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I am submitting herewith a thesis written by Linda Shane Arvin entitled "A Study of Top Quark Production and Decay in Positron-Electron Annihilation." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Physics.

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Associate Vice Chancellor and
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A STUDY OF TOP QUARK PRODUCTION AND DECAY IN POSITRON - ELECTRON ANNIHILATION

A Thesis
Presented for the
Master of Science
Degree
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Linda Shane Arvin
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I would like to thank my most patient husband Jay Sullivan for enduring my ranting and raving as I worked on this thesis. This work seemed like such a pain and sacrifice at the time of writing and only now I remember why I started on this path of torture (kidding). I realize this is one of the greatest gifts I can give myself. I am proud because I endured and because I finished something that I started. Something I thought at times impossible but know now nothing is impossible, even for me. In order to grow, one sometimes has to take leaps. I took a leap not to bog myself down with self-criticism and the questioning of my abilities. I took a leap by not letting myself think that the limits that I have on what I can do are small. Quite to my amazement at times I found strength to continue and that would not have been so easy without my very good friends and family. They make me want to try harder. I would also like to thank the committee members for their understanding and support as they helped me find my way to relate this thesis.

I would like to thank God for making it possible and that it's over. When I finally asked for your help God, you made it incredibly easier for me to do this work. We are truly blessed by your wisdom.

ABSTRACT

In this study a program is developed that accounts for gluon effects in top quark production and decay at a positron-electron high energy collider. The program is based on C++ code written by C. Schmidt who studied gluon effects in the limit where gluon interference can be neglected. The results show that the gluon radiation patterns are sensitive to top quark parameters, in agreement with the semi-classical analysis of Khoze, et. al in the soft gluon regime. Thus an experimental study of gluon radiation in top quark production will shed light on the properties of the top quark.

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1. Introduction

The top quark is a story waiting to be unraveled. It, unlike the other quarks, is unusual because of its large mass, discovered at the Fermilab proton-antiproton collider to be ~180 GeV [1]. Compared to all the other quarks, which can form stable bound states, the top quark is unique, because its lifetime, $1/\Gamma$ (where $\Gamma = 0.18(m_t / m_W)^3$), is much shorter than the hadronization time, $1/\Lambda_{QCD}$ ($\Lambda_{QCD} = 200\text{MeV}$). So the top decays before it has a chance to hadronize. A thorough investigation of the top quark could reveal some very interesting standard or non-standard physics. Since current beam energies do not suffice to probe top quark physics, new accelerators need to be built. The two possible types of accelerators are e^+e^- and hadron colliders. Energies of 400 GeV are sufficient for e^+e^- colliders but hadron colliders would require energies much higher, at least a few TeV. This enormous energy needed at the hadron collider is due to the fact that the energies of the quarks within the hadrons are much less than the energies of the hadrons. Positron-electron annihilation is an excellent place to study top quarks. For one, the annihilation can produce energies large enough to form the top anti-top pairs. Even better, since the electron and positron are not composed of quarks, they can provide a very clean environment for event reconstruction, i.e., since there is no initial state gluon radiation, there can be no gluon interference between initial and final states. Also, the positron and electron can be polarized, thereby providing an additional degree of freedom

for studying the top quark. Experimentally, polarized electron beams can be produced at 80% or better [2].

The possible interactions that can take place involving the top quark are

$t\bar{t} \to b\bar{b}W^+W^- \to b\bar{b}q\bar{q}q'\bar{q}'$ (Branching Ratio (BR) = 36/81),

$t\bar{t} \to b\bar{b}W^+W^- \to b\bar{b}q\bar{q}\ell\nu$ (BR = 36/81), or $t\bar{t} \to b\bar{b}W^+W^- \to b\bar{b}\ell^+\ell^-\nu\bar{\nu}$ (BR = 9/81).

In the first two processes, we see W^+ and W^- can both decay to quarks or one W can decay to quarks and the other one to a lepton and a neutrino (or anti-neutrino). In the last process W^+ can decay to an anti-lepton and neutrino and the W^- can decay to a lepton and anti-neutrino. In particular, the interaction $e^+e^- \to t\bar{t} \to b\bar{b}W^+W^- \to b\bar{b}\ell^+\ell^-\nu\bar{\nu}$ is the focus of my research. We want to simulate what happens in this reaction, theoretically. Although its branching ratio is the smallest, it can possibly provide precise measurements of the top, especially since right-handed electron beams can virtually reduce the background to zero. It will also be cleaner to construct this reaction since we only have to consider gluon emission from t, $\bar{t}$, $\bar{b}$ and b, whereas in other reactions we have to consider gluon contributions coming from the quarks that result from W^+ and W^- decays. These quarks form jets, making it more difficult to consider them in data analysis. Radiative (quantum) corrections due to the strong force have been studied to different extents [3-10]. Considering the contributions that they have to the overall cross section, estimated to be 10%, it is important to understand how to accurately determine their significance and how to include them in simulating events. Studies have been done

in the soft gluon regime [2] (classical limit) and threshold region [10] where the top is produced with little kinetic energy. Given this problem of how to simulate the interaction, we set out to tackle it programmatically. A partial study has been done by Schmidt [8]. He has treated gluon radiation in the production and decay of top quarks in the limit where the top quark is a stable particle. We use his programs, treeTops v 1.1 and ragTops v 1.0, and further develop it to do a complete study of gluon radiation.

Approach to Problem

We first started this project setting out to simulate the results of Schmidt's work by developing a program in Fortran that used a random number generator. After we accomplished simulating his results to zeroth order, we received Schmidt's programs. We found the programs could be useful since he had a better generator and it worked out first order corrections. Schmidt's programs were written in C++. After understanding the structure of his programs, we implemented our routines written in C++, which use different aspects of Schmidt's programs in order to accomplish the physics objectives. In Section 2, we describe the physics our work is based on. In Section 3 we explain the numerical method of the programs. In Sections 4 and 5 we discuss the structure of Schmidt's programs. Section 6 details our work based on these programs. The results and conclusions are discussed in Section 7.

2. The Interactions

The strong interaction is an interaction between quarks mediated by the gluon. Color, which is a property of the quarks and gluons, gives rise to the strong force. There are three possibilities of color (red, blue, and green) and color is a conserved quantity. Remarkably, the quarks are permanently confined inside colorless (a mixture of red, blue, and green) particles. The theory that explains this color force is quantum chromodynamics (QCD). Inside colorless particles, quarks behave as though they are free, but as they try to separate, the interaction strength increases with distance. Hence, the potential energy between the quarks will grow so large that further quarks and anti-quarks will pop out of the vacuum. These newly formed quarks will attract to the original quarks and thus will form jets of hadrons. This unusual behavior arises from an important principle, called color confinement.

The electromagnetic force does not affect color. Instead, it is an interaction between electrically charged particles, where there is only one kind of charge. Photons are transferred from one electrically charged particle to the other resulting in an acceleration of the charge due to the energy absorbed. Unlike the gluons, photons have no charge and thus do not experience the electromagnetic force.

Another interaction is the weak force, mediated by the massive neutral Z (92 GeV) or charged W (82 GeV) bosons. Thus, the weak force usually requires enormous energies to happen because of these massive bosons. These massive bosons also give rise to the fact

that this weak interaction is a short-range force, where the potential goes as e^{-mr}/r. Consequently, weak forces are not very probable at low energies, but at high energies the chance of a W or Z being transmitted is quite similar to the transmission of a photon between charged particles. When there is a weak interaction, a change of flavor (i.e., top changes to a bottom) can be involved. Also note that the weak force does not conserve parity and does not affect color. Theorists have been successful in describing both the electromagnetic and weak forces in terms of a unifying concept, the electro-weak theory. The electro-weak theory coupled with QCD comprises the Standard Model. As we shall see, all the interactions discussed take place in the reaction we are studying.

The interactions that take place in $e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}W^+W^- \rightarrow b\bar{b}\ell^+\ell^-\nu\bar{\nu}$ have varying degrees of impact on the observables. In Figure 1 the Feynman diagram (a diagram that has a set of mathematical rules for the different pieces) illustrates the process at leading order (classical process). What happens first is that the electron and positron collide and annihilate. This is represented in the Feynman diagram by the point where the legs (or lines) of the e^+ and e^- join (also called a vertex). The energy from this annihilation re-congeals into either a photon or a Z. The probability of this electromagnetic or weak interaction happening is given by form factors [8]. The photon or Z will decay into a top anti-top pair. The next part of the process exhibits the charged weak interactions responsible for the change of flavors of the top to bottom and anti-top to anti-bottom. In the Standard Model t decays weakly to bW^+ ~100% of the time. The b and $\bar{b}$ quarks hadronize and form jets. The W^+ boson goes on to decay to an anti-lepton and neutrino

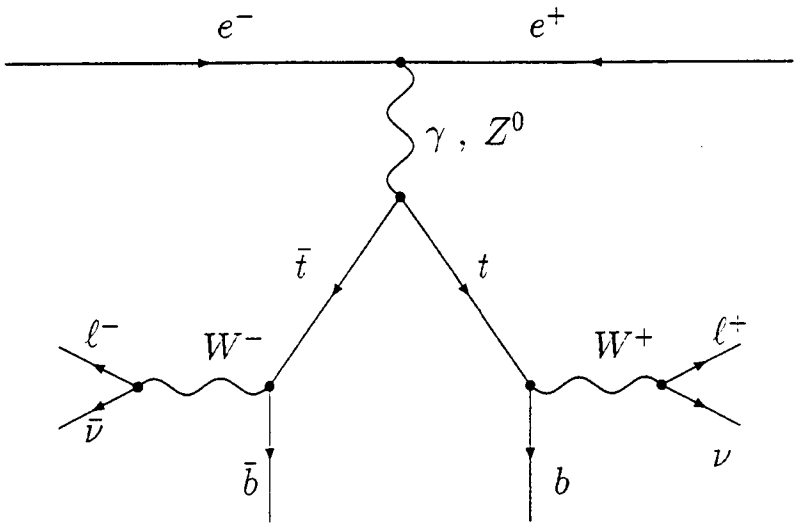


Figure 1. The Feynman Diagram of the Zeroth Order Process.

and the W^- decays to a lepton and anti-neutrino. We study this process at energies of at least 400 GeV and the mass of the top equal to 175 GeV. Therefore, we are enough above the threshold such that the top does not form a bound state with the anti-top. Instead it has enough kinetic energy to escape this effect.

Gluons emitted from a top quark interaction will definitely have an effect on the cross section calculation. However, where the gluon is emitted from and whether it is real (part of the final-state) or virtual (emitted and reabsorbed in the reaction) will give a different impact on the cross section and the angular distribution of the final-states. The gluon can be emitted from the production of the top or anti-top or from the decay of top or anti-top. It is not easy to tell the difference when it comes to reconstructing the event. Production stage emission occurs before the top or anti-top quark goes on shell. In this case, it would be considered off shell ($P^2 \neq m^2$) which is a quantum effect (Uncertainty Principle) where the energy of the top can deviate from the classical result. Decay stage emission occurs only after the top (or anti-top) quark goes on shell ($P^2 = m^2$). If it is a production stage emission the W^+ and b (or W^- and $\bar{b}$ for anti-top) will combine to give the top momentum but if it is a decay stage emission then the gluon momentum must be added as well. In general, these stages are not independent and may be interconnected by radiative interference effects. In basic terms, interference is considering the amplitude of a gluon being emitted by two different legs (e.g., consider the legs of $\bar{b}$ and t in (a) and (c) of Figure 2) at two different vertices of the diagram. The amplitudes of the different legs are added and then squared to get the probability. It is important to note that Schmidt's

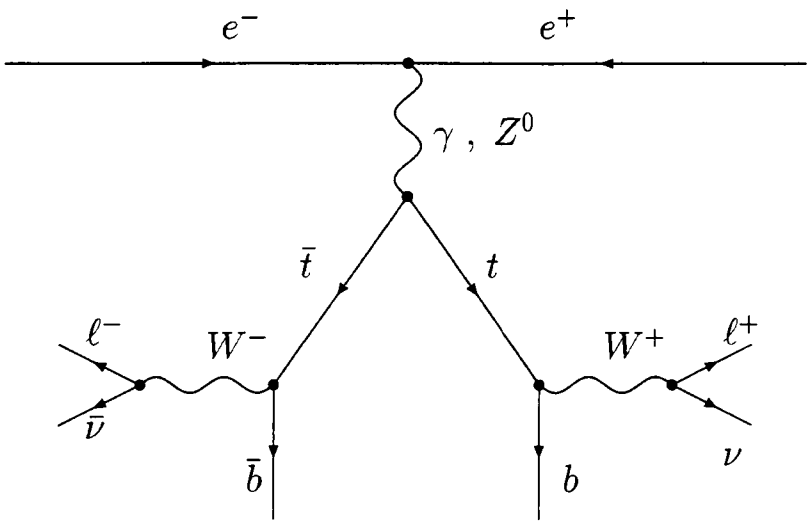


Figure 1. The Feynman Diagram of the Zeroth Order Process.

study is in the limiting case of the narrow width approximation (stable top quark), allowing him not to have to worry about interference contributions between the production and decay stages. Instead he can assign the gluon to correspond to one vertex. However, finite top widths can give rise to interference and can cause mis-assigned gluons. It is our goal to incorporate these interference effects into the calculations. The amount of effect gluon radiation has to this process is to order (α_s) approximately 0.15, or first order in the strong force coupling.

3. Monte Carlo Event Generator

Monte Carlo integration is a numerical technique used to simulate our interaction. First consider the following differential cross section

$d\sigma_0 = (1+\cos\theta)^2 \sin\theta d\theta$.

Integrating over the phase space, we get $\sigma_0 = \int d\sigma_0 = \int_0^{\pi} |(1+\cos\theta)|^2 \sin\theta d\theta$. Setting $x = \cos\theta$ obtains $\sigma_0 = \int_{-1}^{1} (1+x)^2 dx = \frac{8}{3}$. In terms of a uniform random variable r over the interval [0,1], we find $\sigma_0 = \sigma_0 \int_0^1 dr = \frac{8}{3}$. In order to get our probability distribution in terms of r's, we consider the following indefinite integral, $\sigma_0 \int_0^r dr' = \int_{-1}^{x} (1+x')^2 dx'$.

Integrating both sides of this equation obtains $\frac{8}{3}r = \frac{1}{3}(1+x)^3$, from which we can solve to get $x = 2r^{1/3} - 1$, which is the desired equation relating x to r.

With the above simple example in mind let's apply the same technique to a more complicated integral. We now consider the same differential, $d\sigma = F(1+\cos\theta)^2 \sin\theta d\theta$ but now with a more general right hand side, $F = 1 + ge^{-\cos^2\theta}$. The function at hand is more complicated and may not be integrated analytically or at all. With the substitution of $x = \cos\theta$, we get $\sigma = \int_{-1}^{1} \left(1 + ge^{-x^2}\right)(1+x)^2 dx$. Since $x = 2r^{1/3} - 1$, our integral

becomes $\sigma = \sigma_0 \int_0^1 \left(1 + g e^{-(2r^{\frac{1}{3}}-1)^2} \right) dr$. This integral is converted to a summation over the generated random numbers, $\sigma = \frac{\sigma_0}{N} \sum_{i=1}^{N} F(r_i)$, where N is the total number of r_i's. The quantity in the above equation, which multiplies σ_0, we refer to as our weight. This Monte Carlo technique allows us to account for first order effects and will be especially important, since in practice we have multi-dimensional phase space integrations. Thus, we can use this technique to find all our variables in terms of random numbers and perform our calculation.

4. The Event at Tree-Level[1]

Schmidt's program treeTops, is a Monte Carlo event generator to describe the event at tree-level. We will discuss the method of analysis that he employs and how this is incorporated into the program.

The amplitudes can be calculated by considering the helicities[2] of the incoming particles and the outgoing particles. The program exploits this fact and is able to then calculate matrix elements corresponding to the different combinations of helicity [8]. Also, the strength (which is a function of the vertex factors) of the interaction at each stage of the event is considered. For the first process, matrices, which are described by the polar angle θ (see Fig. 3) and the azimuthal angle ϕ of the top in the e^+e^- center of momentum (lab) frame are obtained. The anti-top is similarly described.

The decays $t \to bW^+$ and $\bar{t} \to \bar{b}\ W^-$ can be described well by helicity amplitudes since the spin of the top is transferred to the bW^+ system. In the top frame the helicity angles to describe this process are the polar angle χ_t (see Fig. 3) and the azimuthal angle ψ_t. A couple of interesting things to note are that in the limit of massless b[3] the Standard Model predicts that the top can only decay to left-handed b quarks and thus it can not decay to right polarized W^+'s.

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[1] Tree-level is to zeroth order (no radiative corrections).
[2] Helicity is the spin angular momentum along the direction of propagation.
[3] Helicity is a Lorentz invariant quantity for massless particles.

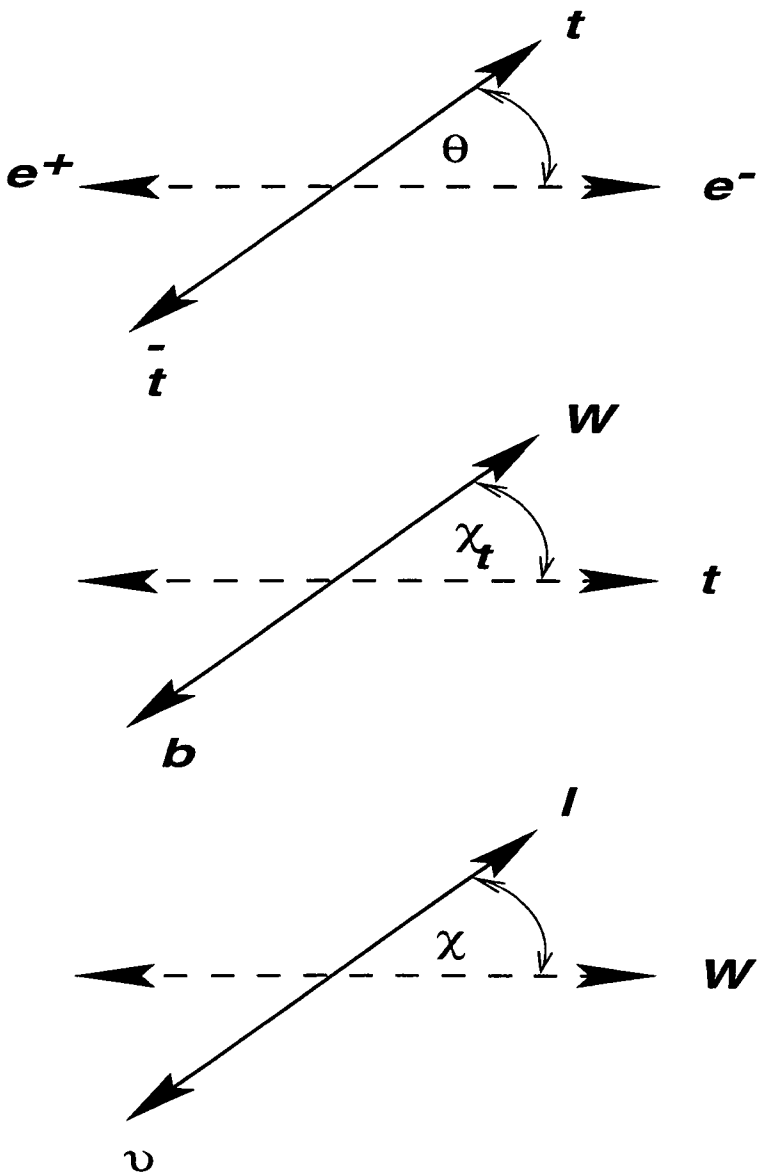


Figure 3. Helicity Angles of the Event.

As we have mentioned before the weak force does not respect parity. The last stage in the event is $W \to \ell \nu$ and can be described by the helicity angles of the charged lepton in the W rest frame.

The program performs at 100% efficiency by using the knowledge of the distribution of angles. For instance, when one of the eight cases involving $e^+e^- \to t\bar{t}$ is chosen the program assign a value for the angle θ. This angle is chosen from a distribution that has the same probability as the matrix elements predict. After the particular case for $e^+e^- \to t\bar{t}$ is chosen with a particular θ and particular helicities, the next step is to choose from one of the possibilities for $t \to bW^+$ and for $\bar{t} \to \bar{b}\ W^-$ and to choose its polar decay angles χ_t and $\bar{\chi}_{\bar{t}}$, respectively. Lastly, the angles for $W^+ \to \ell^+ \nu$ and $W^- \to \ell^- \bar{\nu}$ are chosen from one of the two possibilities for each W. An important feature of the cases that are chosen is that they are weighted by the (a weight class is defined that randomly chooses between the weighted cases) relative amplitudes. Now, the azimuthal production and decay angles are calculated using the already determined polar angles. As mentioned before, Schmidt has previously worked out the angular correlations and thus has a class (called random) of various subroutines that incorporate the different possibilities for the distributions. As well, all matrix factors for the angles chosen are calculated. A weight is then calculated for the event by squaring the probability amplitude. After a number of events are called, the weights are summed from every event and are divided by the number of events to get the total cross section.

Another facet of the program is the calculation of the final-state four momenta for each of the particles in the lab frame as well as $\bar{t}$ and $\bar{t}$'s momenta in the lab frame. It accomplishes this using conservation of energy and by knowledge of the angles for each particle in the corresponding rest frames. By the use of some subroutines, the program thus determines the four momenta in the proper frames and boosts each one of the particles into the e^+e^- lab frame. Conveniently, the final-states may be called in any subroutines and utilized.

The polarization of the electron beam can be set with a declaration of a function that is defined. This can give insight into the behavior of the asymmetric nature of the weak force. As an example, we have shown in Figure 4 the distribution of the production angle of various helicity states for left and right-handed electron beam polarization, using a sample of 100,000 events. Other useful input parameters that may be set are the mass of the top quark and the beam's energy.

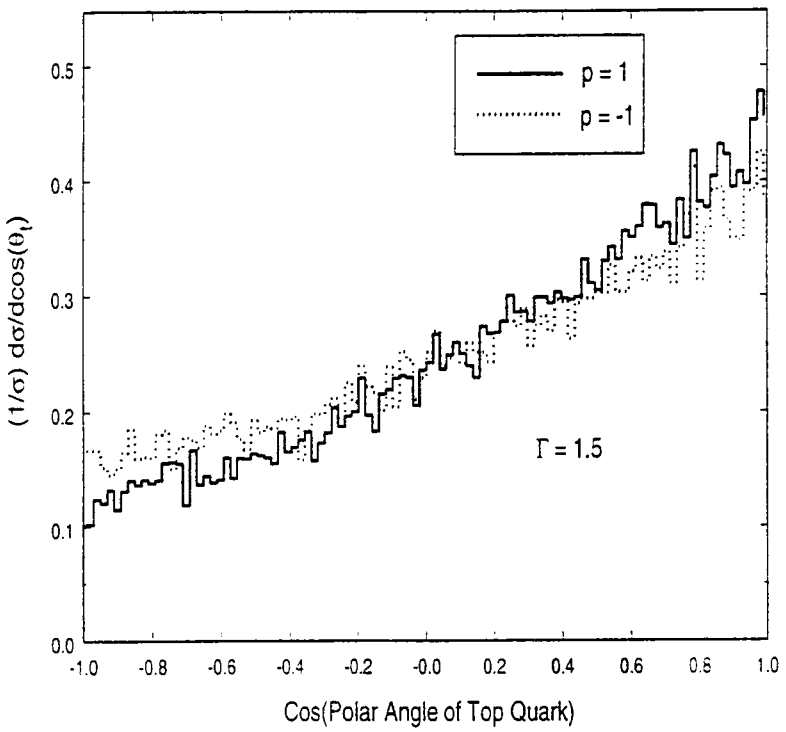


Figure 4. Distribution of the Polar Production Angle for Different Polarizations.

5. The Event at First Order

The program ragTops, uses treeTops as a base class. The ragTops generator extends the goal of treeTops to include first order contributions, namely gluon production, to the process while utilizing the angular correlations calculated in treeTops. We will explain the aspects of this extended version of the program.

For convenience the program separates the contributions of the total cross section into four channels. These are a real gluon in production (Figure 2 (b) and (c)), in decay of t (Figure 2 (c) and (d)), in decay of $\bar{t}$ (Figure 2 (a) and (b)), and one channel including virtual (Figure 5 (a), (b) and (c)) and soft (real gluons with low energies) contributions. Schmidt assigns equal probability for each one of these channels. However if desired one can weight each of these channels by calling a function. This function does a trial run of the generator, calculates the weight for each of the channels and then changes the probability of the channel being called according to these weights. When an event call is made, one of the channels will be chosen and the channel will calculate the matrix elements for that particular case and generate the momenta for the final-states. These matrix elements have been calculated by Schmidt [8].

The program introduces some artificial cutoffs (on the gluon energy) which distinguish gluons that are soft and gluons that are hard. Hard gluons are treated with exact kinematics and soft gluons are combined with virtual gluons and integrated over. The distinction between soft and hard gluons is not a physical one. The cutoff for soft gluons

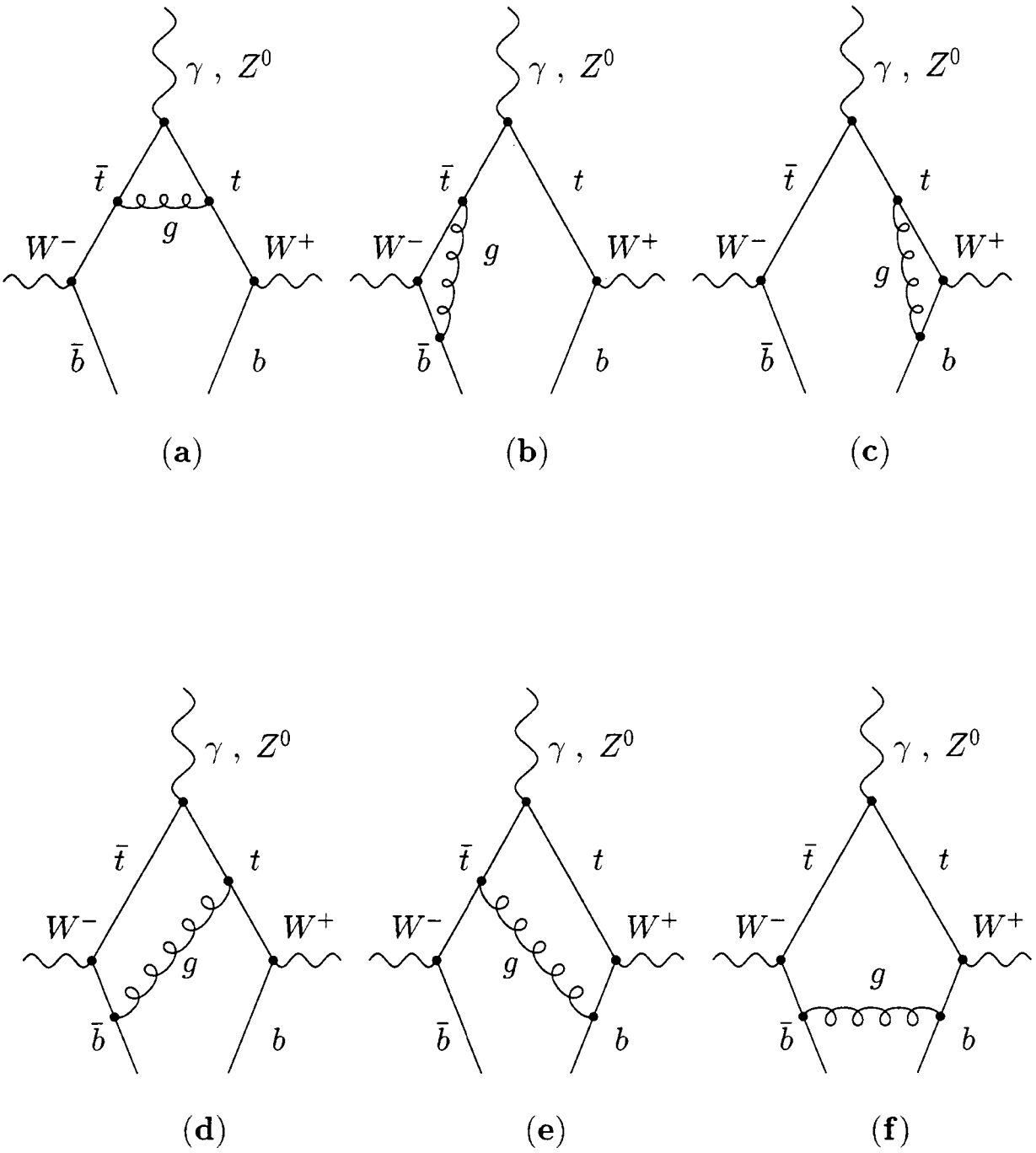


Figure 5. The Feynman Diagrams at First Order.

must be well below the physical cutoff, which is determined by detector specifications. If the cutoff is too low there will be numerical instabilities. Otherwise they are completely arbitrary and no physical results depend on them. The program has defined these artificial cutoffs as x_0, y_0 and z_0. The cutoff x_0 is a cutoff of the gluon energy in the production. The cutoffs y_0 and z_0, are cuts on the gluon energy in the top rest frame and the angle between the gluon and the b quark in the W center of momentum frame. These cutoffs have default values but can be changed by the user. If x or y and z fall below the cutoffs, they are considered soft. As mentioned before, these soft gluons are integrated out analytically and are added to the virtual gluon contributions. If x or y and z are above the cutoffs, then there is a real gluon in the final-state and it must be included in the reconstruction of the event by considering its momentum and its contribution to the matrix elements.

If the channel for virtual gluon is chosen, then the program corrects for this contribution by correcting the form factors for the production and the decay by summing the virtual and soft gluons and adding them to the corresponding form factor. The momenta are generated the same as previously described for the tree-level event. If the channel is the production channel, then the program generates the gluon energy with a particular distribution above the cutoff x and an azimuthal angle for the gluon. These are put into the helicity amplitudes to calculate the effect of the gluon in the production. Also, the program is able to generate the momenta of all the final-states including the gluon and boost them to the lab frame. If the channel is one of the decay channels, then it

generates gluon kinematics above the cutoffs, y and z, calculates matrix elements, prepares the particle four vectors, and gives the event a weight. Therefore, the program is able to calculate the cross section for this process.

The energies of the t, $\bar{t}$, and gluon are dependent on what kind of an event is taking place. If there is a gluon in the production, then the t, $\bar{t}$, and gluon will get a fraction (this fraction is calculated according to a predetermined distribution found by Schmidt) of the beam's energy but the total energy will be conserved. If the gluon is emitted in the decay stage, the t and $\bar{t}$ will receive half of the beam's energy and the gluon receives a fraction (again, this fraction is determined by Schmidt) of the t or $\bar{t}$'s energy. This depends on whether the gluon is in the decay of t or $\bar{t}$. The boost parameters (Lorentz tranformation factors) are generated as well with the kinematics. These boost parameters will be different depending on where the gluon is emitted.

ISR or initial state radiation is the effect of photons coming off the electron and positron or in other words the radiation from the initial state. However, this important effect represents a purely longitudinal boost and the program allows for this option by a switch which can turn on the radiation.

Breit-Wigner resonances (includes width effects) shift the resonances of the top and W and it gives the bottom quark a non-zero mass as well. The energies of the particles are shifted to ensure momentum conservation. The program also has a switch that can turn this effect on or off.

In order to get a more realistic view of what an experimentalist might see as far as top quark production, the program introduces a cutoff called μ. This fixes the decay cutoffs and imposes cuts on what an experimentalist can 'see' determined by detector resolution. An experimentalist will be able to 'see' the mesons that result from the decay of a gluon. This cluster of mesons is called a jet. By summing the momentum of the jet particles, the gluon momentum is reconstructed. A detector will only be so good as to distinguish two jets separated by some angle. μ defines the angle between two separate final-state particles. If for example we are interested in events containing a gluon, and the momentum of the b quark plus the momentum of the gluon squared are less than μ^2 then this event will have to be rejected otherwise we can include the event and perform exact kinematics.

6. Interference Effects

Our specific research entails gluon emission effects as well as their interference effects (see Feynman Diagrams in Figures 2 and 5). Figure 2 shows the diagrams with a real gluon being emitted. The differences between what Schmidt and what we consider is that Schmidt imposes that the leg before or after the gluon is emitted as on shell. This approach ignores coupling between quarks in different frames. We do not make this requirement. In Figure 5, we consider the virtual contributions that Schmidt does (a), (b) and (c) and we also consider the others listed (d), (e) and (f).

Consider as in the Feynman Diagrams of Figure 2, that we have a wave function ψ_1 for the gluon in the t decay (there are two possibilities) and a wave function ψ_2 representing a gluon coming from another leg such as the gluon in production of $\bar{t}$. Then we would have to add the two wave functions and then square them, thus resulting in an interference term. Interference contributions come from interference between t and $\bar{t}$ decay stages or between radiation in $t\bar{t}$ production and radiation in either decay. So for each channel that Schmidt has, we have a corresponding channel with our own definition of the channel. This is done by the introduction of $\eta_1 = (P_t^2 - m_t^2)/m_t^2$ and $\eta_2 = (P_{\bar{t}}^2 - m_t^2)/m_t^2$ where P_t, $P_{\bar{t}}$ and m_t are the four momentum of the top, four momentum of the anti-top and mass of the top or anti-top, respectively. This determines whether the top or anti-top is off shell or not and by how much. We also check if the event is in the

phase space region for each channel. Given the matrix elements [9] for different interference effects, our goal was to use the event generator provided by Schmidt and to compute the interference terms, generating the cross section for each of the possible channels.

Our method of calculating the cross section with interference relies on helicity products of the four momenta and thus a subroutine was made which calculates helicity products. In our formalism the four momenta must all be evaluated in the same frame. The helicity product is a Lorentz invariant quantity that allows us to compute amplitudes describing the event. These amplitudes are also dependent on the quantities, η_1 and η_2. We were able to use Schmidt's program to get final-state particles and perform a similar cross section calculation.

One main problem we had was learning C++ and then learning how Schmidt had approached this problem. Eric Conner, a Science Alliance student, was able to start creating the class (called newwgt in the newwgt.h file) of functions we needed in order to recreate the tree-level process with our new method of calculation. He was able to verify our method worked because it reproduced the results at zeroth order. This left us in a good position to begin the radiative effects. The header file newwgt.h is where we declared our subroutines and even have some small subroutines stored. The driver program rag2.C was the main function, where all our routines are called to do our calculation. Since we had a large number of matrix elements that needed to be calculated, subroutines were created that would return each one of the matrix elements. We stored

them in the files corresponding to the particular channel. We were then able to call these functions from corresponding subroutines (newsum2(p), newsum4(p), newsum6(p), newsum8(p)) that calculated the cross section for that particular channel. These subroutines would output the cross section when called in the driver program. The driver is where the subroutines were initialized and manipulated. (We have outlined the logistics of the driver program in Figure 6). The header file newwgt.h is included and the ragTops generator is initialized in the driver. The polarization, beam energy, mass of the top, the value of μ and the number of events are set in the driver as seen in box 1 of Figure 6. If we want to set the value of Γ, we have to go into treeTops.C and actually comment out the one Schmidt has assigned and assign our own value for Γ. Also, we initialize the final-states class (finalStates p) and our classes newwgt and newrags and call Schmidt's function that calculates the weight by his method (box 2 of Figure 6). In the loop over the number of events, we call a function, which gets the event channel as seen in box 3 of Figure 6. Essentially, this tells which of Schmidt's channel we have. So now we have four cases that respond to what channel it is. After box 3 the rest of the program is what we have done to extend Schmidt's program. If it is a virtual channel, then we call Schmidt's weight for that event, and we call the weight that we calculated for the event (We have not included our effects for the virtual case in this paper), do the appropriate mathematics and bin this weight. If we have one of the other channels then we have an

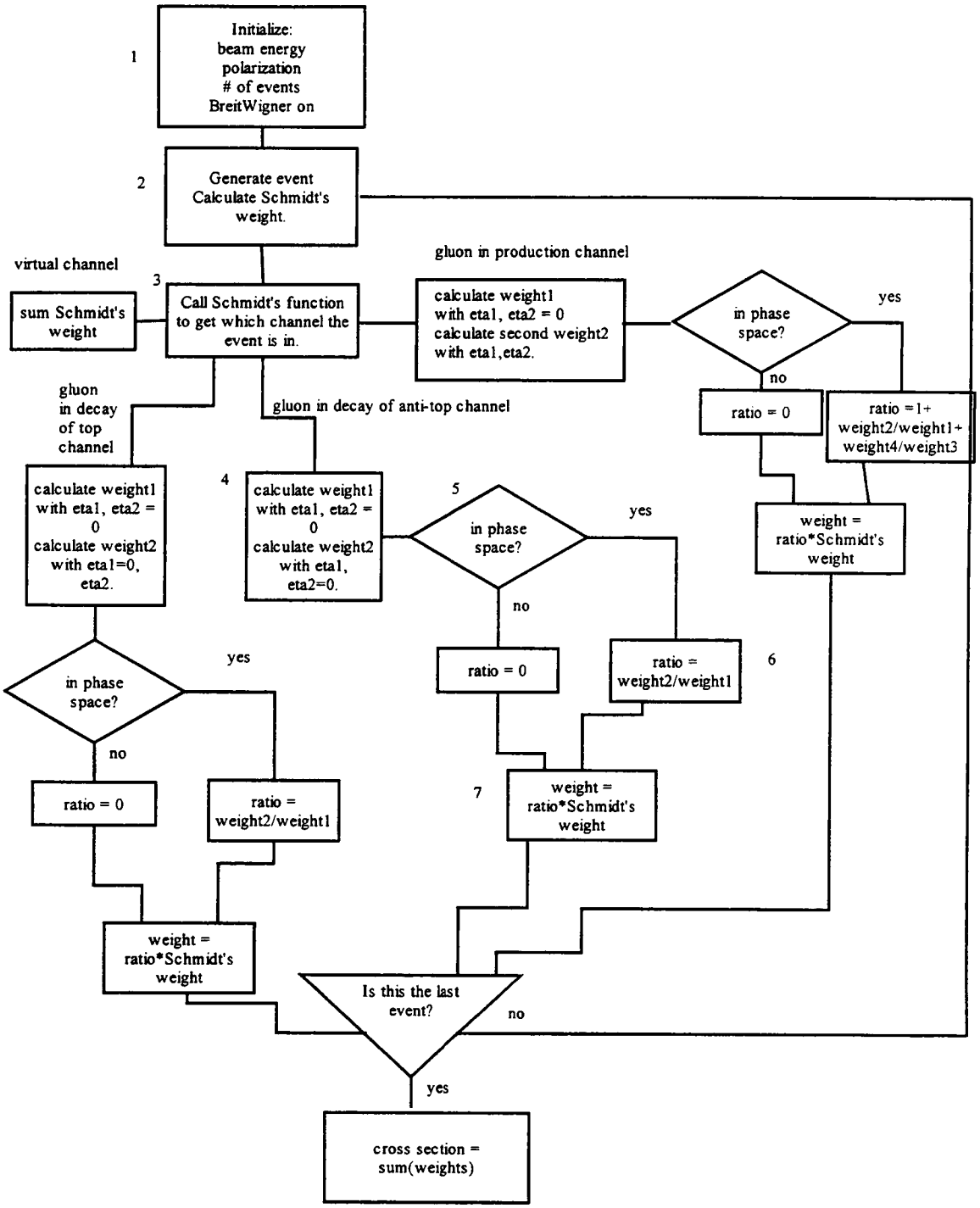


Figure 6. Flow Chart of the Driver program.

extra particle in the final-state, the gluon. Therefore, we take this into account and have a statement that checks whether we have 6 or 7 final-state particles. Otherwise, if we try to access a gluon that isn't in the final-state, we will get an error. If we have seven final-state particles, then the program continues and initializes a subroutine where variables are defined that depend on this gluon momentum. The next step is for the program to choose a case corresponding to one of the three cases involving the particular channel for the event. Essentially these channels do the same type of calculations but the minor differences are within the subroutines we call for the channel as will be explained next. Since the numbers η_1 and η_2 depend on the momenta of t and $\bar{t}$ and their momenta is different depending on what channel we are in, we created functions in the newwgt.h file for each one of the channels. If we are in the production channel, then the momenta for the top and anti-top do not include the momentum of the gluon. If we are in the decay of t then we include the momentum of the gluon in the definition of the momentum of t. Similarly, we can describe the momentum of $\bar{t}$. There is a corresponding function that sets η_1 and η_2 equal to zero. Then in the driver program we will call the cross section function with these values of η_1 and η_2, which represent the cross section with no interference (box 4 of Figure 6). After this is done, a corresponding function that calculates η_1 and η_2 or just η_1 or η_2 will be called and the cross section function called again but now it will have the interference effects included (box 4 of Figure 6). After the two different cross sections are calculated, we divide the one with interference by the one

with no interference in the driver program and obtain a ratio (box 4 of Figure 6). The next step in the program is a call to functions that we have for each channel (box 5 of figure 6). These functions depend on η_1 and η_2 and are used to determine if the event lies within the phase space defined for that channel. If the two functions for each of the channels are less than 1 then we are in the region of phase space (box 6 of Figure 6) defined for that channel. If the event is in the phase space, we take the accepted ratios and multiply them to Schmidt's weights (box 7 of Figure 6). This represents the weight with interference. If the event does not pass the tests for the phase space, then the ratio is set to zero and multiplied to Schmidt's weight. These new weights can then be binned.

In order to compile this program with the g++ compiler we have on our machines, the files must be linked to the appropriate math and C++ libraries.

Problems that had to be overcome are the size of the matrix elements, the number of calls they make to other functions in the class and getting Schmidt's programs to run on our UNIX machines. We defined some variables for some function calls so they would only have to be called once. When the matrix elements were too large, we separated them into more than one file. As far as getting the program to run, we used trial and error.

Future goals for this program is to reduce run time. We have some suggestions now that we have some hindsight as to what the outcomes need to be. For one, the different cases that we have for the channels, could all have their own classes defined. That way, only the matrix elements for that class will have to be initialized and all of the if

statements in the driver program would not be needed. A function could also be created to change the value of Γ in the driver program instead of having to change it in treeTops. Analysis may be done by means of a histogramming package that Schmidt includes with his programs. This allows one to get a set of data from the program. If someone wants to graph the number of gluons versus the energy of the gluons, there are a number of steps to take. First, one would include the file that has the histogram class and initialize the class. One can set the limits on the number of bins and the limits on the x and y data. Calling the function h.bin(σ, Eg), σ being the cross section and Eg being the energy of the gluon, the histogram class bins every call or point into the proper bin. As well it will average all the points in the bin and give each a standard deviation. The set of data will look like equally spaced bins between the limits set on the x and y and each bin has two corresponding numbers, one which is the average of all the numbers in the bin and the other number is the standard deviation within that bin. This is done after all the events are called. Now the generated set of data is ready for a graphing program such as Gnuplot.

7. Results and Conclusions

The effects of gluons can surface in many aspects of the physics. Some effects surface more than others depending on what is being measured and the initial conditions that are set. Schmidt has shown the effects of gluons to the process but neglects interference contributions. The present work includes all the contributions that Schmidt considers and also goes further to account for interference effects. To illustrate the impact of including interference, we compare the present work to Schmidt's results. In most of the plots, we have zero polarization and beam energy of 400 GeV unless otherwise specified. The number of events for all the plots is set to 100,000. Also set is the value of μ and the value of the decay width. We have compared our effects of gluons on the gluon polar angle with interference to Schmidt (no interference) in Figure 7. There is not a major difference between the two and the effect of interference is approximately 1%. The interference contributions do not skew the angular dependence and only impact the intensity slightly. Further analysis with an investigation of the impact of the initial conditions should be performed. However, in Figure 8 we have plotted the distribution of gluon energy with and without interference. Around energies of 2-3 GeV, there is an obvious difference. In this particular energy range, gluon interference effects give rise to a strong suppression of the distribution function. From this plot, one can infer that fewer

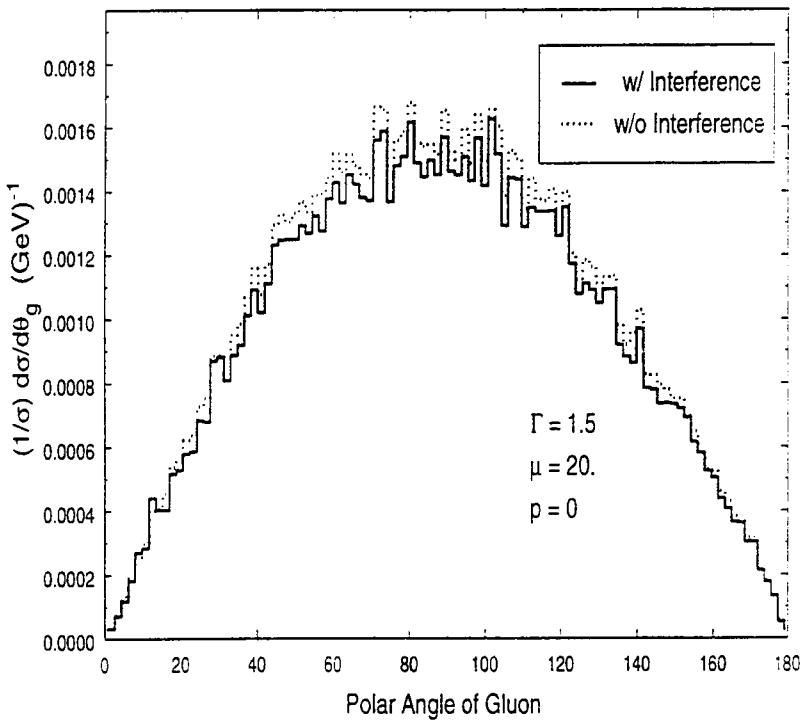


Figure 7. Comparison of Gluon Polar Angle Distribution with and without Interference for $\Gamma = 1.5$, $\mu = 20$ and Polarization = 0.

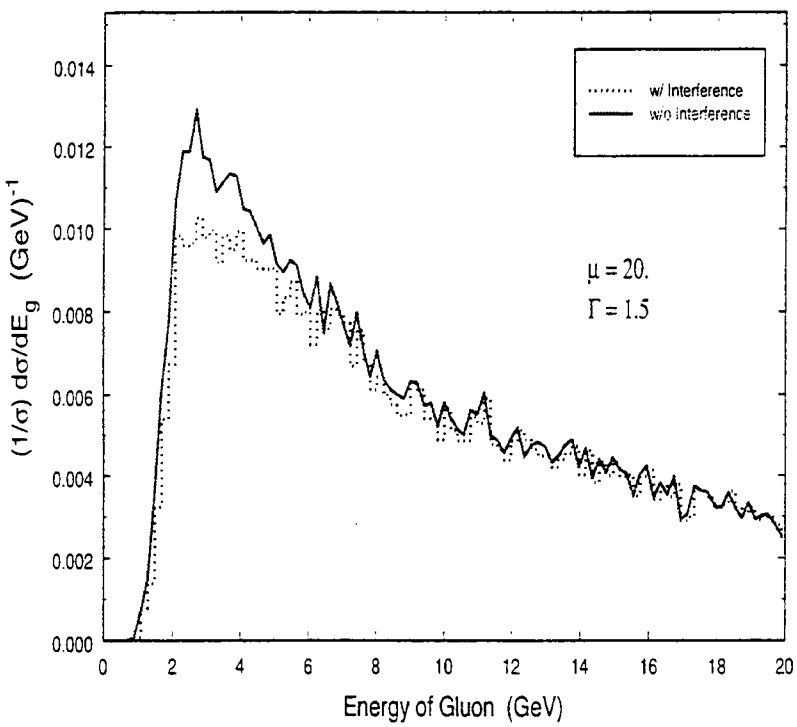


Figure 8. Comparison of Gluon Energy with and without Interference for $\Gamma = 1.5$. $\mu = 20$ and Polarization = 0.

gluons contribute to the physical process when one accounts for (real) interference effects exactly. We have plotted the distribution of the angle of the gluon with various μ s and Γ s in Figures 9 and 10. Figure 9 shows the powerful effect of the detector cutoff, μ, which influences our radiation pattern tremendously. What is 'seen' at the detector is heavily weighed on this detector's resolution and this result has obvious consequences for experimentalist searching for a signature radiation pattern. The results in Figure 10 implies the number of gluons coming off at an angle around 90 degrees is more predominant at smaller values of the decay width. The decay width of the top is an important parameter that is experimentally unknown but has a predicted value given by the Standard Model and so these results provide the possibility to measure the top quark lifetime. Also plotted is the distribution of the energy of the gluon in Figure 11 for different values of μ and in Figure 12 for different values of Γ. The impact of μ is tremendous. Consistent with the results of Figure 9, we see from Figure 11 that detector resolution will be an important parameter in the efficiency of the experiment. Figure 12 illustrates once again the sensitivity of the radiation patterns to the decay width. Around a few GeV there is a dramatic difference in the number of gluons that are produced. When the value of the decay width increases, the suppression of gluons becomes evident. This result is consistent with our physical intuition, since the larger decay width gives a smaller top quark decay time and thus the quarks do not have as much time to radiate. In Figure 13 we plot the energy of the gluon for different beam energies. The increase in

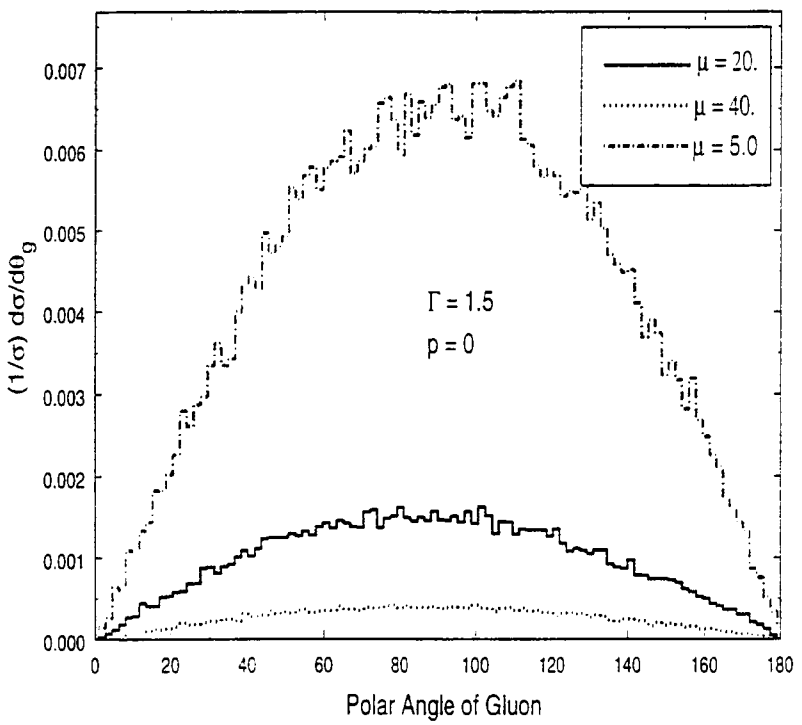


Figure 9. Distribution of Polar Angle of Gluon Including Interference for Different Values of μ with $\Gamma = 1.5$.

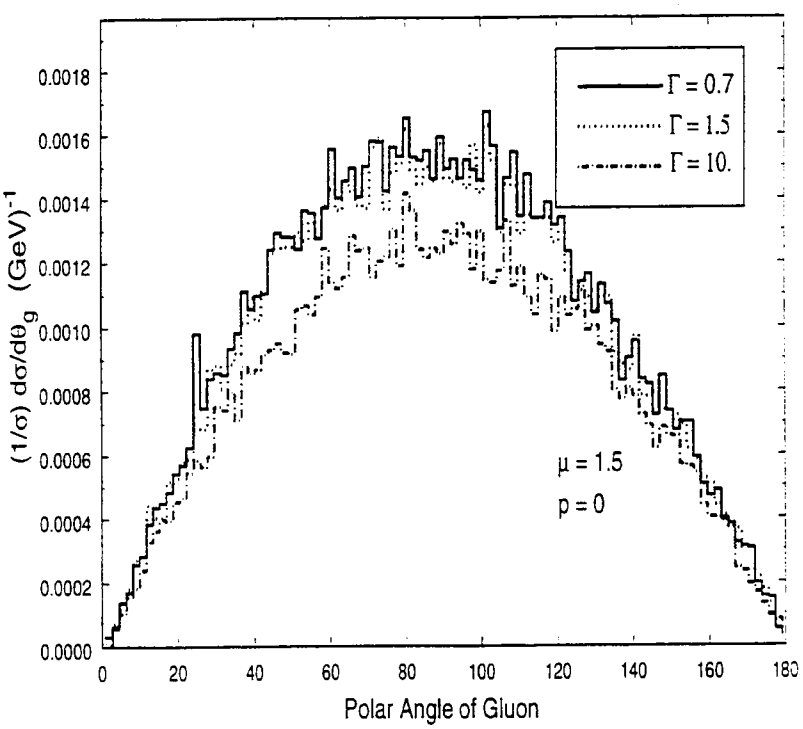


Figure 10. Distribution of Gluon Polar Angle Including Interference for Different Values of Γ with Polarization = 0 and $\mu = 20$.

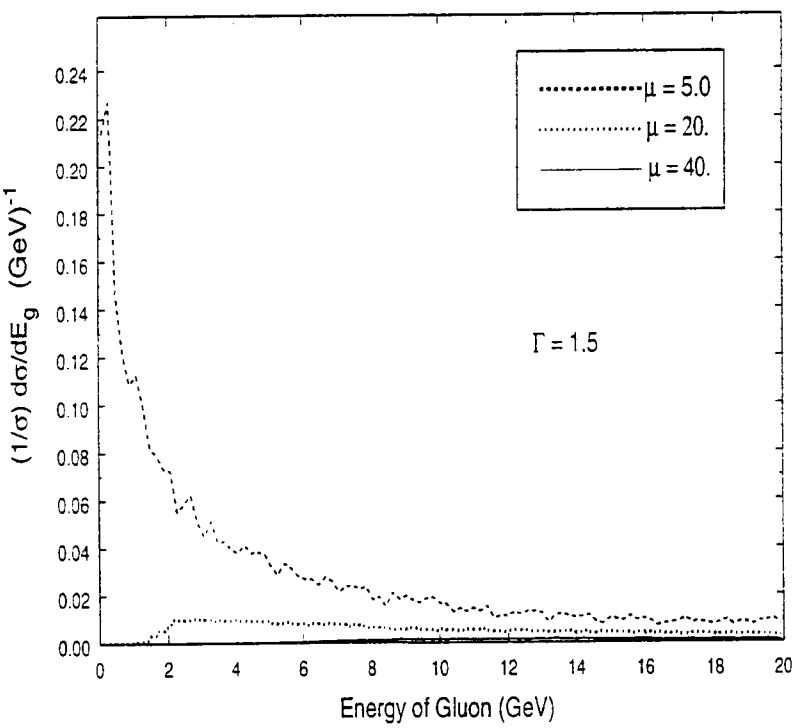


Figure 11. Distribution of Gluon Energy Including Interference for Different Values of μ with $\Gamma = 1.5$ and Polarization = 0.

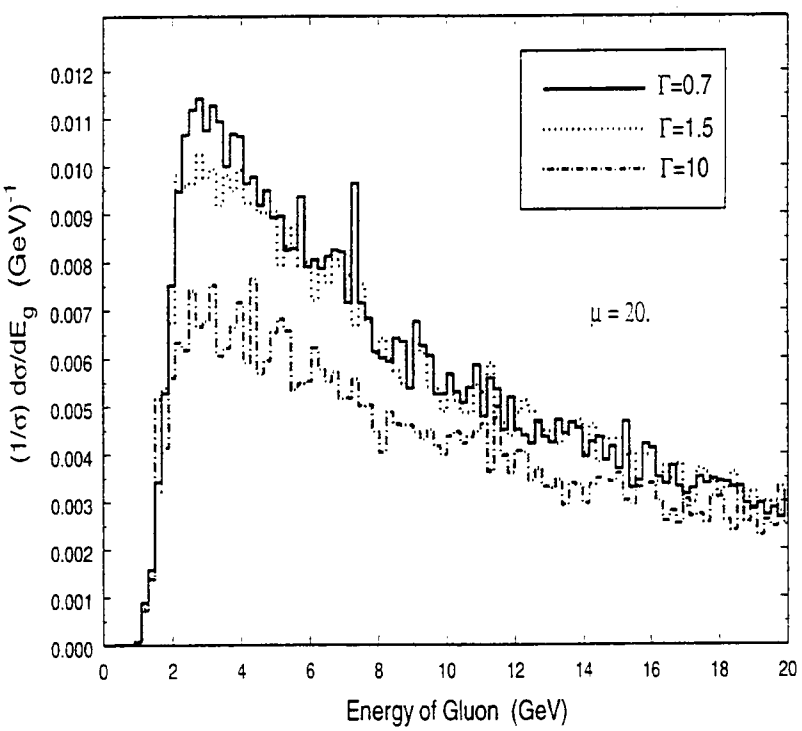


Figure 12. Distribution of Gluon Energy Including Interference for Different Values of Γ with Polarization = 0 and $\mu = 20$.

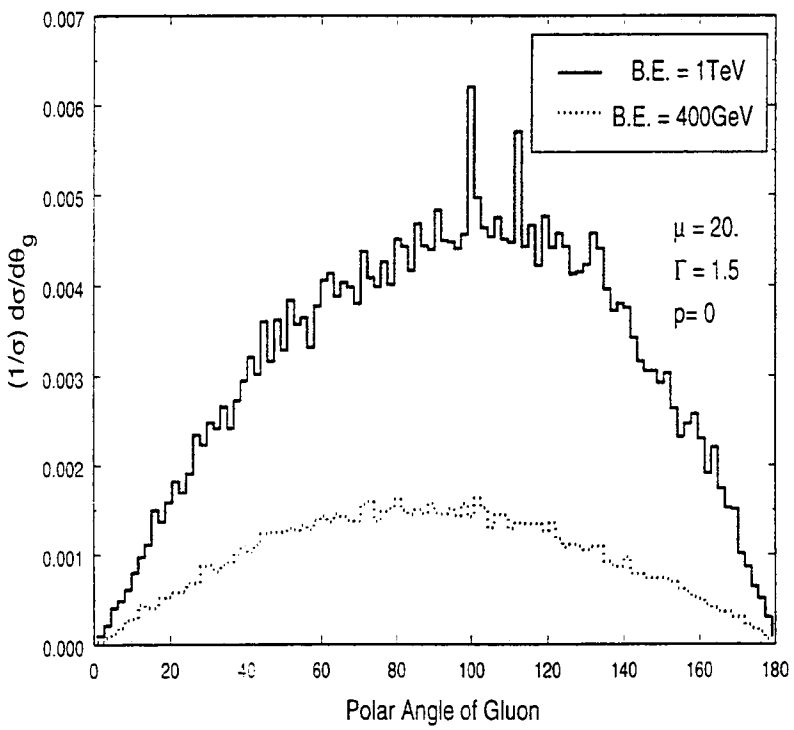


Figure 13. Distribution of Gluon Polar Angle Including Interference for Different Beam Energies with $\mu = 20$, $\Gamma = 1.5$ and Polarization = 0.

beam energy greatly increases the number of gluons that can be seen. There is an obvious change in the radiation pattern. This can be very well justified since we are far above threshold. The higher beam energy can have more leftover energy for the gluon to be produced. This leads to a higher probability of gluons being emitted at angles larger than 90 degrees. Figure 14 shows the impact of beam polarization, which should be possible to observe experimentally. When the beam is polarized the peak of the distribution shifts by quite a few degrees to higher angles which is consistent with conservation of angular momentum arguments. In Figure 15 we have plotted the distribution of the transverse momentum of the gluon. In this figure, one can see that there are a large number of gluons with a transverse energy of 1 GeV. Also, in this figure, we see that decay width doesn't seem to have much of an impact on the results.

There are often difficulties in reconstructing events when it comes to the actual experiments. For one, detector resolution may not be acceptable. At the least, it will give uncertainties in the energy of the final-states. It is estimated detector effects are 3-4 GeV [6]. Another essential problem is the number of $t\bar{t}$ produced. It is expected at the NLC with a center of mass energy of 500 GeV, there will be a corresponding 25000 $t\bar{t}$ s produced in a running year [6]. Background is another difficulty that the experimentalists must consider.

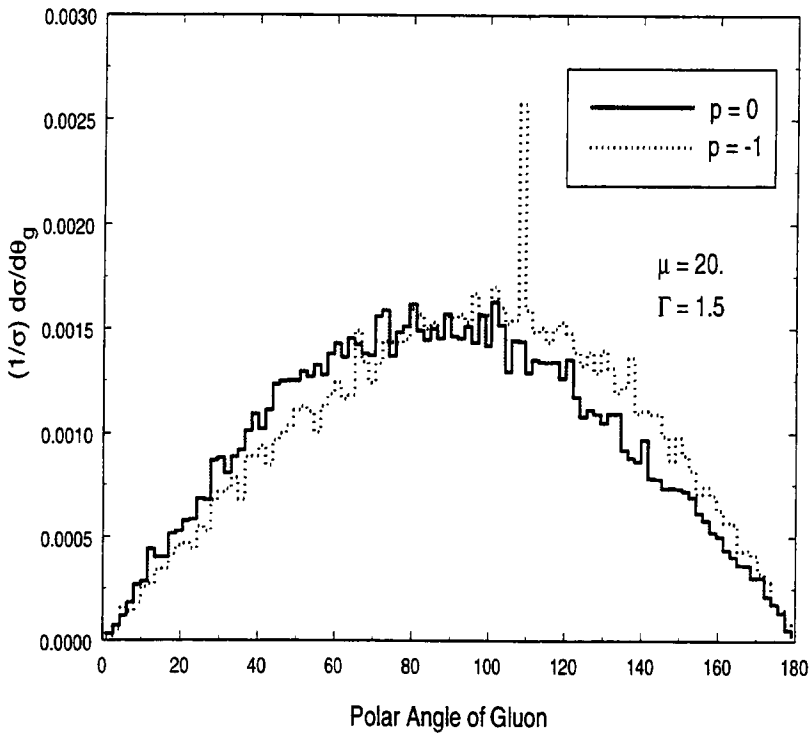


Figure 14. Distribution of Gluon Polar Angle Including Interference for Different Polarizations with $\mu = 20$ and $\Gamma = 1.5$.

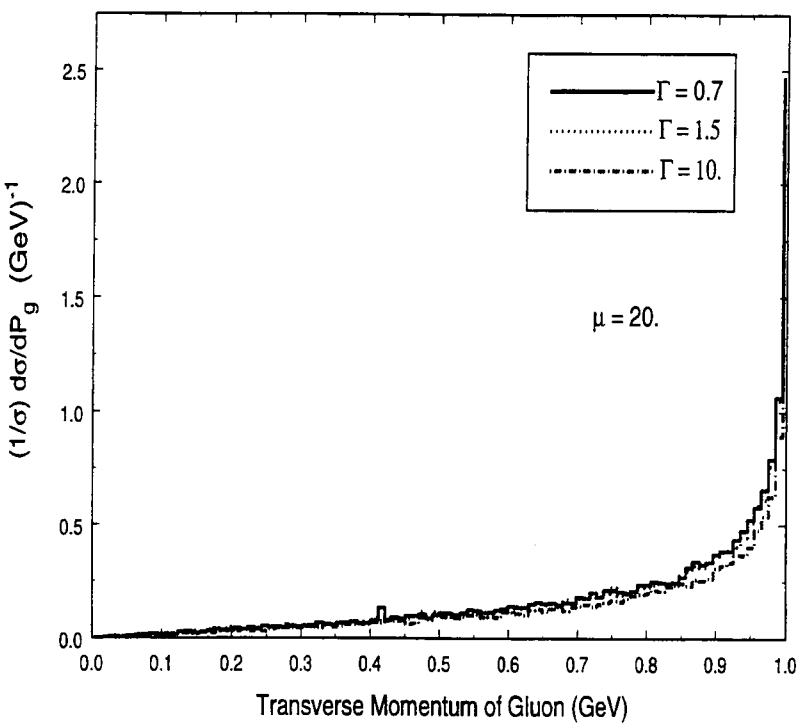


Figure 15. Distribution of Transverse Momentum Including Interference for Different Γ Values with $\mu = 20$ and Polarization = 0.

The future of top physics amounts to exactly calculating next to leading order in QCD at not only e^+e^- colliders, but hadron colliders as well. This assignment of the gluon will be different at higher energies. There the top will radiate more in the production. Energies not far above threshold will result in radiation predominantly in the decay. Further analysis needs to be done with a careful study of the impact of the unknown parameters, e.g. Γ and μ, in order to have available a large database of information with which experimentalist can compare. However, it has been visible in our results that the top quark parameter Γ affects the results of the gluon radiation patterns in a significant way. A detailed study of final states coupled with careful reconstruction analyses, will allow experimentalist to "see" gluon radiation effects. If we can compare our theoretical calculations with measured gluon radiation patterns, then we can learn much about the properties of the top quark. Correctly modeling the radiation will help determine top production cross section and help our theoretical understanding of the top. The precise determination of Standard Model parameters will enable us to further our understanding of high energy physics.

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VITA

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