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Horace W. Crater, Major Professor

We have read this thesis  
and recommend its acceptance:

Christian G. Parigger

Lloyd M. Davis

Accepted for the Council:

Carolyn R. Hodges  
Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

# Experimental Determination of Emissivity and Resistivity of Yttria Stabilized Zirconia at High Temperatures

A Thesis  
Presented for the  
Master of Science  
Degree  
The University of Tennessee, Knoxville

Howard Mosley Frederick  
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# Dedication

This thesis is dedicated to my wife  
Carolyn C. Frederick

and to my parents  
William Mosley Frederick  
and  
Nell S. Frederick

who have always encouraged and supported me in all my endeavors.

# Acknowledgments

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# Abstract

The emissivity and resistivity of yttria stabilized zirconia (YsZ),  $\text{Y}_2\text{O}_3\text{-ZrO}_2$ , a ceramic material that has been specially fabricated for use as an electrically powered high temperature air heater are determined for temperatures above 2000 K. Published results of measurements of emissivity and resistivity of yttria, zirconia, and their alloys at temperatures above 2000 K are rare in the technical literature. A survey of literature describing the properties of YsZ, particularly literature concerning the ionic conduction process at high temperatures, is assembled to provide a theoretical basis for the formulation of a temperature and resistivity relationship.

A specialized test apparatus, instrumentation, and control system which acquires data while controlling the YsZ temperature over the range of interest is described. A unique feature of this test apparatus is the use of joule heating of the material itself to produce the high temperature environment for obtaining the measurements. Radiative properties of the test configuration are analyzed and a methodology is established for determining emissivity and temperature. A method for determining electrical power input to the material and its resistivity are derived from an analysis of the heat transfer properties of the YsZ material and test apparatus. Data from a test that was conducted over the range of temperatures from 2000 K to 2300 K is analyzed to provide the final measured values of emissivity and resistivity. The results of this study provide material property data that can be used in the design of a YsZ air heater for aerospace testing applications.

# Contents

|          |                                                                    |           |
|----------|--------------------------------------------------------------------|-----------|
| <b>1</b> | <b>Introduction</b>                                                | <b>1</b>  |
| <b>2</b> | <b>Zirconia and Yttria Stabilized Zirconia</b>                     | <b>3</b>  |
| 2.1      | Zirconia                                                           | 3         |
| 2.2      | Yttria Stabilized Zirconia                                         | 3         |
| 2.3      | Electrical Properties of Yttria Stabilized Zirconia                | 4         |
| <b>3</b> | <b>The YsZ Heater Element, Test Apparatus, and Instrumentation</b> | <b>8</b>  |
| 3.1      | The YsZ Heater Element                                             | 9         |
| 3.2      | Apparatus                                                          | 9         |
| 3.3      | Instrumentation                                                    | 12        |
| 3.4      | Data Acquisition, Data Recording, and Control                      | 15        |
| 3.5      | Data Reduction and Filtering                                       | 18        |
| <b>4</b> | <b>Radiative Properties</b>                                        | <b>21</b> |
| 4.1      | Introduction                                                       | 21        |
| 4.2      | Thermal Radiation Heat Transfer                                    | 21        |
| 4.3      | Radiometric Measurements and Calibration                           | 25        |
| 4.4      | Application of the Modified Ratio Thermometer                      | 26        |
| <b>5</b> | <b>Resistivity</b>                                                 | <b>38</b> |
| 5.1      | Theory                                                             | 38        |
| 5.2      | Determination of High Temperature Resistivity                      | 40        |
| <b>6</b> | <b>Conclusions</b>                                                 | <b>45</b> |
|          | <b>Bibliography</b>                                                | <b>47</b> |
|          | <b>Appendices</b>                                                  | <b>50</b> |
| <b>A</b> | <b>RMS Voltage Equation for Heater Power Supply</b>                | <b>51</b> |
| <b>B</b> | <b>Computer Programs and Scripts</b>                               | <b>53</b> |
| B.1      | Data Reduction and Filtering - BASIC Language                      | 53        |
| B.2      | Calculate and Plot Temperature and Emissivity - Octave Script      | 57        |
| <b>C</b> | <b>Temperature Versus Emissivity Graphs</b>                        | <b>59</b> |

|                       |           |
|-----------------------|-----------|
| <b>Vita</b> . . . . . | <b>83</b> |
|-----------------------|-----------|

# List of Tables

|     |                                                                                 |    |
|-----|---------------------------------------------------------------------------------|----|
| 3.1 | Resultant voltages for controller conduction angles . . . . .                   | 13 |
| 3.2 | Typical test sequence . . . . .                                                 | 19 |
| 4.1 | Calibration of the ratio thermometer for absolute radiation measurements .      | 28 |
| 4.2 | Calculated emissivities - not corrected for reflection . . . . .                | 34 |
| 4.3 | Emissivities and temperatures - corrected for reflection of irradiance . . . .  | 35 |
| 5.1 | Total emissivity . . . . .                                                      | 42 |
| 5.2 | Summary of calculations of heat transfer, resistance, and resistivity . . . . . | 43 |

# List of Figures

|      |                                                                                |    |
|------|--------------------------------------------------------------------------------|----|
| 2.1  | Phase diagram of nontransformable tetragonal zirconia . . . . .                | 5  |
| 3.1  | YsZ heater element . . . . .                                                   | 10 |
| 3.2  | Gas Sample Cell apparatus . . . . .                                            | 10 |
| 3.3  | Top view of dual-chamber furnace . . . . .                                     | 11 |
| 3.4  | Waveform of the maximum output voltage . . . . .                               | 13 |
| 3.5  | Electrode connections to the YsZ heater element . . . . .                      | 14 |
| 3.6  | Recorded voltage waveform . . . . .                                            | 16 |
| 3.7  | Fourier transform of recorded voltage waveform . . . . .                       | 17 |
| 3.8  | Operator screen on control computer . . . . .                                  | 17 |
| 3.9  | PID controller and sequencer screen on control computer . . . . .              | 19 |
| 4.1  | Output signal from the ratio thermometer . . . . .                             | 27 |
| 4.2  | Calibration curve for radiometer 0.90 micron output . . . . .                  | 30 |
| 4.3  | Calibration curve for radiometer 0.81 micron output . . . . .                  | 30 |
| 4.4  | Graphical solution of emissivity and temperature without irradiance correction | 33 |
| 4.5  | Graphical solution of emissivity and temperature with irradiance correction    | 33 |
| 4.6  | YsZ temperature versus emissivity . . . . .                                    | 37 |
| 5.1  | Temperature versus total emissivity . . . . .                                  | 42 |
| 5.2  | Temperature versus resistivity . . . . .                                       | 44 |
| C.1  | Data point 1 (2023 K) . . . . .                                                | 60 |
| C.2  | Data point 1 (2023 K) . . . . .                                                | 60 |
| C.3  | Data Point 2 (2073 K) . . . . .                                                | 61 |
| C.4  | Data Point 2 (2073 K) . . . . .                                                | 61 |
| C.5  | Data Point 3 (2123 K) . . . . .                                                | 62 |
| C.6  | Data Point 3 (2123 K) . . . . .                                                | 62 |
| C.7  | Data Point 4 (2173 K) . . . . .                                                | 63 |
| C.8  | Data Point 4 (2173 K) . . . . .                                                | 63 |
| C.9  | Data Point 5 (2193 K) . . . . .                                                | 64 |
| C.10 | Data Point 5 (2193 K) . . . . .                                                | 64 |
| C.11 | Data Point 6 (2213 K) . . . . .                                                | 65 |
| C.12 | Data Point 6 (2213 K) . . . . .                                                | 65 |
| C.13 | Data Point 7 (2233 K) . . . . .                                                | 66 |
| C.14 | Data Point 7 (2233 K) . . . . .                                                | 66 |
| C.15 | Data Point 8 (2253 K) . . . . .                                                | 67 |
| C.16 | Data Point 8 (2253 K) . . . . .                                                | 67 |

|                                       |    |
|---------------------------------------|----|
| C.17 Data Point 9 (2273 K) . . . . .  | 68 |
| C.18 Data Point 9 (2273 K) . . . . .  | 68 |
| C.19 Data Point 10 (2293 K) . . . . . | 69 |
| C.20 Data Point 10 (2293 K) . . . . . | 69 |
| C.21 Data Point 11 (2313 K) . . . . . | 70 |
| C.22 Data Point 11 (2313 K) . . . . . | 70 |
| C.23 Data Point 12 (2333 K) . . . . . | 71 |
| C.24 Data Point 12 (2333 K) . . . . . | 71 |
| C.25 Data Point 13 (2293 K) . . . . . | 72 |
| C.26 Data Point 13 (2293 K) . . . . . | 72 |
| C.27 Data Point 14 (2253 K) . . . . . | 73 |
| C.28 Data Point 14 (2253 K) . . . . . | 73 |
| C.29 Data Point 15 (2233 K) . . . . . | 74 |
| C.30 Data Point 15 (2233 K) . . . . . | 74 |
| C.31 Data Point 16 (2213 K) . . . . . | 75 |
| C.32 Data Point 16 (2213 K) . . . . . | 75 |
| C.33 Data Point 17 (2193 K) . . . . . | 76 |
| C.34 Data Point 17 (2193 K) . . . . . | 76 |
| C.35 Data Point 18 (2173 K) . . . . . | 77 |
| C.36 Data Point 18 (2173 K) . . . . . | 77 |
| C.37 Data Point 19 (2153 K) . . . . . | 78 |
| C.38 Data Point 19 (2153 K) . . . . . | 78 |
| C.39 Data Point 20 (2133 K) . . . . . | 79 |
| C.40 Data Point 20 (2133 K) . . . . . | 79 |
| C.41 Data Point 21 (2113 K) . . . . . | 80 |
| C.42 Data Point 21 (2113 K) . . . . . | 80 |
| C.43 Data Point 22 (2093 K) . . . . . | 81 |
| C.44 Data Point 22 (2093 K) . . . . . | 81 |
| C.45 Data Point 23 (2073 K) . . . . . | 82 |
| C.46 Data Point 23 (2073 K) . . . . . | 82 |

# Nomenclature

|             |                                                                                          |
|-------------|------------------------------------------------------------------------------------------|
| $A$         | area                                                                                     |
| $\alpha$    | absorptivity                                                                             |
| ac          | alternating current                                                                      |
| $c$         | speed of light                                                                           |
| dc          | direct current                                                                           |
| $\delta$    | conduction angle                                                                         |
| $\epsilon$  | emissivity                                                                               |
| $E$         | irradiance                                                                               |
| f           | frequency, Hertz                                                                         |
| h           | Planck's constant                                                                        |
| $J$         | radiosity                                                                                |
| i           | electric current                                                                         |
| j           | electric current density                                                                 |
| k           | Boltzmann's constant                                                                     |
| K           | Kelvin temperature                                                                       |
| $\lambda$   | wavelength                                                                               |
| $M$         | radiant emittance                                                                        |
| $M_\lambda$ | spectral radiant emittance at frequency range $\lambda + d\lambda$                       |
| mV          | millivolt                                                                                |
| MW          | megawatt                                                                                 |
| mol         | mole (Avogadro's number of an entity)                                                    |
| $\dot{Q}$   | heat transfer rate                                                                       |
| $\varrho$   | reflectivity                                                                             |
| $\rho$      | resistivity                                                                              |
| $R$         | resistance                                                                               |
| RMS         | root mean square                                                                         |
| $\sigma$    | conductivity                                                                             |
| $\varsigma$ | Stefan-Boltzmann constant                                                                |
| $\tau$      | transmittance                                                                            |
| T           | absolute temperature                                                                     |
| YsZ         | yttria-stabilized zirconia (partial stabilization is implied unless otherwise indicated) |

# Chapter 1

## Introduction

Measurements of the emissivity and conductivity of the ceramic material yttria-stabilized zirconia (YsZ),  $\text{Y}_2\text{O}_3\text{-ZrO}_2$ , at temperatures exceeding 2000 K are rare in technical publications. Studies of the properties of YsZ for two applications, thermal barrier coatings and solid oxide fuel cells, have provided much of the knowledge of YsZ's high temperature properties. Research into the application of YsZ as a thermal barrier coating has provided detailed spectral emissivity and reflectivity data for thin films of YsZ over the temperature range of 900 K to 1400 K [1] [2]. In support of research on thermal barrier coatings in solar applications, the emissivity of YsZ has been measured over the temperature range of 1300 K to 2200 K [3]. High temperature solid oxide fuel cell research has provided the impetus for measurements of the conductivity of fully stabilized YsZ to 1660 K [4]. Although conductivity studies have not been published for temperatures above 2000 K, studies of the effects on conductivity of mixing dopants have been published for temperatures up to 1100 K [5] [7]. The potential use of YsZ in high temperature sensors is also stimulating conductivity research [8].

Measurements of surface temperatures above 2000 K in air must be made through non-contact radiometric methods. In the study of thermal barrier coatings potential sources for measurement errors, such as reflected radiation, have been identified [9] [10]. In this work, a unique high-temperature furnace and specialized instrumentation and data collection techniques are used to make measurements of YsZ electrical conductivity and emissivity at temperatures between 2000 K and 2300 K.

As a source of radiant energy, emissive elements made of zirconia stabilized with yttria and erbium have been applied in the form of the Nernst glower for over 100 years. In Nernst glowers, YsZ filaments reach temperatures approaching 2000 K [11] as a result of joule heating from ionic conduction. Although YsZ remains a stable solid in oxygen atmospheres at temperatures approaching its melting point of 2950 K, the metal electrodes in the Nernst glower limit the upper temperature. Through new approaches in the fabrication and implementation of YsZ conduction heater elements, nonuniform temperature profiles are being produced that create hot sections of heater elements that reach temperatures in excess of 2300 K. The ability of a YsZ heater element to produce high temperature thermal radiation in an oxygen atmosphere makes it a good candidate for a heat source in an aerospace ground test facility. Quantification of the electrical conductivity and emissivity

of YsZ at high temperatures is a necessary step in the research of this potential application.

Aerospace ground test facilities that are capable of testing hypersonic vehicles (vehicles capable of traveling at speeds of Mach 5 or faster) must produce extremely high temperature air streams. Existing means for heating air to the required high temperatures produce undesirable changes in the properties of the air stream (e.g., electric arc heaters produce nitrous oxide and combustion-based heaters add species from the combustion products). Moreover, these are not adaptable to a continuous-flow wind tunnel test facility. To minimize vitiation, a radiant heater, powered by electricity and capable of producing high heat transfer to air at high temperatures is desirable. Electric heater technology based on YsZ is being developed at the Air Force Arnold Engineering Development Center (AEDC) in Tennessee to provide this capability.

Operation of YsZ heaters in an oxygen atmosphere at temperatures exceeding 2000 K creates several difficulties that must be overcome to make measurements of electrical properties. The metallic conductors that one would normally use to connect specimens to instrumentation are not solids at this temperature. Non-metallic alloys that would normally be used as insulators become good conductors. All materials that are used must be evaluated for their ability to not melt, degrade, or react with other materials.

This thesis documents measurements of the emissivity and resistivity of YsZ at temperatures above 2000 K and the methods used to obtain these measurements. Chapter 2 presents a survey of the literature on the properties of zirconia and stabilized zirconia. Chapter 3 describes the furnace, instrumentation, and methods used to make high temperature measurements. Chapter 4 describes the temperature and emissivity measurements. Chapter 5 describes the resistivity measurements. Appendix A contains the derivation of the RMS voltage equation for the phase-angle controlled heater power supplies. Appendix B contains a computer program that was written to filter data and a script that was written to solve equations and produce graphs. Appendix C contains graphs produced from the analyses of the radiometric data.

## Chapter 2

# Zirconia and Yttria Stabilized Zirconia

### 2.1 Zirconia

Zirconia (zirconium dioxide,  $\text{ZrO}_2$ ) is a ceramic compound which has a monoclinic structure at temperatures up to 1440 K. Its density in the monoclinic state is  $5.6 \text{ g/cm}^3$ . At temperatures between 1440 K and 2640 K, zirconia has a tetragonal structure and a density of  $6.1 \text{ g/cm}^3$ . Above 2640 K, zirconia has a cubic form and a density of  $6.27 \text{ g/cm}^3$ . The melting temperature of zirconia is 2950 K [12]. The significant differences of the densities (particularly for the transition between monoclinic and tetragonal phases) correspond to significant changes in volume for the molecular structure with the monoclinic phase having more than 6 percent greater volume than the tetragonal phase [13]. When zirconia is cooled so that it transitions from the tetragonal to the monoclinic phase the changes in the molecular structure's volume create internal stresses that exceed the material's elastic and fracture limits and result in cracking [12].

Pure zirconia is a poor electronic conductor. The band gap between the valence and conduction bands is greater than 5 eV causing zirconia to be considered an insulator at temperatures less than 1000 K [13]. Zirconia is also a poor ionic conductor.

### 2.2 Yttria Stabilized Zirconia

By doping zirconia with yttria (yttrium(III) oxide,  $\text{Y}_2\text{O}_3$ ), a substitutional cation, additional oxygen vacancies are provided in the molecular structure that increase ionic electrical conductivity. In addition to increasing the electrical conductivity, yttria, when alloyed with zirconia in concentrations from 8 percent to 40 percent (molar), fully stabilizes the material's crystalline structure in the cubic form at normal conditions [14]. Because it does not transition between cubic and monoclinic phases, fully stabilized zirconia does not experience microcracking when subjected to wide ranging temperatures. However, fully stabilized zirconia is not a particularly strong material.

By careful control of the feedstock materials and the manufacturing process, useful ceramic materials of zirconia and yttria containing between 2.5 and 10 molar percent yttria can be produced that have superior strength to fully stabilized zirconia. One form of the material, called partially stabilized zirconia (YsZ), is a mixture that consists of cubic, tetragonal, and monoclinic phases. Another form, tetragonal zirconia polycrystals, consists of a submicron tetragonal grain structure [15]. Both materials exhibit enhanced ionic conductivity. Partially stabilized zirconia can be manufactured by a process known as transformation toughening that produces a strong, temperature-shock resistant ceramic. A form of partially stabilized zirconia, called nontransformable tetragonal YsZ, which is produced by methods that enhance the material's ability to retain a predominately tetragonal crystal form through a wide range of temperatures, is the subject material of this investigation.

As shown in the phase diagram for nontransformable tetragonal yttria stabilized zirconia, Fig. 2.1, phase transformations do occur as the temperature changes. To limit microcracking, care is taken to avoid exposing the material to large, rapid temperature changes.

## 2.3 Electrical Properties of Yttria Stabilized Zirconia

From an electrical point of view, the material partially stabilized zirconia (YsZ) is best described as a solid electrolyte. Like pure zirconia, the band gap between the valence and conduction bands is large (typically greater than 5 eV), so YsZ is a good insulator at ordinary temperatures. At high temperatures, vacancies are formed in the lattice of the material allowing ionic conduction [16]. The dopant, yttria, is a substitutional cation in the molecular structure that enhances ionic conduction by providing additional oxygen vacancies. Conduction by electrons in YsZ has been shown to be a small fraction of the total conduction even at temperatures below 900 K [17]. Electron conduction would produce significantly less chemical change in the material than ionic conduction, but it is unlikely that electron conduction is significant at high temperatures. Peak electrical conductivity in YsZ is achieved with an 8 mol percent yttria concentration [18]. However, to produce the nontransformable tetragonal crystalline structure with the desired structural properties for the heater elements, the YsZ material used will probably contain less than 5 mol percent yttria dopant. At this concentration, very high temperatures are required to initiate the ion mobility needed for useful electrical current flow.

The mechanism for ionic conductivity in YsZ is the hopping of mobile ions to vacant sites in the crystal lattice. Conductivity can be expressed in terms of the concentration of charge carriers,  $c_i$ , their charge,  $q$ , and their mobility,  $u_i$ :

$$\sigma_i = c_i q u_i. \quad (2.1)$$

$c_i$  is given by  $\frac{\text{carriers}}{m^3}$ ,  $q$  is equal to  $\frac{\text{coul}}{\text{carrier}}$ , and  $u_i$  equals  $\frac{m/s}{V/m}$ .

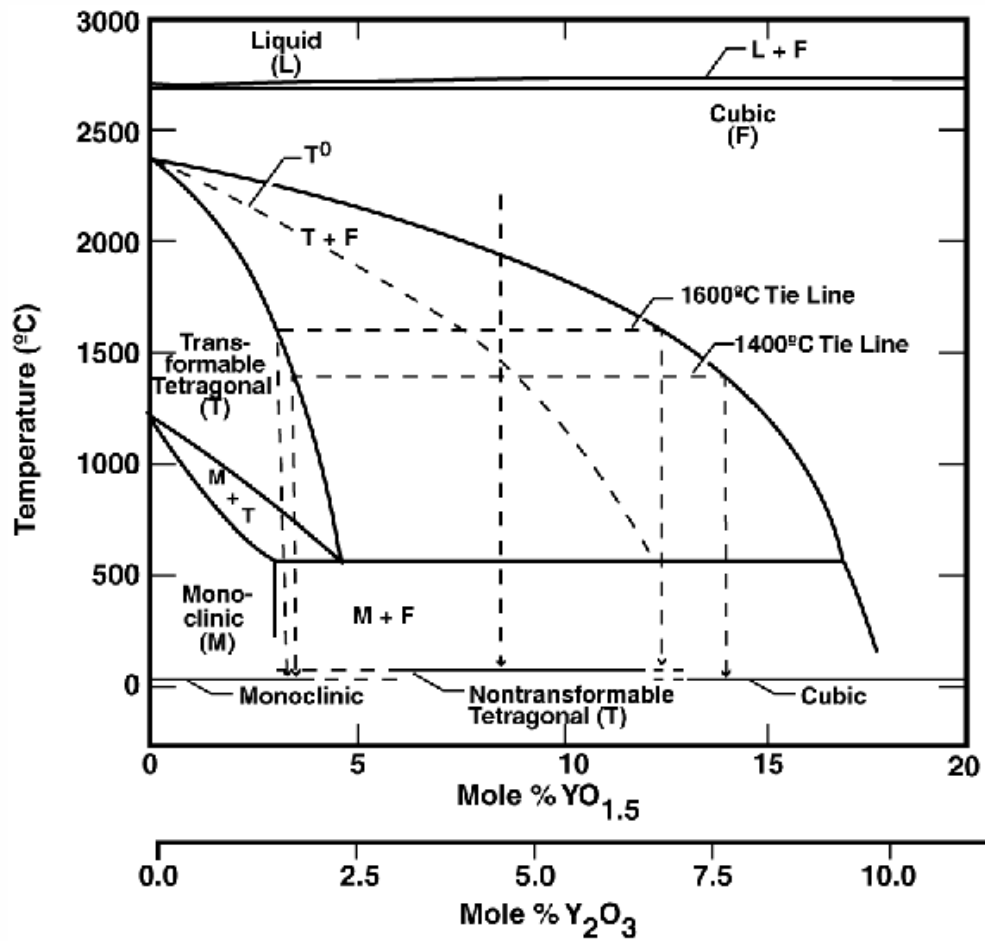


Figure 2.1: Phase diagram of nontransformable tetragonal zirconia [6]

Oxygen vacancies, provided by the yttria dopant, directly relate to the charge concentration,  $c_i$ . The mobility,  $u_i$ , of oxygen ions in YsZ is an interesting study in its own right. In a low-temperature solid electrolyte with a regular crystalline structure, the energy required to move an oxygen ion from vacancy to vacancy through the interstitial sites is called the activation energy for conduction,  $\Delta H_m$ . Maxwell-Boltzmann statistical mechanics can be used to show that the fraction of ions which have kinetic energy greater than  $\Delta H_m$ , enabling motion through the lattice, is an exponential function of temperature and is proportional to  $\exp(-\Delta H_m/kT)$ . The relationship between temperature, activation energy, and the rate at which a physical process occurs was first interpreted by Svante Arrhenius and equations expressing this relationship are often called Arrhenius equations.

An Arrhenius equation which gives ionic conductivity of YsZ to a first approximation can be written as

$$\sigma = A \exp(-\Delta H_m/kT), \quad (2.2)$$

where  $A$ , the prefactor, is an empirically determined constant equal to the conductivity in the limit of high temperature,  $T$ . Using this approximate model of conduction, a typical activation energy for YsZ is approximately 0.90 eV [19].

In reality, YsZ is a polycrystalline material with an irregular structure. From precise resistivity measurements, Bauerle and Brizo postulated the existence of grain boundaries between crystals in YsZ in 1968 [20]. These grain boundaries are thin (approximately 500 angstroms) and contain a higher proportion of yttria than the bulk material. The properties of the grain boundaries within the material affect the conductivity as do the bulk material properties. The activation energy for conduction in the bulk material,  $\Delta H_1$ , was shown to be on the order of half the activation energy of the grain boundary material,  $\Delta H_2$ . Using a variant form of the Arrhenius equation, in which the temperature is separated from the prefactor, a mathematical expression for the total resistivity was created by combining Arrhenius equations for the two sources of resistivity,

$$\frac{\rho}{T} = A_1 \exp(\Delta H_1/kT) + A_2 \exp(\Delta H_2/kT), \quad (2.3)$$

where  $\rho$  is the resistivity (reciprocal of conductivity). Results of the experiments conducted by Bauerle and Brizo [20] established that:

$$\begin{aligned} A_1 &= 9.056 \times 10^{-6} \, \Omega \, \text{cm} \, \text{deg}^{-1}, \\ A_2 &= 4.572 \times 10^{-8} \, \Omega \, \text{cm} \, \text{deg}^{-1}, \\ \Delta H_1 &= 0.682 \, \text{eV}, \text{ and} \\ \Delta H_2 &= 1.168 \, \text{eV}. \end{aligned}$$

This work by Bauerle and Brizo was performed on YsZ samples consisting of 10 molar percent yttria. YsZ is fully stabilized in the cubic form for this yttria concentration. Therefore, their study would not have identified any changes in activation energy due to transformations between monoclinic, tetragonal, and cubic phases.

Another interesting area in the study of the electrical properties of YsZ is the polarization characteristic. Knowledge of time dependent physical phenomena such as the diffusion of charge carriers at the electrode interfaces would be useful in future designs that use this material. However, this subject matter is outside the scope of this thesis.

## Chapter 3

# The YsZ Heater Element, Test Apparatus, and Instrumentation

Three very difficult challenges must be overcome if one is to study the properties of YsZ as a solid electrolyte conductor at temperatures above 1500 K. The first challenge relates to the inverse relationship between temperature and electrical conductivity. It is clear that the test apparatus must include a means to pre-heat the YsZ test specimen to approximately 1400 K to reduce the electrical resistivity; otherwise, an extremely high electrical field strength will be required to cause ionic conduction. Building a pre-heater that is able to operate at 1400 K and higher is an interesting problem in its own right.

The second challenge is in making the electrical connections to the YsZ specimen for conducting electrical current. Tackling this problem requires using a heater element geometry that creates uneven heating. The test apparatus must also implement separating chambers to control the uneven heating.

The third challenge is controlling the rate of temperature change for the high temperature ceramic materials. Sudden changes in temperature create stress concentrations which can result in fractured specimens and test apparatus components. For the size apparatus and YsZ heater elements that were tested, the time required to raise the temperature of the apparatus from room temperature to the desired test temperature, run the test, and cool back down to room temperature was more than a day.

Another difficulty that is encountered while running tests at high temperatures is controlling the path of electrical current flow. In the high temperature regime, many ceramics that would normally be considered insulators become good conductors. Electric currents can flow through "sneak" circuit paths in the ceramic walls and cause erroneous readings in some instrumentation. Experience has shown that stray current flow can also result in heating of ceramic supports to the point of melting and failure. Though not as troublesome as inadvertent current flow through ceramics, it should be noted that metals, which are good conductors at normal temperatures, become more resistive at high temperatures.

### 3.1 The YsZ Heater Element

Metallic electrodes in air would not survive at temperatures in the range of interest (2000 K to 2300 K). Platinum alloys provide the highest practical temperature range for suitable electrodes, but even the alloy of 70% platinum and 30% rhodium has a melting point of approximately 2140 K. To enable the use of electrodes that cannot withstand the maximum YsZ temperature, the geometry of the element has been selected to enable different surface temperatures to exist along the YsZ element. Fig. 3.1 shows a YsZ heater element prior to testing. The YsZ heater element reaches its highest temperatures in the slender cylindrical section at the center of the element - where the electrical resistance is highest. The two outer cylinders provide structure for electrical connection and mechanical support. Transitions between the center section and the two outer sections are conical. A multi-chamber furnace, to be described in the next section, isolates the inner cylindrical section. The wall thickness of the YsZ heater element is uniform throughout the test sample. The yttria to zirconia concentration is also uniform throughout the sample. In the figure, the platinum/rhodium electrodes that are connected to the outer cylinders are covered with zirconia paste.

Alternating current is applied to the YsZ heater element in the experiments. Direct current is not used because of deleterious effects at the electrode connections. If a dc voltage were applied to a YsZ heater element at high temperatures, oxygen would be absorbed at the cathode and  $O^-$  ions would be transported through the heater element. This unidirectional ionic current would cause corrosion at the electrical connections.

### 3.2 Apparatus

A specialized test apparatus, called the Gas Sample Cell, which was originally purchased by the US Air Force for testing gas chemistry at high temperatures, has been adapted for testing heating elements made of Yttria-stabilized Zirconia (YsZ). In its current configuration the test apparatus is sometimes called HOTSTuF - *High Operating Temperature Semiconductor Testing Facility*. The apparatus is shown in Fig. 3.2.

Three main components make up the Gas Sample Cell. These components are: a Computer Cabinet for the data acquisition and sequence controller computer, a Control Cabinet which houses the electric power controllers and indicators, and the High Temperature Furnace. The High Temperature Furnace consists of three sub assemblies: a dual-chamber furnace, a furnace base and support unit, and an environmental chamber (not used in these tests).

The dual-chamber furnace, as shown in Fig. 3.3, provides concentric chambers that operate at different temperatures. The inner chamber (called the working chamber) is made of precast zirconia sections. The working chamber houses the highest temperature section of the YsZ heater element under test. The working chamber is supported above the floor of the outer chamber (called the host chamber) by zirconia blocks so that it is completely enveloped by the host chamber. The host chamber houses eight molybdenum disilicide heater elements. The molybdenum disilicide heaters are used to preheat the furnace to above 1400

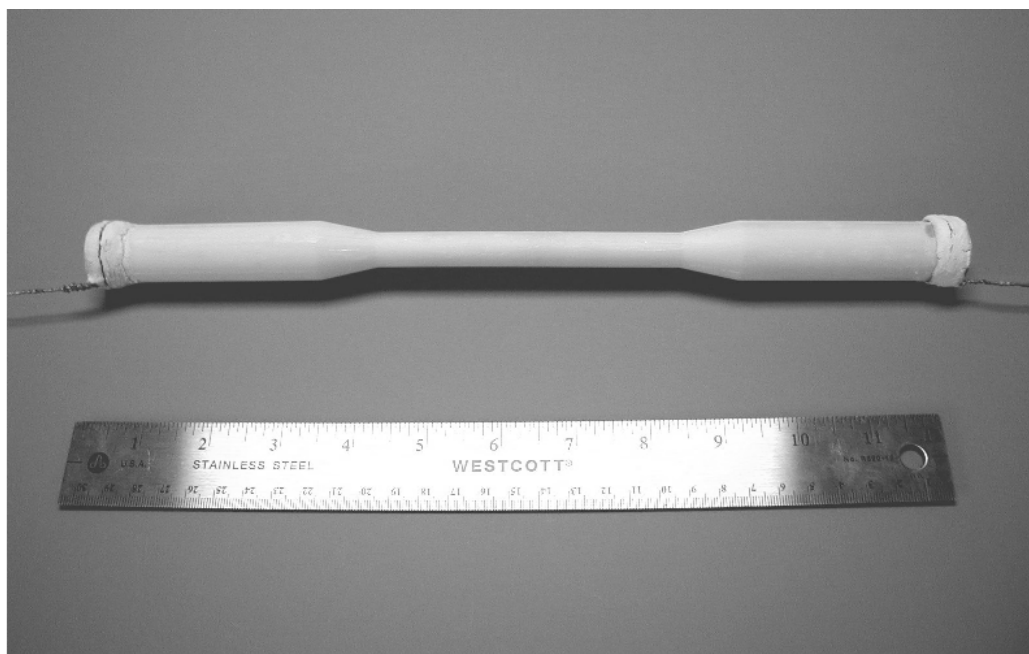


Figure 3.1: A YsZ heater element test specimen. This heater element was made by Materials and Systems Research, Inc. of Salt Lake City, Utah.

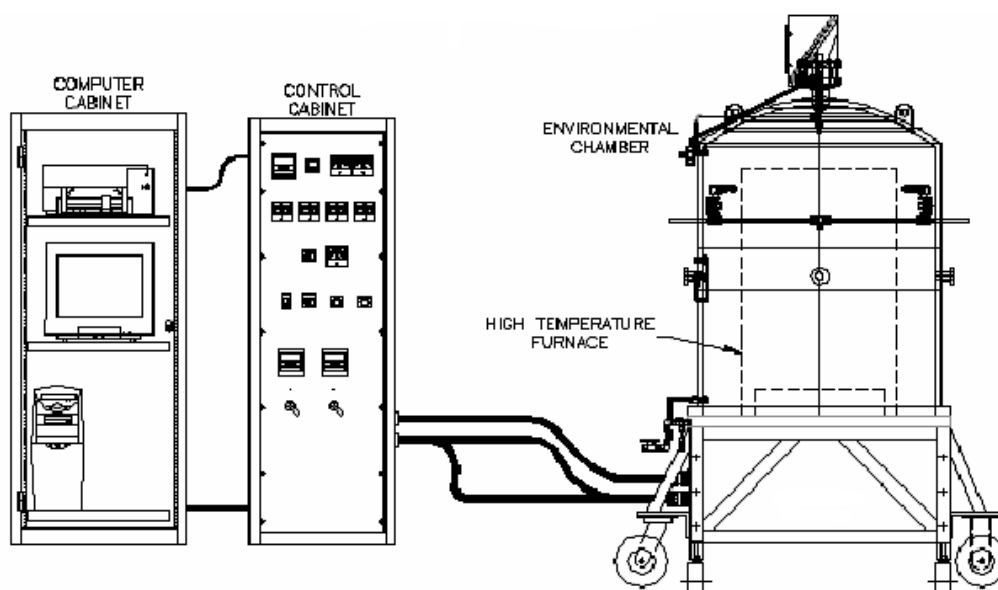


Figure 3.2: Gas Sample Cell apparatus

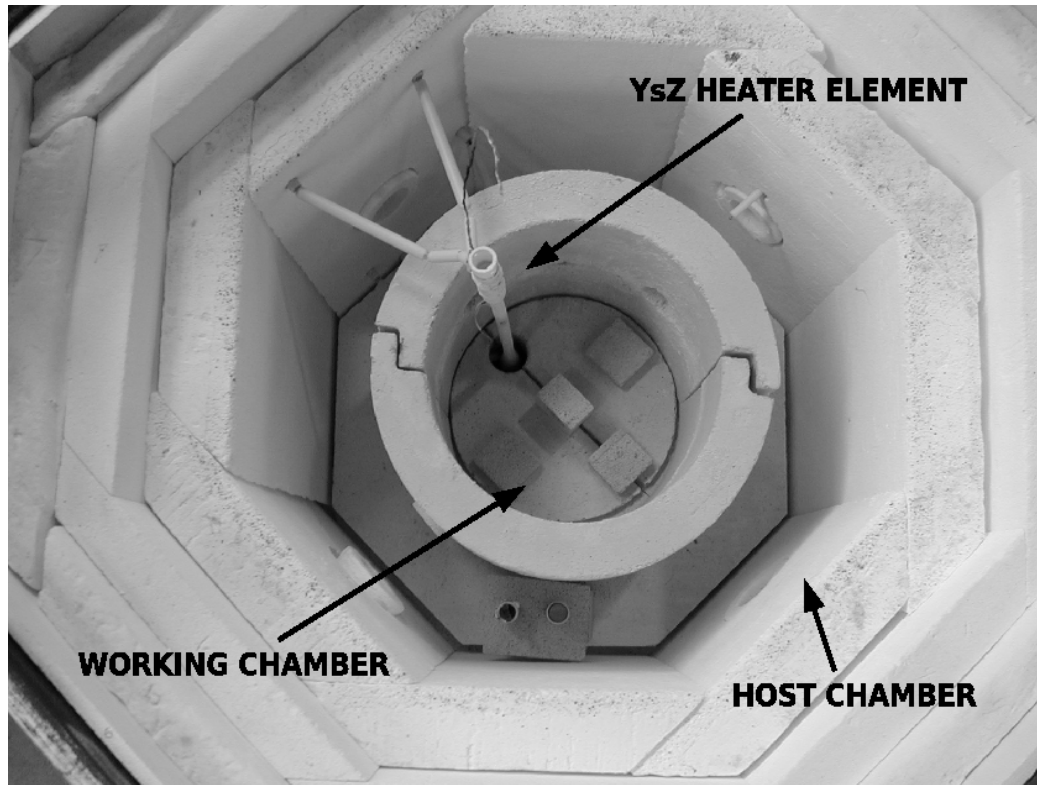


Figure 3.3: Top view of dual-chamber furnace. The tops of the working and host chambers have been removed along with the molybdenum disilicide heaters to reveal the interior of the working chamber. Thermocouple wires are contained in the ceramic tubes that extend from the YsZ heater element to the host chamber wall.

K - a temperature where the resistivity of the YsZ heater element is low enough to allow electrical conductivity with practical voltages. The outside walls of the host chamber are constructed from refractory materials including alumina and magnesium-oxide which provide thermal insulation to allow the host chamber to reach and withstand temperatures of approximately 1500 K. Two small view ports penetrate the insulation and allow viewing through the chambers. Small, tubular, ceramic feed-thrus are used for thermocouple wire routing.

The working chamber of the furnace must have the physical characteristics of a cavity radiator for it to be possible to determine the resistivity of the YsZ heater element from measurements of temperatures and electrical power transferred to the heater element. To ensure that the interior temperature is uniform, the working chamber is made of high-density cast zirconia. This material has a low thermal conductivity which aids in establishing steady temperatures within the working chamber. The rough surface of this material provides diffuse transmission of thermal radiation, which creates a uniform temperature environment. Heat transfer to the outside environment through convection is minimized by the use of

small view ports, which open into the host chamber, which is also at high temperature. The temperature of the inner wall of the working chamber is measured at a point midway up the cylindrical surface by the use of a thermocouple.

The control cabinet houses phase-controlled ac power supplies made by Halmar Robicon. The desired voltage output of the controllers can be set electronically by remote circuitry. Either a computer or a stand-alone electronic controller can be used to provide closed-loop feedback control. One of the power supplies is used to energize the molybdenum disilicide heaters. Another power supply is used for the control of power to the YsZ heater element.

The maximum output that the YsZ phase-controlled power supply can produce is a 60 Hz sinusoidal voltage with a maximum peak value of 1358 volts. To reduce the RMS output voltage, the controller delays operation of its output thyristors by a conduction angle,  $\delta$ , whenever the output crosses a voltage zero. Control of the voltage conduction angle is the only means used to control the amount of power supplied to the YsZ heater element. Anti-parallel thyristors are used so that both the positive and negative half-cycles of the voltage can be controlled. The controller is set so that the maximum allowed RMS voltage is 600 V RMS, which requires that the minimum conduction angle  $\delta$  is 1.74 rad (100 degrees), as can be calculated from

$$V_{RMS}^2 = \frac{V_p^2}{2\pi} \left( \pi - \delta + \frac{1}{2} \sin 2\delta \right). \quad (3.1)$$

Eq. 3.1 is derived in Appendix A.

The resultant waveform of the maximum allowed voltage is shown in Fig. 3.4. The non-sinusoidal shape has a harmonic content that could be helpful in studying the capacitance of the YsZ heater element, but otherwise causes electrical noise problems on instrumentation signals. Table 3.1 gives the RMS and peak voltages for typical conduction angles.

### 3.3 Instrumentation

Three contact measurements of voltages and two contact measurements of temperature are made on both ends of the YsZ heater element. The ends of the YsZ heater element are maintained below the melting temperature of platinum-rhodium alloy, which allows the material to be used in direct contact with the YsZ element. Fig. 3.5 shows how three platinum wire bands encircle each heater element at measured distances along the ends. The first bands from the ends are the power lead connections. A tap conductor is also connected to this junction to enable measuring the voltage at the junction. The remaining wire bands are connections for voltage measurement leads and thermocouples. The voltage measurement leads and thermocouple wires are fed through tiny ceramic ducts to the outside of the host chamber where they are connected to measuring instruments.

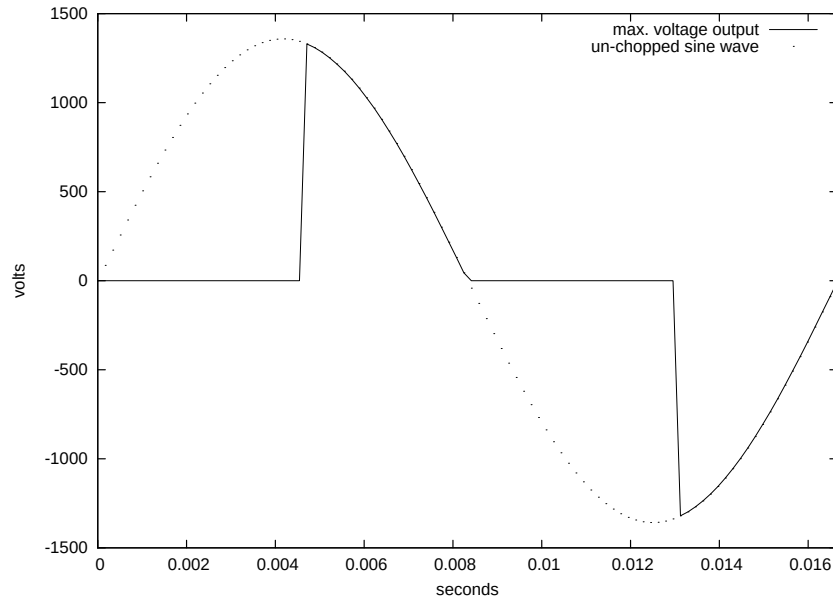


Figure 3.4: “Chopped” waveform of a single cycle of the maximum allowed output voltage from the power controller

Table 3.1: Resultant voltages for controller conduction angles

| $\delta(\text{degrees})$ | $V_{RMS}$     | $V_{peak}$ |
|--------------------------|---------------|------------|
| 100                      | $600 V_{RMS}$ | $1337 V_p$ |
| 112                      | $500 V_{RMS}$ | $1259 V_p$ |
| 123                      | $400 V_{RMS}$ | $1139 V_p$ |
| 134                      | $300 V_{RMS}$ | $977 V_p$  |

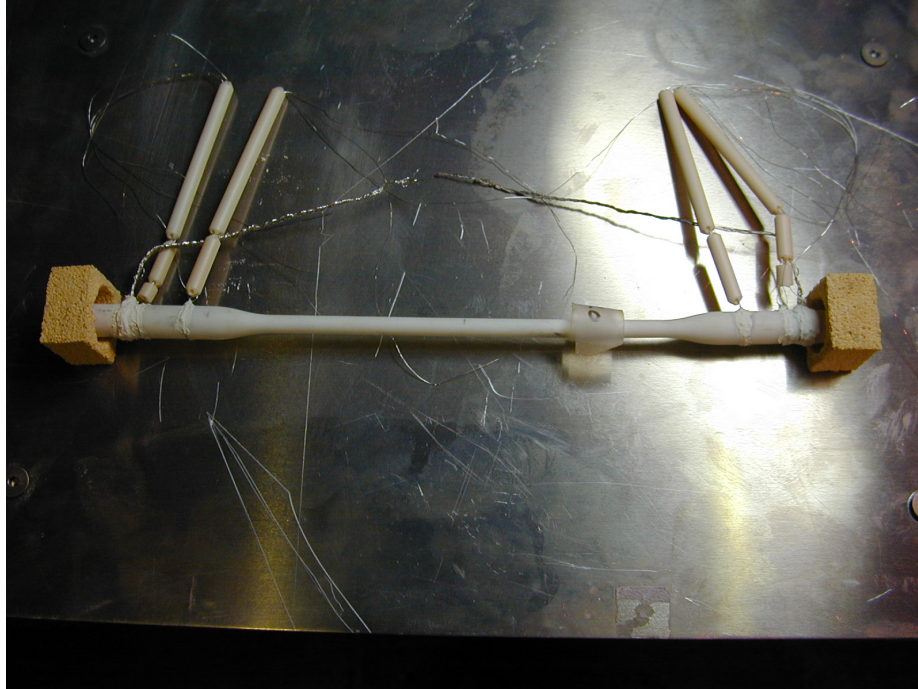


Figure 3.5: Electrode connections to the YsZ heater element

Because the temperatures to be measured are so high and the thermocouples are exposed to air, platinum-based thermocouples must be used. Two Land-Jewel thermocouples (with positive leads consisting of platinum-40% rhodium and negative leads consisting of platinum-20% rhodium) that were supplied as part of the original Gas Sample Cell instrumentation, are used in the host chamber. One Land-Jewel thermocouple serves as the feedback source for the host chamber molybdenum disilicide heater controller. The other measures temperature for the redundant over-temperature shutdown controller. ASTM type B thermocouples, which have a maximum usable temperature of about 2000 K, are used for the other surface and air temperature measurements inside the furnace. Type B thermocouples are made with a positive lead consisting of platinum-30% rhodium and a negative lead consisting of platinum-6% rhodium. The Seebeck Coefficient (the rate of change of thermal electro-motive force with temperature) is low for the type B thermocouple which makes the resolution of the thermocouple less than most other types [26]. There are other types of thermocouples that have higher usable temperatures than the platinum-based thermocouples; the ASTM Type C (tungsten-rhenium) thermocouple is an example, but none can be used at the required temperatures in an air environment.

The thermocouples that are directly connected to the YsZ element are subjected to a high common-mode voltage. The signal conditioning electronics that convert the thermocouple input to a linear output have a high degree of common-mode signal rejection, but voltage spikes do make it through the conditioners and reach the data acquisition system. With the data acquisition system recording two data points a second, there are occasions

when the data is corrupted by a voltage spike. The corrupted data points are obvious because their values are often seen as full-scale outputs from the signal conditioners.

Digital meters are connected to the voltage probes that are connected along the outer cylinders of the heater element. The meters calculate the RMS value of the voltage. To reduce the electrical noise that is present in the signals, the meters use digital filter circuits and also calculate the RMS values over a period of several cycles. The meters are also transmitters that output to the data acquisition system voltage signals that are proportional to the sensed RMS voltages.

A 0.1 ohm, 1 % tolerance resistor connected in series with the YsZ heater element provides the means to measure the electrical current. Using Ohm's Law, a signal conditioner measuring the RMS voltage across the precision resistor calculates the RMS value of current flowing through the YsZ element.

Electromagnetic interference (EMI) is severe in the HOTSTuF furnace area. A troublesome source of EMI is the heater circuit for the molybdenum disilicide heater elements. Good practice for minimizing the electric and magnetic fields emanating from the conductors would require that the supply and return conductors be routed near each other. However, the conductors are separated before they reach the furnace and the geometry of the heaters does not take advantage of any field reduction techniques. The current through the molybdenum disilicide heater elements is typically around 70 amps RMS, and this current is also characterized by large spikes. Another source of EMI is the voltage on the YsZ element, which as mentioned earlier, is characterized by spikes that can approach 2600 volts peak-to-peak. Post-run data reduction and filtering techniques which are used to remove the electrical noise are described in Section 3.5

A Williams TempMatic 8000 IR radiation ratio thermometer (sometimes called a two-color pyrometer) is used to measure the surface temperature of the YsZ heater. The radiation ratio thermometer, which is mounted outside the furnace, is aimed through a view port opening at the mid-point of the YsZ element. Care is taken to ensure that the focusing optics and aperture are aligned so that only the YsZ heater element, and not the inner working chamber wall, is in the field of view of the thermometer.

### 3.4 Data Acquisition, Data Recording, and Control

The data that is measured by the instrumentation is recorded digitally. The rate at which the measured parameter changes determines the data sampling rate that is used for the digital recording. Parameters, such as temperature, that change slowly enough to be recorded at a 2 samples-per-second rate are considered static data. All other parameters are considered dynamic and are recorded with a digital storage oscilloscope.

A LeCroy WaveSurfer four channel digital storage oscilloscope was used to make high resolution, short duration data recordings on demand during tests. This 350 MHz oscilloscope was set to record data at a rate of five million samples per second. Recordings of

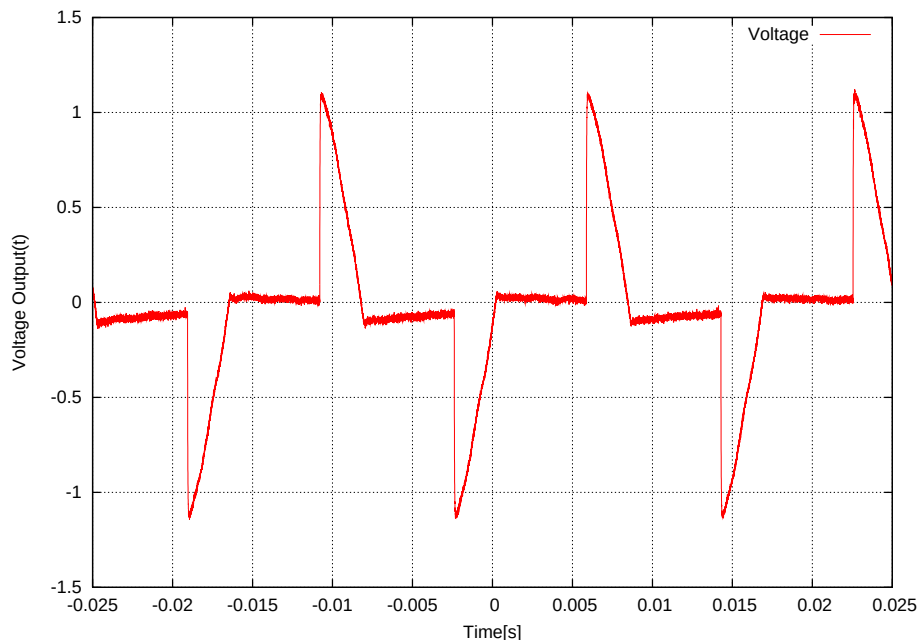


Figure 3.6: A voltage waveform recorded on the digital storage oscilloscope during a test.

voltage waveforms and signals from the internal electronics of the ratio thermometer were captured. For example, Fig. 3.6 shows a voltage waveform that was captured with the

digital storage oscilloscope on a test run while the YsZ heater element was 1970 K. This recorded voltage waveform was analyzed to determine the harmonic spectrum of the frequency components of the chopped wave. Fig. 3.7, which depicts the harmonic components of the waveform in Fig. 3.6, was produced by analysis with the fast Fourier transform function in the Octave software package.

A single portable computer is used to both record the static data and control the electrical current flowing through the YsZ heater element. Inputs to the computer are provided by a maximum of thirty-two analog-to-digital signal conditioners. Up to sixteen data acquisition channels are available through an Analog Devices 6B series analog-to-digital signal conditioner via the computer's RS-232 COMM port. A National Instruments' input/output module provides an additional sixteen input channels of analog-to-digital conversion and two output channels of digital-to-analog conversion via the computer's USB port. LabVIEW software is used to acquire and display measurements from the voltage transmitters, current transmitter, thermocouple transmitters, and radiation ratio thermometer. These measurements are sampled and recorded at a rate of two samples per second. Fig. 3.8 shows the main LabVIEW computer screen where the measurements are monitored.

The control routine is also written with LabVIEW software. The controlled parameter is the electric current through the YsZ element. The control routine includes a sequencer

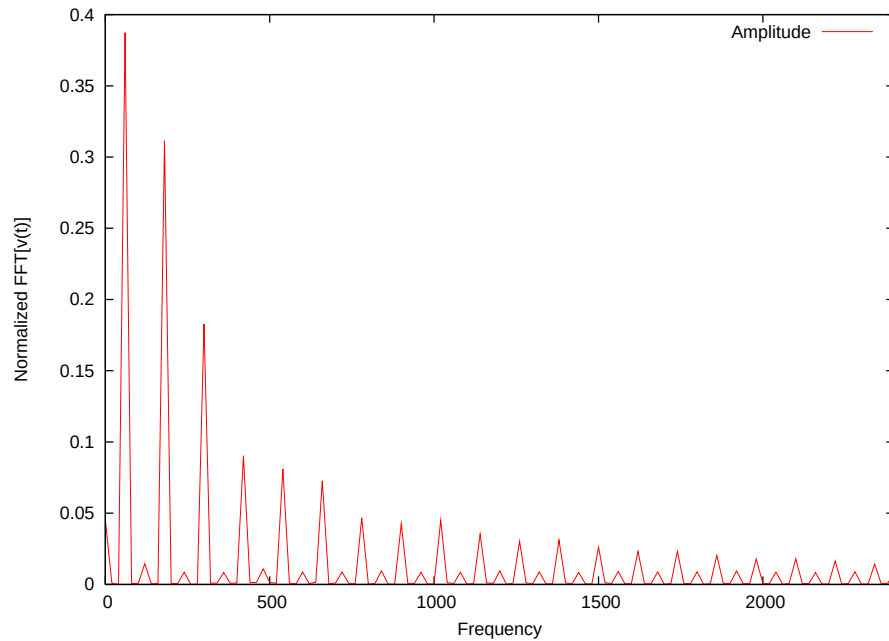


Figure 3.7: Fourier transform of the voltage waveform in Fig. 3.6. The component of the voltage with the fundamental frequency of 60 Hz is shown to be less than 40 percent of the total waveform.

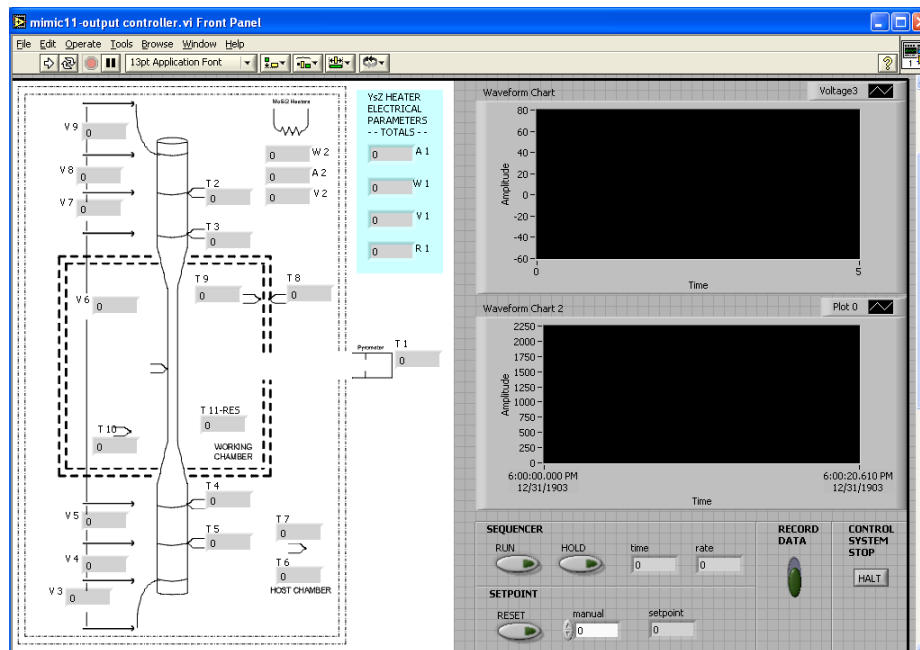


Figure 3.8: Operator screen on control computer for displaying temperature and electrical parameters

and a feedback controller. The set point for the YsZ current is set by the sequencer routine which linearly ramps the set point at rates and for durations predetermined for the operation sequence. The sequencer can manually be placed in “hold” to allow long data acquisition periods. The feedback controller uses a proportional-integral-derivative (PID) algorithm to maintain the electric current output to the YsZ heater element at the set point. The proportional gain, integral time, and differential time parameters that are used in the PID algorithm have been determined by experiment. Fig. 3.9 shows the LabVIEW computer screens that are used for the controller and sequencer inputs. A typical operation sequence is shown in Table 3.2

An additional stand-alone controller is used for the host chamber molybdenum disilicide heaters. For this controller, the host chamber temperature is the controlled parameter. The stand-alone controller ramps the temperature of the host chamber to the temperature required for the test sequence and maintains the host chamber at the required temperature for the duration of the test. At the end of the test, the controller ramps the host chamber temperature back to room temperature.

A stand-alone safety over-temperature relay is connected to the control circuits of the heaters to protect the furnace from dangerous high temperatures. Operation of the over-temperature relay results in an electric power trip to the heaters. This safety feature is not intended to be used in normal testing operation because tripping the heaters would result in uncontrolled cooling of the test article and furnace.

### 3.5 Data Reduction and Filtering

With the exception of using a low-pass filter while recording dynamic data, all of the data filtering and reduction was performed after the test on recorded data. The static data and the dynamic data that were recorded during the test runs were processed differently. The static data was to be shared with other engineers and scientists who had requested that the data be provided in the Microsoft Excel spreadsheet format. To satisfy this request, a Visual Basic macro that ran within the Excel program was written to filter and reduce the static data. For processing the dynamic data, a filter routine that ran in BASIC was written. Several Octave (an open source alternative to MATLAB) scripts were written for data analysis and to produce the charts that are presented in this thesis.

The most significant problem to overcome in processing the static data was filtering out interference caused by common-mode electronic noise on the thermocouple instrumentation. Because the thermocouples are connected directly to the YsZ heater elements, the voltage that is present on the heater element is also present on both leads of the thermocouple. The non-sinusoidal waveform of the YsZ voltage creates periodic common-mode spikes on the signal conditioners, which in turn, saturate and create periodic spikes of maximum output. If a spike is being passed by the signal conditioner at the instant that the analog to digital converter is sampling the signal, an erroneous high temperature is recorded for the sample.

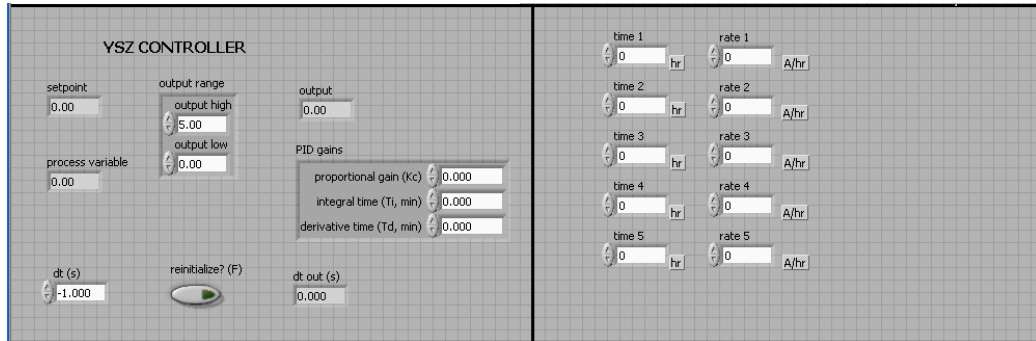


Figure 3.9: The gains for the PID control routine are set via the left-hand portion of this LabVIEW screen. The right-hand side is the input screen for the sequencer times and rates of change.

Table 3.2: Typical test plan sequence showing ramp rates and time delays in the computer controller.

| Time  | Event                                                           | YsZ Controller | MoSi <sub>2</sub> Controller |
|-------|-----------------------------------------------------------------|----------------|------------------------------|
| 00:00 | Heat Host Chamber to 1600 K in 4.5 hours                        | off            | ramp                         |
| 04:30 | Pause 30 minutes to check instrumentation before proceeding     | dwel           | dwel                         |
| 05:00 | Heat YsZ heater element from 1600 K to 2300 K in 3.5 hours      | ramp           | dwel                         |
| 08:30 | Hold YsZ temperature at 2300 K for 1 hour                       | dwel           | dwel                         |
| 09:30 | Cool YsZ heater element from 2300 K to 1600 K in 3.5 hours      | ramp           | dwel                         |
| 01:00 | Cool Host Chamber to room temperature in no less than 4.5 hours | off            | ramp                         |

Because temperatures change fairly slowly on the YsZ heater element, a review of the sampled data quickly shows a random-looking number of samples that are obviously erroneous.

A statistical filtering routine was written for post-processing the static data to remove the noise spikes. This statistical filter stepped through the data and determined which samples were “outliers” by comparing the value of the current sample with a moving window consisting of the average value of the preceding 40 samples. If the current sample exceeded a variation of two standard deviations from the previous 40 samples, the value was replaced by the average value. This statistical filter routine is a type of low-pass filter that works very well at detecting and removing random bad data points from static data. To remove the noise spikes from the data, the filter routine was typically run five to ten times on each channel of thermocouple data.

Because the static data changed slowly with respect to the rate at which it was recorded, reduced data sets could be created easily from the filtered data without causing aliasing errors. For example, over periods on the order of five seconds, usually only minor changes would be perceptible in the data. For reviewing the data, it was acceptable to reduce the data by selecting every twentieth data point. Once transients or other interesting transitions were detected in the reduced data set, it was a simple matter to use the time correlation between the reduced data set and the original data recording to zoom in on the event to be analyzed.

However, the dynamic nature of the data recorded with the digital storage oscilloscope necessitated that greater care be taken in its filtering and data reduction to avoid aliasing errors. The oscilloscope’s sampling rate was  $5 \times 10^6$  samples per second - a far faster sampling rate than necessary to capture the interesting frequency components of the waveform. The Nyquist frequency of this sampling rate is  $2.5 \times 10^6$  Hz. This sample rate produces an oversampled file (the frequencies of interest are less than  $1 \times 10^4$  Hz). To downsample the file without losing information, a three-pass, overlapping moving average filter was written. The moving average filter is a type of low-pass filter. By post-processing the data file through three passes of the filter, the Nyquist frequency was reduced to an eighth ( $1/2^3$ ) of its original value and the number of samples per cycle was reduced from over 83,000 samples per cycle to about 10,400 samples per cycle.

A copy of the overlapping, moving average filter program, written in BASIC, is attached in Section B.1. This program reduces the data samples by half on each pass - giving a final reduction to one eighth of the original number of samples. The kernel of the filter is a convolution of the sampled data and a moving four point block that has a value of 0.25 for each of the four points. Use of a four point block for the convolution on every other sample creates the overlap. The kernel is looped through the data three times to achieve the total data reduction to one eighth of the original number of samples.

## Chapter 4

# Radiative Properties

### 4.1 Introduction

The Gas Sample Cell and the YsZ heater element are designed to minimize the amount of heat that is transferred from the YsZ element by conduction and convection. Radiation is the dominant heat transfer mechanism, and when allowed to reach temperature equilibrium, the working chamber approximates an ideal cavity radiator. The cylindrical side walls of the inner working chamber (made of dense zirconia) provide a uniform temperature environment. Eight molybdenum disilicide heater elements in the surrounding host chamber maintain a controlled temperature differential across the working chamber walls, allowing the heat to be transferred uniformly from the YsZ heater element to the working chamber walls and then out of the apparatus. The temperatures of the inner and outer working chamber wall are measured with type B thermocouples and recorded twice per second during operation of the Gas Sample Cell. A small view port through the host and working chamber walls allows viewing of the hottest part of the exterior of the YsZ element during Gas Sample Cell operation. A radiation ratio thermometer, loosely called a "two-color pyrometer", is used to make radiometric measurements of the temperature of the YsZ element through the view port.

Measurements of the emissivity of YsZ at high temperatures are not available in published literature. Even if published data of the basic emissivity of YsZ did exist, it might not be useful for characterizing the emissivity of the heater elements. This is because emissivity is not a purely intrinsic property of a material but is affected by surface finish and shape. Emissivity can change over time as chemical changes occur in the ceramic material. In this investigation, measurements of temperature of the YsZ heater element, the inner wall of the working chamber, and thermal radiation from the YsZ heater element are used to calculate the high temperature emissivity of the YsZ heater element.

### 4.2 Thermal Radiation Heat Transfer

It is important to use a consistent set of symbols, terminology, and units to describe thermal radiation and radiometric quantities. In this paper, the International System of Units (SI)

as described in Publication 811, Guide for the Use of the International System of Units, [24] of The US National Institute of Standards and Technology is used. The pertinent SI radiometric quantities are:

$M$  = Radiant Emittance (power per unit area emitted from a surface),  
 $E$  = Irradiance (power per unit area incident on a surface), and  
 $J$  = Radiosity (emitted plus reflected power per unit area leaving a surface).

The SI units for power per unit area are  $W/m^2$ .

To denote spectral quantities, the subscript  $\lambda$  is used. For example,  $M_\lambda$  denotes the power per unit area emitted from a surface in the interval  $d\lambda$  at the wavelength  $\lambda$ . The units commonly used in thermal radiometry for spectral quantities are  $W/(m^2 \cdot \mu m)$ . The integration of a spectral quantity over all wavelengths gives the associated radiometric quantity. For example,

$$M = \int_0^\infty M_\lambda d\lambda. \quad (4.1)$$

A mathematical description of the properties of a blackbody is needed because emissivity and absorptivity of opaque surfaces are defined in relation to emittance and absorption of a blackbody. A blackbody is a theoretically perfect emitter and absorber of thermal radiation. During the 19<sup>th</sup> century, through thermodynamic considerations, it was realized that a cavity radiator, whose interior is at thermal equilibrium, is effectively a blackbody. This is because for the cavity to be in thermal equilibrium at all locations within the cavity, regardless of the material present at the location, the rate at which energy is emitted must equal the rate at which it is absorbed. By use of small apertures to observe the radiation inside cavity radiators, by the end of the 19th century, experimenters had determined that the radiant emittance of blackbodies is a function of temperature only and that the spectral radiant emittance follows a distribution that is also a function of temperature. In 1901, Max Planck theorized that the source of the radiation was a large number of distinguishable oscillators, each of which absorbed or emitted energy proportional to the frequency of the oscillator,  $\Delta e = h\nu$ . He also postulated that the oscillators could not have arbitrary energy but must occupy one of a discrete set of energy states given by  $e_n = nh\nu$ . Planck used these postulates and Maxwell-Boltzmann statistics to derive the equation for the distribution of electromagnetic radiant emittance for a blackbody as a function of temperature and wavelength [25]. This equation is commonly called Planck's law,

$$M_{B\lambda} = \frac{2hc^2}{\lambda^5 [\exp(hc/\lambda kT) - 1]}, \quad (4.2)$$

where

$M_{B\lambda}$  = blackbody spectral radiant emittance (power per unit area over the wavelength range  $\lambda + d\lambda$ ),

$\lambda$  = wavelength,

$h$  = Planck's constant,  
 $c$  = speed of light in a vacuum,  
 $k$  = Boltzmann's constant, and  
 $T$  = temperature.

With  $c_1 = 2hc^2$ , and  $c_2 = hc/k$ , at short wavelengths ( $hc \gg \lambda kT$ ), Planck's law reduces to Wien's approximation,

$$M_{B\lambda} \approx c_1 \lambda^{-5} \exp(-c_2/\lambda T). \quad (4.3)$$

The Gas Sample Cell's ratio thermometer makes measurements of radiation at the wavelengths 0.90 microns and 0.81 microns. The ratio thermometer is capable of measuring temperatures over the range from 1773 K to 2673 K. The percent error incurred by using Wien's approximation, instead of Planck's law, to calculate spectral radiant emittance over this range of temperatures and wavelengths varies from a maximum of 0.253 percent at 2673 K and 0.90 microns to a minimum of 0.004 percent at 1773 K and 0.81 microns.

For the more realistic case of materials that are not blackbodies, perfect absorption of thermal radiation does not occur (some radiation is reflected and some radiation may be transmitted). Absorptivity ( $\alpha$ ) is defined as the fraction of the total energy of isotropic blackbody radiation that is absorbed, so that the rate at which radiant energy is absorbed by a non-blackbody per second per unit area is  $\alpha E_B$ . Similarly, a non-blackbody emits less total radiant energy than a blackbody at a given temperature. Emissivity ( $\epsilon$ ) is the ratio of the radiant emittance of a non-blackbody to the emittance of a blackbody at the same temperature. The radiant emittance of a non-blackbody is  $\epsilon M_B$ .

Kirchhoff's law of thermal radiation [21] states, for a surface to be at thermal equilibrium, the rate that energy is absorbed must be equal to the rate at which energy is emitted. The radiant emittance of a blackbody at thermal equilibrium is equal to the irradiance, that is  $E_{B\lambda} = M_{B\lambda}$ . Therefore the emissivity ( $\epsilon$ ) of a surface must be equal to its absorptivity,

$$\alpha E_{B\lambda} = \epsilon M_{B\lambda}. \quad (4.4)$$

Radiation that is observed from an opaque body is the summation of three processes: emission from the surface of the body, reflection of irradiation, and absorption by air in the flux path. The rate of energy transfer due to the emission of thermal radiation from the body's surface is determined by the body's temperature and the nature of its surface. At wavelengths of 0.90 and 0.81 microns, the transmittance of air is nearly 1 [22]. The radiation measurements that were made for this study are through short flux paths in air ( $< 0.5\text{m}$ ). Also, calibrations of instruments were made in similar atmospheric environments

to cancel out the effect of air path transmittance. For these reasons, the transmittance of air is neglected for the remainder of this analysis.

When thermal radiation is incident on a body that is not opaque, fractions of the irradiance are absorbed, reflected, and transmitted. Defining  $\varrho$  as the fraction of the irradiance that is reflected and  $\tau$  as the fraction transmitted, the total irradiance is

$$E = \alpha E + \varrho E + \tau E. \quad (4.5)$$

At thermal equilibrium ( $\alpha = \epsilon$ , and  $E = M$ ), the thermal radiation that is observed from the body will include emitted ( $\epsilon M$ ), reflected ( $\varrho M$ ), and transmitted ( $\tau M$ ) components. The sum of the fractions of observed radiation is 1,

$$\epsilon + \varrho + \tau = 1. \quad (4.6)$$

Inside the Gas Sample Cell, the source of all irradiance on the YsZ heater element is the wall of the working chamber. Because the working chamber approximates a cavity radiator, the spectral irradiance and total irradiance are independent of the location of the YsZ element in the chamber [23]. With  $E_\lambda$  equal to the spectral irradiance impinging on the YsZ heater element over the wavelength range  $\lambda + d\lambda$ , and  $T_w$  equal to the absolute temperature of the working chamber wall, the spectral irradiance can be calculated using Wien's approximation,

$$E_\lambda \approx c_1 \lambda^{-5} \exp(-c_2/\lambda T_w). \quad (4.7)$$

This is because the radiant emittance of a cavity radiator is blackbody radiation.

From Kirchhoff's law of thermal radiation, at thermal equilibrium the fraction of irradiance that is absorbed by the YsZ heater element is equal to the element's emissivity. The remainder of the irradiance is either reflected or transmitted. The radiation that is viewed by the ratio thermometer then includes the radiant emittance of the YsZ heater element and the reflected and transmitted irradiance. This is true of the total radiation observed and the spectral components. The emitted radiance of the YsZ heater element is  $\epsilon M_{B\lambda}$ . The irradiance which is not absorbed by the YsZ heater element is  $(1 - \epsilon)E_\lambda$ . The total observed spectral radiosity,  $J_\lambda$ , which includes emitted, reflected, and transmitted radiation is

$$J_\lambda = \epsilon M_{B\lambda} + (1 - \epsilon)E_\lambda. \quad (4.8)$$

### 4.3 Radiometric Measurements and Calibration

Radiation ratio thermometry uses measurements of the emitted radiation of a surface at two different wavelengths to infer the temperature of the surface from the ratio of the two measurements. By assuming that the surface acts as a graybody (i.e. the emissivity is less than unity, and is constant at all wavelengths that are being measured by the ratio thermometer), an equation can be developed to determine the surface temperature from the measured radiant emittances. For spectral radiances of  $M_{\lambda_1}$  and  $M_{\lambda_2}$  measured at wavelengths  $\lambda_1$  and  $\lambda_2$ , Wien's approximation (Eq. 4.3) and Kirchoff's law of thermal radiation (Eq. 4.4) can be combined and used to establish the ratio thermometer equation,

$$\frac{M_{\lambda_2}}{M_{\lambda_1}} = \frac{\epsilon_2 c_1 \lambda_2^{-5} \exp(-c_2/\lambda_2 T)}{\epsilon_1 c_1 \lambda_1^{-5} \exp(-c_2/\lambda_1 T)}, \quad (4.9)$$

from which follows,

$$\ln(M_{\lambda_2}/M_{\lambda_1}) = \ln(\epsilon_2/\epsilon_1) - 5 \cdot \ln(\lambda_2/\lambda_1) + (c_2/T)(\lambda_2 - \lambda_1)/\lambda_1 \lambda_2. \quad (4.10)$$

For  $\epsilon_1 = \epsilon_2$  (graybody assumption) the  $\ln(\epsilon_2/\epsilon_1)$  is zero. Solving the remaining equation for  $T$  gives

$$\frac{1}{T} = \frac{\lambda_1 \lambda_2}{c_2(\lambda_2 - \lambda_1)} [\ln(M_{\lambda_2}/M_{\lambda_1}) + 5 \cdot \ln(\lambda_2/\lambda_1)]. \quad (4.11)$$

It must be noted that the use of the graybody assumption adds uncertainty to the results of the measurements.

In the Gas Sample Cell apparatus, a Williamson model 8000 ratio thermometer is used to make radiometric measurements. Internal to this ratio thermometer are: optics for focusing the radiation to be measured, a radiometer sensor, electronics for amplifying the signal from the sensor, and a rotating filter wheel. The wheel has two windows made of optical bandpass filter materials. One window passes infrared radiation with a wavelength centered at 0.81 microns. The center wavelength of the other filter is 0.90 microns. As the wheel rotates in the beam, the radiation is alternately blocked, filtered so that only the radiation with 0.90 micron wavelength is passed, blocked, and then filtered at 0.81 microns. For typical applications of this device, the internal electronics measure the peak outputs of the radiometer as it is exposed to the two filtered signals, calculate the ratio of the signals, and uses Eq. 4.11 to determine the temperature of the radiating surface.

To be able to determine the emissivity of the YsZ heater element, the electronics of the Williamson 8000 ratio thermometer have been modified so that the radiometer signal is

output to a digital storage oscilloscope. As seen on the oscilloscope, the signal appears as a train of pulses. Fig. 4.1 is a recording of the raw output from the radiometer. For the range of temperatures of interest, the higher pulse corresponds to the output of the radiometer when exposed to the source through the 0.90 micron filter and the lower pulse is the output with the 0.81 micron bandpass filter.

The modified electronics of the Williamson ratio thermometer allow it to be used as an absolute radiometer at the wavelengths of 0.81 and 0.90 microns. A calibration of the outputs at the two wavelengths was obtained in the Precision Measurement Equipment Laboratory (PMEL) at AEDC. In the PMEL, a Mikron M390 precision radiation source was used to provide blackbody radiation at temperatures from 1773 K to 2673 K. A Mikron model M190 infrared thermometer with calibration traceable to NIST was used to accurately determine the temperature at each step of the calibration. Table 4.1 shows the calculated, by use of Eq. 4.3, blackbody spectral radiant emittance at 0.90 and 0.81 microns at each temperature step. The height of each corresponding peak in the measured output of the ratio thermometer is included in the table.

Using the data that was recorded in the calibration of the ratio thermometer and the computation software Octave, second-order polynomial equations were determined for the relationship between the ratio thermometer outputs and the actual blackbody radiation that was irradiating the sensor ( $M_{0.90}$  and  $M_{0.81}$ ). The calibration equations are:

$$M_{0.90} = (-3.298 \times 10^3)x^2 + (1.673 \times 10^8)x - 1.412 \times 10^9, \quad (4.12)$$

$$M_{0.81} = (-6.248 \times 10^3)x^2 + (2.257 \times 10^8)x - 7.828 \times 10^8. \quad (4.13)$$

Figures 4.2 and 4.3 show the the actual calibration points and the calibration curves.

## 4.4 Application of the Modified Ratio Thermometer

With the modified ratio thermometer it is possible to determine the emittance of a radiating surface. For a surface that is not reflecting due to irradiance (i.e., only emitted thermal radiation is present), it is not difficult to solve for emissivity from the ratio of the Wien's approximations for the two spectral radiant emissions (Eq. 4.9). The graybody assumption ( $\epsilon_1 = \epsilon_2 = \epsilon$ ) provides the means to solve the equation for the unknown emissivity.

First, rearranging Wien's approximation to solve for  $1/T$  gives

$$1/T = \frac{-\lambda}{c_2} \cdot \ln \left( \frac{\lambda^5 M_\lambda}{\epsilon c_1} \right). \quad (4.14)$$

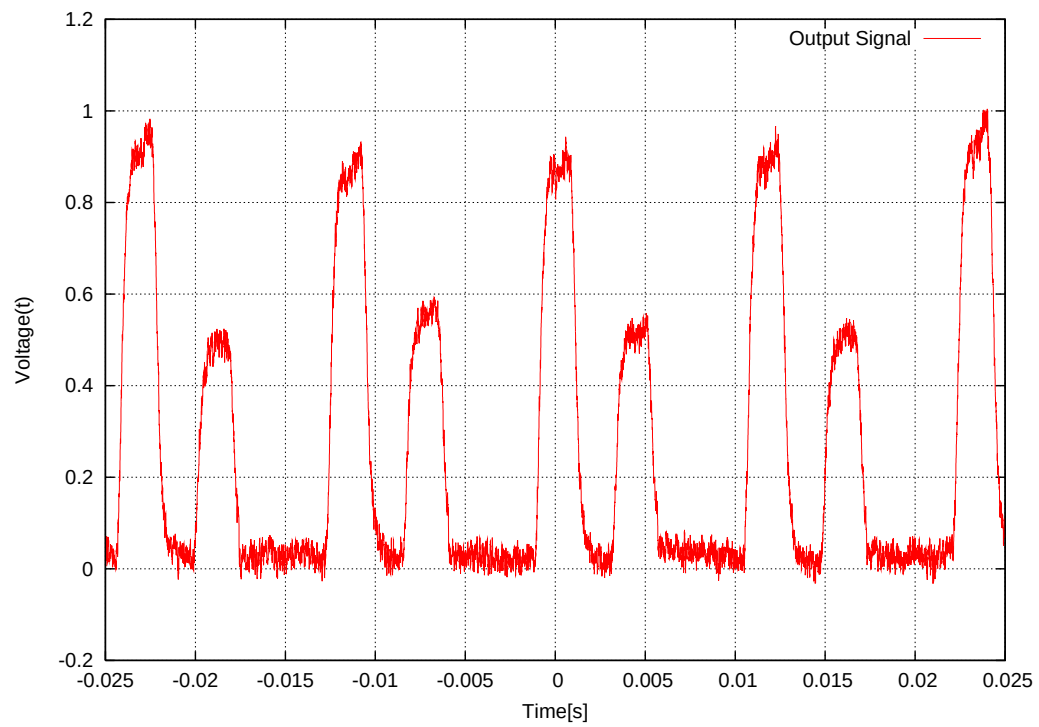


Figure 4.1: Output signal from the ratio thermometer

Table 4.1: Calibration of the Williamson Model 8000 ratio thermometer for absolute radiation measurements

| Temperature<br>K       | Calculated<br>$M_{0.90}$ | Williamson<br>mV Output<br>0.90 micron | Calculated<br>$M_{0.81}$ | Williamson<br>mV Output<br>0.81 micron |
|------------------------|--------------------------|----------------------------------------|--------------------------|----------------------------------------|
| 1773                   | $2.45 \times 10^{10}$    | 151                                    | $1.52 \times 10^{10}$    | 70                                     |
| 1793                   | $2.71 \times 10^{10}$    | 167                                    | $1.70 \times 10^{10}$    | 78                                     |
| 1813                   | $2.99 \times 10^{10}$    | 187                                    | $1.90 \times 10^{10}$    | 87                                     |
| 1833                   | $3.29 \times 10^{10}$    | 203                                    | $2.11 \times 10^{10}$    | 97                                     |
| 1853                   | $3.61 \times 10^{10}$    | 226                                    | $2.35 \times 10^{10}$    | 108                                    |
| 1873                   | $3.96 \times 10^{10}$    | 245                                    | $2.60 \times 10^{10}$    | 118                                    |
| 1893                   | $4.34 \times 10^{10}$    | 268                                    | $2.87 \times 10^{10}$    | 129                                    |
| 1913                   | $4.74 \times 10^{10}$    | 289                                    | $3.17 \times 10^{10}$    | 143                                    |
| 1933                   | $5.16 \times 10^{10}$    | 320                                    | $3.49 \times 10^{10}$    | 158                                    |
| 1953                   | $5.62 \times 10^{10}$    | 346                                    | $3.83 \times 10^{10}$    | 176                                    |
| 1973                   | $6.11 \times 10^{10}$    | 382                                    | $4.20 \times 10^{10}$    | 192                                    |
| 1993                   | $6.62 \times 10^{10}$    | 408                                    | $4.60 \times 10^{10}$    | 212                                    |
| 2013                   | $7.17 \times 10^{10}$    | 442                                    | $5.03 \times 10^{10}$    | 228                                    |
| 2033                   | $7.76 \times 10^{10}$    | 486                                    | $5.48 \times 10^{10}$    | 248                                    |
| 2053                   | $8.37 \times 10^{10}$    | 510                                    | $5.97 \times 10^{10}$    | 270                                    |
| 2073                   | $9.03 \times 10^{10}$    | 554                                    | $6.49 \times 10^{10}$    | 292                                    |
| 2093                   | $9.72 \times 10^{10}$    | 590                                    | $7.04 \times 10^{10}$    | 314                                    |
| 2113                   | $1.04 \times 10^{11}$    | 648                                    | $7.63 \times 10^{10}$    | 344                                    |
| 2133                   | $1.12 \times 10^{11}$    | 678                                    | $8.26 \times 10^{10}$    | 372                                    |
| 2153                   | $1.20 \times 10^{11}$    | 736                                    | $8.92 \times 10^{10}$    | 404                                    |
| 2173                   | $1.29 \times 10^{11}$    | 796                                    | $9.63 \times 10^{10}$    | 440                                    |
| 2193                   | $1.38 \times 10^{11}$    | 852                                    | $1.04 \times 10^{11}$    | 464                                    |
| 2213                   | $1.47 \times 10^{11}$    | 912                                    | $1.12 \times 10^{11}$    | 508                                    |
| 2233                   | $1.57 \times 10^{11}$    | 968                                    | $1.20 \times 10^{11}$    | 548                                    |
| 2253                   | $1.67 \times 10^{11}$    | 1036                                   | $1.29 \times 10^{11}$    | 580                                    |
| 2273                   | $1.78 \times 10^{11}$    | 1092                                   | $1.38 \times 10^{11}$    | 624                                    |
| 2293                   | $1.89 \times 10^{11}$    | 1180                                   | $1.48 \times 10^{11}$    | 676                                    |
| 2313                   | $2.01 \times 10^{11}$    | 1272                                   | $1.58 \times 10^{11}$    | 736                                    |
| 2333                   | $2.13 \times 10^{11}$    | 1328                                   | $1.69 \times 10^{11}$    | 776                                    |
| 2353                   | $2.26 \times 10^{11}$    | 1408                                   | $1.80 \times 10^{11}$    | 824                                    |
| 2373                   | $2.39 \times 10^{11}$    | 1490                                   | $1.92 \times 10^{11}$    | 890                                    |
| 2393                   | $2.53 \times 10^{11}$    | 1550                                   | $2.04 \times 10^{11}$    | 930                                    |
| 2413                   | $2.68 \times 10^{11}$    | 1620                                   | $2.17 \times 10^{11}$    | 980                                    |
| 2433                   | $2.83 \times 10^{11}$    | 1710                                   | $2.31 \times 10^{11}$    | 1020                                   |
| 2453                   | $2.98 \times 10^{11}$    | 1840                                   | $2.45 \times 10^{11}$    | 1110                                   |
| 2473                   | $3.14 \times 10^{11}$    | 1990                                   | $2.60 \times 10^{11}$    | 1200                                   |
| 2493                   | $3.31 \times 10^{11}$    | 2080                                   | $2.75 \times 10^{11}$    | 1250                                   |
| 2513                   | $3.48 \times 10^{11}$    | 2170                                   | $2.91 \times 10^{11}$    | 1350                                   |
| Continued on next page |                          |                                        |                          |                                        |

**Table 4.1 – continued from previous page**

| <b>Temperature<br/><math>K</math></b> | <b>Calculated<br/><math>M_{0.90}</math></b> | <b>Williamson<br/>mV Output<br/>0.90 micron</b> | <b>Calculated<br/><math>M_{0.81}</math></b> | <b>Williamson<br/>mV Output<br/>0.81 micron</b> |
|---------------------------------------|---------------------------------------------|-------------------------------------------------|---------------------------------------------|-------------------------------------------------|
| 2533                                  | $3.66 \times 10^{11}$                       | 2310                                            | $3.08 \times 10^{11}$                       | 1430                                            |
| 2553                                  | $3.85 \times 10^{11}$                       | 2420                                            | $3.25 \times 10^{11}$                       | 1490                                            |
| 2573                                  | $4.04 \times 10^{11}$                       | 2580                                            | $3.43 \times 10^{11}$                       | 1600                                            |
| 2593                                  | $4.24 \times 10^{11}$                       | 2700                                            | $3.62 \times 10^{11}$                       | 1700                                            |
| 2613                                  | $4.44 \times 10^{11}$                       | 2820                                            | $3.81 \times 10^{11}$                       | 1790                                            |
| 2633                                  | $4.65 \times 10^{11}$                       | 2980                                            | $4.02 \times 10^{11}$                       | 1890                                            |
| 2653                                  | $4.87 \times 10^{11}$                       | 3080                                            | $4.22 \times 10^{11}$                       | 1980                                            |
| 2673                                  | $5.10 \times 10^{11}$                       | 3270                                            | $4.44 \times 10^{11}$                       | 2080                                            |
|                                       |                                             |                                                 |                                             |                                                 |

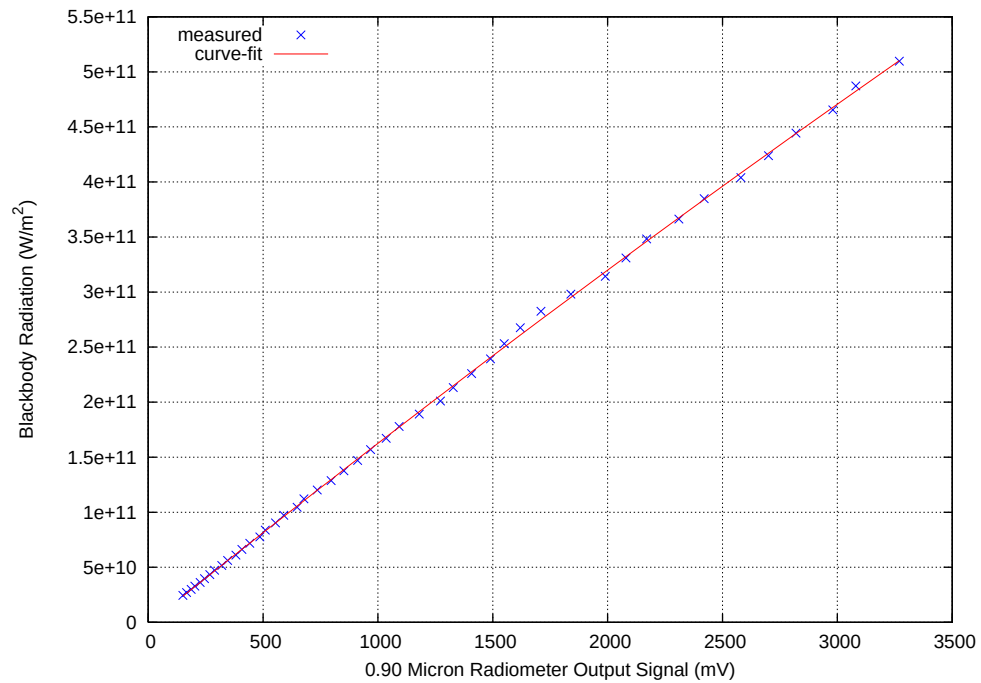


Figure 4.2: Calibration curve for radiometer 0.90 micron output

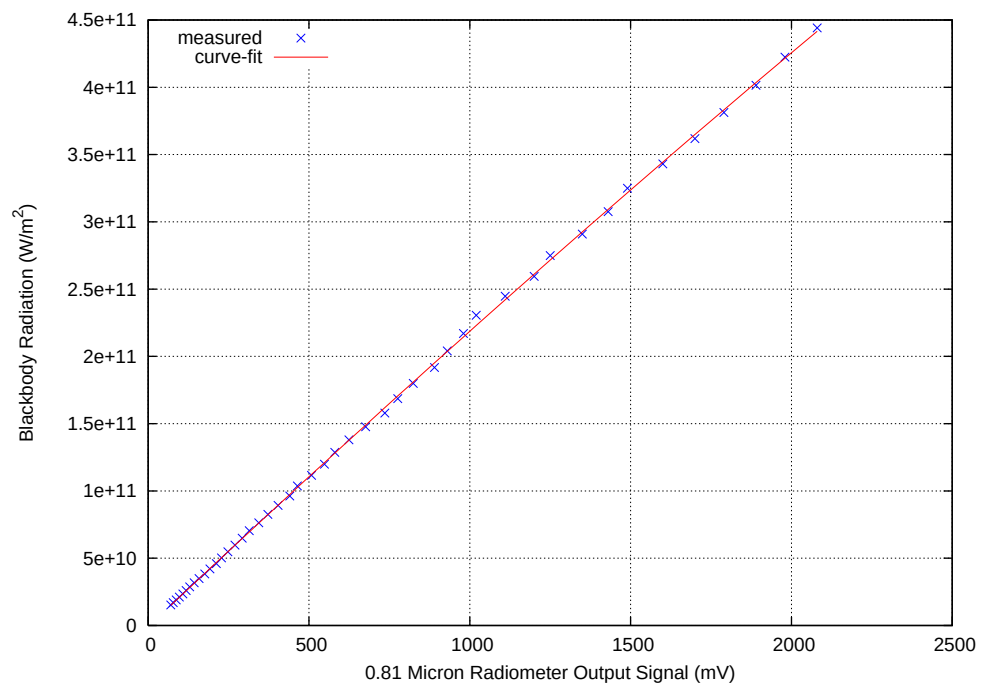


Figure 4.3: Calibration curve for radiometer 0.81 micron output

Knowing that both radiometer measurements are made for a single temperature gives:

$$\begin{aligned}
\frac{-\lambda_1}{c_2} \cdot \ln \left( \frac{\lambda_1^5 M_{\lambda_1}}{\epsilon c_1} \right) &= \frac{-\lambda_2}{c_2} \cdot \ln \left( \frac{\lambda_2^5 M_{\lambda_2}}{\epsilon c_1} \right), \\
-\lambda_1 \cdot \left[ \ln \left( \frac{\lambda_1^5 M_{\lambda_1}}{c_1} \right) - \ln(\epsilon) \right] &= -\lambda_2 \cdot \left[ \ln \left( \frac{\lambda_2^5 M_{\lambda_2}}{c_1} \right) - \ln(\epsilon) \right], \\
\ln(\epsilon) &= \frac{1}{\lambda_1 - \lambda_2} \left[ \lambda_1 \cdot \ln \left( \frac{\lambda_1^5 M_{\lambda_1}}{c_1} \right) - \lambda_2 \cdot \ln \left( \frac{\lambda_2^5 M_{\lambda_2}}{c_1} \right) \right]. \tag{4.15}
\end{aligned}$$

Additional useful information can be obtained from a graphical solution of the temperature-emissivity relationships at the two measured spectral emittances. Inverting Eq. 4.14 provides an equation for the absolute temperature as a function of emissivity. The plots of temperature versus emissivity for a 0.90 micron radiant emittance measurement and an 0.81 micron measurement intersect at the actual temperature and emissivity. Fig. 4.4 demonstrates the graphical solution for measurements  $M_{0.90} = 6.66 \times 10^{10} \text{W/m}^2$  and  $M_{0.81} = 4.86 \times 10^{10} \text{W/m}^2$ . From Fig. 4.4, it should be noted that the plots are nearly parallel and small changes in a measurement of radiant emittance will result in large differences in the intersection of the two curves. The precision with which this measurement can be made affects the uncertainty of the results.

The method for determining the emissivity and temperature of the YsZ heater element with the modified ratio thermometer in the presence of reflected irradiance can be developed from Eq. 4.8. Substituting Wien's approximation for the blackbody spectral radiant emittance provides an equation that can be solved for  $1/T$  as before. The derivation is as follows:

$$\begin{aligned}
J_\lambda &= \epsilon M_{B\lambda} + (1 - \epsilon) E_\lambda, \\
J_\lambda &= \epsilon c_1 \lambda^{-5} \exp(-c_2/\lambda T) + (1 - \epsilon) E_\lambda, \\
\frac{\lambda^5}{\epsilon c_1} \cdot [J_\lambda - (1 - \epsilon) E_\lambda] &= \exp(-c_2/\lambda T), \\
1/T &= \frac{-\lambda}{c_2} \cdot \left[ \ln \left( \frac{\lambda^5}{\epsilon c_1} \right) + \ln(J_\lambda - (1 - \epsilon) E_\lambda) \right]. \tag{4.16}
\end{aligned}$$

Although it is not difficult to solve for  $1/T$ , it is difficult to directly solve for  $\epsilon$ . The analytical technique that was used for the case without irradiance produces a large and cumbersome equation when applied with Eq. 4.16. However, the graphical method that was demonstrated previously works well for solving this system of equations. Fig. 4.5 depicts the same conditions as plotted in Fig. 4.4,  $J_{0.90} = 6.66 \times 10^{10} \text{W/m}^2$  and  $J_{0.81} = 4.86 \times 10^{10} \text{W/m}^2$ ,

with the addition of irradiance from the working chamber wall at a temperature of 1770 K.

The values of  $J_{0.90}$ ,  $J_{0.81}$ , and  $T_w$  that are used to create Fig. 4.4 and Fig. 4.5 are from data taken during operation of the Gas Sample Cell. The figures demonstrate the importance of considering the irradiance in the determination of temperature and emissivity. Fig. 4.4, which does not account for irradiance, concludes that the emissivity is 0.65 instead of 0.32 as shown in Fig. 4.5. The YsZ heater element temperature is determined to be 2110 K when the irradiance is not included in the calculation, but when the irradiance is included the temperature is determined to be 2220 K.

Tables 4.2 and 4.3 include the measurements that were made with the Gas Sample Cell and the ratio thermometer. In Table 4.2 the YsZ temperature, as determined by the internal electronics of the ratio thermometer, and the millivolt outputs of the radiometer at 0.90 and 0.81 microns are listed along with calculations of emissivity and YsZ temperature. These calculations were made with Eq. 4.14 and Eq. 4.15 and do not include the effect of reflected irradiance. Table 4.3 includes the ratio thermometer's measured YsZ temperature and the temperature of the working chamber wall. The emissivity and temperature of the YsZ element are determined by the graphical method and include the effect of reflected irradiance. The graphs of emissivity vs temperature including reflected irradiation were produced with Eq. 4.16. The graphs are included in Appendix C.

In reviewing the data contained in Table 4.3 it is clear that the first data point is inaccurate; the graphical YsZ temperature is more than 200 K greater than the measured YsZ temperature. It is likely that thermal equilibrium had not been reached when this data point was taken. Also, as can be seen on Figures C.1 and C.2 the low signal outputs (shown in Table 4.2) place the intersections of the two equations where the curves are much more vertical than they are for the other data points. Because of the obvious inaccuracy, the first data point will not be used in the remainder of this analysis.

Fig. 4.6 is a scatter diagram of the graphical temperature versus graphical emissivity data from Table 4.3. The second-order polynomial that provides the closest fit by the least squares method is overlaid with the data.

The curve-fit polynomial is:

$$\epsilon = (1.571 \times 10^{-6})T^2 - (5.624 \times 10^{-3})T + 5.244. \quad (4.17)$$

The coefficient of determination,  $R^2$ , for this data and curve-fit equation is 0.76. Over the limited temperature range of the data, the polynomial is an acceptable fit for engineering purposes related to design of an aerospace air heater.

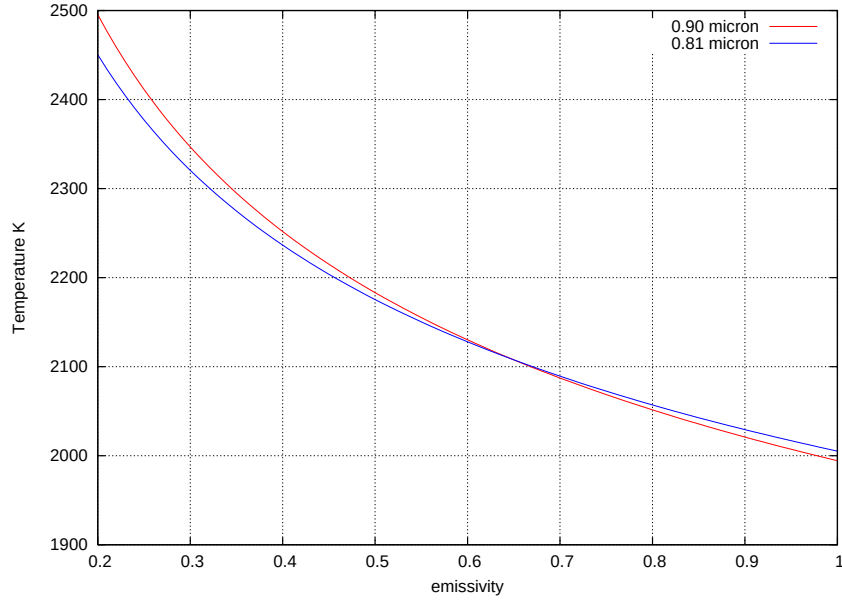


Figure 4.4: Graphical solution of emissivity and temperature using Eq. 4.14. In this figure,  $M_{0.90} = 6.66 \times 10^{10} \text{ W/m}^2$  and  $M_{0.81} = 4.86 \times 10^{10} \text{ W/m}^2$ . There is no reflected irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

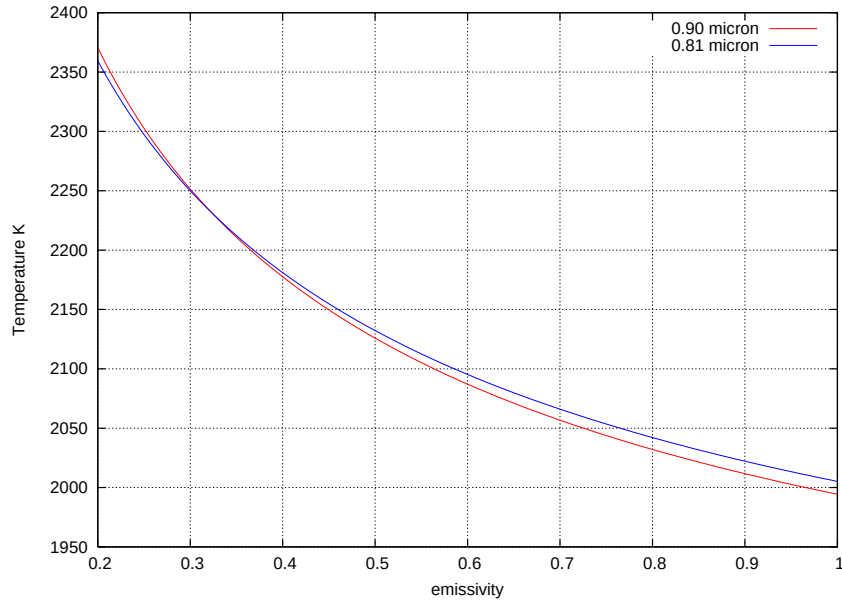


Figure 4.5: Graphical solution of emissivity and temperature using Eq. 4.16. In this figure,  $J_{0.90} = 6.66 \times 10^{10} \text{ W/m}^2$  and  $J_{0.81} = 4.86 \times 10^{10} \text{ W/m}^2$ . The radiosity also includes reflected irradiance from the surrounding wall whose temperature is 1770 K. The intersection of the plots gives the absolute temperature and emissivity of the surface.

Table 4.2: Emissivities as calculated from the output of the ratio thermometer without correcting for reflection of irradiance

|    | Measured<br>YsZ<br>Temperature<br>$K$ | $J_{0.90}$<br>Radiometer<br>Output<br>mV | $J_{0.81}$<br>Radiometer<br>Output<br>mV | Calculated<br>Emissivity | Calculated<br>YsZ<br>Temperature<br>$K$ |
|----|---------------------------------------|------------------------------------------|------------------------------------------|--------------------------|-----------------------------------------|
| 1  | 2023                                  | 240                                      | 125                                      | 0.482                    | 2041                                    |
| 2  | 2073                                  | 360                                      | 190                                      | 0.659                    | 2068                                    |
| 3  | 2123                                  | 465                                      | 250                                      | 0.727                    | 2112                                    |
| 4  | 2173                                  | 610                                      | 335                                      | 0.791                    | 2166                                    |
| 5  | 2193                                  | 660                                      | 365                                      | 0.804                    | 2184                                    |
| 6  | 2213                                  | 725                                      | 405                                      | 0.807                    | 2212                                    |
| 7  | 2233                                  | 780                                      | 440                                      | 0.795                    | 2239                                    |
| 8  | 2253                                  | 865                                      | 490                                      | 0.848                    | 2251                                    |
| 9  | 2273                                  | 920                                      | 525                                      | 0.844                    | 2272                                    |
| 10 | 2293                                  | 1030                                     | 590                                      | 0.911                    | 2283                                    |
| 11 | 2313                                  | 1090                                     | 630                                      | 0.888                    | 2310                                    |
| 12 | 2333                                  | 1170                                     | 680                                      | 0.905                    | 2327                                    |
| 13 | 2293                                  | 1000                                     | 575                                      | 0.855                    | 2294                                    |
| 14 | 2253                                  | 850                                      | 480                                      | 0.857                    | 2242                                    |
| 15 | 2233                                  | 800                                      | 450                                      | 0.836                    | 2231                                    |
| 16 | 2213                                  | 750                                      | 420                                      | 0.816                    | 2219                                    |
| 17 | 2193                                  | 675                                      | 375                                      | 0.789                    | 2197                                    |
| 18 | 2173                                  | 635                                      | 350                                      | 0.797                    | 2176                                    |
| 19 | 2153                                  | 585                                      | 320                                      | 0.786                    | 2156                                    |
| 20 | 2133                                  | 525                                      | 285                                      | 0.754                    | 2136                                    |
| 21 | 2113                                  | 485                                      | 260                                      | 0.779                    | 2105                                    |
| 22 | 2093                                  | 430                                      | 230                                      | 0.702                    | 2100                                    |
| 23 | 2073                                  | 415                                      | 220                                      | 0.734                    | 2078                                    |

Table 4.3: Emissivities and temperatures - corrected for reflection of irradiance

|    | Measured<br>YsZ<br>Temperature<br>$K$ | Measured<br>Wall<br>Temperature<br>$K$ | Graphical<br>Determination<br>of<br>Emissivity | Graphical<br>YsZ<br>Temperature<br>$K$ |
|----|---------------------------------------|----------------------------------------|------------------------------------------------|----------------------------------------|
| 1  | 2023                                  | 1672                                   | 0.17                                           | 2230                                   |
| 2  | 2073                                  | 1716                                   | 0.38                                           | 2160                                   |
| 3  | 2123                                  | 1772                                   | 0.45                                           | 2195                                   |
| 4  | 2173                                  | 1821                                   | 0.55                                           | 2230                                   |
| 5  | 2193                                  | 1845                                   | 0.56                                           | 2247                                   |
| 6  | 2213                                  | 1862                                   | 0.57                                           | 2275                                   |
| 7  | 2233                                  | 1879                                   | 0.55                                           | 2305                                   |
| 8  | 2253                                  | 1902                                   | 0.64                                           | 2300                                   |
| 9  | 2273                                  | 1921                                   | 0.63                                           | 2325                                   |
| 10 | 2293                                  | 1938                                   | 0.77                                           | 2310                                   |
| 11 | 2313                                  | 1952                                   | 0.73                                           | 2350                                   |
| 12 | 2333                                  | 1972                                   | 0.76                                           | 2360                                   |
| 13 | 2293                                  | 1958                                   | 0.63                                           | 2350                                   |
| 14 | 2253                                  | 1924                                   | 0.63                                           | 2295                                   |
| 15 | 2233                                  | 1894                                   | 0.61                                           | 2290                                   |
| 16 | 2213                                  | 1872                                   | 0.58                                           | 2280                                   |
| 17 | 2193                                  | 1859                                   | 0.53                                           | 2270                                   |
| 18 | 2173                                  | 1839                                   | 0.55                                           | 2240                                   |
| 19 | 2153                                  | 1822                                   | 0.53                                           | 2220                                   |
| 20 | 2133                                  | 1804                                   | 0.48                                           | 2220                                   |
| 21 | 2113                                  | 1786                                   | 0.52                                           | 2170                                   |
| 22 | 2093                                  | 1772                                   | 0.39                                           | 2200                                   |
| 23 | 2073                                  | 1753                                   | 0.45                                           | 2155                                   |

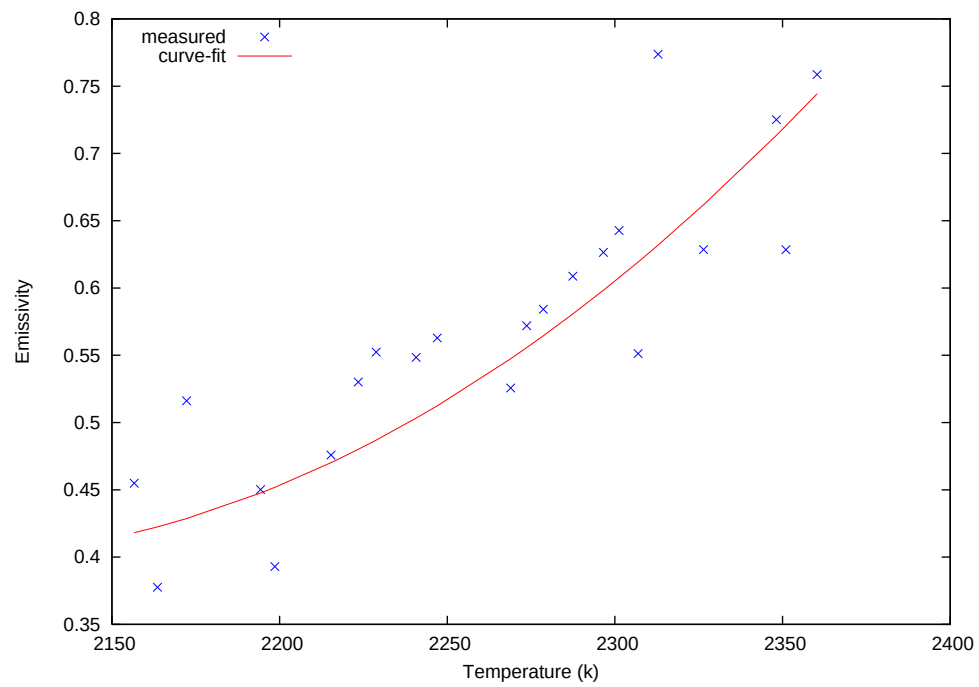


Figure 4.6: Temperature of the YsZ heater element versus emissivity with the correction for the effect of irradiance

## Chapter 5

# Resistivity

### 5.1 Theory

Resistivity is a measure of a material's opposition to the movement of charge carriers in the presence of an electric field. In electronic and ionic current flows, electric fields produce forces which accelerate charge carriers and increase their kinetic energy. Collisions between the charge carriers and the material's molecular structure dissipate the kinetic energy of the carriers and increase the vibratory thermal energy in the material. This effect is known as Joule heating.

In Sec. 2.3, the properties of ionic conductivity (the reciprocal of resistivity) in Yttria-stabilized Zirconia were discussed. In that section, it was shown that the resistivity of YsZ varies exponentially with the inverse of the absolute temperature; increasing temperature results in an increase in the number of ions that have the necessary energy to hop to an adjacent vacancy. The relationship between temperature and resistivity can be expressed in an Arrhenius equation,

$$\rho = A \exp(B/kT). \quad (5.1)$$

As also shown in Sec. 2.3, it is known that a single Arrhenius equation with one set of coefficients does not adequately model the resistivity of YsZ over the entire temperature range up to its melting point. Measurements of the resistivity of YsZ in the temperature range from 1900 K to 2300 K have not been published. It is not known if temperature related changes in the properties of YsZ might result in a divergence from Arrhenius relationships that have been established for YsZ at lower temperatures. For this study, resistivity will be measured as a function of temperature and curve fitting techniques will be applied to determine the coefficients for an Arrhenius equation and evaluate the quality of the fit of the data to the resulting equation.

Two obstacles must be surmounted to determine the coefficients of the Arrhenius equation for resistivity in the temperature range of 1900 K to 2300 K. The first is the nonuniform

temperature and geometry of the YsZ heater element. To make a mathematical model that takes into account the voltage, current, and temperature gradients in the complex shape of the YsZ heater element as it extends through three separate environments is beyond the scope of this analysis. It should also be noted that the temperature of the upper and lower sections of the working chamber, where the electrodes are connected to the heater element, are below the temperature range that is of interest in this study. The second obstacle is the impossibility of connecting probes to the high temperature section of the YsZ heater element to measure the voltage potential across the hot section.

Within the Gas Sample Cell furnace, the hottest section of the YsZ heater element is completely contained in the working chamber. In fact, all heat that is transferred from the YsZ heater element to the working chamber wall is from the high temperature section of the heater element. It is clear that the total energy emitted by a blackbody can be determined by integrating Planck's equation (Eq. 4.2) over all wavelengths,

$$M_B = \int_0^\infty \frac{2hc^2}{\lambda^5 [\exp(hc/\lambda kT) - 1]} d\lambda. \quad (5.2)$$

The solution of Eq. 5.2 yields the Stefan-Boltzmann law,

$$M_B = \varsigma T^4, \quad (5.3)$$

where the constant of proportionality,  $\varsigma$ , is called the Stefan-Boltzmann constant.  $\varsigma$  is the ratio of fundamental constants  $h$ ,  $c$ , and  $k$ ,

$$\varsigma = \frac{2\pi^5 k^4}{15h^3 c^2}. \quad (5.4)$$

The Stefan-Boltzmann constant has been measured to be  $5.670400 \times 10^{-8} \text{ W}/(\text{m}^2 \text{K}^4)$  with an uncertainty of  $0.000040 \times 10^{-8} \text{ W}/(\text{m}^2 \text{K}^4)$  [27].

The heat transferred by thermal radiation from a body within a constant temperature cavity is equal to the difference between the energy of the radiation which is radiated and the radiation that is absorbed [21]. For a body whose surface area is  $A$ , and temperature is  $T$ , within a cavity at temperature  $T_w$ , the rate of energy transfer is

$$\dot{Q} = \frac{dQ}{dt} = A[M(T) - \alpha M_B(T_w)]. \quad (5.5)$$

Substitution of the Stefan-Boltzmann law into Eq. 5.5 produces

$$\dot{Q} = A[\epsilon_{\zeta}T^4 - \alpha_{\zeta}T_w^4]. \quad (5.6)$$

Kirchhoff's law of thermal radiation ( $\alpha = \epsilon$ ) can be used to restate Eq. 5.6 as

$$\dot{Q} = A\epsilon_{\zeta}[T^4 - T_w^4], \quad (5.7)$$

where the emissivity,  $\epsilon$ , is the total emissivity over all wavelengths of radiated energy.

The source of  $\dot{Q}$  is the electrical power that is dissipated in the high temperature section of the heater element. For a resistive circuit, the power dissipated,  $P$ , is equal to the product of the square of the current,  $I$ , and the resistance,  $R$ . Rearranging this equation gives an expression for the resistance of the high temperature section in terms of the rate of energy transfer and the current flowing through the element:

$$\begin{aligned} \dot{Q} &= P = I^2 R, \\ R &= \dot{Q}/I^2 \end{aligned} \quad (5.8)$$

The resistivity of YsZ can be determined from the geometry of the high temperature section of the heater element and the resistance. Using the identities: (1) resistivity is defined as the ratio of the electric field,  $E$ , to the electric current density,  $J$ , (2) the electric field is the change in potential per unit length. and (3) the electric current density is the electric current per unit cross-section area, the resistivity and resistance can be related by the dimensions of the conductor:

$$\begin{aligned} \rho &= \frac{E}{J}, \\ E &= \frac{V}{l}, \text{ and } J = \frac{I}{A}, \\ \rho &= \frac{V/l}{I/A} = R \frac{A}{l}. \end{aligned} \quad (5.9)$$

## 5.2 Determination of High Temperature Resistivity

To use the technique established in Section 5.1 to determine the resistivity of the high temperature section of the YsZ heater element, several measurements are needed. The data that was collected from a test on March 13, 2007 provides all the needed data except for

the emissivity of the material over all temperatures. In Section 4.4, this data was used to determine the temperature and spectral emissivity of the YsZ heater element. Also collected on that date were measurements of the temperature of the working chamber wall and the RMS current flow through the heater element. Dimensional measurements of the YsZ heater element were made before the test. Those measurements determined that the high temperature section of the heater element was 0.161 meters long and had a diameter of 0.00737 meters. Therefore, its surface area was  $0.00374 \text{ m}^2$  and its cross-sectional area was  $2.18 \times 10^{-5} \text{ m}^2$ .

Although the total emissivity has not been determined in this thesis, there is a heat transfer model for the heater element at temperatures above  $2000 \text{ K}$  that has been investigated by Dr Frank Steinle at AEDC that includes the total emissivity [28]. With data from a December 2006 test, Dr Steinle's three dimensional model uses an iterative method to determine the material's resistivity as a function of temperature. The data includes temperature measurements along the entire heater element. The math behind the model iterates to find the resistivity and emissivity values that balance the energy transfer. The total emissivity values that were determined by Dr Steinle for the December 2006 test points are given in Table 5.1.

To interpolate a value for the total emissivity to be used in calculating the resistivity from the March data, a second-order polynomial curve-fit is created. Fig. 5.1 shows temperature versus emissivity for each data entry in Table 5.1 and the curve-fit equation

$$\epsilon = (-1.7003 \times 10^{-6})T^2 + (8.3005 \times 10^{-3})T - 9.5113. \quad (5.10)$$

The March 13, 2007 data, the calculated total emissivity, and the results of the calculations of  $\dot{Q}$ , resistance, and resistivity are listed in Table 5.2.

The graph of  $1/T$  vs. resistivity shows the exponential nature of the relationship. Fig. 5.2 shows temperature versus resistivity for each data entry in Table 5.2. An exponential equation is fit to the data using a least-squares method for determining the coefficients. This curve-fit equation, formulated to separate the Boltzmann constant, is Eq. 5.11. The coefficient of determination,  $r^2$ , (where  $r$  is the coefficient of correlation) for this curve-fit equation is 0.87.

$$\rho = 1.86 \times 10^{-7} \exp(3.42 \times 10^{-19}/kT) \quad (5.11)$$

Table 5.1: Total emissivity as calculated by Frank Steinle

| YsZ<br>Temp.<br>$K$ | Total<br>$\epsilon$ |
|---------------------|---------------------|
| 2066                | 0.383               |
| 2125                | 0.442               |
| 2201                | 0.528               |
| 2270                | 0.572               |
| 2289                | 0.577               |
| 2316                | 0.586               |
| 2340                | 0.598               |
| 2365                | 0.612               |
| 2392                | 0.617               |
| 2416                | 0.623               |
| 2442                | 0.629               |
| 2468                | 0.607               |

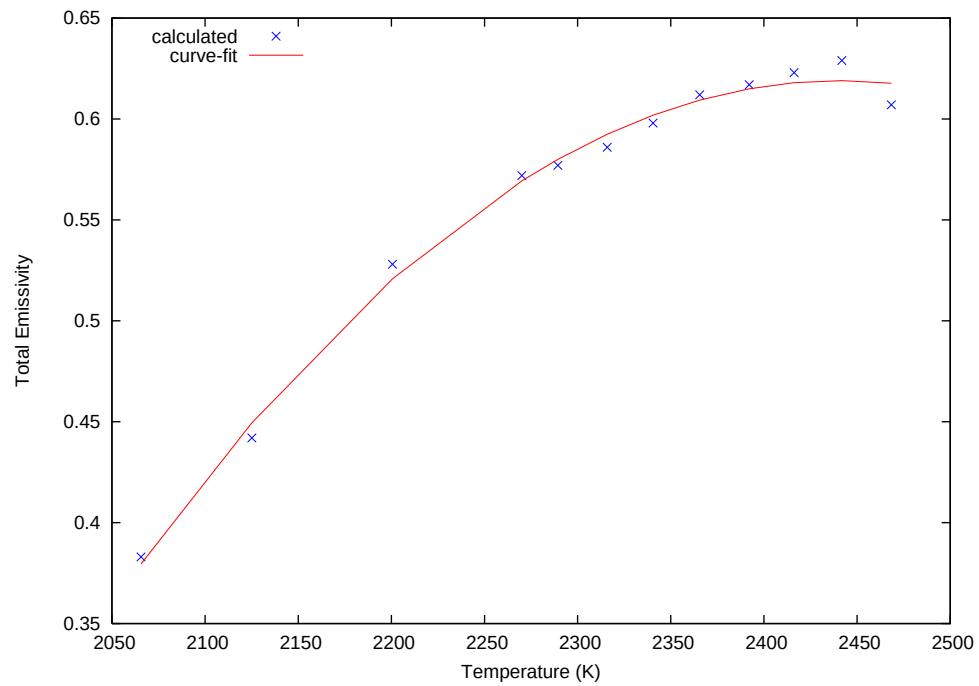


Figure 5.1: Temperature versus total emissivity for December, 2006 data points and quadratic curve-fit, Eq. 5.10

Table 5.2: Calculated values of  $\dot{Q}$ , resistance, and resistivity from March 13, 2007 data

|    | YsZ<br>Temp.<br>$K$ | Wall<br>Temp.<br>$K$ | $\epsilon$ | $\dot{Q}$<br>W | Current<br>A | Resistance<br>$\Omega$ | Resistivity<br>$\Omega \cdot m$ |
|----|---------------------|----------------------|------------|----------------|--------------|------------------------|---------------------------------|
| 2  | 2160                | 1716                 | 0.48       | 1060           | 2.95         | 121                    | 0.0164                          |
| 3  | 2195                | 1772                 | 0.52       | 1270           | 3.68         | 94.1                   | 0.0127                          |
| 4  | 2230                | 1821                 | 0.54       | 1600           | 4.36         | 84.3                   | 0.0114                          |
| 5  | 2250                | 1845                 | 0.56       | 1670           | 4.59         | 79.1                   | 0.0107                          |
| 6  | 2275                | 1862                 | 0.57       | 1780           | 4.83         | 76.5                   | 0.0104                          |
| 7  | 2305                | 1879                 | 0.58       | 1840           | 5.11         | 70.4                   | 0.00953                         |
| 8  | 2300                | 1902                 | 0.59       | 2020           | 5.41         | 69.1                   | 0.00935                         |
| 9  | 2325                | 1921                 | 0.60       | 2080           | 5.70         | 64.2                   | 0.00868                         |
| 10 | 2310                | 1938                 | 0.59       | 2350           | 5.98         | 65.6                   | 0.00888                         |
| 11 | 2350                | 1952                 | 0.60       | 2470           | 6.24         | 63.5                   | 0.00860                         |
| 12 | 2360                | 1972                 | 0.61       | 2560           | 6.50         | 60.6                   | 0.00821                         |
| 13 | 2350                | 1958                 | 0.60       | 2110           | 5.83         | 62.1                   | 0.00841                         |
| 14 | 2295                | 1924                 | 0.58       | 1880           | 5.26         | 67.8                   | 0.00917                         |
| 15 | 2290                | 1894                 | 0.58       | 1890           | 5.05         | 74.2                   | 0.0101                          |
| 16 | 2280                | 1872                 | 0.58       | 1810           | 4.77         | 79.7                   | 0.0108                          |
| 17 | 2270                | 1859                 | 0.57       | 1640           | 4.54         | 79.7                   | 0.0108                          |
| 18 | 2240                | 1839                 | 0.55       | 1600           | 4.30         | 86.7                   | 0.0117                          |
| 19 | 2220                | 1822                 | 0.54       | 1490           | 4.05         | 90.9                   | 0.0123                          |
| 20 | 2220                | 1804                 | 0.54       | 1390           | 3.82         | 95.5                   | 0.0129                          |
| 21 | 2170                | 1786                 | 0.49       | 1320           | 3.50         | 108                    | 0.0146                          |
| 22 | 2200                | 1772                 | 0.52       | 1120           | 3.25         | 106                    | 0.0144                          |
| 23 | 2155                | 1753                 | 0.48       | 1160           | 3.06         | 124                    | 0.0167                          |

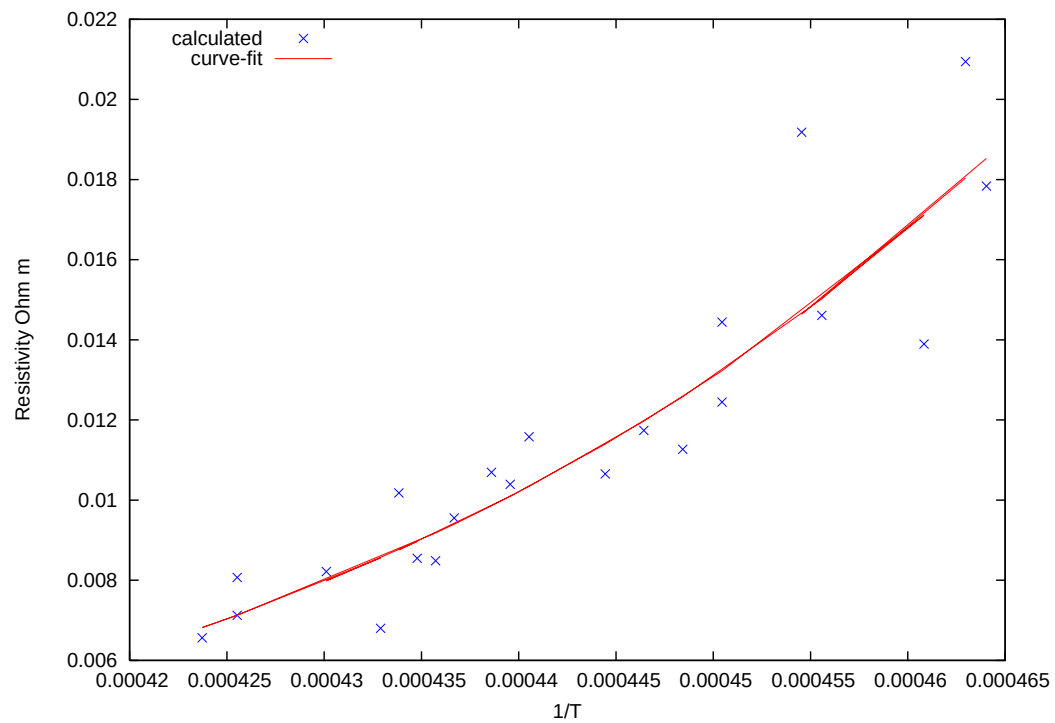


Figure 5.2: Temperature versus resistivity with exponential curve-fit equation Eq. 5.11

## Chapter 6

# Conclusions

The main purposes of this thesis have been achieved with (1) the measurement of the emissivity of the YsZ heater element material at temperatures in the range of 2000 K to 2300 K, and (2) the determination of the material's temperature versus resistivity relationship in the same temperature range. Analytical methods have been developed from physics principles. These analytical methods enabled the determination of the properties within the testing capabilities that were available through the Gas Sample Cell apparatus and its instrumentation.

With the information that has been gained in this study, one can return to its purpose and reconsider the questions about the potential uses for YsZ in a high temperature air heater. Because of the cost of building a full-scale YsZ air heater for an aerospace ground test facility, several mathematical models of the facility will be created prior to the first construction effort. With the calculated emissivity from this effort, the energy balance models of heat transfer will be constructed. With the calculated resistivity, an optimized power supply can be designed.

However, there were limitations in the accuracy of the instrumentation that was available for making some of the measurements in this study. In the development of the graphical method for determining temperature and emissivity in the presence of irradiance (Sec. 4.4), the importance of making highly accurate measurements of radiosity from the YsZ element was shown. Probably the best way to improve the accuracy of the measurements of temperature and emissivity using the method developed in this study is to improve the quality of the radiometric measurement instrument(s).

The potential usefulness of additional studies should be mentioned. Polarization characteristics of the material were not studied in this analysis. This area could provide information that would be useful in designing electrode attachments for the heater elements. Also, although 60 Hertz ac power was used in the test apparatus, there have been no studies to determine if there is an optimum frequency for power transfer to the material.

Through history, the study of basic material properties has been the path to understanding nature. Science has not proceeded in a straight line from ideas to actualizations but through the winding path of discovery. There should be no surprise if the methods

used to make the measurements and the results determined in this study lead to unrelated enabling technologies.

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# Appendices

## Appendix A

# RMS Voltage Equation for Heater Power Supply

The equation for the RMS value of the output voltage of the heater power supply is derived below.

The equation for the RMS value of a sinusoidal voltage is

$$V_{RMS}^2 = \frac{1}{T} \int_0^T V_p^2 \sin^2 \omega t dt, \quad (\text{A.1})$$

where

$T$  = period ,

$V_p$  = peak voltage, and

$\omega$  = angular frequency.

The integral can be evaluated over a single cycle,  $T = 2\pi/\omega$ , and Eq. A.1 can be written

$$V_{RMS}^2 = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} V_p^2 \sin^2 \omega t dt. \quad (\text{A.2})$$

For the heater power supply, the voltage output is zero during time intervals following zero-crossings of the voltage wave. These time intervals are equal to the conduction angle,  $\delta$ , divided by the angular frequency. Eq. A.2 can be rewritten to remove the “chopped” portion of the wave as

$$V_{RMS}^2 = \frac{V_p^2}{2\pi} \left[ \int_{\delta/\omega}^{\pi/\omega} \omega \sin^2 \omega t dt + \int_{(\pi+\delta/\omega)}^{2\pi/\omega} \omega \sin^2 \omega t dt \right]. \quad (\text{A.3})$$

Through the use of a trigonometric identity, Eq. A.3 can be expanded,

$$V_{RMS}^2 = \frac{V_p^2}{2\pi} \left[ \int_{\delta/\omega}^{\pi/\omega} \frac{\omega}{2} (1 - \cos 2\omega t) dt + \int_{(\pi+\delta)/\omega}^{2\pi/\omega} \frac{\omega}{2} (1 - \cos 2\omega t) dt \right], \quad (\text{A.4})$$

and then solved,

$$V_{RMS}^2 = \frac{V_p^2}{2\pi} \left( \pi - \delta + \frac{1}{2} \sin 2\delta \right). \quad (\text{A.5})$$

## Appendix B

# Computer Programs and Scripts

### B.1 Data Reduction and Filtering - BASIC Language

```
10 rem MOVING AVERAGE FILTER
20 rem This program provides a three-pass, overlapping moving average filter.
30 rem Each pass downsamples the original file by approximately 1/2.
40 rem "filelist.txt" names the input and 3 output files.
50 rem by- Howard Frederick, January 1, 2007
60 rem
70 rem Initial Conditions
80 DIM W$(4): rem W$ contains the names of the original file and the 3 filtered outputs
90 H=0: rem H is the outer loop counter
100 OPEN 3, "filelist.txt", "r"
110 rem
120 rem
130 LABEL zero: rem This is the beginning of the outer loop which steps through the files
to be filtered
140 H = H+1
150 IF (H>3) THEN GOTO two: ENDIF: rem compares H with number of entries in
"filelist.txt"
160 LINE INPUT #3 b$: rem b$ contains 4 file names
170 NUM=SPLIT(b$,W$(),"")
180 rem
190 rem
200 rem MOVING AVERAGE FILTER #1
210 rem This program applies a 4 point moving-average filter to 250,002 samples of the
data file.
220 rem An average output value is associated with every other time value, starting with
the third time entry.
230 rem The total size of the output file is  $250,002/2 - 1 = 125,000$  samples.
240 rem
250 rem Initialize Arrays and Open Files
260 DIM T(250002): rem T() is time
270 DIM X(250002): rem X() is the input signal
```

```

280 DIM Y(125000): rem Y() is the output signal
290 DIM H(4): rem H() is the filter kernel
300 DIM Z$(2): rem Z$() buffers the input from the data file
310 OPEN 1, W$(1), "r": rem FOR READING AS #1
320 OPEN 2, W$(2), "w"
330 rem
340 rem Load T() and X() With Data
350 FOR K = 1 TO 250007
360 LINE INPUT #1 a$: rem a$ contains 2 data
370 IF (K\6) THEN GOTO one: ENDIF: rem skip first 5 entries
380 NUM=SPLIT(a$,Z$(),",")
390 T(K-5) = val(Z$(1))
400 X(K-5) = val(Z$(2))
410 LABEL one
420 NEXT K
430 rem
440 rem Load the Filter Kernel
450 FOR I = 1 to 4
460 H(I) = .25
470 NEXT I
480 rem
490 rem Filter the Signal (Convolution)
500 FOR J = 3 TO 250001 step 2
510 M=INT((J-1)/2)
520 Y(M) = 0
530 FOR I = 1 to 4
540 Y(M) = Y(M) + X(J+I-3) * H(I)
550 NEXT I
560 print #2 T(J),",",Y(M)
570 NEXT J
580 rem
590 CLOSE #1
600 rem
610 CLOSE #2
620 rem
630 rem MOVING AVERAGE FILTER #2
640 rem This part applies a 4 point moving-average filter to the 125,000 samples of a data
file.
650 rem An average output value is associated with every other time value, starting with
the third time entry.
660 rem The total size of the output file is  $125,000/2 - 1 = 62,499$  samples.
670 rem
680 rem Initialize Arrays and Open Files
690 DIM T(125000): rem T() is time
700 DIM X(125000): rem X() is the input signal
710 DIM Y(62499): rem Y() is the output signal

```

```

720 DIM H(4): rem H() is the filter kernel
730 DIM Z$(2): rem Z$() buffers the input from the data file
740 OPEN 1, W$(2), "r": rem FOR READING AS #1
750 OPEN 2, W$(3), "w"
760 rem
770 rem
780 rem Load T() and X()
790 FOR K = 1 TO 125000
800 LINE INPUT #1 a$
810 NUM=SPLIT(a$,Z$(),",",",")
820 T(K) = val(Z$(1))
830 X(K) = val(Z$(2))
840 NEXT K
850 rem
860 rem Load the Filter Kernel
870 FOR I = 1 to 4
880 H(I) = .25
890 NEXT I
900 rem
910 rem Filter the Signal (Convolution)
920 FOR J = 3 TO 124999 step 2
930 M=INT((J-1)/2)
940 Y(M) = 0
950 FOR I = 1 to 4
960 Y(M) = Y(M) + X(J+I-3) * H(I)
970 NEXT I
980 PRINT #2 T(J),",",",Y(M)
990 NEXT J
1000 rem
1010 CLOSE #1
1020 rem
1030 CLOSE #2
1040 rem
1050 rem
1060 rem MOVING AVERAGE FILTER #3
1070 rem This part applies a 4 point moving-average filter to 62,498 samples of a data file.
1080 rem An average output value is associated with every other time value, starting with
the third time entry.
1090 rem The total size of the output file is  $62,498/2 - 1 = 31,248$  samples.
1100 rem
1110 rem Initialize Arrays and Open Files
1120 DIM T(62498): rem T() is the time
1130 DIM X(62498): rem X() is the input signal
1140 DIM Y(31248): rem Y() is the output signal
1150 DIM H(4): rem H() is the filter kernel
1160 DIM Z$(2): rem Z$() buffers the input from the data file

```

```

1170 OPEN 1, W$(3), "r": rem FOR READING AS #1
1180 OPEN 2, W$(4), "w"
1190 rem
1200 rem
1210 rem Load T() and X()
1220 FOR K = 1 TO 62498
1230 LINE INPUT #1 a$
1240 NUM=SPLIT(a$,Z$(1),",")
1250 T(K) = val(Z$(1))
1260 X(K) = val(Z$(2))
1270 NEXT K
1280 rem
1290 rem Load the Filter Kernel
1300 FOR I = 1 to 4
1310 H(I) = .25
1320 NEXT I
1330 rem
1340 rem Filter the Signal (Convolution)
1350 FOR J = 3 TO 62497 step 2
1360 M=INT((J-1)/2)
1370 Y(M) = 0
1380 FOR I = 1 to 4
1390 Y(M) = Y(M) + X(J+I-3) * H(I)
1400 NEXT I
1410 PRINT #2 T(J),",",Y(M)
1420 NEXT J
1430 rem
1440 CLOSE #1
1450 rem
1460 CLOSE #2
1470 GOTO zero
1500 LABEL two
1550 CLOSE #3
1600 END

```

## B.2 Calculate and Plot Temperature and Emissivity - Octave Script

```
% — Octave Script to Determine and Plot Temperature and Emissivity
% — Howard Frederick 1/12/2008
%
% — Initialization
h = 6.626076E-34; % Planck's Constant J*s
c = 299792458; % Speed of light m/s
k = 1.380658E-23; % J/K
c1 = 2*h*c^2; % First Radiation Constant
c2 = h*c/k; % Second Radiation Constant
%
l1 = 0.9E-6; % First wavelength (0.90 microns)
l2 = 0.81E-6; % Second wavelength (0.81 microns)
%
% — User Inputs
%
Out1 = 800; % Pyrometer Output @ 0.90 microns
Out2 = 450; % Pyrometer Output @ 0.81 microns
Tw = 1894; % The working chamber wall temperature
%
% — Calculate T without Irradiance for range of Emissivities
%
M1 = (-3.2981e+03)*Out1^2 + 167260000*Out1 - 1412400000 % W/m^2 @ 0.90 microns
M2 = (-6248)*Out2^2 + 225700000*Out2 - 782840000 % W/m^2 @ 0.81 microns
%
E1 = 0;
E2 = 0;
%
e=0.5:0.01:1; % emissivity ranges from 0 to 1 (100 samples)
%
T2 = 1 ./ (-l2/c2 * ( log(l2^5 ./ (c1 * e)) +log(M2 - (1-e)*E2)));
%
T1 = 1 ./ (-l1/c2 * ( log(l1^5 ./ (c1 * e)) +log(M1 - (1-e)*E1)));
%
%
% —Plot T without Irradiance
%
figure;
grid on;
plot(e,T1, "-;0.90 micron;", e, T2, "-;0.81 micron;")
%
xlabel('emissivity');
ylabel('Temperature K');
print('211960C.eps','-depse')
```

```

%
%
%—Calculate T with Irradiance for range of Emissivities
%
E1 = c1 * l1^5 * exp(-c2/(l1*Tw)) % Radiant emittance from working chamber wall at 0.90
micron wavelength
E2 = c1 * l2^5 * exp(-c2/(l2*Tw)) % Radiant emittance from working chamber wall at 0.81
micron wavelength
%
e=0.1:0.01:1; % emissivity ranges from 0 to 1 (1000 samples)
%
T2 = 1 ./ (-l2/c2 * ( log(l2^5 ./ (c1 * e)) +log(M2 - (1-e)*E2)));
%
T1 = 1 ./ (-l1/c2 * ( log(l1^5 ./ (c1 * e)) +log(M1 - (1-e)*E1)));
%
%
% —Plot T with Irradiance
%
figure;
grid on;
plot(e,T1, "-;0.90 micron;", e, T2, "-3; 0.81 micron;")
%
xlabel('emissivity');
ylabel('Temperature K');
print('211960CH.eps','-depsc')
%
%
% End

```

## Appendix C

# Temperature Versus Emissivity Graphs

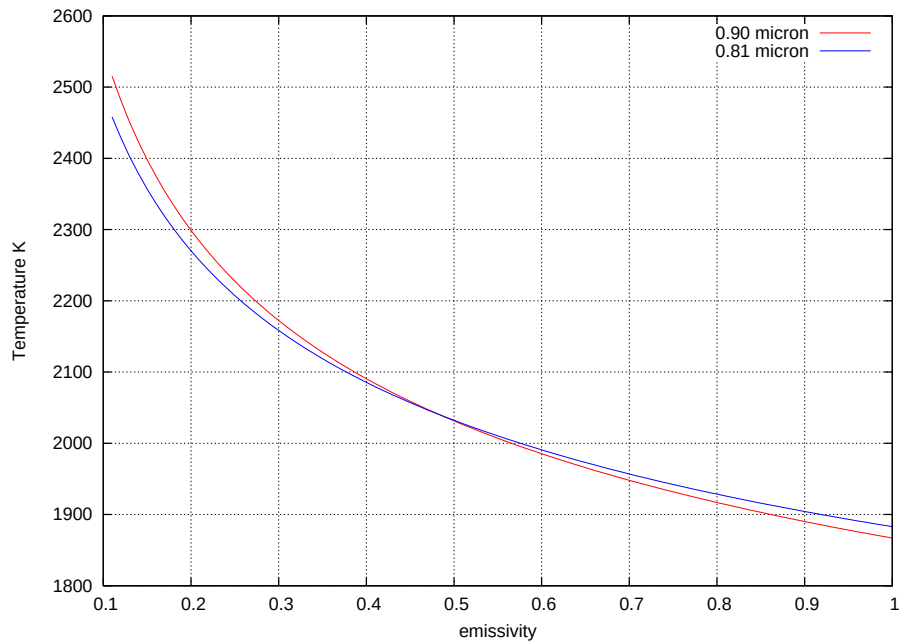


Figure C.1: Data point 1 (ratio thermometer reading 2023 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

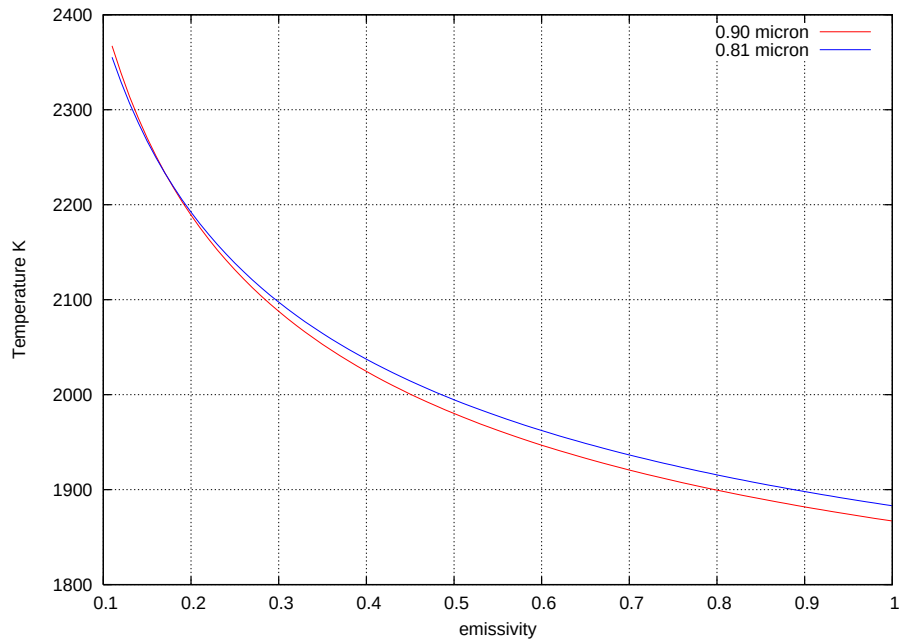


Figure C.2: Data point 1 (ratio thermometer reading 2023 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

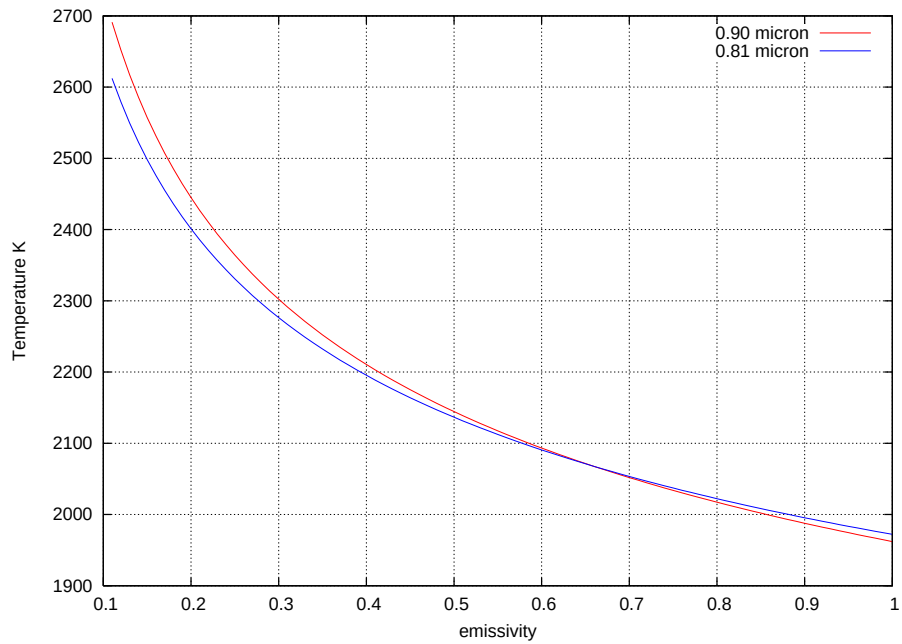


Figure C.3: Data Point 2 (ratio thermometer reading 2073 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

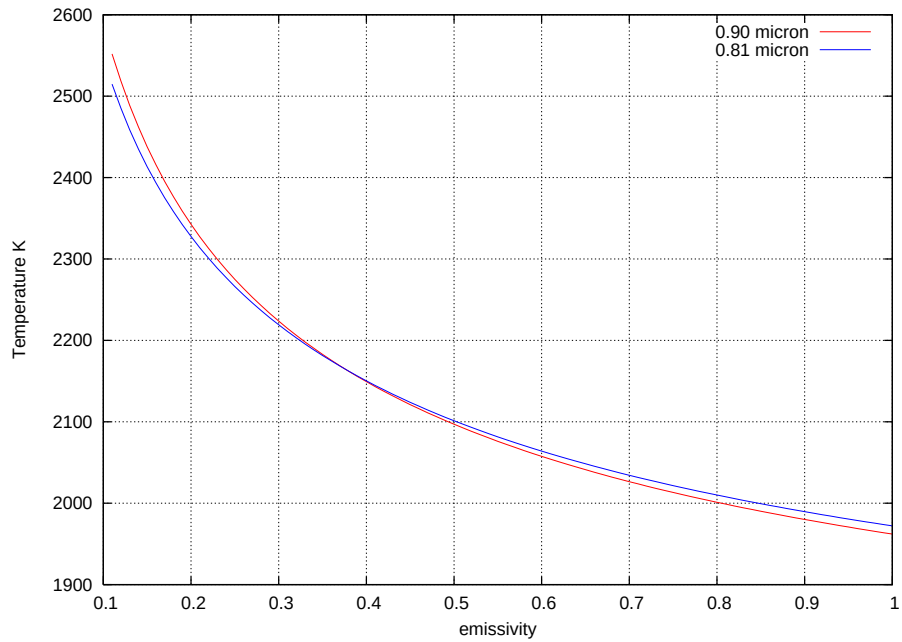


Figure C.4: Data Point 2 (ratio thermometer reading 2073 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

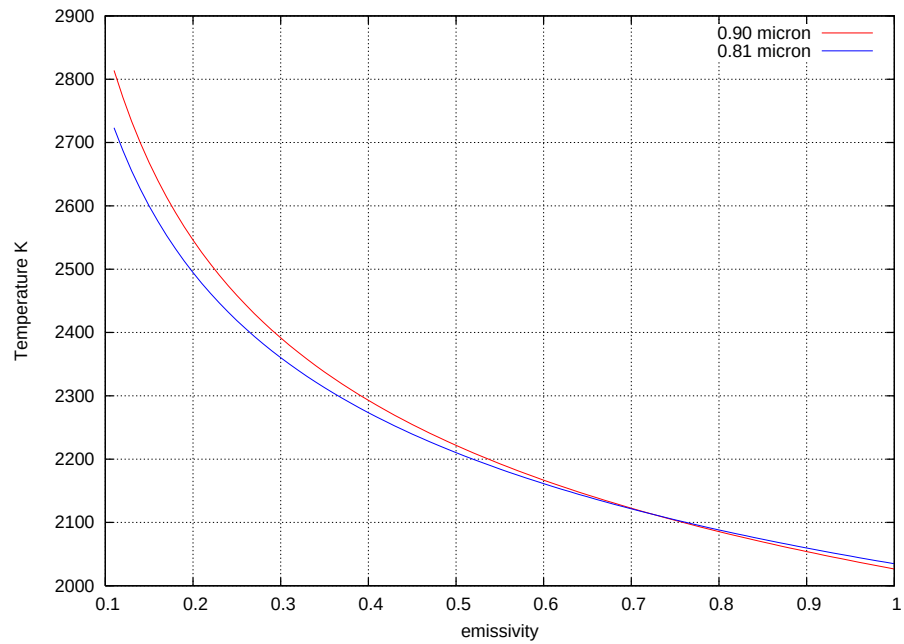


Figure C.5: Data Point 3 (ratio thermometer reading 2123 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

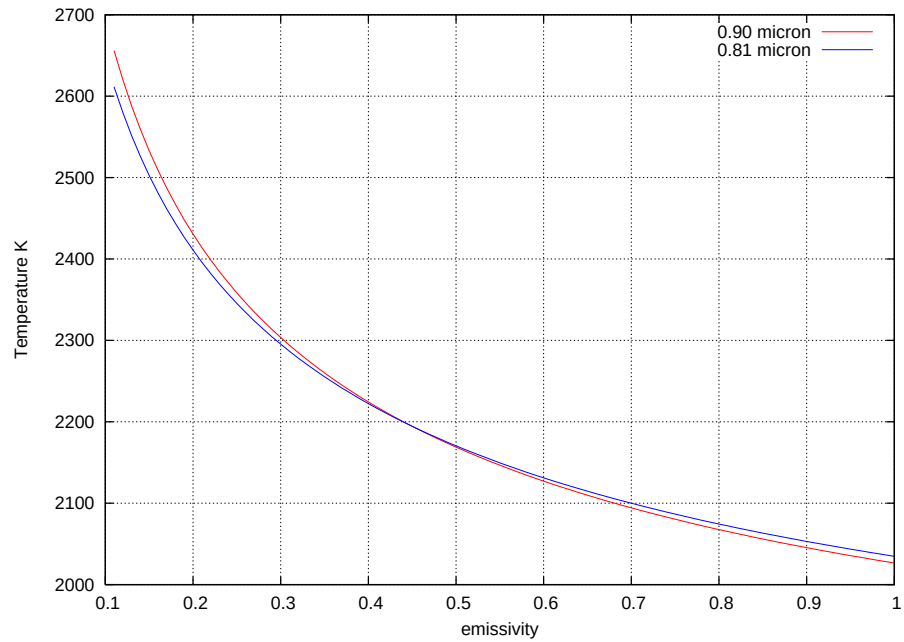


Figure C.6: Data Point 3 (ratio thermometer reading 2123 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

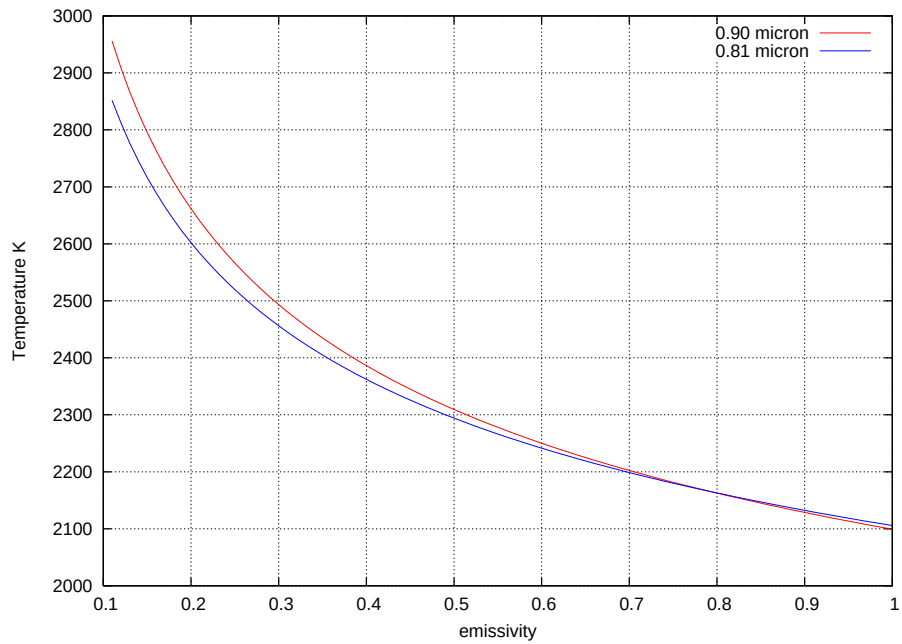


Figure C.7: Data Point 4 (ratio thermometer reading 2173 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

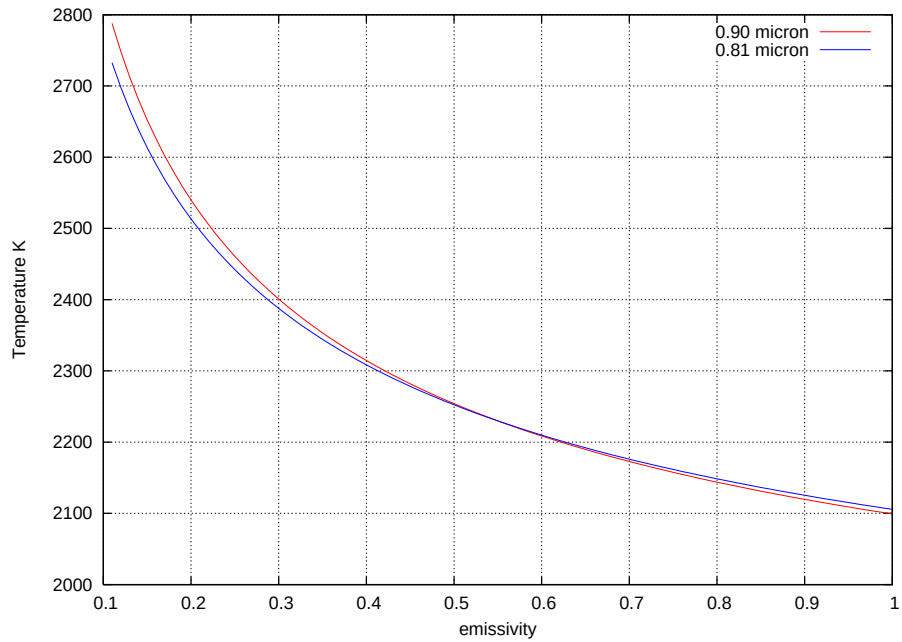


Figure C.8: Data Point 4 (ratio thermometer reading 2173 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

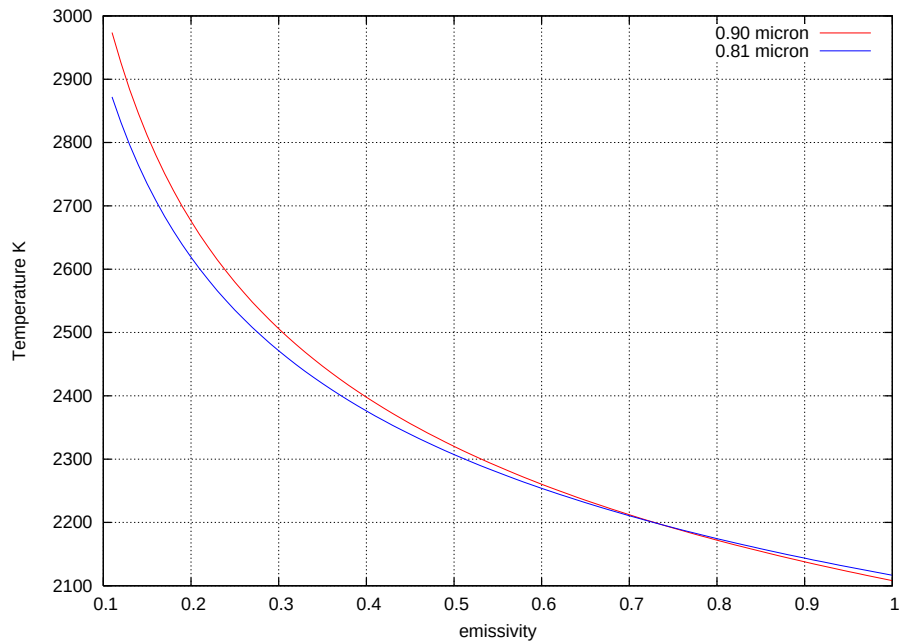


Figure C.9: Data Point 5 (ratio thermometer reading 2193 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

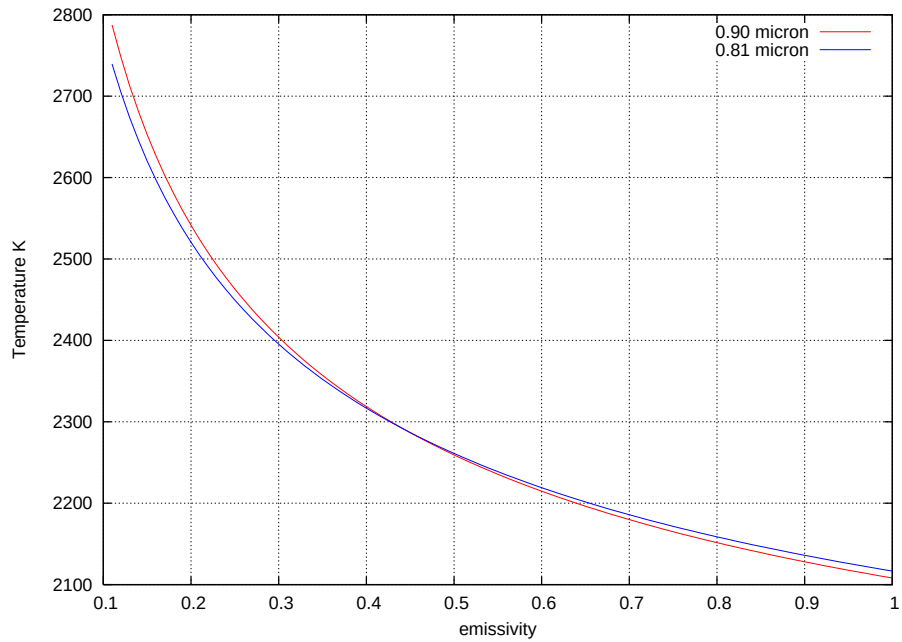


Figure C.10: Data Point 5 (ratio thermometer reading 2193 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

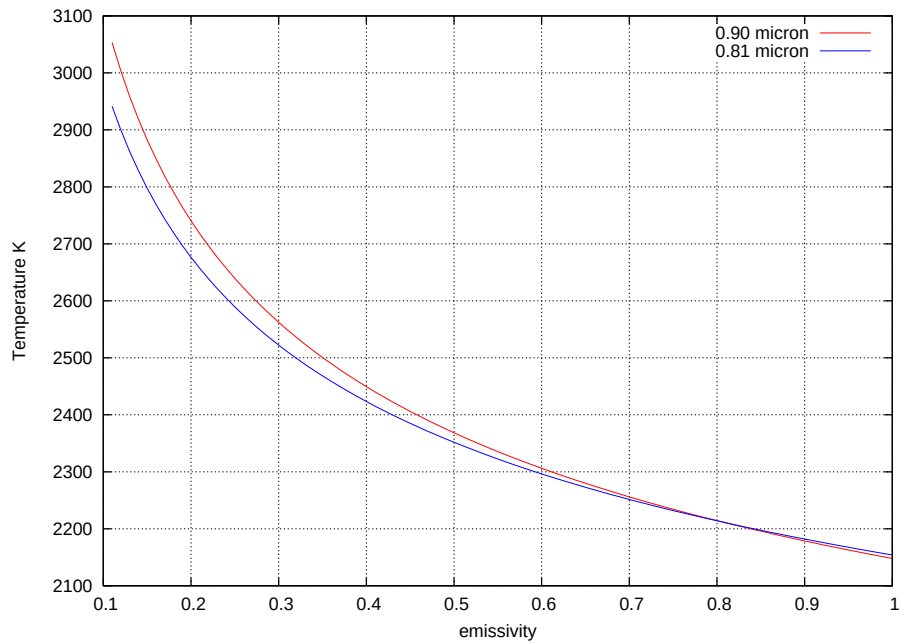


Figure C.11: Data Point 6 (ratio thermometer reading 2213 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

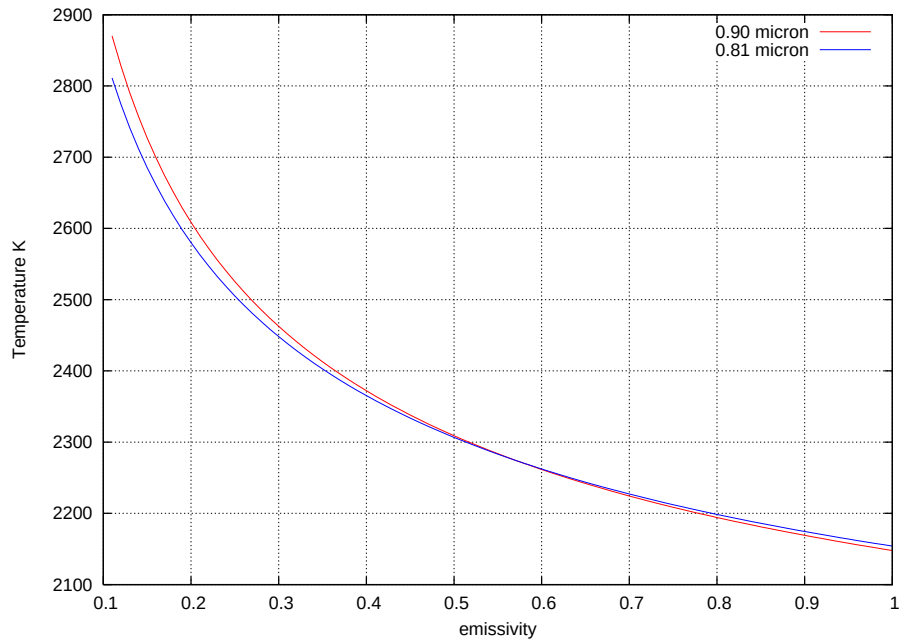


Figure C.12: Data Point 6 (ratio thermometer reading 2213 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

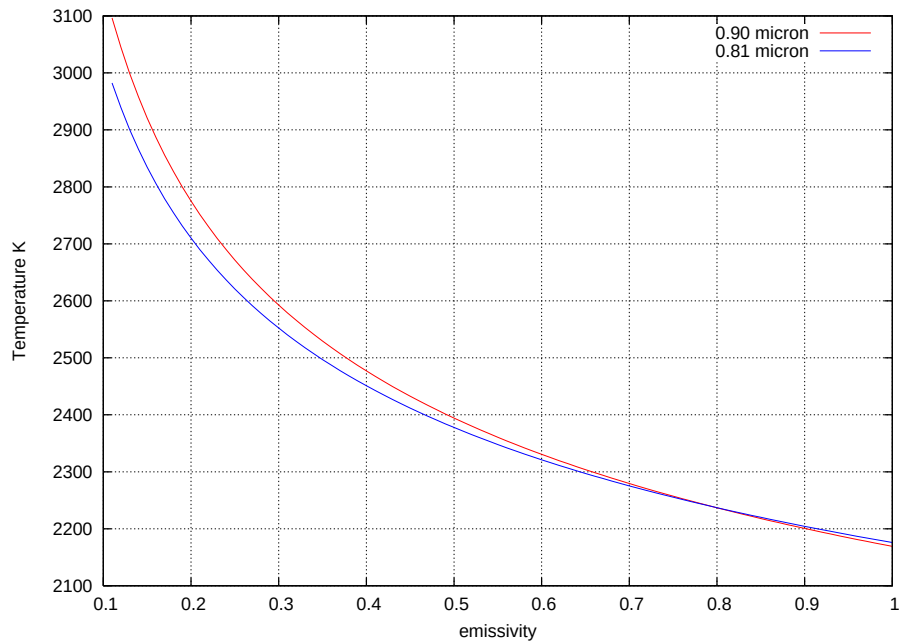


Figure C.13: Data Point 7 (ratio thermometer reading 2233 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

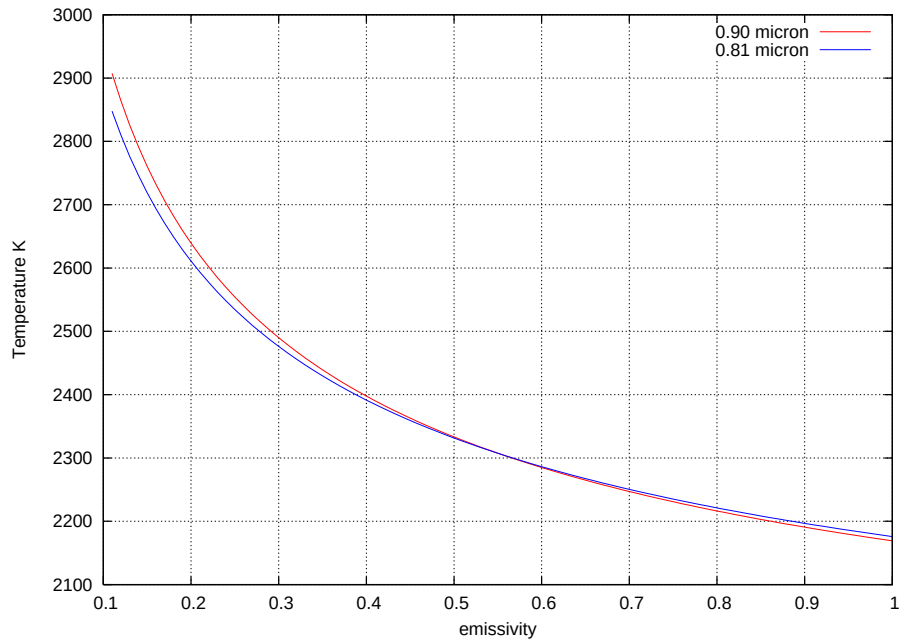


Figure C.14: Data Point 7 (ratio thermometer reading 2233 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

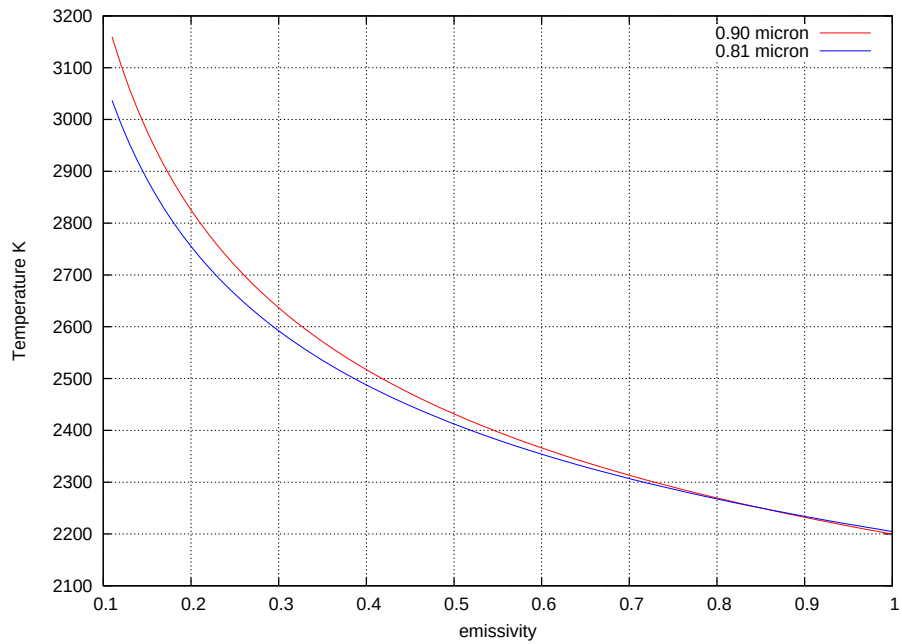


Figure C.15: Data Point 8 (ratio thermometer reading 2253 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

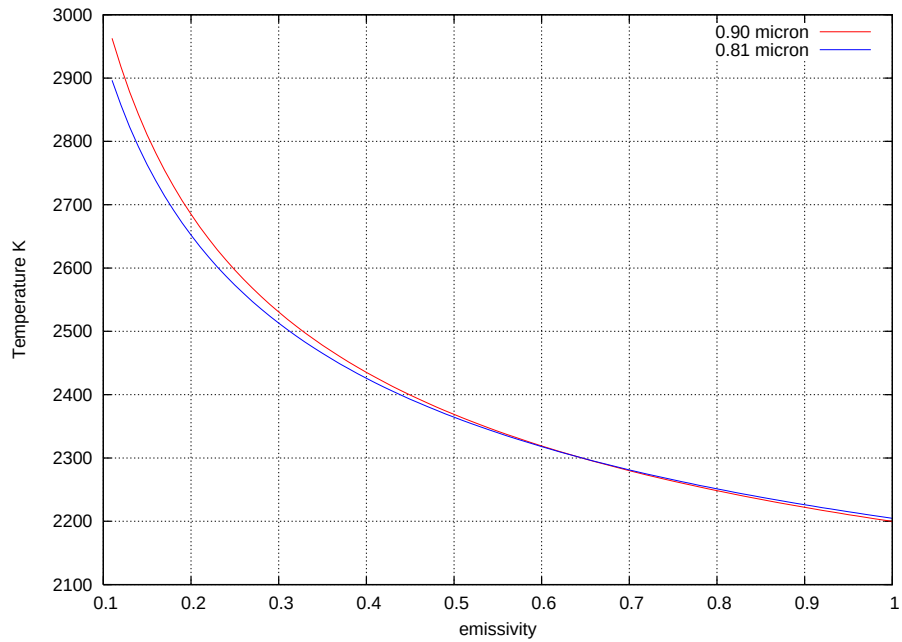


Figure C.16: Data Point 8 (ratio thermometer reading 2253 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

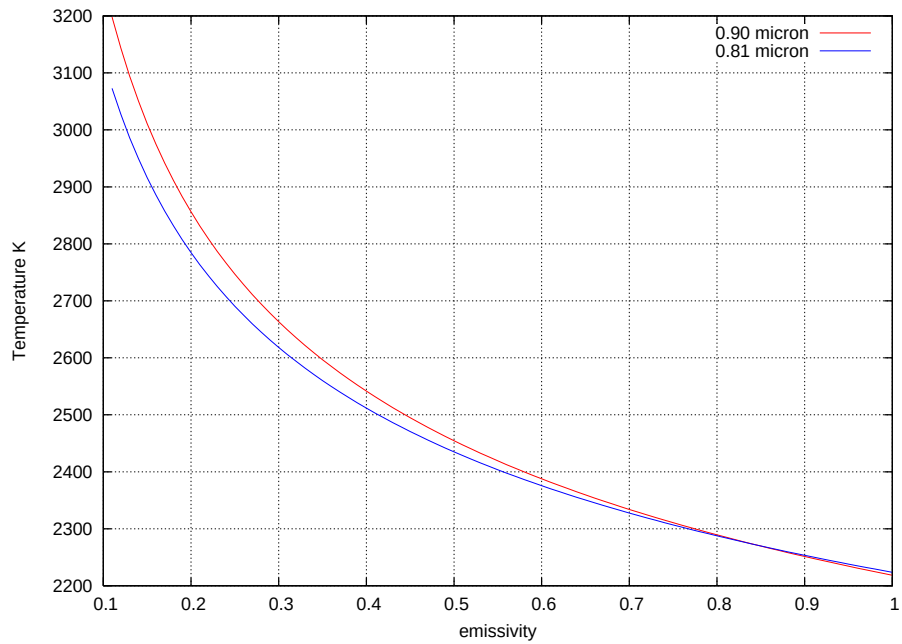


Figure C.17: Data Point 9 (ratio thermometer reading 2273 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

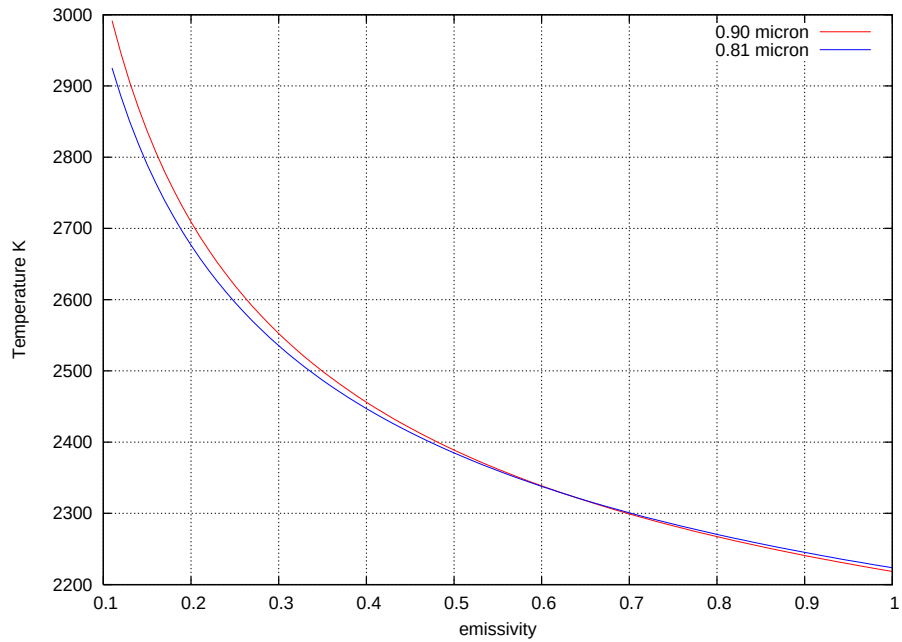


Figure C.18: Data Point 9 (ratio thermometer reading 2273 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

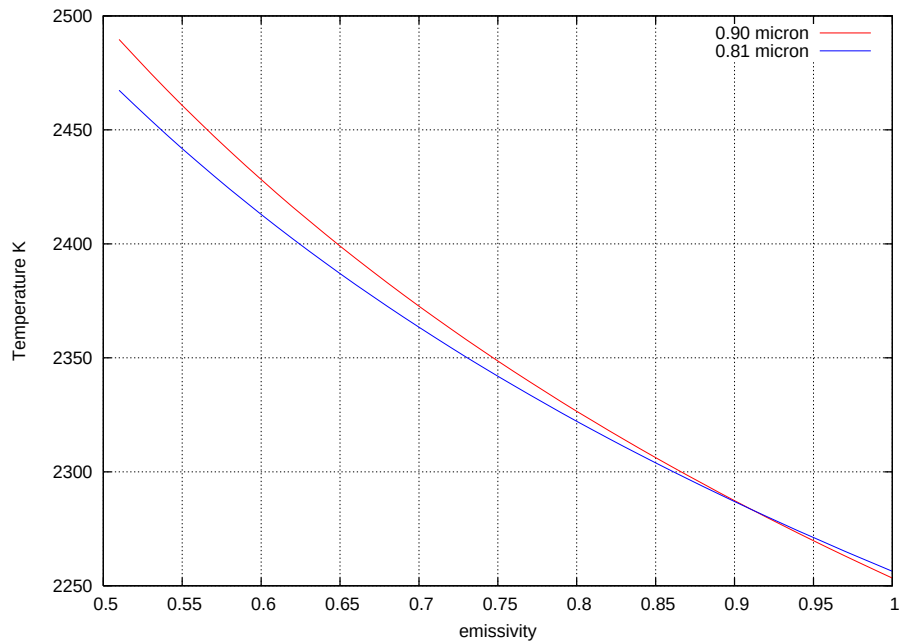


Figure C.19: Data Point 10 (ratio thermometer reading 2293 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

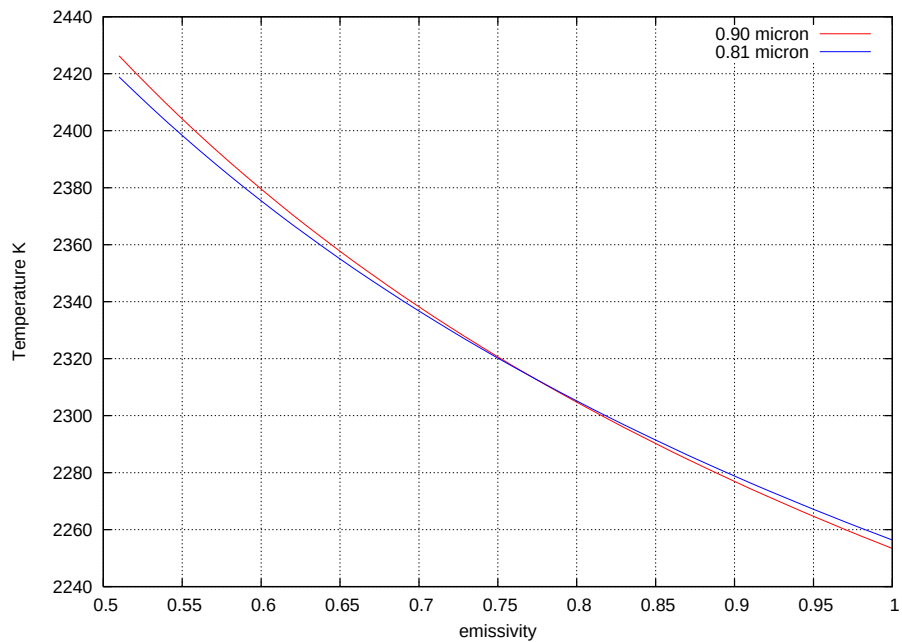


Figure C.20: Data Point 10 (ratio thermometer reading 2293 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

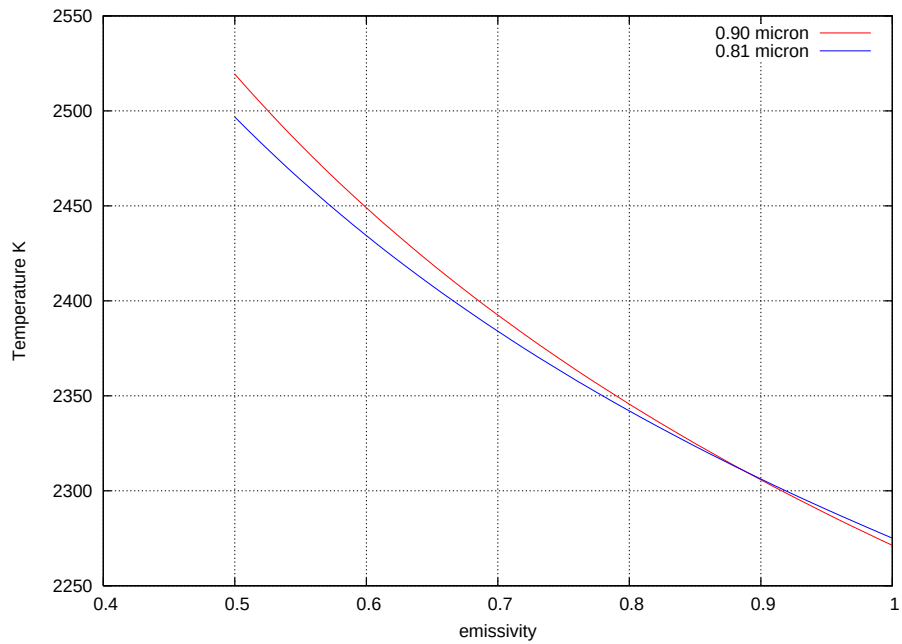


Figure C.21: Data Point 11 (ratio thermometer reading 2313 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

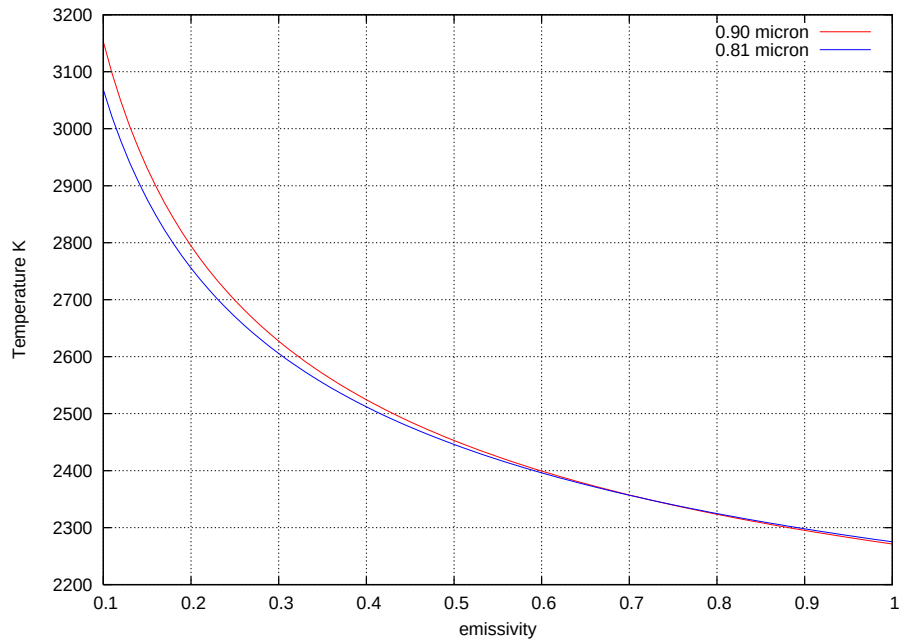


Figure C.22: Data Point 11 (ratio thermometer reading 2313 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

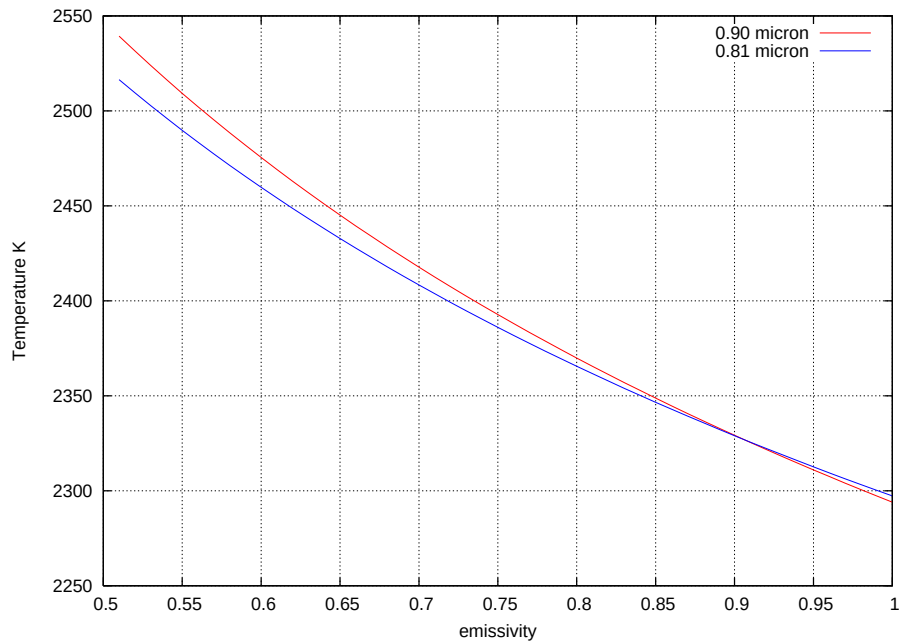


Figure C.23: Data Point 12 (ratio thermometer reading 2333 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

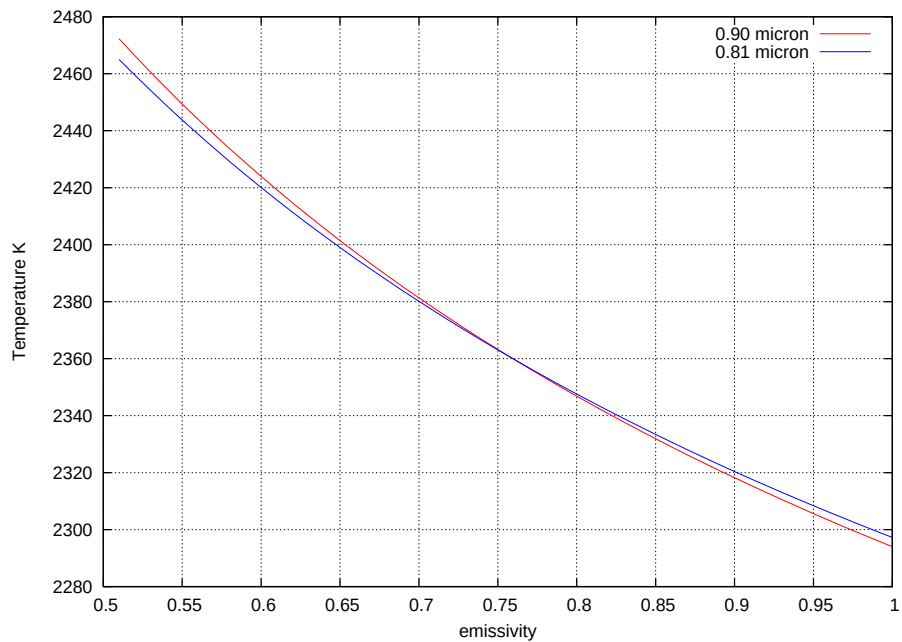


Figure C.24: Data Point 12 (ratio thermometer reading 2333 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

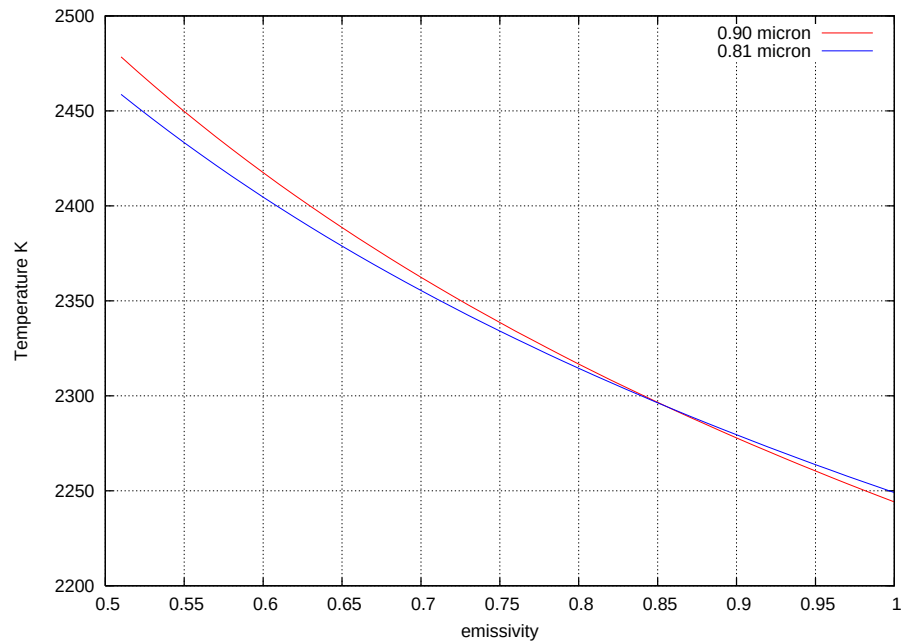


Figure C.25: Data Point 13 (ratio thermometer reading 2293 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

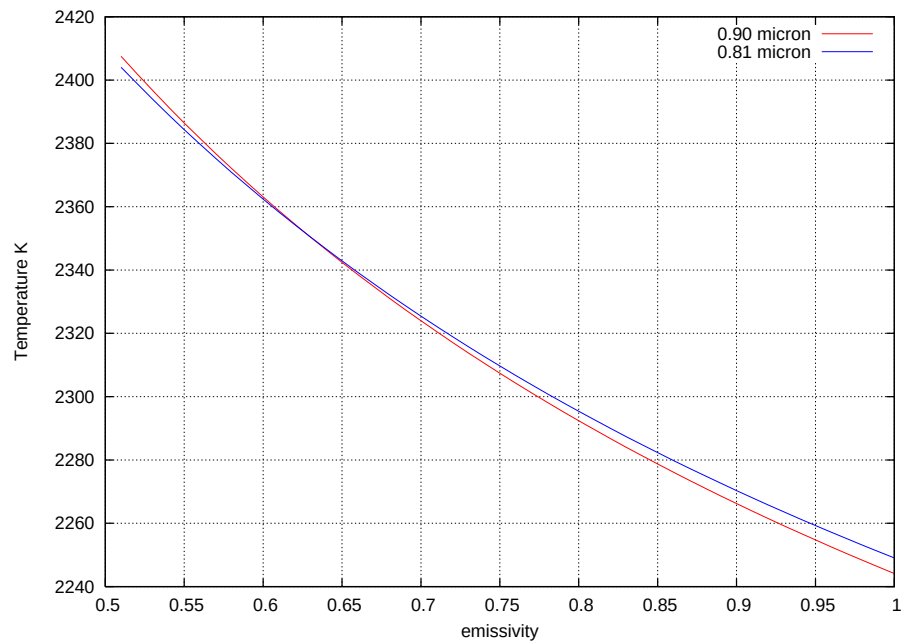


Figure C.26: Data Point 13 (ratio thermometer reading 2293 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

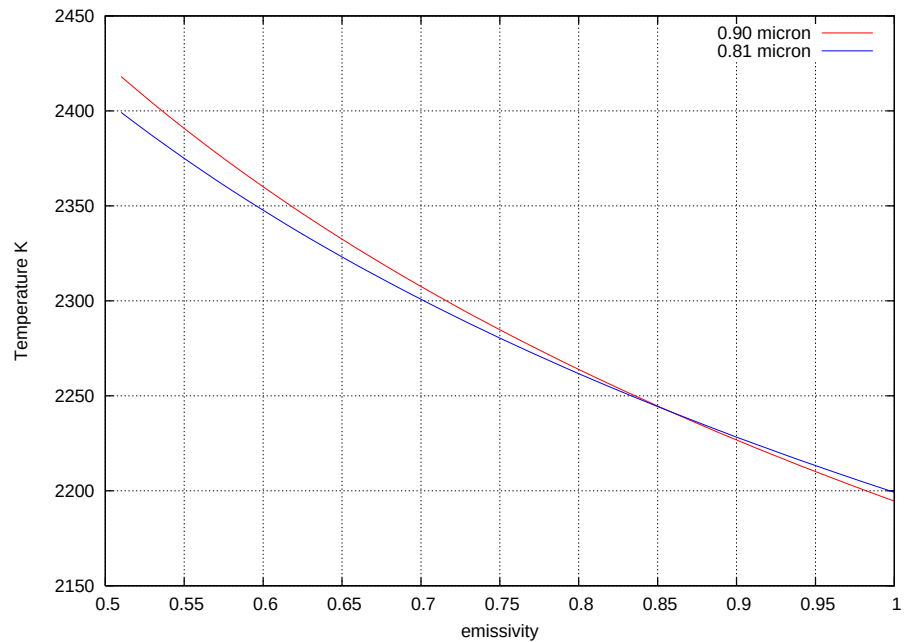


Figure C.27: Data Point 14 (ratio thermometer reading 2253 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

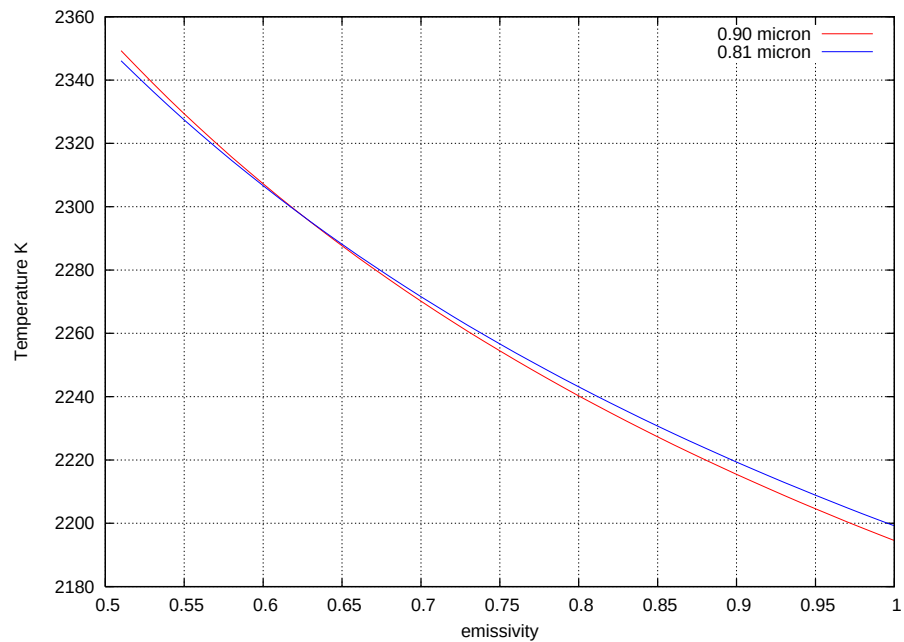


Figure C.28: Data Point 14 (ratio thermometer reading 2253 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

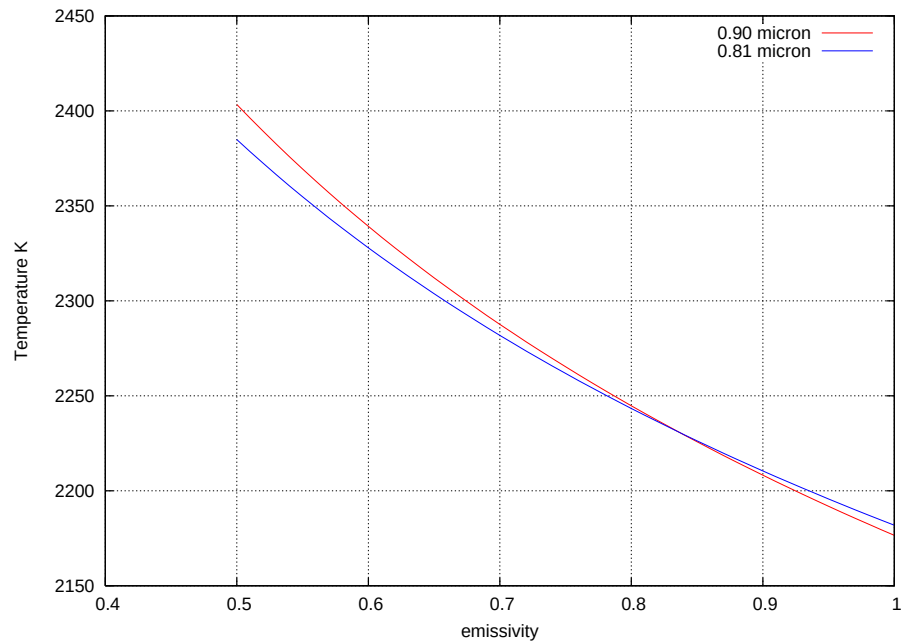


Figure C.29: Data Point 15 (ratio thermometer reading 2233 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

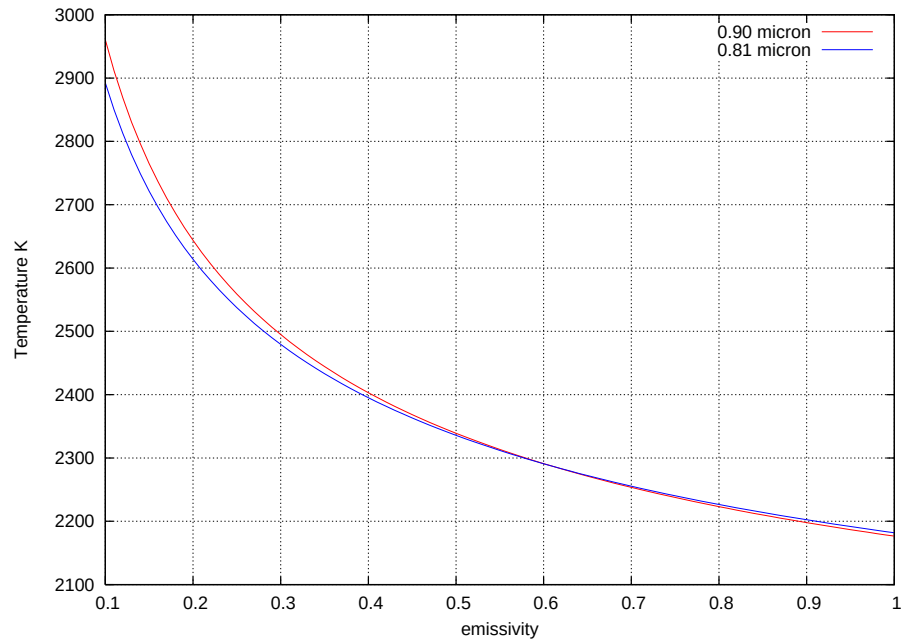


Figure C.30: Data Point 15 (ratio thermometer reading 2233 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

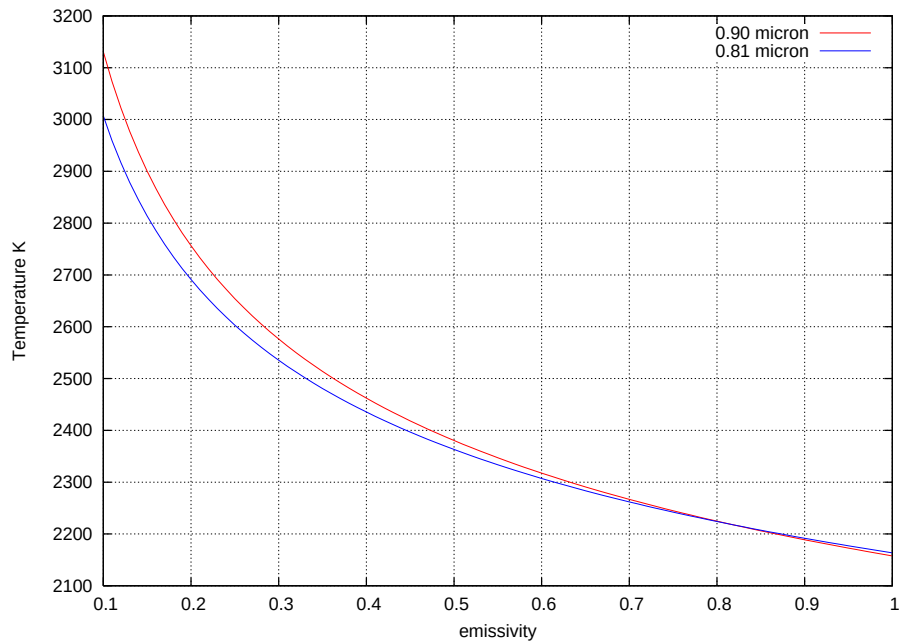


Figure C.31: Data Point 16 (ratio thermometer reading 2213 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

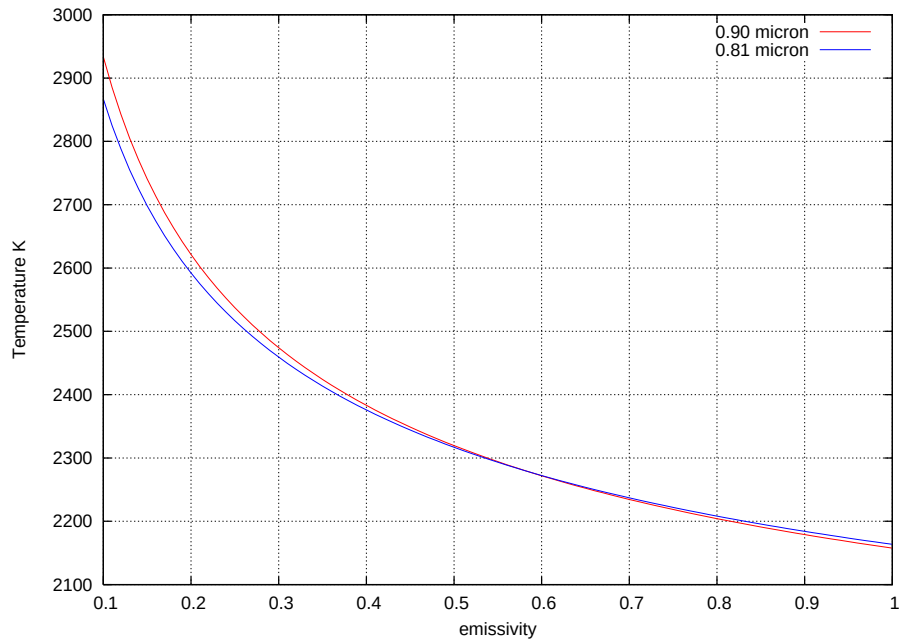


Figure C.32: Data Point 16 (ratio thermometer reading 2213 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

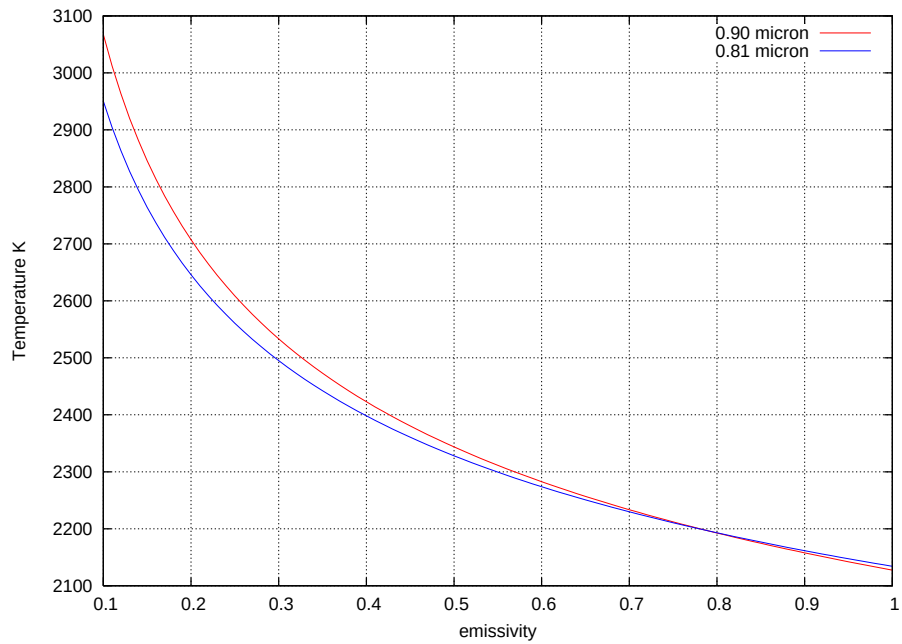


Figure C.33: Data Point 17 (ratio thermometer reading 2193 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

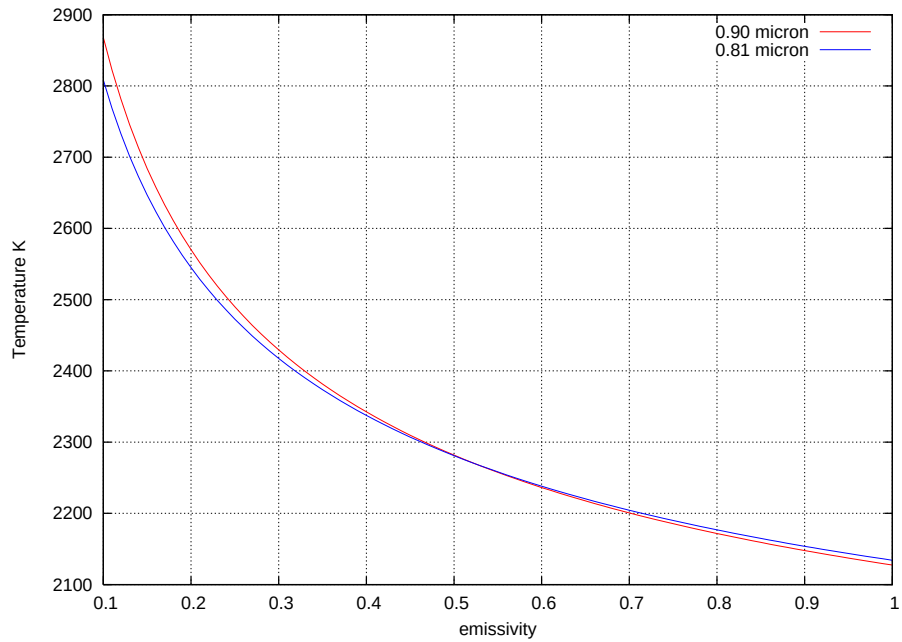


Figure C.34: Data Point 17 (ratio thermometer reading 2193 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

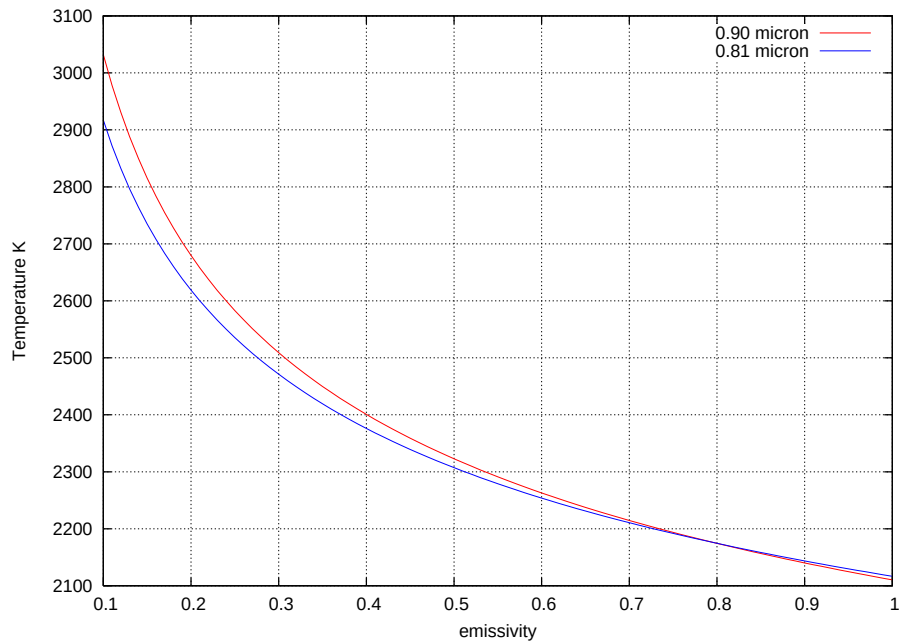


Figure C.35: Data Point 18 (ratio thermometer reading 2173 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

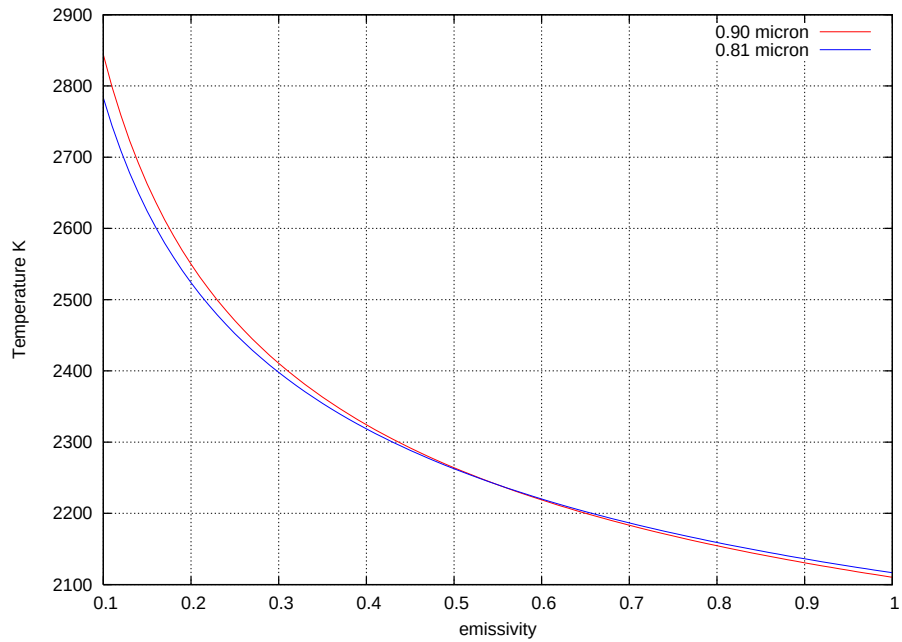


Figure C.36: Data Point 18 (ratio thermometer reading 2173 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

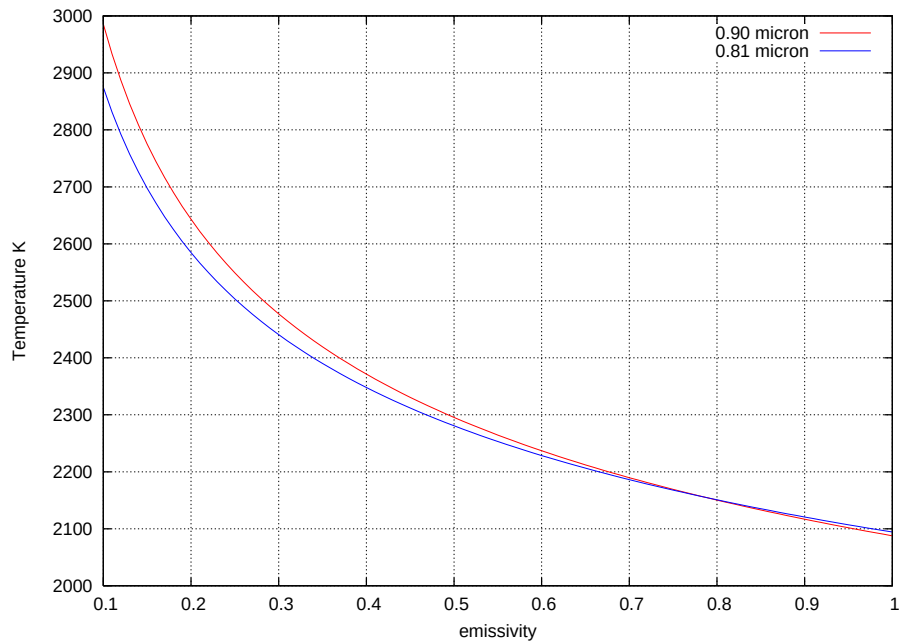


Figure C.37: Data Point 19 (ratio thermometer reading 2153 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

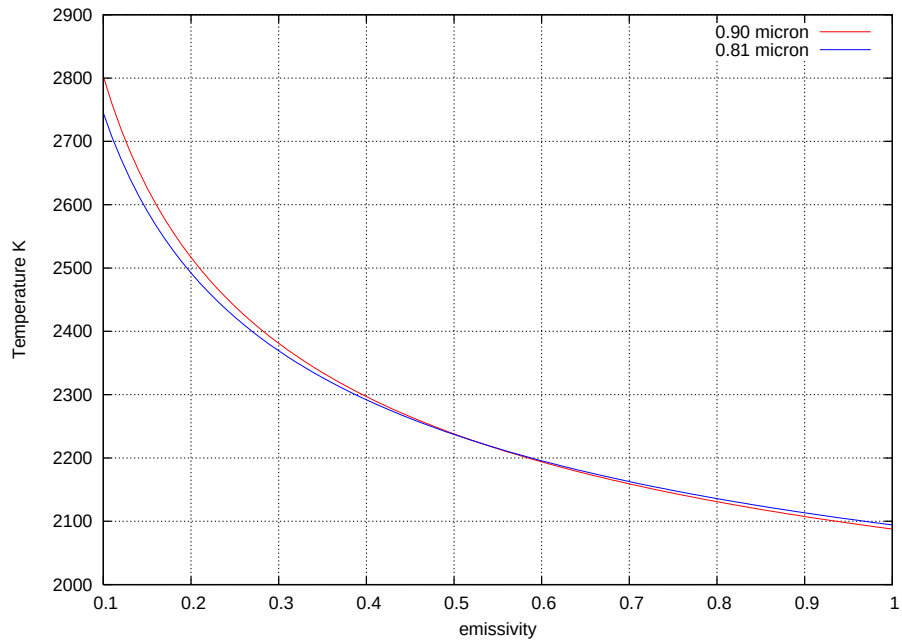


Figure C.38: Data Point 19 (ratio thermometer reading 2153 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

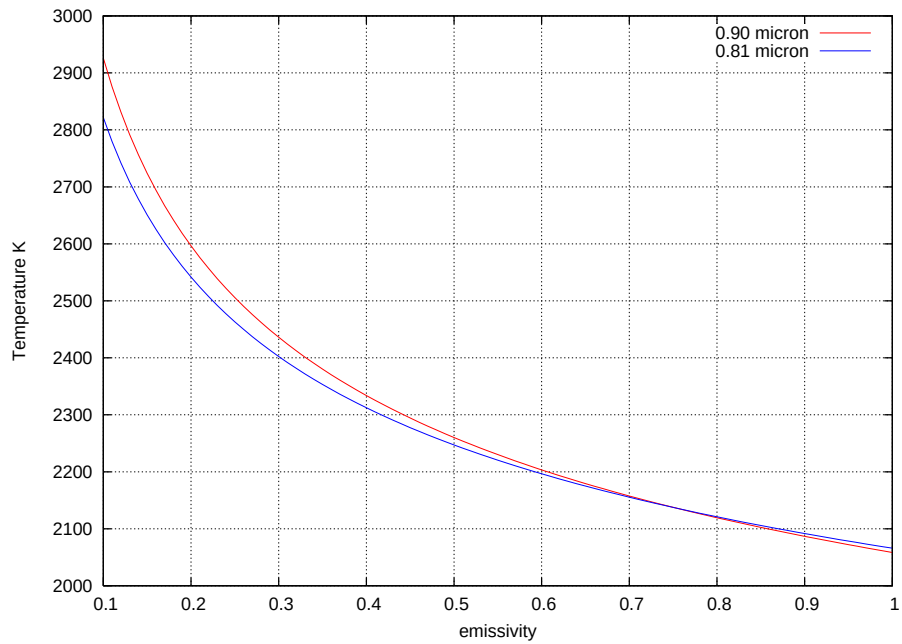


Figure C.39: Data Point 20 (ratio thermometer reading 2133 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

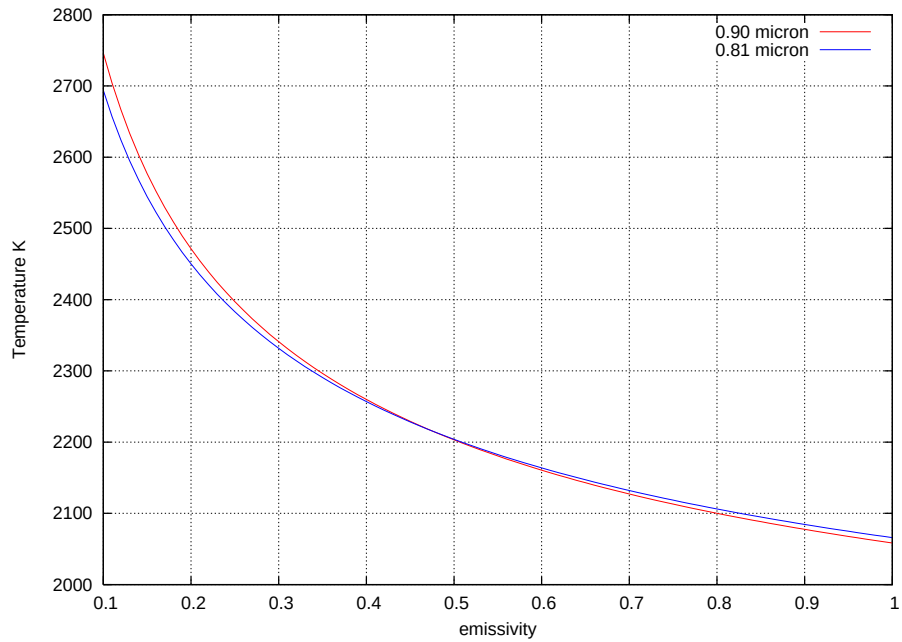


Figure C.40: Data Point 20 (ratio thermometer reading 2133 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

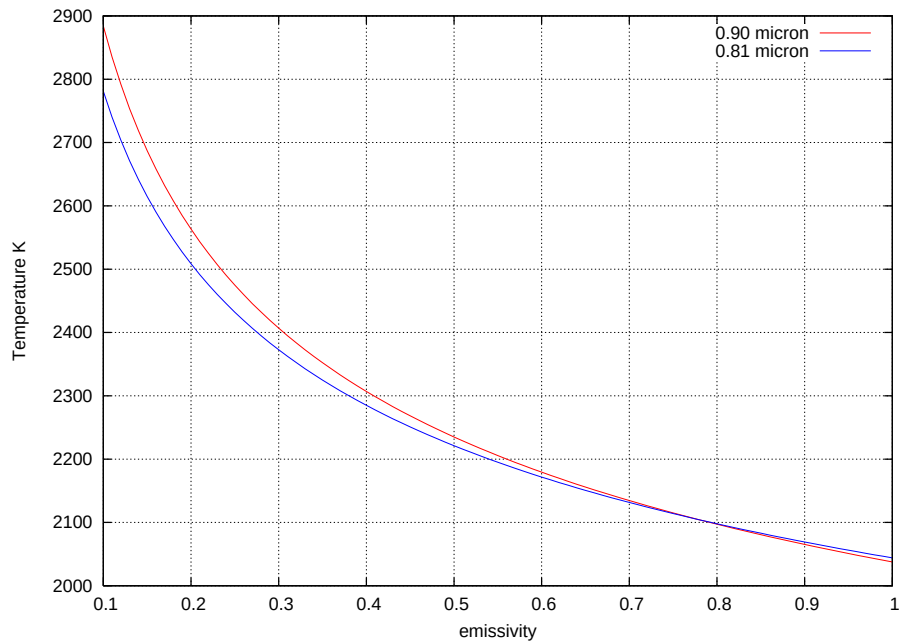


Figure C.41: Data Point 21 (ratio thermometer reading 2113 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

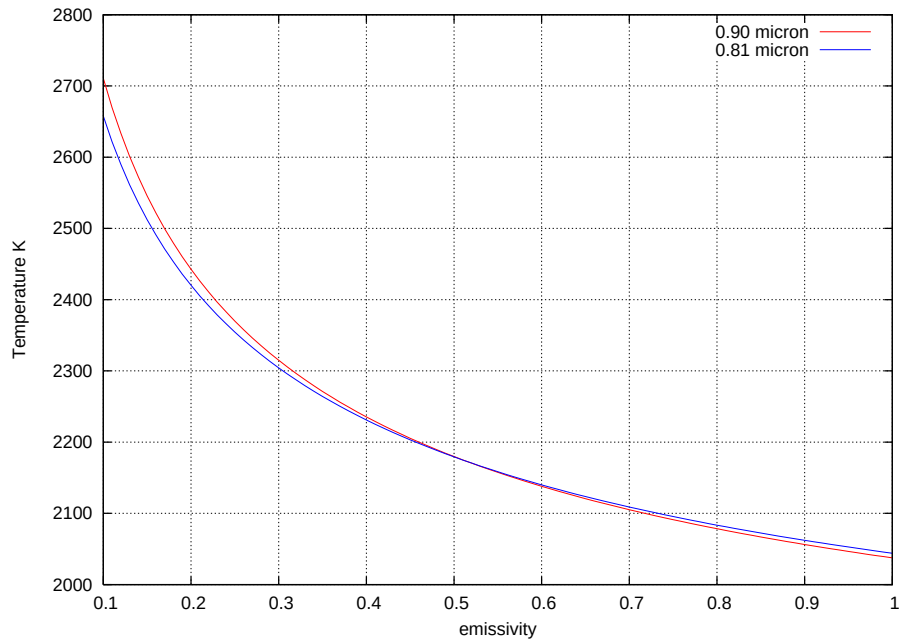


Figure C.42: Data Point 21 (ratio thermometer reading 2113 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

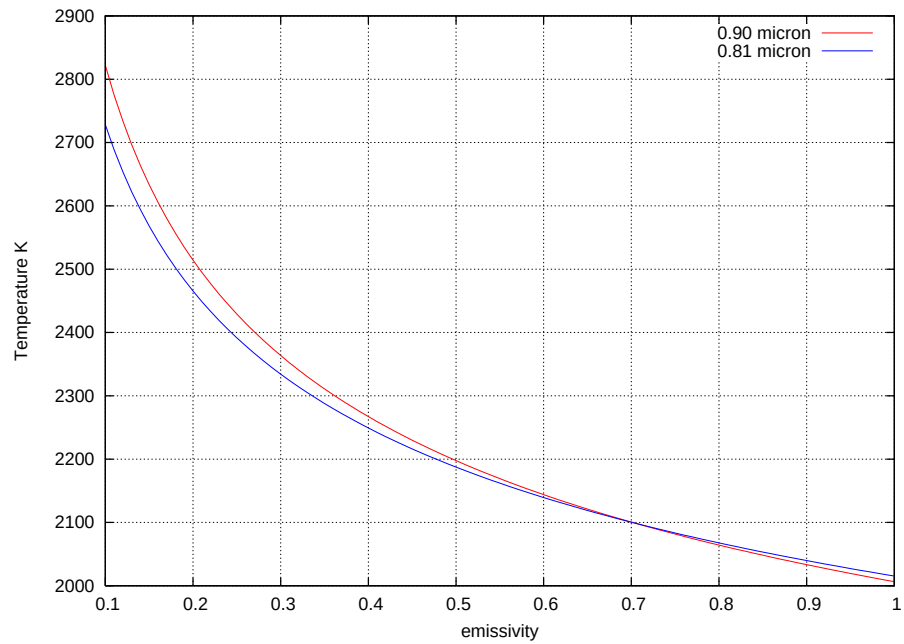


Figure C.43: Data Point 22 (ratio thermometer reading 2093 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

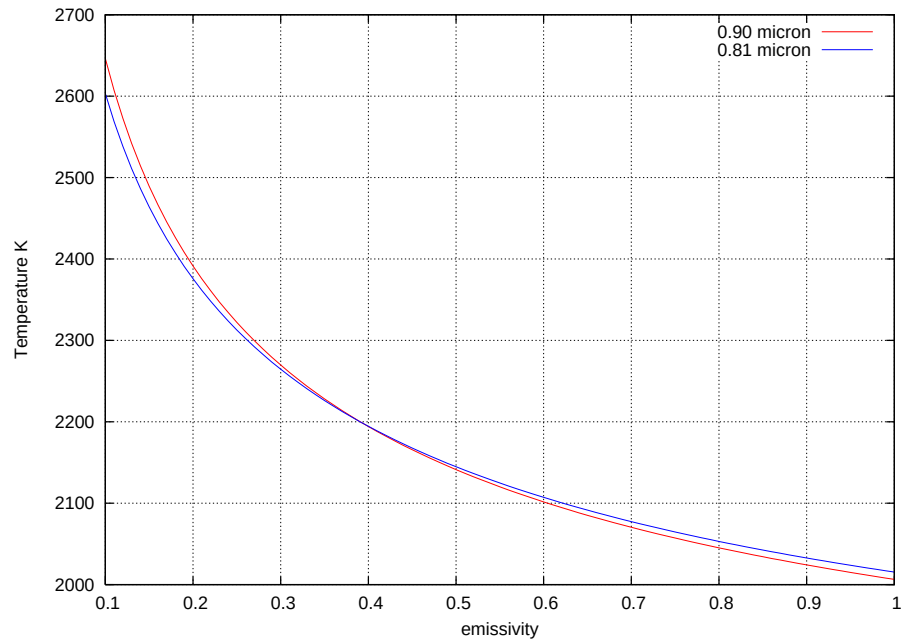


Figure C.44: Data Point 22 (ratio thermometer reading 2093 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

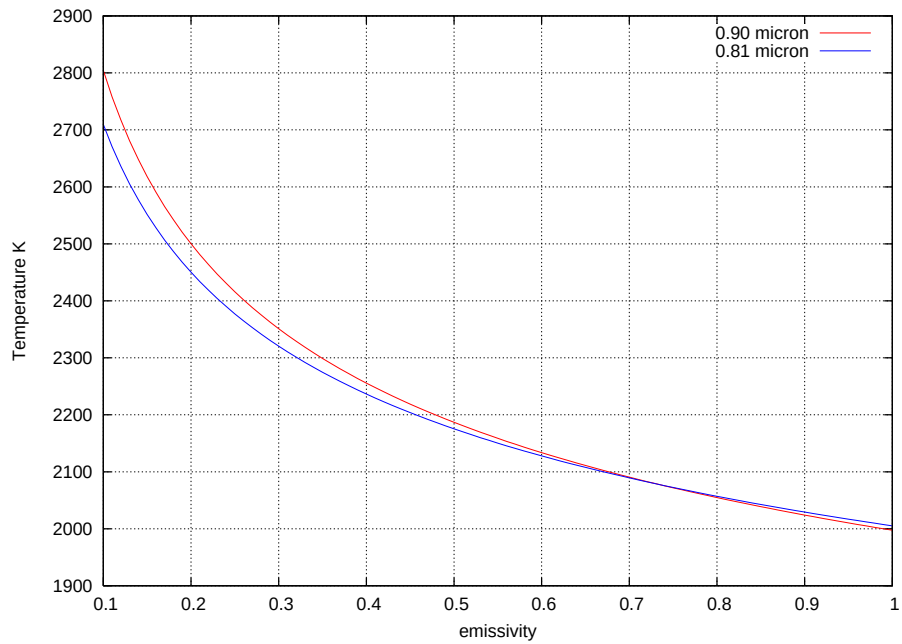


Figure C.45: Data Point 23 (ratio thermometer reading 2073 K) - graphical solution of emissivity and temperature neglecting the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

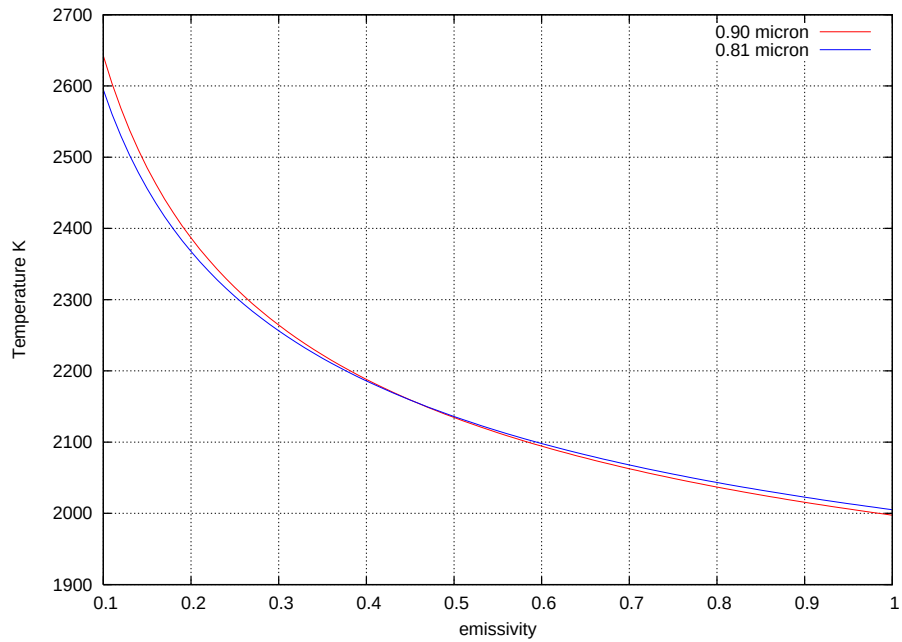


Figure C.46: Data Point 23 (ratio thermometer reading 2073 K) - graphical solution of emissivity and temperature including the effect of irradiance. The intersection of the plots gives the absolute temperature and emissivity of the surface.

# Vita

Howard Mosley Frederick was born in Hopkinsville, Kentucky on May 24, 1956. He was raised in Hopkinsville and attended Highland Elementary, Lacy, and Christian County Middle schools. He graduated from Christian County High School in 1974. From there he went to the University of Louisville and received a B.S. degree in 1977.

Howard has been employed as an electrical engineer for 31 years and is a licensed Professional Engineer in the state of Tennessee. He is currently pursuing a master's degree in physics at the University of Tennessee Space Institute.