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Passive Control Architectures for Collaborative Virtual Haptic Interaction and Bilateral Teleoperation over Unreliable Packet-Switched Digital Network

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Abstract

This PhD dissertation consists of two major parts: collaborative haptic interaction (CHI) and bilateral teleoperation over the Internet. For the CHI, we propose a novel hybrid peer-to-peer (P2P) architecture including the shared virtual environment (SVE) simulation, coupling between the haptic device and VE, and P2P synchronization control among all VE copies. This framework guarantees the interaction stability for all users with general unreliable packet-switched communication network which is the most challenging problem for CHI control framework design. This is achieved by enforcing our novel *passivity condition* which fully considers time-varying non-uniform communication delays, random packet loss/swapping/duplication for each communication channel. The topology optimization method based on graph algebraic connectivity is also developed to achieve optimal performance under the communication bandwidth limitation. For validation, we implement a four-user collaborative haptic system with simulated unreliable packet-switched network connections. Both the hybrid P2P architecture design and the performance improvement due to the topology optimization are verified.

In the second part, two novel hybrid passive bilateral teleoperation control architectures are proposed to address the challenging stability and performance issues caused by the general Internet communication unreliability (e.g. varying time delay, packet loss, data duplication, etc.). The first method—Direct PD Coupling (DPDC)—is an extension of traditional PD control to the hybrid teleoperation system. With the assumption that the Internet communication unreliability is upper bounded, the

passive gain setting condition is derived and guarantees the interaction stability for the teleoperation system which interacts with unknown/unmodeled passive human and environment. However, the performance of DPDC degrades drastically when communication unreliability is severe because its feasible gain region is limited by the device viscous damping. The second method–Virtual Proxy Based PD Coupling (VPDC)–is proposed to improve the performance while providing the same interaction stability. Experimental and quantitative comparisons between DPDC and VPDC are conducted, and both interaction stability and performance difference are validated.

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Chapter 1

Introduction

Collaborative haptic interactive (CHI) system and bilateral teleoperation are two attractive and challenging areas in robotic research. Both areas have found numerous important applications in many scenarios. However, designing the control architectures that guarantee the interaction stability under general Internet unreliability (e.g. varying time delay, packet loss, data duplication/swapping) has been a challenging and open problem in both fields. In this chapter, I will introduce the current status of CHI and bilateral teleoperation research along with the research objectives of my doctoral study.

1.1 Collaborative Haptic Interaction (CHI)

1.1.1 Background and Research Objectives

The local haptic interaction with the virtual environment (VE)—an increasingly common modality in human-computer interfacing—has great practical value in many areas like virtual surgical training, immersive 3D product design (Fig. 1.1). Moreover, collaborative haptic interaction (CHI) allows multiple users to operate in a shared virtual environment (SVE) simultaneously. They do not only interact with the

VE but can also feel the operations done by all other users (both visually and haptically). This feature is important for simulating collaborative virtual tasks, like surgery training, where the force cooperation is important to the efficacy of the virtual task. Furthermore, if we can extend the CHI from local users to remote users over the Internet, many promising applications like remote virtual collaborative surgical training, collaborative haptic evaluation of CAD designs, virtual sculpting among remote artists, and haptics-powered networked computer games, can be implemented and maybe change the way of interacting with others in the cyberspace, by complementing the widely-used vision and audio interaction modalities for virtual reality.

To deliver a real-world CHI system, there are three desired characteristics: 1) stability (for immersiveness and safety); 2) SVE synchronization (for consistent perception among users); 3) force cooperation (let users feel each other). In this chapter, we propose a novel P2P control architecture and the CHI system based on this architecture to achieve all these goals. More specifically, our architecture will 1) guarantees the interaction stability for any passive human operators and environments, 2) passifies the instability induced by any communication unreliability of packet-switched network, 3) provide configuration coordination among all SVE copies, and 4) each user can feel the resultant force from all other users. The real-world communication topology could be very complicated and the communication rate is slower than the VE update rate. So, we also consider arbitrary connected undirected information graph (for flexibility and scalability), interconnecting fast-updated VE (to make the simulation of complicated VE possible) with slow-updated communication, optimizing the network topology (to improve the performance), reducing the usage of network bandwidth (enable large-scale implementation).

Although the CHI is a relatively new research area, many significant research work that have been done intend to achieve the aforementioned objectives. The existing works, depending on their research perspectives, can be further categorized into 1) purely implemental works [CMZ01, CCR97, GJF⁺07, GHS⁺05, IHT04, KHK⁺07]



Figure 1.1: Example of haptics applications (both pictures are from [Sen]).

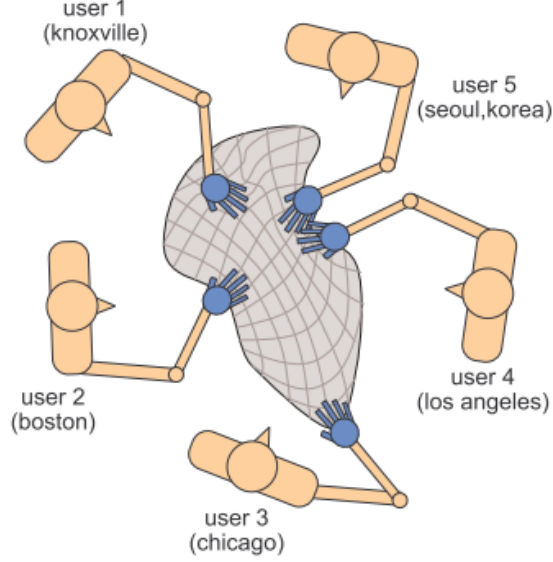


Figure 1.2: CHI over the packet-switched network.

and [PLC06, SSMR95, SDWET06] 2) experimental and qualitative works including [CCR97, LC09], and [SH08b, SH08a, SH06, SZESG04], and 3) theoretical works [CO04, CNK09, FSC07]. We will give a brief introduction of these works from different angles and compare our work with closely-related ones (i.e. theoretical works) in the following subsection.

1.1.2 Literature Review

Experimental Research Work on CHI

In 1989, a string-based haptic device called SPIDAR [HS92] was developed in Tokyo Institute of Technology. Later on, it was extended to allow two users simultaneously grasp a shared virtual object [Ish94], and became the first successful collaborative haptic implementation [BOH97]. After that, many practical systems have been proposed or actually implemented. For example, in [CMZ01], the authors gave a positive outlook for utilizing the collaborative haptics idea to improve the military training. In [KHK⁺07], a virtual multi-player table tennis game was developed which is a promising trial of introducing the haptics into online network gaming. In

[SSMR95], a situational training system was developed. Through these very promising application works, the collaborative haptics has been proven to be an effective way to improve the training performance, e.g. [SSMR95, CMZ01], or the trend of the next generation of online computer games [KHK⁺07]. However, many critical problems like stability, performance, effects of communication problem, effects of the variations of users and environments, and the network topology issue were ignored or solved by trail-and-error in these works.

The work of Buttolo, et al. [BOH97] systematically gave three different types of collaborative haptic system design, according to different cooperation types among users (static, collaborative and cooperative). Other detailed problems like haptic rendering, graphic rendering, communication delays were discussed. However, there were no thoroughly theoretical discussion on these issues, and the performance was poor when the time delay is large. Choi et al. also proposed the CHI idea in [CCR97]. In this work, the authors use the network to connect multiple remote users. However, communication time delay was ignored which weakened its practical value.

In [SZESG04], the authors aimed at developing a heterogeneous scalable architecture for large SVEs where a number of potential users can participate with different kinds of haptic devices. This work proposed the approaches based on centralized and P2P architectures respectively. The sluggish response for the centralized architecture due to the communication delay was observed and a predictor was proposed to reduce the sluggishness. However, this delay compensation mechanism provided no guarantee of improving the sluggish response.

Sankaranarayanan, et al. were also active in this area from the experimental/quantitative perspective. In [SH06], three different control schemes of two-user peer-to-peer (P2P) haptic interaction system were provided, which were all PD-based essentially. With the further consideration of compensating the time delays, PD-based, wave, PO/PC methods were tested and compared in [SH08b]. P2P and client-server topologies are also experimentally studied in [SH08a]. However, all the schemes

provided in [SH06, SH08b, SH08a] only consider two users, so the extendability for these schemes were unknown from these works.

The common issue with all these works is lacking of the thorough analysis of stability, which is understandable since these works focused on the application perspective. Moreover, the stability issue within the scope of CHI system is very complicated because it involves many factors. In the following subsection, we will summarize the theoretical achievement in this area.

Theoretical Research Work on CHI

As the interests in CHI applications growing, people started seeking the answers to many critical and practical questions like guaranteed stability with communication delay, extendability and performance optimization.

Although these questions were not explicitly answered in [CO04], Carignan et al. first introduced the well-known *wave variable method* into CHI research. The wave variable method had been well developed at that time for solving the instability issue induced by communication time delay, a more detailed review on wave variable will be given later. An admittance control was also proposed for low communication delay in that paper.

Fotoohi et al., presented novel and solid control architectures in [FSC07]. Several interesting problems were considered in that work. First, both centralized and P2P frameworks were formulated as a linear digital control problem, so, the stability analysis can be easily solved. Second, the formulation for the VE simulation and the communication were in discrete-time domain, which reflects the practical cases. Third, the ZOH and sampling were considered in the stability analysis. Fourth, multi-rate problem, i.e. fast-updated VE and sampling connecting with slow-updated communication channels, is investigated. [FSC07] is the first systematic work that touch many detailed aspects in CHI research. However, the downsides of this work are also obvious: 1) the entire system, including the human and haptic devices were assumed to be linear and time-invariant (LTI); 2) time delay is known, constant

and very small (within 0-2 update intervals); 3) jitter and packet-loss, due to the assumption that the system runs on Local Area Network (LAN) or Metropolitan Area Network (MAN), are ignored in the analysis. Here, need to point out that, the theoretical approach used in this work, i.e. digital control theory for linear system, cannot be extended to handle jitter, packet-loss and duplication.

Another representative work is [CNK09]. In this paper, Cheong first propose a unique control framework based on the concept of *natural dynamics* for a two-user linear time invariant (LTI) CHI system. To better understand this framework, let us consider the following LTI damped system with two identical subsystems:

$$\begin{aligned} m\ddot{x}_1(t) + b\dot{x}_1(t) &= f_1(t) + f_2(t - T_2) \\ m\ddot{x}_2(t) + b\dot{x}_2(t) &= f_1(t - T_1) + f_2(t) \end{aligned} \tag{1.1}$$

where $m > 0$ is the inertia, $b \geq 0$ is the damping coefficient. For this system, if the b is nonzero, i.e. $b > 0$, the stability and the synchronization error, defined by $e(t) := x_1(t) - x_2(t)$ becomes zero in steady state. The dynamics shown in (1.1) is called *natural dynamics*, which is the core idea of this framework. In ideal case, if (1.1) is precisely preserved, there is no need to design any control algorithm. However, this natural dynamics is vulnerable to disturbance (e.g. noisy force measurement) and initial states error (permanent position drift). To address these issues, the authors proposed a novel controller which is only triggered by the aforementioned conditions, i.e. disturbance and initial errors. The control design was then extended to N subsystems. However, the main limit for this extension is that the network topology has to be ring-type. It is yet not clear that if this extended multiuser control framework can be further extended to general connected communication graph. But it is clear that the following obstacles need to be addressed for such extension: 1) the current framework is one-directional, i.e. each circles among all subsystems in a pre-defined sequence; 2) i^{th} subsystem only accepts the position data from $(i - 1)^{\text{th}}$ subsystem, not from multiple subsystems. The limitation

caused by ring-type topology is obvious. First, the topology is not scalable since the dynamics and sequence of the connected subsystems must be known before the algorithms executed. Second, the time delay, comparing to general connected graph, is unnecessarily increased. Third, communication failure in one subsystem is fatal to the whole system. Even suppose this control framework can be extended to general connected graph, there are still limitations. The subsystems' dynamics have to be exactly known and LTI (or can be converted into LTI forms). Moreover, the time delays have to be known and constant. Furthermore, VE simulation, in practice, is simulated in digital computer discretely. This fact is ignored by this work also.

In this dissertation, we propose a novel and complete CHI control framework which possesses the following features at the same time:

- General communication unreliability: this includes varying time delay, packet-loss, data duplication/swapping.
- Scalability: P2P architecture with arbitrary underline undirected connected information graph.
- Passive deformable VE: the VE simulation is deformable and discrete passive.
- Nonlinear haptic device: any haptic devices (linear or nonlinear) can be connected to the CHI system.
- Closed-loop passivity: the closed-loop system is passive under item 1–4.
- SVE synchronization & force cooperation: SVEs are synchronized in the absence of external force and each user can feel the summation of the force from all other users in steady state.
- Topology optimization: a optimization strategy to achieving nearly best performance based on control gains and network conditions.

To our best knowledge, there are existing no works on CHI that can achieve these features simultaneously like ours.

1.1.3 Outline

The CHI control architecture development, optimization and experimental validation are presented in Chapter 2 which is organized as follows. In Sec.2.1, basic notations and results of graph theory will be introduced. Then, we extend graph Laplacian, which is an important tool of analyzing graph topology, to multiple degree of freedom (DOF). In Sec.2.2, the P2P control architecture for CHI is proposed and the essential *passivity condition* is provided and rigorously proved. We also present a novel simple graph topology optimization method in Sec.2.3. A 4-user CHI system is implemented for validating aforementioned control objectives and topology optimization. The conclusion remarks for CHI will be given in Sec. 2.5 and supplementary mathematical proofs can be found in Sec. 2.6.

1.2 Bilateral Teleoperation

1.2.1 Background and Research Objectives

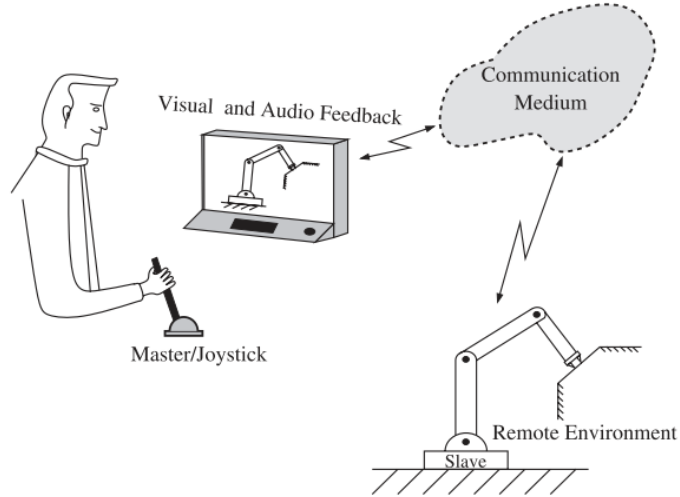


Figure 1.3: Bilateral teleoperation system [HS06].

Bilateral teleoperation is among the most traditional robotic research areas (over 50 years [HS06]) and very challenging [VC86]. In bilateral teleoperation as shown in

Fig.1.3, a human operator conducts a task in a remote environment through master and slave manipulators. Here, ‘bilateral’ means not only the human commands the slave manipulator to interact with the remote environment but the environment is also reflected to the user by displaying the contact force with/without visual displays. It has been recognized that force feedback can improve the teleoperation task performance [HS06]. However, it also raises the challenging instability problem when delay exists in the communication channel.

Such instability problem has attracted many research efforts and a lot of results have been published. For purely continuous case (i.e. the robot control and communication channels are assumed to be continuous-timed), *Scattering approach* [AS89] and its reformulation *wave method* [NS91] are one of the main streams in delayed teleoperation research. The main idea is using scattering approach to make the transmission line passive independent of the amount of time delay. This method was originally designed for constant time delay and was then extended for time-varying delay [LCS02]. The methods other than scattering approach include *adaptive scheme* [CS04], PD-like control law [LS06b, NOBB08, NBOS09], and H_∞ and μ -synthesis based method [LFA95]. For purely discrete-time case, the scattering approach was extended by [KMT96]. Also the passivity controller and observer (PO/PC) method was also proposed in [RKH02, RKH04, RAP10].

The downside of aforementioned methods is clear. First, the aforementioned methods are designed only for purely continuous or discrete case. Both put unrealistic assumption on the real bilateral teleoperation systems. However, in practice, the bilateral teleoperation systems usually consist of continuous robot dynamics and discrete control and communication which defines a *hybrid system*. Second, as mentioned in multiuser haptics part, the unreliability of Internet is complicated and often mixed together. So far, none of these methods can handle such complicated communication unreliability.

In our group, we proposed *passive set-position modulation* (PSPM) method which can fulfill the gaps, which may be the only answer to the Internet bilateral teleoperation so far.

In this dissertation, we focus on designing a novel and effective control law to guarantee the closed-loop passivity and other teleoperation performance requirements (position coordination and force reflection) under complicated Internet unreliability. The hybrid nature will be fully considered and embedded in the design and analysis. How to improve the performance will also be taken into account.

1.2.2 Literature Review

Teleoperation is a major branch of robotics research, and there are many research topics within this area. In our work, we mainly focus on the most challenging *delayed* bilateral teleoperation problem.

Before [AS89], there are virtually no theoretical result on the bilateral teleoperation system with time delay. In this very first work, the scattering theory for networks [Chi68] was introduced to bilateral teleoperation problem aiming to make the delayed communication channel passive. This method was quickly appreciated by the teleoperation research community and many successors were proposed. In [NS91], the scattering approach was reformulated into *wave method*, which takes an easier form than scattering approach. However, since there is no explicit position signal transmitted, the original wave method has the unrecoverable position drift (accumulated over the operation time). To fix this problem, Niemeyer proposed two cures named *wave integral* [NS98] and *adjusting the wave command* [NS04], where the signal transmitted contains explicit position signal or the drift error. The wave method was also extended for varying delay in [LCS02]. Moreover, the position drift problem for varying-delayed wave method was addressed in [NS98, CSHB03].

In continuous-time domain, there are also passivity-based control frameworks other than wave method. In [CS05], a method based on adaptive control laws was

proposed for constantly delayed teleoperation system. In [LS05, LS06b, LS06a], a PD-type control law was given which shed new light that even simple PD-type control can guarantee the passivity for the closed-loop teleoperation system with some damping injection.

The Internet teleoperation started from mid 1990s. Since it is a packet-switching network medium, the already established (aforementioned) time-delay analysis is with difficulties due to the complex communication unreliability such as: randomly varying delays, packet-loss, data duplication/swap and connection blackouts. Furthermore, the need to deal with discrete-time stability arises. To fill this gap, the discrete-time scattering approach was presented in [KM97] which can only handle the constant time delay. An important time-domain passivity based method was developed for pure discrete-time system in [RKH02, RKH04, RP07, RAP10]. The authors use passivity observer (PO) and passivity controller (PC). The PO measures the energy level at both ports (master/slave power ports). The basic idea with PO/PC method is that if the passivity-breaking energy is detected, the PC will dissipate it by damping injection. However, this work only focuses on passifying the communication network and ignores the hybrid nature of Internet teleoperation system. The passive set-position modulation (PSPM) [LH08b, Lee09, LH10a] is another time-domain based control architecture with the ability to passively handle any communication unreliability while considering the hybrid nature ignored by other works. The basic idea of PSPM is that, selectively modulating the (aggressive jumped) received position signals according to the available energy (harvest from local damping injection). By this way, arbitrary position signal sequence can be passively modulated. Hence, PSPM is able to deal with any communication unreliability. However, the PSPM framework requires part of the control to be continuous. This requires the sampling rate to be fast enough for the robots only with sampled-data control.

In most of the existing works on Internet bilateral teleoperation (except for PSPM), the hybrid nature (defined by discrete Internet, sampled-data control and continuous robot's dynamics) has not been thoroughly considered. This is a

unavoidable problem since most modern haptic devices are sampled-data controlled and the Internet is not a continuous communication medium. Also, how to guarantee the interaction stability (through passivity) of the closed-loop system under complex Internet unreliability is still a very challenging and open problem. In this dissertation, our goal is to provide an answer to both questions simultaneously, which is a solid contribution to the teleoperation research area and may change the way how researchers consider Internet bilateral teleoperation system (hybrid nature, unreliable packet-switching Internet, etc.).

1.2.3 Outline

The content regarding Internet bilateral teleoperation is presented in Chapter 3, which is organized as follows. In Sec.3.1, the modelling for teleoperation system and the unreliable packet-switched communication network are presented. The novel DPDC and VPDC methods are proposed in Sec.3.2 and 3.3 along with their passivity conditions. The experimental results are provided in Sec.3.4 for validating the control designs under unreliable communication network. The performance comparison between DPDC and VPDC using simplified models is conducted in Sec.3.5. Some conclusion remarks are given in Sec.3.6.

Chapter 2

Collaborative Haptic Interaction over the Internet

Since the CHI system has been introduced in [1](#), in this chapter, we focus on the theoretical development and experimental validation of our novel CHI architecture. The necessary graph theory notations and properties along with our new multi-dimensional extension to traditional graph Laplacian will be introduced in [Sec.2.1](#). Then, the detailed P2P control architecture will be presented in [Sec. 2.2](#) and the network optimization under bandwidth limitation will be given in [Sec. 2.3](#). Experimental results obtained from a four-user CHI system will be shown in [Sec. 2.4](#). Supplementary mathematical proofs for [Sec. 2.2](#) can be found in [Sec. 2.6](#).

2.1 Graph Theory

2.1.1 Basic Notations and Properties

We use graph theory [[Big93](#)] to describe the communication topology among N users over the Internet. For this, we define $\mathcal{G}(\mathcal{V}, \mathcal{E})$ to be a graph where $\mathcal{V} := \{v_1, \dots, v_N\}$

and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ are respectively the set of N vertexes^{*} (i.e., users or their VOs) and the set of N_e edges connecting them (i.e., information flow). Each directed edge of $\mathcal{E}$ can then be identified either by e_{ij} with v_i and v_j being the head and tail of e_{ij} (e.g., v_i receives information from v_j); or by e_l with $l \in \mathcal{E}_C := \{1, 2, \dots, N_e\}$. In fact, we can define a bijective map between $l \in \mathcal{E}_C$ (i.e., *one-tuple* enumeration) and $(i, j) \in \mathcal{E}_P := \{(i, j) \mid e_{ij} \in \mathcal{E}, v_i, v_j \in \mathcal{V}\}$ (i.e., *two-tuple* enumeration). We will denote this equivalence between $\mathcal{E}_P$ and $\mathcal{E}_C$ by

$$l \approx (i, j) \quad \text{if} \quad e_l = e_{ij} \in \mathcal{E}.$$

In this dissertation, we assume $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is simple (i.e., no self-loops) and undirected (i.e., $e_{ij} \in \mathcal{E} \leftrightarrow e_{ji} \in \mathcal{E}$). We also define the information neighbours of v_i s.t.,

$$\mathcal{N}_i := \{v_j \in \mathcal{V} \mid e_{ij} \in \mathcal{E}\}$$

i.e., the set of users, from which v_i receives information. We will also use the following facts: for any $\varepsilon : \mathcal{E}_P \rightarrow \mathbb{R}^m$,

$$\sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \varepsilon_{ij} = \sum_{l=1, (p,q) \approx l}^{N_e} \varepsilon_{pq} \quad (2.1)$$

and, further, if $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is undirected,

$$\sum_{l=1, (p,q) \approx l}^{N_e} \varepsilon_{pq} = \frac{1}{2} \sum_{l=1, (p,q) \approx l}^{N_e} (\varepsilon_{pq} + \varepsilon_{qp}). \quad (2.2)$$

^{*}We use the term, vertexes, to describe the communication graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, while the term, nodes, are the basic elements of deformable VO.

For $\mathcal{G}(\mathcal{V}, \mathcal{E})$, the incidence matrix $\mathcal{D} := \{d_{il}\} \in \mathbb{R}^{N \times N_e}$ is defined by

$$d_{il} := \begin{cases} 1 & \text{if } v_i \text{ is the head of } e_l \\ -1 & \text{if } v_i \text{ is the tail of } e_l \\ 0 & \text{otherwise} \end{cases} \quad (2.3)$$

and the graph Laplacian matrix $\mathcal{L} = \{l_{ij}\} \in \mathbb{R}^{N \times N}$ by

$$l_{ij} := \begin{cases} \deg(v_i) & \text{if } i = j \\ -1 & \text{if } e_{ij} \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases} \quad (2.4)$$

where $\deg(v_i)$ is the degree (i.e., number of incoming edges) of v_i . For undirected $\mathcal{G}(\mathcal{V}, \mathcal{E})$, we then have [Big93, Prop.4.8]

$$\mathcal{L} = \mathcal{D}\mathcal{D}^T \quad (2.5)$$

and, moreover, if $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is connected as well, $\mathcal{L}$ has a zero eigenvalue at the origin with the eigenvector $\mathbf{1}_N := [1, \dots, 1]^T \in \mathbb{R}^N$, and all the other eigenvalues are strictly positive real. See [OSM04, RB05, Big93] for more details.

2.1.2 Multi-Dimensional Graph Laplacian: Stiffness Matrix

$\mathcal{P}$

To attain the consistency, our P2P architecture in Fig. 2.1 will establish PD-type consensus control among N VO local copies over the communication graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. In contrast to the usual consensus results (e.g., [LJ09, TJP07, RC08, LS06a]), here, we are interested in using n -dimensional consensus gains $P_{ij} \in \mathbb{R}^{n \times n}$ (i.e., full $n \times n$ matrix), rather than 1-dimensional (scalar) P_{ij} (i.e., $P_{ij} = p_{ij}I_n$, with a scalar $p_{ij} > 0$ and identity matrix $I_n \in \mathbb{R}^{n \times n}$), where $n > 0$ is the dimension of the deformable VO

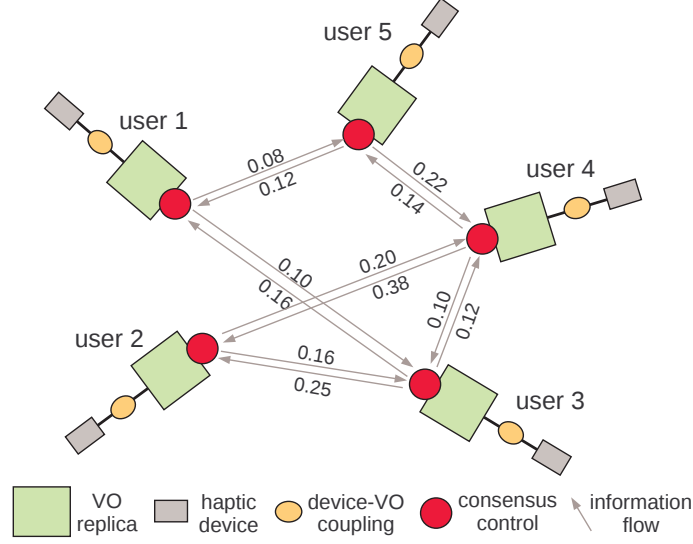


Figure 2.1: Peer-to-peer (P2P) multiuser haptic interaction architecture.

configuration. This is because: 1) some nodes of the VO may need stronger consensus coupling than others (e.g., nodes with heavier mass); and 2) cross-coupling among different nodes between two VOs may improve consensus performance [Li99].

Consensus state among N VO local replicas will then be specified by the following multi-dimensional graph Laplacian, or *stiffness matrix* $\mathcal{P} \in \mathbb{R}^{nN \times nN}$, defined similar to $\mathcal{L}$ (2.4) s.t.,

$$\mathcal{P}_{ij} = \begin{cases} \sum_{k \in \mathcal{N}_i} P_{ik} & i = j \\ -P_{ij} & i \neq j \text{ and } e_{ij} \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases} \quad (2.6)$$

where $P_{ij} \in \mathbb{R}^{n \times n}$ is the symmetric and positive-definite consensus P-gain matrix assigned on $e_{ij} \in \mathcal{E}$. To our knowledge, consensus property of this stiffness matrix $\mathcal{P}$ has not been established. In the following, we show that this $\mathcal{P}$ indeed possesses consensus property similar to that of the (scalar) graph Laplacian $\mathcal{L}$ in (2.4), thereby, extending the current results on the 1-dimensional Laplacian of undirected graphs [OSM04, LJ09, RC08, LS07] to the case of multi-dimensional consensus.

Lemma 1. Suppose that $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is undirected and P_{ij} in (2.6) is symmetric, positive-definite and $P_{ij} = P_{ji}$. Then, we can write the stiffness matrix $\mathcal{P}$ in (2.6) s.t.

$$\mathcal{P} = \frac{1}{2}(\mathcal{D} \otimes I_n)P_d(\mathcal{D} \otimes I_n)^T \quad (2.7)$$

where $\mathcal{D} \in \mathbb{R}^{N \times N_e}$ is the incidence matrix (2.3), $\otimes$ is the Kronecker product, and $P_d := \text{diag}(P_1, P_2, \dots, P_{N_e}) \in \mathbb{R}^{nN_e \times nN_e}$, with $P_l \in \mathbb{R}^{n \times n}$ being the P -gain matrix assigned on $e_l \in \mathcal{E}$, $l \in \mathcal{E}_C = \{1, \dots, N_e\}$. with $l \approx (p, q)$.

Proof. Define $\bar{D} := (\mathcal{D} \otimes I_n)P_d^{\frac{1}{2}} \in \mathbb{R}^{nN \times nN_e}$. Then, its il -th block matrix $\bar{d}_{il} \in \mathbb{R}^{n \times n}$ is given by

$$\bar{d}_{il} = \begin{cases} P_l^{\frac{1}{2}} & \text{if } v_i \text{ is the head of } e_l \\ -P_l^{\frac{1}{2}} & \text{if } v_i \text{ is the tail of } e_l \\ 0 & \text{otherwise} \end{cases}$$

following the structure of $\mathcal{D}$ (2.3). Define also $\mathcal{E}_k := \{l \in \mathcal{E}_C \mid \exists v_r \in \mathcal{V} \text{ s.t., } l \approx (k, r) \text{ or } l \approx (r, k)\}$, that is, the set of any edges connecting/connected to the vertex v_k . We then have $\bar{d}_{kl} = \pm P_l^{\frac{1}{2}} \neq 0$ if $l \in \mathcal{E}_k$, or $\bar{d}_{kl} = 0$ otherwise.

The $n \times n$ diagonal block of $\bar{D}\bar{D}^T$ is then given by: for $i = 1, \dots, N$,

$$\sum_{l=1}^{N_e} \bar{d}_{il}^2 = \sum_{l \in \mathcal{E}_i} P_l = 2 \sum_{j \in \mathcal{N}_i} P_{ij}$$

since $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is undirected, $\mathcal{E}_i$ includes both the incoming and outgoing edges of v_i , and $P_{ji} = P_{ij}$. Also, the $n \times n$ off-diagonal block of $\bar{D}\bar{D}^T$ is given by: $i, j \in \{1, 2, \dots, N\}$, $i \neq j$,

$$\sum_{l=1}^{N_e} \bar{d}_{il} \bar{d}_{jl} = \sum_{l \in \mathcal{E}_i \cap \mathcal{E}_j} \bar{d}_{il} \bar{d}_{jl} = \sum_{l \approx (i,j), l \approx (j,i)} \bar{d}_{il} \bar{d}_{jl} = -2P_{ij}$$

since, with $i \neq j$, 1) if $l \notin \mathcal{E}_i \cap \mathcal{E}_j$, $\bar{d}_{il}\bar{d}_{jl} = 0$; 2) $l \in \mathcal{E}_i$ and $l \in \mathcal{E}_j$ implies that $l \approx (i, j)$ or $l \approx (j, i)$; and 3) $P_{ij} = P_{ji}$. This shows $2\mathcal{P} = \bar{D}\bar{D}^T$, which completes the proof. $\square$

Using Lem. 1, we now show that the multi-dimensional stiffness matrix $\mathcal{P}$ possesses consensus property similar to that of the 1-dimensional (scalar) $\mathcal{L}$ of (2.4).

Proposition 1. *Suppose that $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is undirected and connected; and $P_{ij} \in \mathbb{R}^{n \times n}$ for (2.6) is symmetric, positive-definite and $P_{ij} = P_{ji}$. Then, we have*

$$\text{rank}(\mathcal{P}) = n(N - 1)$$

with the n -dimensional kernel space of $\mathcal{P}$ given by

$$\ker(\mathcal{P}) = \text{span}\{1_N \otimes a_1, \dots, 1_N \otimes a_n\} \quad (2.8)$$

where $\text{span}\{a_1, a_2, \dots, a_n\} = \mathbb{R}^n$.

Proof. By using Lem. 1, properties of rank and $\otimes$, K_d being nonsingular, and (2.5), we have,

$$\begin{aligned} \text{rank}(\mathcal{P}) &= \text{rank}\left(\left[(\mathcal{D} \otimes I_n)P_d^{\frac{1}{2}}\right]^T \left[(\mathcal{D} \otimes I_n)P_d^{\frac{1}{2}}\right]\right) \\ &= \text{rank}(P_d^{\frac{1}{2}}(\mathcal{D}^T \mathcal{D} \otimes I_n)P_d^{\frac{1}{2}}) \\ &= \text{rank}(I_n)\text{rank}(\mathcal{D}^T \mathcal{D}) = n \text{rank}(\mathcal{L}) \end{aligned}$$

where, if $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is connected, $\text{rank}(\mathcal{L}) = N - 1$. Thus, we have $\text{rank}(\mathcal{P}) = n(N - 1)$, or equivalently, $\dim(\ker(\mathcal{P})) = n$. We can also verify that

$$\mathcal{P}(1_N \otimes a) = 0 \quad \forall a \in \mathbb{R}^n$$

which, with $\dim(\ker(\mathcal{P})) = n$, implies (2.8). $\square$

In the following sections, Proposition 1 will play the critical role in proving the multi-DOF consensus among VO copies.

2.2 Peer-to-Peer Control Architecture for Multiuser Haptic Interaction over the Internet

We now present our novel P2P control architecture, composed of N local simulation of the shared deformable VO, PD-type consensus control among these VOs over the Internet, and local passive device-VO coupling. See Fig. 2.1. The main emphasis of this result is on the discrete-time portion of the P2P architecture, that is, N local VO simulations and their consensus control over the Internet. Once the discrete-time passivity of this portion is enforced, we may then simply use some available techniques to passively couple each VO with its haptic device (e.g., PSPM [LH10a] or virtual coupling [CSB95, Lee09, HL11a]), thereby, can guarantee (continuous-time) robust interaction stability, portability and scalability against heterogeneous users and devices.

2.2.1 Passive Local Simulation of Shared VO

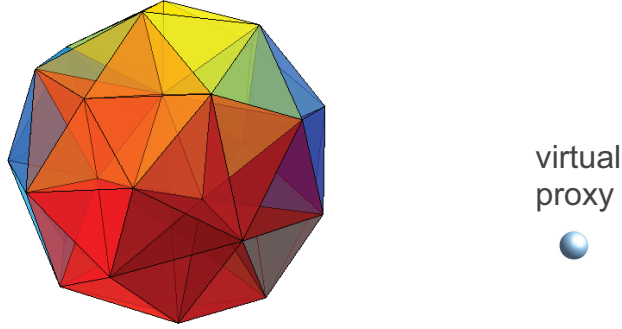


Figure 2.2: 3D deformable virtual object (VO) with 33 nodes and 87 tetrahedron meshes and virtual proxy (VP).

We consider linear deformable objects as the shared VO. See Fig. 2.2 for an example. For our P2P architecture in Fig. 2.1, each user then simulates their own local copy of this shared VO, while the VOs' configurations are connected via a PD-type consensus control over the Internet with communication unreliability and distributed topology $\mathcal{G}(\mathcal{V}, \mathcal{E})$. To enforce passivity of this VO local simulation, we

particularly utilize our recently-proposed non-iterative passive mechanical integrator (NPMI [LH08a]), which can be written as follows: for $i = 1, \dots, N$,

$$\begin{aligned} \frac{1}{N} [Ma_i(k) + B\hat{v}_i(k) + K(\hat{x}_i(k) - x_d)] &= \tau_i(k) + f_i(k) \\ a_i(k) &:= \frac{v_i(k+1) - v_i(k)}{T_i^k} \\ \hat{v}_i(k) &:= \frac{v_i(k+1) + v_i(k)}{2} = \frac{x_i(k+1) - x_i(k)}{T_i^k} \\ \hat{x}_i(k) &:= \frac{x_i(k+1) + x_i(k)}{2} \end{aligned} \quad (2.9)$$

where $k \geq 0$ is the discrete-time index; $T_i^k > 0$ is the update interval; $x_i, v_i, a_i \in \mathbb{R}^{3n}$ are respectively the (combined) configuration, velocity and acceleration of the n -nodes of the i^{th} -user's VO (i.e., $x_i(k) := [x_i^1(k); x_i^2(k); \dots; x_i^n(k)]$, with $x_i^r(k) \in \mathbb{R}^3$ being the displacement of the r^{th} -node of the VO: similar also holds for v_i, a_i); $x_d \in \mathbb{R}^{3n}$ specifies the VO's un-deformed shape and also its mechanical ground; $f_i \in \mathbb{R}^{3n}$ is the device-VO interaction force (e.g., interaction with user-controlled virtual proxy: see Sec. 2.2.3); and $\tau_i(k) \in \mathbb{R}^{3n}$ is to embed consensus control (see Sec. 2.2.2) to coordinate N VO local simulations over the unreliable Internet with topology $\mathcal{G}(\mathcal{V}, \mathcal{E})$.

Also, in (2.9), $M \in \mathbb{R}^{3n \times 3n}$ is the symmetric and positive-definite mass matrix for the n -nodes of VO; and $B, K \in \mathbb{R}^{3n \times 3n}$ are the symmetric damping and spring structure matrices, often decomposable by

$$B := B_{\text{int}} + B_{\text{gnd}}, \quad K := K_{\text{int}} + K_{\text{gnd}} \quad (2.10)$$

where $\star_{\text{int}}$ defines the inter-nodes coupling among the n -nodes of the VO, with its structure similar to that of the stiffness matrix $\mathcal{P}$ in Sec. 2.1.1; while $\star_{\text{gnd}}$ is a positive diagonal matrix, binding some nodes of the VO to the mechanical ground x_d .

More precisely, similar to $\mathcal{P}$ in (2.6), the 3×3 block matrix $K_{\text{int}}^{rs} \in \mathbb{R}^{3 \times 3}$ of K_{int} , $r, s \in \{1, 2, \dots, n\}$, is given by $K_{\text{int}}^{rs} = -K_{rs}$ if $r \neq s$; or $K_{\text{int}}^{rs} = \sum_{k \in \mathcal{N}_r^{\text{VO}}} K_{rk}$ if $r = s$, where $K_{rs} \in \mathbb{R}^{3 \times 3}$ defines the spring connection between the two nodes, $x_i^r(k)$ and

$x_i^s(k)$, over the undirected *structure graph* $\mathcal{G}_{VO}(\mathcal{X}, \mathcal{K})$, with $\mathcal{X} := \{x_i^1, x_i^2, \dots, x_i^n\}$ and $\mathcal{K} \in \mathcal{X} \times \mathcal{X}$ respectively being the sets of the n -nodes of the VO and the K_{rs} -spring connection among them. Then, from the structural similarity between K_{int} and $\mathcal{P}$, if $K_{\text{int}}(x_i - x_d) \rightarrow 0$ for (2.9), following Prop. 1, we would have

$$x_i - x_d \rightarrow \ker(K_{\text{int}}) \approx \mathbf{1}_n \otimes z, \quad z \in \mathbb{R}^3$$

i.e., K_{int} enforces the n -nodes of the VO to make the (un-deformed) shape x_d , which yet can still float by any arbitrary translation $z \in \mathbb{R}^3$ (i.e., symmetry in $E(3)$ [OFL04]). On the other hand, K_{gnd} is given by $K_{\text{gnd}} := \text{diag}[k_{\text{gnd}}^1 I_3, \dots, k_{\text{gnd}}^n I_3]$, where $k_{\text{gnd}}^r > 0$ if the node x_i^r of VO is attached to the mechanical ground x_d^r via $K_{\text{gnd}}(x_i - x_d)$ in (2.9); or $k_{\text{gnd}}^r = 0$ otherwise (i.e., symmetry breaking in $E(3)$ [OFL04]). Similar can also be said for B as well.

Now, suppose that the consensus via $\tau_i(k)$ in (2.9) is perfect (i.e., all the VO replicas' configurations are exactly the same). Then, if a single user tries to deform its own local VO copy with all the other users not touching their copies, this user needs to make the same deformation across all the N local copies. This implies that, the larger the number of VOs (i.e., N) is, the more difficult for each user to move/deform the (shared) VO, since it is distributed among the N users. To address this scaling effect, similar to [SH06, LH10b], here, we scale down (2.9) by N . Note also from (2.9) that, since each user simulates the single *shared* VO, the same M, B, K, x_d are used by all the users.

The NPMI algorithm in (2.9) can be used to simulate VO models, either derived from mass-spring-damper modeling [Del98] or finite element method (FEM [Hug00]), as long as they are linear. The NPMI algorithm in (2.9) is also implicit (e.g., $v_i(k+1), x_i(k+1)$ together showing up in the left hand side of (2.9)), yet, still non-iterative[†] (i.e., (2.9) can be converted into a linear equation with $\star_i(k+1)$ solely dependent on

[†]This is still true even with the consensus control $\tau_i(k)$ (2.14) and the VO-VP interaction force $f_i(k)$ in Sec. 2.2.3.

$\star_i(k)$), thus, can be simulated haptically fast. Furthermore, unlike other integrators frequently used in haptics (e.g., explicit Euler [BC98]), this NPMP enforces the open-loop two-port discrete-time passivity of (2.9). That is, directly using (2.9), we can easily show that, $\forall \bar{M} \geq 0$,

$$\sum_{k=0}^{\bar{M}} [f_i(k) + \tau_i(k)]^T \hat{v}_i(k) T_i^k \geq -E_i(0) =: -d_i^2 \quad (2.11)$$

where $E_i(k) := 1/N \times (\|v_i(k)\|_M^2/2 + \|x_i(k) - x_d\|_K^2/2)$ is the (scaled) total energy of the i^{th} -user's VO, with $\|y\|_A := \sqrt{y^T A y}$ for $y \in \mathbb{R}^m$ and positive-definite/symmetric $A \in \mathbb{R}^{m \times m}$. See [LH08a] for more details on the NPMP algorithm.

Now, let us define the discrete-time N -port closed-loop passivity of the P2P architecture in Fig. 2.1 as follows: $\forall \bar{M} \geq 0, \exists d \in \mathbb{R}$, s.t.,

$$\sum_{k=0}^{\bar{M}} \sum_{i=1}^N \hat{v}_i(k)^T f_i(k) T_i^k \geq -d^2 \quad (2.12)$$

i.e., the maximum extractable energy from the N device-VO interaction ports $(f_i(k), \hat{v}_i(k))$ is bounded. If we attain this discrete-time N -port closed-loop passivity (2.12), we would also be able to enforce *continuous-time* N -port passivity of our P2P architecture (as experienced by the N users), by using some passive (hybrid) device-VO coupling (e.g., PSPM [LH10a]; virtual coupling [CSB95, Lee09, HL11b]) and passive haptic devices. See Sec. 2.2.3.

The next Prop. 2 shows that, similar to the continuous-time case [LL05, LS06b], with the open-loop two-port passivity of each local VO simulation (2.9), we can reduce the problem of enforcing the N -port closed-loop passivity of the P2P architecture (2.12) into that of the N -port consensus controller passivity: $\forall \bar{M} \geq 0, \exists c \in \mathbb{R}$, s.t.,

$$\sum_{k_i=0}^{\bar{M}} \sum_{i=1}^N \hat{v}_i(k)^T \tau_i(k) T_i^k \leq c^2 \quad (2.13)$$

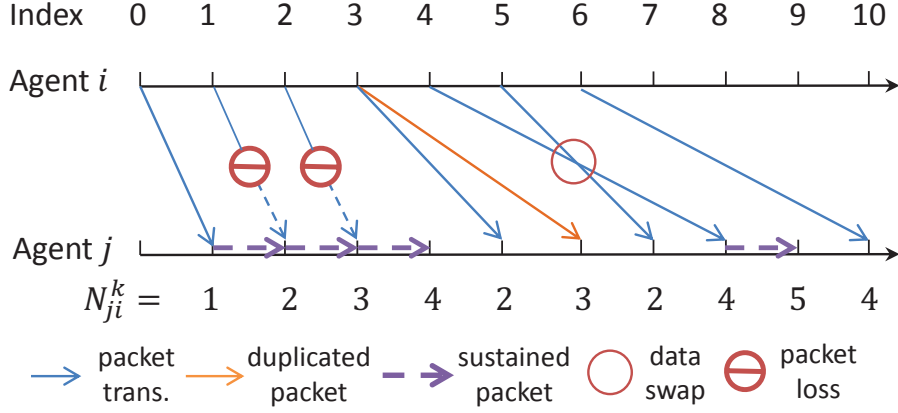


Figure 2.3: Indexing delay N_{ji}^k can capture various communication defects.

i.e., the maximum extractable energy from the N consensus control ports $(\tau_i(k), \hat{v}_i(k))$ is bounded. It is usually simpler to prove this N -port controller passivity (2.13) than the N -port closed-loop passivity (2.12), as the former involves only (often linear) $\tau_i(k)$ and not the VO dynamics. In the next Sec. 2.2.2, we will design the consensus control $\tau_i(k)$ to satisfy this N -port controller passivity (2.13), even if the Internet is unreliable with varying delay, packet loss, etc., and their topology is only partially-connected.

Proposition 2. *Suppose each local VO simulation possesses open-loop two-port discrete-time passivity (2.11). Then, discrete-time N -port consensus controller passivity (2.13) implies discrete-time N -port closed-loop passivity of the P2P architecture (2.12).*

Proof. Substituting (2.13) into (2.11), we can obtain (2.12) with $d^2 := \sum_{i=1}^N d_i^2 + c^2$. $\square$

2.2.2 Passive VO Consensus over the Internet

We design the consensus control $\tau_i(k)$ in (2.9) to be composed of PD-type coupling (distributed over the communication graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$) and local damping injection s.t.,

$$\tau_i(k) := -B_i \hat{v}_i(k) - \underbrace{\sum_{j \in \mathcal{N}_i} D_{ij} [\hat{v}_i(k) - \delta_{ij}^k \hat{v}_j(k - N_{ij}^k)]}_{\text{D-coupling}} - \underbrace{\sum_{j \in \mathcal{N}_i} P_{ij} [\hat{x}_i(k) - \hat{x}_j(k - N_{ij}^k)]}_{\text{P-coupling}} \quad (2.14)$$

where $\mathcal{N}_i$ is the information neighbors of i^{th} -user; $B_i, P_{ij}, D_{ij} \in \mathbb{R}^{n \times n}$ are respectively the symmetric/positive-definite local damping, P and D coupling gain matrices, with P_{ij}, D_{ij} defined on the edge e_{ij} over $\mathcal{G}(\mathcal{V}, \mathcal{E})^{\dagger}$; and $N_{ij}^k \geq 0$ is the time-varying (integer) indexing delay, from the j^{th} -user to the i^{th} -user at the discrete-time index k . We also assume

$$P_{ij} = P_{ji}, \quad D_{ij} = D_{ji} \quad (2.15)$$

i.e., symmetric P and D couplings on the edge e_{ij} , although we allow $N_{ij}^k \neq N_{ji}^k$ (e.g., asymmetric delays) and $P_{ij} \neq P_{pq}, D_{ij} \neq D_{pq}$ if $(i, j) \neq (p, q)$ (i.e., non-uniform PD couplings).

As shown in Fig.2.3, the indexing delay N_{ij}^k in (2.14) can capture various imperfectness of the Internet communication, including varying delay, data swapping, and packet duplication. Note that, for the case of packet loss, N_{ij}^k is not well-defined, since there is no information reception, to extract N_{ij}^k from which. For this case of packet loss, we set N_{ij}^k s.t., $N_{ij}^k \Leftarrow N_{ij}^{k-1} + 1$, that is, “sustain” (or hold) the previous data packet - see Fig. 2.3. With this packet sustainment into the definition of N_{ij}^k , we assume there exists an upper bound $\bar{N}_{ij}$, s.t., $\forall k \geq 0, \bar{N}_{ij} \geq N_{ij}^k$.

[†]The results here can be easily extended to asymmetric B_i or positive semi-definite D_{ij} . The VO damping $(1/N)B$ in (2.9) may also serve the role of local damping (i.e., $B_i := (1/N)B$ for (2.17)): we use B_i here to “modularize” the consensus control (2.14) from the VO simulation (2.9).

For the P-action in (2.14), it is in fact often desirable to sustain the previous set-position data $\hat{x}_j$, when the packet is missing. As shown later in Th. 1 and its proof in Sec.2.6, this set-position holding does not at all jeopardize the N -port passivity of the consensus control (2.13). On the other hand, such packet sustainment for the set-velocity signal $\hat{v}_j$ in (2.14) can compromise passivity (2.13). To prevent this, we utilize duplication avoidance function δ_{ij}^k in (2.14) as defined by

$$\delta_{ij}^k := \begin{cases} 0 & \text{if } \hat{v}_j(k - N_{ij}^k) \text{ is duplicated} \\ 1 & \text{otherwise} \end{cases} \quad (2.16)$$

where the condition of the first line includes both the “real” duplication (i.e., due to the communication itself) as well as the “artificial” duplication (i.e., from packet sustainment). This δ_{ij}^k can be easily implemented by using some packet numbering mechanisms.

We now present the main result of this paper in the following Th. 1, which shows that, under a certain gain setting condition, even if the Internet communication is unreliable and its topology $\mathcal{G}(\mathcal{V}, \mathcal{E})$ only partially-connected, our P2P architecture can guarantee the N -port closed-loop passivity (2.12), configuration consensus among N VO local copies when released from the users, and force balance among the users as if they are physically manipulating the VO together.

Theorem 1 (Main result). *Consider N VO local copies (2.9) with the consensus control (2.14) over imperfect Internet communication with undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. Suppose we set the gains B_i, P_{ij}, D_{ij} , s.t.,*

$$B_i \succeq \sum_{j \in \mathcal{N}_i} \left(\frac{\bar{N}_{ij} + \bar{N}_{ji}}{2} T_j^{max} P_{ij} + \frac{1}{2} \left[\frac{T_j^{max}}{T_i^{min}} - 1 \right] D_{ij} \right) \quad (2.17)$$

for $i = 1, 2, \dots, N$, where $T_j^{max} := \max_k(T_j^k)$, $T_j^{min} := \min_k(T_j^k) > 0$, $v_i(k) = 0, \forall k \leq 0$, and $A \succeq B$ (or $A \succ B$, resp.) implies $A - B$ is positive semi-definite (or definite, resp.). Then,

1. the P2P architecture possesses the discrete-time N -port closed-loop passivity (2.12);

2. if B_i is augmented by an extra positive-definite damping $B_i^e \in \Re^{3n \times 3n}$ ($B_i^e \succ 0$) and $f_i(k) = 0$,

$$\left[\mathcal{P} + I_N \otimes \frac{K}{N} \right] (x(k) - 1_N \otimes x_d) \rightarrow 0 \quad (2.18)$$

where $x(k) = [x_1(k); x_2(k); \dots; x_N(k)] \in \Re^{3nN}$, and $\mathcal{P} \in \Re^{3nN \times 3nN}$ is the stiffness matrix (2.6);

3. if $v_i(k) \rightarrow 0$ for all the users,

$$\sum_{i=1}^N f_i(k) \rightarrow K(\bar{x}(k) - x_d) \quad (2.19)$$

where $\bar{x}(k) := (x_1(k) + x_2(k) + \dots x_N(k)) / N \in \Re^{3n}$.

Proof. With the proof given in Sec. 2.6 and the resulting inequality (2.33), under the condition (2.17), the consensus control $\tau_i(k)$ satisfies N -port controller passivity (2.13) with $c^2 = \sum_{l=1}^{N_e} \varphi_{pq}(0)|_{(p,q) \approx l}$, where $\varphi_{pq}(k) := \|\Delta x_{pq}^k\|_{P_{pq}}^2 / 4$ with $\Delta x_{pq}^k := x_p(k) - x_q(k)$, i.e., half of the energy stored in P_{pq} on the edge e_{pq} . The discrete-time N -port closed-loop passivity (2.12) then follows from Prop. 2 with $d^2 := \sum_{i=1}^{N_e} E_i(0) + \sum_{l=1}^{N_e} \varphi_{pq}(0)$, $(p, q) \approx l$.

For the second item, note that, with the extra B_i^e (i.e., $B_i + B_i^e$ instead of B_i for (2.14) with B_i satisfying (2.17)), we have, instead of (2.33), $\sum_{k=0}^{\bar{M}} s_E(k) \leq \sum_{l=1}^{N_e} \varphi_{pq}(0) - \sum_{k=0}^{\bar{M}} \sum_{i=1}^N \|\hat{v}_i(k)\|_{B_i^e}^2 T_i^k$. Combining this with (2.11), we can then show that, $\forall \bar{M} \geq 0$,

$$\sum_{i=1}^N \sum_{k=0}^{\bar{M}} \hat{v}_i(k)^T f_i(k) T_i^k \geq -d^2 + \sum_{k=0}^{\bar{M}} \sum_{i=1}^N \|\hat{v}_i(k)\|_{B_i^e}^2 T_i^k$$

i.e., the system is still N -port closed-loop passive (2.12), with the extra dissipation due to B_i^e . If $f_i(k) = 0$, we then have

$$d^2 \geq \sum_{k=0}^{\bar{M}} \sum_{i=1}^N \|\hat{v}_i(k)\|_{B_i^e}^2 T_i^k, \quad \forall \bar{M} > 0$$

implying that, with d bounded and $T_i^k > 0$, $\hat{v}_i(k) \rightarrow 0$.

From (2.9) with $\hat{v}_i(k) \rightarrow 0$, we have $x_i(k+1) \rightarrow x_i(k)$ and $\hat{x}_i(k+1) \rightarrow \hat{x}_i(k)$. This then implies that $K(\hat{x}_i(k+1) - x_d) \rightarrow K(\hat{x}_i(k) - x_d)$ for (2.9) and also $\tau_i(k+1) \rightarrow \tau_i(k)$ for (2.14), with $\hat{x}_j(k+1) \rightarrow \hat{x}_j(k) \dots \rightarrow \hat{x}_j(k+1 - N_{ij}^k) \rightarrow \hat{x}_j(k - N_{ij}^k)$. Applying this observation to (2.9) for T_i^k and T_i^{k+1} integration steps, we can achieve

$$\frac{v_i(k+1) - v_i(k)}{T_i^k} \rightarrow \frac{v_i(k+2) - v_i(k+1)}{T_i^{k+1}}$$

where $v_i(k+2) \rightarrow v_i(k)$ from $\hat{v}_i(k+1) \rightarrow \hat{v}_i(k) \rightarrow 0$. Thus, we have $(1/T_i^k + 1/T_i^{k+1})(v_i(k+1) - v_i(k)) \rightarrow 0$, that is, $v_i(k+1) \rightarrow v_i(k)$, which, with $\hat{v}_i(k) \rightarrow 0$, further implies $v_i(k) \rightarrow 0$. Reflecting this back into (2.9) with $f_i(k) = 0$, we can then achieve

$$\frac{K}{N}(x_i(k) - x_d) + \sum_{j \in \mathcal{N}_i} P_{ij}(x_i(k) - x_d - x_j(k) + x_d) \rightarrow 0$$

$\forall i \in \{1, 2, \dots, N\}$, which can be written as (2.18).

For the third item, similar to the above derivation, using $v_i(k) \rightarrow 0$ for (2.9), we have $\hat{x}_i(k) \rightarrow x_i(k)$ and

$$\frac{K}{N}(x_i(k) - x_d) + \sum_{j \in \mathcal{N}_i} P_{ij}(x_i(k) - x_j(k)) \rightarrow f_i(k).$$

Summing this up, we can have

$$\sum_{i=1}^N f_i(k) \rightarrow K(\bar{x}(k) - x_d) + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} P_{ij}(x_i(k) - x_j(k))$$

where the most right term is zero, since, from $\mathcal{G}(\mathcal{V}, \mathcal{E})$ being undirected and $P_{ij} = P_{ji}$, for each $P_{ij}(x_i(k) - x_j(k))$, we also have $P_{ji}(x_j(k) - x_i(k))$, with their sum being zero. $\square$

By enforcing the discrete-time N -port closed-loop passivity (2.12), with some suitable passive (hybrid) device-VO coupling (e.g., Sec. 2.2.3), we can then achieve the continuous-time N -port passivity of the total P2P architecture in Fig. 2.1, and, consequently, can guarantee interaction stability with *any* multiple passive haptic device and human users, regardless how many, unknown, uncertain, complicated, or heterogeneous they are (i.e., user/device-invariant stability and scalability). This discrete-time N port passivity (2.12) also allows us to separate the VO local simulation from the haptic device servo-loop [BC97]. This means that our P2P architecture can be used with any haptic device without being needed to be tuned according to a specific chosen device (i.e., portability across different devices).

The consensus property (2.18) implies that, when all the VO copies are released, their collective configuration $x(k) = [x_1(k); x_2(k); \dots; x_N(k)] \in \mathbb{R}^{3nN}$ will converge to the set $\ker(\mathcal{P}) \cap \ker(I_N \otimes K)$, where the former captures the action of the consensus P-action P_{ij} over $\mathcal{G}(\mathcal{V}, \mathcal{E})$, while the latter the effect of VO spring structure $K = K_{\text{int}} + K_{\text{gnd}}$ (2.10). For instance: 1) if $K = 0$ for (2.9) with connected $\mathcal{G}(\mathcal{V}, \mathcal{E})$, following Prop. 1, the consensus action $\tau_i(k)$ will push $x_i(k) \rightarrow x_j(k) \rightarrow a$ with unspecified $a \in \mathbb{R}^{3n}$, implying that all the VO local copies will consensus to the same, yet, unspecified shape a ; 2) if the spring connections K_{int} inside the VO are built according to an undirected/connected structure graph $\mathcal{G}_{VO}(\mathcal{X}, \mathcal{K})$ (see Sec. 2.2.1) with $K_{\text{gnd}} = 0$, we will have $x_i(k) \rightarrow x_j(k) \rightarrow x_d + I_n \otimes z$ with unspecified $z \in \mathbb{R}^3$, i.e., all the local copies will attain the same (un-deformed) shape given by x_d , yet, also with the same unspecified $E(3)$ -translation z (i.e., symmetry in $E(3)$ [OFL04]); and 3) if $K_{\text{gnd}} \neq 0$ with the above K_{int} , K will become positive definite, $x_i(k) \rightarrow x_d$, and all the VO copies will converge to the same shape with the same $E(3)$ -location (i.e., breaking symmetry in $E(3)$ [OFL04]).

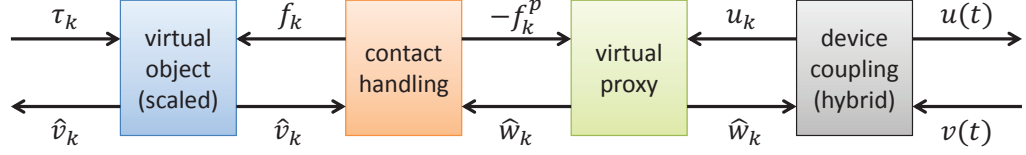


Figure 2.4: Network representation of device-VO coupling.

On the other hand, the force balance property (2.19) shows that our P2P architecture captures the peculiarity of multiuser shared haptic interaction, i.e., through the P2P architecture, the (possibly geographically) distributed N users can interact with each other *as if* they are physically manipulating a common deformable object together. For instance: 1) multiple users together induce the average deformation $\bar{x}(k)$ on the shared deformable VO; 2) if they somehow balance with $\bar{x}(k) = x_d$ (e.g., pushing the VO in opposite directions), their force sum will be zero (e.g., statically holding the VO together); and 3) if only the i^{th} -user pushes the object with the perfect consensus among the N local copies (i.e., $x_i(k) = x_j(k)$), $f_i(k) \rightarrow K(x_i(k) - x_d)$, just as in the case of single user haptic interaction.

Although the results of Th. 1 are for the discrete-time domain, we would likely be able to transfer them to the continuous-time domain (i.e., as experienced by the real N users), if we use some suitable hybrid device-VO coupling of Fig. 2.1, that can passively connect the (passive) device and the VO local simulation and also shares the PD-structure with the consensus control (2.14). See Sec. 2.2.3 for more details.

2.2.3 Passive Device-VO Coupling

The problem of passive hybrid device-VO coupling is in fact a standard problem in haptics (e.g., virtual coupling [CSB95, Lee09]; PSPM [LH10a]). Although it is not the main focus of this dissertation, for completeness and expedited implementation, here, we briefly discuss this device-VO coupling.

At the local site of each user, we implement *hybrid* device-VO coupling as shown in Fig. 2.4, where the virtual proxy (VP), connected to the device through the device-VP coupling, interacts with the VO local copy via the VO-VP contact block. To enforce passivity of the VP, similar to (2.9), we also utilize NPMI algorithm [LH08a]: for the i^{th} -user,

$$\begin{aligned} m_i \frac{w_i(k+1) - w_i(k)}{T_i^k} &= u_i(k) + \sum_{p \in \mathcal{C}_i} f_i^p(k) \\ \hat{w}_i(k) &:= \frac{w_i(k+1) + w_i(k)}{2} = \frac{y_i(k+1) - y_i(k)}{T_i^k} \\ \hat{y}_i(k) &:= \frac{y_i(k+1) + y_i(k)}{2} \end{aligned} \quad (2.20)$$

where $m_i > 0$, $y_i(k), w_i(k) \in \mathbb{R}^3$ are respectively the mass, position, and velocity of VP; $u_i(k) \in \mathbb{R}^3$ is the device-VP coupling (to be explained below); T_i^k is the integration step in (2.9); and $f_i^p(k) \in \mathbb{R}^3$ is the VO-VP contact force defined by

$$f_i^p(k) := \begin{cases} -b_c[\hat{w}_i(k) - \hat{v}_i^p(k)] & \text{if } p \in \mathcal{C}_i^k \\ -k_c[\hat{y}_i(k) - \hat{x}_i^p(k)] & \\ 0 & \text{otherwise} \end{cases}$$

where $\mathcal{C}_i^k \in \mathcal{X}$ is the set of “contact nodes” of the VO for the i^{th} -th user at the time index k . See Fig. 2.5. Among the VO’s nodes, only these contact nodes interact with the VP. This implies that, for $f_i(k) = [f_i^1(k); f_i^2(k); \dots; f_i^n(k)] \in \mathbb{R}^{3n}$ in (2.9), $f_i^j(k) = 0$ if $j \notin \mathcal{C}_i^k$.

Now, we make the following two assumptions for our device-VO coupling in Fig. 2.4: 1) the VO-VP contact block is discrete-time two-port passive: $\forall \bar{M} \geq 0, \exists c_{ci} \in \mathbb{R}$,

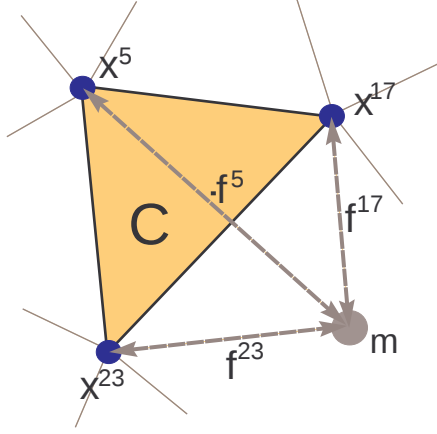


Figure 2.5: Contact mesh $\mathcal{C}$, contact node $x^\star$, contact force $f^\star$ and virtual proxy m .

s.t.,

$$\sum_{k=0}^{\bar{M}} [\hat{v}_i^T(k) f_i(k) + \sum_{p \in \mathcal{C}_i^k} \hat{w}_i^T(k) f_i^p(k)] T_i^k \leq c_{ci}^2 \quad (2.21)$$

for all $i \in \{1, 2, \dots, N\}$; and 2) the device-VP coupling block is hybrid two-port passive [LH10a, Lee09]: $\forall \bar{t} \geq 0, \exists \bar{M} \geq 0$ and $c_{hi} \in \Re$ s.t.

$$\int_0^{\bar{t}} u_i^T(t) \dot{q}_i(t) dt + \sum_{k=0}^{\bar{M}} \hat{w}_i^T(k) u_i(k) T_i^k \leq c_{hi}^2 \quad (2.22)$$

for all $i \in \{1, 2, \dots, N\}$, where $\dot{q}_i(t), u_i(t) \in \Re^3$ are the velocity and control of the haptic device, and $\bar{M}$ is defined s.t. $T_i^{\bar{M}}$ is contained within $[0, \bar{t})$, yet, $T_i^{\bar{M}+1}$ is not.

The hybrid passivity of device-VP coupling (2.22) can be achieved, e.g., by using PSPM [LH10a] or virtual coupling (e.g., [CSB95, Lee09, HL11b]), with some condition imposed *only* on the physical device damping b_{dev} , device-VP coupling spring/damper gains k_i, b_i and their servo-rate T_i^k ; they impose no restriction whatsoever on the VO local simulation's structural parameters, M, B, K in (2.9) (e.g., arbitrary M can be chosen regardless of devices' servo-rates [BC98]), implying that *any* passive devices can be used/replaced without re-tuning the shared VO's parameters while maintaining the N -port passivity (i.e., portability). Actually, the passive device-VP

coupling is an essential part for hybrid VP-based bilateral teleoperation framework. The detailed device-VP coupling and passivity condition will be presented in Sec.3.3.3.

On the other hand, passively rendering the VO-VP contact (2.21), particularly when the contact switches on and off, is still an open problem in haptics. We may plan to solve it as an independent problem in future, but for now let us just simply assume (2.21), which is more or less reasonable when the update rate T_i^k is fast enough. Note that, if the contact is always on with the same contact nodes, we can easily prove (2.21) with $c_{hi}^2 := \sum_{p \in \mathcal{C}_i} \|x_i^p(0) - y_i(0)\|^2/2$ using (2.9) and (2.20). See also [LH08a] for passive VO-VP contact when the VO is 1-dimensional virtual-wall.

Then, combining (2.21) and (2.22) with the open-loop passivity of the VP, i.e., similar to (2.11), from (2.20),

$$\sum_{k=0}^{\bar{M}} \hat{w}_i^T(k)[u_i(k) + \sum_{p \in \mathcal{C}_i^k} f_i^p(k)]T_i^k \geq -\kappa_i(0)$$

$\forall \bar{M} \geq 0$, where $\kappa_i(k) := \|w_i(k)\|_{m_i}^2/2$, we can achieve *hybrid* passivity of the device-VO coupling in Fig. 2.4: $\forall \bar{t} \geq 0, \exists \bar{M} \geq 0$ s.t.

$$\int_0^{\bar{t}} u_i^T(t)\dot{q}_i(t)dt + \sum_{k=0}^{\bar{M}} \hat{v}_i^T(k)f_i(k)T_i^k \leq \kappa_i(0) + c_{ci}^2 + c_{hi}^2$$

$\forall i \in \{1, 2, \dots, N\}$. Also, combining this with the discrete-time N -port closed-loop passivity (2.12), we can further achieve continuous-time N -port passivity of our P2P architecture. With this passive energetics and the PD-structure of the device-VO coupling, we would also be able to duplicate Th. 1 to the continuous-time domain (with passive haptic devices), the detail of which is omitted here, since it would involve derivations/arguments rather very similar to that for the proof of Th. 1 (e.g., with enough damping, $(\ddot{q}_i, \dot{q}_i) \rightarrow 0$ and the resultant dynamics given by merely a combination of discrete/continuous-time spring couplings).

2.3 Network Topology Optimization

Performance of our P2P architecture strongly depends on how fast the convergence of the consensus control is, or equivalently, how fast information propagates among the users over the Internet via the consensus control. This information propagation would likely be fastest if all the users communicate with all the others. Such all-to-all communication (i.e., fully-connected $\mathcal{G}(\mathcal{V}, \mathcal{E})$), yet, is often infeasible or prohibitively expensive particularly when the number of users (N) and/or the dimension of VO ($3n$) are large (e.g., bandwidth limitation). It is rather more reasonable, under the current Internet technology, to assume only few communication links are possible for each user or only a limited number of links available for the whole P2P architecture.

Then, the question would be which graph we should choose from the set of such practically-feasible network topologies $\mathcal{G}_F := \{\mathcal{G}(\mathcal{V}, \mathcal{E}_1), \mathcal{G}(\mathcal{V}, \mathcal{E}_2), \dots, \mathcal{G}(\mathcal{V}, \mathcal{E}_m)\}$ to maximize the speed of information propagation (i.e., fastest mixing graph [BDX04]). For this, we assume that a simple first-order consensus model can adequately capture the information propagation among the users through the consensus control, with P_{ij} defining the information mixing strength and the (constant) average of $N_{ij}^k T_i^k$ specifying the information propagation delay for each link e_{ij} .

More precisely, we use the following widely-used first-order consensus equation to model the information propagation among N users:

$$p_i(k+1) = \left(\sum_{j \in \mathcal{N}_i} P_{ij} \right)^{-1} \sum_{j \in \mathcal{N}_i} P_{ij} p_j(k+1 - N_{ij})$$

where $p_i(k) \in \mathbb{R}^n$ defines an abstract state of information of the i^{th} -user at the time-index k , and N_{ij} represents the averaged delaying effect of $N_{ij}^k T_i^k$ as defined by

$$N_{ij} := \mathcal{Z} \left[\frac{\sum_{k=1}^{\bar{M}} N_{ij}^k T_i^k / \bar{M}}{\sum_{i=1}^N \sum_{k=1}^{\bar{M}} T_i^k / (\bar{M}N)} \right]$$

where $\mathbb{Z}[\star]$ is an integer closest to $\star \in \mathfrak{R}$ with $\bar{M} > 0$ a large enough averaging interval. consensus protocol brings all $p_i(k)$ to the same value, we may then say the information is fully propagated among the N users. Moreover, the optimal network topology would be the one, that achieves this consensus with the fastest convergence.

This first-order information mixing model can then be “lifted” into the following form:

$$\begin{bmatrix} p(k+1) \\ p(k) \\ \vdots \\ p(k+1-\bar{N}) \end{bmatrix} = J(\mathcal{G}, P_{ij}, N_{ij}) \begin{bmatrix} p(k) \\ p(k-1) \\ \vdots \\ p(k-\bar{N}) \end{bmatrix}$$

where $p(k) := [p_1(k); p_2(k) \dots; p_N(k)] \in \mathfrak{R}^{nN}$; $\bar{N} = \max_{ij}(N_{ij})$,

$$J(\mathcal{G}, P_{ij}, N_{ij}) := \begin{bmatrix} A_0 & A_1 & \dots & A_{\bar{N}-1} & A_{\bar{N}} \\ I & 0 & \dots & 0 & 0 \\ 0 & I & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & I & 0 \end{bmatrix}$$

and $A_k \in \mathfrak{R}^{nN \times nN}$ has its $n \times n$ ij -th component as given by

$$A_k^{ij} = \begin{cases} \left(\sum_{j \in N_i} P_{ij} \right)^{-1} P_{ij} & \text{if } k = N_{ij} \text{ and } e_{ij} \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}.$$

The matrix $J(\mathcal{G}, P_{ij}, N_{ij}) \in \mathfrak{R}^{nN(\bar{N}+1) \times nN(\bar{N}+1)}$ then defines the information propagation among the users. We also found that, with connected $\mathcal{G}(\mathcal{V}, \mathcal{E})$, $J(\mathcal{G}, P_{ij}, N_{ij})$ has n -eigenvalues at 1 with eigenvectors $1_{\bar{N}+1} \otimes 1_N \otimes a$ with arbitrary $a \in \mathfrak{R}^n$ and all the other eigenvalues strictly within the unit circle. That is, the

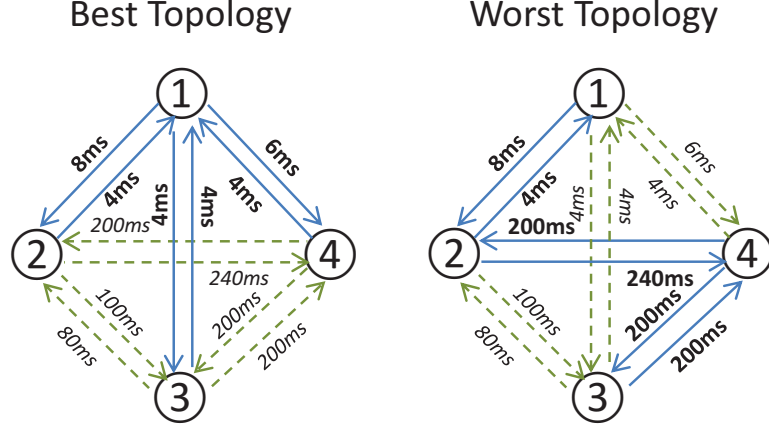


Figure 2.6: Best and worst network topologies with average delays.

n -eigenvalues at 1 correspond to the (steady-state) information consensus state (i.e., $p_i(k) \rightarrow p_j(k) \rightarrow a$), while the remaining $nN(\bar{N}+1) - n$ eigenvalues are related to the (transient) non-consensus residual, that is also vanishing, since the spectral radius of all these eigenvalues < 1 . This suggests the optimal network topology to be the one from $\mathcal{G}_F$, with the minimum $(n+1)$ -th largest spectral radius λ_{n+1} , i.e., the solution of the following optimization problem:

$$\min_{\mathcal{G} \in \mathcal{G}_F} |\lambda_{n+1}(J(\mathcal{G}, P_{ij}, N_{ij}))|.$$

If $P_{ij} = p_{ij}I_n$ with a scalar $p_{ij} > 0$, this network optimization can be further simplified with: scalar information state $p_i(k) \in \mathbb{R}$, reduced-dimension of $J(\mathcal{G}, P_{ij}, N_{ij}) \in \mathbb{R}^{N(\bar{N}+1)}$ with only one eigenvalue at 1 and all the others strictly within the unit circle, and λ_{n+1} replaced by λ_2 (*algebraic connectivity* [KM06]).

2.4 Experiments

We use our P2P control architecture to implement a four-user haptic interaction system over the Internet. Three Phantom Omnis and one Phantom Desktop from Sensable[®] are used as the haptic devices. A ball-like deformable VO, shown in Fig.

2.2, is chosen as the shared VO, with its mass matrix $M = 0.04I_{3n}[\text{kg}]$ and inter-node spring connection $K_{\text{int}}^{rs} = 100I_3[\text{N/m}]$, over undirected/connected structure graph $\mathcal{G}_{VO}(\mathcal{X}, \mathcal{K})$, with the 33 nodes connected via surface and internal meshes. We also set $B = 0$ (i.e., no VO structural damping) and $K_{\text{gnd}} = 0$ (i.e., no mechanical ground).

We set the update rate $T_i^k =: T$ to be 2[ms]. We also use a stochastic model to emulate Internet like communication as shown in Fig. 2.7, and achieve the average communication delays as follows:

$$[N_{ij}T] = \begin{bmatrix} 0 & 4 & 4 & 4 \\ 8 & 0 & 80 & 200 \\ 4 & 100 & 0 & 200 \\ 6 & 240 & 200 & 0 \end{bmatrix} [ms]$$

where the ij^{th} -element is the average delay from j^{th} -user to i^{th} -user. The actual delay $N_{ij}^k T$ randomly varies between 50% to 150% of these average values. The packet loss and duplication rate are 5% and 1% respectively. For the consensus control (2.14), we also set the gains s.t.

$$P_{ij} = P_{ji} = 1000 \left[\frac{I_{3n}}{2} + \frac{K_{\text{int}}}{2\lambda_{\max}(K_{\text{int}})} \right] [\text{N/m}]$$

$$D_{ij} = D_{ji} = 5I_{3n} [\text{Nm/s}]$$

for all e_{ij} , where we inject into P_{ij} the cross-coupling term $K_{\text{int}}/2\lambda_{\max}(K_{\text{int}})$ to enhance the consensus performance while also possibly strengthening the action of K_{int} in (2.9). Given the consensus PD gains and N_{ij} , we then perform the network topology optimization of Sec. 2.3 under the constraints that only total 3 undirected communication links are possible for the P2P architecture. The best and worst topologies are shown in Fig. 2.6. For each of these topologies, using the condition (2.17), we then set the local damping B_i for each user s.t., 1) (16, 7, 5, 6)[Nm/s] for the best topology; and 2) (5, 230, 210, 450)[Nm/s] for the worst topology.

Experimental results are given in Fig. 2.8 (for the best topology) and Fig. 2.9 (for the worst topology): the top plots show the position/force of the most “representative” node along the axis with the largest force/deformation for each user, while the bottom plots the deformation of the four local VO replicas (only surface nodes/meshes shown with contact nodes (black) and VP (red)). The four users start by making light contact with their own VO local copy from four different directions (see the first row in bottom plots of Figs. 2.8-2.9). Then, they push their VO copies together to make a shared deformation. After the steady-state is attained, all the users release and the four VO copies converge back to the un-deformed ball shape.

From the top plots of Figs. 2.8-2.9, we can see that: 1) when released (after 12.5s in Fig. 2.8 and 64s in Fig. 2.9), the four VO copies reach their configuration consensus (i.e., item 2 of Th. 1), the speed of which is drastically different between the best and the worst topologies; and 2) multiuser force balance (i.e., item 3 of Th. 1) is achieved in the steady-state contact (between 5s-12s in Fig. 2.8 and 25s-64s in Fig. 2.9). The bottom plots of Figs. 2.8-2.9 also vividly show the performance difference between the best and worst case topologies: to achieve similar deformation on the shared VO (with similar human force), it takes much less time with the best topology than the worst topology (e.g., 3s for Fig. 2.8; 29s for Fig. 2.9).

The multiuser haptic interaction in Figs. 2.8-2.9 is also stable, even with the Internet’s communication unreliability (e.g., packet loss, varying delay, etc). This is due to our enforcing N -port passivity (i.e., item 1 of Th. 1). To verify this claim, we intentionally violate the passivity condition (2.17) by reducing the local damping B_i to (7.5, 3, 2, 2.5)[Nm/s] for the best topology. The result is shown in Fig. 2.10, where, with only a small perturbation exerted by one user on his local VO copy around 2.5s, the system becomes unstable, clearly manifesting the importance of the N -port passivity (2.12) and enforcing its condition (2.17). The (slight) shape difference in the last row of the bottom plots in Figs. 2.8-2.9 can be further reduced by increasing the consensus P-gain P_{ij} , which, yet, in turn, requires larger local damping B_i , thereby, degrading the system performance, particularly when the communication is poor.

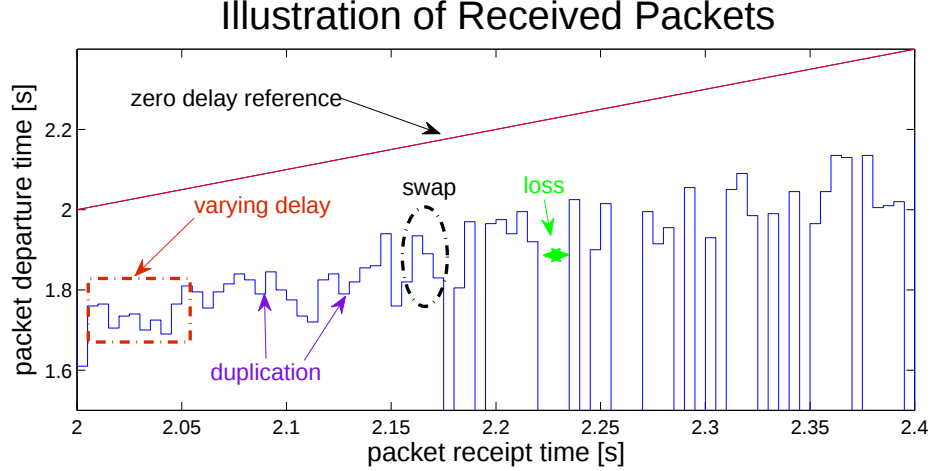


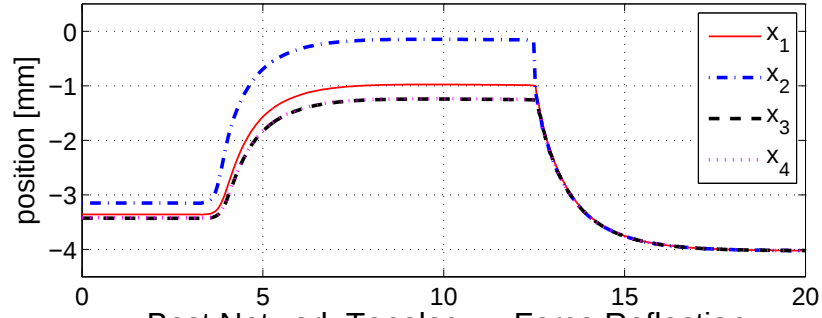
Figure 2.7: Sample pattern of Internet like communication: notice the mismatch between packet departure time and packet reception time, with varying delay, data swap/duplication, and packet loss (i.e., sudden drops).

Such a performance degradation with excessive damping is typical for many “time-invariant” teleoperation/haptics schemes (e.g., PD control [LS06b]; wave approach [NS04]) and may be (possibly significantly) overcome by using other less-conservative schemes (e.g., PSPM [LH10a] with passifying action selectively activated only when necessary).

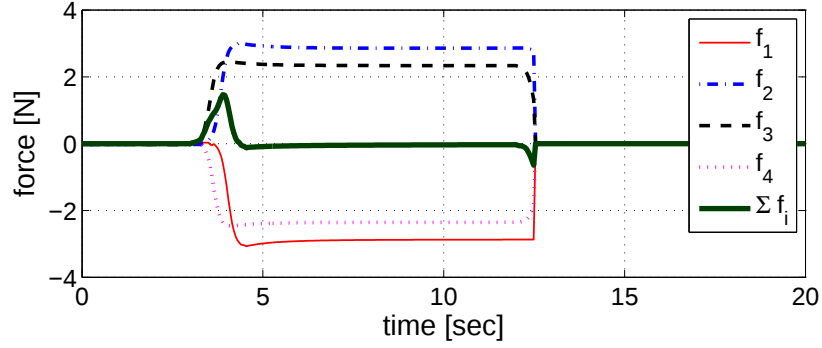
2.5 Conclusion

In this dissertation, a novel and systematic P2P control architecture is presented for the multiuser shared haptic interaction over the Internet. For haptic feedback responsiveness, each user simulates their own local copy of the shared deformable VO and locally interacts with it, while, for haptic experience consistency, these VO local copies are synchronized by the PD-type consensus control over the Internet with undirected, yet, only partially-connected, inter-user communication topology. By extending/utilizing the results of [HL10, LH10b, LH08a], passivity of the P2P architecture is guaranteed, even if the Internet is imperfect (e.g., with varying delay, packet loss, data swapping, etc), thereby, rendering the architecture robust stable,

Best Network Topology – VO Configuration Synchronization



Best Network Topology – Force Reflection



Best Network Topology – VO Snapshots

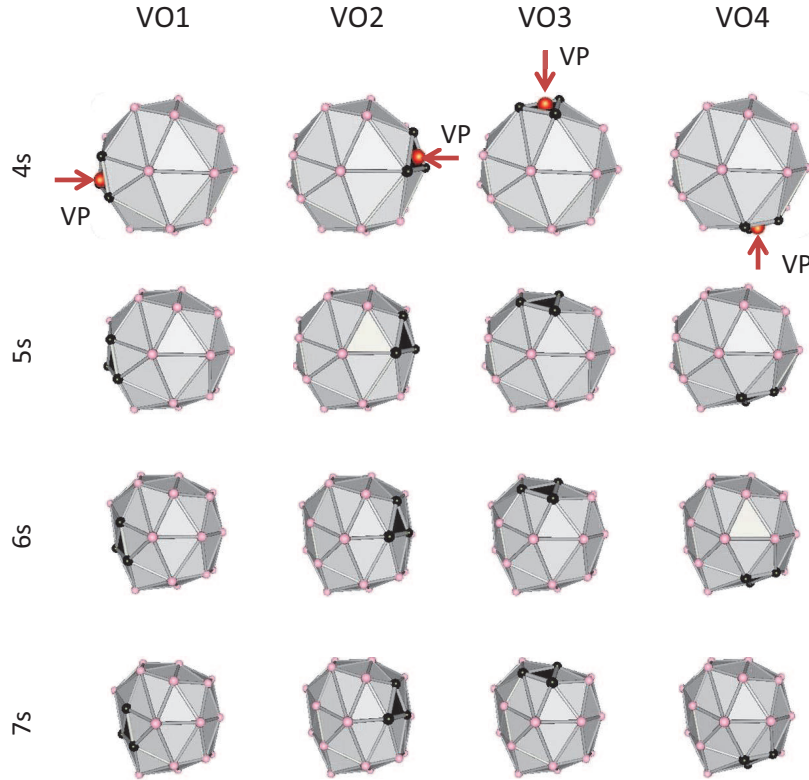
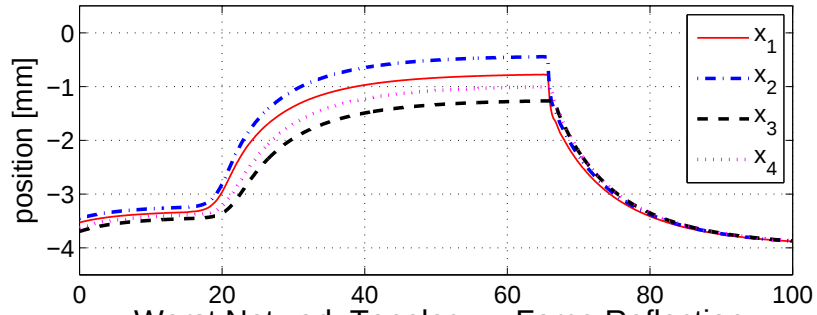
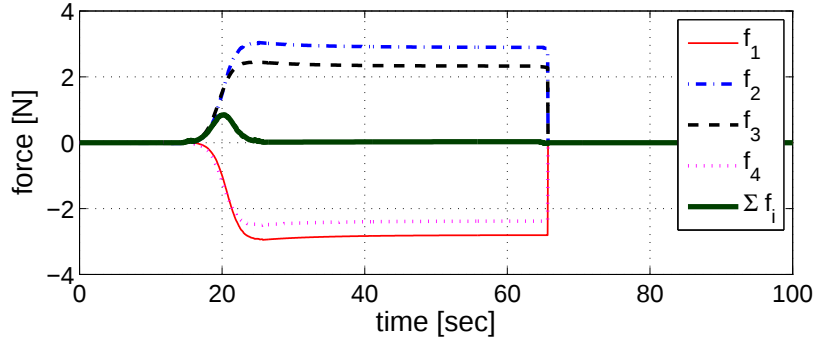


Figure 2.8: Experimental results with best topology: (top) position/force of the representative node; (bottom) deformation of four VOs.

Worst Network Topology – VO Configuration Synchronization



Worst Network Topology – Force Reflection



Worst Network Topology – VO Snapshots

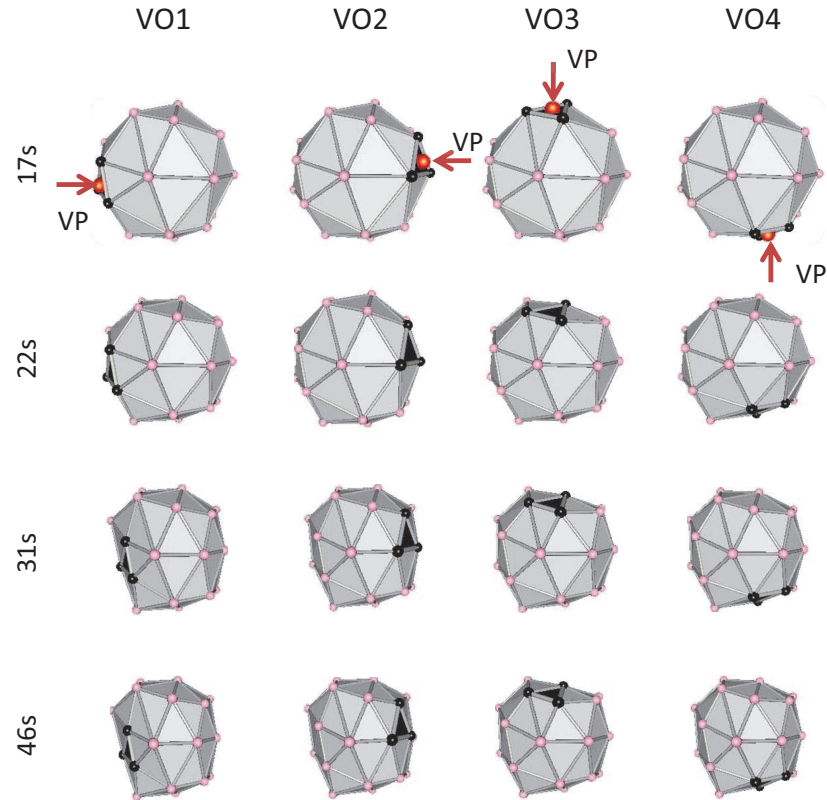


Figure 2.9: Experimental results with worst topology: (top) position/force of the representative node; (bottom) deformation of four VOs.

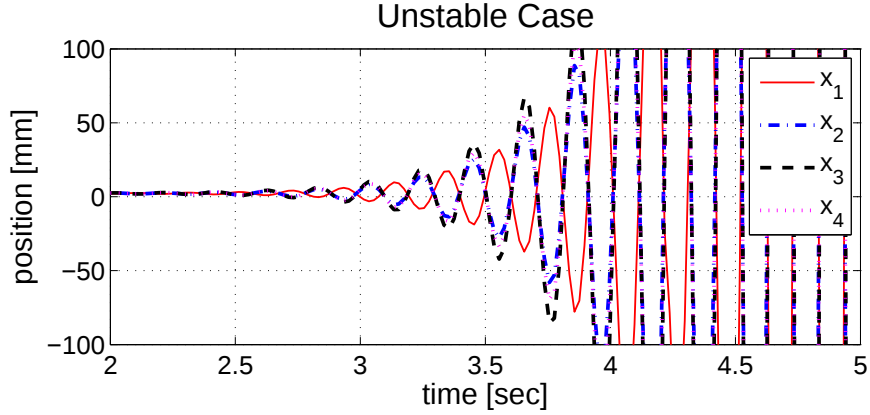


Figure 2.10: Unstable behavior with the condition (2.17) intentionally violated.

portable and scalable against heterogeneous users/devices. Consensus among the VO local copies and the multiuser force balance via the shared deformable VO are shown. Network topology optimization using algebraic connectivity is also proposed along with some experimental results.

Some future research directions include: 1) further improvement of the system performance by using some less conservative consensus schemes (e.g., PSPM [LH10a]) instead of the current (time-invariant) PD-type consensus control; 2) reduction of the amount of data for the VO consensus without compromising human perception (e.g., perception-based data reduction [HHC⁺08]); and 3) application of the result to more interesting and practically-important scenarios and investigate the issue of human perception therein (e.g., collaborative virtual surgical training).

2.6 Supplementary Mathematical Proofs

We first show the N -port consensus controller passivity (2.13). For this, let us denote the energy generated from the N consensus control ports $(\tau_i(k), \hat{v}_i(k))$ during the k^{th}

time-step by

$$\begin{aligned}
s_E(k) &:= \sum_{i=1}^N \hat{v}_i^T(k) \tau_i(k) T_i^k \\
&= - \sum_{i=1}^N \left[\Lambda_i(k) + \sum_{j \in \mathcal{N}_i} \hat{v}_i^T(k) D_{ij} \left(\hat{v}_i(k) - \delta_{ij}^k \hat{v}_j(k - N_{ij}^k) \right) T_i^k \right. \\
&\quad \left. + \sum_{j \in \mathcal{N}_i} \hat{v}_i^T(k) P_{ij} \left(\hat{x}_i(k) - \hat{x}_j(k - N_{ij}^k) \right) T_i^k \right] \\
&= - \sum_{i=1}^N \Lambda_i(k) - \sum_{l=1}^{N_e} \hat{v}_p^T(k) D_{pq} \left[\hat{v}_p(k) - \delta_{pq}^k \hat{v}_q(k - N_{pq}^k) \right] T_p^k \\
&\quad - \sum_{l=1}^{N_e} \hat{v}_p^T(k) P_{pq} \left[\hat{x}_p(k) - \hat{x}_q(k - N_{pq}^k) \right] T_p^k
\end{aligned}$$

where we use the fact (2.1) with $l \approx (p, q)$ and $\Lambda_i(k) := \|\hat{v}_i(k)\|_{B_i}^2 T_i^k$. Here, the last term, related to P_{pq} , can be rewritten as

$$\begin{aligned}
&\sum_{l=1}^{N_e} \hat{v}_p^T(k) P_{pq} \left[\hat{x}_p(k) - \hat{x}_q(k) \right] T_p^k \\
&\quad + \sum_{l=1}^{N_e} \hat{v}_p^T(k) P_{pq} \left[\hat{x}_q(k) - \hat{x}_q(k - N_{pq}^k) \right] T_p^k \\
&= \frac{1}{2} \sum_{l=1}^{N_e} \left(\hat{v}_p(k) T_p^k - \hat{v}_q(k) T_q^k \right)^T P_{pq} (\hat{x}_p(k) - \hat{x}_q(k)) \\
&\quad + \sum_{l=1}^{N_e} \hat{v}_p^T(k) P_{pq} \left[\hat{x}_q(k) - \hat{x}_q(k - N_{pq}^k) \right] T_p^k
\end{aligned}$$

where we use the fact (2.2) over the undirected $\mathcal{G}(\mathcal{V}, \mathcal{E})$.

Define the relative distance $\Delta x_{pq}^k := x_p(k) - x_q(k)$ and the half of the energy stored in P_{pq} on e_{pq} , $\varphi_{pq}(k) := \frac{1}{4} \|\Delta x_{pq}^k\|_{P_{pq}}^2$. Then, using the following derivation:

$$\begin{aligned} & (\hat{v}_p(k)T_p^k - \hat{v}_q(k)T_q^k)^T P_{pq} (\hat{x}_p(k) - \hat{x}_q(k)) \\ &= \frac{1}{2} (\Delta x_{pq}^{k+1} - \Delta x_{pq}^k)^T P_{pq} (\Delta x_{pq}^{k+1} + \Delta x_{pq}^k) \\ &= 2(\varphi_{pq}(k+1) - \varphi_{pq}(k)) \end{aligned}$$

we can further rewrite $s_E(k)$ s.t.

$$\begin{aligned} s_E(k) = & - \sum_{i=1}^N \underbrace{\Lambda_i(k)}_{B_i\text{-dissipation}} - \sum_{l=1}^{N_e} \underbrace{[\varphi_{pq}(k+1) - \varphi_{pq}(k)]}_{\text{energy stored in } P_{pq}} \\ & - \sum_{l=1}^{N_e} \underbrace{\hat{v}_p^T(k) P_{pq} [\hat{x}_q(k) - \hat{x}_q(k - N_{pq}^k)] T_p^k}_{\text{energy generated by P-action}} \\ & - \sum_{l=1}^{N_e} \underbrace{\hat{v}_p^T(k) D_{pq} [\hat{v}_p(k) - \delta_{pq}^k \hat{v}_q(k - N_{pq}^k)] T_p^k}_{\text{energy generated by D-action}} \end{aligned} \quad (2.23)$$

with $(p, q) \approx l$, where the first and second terms are always passivity enforcing, while the third and forth terms may violate passivity due to the Internet's communication unreliability.

We will now show that, under the condition (2.17), those (possibly) passivity-breaking energy generation due to the P and D actions are guaranteed to be dissipated by the local damping injection B_i , thereby, the consensus controller passivity (2.13) and the closed-loop passivity (2.12) are achieved. For this, we first obtain the upper-bound of the energy generation by the P action and that of the D action, and show that the lower-bound of the damping dissipation is larger than their sum under the condition (2.17).

1) *Terms related to delayed P-action:* From (2.23), define

$$\Theta_{pq}(k) := \hat{v}_p^T(k) P_{pq} \left[\hat{x}_q(k) - \hat{x}_q(k - N_{pq}^k) \right] T_p^k \quad (2.24)$$

associated to the energy generation by the P action on the edge e_l with $(p, q) \approx l$ during the k^{th} -th time index. Inserting “telescopic” term, $\sum_{j=k+1-N_{pq}^k}^{k-1} [\hat{x}_q(j) - \hat{x}_q(j+1)]$, between $\hat{x}_q(k)$ and $\hat{x}_q(k - N_{pq}^k)$, we can write $\Theta_{pq}(k)$ as

$$\begin{aligned} \Theta_{pq}(k) &= \hat{v}_p^T(k) P_{pq} \sum_{j=k-N_{pq}^k}^{k-1} [\hat{x}_q(j+1) - \hat{x}_q(j)] T_p^k \\ &= T_p^k \hat{v}_p^T(k) P_{pq} \sum_{j=k-N_{pq}^k}^{k-1} \frac{1}{2} [\hat{v}_q(j+1) T_q^{j+1} + \hat{v}_q(j) T_q^j] \\ &= T_p^k \hat{v}_p^T(k) P_{pq} \sum_{j=k+1-N_{pq}^k}^{k-1} \hat{v}_q(j) T_q^j \\ &\quad + \frac{1}{2} T_p^k \hat{v}_p^T(k) P_{pq} [\hat{v}_q(k) T_q^k + \hat{v}_q(k - N_{pq}^k) T_q^{k-N_{pq}^k}] \end{aligned}$$

where the second equality is obtained by using (2.9), and the third equality by combining the last two terms of the second line. Since P_{pq} is symmetric and positive-definite, we have the following fact:

$$|x^T P_{pq} y| \leq \frac{1}{2} \left(\|x\|_{P_{pq}}^2 + \|y\|_{P_{pq}}^2 \right) \quad (2.25)$$

for any $x, y \in \mathfrak{R}^n$. Using this, we can then show that

$$\begin{aligned}
|\Theta_{pq}(k)| &\leq \frac{1}{2} T_p^k \sum_{j=k+1-N_{pq}^k}^{k-1} T_q^j \left[\|\hat{v}_p(k)\|_{P_{pq}}^2 + \|\hat{v}_q(j)\|_{P_{pq}}^2 \right] \\
&\quad + \frac{1}{4} T_p^k T_q^k \left[\|\hat{v}_p(k)\|_{P_{pq}}^2 + \|\hat{v}_q(k)\|_{P_{pq}}^2 \right] \\
&\quad + \frac{1}{4} T_p^k T_q^{k-N_{pq}^k} \left[\|\hat{v}_p(k)\|_{P_{pq}}^2 + \|\hat{v}_q(k-N_{pq}^k)\|_{P_{pq}}^2 \right] \\
&= \frac{1}{4} \alpha_{pq}(k) \left[\sum_{j=k-N_{pq}^k}^{k-1} T_q^j + \sum_{j=k-N_{pq}^k+1}^k T_q^j \right] \\
&\quad + \frac{1}{4} T_p^k \left[\sum_{j=k-N_{pq}^k}^{k-1} \alpha_{qp}(j) + \sum_{j=k-N_{pq}^k+1}^k \alpha_{qp}(j) \right]
\end{aligned}$$

with $\alpha_{pq}(k) := T_p^k \|\hat{v}_p(k)\|_{P_{pq}}^2 \geq 0$, where the equality is obtained by splitting the two terms in the first line and combine them with the remaining terms.

Since $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is undirected with $P_{pq} = P_{qp}$, we can also define $\Theta_{qp}(k)$ and obtain $|\Theta_{qp}(k)|$ similar to above, with p, q swapped with each other. Summing them up and collecting the terms containing α_{pq} and α_{qp} separately, we can show that

$$|\Theta_{pq}(k)| + |\Theta_{qp}(k)| \leq \Omega_{pq}(k) + \Omega_{qp}(k) \quad (2.26)$$

where

$$\begin{aligned}
\Omega_{pq}(k) &:= \frac{1}{4} \alpha_{pq}(k) \left[\sum_{j=k-N_{pq}^k}^{k-1} T_q^k + \sum_{j=k-N_{pq}^k+1}^k T_q^k \right] \\
&\quad + \frac{1}{4} T_q^k \left[\sum_{j=k-N_{qp}^k}^{k-1} \alpha_{pq}(j) + \sum_{j=k-N_{qp}^k+1}^k \alpha_{pq}(j) \right] \\
&\leq \frac{1}{4} T_q^{\max} \left[2\bar{N}_{pq} \alpha_{pq}(k) + \sum_{j=k-\bar{N}_{qp}}^{k-1} \alpha_{pq}(j) + \sum_{j=k-\bar{N}_{qp}+1}^k \alpha_{pq}(j) \right].
\end{aligned}$$

Then, by summing $\Omega_{pq}(k)$ over the time, we have

$$\begin{aligned}
\sum_{k=0}^{\bar{M}} \Omega_{pq}(k) &\leq \frac{T_q^{\max}}{2} \sum_{k=0}^{\bar{M}} \left[\bar{N}_{pq} \alpha_{pq}(k) + \sum_{j=k-\bar{N}_{qp}+1}^k \alpha(j) \right] \\
&= \sum_{k=0}^{\bar{M}} \frac{\bar{N}_{pq} + \bar{N}_{qp}}{2} T_q^{\max} \alpha_{pq}(k) - \frac{T_q^{\max}}{2} \sum_{k=1}^{\bar{N}_{qp}-1} k \alpha_{pq}(k + \bar{M} - \bar{N}_{qp} + 1) \\
&\leq \sum_{k=0}^{\bar{M}} \frac{\bar{N}_{pq} + \bar{N}_{qp}}{2} T_q^{\max} \alpha_{pq}(k)
\end{aligned} \tag{2.27}$$

where the first inequality is because

$$\begin{aligned}
&\sum_{k=0}^{\bar{M}} \left[\sum_{j=k-\bar{N}_{qp}}^{k-1} \alpha_{pq}(j) + \sum_{j=k-\bar{N}_{qp}+1}^k \alpha_{pq}(j) \right] \\
&= \sum_{k=0}^{\bar{M}} \left[2 \sum_{j=k-\bar{N}_{qp}+1}^k \alpha_{pq}(j) + \alpha_{pq}(k - \bar{N}_{qp}) - \alpha_{pq}(k) \right] \\
&= 2 \sum_{k=0}^{\bar{M}} \sum_{j=k-\bar{N}_{qp}+1}^k \alpha_{pq}(j) + \sum_{k=0}^{\bar{M}} \left[\alpha_{pq}(k - \bar{N}_{qp}) - \alpha_{pq}(k) \right]
\end{aligned}$$

where the last term is always negative, since $\alpha_{pq}(k) \geq 0$ with $\alpha_{pq}(k) = 0$ for $k < 0$. The last inequality of (2.27) is also because $\alpha_{pq}(k) \geq 0$. On the other hand, the equality in the second-line of (2.27) is from the fact that

$$\sum_{k=0}^{\bar{M}} \sum_{j=k-\bar{N}_{qp}+1}^k \alpha_{pq}(j) = \sum_{k=0}^{\bar{M}} \bar{N}_{qp} \alpha_{pq}(k) - \sum_{k=1}^{\bar{N}_{qp}-1} k \alpha_{pq}(\bar{M} - \bar{N}_{qp} + 1 + k) \tag{2.28}$$

which can be shown as follows: 1) write k from 0 to $\bar{M}$ horizontally and each term of $\sum_{j=k-\bar{N}_{qp}+1}^k \alpha_{pq}(j)$ top down from $j = k - \bar{N}_{qp} + 1$ to $j = k$ with $\alpha_{qp}(k) = 0$ for $k < 0$ (i.e., all the terms in the left hand side of (2.28)); 2) append them with terms to collectively make a parallelogram shape with its top and bottom lines respectively consisting of $\bar{N}_{qp} \alpha_{pq}(0)$ and $\alpha_{pq}(\bar{M})$; 3) add all the terms in this parallelogram (i.e., first term in (2.28)); and 4) subtract the terms which were added to make

the parallelogram (i.e., second term in (2.28)).

2) *Terms related to delayed D-action:* This energy can be written as: for the edge $e_l \in \mathcal{E}$ with $(p, q) \approx l$,

$$\begin{aligned}\Upsilon_{pq}(k) &:= \hat{v}_p^T(k) D_{pq} \left[\hat{v}_p(k) - \delta_{pq}^k \hat{v}_q(k - N_{pq}^k) \right] T_p^k \\ &\geq \|\hat{v}_p(k)\|_{D_{pq}}^2 T_p^k - \frac{1}{2} \delta_{pq}^k \left(\|\hat{v}_p(k)\|_{D_{pq}}^2 + \|\hat{v}_q(k - N_{pq}^k)\|_{D_{pq}}^2 \right) T_p^k \\ &\geq \frac{1}{2} \left(\|\hat{v}_p(k)\|_{D_{pq}}^2 T_p^k - \delta_{pq}^k \|\hat{v}_q(k - N_{pq}^k)\|_{D_{pq}}^2 T_p^k \right)\end{aligned}\tag{2.29}$$

where we use (2.25) and the fact that $\delta_{pq}^k = \{0, 1\}$. Then, similar to (2.26), since the graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is undirected with $D_{pq} = D_{qp}$, we can obtain inequality similar to the above for Υ_{qp} . Further, combining those for Υ_{pq} and Υ_{qp} , we can have

$$\Upsilon_{pq}(k) + \Upsilon_{qp}(k) \geq \Psi_{pq}(k) + \Psi_{qp}(k)\tag{2.30}$$

where

$$\Psi_{pq}(k) := \frac{1}{2} \left(\|\hat{v}_p(k)\|_{D_{pq}}^2 T_p^k - \delta_{qp}^k \|\hat{v}_p(k - N_{qp}^k)\|_{D_{pq}}^2 T_q^k \right).$$

Summing this $\Psi_{pq}(k)$ over time then yields:

$$\begin{aligned}\sum_{k=0}^{\bar{M}} \Psi_{pq}(k) &\geq \frac{1}{2} \sum_{k=0}^{\bar{M}} \left(\|\hat{v}_p(k)\|_{D_{pq}}^2 T_p^{\min} - \delta_{qp}^k \|\hat{v}_p(k - N_{qp}^k)\|_{D_{pq}}^2 T_q^{\max} \right) \\ &\geq \frac{1}{2} \sum_{k=0}^{\bar{M}} \left(\|\hat{v}_p(k)\|_{D_{pq}}^2 T_p^{\min} - \delta_{qp}^k \|\hat{v}_p(k)\|_{D_{pq}}^2 T_q^{\max} \right) \\ &\geq \frac{1}{2} \sum_{k=0}^{\bar{M}} \left(T_p^{\min} - T_q^{\max} \right) \|\hat{v}_p(k)\|_{D_{pq}}^2\end{aligned}\tag{2.31}$$

where the first inequality is from the definition of $T_p^{\min}, T_q^{\max}$; while the second and third inequalities are because $\delta_{qp}^k \in \{0, 1\}$ and $\|\hat{v}_p(k)\|_{D_{pq}}^2 \geq 0$, with $\|\hat{v}_p(k)\|_{D_{pq}}^2 = 0$

for $k < 0$.

Now, we prove the N -port consensus controller passivity (2.13) under the condition (2.17). First, using (2.17), we can compute a lower-bound for the energy dissipation by the local damping B_i s.t.,

$$\begin{aligned}
& \sum_{k=0}^{\bar{M}} \sum_{i=1}^N \|\hat{v}_i(k)\|_{B_i}^2 T_i^k \\
& \geq \sum_{k=0}^{\bar{M}} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \left(\frac{\bar{N}_{ij} + \bar{N}_{ji}}{2} T_j^{\max} \|\hat{v}_i(k)\|_{K_{ij}}^2 T_i^k + \frac{1}{2} \left[\frac{T_j^{\max}}{T_i^{\min}} - 1 \right] T_i^k \|\hat{v}_i(k)\|_{D_{ij}}^2 \right) \\
& \geq \sum_{k=0}^{\bar{M}} \sum_{l=1}^{N_e} \left(\frac{\bar{N}_{pq} + \bar{N}_{qp}}{2} T_q^{\max} \alpha_{pq}(k) + \frac{1}{2} [T_q^{\max} - T_p^{\min}] \|\hat{v}_p(k)\|_{D_{pq}}^2 \right) \quad (2.32)
\end{aligned}$$

where $(p, q) \approx l$, $\alpha_{pq}(k) := \|\hat{v}_p(k)\|_{P_{pq}}^2 T_p^k$ and the second inequality is due to the fact 2.1 and $T_p^k/T_p^{\min} \geq 1$.

Then, from (2.23) using (2.24) and (2.29), along with (2.26), (2.27), (2.30) and (3.75), we can achieve the N -port controller passivity (2.13), i.e., $\forall \bar{M} \geq 0$,

$$\begin{aligned}
\sum_{k=0}^{\bar{M}} s_E(k) &= - \sum_{k=0}^{\bar{M}} \sum_{i=1}^N \Lambda_i(k) + \sum_{l=1}^{N_e} [\varphi_{pq}(0) - \varphi_{pq}(\bar{M} + 1)] \\
&\quad - \sum_{k=0}^{\bar{M}} \sum_{l=1}^{N_e} \Theta_{pq}(k) - \sum_{k=0}^{\bar{M}} \sum_{l=1}^{N_e} \Upsilon_{pq}(k) \leq \sum_{l=1}^{N_e} \varphi_{pq}(0) =: c^2 \quad (2.33)
\end{aligned}$$

where, for the inequality, we also use the fact that

$$- \sum_{k=0}^{\bar{M}} \sum_{l=1}^{N_e} \Theta_{pq}(k) \leq \sum_{k=0}^{\bar{M}} \sum_{l=1}^{N_e} |\Theta_{pq}(k)| \leq \sum_{k=0}^{\bar{M}} \sum_{l=1}^{N_e} \Omega_{pq}(k).$$

Chapter 3

Hybrid Control Frameworks for Bilateral Teleoperation over the Internet

3.1 Preliminary

3.1.1 Modeling

Consider the bilateral teleoperator of Fig. 3.1, where the master and slave robots are modeled by n-degree-of-freedom (n-DOF) Lagrange's dynamics with sampled-data controllers, s.t. for $i = 1, 2$ and $t \in [t_k, t_{k+1})$

$$M_i(q_i)\ddot{q}_i(t) + C_i(q_i, \dot{q}_i)\dot{q}_i(t) + B_i\dot{q}_i(t) = u_i(k) + f_i(t) \quad (3.1)$$

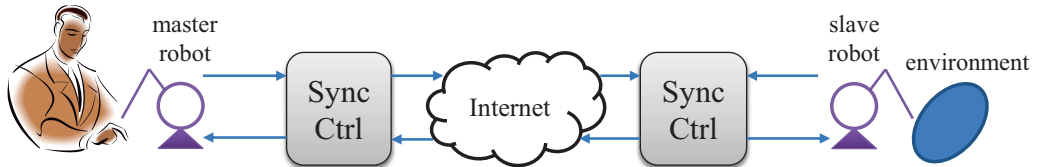


Figure 3.1: Diagram of common Internet bilateral teleoperation system.

where $\star_1, \star_2$ refer to the values of master and slave sides respectively; $q_i(t), \dot{q}_i(t), \ddot{q}_i(t) \in \mathbb{R}^n$ are the robot joint configuration, velocity, and acceleration respectively; $M_i(q_i) \in \mathbb{R}^{n \times n}$ is the positive-definite inertia matrix; $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n \times n}$ is the Coriolis matrix; $B_i \in \mathbb{R}^{n \times n}$ is the positive-definite physical device damping matrix; t_k is k -th update time instant of the sampled-data control; $u_i(k) \in \mathbb{R}^n$ is the sampled-data control which is computed at t_k and being held constantly during $[t_k, t_{k+1})$; and $f_i(t) \in \mathbb{R}^n$ is the human/environmental force acting on robot joints.

For notational simplicity, we define

$$\hat{u}_i(t) := u_i(k) - B_i \dot{q}_i(t) \quad t \in [t_k, t_{k+1}). \quad (3.2)$$

with which we can write (3.1) s.t.

$$M_i(q_i) \ddot{q}_i(t) + C_i(q_i, \dot{q}_i) \dot{q}_i(t) = \hat{u}_i(t) + f_i(t).$$

In this paper, we will use the following well-known properties associated to (3.1) [SHV06]:

$$\lambda_{\min}^i I \preceq M_i(q_i) \preceq \lambda_{\max}^i I \quad (3.3a)$$

$$c_{kj}^i = \sum_{l=1}^n \frac{1}{2} \left\{ \frac{\partial m_{kj}^i}{\partial q_j^i} + \frac{\partial m_{kl}^i}{\partial q_j^i} - \frac{\partial m_{lj}^i}{\partial q_k^i} \right\} \dot{q}_l^i \quad (3.3b)$$

for $i = 1, 2$, where $\lambda_{\min}^i, \lambda_{\max}^i \in \mathbb{R}$ are positive constants, $q_j^i \in \mathbb{R}$ is the j -th element of q_i , $m_{kj}^i \in \mathbb{R}$ and $c_{kj}^i \in \mathbb{R}$ are the kj -th element of $M_i(q_i) \in \mathbb{R}^{n \times n}$ and $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n \times n}$. A direct but important property from (3.3b) is that $\dot{M} - 2C$ is skew-symmetric, which further implies the following two-port open-loop passivity of (3.1): $\forall t > 0, \exists c_i \in \mathbb{R}$ s.t.,

$$\int_0^t \dot{q}_i(\tau)^T [\hat{u}_i(\tau) + f_i(\tau)] d\tau \geq -c_i^2. \quad (3.4)$$

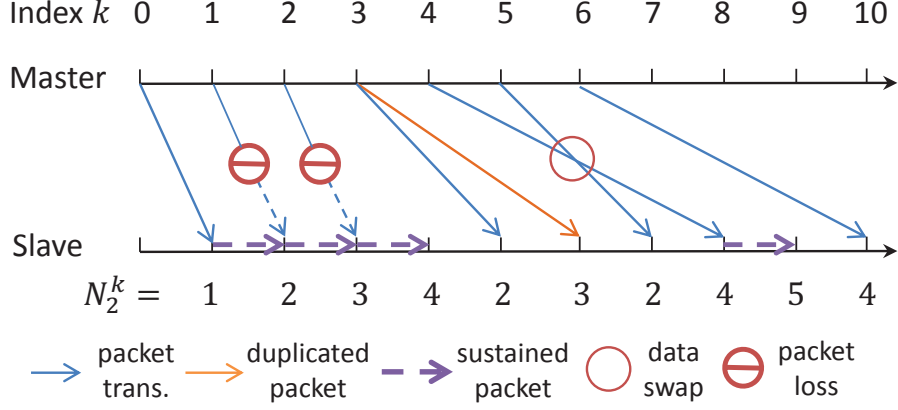


Figure 3.2: Indexing delay N_2^k on local sampling index k (from master to slave) with varying-delay, packet-loss, data duplication and swapping. This is similar to Fig. 2.3, which is for multiple interconnected systems.

meaning that the open-loop robot (3.1) does not produce energy by itself.

In this paper, we consider the case where both the continuous-time master and slave robots are sampled-controlled with their communication established over the discrete-time Internet. We denote their (variable) local sampling rate by $T_k^i > 0$ ($i = 1, 2$). We assume $\exists$ positive constants $T_{\max}, T_{\min}$ s.t. $T_{\min} \leq T_k^i \leq T_{\max}$. This T_k^i can also be written by $T_k^i := t_{k+1}^i - t_k^i$, where $k \geq 0$ is the local device sampling index and t_k^i is the sampling time instance for the master ($i = 1$) or the slave ($i = 2$). Here, we assume that the local sampling rates of the master and slave are similar to each other, i.e., for all $N \geq 0$, $t_N^1 \approx t_N^2$ (e.g., both 1kHz), which also implies that $t_o^1 \approx t_o^2$, that is, the starting time is similar for the master and slave systems. Other than this, we allow asynchronism between the master and slave local samplings (e.g., $t_k^1 \neq t_k^2$, $T_k^1 \neq T_k^2$). Note that, here, we use the same discrete-time index k both for the master and the slave.

On top of this local sampling T_k^i running, some information for generating control is then communicated over the Internet between the master and slave sites. If we denote this signal by $\star$, we can then write this signal received at the discrete-time index k from the i -site (master $i = 1$; slave $i = 2$) by $\star_j(k - N_i^k)$, where $N_i^k > 0$ we call *index delay*, which is defined s.t.: 1) if the packet of $\star_j$ is successfully received

by the i -site, N_i^k is the k -index difference between the i -site reception and the j -site transmission; and 2) if the packet of $\star_j$ is missing from the reception i -site at the index k , $N_i^k := N_i^{k-1} + 1$ with the previous packet being kept (i.e., packet sustainment). See Fig. 3.2. This index delay N_i^k can then capture various communication unreliability of the Internet, including varying-delay (e.g., master $k = 3, 6$ in Fig. 3.2), packet-loss (e.g., master $k = 1, 2$ in Fig. 3.2), data swapping (e.g., master $k = 4, 5$ in Fig. 3.2) and packet duplication (e.g., master $k = 3$ in Fig. 3.2). Our control frameworks, to be proposed below, however do not require us to know (or track) this index delay N_i^k : all they require is that this N_i^k be upper-bounded (with the packet sustainment included) and this upper-bound be known, that is, we can find $\bar{N}_i > 0$ s.t., $\forall k \geq 0, \bar{N}_i \geq N_i^k$, $i = 1, 2$ (e.g., no complete communication blackout).

3.1.2 Control Objectives

For the teleoperator (3.1), we would like to design the sampled-data control $u_i(k)$ to achieve the following control objectives [LS06b]:

- *closed-loop passivity*: $\forall N \geq 0, \exists c \in \mathbb{R}$, s.t.

$$\int_0^{t_N^1} \dot{q}_1(\tau)^T f_1(\tau) d\tau + \int_0^{t_N^2} \dot{q}_2(\tau)^T f_2(\tau) d\tau \geq -c^2 \quad (3.5)$$

- *position coordination*: if $f_i(t) = 0, i = 1, 2$,

$$q_1(t) \rightarrow q_2(t) \quad (3.6)$$

- *force reflection*: with $(\dot{q}_i(t), \ddot{q}_i(t)) \rightarrow 0$,

$$f_1(t) \rightarrow -f_2(t). \quad (3.7)$$

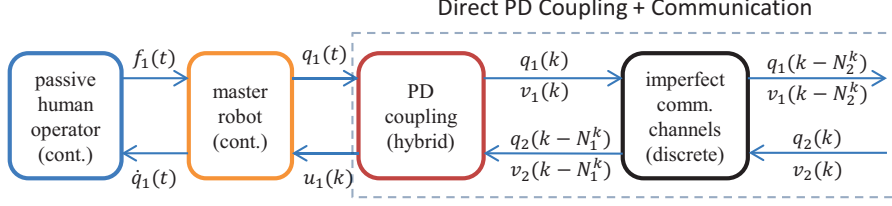


Figure 3.3: DPDC control architecture (only master side shown due to symmetry).

Here, we also recall from [LL05] that the following *controller passivity* implies closed-loop passivity (3.5): $\exists d \in \Re$ s.t.,

$$\int_0^{t_N^1} \dot{q}_1^T(\tau) \hat{u}_1(\tau) d\tau + \int_0^{t_N^2} \dot{q}_2^T(\tau) \hat{u}_2(\tau) d\tau \leq d^2 \quad (3.8)$$

for all $N \geq 0$.

3.2 Direct Sampled-Data PD Control (DPDC)

Following the continuous-time PD-based teleoperation control [LS06b, NBOS09], we design its sampled-data analogue, direct PD-coupling (DPDC) control as shown in Fig. 3.3: for $t \in [t_k, t_{k+1})$ and $(i, j) \in \{(1, 2), (2, 1)\}$,

$$\begin{aligned} u_i(k) &= - \underbrace{K \left[q_i(k) - q_j(k - N_i^k) \right]}_{\text{delayed P-action}} - \underbrace{D \left[v_i(k) - \delta_i^k v_j(k - N_i^k) \right]}_{\text{delayed D-action}} \\ v_i(k) &:= \frac{q_i(k) - q_i(k-1)}{T_{k-1}^i} \end{aligned} \quad (3.9)$$

where $q_i(k) := q_i(t_k)$ is the sampling of $q_i(t)$ at t_k ; $T_k^i := t_{k+1}^i - t_k^i$ is the k -th sampling period; $v_i(k)$ is the numerical estimate of $\dot{q}_i(t)$; $K, D \in \Re^{n \times n}$ are the symmetric and positive-definite control gains; N_i^k is the discrete index delay; and δ_i^k is the avoidance function defined as follows.

For the P-action in (3.9), it is often desirable to sustain the previous set-position signal q_j when there is no packet received instead of using $q_j(k) = 0$. It will be

shown in Th. 2 that this set-position signal sustainment can be incorporated without violating the closed-loop passivity (3.5). However, such sustainment for the velocity signal v_j can compromise the passivity (3.5). To prevent this, we use the duplication avoidance function δ_i^k in (3.9) defined s.t.

$$\delta_i^k := \begin{cases} 0 & \text{if packet received at } k \text{ index is duplicated} \\ 1 & \text{otherwise} \end{cases} \quad (3.10)$$

where the duplication in the first line includes not only “real” duplication by the communication channel, but also the “artificial” duplication caused by the packet sustainment as explained above. This δ_i^k can be easily implemented by some packet numbering mechanism. We now present the first main result in the following theorem.

Theorem 2. *Consider the teleoperator (3.1) under the sampled-data DPDC control (3.9) over the Internet. Suppose the following condition is met*

$$B_i \succeq \left[\frac{\bar{N}_1 + \bar{N}_2}{2} + 1 \right] T_{\max} K + \left[\frac{T_{\max}}{T_{\min}} + 1 \right] D \quad (3.11)$$

for $i = 1, 2$, where B_i is the physical device damping and $T_{\min} \leq T_k^i \leq T_{\max}$. For simplicity, also assume $q_i(k) = 0, \forall k < 0$. Then,

- the closed-loop passivity (3.5) is achieved.
- if $f_i \equiv 0$ and there is un-modeled extra local damping s.t.

$$B_i \succ \left[\frac{\bar{N}_1 + \bar{N}_2}{2} + 1 \right] T_{\max} K + \left[\frac{T_{\max}}{T_{\min}} + 1 \right] D \quad (3.12)$$

the position coordination (3.6) is achieved.

- if $(\dot{x}_i(t), \ddot{x}_i(t)) \rightarrow 0, i = 1, 2$, the static force reflection is achieved s.t.,

$$f_1 \rightarrow -f_2 \rightarrow K[q_1 - q_2]. \quad (3.13)$$

Proof. **1)** Let us denote energy produced by the master and slave controllers and the device physical damping B_i during $[t_k^i, t_{k+1}^i)$ by:

$$\begin{aligned}
\epsilon(k) &= \int_{t_k^1}^{t_{k+1}^1} \hat{u}_1^T(\tau) \dot{q}_1(\tau) d\tau + \int_{t_k^2}^{t_{k+1}^2} \hat{u}_2^T(\tau) \dot{q}_2(\tau) d\tau \\
&= - \sum_{(i,j) \in \mathcal{E}} \left[q_i(k) - q_j(k - N_i^k) \right]^T K \Delta q_i(k) \\
&\quad + \sum_{(i,j) \in \mathcal{E}} \left[v_i(k) - \delta_i^k v_j(k - N_i^k) \right]^T D \Delta q_i(k) \\
&\quad + \sum_{i=1,2} \int_{t_k^i}^{t_{k+1}^i} \|\dot{q}_i^T(\tau)\|_{B_i}^2 d\tau
\end{aligned} \tag{3.14}$$

where $\|b\|_A^2 := b^T A b$ for $b \in \Re^n$ and $A \in \Re^{n \times n}$, $\mathcal{E} := \{(1, 2), (2, 1)\}$, and

$$\Delta q_i(k) := q_i(k+1) - q_i(k).$$

- *Device Damping Dissipation*

First, consider the device damping dissipation in (3.14):

$$\epsilon_b(k) := - \sum_{i=1}^2 \int_{t_k^i}^{t_{k+1}^i} \|\dot{q}_i(\tau)\|_{B_i}^2 d\tau \leq - \sum_{i=1}^2 \frac{1}{T_k^i} \|\Delta q_i(k)\|_{B_i}^2 \tag{3.15}$$

where the inequality is due to Cauchy-Schwartz [[Lee09](#), [LH10a](#)], i.e.,

$$\int_{t_k^i}^{t_{k+1}^i} \|\dot{q}_i^T(\tau)\|_{B_i}^2 d\tau \geq \frac{1}{T_k^i} \|\Delta q_i(k)\|_{B_i}^2. \tag{3.16}$$

- *Energy Generated by P-Action*

We extract the energy generated by the P-action in (3.14) and rewrite it s.t.

$$\begin{aligned}
\epsilon_p(k) &:= - \left[q_1(k) - q_2(k - N_1^k) \right]^T K \Delta q_1(k) - \left[q_2(k) - q_1(k - N_2^k) \right]^T K \Delta q_2(k) \\
&\leq - \left[\hat{q}_1(k) - \hat{q}_2(k) \right]^T K \left[\Delta q_1(k) - \Delta q_2(k) \right] + \|\Delta q_1(k)\|_K^2 + \|\Delta q_2(k)\|_K^2 \\
&\quad - \left[q_2(k) - q_2(k - N_1^k) \right]^T K \Delta q_1(k) - \left[q_1(k) - q_1(k - N_2^k) \right]^T K \Delta q_2(k) \quad (3.17)
\end{aligned}$$

where we use the derivation in Sec.3.7.1 and

$$\hat{q}_i(k) := \frac{q_i(k+1) + q_i(k)}{2}$$

The last two terms of (3.17) are the energy generation due to the communication unreliability, whose upper bounds over time we now establish here.

First, inserting $\sum_{j=k-N_2^k+1}^{k-1} [q_1(j) - q_1(j)] K \Delta q_2(k) = 0$, we have

$$\begin{aligned}
\varepsilon_p^m &:= - \sum_{k=0}^N \left[q_1(k) - q_1(k - N_2^k) \right]^T K \Delta q_2(k) \\
&= - \sum_{k=0}^N \sum_{j=k-N_2^k}^{k-1} \Delta q_1^T(j) K \Delta q_2(k) \\
&\leq \frac{1}{2} \sum_{k=0}^N \sum_{j=k-N_2^k}^{k-1} \left[\|\Delta q_1(j)\|_K^2 + \|\Delta q_2(k)\|_K^2 \right] \\
&\leq \frac{1}{2} \sum_{k=0}^N \sum_{j=k-\bar{N}_2}^{k-1} \left[\|\Delta q_1(j)\|_K^2 + \|\Delta q_2(k)\|_K^2 \right] \\
&= \frac{1}{2} \sum_{k=0}^N \sum_{j=k-\bar{N}_2}^{k-1} \|\Delta q_1(j)\|_K^2 + \frac{\bar{N}_2}{2} \sum_{k=0}^N \|\Delta q_2(k)\|_K^2 \quad (3.18)
\end{aligned}$$

and, similarly,

$$\begin{aligned}\varepsilon_p^s &:= - \sum_{k=0}^N \left[q_2(k) - q_2(k - N_1^k) \right]^T K \Delta q_1(k) \\ &\leq \frac{1}{2} \sum_{k=0}^N \sum_{j=k-\bar{N}_1}^{k-1} \|\Delta q_2(j)\|_K^2 + \frac{\bar{N}_1}{2} \sum_{k=0}^N \|\Delta q_1(k)\|_K^2.\end{aligned}$$

Combing these two inequalities, we then have

$$\begin{aligned}\varepsilon_p^m + \varepsilon_p^s &\leq \frac{1}{2} \sum_{k=0}^N \sum_{j=k-\bar{N}_2}^{k-1} \|\Delta q_1(j)\|_K^2 + \frac{\bar{N}_1}{2} \sum_{k=0}^N \|\Delta q_1(k)\|_K^2 \\ &\quad + \frac{1}{2} \sum_{k=0}^N \sum_{j=k-\bar{N}_1}^{k-1} \|\Delta q_2(j)\|_K^2 + \frac{\bar{N}_2}{2} \sum_{k=0}^N \|\Delta q_2(k)\|_K^2.\end{aligned}\tag{3.19}$$

The following algebraic fact

$$\sum_{k=0}^N \sum_{j=k-\bar{N}_j}^{k-1} \alpha_i(j) = \sum_{k=0}^N \bar{N}_j \alpha_i(k) - \sum_{r=1}^{\bar{N}_j} r \alpha_i(r + N - \bar{N}_j)\tag{3.20}$$

where $\alpha_i(k) := \|\Delta q_i(k)\|_K^2$, which is obtained by collecting $\alpha_i(k)$ and the residual terms of the left-hand side, can be used to rewrite (3.19) such as

$$\begin{aligned}\varepsilon_p^m + \varepsilon_p^s &\leq \frac{\bar{N}_1 + \bar{N}_2}{2} \sum_{k=0}^N \left[\|\Delta q_1(k)\|_K^2 + \|\Delta q_2(k)\|_K^2 \right] \\ &\quad - \frac{1}{2} \sum_{r=1}^{\bar{N}_2} r \|\Delta q_1(r + N - \bar{N}_2)\|_K^2 - \frac{1}{2} \sum_{r=1}^{\bar{N}_1} r \|\Delta q_2(r + N - \bar{N}_1)\|_K^2 \\ &\leq \frac{\bar{N}_1 + \bar{N}_2}{2} \sum_{k=0}^N \left[\|\Delta q_1(k)\|_K^2 + \|\Delta q_2(k)\|_K^2 \right]\end{aligned}\tag{3.21}$$

where the first two terms after the first inequality are from the second and fourth terms of (3.19) and the first term of (3.20), while others from the second term of (3.20).

Define the potential energy of the P-action spring K s.t.,

$$\phi(k) = \frac{1}{2} \|q_1(k) - q_2(k)\|_K^2.$$

It is easy to show that

$$\sum_{k=0}^N \left[\hat{q}_1(k) - \hat{q}_2(k) \right]^T K \left[\Delta q_1(k) - \Delta q_2(k) \right] = \phi(N+1) - \phi(0)$$

thus, combining (3.17) and (3.21), we can show that

$$\sum_{k=0}^N \epsilon_p(k) \leq \left(\frac{\bar{N}_1 + \bar{N}_2}{2} + 1 \right) \sum_{k=0}^N \left[\|\Delta q_1(k)\|_K^2 + \|\Delta q_2(k)\|_K^2 \right] - \phi(N+1) + \phi(0). \quad (3.22)$$

- *Energy Generated by D-Action*

Consider the D-action energy generation: using the definition of $v_i(k)$ in (3.9),

$$\begin{aligned} \epsilon_d(k) &:= - \left[v_1(k) - \delta_1^k v_2(k - N_1^k) \right]^T D \Delta q_1(k) - \left[v_2(k) - \delta_2^k v_1(k - N_2^k) \right]^T D \Delta q_2(k) \\ &= - \sum_{i=1}^2 \frac{1}{T_{k-1}^i} \Delta q_i^T(k-1) D \Delta q_i(k) + \frac{\delta_1^k}{T_{k-N_1^k-1}^i} \Delta q_2^T(k - N_1^k - 1) D \Delta q_1(k) \\ &\quad + \frac{\delta_2^k}{T_{k-N_2^k-1}^i} \Delta q_1^T(k - N_2^k - 1) D \Delta q_2(k) \end{aligned} \quad (3.23)$$

where the first term after the second equality may be thought of as the energy generated by the sampling effect, while the second and third terms by communication

unreliability. Summing up (3.23) over time, we have

$$\begin{aligned}
\sum_{k=0}^N \epsilon_d(k) &\leq \sum_{k=0}^N \left(\frac{\|\Delta q_1(k)\|_D^2}{T_k^1} + \frac{\|\Delta q_2(k)\|_D^2}{T_k^2} \right) + \frac{1}{2} \sum_{k=0}^N \left(\frac{T_k^1}{T_{k-1}^1} + \frac{\delta_1^k T_k^1}{T_{k-N_1^k-1}^1} \right) \frac{\|\Delta q_1(k)\|_D^2}{T_k^1} \\
&\quad + \frac{1}{2} \sum_{k=0}^N \left(\frac{T_k^2}{T_{k-1}^2} + \frac{\delta_2^k T_k^2}{T_{k-N_2^k-1}^2} \right) \frac{\|\Delta q_2(k)\|_D^2}{T_k^2} \\
&\leq \sum_{i=1}^2 \sum_{k=0}^N \left(1 + \frac{T_{\max}}{T_{\min}} \right) \frac{\|\Delta q_i(k)\|_D^2}{T_k^i}
\end{aligned} \tag{3.24}$$

where we use the triangular inequality with the facts that

$$\sum_{k=0}^N \frac{1}{T_{k-1}^i} \|\Delta q_i(k-1)\|_D^2 \leq \sum_{k=0}^N \frac{1}{T_k^i} \|\Delta q_i(k)\|_D^2 \tag{3.25}$$

and

$$\sum_{k=0}^N \delta_i^k \|\Delta q_j(k - N_i^k - 1)\|_D^2 \leq \sum_{k=0}^N \delta_i^k \|\Delta q_j(k)\|_D^2 \leq \sum_{k=0}^N \|\Delta q_j(k)\|_D^2 \tag{3.26}$$

with $q_i(k) = 0 \ \forall k \leq 0$ and $\delta_i^k = 0$ or 1 . Note that the avoidance function δ_i^k is imperative for (3.26) to hold.

- *Controller Passivity Validation*

Combining (3.15), (3.22) and (3.24), with the passivity condition (3.11), we can then

obtain

$$\begin{aligned}
& \int_0^{t_N^1} \hat{u}_1^T(\tau) \dot{q}_1(\tau) d\tau + \int_0^{t_N^2} \hat{u}_2^T(\tau) \dot{q}_2(\tau) d\tau \\
&= \sum_{k=0}^N \epsilon_d(k) + \epsilon_p(k) + \epsilon_b(k) \\
&\leq \sum_{i=1}^2 \sum_{k=0}^N \left[-\frac{\|\Delta q_i(k)\|_{B_i}^2}{T_k^i} + \left(\frac{\bar{N}_1 + \bar{N}_2}{2} + 1 \right) \|\Delta q_i(k)\|_K^2 \right. \\
&\quad \left. + \left(1 + \frac{T_{\max}}{T_{\min}} \right) \frac{\|\Delta q_i(k)\|_D^2}{T_k^i} \right] - \phi(N+1) + \phi(0) \\
&\leq \phi(0) =: d^2
\end{aligned} \tag{3.27}$$

implying the controller passivity (3.8), hence by [LL05], the closed-loop passivity (3.5).

2) With the controller passivity (3.27), the extra damping condition (3.40), and the properties of robot open-loop dynamics (e.g., (3.3a) and (3.3b)), following [LH10a, Th.1], it can be shown that, with $f_i(t) \equiv 0$, $\dot{q}_i(t) \rightarrow 0$ (i.e., the systems eventually stop). Then, the robot dynamics (3.1) with the DPDC control (3.9) are reduced to

$$M_i(q_i) \ddot{q}_i(t) \rightarrow -K[q_i(k) - q_j(k)]. \tag{3.28}$$

The next step is to show $q_i(t) \rightarrow q_j(t)$, which we will prove via contradiction following a similar argument as that in [LH10a, Th.1]. In (3.28), suppose $q_i(k)$ does not converge to $q_j(k - N_j^k)$, that is, for some $\epsilon > 0$, $\forall N_a \geq 0$, there exists $k_\epsilon \geq N_a$ s.t.

$$\|K[q_i(k_\epsilon) - q_j(k_\epsilon - N_j^{k_\epsilon})]\| > \epsilon \tag{3.29}$$

Since $\dot{q}_i(t) \rightarrow 0$ and C_i, B_i, D are bounded, there exists $N_b \geq 0$ s.t. $\forall k \geq N_b$ and $t \in T_k^i := [t_k^i, t_{k+1}^i)$,

$$\|C_i \dot{q}_i(t) + B_i \dot{q}_i(t) + D[v_i(k) - \delta_i^k v_j(k - N_j^k)]\| \leq \frac{\epsilon}{2} \quad (3.30)$$

with some arguments omitted for brevity. Hence, by choosing $N_a \geq N_b$, there exists $k_\epsilon \geq N_a$ which makes (3.64) and (3.65) hold at the same time. Let us denote $\rho(k_\epsilon) := -K[q_i(k_\epsilon) - q_j(k_\epsilon - N_j^{k_\epsilon})]$. Then, from (3.1) with (3.9), (3.64), and (3.65), we have: $\forall t \in [t_{k_\epsilon}^i, t_{k_\epsilon+1}^i)$

$$M_i(q_i) \ddot{q}_i(t) \in \left\{ y \in \mathbb{R}^n \mid \|y - \rho(k_\epsilon)\| \leq \frac{\epsilon}{2} \right\}$$

where $\rho(k_\epsilon)$ during $[t_{k_\epsilon}^i, t_{k_\epsilon+1}^i)$ is constant and outside of $B_\epsilon^c := \{y \in \mathbb{R}^n \mid \|y\| \leq \epsilon\}$ from (3.64). Thus, we have

$$\lambda_{\max}(M_i) \|\dot{q}_i(t_{k_\epsilon+1}) - \dot{q}_i(t_{k_\epsilon})\| \geq \left\| \int_{t_{k_\epsilon}^i}^{t_{k_\epsilon+1}^i} M_i(q_i) \ddot{q}_i(t) dt \right\| > \frac{\epsilon T}{2}$$

which contradicts to $\lim_{t \rightarrow \infty} \dot{q}_i(t) = 0$. Hence, $q_i(k) \rightarrow q_j(k - N_j^k)$, implying that $q_i(t) \rightarrow q_j(t)$.

3) With $\dot{q}_i(t) \rightarrow 0, \ddot{q}_i(t) \rightarrow 0$, (3.1) with (3.9) reduce to

$$f_i(t) \rightarrow K[q_i(k) - q_j(k - N_j^k)] \rightarrow K[q_i(k) - q_j(k)]$$

with $q_j(k) \rightarrow q_j(k - N_j^k)$. □

Note from Th. 2 that, by using the index delay N_i^k , under the passivity condition (3.11), the DPDC control (3.9) can enforce the passivity, while guaranteeing steady-state/static position and force coordination, even in the presence of the general communication unreliability of the Internet. This DPDC control can also address the multi-rate problem [FSC07] (i.e., device local sampling rate is faster than

communication rate), simply by using the packet sustainment (see Fig.3.2) when there is no data reception from the (slower) communication network. For simplicity, in Th. 2, we also assume $q_i(k) = 0 \forall k < 0$. This can be achieved in practice by suitable initialization/registration; or relaxed to the case of $q_i(k) \neq 0$ for $k < 0$ with the passivity bounds (e.g., d^2 in (3.27)) increased due to the non-zero initial condition.

The passivity condition for DPDC control (3.11) may also be written in a similar form as that for VPDC control (3.34) and (3.39) in Sec. 3.3 s.t.:

$$B_i \succeq \left[\frac{\bar{N}_1 + \bar{N}_2}{2} \right] T_{\max} K + \frac{1}{2} \left[\frac{T_{\max}}{T_{\min}} - 1 \right] D + \frac{1}{2} \left[\frac{T_{\max}}{T_{\min}} + 3 \right] D + T_{\max} K$$

where the first line is to passify the Internet's communication unreliability, while the second line the sampling effect. Note that, if $T := T_k^i \rightarrow 0$ (with $N_i^k \rightarrow \infty$), the first line above becomes the well-known passivity condition of the constant delay teleoperation [LS06b, NBOS09] (with $\bar{N}_i^k T$ approximating communication delay), while the second line will possess a form similar (i.e., twice) to the well-known Colgate's passivity condition for haptics [CS94, Lee09].

As shown in (3.11), the DPDC control (3.9) relies solely on the device physical damping B_i to passify both the communication unreliability and the sampling effect. This device damping B_i is, yet, often small for high-performance haptic device and not tunable. Thus, if the Internet's communication unreliability is severe (i.e., large $\bar{N}_i$ in (3.9)), this device damping B_i may be too small to produce desired K, D gains for the DPDC control (3.9) or, conversely, given a small level of B_i , we may need to reduce K, D too much for the direct PD-coupling in the DPDC control (3.9) to be useful (see Sec. 3.4). To overcome this issue of DPDC when the communication unreliability is severe, we present a virtual-proxy based PD (VPDC) control in the next Sec. 3.3. Of course, if the communication is reasonably good (as true for many practical situations), the DPDC control (3.9) would be a better solution than VPDC due to its simple structure.

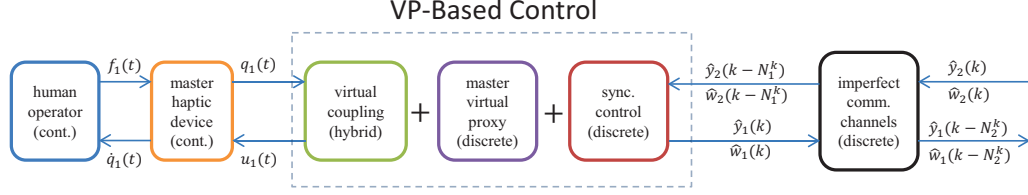


Figure 3.4: VPDC control framework over the Internet (only the master side is shown due to the symmetry).

3.3 Virtual-Proxy Based PD Control (VPDC)

The main goal of VPDC framework is to provide flexibility to adopt user-specific virtual damping, when the physical device damping is insufficient to passify communication unreliability. This is done by inserting a passive VP between the robot and the PD control over the communication. The VP’s discrete-time virtual damping dissipates undesired energy produced by communication unreliability. Hence, the device damping is only required for passifying the sampling effect, which is small due to the fairly fast update rate in practical teleoperation systems. The structure of the teleoperation system using VPDC control, as shown in Fig. 3.4, consists of continuous-time master/slave robots, VP, hybrid virtual coupling (VC) between device and VP, and discrete-time PD coupling between VPs over the communication. Each component will be explained in this section.

3.3.1 Passive Simulation of Virtual Proxies (VPs)

In VPDC framework, the two-port open-loop passivity of VP simulation is desired in order to achieve closed-loop passivity. Moreover, the VP simulation is required to be haptically-fast to provide high-fidelity haptic feedback. In fact, this haptically-fast and passive mechanical simulation has been an open problem since the seminar work [BC98]. For this, we utilize our recently-proposed non-iterative passive mechanical integrator (NPMI) [LH08a], which then suggests the following integration step for the

VP dynamics:

$$\begin{aligned} H_i \frac{w_i(k+1) - w_i(k)}{T_k^i} &= \tau_i^c(k) + \tau_i^s(k) \\ \hat{w}_i(k) &\triangleq \frac{w_i(k+1) + w_i(k)}{2} = \frac{y_i(k+1) - y_i(k)}{T_k^i} \end{aligned} \quad (3.31)$$

$H_i \in \Re^{n \times n}$ is the symmetric positive-definite VP inertia matrix; $y_i(k), w_i(k) \in \Re^n$ are the VP virtual position and velocity respectively; $\tau_i^c(k) \in \Re^n$ is the virtual coupling force acting on the VP; and $\tau_i^s(k) \in \Re^n$ will embed PD-like synchronization control between the master and slave VPs over the communication network. Furthermore, the discrete-time VP dynamics (3.31) is implicit, i.e. the update depends on the future state. But, it can be quickly updated without iterations by solving a linear system derived from (3.31), (3.33) and (3.37). Following [LH08a], we can show that the VP possesses the following discrete-time open-loop passivity: $\forall N > 0, \exists \bar{c}_i \in \Re$, s.t.,

$$\sum_{k=0}^{N-1} \left(\tau_i^c(k) + \tau_i^s(k) \right)^T \hat{w}_i(k) T_k^i \geq -\bar{c}_i^2, \quad (3.32)$$

which is essential for achieving closed-loop passivity. For more details of the NPMI and discrete-time passivity, please see [LH08a].

3.3.2 Passive PD-Coupling $\tau_i^s(k)$ between VPs over the Communication Network

To connect the discrete-time master and slave VPs over the unreliable packet-switched network, we use the following PD-like coupling: during $[t_k, t_{k+1})$,

$$\tau_i^s(k) = -B_i^p \hat{w}_i(k) - D \left(\hat{w}_i(k) - \delta_i^k \hat{w}_j(k - N_i^k) \right) - K \left(\hat{y}_i(k) - \hat{y}_j(k - N_i^k) \right) \quad (3.33)$$

where $(i, j) \in \{(1, 2), (2, 1)\}$, $K, D \in \mathbb{R}^{n \times n}$ are respectively symmetric positive-definite P and positive semi-definite D gains; $B_i^p \in \mathbb{R}^{n \times n}$ is the positive definite local discrete-time virtual damping; $\hat{y}_i(k) \triangleq (y_i(k) + y_i(k+1))/2$ with $\hat{w}_i(k)$ also defined in (3.31); and δ_i^k is the duplication avoidance function defined in (3.10).

Now we provide the gain setting condition for assuring the discrete-time two-port (controller) passivity of this PD-coupling over the Internet and the position coordination between master/slave VPs.

Proposition 3. *Consider the PD-like coupling (3.33) over the Internet. Suppose we set B_i^p, K, D , s.t.,*

$$B_i^p \succeq \left[\frac{\bar{N}_1 + \bar{N}_2}{2} \right] T_{\max} K + \frac{1}{2} \left[\frac{T_{\max}}{T_{\min}} - 1 \right] D \quad (3.34)$$

for $i = 1, 2$, where $\bar{N}_i := \max_k(N_i^k)$, $T_{\max} := \max_k(T_k^i)$, $T_{\min} := \min_k(T_k^i)$, and $w_i(k) = 0$, $\forall k \leq 0$. Then, the PD-like coupling (3.33) possesses the following (controller) passivity [LL05]: $\forall N > 0$, $\exists \bar{d} \in \mathbb{R}$, s.t.,

$$\sum_{k=0}^{N-1} \left[\tau_1^s(k)^T \hat{w}_1(k) + \tau_2^s(k)^T \hat{w}_2(k) \right] T_k^i \leq \bar{d}^2. \quad (3.35)$$

Further suppose $\hat{w}_i(k) \rightarrow \mathbf{0}$ and $\tau_i^c \rightarrow \mathbf{0}$, the position coordination between VPs s.t.

$$y_i(k) \rightarrow y_j(k) \quad (3.36)$$

is achieved.

The proof of Prop.3, due to its similarity to the proof of Th.1 in [HL10], will be presented in a brief manner in Sec.3.7.3.

3.3.3 Passive Virtual Coupling between VP and Haptic Device

Virtual coupling (VC), which is essentially a hybrid spring-damper connection, is widely used in haptics. In VPDC, we utilize the VC to connect the continuous-time haptic device and the discrete-time VP dynamics. It is well known that the VC is not passive due to the sample-and-hold effect. In [CS94], the passivity criterion $b > KT/2 + B$ (b is device damping; T is sampling rate; K, B are virtual spring and damper constants respectively) was provided. This condition was further extended to variable-rate haptics in [Lee09]. In Prop. 4, we derive a slightly different passivity condition to [Lee09] since the VC algorithm has to be modified for connecting the implicit VP update law.

Consider the following VC law: during $[t_k, t_{k+1})$,

$$\begin{aligned} u_i(t) &= -B_i^c \left(v_i(k) - \hat{w}_i(k-1) \right) - K_i^c \left(q_i(k) - y_i(k) \right) \\ \tau_i^c(k) &= -b_i^p \hat{w}_i(k) - B_i^c \left(\hat{w}_i(k) - v_i(k) \right) - K_i^c \left(y_i(k+1) - q_i(k) \right) \end{aligned} \tag{3.37}$$

where $i = 1, 2$; $q_i(k), w_i(k)$ follow the definition in (3.1) and (3.9); $b_i^p \in \Re^{n \times n}$ is the positive semi-definite discrete-time virtual damping matrix; $B_i^c \in \Re^{n \times n}$ is the positive semi-definite VC damping matrix; and $K_i^c \in \Re^{n \times n}$ is symmetric positive-definite VC spring matrix.

The main difference to VC in [Lee09], is the future information $\hat{w}_i(k)$ and $y_i(k+1)$ are included in $\tau_i^c(k)$, whose implementation requires solving the closed-form linear system obtained from (3.31), (3.33) and (3.37).

The passivity condition is summarized in the following proposition.

Proposition 4. *The VC (3.37) is two-port hybrid passive, s.t. $\forall N > 0, \exists d_i \in \mathfrak{R}$,*

$$\int_0^{t_N^i} \hat{u}_i(t)^T \dot{q}_i(t) dt + \sum_{k=0}^{N-1} \tau_i^c(k)^T \hat{w}_i(k) T_k^i \leq d_i^2, \quad (3.38)$$

if the following gain setting condition holds,

$$B_i \succeq B_i^c \left[1 + \frac{T_k^i}{T_{k-1}^i} \right] + K_i^c T_k^i, \quad b_i^p \succeq \frac{B_i^c}{2} \left[\frac{T_k^i}{T_{k-1}^i} - 1 \right] \quad (3.39)$$

where $\hat{u}_i(t)$ is defined in (3.2).

The essential idea of the proof is similar to the one in [Lee09]. The complete proof can be found in Sec.3.7.4.

The VC passivity condition (3.39) indicates three meaningful points. First, enough (i.e. determined by (3.39)) device local damping B_i can passify the excessive energy produced by VC's sampling and ZOH effects. This had been revealed in [CS94, Lee09]. Second, at the VP side, the virtual damping b_i^p is only responsible for passifying the update rate variation, which is ignorable in most real applications. To our best knowledge, this has not been revealed in any other papers. Third, the device damping determined by (3.39) is double as much as the condition in [Lee09]. We believe this increase is because we utilize implicit NPMI to achieve the VP's passivity while it was simply assumed to be passive in [Lee09].

3.3.4 Main Result

Theorem 3. *Consider the bilateral teleoperator in Fig.3.4 with master and slave devices (3.1), virtual coupling (3.37), passive VPs simulation (3.31) and the PD-type coupling over the Internet (3.33). We further suppose the gains are chosen according to the gain setting conditions (3.34) and (3.39). Also suppose $q_i(k), y_i(k) = 0, \forall k < 0$. Then,*

- *the closed-loop passivity (3.5) is achieved.*

- further if external force $f_i = 0, \forall t \geq 0$ and extra local damping is available s.t., for $k \geq 0$

$$B_i \succ B_i^c \left[1 + \frac{T_k^i}{T_{k-1}^i} \right] + K_i^c T_k^i. \quad (3.40)$$

The position coordination between master and slave robots (3.6) is achieved.

- if $(\dot{q}_i(t), \ddot{q}_i(t), w_i(k)) \rightarrow \mathbf{0}, i = 1, 2$, and $T_{\min} > 0$, the force reflection (3.7) is achieved.

Proof. Since we have shown that VPs (3.31) are natively open-loop passive (3.32), the PD-like coupling (3.33) over the Internet is passive (3.35) under condition (3.34), and the hybrid VC (3.37) is passive (3.38) under condition (3.39), the proof for closed-loop passivity becomes straightforward. Combing (3.32), (3.35), (3.38), and (3.4) yields $\forall \bar{t} \geq 0$,

$$\int_0^{t_N^1} f_1^T(t) \dot{q}_1(t) dt + \int_0^{t_N^2} f_2^T(t) \dot{q}_2(t) dt \geq -(c^2 + d^2 + \bar{c}^2 + \bar{d}^2)$$

where $c^2 := c_1^2 + c_2^2$, $d^2 = d_1^2 + d_2^2$, $\bar{c}^2 := \bar{c}_1^2 + \bar{c}_2^2$, $\bar{d}^2$ is from (3.35). This proves the desired closed-loop passivity (3.5).

For the position coordination, if we set $\tilde{B}_i = B_i^c [1 + T_k^i / T_{k-1}^i] + K_i^c T_k^i$, then following very similar procedure as in the proof of Thm. 2, we can show that,

$$\dot{q}_i(t) \rightarrow \mathbf{0}, \quad q_i(k) \rightarrow y_i(k), \quad i = 1, 2$$

as k large. Hence, $\hat{w}_i(k) \rightarrow \mathbf{0}$, which further indicates $\tau_i^c(k) \rightarrow \mathbf{0}$. Then, following Prop. 3, we have $y_i(k) \rightarrow y_j(k)$, and it is straightforward to conclude $q_i(t) \rightarrow q_j(t)$.

We now show the force reflection (3.7). By the assumptions $(\dot{q}_i, \ddot{q}_i) \rightarrow \mathbf{0}$, (3.1) can be rewritten as $f_i(t) \rightarrow -u_i(k)$. Moreover, since we assume $w_i(k) \rightarrow \mathbf{0}$, $u_i(t) \rightarrow -\tau_i^c(k)$ can be derived from (3.37). By (3.31), $w_i(k) \rightarrow \mathbf{0}$, and $T_{\min} > 0$, we have $\tau_i^c(k) \rightarrow$

$-\tau_i^s(k)$. Then, from (3.33) and $w_i(k) \rightarrow \mathbf{0}$, we have $\tau_i^s(k) \rightarrow u_j^p(k)$. Chaining these equalities together, we finally conclude that $f_i(t) \rightarrow -f_j(t)$. $\square$

From Th. 3, it is clear that all the control objectives listed in Sec. 3.1.2 including closed-loop passivity (3.5), position coordination (3.6), and force reflection (3.7) are achieved. Actually, with the closed-loop passivity enforced at all time, position coordination and force reflection are simply the outcomes of the PD-type control. Moreover, same packet sustainment mechanism in DPDC is utilized by VPDC. Hence, the VPDC equally addresses the multi-rate problem as DPDC. Furthermore, we assume $q_i(k) = 0, \forall k < 0$ which can be achieved by suitable initialization/registration or relaxed to the case of increased passivity bounds d^2 due to the non-zero initial condition, which is the same argument given in Sec. 3.2.

Another interesting point is that the passivity condition is separated into two conditions (3.34) and (3.39) because of the insertion of NPMI-powered VP. Hence, from the perspective of control design, the communication unreliability does not affect VC and the haptic devices. Then, the device damping is only required to passify VC's sampling and ZOH effects which is usually very small due to the high update rate provided by modern digital controllers. On the other hand, the VP's virtual damping B_i^p can be set arbitrarily without compromising passivity. Then, even with severe communication unreliability, the closed-loop passivity and all other control objectives are still guaranteed because (3.34) can always be enforced. This is impossible for DPDC (see its passivity condition (3.11)). Actually, this is a novelty of our work since the unstable virtual damping problem has bothered some discrete-time teleoperation frameworks relying on virtual damping to passify the system, e.g. PO/PC [RP07] and two-layer method [FSM⁺11].

In practice, the inertia of NPMI-powered VP can be chosen arbitrarily small without breaking the passivity, i.e. free from the well-known minimum mass limitation [BC98]. So, the intermediate dynamics introduced by VP and its effects on the performance can be almost ignored.

Like DPDC, VPDC is a time-invariant control technique, and the common problem with this kind of passivity-based time-invariant control frameworks (e.g. [LS06b, NBOS09]) is conservativeness. This is because the control gains are chosen according to the ‘worst’ communication condition. This conservativeness may be substantially reduced by using (time-variant) PSPM [LH10a], which only selectively activates the passifying action when necessary. Need to mention that, VPDC is also possible to be extended as a time-variant framework since the ‘active’ energy produced by communication unreliability can be tracked (in theory) in time domain. Following this argument, we may be able to adjust the control gains according to the communication unreliability in real-time to relax the conservativeness on control parameters.

3.4 Experiments

The purpose of the experiments is two-fold. The first goal is to validate Thm. 2 and 3, and the second is to show that the performance of VPDC (i.e. position coordination, transparency) is similar to DPDC’s when delay is slight, but VPDC provides much better performance than DPDC’s when time delay is long. So, the experiments are conducted for the Internet with short and long delays respectively.

For each scenario, a standard hard contact teleoperation task is performed. A stiff wall is set in the slave side. At the beginning, the human operator does not hold the master device and the whole system is in steady state. Then, the master robot is moved by the operator towards the wall (located in slave side). After the operator perceives the wall, he/she stays still for a while and then retracts to the origin. Once the master robot passes the origin, the master device is released and another position coordination between the master and slave robots is performed.

For the robots, a Sensable® Phantom Omni is used as the master robot and a Phantom Desktop is utilized as the slave robot. Their update rate is set to be $T = 1ms$. Phantom Omni and Desktop are equipped with three actuated

revolute joints which determine the Cartesian coordinate of the end-effector, and there are three other unactuated revolute joints which determine the orientation of the end-effector. In our experiments, we only focus on the three actuated joints. The associated device damping of the three actuated joints are estimated by well-known Colgate's condition [CS94]. For our devices, the damping are approximately (65, 40, 55)mNm·s/rad (Omni), and (26, 55, 8)mNm·s/rad (Desktop), respectively.

3.4.1 Exp. 1: Short Delay ($\text{RTT} \leq 0.1s$)

For this case, we assume the Internet communication channels with the time delay varying from 0.01s to 0.05s (master→slave), and 0.02s to 0.04s (slave→master). There are 5% packet-loss and 1% duplication. The round-trip delay upper bound is $(\bar{N}_1 + \bar{N}_2) T \approx 0.1s$.

Substituting the upper bound of round-trip delay and device damping into passivity condition (3.12) we design the control gains for DPDC as

$$K = \text{diag}(430, 660, 110)\text{mNm/rad}, D = \text{diag}(2, 2, 1)\text{mNm·s/rad},$$

where $\text{diag}(a_1, \dots, a_n)$ represents a diagonal matrix whose diagonal entries starting in the upper left corner are $a_1, \dots, a_n$. For VPDC, following passivity condition (3.34) and (3.39), we choose the control parameters as follows:

1. VP inertia: $H_1 = H_2 = 0.001I_{3 \times 3}\text{kgm}^2$;
2. VC gains: $K_1^c = \text{diag}(5.5 \times 10^4, 3.5 \times 10^4, 4.4 \times 10^4)\text{mNm/rad}$, $B_1^c = \text{diag}(5, 2, 5)\text{mNm·s/rad}$, $K_2^c = \text{diag}(2.15 \times 10^4, 4.4 \times 10^4, 6.5 \times 10^3)\text{mNm/rad}$, $B_2^c = \text{diag}(2, 5, 0.5)\text{mNm·s/rad}$;
3. VP local virtual damping (for passifying VC): $b_1^p = b_2^p = 0\text{mNm·s/rad}$;
4. VP local virtual damping (for passifying PD coupling):
 $B_1^p = B_2^p = 100I_{3 \times 3}\text{mNm·s/rad}$;

5. PD coupling gains: $K = 2000I_{3 \times 3} \text{mNm/rad}$, $D = 10I_{3 \times 3} \text{mNm}\cdot\text{s/rad}$.

The experimental results are shown in Fig. 3.5 and 3.6. For clarity, we only show the position and force of the first joint of the device, which performs the most significant displacement over other two joints due to the horizontal motion of the operator.

3.4.2 Exp. 2: Long Delay ($\text{RTT} \leq 1\text{s}$)

The Internet communication channels are assumed with time delay varying from 0.3s to 0.45s (master→slave), and 0.35s to 0.5s (slave→master). 5% packet loss and 1% duplication are also included. Then, the round-trip is upper bounded by $(\bar{N}_1 + \bar{N}_2) T \approx 1\text{s}$.

Similar to previous case, we design the control parameters according to the passivity conditions. For DPDC, we have $K = \text{diag}(50.85, 77.84, 14.97) \text{mNm/rad}$, $D = \text{diag}(2, 2, 1) \text{mNm}\cdot\text{s/rad}$. For VPDC, we choose same VP mass M_i^p , and VC parameters K_i^c, B_i^c, b_i^p as the slight delay case. The corresponding PD coupling gains are chosen as $K = 2000I_{3 \times 3} \text{mNm/rad}$, $D = 10I_{3 \times 3} \text{mNm}\cdot\text{s/rad}$, $B_1^p = B_2^p = 1000I_{3 \times 3} \text{mNm}\cdot\text{s/rad}$. The results are given in Fig. 3.7 and 3.8.

3.4.3 Discussion

As shown in Fig. 3.5-3.8, we introduce the passivity measurement $\Psi(k), k \geq 0$ to validate that closed-loop passivity of the teleoperator. This is because Phantom Desktop and Omni are not equipped with force sensor, hence the energy exchange between the teleoperator and the human/environment cannot be captured. The idea of passivity measurement is stated in the following lemma.

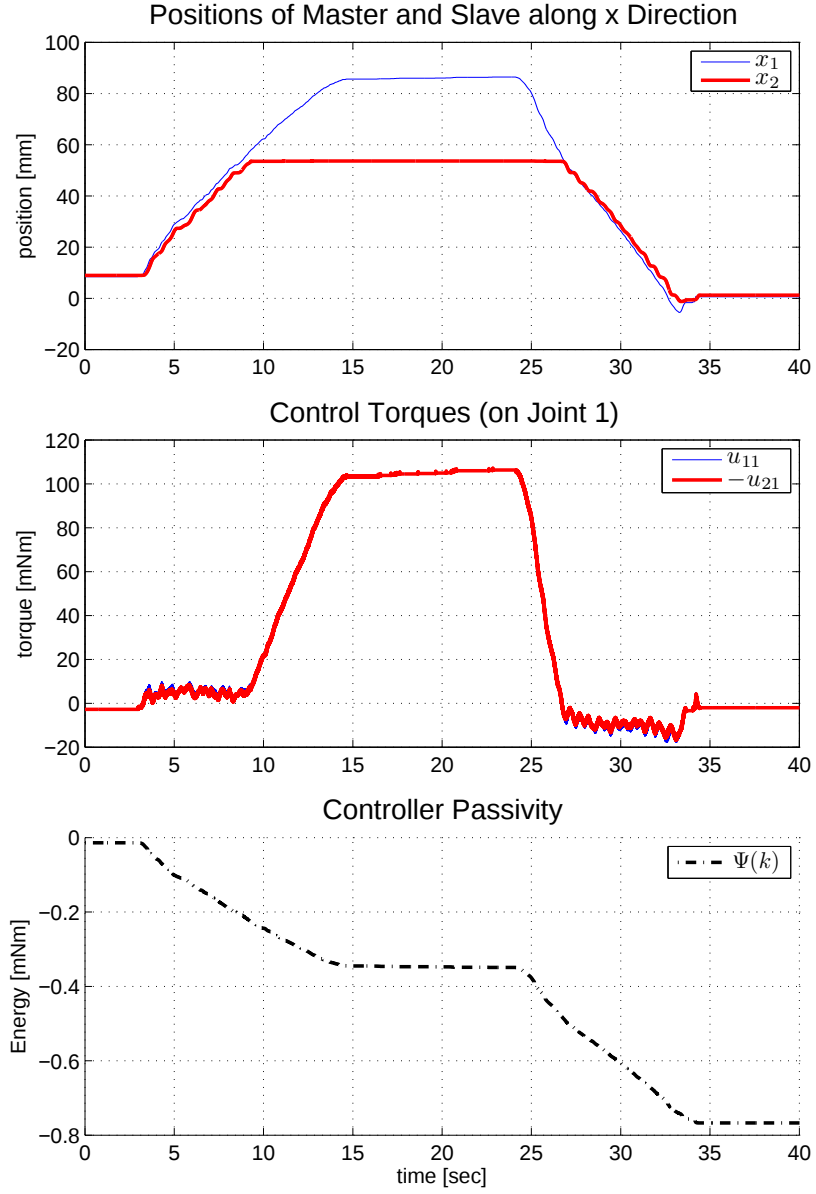


Figure 3.5: DPDC with $\text{RTT} \leq 0.1\text{s}$, where $\Psi(k)$ is the passivity measurement defined in (3.41).

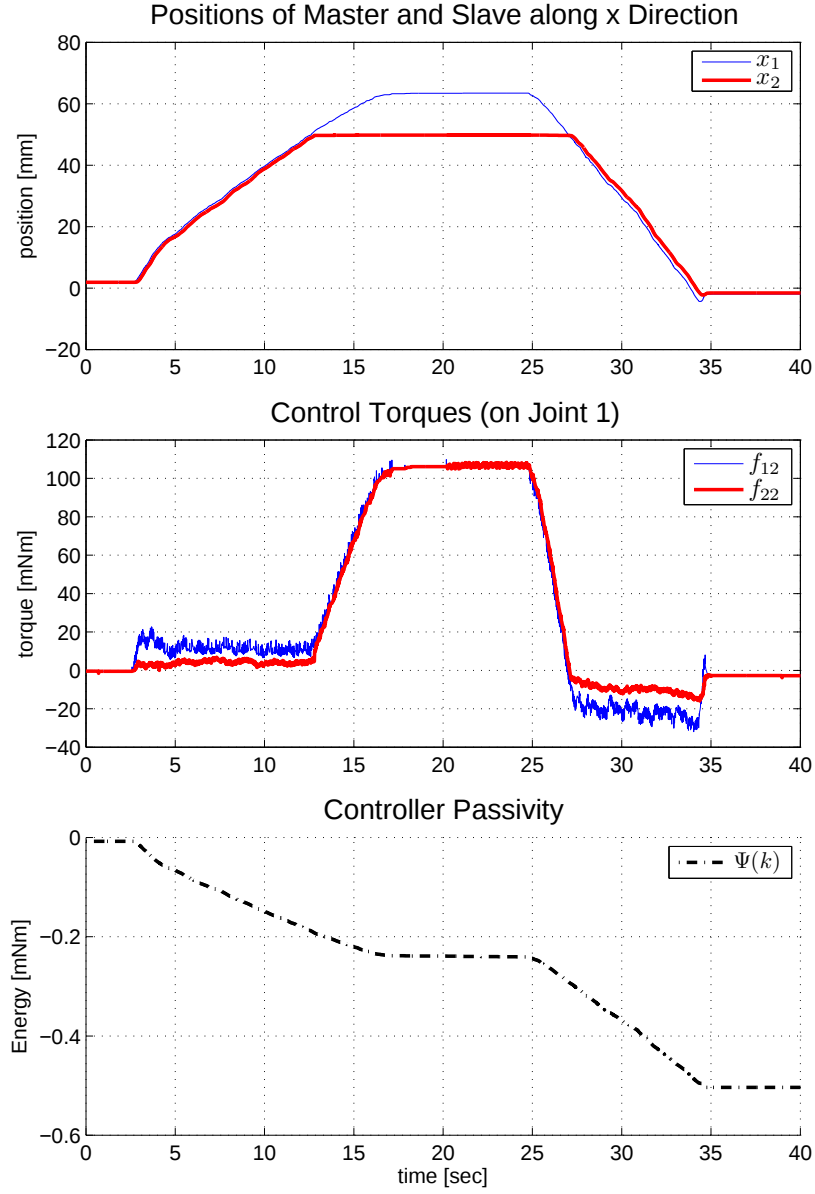


Figure 3.6: VPDC with $\text{RTT} \leq 0.1\text{s}$, where $\Psi(k)$ is the passivity measurement defined in (3.41).

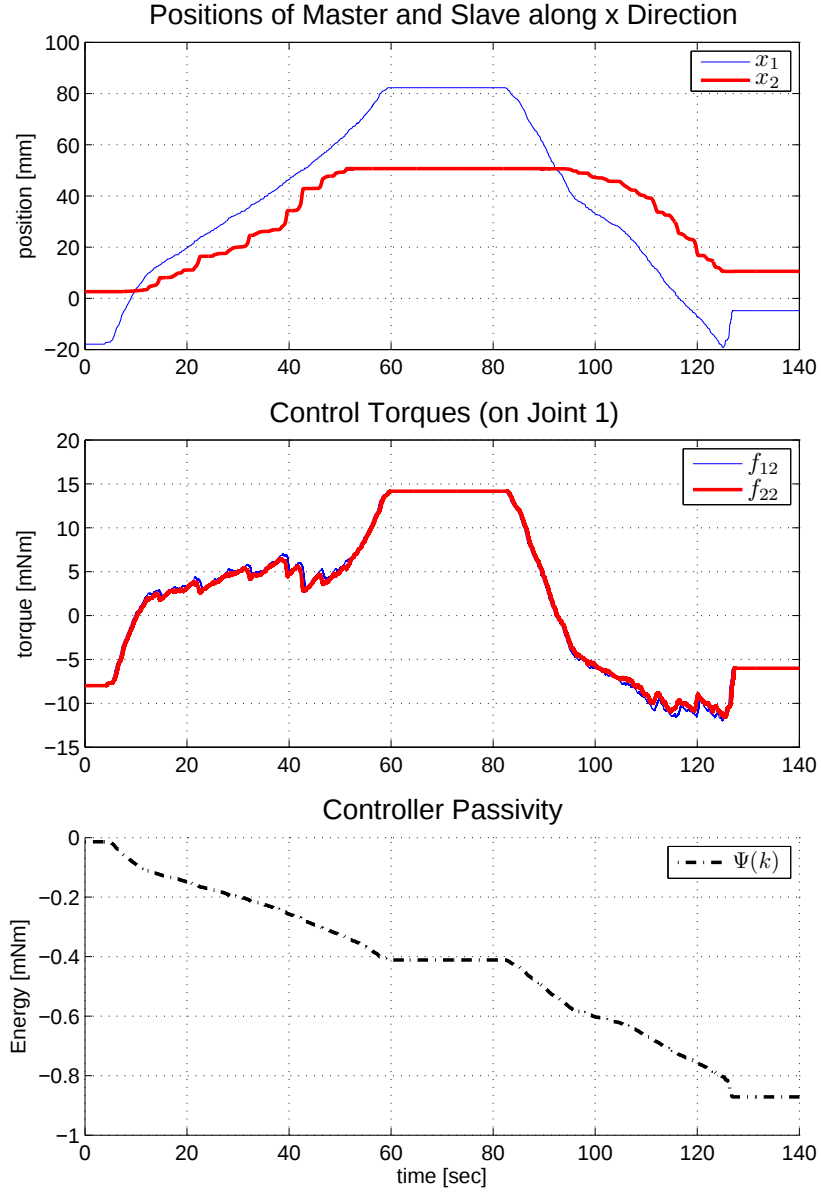


Figure 3.7: DPDC with $\text{RTT} \leq 1\text{s}$, where $\Psi(k)$ is the passivity measurement defined in (3.41).

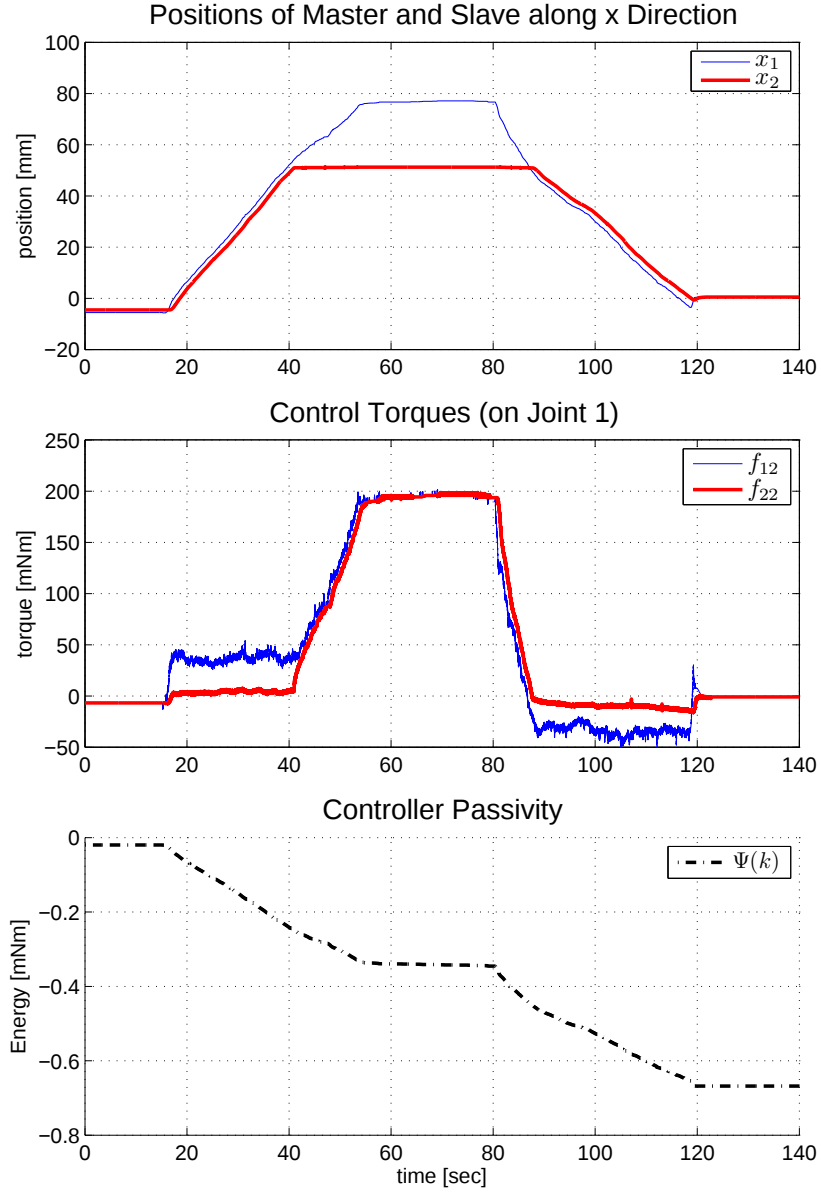


Figure 3.8: VPDC with $\text{RTT} \leq 1\text{s}$, where $\Psi(k)$ is the passivity measurement defined in (3.41).

Lemma 2. *The teleoperation system (3.1) is closed-loop passive if the following condition holds, for any $N \geq 0$*

$$\Psi(N) := \sum_{k=0}^N \sum_{i=1}^2 \Delta q_i(k)^T u_i(k) - \frac{1}{T_k} \|\Delta q_i(k)\|_{B_i}^2 \leq c^2 \quad (3.41)$$

where c is a constant.

Proof. Following [LL05], we only need to show the controller passivity (3.8) holds.

$$\begin{aligned} \int_0^t \sum_{i=1}^2 \dot{q}_i^T(\tau) \hat{u}_i(\tau) d\tau &= \\ &= \sum_{k=0}^{\bar{M}} \sum_{i=1}^2 \Delta q_i^T(k) u_i(k) - \int_0^t \sum_{i=1}^2 \|\dot{q}_i(\tau)\|_{B_i}^2 d\tau \\ &\leq \sum_{k=0}^{\bar{M}} \sum_{i=1}^2 \Delta q_i^T(k) u_i(k) - \frac{1}{T_k} \|\Delta q_i(k)\|_{B_i}^2 \leq c^2 \end{aligned}$$

where the inequality is due to (3.15). $\square$

As shown in Fig. 3.5-3.8, all experiments demonstrate the passivity measurement $\Psi(k) \leq 0$ which means the teleoperation system is closed-loop passive according to Lem. 2. This is mainly due to the passivity conditions ((3.11) for DPDC, (3.34) and (3.39) for VPDC) are enforced.

From Fig. 3.5 and 3.6, we can see that DPDC and VPDC have comparable performance when the delay is slight. First, for both control frameworks, the human operator can perceive the wall from the substantial torque change from moving in free space (3.5 – 9.5s in Fig. 3.5 with peak torque feedback around 15.8mNm, and 2.7 – 12.5s in Fig. 3.6 with peak torque feedback around 30mNm) to steadily contacting with the wall (14.5–24.2s in Fig. 3.5 with about 105mNm torque feedback, and 17.1 – 24.8s in Fig. 3.6 with about 106mNm torque feedback). Second, VPDC shows slightly better performance in some aspects. For example: 1) better position coordination (e.g. less position tracking error during free motion); 2) more stiff wall is displayed

by VPDC (to achieve similar contact force, the penetration is 13.57mm for VPDC but 32.36mm for DPDC). These are mainly due to the strong $K = 2000I_{3 \times 3}\text{mNm/rad}$ for VPDC which cannot be achieved by DPDC because of the limited device damping.

As shown in Fig. 3.7 and 3.8, the VPDC yields much better performance than DPDC when delay is long. In Fig. 3.7, there is substantial steady-state position error between master/slave position (e.g. 127 – 140s). This is mainly because that the upper bound of coordination gains K is severely restricted by device damping and long time delay. In our case, the small allowable K (less than 80mNm/rad) cannot generate enough control torque to conquer the device Coulomb friction. Therefore, large steady-state error appears. The small K also significantly affects the transient position tracking performance of DPDC during the free motion (e.g. large position tracking error during 10 – 60s and 95 – 125s as shown in Fig. 3.7). Moreover, the VPDC lets the human perceive much stiffer wall (200mNm comparing to 13.6mNm with similar penetration on the master side).

3.5 Performance Comparison between DPDC and VPDC

The major difference between DPDC and VPDC frameworks is the insertion of VP dynamics. The VP, regardless its unique passive property, is essentially an n-DOF virtual mass. Intuitively, this intermediate dynamics would degrade the transparency of the controller. But, there is a hidden assumption associated with this argument, i.e. similar control gains can be chosen for DPDC and VPDC simultaneously. However, this assumption does not hold especially when considering the difference between passivity conditions (3.11) (DPDC) and (3.34), (3.39) (VPDC). Moreover, both passivity conditions depend on the communication unreliability and device damping. Therefore, for the sake of fairness, we will compare these two control frameworks under the same device setting and communication profile, and discuss the effects of

VP intermediate dynamics. For simplicity, in this section, we only consider 1-DOF linear master/slave robots, and the only communication unreliability is asynchronous constant time delays.

Transparency is the most important—if not the only—performance measure for bilateral teleoperation system. [Law93] first proposed this idea. Essentially, the concept of transparency reflects the relation between the environmental impedance Z_e and the transmitted impedance Z_t . If the teleoperation system can make the transmitted impedance Z_t close to the environmental impedance Z_e , we say the performance of this teleoperator is good. However, the original transparency definition is hard to be used as a quantitative indicator for the teleoperator's performance. [KC07] extended the transparency concept to a quantitative performance measure in which the trackability and immersivity are included. In the following we will conduct the comparison based on the performance measure [KC07]. Moreover, since the concept of transparency was only defined for continuous-time system, all the discussion in this section will be within continuous-time domain if there is no further explanation.

3.5.1 Transparency for DPDC

Consider the following linear mass-type master/slave robot, for $i = 1, 2$

$$m_i \ddot{q}_i(t) + b_i \dot{q}_i(t) = -K (q_i(t) - q_j(t - \tau_i)) - B (\dot{q}_i(t) - \dot{q}_j(t - \tau_i)) + f_i(t) \quad (3.42)$$

where the notations follow (3.9). The *hybrid matrix* formulation [Han89, RVS89, YY94] for this LTI system can be represented as

$$\begin{bmatrix} F_1(s) \\ V_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} V_1(s) \\ -F_2(s) \end{bmatrix} \quad (3.43)$$

where $V_i(s), F_i(s)$ are the Laplace transformation of $\dot{q}_i(t), f_i(t)$ respectively. The environmental and transmitted impedances are further defined as

$$Z_e(s) := -\frac{F_2}{V_2}(s) \quad (3.44)$$

$$Z_t(s) := \frac{F_1}{V_1}(s) \quad (3.45)$$

For a bilateral teleoperation system, the ideal transparency condition is originally defined in [Law93] as

$$Z_e(s) \equiv Z_t(s). \quad (3.46)$$

This condition means the human operator can perceive the actual dynamics of the remote environment. However there is no guarantee of position coordination between the master and slave robots neither force reflection. Hence, a stronger condition for ideal transparency is defined as,

$$\begin{aligned} H_{11}(s) &= H_{22}(s) = 0 \\ H_{12}(s) &= H_{21}(s) = 1 \end{aligned} \quad (3.47)$$

For the DPDC model (3.42), we can easily get the analytical form of hybrid matrix, s.t.

$$\begin{aligned} H_{11} &= (P_1 + G) - e^{-s(\tau_1 + \tau_2)}(P_2 + G)^{-1}G^2, \\ H_{12} &= e^{-s\tau_1}(P_2 + G)^{-1}G, \\ H_{21} &= e^{-s\tau_2}(P_2 + G)^{-1}G, \\ H_{22} &= -(P_2 + G)^{-1} \end{aligned} \quad (3.48)$$

where

$$P_i := m_i s + b_i, \quad G := B + \frac{K}{s}$$

It is clear that there are several factors preventing the DPDC achieving the ideal transparency (3.47):

1. *Time delay*: By observing H_{11}, H_{12}, H_{21} , it is clear that it is impossible to achieve 1 with nonzero time delay at nonzero frequency range. Since no existing delay compensation mechanism can perfectly cancel the time delay if there is unknown external input, it indicates that time delay is the fundamental limitation of teleoperation performance.
2. *Robot's dynamics*: Assuming there is no time delay, i.e. $\tau_i = 0, i = 1, 2$, to make H_{12} and H_{21} close to 1, the robot's inertia and damping should be close to 0. Also H_{11}, H_{22} are close to 0 if the robot's inertia and damping are close to 0. The effect of inertia is especially significant in high frequency range, and the effect of device damping is constant for all frequency range.
3. *PD-Controller*: It is clearly reflected in H_{11}, H_{12}, H_{21} that the transparency degrades significantly as frequency increases even without time delay. Large control gains B, K would improve the transparency but may violate the stability condition or make the system less robust to disturbance and uncertainties.

3.5.2 Transparency for VPDC

To incorporate the transparency analysis, the system model for VPDC is simplified as with linear 1-DOF master/slave robots, continuous-time VP update and synchronization control. The constant time delay is the only communication unreliability. The system model (including the robot, VP, VC, synchronization control) is given as follows,

$$\begin{aligned} \left(m_i s + b_i + B_c + \frac{K_c}{s}\right) V_i &= \left(B_c + \frac{K_c}{s}\right) W_i + F_i \\ \left(m_p s + b_p + B_c + B + \frac{K_c + K}{s}\right) W_i &= \left(B_c + \frac{K_c}{s}\right) V_i + \left(B + \frac{K}{s}\right) e^{-s\tau_i} W_j \end{aligned} \quad (3.49)$$

where m_p, b_p are the VP mass and damping respectively, and W_i is the Laplace transformation of i^{th} VP's velocity. Moreover, $K_c, B_c \geq 0$ are the VC gains, and $K, B \geq 0$ are the synchronization control gains. After some algebraic manipulations, we can have the analytical presentation of the hybrid matrix H , s.t.

$$\begin{aligned}
H_{11} &= \frac{(G^2 \bar{P}_1 \bar{P}_2 - e^{s(\tau_1 + \tau_2)} (G_c^2 - \bar{P}_1 \bar{P}_p) (G_c^2 - \bar{P}_2 \bar{P}_p))}{G^2 \bar{P}_2 + e^{s(\tau_1 + \tau_2)} \bar{P}_p (G_c^2 - \bar{P}_2 \bar{P}_p)}, \\
H_{12} &= -\frac{e^{s\tau_2} G G_c^2}{G^2 \bar{P}_2 + e^{s(\tau_1 + \tau_2)} \bar{P}_p (G_c^2 - \bar{P}_2 \bar{P}_p)}, \\
H_{21} &= -\frac{e^{s\tau_1} G G_c^2}{G^2 \bar{P}_2 + e^{s(\tau_1 + \tau_2)} \bar{P}_p (G_c^2 - \bar{P}_2 \bar{P}_p)}, \\
H_{22} &= -\frac{(G^2 - e^{s(\tau_1 + \tau_2)} \bar{P}_p^2)}{G^2 \bar{P}_2 + e^{s(\tau_1 + \tau_2)} \bar{P}_p (G_c^2 - \bar{P}_2 \bar{P}_p)}.
\end{aligned} \tag{3.50}$$

where

$$\begin{aligned}
P_i &:= m_i s + b_i, & G_c &:= B_c + \frac{K_c}{s}, \\
G &:= B + \frac{K}{s}, & P_p &:= m_p s + b_p, \\
\bar{P}_i &:= P_i + G_c, & \bar{P}_p &:= P_p + G_c + G.
\end{aligned}$$

Apparently, the hybrid matrix for VPDC, due to the insertion of VP dynamics, is more involved than the DPDC (3.48). But the reasons for preventing it achieving ideal transparency are still time delay, haptic devices' dynamics, and VC/PD controller. To be specific, if 1) there is no time delay; 2) haptic devices's dynamics are perfectly canceled; 3) control gains for VC and PD are infinitely large, the ideal transparency can be achieved. However, this sufficient condition is physically unrealistic.

3.5.3 Quantitative Performance Measure based on Transparency [KC07]

In this sub-section, we briefly introduce the quantitative performance measure proposed by Kim and Chang. For more details please refer to the article [KC07].

The two fundamental performance requirements for bilateral teleoperation system, which are aligned with our strong condition (3.47), are trackability and immersivity. The conditions for the trackability and immersivity are,

$$\text{Trackability:} \quad V_2(s) \equiv V_1(s) \quad (3.51)$$

$$\text{Immersivity:} \quad Z_t(s) \equiv Z_e(s) \quad (3.52)$$

Moreover, the trackability function is defined as,

$$G_T(s) := \frac{V_2(s)}{V_1^*(s)} = \frac{V_2(s)}{e^{-\alpha\tau_2s}V_1(s)} \quad (3.53)$$

and the immersivity function is similarly defined as,

$$G_I(s) := \frac{Z_t(s)}{Z_e^*(s)} = \frac{Z_t(s)}{e^{-\beta\tau_1s}Z_e(s)} \quad (3.54)$$

where α, β are integers that make G_T and G_I 's phase within the range of $-\pi$ to π for any time delay τ_i , i.e.

$$\begin{aligned} \alpha &\in \{a \in \mathbb{Z} | \angle G_T(j\omega) \in [-\pi, \pi), \forall \omega \geq 0, \forall \tau_2 \geq 0\} \\ \beta &\in \{b \in \mathbb{Z} | \angle G_I(j\omega) \in [-\pi, \pi), \forall \omega \geq 0, \forall \tau_1 \geq 0\}. \end{aligned}$$

α, β are called transmission delay constants of trackability and immersivity respectively. By substituting (3.43), (3.44) and (3.45) into (3.53) and (3.54) we have the

following practical formats of G_T and G_I , s.t.

$$G_T(s) = \frac{H_{21}}{e^{-\alpha\tau_2 s}(1 + H_{21}Z_e)} \quad (3.55)$$

$$G_I(s) = \frac{H_{11} + (H_{11}H_{22} - H_{12}H_{21})Z_e}{e^{-\beta\tau_1 s}Z_e(1 + H_{22}Z_e)}. \quad (3.56)$$

Trackability Index:

The trackability index is defined as follows,

$$Q_T := \frac{1}{u_{\max} - u_{\min}} \int_{u_{\min}}^{u_{\max}} \frac{\sqrt{A_T^2(\omega) - 2A_T(\omega) \cos(\theta_T(\omega)) + 1}}{A_T(\omega) + 1} du \quad (3.57)$$

where $A_T(\omega) := |G_T(j\omega)|$, $\theta_T(\omega) := \angle G_T(j\omega)$; $u_{\max} := \log_{10}(\omega_{\max})$, $u_{\min} := \log_{10}(\omega_{\min})$, $u := \log_{10}(\omega)$.

Immersivity Index:

The immersivity index is defined as follows,

$$Q_I := \frac{1}{u_{\max} - u_{\min}} \int_{u_{\min}}^{u_{\max}} \frac{\sqrt{\bar{A}_I^2(\omega) - 2\bar{A}_I(\omega) \cos(\theta_I(\omega)) + 1}}{\bar{A}_I(\omega) + 1} du \quad (3.58)$$

where $A_I(\omega) := |G_I(j\omega)|$, $\theta_I(\omega) := \angle G_I(j\omega)$; and

$$\bar{A}_I(\omega) := \begin{cases} 1, & \text{if } |A_I(\omega) - 1| < Z_{JND} \\ A_I(\omega), & \text{if } |A_I(\omega) - 1| \geq Z_{JND} \end{cases}$$

is the altered magnitude of immersivity distortion with consideration of just-noticeable difference (JND) which is related to human perception [BB96].

Essentially, the trackability and immersivity indexes are defined for specific environmental impedance Z_e . In teleoperation, contact with stiff wall is a standard environment for performance evaluation. In the following sub-section, we model the stiff wall as a mass-spring-damper system with small inertia, damper but very strong spring.

3.5.4 Comparison on Transparency between DPDC and VPDC

It has been shown in Sec. 3.4 that the tracking and force reflection performance between DPDC and VPDC are similar for small time delay but VPDC performs much better when the time delay is large. In this section, we adopt the LTI haptic device model due to the limitation of transparency definition, and conduct the comparison between DPDC and VPDC for both small time delay and large time delay. The aforementioned quantitative performance measure will be used as a simple index for comparing the performance (trackability and immersivity).

Let us consider a simple haptic device with one revolute joint [CNK09]. The inertia $m_i = 0.1148\text{kg} \cdot \text{m}^2, i = 1, 2$ and device local damping $b_i = 0.1912\text{N} \cdot \text{m}^2, i = 1, 2$. The update rate of digital controller $T = 1\text{ms}$.

Setting 1: Small communication time delay ($\tau_1 = \tau_2 = 10\text{ms}$)

Let us consider the communication channels with time delay such that $\tau_1 = \tau_2 = 10\text{ms}$. For the DPDC gain tuning, we first obtain the feasible gain region according to the passivity condition (3.11), and numerically solve for the following optimization problem,

$$\begin{aligned} \min_{K, B} \quad & aQ_T + bQ_I \\ \text{sbj.} \quad & \text{passivity condition (3.11)} \end{aligned} \tag{3.59}$$

where Q_T, Q_I are the trackability and immersivity index defined in (3.57) and (3.58) respectively; $a + b = 0.5, a, b \geq 0$ are the weight coefficients for trackability and immersivity respectively.

To evaluate the Q_T and Q_I and further solve the optimization problem (3.59), we choose $Z_e = 0.0087s + 7.4 + 1225/s, \alpha = 1, \beta = 0$ following [KC07]. The weights in (3.59) are set to be equal, i.e. $a = b = 0.25$. Then we solve (3.59) numerically.

The optimal control gains for DPDC are $K = 17.38\text{N}\cdot\text{m}/\text{rad}$, $B = 0\text{N}\cdot\text{m}\cdot\text{s}/\text{rad}$, and $Q_T = 0.9951$, $Q_I = 0.9803$.

To simplify the gain tuning procedure for VPDC, we need to first choose system parameters and partial control gains as $m_p = 0.0001\text{kg}\cdot\text{m}^2$, $K_c = 151.2\text{N}\cdot\text{m}/\text{rad}$, $B_c = 0.02\text{N}\cdot\text{m}\cdot\text{s}/\text{rad}$, and maximum VP damping is set to be $b_p = 2\text{N}\cdot\text{m}\cdot\text{s}/\text{rad}^*$. Then there are only two control gains K, B left for determination. A simplified control gain tuning optimization problem is defined for VPDC as,

$$\begin{aligned} \min_{K,B} \quad & aQ_T + bQ_I \\ \text{sbj.} \quad & \text{passivity condition (3.34)} \end{aligned} \tag{3.60}$$

Numerically solving (3.60) yields $K = 200\text{N}\cdot\text{m}/\text{rad}$, $B = 20\text{N}\cdot\text{m}\cdot\text{s}/\text{rad}$, and $Q_T = 0.9714$, $Q_I = 0.8926$.

By comparing the trackability and immersivity indexes, it is clear that under small delay case, the VPDC, due to its extended feasible region for control gains, yields better performance over the DPDC on the stiff wall contact task. Another intuitive yet less quantitative way of comparing transparency is plotting Bode plots of hybrid matrix (3.43). As shown in Fig. 3.9, from the Bode plot of H_{12} and H_{21} , VPDC stays much closer to 0dB than DPDC, and VPDC phase shift is closer to 0 than DPDC too. Hence, the tracking capability of VPDC is better than DPDC. Also, from Bode plot of H_{22} , it is clear that VPDC's magnitude is less than DPDC's which means the environmental force has less effects on the velocity tracking distortion. But, it is hard to draw conclusion from Bode plot of H_{11} because DPDC has better magnitude but worse phase shift than VPDC. This is the reason for choosing the quantitative performance measure.

Setting 2: Large communication time delay ($\tau_1 = \tau_2 = 1000\text{ms}$)

*Theoretically, there is no upper limit on the VP damping. But to avoid over sluggish motion in free space, we should choose a realistic value. In current stage, this is chosen through trail and error. We will investigate the theoretical way to determine the best value for this upper limit.

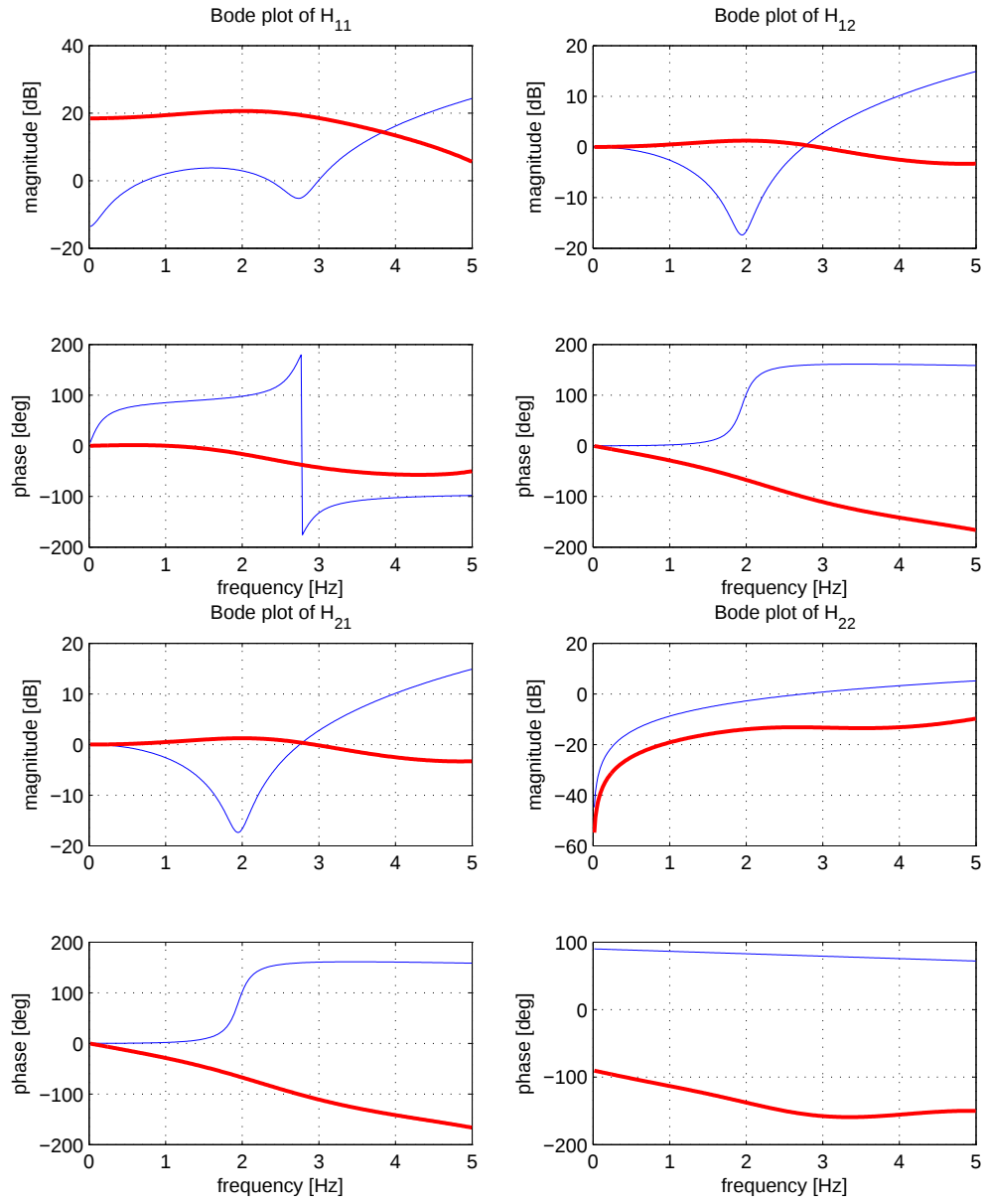


Figure 3.9: Bode plot of the hybrid matrices for DPDC and VPDC in 10ms time delay case. Blue thin line refers to the DPDC, and red thick line refers to the VPDC.

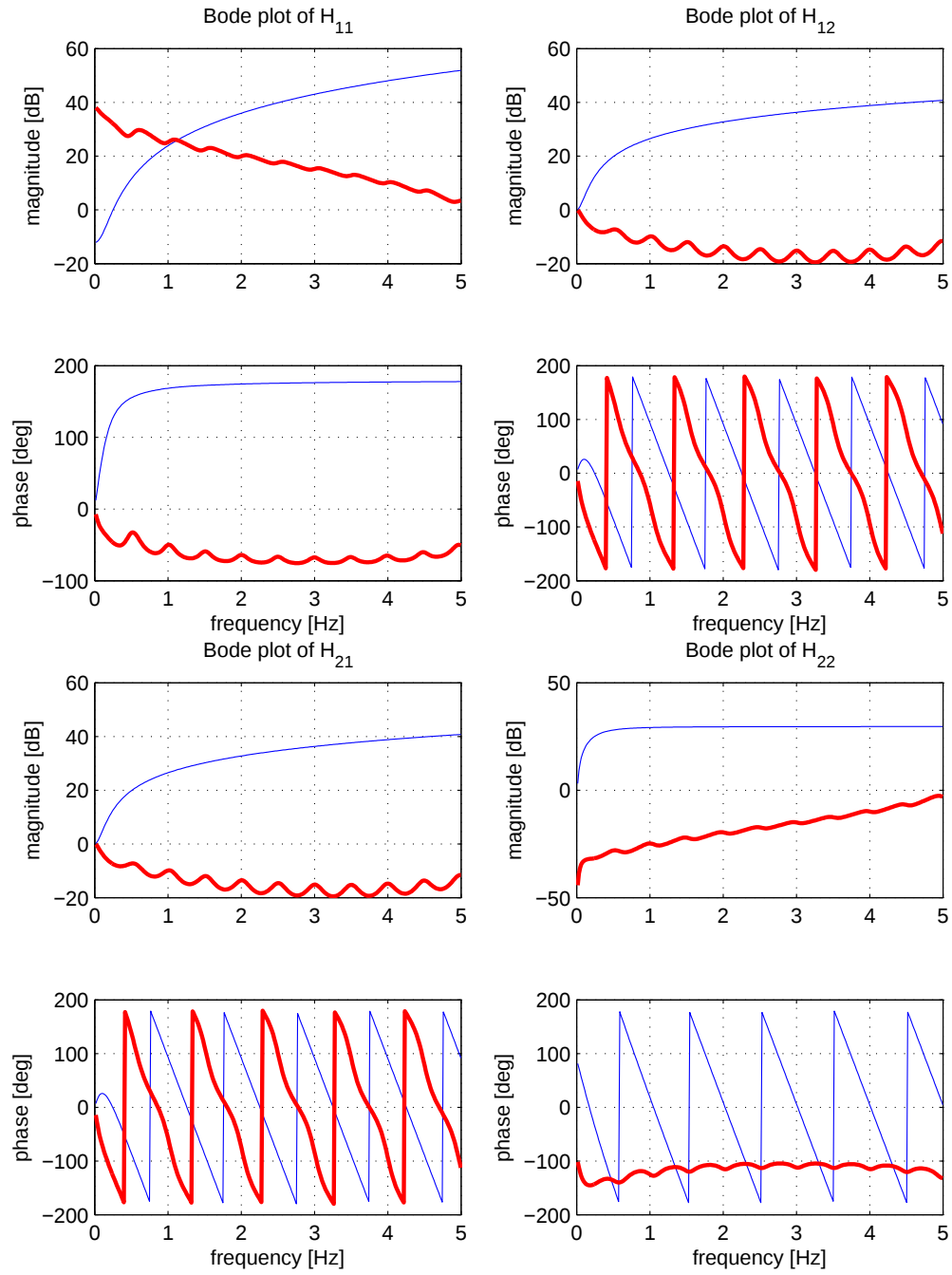


Figure 3.10: Bode plot of the hybrid matrices for DPDC and VPDC in 1s time delay case. Blue thin line refers to the DPDC, and red thick line refers to the VPDC.

In the second setting, we consider the communication channels with time delay such that $\tau_1 = \tau_2 = 1000\text{ms}$. For the DPDC, the solution of optimization problem (3.59) is $K = 0.07\text{N}\cdot\text{m}/\text{rad}$, $B = 0.033\text{N}\cdot\text{m}\cdot\text{s}/\text{rad}$, and $Q_T = 0.9923$, $Q_I = 0.9997$. Next, the solution of optimization problem (3.60) is $K = 20\text{N}\cdot\text{m}/\text{rad}$, $B = 20\text{N}\cdot\text{m}\cdot\text{s}/\text{rad}$, and $Q_T = 0.9746$, $Q_I = 0.8276$. So, DPDC has better trackability and immersivity in large time delay case.

The Bode plots are shown in Fig. 3.10. From which, one can easily tell VPDC provides significantly better transparency than DPDC. This conclusion is consistent with the trackability and immersivity measures.

By these two scenarios, it is clear that VPDC is a better control architecture over DPDC, which is mainly due to the expansion of feasible gain region. This benefit is more significant when the time delay is large. In this case, we can still enjoy the unlimited feasible gain region for $K, B^\dagger$.

3.6 Conclusion and Future Works

In this thesis, we present two novel PD-based hybrid control frameworks—DPDC and VPDC—for the bilateral teleoperation over imperfect packet-switched network with arbitrary varying-delay, packet-loss, data duplication/swapping, etc. By exploiting often ignored hybrid nature of the networked bilateral teleoperation system, both control frameworks achieve the closed-loop passivity under these complex communication conditions. The basic teleoperation performance—position coordination (when there is no external force) and force reflection (in steady state)—are achieved.

However, as communication unreliability increases, the allowable control gains of DPDC decreases. Therefore, DPDC cannot provide acceptable performance for teleoperation system with large delay. The VPDC framework, which uses the

[†]In practice, one needs to decide the upper limit for VP damping to maintain acceptable sluggishness during rapid motion.

numerical damping to dissipate the active energy caused by unreliable communication channels, successfully isolates the device damping requirement from the communication conditions, thereby, significantly enhances control gain design flexibility compared to DPDC. Then, we further compare the performance difference for stiff wall type environment using the quantitative performance index based on the transparency measure. From this index, the VPDC can provide better performance than DPDC especially for large time delay. We also compare the performance through Bode plots of hybrid matrices of both frameworks, which are independent of environmental impedance. The comparison indicates VPDC can provide similar or better (depending on the choice of gains and time delay) performance, which is aligned with the former index.

As time-invariant control frameworks, control gains of DPDC and VPDC are chosen according to the worst communication conditions, which makes these two control frameworks conservative. Our future work is to relax this conservatism by extending DPDC and VPDC to time-variant control frameworks, which would adjust the control gains according to the real-time communication condition and the actual motion of master/slave robots. We believe, such extension could significantly improve the system performance for the Internet with substantial varying delays.

3.7 Supplementary Mathematical Proofs

3.7.1 Proof of Inequality (3.17)

$$\begin{aligned}
\epsilon_p(k) &= - \left[q_1(k) - q_2(k - N_2^k) \right]^T K \Delta q_1(k) - \left[q_2(k) - q_1(k - N_1^k) \right]^T K \Delta q_2(k) \\
&= - \left[q_2(k) - q_2(k - N_2^k) \right]^T K \Delta q_1(k) - \left[q_1(k) - q_1(k - N_1^k) \right]^T K \Delta q_2(k) \quad (3.61) \\
&\quad - \left[q_1(k) - q_2(k) \right]^T K \Delta q_1(k) - \left[q_2(k) - q_1(k) \right]^T K \Delta q_2(k)
\end{aligned}$$

The last two lines can be further written as

$$\begin{aligned}
& -\left[q_1(k) - q_2(k)\right]^T K \left[\Delta q_1(k) - \Delta q_2(k)\right] \\
& = -\left[\hat{q}_1(k) - \hat{q}_2(k)\right]^T K \left[\Delta q_1(k) - \Delta q_2(k)\right] \\
& \quad + \frac{1}{2} \left[\Delta q_1(k) - \Delta q_2(k)\right]^T K \left[\Delta q_1(k) - \Delta q_2(k)\right]
\end{aligned} \tag{3.62}$$

where the last line is upper bounded by

$$\Delta q_1^T(k) K \Delta q_1(k) + \Delta q_2^T(k) K \Delta q_2(k) \tag{3.63}$$

Substituting (3.62) and (3.63) into (3.61) yields (3.17).

3.7.2 Complementary Proof of Position Coordination

Following (3.28) and assuming $q_i(k)$ does not converge to $q_j(k - N_j^k)$, i.e. for some $\epsilon > 0$, $\forall N_a \geq 0$, there exists $k_\epsilon \geq N_a$ s.t.

$$\|K[q_i(k_\epsilon) - q_j(k_\epsilon - N_j^{k_\epsilon})]\| > \epsilon \tag{3.64}$$

Also, by $\dot{q}_i(t) \rightarrow 0$ and boundedness of C_i, B_i, D , there exists $N_b \geq 0$ s.t. $\forall k \geq N_b$ and $t \in T_k := [t_k, t_{k+1})$

$$\|C_i(q_i, \dot{q}_i)\dot{q}_i(t) + B_i\dot{q}_i(t) + D[v_i(k) - \delta_i^k v_j(k - N_j^k)]\| \leq \frac{\epsilon}{2}. \tag{3.65}$$

Hence, by choosing $N_a \geq N_b$ there exists $k_\epsilon \geq N_a$ which makes (3.64) and (3.65) hold at the same time. Let us denote $\rho(k_\epsilon) := -K[q_i(k_\epsilon) - q_j(k_\epsilon - N_j^{k_\epsilon})]$. Then, by (??) and (3.64)-(3.65), it is clear that $\forall t \in T_{k_\epsilon}$

$$M_i(q_i)\ddot{q}_i(t) \in \left\{y \in \mathbb{R}^n \mid \|y - \rho(k_\epsilon)\| \leq \frac{\epsilon}{2}\right\}$$

Note that $\rho(k_\epsilon)$ is constant during $[t_{k_\epsilon}, t_{k_\epsilon+1})$ and is outside of the closed ball $B_\epsilon^c := \{y \in \mathfrak{R}^n \mid \|y\| \leq \epsilon\}$. It is clear that

$$\lambda_{\max}(M_i) \|\dot{q}_i(t_{k_\epsilon+1}) - \dot{q}_i(t_{k_\epsilon})\| \geq \left\| \int_{T_{k_\epsilon}} M_i(q_i) \ddot{q}_i(t) dt \right\| > \frac{\epsilon T}{2}$$

Note that k_ϵ can be arbitrarily large. This inequality then contradicts to $\lim_{t \rightarrow \infty} \dot{q}_i(t) = 0$. Hence, $q_i(k) \rightarrow q_j(k - N_j^k)$, which further implies $q_i(t) \rightarrow q_j(t)$ due to $\dot{q}_i(t), \dot{q}_j(t) \rightarrow 0$.

3.7.3 Proof of Proposition 3:

Notation: In the following derivation, we define the norm $\|a\|_L^2 \triangleq a^T L a$, where $a \in \mathfrak{R}^n$ and $L \in \mathfrak{R}^{n \times n}$.

Energy Generation by PD Coupling: Denote the energy generated by the PD synchronization control (3.33) during k^{th} update interval as

$$\begin{aligned} s_E(k) \triangleq \sum_{i=1}^2 \hat{w}_i(k)^T \tau_i^s(k) T_k^i &= - \sum_{i=1}^2 \left[\Lambda_i(k) + \hat{w}_i(k)^T K \left(\hat{y}_i(k) - \hat{y}_j(k - N_i^k) \right) T_k^i \right. \\ &\quad \left. + \hat{w}_i(k)^T D \left(\hat{w}_i(k) - \delta_i^k \hat{w}_j(k - N_i^k) \right) T_k^i \right] \end{aligned} \quad (3.66)$$

where $(i, j) \in \{(1, 2), (2, 1)\}$ and $\Lambda_i(k) \triangleq \|\hat{w}_i(k)\|_{B_i^p}^2 T_k^i$ is the discrete damping dissipation during T_k^i on site i ; and the second term in the bracket can be rewritten as

$$\begin{aligned} &\sum_{i=1}^2 \left[\hat{w}_i(k)^T K \left[\hat{y}_j(k) - \hat{y}_j(k - N_i^k) \right] T_k^i + \hat{w}_i(k)^T K \left[\hat{y}_i(k) - \hat{y}_j(k) \right] T_k^i \right] \\ &= \sum_{i=1}^2 \hat{w}_i(k)^T K \left[\hat{y}_j(k) - \hat{y}_j(k - N_i^k) \right] T_k^i + [\hat{w}_1(k) T_k^1 - \hat{w}_2(k) T_k^2]^T K [\hat{y}_1(k) - \hat{y}_2(k)]. \end{aligned} \quad (3.67)$$

Denote the potential energy stored in the spring K as $\varphi(k) := \frac{1}{2} \|y_1(k) - y_2(k)\|_K^2$. Then, the last terms of (3.67) can be simplified as, with $e_y(k) \triangleq y_1(k) - y_2(k)$,

$$\begin{aligned} & [\hat{w}_1(k)T_k^1 - \hat{w}_2(k)T_k^2]^T K [\hat{y}_1(k) - \hat{y}_2(k)] \\ &= \frac{1}{2} [e_y(k+1) - e_y(k)]^T K [e_y(k+1) + e_y(k)] \\ &= \varphi(k+1) - \varphi(k). \end{aligned}$$

Then, we can rewrite $s_E(k)$ s.t.

$$\begin{aligned} s_E(k) = & \underbrace{- \left[\varphi(k+1) - \varphi(k) \right]}_{\text{spring energy storage}} - \sum_{i=1}^2 \underbrace{\Lambda_i(k)}_{\text{local damping}} \\ & - \sum_{i=1}^2 \underbrace{\hat{w}_i(k)^T K \left[\hat{y}_j(k) - \hat{y}_j(k - N_i^k) \right] T_k^i}_{\text{delayed spring term}} \\ & - \sum_{i=1}^2 \underbrace{\hat{w}_i(k)^T D \left[\hat{w}_i(k) - \delta_i^k \hat{w}_j(k - N_i^k) \right] T_k^i}_{\text{delayed damper term}} \end{aligned} \quad (3.68)$$

which clearly shows that the (unwanted) energy generation caused by the communication unreliability can be decomposed into those of the delayed spring and the delayed damper. The essential idea of the following proof is to show that, under the passivity condition (3.34), this unwanted energy generation is guaranteed to be dissipated by the local VP damping dissipation Λ_i .

Energy Generated by Delayed Spring Term: Let us denote the delayed spring energy term as

$$\Theta_i(k) := \hat{w}_i(k)^T K \left[\hat{y}_j(k) - \hat{y}_j(k - N_i^k) \right] T_k^i. \quad (3.69)$$

Inserting dummy terms $\sum_{l=k+1-N_i^k}^{k-1} [\hat{y}_j(l) - \hat{y}_j(l)]$ between $\hat{y}_j(k)$ and $\hat{y}_j(k - N_i^k)$, we can rewrite $\Theta_i(k)$ as

$$\begin{aligned}\Theta_i(k) &= T_k^i \hat{w}_i(k)^T K \sum_{l=k-N_i^k}^{k-1} \left(\hat{y}_j(l+1) - \hat{y}_j(l) \right) \\ &= T_k^i \hat{w}_i(k)^T K \sum_{l=k-N_i^k}^{k-1} \frac{1}{2} \left[\hat{w}_j(l+1) T_{l+1}^j + \hat{w}_j(l) T_l^j \right] \\ &= T_k^i \hat{w}_i(k)^T K \left[\sum_{l=k+1-N_i^k}^{k-1} \hat{w}_j(l) T_l^j + \frac{1}{2} \left(\hat{w}_j(k) T_k^j + \hat{w}_j(k - N_i^k) T_{k-N_i^k}^j \right) \right]\end{aligned}$$

where the second line is due to (3.31). Since K is symmetric and positive-definite, we have the following fact s.t.

$$|a^T K b| \leq \frac{1}{2} \left(\|a\|_K^2 + \|b\|_K^2 \right), \quad \forall a, b \in \mathbb{R}^n. \quad (3.70)$$

Using this, we can then show that

$$\begin{aligned}|\Theta_i(k)| &\leq \frac{1}{2} T_k^i \sum_{l=k+1-N_i^k}^{k-1} T_l^j \left[\|\hat{w}_i(k)\|_K^2 + \|\hat{w}_j(l)\|_K^2 \right] + \frac{1}{4} T_k^i T_k^j \left[\|\hat{w}_i(k)\|_K^2 + \|\hat{w}_j(k)\|_K^2 \right] \\ &\quad + \frac{1}{4} T_k^i T_{k-N_i^k}^j \left[\|\hat{w}_i(k)\|_K^2 + \|\hat{w}_j(k - N_i^k)\|_K^2 \right] \\ &= \frac{1}{2} \alpha_i(k) \left[\frac{1}{2} T_k^j + \frac{1}{2} T_{k-N_i^k}^j + \sum_{l=k+1-N_i^k}^{k-1} T_l^j \right] \\ &\quad + \frac{1}{2} T_k^i \left[\frac{1}{2} \alpha_j(k) + \frac{1}{2} \alpha_j(k - N_i^k) + \sum_{l=k+1-N_i^k}^{k-1} \alpha_j(l) \right]\end{aligned}$$

where $\alpha_i(k) := T_k^i \|\hat{w}_i(k)\|_K^2 \geq 0$. We can further obtain,

$$\begin{aligned}
\sum_{i=1}^2 |\Theta_i(k)| &\leq \sum_{i=1}^2 \left[\frac{1}{2} \alpha_i(k) \left[\frac{1}{2} T_k^j + \frac{1}{2} T_{k-N_i^k}^j + \sum_{l=k+1-N_i^k}^{k-1} T_l^j \right] \right. \\
&\quad \left. + \frac{1}{2} T_k^i \left[\frac{1}{2} \alpha_i(k) + \frac{1}{2} \alpha_i(k - N_j^k) + \sum_{l=k+1-N_j^k}^{k-1} \alpha_i(l) \right] \right] \\
&= \sum_{i=1}^2 \left[\frac{1}{2} \alpha_i(k) \left[\frac{1}{2} T_k^j + \frac{1}{2} T_{k-N_i^k}^j + \sum_{l=k+1-N_i^k}^{k-1} T_l^j \right] \right. \\
&\quad \left. + \frac{1}{4} T_k^j \left[\sum_{l=k-N_j^k}^{k-1} \alpha_i(l) + \sum_{l=k-N_j^k+1}^k \alpha_i(l) \right] \right] \\
&\leq \sum_{i=1}^2 \left[\frac{\bar{N}_i}{2} T_{\max} \alpha_i(k) + \frac{1}{4} T_{\max} \left[\sum_{l=k-\bar{N}_j}^{k-1} \alpha_i(l) + \sum_{l=k-\bar{N}_j+1}^k \alpha_i(l) \right] \right]. \tag{3.71}
\end{aligned}$$

By summing (3.71) over the time, we have

$$\begin{aligned}
\sum_{k=0}^{N-1} \sum_{i=1}^2 |\Theta_i(k)| &\leq \sum_{k=0}^{N-1} \sum_{i=1}^2 \left[\frac{\bar{N}_i}{2} T_{\max} \alpha_i(k) + \frac{1}{2} T_{\max} \sum_{l=k-\bar{N}_j+1}^k \alpha_i(l) \right] \\
&= \sum_{k=0}^{N-1} \sum_{i=1}^2 \left[\frac{\bar{N}_i}{2} T_{\max} \alpha_i(k) + \frac{\bar{N}_j}{2} T_{\max} \alpha_i(k) \right] - \frac{1}{2} T_{\max} \sum_{i=1}^2 \sum_{k=2}^{\bar{N}_j} (k-1) \alpha_i(k + N - 1 - \bar{N}_j) \\
&\leq \sum_{k=0}^{N-1} \sum_{i=1}^2 \frac{\bar{N}_i + \bar{N}_j}{2} T_{\max} \alpha_i(k) \tag{3.72}
\end{aligned}$$

where the equality is due to the following fact

$$\sum_{k=0}^{N-1} \sum_{l=k-\bar{N}_j+1}^k \alpha_i(l) = \sum_{k=0}^{N-1} \bar{N}_j \alpha_i(k) - \sum_{k=2}^{\bar{N}_j} (k-1) \alpha_i(k + N - 1 - \bar{N}_j) \tag{3.73}$$

with the proof omitted here due to the space limitation. The inequality (3.72) then shows that the possible energy generation via the delayed spring term is upper bounded.

Energy Generated by Delayed Damper Term: Let us denote the delayed damper term as

$$\Upsilon_i(k) := \hat{w}_i(k)^T D \left[\hat{w}_i(k) - \delta_i^k \hat{w}_j(k - N_i^k) \right] T_k^i. \quad (3.74)$$

Following fact (3.70), we then have

$$\begin{aligned} \Upsilon_i(k) &\geq \|\hat{w}_i(k)\|_D^2 T_k^i - \frac{1}{2} \delta_i^k \left(\|\hat{w}_i(k)\|_D^2 + \|\hat{w}_j(k - N_i^k)\|_D^2 \right) T_k^i \\ &\geq \frac{1}{2} \left(\|\hat{w}_i(k)\|_D^2 - \delta_i^k \|\hat{w}_j(k - N_i^k)\|_D^2 \right) T_k^i \end{aligned}$$

therefore,

$$\sum_{i=1}^2 \Upsilon_i(k) \geq \sum_{i=1}^2 \frac{1}{2} \left(\|\hat{w}_i(k)\|_D^2 - \delta_j^k \|\hat{w}_i(k - N_j^k)\|_D^2 \right) T_k^i.$$

Summing the inequality above over the time then yields

$$\sum_{k=0}^{N-1} \sum_{i=1}^2 \Upsilon_i(k) \geq \frac{1}{2} \sum_{k=0}^{N-1} \sum_{i=1}^2 \left(\|\hat{w}_i(k)\|_D^2 T_{\min} - \delta_j^k \|\hat{w}_i(k - N_j^k)\|_D^2 T_{\max} \right). \quad (3.75)$$

Local Damping Dissipation: From passivity condition (3.34), the local energy dissipation via B_i^p can be computed as

$$\sum_{i=1}^2 \sum_{k=0}^{N-1} \Lambda_i(k) \geq \sum_{i=1}^2 \sum_{k=0}^{N-1} \left[\frac{\bar{N}_1 + \bar{N}_2}{2} T_{\max} \|\hat{w}_i(k)\|_K^2 T_k^i + \|\hat{w}_i(k)\|_D^2 (T_{\max} - T_{\min}) \right] \quad (3.76)$$

Controller Passivity: Then, combining (3.72), (3.75) and (3.76), we can rewrite (3.68) s.t.: $\forall \bar{M} \geq 0$,

$$\sum_{k=0}^{N-1} s_E(k) = - \sum_{i=1}^2 \sum_{k=0}^{N-1} \left(\Lambda_i(k) + \Theta_i(k) + \Upsilon_i(k) \right) + \varphi(0) - \varphi(N+1) \leq \varphi(0) =: \bar{d}^2 \quad (3.77)$$

where d_2 is a bounded constant. The inequality in (3.77) is obtained by the following facts:

$$\begin{aligned} - \sum_{i=1}^2 \sum_{k=0}^{N-1} \Theta_i(k) &\leq \sum_{i=1}^2 \sum_{k=0}^{N-1} |\Theta_i(k)| \\ \sum_{k=0}^{N-1} \|\hat{w}_i(k)\|_D^2 &\geq \sum_{k=0}^{N-1} \delta_j^k \|\hat{w}_i(k - N_j^k)\|_D^2 \end{aligned}$$

and (3.72), (3.75). This proves the controller passivity (3.35).

The proof for VPs' position coordination (3.36) follows the similar procedure presented in [HL10]. From (3.31), $\hat{w}_i(k) \rightarrow \mathbf{0}$ does not directly imply that $w_i(k) \rightarrow \mathbf{0}$ since the oscillation at the update frequency, i.e. $w_i(k) \rightarrow -w_i(k+1), \forall k \geq 0$, could happen. The position coordination cannot be achieved if such oscillation exists. Hence, the first step is to show the oscillation does not exist, i.e. $w_i(k) \rightarrow \mathbf{0}$. Since $\tau_i^p(k) \rightarrow \mathbf{0}$ and $\hat{w}_i(k) \rightarrow \mathbf{0}$, the VP dynamics can be rewritten as,

$$H_i \frac{w_i(k+1) - w_i(k)}{T_k^i} \rightarrow u_i^p(k) \rightarrow -K \left(\hat{y}_i(k) - \hat{y}_j(k - N_i^k) \right).$$

By the definition of $\hat{w}_i(k)$ in (3.31), $\hat{w}_i(k) \rightarrow \mathbf{0}$ implies $y_i(k+1) \rightarrow y_i(k)$. This further implies $\hat{y}_j(k) \rightarrow \hat{y}_j(k - N_i^k)$. We then have

$$H_i \frac{w_i(k+1) - w_i(k)}{T_k^i} \rightarrow -K \left(\hat{y}_i(k) - \hat{y}_j(k) \right) \quad (3.78a)$$

$$H_i \frac{w_i(k+2) - w_i(k+1)}{T_{k+1}^i} \rightarrow -K \left(\hat{y}_i(k+1) - \hat{y}_j(k+1) \right) \quad (3.78b)$$

where $(i, j) \in \{(1, 2), (2, 1)\}$. Again, since $\hat{w}_i(k) \rightarrow \mathbf{0}$, $\hat{y}_i(k+1) \rightarrow \hat{y}_i(k)$, which implies

$$H_i \frac{w_i(k+1) - w_i(k)}{T_k^i} \rightarrow H_i \frac{w_i(k+2) - w_i(k+1)}{T_{k+1}^i}. \quad (3.79)$$

Since $\hat{w}_i(k) \rightarrow \mathbf{0}$, $w_i(k+2) \rightarrow w_i(k)$. Hence, (3.79) becomes

$$H_i \frac{w_i(k+1) - w_i(k)}{T_k^i} \rightarrow H_i \frac{w_i(k) - w_i(k+1)}{T_{k+1}^i}.$$

This can only happen when $w_i(k+1) \rightarrow w_i(k)$ since $T_k^i > 0$ and H_i is symmetric positive-definite matrix. Together with $\hat{w}_i(k) \rightarrow \mathbf{0}$, it implies $w_i(k) \rightarrow \mathbf{0}$. Substituting this into (3.78a) yields $\hat{y}_i(k) \rightarrow \hat{y}_j(k)$. Then, because $y_i(k+1) \rightarrow y_i(k)$, we conclude the position coordination between VPs, i.e. $y_i(k) \rightarrow y_j(k)$.

□

3.7.4 Proof of Proposition 4

The essential idea of this proof is similar to the proof of Prop. 3. *Energy Generation by VC* The energy produced by the VC (3.37) during T_k^i is

$$s_{vc}(k) := \hat{w}_i^T(k) \tau_i^c(k) T_k^i + \int_{t_k^i}^{t_{k+1}^i} \dot{q}_i^T(t) \hat{u}_i(t) dt. \quad (3.80)$$

By substituting (3.37), (3.80) is equivalent to

$$\begin{aligned} s_{vc}(k) = & - \int_{t_k^i}^{t_{k+1}^i} \dot{q}_i^T(t) B_i \dot{q}_i(t) dt - \Delta y_i^T(k) \frac{b_i^p}{T_k^i} \Delta y_i(k) \\ & - \Delta q_i^T(k) \frac{B_i^c}{T_{k-1}^i} \Delta q_i(k-1) + \Delta q_i^T(k) \frac{B_i^c}{T_{k-1}^i} \Delta y_i(k-1) \\ & - \Delta y_i^T(k) \frac{B_i^c}{T_k^i} \Delta y_i(k) + \Delta y_i^T(k) \frac{B_i^c}{T_{k-1}^i} \Delta q_i^T(k-1) \\ & - \Delta q_i^T(k) K_i^c (q_i(k) - y_i(k)) - \Delta y_i^T(k) K_i^c (y_i(k+1) - q_i(k)) \end{aligned}$$

where the first line is damping dissipation, the second and third lines are due to VC damper terms, and the last line is due to the spring terms. The following derivation will analyze these three categorizes of energy respectively.

Damping Dissipation: We denote the damping dissipation as,

$$\begin{aligned}\Lambda_{vc}(k) &:= - \int_{t_k^i}^{t_{k+1}^i} \dot{q}_i^T(t) B_i \dot{q}_i^T dt - \Delta y_i^T(k) \frac{b_i^p}{T_k^i} \Delta y_i(k) \\ &\leq - \left(\|\Delta q_i(k)\|_{B_i}^2 + \|\Delta y_i(k)\|_{b_i^p}^2 \right) / T_k^i\end{aligned}\quad (3.81)$$

where the inequality is due to (3.16).

Energy Contributed by Damper Terms: The energy produced by the damper terms during T_k^i is denoted as,

$$\begin{aligned}\Upsilon_{vc}(k) &:= - \Delta q_i^T(k) \frac{B_i^c}{T_{k-1}^i} \Delta q_i(k-1) + \Delta q_i^T(k) \frac{B_i^c}{T_{k-1}^i} \Delta y_i(k-1) \\ &\quad - \Delta y_i^T(k) \frac{B_i^c}{T_k^i} \Delta y_i(k) + \Delta y_i^T(k) \frac{B_i^c}{T_{k-1}^i} \Delta q_i^T(k-1) \\ &\leq \|\Delta q_i(k)\|_{B_i^c}^2 / T_{k-1}^i + \|\Delta q_i(k-1)\|_{B_i^c}^2 / T_{k-1}^i \\ &\quad + \frac{1}{2} \|\Delta y_i(k-1)\|_{B_i^c}^2 / T_{k-1}^i + \frac{1}{2} \|\Delta y_i(k)\|_{B_i^c}^2 / T_{k-1}^i \\ &\quad - \|\Delta y_i(k)\|_{B_i^c}^2 / T_k^i.\end{aligned}\quad (3.82)$$

Energy Contributed by Spring Terms: The energy produced by the spring terms during T_k^i can be written as,

$$\begin{aligned}\Theta_{vc}(k) &:= - \Delta q_i^T(k) K_i^c \left(q_i(k) - y_i(k) \right) \\ &\quad - \Delta y_i^T(k) K_i^c \left(y_i(k+1) - q_i(k) \right) \\ &= - \left(\Delta q_i(k) - \Delta y_i(k) \right)^T K_i^c \left(\hat{q}_i(k) - \hat{y}_i(k) \right) \\ &\quad + \frac{1}{2} \|\Delta q_i(k)\|_{K_i^c}^2 + \Delta q_i^T(k) K_i^c \Delta y_i(k) - \frac{1}{2} \|\Delta y_i(k)\|_{K_i^c}^2\end{aligned}$$

where in the last equality, the first line is potential energy storage, and the second line is the energy generation caused by the spring terms. Let us define the potential energy of VC as $\varphi_{vc}(k) := \frac{1}{2} \|q_i(k) - y_i(k)\|_{K_i^c}^2$. Then by Cauchy-Schwarz inequality, it is easy to obtain

$$\Theta_{vc}(k) \leq -\left(\varphi_{vc}(k+1) - \varphi_{vc}(k)\right) + \|\Delta q_i(k)\|_{K_i^c}^2. \quad (3.83)$$

Controller Passivity of VC: Combining (3.81), (3.82) and (3.83) and summing over the time yields

$$\begin{aligned} \sum_{k=0}^{N-1} s_{vc}(k) &= \sum_{k=0}^{N-1} (\Lambda_{vc}(k) + \Upsilon_{vc}(k) + \Theta_{vc}(k)) \\ &\leq -\sum_{k=0}^{N-1} \left(\|\Delta q_i(k)\|_{B_i}^2 + \|\Delta y_i(k)\|_{b_i^p}^2 \right) / T_k^i + \sum_{k=0}^{N-1} \|\Delta q_i(k)\|_{B_i^c}^2 \left(\frac{1}{T_k^i} + \frac{1}{T_{k-1}^i} \right) \\ &\quad + \frac{1}{2} \sum_{k=0}^{N-1} \|\Delta y_i(k)\|_{B_i^c}^2 \left(\frac{1}{T_{k-1}^i} - \frac{1}{T_k^i} \right) + \varphi_{vc}(0) + \sum_{k=0}^{N-1} \|\Delta q_i(k)\|_{K_i^c}^2 \end{aligned} \quad (3.84)$$

with the passivity condition (3.39) satisfied, we further have,

$$\sum_{k=0}^{N-1} s_{vc}(k) \leq \varphi_{vc}(0). \quad (3.85)$$

which implies the controller passivity (3.38).

□

Chapter 4

Conclusion and Future Works

4.1 Conclusion

In this dissertation, we propose two major contributions: 1) proposing a complete hybrid P2P architecture for Internet collaborative haptic interaction system; 2) proposing two novel PD-based bilateral teleoperation control frameworks that can handle complex communication unreliability.

In P2P CHI architecture, we consider local deformable VE simulation, VP-VE interconnection, device-VP coupling and the synchronization control among geographically separated SVEs. The usually-ignored but most important interaction stability problem, which is mainly due to the communication unreliability and hybrid device-VE coupling, is rigourously addressed by the proposed architecture through enforcing the passive gain setting conditions. Such guaranteed interaction stability remains true even under partial-connected network topology, asynchronousness between fast VE simulation rate and slow packet transmission rate. Regarding improving the overall system performance, we design a novel and simple network topology optimization mechanism based on algebraic connectivity. A 4-user CHI system consisting of deformable SVEs and simulated unreliable packet-switched

communication network has been built and used for validating the proposed CHI architecture and topology optimization.

For challenging delayed bilateral teleoperation problem, we have taken a brand new hybrid perspective and provided two solutions to and beyond that problem. The first DPDC is a similar control framework to widely-used PD control with the difference lies in the hybrid formulation and packet sustainment. DPDC is able to passify the complex communication unreliability including varying delay, packet loss, data duplication/swapping using the device viscous damping. However, the control gains are limited by the unadjustable device damping and extent of communication unreliability. In practice, DPDC cannot provide acceptable performance when time delay is large given the interaction stability is guaranteed. To address this limitation and make PD-type control usable for large delay case, we have proposed the new VPDC control architecture. A VP is inserted between the device and the communication channel. Thanks to the unique discrete-time passive property of VP simulation, the VP's virtual damping is used to passify the active energy produced by the imperfect communication channel and the device damping requirement is separated from the communication unreliability and significantly reduced. Since there is no upper bound for the virtual damping in theory, the upper bound for gains of the PD-like control over the communication is lifted, which much larger control gains than DPDC can be chosen under same scenario. However, due to the insertion of intermediate VP dynamics, the performance difference between DPDC and VPDC is not obvious. We conduct the comparison through two ways: 1) quantitative performance measure based on transparency concept; 2) Bode plots of hybrid matrix. By two design examples, both indexes show that VPDC can provide better performance over DPDC.

4.2 Future Works

Some future research directions for collaborative haptic interaction include: 1) further improvement of the system performance by using some less conservative consensus schemes (e.g., PSPM [LH10a] or other passivity-based time-variant consensus controls) instead of the current (time-invariant) PD-type consensus control; 2) reduction of the amount of data for the VE consensus without compromising human perception (e.g., perception-based data reduction [HHC+08]); and 3) application of the result to more realistic, interesting and practically-important scenarios and investigate the issue of human perception therein (e.g., collaborative virtual surgical training).

For bilateral teleoperation, the future research plan is to release the conservatism introduced by the time-invariant control which is design for the ‘worst’ scenario. Such conservatism could be reduced by adapting the control gains in real-time according to the communication condition and actual robots’ motion. We believe, such online control gain adaption will contribute to the booming research direction of time-domain bilateral teleoperation control design (e.g., [LH10a, RAP10, FSM+11]).

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Vita

Ke Huang was born in Maanshan, Anhui Province, China on March 12, 1983. After graduating from high school in 2001, he began pursuit of higher education at University of Technology and Science of China (USTC) in Hefei, Anhui Province. Here he met his future wife, Yishu Zou, who has been giving selfless love utmost support to his study and life. At USTC, his study focuses on control engineering and image/video processing. In July 2005, he received Bachelors of the Engineering with outstanding undergraduate thesis rewards. Upon completion of his undergraduate study, he continued his research work on digital watermark with Dr. Zengfu Wang at USTC for another half year. After that, he worked for a software development company in Dalian, Liaoning Province for one year. In August 2007, he became the Ph.D. student under Dr. Dongjun Lee at University of Tennessee and his major research interests are networked collaborative haptic interaction and bilateral teleoperation over unreliable communication. Since January 2012, he has been working with Rensselaer Polytechnic Institute as research specialist, and his research focuses on developing high fidelity laparoscopic cholecystectomy surgical simulation with cognitive feedback & fidelity.