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# Monitoring Additive Manufacturing Machine Health

A Dissertation Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Jeremy Hale

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# Abstract

Additive manufacturing (AM) allows the production of parts and goods with many benefits over more conventional manufacturing methods. AM permits more geometrically complex designs, custom and low-volume production runs, and the flexibility to produce a wide variety of parts on a single machine with reduced pre-production cost and time requirements. However, it can be difficult to determine the condition, or health, of an AM machine since complex designs can increase the variability of part quality. With fewer parts produced, destructive testing is less desirable and statistical methods of tracking part quality may be less informative. Combined with the relatively more complex nature of AM machines, qualifying AM machines and monitoring their health to perform maintenance or repairs is a challenging task.

We first present a case study that demonstrates the difficulty of monitoring the qualification of an AM machine. We then discuss some unique challenges AM presents when calibrating and taking measurements of laser power, and we demonstrate the relative insufficiency of this method in tracking the qualification status of an AM machine and the quality of the parts produced.

Next, we present a framework that reverses the directionality of monitoring AM machine health. Rather than monitoring machine subsystems and intermediate metrics reflective of part quality, we instead directly monitor part quality through a combination of witness builds and witness parts that provide observational data to define the health status of a machine. Witness builds provide more accurate data separated from the noisy influence of parts and parameter settings, while witness artifacts provide more timely data but with less accuracy.

Finally, machine health is modeled as a partially observed Markov decision process using the witness parts framework to maximize the long-term expected value per build. We show the superiority of this model by comparison to two less complex models, one that uses no use no witness parts and another that uses only witness builds. A case study shows the benefits of implementing the model, and a sensitivity analysis is performed to show relevant insights and considerations.

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# Chapter 1

# Introduction

We first provide an introduction and background on the topic of additive manufacturing, as well as some more general information on the challenges associated with the technology. We also reference some literature and work being done in the area that are motivate the topic of the remaining chapters of this dissertation.

## 1.1 Background on Additive Manufacturing

Additive manufacturing (AM) is a "process of joining materials to make parts from 3D model data, usually layer upon layer, as opposed to subtractive manufacturing and formative manufacturing methodologies" (ISO/ASTM52900-15, 2015). This process contrasts with the conventional manufacturing (CM) methods of molding, casting, shaping, forming, or other material-subtractive and formative methods that involve removing or reshaping material, respectively, to form a final product (ISO/ASTM52900-15, 2015).

The relative differences between AM and CM permits separate use cases. AM gives manufacturers increased freedom to rapidly produce parts across a variety of sizes, shapes, and materials on a single machine with reduced lead time. As a result, AM production allows simpler, faster, and cheaper production of geometrically complex products, one-time produced products, or otherwise low-volume and customized products, which may be otherwise impossible with CM. Additionally, AM processes may be easily changed and

modified to produce different parts without changing machines, allowing the manufacturing of multiple part designs using a single machine. Multiple part designs may be constructed within a single build or from build-to-build with minimal changeover time. As a result, AM is often used for small batch manufacturing, custom part manufacturing, or producing a variety of different parts on a single machine (Frazier, 2014). Increased industry adoption of AM relies upon the ability to produce parts repeatably and consistently across within the various locations of a single build, between different build instances on a single machine, and between multiple machines of the same model (Huang et al., 2015).

## 1.2 Qualification and Standards for Additive Manufacturing

Qualification of AM processes and machines has been a challenging topic due in-part and largely to the unique aspects of the AM process compared to CM, and stands as a major impedence to solving the problems of high variability in part, machine, and process performance. Alternatives to the qualification processes typically used for CM are needed, and may include probabilistic techniques or the development of closed-loop, in-situ sensors and controls (Frazier, 2014).

CM methods often rely on statistically-based qualification schema that often rely on significant initial testing regimen, fixing parameters and processes, then requalifying based on deviations found during on-going quality control processes. This method is potentially appropriate for AM production environments which produce high-volume of a single part in which AM is simply being used for its ability to produce complex designs. However, using the full potential and capability of AM production, that is, low-volume, custom, flexible production with multiple designs and feedstocks, this qualification process can become extremely costly, time-consuming, or otherwise impossible to implement because of a lack of parts on which to perform analyses.

One framework for qualification has been for the framework of "certify-as-you-build", in which a single part is considered qualified by closely monitoring the live data of a build as a synthesis of various indicators, often focusing on melt pool and defect monitoring (Mazumder, 2015). Similar "qualify as you go" paradigms have been suggested which consider holistic perspectives of quality control processes and rely on using pre-process, in-process, and post-process data to qualify a part may be better suited for AM (Seifi et al., 2016). Qualification procedures by considering both machine and part performance and has proven to be a difficult, costly, and time-consuming task, and many of the developed standards and qualification procedures are often proprietary (Seifi et al., 2017).

National Institute of Standards and Technology (NIST) has noted that other alternatives to statistical-based qualification for AM may include equivalence-based modeling, which compares the performance of newly qualified processes and materials to previously qualified versions, or model-based qualification, in which minimal testing verifies computational simulations and models (NIST, 2020).

Many agencies have put forth standards relating to AM processes and qualification. The 2018 document entitled Standardization Roadmap for Additive Manufacturing: Version 2.0 was created in collaboration between America Makes and ANSI AMSC (American National Standards Institute's Additive Manufacturing Standardization Collaborative) to provide a guide the development of standards and specifications for AM, inform about the existence of relevant research and work already performed, and identify areas in need of research (American Makes and ANSI, 2018).

Much of the research has been centered around industries and critical use cases that demand a high-degree of quality and consistency where the needs are greater and costs can be more justified. Exemplifying this, at the time of the document's release, the FDA was working on multiple areas of research related to the use of AM in its various medical uses, and have produced a technical document outlining considerations when for device design, material controls, and device testing (FDA, 2017) . A pair of documents released by NASA MSFC include a set of standards (MSFC-STD-3716, 2017) and specifications (MSFC-SPEC-3717, 2017) which together outline a framework for the use of AM in the spaceflight

industry. Specifically, the documents provide guidance for the development, production, and evaluation of parts produced using laser powder bed fusion (L-PBF) AM machines by considering part classification, part process control, part inspection, and part acceptance as well as qualification requirements for the L-PBF metallurgical processes, equipment process control, and personnel training.

The F42 Committee of ASTM has released multiple documents which develop specifications and standards relating to assessing the geometry of test parts and benchmarking AM systems (ISO/ASTM52902-19, 2019), terminology (ISO/ASTM52900-15, 2015), part design (ISO/ASTM52910-18, 2018), reporting test specimen data (ASTM F2971-13, 2013), evaluating mechanical properties of metal AM parts (ASTM F3122-14, 2014), and production control for process characteristics and performance of powder bed fusion AM in aerospace and medical applications (ASTM F3303-18, 2018). One additional particularly relevant document describes the qualification of L-PBF equipment for installation, operation, and performance qualification (IQ/OQ/PQ) (ASTM F3434-20, 2020). This document outlines a guide for process validation and machine qualification strategies to monitor and ensure quality as well as the situations in which to revalidate. While this document does give some guidelines on what maintenance plans to implement, which process parameters are known to affect quality, and key variables of the machine and parts which can effect and reflect quality, it does not necessarily dictate any particular models to implement or help determine an optimal strategy for their use.

Though these standards are helpful, other standards gaps have been identified that receive relatively less attention in the areas of machine calibration and preventative maintenance, machine health monitoring, and machine qualification. Though the identified gap is for a lack of standards per se, some of this slow development is a result of the need for research and the difficulty in implementing certain strategies and techniques in AM. Indeed, the need for research into these areas is explicitly noted in the Standardization Roadmap.

## 1.3 Machine Health Monitoring

Much of the focus on machine calibration and preventative maintenance, health monitoring, and qualification has centered on implementing more directly in-process monitoring of the machine using in-situ sensors, and feedbacks loops in order to gather information on the health, status, and operational capability of the machine. This requires not only knowledge of the effects of the various influential processes and control over them, but may also be difficult or otherwise cost-prohibitive to implement. Moreover, many of these control systems do not necessarily provide information on the connection between part quality and machine performance, or allow any sort of optimization of preventative maintenance. Often, the implementation of these strategies also relies heavily on the need for testing and evaluation of parts, which often means using limiting non-destructive evaluation techniques given previously mentioned low-volume and costly parts often produced using AM.

Condition-based monitoring (CBM) models and machine health monitoring techniques have long existed in the area of CM, and have been implanted in various modern industries. The need for process control in the semiconductor industry has been noted as one strategy that can allow the reduction of waste and faulty parts, reduction of time between failures, detection and performance of repair operations, efficient scheduling of preventative maintenance, increased flexibility to produce various designs, and more efficient machine use (Butler, 1995). At least one strategy of implementing condition-based maintenance using Bayesian networks and hidden Markov models has been proposed for semiconductor wafer fabrication (Liu, 2008). Bayesian networks have also been proposed as one strategy of describing the relationships between input variables, quality metrics, and manufacturing decisions for AM (Panicker et al., 2019).

The many similarities between AM and semiconductor industry for wafer fabrication and machine qualification make the strategies implemented there likely good candidates for use in AM. Still, some differences exist. AM machines may require greater capability for flexibility than the wafer fabrication machines given their all-in-one nature and usefulness to produce a greater number of build designs. Feedstock, designs, and costs have the

potential to vary wildly while still being produced on a single, all-in-one AM machine, while wafer fabrication may have a system of multiple machines and are often qualified for a particular wafer "recipe." Testing recipes may be relatively cheaper than testing parts for AM, and the importance of implementing strategies from the semiconductor industry may rely on the availability to produce test parts that reflect the AM machine and its processes. Still, the ability to monitor and qualify the machines in this industry have enabled further more complicated mathematical models to optimize production, including a stochastic programming for managing machine qualification (Chang and Dong, 2017), condition-based monitoring of semiconductor machines (Cholette et al., 2013), stochastic optimization of machine-product qualifications (Fu et al., 2015), frameworks for maintenance by predicting machine quality degradation (Luo et al., 2015), optimization of machine throughput and costs (Klemmt et al., 2010), optimizing production flexibility and machine utilization (Johnzen et al., 2009), and optimal planning of machine qualification using relevant indicators (Rowshannahad et al., 2015). These examples provide some goalposts of the sort of improved machine monitoring and production that can be achieved by taking lessons from the semiconductor industry.

Much work research has already endeavored to form a connection between various AM design, machine, and process parameters to part defects. Bayesian networks can be used to predict the effects of these parameters on defects and to determine influential variables that affect the quality of outcome (Mokhtarian et al., 2019). Bayesian networks can also be used to determine connections between manufacturing decisions and production cost indicators (Panicker et al., 2019). Diagnostic agents that indicate machine degradation patterns can be linked to known root causes of machine faults through to implement condition-based maintenance (Rusu et al., 2019). The growing prevalence of data may allow some more complex machine and deep learning techniques to classify patterns for inferring manufacturing machine health (Yan et al., 2019).

For AM, these probabilistic fault networks often rely on direct indicators of machine performance between the separation of the indicators for part quality can be incredibly difficult to separate from the various fault causes. Machine operators and AM producers

are concerned primarily with certain quality metrics to reflect the quality of parts produced. Density, color, tensile strength, fracture strength, dimensional accuracy, and microstructure are just a few of the common metrics. Each of these can be further determined by a variety of testing procedures ranging from simple measurements, such as basic mass and volume calculations for density, to more complex and high-precision methods, such as blue light scanning for dimensional accuracy. While these metrics and measurements give a direct measurement of the quality of the manufactured part, they only give indirect information on the AM processes and machine which produced such a part.

Thus, separating the effects of machine health from other influential factors can further prove to be a difficult task, particularly considering the complex machines and processes which constitute AM. Feedstock, design, parameter selection, machine health, and human factors may contribute to changes in the quality metrics. The ability to determine the degree to which each of these contributes to changes in quality would not only help to improve the ability to produce parts of higher quality, but also reduce the costs associated with achieving consistent quality in production over time. In order to monitor AM machine health through these part metrics, it will be necessary to separate the source of various faults.

## 1.4 Research Overview

The remaining chapters of the research in this dissertation will work within the subjects introduced here. First, Chapter 2 presents a case study of a machine which presents with a change in quality and the difficulties, and attention to detail, that are required in order to correct the issue. We present this as an example of the sort of work that exists in the paradigm that can somewhat be alleviated by the shift in perspective and monitoring framework introduced in subsequent chapters. Chapter 3 presents a modified machine health monitoring framework for AM that utilizes and incentivizes a system of witness parts. Under this framework, information can be gathered about the health status of an AM machine in a way that separates it from the noisier information of other fault modes, thus enabling the use of a new set of condition-based monitoring solutions. In Chapter 4 we present a

model in the form of a partially observed Markov decision process for optimally planning maintenance and repair on an AM machine. We show how this model can be implemented through a case study and consider an analysis of the model to determine relevant managerial insights. Finally, we conclude in Chapter 5 by discussing the implications the research and its results.

# Chapter 2

# A Case Study in Calibrating an Additive Manufacturing Machine

As discussed in the previous chapter, the absence of a comprehensive machine health monitoring model requires the use of identifying, monitoring, and controlling the individual processes and root causes of an AM machine associated with degrading machine health and performance. This can be a difficult process to identify and standardize for many reasons. What these standards encompass and how tightly they are controlled and monitored can depend heavily on the particular AM process, machine, application, and production goals. For example, an extrusion process of fused-deposition modeling using ABS plastic for rapid prototyping will have very different requirements from a laser powder bed fusion machine in aluminum alloys for aerospace applications. Often the standards of quality to which AM production is held can be influenced by the qualification process used, since the two are in a feedback loop. For this reason, the qualification process itself can be highly inefficient, insufficient, time-consuming, and costly. To demonstrate this and the difficulties involved therein, the remainder of this chapter is devoted to discussing a case study of an effort to detect, monitor, and control the laser power on a laser powder bed fusion AM machine. This content is taken in part or in whole from other sources, and is not original to this dissertation.

## 2.1 Abstract

Purpose: The importance of the accuracy of laser power used in an additive manufacturing (AM) setting has long been emphasized in standards and literature. Despite the existence of standards for measuring laser power, the influence of the unique processes, restrictions, and requirements of AM have not been fully considered in laser power calibration. Compared with other industrial laser applications, AM may require greater precision and accuracy or increased control over the measurement process.

Methods: A procedure, explained through a case study, is presented for laser power calibration on an AM machine in which the length of time to take a measurement using a power meter is determined, and a simple method of compensation for discrepancies using the capabilities of the machine is proposed.

Results: Differences in measured power over time are observed and determined likely to be an effect of calibration technique rather than laser or optics hardware degradation.

Conclusion: This paper presents one method of implementing a qualified process to calibrate laser power for AM.

## 2.2 Introduction

Additive manufacturing (AM) is a process of joining materials to make parts from 3D model data, usually layer upon layer, as opposed to subtractive manufacturing and formative manufacturing methodologies" (ISO/ASTM52900-15, 2015). This process contrasts with conventional manufacturing (CM) methods of moulding, casting, shaping, forming, or other material-subtractive methods. AM gives manufacturers increased freedom to rapidly produce parts across a variety of sizes, shapes, and materials on a single machine with reduced lead time.

Calibrations of the laser and optics systems are performed on most AM machines, and the accuracy and precision of these results can affect part quality and parameter settings. Control of the laser power is critical to achieving material and product specifications using

AM and replicating results across various manufacturing instances, machines, and production environments. Parameters for laser power in AM may need to be more precise than those optimized for CM applications such as cutting where stable, optimized parameters are desired, but the acceptable range is on the order of hundreds of watts (Rahman Rashid et al., 2014). Some efforts have attempted to directly control power variation using active, closed-loop systems to compensate for irregularities (Reutzel and Nassar, 2015). Other methods attempt to directly stabilize physical laser power generation by electronic and physical means in the laser source hardware (Wyss et al., 2002). However, not all machines are equipped with such feedback and control systems, and those that are must still be qualified and validated for the AM process. Proper qualification of AM machines should consider all such systems and verify that the power output from laser optics systems is stable and controlled across the broad range of power levels required for the flexible production capabilities provided by AM. For industries such as defence, biomedicine, and aerospace where quality is critical and more exotic materials are used, it is imperative to maintain tight control, calibration, verification, and qualification of laser power systems.

The results of laser power control can have cascading effects such as necessitating calibration in other areas such as beam offset (Wang, 1999). The effects can also be seen in part quality, material deposition efficiency and waste (Mahamood and Akinlabi, 2016), and energy density (King et al., 2014). Knowledge of laser power can influence parameter development for use on other AM machines and in future production, and therefore the consistent accuracy over time is similarly of critical importance. Laser manufacturers may provide documentation at installation outlining any initial irregularities, imprecision, and inaccuracies, but ongoing qualification must still be performed. Laser power qualification procedures use power meters, which must themselves be calibrated. The National Institute for Standards and Technology (NIST) produced a standard for calibrating these sensors (Li et al., 2008), an updated version of an earlier method (West, 1976), which builds on the physical theory of calorimetry for laser power and energy measurements (West and Churney, 1970).

The requirements of AM to have high precision at individual power levels for quality while also providing the flexibility to use a wide range of power levels means power meter calibrations could have amplified effects for AM applications compared to other industries. Indeed, calorimetry is difficult at elevated temperatures and can have effects on material properties such as absorptivity (Indhu et al., 2018). The enclosed chambers present in some AM machines may limit the ability to provide cooling and airflow, leading to environmental conditions for which the meter was not calibrated. This may be an issue particularly for AM machines that may require lengthy measurement times at numerous power levels for multiple optics systems. Long measurement times or additional cooling times for such a scenario could be costly and time-consuming for some AM applications, particularly those processes involving high-cost machines, high-cost exotic materials, high-quality requirements, or critical industrial applications previously mentioned such as defence, biomedicine, and aerospace. Power meter manufacturers have attempted to reduce measurement times by adding "ramp-up" features, described in Section 2.3, to reach a stable measurement value quickly, and it is essential to ensure continued accuracy for use in the unique requirements and applications of AM. Although removing these ramp-up systems would yield raw, unfiltered data, it would also increase the time required for regular maintenance of the optics systems. As a result, determining an optimal measurement time using a calibrated power meter permits fast and low-cost recording of measurements while still maintaining accuracy and precision tailored to AM. These techniques can then be implemented as part of a machine qualification strategy.

Accurate and precise calibration is crucial not only for the quality of parts produced but also for research into future materials and parameter development, which can also be a costly and time-consuming process. In practice, calibrated laser power meters may have an acceptable tolerance level wider than desired by AM operators. Depending on the feedstock material being used, this may be imprecise enough to result in negative product quality including porosity or low density, poor material properties, discolouration, or wasted energy and material. Drawing conclusions about the proper power level for various part designs, build material, or environmental conditions may be further complicated if the

power level is imprecise. This effect may be further amplified as power levels increase. Knowledge of the mechanics for power compensation also allows for calibration of machines in-house, thereby enhancing production site security, reducing costs for outside contracting, and potentially increasing control over the precision and accuracy of part quality. Further, manufacturers that produce high-criticality, high-cost, or otherwise security-critical parts may wish to establish their own qualification standards. The in-house calibration and standards may allow tighter control over the process than is provided by the original equipment manufacturer (OEM).

One possible challenge to monitoring laser power is a dearth of records over time for analyzing the effects of laser power on machine and process qualification, hardware degradation, parameter selection, or build quality. Industry practitioners may purchase service contracts with machine manufacturers to control calibration practices, or may not closely monitor laser power and instead choose to control and optimize parameters on a build-by-build basis to compensate for observed undesirable quality. Machine operators may not calibrate laser power beyond an initial installation calibration unless an issue with production quality is exposed, sensitive parts are built, or a comprehensive qualification process is imposed. Further, the calibration methods and procedures used by OEMs or manufacturers may be subject to proprietary restrictions and not available to the public or other practitioners.

The physical properties of laser power, the operation of power meters, the calibration of the meters, and the method by which measurements are taken are generally well-understood and available in literature and standard in the manufacturing industry. Though standards exist for broad qualification and verification of laser power measurements, AM presents complicating factors and requirements that must be considered. No guideline or case study exists for measuring, controlling, and compensating for laser power on AM machines despite literature emphasizing the importance of AM machine qualification and laser power control. Thus, a literature gap exists in developing a method to calibrate laser power that takes into account specific concerns of an AM machine.

## 2.3 Method of Laser Power Measurements for an AM Machine

Discrepancies in the laser power control can occur in one of two ways: (1) the laser itself produces power differently than nominally calibrated, and (2) measurement of the laser power produced is inaccurate. This method will consider laser power control on an SLM 280HL powder bed fusion machine, which influences laser power in four stages and ultimately results in a de facto fifth stage in the form of an upstream feedback loop for parameter development for machines, parts, and materials. Figure 2.1 shows the locations where the measured output of the laser (at the build plate) can be influenced.

A nominal laser power value can be individually set according to build, geometric feature, and part location. This nominal value is filtered through an offset compensation stage, which can be controlled and calibrated on the AM machine software. Next, the offset value is given to the laser source which produces an electrical signal according to its own correlated values set by an initial calibration. Finally, the physical output of the laser is delivered to the part having been attenuated by factors such as gas flow, mirror and laser glass energy absorption, or other physical factors. On the SLM 280HL machine, laser power is primarily controlled and calibrated at the offset compensation stage by using a "look-up table" that correlates and corrects for discrepancies in power between the nominal parameter values and measured physical output as delivered to the location of the build platform. Calibrations of the look-up table are typically performed on a yearly basis along with calibrations of the power meters and other aspects of the optics system such as focus offset, beam profile, optics alignment, and any other relevant tools.

Standards for calibrating a laser power meter are defined by NIST (Li et al., 2008) and allows for a stated precision and accuracy compared to known laser emission sources. Using these standards, any measurement taken on these power meters will result in a thermal power (detector output) curve, as shown in Figure 2.2, because of the thermodynamic properties associated with the meter. An initial and final rating period measures the baseline values before and after the "injection period," the period of time when the shutter is open and

exposes the meter (detector) to the power source. The measurement during the injection period approaches an exponential curve composed of a rising "settling period" and a stable "measurement period," the latter of which is averaged to determine the measured value of the power source after the rating period values are subtracted.

The exponential portion of the power meter measurement is expressed (West, 1976) as

$$T(t) = (T_0 - T_\infty)\, e^{-\epsilon(t-t_0)} + T_\infty \tag{2.1}$$

where $T(t)$ is the temperature measured by the meter at time t, with $T_0 = T(t_0)$ and $T_\infty = \lim_{t\to\infty} T(t)$ and $\epsilon$ is the exponential rate constant that typically reflects the cooling rate of the meter. Power meter manufacturers often implement internal proportional-integral-derivative (PID) systems to compensate for potentially long initial settling periods, as shown in Figure 2.2. As a result, users do not see the raw value of $T(t)$ and instead only see a modified value $u(t)$ given by

$$u(t) = k_P T(t) + k_I \int_{t_0}^{t} T(t')dt' + k_D \frac{dT(t)}{dt} \tag{2.2}$$

where $k_P$, $k_I$, and $k_D$, are the PID parameters set by the power meter manufacturer during calibration. Substituting the thermal measurement function from Equation 2.1, a power meter measurement observed by a user can be expressed by the function

$$u(t) = T_f + k_P\left((T_0 - T_\infty)\, e^{-\epsilon(t-t_0)}\right) + k_I\left(T_\infty t - \frac{T_0 - T_\infty}{\epsilon} e^{-\epsilon(t-t_0)} - (T_0 - T_\infty)\right) + k_D\left(-\epsilon t\,(T_0 - T_\infty)\, e^{-\epsilon(t-t_0)}\right) \tag{2.3}$$

where $T_f = k_P T_\infty$ for convenience since the parameter $k_P$ also scales the raw signal value of $T_\infty$ for numerical accuracy. Writing this as a separate term, therefore, allows the ultimate stable measurement to be gathered by simply reading $T_f$. As $t$ increases, the function becomes dominated by the first and third terms in Equation 2.3, with the third term approaching a linear function for some sufficiently large $t$. As a result, the meter may

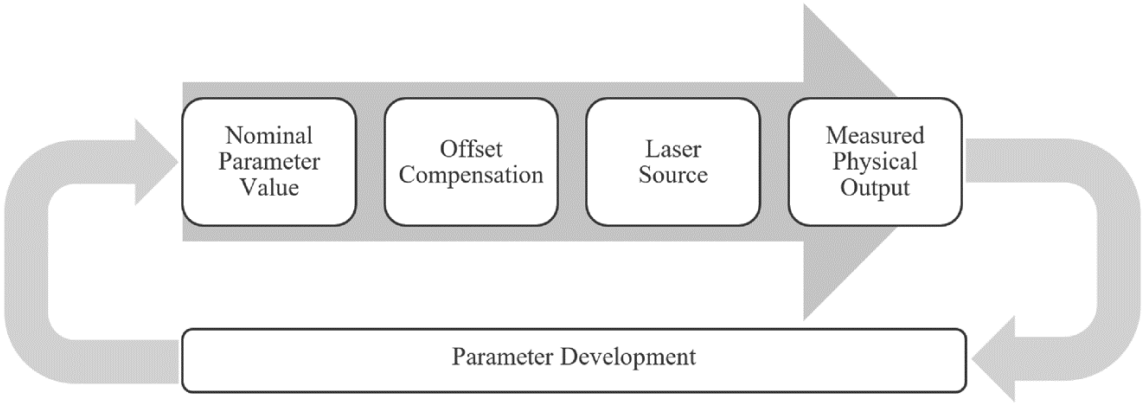



**Figure 2.1:** Feedback loop for laser power control

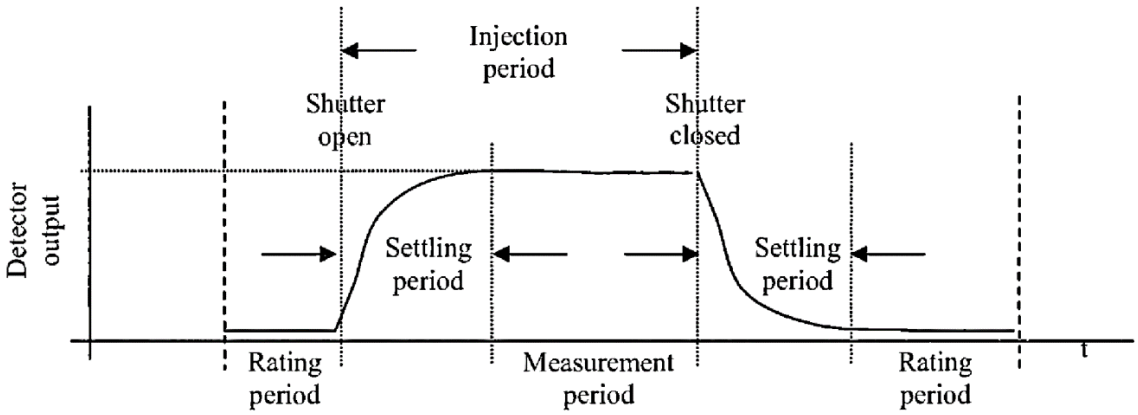



**Figure 2.2:** Sample power meter measurement (Li et al., 2008)

demonstrate some linear increase in value for sufficiently large $t$ according to the parameters chosen during calibration.

## 2.4 Procedure with Case Results

A procedure for calibration of laser power on an AM machine is described here with a case study. First, individual measurements across the entire operational range of each optic are performed, interpolated, and recorded. These measurements are taken according to the method and equations of AM laser power control described in Section 2.3, which includes the calibration feedback loop on the AM machine, the physics of laser calibration from literature, and the mathematics of power meter calibration. Second, this set of measurements is used to generate a single power curve for each optic to determine the difference from nominal values and any required offset compensation. Finally, historical calibration results on the AM machine taken at installation and after 23 months, 32 months, and 44 months are compared with the results of this calibration procedure performed at 52 months.

The procedure was performed on an SLM 280HL machine that includes two 1064nm lasers that each nominally produce 400W of power. Measurements were taken using a thermopile power meter that meets calibration standards set by NIST for a 1064nm laser to give an accurate reading to within ±2.6% of true value. The meter was used to take measurements across the entire power curve in 5% increments from 10% to 100% of nominal power for 5 minutes per reading.

### 2.4.1 Power Meter Measurements

Following the procedure as described in the introduction of Section 3, it was attempted to record individual measurements on a power meter. Initial measurement attempts demonstrated a slow increase over time and only somewhat approached a stable measurement. This is expected and accounted for per the analysis in Section 2.3. Nonetheless, some attempts to minimize this effect were undertaken. Since the meter had only been previously calibrated

using four measurements below 100-watts, the meter was recalibrated by the power meter manufacturer at power levels 50, 100, 200, 300, and 400-watts according to NIST standards of accuracy within ±2.6% of nominal power after 30 seconds of measurement. Despite this altered calibration, some instability in measurement over time remained. Using this altered power meter calibration, a time series of data points from the power meter was interpolated by regression using Equation 2.3, and a sample is shown in Figure 2.3. In this case, a data point was collected every second. A slow increase in measured power is visible as a linear increase in power over time, differentiating this measurement from the ideal exponential curve expected. However, repeating this regression on a truncated subset of data points $\{1, \ldots, i\}$ for elapsed times between time 0 and the moment when the $i^{th}$ data point is collected yields the function $u_i(t)$ and corresponding value $T_{f,i}$, which stabilizes as $i$ increases.

In practice, operators desire to record a single, stable value from the power meter as the accepted power measurement. According to NIST standards, the recorded value should be the average of the "measurement period" as shown in Fig. 2.2. To meet both requirements, it is desirable to find the earliest data point that closely matches a stable, average value of $T_{f,i}$.

If the data points of a single power level measurement are given by $P = \{P_1, \ldots, P_i, \ldots, P_{300}\}$, the estimated power determined by regression at point $i$ is

$$E(i) = \frac{1}{300 - i + 1} \sum_{j=i}^{300} T_{f,j} \tag{2.4}$$

As mentioned previously, the series $\{P_i\}$ usually increases, almost linearly, after $i$ is larger than $i_0$ (i.e., $i_0$ is about 20 seconds based on the experimental results) while $E(i)$ tends to converge to a constant. The deviation between the measurement and estimated power is defined by

$$A(i) = P_i - E(i). \tag{2.5}$$

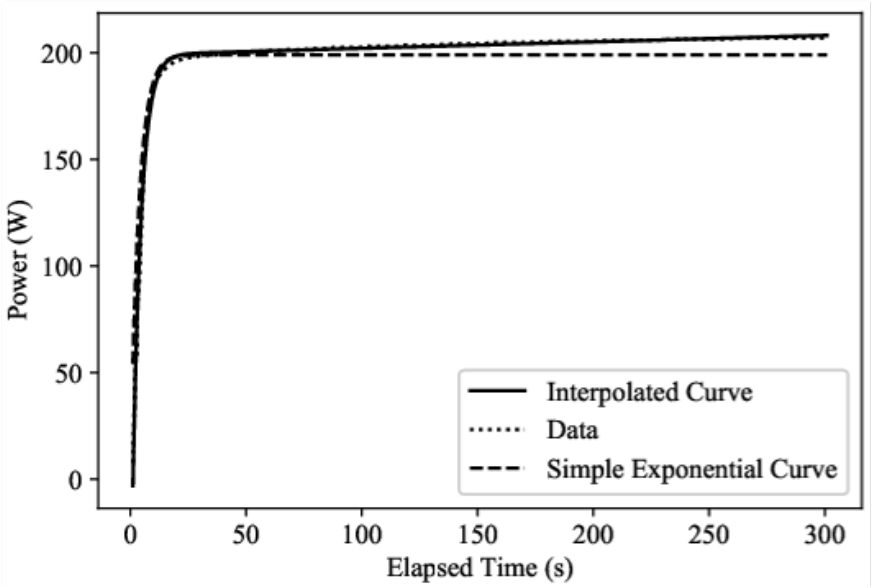



**Figure 2.3:** Measurement on single laser optic at 50% nominal power

The value $N$ is defined as the moment that the recorded value of $P_N$ yields the smallest deviation from $E(i)$, that is,

$$N = \arg\min_{i \geq i_0} |A(i)|. \tag{2.6}$$

The indices of the summation in Equation 2.4 are chosen to be $i \leq j \leq 300$, rather than $0 \leq j \leq i$, so that the recorded value of $P_N$ is close to the average of $T_{f,i}$ if the measurement elapsed up to 300 seconds. Therefore, $N$ is around the point of intersection between the two series of $P_i$ and $E(i)$ and provides the earliest moment that these two series are close to each other. Additionally, this allows N to correspond to the boundary between the "settling period" and "measurement period" as shown in Figure 2.2, and the average of $T_{f,i}$ corresponds to the average of recorded values during the measurement period.

Following this method, the measurement data for either laser optics and each power level considered were repeatedly fitted to find the value of N for each combination. Values of $T_{f,i}$ for $i < 20$ were observed to be erratic for all power levels, likely due to regression with data points that were still sharply increasing, and therefore were not considered. Therefore, Equation 2.6 can be rewritten as $N = \arg\min_{i \geq 20} |A(i)|$. The mean value of $N$ for all power levels and both optics was approximately 51 seconds. However, for both laser optics, the values of $A(N)$ for 20% nominal power were clear outliers, and removing these data points reduced the mean value of $N$ to 45 seconds. Figure 2.4 shows the error $|A(N)(P_N - A(N))|$ at $N = 45$ as a percentage for both optics and all power levels.

The outlier data points around 20% nominal power are likely a result of the meter's two operating modes: one for measured power values below 60 watts, and another for values between 60 and 600 watts. Since the 20% nominal power measurement reached a stabilized value of $T_f$ at approximately 62 watts, the meter underwent an automated mode change near enough to interfere with the regression but not with the measured value of $P_N$. Therefore, measurements for both laser optics and all powers will be allowed to elapse for 45 seconds and $P_45$ recorded as the measured power.

For qualification purposes, this process for determining the optimal time elapsed may need to be repeated over time if the power meter is recalibrated, it is sufficiently far past recent calibration, or a different power meter is used.

### 2.4.2 Offset Compensation

Next, the set of measurements determined in Section 2.4.1 are used to generate a single power curve for each optic and to any necessary offset compensation. The machine control system (MCS) on the SLM 280HL, like many other AM machines, provides the ability to compensate for laser power using a look-up table generated by three or more data points. In practice, the recommendations for the number of data points to use, which data points to use, and how to record a measurement have changed over time. Most recently, the AM machine OEM recommends using three data points at nominal power levels of 15%, 25%, and 100%, with an implied data point at 0%. These choices are not necessarily held constant over time leading to potential perceived changes in delivered power that can be incorrectly attributed to hardware degradation or changes in measurement values which may not exist. For qualification purposes, and to perform any meaningful future analysis of potential changes, it is necessary to decide the quantity and value of data points to be used.

Though the power curve fitting the measured data would ideally be a simple linear function producing zero watts at 0% power, it was observed, and known by the OEM, that some power attenuation exists at the lowest power levels, typically below 20% nominal power. While a hyperbolic curve, cubic spline, or simple linear or affine function could compensate for this reduction at low power levels, a piecewise linear function can provide a simple solution for direct control over the number and value of points used without necessitating more complex calculations or curve fitting algorithms. Once these points are decided, future calibrations would only need to verify the measurements. Although the actual method used by the machine OEM to generate the power curve remains proprietary, this paper will demonstrate the benefits if only a simple piecewise linear function.

The measurements at 10% and 100% are the minimum and maximum achievable power levels, respectively, while also including a data point of 0-watts at 0% power. One point will be further chosen at 45% near the known operating power of the 316L feedstock used on the machine, and one additional point at 90% since the 95% and 100% measurements showed some non-linearity with the other values. Thus, the measurements used are 10%, 45%, 90%, and 100%, along with the implied 0%. Figure 2.5 shows the power curves generated by using these five data points.

The power attenuation at low power levels is accounted for by this choice of data points since powers below 10% are not achievable on the machine optics, and the biggest non-linearity appears to occur below 10%. The non-linearity observed between 95% and 100% power levels is possibly a result of saturation due to hardware limitations or calibration of the power meter, but ultimately the cause is unknown. More generally, the apparent reduced linearity observed above 50% power could be a result of the calibration process for the power meter and is discussed in Section 2.5.

Ultimately, many more points could be chosen, but minimizing the number of measurements taken saves time by reducing the need for multiple measurements. While additional measurements can easily be included, the benefits of that option have not yet been investigated or weighed against the risk of overfitting, time, or costs.

Once values are input into the AM machine to complete the offset compensation stage of laser power control, verification can be performed to ensure values expected by the machine match the values measured by the power meter. Similar verification could be used to show whether additional data points are needed in the offset compensation table of the AM machine to match qualification requirements.

### 2.4.3 Calibration Values Over Time

The results from the historical calibrations on the AM machine can be compared with the power curve results of the method proposed in this study. Historical measurements at installation and following installation by 23 months, 32 months, and 44 months were

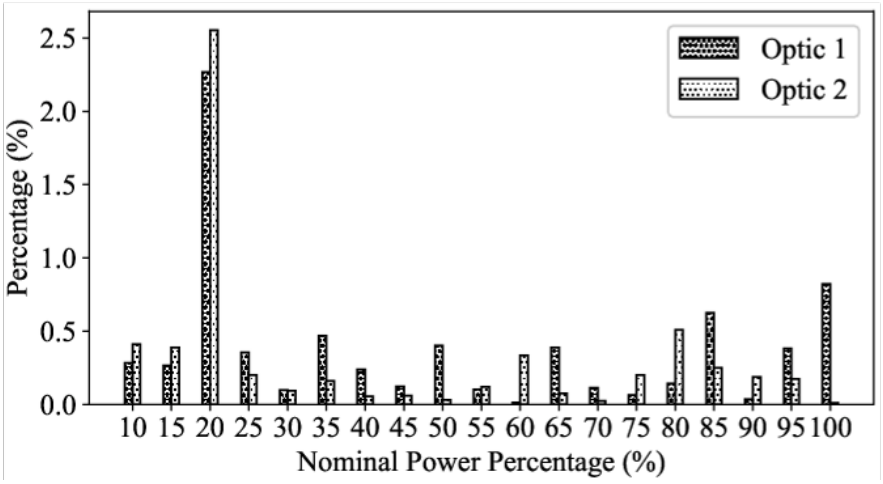



**Figure 2.4:** Percent error between power meter measurement and value determined by regression after 45 seconds of measurement

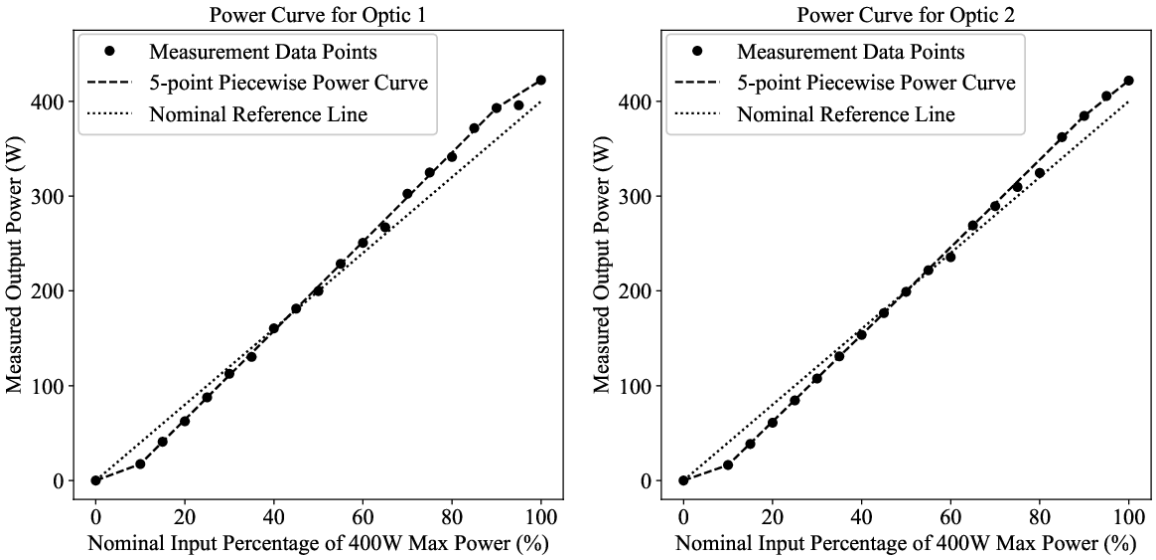



**Figure 2.5:** Power curves for both optics, using power levels 10%, 45%, 90%, and 100%

compared to the power curves generated as described in Section 2.4.2, which were performed 52 months after installation. Figure 2.6 shows a comparison over time for both optics systems.

Measurements were not performed in the same way at each calibration. The measurements taken at installation were performed by the laser manufacturer, subsequent measurements performed by the AM machine OEM, and the measurements at 52 months following installation were performed following the procedure described in this study. Thus, all power levels cannot be compared directly or equally. The top two charts compare measurements taken across five separate calibration instances at a limited set of nominal values of 10%, 25%, and 100%. The bottom pair of charts compare a wider range of measured nominal powers but for only three calibration instances: the factory calibration at installation of the machine, the most recent 44-month calibration, and the calibration performed as part of this study at 52-months. Values are given as difference from expected nominal power in Watts.

The measurements resulting from this investigation (52-month in Figure 2.6 more closely match the initial measurements at installation taken by the laser manufacturer at all nominal power levels and for both optics than more recently recorded measurements. This implies that the apparent changes in measured power at previous calibrations (i.e., 23-, 31-, and 44-month calibrations) were more likely a result of inconsistent calibration techniques rather than optics hardware degradation as previously believed.

## 2.5 Discussion

Other solutions for calibrating laser power on an AM machine are possible. The necessity of regression to fit each power measurement could be avoided by using a power meter without the ramp-up time compensation effects of PID parameters or otherwise disabling those features. More process control during the measurements, such as ensuring better cooling of the build chamber and power meter, may reduce some of the slow linear increase in value as measurement time increases or as multiple consecutive measurements are taken. However, this process control may unacceptably increase the time required to perform calibrations. For

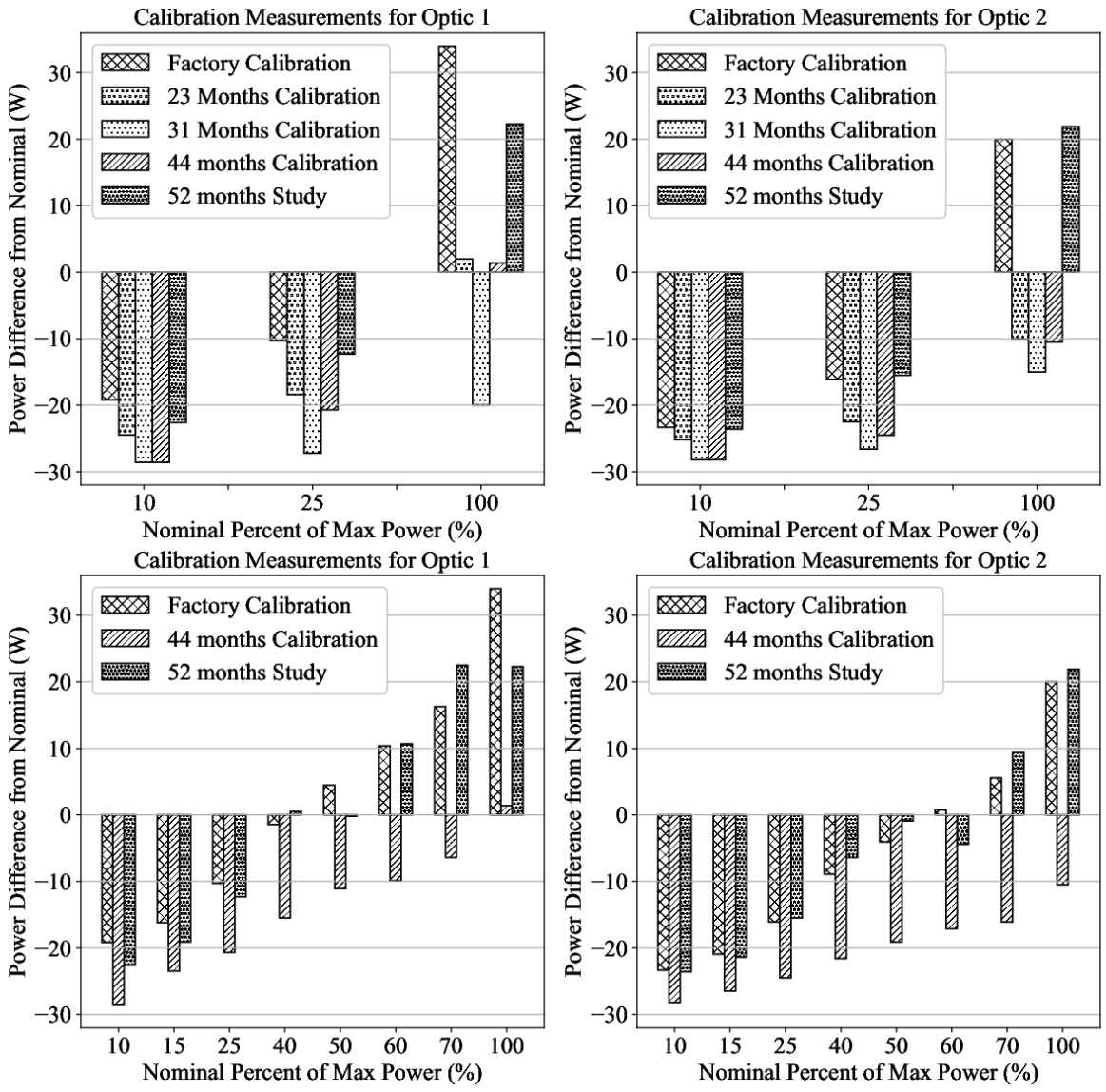



**Figure 2.6:** Measurements recorded over time at selected power levels

measurements above 50% power where less linearity of the power curve was observed, greater precision and accuracy may be achieved by using multiple power meter modes described by additional sets of PID parameters, controlling and varying the exponential rate constant $\epsilon$, or calibrating the power meter itself more tightly at a greater number of power levels. Standards for calibrating the power meters themselves for use with AM machines may need to be established, and these requirements for the power meter should be included as part of a qualified procedure for laser power calibration on the AM machine. This study provides a procedure for standardization.

For regression, an assumption is made that the power measurements will follow physical laws according to Equation 2.3, but other equations, such as rational polynomials, could also reflect asymptotic behaviour of the measurements. While these would not necessarily reflect the physical laws governing these processes, they may provide better results by incorporating other effects not accounted for by Equation 2.3, such as slow thermal warming of the power meter due to multiple, successive uses. It is not examined here whether these mathematical tools are a better solution than increased process control previously mentioned. However, mathematical tools may prove to be more practical due to time constraints and cost requirements associated with increased calibration and measurement times.

Unfortunately, the comparison of power measurements over time is hindered by a lack of consistent data caused by measuring different nominal power levels each year. Additionally, the accuracy of the factory-installation calibration measurements is unknown. Various methods were used to take measurements each year by the machine OEM, and it is possible that much of the inconsistency in values each year can be attributed to the lack of a standard, qualified process for these laser power calibrations.

## 2.6 Conclusion

The unique application of lasers within AM processes requires more careful examination of the qualification and calibration techniques used than in other industrial processes. This investigation demonstrates the importance of controlling the laser power calibration process

for AM machines. A proposed method of power calibration and compensation for application on AM is presented and the results of a case study performed on an SLM 280 HL laser powder bed fusion machine with two optics systems.

Two optics systems were used in this study for the purposes of calibrating both the optics on a single machine. Other AM machines and processes may use different types, quantities, or configurations of optics system. As such, the proposed calibration process presented here may not be applicable across a wide variety of AM machines, and no conclusions about general applicability of this particular calibration procedure are drawn for use on other AM machines. Therefore, this research only provides one proposed method of calibrating power for the particular application of AM. The process presented provides only one method of calibrating power for the unique concerns of AM by considering one particular AM machine and process. Future research may involve testing the necessity and robustness of the calibration process for other AM machines and processes. Standardizing this calibration process for broader AM application would permit greater understanding of the laser power and greater control of part quality and AM process qualification as a whole. Short of a standard procedure for all of AM, establishing and maintaining a consistent calibration procedure for a particular AM machine and process provides valuable information and changes to this process should be well-tracked and recorded for valid comparison as part of a comprehensive qualification for AM machines and processes.

Prior to this study, the end-user had suspected degradation of the laser optics hardware caused decreases in power output over time. However, the results of this study indicate that the perceived degradation may, instead, be due to inconsistencies in measurement technique or the calibration technique being unsatisfactorily qualified for AM machines and processes. Though it is possible that the effects of these inconsistencies resulted in changes in AM part quality, such as a reduction in part density or discolouration, the connection has not necessarily been established beyond the theoretical parameter development feedback loop and may warrant future investigation. Other topics of future research may include determining when to perform calibration of the laser or investigating the effects of regular calibrations of the meter or laser. Additionally, it may be possible to optimize costs associated with

the increased calibration regularity balanced with the accuracy of output. The accuracy of alternate power curves or regression techniques could also be considered.

## 2.7 Declarations and Funding

Jeremy Hale worked as a cooperative graduate researcher with the U.S. Department of Energy (DOE) by Consolidated Nuclear Security, LLC (CNS) at Y-12 National Security Complex (NSC). This research was conducted as part of a project funded by CNS Y-12. Data, processes, operations, and machines were provided by CNS Y-12 for analysis and use. Derek Morin, Michael Boice, and Kevin Lamb were employed by CNS Y-12 at the time of the research.

# Chapter 3

# A Framework Using Witness Parts

## 3.1 Introduction and Problem Description

Additive manufacturing (AM) uses 3D model data to make parts by joining material, often layer upon layer, and contrasts with subtractive and formative manufacturing (SFM) methodologies of creating products often by molding, casting, forming, or otherwise removing or reshaping material (ISO/ASTM52900-15, 2015). This distinctive process gives AM distinctive properties and unique production capabilities that permit manufacturers to use a wide range of materials and explore more complex designs not previously possible. AM is an ideal candidate for the production geometrically complex designs and low-volume parts with the added benefits of the flexibility to change materials between production runs, use different process parameters on a single build or build-to-build, combine a variety of parts in a single build, or otherwise modify production processes and materials on a single machine. Though these properties make AM an attractive candidate for production, they can also introduce challenges that make it difficult to achieve the same level of reliability and repeatability possible with SFM. AM machines can be highly complex and sensitive such that a relationship between machine performance and part quality is difficult to qualify and quantify. Therefore, much of the efforts to increase AM performance have focused on determining a causal relationship between the properties of finished parts and specific

processes of AM production, such as feedstock properties, melt pool characterization, build process parameter optimization, and design optimization.

While processes that produce high-volume parts using AM solely to produce complex designs can borrow more heavily from the statistical methods used for SFM, these methods are not easily applied to the low-volume or stochastic production for which AM is relatively well-suited. As a result, monitoring an AM machine has been largely ignored in favor of researching the metallurgical processes. National Aeronautics and Space Administration's Marshall Space Flight Center (NASA MSFC) provides one such example of this perspective in a pair of specifications (MSFC-SPEC-3717, 2017) and standards (MSFC-STD-3716, 2017), which outline a qualification procedure for laser powder bed fusion of metals in AM by separating qualification of machines and parts according to varying classifications. Additional methods, standards, and specifications may exist but are mostly unavailable to the broader body of practitioners and academics, or may be otherwise insufficient for broad AM applications. To begin addressing this shortcoming, America Makes and American National Standards Institute's (ANSI) Additive Manufacturing Standardization Committee (AMSC) have released a document entitled Standardization Roadmap for Additive Manufacturing: version 2.0, which outlines the progress towards standardization of AM and the current gaps in standardization knowledge (American Makes and ANSI, 2018). The document identifies a standards gap, among many others, in the area of machine health monitoring whereby a machine is monitored for changes that may indicate a fault so that maintenance and repairs can be performed.

Previous efforts to overcome variable product quality in AM have focused on controlling the effects of material and mechanical properties such as melt pool properties, powder characterization and packing, heat sources, solidification, and material stress (Hu and Mahadevan, 2017). In situ monitoring for defects using specially designed artifact parts and measurement techniques have been designed to allow corrective procedures in real-time when detected (Xu et al., 2019). In process data has been used with machine learning techniques to predict part quality as measured by tensile strength (Zhang et al., 2019). Process parameters and design decisions have been correlated to curling of the printed parts using Bayesian

inference (Mokhtarian et al., 2019). Aggregate measurements, such as vibration, have been shown to accurately reflect the functional status of an AM process by using condition-based monitoring and Bayesian networks (Rusu et al., 2019). Some key performance indicators have been identified for cost estimation and decision-making in evaluating AM production processes (Panicker et al., 2019). However, the condition of the AM machine itself plays a critical role in determining the quality of produced parts and reducing inconsistencies in output (Yan et al., 2019). Therefore, models that can reflect the health of the machine itself and quantify its effect on product quality are preferable.

The ability to monitor AM machine health directly from the quality of parts produced is more difficult for AM than SFM and therefore has been limited and less strongly developed in research, industry application, or standards documentation. A holistic perspective and framework of witness parts designed to both exploit and sidestep the unique aspects of AM machines and processes would allow the benefits of cheap, efficient techniques from other industries and production processes to be applied to AM production-oriented environments. Additionally, this framework will also be a helpful step towards developing specific AM machine health monitoring standards and will aid in the filling several other standardization gaps including parameter control, machine qualification, environmental effects on machines and materials, in-process monitoring of build quality, part classification and criticality, and non-destructive evaluation or parts. This conceptual paper endeavors to present one such witness parts framework that permits a revised perspective for the way machine health can be monitored in AM.

## 3.2 Revised Framework for AM Machine Health Monitoring

Much research has been done with the goal of using data-driven methods of in-situ sensor data during AM processes to detect and diagnose machine faults. Mechanical machine faults, and therefore reductions in quality, can be detected using a combination of piezoelectric

and acoustic sensors (Yoon et al., 2014), vibration sensors (Li et al., 2019), accelerometers, thermocouples, and acoustic emission sensors (Nam et al., 2020), accelerometers and acoustic emission sensors (Kim et al., 2018), or velocity and angle sensors that are trained using only data only from healthy machine processes for machine learning training data (Li et al., 2021). More generally, a framework for a centralized system to monitor fleets of AM machines has been proposed to optimize quality, throughput, and cycle time for production by monitoring various in-situ and post-process sensors (Balta et al., 2018). However, these models rely heavily on the implementation in-situ sensors and complex monitoring systems, which can be costly or difficult to implement. Additionally, this type of framework may be better suited to AM processes like fused deposition modelling, where machine faults are often mechanically manifest to effect product quality. Complex systems may be appropriate for detailed, research-based applications of AM but less well-suited for production-based application of AM looking for quick, cheap, and easily implementable health monitoring models. Depending on the needs of the production environment, the necessity to detect small changes in quality may be difficult without extremely specific and specialized sensors. As a result, the ability to detect changes in quality may be limited for certain AM processes, machines, and production requirements.

Bottom-up fault detection that finds faults using sensors directly on the machine or monitoring the building process directly has comparative weaknesses compared to top-down fault detection that finds faults by monitoring issues in the output of the parts themselves. Top-down monitoring methods, which typically directly monitor the quality of parts produced by a machine, require statistical or probabilistic data from production. However, these methods, often used in SFM, have been less readily applied to AM because of the unique properties of AM. Low-volume, custom parts, or parts with high production costs often produced with AM may be too valuable to produce or to test destructively. There may be an insufficient quantity of parts produced to derive meaningful statistical confidence, or it may otherwise be cost-inefficient to test every new design or production run on the machine. Further, the physics of additively building parts combined with the flexible

production possibilities of AM makes the effects of design, process parameters, feedstock, or machine health difficult to separate by simple testing of produced parts.

Witness parts, sometimes alternatively called "benchmark artifacts," "test parts," or "test artifacts," have been specially designed and used to evaluate the performance of AM processes by evaluating the dimensional accuracy of a built part (Rebaioli and Fassi, 2017). The capabilities of an AM machine can be evaluated by examining the dimensional accuracy of geometric features on standardized parts (Weaver et al., 2018). Producing these witness parts at regular intervals over time would provide consistent referential data by which the quality could be tracked. Building witness parts as a standard practice would also provide increased data for performing other analyses and tracking cost and quality of parts production, as opposed to research, environments. Witness parts can be implemented in two ways: (1) witness builds, which are builds that contain only witness parts, such as the NIST standard part (Moylan et al., 2014), or other parts and features constructed specifically to be analyzed, and (2) witness artifacts, which are parts or features made as part of every build alongside normal production parts, and are sometimes referred to as "test coupons." Figure 3.1 shows a model of such a production order for AM when including both types of witness parts.

Each type of witness part carries some benefit. Witness builds generate data that is comparatively more representative of machine health by virtue of maintaining consistent design, process parameters, and feedstock for each witness build over time. Witness artifacts provide regular, actionable data points for each build but may imbue less confidence about the health of a machine because of changing designs, process parameters, and feedstock used for normal production parts. While normal production parts can still be validated separately, they may be too valuable to destroy, particularly for small production quantities possible with AM. By contrast, witness parts can be tested often and even by destructive means. Further, the flexibility of AM, rooted in digital manufacturing and low change-over effects between builds, permits witness builds and artifacts to be integrated more readily into a production schedule compared with SFM. This generates relatively minimal interruption and disturbance to build designs or production planning while keeping costs and

time requirements low. Additionally, the separation of production and witness parts may provide more detailed information about the root causes of faults or issues observed during production. For example: if a newly produced part is observed to have reduced density but the witness parts present no changes, manufacturers now have higher confidence that the cause for the issue likely lies with the design being unsuitable for the machine or feedstock under current qualification requirements.

Once data can be collected from the AM production process using witness parts, other analyses to track machine health can be performed. Condition-based monitoring has been suggested as one such possible method for monitoring machine health (American Makes and ANSI, 2018) and has many existing standards, research applications, and examples readily available to be applied to AM to permit corrective machine maintenance. Though condition-based monitoring has already been applied to AM machine health monitoring using data-driven methods and live monitoring systems using in-situ sensors, data from witness parts can augment these monitoring systems, completely replace them, or to implement alternative models that were otherwise difficult to use without regular witness data. Figure 3.2 shows the flowchart of data in an AM production environment that considers machine health.

## 3.3 Monitoring Using the Framework

All AM processes may not be able to take full advantage of the features of a witness parts framework or may otherwise be unsuitable for its implementation. AM machines and processes with fault modes that can be more easily and directly monitored may be better suited by directly monitoring those particular systems or sensors that reflect those faults developing. Physical restrictions of a particular process, the quality metrics of concern by the manufacturer, and the requirements of any monitoring models built on top of the witness parts framework must be weighed against other possible health monitoring methods.

For example, processes that use filament extrusion or lamination techniques may be suitably reflected by extrusion temperature or nozzle alignment. Further, processes with significant post-processing, such as furnace heating to remove binding agents or surface finish

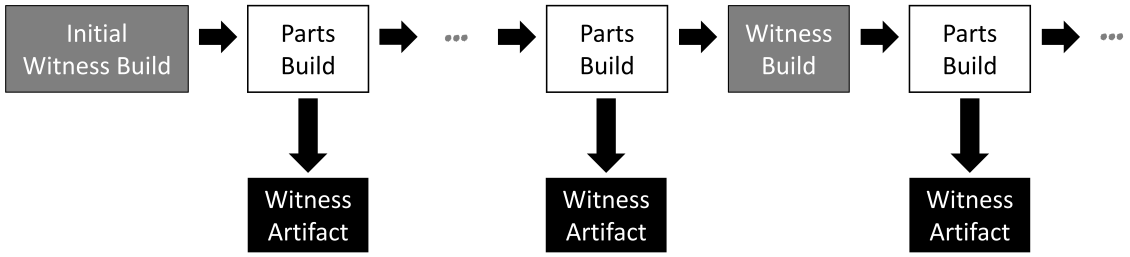



**Figure 3.1:** Production order for AM machine using witness builds and witness artifacts

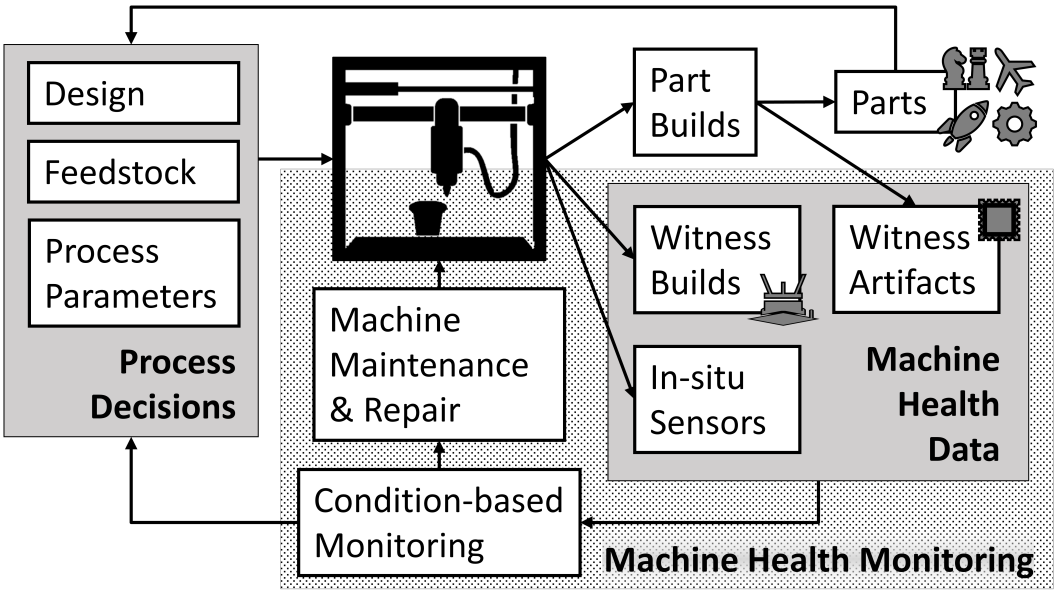



**Figure 3.2:** Revised AM production environment using condition-based machine health monitoring

treatments, may be may make witness parts poor reflectors of part quality or machine health. Some processes may not provide sufficient build volume to construct witness artifacts, such as some direct metal sintering, or may produce parts sufficiently rapidly and inexpensively, such as some forms of stereolithography or vat polymerization, that it is more financially feasible to either produce only witness builds or produce extra parts for direct quality verification. Therefore, a witness parts framework will add value to machines with relatively complex systems whose effects can be measured in the quality of parts. Selective laser melting processes and laser powder bed fusion, particularly of metals, for use in high precision and quality-critical applications such as those in the defense, biomedical, or aerospace industries are among the most ideal processes for the benefits of the framework.

By systematically measuring the quality of witness parts, the influence of machine health on part quality can be observed in detail over time. Many of the quality metrics are already the concern of manufacturers including dimensional accuracy, density, surface finish, color, tensile strength, fracture mechanics, compression strength, and microstructure. Each of these can tested using methods of varying complexity such as comparison charts for color or blue light imaging for dimensional accuracy. Since quality is being measured directly, it is desirable to use witness part designs and tests that are sensitive to changes in quality and machine health, and therefore both witness part designs and tests should be chosen to provide data that is sensitive to changes in part quality, sensitive to changes in machine health, and reflective of machine health. Since these specific decisions must likely be made through case studies and experimentation, they are not defined here. However, many standards already exist for part designs for testing metrics like tensile and compression strength that can inform practical decisions. As the connection between quality metrics and machine health is established, standard part designs unique for monitoring AM machine health may be designed. For example, complex lattice structures that are often manufactured using AM and should be prioritized as a unique design capability of AM. Further, by designing parts at the limits of the machine capabilities, it may be more sensitive to changes.

The rise of AM has also corresponded with a rise in big data processing and industry 4.0, making bottom-up health monitoring methods possible. The inherent properties of AM that

make the differentiation of machine health from other influential factors when analyzing the quality of produced parts, and previous research been focused on efforts to "qualify as you go" by monitoring and characterizing in-process data collected during AM production (Seifi et al., 2016). In-situ sensors in this paradigm must directly monitor systems for gas control, optics, feedstock deposition, or software, or can indirectly monitor the systems through compositive sensors that reflect changes in the performance of these systems.

However, the top-down monitoring strategy permitted by a witness parts framework allows the implementation of simple other useful models, commonly used in SFM and other industries, to optimize the cost and quality associated with long-term machine health and maintenance strategies. Maintenance and repair needs can be identified, diagnosed, predicted, and performed in to optimize production throughput, product quality, costs, and time for production schedules. These sorts of health monitoring models may be more suited for AM as the technology leaves a more quality-optimizing research level and enters more cost-optimizing production level of usage.

Similar models have shown success in the semiconductor industry, in which indicators of equipment health and wafer quality can be used to schedule equipment use and determine qualification status (Yugma et al., 2015). In particular, hidden Markov models and Bayesian inference have been implemented in the semiconductor industry to provide condition-based health monitoring with consideration of the machine health (Liu, 2008). Hidden Markov decision processes are one method often used for this task, which would treat machine health as a hidden state and production and maintenance decisions made at discrete time intervals.

Initially the root causes of machine faults will require either guesses from expert knowledge or the use of other sensors and in-process data to guide the source of any issues. However, as the witness parts framework and health monitoring models are implemented, they can facilitate the identification of more complex modes of failure as data are collected for maintenance, part quality, and machine health. This data can help establish conditional probability relationships, potentially using Bayesian networks as previously mentioned, to determine the critical health variables and quality metrics.

## 3.4 Conclusion

In order to implement condition-based health monitoring of AM machines, a framework is proposed that uses two types of witness parts to monitor part quality and machine health. This framework may permit the collection of data that reflects AM machine performance while separating the confounding influence of design, feedstock, and process decisions. Though some research exists on creating a standardized part for witness builds, such parts have primarily been used to observe machine capabilities and production process quality. While the data from these standard parts will be useful, incorporating witness parts, both builds and artifacts, into a condition-based machine health monitoring framework may inform the addition or removal of features from this standard part design, and multiple standard parts may be required and developed as the importance of monitoring certain key features is determined. Implementation of this framework may also allow future research into determining root causes for the failure of a machine and how they are indicated in the machine health data. While only one possible method of implementing the framework is suggested here, other methods may be readily implemented as a more top-down monitoring framework is made possible. The effects of machine health on production throughput, production scheduling, and costs can subsequently be considered and optimized using common methods from other industries and manufacturing processes. Ultimately, the framework may permit cheaper monitoring of machine health without the high costs of sensors and monitoring systems that may not be easily implemented into all AM machines and processes. By shifting to a top-down monitoring perspective using witness parts, the focus can be placed directly on monitoring machine performance and part quality, which can be particularly for aerospace, defense, and biomedical industries.

# Chapter 4

# Modeling AM Machine Health and Maintenance

Having implemented witness parts, a model of monitoring AM machine health can be implemented. Such a model will use the observations emitted by a combination of witness artifacts and witness builds as well as the probabilistic values and calculated costs for a particular AM machine. Using this data, a model of a partially observed Markov decision process can be created. While the solution to such a model is itself informative in regard to providing information about the costs and maintenance schedule for a particular situation, a sensitivity analysis of the POMDP model can give further information about the effects on costs, maintenance schedules, and help guide decision-making on how to implement an AM production environment. The first section of this chapter introduces some background on witness parts and how the framework lends itself to implementing a POMDP, as well as some historical and mathematical background on POMDPs. The second section describes some methods and methodology that will be used to implement the POMDP and analyze its utility. We then analyze the results for a particular case study, and then provide a discussion of this model and a sensitivity analysis of the model. Finally, we provide some managerial insights on the model, sensitivity analysis, and how it may influence decision-making on AM

production processes. A conclusion summarizes some of the highlights from the POMDP model and application.

Maintenance for machine health can be performed in a variety of ways given the ability to implement and make use of witness parts. In this section, we will model three methods of monitoring machine health and performing maintenance. First, a simple model is shown that uses no witness parts and reflects a machine that is allowed to degrade indefinitely. Second, we show a renewal-reward model that produces witness builds on some regular interval to monitor the health of the machine. Finally, we model a POMDP using a combination of witness builds and witness artifacts.

There are two other potential models that are not explicitly considered. The first is a model that exclusively uses witness artifacts but not witness builds. We exclude this model because the benefits of witness parts are greatest when used in tandem for all the reasons described in Chapter 3. Moreover, the optimal policy described by a solution to a POMDP will describe the optimal times to construct a witness build, if at all. The second model not considered is one in which machine health is monitored via a combination of machine health indicators, e.g. vibrations, rather than witness parts, that are known to reflect machine health. Principally, this model is not considered because it does not make use of witness parts and is outside the scope of this research. Additionally, this bottom-up monitoring model requires more in-depth understanding of the root causes of machine failure in AM. Given the wide variety of AM machines and the difficulty in creating such a model for AM machine health presently, it is unsuitable for equal comparison to the other models that are considered in this research and would be relatively uninformative. However, we note, as in Section 3.2 that such indicators could be used in tandem with witness parts for a more complex, tailored model for a given AM process, machine, and application.

## 4.1 Model of an Unmonitored Machine

Under this model, neither witness builds nor witness artifacts are constructed. The health of the machine is allowed to degrade, undetected and unrepaired, for a number of builds, $n$.

Operationally, this corresponds performing maintenance and repairs on some finite, regularly-spaced interval irrespective of the machine health condition. Once it degrades, the machine remains unhealthy and can only be improved with the maintenance and repair procedures.

The model is described by a stochastic process represented by $X_j$ for $j = 1, 2, \ldots$ with all possible values of $X_j$ constituting the state space $S = \{h, u\}$, which includes a "healthy" machine state, $h$, and an "unhealthy" machine state, $u$. The initial machine state is healthy, $X_0 = h$, with the probability of becoming unhealthy before the next build given by $p$. If $R_j$ is the percentage of healthy builds for the first $j$ builds, the expected percentage of healthy builds produced with a healthy machine by $j$ builds is given by $E(R_j) = \frac{1}{j}\sum_{m=1}^{j}(1-p)^m$ . Note that $\lim_{j\to\infty} E(R_j) = 0$. Therefore, this particular model of machine health degradation is only useful on some finite interval. It is assumed that maintenance and repairs are conducted every $k$ builds and guarantee a healthy state (i.e., $X_k = h$).

Further, it is assumed that maintenance costs $c$ and takes time, $t$, where $t$ is normalized by build time (i.e., $t = 2$ means the time for a maintenance is equivalent to the time to two builds). Note that this implies $X_{2k} = X_{3k} = \cdots = h$. Good quality builds completed on a healthy machine have value of $r_h$, while bad quality builds completed on an unhealthy machine have value of $r_u$. Each build incurs a cost of $\beta$ irrespective of its quality. Without loss of generality, the cost and value parameters $c$, $r_h$, and $r_u$ can be normalized in terms of the unit build cost (i.e., $\beta = 1$). For example, $r_u = 0$ and $c = 2$ mean that an unacceptable-quality build has zero value and the cost of maintenance is equivalent to the cost of two builds.

Let $V_N(k)$ be the long-run average value per build with maintenance cycle time $k$. Then $V_N(k)$ is given by

$$V_N(k) = \frac{r_u\left(k - \sum_{m=1}^{k}(1-p)^m\right) + r_h\left(\sum_{m=1}^{k}(1-p)^m\right) - k\beta - c}{k+t}. \tag{4.1}$$

Mathematically, the summation is a geometric series, and the equation can be written and simplified to

$$V_N(k) = \frac{(r_h - r_u)(1 - p - (1-p)^{k+1}) + pk(r_u - \beta) - pc}{p(k+t)}. \tag{4.2}$$

The optimal value is $V_N^* = V_N(k^*) = \max_k\{V_N(k)\}$, and the optimal maintenance cycle time is $\kappa_N = k^*$ for this model. The importance of this distinction between $\kappa_N$ and $k^*$ will become clearer after considering the implications of the next model.

## 4.2 Model Using Only Witness Builds

Rather than periodic maintenance, witness builds can be used to determine the machine health state and perform condition-based maintenance, which may help the machine return to a back a healthy state more quickly. Similar to the unmonitored model, repairs are assumed to be performed prior to the next parts build and with perfect success. Further, the witness builds are be assumed to perfectly sensitive and specific to the health of the machine. Mathematically, this can be modeled by augmenting the stochastic process described in the previous section into a renewal-reward process, which continues the assumptions that the process is memoryless and stationary. Witness builds are constructed on regular intervals after $k$ builds, meaning the $k+1$ build is a witness build. The value of witness builds are given by $r_d$, which may be different than $r_h$ or $r_u$ since constructing a witness build consumes build time and feedstock but yields no quality parts.

Let $V_D(k)$ be the long-run average value per build in which a witness build is produced and tested (without delay) every $k+1$ builds as the build immediately following $k$ normal production builds. Then

$$V_D(k) = \frac{(r_h - r_u)\sum_{m=1}^{k}(1-p)^m + k(r_u - \beta) + (r_d - \beta) - c(1-(1-p)^{k+1}))}{k+1+t(1-(1-p)^{k+1})} \tag{4.3}$$

$$= \frac{(r_h - r_u)\frac{1-p-(1-p)^{k+1}}{p} + k(r_u - \beta) + (r_d - \beta) - c(1-(1-p)^{k+1}))}{k+1+t(1-(1-p)^{k+1})} \tag{4.4}$$

The optimal value is given by $V_D^* = V_D(k^*) = \max_k\{V_D(k)\}$. However, under this model, the optimal cycle $k^*$ represents the interval on which witness builds are produced as the $k^* + 1$ build. Maintenance is only performed when the witness build indicates the machine is in an unhealthy condition, but the expected *maintenance* cycle time is found by

$$\kappa_D = (k+1)\left(1-(1-p)^{k+1}\right)\sum_{n=0}^{\infty} n\,(1-p)^{(k+1)(n+1)}\,. \tag{4.5}$$

## 4.3 Model Using Witness Artifacts (and Witness Builds)

In order to consider the third model in which witness artifacts, and possibly witness builds, are used, we will consider a partially observed Markov decision process, or POMDP. We first define and describe POMDPs. This is followed by a description of some solution methods. Last, we model the problem of monitoring AM machine health as a POMDP.

### 4.3.1 Description of POMDPs

A Markov chain can refer to a stochastic process for a sequence of stages of states of the process that change according to a Markov property, or memoryless property, in which the probability a state transition, or the probability of the next state in the process, requires knowing only the current state of the process. A Markov decisions process, then, is a stochastic process in which the particular strategy for decision-making strategy, or policy, at each discrete stage of the process is optimized. A hidden Markov model describes a Markov chain, or process, in which the underlying state, or events, are hidden or not directly observable, but rather inferred through some other observable process with a probabilistic correspondence to reflecting the underlying hidden states. Finally a decision process for such a model with hidden states is referred to as a partially observed Markov decision process, or POMDP.

While the state space of such a model can be continuous, we will limit our discussion here to only finite, discrete state spaces. However, for a POMDP, this underlying set of spaces are projected into a continuous space, or belief space, representing some probability distribution

across the underlying states. Since these states are unobserved, actions are chosen based on a particular policy after making an observation, which itself has some likelihood of being emitted. In this way, observation histories are mapped to the belief space, and the policy is a mapping of the belief space to actions.

Mathematically, we define a POMDP by a tuple $(S, A, Z, T, \Omega, R)$ which defines the characteristic parameters of a particular POMDP model. Here, we have the set of all underlying states $S$ with $s, s' \in S$, the set of all possible actions $A$ that can be taken at each decision stage with $a \in A$, and the set of all possible observations $Z$ that can be made. The transition probability, $T$ is the probability of transitioning from a given state, $s$, to a subsequent state, $s'$ after taking an action $a$, so that $T(s'|s, a)$. The emission probability, $\Omega$, is the probability of receiving a particular observation $z$ when in state $s'$ after taking action $a$, such that $\Omega(z|a, s')$. Finally, the reward, $R$ is the value accumulated when taking an action, $a$, while in a particular state, $s$.

Since a belief state is a probability distribution across $S$, any belief state $b$ is an element of the $|S|$-dimensional belief space $B$ with $\sum_{s \in S} b_s = 1$. As the decision process progresses, the belief state for any decision stage $j$ can be updated by considering the Markov property of the model, thereby requiring only the belief state $b_{j-1}$ from the previous decision stage and the observation $z$ received after action $a$ was taken. The *belief update* is a Markov operation evaluated by:

$$b_j(\bar{s}) = \frac{\Omega(z, \bar{s}, a) \sum_{s \in S} T(h, s, a) b_{j-1}(s)}{\sum_{s' \in S} \sum_{s \in S} \Omega(z, s', a) T(s', s, a) b_{j-1}(s)}$$

for all $\bar{s} \in S$. We note that the denominator is a normalizing constant as the sum of all possible observation-transition probabilities from the previous belief state.

### 4.3.2 Solution Methods for POMDPs

Solutions for POMDPs consist of determining a policy and the optimal long-run expected value associated with the policy over its time horizon. The algorithm by which this solution is found depends partially upon the particular properties of the problem being modeled, the complexity of the solution, the speed with which a solution must be determined, and the

need for precision or accuracy of the solution. We will briefly describe, here, some of the options and the various considerations when choosing a solution method.

The POMDP can be modeled over a finite or infinite horizon, that is, the number of stages in the decision process. If the process being modeled is most concerned with some set period of time, a finite horizon may suffice. Indeed, condition-based machine maintenance can be modeled using a finite horizon POMDP on a machine with a finite lifespan (Byon and Ding, 2010). Applied to AM, relatively rapidly-evolving technology in the nascent stages of adoption as well as a predisposition towards using machines for research and development can mean shorter lifespans. However, as AM technology begins to enter maturity and machines are being included as part of long-term, stable production processes, the lifespans become longer and indefinite. We therefore become more concerned with models that are reflective of a longer-term, indefinite horizon for decision processes as applied to AM machines. A POMDP that models an infinite horizon can be appropriately applied in this scenario for AM. An infinite horizon, however, requires the use of a discount factor, $\gamma$ wherein the reward accumulated for each subsequent process stage is reduced by a factor of $1-\gamma$. As a result, this discount factor is sometimes included in the mathematical tuple of the POMDP definition, since its selection can have some effect on the determined value or policy, and can be reflective of some real, rather than simply mathematical, requirements of a particular POMDP model. Some implications and tuning of this parameter are included and further detailed in Section 4.6.3.

Algorithms for determining a solution for a POMDP are can be done in one of two iterative methods: value iteration, or policy iteration. Value iteration iteratively updates the value and selecting the policy which reflects an improved value. By contrast, policy iteration iteratively repeats steps of policy evaluation and policy improvement. The original one-pass iteration algorithm developed for policy iteration (Sondik, 1978), however, is relatively complicated and in practice was not widely used. The introduction of a finite-state controller (Hansen, 1997) made the use of policy iteration much simpler to implement and more practical to use. In addition, policy iteration tends to outperform value iteration (Littman et al., 1995) particularly for the most complex or computationally difficult problems. For

this and other reasons policy iteration has several benefits such as increased solution speed for many problems and finding optimal policies for all starting states of the decision process.

Each of these methods, value iteration and policy iteration, can be calculated by exact or approximate solution strategies, and the selection between these is often a factor of the complexity of the particular problem and the trade off of solution time against solution accuracy.

Algorithms of various efficiencies exist to include in the dynamic programming step of the overall solution algorithm including the one-pass algorithm (Sondik, 1978), witness algorithm (Littman et al., 1995), relaxed region algorithm (Cheng, 1988), incremental pruning (Zhang and Liu, 1997), witness algorithm with incremental pruning (Cassandra et al., 1997), and a best-first heuristic search (Lark et al., 1995). For policy iteration, exact algorithms use dynamic programming for the policy improvement step, but this is the performance bottleneck, and heuristic policy search algorithms for larger problems have been developed (Hansen, 1998). For problems of close to two states or two observations, most algorithms perform approximately the same, with the witness algorithm typically outperforming the others for larger problems (Cassandra et al., 1997).

Even for small quantities of states, actions, and observations, the dynamic programming update can become intractable or severely increase the algorithm runtime. Indeed, at only 5 observations, generating an exhaustive set of possible nodes can be intractable (Hansen, 1998). Incremental pruning reduces the size of this set of possible nodes by pruning during the process of the dynamic programming one-step backup, rather than completing the backup and performing pruning as a separate subsequent step. In this way, the number of new nodes that must be found is greatly reduced, limiting the number of nodes that would otherwise have to be considered. This has an overall effect of drastically improving the time required for the DP backup.

For the sake of potential expansion to larger problem sizes and the application of theoretically superior properties, the solution algorithm chosen in this research is policy iteration, and further details of algorithm provided in the rest of this section. Given the relatively small size of the state, observation, and action spaces for this model described in

this paper, a simple version of exact policy iteration using a finite state controller (Hansen, 1997) will be used as well as incremental pruning (Cassandra et al., 1997) in the dynamic programming step for improvements in speed, which will be particularly useful for the multiple POMDP solutions found for the analysis in Section 4.6.2.

As described previously, policy evaluation is performed by determining a set of vectors $V_i \subset \mathbb{R}^{|S|}$ with $v_i^n \in V_i$ for $n = \{1, \ldots, |N_i|\}$. These value vectors a calculated by solving for the variables $v_i^n(s)$ in the system of equations defined by

$$v_i^n(s) = R\left(s, a_i^n\right) + \gamma \sum_{s',z} T\left(s', s, a_i^n\right) \Omega\left(z, s', a_i^n\right) v_i^{\phi}(s').$$

Here, $v_i^{n'}(s')$ is the value for state $s'$ of the successor node, $n' \in N_i$, from node $n \in N_i$ for an observation, $z$, according to current FSC, $\phi_i(n, z) = n'$. Note that this representation of the system of equations is for each state, meaning the system involves $|N_i||S|$ number of equations as well as $|N_i||S|$ unknown variables.

Policy improvement uses a dynamic programming algorithm to perform a one-step backup of the decision process to find an improved set of value vectors, $V_{i+1}$. First a superset $\bar{V}_{i+1}$ with $\bar{v}_{i+1}(a, s) \in \bar{V}_{i+1}$ and $\bar{v}_{i+1}(a, s)$ is calculated directly using

$$\bar{v}_{i+1}(a, s) = R\left(s, a\right) + \gamma \sum_{s',z} T\left(s', s, a\right) \Omega\left(z, s', a\right) v_i^{\bar{\phi}}(s')$$

where $v_i^{\bar{\phi}}(s')$ is the associated value of the vector component of the successor node $n' \in N_i$ after action $a$ and observation $z$ according to observation strategy $\bar{\phi}_i\left(z, a\right)$. Next, the minimal set $V_{i+1}$ is found by pruning $\bar{V}_{i+1}$ of all dominated vectors using linear programming. Finally, the improved FSC, $\phi_{i+1}$, and therefore a new set of nodes $N_{i+1}$ and improved policy $\pi_{i+1}$, is found by performing a transformation between $V_i$ and $V_{i+1}$ described by Hansen (Hansen, 1997).

This algorithm is repeated until convergence, determined once the Bellman residual is less than $\epsilon$. The Bellman residual is simply the greatest improvement in value at any belief

state after each iteration, or $\max_b |V_{i+1}(b) - V_i(b)|$. This can be done for piecewise linear and convex functions in polynomial time with linear programming (Littman et al., 1995).

### 4.3.3 Applying a POMDP to AM

We will use a POMDP to model AM machine health. It is assumed that witness artifacts are constructed alongside every normal production build for $k$ builds, and that a witness artifact indicates an action on the $k+1$ build to either (1) construct a witness build for analysis, or (2) directly perform machine repairs. The machine may actually be in a "healthy" or "unhealthy" state at the time of the $k+1$ build, and both the transition and emission probabilities are the same for witness builds as for normal production builds since the machine could become unhealthy over the same course of time. The implementation of witness artifacts permits the health status of the machine to be monitored per-build. Witness artifacts are constructed alongside every normal production build and a decision made after each to either (1) continue normal production parts builds, each of which includes a witness artifact, (2) construct a witness build, or (3) perform machine maintenance and repairs.

Since witness artifacts can be affected by factors other than machine health and do not perfectly or directly reflect the status of the machine, the machine health state is represented as a probability distribution across the two health states, that is $b = [b_h, b_u] = [Pr(X_j = h), Pr(X_j = u)]$ such that $b_h + b_u = 1$. Further, cheaper witness artifacts provide less detailed information about the machine state and but provide some update to the belief state, but the relatively more costly witness builds can perfectly reflect the machine state. Similar to the first two models, the machine can change states during the course of any build, whether a witness build or production parts build with a witness artifact. At the end of this build, an observation is received from the available witness part, and a decision is made about the next action to take.

First we choose the underlying, hidden states of the POMDP to be the two health states of the AM machine, healthy and unhealthy, such that $S = \{s_h, s_u\}$. We the set of possible actions each stage of the decision process as $A = \{a_f, a_d, a_m\}$ where $a_f$ is constructing a

normal production with an included witness artifact, $a_d$ is constructing a witness build, and $a_m$ is performing machine maintenance and repairs. Observations are chosen as $Z = \{z_1, z_2\}$ as acceptable, $z_1$, or unacceptable, $z_2$. For these first three sets, we note that there are many ways of choosing their definitions, but the sets described here represent a simple, minimum, and intuitive way of defining them that easily match the practical descriptions of a machine and the quality of the observations. With these sets defined, the remaining we define the transition probabilities as the probability that the underlying machine health state transitions from $s$ to $s'$ when taking action $a$. Maintaining the assumptions from Sections 4.1 and 4.2, our assumptions are that a healthy machine transitions to an unhealthy state by the end of any build (that is, for action $a_f$ or $a_d$) with probability $p$, and that an unhealthy machine will remain unhealthy until maintenance, $a_m$, is performed. Further, machine maintenance and repairs always return the machine to a healthy state. Our transition matrix for the three actions are given below.

$$T_{s',s}(a_f) = \begin{bmatrix} 1-p & p \\ 0 & 1 \end{bmatrix} \tag{4.6}$$

$$T_{s',s}(a_d) = \begin{bmatrix} 1-p & p \\ 0 & 1 \end{bmatrix}$$

$$T_{s',s}(a_m) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

Emission parameters are defined in three parts, as a function of the relevant actions. Since we assume machine maintenance and repair, $a_m$, always returns the machine to a healthy state, we further assume that maintenance and repair emits and observation perflectly reflective of that transition. Similarly, we assume that witness builds are perfectly reflective of machine health and emit the corresponding observations. Finally, however, the witness artifacts emit observations with some probability determined by some process, as discussed in more detail in Section 4.5. We denote the probability that a machine emits an "acceptable" quality

observation when in a healthy state by $\omega^f_{h,1}$ and the probability that a machine emits an "unacceptable" quality observation when in an unhealthy state by $\omega^f_{u,2}$. In total, the emission parameters are defined as shown below.

$$\omega_{s',z}(a_f) = \begin{bmatrix} \omega^f_{h,1} & 1-\omega^f_{h,1} \\ 1-\omega^f_{u,2} & \omega^f_{u,2} \end{bmatrix} \tag{4.7}$$

$$\omega_{s',z}(a_d) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\omega_{s',z}(a_d) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Rewards in the POMDP are similarly defined to be comparable to the other two models and making the same assumptions. However, given that the POMDP considers all actions in stages, we do not have to explicitly includes the time parameters. The rewards are modeled as shown below.

$$R_{a,s} = \begin{bmatrix} r_h - \beta & r_u - \beta \\ -c & -c \\ r_d - \beta & r_d - \beta \end{bmatrix} \tag{4.8}$$

The solution to the POMDP model differs from the solution to the previous models in Sections 4.1 and 4.2 in two important ways. First, rather than a maintenance cycle, the solution to the POMDP is a function of a policy, $\pi$, that maps belief states to actions $\pi : B \rightarrow A$. This is exactly the benefit of using a POMDP and witness artifacts since maintenance can be tightly tailored to machine health after each build. As a result, an optimal policy, $\pi^*$, is the POMDP analog of the optimal process cycle, $k^*$, of the previous two models. Second, the value function for an optimal solution to a POMDP is represented as a set of $|S|$-dimensional vectors, $V$, where each vector $v_n \in V$ represents the expected long-run accumulated value when beginning the decision process in a belief state mapped to that vector. We can consider the expected long-run value per unit-build-time as the sum

of a geometric series with common ratio $\gamma$ and a coefficient term representing the long-run expected value per build, so that $E[V^*] = V_F + \gamma V_F + \gamma^2 V_F + \cdots = \frac{V_F}{1-\gamma}$, and the expected long-run value per unit-build-time is given by

$$V_F = (1-\gamma)\, v^*. \tag{4.9}$$

Though there are potentially several ways to calculate expected maintenance cycle time, $kappa_F$, in this model, we have decided to use of a short simulation to determine the expected maintenance cycle time. This has the added effect of also giving a simulated calculation of expected long-run value per unit-build-time, as well. The simulation is run for some minimum number of decision process stages, calculating the average of each $V_F$ and $\kappa_F$ with no discount factor considered. While $V_F$ for the pomdp model will be determined using Equation 4.9, the difference between this value and the simulated calculation of $V_F$ will provide some confidence in the accuracy of $V_F$ as calculated by the POMDP as well as the value of $\kappa_F$ found by the simulation.

## 4.4 Choosing Witness Parts

Several decisions must be made in choosing witness parts. As stated in Chapter 3, witness builds should represent whether a machine is meeting the qualification requirements set out for the machine. We recall that machine health, in this model, is being defined as the ability of a to build production parts within the defined qualification parameters. Therefore, the simplest and most direct way to choose witness parts is to select designs that directly measure the quality metrics required for the final production parts. While these will vary depending on the production requirements and the type of machine chosen, some suggestions are density, surface finish, dimensional accuracy, dimensional precision, color, or mechanical properties such as tensile strength or heat resistance. Indeed, these are some of the properties included in the NIST standard test artifact (Moylan et al., 2014) often used for benchmarking and comparing the capabilities of an AM machine or process, and such an artifact can be

used to determine the accuracy of metals AM processes (Weaver et al., 2018). Including all of the relevant metrics provides a direct measurement of the health of the machine that can be determined by observations of the witness part. However it is not strictly required to include all or even only those metrics. One could choose only a subset of those metrics if they are dependent upon one another, for example, or very insensitive metrics and therefore not economical to test compared to other metrics. If done for witness builds, this option could also necessitate modifying the emission parameters to be less than 1, if they no longer perfectly reflect machine health. Alternatively, one could include more metrics that are not direct quality requirements of the witness part but instead are more sensitive to changes in machine health or otherwise provide early detection of changes to production part quality or machine health. Measuring properties of the microstructure of parts provides one such example (Kok et al., 2018).

Despite all of these possibilities, it is likely advisable, still, to choose witness parts that directly measure the quality metrics and requirements of a production part. In this way, we (1) gather measurements of the production quality over time, (2) have a one-to-one relationship between machine health and the witness builds, and (3) do not have to establish more complex definitions of machine health or relationships between the machine, production parts, and witness parts.

In this research, we recommend the use of a lattice structure, which is a design relatively unique to AM compared to CM processes. Additionally, lattice structures can be built on a variety of AM machines and processes and some guidance already exists on how they can be applied in appropriate ways across these different applications (Uribe-Lam et al., 2021). Much work has been done on the various properties of lattices in AM production, the manufacturability of such structures, and the types of properties they can reflect (**?**). For metals laser powder bed fusion, a single lattice cube can provide information about density, geometric accuracy, dimensional precision (e.g., how small the struts of the lattice can be), compression strength, surface finish or roughness, and other various properties. In this way, it represents a single, simple feature for the measurement of multiple quality metrics often considered when measuring the quality of production parts.

A lattice structure designed as a small cube, a lattice cube has some desirable features that further make it a desirable choice for witness parts. As a witness artifact, a lattice cube can be designed to be relatively small, taking up little space in a build and therefore allowing increased profits per build, increased throughput, and reduced time to build. For witness builds constructed in metals laser powder bed fusion machines, a grid of lattice cubes across the entire build plate can provide the ability to see how measurements vary across the build plate, making it more sensitive to changes in machine health that may only effect quality in the extremes of the build plate. Its small size also makes it simple to supplement a witness build with other features, such as tensile bars, density cubes, or other pieces which may require some standard design for testing or statistical significance and therefore must be built separately.

Some care should be given to selecting the particular lattice design used, the machine for which it is being designed, the feedstock used in the process, and the parameters ultimately chosen for the qualified process. For example, aluminum alloy feedstock used for an selective laser melting AM process can be sensitive to humidity, which would ultimately influence the range of measurements achievable on a particular lattice witness part. Other lattice designs may be too geometrically intricate, negatively affecting the measurements of the quality metrics simply because of poor heat dissipation or by attempting to construct a lattice cube with struts too detailed given the diameter of the laser beam. The sensitivity inherent in the lattice design is both a concern and a strength of using lattice structures for witness parts. Making decisions about which lattice design to use, how many and what diameter struts, how tall of lattice cubes, and other details will be unique to each AM machine, process, and production requirements. While a detailed outline of the selection process for all AM processes is not the subject of this research and therefore is not provided, lattice cubes should, in general, reflect the capabilities and requirements for quality metrics in a qualified production process of a while also being sensitive to changes in that quality. This is likely best done by designing lattice cubes at the practical edges of the production capabilities of a machine within a given qualified process.

## 4.5 Parameter Estimation and Data Collection

An initial sequence of witness builds is produced, setting the values for standard operating values of the quality metrics with distributions. Witness builds may take several builds to complete, depending on the type of data being collected, and will be accounted for in costs. Next, a normal production build is begun. Data is recorded each time period, that is, after each normal production build or sequence of witness builds, and a decision is subsequently made whether to take one of several possible actions.

## 4.6 Numerical Results

The numerical results are shown in three parts. First, a case study shows a comparison of the three machine health monitoring models, ultimately showing the relative benefits of a POMDP. Second, a sensitivity analysis shows where which parameters have the strongest effects and how they influence one another. Finally, we include a discussion that includes additional insights and considerations of the model from both mathematical and pragmatic standpoints.

### 4.6.1 Case Study

The parameters used for in this case study are given in Appendix Table 1. Given that this model is designed partially to demonstrate the benefits of a model combining witness parts and machine health monitoring when building a variety of part designs in a production-oriented environment, few, if any, applications of AM exist that meet all these requirements from which parameters can be derived. As such, parameters are chosen by considering multiple sources.

The case study considers an SLM 280 HL powder bed laser fusion machine used for research-and-development purposes to build a variety of part designs. The feedstock used is 316L stainless steel and is itself monitored using a qualification process that is separate from this machine health monitoring model. This ensures that any observed changes in witness

part quality are not the result of changes to feedstock quality, as those changes are assumed to be detected via these separate processes.

Historical data from this machine were collected over a 24-month period, which were used to determine transition probabilities and witness artifact emission probabilities. During this time, the machine was used to build a variety of part designs for both production and research-and-development purposes.

Values used for $r_h$, the value of a healthy production build, and $\beta$, were chosen according to the cost model in (Griffin et al., 2022). Subsequently, all values and costs are normalized to $\beta$. Similarly, the corresponding time required for a single production build, $\theta$, represents an implicit parameter to which maintenance time, $t$ is normalized.

For this machine, maintenance and repair costs were performed on a contractual basis with the machine OEM and did not differentiate the type of maintenance or the individual details of the maintenance procedure performed. Therefore costs of a single repair were calculated as the ratio of annual maintenance contract costs to repair time (inclusive of machine downtime waiting to be repaired). This cost was then normalized to $\beta$. Parts constructed on an unhealthy machine as well as witness parts are assumed to have zero scrap value and zero disposal costs, so $r_u = 0$ and $r_d = 0$.

A few strong assumptions are made in this case study for the purposes of simplifying the model, and the resulting analysis, as well as accommodating a lack of detailed historical data collection largely because collection was unnecessary without a model, such as this POMDP model, in which to utilize the data.

In this case study, density is the only quality metric considered. Further, only two observations of density are made: either "acceptable" or "unacceptable". What constitutes "acceptable" quality is determined via separate qualification processes set by the production requirements. We note that, for only two observations, the model results will hold regardless of where the numerical distinction is set between the two levels. Additionally, while it is possible to consider the observations of witness parts quality on a different scale from the quality of production parts, it is not a distinction made here. That is, acceptable quality of witness parts is assumed to be the same acceptable quality of production parts. The testing

process is considered to be instantaneous for the purposes of these models and the cost of the test considered to be 0. We discuss the deviations and caveats from these assumptions in Section 4.7.

Maintenance time, $t$, is assumed to be the same as one production build or one stage of the POMDP and is done for several reasons. First, detailed data on how long machine maintenance took was unavailable, often including what repairs were performed and when the normal builds started and stopped. In practice, this meant that most maintenance times were simply the length of one production build in the data. Second, this assumption helps to simplify the POMDP model by allowing all stages of the decision process to be the same length. Deviating from this assumption is possible and would provide several benefits to the model and, again, is discussed further in 4.7.

Machine health was considered a function of density. However, a builds often failed due to complex part designs and other other non-machine health causes. In these situations, the machine was considered to have remained healthy. Density samples were collected at intervals across the 24-month period, and all builds occurring on the machine during the period when density was below acceptable levels were considered unhealthy. Since witness builds in this model are assumed to be perfect indicators of machine health, the associated emission probabilities of a witness build are assumed to be $\Omega\left(z_1, s_h, a_d\right) = \Omega\left(z_2, s_u, a_d\right) = 1$ and $\Omega\left(z_1, s_u, a_d\right) = \Omega\left(z_2, s_h, a_d\right) = 0$.

For the first model as described in Section 4.1, we find $V_N(k) = \frac{10-k-12(0.8)^k}{k+2}$, which has a global maximum optimal value, and can be solved analytically using the derivative. This is shown graphically in Figure 4.1. We see that $V_N'(k) = \frac{4 \cdot \ln 0.8 (0.8)^k (k-2) - (4-4(0.8)^k)}{(k+2)^2}$ gives the maximum value of $V_N(k) \approx 0.1833$ at $k \approx 3.65986$. Since maintenance cycle must be after discrete builds, Table 4.1 shows the integer values of $k$ with the highest value of $V_N(k)$ occurring at $k = 4$, meaning the optimal maintenance cycle under this first model is $k^* = 4$ with a long-run average value per build of $V_N^*(k^*) = 0.1808$. Therefore $\kappa_N = 4$.

For the second model described in 4.2, we see that optimal value of $V_D$ occurs at $k^* = 6$. This result is shown in comparison to the first model both numerically in

Table 4.1 and graphically in Figure 4.1. Therefore, the expected maintenance cycle is $\kappa_D^* = (7)\,(1-0.95^7)\sum_{n=1}^{\infty} n\,(0.95^7)^{(}\,n-1) \approx 23.2$ builds (including the witness builds).

For the third model using a POMDP, the parameters are defined for the purposes of a useful comparison using the same values from the first two models, shown in Appendix Table 1, and augmented where necessary to incorporate witness artifacts and emission probabilities. The full parameters used for the POMDP are defined in Appendix Table 2, including all search, or tuning, parameters of the POMDP. A discussion of these tuning parameters, their effects, and any changes for purposes of analysis is given in the Discussion in Section 4.6.3.

For this particular case study, we have chosen to simply consider the POMDP results across the range of emission parameters. While an experimental setup is possible, as mentioned in Section 4.5, this approach highlights the benefits of the POMDP without requiring great detail on the witness artifacts, how they should be produced, or what quality metrics they should monitor. This range of results of the POMDP is compared to the singular results from both the unmonitored and witness build-only models. We find that the use of a POMDP improves value over the witness build-only model by an average of 34%, in a range of $13\% - 73\%$, and an increase in maintenance cycle length by an average of 5.6 builds or 65%, in a range of 0-14.9 builds or $0\% - 173\%$. Since the witness build-only model itself is an improvement in value and increase in maintenance cycle time compared to the unmonitored model, the POMDP represents an improvement over the unmonitored model, as well.

The expected long-run value per unit-build-time, $V_F$, is shown as a percent marginal difference to $V_D$ in Figure 4.2 for combinations of the witness artifact emission probabilities $\omega_{h,1}^f$ and $\omega_{u,2}^f$. This value shows an improvement in value for all combinations of witness artifact emission probabilities, and, given $V_D > V_N$, also an improvement compared to $V_N$. The expected value is higher near the extremes of the emission parameters, i.e. $\omega_{h,1}^f = 0.9$ or $\omega_{u,2}^f = 0.1$, which is to be expected as the artifacts are more, or less, definitive in true reflection of machine health. We also observe a symmetry in the value function across these observation parameters such that the value between, for example, the case when $\omega_{h,1}^f = \omega_{u,2}^f = 0.5$ and the case when $\omega_{h,1}^f = 0.1$ and $\omega_{u,2}^f = 0.9$ is similar. In this way, the value

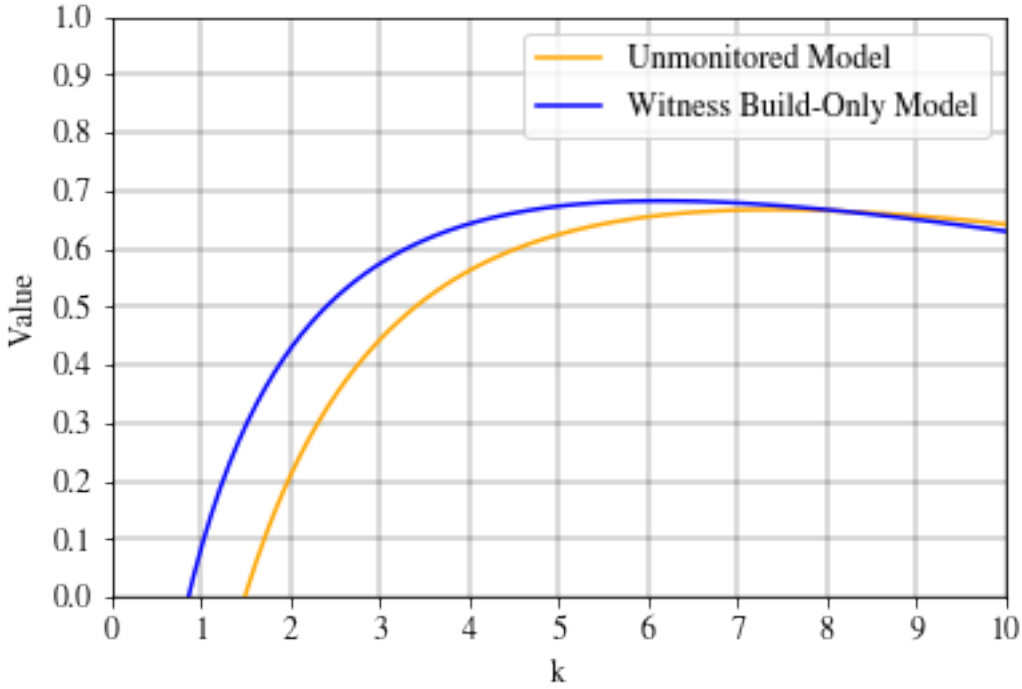



**Figure 4.1:** Plot of $V_N(k)$ and $V_D(k)$.

**Table 4.1:** Expected long-run value per build, $V_N(k)$ and $V_D(k)$, for each maintenance cycle, $k$.

| $k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $V_N(k)$ | -0.3125 | 0.2104 | 0.4437 | 0.5622 | 0.6242 | 0.6547 | 0.6661 | 0.6653 |
| $V_D(k)$ | 0.0858 | 0.4283 | 0.5743 | 0.6426 | 0.6729 | 0.6820 | 0.6784 | 0.6668 |

is likely a simple function sum of the artifact emission parameters $\omega^f_{h,1} + \omega^f_{u,2}$. For values of emission probability near 0.5, the value is reasonably lowest as these are the emission parameters that give very little consistent observations in one direction or the other of machine health. However, the symmetry arises from the fact that probabilities close to zero, similar to those close to one, consistently indicate a particular machine health state. In this way, a consistently "bad" indicator serves a valuable purpose as an indicator of machine health in the opposite direction of the observation.

Figure 4.3 shows the percent marginal difference in expected long-run maintenance cycle time between the POMDP model and witness build-only model. We see that the maintenance cycle time decreases nearly everywhere except outside of a central ridge. These cases in the central ridge correspond to cases in which the POMDP used witness builds. As a result, we find that the POMDP maintenance cycle time for these policies (and parameter sets) approaches the one identified by the witness build-only model. By contrast, at the valleys of the surface, the maintenance cycle time decreases and approaches the maintenance cycle time found in the unmonitored model. Figure 4.4 shows the percent marginal difference for the maintenance cycle time between when comparing the POMDP model to the unmonitored model, and we see the valleys approach 0% change. At the corners of the surface where the emission probabilities are near equal, we see the benefits of the witness artifacts as value increases crastically and maintenace cycle time increases again. By contrast, the corners of the surface where the emission probabilities are at extremes, such as $\omega^f_{h,1} = 1 - \omega^f_{u,2}$, we see that the ridge stays high and the POMDP policy makes use of witness builds. By intuition, these represent areas where the observations are contradictory and the witness builds provide more reliable information.

Figure 4.5 compares the value to maintenance cycles as well as which cases included witness builds. We, again, see here that the witness builds are used in particularly those cases where values are the lowest, with large increases in maintenance cycle time. There is some ambiguity of whether these models use witness builds, but nonetheless these cases all occur on the edge cases of low-value, low-maintenance cycle times. The ambiguity could be simple, direct result of the value function, but we also acknowledge that it may be possible

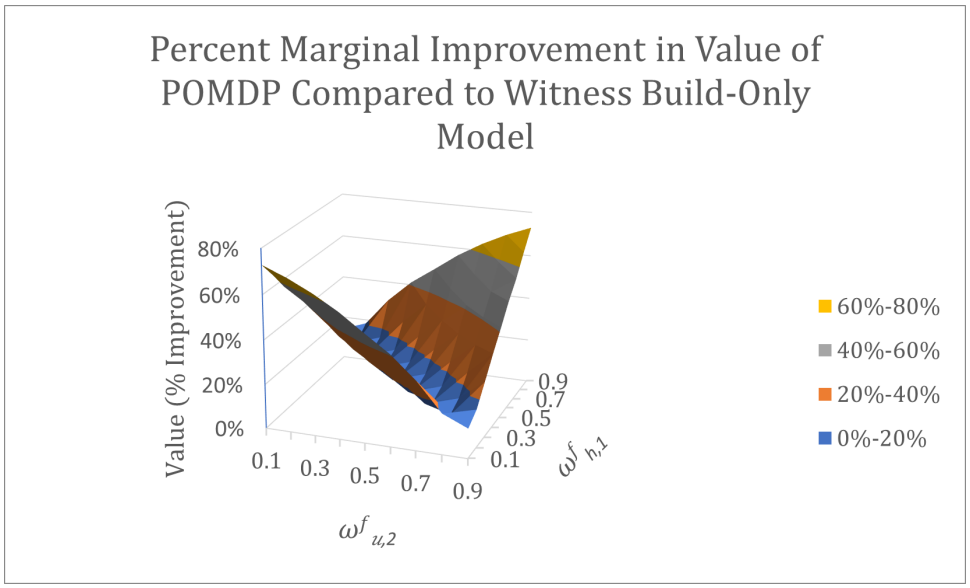



**Figure 4.2:** The percent marginal difference between the expected long-run value per unit-build-time for the POMDP model compared to the witness build-only model.

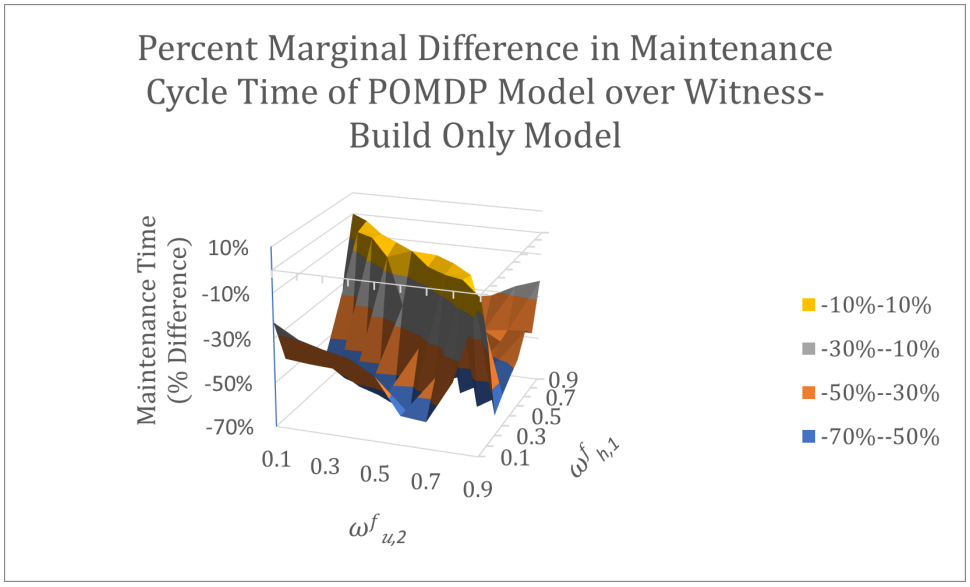



**Figure 4.3:** The percent marginal difference between the long-run expected maintenance cycle time for the POMDP model compared to the witness build-only model.

to discern a more distinct boundary between the cases that use, or do not use, witness builds by tightening the POMDP search parameters to allow a longer and deeper searches and more iterations.

### 4.6.2 Sensitivity Analysis

Sensitivity analysis for the POMDP model is performed in two ways. First, we perform an analysis of variance (ANOVA) on all of the relevant parameters to show compare the strength of their effects and identify dependencies. Second, we show a deeper analysis of parameters of interested identified in the ANOVA and provide context for their significance.

The ANOVA was performed using a reduced set of values for each the parameters $p$, $c$, $r_h$, $\omega^f_{u,2}$, and $\omega^f_{h,1}$. All other parameters, such as $r_u$ or $t$, were held constant, as defined by the assumptions of the models. A full description of the values used is shown in Appendix Table 3. A two-way ANOVA analysis was chosen for analyzing all pairwise combinations of the five selected parameters using a standard significance parameter of 0.05. The ANOVA was performed for each of several dependent variables. We first consider directly the effect on $V_F$ and $\kappa_F$. We then consider effect on value and maintenance cycle times of the POMDP model relative to the witness build-only model in terms of both absolute and percent marginal difference. Last, we show the effect on value and maintenance cycle in absolute terms only when comparing the POMDP model to the unmonitored model. For the unmonitored model, we choose to omit the percent marginal difference since some parameter combinations of the unmonitored model had such low maintenance cycles that the percent difference became infinite making the comparison unhelpful. Instead, we will look at these cases on an individual level.

Table 4.2 shows the results of the ANOVA directly against $V_F$ and $\kappa_D$. We see that $p$, $r_h$, and $c$ were the significant main factors contributing to the value $V_F$, with the strongest effect resulting from $r_h$. There was a significant interaction effect occurring between $p$ and $r_h$. These are somewhat expected when analyzed against $V_F$ as the dependent variable, given that healthy machines, more-valuable builds, and reduced maintenance costs intuitively lead

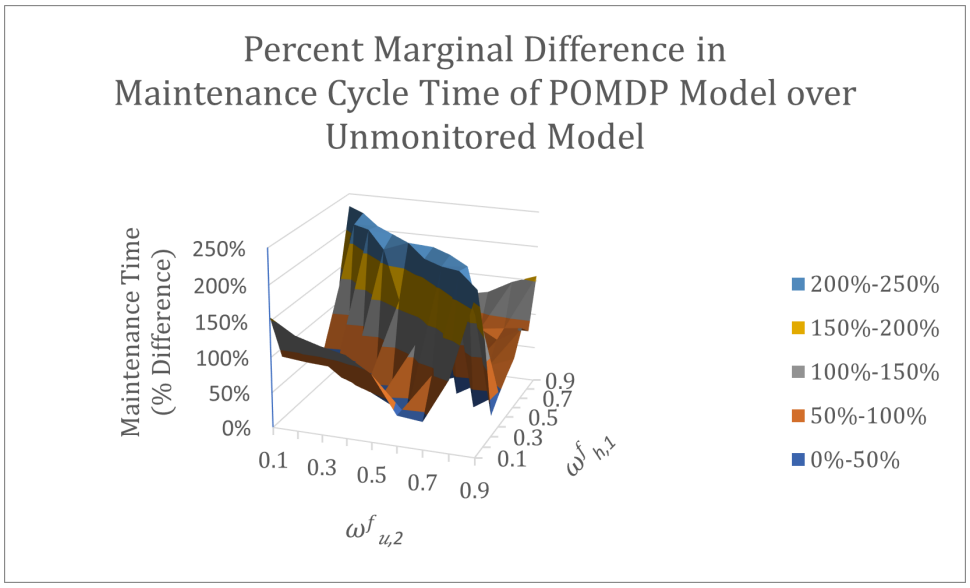



**Figure 4.4:** The percent marginal difference between the long-run expected maintenance cycle time for the POMDP model compared to the unmonitored model.

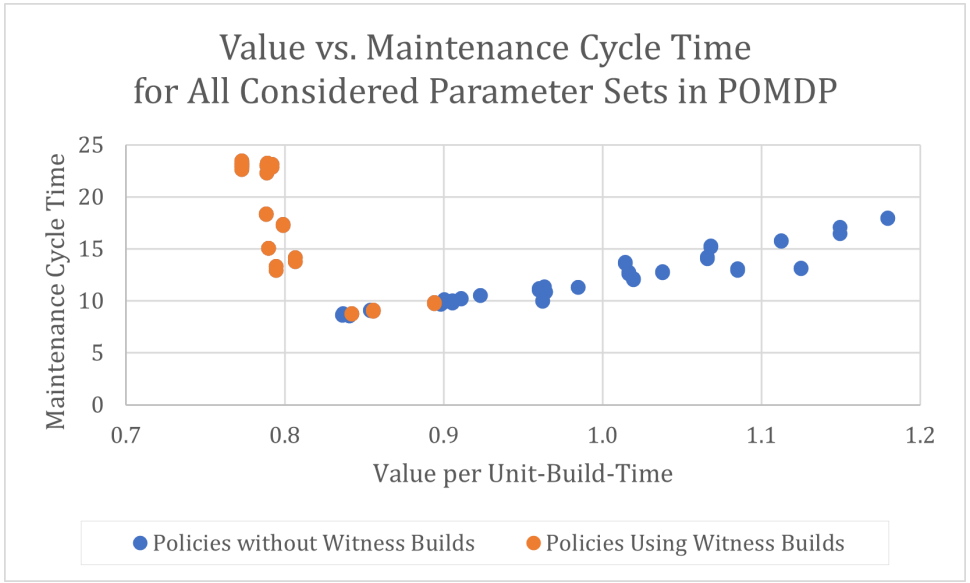



**Figure 4.5:** A comparison of expected long-run value per unit-build-time with expected maintenance cycle time, and the cases that make use of witness builds.

to more value per build in the long run. For maintenance cycle time, $p$ and $c$ were the main factors with significant interaction between the two, and the more significant interaction due to $p$. Again, this is somewhat intuitive given that a healthy machine requires less maintenance and cheaper maintenance is more likely to be performed more frequently.

Table 4.3 shows the results of the ANOVA against the absolute difference in value and maintenance cycle time between the POMDP and witness build-only models. Whereas the previous figure describes direct effects to value and maintenance cycle time and provides some information therein, we now consider which parameters drive the improvements in value and maintenance cycle time by the implementation of the POMDP model over the use of the less complex witness build-only model. For this ANOVA, we see that all parameters represent some significant influence on the improvement of the value achieved by the POMDP model over the witness build-only model as well as the addition of a significant interaction between $p$ and $c$. Notably, we see the addition of the $\omega^f_{h,1}$ and $\omega^f_{u,2}$ parameters as well as their interaction. Again, this makes some intuitive sense as the reliability of the observation from witness artifacts allows for more timely maintenance and repairs for a unhealthy machine while avoiding unnecessary and costly witness builds of the witness build-only model. However, by viewing the F-statistic, we can intuit that the effect of the emission parameters on overall value, while significant, is still relatively weak when compared to the other parameters. For the ANOVA of absolute difference in maintenance cycle time, we see the addition of the significant interaction between $p$ and $r_h$ as well as $c$ and $r_h$. Importantly, however, we do not see the witness artifact emission parameters as significant. This can be interpreted not only the relatively more significant impact of $p$, $c$, and $r_h$, but also the effect of the addition of witness builds into the optimal POMDP policy for parameters sets resulting in low value. This reflects the observations in the case study, which shows increases maintenance cycle times for low-value parameters sets that include witness builds despite the witness artifact emission parameters.

For much of the ANOVA data, the significant factors tend to follow along with the parameters $p$, $c$, and $r_h$. This is unsurprising for the reasons mentioned previously. Notably, however, we see the significance effects from the artifact emission probabilities $\omega^f_{a,1}$ and

**Table 4.2:** Two-way ANOVA results for parameters analyzed against $V_F$ and $\kappa_F$ by parameter.

| Dependent Variable | Factor1 | Factor2 | p-value | | | F-statistic | | |
|---|---|---|---|---|---|---|---|---|
| | | | Factor1 | Factor2 | Int | Factor1 | Factor2 | Int |
| $V_F$ | $p$ | $c$ | 0.000 | 0.000 | 0.092 | 67.759 | 10.329 | 1.576 |
| | $p$ | $r_h$ | 0.000 | 0.000 | 0.000 | 950.472 | ###### | 39.646 |
| | $p$ | $\omega^f_{h,1}$ | 0.000 | 0.444 | 1.000 | 65.019 | 0.893 | 0.018 |
| | $p$ | $\omega^f_{u,2}$ | 0.000 | 0.858 | 1.000 | 64.917 | 0.255 | 0.009 |
| | $c$ | $r_h$ | 0.000 | 0.000 | 1.000 | 46.945 | ###### | 0.109 |
| | $c$ | $\omega^f_{h,1}$ | 0.000 | 0.498 | 1.000 | 8.797 | 0.793 | 0.003 |
| | $c$ | $\omega^f_{u,2}$ | 0.000 | 0.878 | 1.000 | 8.785 | 0.227 | 0.000 |
| | $r_h$ | $\omega^f_{h,1}$ | 0.000 | 0.011 | 0.981 | ###### | 3.731 | 0.276 |
| | $r_h$ | $\omega^f_{u,2}$ | 0.000 | 0.365 | 1.000 | ###### | 1.060 | 0.072 |
| | $\omega^f_{h,1}$ | $\omega^f_{u,2}$ | 0.503 | 0.879 | 0.067 | 0.784 | 0.224 | 1.782 |
| $\kappa_F$ | $p$ | $c$ | 0.000 | 0.000 | 0.000 | ###### | 304.467 | 155.721 |
| | $p$ | $r_h$ | 0.000 | 0.025 | 0.446 | 714.033 | 3.118 | 0.991 |
| | $p$ | $\omega^f_{h,1}$ | 0.000 | 0.998 | 1.000 | 704.053 | 0.012 | 0.035 |
| | $p$ | $\omega^f_{u,2}$ | 0.000 | 0.986 | 1.000 | 704.082 | 0.049 | 0.028 |
| | $c$ | $r_h$ | 0.000 | 0.275 | 0.720 | 37.295 | 1.295 | 0.733 |
| | $c$ | $\omega^f_{h,1}$ | 0.000 | 0.999 | 0.998 | 36.997 | 0.005 | 0.207 |
| | $c$ | $\omega^f_{u,2}$ | 0.000 | 0.996 | 1.000 | 36.938 | 0.020 | 0.035 |
| | $r_h$ | $\omega^f_{h,1}$ | 0.326 | 1.000 | 1.000 | 1.154 | 0.005 | 0.021 |
| | $r_h$ | $\omega^f_{u,2}$ | 0.326 | 0.997 | 1.000 | 1.154 | 0.018 | 0.030 |
| | $\omega^f_{h,1}$ | $\omega^f_{u,2}$ | 1.000 | 0.997 | 0.989 | 0.005 | 0.018 | 0.236 |

**Table 4.3:** Two-way ANOVA results for parameters analyzed against the absolute difference in value and maintenance cycle time between the POMDP and witness build-only models.

| Dependent Variable | Factor1 | Factor2 | p-value | | | F-statistic | | |
|---|---|---|---|---|---|---|---|---|
| | | | Factor1 | Factor2 | Int | Factor1 | Factor2 | Int |
| Absolute Difference in $V_F$ to $V_D$ | $p$ | $c$ | 0.000 | 0.000 | 0.000 | 227.558 | 27.335 | 3.098 |
| | $p$ | $r_h$ | 0.000 | 0.000 | 0.000 | 483.785 | 466.608 | 36.267 |
| | $p$ | $\omega^f_{h,1}$ | 0.000 | 0.000 | 0.984 | 211.196 | 12.999 | 0.265 |
| | $p$ | $\omega^f_{u,2}$ | 0.000 | 0.012 | 0.999 | 206.457 | 3.637 | 0.131 |
| | $c$ | $r_h$ | 0.000 | 0.000 | 0.994 | 26.130 | 209.804 | 0.263 |
| | $c$ | $\omega^f_{h,1}$ | 0.000 | 0.000 | 1.000 | 17.778 | 9.110 | 0.030 |
| | $c$ | $\omega^f_{u,2}$ | 0.000 | 0.053 | 1.000 | 17.503 | 2.567 | 0.001 |
| | $r_h$ | $\omega^f_{h,1}$ | 0.000 | 0.000 | 0.482 | 201.115 | 12.834 | 0.948 |
| | $r_h$ | $\omega^f_{u,2}$ | 0.000 | 0.014 | 0.988 | 195.882 | 3.578 | 0.244 |
| | $\omega^f_{h,1}$ | $\omega^f_{u,2}$ | 0.000 | 0.034 | 0.000 | 10.125 | 2.898 | 23.013 |
| Absolute Difference in $\kappa_F$ to $\kappa_D$ | $p$ | $c$ | 0.000 | 0.000 | 0.000 | ###### | 293.683 | 158.698 |
| | $p$ | $r_h$ | 0.000 | 0.044 | 0.358 | 300.607 | 2.701 | 1.102 |
| | $p$ | $\omega^f_{h,1}$ | 0.000 | 0.998 | 1.000 | 296.462 | 0.012 | 0.035 |
| | $p$ | $\omega^f_{u,2}$ | 0.000 | 0.986 | 1.000 | 296.474 | 0.049 | 0.028 |
| | $c$ | $r_h$ | 0.000 | 0.131 | 0.278 | 60.470 | 1.882 | 1.199 |
| | $c$ | $\omega^f_{h,1}$ | 0.000 | 0.999 | 0.980 | 59.722 | 0.009 | 0.347 |
| | $c$ | $\omega^f_{u,2}$ | 0.000 | 0.992 | 1.000 | 59.562 | 0.034 | 0.059 |
| | $r_h$ | $\omega^f_{h,1}$ | 0.195 | 0.999 | 1.000 | 1.569 | 0.007 | 0.033 |
| | $r_h$ | $\omega^f_{u,2}$ | 0.195 | 0.993 | 1.000 | 1.570 | 0.029 | 0.048 |
| | $\omega^f_{h,1}$ | $\omega^f_{u,2}$ | 0.999 | 0.993 | 0.949 | 0.007 | 0.029 | 0.372 |

**Table 4.4:** Two-way ANOVA results for parameters analyzed against the percent marginal difference in value and maintenance cycle time between the POMDP and witness build-only models.

| Dependent Variable | Factor1 | Factor2 | p-value | | | F-statistic | | |
|---|---|---|---|---|---|---|---|---|
| | | | Factor1 | Factor2 | Int | Factor1 | Factor2 | Int |
| % Marginal Difference in $V_F$ to $V_D$ | $p$ | $c$ | 0.000 | 0.000 | 0.000 | 40.8 | 7.4 | 12.2 |
| | $p$ | $r_h$ | 0.000 | 0.000 | 0.000 | 46.8 | 16.8 | 37.0 |
| | $p$ | $\omega^f_{h,1}$ | 0.000 | 0.491 | 0.998 | 36.1 | 0.8 | 0.2 |
| | $p$ | $\omega^f_{u,2}$ | 0.000 | 0.862 | 1.000 | 36.0 | 0.2 | 0.1 |
| | $c$ | $r_h$ | 0.000 | 0.000 | 0.000 | 7.1 | 14.0 | 13.9 |
| | $c$ | $\omega^f_{h,1}$ | 0.000 | 0.520 | 1.000 | 6.1 | 0.8 | 0.1 |
| | $c$ | $\omega^f_{u,2}$ | 0.000 | 0.873 | 1.000 | 6.1 | 0.2 | 0.0 |
| | $r_h$ | $\omega^f_{h,1}$ | 0.000 | 0.515 | 1.000 | 12.2 | 0.8 | 0.1 |
| | $r_h$ | $\omega^f_{u,2}$ | 0.000 | 0.871 | 1.000 | 12.2 | 0.2 | 0.0 |
| | $\omega^f_{h,1}$ | $\omega^f_{u,2}$ | 0.523 | 0.874 | 0.133 | 0.7 | 0.2 | 1.5 |
| % Marginal Difference in $\kappa_F$ to $\kappa_D$ | $p$ | $c$ | 0.819 | 0.000 | 0.000 | 0.3 | 549.2 | 20.4 |
| | $p$ | $r_h$ | 0.955 | 0.000 | 0.645 | 0.1 | 9.1 | 0.8 |
| | $p$ | $\omega^f_{h,1}$ | 0.957 | 0.808 | 0.999 | 0.1 | 0.3 | 0.1 |
| | $p$ | $\omega^f_{u,2}$ | 0.957 | 0.956 | 1.000 | 0.1 | 0.1 | 0.1 |
| | $c$ | $r_h$ | 0.000 | 0.000 | 0.000 | 571.3 | 26.9 | 18.8 |
| | $c$ | $\omega^f_{h,1}$ | 0.000 | 0.486 | 0.001 | 472.7 | 0.8 | 2.8 |
| | $c$ | $\omega^f_{u,2}$ | 0.000 | 0.851 | 0.998 | 460.7 | 0.3 | 0.2 |
| | $r_h$ | $\omega^f_{h,1}$ | 0.000 | 0.803 | 0.992 | 9.0 | 0.3 | 0.2 |
| | $r_h$ | $\omega^f_{u,2}$ | 0.000 | 0.954 | 1.000 | 9.0 | 0.1 | 0.1 |
| | $\omega^f_{h,1}$ | $\omega^f_{u,2}$ | 0.808 | 0.955 | 0.842 | 0.3 | 0.1 | 0.5 |

**Table 4.5:** Two-way ANOVA results for parameters analyzed against the absolute difference in value and maintenance cycle time between the POMDP and unmonitored models.

| Dependent Variable | Factor1 | Factor2 | p-value | | | F-statistic | | |
|---|---|---|---|---|---|---|---|---|
| | | | Factor1 | Factor2 | Int | Factor1 | Factor2 | Int |
| Absolute Difference in $V_F$ to $V_N$ | $p$ | $c$ | 0.000 | 0.000 | 0.016 | 141.001 | 9.564 | 2.074 |
| | $p$ | $r_h$ | 0.000 | 0.000 | 0.000 | 295.948 | 412.472 | 30.636 |
| | $p$ | $\omega^f_{h,1}$ | 0.000 | 0.000 | 0.960 | 140.416 | 16.854 | 0.344 |
| | $p$ | $\omega^f_{u,2}$ | 0.000 | 0.003 | 0.997 | 136.358 | 4.685 | 0.169 |
| | $c$ | $r_h$ | 0.000 | 0.000 | 1.000 | 10.789 | 221.689 | 0.132 |
| | $c$ | $\omega^f_{h,1}$ | 0.000 | 0.000 | 1.000 | 7.275 | 12.873 | 0.042 |
| | $c$ | $\omega^f_{u,2}$ | 0.000 | 0.013 | 1.000 | 7.116 | 3.605 | 0.002 |
| | $r_h$ | $\omega^f_{h,1}$ | 0.000 | 0.000 | 0.164 | 226.939 | 19.544 | 1.444 |
| | $r_h$ | $\omega^f_{u,2}$ | 0.000 | 0.001 | 0.951 | 218.068 | 5.375 | 0.366 |
| | $\omega^f_{h,1}$ | $\omega^f_{u,2}$ | 0.000 | 0.003 | 0.000 | 16.029 | 4.588 | 36.433 |
| Absolute Difference in $\kappa_F$ to $\kappa_N$ | $p$ | $c$ | 0.000 | 0.000 | 0.000 | ###### | 176.962 | 137.774 |
| | $p$ | $r_h$ | 0.000 | 0.390 | 0.426 | 570.362 | 1.004 | 1.015 |
| | $p$ | $\omega^f_{h,1}$ | 0.000 | 0.997 | 1.000 | 565.128 | 0.016 | 0.044 |
| | $p$ | $\omega^f_{u,2}$ | 0.000 | 0.980 | 1.000 | 565.157 | 0.061 | 0.036 |
| | $c$ | $r_h$ | 0.000 | 0.706 | 0.692 | 28.941 | 0.466 | 0.760 |
| | $c$ | $\omega^f_{h,1}$ | 0.000 | 0.999 | 0.991 | 28.782 | 0.007 | 0.292 |
| | $c$ | $\omega^f_{u,2}$ | 0.000 | 0.994 | 1.000 | 28.717 | 0.029 | 0.050 |
| | $r_h$ | $\omega^f_{h,1}$ | 0.735 | 0.999 | 1.000 | 0.426 | 0.007 | 0.030 |
| | $r_h$ | $\omega^f_{u,2}$ | 0.735 | 0.994 | 1.000 | 0.426 | 0.026 | 0.043 |
| | $\omega^f_{h,1}$ | $\omega^f_{u,2}$ | 0.999 | 0.994 | 0.961 | 0.007 | 0.026 | 0.341 |

$\omega^f_{u,2}$ when considering the absolute difference in value from the POMDP to each of the unmonitored and witness build-only models observed in Table 4.3 and Table 4.5. While the effect of the witness artifact emission probabilities is certainly outweighed by the effects of $p$, $c$, and $r_h$ when looking at value overall, this does provide some insight into the ability of emission parameters to affect the maintenance policy in such a way that they do provide significant increased value. The significant interaction effect between $\omega^f_{a,1}$ and $\omega^f_{u,2}$ further indicates the benefits of having observations that agree with one another, creating compounding benefits in overall achieved value. It is plausible that this effect does not appear significant when looking at maintenance cycle time simply because the type of model, or whether witness builds are used plays a larger role, despite the value always being improved by the use of the POMDP model.

Last, we consider the effects of the emission parameters. While having no significant effect directly on value or directly on maintenance cycle time, we find some significant effects on the margin for both the absolute and percent differences of the POMDP value when compared to the unmonitored and witness build-only models. We note the effects of the witness artifact emission parameters is significant for changes in absolute difference in value shown in both Table 4.3 and Table 4.5 demonstrating that, on the margin, changes in these parameters do have some significant effect as we might expect in having better or worse observations. Numerically, this is because the value of the witness build-only models is much higher than the value of the unmonitored model. But this can also be understood as the witness build-only model having already incorporated some increases to value by using regular witness builds. By contrast, when witness artifact emission parameters are considered as a percent marginal difference, as in Table 4.4 and Table 4.6, we see that only the interaction effect is significant and only for the latter, that is, the percent marginal difference over the unmonitored model. Again, numerically, we can understand this by recognizing that the value of the unmonitored model is much lower compared to the absolute improvements in value gained by implementing a POMDP. Therefore the observation emission parameters are significant based on their relative values to one another, and we can suppose this corresponds to certain combinations of emission artifacts, such as those in "agreement"

like high probabilities for both $\omega^f_{a,1}$ and $\omega^f_{u,2}$, compared to those in "conflict" like high probability for $\omega^f_{a,1}$ and low probability for $\omega^f_{u,2}$. Figure 4.6 shows trends for the percent marginal difference in value of the POMDP model compared to the unmonitored model. Each trendline represents a single parameter of $\omega^f_{u,2}$ plotted across the entire range of $\omega^f_{a,1}$, noting that we exclude values of $\omega^f_{u,2} < 0.5$ because of the symmetry compared to $\omega^f_{u,2} > 0.5$. We note that the value percent marginal value achieved by two probabilities in "agreement" yield higher results than those in "conflict."

Next we look in more detail at the effects of these parameters on and attempt to gain some additional insights. We first build off the information from the ANOVA and consider a set of parameters which mirror those chosen for the case study, but only varying a single parameter. Note that since there is no single witness artifact emission parameter, however, that each parameter set is will repeated across a small combination of witness artifact parameters. The list of parameters used is included in Appendix Table 4.

Across modifications of $p$, we consider the effect on value for $V_N$, $V_D$, and the values of $V_F$ for when the witness artifact emission parameters are at two values: $(\omega^f_{a,1}, \omega^f_{u,2}) = (0.9, 0.9)$ and $(\omega^f_{a,1}, \omega^f_{u,2}) = (0.65, 0.35)$. The results are shown in Figure 4.7. These combinations of emission parameters were chosen to give an idea for the best and worst values of the POMDP, and fall on the extremes of the surface shown in the case study in Figure 4.2. Other combinations of emission parameters simply fall in the middle, and due to the symmetry of value function, several values were equivalent, such that the value for $(\omega^f_{a,1}, \omega^f_{u,2}) = (0.9, 0.1)$ was equivalent to the value for $(\omega^f_{a,1}, \omega^f_{u,2}) = (0.65, 0.35)$. Values for $(\omega^f_{a,1}, \omega^f_{u,2}) = (0.5, 0.)$ would be somewhat approaching the values of $V_D$ or $V_N$, depending whether witness builds are used in the optimal POMDP policy. We note here that in the extreme case of $p = 0.19$, the unmonitored and witness build-only models yield slightly negative values, while the POMDP yields a profit for even for the moderate values of the emission parameters.

The percent difference in marginal value for $c$ is shown in Figure 4.8. While the POMDP value is always improved over the unmonitored and witness build-only models, we see that there are increasing improvements in value as $c$ increases over the unmonitored model, but flat or diminishing improvements for increasing $c$ over the witness build-only model.

**Table 4.6:** Two-way ANOVA results for parameters analyzed against the percent marginal difference in value between the POMDP and unmonitored models.

| Dependent Variable | Factor1 | Factor2 | p-value | | | F-statistic | | |
|---|---|---|---|---|---|---|---|---|
| | | | Factor1 | Factor2 | Int | Factor1 | Factor2 | Int |
| % Marginal Difference in $V_F$ to $V_N$ | $p$ | $c$ | 0.000 | 0.000 | 0.000 | 69.065 | 12.723 | 7.793 |
| | $p$ | $r_h$ | 0.000 | 0.000 | 0.000 | 76.790 | 9.696 | 29.822 |
| | $p$ | $\omega^f_{h,1}$ | 0.000 | 0.292 | 0.997 | 62.419 | 1.245 | 0.170 |
| | $p$ | $\omega^f_{u,2}$ | 0.000 | 0.768 | 1.000 | 62.247 | 0.379 | 0.071 |
| | $c$ | $r_h$ | 0.000 | 0.000 | 0.000 | 11.940 | 8.184 | 15.010 |
| | $c$ | $\omega^f_{h,1}$ | 0.000 | 0.342 | 1.000 | 10.302 | 1.116 | 0.037 |
| | $c$ | $\omega^f_{u,2}$ | 0.000 | 0.797 | 1.000 | 10.280 | 0.340 | 0.009 |
| | $r_h$ | $\omega^f_{h,1}$ | 0.000 | 0.347 | 1.000 | 6.972 | 1.102 | 0.026 |
| | $r_h$ | $\omega^f_{u,2}$ | 0.000 | 0.800 | 1.000 | 6.959 | 0.335 | 0.009 |
| | $\omega^f_{h,1}$ | $\omega^f_{u,2}$ | 0.347 | 0.799 | 0.013 | 1.102 | 0.336 | 2.339 |

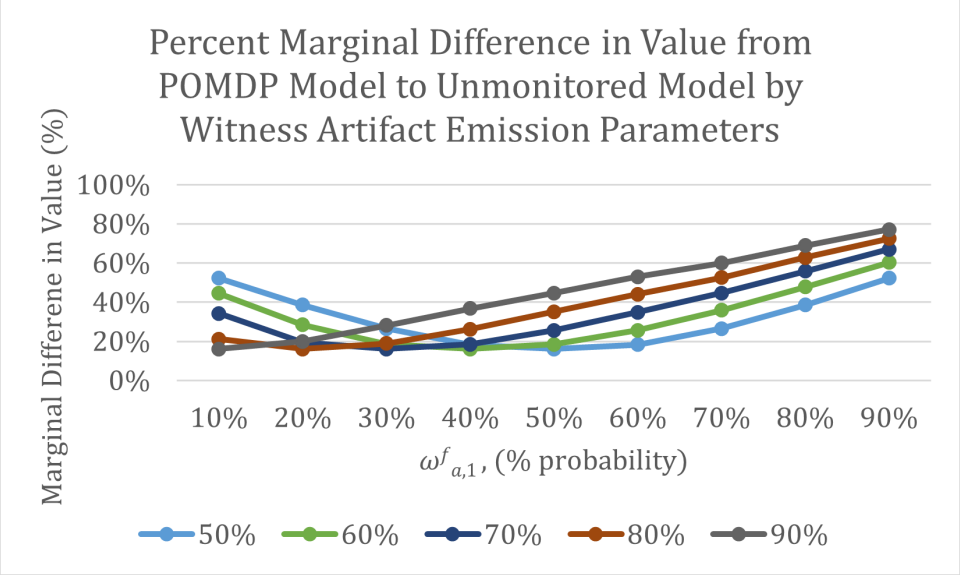



**Figure 4.6:** Percent marginal difference in value of POMDP model compared over unmonitored model for ranges of $\omega^f_{u,2}$ from 50% to 90%.

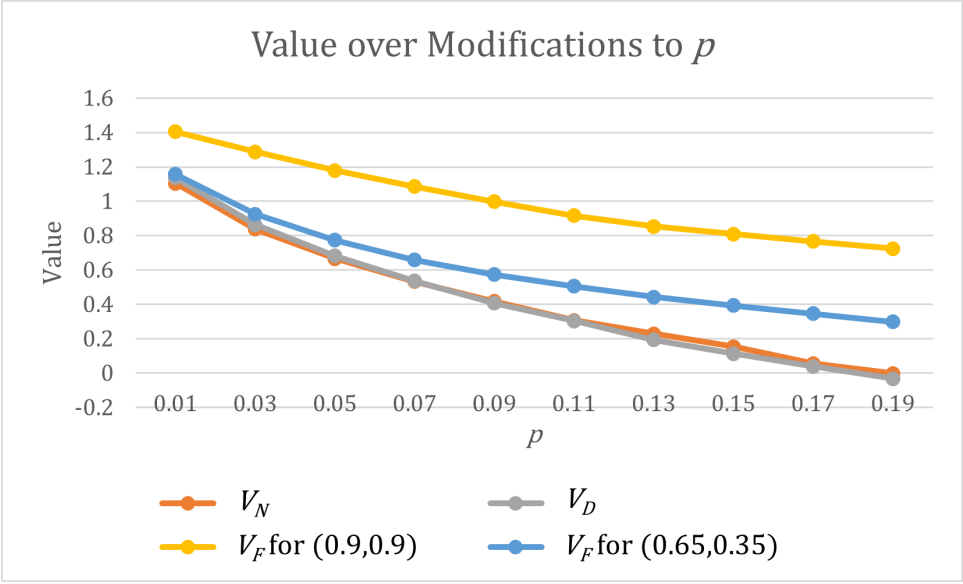



**Figure 4.7:** Value over modifications to $p$ for the shown models and witness artifact emission parameter pairs.

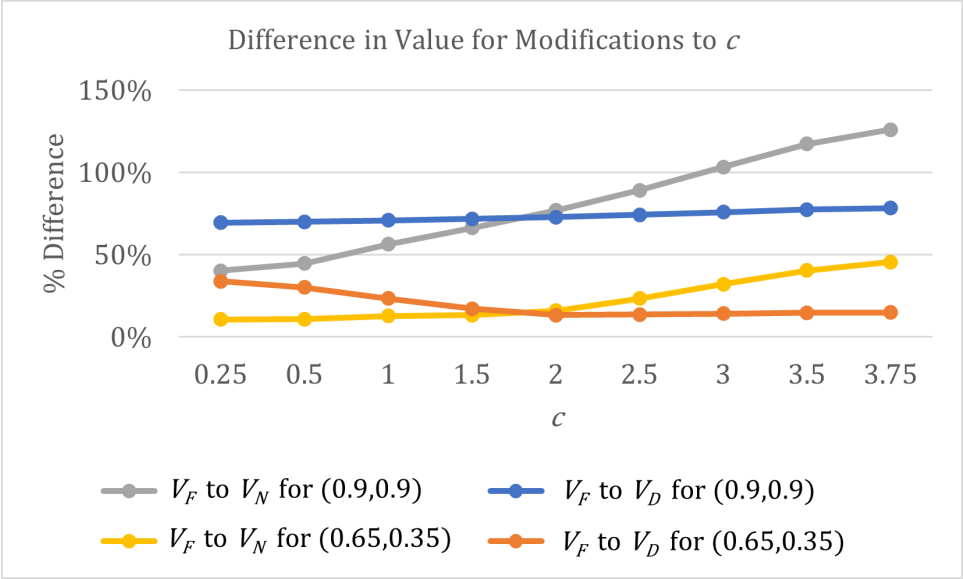



**Figure 4.8:** The percent marginal difference in value of $V_F$ compared to the $V_N$ and $V_D$ for the shown witness artifact emission parameter pairs.

### 4.6.3 Discussion

There are two considerations when finding solutions to a POMDP. First is the accuracy of the solution, and second is the speed, or convergence rate, of the solution. For the case study, the speed is less important than accuracy. For the sensitivity analysis, however, some considerations must be made for convergence rate and speed in order to gather enough data for consideration.

The parameters associated with cost and time were chosen several reasons. First, they are the parameters that are easily changed for a given AM production process, and provide key insights into the selection of a AM production processes, machines, investment, and uses. Second, they are the parameters which most significantly effect the results of the POMDP, as shown in Section 4.6.1.

The parameters associated with probability were chosen primarily because they closely reflect potential changes to the model without modifying the assumptions of the model. That is emission probabilities of the witness artifact may readily change according to the relevant quality metrics chosen by the machine operator. By contrast, emission probabilities associated with the witness builds more readily reflect changing to the root cause of machine failure, and are not analyzed in this research.

The tuning, or search, parameters of the POMPD including $\rho$, $\gamma$, and $\epsilon$ were chosen to give some reasonable accuracy and precision of the value determined by the POMDP while also balancing the time required to find an optimal POMDP solution. To the extent that there is some error in these values, we note that increasing these parameters should only increase the ultimate value as longer, deeper, and more sensitive searches are performed given that the dynamic programming step will only change with improvements in value. For the simulation used in calculating maintenance cycle time, it is possible that the cycle time decreases with longer simulations, but this is largely unlikely since this also generally corresponds with longer, and less probable, observation histories. Therefore, all numbers found for the POMDP generally represent a floor on the underlying, true optimal values,

and, as shown in the remainder of this section, would only further increase the improvements gained by utilizing a POMDP.

Given further that value functions can be particularly complex or converge at a relatively slower rate than others, we choose more aggressive tuning parameters and loosen others for the sensitivity analysis in order to reduce the total time required to complete the sensitivity analysis. In particular, we increase $\rho$ and $\epsilon$, but endeavor to keep $\gamma$ the same for some reasonable comparison. Given the $\gamma$ parameter is a discount factor, it can play a role in the accuracy of the value-per-build. The value becomes more accurate as $\gamma$ approaches one, and as a result we chose a parameter $\gamma = 0.9999$ to achieve accuracy in the POMDP. Since the simulation used no discount factor, and the error between the value found by the POMDP and that found by the simulation is generally on the order of 0.01 or less, there is some reasonable confidence that the discount factor had minimal impact on the value determined by the POMDP relative to the other tuning parameters. Anecdotally, the error between the POMDP-determined value and the simulation-determined value remained unchanged when using $\gamma = 0.999$ and 0.99, lending further credibility to this hypothesis.

The other two parameters of concern are the precision parameter $\rho$ and the convergence parameter $\epsilon$. Since we end the cycle of POMDP iterations according to the Bellman residual (which is a function of $\epsilon$), epsilon functions more as a parameter than a max error parameter. Moreover, given our choice of $\gamma = 0.9999$ for accuracy, we find that $\epsilon$ can be as large as 100 and still provide a reasonable estimates of the value.. Finally, $\rho$ represents a minimum improvements in the DP backup stage that shifts the POMDP solution algorithm from an theoretical exact solution algorithm closer to an approximate solution algorithm. In practice, however, a choice of $\rho$ up to machine precision has minimal effect on the accuracy of the cost-per-build, but greatly decreases the number of nodes in the FSC, thereby exponentially improving the speed of the DP backup and ultimately the convergence to a solution.

The assumptions of the case study, and the values used, may be reflective of a production-focused machine rather than an research and development machine. Still, other assumptions may be better estimated with more detailed historical maintenance data or on a machine which does not make use of a maintenance service contract. In particular, the effects of these

assumptions are reflected in the values chosen for $p$, $r_h$, $\beta$, $\theta$, $c$, and $t$. Since one goal of this model is to permit and ease a transition from research- to production-focused models, these may be strong but nonetheless helpful assumptions. To the extent that these assumptions are inaccurate, the sensitivity analysis provides some information on how the model might change as a result.

The inclusion and implementation of a POMDP to monitor machine health and perform maintenance has some effects on the AM production itself and the decision-making process of choosing which AM processes to choose, which parts to produce, and which machines to choose. It will also have a feedback loop with the witness parts and which quality indicators are important to monitor.

There are many implicit assumptions of the model that we do not consider. This is done partially simplicity of the model at this introductory stage to provide a clear analysis of the benefits of a POMDP and witness parts. Many of these can be included by adding additional parameters and mathematical factors into the model. No consideration of scrap rate or disposal costs were considered, which may be particularly applicable for $r_d$ and $r_u$, which in this model were simply considered to be zero. We also did not consider various lengths of $t$, the maintenance time, instead simply setting it equal to one stage of the build process. No differentiation was made for the source of maintenance cost. In practice, much of the maintenance cost is overhead, administrative, or otherwise time-related costs and distinguishing the various sources of maintenance costs would be necessary if variations in $t$ were considered. Additionally, witness artifacts were considered not to reduce the value (due to space or time effects) when being included into a witness build. The difference in cost of between a witness artifact and a witness build could also be considered, since the construction of a witness build takes a different amount of time and consumes feedstock.

Ultimately, as the various root causes and influential factors for machine health are identified for particular AM machines and processes, the use of this model simplifies and incentivizes the inclusion of additional and more complex factors, assumptions, and costs into the model.

## 4.7 Potential Extensions and Model Expansions

The model and its various assumptions were chosen to help simplify the model so that insights could be provided on the benefits of the POMDP model in the application of monitoring AM machine health. However, there are many places the model can be expanded to more accurately and precisely reflect the requirements of real-world production models. While these would add utility to the model, they are perhaps too complex to consider at this introductory stage. We point out, here, some of the limitations and considerations of the assumptions of the model, where they offer room to expand on the model, and where further work could be focused.

This model considers only two tiers of observations: acceptable and unacceptable, both of which are assumed to be defined according the same criteria and for both witness builds and witness artifacts. This a straightforward and natural way to define the set of observations given that machine qualification and standards often include some minimal requirements for quality of parts and machine. However, the modification of where line between acceptable and unacceptable is defined may have effects on the model. If the delineation is too high or too low, the reliability of both witness artifacts and witness builds could be affected due to variability or being otherwise poorly tuned indicators. Alternatively, additional observation tiers could mitigate this effect and provide some benefits if some middle tier representing a less reliable observation is defined. Further, observations could be defined differently for witness builds and witness parts. Similar to the changes just mentioned, this could be done either by defining more observation levels for one type of witness part compared to the other, or by defining differently between the two types of witness parts what constitutes a particular observation level. For example, acceptable observation on a witness build may be more strict than what defines an acceptable observation on a witness artifact, or we may have three observation levels for witness artifacts and only two levels for witness builds. The effects or usefulness of each of these potential changes to the model are unknown, but they may provide some potential benefits if the observations from witness parts are noisy or unequally distributed. However, producing witness parts of the various qualities required to

correlate the data to machine health may prove difficult, and the benefits may ultimately be outweighed simply by working with the other parameters as discussed in Section 4.6.2.

Witness build emissions are considered to be perfectly reliable and perfectly corresponding to the health of the machine. We do this by defining machine health as the ability to produce the witness part to the standards set. However, this may be a strong assumption if other factors affect the quality of the build but not the health of the machine. In our assumptions, we account for feedstock by assuming a separate, controlled qualification process exists, but environmental factors could be one example of a root cause for variability in witness builds not directly tied to machine health. The consideration of such factors would require knowing their probable effects and lowering the witness build emission parameters to less than one. Alternatively, additional indicators could be implemented, such as sensors, that provide an observation of these other factors. Again, a more complex analysis of the fault modes for the machine and the quality of the witness builds would be required.

We do not consider the time or cost of testing the witness parts to gain the observations or the real-world ways in which this may actually be done. In practice, a particular production may have some lag in their testing results and may not receive the information from an observation for several stages of the decision process. Alternatively, there may be some batching of part testing, in which witness artifacts are delayed and tested simultaneously after several stages of the decision process have passed. The effects of the time and costs of these tests are not considered in this model, but such an analysis has the potential to add insights to the model. Analyzing their effects may provide some insights into which quality metrics to consider if some tests are particularly expensive or time-consuming. These effects may also differ wildly for different AM processes, machines, or production goals.

Maintenance and repairs were considered to be indistinguishable from one another. In reality, there are many maintenance and repair operations that differ in likelihood, time, and costs. Identifying these common operations and their fault modes would make the model more accurate. Further, it would provide some upstream feedback on the types of quality metrics to monitor.

## 4.8 Conclusions for Modeling

We have presented a model of condition-based health monitoring for AM machines by the implementation of a POMDP. The benefits of the POMDP were analyzed by primarily considering the effects to value, but also looking at the effects to maintenance cycles. A comparison to two other models was made as the baseline. The first model considered an unmonitored machine and determined optimal maintenance scheduling as well as the costs. The second model considered a machine monitored using only witness builds, which were constructed on an optimal schedule so that maintenance was only performed when the machine was determined to be in an unhealthy state. Ultimately, the POMDP was shown to have improved value over both of these models even in the worst-case scenario of largely uninformative observations of the witness artifacts. Thus, any practical uselessness of the witness artifacts by virtue of an unavailability emission probabilities, sparse or unreliable data, or poorly designed witness artifacts, is mitigated by virtue that the POMDP model collapses to one of the first two models. By comparison, the use of witness builds is decided simply in the process of solving the POMDP model, again collapsing to one of the first two models in the worst case scenario.

Many of the benefits of the POMDP model reflect intuitive results as well as those from elsewhere in literature. Any efforts that can increase the value of a witness build, reduce costs of machine use or maintenance, or improve the accuracy of the information provided by witness artifacts will ultimately further increase the incentives to use the POMDP model. However, much work still needs to be done in order to show the effects of the POMDP model in the various applications. Among these needs are the consideration of more complex AM machine operational models including test batching or information lag, a more detailed description of the costs for witness parts and the costs of machine operation and maintenance, and a more complex modeling of the POMDP that allows for the relaxation of assumptions that limited analysis of some parameters in this research. More case studies would also provide some examples of the utility of the application of a POMDP across a variety of AM machine and AM processes.

# Chapter 5

# Conclusion

The aim of this research is to providing frameworks, models, tools, and evidence to enable using AM in the same tools often used for conventional manufacturing. This has been thus far been a challenging task given the unique properties of AM and its ideal, potential use cases as a manufacturing technology. Complex parts make determining the cause of failure more complicated and low or reduced volume production makes statistical data difficult to gather and analyze. While AM is a flexible technology useful for these edge cases in production, it is these same properties that make it harder to model, harder to optimize maintenance and production, and harder to maintain quality in production over time.

By providing a witness parts framework, the collection of data is incentivized on the status of the machine and the quality of the parts produced at any point in the production process. A POMDP is one such model that can make use of such data and both permits the monitoring of machine health and the optimal scheduling and performance of maintenance. Further, however, the use of the POMDP model not only benefits from the introduction of the concept of witness parts, but it simultaneously provides upstream incentives to begin producing witness parts themselves, collecting machine health data, and better describing the causal connection between various AM machine fault modes and quality metrics of the produced parts.

We have shown the benefits of the implementation of the POMDP over an unmonitored maintenance model with an approximated maintenance schedule as well as a witness build-only maintenance model that tests the status of the machine health on some interval and determines at that point whether to perform repairs. The POMDP is flexible in the use of witness parts, only having them included in the optimal policy when necessary to achieve the highest value. In this way, we also showed the benefits of the witness artifacts, but also the existence of the benefits in using them at their early stages when data is rare or unreliable. Given that this research does not consider what the best witness artifacts are or how to design them, this is a useful property so that the benefits of a POMDP and witness parts are realized even in the case of poorly designed witness artifacts as may likely be the case in the early stages of using this model.

As the use of witness parts is incentivized and more complex and precise models are implemented, the benefits are likely to be synergistic in increasing value and improving the quality and reliability of AM machines, AM production environments, and AM-produced parts. This provides one more step in the direction of shifting AM from its previous status as a nascent, research and development technology more suited for unique edge-case applications to a position as a robust and reliable manufacturing technology prepared for use in full production environments in the long-term.

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# Appendix

**Table 1:** A list of the parameters used for calculating unmonitored and witness build-only models.

$$
\begin{aligned}
p &= 0.05 \\
c &= 2 \\
r_h &= 2.5 \\
q &= 1 - p \\
t &= 1 \\
\beta &= 1 \\
r_u &= 0 \\
r_d &= 0
\end{aligned}
$$

**Table 2:** A list of the sets of parameters chosen for the case study. Each parameter set considered all combinations of $\omega^f_{a,1}$, and $\omega^f_{u,2}$.

$$
\begin{aligned}
p &= 0.05 \\
c &= 2 \\
r_h &= 2.5 \\
\omega^f_{a,1} &\in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\} \\
\omega^f_{u,2} &\in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\} \\
q &= 1 - p \\
t &= 1 \\
\beta &= 1 \\
r_u &= 0 \\
r_d &= 0 \\
\gamma &= 0.9999 \\
\epsilon &= 100 \\
\rho &= 0.0001 \\
\omega^d_{a,1} &= 1 \\
\omega^d_{u,2} &= 1
\end{aligned}
$$

**Table 3:** A list of the sets of parameters chosen for the ANOVA. All combinations of parameters were considered for each of $p$, $c$, $r_h$, $\omega^f_{a,1}$, and $\omega^f_{u,2}$.

$$
\begin{aligned}
p &\in \{0.01, 0.05, 0.1, 0.25\} \\
c &\in \{0.25, 1, 2, 3, 4\} \\
r_h &\in \{1.1, 2, 3, 4\} \\
\omega^f_{a,1} &\in \{0.1, 0.35, 0.65, 0.9\} \\
\omega^f_{u,2} &\in \{0.1, 0.35, 0.65, 0.9\} \\
q &= 1 - p \\
t &= 1 \\
\beta &= 1 \\
r_u &= 0 \\
r_d &= 0 \\
\gamma &= 0.9999 \\
\epsilon &= 100 \\
\rho &= 0.001 \\
\omega^d_{a,1} &= 1 \\
\omega^d_{u,2} &= 1
\end{aligned}
$$

**Table 4:** A list of the sets of parameters chosen for the sensitivity analysis. For parameters $p$, $c$, and $r_h$, each was first set to the equality-defined values from the case study, and then only one parameter was modified at a time. Each parameter resumed the equality-defined value before the next parameter was modified. Each of those parameter sets considered all combinations of values for $\omega^f_{a,1}$ and $\omega^f_{u,2}$.

$$
\begin{aligned}
p &= 0.05 \\
&\text{when modified, } p \in \{0.01, 0.03, 0.05, 0.07, 0.09, 0.11, 0.13, 0.15, 0.17, 0.19\} \\
c &= 2 \\
&\text{when modified, } c \in \{0.25, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 3.75\} \\
r_h &= 2.5 \\
&\text{when modified, } r_h \in \{1.1, 1.45, 1.8, 2.15, 2.5, 2.85, 3.2, 3.55, 3.9\} \\
\omega^f_{a,1} &\in \{0.1, 0.35, 0.65, 0.9\} \\
\omega^f_{u,2} &\in \{0.1, 0.35, 0.65, 0.9\} \\
q &= 1 - p \\
t &= 1 \\
\beta &= 1 \\
r_u &= 0 \\
r_d &= 0 \\
\gamma &= 0.9999 \\
\epsilon &= 100 \\
\rho &= 0.001 \\
\omega^d_{a,1} &= 1 \\
\omega^d_{u,2} &= 1
\end{aligned}
$$

# Vita

Jeremy Hale grew up in Dallas, Georgia. After high school, he attended Berry College in Mount Berry, Georgia and received a Bachelor of Science degree in Mathematics and Physics in 2011. He continued his education at the University of Tennessee at Chattanooga in Chattanooga, Tennessee, and received a Master of Science degree in Mathematics in 2013. In 2017, he began pursuing a Doctor of Philosophy degree in Industrial and Systems Engineering at the University of Tennessee in Knoxville, Tennessee. During this time, he also worked as a research assistant in partnership with the United States Department of Energy at Y-12 National Security Complex in Oak Ridge, Tennessee working in the area of research and development of additive manufacturing.