BLT settings; dashed lines are empirical settings.

Figure 3.5. The results showed a significant improvement over the empirical (Zeigler-Nicols) tuning. The method is however rather complex and difficult to apply.

3.1.3 Weischedel and McAvoy Column (column C)

The third system studied was the Weischedel and McAvoy column (column C). Since Weischedel and McAvoy presented this highly coupled methanol-ethanol system it has been used frequently as an example in process control design. It has the following plant transfer function matrix:

$$\mathbf{G(s)} = \begin{bmatrix} \frac{0.471e^{-1.0s}}{(30.7s+1)^2} & \frac{0.495e^{-2.0s}}{(28.5s+1)^2} \\ \frac{0.749e^{-1.7s}}{(57.0s+1)^2} & \frac{-0.832e^{-1.0s}}{(50.5s+1)^2} \end{bmatrix}. \tag{3.3}$$

In the study by Weischedel and McAvoy (1980), the column control system responses to a setpoint change in X_B were compared using simplified decoupling (Fig 3.6) and no decoupling. The simulation response plots for each are shown in Figure 3.7 and Figure 3.8 respectively. The conclusion from this study was that the addition of decouplers actually deteriorates the control system responses. This confirmed earlier predictions that the decoupling of highly interactive systems is not feasible.

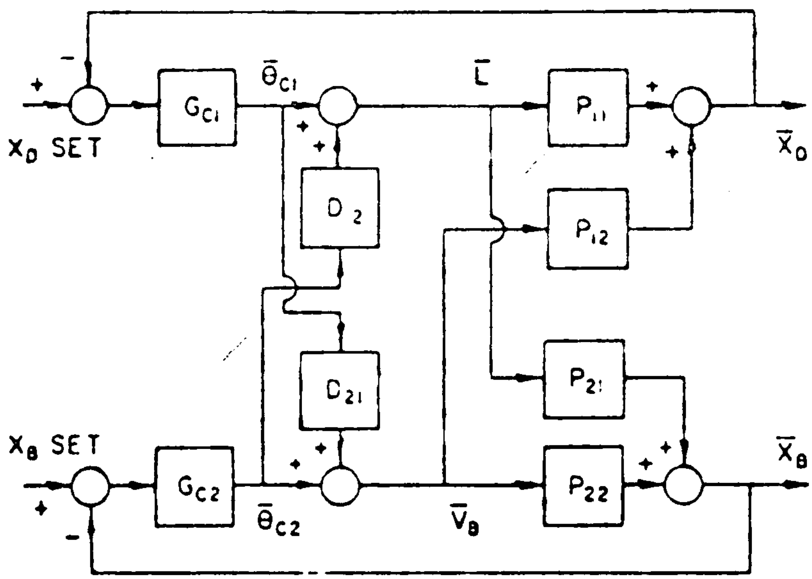


Figure 3.6: Simplified Decoupling.

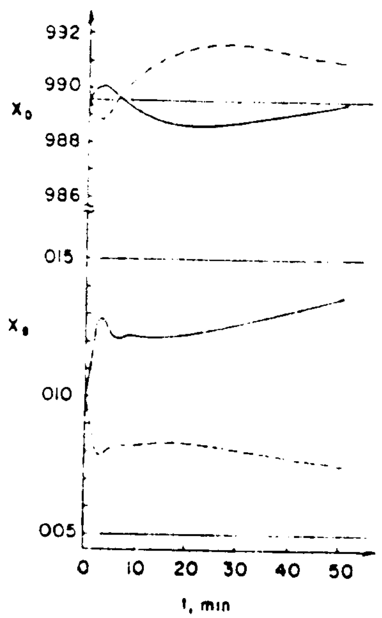


Figure 3.7: Response for Column C (simplified decoupling). Solid lines represent a positive setpoint change in X_B; dashed lines represent a negative change.

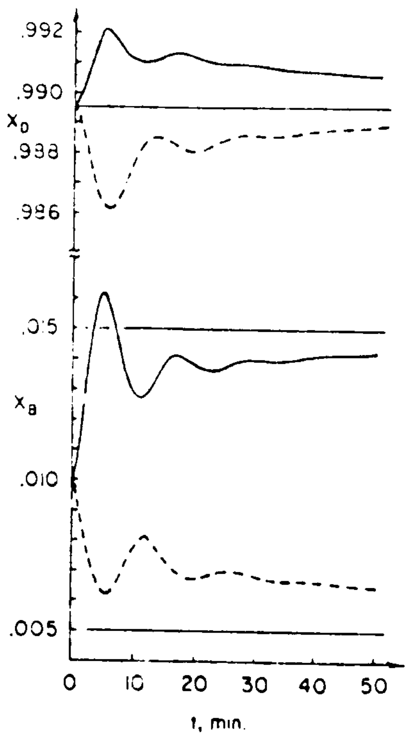


Figure 3.8: Response for Column C (no decoupling). Solid lines represent a positive setpoint change in X_B; dashed lines represent a negative change.

CHAPTER 4

CONTROLLER DESIGN RESULTS AND STABILITY ANALYSIS

4.1 IMC-PID Controller Design

Using the Internal Model Control (IMC) procedure (Rivera, *et al.*, 1986), diagonal proportional-integral-derivative (PID) controllers were designed for each of the three systems using the decentralized system description. The IMC-PID controller design procedure is very attractive because it results in only one adjustable parameter, ϵ, the closed-loop time constant. This simplifies controller tuning considerably.

When designing controllers for any decoupled multivariable system, it is very important to correctly pair the input/output variables to give the least interactive structure. In using the μ-Interaction Measure analysis, Grosdidier and Morari (1987) have shown,based on Equation 2.10, that only if $\mu(\mathbf{E}(0)) \leq 1.0$ is one assured that the decentralized controller can incorporate the integral mode. Hence, one should make every effort to design a control structure that fulfills this requirement. At present, the only way of selecting the least interactive control structure (*ie.* the highest $\mu^{-1}(\mathbf{E}(j\omega))$ vs ω curve), is by successively trying all possible alternatives (Grosdidier and Morari, 1987). In other words, traditional means of pairing variables do not necessarily give the best combination with respect to this requirement. It has been shown that the relative gain array may give one "best" pairing while other methods will give a completely different "best" combination. Thus the only way to successfully get the least interactive pairing is to try all the possibilities. For 2x2 systems, however, the relative gain array does seem to give, if not the "best" pairing, at least the "safest". By this, it is meant that by totally, if possible, avoiding the pairing on negative relative gains one can produce systems with integrity (systems that remain stable when one or more of the loops are in manual mode, Morari, 1986). By using the RGA as a means of determining the variable pairings, one also satisfies the guidelines set forth by Grosdidier and Morari (1987), *ie.* that the pairing should be so that $|\kappa(0)| < 1.0$.

4.1.1 Wood and Berry Column

The relative gain array for the Wood and Berry column is:

$$\mathbf{RGA} = \begin{bmatrix} \boxed{2.0094} & -1.0094 \\ -1.0094 & \boxed{2.0094} \end{bmatrix}. \tag{4.1}$$

Thus, avoiding pairing on a negative relative gain array index, the input/output variables will be paired as (1,1), (2,2). The output, y_1, will be in a feedback loop with input, r_1, and the output, y_2, will be in a feedback loop with input, r_2. The off-diagonal elements are then "dropped" and the nominal diagonal plant transfer function, $\tilde{\mathbf{G}}(s)$ becomes:

$$\tilde{\mathbf{G}}(\mathbf{s}) = \begin{bmatrix} \frac{12.8e^{-1.0s}}{(16.7s+1)} & 0 \\ 0 & \frac{-19.4e^{-3.0s}}{(14.4s+1)} \end{bmatrix}. \tag{4.2}$$

A decentralized controller, $\mathbf{C}(s)$, with diagonal PID controllers is then designed, using the IMC-PID controller design procedure, based on the diagonal plant, $\tilde{\mathbf{G}}(s)$.

Rivera, *et al.*, (1986) provide a table by which, knowing the individual system's nominal transfer function structure, one can systematically design a PID controller for that system. The method does not, however, accommodate dead-time in the mathematical structure. Therefore, a Padé approximation must be used. For the general first order transfer function

$$g(s) = \frac{ke^{-\theta s}}{\tau s + 1}, \tag{4.3}$$

the first order Padé approximation yields

$$g'(s) \approx \frac{k(\frac{-\theta}{2}s + 1)}{(\tau s + 1)(\frac{\theta}{2}s + 1)} \tag{4.4}$$

So that the first order Padé approximation of the diagonalized Wood and Berry column model gives the following plant transfer function matrix:

$$\tilde{\mathbf{G}}'(s) \approx \begin{bmatrix} \frac{12.8(-0.5s+1)}{(16.7s+1)(0.5s+1)} & 0 \\ 0 & \frac{-19.4(-1.5s+1)}{(14.4s+1)(1.5s+1)} \end{bmatrix}. \tag{4.5}$$

The results of the IMC-PID design, using a value of $\epsilon = \beta$ for optimal integral of the squared error (ISE) are shown in Table 4.1.

Table 4.1: IMC-PID Design Parameters For The Wood and Berry Column ($\epsilon = \beta$)

Transfer Function	Model	K_cK_p	K_c	τ_I(min)	τ_D(min)
General	$\frac{k(-\beta s+1)}{\tau^2 s^2+2\zeta \tau+1}$	$\frac{2\zeta \tau}{\beta+\epsilon}$	-	$2\zeta \tau$	$\frac{\tau}{2\zeta}$
$G'_{11}(s)$	$\frac{12.3(-0.5s+1)}{8.35s^2+17.2s+1}$	17.2	1.398	17.2	0.485
$G'_{22}(s)$	$\frac{-19.4(-1.5s+1)}{21.15s^2+15.6s+1}$	5.20	-0.268	15.6	1.36

4.1.2 Vinante and Luyben Column

The relative gain array for the Vinante and Luyben column model is:

$$\mathbf{RGA} = \begin{bmatrix} \boxed{1.6254} & -0.6254 \\ -0.6254 & \boxed{1.6254} \end{bmatrix}. \tag{4.6}$$

Thus the suggested input/output variable pairing is (1,1), (2,2). The off-diagonal elements of the plant transfer function matrix are dropped and the diagonal plant, $\tilde{G}(s)$, becomes:

$$\tilde{\mathbf{G}}(s) = \begin{bmatrix} \frac{-2.2e^{-1.0s}}{(7.0s+1)} & 0 \\ 0 & \frac{4.3e^{-0.35s}}{(9.2s+1)} \end{bmatrix}. \tag{4.7}$$

Using the first order Padé approximation gives the following:

$$\tilde{\mathbf{G}}'(s) \approx \begin{bmatrix} \frac{-2.2(-0.5s+1)}{(7.0s+1)(0.5s+1)} & 0 \\ 0 & \frac{4.3(-0.1705s+1)}{(9.2s+1)(0.1705s+1)} \end{bmatrix}. \tag{4.8}$$

Then, using the IMC-PID design procedure, much like the Wood and Berry column model, a PID controller is designed. The parameters are shown in Table

4.2 (again using $\epsilon = \beta$ for optimal ISE.)

Table 4.2: IMC-PID Design Parameters For The Vinante and Luyben Column ($\epsilon = \beta$)

Transfer Function	Model	$K_c K_p$	K_c	τ_I(min)	τ_D(min)
General	$\frac{k(-\beta s+1)}{\tau^2 s^2+2\zeta \tau+1}$	$\frac{2\zeta \tau}{\beta+\epsilon}$	-	$2\zeta \tau$	$\frac{\tau}{2\zeta}$
$G'_{11}(s)$	$\frac{-2.2(-0.5s+1)}{3.5s^2+7.5s+1}$	7.5	-3.409	7.5	0.467
$G'_{22}(s)$	$\frac{4.3(-0.1705s+1)}{1.57s^2+9.37s+1}$	27.481	6.391	9.371	0.167

4.1.3 Weischedel and McAvoy Column

The relative gain array for the Weischedel and McAvoy column is:

$$\mathbf{RGA} = \begin{bmatrix} 0.5168 & 0.4862 \\ 0.4862 & 0.5168 \end{bmatrix}. \tag{4.9}$$

The relative gain array indices indicate that this is a highly coupled process. It is thus not very practical to pair input/output variables based only on the RGA. In practice by inspecting the RGA, most systems designers would advise that it really doesn't matter which pairing one chooses. Hence, what is attempted here is to use both pairings (*ie.*, (1,1),(2,2) and (1,2),(2,1)) and see if the μ-IM analysis provides any insight in pairing of highly interactive systems.

Because of the relatively slow dynamics associated with the large plant time constants, the relatively small dead times are rather negligible and are dropped and PID controllers are designed based on all plant transfer function matrix indices. Using the IMC-PID design procedure, the PID controllers are designed using an initial value of $\epsilon = K_p/2\tau$. The associated tuning controller parameters are displayed in Table 4.3.

Table 4.3: IMC-PID Design Parameters For The Weischedel and McAvoy Column $\epsilon = K_p/2\tau$.

Transfer Function	Model	K_cK_p	K_c	τ_I(min)	τ_D(min)
General	$\frac{k}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2}{\epsilon}$	-	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$
$G'_{11}(s)$	$\frac{0.471}{(30.7s+1)^2}$	0.471	1.0	61.4	15.35
$G'_{22}(s)$	$\frac{-0.832}{(50.5s+1)^2}$	0.832	-1.0	101.0	25.25
$G'_{12}(s)$	$\frac{0.495}{(28.5s+1)^2}$	0.495	1.0	-	14.25
$G'_{21}(s)$	$\frac{0.749}{(57.0s+1)^2}$	0.749	1.0	-	28.50

4.2 Stability Analysis Using The μ-Interaction Measure

Using the μ-Interaction Measure (Grosdidier and Morari, 1987), one can indicate under what conditions the stability of the closed-loop diagonal system will imply the stability of the closed-loop original system. Mathematically, the interaction measure is employed as follows. A diagonal PID controller,

$$\mathbf{C}(s) = diag(C_1, C_2, \ldots, C_n), \tag{4.10}$$

is designed, using the IMC design procedure, for the decentralized plant transfer function,

$$\tilde{\mathbf{G}}(s) = diag(G_{11}(s), G_{22}(s), \ldots, G_{nn}(s)), \tag{4.11}$$

such that the closed-loop, block-diagonal system,

$$\tilde{\mathbf{H}}(s) = \tilde{\mathbf{G}}(s)\mathbf{C}(s)(\mathbf{I} + \tilde{\mathbf{G}}(s)\mathbf{C}(s))^{-1}, \tag{4.12}$$

is stable. The μ-Interaction Measure then provides an upper bound on the magnitude of $\tilde{\mathbf{H}}(s)$ such that the full-block, closed-loop transfer function,

$$\mathbf{H}(s) = \mathbf{G}(s)\mathbf{C}(s)(\mathbf{I} + \mathbf{G}(s)\mathbf{C}(s))^{-1}, \tag{4.13}$$

is also stable.

For 2 x 2 systems, the stability criteria, in the magnitude-frequency plot sense, becomes:

$\mathbf{H}(s)$ *is stable if* $\mathbf{G}(s)$ *and* $\tilde{\mathbf{H}}(s)$ *are stable and*

$$|\tilde{h}_i(j\omega)|_{i=1,2} < \mu^{-1}(\mathbf{E}(j\omega)) \qquad \forall \qquad \omega \tag{4.14}$$

4.2.1 Wood and Berry Column

Since the μ-IM is displayed very conveniently on a magnitude-frequency diagram and since deadtime has no effect on the magnitude (only the phase) of the system, the deadtime is not used in the calculation of μ. The deadtime is, however, used later in the simulation studies. For the following plant transfer function matrix,

$$\mathbf{G}'(s) = \begin{bmatrix} \frac{12.8}{(16.7s+1)} & \frac{-18.9}{(21.0s+1)} \\ \frac{6.6}{(10.9s+1)} & \frac{-19.4}{(14.4s+1)} \end{bmatrix}, \tag{4.15}$$

the Rijnsdorp Interaction Quotient, using the (1,1), (2,2) pairing is

$$\kappa(s) = \frac{0.5023(16.7s+1)(14.4s+1)}{(10.9s+1)(21.0s+1)} \tag{4.16}$$

and the inverse is

$$\kappa^{-1}(s) = \frac{1.991(10.9s+1)(21.0s+1)}{(16.7s+1)(14.4s+1)}. \tag{4.17}$$

So $\mu^{-1}(\mathbf{E}(j\omega))$ is found by simply plotting the absolute value of the square root of $\kappa^{-1}(s)$ for all values of frequency (ω). This $\mu^{-1}(\mathbf{E}(j\omega))$ versus ω is shown in Figure 4.1.

The closed-loop diagonal matrix, $\tilde{\mathbf{H}}(s)$, is defined as

$$\tilde{\mathbf{H}}(s) = \tilde{\mathbf{G}}(s)\mathbf{C}(s)(\mathbf{I} + \tilde{\mathbf{G}}(s)\mathbf{C}(s))^{-1} \tag{4.18}$$

and has the form

$$\tilde{\mathbf{H}}(s) = \begin{bmatrix} \frac{G_{11}(s)C_1(s)}{(1+G_{11}(s)C_1(s))} & 0 \\ 0 & \frac{G_{22}(s)C_2(s)}{(1+G_{22}(s)C_2(s))} \end{bmatrix} \tag{4.19}$$

where $C_i(s)_{i=1,2}$ is a PID controller of the form

$$C_i(s) = K_{c_i}(1 + \frac{1}{\tau_{I_i}s} + \tau_{D_i}s). \tag{4.20}$$

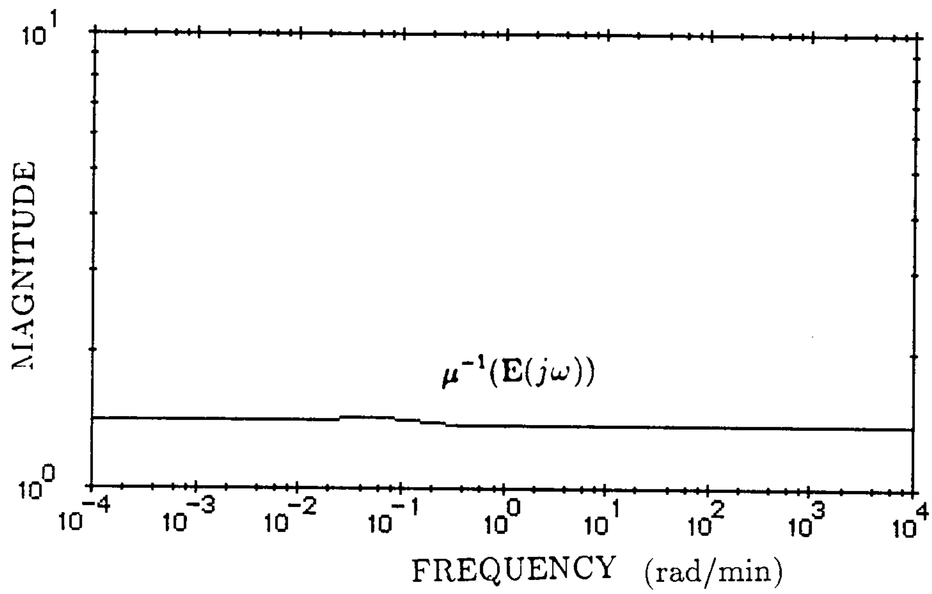


Figure 4.1: $\mu^{-1}(\mathbf{E}(j\omega))$ vs. Frequency (ω) For The Wood and Berry Column.

The values of K_c, τ_I, and τ_D for each controller of the Wood and Berry column are shown in Table 4.1. Using these values, the plot of $|\tilde{h}_i(j\omega)|_{i=1,2}$ and $\mu^{-1}(\mathbf{E}(j\omega))$ versus frequency is shown in Figure 4.2.

The open-loop response for a step input in r_1 is shown in Figure 4.3, and the closed-loop full-block system response to a step input in r_1 is shown in Figure 4.4. Figure 4.4 shows that since $\mu^{-1}(j\omega) > |\tilde{h}_i(j\omega)|_{i=1,2}$ (in Figure 4.2) for all values of frequency (ω) the closed-loop full-block system is indeed stable as predicted.

While the constraints placed by the μ-Interaction Measure guarantee stability of the full closed-loop system, the performance may be arbitrarily poor. However, it is difficult to determine sometimes what is "poor" performance. For instance, in Figure 4.4, the step response for the closed-loop system was quite fast; but there was also a large overshoot. If the value of ϵ, in the IMC designed controller, is raised (and, in the case of minimum-phase models the controller gain is lowered) the value of $|\tilde{h}_i(j\omega)|_{i=1,2}$ is lowered as is not as close to the $\mu^{-1}(\mathbf{E}(j\omega))$ curve (Figure 4.5) and the performance is altered (Figure 4.6). The response in Figure 4.6 shows a different performance. The dynamics are slower but there is no overshoot. In this case, one would need to decide which is more attractive to the operation of a particular process: fast dynamics or minimal overshoot. The problem that becomes quite apparent in this analysis is that one can't pick an ϵ to meet a desired performance characteristic. The process is more or less trial and error. A means of selecting a value of ϵ analytically which gives preset performance characteristics is a current subject of research. How-

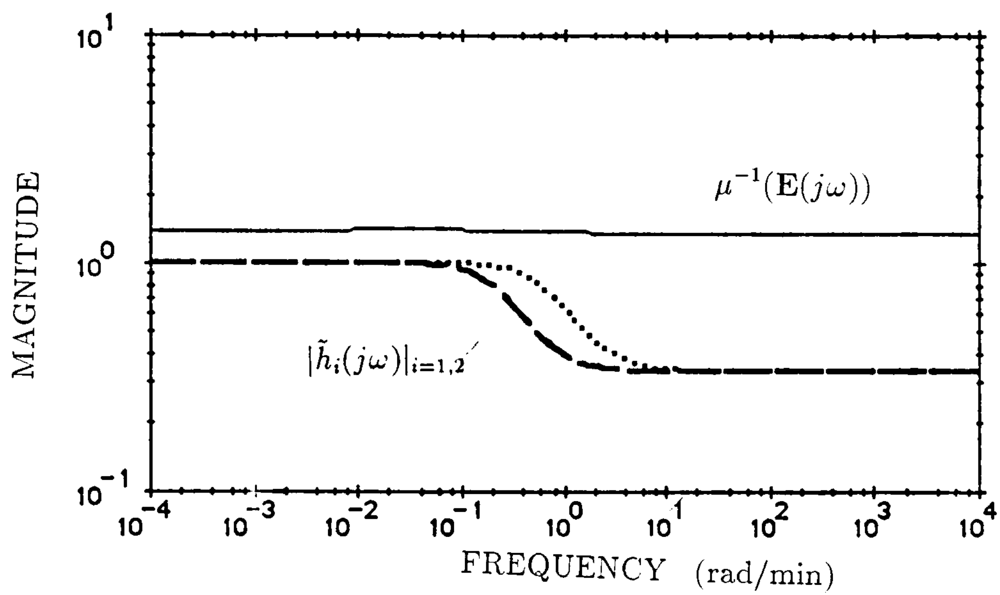


Figure 4.2: $\mu^{-1}(\mathbf{E}(j\omega))$ and $|\tilde{h}(j\omega)|_{i=1,2}$ versus Frequency (ω) for the Wood and Berry Column.

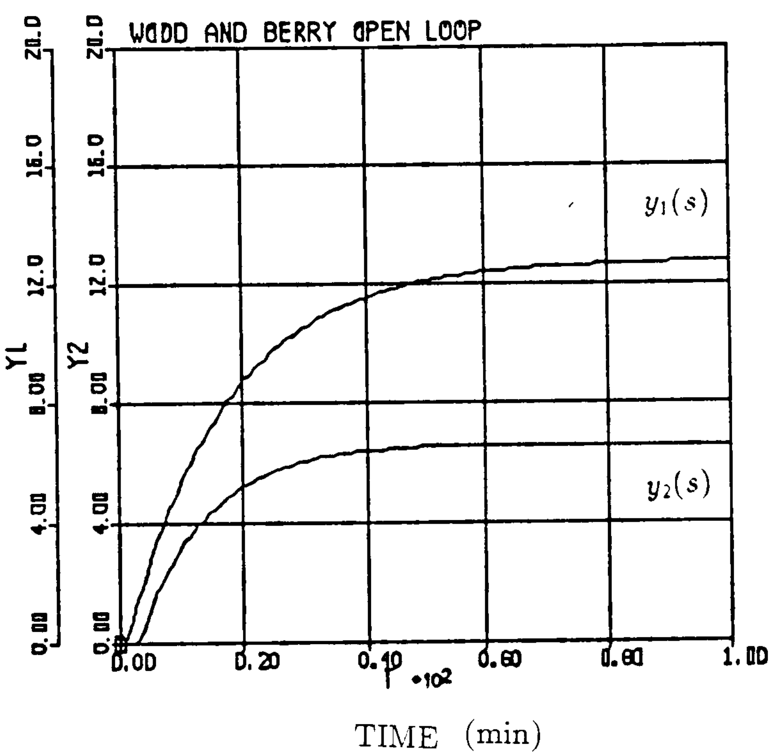


Figure 4.3: Open-loop Response for a Step Input in r_1 for the Wood and Berry Column.

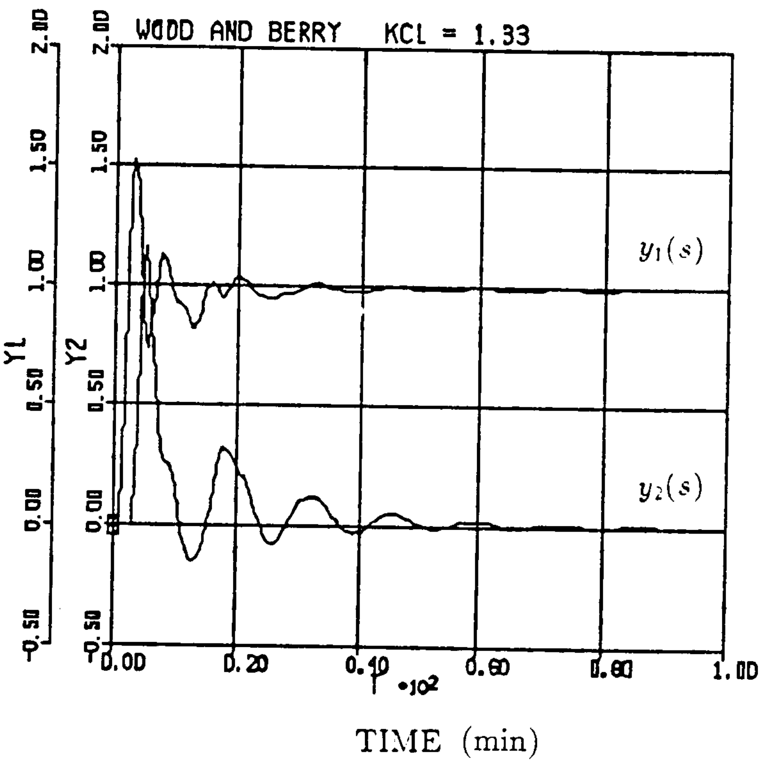


Figure 4.4: Response to a Step Input in r_1 for the Closed-loop Full-block System, H(s), for the Wood and Berry Column.

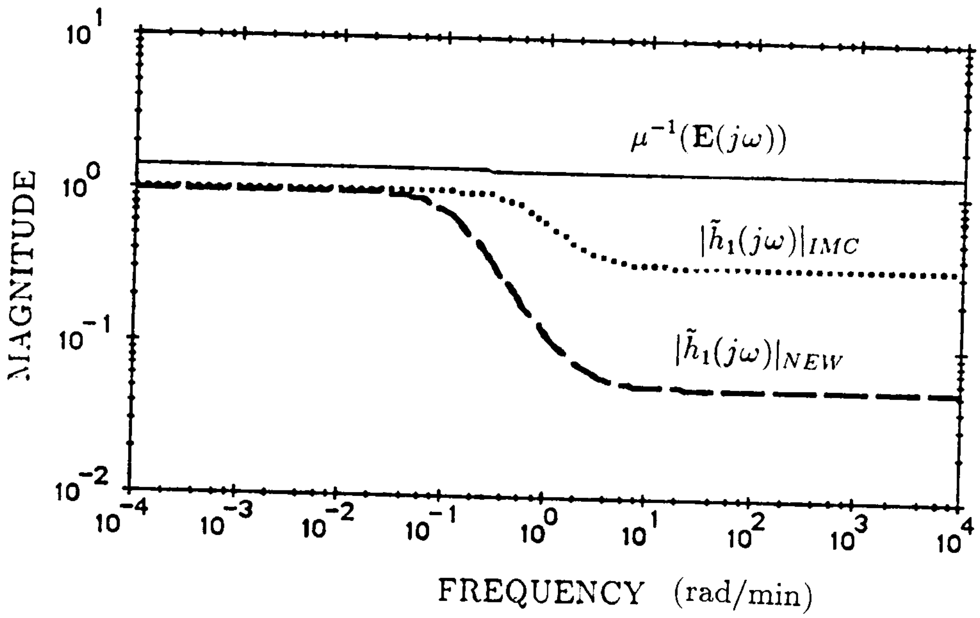


Figure 4.5: $\mu^{-1}(\mathbf{E}(j\omega))$ and $|\tilde{h}_1(j\omega)|_{IMC}$ and $|\tilde{h}_1(j\omega)|_{NEW}$ versus Frequency (ω) for the Wood and Berry Column. (*NEW* refers to an adjusted value of ϵ.)

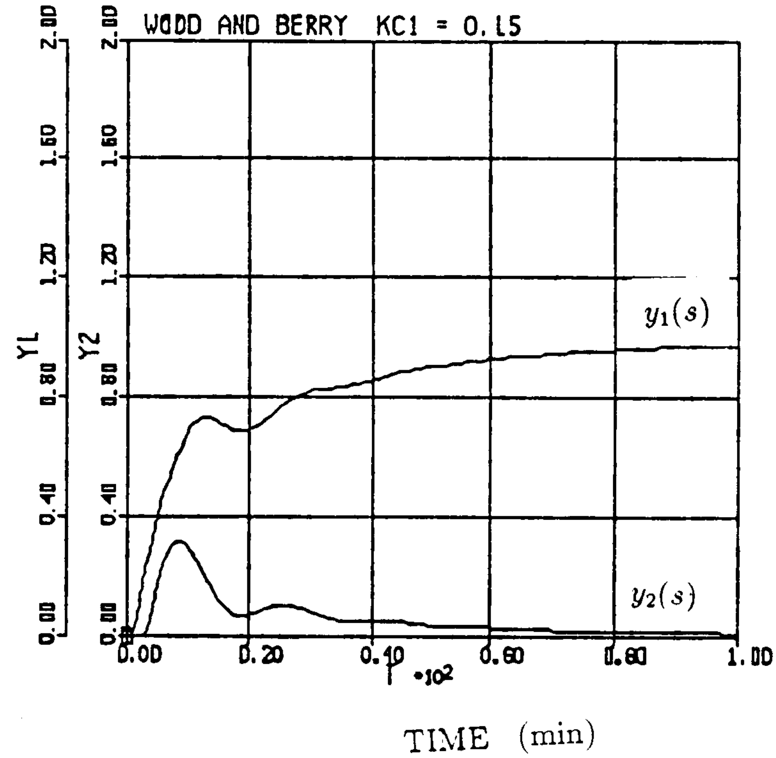


Figure 4.6: Response of Closed-loop Full-block System, $\mathbf{H}_{\mathbf{new}}(s)$, to a Step Input in r_1 for the Wood and Berry Column. (*NEW* refers to an adjusted value of ϵ.)

ever, despite having to use trial and error to achieve desired performance, the fact that IMC-PID controller has only one parameter, as opposed to three for the classical PID controller, makes the trial and error less timeconsuming. Using this (IMC) approach is also more reliable in a real-world control configuration as opposed to the non-interactive scheme proposed by Wood and Berry (1973) which required that the process model be exactly known in order to incorporate the inverse-based compensator. Exact mathematical models of systems are rarely known in the real world.

We have thusfar shown that the performance is affected when the closed-loop time constant, ϵ, is increased. If ϵ is decreased (and K_c is increased) such that $|\tilde{h}_i(j\omega)|_{i=1,2} > \mu^{-1}(\mathbf{E}(j\omega))$ as in Figure 4.7, then, according to the stability guidelines set forth for the μ-Interaction Measure by Grosdidier and Morari (1987), the system should be unstable. This is confirmed in a plot of the response to a step input in $r_1(s)$ shown in Figure 4.8. The system is indeed unstable.

4.2.2 Vinante and Luyben Column

As in the Wood and Berry model, the deadtime is not used in the calculation of μ but is used later in the simulation studies of the system.

For the following plant transfer function matrix,

$$\mathbf{G}'(\mathbf{s}) = \begin{bmatrix} \frac{-2.2}{(7.0s+1)} & \frac{1.3}{(7.0s+1)} \\ \frac{-2.8}{(9.5s+1)} & \frac{4.3}{(9.2s+1)} \end{bmatrix}, \tag{4.21}$$

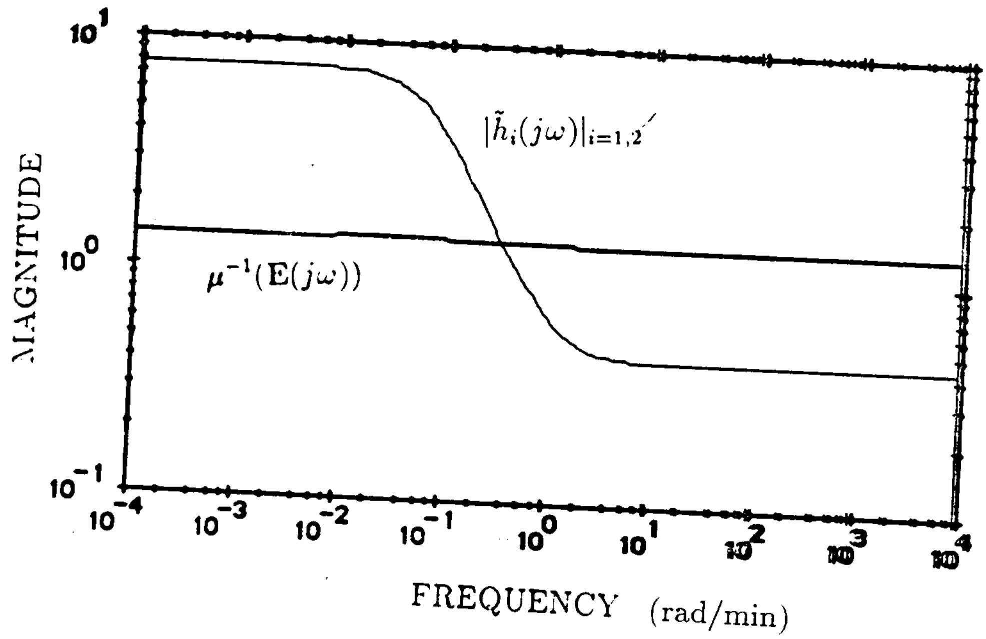


Figure 4.7: $\mu^{-1}(\mathbf{E}(j\omega)) < |\tilde{h}_1(j\omega)|$ versus Frequency (ω) for the Wood and Berry Column.

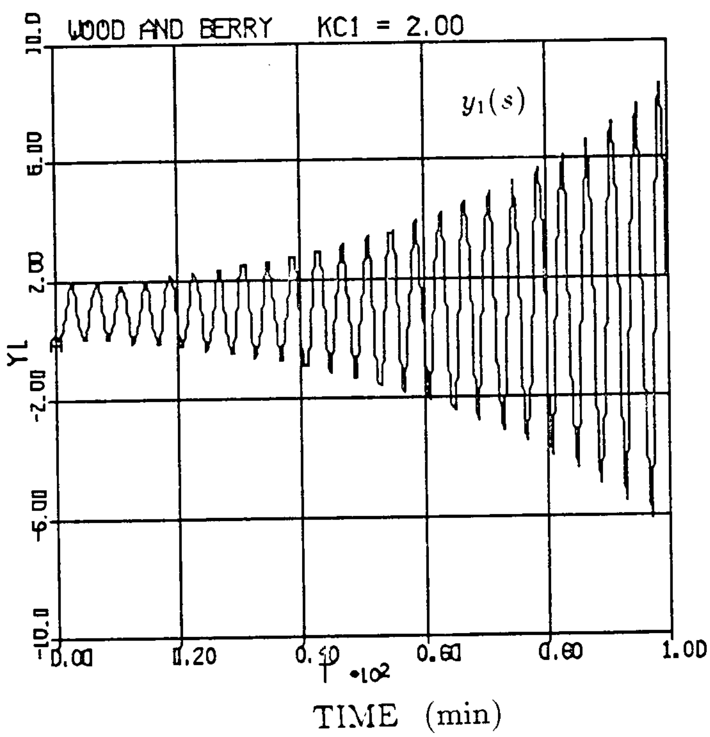


Figure 4.8: System Response to a Step Input in $r_1(s)$ for the Wood and Berry Column, Showing Instability When $|\tilde{h}_i(j\omega)|_{i=1,2} > \mu^{-1}(\mathbf{E}(j\omega))$.

the Rijnsdorp Interaction Quotient (1965), using the (1,1), (2,2) pairing is

$$\kappa(s) = \frac{0.38478(7.0s+1)(9.2s+1)}{(7.0s+1)(9.5s+1)} \tag{4.22}$$

and the inverse is

$$\kappa^{-1}(s) = \frac{2.5989(9.5s+1)}{(9.2s+1)} \tag{4.23}$$

So, as with the Wood and Berry column model, $\mu^{-1}(\mathbf{E}(j\omega))$ is found by plotting the absolute value of the square root of $\kappa^{-1}(j\omega)$ (*ie.* $|\kappa^{-1}(j\omega)|^{1/2}$) for values of frequency (ω). This plot is shown in Figure 4.9.

The diagonal PID controller, $\mathbf{C}(s)$, is then designed using the IMC-PID design procedure. The tuning parameters (K_c, τ_I, and τ_D) are shown in Table 4.2. The indices of the diagonal closed-loop system, $|\tilde{h}_i(j\omega)|_{i=1,2}$, are plotted with $\mu^{-1}(\mathbf{E}(j\omega))$ versus ω and are shown in Figure 4.10.

The simulation of the closed-loop full-block system response to a step input in $r_1(s)$ is shown in Figure 4.11. This plot shows that, as predicted by the diagram where $\mu^{-1}(\mathbf{E}(j\omega)) > |\tilde{h}_i(j\omega)|_{1=1,2}$ for all values of frequency, the system is indeed stable. As with the Wood and Berry model, the performance characteristics can again be changed by adjusting the value of ϵ (which, as with the Wood and Berry model, adjusts the controller gain, K_c). For an adjusted value of K_{c1} (which was arbitrarily changed from a calculated value of -3.409 to -1.0), the plot of $\mu^{-1}(\mathbf{E}(j\omega))$, an IMC value of $|\tilde{h}_1(j\omega)|$, and a "new" (or adjusted) value of $|\tilde{h}_1(j\omega)|$ are all plotted versus frequency. This is shown in Figure 4.12. The corresponding "new" simulation to a step input in $r_1(s)$ is then shown in Figure

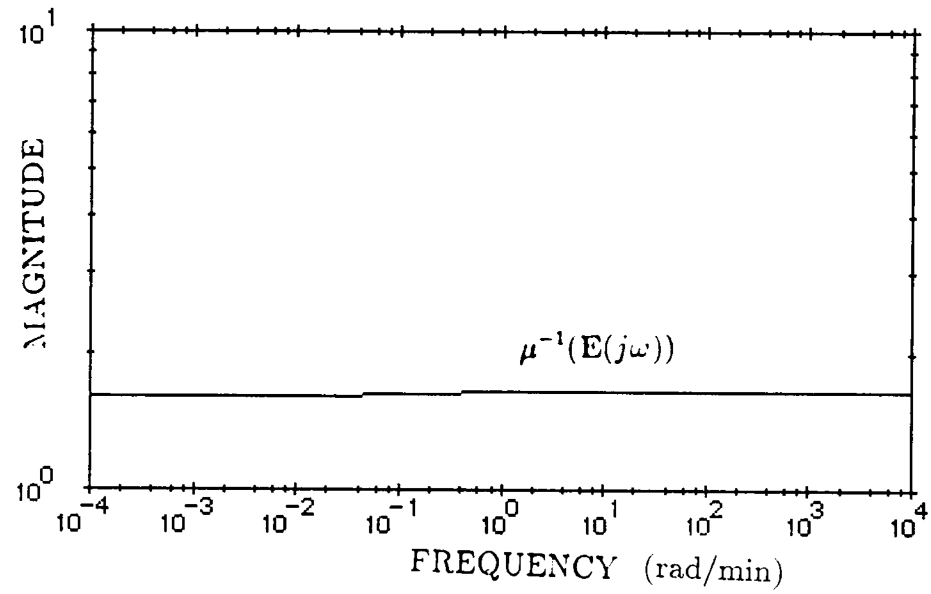


Figure 4.9: $\mu^{-1}(\mathbf{E}(j\omega))$ versus Frequency (ω) for the Vinante and Luyben Column.

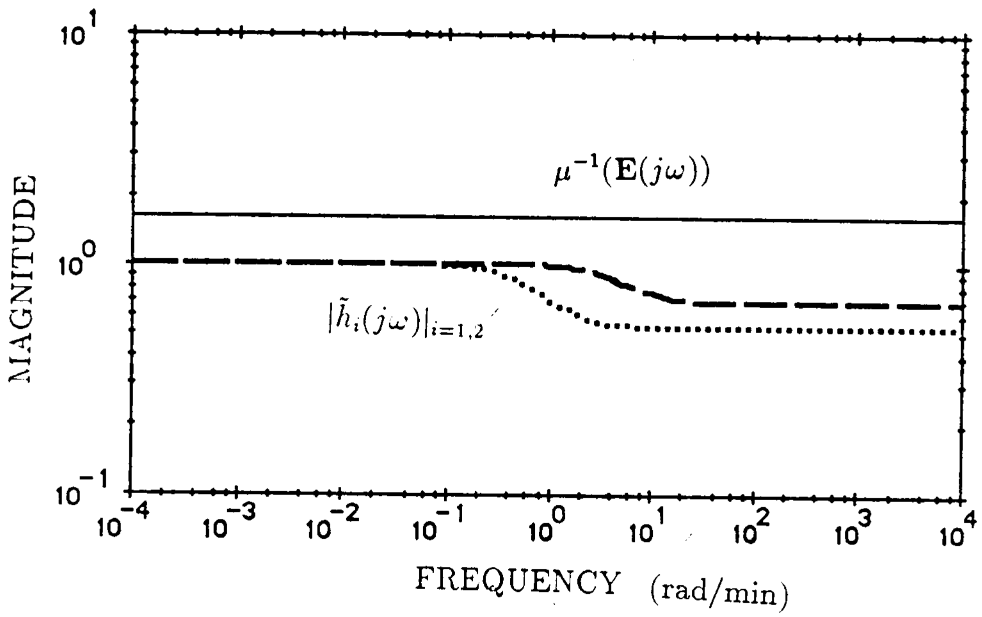


Figure 4.10: $\mu^{-1}(\mathbf{E}(j\omega))$ and $|\tilde{h}_i(j\omega)|_{i=1,2}$ versus Frequency (ω) for the Vinante and Luyben Column.

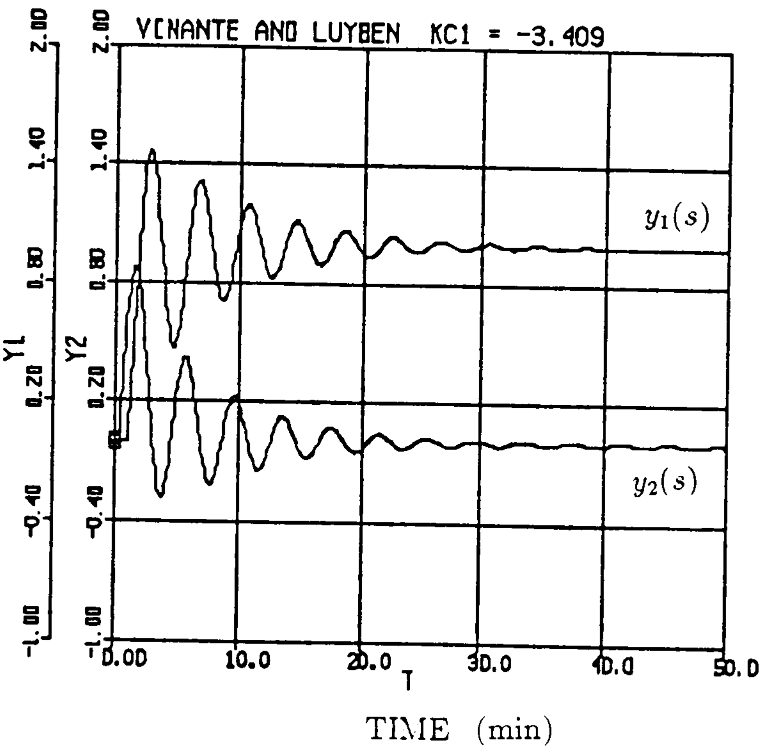


Figure 4.11: Response to a Step Input in $r_1(s)$ for the closed-loop full-block system, $\mathbf{H}(s)$, for the Vinante and Luyben Column.

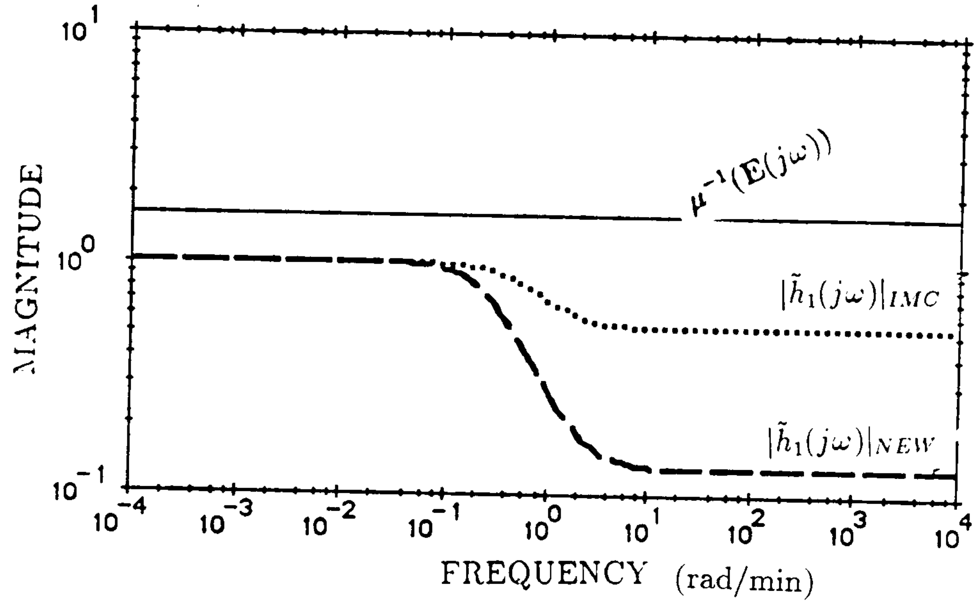


Figure 4.12: $\mu^{-1}(\mathbf{E}(j\omega))$, $|\tilde{h}_1(j\omega)|_{IMC}$, and $|\tilde{h}_1(j\omega)|_{NEW}$ versus Frequency (ω) for the Vinante and Luyben Column.

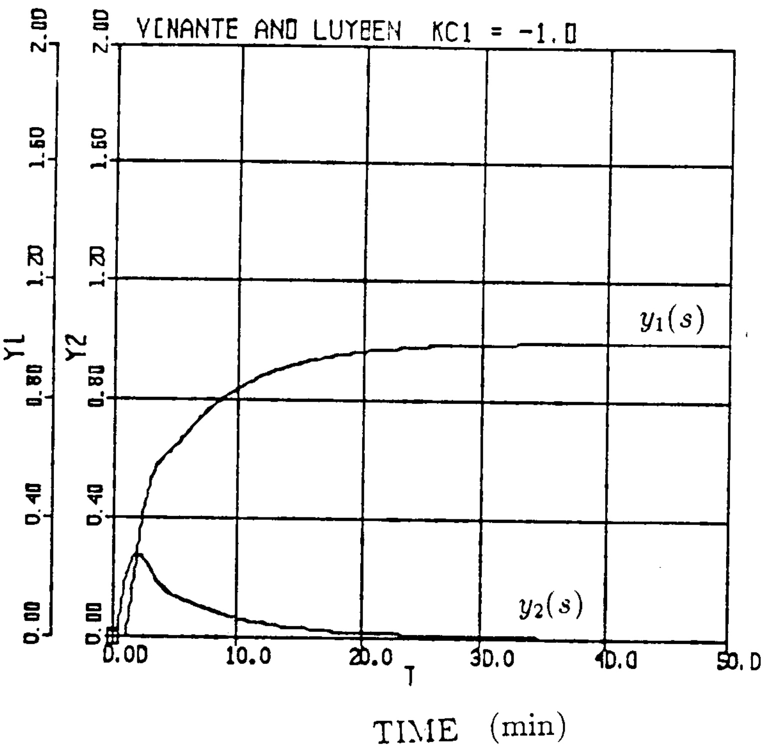


Figure 4.13: Response to Step Input in $r_1(s)$ with an Adjusted Value of ϵ for the Vinante and Luyben Column.

4.13. This plot shows a much smoother but slower response than that of Figure 4.11. Again, to determine the "best" controller setting, one must determine what performance characteristic is most suitable for a particular system. In general one sacrifices response speed for a smoother (less overshoot) response and *vise versa.* For every process the system designer should decide in which direction he can most afford to sacrifice. Usually minimal trial and error can bring a system to a "middle ground" in which neither rise time nor response smoothness are optimal but both are acceptable. The results obtained here compared quite closely with those found in the study by Luyben (1986). The performance characteristics in Figure 4.13 are very similar to those obtained by tuning using the BLT method (Figure 3.5). The tuning method used here (IMC) is; however, much less complex and time-consuming than the BLT method used by Luyben.

4.2.3 Weischedel and McAvoy Column

As mentioned earlier, the deadtimes associated with the individual transfer functions are much smaller than the plant time constants and are, therefore, dropped. So the transfer matrix,

$$\mathbf{G}'(s) = \begin{bmatrix} \frac{0.471}{(30.7s+1)^2} & \frac{0.495}{(28.5s+1)^2} \\ \frac{0.749}{(57.0s+1)^2} & \frac{-0.832}{(50.5s+1)^2} \end{bmatrix}, \tag{4.24}$$

is used to represent the system. In the earlier controller design procedure, it was noted that with the relative gain array indices so close to 0.5, it is not appropriate to base the input/output pairing on the RGA. The "rule-of-thumb" of RGA use would indicate that it really doesn't matter which variables are paired since they are so interactive. This being the case, we will apply the μ-IM analysis to both pairings (*ie.* (1,1),(2,2) and (1,2),(2,1)).

For the (1,1),(2,2) pairing, the Rijnsdorp IQ is

$$\kappa_{11,22}(s) = \frac{-0.9461(30.7s+1)^2(50.5s+1)^2}{(57.0s+1)^2(28.5s+1)^2} \tag{4.25}$$

and for the (1,2),(2,1) pairing, the Rijnsdorp IQ would simply be the inverse of the (1,1),(2,2) case, so

$$\kappa_{12,21}(s) = \frac{-1.057(57.0s+1)^2(28.5s+1)^2}{(30.7s+1)^2(50.5s+1)^2}. \tag{4.26}$$

Values of $\mu^{-1}(\mathbf{E}(j\omega))$ for both pairings are found *via* the algorithm provided by Grosdidier and Morari (1987) and are plotted versus frequency in Figure 4.14. This plot shows no significant difference in the magnitude of their respective $\mu^{-1}(\mathbf{E}(j\omega))$ curves. However, the curve for the (1,1),(2,2) pairing lies just above 1.0 while that of the (1,2),(2,1) lies just below the value of 1.0. It would then seem that if one follows the recommendations of Grosdidier and Morari (1987), the input/output variables would be paired as (1,1),(2,2) in order to comply with the provision of $\mu(\mathbf{E}(j\omega)) \leq 1.0$ to ensure the incorporation of an integral mode in the controller, $\mathbf{C}$(s).

The diagonal controllers designed by the IMC-PID procedure were used to compute the diagonal closed-loop system, $\tilde{\mathbf{H}}(s)$, for both pairings. The magnitude of the indices, $|\tilde{h}_i(j\omega)|_{i=1,2}$ were plotted along with $\mu^{-1}(\mathbf{E}(j\omega))$ against fre-

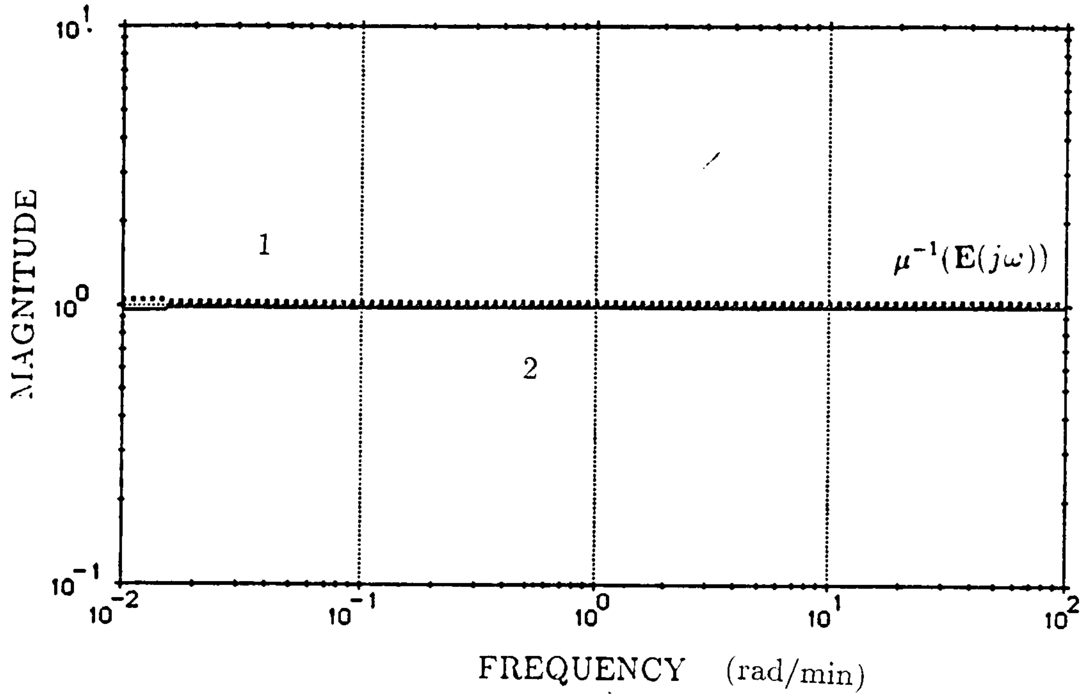


Figure 4.14: $\mu^{-1}(\mathbf{E}(j\omega))$ versus Frequency (ω) for the Weischedel and McAvoy Column. (Line-1 is for the (1,1),(2,2) pairing, line-2 is for the (1,2),(2,1) pairing.)

quency for both variable pairings. Figure 4.15 shows this plot for the (1,1),(2,2) pairing, while the case of the (1,2),(2,1) pairing is shown in Figure 4.16. However, these diagram don't tell much about which pairing would indeed produce the most desireable results. The plot of $|\tilde{h}_i(j\omega)|_{i=1,2}$ in Figure 4.16 does seem to have a slightly larger margin from the plot of $\mu^{-1}(\mathrm{E}(j\omega))$ than that of Figure 4.15. This indicates that the response using the (1,2),(2,1) pairing should be slightly more sluggish than that of the (1,1),(2,2) pairing.

The response of each to a unit step input of $r_1(s)$ is shown, for the (1,1),(2,2) pairing in Figure 4.17, and for the (1,2),(2,1) pairing in Figure 4.18. The response using the (1,2),(2,1) pairing is, as predicted, indeed more sluggish at these controller settings and because of the inability of the system to incorporate an integral mode, the system has a degree of offset. This difference would lead most systems designers to pair on the (1,1), (2,2) setting. The response of the preferred pairing (Figure 4.17) shows more desireable performance characteristics than that of the study done by Weischedel and McAvoy (Figure 3.8). The controller settings for that study were most likely done by some empirical approach (Zeigler-Nicols, *etc*) which is more time-consuming than the IMC-PID tuning applied in this study.

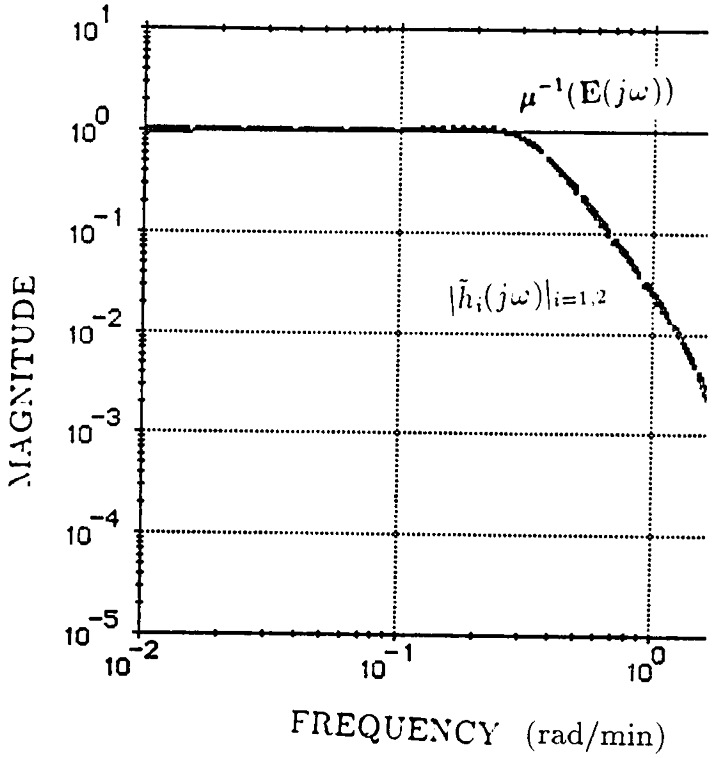


Figure 4.15: $\mu^{-1}(\mathbf{E}(j\omega))$ and $|\tilde{h}_i(j\omega)|_{i=1,2}$ versus Frequency (ω) for (1,1),(2,2) pairing on the Weischedel and McAvoy Column.

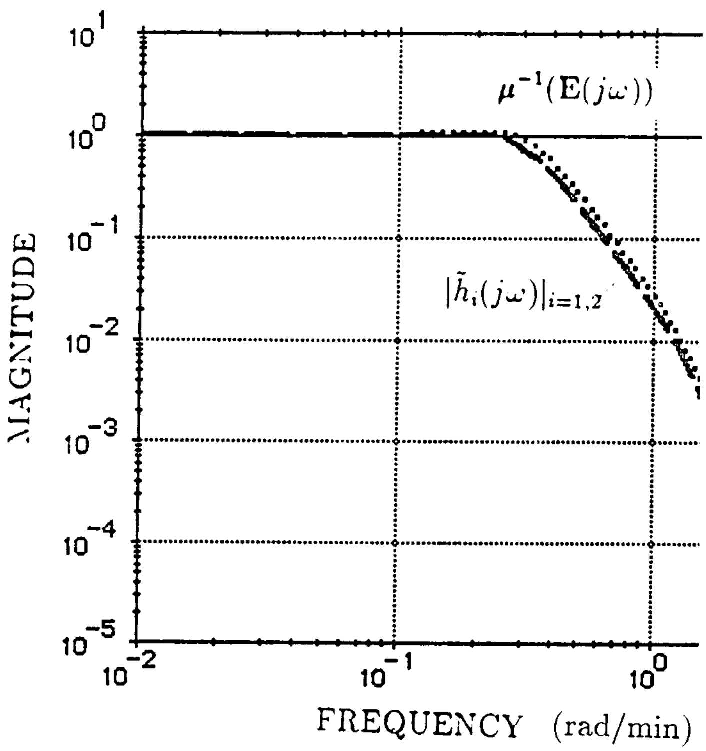


Figure 4.16: $\mu^{-1}(\mathbf{E}(j\omega))$ and $|\tilde{h}_i(j\omega)|_{i=1,2}$ versus Frequency (ω) for (1,2),(2,1) pairing on the Weischedel and McAvoy Column.

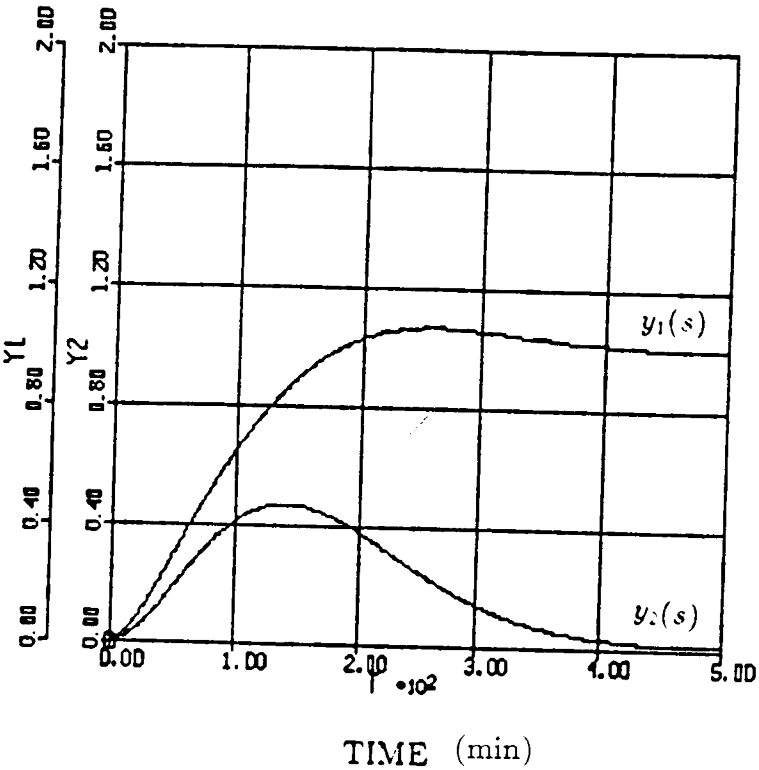


Figure 4.17: Response to a Step Input in $r_1(s)$ for the Closed-loop Full-block System, $\mathbf{H}$(s), for the Weischedel and McAvoy Column Using the (1,1),(2,2) Pairing.

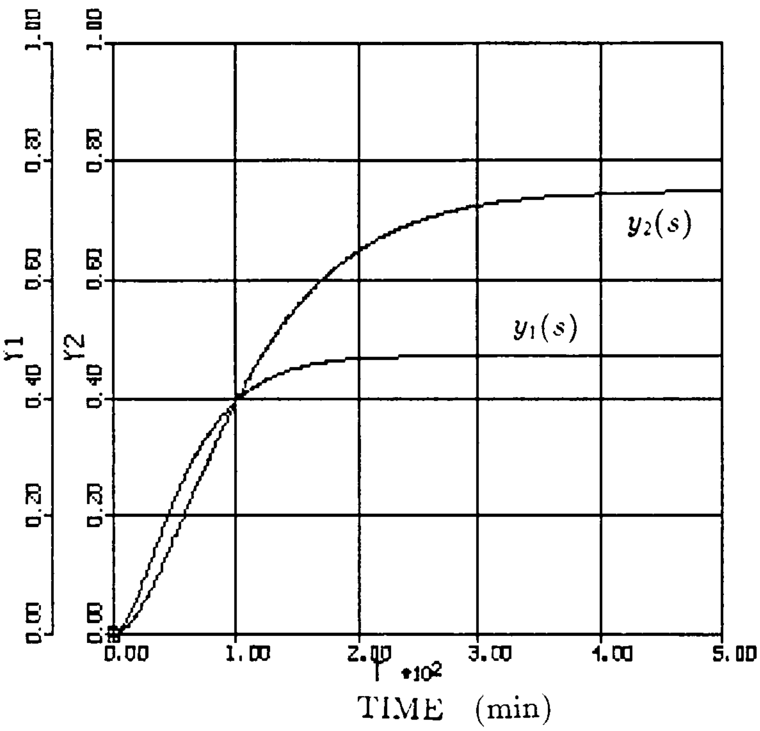


Figure 4.18: Response to a Step Input in $r_1(s)$ for the Closed-loop Full-block System, $\mathbf{H}(s)$, for the Weischedel and McAvoy Column Using the (1,2),(2,1) Pairing.

CHAPTER 5

USING μ-INTERACTION MEASURE ANALYSIS FOR SYSTEMS WITH UNCERTAINTY

5.1 Incorporation of an Uncertainty Bound for Design Using the μ-Interaction Measure

In using the μ-Interaction Measure stability analysis approach thusfar, we have only considered systems which have no plant/model mismatch. We have assumed that the models were known exactly and thus that the resulting plot of $\mu^{-1}(\mathbf{E}(j\omega))$ provides an absolute bound on the stability of $\tilde{\mathbf{H}}(s)$ which also guarantees the stability of $\mathbf{H}(s)$. However, for the most part, plant models are not known exactly and thus there is usually a mismatch in the actual dynamics of the system and the modeled dynamics of the system.

In the use of the μ-IM stability analysis procedure, if there is a degree of uncertainty in the modeled system, then $\mu^{-1}(\mathbf{E}(j\omega))$ no longer represents the absolute bound on $\tilde{\mathbf{H}}(j\omega)$. If the model is not known exactly then the $\sigma_{max}(\tilde{\mathbf{H}}(j\omega))$ may lie below the curve of $\mu^{-1}(\mathbf{E}(j\omega))$, where $\mathbf{E}$ represents the error between the nominal diagonal and nominal full block transfer function, and yet the closed-loop system, $\mathbf{H}(s)$, could still be unstable, contradictory to the stability criteria of the μ-IM.

When there is plant/model mismatch, or the possibility of such, then an uncertainty bound (Figure 5.1) should be incorporated into the μ-IM analysis. This uncertainty bound provides a new bound on the magnitude of $\tilde{\mathbf{H}}(s)$ to imply the stability of $\mathbf{H}(s)$. In other words, if the maximum singular value of the nominal diagonal closed-loop system, $\tilde{\mathbf{H}}(j\omega)$, lies below the lowest $\mu^{-1}(\mathbf{E_{real}}(j\omega))$ bound then the closed-loop full-block real plant system, $\mathbf{H}_{real}(s)$, is stable, where $\{\mathbf{E_{real}}\}$ represents the set of $\mathbf{E}$'s derived from the set of possible real plants, $\mathbf{G}$ (see Section 5.2 for the calculation of $\sigma_{max}(\mathbf{E}(j\omega))$ as a function of frequency).

A problem arises in the mathematical calculation of this bound. Ideally, one would calcualte the $\mu^{-1}(\mathbf{E}(j\omega))$ based on the nominal plant, $\mathbf{G}(s)$, where $\mathbf{E}(s)$ is defined as

$$\mathbf{E}(s) = (\mathbf{G}(s) - \tilde{\mathbf{G}}(s))\tilde{\mathbf{G}}^{-1}(s). \tag{5.1}$$

$\tilde{\mathbf{G}}(s)$ is the diagonalized nominal plant. For 2x2 systems, Grosdidier and Morari (1987) provide an algorithm to calculate $\mu^{-1}(\mathbf{E}(j\omega))$. This algorithm is applicable to systems where the corresponding diagonal elements of $\mathbf{G}(s)$ and $\tilde{\mathbf{G}}(s)$ are the same. In the case where $\mathbf{G}(s)$ is not known exactly, then the corresponding diagonal elements of $\mathbf{G_{real}}(s)$ and $\tilde{\mathbf{G}}(s)$ may or may not be equal. Thus the just mentioned algorithm can not be used. One can, however, approximate the μ value and illustrate the approach by using the fact that the structured singular value (μ) of a matrix is bounded above and below as follows (Doyle, 1982):

$$\rho(\mathbf{A}) \leq \mu(\mathbf{A}) \leq \sigma_{max}(\mathbf{A}) \tag{5.2}$$

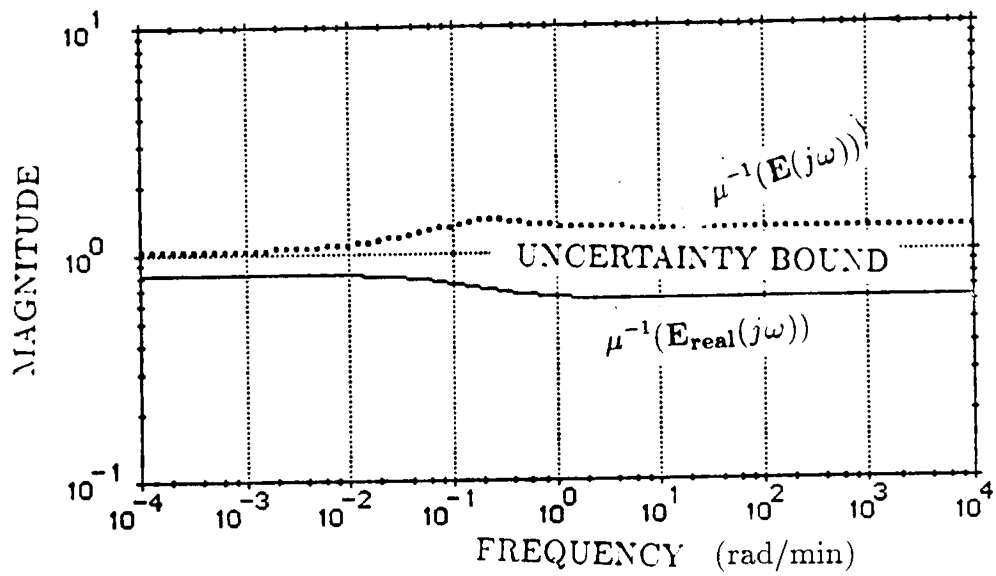


Figure 5.1: Uncertainty Bound Incorporated into the μ-Interaction Measure Stability Analysis.

which can also be written as

$$\rho^{-1}(\mathbf{A}) \geq \mu^{-1}(\mathbf{A}) \geq \sigma_{max}^{-1}(\mathbf{A}) \tag{5.3}$$

where $\sigma_{max}(\mathbf{A})$ is the maximum singular value of matrix $\mathbf{A}$ and $\rho(\mathbf{A})$ is the spectral norm of $\mathbf{A}$ (the largest absolute eigenvalue of $\mathbf{A}$).

This analogy would imply that $\sigma_{max}^{-1}(\mathbf{E}(j\omega))$ would lie below the curve of $\mu^{-1}(\mathbf{E}(j\omega))$ for all values of frequency. Hence, one can calculate $\sigma_{max}^{-1}(\mathbf{E}(j\omega))$ of the perturbed system and this curve for the perturbed system would actually lie below the desired uncertainty bound, *ie.* $\mu^{-1}(\mathbf{E}(j\omega))$ of the perturbed system, and thus is conservative.

Mathematically, $\sigma_{max}^{-1}(\mathbf{E}(j\omega))$ is employed in the following manner. A diagonal controller, $\mathbf{C}(s)$, is designed based on a nominal diagonalized plant transfer function matrix, $\tilde{\mathbf{G}}(s)$, for the decentralized control of a nominal model-based plant transfer function matrix, $\mathbf{G}(s)$. This model, $\mathbf{G}(s)$, has uncertainty and thus does not represent the true plant, $\mathbf{G}_{\mathbf{real}}(s)$.

The stability criteria for using the μ-IM approach then becomes:

Assuming $\tilde{\mathbf{H}}(s)$ and $\mathbf{G}(s)$ are stable, then, the full-block closed-loop system, $\mathbf{H}(s)$, is stable if

$$\sigma_{max}(\tilde{\mathbf{H}}(j\omega)) < \sigma_{max}^{-1}(\mathbf{E}_{\mathbf{real}}(j\omega)) \quad \forall \quad \omega \tag{5.4}$$

where $\sigma_{max}(\mathbf{A})$ is the maximum singular value of $\mathbf{A}$ and $\mathbf{E}_{\mathbf{real}}(s)$ is defined as

$$\mathbf{E}_{\mathbf{real}}(s) = (\mathbf{G}_{\mathbf{real}}(s) - \tilde{\mathbf{G}}(s))\tilde{\mathbf{G}}^{-1}(s) \tag{5.5}$$

and $\mathbf{G}_{\mathbf{real}}(s)$ is the real plant representing the system's true dynamics and $\tilde{\mathbf{G}}(s)$ is the diagonalized nominal plant transfer function.

5.2 Wood and Berry Column

The nominal model-based plant transfer function matrix, $\mathbf{G}(s)$, for the Wood and Berry column is

$$\mathbf{G}(s) = \begin{bmatrix} \frac{12.8e^{-1.0s}}{(16.7s+1)} & \frac{-18.9e^{-3.0s}}{(21.0s+1)} \\ \frac{6.6e^{-7.0s}}{(10.9s+1)} & \frac{-19.4e^{-3.0s}}{(14.4s+1)} \end{bmatrix}. \qquad (5.6)$$

For this analysis, it was assumed that the process gains, K_p's, and the process time constants, τ_p's, could vary $\pm 20\%$ in each index of the matrix, $\mathbf{G}(s)$. Each combination of ($K_p \pm 20\%$ and $\tau_p \pm 20\%$) variation gives a different perturbed system, $\mathbf{G_{real}}(s)$. Therefore for all cases, an error ($\mathbf{E_{real}}(s)$) between the real plant and the nominal diagonalized plant transfer function is computed and the largest of the maximum singular values for this error function at each frequency is found and plotted for all values of frequency. To get the worst case, we must search for the lowest value of $\sigma_{max}^{-1}(\mathbf{E_{real}}(j\omega))$ for all values of frequency.

A family of $\sigma_{max}^{-1}(\mathbf{E_{real}}(j\omega))$'s for the varying $\mathbf{E_{real}}(s)$'s were calculated and the spectrum is shown in Figure 5.2.

This diagram shows that there is a single worst case (*ie.* lowest $\sigma_{max}^{-1}(\mathbf{E_{real}}(j\omega))$ at all values of frequency). This occured, for this model, for the case where all process gains, K_p, varied +20%, and when all process time constants, τ_p, varied -20%. This worst case then provides the new magnitude constraint on $\tilde{\mathbf{H}}(s)$ in order to also imply the stability of $\mathbf{H}(s)$. Figure 5.3 compares the plot of $\sigma_{max}^{-1}(\mathbf{E_{real}}(j\omega))$ (the worst case) and $\sigma_{max}^{-1}(\mathrm{E}(j\omega))$ (the nominal case) versus

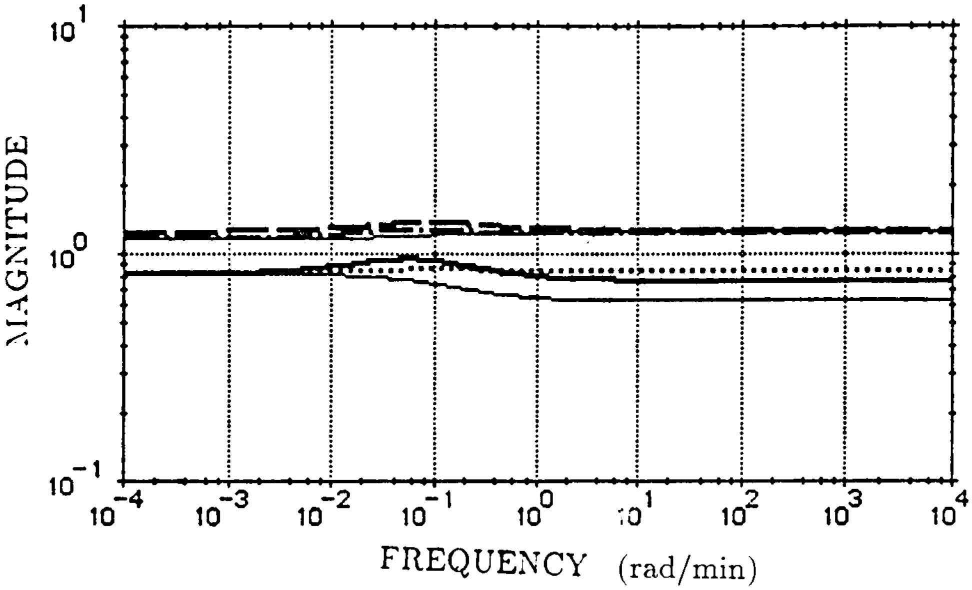


Figure 5.2: Spectrum of $\sigma_{max}^{-1}(\mathbf{E_{real}}(j\omega))$ for Varying Perturbed Plant Transfer Function Matrix, $\mathbf{G_{real}}(s)$, for the Wood and Berry Column. Process Parameters, K_p and τ_p, vary $\pm 20\%$.

frequency.

One important aspect to consider in the analysis of these types of systems is that if one guarantees robustness, by lowering the upper bound on $\sigma_{max}(\tilde{\mathbf{H}}(j\omega))$, one can also decrease the performance of the system. As noted earlier, in general, the greater the margin between $\sigma_{max}(\tilde{\mathbf{H}}(j\omega))$ and $\mu^{-1}(\mathbf{E}(j\omega)$, the more sluggish the system.

When using the diagonal controller designed for the mathematical model of the plant to control the "real" plant, the system is unstable (Fig. 5.4). The controller originally maintained a stable system (Fig. 4.4) since the plot of $\mu^{-1}(\mathbf{E}(j\omega)) > |\tilde{h}_i(j\omega)|_{i=1,2}$ for all values of frequency. However the plot of $\mu^{-1}(\mathbf{E_{real}}(j\omega))$ lies below the plot of $\mu^{-1}(\mathbf{E}(j\omega))$ (Fig. 5.1), and since the "real" system is unstable under feedback control, obviously the plot of $|\tilde{h}_i(j\omega)|_{i=1,2}$ does not lie below the plot of $\mu^{-1}(\mathbf{E_{real}}(j\omega))$ for all values of frequency. When one reduces the controller gains to ensure $|\tilde{h}_i(j\omega)|_{i=1,2} < \mu^{-1}(\mathbf{E_{real}}(j\omega))$ (or according to Equation 5.4 , $\sigma_{max}^{-1}(\mathbf{E_{real}}(j\omega)))$ the system becomes stable (Fig. 5.5).

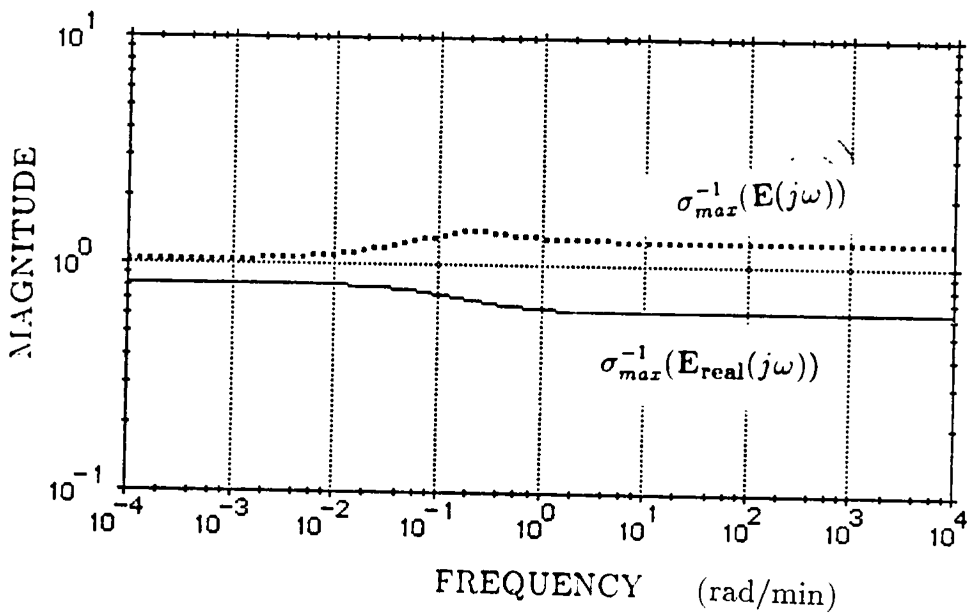
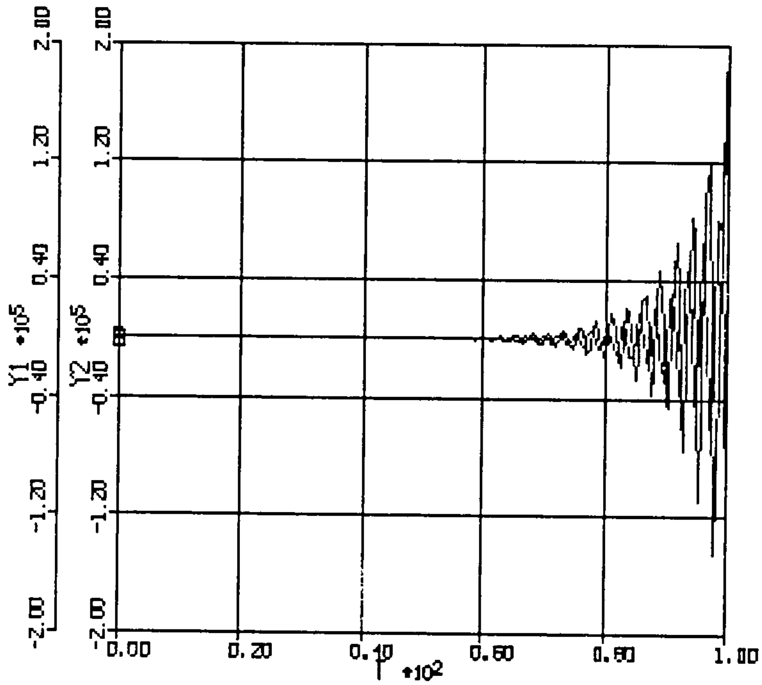


Figure 5.3: $\sigma_{max}^{-1}(\mathbf{E}_{\text{real}}(j\omega))$ (the worst case) and $\sigma_{max}^{-1}(\mathbf{E}(j\omega))$ (the nominal case) versus Frequency Showing the Uncertainty Bound for the Wood and Berry Column.



TIME (min)

Figure 5.4: Response to a Step Input in r_1 for the Closed-loop Full-block System, $\mathbf{H}_{\text{real}}(s)$, for the Wood and Berry Column.

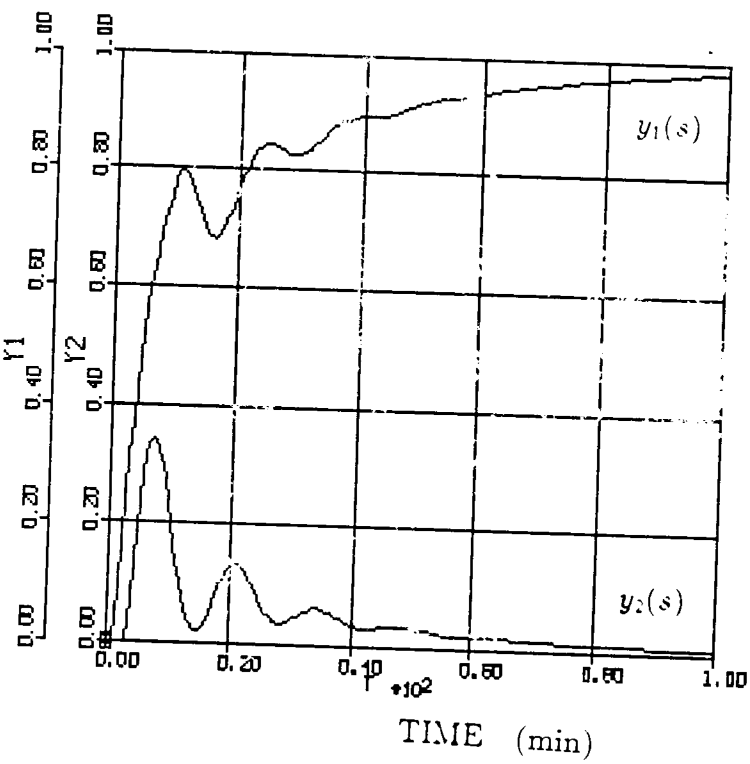


Figure 5.5: Response to a Step Input in r_1 for the Closed-loop Full-block System with an Adjusted Value of $\mathbf{H}_{\mathbf{real}}(s)$, for the Wood and Berry Column.

CHAPTER 6

CONCLUSIONS

The following conclusions can be made about using the μ-Interaction Measure in the stability analysis of decentralized control structures:

1. The chemical processing industry tends to favor using diagonal controllers due to their simplicity; thus the μ-IM gives a good handle on assessing stability in a sound way.

2. The μ-Im approach renders a simple and direct method of arriving at the tuning parameters; much simplified compared to the older empirical plus trial-and-error tuning procedures.

3. The μ-IM can be used in simple (decentralized), systematic controller design to guarantee the stability of the full-block system.

4. The μ-IM is conviently displayed on a magnitude-frequency diagram.

5. The μ-IM analysis approach can be applied to any decentralized controller design. The Internal Model Control (IMC) design procedure was used in this research.

6. While the μ-IM guarantees stability of the closed-loop system, the performance may be poor.

7. In the 2x2 case, the μ-IM is closely related to the Rijnsdorp Interaction Quotient and is calculated quite easily. However, computation for higher order systems is under current research by the systems community.

8. An uncertainty bound can be incorporated into the stability analysis of a control design based on a nominal diagonal plant transfer function.

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