

Predictive Capabilities of Laminar-Turbulent Transition Models for Aerodynamics Applications

A Dissertation Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Jared Alexander Carnes

August 2022

© by Jared Alexander Carnes, 2022
All Rights Reserved.

To my fiancée, Meghan

Acknowledgments

I would like to thank my advisor and committee chair, Dr. James Coder for his guidance and encouragement throughout my graduate studies, without which this work would not be possible. I would also like to thank Dr. Ryan Glasby, Dr. Kivanc Ekici, and Dr. Devina Sanjaya for serving on my committee throughout my career in graduate school.

I would like to thank the Pennsylvania State University Vertical Lift Research Center of Excellence for its support throughout the last four years. I would also like to thank Dr. Pieter Buning for his advice and expertise in performing OVERFLOW simulations, Dr. Balaji Venkatachari for fruitful discussions on boundary-layer transition modeling, Dr. Anthony Gardner, Dr. Christian Wolf, and Dr. Armin Weiss of DLR for their aid in the RTG and DSA-9A simulations, and Mr. Rohit Jain for his help in rotorcraft CFD simulations through the years. I would like to thank all of the students in the volAIR research lab for their insights on this research. Thanks go to my friends, family, and loved ones for helping me see this research through.

This research was partially funded by the Government under Agreement No. W911W6-17-2-0003. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Aviation Development Directorate or the U.S Government.

Some images in this work were generated using FieldView, licenses for which were provided through their University Partners Program.

Abstract

Laminar-turbulent boundary-layer transition has a demonstrable impact on the performance of aerospace vehicles. The ability to accurately predict transition is integral to properly capturing relevant flow physics. Traditionally, computational fluid dynamics simulations are performed fully turbulent, meaning that laminar flow is neglected. This, however, can result in errant predictions of vehicle performance as quantities such as skin-friction drag may be overpredicted. Resultingly, development of Reynolds-averaged Navier-Stokes transition models has seen significant attention over the last decades in order to model transition and realize the performance improvements of laminar flow.

In this work, the behaviors of several different transition-prediction methods are analyzed both for their ability to predict transition and vehicle performance. The popular local-correlation transition model is assessed analytically and numerically. It is found that there exists a singularity near the wall after transition to turbulence. This results in singular-like behavior of the destruction term of the turbulent kinetic energy equation and prevents the model from ever achieving asymptotic grid convergence. A turbulence index was developed to more robustly and accurately detect transition relative to other quantities such as turbulent intermittency.

The behavior of the amplification factor transport (AFT) model was examined for a four-bladed helicopter rotor undergoing dynamic pitching conditions. The transition front predicted by the AFT model agreed well with experiments, with exception to that during the upstroke of the pitch cycle. The effect of freestream turbulence intensity on transition was examined by varying the critical N -factor, finding that as turbulence intensity increased, the transition front moved increasingly further upstream throughout the pitch cycle.

Additionally, large eddy simulations were performed for a rotorcraft airfoil undergoing dynamic pitching conditions. A laminar separation bubble was found to be the primary mechanism of transition, finding also that the length of the separation bubble decreased as the pitch angle increased. A Kelvin-Helmholtz-like instability was identified near the aft end of the separation bubble which drives transient bursting of the bubble and is partially responsible for the behavior of the transition front. An additional investigation using the AFT model found that predicted transition agreed well, but predicted a shorter separation bubble.

Table of Contents

1	Introduction	1
1.1	Research Goals	6
2	Background	8
2.1	Physics of Boundary-layer Transition	8
2.1.1	Natural Transition	9
2.1.2	Bypass Transition	10
2.1.3	Crossflow Transition	11
2.1.4	Separation-induced Transition	11
2.1.5	Other Transition Mechanisms	12
2.2	Overview of Transition-Prediction Methods	13
2.2.1	Linear Stability Methods	13
2.2.2	Local-correlation Methods	16
2.2.3	PDE Methods	19
2.2.4	Large Eddy Simulation	22
2.2.5	Hybrid RANS/LES Methods	24
2.3	Contemporary Use of Transition Models	25
2.3.1	Inviscid/Viscous Segregated Flow Solvers	25
2.3.2	RANS-based Transition Prediction	27
2.3.3	LES-based Transition Prediction	32
3	Computational Methodology	36

3.1	Description of Turbulence Models	36
3.1.1	The Shear Stress Transport Model	38
3.1.2	The Spalart-Allmaras Model	43
3.2	Description of Transition Models	45
3.2.1	The Langtry-Menter Local Correlation Model	46
3.2.2	The Amplification Factor Transport Model	53
3.3	Navier-Stokes Solution Techniques	59
3.3.1	Central-Difference Schemes	62
3.3.2	WENO Flux Reconstruction	64
3.3.3	Roe Flux-Difference Splitting	66
3.3.4	The Pulliam-Chaussee Scheme	67
3.3.5	The Symmetric Successive Over-Relaxation Scheme	69
4	Near-Wall Behavior of the Langtry-Menter Model	71
4.1	Introduction	71
4.2	Governing Equations Near the Wall	74
4.3	Near-wall Analysis	76
4.3.1	Turbulence Quantities Near the Wall	76
4.3.2	New Turbulence Index	79
4.4	Validation for Practical Aerodynamic Cases	80
4.4.1	T3A Zero-Pressure-Gradient Flat Plate	81
4.4.2	TMR Zero-Pressure-Gradient Flat Plate	83
4.4.3	PSU 94-097 Winglet Airfoil	84
4.4.4	RAE 2822 Airfoil in Transonic Conditions	86
4.4.5	TU Braunschweig Sickle Wing	87
4.5	Summary and Discussion	89
5	DLR RTG Rotor in Dynamic Pitching Conditions	91
5.1	Description of Test Case	91
5.2	Rotor Geometry and Grid System	93

5.3	Results	95
5.3.1	Determination of Transition Location	96
5.3.2	Predicted Transition Locations	97
5.3.3	Effect of Freestream Turbulence Intensity	101
5.3.4	Effect of Rotor Thrust Level	104
5.4	Summary and Discussion	104
6	Implicit Large Eddy Simulation of the DSA-9A Airfoil	107
6.1	Methodology	108
6.1.1	Test Case	108
6.1.2	Grid Generation	110
6.1.3	Numerical Schemes	112
6.2	Results	114
6.3	Summary and Discussion	122
7	Summary and Conclusions	125
	References	129
	Appendices	156
A	Chapter 1 Figures and Tables	156
B	Chapter 2 Figures and Tables	157
C	Chapter 3 Figures and Tables	162
D	Chapter 4 Figures and Tables	163
E	Chapter 5 Figures and Tables	173
F	Chapter 6 Figures and Tables	184
	Vita	194

List of Tables

4.1	T3A grid dimensions	163
4.2	PSU 94-097 grid dimensions	163
5.1	Test conditions for Case II	173

List of Figures

1.1	General Atomics MQ-9A Reaper	156
1.2	PSP Rotor figure of merit	156
2.1	Breakdown of natural transition	157
2.2	Flowfield near a laminar separation bubble	157
2.3	Neutral stability curve for a Blasius boundary layer	158
2.4	Predicted contours of $\widetilde{Re}_{\theta,t}$ about the DSA-9A airfoil	158
2.5	Performance of the PSU 94-097 airfoil at $Re = 1 \times 10^6$	159
2.6	Predicted surface pressure on the CRM-NLF using two different transition models	159
2.7	Predicted skin friction distributions on the 6:1 inclined prolate spheroid . . .	160
2.8	LES-predicted isosurfaces of the Q-criterion for a flat plate boundary layer, showing H-type natural transition	160
2.9	Predicted and experimentally determined Reynolds shear stress about the SD7003 at $\alpha = 11^\circ$	161
3.1	Modified wavenumber analysis of central-difference schemes and 5 th -order WENO	162
4.1	Comparison of turbulent kinetic energy budgets for the Langtry-Menter and SST models on the T3A flat plate	164
4.2	Comparison of turbulence index on the wall for the Langtry-Menter and SST models on the T3A flat plate	165
4.3	Comparison of intermittency on the wall for the Langtry-Menter model on the T3A flat plate	166

4.4	Surface quantity distributions for the TMR case	166
4.5	Predicted and measured pressure distribution on the PSU 94-097 airfoil at $Re = 2.4 \times 10^5$	167
4.6	Skin friction and turbulence index on the PSU 94-097 airfoil at $\alpha = 5^\circ$ and $Re = 2.4 \times 10^5$	168
4.7	Comparison of turbulent kinetic energy budgets for the Langtry-Menter and SST models on the PSU 94-097 airfoil at $\alpha = 2^\circ$, Fine grid (513×129) . . .	169
4.8	Comparison of predicted pressure distribution on the RAE 2822 airfoil at $\alpha = 2.79^\circ, M = 0.734, Re = 6.5 \times 10^6$	170
4.9	Turbulence index on the RAE 2822 airfoil at $\alpha = 2.79^\circ, M = 0.734, Re =$ 6.5×10^6	170
4.10	LM2015 predicted transition on the sickle wing (Case A)	171
4.11	Turbulence index on the sickle wing at select spanwise stations for Case A .	172
5.1	DLR RTG blade planform	173
5.2	DLR RTG blade structured, overset, baseline grid	173
5.3	DLR RTG off-body, Cartesian, overset grid system	174
5.4	Thrust and torque history over the final rotor revolution, baseline grid	174
5.5	Predicted transition visualized via Spalart's turbulence index, fine grid (blue - laminar, red - turbulent)	175
5.6	Predicted skin friction contours, fine grid	175
5.7	Transition location as a function of pitch cycle and blade radius	176
5.8	Comparison of predicted transition location as a function of pitch cycle at $r/R = 0.615$	177
5.9	Comparison of predicted transition location as a function of pitch cycle at $r/R = 0.723$	177
5.10	Comparison of predicted transition location as a function of pitch cycle at $r/R = 0.908$	178
5.11	Comparison of predicted transition location as a function of pitch angle at $r/R = 0.615$	178

5.12	Comparison of predicted transition location as a function of pitch angle at $r/R = 0.723$	179
5.13	Comparison of predicted transition location as a function of pitch angle at $r/R = 0.908$	179
5.14	Predicted pressure distribution at select radial stations at $t/T = 0.5$, baseline grid	180
5.15	Predicted transition location using AFT2019b for $Tu_\infty = 0.2\%$	180
5.16	Predicted transition location using AFT2019b for $Tu_\infty = 0.5\%$	181
5.17	Predicted transition location using AFT2019b for $Tu_\infty = 0.9\%$	181
5.18	Comparison of predicted transition location as a function of pitch cycle and FSTI at $r/R = 0.615$, fine grid	182
5.19	Comparison of predicted transition location as a function of pitch angle and FSTI at $r/R = 0.908$, fine grid	182
5.20	Predicted transition location using AFT2019b after stepping from $Tu_\infty = 0.9\%$ to $Tu_\infty = 0.09\%$	183
5.21	Transition locations as a function of pitch cycle and blade radius for the same thrust level	183
6.1	Near-body grids for the DSA-9A airfoil	184
6.2	Overlap regions for the DSA-9A near-body grids	184
6.3	DSA-9A off-body grids	184
6.4	ILES-predicted separation and reattachment for the DSA-9A airfoil	185
6.5	Instability leading to bubble burst on the DSA-9A airfoil	186
6.6	Surface pressure distributions predicted by ILES during a bursting event	187
6.7	Surface pressure distributions predicted by ILES for $0 \leq t/T \leq 0.3$	188
6.8	Surface pressure distributions predicted by ILES for $0.35 \leq t/T \leq 0.5$	189
6.9	Comparison of predicted surface-pressure distributions on the DSA-9A airfoil	190
6.10	Comparison of predicted skin-friction distributions on the DSA-9A airfoil	191
6.11	Comparison of predicted lift coefficient on the DSA-9A airfoil	192
6.12	Comparison of predicted drag coefficient on the DSA-9A airfoil	192

6.13	Comparison of predicted drag components on the DSA-9A airfoil	193
6.14	Comparison of predicted pitching-moment coefficient on the DSA-9A airfoil .	193

Nomenclature

A_0	initial wave amplitude
A	flux Jacobian in ξ
B	flux Jacobian in η
C	flux Jacobian in ζ
C_D	drag coefficient
C_d	sectional drag coefficient
C_f	skin-friction coefficient
C_L	lift coefficient
C_l	sectional lift coefficient
$C_{l\alpha}$	sectional lift coefficient slope
C_m	sectional pitching-moment coefficient
C_p	pressure coefficient
C_T	thrust coefficient
c	chord
c_{ref}	reference chord

d, y	wall distance
$\tilde{d}$	delayed, detached eddy simulation length scale
E	inviscid flux in ξ
E_v	viscous flux in ξ
e_0	total internal energy
F	inviscid flux in η
F_v	viscous flux in η
FM	rotor figure of merit
G	inviscid flux in ζ
G_v	viscous flux in ζ
H_{12}	boundary-layer shape factor
$H_{crossflow}$	crossflow shape factor
H_L	local boundary-layer shape factor
h_0	total enthalpy
i_t	turbulence index
$i_{t,LM}$	Langtry-Menter turbulence index
$i_{t,SA}$	Spalart-Allmaras turbulence index
$i_{t,SST}$	SST turbulence index
J	spatial coordinate Jacobian
k	turbulent kinetic energy, reduced frequency

k^+	non-dimensional turbulent kinetic energy
M	Mach number
M_{tip}	tip Mach number
N	amplification factor
N_{crit}	critical amplification factor
n	wall-normal direction
$\tilde{n}$	transported amplification factor
P	pressure
q	vector of conserved variables
R	radius
R_T	turbulent Reynolds number
Re	Reynolds number
Re_c	chord-based Reynolds number
Re_{SCF}	stationary-crossflow Reynolds number
Re_{tip}	tip-velocity Reynolds number
Re_V	strain Reynolds number
Re_v	vorticity Reynolds number
Re_x	streamwise Reynolds number
Re_y	wall-distance Reynolds number
Re_θ, Re_{δ_2}	momentum-thickness Reynolds number

$Re_{\theta,t}$	transitional momentum-thickness Reynolds number
Re_{ω}	turbulent dissipation Reynolds number
$\widetilde{Re}_{\theta,t}$	transported, transitional momentum-thickness Reynolds number
RHS	right-hand-side of the Navier-Stokes equations
r	radial location
r_d	delayed, detached eddy simulation distance function
S	strain rate magnitude
S_{ij}	strain rate tensor
s	streamwise direction
T	period
Tu	turbulence intensity
Tu_{∞}	freestream turbulence intensity
t	time
U	contravariant x-velocity, velocity magnitude
u	x-velocity
u_i	velocity vector
u_i'	fluctuating velocity vector
u_{∞}	freestream velocity
u_{τ}	friction velocity
u^+	non-dimensional velocity

$\overline{u'_i u'_j}$	Reynolds stress tensor
$\overline{u'v'}$	Reynolds shear stress
V	contravariant y-velocity
v	y-velocity
W	contravariant z-velocity
w	z-velocity
w_∞	rotor inflow velocity
x	streamwise distance
x_i	Cartesian coordinate vector
$x_{tr}, (x/c)_{tr}$	boundary-layer transition location
y^+	non-dimensional wall coordinate
α	angle of attack, wave number
γ	turbulent intermittency, ratio of specific heats
$\tilde{\gamma}$	natural logarithm of turbulent intermittency
$\Delta H_{crossflow}$	crossflow strength shifting function
Δt	timestep
Δx^+	non-dimensional x-coordinate, based on u_τ
Δz^+	non-dimensional z-coordinate, based on u_τ
δ, δ_{99}	boundary-layer thickness
δ_{ij}	Kronecker delta

ϵ	turbulent dissipation rate
θ, δ_2	momentum thickness
θ	instantaneous collective pitch
θ_0	average collective pitch
$\hat{\theta}$	collective pitch amplitude
κ	von Kármán constant
λ	rotor inflow ratio
$\lambda_\theta, \lambda_{\delta_2}$	Thwaites parameter
μ	dynamic viscosity
μ_t	turbulent eddy viscosity
μ_∞	freestream dynamic viscosity
ν	kinematic viscosity
ν_t	turbulent eddy viscosity
$\tilde{\nu}$	transported eddy viscosity, Spalart-Allmaras variable
ξ, η, ζ	computational-space coordinates
ρ	density
σ	rotor solidity, turbulence-variable Prandtl number
τ	turbulence intensity limiter
τ_{ij}	shear stress tensor
τ_w	wall shear stress

ϕ	phase angle, Orr-Sommerfeld basis function, generic scalar
ψ	streamline
Ω	vorticity magnitude, rotor rotation rate
Ω_{ij}	vorticity tensor
ω	specific turbulent dissipation rate
ω^+	non-dimensional specific turbulent dissipation rate

Chapter 1

Introduction

The performance and flight capabilities of aerospace vehicles is closely tied to the aerodynamic design of the vehicle. Improving aerodynamic efficiency has been at the forefront of aerospace design since the Wright brothers' first powered flight in 1903. In 1940, the design of the P-51 Mustang fighter sought to use elliptic wings, the optimal planform to minimize induced drag [1], along with a laminar flow airfoil designed by the NACA to reduce skin-friction drag. In 1952, Whitcomb [2] discovered that by keeping the cross-sectional area of an aircraft approximately constant along its length greatly decrease the wave drag at transonic speeds. Such means of drag reduction range from manufacturing methods like reducing the height of exposed rivets on the surface to first-principles methods designed to control the pressure gradient on the wing to achieve long runs of laminar flow.

One means of reducing drag comes with the control of the boundary layer state. Specifically, longer runs of laminar flow come with reduced skin-friction drag in comparison to if the boundary layer were turbulent. This reduction in drag reduces overall fuel burn, as the required thrust decreases with drag in steady, level flight (e.g., cruise). Understanding where and how the boundary layer transitions from laminar to turbulent is integral to harnessing the benefit of laminar flow. Computational fluid dynamics (CFD) methods offer a means of predicting this laminar-turbulent transition. By solving the Reynolds-averaged Navier-Stokes (RANS) equations in conjunction with a turbulence-closure model, high-fidelity predictions can be made for complex configurations in order to assess design

changes and potential performance improvements. Typically, the turbulence closure problem is solved by employing a fully turbulent model, one which does not account for laminar flow nor the transition to turbulence [3, 4]. These fully turbulent models are well posed for high-Reynolds number flows, such as for commercial aircraft, where the transition to turbulence occurs near the leading edge of the wing. More recently, turbulence closure models have been extended with laminar-turbulent transition in mind [5–7] such that additional flow physics may be modeled, as the inclusion of laminar flow may have a significant impact on drag [5] and therefore on fuel consumption. These laminar-turbulent transition models are in constant development for flow regimes ranging from low-Reynolds number ($10^3 - 10^5$), high-Reynolds number, transonic, and hypersonic flows.

Recent aircraft have been designed on the basis of maximizing laminar flow, such as the HondaJet [8], which utilizes a natural laminar flow (NLF) airfoil on the upper surface of the wing. At flight-relevant lift coefficients of $C_L \approx 0.2 - 0.3$ at Reynolds numbers of $11 - 17 \times 10^7$, the upper surface of the wing was able to achieve 50% natural, laminar flow before transition occurred, lowering overall profile drag. The MIT D8.5 “Double Bubble” was designed to achieve 60% laminar flow on the lower surface of the wing for drag reduction [9]. The airfoil was designed using the MSES code [10] with a mildly adverse pressure gradient on the upper surface and a favorable pressure gradient on the lower surface, up to the shock location, at transonic speeds in order to delay transition. More recent concepts such as the transonic, truss braced wing (TTBW) [11] designed jointly by Boeing and NASA utilize slotted, natural laminar flow (SNLF) airfoils [12]. The class of SNLF airfoils allows for longer runs of laminar flow on both surfaces of the airfoil, taking advantage of the dumping velocity [13] to enable favorable pressure gradients on the fore element of the airfoil. Similarly, since the aft element of SNLF airfoils experiences a “fresh” boundary layer, it too can support significant runs of laminar flow, save for portions of the upper surface where the fore-element flow is “dumped” onto the aft element [14]. The NASA Common Research Model (CRM) was redesigned as a natural laminar flow configuration [15] for the upper surface of the wing. Approximately 60% of the upper surface experienced laminar flow, and the collected data has provided an excellent, publicly available dataset for analyzing the predictive capabilities

of transition models [16–18]. Vehicles like the General Atomics MQ-9A Reaper, shown in Fig. 1.1 [19], that are designed for endurance flight often utilize laminar flow to reduce drag and extend the overall mission profile. These vehicles that are designed for endurance function similarly to gliders, featuring high-aspect ratio wings for maximum span efficiency and airfoils designed for significant runs of laminar flow at flight-relevant Reynolds numbers [20].

In the realm of rotary wing vehicles, experimental studies by Overmeyer and Martin [21] have shown that fully turbulent rotorcraft blades exhibit lower rotor-hover figure of merit (FM), the ratio of ideal rotor power required to actual rotor power required, in comparison to rotorcraft blades in which transition from laminar to turbulent occurs naturally, shown in Fig. 1.2. Overmeyer and Martin’s results illustrate experimentally that accounting for natural transition improves figure of merit by 4 to 7 counts depending on the thrust condition. To put that in perspective, one count of FM (0.01) correlates to approximately 170 pounds of additional payload during hover for a UH-60-type helicopter [22]. Therefore, at high thrust, the 5 count improvement in figure of merit corresponds to approximately 850 pounds of additional payload during hover. A computational study by Heister [23] illustrated that accounting for laminar flow reduces rotor power required by approximately 5% in comparison to a fully turbulent assumption in forward flight and 7% in hover, corroborating the experimental studies of Overmeyer and Martin.

The benefits of laminar flow in aircraft design are numerous, such as reducing drag and fuel burn, along with an increase in available payload. As a result, the aerodynamic-design process must be able to predict the transition to turbulence to properly navigate the design trade space. The means to accurately predict transition in a physics-based manner in the context of CFD has been identified as a key technology in NASA’s CFD 2030 Vision [24]. Currently, a range of RANS-CFD-compatible transition models exist of varying fidelity; however, these RANS-based models are local and correlation-based, rather than physics-based, and do not wholly model the physics of transition.

Physics-based transition models exist and are well documented but can be difficult to integrate with RANS CFD solvers. One popular physics-based model developed by van

Ingen [25] tracks the growth rate of instabilities in the boundary layer that are responsible for transition, the so-called e^N method. For a linear disturbance in a laminar boundary layer, once the amplitude of the disturbance has grown by a factor of e^N , where $N \approx 9$ for free-air conditions, then transition is considered to have occurred. Such stability methods have been used in conjunction with RANS CFD solvers, but must be externally coupled to the RANS solver. With this coupling method, the RANS CFD solver is responsible for solving the mean flowfield. Then, the mean flow is passed to the linear stability solver (or other external integral boundary layer solver) to predict transition. This, however, requires spatial marching and integration along streamlines in a non-local manner, which can be difficult to implement in a parallel computing architecture [26]. Once the external boundary layer solver predicts the transition front, then transition locations can be imposed as a boundary condition in the RANS solution. Then RANS predictions are made with the imposed transition front, fed into the external solver, and repeated until the transition front converges. Drawbacks from these methods come with the uncertainty for what values of N correlate to transition, difficulties in predicting separation-induced transition, and the need for a large number of grid points within the boundary layer of the RANS solution to facilitate accurate stability computations.

The class of RANS-compatible transition models offers many benefits over the aforementioned external-coupling method. Particularly, RANS transition models offer *in-situ* transition predictions, meaning that transition is predicted in conjunction with the mean flow, reducing computational expense comparatively. Additionally, the models are locally defined, requiring only local quantities to predict transition. As a result, some non-local quantity must be transported throughout the domain in the form of a partial differential equation (PDE) in order to predict transition [27]. These transition models can be calibrated for different flow regimes (wall-bounded shear flows, separated flows, etc.) and can be modified to account for a range of transition mechanisms [7]. Compared to external-coupling methods, the grid requirements are significantly lesser, both in the streamwise and wall-normal directions, reducing overall computational cost while maintain a high-fidelity flow solution. RANS-based transition modeling has gained significant attention in the last 15

years since the advent of the local-correlation transition model of Langtry and Menter [6]. Since, numerous transition models have been developed for RANS CFD that can be better applied to external aerodynamic flows [5, 28, 29] and turbomachinery [6], while other model variations seek to reduce computational cost for the same solution fidelity [7, 29–31]. These RANS transition models are in a state of continuous development, with a multitude of researchers tweaking the transition models to their particular flavor of aerodynamic problem [32, 33]. There are still outstanding shortcomings of these RANS models, such as poor prediction of transition induced by laminar separation and poor prediction of the maximum lift coefficient before stall for many airfoil cases [34].

There are means to more accurately predict transition without a turbulence closure model, as is done in large eddy simulation (LES) and direct numerical simulation (DNS). Both of these methods are of significantly greater computational cost than a RANS simulation [35], as they seek to resolve the turbulent scales of the problem rather than model them. Large eddy simulations resolve all but the smallest scales of turbulent structures and rely on either a subgrid stress model [36] to dissipate energy into scales smaller than the smallest grid, or careful selection of numerical scheme to dissipate energy into the smallest scales. The latter approach is that of implicit, large eddy simulation (ILES) which can accurately predict laminar-turbulent transition without the use of a subgrid stress model or turbulence model [37]. The ILES approach has risen to prominence within the last decade as computing power has increased; however, ILES in-practice is generally limited to airfoil simulations [38, 39] or portions of a full flowfield domain [40]. Large eddy simulations with a subgrid stress model are especially useful in free shear flows such as turbulent jets [41], as the smallest scales are less important for such a problem in comparison to wall-bounded flows, which have an inherent length scale (the wall distance). The wall-modeled large eddy simulation (WMLES) approach blends the high-fidelity nature of LES methods away from the wall with reduced-cost modeling near the wall and has been used for a full aircraft simulation [42] as a step toward NASA’s CFD 2030 Vision [24]. In the even-higher-fidelity regime lies DNS, which requires that all turbulent scales be directly resolved. This, in turn, necessitates an extremely fine grid capable of resolving scales from reference chord lengths

all the way down to the Kolmogorov scales [43]. The size of the grid scales with $Re^{13/7}$, rendering DNS as prohibitively expensive for all but the simplest, low-Reynolds number flow flows. This includes flat plate cases, free shear flows, and backward-facing steps [44] as model problems that are often used as a “truth” answer to compare against when investigating and calibrating LES models and RANS models. Often, LES and DNS require too large of computational overhead to justify their use for practical aerodynamic problems, highlighting the necessity to continually improve the capabilities of RANS-compatible modeling.

1.1 Research Goals

The present work seeks to investigate the behavior of various Reynolds stress closure methods, with particular attention paid to laminar-turbulent boundary-layer transition models. Both the Langtry-Menter transition model [6, 45] and the Amplification Factor Transport (AFT) transition modeling framework [5, 28, 46, 47] are analyzed using CFD. Their behavior and predictive capabilities are assessed for a suite of common CFD test cases. Additionally, large eddy simulations (LES) of a rotorcraft airfoil are performed in order to predict laminar-turbulent transition without a turbulence closure method. Predictions will be compared to results obtained using the AFT model.

1. Analyze the near-wall behavior of the Langtry-Menter transition model to illuminate shortcomings of the transition model. Investigate the transition behavior and develop a robust metric to identify the transition point.
2. Employ the AFT transition model to make predictions on a model-scale rotor in climbing conditions with a prescribed pitch cycle. Carry out a grid-resolution study to determine the effects of streamwise spacing on the solution. Additionally, perform a parameter sweep on freestream turbulence intensity to show its effect on predicted transition behavior.
3. Perform LES simulations of the DSA-9A rotorcraft airfoil at low-Reynolds number conditions and determine the predicted transition behavior. Then, perform RANS

simulations using the AFT model to compare to LES results and identify any limitations of the RANS-based transition models.

During the pursuit of these goals, additional attention is paid to uncertainties in transition predictions that are often ignored or unaddressed. Pertaining to the first goal, it is found that the Langtry-Menter model exhibits singular behavior in turbulent dissipation very near the wall. This has the effect of preventing convergence of the model partial differential equations, as the analytical solution diverges near the wall. This implies that the governing equations of the Langtry-Menter model are poorly posed near the wall and can never converge. Involving the second goal, it is shown that there is a sensitivity to the streamwise grid spacing as transition predictions show large differences between two grids with different spacing. Specifically, the grid designed following current best practices exhibits poorer solution quality relative to a finer grid, highlighting the need to tailor the design of the numerical experiment to the specific problem. Additionally, in the rotating reference frame, as is used for rotor simulations, there is a lack of a distinct reference velocity, the choice of which can greatly affect the level of the modeled freestream turbulence intensity. This, in turn, results in different transition predictions, without a clear answer as to which is closer to the “truth” answer. Finally, for the third goal, large discrepancies in transition predictions and performance predictions are found when comparing RANS with LES. These discrepancies were primarily a result of differences in the predicted length of a laminar separation bubble. The aforementioned uncertainties, and those similar, are often unaddressed in CFD simulations. The present work seeks to expound upon and address some of these uncertainties and highlight their effects on the predictive capabilities resulting from common CFD modeling approaches.

Chapter 2

Background

2.1 Physics of Boundary-layer Transition

The transition to turbulence was first documented by Reynolds [48], resulting in the now ubiquitous non-dimensional parameter of the Reynolds number to characterize the relative importance of inertial effects to viscous effects in fluid flow. Reynolds posited his similarity law, which states that flows with the same ratio of inertial effects to viscous effects are essentially similar to each other. He hypothesized that for internal pipe flows, transition to turbulence occurs once a critical threshold of the ratio of inertial effects to viscous effects is achieved (the now called “critical Reynolds number”), approximately 2300 when based on the hydraulic diameter [49]. Since Reynolds’ discovery, the works of Rayleigh in 1892 [50] added on to (and first mentioned) Reynolds’ similarity law, while later Prandtl [51] postulated the concept of a critical Reynolds number for transition and illustrated that drag is greatly dependent on the Reynolds number. Blasius [52] similarly expanded upon Reynolds’s theories, from whom one of the first usages of the term “Reynolds number” appears (in fact, Sommerfeld used that term first in 1908). Later, von Kármán highlighted the true importance to the Reynolds number for aerodynamic applications [53].

Reynolds’ assumption that transition is merely a function of the Reynolds number is unfortunately an incomplete description of the problem of laminar-turbulent transition. The breakdown of the laminar boundary layer and its eventual transition to turbulence is a

complex process and can take many paths to turbulence, but is still heavily dependent on Reynolds number. These transition mechanisms, or paths, can be modeled and predicted through the use of linear stability theory or local-correlation-based methods, or can be fully resolved using the Navier-Stokes equations in the form of LES or DNS [38, 54]. In order to understand the transition to turbulence it is prudent to understand the mechanisms by which transition occurs, how these mechanisms arise, and what can be done to control these mechanisms.

The process of transition begins with the problem of receptivity [55], which describes the entrainment of disturbances into the laminar boundary layer and the subsequent response to those disturbances. Reynolds first noted that, in pipe flow, transition occurs increasingly upstream as the strength of these disturbances increases [48]. In the freestream, this comes in the form of naturally occurring vortical, acoustic, and entropy disturbances, to which the boundary layer may be receptive. Additionally, surface roughness and surface vibration can also be included in the receptivity problem [55]. The reception of these disturbances in the boundary layer can be cast as a spectral problem [56] where both wavelength and frequency become important, forming the basis of the development and application of stability theory as a means to predict the influence of these disturbances.

2.1.1 Natural Transition

The class of instabilities attributed to the interactions of the laminar boundary layer with freestream acoustic disturbances are known as Tollmien-Schlichting instabilities, named after students of Prandtl who first determined the neutral-stability curve [57] and described the growth rate of the instabilities [56], respectively. The transition to turbulence due to these instabilities is referred to as “natural transition”, in which low-amplitude, linear, freestream disturbances incite the boundary layer to turbulence at some point downstream of the leading edge of a solid surface, in the absence of some artificially forced instability. Tollmien-Schlichting waves are the primary instability for two-dimensional flows (e.g. flat plates and airfoils) and their amplitude and frequency can be correlated to a function of the freestream turbulence intensity [58]. The turbulence intensity is defined as the ratio

of the root-mean-square of turbulent fluctuations to some reference velocity, typically the freestream velocity. For freestream turbulence intensities less than approximately 1% for a two-dimensional flow, transition can be best described by the growth of Tollmien-Schlichting instabilities [59]. The growth of these instabilities can be described by linear-stability theory and the Orr-Sommerfeld equation [60–62], which will be discussed in subsequent sections.

The process of natural transition is depicted visually in Fig. 2.1. Initially, the Tollmien-Schlichting waves are two-dimensional upon reception into the boundary layer. Once the amplitude of the root-mean-square of velocity fluctuations in the boundary layer reaches approximately 1% the reference velocity, these two-dimensional waves break down into a three-dimensional wave, inducing a spanwise vorticity component along the surface [63]. This process occurs over approximately five Tollmien-Schlichting wavelengths. The ensuing subsequent breakdown of the three-dimensional waves occurs over approximately one Tollmien-Schlichting wavelengths [63]. This three-dimensional breakdown results in the formation of locally turbulent regions, called turbulent spots [64], that eventually incite the full breakdown to turbulence as the turbulent spots spread out downstream. In fact, the rate of turbulent spot formation and the rate of spreading of these spots was used by Narasimha et al. [65] as the basis of one of the first algebraic transition models. Fortunately, Tollmien-Schlichting instabilities can be quelled by careful control of the pressure gradient, as these instabilities are damped by favorable pressure gradients [56]. This is the central idea behind, and the crux of, NLF airfoil design [12, 66, 67].

2.1.2 Bypass Transition

Bypass transition is the most prevalent transition mechanism when the freestream turbulence intensity is sufficiently large that the linear growth of Tollmien-Schlichting waves in the boundary layer is bypassed by the high-amplitude freestream disturbances [68]. These disturbances are sufficiently large that they immediately break down to secondary instabilities upon reception in the boundary layer, quickly forming turbulent spots. These disturbances manifest as low-frequency, high-amplitude oscillations in streamwise velocity, which grow linearly with downstream distance, whereas Tollmien-Schlichting instabilities

grow exponentially with downstream distance. The bypass mechanism is especially prevalent in turbomachinery flows [6], as the turbine blades operate in the turbulent wake of preceding blades in the turbine.

2.1.3 Crossflow Transition

In flows where the surface is oriented obliquely to the freestream (e.g. swept wings), crossflow instabilities can form and incite transition [69]. For such flows, a surface streamline has both a chordwise and spanwise velocity component, permitting instabilities in both directions. Similar to Tollmien-Schlichting waves, crossflow instabilities can be described by linear stability theory and are amplified exponentially with downstream distance [56] at characteristic wavelengths and frequencies. Typically, crossflow instabilities arise in the form of stationary (zero-frequency) vortices on the surface, that lead to the formation of turbulent spots [70]. As such, the frequency of the crossflow vortices is much lesser than that of Tollmien-Schlichting instabilities. Practically, the stationary crossflow instability is amplified in the presence of favorable pressure gradients and damped in the presence of adverse pressure gradients, making crossflow transition the more common transition mechanism on the lower surface of swept-wing flows [71]. For many swept-wing flows, both stationary-crossflow transition and Tollmien-Schlichting transition play a role, requiring that both mechanisms be considered in order to accurately described transition [72].

2.1.4 Separation-induced Transition

At low Reynolds numbers, laminar-turbulent transition commonly occurs by means of laminar separation bubbles (LSBs), as opposed to natural or bypass transition [49]. Due to the lesser kinetic energy present in the laminar boundary layer, the flow tends to separate from the surface in the presence of an adverse pressure gradient [73], which typically occurs near the trailing edge of the aerodynamic body at low angles of attack [74]. This separated shear layer entrains high-momentum fluid from the outer flow and can reattach downstream as a turbulent boundary layer. A schematic of laminar separation and

subsequent reattachment is given in Fig. 2.2. This separation and transition mechanism also occurs for airfoils at moderate-to-high-Reynolds numbers, on the order of $10^4 - 10^6$ [20, 67, 75], where the length of the LSB is shorter than that of a similar low-Reynolds number flow. At higher Reynolds numbers, the separated shear layer is able to more quickly entrain more energetic flow from outside the bubble, leading to a shorter overall bubble length [76]. At moderate-to-high angles of attack, the turbulent boundary layer can separate again (after the flow had already reattached) near the trailing edge of the surface, increasing pressure drag and decreasing lift [77]. In low-Reynolds number flows, the LSB formed about the leading edge can “burst” and rapidly increase in size as angle of attack increases [74]. In this condition, the airfoil still generates lift up to some angle of attack at which the separation bubble becomes sufficiently long (approximately 15 – 40% chord per Winslow et al. [73]) and the airfoil stalls. Near this condition, the surface flow on the airfoil separates and reattaches, and the separates again and continues in this hysteresis loop with respect to time; this is often referred to as dynamic stall [78].

Another separation transition mechanism is that of shock-induced separation, in which a laminar boundary layer encounters a shock wave, separates from the surface, and reattaches as turbulent downstream of the shock [79]. Shock-induced separation is highly complex and is relatively common in hypersonic flows. When the laminar boundary layer separates from the surface through an impinging shock, a separation bubble forms, which later encounters a recompression shock and reattaches to the surface as a turbulent boundary layer.

2.1.5 Other Transition Mechanisms

In curved channel flows, Dean vortices are the convalescence of a streamwise vortex instability over a convex region of the channel [80]. These Dean vortices twist as they move downstream, inciting secondary instabilities and the formation of turbulent spots. For boundary layers over a convex surface, the Görtler instability becomes important [81]. Similar to Dean vortices, Görtler vortices are counter-rotating vortices that are oriented in the streamwise direction. If the boundary layer thickness is on the same order of magnitude as the radius of curvature of the concave region, the boundary layer experiences a pressure gradient

in the wall-normal direction, destabilizing the boundary layer and causing breakdown to turbulence. Transition due to the Görtler instability can occur, for example, in supersonic nozzles where the wall is locally concave [55] and through the slot and cove of multi-element airfoils [12]. Calculation of these instabilities is an eigenvalue problem, similar to that of Tollmien-Schlichting and crossflow instabilities [56].

Roughness elements on the surface can also accelerate the transition process as well [56]. Large amplitudes of surface roughness can incite larger amplitudes of Tollmien-Schlichting instabilities and crossflow instabilities [69], with a particularly strong effect on the formation of stationary crossflow vortices. In a sense, roughness can function similarly to bypass transition by reducing the critical Reynolds number for which transition occurs, necessitating that aerodynamic surfaces be as smooth as possible with few surface imperfections.

2.2 Overview of Transition-Prediction Methods

2.2.1 Linear Stability Methods

Linear stability theory (LST) relies on tracking the amplitude and frequency of instabilities in a flowfield using a reduced-order formulation of the Navier-Stokes equations. LST methods do not account for all physics relevant to transition and flow instability, but nonetheless serve as a strong basis for such predictions of instabilities. The field of stability theory for fluid dynamics began in the early 1900s, when Orr [60, 61] and Sommerfeld [62] independently arrived to the same equation when linearizing the two-dimensional Navier-Stokes equations about a base flow and a perturbation, for which the linearization decomposes the velocity as,

$$u_i = \overline{u_i} + u_i' \quad (2.1)$$

where $\overline{u_i}$ represents the base-flow (average) velocity and u_i' represents a velocity perturbation about the base flow. The same decomposition is done for pressure as well. By taking the curl of the linearized form of Navier-Stokes, the equations collapse down into a streamline-velocity formulation such that the growth of instabilities can be tracked along a streamline [55].

Additionally, the instabilities are assumed to be solutions to the wave equation, such that the perturbation streamlines can be represented as, not considering spanwise instabilities,

$$\psi' = \phi(y)A_0e^{i(\alpha x - \omega t)} \quad (2.2)$$

for which $\phi(y)$ is the to-be-determined solution to the Orr-Sommerfeld equation, A_0 is some unknown coefficient, α is the streamwise wave number, ω is the frequency of the wave, and x and y are streamwise and wall-normal coordinates, respectively. Resultingly, the definition of u' and v' on a streamline can be invoked to give,

$$u' = \frac{d\phi}{dy}A_0e^{i(\alpha x - \omega t)} \quad (2.3)$$

$$v' = -i\alpha\phi(y)A_0e^{i(\alpha x - \omega t)} \quad (2.4)$$

Then, the full form of the Orr-Sommerfeld equation is,

$$\left(\bar{u} - \frac{\omega}{\alpha}\right)\left(\frac{d^2\phi}{dy^2} - \alpha^2\phi\right) - \frac{d^2\bar{u}}{dy^2}\phi = \frac{-i\nu}{\alpha}\left(\frac{d^4\phi}{dy^4} - 2\alpha^2\frac{d^2\phi}{dy^2} + \alpha^4\phi\right) \quad (2.5)$$

which is a fourth-order, homogeneous, ordinary differential equation. The wavenumber α and frequency ω are eigenvalues that must be found such that the homogeneous boundary conditions are satisfied [55]. Note that the quantity ω/α can be thought of as a complex wave speed and accounts for the speed of information propagation in the complex plane.

The solution to the Orr-Sommerfeld equation determines whether a given instability will grow or decay, based on its wavenumber and the local displacement-thickness Reynolds number. As such, solutions are typically plotted in a wavenumber versus displacement-thickness Reynolds number space, with curves of neutral stability that correspond to various pressure gradients or growth rates. Depicted in Fig. 2.3 is the solution to the Orr-Sommerfeld equation for a Blasius boundary layer [82], where the contour lines correspond to constant instability growth rates. The shaded region represents the portion of phase space for which unstable modes may exist where the instability growth rates are positive. The leftmost portion of this shaded region corresponds to the minimum Reynolds number for which

instabilities may grow, which is often referred to as the critical Reynolds number [55], below which instabilities are only damped. For Blasius flow, the critical Reynolds number is 519.4 [55].

The umbrella term of “linear stability theory” encompasses solutions to the Orr-Sommerfeld equation for various flow instabilities. In the formulation depicted in Eq. 2.5, the stability of Tollmien-Schlichting waves can be predicted [55]. By extending Orr-Sommerfeld to three-dimensions, the stability of crossflow waves can be predicted [83]. Orr-Sommerfeld can be similarly extended to other linear disturbances, such as Görtler vortices [55].

Another stability-based method comes from the parabolized stability equations (PSE), first introduced for Floryan and Saric [84]. This set of equations allows for prediction of linear and nonlinear instabilities in a flow that is weakly nonparallel [85]. PSE is designed such that the growth of an instability on a streamline is tracked, which was initially specified at some streamwise-upstream location. As such, PSE assumes that a given disturbance varies slowly in the streamwise direction and can be tracked with a single wavenumber [55]. The form of the PSE equations is parabolic, allowing a numerical procedure based on downstream marching, which is amenable for solving the parabolized stability equations, allowing for rapid solutions. For PSE solutions, the velocity is decomposed into an amplitude portion and exponential portion as,

$$\hat{u}_i = \tilde{u}_i \exp\left(i \int_{x_0}^x \alpha(\xi) d\xi - i\omega t\right) \quad (2.6)$$

$$u'_i = \sum_{\omega} \hat{u}_i e^{-i\omega t} \quad (2.7)$$

This decomposition yields a partial differential equation that is capable of predicting the growth of instabilities, both linear and nonlinear, for flows with small curvature over a range of wavenumbers and Reynolds numbers. The PSE method, similar to Orr-Sommerfeld, can be extended to three-dimensional flows to account for crossflow and supersonic flows to account for second-mode instabilities, like the Mack mode [86].

Now that flow instabilities can be predicted via the Orr-Sommerfeld, it remains that transition must be predicted in some manner based on this family of computed instabilities.

One method of predicting transition is that developed by Smith and Gamberoni [87] and van Ingen [25], the so-called e^N method. This method leverages linear stability theory to predict transition via Tollmien-Schlichting waves. The N -factor tracks the exponential growth of instabilities on a streamline as,

$$N(s) = \ln\left(\frac{A(s)}{A_0(s)}\right) = \int_{s_0}^s -\alpha_i ds \quad (2.8)$$

Integration begins when a given instability at wavenumber α_i crosses the neutral stability point (subscript 0) from the Orr-Sommerfeld solution. Transition is considered to have occurred once the amplification of the Tollmien-Schlichting wave has increased by a factor of $e^{N_{crit}}$ from its crossing of the neutral stability point. Both Smith and Gamberoni [87] and van Ingen [25] suggest that a critical N -factor, N_{crit} , in the range of 8 to 11 can accurately predict transition in low-freestream-disturbance environments, where the lower value corresponds to environments with higher freestream turbulence intensity.

The e^N method is very accurate for flows where Tollmien-Schlichting-type transition is expected, which is common for many two-dimensional flows such as flat plates and airfoils. This method is unfortunately difficult to pair with a CFD solver, as it requires non-local integration across the boundary layer and streamline [27]. As mentioned in Chapter 1, some RANS solvers couple an e^N solver externally to the CFD solver for boundary layer predictions.

Since its inception, the e^N method has been extended for suction and blowing [88], along with extensions allowing predictions of laminar separation bubbles [89]. Similarly, the e^N method has also been simplified, such as that done by Drela and Giles [90].

2.2.2 Local-correlation Methods

One attractive means of predicting transition is that of the local correlation methods. These methods sacrifice knowledge of the upstream flow history that is required in LST or PSE methods in favor of using local quantities to evaluate the state of the boundary layer. The earliest of these local correlation methods were algebraic, meaning that no additional

partial differential equations (PDE) need be solved to evaluate transition. An immediate drawback, however, is that the local nature of the correlations renders them unable to account for upstream flow history. Nevertheless, these methods provide an accurate first-pass on transition predictions.

One of the simplest methods is that proposed by Michel [91], which approximates the momentum-thickness Reynolds number at transition, $Re_{\delta_2,t}$, as a function of the streamwise Reynolds number,

$$Re_{\delta_2,t} = 2.9Re_{x,t}^{0.4} \quad (2.9)$$

This requires that the momentum-thickness Reynolds number be computed from some other method, such as Thwaites' method [92]. Transition is considered to have occurred once the computed Re_{δ_2} exceeds the value predicted by Eq. 2.9. This method tends to work well for flows in the presence of adverse pressure gradients.

The method of Granville [93] requires knowledge of the local pressure gradient and the streamwise location for which instabilities begin to grow. Over the streamwise distance from instability onset to transition, a “mean” Thwaites' parameter is computed as,

$$\overline{\lambda_{\delta_2}} = \frac{1}{x_t - x_0} \int_{x_0}^{x_t} \lambda_{\delta_2}(x) dx \quad (2.10)$$

where the actual Thwaites' parameter is [92],

$$\lambda_{\delta_2}(x) = \frac{\delta_2(x)^2}{\nu} \frac{dU}{dx} \quad (2.11)$$

Then, the transitional momentum-thickness Reynolds number is computed as,

$$Re_{\delta_2,t} = Re_{\delta_2}(x_0) + 450 + 400\exp(60\overline{\lambda_{\delta_2}}) \quad (2.12)$$

For mean, adverse pressure gradients of $\overline{\lambda_{\delta_2}} \leq -0.1$, the exponential term is small and transition occurs very close to the location of instability onset. Conversely, for mean, favorable pressure gradients, the exponential term is large and transition occurs much further downstream from the onset of instabilities.

Wazzan et al. [94] developed a local correlation based on the change of Re_{δ_2} with respect to the shape factor H_{12} determined from e^N theory for $N = 9$. By computing H_{12} from some boundary-layer method such as Falkner-Skan [95] or Thwaites [92], the transitional Reynolds number can be approximated as,

$$\log_{10}(Re_{x,t}) = -40.4557 + 64.8066H_{12} - 26.7538H_{12}^2 + 3.3819H_{12}^3 \quad (2.13)$$

It is of note that the Michel criterion and Granville criterion were calibrated against Schubauer and Skramstad's data [96], whereas that of Wazzan et al. was calibrated against an experiment performed by Wells [97] which eliminated acoustic disturbances apparent in Schubauer and Skramstad's experiments. As a result, the criterion of Wazzan et al. typically predicts a transitional Reynolds number much larger than that of both Michel and Granville.

Abu-Ghannam and Shaw (AGS) [98] developed a correlation as a function of the freestream turbulence intensity and local pressure gradient parameter. The AGS relation is commonly used for high-turbulence-intensity flows, such as turbomachinery flows, and is generally accurate for predictions of bypass transition. The AGS relation is as follows,

$$Re_{\delta_2,t} = 163 + \left(F(\lambda_{\delta_2}) - \frac{F(\lambda_{\delta_2})}{6.91} Tu_{\infty} \right) \quad (2.14)$$

$$F(\lambda_{\delta_2}) = \begin{cases} 6.91 + 12.75\lambda_{\delta_2} + 63.64\lambda_{\delta_2}^2 & \text{if } \lambda_{\delta_2} \leq 0 \\ 6.91 + 2.48\lambda_{\delta_2} - 12.27\lambda_{\delta_2}^2 & \text{if } \lambda_{\delta_2} > 0 \end{cases} \quad (2.15)$$

These correlation-based methods are accurate for the flow regimes against which they were calibrated; however, these methods are not generally applicable to all flow regimes, especially those which have complex interactions and are largely three-dimensional. Additionally, the functional dependence on the Thwaites parameter and momentum-thickness Reynolds number makes them difficult to couple with CFD methods, as they too would require non-local integration techniques to compute the momentum thickness.

2.2.3 PDE Methods

The class of PDE-based transition-prediction methods is attractive for RANS CFD applications, as most CFD solvers already solve the Navier-Stokes equations in the form of a PDE. These methods tend to be locally correlated, meaning that they are amenable to parallelization, as there is not a need for communication across portions of the CFD domain. One of the first, PDE-based transition models is that developed by Walters and Leylek [99]. This model takes the form of an additional equation that is coupled to a $k - \epsilon$ turbulence model. The transport equation for turbulent kinetic energy k is bifurcated into a turbulent portion and a laminar portion, each of which gets its own PDE. The laminar kinetic energy mechanism models the wall-normal instabilities in the boundary layer “splating” against the wall and propagating into the streamwise direction, which is used to trigger transition to turbulence. This model is capable of predicting both natural and bypass transition and has shown good agreement with experiments for turbine flows [99].

Debatably, the most popular transition model is the local-correlation transition model developed by Langtry and Menter [6], referred to herein as the “Langtry-Menter model” or “LM2009”. The Langtry-Menter model was developed to couple with the popular Shear Stress Transport (SST) turbulence model of Menter [3] and was designed to work “out of the box”, requiring no calibration to the model equations and with minimal modification to the SST model. The correlations employed are largely algebraic and can account for natural, bypass, and separation-induced transition mechanisms [6], making the model easy to incorporate with a given CFD solver. The model seeks to transport two scalar quantities in addition to those already transported by the base SST model, the local turbulent intermittency and the transitional, momentum-thickness Reynolds number. In the context of CFD, the intermittency serves as an indicator of laminar or turbulent flow, where a value of 0 indicates laminar flow and a value of 1 indicating turbulent flow, with intermediate values representing points along the transition length. The transitional, momentum-thickness Reynolds number serves as a similar transition-triggering criterion as that done by Michel [91], Granville [93], and Abu-Ghannam and Shaw [98]. The Langtry-Menter model takes a

form similar to that of all CFD-compatible transport equations,

$$\frac{\partial(\rho\gamma)}{\partial t} + \frac{\partial(\rho u_j \gamma)}{\partial x_j} = P_\gamma - D_\gamma + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_f} \right) \frac{\partial \gamma}{\partial x_j} \right] \quad (2.16)$$

$$\frac{\partial(\rho \widetilde{Re}_{\theta,t})}{\partial t} + \frac{\partial(\rho u_j \widetilde{Re}_{\theta,t})}{\partial x_j} = P_{\widetilde{Re}_{\theta,t}} - D_{\widetilde{Re}_{\theta,t}} \quad (2.17)$$

$$+ \frac{\partial}{\partial x_j} \left[\sigma_{\theta,t} \left(\mu + \mu_t \right) \frac{\partial \widetilde{Re}_{\theta,t}}{\partial x_j} \right] \quad (2.18)$$

where P and D are the production terms and destruction terms for each equation.

The Langtry-Menter model has seen significant success in predicting transition accurately for a wide range of flows [6] and has been extended to account for crossflow instabilities as well [45]. It is of note that the set of γ and $\widetilde{Re}_{\theta,t}$ transport equations are typically referred to as the “ $\gamma - Re_\theta$ model”, whereas the “Langtry-Menter” model strictly refers to the coupling of the $\gamma - Re_\theta$ model to SST.

One of the primary drawbacks of the Langtry-Menter model is its lack of Galilean invariance, as the correlations of the model explicitly use the velocity vector [6]. This introduces additional uncertainties in flows with rotating reference frames, such as rotary-wing flows. Jain [100] recently investigated the effects of Galilean invariance for a rotorcraft airfoil by considering both a case of a stationary airfoil and an airfoil translating into a stagnant freestream, as shown in Fig. 2.4. This lack of Galilean-invariance illustrates that the transition-prediction criteria can vary significantly for different flow regimes. Jain [100] proposed that since the transition correlations were developed around stationary walls with a moving freestream, then the velocity of the freestream relative to the wall should be used as the velocity scale (i.e., $u_i = u_{i,\infty} - u_{i,grid}$). This modification does indeed render the model as Galilean-invariant.

In 2015, Menter et al. [7] recast the Langtry-Menter model as a one-equation model that eliminated the $\widetilde{Re}_{\theta,t}$ equation, known as the γ -model. With some additional modifications, the model was rendered as Galilean-invariant by replacing the explicit use of the velocity vector. Where the Langtry-Menter model uses U , the γ -model uses ωd_{wall} , and where

Langtry-Menter uses dU/ds , the γ -model uses the wall-normal component of the wall-normal velocity gradient, dv/dn . The formulation of the intermittency equation is largely unchanged from Langtry-Menter, with exception to correlations that include $\widetilde{Re}_{\theta,t}$, which have been reduced to either constants or functions of the local turbulence intensity or local pressure gradient parameter. The transition model has seen limited use, but has been used more exploritatively for rotorcraft simulations, such as that done by Lee and Baeder [101]. Due to the modular framework of the γ -model, it has also been successfully coupled to the Spalart-Allmaras turbulence model [4] by both Nichols [30] and Lee [31].

In efforts to reduce the computational expense of the Langtry-Menter model, Medida and Baeder [29] coupled the $\gamma - Re_\theta$ model to the one-equation Spalart-Allmaras model. The so-called Medida-Baeder model overcame many shortcomings of the Langtry-Menter model by developing improved correlations for $\widetilde{Re}_{\theta,t}$ and by ensuring that the turbulent intermittency convects all the way to the wall (this phenomenon will be discussed in-depth in Chapter 4). This modification, however, requires non-local operations across the height of the boundary layer in order to improve the behavior of intermittency in the turbulent boundary layer [29]. While this modification remains cheap and simple to implement for structured grids, its application to unstructured grids is enigmatic and expensive. The model still lacks Galilean-invariance, however, again due to the explicit use of the velocity vector. Additionally, the Medida-Baeder model can account for crossflow instabilities based on the crossflow criteria of Seyfert and Krumbein [26]. Similar to the γ -model and the Langtry-Menter model, the Medida-Baeder model is equally unable to properly account for upstream flow history.

Another PDE-based transition model is the two-equation, amplification factor transport (AFT) transition modeling framework of Coder [28] and Coder and Maughmer [5] that couples with the widely implemented Spalart-Allmaras one-equation turbulence model [4]. The AFT model is based on the approximate envelope e^N method developed by Drela and Giles [90], which itself is based on linear stability theory, meaning the AFT model is more physically founded in comparison to other correlation-based models. A single transport equation is able to account for upstream flow history by approximating the growth of the N factor of the linearly amplified instabilities. Once this value exceeds a certain critical value

N_{crit} , transition is deemed to have occurred and the turbulence model is activated via the second transport equation for turbulent intermittency. This model is attractive for CFD methods as it accounts for upstream flow history, relies on locally computed quantities, and can be integrated with finite-element CFD methods [18].

2.2.4 Large Eddy Simulation

A separate way to predict transition can be achieved by departing from a RANS framework to an LES framework [102], which seeks to resolve the large relevant scales rather than modeling them. In essence, the LES framework functions as a filter to the Navier-Stokes equations, filtering out eddies that are smaller than the smallest grid spacing [36]. Consequently, the smallest-scale turbulent eddies are modeled using subgrid-scale models (e.g. Smagorinsky [36] and Lilly [103]) while larger-scale turbulent eddies are resolved by the grid, which is much finer than would be used for a RANS computation. The popular Smagorinsky model is based on Prandtl’s mixing length concept [104] by approximating the subgrid eddy viscosity as,

$$\mu_t = \rho L_s^2 S \quad (2.19)$$

where L_s is the subgrid-scale mixing length and S is the strain rate magnitude. The mixing length is generally taken as $L_s = C_s \Delta$, with κ being von Kármán’s constant, C_s the Smagorinsky constant, and Δ a grid-based length scale. The length scale Δ is usually implemented as either the length of the longest side of the grid cell [36] or the cube root of the cell volume [105]. For example, in ANSYS Fluent [43], $L_s = \min(\kappa d, C_s \Delta)$. This subgrid-scale eddy viscosity is similarly applied to hybrid RANS/LES methods, like detached eddy simulations (DES) [106] and delayed, detached eddy simulations (DDES) [107]. The value of C_s appearing in the eddy viscosity is enigmatic and typically varies based on the implementation. Lilly [103] derived a value of $C_s = 0.173$ (cf. Pope [43]) based on homogeneous, isotropic turbulence; however, this value is often found to be too dissipative for shear flows. The choice of the Smagorinsky constant, C_s , is still debated in the LES community [108]. There are other available subgrid scale models that seek to improve on

Smagorinsky’s eddy viscosity. For example, the WALE model [109] modifies the velocity scale used in the strain rate tensor, while the Van Driest Damping Dynamic Smagorinsky model [110] modifies the length scale used to compute eddy viscosity. Other subgrid-scale models exist, such as the renormalization group (RNG) subgrid-scale model [111, 112], which seeks to blend the Smagorinsky model with a less-dissipative subgrid-scale model for low-Reynolds number flows. Specifically, in flows where $\mu_t \gg \mu$ the RNG subgrid-scale model returns to the Smagorinsky model with a different value of C_s , whereas in low-Reynolds-number regions of the flow, $\mu_t \rightarrow \mu$ in hopes to better treat near-wall regions, where Smagorinsky is known to predict poorly.

As opposed to traditional LES, implicit large-eddy simulations can be performed using a RANS code by disabling the turbulence model and resolving the large turbulence scales without the use of a subgrid-scale model. This method uses the numerical dissipation inherent to the computational scheme to function implicitly as a subgrid-scale model, lending to the method being called Implicit LES (ILES) [37]. This ILES approach is greatly dependent on the sizing and spacing of the numerical grid in order to label the results as “ILES predictions”. If the grids are too coarse, then there will be significant grid-induced dissipation that will compound that of the numerical scheme selected, which may result in errant predictions [37]. The wall-normal, streamwise, and spanwise spacing must be sufficiently small to capture the largest eddies, while not being so small as to errantly perform direct numerical simulation. When running simulations in ILES mode, which is time-accurate by definition, it is often prudent to initialize the flowfield from a steady-state RANS solution using a turbulence model, as to aid numerical stability [40, 113].

One drawback of LES methods is the treatment of farfield boundary conditions. In RANS turbulence models, turbulent quantities such as eddy-viscosity ratio and turbulent kinetic energy are set at the farfield boundary; however, in the LES framework, such quantities cannot be imposed at the farfield [54]. In many cases, no turbulence quantities are specified at the farfield, or the farfield is prescribed as having some type of “synthetic” turbulence [114]. In the case of unspecified turbulence quantities, predictions may be more difficult to compare with linear-stability predictions, as the freestream turbulence intensity sets the

critical N -factor that determines transition [38, 39]. On the other hand, the method of synthetic turbulence generation developed by Shur et al. [114] prescribes a randomly oriented distribution of velocity fluctuations for various wavenumbers at the farfield in order to recover the target Reynolds stresses at the inlet. Another method is that of recycling/rescaling, which requires iterations over many simulations [115]. In this method, a simulation is run as “clean” with no turbulence variables imposed at the farfield. Then, after some simulation time has passed, a slice of the flowfield is extracted from some region in the simulation, and is rescaled and reimposed at the inlet of the subsequent simulation. While this approach avoids the implementation of a synthetic turbulence generation method and does not require as long of an inlet domain, it can quickly become computationally expensive due to the number of simulation restarts required [115].

2.2.5 Hybrid RANS/LES Methods

In between the fidelity of RANS and LES lies the class of hybrid RANS/LES methods. These methods seek to blend the reduced expense of RANS turbulence models with the scale-resolving capabilities of LES [106]. In cases of flow separation, where RANS models offer poor predictions, hybrid RANS/LES methods leverage the improved separation predictions incumbent with LES. Spalart introduced the concept of detached eddy simulation [106], in which the boundary layer is treated and computed using a RANS turbulence model (RANS branch) and regions outside the boundary layer are resolved using a modification to the turbulence model that mirrors an LES method (LES branch) [106]. Specifically, outside the boundary layer, the turbulent length scale is used as the grid-cell size Δ rather than the wall distance, reducing the dissipation of the turbulence model and allowing those scales far from the wall to be resolved rather than modeled. This, in turn, mimics a subgrid-scale-like model for detached flows, such as flow separation or free shear flows.

In some cases, as in DES, fine grids can inadvertently cause a switch to the LES branch in attached boundary layers, resulting in modeled-stress depletion [34]. To overcome this, the method of delayed, detached eddy simulation (DDES) was developed [107]. The DDES framework shields the attached boundary layer by using an internal length scale to

approximate where in the boundary layer that a grid point lies. This formulation results in improved predictions in flows with attached boundary layers, but is known to be sensitive to its internal constants [107]. Additionally, the use of an internal length scale to approximate an external length scale (i.e., the boundary layer height) is poorly posed and remains an issue for many RANS-based CFD methods [107, 116, 117]. Care must be taken when employing hybrid RANS/LES simulations for transitional flows, as they cannot distinguish between laminar boundary layer and separated wakes [34]. For intermittency-based transition models, this fix is straightforward and can be implemented by including the turbulent intermittency in the DDES sensor function.

2.3 Contemporary Use of Transition Models

The use of transition modeling for CFD has become more commonplace in the last several decades. When possible, physics-based prediction methods are preferred; however, in many cases, flows are too complex to employ methods like LST or PSE. To overcome this, more practical transition-prediction methods are employed and have seen significant success.

2.3.1 Inviscid/Viscous Segregated Flow Solvers

The class of inviscid/viscous segregated flow solvers solves the inviscid flowfield separately from the viscous flowfield. This allows for rapid computation of the inviscid flowfield which can then be passed on to a boundary-layer solver to perform viscous computations. Then, the two flowfields are coupled back together and solved in-tandem until iterative convergence is achieved.

One of the most popular, publicly available, segregated solvers with a transition-prediction tool is that of XFOIL [118]. The XFOIL code uses an inviscid panel method to compute the flow about an airfoil. Transition predictions are made separately by approximating the envelope of instability amplification from e^N theory. First, the code computes integral boundary layer parameters such as δ_2 and H_{12} , which are then used to approximate the streamwise growth of the amplification factor N [90] in order to determine

the transition location. Then, the predicted viscous flowfield is imposed back on to the inviscid flowfield and the process continues until residual convergence is satisfied. XFOIL also offers inverse design capabilities, allowing the user to modify the shape of the airfoil based on user-specified design requirements.

XFOIL has been used both for design and validation studies. A primary advantage of XFOIL is that it employs accurate, physics-based predictions, with a turnaround time on the order of a few seconds. Metkowsky et al. [119] used XFOIL as a validation tool for flow about the PSU 94-097 airfoil at low Reynolds numbers. The drag polar and lift-curve slope predictions from XFOIL were compared to experiment, as shown in Fig. 2.5, showing excellent agreement with experiments. The XFOIL predictions slightly overpredict the maximum lift coefficient for the older version of XFOIL, and greatly overpredict the maximum lift coefficient for the updated version. The XFOIL predictions similarly agreed well with experiments for the surface pressure distribution over a range of Reynolds numbers, even in the presence of a laminar separation bubble. Coder and Maughmer [120] investigated XFOIL’s predictive capabilities for a suite of airfoils, citing that XFOIL predicts performance well in low-drag regions of the drag polar, but shows larger deviations from experiments at higher angles of attack and in higher-drag regions. Kinzel et al. [121] investigated the predictive performance over an airfoil with a Gurney flap, illustrating that XFOIL’s performance predictions agreed very well with experiments, further demonstrating the power of the tool.

Whereas XFOIL can only make predictions about a single-element airfoil, its sister code MSES [10] was developed for use with multi-element airfoils. Rather than employing a panel method, MSES solves the Euler equations coupled to an approximate envelope e^N method [90]. MSES has been widely used for high-lift, multi-element airfoil design and validation. Coder and Somers [12] leveraged MSES to predict the performance of an SNLF airfoil in a high-lift configuration. Weiss et al. [122] employed MSES to make transition predictions on sections of a rotor blade, finding that MSES predictions agreed well with experimental data even though MSES cannot take into account the three-dimensionality of a rotor blade.

Similarly, Gardner et al. [123] used MSES to make transition predictions for a rotor in forward flight, again finding good agreement with experimental data.

2.3.2 RANS-based Transition Prediction

While the methods used in XFOIL and MSES provide accurate predictions for two-dimensional (and some quasi-three-dimensional) flowfields, other methods are needed to predict transition and the viscous flowfield for complex configurations. For these complex flows, RANS-based transition models are often employed, as they can automatically predict laminar-turbulent transition with minimal additional user burden. The aforementioned Langtry-Menter [6], AFT [5, 28], and γ -model [7] are among the most commonly used transition models in RANS frameworks.

Before the advent of transition modeling, CFD simulations were performed as fully turbulent, neglecting laminar flow and the subsequent performance improvements of laminar flow. In order to confidently apply transition models during the aerospace-design process, there is an initial burden of test-case validation, often done by the developers of the transition models. Consequently, a significant portion of transition model development focuses on the validation of these models, as many researchers recommend calibration of the models to specific flow regimes [7, 28, 29, 45]. These validation cases generally take the form of comparisons against experimentally acquired data or DNS data. There is a significant lack of high-quality experimental data sets with respect to transition, some of which are bereft even of transition locations; however, these data sets still remain a valuable resource for transition model validation, calibration, and comparisons, as they highlight the links between modeled behavior and physical behavior, especially with respect to aerodynamic performance.

One popular, fixed-wing data set used for validation is that of NASA’s Natural Laminar Flow Common Research Model (CRM-NLF) [15]. The CRM-NLF experiment sought to reshape a transonic, transport wing design in order to achieve significant runs of laminar flow on the upper surface [15]. This experiment aimed to demonstrate the performance improvements that come with laminar flow, along with providing a high-quality data set for transition model validation. The profile of the wing was designed using a combination

of airfoil design tools and stability-based transition tools, which used a conical flow approximation to account for the sweep of the wing [15]. Based on the design of the wing, the upper surface experiences both transition due to Tollmien-Schlichting instabilities and crossflow instabilities. Transition locations were determined experimentally using infrared thermography of the wing, which was coated in a temperature-sensitive paint [15]. Additionally, surface pressure was determined at select semi-span locations.

Venkatachari et al. [16] examined transition predictions on the CRM-NLF using NASA’s OVERFLOW code [124]. When using the Langtry-Menter model with crossflow (LM2015), freestream sustaining terms were used to combat the freestream turbulence decay. These sustaining terms take the form of that proposed by Spalart and Rumsey [116] which seeks to cancel out turbulent destruction in the freestream by including additional turbulent source terms; however, this has the additional effect of introducing a turbulent source, albeit low-magnitude, into the boundary layer, the effect of which is not well quantified for the Langtry-Menter model. For LM2015, predicted surface pressure distributions did not agree well with experiment. For some portions of the wing, the LM2015 model predicted shocks significantly further upstream than was seen in experiment, as shown in Fig. 2.6, resulting in a significant loss of lift. Conversely, surface pressure predicted by the AFT2017b model [47] agreed very well with experiments, accurately predicting the shock locations and pressures about the leading edge. At higher angles of attack, however, the AFT2017b model predicts a much stronger shock on the wing surface compared to that seen in experiments. Both LM2015 and AFT2017b predicted that transition occurs too far upstream compared to experiments over all angles of attack investigated. By reducing the freestream turbulence intensity in the simulations, the LM2015 model predicted the pressure distributions much better, and both the LM2015 and AFT2017b models more accurately predicted the transition locations. This discrepancy highlights the need for accurate characterization of wind tunnel conditions, particularly with respect to freestream turbulence intensity, as such parameters are highly influential in transition model development and validation.

Barrouillet et al. [125] simulated the CRM-NLF using a modified version of the Langtry-Menter model using a custom crossflow implementation, finding that surface pressures

agreed well with experiment. To deal with freestream turbulence decay, a large value of turbulent eddy viscosity was imposed at the inlet rather than using freestream sustaining terms. Transition predictions compared poorly with experiment, however, occurring near the leading edge with little apparent influence of crossflow. Paredes et al. [126] imposed transition locations predicted by a linear-stability solver onto a RANS simulation, finding good agreement in predicted transition compared to experiment. Venkatachari et al. [17] later revisited the CRM-NLF, this time modifying both LM2015 and AFT2017b with compressibility corrections. Specifically, the production terms of $\widetilde{Re}_{\theta,t}$ and $\tilde{n}$, respectively, were modified to account for the Mach number at the boundary-layer edge. Predicted transition agreed much better with experiment for both models, but the predicted performance agreed more poorly when including the compressibility corrections. Piotrowski and Zingg [33] sidestepped the turbulence decay problem of Langtry-Menter by coupling the $\gamma - Re_{\theta}$ model to the Spalart-Allmaras model, showing improved surface pressure predictions over LM2015. Denison [127] identified that for the Langtry-Menter model, the SST turbulence index (a post-processing metric to detect transition) is not valid when applied to the Langtry-Menter model, as evidenced by the grid-dependent behavior when applied to CRM-NLF predictions. This SST turbulence index was similarly used by Venkatachari et al. [16] and Venkatachari et al. [17], which may explain poor transition predictions as due to a post-processing error. Zore et al. [17] highlighted the impact of grid topology, finding significant differences between unstructured solutions for tetrahedral grids and hexahedral grids. Lee et al. [128] examined the predictive capabilities of the Medida-Baeder model [29] and the $SA - \gamma$ model. Both models predicted transition far upstream of experiments, citing the strong dependence on the calibrations innately built-in to the transition correlations. Krimmelbein and Krumbein [72] highlighted the sensitivity to angle of attack for the CRM-NLF, showing significantly different surface-pressure distributions for small changes in angle of attack, primarily due to the predicted shock location. In general, custom variations of these transition models can greatly affect their predictive capabilities. Thus, great care must be taken when selecting the transition model for a given application.

Airfoil test cases are also of great use to transition model validation and prediction, as airfoil predictions represent an earlier portion of the design stage (relative to wings) and can illustrate improvements that come with laminar flow. Coder [129] suggested that the S809 [67] and NLF(1)-0416 [66] airfoils be standard test cases for transition models due to their plethora of available transition data, pressure data, and performance data. These airfoils have been used by a plethora of transition-modeling researchers and participants in the AIAA CFD Transition Modeling Group [127, 130]. Medida and Baeder [29] numerically investigated transition about three airfoils known to undergo transition. Generally, skin-friction distributions compared well with experiments, with exception to regions with separation-induced transition, suggesting that a hybrid RANS/LES method may be needed to improve such predictions. Ortiz-Melendez et al. [131] simulated an SNLF airfoil using the AFT model with a hybrid RANS/LES method. This work served as pre-predictions for an upcoming (at the time) wind-tunnel test for a high-lift airfoil configuration and illustrated the benefits of using hybrid RANS/LES methods when flow separation is expected.

Transition is also of interest for fuselage-like configurations, as skin-friction drag plays a large role in fuselage performance. Jung and Baeder [132] investigated the predictive capabilities of the Medida-Baeder model with a crossflow extension on the ROBIN mod7 helicopter fuselage [133]. Transitional simulations showed improved pressure predictions for the “free-transition” experimental case. The 6:1 inclined prolate spheroid tested by DLR [134–136] serves as a simplification of a generic, slender, fuselage-like body. Transition about the prolate spheroid is known to occur via a mix of Tollmien-Schlichting-type and stationary-crossflow-type instabilities [134–136] and is an excellent stress test for a model’s crossflow-prediction capabilities. Coder [129] simulated the prolate spheroid as a validation case for the 2019 variant of the AFT model, showing that transition due only to Tollmien-Schlichting instabilities is not sufficient for accurate predictions. Xu et al. [137] extended the AFT2014 model by developing an equation for the crossflow amplification factor, which showed improved transition predictions for the prolate spheroid test case compared to AFT2014. Jung and Baeder [132] also investigated the SA- γ model with crossflow for the prolate spheroid, resulting in good transition predictions for strong crossflow (i.e., high

angle of attack) and comparatively poorer results for weak crossflow. Grabe et al. [138] investigated two different crossflow extensions for the Langtry-Menter model, where both extensions agreed relatively well with experiments; however, it was unclear which extension was better for the prolate spheroid case, as illustrated in Fig. 2.7. Other crossflow test cases include that of the NLF(2)-0415 infinitely swept wing [69, 139], which was used in the calibration of the crossflow model developed for use with Langtry-Menter [45], and the TU Braunschweig sickle wing [71]. These two test cases have undergone numerous numerical studies [129, 132, 138, 140] to better evaluate the influence of crossflow in transition modeling.

In the rotating reference frame, rotorcraft have seen increasing use of transition models. The four-bladed PSP rotor was tested experimentally by Overmeyer and Martin [21] and is the subject of numerous verification studies performed as a part of the AIAA Hover Prediction Workshop [141] as it features both rotor performance data and transition locations. The experimental studies were carried out with the rotor installed on a ROBIN mod7 [133] fuselage, whereas many numerical studies typically focused on the isolated rotor when making transition predictions. Jain [142] investigated the PSP rotor in the installed and isolated configuration by using both fully turbulent models and transitional models. Predicted thrust was found to be negligibly different between fully turbulent and transitional models, while the predicted torque was significantly reduced when using the transition model. As a result, the fully turbulent predictions underpredicted the hover figure of merit by three to seven counts ($0.03 - 0.07$) in comparison to the transitional predictions. Vieira et al. [143] simulated the PSP rotor in hover using the SST-based variant of the AFT model [27], finding accurate predictions of transition locations when compared to experiments. Sheng et al. [144] identified that transition on rotors tends to occur due to laminar separation, and included a stall modification to the Langtry-Menter model, aiming to account for Coriolis effects. Consequently, predicted transition locations compared better to experiments when the stall modification was used. In hover, crossflow instabilities are less important but may still play a role in transition. Lee et al. [145] simulated the PSP rotor in hover using a transition model with and without crossflow. While the crossflow model resulted in shorter runs of laminar flow on the surface, differences in predicted performance were negligible. Carnes

and Coder [140] implemented the crossflow model of Langtry et al. [45] with the AFT2019b model, finding that while crossflow reduces the overall extent of laminar flow, the impacts on predicted performance are small. Similarly, the modeled crossflow instabilities appear to erroneously cause upstream transition in regions where a tip vortex impacts the surface of the blade. Jain [146] used the Galilean-invariant form of Langtry-Menter for hover predictions of the PSP rotor and found that transition agrees better with experiments when using a Galilean-invariant model. Additionally, the inclusion of crossflow reduced the predicted figure of merit by several counts, compared to the model without crossflow, and agreed very well with experimentally determined performance. The rotor was also simulated as elastic by coupling the CFD solution with a computational structural dynamics solver; however, the elastic rotor did not offer significantly different predictions compared to the rigid rotor. Similar transition predictions were made for the PSP rotor in forward flight [100, 147]. No transition model performed particularly well with respect to accurately predicting transition, underlining the need to calibrate these transition models for different flows.

In the low-Reynolds number regime, Koning et al. [148] and Koning et al. [149] used the AFT2017b model [47] as part of the design of the rotor for the Mars Helicopter “Ingenuity”. Due to the low-density, low-temperature, Martian environment, the blade-tip Reynolds number seldom exceeds 1.2×10^4 , rendering fully turbulent models inappropriate for performance predictions, even though they may not fully activate as turbulent at such low Reynolds numbers (cf. Rumsey and Spalart [150]). At the same time, however, the low Reynolds number means the boundary layer may be more likely to separate and potentially reattach as turbulent, a niche which transition models can accurately predict.

2.3.3 LES-based Transition Prediction

Unlike their RANS counterparts, large eddy simulations are capable of predicting the physics of transition without the requirement of a RANS turbulence model [54]. In the past, LES predictions tended to focus on smaller scale problems such as flow over a flat plate, airfoils, turbine cascades, or free shear layers due to the computational limitations.

Some of the simplest, transitional LES predictions come in the form of a free shear layer, typically a jet [41]. While these jet flows may not have long runs of laminar flow, they provide a canonical case in which to investigate shear-layer instabilities and the transition to turbulence. Raman et al. [151] and Antonia et al. [152] independently hypothesized that jets that are initially laminar or transitional, when exiting the jet nozzle, tend to fully develop faster. This means that the jets approach self-similar behavior over a shorter distance compared to jets that are initially fully turbulent, which is likely due to the faster decay in velocity for initially laminar and transitional shear flows [41]. Insight into the shear-layer behavior is particularly useful for nozzle flows of jet engines and rotorcraft [153] for which turbulent shear layers are known to produce much higher noise levels compared to laminar shear layers.

The next step-up in complexity comes with that of flat plates, a popular test case for LES models. Voke and Yang [154] studied bypass transition on a flat plate using a low-Reynolds number modification to the Smagorinsky model [36], finding that predictions agreed well with experiment. Huai et al. [155] simulated a flat-plate boundary layer using the dynamic Smagorinsky model [156], which seeks to dynamically modify the Smagorinsky constant in hopes of dissipating energy to subgrid scales more accurately. The model was able to accurately predict both amplitude and phase of subharmonic instabilities, but slightly underpredicted skin friction in the transition region. Sayadi and Moin [157] simulated H-type [158] and K-type [63] natural transition on a flat plate. Tollmien-Schlichting waves were introduced into the domain by a “disturbance strip” with a suction/blowing actuation near the leading edge of the plate, a computational surrogate to Kachanov and Levchenko’s vibrating ribbon experiment [159]. Most of the subgrid-scale closures investigated greatly underpredicted the wall-normal distribution of the Reynolds shear stress $(\overline{u'v'})$ post-transition. Kim et al. [160] used PSE to predict the transition location for a flat plate, which was used to force transition at that point in an LES simulation. Disturbance mode shapes of the Tollmien-Schlichting wave amplitudes obtained from PSE were imposed at the transition point of the computational domain, shortening the length of the domain needed to compute LES-predicted turbulence. As a result, the LES predictions exhibited

H-type natural transition, as seen in Fig. 2.8, similar to that predicted by PSE, insinuating that LES accurately predicted the breakdown mechanism of transition.

While the LES method is attractive to employ since a turbulence or transition model is not needed, one must take care in designing the numerical experiment as the computational expense can become quite large due to the extremely fine grids and small timesteps needed [35, 161], especially as the flowfield-of-interest becomes increasingly complex. Nevertheless, many researchers have employed LES for transition prediction about, for example, airfoils at low-Reynolds numbers [39, 162], reporting that LES is capable of predicting transition for many different transition mechanisms such as Tollmien-Schlichting instabilities and LSBs, and can predict non-linear effects like three-dimensional interactions. Breuer [163] simulated a micro-air-vehicle airfoil at a Reynolds number of 60,000 in an LES framework. The methodology was able to capture the laminar separation bubble on the upper surface of the airfoil. As the specified value of freestream turbulence intensity (FSTI) increased, laminar separation occurred increasingly upstream on the airfoil until the FSTI was sufficiently large to trip to turbulence before separation could occur. Uranga et al. [164] investigated crossflow-induced transition on an airfoil using LES, finding that for sweep angles between 10° and 40° , crossflow instabilities interact with Tollmien-Schlichting instabilities during separation-induced transition. This results in transition due to non-linear effects, suggesting that the two-dimensional strip-theory assumption used in RANS-compatible transition models may be inaccurate for transition predictions for swept-wing flows. On the other hand, for sweep angles outside of the aforementioned range it was found that crossflow and Tollmien-Schlichting instabilities did not interact with each other, meaning they can be treated independently. Therefore, there may be a range of sweep angle where the non-linear interactions between these two instabilities must be considered. Galbraith and Visbal [39] simulated transitional flow over the SD7003 airfoil using ILES, for which transition is separation-induced, using a Padé-type filter instead of a subgrid scale model. The lower surface of the airfoil utilizes a coarse grid, as the positive angles of attack that were investigated result in fully laminar flow on the lower surface. The upper surface was refined significantly and was used as the “LES grid”. The spanwise extent of the airfoil was set

to $z/c = 0.1, 0.2$, and 0.3 in order to investigate its effects on transition. Too small of a spanwise extent will function as an effective filter for low-wavenumber instabilities; too large of a spanwise extent may result in too large of a computational cost for the same results. Transition was said to have occurred once $\overline{u'v'} = 0.001u_\infty^2$ as suggested by Ol et al. [165] and Radespiel et al. [166]. Compared to experimental data, the transition point and reattachment point were predicted well using ILES; however, the separation point was too far upstream compared to experimental results. Depicted in Fig. 2.9 are predicted Reynolds stresses and streamlines about the SD7003 airfoil at $\alpha = 11^\circ$. The size of the LSB predicted by ILES is much larger than that in the experiments, which is likely due to the more upstream separation point. Some of this discrepancy may be due to the treatment of freestream turbulence intensity in the ILES predictions, which was set to zero.

In general, transition models still offer significant benefits over fully turbulent models, especially with respect to performance predictions and separation predictions. Nonetheless, these models do have their drawbacks. There is unfortunately no purely physics-based model currently available that is fully compatible with RANS CFD. These models still rely on some degree of correlations (empirical or theoretical) or assumptions built-in to these models, requiring tailoring by the end-user for their specific flow regime. Often times these models are treated as a “black box”, where the predictions are simply accepted without investigating the inner workings of the models. Conversely, the use of LES is likely to provide more accurate transition prediction and can better account for the flow physics at-hand, but becomes prohibitively expensive for large aerodynamic bodies and domains, whereas RANS models remain relatively cheap at the same scale.

Chapter 3

Computational Methodology

3.1 Description of Turbulence Models

Turbulence and transition models that are compatible with the RANS equations are a means of closing the turbulence problem introduced by the Reynolds-averaging decomposition. The idea behind the need for a turbulence closure comes by inspection of, for simplicity, the incompressible RANS equations, which are given as,

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j} = \frac{-1}{\rho} \frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \overline{u_i}}{\partial x_j} - \overline{u'_i u'_j} \right) \quad (3.1)$$

Computing and predicting the Reynolds stress term $\overline{u'_i u'_j}$ is nebulous in the context of RANS CFD, as RANS methods are not capable of predicting the time variation of velocity perturbations u'_i , as only the average values can be recovered. One common remedy is that which Boussinesq hypothesized in 1877 [167], that the momentum transfer of turbulent eddies can be modeled by a turbulent eddy viscosity; that is that, the turbulent stresses act similarly to viscous stresses. More completely, the Boussinesq approximation states the Reynolds stresses due to turbulent fluctuations vary linearly with the deviatoric (trace-less) strain rate tensor,

$$\tau_{ij} = 2\mu_t S_{ij} - \frac{2}{3} \overline{\rho u'_i u'_i} \delta_{ij} \quad (3.2)$$

or in terms of the Reynolds stresses only,

$$-\overline{\rho u'_i u'_j} = \mu_t \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} - \frac{2}{3} \frac{\partial \overline{u_k}}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \overline{\rho u'_i u'_i} \delta_{ij} \quad (3.3)$$

Turbulence-closure methods seek to approximate the quantity μ_t in Eq. 3.3, the turbulent eddy viscosity. Prandtl's mixing length concept [104] is one of the simplest closure methods, which assumes that the eddy viscosity is linear proportional to the square of the strain rate and the square of a turbulent-eddy mixing length. This mixing length can be difficult to approximate, but can be related to some characteristic length scale like distance to the wall for a wall-bounded flow or the diameter of a turbulent jet [168]. In the near-wall region of a turbulent boundary layer, this length scale goes with κy as the Reynolds stresses are approximately constant in the log-layer. Generally, however, Prandtl's mixing length concept is insufficient for making predictions of turbulent flow physics due to the wide variety of flowfields that experience turbulence.

Various other turbulence models exist, with increasing complexity and with wider ranges of applicability. Most turbulence models elect to transport the eddy viscosity throughout the domain, which takes the general form of,

$$\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\rho u_j \phi)}{\partial x_j} = P_\phi + D_\phi + \frac{\partial}{\partial x_j} \left[(\mu + \sigma \mu_t) \frac{\partial \phi}{\partial x_j} \right] \quad (3.4)$$

where P_ϕ and D_ϕ represent production and destruction, respectively, of the scalar quantity ϕ . The class of zero-equation (algebraic) turbulence models does not use a transport equation and relies on only the available mean flow [169, 170]. These algebraic models typically split the boundary layer into an inner region for which Prandtl's mixing length concept holds and an outer region for which the mixing length becomes more difficult to compute.

Turbulence models that utilize transport equations can properly account for upstream flow history and are generally more robust in terms of approximating the Reynolds stresses [168]. One of the first, widely used transport-equation-based models is that of Baldwin and Barth [171]. The Baldwin-Barth model transports the turbulent Reynolds number, μ_t/μ , which is then used to compute the turbulent eddy viscosity by means of an additional

algebraic relation. One of the most popular one-equation models is the Spalart-Allmaras model [4], which transports a modified form of the eddy viscosity and was developed specifically for external aerodynamic flows. This model will be discussed in-depth in the forthcoming sections. Arguably the most popular two-equation model is the shear stress transport model developed by Menter [3], which transports the turbulent kinetic energy and specific turbulent dissipation rate. Other popular two-equations models generally fall under the umbrella of $k - \epsilon$ models which transport turbulent kinetic energy and the dissipation rate [172–174] and $k - \omega$ models which transport turbulent kinetic energy and the specific dissipation rate [175–177]. Both models have their own shortcomings that must be considered when applying them to a given flow.

3.1.1 The Shear Stress Transport Model

The shear stress transport (SST) model developed by Menter blends the popular $k - \epsilon$ and $k - \omega$ models in order to develop a model that is less sensitive to freestream conditions and accurately predicts shear-flow behavior in the presence of adverse pressure gradients [3]. As such, the SST model blends $k - \epsilon$ [172–174] and $k - \omega$ [175–177] so that model follows the “ $k - \omega$ branch” in the viscous sublayer and portions of the log-law region. Higher up in the boundary layer, the model switches to the “ $k - \epsilon$ branch”, where the Reynolds stresses dominate and the local velocity profile mimics that of a free shear flow, for which $k - \epsilon$ models are known to be accurate [3]. Before the advent of the SST model, the inclusion of a so-called “shear stress transport” term (cf. Johnson and King [178]) in turbulence modeling was known to improve predictions in the presence of adverse pressure gradients. This concept serves as one of the cornerstones of Menter’s model.

Consider a two-dimensional boundary layer about a flat plate with zero pressure gradient. The shear stress as a function of distance from the wall can be written as,

$$\tau = \mu \Omega - \rho \overline{u'v'} \quad (3.5)$$

where $\Omega = \partial u / \partial y$. In the viscous sublayer, the Reynolds stress term goes to zero as both u' and v' approach zero, enforced by the no-slip condition. In the log layer, the Reynolds stress term dominates the laminar shear stress term. By finding a means to model the term $\overline{u'v'}$ in the framework of a RANS turbulence model, the turbulent shear stress can be accounted for. The approach used by Menter in his formulation of SST relies on Bradshaw's assumption [179] which states that the shear stress is directly proportional to the turbulent kinetic energy,

$$\tau = \rho a_1 k \quad (3.6)$$

or more specifically,

$$a_1 = \frac{\overline{u'v'}}{k} \quad (3.7)$$

where a_1 is commonly referred to as Bradshaw's constant. Bradshaw suggests a value of 0.31 [179] while other studies suggest a value of 0.30 [175, 180]. Following Boussinesq, the turbulent shear stress is computed as,

$$\tau = \mu_t \Omega \quad (3.8)$$

which can be re-written as,

$$\tau = \rho a_1 k \sqrt{\frac{P_k}{D_k}} \quad (3.9)$$

In equilibrium (i.e., regions of constant shear stress), the ratio of the production term P_k and the destruction term D_k is unity, and Eq. 3.6 is recovered. In flows with adverse pressure gradients, the production term is larger than the destruction term and Eq. 3.9 overpredicts the shear stress. Following this line of thought, Menter elected to redefine the eddy viscosity as,

$$\mu_t = \frac{\rho a_1 k}{\Omega} \quad (3.10)$$

such that the eddy viscosity (and therefore the shear stress) cannot grow faster than $a_1 k$. This formulation in-and-of-itself is unsatisfactory, as near the wall $\Omega \rightarrow 0$. Thus, the eddy viscosity is computed as a blend of Eq. 3.10 and the typical formulation for two-equation

models ($\rho k/\omega$),

$$\mu_t = \frac{\rho a_1 k}{\max(a_1 \omega, \Omega)} \quad (3.11)$$

In regions where the turbulence dissipation rate ω dominates (e.g. the viscous sublayer), the eddy viscosity computed via Eq. 3.11 goes to $\rho k/\omega$, which is typical for $k - \omega$ models. In regions of high vorticity where the dissipation rate is smaller (e.g. the log layer), the eddy viscosity is computed as $\rho a_1 k/\Omega$, which follows Bradshaw's assumption and functions as a pseudo-transport of the shear stress, ergo "shear stress transport".

Strictly speaking, Bradshaw's constant a_1 is only constant in the log layer, where the relation between Reynolds stress and turbulent kinetic energy is linear. For separated flows, the constant is often lower as the equilibrium is between the convection terms and destruction terms (rather than production and destruction), as production is small [168]. Thus, Eq. 3.11 must be limited to wall-bounded flows only to prevent overpredictions of the Reynolds stress outside the boundary layer where Bradshaw's assumption breaks down. In the sense of CFD, it is impossible to distinguish between the log layer and a turbulent, separated flow, giving reasoning to disable Bradshaw's assumption outside the boundary layer to prevent excessive turbulent stresses in the wake. This is achieved by multiplying the vorticity in Eq. 3.11 by a blending function F_2 that is designed to be zero near the wall (to ensure $a_1 \omega > \Omega$), unity in the log layer, and returning to zero outside the boundary layer. The function F_2 is defined as,

$$F_2 = \tanh(\arg_2^2) \quad (3.12)$$

where

$$\arg_2 = \max\left(2 \frac{\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2 \omega}\right) \quad (3.13)$$

Finally, the eddy viscosity is re-formulated as,

$$\mu_t = \frac{\rho a_1 k}{\max(a_1 \omega, \Omega F_2)} \quad (3.14)$$

Now, with this formulation of eddy viscosity in mind, the transport equations of the SST model can be defined. As aforementioned, the SST model takes advantage of the freestream

independence of $k - \epsilon$ to accurately predict the outer regions of the boundary layer, while using the $k - \omega$ formulation near the wall to accurately predict the near-wall behavior. The $k - \epsilon$ formulation can be converted to a $k - \omega$ formulation by a change of variables from ϵ to ω , adding a cross-diffusion term, and modifying the model constants. Then, the $k - \omega$ formulation is multiplied by another blending function F_1 while the transformed $k - \epsilon$ formulation is multiplied by $(1 - F_1)$ and are then added together. The blending function F_1 is designed to be unity near the wall to ensure the $k - \omega$ branch is active low in the boundary layer and zero higher up in the boundary layer to activate the $k - \epsilon$ branch. From this, the SST transport equations are,

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = \mu_t S^2 - \beta^* \rho k \omega + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right] \quad (3.15)$$

$$\begin{aligned} \frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho u_j \omega)}{\partial x_j} = & \alpha S^2 - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_j} \right] \\ & + 2(1 - F_1) \frac{\rho \sigma_{\omega 2}}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \end{aligned} \quad (3.16)$$

The constants $\alpha, \beta, \sigma_k, \sigma_\omega$ are blended using the F_1 function, depending on whether a given grid point is in the $k - \omega$ or $k - \epsilon$ branch, as,

$$\phi = F_1 \phi_1 + (1 - F_1) \phi_2 \quad (3.17)$$

where $\phi = [\alpha, \beta, \sigma_k, \sigma_\omega]$ and ϕ_1 is the value of the constant in the $k - \omega$ branch and ϕ_2 is the value in the $k - \epsilon$ branch. The function F_1 is designed to be unity up to approximately $y/\delta_{99} = 0.4$, above which the term goes to zero.

$$F_1 = \tanh(arg_1^4) \quad (3.18)$$

$$arg_1 = \min \left[\max \left(\frac{\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2 \omega} \right), \frac{4\rho \sigma_{\omega 2} k}{CD_{k\omega} y^2} \right] \quad (3.19)$$

$$CD_{k\omega} = \max\left(2\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}, 10^{-20}\right) \quad (3.20)$$

The model constants take on the values,

$$\begin{aligned} \alpha_1 &= \frac{1}{\nu_t}\left(\frac{\beta^*}{\beta_1} - \frac{\sigma_{\omega 1}\kappa^2}{\sqrt{\beta^*}}\right) & \alpha_2 &= \frac{1}{\nu_t}\left(\frac{\beta^*}{\beta_1} - \frac{\sigma_{\omega 2}\kappa^2}{\sqrt{\beta^*}}\right) \\ \sigma_{k1} &= 0.85 & \sigma_{k2} &= 1.0 \\ \beta_1 &= 0.075 & \beta_2 &= 0.0828 \\ \sigma_{\omega 1} &= 0.5 & \sigma_{\omega 2} &= 0.856 \\ \beta^* &= 0.09 & \kappa &= 0.41 \\ a_1 &= 0.31 \end{aligned}$$

On the wall, the no-slip condition mandates that $k = 0$. However, the ω boundary condition is nebulous, as the wall boundary condition is theoretically infinite. Wilcox [175] derived a functional limit to ω at the wall. On the wall, the production term of Eq. 3.16 goes to zero, and thus the destruction term and the diffusion term are balanced. Near the wall in a steady, two-dimensional, zero-pressure-gradient boundary layer, the ω equation reduces to an ordinary differential equation,

$$\nu \frac{\partial^2 \omega}{\partial y^2} = \beta_1 \omega^2 \quad (3.21)$$

By performing a perturbation analysis, it can be assumed that $\omega = cy^p$, which gives,

$$\beta_1 c^2 = \nu p(p-1)y^{-2-p} \quad (3.22)$$

Setting $p = -2$ causes the y term to vanish, giving the constant as,

$$c = \frac{6\nu}{\beta_1} \quad (3.23)$$

Thus, at $y = 0$, Wilcox suggests that wall value of ω be

$$\omega_{wall} = \frac{6\nu}{\beta_1 \Delta y_1^2} \quad (3.24)$$

Menter suggests that the constant in Eq. 3.24 be 60 rather than 6 [3], as this improves robustness near the wall for solvers without appreciably changing the solution.

3.1.2 The Spalart-Allmaras Model

The Spalart-Allmaras (SA) model [4] is a one-equation turbulence model that, unlike other one-equation models such as Bradshaw's model [179], is of a purely local formulation. Rather than formulate the transport equation using physical quantities used to compute the eddy viscosity, such as k , ω , or ϵ , the eddy viscosity itself is transported using a change of variables. This transported variable $\tilde{\nu}$ is designed around the concept of eddy viscosity in the log layer, where the eddy viscosity is equal to $u_\tau \kappa y$. As a result, the modified eddy viscosity is prescribed as $\tilde{\nu} = u_\tau \kappa y$. This formulation holds true in the log layer but does not hold in the viscous sublayer (i.e., low-Reynolds-number regions), as the contribution of Reynolds stresses near the wall is negligible [4]. As a result, the actual eddy viscosity is computed using a mapping of the modified eddy viscosity,

$$\nu_t = \tilde{\nu} f_{v1} \quad (3.25)$$

where the damping function f_{v1} is found using,

$$f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3} \quad (3.26)$$

with

$$\chi = \frac{\tilde{\nu}}{\nu} \quad (3.27)$$

The transport equation is designed in a piece-wise manner. Firstly, the production terms and diffusion terms were designed with homogeneous turbulence in mind. Next, the

destruction term was designed based on the blocking effect of the wall. This destruction term varies with the inverse square of distance from the wall and thus is negligible far from solid walls. When near the wall, the destruction term is corrected in order to more accurately predict skin friction by means of an additional damping function. Similarly to the modified eddy viscosity, the strain rate in the production term is additionally modified to take into account the damping near the wall, such that $\tilde{S} = u_\tau/(\kappa y)$ all the way to the wall. Additional “trip” terms are added so that the user may specify a turbulent trip location. The full SA model is written as,

$$\frac{D\tilde{\nu}}{Dt} = c_{b1}\tilde{S}\tilde{\nu}(1 - f_{t2}) - \left(c_{w1}f_w - \frac{c_{b1}}{\kappa^2}f_{t2}\right)\left(\frac{\tilde{\nu}}{d}\right)^2 + \frac{1}{\sigma}\left[\frac{\partial}{\partial x_j}\left((\nu + \tilde{\nu})\frac{\partial\tilde{\nu}}{\partial x_j}\right) + c_{b2}\frac{\partial\tilde{\nu}}{\partial x_j}\frac{\partial\tilde{\nu}}{\partial x_j}\right] \quad (3.28)$$

The functions contained within the transport equation are computed as,

$$\tilde{S} = \Omega + \frac{\tilde{\nu}}{\kappa^2 d^2} f_{v2} \quad (3.29)$$

$$f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}} \quad (3.30)$$

$$f_w = g \left[\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right]^{1/6} \quad (3.31)$$

$$g = r + c_w(r^6 - r) \quad (3.32)$$

$$r = \min \left[\frac{\tilde{\nu}}{\tilde{S}\kappa^2 d^2}, 10 \right] \quad (3.33)$$

$$f_{t2} = c_{t3} \exp(-c_{t4}^2 \chi^2) \quad (3.34)$$

The functions f_w and r are designed to be unity in the log-layer and decrease in the outer regions of the boundary layer, while g is simply a limiter preventing large values of f_w and therefore overpredicting destruction. The damping function f_{v2} is designed similarly to f_{v1} , with application only to ensure that $\tilde{S} = u_\tau/(\kappa y)$ to the wall. The model constants are given

as,

$$\begin{aligned}
c_{b1} &= 0.1355 & c_{b2} &= 0.622 \\
\sigma &= 2/3 & \kappa &= 0.41 \\
c_{w2} &= 0.3 & c_{w3} &= 2 \\
c_{t3} &= 1.2 & c_{t4} &= 0.5 \\
c_{w1} &= \frac{c_{b1}}{\kappa^2} + \frac{1 + c_{b2}}{\sigma}
\end{aligned}$$

The subscript “ b ” refers to the basic form of the model (i.e. free shear flow), the “ w ” subscript refers to functions active to maintain law-of-the-wall behavior, and subscript “ v ” refers to functions active in the viscous sublayer and the buffer layer.

A useful post-processing function is that of the turbulence index [4]. Given that the model prescribes $\tilde{\nu} = \kappa u_\tau y$, then the wall-normal derivative of $\tilde{\nu}$ should attain the value κu_τ . If the wall-normal derivative evaluated at the wall is greater than or equal to κu_τ , then the boundary layer is fully turbulent. This is a useful metric for determining the transition point for SA-based transition models. The turbulence index is defined as,

$$i_{t,SA} = \frac{1}{\kappa u_\tau} \frac{\partial \tilde{\nu}}{\partial n} = \frac{1}{\kappa \sqrt{\nu \Omega}} \frac{\partial \tilde{\nu}}{\partial n} \quad (3.35)$$

where a value of 1 indicates turbulent flow and a value of 0 indicates laminar flow.

3.2 Description of Transition Models

The concept behind transition models lies in the fact that fully turbulent models are inappropriate for predicting transitional flows. While fully turbulent models exhibit laminar-like behavior at low Reynolds numbers [150], their predictive capabilities of transition are poor. Developing additional equations for transition allows for more accurate predictions. Typically, these models transports scalar quantities and couple directly with a fully turbulent model. In this section, the Langtry-Menter local correlation model [6] will be introduced and

laid out in its full form, in addition to the amplification factor transport transition modeling framework [28].

3.2.1 The Langtry-Menter Local Correlation Model

The Langtry-Menter local correlation model is a two-equation transition modeling framework that couples with the SST turbulence model [6]. The model seeks to account for multiple transition mechanisms, to be formulated locally, and to be flexible enough to calibrate the correlations for transition onset. The chief idea of the formulation comes from Van Driest and Blumer's [181] observation that the momentum thickness Reynolds number in the boundary layer is directly proportional to the maximum vorticity Reynolds number in the boundary layer. Using this criterion in combination with transported variables based on experimentally calibrated transition correlations allows for a CFD-compatible transition model that is applicable to a range of flow regimes. This led to the design of two transport equations. This first is that of the intermittency γ . In the CFD-sense, the intermittency is an indicator of the flow regime, where a value of 0 indicates laminar flow and a value of 1 indicates turbulent flow. The second transport equation is that of the transitional momentum thickness Reynolds number $Re_{\theta,t}$, which is based on the momentum thickness at the onset of transition. The two transport equations are written as,

$$\begin{aligned} \frac{\partial(\rho\gamma)}{\partial t} + \frac{\partial(\rho u_j \gamma)}{\partial x_j} &= F_{length} c_{a1} \rho S [\gamma F_{onset}]^{0.5} (1 - c_{e1} \gamma) \\ &\quad - c_{a2} \rho \Omega \gamma F_{turb} (c_{e2} \gamma - 1) + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_f} \right) \frac{\partial \gamma}{\partial x_j} \right] \end{aligned} \quad (3.36)$$

$$\begin{aligned} \frac{\partial(\rho \widetilde{Re}_{\theta,t})}{\partial t} + \frac{\partial(\rho u_j \widetilde{Re}_{\theta,t})}{\partial x_j} &= c_{\theta,t} \frac{\rho}{T} \left(Re_{\theta,t} - \widetilde{Re}_{\theta,t} \right) (1 - F_{\theta,t}) \\ &\quad + \frac{\partial}{\partial x_j} \left[\sigma_{\theta,t} \left(\mu + \mu_t \right) \frac{\partial \widetilde{Re}_{\theta,t}}{\partial x_j} \right] \end{aligned} \quad (3.37)$$

The subfunctions of the production and destruction terms of the γ equation are,

$$F_{onset,1} = \frac{Re_v}{2.193Re_{\theta,c}} \quad (3.38)$$

$$Re_v = \frac{\rho\Omega d^2}{\mu} \quad (3.39)$$

$$F_{onset,2} = \min\left(\max\left[F_{onset,1}, F_{onset,1}^4\right], 2.0\right) \quad (3.40)$$

$$R_T = \frac{\rho k}{\mu\omega} \quad (3.41)$$

$$F_{onset,3} = \max\left[1 - \left(\frac{R_T}{2.5}\right)^3, 0\right] \quad (3.42)$$

$$F_{onset} = \max(F_{onset,2} - F_{onset,3}, 0) \quad (3.43)$$

where F_{onset} determines whether the transition from laminar to turbulent has begun. Before this transition onset, F_{onset} remains approximately zero and production of intermittency is small. Next, the function F_{length} dictates the streamwise length of the transition region. In the viscous sublayer, the function F_{length} takes on its maximum value of 40 in order to prevent an undesired decrease of intermittency in the viscous sublayer when $\widetilde{Re}_{\theta,t}$ sharply decreases post-transition.

$$F_{length} = F_{length,1}(1 - F_{sublayer}) + 40.0F_{sublayer} \quad (3.44)$$

$$F_{length,1} = \begin{cases} 39.8189 + (-119.270 \times 10^{-4} \widetilde{Re}_{\theta,t}) \\ \quad + (-132.567 \times 10^{-6} \widetilde{Re}_{\theta,t}^2), & \text{if } 400 > \widetilde{Re}_{\theta,t} \\ 263.404 + (-123.939 \times 10^{-2} \widetilde{Re}_{\theta,t}) \\ \quad + (194.548 \times 10^{-5} \widetilde{Re}_{\theta,t}^2) \\ \quad + (-101.695 \times 10^{-8} \widetilde{Re}_{\theta,t}^3), & \text{if } 400 \leq \widetilde{Re}_{\theta,t} < 596 \\ 0.5 - (3.0 \times 10^{-4})(\widetilde{Re}_{\theta,t} - 596.0), & \text{if } 596 \leq \widetilde{Re}_{\theta,t} < 1200 \\ 0.3188 & \text{if } 1200 \leq \widetilde{Re}_{\theta,t} \end{cases} \quad (3.45)$$

$$F_{sublayer} = \exp \left[- \left(\frac{Re_\omega}{200} \right)^2 \right] \quad (3.46)$$

$$F_{turb} = \exp \left[- \left(\frac{R_T}{4} \right)^4 \right] \quad (3.47)$$

$$Re_\omega = \frac{\rho \omega d^2}{\mu} \quad (3.48)$$

where F_{turb} is used to locally disable the relaminarization source term in regions of laminar flow or the viscous sublayer. The critical momentum thickness Reynolds number is computed as a correlation of the transported, transition momentum thickness Reynolds number,

$$Re_{\theta,c} = \begin{cases} -396.035 \times 10^{-2} + (10120.656 \times 10^{-4} \widetilde{Re}_{\theta,t}) \\ \quad + (-868.230 \times 10^{-6} \widetilde{Re}_{\theta,t}^2) + (696.506 \times 10^{-9} \widetilde{Re}_{\theta,t}^3) \\ \quad + (-174.105 \times 10^{-12} \widetilde{Re}_{\theta,t}^4), & \text{if } 1870 \geq \widetilde{Re}_{\theta,t} \\ \widetilde{Re}_{\theta,t} - (593.11 + 0.482(\widetilde{Re}_{\theta,t} - 1870)) & \text{if } 1870 \leq \widetilde{Re}_{\theta,t} \end{cases} \quad (3.49)$$

Next, the production and destruction terms of the $\widetilde{Re}_{\theta,t}$ equation are written as,

$$T = \frac{500}{\nu U} \quad (3.50)$$

$$U = \sqrt{u_k u_k} \quad (3.51)$$

$$F_{\theta,t} = \min \left[\max \left(F_{wake} \exp \left[- \frac{d^4}{\delta^4} \right], 1.0 - \left(\frac{c_{e2}\gamma - 1}{c_{e2} - 1} \right)^2 \right), 1.0 \right] \quad (3.52)$$

$$F_{wake} = \exp \left[- \left(\frac{Re_\omega}{10000} \right)^2 \right] \quad (3.53)$$

$$\delta = \frac{375 \Omega \widetilde{Re}_{\theta,t} d}{\nu U^2} \quad (3.54)$$

The blending function $F_{\theta,t}$ disables the production term within the boundary layer and allows $\widetilde{Re}_{\theta,t}$ to diffuse outside the boundary layer. Similarly, F_{wake} serves to disable $F_{\theta,t}$ in wake regions as well, allowing for free diffusion of $\widetilde{Re}_{\theta,t}$. The local (non-transported) transition momentum thickness Reynolds number is a function of the local turbulence intensity and

local pressure gradient. Its formulation is superficially and functionally similar to the Abu-Ghannam and Shaw relation [98].

$$Re_{\theta,t} = \begin{cases} (1173.51 - 589.428Tu + 0.2196Tu^{-2})F(\lambda_\theta) & \text{if } Tu \leq 1.3 \\ 331.50(Tu - 0.5658)^{-0.671}F(\lambda_\theta) & \text{if } Tu > 1.3 \end{cases} \quad (3.55)$$

$$F(\lambda_\theta) = \begin{cases} 1 + (12.986\lambda_\theta + 123.66\lambda_\theta^2 + 405.689\lambda_\theta^3)\exp\left(-\left(\frac{Tu}{1.5}\right)^{1.5}\right) & \text{if } \lambda_\theta \leq 0 \\ 1 + 0.275(1 - \exp[-35.0\lambda_\theta])\exp\left(\frac{-Tu}{1.5}\right) & \text{if } \lambda_\theta > 0 \end{cases} \quad (3.56)$$

The parameter λ_θ is similar to the Thwaites parameter [92] except that it is evaluated at the transition onset location. The additional functions that define the equilibrium momentum thickness Reynolds number $\widetilde{Re}_{\theta,t}$ and the pressure gradient function $F(\lambda_\theta)$ are,

$$\lambda_\theta = \frac{\theta_t^2}{\nu} \frac{dU}{ds} \quad (3.57)$$

$$\frac{dU}{ds} = \frac{u_m u_n}{U^2} \frac{\partial u_m}{\partial x_n} \quad (3.58)$$

$$Tu = 100 \sqrt{\frac{2}{3}} \frac{k}{U^2} \quad (3.59)$$

The value of the transitional momentum thickness θ_t must be found at every grid point by using a root finding technique such as Newton's method. The calibration constants are,

$$\begin{aligned} c_{a1} &= 2.0 & c_{a2} &= 0.06 \\ c_{e1} &= 1.0 & c_{e2} &= 50 \\ c_{\theta,t} &= 0.03 & \sigma_f &= 1.0 \\ \sigma_{\theta,t} &= 2.0 \end{aligned}$$

The transition modeling framework couples with the SST model by modifying the production and destruction terms of the turbulent kinetic energy equation as,

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = \gamma_{eff} \mu_t S^2 - \gamma_{lim} \beta^* \rho k \omega + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right] \quad (3.60)$$

with

$$\gamma_{lim} = \min \left[\max(\gamma_{eff}, \gamma_{min}), 1.0 \right] \quad (3.61)$$

$$\gamma_{eff} = \max(\gamma, \gamma_{sep}) \quad (3.62)$$

$$\gamma_{sep} = \min \left(2 \max \left[0, \left(\frac{Re_v}{3.235 Re_{\theta,c}} \right) - 1 \right] F_{reattach}, 2 \right) F_{\theta,t} \quad (3.63)$$

$$F_{reattach} = \exp \left[- \left(\frac{R_T}{20} \right)^4 \right] \quad (3.64)$$

The variable γ_{lim} serves to ensure that destruction of turbulent kinetic energy exceeds production in the viscous sublayer, achieved by the floor value γ_{min} , which is suggested to be 0.1. The consequences of this floor value will be discussed further in Chapter 4. Separation-induced transition is accounted for via γ_{sep} , for which its function argument is notionally similar to F_{onset} . The value of γ_{sep} is restricted to a maximum value of 2. Although $\gamma > 1$ is not physical, values above 1 aid in reattaching separated flows to the surface by increasing turbulent production. This is particularly useful in the case of laminar separation bubbles and their subsequent reattachment. The constant 3.235 in Eq. 3.64 is the ratio of Re_v to $Re_{\theta,c}$ at the separation point (i.e., where $H_{12} = 3.5$). The SST blending function F_1 is modified to prevent activation of turbulent source terms in laminar boundary layers.

$$F_1 = \max(F_{1,SST}, F_3) \quad (3.65)$$

$$F_3 = \exp \left[- \left(\frac{Re_y}{120} \right)^8 \right] \quad (3.66)$$

$$Re_y = \frac{d\sqrt{k}}{\nu} \quad (3.67)$$

This modification ensures that F_1 is unity throughout the laminar boundary layer to maintain $k-\omega$ -like behavior. The constant 120 is troublesome, as if this constant is too large, $F_1 \rightarrow 0$ too low in the boundary layer and can result in premature transition. The current value, however, causes F_1 to maintain unit behavior far outside the boundary layer due to the F_3 function ($\approx 2\delta_{99}$ for some problems) which only permits the $k-\omega$ solution in this region and

may result in increased sensitivity to the freestream in regions just outside the boundary-layer edge. Modification of this constant is beyond the scope of this work and the original formulation is used herein.

In its formulation, the Langtry-Menter model is attracted to the solution of $\gamma = 0$ (i.e., fully laminar). As such, the farfield value of intermittency must be unity or the model will never predict transition. This has the additional effect of maintaining the SST equations' intended turbulent decay rate [6]. In this sense, Langtry-Menter laminarizes a turbulent solution rather than starting from a laminar solution with eventual transition to turbulence. The recommended boundary conditions for the model are,

$$\left. \frac{\partial \gamma}{\partial n} \right|_{wall} = 0 \quad (3.68)$$

$$\left. \frac{\partial \widetilde{Re}_{\theta,t}}{\partial n} \right|_{wall} = 0 \quad (3.69)$$

$$\gamma_{\infty} = 1 \quad (3.70)$$

$$\widetilde{Re}_{\theta,t,\infty} = \begin{cases} (1173.51 - 589.428Tu + 0.2196Tu_{\infty}^{-2}) & \text{if } Tu_{\infty} \leq 1.3 \\ 331.50(Tu_{\infty} - 0.5658)^{-0.671} & \text{if } Tu_{\infty} > 1.3 \end{cases} \quad (3.71)$$

Like the SST model, the Langtry-Menter is subject to freestream turbulence decay. As the turbulence intensity decays from the freestream to the surface of interest, the value of $Re_{\theta,t}$ increases and will delay laminar-turbulent transition. It is thus up to the user to select proper inflow conditions to ensure accurate transition predictions. This turbulence decay can be prevented by adding turbulence sustaining terms [116] but these terms are active within the laminar boundary layer and their effect on transition is still not wholly understood [16, 182, 183]. A study on their effect, and a potential solution to the turbulence decay problem, for Langtry-Menter is worthy of a dissertation in and of itself.

The Langtry-Menter model can be further modified to account for crossflow transition [45]. For swept-wing flows, the stationary crossflow instability becomes increasingly important with more aggressive sweep and rougher surfaces and is the primary crossflow mode responsible for transition. Additional correlations were developed by Langtry et al.

[45] that added a stationary crossflow sink term to the $\widetilde{Re}_{\theta,t}$ equation, which reduces the value of $\widetilde{Re}_{\theta,t}$ where crossflow strength is high in order to initiate transition further upstream for strong crossflow. This sink term modifies the $\widetilde{Re}_{\theta,t}$ -equation as,

$$\begin{aligned} \frac{\partial(\rho\widetilde{Re}_{\theta,t})}{\partial t} + \frac{\partial(\rho u_j \widetilde{Re}_{\theta,t})}{\partial x_j} = c_{\theta,t} \frac{\rho}{T} \left(Re_{\theta,t} - \widetilde{Re}_{\theta,t} \right) (1 - F_{\theta,t}) + D_{SCF} \\ + \frac{\partial}{\partial x_j} \left[\sigma_{\theta,t} (\mu + \mu_t) \frac{\partial \widetilde{Re}_{\theta,t}}{\partial x_j} \right] \end{aligned} \quad (3.72)$$

The crossflow destruction term D_{SCF} is only active where the stationary crossflow Reynolds number Re_{SCF} is lesser than $\widetilde{Re}_{\theta,t}$ such that crossflow transition does not occur unless it is the predominant instability,

$$D_{SCF} = c_{\theta,t} \frac{\rho}{t} c_{SCF} \min(Re_{SCF} - \widetilde{Re}_{\theta,t}, 0) F_{\theta,t2} \quad (3.73)$$

where $c_{SCF} = 0.6$. The function $F_{\theta,t2}$ ensures that the effect of crossflow is felt in the upper portions of the boundary layer by including the wake function,

$$F_{\theta,t2} = \min \left[F_{wake} \exp \left(- \left(\frac{y}{\delta} \right)^4 \right), 1.0 \right] \quad (3.74)$$

The stationary crossflow Reynolds number is computed based on correlations derived from the NLF(2)-0415 infinitely swept wing experiments performed by Refs. [184, 185]. These correlations are primarily a function of the crossflow shape factor, defined as,

$$H_{crossflow} = \frac{y \left| \vec{U} \cdot \vec{\Omega} \right|}{U} \quad (3.75)$$

The quantity $\vec{U} \cdot \vec{\Omega}$ is the helicity, which has been shown to be a good indicator of stationary crossflow strength [186]. It is noteworthy that this quantity is not Galilean-invariant. In the case of grid motion, $\vec{U}$ can be replaced by $\vec{U} - \vec{U}_{grid}$ and U can be replaced by $U - U_{grid}$ to recover the intended effect. This crossflow shape factor then is used to compute the crossflow

strength function,

$$\Delta H_{crossflow} = H_{crossflow} \left(1.0 + \min(R_T, 0.4) \right) \quad (3.76)$$

Given that the experimental data used for calibration of Eq. 3.76 was for a specific data point with a specific stationary crossflow strength, additional upshift and downshift functions were developed to account for lower and higher crossflow strength [45], respectively, in the context of CFD.

$$+ \Delta H_{Crossflow} = \max \left[(0.1066 - \Delta H_{Crossflow}), 0 \right] \quad (3.77)$$

$$f(+\Delta H_{Crossflow}) = 6200 (+\Delta H_{Crossflow}) + 50000 (+\Delta H_{Crossflow})^2 \quad (3.78)$$

$$- \Delta H_{Crossflow} = \max \left[- (0.1066 - \Delta H_{Crossflow}), 0 \right] \quad (3.79)$$

$$f(-\Delta H_{Crossflow}) = 75 \tanh \left(\frac{-\Delta H_{Crossflow}}{0.0125} \right) \quad (3.80)$$

Finally, the stationary crossflow Reynolds number can be computed as,

$$Re_{SCF} = \frac{\theta_t \rho \left(\frac{U}{0.82} \right)}{\mu} = -35.088 \ln \frac{h}{\theta_t} + 319.51 + f \left(+\Delta H_{crossflow} \right) - f \left(-\Delta H_{crossflow} \right) \quad (3.81)$$

which must be solved as a transcendental equation given that θ_t appears on both sides.

3.2.2 The Amplification Factor Transport Model

The Amplification Factor Transport (AFT) transition modeling framework designed by Coder [27, 46] and Coder and Maughmer [5] is a two-equation transition model based on linear stability theory. The model is based on approximate e^N envelope theory [90], a simplified approach to stability theory that tracks the envelope of the amplification of Tollmien-Schlichting instabilities. In the AFT model, this amplification factor is transported via the variable $\tilde{n}$, meaning that the AFT model can predict transition only due to two-dimensional instabilities. The transport equation for modified intermittency [47], the natural log of actual intermittency, improves coupling with the SA model and facilitates the production of eddy viscosity. This intermittency equation was based on that of Menter

et al. [7] which, when incorporated into AFT2017b, rendered the AFT model as Galilean-invariant. Used herein is the 2019b variant of the AFT model [28], dubbed “AFT2019b”. The equations for the transported amplification factor and the transported modified intermittency are written as, respectively,

$$\frac{\partial(\rho\tilde{n})}{\partial t} + \frac{\partial(\rho u_j \tilde{n})}{\partial x_j} = \rho \Omega F_{crit} F_{growth} \frac{d\tilde{n}}{dRe_\theta} + \frac{\partial}{\partial x_j} \left[\sigma_n (\mu + \mu_t) \frac{\partial \tilde{n}}{\partial x_j} \right] \quad (3.82)$$

$$\begin{aligned} \frac{\partial(\rho\tilde{\gamma})}{\partial t} + \frac{\partial(\rho u_j \tilde{\gamma})}{\partial x_j} = & c_1 \rho S F_{onset} \left[1 - \exp(\tilde{\gamma}) \right] + c_2 \rho \Omega F_{turb} \left[c_3 \exp(\tilde{\gamma}) - 1 \right] \\ & + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_y} \right) \frac{\partial \tilde{\gamma}}{\partial x_j} \right] \end{aligned} \quad (3.83)$$

Once $\tilde{n}$ exceeds a critical amplification factor N_{crit} , transition is considered to have occurred. Given that $\tilde{n}$ is a global quantity and increases with the age of the boundary layer, there is no destruction term in Eq. 3.82. The source terms of Eq. 3.82 rely on an estimation of the integral boundary layer shape factor, H_{12} . Such integral quantities are difficult to compute in CFD, as it requires integration across a global domain and can become particularly difficult if the CFD code is parallelized and the grid is split across many processors. Thus, H_{12} is estimated based on a local shape factor suggested by Menter et al. [7] as,

$$H_L = \frac{d^2}{\nu} \left[\frac{\partial}{\partial x_i} \left(u_j \frac{\partial d}{\partial x_j} \right) \frac{\partial d}{\partial x_i} \right] \quad (3.84)$$

which functions similarly to the Thwaites parameter and the λ_θ parameter of the Langtry-Menter model. and ia an indicator of the pressure gradient imposed by the freestream. The integral shape factor is then estimated as,

$$H_{12} = 0.26 H_L + 2.4 \quad (3.85)$$

In linear stability theory, growth of N does not begin until instabilities begin to form. Analogously, production of $\tilde{n}$ does not begin until the modeled instabilities begin, indicated by some critical function. This instability-onset indicator is formulated to be a function of

the vorticity Reynolds number at the onset of instabilities, momentum thickness Reynolds number at the onset of instabilities, and the shape factor. Once the local vorticity Reynolds number exceeds that at the onset of instabilities, the production terms are activated and $\tilde{n}$ grows. This function F_{crit} is computed as,

$$F_{crit} = \begin{cases} 0, & Re_V < Re_{V,0} \\ 1, & Re_V \geq Re_{V,0} \end{cases} \quad (3.86)$$

where Re_V is the local vorticity Reynolds number and $Re_{V,0}$ is the vorticity Reynolds number at the onset of instability growth. The additional functions for F_{crit} are,

$$Re_V = \frac{\rho S d^2}{\mu + \mu_t} \quad (3.87)$$

$$Re_{V,0} = k_V Re_{\theta,0} \quad (3.88)$$

$$k_V = 0.246175 H_{12}^2 - 0.141831 H_{12} + 0.008886 \quad (3.89)$$

$$\log_{10}(Re_{\theta,0}) = 0.7 \tanh\left(\frac{14}{H_{12} - 1} - 9.24\right) + \frac{2.492}{(H_{12} - 1)^{0.43}} + 0.62 \quad (3.90)$$

The function k_V is the ratio of the vorticity Reynolds number to the momentum thickness Reynolds number where H_L is at a maximum in the boundary layer. Note that the formulation of Re_V includes the eddy viscosity in the denominator. This inclusion has no effect on laminar boundary layers, where eddy viscosity is small, but prevents large production of $\tilde{n}$ in turbulent boundary layers in the vicinity of flow separation. This ensures that the relaminarization of the boundary layer after separation is a permissible solution. The physical growth of the boundary layer is accounted for with the function F_{growth} ,

$$F_{growth} = D(H_{12}) \frac{1 + m(H_{12})}{2} l(H_{12}) \quad (3.91)$$

$$D(H_{12}) = \frac{2.4 H_{12}}{H_{12} - 1} \quad (3.92)$$

$$l(H_{12}) = \frac{6.54 H_{12} - 14.07}{H_{12}^2} \quad (3.93)$$

$$m(H_{12}) = \frac{1}{l(H_{12})} \left[0.058 \frac{(H_{12} - 4)^2}{H_{12} - 1} - 0.068 \right] \quad (3.94)$$

The functions $l(H_{12}), m(H_{12})$ were designed by Drela and Giles [90] to relate the growth of the momentum thickness Reynolds number to distance from the leading edge. Finally, the growth in the amplification factor with respect to momentum thickness Reynolds number (a tenet of approximate-envelope e^N theory) is estimated using H_{12} as,

$$\frac{d\tilde{n}}{dRe_\theta} = 0.028(H_{12} - 1) - 0.0345 \exp \left[- \left(\frac{3.87}{H_{12} - 1} - 2.52 \right)^2 \right] \quad (3.95)$$

Unlike Eq. 3.82, the transport equation for modified intermittency has both production terms to permit a laminar-turbulent transition solution and also destruction terms to permit relaminarization of a turbulent boundary layer. First, the production of $\tilde{\gamma}$ is controlled using the F_{onset} function, which begins production once transition onset has occurred. Specifically, once $\tilde{n}$ exceeds N_{crit} , production of modified intermittency begins.

$$F_{onset,1} = \frac{\tilde{n}}{N_{crit}} \quad (3.96)$$

The critical amplification factor N_{crit} is estimated as a function of freestream turbulence intensity and is a global parameter. This formulation is suggested by Mack [58] and Drela and Youngren [187] as,

$$N_{crit} = -8.43 - 2.4 \ln \left(\frac{\tau}{100} \right) \quad (3.97)$$

The parameter τ is a turbulence intensity limiter, based on that suggested by Ref. [187]. This limiter is purely numerical, intended to prevent negative values of N_{crit} for large turbulence intensities.

$$\tau = 2.5 \tanh \left(\frac{Tu(\%)}{2.5} \right) \quad (3.98)$$

The $F_{onset,1}$ function is limited in order to prevent excess production of intermittency, as suggested by Menter and Smirnov [7], and is,

$$F_{onset,2} = \min \left(F_{onset,1}, 2 \right) \quad (3.99)$$

An additional function $F_{onset,3}$ is designed to maintain fully turbulent levels of intermittency in turbulent boundary layers and laminar values of intermittency in laminar boundary layers. Consequently, the function is designed to be zero in turbulent boundary layers and unity in laminar boundary layers and is.

$$F_{onset,3} = \max\left[1 - \left(\frac{R_T}{3.5}\right)^3, 0\right] \quad (3.100)$$

$$R_T = \frac{\mu_t}{\mu} \quad (3.101)$$

The full function F_{onset} is given as,

$$F_{onset} = \max\left[F_{onset,2} - F_{onset,3}, 0\right] \quad (3.102)$$

which has a value of zero upstream of transition and a value of unity downstream of transition onset. Note that production is disabled once transition onset has occurred by the term $(1 - \exp(\tilde{\gamma}))$, as $\gamma \rightarrow 1$.

The destruction term is controlled by a single function F_{turb} , which prevents destruction of intermittency in turbulent boundary layers, where the characteristic turbulent Reynolds number R_T is large.

$$F_{turb} = \exp\left[-\left(\frac{R_T}{2}\right)^4\right] \quad (3.103)$$

Finally, the AFT model couples with the SA model via a modification to the f_{t2} term,

$$f_{t2} = c_{t3}[1 - \exp(\tilde{\gamma})] \quad (3.104)$$

which ramps up production of $\tilde{\nu}$ as intermittency increases. The constants of the AFT model are,

$$\begin{array}{ll} \sigma_n = 1.0 & c_1 = 100 \\ c_2 = 0.6 & c_3 = 50 \\ \sigma_y = 1.0 & c_{t3} = 1.2 \end{array}$$

with the boundary conditions,

$$\tilde{n}_\infty = 0 \quad (3.105)$$

$$\tilde{\gamma}_\infty = 0 \quad (3.106)$$

$$\left. \frac{\partial \tilde{n}}{\partial y} \right|_{wall} = 0 \quad (3.107)$$

$$\left. \frac{\partial \tilde{\gamma}}{\partial y} \right|_{wall} = 0 \quad (3.108)$$

The farfield boundary condition for $\tilde{n}$ is zero as there are assumed to be no disturbances in the freestream with $\tilde{\gamma}$ is set to zero as well (i.e., $\gamma = 1$). The Neumann conditions at the wall are both zero, as $\tilde{n}$ is a global parameter within the boundary layer and intermittency is constant within the viscous sublayer.

To detect transition onset, Spalart's turbulence index is used, as outlined in Eq. 3.35. This is purely a post-processing step. This metric is used herein when discussing the transition point for the AFT model.

For unsteady simulations, it is advantageous to use a hybrid RANS/LES method, as the boundary layer is modeled and maintains RANS-like behavior while the wake and off-body features are directly resolved [107]. For the AFT model, hybrid RANS/LES behavior is achieved by using the formulation of delayed, detached eddy simulation (DDES) [34]. In the boundary layer, the relevant length scale is maintained as the wall distance. Conversely, for wakes and separated regions, this length scale switches to a grid-based length scale. The switching function designed by Coder and Ortiz-Melendez [34] uses intermittency as the functional parameter, as traditional DDES sensor functions (i.e., designed for fully turbulent models) cannot distinguish a laminar boundary layer from a wake. This transitional DDES sensor is formulated as,

$$\tilde{d} = d - \gamma_{lim} f_d \max[0, d - C_{DES} \Delta] \quad (3.109)$$

$$\gamma_{lim} = \min[\exp(\tilde{\gamma}), 1] \quad (3.110)$$

where $C_{DES} = 0.65$, Δ is the maximum dimension of the grid cell in x, y, z , and f_d is a sensor function to determine approximately where a given grid point is in the boundary layer and

is formulated as,

$$f_d = 1 - \tanh[(8r_d)^3] \quad (3.111)$$

$$r_d = \frac{\nu + \nu_t}{\sqrt{u_{ij}u_{ij}}\kappa^2 d^2} \quad (3.112)$$

The DDES distance function r_d is designed to be large near the wall, unity in the log-layer, and to approach zero in the defect layer and outside the boundary layer, driving $f_d \rightarrow 1$ and switching to LES-like behavior. The constant appearing in the function f_d has been a subject of debate (cf. [117, 188, 189]) but remains 8 here for consistency with Ref. [34]. Additionally, the validity of r_d is questionable in the context of unsteady (and developing) boundary layers, as it relies on the assumption of constant stress in the log-layer. This may not necessarily be true for unsteady boundary layers and requires future investigation.

3.3 Navier-Stokes Solution Techniques

The accuracy of aerodynamic predictions depends greatly on the accuracy of the numerical methods used to discretize and solve the Navier-Stokes equations. Often, the methods used for RANS may be inappropriate for use with LES, and *vice versa*. While some schemes may have higher numerical dissipation that improves stability for RANS predictions, enabling larger timesteps, such large dissipation may not be desired for scale-resolving simulation, such as LES.

The present work utilizes the OVERFLOW solver [124], a structured, overset, density-based, RANS code designed and maintained by NASA. The OVERFLOW code solves the Navier-Stokes equations in curvilinear coordinates, which can be written in the form [190],

$$\frac{\partial \vec{q}}{\partial t} + \frac{\partial}{\partial \xi}(\vec{E} - \vec{E}_v) + \frac{\partial}{\partial \eta}(\vec{F} - \vec{F}_v) + \frac{\partial}{\partial \zeta}(\vec{G} - \vec{G}_v) = 0 \quad (3.113)$$

where $\vec{q}$ is the vector of conserved variables, $\vec{E}$, $\vec{F}$, and $\vec{G}$ are the inviscid fluxes, $\vec{E}_v$, $\vec{F}_v$, and $\vec{G}_v$ are the viscous fluxes, and ξ, η, ζ represent the curvilinear coordinate system. The

aforementioned vectors are,

$$\begin{aligned}
\vec{q} = J^{-1} \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e_0 \end{bmatrix}, \quad \vec{E} = J^{-1} \begin{bmatrix} \rho U \\ \rho u U + \xi_x P \\ \rho v U + \xi_y P \\ \rho w U + \xi_z P \\ U(\rho e_0 + P) - \xi_t P \end{bmatrix}, \quad \vec{F} = J^{-1} \begin{bmatrix} \rho V \\ \rho u V + \eta_x P \\ \rho v V + \eta_y P \\ \rho w V + \eta_z P \\ V(\rho e_0 + P) - \eta_t P \end{bmatrix}, \\
\vec{G} = J^{-1} \begin{bmatrix} \rho W \\ \rho u W + \zeta_x P \\ \rho v W + \zeta_y P \\ \rho w W + \zeta_z P \\ V(\rho e_0 + P) - \zeta_z P \end{bmatrix}
\end{aligned} \tag{3.114}$$

The velocities U, V , and W are the contravariant velocities, the equivalent of cell face-normal velocities, and are the velocity components in computational space. These can be computed from the physical-space velocities u, v , and w , along with the coordinate transformations between physical and computational space, written as,

$$\begin{aligned}
U &= \xi_t + \xi_x u + \xi_y v + \xi_z w \\
V &= \eta_t + \eta_x u + \eta_y v + \eta_z w \\
W &= \zeta_t + \zeta_x u + \zeta_y v + \zeta_z w
\end{aligned} \tag{3.115}$$

The viscous flux terms can be written as,

$$\begin{aligned}
\vec{E}_v = J^{-1} \begin{bmatrix} 0 \\ \xi_x \tau_{xx} + \xi_y \tau_{xy} + \xi_z \tau_{xz} \\ \xi_x \tau_{yx} + \xi_y \tau_{yy} + \xi_z \tau_{yz} \\ \xi_x \tau_{zx} + \xi_y \tau_{zy} + \xi_z \tau_{zz} \\ \xi_x \beta_x + \xi_y \beta_y + \xi_z \beta_z \end{bmatrix}, \quad \vec{F}_v = J^{-1} \begin{bmatrix} 0 \\ \eta_x \tau_{xx} + \eta_y \tau_{xy} + \eta_z \tau_{xz} \\ \eta_x \tau_{yx} + \eta_y \tau_{yy} + \eta_z \tau_{yz} \\ \eta_x \tau_{zx} + \eta_y \tau_{zy} + \eta_z \tau_{zz} \\ \eta_x \beta_x + \eta_y \beta_y + \eta_z \beta_z \end{bmatrix}, \\
\vec{G}_v = J^{-1} \begin{bmatrix} 0 \\ \zeta_x \tau_{xx} + \zeta_y \tau_{xy} + \zeta_z \tau_{xz} \\ \zeta_x \tau_{yx} + \zeta_y \tau_{yy} + \zeta_z \tau_{yz} \\ \zeta_x \tau_{zx} + \zeta_y \tau_{zy} + \zeta_z \tau_{zz} \\ \zeta_x \beta_x + \zeta_y \beta_y + \zeta_z \beta_z \end{bmatrix} \tag{3.116}
\end{aligned}$$

with the corresponding stress terms,

$$\begin{aligned}
\tau_{xx} &= \lambda(u_x + v_y + w_z) + 2\mu u_x & \tau_{xy} &= \tau_{yx} = \mu(u_y + v_x) \\
\tau_{xz} &= \tau_{zx} = \mu(u_z + w_x) & \tau_{yy} &= \lambda(u_x + v_y + w_z) + 2\mu v_y \\
\tau_{yz} &= \tau_{zy} = \mu(v_z + w_y) & \tau_{zz} &= \lambda(u_x + v_y + w_z) + 2\mu w_z \\
\beta_x &= \gamma \kappa P r^{-1} \partial_x e_1 + u \tau_{xx} + v \tau_{xy} + w \tau_{xz} & \beta_y &= \gamma \kappa P r^{-1} \partial_y e_1 + u \tau_{yx} + v \tau_{yy} + w \tau_{yz} \\
\beta_z &= \gamma \kappa P r^{-1} \partial_z e_1 + u \tau_{zx} + v \tau_{zy} + w \tau_{zz} & e_1 &= e \rho^{-1} - \frac{1}{2}(u^2 + v^2 + w^2)
\end{aligned}$$

The λ term represents the bulk viscosity, which is set to $-2/3\mu$, often called ‘‘Stokes’ hypothesis’’. Note that κ here is the thermal conductivity, not the von Kármán constant.

The ideal gas law closes the set of equations and is,

$$P = (\gamma - 1) \left[e - \frac{1}{2} \rho (u^2 + v^2 + w^2) \right] \tag{3.117}$$

The Jacobian J is used to facilitate the transformation between physical space and computational space, for which its inverse is,

$$J^{-1} = x_\xi y_\eta z_\zeta + x_\zeta y_\xi z_\eta + x_\eta y_\zeta z_\xi - x_\xi y_\zeta z_\eta - x_\eta y_\xi z_\zeta - x_\zeta y_\eta z_\xi \quad (3.118)$$

with the transformation derivatives, or grid metrics,

$$\begin{aligned} \xi_x &= J(y_\eta z_\zeta - y_\zeta z_\eta) & \zeta_x &= J(y_\xi z_\eta - z_\xi y_\eta) \\ \xi_y &= J(z_\eta x_\zeta - x_\eta z_\zeta) & \zeta_y &= J(x_\eta z_\xi - x_\xi z_\eta) \\ \xi_z &= J(x_\eta y_\zeta - y_\eta x_\zeta) & \zeta_z &= J(x_\xi y_\eta - y_\xi x_\eta) \\ \eta_x &= J(z_\xi y_\zeta - y_\xi z_\zeta) & \xi_t &= -x_t \xi_x - y_t \xi_y - z_t \xi_z \\ \eta_y &= J(x_\xi z_\zeta - x_\zeta z_\xi) & \eta_t &= -x_t \eta_x - y_t \eta_y - z_t \eta_z \\ \eta_z &= J(y_\xi x_\zeta - x_\xi y_\zeta) & \zeta_t &= -x_t \zeta_x - y_t \zeta_y - z_t \zeta_z \end{aligned}$$

3.3.1 Central-Difference Schemes

When employing LES, or any other scale-resolving simulation, particular attention should be paid to the selection of spatial and flux discretization scheme (i.e., the right-hand side of the Navier-Stokes equations). In the present work, a high-order central-difference scheme is used for discretization of the inviscid fluxes [191] when making ILES predictions. For simplicity, consider the one-dimensional Euler equation, with vector symbols dropped for convenience,

$$\frac{\partial q}{\partial t} + \frac{\partial E}{\partial x} = 0 \quad (3.119)$$

The differential operator $\partial/\partial x$ can be discretized using a central, finite-difference stencil to achieve a high order of accuracy. In OVERFLOW, this operator can be specified as 2^{nd} , 4^{th} , or 6^{th} order accurate in space. In the present work, the 6^{th} -order central-difference operator is used,

$$\frac{\partial E}{\partial x} \approx \frac{E_{j+3} - 9E_{j+2} + 45E_{j+1} - 45E_{j-1} + 9E_{j-2} - E_{j-3}}{60\Delta x} \quad (3.120)$$

Central difference schemes are known to experience odd-even decoupling, wherein the odd grid points and even grid points are solved separately by consequence of taking the central difference of a central difference. This can be overcome by using staggered grids, coupling the boundary conditions for odd and even points, or employing artificial dissipation. In OVERFLOW, artificial dissipation is used to overcome the odd-even decoupling problem. Considering only a one-dimensional grid in curvilinear coordinates, the artificial dissipation term takes the form [191],

$$\nabla_\xi(\sigma_j J_j^{-1} + \sigma_{j+1} J_{j+1}^{-1})(\epsilon_j^{(2)} \Delta_\xi q_j - \epsilon_j^{(4)} \Delta_\xi \nabla_\xi \Delta_\xi q_j + \epsilon_j^{(6)} \Delta_\xi \nabla_\xi \Delta_\xi \nabla_\xi \Delta_\xi q_j) / \Delta \xi \quad (3.121)$$

where ∇_ξ and Δ_ξ are differencing operators such that,

$$\nabla_\xi q_j = q_j - q_{j-1} \quad (3.122)$$

$$\Delta_\xi q_j = q_{j+1} - q_j \quad (3.123)$$

The dissipation terms are,

$$\epsilon_j^{(2)} = \epsilon_2 \max(\Upsilon_{j+1}, \Upsilon_j, \Upsilon_{j-1}) \quad (3.124)$$

$$\Upsilon_j = \frac{|P_{j+1} - 2P_j + P_{j-1}|}{|P_{j+1} - 2P_j + P_{j-1}|} \quad (3.125)$$

$$\epsilon_j^{(4,6)} = \max(0, \max(\epsilon_4, \epsilon_6) - \epsilon_j^{(2)}) \quad (3.126)$$

where the OVERFLOW user can specify ϵ_2 and either ϵ_4 or ϵ_6 , depending on the desired order of accuracy. The pressure switch function Υ_j is similar to the Laplacian of pressure and is used to scale the second-order dissipation, which increases the dissipation in the vicinity of a shock. Note that this form of artificial dissipation is that of scalar dissipation, rather than matrix dissipation, such as that seen in the Roe scheme [192]. The term σ_j is a spectral radius scaling and is given by,

$$\sigma_j = |U| + a \frac{\partial \xi}{\partial x} \quad (3.127)$$

and is the spectral radius of the inviscid flux Jacobian $\partial E/\partial q$. Setting different values of ϵ enables, or disables, which of the higher-order dissipation terms are active. OVERFLOW allows the user to specify **DIS2** (ϵ_2) regardless of the desired spatial accuracy (since 2^{nd} -order in space is the lowest-order option). The user can also specify **DIS4**, which is either ϵ_4 or ϵ_6 depending on the desired order of artificial dissipation [124, 191]. If 3^{rd} -order dissipation is desired, $\epsilon_4 \neq 0$ and $\epsilon_6 = 0$, where **DIS4** corresponds to ϵ_4 . If 5^{th} -order dissipation is desired, $\epsilon_4 = 0$ and $\epsilon_6 \neq 0$, where **DIS4** corresponds to ϵ_6 . For scale-resolving simulations, such as the implicit large eddy simulation in the present work, it is prudent to use the highest order of accuracy available with the highest order of artificial dissipation. In OVERFLOW, this corresponds to 6^{th} -order spatial differencing with a 5^{th} -order artificial dissipation term (**FSO** = 5), rendering the scheme formally 5^{th} -order accurate.

3.3.2 WENO Flux Reconstruction

Departing from finite difference schemes, finite volume schemes allow for improved flexibility in the vicinity of discontinuities, such as shocks, without requiring large amounts of artificial dissipation that may deteriorate solution accuracy. Considering the one-dimensional Euler equation in curvilinear coordinates, the equation can be recast into a hyperbolic conservation law as,

$$\frac{\partial q}{\partial t} + \hat{A} \frac{\partial q}{\partial \xi} = 0 \quad (3.128)$$

where $\hat{A}$ is the flux Jacobian, $\partial E/\partial q$. This formulation splits the fluxes into left-running and right-running components that are discontinuous, piecewise constant across a cell face. As these data are discontinuous across the cell face, reconstructing the data across the cell face is required in order to achieve a valid solution. Fortunately, several flux reconstruction methods exist with varying order of accuracy. When employing a higher order flux reconstruction, oscillations can occur in regions with high gradients. The weighted essentially non-oscillatory (WENO) [193] scheme combines several lower order reconstruction schemes, each weighted carefully such that oscillations are minimized while maintaining a high-order reconstruction. In OVERFLOW, a 5^{th} -order WENO reconstruction is achieved

by a weighted sum of three parabolic reconstructions of the fluxes at the left side and right side of the cell interface. Following Jiang and Shu [194] these three reconstructions are,

$$\begin{aligned} f_{j+\frac{1}{2}}^{(1)} &= \frac{1}{3}f_{j-2} - \frac{7}{6}f_{j-1} + \frac{11}{6}f_j \\ f_{j+\frac{1}{2}}^{(2)} &= \frac{-1}{6}f_{j-1} - \frac{5}{6}f_j + \frac{1}{3}f_{j+1} \\ f_{j+\frac{1}{2}}^{(3)} &= \frac{1}{3}f_j - \frac{5}{6}f_{j+1} + \frac{-1}{6}f_{j+2} \end{aligned} \quad (3.129)$$

At the left side of the cell interface, the flux is reconstructed and weighted by the weighting terms ω_k as,

$$f_{j+\frac{1}{2}}^L = \omega_1 f_{j+\frac{1}{2}}^{(1)} + \omega_2 f_{j+\frac{1}{2}}^{(2)} + \omega_3 f_{j+\frac{1}{2}}^{(3)} \quad (3.130)$$

$$\omega_k = \frac{\tilde{\omega}_k}{\sum \tilde{\omega}_k} \quad (3.131)$$

$$\tilde{\omega}_k = \frac{\gamma_k}{\epsilon + \beta_k^2} \quad (3.132)$$

$$\gamma_1 = \frac{1}{10}, \quad \gamma_2 = \frac{3}{5}, \quad \gamma_3 = \frac{3}{10}$$

$$\beta_k^{(1)} = \frac{13}{12} \left(f_{j-1} - 2f_j + f_{j+1} \right)^2 + \frac{1}{4} \left(f_{j-1} - 4f_j + 3f_{j+1} \right)^2 \quad (3.133)$$

$$\beta_k^{(2)} = \frac{13}{12} \left(f_{j-1} - 2f_j + f_{j+1} \right)^2 + \frac{1}{4} \left(f_j - f_{j+2} \right)^2 \quad (3.134)$$

$$\beta_k^{(3)} = \frac{13}{12} \left(f_{j+1} - 2f_{j+2} + f_{j+3} \right)^2 + \frac{1}{4} \left(3f_{j+1} - 4f_{j+2} + 3f_{j+3} \right)^2 \quad (3.135)$$

The smoothness indicators β_k account for contributions of the fluxes at each neighboring cell center and passively adapt the weighting terms ω_k . A symmetric procedure is performed at the right side of the cell interface. After this reconstruction process, the fluxes can be passed to a Riemann solver in order to solve Eq. 3.128.

ILES simulations in this work employ a 5th-order central-difference scheme rather than the upwinded, 5th-order WENO. While both schemes are 5th-order accurate, the 5th-order central-difference scheme as implemented in OVERFLOW features less dissipation in comparison to WENO. By selecting the **DIS2** and **DIS4** parameters in conjunction with **FSO = 5** in OVERFLOW, the user can directly control the dissipation of the scheme,

where in WENO the dissipation is automatically adapted based on the weighting terms. By performing a modified-wavenumber analysis, as shown in Fig. 3.1, it can be seen that while 5th-order central-difference and 5th-order WENO have the same dispersion error, 5th-order central-difference exhibits less dissipation error, for the recommended value of **DIS4** = **0.005** and no 2nd-order dissipation (as used in ILES in this work). Due to the lesser dissipation of the scheme, the 5th-order central-difference scheme was used for ILES in this work, as smaller scales can be more accurately resolved, leading to a higher quality solution compared to WENO. Additionally, the 5th-order central-difference scheme has the benefit of less computational time per iteration, reducing computational overhead for a given simulation.

3.3.3 Roe Flux-Difference Splitting

In the present work, full rotor computations utilize the Roe flux-difference splitting scheme [192] in order to solve the Riemann problem of Eq. 3.128. The Roe scheme takes the fluxes at the left side and the right side of a cell interface, computed by a reconstruction such as WENO, and locally linearizes the Riemann problem through the cell interface. In the context of the Roe scheme, the flux Jacobian $\hat{A}$ is the “Roe-averaged” form of the flux Jacobian and is constant between adjacent cells. The flux Jacobian is diagonalizable and takes the form,

$$\hat{A} = \hat{T} \hat{\Lambda} \hat{T}^{-1} \quad (3.136)$$

The Roe-averaged states are used to compute $\hat{T}$, $\hat{\Lambda}$, and $\hat{T}^{-1}$, which in one dimension are,

$$\hat{\rho} = \sqrt{\rho_L \rho_R} \quad (3.137)$$

$$\hat{u} = \frac{\sqrt{\rho_L} u_L + \sqrt{\rho_R} u_R}{\sqrt{\rho_L} + \sqrt{\rho_R}} \quad (3.138)$$

$$\hat{h}_0 = \frac{\sqrt{\rho_L} h_{0L} + \sqrt{\rho_R} h_{0R}}{\sqrt{\rho_L} + \sqrt{\rho_R}} \quad (3.139)$$

The flux at the cell interface is computed using a linear combination of the left-running-wave and right-running-wave fluxes with an additional dissipation term for stability,

$$f_{j+\frac{1}{2}} = \frac{1}{2}(f_L + f_R) - \frac{1}{2}|\hat{A}|(q_R - q_L) \quad (3.140)$$

and is repeated for both the left and the right side of the cell in question. Finally, the solution can be updated by integrating a finite difference equation and solving for the flux at the cell center, such as,

$$\Delta q = \frac{f_{j+\frac{1}{2}}^R - f_{j+\frac{1}{2}}^L}{\Delta x} \Delta t \quad (3.141)$$

3.3.4 The Pulliam-Chaussee Scheme

In order to march in time, the Navier-Stokes equations are solved in the general form of a matrix equation, $Ax = b$. However, inverting “ A ” is particularly expensive as the size of the grid increases. As such, this requires clever numerical techniques to efficiently invert, or factor, the left-hand side A . The Pulliam-Chaussee scheme [195] is one such left-hand side scheme for the Navier-Stokes equations that seeks to more efficiently approximately factorize the partial differential equations. In essence, the Pulliam-Chaussee scheme is a scalar, pentadiagonal version of the block, tridiagonal Beam-Warming scheme [196], rendering it much faster in terms of solution time when used in conjunction with 2^{nd} -order and 4^{th} -order artificial dissipation. The Navier-Stokes equations from Eq. 3.113 can be written in a linearized, implicit form as,

$$\begin{aligned} & \left[I + \frac{\Delta t}{(1+\theta)\Delta\tau} + \frac{\Delta t}{1+\theta} \left(\frac{\partial A}{\partial \xi} + \frac{\partial B}{\partial \eta} + \frac{\partial C}{\partial \zeta} \right) \right] \Delta q^{n+1,m+1} = \\ & - \left[\left(q^{n+1,m} - q^n \right) - \frac{\theta}{1+\theta} \Delta q^n + \frac{\Delta t}{1+\theta} RHS^{n+1,m} \right] \end{aligned} \quad (3.142)$$

where A, B , and C are the flux Jacobians in the directions ξ, η , and ζ , respectively, n represents the index in time, m represents the index of the subiterations, and RHS contains the right-hand-side of Eq. 3.113. The parameter θ controls the order of accuracy in time, where a value of 0 gives first-order and a value of 1/2 gives second-order. Inverting the

left-hand side of Eq. 3.142 is particularly cumbersome. This can be sidestepped by factoring in space using a five-point stencil as [197],

$$\begin{aligned} & \left[I + \frac{\Delta t}{1+\theta} \frac{\partial A}{\partial \xi} \right] \left[I + \frac{\Delta t}{1+\theta} \frac{\partial B}{\partial \eta} \right] \left[I + \frac{\Delta t}{1+\theta} \frac{\partial C}{\partial \zeta} \right] \Delta q^{n+1,m+1} = \\ & - \left[\left(q^{n+1,m} - q^n \right) - \frac{\theta}{1+\theta} \Delta q^n + \frac{\Delta t}{1+\theta} RHS^{n+1,m} \right] + Error \end{aligned} \quad (3.143)$$

where the term *Error* is the factorization error and is ignored here, resulting in an approximate factorization. Note that ignoring this error term reduces the CFL condition for stability relative to unfactored schemes. This factorization results in a so-called alternating direction implicit (ADI) scheme, wherein the matrices A , B , and C can be solved independently of each other, assuming the matrices do not commute. The matrices then undergo an eigenvalue decomposition, for which,

$$A = X_A \Lambda_A X_A^{-1} \quad (3.144)$$

$$B = X_B \Lambda_B X_B^{-1} \quad (3.145)$$

$$C = X_C \Lambda_C X_C^{-1} \quad (3.146)$$

By clever manipulation of this eigenvalue decomposition, a scalar pentadiagonal matrix can be written in each direction when a mix of 2^{nd} -order and 4^{th} -order smoothing are employed, as used in the aforementioned central differencing schemes. This form of the scheme was derived by Pulliam and Chaussee [195] and is a diagonal ADI (DADI) scheme. The Pulliam-Chaussee scheme is used in the present work when using central differencing schemes for scale-resolving simulations. The full, implicit form of the scheme can be written as,

$$\begin{aligned} & X_A \left[I + \frac{\Delta t}{1+\theta} \frac{\partial \Lambda_A}{\partial \xi} \right] X_A^{-1} X_B \left[I + \frac{\Delta t}{1+\theta} \frac{\partial \Lambda_B}{\partial \eta} \right] X_B^{-1} X_C \left[I + \frac{\Delta t}{1+\theta} \frac{\partial \Lambda_C}{\partial \zeta} \right] X_C^{-1} \Delta q^{n+1,m+1} = \\ & - \left[\left(q^{n+1,m} - q^n \right) - \frac{\theta}{1+\theta} \Delta q^n + \frac{\Delta t}{1+\theta} RHS^{n+1,m} \right] \end{aligned} \quad (3.147)$$

Separating Eq. 3.147 into diagonal and off-diagonal terms and factoring results in the diagonally dominant ADI scheme (DDADI). Diagonalizing DDADI gives the diagonalizable

diagonally dominant ADI scheme (D3ADI) [198], which has the additional benefit of not requiring artificial dissipation for stability.

It should be noted that one drawback of the Pulliam-Chaussee scheme, along with DDADI and D3ADI, is that it is very difficult to treat boundary conditions implicitly. During the solution process, each factor is treated independently (explicitly) and thus their respective boundary conditions are treated independently. In this context, solution convergence may be slowed in comparison to a scheme that can treat boundary conditions implicitly during the solution process.

3.3.5 The Symmetric Successive Over-Relaxation Scheme

In OVERFLOW, a symmetric successive over-relaxation (SSOR) scheme is implemented and has the benefit of eliminating the need for a factorization of Eq. 3.142 [197]. While the factorization error is eliminated, the computational expense is larger, as the entire implicit flux Jacobian must be stored in memory. Additionally, approximately 10 SSOR sweeps are required in order to converge the procedure.

SSOR sweeps are performed over each grid index j, k, l , with a forward sweep over the j index, and symmetric sweeps performed over the k and l indices. The tridiagonal Jacobians A , B , and C are broken up into their respective left blocks and right blocks and premultiplied by the inverse of the diagonal matrix $A_D + B_D + C_D$, each of which contains the diagonal blocks of A , B , and C . The SSOR scheme as implemented in OVERFLOW is,

$$\begin{aligned} \Delta q_{j,k,l}^{mm+1} = & (1 - \Omega) \Delta q_{j,k,l}^{mm} + \Omega (RHS - A_L \Delta q_{j-1,k,l}^{mm-1} - A_R \Delta q_{j+1,k,l}^{mm-1} \\ & - B_L \Delta q_{j,k-1,l}^{mk,1} - B_R \Delta q_{j,k+1,l}^{mk,2} - B_L \Delta q_{j,k,l-1}^{ml,1} - B_R \Delta q_{j,k,l+1}^{ml,2}) \end{aligned} \quad (3.148)$$

where Ω is the relaxation factor, for which the default value is 0.9. Following the notation of Nichols [197], the superscripts above Δq indicate the update levels in k, l during the solution

procedure. Forward sweeps in k and l gives,

$$mk,1 = mm + 1$$

$$mk,2 = mm$$

$$ml,1 = mm + 1$$

$$ml,2 = mm$$

and for backwards sweeps,

$$mk,1 = mm$$

$$mk,2 = mm + 1$$

$$ml,1 = mm$$

$$ml,2 = mm + 1$$

Note that one symmetric sweep encompasses a forward then a backward sweep. This SSOR method is quite robust in solving the implicit formulation of Eq. 3.142, but is much more expensive in comparison to the Pulliam-Chaussee scheme or D3ADI, for example. The SSOR method, however, implicitly treats the boundary conditions for overset grids, which aids in residual convergence.

Chapter 4

Near-Wall Behavior of the Langtry-Menter Model

Data source statement

The analysis of the near-wall behavior of the Langtry-Menter model was originally conducted as a publication in Flow, Turbulence and Combustion (cf. Flow, Turbulence and Combustion, Vol. 108, No. 3, March 2022, pp. 683-715). Contributions by the co-author, Dr. James Coder, have been omitted as they were not made by the present author. The contributions of the present author have been adapted for inclusion here.

4.1 Introduction

When employing a boundary-layer transition model for aerodynamic predictions, it is prudent to understand the behavior of the model in the near-wall regions of the boundary layer. The near-wall region of the boundary layer is generally defined as the region in which shear stress is constant [168]. In the viscous sublayer, the shear stress is dominated by the viscous stress. In the log-layer, the shear stress is dominated by the Reynolds shear stress and the influence of the viscous stress is diminished. While fully turbulent models must account for this in order to maintain accurate predictions of turbulent boundary layers, this may not be the case for transition models. When transition models are coupled to turbulence

models, this coupling often takes the form of some *ad hoc* modification under the assumption or expectation that once transition occurs, the model returns to the behavior of the fully turbulent model to which it is coupled. These modifications, however, do not preclude the presence unintended behavioral changes in the fully turbulent model [16, 127].

The $\gamma - Re_\theta$ model of Langtry and Menter [6] is a transition modeling framework that most often is coupled to Menter’s shear stress transport model [3]. In this form, the model is referred to as the “Langtry-Menter” model. The Langtry-Menter model couples to the SST model through the intermittency variable γ , which directly modifies the production term and destruction term of the turbulent kinetic energy equation of SST (cf. Eq. 3.60). This coupling to the SST model was intended to eliminate the influence of the γ and Re_θ equations on the fully turbulent solution. Assuming that transition has occurred already, the Langtry-Menter model was designed such that when $\gamma \rightarrow 1$ the production of turbulent kinetic energy returns to that intended by SST. While this holds true post-transition in the upper portions of the boundary layer and regions of homogeneous turbulence, where diffusion far outweighs production and destruction, it does not necessarily hold true near the wall.

One behavior of the Langtry-Menter model is that, given its local correlations, it cannot distinguish between a laminar boundary layer and the viscous sublayer of a turbulent boundary layer. To overcome this, the Langtry-Menter model ensures that in regions with zero turbulent kinetic energy production the destruction term outweighs the production term regardless if the boundary layer is laminar or turbulent. Specifically, the floor value of the destruction term in the turbulent kinetic energy equation is taken as 10% of that of the original SST model, which can be written generally as,

$$D_{k,LM} = \gamma_{lim} \beta^* k \omega \quad (4.1)$$

where the floor value of γ_{lim} is $\gamma_{min} = 0.1$. This ensures that very near the wall, turbulent kinetic energy is destroyed, which is consistent with the physics of the viscous sublayer. Other implementations of the Langtry-Menter model have investigated the effect of different values of γ_{min} on the behavior of transition. Barrouillet et al. [32] suggest that decreasing

the value from the suggested 0.1 to a value between 0.06 and 0.08 improves predictions of skin friction without substantially altering the predicted transition location. This does have the side effect of causing the wall-normal velocity profiles to deviate further from that intended by SST. Specifically, modifying γ_{min} changes the law-of-the-wall offset constant to a value lower than the intended value of 5.1 from SST [27, 199]. Additionally, in the near-wall region for a boundary layer in equilibrium, production and destruction of turbulent kinetic energy must be balanced with each other and zero-out the so-called “turbulent kinetic energy budget” near the wall. The modification of the destruction term of SST inherent to Langtry-Menter may violate this balance between production and destruction and is of interest in this investigation.

Aside from the destruction term, the production term of turbulent kinetic energy is also modified as it is multiplied by the effective intermittency, and takes the form,

$$P_{k,LM} = \gamma_{eff} \nu_t S^2 \quad (4.2)$$

This modification is intended to suppress turbulent kinetic energy production in the pre-transition boundary layer. Unlike the destruction term, the floor value of γ_{eff} is zero, ensuring that there is no production in the laminar boundary layer or the viscous sublayer.

Detecting transition accurately is a useful post-processing step for any transitional CFD solution. Transition models that couple with the Spalart-Allmaras model may use Spalart’s turbulence index [4] to detect the transition point. There exists a turbulence index analog for the SST model that was determined empirically by Strelets* and was designed such that a value of 0 indicates a laminar flow and a value of 1 indicates a turbulent flow. For the Langtry-Menter model, the use of the SST turbulence index is not particularly robust in detecting transition. For example, Denison [127] reported that the value of the SST turbulence index varies post-transition for a Langtry-Menter solution, with respect to grid resolution. Venkatachari et al. [16] reported that in the laminar boundary layer, SST turbulence index maintains a value near 0.5 for a few transitional-flow test cases. A value

*Private communication with Dr. Michael Strelets (NTS) and Dr. Philippe Spalart (now FlexCompute)

of 0.5 would suggest that the flow is transitioning at the grid point; however, it is known by skin-friction distributions that the boundary layer is fully laminar at those grid points. Thus, the Langtry-Menter solution lacks a robust definition of a turbulence index. Most often, the transition point is reported in literature by analyzing surface contours of intermittency. In the laminar boundary layer, the floor value of intermittency on the surface is $\gamma = 1/c_{e2}$, which has a value of 0.02. The floor value of intermittency on the surface for a fully turbulent boundary layer from Langtry-Menter is nebulous, as its value is dependent on the value of $Re_{\theta,t}$, but is generally between 0.04 and 0.1. Sheng et al. [144] used intermittency to determine transition on a rotor in hover, reporting that a value of 0.7 indicated a fully turbulent boundary layer, which is antithetical to the concept that $\gamma = 1$ indicates a fully turbulent flow. Skin friction is another commonly used metric for determining the transition location for the Langtry-Menter model [200]. Using skin friction, however, can prove difficult as the magnitude of skin friction that indicates transition will vary based on the problem. In this investigation, a Langtry-Menter-compatible turbulence index will be derived based on the behavior of the Langtry-Menter model near the wall such that the predicted transition location may be accurately and robustly determined.

4.2 Governing Equations Near the Wall

This study focuses on the behavior of the Langtry-Menter model in the near-wall region of a turbulent boundary layer, which encompasses the viscous sublayer, the buffer layer, and the logarithmic layer. Thus, it is prudent to analyze the model equations in near-wall coordinates by means of a coordinate transformation. Assuming a steady-state, two-dimensional, incompressible boundary layer, the advection terms of the left-hand sides of Eq. 3.36, 3.37, and 3.60 may be assumed negligible relative to the production, destruction, and diffusion terms on the right-hand side. Additionally, any streamwise derivatives vanish as the wall-normal derivatives dominate near the wall. Near the wall, production of $\widetilde{Re_{\theta,t}}$ goes to zero, meaning that the $\widetilde{Re_{\theta,t}}$ transport equation drops out and $\widetilde{Re_{\theta,t}}$ becomes simply

a free parameter of the flow that will affect the intermittency distribution in the boundary layer.

The following relations apply in the near-wall region and will be used to derive the near-wall formulation of the model equations, where a “+” indicates a near-wall quantity,

$$\begin{aligned} u_\tau &= \sqrt{\frac{\tau_w}{\rho}} & y^+ &= \frac{yu_\tau}{\nu} & u^+ &= \frac{u}{u_\tau} \\ \omega^+ &= \frac{\omega\nu}{u_\tau^2} & k^+ &= \frac{k}{u_\tau^2} & Re_T &= \frac{k^+}{\omega^+} \end{aligned} \quad (4.3)$$

The assumption of constant stress near the wall gives the additional relation that,

$$\frac{du^+}{dy^+} = \frac{1}{1 + Re_T} \quad (4.4)$$

Using the quantities in Eq. 4.3, the production, destruction, and diffusion terms of the k -equation of the SST model become,

$$P_k = \gamma_{eff}\nu_t \left(\frac{du}{dy}\right)^2 \rightarrow P_k^+ = \frac{u_\tau^4}{\nu} \left[\gamma_{eff} Re_T \left(\frac{du^+}{dy^+}\right)^2 \right] \quad (4.5)$$

$$D_k = \gamma_{lim}\beta^*k\omega \rightarrow D_k^+ = \frac{u_\tau^4}{\nu} \gamma_{lim} \left[\beta^*k^+\omega^+ \right] \quad (4.6)$$

$$\frac{d}{dy} \left[\left(\nu + \sigma_k \nu_t \right) \frac{dk}{dy} \right] \rightarrow \frac{u_\tau^4}{\nu} \frac{d}{dy^+} \left[\left(1 + \sigma_k Re_T \right) \frac{dk^+}{dy^+} \right] \quad (4.7)$$

The same can be done for the ω -equation to get,

$$P_\omega = \alpha \left(\frac{du}{dy}\right)^2 \rightarrow P_\omega^+ = \frac{u_\tau^4}{\nu^2} \alpha_1 \left(\frac{du^+}{dy^+}\right)^2 \quad (4.8)$$

$$D_\omega = \beta\omega^2 \rightarrow D_\omega^+ = \frac{u_\tau^4}{\nu^2} \beta_1 \omega^{+2} \quad (4.9)$$

$$\frac{d}{dy} \left[\left(\nu + \sigma_\omega \nu_t \right) \frac{d\omega}{dy} \right] \rightarrow \frac{u_\tau^4}{\nu^2} \frac{d}{dy^+} \left[\left(1 + \sigma_{\omega 1} Re_T \right) \frac{d\omega^+}{dy^+} \right] \quad (4.10)$$

Finally, the intermittency equation becomes,

$$P_\gamma = F_{length} c_{a1} \frac{du}{dy} \sqrt{\gamma F_{onset}} (1 - c_{e1} \gamma) \rightarrow P_\gamma^+ = \frac{u_\tau^2}{\nu} F_{length} c_{a1} \frac{du^+}{dy^+} \sqrt{\gamma F_{onset}} (1 - c_{e1} \gamma) \quad (4.11)$$

$$D_\gamma = c_{a2} \frac{du}{dy} \gamma F_{turb} (c_{e2} \gamma - 1) \rightarrow D_\gamma^+ = \frac{u_\tau^2}{\nu} c_{a2} \frac{du^+}{dy^+} \gamma F_{turb} (c_{e2} \gamma - 1) \quad (4.12)$$

$$\frac{d}{dy} \left[\left(\nu + \frac{\nu_t}{\sigma_f} \right) \frac{d\gamma}{dy} \right] \rightarrow \frac{u_\tau^2}{\nu} \frac{d}{dy^+} \left[\left(1 + \frac{Re_T}{\sigma_f} \right) \frac{d\gamma}{dy^+} \right] \quad (4.13)$$

These near-wall modifications reduce the Langtry-Menter model to a set of three ordinary differential equations for k^+ , ω^+ , and γ in the wall-normal direction,

$$\gamma_{eff} Re_T \left(\frac{du^+}{dy^+} \right)^2 - \gamma_{lim} \beta^* k^+ \omega^+ + \frac{d}{dy^+} \left[\left(1 + \sigma_k Re_T \right) \frac{dk^+}{dy^+} \right] = 0 \quad (4.14)$$

$$\alpha_1 \left(\frac{du^+}{dy^+} \right)^2 - \beta_1 \omega^{+2} + \frac{d}{dy^+} \left[\left(1 + \sigma_{\omega,1} Re_T \right) \frac{d\omega^+}{dy^+} \right] = 0 \quad (4.15)$$

and

$$F_{length} c_{a1} \frac{du^+}{dy^+} \sqrt{\gamma F_{onset}} (1 - c_{e1} \gamma) - c_{a2} \frac{du^+}{dy^+} \gamma F_{turb} (c_{e2} \gamma - 1) + \frac{d}{dy^+} \left[\left(1 + \frac{Re_T}{\sigma_f} \right) \frac{d\gamma}{dy^+} \right] = 0 \quad (4.16)$$

where $\widetilde{Re}_{\theta,t}$ becomes a parameter that influences the solution of γ via Re_{θ_c} , F_{length} , and F_{onset} . Note that a smaller value of $\widetilde{Re}_{\theta,t}$ corresponds to transition occurring for a young boundary layer (rather than occurring further downstream for an old boundary layer).

4.3 Near-wall Analysis

4.3.1 Turbulence Quantities Near the Wall

Recalling that the Langtry-Menter model cannot distinguish between the viscous sublayer and the laminar boundary layer, the value of intermittency very near the wall ($y^+ \leq 20$) decays even for a fully turbulent boundary layer when moving toward the wall. Whereas

for a fully turbulent boundary layer the intermittency may be unity in the log layer and defect layer, the value of intermittency will approach approximately $0.04 - 0.1$ on the wall, where the exact value depends on $\widetilde{Re}_{\theta,t}$. This behavior originates in the source terms of the γ -equation (Eq. 3.36) where the function F_{onset} vanishes and the term F_{turb} is near 1. Thus, the value of intermittency on the wall is attained by a balance of the destruction term and the diffusion term. This same behavior of intermittency decay was identified by Medida and Baeder [29] and was alleviated by performing non-local integration across the boundary layer to enforce $\gamma = 1$ in the viscous sublayer. This behavior of the intermittency is the impetus behind the differences in near-wall scaling for the Langtry-Menter model compared to SST and will be expounded upon in the upcoming sections.

It is first necessary to analyze the near-wall behavior of the SST model, such that quantitative comparisons may be made with the Langtry-Menter model. The dominant behavior of k and ω may be determined by performing a perturbation analysis near the wall, where the destruction term and the diffusion term are balanced. For the specific turbulent dissipation, it was shown in Chapter 3 in Eq. 3.24 that $\omega \sim y^{-2}$ on the wall. The same analysis can be done for k , where on the wall Eq. 3.15 reduces to,

$$\beta^* \omega k = \nu \frac{\partial^2 k}{\partial y^2} \quad (4.17)$$

for which it is assumed that $k \sim cy^n$. The wall value of specific turbulence dissipation is $\omega = 6\nu y^{-2}/\beta_1$ and may be substituted in, arriving to,

$$n(n-1) = 6 \frac{\beta^*}{\beta} \quad (4.18)$$

giving the solutions $n = -2.2295, 3.2295$. Only the positive value is taken to be physical, as if $k \sim y^{-2.2295}$ then the wall boundary condition of $k = 0$ would be violated. Thus, for SST, $k \sim y^{3.2295}$ and therefore the eddy viscosity ratio $Re_T \sim y^{5.2295}$. Wilcox argues that a more physical interpretation of turbulent kinetic energy is that $k \sim y^4$ or $k \sim y^2$ which can be achieved by varying β^* as a function of Re_T [175]. In the near-wall region, however, the

differences between $k \sim y^2$, $k \sim y^{3.2295}$, and $k \sim y^4$ are minimal, as y is still of $\mathcal{O}(\epsilon)$ in this region, regardless of the imposed value of n .

Now, the same analysis can be done for the Langtry-Menter model. For Langtry-Menter, the ω -equation is unchanged, meaning that $\omega \sim y^{-2}$. For the k -equation, however, this formulation is modified relative to SST. Performing another perturbation analysis starting with the balance between destruction and diffusion on the wall gives,

$$\gamma_{min}\beta^*\omega k = \nu \frac{\partial^2 k}{\partial y^2} \quad (4.19)$$

recalling that $\gamma_{min} = 0.1$. Assuming again that $k \sim cy^n$ results in,

$$n(n-1) = \gamma_{min}6\frac{\beta^*}{\beta} \quad (4.20)$$

which allows the values $n = -0.4849, 1.4849$. Again, only positive values of n are permitted for consistency with the wall boundary condition. Thus, for Langtry-Menter, $k \sim y^{1.4849}$ and therefore $Re_T \sim y^{3.4849}$. This near-wall behavior is substantially different from that of SST and the value of the exponent is dependent on the value of γ_{min} as well. Near the wall for $y^+ < 5$, k^+ is less than 1, meaning that the larger exponent of SST will result in slower production of turbulent kinetic energy in the viscous sublayer. Conversely, the smaller value of the exponent for Langtry-Menter will result in more rapid production of turbulent kinetic energy, greatly departing from that intended by SST.

An additional consequence comes from the behavior of the turbulent dissipation ϵ . Recalling that $\epsilon = \omega k$, the scaling of ϵ for Langtry-Menter gives $\epsilon \sim y^{-0.5151}$ whereas for SST $\epsilon \sim y^{1.2295}$. This insinuates that diffusion for the Langtry-Menter model is singular at the wall, whereas for SST $\epsilon \rightarrow 0$ as is physically intended. A true singularity can never be achieved in a Langtry-Menter solution by virtue of the wall boundary condition of ω in Eq. 3.24. It still remains, however, that the Langtry-Menter model will produce singular-like behavior on the wall, resulting in excessively high values of turbulent kinetic energy destruction and diffusion.

The singularity in ϵ can be removed if $n \geq 2$, satisfying that $\epsilon \sim y^{n-2}$ remains greater than or equal to 0. This requirement greatly modifies the value of γ_{min} , however, because for $n = 2$, $\gamma_{min} = 0.2\bar{7}$. Coder [27] has previously shown that for this value of γ_{min} the log-layer intercept increases greatly to approximately 33. This has the implication that the buffer layer and the log-layer are artificially laminarized as γ_{min} is increased. As such, the risk of relaminarizing the boundary layer increases for increasing values of γ_{min} . It is of note that as γ_{min} increases, the “strength” of the singularity on the wall decreases, but the risk of laminarization increases accordingly. For this analysis, the value of γ_{min} is kept at the suggested value of 0.1.

4.3.2 New Turbulence Index

The form of a turbulence index of a $k - \omega$ -type model can be generalized as,

$$i_t = C \frac{d(k^+)^m}{dy^+} \quad (4.21)$$

which can be rewritten in global coordinates as, recalling Eq. 4.3,

$$i_t = \frac{C}{u_\tau^{2m+1}} \frac{d(\nu k^m)}{dy} \quad (4.22)$$

The value of the exponent m is selected such that the derivative is linear in y on the wall, meaning that $m \equiv 1/n$. This is maintained in order to enforce (in the post-processing sense) that the variation of k^+ is approximately linear with y^+ on the wall. The value of the constant C is chosen during the derivation of the turbulence index, for which its value is calibrated to achieve $i_t = 1$ for the fully turbulent boundary layer. With this in mind, the turbulence index for SST is found to be,

$$i_{t,SST} = \frac{7.4}{u_\tau^{1.62}} \frac{d(\nu k^{0.31})}{dy} \quad (4.23)$$

which is consistent with that determined by Strelets. The Langtry-Menter turbulence index can be similarly derived to give,

$$i_{t,LM} = \frac{6.1}{u_\tau^{2.346}} \frac{d(\nu k^{0.673})}{dy} \quad (4.24)$$

Both Eq. 4.23 and 4.24 are calibrated such that a value of 0 indicates a laminar flow and a value of 1 indicates a turbulent flow, akin to that of Spalart’s turbulence index. For Langtry-Menter, the “true” value of C varies logarithmically with the value of $\widetilde{Re}_{\theta,t}$, but a value of 6.1 is sufficient to attain $i_{t,LM} = 1$ at the transition point.

4.4 Validation for Practical Aerodynamic Cases

A suite of two-dimensional and three-dimensional aerodynamic test cases are used to analyze the behavior of the Langtry-Menter model in the near-wall region. Validation was performed based on CFD solutions obtained from NASA’s OVERFLOW 2.3 solver [124], a structured, overset, RANS CFD code. Test cases were selected to cover a range of transition mechanisms for which the Langtry-Menter model is capable of predicting: bypass transition on a zero-pressure-gradient flat plate, natural transition on a zero-pressure-gradient flat plate at high-Reynolds number, an airfoil with a laminar separation bubble, a three-dimensional swept wing undergoing transition due to crossflow instabilities, and a transonic airfoil experiencing shock-induced separation. This provides a wide array of flow regimes against which to evaluate the newly developed Langtry-Menter turbulence index.

Three different turbulence closures are employed for each test case: the fully turbulent SST model [3], the Langtry-Menter model [6], and a variant of the Langtry-Menter model with the transitional momentum thickness Reynolds number fixed at 20 (i.e. $\widetilde{Re}_{\theta,t} = 20$). By enforcing a constant $\widetilde{Re}_{\theta,t} = 20$, this causes the Langtry-Menter model to transition to turbulence very early, which functions more like a fully turbulent version of the model. This gives a meaningful baseline to compare SST to Langtry-Menter in its fully turbulent mode. For the case with crossflow, the Langtry-Menter model with the crossflow modification of Langtry et al. [45] is used. In the data presented, these models are shortened to read

as “SST”, “LM2009”, and “LM2009, $Re_{\theta,t} = 20$ ”, respectively, unless specified otherwise. Similarly, the model with crossflow transition is shortened to “LM2015”.

Transition locations are determined using the turbulence indices of Eq. 4.23 and 4.24. In the data presented, the turbulence index for SST is referred to as $i_{t,SST}$ and the turbulence index for Langtry-Menter is referred to as $i_{t,LM}$.

4.4.1 T3A Zero-Pressure-Gradient Flat Plate

The ERCOFTAC T3-series provides a suite of flat-plate tests cases for transition model validation. The T3A case was selected for validation here due to its commonplace usage in transitional model validation and verification among the CFD community [201–203]. The turbulence intensity was specified at the inlet as 5.855% with a freestream eddy viscosity ratio of $\mu_t/\mu_\infty = 11.9$, as recommended by Coder [129], in order to match the experimental value of turbulence intensity at the leading edge and the decay rate of turbulence quantities across the plate. The freestream Mach number was set to 0.2, a value low enough to maintain incompressibility but large enough to avoid the need for low-Mach preconditioning. A grid family was provided by the AIAA CFD Transition Modeling Discussion Group[†], as the T3A case is featured as a standard test case among the group’s participants. The Fine (F) grid and the Extra Fine (X) grid were used here, as the less refined grids are known to provide poor predictions of transition due to their coarse spacings. The dimensions of the fine and extra fine grids are given in Table 4.1. Each grid was simulated using the SST model, the Langtry-Menter model, and the Langtry-Menter model with the fixed value of $\widetilde{Re}_{\theta,t} = 20$. Solutions were carried out until residuals decreased to near-machine-zero precision, which was a drop of approximately 15 orders of magnitude.

Turbulent kinetic energy budgets are given in Fig. 4.1 for the production term and destruction term of the near-wall formulation of the turbulent kinetic energy equation (Eq. 4.14). These wall-normal profiles were extracted at $Re_x = 3.0 \times 10^6$, or 75% of the length of the flat plate, where the boundary layer is considered fully turbulent. Note that the case of $\widetilde{Re}_{\theta,t} = 20$ triggers transition near the leading edge of the plate. The production term of

[†]<https://transitionmodeling.larc.nasa.gov/workshop.i/>

both variants of Langtry-Menter remain bounded, as expected since $k \sim y^{1.4849}$. There is, however, a large defect in turbulent kinetic energy production near the wall as a result of the near-wall behavior of Langtry-Menter. For the destruction term, there is singular-like behavior for both variants of Langtry-Menter as $y^+ \rightarrow 0$, confirming the previous near-wall analysis. Mathematically, the Langtry-Menter model can never grid converge, as singular-like behavior will always occur at the wall. Consequently, error-based grid adaptation using the Langtry-Menter model will be difficult as the error will always be subject to this near-wall singular-like behavior. The effect of $\widetilde{Re}_{\theta,t}$ is noticeable, as there is a greater defect in the destruction term for the Langtry-Menter case with free transition. This highlights the dependency on the value of $\widetilde{Re}_{\theta,t}$ in the near-wall behavior of the model.

The turbulence index is plotted in Fig. 4.2 against the streamwise Reynolds number for the Fine grid and Extra Fine grid for each model investigated. Additionally, skin friction is plotted as a reference. For the SST solution, the SST turbulence index attains a value of unity near the leading edge of the plate, as expected given its fully turbulent formulation. This behavior is confirmed in the skin friction distribution. The Langtry-Menter turbulence index accurately picks out the transition point, reaching a value of near 1 in tandem with the rise in skin friction. If the SST turbulence index were used with a Langtry-Menter solution, as shown in Fig. 4.2c, it is evident that there is a grid dependency on the value of the turbulence index at transition. Thus, the use of the SST turbulence index with a Langtry-Menter solution results in meaningless data for inferring the transition location predicted by the Langtry-Menter model. For the case of $\widetilde{Re}_{\theta,t} = 20$, the distribution of the turbulence index for the Fine grid is “choppy” near the leading edge. This is due to insufficient resolution in the wall-normal direction, wherein the wall-normal distribution of intermittency γ shows this same behavior. The near-wall behavior is greatly influenced by the point where the intermittency limiter γ_{lim} switches to 0.1, which may be a contributing factor for this behavior. As the wall-normal resolution is increased for the Extra Fine grid, these “kinks” vanish, suggesting there is a grid dependency on the distribution of intermittency. This is confirmed by examining the surface distribution of intermittency in Fig. 4.3, showing that the kinks smooth out as the grid is better resolved.

These kinks in both the intermittency distribution and the turbulence index distribution raise questions as to the uniqueness of the solution to the discrete equations of the Langtry-Menter model. To verify uniqueness, a solution of each variation of the Langtry-Menter model was restarted from the other variant. That is, the case using the baseline Langtry-Menter model was restarted from the fully converged solution file of the variation of Langtry-Menter with $\widetilde{Re}_{\theta,t} = 20$ as a starting point, and vice versa. These solutions converged to near-machine-zero and their respective flowfields converged to the same flowfield as if the simulation had started from scratch. This verifies that the solution to the discrete equations is indeed unique and the initial flowfield does not give different solution behavior for the same variation of the Langtry-Menter model.

4.4.2 TMR Zero-Pressure-Gradient Flat Plate

As a high-Reynolds number companion to the T3A case, the TMR zero-pressure-gradient cases is investigated, referred to here as the “TMR case”. The TMR case is a flat-plate test case that is featured on NASA’s Turbulence Modeling Resource (TMR) [204]. The TMR case features a chord-Reynolds number of 10×10^6 with a freestream turbulence intensity of 0.07% and a freestream eddy viscosity ratio of $\mu_t/\mu_\infty = 2 \times 10^{-7}$. The test case is intended to be incompressible; thus, the freestream Mach number was set to 0.2. The grid was provided on the NASA TMR webpage and features 273 points chordwise and 193 points wall-normally. Simulations were performed with each of the three turbulence closures. Solutions were carried out until residuals dropped to near-machine-zero, a drop of approximately 15 orders of magnitude.

Surface distributions of the momentum thickness Reynolds number and skin friction are plotted in Fig. 4.4. In comparison to the SST results, the Langtry-Menter variant with $\widetilde{Re}_{\theta,t} = 20$ overpredicts skin friction and the momentum thickness Reynolds number, a consequence of the near-wall scaling post-transition. This overprediction in skin friction was similarly reported by Barrouillet et al. [32], recommending that a lower value of γ_{min} be specified to alleviate this overprediction. In terms of the momentum thickness Reynolds

number, both Langtry-Menter models arrive to the same distribution post-transition, raising uncertainties in the actual integrated skin friction drag on the body.

4.4.3 PSU 94-097 Winglet Airfoil

The PSU 94-097 airfoil was designed for use as a winglet for high-performance sailplanes [20]. The airfoil has been tested extensively at the Pennsylvania State University’s Low-Speed Low-Turbulence Wind Tunnel for chord-Reynolds numbers between 2.4×10^5 and 1×10^6 . At low-to-moderate angles of attack, the upper surface of the airfoil is known to form a laminar separation bubble through which transition occurs. This provides an insightful test case into the Langtry-Menter model applied to separation-induced transition. Grids were generated at two different resolutions, denoted Medium (M) and Fine (F), with key grid parameters outline in Table 4.2. The grids were generated as O-type grids with a farfield distance corresponding to $1000c$ from the airfoil surface. The airfoil was simulated the 2° and 5° angle of attack at a chord-Reynolds number of 2.4×10^5 . The 5° case was simulated with both grids to investigate grid dependence, whereas the 2° case was simulated using only the Fine grid. For all simulations, the freestream eddy viscosity ratio was set to $\mu_t/\mu_\infty = 10$ with a freestream turbulence intensity of 0.07%, along with a freestream Mach number of 0.2. Predicted pressure distributions on the surface for both angles of attack are plotted in Fig. 4.5 along with data reported from experiments by Ref. [20]. The baseline Langtry-Menter model correctly predicts a laminar separation bubble on the upper surface at both angles of attack. For the case of $\alpha = 5^\circ$, the Fine grid predicts a slightly longer separation bubble than the Medium grid. Generally, the pressure distributions predicted by the baseline Langtry-Menter model agree well with that obtained from experiment.

Upper-surface surface distributions of skin friction and the turbulence indices are plotted in Fig. 4.6 for the $\alpha = 5^\circ$ case. The lower surface is fully laminar and excluded here for brevity. The skin-friction distributions of the Langtry-Menter solution confirm the existence of a laminar separation bubble. Note that the magnitude of skin friction is shown, meaning that the first inflection point is where the wall-normal gradient of streamwise velocity becomes negative, and the second inflection point is where it switches back to positive.

The distance between these two inflections is referred to as the “bubble length”. The SST turbulence index applied to the baseline Langtry-Menter solution, plotted in Fig. 4.6b, shows a spike near $x/c = 0.41$. This is approximately where laminar separation occurs but is not the transition point. Thus, the SST turbulence index does not correctly detect the transition point. On the other hand, the Langtry-Menter turbulence index does correctly indicate the transition location as further downstream. Additionally, the Langtry-Menter turbulence index maintains a value of approximately 2 after transition. It is expected that post-transition the turbulence index will be unity, but this assumes an equilibrium boundary layer, which is not necessarily true for all test cases. It remains, however, that a value of 1 still corresponds to the transition location. For the Langtry-Menter $\widetilde{Re}_{\theta,t} = 20$ solution in Fig. 4.6c and Fig. 4.6d, the Langtry-Menter turbulence index indicates transition at the same location as that given by its skin-friction distributions, $x/c \approx 0.18$. A similar result is achieved when examining the SST solution, where the transition location is slightly further upstream at $x/c \approx 0.15$.

Turbulent kinetic energy budgets at $\alpha = 2^\circ$ are plotted in Fig. 4.7 at two chordwise stations. The station $x/c = 0.70$ is in the middle of the laminar separation bubble and the station $x/c = 0.85$ is during turbulent reattachment. At the station $x/c = 0.70$, the Langtry-Menter production term increases in the buffer layer as the boundary layer separates from the surface. The destruction term is an order of magnitude larger than the production term (note the vertical axis ticks) as the Langtry-Menter model attempts to relaminarize the boundary layer. As $y^+ \rightarrow 0$, the destruction term becomes nearly singular for the Langtry-Menter model both in its baseline form and with forced $\widetilde{Re}_{\theta,t} = 20$. During turbulent reattachment at the station $x/c = 0.85$, the Langtry-Menter destruction term is still an order of magnitude larger than its production-term counterpart, exhibiting singular-like behavior at the wall as expected. This verifies that the singular-like behavior of the destruction term can be seen for both a flat-plate test case and an airfoil test case.

4.4.4 RAE 2822 Airfoil in Transonic Conditions

The RAE (Royal Aircraft Establishment) 2822 airfoil [205] is a rear-loaded, subcritical airfoil with a rooftop pressure distribution at the design condition, $M_\infty = 0.66$ and $C_l = 0.56$. The airfoil features a strong shock on the upper surface with shock-induced separation at transonic Mach number and moderate angles of attack. This test case features a shock-boundary-layer interaction and represents an extreme case against which to test the Langtry-Menter turbulence index. Based on Case 9 as specified by Cook et al. [205], the freestream Mach number was specified as 0.734 at a chord-Reynolds number of 6.5×10^6 and an angle of attack of $\alpha = 2.79^\circ$, which is corrected for tunnel effects [79].

The grid was generated using an in-house conformal mapping tool for airfoil C-grids based on input coordinates with a closed trailing edge. The grid features 513 chordwise points with an additional 129 points along the C-grid wake cut. The grid features 129 points in the wall-normal direction based on an initial wall spacing corresponding to $y^+ = 0.67$. Three different turbulence closure methods were investigated: the SST model [3], the Spalart-Allmaras model [4], and the baseline Langtry-Menter model [6]. The Spalart-Allmaras model was employed in order to provide a benchmark using the well validated Spalart's turbulence index, against which the SST and Langtry-Menter turbulence indices may be compared. The freestream eddy viscosity ratio was set to $\mu_t/\mu_\infty = 2.8 \times 10^{-7}$ for all simulations with a freestream turbulence intensity of 0.07%. Solutions were carried out until residuals decreased by at least 8 orders of magnitude.

Predicted surface pressure distributions are given in Fig. 4.8 for each simulation. Predictions are compared to experimental data digitized from Cook et al. [205]. The Langtry-Menter model predicts the upper surface shock downstream of that of the fully turbulent models. This a consequence of the thinner boundary layer due to the modeling of the laminar boundary layer, pushing the upper surface shock further downstream by the action of a reduced displacement thickness, reducing the effective angle of attack. This does, however, culminate in a stronger shock to facilitate the correct pressure recovery and exhibits more drastic separation through the shock. Thus, the shock location predicted

by the Langtry-Menter model may be more-so an affectation of the global behavior of the model rather than its near-wall behavior. Additionally, the Langtry-Menter model exhibits a laminar separation bubble on the lower surface at approximately $x/c = 0.5$, which was not seen in experiments.

The turbulence index for each turbulence closure is plotted for both the upper surface and lower surface in Fig. 4.9. For the Spalart-Allmaras solution, the turbulence index used is that of Eq. 3.35, referred to here and in the figures as $i_{t,SA}$. On the upper surface, both fully turbulent indices immediately give a value near 1. Conversely, the Langtry-Menter turbulence index is near zero, suggesting the boundary layer is initially laminar. Through the shock, each turbulence index greatly increases, as the behavior of the boundary layer deviates from the equilibrium conditions from which each turbulence index was derived. The transient peaks and troughs along the surface indicate regions of shock-induced separation, which is particularly pronounced for the Langtry-Menter solution. The Langtry-Menter solution attains a value of approximately 1.2 after turbulent reattachment and maintains a good metric for determining the boundary layer state. The lower surface does not experience a shock but does exhibit transition at $x/c = 0.5$ for the Langtry-Menter solution.

4.4.5 TU Braunschweig Sickle Wing

The TU Braunschweig Sickle Wing was designed by Petzold and Radespiel [71] to investigate spanwise variations in sweep angle and how these variations affect transition due to stationary crossflow instabilities. The wing was designed to damp Tollmien-Schlichting instabilities while amplifying stationary crossflow instabilities, rendering it a difficult test case for transition model validation. The wing is divided into four sections of increasing sweep angle. The root section has no sweep and serves to keep the wing out of the tunnel-wall boundary layer. The sweep breaks between each spanwise section introduce deviations from the semi-infinite-wing assumption, upon which many transition models are built.

The sickle wing is featured in the suite of test cases as provided with of the OVERFLOW 2.3 solver. The near-body grids were provided as a part of this test case. The wing is comprised of a main planform grid and a tip cap grid. The main planform grid features 497

points chordwise with an additional 61 points along the C-grid-type wake cut, 233 points spanwise, and 59 points in the wall-normal direction. The initial wall-normal spacing was 1.6×10^{-6} inches, based on a nominal $y^+ = 0.33$ at a chord-Reynolds number of 4.43×10^6 . The tip cap grid is comprised of 279 points around the tip with 73 points wrapping from the lower surface to the upper surface, with 59 points in the wall-normal direction with the same spacings as that of the main planform grid. The farfield was captured using a nested, Cartesian grid system with an isotropic cell spacing of 0.05 inches ($0.05c_{ref}$) for the nearest off-body brick. The full grid system (near-body and off-body grids) contains approximately 9.2 million points.

In this analysis, Case A of the sickle wing was simulated, featuring an angle of attack of -2.6° , a chord-Reynolds number of 2.75×10^6 , and a freestream Mach number of 0.156. The variant of Langtry-Menter used here is LM2015, which features a crossflow modification. From experiments, the peak-to-peak surface roughness was found to be $9.78\mu\text{m}$ and was used as the surface roughness input the the LM2015 model. Additional simulations were performed using LM2015 with a prescribed $\widetilde{Re}_{\theta,t} = 20$ in order to force early transition and examine the effects of the Langtry-Menter model on near-wall quantities. The freestream eddy viscosity ratio was set to $\mu_t/\mu_\infty = 5$ with a freestream turbulence intensity of 0.17%. Solutions were carried out until residuals decreased by at least 8 orders of magnitude.

Contours of transition quantities as predicted by the LM2015 model are plotted in Fig. 4.10. Contours of skin friction in Fig. 4.10a and 4.10b reveal a transition front that agrees well with the transition front determined from experiment. Additionally plotted in Fig. 4.10c and Fig. 4.10d are contours of the SST turbulence index applied to the LM2015 solution. With this metric, the transition front is predicted too far upstream compared to that of experiment and that shown by skin friction. On the upstream portions of the wing, the SST turbulence index gives intermediate values greater than 0 but less than 1, which raises the question as to whether this indicates laminar flow or a transition region. Similar results using the SST turbulence index have been reported by Venkatachari et al. [17]. The Langtry-Menter turbulence index in Fig. 4.10e and Fig. 4.10f accurately shows the predicted transition front and agrees well with experimentally reported transition on

both surfaces as well as with the front given by skin friction, suggesting the turbulence index correctly indicates predicted transition.

Line plots along the chord of the turbulence index are given for two spanwise stations in Fig. 4.11. The SST turbulence index, $i_{t,SST}$, and the Langtry-Menter turbulence index, $i_{t,LM}$, are plotted for LM2015 and the variation of LM2015 with prescribed $\widetilde{Re}_{\theta,t} = 20$. For the LM2015 solution, the Langtry-Menter turbulence index attains a value of 1 at the transition point and 0 upstream of transition. The SST turbulence index, however, has a value near 0.2 upstream of transition, which would suggest otherwise that the boundary layer may be in a state of transition, but it is known that this is not the case. At both stations, the SST turbulence index reaches unity far upstream of the actual transition location. It remains that the Langtry-Menter turbulence index is the most robust metric to post-process the transition location for a Langtry-Menter solution.

4.5 Summary and Discussion

The near-wall behavior of the Langtry-Menter model was investigated analytically and numerically. The near-wall scaling of turbulent kinetic energy for Langtry-Menter was found to differ from that intended by the SST model for a fully turbulent boundary layer, leading to singular behavior in the destruction term and the diffusion term when approaching the wall. As a result, the destruction term is significantly larger than the production term in the viscous sublayer. While this may be amenable to the Langtry-Menter model pre-transition, as the model cannot distinguish between a laminar boundary layer and the viscous sublayer, this singular behavior raises questions as to the state and health of the boundary layer post-transition. Numerically, this singular behavior is avoided by virtue of the wall boundary condition for the specific turbulent dissipation. This boundary condition, however, is dependent on the wall-normal distance from the wall to the first off-wall grid point, which can exacerbate the singular-like behavior as the grid is increasingly refined in the wall-normal direction. This singular behavior can be weakened by increasing the floor value of the intermittency limiter in the destruction term of the turbulent kinetic energy equation.

Unfortunately, this runs the risk of errant relaminarization and further alters the velocity profile of the turbulent boundary layer by increasing the log-layer intercept. In essence, this singular-like behavior on the wall will prevent the Langtry-Menter model from ever reaching asymptotic convergence, and introduces significant uncertainties in the solution of the model equations.

Several practical aerodynamic test cases were used to confirm the analytical scaling analysis. Analyzing the predicted turbulent kinetic energy budgets for the T3A flat plate and the PSU 94-097 airfoil confirmed the singular-like behavior in the destruction term. A new turbulence index compatible with the Langtry-Menter model was derived based on the near-wall behavior of the model. The index is designed such that a value of 0 indicates a laminar flow and a value of 1 indicates a turbulent flow. The new, Langtry-Menter turbulence index was applied to Langtry-Menter solutions and accurately detected the transition position on the TU Braunschweig sickle wing, correlating very well skin predicted skin friction distributions and experimentally determined transition positions. Applying the existing SST turbulence index to these same Langtry-Menter solutions led to erroneous interpretations of the transitional solution, particularly that the value of the index can exceed unity for a laminar boundary layer and shows a strong grid dependency. For a Langtry-Menter solution, if using automated analysis tools to detect transition as the point where the turbulence index first reaches unity, the SST turbulence index will not give the correct transition point, whereas the newly proposed Langtry-Menter turbulence index will.

The presented analysis procedure is generally applicable to other transition models that couple with two-equation turbulent models. This includes the numerous custom variants within the $\gamma - Re_\theta$ framework and one-equation, intermittency-based models. It is recommended that transition model developers verify the intended near-wall behaviors and adapt the turbulence index function to match the near-wall behavior of turbulent kinetic energy with an appropriate calibration coefficient.

Chapter 5

DLR RTG Rotor in Dynamic Pitching Conditions

Data Source Statement

The analysis of the DLR RTG rotor in dynamic pitching conditions was originally conducted for a paper submission at the Vertical Flight Society Forum 76 in 2020. This analysis was extended and accepted for publication in the Journal of the American Helicopter Society (cf. Journal of the American Helicopter Society, Vol. 67, No. 1, January 2022, pp. 012003-1 - 012003-14). These two works have been adapted for inclusion here.

5.1 Description of Test Case

The DLR (German Aerospace Center) RTG (Rotor Test Cell Göttingen) rotor blade was designed by Schwermer et al. [206] in order to experimentally investigate dynamic stall on rotor blades. The RTG is an open, Eiffel-type tunnel with a test section that is 5.25 ft $\times$ 13.1 ft, allowing ample room for experimental equipment. The blade of Schwermer et al. [206] was designed to represent a generic, Mach-scaled, and simplified rotor planform. The so-called RTG blade utilizes a single airfoil throughout its span, the DSA-9A airfoil [207, 208], and features a linear twist throughout the blade span. The DSA-9A airfoil is simply representative of a rotor airfoil that is known to undergo dynamic stall with a thickness

of 9% (i.e., Dynamic Stall Airfoil variant A with 9% thickness). In more recent years, the RTG rotor has been the subject of numerous experimental works, such as temperature-sensitive paint for a rotor in climb conditions [122, 209]. Similarly, the DSA-9A airfoil has been the subject of experiments on hot-film anemometry for a pitching airfoil [210] and differential infrared thermography (DIT) of a pitching airfoil [211].

The present investigation numerically expounds upon the experiments of Weiss et al. [212] on the DLR RTG rotor in pitching conditions. Weiss et al. [212] investigated unsteady boundary-layer transition on the RTG rotor undergoing a prescribed pitch cycle. The nonintrusive DIT method was used to determine boundary layer transition. The DIT camera system was fixed on a specific region of space where the camera system had a field-of-view corresponding to 18° of rotor azimuth. The camera system was mounted to a rotating mirror assembly which allowed capturing of images with the camera moving in the same direction as the rotor, reducing motion blur. During blade motion, the camera system took two images, one at each edge of the 18° -field-of-view. This allowed for capturing the change in transition locations throughout the pitch cycle and had the additional effect of providing time-correlated images such that the blade images can be de-warped during post-processing. The rotor swashplate was slowly rotated during the pitch cycle (approximately 1° each revolution) to facilitate the capturing of image pairs at every rotor azimuth of interest. Images were taken every three rotor revolutions. The DIT experiment performed sweeps on rotor frequency, tip Mach number, axial inflow rates, and pitching rate. For each case, an axial inflow was specified in order to prevent recirculation of tip vortices. In this sense, the rotor experiences flight conditions akin to that of climb. The tunnel freestream turbulence intensity was determined from previous experiments by Weiss et al. [122], computed by correlating transition locations at the 77% rotor radius with that predicted by linear stability theory. The associated critical N -factor was used to compute the freestream turbulence intensity via Mack's relation [58]. Based on the tangential velocity at the 77% rotor radius, the freestream turbulence intensity was determined to be 0.09%, or $N_{crit} \approx 8.4$. From these relations, Weiss et al. [212] performed numerical simulations of the aforementioned test cases using the DLR TAU solver [213] which will be used for additional comparisons.

In this study, Case II from the experiments of Weiss et al. [212] was simulated using NASA’s OVERFLOW 2.3 solver [124] with transition modeling [28] and hybrid RANS/LES enabled [34]. The test conditions of Case II are given in Table 5.1. Laminar-turbulent boundary layer transition predictions are compared to that reported by Weiss et al. [212]. The collective pitch was prescribed as 10.1° with an amplitude of 2.9° , with the pitch cycle prescribed such that it begins at the minimum pitch condition and is,

$$\theta = \theta_0 - \hat{\theta} \cos(2\pi t/T) \quad (5.1)$$

5.2 Rotor Geometry and Grid System

The DLR RTG blade features the DSA-9A airfoil throughout its span and features a chord length of 2.83 inches. The blade is twisted linearly by -9.3° from $0.25 < r/R < 1.0$ with a radius of 25.59 inches. The blade tip begins at $r/R = 0.91$ (i.e., the sweep break) and tapers parabolically to the tip [214], resulting in a taper ratio of 1/3. The four-bladed RTG rotor has a rotor solidity of 0.141, which is the ratio of rotor-planform area to the area swept by the rotor during rotation ($N_{blades} c_{ref}/(\pi R)$). A diagram of the rotor planform is shown in Fig. 5.1.

The structured, near-body, overset, surface grids were generated from CAD data of the RTG rotor as provided by Weiss et al. [212]. Each blade grid is comprised of three surface grids: a main planform grid, a root cap grid, and a tip cap grid. Initial wall spacing is based on $y^+ = 0.67$, from which the physical spacing was determined from flat-plate boundary-layer theory at a tip Mach number of 0.143 and a Reynolds number of 82,000 per inch. Volume grids were generated using Chimera Grid Tools [215] and were extruded 3 inches ($1.06c$) in the wall-normal direction using 65 wall-normal points, as shown in Fig. 5.2. Two different grids were generated in order to investigate predicted transition with respect to grid resolution. The “baseline” grid features 537 points in the chordwise direction, where 25 of those points encompass the blunt trailing edge. The “fine” grid features 50% more grid points in the chordwise direction, which is a 1/3 reduction in nominal grid spacing, with

805 points chordwise, where 37 of those points encompass the trailing edge. The spanwise grid spacing and wall-normal spacing are the same for both grid levels. Both the root cap grid and the tip cap grid were refined in the chordwise direction. Each volume grid contains approximately 11.9 million points and 17.9 million points for the baseline grid and fine grid, respectively.

In order to capture the rotor velocity field and pressure field, a nested, Cartesian, off-body grid system was designed. The OVERFLOW solver is capable of automatically generating off-body grids based on the user’s specification of the nearest, most-refined off-body grid. This nearest grid was specified to have an isotropic spacing of 0.2 inches ($0.07c$) in order to match the cell size at the edge of the blade volume grids. The nearest off-body grid is designed as a rectangular grid in the $x - y$ plane with extension into the z -direction. In both the x -direction and y -direction, which corresponds to the rotor plane, the grid extends from $-1.13R$ to $1.13R$. In the z -direction, which is below and above the rotor disk, the grid extends from $-0.98R$ to $0.31R$, as depicted in Fig. 5.3. In order to reduce the influence of the farfield boundary, the farfield distance was specified as 600 inches ($23.4R$) away from the center of rotation. Thus, the full grid system contains approximately 69.1 million and 93 million points for the baseline grid and the fine grid, respectively.

All simulations of the RTG rotor utilized NASA’s OVERFLOW solver [124], a structured, overset, RANS CFD code designed and maintained by NASA. OVERFLOW is capable of simulating moving bodies with high-order discretizations of convective fluxes. Roe upwind fluxes [192] were used for all grids, paired with a fifth-order WENO reconstruction [193] of the convective fluxes. The improved-SSOR scheme of Derlaga [216] was used for inversion of the left-hand side, as it efficiently inverts the system with better subiteration convergence compared to other left-hand side schemes. The AFT2019b model [28] was used for turbulence closure in conjunction with the negative variant of the Spalart-Allmaras model [217], as it provides accurate predictions of laminar-turbulent boundary-layer transition. Additionally, the rotation/curvature correction of Shur et al. [218] was used, as it more accurately predicts eddy viscosity in regions of solid-body-type rotation. Hybrid RANS/LES was used [34] in order to accurately predict off-body flow features. This turbulence closure is more formally

referred to as “SA-neg-RC-DDES-AFT2019b”, but will be referred to in this chapter as the “AFT” model or the “AFT2019b” model.

5.3 Results

Transition predictions were made using the 2019b version of the amplification factor transport transition modeling framework [28, 140] using NASA’s OVERFLOW 2.3 solver. In order to set up the velocity field and pressure field and convect initial transients out of the domain, 10 start-up revolutions were used. For these start-up revolutions, a dual time-stepping approach was used as it can setup the flowfield and advect transients from the domain with a reduced computational expense. During start-up, a maximum CFL number of 10 and a minimum CFL number of 1 was used in order to balance stability and convergence. A timestep corresponding to 1° of rotor rotation was used with 20 Newton-type subiterations. After the 10 start-up revolutions, the time-stepping scheme was switched to global timestepping, with a timestep corresponding to 0.25° of rotor rotation with 20 Newton-type subiterations. Solutions were carried out for an additional 6 rotor revolutions, until thrust, torque, and residual convergence did not appreciably change from revolution-to-revolution. During each timestep, the unsteady residual dropped approximately 1 order of magnitude during the subiterations. The temporal error estimates were approximately 1×10^{-6} for the mean flow and 7×10^{-4} for the combined turbulence and transition equations.

A sample time history of thrust and torque is shown in Fig. 5.4. Experimental thrust was reported as 20 N [212], corresponding to a thrust level of $C_T/\sigma = 0.037$. The present simulations, for both the baseline and fine grid, predicted a thrust level of $C_T/\sigma = 0.052$, averaged over the final rotor revolution. Although the predicted thrust is higher than that of experiment, the behavior of the boundary layer is primarily a function of the pitch cycle, and thus thrust prediction is of secondary import here. To confirm this, an additional simulation was performed in order to match the integrated rotor thrust to experiment and examine the transition predictions. This will be discussed later. It is of note that rotor thrust is more difficult to measure in experiments than the blade-root angle of attack. Thus, there is likely

higher uncertainty in the thrust measurement than the pitch-cycle measurement *. Similar overpredictions in thrust were reported by Jain [100] using two different CFD codes. Given that rotor performance prediction is not the primary objective of this study, the pitch cycle prescribed is kept the same as that of experiments.

5.3.1 Determination of Transition Location

In order to compute the predicted transition locations, Spalart’s turbulence index was used [4], shown previously in Eq. 3.35. Surface contours of the turbulence index were plotted every 1° of rotor rotation, digitized, and extracted to report $(x/c)_{tr}$ as a function of r/R and t/T . Sample contours of Spalart’s turbulence index are given in Fig. 5.5.

When post-processing the transition location from a transitional simulation, it is common to use intermittency or skin friction as a metric to determine the transition location. For an intermittency-based model such as Langtry-Menter [6] or the γ -model, the value of intermittency on the wall is $1/c_{e2}$ (0.02) for a laminar flow, but the wall value for a turbulent flow is not always so consistent. Additionally, the intermittency variable does not diffuse down to the wall. When using skin friction, surface values can be misleading during laminar separation or regions where the boundary layer relaminarizes. Illustrated in Fig. 5.6 are sample skin friction contours at the same portion of the pitch cycle as was shown for the turbulence index in Fig. 5.5. Early in the pitch cycle, the skin friction is a relatively good indicator of the transition point, as the skin friction clearly increases near the trailing edge and exhibits a clear transition front. At the maximum pitch condition at $t/T = 0.5$, skin friction would suggest that the flow in the vicinity of the sweep break is fully laminar; however, this is not the case. as the eddy viscosity grows in the wall-normal direction near the 80% chord. On the other hand, Spalart’s turbulence index does predict transition correctly here. In the same vein, at $t/T = 0.75$, the skin friction contours suggests that near the 80% radius transition occurs at approximately the mid-chord. This is not the case, however, since transition occurs at the 40% chord. Therefore, Spalart’s turbulence index is used to

*Private communication with Dr. Anthony Gardner of DLR

post-process the transition location as it accurately determines where the turbulence model is fully active.

5.3.2 Predicted Transition Locations

Predicted transition locations as a function of the blade radius and the pitch cycle are given in Fig. 5.7 for both grid levels along with that measured in experiments [212]. In the polar plots, the pitch cycle begins at $t/T = 0$ for which the collective pitch is at its minimum. The left half of the polar corresponds to the blade in upstroke (i.e., pitching up) and the right half of the polar corresponds to the blade in downstroke (i.e., pitching down). During the first quarter of the pitch cycle ($0 < t/T < 0.25$), the blade experiences long runs of laminar flow for both grid levels. Specifically, both grid levels predicted transition occurs approximately 5% further upstream than that determined by experiments. The inboard regions of the blade are fully laminar due to the locally low Reynolds number. Although there is a root vortex system near this region, it does not appreciably affect the surface flow as it is convected below the rotor due to the axial inflow velocity. Between the stations $r/R = 0.45$ and $r/R = 0.90$, transition is predicted to occur near the 70% chord, whereas experiments show transition occurs near the 83% chord. The blade tip is predicted to be primarily laminar, where the fine grid predicts that the tip region is laminar for longer. During the second-half of the upstroke ($0.25 < t/T < 0.50$), the predicted transition location begins to move upstream in the middle regions of the blade. As the rotor pitches upward, the transition front moves upstream first from the outboard sections of the blade, and then the inboard sections. Near $t/T = 0.33$, the transition front is predicted to move upstream rapidly for both grid levels. Experimental data suggest that this upstream motion occurs more slowly during this portion of the upstroke. This could be a result of how the AFT model responds to changes in pressure gradient. As the upper surface pressure gradient becomes increasingly adverse as collective pitch increases, AFT2019b appears to respond slowly to these changes, suggesting that the model does not “feel” the changes in pressure gradient rapidly enough. In the region near the sweep break, specifically $0.88 < r/R < 0.95$, transition is predicted further downstream compared to other radial stations on the blade, suggesting there may be some additional

physics delaying transition here. In fact, there is a streamwise vortex released at the sweep break that affects the boundary layer. This will be discussed further in this section. Inboard of the sweep break region, the transition front moves upstream rapidly during the upstroke; however, the tip regions appear to be unaffected by this motion, suggesting that the inboard region may have been tripped to turbulence. During the downstroke, the boundary layer recovers to a larger extent of laminar flow. The initial downstream motion of the transition front occurs slightly earlier in the pitch cycle compared to experiments. Generally, the fine grid captures this downstream motion better than the baseline grid. Additionally, the fine grid accurately predicts the curvature of the isolines on the polar. Contours obtained from the fine grid solution are smoother than that of the baseline grid solution, suggesting that the additional chordwise refinement was better able to capture the relevant length scales in the boundary layer. In experiments, between $0.7 < t/T < 0.95$ the transition front moves downstream and eventually settles to a more steady boundary layer. In this region, the local angular velocity $d\theta_0/dt$ is small and does not greatly affect the transition location. This trend is captured well in both the baseline and fine grid solutions. This relaminarization of the boundary layer during the downstroke can be seen in Fig. 5.5d, where there are some locally relaminarized features downstream of the apparent transition line. Predictions using AFT2019b show that this relaminarization process has completed by $t/T = 0.875$, whereas this behavior does not occur until $t/T = 0.95$ in experiments.

To further investigate the unsteady behavior of the boundary layer, transition locations at select radial stations are plotted in Fig. 5.8 - 5.10 as a function of the pitch cycle t/T . Shown in each line plot are results obtained from experiments [212] and predictions obtained from the DLR TAU solver [212] using the AHD transition criterion [219]. Given in Fig. 5.8 is predicted transition as a function of the pitch cycle for both grid levels, experiments, and that from DLR TAU at the $r/R = 0.615$ station, which is representative of the midspan region of the blade. During the first quarter of the pitch cycle, both AFT2019b solutions predict little motion of the transition front, agreeing well with experiment. Near the mean pitch in upstroke at $t/T = 0.25$, experiments show the transition front begins to move upstream. AFT2019b predictions show the upstream motion, but much more rapidly, as was seen in

the polar plots as well. The transition location is predicted to be nearly constant at $0.4 < t/T < 0.7$ for both AFT2019b solutions and DLR TAU solutions. Experiments suggest that transition is further downstream at the maximum pitch condition than was predicted. Given that both AFT2019b and the AHD criterion are stability-based transition models, there may be some additional transition physics that are not accounted for near maximum pitch. During the upstroke, AFT2019b predictions match very well with experiments, especially so for the fine grid. When examining the 72% radius in Fig. 5.9 many of the same phenomena can be seen, as this radial station experiences a flowfield similar to that of the 62% station shown previously. Again, both AFT2019b solutions exhibit a rapid upstream motion of the transition front with near-constant transition at the maximum pitch, settling near the 25% chord. This suggests that there may be some phenomenon occurring here to trip the laminar boundary layer. In fact, there is laminar separation just upstream of the 25% chord and the boundary layer quickly reattaches as turbulent. This can be visualized in Fig. 5.6c near the leading edge, where the skin friction locally decreases through the separation region and rapidly increases after reattachment. Analyzing the region just inboard of the sweep break in Fig. 5.10 reveals additional phenomena not seen in experiments. The station $r/R = 0.908$ is the most outboard station for which there are experimental data. At the sweep break at $r/R = 0.91$, a streamwise vortex is released, which has increasingly higher circulation as the pitch angle increases. This streamwise vortex does indeed interact with the unsteady boundary layer, having a pronounced effect near the maximum pitch condition. In this region, $0.4 < t/T < 0.6$, the transition location reflexes downstream, rather than moving upstream as previously seen. This does not agree well with experiments nor with DLR TAU predictions. The presence of this vortex appears to stabilize the laminar boundary layer near maximum pitch. For lower pitch angles, this streamwise vortex has seemingly little effect. These errant predictions highlight the need to accurately predict the vortex dynamics, as this has an impact on the predicted rotor power required and on the stability of the boundary layer. Jain [100] did not explicitly report this behavior, but the transition polars included in his work appear to show this same behavior. Additionally, he reported that structural dynamics did not play a large role in predictions on the RTG rotor, due to

the low thrust level and tip Mach number, suggesting that this vortex interaction is not due to the rigid-rotor assumption inherent to the present work.

Transition locations can also be plotted as a function of pitch angle to reveal hysteresis in the predicted transition location. Shown in Fig. 5.11 are predicted transition locations at $r/R = 0.615$ as a function of the collective pitch θ_0 for AFT2019b, DLR TAU, and experimental data. During the upstroke, the shape of the hysteresis loop matches well with experiments, although it is shifted slightly downward due to the more upstream predicted transition. The rapid upstream motion of the predicted transition front can be again seen near maximum pitch. The downstroke region agrees well with experiments, at least in terms of the trendline, near the mean pitch angle. In that region, differences between experimentally determined transition and predictions using the baseline grid are minimal. Generally, the area bounded by the hysteresis loop for AFT2019b is lesser than that of experiments, meaning that the AFT2019b predictions exhibit a more rapid rate of change in transition location throughout the cycle. When examining the station $r/R = 0.723$ in Fig. 5.12, the rapid upstream motion of the transition front is again seen in the upstroke. At the beginning of the downstroke, the transition front is slow to respond to the change in pitch. This is likely an affectation of the laminar separation near the leading edge at maximum pitch, attracting transition to occur near the 25% chord and requiring more aggressive changes in pitch angle to move the transition front. Beginning at approximately 11.5° pitch in the downstroke, the transition point begins to move downstream, although this motion is much slower than that seen during the upstroke. This slower response agrees better with experiments, suggesting that AFT2019b can more accurately respond to changes in the boundary layer due to an increasingly favorable pressure gradient. Near the 9.5° collective pitch in downstroke the boundary layer begins to recover to laminar flow more quickly, which correlates very well with experiments. About the sweep break at $r/R = 0.908$ in Fig. 5.13, the impact of the streamwise vortex has a significant effect on the shape of the hysteresis loop for the AFT2019b solutions. In the regions near maximum pitch, predicted transition does not agree well with experiments; however, transition does agree well at low collective pitches. Predictions made by DLR TAU also do not agree well with experiments here. The

TAU solutions show a “jagged” behavior in the transition location, suggesting there are some additional physics here for which the AHD criterion does not account. It remains, however, that the impact of the streamwise vortex must be accurately predicted and does indeed require further investigation in both computations and experiments.

The effect of transition on the blade performance can be seen by plotting sectional surface pressure distributions for multiple radial stations. In this case, three radial stations are plotted in Fig. 5.14 for the baseline grid solution at the maximum pitch condition. Two stations are inboard of the sweep break and one station is on the sweep break itself. For the two inboard stations, $r/R = 0.804$ and $r/R = 0.881$, the predicted transition location is approximately the same at $x/c = 0.25$, evidenced by the local increase in surface pressure. These two stations likely experience laminar-separation-induced transition. At the sweep break at $r/R = 0.91$, the pressure does not see a rapid increase until $x/c = 0.82$. Although there is a small kink in the pressure distribution at $x/c = 0.3$ for $r/R = 0.91$ that appears to indicate some separation, transition does not occur here. This suggests that the streamwise vortex aids in preventing laminar separation, but does not have so strong of circulation as to trip the boundary layer to turbulence until much further downstream. This phenomenon does require further investigation and highlights the need for accurate vortex predictions.

5.3.3 Effect of Freestream Turbulence Intensity

The definition of freestream turbulence intensity is poorly posed for rotary wings as they lack an intrinsic velocity scale against which the local perturbations may be related. Bodies in the inertial frame simply use the freestream velocity as the velocity scale for turbulence intensity. For bodies in a rotating frame, the local freestream velocity is the product of the angular velocity and the distance to the center of rotation, which introduces a radial component of velocity into the definition of freestream turbulence intensity. The selection of this velocity scale introduces uncertainties into rotorcraft CFD solutions, especially when transition is of import. As aforementioned, Weiss et al. [122] suggest the tangential velocity at the 77% station be used for the velocity scale in turbulence intensity.

The AFT model incorporates the freestream turbulence intensity in its transition criterion N_{crit} . For larger turbulence intensities, N_{crit} decreases and transition is expected to occur increasingly further upstream. To investigate the effect of freestream turbulence intensity on the RTG rotor, three additional simulations were performed on both grids, with each simulation using a larger turbulence intensity. The turbulence intensities were selected based on fractions of $N_{crit} = 8.4$ such that the highest turbulence intensity is one order of magnitude larger than that suggested by Weiss et al. [122]. The selected freestream turbulence intensities were specified as 0.2%, 0.5%, and 0.9%.

A freestream turbulence intensity of 0.2%, as shown in Fig. 5.15, results in a decrease of overall laminar area. During the upstroke, the upstream motion of the transition front is slower than for the 0.09% case (cf. Fig. 5.7). In the downstroke, the boundary layer takes longer to recover to laminar flow. Again, the fine grid solution shows smoother contours than the baseline grid solution. Generally, at this turbulence intensity, the rate of upstream motion of the transition front correlates marginally better with experiments, but still does not accurately predict transition during the upstroke. Increasing the turbulence intensity to 0.5%, as shown in Fig. 5.16, the blade tip becomes primarily turbulent for $0.37 < t/T < 0.67$, suggesting that the higher turbulence intensity overrides the stabilizing effect of the streamwise vortex. Over the full pitch cycle, both the upstream motion and downstream motion is slower than that of the original simulations. In the upstroke, the outboard regions transition further upstream compared to the inner regions, suggesting that transition is sensitive to the twist distribution at this turbulence intensity. The lesser extent of laminar flow is further exacerbated at a turbulence intensity of 0.9%, shown in Fig. 5.17. The increase of turbulence intensity appears to reduce the extent of laminar flow, which is expected in the context of the AFT model. The overall upstream and downstream motion of the transition front is slower for increasing turbulence intensities, which may be an effect of the time scales intrinsic to the AFT model.

Sectional transition locations for the turbulence intensity sweep are given in Fig. 5.18 at the 61.5% radial station for the fine grid solution. As freestream turbulence intensity increases, the overall motion of the transition front decreases, meaning that the boundary

layer transitions increasingly further upstream throughout the pitch cycle. Near the maximum pitch condition in the middle of the pitch cycle, the predicted transition location is nearly the same for each turbulence intensity, suggesting the existence of a laminar separation region on the surface of the blade. For the 0.09%, 0.2%, and 0.5% cases, the recovery rate of the boundary layer is nearly the same during the last quarter of the pitch cycle. Conversely, the 0.9% case exhibits a much slower recovery rate, suggesting that this recovery rate may be fixed by the geometry of the blade and its pitch cycle, and that the recovery rate responds only weakly to turbulence intensity. The hysteresis loop at the 90.8% station can be plotted, as in Fig. 5.19. As the freestream turbulence intensity increases, the area bounded by the loop becomes increasingly smaller, indicating less overall motion of the transition front. Additionally, the influence of the streamwise vortex released at the sweep break diminishes with increasing turbulence intensity. The higher turbulence intensity partially nullifies the stabilizing effect of the boundary layer due to the presence of the vortex, introducing additional uncertainties into the value of turbulence intensity on the blade.

An additional simulation was performed to investigate whether stepping down from high turbulence intensity to low turbulence intensity affects the transition predictions at low turbulence intensity. To realize this, simulations were performed based on a restart from the 0.9% turbulence intensity solution for both grid levels. The turbulence intensity was decreased to 0.09%, that of the original simulation, and the simulation was carried out for two additional rotor revolutions. If these predictions are the same as the original simulation, then this verifies that the solution is indeed unique. Plotted in Fig. 5.20 are predicted transition polars for both grid levels after the stepdown from a turbulence intensity of 0.9% to 0.09%. There are some minor differences with the predictions of the original solution. For example, near the 45% radial station at $t/T = 0.8$, there is a greater extent of laminar flow. Additionally, the recovery of the boundary layer to laminar flow takes slightly longer during the downstroke. These differences, however, are more likely to be a result of transient effects rather than an affectation of a non-unique solution.

5.3.4 Effect of Rotor Thrust Level

In many cases, especially for fixed-wing systems, matching the thrust coefficient or the lift coefficient is desired in order to best replicate experiments. For the RTG rotor, blade element momentum theory shows that reducing the collective pitch from 10.1° to 8.6° should decrease the predicted thrust to $C_T/\sigma = 0.037$, the experimental value. As such, a simulation was performed with this reduced collective pitch. The predicted thrust level was $C_T/\sigma = 0.038$, which is slightly higher than experiments but allows for comparison of predicted transition at nearly the same thrust level as experiments. Shown in Fig. 5.21 are predicted transition locations at this lower collective, based on the baseline grid. It is immediately obvious that the predicted transition locations agree poorly with experiments. This reiterates that for this test case, the relevant basis of comparison between experiments and simulations lies in matching the pitch cycle, rather than the thrust level.

5.4 Summary and Discussion

The four-bladed DLR RTG rotor with a prescribed pitch cycle was used as a test case for unsteady boundary layer transition predictions. Transitional solutions were obtained via the OVERFLOW 2.3 RANS solver, used in conjunction with the SA-neg-RC-DDES-AFT2019b transition model. Two different grid levels were employed to investigate transition predictions with respect to grid spacing. Transition predictions were compared to those obtained experimentally at the German Aerospace Center. Rotor thrust was greatly overpredicted in comparison to experiments. Predicted transition agreed relatively well with experiments for portions of the upstroke and the downstroke. During the upstroke, predictions only match well with experiments in that the transition front moves upstream. This upstream motion of the transition front was found to occur much faster than seen in experiments and was seen for both grid levels. This suggested that the AFT2019b model may not respond correctly to increasingly adverse pressure gradients. Near the maximum pitch condition, the transition model underpredicted the extent of laminar flow, as laminar separation on the surface fixed transition near the 25% chord near maximum pitch, which was seen when

examining sectional transition positions as a function of the pitch cycle. In the downstroke, the transition predictions agree very well with experiments, both in the transition location and the rate at which the boundary layer recovers laminar flow. In the region about the sweep break, the presence of a streamwise vortex appeared to stabilize the boundary layer, extending the runs of laminar flow to near the trailing edge. This phenomenon was not seen in the experimental data but was similarly reported in computations by Jain [100].

The effects of freestream turbulence intensity on transition predictions were examined by performing simulations with three increasingly larger values of turbulence intensity. As turbulence intensity increased, transition occurs increasingly further upstream and the overall integrated laminar area decreased. Additionally, during the downstroke, the rate at which the boundary layer recovers to laminar flow decreases with increasing turbulence intensity. The influence of the vortex at the sweep break was reduced, with respect to transition, as the freestream turbulence intensity increased.

Finally, a simulation was performed for one grid level in order to investigate transition predictions if the thrust matched that of experiment. This resulted in decreasing the collective pitch by 2.5° . Predicted transition agreed poorly with experiments throughout the pitch cycle, as the extent of laminar flow was greatly overpredicted.

Overall, the AFT2019b model was able to predict transition accurately for a rotor known to undergo unsteady boundary layer transition. It still remains, however, that the predictions were poor during portions of the upstroke, specifically with respect to the rate of upstream motion of the transition front.

It should be noted that there remains uncertainty in the ability of the RANS models to accurately capture the time scales and spatial scales with respect to this unsteady transition case. For example, the baseline grid was designed based on current best practices for rotorcraft simulation; however, the fine grid resulted in higher quality predictions, but did exhibit quantitative differences relative to the fine grid. It appears, however, that finer streamwise grid spacing, relative to best practices, more accurately captures the scales of this problem. There is additional uncertainty in the specification of the freestream turbulence intensity, an ambiguous quantity in the rotating reference frame. Based on results in this

work, there are only minor quantitative differences in predictions for $Tu_\infty \leq 0.2\%$. It must be noted, however, that the choice of Tu_∞ greatly affects N_{crit} , to which transition may be more sensitized for other problems.

Chapter 6

Implicit Large Eddy Simulation of the DSA-9A Airfoil

The DSA-9A airfoil is an airfoil designed by the German Aerospace Center (DLR) as a main-rotor-blade airfoil for a generic helicopter [207, 208]. The airfoil has been the subject of numerous experimental studies on boundary-layer transition, particularly unsteady transition studies that seek to measure the movement of the transition front during a prescribed pitch cycle. Gardner and Richter [220] have measured transition on the DSA-9A airfoil using three methods: hot-film anemometry, pressure-sensor analysis, and infrared imaging. Schwermer et al. [206] used the DSA-9A in the design of the DLR RTG rotor blade that used airfoil throughout its span. This design served to extend transition measurements on the DSA-9A to full, three-dimensional, pitching rotors, in addition to measurements of dynamic stall. Transition on both the DSA-9A airfoil and the four-blade RTG rotor has been measured using temperature-sensitive paint [122, 209], hot-film anemometry [210], infrared thermography [220], and differential infrared thermography [211, 212, 221].

The work of Weiss et al. in Ref. [122] explored computational predictions of boundary-layer transition on a rotor in order to examine whether two-dimensional strip theory analyses can accurately prediction transition on a rotating blade. Although the rotational forces on the rotor blade induce a crossflow component, two-dimensional strip theory was found to accurately predict transition when compared to experimental data obtained from

temperature-sensitive paint. This suggests that using transition models calibrated for two-dimensional instabilities may be employed to predict transition on the three-dimensional RTG rotor, which was previously explored in Chapter 5.

From the work in Chapter 5, it was shown that during the upstroke of the blade, the upstream motion of the transition front lagged that of experiment. The present work seeks to investigate this lag by performing high-fidelity, large eddy simulations (LES) of a radial section of the RTG blade (i.e., the DSA-9A airfoil). These large eddy simulations will not only provide high-quality transition predictions, but may also highlight deficiencies of current transition models or current modeling techniques with respect to rotorcraft transition predictions.

6.1 Methodology

Large eddy simulations often employ subgrid-scale modeling [54] to compute viscous stresses for the scales below that of the smallest grid cell size. In this work, the approach of implicit large eddy simulations (ILES) is used. Rather than employ subgrid-scale models, ILES relies on the inherent dissipation of the numerical schemes employed to function implicitly as a subgrid model. More accurately, the leading-order truncation error due to numerical dissipation for an ILES method closely mimics the dissipation to smaller scales modeled by a subgrid stress model [222]. ILES serves as an intermediary between RANS modeling and direct numerical simulation (DNS), providing high-quality solutions of larger scale flow phenomena (i.e., above the Kolmogorov scale) relevant to boundary-layer transition.

6.1.1 Test Case

For this study, the primary focus is to investigate ILES-predicted transition about the DSA-9A rotorcraft airfoil using the OVERFLOW 2.3d solver [124] and compare predictions to that obtained from a RANS solution using the AFT2019b transition model. The same pitch cycle of Case II presented in Chapter 5 is used here. For the full-rotor simulations of Case II, during the upstroke it was found that the RANS models do not accurately predict the

upstream movement of the transition front for all radial stations examined. Thus, this ILES study seeks to follow the same prescribed pitch cycle to investigate the unsteady transition behavior.

The radial station $r/R = 0.5462$ was chosen as it is the innermost radial station for which both experimental and RANS results are available. This has the benefit of reducing computational cost, which scales with the reduced frequency ($k = \frac{cR}{2r}$), for which $k = 0.1$ at this station. Furthermore, the local Reynolds number of 1.27×10^5 reduces the number of points needed in the streamwise direction to maintain streamwise spacings consistent with LES [54]. To decrease computational costs again, only the first half of the pitch cycle is simulated, as the lag in transition predictions for the full rotor was found to be most prevalent during the upstroke. From the twist distribution of the RTG blade, the local angle of incidence at $r/R = 0.5462$ is 9.75° at the minimum collective pitch condition.

Given that only a section of the blade is simulated, the angle of attack must be corrected to account for the axial inflow, w_∞ , prescribed in the experiments and the induced velocity at the local radial station, as the rotor operates in its own wake. The reduction in local angle of attack can be estimated using blade element momentum theory as [223],

$$\alpha = \theta - \tan^{-1}(\lambda) \quad (6.1)$$

where the inflow ratio λ is estimated as a function of radial location and axial (climb) velocity,

$$\lambda(r/R, w_\infty) = \sqrt{\left(\frac{\sigma C_{l_\alpha}}{16} - \frac{w_\infty}{2\Omega R}\right)^2 + \frac{\sigma C_{l_\alpha} \theta r}{8R}} - \left(\frac{\sigma C_{l_\alpha}}{16} - \frac{w_\infty}{2\Omega R}\right) \quad (6.2)$$

Based on estimations of the lift-curve slope, C_{l_α} , from XFOIL [118], the inflow ratio reduces the effective angle of attack at the minimum pitch condition (i.e., $t/T = 0$) at station $r/R = 0.5462$ from 9.75° to 5.75° . The non-dimensional time rate of change of angle of attack is then specified, in general form, as

$$\frac{\partial \alpha}{\partial t} = 2k\hat{a}\cos(2kt + \phi) \quad (6.3)$$

where the non-dimensional period of the pitch cycle is $T = \pi/k$. To promote convergence at the beginning of the simulated pitch cycle, the pitch cycle is shifted by $3\pi/2$ such that the initial angular velocity is zero. This pitch cycle is then specified in OVERFLOW as,

$$\frac{\partial\alpha}{\partial t} = 0.2(2.9\pi/180)\cos(0.2t + 3\pi/2) \quad (6.4)$$

where $2.9\pi/180$ is the pitch amplitude. OVERFLOW then integrates to find the angle of attack as a function of time, with the initial condition specified as $\alpha(t = 0) = 5.75^\circ$.

6.1.2 Grid Generation

In this work, structured, overset grids were generated using a combination of Pointwise [224] and Chimera Grid Tools [215]. The grid system features two near-body grids and two O-type, off-body grids, designed in a similar manner to Galbraith and Visbal [38, 39]. Rather than use a single grid for the near-body of the airfoil, there are two individual grids: one for the upper surface and one for the lower surface, as shown in Fig. 6.1. Note that the grids are shown at the angle of incidence corresponding to the minimum pitch condition.

The upper surface grid was refined such that it meets or exceeds the requirements of LES grid-spacing as recommended by Georgiadis et al. [54] and is referred to as the “LES region”. The LES region begins at $x/c = 0.1$ on the lower surface, and wraps clockwise to the trailing edge on the lower surface. This ensures that at positive angles of attack, the stagnation point lies in the LES region. The LES region is comprised of 1612 points in the streamwise direction: 1548 points on the surface (11 of which facilitate the overlap with the lower surface grid) and 65 points on the trailing edge, as shown in Fig. 6.2. The maximum streamwise spacing corresponds to $\Delta x^+ \approx 12$, based on flat-plate boundary-layer theory and a chord Reynolds number of 1.27×10^5 . In the wall-normal direction, the surface grid was extruded hyperbolically to a distance of $0.1c$ using 193 points with a maximum stretching ratio of 1.04. The initial wall-spacing was specified as $1.0 \times 10^{-5}c$ which corresponds to $y^+ = 0.1$. The first three points are specified to have constant spacing in order to better resolve the viscous sublayer. The grid extends $0.2c$ in the spanwise direction as suggested by Galbraith

and Visbal [38, 39] using 129 evenly spaced points, which corresponds to $\Delta z^+ \approx 14$. Due to the positive angle of attack, the lower surface serves primarily as a pressure surface and does not require as fine of spacing compared to the upper surface. As such, the lower surface is comprised of 289 points streamwise (11 of which facilitate the overlap with the upper surface) with 25 points on the trailing edge. The maximum streamwise spacing corresponds to $\Delta x^+ \approx 43$. The initial wall-normal spacing is set to $7.3 \times 10^{-5}c$, which corresponds to $y^+ = 0.67$ and then extends $0.1c$ in the wall-normal direction using 65 points, providing a maximum stretching ratio of 1.07. The spanwise extent is set to $0.2c$ with 65 evenly spaced spanwise points, corresponding to $\Delta z^+ \approx 29$.

To capture off-body flow features, two O-type off-body grids were designed. The nearest off-body grid extends $2c$ into the wall-normal direction using 65 wall-normal points and 65 evenly spaced spanwise points. In the streamwise direction there are 385 points, 257 of which overlap with the LES region, 113 that overlap with the lower surface, and 17 points that overlap with the trailing edge. This ensures better refinement on the upper surface, such that off-body flow features emanating from the LES region may be more accurately captured. The farthest off-body grid features 97 points in the wall-normal direction and extends $1000c$ into the farfield to reduce the influence of the inflow boundary. The grid utilizes 51 evenly spaced points in the spanwise direction. In the streamwise direction, there are 193 points, where there are 91 points on the upper surface, 91 points on the lower surface, and 13 points on the trailing edge. To ensure that the off-body grids do not intersect with the surface of the airfoil, both grids were truncated such that the first point in the wall-normal direction lies $0.02c$ above the airfoil surface, shown in Fig. 6.3. As such, hole-cutting routines are not required during airfoil motion, as all grids are prescribed to move with the near-body grids. Thus, during prescribed pitching motions, both the near-body and off-body grids move at the same rate, reducing computational expense by eliminating the need to recompute domain connectivity during grid motion.

6.1.3 Numerical Schemes

For this work, the OVERFLOW 2.3d solver [124] was used. In order to run OVERFLOW in ILES mode, several changes must be made compared to a typical RANS or DDES run. First, the turbulence model must be disabled. There is no way to fully disable a turbulence model in OVERFLOW; however, this limitation can be sidestepped by limiting the eddy viscosity of the turbulence model to near-machine-zero values in the ILES regions-of-interest, or $\mu_t/\mu_\infty = 1.0 \times 10^{-15}$ in this case. This ensures that eddy viscosity cannot grow in the ILES region. Second, a sufficiently small timestep must be chosen in order to accurately resolve the scales relevant boundary layer. To achieve this, the global, non-dimensional timestep was specified as $\Delta t = 1.0 \times 10^{-4}$. In OVERFLOW, this is interpreted as the distance that a fluid element is advected by the freestream along the x -direction during each timestep (relative to the reference length). This can be interpreted as a fluid element travelling one chord-length in 10,000 iterations. This time scale is referred to here as a “time unit”. Each global timestep featured three Newton-type subiterations to improve temporal convergence. Third, the numerical scheme must be selected such that the amount of dissipation is appropriate for an ILES-quality solution. These schemes must be chosen such that the numerical dissipation is small enough to implicitly effect an energy cascade into sub-grid scales, but not large enough to overdamp large-scale flow features. For this work, the right-hand side of the Navier-Stokes equations was solved using 6th-order central differencing in space in addition to 5th-order artificial dissipation. The left-hand side was solved using the Pulliam-Chaussee scheme [195]. The artificial dissipation of the left-hand side was controlled by the built-in parameter **DIS4**, which modifies the numerical smoothing coefficient as per Eq. 3.121. Numerical experimentation found that, for ILES, a value of **DIS4** = 0.005 was small enough to maintain stability of the solution, but not too large to damp out scales of interest. Finally, a sufficiently resolved grid is required, which was discussed in-depth in previous sections. It is of note that the lower surface grid and the farfield grids are not subject to as-stringent of scheme choices as the upper surface grids as they need not fully resolve the scales of interest and thus do not operate in ILES mode.

Consequently, the lower surface grid and farfield grids were not subjected to a limit on the eddy viscosity. Additionally, the right-hand side of the Navier-Stokes equations was solved using a 3^{rd} -order central-differencing scheme for these grids, as it improved solution stability over the 5^{th} -order scheme due to the larger grid spacing and subsequent larger, local, CFL number. The freestream Mach number was set as 0.1 to maintain an incompressible flow.

The full solution process was broken up into multiple parts as to set up the initial flowfield before beginning the pitching motion and is given as follows,

1. Initialize as a RANS solution at the minimum pitch condition using 8,000 dual-time steps to develop the initial velocity field
2. Switch to ILES mode for 192,000 timesteps (19.2 time units) at the minimum pitch condition, allowing for a long run in ILES mode to advect initial transients out of the domain
3. Maintain ILES mode and begin the pitch cycle (15.7 time units)

A similar process was done for a solution using the AFT2019b model. This solution was performed in an unsteady RANS (URANS) framework and serves as an additional data set against which ILES predictions may be compared. For the AFT2019b simulation, the freestream eddy viscosity was set to $\mu_t/\mu_\infty = 2.8 \times 10^{-7}$ with a freestream turbulence intensity of 0.09%, identical to that used in the DLR RTG simulations in Chapter 5. The right-hand side was solved using 6^{th} -order central differencing in space with 5^{th} -order artificial dissipation for the upper surface, and 3^{rd} -order central differencing in space for the lower surface and farfields grids. Compared to ILES, the global timesteps required for URANS can be larger, as the turbulent scales in the boundary layer are modeled rather than resolved. As such, the non-dimensional timestep was increased to $\Delta t = 5.0 \times 10^{-4}$, meaning one time unit corresponds to 2,000 global iterations. Additionally, the artificial dissipation was increased relative to the ILES case. The parameter **DIS4** was increased to 0.04 and the parameter **DIS2** was set to 2.0 (cf. $\epsilon_j^{(2)}$ in Eq. 3.121). Local timestep scaling was used, with a specified minimum CFL number of 5.0 with a floating maximum CFL number. Each global timestep featured five Newton subiterations to improve solution convergence. It was found through

numerical experimentation that three subiterations, as used in the ILES simulation, was insufficient for temporal convergence and resulted in poor solution behavior. As with the ILES simulation, the Pulliam-Chaussee scheme [195] was used for solving the left-hand side. The process for the AFT2019b URANS solution is as follows,

1. Initialize as RANS at minimum pitch condition using 10,000 dual-time steps to develop the initial velocity field
2. Switch to URANS mode for 40,000 timesteps (20 time units) at the minimum pitch condition.
3. Maintain URANS mode and begin the pitch cycle (15.7 time units)

The URANS solution can be post-processed using Spalart’s turbulence index to determine the transition location predicted by the AFT2019b model.

6.2 Results

Due to the high-fidelity nature of an ILES solution, care must be taken when managing solution data. A full solution file at a single timestep is approximately 4 GB, which contains the value of each conserved variable at each point for each grid in the solution. For simulations encompassing large amounts of simulated time, this size of solution file becomes prohibitively expensive to write, as the input/output segment of the executable will result in significant computational burden and may slow down the solver. To reduce the overall data burden in the present simulations, the full solution file was written every 10,000 iterations (one time unit), whereas the solution file for centerplane of the LES grid, coinciding with the midspan, was written every 5 iterations. This reduced-size solution file was approximately 16 MB, significantly reducing the input/output burden. For the AFT2019b URANS solutions, the same process was performed, with the centerplane solution written every 5 timesteps. The size of this file was approximately 24 MB, as the solution file also stores the turbulence and transition variables, $\tilde{\nu}$, $\tilde{\gamma}$, and $\tilde{n}$. For both simulations, force and moment coefficients were computed and output every iteration.

Determining the transition onset position is difficult in an ILES solution, as there is no robust metric or turbulence index to indicate transition onset. In this case, however, the DSA-9A airfoil was found to undergo transition due to a laminar separation bubble (LSB). This mechanism is common for rotorcraft airfoils, especially at low Reynolds numbers. The transition point can be determined by searching along the airfoil surface for the reattachment point downstream of laminar separation. Here, the streamwise velocity was computed along a constant wall-tangent line for the first point off the wall. The first point for which the streamwise velocity becomes negative (downstream of stagnation) can be considered the laminar-separation point. Then, marching along this same wall-tangent line, the point where the streamwise velocity becomes positive again can be considered as the turbulent-reattachment point. This point then indicates the first point along the airfoil surface for which the flow is turbulent and is therefore the transition point. It was found that marching from the leading edge to the trailing edge was an acceptable method to find the laminar-separation point; however, finding the turbulent-reattachment point was more lucrative. Just downstream of laminar separation, the boundary layer locally reattaches for a short streamwise distance (on the order of $0.002c$) as the instability mechanism responsible for the separation bubble can propagate upstream and effect local, transient reattachment just after separation. This causes the formation of small, streamwise rollers that make difficult the search for the reattachment point. Finding the reattachment point is additionally difficult due to the trim tab on the airfoil, where there is turbulent separation in the vicinity of the trailing edge, suggesting that marching from the trailing edge to the leading edge will give incorrect reattachment positions. As a result, the location of turbulent reattachment was found by marching from the 50% chord station toward the leading edge. Through numerical experimentation, this method was found to more robustly determine the turbulent-reattachment point, as this dodges the local reattachment just downstream of separation. For the AFT2019b URANS solution, Spalart's turbulence index was used to explicitly determine the transition location. The same search algorithm was used to determine the separation point and reattachment point for the URANS result. Note that the location where Spalart's turbulence index reaches unity on the surface coincides exactly with the reattachment point

for the URANS case. Thus, this point is both the transition point and the reattachment point.

The numerically predicted laminar-separation positions and turbulent-reattachment (transition) positions as a function of time are shown in Fig. 6.4 for both the ILES predictions and URANS predictions using the AFT2019b model. The horizontal axis covers $t/T = [0, 0.5]$ as this corresponds to the upstroke half of the prescribed pitch cycle.

For the ILES simulation, the LSB first forms at approximately $x/c = 0.11$, illustrating the strong influence of low-Reynolds-number effects for this case and effecting separation near the suction peak. The separation point slowly moves upstream and occurs very close to the leading edge at the maximum pitch condition (i.e., at $t/T = 0.5$). This upstream creep of the separation point is relatively slow in comparison to the upstream creep of the turbulent-reattachment point, that is that the length of the LSB decreases as pitch increases. Initially, at lower angles of attack, the reattachment point is approximately $0.2c$ to $0.3c$ downstream of the separation point, indicating a long separation bubble. This is consistent with the expected behavior of the LSB at low angles of attack, for which the airfoil experiences comparatively weaker adverse pressure gradients and the bubble is expected to be longer [66]. The reattachment point varies greatly at any given point in time. This is a function of a flow instability present at the aft end of the separation bubble which causes transient reattachment and separation along the surface. The length of the separation bubble, and locations of separation and reattachment, positively correlates with the suction peak of the airfoil pressure coefficient, wherein separation closer to the leading edge will likely occur after large suction at the leading edge. This will be shown in the forthcoming sections.

For the URANS simulation, the separation point occurs slightly downstream of that of the ILES results. Additionally, the reattachment point (the transition point) occurs upstream of that predicted by ILES. This suggests that the predicted length of the LSB is shorter for URANS, which may have an effect on the integrated forces. Unlike the ILES case, the reattachment point predicted via URANS does not exhibit strong oscillations in position, as the boundary layer does not intermittently separate along the surface for URANS. Specifically, the lack of the need to resolve the turbulent scales with the URANS approach

damps any instabilities present at the aft end of the separation bubble. This resulted in few-to-no oscillations in the reattachment point of the separation bubble. Interestingly, the predicted reattachment point is seldom downstream of that predicted by ILES, despite the large oscillations seen in ILES. The comparatively shorter separation bubble may be a function of the specified freestream turbulence intensity of 0.09%, for which there was no specified freestream turbulence intensity for the ILES simulation. It is well known that higher freestream turbulence intensities can reduce the length of the separation bubble, as there is a larger “pool” of high-momentum flow outside the separation bubble from which the separated region may draw and effect reattachment [225]. The lack of freestream disturbances in the ILES simulation (modeled or resolved) may slightly affect the separation point and will certainly affect the reattachment point. This discrepancy in freestream conditions may partially explain the differences in separation and reattachment positions.

Comparing these two-dimensional separation and reattachment results for the DSA-9A airfoil to the three-dimensional DDES results for the RTG rotor (cf. Fig. 5.7b), it is evident that transition predicted for the airfoil does not agree well with transition predicted for the full rotor. Specifically, the full-rotor RTG simulation exhibits transition much further downstream of that predicted by both ILES and URANS. This is likely due to the phenomenon of “stall delay”, where the rotation of the blade results in a Coriolis force that causes a more favorable pressure gradient in the chordwise direction, which has a pronounced effect on the pressure distribution near the leading edge [226, 227]. This reduces the suction peak at the leading edge of the blade section, delaying separation by a comparatively less adverse pressure gradient. The Coriolis effect on the pressure distribution is strongest at the most inboard sections of the blade, where $v_{tip}/(\Omega r)$ is large. The present results suggest that transition predicted by two-dimensional simulations do not correlate to the transition locations for a full rotor blade as the three-dimensional effects are not accounted for. While three-dimensional effects can be included in a two-dimensional simulation by means of a quasi-three-dimensional Navier-Stokes formulation [228], no such quasi-three-dimensional studies have been performed in the context of dynamic stall and boundary-layer transition. The inclusion of these effects in a two-dimensional simulation may improve predictions.

During the pitch up for the ILES simulation there are several bubble “bursting” events, such as that at $t/T \approx 0.09$, where the reattachment point rapidly moves upstream, in the transient sense, but quickly moves downstream again. This bursting event indicates that there is a streamwise instability present near the trailing end of the separation bubble that is capable of moving upstream within the recirculation region of the bubble. This instability aids in entraining high-momentum flow by injecting high-vorticity fluid low into the LSB, temporarily inducing turbulent reattachment further upstream; however, this reattachment point moves downstream very soon after this intermittent bubble bursting. A brief time history of this bursting event can be seen in Fig. 6.5, which depicts the presence of the streamwise instability near the trailing end of the bubble, the subsequent upstream position of the reattachment point, and the reattachment point moving further downstream, highlighting the transient behavior of the bursting event. The effects of this event can be seen by examining the pressure distributions in Fig. 6.6. From $t/T = 0.08312$ to $t/T = 0.08535$, the amplitude of the oscillation in surface pressure grows. This culminates in a further-upstream transition point at $t/T = 0.08822$; however, shortly after, this transition point moves downstream again. Thus, this bursting mechanism seems to be driven by simply a transient instability.

Distributions of surface pressure predicted by ILES throughout the first 30% of the pitch cycle are shown in Fig. 6.7. The presence of the separation bubble has a marked effect on the pressure distribution, where the surface pressure is approximately constant over the length of the bubble. At the aft end of the bubble, a spike in pressure indicates reattachment. This is clearly seen at $t/T = 0$. Downstream of reattachment, there are local, streamwise rollers that form and impact the pressure distribution, which causes repeated local separation and reattachment. This is most evident at $t/T = 0.3$, where three distinct pressure spikes are present, with a peak-to-peak wavelength of approximately $0.05c$, indicating the length of this additional separation region. The transient peaks and troughs serve as additional evidence of a streamwise instability present in the boundary layer aft of the separation bubble, specifically, a Kelvin-Helmholtz-like instability. In fact, during the upstream creep of this instability, there are large variations in the spanwise velocity that creep upstream of the

reattachment point. This suggests that the streamwise instability has a spanwise wavelength to which the boundary layer is receptive and is likely on the order of the boundary-layer thickness. Unfortunately, since this study only examines the centerplane of the airfoil, this hypothesis cannot be examined thoroughly and is a subject of suggested future work.

During the next 20% of the pitch cycle, as shown in Fig. 6.8, the length of the LSB decreases significantly as the pressure gradient becomes increasingly adverse. Post-transition, the surface-pressure distribution is nearly the same for all portions of the pitch cycle as the turbulent scales are damped moving downstream after reattachment. This, in turn, renders the surface pressure distribution on the surface to be nearly the same downstream of reattachment. For lower pitch angles (i.e., smaller values of t/T), this same behavior is observed, but not as drastically so, as the lower pitch angles do not support such scales.

The surface-pressure distributions predicted by URANS for select timesteps are compared in Fig. 6.9 with that predicted by ILES. At each timestep, the minimum surface pressure (the suction peak) is nearly the same for both ILES and URANS. Conversely, the URANS simulation exhibits a pressure recovery that occurs more upstream of that exhibited by ILES, a reflection of the shorter LSB predicted by URANS. At $t/T = 0$, the Kelvin-Helmholtz-like instability has a pronounced effect on the pressure predicted by ILES, which results in pressure recovery that is far more rapid than that predicted by URANS. This rapid recovery results in intermittent separation and reattachment downstream, which was not seen in URANS. As time progress and the pitch angle increases, the predicted pressure distribution downstream of reattachment agrees well across both methods. This insinuates that both methods predict nearly the same aft loading of the airfoil. The predicted bubble length is the main source of difference between predictions, as are the oscillations in surface pressure as predicted by ILES. The modeled freestream turbulence intensity for the URANS simulation contributes partially to this difference, as turbulence intensity has a 2^{nd} -order effect on the pressure distribution.

Distributions of skin friction are given in Fig. 6.10 for both the ILES solution and URANS solution at identical timesteps. Skin friction indicates that the separation point is approximately the same at each timestep examined, as was seen previously. The minimum

skin friction is typically thought of as the “turning point”, where the boundary layer begins to form again by means of entrainment of the outer flow. This minimum skin friction is clearly identifiable for the URANS results, but is not for the ILES results. The instability present in ILES appears to cause drastic changes in vorticity near the wall that makes unclear whether the point of minimum skin friction is indeed this “turning point”. Downstream of reattachment, ILES predictions exhibit drastic spikes in skin friction, caused by the Kelvin-Helmholtz-like instability introducing parcels of high-vorticity flow near the wall. Far downstream of reattachment, predicted skin friction agrees relatively well between both methods as the boundary layer is attached and turbulent. Additionally, the URANS predictions somewhat mimic a moving average of that predicted by ILES. At maximum pitch at $t/T = 0.5$, the skin friction downstream of reattachment predicted by ILES exceeds that predicted by URANS, which may cause an overprediction of skin-friction drag. Generally speaking, skin-friction distributions agree well for both models, where the ILES predictions exhibit slightly higher skin friction overall.

Given the differences in surface-pressure distributions, skin-friction distributions, separation, and transition, it is expected that there be differences between the predicted force and moment coefficients. In fact, the difference in predicted bubble length should have a demonstrable impact on integrated drag [73]. Additionally, the unsteady separation and reattachment predicted by ILES may affect the integrated pitching moment.

The lift as predicted by both ILES and URANS is shown in Fig. 6.11 throughout the simulated pitch cycle. Predictions made using URANS exhibit slightly larger lift, which can likely be attributed to the shorter separation bubble in comparison with ILES. Longer separation bubbles are more detrimental to lift than are shorter separation bubbles, as separated flow cannot provide the same level of circulation about the airfoil surface as can reattached flow. Near maximum pitch at $t/T < 0.4$, the ILES-predicted lift begins to taper off earlier than URANS. This is caused not only by more upstream and more drastic separation, but also by a more rapid recovery in surface pressure (cf. Fig. 6.9f). This comparatively rapid pressure recovery decreases the pressure differential between the upper and lower surface, which may partially explain the tapering off of the lift curve. In the

context of rotorcraft aerodynamics, this pitch condition would most align with a blade on the retreating side of the rotor. If only ILES results were considered, it would seem that the airfoil exhibits a lower lift coefficient at such a condition, which would affect the trim state of the rotor and may exhibit a more “narrow” flight envelope compared to other similar methods. Here, not only do URANS and ILES predictions give different airfoil performance, but these differences may affect decisions made in the design stage of aerospace vehicles. It is thus important to most accurately model, or resolve, the physics at-hand in order to provide the most accurate predictions. Generally speaking, both the ILES and URANS predictions agree quite well in terms of lift production. The primary difference lies in a small offset in C_l at each point in time, which is likely due to the difference in the predicted length of the separation bubble. This offset maybe a function of flow history. In the present simulations, no previous pitch cycles were simulated, which may affect the performance of the airfoil throughout the pitch cycle. Only simulating a “cold start” of the pitch cycle may improperly account for the pressure distribution at $t/T = 0$, since the effects of the downstroke are not included. It is suggested that for other ILES or URANS simulations of airfoils in dynamic pitching conditions, bar computational costs, that at least one pitch cycle be performed before data collection begins.

Integrated drag is plotted in Fig. 6.12. It is evident that the ILES results exhibit larger drag. This is due to the larger jump in boundary-layer-edge velocity from the ILES predictions, which results in a rapid, large increase in momentum thickness, which makes the boundary layer more likely to separate. This, in turn, results in a longer separation bubble and subsequently higher drag. The slope of the drag curve is nearly the same between ILES and URANS. The primary difference lies in the drag difference between the two, which is relatively constant throughout the pitch cycle.

The differences in predicted drag can be further investigated by examining the pressure component of drag and the viscous component of drag, as done in Fig. 6.13. The offset between drag predictions is again evident here. ILES predictions exhibit a larger pressure component of drag compare to URANS, a direct consequence of the longer separation bubble and more drastic pressure recovery. Additionally, ILES predictions show a larger viscous

component of drag, comparatively, which correlates well with the skin-friction distributions shows previously. Thus, it appears that the primary driver of differences in drag prediction can be ascribed to the length of the separation bubble and the more drastic pressure recovery seen in ILES predictions.

Pitching moment as a function of time is plotted in Fig. 6.14 for both ILES and URANS. While URANS predicts a smooth, concave-up pitching moment as a function of time, ILES predictions exhibit rapid, high-magnitude oscillations in pitching moment over time. This is a direct result of the temporal shortening and lengthening of the separation bubble throughout the pitch cycle. Specifically, the Kelvin-Helmholtz-like instability transiently “shortens” the bubble by locally reattaching the flow, which, in turn, affects the loading distribution on the airfoil surface. At higher angles of attack, ILES appears to predict more positive pitching moments than URANS, occurring at approximately the same time that lift begins to taper off. For rotorcraft, a pitching moment closer to zero is more desirable as this reduce the loading on the pitch links. Comparatively, the more negative URANS-predicted pitching moments would result in larger magnitude pitch-link loads, which may affect the design of a blade using the DSA-9A airfoil or the design of the rotor hub.

6.3 Summary and Discussion

The DSA-9A airfoil was simulated in dynamic pitching conditions using an implicit large eddy simulation framework in order to predict boundary-layer transition and airfoil performance. Simulations were performed using the OVERFLOW solver by disabling the turbulence model with timesteps and grid resolution consistent with that required by ILES. An additional simulation was performed using the AFT2019b transition model in an unsteady RANS framework. The airfoil was pitched at a reduced frequency of $k = 0.1$ at a chord Reynolds number of 1.27×10^5 .

Transition was found to occur by means of a laminar separation bubble, with incipient separation near $0.1c$ at the beginning of the pitch cycle. ILES predictions exhibit laminar separation further upstream compared to URANS predictions, with an accompanying

reattachment point that is downstream of that predicted by URANS. Thus, the separation bubble predicted by ILES was found to be longer than that predicted by URANS. ILES results show the presence of a Kelvin-Helmholtz-like instability near the aft end of the separation bubble. This instability causes unsteady reattachment and re-separation of the separated shear layer. This leads to large oscillations in the reattachment point of the boundary layer, in addition to surface pressure and skin friction in the vicinity of reattachment. Both ILES and URANS predictions exhibit boundary-layer separation near the trailing edge, where the trim-tab of the airfoil causes a rapid change in local curvature, for which the boundary layer must separate in order to recover pressure. Additionally, transition predicted by ILES and URANS does not correlate well with that predicted by three-dimensional, full-rotor simulations. In this case, the rotation of the blade likely has a strong effect on the separation and reattachment points, as the resulting Coriolis acceleration induces a more favorable chordwise pressure gradient, which will delay the boundary-layer separation point compared to a non-rotating airfoil. Performing such two-dimensional ILES simulations in a quasi-three-dimensional framework that includes rotational effects may improve these predictions.

Predicted surface-pressure distributions agree well across ILES and URANS, especially so downstream of turbulent reattachment. The largest differences in predicted surface pressure are due to differences in separation-bubble length, which tends to be longer for ILES. Large oscillations in surface pressure occur about the aft end of the separation bubble in ILES predictions, indicative of the Kelvin-Helmholtz-like instability that is present. When examining skin friction, ILES predicts a more negative minimum skin friction through the separation bubble than does URANS. Downstream of this minimum skin friction, ILES results exhibit intermittent surface separation and reattachment, whereas URANS results do not exhibit this same behavior. At higher pitch angles, ILES predicts higher skin friction near the trailing edge of the airfoil, resulting in a larger contribution of skin-friction drag, comparatively.

Predicted force and moment coefficients agreed well between ILES and URANS. ILES predictions exhibit large oscillations in pitching moment relative to URANS; however, the

qualitative trend appears to agree well with URANS. Predicted lift is slightly higher for URANS, as the shorter bubble length results in an improvement to the airfoil's ability to produce lift. Predictions of drag differ significantly between ILES and URANS, particularly that ILES predicts higher drag due to the longer separation bubble. Examining the pressure component and viscous component of drag, it was found that ILES predicts higher contributions of both, relative to URANS. Respectively, this is due to the longer separation bubble on the surface and higher skin friction downstream of reattachment.

Despite the fact that the URANS framework is incapable of predicting the exact transition mechanisms seen in ILES, the separation point and transition point predicted using URANS agree well with ILES. This suggests that, for this airfoil, URANS used in conjunction with the AFT2019b model can indeed model the two-dimensional boundary-layer transition point accurately. It must not be understated, however, that it is important to select models that best represent the physics at-hand in order to make the most accurate predictions. Future work should investigate the use of hybrid RANS/LES methods along with other transition models in order to assess predictive capabilities and compare to ILES and URANS computations. Additionally, there is uncertainty in the level of artificial dissipation needed for this problem that should be investigated. It is also suggested that future work capture at least one full pitch cycle before beginning data collection in order to properly account for hysteresis effects in the domain.

Chapter 7

Summary and Conclusions

Three different analyses of laminar-turbulent transition modeling methods were presented. These analyses focus not only on predictive capabilities, but highlight deficiencies in the investigated models, make suggestions for future improvements, and provide a cautionary tale for the application of these models. While these models are often designed to work “out of the box”, it is prudent to understand the inner workings of these models to identify both situations in which they perform poorly and situations in which they perform well.

The near-wall behavior of the Langtry-Menter model was investigated analytically and numerically. It was shown that there exists singular behavior in the destruction term and diffusion term of the turbulent kinetic energy equation in the limit of $y^+ \rightarrow 0$ after transition. In fact, the magnitude of the destruction term outweighs that of the production term by several orders of magnitude in the viscous sublayer, where production and destruction are expected to be equivalent. This behavior introduces uncertainties in the predictive capabilities of the model, as the risk of errant relaminarization is greatly increased due to the physically inaccurate turbulent kinetic energy budget near the wall. This singular-like behavior can be mitigated by increasing the floor value of intermittency in the destruction term; however, the predicted velocity profile deviates greatly from that intended by the base, shear stress transport turbulence model. The singular behavior near the wall was confirmed for several, practical aerodynamics cases by performing CFD simulations using the OVERFLOW solver. Due to this singular-like behavior, the Langtry-Menter model’s

transport equations can never truly converge, regardless of the level of grid refinement. Additionally, the analysis of the near-wall behavior guided the development of a turbulence index compatible with the Langtry-Menter model. This turbulence index was designed such that a value of 0 indicates a laminar boundary layer and a value of 1 indicates a turbulent boundary layer. This metric was found to more robustly detect the transition position in comparison to skin friction or intermittency and serves as a simple means to post-process the transition front predicted by the Langtry-Menter model.

The predictive capabilities of the amplification factor transport model were investigated with application to a test case on the four-bladed RTG rotor designed at the German Aerospace Center. This rotor was prescribed to undergo dynamic pitching conditions with weak axial inflow for which experimental transition positions had been measured. This test case serves as a difficult case of unsteady boundary layer transition, for which little experimental data exist for rotorcraft applications. During the upstroke of the pitch cycle, the predicted transition front moved upstream faster than that seen in experiment. As the pressure gradient becomes increasingly adverse during the upstroke, the AFT model appears to respond to these changes more slowly than that seen in experiment. Near the maximum pitch condition, laminar separation on the surface fixed transition near $0.25c$, which was upstream of that determined experimentally. During the downstroke, the AFT model accurately predicted transition across the radius of the blade. Additional studies were performed for increasingly larger values of freestream turbulence intensity, which was expected to shift the transition front further upstream for all portions of the pitch cycle. This was indeed the case, as the transition front was further upstream at all points of the pitch cycle. Interestingly, as the freestream turbulence intensity increased, the rate of upstream movement of the transition front decreased during the upstroke. There is great uncertainty in the specification of the freestream turbulence intensity, an ambiguous quantity in the rotating reference frame. Based on the RTG results, there are only relatively minor differences in predictions for $Tu_\infty \leq 0.2\%$ for this problem. It must be noted, however, that the choice of Tu_∞ greatly affects the critical amplification factor, to which transition may be more sensitized for other problems. Overall, the AFT model accurately predicted transition

for a dynamically pitching rotor, with exception to transition predictions during a portion of the upstroke.

Based on the behavior of the AFT model during the upstroke of the RTG rotor, laminar-turbulent transition on the DSA-9A airfoil (that of the RTG blade) was investigated using implicit, large eddy simulations. The airfoil was simulated as a wing with a spanwise extent of $0.2c$ and underwent a pitch cycle consistent with that of the upstroke of the RTG blade. This upstroke portion was simulated at a Reynolds number and angle of attack corresponding to the 54.62% radial station of the RTG blade. The large eddy simulation was performed using the OVERFLOW solver with the turbulence model disabled in conjunction with a sufficiently fine grid to follow best-practice guidelines for LES-compatible grids. During the upstroke, a laminar separation bubble forms on the upper surface within the first 15% chord, where the location of laminar separation moves upstream as the pitch angle increases. The point of turbulent reattachment was prescribed as the laminar-turbulent transition point as it is the first point for which a fully turbulent boundary layer exists on the surface of the airfoil. This separation bubble was the primary driver of transition for this test case. A Kelvin-Helmholtz-like instability near the aft end of the separation bubble drove the reattachment point upstream during portions of the pitch cycle. This instability is partially responsible for the constantly changing reattachment position, whereas the separation point moves upstream with time. Qualitatively comparing the ILES-predicted and URANS-predicted transition locations reveals that they do not agree with that of the full-rotor RTG computation. This is likely due to the lack of including three-dimensional, or quasi-three-dimensional, effects that arise from the rotation of the blade. A companion unsteady RANS simulation was performed with the AFT model for transition predictions. Results from both URANS and ILES generally agreed well in the separation position and reattachment position as a function of time. The primary difference between the two approaches lay in the predicted length of the laminar separation bubble. When examining surface-pressure distributions, both ILES and URANS predicted nearly the same suction peak value, although the length of the bubble is different. Similarly, both models predicted nearly the same pressure distribution downstream of reattachment. Skin-friction distributions revealed that ILES predicts a more

negative minimum skin friction in the separation bubble, while both ILES and URANS predict similar skin friction downstream of reattachment. In terms of airfoil performance, the longer separation bubble predicted by ILES slightly reduces the lift relative to URANS, but increases the integrated drag throughout the pitch cycle. The differences between the ILES and URANS results highlight the importance of accurately modeling or resolving the physics at-hand. Understanding the potential differences between different simulation approaches is integral to providing the most accurate predictions as possible. Simply relying on one approach may result in deleterious predictions in comparison to another model, and may, in turn, stray further from that which is physical.

Laminar-turbulent transition models offer a means to model additional physics pertinent to aerodynamic flows compared their fully turbulent counterparts, especially at low-to-moderate Reynolds number ($\mathcal{O}(10^4) - \mathcal{O}(10^6)$). Each transition model, and the individual variants of each transition model, is sensitive to boundary conditions in the near-wall region, in addition to aerodynamic boundary conditions such as freestream turbulence intensity of eddy-viscosity ratio. These models are also sensitive to grid spacing, the choice of numerical scheme, and timestep, among others. These limitations make it difficult to provide a blanket “best practices” for a given transition model, especially when applied to various flow regimes. Common test cases, some of which were presented in this work, seek to bridge this gap and provide a basis against which different modeling approaches may be compared. Each flow regime and its expected transition mechanisms will have its own set of best practices, such as the case of modeling the length of a laminar separation bubbles. It remains, however, that the base forms of the examined RANS-based transition models tend to provide accurate prediction and modeling of the physics of transition.

References

- [1] Bertin, J. J., and Cummings, R. M., *Aerodynamics for Engineers*, 6th ed., 2021. <https://doi.org/10.1017/9781009105842>.
- [2] Whitcomb, R. T., and Kelly, T. C., “A Study of the Flow over a 45-degree Sweptback Wing-Fuselage Combination at Transonic Mach Numbers,” *NACA RM L52DO1*, 1952.
- [3] Menter, F. R., “Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications,” *AIAA Journal*, Vol. 32, No. 8, 1994, pp. 1598-1605. <https://doi.org/10.2514/3.12149>.
- [4] Spalart, P. R., and Allmaras, S. R., “A One-Equation Turbulence Model for Aerodynamic Flows,” *Recherche Aerospatiale*, No. 1, 1994, pp. 5-21.
- [5] Coder, J. G., and Maughmer, M. D., “Computational Fluid Dynamics Compatible Transition Modeling Using an Amplification Factor Transport Equation,” *AIAA Journal*, Vol. 52, No. 11, 2014, pp. 2506-2512. <https://doi.org/10.2514/1.J052905>.
- [6] Langtry, R. B., and Menter, F. R., “Correlation-Based Transition Modeling for Unstructured Parallelized Computational Fluid Dynamics Codes,” *AIAA Journal*, Vol. 47, No. 12, 2009, pp. 2894–2906. <https://doi.org/https://doi.org/10.2514/1.42362>.
- [7] Menter, F. R., Smirnov, P. E., Liu, T., and Avancha, R., “A One-Equation Local Correlation-Based Transition Model,” *Flow, Turbulence and Combustion*, Vol. 95, No. 4, 2015, pp. 583-619. <https://doi.org/10.1007/s10494-015-9622-4>.
- [8] Fujino, M., “Design and Development of the HondaJet,” *Journal of Aircraft*, Vol. 42, No. 3, 2005, pp. 755–764. <https://doi.org/10.2514/1.12268>.
- [9] Greitzer, E. M., “N+3 Aircraft Concept Designs and Trade Studies,” *NASA CR-2010-216794*, 2010.
- [10] Drela, M., “A User’s Guide to MSES 3.05,” MIT Department of Aeronautics and Astronautics, July 2007.
- [11] Bradley, M. K., and Droney, C. K., “Subsonic Ultra Green Aircraft Research,” *NASA CR-2011-216847*, 2011.

- [12] Coder, J. G., and Somers, D. M., “Design of a slotted, natural-laminar-flow airfoil for commercial transport applications,” *Aerospace Science and Technology*, Vol. 106, 2020, p. 106217. <https://doi.org/https://doi.org/10.1016/j.ast.2020.106217>.
- [13] Smith, A. M. O., “High-Lift Aerodynamics,” *Journal of Aircraft*, Vol. 12, No. 6, 1975, pp. 501–530. <https://doi.org/10.2514/3.59830>.
- [14] Somers, D., “Design of a Slotted, Natural-Laminar-Flow Airfoil for a Transport Aircraft,” *NASA/CR-2019-220403*, 2019.
- [15] Lynde, M. N., and Campbell, R. L., “Computational Design and Analysis of a Transonic Natural Laminar Flow Wing for a Wind Tunnel Model,” *35th AIAA Applied Aerodynamics Conference*, AIAA Paper 2017-3058, Denver, CO, June 2017. <https://doi.org/10.2514/6.2017-3058>.
- [16] Venkatachari, B. S., Paredes, P., Derlaga, J. M., Buning, P. G., Choudhari, M. M., Li, F., and Chang, C.-L., “Assessment of RANS-based Transition Models based on Experimental Data of the Common Research Model with Natural Laminar Flow,” *AIAA SciTech 2021 Forum*, AIAA Paper 2021-1430, Virtual, January 2021. <https://doi.org/10.2514/6.2021-1430>.
- [17] Venkatachari, B. S., Paredes, P., Choudhari, M. M., Li, F., and Chang, C.-L., “Transition Analysis for the CRM-NLF Wind Tunnel Configuration using Transport Equation Models and Linear Stability Correlations,” *AIAA SciTech 2022 Forum*, AIAA Paper 2022-1542, San Diego, CA, January 2022. <https://doi.org/10.2514/6.2022-1542>.
- [18] Stefanski, D. L., Glasby, R. S., Erwin, T. J., and Coder, J. G., “Improvements to the Amplification Factor Transport Transition Model for Finite-Element Implementations,” *AIAA SciTech 2019 Forum*, AIAA Paper 2019-0040, San Diego, CA, January 2019. <https://doi.org/10.2514/6.2019-0040>.
- [19] Force, U. S. A., “MQ-9 Reaper,” retrieved April 2022. URL <https://www.af.mil/About-Us/Fact-Sheets/Display/Article/104470/mq-9-reaper/>.
- [20] Maughmer, M. D., Swan, T. S., and Willits, S. M., “Design and Testing of a Winglet Airfoil for Low-Speed Aircraft,” *Journal of Aircraft*, Vol. 39, No. 4, 2002, pp. 654–661. <https://doi.org/10.2514/2.2978>.

- [21] Overmeyer, A. D., and Martin, P. B., “Measured Boundary Layer Transition and Rotor Hover Performance at Model Scale,” *55th AIAA Aerospace Sciences Meeting*, AIAA Paper 2017-1872, Grapevine, TX, January 2017. <https://doi.org/10.2514/6.2017-1872>.
- [22] Hariharan, N. S., Egolf, T. A., Narducci, R., and Sankar, L. N., “Helicopter Rotor Aerodynamic Modeling in Hover: AIAA Standardized Hover Evaluations,” *53rd AIAA Aerospace Sciences Meeting*, AIAA Paper 2015-1241, Kissimmee, FL, January 2015. <https://doi.org/10.2514/6.2015-1242>.
- [23] Heister, C. C., “Laminar-turbulent transition prediction for helicopter rotors in hover and forward flight - a RANS based investigation of transition mechanisms using empirical criteria,” *38th European Rotorcraft Forum*, Amsterdam, The Netherlands, September 2012.
- [24] Slotnick, J., Khodadoust, A., Alonso, J., Darmofal, D., Gropp, W., Lurie, E., and Mavriplis, D. J., “CFD Vision 2030 Study: A Path to Revolutionary Computational Aerosciences,” *NASA/CR-2014-218178*, 2014.
- [25] van Ingen, J. L., “A suggested Semi-Empirical Method for the Calculation of the Boundary Layer Transition Region,” *Department of Aerospace Engineering, Delft University of Technology, Report VTH-74*, 1956.
- [26] Seyfert, C., and Krumbein, A., “Evaluation of a Correlation-Based Transition Model and Comparison with the e^N Method,” *Journal of Aircraft*, Vol. 49, No. 6, 2012, pp. 1765–1773. <https://doi.org/10.2514/1.C031448>.
- [27] Coder, J. G., “Development of a CFD-compatible Transition Model Based on Linear Stability Theory,” *Ph.D. Dissertation*, Department of Aerospace Engineering, The Pennsylvania State University, State College, PA, 2014.
- [28] Coder, J. G., “Further Development of the Amplification Factor Transport Transition Model for Aerodynamic Flows,” *AIAA SciTech 2019 Forum*, AIAA Paper 2019-0039, San Diego, CA, January 2019. <https://doi.org/10.2514/6.2019-0039>.
- [29] Medida, S., and Baeder, J., “Application of the Correlation-based Gamma-Re Theta t Transition Model to the Spalart-Allmaras Turbulence Model,” *20th AIAA Computational*

- Fluid Dynamics Conference*, AIAA Paper 2011-3979, Honolulu, HI, June 2011. <https://doi.org/10.2514/6.2011-3979>.
- [30] Nichols, R. H., “Addition of a Local Correlation-Based Boundary Layer Transition model to the CREATETM-AV Kestrel Unstructured Flow Solver,” *AIAA SciTech 2019 Forum*, AIAA Paper 2019-1343, San Diego, CA, January 2019. <https://doi.org/10.2514/6.2019-1343>.
- [31] Lee, B., “Development of a laminar-turbulent transition model and blended time-marching schemes for rotorcraft CFD application,” *Ph.D. Dissertation*, Department of Aerospace Engineering, University of Maryland, College Park, MD, 2021.
- [32] Barrouillet, B., Laurendeau, E., and Yang, H., “On the calibration of the transitional $k - \omega - \gamma - Re_{\theta t}$ turbulence model,” *2021 AIAA SciTech Forum*, AIAA 2021-0629, Virtual, January 2021. <https://doi.org/10.2514/6.2021-0629>.
- [33] Piotrowski, M. G., and Zingg, D. W., “Numerical Behaviour of a Smooth Local Correlation-based Transition Model in a Newton-Krylov Flow Solver,” *AIAA SciTech 2022 Forum*, AIAA Paper 2022-0909, San Diego, CA, January 2022. <https://doi.org/10.2514/6.2022-0909>.
- [34] Coder, J. G., and Ortiz-Melendez, H. D., “Transitional Delayed Detached-Eddy Simulation of Multielement High-Lift Airfoils,” *Journal of Aircraft*, Vol. 56, No. 4, 2019, pp. 1303-1312. <https://doi.org/10.2514/1.C035161>.
- [35] Piomelli, U., “Large eddy simulations in 2030 and beyond,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, Vol. 372, No. 2022, 2014, pp. 20130320. <https://doi.org/10.1098/rsta.2013.0320>.
- [36] Smagorinsky, J., “General Circulation Experiments with the Primitive Equations,” *Monthly Weather Review*, Vol. 91, No. 3, 1963, pp. 99–164. [https://doi.org/10.1175/1520-0493\(1963\)091<0099:GCEWTP>2.3.CO;2](https://doi.org/10.1175/1520-0493(1963)091<0099:GCEWTP>2.3.CO;2).
- [37] Grinstein, F. F., Margolin, L. G., and Rider, W. J., *Implicit Large Eddy Simulation: Computing Turbulent Fluid Dynamics*, 1st ed., Cambridge University Press, 2007.

- [38] Galbraith, M. C., and Visbal, M. R., “Implicit Large Eddy Simulation of Low Reynolds Number Flow Past the SD7003 Airfoil,” *46th AIAA Aerospace Sciences Meeting and Exhibit*, AIAA Paper 2008-225, Reno, NV, January 2008. <https://doi.org/10.2514/6.2008-225>.
- [39] Galbraith, M. C., and Visbal, M. R., “Implicit Large Eddy Simulation of Low-Reynolds-Number Transitional Flow Past the SD7003 Airfoil,” *40th Fluid Dynamics Conference and Exhibit*, AIAA Paper 2010-4737, Chicago, IL, June 2010. <https://doi.org/10.2514/6.2010-4737>.
- [40] Vogel, E. A., and Coder, J. G., “Entropy-Based Modal Decomposition of High-Speed, Vortex-Dominated Flows,” *AIAA SciTech 2021 Forum*, AIAA Paper 2021-1674, Orlando, FL, January 2021. <https://doi.org/10.2514/6.2021-1674>.
- [41] Bogey, C., and Bailly, C., “Large-eddy simulations of round jets: effects of initial conditions on self-similarity region,” *19th French Congress of Mechanics*, Marseille, France, August 2009.
- [42] Lozano-Durán, A., Bose, S. T., and Moin, P., “Performance of Wall-Modeled LES with Boundary-Layer-Conforming Grids for External Aerodynamics,” *AIAA Journal*, Vol. 60, No. 2, 2022, pp. 747–766. <https://doi.org/10.2514/1.J061041>.
- [43] Pope, S. B., *Turbulent Flows*, 1st ed., Cambridge University Press, Cambridge, United Kingdom, 2006.
- [44] Pont-Vílchez, A., Trias, F. X., Gorobets, A., and Oliva, A., “Direct numerical simulation of backward-facing step flow at $Re_\tau = 395$ and expansion ratio 2,” *Journal of Fluid Mechanics*, Vol. 863, 2019, p. 341–363. <https://doi.org/10.1017/jfm.2018.1000>.
- [45] Langtry, R. B., Sengupta, K., Yeh, D. T., and Dorgan, A. J., “Extending the γ - $Re_{\theta t}$ Local Correlation based Transition Model for Crossflow Effects,” *45th AIAA Fluid Dynamics Conference*, AIAA Paper 2015-2474, Dallas, TX, June 2015. <https://doi.org/10.2514/6.2015-2474>.
- [46] Coder, J. G., “Enhancement of the Amplification Factor Transport Transition Modeling Framework,” *55th AIAA Aerospace Sciences Meeting*, AIAA Paper 2017-1709, Grapevine, TX, January 2017. <https://doi.org/10.2514/6.2017-1709>.

- [47] Coder, J. G., Pulliam, T. H., and Jensen, J. C., “Contributions to HiLiftPW-3 Using Structured, Overset Grid Methods,” *AIAA SciTech 2018 Forum*, AIAA Paper 2018-1039, Kissimmee, FL, January 2018. <https://doi.org/10.2514/6.2018-1039>.
- [48] Reynolds, O., “An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous, and of the law of resistance in parallel channels,” *Philosophical Transactions of the Royal Society*, Vol. 174, 1883, pp. 935–982. <https://doi.org/10.1098/rstl.1883.0029>.
- [49] White, F., *Viscous Fluid Flow*, 3rd ed., McGraw-Hill Education, 2011.
- [50] Rayleigh, L., “On the question of the stability of the flow of fluids,” *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Vol. 34, No. 206, 1892, pp. 59–70. <https://doi.org/10.1080/14786449208620167>.
- [51] Prandtl, L., “Der Luftwiderstand von Kugeln,” *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen*, Vol. 5, 1914, pp. 177–190.
- [52] Blasius, H., “Das Aehnlichkeitsgesetz bei Reibungsvorgängen,” *Z Vereines Deutsch Ing*, Vol. 56, No. 16, 1912, pp. 639–643.
- [53] von Kármán, T., “Dimensionslose Größen in Grenzgebieten der Aerodynamik,” *Zeitschrift für Flugwissenschaften und Weltraumforschung*, Vol. 4, 1956, pp. 3–5.
- [54] Georgiadis, N. J., Rizzetta, D. P., and Fureby, C., “Large-Eddy Simulation: Current Capabilities, Recommended Practices, and Future Research,” *AIAA Journal*, Vol. 48, No. 8, 2010, pp. 1772–1784. <https://doi.org/10.2514/1.J050232>.
- [55] Schmid, P. J., and Henningson, D. S., *Stability and Transition in Shear Flows*, 1st ed., Springer, 2001.
- [56] Schlichting, H., *Boundary-Layer Theory*, 6th ed., McGraw-Hill, 1966.
- [57] Tollmien, W., “Grenzschichttheorie,” *Handbuch der Experimentalphysik*, Vol. 4, No. 1, Leipzig, Germany, pp. 239–287.
- [58] Mack, L. M., “Transition and Laminar Instability,” *NASA CR-153203*, 1977.

- [59] Klebanoff, P. S., “Effect of free-stream turbulence on a laminar boundary layer,” *Bulletin of the American Physical Society*, Vol. 16, 1971.
- [60] Orr, W. M., “The Stability or Instability of the Steady Motions of a Perfect Liquid and of a Viscous Liquid, Part I: A Perfect Liquid,” *Proceedings of the Royal Irish Academy, Section A: Mathematical and Physical Sciences*, Vol. 27, 1907, pp. 9-68.
- [61] Orr, W. M., “The Stability or Instability of the Steady Motions of a Perfect Liquid and of a Viscous Liquid, Part II: A Viscous Liquid,” *Proceedings of the Royal Irish Academy, Section A: Mathematical and Physical Sciences*, Vol. 27, 1907, pp. 69-138.
- [62] Sommerfeld, A., “Ein Beitrag zur hydrodynamische Erklärung der turbulenten Flüssigkeitsbewegungen,” *Proceedings of the 4th International Congress of Mathematicians*, Vol. III, Rome, Italy, 1909, pp. 116-124.
- [63] “The three-dimensional nature of boundary-layer instability,” *Journal of Fluid Mechanics*, Vol. 12, No. 1, 1962, p. 1–34. <https://doi.org/10.1017/S0022112062000014>.
- [64] Emmons, H. W., “The Laminar-Turbulent Transition in a Boundary Layer-Part I,” *Journal of the Aeronautical Sciences*, Vol. 18, No. 7, 1951, pp. 490–498. <https://doi.org/10.2514/8.2010>.
- [65] Narasimha, R., “The laminar-turbulent transition zone in the boundary layer,” *Progress in Aerospace Sciences*, Vol. 22, No. 1, 1985, pp. 29–80. [https://doi.org/https://doi.org/10.1016/0376-0421\(85\)90004-1](https://doi.org/https://doi.org/10.1016/0376-0421(85)90004-1).
- [66] Somers, D., “Design and Experimental Results for a Natural-Laminar-Flow Airfoil for General Aviation Applications,” *NASA TP-1861*, 1981.
- [67] Somers, D., “Design and Experimental Results for the S809 Airfoil,” *NREL/SR-440-6918*, 1997.
- [68] Morkovin, M. V., “On the Many Faces of Transition,” *Viscous Drag Reduction*, 1969, pp. 1-31.
- [69] Dagenhart, J. R., Saric, W. S., Mousseux, M. C., and Stack, J. P., “Crossflow-vortex Instability and Transition on a 45-degree Swept Wing,” *20th Fluid Dynamics, Plasma*

- Dynamics and Lasers Conference*, AIAA Paper 89-1892, Buffalo, NY, June 1989. <https://doi.org/10.2514/6.1989-1892>.
- [70] Malik, M. R., Li, F., Choudhari, M. M., and Chang, C.-L., *Journal of Fluid Mechanics*, Vol. 399, 1999, p. 85–115. <https://doi.org/10.1017/S0022112099006291>.
- [71] Petzold, R., and Radespiel, R., “Transition on a Wing with Spanwise Varying Crossflow and Linear Stability Analysis,” *AIAA Journal*, Vol. 53, No. 2, 2015, pp. 321–335. <https://doi.org/10.2514/1.J053127>.
- [72] Krimmelbein, N., and Krumbein, A., “Transition prediction for the NASA CRM-NLF with the e^N -method using incompressible and compressible linear stability analysis,” *AIAA SciTech 2022 Forum*, AIAA Paper 2022-1953, San Diego, CA, January 2022. <https://doi.org/10.2514/6.2022-1953>.
- [73] Winslow, J., Otsuka, H., Govindarajan, B., and Chopra, I., “Basic Understanding of Airfoil Characteristics at Low Reynolds Numbers ($10^4 - 10^5$),” *Journal of Aircraft*, Vol. 55, No. 3, 2018, pp. 1050–1061. <https://doi.org/10.2514/1.C034415>.
- [74] Horton, H. P., “Laminar Separation Bubbles in Two and Three Dimensional Incompressible Flow,” *PhD Disseration*, Queen Mary College, University of London, 1968.
- [75] McGhee, R. J., Walker, B. S., and Millard, B. S., “Experimental Results for the Eppler 387 Airfoil at Low reynolds Number in the Langley Low Turbulence Pressure Tunnel,” *NASA TM 4062*, 1988.
- [76] Mueller, T. J., and DeLaurier, J. D., “Aerodynamics of Small Vehicles,” *Annual Review of Fluid Mechanics*, Vol. 35, 2003, pp. 89–111. <https://doi.org/10.1146/annurev.fluid.35.101101.161102>.
- [77] McGhee, R. J., and Walker, B. S., “Performance Measurements of an Airfoil at Low Reynolds number,” *Low Reynolds Number Aerodynamics Conference*, Proceedings of the Conference, Notre Dame, IN, June 1989, pp. 131–145.

- [78] Ghia, K. N., Osswald, G., and Ghia, U., “Study of Low-Reynolds Number Separated Flow Past the Wortmann FX 63-137 Airfoil,” *Low Reynolds Number Aerodynamics Conference*, Proceedings of the Conference, Notre Dame, IN, June 1989, pp. 58–69.
- [79] Lee, C., Koo, D., Telidetzki, K., Buckley, H., Gagnon, H., and Zingg, D., “Aerodynamic Shape Optimization of Benchmark Problems Using Jetstream,” *AIAA SciTech Forum 2015*, AIAA Paper 2015-0262, Kissimmee, FL, January 2015. <https://doi.org/10.2514/6.2015-0262>.
- [80] Dean, W., “XVI. Note on the motion of fluid in a curved pipe,” *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Vol. 4, No. 20, 1927, pp. 208–223. <https://doi.org/10.1080/14786440708564324>.
- [81] Görtler, H., “On the Three-Dimensional Instability of Laminar Boundary Layers on Concave Walls,” *NACA TM-1375*, 1954.
- [82] Blasius, H., “Grenzschichten in Flüssigkeiten mit Kleiner Reibung,” *Zeitschrift für angewandte Mathematik und Physik*, Vol. 56, 1908, pp. 1–37.
- [83] Dagenhart, J. R., “Amplified crossflow disturbances in the laminar boundary layer on swept wings with suction,” *NASA TP 1902*, 1981.
- [84] Floryan, J. M., and Saric, W. S., “Stability of Gortler Vortices in Boundary Layers,” *AIAA Journal*, Vol. 20, No. 3, 1982, pp. 316–324. <https://doi.org/10.2514/3.51076>.
- [85] Towne, A., Rigas, G., and Colonius, T., “A critical assessment of the parabolized stability equations,” *Theoretical and Computational Fluid Dynamics*, Vol. 33, No. 3, 2019, pp. 359–382. <https://doi.org/10.1007/s00162-019-00498>.
- [86] Mack, L. M., “Linear Stability Theory and the Problem of Supersonic Boundary-Layer Transition,” *AIAA Journal*, Vol. 13, No. 3, 1975, pp. 278–289. <https://doi.org/10.2514/3.49693>.
- [87] Smith, A. M. O., and Gamberoni, N., “Transition Pressure Gradient and Stability Theory,” *Douglas Aircraft Company, Report ES 26388*, 1956.

- [88] van Ingen, J. L., “Theoretical and Experimental Investigations of Incompressible Laminar Boundary Layers with and without Suction,” *Department of Aerospace Engineering, Delft University of Technology, Report VTH-124*, 1965.
- [89] van Ingen, J. L., “Research on Laminar Separation Bubbles at Delft University of Technology,” 1990.
- [90] Drela, M., and Giles, M. B., “Viscous-inviscid Analysis of Transonic and Low Reynolds Number Airfoils,” *AIAA Journal*, Vol. 25, No. 10, 1987, pp. 1347–1355. <https://doi.org/10.2514/3.9789>.
- [91] Michel, R., “Etude de la Transition et Calcul de la Trainee des Profiles d’aile en Incompressible,” *ONERA Publ. 58*, 1951.
- [92] Thwaites, B., “Approximate Calculation of the Laminar Boundary Layer,” *Aeronautical Quarterly*, Vol. 1, No. 3, 1949, p. 245–280. <https://doi.org/10.1017/S0001925900000184>.
- [93] Granville, P. S., “The Calculation of the Viscous Drag of Bodies of Revolution,” *David W. Taylor Model Basin Report 849, Navy Department*,, 1953.
- [94] Wazzan, A., Gazley, C., and Smith, A., “Tollmien-Schlichting waves and transition: Heated and Adiabatic Wedge Flows with Application to Bodies of Revolution,” *Progress in Aerospace Sciences*, Vol. 18, 1979, pp. 351–392. [https://doi.org/https://doi.org/10.1016/0376-0421\(77\)90012-4](https://doi.org/https://doi.org/10.1016/0376-0421(77)90012-4).
- [95] Falkner, V., and Skan, S., “Some Approximate Solutions of the Boundary Layer Equations,” *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, Vol. 12, No. 80, 1931, pp. 865–896. <https://doi.org/10.1080/14786443109461870>.
- [96] Schubauer, G. B., and Skramstad, H. K., “Laminar Boundary-Layer Oscillations and Stability of Laminar Flow,” *Journal of the Aeronautical Sciences*, Vol. 14, No. 2, 1947, pp. 69–78. <https://doi.org/10.2514/8.1267>.
- [97] Wells, C. S., “Effects of freestream turbulence on boundary- layer transition.” *AIAA Journal*, Vol. 5, No. 1, 1967, pp. 172–174. <https://doi.org/10.2514/3.3931>.

- [98] Abu-Ghannam, B. J., and Shaw, R., “Natural Transition of Boundary Layers: The Effects of Turbulence, Pressure Gradient, and Flow History,” *Journal of Mechanical Engineering Science*, Vol. 22, No. 5, 1980, pp. 213-228.
- [99] Walters, D. K., and Leylek, J. H., “A New Model for Boundary Layer Transition Using a Single-Point RANS Approach,” *Journal of Turbomachinery*, Vol. 126, No. 1, 2004, pp. 193-202. <https://doi.org/10.1115/1.1622709>.
- [100] Jain, R., “Computational Fluid Dynamics Transition Models Validation for Rotors in Unsteady Flow Conditions,” *Journal of Aircraft*, Vol. Articles in advance, 2022. <https://doi.org/10.2514/1.C036580>.
- [101] Lee, B., and Baeder, J. D., “Prediction and validation of laminar-turbulent transition using SA- γ transition model,” AIAA Paper 2021-1532, Virtual, January 2021. <https://doi.org/10.2514/6.2021-1532>.
- [102] Di Pasquale, D., Rona, A., and Garrett, S. J., “A selective review of CFD transition models,” *39th AIAA Fluid Dynamics Conference*, AIAA Paer 2009-3812, San Antonio, TX, June 2009. <https://doi.org/10.2514/6.2009-3812>.
- [103] Lilly, D. K., “On the Application of the Eddy Viscosity Concept in the Inertial Sub-range of Turbulence,” University Corporation for Atmospheric Research, 1966. <https://doi.org/10.5065/D67H1GGQ>.
- [104] Prandtl, L., “7. Bericht über Untersuchungen zur ausgebildeten Turbulenz,” *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 5, No. 2, 1925, pp. 136–139. <https://doi.org/10.1002/zamm.19250050212>.
- [105] ANSYS, “ANSYS Fluent User’s Guide,” ANSYS Inc., 2022.
- [106] Spalart, P., “Comments on the feasibility of LES for wing and on a hybrid RANS/LES approach,” *1st ASOSR CONFERENCE on DNS/LES*, 1997.
- [107] Spalart, P. R., Deck, S., Shur, M. L., Squires, D., K., Strelets, Kh., M., and Travin, A., “A New Version of Detached-Eddy Simulation, Resistant to Ambiguous Grid Densities,”

- Theoretical and Computational Fluid Dynamics*, Vol. 20, No. 2, 2006, pp. 181-195. <https://doi.org/10.1007/s00162-006-0015-0>.
- [108] Uzun, A., Blaisdell, G. A., and Lyrintzis, A. S., “Sensitivity to the Smagorinsky Constant in Turbulent Jet Simulations,” *AIAA Journal*, Vol. 41, No. 10, 2003, pp. 2077–2079. <https://doi.org/10.2514/2.1900>.
 - [109] Ducros, F., Nicoud, F., and Poinso, T., “Wall-Adapting Local Eddy-Viscosity models for simulations in complex geometries,” 1998, pp. 293–299.
 - [110] Majander, P., and Siikonen, T., “Evaluation of Smagorinsky-based subgrid-scale models in a finite-volume computation,” *International Journal for Numerical Methods in Fluids*, Vol. 40, No. 6, 2002, pp. 735–774. <https://doi.org/10.1002/fld.374>.
 - [111] Forster, D., Nelson, D. R., and Stephen, M. J., “Large-distance and long-time properties of a randomly stirred fluid,” *Phys. Rev. A*, Vol. 16, 1977, pp. 732–749. <https://doi.org/10.1103/PhysRevA.16.732>.
 - [112] Fournier, J. D., and Frisch, U., “Remarks on the renormalization group in statistical fluid dynamics,” *Phys. Rev. A*, Vol. 28, 1983, pp. 1000–1002. <https://doi.org/10.1103/PhysRevA.28.1000>.
 - [113] Sherer, S. E., Morgan, P. E., and Visbal, M. R., “Large Eddy Simulations of Wall-Mounted Cylinder in Supersonic Flow Using OVERFLOW,” *AIAA SciTech 2017 Forum*, AIAA Paper 2017-0309, Grapevine, TX, January 2017. <https://doi.org/10.2514/6.2017-0309>.
 - [114] Shur, M. L., Spalart, P., Strelets, M. K., and K., T. A., “Synthetic Turbulence Generators for RANS-LES Interfaces in Zonal Simulations of Aerodynamic and Aeroacoustic Problems,” *Flow, Turbulence and Combustion*, Vol. 93, No. 1, 2014, pp. 63–91. <https://doi.org/10.1007/s10494-014-9534-8>.
 - [115] Spalart, P. R., “Direct simulation of a turbulent boundary layer up to $Re_\theta = 1400$,” *Journal of Fluid Mechanics*, Vol. 187, 1988, p. 61–98. <https://doi.org/10.1017/S0022112088000345>.

- [116] Spalart, P. R., and Rumsey, C. L., “Effective Inflow Conditions for Turbulence Models in Aerodynamic Calculations,” *AIAA Journal*, Vol. 45, No. 10, 2007, pp. 2544–2553. <https://doi.org/https://doi.org/10.2514/1.29373>.
- [117] Baugher, S. K., and Bisek, N. J., “Application of Hybrid Reynolds Averaged Navier–Stokes and Large-Eddy-Simulation Eddy Viscosity Blending Methods in OVERFLOW,” *AIAA Journal*, Vol. Articles in Advance, 2021, pp. 1–14. <https://doi.org/10.2514/1.J060398>.
- [118] Drela, M., “XFOIL 6.9 User Primer,” MIT, Nov 2001.
- [119] Metkowsky, L. P., Axten, C. J., and Maughmer, M. D., “A Computational and Experimental Performance Comparison for a Low-Reynolds Number Airfoil,” *AIAA SciTech 2022 Forum*, AIAA Paper 2022-1530, San Diego, CA, January 2022. <https://doi.org/10.2514/6.2022-1530>.
- [120] Coder, J. G., and Maughmer, M. D., “Comparisons of Theoretical Methods for Predicting Airfoil Aerodynamic Characteristics,” *Journal of Aircraft*, Vol. 51, No. 1, 2014, pp. 183–191. <https://doi.org/10.2514/1.C032232>.
- [121] Kinzel, M., Maughmer, M., Lesieutre, G., and Duque, E., “Numerical Investigation of Miniature Trailing-Edge Effectors on Static and Oscillating Airfoils,” *43rd AIAA Aerospace Sciences Meeting and Exhibit*, AIAA Paper 2005-1039, Reno, NV, January 2005. <https://doi.org/10.2514/6.2005-1039>.
- [122] Weiss, A., Gardner, A. D., Schwermer, T., Klein, C., and Raffel, M., “On the Effect of Rotational Forces on Rotor Blade Boundary-Layer Transition,” *AIAA Journal*, Vol. 57, No. 1, 2018, pp. 252–266. <https://doi.org/10.2514/1.J057036>.
- [123] Gardner, A., Weiss, A., Heineck, J. T., Overmeyer, A., Spooner, H., Jain, R., Wolf, C., and Raffel, M., “Boundary Layer Transition Measured by DIT on the PSP Rotor in Forward Flight,” *Journal of the American Helicopter Society*, Vol. 66, No. 022008, 2021, pp. 1–10. <https://doi.org/10.4050/JAHS.66.022008>.
- [124] Nichols, R. H., and Buning, P. G., “User’s Manual for OVERFLOW 2.3,” NASA, October 2019.

- [125] Barrouillet, B., Laurendeau, E., and Yang, H., “Optimized cross flow transition model for Reynolds Averaged Navier Stokes applications,” *AIAA Aviation 2021 Forum*, AIAA Paper 2021-2714, Virtual, August 2021. <https://doi.org/10.2514/6.2021-2714>.
- [126] Paredes, P., Venkatachari, B. S., Choudhari, M. M., Li, F., Hildebrand, N. J., and Chang, C.-L., “Transition Analysis for the CRM-NLF Wind Tunnel Configuration,” *AIAA SciTech 2021 Forum*, AIAA Paper 2021-1431, Virtual, January 2021. <https://doi.org/10.2514/6.2021-1431>.
- [127] Denison, M., “1st AIAA CFD Transition Modeling and Prediction Workshop: OVERFLOW Results for the SST and Langtry-Menter Models,” *AIAA SciTech 2022 Forum*, AIAA Paper 2022-0908, San Diego, CA, January 2022. <https://doi.org/10.2514/6.2022-0908>.
- [128] Lee, B., Jung, Y. S., and Baeder, J. D., “Validation of SA- γ - $Re_{\theta t}$ and SA- γ turbulence/transition model,” *AIAA SciTech 2022 Forum*, AIAA Paper 2022-0910, San Diego, CA, January 2022. <https://doi.org/10.2514/6.2022-0910>.
- [129] Coder, J. G., “Standard Test Cases for Transition Model Verification and Validation in Computational Fluid Dynamics,” *2018 AIAA Aerospace Sciences Meeting*, AIAA Paper 2018-0029, Kissimmee, FL, January 2018. <https://doi.org/10.2514/6.2018-0029>.
- [130] Zastawny, M., and Lardeau, S., “Validation of Intermittency-based Transition Prediction Models,” *AIAA SciTech 2022 Forum*, AIAA Paper 2022-0568, San Diego, CA, January 2022. <https://doi.org/10.2514/6.2022-2568>.
- [131] Ortiz-Melendez, H. D., Coder, J. G., and Shmilovich, A., “High-Lift Computational Strategy for Slotted, Natural-Laminar-Flow Airfoils,” *Journal of Aircraft (Articles in Advance)*, 0, pp. 1–15. <https://doi.org/10.2514/1.C036550>.
- [132] Jung, Y. S., and Baeder, J., “ $\gamma - \overline{Re_{\theta t}}$ Spalart–Allmaras with Crossflow Transition Model Using Hamiltonian–Strand Approach,” *Journal of Aircraft*, Vol. 56, No. 3, 2019, pp. 1040–1055. <https://doi.org/10.2514/1.C035149>.
- [133] Schaeffler, N., Allan, B., Lienard, C., and Le Pape, A., “Progress Towards Fuselage Drag Reduction via Active Flow Control: A Combined CFD and Experimental Effort,” *36th European Rotorcraft Forum*, Paris, France, September 2010.

- [134] Kreplin, H.-P., Meier, H. U., and Maier, A., “Wind Tunnel Model and Measuring Techniques for the Investigation of Three-Dimensional Turbulent Boundary Layers,” *10th AIAA Aerodynamic Testing Conference*, AIAA Paper 78-781, San Diego, CA, April 1978. <https://doi.org/10.2514/6.1978-781>.
- [135] Kreplin, H.-P., Vollmers, H., and Meier, H. U., “Wall Shear Stress Measurements on an Inclined Prolate Spheroid in the DFVLR 3 m x 3 m Low Speed Wind Tunnel,” *Rept. IB 22-84 A 33*, DFVLR-AVA, Göttingen, 1985.
- [136] Krimmelbein, N., and Krumbein, A., “Automatic Transition Prediction for Three-Dimensional Configurations with Focus on Industrial Application,” *Journal of Aircraft*, Vol. 48, No. 6, 2011, pp. 1878-1887. <https://doi.org/10.2514/1.C031230>.
- [137] Xu, J., Han, X., Qiao, L., Bai, J., and Zhang, Y., “Fully Local Amplification Factor Transport Equation for Stationary Crossflow Instabilities,” *AIAA Journal*, Vol. 57, No. 7, 2019, pp. 2682-2693. <https://doi.org/10.2514/1.J057502>.
- [138] Grabe, C., Shengyang, N., and Krumbein, A., “Transport Modeling for the Prediction of Crossflow Transition,” *AIAA Journal*, Vol. 56, No. 8, 2018, pp. 3167–3178. <https://doi.org/10.2514/1.J056200>.
- [139] Radeztsky, R. H., Jr., M. S., Reibert, and Saric, W. S., “Effect of Micron-Sized Roughness on Transition in Swept-Wing Flows,” *31st Aerospace Sciences Meeting*, AIAA Paper 1993-0076, Reno, NV, January 1993.
- [140] Carnes, J. A., and Coder, J. G., “Effect of Crossflow Transition on the Pressure-Sensitive Paint Rotor in Hover,” *Journal of Aircraft*, Vol. 59, No. 1, 2022, pp. 29–46. <https://doi.org/10.2514/1.C036121>.
- [141] Hariharan, N., Abras, J., Jain, R., Narducci, B., and Wake, B., “AIAA Rotorcraft Hover Prediction Workshop Website,” 2019. URL <https://aiaahover.wixsite.com/website-6/file-share>.
- [142] Jain, R., “CFD Performance and Turbulence Transition Predictions on an Installed Model-scale Rotor in Hover,” *55th AIAA Aerospace Sciences Meeting*, AIAA Paper 2017-1871, Grapevine, TX, January 2017. <https://doi.org/10.2514/6.2017-1871>.

- [143] Vieira, B. A. O., Kinzel, M. P., and Maughmer, M. D., “CFD Hover Predictions Including Boundary-Layer Transition,” *55th AIAA Aerospace Sciences Meeting*, AIAA Paper 2017-1665, Grapevine, TX, January 2017. <https://doi.org/10.2514/6.2017-1665>.
- [144] Sheng, C., Wang, J., and Zhao, Q., “S-76 Rotor Hover Predictions Using Advanced Turbulence Models,” *53rd AIAA Aerospace Sciences Meeting*, AIAA Paper 2015-1715, Kissimmee, FL, January 2015. <https://doi.org/10.2514/6.2015-1715>.
- [145] Lee, B., Jung, Y. S., Jude, D., and Baeder, J. D., “Turbulent Transition Prediction of PSP Hovering Rotor Using $\gamma - Re_{\theta t} - SA$ with Crossflow Transition Model,” *AIAA SciTech 2019 Forum*, AIAA Paper 2019-0286, San Diego, CA, January 2019. <https://doi.org/10.2514/6.2019-0286>.
- [146] Jain, R., “CFD Hover Performance and Transition Predictions on the PSP and HVAB Rotors using CREATETM-AV Helios,” *AIAA SciTech 2022 Forum*, AIAA Paper 2022-1549, San Diego, CA, January 2022. <https://doi.org/10.2514/6.2022-1549>.
- [147] Carnes, J. A., “Computational Fluid Dynamics Assessment of Laminar-Turbulent Boundary Layer Transition and its Application to Rotorcraft,” *M.S. Thesis*, Department of Mechanical, Aerospace, and Biomedical Engineering, The University of Tennessee, Knoxville, TN, 2020.
- [148] Koning, W. J. F., Johnson, W., and Grip, H. F., “Improved Mars Helicopter Aerodynamic Rotor Model for Comprehensive Analyses,” *AIAA Journal*, Vol. 57, No. 9, 2019, pp. 3969–3979. <https://doi.org/10.2514/1.J058045>.
- [149] Koning, W. J., Romander, E. A., and Johnson, W., “Optimization of Low Reynolds Number Airfoils for Martian Rotor Applications Using an Evolutionary Algorithm,” *AIAA SciTech 2020 Forum*, AIAA Paper 2020-0084, Orlando, FL, January 2020. <https://doi.org/10.2514/6.2020-0084>.
- [150] Rumsey, C. L., and Spalart, P. R., “Turbulence Model Behavior in Low Reynolds Number Regions of Aerodynamic Flowfields,” *AIAA Journal*, Vol. 47, No. 4, 2009, pp. 982–993. <https://doi.org/10.2514/1.39947>.

- [151] Raman, G., Rice, E. J., and Reshotko, E., “Mode spectra of natural disturbances in a circular jet and the effect of acoustic forcing,” *Experiments in Fluids*, Vol. 17, 1994, pp. 415–426. <https://doi.org/10.1007/BF01877044>.
- [152] Antonia, R. A., and Zhao, Q., “Effect of initial conditions on a circular jet,” *Experiments in Fluids*, Vol. 31, 2001, pp. 319–323. <https://doi.org/10.1007/s003480100289>.
- [153] George, A. R., “Analyses of Broadband Noise Mechanisms of Rotors,” *Recent Advances in Aeroacoustics*, 1986, pp. 87–127. https://doi.org/10.1007/978-1-4612-4840-8_4.
- [154] Voke, P. R., and Yang, Z., “Numerical study of bypass transition,” *Physics of Fluids*, Vol. 7, No. 9, 1995, pp. 2256–2264. <https://doi.org/10.1063/1.868473>.
- [155] Huai, X., Joslin, R., and Piomelli, U., “Large-Eddy Simulation of Transition to Turbulence in Boundary Layers,” *Theoretical and Computational Fluid Dynamics*, 1997, pp. 149–163. <https://doi.org/10.1007/s001620050037>.
- [156] Germano, M., Piomelli, U., Moin, P., and Cabot, W. H., “A dynamic subgrid-scale eddy viscosity model,” *Physics of Fluids A: Fluid Dynamics*, Vol. 3, No. 7, 1991, pp. 1760–1765. <https://doi.org/10.1063/1.857955>.
- [157] Sayadi, T., and Moin, P., “Predicting natural transition using large eddy simulation,” *Stanford Center for Turbulence Research Annual Research Briefs*, 2011, pp. 97–108.
- [158] Herbert, T., “Secondary Instability of Boundary Layers,” *Annual Review of Fluid Mechanics*, Vol. 20, No. 1, 1988, pp. 487–526. <https://doi.org/10.1146/annurev.fl.20.010188.002415>.
- [159] Kachanov, Y. S., and Levchenko, V. Y., “The resonant interaction of disturbances at laminar-turbulent transition in a boundary layer,” *Journal of Fluid Mechanics*, Vol. 138, 1984, p. 209–247. <https://doi.org/10.1017/S0022112084000100>.
- [160] Kim, M., Lim, J., Kim, S., Jee, S., Park, J., and Park, D., “Turbulent Transition Prediction Using Large-Eddy Simulation with the Stability Theory,” *Tenth International Conference on Computational Fluid Dynamics (ICCFD10)*, Barcelona, Spain, July 2018.

- [161] Spalart, P. R., Jou, W.-H., Strelets, M., and Allmaras, S. R., “Comments on the Feasibility of LES for Wings and on a Hybrid RANS/LES Approach,” *Proceedings of the First AFOSR International Conference on DNS/LES*, Ruston, LA, August 1997, pp. 137–147.
- [162] Yuan, W., Khalid, M., Windte, J., Scholz, U., and Radespiel, R., “An Investigation of Low-Reynolds-Number Flows Past Airfoils,” *23rd AIAA Applied Aerodynamics Conference*, AIAA Paper 2005-4607, Toronto, Ontario, CA, June 2005. <https://doi.org/10.2514/6.2005-4607>.
- [163] Breuer, M., “Effect of Inflow Turbulence on an Airfoil Flow with Laminar Separation Bubble: An LES Study,” *Flow, Turbulence and Combustion*, Vol. 101, 2018, pp. 433-456. <https://doi.org/10.1007/s10494-017-9890-2>.
- [164] Uranga, A., Persson, P.-O., Drela, M., and Peraire, J., “Preliminary Investigation Into the Effects of Cross-Flow on Low Reynolds Number Transition,” *20th AIAA Computational Fluid Dynamics Conference*, AIAA Paper 2011-3558, Honolulu, HI, June 2011. <https://doi.org/10.2514/6.2011-3558>.
- [165] Ol, M. V., McAuliffe, B. R., Hanff, E. S., Scholz, U., and Kahler, C., “Comparison of Laminar Separation Bubbles Measurements on a Low Reynolds Number Airfoil in Three Facilities,” *35th AIAA Fluid Dynamics Conference*, AIAA Paper 2005-5149, Toronto, ON, CA, June 2005. <https://doi.org/10.2514/6.2005-5149>.
- [166] Radespiel, R., Windte, J., and Scholz, U., “Numerical and Experimental Flow Analysis of Moving Airfoils with Laminar Separation Bubbles,” *AIAA Journal*, , No. 6, 2006, pp. 1346–1356. <https://doi.org/10.2514/1.25913>.
- [167] Boussinesq, J., “Essai sur la théorie des eaux courantes,” *Mémoires présentés par divers savants à l’Académie des Sciences XXIII*, , No. 1, 1877, pp. 1–680.
- [168] Cebeci, T., *Analysis of Turbulent Flows*, 2nd ed., 2004.
- [169] Smith, A. M. O., and Cebeci, T., “Numerical Solution of the Turbulent-Boundary-Layer Equations,” *DAC Report No. 33735*, 1967.

- [170] Baldwin, B. S., and Lomax, H., "Thin-layer Approximation and Algebraic Turbulence Model for Separated Turbulent Flows," *AIAA 16th Aerospace Sciences Meeting*, AIAA 1978-257, Huntsville, AL, January 1978.
- [171] Baldwin, B. S., and Barth, T. J., "A One-Equation Turbulence Transport Model for High Reynolds Number Wall-Bounded Flows," *NASA TM 102847*, 1990.
- [172] Launder, B. E., and Sharma, B. I., "Application of the energy-dissipation model of turbulence to the calculation of flow near a spinning disc," *Letters in Heat and Mass Transfer*, Vol. 1, No. 2, 1974, pp. 131-137.
- [173] Shih, T.-H., Liou, W. W., Shabbir, A., Yang, Z., and Zhu, J., "A new k - ϵ eddy viscosity model for high reynolds number turbulent flows," *Computers & Fluids*, Vol. 24, No. 3, 1995, pp. 227-238.
- [174] Yakhot, V., Orzsag, S. A., Thangam, S., Gatski, T. B., and Speziale, C. G., "Development of turbulence models for shear flows by a double expansion technique," *Physics of Fluids A: Fluid Dynamics*, Vol. 4, No. 7, 1992, pp. 1510-1520.
- [175] Wilcox, D. C., "Reassessment of the Scale-Determining Equation for Advanced Turbulence Models," *AIAA Journal*, Vol. 26, No. 11, 1988, pp. 1299-1310.
- [176] Wilcox, D. C., "Formulation of the k - ω Turbulence Model Revisited," *AIAA Journal*, Vol. 46, No. 11, 2008, pp. 2823-2838.
- [177] Launder, B. E., and Spalding, D. B., *Mathematical Modeling of Turbulence*, 1972.
- [178] Johnson, D. A., and King, L. S., "A mathematically simple turbulence closure model for attached and separated turbulent boundary layers," *AIAA Journal*, Vol. 23, No. 11, 1985, pp. 1684-1692. <https://doi.org/10.2514/3.9152>.
- [179] Bradshaw, P., Ferriss, D. H., and Atwell, N. P., "Calculation of boundary-layer development using the turbulent energy equation," *Journal of Fluid Mechanics*, Vol. 28, No. 3, 1967, p. 593-616. <https://doi.org/10.1017/S0022112067002319>.

- [180] Harsha, P. T., and Lee, S. C., “Correlation Between Turbulent Shear Stress and Turbulent Kinetic Energy,” *AIAA Journal*, Vol. 8, No. 8, 1970, pp. 1508–1510. <https://doi.org/10.2514/3.5932>.
- [181] Van Driest, E. R., and Blumer, C. B., “Boundary Layer Transition: Freestream Turbulence and Pressure Gradient Effects,” *AIAA Journal*, Vol. 1, No. 6, 1963, pp. 1303–1306. <https://doi.org/10.2514/3.1784>.
- [182] Nichols, R. H., “Addition of a Local Correlation-Based Boundary Layer Transition model to the CREATETM-AV *Kestrel* Unstructured Flow Solver,” *AIAA SciTech Forum 2020*, AIAA Paper 2019-1343, San Diego, CA, January 2019. <https://doi.org/10.2514/6.2019-1343>.
- [183] Nie, S., “Extension of Transition Modeling by a Transport Equation Approach,” *Ph.D. Dissertation*, Faculty of Mechanical Engineering, Technical University - Braunschweig, Braunschweig, Germany, 2017.
- [184] Dagenhart, J. R., Saric, W. S., Mousseux, M. C., and Stack, J. P., “Crossflow-vortex Instability and Transition on a 45-degree Swept Wing,” *20th Fluid Dynamics, Plasma Dynamics and Lasers Conference*, AIAA Paper 89-1892, Buffalo, NY, June 1989. <https://doi.org/10.2514/6.1989-1892>.
- [185] Radeztsky, R. H., Jr., M. S., Reibert, and Saric, W. S., “Effect of Micron-Sized Roughness on Transition in Swept-Wing Flows,” *AIAA Paper 1993-0076*, 1993. <https://doi.org/10.2514/6.1993-76>.
- [186] Muller, C., and Herbst, F., “Modelling of Crossflow-induced Transition based on Local Variables,” *6th European Conference on Computational Fluid Dynamics*, Barcelona, Spain, July 2014.
- [187] Drela, M., and Youngren, H., “A User’s Guide to MISES 2.63,” MIT Aerospace Computational Design Laboratory, February 2008.
- [188] Gritskevich, M. S., Garbaruk, A. V., Schütze, J., and Menter, F. R., “Development of DDES and IDDES Formulations for the $k - \omega$ Shear Stress Transport Model,” *Flow, Turbulence and Combustion*, Vol. 88, 2012, pp. 431-449. <https://doi.org/10.1007/s10494-011-9378-4>.

- [189] Reddy, K., Ryon, J., and Durbin, P., “A DDES Model with a Smagorinsky-type Eddy Viscosity Formulation and Log-layer Mismatch Correction,” *International Journal of Heat and Fluid Flow*, Vol. 50, 2014, pp. 103–113. <https://doi.org/10.1016/j.ijheatfluidflow.2014.06.002>.
- [190] Pulliam, T. H., and Steger, J. L., “Implicit Finite-Difference Simulations of Three-Dimensional Compressible Flow,” *AIAA Journal*, Vol. 18, 1980, pp. 159–167. <https://doi.org/10.2514/3.50745>.
- [191] Pulliam, T., “High Order Accurate Finite-Difference Methods: as seen in OVERFLOW,” *20th AIAA Computational Fluid Dynamics Conference*, AIAA Paper 2011-3851, June 2011, Honolulu, HI. <https://doi.org/10.2514/6.2011-3851>.
- [192] Roe, P. L., “Approximate Riemann solvers, parameter, vectors, and difference schemes,” *Journal of Computational Physics*, Vol. 43, No. 2, 1981, pp. 357–372. [https://doi.org/10.1016/0021-9991\(81\)90128-5](https://doi.org/10.1016/0021-9991(81)90128-5).
- [193] Nichols, R. H., Tramel, R. W., and Buning, P. G., “Evaluation of Two High-Order Weighted Essentially Nonoscillatory Schemes,” *AIAA Journal*, Vol. 46, No. 12, 2008, pp. 3090–3102. <https://doi.org/10.2514/1.36849>.
- [194] Jiang, G.-H., and Shu, C.-W., “Efficient Implementation of Weighted ENO Schemes,” *Journal of Computational Physics*, Vol. 126, 1996, pp. 202–228.
- [195] Pulliam, T. H., and Chaussee, D. S., “A Diagonal Form of an Implicit Approximate-Factorization Algorithm,” *Journal of Computational Physics*, Vol. 39, 1981, pp. 347–363.
- [196] Warming, R. F., and Beam, R. M., “On the construction and application of implicit factored schemes for conservation laws,” *SIAM-AMS Proc* 1978, pp. 85–129.
- [197] Nichols, R., Tramel, R., and Buning, P., *Solver and Turbulence Model Upgrades to OVERFLOW 2 for Unsteady and High-Speed Applications*, AIAA Paper 2006-2824, June 2006, San Francisco, CA. <https://doi.org/10.2514/6.2006-2824>.
- [198] Klopfer, G. H., Van der Wijngaart, R. F., Hung, C. M., and Onufer, J. T., “A Diagonalized Diagonal Dominant Alternating Direction Implicit (D3ADI) Scheme and Subiteration

- Correction,” *29th AIAA Fluid Dynamics Conference*, AIAA Paper 1998-2824, Albuquerque, NM, June 1998.
- [199] Wilcox, D. C., “Reassessment of the Scale-Determining Equation for Advanced Turbulence Models,” *AIAA Journal*, Vol. 26, No. 11, 1988, pp. 1299–1310. <https://doi.org/10.2514/3.10041>.
- [200] Denison, M., and Pulliam, T. H., “Implementation and Assessment of the Amplification Factor Transport Laminar-Turbulent Transition Model,” *AIAA Aviation 2018*, 2018-3382, Atlanta, GA, June 2018. <https://doi.org/10.2514/6.2018-3382>.
- [201] Nie, S., Krimmelbein, N., Krumbein, A., and Grabe, C., “Coupling of a Reynolds Stress Model with the $\gamma - Re_{\theta,t}$ Transition Model,” *AIAA Journal*, Vol. 56, No. 1, 2018, pp. 146–157. <https://doi.org/10.2514/1.J056167>.
- [202] Çakmakçıoğlu, S. C., O., B., Mura, R., and Kaynak, U., “A Revised One-Equation Transitional Model for External Aerodynamics,” *AIAA Aviation 2020 Forum*, AIAA Paper 2020-2706, Virtual, June 2020. <https://doi.org/10.2514/6.2020-2706>.
- [203] Wang, S., Ge, M., Deng, X., Yu, Q., and Wang, G., “Blending of Algebraic Transition Model and Subgrid Model for Separated Transitional Flows,” *AIAA Journal*, Vol. 57, No. 11, 2019. <https://doi.org/10.2514/1.J058313>.
- [204] Rumsey, C. L., Smith, B. R., and Huang, G. P., “Description of a Website Resource for Turbulence Modeling Verification and Validation,” *40th AIAA Fluid Dynamics Conference*, AIAA Paper 2010-4742, Chicago, IL, June-July 2010. <https://doi.org/10.2514/6.2010-4742>.
- [205] Cook, P., McDonald, M. A., and P., F. M. C., “Aerofoil RAE 2822 - Pressure Distributions, and Boundary Layer and Wake Measurements,” *AGARD Advisory Report 138*, May 1979.
- [206] Schwermer, T., Richter, K., and Raffel, M., “Development of a Rotor Test Facility for the Investigation of Dynamic Stall,” *New Results in Numerical and Experimental Fluid Mechanics X, Vol. 132. Notes on Numerical Fluid Mechanics and Multidisciplinary Design.*, 2016, pp. 663–673. https://doi.org/10.1007/978-3-319-27279-5_58.

- [207] Richter, K., Koch, S., Goerttler, A., Lütke, B., Wolf, C. C., and Benkel, A., “Unsteady Boundary Layer Transition on the DSA-9A Rotor Blade Airfoil,” *41st European Rotorcraft Forum*, 2015.
- [208] Schwermer, T., Gardner, A. D., and Raffel, M., “A Novel Experiment to Understand the Dynamic Stall Phenomenon in Rotor Axial Flight,” *Journal of the American Helicopter Society*, Vol. 64, 012004 (2019). <https://doi.org/10.4050/JAHS.64.012004>.
- [209] Weiss, A., Gardner, A. D., Klein, C., and Raffel, M., “Boundary-layer transition measurements on Mach-scaled helicopter rotor blades in climb,” *CEAS Aeronautical Journal*, Vol. 8, No. 4, 2017, pp. 613–623. <https://doi.org/10.1007/s13272-017-0263-2>.
- [210] Richter, K., Wolf, C. C., Gardner, A. D., and Merz, C. B., “Detection of Unsteady Boundary Layer Transition Using Three Experimental Methods,” *54th AIAA Aerospace Sciences Meeting*, 2016. <https://doi.org/10.2514/6.2016-1072>.
- [211] Gardner, A. D., Eder, C., Wolf, C. C., and Raffel, M., “Analysis of differential infrared thermography for boundary layer transition detection,” *Experiment in Fluids*, Vol. 58, No. 112, 2017. <https://doi.org/10.1007/s00348-017-2405-z>.
- [212] Weiss, A., Wolf, C. C., Kaufmann, K., Braukmann, J. N., Heineck, J. T., and Raffel, M., “Unsteady boundary-layer transition measurements and computations on a rotating blade under cyclic pitch conditions,” *Experiments in Fluids*, Vol. 61, No. 2, 2020. <https://doi.org/10.1007/s00348-020-2899-7>.
- [213] Schwamborn, D., Gerhold, T., and Heinrich, R., “The DLR TAU-code: Recent applications in research and industry,” *ECCOMAS CFD 2006*, 2006.
- [214] Vuillet, A., Allongue, M., Philippe, J., and Desopper, A., “Performance and Aerodynamic Development of the Supper Puma Mk II Main Rotor with New SPP8 blade tip design,” *15th European Rotorcraft Forum*, 1989.
- [215] Chan, W. M., Rogers, S. E., Pandya, S. A., Kao, D. L., Buning, P. G., Meakin, R. L., Boger, D. A., and Nash, S. M., “Chimera Grid Tools User’s Manual,” Moffett Field, CA, 2010.

- [216] Derlaga, J. M., Jackson, C. W., and Buning, P. G., “Recent Progress in OVERFLOW Convergence Improvements,” *AIAA SciTech 2020 Forum*, AIAA Paper 2020-1045, January 2020, Orlando, FL. <https://doi.org/10.2514/6.2020-1045>.
- [217] Allmaras, S. R., Johnson, F. T., and Spalart, P. R., “Modifications and Clarifications for the Implementation of the Spalart-Allmaras Turbulence Model,” *7th International Conference on Computational Fluid Dynamics*, 2012.
- [218] Shur, M. L., Strelets, M. K., Travin, A., and Spalart, P. R., “Turbulence Modeling in Rotating and Curved Channels: Assessing the Spalart-Shur Correction,” *AIAA Journal*, Vol. 38, No. 5, 2000, pp. 784–792. <https://doi.org/10.2514/2.1058>.
- [219] Arnal, D., Habiballah, M., and Coustouls, E., “Laminar instability theory and transition criteria in two and three-dimensional flow,” *La Recherche Aeronautique (English Edition)*, , No. 2, 1984, pp. 45–63.
- [220] Gardner, A. D., and Richter, K., “Boundary layer transition determination for periodic and static flows using phase-averaged pressure data,” *Experiment in Fluids*, Vol. 56, No. 119, 2015. <https://doi.org/10.1007/s00348-015-1992-9>.
- [221] Wolf, C. C., Mertens, C., Gardner, A. D., Dollinger, C., and Fischer, A., “Optimization of differential infrared thermography for unsteady boundary layer transition measurement,” *Experiment in Fluids*, Vol. 60, No. 19, 2019. <https://doi.org/10.1007/s00348-018-2667-0>.
- [222] Grinstein, F. F., and Fureby, C., “On Flux-Limiting-Based Implicit Large Eddy Simulation,” *Journal of Fluids Engineering*, Vol. 129, No. 12, 2007, pp. 1483–1492. <https://doi.org/10.1115/1.2801684>.
- [223] Leishman, J. G., *Principles of Helicopter Aerodynamics*, 2nd ed., Cambridge University Press, New York, 2006.
- [224] Cadence Design Systems, I., “Pointwise User Manual,” retrieved April 2022. URL <https://www.pointwise.com/doc/user-manual/>.

- [225] Istvan, M. S., Kurelek, J. W., and Yarusevych, S., “Turbulence Intensity Effects on Laminar Separation Bubbles Formed over an Airfoil,” *AIAA Journal*, Vol. 56, No. 4, 2018, pp. 1335–1347. <https://doi.org/10.2514/1.J056453>.
- [226] McCroskey, W. J., and Dwyer, H. A., “Methods of analyzing propeller and rotor boundary layers with crossflow,” *Analytic Methods in Aircraft Aerodynamics*, October, 1969, Moffett Field, CA.
- [227] Glauert, H., *Aerodynamic Theory A General Review of Progress*, 1935, Vol. 4.
- [228] Snel, H., Houwink, R., and Piers, W. J., “Sectional Prediction of 3D Effects for Separated Flow on Rotating Blades,” *NLR TP 92409 U*, 1992.

Appendices

Appendix A: Chapter 1 Figures and Tables



Figure 1.1: General Atomics MQ-9 Reaper, Ref. [19]

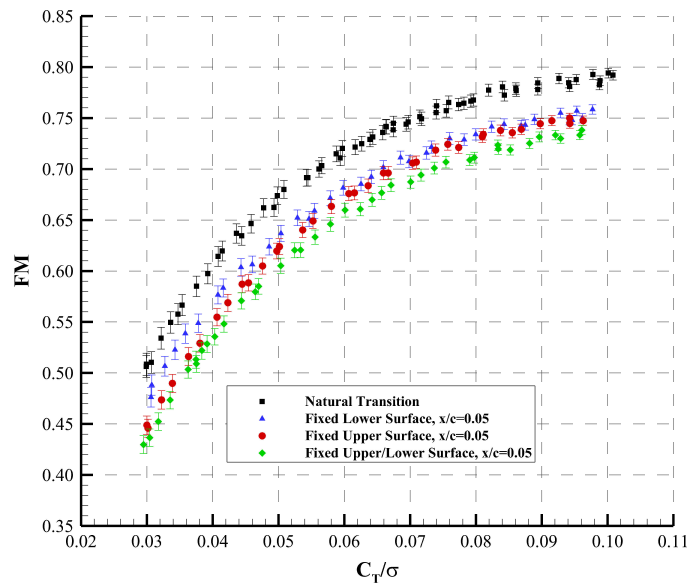


Figure 1.2: PSP Rotor figure of merit, Ref. [21]

Appendix B: Chapter 2 Figures and Tables

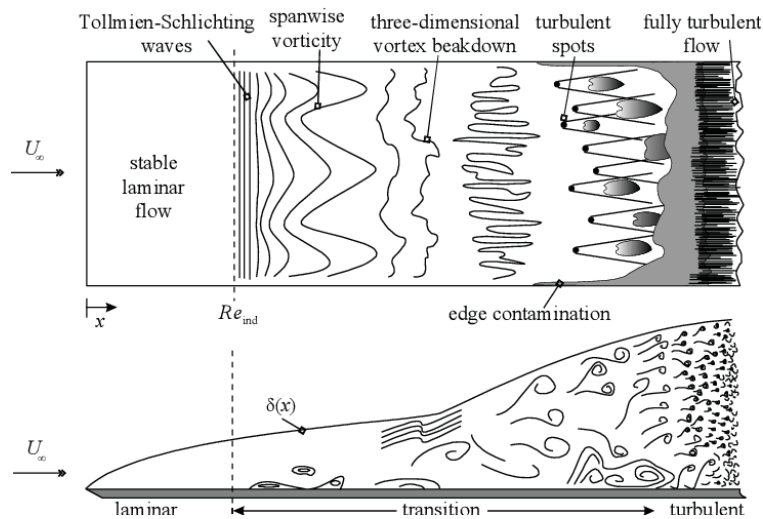


Figure 2.1: Breakdown of natural transition, Ref. [49]

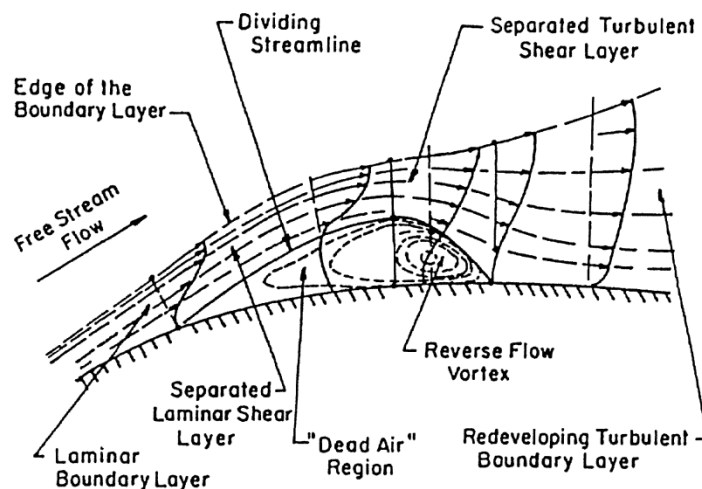


Figure 2.2: Flowfield near a laminar separation bubble (adapted from Horton [74])

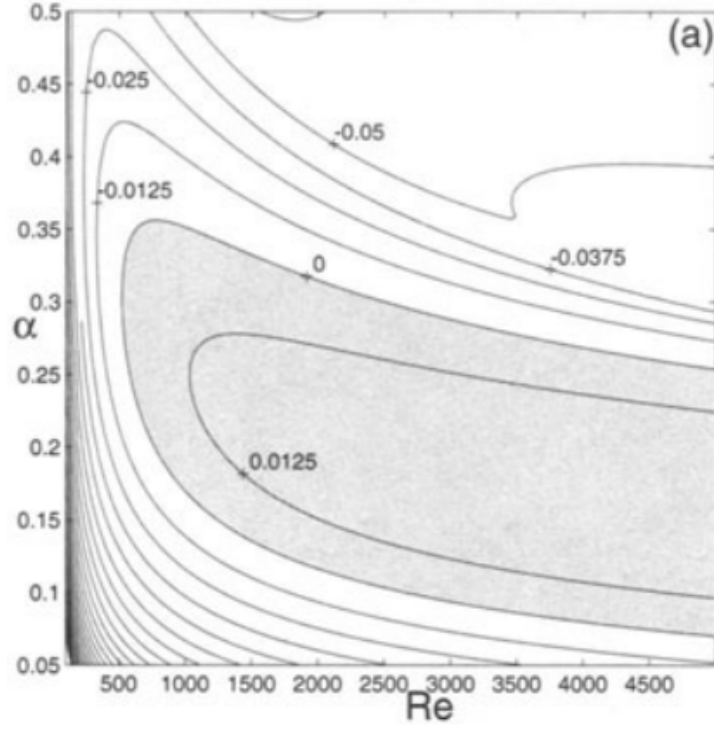


Figure 2.3: Neutral stability curve for a Blasius boundary layer [55]

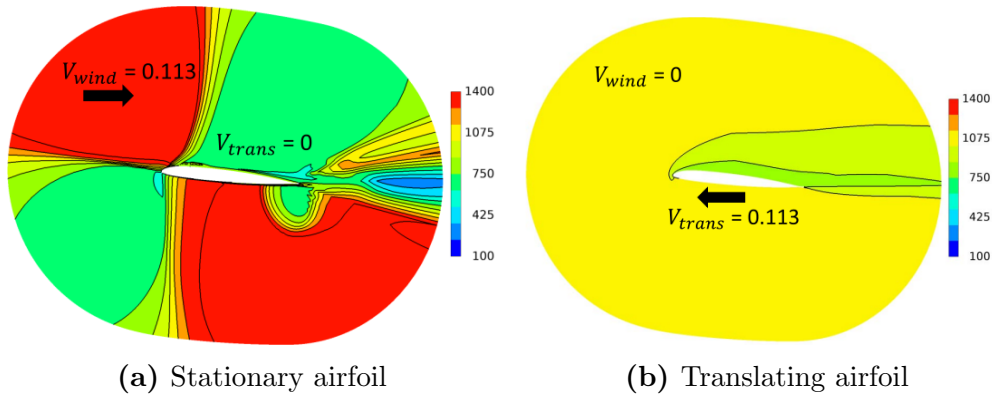


Figure 2.4: Predicted contours of $\widetilde{Re}_{\theta,t}$ about the DSA-9A airfoil (adapted from Ref. [100])

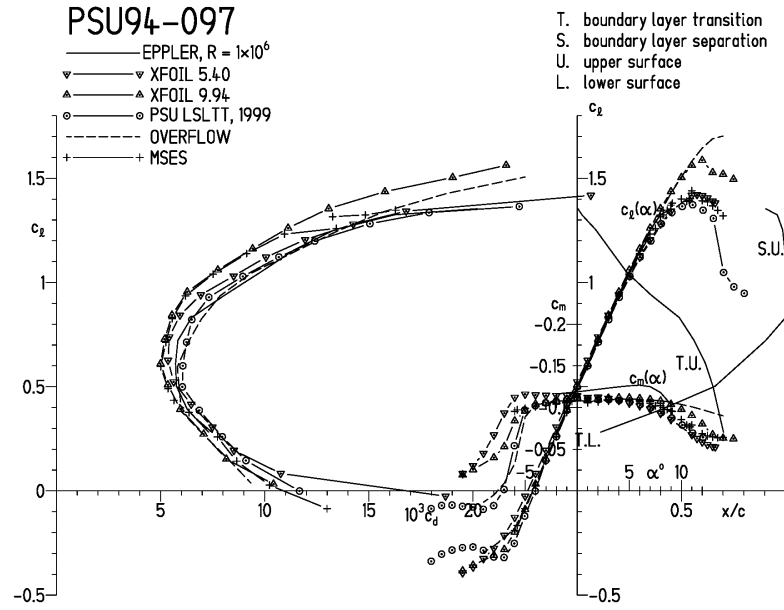


Figure 2.5: Performance of the PSU 94-097 airfoil at $Re = 1 \times 10^6$, Ref. [119]

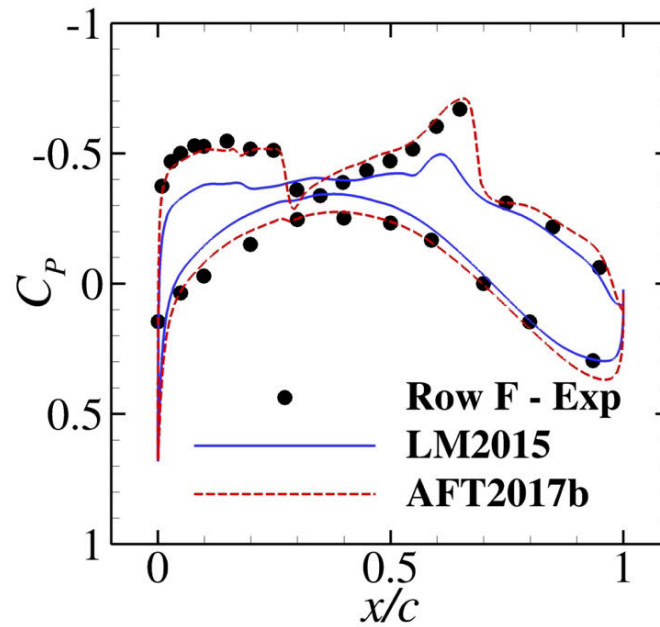


Figure 2.6: Predicted surface pressure on the CRM-NLF using two different transition models (adapted from Ref. [17])

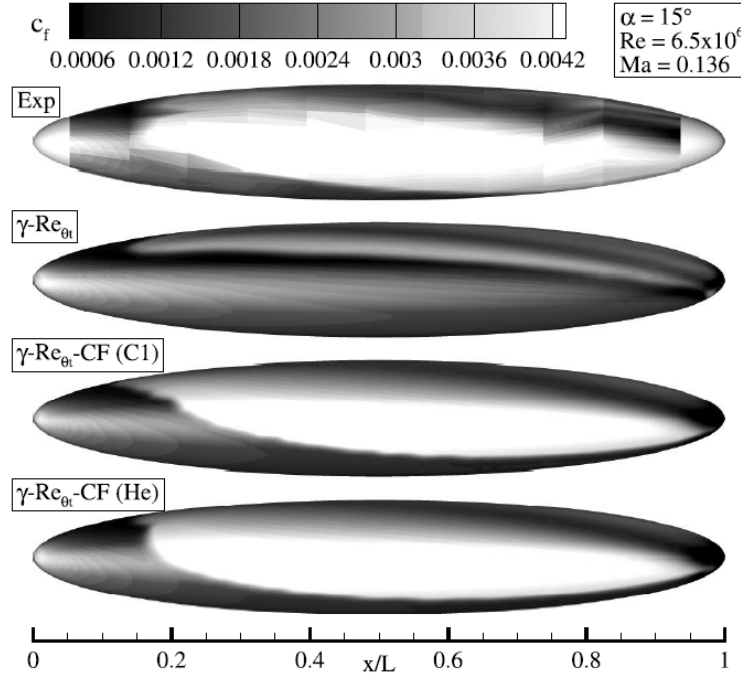


Figure 2.7: Predicted skin friction distributions on the 6:1 inclined prolate spheroid [138]

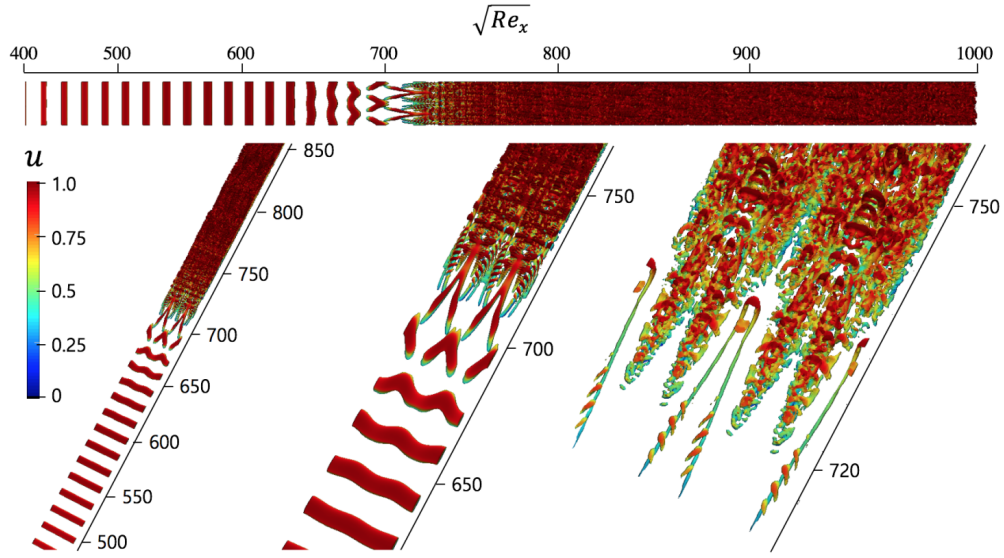


Figure 2.8: LES-predicted isosurfaces of the Q-criterion for a flat plate boundary layer, showing H-type natural transition [160]

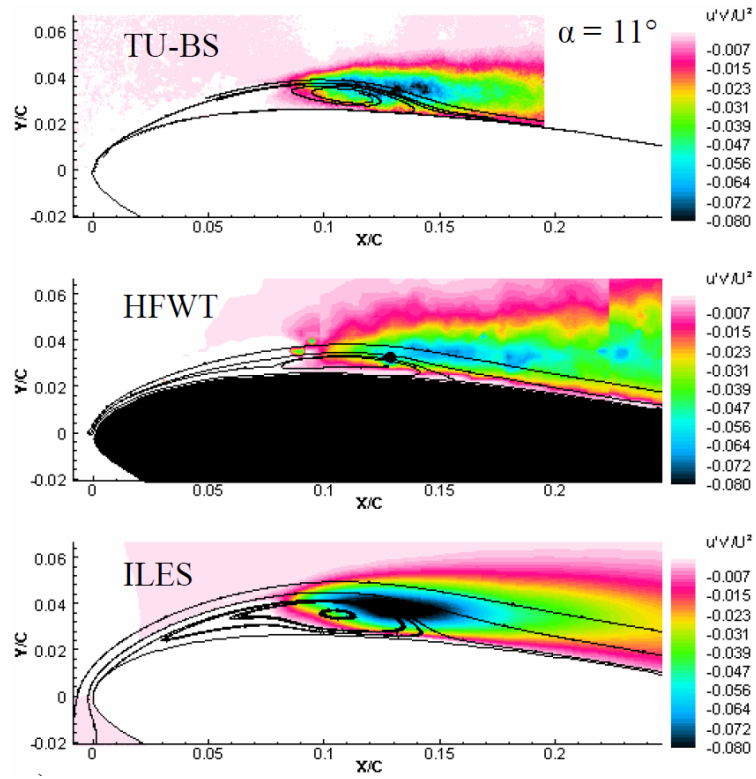
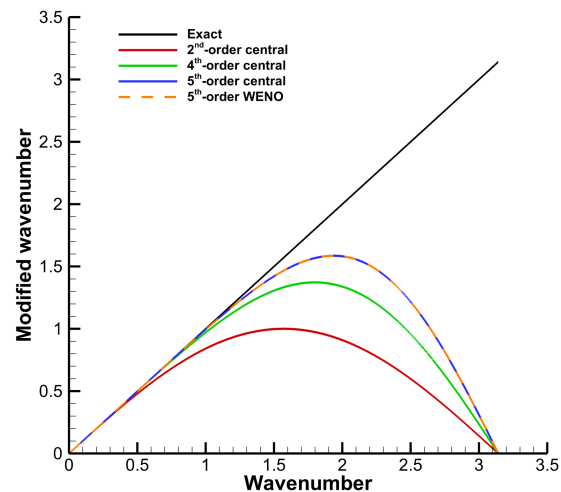
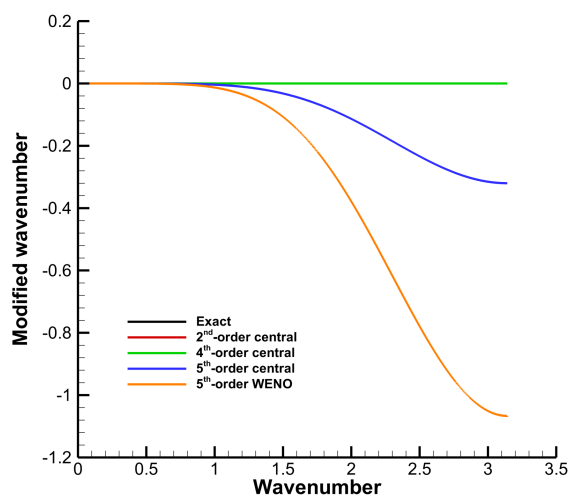


Figure 2.9: Predicted and experimentally determined Reynolds shear stress about the SD7003 at $\alpha = 11^\circ$ [39, 165, 166]

Appendix C: Chapter 3 Figures and Tables



(a) Dispersion



(b) Diffusion error

Figure 3.1: Modified wavenumber analysis of central-difference schemes and 5th-order WENO

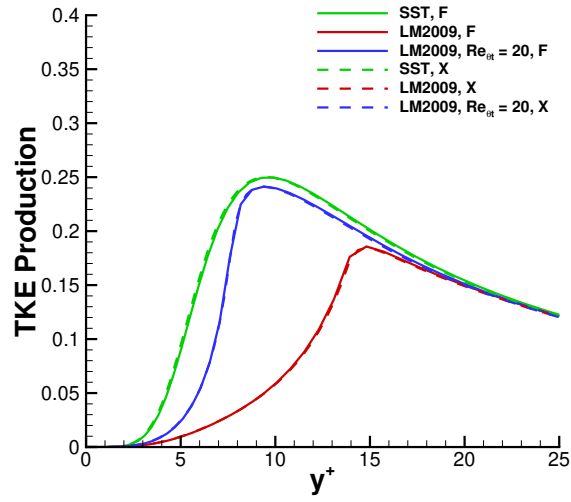
Appendix D: Chapter 4 Figures and Tables

Table 4.1: T3A grid dimensions

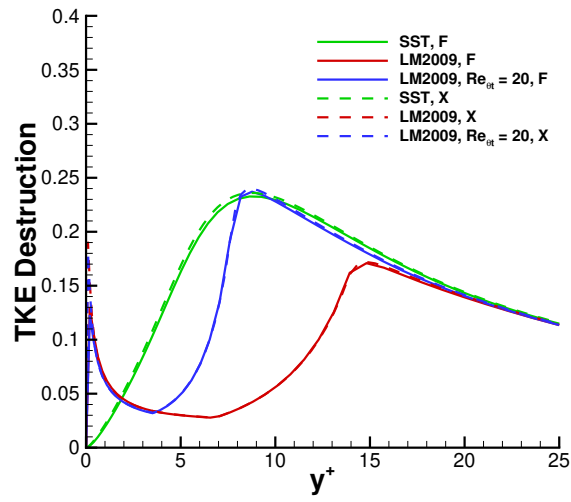
Grid Level	Dimensions	Points Upstream of Plate	Total Points
Fine (F)	353×193	97	68,129
Extra Fine (X)	705×385	193	271,425

Table 4.2: PSU 94-097 grid dimensions

Grid Level	Near-Wall Spacing	Nominal y^+	Dimensions	Trailing-Edge Points
Medium	$3.1 \times 10^{-5}c$	0.50	403×97	19
Fine	$2.1 \times 10^{-5}c$	0.33	537×129	25



(a) Production



(b) Destruction

Figure 4.1: Comparison of turbulent kinetic energy budgets for the Langtry-Menter and SST models on the T3A flat plate

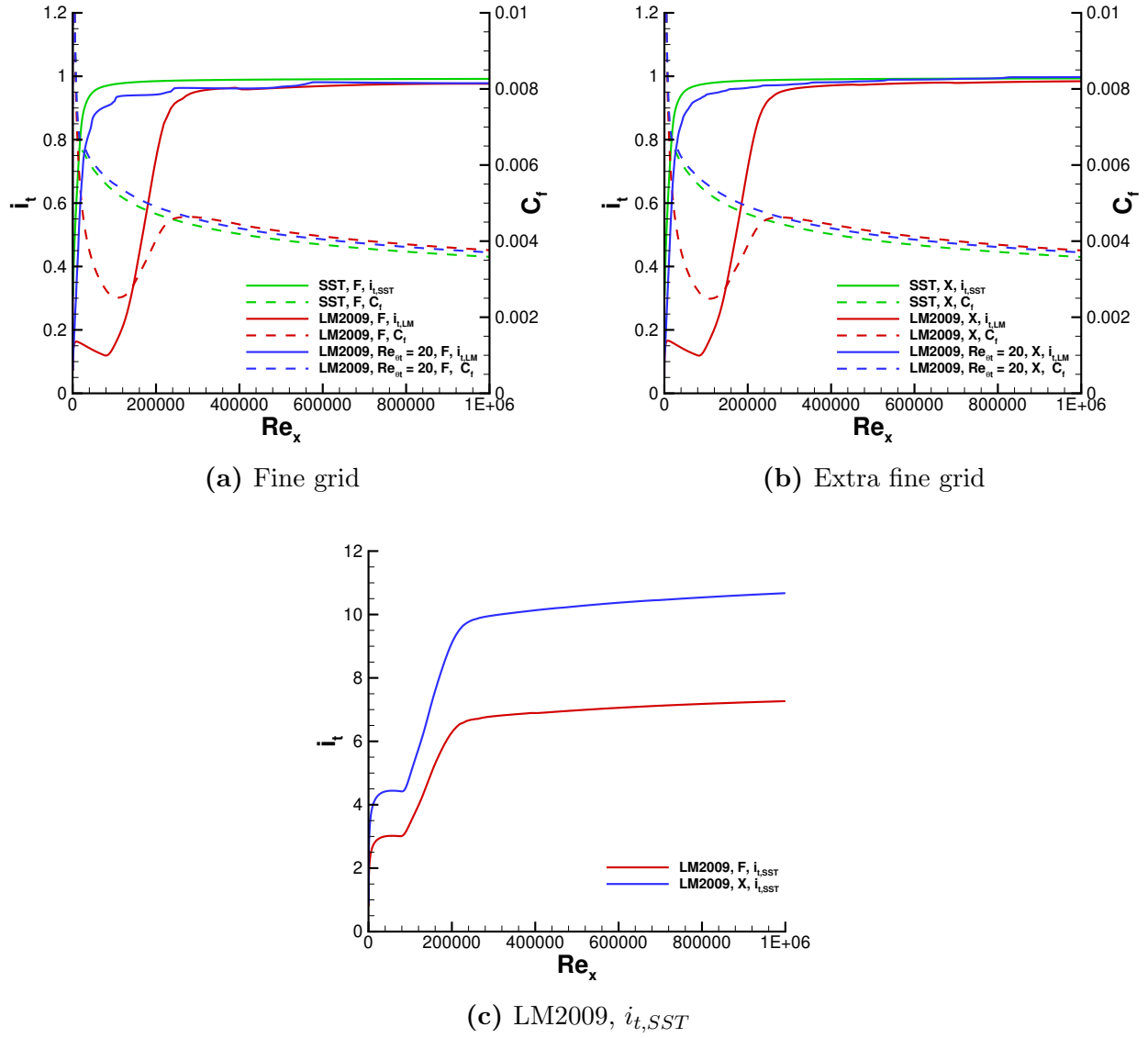
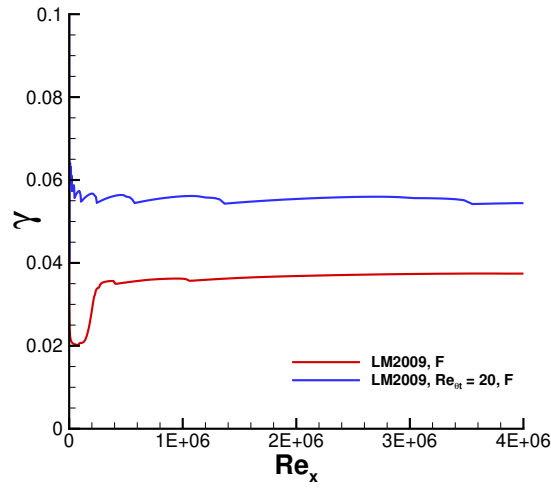
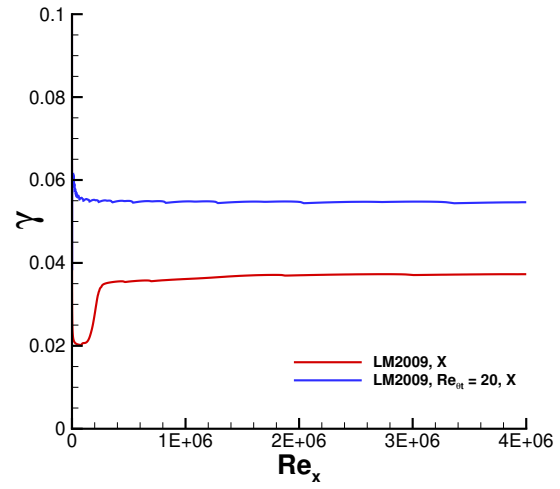


Figure 4.2: Comparison of turbulence index on the wall for the Langtry-Menter and SST models on the T3A flat plate

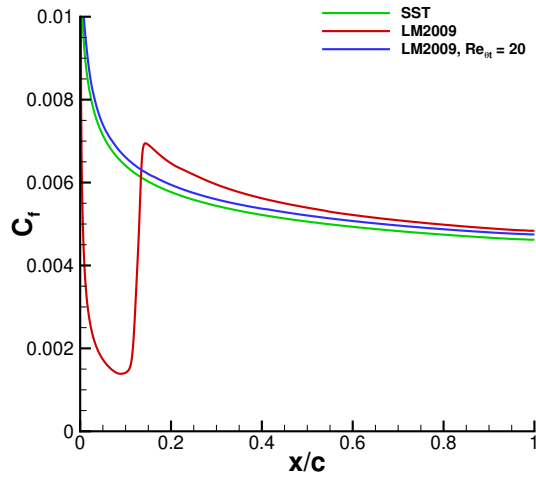


(a) Fine grid

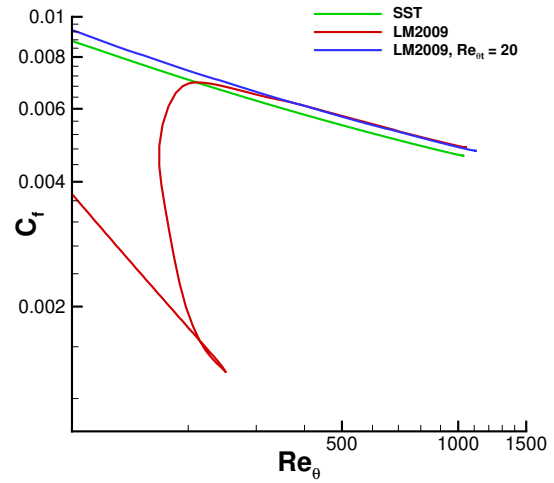


(b) Extra fine grid

Figure 4.3: Comparison of intermittency on the wall for the Langtry-Menter model on the T3A flat plate

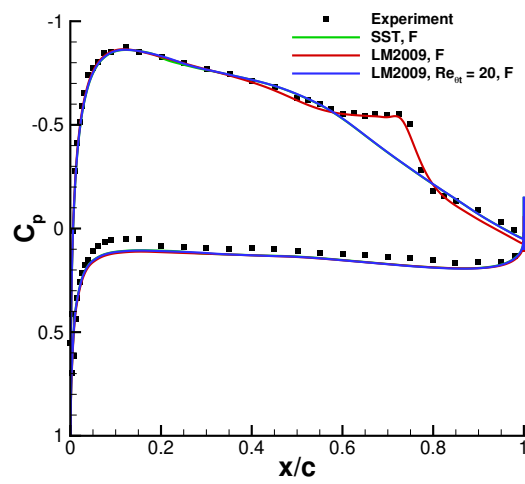


(a) Skin friction distribution

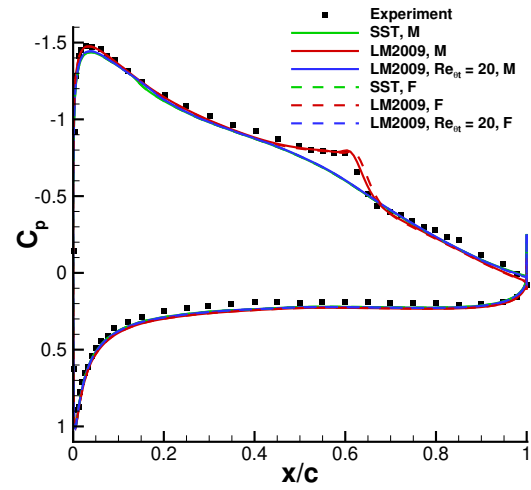


(b) Momentum-thickness Reynolds number distribution

Figure 4.4: Surface quantity distributions for the TMR case

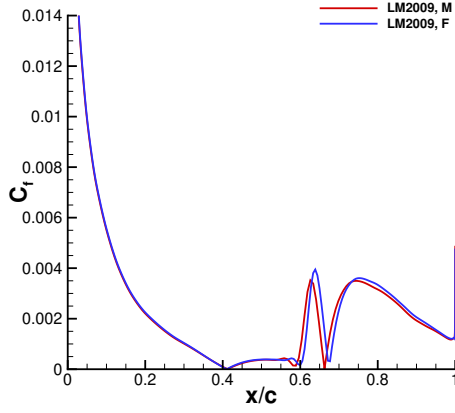


(a) $\alpha = 2^\circ$

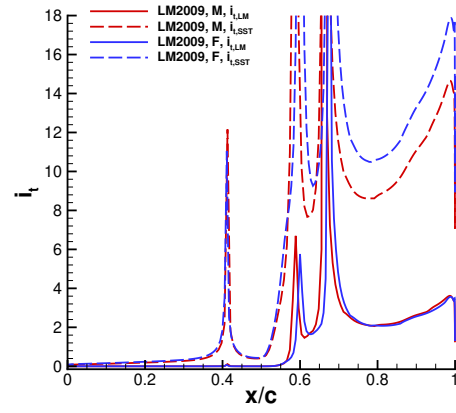


(b) $\alpha = 5^\circ$

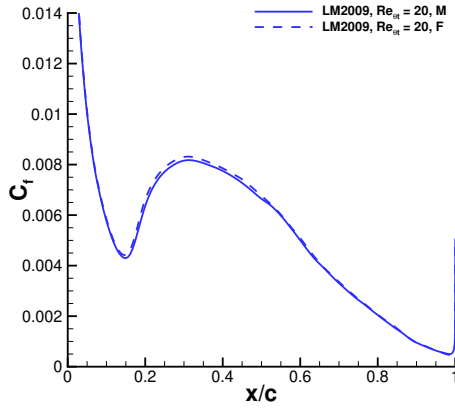
Figure 4.5: Predicted and measured [20] pressure distribution on the PSU 94-097 airfoil at $Re = 2.4 \times 10^5$



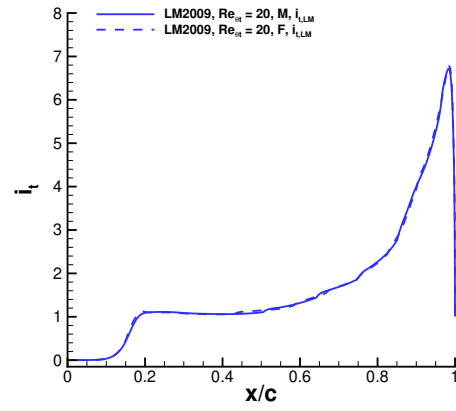
(a) Skin friction distribution for the Langtry-Menter solution



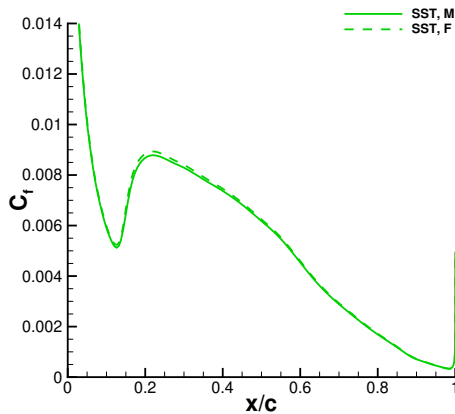
(b) Turbulence indices for the Langtry-Menter solution



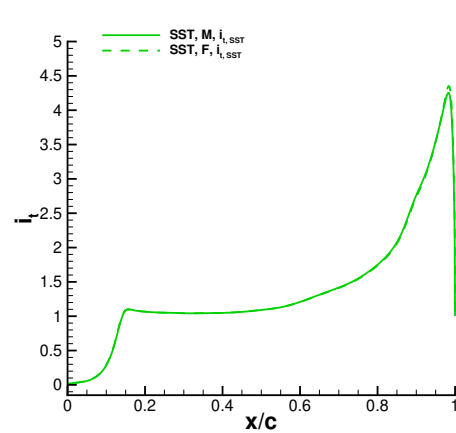
(c) Skin friction distribution for the Langtry-Menter, $\widetilde{Re}_{\theta,t} = 20$ solution



(d) Langtry-Menter turbulence index for the Langtry-Menter, $\widetilde{Re}_{\theta,t} = 20$ solution

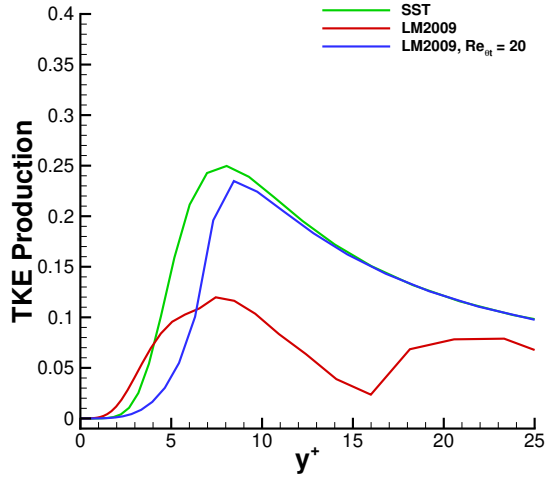


(e) Skin friction distribution for the SST solution

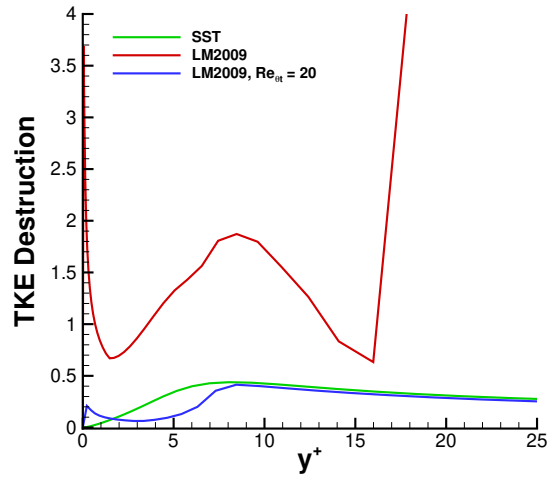


(f) SST turbulence index for the SST solution

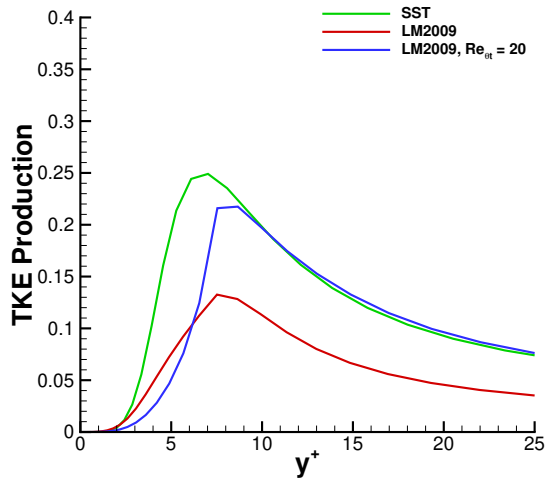
Figure 4.6: Skin friction and turbulence index on the PSU 94-097 airfoil at $\alpha = 5^\circ$ and $Re = 2.4 \times 10^5$



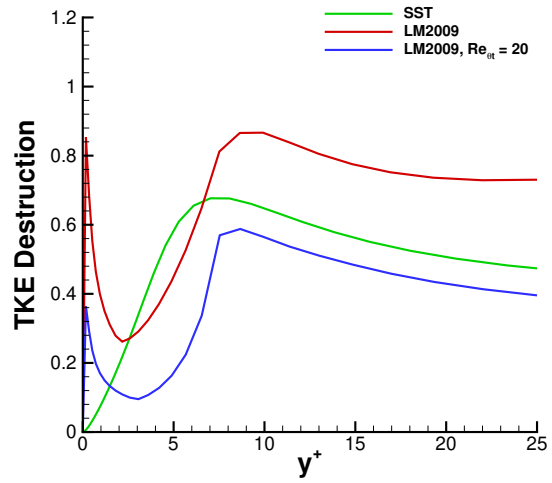
(a) Production at $x/c = 0.70$



(b) Destruction at $x/c = 0.70$



(c) Production at $x/c = 0.85$



(d) Destruction at $x/c = 0.85$

Figure 4.7: Comparison of turbulent kinetic energy budgets for the Langtry-Menter and SST models on the PSU 94-097 airfoil at $\alpha = 2^\circ$, Fine grid (513×129)

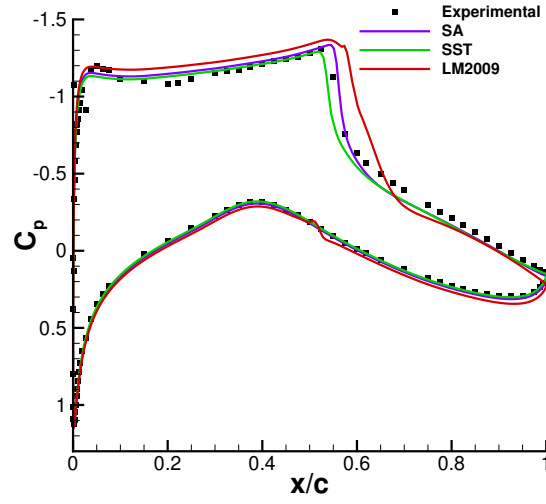


Figure 4.8: Comparison of predicted pressure distribution on the RAE 2822 airfoil at $\alpha = 2.79^\circ$, $M = 0.734$, $Re = 6.5 \times 10^6$

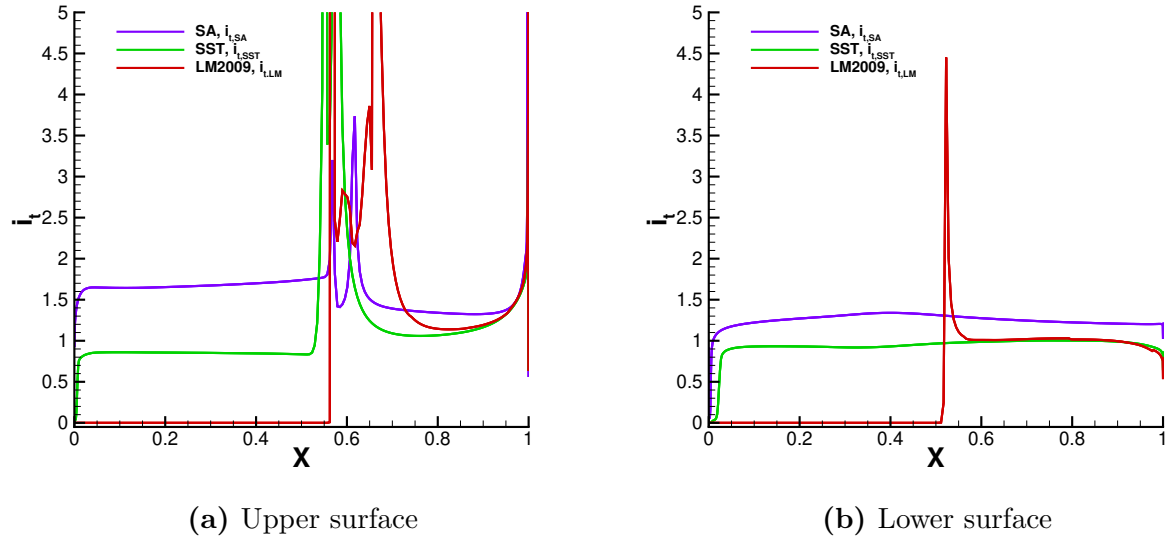
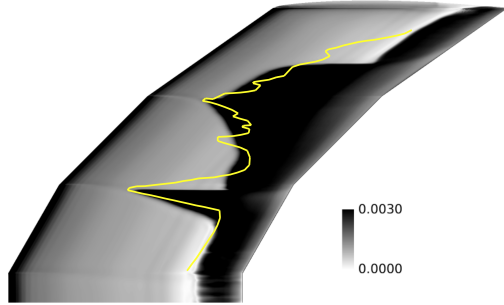
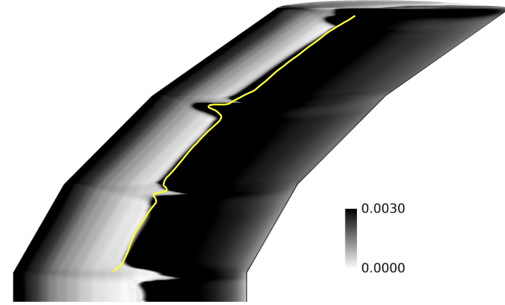


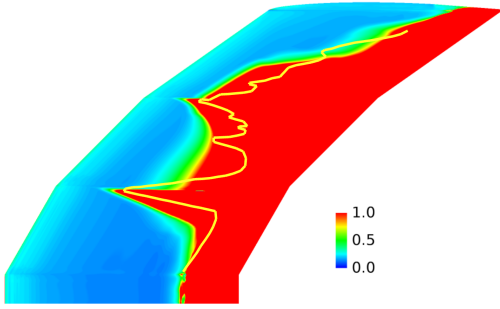
Figure 4.9: Turbulence index on the RAE 2822 airfoil at $\alpha = 2.79^\circ$, $M = 0.734$, $Re = 6.5 \times 10^6$



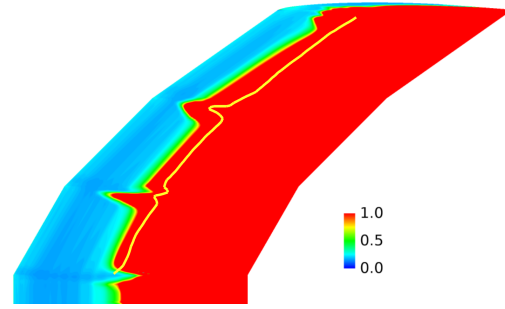
(a) Upper surface skin friction



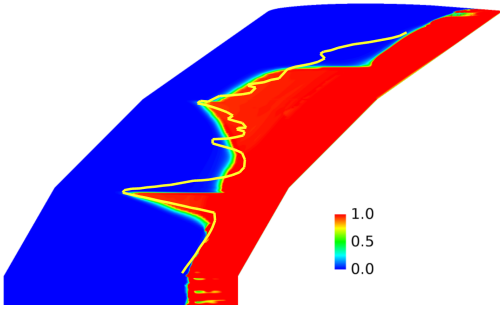
(b) Lower surface skin friction



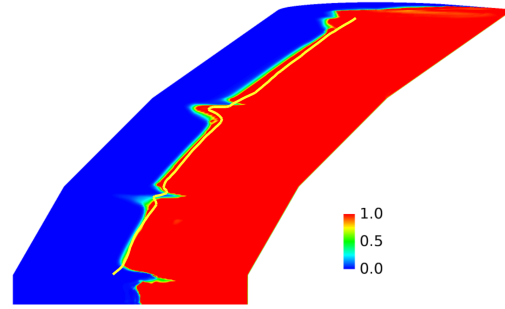
(c) Upper surface SST turbulence index
(blue - laminar, red - turbulent)



(d) Lower surface SST turbulence index
(blue - laminar, red - turbulent)

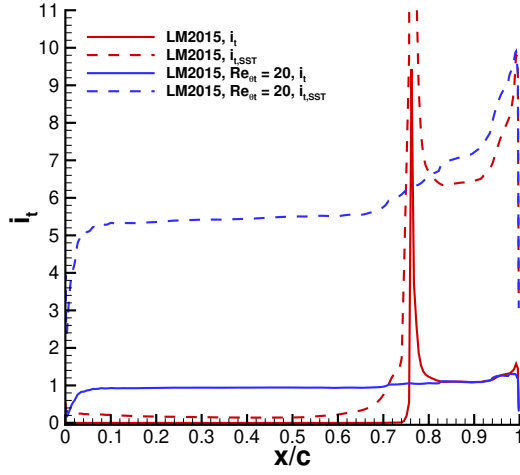


(e) Upper surface Langtry-Menter turbulence index (blue - laminar, red - turbulent)

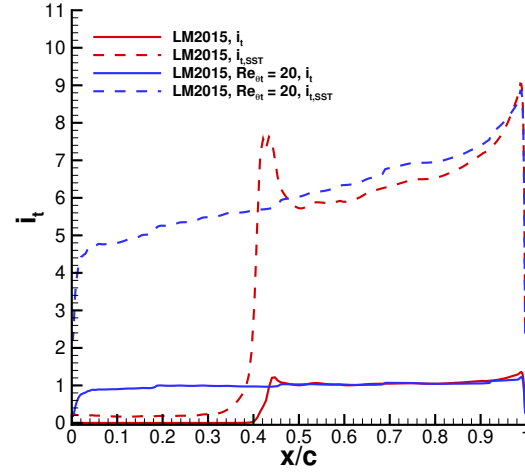


(f) Lower surface Langtry-Menter turbulence index (blue - laminar, red - turbulent)

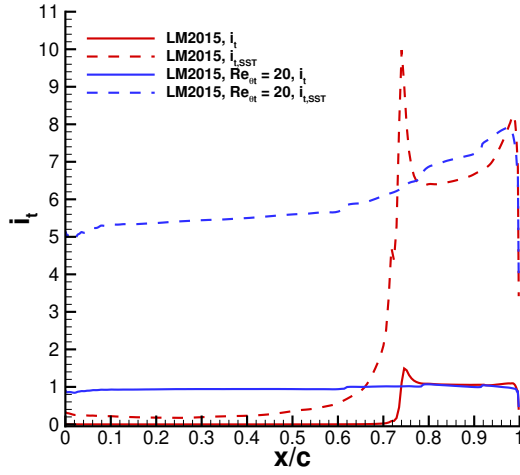
Figure 4.10: LM2015 predicted transition on the sickle wing (Case A), yellow - measured transition locations [71]



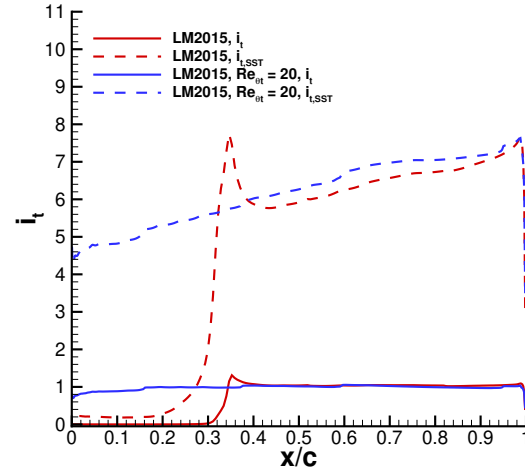
(a) Upper surface at $\eta = 0.233$



(b) Lower surface at $\eta = 0.233$



(c) Upper surface at $\eta = 0.80$



(d) Lower surface at $\eta = 0.80$

Figure 4.11: Turbulence index on the sickle wing at select spanwise stations for Case A

Appendix E: Chapter 5 Figures and Tables

Table 5.1: Test conditions for Case II (Ref. [212])

Parameter	Value
θ_0, deg	10.1
$\hat{\theta}, deg$	2.9
Re_{tip}	2.32×10^5
M_{tip}	0.143
$w_\infty, ft/s$	7.22
$Tu_\infty(\%)$	0.09



Figure 5.1: DLR RTG blade planform

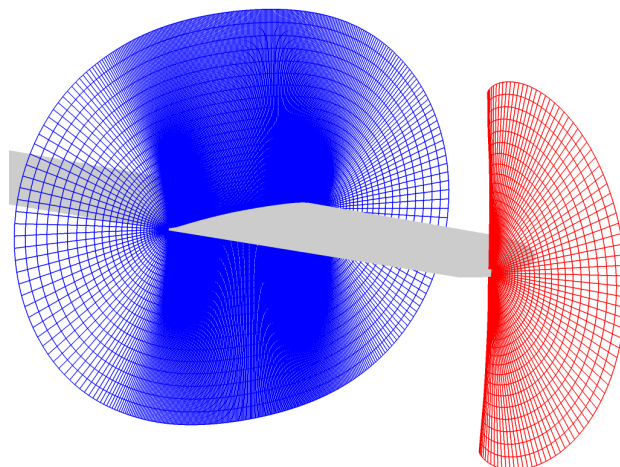


Figure 5.2: DLR RTG blade structured, overset, baseline grid

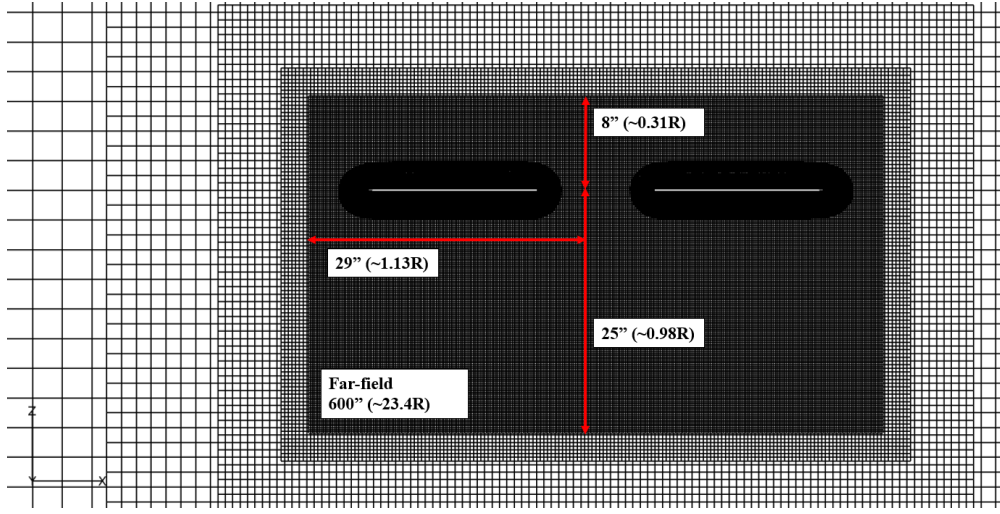


Figure 5.3: DLR RTG off-body, Cartesian, overset grid system

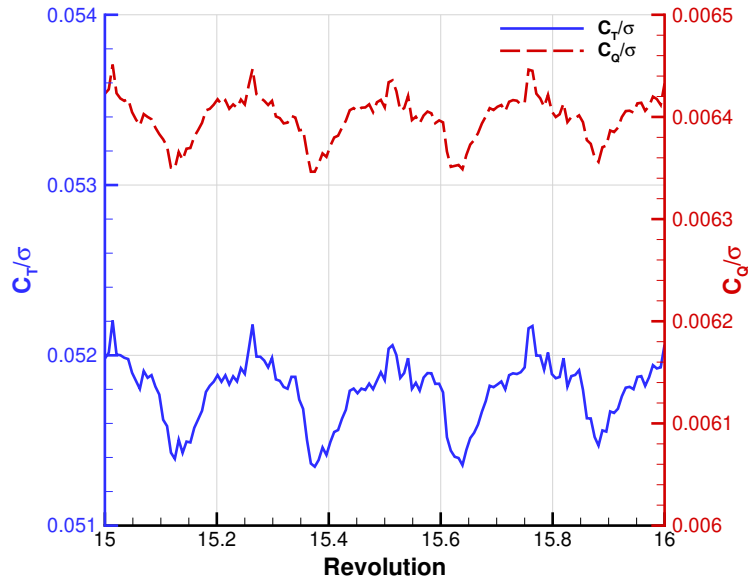


Figure 5.4: Thrust and torque history over the final rotor revolution, baseline grid

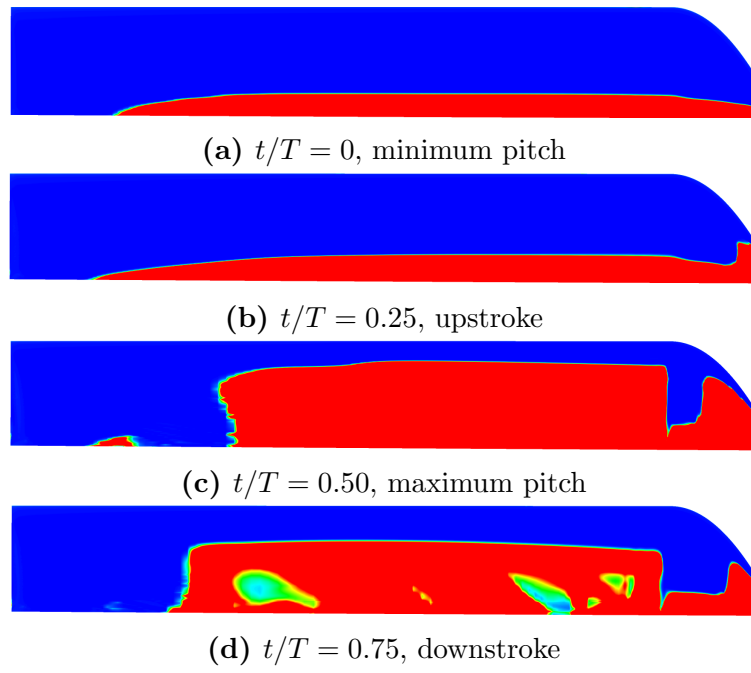


Figure 5.5: Predicted transition visualized via Spalart's turbulence index, fine grid (blue - laminar, red - turbulent)

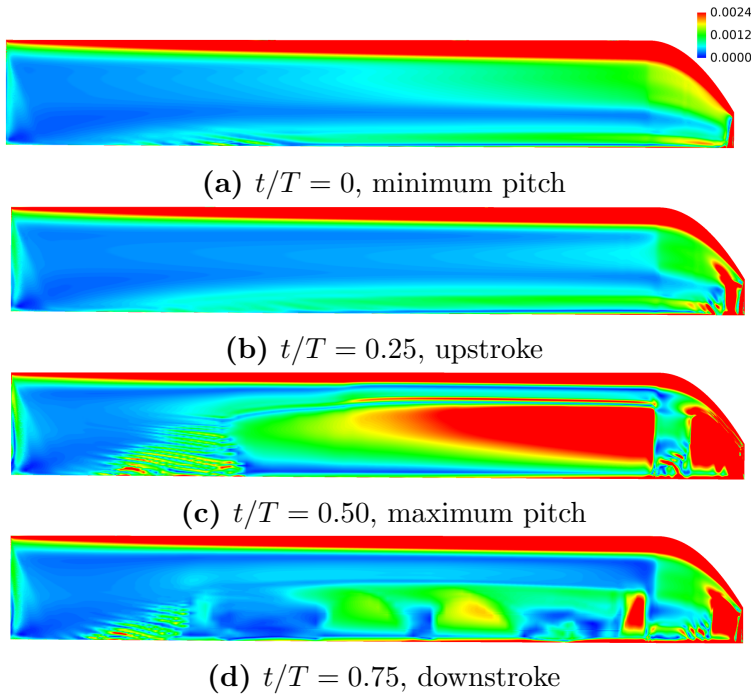


Figure 5.6: Predicted skin friction contours, fine grid

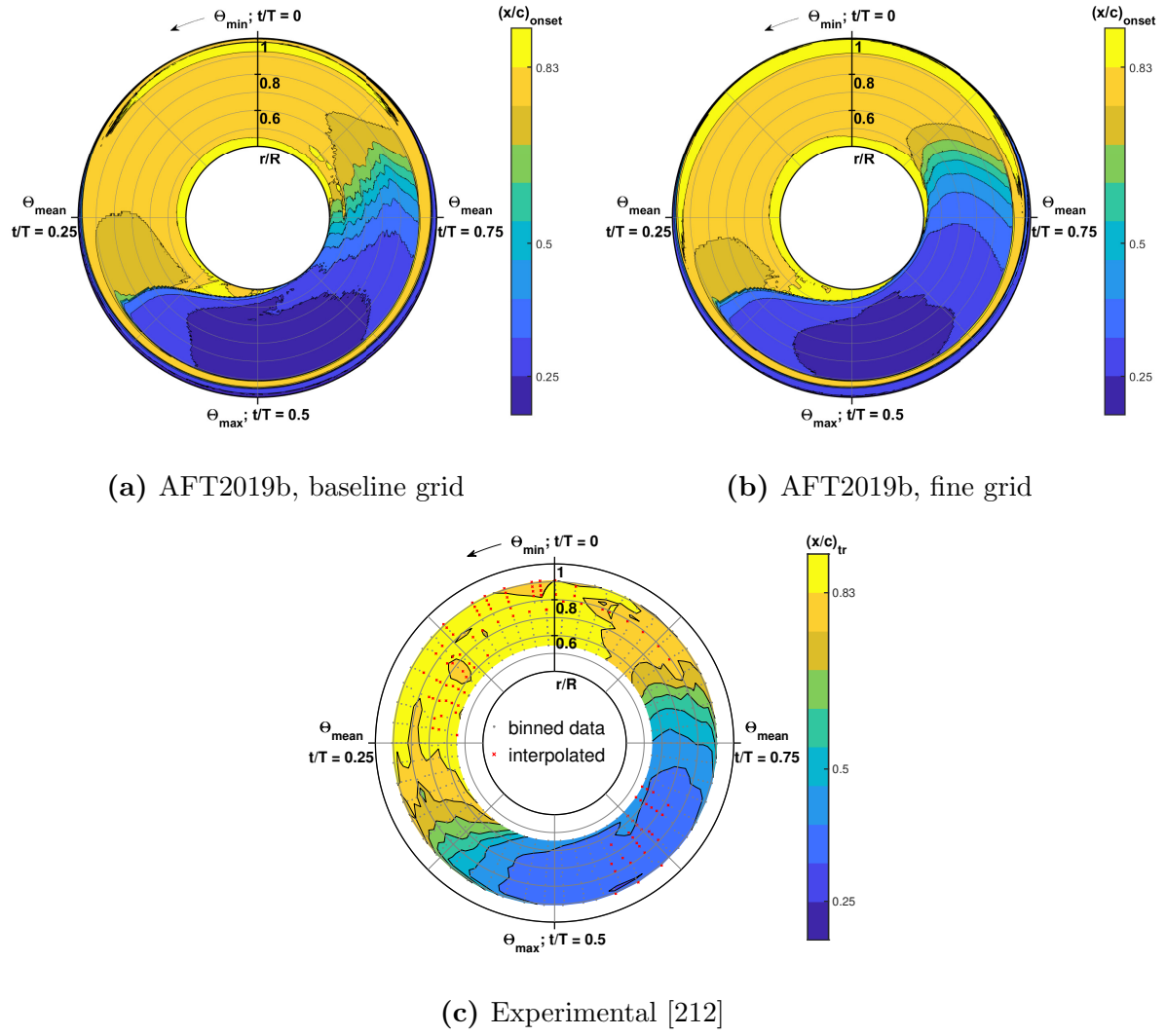


Figure 5.7: Transition location as a function of pitch cycle and blade radius

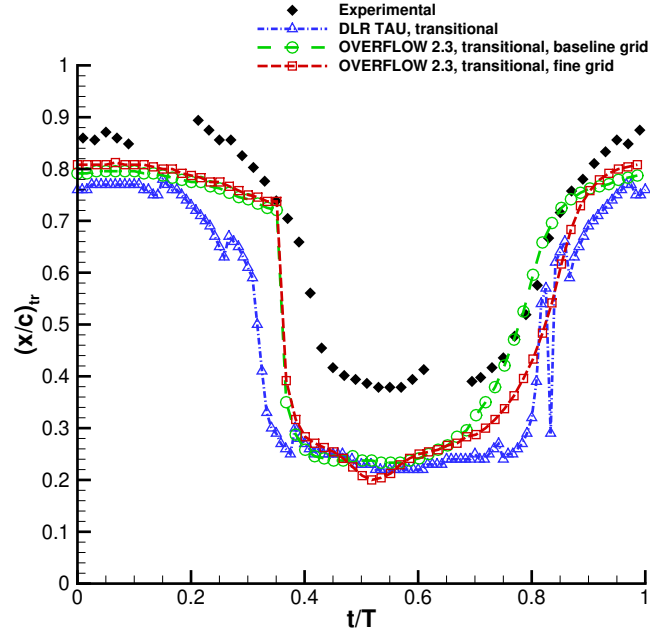


Figure 5.8: Comparison of predicted transition location as a function of pitch cycle at $r/R = 0.615$

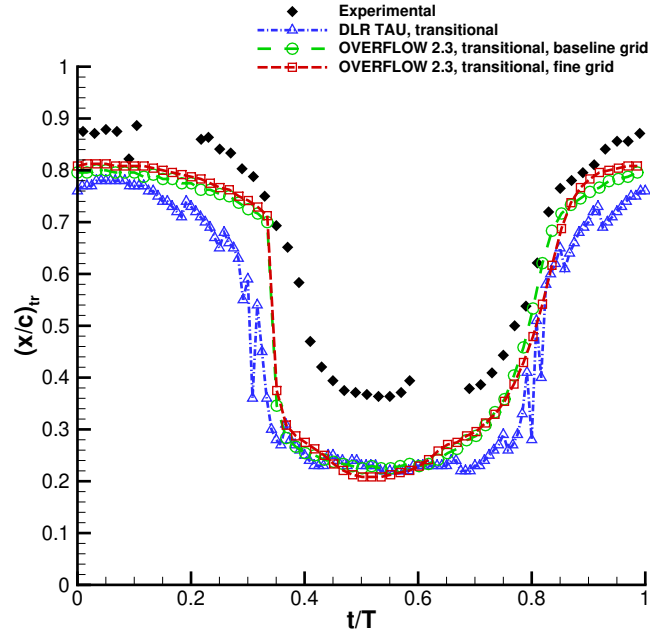


Figure 5.9: Comparison of predicted transition location as a function of pitch cycle at $r/R = 0.723$

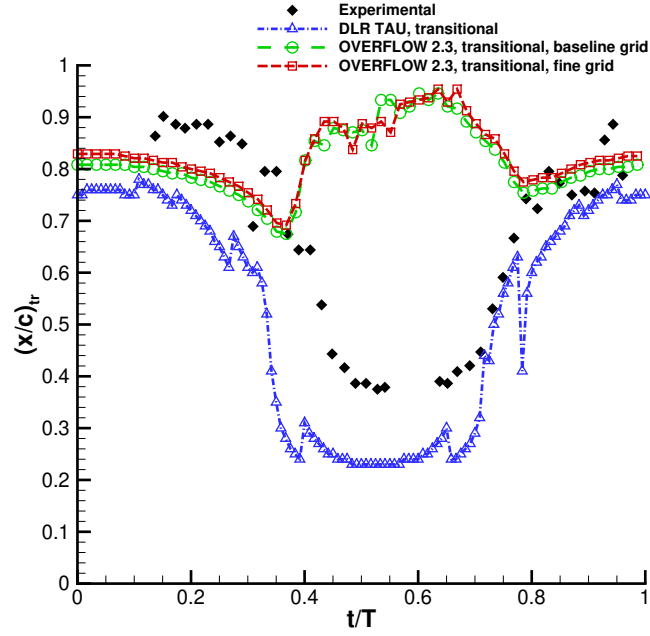


Figure 5.10: Comparison of predicted transition location as a function of pitch cycle at $r/R = 0.908$

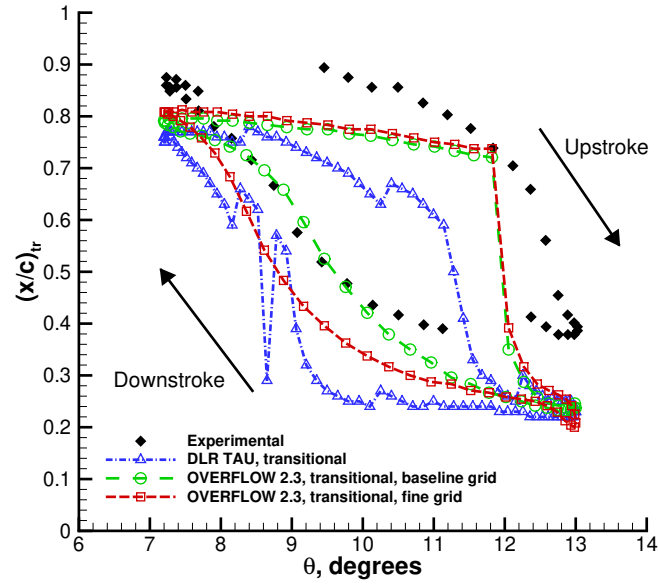


Figure 5.11: Comparison of predicted transition location as a function of pitch angle at $r/R = 0.615$

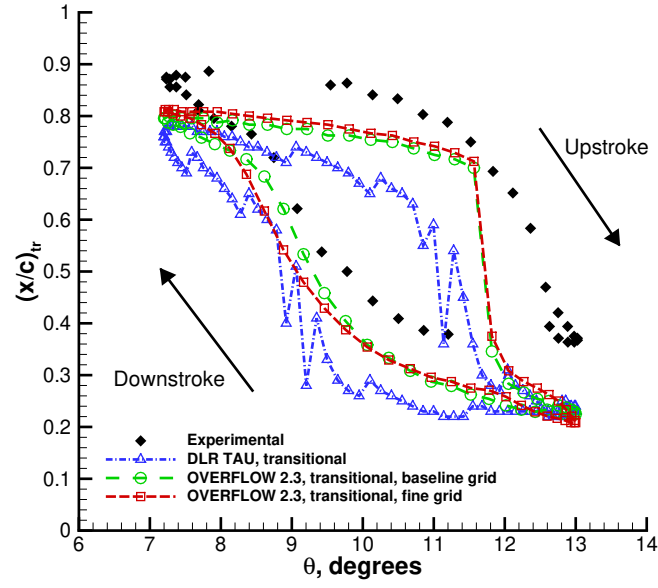


Figure 5.12: Comparison of predicted transition location as a function of pitch angle at $r/R = 0.723$

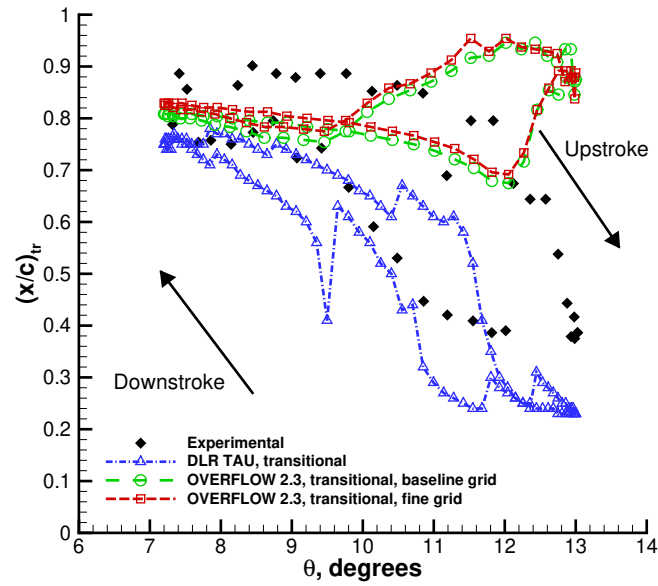


Figure 5.13: Comparison of predicted transition location as a function of pitch angle at $r/R = 0.908$

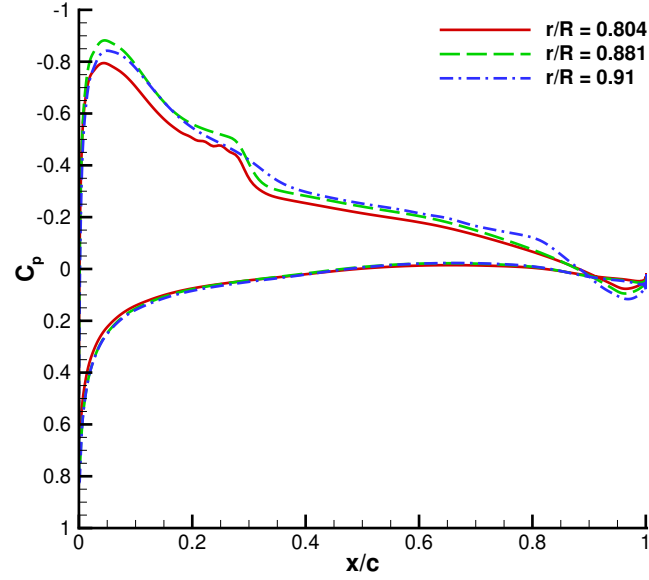


Figure 5.14: Predicted pressure distribution at select radial stations at $t/T = 0.5$, baseline grid

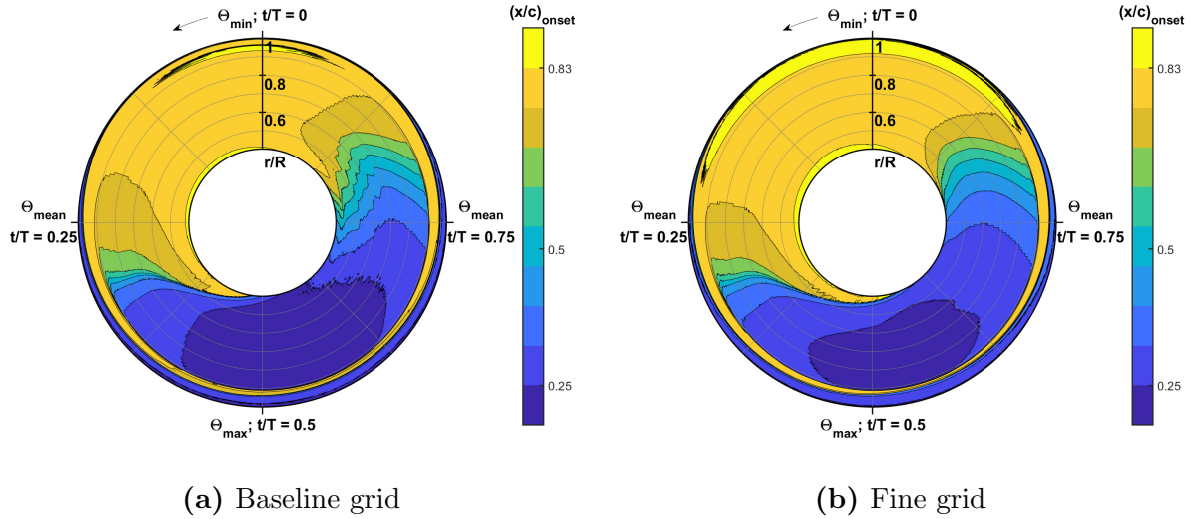
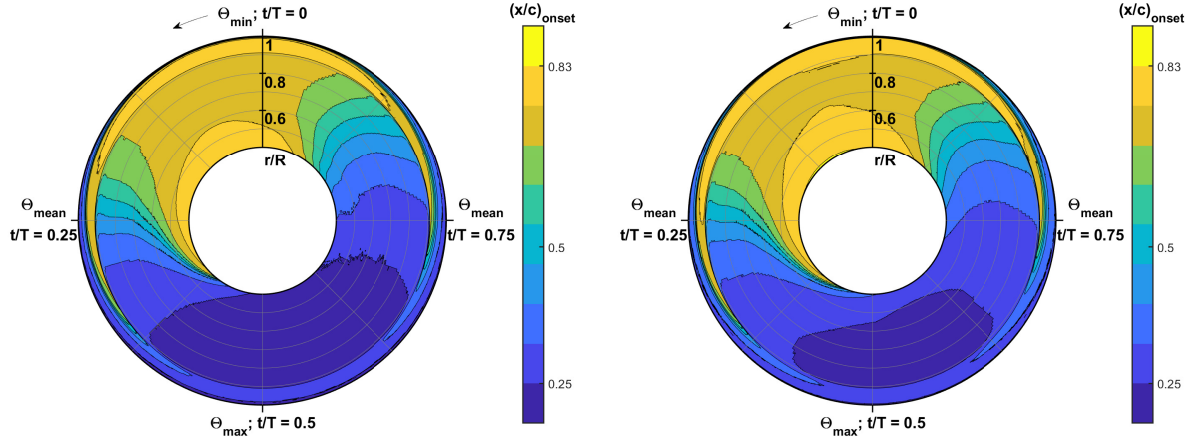


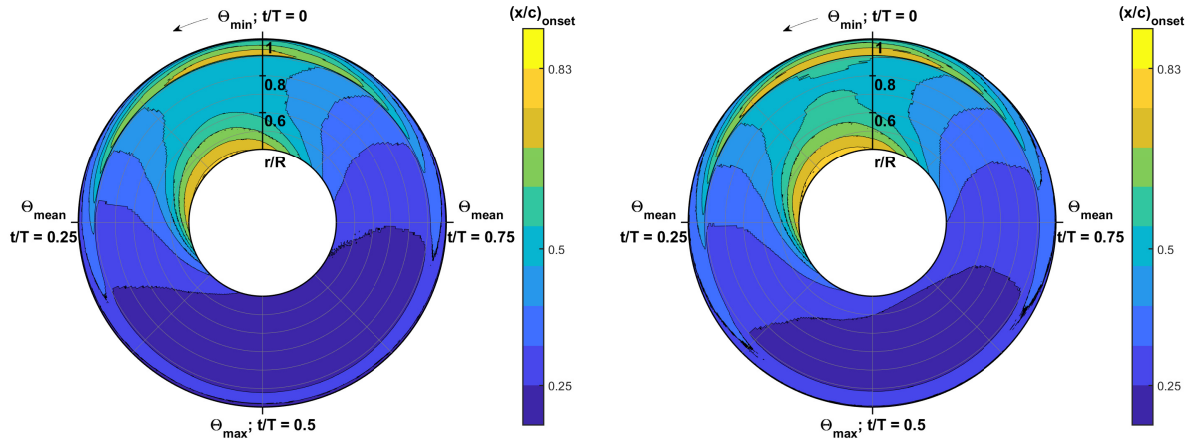
Figure 5.15: Predicted transition location using AFT2019b for $Tu_\infty = 0.2\%$



(a) Baseline grid

(b) Fine grid

Figure 5.16: Predicted transition location using AFT2019b for $Tu_\infty = 0.5\%$



(a) Baseline grid

(b) Fine grid

Figure 5.17: Predicted transition location using AFT2019b for $Tu_\infty = 0.9\%$

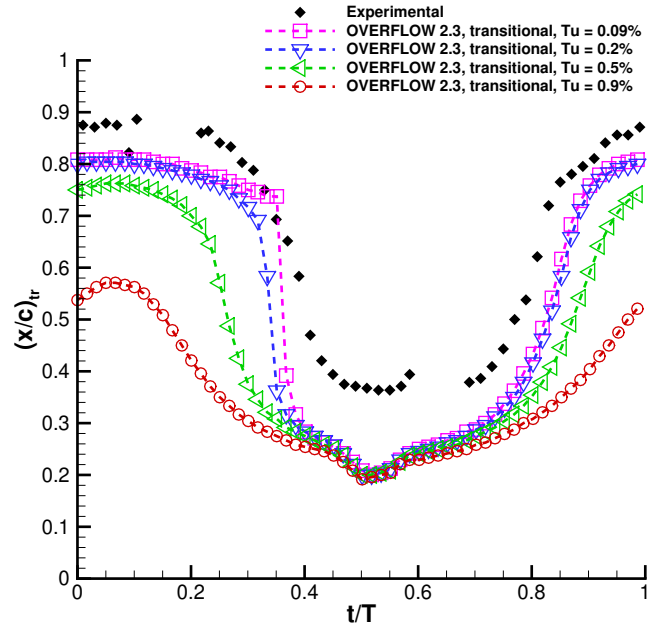


Figure 5.18: Comparison of predicted transition location as a function of pitch cycle and FSTI at $r/R = 0.615$, fine grid

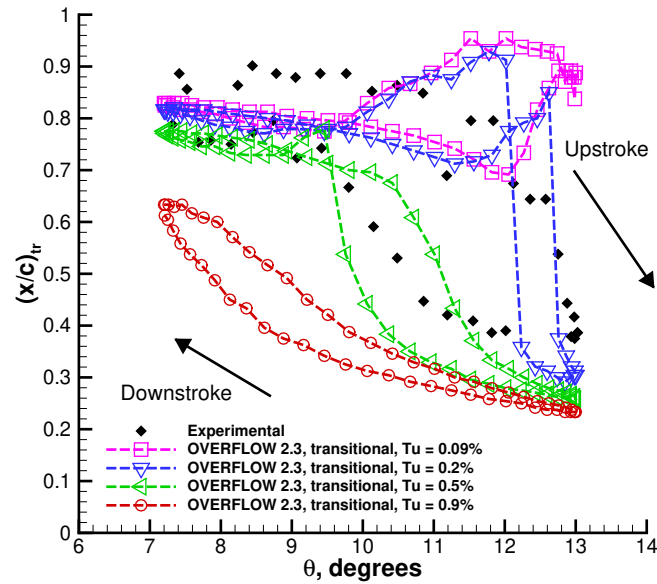


Figure 5.19: Comparison of predicted transition location as a function of pitch angle and FSTI at $r/R = 0.908$, fine grid

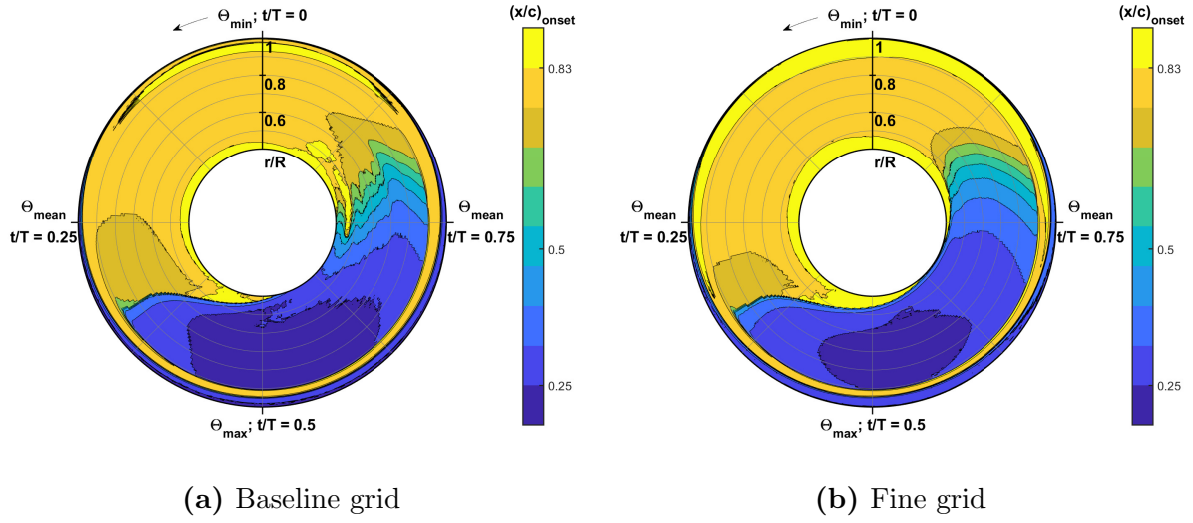


Figure 5.20: Predicted transition location using AFT2019b after stepping from $Tu_\infty = 0.9\%$ to $Tu_\infty = 0.09\%$

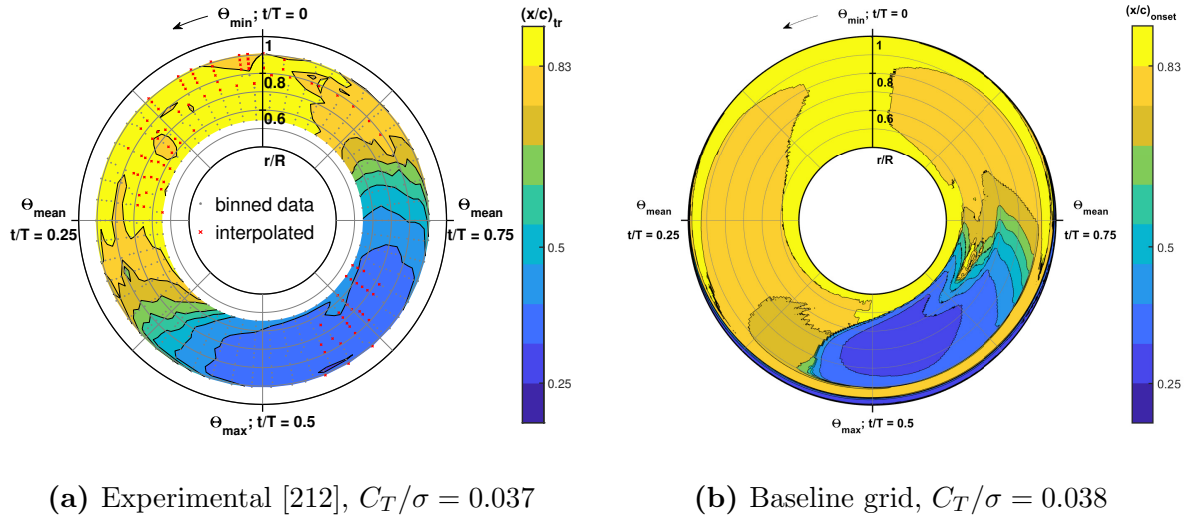


Figure 5.21: Transition locations as a function of pitch cycle and blade radius for the same thrust level

Appendix F: Chapter 6 Figures and Tables

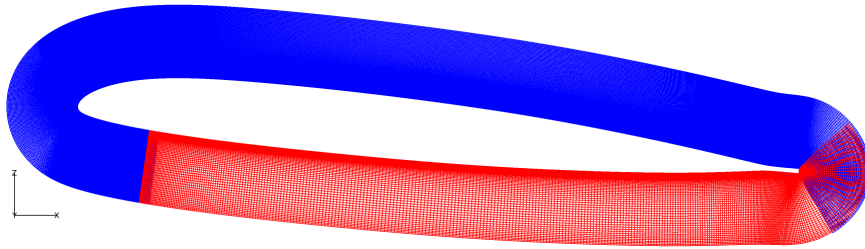
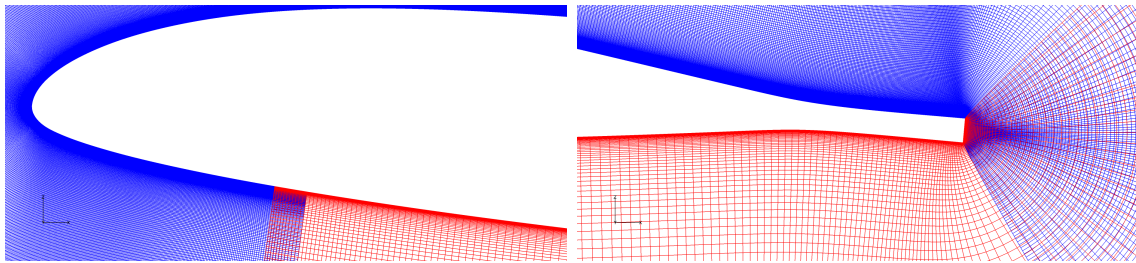


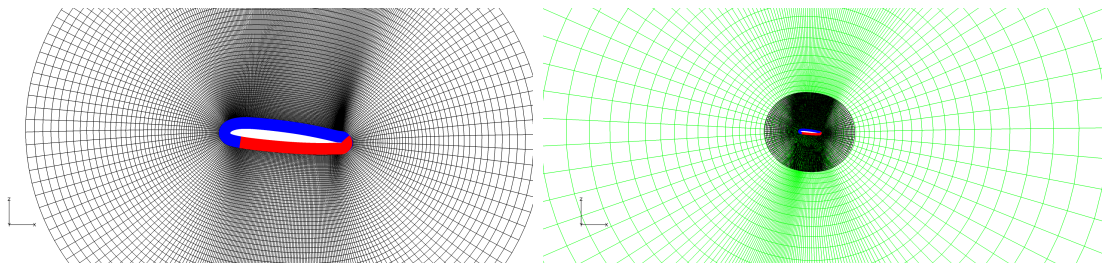
Figure 6.1: Near-body grids for the DSA-9A airfoil (blue - LES region)



(a) About the lower surface

(b) About the trailing edge

Figure 6.2: Overlap regions for the DSA-9A near-body grids



(a) Nearest off-body grid (black)

(b) Full grid system

Figure 6.3: DSA-9A off-body grids

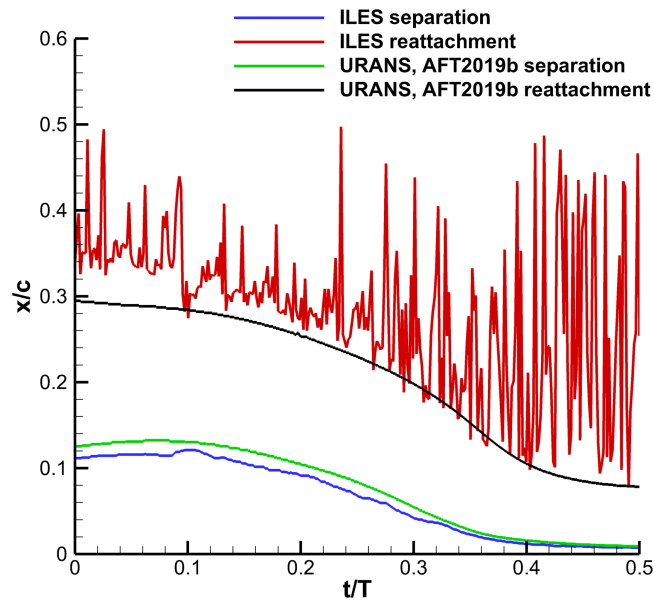


Figure 6.4: ILES-predicted separation and reattachment for the DSA-9A airfoil

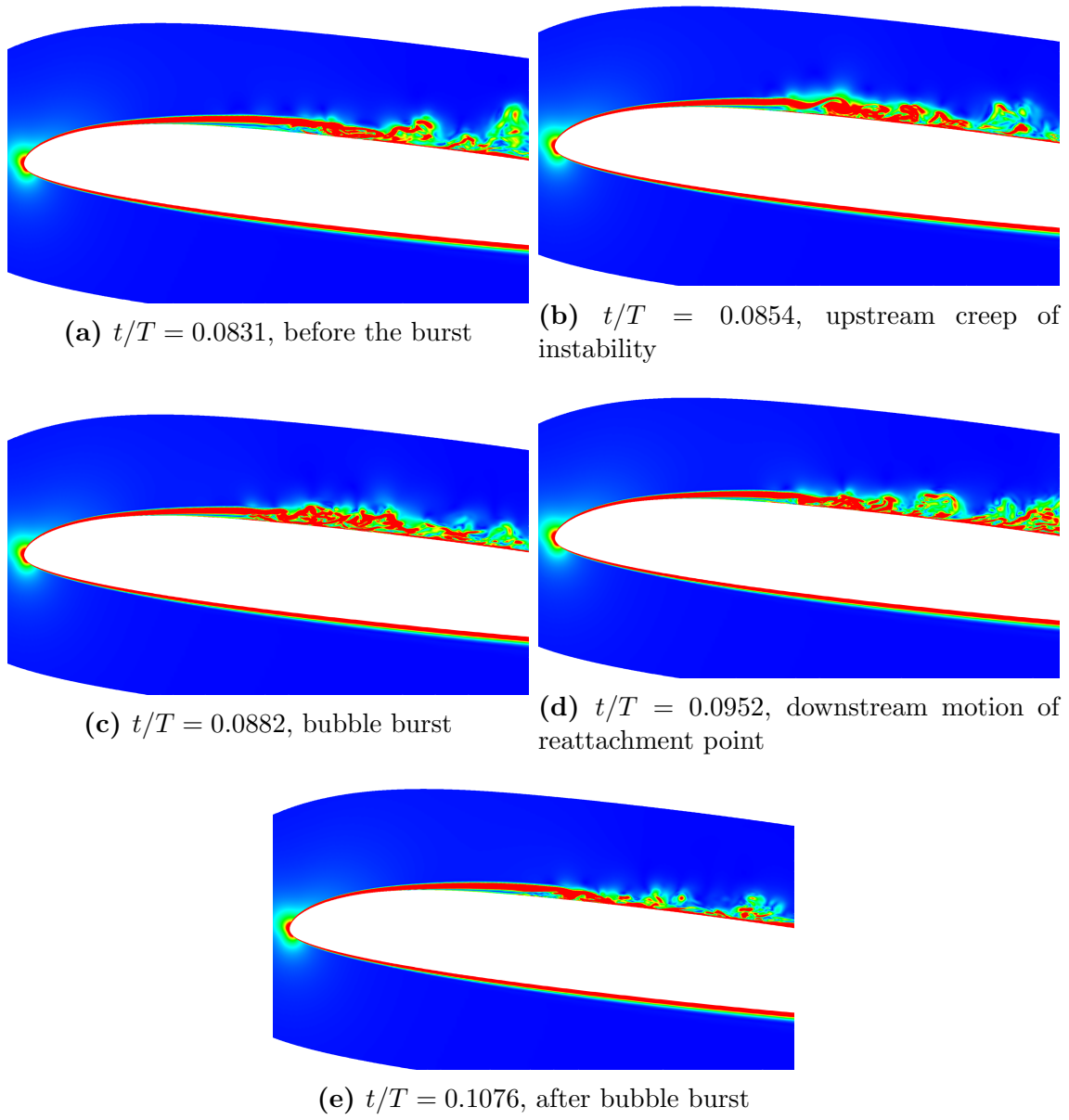


Figure 6.5: Instability leading to bubble burst on the DSA-9A airfoil

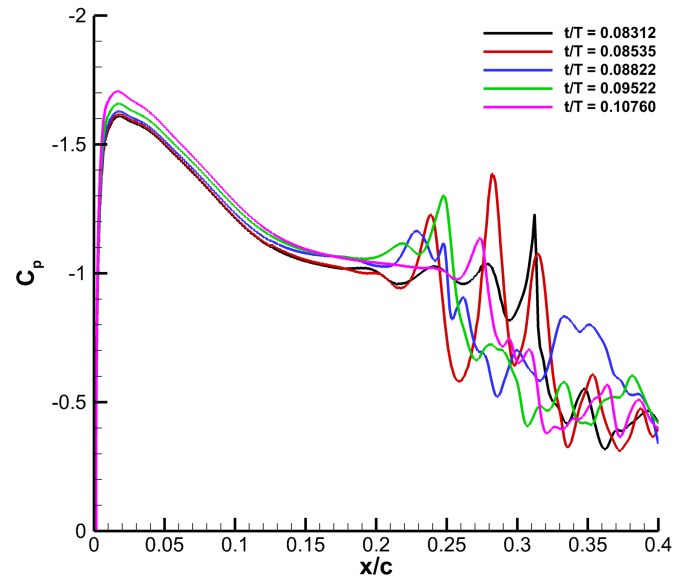
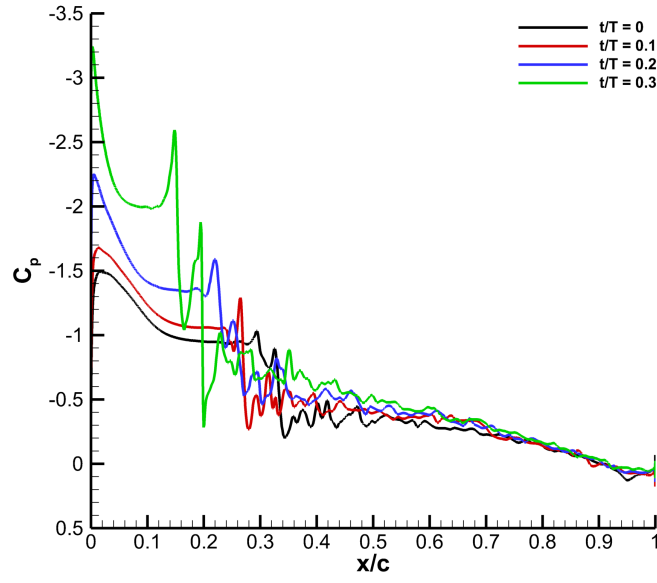
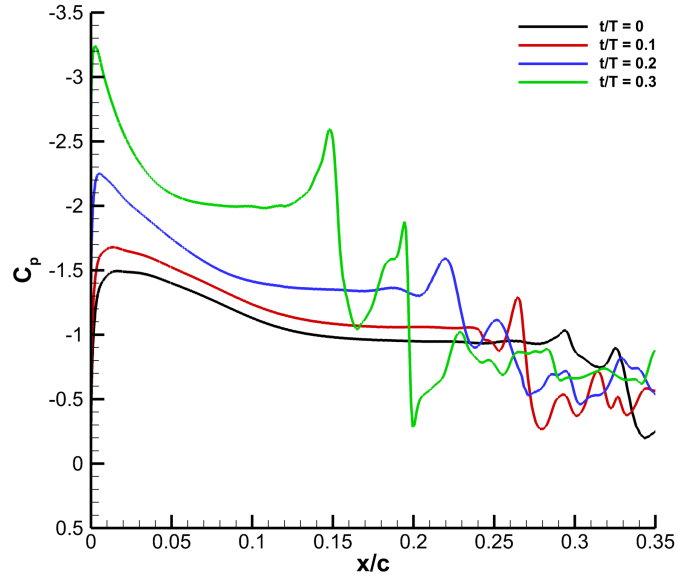


Figure 6.6: Surface pressure distributions predicted by ILES during a bursting event

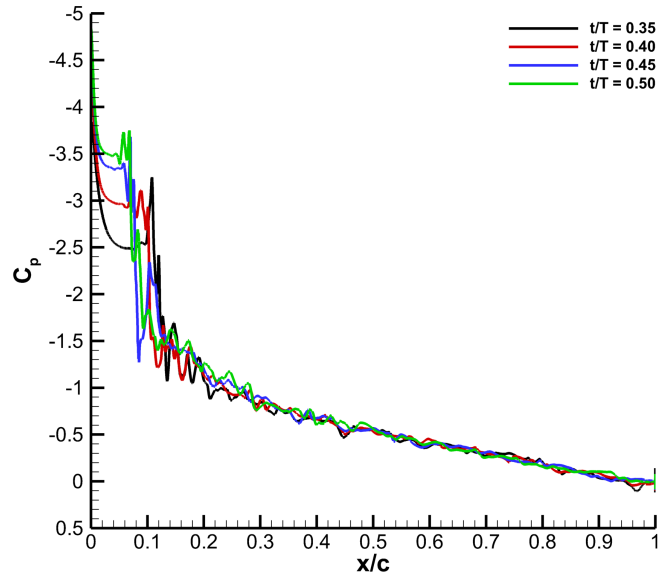


(a) Surface pressure distributions in the first-quarter of the pitch cycle

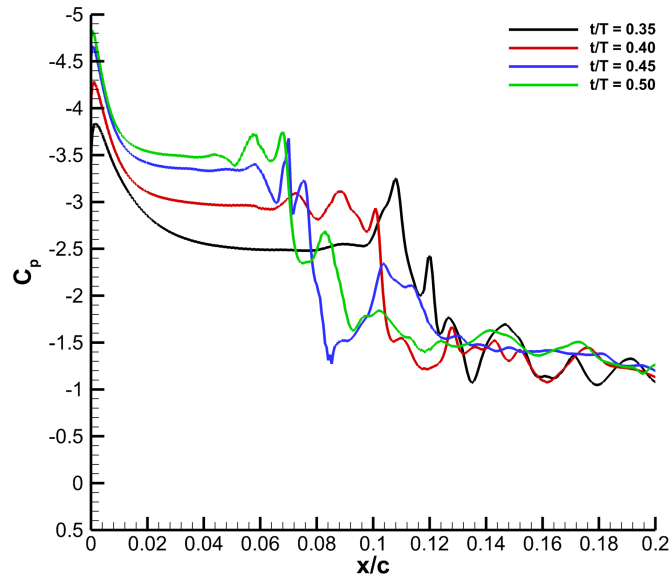


(b) Surface pressure distributions, zoomed up to $0.35c$

Figure 6.7: Surface pressure distributions predicted by ILES for $0 \leq t/T \leq 0.3$

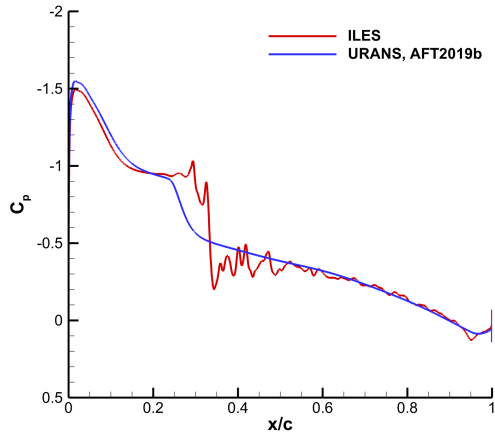


(a) Surface pressure distributions in the second-quarter of the pitch cycle

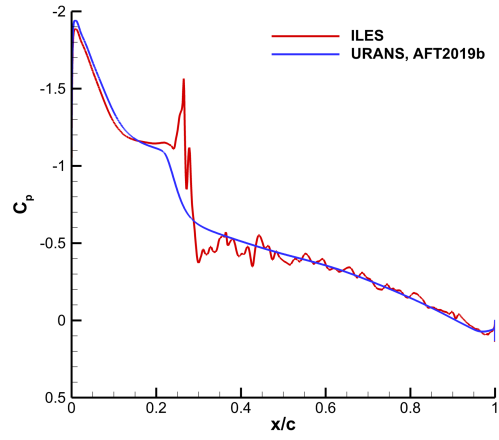


(b) Surface pressure distributions, zoomed up to $0.20c$

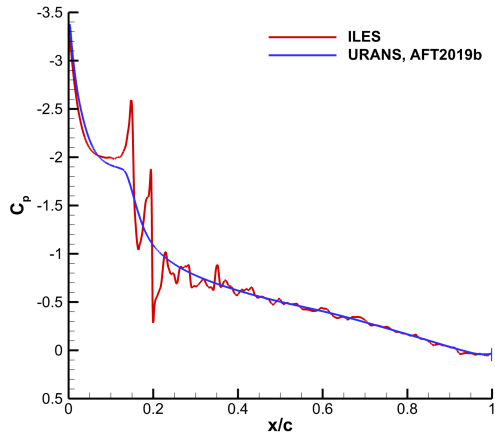
Figure 6.8: Surface pressure distributions predicted by ILES for $0.35 \leq t/T \leq 0.5$



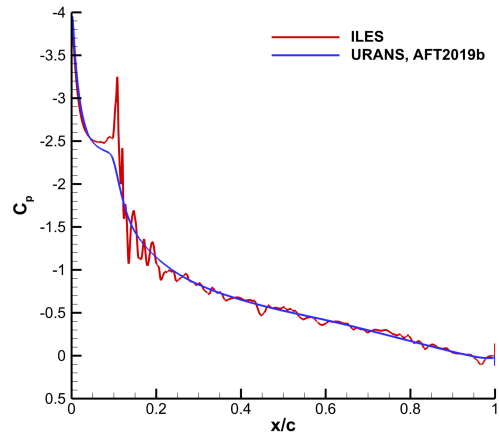
(a) $t/T = 0.0$



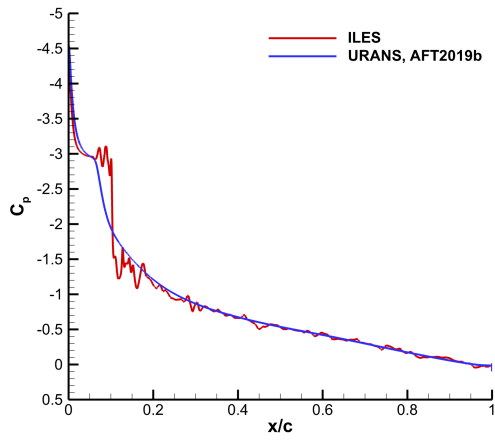
(b) $t/T = 0.15$



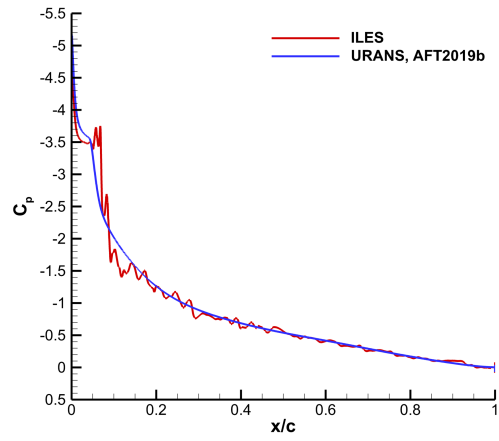
(c) $t/T = 0.3$



(d) $t/T = 0.35$

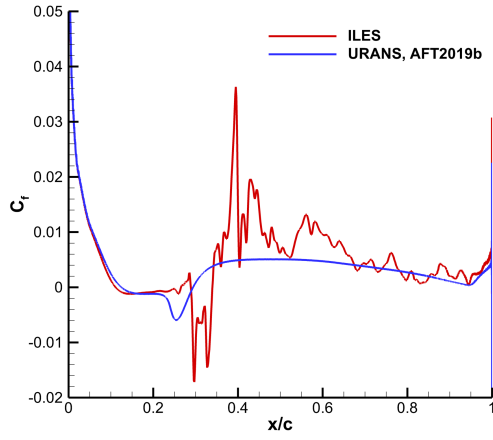


(e) $t/T = 0.40$

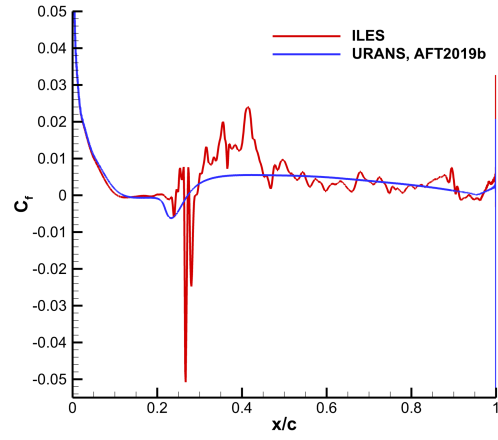


(f) $t/T = 0.5$

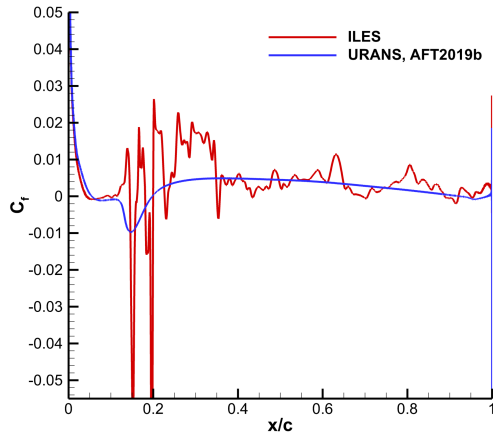
Figure 6.9: Comparison of predicted surface-pressure distributions on the DSA-9A airfoil



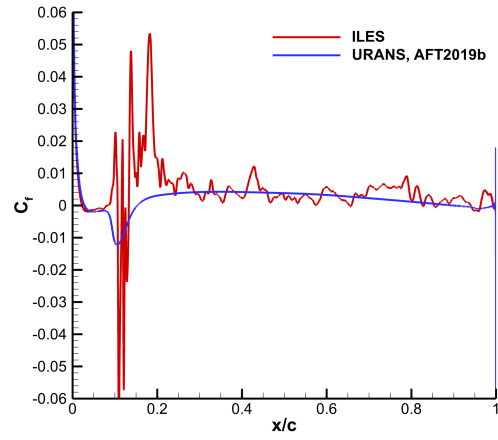
(a) $t/T = 0.0$



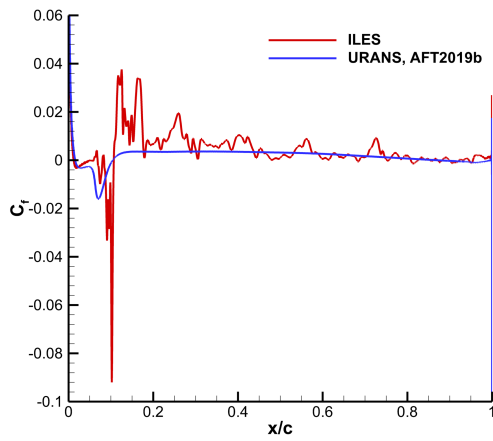
(b) $t/T = 0.15$



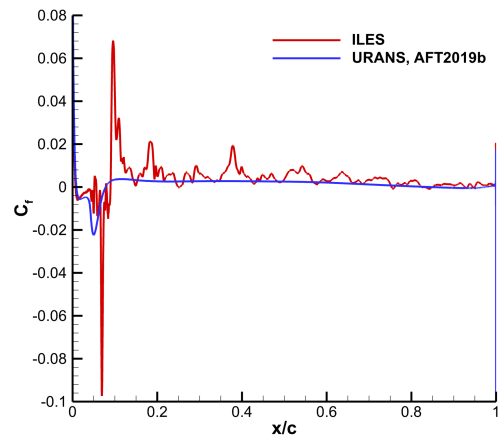
(c) $t/T = 0.3$



(d) $t/T = 0.35$



(e) $t/T = 0.40$



(f) $t/T = 0.5$

Figure 6.10: Comparison of predicted skin-friction distributions on the DSA-9A airfoil

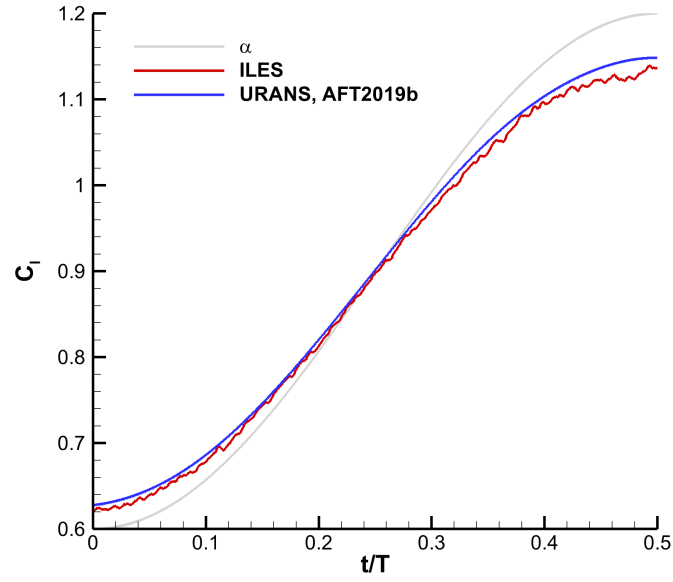


Figure 6.11: Comparison of predicted lift coefficient on the DSA-9A airfoil

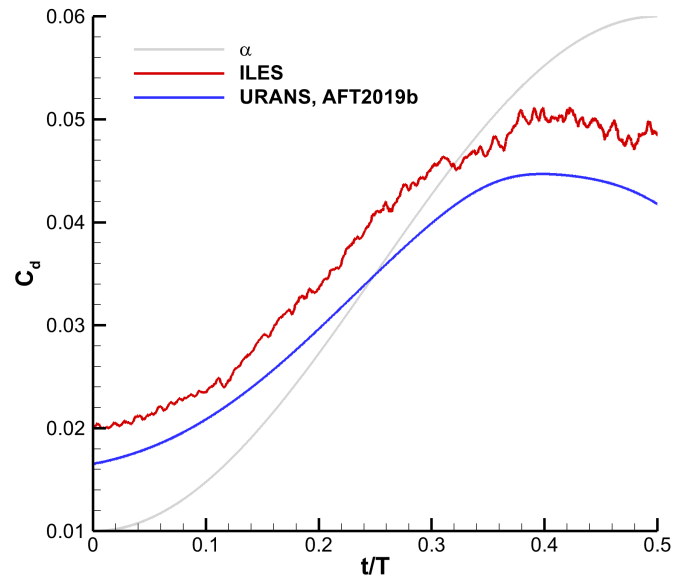


Figure 6.12: Comparison of predicted drag coefficient on the DSA-9A airfoil

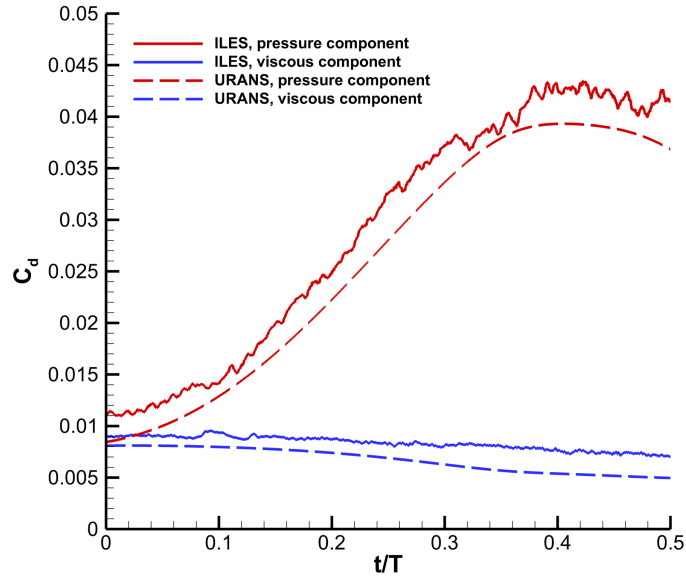


Figure 6.13: Comparison of predicted drag components on the DSA-9A airfoil

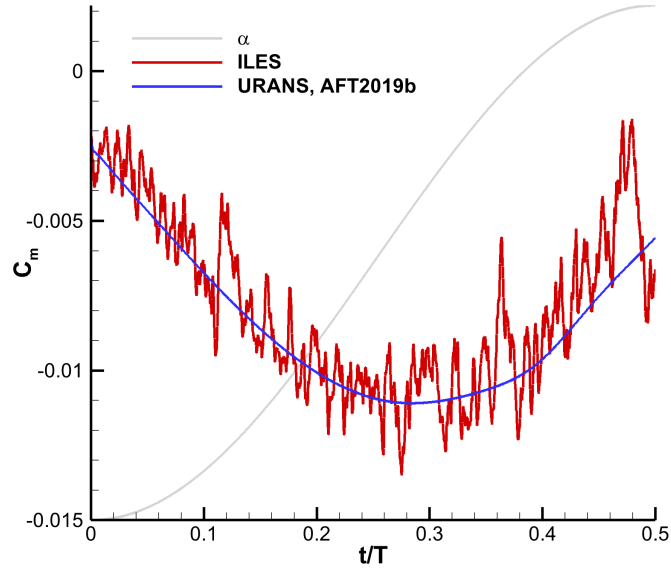


Figure 6.14: Comparison of predicted pitching-moment coefficient on the DSA-9A airfoil

Vita

Jared Carnes was born in 1995 in Dallas, Texas to the parents of Ed Carnes and Teresa Carnes. In 2001, he moved to Franklin, Tennessee and lived there until he graduated high school in 2014. He attended the University of Tennessee, majoring in Aerospace Engineering and minoring in Mechanical Engineering, receiving his Bachelor's Degree summa cum laude in May 2018. He continued his education at the University of Tennessee, graduating with a Master of Science degree in May 2020 under advisor Dr. James Coder and continued on to pursue a doctorate in Aerospace Engineering. Outside of aerodynamics, Jared enjoys weightlifting, rock climbing, and watching hockey.