To the Graduate Council:

I am submitting herewith a thesis written by Gary Dwight Ledbetter entitled "Investigation of the Application of Statistical Analysis Tools to Commercial Aircraft Engine Performance and Design". I have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.

Roy J. Schulz

Roy James Schulz, Major Professor

We have read this thesis
and recommend its acceptance:

Roger Crawford

Ahmad D. Vakili

Accepted for the Council:

C W Minkel

Vice Provost
and Dean of The Graduate School

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a Master's degree at The University of Tennessee, Knoxville, I agree that the Library shall make it available to borrowers under rules of the Library. Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of the source is made.

Permission for extensive quotation from or reproduction of this thesis may be granted by my major professor, or in his absence, by the Head of Interlibrary Services when, in the opinion of either, the proposed use of the material is for scholarly purposes. Any copying or use of the material in this thesis for financial gain shall not be allowed without my written permission.

Signature Gary D. Ledbetter

Date November 30, 1990

INVESTIGATION OF THE APPLICATION OF STATISTICAL ANALYSIS TOOLS TO COMMERCIAL AIRCRAFT ENGINE PERFORMANCE AND DESIGN

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Gary Dwight Ledbetter

December 1990

ACKNOWLEDGMENTS

Every graduate student knows that the composition of a thesis is a labor of patience that comes to fruition only with the help of many friends. Even so, the list of indebtedness for this effort is an especially long one.

Initial inspiration for the topic was provided by Mr. H. Thomas Bentley III. Mr. and Mrs. Gregory and Janice Denny lent untiring and wide-ranging assistance, and provided commiseration through the memories of their own University of Tennessee theses. One could accurately describe their help as being both selfless and insistent. Dr. R. L. Iman, of Sandia National Laboratories, kindly provided the statistical sampling and regression analysis computer codes. Dr. H. J. Wilkerson, professor at the University of Tennessee Knoxville campus, helped to instill an interest in airbreathing propulsion during the author's now-distant undergraduate senior year of 1975. Ms. Becky Stines provided initial manuscript formatting, and helped in many other ways with a friendliness and efficiency that are unusual to find. Brad Winkleman lent the use of his UTSI office space during the final hectic weeks of composition and revision. Ms. Ann Lacava, the official thesis consultant, made her home and office telephones available at all hours of the day and night. Mr. Alan Hale provided constant general encouragement.

However, the most profound help given was that provided by the author's mother, Mrs. Sue Carol Heady. Her lifetime of selfless giving has reserved the formal dedication of this thesis for her.

ABSTRACT

This thesis is an investigation of the use of a statistical sampling code and a regression analysis code to develop a simple mathematical function that represents the relationship between a given engine's thrust and its operating parameters. The analysis has direct application to the general design and performance analysis of commercial aircraft engines. The statistical sampling code was written by Department of Energy personnel. It employs a Latin Hypercube sampling (LHS) technique which permits accurate sampling of engine behavior over an input parameter list while using a minimum amount of data. The regression analysis code (STEP) yields information on engine parameter interrelationships, and was used to perform multivariate regression analysis. Two thermodynamic cycle decks were also used in this investigation in order to obtain the thrust values of a specific airbreathing engine operating at specified flight conditions. These cycle decks, written by faculty of the United States Air Force Academy, address on-design (ONX) and off-design (OFFX) engine behavior. All of the codes used in this thesis are publicly available with unlimited distribution.

The representative scenario that was chosen for this thesis was the determination of the change in takeoff thrust of a specific engine at a variety of airport elevations and over a range of ambient temperatures. This type of scenario provided important information concerning the upper limit of temperature that will safely permit takeoff from a given airport. First, the LHS program was used to obtain the sample set of takeoff conditions (altitudes and temperatures). In order to obtain engine performance data, the ONX and OFFX programs were then exercised at the conditions specified by LHS. Finally, the regression analysis was conducted using STEP in order to determine a mathematical function that expressed takeoff thrust for the given engine at any elevation and temperature.

A comparison is presented for engine math model (i.e., cycle deck) data vs. the thrust predicted by the function derived from LHS and STEP. The main conclusion of the research was that remarkably accurate functional relations could be achieved using LHS and STEP. A suitable amount of background theory is included for the computational methods underlying each code.

The results of the application of these programs showed that they represent useful, viable tools for the investigation of the performance of aircraft engines, and that they do so in a manner that legitamitely permits the use of a minimal amount of data via the Latin Hypercube sampling technique. As the costs increase to obtain today's engineering test data (or model predictions), so does the importance of the application of such methods, because they permit fewer tests to be performed in order to obtain a predictive operational database.

TABLE OF CONTENTS

CHAPTER PAGE

I. INTRODUCTION 1

A. Background and Problem Statement 1
B. Approach 3

II. LHS COMPUTER PROGRAM 4

A. Supporting Theory 4
B. LHS Code Features 7
C. Application 9
D. Sample Size Requirements 11
E. Results 11

III. ONX AND OFFX COMPUTER PROGRAMS 13

A. Supporting Theory 13
B. ONX and OFFX Code Features 14
C. Application 15
D. Engine Design Point Specification 16
E. Results 17

IV. STEP COMPUTER PROGRAM 18

A. Supporting Theory 18
B. STEP Code Features 21
C. Application 25
D. Regression Analysis Accuracy 27
E. Results 28

V. SUMMARY 31

A. Conclusions 31
B. Recommendations 32

LIST OF REFERENCES 33

APPENDICES 35

A. Tables 36
B. Figures 41
C. Governing Equations for a Turbofan, Bypass Ratio Prescribed 53

NOMENCLATURE 58

VITA 63

LIST OF TABLES

TABLE PAGE

1. Summary of the Fifteen Candidate Latin Hypercube Samples 37
2. Input File for ONX Program Execution 38
3. Engine Off-Design Performance at Aircraft Takeoff Conditions Specified by the Latin Hypercube Sample 39
4. Comparison of Actual and Predicted Thrust Values 40

LIST OF FIGURES

FIGURE PAGE

1. A Typical Use for Thrust Lapse Rate Models 42
2. Solution Approach 43
3. Intervals Used with a LHS of Size n=5 in Terms of the Density Function and Cumulative Distribution Function for a Normal Random Variable...44
4. Definition of Normal and Lognormal Distributions..................................45
5. Definition of Uniform, Modified Uniform, and Loguniform Distributions.......46
6. Definition of Triangular Distribution..47
7. Definition of Beta Distribution..48
8. Example of User-Supplied Discrete Probability Distribution 49
9. LHS Input File for Takeoff Thrust Lapse Rate Investigation.......................50
10. LHS Program Output for Takeoff Thrust Lapse Rate Investigation 51
11. STEP Input File for Takeoff Thrust Lapse Rate Investigation.....................52

I. INTRODUCTION

BACKGROUND AND PROBLEM STATEMENT

As the size and complexity of modern commercial aircraft engines continue to grow, it becomes ever more important to control testing costs. The workhorse long-haul aircraft of the 1990's will in all probability continue to be the MD-11, L-1011, Airbus 340, and 747-class aircraft, with their high thrust requirements satisfied by new generations of high-bypass turbofan engines. One way to control the costs associated with this increasingly expensive hardware is to minimize the engineering test requirements, both for ground testing and flight testing. One experimental design technique that has been used in the past for a variety of engineering subsystems (and with just such a purpose in mind) is a statistical sampling technique known as Latin Hypercube Sampling (LHS). Having judiciously obtained the sample set of flight conditions at which ground testing or in-flight testing is to occur, one must consider how best to represent the data to allow for interpolation or extrapolation to off-test flight conditions. A commonly employed method is multivariate regression analysis (i.e., curvefitting).

The specific problem application chosen for this thesis is the determination of turbofan engine thrust-lapse characteristics at takeoff for a variety of ambient pressures (ergo, altitudes) and temperatures. The outcome of this operational scenario thus defines the upper limit of ambient temperature that will safely permit takeoff from a given airport. Gross takeoff weight and runway length were assumed to be known constant values, and did not enter into this engine performance-oriented analysis. Thus, the statement of the problem may be formally worded as follows: obtain an empirical funtional relationship for commercial aircraft engine takeoff thrust which can be used to predict the thrust lapse rate expected for any combination of ambient temperature and airport elevation (i.e., ambient pressure). It is also required, in the interest of cost control, to gather and use the smallest feasible database.

Numerous precedents exist that give justification to the validity of this thesis topic's connection with real-world engineering and design applications. One engine performance investigation which represents both ground test and in-flight thrust measurement (a logical extension to this thesis) is the thrust measurement process illustrated in Figure 1[1]. This illustration, taken from p. 254 of Reference 1, mentions the use of ground testing to construct a calibrated (i.e., validated) lapse rate model (such as a regression analysis) that can be used to calculate thrust at other flight conditions. Indeed, the preface of Reference 1 specifically states the aircraft engine-related goal of "interpolation of measured thrust . . . over a range of flight conditions by validation and development of analytical models". There is no implication, however, that one should always choose a small but representative Latin Hypercube sample of test conditions. Some engineering subsystem developments require larger test matrices. As stated in section 5.2.2.1 of Reference 1, convergent-divergent nozzle scale-model test requirements for component-level performance (such as nozzle thrust coefficient and discharge coefficient) are based on an evaluation of aircraft mission and engine operation within the flight envelope. The results of such an evaluation define the types of testing and the data-point test matrix required to provide the data necessary to develop the exhaust system nozzle performance characteristics. In that instance, the scale-model test program must be structured to generate data which completely bound the range of nozzle pressure ratio produced in flight.

---

[1] All tables and figures are listed in Appendix A and Appendix B, respectively.

APPROACH

A diagram of the solution approach is presented in Figure 2. The Latin Hypercube sampling (LHS) program was used to obtain the matrix of engine test conditions (altitudes and ambient temperatures). This code was produced by Department of Energy personnel at Sandia National Laboratories in Albuquerque, New Mexico. Next, the engine thrust values were determined at the operating conditions specified by the LHS code. In order to generate these engine performance data sets, two generalized engine performance computer programs were employed. These codes represent thermodynamic cycle analyses, and were written by faculty of the United States Air Force Academy. They provide for both on-design (program ONX) and off-design (program OFFX) engine performance for a variety of engine cycles, including the high-bypass turbofan considered herein. Finally, a regression analysis (i.e., curvefit) was performed in order to construct a mathematical function that expresses one specific turbofan engine's takeoff thrust at any airport elevation and ambient temperature. The regression code used was the STEP computer program, which was also obtained from Sandia. All of the computer programs are publicly available with unlimited distribution.

II. LHS COMPUTER PROGRAM

LHS is a VAX 11/780-based FORTRAN code written by Sandia National Laboratories personnel for the generation of multivariate samples either completely at random or by a constrained (stratified) randomization technique known as Latin Hypercube Sampling. It requires the user to specify, among other things, the sample size, the number of samples to be drawn, and the independent parameter statistical distributions (i.e., density functions). This chapter describes the supporting theory associated with the code, the major features and options it offers, and how it was applied to the current investigative effort. Reference is made to three principal documents that offer an expanded description for program users.

SUPPORTING THEORY

What is the underlying reason for the use of a statistical sampling code that uses any method besides the simple, straightforward random sampling technique? The generalized answer given in Reference 2 by the principal author of the LHS program is that "some physical processes are difficult to study directly, for various reasons, and are therefore observed indirectly through the use of mathematical models. The mathematical models are often so complex that they are amenable to solution only through the use of numerical methods on a computer. Even then the solution may involve a considerable amount of computer time, so care is needed in selecting the input variables in such a way that the most important information is obtained concerning the output variable. This suggests the use of efficient statistical methods for the design and analysis of pseudo-data generated through the use of such models". The more specific reason for using LHS in connection with this thesis topic is that it permits the use of a small but representative sample set of test conditions at which the engine thrust values are to be obtained. Thus, through the use of an LHS sample it is possible to employ the smallest number of test conditions that will constitute a representative sample of the independent parameters of interest that control, characterize, or mark the system's performance. This satisfies the portion of the formal problem statement that mentions the need to control costs through the use of a small database.

This idea of cost control through planned minimization of data is certainly not a new one. Several noteworthy references exist on this subject. Canavos discusses in Reference 3 the criteria for optimal design of experimental tests. Through the use of matrix manipulation techniques, optimal approximating functions are obtained to relate a set of controlled variables to a measurable response. In Reference 4, Box and Wilson have contributed a discussion concerning the "experimental attainment of optimum conditions". In a similar line of thought, Lipson and Sheth devote an entire textbook on the subject of statistical design and analysis of engineering experiments (Reference 5). One of the simplest approaches to this subject, however, is the method used in this thesis -- the use of a Latin Hypercube sample set of conditions.

How does a Latin Hypercube sample set of engine operating conditions differ from a set of conditions that are simply chosen at random? The most concise answer is that the conditions in a Latin Hypercube sample are spread over the entire range of each engine operating variable to be considered (no clustering of values can occur). See, for example, the interval spread in Figure 3 associated with a normally distributed parameter using a sample size of five (i.e., five subdivisions indicated by the dashed lines). This distribution of parameter values is unlike a random sample of values, which could produce clusters of observations anywhere within the range of each engine operating variable. Can we be sure that a Latin Hypercube Sample will always produce better results? To quote Iman

(Reference 2, p. 19), "it is difficult to prove analytically that either method of sampling is better than the other. However, some comparisons of the two sampling methods have been made with actual models, and the Latin Hypercube samples appear to give much better results . . . ". In other words, although no closed-form solution proof can be given, it is highly likely that the Latin Hypercube technique is more accurate; and this has direct implications on the number of test conditions that are needed (more will be said about this later).

A more formal description of the Latin Hypercube sampling procedure is now given. The initial step is the selection of n different values (n is the sample size) for each of k variables $x_1, x_2, \ldots, x_k$ in the following manner. First the range of each variable is divided into n nonoverlapping intervals (again, see Figure 3). Next, one value within each of these intervals is selected in a purely random fashion. This is done for every variable. Then, each one of the n values thus obtained for variable x_1 is paired in a random manner (i.e., using equally likely combinations) with one of the n values of variable x_2. Of course, each value must be used once and only once in this pairing procedure. In the same manner, each of these n pairs is then combined in a random fashion with the n values of variable x_3 to form n triplets, and so on, until n k-tuplets are formed. This is the Latin Hypercube sample. It can correctly be viewed as forming an $n \times k$ matrix of test conditions (or computer model inputs) where the i^{th} row contains specific values of each of the k variables to be used for the i^{th} test condition (or the i^{th} run of a computer model). See p. 1 of Reference 6 for a concrete example of how a Latin Hypercube sample is drawn. For the interested reader who desires practice in drawing a Latin Hypercube sample manually (i.e., without using the LHS program), pp. 31 - 42 of Reference 2 give clear step-by-step instructions, along with worksheets for a practice problem. The author has worked through this problem, and can vouch for its usefulness.

The Latin Hypercube sample in this thesis defines the set of altitudes and ambient temperatures to be used as test conditions. The off-design program OFFX, discussed below in Chapter III, was executed at each of these test conditions in order to obtain the engine thrust values.

LHS CODE FEATURES

References 2, 6, and 7 provide exhaustive documentation of the LHS code. The more salient features will now be discussed.

Parameter Distributions Allowed

The LHS code requires the user to specify a statistical distribution for each test parameter of interest (altitude and temperature in our instance). The next section of this chapter explains which distribution functions were chosen for our independent parameters. Another requirement is the specification of each parameter's lower and upper limits (i.e., the range). The choice of distribution has a direct effect on the parameter values that will be included in the Latin Hypercube sample, so it is a very important consideration. Later in this chapter, a justification is given for the distributions that were chosen to represent altitude and ambient temperature.

The parameter distributions allowed by LHS are: normal, lognormal, uniform, modified uniform (designated UNIFORM*), loguniform, triangular, and beta. In addition, the code allows for any user-supplied discrete (i.e., discontinuous) distribution. All distributions are illustrated and mathematically described in Figures 4 through 8.

Whenever a parameter distribution cannot be appropriately represented by standard distributions such as normal, lognormal, uniform, etc., then the beta distribution should be utilized. With the beta distribution, any continuous distribution for an independent parameter can be represented (see Figure 7). The beta distribution function equations listed in the LHS User's Guide (equations #3b and #4b on p. 23 of Reference 6) are as follows:

$$\mu = \frac{a \cdot q + b \cdot p}{p + q}$$

$$\upsilon = \frac{(b - a)^2 \cdot p \cdot q}{(p + q)^2 \cdot (p + q + 1)}$$

a = lower limit,
b = upper limit,
μ = arithmetic mean,
υ = variance = (std. dev.)^2
p and q are constants.

These are not the most useful forms of the Beta distribution equations. Values for a, b, μ, and υ are generally known, from which p and q are solved (two equations and two unknowns). The equations have been recast to form the following expressions for p and q:

$$\text{define } K = \frac{(\mu - a)}{(b - \mu)}$$

$$\text{then, } p = q \cdot K$$

$$\text{and } q = \frac{(b - a)^2 \cdot K - \upsilon \cdot (K + 1)^2}{\upsilon \cdot (K + 1)^3}$$

General Features

There is no limit placed on the sample size. Another feature of the code is its provision for an input correlation structure among the independent parameters, using the restricted pairing technique of Iman and Conover (pp. 311 - 334 of Reference 8). However, the code contains no provision for the initial generation of such parameter correlations from tabulated input data. That function can be performed using the STEP regression analysis code. For that reason, a detailed discussion of parameter correlation is presented in Chapter IV. LHS does, however, output the correlation matrix associated with each Latin Hypercube sample chosen.

If desired, multiple samples can be chosen within one LHS run. If several samples are specified, and if a parameter correlation matrix was input, then the "best" sample to use would be the one whose output correlation matrix most closely approximated the input matrix.

APPLICATION

In order to choose the commercial aircraft engine test conditions for the investigation of takeoff thrust lapse rates, each independent parameter must be defined in terms of its distribution function. The choice of a distribution function is an important part of the investigation. If an inappropriate distribution is haphazardly selected, a representative sample will not result. The loguniform distribution was chosen for altitude because more airports are located at lower altitudes (near sea-level, the lower limit of the distribution) than at higher altitudes (near 6000 ft., the upper limit of the distribution). A loguniform distribution emphasizes the lower values within its range (Figure 5). At sea-level the ambient pressure is 14.696 psi, and the value is 11.777 psi at 6000 feet (Reference 9). The normal distribution was chosen for ambient temperature because over vast ranges of land (latitudinal and longitudinal variation) and over long periods of time (any season of the year) the temperature at any altitude will be as likely to be cooler than normal as it is to be warmer than normal. Using a lower limit of -15 °F (445 °R) and an upper limit of 125 °F (585 °R) yields a distribution mean of 55 °F, which is a reasonable assumption for the global average yearly temperature. It is true that lower temperatures commonly occur at many airports, but using a lower interval minimum would skew the mean temperature toward a falsely low value. Besides, engine thrust will be lower at higher temperatures, so these are of generally greater interest. The source of engine data used for the thrust lapse rate investigation did not incorporate the controls system that is used on some engines with the intent of negating or limiting thrust lapse due to temperature effects.

The decision was made to ignore compressibility effects; in other words, to assume that engine inlet conditions were identical to ambient conditions. Otherwise, thrust is not constant during takeoff. As a measure of the inaccuracies this assumption induces, the following calculations were made, using a takeoff velocity of 150 mph at 70 °F:

$$v = 150 \text{ mph}$$

$$\text{therefore, } M = (150 \text{ mph})(88 \text{ fps}/60 \text{ mph})\Big/\left(49.01 \text{ fps}°\text{R}^{-1/2}\right)\sqrt{530\,°\text{R}} = 0.195$$

$$\frac{P_t}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma - 1)} = (1 + 0.2M^2)^{3.5} = 1.027$$

$$\frac{T_t}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right) = (1 + 0.2M^2) = 1.008$$

This gives a difference of only 0.8% in temperature and 2.7% in pressure, so the assumption is reasonable.

The LHS input file is listed in Figure 9, and the output file is depicted in Figure 10. The input file consists of the following items: a title card; a random seed; specification of the number of observations (NOBS, the sample size); the number of samples to be drawn (NREPS, the number of replications); the independent parameter ranges and distributions; and an output control card. For the purpose of brevity, the program output is shown for replication #15 only (the sample eventually selected as the best).

SAMPLE SIZE REQUIREMENTS

The question naturally arises: how large must a Latin Hypercube sample be in order to fully represent the parameter populations involved? A discussion of this subject may be found on p. 59 of Reference 2. Briefly, the answer presented there calls for a minimum number of runs (i.e., the sample size) equivalent to 25% more than the number of variables involved (note: this recommendation is based solely on the observations and experience of the Reference 2 author, not on theoretical considerations). Although this might suffice in instances where numerous independent parameters are involved (the code was originally intended for such applications), it would be inappropriate to the present application. For best results, the number of runs should be about two or three times the number of variables involved if time and money permit. It should be kept in mind, though, that one of the main advantages of the Latin Hypercube sampling procedure is that the number of runs (test conditions in our instance) can be much smaller than the number needed when using a purely random sampling scheme. Ten flight conditions were selected for this investigation. The reasons for selecting these ten conditions are discussed below.

RESULTS

Although minimal input is required from the user to execute the LHS code, it is not a blindly automatic process to select a good LHS sample. This is the reason behind the code's option to generate multiple samples during one execution period. It is always a good idea to request many samples and compare them. Fifteen LHS samples were analyzed during this investigation for the minimum, maximum, and mean values for each distribution parameter. These are summarized in Table 1. One of the desired attributes used to identify the "best" sample is a good range (i.e., one that includes values very near each endpoint). Another desired attribute is a representative mean value. This is guaranteed by definition for the normal distribution on temperature, but the mean of the loguniform altitude distribution exhibited some variation. Most of the mean altitudes of the LHS samples were less than the best estimate of the true mean altitude for airports. The sample must also pair the temperatures and pressures so that no physically unrealistic combination occurs. Also, the mean deviation of temperature from standard atmospheric values expected at the sample altitudes should be as close to zero as possible. When all these requirements were considered, sample #15 was found to be the best. Figure 10 (program output listing for the fifteenth sample only) gives the resulting Latin Hypercube sample. In this figure, X(1) denotes altitude and X(2) denotes temperature. The ranks of the sample input vectors are also listed in Figure 10. These represent ranking of the numerical values from smallest to largest, and are used with an optional feature of the LHS code called "rank regression". Rank regression can sometimes produce more accurate regression analyses if the parameter functions are known to be monotonic. It was not used in the current investigation.

Altitudes range from 2 ft. to 5992 ft. while the temperatures range from 459 °R to 580 °R (-1 °F to +120 °F). The mean altitude is 939 ft., and the mean temperature is 516 °R (+56 °F). As mentioned earlier in this chapter, this is the value used for the global average yearly temperature. Each of the ten flight conditions in the sample is a realistic pressure vs. temperature combination, and the mean temperature deviation from standard values was very close to zero (differed by only one degree!). Thus, it may be concluded that sample #15 is an excellent one to use. The set of temperatures and pressures in sample #15 will be those used to generate the engine thrust levels needed in the ensuing regression analysis.

III. ONX AND OFFX COMPUTER PROGRAMS

ONX and OFFX are personal computer-based codes (for IBM PC-type machines) written by faculty of the United States Air Force Academy. They give generic performance information for a wide variety of airbreathing engines, and were used to obtain the thrust performance data at the ambient conditions specified by the output of the LHS code. This chapter addresses the basic performance equations, the code features and options, and the results of the application to the thesis topic.

SUPPORTING THEORY

In Reference 10 the systematic process termed "cycle analysis" has been applied to several different engine types in order to obtain a collection of performance equations that are very useful for the design and analysis of airbreathing engines. This reference textbook gives a clear and concise description of both ideal and real cycle analysis, and is an excellent source for the equations describing a wide variety of engine cycles. The equations applicable for a turbofan with losses, and with the bypass ratio prescribed, may be found in Appendix C of this thesis.

These performance equations are encoded in a manner which allows the evaluation of gaspath parameters in a sequential manner from the engine inlet to the nozzle exhaust plane. The underlying thermodynamic cycle for this sequence is the open-ended Brayton cycle. A turbine/compressor work balance is maintained, which means the engine behavior is assumed to be steady-state (no transient or dynamic effects are included). The gaspath pressures and temperatures are evaluated through the use of one-dimensional compressible flow equations (with conservation of mass, momentum, and energy). Where appropriate, Rayleigh flow and fanno flow considerations are made (flow with heat addition and friction, respectively). Ultimately, ONX and OFFX are designed to calculate the engine gross thrust and specific fuel consumption.

ONX AND OFFX CODE FEATURES

The on-design-point code ONX will accomodate the following engine cycles:

turbojet (with and without afterburning)
turbofan with separate exhaust
turbofan with mixed exhaust (with and without afterburning)
turboprop
ramjet

It has the ability to automatically iterate on the following design parameters:

Mach number
combustor exit temperature
augmentor exit temperature
fan pressure ratio
compressor pressure ratio
bypass ratio

The program is interactive yet conveniently reads from and writes to external storage files. This avoids laborious re-specification of run parameters for each computer session. Also, the user has a choice of output device (screen and/or printer). Finally, ONX features the ability to communicate the design point conditions to the off-design-point code, OFFX.

Program ONX must be executed before OFFX can be used. After reading the ONX output file, OFFX will allow for automatic iteration on the following design parameters:

Mach number
altitude
ambient pressure
ambient temperature
combustor exit temperature
augmentor exit temperature (if applicable)
exhaust pressure ratio

Again, there is a choice of output device. OFFX produces steady-state performance of an engine operating at conditions different from its optimal, on-design operating condition.

Neither code has a graphics package. No simulated engine controls or component maps are included, so no information can be gained about such phenomena as compression system stall margin, combustor blowout/relight capability, or transient performance response. Because no engine controls or engine aging effects are included, the results obtained using this code will be unrealistically repeatable. Data scatter that might be expected during an experimental program will therefore be absent, and this will lead to a regression analysis with optimistic accuracy.

The user's manual, Reference 11, provides ample program documentation. Reference 12 is the formal textbook containing the aircraft engine design theory upon which ONX and OFFX are based.

APPLICATION

Due to the highly competitive nature of airbreathing propulsion manufacturing, there is no openly available source of commercial engine performance parameters, except for Jane's All the World's Aircraft Engines. Engine test data and detailed engine-specific cycle decks are closely guarded military and corporate treasures. For this reason, ONX and OFFX were used to obtain performance estimates. The engine cycle chosen was the high-bypass turbofan without afterburner, and with separate exhaust streams. This engine cycle represents the engines most commonly used on 747-class commercial aircraft.

ENGINE DESIGN POINT SPECIFICATION

Because the performance levels of thrust and specific fuel consumption will change radically from one flight condition to another, the design point for an airbreathing engine is naturally considered to be the condition at which it most frequently operates. Typically, long-haul commercial aircraft will cruise at, say, 39,000 ft. altitude, with an airspeed of about 550 miles per hour. At that altitude, the ambient temperature is 390 °R, so that the corresponding Mach number is:

$$M = (550)(88 \text{ fps}/60 \text{ mph})/49.01\sqrt{390\ ^{\circ}\text{R}} = 0.833$$

The turbomachinery design for the engine was chosen to be: fan pressure ratio, $\pi_{c'} = 1.4$; compressor pressure ratio, $\pi_c = 32.0$; and bypass ratio, $\beta = 5.8$. These values are quoted for a representative engine currently in use (Reference 10, p. 8). Table 2 is a listing of all other engine specifications needed to execute ONX and OFFX.

The ONX and OFFX computer codes assume conditions of steady one-dimensional flow (i.e., fixed throttle position, constant altitude and Mach number, and with no radial or circumferential flow variations included). They were written primarily as trend analysis tools (e. g., how will specific fuel consumption vary with compressor pressure ratio), and are intended to yield accurate estimates for gross thrust and specific fuel consumption.

RESULTS

Table 3 is a summary of the engine performance calculated by OFFX at the off-design aircraft takeoff conditions that comprise the LHS sample (i.e., those of Figure 10). The design value chosen for turbine inlet total temperature ratio, τ_λ ,was 2900/390 = 7.44. At the on-design point of 39K/0.833/390 °R, the thrust was 10,000 lbf. This represents the set value chosen by the user as a pre-specified design parameter. In the present thesis, this value was arbitrarily selected. In real aircraft design, however, the thrust requirement is set by the aircraft drag at steady cruising conditions, and by the number of engines on the aircraft. A predetermined value for thrust or airflow may be specified by the user.

Note the wide range of thrust values found in Table 3 (from 31931 lbf to 55494 lbf). This very desirable attribute is yet another sign that the sample set of test conditions chosen by the LHS program (Chapter II) was representative of a large range of engine operating conditions. If the range of takeoff thrust had instead been very small, it would have been necessary to start over from the beginning and consider an alternative LHS set of test conditions. Otherwise, our goal of obtaining an approximating function for thrust would have been compromised. A function derived from a narrow range of thrust would be a tool for data extrapolation beyond the test condition limits (risky!), rather than interpolation within the limits (safer).

IV. STEP COMPUTER PROGRAM

STEP is a VAX 11/780-based FORTRAN code written by Sandia National Laboratories personnel for multivariate regression analysis. It was used to analyze the engine takeoff thrust lapse data produced by the ONX and OFFX computer programs (described in Chapter III) at the simulated test conditions specified by the LHS program (described in Chapter II). This chapter describes these efforts, along with some supporting theory of regression analysis. Examples of similar investigations utilizing regression analyses were located in Reference 1 to lend credence to the use of STEP for airbreathing engine analysis.

SUPPORTING THEORY

Regression methods are used to identify or define the relationship between some dependent variable of interest (engine thrust in our instance) and one or more independent parameters that can be either directly observed (measured) or indirectly observed (calculated). The method employed by STEP is the method of least squares, probably the most popular of all methods today. It is mathematically described as follows (Reference 13, p. 364):

The general problem of fitting the best least-squares line to a collection of data involves minimizing $\sum_{i=1}^{n} [y_i - (a x_i + b)]^2$ with respect to the parameters a and b. For a minimum to occur, it is necessary that

$$0 = \frac{\partial}{\partial a} \sum_{i=1}^{n} [y_i - (a x_i + b)]^2 = 2 \sum_{i=1}^{n} (y_i - a x_i - b)(-x_i)$$

and

$$0 = \frac{\partial}{\partial b} \sum_{i=1}^{n} (y_i - a x_i - b)^2 = 2 \sum_{i=1}^{n} (y_i - a x_i - b)(-1).$$

These equations simplify to what are known as the normal equations:

$$a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i \quad \text{and} \quad a\sum_{i=1}^{n} x_i + b\cdot n = \sum_{i=1}^{n} y_i .$$

The solution to this system of equations is

$$a = \frac{n\left(\sum_{i=1}^{n} x_i y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$b = \frac{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i\right) - \left(\sum_{i=1}^{n} x_i y_i\right)\left(\sum_{i=1}^{n} x_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2} .$$

The general problem of approximating a set of data, $\{(x_i, y_i) | i = 0, 1, ..., M\}$ with a polynomial $P_n(x) = \sum_{k=0}^{n} a_k x^k$ of degree $n < M$ using the least-squares procedure is handled in a similar manner and requires choosing the constants $a_0, a_1, ..., a_n$ to minimize the least-squares error

$$\begin{aligned}
E &= \sum_{i=0}^{M} (y_i - P_n(x_i))^2 \\
&= \sum_{i=0}^{M} y_i^2 - 2\sum_{i=0}^{M} P_n(x_i) y_i + \sum_{i=0}^{M} (P_n(x_i))^2 \\
&= \sum_{i=0}^{M} y_i^2 - 2\sum_{i=0}^{M} \left(\sum_{j=0}^{n} a_j x_i^j\right) y_i + \sum_{i=0}^{M} \left(\sum_{j=0}^{n} a_j x_i^j\right)^2 \\
&= \sum_{i=0}^{M} y_i^2 - 2\sum_{j=0}^{n} a_j \left(\sum_{i=0}^{M} y_i x_i^j\right) + \sum_{j=0}^{n}\sum_{k=0}^{n} a_j a_k \left(\sum_{i=0}^{M} x_i^{j+k}\right).
\end{aligned}$$

As in the linear case, for E to be minimized, it is necessary that $\partial E / \partial a_j = 0$ for each $j = 0, 1, \ldots, n$. Thus, for each j,

$$0 = \frac{\partial E}{\partial a_j} = -2 \sum_{i=0}^{M} y_i x_i^j + 2 \sum_{k=0}^{n} a_k \sum_{i=0}^{M} x_i^{j+k}.$$

This gives $n+1$ equations in the $n+1$ unknowns, a_j, called the normal equations,

$$\sum_{k=0}^{n} a_k \sum_{i=0}^{M} x_i^{j+k} = \sum_{i=0}^{M} y_i x_i^j, \qquad j = 0, 1, \ldots, n.$$

It is helpful to write the equations as follows:

$$a_0 \sum_{i=0}^{M} x_i^0 + a_1 \sum_{i=0}^{M} x_i^1 + a_2 \sum_{i=0}^{M} x_i^2 + \cdots + a_n \sum_{i=0}^{M} x_i^n = \sum_{i=0}^{M} y_i x_i^0$$

$$a_0 \sum_{i=0}^{M} x_i^1 + a_1 \sum_{i=0}^{M} x_i^2 + a_2 \sum_{i=0}^{M} x_i^3 + \cdots + a_n \sum_{i=0}^{M} x_i^{n+1} = \sum_{i=0}^{M} y_i x_i^1$$

$$\vdots$$

$$a_0 \sum_{i=0}^{M} x_i^n + a_1 \sum_{i=0}^{M} x_i^{n+1} + a_2 \sum_{i=0}^{M} x_i^{n+2} + \cdots + a_n \sum_{i=0}^{M} x_i^{2n} = \sum_{i=0}^{M} y_i x_i^n$$

It can be shown that the normal equtions always have a unique solution provided that the x_i for $i = 0, 1, \ldots, M$, are distinct (i.e., no repeated values).

STEP uses three separate multiple regression procedures. These are best explained in the following description (adapted from Reference 2, p. 89):

The forward procedure: The regression model begins with the first variable listed. Variables are then added to the regression model one at a time until a decision is reached that no new variables will contribute significantly to the improvement of the fit of the model to the data. The input file contains a constant named SIGIN, which controls whether or not a proposed independent parameter should be added to the list of parameters already in the model. (In this thesis, the value chosen for SIGIN was 0.05, meaning that any proposed parameter would be included in the regression model parameter list only if its inclusion would further reduce the sums of squares of the residuals by at least 5%.)

The backward procedure: The regression model starts with all of the variables in it. Then variables which are considered not making a significant contribution to the fit are dropped one by one until all insignificant variables have been removed from the model. In a manner similar to the SIGIN constant mentioned in the forward procedure, another control constant named SIGOUT is used to decide whether or not a particular independent parameter should be deleted from the list of model parameters. (In this thesis, the value chosen for SIGOUT was 0.10, meaning that any parameter in question would be deleted from the regression model parameter list only if its absence would further reduce the sums of squares of the residuals by at least 10%.)

The stepwise procedure: The regression model begins with the first variable listed. Variables are then added, one at a time, as in the forward procedure. Each time a variable is added, the backward procedure is used to delete all variables previously added to the model but which are now considered to be insignificant in the presence of the new variable. This combination of the forward and backward procedures is the most commonly used method of determining which set of variables belongs in the model. Both SIGIN and SIGOUT (discussed in the forward and backward procedures) are used in the stepwise procedure.

Reference 14 discusses these procedures in more detail.

STEP CODE FEATURES

Reference 15 was written by the program authors, and gives definitive documentation. Certain applicable code features will now be discussed.

General Features

STEP allows for the use of up to 179 independent parameters, and will automatically retain only those variables found to contribute significantly to the regression. A parameter codenamed PRESS (PRedicted Error Sum of Squares), taken from Reference 16, is used to determine the adequacy of a prediction model. If a regression model has k variables and n observations PRESS is computed by first deleting the i^{th} observation (i = 1, 2, 3, ..., n) from the original set of observations, and then constructing a regression model from the remaining n - 1 observations. Using this model, define the value of $\widehat{Y}_k(i)$ as the estimate for the deleted observed value of Y_k. Then:

$$\mathrm{PRESS}_k = \sum_{i=1} \left(Y_i - \widehat{Y}_{k(i)}\right)^2$$

If PRESS is small, the fit will not be significantly changed by the addition of a new observation (i.e., taking another data point). In general, the regression model having the smallest PRESS value is preferred when choosing between competing models (Reference 15, p. 9).

The STEP program user's manuals mention the program's capability to automatically compute, from input file specifications, new independent parameters that are transformations of other input parameters (limited to simple products, quotients, differences, and/or sums). Although this feature was not necessary to conduct the thesis investigation, an unrelated attempt was made to verify the capability. The subsequent investigation revealed that it did not work properly. In an effort to eliminate this shortcoming, the thesis author conducted a telephone conversation in January, 1990 with the principal author of the original Department of Energy source code. It was concluded that the actual FORTRAN calculations must be placed within subroutine TRANS. This is the only way to avoid manually calculating the new terms and listing them explicitly in the input file. Of course this necessitates the disadvantage of a re-compilation of the source code. On the other hand, one big advantage to using subroutine TRANS is its ability to formulate a wider range of more complicated independent parameter terms.

Parameter Correlations

Although the primary feature used in the current investigation was regression analysis, STEP may also be used to identify which input variables are the most influential on the dependent parameter of interest. One potential use of this feature would be to investigate the relative effects on specific fuel consumption (SFC) brought about by perturbations (during engine testing) in key engine design parameters whose control schedules can be manually altered, using a special "breadboard" control arrangement. Analysis of STEP's parameter correlation matrix, using data obtained from such testing, would reveal which program control parameters were most closely interrelated with SFC. This procedure is known as sensitivity analysis.

The strength of a simple linear relationship between two variables is usually measured using Pearson's product moment correlation coefficient (or, the "correlation coefficient"). The correlation coefficient between two parameters, x and y, may be defined as (Reference 2, p. 86):

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \cdot \sum (y_i - \bar{y})^2}}$$

for paired observations $(x_1,y_1), (x_2,y_2), \ldots, (x_n,y_n)$, where $\bar{x}$ and $\bar{y}$ are the sample means. A correlation coefficient should not be confused with a partial deriviative. For the situation involving the correlation of two independent parameters, as opposed to the above correlation between the dependent parameter and one independent parameter, subscripts are used to notate which variables are being correlated. For example, r_{12} refers to the correlation between independent parameter 1 and independent parameter 2; and is defined as:

$$r_{12} = \frac{\sum (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2)}{\sqrt{\sum (x_{1i} - \bar{x}_1)^2 \cdot \sum (x_{2i} - \bar{x}_2)^2}}$$

Sometimes the apparent correlation between two variables may be due in part to an indirect influence they each experience from a third variable. STEP can be used to calculate such partial correlation coefficients; the interested reader should consult pp. 87 - 89 of Reference 2.

The program utilizes partial correlation coefficients as part of the procedure for successively adding independent parameters to the regression model. The largest partial correlation coefficient is found, and that variable is examined using a test of significance (whose magnitude is conveniently under the user's control) to see if including it in the model reduces the variation of the dependent parameter by a significant amount. If it is judged to be insignificant, it is not included in the model.

Program Checkout

Before the program was used, a quick checkout was performed to investigate one of its claimed capabilities. This checkout merely involved the selection of a sample polynomial function, and the use of a programmable calculator to formulate an input file containing tabulated data for that function. In addition, some unrelated data was intentionally included in the input file for "bogus" or "dummy" mathematical terms that did not form a part of the sample polynomial. When STEP was applied to this simple checkout problem, it correctly calculated the coefficients of all pertinent terms, and managed to successfully ignore each bogus term. This very important result provided clear proof that STEP does indeed possess the ability to permit the simultaneous inclusion of many potentially useful independent parameters (even those based on a hunch or a guess!) without fear of minimizing or cancelling the truly important independent parameters. Its potential as a regression analysis tool is thereby greatly enhanced.

One potential limitation discovered is that STEP permits one and only one form of the equation used to execute the data regression. This form is described by:

$$Y = C_1 \cdot [\text{term 1}] + C_2 \cdot [\text{term 2}] + \ldots + C_n \cdot [\text{term n}] + K$$

where Y is the dependent parameter, C_n values are constant coefficients, [term n] is any algebraic expression for the independent parameters, and K is the intercept constant. Note that this required form of independent parameter expression implies that all constants determined by the regression must be expressed as term coefficients (i.e., they cannot be embedded within an algebraic expression, such as $\sin\{\Theta + C_1 \cdot x\}$. This inconvenience might be a serious limitation in certain circumstances, but it can be circumvented via the use of a separate computer analytical tool -- EUREKA, a product of Borland software (Reference 17). EUREKA will solve for any unknown constant, regardless of where it appears in an expression. Furthermore, it does not demand that the expression be recast (rewritten to isolate the parameter of interest). EUREKA was not actually used in conjunction with the current effort, however, so no further discussion is provided. This information is included merely for the reader's benefit.

APPLICATION

It is a justifiable and common engineering practice to express certain airbreathing engine design parameters as a polynomial function of other engine variables. For example, SAE Committee E-33 (in-flight propulsion measurement) published an official document (Reference 1, p. 406) containing numerous instances of this. One such instance was:

$$z = A_0 + A_1 x + A_2 x^2 + B_1 y + B_2 y^2$$

with x, y, and z representing nozzle pressure ratio, ram pressure ratio, and nozzle velocity coefficient, respectively.

In our instance, to obtain thrust, τ, as a function of pressure and temperature, it was recognized that $\tau \propto \dot{m}\ (\Delta v)$, so $\tau = f(\dot{m}) = f(\rho A\ v)$

For a given engine size or cross-sectional area, then, $\tau = f(\rho\ v)$

From the perfect gas equation of state: $P = \rho RT$; $\rho = f(P/T)$.

Also, from the Mach number / velocity relation for air, $v = 49.01(M)(T)^{1/2}$.

Combining the function of density and the function of velocity, one gets $\tau = f(P/(T)^{1/2})$, or in terms of nondimensional parameters δ and Θ

$$\tau = f(\delta, (\Theta)^{-1/2}).$$

Here, the nondimensional pressure is $\delta = (P/14.696)$, and the nondimensional temperature is $\Theta = (T/518.67)$.

Thus, the input file to STEP for a first-order regression contains tabulations of τ, δ, and $(\Theta)^{-1/2}$ that represent the conditions for thrust lapse rates that are to be examined by regression analysis; so that

$$\tau = A\delta + B(\Theta)^{-1/2} + C.$$

An alternate approach would be the second-order form

$$\tau = A\delta + B(\Theta)^{-1/2} + C\delta^2 + D(\Theta)^{-1} + E.$$

In our instance, the pressures and temperatures used were those output from program OFFX (chapter III) at the ambient conditions specified by the Latin Hypercube sample (chapter II). When used as a regression analysis parameter, the combined parameter $\delta(\Theta)^{-1/2}$ gave unacceptably large residuals.

Although the second-order form with constant coefficients proved to be effective in this investigation (see discussion below), sometimes it becomes necessary to use non-constant coefficients that are themselves functions of one or more independent parameters. Although there is no automatic provision for this in the program, an illustration of how this may be done is:

given: $z = ax^2 + bx + c$

where $a = f_1(y)$; $b = f_2(y)$; $c = f_3(y)$

then $z = f(x,y)$.

Substituting, $z = f_1(y)x^2 + f_2(y)x + f_3(y)C$.

How shall f_1, f_2, and f_3 be determined? First, use STEP to find the coefficients a, b, c, for each individual curve one at a time to obtain

$$
\begin{aligned}
y_1 &= a_1x^2 + b_1x + c_1 \\
y_2 &= a_2x^2 + b_2x + c_2 \\
&\vdots \\
y_n &= a_nx^2 + b_nx + c_n
\end{aligned}
$$

Then use STEP once again in the same manner to obtain

$$
\begin{aligned}
a &= f_1(y) = \alpha_1 y^2 + \beta_1 y + \delta_1 \\
b &= f_2(y) = \alpha_2 y^2 + \beta_2 y + \delta_2 \\
c &= f_3(y) = \alpha_3 y^2 + \beta_3 y + \delta_3
\end{aligned}
$$

Finally,

$$z = (\alpha_1 y^2 + \beta_1 y + \delta_1)x^2 \; + \; (\alpha_2 y^2 + \beta_2 y + \delta_2)x \; + \; (\alpha_3 y^2 + \beta_3 y + \delta_3)$$

REGRESSION ANALYSIS ACCURACY

Because test measurements are not exact, regression models based on test data will also be inexact. Ground-testing community measurement uncertainties for engine inlet pressures and temperatures are known typically to within a few percentage points of nominal values. The only other error inherent to the regression-based model would come from computer roundoff error, which can be minimized by specifying double precision computer arithmetic. The DEC VAX family of computers even offers the option of quadruple precision computer arithmetic!

RESULTS

Since the regression analysis produces the function that approximates engine thrust, the output of the STEP program represents the final result of the investigation. One option is to choose an approximating function based on ambient conditions themselves as independent parameters. This would be a perfectly valid approach if engine performance was always nominal. However, if reduced performance was to be expected for any reason (e. g., engine aging effects, or even a potential bird strike), then an approximating function based on ambient conditions alone would be inappropriate. In that case, a better choice of a set of independent parameters would be to include the engine's fan pressure ratio and fan temperature ratio. To provide a more complete analysis for this investigation, both types of takeoff thrust functions were obtained by using two different input files to the STEP regression analysis program.

When ambient conditions were used as the only independent parameters, the following approximating polynomial resulted:

$$\tau = 94{,}755(\delta) - 504{,}941(\Theta)^{-1/2} - 27{,}588(\delta)^2 + 351{,}799(\Theta)^{-1} + 127{,}698\ .$$

The STEP program tabulates the numerical differences between the "observed" values of thrust (the actual input value) and the "predicted" value (the value that is obtained using the approximating function). These differences are termed "residuals". The largest percentage residual obtained for the ten input conditions was a scant 0.12% when using ambient conditions as independent parameters. In other words, an engine's takeoff thrust could be predicted at any one of the test conditions to within less than two-tenths of one percent if nominal operation could be correctly assumed, and if the ambient pressure and temperature could be measured with complete accuracy.

However, because off-nominal engine performance is sometimes encountered, the better choice of independent parameters is the use of fan outlet pressure and temperature. The presence of certain abnormal engine operating conditions will be reflected in lower values for these parameters (e. g., a partially lit combustor, or fan blade damage resulting from a bird strike or other foreign object ingestion). When fan outlet conditions were used as the independent parameters, the following approximating polynomial resulted:

$$\tau = 68{,}281(\delta) - 834{,}347(\Theta)^{-1/2} - 14{,}351(\delta)^2 + 542{,}478(\Theta)^{-1} + 279{,}454\,.$$

The input file to STEP for this case is shown in Figure 11. Note that the MODEL card is used to relate the independent parameters and the dependent parameter. The resulting output file tabulation of observed (input) thrust values, predicted (calculated) thrust values, and their differences (the residuals) are listed in Table 4. Manually calculated residual percentages are also given in this table. The largest percentage thrust residual encountered for the ten specified fan outlet conditions was only 0.18%. Thus, very little accuracy of fit was sacrificed by choosing an approximation based on nondimensional fan outlet conditions instead of one based on nondimensional ambient conditions. A checkout evaluation of this thrust function was performed at randomly chosen conditions different from those that comprised the LHS set. This checkout revealed accuracies no worse than 4.0%, which occurred at 2,500 ft/32 °F. The accuracy associated with a "high and hot" day (i.e., one that tends to minimize takeoff thrust) was only 1.5% (at 5,280 ft./100 °F).

As mentioned earlier, part of the reason that such an accurate fit was possible in this instance is that the thrust values were obtained from the ONX/OFFX cycle decks instead of a real engine experimental program. In other words, it would be unreasonable to expect this level of accuracy when using actual experimental test data. Nonetheless, the method of analysis upon which this thesis is based remains quite sound.

An inaccurate measurement of independent parameter values will be reflected in the approximating polynomial. Normally, fan outlet conditions can be measured very accurately with current instrumentation technology; therefore, these small inaccuracies are not considered a hindrance to this type of analysis for takeoff thrust prediction.

One final comment about the choice of independent parameters is that using more detailed information about engine internal behavior will usually lead to a better (more accurate and more thorough) model for thrust, or for any other dependent parameter of interest. For example, if information were available on rotor speeds, turbine inlet temperature, nozzle area, etc., then inclusion of these in the list of independent parameters could only increase the chances of obtaining a useful regression analysis. In our case, the throttle was assumed to be at a fixed position (hence, turbine inlet temperature was constant), and no information was available from OFFX for parameters such as rotor speeds or nozzle area.

V. SUMMARY

Four unclassified computer programs were used to construct two regression models of takeoff thrust lapse. One model was a function of nondimensional ambient conditions, and the other was based on nondimensional fan exit conditions. The thrust system under consideration was the high thrust, high-bypass nonafterburning turbofan with unmixed exhaust (chosen because it is the most common type of powerplant installed in 747-class commercial aircraft). A Latin Hypercube sample set of ambient conditions was obtained for airport elevations that ranged from sea-level to 6000 ft., with the ambient temperatures ranging from -15 °F to +125 °F.

It is important to realize that the selection of a Latin Hypercube sample (as opposed to a purely random sample) permits the legitimate use of a smaller set of conditions, thereby helping to control the cost of any experimental or computer-based analysis.

CONCLUSIONS

The statistical sampling code that was employed to define the engine test conditions proved capable of providing Latin Hypercube samples that were thoroughly representative of the altitude and temperature distributions. Latin Hypercube samples are superior to simple random samples because they permit smaller samples (hence, lower testing costs) without sacrificing accuracy. Execution of the on-design-point (39K/0.833/390 °R) and off-design-point cycle analysis codes successfully yielded engine performance data at each of the conditions in the sample. These codes were utilized in lieu of actual commercial engine test data because such data is generally proprietary. Fortunately this did not impact the validity of the investigation in any way. The ensuing regression analysis based on minimal input data sets produced a curvefit of engine takeoff thrust that was accurate to within a remarkable ±0.18% at all conditions calculated within the regression. A checkout evaluation of this thrust function at randomly extrapolated conditions revealed an error no worse than 4.0%.

RECOMMENDATIONS

If the method of takeoff thrust prediction used in this thesis is actually employed in practice, it would be advisable to use a different distribution for altitude. The loguniform distribution should be replaced with a beta distribution calculated from an accurate compilation of the elevations at every prospective airport for which the model is intended. The form of the equations derived in Chapter II may be used for calculating the necessary beta distribution constants.

Also, if computer model engine performance predictions are to be used in lieu of actual test data, then programs ONX and OFFX should be abandoned in favor of an engine-specific performance deck supplied by the manufacturer.

LIST OF REFERENCES

1. "In-Flight Thrust Determination and Uncertainty". SAE Publication SP-674, Aerospace Technology Conference and Exposition, Long Beach, California, October 13-16, 1986.

2. Iman, Ronald L., and Conover, W. J., "Sensitivity-Analysis Techniques: Self-Teaching Curriculum". Sandia National Laboratories SAND81-1978; Nuclear Regulatory Commission NUREG/CR-2350, June 1982.

3. Canavos, George C., "Criteria for the Optimal Design of Experimental Tests". NASA TM X-2663, November 1972.

4. Box, G. E. P., and Wilson, K. B., "On the Experimental Attainment of Optimum Conditions". Journal of the Royal Statistical Society, Series B, Volume 13, 1951.

5. Lipson, Charles, and Sheth, Narendra J., Statistical Design and Analysis of Engineering Experiments. McGraw-Hill Book Company, New York, 1973.

6. Iman, Ronald L., and Shortencarier, Michael J., "A FORTRAN 77 Program and User's Guide for the Generation of Latin Hypercube and Random Samples for Use with Computer Models". Sandia National Laboratories SAND83-2365; Nuclear Regulatory Commission NUREG/CR-3624, March 1984.

7. Iman, Ronald L., Davenport, James M., and Zeigler, Diane K., "Latin Hypercube Sampling (Program User's Guide)". Sandia National Laboratories SAND79-1473, January 1980.

8. Iman, Ronald L., and Conover, W. J., "A Distribution-Free Approach to Inducing Rank Correlation Among Input Variables". Communications in Statistics, B11 (3), 1982.

9. U. S. Standard Atmosphere, 1976. Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., October 1976.

10. Oates, Gordon C., Aerothermodynamics of Gas Turbine and Rocket Propulsion (Revised and Enlarged). American Institute of Aeronautics and Astronautics, Inc., Washington, D. C., 1988.

11. Mattingly, Jack D., "On-design and Off-design Cycle Analysis Computer Programs". American Institute of Aeronautics and Astronautics, Inc., Washington, D. C., January 1987.

12. Mattingly, Jack D., Heiser, William H., and Daley, Daniel H., Aircraft Engine Design. American Institute of Aeronautics and Astronautics, Inc., Washington, D. C., 1987.

13. Burden, Richard L., and Faires, J. Douglas, Numerical Analysis. Third edition, Prindle, Weber, & Schmidt, Boston, 1985.

14. Draper, N. R., and Smith, H., Applied Regression Analysis. Second edition, John Wiley & Sons, Inc., New York, 1981.

15. Iman, Ronald L., et al., "Stepwise Regression with PRESS and Rank Regression (Program User's Guide)". Sandia National Laboratories SAND79-1472, January 1980.

16. Allen, D. M., "The Prediction Sum of Squares as a Criterion for Selecting Predictor Variables". Technical Report #23, Department of Statistics, University of Kentucky, 1971.

17. "EUREKA: The Solver (Owner's Handbook)". ISBN 0-87524-156-5, Borland International Software, Scotts Valley, California, 1987.

APPENDICES

APPENDIX A

TABLES

Table 1. Summary of the Fifteen Candidate Latin Hypercube Samples

Sample #	Altitude, ft. (1 - 6000) min/max/mean	Temperature, R (445 - 585) min/max/mean
1	1/3449/563	473/552/514
2	1/3022/569	478/545/514
3	1/4129/765	464/553/514
4	1/5389/868	460/547/514
5	2/3927/708	485/550/516
6	1/3166/705	479/548/514
7	2/5059/801	468/555/515
8	2/4993/748	478/573/516
9	1/4613/779	477/560/516
10	2/3072/625	457/547/512
11	1/4922/715	478/554/515
12	2/2932/589	480/570/517
13	2/4412/700	480/553/515
14	2/2572/565	480/554/515
15 (*)	2/5992/939	459/580/516

* Sample # 15 is the set of test conditions that was selected for input to program OFFX.

Table 2. Input File for ONX Program Execution

Input Parameter	Value
MACH NUMBER	0.833
ALTITUDE	39000.0 ft.
AMBIENT VALUES:	
TEMPERATURE	390.0 deg. R
PRESSURE	2.864 psia
DENSITY	0.0006161 slug/ft**3
CP C	0.2380 btu/lbm-R
GAMMA C	1.40
CP T	0.2950 btu/lbm-R
GAMMA T	1.30
FUEL HEATING VALUE	18000.0 btu/lbm
MAX TEMP (TT4) LEAVING COMBUSTOR	2900.0 deg. R
BLEED AIR FLOW	1.0 %
COOLING AIR FLOW	5.0 %
POWER TAKE-OFF (CTO)	0.010
PI DIFFUSER (MAX)	0.970
PI BURNER	0.970
PI NOZZLE	0.980
POLYTROPIC EFFICIENCIES:	
LP COMPRESSOR (FAN), EC'	0.890
HP COMPRESSOR, ECH	0.900
HP TURBINE, ETH	0.890
LP TURBINE, ETL	0.910
COMPONENT EFFICIENCIES:	
BURNER	0.980
MECHANICAL (HIGH PRESSURE SPOOL)	0.980
MECHANICAL (LOW PRESSURE SPOOL)	0.990
MECHANICAL (POWER TAKE-OFF)	0.980
P0/P9	1.000
MACH NUMBER AT STATION # 5	0.400
DESIGN CONDITIONS:	
PI C (COMPRESSOR)	32.00
PI C' (FAN)	1.40
BYPASS RATIO (ALPHA or BETA)	5.80

Table 3. Engine Off-Design Performance at Aircraft Takeoff Conditions Specified by the Latin Hypercube Sample

data point	altitude (ft.)	ambient press. (psia)	ambient temp. (deg. R)	bypass ratio	SFC	thrust (lbf)
1	7.	14.692	580.	6.44	0.5158	31931.
2	228.	14.577	511.	5.97	0.5101	42881.
3	5.	14.693	498.	5.86	0.5097	45953.
4	2.	14.695	522.	6.05	0.5104	41103.
5	5992.	11.782	506.	5.91	0.5119	35326.
6	40.	14.675	492.	5.82	0.5097	47230.
7	28.	14.681	534.	6.14	0.5111	38902.
8	2375.	13.480	520.	6.03	0.5111	37985.
9	601.	14.382	538.	6.16	0.5116	37419.
10	116.	14.635	459.	5.53	0.5107	55494.

Table 4. Comparison of Actual and Predicted Thrust Values

data point	observed value	predicted value	residual (lbf)	residual (%)
1	31931.0	31937.0	-6.0	-0.02
2	42881.0	42903.4	-22.4	-0.05
3	45953.0	45955.0	-2.0	-0.004
4	41103.0	41125.7	-22.7	-0.06
5	35326.0	35324.9	1.1	0.003
6	47230.0	47201.1	28.9	0.06
7	38902.0	38926.2	-24.2	-0.06
8	37985.0	38002.9	-17.9	-0.05
9	37419.0	37351.7	67.3	0.18
10	55494.0	55496.1	-2.1	-0.004

APPENDIX B

FIGURES

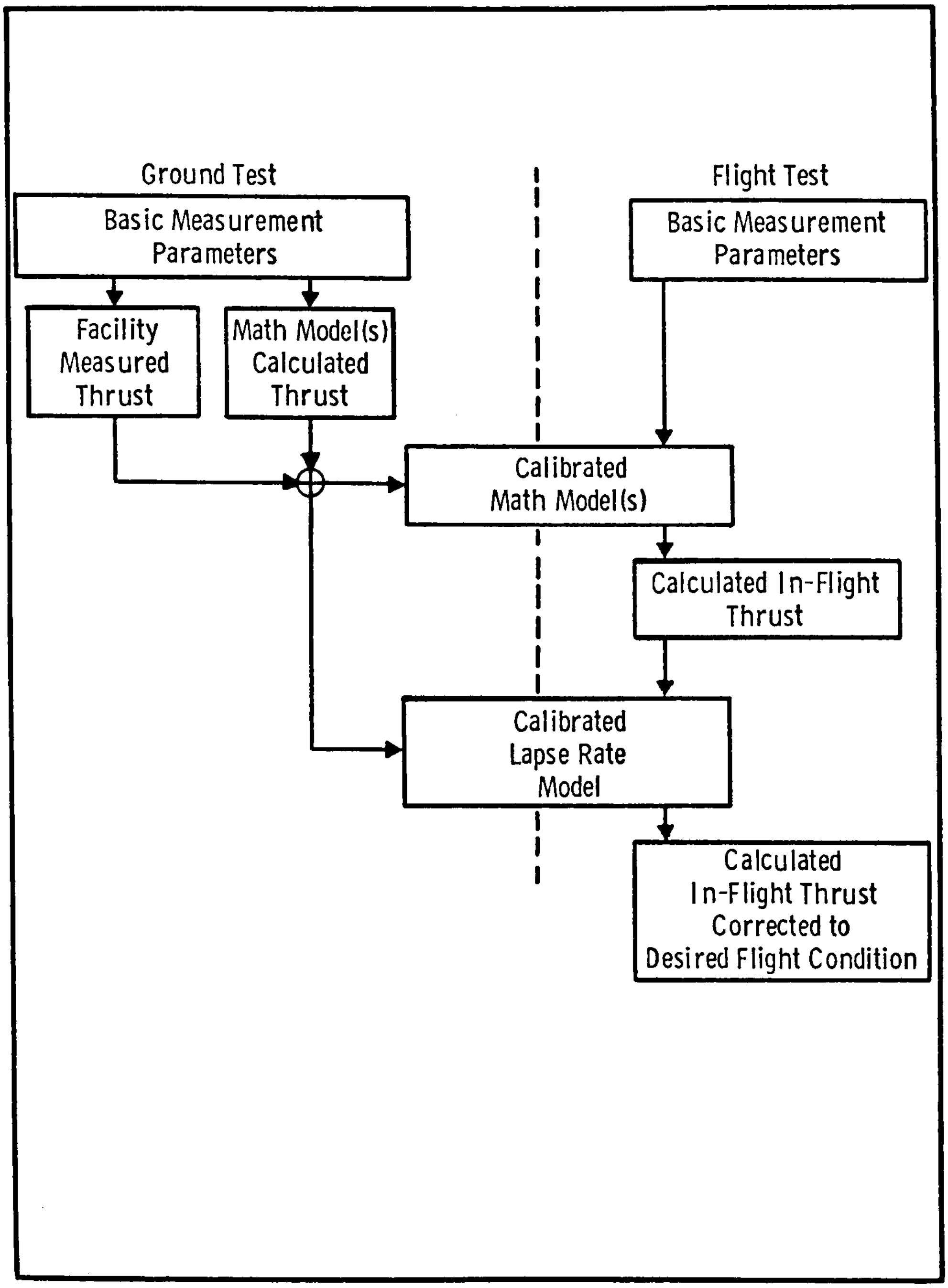


Figure 1. A Typical Use for Thrust Lapse Rate Models

- o UTILIZE A STATISTICAL SAMPLING CODE TO OBTAIN A LATIN HYPERCUBE SAMPLE OF TEST CONDITIONS (ELEVATION AND AMBIENT TEMPERATURE)
- o OBTAIN DATA AT THESE CONDITIONS USING AN ENGINE CYCLE DECK
- o PERFORM MULTIVARIATE REGRESSION ANALYSIS TO OBTAIN THE EMPIRICAL THRUST LAPSE RATE FUNCTION

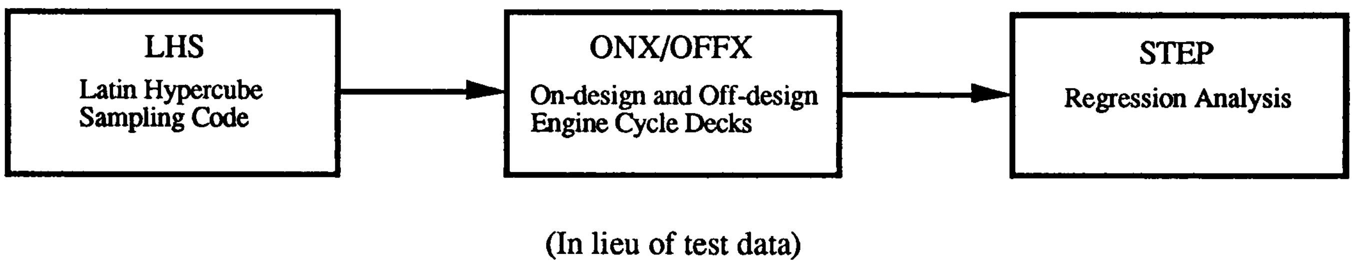


Figure 2. Solution Approach

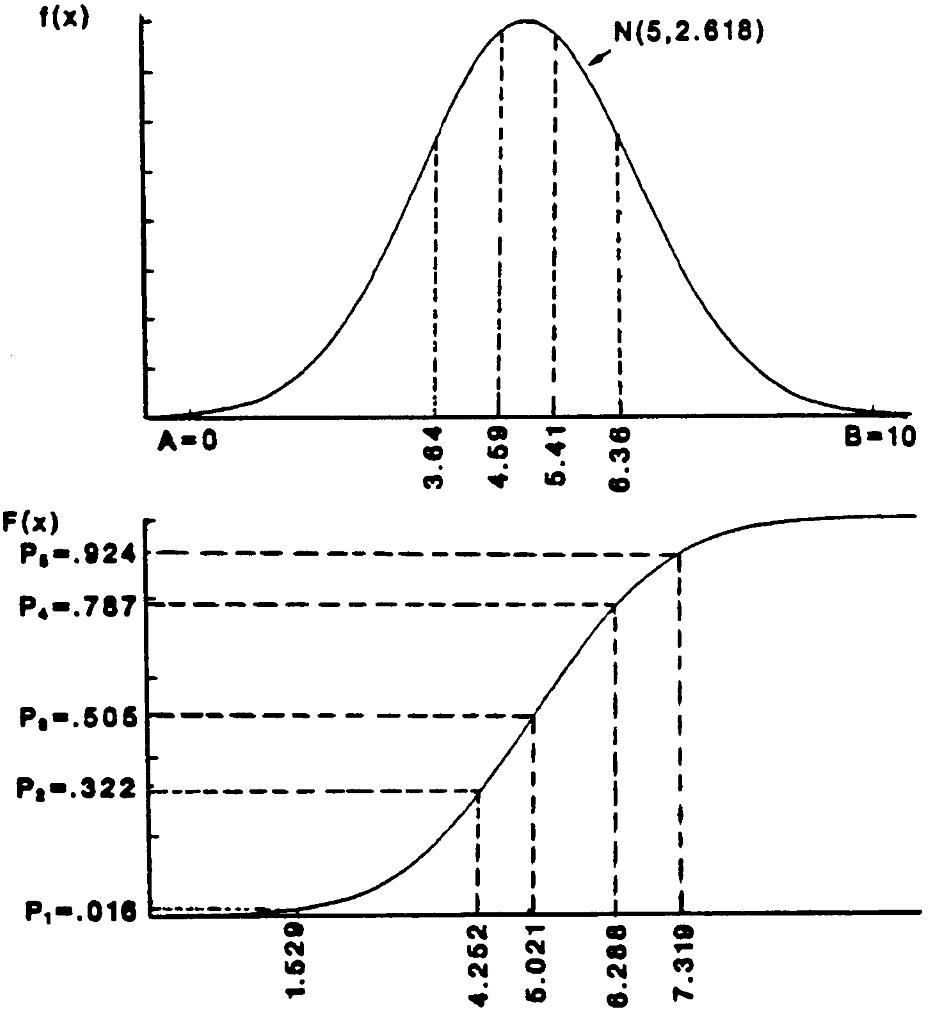
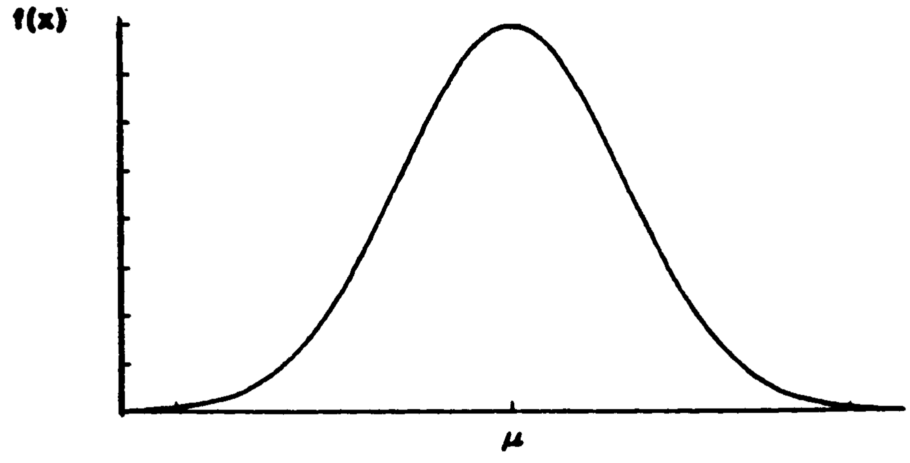


Figure 3. Intervals Used with a LHS of Size n=5 in Terms of the Density Function and Cumulative Distribution Function for a Normal Random Variable

Normal Distribution

$f(x) = 1/(\sigma\sqrt{2\pi})\ \exp\{-(x-\mu)^2/2\sigma^2\}$



Lognormal Distribution

$f(x) = 1/(x\sigma\sqrt{2\pi})\ \exp\{-(\ln x - \mu)^2/2\sigma^2\}$

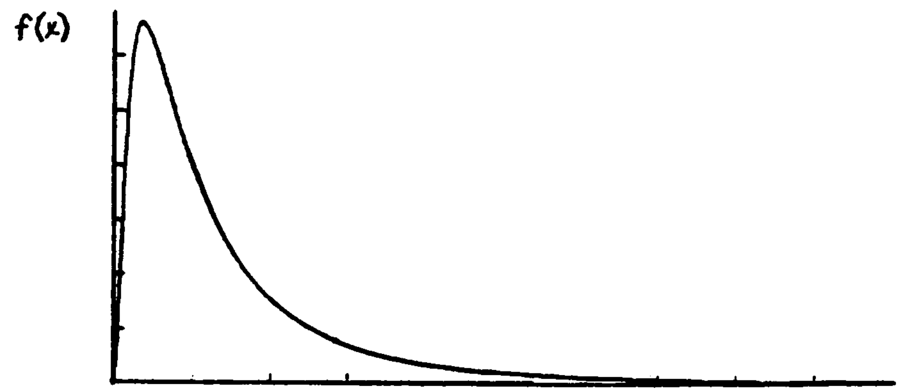


Figure 4. Definition of Normal and Lognormal Distributions

Uniform Distribution

$f(x) = \text{constant}$

Modified Uniform Distribution

$f(x)$ from a histogram

Loguniform Distribution

$f(x) = 1/x \ \ (\ln B - \ln A)$

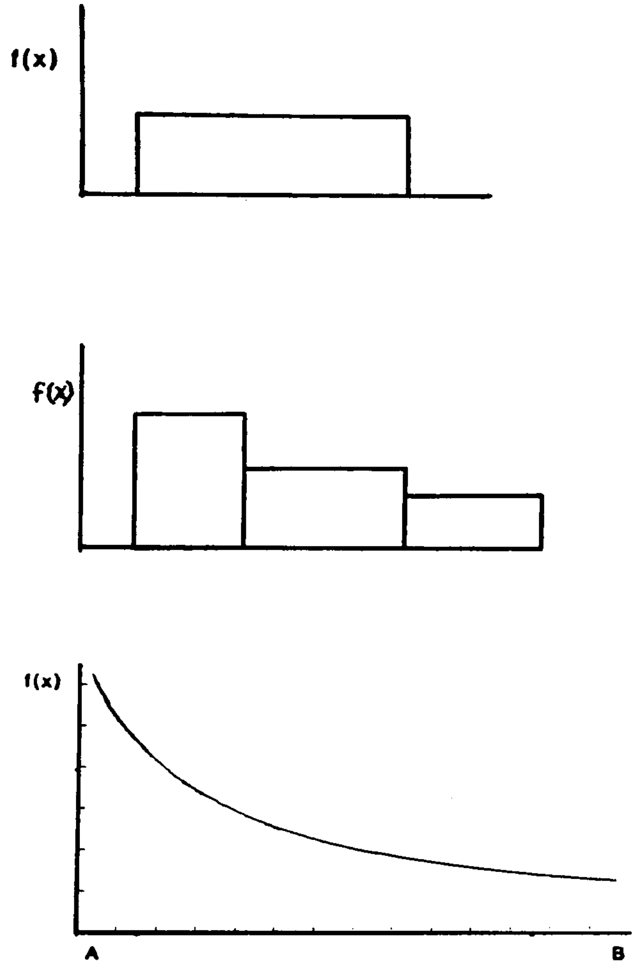


Figure 5. Definition of Uniform, Modified Uniform, and Loguniform Distributions

Triangular Distribution

$$f(x) = \frac{2(x-a)}{(c-a)\,(b-a)} \quad ; \ a \leq x \leq b$$

$$f(x) = \frac{2(c-x)}{(c-a)\,(c-b)} \quad ; \ b \leq x \leq c$$

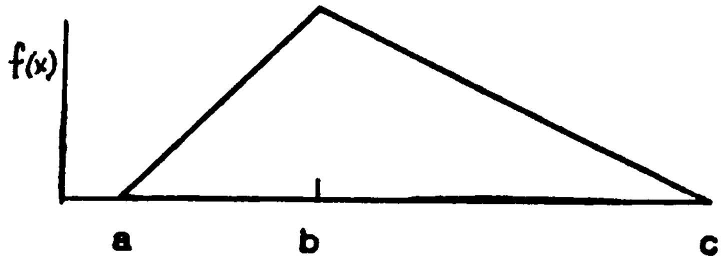


Figure 6. Definition of Triangular Distribution

<u>Beta Distribution</u>

$$\text{define } K = \frac{(\mu - a)}{(b - \mu)}$$

$$\text{then, } p = q^{*}K$$

$$\text{and } q = \frac{(b-a)^{2}{*}K - v^{*}(K+1)^{2}}{v^{*}(K+1)^{3}}$$

$$\mu = \frac{a^{*}q + b^{*}p}{p + q}$$

$$v = \frac{(b-a)^{2}{*}p^{*}q}{(p+q)^{2}{*}(p+q+1)}$$

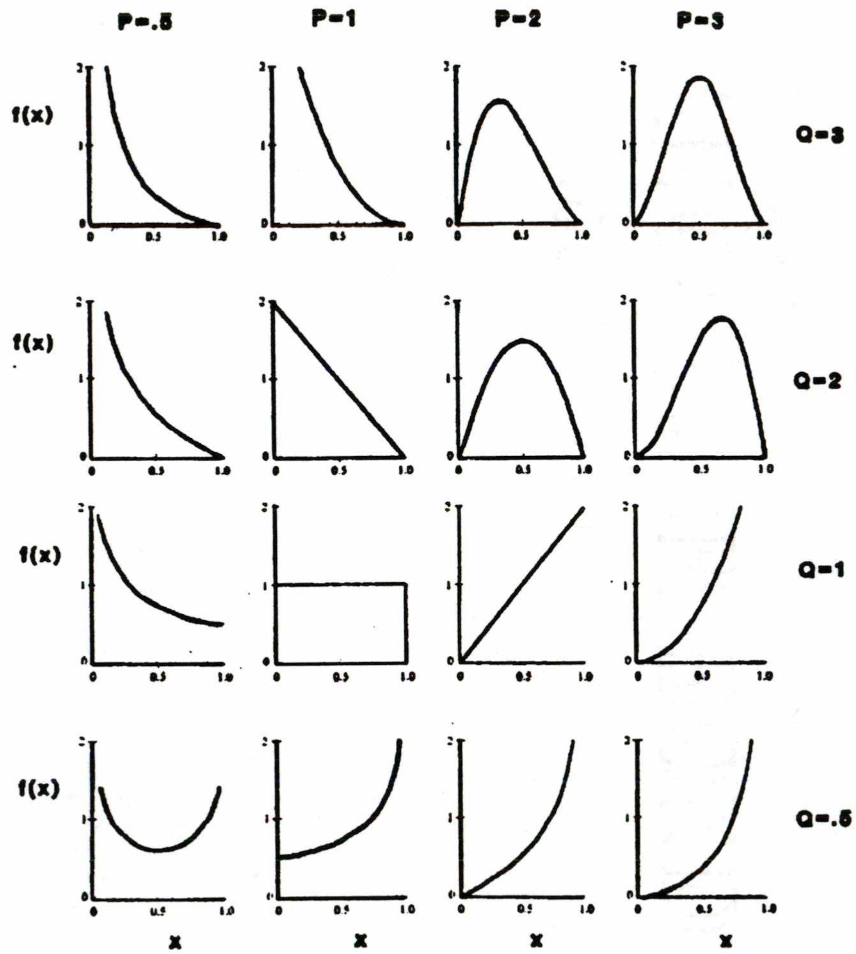


Figure 7. Definition of Beta Distribution

User-supplied
<u>Discrete Probability Distribution</u>

$F(x) = 0.2$ @ $x=0.9$;
$F(x) = 0.4$ @ $x=1.4$;
$F(x) = 0.6$ @ $x=1.9$;
etc.

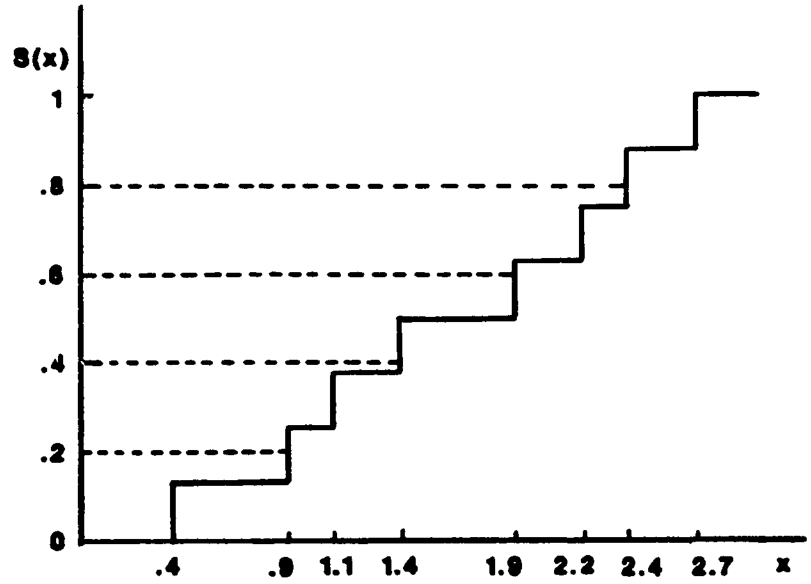


Figure 8. Example of User-Supplied Discrete Probability Distribution

```
TITLE - INPUT FILE FOR LHS SAMPLE FOR THRUST LAPSE
RANDOM SEED   -3527346
NOBS        10
NREPS       15
LOGUNIFORM       ALTITUDE, FT.
        1.0       6000.0
NORMAL           TEMPERATURE (DEG. R)
      445.0        585.0
OUTPUT  HIST CORR DATA
```

Figure 9. LHS Input File for Takeoff Thrust Lapse Rate Investigation

TITLE - ....
RANDOM SEED = -1399508694
NUMBER OF VARIABLES = 2
NUMBER OF OBSERVATIONS = 10
REPLICATION NUMBER 15 OF 15 REPLICATIONS
THE SAMPLE INPUT VECTORS WILL BE PRINTED
ALONG WITH THEIR CORRESPONDING RANKS

LATIN HYPERCUBE SAMPLE INPUT VECTORS

RUN NO.	X(1)	X(2)	STD. DAY TEMP., ^oR	DEVIATION FROM STD. DAY
1	6.95	580.	519.	+61.
2	228.	511.	518.	-7.
3	5.23	498.	519.	-21.
4	2.32	522.	519.	+3.
5	5.992E+03	506.	497.	+9.
6	40.0	492.	519.	-27.
7	27.7	534.	519.	+15.
8	2.375E+03	520.	510.	+10.
9	601.	538.	517.	+21.
10	116.	459.	518.	-59.

RANKS OF LATIN HYPERCUBE SAMPLE INPUT VECTORS

RUN NO.	X(1)	X(2)
1	3.	10.
2	7.	5.
3	2.	3.
4	1.	7.
5	10.	4.
6	5.	2.
7	4.	8.
8	9.	6.
9	8.	9.
10	6.	1.

Figure 10. LHS Program Output for Takeoff Thrust Lapse Rate Investigation

```
TITLE, NONDIMENSIONAL FAN OUTLET CONDITIONS
DATA,5,0,0.
LABEL(1)=P,1/ROOT(T),P**2,1/T,THRUST
MODEL,5=1+2+3+4.
STEPWISE,SIGIN=.05,SIGOUT=.10
PRESS
OUTPUT,CORR,STEPS,RESIDUALS
END OF PARAMETERS
 (F7.5,2X,F6.4,2X,F7.5,2X,F6.4,2X,F7.1)
1.24224  0.9087  1.54317  0.8258  31931.0
1.29892  0.9602  1.68721  0.9219  42881.0
1.32573  0.9704  1.75757  0.9417  45953.0
1.29682  0.9512  1.68173  0.9049  41103.0
1.05294  0.9645  1.10868  0.9302  35326.0
1.33186  0.9755  1.77385  0.9515  47230.0
1.28300  0.9422  1.64609  0.8878  38902.0
1.19134  0.9531  1.41930  0.9084  37985.0
1.25211  0.9392  1.56778  0.8821  37419.0
1.37847  1.0040  1.90018  1.0081  55494.0
```

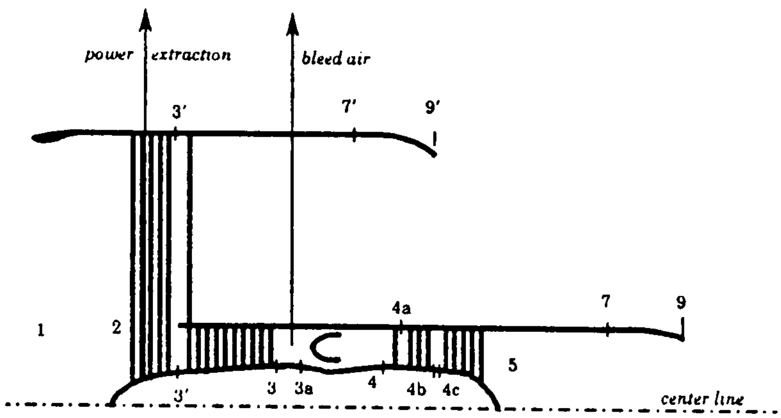
Figure 11. STEP Input File for Takeoff Thrust Lapse Rate Investigation

APPENDIX C

GOVERING EQUATIONS FOR A TURBOFAN, BYPASS RATIO PRESCRIBED

This appendix contains a listing of the governing equations (Reference 10, pp. 240 - 243) used by the on-design cycle deck ONX for a turbofan with losses, and with bypass ratio prescribed. The presence of losses implies a real cycle analysis, as opposed to a less accurate ideal cycle analysis. Using these equations, ONX calculates the solution values of gross thrust and specific fuel consumption. The inclusion of these equations in this appendix provides the reader with an idea of the compressible flow and thermodynamics that is contained within the cycle decks. The sketch below (taken from Reference 9, Appendix G) provides a handy definition of the engine station locations that are indicated by the equation subscripts. Symbol definitions are listed in the thesis nomenclature.

Station	Location	Station	Location
0	Far upstream or freestream	4a	High pressure turbine entry for τ_{tH} definition
1	Inlet or diffuser entry	4b	High pressure turbine exit Modeled coolant mixer 2 entry
2	Inlet or diffuser exit Fan entry	4c	Coolant mixer 2 exit Low pressure turbine entry
3'	Fan exit High pressure compressor entry	5	Low pressure turbine exit Core stream mixer entry
3	High pressure compressor exit	7	Core exhaust nozzle entry
3a	Burner entry	9	Core exhaust nozzle exit
4	Burner exit Nozzle vanes entry Modeled coolant mixer 1 entry High pressure turbine entry for π_{tH} definition	7'	Bypass exhaust nozzle entry
4a	Nozzle vanes exit Coolant mixer 1 exit	9'	Bypass exhaust nozzle exit



$$R_c = \frac{\gamma_c - 1}{\gamma_c} C_{p_c} \ \text{m}^2/\text{s}^2\cdot\text{K}$$

$$R_c = \frac{\gamma_c - 1}{\gamma_c} C_{p_c} \ (2.505)(10^4) \ \text{ft}^2/\text{s}^2\cdot{}^\circ\text{R}$$

$$a_0 = \sqrt{\gamma_c R_c T_0} \ \text{m/s [ft/s]}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c - 1)}$$

$$\tau_c = \pi_c^{(\gamma_c - 1)/\gamma_c e_c}$$

$$\tau_{c'} = (\pi_{c'})^{(\gamma_c - 1)/\gamma_c e_{c'}}$$

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{(h \eta_b / C_{p_c} T_0) - \tau_\lambda}$$

$$\tau_t = 1 - \frac{1}{\eta_m (1 + f)} \frac{\tau_r}{\tau_\lambda} [(\tau_c - 1) + \alpha(\tau_{c'} - 1)]$$

$$\pi_t = \tau_t^{\gamma_t/(\gamma_t - 1)e_t}$$

$$\frac{p_{t9}}{p_9} = \frac{p_0}{p_9} \pi_r \pi_d \ \pi_c \pi_b \pi_t \pi_n$$

$$\frac{T_9}{T_0} = \frac{(C_{p_c}/C_{p\,AB})\tau_{\lambda AB}}{(p_{t9}/p_9)^{(\gamma_{AB} - 1)/\gamma_{AB}}}$$

(Note: In our instance, no afterburning is present, so $\tau_{\lambda_{AB}}$ should be put equal to $\tau_\lambda \tau_t$. Also, $C_{p_{AB}}$ and γ_{AB} should be set equal to C_{p_t} and γ_t .)

$$M_0 \frac{u_9}{u_0} = \left\{ \frac{2}{\gamma_c - 1} \tau_{\lambda_{AB}} \left[1 - \left(\frac{p_{t_9}}{p_9} \right)^{-[(\gamma_{AB} - 1)/\gamma_{AB}]} \right] \right\}^{\frac{1}{2}}$$

$$\frac{p_{t_{9'}}}{p_{9'}} = \frac{p_0}{p_{9'}} \pi_r \, \pi_d \, \pi_c{}' \, \pi_n{}'$$

$$\frac{T_{9'}}{T_0} = \frac{(C_{p_c}/C_{p_{AB'}})\tau_{\lambda_{AB'}}}{(p_{t_{9'}}/p_{9'})^{(\gamma_{AB'} - 1)/\gamma_{AB'}}}$$

(Note: In our instance, no afterburning is present, so $\tau_{\lambda_{AB'}}$ should be put equal to $\tau_r \tau_{c'}$. Also, $C_{p_{AB'}}$ and $\gamma_{AB'}$ should be set equal to C_{p_c} and γ_c.)

$$M_0 \frac{u_{9'}}{u_0} = \left\{ \frac{2}{\gamma_c - 1} \tau_{\lambda_{AB}} \left[1 - \left(\frac{p_{t_{9'}}}{p_{9'}} \right)^{-[(\gamma_{AB'} - 1)/\gamma_{AB'}]} \right] \right\}^{\frac{1}{2}}$$

$$f_{AB} = (1 + f\ ) \frac{\tau_{\lambda_{AB}} - \tau_\lambda \tau_t}{(h\, \eta_{AB}/C_{p_c} T_0) - \tau_{\lambda_{AB}}}$$

$$f_{AB'} = \frac{\tau_{\lambda_{AB'}} - \tau_r \tau_c{}'}{(h\, \eta_{AB'}/C_{p_c} T_0) - \tau_{\lambda_{AB'}}}$$

$$\frac{F_A}{\dot{m}_c + \dot{m}_F} = \frac{a_0}{1+\alpha}\left\{(1+f+f_{AB})\left(M_0\frac{u_9}{u_0}\right) - M_0 + (1+f+f_{AB})\right.$$

$$\times \frac{1}{\gamma_c\left[M_0(u_9/u_0)\right]}\frac{T_9}{T_0}\left(1-\frac{p_0}{p_9}\right)$$

$$+\alpha\left[(1+f_{AB'})\left(M_0\frac{u_{9'}}{u_0}\right) - M_0\right.$$

$$\left.\left.+ (1+f_{AB'})\frac{1}{\gamma_c\left[M_0(u_{9'}/u_0)\right]}\frac{T_{9'}}{T_0}\left(1-\frac{p_0}{p_{9'}}\right)\right]\right\}$$

(Note: To obtain British units of lbf/lbm/s, replace a_0 in meters/second with $a_0/32.174$ where a_0 should be given in feet/second.)

$$S = \frac{(f + f_{AB} + \alpha f_{AB'})}{(1+\alpha)\left[F_A/(\dot{m}_c + \dot{m}_F)\right]}(10^6) \qquad \left[S = \frac{3600(f + f_{AB} + \alpha f_{AB'})}{(1+\alpha)\left[F_A/g_0(\dot{m}_c + \dot{m}_F)\right]}\right]$$

NOMENCLATURE

Roman numerals in parentheses indicate the appropriate chapter for each symbol.

a	Lower limit of independent parameter distribution (II) ; Normal equations coefficient, or Speed of sound (IV)
A	Lower limit of independent parameter distribution (II) ; Cross-sectional area, or Constant coefficient (IV)
b	Upper limit of independent parameter distribution (II) ; Normal equations coefficient (IV)
B	Upper limit of independent parameter distribution (II) ; Constant coefficient (IV)
C	Constant-pressure specific heat, when subscripted with p (III); Constant coefficient (IV)
D	Constant coefficient (IV)
e	Polytropic efficiency (III)
E	Least squares error (IV)
f	Fuel-to-air ratio (III)
F	Gross thrust (III)
g	Gravitational constant (III)
h	Fuel lower heating value (III)
K	Beta distribution constant (II) ; Intercept constant (IV)
$\dot{m}$	Mass flowrate (IV)
M	Mach number (II, III, IV)

n	Sample size (II); Polynomial degree (IV)
p	Beta distribution constant (II)
P	Pressure (II, III, IV)
q	Beta distribution constant (II)
r	Correlation coefficient (IV)
R	Gas constant for air (IV)
S	Specific fuel consumption (III)
T	Temperature (II, III, IV)
u	Velocity (III)
v	Velocity (II)
x	Generalized independent variable (II, IV)
$\bar{x}$	Mean value of independent variable x (IV)
y	Generalized independent variable (IV)
$\bar{y}$	Mean value of independent variable y (IV)
Y	Observed parameter value (IV)
$\hat{Y}$	Predicted parameter value (IV)
z	Generalized mathematical function (IV)

Greek Symbols:

α	Constant used for variable coefficient calculation (IV); Bypass ratio (III)
β	Bypass ratio (III); Constant used for variable coefficient calculation (IV)
γ	Ratio of specific heats (II, III, IV)
δ	Nondimensional pressure ratio (IV)
Δ	Differential amount (IV)
η	Component efficiency (III)
Θ	Nondimensional temperature ratio (IV)
μ	Arithmetic mean (II)
π	Engine component stagnation pressure ratio (III)
ρ	Density (IV)
Σ	Mathematical summation notation (IV)
τ	Engine component stagnation temperature ratio (III); Gross thrust (IV)
υ	Variance (II)
$\frac{\partial f}{\partial x}$	Partial differentiation notation (here, denotes partial derivative of function f with respect to x) (IV)

Subscripts:

A Denotes actual (gross) thrust, when subscripted to F (III)

AB Afterburner (III)

c Compressor (III)

c' Fan (III)

d Diffuser (III)

F Denotes fan, when subscripted to $\dot{m}$ (III)

i Index (IV)

j Index (IV)

k Index (II, IV)

m Mixer (III)

n Nozzle (III)

r Recovery (III)

t Total, or stagnation, flow conditions (II, III, IV); Turbine (III)

λ Denotes turbine inlet stagnation temperature divided by ambient temperature, when subscripted to τ (III)

0 Ambient conditions (III)

VITA

Mr. Gary Dwight Ledbetter was born on November 30, 1952 in Livingston (Overton County) Tennessee. His elementary education was at nearby Monroe Elementary School. His high school years were spent at Livingston Academy, where he graduated in 1970. He attended the University of Tennessee, Knoxville, from 1970 to 1975, graduating with a Bachelor of Science degree in Mechanical Engineering. In 1970 and 1971 he was a trumpet-playing member of the University of Tennessee Pride of the Southland Marching Band. Shortly after graduation, five years were spent in sunny Southern California with the Rocketdyne Division of Rockwell International. There he helped perform engineering analysis of America's Space Shuttle. Mr. Ledbetter is a Senior Member of AIAA. He is currently employed at the Arnold AFB division of Sverdrup Technology.