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I am submitting herewith a dissertation written by Lawrence Peter Shao entitled "Mortgage Backed Securities, Duration and Implied Prepayments." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Finance.

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MORTGAGE BACKED SECURITIES, DURATION AND IMPLIED PREPAYMENTS

A Dissertation

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Degree

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Although this dissertation has required a tremendous amount of time and energy to complete, it has been one of the most rewarding experiences of my life. As a doctoral student, I experienced the entire gamut of emotions, from intense frustration to extreme elation. With the many demands placed upon me, I often reminded myself that perseverance was the key to success.

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different and have special qualities I deeply respect. My father, Dr. Stephen P. Shao has always been my mentor. Throughout my life he has stressed the importance of both a good education and a committment to family. Having served the dual role as both my father and mother, I will never be able to thank him enough for his total support and endless encouragement. My step mother, Priscilla G. Shao has been a dedicated and loving member of our family. Finally, I would like to remember my late mother, Betty Lucille Outen Shao. Although I only knew her for 8 short years, the memories of her will always be dear to my heart. I proudly dedicate this dissertation to my father and mother to express my love for them.

In closing, I hope my family and friends will have continued success and I pray God will generously bless and enrich their lives as he has so richly done to mine.

ABSTRACT

The primary purposes of this study are to provide an accurate measure of risk for mortgage backed securities and to infer the market's expectations regarding future prepayment rates from actual mortgage backed security price data. The first major objective is to accurately estimate the implied duration of a mortgage backed security, using an elasticity concept. The results of this study clearly show that the proposed multiple regression model does a significantly better job at estimating implied duration than the proposed simple regression model. The multiple regression model provides an extremely strong relationship between Macaulay's duration and total return. This finding should be of interest to investors, since it provides a means of measuring the risk of mortgage backed securities.

The second major objective is to use implied duration to estimate prepayments. Using an estimate of implied yield, an iterative process is employed to find the prepayment rate that provides the implied duration. With this procedure, prepayment rates are implied by market prices. While the results of the study indicate the multiple regression model generally works better than the simple regression model, neither model seems to work well in predicting prepayments. Implied prepayment is found to be a

biased predictor of actual prepayment, indicating both models overestimate the prepayment rate.

Although the multiple regression model approach is effective at estimating the risk of a mortgage backed security, it should not be used to estimate constant prepayments associated with the mortgage pool. None of the models do well in predicting prepayments, indicating that the constant prepayment rate assumption is, in all likelihood, incorrect. During times of volatile interest rate movements, which is the time of this study, mortgage prepayments probably follow some nonconstant prepayment pattern.

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CHAPTER I

INTRODUCTION

There has been phenomenal growth in the market for mortgage backed securities (MBS) during recent years. By 1986 over $529 billion of mortgage debt had already been securitized. With continued interest and activity in the MBS market the potential for future growth is considerable. The two most common types of MBS that have received the most attention from investors are mortgage pass through securities (MPTs) and mortgage backed bonds (MBBs). Financial institutions that issue mortgage pass through securities are essentially selling assets to investors. Institutions that market mortgage backed bonds recognize these securities as actual debt obligations. The Government National Mortgage Association (GNMA) has issued about half of all mortgage pool debt that is currently outstanding. Recent figures show that of the more than $529 billion of mortgage pool debt outstanding, over $260 billion these securities were issued by GNMA.[1]

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[1]According to 1986 figures provided by the Federal Reserve Bulletin, the FHLMC issued $171.37 billion of mortgage pool debt while the FNMA has issued $97.17 billion.

Although there has been unprecedented growth in this market, valuation problems exist for individuals and firms that invest in MBS. MBS possess distinct features that are unlike those of conventional corporate and government debt securities. MBS valuation is more complex due to the fact that the amount and timing of payments are not known with certainty. The primary risk associated with these instruments occurs as a result of the prepayment option that is granted to mortgage borrowers.

A GNMA pass through mortgage security is backed by a pool of mortgages; any individual mortgage in the pool may be prepaid prior to maturity by the homeowner. Federal legislation regarding Federal Housing Authority (FHA) insured and Veterans Administration (VA) guaranteed mortgages prevents any charges to the homeowner associated with prepayment penalties.[2] The prepayment option is essentially an unrestricted call option, similar to that found on other callable bonds. In summary, it is the potential mortgage prepayments that cause MBS valuation to be more difficult than valuation of traditional fixed income securities.

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[2]The Federal Housing Authority guarantees 100% of the value of the mortgage loan. The Veterans Administration guarantees only the top 60% of the loan amount.

Purpose of Dissertation

Traditional fixed income securities such as treasury bonds and noncallable debt can be priced relatively easily since the amount and timing of the cash flows are known with certainty. As long as coupon payments are made to investors at specified intervals over the entire investment horizon and the principal is paid at maturity, the market value of the debt issue can be accurately calculated as the present value of the contracted cash flows.

These types of securities are exposed to two types of risk, both of which result from changes in interest rates. Price risk occurs when the bond is sold at some price other than the expected sale price. Coupon reinvestment risk occurs when the coupon payments are reinvested at rates that are different from the interest rates that prevailed when the securities were originally issued. It should be noted that these two risk types vary in opposite direction as interest rates change. If interest rates fall, bond prices rise while reinvestment income is reduced. The sum of both types represent the total risk of a fixed income security investment during the investment horizon.

Mortgage backed securities (MBSs) are exposed to the same types of interest rate risk as the traditional fixed income securities; however, there is a higher degree of

total risk associated with the mortgage security investments. MBSs have consistently provided a higher rate of return than comparable government bonds. During 1986, GNMA securities provided an average yield of 9.30% compared to Treasury bonds which yielded 7.68%.[3] These figures indicate a higher degree of uncertainty associated with mortgage security investments. One possible reason may be due to the frequency of MBS payments. The coupon reinvestment risk can potentially be greater because mortgage securities provide monthly, rather than semiannual, payments to their investors. As interest rates decline, the investor receives less return from reinvesting coupon payments. This leaves security holders with more frequent reinvestment decisions during the investment horizon.[4]

The greatest source of risk associated with mortgage backed securities occurs because most mortgage investors have the right to prepay their loans without penalty. Therefore, the expected cash flows of mortgage backed securities are influenced to a large extent by the

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[3]According to the Federal Reserve Bulletin, the 9.6% figure represents the average net yield on GNMA guaranteed, mortgage backed, fully modified pass-through securities, assuming prepayment in 12 years on pools of 30-year FHA/VA mortgages.

[4]A portion of the monthly cash flow includes amortized payment of interest and principal supplemented by principal prepayments.

prepayment rate of the mortgage pool. Valuation models that are used to price mortgage backed securities must incorporate the prepayment option feature. It is very important to investors in mortgage backed securities that future prepayment rates be accurately estimated; otherwise, the price of a mortgage security cannot be determined. The current methodologies that are utilized to predict mortgage prepayment rates will be discussed in this study.

Despite the fact that accurate prepayment rates are crucial for calculating the value of mortgage securities, there have been few attempts to develop viable prepayment estimation models. Typically, investors have adopted rules of thumb such as the 12 year life assumption, the multiple FHA experience, and the conditional prepayment rate associated with mortgage pools. These methods are essentially estimates of future prepayments rates based on past prepayment patterns and expected future interest rate changes. Since the mortgage securities market has developed rapidly in the last decade, the need for more effective prepayment models has also increased. Mortgage prepayment rate is an important variable used in mortgage backed security pricing models. As the mortgage security market

continues to growth, prepayment models will be applied to larger data bases.[5]

In this study, it is assumed that efficient capital markets do exist. In an efficient market scenario, information is widely known to all market participants so that security prices can adjust to any announcement of new information; prices instantaneously reflect all relevant information. The efficient capital market concept is mentioned because this concept has several important implications that effect mortgage backed security analysis.

It should be noted that important relationships will be developed between U.S. Treasury bonds and mortgage backed securities. These securities have several common features; both instruments have the same default risk as a result of the government guarantee associated with each investment. It is also known that both Treasury bonds and mortgage securities have similar liquidity risk since there exists very active secondary markets for each security. The remaining risk variable associated with the mortgage security must involve prepayment risk.

An important assumption in this study is that Treasury bonds and mortgage backed securities are priced correctly in

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[5]Green and Shoven (1986) spent a large amount of their time collecting prepayment data. They examined the prepayment rates of almost 4,000 mortgages of two savings and loan associations for an 8 year period.

their respective markets. This assumption implies that both markets are efficient. Although the mortgage backed securities market is not generally perceived as totally efficient, it is assumed that the market does price its securities correctly. Thus, the risk associated with the prepayment option can be thoroughly examined in this dissertation; certain results of this study are based on the efficient capital market assumption.

The purpose of this study is to infer the market's expectations regarding future prepayment rates from actual market price data. The empirical model that is proposed is based on market prices and recognizes the fact that prepayment risk is not constant across various mortgage coupons. There exist a larger probability of prepayment for premium bonds than for discount bonds when interest rates fall; in this case, investors are able to refinance their debt at lower interest rates. The application of a proposed market price bond model should provide investors with a more efficient way to estimate mortgage security prepayment rates.

Organization of Dissertation

This dissertation is divided into eight sections. Chapter II provides a review of the mortgage backed security

market and a discussion of changes that the mortgage market is currently experiencing. Unique features associated with mortgage backed securities are identified and a thorough description is provided for mortgage pass through securities. Chapter III is a summary of alternative mortgage backed security pricing models; a comprehensive review of the mortgage backed security pricing literature is also provided. A review of the mortgage prepayment literature is discussed in Chapter IV.

Chapter V is an analysis of the systematic development of duration theory; the literature is divided into two major segments, immunization and bond price volatility. Chapter VI presents an application of the duration measure to mortgage security analysis and illustrates how implied duration can be used to estimate mortgage prepayment rates. Chapter VII describes the data that is used in this study and also provides in detail the specific methodology that is employed in the empirical portion of this study. In Chapter VIII a comprehensive analysis of the empirical findings is performed. Lastly, Chapter IX summarizes the primary results and major conclusions of this dissertation.

CHAPTER II

INSTITUTIONAL FEATURES OF MORTGAGE BACKED SECURITIES

Creation of the Mortgage Backed Security Market

This chapter explains why mortgage backed securities were created and how agencies issue these securities to investors. A review is provided on current changes that are occurring within the mortgage market. It is important that investors are familiar with the common features associated with different types of mortgage pass through securities; in addition, a comparison of pass throughs and treasury securities will also be addressed is this chapter.

Thrift institutions, primarily savings and loan associations, were unable to provide for the increased demands of mortgage money during the early 1970s; as a result, nontraditional mortgage investors become more active in the mortgage financing market. It was this increased demand for additional sources of funds for mortgage lending that stimulated the growth of the MBS market. When the demand for mortgage credit exceeded the ability of traditional sources to supply funds, mortgage securities were created to fill the need for mortgage funds. The mortgage backed security market was established mainly to

allow for easier exchanges of mortgage related securities and to stimulate more active participation in mortgage investments by market investors.[6]

Mortgage backed securities are investments that are backed by government agencies and by mortgage originators such as saving and loan associations, commercial banks, mortgage companies, and thrift institutions. Mortgage securities are divided into two major groups. The first group, pass through certificates, are created when mortgages are grouped together and undivided interests in the mortgage pools are sold to investors. The cash flow from the underlying mortgages is passed through to the security investors in the form of monthly payments that include interest, principal, and also principal prepayment. The second group, mortgage backed bonds, provide scheduled coupon payments to investor and are considered general debt obligations of the issuing firm.[7] The sale of a mortgage backed bond does not represent the sale of assets, as is the case with pass through certificates.

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[6]The rapid development of the secondary mortgage market has enabled institutional investors to become a significant source of capital for the housing market.

[7]To be considered for a AAA rating, Standard and Poor's requires mortgage bond issuers to maintain a minimum collateralization level of 150% of the bond's par value.

Government National Mortgage Association (GNMA) is a government agency which is actively involved in issuing large amount of mortgage backed securities. GNMA pass through certificates are structured so that the amount of investment risk due to default is virtually eliminated; this type of mortgage security is guaranteed by the U.S. government. The U.S. Treasury department ultimately guarantees the timely payment of principal and interest each month.[8] Even if an individual mortgage underlying a GNMA pool defaults, the investor is assured of timely payments each month. The government guarantee provides protection so that investors do not have to concern themselves with the credit quality of the individual mortgage loans. Three government sponsored agencies including GNMA, FHLMC, and FNMA will be discussed in the next section.

Common Features of Mortgage Pass Through Securities

By far the largest group of mortgage securities that are traded in the market are the mortgage pass through securities (MPT). These securities have become very popular since federally supported agencies, such as GNMA, FNMA, and

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[8]Fabozzi (1987) cites that the GNMA guaranty, which is backed by the full faith and credit of the United States Treasury, is considered to be free of default risk.

FHLMC, have constructed mortgage pools that are used as collateral to back the MPT. Market investors view MPT as very safe investments because of the government backed guarantee of cash flow payments and the fact that only mortgages that meet certain specific features are included in the mortgage pools.

A general procedure is employed when mortgage pass through securities are actually created by lending institutions. These securities are created when a group of home mortgages are pooled together and specific claims to the mortgage pool are bought by investors. The actual sale of a MPT reflects a sale of assets and does not represent a debt claim. Cash flow payments collected from mortgages within the pool are passed through to the MPT investors in the form of monthly payments of interest and principal including prepayments. Prepayments will result when the mortgage holder prepays either part or all of the outstanding principal balance before the final scheduled cash flow payment. Four types of mortgage pass through securities that are often traded include GNMA Pass Throughs, FHLMC Participation Certificates, FNMA Mortgage Backed Securities, and Private Pass Throughs. The common features associated with these securities are described in the following section. It is apparent that understanding the specific features of pass through securities are crucial to accurate pricing of these financial instruments.

Government National Mortgage Association

Pass Through Securities

The most widely issued group of pass through securities (PTs) is guaranteed by the Government National Mortgage Association (GNMA), also known as Ginnie Mae.[9] GNMA PTs are backed by pools of FHA insured or VA guaranteed mortgage loans that are packaged into securities by mortgage lenders. These instruments are backed by the full faith and credit of the United States government. Part of the cash flow that is paid by the mortgage is passed through the issuer of the GNMA to the ultimate investor who purchased the security. The scheduled cash flow of principal and interest from the underlying mortgage is paid to the investor on the fifteenth day of each month. The investor will also receive any unscheduled payments of principal that are made when mortgage holders prepay the outstanding principal balance on their mortgage loans.

Many reasons are often given to explain why homeowners prepay their mortgage loans. Prepayments often occur when borrowers change residence or refinanced their loans at lower interest rates. Another reason for unscheduled principal payments is the treatment of loans that are in

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[9]In 1968, GNMA was established as a wholly owned U.S. government corporation within the Department of Housing and Urban Development.

default. The FHA or VA will guarantee the ultimate payment of principal on loans that have defaulted. Mortgage issuers make timely scheduled payments to GNMA security holders even if the payments have not been received from the mortgage holders. The GNMA provides the highest level of cash flow protection to its investors.

Besides the guarantee of cash flows, GNMAs possess several other general characteristics. These mortgage securities have consistently offered the highest yield of all government guaranteed securities, which helps explain their rapid acceptance into the market. Table 1 shows that during 1986, the average yield on GNMA was about 1.6% greater than the average yield on 10 year Treasury bond. There is an active secondary market for GNMA PTs. Another characteristic of GNMA PTs is that they are considered homogenous investments, since all mortgages within the pool have the same coupon rate and maturity. By 1986, a total of $260.86 billion of GNMA PTs had been issued. Table 1 provides information on the issuance of mortgage securities from 1970 to 1986.

GNMA pass through securities are classified as fully modified issues which means that even if the mortgage payment is not made, the pass through security investor will always receive full and timely payment of principal and interest. A fully modified pass through guarantees

Table 1

Mortgage Pool Debt Outstanding
(billions of dollars)

YEAR	GNMA	FHLMC	FNMA	FHA*	TOTAL	GNMA YIELD (%)	GOV'T BOND (%)
1988	340.52	224.97	178.25	00.10	773.84	9.83	8.85
1987	319.36	212.11	139.96	00.32	671.75	9.42	8.39
1986	262.69	171.37	97.17	00.35	531.59	9.30	7.68
1985	212.14	100.38	54.98	47.52	415.04	11.61	10.62
1984	179.98	070.82	36.21	45.03	332.05	13.13	12.44
1983	159.85	057.89	25.12	42.20	285.07	12.25	11.10
1982	118.94	042.96	14.45	40.30	216.65	14.68	13.00
1981	105.79	019.85	00.71	36.64	163.00	15.29	13.91
1980	093.87	016.85	n.a.	31.53	142.25	12.55	11.46
1979	076.40	015.18	n.a.	27.69	119.27	10.22	09.44
1978	054.34	011.89	n.a.	22.39	088.63	08.98	08.41
1977	044.89	06.61	n.a.	18.78	070.28	08.04	07.42
1976	030.57	02.67	n.a.	16.55	049.80	08.17	07.61
1975	018.25	01.59	n.a.	14.28	034.13	08.52	07.99
1974	011.76	00.75	n.a.	11.27	023.79	n.a.	06.99
1973	007.89	00.76	n.a.	09.38	018.04	n.a.	06.30
1972	005.81	n.a.	n.a.	n.a.	005.81	n.a.	05.63
1971	003.15	n.a.	n.a.	n.a.	003.15	n.a.	06.59
1970	000.45	n.a.	n.a.	n.a.	000.45	n.a.	06.59

* Farmers Home Administration
Source: Federal Treasury Bulletin (1970-1988)

investors that they will receive their cash flow payments even if some mortgages within the pool default. It is not surprising that the GNMA pass through is the most common type of pass through security.

Federal Home Loan Mortgage Corporation Participation Certificates

The second most widely issued type of pass through is the Federal Home Loan Mortgage Corporation Participation Certificate (FHLMC PCs). These securities are backed by large pools of conventional mortgages and are not guaranteed by FHA or VA mortgages. PCs are relatively liquid investments and are traded in active secondary markets. The most striking difference between FHLMC PCs and government backed GNMA PTs relates to principal payback. Although FHLMC guarantees the timely payment of interest, it provides only for the eventual payment of principal on all conventional mortgages that comprise the investment pool.

Mortgage payments are guaranteed against losses by FHLMC and not by the U.S. government. Many conventional mortgages in the FHLMC mortgage pool are insured by private mortgage insurance companies. The additional risk associated with FHLMC PCs may explain why these securities

generally trade at higher yields than GNMAs.[10] By 1986, FHLMC had issued a total of $171.37 billion of PCs.

Federal National Mortgage Association

Mortgage Backed Securities

A third type of pass through security is referred to as Federal National Mortgage Association Mortgage Backed Securities (FNMA MBS). FNMA is a federally chartered private corporation that maintains a line of credit with the U.S. Treasury. This quasi-government agency entered the MBS market in 1981 when it offered mortgage securities backed by conventional mortgages. FNMA guarantees the timely payment of principal; the corporation also serves as both the issuer and guarantor for all mortgage securities that it issues.[11]

FNMA pools that contain conventional mortgages are not backed by the full faith and credit of the U.S. government; however, most investors view these securities as risk free investments because of the close government affiliation. The yields associated with FNMA MBS are similar to those of

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[10]The FHLMC is owned by the 12 members of the Federal Home Loan Bank Board. It is the youngest of the three large agencies and often competes with FNMA.

[11]The FNMA was originally owned by the U.S. government. The agency was privatized in 1968 and is now listed on the NYSE. It is still viewed as a government sponsored institution as a result of continued government regulation of its operations.

found on FHLMC PCs; however, FNMA mortgage backed securities do provide slightly higher yields than those of GNMAs. FNMA MBSs are also considered to be relatively liquid investments and are traded in an active secondary market.

Private Mortgage Pass Through Securities

Another type of pass through security involves those issued by private financial institutions. The first privately issued pass through security was made by the Bank of America in 1977; the total mortgage pool amounted to $150 million. Other private issues have included First Federal Savings and Loan of Chicago, Northwest Mortgage, and Banco Mortgage Company. The total volume of private pass throughs traded is small compared to the three government related agencies and no single private issuer controls a major share of the mortgage market.

Privately issued pass through securities have several features.[12] Private issuer pools may include a variety of mortgage types with varying interest rates and loan sizes. The total number of loans in private pools is often less than in government pools. Generally, the total issue size

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[12]Although privately placed PTs have similar features of the government related programs, there are basic differences in their composition. Also, the availability of public information related to these issued is more limited than the three government sponsored programs.

of private pools is less than for government pools. The mortgages backing privately placed pass throughs are guaranteed by private mortgage insurance companies, instead of by government agencies.

Comparison of Mortgage Pass Throughs with Treasuries

Treasury securities are considered default free investments since they are guaranteed by the federal government. These instruments are traded in a very active secondary market and possess excellent liquidity. Similarly, some MPTs are also guaranteed, either explicitly or implicitly with the government and have a liquid secondary market. However, there are several important differences between Treasury and MPT securities.[13] The most important difference is the prepayment feature of the MPT securities. As previously noted, the prepayment option exposes investors to higher levels of risk since they are unable to know with certainty the exact pattern of cash flows. This problem does not exist for government securities.

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[13]These securities are similar with respect to their high quality credit risk assessment, high liquidity rating, and full range of coupon offerings.

The pattern of cash flows received from a pass through security can be greatly affected by the prepayments that are paid to the investors. The actual effect of a given prepayment rate on the return paid by GNMA securities based on Federal Housing Authority (FHA) experience can be easily determined. When the prepayment rate is increased, the maturity of the investment is reduced and the cash flow payments are received earlier than expected. For a pass through security selling at a premium, an increase in prepayment rates causes the principal amount to be returned earlier than expected.

Another difference between MPTs and Treasuries is the frequency of payments made to investors. Pass through securities provide payments each month while Treasury bonds have semiannual payments. The monthly payment feature associated with pass through securities provides investors with an advantage of monthly compounding opportunities which are not available to government security investors. It also exposes them to greater variability in possible reinvestment rates.

In this study, it is assumed that efficient capital markets do exist. In an efficient market scenario, information is widely known to all market participants so that security prices can adjust to any announcement of new information; current stock prices instantaneously reflect

all relevant information. This efficient capital market concept is mentioned because there are several important implications from this hypothesis that have a direct effect on mortgage backed security analysis. Capital markets are considered efficient since there are many rational investors who will immediately react to the announcement of new information.

It should be noted that important relationships will be developed between U.S. Treasury bonds and mortgage backed securities. Both instruments have the same default risk as a result of the government guarantee associated with each investment. It should also be realized that both Treasury bonds and mortgage securities have similar liquidity risk since there exists very active secondary markets for each security. The remaining risk variable associated with the mortgage security involves prepayment risk. The risk associated with the prepayment option will be thoroughly examined in this dissertation; certain results are based on the efficient capital market assumption.

Current Changes in the Mortgage Security Market

A wide variety of nontraditional financial instruments have been structured to meet the needs of market investors. A description of the features found on such issues as

collateralized mortgage obligations, mortgage backed bonds, and stripped mortgage backed securities will be provided to enhance the understanding of these newly created issues. This study, however, will focuses primarily on the analysis of Federal Home Loan Mortgage Corporation Participation Certificates (FHLMC PCs).

Collateralized Mortgage Obligations

Collateralized Mortgage Obligations (CMOs) are debt obligations that are backed by whole loan mortgages or mortgage pass through securities. CMOs, which were originally issued by FHLMC in June 1983, have become a very popular type of mortgage security investment; investors recognize that this security provides predictable cash flow payments. These instruments differ from traditional PTs primarily by the manner in which principal is retired. As previously stated, investors in pass throughs receive part of their principal each month; full principal payback does not occur until the final mortgage in the pool is retired.

The structure of CMOs is unique in that a sequential bond retirement process is employed instead of a pro rata principal process.[14] Funds that are raised from the assets

[14]For a comprehensive presentation of the structural characteristic of the CMO and an analysis of CMO cash flows refer to Richard Roll's 1986 study.

in the collateral pool are used to pay the interest and the principal to a specific group of bondholders or tranches. Interest payments are always made to each of the bondholders within the obligations. Any excess funds that remain after all interest payments are made to all bondholders are paid only to the shortest maturity bondholders. After the first group is paid, the next tranche then becomes the sole receiver of any principal payments. After this group of bonds is retired, the next tranche in the sequence becomes the sole receiver of the principal payments. This process continues until the outstanding principal owed to the last group of bondholders is retired.

CMOs are designed so that prepayment risk can be reduced. Short term maturity investors receive predetermined interest and principal payments and any unplanned prepayments that are made by mortgage holders. Both intermediate term and long term investors receive only interest payments until all short term bondholder obligations are fully paid. Since all short term maturity debt obligations must be retired before any other investors can receive principal payments, the long term investors are protected from the prepayment option. Short term investors accept the prepayment risk for the long term investors; however, the short term bondholders are protected from exposure to interest rate risk.

Although FHLMC first introduced CMOs with whole loans as collateral, almost half of all CMOs have been collateralized by GNMA MBS. CMOs generally offer yields higher than similar Treasury bonds; in 1986, CMOs yielded between 8% to 10% while Treasury bonds yielded between 7% to 9%. By 1985, a total of $30 billion of CMOs had been issued since their inception in June 1983.[15]

Mortgage Backed Bonds

Mortgage backed bonds (MBBs) are an increasingly important type of mortgage security being traded in the capital markets. These instruments are general obligation debts of issuing institutions and do not represent the sale of assets.[16] The debt of a lending institution is backed by a pool of mortgages that generally maintains collateralization of approximately twice the market value of the secured debt. To obtain a favorable rating, issuers must maintain collateralization that is far in excess the the actual face value of the bond principal amount. This distinct collateralization feature enables these investments to

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[15]See Euromoney June 1986 p30.

[16]In 1975, California Federal Savings and Loan was the first institution to issue a mortgage backed bond. Roughly 80 mortgage backed bond issues have been sold by various institutions since the initial offering.

possess high credit ratings regardless of size and creditworthiness of the issuing institutions.

The process of issuing MBBs is very similar to that of corporate bond issuance. While interest payments are paid semiannually, principal payments are made either at maturity or at periodic points in time. Financial institutions are increasing the issuance of MBBs because of the low rates that investors require for this low risk investments; this allows the issuer to borrow funds at very affordable rates for an intermediate time period. The more recent MBBs have used guaranteed mortgage PTs as well as single family loans as backing for the mortgage bonds. The general size of the offering is about $100 million.

Stripped Mortgage Backed Securities

Stripped Mortgage Backed Securities (SMBs) represent claims to unequal portions of payments derived from a mortgage pool; the coupon payments are removed or stripped from the principal. This type of security is created by separating portions of the principal and interest payments of a mortgage pool into different groups of securities. Although CMOs separate principal receipts among groups of securities with different maturities, SMBs separate both principal and interest payments among groups of securities

with the same maturity.[17] The two forms of SMBs currently offered by FNMA include one class mortgage pools backed by discount MBSs and another class backed by mortgage pools with premium MBSs.

The SMBs supported by mortgage pools with discount mortgage backed securities are backed by relatively low coupon mortgages with low prepayment rates. These mortgages are used to create both a low coupon, discount security and a high coupon, premium security. While principal payments are equally divided between each class, interest payments are not divided equally between each class of mortgage security.

The SMBs supported by mortgage pools with premium mortgage backed securities are backed by relatively high coupon mortgages with high prepayment rates. This type of SMBs includes both a low coupon discount issue and an interest issue. The bulk of the payments received by the low coupon discount security are derived from the principal generated by the mortgage pool. The majority of the payments received by the interest security are provided by the interest generated from the mortgage pool.

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[17]Richard Roll's study in 1986 provides a discussion on the structure of FNMA stripped mortgage bonds and the price sensitivity of these securities.

CHAPTER III

MORTGAGE BACKED SECURITY PRICING

Introduction to Mortgage Security Pricing

A mortgage backed security is backed by a collection of mortgages that are grouped together to form a pool. All cash flows received from the mortgages are paid to the mortgage security investors on a monthly basis. Although a MBS is considered a fixed income security, it is different from a traditional fixed income security. Conventional fixed income securities offer a constant semiannual payment of coupon interest and a single payment of principal on the bond's maturity date. This payment process enables the pricing of conventional fixed income securities to be a fairly straightforward process.

MBS provide variable cash flows that consist of predetermined amortized principal and interest payments. It is the potential of principal prepayments which complicates the cash flow patterns of MBS. If mortgage prepayments exist but are assumed to be static and unaffected by changes in interest rates during the investment horizon, the standard bond valuation Equation 1 can be modified to price mortgage securities.

$$(1)\quad P_o = \sum_{t=1}^{n} [I/12(1/(1+k_d/12)^t]$$

where I is the scheduled interest, principal, and prepayments; k_d is the required rate of return, and n is the time at which the bond matures. Since homeowners usually do have the right to prepay without penalty, MBS are difficult to value because of the uncertain pattern of their cash flows. Thus, the majority of the concern about pricing mortgage securities has focused on the tendency of mortgagors to prepay their loans prior to maturity.

Review of Mortgage Backed Security Pricing Theory

Deriving the price of a mortgage backed security is a complex process, due largely to the variability of payments. The purpose of this section is to describe the methodologies that are currently being used to price mortgage backed securities and to summarize the empirical tests of the various models. In recent years, there have been several important studies that examine the process of correctly pricing mortgage backed securities; Curley and Guttentag (1977), Dunn and Singleton (1983), and Dunn and McConnell (1981A), (1981B). The primary goal of these research

efforts is to estimate prices based on different assumptions of security prepayment rates and market interest rates.

Curley and Guttentag (CG) (1977) consider the significance of the mortgage prepayment feature in the process of pricing GNMA securities.[18] They assume that the market price of any financial security must be based on a correct calculation of the expected yield which the investor associates with the market price. CG claim that mortgage securities have not been correctly priced; the realized yield of mortgage backed securities have not been equal to the expected yield that is implied by the transaction price. Their study is motivated by the belief that the existing procedure used to price outstanding mortgages is inappropriately designed and tends to provide incorrect prices.

In a review of the main variables that are used to determine expected yield on mortgages, CG assert that since most mortgages are purchased at prices different from par, the expected yield is dependent on when the loan is prepaid. The policy of practitioners at the time of their study was to assume that the entire remaining loan balance would be prepaid after a period equal to the average life of a portfolio of comparable mortgages. The specific procedure

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[18]They provide a method for predicting the value of an outstanding residential mortgage and examine the difference between their method and traditional methods.

was to assume that a 30 year mortgage would be prepaid in 12 years. CG demonstrate that this assumption tends to produce biased estimates of expected yields since the true mortgage yield and the mortgage life maintain a nonlinear relationship.

To effectively calculate mortgage yield, CG propose that an entire distribution of prepayment probabilities should be used in place of an average life value and that the mortgage yield must incorporate reinvestment of prepaid balances in the portfolio based on the same probability values. They develop a linear regression model in which the prepayment rate is a function of the interest rate differential, prevailing market discount rates, and mortgage pool age. CG use the linear regression model and actual data to estimate the distribution of termination rates.

Simulation analysis is used to compare the mortgage yields derived from the CG model with those derived from the traditional average life model. The results of using the CG method to determine mortgage yields compared to the conventional rule of thumb method were much more robust when outstanding mortgage data was utilized than when new mortgage data was used. Specifically, CG examine the difference between the value obtained with their procedure and the value obtained with a rule of thumb method which assumes termination occurs after a period equal to half the

remaining life. As expected significant errors in valuation resulted when the rule of thumb method was considered.

CG conclude that their model, which uses a distribution of termination or prepayment probabilities, provides more accurate values of outstanding mortgages than any rule of thumb method based upon point estimates of expected remaining life. Their approach enables researchers to test the level of possible error in termination rates and analyze risk in a more effective manner than has previously been available to mortgage investors.

Dunn and McConnell (May 1981) use simulation analysis in order to test whether there exists significant differences in the prices derived from several widely used GNMA mortgage backed valuation models.[19] Dunn and McConnell use three distinct valuation models including the traditional average life method, the Curley and Guttentag model, and the Dunn and McConnell interest contingent claims model to price GNMA mortgage backed securities. They justify the selection of these valuation models since each takes a unique approach to incorporating the prepayment effect on the price of mortgage backed securities. As previously mentioned in the literature, the main area of confusion in pricing MBSs

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[19]If the alternative models they test all generate similar values, investors can use the most straightforward and economical approach. This is one reason that is used to justify the importance of study.

involves the tendency of mortgage holders to prepay their loans prior to actual maturity.

The traditional average life model assumes that amortized payments of interest and principal are paid to investors during an average life period that is associated with a portfolio of similar mortgages. At the end of the average life period, typically 12 years, the entire remaining principal balance of the loan is repaid. Unfortunately, the average life procedure does not take into account the prepayments that can occur either before or after the expected average life.

To eliminate the obvious limitation of the average life model, Curley and Guttentag include an entire time distribution of prepayment probabilities in their valuation model. Assuming that the yield to maturity for all GNMA securities is the same, a modified version of the Curley and Guttentag method utilize the yield to maturity for new GNMA securities to price old GNMA. Dunn and McConnell also test a contingent claims model which they developed specifically for valuing GNMA securities and largely based on the generic Cox Ingersoll Ross model.

In addition to the different manner in which each of the models incorporate the prepayment effect on the valuation of GNMA securities, there are other distinctions among the models. Both the average life model and the

Curley and Guttentag model represent static certainty approaches which assume a constant horizontal term structure of interest rates. The Dunn and McConnell contingent claims model represents a dynamic uncertainty approach which is based on a general equilibrium theory of the term structure of interest rates.

Several results can be drawn from the simulations performed in this study. Prices are derived from each of the models under several different market scenarios. They examine market conditions when the term structure is ascending and when the term structure is descending. When new GNMA security prices are close to par, the models on average provide smaller price differentials. When the remaining time to maturity is greater than 25 years, the average life model generates lower price values than the Curley and Guttentag model or the contingent claims model. For both upward and downward sloping term structures, the models provided greater price differentials for those issues with lower coupon rates than for those with higher coupon rates.

In summary, Dunn and McConnell use simulation analysis to determine whether several GNMA mortgage backed pricing models provide significantly different prices under various market conditions. They compare prices derived with a contingent claims model, a traditional average life model,

and a variation of the Curley and Guttentag model. For certain situations, price differences are evident but the authors are unable to conclude that one model is always superior to another model.

Dunn and McConnell (June 1981) note that GNMA securities are simply instruments that are backed by amortizing, callable mortgage loans. Their primary concern is to develop a valuation model for GNMA securities which will allow them to gauge the effect of the amortization, call, and prepayment features on the price of GNMA mortgage pass through securities. To accomplish their objective of deriving a more effective pricing model to value GNMA securities, they rely heavily on the generic model for pricing interest contingent securities developed by Cox, Ingersoll, and Ross (CIR) (1978).[20]

Dunn and McConnell systematically describe the basic assumptions and the valuation equation derived by CIR for the value of any contingent claim and then extend this generic bond pricing model to include the unique features of GNMA mortgage backed PT securities. They note that since the price of a GNMA security must incorporate any

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[20]Cox, Ingersoll, and Ross (1978) develop a model that determined the equilibrium price of fixed income securities. Dunn and McConnell (1981) used this model along with the assumption that investors display logarithmic utility to calculate prices of GNMA MBSs.

possibility of loan prepayments an additional term must be added to the generic pricing model.

The results from their study indicate that the amortization and prepayment characteristics of GNMA securities tend to increase the price of GNMAs while the callability feature tends to decreases the price of the issues when the time to maturity is long. The callability feature has more of an effect on the value of the security than either the amortization or the prepayment feature; when the time to maturity is short, the amortization feature has more of an influence on security price. In the aggregate, the impact of all three features actually lowers the interest rate risk and expected return of GNMA securities relative to other issues that do not possess the typical characteristics of GNMAs.

DM developed their GNMA valuation model in an effort to improve the pricing and ultimate performance of GNMA issues. They realize that there still exists the need for a thorough empirical test of their valuation model against other popular MBS valuation models.

Dunn and Singleton (1983) examine a consumption based model of prices for both GNMA securities and Treasury bonds, where few restrictions are placed on the stochastic process

that determines consumption and return values.[21] They estimate parameters describing preferences and test the price behavior of these securities. It should be noted that the aggregate return relation of their model differs from previously developed mortgaged backed security pricing models.

Specifically, DS develop a general conditioning information set that considers more information than previous models have allowed. An interesting twist to their approach occurs since they do not explicitly model the conditional probabilities of prepayments. The empirical results of the DS study show that the restrictions on the return behavior of on GNMA securities are consistent with the data. The tests did not indicate rejection of the model. The evidence from their analysis provides supports to the position that GNMA pass through securities are correctly priced relative to long-term Treasury bonds.

Summary of Mortgage Pricing Literature

Several observations can be made from this review of the mortgage backed securities pricing literature. To

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[21]The issue of whether the relative price volatility of GNMAs and Treasury bonds are consistent with certain equilibrium models is tested by using the cross sectional restrictions on the prices.

correctly price a MBS, the valuation model must incorporate the prepayment option. The literature has shown that the prepayment feature has a significant effect on the pricing of MBS. Conditional probabilities of prepayments and FHA termination data are used in the models; the prepayment issue is not explicitly considered in the valuation models. The primary goal of these studies has been to examine the valuation problem from an equilibrium pricing perspective and explicitly incorporate the prepayment option effect. Although the researchers do recognize the importance of the prepayment option, they have tested their models using simulated data but not actual market data.

The next chapter focuses on the estimation of prepayments for mortgages that are used to back MBS. Since mortgage backed security pricing models include a prepayment component, a comprehensive discussion of the prepayment feature is provided in this study.

CHAPTER IV

CURRENT METHODS OF ESTIMATING PREPAYMENTS OF MORTGAGE BACKED SECURITIES

Ways to Measure Mortgage Prepayments

In order to apply any mortgage security pricing model, the cash flow pattern of the security must be considered. To obtain a more thorough understanding of mortgage security cash flow behavior, one has to focus on the factors that influence prepayments. Similar to the feature found on callable corporate bonds is the prepayment option associated with mortgage security investments.

Although the call option on corporate bonds is typically carried out whenever management decides to refinance their debt at lower interest rates, the prepayment option may be exercised during different changes in interest rates. Mortgages included in a pool come from a fairly large population of mortgage holders each of whom is motivated by many different reasons to prepay their loans. Many of these factors, which will be described in section II, are extremely difficult to quantify. As would be expected, trying to price mortgage securities by directly

valuing each of the prepayment options is a difficult process.

The previous chapter on mortgage backed security pricing theory stressed that prepayment rates must be included in the valuation of mortgage backed securities. In review of the methods that have been used to estimate mortgage prepayment rates, three popular techniques include point estimate, FHA experience, and conditional prepayment rate (CPR).

Average Life Estimate

The first and most straightforward technique used to estimate prepayments on mortgage backed securities, the average life estimate, assumes the occurrence of prepayment at a single point in time.[22] The 12 year life value is the most common time period used by investors and was determined from past prepayment behavior where most FHA mortgages prepaid after twelve years. The 12 year life convention assumes that all mortgages in a pool return scheduled principal and interest payments up to the 12th year; on the 12th year all mortgages in the pool are entirely prepaid. It is evident that this arbitrarily chosen process is not

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[22] A review of actual mortgage pool prepayment rates shows that prepayments generally occur throughout the entire life of the mortgage pool.

very effective for evaluating mortgage securities particularly during times of rapidly changing interest rates. The main problem associated with this technique is that prepayments do not necessarily occur at a single point in time. Generally, prepayments are made throughout the life of the mortgage pool.

Federal Housing Authority Experience

The most widely used technique for prepayments estimation on mortgage backed securities is the FHA experience method.[23] The FHA provides investors with information related to the percentage of mortgages expected to prepay during each year in monthly intervals. By using a multiple of FHA experience cash flow patterns are provided over the life of the mortgage pool.[24] This results in a specific part of the mortgage in the pool prepaying during various periods, over the life of the investment, instead of the entire pool prepaying at a set time period.

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[23]This traditional method for estimating prepayments is based on survival rate data gathered for mortgages that are insured by the Federal Housing Authority.

[24]A multiple such as 100% FHA experience represents the normal prepayment rate that is suggested by the published FHA table. A 300% FHA experience refers to a pool that is expected to maintain prepayments at 3 times the rate of those obtained from the FHA table.

Unfortunately, there exist serious problems associated with the FHA experience method. As would be expected, FHA experience is based only on the age factor of a mortgage and fails to consider other important variables such as the coupon rate and the year of mortgage origination. A further problem involves the restrictive application of FHA experience since only assumable FHA and VA mortgages are considered. Most conventional mortgages cannot be assumed by the purchasers of a home.

Another shortcoming associated with the FHA experience measure occurs when future prepayment rates are estimated. If a multiple of FHA experience is used to predict future prepayment rates, there exists the assumption that a particular prepayment pattern will continue indefinitely. This assumption may not hold. Normally, mortgage pools do not follow a predictable constant prepayment pattern. Fast paying pools do not necessarily increase prepayments proportionately across policy years. It should be noted that in spite of the shortcoming that have been listed, the FHA experience technique continues to be widely used throughout the mortgage backed securities market.

Conditional Prepayment Rate

An increasingly popular technique used for prepayment estimation on mortgage backed securities is the conditional

prepayment rate (CPR) measure. The CPR is derived from actual mortgage pool experience and is defined as the percentage of principal outstanding at the beginning of a period that prepays during the particular period. By incorporating the prepayment calculation on the principal balance of mortgages that have endured until the beginning of the period, CPRs take into consideration the age of the security.[25] The CPR technique assumes that principal is prepaid at some constant rate.

There are several distinct advantages associated with the CPR approach. This technique represents the actual prepayment experience of the mortgage issues and is not an arbitrary number. The cash flow distribution derived from the CPR technique incorporate a more efficient reflection of the actual prepayment on the mortgage security. The attractiveness of this constant measure also results from its simplistic calculations, its easily interpretable rate, and the fact that it can be easily adjusted to reflect current prepayment conditions.

A definite drawback in using the CPR technique to estimate future prepayments on mortgage backed securities involves the fact that a single prepayment rate estimate

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[25]This approach has also been referred to as a constant prepayment rate since it assumes a constant fraction of outstanding principal is prepaid each month.

applied against the entire remaining investment period may not effectively describe the mortgage aging effect on prepayments during the early period of the pool's life. When interest rates are fairly stable, the prepayment pattern for mortgages pools are expected to increase during the early years of its life primarily as a result of mortgage age and then tend to level off. One way to consider the age factor is to use a CPR which employs a time weighted average for a series of varying CPRs over time so that a greater weight is placed on the earlier years' expected prepayment experience.

Review of Mortgage Prepayment Theory

By examining mortgage backed pricing models, it becomes apparent that prepayment rates have an important effect on the valuation of mortgage securities. The amount of cash received from a mortgage backed security depends on the prepayment rate of the individual mortgages included in the mortgage pool. If the prepayment option is thought of as an unrestricted call provision, a mortgage borrower has the right to prepay the loan any time prior to maturity without incurring a prepayment penalty. The implication is that cash flow payments of mortgage securities are determined primarily by prepayment rates of underlying mortgages.

Recently there have been several studies that have focused on the important issue of effectively estimating mortgage prepayment rates. When mortgage prepayment rates become unstable, mortgage cash flows are more unpredictable. Since the actual cash flow received from an investment in mortgage securities is not known with certainty, researchers have recognized the importance of accurately estimating future prepayment rates for mortgage security investors.

Navratil (1985) analyzes the extent to which prepayments can influence the value of mortgage backed securities. He tests whether unexpected changes in prepayments result in large price changes for MBS investors. To examine the hypothesis, he employs a nonlinear asymmetric relationship to model prepayment rates on GNMA mortgage pools; the rates are assumed to be a function of the interest rate differential between the current market interest rate for mortgages and the contract rate at which the mortgage pool is first issued.[26]

Navratil suggests that not only should prepayments be inversely related to the interest rate differential but the relationship should also be asymmetrical. He claims that a proportional change in interest rates should have more of an

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[26]Navratil uses prepayment rate data that was presented in Pinkus and Firestone (1983). Each observation is based on several hundred GNMA pools.

effect on prepayments when the interest rate differential is positive than when it is negative. By this Navratil means that prepayments are expected to increase at a faster pace when current market rates are below mortgage contract rate. Prepayment rates are not expected to decrease as fast when current rates are above mortgage contract rates.

Several interesting results are provided that relate to the prepayment effect. Point estimates of prepayment rates are used to enhance the analysis of the differential prepayment relationship. On average, prepayment rates decline as the pool contract rate increase when the interest rate differential is negative. When pool contract rates equals current market rates, the payment rates averaged about 4.3%. Prepayment rates decline slightly as the mortgage pool contract rate increases when the interest rate differential is positive.

The estimated elasticities for various interest rate differentials increase as the mortgage pool contract rate increases for both negative and positive interest rate differentials. For a zero differential, the elasticity measure is consistently at 0.20. The greatest elasticity value occurs at the -300 basis point level; on average the elasticities tend to be slightly more volatile when the interest rate differential is negative.

Navratil performs an additional test to examine the effects of interest rate changes on the duration of mortgage pools. The durations of the mortgages are used to examine the effect that changes in prepayment rates have on the pricing of MBS. As expected the duration of a mortgage is inversely related to both interest rates and prepayment rates. For discount mortgages, estimated durations tend to be greater when the interest rate differential is positive and the prepayment rate is relatively low. The duration for a mortgage pools is greatest when the interest rate differential is +100 basis points. Upon observation, it is apparent that the discount mortgages have intermediate term duration measures that range from 4 to 6 years. Therefore, discount mortgages tend to have durations similar to intermediate investments; i.e., 5 to 10 years to maturity.

For premium mortgages, estimated duration is lower when the interest rate differential is negative and the prepayment rate is relatively high. Premium mortgages are prepaid so fast that their durations mirror those similar to short term investments. The results of his study clearly reveals the asymmetrical risks to which mortgage security investors are exposed as a result of the prepayment option.

Clayton and Goldstein (1986) propose a different approach for estimating prepayment rates of mortgage backed securities. Their technique employs interest rates,

security prices, and cash flows for mortgage securities in order to estimate prepayment rates.[27] They assume that expected rate of prepayment can be determined from the market prices of mortgage backed securities and note that prepayment risk is not expected to be constant across coupons. Discount securities tend to have lower prepayment risk than premium priced issues. The highest level of prepayment risk occurs for securities priced at par since any relatively small change in market rates can potentially have major effects on mortgage prepayments.

In their study, monthly data for 125 FHLMC PCs are used. For various prepayment rates, they estimate cash flows. The cash flows and security prices are then used to determine yield to maturity values for each possible prepayment rate. From the data Clayton and Goldstein observe that the yield to maturity on premium priced bonds decreases, when higher prepayment rates are used, while the yield to maturity on discount bonds increases, when higher prepayment rates are used.

Both duration and prepayment rates for PCs with various coupons are estimated during the market conditions that prevailed during the period of July 1983 to June 1984. The

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[27]Their prepayment approach can provide valuable information to savings and loan associations, savings banks, mortgage lending agencies, and other institutions that attempt to control their to interest rate risk.

estimated prepayment patterns appear to be fairly predictable. Discount mortgage securities consistently had low expected prepayment rates and a duration of about 4.5 years; premium priced securities had high expected prepayment rates and an expected duration of 1 to 3 years. Both the prepayment rates and durations for these premium securities appear to be relatively unstable.

Clayton and Goldstein also employ a procedure similar to the one that is actually used in the market to quote yields on mortgage backed securities. They discover that the durations estimated by this technique follow a predictable pattern and remain relatively constant across PC, with different coupons. The duration estimates are all close to 4.5 years. Under the 12 year prepayment assumption, the estimated yields to maturity follow a U shaped pattern such that the yields are relatively high for low and high coupon securities and bottom out for current coupon issues.

Generally, investors would expect the yields for equal duration bonds to be relatively constant across coupons. Prepayment risk should be higher for par value securities than for premium or discount issues, which implies higher yields for current coupon securities. A comparison is made between the estimated prepayment rates and the actual prepayment rates; they conclude that their procedure

provides fairly accurate prepayment estimates, though some values are biased upward.

Green and Shoven (1986) estimate the effects of interest rates on mortgage prepayments.[28] They point out a serious problem that has limited the amount of work related to mortgage analysis, namely, data availability. They expend a substantial amount of their efforts in collecting mortgage data. Specifically, they gathered data on individual mortgages and analyzed the related prepayment periods. Prepayment data was collected for over 3900 mortgages from several savings and loan associations during the period of 1975 to 1982.

Green and Shoven make use of a proportional hazards model in order to estimate the percentage reduction in prepayment probability associated with interest rate changes. Instead of simply using an interest rate differential variable as previous studies have used, they employ a measure of the lock in effect that results from the interest rate differential between that existing in the market and the contracted rate of the mortgage. Prepayments should be influenced by the percentage of the lock in

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[28]Green and Shoven (1986) note that savings and loan associations have been hurt as a result of the term structure of their assets and liabilities. These institutions held long term fixed rate mortgage assets and short term liabilities. As interest rates rose, the net worth of many institutions actually became negative.

relative to the market value of the house. This lock in effect is similar to real estate brokers fees and can be thought of as a transaction cost of moving.

In the aggregate, Green and Shoven calculate the lock in measure for each mortgage each year for the period of 1975 to 1982. They discover a significant inverse relationship between the lock in variable and the prepayment rate, which implies that the interest rate lock in is an important factor of prepayment occurrences. They determined that a modest 10% lock in reduces prepayment occurrences by 35% during 1979-1982, when due on sale clauses were allowed. Prepayments were reduced by 63% in 1979-1982, when due on sale clauses were unenforceable. The results indicate that mortgage prepayment probabilities depend on the relationship between mortgage contract rates and prevailing market rates.

Milonas and Lacey (1987) propose that many factors may induce homeowners to prepay their debt; they examine the prepayment rates of GNMA pools by incorporating both economic factors and socioeconomic factors. It is widely accepted that there exists an economic incentive to prepay mortgage loans if prevailing mortgage rates fall below the original mortgage contract rate. It is also recognized that socioeconomic factors which include marital status, population mobility, job relocation, mortgage owner age, and regional economic conditions are likely to cause homeowners to prepay their loans.

These researchers realize that problems are encountered when all factors are considered. Although the economic factors can be accurately measured, the various socio-economic factors are extremely difficult to directly measure.[29] Upon close scrutiny, it is learned that the socioeconomic factors tend to vary across different GNMA pools. Milonas and Lacey demonstrate that when mortgage pool age serves as a proxy for the socioeconomic factors, prepayment rates increase through time since pool age increases as time progresses. They notice that investors associated with older GNMA pools prepay their loans at a quicker pace than investors associated with newer GNMA pools as a direct result of the socioeconomic factors.

The researchers also suggest that prepayment experience can be perceived as a sequence of higher order age variables. Assuming the pool age effectively incorporates socioeconomic factors, pool age may follow a cyclical pattern of the socioeconomic factor effects. It is suggested that low mortgage prepayments occur during the early years of the mortgage pools followed by a cyclical pattern of high mortgage prepayments as time progresses.[30]

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[29]They point out that prepayments may also be related to the joint effect of both economic and socioeconomic factors, which maybe difficult to separate.

[30]Milonas and Lacey found that the prepayment option of GNMAs is significantly affected by both the changes in interest rates and the shape of the term structure.

Milonas and Lacey include in their analysis an examination of the economic advantage of investors prepaying their loans.

Several important results are derived from the study. In comparing the prepayment experience with the mortgage pool rate, they discover significant differences in the prepayment rate among various mortgage pool rates. The authors note that no general conclusions can be made, since it is possible that the mortgage pool rate variable is also a proxy for the age of the mortgage pool. They observe from their data that high age pools tend to have low pool rates while low age pools are associated with high pool rates.

When they compare prepayment rates and mortgage pools age, their tests show significant differences between mortgage age and prepayment experience for the period of September 1983 to May 1985. Both the small and large mortgage age groups possess higher prepayment experience than the intermediate mortgage pool age groups. Again the authors note that general conclusions cannot be drawn since it is possible for the age variable to serve as a proxy for the mortgage pool rate. They observe from the data that high pool rates are matched with low mortgage age while low pool rates are associated with high age mortgages. In testing whether age alone can explain a significant portion of the prepayment experience, the authors learn that the

explanatory power of the regression model is high for all of the mortgages except for the low and the high pool rates.

When Milonas and Lacey combine the effects of interest rates and age on prepayment experience, they discover from the regression model that high pool rates provide fairly high explanatory power since the economic incentive to prepay is greatest for the higher mortgage pool rates. In a related test, the current market interest rates did represent an important factor in predicting mortgage prepayment rates. When the mortgage pool age variable is held constant, the researchers demonstrate that prepayment decisions are driven by increased value of the prepayment option.

In review of the material relating to the mortgage prepayment literature, several important observations can be made. Generally, analysts agree that mortgage prepayment rates depend on the relationship between mortgage contract rates and prevailing market rates. It is also shown that socioeconomic variables also effect mortgage prepayment rates. The studies that have been cited have focused on the estimation of prepayments for mortgage backed security pricing models based on such factors as market rates of interest, socioeconomic factors, and current market prices. All of these variables are very important for estimating prepayment rates and should be included in mortgage valuation models.

The prepayment literature can be grouped into two parts. The first section values the prepayment option effect and does not explicitly address the prepayment issue. The second section explicitly addresses the prepayment issue by either directly or indirectly estimating prepayment rates. Prepayment rates are directly estimated by regressing historical prepayment data against other independent variables. To estimate prepayment rates indirectly, market based data is used to infer prepayments. Prepayment rates are determined by equating future cash flows with current observed prices. Estimated prepayments are then compared with observed prepayments.

CHAPTER V

A REVIEW OF DURATION THEORY

Review of Duration Analysis

In the last decade, duration analysis has been used extensively by fixed income managers to determine and manage the risk of their portfolios. Fredrick Macaulay developed the concept of duration in 1938 after he became dissatisfied with the concept that was currently being used to define risk. In the bond market, term to maturity was the primary measure of investment risk. Maturity is an inadequate measure because it considers only the final payment and does not take into account the amount or timing of interim payments. Macaulay claimed that duration was a superior measure of risk since it incorporates the size and timing of all payments.[31]

The objectives of this section are to examine the basic concept of duration, to review its development process, and to show alternative ways to measure duration based on different assumptions about the term structure. Increased

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[31]While most researchers agreed that Macaulay's measure was theoretically superior to maturity, duration received very little attention and was virtually disregarded until the early 1970's.

research efforts involving fixed income securities have led to two important applications of duration. Specifically, the primary uses of duration involve the analysis of bond price volatility and the immunization of bond portfolios.

Macaulay defined as duration the weighted average maturity of a fixed income security. The traditional duration formula is provided in the equation below.

$$(1) \quad D1 = \sum_{n=1}^{m} [C_n/(1+i)^n + P_m/(1+i)^m] \Big/ \sum_{n=1}^{m} [C/(1+i)^n + P/(1+i)^m]$$

$$(1a) \quad D1 = f(C,P,i,n,m),$$

where C is coupon payment, P is maturity payment, n is number of periods to the coupon payment, m is number of periods to maturity, and i is yield to maturity. It should be noted that the duration of a zero coupon bond is always equal to the term to maturity, since the entire cash flow is made at maturity. For the more common coupon bond, the duration will always be less than the term to maturity because of the interim cash flow payments.

The duration measure is a function of several variables. In examination of Equation 1, it is apparent that duration is inversely related to the coupon of a bond. High coupon payments cause a larger proportion of the total

return to be received in the form of interim cash flows; this leads to a shorter duration. Duration is also inversely related to changes in market interest rates; as market rates increase, the duration period will decrease. Generally, duration is directly related to the term to maturity. For bonds selling at par or premium, duration increases at a decreasing rate with maturity. For bonds selling at a discount, duration increases at a decreasing rate with maturity and eventually starts to declines.

The definition of duration depends upon the assumed shape of the term structure and the nature of the shock to the term structure.[32] An additive shock to interest rates causes all spot rates to change by an equal amount, while a multiplicative shock causes all spot rates to change by an equal percentage. Macaulay's duration is based on the assumption of a flat term structure and an additive shock. The derivation of the duration measure for an additive stochastic process is provided in Appendix A.

Duration and Immunization

Bond portfolio managers are often required to provide their investors with a certain rate of return at the end of

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[32]Several different duration measures will be provided in this chapter based on the assumed position of the term structure and the type of shock to the term structure.

an investment period. Changes in interest rates cause the term structure to shift; this change in market conditions exposes investors to interest rate risk during the time of their investment period. Interest rate risk occurs when portfolio manager cannot guarantee investors their ending wealth position because of changes in market rates during the investment period. Interest rate risk is made up of two risk factors.[33]

The first factor involves price risk. If interest rates change during the investment period, which usually happens, realized prices at the end of the period will differ from the prices that were expected to prevail. As interest rates rise, the realized price will be below the expected price; if rates fall, the realized price will be above the expected price.

The second factor involves coupon reinvestment risk. This type of risk also occurs when market interest rate change; coupon payments are reinvested at different market rates. If interest rates rise after the bond is purchased, coupon payments will be reinvested at higher rates, which causes their portion of the ending wealth to be above expectations.

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[33]Van Horne provides an example of reducing interest rate risk in his discussion involving immunization of bond portfolios.

From this analysis it can be concluded that price risk and reinvestment risk have opposite effects on the ending wealth position. Portfolio managers often attempt to immunize their investments by eliminating these two risks factors. Bond portfolios can be protected from interest rate risk if the portfolio's duration, based upon appropriate assumptions of the term structure, equals the investment horizon. Duration can be thought of as the length of time which the price risk and the reinvestment risk have equal size but opposite positions.

Review of Duration Literature

There are several important studies that provide empirical tests of the relationship between duration and immunization. A review of the literature that has directly contributed to the application of duration analysis and bond portfolio immunization is discussed in this chapter. Examination of the original Macaulay duration measure, D1, shows that it can only be used to immunize a bond portfolio if the yield curve is assumed to be horizontal and limited to parallel shifts. This restrictive assumption of a flat term structure constrained only to parallel shifts caused researchers to actively examine other interest rate scenarios.

Fisher and Weil (1971) were the first to perform extensive simulation tests on bond immunization.[34] They compared their duration based, or immunization, strategy with a maturity based, or naive, strategy. Bonds were purchased with either a duration or maturity equal to a specific holding period. Their simulations provide wealth ratios for the alternative portfolio strategies over 5, 10, and 20 year investment horizons; they compare these wealth ratios with expected wealth ratios assuming the yield to maturity is realized.

Their first step was to create a portfolio of default free coupon bonds that is immunized against unexpected changes in interest rates; this ensures that investors are able to earn at least the return they expect at the time of purchase. Immunization is achieved by creating a portfolio whose average duration equals the investor's holding period.

The definition of duration that produces immunization is dependent on the nature of the assumed stochastic interest rate shocks. The stochastic process describes the way in which the term structure is assumed to shift. An additive stochastic shock causes the term structure to move

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[34]They define immunization as the elimination of interest rate risk. If a portfolio of bonds is able to maintain a certain value, regardless of changes in interest rates during the holding period, it is protected from interest rate risk.

up or down by the same random amount for each level of maturity. Fisher and Weil did not restrict their analysis to a flat term structure, but they did assumed interest rates changed in an additive pattern. The Fisher and Weil duration measure, D2, is defined below.

$$(2)\quad D2 = \sum_{n=1}^{m} [Cn/\ \pi_{t=1}^{n}(1+r_t) + Pm/\ \pi_{t=1}^{m}(1+r_t)]\ /\ \sum_{n=1}^{m} [C/\ \pi_{t=1}^{n}(1+r_t) + P\ /\ \pi_{t=1}^{m}(1+r_t)]$$

where C, P, n, m are defined as before and r_t is the one period discount rate (forward rate) in period t. If all forward rates are the same, implying a flat yield curve, D1 and D2 are the same.

Specifically, Fisher and Weil examined whether their duration strategy is more effective in immunizing a portfolio of bonds than a naive maturity strategy. The maturity strategy consists of buying and holding bonds with maturities equal to the length of the investment horizon and reinvesting the coupon payments in bonds with maturities equal to the remaining time to the end of the investment period. The data used in the study included interest rates for the period 1925 to 1968. During this time there were 39 overlying 5 year holding periods, 34 overlapping 10 year investment periods, and 24 overlapping 20 year periods.

The duration strategy was implemented at the beginning of the investment period and was based on the term structure that existed during the particular time period. The portfolio investment was rebalanced each year in order to maintain a portfolio duration equal to the remaining length of the investment period.

Their duration strategy out performed the maturity strategy over 70% of the time for several different holding periods; it provided consistently lower variances for the difference between realized and expected returns. The duration strategy results were generally closer to the expected yield; however, the values were not perfect matches, which implies that the duration based portfolios were not perfectly immunized. Fisher and Weil concluded that it is possible to select a portfolio of long-term bonds that is essentially riskless if investors choose a bond portfolio with duration equal to the time horizon.

There have been several other studies that have examined the immunization process using measures of duration that were different from the traditional Macaulay measure, since the duration measure used to immunize a bond portfolio should be a function of the nature of the shock of the term structure. Bierwag and Kaufman (1977) noted the possible existence of an additive shock to the yield curve in which all interest rates are not changed by the same nominal

amount; it is possible that the yield curve could experience a multiplicative shock in which all interest rates change by the same percentage. In their study, Bierwag and Kaufman assume that changes in the term structure occur by a multiplicative process rather than on additive manner, as Fisher and Weil consider in their study.[35]

Bierwag and Kaufman discuss several ways to specify duration. Macaulay's duration measure, D1, discounts all cash flows by the prevailing average yield to maturity on the bond being measured. The Fisher and Weil duration measure, D2, uses the appropriate spot rates to discount the future flows. Depending on the particular shift in the term structure, the two definitions will generally provide different duration values. It should be noted that the two duration measures will be equal only in the case of a flat yield curve. The Bierwag and Kaufman duration measure, D3, is defined below.

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[35]Bierwag and Kaufman realize that the duration measures used to immunize a portfolio should be a function of the nature of the change to the term structure.

$$(3)\quad D3 = 1/h(0,D3)^{*} \sum_{n=1}^{m} \left[Cnh(0,n) / \prod_{t=1}^{n} (1+r_t) + Pmh(0,m) / \prod_{t=1}^{m} (1+r_t) \right] \Big/ \sum_{n=1}^{m} \left[C / \prod_{t=1}^{n} (1+r_t) + P / \prod_{t=1}^{m} (1+r_t) \right]$$

where C, P, n, m are defined as before and h(0,n) is the n period yield.

Bierwag and Kaufman determined the duration values for a group of bonds using three definitions of duration D1, D2, D3 and concluded that the values did not significantly differ; all three values are highly correlated. They suggest that D1 may be used as an estimate for either D2 or D3. D1 is easier to calculate, since it is a function of the readily observed yield to maturity of the bond. As noted by Bierwag and Kaufman, the calculation of D1 requires neither a forecast of the stream of one period forward rates over the maturity of the bond nor a specific assumption about the nature of the random shocks is required.

Bierwag (1977) demonstrated that immunization is possible for various stochastic processes of the term structure but notes that the duration measure may differ for each case considered. He analyses additive, multiplicative, and combined shocks to the term structure and modifies the traditional Macaulay duration measure in order to provide

immunization strategies for non-flat term structures.[36] Bierwag successfully tests several measures of duration that achieve immunization based on different assumptions about the stochastic changes in interest rates.

Bierwag succinctly summarizes his results in several theorems. First, if there is a random additive shock to the future holding period yields, an investor who has a planning period of length m may fully immunize his return by choosing a coupon rate, so that the duration of the bond is equal to the length of the planning period. Second, if there is a random multiplicative shock to the future holding period yields, an investor who has a planning period of length M may fully immunize his return by choosing a coupon rate, so that the duration of the bond is equal to the length of the planning period.

Third, an investor can fully immunize his investment from additive and multiplicative random shocks by choosing a coupon rate, so the duration is equal to the length of the planning period, provided the initial yield structure makes this possible. Bierwag notes that this can hold only for a particular class of yield structures. Finally, an investor may be able to immunize his investment by diversifying among

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[36]He shows that the optimal duration measure for portfolio immunization is a function of the change to the term structure of interest rates.

different coupon bonds so that each relevant measure of duration for the income stream, taken as a whole, equals the length of the planning period.

Bierwag, Kaufman and Khang (1978) provide an overview of duration and bond portfolio analysis. In their review paper, they discuss the proper uses of duration in the analysis of security portfolios. They show that there is not any one unique measure of duration and equate duration to an index with a large number of alternative specifications each with its own purpose. Duration is viewed as a powerful tool but only within the restrictions of the appropriate model. Although Bierwag, Kaufman, and Khang limited their discussion to default free bonds, they claim that duration holds much promise of being applicable for bonds subject to default risk and for equity securities. Other potential areas of duration analysis include development of multiperiod models, measures of risk for financial institutions, and alternative optimization models.

Ingersoll, Skelton, and Weil (1978) note that much of the work prior to their study used duration as a proxy for interest rate risk. They systematically demonstrate that duration can't be a risk proxy for markets in competitive equilibrium except under very restrictive conditions.[37]

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[37]They summarize the alternative way to value duration and the market conditions associated with each measure.

If yield to maturity is used in the present value calculations instead of spot interest rates, Macaulay's definition of duration is not a general risk proxy. In their first theorem, they show that yields to maturity on assets with known fixed payments can change by the same amount only if the yield curve is flat and makes a parallel shift. Ingersoll, Skelton, and Weil point out that this result is not a dynamic relationship which can be used to measure security risk; instead, it must be considered only a comparative statics result for a one time change in the level of a horizontal term structure.

The second issue they examine is whether it is appropriate to use Macaulay's definition of duration for risk measurement when the yield curve is not flat but that all changes in the yield curve are constant uniform infinitesimal shifts. Here they are able to justify the use of the Macaulay duration measure in risk measurement for a special case. Ingersoll, Skelton, and Weil prove in their second theorem that for small shifts in the yield curve, the percentage change in value of assets with fixed payments is proportional to Macaulay's duration measure only if the entire yield curve experiences a uniform additive change.

Similar results can also be obtained for infinitesimal multiplicative shifts in the yield curve; as cited in their third theorem, the percentage change in the value of any

asset is proportional to D3 only if the yield curve experiences a multiplicative shift.

Finally, they point out that the occurrence of noninfinitesimal uniform changes in the yield curve implies arbitrage opportunities. Since there cannot exist arbitrage possibilities in the market place, they claim that the term structure of interest rates cannot shift in this manner. Therefore, noninfinitesimal uniform changes in interest rates cannot occur in a competitive equilibrium.

Khang (1979) realized that in examining interest rate movements certain patterns were clearly evident. It is well known that short term rates have fluctuated more rapidly than long-term rates and that the yield curve on the average has been upward sloping.[38] Since market data supports these observations, it becomes apparent that neither the additive nor the multiplicative interest rate shocks are consistent with these empirical findings. Khang noted that the duration values derived from these assumptions are unlikely to produce immunization for bond portfolio managers in the real world market environment.

In order to provide a duration measure that reflects actual market conditions, Khang developed a maturity

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[38]The traditional yield curve was upward sloping from 1930-1966. The pattern became cyclical from about 1975-1982. The current shape has been upward sloping.

dependent stochastic process. This provides a more effective measure of interest rate changes than either the additive or multiplicative assumptions of Bierwag's 1977 study. He specifies alternative duration measures that are valid for an additive maturity dependent process and a multiplicative maturity dependent process.

(4) $D4 = [\exp[1/P(m)]S\ C(t)\ln(1+\alpha t)\exp[-R(t)t]dt]-(1/\alpha)$

where P(m) is the market value of a bond with maturity m at some instant of time, C(t) is the cash flow received at t, ln is the natural logarithm, R(t) is the spot rate associated with each cash flow, and α is a measure of the variability of long term yields relative to short term yields. Thus, Khang's duration measure produces immunization for a stochastic process in which short term rates fluctuates more than long-term rates.

In an effort to test the relative effectiveness of several immunization strategies Bierwag, Kaufman, Toevs, and Schweitzer (1981) examine several duration measures, each based on assumptions of different stochastic shifts in the term structure of interest rates. To analyze the immunization strategies under various market environments, they divide the 1925-1978 period into 3 subperiods, with each subperiod characterized by particular market conditions. Subperiod one includes years 1925 to 1949, during which interest rates declined and remained at relatively low

levels. Subperiod two includes years 1940 to 1963, when interest rates began to move upward. Subperiod three includes years 1954 to 1978, during which interest rate levels increased at a rapid pace and were quite volatile.

They use a total of eight bond portfolios in the study.[39] Tests are conducted for five passive duration matching immunization strategies, two active immunization strategies, and one yardstick maturity matching immunization strategy. The five passive duration immunization strategies each incorporate a distinct assumption about the stochastic shift in interest rates. In particular, the first strategy, ID1, uses Macaulay's duration, assuming a flat yield curve and equal interest rate changes for all maturities. The second strategy, ID2, utilizes and is based on an additive stochastic process; it is assumed that any interest rate changes affect all maturities by an equal amount. This implies that the shape but not the location of the yield curve remains constant after the interest rate shock has occurred. The third strategy, ID3, is similar to the Fisher and Weil duration measure and is based on a multiplicative stochastic process; it assumes that the interest rate

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[39]Realized yields will always equal promised yields for default free zero coupon bonds with maturities equal to the holding period. Thus, these researchers examine coupon paying bonds.

changes cause the rate on each maturity to differ by a multiplicative value of the rate change.

The fourth and fifth immunization strategies, ID4(1.0) and ID4(0.1), assume that short term rates always change by more than long-term rates, an assumption clearly supported by empirical data. The two strategies differ by the proportion that the short term rates fluctuate relative to the long-term rates. The ID4(1.0) strategy assumes greater changes in short term rates relative to long-term rates than does the ID4(0.1) strategy.

Under the active rollover strategy, the portfolio is composed of a series of one year bonds that are rolled over each year. With the active long bond strategy, the portfolio includes a 20 year maturity bond that is maintained so that its maturity will decline to 10 years at the end of the investment planning period.

The yardstick maturity matching strategy is comprised of a portfolio made up of one bond with maturity equal to the remaining planning period. All coupon payments are reinvested in bonds having maturities equal to the length of time left in the planning period.

The results from the Bierwag et al. study are very interesting. Generally, the actual returns from the duration matching immunization strategies, ID1-ID3, were closer to the average annual promised returns than the

average annual actual returns from the other investment strategies. For both the total period and the subperiods, strategies ID1-ID3 provide almost identical realized returns. During the total time period, ID1-ID3 provided return values closer to the promised realized return than those promised by the maturity strategy 87% of the time. During the different subperiods, the effective rate of these immunization strategies ranged from 78% to 93%. ID1-ID3 provide return values within 5 basis points of the promised returns much more often than the other three strategies.

ID4(1.0) and ID4(0.1) consistently provide different responses from each other. The realized return for ID4(1.0) is on average lower than for ID4(0.1). It is also obvious that ID4(0.1) immunizes better than the maturity strategy only during the subperiod of 1925 to 1949.

The major conclusions from their research efforts are reasonably unambiguous. First, the empirical evidence suggests that portfolio immunization strategies based on the simple Macaulay duration measure ID1 immunize investments just as well as the more complex strategies. Given the inherent simplicity of ID1, this seems to imply that ID1 is the preferred strategy. Second, the duration matching immunization strategies produce realized returns closer to the promised returns than do the maturity matching strategies. Third, the duration strategies produce returns slightly less than the promised returns.

Bierwag, Kaufman, and Toevs (1982) recognize that many single factor duration (SFD) models used in pricing debt issues are not derived from a comprehensively defined theoretical basis.[40] They attempt to derive the theoretical foundations of a general bond valuation model; they also include in their discussion the primary conditions that must exist for the SFD model to satisfy general equilibrium conditions.

They began by noting that there have been a number of criticisms of the SFD models. The most popular criticism prior to the Fisher and Weil study was that duration could only be used when the term structure was horizontal. The development of duration measures based on different stochastic processes solved this issue. A second criticism focused on the point that most investors are not necessarily completely averse to interest rate risk, implying that immunization addresses only one part of the risk issue.

It is known that many financial institutions choose not to be totally immunized from interest rate risk in order to increase potential levels of profit. Finally the models have been criticized because they are deterministic in nature and do not allow random error terms.

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[40]They feel this may partially explained why practitioners have accepted the duration models for valuing debt securities much quicker than academicians.

In spite of the criticisms, bond portfolio managers have used the SFD models quite extensively; the models have also received a considerable amount of interest from individual investors. The major reasons for their prevalent usage are because the SFD models are fairly inexpensive to employ and empirical tests of the simple models provide results comparable to those obtained by the more advanced models. Several primary economic assumptions upon which the SFD models are based are highlighted by the authors. Investors are assumed to maximize returns for a given level of risk, a given stochastic process, and a given investment period. Only default-free debt securities are used in the analysis.

Duration and Bond Price Volatility

Under certain conditions, a direct relationship exists between the duration and the price volatility of a bond. The model that represents this direct relationship is provided in equation below.

(5) $dP/P = -D * [dr(1+r)/(1+r)]$,

where dP/P is the percentage change in price for the bond, D is the duration of the bond, and dr is the change in the market yield. A thorough understanding of this proportional relationship between duration and bond price volatility is

extremely important for managers who may need to adjust their portfolios as market rates change. If interest rates are expected to fall, the portfolio's duration should be maximized; the opposite is true for expected increases in interest rates. Price volatility is often used as a gauge for investment risk. Since there exists a direct relationship between duration and bond price volatility, duration can be considered an effective proxy for the risk of a bond portfolio.

Duration and immunization have received extensive attention in the past 20 years. Interest in duration and price volatility is relatively recent but appears to be increasing. A review of the studies that have assisted in the analysis of the duration and price volatility relationship is provided in this section. This relationship is crucial to the proposed empirical work of this dissertation.

Hopewell and Kaufman (1973) were the first to use duration to explain the relation between bond price volatility and changes in market interest rates. They systematically derive a relationship on which the percentage change of bond price equals the product of bond duration and the change in market rates (see Equation 5).[41] From this

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[41]If interest rates rise from 11% to 12%, the price of a bond with a 10 year duration and a 9 year adjusted duration, duration should decrease by 9%.

$$dP/P=[10/1.11][1]=-9.0\%$$

relationship, it is readily apparent that price sensitivity is proportionately related to the duration of the bond, given a small change in market interest rates. To derive the bond price equation, they take the differential of bond price with respect to interest rates. This process is demonstrated in Appendix B.

As Hopewell and Kaufman note in their study, a common generalization of bond price behavior is that for changes in interest rates, bond prices fluctuate more for long maturity bonds than for short maturity bonds. They show that this proposition, however, does not necessarily hold for all securities.

In particular, this behavior may not hold for long term discount bonds. There may exist conditions such that long term securities are less sensitive to changes in market interest rates than are shorter term securities. In this case we would observe an inverse relationship between maturity and bond price changes. Hopewell and Kaufman felt further research was needed in order to provide a more complete understanding of bond price behavior.

In summary, Hopewell and Kaufman use a price volatility relationship to demonstrate that for changes in interest rates, the relative change in prices of par and premium bonds is larger for long term maturity bonds. This is true because duration increases monotonically with maturity,

although at a decreasing rate, for par and premium bonds. For discount bonds, this relationship is not true, since duration increases with maturity up to a point and then decreases. The implication is that for discount bonds with maturities beyond a certain maximum level, the relative change in bond prices could be lower for long term maturity bonds.

Thus, Hopewell and Kaufman are able to explain both the common case where a change in interest rates causes the relative change in bond price to be greater for long term bonds and the perplexing case where a change in interest rates may cause the relative change in bond price to be smaller for certain long term discount bonds.

Haugen and Wichern (1974) provide a lucid description of the elements which have an impact on price volatility.[42] They note that not enough information is known about the theoretical determinants of bond price volatility. They demonstrate how the elasticity of price is positively related to the duration of the bond. In analyzing the relative interest elasticities of risky and risk free bonds, they show that the interest elasticity of risky bonds is less than for risk free bond issues with the same maturity.

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[42]Another concern of Haugen and Wichern relate to why the price of a growth stock is highly responsive to changes in the general level of stock prices.

The risky coupon bonds are less sensitive to interest rate changes because they possess shorter duration values. Generally, the variability of the coupon bonds tends to be an increasing function of maturity, but increases at a diminishing rate. Haugen and Wichern conclude that interest elasticity may increase with maturity and beyond some maximum point decrease monotonically.

Yawitz (1977) elucidates the importance of yield volatility and duration in determining bond price volatility.[43] He shows how yield volatility and duration should be used to determine bond price volatility. The nonlinearity of price volatility with respect to maturity comes from several factors. For a constant change in yield values, duration must be used to properly assess the price volatility effect. For the more typical case in which there are unequal change in yield rates, price volatility must be used in order to correctly examine the price volatility effect.

Cox, Ingersoll, and Ross (1979) analyze Macaulay's duration measure and focus on the conditions under which it is proper to use it in measuring basis risk, defined as the price volatility, of securities. Their objective is to

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[43]Although his analysis is restricted to default free government securities, Yawitz claims his results can be applied to any fixed income asset.

develop a dynamic duration measure that will gauge risk when multiple shocks affect interest rates and when the yield curve can change both its shape and location. Noting that previous literature has shown that Macaulay's duration is a valid measure of basis risk only under conditions which are empirically unrealistic, they attempt to study duration under more realistic market conditions.

Cox et al. develop a theoretical description of the term structure and use it to define a highly sophisticated duration measure, one they refer to as stochastic duration.[44] They assume that the term structure is not expected to change according to any constant or known pattern; otherwise arbitrage opportunities would obviously result. They develop a general equilibrium model of the term structure and define a measure of basis risk. The stochastic duration measure considers both the timing and size of coupons; it also incorporates all relevant information about the equilibrium term structure.

The stochastic duration measure is derived by assuming that the spot rate, dr, follows a first order autoregressive process.

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[44]Cox, Ingersoll, and Ross compare the properties of their stochastic duration measure to those of the Macaulay's traditional measure to demonstrate the superiority of the stochastic measure.

(6) $dr = \beta(\mu-r)dt+\sigma r^{1/2}dz$,

where $\beta(\mu-r)$ is the instantaneous drift term, μ is the long run mean, and $\sigma^2 r$ is the instantaneous variance. The stochastic duration measure, D5, is based on the spot rate and is defined below.

(7) $D5 = G^{-1}*[\Sigma C(t)P(t)G(t)/C(t)P(t)]$,

where $G^{-1}[x]=2/\delta\coth^{-1}(2/\delta x+\pi-\beta/\delta)$, $P(t)=F(t)\exp[-rG(t)]$, $G(t)=2/[\beta-\pi+\delta\coth(\delta t/2)]$, $\delta=[(\beta-\pi)^2+2\sigma^2]^{1/2}$, π is the liquidity premium. The measure of stochastic duration will depend upon the parameter of the specific interest rate process and the liquidity preferences of individuals. A concern in the use of stochastic duration measure is the amount of computation that is required; Equation 5 requires the estimation of several complex variables. Furthermore, advanced statistical methods are required to estimate the parameters for the assumed spot process and to estimate liquidity premiums.

Several duration measures, for different bonds, are compared by assuming that interest rates follows a driftless random walk and that investors display liquidity neutrality. In general, the stochastic duration values are much smaller than the Macaulay duration values.

Hilliard (1984) broadly defines immunization as a process by which interest rate risk is virtually eliminated. When a very short holding period is assumed, this concept is

more similar to the price elasticity use of duration than the traditional immunization, which assumes a longer investment horizon. He notes that interest rate futures have been used to immunize bond portfolio investments.[45] Hilliard develops a multivariate hedging model based upon Taylor Series approximations; he tests the model for effectiveness in reducing interest rate risk for a fixed portfolio of assets. The general portfolio yield equation that is used in this study is provided below.

(8) $r_p = E[r_p]+\alpha' r_u+\delta' f,$

where r_p is the portfolio yield equation, α are spot portfolio weights, r_u is the unexpected holding period yield component, δ are futures weights, and f are future prices. Hilliard derives the variance of the portfolio yield as shown in the equation below.

(9) $var(r_p) = \alpha' R_u \alpha - \delta' R_f \delta,$

where R_u is the (nxn) covariance matrix of unexpected spot holding period yields, and R_f is the (mxm) covariance matrix of futures price yields. It is apparent that the variance of the optimal portfolio is defined as the variance of the spot portfolio yield less the variance of the futures portfolio yield.

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[45]Recognizing that interest rate fluctuations can have a significant impact on the market value of bond portfolios, Hilliard sets out to develop a technique for reducing interest rate risk in a fixed spot portfolio.

A solution for a vector of hedge ratios is determined by using a vector of asset weights, a matrix of covariances between spot and future yield, and a covariance matrix of futures price yields.

(10) $\delta' = -\alpha' R_{uf} R_f^{-1}$,

where R_{uf} is the (nxm) matrix of covariances between spot and futures yields. Unfortunately, the covariance matrix of spot yields and futures price yields is not directly observable.

In an effort to provide further application of his results, Hilliard makes use of the bond price volatility model (see Equation 3). Models for both the unexpected holding period yield component due to a change in spot rates and for futures one period price yield are utilized. In fact Hilliard uses a matrix formulation for the portfolio of spot and future positions to derive an optimal hedge as defined in the equation below.

(11) $\delta' = -\alpha' A R_r C' [C R_r C']^{-1}$,

where A are default free cash flow positions, and C are promised default free cash flows.

Hilliard used portfolios made up of Treasury bills with maturities of 3, 6, 9, and 12 months. The futures contracts include 3 months Treasury bills contracts. Treasury bill data from January 1970 to October 1982 were recorded for use in this study. An examination of the results reveals that

for all applications, his hedge model yielded better results than those obtained with other competing short term duration hedging models used in his study. An important practical contribution from this work is the extension of a univariate analysis, in which only a single value of bond risk is considered, to a more realistic multivariate analysis, in which matrix values for both duration and spot rates are utilized. Hilliard is able to take a single factor duration concept, and extend it to a multivariate environment.

Hilliard and Jordan (1987) address several important issues that were not specifically considered in an earlier study by Hilliard (1984).[46] An important assumption of his full information model is that the covariance matrix of unexpected changes in interest rates provides an indication of the level of risk associated with cash flow values. A serious problem may occur if the covariance matrix is nonstationary.

An approach is used to determine the covariance matrix of unexpected changes in interest rates. First, the term structure of interest rates is estimated using an exponential spline procedure; this is repeated for each month in the study period. Significant negative autocorrelation

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[46]In his 1984 study, Hilliard developed a multi-factor interest rate model based on the term structure. He claims that the full information model is clearly superior to the duration models in short term hedging.

estimates are discovered. Hilliard and Jordan employ filtering theory to develop more accurate values of the unexpected changes in interest rates. They find that a three filter term is able to remove the correlation from successive unexpected changes.

The covariance matrix is estimated from unexpected changes in rates with maturities of one to ten years, using both the raw and the filtered data. The stationarity of the covariance matrix is important, since several variables are estimated from past data. The Hilliard and Jordan data indicate that the covariance matrix is nonstationary. The use of the filtering technique improves the results only slightly, indicating that the covariance matrix of unexpected rate changes is not stable over time.

It should be noted that they develop another model based on an assumption of efficient market behavior. In theory, this model simplifies the calculation of the covariance matrix. The model is tested on both filtered and unfiltered data; it is generally not supported by the data used in the study. The efficient market concept, however, does provide a basis for development of future models of the term structure. Unfortunately, Hilliard and Jordan were unable to accurately estimate the covariance matrix by specifying a structure derived from his market efficiency concept.

In a comprehensive empirical study, Gultekin and Rogalski (1984) test several different measures of duration in an effort to identity whether a particular measure is superior in explaining price volatility than the other duration measures.[47] There are three hypotheses that are used to test the relative effectiveness of duration measures D1, D3, D4, and D5. The first hypothesis, the linearity hypothesis, states that a linear relationship exists between duration and security price changes. The completeness hypothesis states that duration is considered a complete measure of risk, since duration includes the effect of maturity and coupon differences on bond price volatility. The efficiency hypothesis tests whether capital markets are efficient.

Gultekin and Rogalski develop a model of period by period changes and use it to monitor average price changes in order to test the three hypotheses.

(12) $r_t(m) = a_t+b_tD_{k,t-1}(m)+c_tD^2_{k,t-1}(m)+d_tC_t(m)+e_t(m)$

where $r_t(m)$ is the one period continuously compounded price change on security m in period t, $D_{k,t-1}(m)$ is the duration measure k calculated for the security in the previous period t-1, $C_t(m)$ is the coupon of the security, and $e_t(m)$ is a random disturbance term.

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[47]There has been many studies and commercial programs that have claimed superior performance of a particular duration measure.

Each hypothesis can be tested by examination of the estimated coefficients. For the completeness hypothesis to be correct, b_t should be statistically significant and d_t should equal zero. For the linearity hypothesis to be true, c_t should equal zero. The capital market efficiency hypothesis implies that a_t, b_t, c_t, d_t, and e_t should be uncorrelated through time.

Macaulay's duration, D1, is the first measure that is used in testing the hypotheses. Neither the linearity nor the completeness hypotheses are supported by the data; the efficiency hypothesis is accepted. They also claim that simple duration overstates the basis risk for long term securities. In order to examine the robustness of Macaulay's duration for different holding periods, additional tests were conducted. They show that the length of the holding period does not alter their original results.

They repeat the tests for each of the other duration measures. It is interesting to note that the results for duration measures D3 through D5 are similar to those of D1. This result leads Gultekin and Rogalski to conclude that none of the seven duration measures could explain price volatility any better than the maturity measure. There has been much support in the immunization literature for the claim that duration is a better measure of risk than maturity. The implication from their findings is that

maturity provides just as effective a measure of basis risk as any of the other duration measures.

No significant difference in explaining price volatility is observed from the various duration measures. In an effort to explain this conclusion, they examine the degree of correlation among the duration measures. In observing their data, it is obvious that the average correlation coefficients among four duration measures are quite large (90%), implying that duration measures are not independent of one another.

If duration is an effective measure of basis risk, then portfolios with a different number of bonds but with the same initial average duration should have equal expected returns. However, their results show that portfolios with the same initial duration often have very different returns at the end of a 3 month holding period; this suggest that duration measures may not be useful in developing immunization strategies.

Bierwag, Kaufman, Latta, and Roberts (BKLR) (1987) directly respond to the critical analysis of duration made by Gultekin and Rogalski (1984). They demonstrate that the G&R conclusion is based on obvious errors in their theoretical and empirical analysis. BKLR claim the single

factor duration models are useful for interest rate risk management.[48]

BKLR note that Gultekin and Rogalski base their analysis on the standard price volatility concept. Equation (3) shows how the percentage change in the price of a bond is related to the product of the duration of the bond and the interest rate change. According to Gultekin and Rogalski, the linearity, completeness, and efficiency hypotheses can each be tested using equation (12).

Gultekin and Rogalski claim that both the linearity and completeness hypotheses are rejected, while the market efficiency hypothesis is accepted. They find, despite what is often cited in the duration literature, that portfolios containing bonds with equal durations and different maturities do not generate the same return. BKLR claim that this occurrence is expected. The application of duration measures must be consistent with the actual stochastic process that affects interest rates. BKLR note that this condition is typically not satisfied in empirical work since the actual process is not known with any degree of certainty. Since the condition is not met, the equal duration portfolios are different and will not provide equal

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[48]They focus on the Gultekin and Rogalski (1984) paper and claim it provides a representative study of criticisms of duration analysis among academicians.

returns. Thus the linearity hypothesis is not true by definition.

It is also apparent that Gultekin and Rogalski do not correctly examine the duration measures when they consider only one strategy in isolation. An effective methodology requires a comparison of one strategy with an alternative strategy; in this case, the results of a duration strategy should be compared with those of a maturity strategy. Thus, BKLR show that criticisms directed at duration theory are not substantiated by either the theoretical or empirical evidence. They remain adamant in their stance that duration analysis is an important device that can be used to manage interest rate risk.

In review of duration theory, several observations are readily apparent. Although duration is not a new concept, it has received a tremendous amount of attention in the past 20 years. This risk measure has become very popular with practitioners. The primary use of duration analysis includes immunization of bond portfolios and estimation of the volatility of the bond prices caused by changes in interest rates.

This dissertation relies heavily on the relationship between bond price volatility and duration; the concept is applied to mortgage backed security analysis. Based on the previous empirical research, duration is an effective

measure of risk for a bond portfolio. Therefore, duration can be effectively employed to obtain a measure of risk associated with mortgage backed securities.

CHAPTER VI

APPLYING DURATION TO MORTGAGE BACKED SECURITIES

Duration of Mortgage Backed Securities

The traditional Macaulay duration measure has been widely accepted by portfolio managers of standard fixed income securities as an effective gauge of interest rate risk, when risk is defined in terms of relative price volatility. As discussed in chapter V, a proportional relationship exists between duration and price volatility. As the duration of a security increases, the price volatility of the security, and therefore its risk, also increases.

The duration for traditional fixed income securities such as treasury bonds, or zero coupon bonds, is extremely easy to determine; however, researchers have experienced difficulty in calculating the duration for both mortgage backed securities and callable bonds. The calculation of duration for any security requires that the present value of each future cash flow be known; therefore, it is imperative that the cash flows be known with certainty. Unfortunately, the actual future cash flows of mortgage backed securities are not observable, a result of the prepayment option.

Pinkus and Chandoha (1986) demonstrate the inadequacy of the traditional duration measure as a basis for determining the relative price risk of mortgage backed securities. They compare discount GNMA securities with par and premium GNMA securities. For their sample, the traditional measures of duration for the premium securities are more than the durations of the other securities. Based on the relative values of duration, this would imply that premium bond prices are more volatile than other bond prices. Pinkus and Chandoha monitor the historical price movements of the bonds from January 1, 1984 to September 1, 1985, and find that the daily price changes of discount bonds were three times as volatile as the premium bonds.[49]

Their finding is consistent with the assertion of practitioners that premium mortgage backed securities are less responsive to changes in interest rates than discount mortgage backed securities. This is because most prepayments occur on discount securities; there is little incentive to prepay a mortgage when current market rates are greater than the contract rate on the mortgage. The effective maturities of premium bonds are actually much shorter than the nominal maturities because of expected

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[49]These researchers examine generic based MBSs where the specific pools not identified at the time of trade. This differs from pool specific MBSs which contain specific pools that are identified at the time of trade.

prepayments; hence, these premium bonds actually have durations shorter than the traditional measure of duration. Because the actual durations are smaller, the premium bonds are less volatile than expected.

Pinkus and Chandoha estimate the relative price volatilities (RPVs) between GNMAs trading in various price ranges by using regression analysis. Regression equation (1) compares price volatilities of GNMAs trading at prices between P1 to P2 to the current coupon GNMA. Daily price data was used from January 1, 1984 to September 30, 1985.

(1) $\% \Delta G_{P1-P2} = b * \% \Delta G_C$

The dependent variable, $\% \Delta G_{P1-P2}$, is defined as the time series of daily percentage price changes of GNMAs trading in the P1 to P2 price range on a given day. The independent variable, $\% \Delta G_C$, the current coupon yield, is defined as the percentage price change of the GNMA trading closest to but below par on a given day. The estimated coefficient, b, is the measure of RPV.

The use of time series data to estimate RPVs has theoretical shortcomings. The RPV of any bond, including mortgage backed securities, is a function of duration. Duration is itself a function of the current level of interest rates and the time to maturity. When time series data is used to estimate the RPV using equation 1, it is implicitly assumed that the coefficient, b, is stationary.

Since b is a function of duration, this assumption is not warranted.

There is also a serious problem with the sampling procedure because the independent and dependent variables change in composition over time. As interest rates change, bonds with different coupon rates are included in the various price ranges. Thus, the dependent variable represents a group of bonds with different coupons and different maturities. A similar problem exists with the independent variable.

Even if it is the same bonds that are included in the price ranges, there is also a problem with the attenuation of maturity. Suppose price volatility is measured over a 2 year period and that the bond initially has 10 years until maturity. At the end of the period, the bond will have 8 years to maturity. Since the maturity of a bond is changing over time, the bond at the beginning of the period and the bond at the end are actually two different securities.

Despite the fact that their basic price volatility equation is seriously flawed, Pinkus and Chandoha develop an implied duration measure for mortgage backed securities that is analogous to the traditional duration measure. The major advantage of the implied duration approach is that this measure implicitly values the prepayment risk associated with mortgage backed securities. The implied duration

measure is based on actual market data and allows investors to compare the interest rate sensitivities of mortgage securities to those of other fixed income securities.

To determine the implied durations for mortgage backed securities, Pinkus and Chandoha first calculate the relative price volatilities (RPVs) between GNMAs trading in various price ranges by using regression analysis. From equation 1, the estimated coefficient, b, is used to measure the relative price volatility between GNMAs and the current coupon bond.

(2) $b = \% \Delta G_{P1-P2} / \% \Delta G_C$,

where b is defined as the relative price volatility of GNMA securities trading in the P1 to P2 price range relative to the price volatility of the current coupon GNMA, $\% \Delta G_{P1-P2}$, is defined as the time series of daily percentage price changes of GNMAs trading in the P1 to P2 price range on a given day, and $\% \Delta G_C$ is defined as the percentage price change of the current coupon GNMA.

They use the following procedure to calculate implied durations. RPV_{gm} represents an estimated coefficient and is derived below.

(3) $RPV_{gm} = D_g / ID_m$

where RPV_{gm} is defined as the relative price volatility of the Treasury bond to a GNMA security, D_g is defined as the duration of the Treasury bond, and ID_m is defined as the

implied duration of the GNMA security.[50] Pinkus and Chandoha estimate the implied durations for GNMAs trading at various price levels from the time series regression; from this data, they plot an implied duration curve for GNMAs as a function of different price levels.

Pinkus and Chandoha compared the implied durations with standard durations for various mortgage securities. While the standard durations incorrectly indicated that higher coupon (premium) securities were less volatile than lower (discount) coupon issues, the implied durations correctly indicated that higher (premium) coupon securities were more volatile. For discount coupon mortgage issues, which have a low probability of prepayment, the two measures provided similar durations. Only when the mortgage security was priced near par or at a premium did the implied duration measure significantly differ from the standard duration measure.

Although the standard Macaulay duration is not an effective measure of the relative price risk of mortgage backed securities, Pinkus and Chandoha demonstrate that the concept of duration can still be used to provide valuable information to investors in mortgage backed securities.

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[50]Pinkus and Chandoha acknowledge that government bonds and MBS prices are not as highly correlated as the prices of two MBSs. This can present problems with measuring the price volatility link between the issues.

Their important contribution is that the duration for mortgage backed securities cannot be directly observed. While their basic idea is valid, the approach they use is clearly deficient. By using an invalid sampling procedure and an inappropriate use of time series data, their implied durations are calculated improperly; however, Pinkus and Chandoha are innovative in that they estimate duration for mortgage securities using an indirect approach.

CHAPTER VII

DATA AND METHODOLOGY

The Data

The data for mortgage backed securities comes from the Federal Home Loan Mortgage Corporation (FHLMC); this is often referred to as "Freddie Mac", which is division of the Federal Home Loan Bank System. In order to provide a more complete description of the type of market data that will be used in this study, a brief discussion of the FHLMC agency is presented. The FHLMC was established in 1970 to provide assistance to savings and loan associations which are the primary issuers of conventional mortgage loans.[51] It was created to provide a more active secondary market for conventional mortgage loans which did not possess the government guarantee feature. An active secondary market was needed to help generate funds in the conventional mortgage market.

The FHLMC is a major issuer of participation certificates (PCs). PCs are similar to GNMA pass throughs in that

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[51]Although the FHLMC does not offer securities backed by the U.S. government, it does have implied credit rights with the federal reserve.

both types of securities are backed by pools of mortgages and pass through all principal and interest payments made on the individual mortgages in those pools. However, there exist several unique features that distinguish FHLMC PCs from GNMA pass throughs. PCs include only conventional mortgages, which are guaranteed by the FHLMC, a privately managed institution; they are not directly guaranteed by the federal government. FHLMC PCs offer a slightly higher yield than GNMA issues. The pools may include individual mortgages with different interest rates and are generally put together by the FHLMC instead of by private sector originators.

The data base used in this study contains market price information for 125 FHLMC participation certificate pools from January 1979 to May 1984. The mortgage pools consist of traditional, fixed rate mortgage loans for residential dwellings. Each individual mortgage pool has an original unpaid principal balance of at least $50 million and a maximum original maturity of 30 years. Over 500 mortgage loans are included in each pool. Specific data that is provided for each individual pool includes the average coupon rate on the mortgage pool at the time of issue and the original mortgage pool balance. The prices of individual mortgage pools and the amount of the original pool balance outstanding are reported each month.

Methodology

Through an extension of the implied duration concept developed by Pinkus and Chandoha (1986), this study will indirectly infer duration for mortgage backed securities. Although Pinkus and Chandoha note that duration cannot be directly observed for mortgage backed securities, they do show that it is possible to indirectly infer duration by comparing the relative price volatilities of MBSs with respect to government bonds. However, as shown in chapter VI, the methodology of Pinkus and Chandoha is flawed due to the inappropriate use of time series data.

Implied Duration

This dissertation has two major objectives. The first is to accurately estimate the implied duration of a mortgage backed security. Since duration is a measure of risk, this result will be valuable to investors. The concept of elasticity developed by Hopewell and Kaufman (1971) will be employed to estimate the implied duration of a bond. They show that the price volatility of a fixed income security is equal to the product of duration and a measure of interest rate changes. This is consistent with market observations in which changes in interest rates cause bonds with large durations to experience significant price changes relative to bonds with small durations.

The price elasticity of a bond with respect to a change in interest rates is shown in equation (1). Price volatility is a function of duration and the change in interest rates, as shown below.

(1) $R_i = -D_i * [d(1+r)/(1+r)]$,

where R_i is defined as the instantaneous rate of return (dP/P), or price volatility, of bond i, and D_i is defined as the duration of bond i. The instantaneous change in interest rates is defined by d(1+r)/(1+r).

In the first stage, cross sectional regressions will be run using government bond data. The following equation will be estimated for each month in the study period.

(2) $D_{gi} = a_o + b_1 * R_{gi}$,

where D_{gi} is the Macaulay duration of treasury bond i and R_{gi} is the observed return on treasury bond i. The estimated coefficient b_1 is a measure of changes in interest rates. In summary, government bonds will be used in the first stage to estimate or infer the values of the interest rate changes.

For mortgage backed securities, the following equation describes the risk-return relationship.

(3) $D_{mj} = a_o + b_1 * R_{mj}$,

where D_{mj} is the duration of mortgage backed security j and R_{mj} is defined as the return on mortgage backed security j. As in equation (1), b_1 is the change in interest rates.

Although the value of variable R_{mj} is directly observable, D_{mj} is not directly observed. However, equation (3) can be modified, establishing an important link between government bonds and mortgage backed securities.

(4) $ID_{mj} = a_o + b_1 * ID_{mj}$,

ID_{mj} represents the inferred duration of mortgage backed security j. Values for R_{mj} are directly observable; an estimate of b_1 can be obtained from the government bond data. These will be used to determine the value of ID_{mj}.

To validate this approach, the following test will be conducted. A sample of government bonds will be collected and divided into two groups. A cross sectional regression (defined by equation 2) will be run on the first group to determine b_1, a measure of interest rate changes. Given this estimate of b_1 and the actual returns from the second group of bonds, durations for the second group can be estimated by the previously described process. These estimated durations will be compared with the actual durations to determine the accuracy of this model. After the method is validated using government bonds, it will be applied to mortgage backed securities.

Implied Prepayment Rates

The second major objective of this dissertation is to use implied duration to estimate prepayments. A serious

problem with investing in mortgage backed securities is the difficulty of estimating the yield for these securities. The prepayment rates associated with mortgage pools are not known with certainty. This implies that the yields of MBSs cannot be directly determined, since the calculation of the yield requires certain cash flow streams.

The previous section discussed how the implied duration of a MBS can be inferred by using observed market returns of MBSs and the observable durations of government bonds. If the implied durations are significantly different from the standard durations, then investors must be incorporating their expectations of prepayments into the pricing of mortgage backed securities. Implied duration, which serves as a measure of risk, can then be used to estimate the implied yield of a MBS.

Assuming that financial markets operate efficiently, two instruments with the same risk (duration), should provide the same yield. Therefore, the implied yield of a MBS should be equal to the observable yield of a government bond with the same duration. Using this estimate of yield, an iterative process is employed to find the prepayment rate that provides the implied duration. With this procedure, prepayment rates are implied by market prices. Specifically, the following procedure will be employed.

The duration of a mortgage backed security is a function of the size of the cash flows, the timing of the cash flows, and the yield.

$$(5)\quad ID_m=\sum_{t=1}^{n}[C_{mt}*t*1/(1+Y_m)^t]/[P],$$

where ID_m is the implied duration, C_{mt} is the cash flow of the MBS, Y_m is the implied yield of the MBS, and P is the price of the security. ID_m is calculated in the previous section, Y_m is obtained via comparison with a government bond of the same duration, and P is obtained from the MBS data base. Given a prepayment rate, the cash flow of a MBS can be calculated with the following equation.

$$(6)\quad C_{mt}=F_t*B_{t-1}+R_t,$$

where F_t is the amortization factor, B_t is the outstanding balance at time t, and R_t is the prepayment amount. The amortized factor is determined using the equation below.

$$(7)\quad F_t=[(1+i)^{Tm-t}-1]/[(1+i)^{Tm-t}*i],$$

where i is the constant prepayment rate associated with the mortgage pool. The outstanding balance of the mortgage pool at time t-1 will be determined using equation (8).

$$(8)\quad B_{t-1}=B_{t-2}-R_t-N_t,$$

where N_t is the scheduled amortized payment of principal. The monthly prepayment amount is based on the outstanding pool balance and the constant prepayment rate.

(9) $R_t=B_{t-1}*i,$

The scheduled amortized payment of principal is calculated by the following equation.

(10) $N_t=C_{mt}-P_t-I_t,$

where I_t represents the interest payment and can be determined using the following equation.

(11) $I_t=B_{t-1}*C,$

where C is the coupon rate associated with the MBS.

Given a prepayment rate, the cash flow of a MBS can be defined by equation (12).

(12) $C_{mt}=F_t*B_{t-1}+B_{t-1}*i,$

This can be rewritten as equation (13).

(13) $C_{mt}=(F_t+i)*B_{t-1},$

The initial balance, B_o, is observable. Given a prepayment rate, equation (13) can be used to find the cash flow at time 1. The balance at time 1 can be calculated, leading to the cash flow at time 2. In this sequential manner, the cash flows for all payment dates can be calculated.

Tables 2-6 show how an iterative process can use the mortgage prepayment rate to estimate the implied duration. Assume a monthly interest rate of 1.0%, a maturity of 36 months, and a mortgage pool balance of $100,000,000. When different prepayment rates are used in the process, different durations are calculated. Prepayment rates of 0%, 3%, 6%, 12%, and 20% provide durations of 16.99, 12.46,

Table 2

Estimation of Implied Duration*
(for a 0% mortgage prepayment rate)

Term	Balance Remaining	Principal Payment	Interest Payment	Total Payment
0	$1,000.00			
1	$976.79	$23.21	$10.00	$33.21
2	$953.34	$23.45	$9.77	$33.21
3	$929.66	$23.68	$9.53	$33.21
4	$905.74	$23.92	$9.30	$33.21
5	$881.58	$24.16	$9.06	$33.21
6	$857.19	$24.40	$8.82	$33.21
7	$832.54	$24.64	$8.57	$33.21
8	$807.65	$24.89	$8.33	$33.21
9	$782.52	$25.14	$8.08	$33.21
10	$757.13	$25.39	$7.83	$33.21
11	$731.48	$25.64	$7.57	$33.21
12	$705.58	$25.90	$7.31	$33.21
13	$679.43	$26.16	$7.06	$33.21
14	$653.01	$26.42	$6.79	$33.21
15	$626.32	$26.68	$6.53	$33.21
16	$599.37	$26.95	$6.26	$33.21
17	$572.15	$27.22	$5.99	$33.21
18	$544.66	$27.49	$5.72	$33.21
19	$516.89	$27.77	$5.45	$33.21
20	$488.84	$28.05	$5.17	$33.21
21	$460.52	$28.33	$4.89	$33.21
22	$431.91	$28.61	$4.61	$33.21
23	$403.01	$28.90	$4.32	$33.21
24	$373.83	$29.18	$4.03	$33.21
25	$344.35	$29.48	$3.74	$33.21
26	$314.58	$29.77	$3.44	$33.21
27	$284.51	$30.07	$3.15	$33.21
28	$254.15	$30.37	$2.85	$33.21
29	$223.47	$30.67	$2.54	$33.21
30	$192.49	$30.98	$2.23	$33.21
31	$161.20	$31.29	$1.92	$33.21
32	$129.60	$31.60	$1.61	$33.21
33	$97.68	$31.92	$1.30	$33.21
34	$65.45	$32.24	$0.98	$33.21
35	$32.89	$32.56	$0.65	$33.21
36	$0.00	$32.89	$0.33	$33.21

*mortgage pool data:

annual rate = 12.0% ; monthly rate = 1.0%
maturity/months = 36 ; balance due = $1,000
implied duration = 16.9 months

Table 3

Estimation of Implied Duration*
(for a 3% mortgage prepayment rate)

Term	Balance Remaining	Principal Payment	Interest Payment	Total Payment
0	$1,000.00			
1	$946.79	$53.21	$10.00	$63.21
2	$895.66	$51.13	$9.47	$60.60
3	$846.54	$49.12	$8.96	$58.07
4	$799.36	$47.18	$8.47	$55.64
5	$754.06	$45.30	$7.99	$53.29
6	$710.57	$43.49	$7.54	$51.03
7	$668.83	$41.74	$7.11	$48.85
8	$628.77	$40.06	$6.69	$46.75
9	$590.33	$38.43	$6.29	$44.72
10	$553.47	$36.86	$5.90	$42.77
11	$518.12	$35.35	$5.53	$40.88
12	$484.23	$33.89	$5.18	$39.07
13	$451.75	$32.48	$4.84	$37.32
14	$420.63	$31.12	$4.52	$35.64
15	$390.83	$29.81	$4.21	$34.01
16	$362.28	$28.54	$3.91	$32.45
17	$334.96	$27.32	$3.62	$30.94
18	$308.82	$26.14	$3.35	$29.49
19	$283.81	$25.01	$3.09	$28.10
20	$259.90	$23.91	$2.84	$26.75
21	$237.04	$22.86	$2.60	$25.46
22	$215.20	$21.84	$2.37	$24.21
23	$194.35	$20.85	$2.15	$23.01
24	$174.44	$19.90	$1.94	$21.85
25	$155.46	$18.99	$1.74	$20.73
26	$137.35	$18.10	$1.55	$19.66
27	$120.10	$17.25	$1.37	$18.62
28	$103.68	$16.42	$1.20	$17.62
29	$88.06	$15.62	$1.04	$16.66
30	$73.21	$14.85	$0.88	$15.73
31	$59.11	$14.10	$0.73	$14.83
32	$45.75	$13.36	$0.59	$13.95
33	$33.11	$12.64	$0.46	$13.10
34	$21.19	$11.92	$0.33	$12.25
35	$10.01	$11.18	$0.21	$11.39
36	$0.00	$10.01	$0.10	$10.11

*mortgage pool data:

annual rate = 12.00% ; monthly rate = 1.0%
maturity/months = 36 ; balance due = $1,000.00
implied duration = 12.46 months

Table 4

Estimation of Implied Duration*
(for a 6% mortgage prepayment rate)

Term	Balance Remaining	Principal Payment	Interest Payment	Total Payment
0	$1,000.00			
1	$916.79	$83.21	$10.00	$93.21
2	$839.77	$77.01	$9.17	$86.18
3	$768.53	$71.25	$8.40	$79.64
4	$702.64	$65.88	$7.69	$73.57
5	$641.74	$60.90	$7.03	$67.93
6	$585.48	$56.27	$6.42	$62.68
7	$533.52	$51.96	$5.85	$57.81
8	$485.56	$47.96	$5.34	$53.30
9	$441.31	$44.25	$4.86	$49.10
10	$400.51	$40.80	$4.41	$45.21
11	$362.92	$37.60	$4.01	$41.60
12	$328.29	$34.62	$3.63	$38.25
13	$296.42	$31.87	$3.28	$35.15
14	$267.11	$29.31	$2.96	$32.28
15	$240.17	$26.94	$2.67	$29.61
16	$215.43	$24.74	$2.40	$27.15
17	$192.72	$22.71	$2.15	$24.86
18	$171.89	$20.82	$1.93	$22.75
19	$152.82	$19.08	$1.72	$20.80
20	$135.36	$17.46	$1.53	$18.99
21	$119.39	$15.96	$1.35	$17.32
22	$104.81	$14.58	$1.19	$15.77
23	$91.51	$13.30	$1.05	$14.35
24	$79.39	$12.12	$0.92	$13.03
25	$68.37	$11.02	$0.79	$11.82
26	$58.36	$10.01	$0.68	$10.70
27	$49.28	$9.08	$0.58	$9.66
28	$41.06	$8.22	$0.49	$8.71
29	$33.64	$7.42	$0.41	$7.83
30	$26.96	$6.68	$0.34	$7.02
31	$20.96	$6.00	$0.27	$6.27
32	$15.59	$5.37	$0.21	$5.58
33	$10.82	$4.78	$0.16	$4.93
34	$6.60	$4.22	$0.11	$4.33
35	$2.92	$3.68	$0.07	$3.74
36	$.00	$2.92	$0.03	$2.95

*mortgage pool data:
annual rate = 12.00% ; monthly rate = 1.0%
maturity/months = 36 ; balance due = $1,000.00
implied duration = 9.59 months

Table 5

Estimation of Implied Duration*
(for a 12% mortgage prepayment rate)

Term	Balance Remaining	Principal Payment	Interest Payment	Total Payment
0	$1,000.00			
1	$856.79	$143.21	$10.00	$153.21
2	$733.41	$123.38	$8.57	$131.95
3	$627.18	$106.23	$7.33	$113.56
4	$535.78	$91.40	$6.27	$97.67
5	$457.20	$78.58	$5.36	$83.94
6	$389.68	$67.52	$4.57	$72.09
7	$331.72	$57.96	$3.90	$61.86
8	$281.99	$49.72	$3.32	$53.04
9	$239.38	$42.62	$2.82	$45.44
10	$202.89	$36.49	$2.39	$38.89
11	$171.67	$31.22	$2.03	$33.25
12	$144.99	$26.68	$1.72	$28.40
13	$122.22	$22.77	$1.45	$24.22
14	$102.80	$19.42	$1.22	$20.64
15	$86.26	$16.54	$1.03	$17.56
16	$72.20	$14.06	$0.86	$14.93
17	$60.26	$11.94	$0.72	$12.66
18	$50.13	$10.13	$0.60	$10.73
19	$41.56	$8.57	$0.50	$9.07
20	$34.32	$7.24	$0.42	$7.66
21	$28.21	$6.11	$0.34	$6.45
22	$23.07	$5.14	$0.28	$5.42
23	$18.76	$4.31	$0.23	$4.54
24	$15.15	$3.61	$0.19	$3.80
25	$12.14	$3.01	$0.15	$3.16
26	$9.63	$2.51	$0.12	$2.63
27	$7.56	$2.08	$0.10	$2.17
28	$5.84	$1.71	$0.08	$1.79
29	$4.44	$1.41	$0.06	$1.46
30	$3.29	$1.15	$0.04	$1.19
31	$2.36	$0.93	$0.03	$0.96
32	$1.61	$0.75	$0.02	$0.77
33	$1.02	$0.59	$0.02	$0.61
34	$0.56	$0.46	$0.01	$0.47
35	$0.22	$0.35	$0.01	$0.35
36	$.00	$0.22	$.00	$0.22

*1mortgage pool data:
annual rate = 12.00% ; monthly rate = 1.0%
maturity/months = 36 ; balance due = $1,000.00
implied duration = 6.34 months

Table 6

Estimation of Implied Duration*
(for a 20% mortgage prepayment rate)

Term	Balance Remaining	Principal Payment	Interest Payment	Total Payment
0	$1,000.00			
1	$776.79	$223.21	$10.00	$233.21
2	$602.78	$174.00	$7.77	$181.77
3	$467.25	$135.53	$6.03	$141.56
4	$361.78	$105.47	$4.67	$110.14
5	$279.78	$82.01	$3.62	$85.62
6	$216.08	$63.70	$2.80	$66.50
7	$166.65	$49.43	$2.16	$51.59
8	$128.34	$38.31	$1.67	$39.98
9	$98.68	$29.66	$1.28	$30.95
10	$75.74	$22.94	$0.99	$23.92
11	$58.03	$17.71	$0.76	$18.47
12	$44.37	$13.66	$0.58	$14.24
13	$33.85	$10.52	$0.44	$10.96
14	$25.76	$8.09	$0.34	$8.42
15	$19.56	$6.21	$0.26	$6.46
16	$14.80	$4.75	$0.20	$4.95
17	$11.17	$3.63	$0.15	$3.78
18	$8.40	$2.77	$0.11	$2.88
19	$6.29	$2.11	$0.08	$2.19
20	$4.69	$1.60	$0.06	$1.66
21	$3.48	$1.21	$0.05	$1.26
22	$2.57	$0.91	$0.03	$0.95
23	$1.88	$0.69	$0.03	$0.71
24	$1.37	$0.51	$0.02	$0.53
25	$0.99	$0.38	$0.01	$0.40
26	$0.71	$0.28	$0.01	$0.29
27	$0.50	$0.21	$0.01	$0.22
28	$0.34	$0.15	$.00	$0.16
29	$0.23	$0.11	$.00	$0.11
30	$0.15	$0.08	$.00	$0.08
31	$0.10	$0.06	$.00	$0.06
32	$0.06	$0.04	$.00	$0.04
33	$0.03	$0.03	$.00	$0.03
34	$0.02	$0.02	$.00	$0.02
35	$.00	$0.01	$.00	$0.01
36	$.00	$.00	$.00	$.00

*mortgage pool data:
annual rate = 12.00% ; monthly rate = 1.0%
maturity/months = 36 ; balance due = $1,000.00
implied duration = 4.27 months

9.59, 6.34, and 4.27 months respectively. This shows there exists an inverse relationship between prepayment rate and duration. As prepayment rates are increased, durations are reduced considerably.

Different prepayment rates will be tried until the computed duration is equal to the implied duration. This procedure demonstrates how prepayment rates can be inferred by market prices. The accuracy of this procedure will be tested by comparing estimated prepayments with actual prepayments.

Summary

The use of cross sectional data rather than time series data is a major enhancement of the implied duration methodology of Pinkus and Chandoha. Their use of time series data requires the implicit assumption that the duration of a security is constant during the sample period. Given the attenuation in maturity of a bond and shifts in the market interest rates, it is virtually certain that the duration of a bond changes over time. The methodology proposed in this study estimates a measure of duration for each bond at each cross section; it does not impose any requirement that the duration remain constant during the sample period.

In summary, the basic objective of this study is to estimate implied duration and use it to infer the expected prepayment rates on mortgage backed securities. Tests will be conducted to determine if any differences exist between the predicted prepayment rates and the actual prepayment rates. The important contribution of this study will be made by providing a new way to accurately estimate prepayment rates on mortgage backed securities. Since prepayments are included in pricing models of mortgage securities, researchers will be better able to value mortgage backed securities.

Expected Contribution

The market for MBS has become an extremely important option for both corporations and individual investors. Despite the rapid growth of the mortgage backed security market, there are investors in this market who have experienced difficulty when investing in MBSs. The reason is that MBSs have investment characteristics that differ from the traditional debt obligations held in portfolios. Specifically, a MBS is more difficult to value because of the uncertainty of its cash flow. The uncertainty arises because of the prepayment opportunity available to

homeowners and it is this difficulty of assessing the value of MBSs that results in potential mispricing.

Analysts are now realizing that the market has grown so large in such a short time that the need for more sophisticated techniques for evaluating these instruments is clearly evident. The significance of this study will be in providing more accurate estimates of MBS cash flows for use by investors and portfolio managers. The basic hypothesis investigates whether implied duration can be used to accurately measure prepayment rates on MBSs. Since prepayment values are included in the pricing of MBSs, investors will be better able to correctly value these increasing popular instruments. As research efforts provide better ways to determine the true value of MBSs, the potential for even further growth is possible.

CHAPTER VIII

EMPIRICAL FINDINGS

Part I--Results for Simple Regression Model

Implied Duration

The first major objective is to accurately estimate the implied duration of a mortgage backed security, employing the concept of elasticity. The price elasticity of a bond with respect to a change in interest rates, which is also known as duration, is shown in equation (1).

(1) $-D = [dP/P]/[d(1+r)/(1+r)]$,

After rearranging the equation, it can be seen that volatility is a function of duration and the change in interest rates:

(2) $R_i = -D_i * d(1+r)/(1+r)$,

where R_i is defined as the instantaneous rate of return (dP/P, the price volatility) of bond i, and D_i is defined as the duration of bond i. The instantaneous change in interest rates is defined by d(1+r)/(1+r).

a. A Simple Regression Model of Implied Duration

The following equation, which expresses duration as a function of total return, is estimated for each month in the study period.

(3) $D_{gi} = a_o + b_1 * R_{gi}$,

where D_{gi} is the Macaulay duration of treasury bond i and R_{gi} is the observed return on treasury bond i. Notice that duration is a function of an observable variable, return. The estimated coefficient b_1 is a measure of changes in interest rates.

The theoretical assumptions underlying equation (3) are: (1) returns are defined as price changes, (2) the term structure is horizontal, and (3) the term structure only changes by a constant upward or downward shift. To empirically test equation (3), however, these assumptions are relaxed. In particular, returns represent price changes and accrued interest and the term structure is not assumed to be horizontal with constant shifts. If good results are obtained using total actual returns, i.e., small pricing errors are observed, then total returns will be considered an acceptable measure. If large pricing errors are observed, the return variable in equation (3) will be redefined to include price changes only. Thus, the results based on this simple approach are conservative in that a more complicated model might provide stronger results. The

simplicity of the proposed approach, however, is one of its more appealing features.

To determine whether equation (3) can be used to infer durations for other bonds a data base containing market price and duration information on selected government bonds is created from the CRSP Government Bonds Tapes. Flower bonds, bonds that have not yet traded or that have finished trading before the study period, treasury bills, callable bonds, bonds with abnormal tax codes, and bonds which are not eligible for banks are excluded from the sample. All remaining bonds are included in the sample.

An average of just over 105 bonds were chosen per month from January 1979 to May 1984. For each month, the sample of government bonds is collected and sorted by return. The sample is divided into two groups; every eighth bond is placed in a holdout sample. The remaining bonds comprise the estimation sample. Both groups contain specific information on each bond such as Macaulay's duration, market price, and coupon rate.

A cross sectional regression (defined by equation 3) is run on the first group, the estimation sample. Table 7 provides summary measures for this regression. The results support the model; the estimated coefficient,b, is significant in 63 of the 64 regressions. As can be seen, the model provides a reasonably good fit with a mean R^2 of

Table 7

Summary of Cross-Sectional Results
for Simple Regression Model

R^2 within various ranges:

Range	Number	Percentage	Cumulative Percentage
$90 < R_2^2 < 100$	13	20	20
$80 < R_2^2 < 90$	16	25	45
$70 < R_2^2 < 80$	9	14	59
$60 < R_2^2 < 70$	5	08	67
$50 < R_2^2 < 60$	1	02	69
$40 < R_2^2 < 50$	8	13	82
$30 < R_2^2 < 40$	3	05	87
$20 < R_2^2 < 30$	1	02	89
$10 < R_2^2 < 20$	2	03	92
$0 < R^2 < 10$	6	08	100

Specification:	$Dgi = a_0 + b_1 R_{gi}$
Average number of bonds per regression:	92.48
Number of regressions:	64
Number of significant estimates b_i:	Positive: 27 Negative: 36 Total: 63
Mean value of b_1: t statistic	-16.956E3 (-1.63)
Mean R^2:	65.87

.6587; over two thirds have an R2 greater than .60. Although there is no evidence of a non-zero ex ante risk premia (based on the insignificant t-statistic of the mean value of b), the model does explain ex post returns quite well.

These results are analogous to those found in the tests of CAPM, capital asset pricing model. Black, Jensen, and Scholes (1972) tested over two year intervals and found that for many periods, the mean regression slopes were zero or negative. Fama and McBeth (1973) used monthly data and found that the mean regression slopes were negative for many cases. The difference between the CAPM results and the results found here are as follow. With CAPM, when all the slopes were summed together, on average, they were positive, implying a positive relationship between risk and return. With the results found in this study, since the regression coefficient is not significantly different from zero, on average, a specific relationship does not exist.

The model coefficients from the estimation sample are used with the returns from the hold-out group of bonds to infer durations for the hold-out sample. These implied durations are compared with the actual durations to determine the predictive accuracy of the model.

The evidence in Table 8 indicates the model has significant predictive power. A total of 953 bonds are used

Table 8

Tests of Predictive Accuracy
for Simple Regression Model

Total number of bonds in the hold-out sample:	953
Total number of bonds for which the estimate is outside the 95% confidence interval:[a]	0
Mean actual duration:	1084.80
Mean estimated duration:	1056.46
Mean estimation error:	-28.33
t statistic for test of significance:	-1.70
Regression equation:[b]	$D_g = \underset{(2.96)}{92.58} + \underset{(37.39)}{.93}\ ID_g$
Confidence Interval:	$.5969 < b_1 < .6631$
	$R^2 = 59.51$

[a]The following formula is used to find the prediction confidence interval:

$$y + t_{\alpha/2}\, S[1 + 1/n + ((X_p - x)^2/SS_{XX})]^{1/2}$$

[b]ID_g represents the expected duration that is predicted from the regression model. This variable is dependent on observed duration. D_g represents the actual duration that is observed for each bond in the sample. This predictor variable is used in the calculation of the estimated durations.

in the hold-out sample; none of the implied durations lie outside the 95% prediction confidence interval range.

The mean implied duration is very close to the mean actual duration in the sample; the actual average duration for the hold-out sample is 1084.80 days while the mean implied duration is 1056.46 days. This mean error is only -28.33 and is not significantly different from zero.

The following regression is conducted:

(4) $ID_g = a_o + b_1 D_g$,

where ID_g is the implied duration and D_g is the actual duration. The regression equation has a mean R^2 of about 60%, implying a strong relationship between estimated and actual duration. Both a_o and b_1 are significantly different from zero. The slope coefficient ,b_1, is .63 with a t-statistic of 37.39. Although these coefficients are different from the theoretical values of 0 and 1, the estimated and actual duration appear to be highly correlated.

In summary, these tests are encouraging, indicating that equation (2) provides a reasonable relationship between Macaulay's duration and total return.

b. Implied Duration for Mortgage Backed Securities

The data base used in this study is obtained from the Federal Home Loan Mortgage Corporation (FHLMC) and contains

market price information for 125 FHLMC participation certificate pools from January 1979 to May 1984. The mortgage pools consist of traditional, fixed rate mortgage loans for residential dwellings. Each individual mortgage pool has an original unpaid principal balance of at least $50 million and a maximum original maturity of 30 years. Over 500 mortgage loans are included in each pool. Specific data that is provided for each individual pool includes the average coupon rate on the mortgage pool at the time of issue and the original mortgage pool balance. The prices of individual mortgage pools and the amount of the original pool balance outstanding are reported each month.

For mortgage backed securities, equation (5) is used to describe the risk-return relationship. For an explanation on the price elasticity of a bond with prepayments refer to Appendix 3.

(5) $D_{mj} = a_o + b_1 * R_{mj}$,

where D_{mj} is the duration of mortgage backed security j and R_{mj} is defined as the return on mortgage backed security j. As in equation (3), b_1 is the change in interest rates. Although the value of variable R_{mj} is directly observable, neither D_{mj} nor b_1 can be directly observed.

Mortgage backed securities and government bonds are similar securities since they have common characteristics; both securities are risk-free and have government guarantee

protection. If two instruments have the same duration (risk), they should provide the same yield. The duration of a security is a function of the size of the cash flow, the timing of the cash flow, and the yield. With government bonds, all variables, i.e., cash flows, timing of cash flows, and yield, are known by the investor. With mortgage backed securities, the cash flows can be estimated given a prepayment rate. Recognizing the close association between mortgage backed securities and government bonds, equation (5) can be modified; an important link between the two securities is established. The connection between the two securities is plausible since duration is considered an acceptable measure of risk and both government bonds and mortgage backed securities have similar duration (risk) features.

(6) $ID_{mj} = a_o + b_g * R_{mj}$,

where ID_{mj} represents the inferred duration of mortgage backed security j. Values for R_{mj} are directly observable; an estimate of b_g is obtained from the government bond data. These values can be used to compute ID_{mj} for mortgage backed security j. The inferred durations are determined for each pool in each month.

c. Implied Yields, Prepayments, and Prices

The implied yield of a MBS can be inferred by using implied duration. Assuming that financial markets operate efficiently, two instruments with the same risk (duration), should provide the same yield. Therefore, the implied yield of a MBS should be equal to the observable yield of a government bond with the same duration. The duration and yield of the closest government bonds above and below the implied duration of the MBS bond are identified and the implied yield is estimated by interpolation.

Using the estimate of implied yield, an iterative process is employed to find the implied prepayment rate that provides the implied duration. With this procedure, prepayment rates are implied by market prices. Specifically, the following procedure is employed.

The duration of a mortgage backed security is a function of the size of the cash flows, the timing of the cash flows, and the yield:

$$(7)\quad ID_m = \sum_{t=1}^{n} [C_{mt} * t * 1/(1+Y_m)^t]/[P],$$

$$(7a)\quad ID_m = f(C_{mt}, t, Y_m),$$

where ID_m is the implied duration, C_{mt} is the cash flow of the MBS, Y_m is the implied yield of the MBS, and P is the price of the security. ID_m is calculated in the previous

section, Y_m is obtained via comparison with a government bond of the same duration, and P is obtained from the MBS data base.

In order to equate computed duration with implied duration, the total cash flow in each period must be estimated. The total cash flow of a MBS is a function of both observed variables (scheduled coupon and principal payments) and an unobserved variable (mortgage prepayment). The scheduled total payment is determined by multiplying outstanding pool balance by an amortization factor, which is based on the number of months remaining in the life of the pool and the monthly coupon interest rate. The scheduled coupon payment is calculated by multiplying outstanding pool balance by the coupon rate; the scheduled principal payment is determined by subtracting scheduled coupon payment from scheduled total payment. The amount of prepayment is determined by multiplying the outstanding pool balance by a prepayment rate. Total cash flow is calculated by taking the sum of the scheduled total payment and the prepayment amount. Once the total cash flow is determined, the duration can be computed.

Using implied yield, an iterative process is employed to find the prepayment rate that equates the left and right hand sides of equation (7). Different mortgage prepayment rates are used in equation (7) until the computed duration

is equal to the implied duration. This procedure is used to infer prepayment rates from market prices.

A test is performed to determine whether implied price, based on implied yield, is significantly different from actual price. Implied price is determined based on implied yield and implied prepayment; i.e., the implied prepayment is used to define cash flows which are discounted at the implied yield. The implied price is compared with actual price to examine whether the prices are significantly different from each other.

In equation (8), a test is conducted to determine whether implied price is an unbiased estimator of actual price. There is a significant relationship between implied pool price and actual pool price, with an F of 31,821. The R2 is also very high, indicating a high degree of explanatory power. However, the intercept is different from zero and the slope coefficient is different from 1. Therefore, the implied price is a biased predictor. This is confirmed by examining the mean error.

(8) ACP = 10.277E+6 + .7477 * IMPPRI R2 = .875

(20.45) (177.05)

IP1ERR = ACP - IMPPRI

mean estimation error -16.916E+6

(-59.22)

Since the mean estimation error is statistically significant, there does appear to be a difference between ACP and IMPPRI. Implied pool price is greater than actual pool price indicating the model overestimates price.

Additional tests are performed to determine whether certain types of bonds are priced accurately while other types of bonds are not priced accurately. For premium bonds, the market interest rates are below the coupon rate; this should lead to high prepayments, shorter durations and low levels of risk. With discount bonds the interest rates have risen, which should lead to low prepayments, longer durations and high levels of risk.

The results from the regression equations in Table 9 show that for both discount and premium bonds, there does appear to exist a significant relationship between actual price and implied price. In all cases, both variables are significantly correlated, indicating that IMPPRI is a good estimator of ACP. When actual prices are tested against implied prices the model seems to be reasonably accurate.

When discount and premium bonds are analyzed separately, the results indicate a significant relationship between implied price and actual price for both bond types, with an average F of 8,469 and 639 respectively. The R2s are also high, indicating a high degree of explanatory power. However, the intercept is different from zero and the slope

Table 9

Test of Predictive Accuracy:
Implied Price as a Predictor of Actual Price
for Simple Regression Model

Bond Category	No. of Bonds	Regression Equation	R2	F
1 (D)	215	ACP = 2.095E+6 + .8025*IMPPRI (2.08) (63.50)	.9498	4032.10
2 (D)	943	ACP = 4.629E+6 + .7928*IMPPRI (7.34) (133.53)	.9498	17829.66
3 (D)	1371	ACP = 9.678E+6 + .7550*IMPPRI (12.38) (108.82)	.8963	11842.38
4 (D)	1099	ACP = 14.04E+6 + .7354*IMPPRI (11.75) (82.96)	.8625	6882.57
5 (D)	278	ACP = 16.78E+6 + .7080*IMPPRI (5.28) (30.19)	.7676	911.72
6 (Par)	147	ACP = 15.30E+6 + .7500*IMPPRI (5.40) (34.20)	.8897	1169.31
7 (P)	242	ACP = 26.63E+6 + .5980*IMPPRI (8.00) (22.44)	.6773	503.62
8 (P)	146	ACP = 22.66E+6 + .5449*IMPPRI (5.35) (15.67)	.6302	245.41

Regression Equation: $ACP = a_0 + b_1$ IMPPRI

Overall Equation:
ACP = 10.2E+6 + .7477*IMPPRI R2=.876
(20.45) (177.1)

NOTE:
Bond Categories 1-5 represent discount bonds
Bond Category 6 represents par bonds
Bond Categories 7-8 represent premium bonds

Bond Category 1 has prices less than .60
Scale: .60 * 1000 = $600 price range
Bond Category 2 has prices between .60 and .70
Bond Category 3 has prices between .70 and .80
Bond Category 4 has prices between .80 and .90
Bond Category 5 has prices between .90 and .98
Bond Category 6 has prices between .98 and 1.01
Bond Category 7 has prices between 1.01 and 1.05
Bond Category 8 has prices greater than 1.05

coefficient is different from 1. Therefore, the implied price is a biased predictor. This is confirmed by examining the mean errors found in Table 10. It should be noted that the intercept is closer to zero and the slope is closer to 1 for the deep discount bonds, so the model works best for them. The deep discount bonds in category 1 have the smallest intercept and largest slope coefficient of all the groups in the study.

Since the mean estimation errors are statistically significant, there does appear to be differences between ACP and IMPPRI. The implied price appears to be a biased estimator of actual price; in all cases, implied price is consistently greater than actual price.

One possibility for this phenomenon is an error in estimating prepayment rates. The assumption that prepayments occur as a constant percentage throughout the life of the mortgage pool may not be valid. For example, a nonlinear prepayment schedule may be a more realistic pattern for prepayments. When interest rates fall to a certain level, some mortgage holders will prepay their mortgage loans and refinance their debt at a lower rate. Other homeowners may wait in anticipation of further drops in mortgage rates; their delay may also be a function of transaction costs. This scenario creates a condition where actual prepayments may be less than theoretical prepayments.

Table 10

Mortgage Pool Price Data
by Bond Price Relative to Outstanding Balance
for Simple Regression Model

Bond Category	N	Mean Actual Price	Mean Implied Price	Mean Estimation Error	T
1	215	60.71E+6	73.03E+6	-12.33E+6	-20.86
2	943	81.12E+6	96.48E+6	-15.36E+6	-38.59
3	1371	86.47E+6	101.72E+6	-15.23E+6	-32.81
4	1099	104.19E+6	122.58E+6	-18.39E+6	-27.55
5	278	107.57E+6	128.22E+6	-20.64E+6	-16.02
6	147	106.03E+6	120.96E+6	-14.93E+6	-10.96
7	242	96.15E+6	116.23E+6	-20.08E+6	-11.76
8	146	83.37E+6	111.40E+6	-28.03E+6	-11.17

Mean Estimation Error = Actual Price - Implied Price

Eventually, when a substantial part of the remaining homeowners feel that rates will not fall any further, there may be an increase in prepayments. This causes a condition where actual prepayments are greater than theoretical prepayments.

In such a scenario, there would be differences between implied prices and actual prices. The incorporation of a nonlinear prepayment schedule is beyond the scope of this dissertation and is reserved for future study. A direct comparison of implied prepayments and subsequent prepayments will be addressed later in the study.

Comparison with Other Durations

The duration of a MBS can be calculated in many ways. In order to show that implied duration, the true measure of duration, provides the most effective measure of the riskiness of a MBS, several other candidate measures of duration are estimated. Specifically, implied duration is compared with nominal duration, historical duration, and twelve-year conventional duration.

a. Nominal Duration as a Risk Measure

To calculate nominal duration, nominal yield must first be estimated. Assuming no prepayments, different values for yield are used to discount each scheduled cash payment until

estimated price equaled actual price. Nominal yield is calculated in this manner for each pool in each month of the study period.

Nominal duration, assuming no prepayments, is calculated for each pool using the standard Macaulay duration equation, adjusted on a monthly basis. Nominal yield is used in equation (9) to estimate nominal duration. Actual price is divided into the sum of each scheduled cash payment discounted by nominal yield to determine nominal duration for each month.

(9) $ND_m=\Sigma[C_{mt}*t*1/(1+Y_m)^t]/[P]$

where ND_m is the nominal duration, C_{mt} is the cash flow of the MBS, Y_m is the nominal yield of the MBS, and P is the price of the security. The nominal duration of a mortgage backed security is a function of the size of the cash flows, the timing of the cash flows, and the nominal yield.

In equation (10), nominal duration is compared with implied duration. There does exist a statistically significant, although weak, relationship between NODUR and IMPDUR.

(10) IMPDUR = 3876.67 + (-.881) * NODUR R2 = .017

(16.54) (-8.96)

NDERR = IMPDUR - NODUR

mean estimation error -594.74

(-32.23)

Since the mean estimation error is statistically significant, there does appear to be a difference between NODUR and IMPDUR. Nominal duration is much greater than the implied duration, indicating that nominal duration overestimates risk.

If no prepayments are assumed to exist, a bond with no prepayments will have a longer duration and a higher level of risk. Thus, this result supports the hypothesis that nominal duration should be greater than implied duration.

In equation (11), nominal yield is estimated using implied yield. There does exist a significant relationship between NOMYLD and IMPYLD. Both variables are positively correlated, indicating that IMPYLD can be used to estimate NOMYLD.

(11) IMPYLD = .0008 + .7796 * NOMYLD R2 = .4080

(4.57) (55.31)

NYERR = IMPYLD - NOMYLD

mean estimation error -.0018

(-84.22)

Since the mean estimation error is statistically significant, there does appear to be a difference between NOMYLD and IMPYLD. Nominal yield is greater than the implied yield indicating the model overestimates yield. If no prepayments are assumed to exist, the bond with no prepayments will have a larger yield than it actually

provides. Thus, this result supports the position that nominal yield should be greater than implied yield.

In summary, implied duration, the true measure of duration, provides a more effective measure of the riskiness of a MBS bond than does nominal duration. Nominal duration assumes that no prepayments exist; this leads to upwardly biased estimates of duration and higher risk. As expected, nominal duration is consistently greater than implied duration, indicating that nominal duration overestimates the true level of risk.

b. Historical Duration as a Risk Measure

To calculate historical duration, historical yield must be estimated. Assuming future prepayments occur at the same rate as the previous month's prepayment, different values for yield are used to discount each scheduled cash payment until estimated price equaled actual price. Historical yield is calculated for each pool for each month of the study period.

Historical duration, assuming previous month's prepayment, is calculated for each pool using the standard Macaulay duration equation. Historical yield is used in the duration equation to estimate historical duration. Actual price is divided into the sum of each scheduled cash payment discounted by historical yield to determine historical duration for each month.

In equation (12), historical duration is estimated using implied duration. There does not appear to exist a significant relationship between HISDUR and IMPDUR, indicating that HISDUR is not an effective estimator of IMPDUR.

(12) IMPDUR = 1780.85 + .0012 * HISDUR R2 = .00004

(94.45) (.40)

HDERR = IMPDUR - HISDUR

mean estimation error -199.87

(-2.18)

Since the mean estimation error is statistically significant, there does appear to be a difference between HISDUR and IMPDUR. Historical duration is significantly greater than implied duration, indicating that historical duration overestimates risk.

In equation (13), historical yield is estimated using implied yield. There does exist significant relationship between HISYLD and IMPYLD. Both variables are positively correlated, indicating that IMPYLD can be used to estimate HISYLD.

(13) IMPYLD = .0008 + .7501 * HISYLD R2 = .4263

(5.02) (57.44)

HYERR = IMPYLD - HISYLD

mean estimation error -.0022

(-103.33)

Since the mean estimation error is statistically significantly, there does appear to be a difference between HISYLD and IMPYLD. Historical yield is greater than the implied yield indicating the model overestimates yield.

In summary, implied duration, the true measure of duration, provides a more effective measure of the riskiness of a MBS bond than does historical duration. Historical duration assumes future prepayments occur at the same rate as the previous month's prepayment. There exists no significant relationship between historical duration and implied duration. Historical duration is consistently greater than implied duration, indicating that historical duration overestimates the true level of risk.

c. Twelve-Year Convention as a Risk Measure

To calculate twelve-year conventional duration, conditional yield must be estimated. Assuming no prepayments until the 12th year at which time all payments occur, different values for yield are used to discount each scheduled cash payment until estimated price equaled actual price. Conditional yield is calculated for each month of the study period.

Conventional duration, assuming no prepayments until the 12th year, is calculated for each pool using the standard Macaulay duration equation, adjusted on a monthly

basis. Conventional yield is used in the duration equation to estimate conventional duration. Actual price is divided into the sum of each scheduled cash payment discounted by conventional yield to determine conventional duration for each month.

In equation (14), conditional duration is estimated using implied duration. Although there does exist a significant but weak relationship between CONDUR and IMPDUR, the estimator is not particularly accurate.

(14) IMPDUR = 4826.54 + (-1.71) * CONDUR R2 = .0168

(13.82) (-8.73)

CDERR = IMPDUR - CONDUR

mean estimation error -1.39

(-.08)

Since the mean estimation error is not statistically significant, there does not appear to be an average difference between CONDUR and IMPDUR. The 12 year conditional duration is similar to the implied duration. Surprisingly, the 12 year adjustment works fairly well, on average. However, the error can be extremely large for individual bonds.

In equation (15), conditional yield is estimated using implied yield. There does exist significant relationship between CONYLD and IMPYLD. Both variables are positively correlated, indicating that CONYLD can be used to estimate IMPYLD.

(15) IMPYLD = .0044 + .6555 * CONYLD R2 = .4601

(48.63) (61.51)

CYERR = IMPYLD - CONYLD

mean estimation error .0015

(69.78)

Since the mean estimation error of monthly yields is statistically significant, there does appear to be a difference between CONYLD and IMPYLD. Conditional yield is less than the implied yield indicating the model underestimates yield.

In summary, implied duration, the true measure of duration, and twelve year duration both provide an effective measure of the riskiness of a MBS bond. Twelve year duration assumes no prepayments until the 12th year at which time all payments occur. Surprisingly, the twelve year duration works fairly well, on the average, in estimating the true level of risk.

Estimation of Implied Prepayment Rates

The second primary objective of this study is to use implied duration to estimate prepayments. A serious problem with investing in mortgage backed securities is the difficulty of estimating the yield for these securities. The future prepayment rates associated with mortgage pools are not known with certainty. This implies that the yields

of MBSs cannot be directly determined, since the calculation of the yield requires certain cash flow streams.

As shown in this study, the implied duration of a MBS can be inferred by using observed market returns of MBSs and the observable durations of government bonds. If the implied durations are significantly different from the standard durations, then investors must be incorporating their expectations of prepayments into the pricing of mortgage backed securities. Implied duration can then be used to estimate the implied yield of a MBS.

In (16), a test is conducted to determine whether implied prepayment is an accurate estimator of actual prepayment. There does appear to be a significant relationship between implied prepayment and actual prepayment, with a statistically significant F of 88. However, the estimator is not particularly accurate. The R2 is very low, indicating a low degree of explanatory power. The intercept is different from zero and the slope coefficient is different from 1. Therefore, the implied prepayment is a biased predictor of actual prepayment. This is confirmed by examining the mean error.

(16) PRERP1 = .0034 + .1060 * IMPPRE R2 = .034

(12.23) (9.39)

PRERT = PRERP1 - IMPPRE

mean estimation error	-.0113
	(-28.85)
mean of PRERP1	.0052
mean of IMPPRE	.0163

Since the mean estimation error is statistically significant, there does appear to be a difference between PRERP1 and IMPPRE. Implied prepayment is greater than actual prepayment indicating the model overestimates prepayment.

A test is performed to determine whether prepayments on certain types of bonds are estimated accurately while prepayments on other types of bonds are not estimated accurately. With premium bonds the interest rates have fallen, which should lead to high prepayments, shorter durations and low levels of risk. With discount bonds the interest rates have risen, which should lead to low prepayments, longer durations and high levels of risk.

The results from regression equations Table 11 show that for discount bonds (categories 1 through 5), there does not appear to exist a significant relationship between actual prepayment and implied prepayment. In all cases, the variables are not significantly correlated, indicating that IMPPRE is not a good estimator of PRERP1. The R2's are low, indicating a low degree of explanatory power. For discount bonds, a relatively low level of prepayments occurs, resulting in the model not working well.

Table 11

Test of Predictive Accuracy:
Actual Prepayment as a Predictor
of Implied Prepayment
for Simple Regression Model

Bond Category	No. of Bonds	Regression Equation	R2	F
1 (D)	36	PRERP1 = .0010 + .0020*IMPPRE (3.93) (0.02)	.0012	0.04
2 (D)	447	PRERP1 = .0009 + (-.0050)*IMPPRE (18.52) (-1.91)	.0082	3.66
3 (D)	748	PRERP1 = .0028 + (-.0018)*IMPPRE (24.2) (-.39)	.0002	0.15
4 (D)	731	PRERP1 = .0036 + .0002*IMPPRE (21.83) (0.03)	.0000	0.00
5 (D)	192	PRERP1 = .0052 + .0127*IMPPRE (10.94) (0.45)	.0011	0.20
6 (Par)	103	PRERP1 = .0048 + .1084*IMPPRE (6.40) (3.01)	.0822	9.04
7 (P)	164	PRERP1 = .0141 + .2308*IMPPRE (5.44) (3.37)	.0654	11.3
8 (P)	82	PRERP1 = .0279 + .1795*IMPPRE (5.75) (1.28)	.0199	1.63

Regression Equation: $PRERP1 = a_0 + b_1\ IMPPRE$

Overall Equation:
PRERP1 = .0034 + .1060*IMPPRE R2=.0340
(12.23) (9.39)

NOTE:
Bond Categories 1-5 represent discount bonds
Bond Category 6 represents par bonds
Bond Categories 7-8 represent premium bonds

Bond Category 1 has prices less than .60
Scale: .60 * 1000 = $600 price range
Bond Category 2 has prices between .60 and .70
Bond Category 3 has prices between .70 and .80
Bond Category 4 has prices between .80 and .90
Bond Category 5 has prices between .90 and .98
Bond Category 6 has prices between .98 and 1.01
Bond Category 7 has prices between 1.01 and 1.05
Bond Category 8 has prices greater than 1.05

The results for premium bonds (categories 6 through 8) generally show that there does exist a significant relationship between implied prepayment and actual prepayment. For premium bonds, a relatively high level of prepayments occurs, resulting in the model working well.

In Table 12 it is shown that implied prepayments are generally greater than actual prepayments. The simple regression model does a fair job estimating prepayments, but it tends to overestimate prepayments.

Implications of the Simple Regression Model

Since the simple regression model does not seem to work well in predicting prepayments, several implications can be made. The results indicate that next periods prepayments are different from anticipated prepayments. If actual prepayments are different from expected prepayments, future prepayments are not very predictable. If actual prepayments are equal to expected prepayments, this effect would be reflected in today's price. Thus, it appears that today's price doesn't really provide much information about what next periods prepayments will be under the assumption of constant prepayment rates.

Another important implication involves the assumption of constant prepayment rates. The results from the simple regression model test seem to indicate that the constant

Table 12

Pool Prepayment Mean Estimation Error
by Industry
for Simple Regression Model

Bond Category	N	Mean Actual Prepmt	Mean Implied Prepmt	Mean Estimation Error	T	Mean Implied Duration
1	213	.0011	.0227	-.0216	-9.63	2677.92
2	908	.0008	.0135	-.0128	-19.65	2181.73
3	1297	.0034	.0155	-.0128	-17.34	1758.07
4	1080	.0039	.0165	-.0127	-19.02	1726.63
5	272	.0054	.0107	-.0057	-5.83	1464.69
6	133	.0068	.0141	-.0089	-6.73	1340.16
7	242	.0220	.0292	-.0083	-3.78	951.99
8	146	.0367	.0259	.0066	1.75	936.63

Mean Estimation Error = Actual Prepayment - Implied Prepayment

prepayment rate assumption may be incorrect. During times of volatile interest rate movements, which is the time of this study, mortgage prepayments have often followed a nonlinear prepayment pattern; this describes a more realistic picture of actual mortgage pool prepayments.

Part II--Results for Multiple Regression Model

Replication of Study

After the single regression model tests are complete, the study is replicated using a multiple regression model which incorporates three month period returns. In this case, three coefficients are used, instead of one coefficient, to determine whether a measure of implied duration can be obtained.

a. A Multiple Regression Model of Implied Duration

The following equation, which expresses duration as a function of total return over a three month period, is estimated for each month in the study period.

(1) $D_{gi} = a_o + b_1 * R_{git} + b_2 * R_{git-1} + b_3 * R_{git-2}$,

where D_{gi} is the Macaulay duration of treasury bond i and R_{gi} is the observed return on treasury bond i in period t, R_{git-1} is the observed return on treasury bond i in period t-1, and R_{git-2} is the observed return on treasury bond i

in period t-2. Duration is a function of observable variables, returns. An assumption is made that D_{gi} is constant over the last three periods; this is reasonable since, in the short run, duration generally does not change significantly. The estimated coefficients b_1, b_2, and b_3 represent changes in interest rates.

The theoretical assumptions based on equation (1) are the same as in the simple regression model. To empirically test equation (1), however, these assumptions are relaxed. Specifically, returns represent price changes and the term structure is not assumed to be flat with constant shifts. The results based on this multiple regression approach are, again, conservative in that a more complex model may be able to provide more robust results.

To determine whether equation (1) can be used to estimate durations for other bonds, the following test, which is similar to the one used in the simple regression approach, is conducted. An average of about 117 bonds were chosen per month from January 1979 to May 1984. The sample is divided into two groups; every eighth bond is placed in a holdout sample. The remaining bonds make up the estimation sample.

As in the simple regression model approach, a cross sectional regression (defined by equation 1) is run on the first group, the estimation sample. Table 13 provides

Table 13

Summary of Cross-Sectional Results
for Multiple Regression Model

R^2 within various ranges:

Range	Number	Percentage	Cumulative Percentage
$90 < R_2^2 < 100$	46	74.19	74.19
$80 < R_2^2 < 90$	11	17.74	91.93
$70 < R_2^2 < 80$	3	4.84	96.77
$60 < R_2^2 < 70$	1	1.61	98.38
$50 < R_2^2 < 60$	1	1.61	100.00
$0 < R^2 < 50$	0	0	

Specification:	$Dgi=a_0 + b_1 R_{gi} + b_2 R_{gi-1} + b_3 R_{gi-2}$
Average number of bonds per regression:	102.69
Number of regressions:	62
Number of significant estimates b_1,b_2,b_3:	Positive: 24,30,33 Negative: 38,32,29 Total: 62,62,62
Mean value of b_1,b_2,b_3: t statistic	-8,761.79 -4,828.65 -3,231.87 (-1.69) (-1.02) (-0.71)
Mean R^2:	91.51

summary measures for this regression. The results support the multiple regression model; the estimated coefficients (b_1,b_2,b_3) are significant in 62 of the 62 regressions. As is evident in Table 13, the multiple regression model provides an extremely good fit with a mean R^2 of .9151; about three fourths have an R2 greater than .90. Again, there is no evidence of a non-zero ex ante risk premia (based on the insignificant t-statistic of the mean value of b); the multiple regression model does explain ex post returns quite well.

The multiple regression model coefficients from the estimation sample are used with the returns from the hold-out group of bonds to infer durations for the hold-out sample. These implied durations are compared with the actual durations to determine the predictive accuracy of the model.

The values found in Table 14 indicate that the model has significant predictive power. A total of 888 bonds are used in the hold-out sample; none of the implied durations lie outside the 95% prediction confidence interval range.

Upon observation of the data in Table 14, the mean implied duration is very close to the mean actual duration in the sample; the actual average duration for the hold-out sample is 1088.72 days while the mean implied duration is 1091.22 days. This mean error is only 2.51 and is not significantly different from zero.

Table 14

Tests of Predictive Accuracy
for Multiple Regression Model

Total number of bonds in the hold-out sample:	888
Total number of bonds for which the estimate is outside the 95% confidence interval:[a]	0
Mean actual duration:	1088.72
Mean estimated duration:	1091.22
Mean estimation error:	2.51
t statistic for test of significance:	0.289
Regression equation:[b]	$D_g = \underset{(0.47)}{7.039} + \underset{(89.28)}{.9913}\ ID_g$
Confidence Interval:	$.9694 < b_1 < 1.013$
	$R^2 = 89.99$

[a]The following formula is used to find the prediction confidence interval:

$$y + t\alpha/\alpha\ S[1 + 1/n + ((X_p - x)^2/SS_{XX})]^{1/2}$$

[b]ID_g represents the expected duration that is predicted from the regression model. This variable is dependent on observed duration. D_g represents the actual duration that is observed for each bond in the sample. This predictor variable is used in the calculation of the estimated durations.

The following regression was conducted:

(2) $ID_g = a_o + b_1 D_g$,

where ID_g is the implied duration and D_g is the actual duration. In this case, the regression equation has an average R^2 of about 90%, implying an extremely strong relationship between estimated and actual duration. While a_o is not different from zero, b_1 is significantly different from zero. The slope coefficient ,b_1, is .99 with a t-statistic of 89.28. The coefficients are almost equal to the theoretical values of 0 and 1, indicating the estimated and actual duration are highly correlated.

In summary, these results clearly show that the multiple regression model does a significantly better job at estimating implied duration than the simple regression model. These tests are extremely encouraging, indicating that equation (1) provides a reasonable relationship between Macaulay's duration and total return.

b. Implied Duration for Mortgage Backed Securities

For mortgage backed securities, the following equation describes the risk-return relationship:

(3) $D_{mj} = a_o + b_1*R_{mjt} + b_2*R_{mjt-1} + b_3*R_{mjt-2}$,

where D_{mj} is the duration of mortgage backed security j and R_{mjt} is the return on mortgage backed security j in period t, R_{mjt-1} is the return on mortgage backed security j in

period t-1, and R_{mjt-2} is the return on mortgage backed security j in period t-2. As in equation (1), b_1, b_2 and b_3 represent changes in interest rates. Although the value of variables R_{mjt}, R_{mjt-1}, and R_{mjt-2} are directly observable, neither D_{mj} nor coefficients b_1, b_2 and b_3 can be directly observed. However, equation (3) can be modified, establishing an important link between government bonds and mortgage backed securities. As stated before, the connection between the two securities is plausible since duration is considered a measure of risk and government bonds and mortgage backed securities have similar risk characteristics.

(4) $ID_{mj} = a_o + b_{g1}*R_{mjt} + b_{g2}*R_{mjt-1} + b_{g3}*R_{mjt-2}$

ID_{mj} represents the inferred duration of mortgage backed security j. Values for R_{mjt}, R_{mjt-1}, and R_{mjt-2} are directly observable; an estimate of b_{g1}, b_{g2}, and b_{g3} are obtained from the government bond data. These values can be used to compute ID_{mj} for mortgage backed security j. The inferred durations are determined for each pool in each month.

c. Implied Yields, Prepayments, and Prices

In Part I it is shown that the duration of a mortgage backed security is a function of the size of the cash flows, the timing of the cash flows, and the yield. ID_m is calculated in the previous section, Y_m is obtained via

comparison with a government bond of the same duration, and P is obtained from the MBS data base. An iterative process, similar to the process used in the simple regression approach, is then used to find the prepayment rate. Different mortgage prepayment rates are used until the computed duration is equal to the implied duration. Thus, the procedure is used to infer prepayment rates from market prices.

With the multiple regression approach, a test is performed to see whether implied price, based on implied yield, is significantly different from actual price. As in the simple regression approach, implied price is determined based on implied yield and implied prepayment; i.e., the implied prepayment is used to define cash flows which are discounted at the implied yield. The implied price is compared with actual price to examine whether the prices are significantly different for each other.

Equation (6) is used to test whether implied price is an unbiased estimator of actual price. There does appear to be a significant relationship between implied pool price and actual pool price, with an F of 47,048. The R2 is also very high, indicating a high degree of explanatory power. However, the intercept is different from zero and the slope coefficient is different from 1. Therefore, the implied price is a biased predictor. This is confirmed by examining the mean error.

(6) ACP = 6.34E+6 + .8104*IMPPRI R2 = .918

(14.77) (216.9)

IP1ERR = ACP - IMPPRI

mean estimation error -13.488E+6

(-59.56)

Since the mean estimation error is statistically significant, there does appear to be a difference between ACP and IMPPRI. Implied pool price is greater than actual pool price indicating the model overestimates price.

Further tests are performed to determine whether certain types of bonds are priced accurately while other types of bonds are not priced accurately. With discount bonds, the market interest rates are above the coupon rate; this should lead to low prepayments, longer durations and high levels of risk. With premium bonds, the interest rates have fallen, which should lead to high prepayments, shorter durations and low levels of risk.

The figures from the regression equations in Table 15 show that for both discount and premium bonds, there does appear to exist a significant relationship between actual price and implied price. In most cases, both variables are significantly correlated, indicating that IMPPRI is a good estimator of ACP. When actual prices are tested against implied prices the model seems to be fairly accurate.

Table 15

Test of Predictive Accuracy:
Implied Price as a Predictor of Actual Price
for Multiple Regression Model

Bond Category	No. of Bonds	Regression Equation	R2	F
1 (D)	182	ACP = .615E+6 + .8632*IMPPRI (0.97) (103.52)	.9835	10716.88
2 (D)	867	ACP = 2.323E+6 + .8441*IMPPRI (4.54) (170.07)	.9716	28923.37
3 (D)	1292	ACP = 2.322E+6 + .8645*IMPPRI (6.01) (236.34)	.9774	55858.44
4 (D)	1075	ACP = 3.151E+6 + .8558*IMPPRI (5.68) (198.63)	.9735	39454.26
5 (D)	270	ACP = 26.655E+6 + .6446*IMPPRI (8.59) (28.00)	.7452	783.88
6 (Par)	141	ACP = 2.927E+6 + .8743*IMPPRI (2.08) (77.61)	.9774	6022.55
7 (P)	239	ACP = 24.376E+6 + .6242*IMPPRI (7.29) (22.96)	.6897	526.97
8 (P)	146	ACP = 33.052E+6 + .4353*IMPPRI (6.80) (11.39)	.4738	129.67

Regression Equation: $ACP = a_0 + b_1\ IMPPRI$

Overall Equation:
ACP = 6.34E+6 + .8104*IMPPRI R2=.918
(14.77) (216.9)

NOTE:
Bond Categories 1-5 represent discount bonds
Bond Category 6 represents par bonds
Bond Categories 7-8 represent premium bonds

Bond Category 1 has prices less than .60
Scale: .60 * 1000 = $600 price range
Bond Category 2 has prices between .60 and .70
Bond Category 3 has prices between .70 and .80
Bond Category 4 has prices between .80 and .90
Bond Category 5 has prices between .90 and .98
Bond Category 6 has prices between .98 and 1.01
Bond Category 7 has prices between 1.01 and 1.05
Bond Category 8 has prices greater than 1.05

When premium and discount bonds are analyzed separately, the results indicate a significant relationship between implied price and actual price for both bond types, with an average F of 27,147 and 2,226 respectively. The R2s are also high, indicating a high degree of explanatory power. As in the simple regression approach, the intercept is different from zero and the slope coefficient is different from 1. Therefore, the implied price is a biased predictor. This is also confirmed by examining the mean errors found in Table 16. Note that the intercept is, again, closer to zero and the slope is closer to 1 for the deep discount bonds, so the model works best for these relatively low priced bonds. The deep discount bonds in category 1 have the smallest intercept and largest slope coefficient of all the groups in the study.

Since the mean estimation errors are statistically significant, there do appear to be differences between ACP and IMPPRI. The implied price appears to be a biased estimator of actual price; in all cases, implied price is consistently greater than actual price.

A possible explanation for this phenomenon involves errors in estimating prepayment rates. With the multiple regression approach, the assumption that prepayments occur as a constant percentage throughout the life of the mortgage pool may not be valid. As mentioned previously, it appears

Table 16

Mortgage Pool Price Data
by Bond Price Relative to Outstanding Balance
for Multiple Regression Model

Bond Category	N	Mean Actual Price	Mean Implied Price	Mean Estimation Error	T
1	182	61.24E+6	70.23E+6	-8.99E+6	-22.74
2	847	81.53E+6	93.83E+6	-12.30E+6	-39.30
3	1292	85.58E+6	96.30E+6	-10.72E+6	-47.04
4	1075	104.23E+6	118.09E+6	-13.87E+6	-43.80
5	270	107.78E+6	125.85E+6	-18.06E+6	-11.84
6	141	105.93E+6	117.81E+6	-11.87E+6	-18.65
7	239	95.93E+6	114.62E+6	-18.69E+6	-11.53
8	146	83.37E+6	115.58E+6	-32.21E+6	-10.04

Mean Estimation Error = Actual Price - Implied Price

that a nonlinear prepayment schedule may be a more realistic pattern for prepayments.

Comparison with Other Durations

Recall that the duration of a MBS can be calculated in many ways. In order to show that implied duration, the true measure of duration, provides the most effective measure of the riskiness of a MBS, several additional measures of duration are determined. In particular, implied duration is, again, compared with nominal duration, historical duration, and twelve-year conventional duration.

a. Nominal Duration as a Risk Measure

As previously defined in Part I, nominal duration assumes no prepayments and is calculated for each pool using the standard Macaulay duration equation, adjusted on a monthly basis. Nominal yield is used in the standard duration equation to estimate nominal duration. Actual price is divided into the sum of each scheduled cash payment discounted by nominal yield to determine nominal duration for each month.

In equation (7), nominal duration is compared with implied duration. There does exist a statistically significant, although weak, relationship between NODUR and IMPDUR.

(7) IMPDUR = 5092.96 + (-1.28) * NOMDUR R2 = .053

(25.05) (-15.04)

NDERR = IMPDUR - NODUR

mean estimation error -342.50

(-20.89)

Since the mean estimation error is statistically significant, there does appear to be a difference between NODUR and IMPDUR. In this case, nominal duration is always much greater than the implied duration, indicating that nominal duration overestimates risk. Again, this result supports the hypothesis that nominal duration should be greater than implied duration.

In equation (8), nominal yield is estimated using implied yield. There does exist a significant relationship between NOMYLD and IMPYLD. Both variables are positively correlated indicating that IMPYLD can be used to estimate NOMYLD.

(8) IMPYLD = .0027 + .6306 * NOMYLD R2 = .3890

(19.09) (51.65)

NYERR = IMPYLD - NOMYLD

mean estimation error -.0016

(-78.81)

Since the mean estimation error is statistically significant, there does appear to be a difference between NOMYLD and IMPYLD. It is observed that nominal yield is

greater than the implied yield indicating the model overestimates yield. Thus, this result supports the position that nominal yield should be greater than implied yield.

In summary, implied duration, the true measure of duration, provides a more effective measure of the riskiness of a MBS bond than does nominal duration. Nominal duration assumes that no prepayments exist; this leads to longer duration and higher risk. As expected, nominal duration is consistently greater than implied duration, indicating that nominal duration overestimates the true level of risk.

b. Historical Duration as a Risk Measure

As defined in Part I, historical duration uses only the previous month's prepayment and is calculated for each pool using the standard Macaulay duration equation. Historical yield is used in the duration equation to estimate historical duration. Actual price is divided into the sum of each scheduled cash payment discounted by historical yield to determine historical duration for each month.

In equation (9), historical duration is estimated using implied duration. With this approach, there does not appear to exist a significant relationship between HISDUR and IMPDUR, indicating that HISDUR is not an effective estimator of IMPDUR.

(9) IMPDUR = 2036.08 + .0034 * HISDUR R2 = .0004

(124.7) (1.35)

HDERR = IMPDUR - HISDUR

mean estimation error 57.69

(0.60)

Since the mean estimation error is not statistically significant, there does not appear to be a difference between HISDUR and IMPDUR.

In equation (10), historical yield is estimated using implied yield. There does exist significant relationship between HISYLD and IMPYLD. Both variables are highly correlated indicating that IMPYLD can be used to estimate HISYLD.

(10) IMPYLD = .0009 + .7629 * HISYLD R2 = .5964

(7.49) (78.69)

HYERR = IMPYLD - HISYLD

mean estimation error -.0019

(-126.53)

Since the mean estimation error is statistically significantly, there does appear to be a difference between HISYLD and IMPYLD. Historical yield is greater than the implied yield indicating the model overestimates yield.

In summary, a definite conclusion, similar to the one found in the simple regression model approach, is reached in the multiple regression model approach. It is evident in

Part II that implied duration, the true measure of duration, provides a more effective measure of the riskiness of a MBS bond than does historical duration. Historical duration assumes future prepayments occur at the same rate as the previous month's prepayment. In this study it is shown that there exists no significant relationship between historical duration and implied duration. Historical duration is consistently greater than implied duration indicating, that historical duration overestimates the true level of risk.

c. Twelve-Year Convention as a Risk Measure

Conventional duration, which assumes no prepayments until the 12th year, is calculated for each pool using the standard Macaulay duration equation, adjusted on a monthly basis. Conventional yield is used in the duration equation to estimate conventional duration. Actual price is divided into the sum of each scheduled cash payment discounted by conventional yield to determine conventional duration for each month.

In equation (11), conditional duration is estimated using implied duration. While there does exist a significant but weak relationship between CONDUR and IMPDUR, the estimator is not very accurate.

(11) IMPDUR = 7043.20 + (-2.79) * CONDUR R2 = .0615

(23.31) (-16.57)

duration assumes no prepayments until the 12th year at which time all payments occur; the twelve year duration is different from the implied duration.

Estimation of Implied Prepayment Rates

As previously shown in this study, the implied duration of a MBS can be inferred by using observed market returns of MBSs and the observable durations of government bonds. If a situation occurs such that the implied durations are significantly different from the standard durations, it can be postulated that investors are incorporating their expectations of prepayments into the pricing of mortgage backed securities. Implied duration can then be used to estimate the implied yield of a MBS.

In (13), a test is performed to see whether implied prepayment is an accurate estimator of actual prepayment. There does appear to be a significant relationship between implied prepayment and actual prepayment, with a statistically significant F of 158.94. However, the estimator is not particularly accurate. The R2 is very low, indicating a low degree of explanatory power. The intercept is different from zero and the slope coefficient is different from 1. Therefore, the implied prepayment is a biased predictor of actual prepayment. This is confirmed by examining the mean error.

(13) PRERP1 = .0038 + .1996 * IMPPRE R2 = .0590

(13.34) (12.61)

PRERT = PRERP1 - IMPPRE

mean estimation error -.0048

(-15.07)

mean of PRERP1 .0054

mean of IMPPRE .0106

Since the mean estimation error is statistically significant, there does appear to be a difference between PRERP1 and IMPPRE. Implied prepayment is greater than actual prepayment indicating the model overestimates prepayment.

A further test is used to see whether prepayments on certain types of bonds are estimated accurately while prepayments on other types of bonds are not estimated accurately. With discount bonds the interest rates have risen, which should lead to low prepayments, longer durations and high levels of risk. The opposite effects should occur with premium bonds.

The results from regression equations Table 17 show that for discount bonds (categories 1 through 5), there does not appear be a significant relationship between actual prepayment and implied prepayment. In many cases, the variables are not significantly correlated, indicating that IMPPRE can not be used to estimate PRERP1. The R2's are low, indicating a low degree of explanatory power.

Table 17

Test of Predictive Accuracy:
Actual Prepayment as a Predictor
of Implied Prepayment
for Multiple Regression Model

Bond Category	No. of Bonds	Regression Equation	R2	F
1 (D)	25	PRERP1 = .0009 + (-.0068)*IMPPRE (5.27) (-0.59)	.0149	0.35
2 (D)	252	PRERP1 = .0010 + (-.0108)*IMPPRE (16.08) (-3.03)	.0354	9.19
3 (D)	838	PRERP1 = .0033 + (.0628)*IMPPRE (27.62) (7.71)	.0664	59.46
4 (D)	845	PRERP1 = .0040 + (.0056)*IMPPRE (25.59) (0.58)	.0004	0.33
5 (D)	218	PRERP1 = .0054 + .0429*IMPPRE (12.60) (1.85)	.0155	3.41
6 (Par)	109	PRERP1 = .0054 + .0868*IMPPRE (7.08) (2.08)	.0388	4.32
7 (P)	172	PRERP1 = .0091 + .4573*IMPPRE (3.44) (5.28)	.1409	27.89
8 (P)	76	PRERP1 = .0386 + (-.567)*IMPPRE (5.67) (-0.25)	.0008	0.06

Regression Equation: $PRERP1 = a_0 + b_1\ IMPPRE$

Overall Equation:
PRERP1 = .0038 + .1996*IMPPRE R2 = .0590
(13.34) (12.61)

NOTE:
Bond Categories 1-5 represent discount bonds
Bond Category 6 represents par bonds
Bond Categories 7-8 represent premium bonds

Bond Category 1 has prices less than .60
Scale: .60 * 1000 = $600 price range
Bond Category 2 has prices between .60 and .70
Bond Category 3 has prices between .70 and .80
Bond Category 4 has prices between .80 and .90
Bond Category 5 has prices between .90 and .98
Bond Category 6 has prices between .98 and 1.01
Bond Category 7 has prices between 1.01 and 1.05
Bond Category 8 has prices greater than 1.05

The results for premium bonds (categories 6 through 8) generally show that there does exist a significant relationship between implied prepayment and actual prepayment. For premium bonds, a relatively high level of prepayments occurs, resulting in the model working well.

In Table 18, it can be seen that implied prepayments are consistently greater than actual prepayments. The multiple regression model does a better job than the simple regression model at estimating prepayments, but it still tends to overestimate prepayments.

Implication of the Multiple Regression Model

While the results of the study indicate the multiple regression model works better than the simple regression model, the model still does not seem to work well in predicting prepayments. An important implication, similar to those made with the simple regression model approach, can be made. The multiple regression approach does appear to be good at estimating security risk (duration), however, it does not appear to be very effective at finding a constant prepayment rate. As mentioned in the simple regression approach, a constant prepayment pattern may not exist. The results from this study seem to indicate that the assumption that prepayments occur at a constant percentage throughout the life of the mortgage pool may not be valid. Again, a

Table 18

Pool Prepayment Mean Estimation Error
by Industry
for Multiple Regression Model

Bond Category	N	Mean Actual Prepmt	Mean Implied Prepmt	Mean Estimation Error	T	Mean Implied Duration
1	25	.0011	.0079	-.0069	-2.42	3328.56
2	281	.0008	.0111	-.0103	-11.26	2676.84
3	898	.0036	.0077	-.0043	-10.80	2070.06
4	861	.0040	.0092	-.0053	-11.22	1899.98
5	224	.0055	.0112	-.0054	-5.44	1643.98
6	122	.0068	.0141	-.0069	-5.93	1403.06
7	172	.0222	.0242	-.0040	-2.24	925.94
8	76	.0367	.0224	.0149	2.94	756.68

Mean Estimation Error = Actual Prepayment - Implied Prepayment

nonlinear prepayment schedule may provide a more realistic description of actual prepayment patterns.

In summary, the multiple regression model approach is effective at estimating the risk of a mortgage backed security, but should not be used to estimate constant prepayments associated with the mortgage pool.

Part III--Results from Simultaneous Determination Model

Simultaneous Determination Approach

The simultaneous determination model uses two variables, prepayment and yield, to estimate the duration of a mortgage backed bond. It is known that D_g, the duration of a government bond, Y_g, the yield of a government bond, and P_{mb}, the price of a mortgage backed bond are observable variables. D_g, Y_g, P_{mb}, and a particular prepayment rate can be used to calculate PIYLD, the yield of a mortgage backed bond. Once PIYLD is determined, it can be used to calculate PIDUR, the duration of a mortgage backed bond.

The duration of a mortgage backed bond is estimated using the following procedure. To calculate PIDUR, PIYLD must be estimated. After selecting a prepayment rate, different values for yield are used to discount each scheduled cash payment until estimated price equals actual price. PIYLD is calculated for each pool for each month in

the study period. Once PIYLD is estimated, it is used in the standard duration equation to estimate PIDUR for each pool. Actual price is divided into the sum of each scheduled cash payment discounted by PIYLD to determine PIDUR for each month. PIDUR is then matched with government bond duration to determine whether PIYLD is equal to the yield of a comparable government bond.

Estimation of Mortgage Prepayments

In equation (1), a test is conducted to determine whether PITRY is an accurate estimator of PRER1, actual prepayment rate. There does appear to be significant relationship between PITRY and PRER1, with a statistically significant F of 928.43. However, the estimator is not particularly accurate. The R2 is very low, indicating a low degree of explanatory power. The intercept is different from zero and the slope coefficient is different from one. Thus, the PITRY is a biased predictor of actual prepayment. This confirmed by examining the mean error.

(1) PRER1 = .0041 + .0291 * PITRY R2 = .180

(24.26) (30.47)

ERROR = PRER1 - PITRY

mean estimation error -.036

(-13.79)

mean PRERP1	.0052,	mean IMPPRE	.0163
mean PITRY	.0391,	mean DIFYLD	.0015

Since the mean estimation error is statistically significant, there does appear to be a difference between PRER1 and PITRY. Both PITRY and IMPPRE are greater than PRER1 (actual prepayment) indicating that both variables overestimate prepayments.

Further evidence supports the position that the assumption of prepayments occurring at a constant percentage throughout the life of the mortgage pool may not be appropriate. DIFYLD, the mean difference between PITRY and PRER1, equals 180 basis points. This value indicates that the average difference between government bond yield and mortgage back bond is quite large. Since the mean difference between government bond yield and mortgage back bond yield is not small, the security prices do not appear to be consistent with constant prepayment rates. Two securities with similar risk features should have similar prices; the mean yield difference should be small. This is not the result that is obtained with DIFYLD.

Implication of the Simultaneous Determination Model

Since the simultaneous determination model does not seem to work well in predicting prepayments, an important implication can be made. Similar to the simple regression and the multiple regression models, the simultaneous determination model does not appear to be very effective at

estimating a constant prepayment rate associated with a mortgage pool. Again, it should be mentioned that a constant prepayment pattern may not exist. The results of this approach indicate that the assumption that prepayments occur at a constant percentage throughout the life of the mortgage pool may not be valid. As stated previously, a nonlinear prepayment schedule may provide a more realistic description of actual prepayment patterns.

CHAPTER IX

SUMMARY AND CONCLUSIONS

Review of Estimation Models

In this study three approaches are used to estimate future prepayment rates from actual mortgage backed security price data. The simple regression model, which is the first approach that is used, expresses duration as a function of total return over a one month period. The multiple regression model, which is the second approach that is used, expresses duration as a function of total return over a three month period. The simultaneous determination model, which is the final approach that is used, uses two variables simultaneously to estimate prepayment rates.

Summary of Simple Regression Model

The primary purposes of this study are to provide an accurate measure of risk for MBSs and to infer the market's expectations regarding future prepayment rates from actual mortgage backed security price data. The approach is to estimate implied duration and subsequently use it to infer the expected prepayment rates on mortgage backed securities.

After prepayments are estimated, the accuracy of this procedure is tested by comparing estimated prepayments with actual prepayments.

The first major objective is to accurately estimate the implied duration of a mortgage backed security, using an elasticity concept. Since duration is a widely recognized measure of risk, this result will be valuable to investors.

Cross sectional regressions are run on an estimation sample using government bond data to estimate relationship between bond returns and interest rate changes. Summary measures for these regressions support the model; the monthly estimated coefficients are statistically significant and the mean R^2 provides a fairly good fit.

The simple regression model coefficients are used with returns from a hold-out group of bonds to infer implied durations for the hold-out sample. The evidence indicates the model has significant predictive power; i.e., the mean implied duration is very close to the mean actual duration and the mean error is not significantly different from zero.

Actual durations are regressed against implied durations; although the regression coefficients for the intercept and slope are significantly different from the theoretical values of 0 and 1, the estimated and actual duration appear to be highly correlated.

Estimated prices are obtained using the implied prepayment rate and the yield from a government bond with a similar duration. When actual prices are compared with estimated prices, results show that for both discount and premium bonds there does appear to exist a significant relationship between actual price and implied price. The implied price, however, is found to be a biased predictor; implied price is greater than actual price, indicating the model overestimates price.

In order to show that implied duration, the true measure of duration, provides the most effective measure of the riskiness of a MBS bond, several other candidate measures of duration are estimated. Implied duration provides a better measure of the riskiness of a MBS bond than does nominal duration. The nominal duration measure, which assumes that no prepayments exist, leads to a biased estimate of duration. There does exist a statistically significant, but weak, relationship between nominal duration and implied duration. As anticipated, nominal duration is greater than implied duration, indicating that nominal duration overestimates the actual level of risk.

Implied duration provides a more effective measure of the riskiness of a MBS bond than does historical duration, for which it is assumed that future prepayments occur at the same rate as the previous month's prepayment. The results

show there exists no significant relationship between historical duration and implied duration, indicating that historical duration is not an effective estimator of the true level of risk.

Twelve year duration assumes no prepayments until the 12th year, at which time the entire balance is paid. There does exist a statistically significant, but weak, relationship between twelve year duration and implied duration. The twelve year duration works fairly well, on the average, in estimating the true level of risk but there are large differences for individual bonds.

The second major objective is to use implied duration to estimate prepayments. The implied duration of a MBS is inferred by using observed market returns of MBSs and the observable durations of government bonds. Using an estimate of implied yield, an iterative process is employed to find the prepayment rate that provides the implied duration. With this procedure, prepayment rates are implied by market prices.

While it is shown that there exists a significant relationship between implied prepayment and actual prepayment, the estimator is not particularly accurate. The R2 is very low, indicating a low degree of explanatory power. The implied prepayment is found to be a biased predictor of actual prepayment; implied prepayment is

greater than actual prepayment, indicating the model overestimates the prepayment rate.

In estimating prepayments for different types of bonds, it is found that with discount bonds, a relatively low level of prepayments occurs, resulting in the model not working well. With premium bonds, relatively high levels of prepayments occur, resulting in the model working more effectively.

Summary of Multiple Regression Model

The study is replicated using a multiple regression model which incorporates three monthly returns. As in the simple regression model approach, cross sectional regressions are run on an estimation sample using government bond data to estimate the relationship between returns and interest rate changes. Summary measures for this regression strongly support the multiple regression model; the estimated coefficients are statistically significant and the mean R^2 provides an extremely good fit.

The multiple regression model coefficients are used with the returns from the hold-out group of bonds to infer implied durations for the hold-out sample. Results show that the model has significant predictive power. The mean implied duration is extremely close to the mean actual

duration in the sample; the mean error is not significantly different from zero.

When actual duration is regressed against implied duration it is found that the regression equation has high explanatory power, implying an extremely strong relationship between estimated and actual duration. The coefficients are almost equal to the theoretical values of zero and one, indicating that implied duration is an unbiased estimator of actual duration.

As in the simple regression approach, it is shown that implied duration provides a more effective measure of the riskiness of a MBS bond than nominal duration, historical duration, or twelve year duration. Implied duration provides a better measure of the riskiness of a MBS bond than does nominal duration. There does exist a significant, although weak, relationship between nominal duration and implied duration. As expected, nominal duration is consistently greater than implied duration, indicating that nominal duration overestimates the true level of risk.

When implied duration is compared to historical duration, a result similar to the one found in the simple regression model is obtained. There exists no significant relationship between historical duration and implied duration, indicating that historical duration is not a good measure of the true level of risk.

When implied duration is compared to twelve year duration, a different result from the one found in the simple regression model is obtained. There does appear to be an average difference between implied duration and twelve year duration; thus, the twelve year duration does not work well, on average, in providing an effective measure of the riskiness of a MBS bond.

With the multiple regression approach, it is shown that there does exist a significant relationship between implied prepayment and actual prepayment; however, the estimator is not very accurate. There is an extremely a low degree of explanatory power and the implied prepayment is a biased predictor of actual prepayment.

Prepayment rates are estimated for different groups of bonds. With discount bonds, there does not appear be a significant relationship between actual prepayment and implied prepayment. With premium bonds, relatively high levels of prepayments occur, resulting in the model working somewhat better.

Summary of Simultaneous Determination Model

When two variables, prepayment and yield, are used simultaneously to estimate MBS prepayments, similar results are found. While there does appear to be significant

relationship between estimated prepayment and actual prepayment, the estimator is not particularly accurate. The R2 is very low, indicating a low degree of explanatory power. The estimated prepayment is once again an upwardly biased predictor of actual prepayment. In addition, there does not exist a constant prepayment rate such that both the duration and yield of a MBS are comparable to a government bond.

General Conclusions

The results of this study clearly show that the multiple regression model does a significantly better job at estimating implied duration than the simple regression model. The multiple regression model provides an extremely strong relationship between Macaulay's duration and total return. This finding should be of interest to investors, since it provides a means of measuring the risk of MBSs.

While the results of the study indicate the multiple regression model generally works better than the simple regression model, no other model seems to work well in predicting prepayments; both tend to overestimate prepayments. Although the multiple regression model approach is effective at estimating the risk of a mortgage backed

security, it should not be used to estimate constant prepayments associated with the mortgage pool.

None of the models does well in predicting prepayments, indicating that the constant prepayment rate assumption is, in all likelihood, incorrect. During times of volatile interest rate movements, which is the time of this study, mortgage prepayments probably follow some nonconstant prepayment pattern. The rejection of the constant prepayment rate hypothesis is valuable since it provides important insights into future directions of research about prepayments.

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APPENDIXES

APPENDIX A

DERIVATION OF THE DURATION MEASURE FOR AN ADDITIVE STOCHASTIC PROCESS

Let Q = the value of accumulated coupon receipts at time m.

T = the value of the remaining coupon payments and the maturity amount A to be paid at time to.

r_m = interest rate at maturity time m.

r_t = interest rate at investment time t.

proof: $dQ/dX > 0$

$$Q(X)=\Sigma cA(1+r_m)^{m}(1+r_t)^{-t}$$

$$dQ/dX=\Sigma[cAm(1+r_m)^{m-1}(1+r_t)^{-t}+cA(-t)(1+r_m)^{m}(1+r_t)^{-t-1}]$$

$$=cA(1+r_m)^{m}\Sigma[m[(1+r_t)^{-t}/(1+r_m)]^{-t}[(1+r_t)^{-t}/(1+r_t)]$$

Is $m/(1+r_m) > t/(1+r_t)$?

Yes, because of the following assumption. According to Bierwag, the analysis should be confined to a class of additive yield curves for which forward rates and holding period yields are positively related. If one lets [1+h*(0,t),t] be an array of points in a plane, the class of yields considered, here, is the class for which the elasticity between any two such points with respect to t is less than unity (see Bierwag 1977 page 739).

proof: $d^2Q/dX^2 > 0$

$$d^2Q/dX^2=\Sigma[cAm(m-1)(1+r_m)^{m-2}(1+r_t)^{-t}+cAm(1+r_m)^{m-1}(-t)$$
$$(1+r_t)^{-t-1}+cA(-t)(m)(1+r_m)^{m-1}(1+r_t)^{-t-1}+$$
$$cA(-t)(1+r_m)^{m}(-t-1)(1+r_t)^{-t-2}]$$
$$=cA(1+r_m)\Sigma[m(m-1)(1+r_m)^{-2}(1+r_t)^{-t}+m(-t)(1+r_m)^{-1}$$
$$(1+r_t)^{-t-1}+(-t)(m)(1+r_m)^{-1}(1+r_t)^{-t-1}+(-t)(-t-1)$$
$$(1+r_t)^{-t-2}]$$
$$d^2Q/dX^2=cA(1+r_m)^{m}\Sigma[m(m-1)(1+r_m)^{-2}(1+r_t)^{-t}+2mt(1+r_m)^{-1}+(-t)$$
$$(-t-1)(1+r_t)^{-t-2}]$$
$$[m(m-1)/(1+r_m)^2]-[2mt/(1+r_m)(1+r_t)]+[t(t+1)/(1+r_t)^2]$$

Let $A = [[m^2/(1+r_m)^2]-[2mt/(1+r_m)(1+r_t)]+[t^2/(1+r_t)^2]] +$

$B = [[t/(1+r_t)^2]+[-m/(1+r_t)^2]]$

Is A and B positive?

Yes, because of the assumption that the analysis is confined to a class of additive yield curves for which forward rates and holding period yields are positively related (see Bierwag 1977 page 739).

proof: $dT/dX < 0$

$$T(X)=\Sigma cA(1+r_m)^{m}(1+r_t)^{-t}+A(1+r_{to})^{-to}$$
$$dT/dX=\Sigma[cAm(1+r_m)^{m-1}(1+r_t)^{-t}+cA(-t)(1+r_m)^{m}(1+r_t)^{-t-1}]$$
$$+A(-to)(1+r_{to})^{-to-1}$$
$$=cA(1+r_m)^{m}\Sigma[m(1+r_m)^{-1}(1+r_t)^{-t}-t(1+r_t)^{-t-1}]+A(-to)$$
$$(1+r_{to})^{-to-1}$$

Is $[t/(1+r_t)] > [m/(1+r_t)]$?

Yes (see proof below).

proof: $d^2T/dX^2 > 0$

$$d^2T/dX^2=cA(1+r_m)^m\Sigma[m(-1)(1+r_m)^{-2}(1+r_t)^{-t}+m(-t)(1+r_m)^{-1}$$
$$(1+r_t)^{-t-1}+(-t)(-t-1)(1+r_t)^{-t-2}]+cAm(1+r_m)^{m-1}$$
$$\Sigma[m(1+r_m)^{-1}(1+r_t)^{-t}-t(1+r_t)^{-t-1}]+A(-to)(-to-1)$$
$$(1+r_{to})^{-to-2}$$

$$d^2T/dX^2=cA(1+r_m)^m\Sigma(1+r_t)^{-t}[m(-1)(1+r_m)^{-2}+m(-t)(1+r_m)^{-1}$$
$$(1+r_t)^{-1}+t(t-1)(1+r_t)^{-2}+m(1+r_m)^{-1}m(1+r_m)^{-1}+$$
$$m(1+r_m)^{-1}(-t)(1+r_t)^{-1}]$$
$$[-m(1+r_m)^{-2-t}m(1+r_m)^{-1}(1+r_t)^{-1}+t(t+1)(1+r_t)^{-2}$$
$$+m^2(1+r_m)^{-2-t}m(1+r_m)^{-1}(1+r_t)^{-1}]$$
$$[[m^2/(1+r_m)^2]-[2mt/(1+r_m)(1+r_t)]+[t^2/(1+r_t)^2]-$$
$$[m/(1+r_m)^2]+[t/(1+r_t)^2]]$$
$$[[m^2/(1+r_m)^2]-[2mt/(1+r_m)(1+r_t)]+[t^2/(1+r_t)^2]$$
$$-[m/(1+r_m)^2]+[t/(1+r_t)^2]]$$
$$[[m/(1+r_m)]-[t/(1+r_t)]]^2-[[m/(1+r_m)^2]-[t/(1+r_t)^2]]$$

Proof:

(1) $[[m/(1+r_m)]-[t/(1+r_t)]]^2-[[m/(1+r_m)^2]-[t/(1+r_t)^2]]>0$

Given (A) m>t and (B) $[m/(1+r_m)]>[t/(1+r_t)]$, (A) allows the following substitution: m=t+n+1, where n is some nonnegative integer less than or equal to m-2. If m=t, then (1) is true by inspection.

(2) [m-1=t+n] and [t+1=m-n].

(3) (B) allows $[(1+r_t)/(1+r_m)]>(t/m)$

(4) (B) also allows $[(m/t)*(1+r_t)/(1+r_m)]>1$

Expand (1)

(5) $[m^2/(1+r_m)^2]-[2mt/(1+r_m)(1+r_t)]+[t^2/(1+r_t)^2]-[m/(1+r_m)^2]$
$+[t/(1+r_t)^2]>0$

Multiply by $(1+r_m)^2(1+r_t)^2$ and collect terms.

(6) $[(m-1)m(1+r_t)^2]-[2mt(1+r_m)(1+r_t)]+[t(t+1)(1+r_m)^2]>0$

Substitute (2) into (6)

(7) $[(t+n)m(1+r_t)^2]-[2mt(1+r_m)(1+r_t)]+[t(m-n)(1+r_m)^2]>0$

Terms are grouped

(8) $tm[[(1+r_t)^2]-[2(1+r_m)(1+r_t)]+[(1+r_m)^2]]+[mn(1+r_t)^2]$
$-[tn(1+r_m)]>0$

(9) $tm[(1+r_t)-(1+r_m)]^2+[nm(1+r_t)^2]-[tn(1+r_m)^2]>0$

Divide by $t(1+r_m)^2$

(10) $f=m[[(1+r_t)/(1+r_m)]-1]^2+[n(m/t)[(1+r_t)/(1+r_m)]^2-n>0$

(11) from (3) $[(1+r_t)/(1+r_m)]>(t/m)$

This can be substituted into (10).

(12) $g=m[(t/m)-1]^2+[n(m/t)(t/m)]^2-n>0$

If (12) is true (f>g), (10) must be true;hence, (1) is true.

Rewrite (12)

(13) $m[(t-m)/m]^2+[n(t/m)]-n]>0$

Substitute (2) into (13)

(14) $(t+n+1)[-(n+1)/(t+n+1)]^2+[n(t/t+n+1)]-n>0$

(15) $(n^2+2n+1)/(t+n+1)+[nt/(t+n+1)]-n>0$

Divide by n

(16) $[n+2+(1/n)]+[t/(t+n+1)]-1>0$

(17) $[(t+n+1)+(1+(1/n))/(t+n+1)]-1>0$

(18) $[1+(1/n)]/[t+n+1]>0$

(18) is true, so (10) is also true. Thus, (1) is true.

APPENDIX B

DERIVATION OF DURATION AS A VOLATILITY MEASURE UNDER MACAULAY'S ASSUMPTION

The price of a security can be specified as

(1) $P=\Sigma C_t(1+r)^{-t}$

where r is the spot rate for a payment at time t, and C_t is the cash flow promised at time t.

Take the derivative of P with respect to r

(2) $dP=-\Sigma C_t(t)(1+r)^{-t-1}d(1+r)$

(2a) $dP=-\Sigma C_t(t)(1+r)^{-t}d(1+r)(1+r)^{-1}$

Divide equation (2a) by P

(3) $dP/P=[-\Sigma C_t(t)(1+r)^{-t}d(1+r)(1+r)^{-1}]/P$

(3a) $dP/P=[-\Sigma C_t(t)(1+r)^{-t}]/P[d(1+r)(1+r)^{-1}]$

By definition duration equals

(4) $D=[\Sigma C_t(t)(1+r)^{-t}]/P$

Substitute (4) into (3a)

(5) $dP/P=-D^*[d(1+r)(1+r)^{-1}]$

(5a) $dP/P=-D^*[d(1+r)/(1+r)]$

Therefore, the percentage change in price for an instantaneous change in interest rates is represented by the equation above.

APPENDIX C

DURATION AND BOND PRICE VOLATILITY

The price of a security is defined by equation (1).

$$(1) \quad P = \sum_{t=1}^{n} CF_t(1+r)^{-t}$$

where CF_t is the cash flow at time t and r is the appropriate interest rate.

Duration is defined as price elasticity,

$$(2) \quad D = dP/d(1+r)*(1+r) \ / \ P$$

By differentiating P with respect to (1+r), the duration of a bond can be obtained.

$$(3) \quad dP/d(1+r) = \sum_{t=1}^{n} CF_t(-t)(1+r)^{-t-1}$$

Substituting yields:

$$(3a) \quad D = \left(\sum_{t=1}^{n} CF_t(-t)/(1+r)^t \right) / P$$

However, this assumes cash flows are fixed for the current level of (1+r) and will not change as (1+r) changes. If cash flows are sensitive to changes in (1+r), then Macaulay`s duration may be inappropriate.

If cash flows are not fixed then:

$$(4)\quad dP/d(1+r) = \left[\sum_{t=1}^{n} (CF_t(-t)(1+r)^{-t-1}) \right] + \left[\sum_{t=1}^{n} (dCF_t/d(1+r))*(1+r) \right]$$

Cf_t is now defined as the cash flow at time t given the prepayment rate associated with interest rate r. It is not the nominal cash flow; i.e., the cash flow for zero prepayments.

Under the assumption that the prepayment rate is constant for all payments (given a particular value of 1+r):

(4a)

$$dP/d(1+r) = \left[\sum_{t=1}^{n} ((-t)CF_t)/(1+r)(1+r)^t) \right] + \left[\sum_{t=1}^{n} (dCF_t/dII) *(dII/d(1+r))*(1/(1+r)^t) \right],$$

where II is the prepayment rate.

It is apparent that the derivative is composed of two components:

(4b) $dP/d(1+r) = (a) + (b)$,

where (a) and (b) are defined as the bracketed terms in equation (4a).

If equation (4b) is multiplied by (1+r)/P, then Macaulay's duration is identical to term (a). In this study, term (b)

is ignored. To test whether it is appropriate to neglect term (b), the relative size difference between term (a) and term (b) is examined.

The following example shows that this term may safely be neglected; i.e., Macaulay's duration is the appropriate measure of risk for a bond.

The assumptions are:

(1) initial balance = 1,000 ;

(2) number of months, t = 360 months ;

(3) interest rate, r = .01 per month (note: this is comparable to market rates in the study period) ;

(4) prepayment rate, II = .005 per month (note: this is the average prepayment rate actually occurring in the MBS pools) ;

(5) delta, dII = .0005 (note: this term is the change in the prepayment rate) ;

It is necessary to estimate dII/d(1+r). This is estimated from the MBS sample data using two approaches,

The first approach employs a regression analysis:

$$II = a + b\,(1+r)$$

In the example above, the following regression equation is calculated.

$$II = .004 + .029*(1+r) \qquad R2 = .180$$

$$(24.26)\ (30.47)$$

It can be seen that:

$$dII/d(1+r) = b = .029$$

The second approach is to directly compute the change is prepayment rate for each period:

$$dII = II(t) - II(t-1)$$

The change in interest rate is also determined for each period:

$$d(1+r) = r_{t+1} - r_t$$

where r_t is the appropriate interest rate at time t.

$$dII/d(1+r) = b = .0003/.010 = .030$$

Both approaches provide essentially the same result, i.e. several methods can be used to estimate dII/d(1+r).

It is also necessary to compute dCF_t. This is estimated from the MBS sample data using the following approach.

Given a prepayment rate, the cash flow for each period can be calculated with the following equation.

$$CF_t = A_t * B_{t-1} + R_t$$

where A_t is the amortized factor, B_t is the outstanding balance at time t, and R_t is the prepayment amount.

The change in cash flows for each period is determined using the following equation.

$$dCF_t = CF_t(II) - CF_t(II + delta)$$

$$dII = -delta = -.0005$$ (note: both variables are estimated for the hypothetical bond)

Both terms (a) and (b) are estimated using the sample data.

Step 1: Calculation of term (a).

$$\text{Term (a)} = \left[\sum_{t=1}^{n} CF_t * (-t) * (1+r)^{-t-1} \right]$$

Term (a) = 63,795

Step 2: Calculation of term (b).

$$\text{Term (b)} = dII/d(1+r) * \sum_{t=1}^{n} (dCF_t/dII) * (1/(1+r)^t)$$

$$= -2.30 * (1/-.0005)$$

$$= 4600 * .0023$$

$$= 10.70$$

To find duration, these should to be multiplied by (1+r/P). For this hypothetical bond, the duration = 63 months, which is comparable to average MBS duration. The average error from neglecting term b is .01 months. Since this is very small relative to the duration, Macaulay's duration is an appropriate measure of risk.

VITA

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