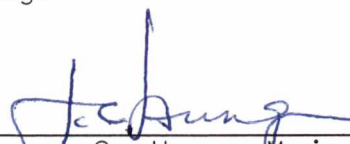


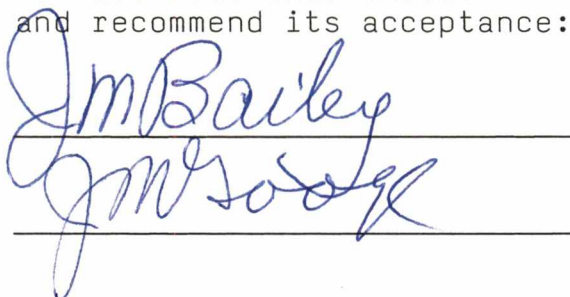
To the Graduate Council:

I am submitting herewith a thesis written by George Shu-xing Cheng entitled "An Automated Least-Squares Parameter Identification Technique and Optimal PID Control." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.



James C. Hung, Major Professor

We have read this thesis
and recommend its acceptance:



Accepted for the Council:



The Graduate School

AN AUTOMATED LEAST-SQUARES PARAMETER
IDENTIFICATION TECHNIQUE AND
OPTIMAL PID CONTROL

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

George Shu-xing Cheng

March 1985

DEDICATED TO
my parents, Dr. and Mrs.
Sheo-chu Cheng
for your encouragement and help
in token of the love I have for you

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ABSTRACT

This thesis presents the result of a study on the PID control theory and application. A so-called automated least-squares parameter identification algorithm is developed and implemented with a HP desktop computer. This technique can automatically model most stable SISO processes in the first-order-lag-plus-delay (FOLPD) or first-order-lag (FOL) form based on the least-squares curve-fitting criterion. The FOL and FOLPD model related PID controller tuning techniques are reviewed and improved based on the commonly preferred control performance specification: the 0.707 damping ratio criterion. Simulations of tuned PID control systems relating to the optimal models and controller settings are performed. Very interesting and encouraging results are obtained.

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CHAPTER 1

INTRODUCTION

In control system applications, one looks for a low cost, efficient, and reliable controller. The Proportional-Integral-Derivative controller (PID controller) meets these requirements and is most widely used in industrial systems. It is simple so that it can be implemented in both analog circuits and in microprocessors. It offers great flexibility by allowing tuning of its parameters over a wide range. Most importantly, it is able to control various kinds of high-order single-input-single-output (SISO) systems with good transient response. These may be the reasons that the PID controller is still very popular even though many powerful advanced control techniques have been developed.

Conventional procedures in PID control application include: 1) parameter identification for the processes, and then 2) design or tuning of controller based on the obtained approximation models.

The most common approximation model used in PID control is that of first-order lag plus a pure time delay. It will be called first-order-lag-plus-delay model (FOLPD model) in this thesis. Experiments indicate that this is one of the simplest approximation models for high-order SISO processes and is sufficiently accurate for most processes, although the

second-order-lag-plus-delay (SOLPD) model may give a better approximation.

The pioneers in the PID control field are Ziegler and Nichols who proposed a graphical parameter identification method for the FOLPD model in 1942 [1]. This method later was improved by Miller [2] and by Smith [3]. They are called graphical methods since they determine the parameters of the model from the process open-loop step response curve.

The first FOLPD model related PID controller tuning formula was also proposed by Ziegler and Nichols [1]. Later versions were introduced by Cohen and Coon [9], Smith and Murrill [10], Lopez [11], Rovira [13], and Shinskey [5]. They are based on different control performance specifications which include: 1) quarter decay ratio criterion [1][9][10]; 2) minimum error integrals criteria [11][13]; and 3) integrated error criterion [5].

The graphical parameter identification methods for the SOLPD model [21][22][23][24] and the SOLPD model related PID controller tuning techniques [25][26] are also available. But they are seldom used due to their complexity and inconvenience.

The PID controller can also be tuned on-line based on the process closed-loop step response [1][27]. In such case, the process approximation model is not required. However, this method has the following disadvantages: 1) it can cause sustained oscillations of the closed-loop system which is not

acceptable for most applications; 2) it works only for some processes; and 3) the control performance specification used is the quarter decay ratio criterion which is not a good criterion for high-order processes. Therefore, the method is not widely used.

A current article [16] introduced a PID controller self-tuning method using a pattern recognition approach. It begins with tuning the PID controller in the closed-loop, then observing the pattern of the response and finally comparing the actual pattern with one that is desired. The controller parameters then can be adjusted accordingly until these two patterns agree closely. However, this method may have the disadvantages mentioned in the last paragraph, since the controller is tuned based on the closed-loop response.

The linear quadratic (LQ) control law of optimal control theory was used to design the PID controller by Zhang [17]. The difficulties of this method include: 1) process models must be obtained by using optimal estimation algorithms; 2) for different models, different computation algorithms for control law must be derived; 3) the weighting matrices in quadratic form are not easy to select; and 4) control performances may be very sensitive to the process models and the weighting matrices. In fact, even though modern optimal control theory has achieved great successes in aerospace and military applications, there is no merit to apply the theory to most conventional industrial systems such as the PID

control system, since this can make simple problems more complicated.

Although various solutions have been proposed over several decades, the PID control theory and application are far from sophisticated; and more effort can still be made towards the goal of making the PID control better in performance and ease of use.

The objectives of this thesis include: 1) to develop an automated parameter identification technique which can allow most high-order SISO processes to be approximated by a FOLPD model in some optimal sense; 2) to improve PID controller tuning techniques so that the control performances can meet more adequate and strict specifications; 3) to simulate PID control systems so that the performances of the optimal FOLPD model and tuning techniques can be evaluated; and 4) to point out the possibility of the development of a new computer-microprocessor-based self-tuning control system.

The remaining chapters are organized as follows:

Chapter 2 discusses the graphical parameter identification methods in somewhat more depth.

Chapter 3 introduces a new automated parameter identification technique for modeling high-order processes in FOLPD form using the least-squares curve-fitting criterion.

Chapter 4 reviews the concepts of the PID controller and its tuning techniques relating to the FOLPD model. A new PI

controller tuning formula relating to the first-order-lag model (FOL model) is derived for a damping ratio of 0.707.

Chapter 5 improves the FOLPD model related PID controller tuning techniques based on the 0.707 damping ratio criterion. The simulation results for five experimental processes are presented.

Chapter 6 concludes the study and recommends a new configuration of a computer-microprocessor-based self-tuning control system.

All computer programs are written in Basic language and some of them are included in the Appendices.

CHAPTER 2

REVIEW OF GRAPHICAL PARAMETER IDENTIFICATION TECHNIQUES

1. PRELIMINARY REMARKS

The first-order-lag-plus-delay model is usually represented by the Laplace transfer function as

$$G_m(s) = \frac{Ke^{-\tau s}}{Ts + 1}, \quad (2-1)$$

where K is the process DC gain, τ the delay time, and T the time constant. Note that the FOLPD reduces to first-order-lag (FOL) when delay time is zero. The FOL model is also useful in situations when processes possess a quick transient response.

In practice, the parameters of the model (2-1) are frequently obtained from the step response curve. For example, consider the open-loop system as shown in Figure 2-1. The unit step response of the process can be recorded as shown in

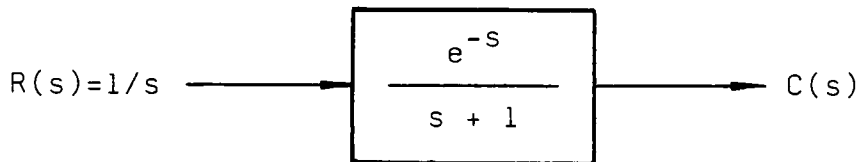


Figure 2-1. Block Diagram of an Open-Loop System

Figure 2-2 when a unit step signal is applied to the process. In the Figure, the step response attains 63.2 percent of its final value in one time constant ($t=T$), and a straight line drawn tangent to the curve at its point of maximum rate of response intersects the 100 percent response line at time equal to the delay time plus the time constant ($t=\tau+T$). These are important properties of the open-loop step response curve of the FOLPD process. The following graphical parameter identification methods are based on these facts.

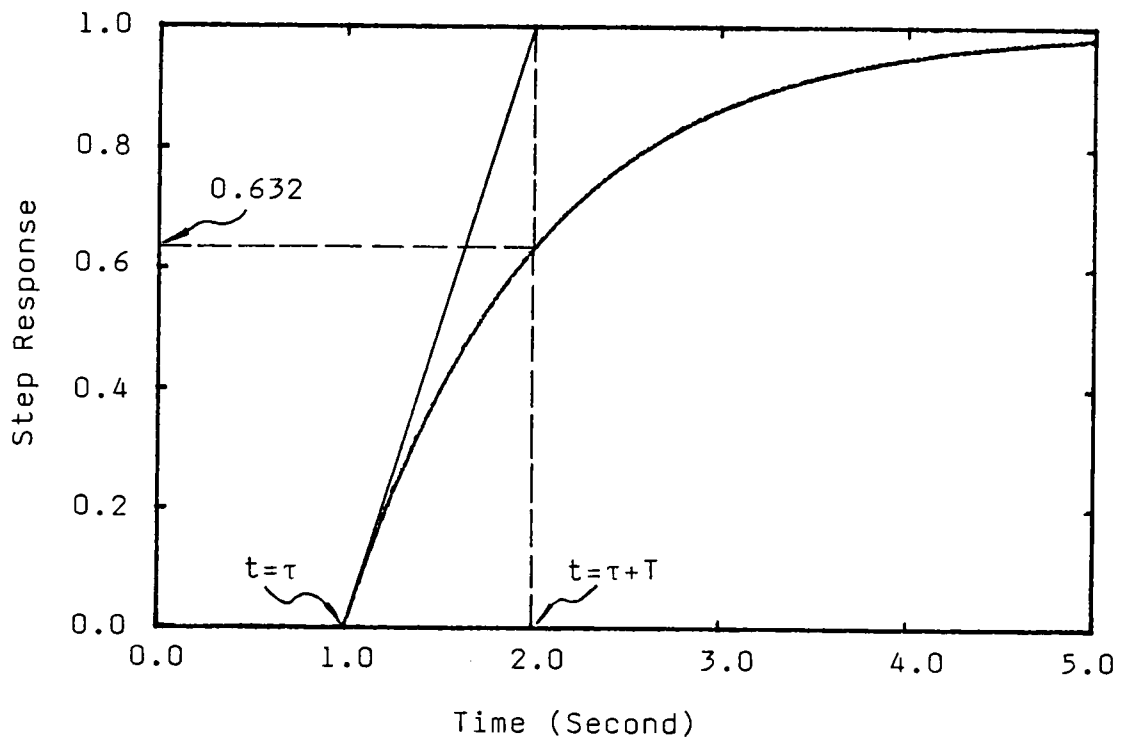


Figure 2-2. Step Response of a FOLPD Process

2. THE GRAPHICAL PARAMETER IDENTIFICATION METHODS FOR FOLPD AND FOL MODEL

An illustration of the graphical parameter identification methods is shown in Figure 2-3 for the high-order process:

$$G_p(s) = \frac{2}{(s + 1)^2(s + 2)(s + 0.5)} \quad (2-2)$$

Fit 1 Model Method

Proposed by Ziegler and Nichols in 1942 [1], this is a well known method. It can be explained in the following steps with the help of Figure 2-3.

1. The process DC gain is first obtained as

$$K = c(\infty) - c(0) , \quad (2-3)$$

where $c(\infty)$ is the steady-state of the unit step response $c(t)$.

2. Draw a straight line tangent to the curve at its inflection point, which is also the point of maximum rate of response.
3. Measure the values of parameters τ_1 and T_1 shown in the Figure.

Although the third step is rather simple, it is always difficult to determine the inflection point in the second step. Therefore, a considerable amount of error will exist.

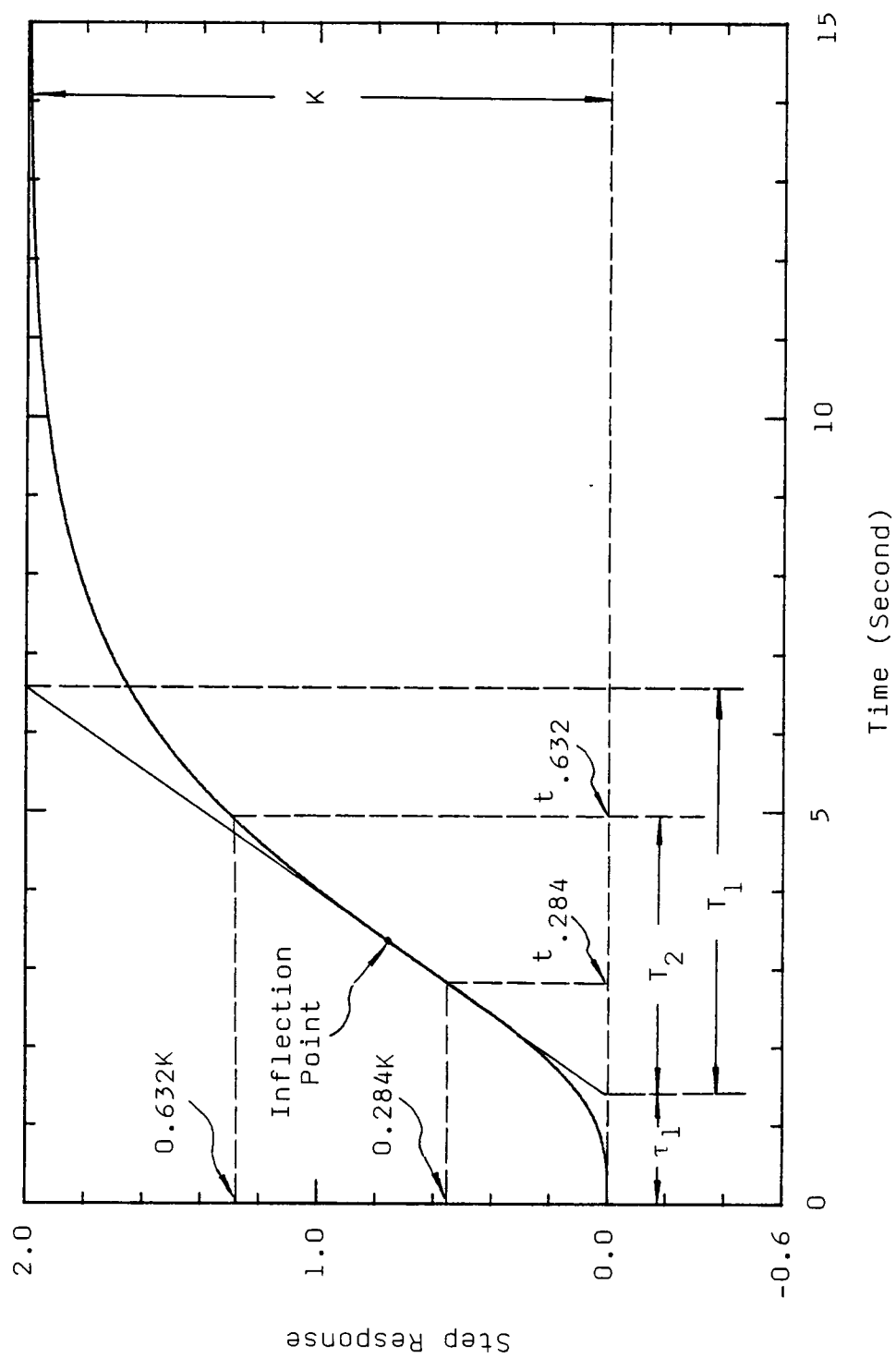


Figure 2-3. Illustration of Fit 1, Fit 2, Fit 3, and FOL Model Method

Fit 2 Model Method

Using the property that the step response of FOLPD process attains 63.2 percent of the final value at $t=\tau+T$, Miller [2] suggested a modified formula based on the Fit 1 model method. As shown in Figure 2-3, the value of $0.632K$ and the corresponding time, $t_{0.632}$, can be found. Then,

$$\tau_2 = \tau_1 \quad (2-4)$$

$$T_2 = t_{0.632} - \tau_2 \quad (2-5)$$

Fit 3 Model Method

Overcoming the disadvantage of drawing the tangent line, this method prefers calculation procedures from the step response of the FOLPD process, which can be represented as

$$c(t) = K[1 - e^{-(1/T)(t-\tau)}]u(t-\tau) \quad (2-6)$$

Introduced by Smith [3], the method starts with the calculation of $c(t)$ at two times, say $\tau+T/3$, and $\tau+T$:

$$c(\tau + T/3) = 0.284K \quad (2-7)$$

$$c(\tau + T) = 0.632K \quad (2-8)$$

Then, the times when the step response attains 28.4 percent and 63.2 percent of the final value can be read from the process step response curve, as illustrated in Figure 2-3:

$$t_{0.284} = 2.88 = \tau + T/3 \quad (2-9)$$

$$t_{0.632} = 4.80 = \tau + T . \quad (2-10)$$

Solve the simultaneous equations (2-9) and (2-10) for results:

$$\tau_3 = 1.92$$

$$T_3 = 2.88 .$$

FOL Model Method

Only two parameters, K and T, need to be known in this case. The DC gain can be found from Equation (2-3), and

$$T = t_{0.632} . \quad (2-11)$$

It must be pointed out that this model is good only for those processes with the maximum rate point of response located at $t=0$. This kind of process must have a quick transient response and need not to be modeled with a pure delay time.

3. RESULTS AND DISCUSSIONS

Results

The parameters obtained from these four graphical identification methods for the process (2-2) are listed in Table 2-1, and the corresponding step response curves are plotted in Figure 2-4. It can be seen that Fit 3 is the best model of the four. Fit 1 and Fit 2 are poor since drawing the tangent line is not a good idea; and FOL is poor since the maximum rate point of response is not located at $t=0$.

TABLE 2-1
COMPARISON OF FIT 1, FIT 2, FIT 3, AND FOL MODEL

Model	DC Gain K	Time Constant T	Delay Time τ
Fit 1	2.0	4.98	1.46
Fit 2	2.0	3.34	1.46
Fit 3	2.0	2.88	1.92
FOL	2.0	4.80	0

Discussions

To evaluate the graphical parameter identification methods, it can be concluded that the advantages of these methods are: 1) the principle is easy to understand; and 2) the manipulation is convenient as long as the process step response curve is given. However, their disadvantages include: 1) inaccuracy; 2) being invalid if the step response is contaminated with noise; and 3) being sensitive to human errors.

In graphical identification methods, accuracy is poor since the methods are only based on some properties of the step response curve and have nothing to do with any optimal sense. The methods cannot work in the noise-contaminated

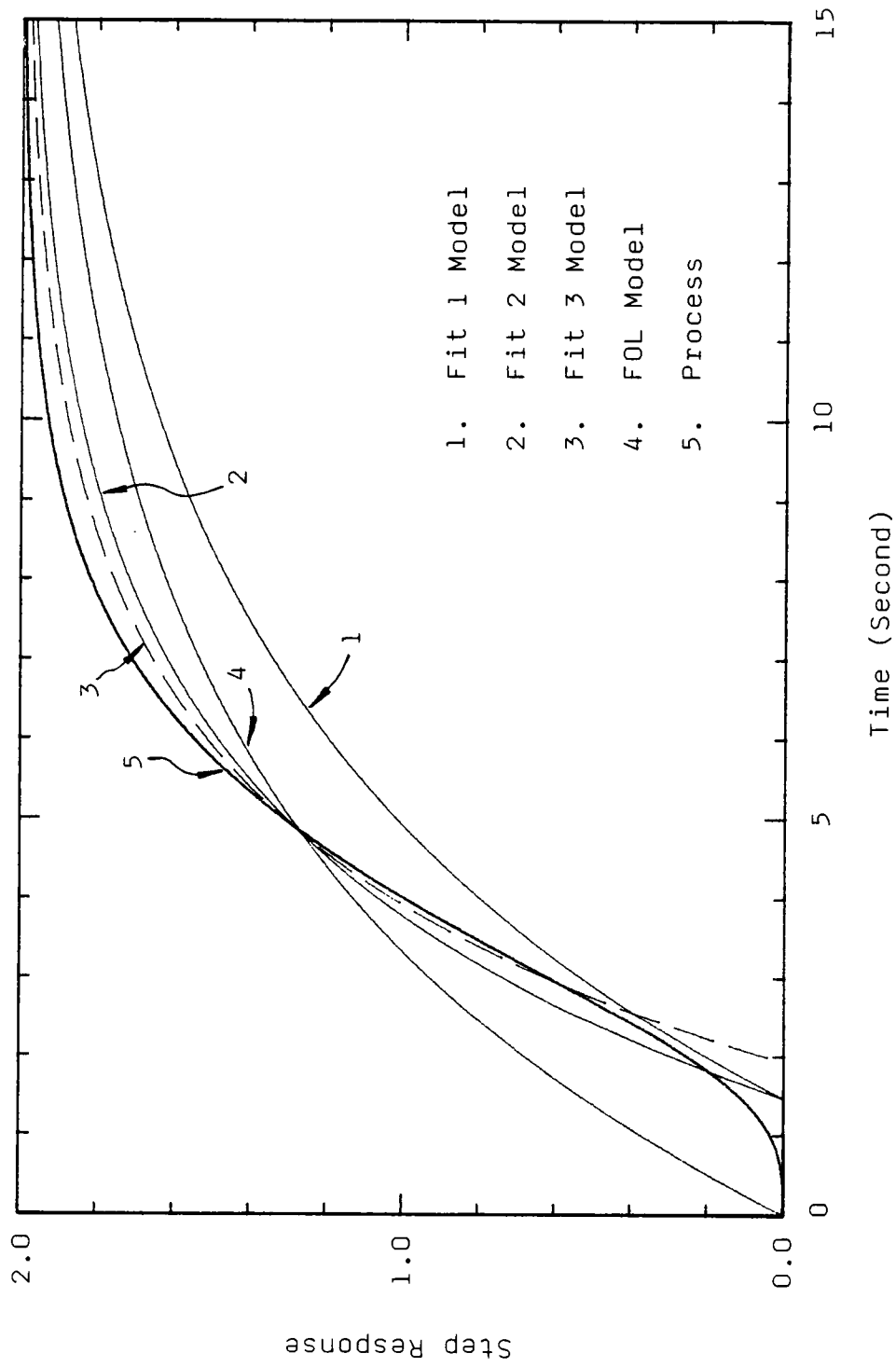


Figure 2-4. Step Response Comparison of Fit 1, Fit 2, Fit 3, and FOL Model.

case since the manipulation can be performed only on a smooth step response curve. The parameters of the model are sensitive to human errors since the procedures of drawing lines and reading data from graphs involve personal interpretation.

As an example, Table 2-2 compares two groups of data for Fit 3 model of process (2-2). Group 1 is from reference [3], and group 2 was derived by this writer. The relative errors for corresponding parameters are not small.

TABLE 2-2
AN EXAMPLE OF HUMAN ERROR SENSITIVITY

Parameter	Data Group 1	Data Group 2	Relative Error
DC Gain K	2.0	2.0	0%
Time Constant T	2.88	3.30	12.7%
Delay Time τ	1.92	1.65	16.4%

CHAPTER 3

THE AUTOMATED LEAST-SQUARES PARAMETER IDENTIFICATION TECHNIQUE

1. PRELIMINARY REMARKS

In the last chapter, graphical parameter identification methods were reviewed. For further studies, it is important to consider the following questions:

1. Is it possible to approximate most SISO high-order processes by the FOLPD or FOL model?
2. If yes, could the parameters of the model be determined in some optimal sense?
3. Could the model be used in practice?

This chapter answers the above questions through developing a so-called automated least-squares parameter identification technique. The principles of the algorithm and the implementation procedures are introduced. Results are given and computer software is attached in the Appendices.

2. BASIC CONCEPTS OF MODERN SYSTEM IDENTIFICATION

The graphical methods discussed earlier are one type of classical methods of system identification. They are called classical in the sense that they have been around for a longer time than the so-called modern methods. With the advent of digital computers, a large variety of modern identification methods have been proposed, both off-line and on-line.

An identification scheme is said to be of the off-line type when one collects a large amount of system input and output data which may be stored in a computer. These data are then processed in a batch to estimate the parameters of the model and obtain the best fit according to a prescribed criterion. An identification method is said to be of the on-line type if it satisfies the following conditions: 1) it does not require a special input; 2) all the data need not be stored; 3) a recursive algorithm is used for adjusting the estimates of the parameter after each sampling instant; and 4) the amount of computation required for "model adjustment" is a fraction of the sampling interval [6].

A so-called model adjustment scheme is widely used in modern system identification. As illustrated in Figure 3-1,

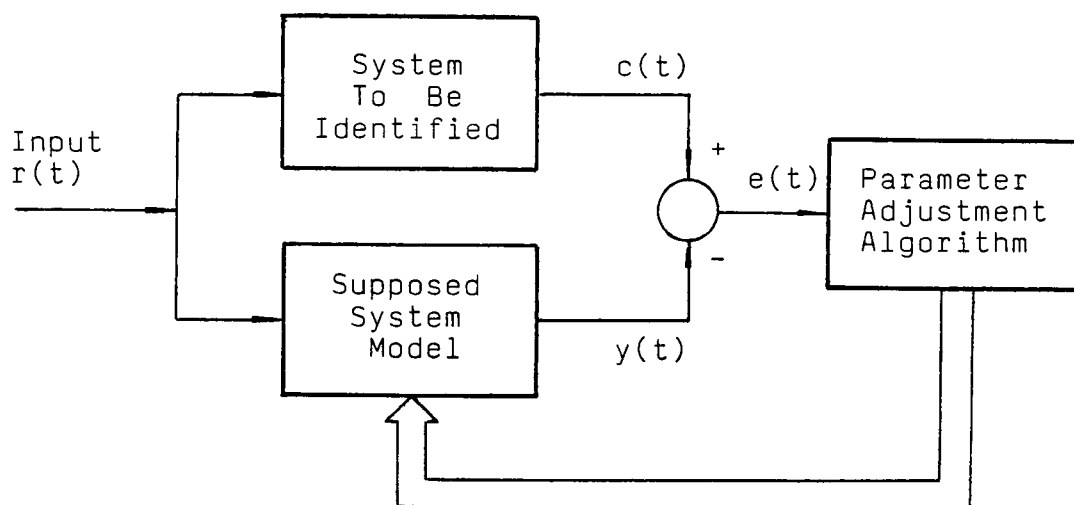


Figure 3-1. Block Diagram of the Model Adjustment Scheme

the technique is based on using a model subject to the same input as the system, and then adjusting the parameters of the model so that the error between the output of the system and that of the model is minimized according to a given criterion. This procedure can be used in on-line identification if the input and output are sampled, and the computation for model adjustment is carried out recursively during each sampling interval [6].

Although a number of modern identification methods have been proposed during the past twenty years, few of them can be used in parameter identification for the FOLPD model. The difficulty of the problem is that the delay time of the FOLPD model is hard to treat. In reference [3], a discrete-time parameter identification algorithm for the FOLPD model is proposed. However, the delay time of the model must be assumed as an integer multiple of the sampling interval. That is, $\tau = nT_s$, where n is an integer, and T_s is the sampling interval. This is indeed a disadvantage of the algorithm since delay time in a model can be any fractional number. In addition, the algorithm is of theoretical interest only since it is not derived in the noise-contaminated measurement case.

An automated least-squares parameter identification algorithm is introduced in the next section. It is a continuous time parameter identification method for the FOLPD model with off-line type, model-adjustment scheme, and least-squares criterion.

3. INTRODUCTION TO THE AUTOMATED LEAST-SQUARES PARAMETER IDENTIFICATION ALGORITHM

The algorithm can be introduced step by step as follows:

1. Suppose the process unit step response $c(t)$ has been recorded at each sampling instant and stored in a computer. The DC gain of FOLPD model (2-1) can be determined by

$$K = c(\infty) - c(0) . \quad (3-1)$$

Although $c(\infty)$ is the process steady-state response which is theoretically unreachable, one value of $c(t)$ can be chosen to approximate $c(\infty)$.

A detailed discussion is presented in section 4.

2. Consider the open-loop step response of the FOLPD model:

$$y(t) = \begin{cases} 0 & \text{when } t < \tau \\ K[1 - e^{-A(t-\tau)}] & \text{when } t \geq \tau , \end{cases} \quad (3-2a)$$

$$(3-2b)$$

where A is the inverse of time constant T .

Substitute the model step response $y(t)$ by the process step response $c(t)$ in Equation (3-2b).

It follows that

$$c(t) = K[1 - e^{-A(t-\tau)}] . \quad (3-3)$$

Rearrange the equation and introduce a function $z(t)$. It becomes

$$z(t) = \ln[1 - c(t)/K] = -A(t - \tau) , \quad (3-4)$$

where $z(t)$ can be determined since $c(t)$ and K are known. By this step, the model equation reduces to a linear equation with two unknowns, A and τ .

3. Assume an initial value for delay time, say $\tau = \tau_1$. Then Equation (3-4) becomes

$$z(t) = -A_1(t - \tau_1) , \quad (3-5)$$

where A_1 denotes the value of A corresponding to τ_1 , which is then the only unknown of the equation.

4. To fit the process step response curve, arrange Equation (3-5) into a matrix form with respect to each sampling instant denoted by t_k :

$$\underline{Z} = \underline{H}A_1 , \quad (3-6)$$

where

$$\underline{Z} = \begin{bmatrix} z(t_1) \\ z(t_2) \\ \vdots \\ z(t_n) \end{bmatrix} \quad (3-7)$$

$$\underline{H} = \begin{bmatrix} \tau_1 - t_1 \\ \tau_1 - t_2 \\ \vdots \\ \tau_1 - t_n \end{bmatrix}, \quad (3-8)$$

and n is the number of sampling instants.

Then, the solution of A_1 can be found according to the least-squares criterion:

$$A_1 = [\underline{H}^T \underline{H}]^{-1} \underline{H}^T \underline{Z}, \quad (3-9)$$

where $\underline{H}^T$ is the transpose of matrix $\underline{H}$, and $[\underline{H}^T \underline{H}]^{-1} \underline{H}^T$ is usually called the pseudo-inverse of matrix $\underline{H}$.

5. Since A_1 and τ_1 may not be the best pair of the parameters, more computation is needed. Thus, determine the least-squares error:

$$E_1 = \sum_{k=1}^n [c(t_k) - y_1(t_k)]^2, \quad (3-10)$$

where $y_1(t)$ is the model step response with parameters K , A_1 , and τ_1 .

6. Repeat steps 3 to 5 by assuming different values of delay time $\tau_2, \tau_3, \dots, \tau_m$, and obtain the corresponding values of $A_2, A_3, \dots, A_m$, and $E_2, E_3, \dots, E_m$. Then, by comparing the least-

squares errors, select the smallest error E and its corresponding A and τ as the final results.

The proposed method is a semi-trial and error method. It must be pointed out that: 1) the way to choose assumed values of delay time, say τ_1 to τ_m , is very important; and 2) different curve-fitting ranges¹ may cause different identification results. These problems are solved and will be discussed in section 4.

4. IMPLEMENTATION OF THE ALGORITHM

The Computer Used for Implementation

The algorithm introduced in the last section was implemented by using a HP System 9835 Desktop Computer. This is a high speed computer with 256K bytes internal Read/Write memory. The program is written in the Basic language, and many array operations are provided.

A Way To Find Process DC Gain On-Line

As mentioned in step 1 of section 3, the process DC gain can be determined by $K=c(\infty)-c(0)$. But, a value of $c(t)$, say $c(t_f)$, must be chosen to approximate $c(\infty)$, where t_f is termed final time. Since different processes can have widely varying settling times, the problem then is how to find $c(t_f)$ in a general way. A possible solution is as the following:

¹The definition of curve-fitting range is given in section 4.

1. When sampling the process step response $c(t)$, find the slope of the response curve at each sampling instant:

$$\text{slope at point } k = \frac{c_k - c_{k-1}}{T_s}, \quad (3-11)$$

where c_k denotes the $c(t)$ at sampling instant t_k , and T_s is the sampling interval. (See Figure 3-2)

2. Compare the difference of two slopes with an index of precision, which can be chosen arbitrarily. In other words, test the change of the slopes of $c(t)$ after each sampling instant until it is smaller than the index s_p :

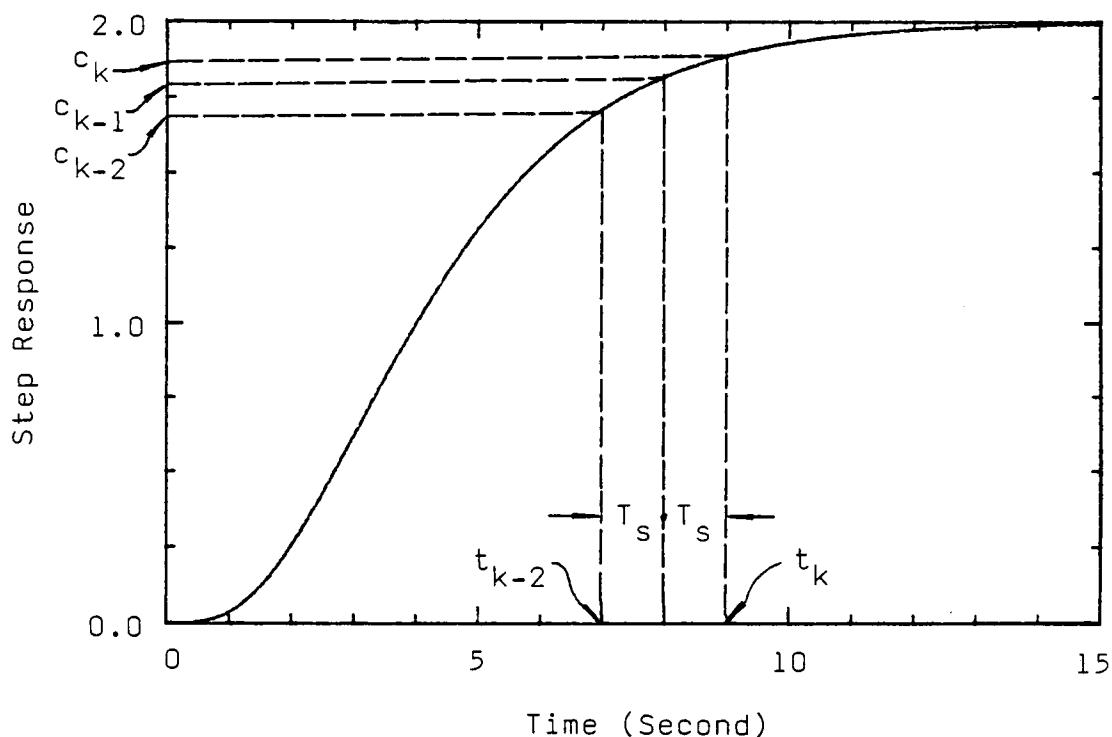


Figure 3-2. Illustration of Slopes of Step Response Curve

$$\left| \frac{c_k - c_{k-1}}{T_s} - \frac{c_{k-1} - c_{k-2}}{T_s} \right| < s_p = 1 \times 10^{-15} . \quad (3-12)$$

Then, choose $c_k = c(t_f) = c(\infty)$.

Note that the sampling process is ended at the final time t_f ; however, only those sampling data that will be used in computation need to be stored.

The experimental computer program indicates that good precision can be achieved in this way. For some processes, DC gain can be obtained without error. Meanwhile, the final times are not large, although they depend on different processes. The experiment results are shown in Table 3-1.

A Way To Assume Delay Time In the Algorithm

One of the remaining questions from the last section is how to assume initial values of delay time in the algorithm so that the program can get suitable results as quickly as possible.

Different approaches were tried, and a best one is introduced as follows, referring to Figure 3-3:

1. Find two values of process step response which are denoted by c_4 and c_6 :

$$c_4 = 0.4K \quad (3-13)$$

$$c_6 = 0.6K . \quad (3-14)$$

Also find the corresponding times t_4 and t_6 .

TABLE 3-1

COMPARISON OF TRUE DC GAIN, IDENTIFIED DC GAIN, AND FINAL TIME FOR FIVE EXPERIMENTAL PROCESSES

NO.	Process	True DC Gain	Identified DC Gain	Relative Error	Final Time t_f (Second)
1.	$\frac{2e^{-s}}{s+1}$	2	2.0	0%	30
2.	$\frac{2}{(s+1)^2(s+2)(s+0.5)}$	2	1.99999...	$5 \times 10^{-12}\%$	53
3.	$\frac{60}{(s+1)(s+2)(s+3)(s+4)}$	2.5	2.5	0%	30
4.	$\frac{5(s+2)(s+4)}{(s+1)(s+3)(s+5)}$	$8/3$	2.66666...	$2.25 \times 10^{-12}\%$	27
5.	$\frac{500(s+4)}{(s+1)(s+10)(s+100)}$	2	2.0	0%	29

2. Choose the first three values of delay time:

$$\tau(1) = 3t_4 - 2t_6 \quad (3-15)$$

$$\tau(2) = \tau(1) + 20D \quad (3-16)$$

$$\tau(3) = \tau(1) + 10D, \quad (3-17)$$

where $D=0.1\tau(1)$ is a constant. In this way, the algorithm can be used in general situations when delay times are different from process to process. Also note that $\tau(3)$ is the middle point of the three.

3. Compare corresponding squares errors $E(1)$, $E(2)$, and $E(3)$ which are obtained through the least-squares computation. Choose one value out of

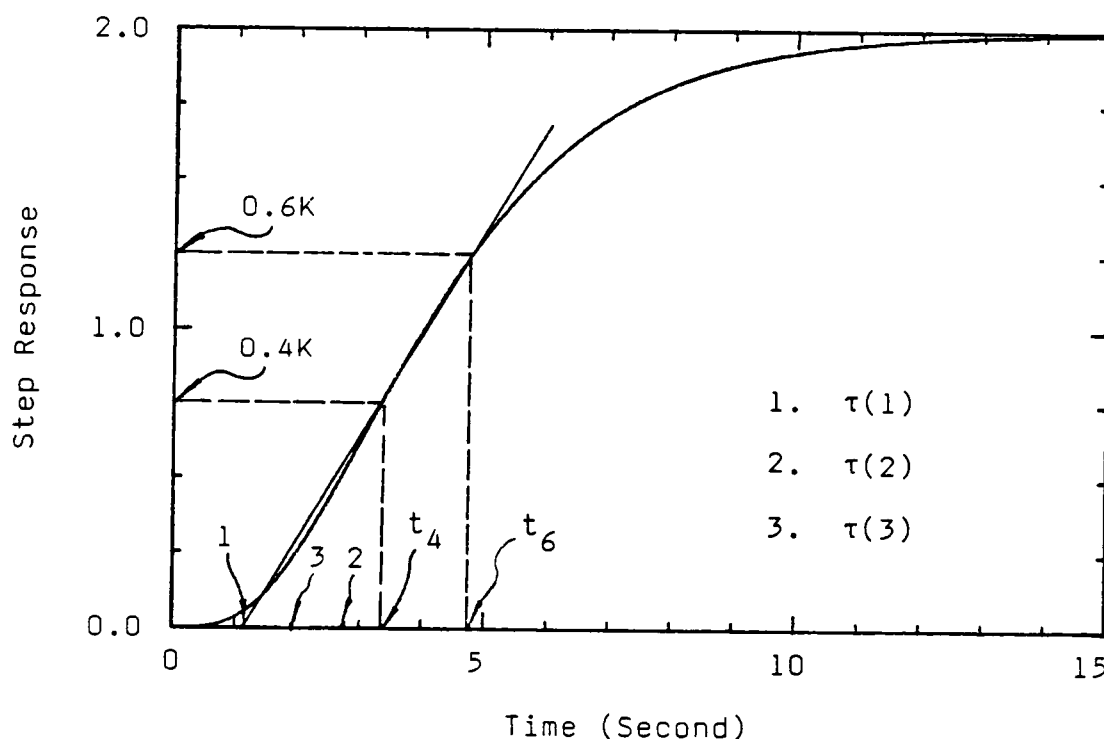


Figure 3-3. Illustration of the Choices of Delay Times

$\tau(1)$, $\tau(2)$, and $\tau(3)$ which has the smallest error of the three. Then, set this value to $\tau(3)$ as the new middle point.

4. In the program, $\tau(1)$, $\tau(2)$, and $\tau(3)$ can be considered as variables. Therefore, reset

$$\tau(1) = \tau(3) - 5D \quad (3-18)$$

$$\tau(2) = \tau(3) + 5D \quad (3-19)$$

5. Calculate corresponding unknowns A and E, and repeat steps 3 to 5 decreasing D by half each time until the results are satisfactory.

The computer program written this way takes about five minutes to run. Accurate results can be obtained. For example, Table 3-2 compares a FOLPD process with its FOLPD model. They agree very closed.

TABLE 3-2
COMPARISON OF A FOLPD PROCESS WITH ITS FOLPD MODEL

Parameter	Process	Model	Relative Error
DC Gain K	2.0	2.0	0%
Time Constant T	1.0	0.99994	$6 \times 10^{-7}\%$
Delay Time τ	1.0	0.99993	$7 \times 10^{-7}\%$

Definition of Curve-Fitting Range

In this thesis a new term, the curve-fitting range (CFR), is defined as the time period during which the least-squares criterion is applied to fitting a curve. As illustrated in Figure 3-4, the curve-fitting range can be chosen arbitrarily between two sampling instants, say from t_i to t_j . Also, it can be measured since the time at sampling instant t_k is

$$t_k = kT_s, \quad (3-20)$$

where T_s is the sampling interval, and $k=0, 1, 2, \dots, n$.

For example, let $T_s=0.05$ (second), $i=0$, and $j=255$. Then the curve-fitting range is from $t_0=0$ to $t_{255}=12.75$ (second).

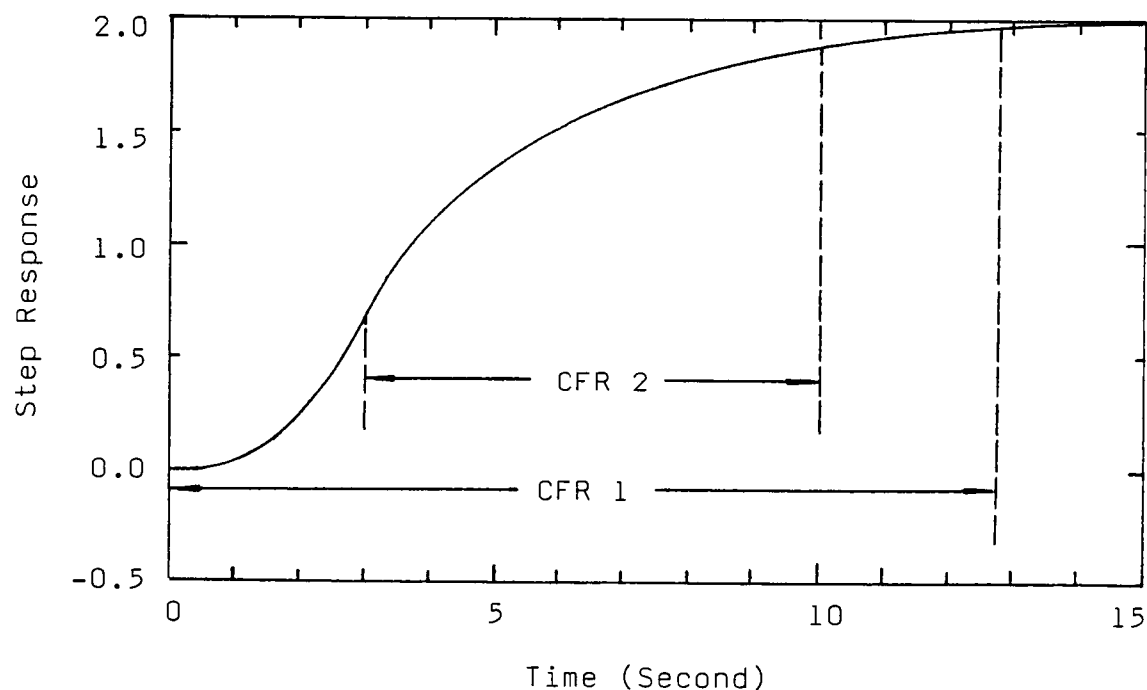


Figure 3-4. Illustration of Curve-Fitting Ranges

A Way To Choose Curve-Fitting Ranges In the Algorithm

As mentioned in the last section, the choice of a curve-fitting range in the automated parameter identification algorithm is extremely important. Experiments indicate that:

1) if the process step response curve is fitted within different CFRs, different parameters of the FOLPD model could be obtained; 2) processes possessing a quick transient response should be identified by using a short CFR, and vice versa; and 3) a too long or too short CFR could result in improper parameters of the model.

In fact, curve-fitting ranges can be selected for various processes according to their delay times. This is because a process possessing a quick transient response usually has a short delay time, and vice versa. Therefore, one can first find an estimated value of delay time for any process. This is done by using the algorithm with a short CFR (0 to 5 seconds, for example). Then, an adequate CFR for that process can be selected according to the value of delay time. In the computer program, these procedures can be done automatically.

Table 3-3 presents a trial selection of curve-fitting ranges according to delay times. They are chosen this way in consideration of the following: 1) the number of sampling instants is kept constant for convenience in working with the algorithm; 2) the sampling interval is increased in small steps to give different CFRs; and 3) that the CFR starts at

$t_0=0$ is suitable in this application. The results shown in this thesis were obtained by applying this selection.

For those processes having very large delay times, more curve-fitting ranges can also be selected through experiments. These are left for further studies.

TABLE 3-3
A TRIAL SELECTION OF CURVE-FITTING RANGES
ACCORDING TO DELAY TIMES

Delay Time (Second)	Sampling Points	Sampling Interval	Curve-Fitting Range (Second)
0	255	0.015	0 - 3.825
0 - 0.5	255	0.020	0 - 5.10
0.5 - 1	255	0.025	0 - 6.375
1 - 1.5	255	0.030	0 - 7.65
1.5 - 2	255	0.040	0 - 10.20
2 - 3	255	0.050	0 - 12.75
3 - 4	255	0.060	0 - 15.30
4 - 5	255	0.070	0 - 17.85

5. RESULTS AND DISCUSSIONS

This section presents the results of automated parameter identification for five experimental processes. These processes were chosen in consideration of their poles and zeros such that they can represent the general characteristics of most SISO high-order processes. It must be pointed out that: 1) processes possessing an unconvergent open-loop step response cannot be identified this way since the algorithm is based on fitting the step response curve; and 2) processes having under-damped complex poles are not suitable for approximation by a FOLPD model since their open-loop step responses are oscillatory.

Table 3-4 shows the transfer functions of experimental processes 1 to 5 and their optimal FOLPD or FOL models.

Figures 3-5 to 3-9 plot the open-loop step responses of these processes and models.

It can be seen that each squares error listed in the table is quite small. The step responses of the process and its model in every figure are very close to each other. These facts obviously indicate that the models obtained by using the automated least-squares parameter identification technique are much better than those obtained by the graphical parameter identification methods.

TABLE 3-4
EXPERIMENTAL PROCESSES AND THEIR OPTIMAL MODELS

NO.	Process	Model DC Gain	Time Const. (Second)	Delay Time (Second)	Squares Error
1.	$\frac{2e^{-s}}{s+1}$	2.0	0.99994	0.99993	7.3×10^{-6}
2.	$\frac{2}{(s+1)^2(s+2)(s+0.5)}$	2.0	2.37609	2.07104	0.36844
3.	$\frac{60}{(s+1)(s+2)(s+3)(s+4)}$	2.50	1.05342	0.98275	0.94996
4.	$\frac{5(s+2)(s+4)}{(s+1)(s+3)(s+5)}$	2.66667	0.88075	0	3.33820
5.	$\frac{500(s+4)}{(s+1)(s+10)(s+100)}$	2.0	0.98002	0	0.14948

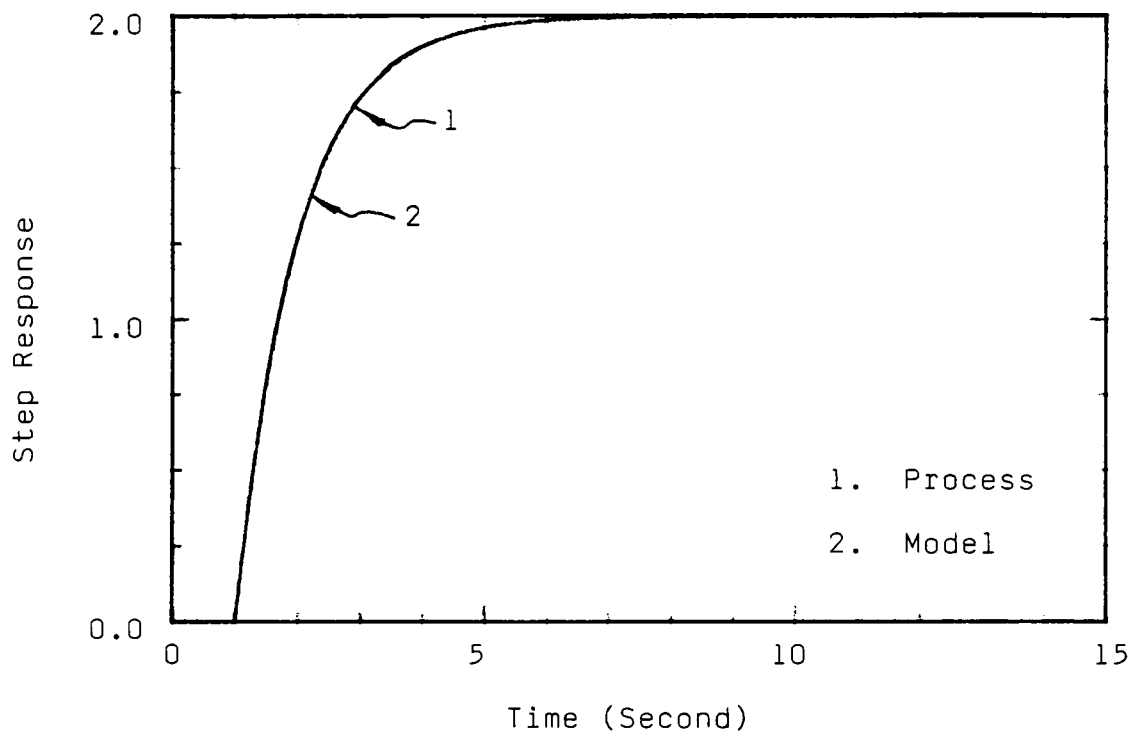


Figure 3-5. Step Response of Process 1 and Its Model

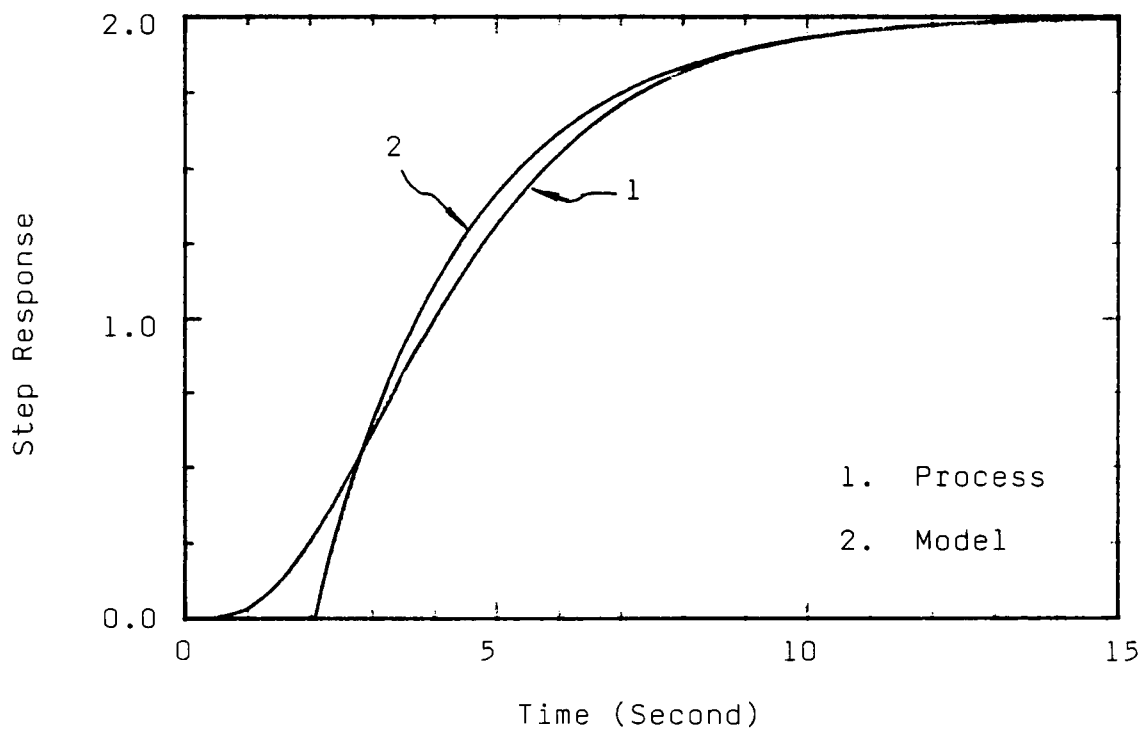


Figure 3-6. Step Response of Process 2 and Its Model

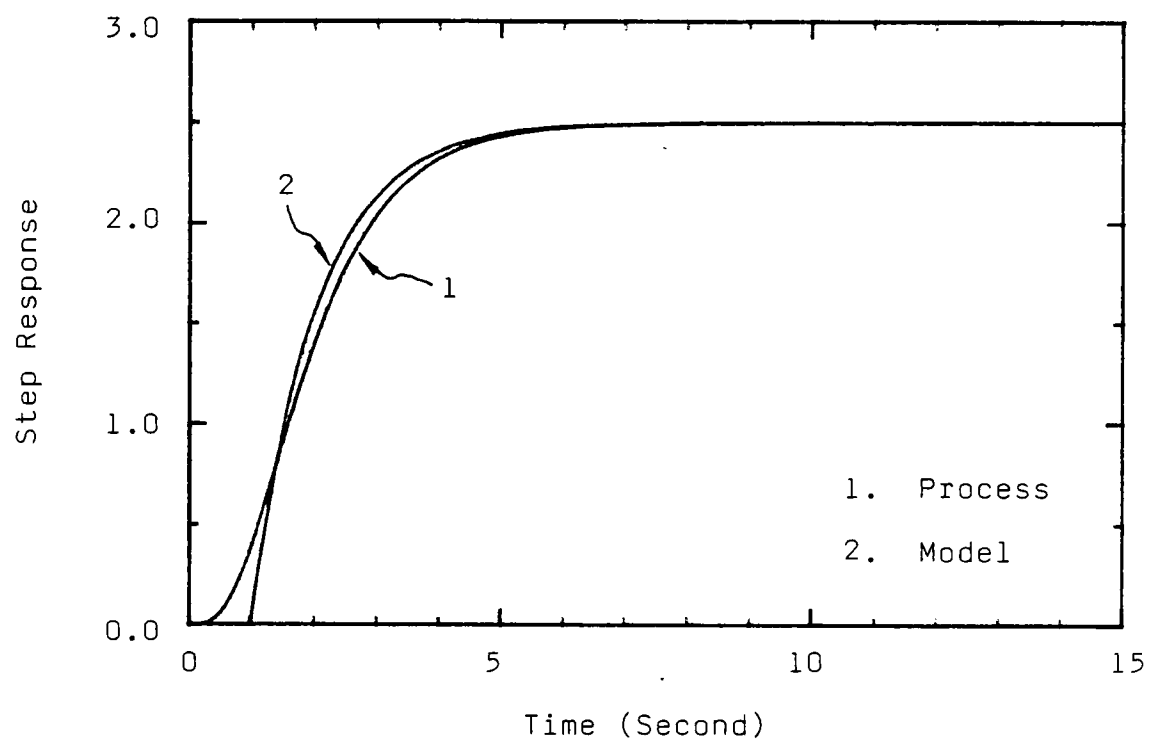


Figure 3-7. Step Response of Process 3 and Its Model

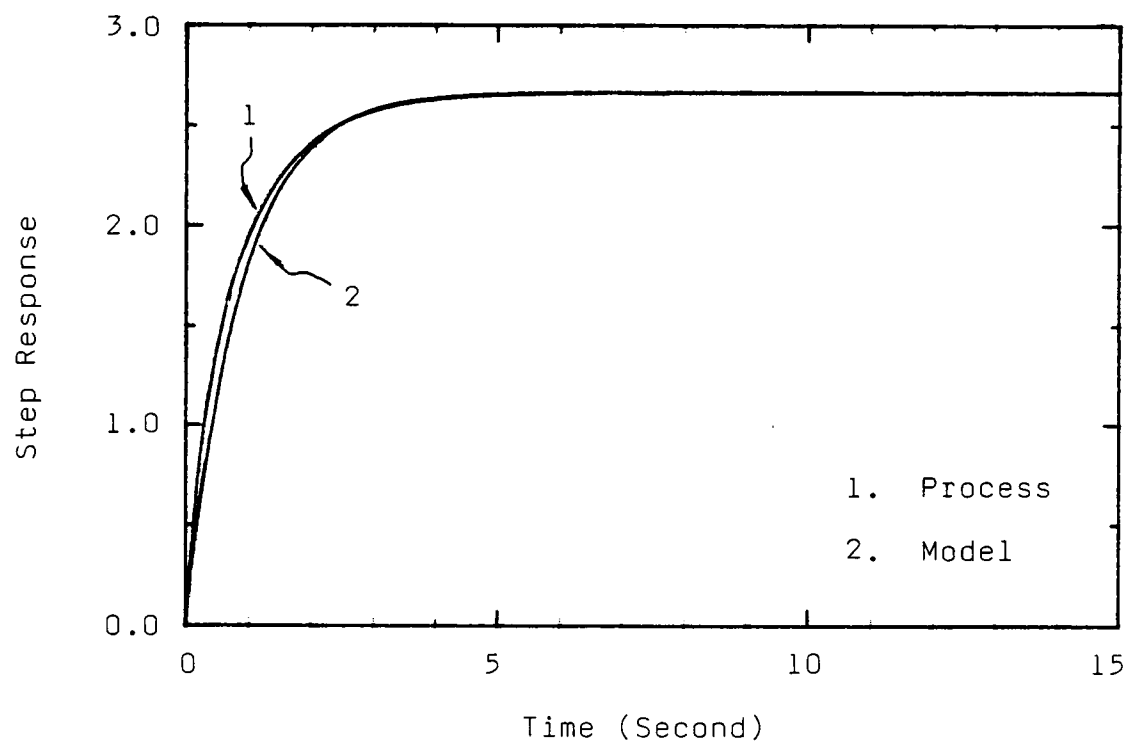


Figure 3-8. Step Response of Process 4 and Its Model

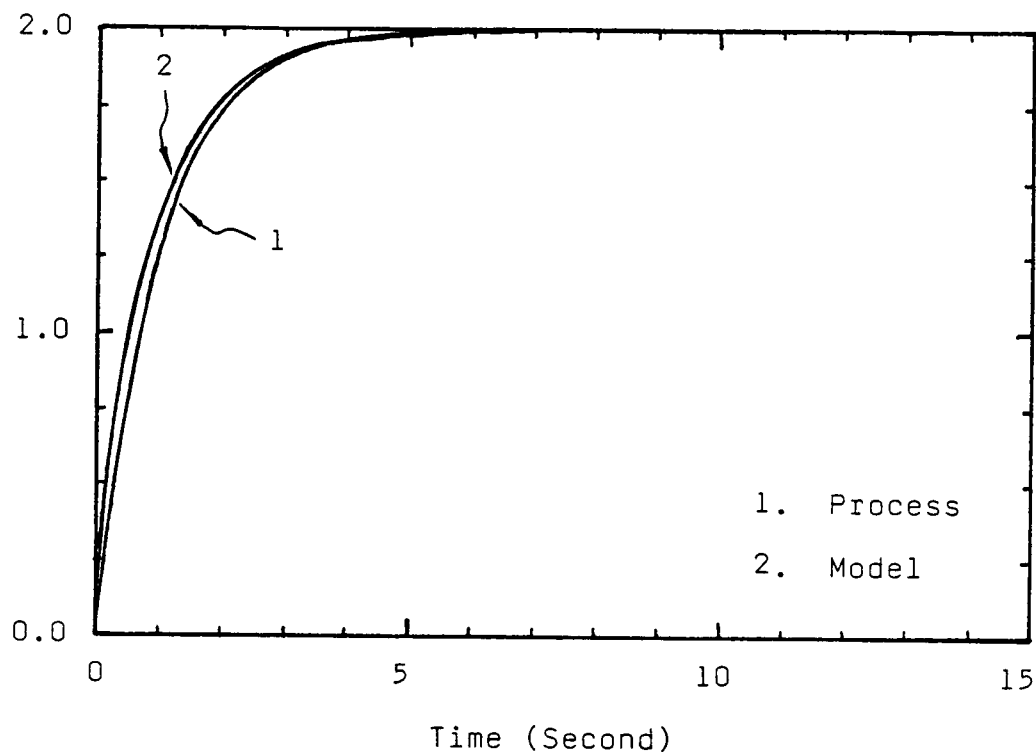


Figure 3-9. Step Response of Process 5 and Its Model

6. IDENTIFICATION FROM NOISE-CONTAMINATED DATA

Noise-Contaminated Measurement

The algorithm introduced earlier is of the theoretical interest only, since the measurements may be contaminated with noise. The object of this section is to measure and reduce the effect of noise in the automated least-squares parameter identification algorithm.

Consider an open-loop system with process $G_p(s)$ and noise $N(s)$ as shown in Figure 3-10. The step response of the system is

$$f(t) = c(t) + n(t) , \quad (3-21)$$

where $c(t)$ is the process step response, and $n(t)$ is the measurements random noise which is assumed white and Gaussian.

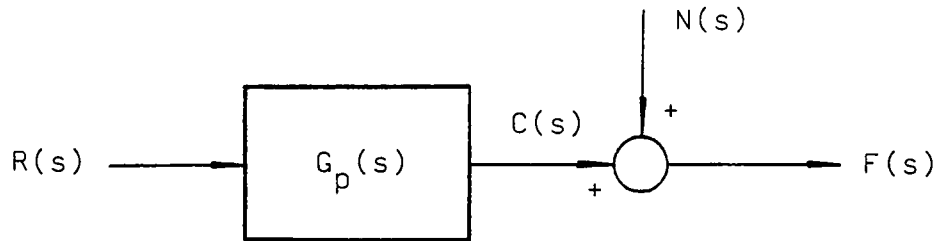


Figure 3-10. Block Diagram of an Open-Loop System with Measurements Random Noise

Improvements On the Identification Algorithm

In the noiseless measurement case, the step response $c(t)$ can never reach its steady-state $c(\infty)$ theoretically. But, in the noise-contaminated measurement case, the system step response $f(t)$ can be equal to or greater than $c(\infty)$, and Equation (3-4) in the algorithm becomes

$$z(t) = \ln[1 - f(t)/K] . \quad (3-22)$$

If $f(t) > c(\infty)$, then $[1 - f(t)/K] < 0$. Since $z(t)$ then is the logarithm of a negative number, it is undefined.

This problem is solved by changing Equation (3-22) to

$$z(t) = \ln|1 - f(t)/K| . \quad (3-23)$$

For the points satisfying $f(t) > c(\infty)$, this is equivalent to setting them to the symmetrical points about the line $c(t) = c(\infty)$. Since this change may affect the accuracy of

fitting the step response curve, it is needed to reduce the curve-fitting range such that there are only a few this kind of points within the CFR. An illustration is shown in Figure 3-11. In practice, the curve-fitting range can still be desired according to the selections in Table 3-3. But, a corresponding smaller CFR should be chosen.

Another problem in the noisy measurement case is the vanish of the way to find process DC gain introduced in section 4. This is because the slopes of the process step response cannot be determined any more.

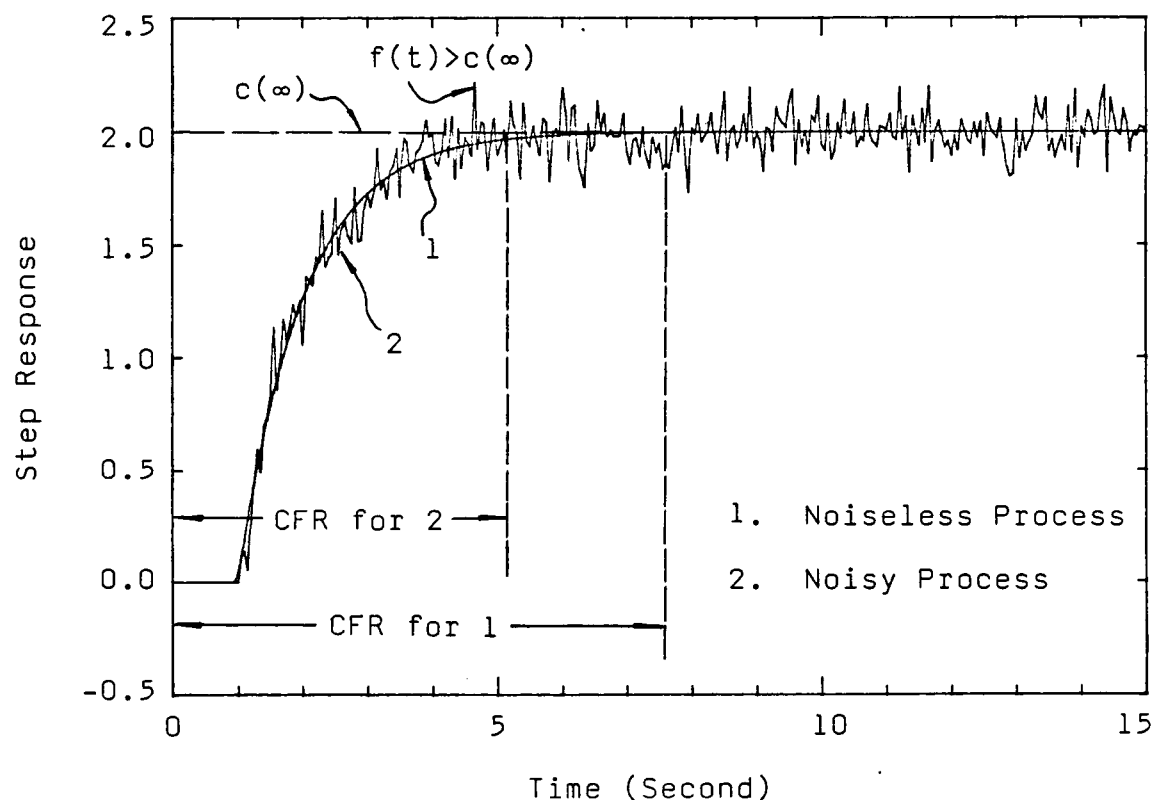


Figure 3-11. Illustration of Noisy Measurement and the Choice of the Corresponding Curve-fitting Range

In general, processes possessing small delay times also have short settling times, and vice versa. In consideration of this fact and the final times obtained in experiments (refer to Table 3-1), one way to find process DC gain in the noisy measurement case is suggested as follows: 1) For those processes having regular delay times ($\tau \leq 5$ seconds), record the system step response from 0 to 100 seconds with an adequate sampling interval ($T_s = 0.02$ seconds, for example). Then make an average for the sampling data which are recorded from 50 to 100 seconds, and choose the average value as $c(\infty)$ for Equation (3-1). 2) For those processes having extra large delay times ($\tau > 5$ seconds), apply the same strategy with a longer period of sampling process. In applications, the general characteristics of the process are usually known. Therefore, selections of the sampling interval and the period of sampling process can be desired by the user.

Identification Results In the Noisy Measurement Case

Experiments were designed to identify five experimental processes with different sizes of noise. The white Gaussian noises with zero mean and different standard deviations were applied to the processes so that the process step responses were noisy. A subroutine was written to transfer the RND (random) function into white Gaussian noise since the HP Desktop computer cannot generate the noise directly. The standard deviation of the noise can be adjusted by a constant proportional to the magnitude of the noise.

The quantitative measure of the effect of noise to signal is commonly represented by signal-to-noise ratio (S/N). In this study, it is defined as:

$$S/N = \frac{\text{The Mean Signal Power } \bar{c}^2}{\text{The Mean Noise Power } \bar{n}^2} = \frac{1/T_f \int_0^{T_f} c^2(t) dt}{1/T_f \int_0^{T_f} n^2(t) dt}, \quad (3-24)$$

where T_f denotes the curve-fitting range. Note that the mean power of white Gaussian noise is equal to the variance of the noise, that is $\bar{n}^2 = \sigma^2$. The mean signal power can be calculated in experiment when $c(t)$ and T_f are known. In practice, although the process step response $c(t)$ is not desired analytically, one can estimate a value to approximate the mean signal power.

Tables 3-5 to 3-9 present the FOLPD or FOL models for the experimental processes in various cases and the corresponding values of signal-to-noise ratio.

Figures 3-12 to 3-14 plot the step responses of three noisy experimental processes and their models.

Comparing the results obtained in both the noiseless case and the noise-contaminated case, it can be concluded that the automated least-squares parameter identification technique has filtering properties and can often cope with the effect of white Gaussian noise in measurements. Therefore, this technique possesses great practical importance.

TABLE 3-5
EXPERIMENTAL IDENTIFICATION RESULT 1^a

Parameter	Noise Size			
	(Standard Deviation)			
	$\sigma=0$	$\sigma=0.1$	$\sigma=0.5$	$\sigma=1.0$
Process DC Gain	2.0	2.0	2.0	2.0
Model DC gain K	2.0	2.01031	2.02488	2.04310
Time Constant T	0.99994	1.08460	0.87837	0.78550
Inverse of T, A	1.00006	0.9220	1.12021	1.25583
Delay Time τ	0.99993	0.960	1.08437	0.99063
Squares Error E	7.3×10^{-6}	2.3640	56.745	231.058
S/N	-	202.9:1	8.116:1	2.029:1

^aThe result is for the experimental process 1:

$$G_p(s) = \frac{2e^{-s}}{s + 1} .$$

TABLE 3-6
EXPERIMENTAL IDENTIFICATION RESULT 2^a

Parameter	Noise Size			
	(Standard Deviation)			
	$\sigma=0$	$\sigma=0.1$	$\sigma=0.5$	$\sigma=1.0$
Process DC Gain	2.0	2.0	2.0	2.0
Model DC Gain K	2.0	2.01030	2.02488	2.04310
Time Constant T	2.37609	2.37660	2.33946	2.17888
Inverse of T, A	0.42086	0.42077	0.42301	0.45442
Delay Time τ	2.07104	2.11875	1.9750	2.10
Squares Error E	0.36844	3.84650	58.312	228.747
S/N	—	228.0:1	9.122:1	2.280:1

^aThe result is for the experimental process 2:

$$G_p(s) = \frac{2}{(s+1)^2(s+2)(s+0.5)} .$$

TABLE 3-7
EXPERIMENTAL IDENTIFICATION RESULT 3^a

Parameter	Noise Size			
	(Standard Deviation)			
	$\sigma=0$	$\sigma=0.1$	$\sigma=0.5$	$\sigma=1.0$
Process DC Gain	2.50	2.50	2.50	2.50
Model DC Gain K	2.50	2.51198	2.52655	2.54477
Time Constant T	1.05342	1.15417	1.02517	0.99764
Inverse of T, A	0.94929	0.86642	0.95951	1.00237
Delay Time τ	0.98275	0.91211	1.02188	0.99063
Squares Error E	0.94996	4.02869	60.528	229.72
S/N	—	338.4:1	13.54:1	3.384:1

^aThe result is for the experimental process 3:

$$G_p(s) = \frac{60}{(s+1)(s+2)(s+3)(s+4)}.$$

TABLE 3-8
EXPERIMENTAL IDENTIFICATION RESULT 4^a

Parameter	Noise Size			
	(Standard Deviation)			
	$\sigma=0$	$\sigma=0.1$	$\sigma=0.5$	$\sigma=1.0$
Process DC Gain	2.66667	2.66667	2.66667	2.66667
Model DC Gain K	2.66667	2.67920	2.69377	2.71199
Time Constant T	0.88075	0.86226	0.79981	1.02838
Inverse of T, A	1.13539	1.15974	1.25029	0.97240
Delay Time τ	0	0	0	0
Squares Error E	3.33820	4.90244	59.184	245.021
S/N	—	378.8:1	15.15:1	3.788:1

^aThe result is for the experimental process 4:

$$G_p(s) = \frac{5(s+2)(s+4)}{(s+1)(s+3)(s+5)} .$$

TABLE 3-9
EXPERIMENTAL IDENTIFICATION RESULT 5^a

Parameter	Noise Size			
	(Standard Deviation)			
	$\sigma=0$	$\sigma=0.1$	$\sigma=0.5$	$\sigma=1.0$
Process DC Gain	2.0	2.0	2.0	2.0
Model DC Gain K	2.0	2.01031	2.02488	2.04310
Time Constant T	0.98002	0.88078	0.90666	0.84830
Inverse of T, A	1.02040	1.13535	1.10295	1.17883
Delay Time τ	0	0	0	0
Squares Error E	0.14948	3.43976	58.9270	231.290
S/N	—	256.7:1	10.26:1	2.567:1

^aThe result is for the experimental process 5:

$$G_p(s) = \frac{500(s+4)}{(s+1)(s+10)(s+100)} .$$

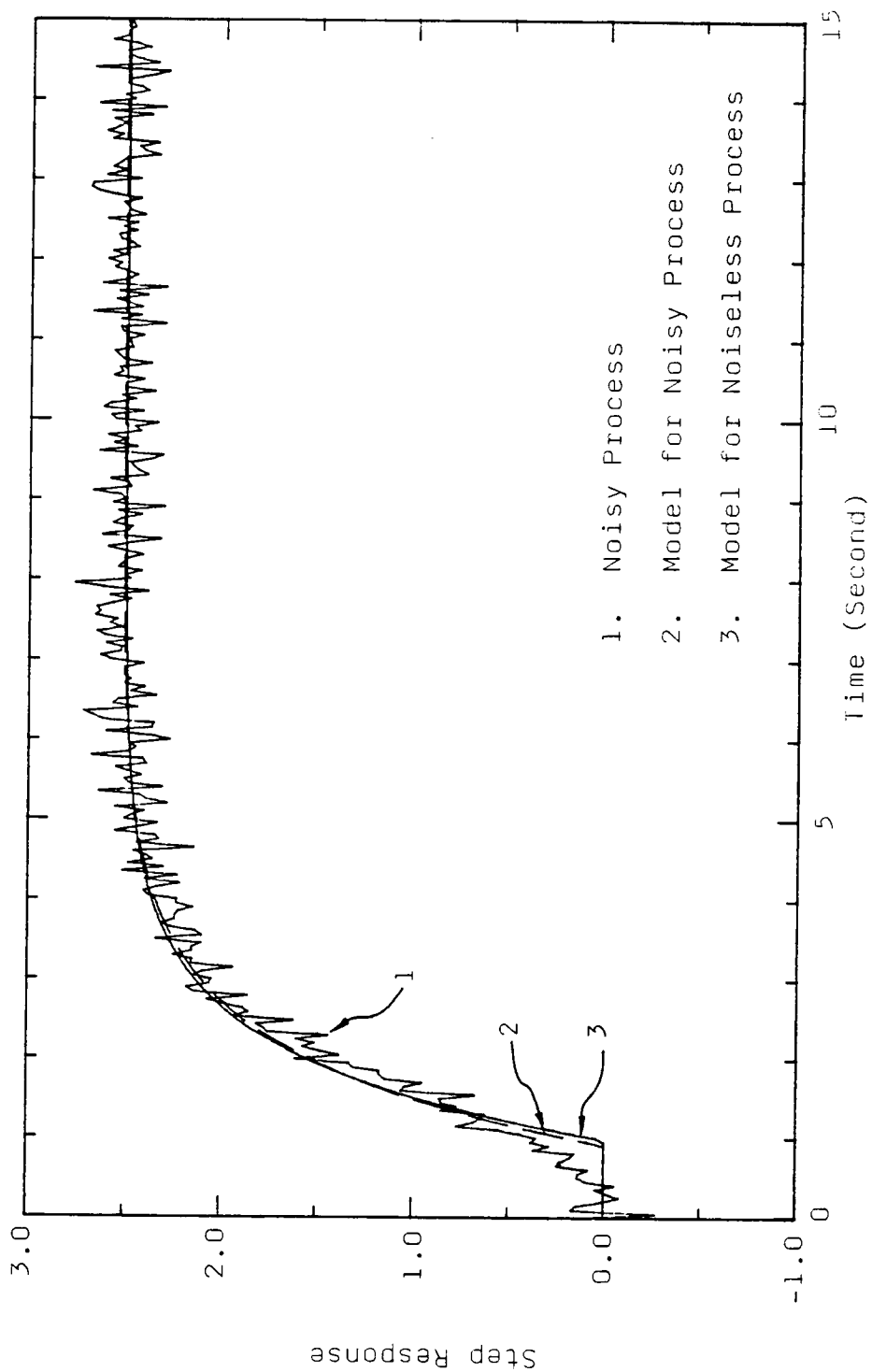


Figure 3-12. Illustration of Identification with Small Noise

Process: $\frac{60}{(s+1)(s+2)(s+3)(s+4)}$ $S/N=338.4:1$

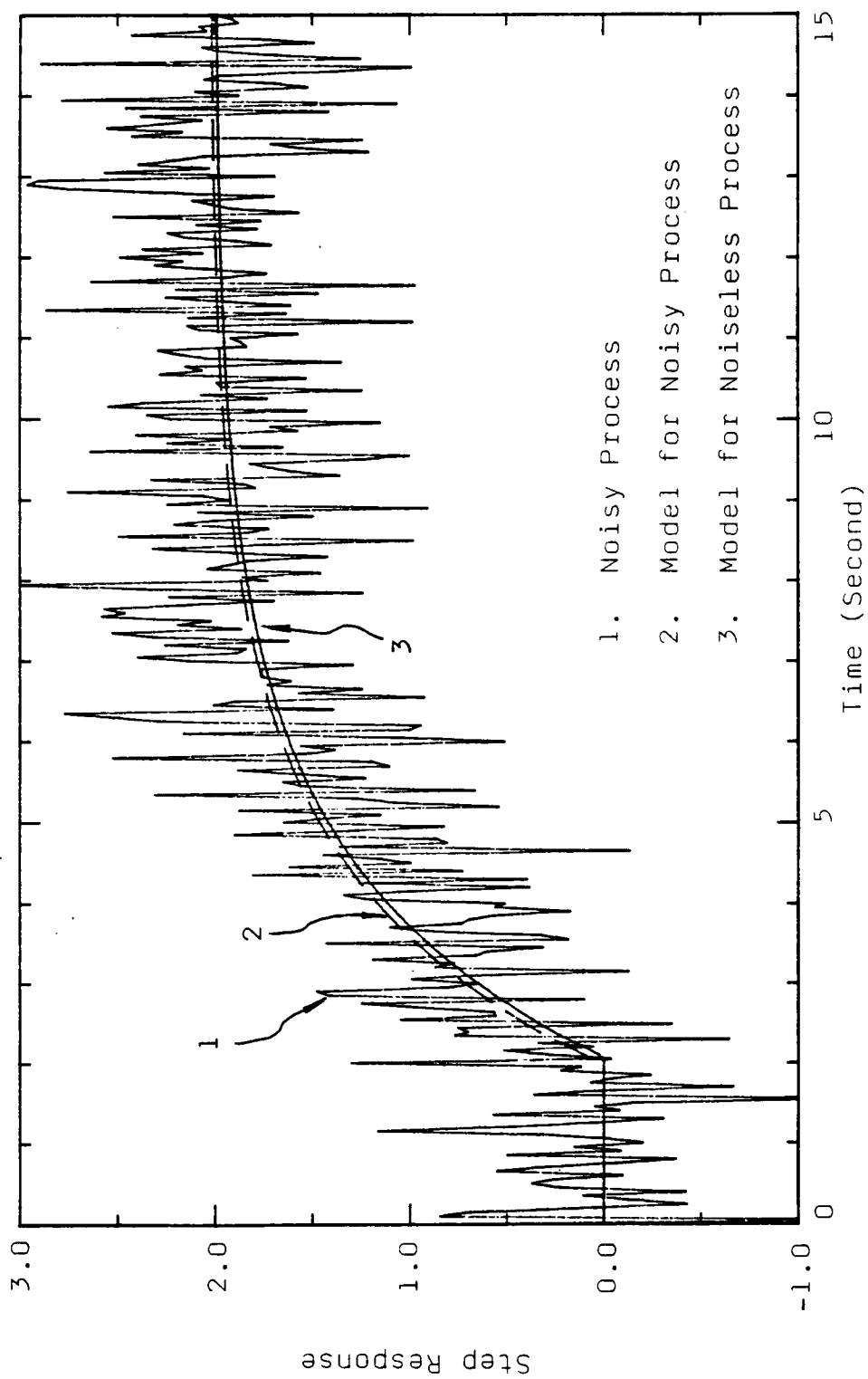


Figure 3-13. Illustration of Identification with Medium Noise

Process: $\frac{2}{(s+1)^2(s+2)(s+0.5)}$ $S/N=9.122:1$

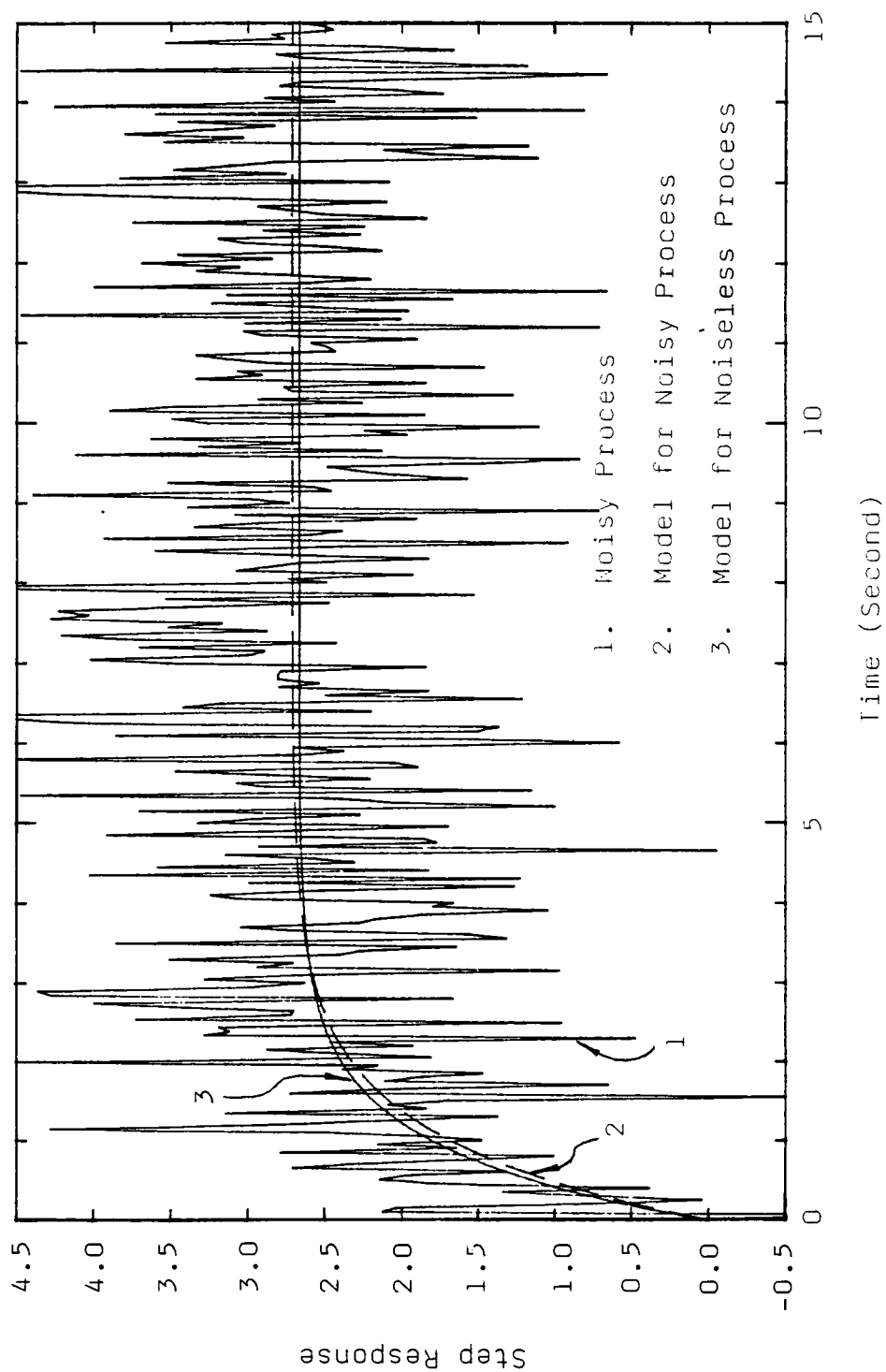


Figure 3-14. Illustration of Identification with Heavy Noise

Process:
$$\frac{5(s+2)(s+4)}{(s+1)(s+3)(s+5)}$$

$S/N=3.788:1$

CHAPTER 4

REVIEW AND IMPROVEMENT OF PID CONTROLLER TUNING TECHNIQUES

1. PRELIMINARY REMARKS

In the last two chapters, graphical and automated parameter identification techniques for the FOLPD or FOL model were discussed.

Classical techniques for tuning PID controller are usually based on the FOLPD model. This is because: 1) high-order processes are usually approximated by FOLPD model using the graphical parameter identification methods reviewed in Chapter 2; and 2) the PID controller can treat the FOLPD and FOL processes reasonably well due to its special characteristics.

With the development of the automated least-squares parameter identification technique, one is able to find those FOLPD or FOL models for most SISO stable processes in the optimal sense of least-squares curve-fitting criterion. Then the question for further study is how the optimal models can be used in PID control.

The purpose of this chapter is to review the basic concepts and functions of PID control and to discuss various PID controller tuning techniques.

2. BASIC CONCEPTS OF PID CONTROLLER

The Proportional-Integral-Derivative controller is usually represented in a Laplace transfer function as

$$G_c(s) = K_c \left[1 + \frac{1}{T_i s} + T_d s \right], \quad (4-1)$$

where K_c is called proportional gain, T_i is termed integral time (or reset time), and T_d is termed derivative time (or rate time). The inverse of T_i is usually called reset rate.

The PID controller is frequently used in single loop feedback control systems. It can solve both tracking and regulating problems. As illustrated in Figure 4-1, tracking means that one sets the input $r(t)$ to a value and wants the output $c(t)$ to track that value. In the regulating case, $r(t)$ is not changed and there is a disturbance or load change, $d(t) \neq 0$; one wants the controller to regulate this change such that $c(t)$ can return to its original point in an adequate time period. Both of these cases will be discussed in this thesis.

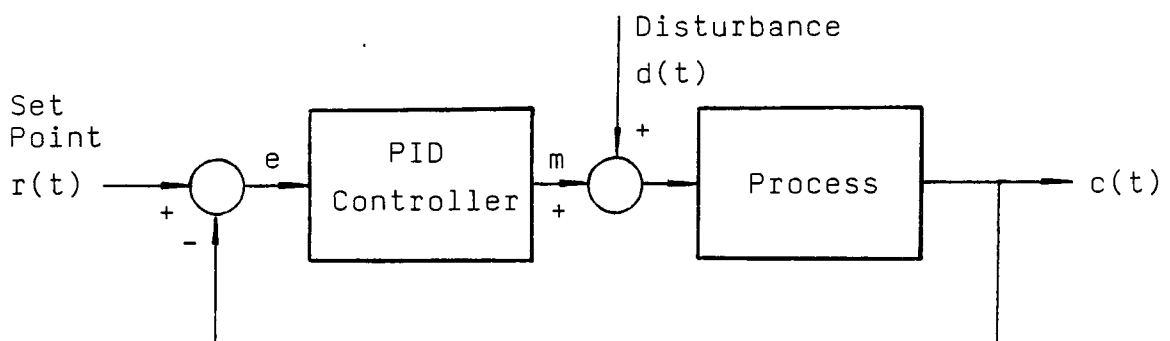


Figure 4-1. Block Diagram of a Single Loop Feedback Control System

The functions of proportional, integral, and derivative are illustrated respectively in Figure 4-2. Generally, increasing K_c will cause: 1) reduction of system sensitivity with respect to variation of process parameters; 2) reduction

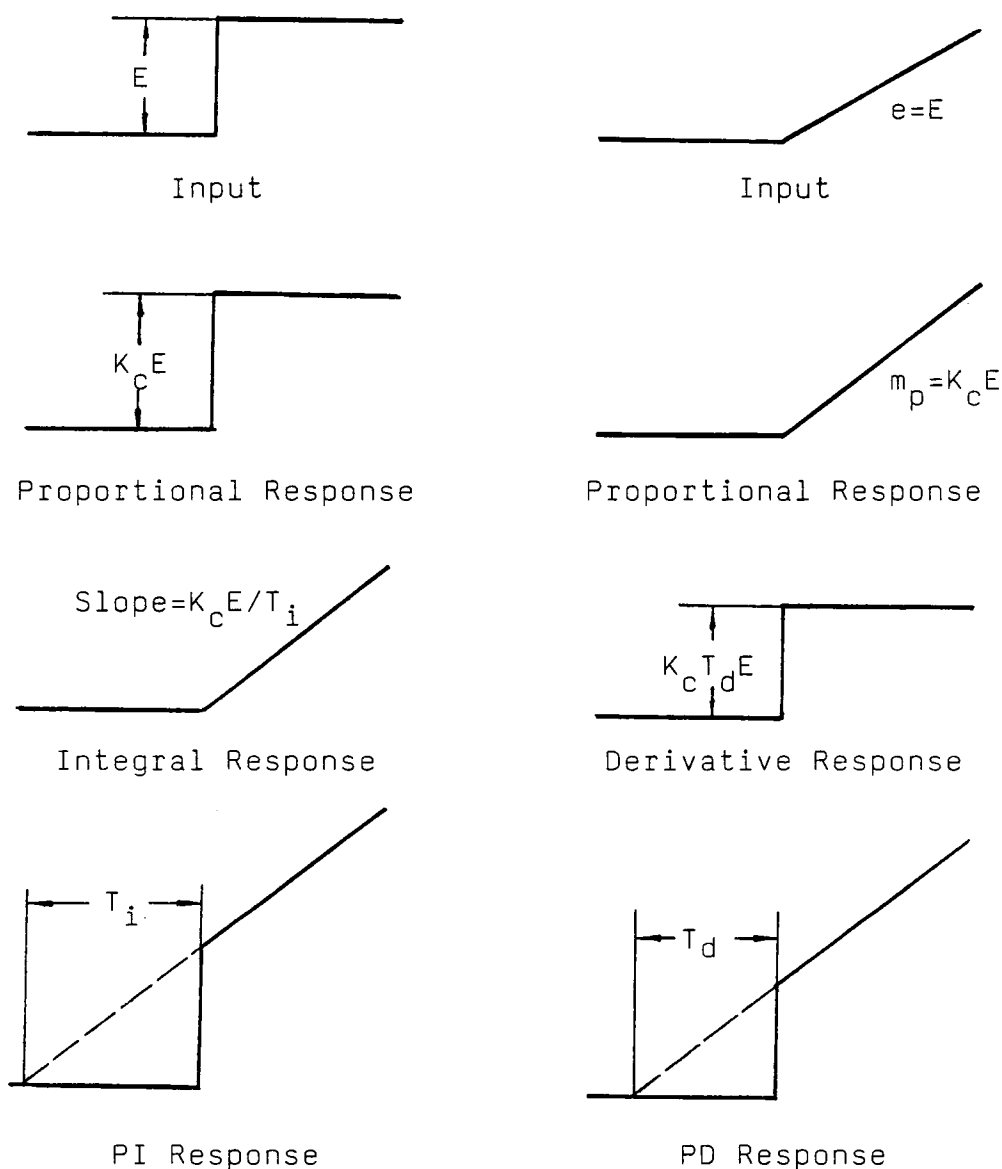


Figure 4-2. Illustration of P, I, and D Functions and Corresponding Parameters

of steady-state system errors; 3) widening of system bandwidth; and 4) quickening of system response. The effect of $1/T_i s$ is to eliminate the steady-state error. It is called "reset" because it removes the "offset". The effect of $T_d s$ is to reduce the time lag property of the control loop. So, it can cause a small overshoot and quick settling time. [8] The PID controller can be reduced to PI, PD or P simply by choosing $T_d=0$, $T_i \rightarrow \infty$, or both, respectively. This is also an attractive property of PID controller when on-line parameter tuning is concerned.

3. FOLPD MODEL RELATED PID CONTROLLER TUNING TECHNIQUES

The control performance specifications commonly used in PID controller design or tuning are: 1) quarter decay ratio criterion; and 2) minimum error-integrals criteria. The techniques based on both of these criteria are discussed in the following paragraphs.

PID Tuning Techniques Based on Quarter Decay Ratio

The so-called quarter decay ratio means that the overshoots of the process closed-loop step response are reduced one quarter the previous one. It was assumed that, in this case, the process closed-loop output will quickly return to the steady-state after any disturbance. However, this is correct only for second-order systems and is not really true for high-order processes or those containing delay times.

Thus, the process controlled in quarter decay ratio condition does not always have a short settling time. Even worse, experiments showed that PID controllers tuned in this way will make the system be close to the unstable region. Therefore, applications of the quarter decay ratio criterion should be avoided.

For the purpose of comparison, the FOLPD model related PID controller tuning formulas based on quarter decay ratio criterion are listed in Table 4-1, which are discussed in Reference [4]. The first of these formulas was proposed by Ziegler and Nichols [1]. The second one was introduced by Cohen and Coon [9], and the last one by Smith and Murrill [10]. The methods to derive these formulas will not be discussed.

PID Tuning Techniques Based on Minimum Error-Integrals

Realizing the disadvantages of the quarter decay ratio criterion, Lopez [11] introduced the so-called minimum error-integrals criteria. It begins with the suggestion of a cost functional

$$J = \int_0^{\infty} F[e(t), t] dt, \quad (4-2)$$

where F is a function of both the output error $e(t)$ and time t . Since $e(t) = c(t) - c(\infty)$, and $c(t)$ is the system closed-loop step response which is related to the controller parameters K_c , T_i , and T_d , the criterion then goes to the selection of a set of K_c , T_i , T_d such that they minimize the cost functional J .

TABLE 4-1

COMPARISON OF ZIEGLER-NICHOLS, COHEN-COON, AND
3C FORMULAS FOR PID CONTROLLER SETTING

Controller	Ziegler-Nichols	Cohen-Coon	3C
P	$KK_C = (\tau/T)^{-1}$	$KK_C = (\tau/T)^{-1} + 0.333$	$KK_C = 1.208(\tau/T)^{-0.956}$
	$KK_C = 0.9(\tau/T)^{-1}$	$KK_C = 0.9(\tau/T)^{-1} + 0.082$	$KK_C = 0.928(\tau/T)^{-0.946}$
PI	$T_i/T = 3.33(\tau/T)$	$T_i/T = \frac{3.33(\tau/T)[1 + (\tau/T)/11]}{1 + 2.2(\tau/T)}$	$T_i/T = 0.928(\tau/T)^{0.583}$
	$KK_C = 1.2(\tau/T)^{-1}$	$KK_C = 1.35(\tau/T)^{-1} + 0.27$	$KK_C = 1.37(\tau/T)^{-0.95}$
PID	$T_i/T = 2.0(\tau/T)$	$T_i/T = \frac{2.5(\tau/T)[1 + (\tau/T)/5]}{1 + 0.6(\tau/T)}$	$T_i/T = 0.74(\tau/T)^{0.738}$
	$T_d/T = 0.5(\tau/T)$	$T_d/T = \frac{0.37(\tau/T)}{1 + 0.2(\tau/T)}$	$T_d/T = 0.365(\tau/T)^{0.95}$

There are several ways to define function F in the cost functional J because what is optimal control is based on judgement or even personal taste. The most commonly used integral criteria are:

1. Integral of the square error (ISE),

$$ISE = \int_0^{\infty} [e(t)]^2 dt ; \quad (4-3)$$

2. Integral of the absolute value of the error (IAE),

$$IAE = \int_0^{\infty} |e(t)| dt ; \quad (4-4)$$

3. Integral of time multiplied by the absolute value of the error (ITAE),

$$ITAE = \int_0^{\infty} t |e(t)| dt . \quad (4-5)$$

For these criteria, article [11] concludes the following: 1) The ISE is relatively insensitive to small errors, but large errors contribute heavily to the value of the integral. This is because errors are squared. Thus, ISE may cause small overshoots but long settling time since small errors occur late in time. 2) The IAE is more sensitive to small errors but less sensitive to large errors than ISE. 3) The ITAE is insensitive to the initial and somewhat unavoidable errors, but it will heavily weight errors occurring late in time. Thus, it can cause short rise time and large overshoot.

To show the derivation of the PID controller tuning formulas based on these error-integrals criteria, consider the feedback control system shown in Figure 4-3.

For tracking problem, the system step response to a change in setpoint is

$$C(s) = \frac{KK_c[T_d s^2 + s + 1/T_i]e^{-\tau s}}{Ts^2 + s + KK_c e^{-\tau s}[T_d s^2 + s + 1/T_i]} R(s) . \quad (4-6)$$

For regulating problem, the system step response to a change of disturbance is

$$C(s) = \frac{Kse^{-\tau s}}{Ts^2 + s + KK_c e^{-\tau s}[T_d s^2 + s + 1/T_i]} D(s) . \quad (4-7)$$

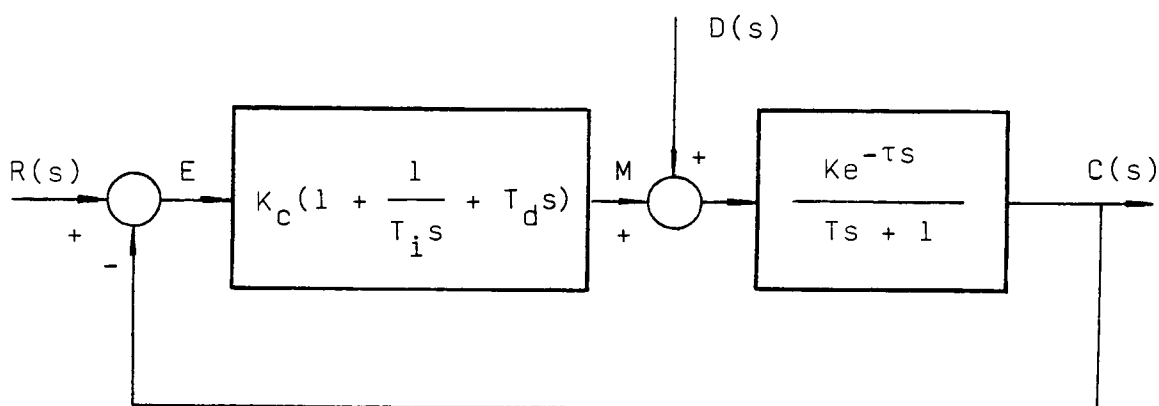


Figure 4-3. Feedback Control System with a PID Controller and FOLPD Process

Since the step response (4-6) and (4-7) are different, two groups of formulas should be derived for both tracking and regulating problems.

Now, consider the regulating problem. Firstly substitute a variable $u=Ts$ into Equation (4-7). This is equivalent to introducing a dimensionless time, $t'=t/T$. Then the system step response in time domain can be written as

$$c(t') = \mathcal{L}^{-1} \left\{ \frac{KT e^{-u\tau/T}}{u^2 + u + KK_c e^{-u\tau/T} [u^2 T_d/T + u + T/T_i]} \right\} . \quad (4-8)$$

To simplify the problem, introduce

$$x(t') = c(t')/KT . \quad (4-9)$$

Then, Equation (4-9) is a function of the dimensionless parameters τ/T , KK_c , T/T_i , and T_d/T .

For regulating problem, if $r(t)=0$, then $e(t)=-c(t)$. Substitute $c(t')$ into Equation (4-3)

$$ISE = K^2 T^2 \int_0^{\infty} [x(t')]^2 dt . \quad (4-10)$$

Note that same procedure can also be done for IAE and ITAE.

To find the optimal values of KK_c , T/T_i , and T_d/T , a severe difficulty is the analytical calculation of $x(t')$ that involves the inverse Laplace transform of Equation (4-8).

However, a technique was suggested in reference [11] which mechanizes the trial and error solution of the minimization problem. The procedures are as follows: 1) simulate the system and compute the integral for repeated choices of parameter values; 2) for each selection of process parameters, find a unique set of controller parameters which minimizes the corresponding cost functional.

A so-called optimum gradient steepest descent method [12] was used by Lopez for these trial calculations. By drawing some curves from the calculation data, Lopez finally suggested a group of formulas and constants which can match these curves so that the relations between the parameters of PID controller and FOLPD model are defined. Some of these curves [11] are shown in Figure 4-4.

In the same method mentioned above, Rovira [13] derived the PID controller tuning relations for the tracking problem.

The FOLPD model related PID controller tuning formulas for regulating problem are as follows:

$$K_c = A_p(\tau/T)^{B_p}/K \quad (4-11)$$

$$T_i = T/[A_i(\tau/T)^{B_i}] \quad (4-12)$$

$$T_d = TA_d(\tau/T)^{B_d}, \quad (4-13)$$

where A_p , B_p , A_i , B_i , A_d , and B_d are constants for the given controller and criterion. They are listed in Table 4-2.

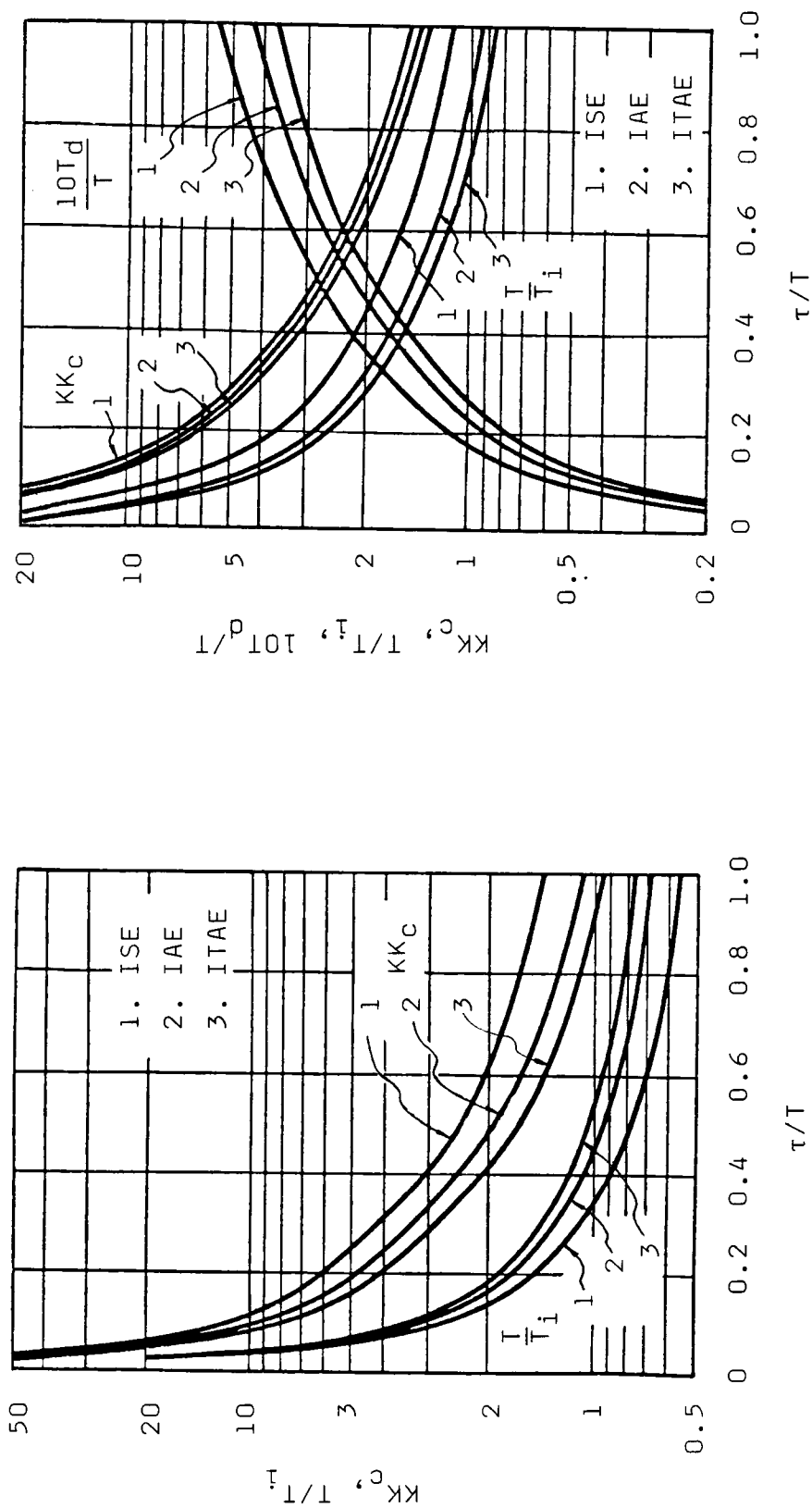


Figure 4-4. Minimum Error-Integrals Based Relations Between PID Controller Parameters and FOLPD Model Parameters

TABLE 4-2

FOLPD MODEL RELATED PID CONTROLLER TUNING RELATIONS
 BASED ON MINIMUM ERROR-INTEGRALS CRITERIA
 FOR REGULATING PROBLEM

Criterion	Mode	A_p	B_p	A_i	B_i	A_d	B_d
IAE	P	0.902	-0.985	—	—	—	—
ISE	P	1.411	-0.917	—	—	—	—
ITAE	P	0.490	-1.084	—	—	—	—
IAE	PI	0.984	-0.986	0.608	-0.707	—	—
ISE	PI	1.305	-0.959	0.492	-0.739	—	—
ITAE	PI	0.859	-0.977	0.674	-0.680	—	—
IAE	PID	1.435	-0.921	0.878	-0.749	0.482	1.137
ISE	PID	1.495	-0.945	1.101	-0.771	0.560	1.006
ITAE	PID	1.357	-0.947	0.842	-0.738	0.381	0.995

The FOLPD model related PID controller tuning relations for tracking problem [13] are shown in Table 4-3. In this case, parameters K_c and T_d are calculated from Equation (4-11) and (4-13), respectively, and

$$T_i = T/[A_i + B_i(\tau/T)] \quad (4-14)$$

TABLE 4-3

FOLPD MODEL RELATED PID CONTROLLER TUNING RELATIONS
BASED ON MINIMUM ERROR-INTEGRALS CRITERIA
FOR TRACKING PROBLEM

Criterion	Mode	A_p	B_p	A_i	B_i	A_d	B_d
IAE	PI	0.758	-0.861	1.020	-0.323	—	—
ITAE	PI	0.586	-0.916	1.030	-0.165	—	—
IAE	PID	1.086	-0.869	0.740	-0.130	0.348	0.914
ITAE	PID	0.965	-0.855	0.796	-0.147	0.308	0.929

4. A FOL MODEL RELATED PI CONTROLLER TUNING FORMULA

As it has been found in the last chapter, there exist some high-order processes which can be approximated simply by the first-order-lag model due to their quick transient responses. In such case, the FOLPD model related PID controller tuning formulas are not valid since zero delay time will cause zero proportional gain in Equation (4-11). In fact, FOL processes can be easily treated by using a PI controller. The reasons include: 1) the transient response of a FOL process is quick enough so that no derivative action is needed; and 2) the feedback control system with a PI controller and a FOL process is a second-order system which is easy to deal with. Therefore, the purpose of this section is to find a FOL model related PI controller tuning formula.

Consider the feedback control system shown in Figure 4-5.

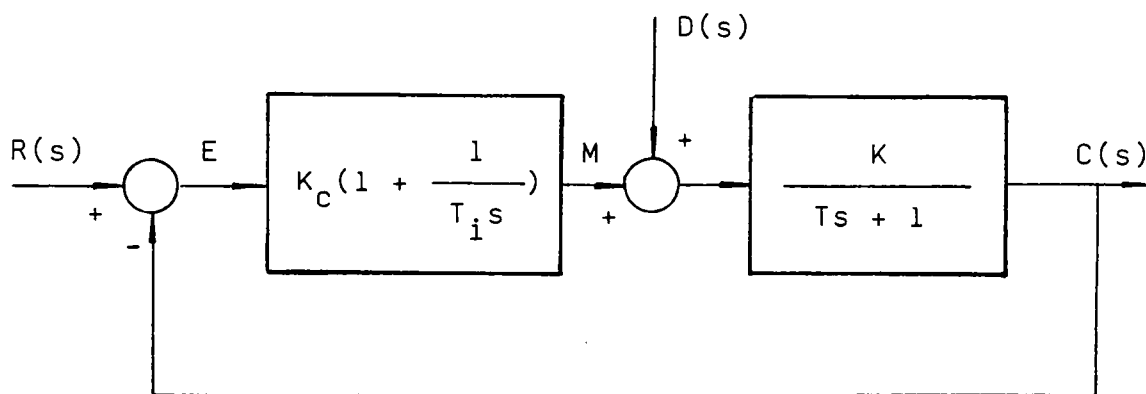


Figure 4-5. Feedback Control System with a PI Controller and FOL Process

For tracking problem, the system closed-loop transfer function can be written as

$$\frac{C(s)}{R(s)} = \frac{KK_c(T_i s + 1)}{T_i T s^2 + (T_i + KK_c T_i)s + KK_c} . \quad (4-15)$$

Then, the characteristic equation of the system can be found

$$\Delta = s^2 + [(1 + KK_c)/T]s + KK_c/T_i T , \quad (4-16)$$

which indicates that this is a second-order system.

Although there is a zero in Equation (4-15), the PI controller can still be designed based upon the characteristic equation (4-16). Like conventional second-order system, the control performance specification can be chosen holding the damping ratio $\zeta=0.707$.

Now, compare Equation (4-16) with the standard second-order characteristic equation

$$\Delta = s^2 + 2\zeta\omega_n s + \omega_n^2 , \quad (4-17)$$

where ζ is the damping ratio, and ω_n is the natural undamped frequency. It follows

$$\begin{cases} 2\zeta\omega_n = (1 + KK_c)/T \\ \omega_n^2 = KK_c/T_i T . \end{cases} \quad (4-18)$$

$$(4-19)$$

Solve the simultaneous equations for $\zeta=0.707$:

$$T_i = 1.414^2 \frac{KK_c T}{(1 + KK_c)^2} . \quad (4-20)$$

This is the PI controller tuning formula relating to the FOL model. Experiment shows that the choice of $K_c=1$ will be satisfactory.

For regulating problem, the system closed-loop transfer function can be written as

$$\frac{C(s)}{D(s)} = \frac{KT_i s}{T_i Ts^2 + (T_i + KK_c T_i)s + KK_c} . \quad (4-21)$$

The characteristic equation is the same as Equation (4-16). Therefore, the previous result, Equation (4-20), can also be used to tune the PI controller in the regulating case.

CHAPTER 5

PERFORMANCES OF TUNED PID CONTROL BASED ON VARIOUS PARAMETER IDENTIFICATION TECHNIQUES AND TUNING FORMULAS

1. PRELIMINARY REMARKS

From previous chapters, various parameter identification techniques and PID controller tuning formulas were reviewed or developed. The object of this chapter then involves simulating tuned PID control systems so that those techniques can be evaluated. In detail, this chapter needs to discuss the following:

1. Look for a group of PID controller settings from various tuning formulas which is most recommended to use in application. If no such a group of settings exists, improvements on these formulas are needed.
2. Use the obtained best settings to check and compare the performances of the developed automated parameter identification technique and those graphical identification methods.
3. Use the FOL model related PI controller tuning formula to evaluate the performances of different identification techniques for the FOL model.
4. For the above items, both tracking and regulating problems are considered.

2. THE RECOMMENDED PID CONTROLLER SETTINGS

Tracking Problem

The commonly preferred control performance specification in time domain is that of damping ratio $\zeta=0.707$. In this case, the rise time, overshoot, and settling time of the transient response of the system will be most satisfactory if a trade-off of these performance indices is concerned. Although the damping ratio was firstly introduced in a second-order system, the control performances of a high-order system can also be qualified by checking its relative damping ratio [19]. This is because one can assume this damping ratio is mainly affected by a couple of dominant complex roots of the system.

For the purpose of comparison, the unit step responses of a second-order system with different damping ratios are shown in Figure 5-1.

The closed-loop step responses of experimental process 2 are plotted in Figure 5-2, where the PID and PI controllers are tuned using the IAE and ITAE formulas relating to the optimal FOLPD model of the process. Comparing Figure 5-2 with Figure 5-1, one may find that no step response in Figure 5-2 has achieved the condition of damping ratio $\zeta=0.707$. Then, improvements over this problem are needed although it is very hard to derive a tuning formula for PID controller settings based on the damping ratio criterion.

Recalling that increasing the proportional gain K_c will

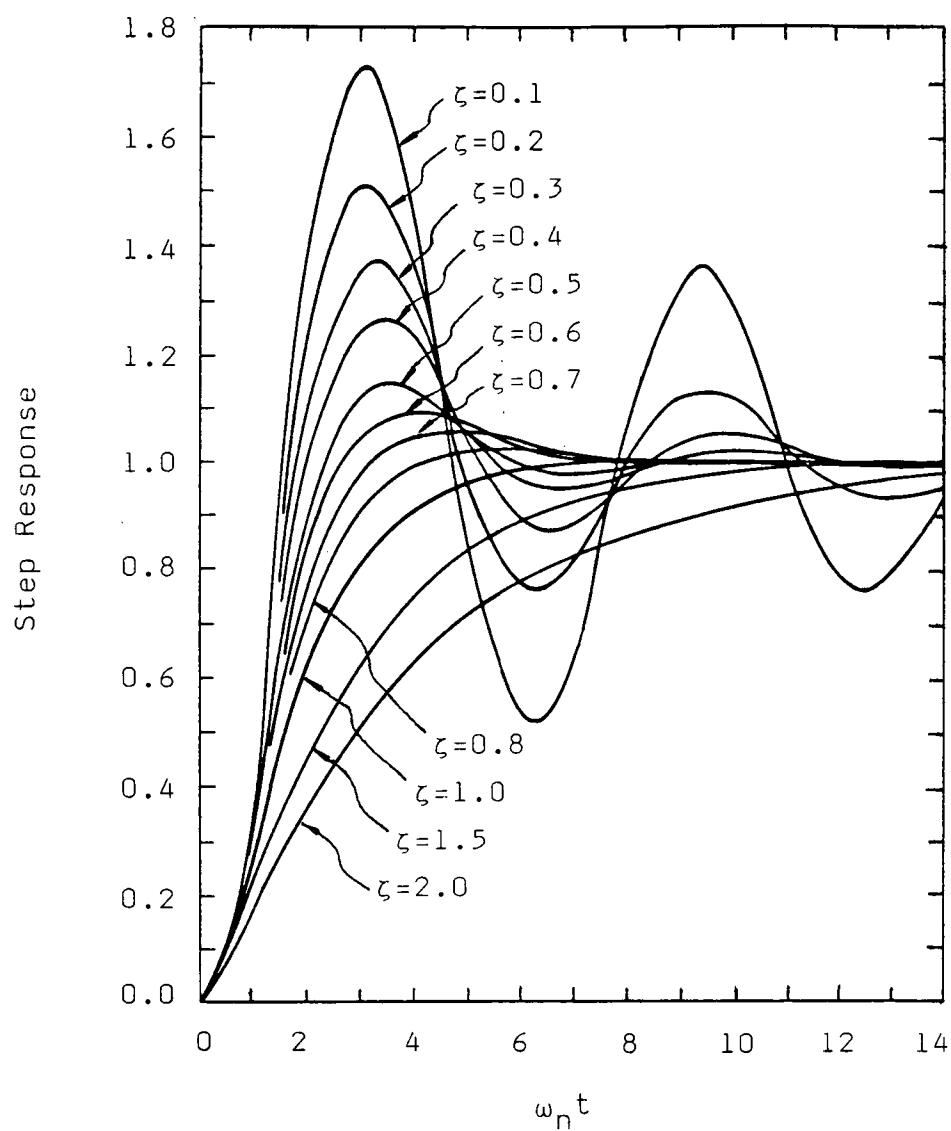


Figure 5-1. Transient Response of a Second-Order System to a Unit Step Input

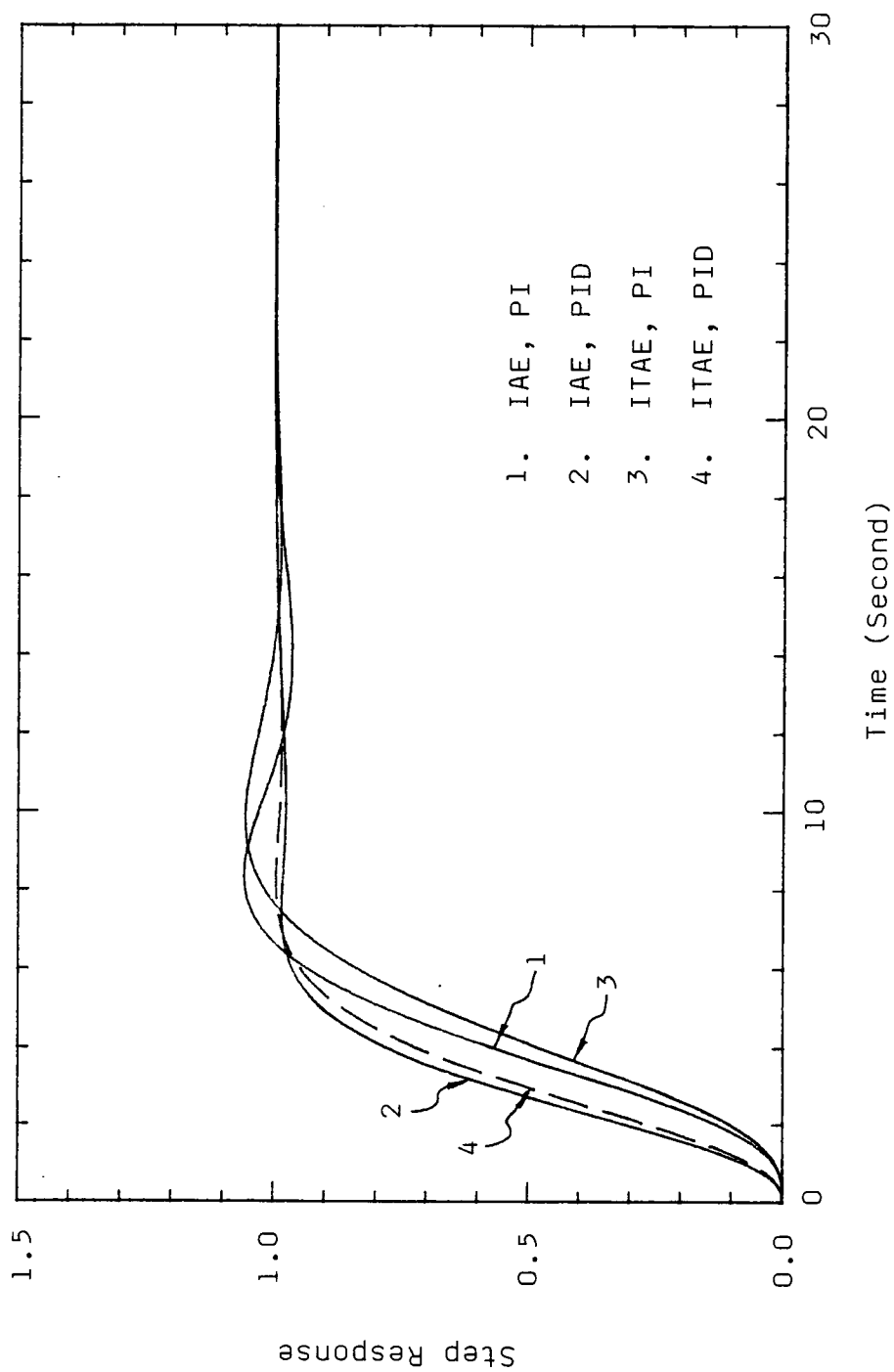


Figure 5-2. Response of Process 2 to a Unit Step Change in Set-Point with Different Controller Settings Relating to the Optimal FOLPD Model of the Process

quicken the transient response, it is wise to try different values of K_c in a considered good tuning formula so that a more adequate value of K_c might be found.

Figure 5-3 shows the closed-loop step response of process 2, where PID controller is tuned using the ITAE setting with different values of constant A_p in Equation (4-11). It can be seen that when $A_p=1.2$, the relative $\zeta=0.707$. Therefore, a considered best setting for PID controller in tracking problem can be recommended. It is termed Modified ITAE and presented in Table 5-1.

Figure 5-4 compares the Modified ITAE with some other settings.

Regulating Problem

Figure 5-5 plots the unit impulses responses of a second-order system with different damping ratios. For second-order systems, the response to a impulse change at set-point is similar to the response to a step change of disturbance. Therefore, Figure 5-5 might be used to show different conditions of damping ratio for the regulating problem.

However, the magnitude of impulse response is proportional to the natural undamped frequency ω_n . Then it is hard to determine the 0.707 damping ratio condition for high-order systems, since the ω_n which is mainly affected by a couple of dominant complex roots is not easy to find. Therefore, the 0.707 damping ratio criterion cannot be applied satisfactorily in the regulating case.

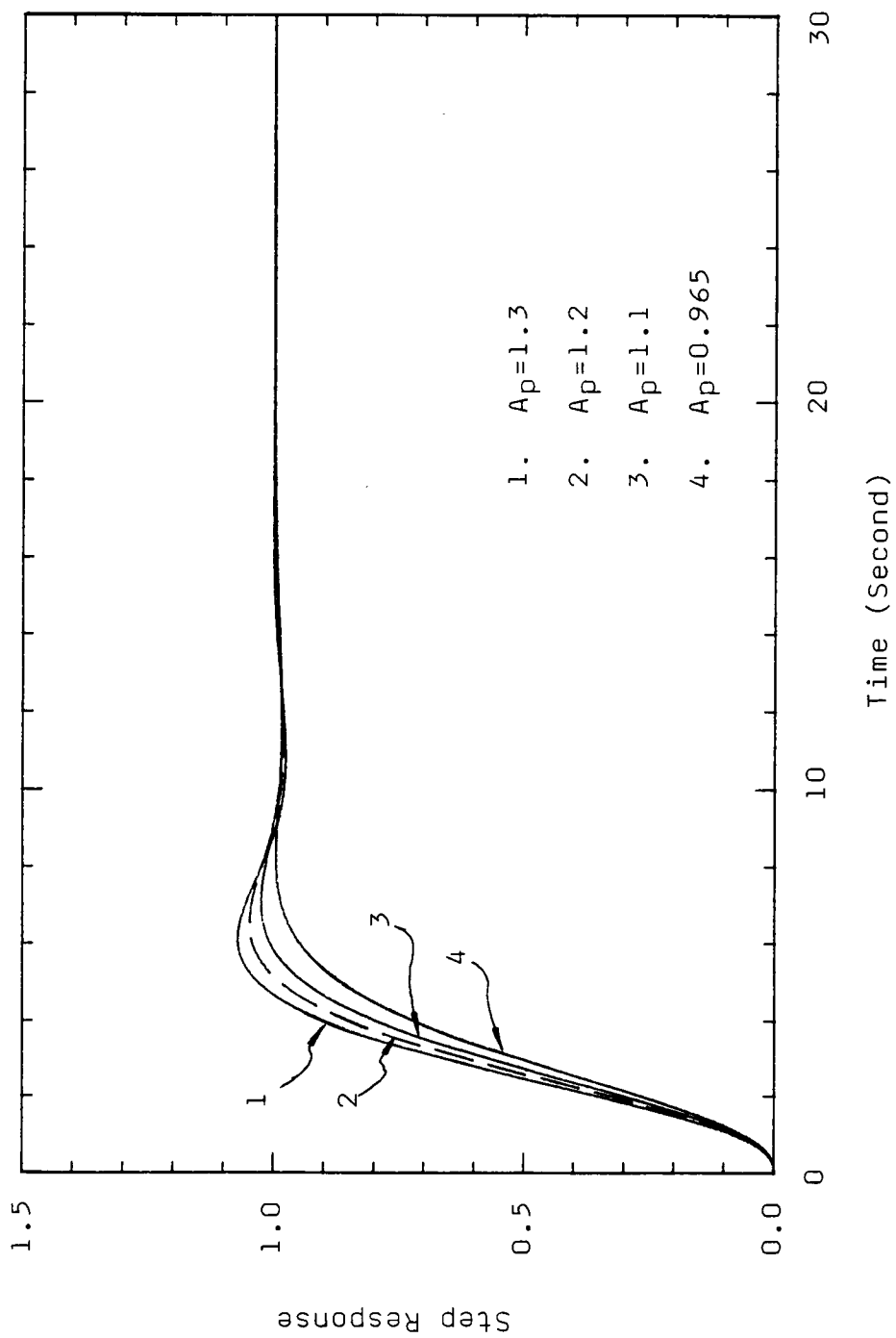


Figure 5-3. Comparison of Response to Set-Point Change for Different Values of Constant A_p Based on ITAE PID Setting (Refer to Table 4-3)

TABLE 5-1
THE RECOMMENDED PID CONTROLLER SETTING RELATING
TO FOLPD MODEL IN TRACKING PROBLEM^a

Criterion	Mode	A_p	B_p	A_i	B_i	A_d	B_d
Modified ITAE	PID	1.20	-0.855	0.796	-0.147	0.308	0.929

^aThe PID controller parameters K_c , T_i , and T_d are calculated from Equations (4-11), (4-14), and (4-13) respectively with the constants in the Table.

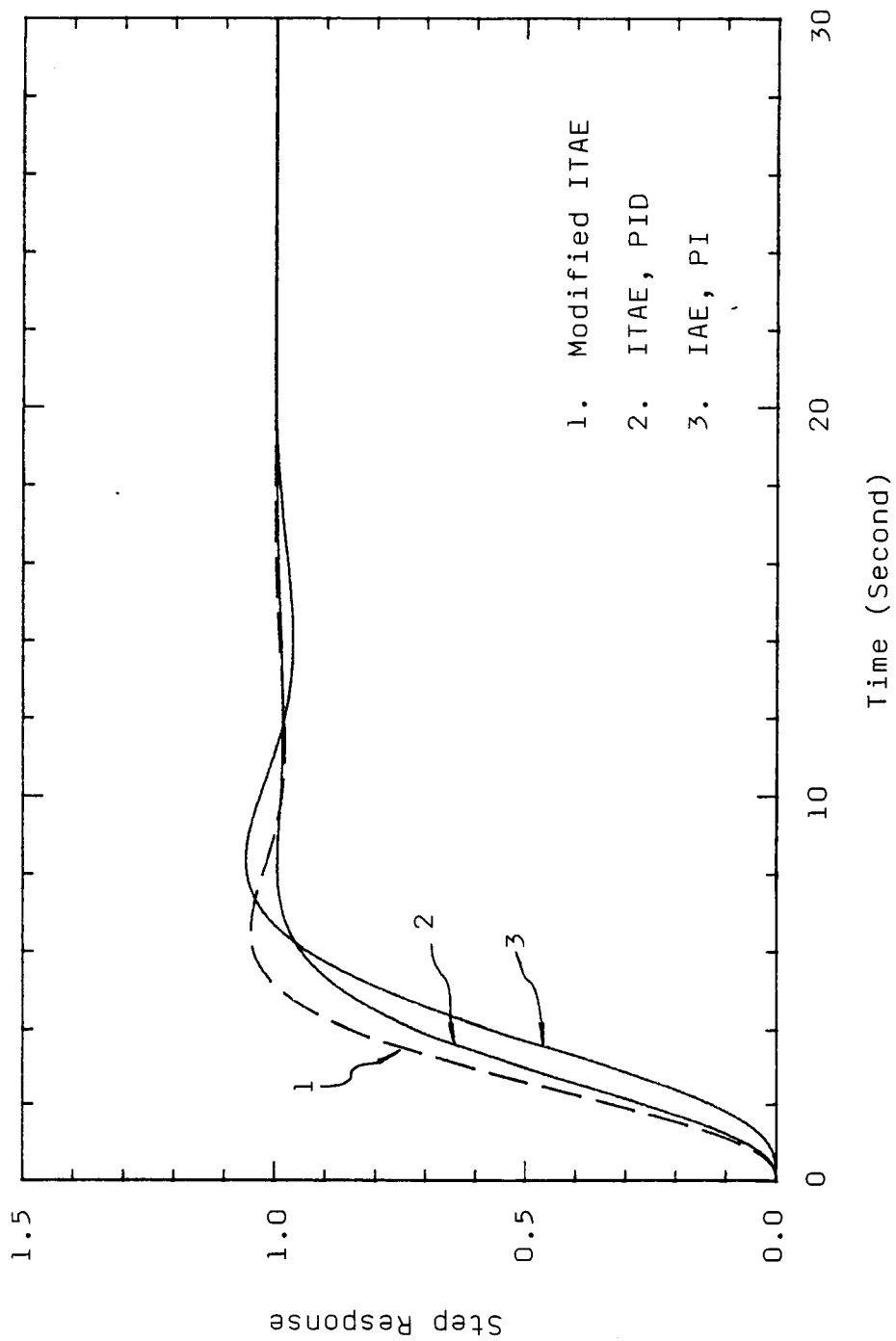


Figure 5-4. Comparison of Response to Set-point Change for Modified ITAE and Some Other Controller Settings

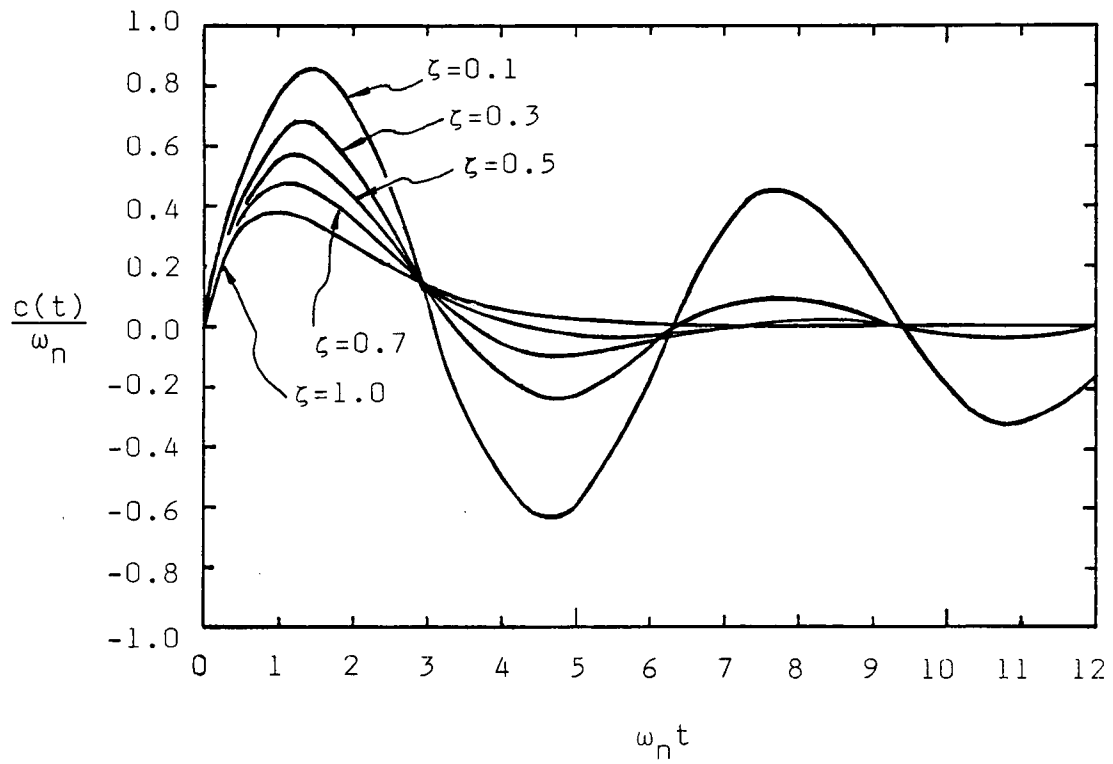


Figure 5-5. Transient Response of a Second-Order System to a Unit Impulse Input

Figure 5-6 shows the closed-loop response of process 2 to a unit step change of disturbance, where the PID controller is tuned by using different formulas relating to the optimal FOLPD model of the process. It can be seen that the IAE PID setting is the best one of the six in the Figure. However, there still exist possibilities to improve the setting for the purpose of decreasing the overshoots of the response.

Similar to the procedures done in the tracking problem, Figure 5-7 compares the disturbance responses for different values of constant A_p in Equation (4-11), where the IAE PID setting and the optimal FOLPD model are used. It is seen that when $A_p=3$, the disturbance response is the most satisfactory one of the six. Then, a considered best PID controller setting in regulating problem can be recommended. Termed Modified IAE, the setting is presented in Table 5-2.

Figure 5-8 compares the Modified IAE with some other settings. The improvement is obvious to see.

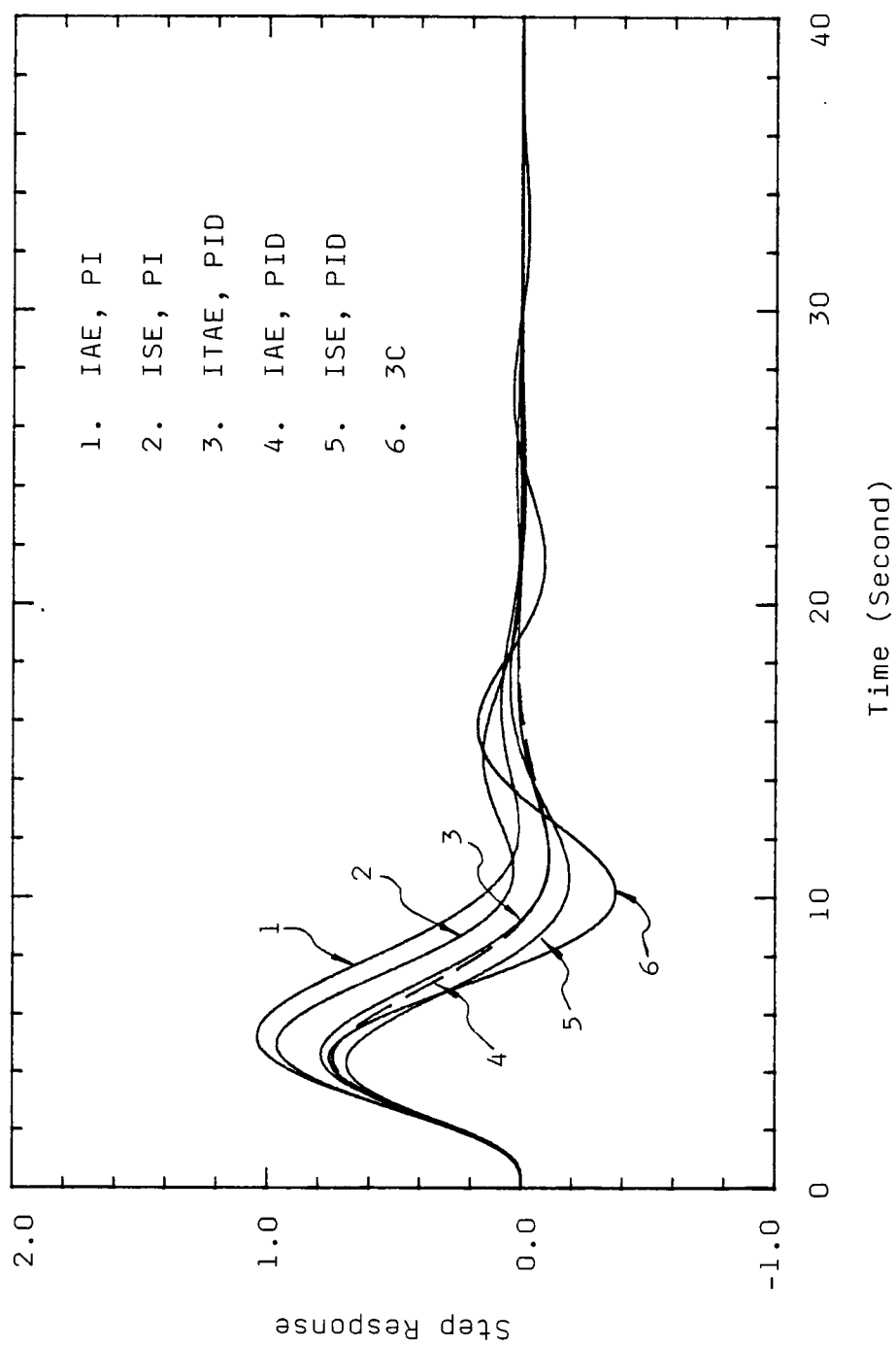


Figure 5-6. Response of Process 2 to a Unit Step Change of Disturbance with Different Controller Settings Relating to the Optimal FOLPD Model of the Process

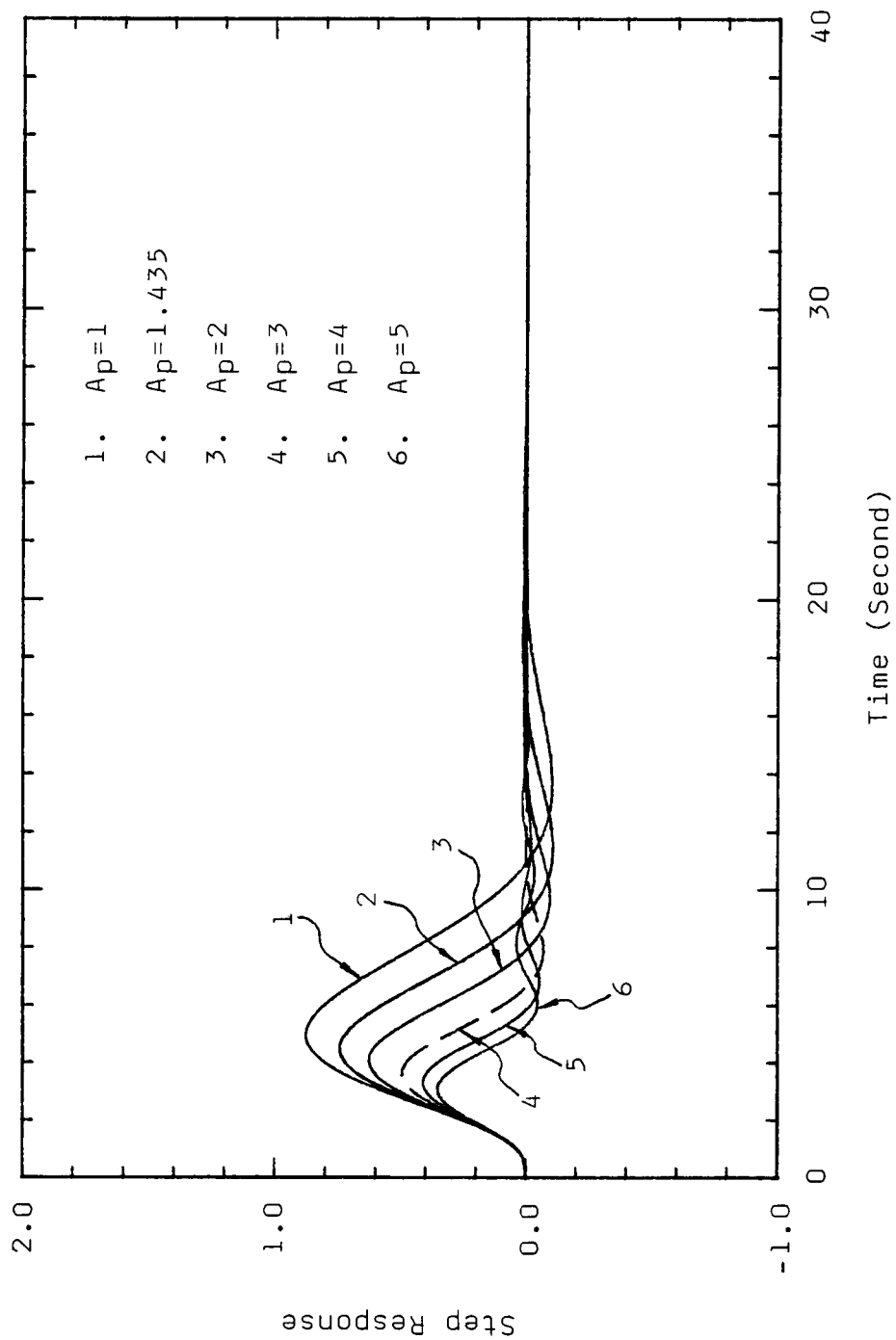


Figure 5-7. Disturbance Response Comparison For Different Values of Constant A_p Based on IAE PID Setting (Refer to Table 4-2)

TABLE 5-2
THE RECOMMENDED PID CONTROLLER SETTING RELATING
TO FOLPD MODEL IN REGULATING PROBLEM^a

Criterion	Mode	A_p	B_p	A_i	B_i	A_d	B_d
Modified IAE	PID	3.0	-0.921	0.878	-0.749	0.482	1.137

^aThe PID controller parameters K_c , T_i , and T_d are calculated from Equations (4-11), (4-12), and (4-13) respectively with the constants in the Table.

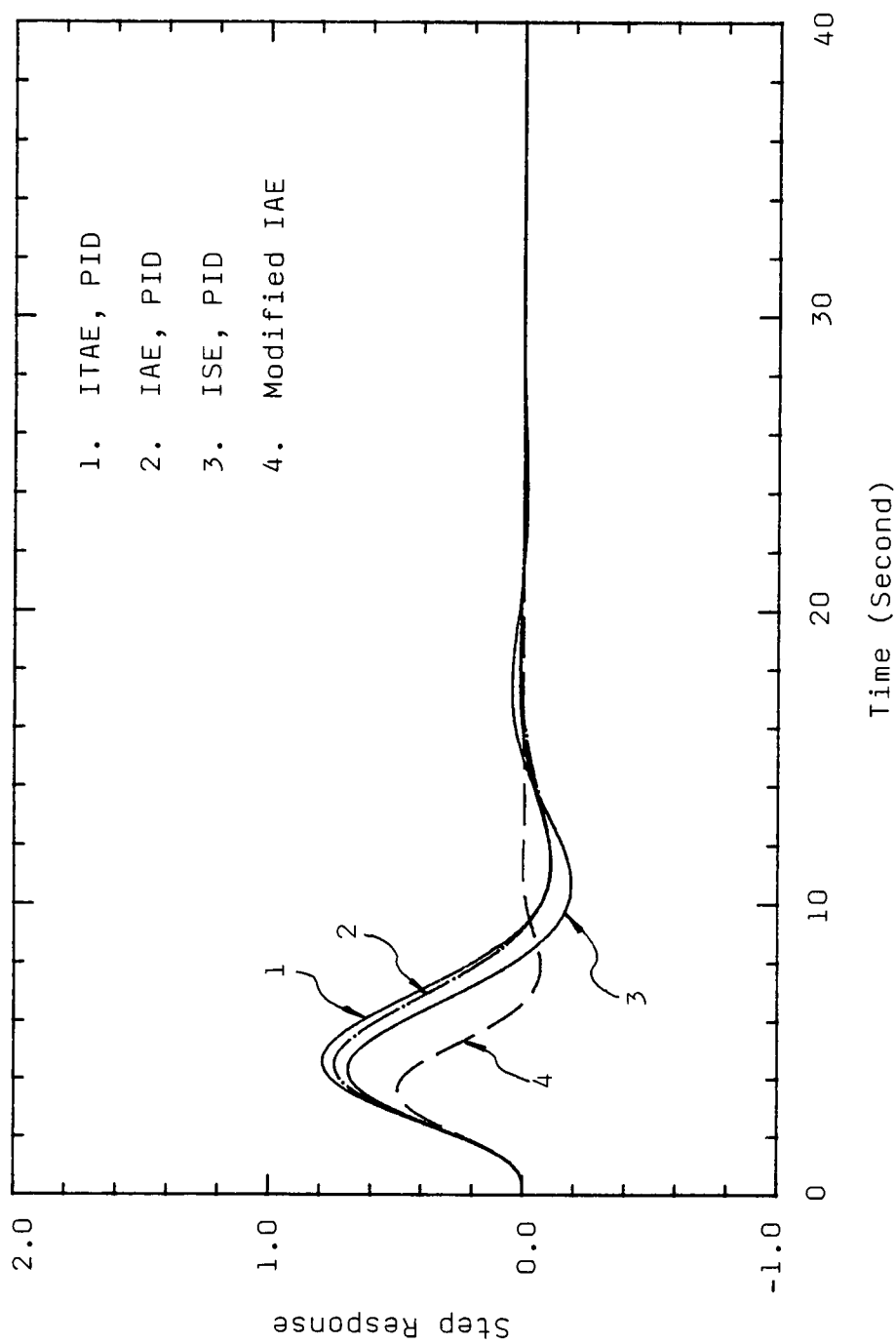


Figure 5-8. Disturbance Response Comparison For Modified IAE and Some Other Controller Settings

3. COMPARISON OF AUTOMATED IDENTIFICATION TECHNIQUE WITH GRAPHICAL IDENTIFICATION METHODS

Tracking Problem

By using the recommended PID controller settings, the control performances of PID control system relating to different FOLPD or FOL models of the process can be evaluated.

In Figure 5-9 and 5-10, responses of experimental process 2 and 3 to a unit step change in set-point are shown. PID controller is tuned using the Modified ITAE which is related to Fit 1, Fit 2, Fit 3, optimal, and noisy process optimal model. The noisy process optimal models used in this section are the models obtained in the heavy noise case (standard deviation $\sigma=1$).

In Figure 5-11 and 5-12, responses of process 4 and 5 to a unit step change in set-point are shown. The PI controller is tuned using the FOL model related tuning formula. The graphical, optimal, and noisy process optimal FOL models are compared.

Regulating Problem

In Figures 5-13 to 5-16, responses of processes 2 to 5 to a unit step change of disturbance are shown. Controllers are tuned using the Modified IAE or the FOL model related tuning formula. The graphical, optimal, and noisy process optimal models are compared correspondingly.

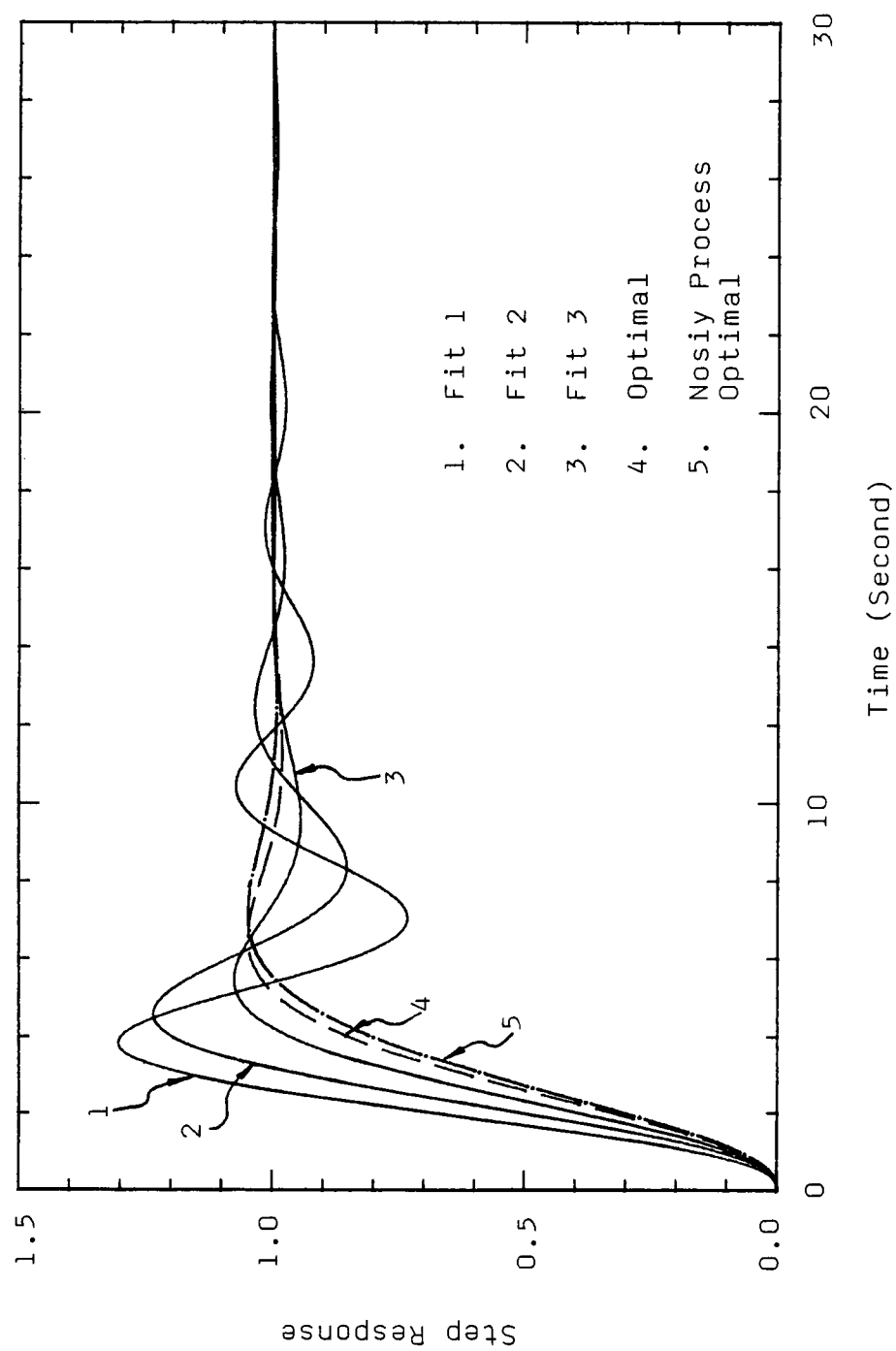


Figure 5-9. Control Performance Comparison for Different FOLPD Models of Process 2 in Tracking Problem

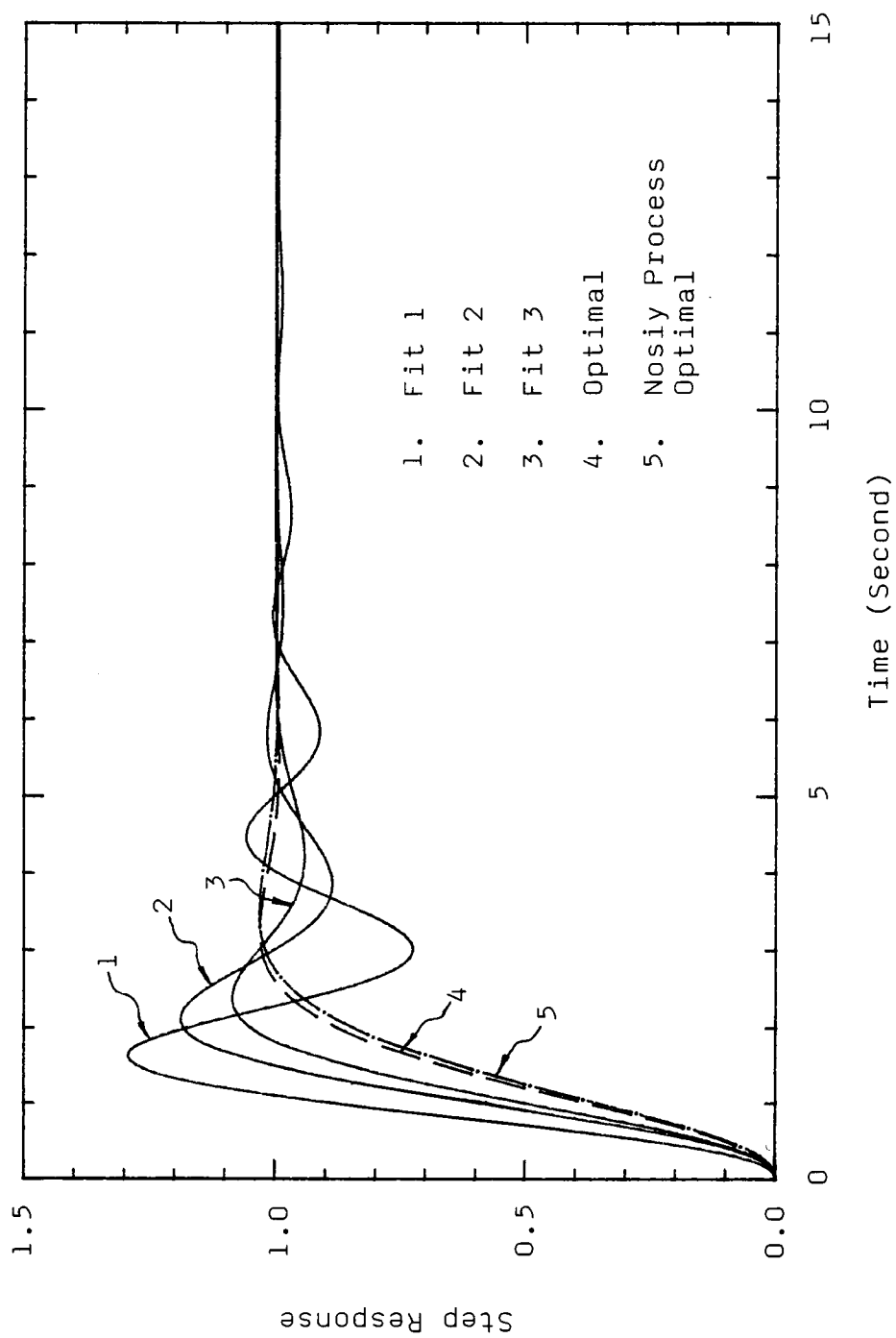


Figure 5-10. Control Performance Comparison for Different FOLPD Models of Process 3 in Tracking Problem

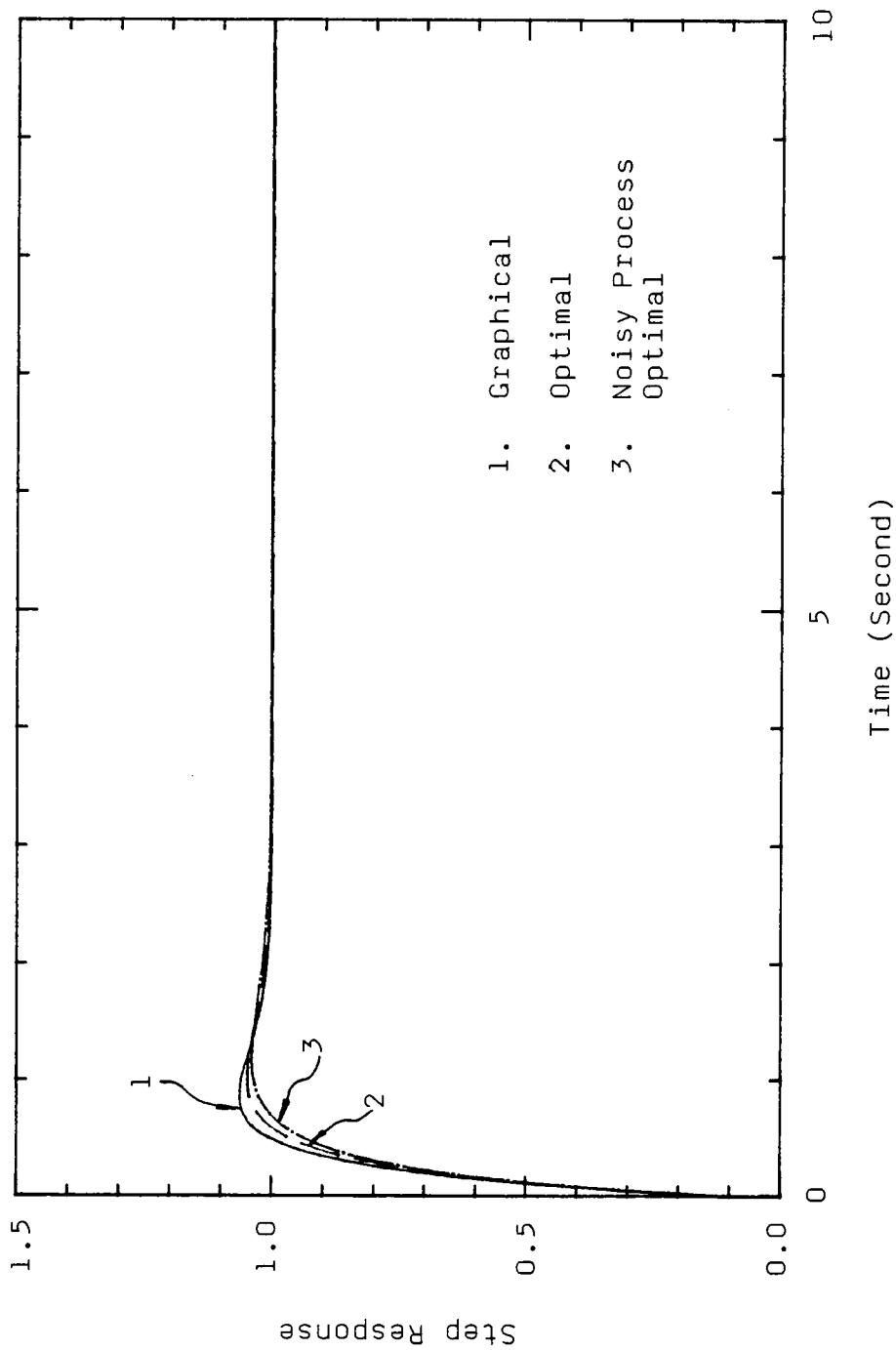


Figure 5-11. Control Performance Comparison for Different FOL Models of Process 4 in Tracking Problem

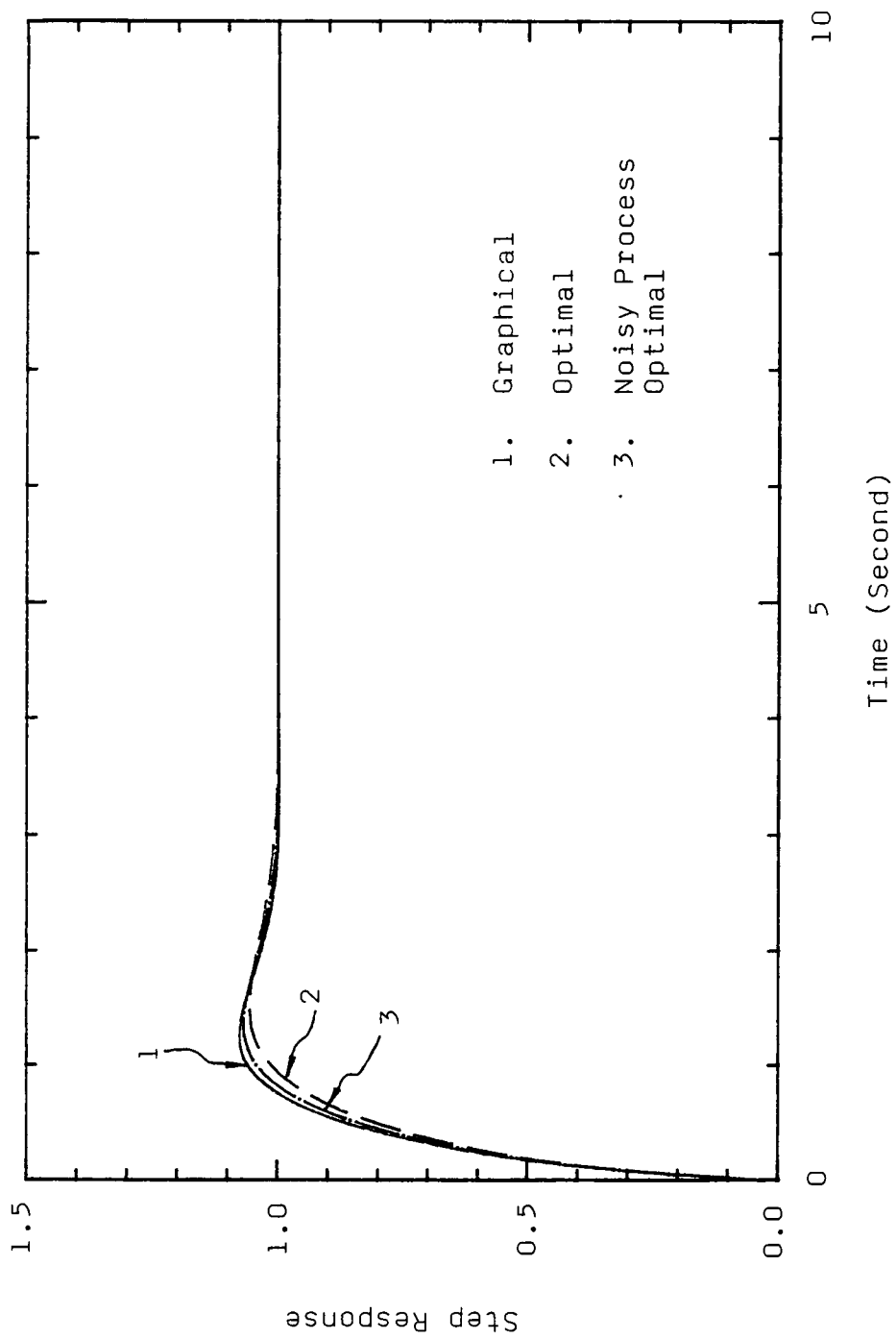


Figure 5-12. Control Performance Comparison for Different FOL Models of Process 5 in Tracking Problem

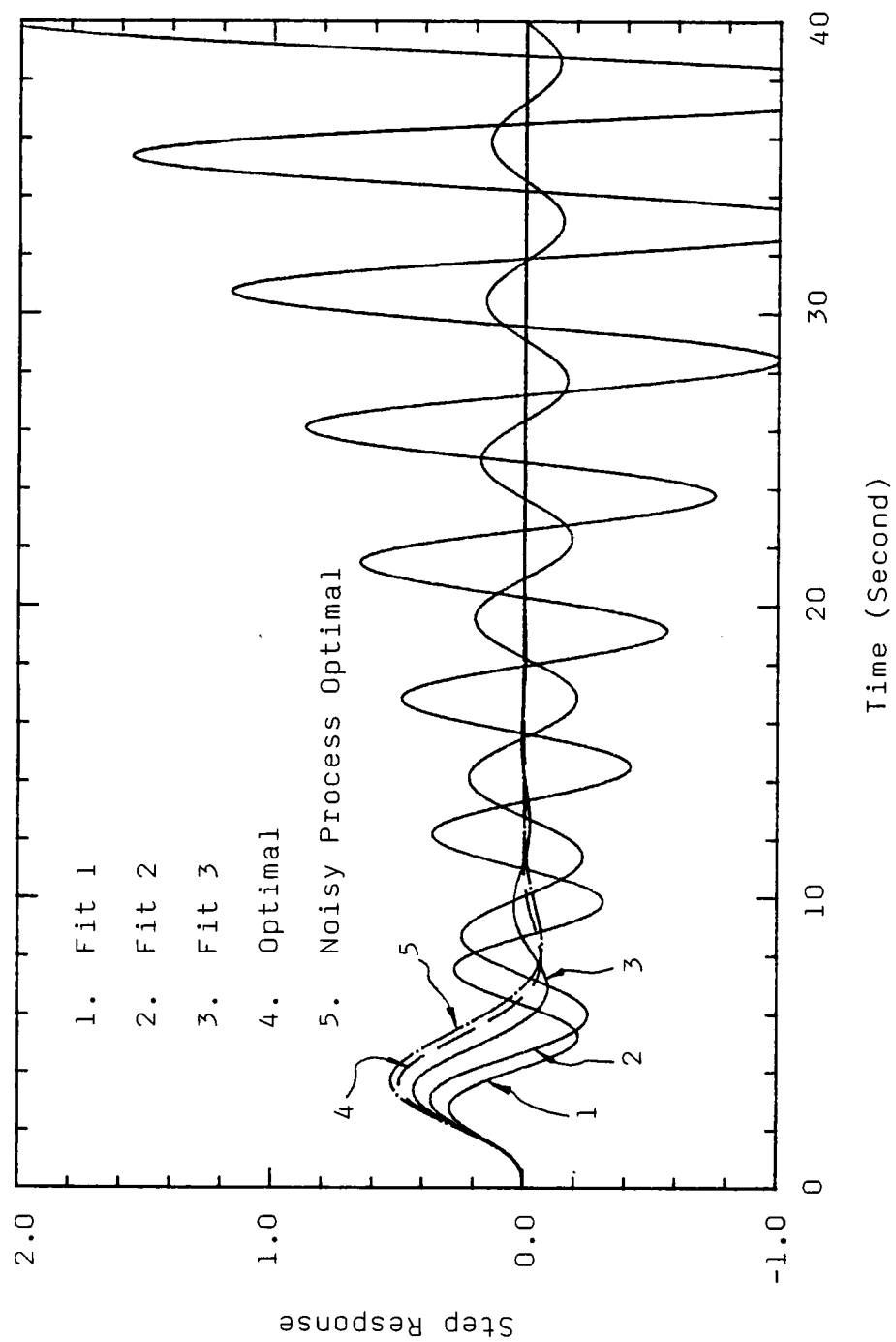


Figure 5-13. Control Performance Comparison for Different FOLPD Models of Process 2 in Regulating Problem

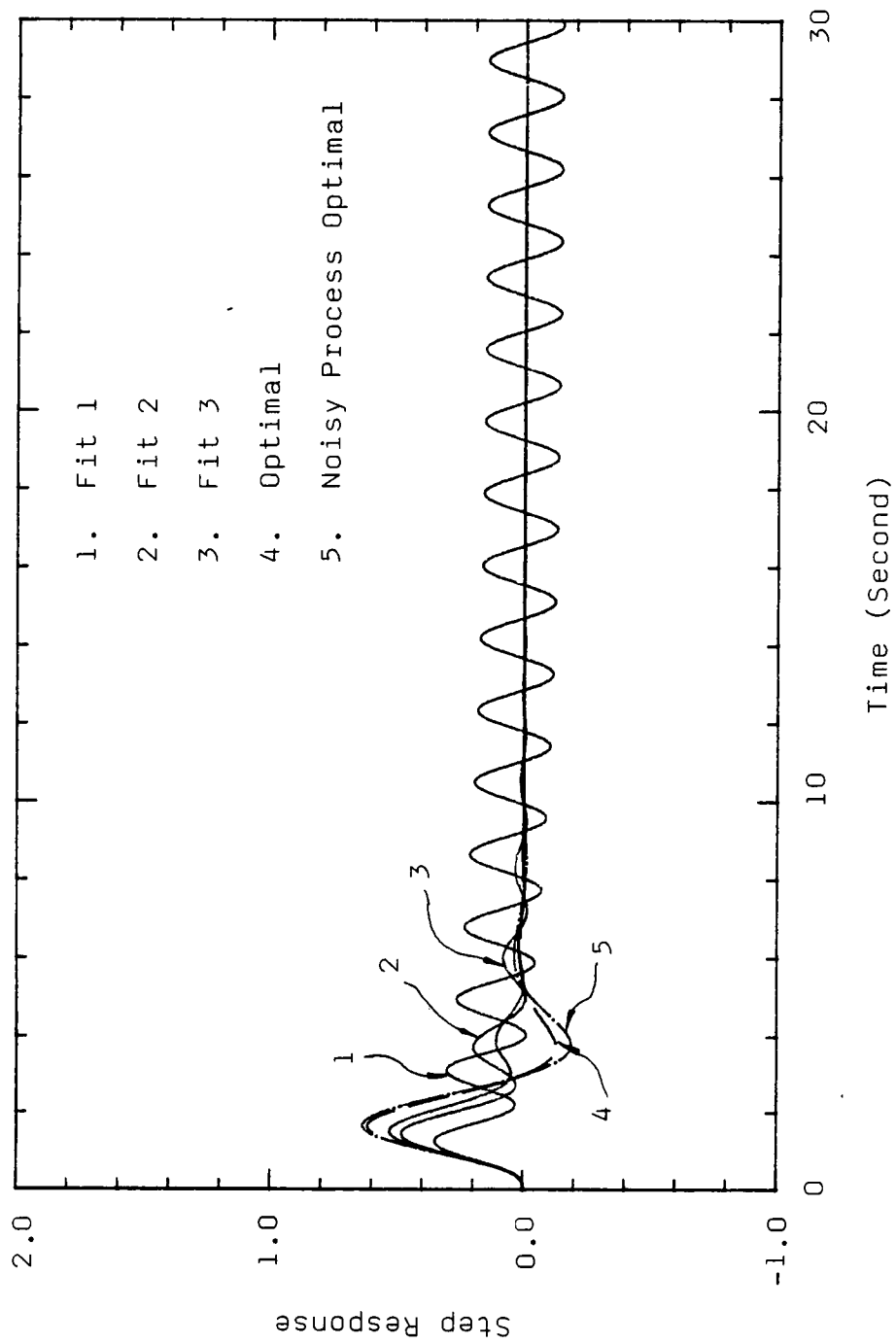


Figure 5-14. Control Performance Comparison for Different FOLPD Models of Process 3 in Regulating Problem

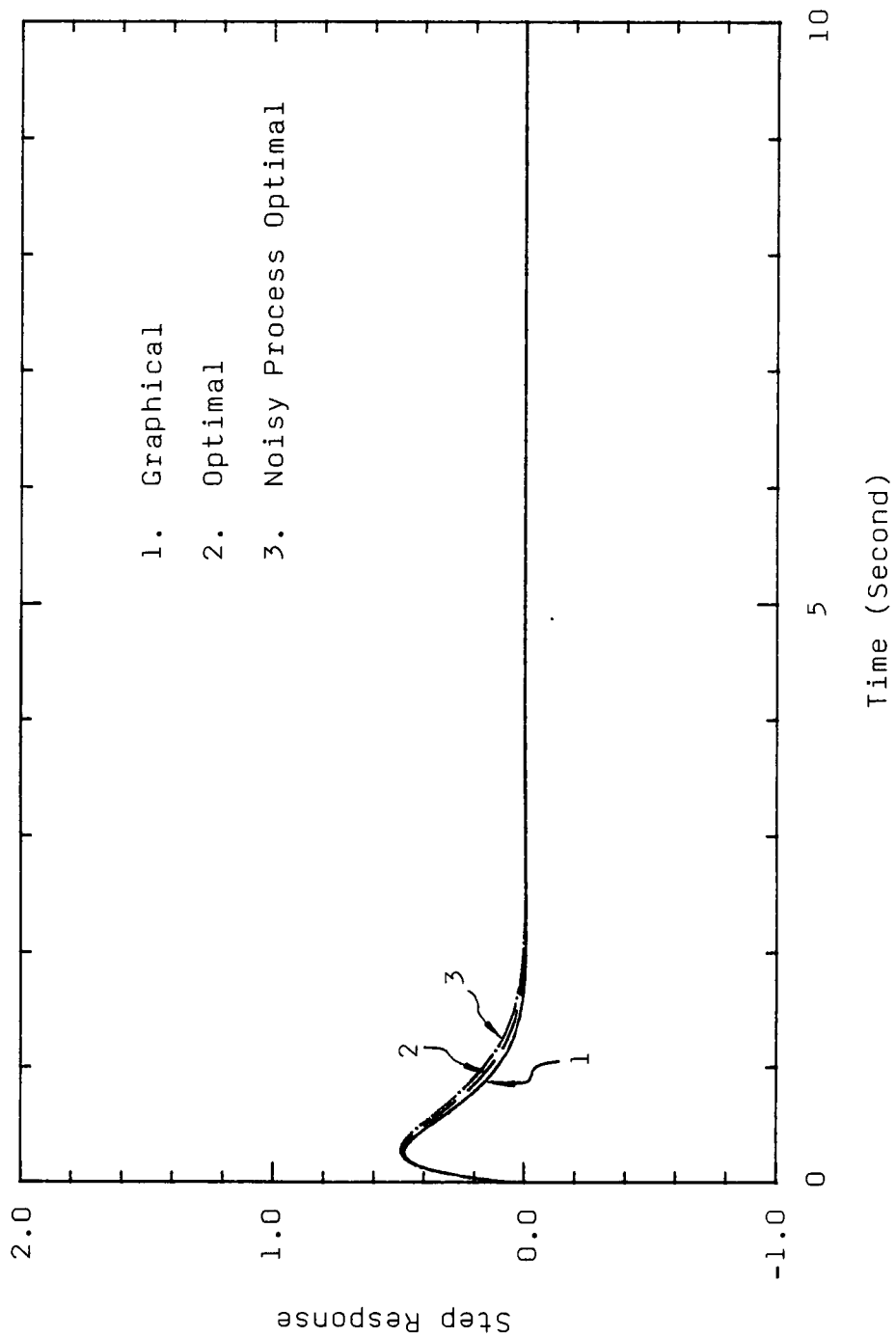


Figure 5-15. Control Performance Comparison for Different FOL Models of Process 4 in Regulating Problem

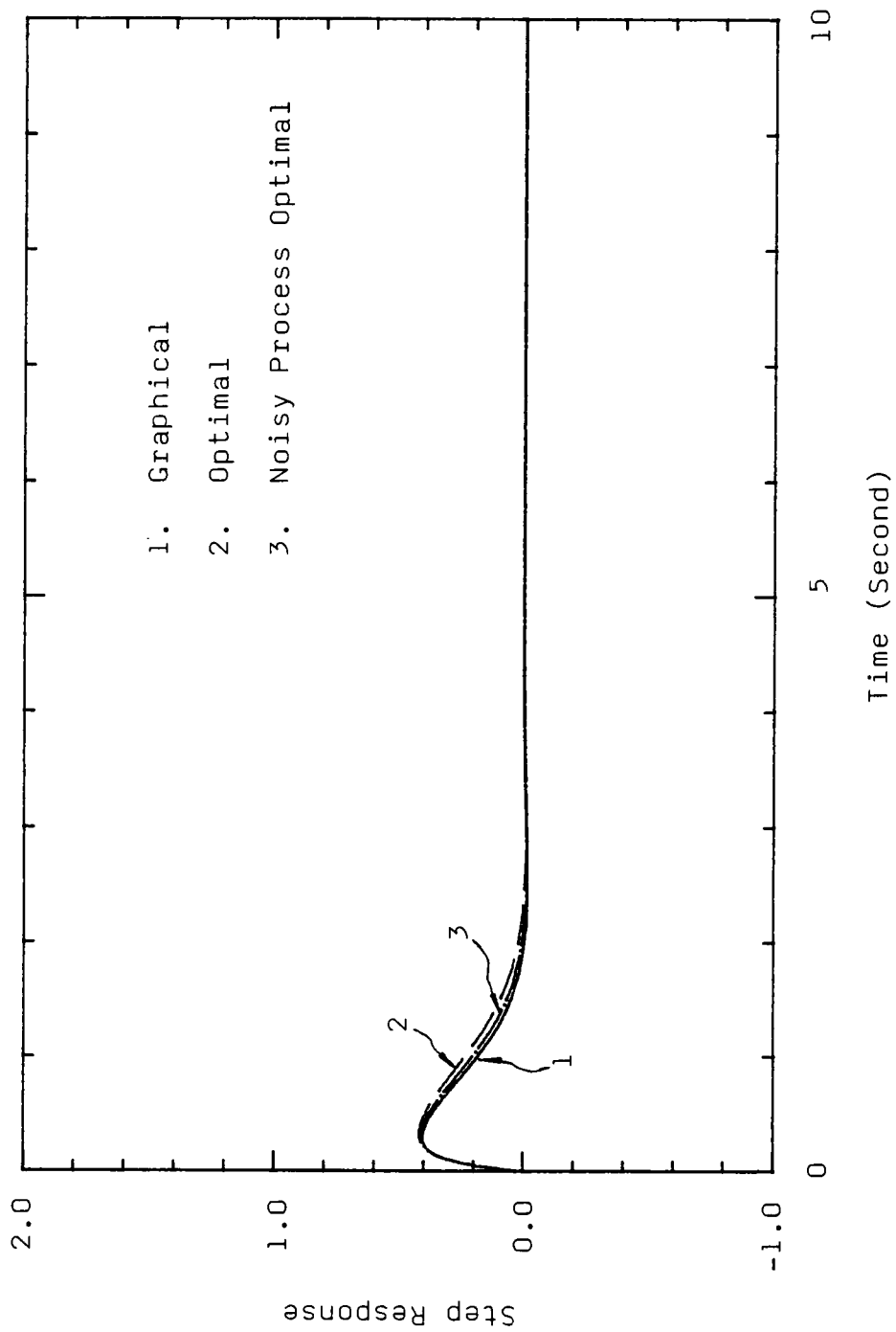


Figure 5-16. Control Performance Comparison for Different FOL Models of Process 5 in Regulating Problem

4. CHAPTER SUMMARY

Through various experiments, it can be concluded that:

- 1) for tracking problem, the PID control systems tuned by using the Modified ITAE setting relating to the optimal FOLPD models will provide an optimal control performance in the sense of relative damping ratio equal 0.707;
- 2) for regulating problem, the PID control systems tuned by using the Modified IAE setting relating to the optimal FOLPD models will provide a much better control performance than that tuned by using any other tuning formulas and FOLPD models;
- 3) for both tracking and regulating problem, the PI control systems tuned by using the FOL model related PI tuning formula will have an optimal control performance in the sense of damping ratio equal 0.707 (for those processes having optimal FOL models only);
- and 4) the steady-state error in each case is very small or equal to zero due to the integral action of PID control.

In addition, it is seen that the performances of graphical models, say Fit 1, Fit 2, and Fit 3, are not satisfactory in most cases even though the optimal controller settings are used. This implies the fact that the parameter identification in control system applications is the first important thing. If the parameters of the approximation model of the process cannot be determined adequately, the PID control performances will be very sensitive to controller settings and variation of process parameters.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

1. CONCLUSIONS

This study has achieved two main developments in PID control theory and application.

In the parameter identification problem, a so-called automated least-squares parameter identification algorithm has been developed and implemented. By using this technique, one can approximate most stable single-input-single-output processes by the first-order-lag-plus-delay or the first-order-lag model. The advantages of this technique include the following: 1) it has no restrictions for the parameters of the model; 2) it optimizes the model based on the least-squares curve-fitting criterion; 3) it has good filtering properties so that the effect of measurements random noise can be eliminated; 4) it can be implemented with a desktop computer and the program execution time is rather short; and 5) it can be used in controller self-tuning fashion.

In the PID controller parameter setting problem, the FOLPD model related PID controller tuning formulas have been improved for both tracking and regulating problems. Moreover, the FOL model related PI controller tuning formula has been proposed. In these formulas, the commonly preferred

control performance specification, the 0.707 damping ratio criterion, has been applied.

Through various experiments, the identification algorithm has been implemented, and control systems have been simulated. The results are very encouraging.

2. RECOMMENDATIONS

With the advent of digital computers during past decades and rapid development of microcomputers, conventional control systems have been changed a lot. Many continuous systems have been replaced by discrete systems and analog controllers have been substituted by microprocessor-based controllers. This is the trend which is expected to continue in the years to come.

The results from this study have revealed a possibility of the development of a computer-microprocessor-based control system. As shown in Figure 6-1, the configuration of this system is quite simple. It is just a general discrete-time control system plus a control computer.

The main points of this thinking are as the following:

- 1) armed with the automated least-squares parameter identification algorithm, the computer can automatically find the optimal FOLPD or FOL models for most stable SISO processes;
- 2) the microprocessor-based PID controller then can be self-tuned by the computer using the proposed PID or PI controller tuning relations based on the 0.707 damping ratio criterion.

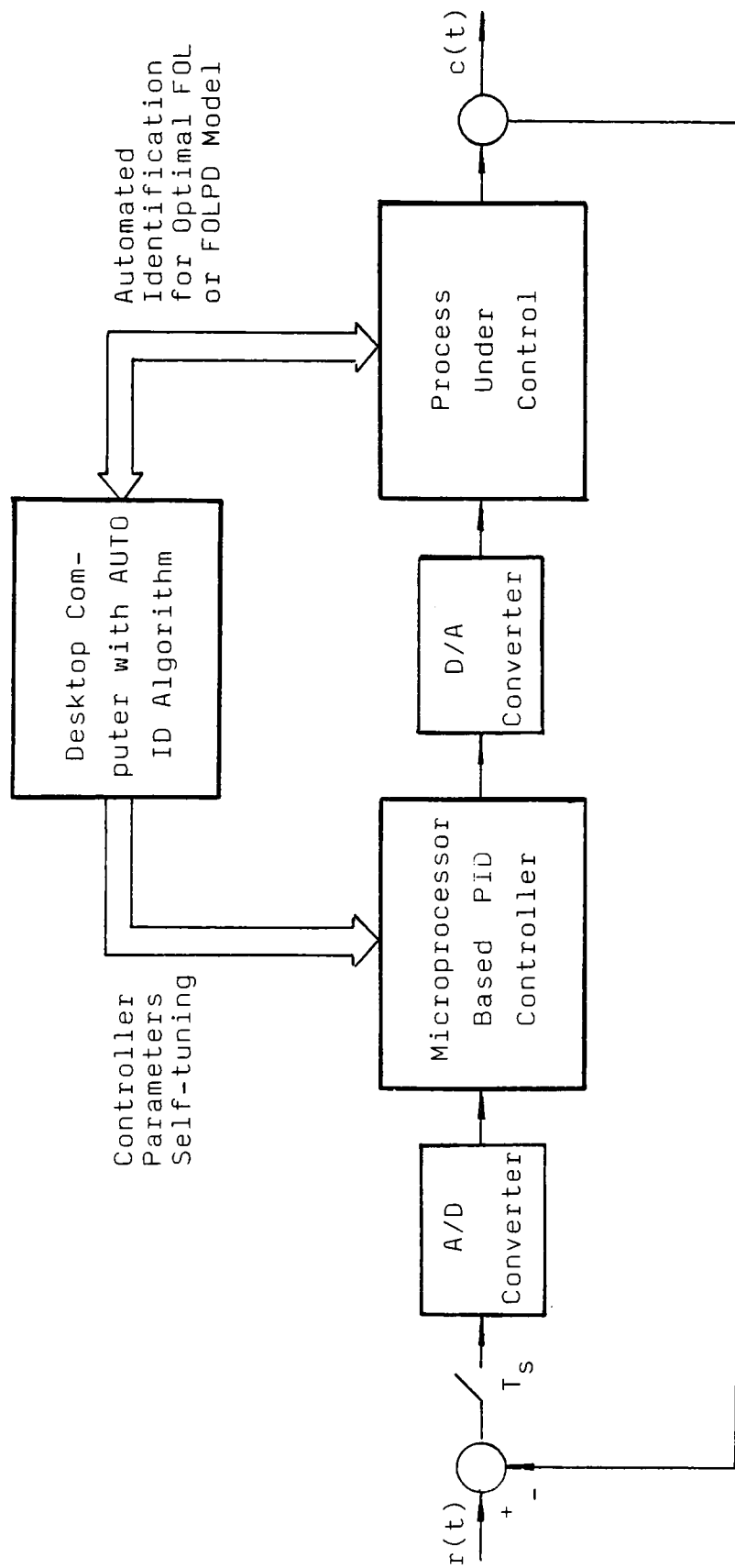


Figure 6-1. Block Diagram of a Computer-Microprocessor-Based Self-Tuning Control System

The prospective advantages of this control scheme are as follows: 1) equipped with one desktop computer and n microprocessor-based PID controllers, n SISO processes can be controlled in this configuration at same time; 2) in the multiloop case, this idea can also be accepted if the decoupling problem is solved; 3) from a commercial point of view, the identification algorithm can be written in a package and the cost for this kind of control system will not be high; and 4) it can be widely used in many areas since the control performances for most high-order processes are good and no human design and tuning is needed.

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APPENDICES


```

300 GOSUB 2010
310 ! *** CALCULATE THE STEP RESPONSE OF THE EXPERIMENTAL PROCESS 2 ***
320 LET T(1)=0
330 FOR I=1 TO 255
340   C(I)=2*(1+(2*T(I)+4)*EXP(-T(I))-16/3*EXP(-.5*T(I))+1/3*EXP(-2*T(I)))
350   C(I)=C(I)+X1(I)
360   T(I+1)=T(I)+.04
370 NEXT I
380 ! *** CALCULATE THE GAIN K VIA SUBROUTINE 1 ***
390 GOSUB 1220
400 PRINT "SAMPLING TIME IS:",T(*)
410 PRINT "THE STEP RESPONSE OF THE PROCESS IS:",C(*)
420 PRINT "K IS:",K
430 LET D=.2
440 ! *** SET THE FIRST THREE ASSUMED VALUES OF DELAY TIME
450 !
460 LET B(1)=.5
470 LET B=B(1)
480 GOSUB 1370
490 LET E(1)=E
500 LET A(1)=A
510 LET B(2)=B(1)+20*D
520 LET B=B(2)
530 GOSUB 1370
540 LET E(2)=E
550 LET A(2)=A
560 LET B(3)=B(1)+10*D
570 LET B=B(3)
580 GOSUB 1370
590 LET E(3)=E
600 LET A(3)=A
610 ! *** LOOP FOR FINDING THE OPTIMAL PARAMETERS ***
620 LET Error=500
630 FOR J=1 TO 6
640   IF E(3)>=E(1) THEN C1
650   IF E(3)>E(2) THEN C2

```

```

660 C3: Base=B(3)
670 GOTO 710
680 C1: Base=B(1)
690 GOTO 710
700 C2: Base=B(2)
710 ! *** TRY THE 1ST POINT ***
720 LET B=Base-5*D
730 GOSUB 1370
740 LET E(1)=E
750 LET A(1)=A
760 LET B(1)=B
770 IF Error<E THEN 810
780 LET Error=E
790 LET Tcr=A
800 LET Delay=B
810 ! *** TRY THE 2ND POINT ***
820 LET B=Base+5*D
830 GOSUB 1370
840 LET E(2)=E
850 LET A(2)=A
860 LET B(2)=B
870 IF Error<E THEN 910
880 LET Error=E
890 LET Tcr=A
900 LET Delay=B
910 ! *** TRY THE 3RD POINT ***
920 IF Base=B(3) THEN 1040
930 LET B=Base
940 GOSUB 1370
950 LET E(3)=E
960 LET A(3)=A
970 LET B(3)=B
980 IF Error<E THEN 1040
990 LET Error=E
1000 LET Tcr=A
1005 LET Delay=B

```

```

1010 IF B>0 THEN 1040
1020 LET Delay=0          ! B<=0, USE FOL MODEL (DELAY TIME IS ZERO)
1030 GOTO 1060
1040 LET D=.5*D
1050 NEXT J
1060 LET T=1/A
1070 ! *** CALCULATIONS ARE FINISHED, PRINT THE RESULTS ***
1080 PRINT "THESE ARE THE PARAMETERS OF THE FOLPD MODEL"
1090 PRINT "DC GAIN K IS:",K
1100 PRINT "A IS:",Tcr
1110 PRINT "TIME CONSTANT T IS:",T
1120 PRINT "DELAY TIME B IS:",Delay
1130 PRINT "LEAST-SQUARES ERROR E IS:",Error
1140 !
1150 PAUSE
1160 !
1170 ! *** GOTO PLOTTING SUBROUTINE ***
1180 GOSUB 1220
1190 !
1200 STOP
1210 !
1220 ! *** SUBROUTINE 1 FOR CALCULATING DC GAIN K ON LINE ***
1230 !
1240 LET Sum=0
1250 LET I=1
1260 FOR Time=50 TO 100 STEP .1
1270   Output=1+(2*Time+4)*EXP(-Time)-16/3*EXP(-.5*Time)+1/3*EXP(-2*Time)
1280   Output=2*Output+X1(I)
1290   I=I+1
1300   Sum=Sum+Output
1310 NEXT Time
1320 K=Sum/500
1330 PRINT "THE IDENTIFIED DC GAIN K IS:",K
1340 RETURN
1350 ! *** THE END OF SUBROUTINE 1 ***
1360 !

```

```

1370 ! *** SUBROUTINE 2 FOR THE LEAST-SQUARES CALCULATIONS ***
1380 !
1390 LET E=0
1400 IF B>=0 THEN 1420
1410 LET B=0
1420 FOR I=1 TO 254
1430 Z(I)=ABS(1-C(I))/K
1440 Z(I)=LOG(Z(I))
1450 H(I)=B-T(I)
1460 IF H(I)<=0 THEN 1480
1470 H(I)=0
1480 S(I,1)=H(I)
1490 U(I,1)=Z(I)
1500 NEXT I
1510 MAT P=TRN(S)
1520 MAT Q=P*S
1530 MAT R=INV(Q)
1540 MAT V=P*U
1550 MAT W=R*V
1560 A=W(1,1)
1570 FOR I=1 TO 254
1580 Y(I)=K*(1-EXP(A*H(I)))
1590 D(I)=(C(I)-Y(I))^2
1600 E=E+D(I)
1610 NEXT I
1620 PRINT "*****"
1630 PRINT "UPDATE A IS:", A
1640 PRINT "UPDATE B IS:", B
1650 PRINT "UPDATE E IS:", E
1660 RETURN
1670 ! *** THE END OF SUBROUTINE 2 ***
1680 !
1690 ! *** SUBROUTINE 3 FOR PLOTTING ***
1700 ! THIS SUBROUTINE PLOTS THE STEP RESPONSE OF THE PROCESS AND ITS MODEL
1710 !
1720 PLOTTER IS 7,5,"9872A"

```

```

1730 LIMIT 50,50+170,60,60+110
1740 FRAME
1750 Xmin=0
1760 Xmax=15
1770 Ymin=-.5
1780 Ymax=2.5
1790 SCALE Xmin,Xmax,Ymin,Ymax
1800 AXES 1,.1,Xmin,Ymin,5,5,5
1810 AXES 1,.1,Xmax,Ymax,5,5,5
1820 MOVE 0,0
1830 LET I=1
1840 FOR X=Xmin TO Xmax STEP .05
1850   Y=2*((1+(2*X+4)*EXP(-X)-16/3*EXP(-.5*X)+1/3*EXP(-2*X)))+X1(I)
1860   DRAW X,Y
1870   I=I+1
1880 NEXT X
1890 MOVE 0,0
1900 FOR X=Xmin TO Xmax STEP .05
1910   IF X>=B THEN 1940
1920   Y=0
1930   GOTO 1950
1940   Y=K*(1-EXP(-A*(X-B)))
1950   DRAW X,Y
1960 NEXT X
1970 PEN 0
1980 RETURN
1990 ! *** THE END OF SUBROUTINE 3 ***
2000 !
2010 ! *** SUBROUTINE 4 ***
2020 ! THIS SUBROUTINE GENERATES A WHITE GAUSSIAN NOISE WITH ZERO MEAN AND
2030 ! NORMAL DEVIATIONS
2040 !
2050 DIM X1(301),X2(301)
2060 PRINT "THE WHITE GAUSSIAN SIGNALS ARE:"
2070 N=300
2080 I=1

```

```

2090 ! *** GENERATE THE WHITE GAUSSIAN NOISE ***
2100 U1=RND
2110 U2=RND
2120 V1=2*U1-1
2130 V2=2*U2-1
2140 F=V1*V1+V2*V2
2150 IF F>1 THEN 2100
2160 F1=-2/F
2170 F2=F1*LOG(F)
2180 F3=SQR(F2)
2190 G=1
2200 X1(I)=G*V1*F3
2210 X2(I)=G*V2*F3
2220 PRINT I, X1(I), X2(I)
2230 I=I+1
2240 IF I<N+1 THEN 2100
2250 ! *** FIND THE MEAN OF NOISE ***
2260 Q1=0
2270 Q2=0
2280 FOR K=1 TO N
2290 Q1=Q1+X1(K)
2300 Q2=Q2+X1(K)
2310 NEXT K
2320 Q1=Q1/N
2330 Q2=Q2/N
2340 PRINT
2350 PRINT "MEAN OF X1 IS:", Q1, "MEAN OF X2 IS:", Q2
2360 ! *** FIND THE STANDARD DEVIATIONS ***
2370 P1=0
2380 P2=0
2390 FOR K=1 TO N
2400 P1=P1+(X1(K)-Q1)^2
2410 P2=P2+(X2(K)-Q2)^2
2420 NEXT K
2430 P1=P1/N
2440 P2=P2/N

```

! CONSTANT FOR ADJUSTING DEVIATIONS

```
2450      P1=SQR(P1)
2460      P2=SQR(P2)
2470 PRINT "S.D. OF X1 IS:",P1,"S.D. OF X2 IS:",P2
2480 RETURN
2490 ! *** THE END OF SUBROUTINE 4 ***
2500 !
2510 !
2520 END
```

APPENDIX 2

THE PROGRAM FOR THE SIMULATION OF THE CLOSED-LOOP PID CONTROL
SYSTEM IN TRACKING PROBLEM

```

10 *****
20 *
30 *
40 *
50 *
60 *
70 *
80 *
90 *
100 *
110 *
120 *
130 *
140 *
150 *
160 *
170 *
180 *
190 *
200 *
210 *****
220 *
230 *
240 *
250 *
260 *
270 *
280 *
290 *

      PROGRAM NAME: SIMU1
      THIS PROGRAM IS USED TO SIMULATE A FEEDBACK CONTROL SYSTEM
      WITH A STABLE SISO PROCESS AND A PID CONTROLLER. THE SYSTEM CLOSED-
      LOOP RESPONSE TO A STEP CHANGE AT SET-POINT CAN BE CALCULATED AND
      PLOTTED. THE PROGRAM IS ABLE TO SIMULATE PROCESSES UP TO 4TH ORDER.
      THE FOLPD OR THE FOL MODEL RELATED PID CONTROLLER TUNING TECHNIQUES
      ARE USED TO TUNE THE CONTROLLER.
      THE FOLLOWING NOTATIONS ARE USED:
      K: PROCESS DC GAIN
      C0, C1, D0, D1, D2, D3, D4: PROCESS PARAMETERS
      Km: DC GAIN OF THE FOLPD MODEL
      Tc: TIME CONSTANT OF THE MODEL
      To: DELAY TIME OF THE MODEL
      Ap, Bp, Ai, Bi, Ad, Bd: CONSTANTS FOR CONTROLLER SETTING
      Kc, Ti, Td: PID PARAMETERS
      R(I): SET-POINT INPUT
      Y(I): STSTEM CLOSED-LOOP RESPONSE

*****
*** DIMENSION STATEMENTS ***
DIM Y(601),R(601)
*** ENTER THE PARAMETERS OF THE PROCESS ***
K=2
C0=1
C1=0
      ! COEFFICIENT OF ZERO-ORDER TERM IN THE NUMERATOR
      ! COEFFICIENT OF 1ST-ORDER TERM

```

```

300 D0=1
310 D1=4.5
320 D2=7
330 D3=4.5
340 D4=1
350 ! *** ENTER THE PARAMETERS OF THE MODEL ***
360 Km=2.04310
370 Tc=2.17888
380 To=2.10
390 IF To=0 THEN 510
400 ! *** CALCULATE PID PARAMETERS BASED ON MODIFIED ITAE SETTING ***
410 Ap=1.2
420 Bp=-.855
430 Ai=.796
440 Bi=-.147
450 Ad=.308
460 Bd=.929
470 Kc=Ap*(To/Tc)^Bp/Km
480 Ti=Tc/(Ai+Bi*(To/Tc))
490 Td=Tc*(Ad*(To/Tc)^Bd)
500 GOTO 560
510 ! *** CALCULATE PI PARAMETERS BASED ON
520 ! THE FOL MODEL RELATED PI TUNING FORMULA ***
530 Kc=1
540 Ti=2*Km*Kc*Tc/(1+Km*Kc)^2
550 Td=0
560 ! *** REARRANGE THE PARAMETERS ***
570 K1=K*Kc*Td
580 K2=K*Kc
590 K3=K*Kc/Ti
600 !
610 E0=K3
620 E1=K3*C1+K2
630 E2=K2*C1+K1
640 E3=K1*C1
650 F0=K3

```

```

660 F1=K3*C1+K2+D0
670 F2=K2*C1+k1+D1
680 F3=K1*C1+D2
690 F4=D3
700 F5=D4
710 !
720 ! *** CHOOSE THE SAMPLING INTERVAL ***
730 T=.05
740 ! *** USE THE TRAPEZOIDAL RULE TO TRANSFER
750      THE CONTINUOUS SYSTEM INTO DISCRETE SYSTEM ***
760 B0=32*F5+16*F4*T+8*F3*T^2+4*F2*T^3+2*F1*T^4+F0*T^5
770 A0=(8*E3*T^2+4*E2*T^3+2*E1*T^4+E0*T^5)/B0
780 A1=(-8*E3*T^2+4*E2*T^3+6*E1*T^4+5*E0*T^5)/B0
790 A2=(-16*E3*T^2-8*E2*T^3+4*E1*T^4+10*E0*T^5)/B0
800 A3=(16*E3*T^2-8*E2*T^3-4*E1*T^4+10*E0*T^5)/B0
810 A4=(8*E3*T^2+4*E2*T^3-6*E1*T^4+5*E0*T^5)/B0
820 A5=(-8*E3*T^2+4*E2*T^3-2*E1*T^4+E0*T^5)/B0
830 B1=(-160*F5-48*F4*T-8*F3*T^2+4*F2*T^3+6*F1*T^4+5*F0*T^5)/B0
840 B2=(320*F5+32*F4*T-16*F3*T^2-8*F2*T^3+4*F1*T^4+10*F0*T^5)/B0
850 B3=(-320*F5+32*F4*T+16*F3*T^2-8*F2*T^3-4*F1*T^4+10*F0*T^5)/B0
860 B4=(160*F5-48*F4*T+8*F3*T^2+4*F2*T^3-6*F1*T^4+5*F0*T^5)/B0
870 B5=(-32*F5+16*F4*T-8*F3*T^2+4*F2*T^3-2*F1*T^4+F0*T^5)/B0
880 !
890 ! *** GENERATE THE STEP RESPONSE ***
900 R(0)=1
910 R(1)=1
920 R(2)=1
930 R(3)=1
940 R(4)=1
950 Y(0)=A0*R(0)
960 Y(1)=-B1*Y(0)+A0*R(1)+A1*R(0)
970 Y(2)=-B1*Y(1)-B2*Y(0)+A0*R(2)+A1*R(1)+A2*R(0)
980 Y(3)=-B1*Y(2)-B2*Y(1)-B3*Y(0)+A0*R(3)+A1*R(2)+A2*R(1)+A3*R(0)
990 Y(4)=-B1*Y(3)-B2*Y(2)-B3*Y(1)-B4*Y(0)+A0*R(4)+A1*R(3)+A2*R(2)+A3*R(1)
1000 Y(4)=Y(4)+A4*R(0)

```

```

1010 FOR I=5 TO 600
1020   R(I)=1
1030   Y(I)=-B1*Y(I-1)-B2*Y(I-2)-B3*Y(I-3)-B4*Y(I-4)-B5*Y(I-5)
1040   Y(I)=Y(I)+A0*R(I)+A1*R(I-1)+A2*R(I-2)+A3*R(I-3)+A4*R(I-4)+A5*R(I-5)
1050 NEXT I
1060 PRINT "THE STEP RESPONSE OF THE CONTROL SYSTEM:"
1070 PRINT
1080 PRINT Y(*)
1090 !
1100 PAUSE
1110 !
1120 ! *** PLOTTING SUBROUTINE ***
1130 !
1140 PLOTTER IS 7,5,"9872A"
1150 LIMIT 60,60+160,65,65+105
1160 Xmin=0
1170 Xmax=30
1180 Ymin=0
1190 Ymax=1.5
1200 SCALE Xmin,Xmax,Ymin,Ymax
1210 AXES 2,.1,Xmin,Ymin,5,5,5
1220 AXES 2,.1,Xmax,Ymax,5,5,5
1230 MOVE 0,0
1240 LINE TYPE 5
1250 I=0
1260 FOR X=Xmin TO Xmax STEP T
1270   X=.05*I
1280   Y=Y(I)
1290   I=I+1
1300   DRAW X,Y
1310 NEXT X
1320 !
1330 END

```

APPENDIX 3

THE PROGRAM FOR THE SIMULATION OF THE CLOSED-LOOP PID CONTROL SYSTEM IN REGULATING PROBLEM

```

10 *****
20 *
30 *
40 *
50 *
60 *
70 *
80 *
90 *
100 *
110 *
120 *
130 *
140 *
150 *
160 *
170 *
180 *
190 *
200 *
210 *****
220 !
230 !
240 !
250 !
260 !
270 !
280 !
290 !

      PROGRAM NAME: SIMU2
      THIS PROGRAM IS USED TO SIMULATE A FEEDBACK CONTROL SYSTEM
      WITH A STABLE SISO PROCESS AND A PID CONTROLLER. THE SYSTEM CLOSED-
      LOOP RESPONSE TO A STEP CHANGE OF DISTURBANCE CAN BE CALCULATED AND
      PLOTTED. THE PROGRAM IS ABLE TO SIMULATE PROCESSES UP TO 4TH ORDER.
      THE FOLPD OR THE FOL MODEL RELATED PID CONTROLLER TUNING TECHNIQUES
      ARE USED TO TUNE THE CONTROLLER.
      THE FOLLOWING NOTATIONS ARE USED:
      K: PROCESS DC GAIN
      C0, C1, C2, D0, D1, D2, D3, D4: PROCESS PARAMETERS
      Km: DC GAIN OF THE FOLPD MODEL
      Tc: TIME CONSTANT OF THE MODEL
      To: DELAY TIME OF THE MODEL
      Ap, Bp, Ai, Bi, Ad, Bd: CONSTANTS FOR CONTROLLER SETTING
      Kc, Ti, Td: PID PARAMETERS
      D(I): DISTURBANCE OR LOAD CHANGE
      Y(I): SYSTEM CLOSED-LOOP RESPONSE
      *****
      *** DIMENSION STATEMENTS ***
      DIM Y(601),D(601)
      ! *** ENTER THE PARAMETERS OF THE PROCESS ***
      K=60
      C0=1
      C1=0
      ! COEFFICIENT OF ZERO-ORDER TERM IN THE NUMERATOR
      ! COEFFICIENT OF 1ST-ORDER TERM

```

```

300 C2=0
310 D0=24
320 D1=50
330 D2=35
340 D3=10
350 D4=1
360 ! *** ENTER THE PARAMETERS OF THE MODEL ***
370 Km=2.5
380 Tc=1.0534
390 To=.98275
400 IF To=0 THEN 520 ! WHEN DELAY TIME IS 0, USE PI CONTROLLER ***
410 ! *** CALCULATE PID PARAMETERS BASED ON THE MODIFIED IAE SETTING ***
420 Ap=3.0
430 Bp=-.921
440 Ai=.878
450 Bi=-.749
460 Ad=.482
470 Bd=1.137
480 Kc=Ap*(To/Tc)^Bp/Km
490 Ti=Tc/(Ai*(To/Tc)^Bi)
500 Td=Tc*(Ad*(To/Tc)^Bd)
510 GOTO 570
520 ! *** CALCULATE PI PARAMETERS BASED ON
530 ! THE FOL MODEL RELATED PI TUNING FORMULA ***
540 Kc=1
550 Ti=2*Km*Kc*Tc/(1+Km*Kc)^2
560 Td=0
570 ! *** REARRANGE THE PARAMETERS ***
580 K1=K*Kc*Td
590 K2=K*Kc
600 K3=K*Kc/Ti
610 !
620 E0=0
630 E1=K
640 E2=K*C1
650 E3=K*C2

```

```

660 F0=K3
670 F1=K3*C1+K2+D0
680 F2=K3*C2+K2*C1+K1+D1
690 F3=K2*C2+K1*C1+D2
700 F4=K1*C2+D3
710 F5=D4
720 !
730 ! *** CHOOSE THE SAMPLING INTERVAL ***
740 T=.05
750 ! *** USE THE TRAPEZOIDAL RULE TO TRANSFER
760 ! THE CONTINUOUS SYSTEM INTO DISCRETE SYSTEM ***
770 B0=32*F5+16*F4*T+8*F3*T^2+4*F2*T^3+2*F1*T^4+F0*T^5
780 A0=(8*E3*T^2+4*E2*T^3+2*E1*T^4+E0*T^5)/B0
790 A1=(-8*E3*T^2+4*E2*T^3+6*E1*T^4+5*E0*T^5)/B0
800 A2=(-16*E3*T^2-8*E2*T^3+4*E1*T^4+10*E0*T^5)/B0
810 A3=(16*E3*T^2-8*E2*T^3-4*E1*T^4+10*E0*T^5)/B0
820 A4=(8*E3*T^2+4*E2*T^3-6*E1*T^4+5*E0*T^5)/B0
830 A5=(-8*E3*T^2+4*E2*T^3-2*E1*T^4+E0*T^5)/B0
840 B1=(-160*F5-48*F4*T-8*F3*T^2+4*F2*T^3+6*F1*T^4+5*F0*T^5)/B0
850 B2=(320*F5+32*F4*T-16*F3*T^2-8*F2*T^3+4*F1*T^4+10*F0*T^5)/B0
860 B3=(-320*F5+32*F4*T+16*F3*T^2-8*F2*T^3-4*F1*T^4+10*F0*T^5)/B0
870 B4=(160*F5-48*F4*T+8*F3*T^2+4*F2*T^3-6*F1*T^4+5*F0*T^5)/B0
880 B5=(-32*F5+16*F4*T-8*F3*T^2+4*F2*T^3-2*F1*T^4+F0*T^5)/B0
890 !
900 ! *** GENERATE THE STEP RESPONSE ***
910 D(0)=1
920 D(1)=1
930 D(2)=1
940 D(3)=1
950 D(4)=1
960 Y(0)=A0*D(0)
970 Y(1)=-B1*Y(0)+A0*D(1)+A1*D(0)
980 Y(2)=-B1*Y(1)-B2*Y(0)+A0*D(2)+A1*D(1)+A2*D(0)
990 Y(3)=-B1*Y(2)-B2*Y(1)-B3*Y(0)+A0*D(3)+A1*D(2)+A2*D(1)+A3*D(0)
1000 Y(4)=-B1*Y(3)-B2*Y(2)-B3*Y(1)-B4*Y(0)+A0*D(4)+A1*D(3)+A2*D(2)+A3*D(1)
1010 Y(4)=Y(4)+A4*D(0)

```

```

1020 FOR I=5 TO 600
1030   R(I)=1
1040   Y(I)=-B1*Y(I-1)-B2*Y(I-2)-B3*Y(I-3)-B4*Y(I-4)-B5*Y(I-5)
1050   Y(I)=Y(I)+A0*D(I)+A1*D(I-1)+A2*D(I-2)+A3*D(I-3)+A4*D(I-4)+A5*D(I-5)
1060 NEXT I
1070 PRINT "THE STEP RESPONSE OF THE CONTROL SYSTEM:"
1080 PRINT
1090 PRINT Y(*)
1100 !
1110 PAUSE
1120 !
1130 ! *** PLOTTING SUBROUTINE ***
1140 !
1150 PLOTTER IS 7,5,"9872A"
1160 LIMIT 60,60+160,65,65+105
1170 Xmin=0
1180 Xmax=30
1190 Ymin=-.5
1200 Ymax=1.5
1210 SCALE Xmin,Xmax,Ymin,Ymax
1220 AXES 1,.1,Xmin,Ymin,5,5,5
1230 AXES 1,.1,Xmax,Ymax,5,5,5
1240 MOVE 0,0
1250 LINE TYPE 5
1260 I=0
1270 FOR X=Xmin TO Xmax STEP T
1280   X=.05*I
1290   Y=Y(I)
1300   I=I+1
1310   DRAW X,Y
1320 NEXT X
1330 !
1340 END

```

VITA

George Shu-xing Cheng was born in Shanghai, China, on April 22, 1957. He received his elementary-and-secondary education at that city. He entered the Automation Department at East China Institute of Textile Science and Technology, Shanghai, in February 1978 and received a Bachelor of Engineering degree in January 1982.

The following September he entered the Graduate School at The University of Tennessee, Knoxville. During the time he worked as a graduate teaching or research assistant. He received a Master of Science degree in Electrical Engineering in March 1985. He is a student member of The Institute of Electrical and Electronics Engineers, and The IEEE Control Systems Society.