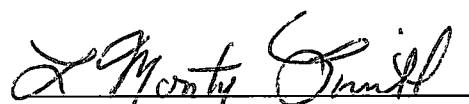
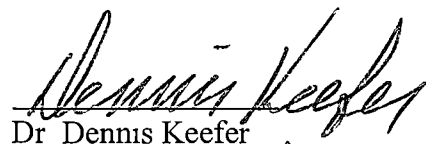


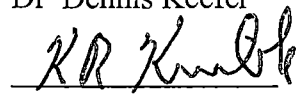
To the Graduate Council: ..

I am submitting herewith a thesis written by Nathan Allan Warvio entitled "A CIRCUIT MODEL OF AN INDUCTIVE PLASMA GENERATOR" I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Engineering Science


Dr Monty Smith, Major Professor

We have read this thesis
and recommend its acceptance


Dr Dennis Keefer


Dr. Kenneth Kimble

Accepted for the Council:


Interim Vice Provost and
Dean of the Graduate School

A CIRCUIT MODEL OF AN INDUCTIVE PLASMA GENERATOR

A Thesis
Presented For The
Master Of Science Degree
University Of Tennessee, Knoxville

NATHAN A. WAIVIO

December 2000

DEDICATION

The author wishes to express thanks to my GOD and FATHER and LORD, Jesus Christ offspring of the FATHER and the SPIRIT who was sent by the SON from the FATHER to dwell within me through the best of times and the worst of times, for giving me this opportunity, I do this as unto Him. The wise man would realize that the greatest work that could be done would be that done for GOD's Kingdom, and that the pursuit of Righteousness, this greatest work, is also the most rewarding.

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There are many people to whom I am grateful for making me who I am. I would like to thank my parents for bearing with me through the early years of dealing with my dyslexia and showing me how to strive. Thanks to my wife for motivation. Thanks to Heard Lowry and Les Crossway for the work here at ADEC. Heard gave me financial support when he could have thrown me out on the street. Thanks to Dr. Monty Smith, Dr. Dennis R. Keefer, and Dr. Kenneth Kimble for direction. I thank them for bearing with me in these latter years with my dyslexia and helping me to produce this thesis.

Lastly, special thanks to the great Christian, Michel Faraday; who's Law I rely so heavily upon.

ABSTRACT

A circuit model for an inductively coupled RF plasma generator was developed. This analysis is restricted to a cylindrical geometry with a solenoid driving coil. The system modeled assumes a cylindrical chamber filled with partially ionized argon gas. The circuit model was developed for temperatures from 1000 to 20000K and pressures from 1 to 760 Torr. The plasma was modeled as an inductor of cylindrical geometry magnetically coupled to the driving solenoid coil with a load resistance. The circuit parameters modeled are the coil and plasma inductance, coil and plasma resistance, and coupling of the plasma to the coil. The inductance, resistance, and coupling are dependent on the conductivity of the plasma and the dimensions of the plasma generator. The coupling and inductance were calculated using standard formulas. The resistance, length, and the radius of the plasma were calculated using techniques developed in this thesis. Three example configurations were analyzed: an estimated model of a low pressure plasma generator with a long solenoid and loose coupling, a single short solenoid with stronger coupling, and two coaxial short solenoids separated by the length of the radius with its respective coupling coefficients. The results are plots of each circuit parameter as a function of temperature of the plasma. We can conclude that the results demonstrate the mechanics behind the E to H mode transition

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NOMENCLATURE

L	Inductance
n	Number of turns
d	Diameter of the coil
F	F-function
k	Coupling coefficient
J	Current
σ	Conductivity
E	Electric field
R	Resistance
A	Area
L_{coil}	Inductance of the coil
R_{coil}	Resistance of the coil
l_{coil}	Length of coil
l_{lead}	Length of the leader on the ends of the solenoid
r_{coil}	Radius of the solenoidal coil
A_{tube}	Area of the tubing
r_{outer}	Outer radius of the tubing
r_{inner}	Inner radius of the tubing
L_p	Inductance of the plasma
r_c	Radius of the container
l_p	Length of the plasma
α	Skin depth term
e	Fundamental electric charge
ω	Frequency of the electric field/current
μ	Magnetic permeability
ν	Collisional frequency
ϵ	Electric permittivity

CHAPTER I

INTRODUCTION

The inductive plasma discharge is generated by high-frequency electric fields. A current path generates these electric fields. The current path is generally of a helical geometry formed by a solenoidal coil. The plasma generator will have two characteristic modes: a starting mode and a Faraday Law induced electric field mode governed by the conductivity, (The two modes are sometimes called an E and an H mode.) It is desired to have an electric circuit to model this plasma generator and its behavior. The electric circuit model could then be used in programs such as P-SPICE to design the power and impedance matching systems.

The inductive plasma generator (IPG) was first invented in the 1880's. It was then used in some research in the 1920's. Later in the 50's it was investigated as a method of heating wind-tunnel flows [1]. In the 80's and 90's it has been used as a plasma torch in materials science and in plasma deposition processes for the electronics industry [2]. This IPG analysis was applied to design a plasma generator that will be used in a ballistics range in connection with Air Force research in the area of plasma drag [3]. Weakly ionized gases have shown larger shock wave stand off than non-ionized gases, corresponding to a temperature much higher than that measured [4]. The exact mechanism behind this phenomena is not completely understood, thus the need for investigation.

The capacitive plasma discharge was used previously in ballistics range testing in connection with the investigation of anomalous plasma effects [4]. Problems with the capacitive plasma discharge were encountered. The problem with the capacitive discharge was that it generated streamers that were far hotter than the surrounding plasma glow discharge. These high temperature, high number density streamers were of some concern in the theoretical calculations. These streamers had to be eliminated, thus the development of the inductive plasma discharge.

The IPG in this case is of the cylindrical type. The cylindrical vacuum vessel is made from a Pyrex or quartz tube with the plasma contained in its respective inner

diameter and is fitted with end caps to hold the vacuum. The coil is made out of copper tubing wound with a certain diameter and number of turns per unit length. The copper tubing can be used for cooling by passing water through the tube. The gas used is argon because of its high conductivity in our temperature range. The plasma forms inside the coils and is sustained by power coupled from the drive coil. Figure 1 is a diagram of a general IPG showing the current path, with its inductance L_{coil} and resistance R_{coil} , spiraling around the pressure vessel. The plasma is inside the pressure vessel with its conductivity σ and the current J flowing in the plasma. The current creates its inductance L_p with the coupling k between the two inductors. Figure 2 shows a photograph from the

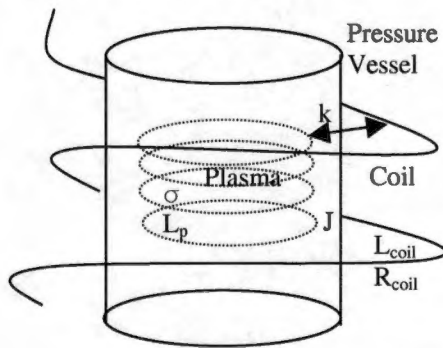


Figure 1. Physical configuration of an inductive plasma generator.

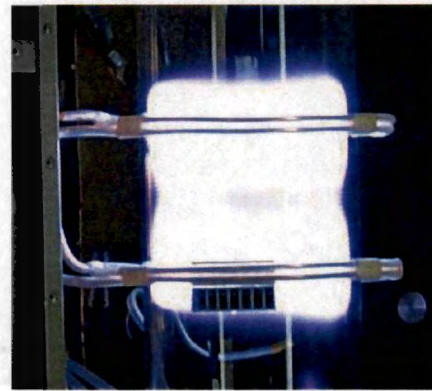


Figure 2. Photograph of an operating two-coil inductive plasma generator.

two-coil design analyzed in this thesis from Lowry [3]. From this photograph one can see that there are two plasmas, one inside of each coil.

The physical setup has a plasma where electrons circulate from the non-conservative electric field generated by the time-varying current. The electrons follow a circular path and they have collisions with atoms and ions dissipating power. The plasma tends to behave like a single turn secondary coil inside the drive coil. Therefore, the IPG is electrically analogous to a circuit with the inductance of the coil coupled to the inductance of the plasma in series with the resistance of the plasma. Figure 3 shows the circuit model for a simple IPG. In chapter 4 the circuit models for the three cases are examined in greater detail.

Modeling of the IPG thus consists of determining the

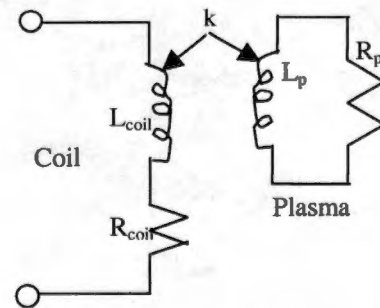


Figure 3. Circuit model developed for the inductive plasma generator.

values for the coil inductance L_{coil} and resistance R_{coil} as well as the plasma inductance L_p , resistance R_p , and the coupling coefficient k .

When we modeled the plasma as circuit element values we made a few assumptions. The inductance was found with standard formulae assuming a cylindrical sheet of current. The coupling coefficient calculations also came from the standard formula [6]. The conductivity, a function of the pressure and temperature of the gas, was determined through theoretical means by DeVoto [5] and we used his results for the conductivity in this thesis. The electric field was assumed to have the same profile as the induced field in an infinite solenoid. The effective radius of the plasma was determined by incorporating the effect of the skin-depth. The skin-depth was calculated from the conductivity of the plasma and the electric field distribution. To model the length of the plasma we equated the inductance of the coil with the inductance of a section of an infinite solenoid, which was assumed to have the effective length of the induced field. The conductivity of the plasma was also used in the calculations for the plasma's resistance. The resistance was calculated by deriving the power dissipated by the plasma, using the conductivity and current density of the plasma. Using the well-known equation for electrical power as a function of current and resistance we solved for the resistance.

The final results are the inductance, resistance, and coupling coefficients for three example plasma generators drawn from Keefer [7], and Lowry [3]. The description of the example configuration drawn from Keefer [7] was an estimated model of a low-pressure plasma generator with a long solenoid and loose coupling. The two example configurations drawn from Lowry [3] were a single short solenoid with stronger coupling, and two coaxial short solenoids separated by the length of the radius with its respective coupling coefficients. These results are calculated over a temperature range from 1000K-20000K and a pressure range from 1 Torr-760 Torr. It can be seen that due to the nonlinearity of the plasma conductivity there is a sharp change in the inductance, resistance, and coupling, as the temperature increases and it is responsible for the rapid mode transition.

In the following chapters the reader will read about the modeling of the IPG. The background chapter discusses the theory utilized in this thesis. Chapter III discusses the

steps taken to implement the approach. Chapter IV presents the results of the thesis. Chapter V discusses the conclusions and possible future work.

CHAPTER II

BACKGROUND

This chapter deals with the background theory developed by other scientists that is used later in this thesis. The subject matter is divided into several sections. Section (A) inductance, coupling, and resistance, Section (B) conductivity, (C) electric field, (D) skin depth, and (E) deals with the characteristic mode transition exhibited in this plasma generator type.

Section A

Inductance, Coupling, and Resistance

Inductance is a measure of the resistance to a change in the current analogous to mass in physical systems resisting a change in velocity (inertia). For this analysis the formula for the single-layer solenoid was used. This formula makes the assumption of a current sheet. The inductance is calculated from the formula from Terman [6]:

$$L = F n^2 d \text{ (}\mu\text{H)} \quad (1)$$

Where n is the number of turns, d is the diameter of the coil (center-to-center of winding), in inches, and F is a function which may be interpolated from the table in Terman and is dependent on the diameter to length ratio shown in Figure 4.

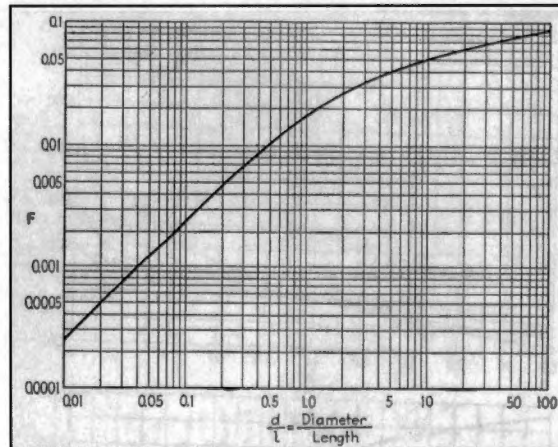


Figure 4. Factor F for a single-layer solenoid (ref 6).

The coupling coefficient is the ratio of the mutual inductance to the maximum mutual inductance. The formula for the coupling coefficient between two inductors with values L_1 and L_2 is:

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad (2)$$

where M is the mutual inductance and the denominator is the maximum possible mutual inductance, the square root of the product of the self-inductances. "The mutual inductance is the number of flux linkages in the second coil per unit current in the first coil, or vice versa [6]." The formula for mutual inductance between single-layer solenoid coaxial coils not concentric (Figure 5), is used for the case of the coupling from coil to coil, plasma to plasma, and plasma to opposite plasma for the two coil case. The other cases only use the formula for coaxial concentric coils in the case where the outer coil is shorter (Figure 6). These formulas are found in Terman [6].

The mutual inductance between single-layer solenoids as shown in Figure 5 is given by:

$$M = 0.02505 \frac{a^2 A^2 n_1 n_2}{4l_x} (K_1 k_1 + K_3 k_3 + K_5 k_5) (\mu H) \quad (3)$$

where a = smaller radius from axis to center of the wire in inches

A = larger radius (in)

$2l$ = length of smaller coil (in)

$2x$ = length of larger coil (in)

$$K_1 = \frac{2}{A^2} \left(\frac{x_2}{r_2} - \frac{x_1}{r_1} \right) \quad k_1 = 2l$$

n_1, n_2 = number of turns each coil

$$x_1 = D - x, \quad r_1 = \sqrt{x_1^2 + A^2}$$

$$x_2 = D + x, \quad r_2 = \sqrt{x_2^2 + A^2}$$

$$K_3 = \frac{1}{2} \left(\frac{x_1}{r_1^3} - \frac{x_2}{r_2^3} \right) \quad k_3 = a^2 l \left(3 - \frac{4l^2}{a^2} \right)$$

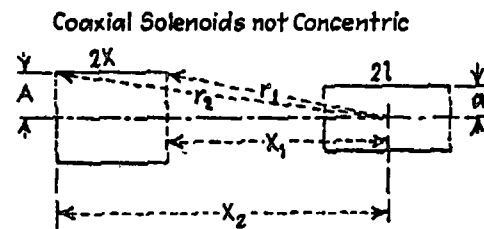


Figure 5 Coaxial solenoids not concentric

Coaxial Concentric Solenoids-Outer Coil the Shorter

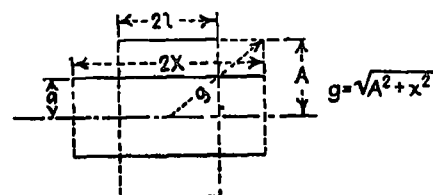


Figure 6 Coaxial concentric solenoids outer coil shorter

$$K_5 = -\frac{A^2}{8} \left[\frac{x_1}{r_1} \left(3 - \frac{4x_1^2}{A^2} \right) - \frac{x_2}{r_2} \left(3 - \frac{4x_2^2}{A^2} \right) \right] \quad k_5 = a^4 \left(\frac{5}{2} - 10 \frac{l^2}{a^2} + 4 \frac{l^2}{a^4} \right)$$

D = axial distance between centers of coils (in)

The mutual inductance, for coaxial concentric solenoids with the outer coil shorter as shown in Figure 6 is given by:

$$M = 0.0501 \frac{a^2 n_1 n_2}{g} \left[1 + \frac{A^2 a^2}{8g^4} \left(3 - 4 \frac{l^2}{a^2} \right) \right] (\mu H) \quad (4)$$

where all of the terms are the same as above and g is defined in Figure 6.

As current flows it will generally encounter a resistance that will dissipate power. The formula for calculating resistance is:

$$R = \frac{l}{\sigma A} \quad (5)$$

where R is resistance, l is length, A is cross sectional area, and σ is the conductivity of the material.

Section B

Plasma Conductivity

Conductivity in this thesis must be calculated over a very wide range of temperatures from 1000K-20000K and pressures from .1 Torr to 1 atm. The equation for conductivity is:

$$\sigma_e = n_e e \mu_e \quad (6)$$

where, n_e is the charge density and μ_e is the electron mobility transport coefficient. If the electron is in an oscillating electric field of frequency ω then the electron mobility transport coefficient is defined as:

$$\mu_e = \frac{e}{m_e(\nu + j\omega)} \quad (7)$$

where ν is the average collisional frequency. Only the electron contribution was considered in these calculations because the ion contribution is much smaller (on the order of the ratio of $m_e/m_i = 1.3638 \times 10^{-5}$ for argon). After separation into its real and

imaginary parts the conductivity σ_e can be written as:

$$\sigma_e = \frac{n_e e^2}{m_e} \left(\frac{\nu}{\nu^2 + \omega^2} - j \frac{\omega}{\nu^2 + \omega^2} \right) \quad (8)$$

This creates an imaginary term in the conductivity that represents the capacitive reactance. It is capacitive reactance because the value is always negative. If one compares the factors contributing to the imaginary part of the conductivity ($\omega \sim 10^7$, $\nu \sim 10^9$) one can see that $\omega/(\nu^2 + \omega^2) \ll 1$ so we may neglect the imaginary part of the conductivity, and the conductivity reduces to the DC conductivity. For this thesis we use the data for DC conductivity presented by deVoto [5]. The assumptions made by deVoto were a one temperature homogeneous plasma, consisting of argon atoms and singly ionized ions. The ionization potential was reduced by the same method used by Drellishak [10] for the one species Saha equation

Section C

Electric Field

To determine an effective radius and resistance for the model we must know the electric field distribution in the plasma. The induced electric field for a short solenoid is not linear nor is it elementary to develop an equation describing it. If one makes the assumption of the infinite solenoid one may simplify the equation so that it will be linear [1]. The equation for the electric field in the solenoid without the skin effect is:

$$E = \frac{E_0}{r_c} r \quad (9)$$

where r_c is the radius, r is the independent variable, and E_0 is the magnitude of the electric field at radius r_c

Section D

Skin Depth

The skin effect is a phenomenon caused by high frequency electric fields which causes a current to flow on the surface of the conductor at high current densities, resulting in an attenuation of the electric field, causing the electric field to fall off exponentially in the conductor. The skin depth is defined as the distance where the field falls off to $1/e$ of

its initial value. The skin depth of the plasma deals with the optical properties of electrically conducting media [8,9]. The skin depth term found in the exponential is α :

$$\alpha = \frac{1}{\delta} = kn\kappa \quad (10)$$

where δ is called the skin depth, k is the real part of the wave number, n is the real part of the index of refraction, and κ is the attenuation index or the extinction coefficient. The formulas for these parameters are:

$$k = \frac{\omega}{c} \quad (11)$$

$$n = \left\{ \frac{\mu\epsilon}{2\mu_0\epsilon_0} \left[\left(1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right)^{1/2} + 1 \right] \right\}^{1/2} \quad (12)$$

$$\kappa = \left(1 + \left(\frac{\omega\epsilon}{\sigma} \right)^2 \right)^{1/2} - \frac{\omega\epsilon}{\sigma} \quad (13)$$

where ω is the frequency, c is the speed of light, μ is the magnetic permeability of the material, μ_0 is the magnetic permeability of free space, ϵ is the electric permittivity of the material, ϵ_0 is the electric permittivity of free space, and σ is the conductivity of the material. This will yield a simple formula for skin depth that works for conductors.

Section E

Characteristic Mode Transition

The plasma will have two characteristic modes: an E and an H mode. The proper modeling of this plasma generator should illustrate this mode transition as the temperature increases.

The E mode will be generated during the initial operation in regions of high electric fields and long path lengths (due to electron avalanche), in the operating regime where the power input and the conductivity of the gas is low. Electron avalanche is the method by which an electron will be accelerated by an electric field and collide with an atom transferring energy to it. When the input power to the system is increased, the thermodynamic equilibrium temperature and the conductivity will increase. When modeling an arbitrary plasma generator buoyancy and convection must be accounted for in some cases. If enough power is supplied this will create a self-reinforcing effect due to

the skin effect. This effect in our case increases the inductance and coupling of the plasma to the coil, while reducing the resistance, causing more current to flow, and more ohmic heating to occur, raising the local gas temperature, and thus the conductivity. This amplifies the effect until a new thermodynamic equilibrium is reached. This causes a very rapid change in modes. The H mode also exhibits hysteresis due to the increased stored energy in the high temperature gas and the skin effect. That is, the larger stored energy and the skin effect will allow the H mode to operate at lower power input than was necessary to trigger the H mode [11].

CHAPTER III

DEVELOPMENT OF THE CIRCUIT MODEL

This chapter contains the method of the analysis that was implemented to obtain the results of the thesis. The calculations were performed using Matlab and the code is included in Appendix A. The work accomplished is divided into several sections, (A) inductance, coupling, and resistance, (B) conductivity, (C) current distribution, (D) spatial parameters for the plasma, and (E) resistance of the plasma.

Section A

Inductance, Coupling, and Resistance

Inductance

The inductance for the coil was calculated by using equation (1) from the previous chapter. Since the plasma was modeled as a single-turn solenoid, the inductance for the plasma was determined using the same formula with the effective radius and length as determined later in this chapter.

Coupling

The coupling coefficients for the system were calculated by using equations (2), (3) and (4) from the previous chapter with the plasma treated as the smaller solenoid. The plasma dimensions were calculated using the methods developed later in this chapter.

Resistance

The resistance of the coil was calculated by using equation (5). The length of the coil was calculated using the equation:

$$l_{coil} = 2\pi nr_{coil} + 2l_{lead} \quad (14)$$

Where n is the number of turns, d is the diameter of the coil, and l_{lead} is the leader length

on both ends of the coil. The cross sectional area was found using equation:

$$A_{\text{tube}} = \pi(r_{\text{outer}}^2 - r_{\text{inner}}^2) \quad (15)$$

where r_{outer} is the outer radius and r_{inner} is the inner radius of the coil tubing. These calculations were for a D.C. resistance. The current will flow in a fraction of the cross sectional area calculated in the D.C. case due to the skin effect and the high conductivity of copper. Neglecting the effect of the “skin effect” in this case should not have a great impact on the modeling of the plasma generator because the conductor is hollow tubing. The resistance of the plasma was calculated from the conductivity and assumed current distribution as described in Section E of this chapter.

Section B

Conductivity

The conductivity is the single most important property in the plasma generator model because the radius and the resistance are dependent on it. In calculating the conductivity we made the assumption that all of the species present were of the same temperature and that it was uniform with no temperature gradients present. The conductivity is found using a two dimensional linear interpolation function that is built into Matlab. The data interpolated was taken from the tables developed by deVoto [5]. The tables contain values of the conductivity of argon pressures of 1, 10, 100, and 760 Torr and from 1000K to 20000K at increments of 1000K. These tables are included in the Appendix B.

Section C

Current Distribution

The current distribution is necessary to find the effective radius and the resistance of the plasma. Here we will derive the current distribution in detail. Ohm's Law relates the current distribution to the electric field:

$$J = \sigma E \quad (16)$$

The exponential skin depth term, equation (10), was developed from a plane wave traveling into a conducting media. In this case the wavelength is longer than the diameter

of the coil and since the wave propagates radially inward, we can use the simplified equation (9) as the profile of the electric field. One can then visualize the electric field profile as a plane wave with the amplitude decreasing linearly to zero starting at a point r_c . One would just multiply the electric field in the normal way with the skin depth exponential. If we visualize this in cylindrical coordinates then the electric field would be like an exponential dropping off radially inward starting at the boundary of the plasma. Following this procedure in cylindrical coordinates yields:

$$E(r,\phi) = \frac{E_0}{r_c} r e^{-\alpha(r_c - r)} \quad (17)$$

where E_0 is the maximum electric field value occurring at the inner radius of the chamber, r_c , from the axis. When calculating α from equations (10), (11), (12), and (13), we made the assumption that the electric permittivity and the magnetic permeability of the plasma were the same as that of vacuum, and that the conductivity of the plasma came from our interpolated values taken from deVoto.

Section D

Spatial Parameters for the Plasma

Effective Radius

The effective radius of the plasma was found by taking the mean radius of the assumed current distribution. The mean radius is found by a 2-D density function based upon the electric field function which varies only in the r -direction.

$$E(r,\phi) = \frac{E_0}{r_c} r e^{-\alpha(r_c - r)} \quad (18)$$

To calculate the mean radius, we defined a 2-D density function with α defined in equation (10) by

$$p(r,\phi) = \frac{\frac{E_0}{r_c} r e^{-\alpha r}}{\int_0^{2\pi} \int_0^{r_c} \frac{E_0}{r_c} r e^{-\alpha r} r dr d\phi} \quad (19a)$$

$$= \frac{r e^{-\alpha r}}{\int_0^{2\pi} \int_0^{r_c} r e^{-\alpha r} r dr d\phi} \quad (19b)$$

To find the mean over the variable r , we multiply the 2-D density function by r and integrate:

$$r_{\text{eff}} = \int_0^{2\pi} \int_0^{r_c} r p(r, \phi) r dr d\phi \quad (20a)$$

$$= \frac{\int_0^{r_c} r^3 e^{\alpha r} dr}{\int_0^{r_c} r^2 e^{\alpha r} dr} \quad (20b)$$

The result of this derivation is the effective radius dependent only on the container radius and the skin depth term α :

$$r_{\text{eff}} = \frac{(\alpha r_c)^3 - 3(\alpha r_c)^2 + 6(\alpha r_c) - 6 + 6e^{-\alpha r_c}}{\alpha((\alpha r_c)^2 - 2(\alpha r_c) + 2 - 2e^{-\alpha r_c})} \quad (21)$$

Length

The estimate of the length of the plasma was found by using the inductance of the solenoidal coil and finding an effective length of the magnetic field produced by it. To accomplish this we determined the length of a section of an infinite solenoid with the same inductance and radius as the coil. We estimate the length of the plasma to be the length of a section of an infinite solenoid with the same radius and inductance as the coil. A finite length section of an infinite solenoid has the inductance:

$$L_{\text{coil}} = \mu n^2 \frac{\pi r_{\text{coil}}^2}{l_p} \quad (22)$$

Where n is the number of turns and is a unitless number, and μ is the magnetic permeability of free space, which is in the same units as the length and area. This formula for inductance gives its value in Henrys. The equation for the inductance of a solenoid used came from equation (1). Equating equation (22) and (1) and collecting terms yields the equation:

$$l_p = 5.01376 \times 10^{-2} \frac{r_{\text{coil}}}{F\left(\frac{2r_{\text{coil}}}{l_{\text{coil}}}\right)} \quad (23)$$

This equation will give the length in inches where r_{coil} is the radius of the coil in inches and F is the tabulated function shown in Figure 4 and is dependent on the diameter and length of the coil

Section E

Resistance of the Plasma

The model of the resistance comes from the conductivity, radius, and length. The model of the resistance draws heavily from the power equation. The power dissipated in a differential volume element is given by:

$$dP = \frac{1}{\sigma} J_{\phi}^2(r, \phi, z) r dr d\phi dz \quad (24)$$

where σ is the conductivity of the element and J_{ϕ} is the current through the element in the ϕ direction. One can see that this is nothing more than the common equation $P = I^2 R$. We assume a current density of the form $J_0 r e^{\alpha r}$ as developed earlier. To find the total current one must integrate the current density over the area of the solenoid.

$$I = \int_{-lp/2}^{lp/2} \int_0^{r_c} J_0 r e^{\alpha r} r dr dz \quad (25)$$

After equation (25) is integrated, we rearranged the formula and solved for J_0 yielding:

$$J_0 = \frac{\alpha^2 I}{l_p [e^{\alpha r_c} (\alpha r_c - 1) + 1]} \quad (26)$$

Equation (24) was used to determine the total power by integrating over the plasma's volume. The equation is prepared for integration by factoring out the constant coefficients, yielding:

$$P = \frac{J_0^2}{\sigma} \int_{-lp/2}^{lp/2} \int_0^{2\pi} \int_0^{r_c} r^2 e^{2\alpha r} r dr d\phi dz \quad (27)$$

After integrating, substituting J_o , setting equal to I^2R , and rearranging we get:

$$R_p = \frac{\pi[4(\alpha r_c)^3 - 6(\alpha r_c)^2 + 6(\alpha r_c) - 3 + 3e^{-2\alpha r_c}]}{4\sigma l_p[(\alpha r_c) - 1 + e^{-\alpha r_c}]^2} \quad (28)$$

which is the resistance for a plasma solenoid in terms of conductivity, radius, and length.

CHAPTER IV

EXAMPLES

The results of this thesis are categorized into similar sections as in the background and method. Section (A) presents the example drawn from Keefer [7], section (B) the one coil case drawn from Lowry [3], and section (C) the two coil case drawn from Lowry [3]. Each section contains the results of the inductance, coupling, and resistance for the examined configurations. In each section there will be a table of the input parameters and a table of the results at a nominal temperature at the operating pressure. This will be followed by figures of the configuration and in case C a figure of the circuit model. The results will also be displayed in plots of the parameter vs. temperature for several pressures. The coils were assumed to be made of one-eighth inch copper tubing with an inner diameter of one-tenth of an inch. Each example has six inches of leader length included in the calculations. The resistivity of copper used was 1.69×10^{-8} ohms per meter. In all of the cases the coil is concentric with the chamber.

Section A

The Case Drawn from Keefer [7]

The system in Keefer [7] is a one coil configuration that has a slightly smaller coupling than the other systems that we examined. The parameters are shown in Table 1. The physical design is depicted in Figure 7. In Keefer the plasma did not touch the actual chamber wall because the plasmoid was flow stabilized. The analysis assumes the

Table 1 Dimensions of the plasma generator design A.

System Parameters	
Number of Coils:	1
Number of Turns.	6
Length of Coil:	4.15"
Diameter of the Coil:	7.63"
Diameter of the Chamber:	4"
Distance between Coils:	N/A
Operating Frequency:	4.5MHz

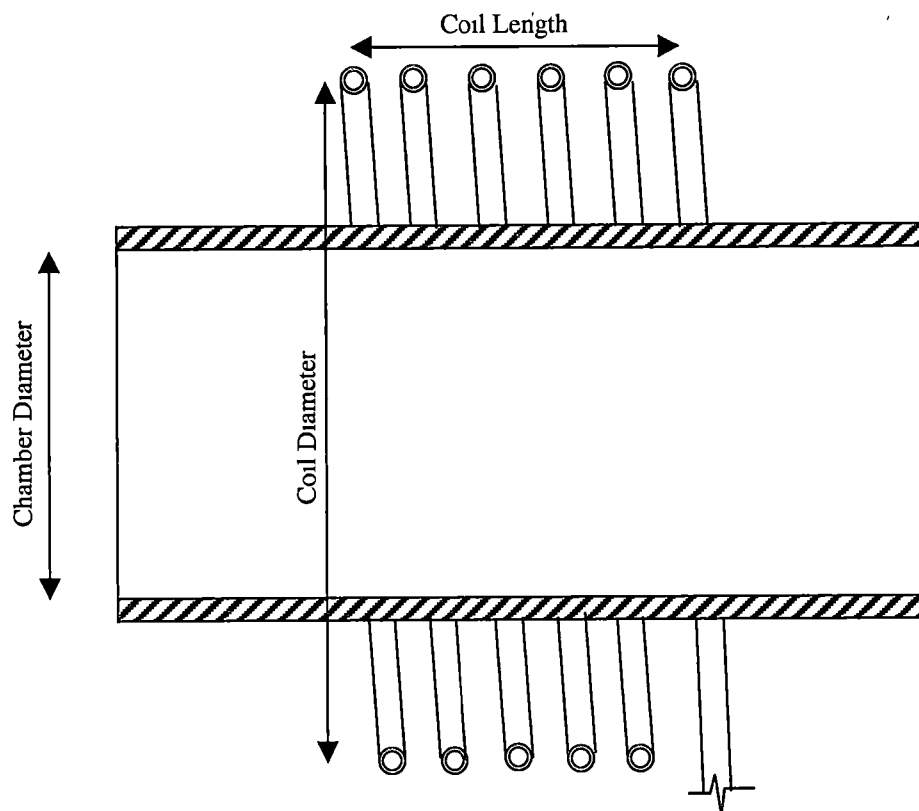


Figure 7 Physical diagram for case A

plasma extends to the chamber wall. The chamber diameter in this case is an effective chamber diameter chosen to be at the boundary of the flow-stabilized plasma.

The results of the calculations are outlined in Table 2, that shows the calculated values for the parameters at a nominal temperature of 7000 K and pressure of 1 Torr. The two calculated coil parameters, the inductance of the coil and the resistance of the coil, are listed first. The other parameters shown are the plasma parameters. The plasma

Table 2 Results for case A

Results	
Inductance of the Coil.	6.9122 μH
Resistance of the Coil	23 769 $\text{m}\Omega$
Temperature:	7000 K
Pressure.	1 Torr
Inductance of the Plasma:	0.0350 μH
Coupling of the Plasma:	0.3580
Resistance of the Plasma:	3.0326 $\text{m}\Omega$
Length of the Plasma:	7.6010"
Eff. Diameter of the Plasma:	3.5812"

parameters are the inductance, which is dependent on the effective diameter and the length. The coupling depends on the effective diameter, length, and inductance of the coil and the plasma. The final parameter, resistance of the plasma, is about seven times less than the resistance of the coil. This describes all of the parameters that are used in our circuit model. We have included plots of the plasma inductance, coupling coefficient, and resistance vs. temperature at several different pressures. These could be used for a dynamic case where the thermodynamics of the problem would be included. Figure 8 is the plot for the inductance of the plasma at 1, 10, 100, 760 Torr. Figure 9 is the plot of the coupling of the plasma to the coil at the same pressures. Figure 10 is the plot of the resistance of the plasma at the same pressures. Note the rapid change in all three parameter values near 6000 K, and that this is reflected in the characteristic mode transition observed in experiments.

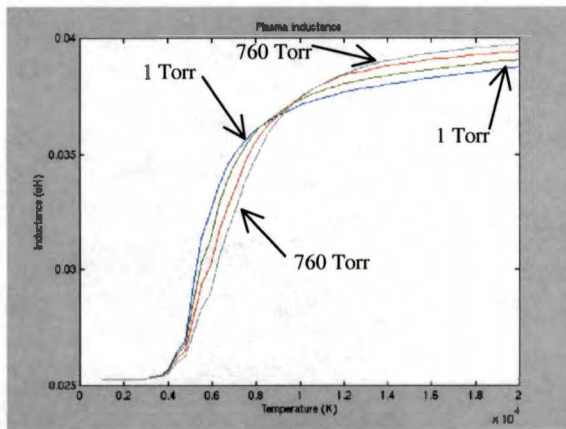


Figure 8. The inductance for case A.

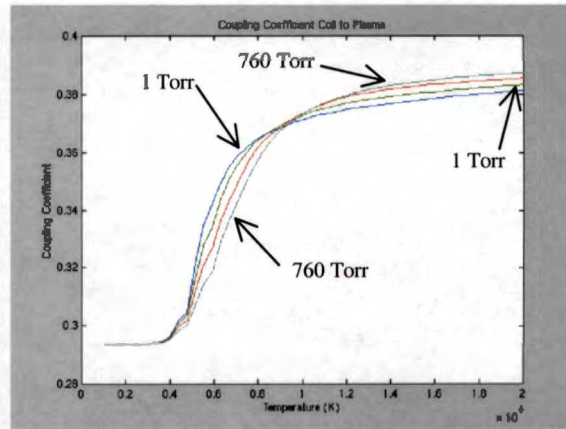


Figure 9. The coupling coefficient for case A

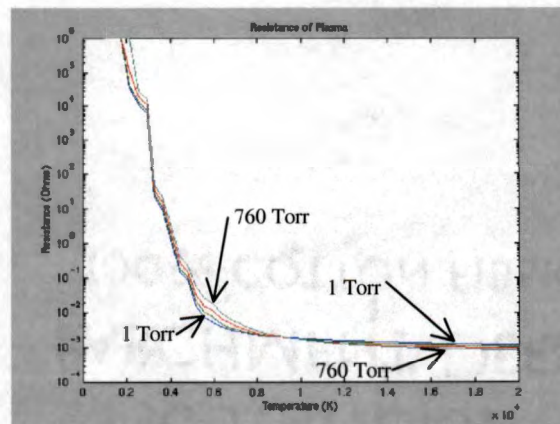


Figure 10. The resistance for case A.

Section B

Single Coil Case from Lowry

The next case is the single coil case drawn from Lowry. The number of turns per unit length is much greater in this case. This helps the plasma generator operate at higher pressure because the field strength is greater. In this case the diameter of the coil is closer to the diameter of the plasma. The dimensions of this plasma generator are found in Table 3. The physical setup is shown in Figure 11. One should notice the small coil length and the closer distance from the coil to the plasma boundary at the cylinder wall. The results are summarized in Table 4. The coupling coefficient at the 30 Torr pressure is just slightly less than the coupling coefficient in the previous example at the lower pressure of 1 Torr. It should be noted that this is because of the conductivity's behavior due to pressure. The behavior of the inductance, the coupling coefficient, and resistance vs. temperature at several different pressures are shown in Figures 12, 13, and 14. See how closely the parameters match the behavior of the conductivities. The plot of the conductivity can be found in Appendix B. Again, note the sharp increase in all three parameter values near 6000 K, and the mode transition.

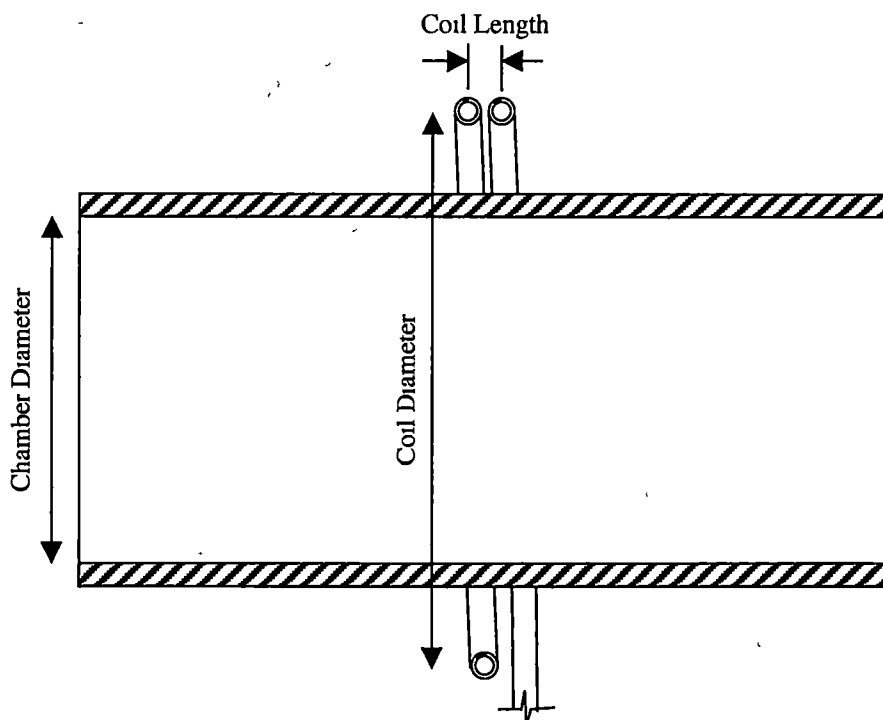


Figure 11 Physical diagram for case B

Table 3. Dimensions of the plasma generator design B.

System Parameters	
Number of Coils:	1
Number of Turns:	1.9
Length of Coil:	0.375"
Diameter of the Coil:	6"
Diameter of the Chamber:	4.125"
Distance Between Coils:	N/A
Operating Frequency:	13.56MHz

Table 4. Results for case B.

Results	
Inductance of the Coil:	1.2661 μ H
Resistance of the Coil:	7.5023 m Ω
Temperature:	7000 K
Pressure:	30 Torr
Inductance of the Plasma:	0.0846 μ H
Coupling of the Plasma:	0.3578
Resistance of the Plasma:	17.820 m Ω
Length of the Plasma:	2.5732"
Eff. Diameter of the Plasma:	3.8103"

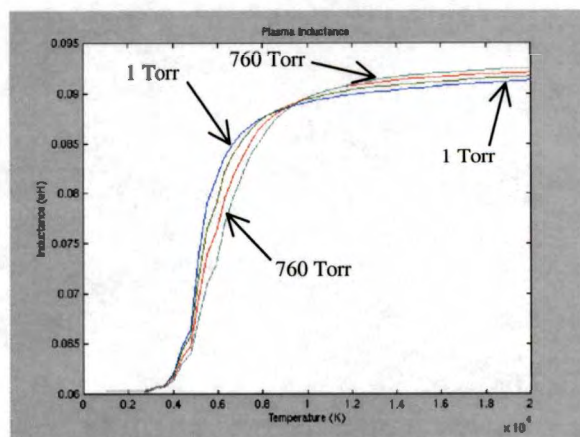


Figure 12. The inductance for case B.

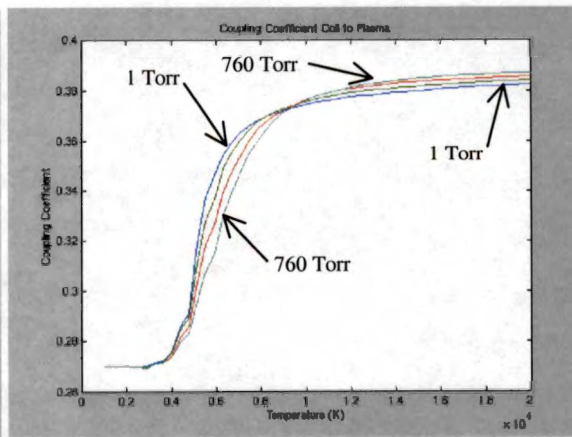


Figure 13. The coupling coefficient for case B.

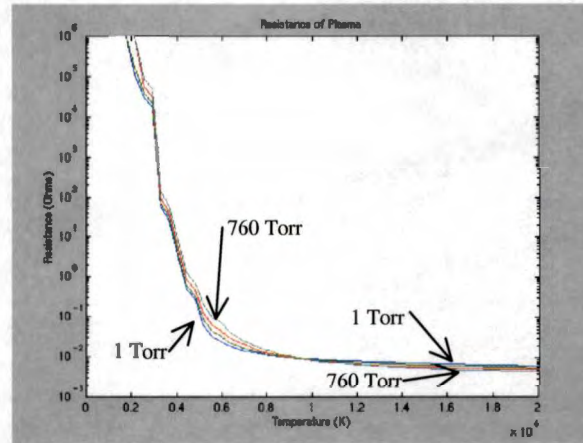


Figure 14. The resistance of case B.

Section C

The Two Coil Case Drawn from Lowry

The previous example can be extended to include a multiple coil case. The two coil case, drawn from Lowry, is the same coil as above. The two coils are separated by a distance equal to the radius of the coil. This configuration is often called the Helmholtz configuration. In the Helmholtz configuration the magnetic field is far more uniform between the coils than even a solenoid of that length. This creates a uniform plasma in the space between the coils. The dimensions of this plasma generator are found in Table 5. The circuit shown in Figure 15 can model this type of plasma generator. The physical system is shown in Figure 16. This circuit is similar to the first circuit model presented but there is another coil with coupling from coil to coil and coil to each plasma and from plasma to plasma. The results at a nominal pressure of 30 Torr and temperature of 7000 K are summarized in Table 6. The results for inductance, the coupling coefficients, and resistance are shown in Figures 17, 18, 19, 20, and 21. Note again the sharp change in values near 6000 K, and how this leads to the observed mode transition.

Table 5 Dimensions of the plasma generator
design C

System Parameters	
Number of Coils:	2
Number of Turns:	1.9 each
Length of Coil:	0.375"
Diameter of the Coil:	6"
Diameter of the Chamber:	4.125"
Distance Between Coils:	3"
Operating Frequency	13.56MHz

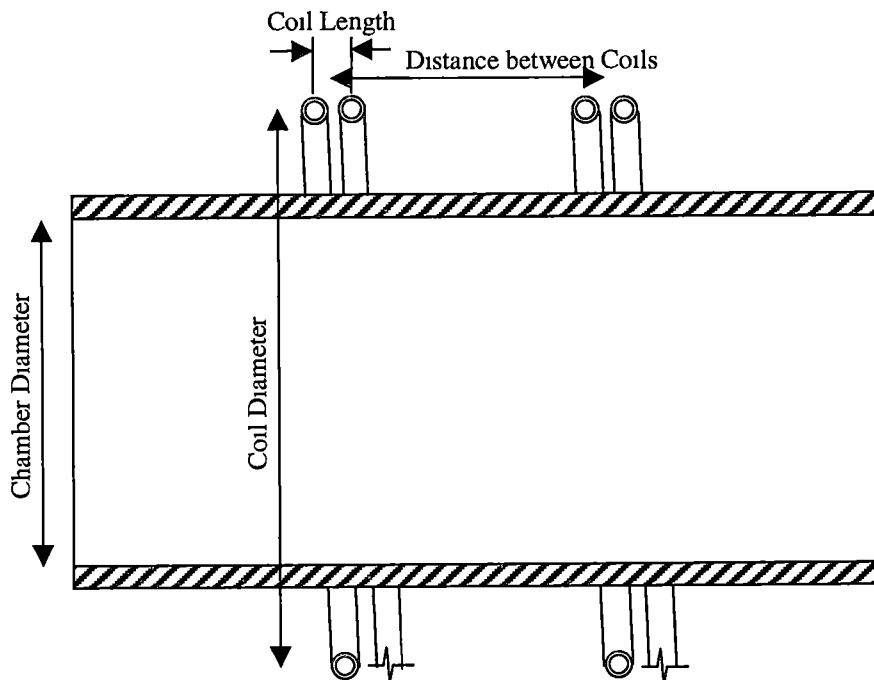


Figure 15 Physical diagram for case C

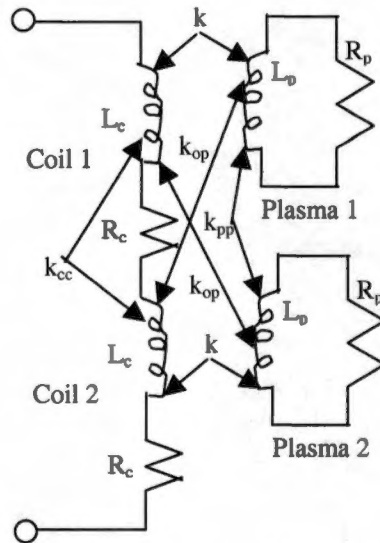


Figure 16. Circuit model developed for the two-coil inductive plasma generator.

Table 6. The results for case C.

Results for Case C	
Inductance of the Coil:	1.2661 μH each
Coupling from Coil to Coil:	0.1873
Resistance of the Coil:	7.5023 $\text{m}\Omega$ each
Temperature:	7000 K
Pressure:	30 Torr
Inductance of the Plasma:	0.0846 μH each
Coupling Coil to Plasma (adjacent):	0.3578
Coupling Coil to Plasma (opposite):	0.1307
Coupling Plasma to Plasma:	0.2266
Resistance of the Plasma:	17.820 $\text{m}\Omega$ each
Length of the Plasma:	2.5732" each
Eff. Diameter of the Plasma:	3.8103"

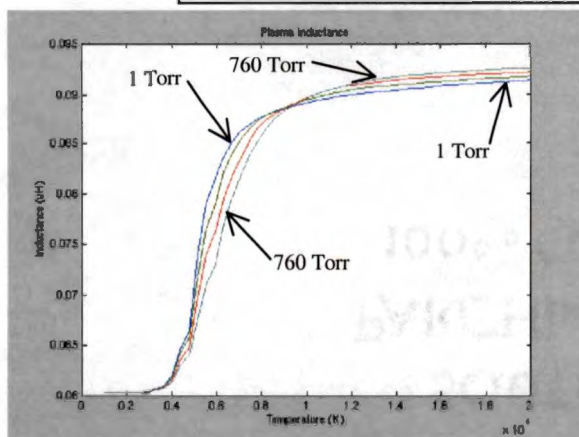


Figure 17. The inductance for case C.

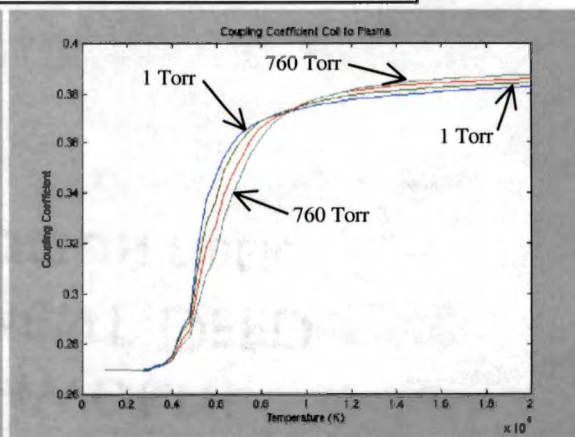


Figure 18. The coupling from the coil to the adjacent plasma for case C.

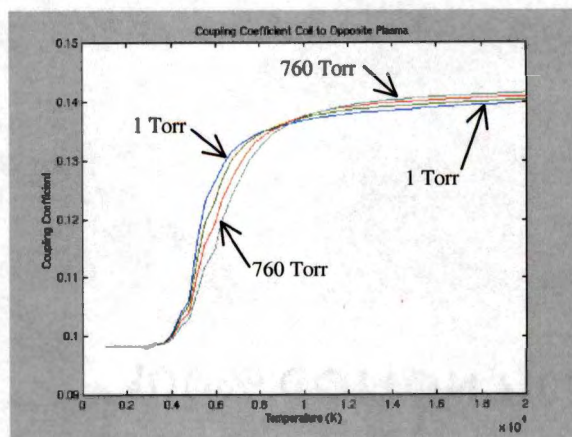


Figure 19. The coupling coefficient from the coil to the opposite plasma for case C.

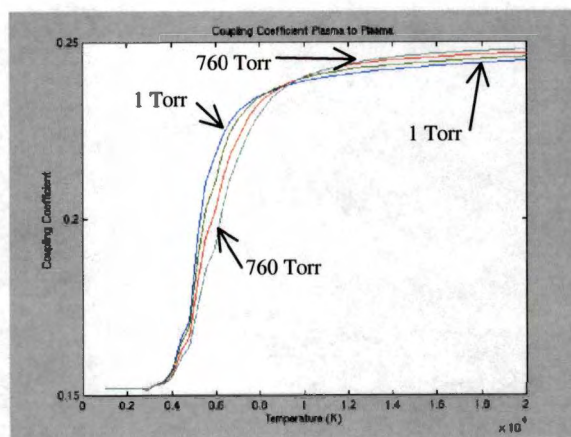


Figure 20. The coupling coefficient from the plasma to the other plasma.

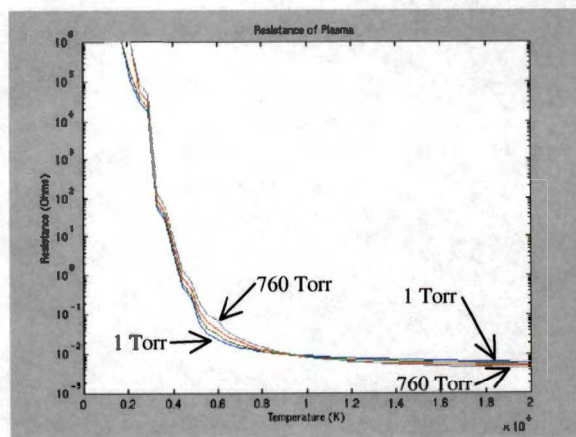


Figure 21. The resistance for case C.

CHAPTER V

CONCLUSIONS

An inductive plasma generator was modeled using an electric circuit model. The model was developed for use in plasma generator and impedance matching network design. The model consists of a drive inductor coil, with its inductance and resistance, coupled to the plasma's inductance. The induced current flows through the plasma's resistance dissipating power.

To perform the calculations a program was written in Matlab. Established formulae for the inductance and coupling were used to calculate these parameters for the coil and the plasma. This program determines the conductivity of the plasma from interpolated tables. Then the program makes use of this information to calculate the effective radius. The conductivity is used in calculating the skin depth, which is used in calculations of the effective radius and the resistance. The program also estimates the length so that with the radius and the length the inductance and coupling coefficient are determined. The resistance is found through the formula developed using the length, skin depth, and radius of the container. The program is applicable for temperatures from 1000 to 20000 and pressures from 1 to 760 Torr.

Results of the inductance, coupling coefficient, and resistance were generated for three representative examples. The results were displayed in tables and figures. This model produces acceptable and effective estimates of the circuit element values in spite of the assumptions and the uncertainty in the plasma conductivity estimate. By examining the results one can see that the skin effect and its effect on the inductance, coupling, and resistance caused the E to H mode transition phenomenon. Which is in good agreement with the observed phenomenon.

For future work the model could be enhanced. One could include the thermodynamics of the plasma generator. One could do this by breaking the plasma into finite elements with each element having a different temperature and therefore conductivity. The conductivity determines the attenuation of the electric field in the plasma. This would account for the effects of the plasma conductivity on the electric

field. The electric field influences the power input, and when the flow of energy out of the element is included a reasonable approximation of the thermodynamic system is set up. One could model the impedance matching network and model the transient behavior of the load for some power input. One could make assumptions closer to reality like the inductance of the plasma not coming from a current sheet but from a current with some profile from the electric field.

Experimental validation could be carried out in the future. This would include measuring the load impedance over a range of operating conditions. From the load one could back out the temperature and conductivity of the plasma. The experiment could also use other measurements like microwave interferometry along the axis of the plasma generator for confirmation of the model's accuracy.

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APPENDICES

APPENDIX A

APPENDIX A

MATLAB CODE

dttotal.m

```

pstart=1,
pfinish=760,
Tstart=1000,
Tfinish=20000,
pdiv=50,
Tdiv=50,

prange=[1 10 100 760]*(101 3e3/760),,[pstart (pfinish-
pstart)/pdiv:pfinish]*(101 3e3/760)
Trange=[Tstart (Tfinish-Tstart)/Tdiv Tfinish],%,

mu0=1 26e-6,
R=8 31,
Na=6 023e23,
V=1;
e=1 60e-19,
me=9 11e-31,
eps0=8 85e-12,
kJ=1 38e-23,
keV=8 62e-5,
hJ=6 63e-34,
heV=4 14e-15,
omega=2*pi*4 5e6,
Ar=40,
mp=1 67e-27,
mi=Ar*mp,

% inductance of the coil all units inches
numturns=6,%1 9
diameter=7 63,%6
lencoil=4 15,% 375
indcoil=inductor(numturns,diameter,lencoil)
%[kcc,mcc]=coupling(indcoil,indcoil,diameter/2,diameter/2,numturns,numturns,lencoil,lenco
il,diameter/2)
%tind=2*indcoil+2*mcc

% plasma properties
od=4,%4 125
doverl=[0 0 08 0 1 0 18 0 2 0 28 0 3 0 38 0 4 0 48 0 5 0 7 0 75 0 95 1 1 2 1 25 1 45 1 5
1 7 1 75 1 95 2 2 4 2 5 2 9 3 3 4 3 5 3 9 4 4 4 4 5 4 9 5 5 8 6 6 8 7 7 8 8 10 14 15 19
20 28 30 50 60 100],
Fvals=[0 001939 002404 004186 004614 006254 006647 008159 008522 009919 01026
01335 01406 01666 01726 01948 02 02193 02238 02408 02448 02599 02635 02898
02957 03177 03228 03417 03461 03626 03665 03811 03846 03978 04009 04240
04293 04489 04534 04705 04745 05097 05633 05743 06116 06197 06732 .0684 0766
0794 0877],
F=spline(doverl,Fvals,diameter/lencoil),

lplasma= 0501376*(diameter/2)/F

% resistance coil res=ro*L/crossectional Area
areacoil=pi*( 125*2 54e-2)^2/4-pi*( 1*2 54e-2)^2/4,
rescoil=(numturns*2*pi*(diameter/2)+2*7)*2 54e-2*1 69e-8/areacoil

countp=1,
for p=prange
    countT=1,
    for T=Trange

```

```

sigmare(countp,countT)=DeVoto(p*(760/101 3e3),T),

% effective radius
khat=sqrt(complex(omega^2*mu0*eps0,omega*mu0*sigmare(countp,countT))),
exponent=imag(khat),
Radius=(od/2)*2 54e-2,

if exponent*Radius<1e-3
    r= 75,
else
    r=((exponent*Radius)^3-3*(exponent*Radius)^2+6*(exponent*Radius)-6+6*exp(-
exponent*Radius))/(exponent*Radius*((exponent*Radius)^2-2*(exponent*Radius)+2-2*exp(-
exponent*Radius))),
    end
    rnorm(countp,countT)=r,

% inductance of plasma all units inches
Radop=(od/2),
reff=r*Radop,
plsmind(countp,countT)=inductor(1,2*reff,lplasma),%plinductance(reff,od,lplasma)

% coupling of single plasma to coil

[kcoef(countp,countT),mutual(countp,countT)]=couplingcon(indcoil,plsmind(countp,countT),d
iameter/2,reff,numturns,1,lencoil,lplasma),
% couplig for two coils

[kkpp(countp,countT),mpp(countp,countT)]=coupling(plsmind(countp,countT),plsmind(countp,c
ountT),reff,reff,1,1,lplasma,lplasma,diameter/2),

[kkaj(countp,countT),maj(countp,countT)]=coupling(indcoil,plsmind(countp,countT),diameter
/2,reff,numturns,1,lencoil,lplasma,diameter/2),

% resistance of plasma
Radius=(od/2)*2 54e-2;
Respl(countp,countT)=pi*(4*(exponent*Radius)^3-
6*(exponent*Radius)^2+6*(exponent*Radius)-3+3*exp(-
2*exponent*Radius))/(4*sigmare(countp,countT)*lplasma*((exponent*Radius)-1+exp(-
exponent*Radius))^2),
    if isnan(Respl(countp,countT)) | Respl(countp,countT)<=0 |
    isnan(Respl(countp,countT)) | Respl(countp,countT)>1e6
        Respl(countp,countT)=1e6,
    end

    countT=countT+1,
end
countp=countp+1,
end

figure
%[c10,h10]=contour(Trange,prange/(101 3e3/760),rnorm,[ 75 752 .755 76 8 85 9 95 96
965 97 973 976 978 979 98 981 9819 99 995 9975 999 9995 9997]),
%clabel(c10,h10)
plot(Trange,rnorm(1,:),Trange,rnorm(2,:),Trange,rnorm(3,:),Trange,rnorm(4,))
title('Normalized Radius or Plasma')
xlabel('Temperature (K)')
ylabel('Normalized Radius')
figure
%[c20,h20]=contour(Trange,prange/(101 3e3/760),plsmind,[.0436 044 045 05 .06 07 08
082 .084 086 088 089 09 0903 0907 091 912 0914]),%[ 0193 .020 025 035 037
.038 .039 0395 04 0402 0404 0406 0408]
%clabel(c20,h20)
plot(Trange,plsmind(1,:),Trange,plsmind(2,:),Trange,plsmind(3,:),Trange,plsmind(4,))
title('Plasma Inductance')
xlabel('Temperature (K)')
ylabel('Inductance (uH)')
figure

```

```

%[c21,h21]=contour(Trange,prange/(101 3e3/760),kcoef,[ 3176 32 33 34 35 36 37 38
385 39 391 .392 393 3935 394 395 396 397 398 399 3995 3999]),%[ 3364 34 35
36 37 38 39 391 392 393 394 395 396 397 398 399 3995 3999]
%clabel(c21,h21)
plot(Trange,kcoef(1,:),Trange,kcoef(2,:),Trange,kcoef(3,:),Trange,kcoef(4,))
title('Coupling Coefficient Coil to Plasma')
xlabel('Temperature (K)')
ylabel('Coupling Coefficient')
figure
%[c22,h22]=contour(Trange,prange/(101 3e3/760),Respl,[1e-3 2 3e-3 2 5e-3 3e-3 4e-3 5e-3
6e-3 7e-3 7 5e-3 8e-3 9e-3 1e-2 1 5e-2 2e-2 3e-2 7e-2 1e-1 1 10 100 1e3 1e4 1e5 1e6]),
%clabel(c22,h22)
semilogy(Trange,Respl(1,:),Trange,Respl(2,:),Trange,Respl(3,:),Trange,Respl(4,))
title('Resistance of Plasma')
xlabel('Temperature (K)')
ylabel('Resistance (Ohms)')

% for two coils
%figure
%[c23,h23]=contour(Trange,prange/(101 3e3/760),kpp,[ 2104 213 217 22 .23 24 25 251
.252 253 254 2545 25475 255 2552]),
%clabel(c23,h23)
%plot(Trange,kpp(1,:),Trange,kpp(2,:),Trange,kpp(3,:),Trange,kpp(4,))
%title('Coupling Coefficient Plasma to Plasma')
%xlabel('Temperature (K)')
%ylabel('Coupling Coefficient')
%figure
%[c24,h24]=contour(Trange,prange/(101 3e3/760),kaj,[ 1155 1156 116 12 13 14 .141
142 143 14325 1435 14375 143875 1440]),
%clabel(c24,h24)
%plot(Trange,kaj(1,:),Trange,kaj(2,:),Trange,kaj(3,:),Trange,kaj(4,))
%title('Coupling Coefficient Coil to Opposite Plasma')
%xlabel('Temperature (K)')
%ylabel('Coupling Coefficient')

```

inductor.m

```

function ind=inductor(n,d,l)

doverl=[0 0 08 0 1 0 18 0 2 0 28 0 3 0 38 0 4 0 48 0 5 0 7 0.75 0 95 1 1 2 1 25 1 45 1 5
1 7 1 75 1 95 2 2 4 2 5 2 9 3 3 4 3 5 3 9 4 4 4 4 5 4 9 5 5 8 6 6 8 7 7 8 8 10 14 15 19
20 28 30 50 60 100],
Fvals=[0 001939 002404 004186 004614 006254 006647 008159 008522 009919 .01026
01335 01406 01666 01726 01948 02 02193 02238 02408 02448 02599 02635 02898
02957 03177 03228 03417 03461 03626 03665 03811 03846 03978 04009 .04240
04293 04489 .04534 04705 04745 05097 05633 05743 06116 06197 06732 .0684 0766
0794 0877],

F=spline(doverl,Fvals,d/l),

ind=F*n^2*d,

```

coupling m

```

function [k,Mut]=coupling(LA,La,A,a,n1,n2,lenX,lenl,D)
X=lenX/2, l=lenl/2,
x1=D-X, x2=D+X,
r1=sqrt(x1^2+A^2),
r2=sqrt(x2^2+A^2),

K1=(2/A^2)*((x2/r2)-(x1/r1)),
k1=2*l,

K3=(1/2)*((x1/r1^5)-(x2/r2^5)),
k3=a^2*l*(3-(4*l^2/a^2));

K5=(-A^2/8)*((x1/r1^9)*(3-(4*x1^2/A^2))-(x2/r2^9)*(3-(4*x2^2/A^2))),
k5=(a^4*l)*((5/2)-10*(l^2/a^2)+4*(l^2/a^4)),

```

```
Mut=0.02505*((a^2*A^2*n1*n2)/(4*l*X))*(K1*k1+K3*k3+K5*k5),
```

```
k=Mut/sqrt(LA*La),
```

couplingcon.m

```
function [k,Mut]=couplingcon(LA,La,A,a,n1,n2,lenX,lenl)
```

```
X=lenX/2, l=lenl/2,
```

```
g=sqrt(A^2+l^2);
```

```
Mut=0.0501*((a^2*n1*n2)/g)*(1+((A^2*a^2)/(8*g^4))*(3-4*(X^2/a^2))),
```

```
k=Mut/sqrt(LA*La),
```

DeVoto.m

```
function sig=DeVoto(p,T)
```

```
% one atm
```

```
t=[1000 2000 3000 4000 5000 6000 7000 8000 9000 10000 11000 12000 13000 14000 15000 16000  
17000 18000 19000 20000],
```

```
con=[3.9216e-23 1.6592e-11 5.4577e-5 6.2885e-2 5.1615e0 7.6821e1 3.508e2 9.9034e2  
1.9358e3 2.990e3 4.0544e3 5.1049e3 6.1243e3 7.0844e3 7.9561e3 8.7361e3 9.447e3 1.0111e4  
1.0728e4 1.1275e4],
```

```
pressure=[1,10,100,760], %pressure in torr
```

```
temperature=[1000 100 20000],
```

```
[temp,press]=meshgrid(temperature,pressure),
```

```
c1=interp1(t,[13e-25 12e-13 15e-7 15e-4 15e-2 4 25 10 5 16 5 21 26 30 34 37 40 43 47 50  
53 56 60],temperature,'linear'),
```

```
c2=interp1(t,[10e-25 8e-13 12e-7 12e-4 12e-2 2 55 8 5 16 22 29 35 40 5 45 49 53 57 61 65  
69 73],temperature,'linear'),
```

```
c3=interp1(t,[7e-25 5e-13 8e-7 9e-4 8e-2 1 5 5.3 13 5 21 30 5 40 49 55 62 67 73 77 82 87  
90],temperature,'linear'),
```

```
connew=interp1(t,con,temperature,'linear'),
```

```
cond=[c1,c2,c3;connew/1e2],
```

```
sig=interp2(temp,press,cond*1e2,T,p,'linear'),
```

APPENDIX B

DE VOTO CONDUCTIVITY

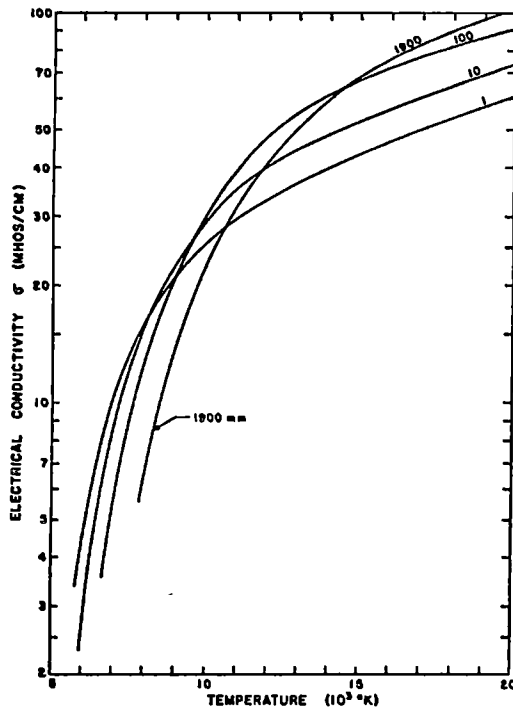


Figure 22 Electrical conductivity of equilibrium Argon from De Voto

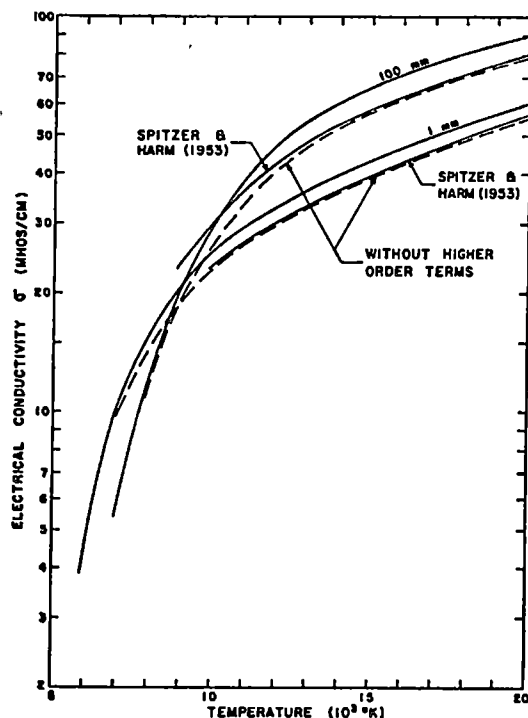


Figure 23 Comparison of De Voto's conductivity and Spitzer's conductivity

Table 7 De Voto's conductivity at atmospheric pressure

T (K)	Density (kg/m ³)	Enthalpy (J/kg)	Sp heat (J/kg K)	Viscosity (Pa·m/s)	Therm cond (W/m K)	Elect cond (A/V m)
ARGON						
8000	4.843E-02	5.563E+00	1.351E+03	2.472E+04	5.474E-01	2.774E+03
8000	4.827E-02	5.561E+00	1.3577E+03	2.4836E-04	6.148E-01	2.624E+03
10000	7.698E-02	9.186E+00	1.3972E+03	2.5855E-04	6.443E-01	3.902E+03
10100	7.715E-02	9.189E+00	1.3924E+03	2.7401E-04	6.740E-01	3.306E+03
10200	6.934E-02	8.182E+00	1.3791E-04	2.758E-05	6.750E-01	3.200E+03
10300	6.151E-02	7.175E+00	1.3744E+03	2.5884E-05	3.200E+03	
10400	5.402E-02	6.168E+00	1.3618E+03	2.7535E-04	7.750E-01	3.415E+03
10500	4.682E-02	5.162E+00	1.3480E+03	2.7431E-04	6.075E-01	3.232E+03
10600	4.017E-02	4.155E+00	1.3347E+03	2.7441E-04	8.805E-01	3.423E+03
10700	3.370E-02	3.1484E+00	1.3191E+03	2.7470E-04	8.200E-01	3.422E+03
10800	2.737E-02	2.1400E+00	1.3027E-04	2.7472E-04	8.200E-01	3.422E+03
10900	2.120E-02	1.1328E+00	1.2850E+03	2.7419E-04	8.095E-01	3.404E+03
11000	1.510E-02	2.025E+00	1.2673E+03	2.7341E-04	7.980E-01	3.386E+03
11100	1.434E-02	1.0170E-02	1.2478E+03	2.7255E+00	7.865E+00	3.368E+03
11200	1.084E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
11300	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
11400	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
11500	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
11600	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
11700	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
11800	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
11900	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12000	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12100	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12200	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12300	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12400	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12500	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12600	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12700	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12800	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
12900	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
13000	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
13100	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
13200	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
13300	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
13400	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
13500	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03
13600	1.081E-02	4.587E+00	1.2321E+03	2.7155E-04	7.840E+00	3.350E+03

(continued)

VITA

My name is Nathan Waivio. I got my Bachelors Degree at Michigan Technological University, in Mechanical Engineering. I am 25 and I am married. May GOD bless you and keep you.