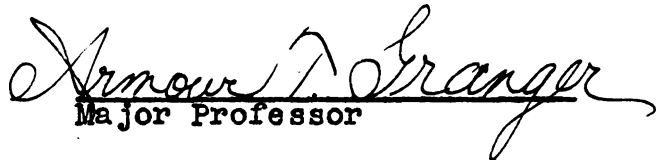


December 11, 1950

To the Committee on Graduate Study:

I am submitting to you a thesis written by Richard E. Whitmore entitled "Experimental Investigation of Stresses in Gusset Plates." I recommend that it be accepted for twelve quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Civil Engineering.

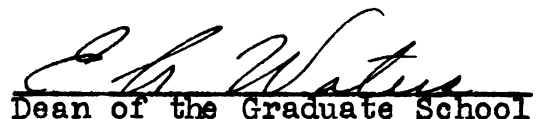

Major Professor

We have read this thesis
and recommend its acceptance:


C. R. Fabian


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Accepted for the Committee


Dean of the Graduate School

EXPERIMENTAL INVESTIGATION OF STRESSES
IN GUSSET PLATES

A THESIS

Submitted to
The Committee on Graduate Study
of
The University of Tennessee
in
Partial Fulfillment of the Requirements
for the degree of
Master of Science

by

Richard E. Whitmore

December, 1950

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CHAPTER I

INTRODUCTION

Purpose

Formerly, it was common practice to design and build trusses with pin connected joints. However, at present this practice has been virtually discontinued, owing to the increasing popularity of riveted connections. It is a reasonable assumption that rivets (or bolts) are used in 90% of all connections in present structural practice.¹

The advantages of the riveted joint are due largely to the inherent strength, stiffness and rigidity of this type of construction. This eliminates much vibration and wear, resulting in a more durable structure.

At present, the usual practice is to rivet or bolt all the members meeting at a joint to a plate (or plates). These plates, commonly called "gusset plates", are subjected to a multiplicity of stresses which fact makes the analytical stress determination quite difficult, if not impossible. The gusset plate must transmit the direct stress in each member (or difference in stress for a member which

¹ Thomas Clark Shedd, Structural Design in Steel (New York: John Wiley and Sons, Inc., 1934), p. 263.

continues through the joint) by direct, bending and shearing stresses. If there is any bending twist (torque), in the members, the gusset plate must also resist these moments, setting up a more complex pattern of stress.

Modern practice is to make the size of the gusset plate, in the plane of the truss, as large as necessary so that all required rivets may be placed properly, and then to select a thickness in accordance with average values used for similar trusses.² The stresses are then analyzed as carefully as possible to make sure that the plates are adequate and economical. This analysis generally requires passing various sections through the plate and analyzing for direct, bending and shearing stresses, all of which necessitates making certain questionable assumptions.

Because of this apparent uncertainty of the distribution of stress, intensity of stress, and a proper approach in investigation of stress analytically, an experimental investigation was suggested.

The purpose of this research is this: to determine, experimentally, the stress distribution in gusset plates, the maximum intensity of stress and its location and, if possible, to develop a straight-forward method of deter-

² Ibid., p. 349.

mining analytically the true stress which exists in gusset plates. To simplify the analysis, the effect of only direct stress in the members will be considered. This requires arranging the experimental apparatus so that all bending moments in the members are eliminated.

Experimental Methods

Of the several experimental methods available, the photoelastic method, brittle coatings and wire bonded strain gages appear to offer the best possibilities for the solution of this problem. Because the problem was too comprehensive for one thesis, it was divided into two separate theses. Another investigator, J. A. Sandel³ chose the photoelastic method, and the writer the brittle coatings and strain gages methods. Finally, the results obtained by the several methods will be compared.

³ J. A. Sandel, "Photoelastic Analysis of Gusset Plates", Knoxville, Tennessee, Unpublished Master's Thesis, University of Tennessee, 1950.

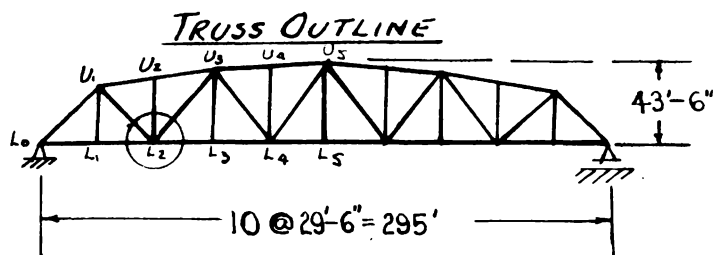
CHAPTER II

PRELIMINARY PLANNING

Selection of Prototype

After it was decided to investigate stresses in gusset plates experimentally, it was immediately concluded that the present investigation could not include gussets in general, but would have to be limited to a single type of joint. It is quite obvious that it would not be possible to test all types of gussets as the joints and plates vary with: type of truss, whether it is single or double plane, whether or not the chord is spliced at the joint, direction of members, stress in each member, etc.

Therefore, it was decided to select a joint from a common type of double plane truss, with several members meeting, but with no splices in the joint. This would give a very typical gusset problem. A design of a through Warren type highway bridge was available. The design included an analytical check of the gusset plate at joint L_2 and it was decided to use this as the prototype (see Figure 1). This is quite appropriate since the Warren truss is the predominant - almost universal - truss type in modern bridge design.



JOINT L2

Scale: $\frac{1}{16}" = 1"$

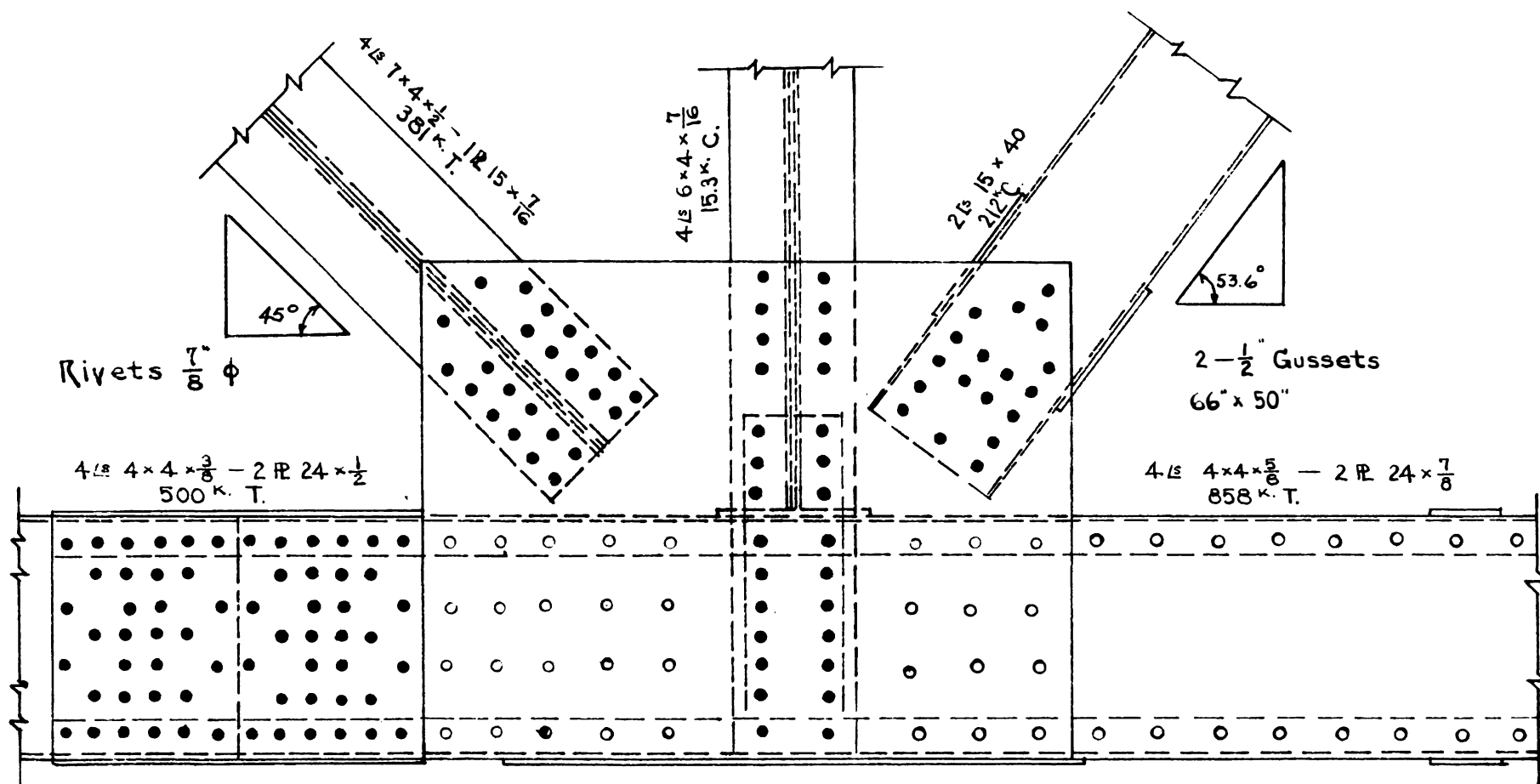


FIGURE 1. GUSSET PLATE SELECTED FOR ANALYSIS.

Relationship of Model to Prototype

Since it was impossible to use a full scale model, a model one fourth the size of the prototype was selected. This resulted in two plates 16.6" x 12.6" x 1/8" which appeared to be of proper laboratory proportions. If the forces in the members are then reduced by the same ratio, stresses of the same magnitude exist in the model as exist in the prototype. As the exact shape of the members is not important for this problem, a simple, solid rectangular section was selected for the members.

Selection of Model Material

The fundamental consideration in the selection of the model material was the fact that the experimental methods to be used would be strain measuring devices. Consequently, large magnitudes of strains in the plate were desired. This necessitated a material with a low modulus of elasticity. Also, it was desired that the model be of reasonably high strength, light in weight, ductile, and in general, possessed of properties similar to those of structural steel. All of these considerations indicated that a high strength aluminum alloy would be the most suitable material. Subsequently, the aluminum alloy selected was 61 ST, which has the following physical and mechanical

properties:

Modulus of elasticity -----	10,000,000 lbs. per sq. in.
Modulus of rigidity -----	3,800,000 lbs. per sq. in.
Poisson's ratio -----	0.33
Yield strength (tension --- and compression)	39,000 lbs. per sq. in.
Ultimate tensile strength -	45,000 lbs. per sq. in.
Per cent elongation in two inches -	15%
Yield strength (shear) ----	26,000 lbs. per sq. in.
Ultimate strength (shear) -	30,000 lbs. per sq. in.
Weight -----	0.098 lbs. per cubic in.

Alloys:

Copper -----	0.25 per cent
Silicon ----	0.6 per cent
Magnesium --	1.0 per cent
Chromium ---	0.25 per cent

This alloy is recommended for structural uses where good strength and resistance to corrosion are desired.¹

Design of the Loading System

It was realized in the early planning stage that loading the model would be a major problem. The facts that the loads would be in the general order of 10,000 pounds and that five members were involved with four different directions added to the complexity of the situation. To simplify matters, it was decided to make the stress in member L₂ U₂ zero. This member would normally carry only a

¹ Alcoa Structural Handbook (Pittsburgh: Aluminum Company of America, 1945), pp. 20-23.

small fraction of the loads in the other members, so it seemed reasonable to neglect it.

It seemed apparent that a rectangular frame large enough to accomodate the model would have to be built. Cross members, perpendicular to the direction of the diagonals, could be attached to the frame. Then the loads could be applied by jacking against the frame or by a draw bar arrangement.

To make the loading as simple as possible, it was decided to load members $L_0 L_2$ and $U_1 L_2$ only. Then with $L_2 L_4$ (tension) attached to the frame and $L_2 U_3$ (compression) resting against the frame, the stresses in these two members would be the correct amounts when the loads in $L_0 L_2$ and $U_1 L_2$ were correct. In other words, since the sum of the horizontal and vertical components of the forces must equal zero, only two members need be loaded if one of the members is a diagonal.

The load would be applied to these two tension members by tightening up on a nut attached to the threaded end of a clevis, which was pin-connected to the members. (See Figure 2).

Rough calculations indicated that a 6 inch American Standard steel beam weighing 14.75 pounds per foot would be sufficient to resist the applied forces. This beam has a section modulus of 1.2 inches cubed when loaded perpen-



FIGURE 2. THE LOADING DEVICE

dicular to the web.

Selection of Strain Measuring Devices

Of the various types of brittle coatings available for experimental use, it appeared that "Stresscoat" would be the best to use for this particular study. It is common knowledge that brittle materials fail when the unit strain reaches the ultimate value, which is quite small when compared with strains of ductile materials. Some brittle coatings fail due to shear at 45° to the axis of principal stress (an example is the "flaking off" of mill scale from hot rolled carbon steel). Other brittle materials fail in tension and the crack which appears is perpendicular to the direction of maximum tension at that point. "Stresscoat" (a brittle lacquer) is such a material. The cracks result, then, from the maximum tension at the point (the principal stress hereinafter identified as "p") and are perpendicular to the direction of "p". Since the other principal stress ("q") is perpendicular to the direction of "p", the cracks in the "Stresscoat" are stress trajectories for "q".

The SR-4 wire bonded strain gage is one of the most useful and sensitive of all the strain measuring devices. The SR-4 gages are available in various sizes and types. Since it was desired to obtain p, q, and θ (direction of

"p") at various points in the plate, strain rosettes were selected for use on the interior of the plate. The rosettes available are of several types, but a simple 45° rectangular rosette (type AR-1) with three gages at 45° with one another was selected. For a given gage length, this type of rosette covers a minimum area. Therefore, such rosettes are particularly suitable where the strain gradient perpendicular to the surface is relatively low and the strain gradient parallel to the surface relatively high.

Experimental stress analysts have long known that a combination of "Stresscoat" and SR-4 gages is desirable. The "Stresscoat" can be used to detect troublesome spots (points of high stress) and also the direction of maximum stresses. This serves as a guide in locating the strain gages. It also is possible to obtain quantitative results from "Stresscoat", which could serve as a rough check on the gage results.

Along the edges of the plate, the direction of the principal stress is known. Therefore, it was unnecessary to use costly rosettes along the periphery and single gages of the type A-1 were used.

A sketch showing the essential parts of the AR-1 and A-1 gages is given in Figure 3. These gages are manufactured by The Baldwin Locomotive Works, Baldwin Southwork Division of Philadelphia, Pennsylvania.

Type A-1

Scale: 2" = 1"

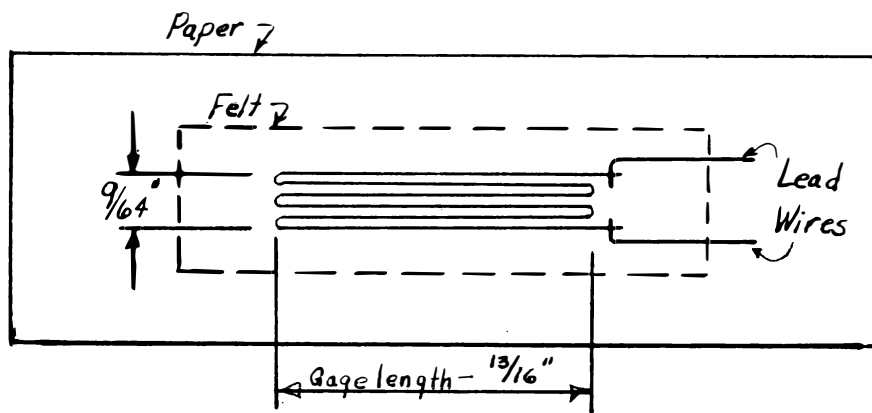
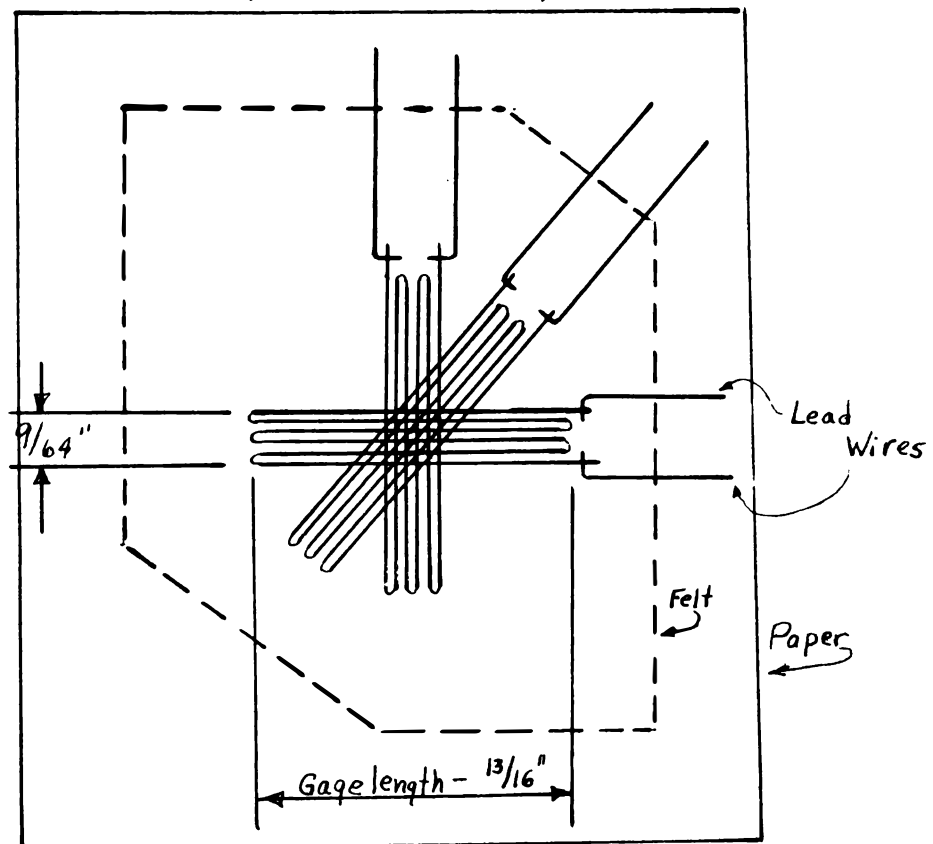
45° Rectangular Rosette - Type AR-1

FIGURE 3. SR-4 STRAIN GAGES

CHAPTER III

CONSTRUCTION OF MODEL AND LOADING FRAME

The Model

A scale drawing of the model was made as a guide for ordering the materials and to aid in the determination of the required size of the loading frame. It was previously determined that all the members would be of solid rectangular cross sections, each of the same thickness, and of sufficient length to cause the stress to be uniformly distributed near the plate. The width of each member was roughly scaled down to one fourth the width of each member of the prototype. Then the stress in each member was calculated and found to be well within the allowable stress.

A full scale drawing of the gusset plate indicated that some rivets would have to be omitted so that adequate plate area would be available for attaching the rosettes. Sandel,¹ working with a smaller plastic model was encountering the same difficulty. Consequently, the rivets in both models were arranged as shown in Figure 4. This arrangement was satisfactory for both studies. The unit shearing and bearing stresses were calculated and found to be within

¹ Sandel, loc. cit.

the allowable values.

The aluminum was ordered from J. M. Tull Metal and Supply Company of Atlanta, Georgia. The material arrived with each member and plate cut to exact size. Considerable length of $1/4$ inch round rod was ordered to be used as pins.

The drawing and materials were given to the laboratory technician for the Civil Engineering Department, who cut the pins to length, drilled and reamed the holes in the plates and members, and assembled the model. To hold the plates securely to the members, about one third of the pins were removed and $1/4$ inch diameter brass bolts were used.

Holes were drilled in each member for a one inch diameter pin, through which the load would be applied to each member.

A drawing of the model giving all pertinent dimensions is shown in Figure 4.

The Loading Frame

To establish the required size of the frame, it was necessary first to decide how the load would be applied. Various methods were considered including jacking, turnbuckles, and others. The device finally selected seemed to be the simplest and required a minimum of working space (See Chapter IV for details).

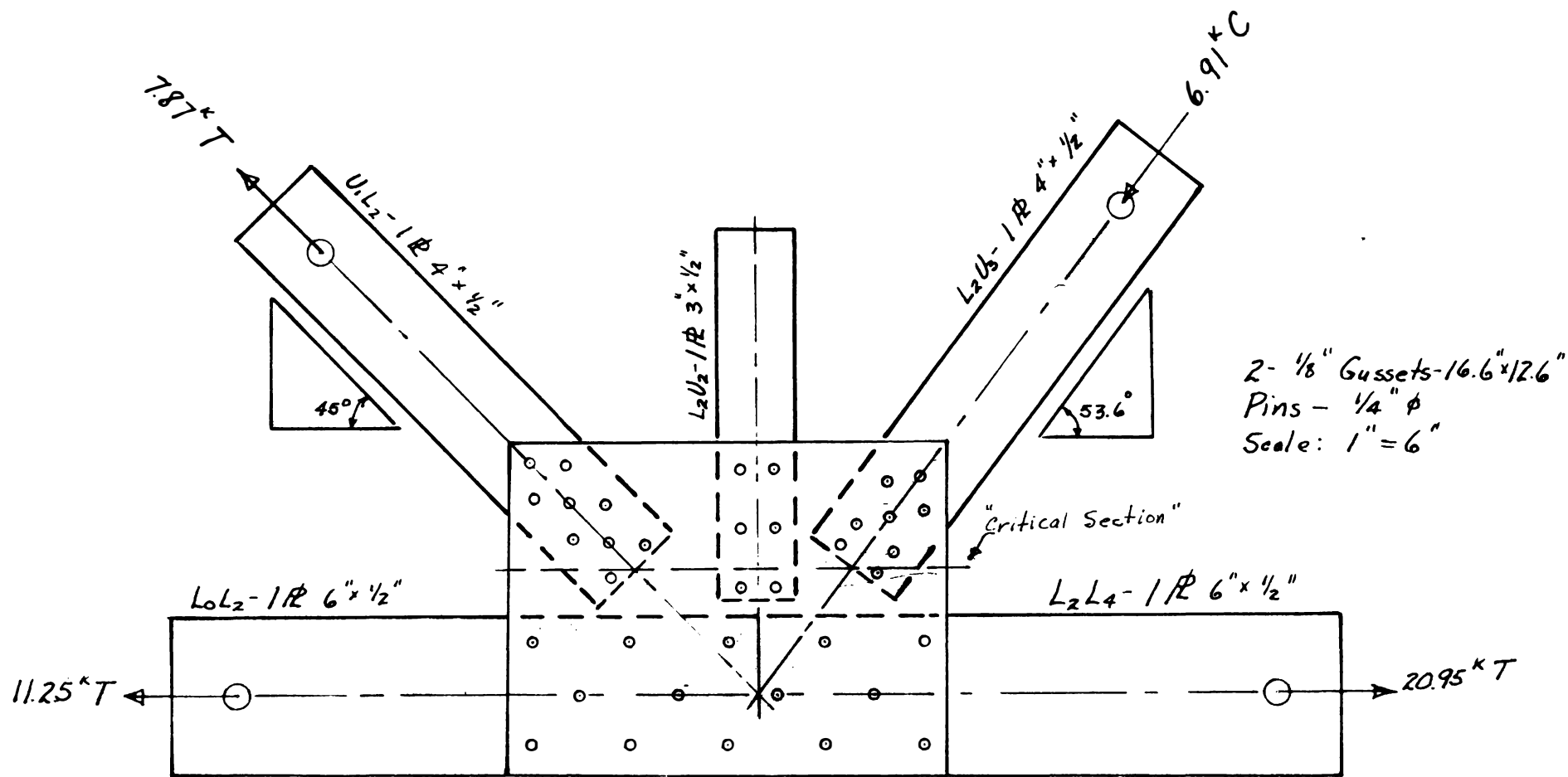


FIGURE 4. THE ALUMINUM MODEL

A sketch of the model and clevises was made, and the frame was merely fitted around this scheme. This fixed the dimensions of the frame. The cross bars to accommodate the diagonals were fixed in direction, as they must be perpendicular to the diagonal members. For rigidity, all connections were to be welded. The original plans specified that 6 inch American Standard steel beams at 14.75 pounds would be used.

The specified beam was not available locally. To eliminate delay, two and one-half inch standard steel pipe was substituted, as sufficient length was in stock. This pipe has a section modulus of 1.06 inches cubed; slightly less than the 6 inch beam. This substitution was later regretted because considerable deformation of the frame occurred when the load was applied to the members.

The pipe was cut to the proper length and welded together in accordance with the plans. The holes to accommodate the clevises were located and then cut with a torch, and one and one-half inch diameter pipe inserted in the cut holes and welded to the frame. This served to reinforce the pipe at the point where the bearing stress would be quite high.

Finally, legs of one and one-half inch pipe were welded to the frame to place the plate at a workable height, and to give a stable base. The legs are about 15 inches

high and the frame roughly 45 inches by 58 inches.

The clevises of one-fourth inch steel plate and one inch diameter cold rolled steel bars were made, the holes drilled, one inch pins cut and the model, clevises and frame assembled. Figure 5 shows the final assembly.

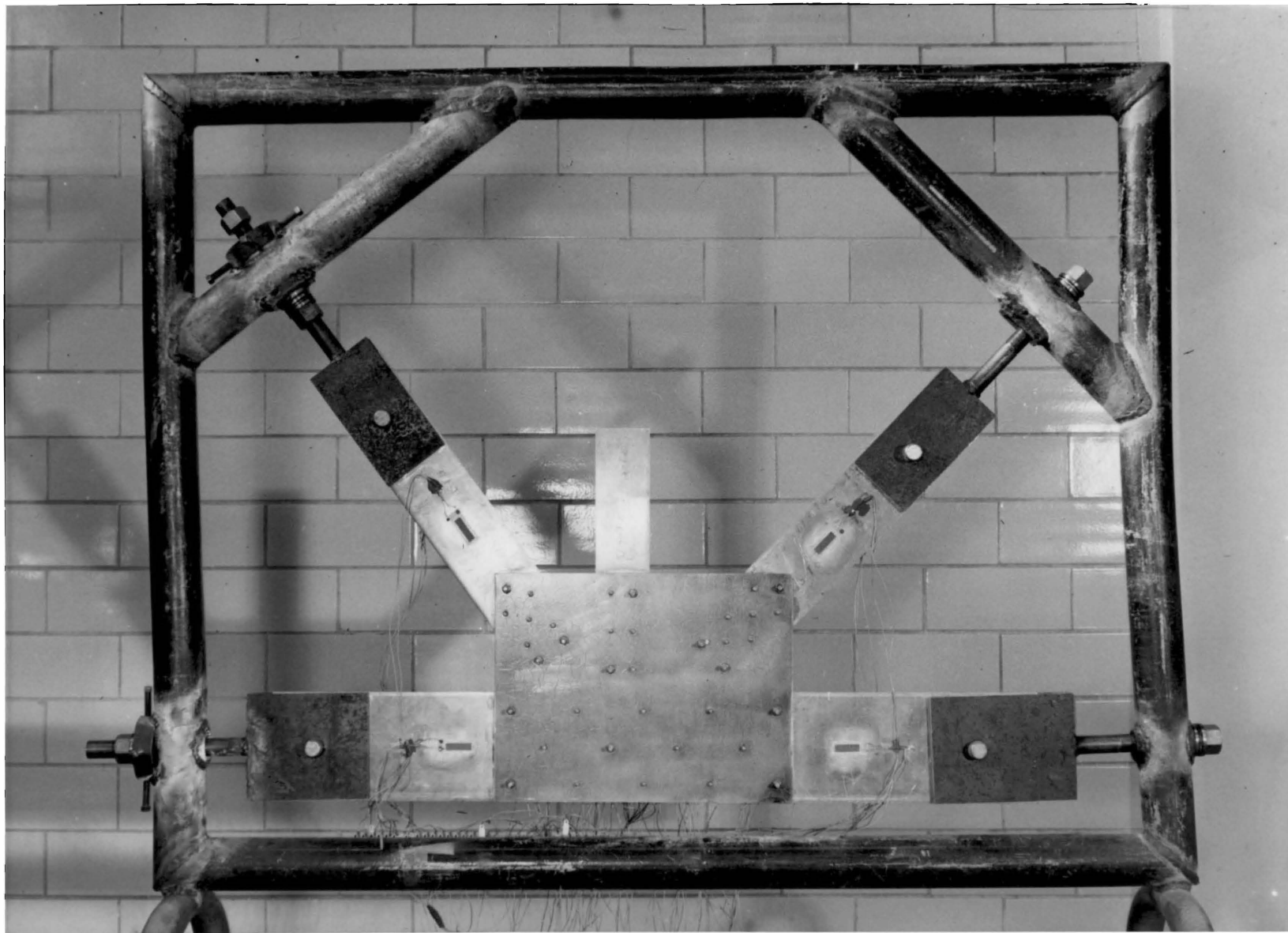


FIGURE 5. GENERAL VIEW OF MODEL, LOADING FRAME AND CLEVISES

CHAPTER IV

LOADING THE MODEL

Applying the Loads

The method of loading the members will be apparent from an examination of Figure 2. By tightening the nut on the outside of the frame, the clevis would be pulled up. The clevis is attached to the truss member by means of a one inch diameter pin. Therefore, as the nut on the outside is tightened, the member is pulled, applying tension to the member.

Moving the nut on the inside of the frame up on the threads, causes the clevis to move downward, pushing on the member (through the pin) which applies compression to the member.

Heavy washers were used to distribute the load to the material in the frame, and ball thrust bearings were used to reduce the friction between nut and washer. A specially made, long handle wrench was used to twist the nuts.

Originally, the clevis arrangement was used only on members $L_0 L_2$ and $U_1 L_2$, the others being attached to the frame. This arrangement was not satisfactory, because this caused the gusset to move to the left and upward. This

changed the direction of the members, which induced bending. Therefore, the clevis arrangement was added to the other members so that the members could be loaded with very little movement of the gusset plate.

In order further to reduce the bending, rings with four set screws were attached to the frame so that the clevises attached to members $L_2 U_3$ and $L_2 L_4$ could be shifted slightly in direction. This device, which is shown clearly in Figure 2, helped to make the loads axial but was not entirely adequate. Due to bending of the frame, the direction of the cross members changed and the location of the holes was shifted. Furthermore, bending perpendicular to the plane of the plate was detected and could not be removed entirely. However, it is believed that the small amount of bending which might have been present had very little effect on the final stresses.

Measuring the Loads

Maximum dead plus live load stress does not occur simultaneously in each member of the prototype. In the analytical check of stresses in the gusset plate, maximum loads in both diagonals are generally used. This situation is impossible to duplicate in the model, because of the fact that the equations of statics must be satisfied. Therefore,

the maximum total stress in each member meeting at joint L_2 was adjusted to make the summation of the horizontal and vertical components equal to zero. These were reduced to the magnitudes shown in Figure 4. It was desired to apply these loads to the members of the model.

Three SR-4, type A-1, gages were attached to each member to be loaded. One gage was located on the axis of the member on one side, and two gages spaced equidistant from the axis were located on the opposite side. All gages were placed in the direction of the member. The purpose of this arrangement was to detect bending in the plane of the truss and, also, perpendicular to it.

The gages were cemented to the surface, following the recommended procedure of the manufacturer, and flexible, light weight lead wires were soldered to the gages. One lead wire from each gage was connected to a Baldwin SR-4 twenty channel switching unit, and the other lead wires brought to a common point and connected to a Baldwin SR-4, Model K strain indicator. To compensate for temperature strain, a similar A-1 gage was attached to an unstrained piece of 61 ST aluminum and connected to the switching unit and indicator. The circuit was then completed by connecting the switching unit to the indicator. The gages were numbered 1 through 12 for identification. An illustration of the general arrangement is shown in Figure 6.

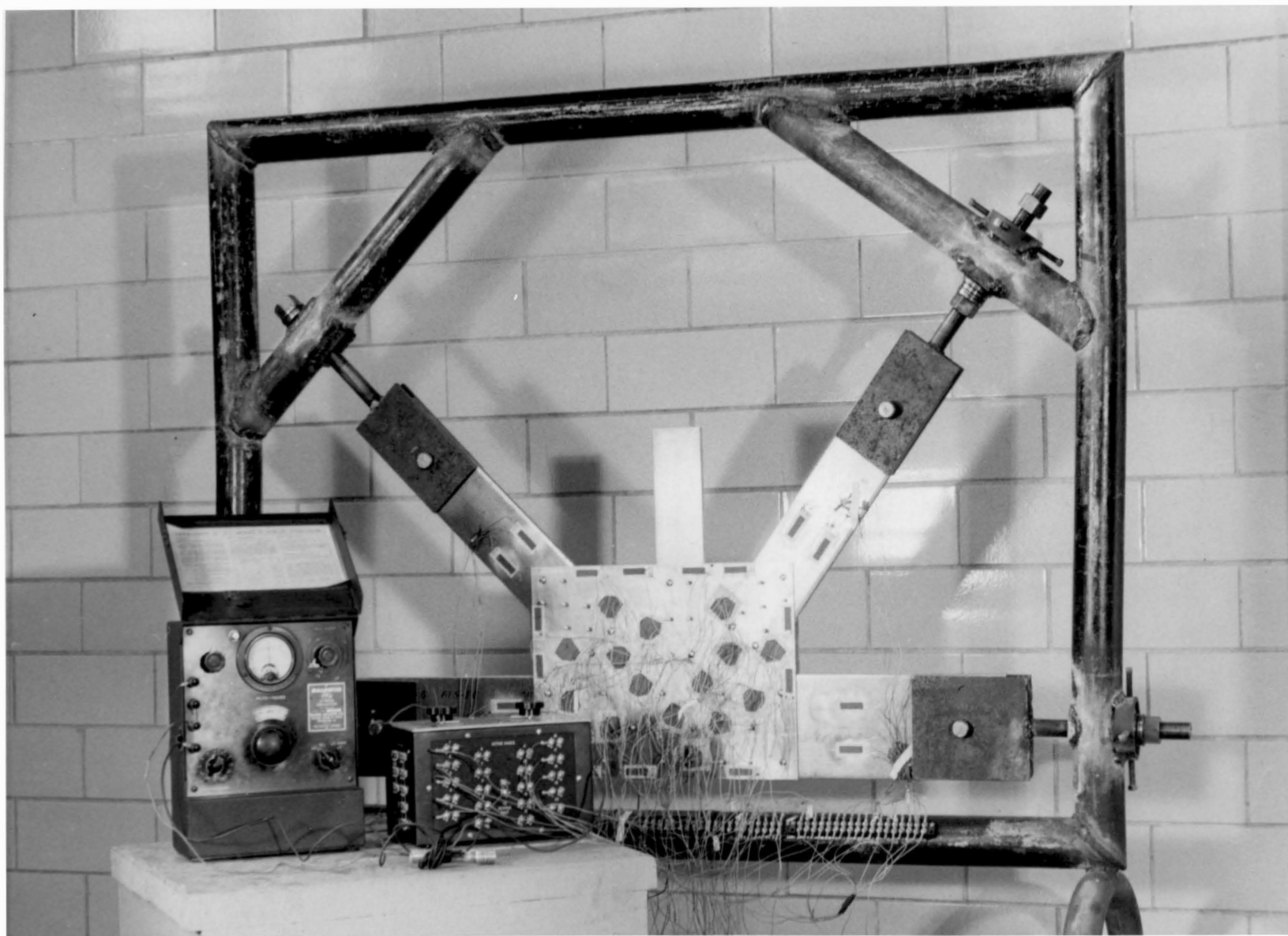


FIGURE 6. GENERAL VIEW OF MODEL, GAGES AND STRAIN MEASURING EQUIPMENT

With zero load applied to the members, the indicator was read for each gage. The indicator reading each gage should have at full load could then be calculated as illustrated in the following example:

Member U1 L2
 Total stress to be applied - 7.87^k tension
 Cross-sectional area - $(4)(1/2) = 2.0$ sq. inches
 Unit stress: $\sigma = P/A = 7.87/2 = 3.93^k$ per sq. inch
 Unit strain: $\epsilon = \sigma/E = 3.93/10,000 = 0.000393$ "/"
 Say, plus 390 micro inches per inch

Gage No. 1

Zero load reading of the indicator 0-10-1460
 Full load reading of the indicator should be
 1460 plus 390 = 1850 or:
 Range extendor position - 0
 Reference switch ----- 10
 Dial reading ----- 1850

This method was followed for each gage and the loads were increased until each gage had the proper indicator reading. The calculated and actual values are listed in Table I. Exact indicator readings were not considered necessary so there are some discrepancies. Furthermore, gage No. 7 was inoperative for some unknown reason, so it is not definitely known whether the bending perpendicular to the truss was entirely removed. However, for all practical purposes, the load in each member was axial and of the desired amount.

TABLE I

COMPARISON OF CALCULATED AND ACTUAL STRAIN
INDICATOR READINGS FOR LOAD MEASURING GAGES

Gage Number	Zero Load Reading	Calc. Strain Micro $\text{"}/\text{"}$	Calc. Full Load Reading	Actual Full Load Reading
1	0-10-1370	390 - -	0-10-1760	0-10-1750
2	0-10-830	390	0-10-1220	0-10-1200
3	0-10-1110	390	0-10-1500	0-10-1520
4	0-10-590	375	0-10-965	0-10-660
5	0-10-590	375	0-10-965	0-10-1000
6	0-10-50	375	0-10-425	0-10-430
7	0-10-60	695	0-10-755	No reading
8	0-9-450	695	0-9-1145	0-9-1120
9	0-9-430	695	0-9-1125	0-9-1110
10	0-9-1870	-345	0-9-1525	0-9-1510
11	0-9-1370	-345	0-9-1025	0-9-1020
12	0-9-1820	-345	0-9-1475	0-9-1470

CHAPTER V

LABORATORY ANALYSIS OF PRINCIPAL STRAINS BY MEANS OF A BRITTLE LACQUER

Procedure

The Magnaflux Corporation of Chicago, Illinois manufactures a highly sensitive brittle lacquer under the trade name of "Stresscoat". It is marketed in complete kits which include a variety of lacquers, aluminum paint, red dye, air pumps, air brush, calibration device and other accessories. This equipment was borrowed from the Mechanical Engineering Department for use in this study, and the essential items used are shown in Figure 7.

The first step in the procedure is to paint the surface to be tested with an aluminum paint to brighten the surface. This was sprayed on one side of the gusset plate. Then the wet and dry bulb temperatures were taken with a sling psychrometer and the proper lacquer for that temperature and humidity selected from a chart furnished by the manufacturer. For the first test, the dry bulb temperature was 69°F and the wet bulb temperature 58°F, so a coating identified as ST-1203 was selected.

With no load on the members, this lacquer was sprayed on the plate and allowed to dry twenty-four hours. The



FIGURE 7. THE STRESSCOAT EQUIPMENT

members were loaded to full load. However, no cracks appeared in the coating, which indicated that the coating was not brittle enough to crack at the small magnitude of strains in the plate. Therefore, a more sensitive coating, ST-1210, was selected, the old coating removed, and the new sprayed on. After drying for twenty-four hours, the model was loaded and again no cracks appeared. Ice water was applied to the surface and finally the red dye etchant applied to bring out the cracks but none were detected.

As it was impossible to load the members higher because of the weakness of the frame, it was decided to test a small plastic model. The model used was one Sandel¹ had previously tested in his photoelastic studies. The plate was of Marblette which had a low modulus of elasticity, yielding strains of high magnitude.

This model was cleaned, coated with aluminum, and sprayed with the proper coating, ST-1209, and tested in the loading frame used by Mr. Sandel. The crack pattern in this case was quite pronounced.

Results

No quantitative results were obtained as it was impossible to hold the temperature and humidity constant.

¹ Sandel, loc. cit.

Temperature fluctuations of plus or minus 5°F prevent quantitative conclusions.² An air-conditioned room for testing is practically a necessity.

However, the qualitative results were quite definite. A sketch of the crack pattern is shown in Figure 8. Since the cracks are orthogonal to the direction of maximum tension, it is quite clear where the serious tensile strains are located. The cracks indicate that the tensile force travels a path from the upper left hand corner ($U_1 L_2$) to the lower right side of the plate ($L_2 L_4$). This, of course, seems an obvious situation when the nature of the forces is carefully considered.

The first cracks appeared near the end of the tension diagonal, indicating that this is the region of maximum tensile strains. For isotropic materials, the principal directions of stress and strain are coincident. Therefore, it can be concluded that the maximum tensile stresses in the plate are near the end of the tension diagonal and parallel to the direction of the member.

² George H. Lee, An Introduction to Experimental Stress Analysis (New York: John Wiley and Sons, Inc., 1950), p. 249.

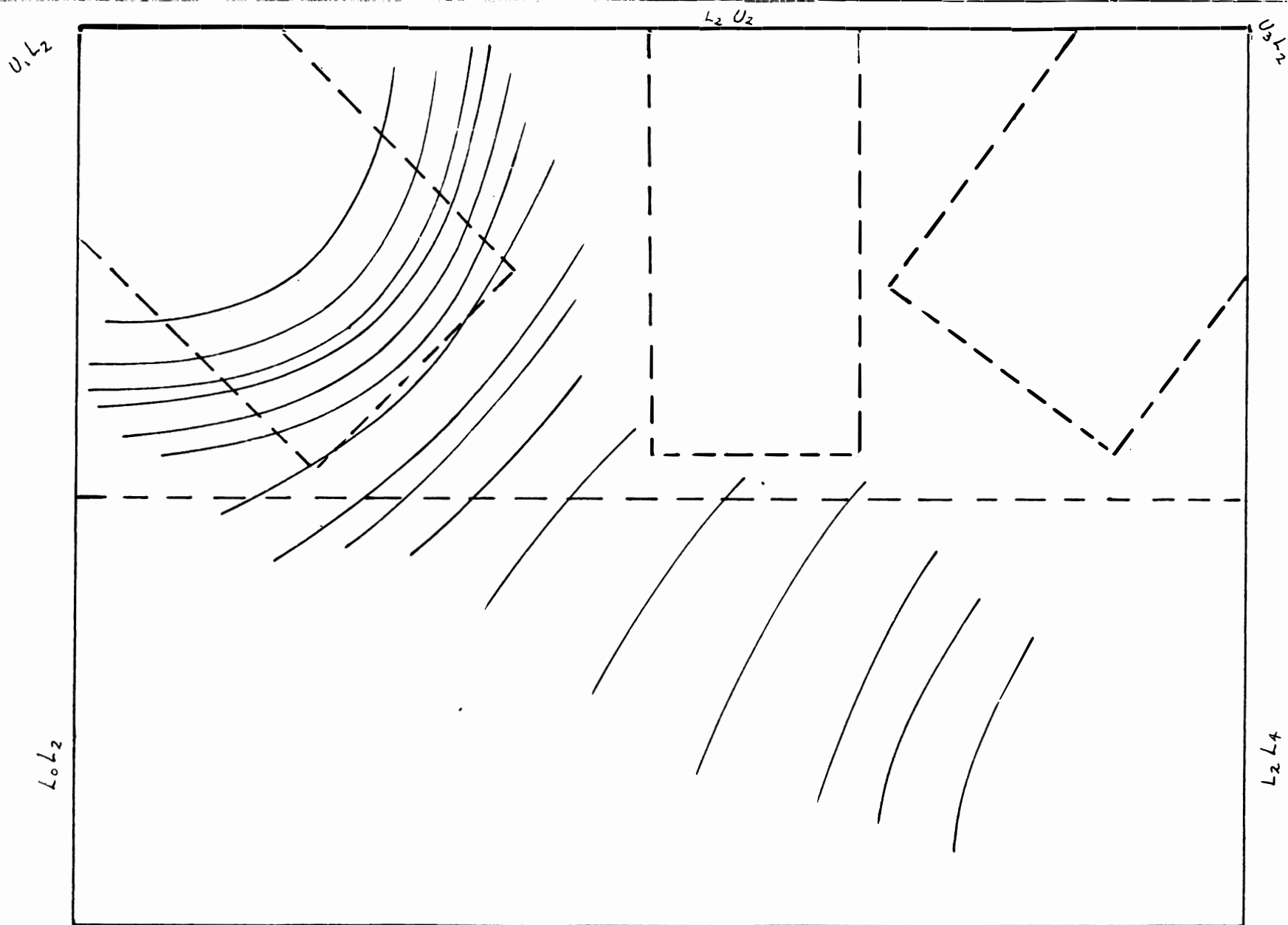


FIGURE 8. TENSION CRACKS IN STRESSCOAT-MARBLETTE MODEL

CHAPTER VI

LABORATORY ANALYSIS OF STRAINS BY MEANS OF WIRE BONDED STRAIN GAGES

Procedure

Fifteen type AR-1, 45° rectangular rosettes, and eleven type A-1, flat grid single gages were used to measure strains in the gusset plate. The rosettes were used on the inner portion of the plate, while the gages were attached to the edges where the direction of the principal strains and the magnitude of one of the principal strains are known.

First, the surface of one of the plates was rubbed with a fine emery cloth, to clean and slightly roughen the surface. The surface was cleaned with acetone and coated with SR-4 pre-coat cement. The area where each gage was to be located was coated with SR-4 cement and the gages attached.

The gages were located in such a manner that the entire plate could be analyzed for stresses, but the regions where the stresses were known to be critical were emphasized in locating the gages. This information was obtained from the "Stresscoat" and analytical studies. In other words, the regions of maximum tension and compression are quite obvious and more gages were used on these areas than

on the other, less highly-stressed areas. To obtain best results, the attempt was made to place the perpendicular gages of the rosettes in the directions of principal strains. The results of the "Stresscoat" analysis aided greatly in this determination. It is known that the stress perpendicular to the edge is zero, and parallel to the edge is one of the principal stresses. Consequently, all of the single gages were located as near the edge as possible, and parallel to the edge.

The location of each gage was determined and the direction of the gages measured and recorded. The rosettes were marked A through O for identification and each gage was identified by the numbers 1, 2 and 3. The edge gages were marked P through Z. Figures 9 and 10 show the location of gages.

After the proper drying period, flexible, standard lead wires were soldered to the lead wires of each gage. One lead wire from each gage was brought to a common point and wires were soldered together. As there were fifteen rosettes and three gages to each rosette, there were forty-five gages in the rosettes alone. Since the switching unit could accommodate only 20 gages at one time, the rosette gages were divided into three groups of fifteen each. This made three common point connections for the rosettes. The eleven A-1 gages were handled as a separate unit and connect-

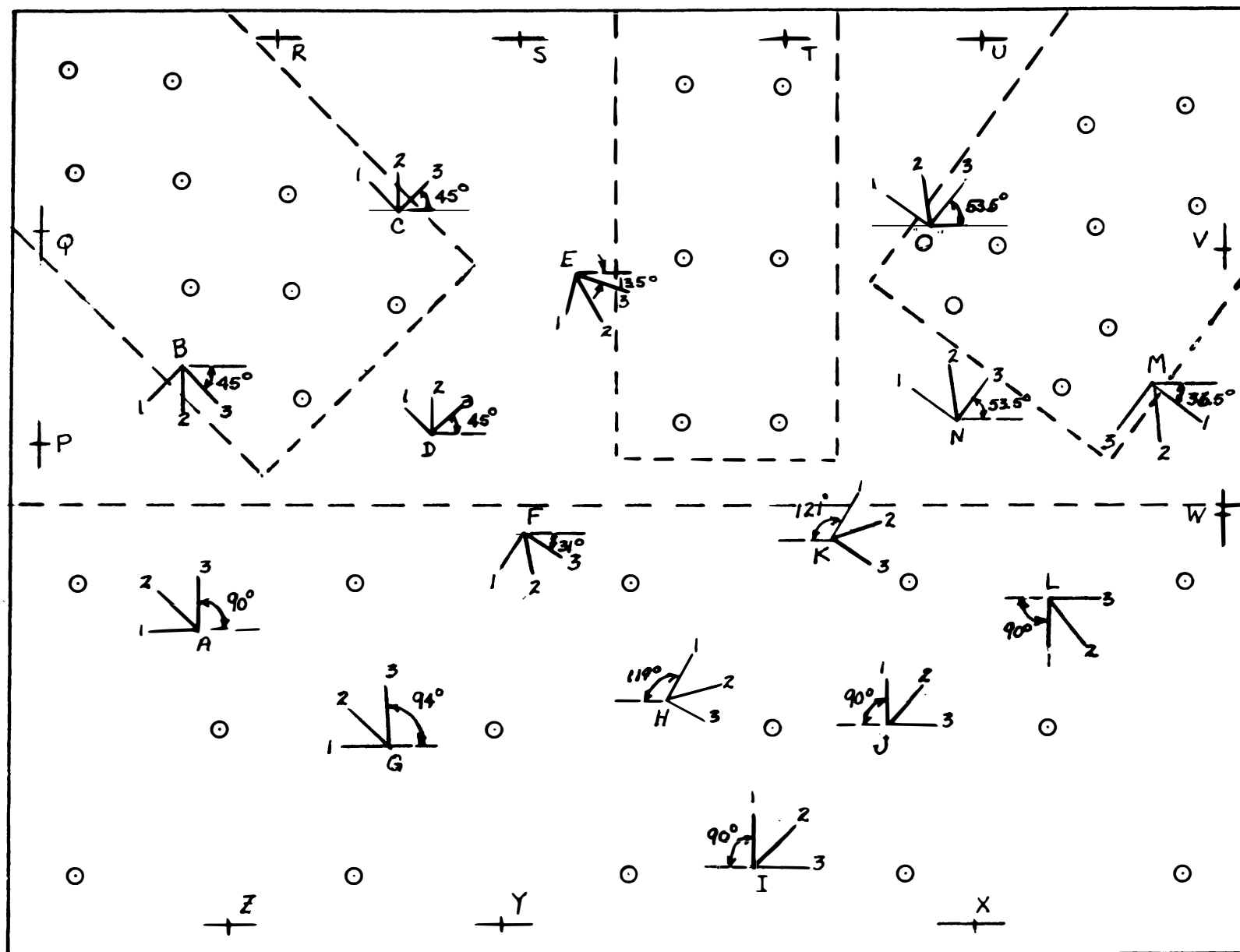


FIGURE 9. LOCATION AND DIRECTION OF GAGES ON GUSSET PLATE

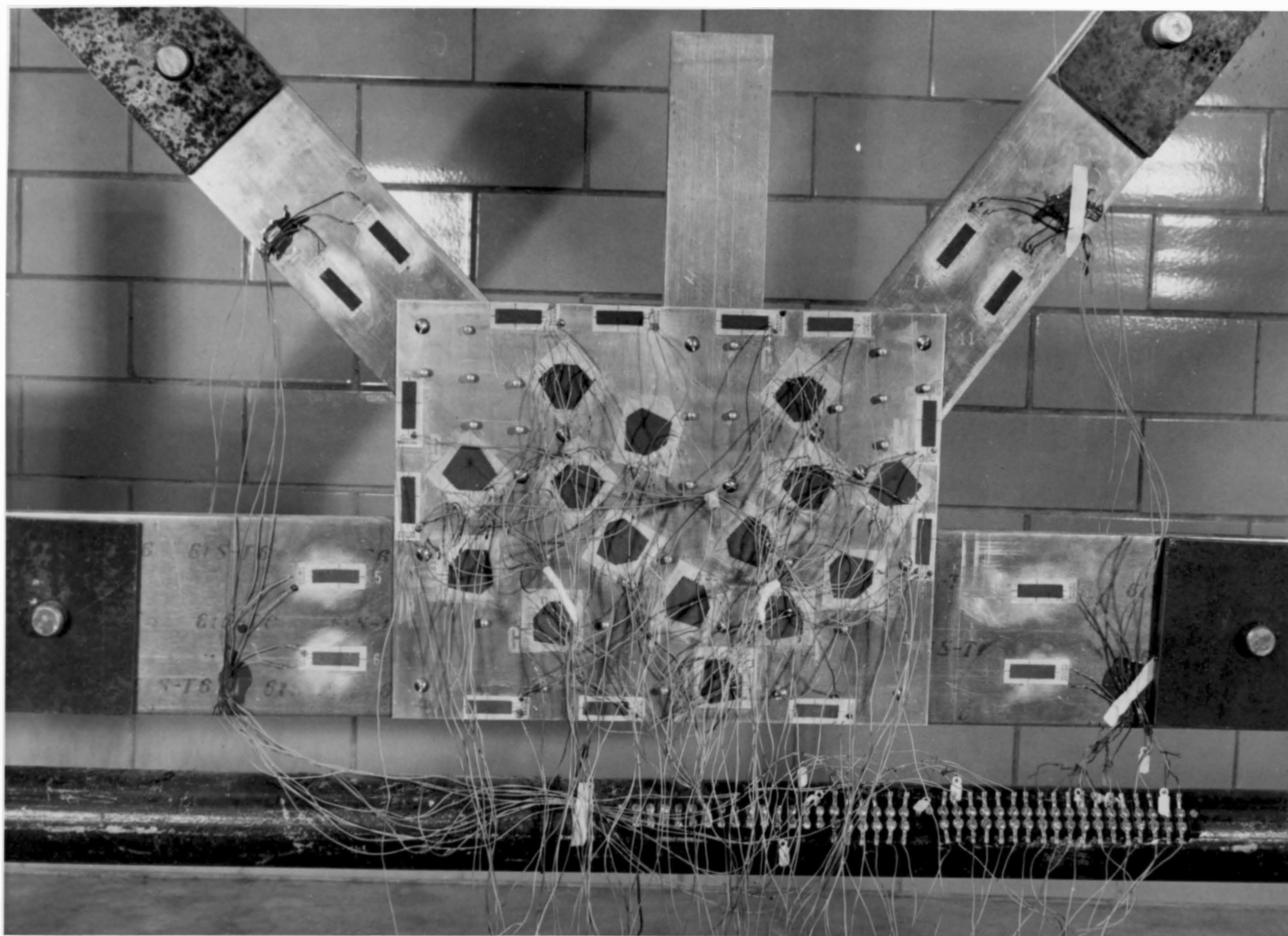


FIGURE 10. CLOSE-UP OF SR-4 GAGES AND LEAD WIRES

ed together at a common point.

With no load on the model, gages A-1 through E-3 were connected to the switching unit to positions 1 through 15. The common point lead wire was connected to the indicator, the dummy gage attached, the circuit closed, and readings of each gage taken and recorded. The wires were disconnected, and gages F-1 through J-3 connected to the switching unit and indicator readings recorded for this set. Then gages K-1 through O-3 were connected and readings of the strain indicator recorded. Next, the single gages P through Z were connected to positions 1 through 11 and readings were similarly recorded. Finally, the gages attached to the members (called gages 1 - 12) were connected and "no load" readings taken.

Calculations similar to those in Chapter V gave the necessary indicator reading for the desired axial load in each member. The model was loaded until gages 1 - 12 read nearly the calculated value. Again, gage number 7 was inoperative.

With the full loads on the gusset plate, gages 1 - 12 were disconnected and, successively, all gages connected to the switching unit and indicator and full load strain readings recorded.

Results

In Table II the no load and full load indicator readings are tabulated. The difference of the two readings is also given for each gage. These differences represent the unit strain measured in micro inches per inch of length. If the difference is plus, the strain is tensile; if it is minus, the strain is compressive.

TABLE II

UNIT STRAINS OF GAGES ATTACHED TO GUSSET PLATE ^a

Gage Reading	Zero Load Indicator Reading	Full Load Indicator Reading	Unit Strain Micro $\frac{1}{2}$ "
A-1	0-8-690	0-7-1720	30
A-2	0-6-835	0-6-980	145
A-3	0-3-1490	0-3-1500	10
B-1	0-4-1010	0-4-1050	40
B-2	0-8-150	0-7-1430	280
B-3	0-8-790	0-8-1050	260
C-1	0-8-1280	0-8-1540	260
C-2	0-8-1385	0-8-1350	-35
C-3	0-7-640	0-7-620	-20
D-1	0-7-1440	0-8-780	340
D-2	0-6-870	0-6-960	90
D-3	0-2-1465	0-2-1290	-175
E-1	0-2-1740	0-3-650	-90
E-2	0-5-1210	0-5-1360	150
E-3	0-7-1210	0-7-1390	180
F-1	0-4-935	0-4-740	-195
F-2	0-7-725	0-7-840	115
F-3	0-7-1260	0-7-1560	300
G-1	0-8-1160	0-8-1230	70
G-2	0-6-710	0-6-845	135
G-3	0-4-645	0-3-1595	-50
H-1	0-8-900	0-8-720	-180
H-2	0-7-1320	0-8-350	30
H-3	0-6-635	0-6-920	285
I-1	0-7-1195	0-7-1125	-70
I-2	0-7-650	0-7-610	-40
I-3	0-2-1250	0-2-1350	100

^a Note: Plus strains are tensile and - strains compressive.

TABLE II

UNIT STRAINS OF GAGES ATTACHED TO GUSSET PLATE (continued)

Gage Reading	Zero Load Indicator Reading	Full Load Indicator Reading	Unit Strain Micro μ / μ
J-1	0-7-625	0-7-490	-135
J-2	0-6-1425	0-6-1335	-90
J-3	0-3-815	0-3-1030	215
K-1	0-7-1085	0-7-855	-230
K-2	0-6-605	0-5-1540	-65
K-3	0-4-1095	0-4-1395	300
L-1	0-4-1050	0-4-1010	-40
L-2	0-8-215	0-7-1370	155
L-3	0-8-565	0-8-610	45
M-1	0-7-1380	0-7-1410	30
M-2	0-6-875	0-6-540	-335
M-3	0-2-1190	0-2-910	-280
N-1	0-7-255	0-6-1455	200
N-2	0-5-1080	0-5-960	-120
N-3	0-2-760	0-1-1370	-390
O-1	0-7-1620	0-8-625	5
O-2	0-8-695	0-8-705	10
O-3	0-6-810	0-5-1585	-225
P	0-7-835	0-7-895	60
Q	0-5-730	0-5-1020	290
R	0-8-835	0-8-1105	270
S	0-8-790	0-8-830	40
T	0-8-960	0-8-810	-150
U	0-9-1150	0-9-890	-260
V	0-8-760	0-8-620	-140
W	0-6-1600	0-6-1530	-70

TABLE II

UNIT STRAINS OF GAGES ATTACHED TO GUSSET PLATE (continued)

Gage Reading	Zero Load Indicator Reading	Full Load Indicator Reading	Unit Strain Micro $\frac{1}{2}$ "
X	0-7-1040	0-7-1180	140
Y	0-9-540	0-9-670	130
Z	0-7-610	0-7-640	30

CHAPTER VII

CALCULATION OF UNIT STRESSES FROM STRAIN GAGE READINGS

Calculation of Principal Strains

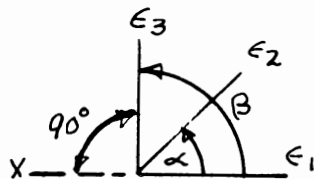
Principal strains may be evaluated from two orthogonal normal strains and the accompanying shearing strain.¹ However, since most strain measuring equipment has been designed to measure normal rather than shearing strains, a means of evaluating the principal strains from normal strains alone is essential. This is quite simple mathematically, and various analytical, semi-graphical and graphical methods have been developed to convert three normal strains measured in known directions relative to one another to principal strains.

To illustrate the procedure, an analytical solution and a graphical solution are given below. The analytical solution (Figure 11) is taken from a method outlined in Murphy². The graphical solution, which is essentially a Mohr circle construction (Figure 12) is taken from a

¹ Glenn Murphy, Advanced Mechanics of Materials (New York: McGraw-Hill Book Co., Inc., 1946), p. 47.

² Ibid., p. 48

Gage "J"



Note that directions 1 and 3 have been interchanged to conform to Murphy's notation.

Equations:

$$1) \epsilon_1 = \frac{\epsilon_u + \epsilon_v}{2} + \frac{(\epsilon_u - \epsilon_v)}{2} \cos 2\theta$$

$$2) \epsilon_2 = \frac{\epsilon_u + \epsilon_v}{2} + \frac{(\epsilon_u - \epsilon_v)}{2} \cos 2\theta \cos 2\alpha - \frac{(\epsilon_u - \epsilon_v)}{2} \sin 2\theta \sin 2\alpha$$

$$3) \epsilon_3 = \frac{\epsilon_u + \epsilon_v}{2} + \frac{(\epsilon_u - \epsilon_v)}{2} \cos 2\theta \cos 2\beta - \frac{(\epsilon_u - \epsilon_v)}{2} \sin 2\theta \sin 2\beta$$

Procedure:

ϵ_1 , ϵ_2 , ϵ_3 , α , and β are known, therefore, the three equations may be solved simultaneously for ϵ_u , ϵ_v and θ (the angle gage line 1 makes with the principal strain.)

Data:

$$\epsilon_1 = +215, \quad \epsilon_2 = -90, \quad \epsilon_3 = -135 \text{ micro"/} \\ \alpha = 45^\circ, \quad \beta = 90^\circ$$

Solution:

$$+215 = \frac{\epsilon_u + \epsilon_v}{2} + \frac{(\epsilon_u - \epsilon_v)}{2} \cos 2\theta$$

$$-90 = \frac{\epsilon_u + \epsilon_v}{2} + \frac{(\epsilon_u - \epsilon_v)}{2} \cos 2\theta \cos 90^\circ - \frac{(\epsilon_u - \epsilon_v)}{2} \sin 2\theta \sin 90^\circ$$

$$-135 = \frac{\epsilon_u + \epsilon_v}{2} + \frac{(\epsilon_u - \epsilon_v)}{2} \cos 2\theta \cos 180^\circ - \frac{(\epsilon_u - \epsilon_v)}{2} \sin 2\theta \sin 180^\circ$$

Results:

$$\epsilon_u = +258 \text{ micro"/}$$

$$\epsilon_v = -178 \text{ micro"/}$$

$$\theta = -18\frac{1}{4}^\circ$$

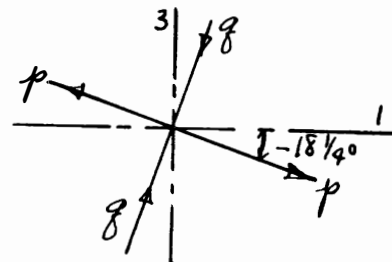


FIGURE 11. MURPHY'S ANALYTICAL SOLUTION FOR PRINCIPAL STRAINS

Gage J

Data:

$$\epsilon_1 = -135, \quad \epsilon_2 = -90, \quad \epsilon_3 = +215 \text{ micro"/"}$$

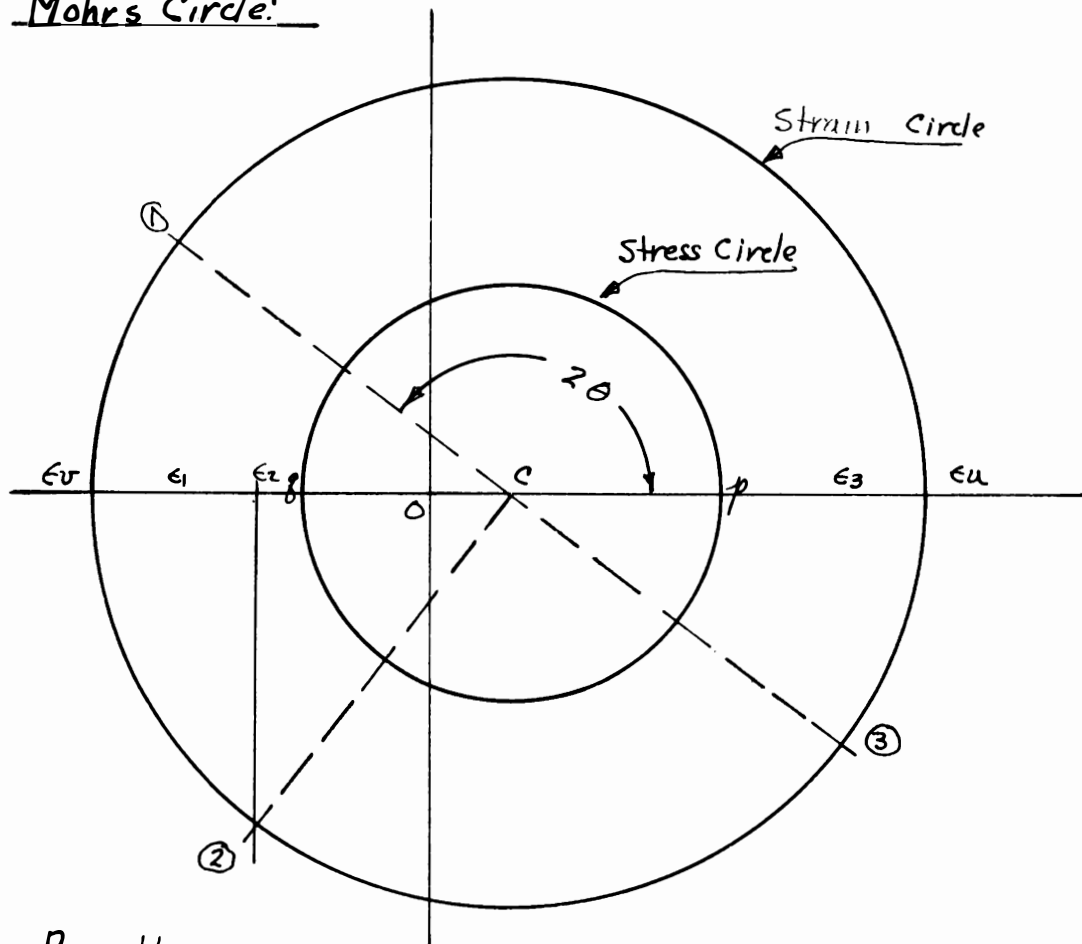
Scales:

Strain circle - $1'' = 100 \text{ micro"/"}$

Stress circle - Radius = $\frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{1}{2}$ radius of strain circle

$$1'' = 100 \left(\frac{E}{\mu} \right) = 100 \left(\frac{10}{0.667} \right) = 1500 \text{ #/sq"} \quad \mu$$

Mohr's Circle:



Results:

$$\epsilon_u = +258 \text{ micro"/"}, \quad \epsilon_v = -177 \text{ micro"/"}$$

$$p = (15)(149) = +2,240 \text{ #/sq"}, \quad q = (15)(-68) = -1,020 \text{ #/sq"} \quad \mu$$

$$2\theta = 144^\circ; \quad \theta = 72^\circ$$

FIGURE 12. GRAPHICAL SOLUTION FOR PRINCIPAL STRESS AND STRAIN

Baldwin Locomotive Company bulletin.³

Principal strains and their directions for each rosette were determined graphically and these values are recorded in Table III.

Calculation of Principal Stresses from Principal Strains

Since the aluminum used in this study can be considered isotropic, the principal direction of stress and strain are coincident.

The relationship between the principal stresses and principal strains has been definitely established by the theory of elasticity. It is known that:

$$\sigma_{\max.} = \frac{E}{1 - \mu^2} (\epsilon_{\max.} + \mu \epsilon_{\min.})$$

$$\sigma_{\min.} = \frac{E}{1 - \mu^2} (\epsilon_{\min.} + \mu \epsilon_{\max.})$$

in which σ = unit normal stress

ϵ = unit normal strain

μ = poisson's ratio

E = the modulus of elasticity

Therefore, with the principal strains calculated and the elastic constants known, it is a simple matter to calculate the principal stresses.

³ F. G. Tatnall, "Concerning the Subject of Testing", SR-4 News Letter, II: 1-8, March 1948.

TABLE III

PRINCIPAL STRAINS AND DIRECTIONS AT EACH
GAGE LOCATION^a

Gage Number	Maximum Normal Strain - ϵ_u	Minimum Normal Strain - ϵ_v	Direction of ϵ_u - θ
A	145	-108	-42.5°
B	320	-23	65°
C	328	-89	24°
D	340	-175	-1°
E	216	-126	71°
F	307	-200	83°
G	150	-130	-32°
H	285	-180	-92.5°
I	117	-87	74°
J	257	-175	71.5°
K	322	-250	69.5°
L	162	-155	52.5°
M	140	-388	27°
N	200	-392	2.5°
O	56	-275	-23°
P	60	0	90°
Q	290	0	90°
R	270	0	0
S	40	0	0
T	0	-150	0
U	0	-260	0
V	0	-140	-90°
W	0	-70	-90°
X	140	0	0
Y	130	0	0
Z	30	0	0

^a Note: Strains are measured in micro "/" and θ for gages A through O are with respect to the "1" direction and for gages P through Z with respect to the horizontal. A plus angle means counterclockwise. Plus strains are tensile and - strains compressive.

The writer found that the graphical method was the most expedient. Mohr's circle for strains had been plotted previously and the stress circle could be plotted with the same center by merely changing the scale of the radius by the factor $\frac{1-\mu}{1+\mu}$. Whatever scale is used for strain must be multiplied by $E/(1-\mu)$ to find the stress.⁴

For certain types of rosettes formulas have been developed for direct calculation of p, q, and θ from the rosette readings. Formulas for direct calculation for 45° rectangular strain rosette are given by Lee.⁵

Figure 13 shows the direct analytical solution and Figure 12 the graphical solution for rosette J. Results for all gages are tabulated in Table IV.

Calculation of Maximum Shearing Stresses

The maximum shearing stress at a point is equal to one half of the difference of principal stresses at that point. Since p and q had previously been calculated for each gage location, it was a simple matter to calculate the maximum stresses.

These values for each gage are tabulated in Table IV.

⁴ Tatnall, loc. cit.

⁵ Lee, op. cit., p. 54.

Gage "J"

Formulas:

$$p = \frac{E}{2} \left[\frac{e_a + e_c}{1 - \mu} + \frac{1}{H\mu} \sqrt{2(e_a - e_b)^2 + 2(e_b - e_c)^2} \right]$$

$$q = \frac{E}{2} \left[\frac{e_a + e_c}{1 - \mu} - \frac{1}{H\mu} \sqrt{2(e_a - e_b)^2 + 2(e_b - e_c)^2} \right]$$

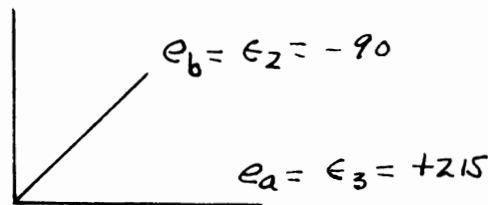
$$\theta = \frac{1}{2} \tan^{-1} \frac{(2e_b - e_a - e_c)}{(e_a - e_c)}$$

Procedure:

Substitute data in formulas and solve directly for "p", "q", and "θ"

Data:

$$e_c = \epsilon_1 = -135$$



$$E = 10$$

$$\mu = 1/3$$

Solution:

$$p = \frac{10}{2} \left[\frac{215 - 135}{0.667} + \frac{1}{1.333} \sqrt{2(215 + 90)^2 + 2(-90 + 135)^2} \right]$$

$$q = \frac{10}{2} \left[\frac{215 - 135}{0.667} - \frac{1}{1.333} \sqrt{2(215 + 90)^2 + 2(-90 + 135)^2} \right]$$

$$\theta = \frac{1}{2} \tan^{-1} \frac{(-180 - 215 + 135)}{(215 + 135)}$$

Results:

$$p = +2240 \text{ #/sq"} \\ q = -1040 \text{ #/sq"} \\ \theta = 18.3^\circ$$

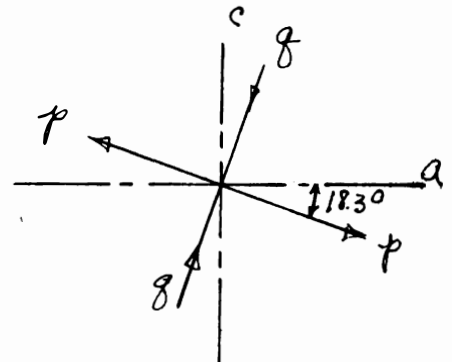


FIGURE 13. LEE'S SOLUTION FOR PRINCIPAL STRESSES

TABLE IV

MAXIMUM NORMAL AND SHEARING STRESSES^b

Gage Number	"p" psi.	"q" psi.	τ max. psi.
A	1260	-630	945
B	3540	960	1290
C	3360	225	1570
D	3170	-680	1925
E	1970	-600	1285
F	2660	-1,050	1855
G	1200	-900	1050
H	2540	-960	1750
I	990	-550	770
J	2250	-1,020	1635
K	2670	-1,620	2145
L	1240	-1,140	1190
M	140	-3,860	2000
N	830	-3,680	2255
O	-370	-2,900	1265
P	600	0	300
Q	2900	0	1450
R	2700	0	1350
S	400	0	200
T	0	-1500	750
U	0	-2600	1300
V	0	-1400	700
W	0	-700	350
X	1400	0	700
Y	1300	0	650
Z	300	0	150

^b Note: A plus stress is tensile and a - stress is compressive, τ is the symbol used to represent shearing stress.

CHAPTER VIII

DISTRIBUTION OF STRESSES IN THE GUSSET PLATE

The state of stress at a point can best be represented by the maximum and minimum normal stresses, the maximum shearing stress, and the direction of these stresses with some established reference axis.

The writer has attempted to illustrate his findings as to the distribution of stresses in the gusset plate by sketching from the experimental results:

- (a) Stress trajectories (lines so drawn that their tangents at every point are in the direction of the principal stress) for p and q (Figure 14). Due to insufficient coverage of the plate in areas which were not considered serious, stress trajectories over the entire plate could not be sketched. Consequently, some of the plate area is left blank.
- (b) Contours of maximum tensile stresses (Figure 15).
- (c) Contours of maximum compressive stresses (Figure 16).
- (d) Contours of maximum shearing stresses (Figure 17).

It should be noted that the contours of normal stresses are for maximum tension and compression and not necessarily for

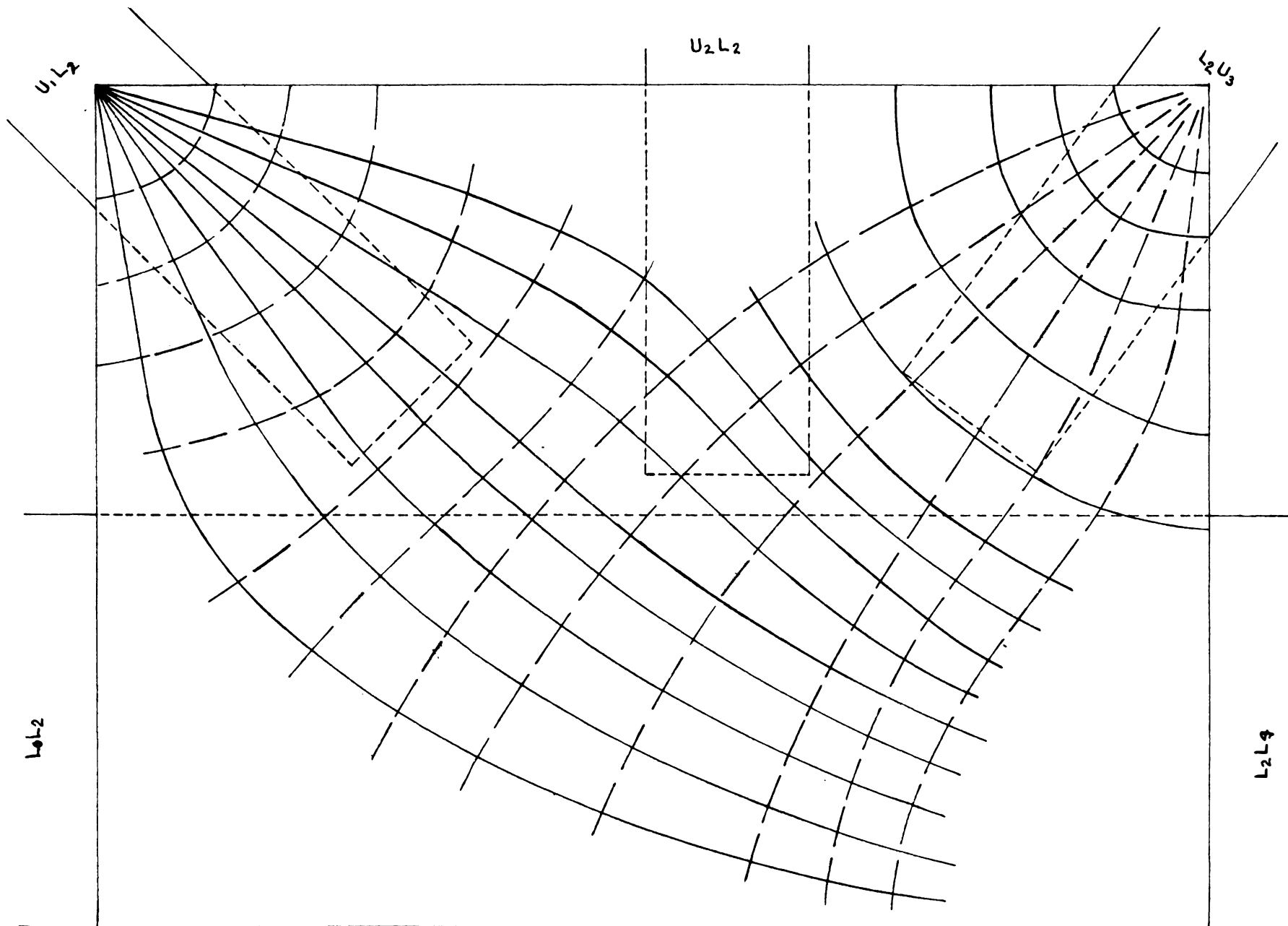


FIGURE 14. STRESS TRAJECTORIES ($p = \text{—}$; $q = \text{---}$)

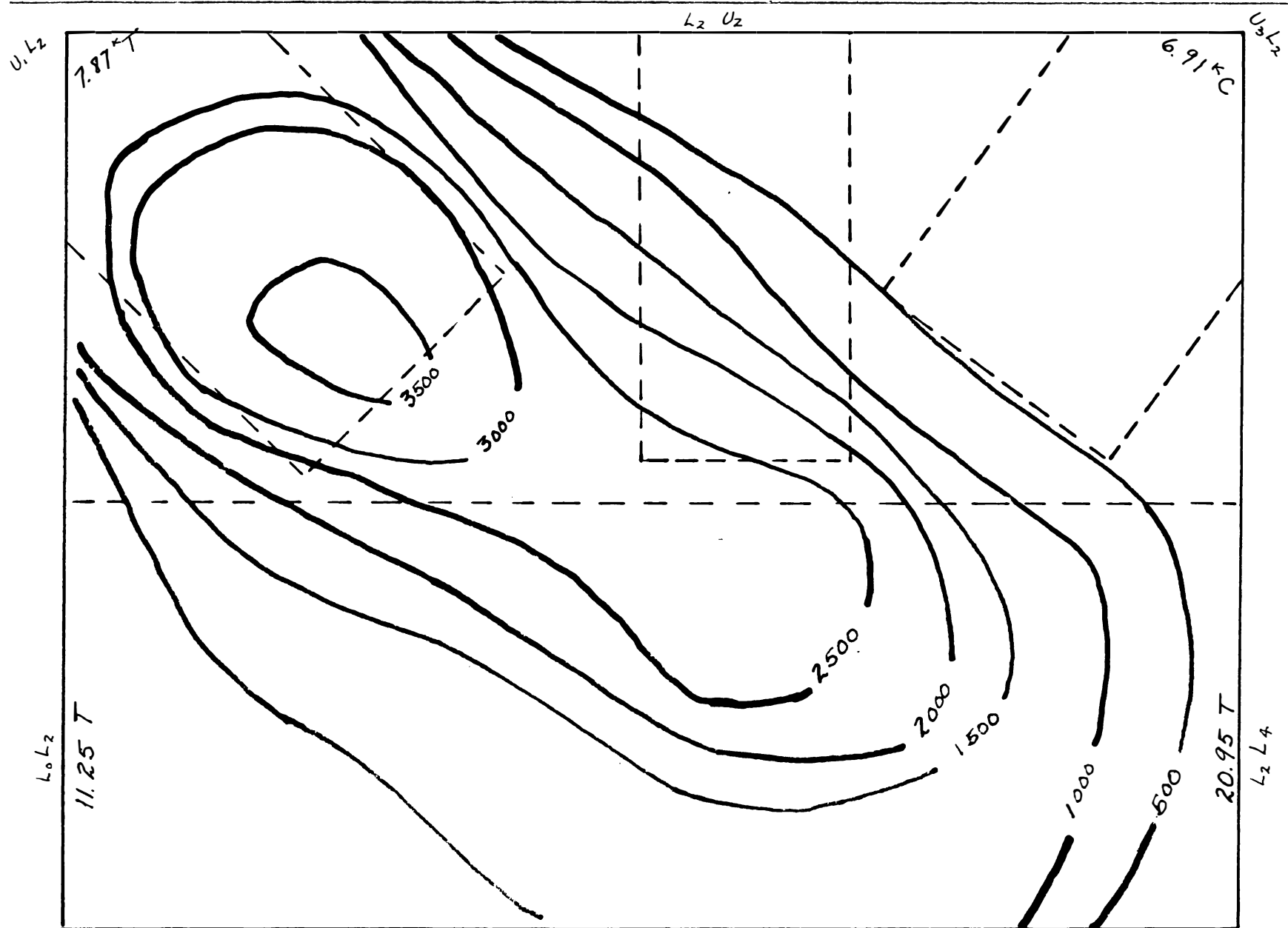


FIGURE 15. MAXIMUM TENSION CONTOURS ($\#/\text{sq}''$)

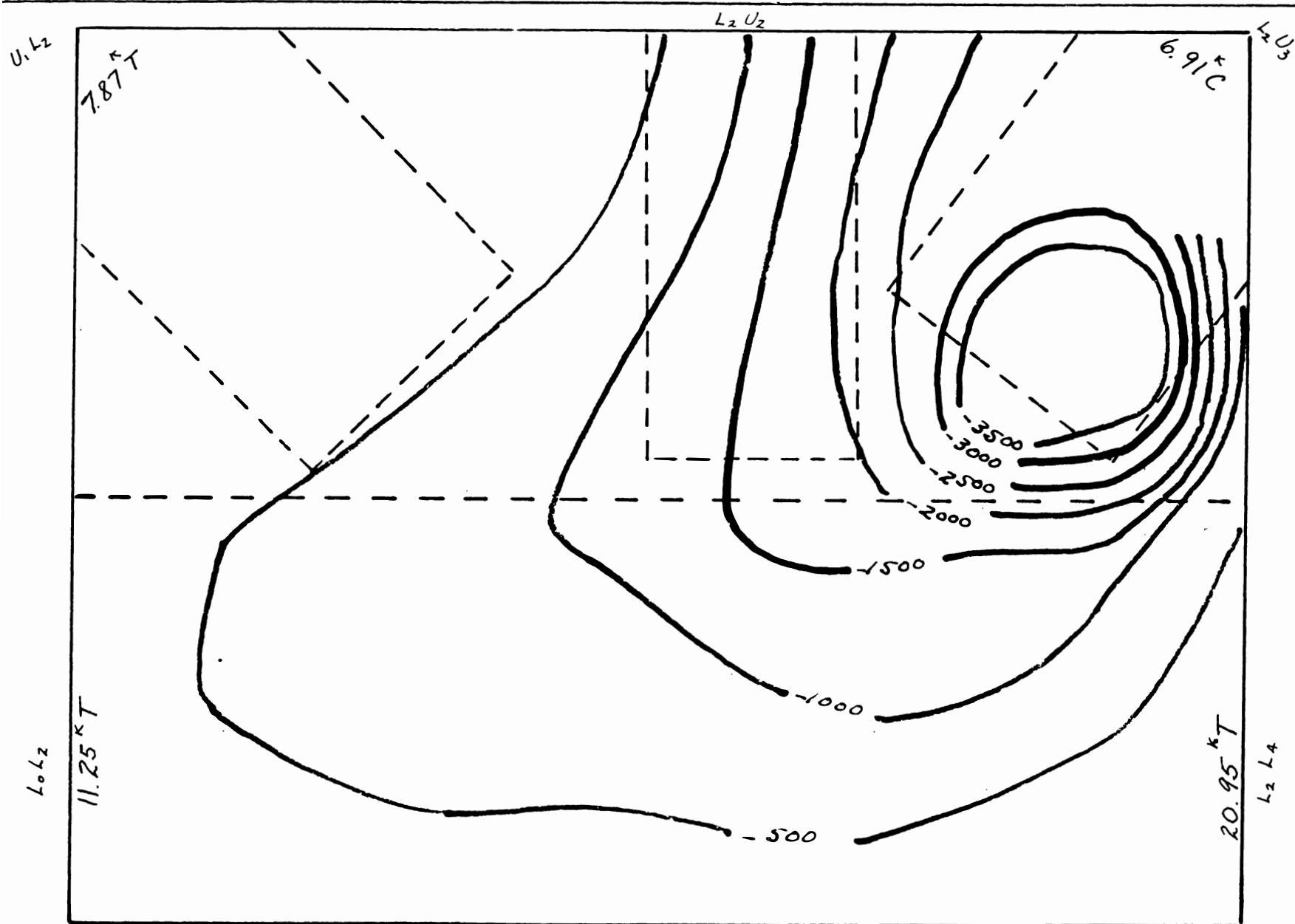


FIGURE 16. MAXIMUM COMPRESSION CONTOURS (#/sq in)

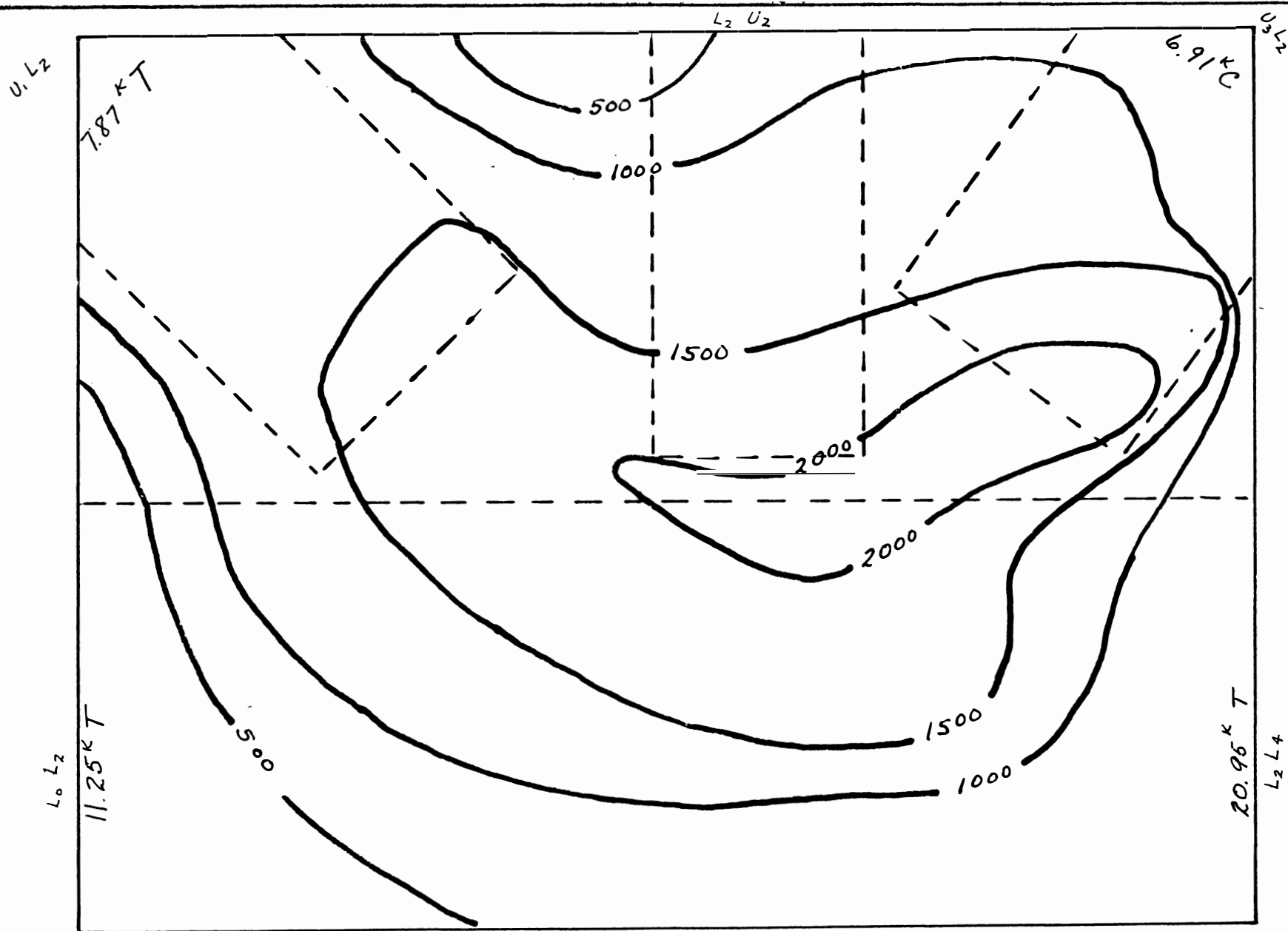


FIGURE 17. MAXIMUM SHEARING STRESS CONTOURS (#/in²)

p and q.

It is believed that this clearly shows the stress distribution in the gusset plate at full experimental load.

The stress contours and stress trajectories were sketched without consideration of the localized disturbance caused by the rivets and rivet holes. It is known that a hole is a "stress raiser" and that the stresses in the immediate vicinity of the holes will be larger than those indicated by the contours.

CHAPTER IX

COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS

Comparison with Photoelastic Results

As has been previously stated, J. A. Sandel² was making at the same time a photoelastic study of a small plastic model using the same prototype as the writer's. Because of different properties of the materials used in the separate studies, and because Sandel was unable to reduce the thickness of his model by the same ratio as the other dimensions, direct quantitative comparisons can not readily be made. However, a qualitative comparison may be made directly. In other words, the comparison will be made as to stress distribution rather than stress magnitude.

Sandel's contours for principal stresses and maximum shearing stresses are reproduced as Figures 18, 19 and 20.

From a comparison of Figures 15 and 18, it is seen that the general distribution of tensile stresses is quite similar. The contours extend diagonally from the upper left hand corner ($U_1 L_2$) to the lower right side ($L_2 L_4$) in each case. Furthermore, the maximum tension contours originate

¹ Sandel, loc. cit.

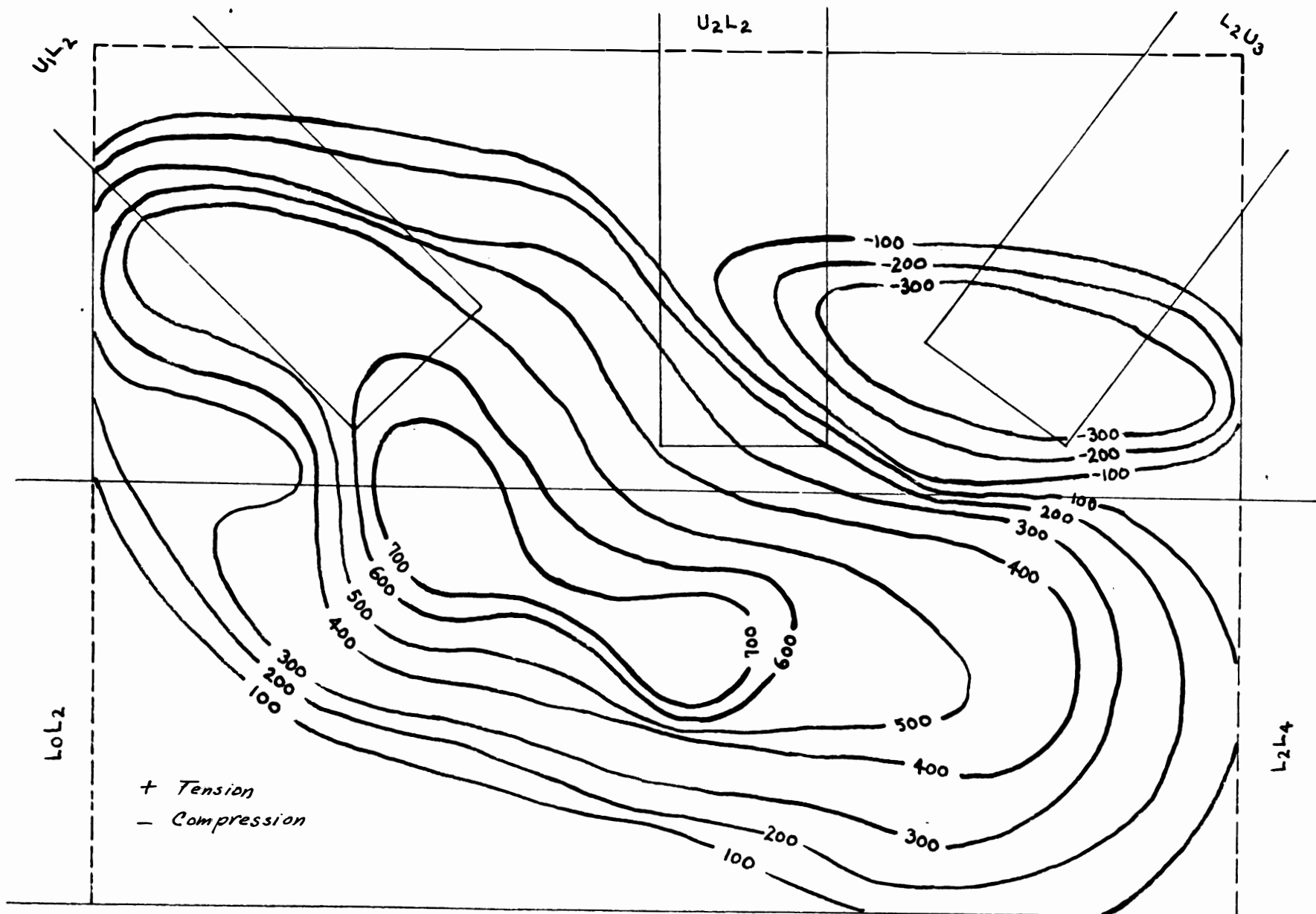


FIGURE 18 . APPROXIMATE CONTOURS FOR "P" STRESSES-MARLETTE MODEL

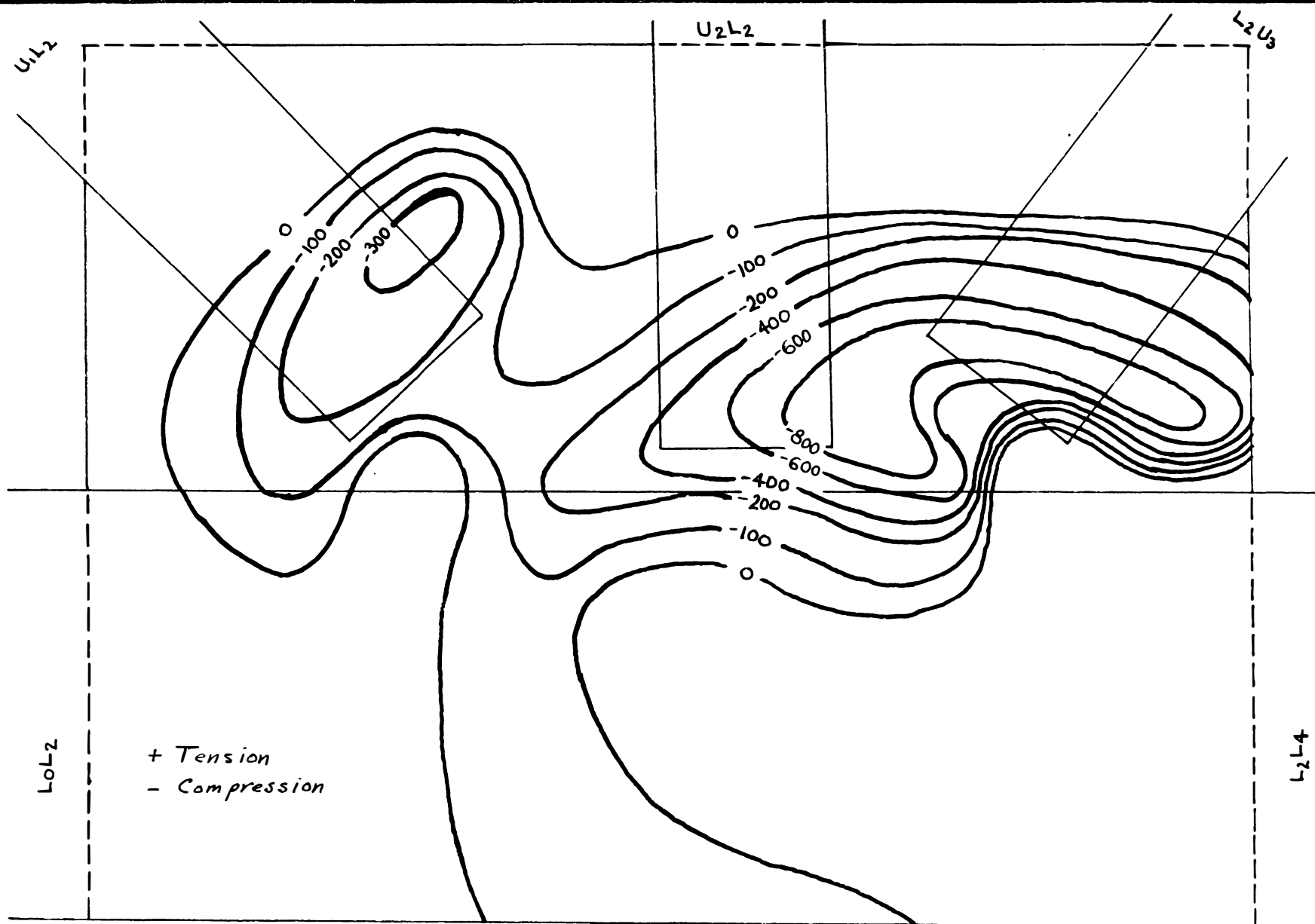


FIGURE 19 . APPROXIMATE CONTOURS FOR "Q" STRESSES- MARBLETTE MODEL

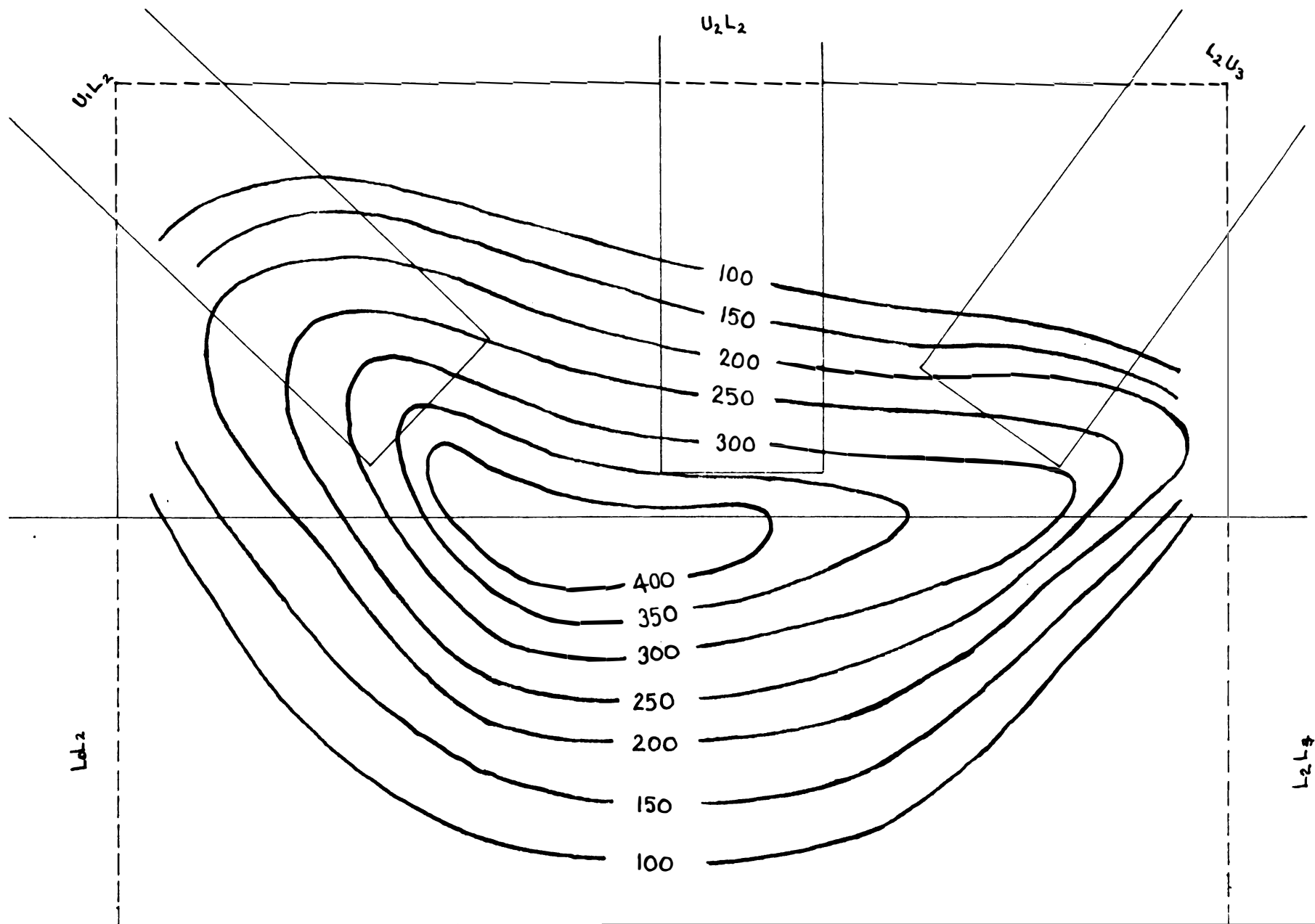


FIGURE 20 APPROXIMATE CONTOURS FOR MAXIMUM "S_s" STRESSES- MARBLETTE MODEL

near the end of the tension diagonal in each figure, although they do cover somewhat different regions of the plate. It is interesting to note that both experimentors found the maximum tensile stress near the lower right hand edge to be one seventh of the maximum in the plate.

Comparison of Figure 16 and 19 does not give the same general agreement obtained for the tensile stresses. The difference could be due to bending in the compression diagonal in the plastic model. However, it is encouraging to note that each test indicates that the maximum compressive stress is located near the end of member $L_2 U_3$.

The general distribution of maximum shearing stresses was found to be quite similar in each study, as shown by Figures 17 and 20. Each figure locates the maximum contour near the center of the plate and shows it extending more or less parallel to the bottom chord. Other contours follow the same general trend and there is fairly good agreement between the two figures.

Comparison with Analytical Results

The usual procedure for checking gusset plates analytically is to test them for tearing out, crushing, bending and direct stresses. For testing of direct and bending stresses, the section should be taken parallel to one of the

intersecting members².

For the joint used in this experiment, the "critical section" is found to be that section parallel to the bottom chord and through the ends of each diagonal. (See Figure 4). On this section, the vertical component of each diagonal contributes to the bending and the horizontal components act in the same direction to increase the shearing stresses. Assuming that the usual beam formulas apply, the bending and shearing stresses may be calculated.

Figure 21 shows the part of the plate above the critical section, the components of the forces, and the calculation of stresses. The stresses are plotted in Figure 22.

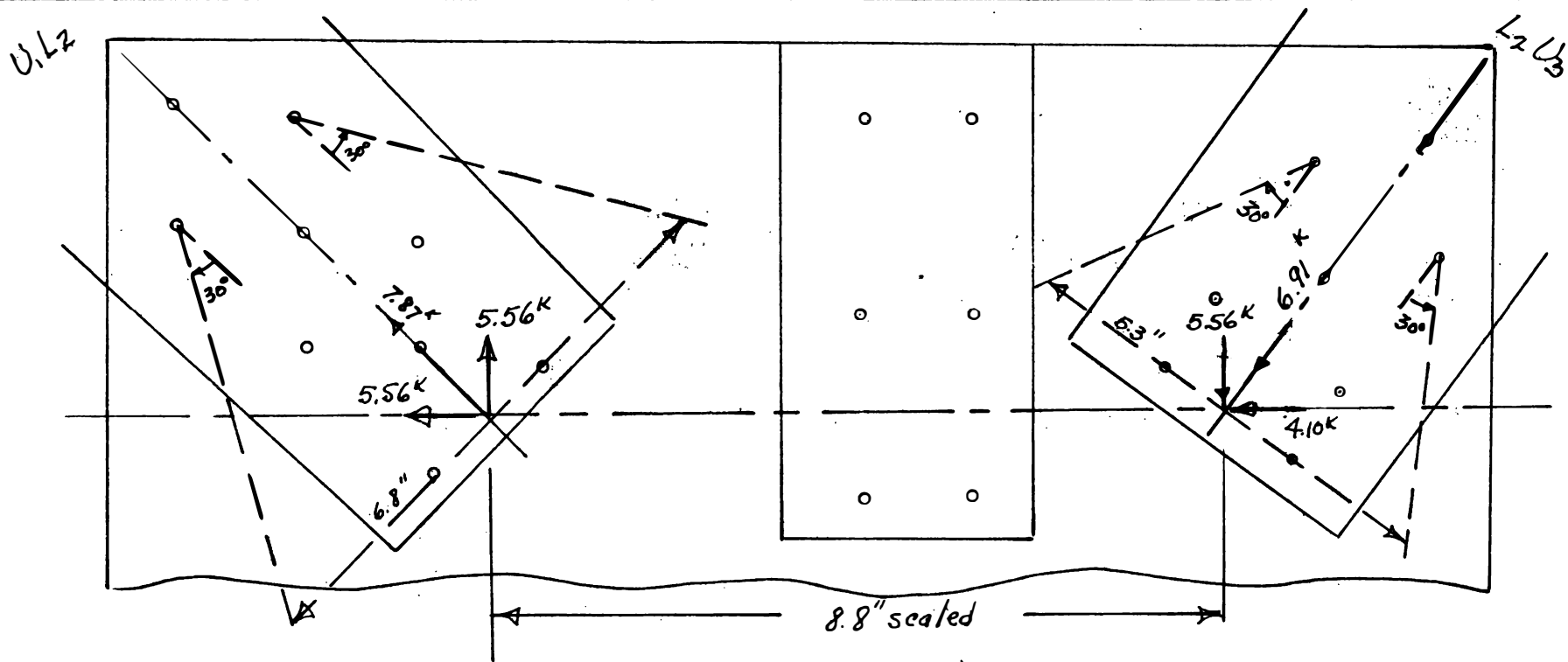
To obtain a comparison with these analytical results, similar stresses were determined from the experimental data. From Figures 14, 15 and 16, values of θ , p , and q were obtained at various locations along the "critical section". Normal and shearing stresses then were calculated by means of the following known relationships³:

$$\sigma_y = \frac{p + q}{2} - \frac{p - q}{2} \cos 2\theta$$

$$\tau_{xy} = \frac{p - q}{2} \sin 2\theta$$

² J.A.L. Waddell, Bridge Engineering (New York: John Wiley and Sons, Inc., 1916), Vol. I, p. 521.

³ P. G. Laurson and W. J. Cox, Mechanics of Materials (New York: John Wiley and Sons, Inc., 1938), p. 274.



Bending Stresses: $\sigma_y = \frac{Mc}{I} = \frac{(5.56)(8.8)(6)}{(0.25)(16.6)^2} = 4.25 \text{ K/in}^2$

Shearing Stresses: $\tau_{xy} (\text{max}) = \frac{3V}{2A} = \frac{3(5.56+4.10)}{2(16.6)(0.25)} = 3.5 \text{ K/in}^2$

Direct Tension: $\sigma = \frac{P}{A} = \frac{7.87}{(6.8)(0.25)} = 4.63 \text{ K/in}^2$

Direct Compression: $\sigma = \frac{P}{A} = \frac{6.91}{(5.3)(0.25)} = 5.22 \text{ K/in}^2$

FIGURE 21. CALCULATION OF STRESSES IN THE MODEL

The values are tabulated in Table V and results plotted in Figure 22.

A check on the experimental results may be obtained from Figure 22 by applying the equations of statics, namely:

$$\sum H = 0$$

$$\sum V = 0$$

$$\sum M = 0$$

For the summation of the horizontal forces to be equal to zero, the area under the analytical shear stress curve should equal the area under the experimental curve. The ratio of the planimetered areas is 1.5 to 1. This, at first, appears to indicate considerable experimental error. However, examination of Figure 21 shows clearly that the entire load in the diagonals has not been transferred to the plate above the assumed "critical section". Assuming that each rivet transfers its proportionate share of the load, this would not account for the large difference of areas shown above. However, it is believed that each rivet is not loaded equally and, furthermore, that the end rivets transfer more load than other rivets. In addition, it is believed that a large portion of the load is transferred to the plates in skin friction and it is noted that a considerable portion of the diagonal members is located below the critical section.

If this is a correct explanation of the differences in horizontal forces, then, it necessitates that there be a

TABLE V

NORMAL AND SHEARING STRESSES
ON THE "CRITICAL SECTION"^a

Distance from Left Edge - In.	θ°	"p" psi.	"q" psi.	σ_y psi.	τ_{xy} psi.
1	75	-2,400	-100	2,230	620
2	64	3,100	-200	2,470	1,300
3	62	3,500	-300	2,660	1,580
4	51	3,500	-400	1,960	1,940
5	44	3,500	-500	1,430	1,990
6	43	3,200	-600	1,170	1,890
7	41	2,700	-800	710	1,730
8	39	2,300	-1,100	250	1,660
9	39	1,900	-1,400	-90	1,610
10	43	1,500	-1,700	-210	1,590
11	49	900	-2,200	-490	1,530
12	45	600	-2,800	-1,100	1,700
13	38	400	-3,500	-2,020	1,890
14	31	300	-3,500	-2,490	1,680
15	28	200	-3,500	-2,680	1,530
16	17	100	-2,000	-1,820	590

^a Note: " θ " is a clockwise angle measured in the second quadrant between the "x" and the "p" directions. Plus stresses are tensile and - stresses compressive.

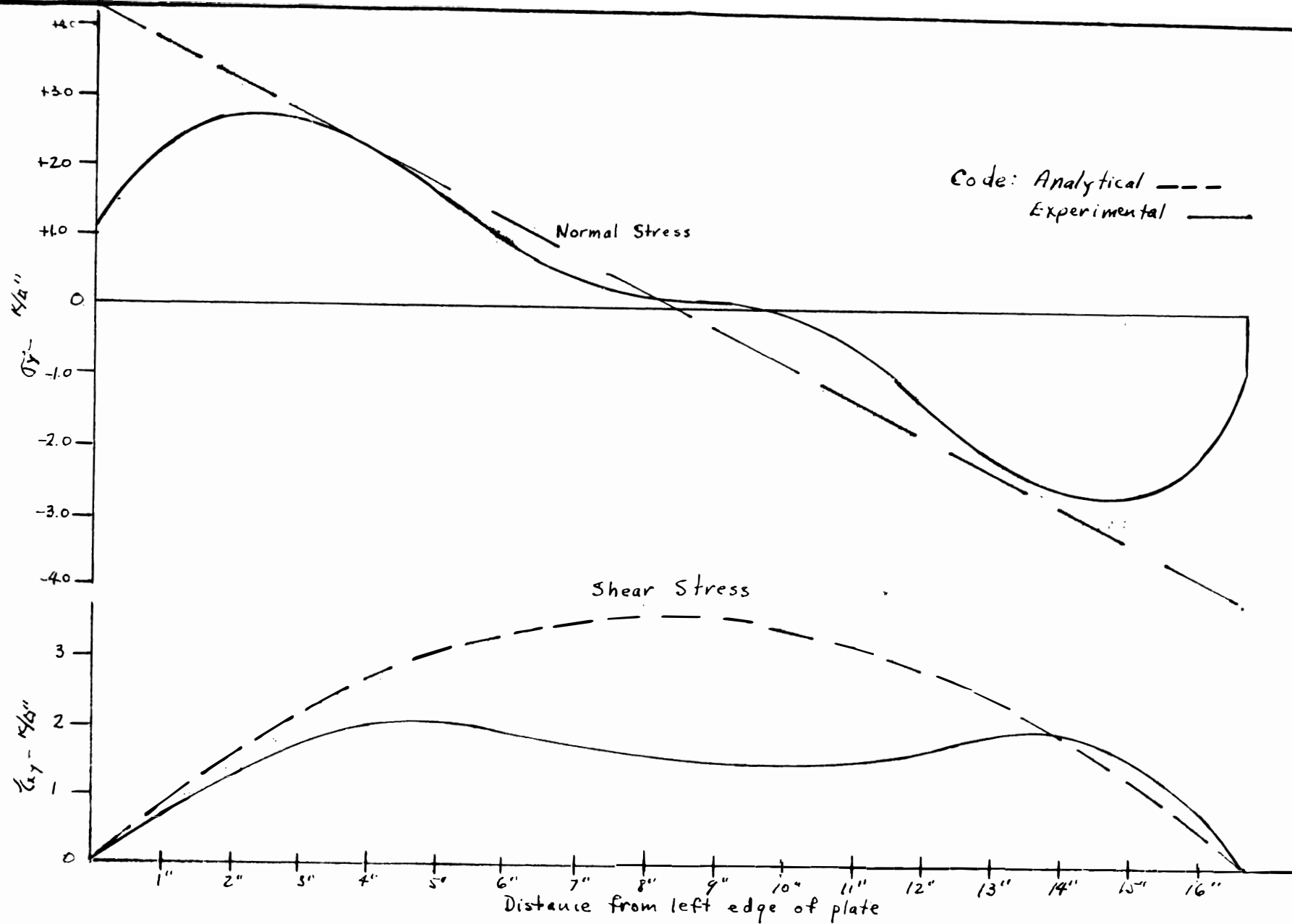


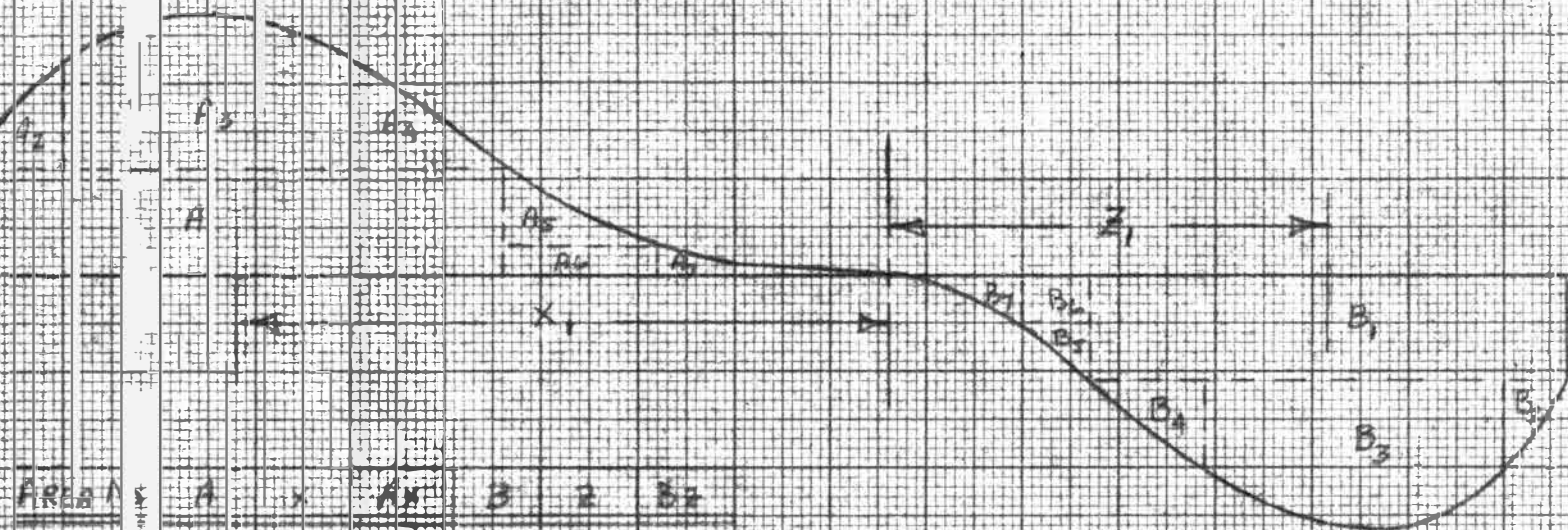
FIGURE 22. COMPARISON OF ANALYTICAL AND EXPERIMENTAL STRESSES ON "CRITICAL SECTION"

similar difference in the vertical components of the forces in the diagonals. This would cause the theoretical and experimental moments to differ by the same proportion. In Figure 23 the experimental moment is calculated. The theoretical moment is $5.56^k \times 8.8" = 50^k$. The ratio of moments is 1.43 to 1. This compares very favorably with the ratio of horizontal forces and apparently verifies the explanation previously given.

The summation of vertical forces is not quite equal to zero - 3.36^k to 2.9^k (See Figure 23). However, due to the previous explanation, some difference is to be expected. The rivets, in the two diagonals, do not transfer the same amount of load and this might well explain the difference in vertical forces. However, since the difference is only about 15%, it would not be alarming if this was attributed entirely to experimental error.

In general it can be stated that the experimental results have been verified by statics.

As a further comparison of results, direct tensile and compressive stresses may be calculated. As illustrated in Figure 21, it is assumed that the forces in the diagonals are distributed over an area of gusset material obtained by multiplying the thickness of the plates by an arbitrary length perpendicular to the diagonals. This length is obtained by extending lines through the first rivets making



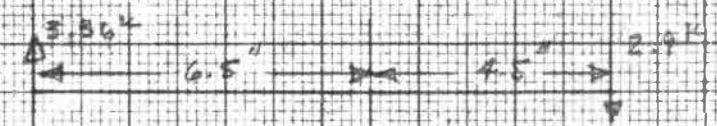
AREAS	A	x	Ax	B	z	Bz
1	615	66	41,000	539	46	24,800
2	60	89	5,300	29	66	1,800
3	440	71	31,200	442	49	21,700
4	18	51	900	60	29	1,700
5	60	94	5,600	21	19	400
6	48	36	1,700	35	18	600
7	36	17	600	0	8	200
Total	1341		87,100	1,154		51,200

$$\bar{x} = \frac{87,100}{1,341} = 65 \quad \bar{z} = \frac{51,200}{1,154} = 45$$

$$I''_{Area} = (2.0^{in}) \left(2 \times \frac{1}{4} \right) = 1''$$

$$400 \text{ squares} / I''$$

$$\therefore \frac{1341}{400} = 3.36'' \quad \& \quad \frac{1154}{400} = 2.9''$$



$$\text{Experimental Moment} = (3.36)(6.5) + (2.9)(4.5)$$

$$= 35.0''$$

FIGURE 23. CALCULATION OF RESISTING MOMENT ON "CRITICAL SECTION"

angles of 30° with the direction of the member until they intersect a perpendicular line drawn through the bottom row of rivets. The distance can then be scaled. This method is frequently used by practicing engineers.

This construction and the resulting stresses are shown in Figure 21. The stresses may be compared with those plotted in Figures 15 and 16.

CHAPTER X

CONCLUSIONS

In drawing conclusions from the results of the experiments performed in this study, it must be remembered that they may be applied directly only to joints of a similar type - namely, joints of a Warren truss in which the chord direction does not change at the joint and in which the chord is not spliced in the joint. However, a great many actual joints are of this type.

Probably the most conclusive information obtained relates to the general distribution of stresses in the plate. The photoelastic, strain gage and Stresscoat methods all show the tensile stresses originating near $U_1 L_2$ and extending down and toward $L_2 L_4$ with the maximum value near the end of the tension diagonal. Both the photoelastic and the strain gage results locate the region of maximum compression near the end of the compression diagonal ($L_2 U_3$) and the maximum shear near the center of the plate.

It has been common practice among engineers to test "critical sections" for direct stress plus bending - assuming that the general expression $\frac{P}{A} \pm \frac{Mc}{I}$ applies, which result in maximum stress values at the edges of the plate. One of the most definite conclusions which may be drawn is that the maximum stresses are not on the edges, but near the

ends of the members which terminate in the plate.

It is, therefore, evident that the beam formulas do not apply to gusset plates of this type and that the stress distribution is quite different, as shown in Figure 22.

The results seem to indicate that the best method of calculating shear stresses is to use the average stress (V/A) on the "critical section" as recommended by Waddell¹. The average shear stress on the section is just over 2^k per square inch which compares favorably with the experimental values. It is indeed interesting to note that the distribution of shear stresses as given by Figure 22 is almost identical with the theoretical shear distribution given by Timoshenko², for a beam loaded with a couple. The b/c ratio for the model is $1/3$ and the ratio of maximum stress compares favorably with the theoretical ratio. This further verifies the results. The parabolic shear stress distribution is given in Figure 22 because it is the type distribution assumed by most practicing engineers.

It should be noted that direct stresses, when computed as illustrated in Figure 21, are considerably larger than actual stresses. However, the method is somewhat

¹ Waddell, op. cit., p. 524.

² S. Timoshenko, Theory of Elasticity (New York: McGraw-Hill Book Co., Inc., 1934), p. 49.

ambiguous since the area (and hence, the stresses) depends on the spacing of the rivets, distance of the first row of rivets from the edge of the plate, etc. Figure 14 gives a clear picture of how the stresses "fan out". Furthermore, Figures 14 and 18 indicate that near the ends of the diagonals the maximum stresses are virtually parallel to the members. These results lend support to the use of the method previously described (Figure 21) for calculating tensile and compressive stresses.

However, for the particular conditions of this experiment, the "spread out" angle of 30° seems to be a little conservative. If an angle of 35° is used instead, the tensile stress is $4.0 \text{ K/square inch}$ (as compared with a maximum experimental tensile stress of $3.6 \text{ K/square inch}$) and the compressive stress is $4.2 \text{ K/square inch}$ (as compared with maximum experimental values of $3.9 \text{ K/square inch}$).

Undoubtedly, further testing of gusset plates is necessary before generalized conclusions may be drawn. This would involve testing joints of other types, sizes and shapes, and studying the effects of secondary stresses in the members in addition to the direct stresses.

However, the results of the tests made on the gusset plate of this thesis are quite definite and could be applied with considerable confidence to the analysis of stresses in gussets similar to the one tested.

BIBLIOGRAPHY

1. *Journal of the American Medical Association*, 1954, 157, 1000-1001.

BIBLIOGRAPHY

- Alcoa Structural Handbook. Pittsburgh, Pennsylvania:
Aluminum Company of America, 1945.
- Hetenyi, M., et al. Handbook of Experimental Stress Analysis. New York: John Wiley and Sons, Inc., 1950
- Laurson, P. G. and Cox, W. J. Mechanics of Materials.
New York: John Wiley and Sons, Inc., 1938.
- Lee, G. H. An Introduction to Experimental Stress Analysis.
New York: John Wiley and Sons, Inc., 1950
- Murphy, G. Advanced Mechanics of Materials. New York:
McGraw-Hill Book Co., Inc., 1946.
- Sandel, J. A. "Photoelastic Analysis of Gusset Plates",
(Unpublished Master's Thesis, The University of
Tennessee, Knoxville, 1950).
- Shedd, T. C. Structural Design in Steel. New York:
John Wiley and Sons, Inc., 1934.
- Tatnall, F. G. "Concerning the Subject of Testing", SR-4
News Letter, II: 1-8, March 1946.
- Timoshenko, S. Theory of Elasticity. New York: McGraw-
Hill Book Co., Inc., 1934.
- Waddell, J. A. L. Bridge Engineering. New York: John
Wiley and Sons, Inc., 1916. Vol. I, pp. 520 - 530.