

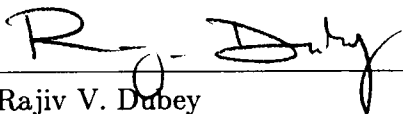
To the Graduate Council:

I am submitting herewith a thesis written by David C. Lennon entitled "Sensor Platform Modeling and Simulation for Unmanned Ground Vehicles on the Move". I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.



William R. Hamel, Major Professor

We have read this thesis and
recommend its acceptance:



Rajiv V. Dubey



J.A.M. Boulet

Accepted for the Council:



Associate Vice Chancellor and
Dean of The Graduate School

SENSOR PLATFORM MODELING AND SIMULATION FOR
UNMANNED GROUND VEHICLES ON THE MOVE

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

David C. Lennon
August 1998

Copyright ©, 1998 by David C. Lennon
All rights reserved.

*This thesis is dedicated to my parents
Thomas and Cecilia Lennon.
The love and support they have given me has been without measure.
God bless the both of you.*

Acknowledgements

I would like to thank the members of my graduate committee, Dr. William R. Hamel, Dr. J.A.M Boulet, and Dr. Rajiv V. Dubey for there continued dedication to the passage of knowledge to their students. I would like to especially thank Dr. Hamel, my graduate advisor. Dr. Hamel has presented me with opportunities over the past two years that I will be forever grateful to have received. I would also like to thank the students who work in the Robotics and Electromechanical Laboratory at The University of Tennessee, Knoxville. It has been my pleasure to be in the company of Chris Widner, Steve Everett, Scott Marland, James Franks, Stacey Mitchell, Yasunobu Isoda, and Sam Richardson. I would especially like to thank Chris Widner and Steve Everett, Ph.D. candidates, who were always available to answer questions and offer technical assistance. This thesis was written in L^AT_EX which would not have been possible without the help of Chris Widner. Most importantly I would like to thank God for His many blessings and guidance throughout my life.

Abstract

Unmanned ground vehicles (UGVs) are being studied for a variety of different military applications. Technology in the area of sensors and high speed computers has motivated this research and will continue to push UGV technology in the future. A particularly important area of research is associated with the stable orientation of sensors and cameras while the vehicle travels over various terrain conditions. This pushes the need to model and simulate the response characteristics of a sensor platform for unmanned ground vehicles.

In order to determine the response characteristics of a sensor platform, a dynamic model was developed. Sensor platform modeling and simulation allows one to obtain important stability and control information concerning sensor platform performance. The platform was modeled as a concentrated mass connected to one end of a simple beam element. The mass represents various sensors and cameras mounted to a pan/tilt head. The full six degrees of freedom of the platform are modeled. Test data, obtained from a series of field tests conducted on a next generation unmanned ground vehicle (NGUV), was used to obtain typical vehicle chassis motion at the to center location of the vehicle. Amplitude spectrum plots were determined for this test run data. These represent translational and rotational displacements of the vehicle chassis as functions of frequency. Forcing functions were determined based on the data analysis and used to excite the platform model. The response characteristics of the platform, when excited with these forcing functions, were then used to make performance predictions concerning sensor pointing control. Pointing algorithms were also developed for the orientation of the platform's sensor suite in order to maintain inertial stability and/or to maintain a desired line of sight (LOS).

Contents

1	Introduction	1
1.1	Scope	1
1.2	Unmanned Ground Vehicles (UGVs)	2
1.2.1	UGV History	2
1.2.2	Current UGV Goals	3
1.3	Reconnaissance, Surveillance, and Target Acquisition (RSTA) and Mobility for Unmanned Ground Vehicles	5
2	Inertial Measurement Systems	8
2.1	Inertial Measurement Devices (Gyroscopes and Accelerometers)	9
2.2	Inertial Platforms	11
2.2.1	Classical Inertial Measurement Systems	12
2.2.2	Strapdown Systems	13
3	Sensor Module Stabilization	14
3.1	Euler Angles	14
3.2	Sensor Platform Stabilization	18
3.2.1	Inertial Stabilization	20
3.2.2	Camera Aiming	24
4	Pan/Tilt Platform Pedestal	28
4.1	Platform Pedestal Model	28
4.1.1	Potential and Kinetic Energy Equations	33
4.2	Equations of Motion	41
5	Platform Pedestal Analysis Approach	49
5.1	Frequency Analysis	54
5.2	Nominal Plant Frequency Domain Response Characteristics	54
5.3	Approximate Frequency Response Sensitivity Based on Variations from Nominal Plant Values	61
5.4	Time Domain Analysis of Measurement Data obtained from Test Conducted on the Next Generation Unmanned Ground Vehicle (NGUV)	63

5.4.1	Beam Deflection based on Test Run Harmonic Excitations	68
5.5	Simulated Impulse Response	75
5.5.1	Simulation Run	78
6	Conclusions	81
	Bibliography	85
	Appendices	87
A	Code for Tracking Angles	88
B	Frequency Analysis Code	92
B.1	Code for the State Space Platform Model	92
B.2	Code for the bode plot	93
B.3	Sensitivity Analysis in the Frequency Domain	97
C	Amplitude Spectrum Code	98
C.1	Plant Model Code	98
D	Harmonic Simulation Code	103
D.1	Plant Model Code	103
E	Amplitude Spectrum for Test Run 4 Data	106
F	Amplitude Spectrum for Test Run 8 Data	110
G	Amplitude Spectrum for Test Run 11 Data	114
H	Amplitude Spectrum for Test Run 15 Data	118
	Vita	122

List of Tables

5.1	Single Input, Single Output open loop transfer functions G_i for various input/output relationships.	51
5.2	Nominal Plant Setup.	54
5.3	SISO transfer functions natural frequencies.	60
5.4	Description of test runs.	65
5.5	Runs considered from the test conducted on the NGUV.	65
5.6	Base excitation driving frequency corresponding to measurement device frequency band.	68
5.7	Maximum displacement relative to ground obtained from simulation response curves when excited with excitations from test run 4 harmonic inputs. . . .	68
5.8	Maximum displacement relative to ground obtained from simulation response curves when excited with excitations from test run 8 harmonic inputs. . . .	71
5.9	Maximum displacement relative to ground obtained from simulation response curves when excited with excitations from test run 11 harmonic inputs. . .	73
5.10	Maximum displacement relative to ground obtained from simulation response curves when excited with excitations from test run 15 harmonic inputs. . .	73
5.11	Simulation parameters	78

List of Figures

2.1	Law of gyroscopes.	9
2.2	Three axis gyroscope.	10
2.3	Classical Inertial Platform.	12
3.1	Camera Aiming	15
3.2	Components of the vehicle's angular velocity.	16
3.3	Control Process for camera stabilization.	19
3.4	Yaw, Pitch, and Roll rotations.	20
3.5	Transformations for sensor stabilization.	21
3.6	Determining angles from range to target.	25
4.1	UGV with mounted sensor platform pedestal.	29
4.2	Cross sectional area of a pipe with differential area $rd\theta dr$	30
4.3	Shear stress distribution for a uniform, cantilever beam.	30
4.4	Deflection due to shear.	32
4.5	Combined translation and rotation due to force and moment applied to free end of beam.	34
4.6	Sign convention.	36
5.1	Physical UGV and sensor platform / Sensor platform model analysis.	50
5.2	Sensor platform analysis concept	53
5.3	Bode plot for SISO open loop transfer function G_1 corresponding to translational input in the vertical direction, translational output in the vertical direction.	55
5.4	Bode plot for SISO open loop transfer function G_2 corresponding to translational input in the transverse direction, translational output in the transverse direction.	55
5.5	Bode plot for SISO open loop transfer function G_3 corresponding to translational input in the longitudinal direction, translational output in the longitudinal direction.	56
5.6	Bode plot for SISO open loop transfer function G_4 corresponding to rotational input about the vertical direction, rotational output about the vertical direction.	56

5.7	Bode plot for SISO open loop transfer function G_5 corresponding to rotational input about the transverse direction, rotational output about the transverse direction.	57
5.8	Bode plot for SISO open loop transfer function G_6 corresponding to rotational input about the longitudinal direction, rotational output about the longitudinal direction.	57
5.9	Bode plot for SISO open loop transfer function G_7 corresponding to translational input in the transverse direction, rotational output about the longitudinal direction.	58
5.10	Bode plot for SISO open loop transfer function G_8 corresponding to translational input in the longitudinal direction, rotational output about the transverse direction.	58
5.11	Bode plot for SISO open loop transfer function G_9 corresponding to rotational input about the longitudinal direction, translational output in the transverse direction.	59
5.12	Bode plot for SISO open loop transfer function G_{10} corresponding to rotational input about the transverse direction, translational output in the longitudinal direction.	59
5.13	Plant Frequency Response for the case where the beam length is 1 ft, 5 ft, and 8 ft.	62
5.14	Plant Frequency Response for the case where the mass is 50 lbs compared to the case where the mass is 150 lbs.	64
5.15	Amplitude Plot for Run 15 Vertical Accelerometer.	67
5.16	Translational displacements of the concentrated mass when excited with harmonic inputs obtained from run 4 data.	69
5.17	Rotational displacements of the concentrated mass when excited with harmonic inputs obtained from run 4 data.	69
5.18	Translational displacements of the concentrated mass when excited with harmonic inputs obtained from run 8 data.	70
5.19	Rotational displacements of the concentrated mass when excited with harmonic inputs obtained from run 8 data.	71
5.20	Translational displacements of the concentrated mass when excited with harmonic inputs obtained from run 11 data.	72
5.21	Rotational displacements of the concentrated mass when excited with harmonic inputs obtained from run 11 data.	72
5.22	Translational displacements of the concentrated mass when excited with harmonic inputs obtained from run 15 data.	74
5.23	Rotational displacements of the concentrated mass when excited with harmonic inputs obtained from run 15 data.	74
5.24	Simulated impulse applied to UGV system.	76
5.25	Mass translational displacement in vertical direction with respect to ground when excited with an impulse.	79

5.26	Mass translational displacement in longitudinal direction with respect to ground when excited with an impulse	79
5.27	Mass rotational displacement about the transverse direction with respect to ground when excited with an impulse.	80
E.1	Amplitude Plot for Run 4 Vertical Accelerometer.	107
E.2	Amplitude Plot for Run 4 Transverse Accelerometer.	107
E.3	Amplitude Plot for Run 4 Longitudinal Accelerometer.	108
E.4	Amplitude Plot for Run 4 Pitch Gyro.	108
E.5	Amplitude Plot for Run 4 Roll Gyro.	109
E.6	Amplitude Plot for Run 4 Yaw Gyro.	109
F.1	Amplitude Plot for Run 8 Vertical Accelerometer.	111
F.2	Amplitude Plot for Run 8 Transverse Accelerometer.	111
F.3	Amplitude Plot for Run 8 Longitudinal Accelerometer.	112
F.4	Amplitude Plot for Run 8 Pitch Gyro.	112
F.5	Amplitude Plot for Run 8 Roll Gyro.	113
F.6	Amplitude Plot for Run 8 Yaw Gyro.	113
G.1	Amplitude Plot for Run 11 Vertical Accelerometer.	115
G.2	Amplitude Plot for Run 11 Transverse Accelerometer.	115
G.3	Amplitude Plot for Run 11 Longitudinal Accelerometer.	116
G.4	Amplitude Plot for Run 11 Pitch Gyro.	116
G.5	Amplitude Plot for Run 11 Roll Gyro.	117
G.6	Amplitude Plot for Run 11 Yaw Gyro.	117
H.1	Amplitude Plot for Run 15 Vertical Accelerometer.	119
H.2	Amplitude Plot for Run 15 Transverse Accelerometer.	119
H.3	Amplitude Plot for Run 15 Longitudinal Accelerometer.	120
H.4	Amplitude Plot for Run 15 Pitch Gyro.	120
H.5	Amplitude Plot for Run 15 Roll Gyro.	121
H.6	Amplitude Plot for Run 15 Yaw Gyro.	121

Chapter 1

Introduction

1.1 Scope

One of the key components of unmanned ground vehicles (UGVs) is the sensor suite that is used for both the reconnaissance, surveillance, and target acquisition (RSTA) mission and vehicle mobility. The UGVs are to be able to maneuver over a variety of hazardous terrains, which makes camera stabilization and aiming crucial objectives. In order for the UGV to maneuver while cameras in a sensor suite perform their operations, it is necessary to actuate pan and tilt motors that will keep the cameras oriented so as to “null out” vehicle rotations and vibrations and keep a desired line of sight (LOS). Feedback, such as gimbal angles and acceleration measurements, from an inertial measurement system (IMS) can be used to aid in sensors suite stabilization. These IMS measurements provide the vehicle’s computerized control system with an inertial position and orientation reference with respect to the maneuvering vehicle. The inertial reference can then serve as a basis for determining the angles necessary to maintain a desired orientation of the sensor suite.

The focus of this research is to determine the robustness and stability of a pan/tilt sensor platform for UGVs via platform modeling and model analysis. The sensor platform was modeled as a simple beam element with a concentrated mass attached to one end. The other end of the beam serves as the base of the sensor platform which would be connected to

the UGV. Since the UGVs are to maneuver over various terrains, the model was developed to show concentrated mass response characteristics for various base excitations. Platform sensitivity information was obtained by varying certain model parameters.

1.2 Unmanned Ground Vehicles (UGVs)

1.2.1 UGV History

UGVs have a vital role in military operations. The motivation for using such vehicles is to place highly mobile and multi-tasked vehicles in hostile environments, which eliminates the need to put human life in danger. The development of unmanned ground vehicle test beds was begun in 1987 by the U.S. Army Laboratory Command (LABCOM) [2]. Initially the UGV was to have a target acquisition package and be used as a weapon for tank destruction. It was later assigned a more passive role. The new goal was to develop a highly mobile, multipurpose vehicle capable of performing functions such as RSTA. This was the beginning of the UGV program which was divided into three demonstrations, Demos I, II, and III. Each demonstration was also broken into subsections which specialized in various UGV subsystems. These demonstrations were scheduled to assess the achievements of the UGV program in the areas of navigation, autonomy, cooperative UGV operations, and RSTA. Demo I was scheduled for May 1992 at Churchville, Maryland.

Demo II was sectioned into four planned UGV demonstrations, A, B, C, and D. The objective of Demo II was stated in [10] as,

“The objective of the UGV / Demo II program is to develop and mature those navigation and automatic target recognition technologies critical for the development and demonstration of supervised, autonomous ground vehicles capable of performing military scout missions with a minimum of human oversight. The intent is to focus on and exploit the artificial intelligence, computer vision, and advanced processor developments sponsored under the Advanced Research Projects Agency’s (ARPA) science and technology program.” (p. 110)

The goals for Demo II C, as found in [10], are stated as

- Multivehicle mission planning

- Mission execution monitoring
- Multivehicle mobility cooperation
- Target detection from moving and stationary platforms
- Obstacle avoidance
- Obstacle map sharing
- Stealthy movements
- Autonomous turnarounds
- Formation control / zone security
- Cooperative RSTA
- Hill cresting

Various groups have worked on issues associated with Demo II. Research concerning target acquisition was addressed at the Army Research Laboratory in Adelphi, MD and is discussed in [11]. That paper addresses the Army's Scout Mission which focused on performing RSTA operations. In that paper it was stated that target acquisition was accomplished only when the target was moving, and the UGV had to be in a stationary position. One of the main conclusions of that paper was that issues related to target acquisition from a moving vehicle should be addressed. Other research activities included the Surrogate Semi-Autonomous Vehicle (SSV) program [4]. In that paper the design issues associated with autonomous navigation were discussed. One of the important design issues discussed in that paper was the ability of the vehicle to operate solely and cooperatively with other UGVs. Demo II was in fact the first demonstration to have two vehicles work cooperatively. Presently the UGV program is in the Demo III phase. The goals of this demonstration are discussed in the next section.

1.2.2 Current UGV Goals

The Demo III phase primarily focuses on UGV mobility and the RSTA mission. One of the goals of Demo III is for the UGV to perform a variety of functions with little or no

help from a human operator. This requires integrating multiple sensor feedback to make semi-autonomous vehicles that are highly mobile with RSTA capability. One of the major goals of Demo III is to enhance the vehicle's ability to perform 'RSTA on the move'.

Mobility is a key issue for UGV operations. The mobility goals for the UGV Demo III, as given in [5] are: 1) The environment in which the UGV is to operate can be unstructured and the terrain over which the vehicle travels can be rough and composed of positive obstacles (i.e. rocks, trees, etc.) and negative obstacles (i.e. ditches, holes, etc.). 2) The vehicle is to be able to operate at any time, day or night, in conditions such as light rain, dust, or smoke. An important issue discussed in [5] related to vehicle mobility is the capability of sensor stabilization and pointing. In [5] the authors state,

"... high resolution images are sensitive to blurring due to roll, pitch, and yaw motions caused by engine vibrations and high speed driving over rough terrain. Tracking of objects and features over multiple frames is severely compromised by high frequency camera rotational motions. A combination of mechanical and electronic stabilization needs to be developed to achieve the most cost effective stabilization of images." (p. 6)

Another important issue for UGV development in the Demo III phase is UGV system modeling. Modeling, model analysis and experimentation are currently being conducted at Mounted Maneuver Battlespace Laboratory (Fort Knox, Kentucky) and at the Army Research Laboratory (Aberdeen Proving Grounds, Maryland) [9]. The efforts of those groups is focused on various modeling and model analysis consisting of, but not limited to the following: 1) optimum configuration for sensors, mobility, and platform performance; 2) soldier in the loop model analysis; 3) battle model analysis.

A number of groups have worked to develop subsystems that can be integrated to produce a multi-task UGV. Many design issues associated with the UGV program have been addressed and are presently being researched. It is hoped that as this research is conducted and as technological advancements are made, the UGV will mature into a highly mobile, multi-tasked, autonomous vehicle.

1.3 Reconnaissance, Surveillance, and Target Acquisition (RSTA) and Mobility for Unmanned Ground Vehicles

Falling into the scope of this research are concerns with the RSTA mission. The goals of the RSTA mission have been identified in [6] and [9], and are as follows.

- Detect a tank-size object at 6 km from a stationary sensor platform
- Detect dismounted soldiers at 2 km
- Operate in day and night, in a variety weather conditions
- Detect, identify, and locate targets

As discussed in the previous section, the UGV is to be capable of these functions solely and in cooperation with other UGVs. The cooperative tasks will increase detection and identification of targets and will increase target position estimation via triangulation using multiple views of the target. Being able to perform these function while the vehicle is in motion is also of primary concern.

As mentioned in the previous section, excessive vehicle motion degrades camera images by disrupting the correspondence from one frame to the next and by causing image blurring. These problems are caused when the cameras scan a scene or when the vehicle is in motion. As discussed in [5], camera stabilization is addressed from a passive and active damping point of view. Passive damping is obtained by the use of physical damping elements in the system while active damping is obtained by the use of IMS measurements. These measurements can be used to generate commands to a pan/tilt head to which the cameras are mounted. The authors of [5] discuss the use of a Vinten pan/tilt head with the following capabilities.

- 75 lbs payload

- Max angular velocity = $\frac{90 \text{ deg}}{\text{sec}}$
- Max angular acceleration = $\frac{360 \text{ deg}}{\text{sec}^2}$

The RSTA mission requires the use of multiple, integrated sensors. The sensors that have been identified as essential, as stated in [6], in the RSTA suite are:

- IR camera,
- visible camera,
- mobility cameras,
- acoustic sensors,
- LADAR,
- NBC,
- flash detector,
- laser range finder, and
- laser designator.

When this sensor suite is excited by vehicle rotations and vibrations it is necessary to actuate the pan/tilt head to stabilize the sensor's orientation. This pushes the need to develop a sensor platform model and evaluate its response characteristics.

RSTA on the Move

One of the goals of Demo III is to be able to accomplish the RSTA mission while the vehicle is in motion. This is a task that could give the UGV a great advantage during its mission. Stopping the vehicle not only slows the mission, it could also prove to be dangerous in certain environments. 'RSTA on the move' is a task that is not easily accomplished since

the vehicle undergoes rotations and vibrations that cause camera images to become blurred and targets to be lost from the camera's field of view (FOV). Therefore 'RSTA on the move' requires fast control algorithms for camera aiming, accurate position and attitude measurement sensors, and fast sensor suite pan/tilt actuators.

RSTA can also have an important role in maintaining the mobility of the UGV. As the vehicle maneuvers along the terrain, it will be necessary for the camera to 'look ahead' or aim the cameras in certain directions. This is necessary when the vehicle is turning. The vehicle will need to be able to view what might be in its path or vicinity and make decisions based on the results. This 'peripheral vision' information will allow the computer system of the UGV to have an extended knowledge of its environment.

Chapter 2

Inertial Measurement Systems

An inertial measurement system (IMS) is made up of inertial measurement units that measure dynamic characteristics of the body to which they are mounted. These measurement units consists of gyroscopes and accelerometers. When six degrees of freedom are needed to describe the position and orientation of a body in motion, the accelerometers and gyroscopes have the role of measuring accelerations and rotation rates along and about three orthogonal axes, respectively. The accelerometer data can be integrated once or twice to obtain velocity and position data, respectively, while rotational angles can be determined from the gyroscopic rotation measurements¹. Together the position and rotation data can be used to describe the position and orientation of a body in motion.

¹Angular rates cannot be integrated directly since the order of rotations used to describe the orientation of a body cannot be chosen arbitrarily. The use of Euler angles is required.

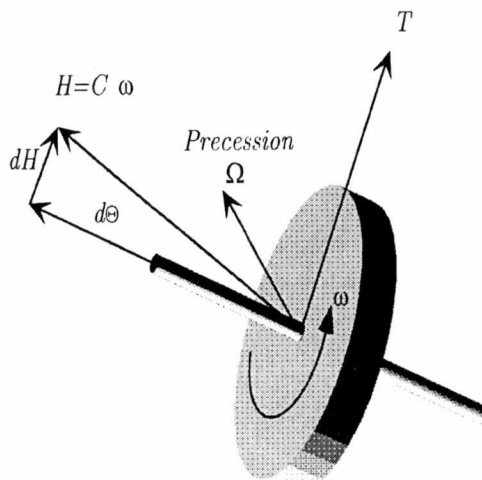


Figure 2.1: Law of gyroscopes.

2.1 Inertial Measurement Devices (Gyroscopes and Accelerometers)

Angular Momentum

By giving a rotor an angular velocity, an angular momentum is produced in the direction of its spin axis, figure 2.1. If an external torque is applied to the rotor, the direction of the angular momentum will tend to align itself with the direction of the applied torque. Angular momentum is related to the applied torque, T (about the mass center), by the equation

$$T = \frac{dH}{dt} = I \frac{d\omega}{dt}, \quad (2.1)$$

where H is the angular momentum, I is the constant, principal moment of inertia of the rotor, and ω is the angular velocity. When an applied torque acts on the rotor, its effect is to cause the rotor to precess about an axis that is normal to both the applied torque and the spin axis of the rotor [7]. Figure 2.1 illustrates this phenomenon.

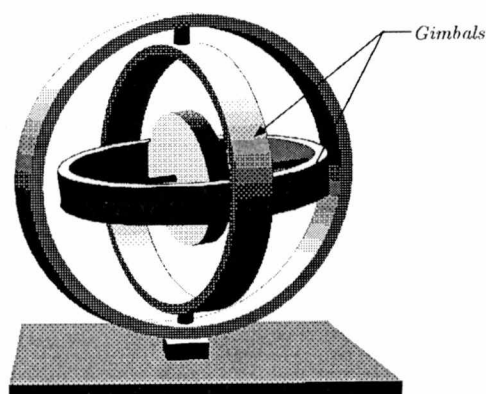


Figure 2.2: Three axis gyroscope.

Mechanical Gyroscope

Mechanical gyroscopes are based on the conservation of angular momentum. A mechanical gyroscope is made up of a rotor and gimbals, figure 2.2. The gimbals are connected by nearly 'friction-free' bearings and tend not to resist any external torques. By mounting the rotor in the gimbals and giving the rotor an angular velocity, the rotor's angular momentum tends to keep the rotor's orientation fixed in space. Gimbal "pickoff" encoders can then measure the rotations experienced between the gimbals. The gimbal "pickoff" angles can then be used describe the attitude of the body to which the gyroscope is mounted. However, one problem associated with the mechanical gyroscope is the small amount of friction in the gimbal bearings. This friction will cause the rotor to drift over time which in turn results in gimbal measurement errors.

Ring Laser Gyroscope

There are many types of gyroscope available that do not require a spinning rotor. Whenever components of a mechanical system are in motion there is a friction introduced. In the mechanical gyroscope this friction results in measurement drift errors. Therefore it would be advantageous to eliminate any or all moving components of a gyroscope to lower friction,

hence reducing the drift errors. A ring-laser gyroscope (RLG) is one in which two light beams, originating from one source, are emitted along two paths which are equal in length but opposite in direction, and are then collected at a detector. If there is no rotation of the device, the two light beams will be collected at the detector at the same time since their effective path lengths will have remained the same (i.e. the output at the detector will be the total constructive interference of the two beams). If the device is subjected to an external torque, the two light signals will reach the detector at different times since the effective path lengths of the two beams will have changed and the velocity of light is constant. The rotational rate experienced normal to the plane of the paths, or in the RLG's sensitive axis, can then be determined based on the net phase difference and destructive interference of the two light beam signals when added at the detector. The resulting amplitude of this combined signal gives an indication of the phase shift between the two signals and can be converted to a rotation rate. The RLG is in fact a gyroscope that requires no moving parts which means no inherent drift errors.

Accelerometers

The theory of an accelerometer is based on the fact that when a mass undergoes acceleration, an "inertia" force is generated. Accelerometers can be fixed to a body and used to measure such forces. When the body is in motion its acceleration can be determined by solving the dynamic equation of motion specific for the individual accelerometer. This measurement can then be integrated once or twice to give the velocity or position, respectively, of the moving body.

2.2 Inertial Platforms

An inertial platform is a platform which maintains a fixed attitude in space with respect to the rotation of the body to which it is mounted. This platform acts as an inertial

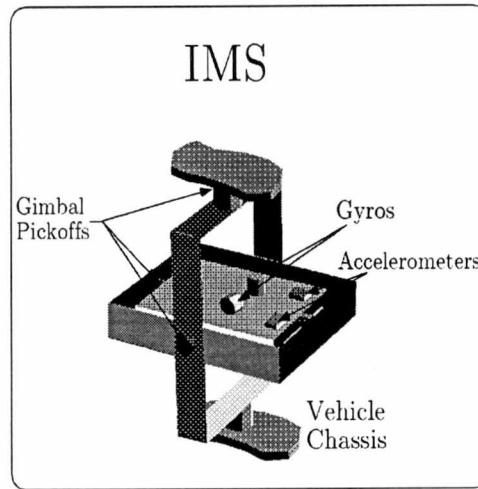


Figure 2.3: Classical Inertial Platform.

orientation reference for the body in motion. This inertial reference is then used as a basis for determining the body's position and attitude while the body is in motion.

2.2.1 Classical Inertial Measurement Systems

Classically, maintaining the fixed attitude of an inertial platform is accomplished by mounting the platform in near 'friction-free' gimbals. This gimbal system is then mounted to the vehicle as shown in figure 2.3. Since the platform is mounted in gimbals, the platform's inertia will tend to keep it in an inertially fixed orientation in space as the vehicle rotates. However, the platform will tend to drift due to the small amount of friction in the bearings between the gimbals. Therefore, three perpendicularly oriented gyros mounted on the platform can be used to compensate for platform drift. As the platform drifts, gyro measurements can be used to command torque motors and "null out" any drift rotation. If the gimbals of the platform are aligned with the vehicle's axes before beginning its journey, the angles between the gimbals, measured by gimbal pickoff encoders, will indicate the rotations experienced by the body in motion. Then, since the platform maintains an inertial reference orientation, the platform can serve to define the directions for the measurements

of acceleration.

2.2.2 Strapdown Systems

A classic IMS consists of a number of moving parts. The major disadvantages with the system are associated with the drift errors caused by the friction between the moving parts. A demand for systems that reduce the number of moving parts has increased and has resulted in systems that take advantage of technological advancements in high speed computing and computer controlled systems.

The strapdown system has evolved in light of the need for faster and more accurate inertial measurement and navigation systems. This system does not require the use of gimbals. Instead, a computer takes the role of the gimbals electronically [7]. In these systems the gyroscopes and accelerometers are rigidly mounted to the vehicle. These systems require that the coordinate rotational transformation matrix, ${}^I[R]_{IMS}$, between an inertial coordinate frame (I) and the IMS unit coordinate frame be determined. It is also required that the constant rotational transformation between the IMS unit and the vehicle's center of gravity be known *a priori*. As the vehicle travels, the gyroscopes measure the rotation rates of the vehicle. These measured rates are sent to the computer and converted to rotational angles which allows ${}^I[R]_{IMS}$ to be determined. The rotational transformation matrix between the vehicle and the inertial reference frame can then be determined from

$${}^I[R]_{IMS} = {}^I[R]_V^V [R]_{IMS}. \quad (2.2)$$

This rotation matrix then allows the accelerometer measurements to be resolved into inertial reference coordinates.

Chapter 3

Sensor Module Stabilization

Sensor Aiming Concept

The concept of performing RSTA as the vehicle is in motion has been viewed from two mission perspectives. These missions are:

- Keeping a target in the camera's field of view,
- Road following via "look ahead" camera pointing.

These two concepts are shown in figure 3.1. In order for these objectives to be completed, the angles required for the pan/tilt head need to be determined. This can be achieved by finding the rotational transformation that can be used to express the orientation of the sensor platform with respect to an inertial reference as the vehicle travels over various terrains.

3.1 Euler Angles

Euler angles are used extensively in describing the orientation of a body in motion [8]. The angular velocity components ω_{b1} , ω_{b2} , and ω_{b3} about a set of body axes can not be directly

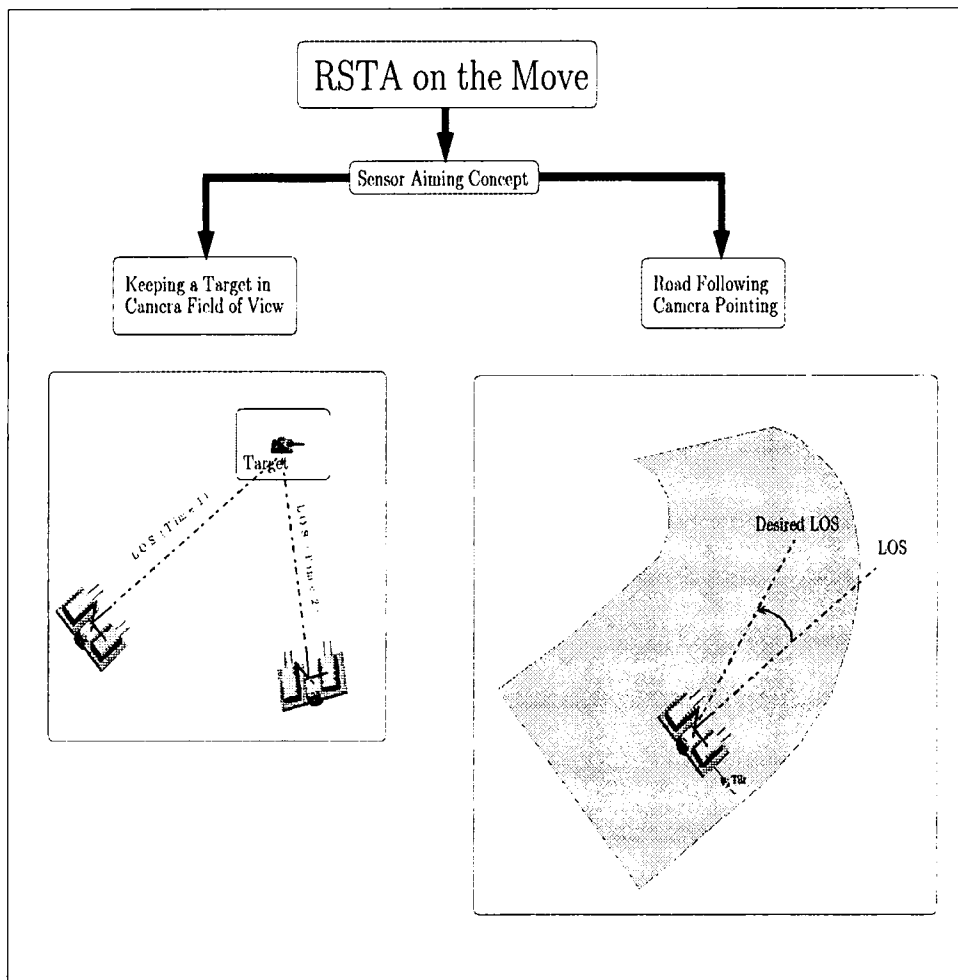


Figure 3.1: Camera Aiming

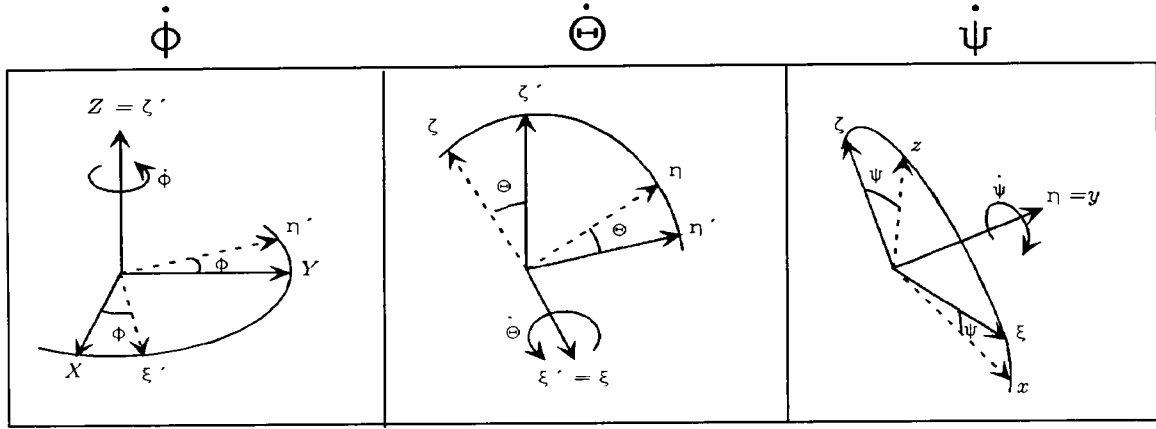


Figure 3.2: Components of the vehicle's angular velocity.

integrated to determine the angular displacements since the order of rotations cannot be arbitrarily chosen. However, the three components of the body's angular velocity can be expressed in terms of Euler's angles and their time derivatives, as shown in figure 3.2. The process of obtaining Euler angles is given next. The notation used is as found in [8].

Initially the body's axes are coincident with a set of Cartesian inertial axes X, Y, Z . A rotation about the Z brings the body axes into coincidence with the axes ξ', η', ζ' . A rotation θ about the ξ' axis brings the body axes into coincidence with the axes ξ, η, ζ . Finally, a rotation ψ about the η axes brings the body axes into coincidence with the actual body axes x, y, z .

There are different methods of determining Euler angles, but this method used here is the same as found in [8]. First the body axes rotate with an angular velocity $\dot{\phi}$ about the

Z axis. The angular velocity vector, in terms of the ξ', η', ζ' system is

$$\omega_{\xi'\eta'\zeta'} = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix}. \quad (3.1)$$

Next the body moves with an angular velocity $\dot{\theta}$ about the ξ' axis. The angular velocity vector, in terms of the ξ, η, ζ system, is then

$$\omega_{\xi\eta\zeta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ 0 \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \sin(\theta) \\ \dot{\phi} \cos(\theta) \end{bmatrix}. \quad (3.2)$$

Again the body also moves relative to the ξ, η, ζ system with an angular velocity $\dot{\psi}$ about η . The angular velocity of the body system in terms of the ξ, η, ζ system is then given by

$$\Omega_{\xi\eta\zeta} = \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \sin(\theta) + \dot{\psi} \\ \dot{\phi} \cos(\theta) \end{bmatrix}. \quad (3.3)$$

One final rotation must be used in order to get to the body axes x, y, z . This rotation is given by

$$\Omega_{xyz} = \begin{bmatrix} \cos(\psi) & 0 & -\sin(\psi) \\ 0 & 1 & 0 \\ \sin(\psi) & 0 & \cos(\psi) \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \sin(\theta) + \dot{\psi} \\ \dot{\phi} \cos(\theta) \end{bmatrix}. \quad (3.4)$$

The angular velocity components, Ω_x , Ω_y , and Ω_z can now be expressed in terms of the

angular velocities $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$ as

$$\begin{Bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{Bmatrix} = \begin{bmatrix} -\sin(\psi) \cdot \cos(\theta) & \cos(\psi) & 0 \\ \sin(\theta) & 0 & 1 \\ \cos(\psi) \cdot \cos(\theta) & \sin(\psi) & 0 \end{bmatrix} \begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix}. \quad (3.5)$$

Euler's angles ϕ , θ , and ψ can now be solved for by integrating

$$\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} -\sin(\psi) \cdot \cos(\theta) & \cos(\psi) & 0 \\ \sin(\theta) & 0 & 1 \\ \cos(\psi) \cdot \cos(\theta) & \sin(\psi) & 0 \end{bmatrix}^{-1} \begin{Bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{Bmatrix}. \quad (3.6)$$

For use in platform stabilization the components of the vector Ω_{xyz} would be obtained from the IMUs mounted the body experiencing the rotation.

3.2 Sensor Platform Stabilization

The basic procedure for sensor suite stabilization begins with measuring the rotation rates and accelerations of the vehicle to which the camera is mounted. As the vehicle travels over arbitrary terrains, the IMS is used to measure these rotation rates and accelerations and feed the measurement data to the control algorithms that are used for camera stabilization and pointing. This process is shown in figure 3.3. The method for the camera stabilization and pointing process is discussed in the next section. The development of the control algorithms is in two parts. First, the angles are determined that “null out” vehicle rotations based on IMS measurements. Second, the angles are determined that orient the cameras in such a way as to maintain a desired LOS.

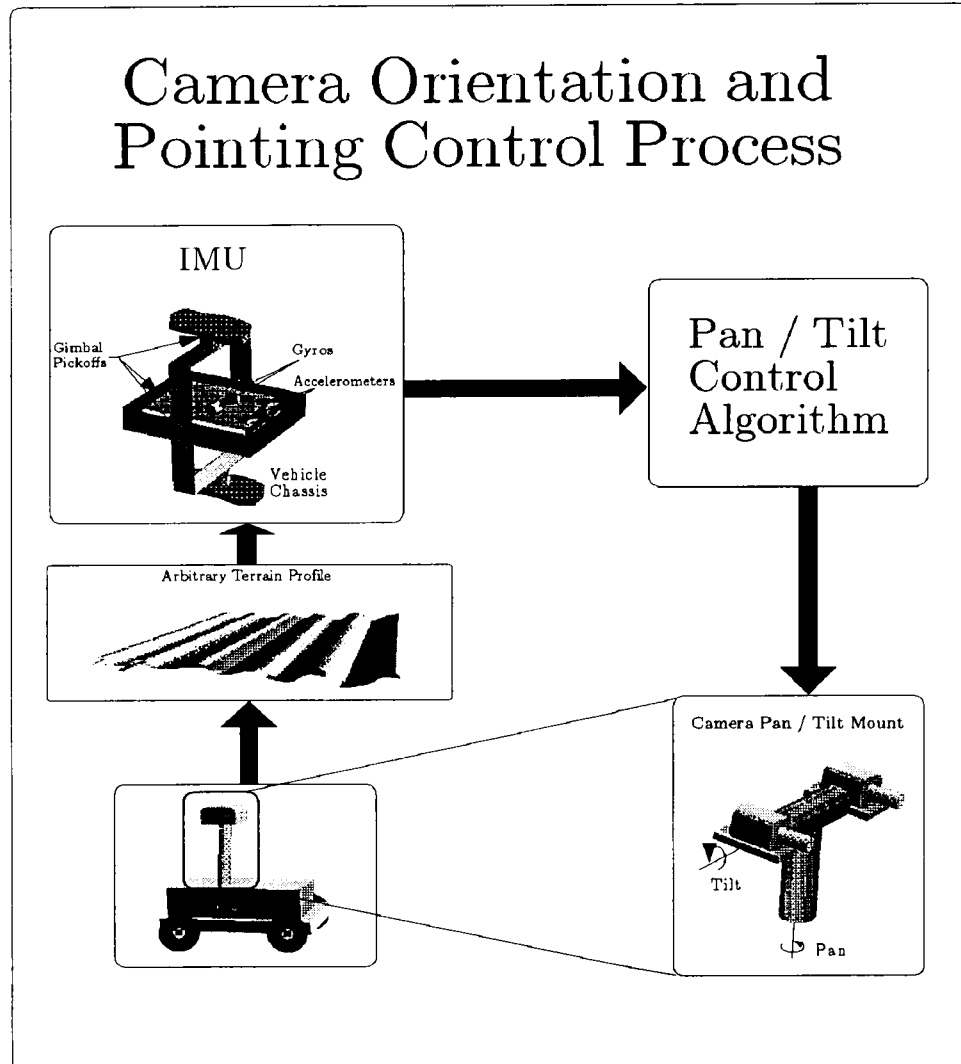


Figure 3.3: Control Process for camera stabilization.

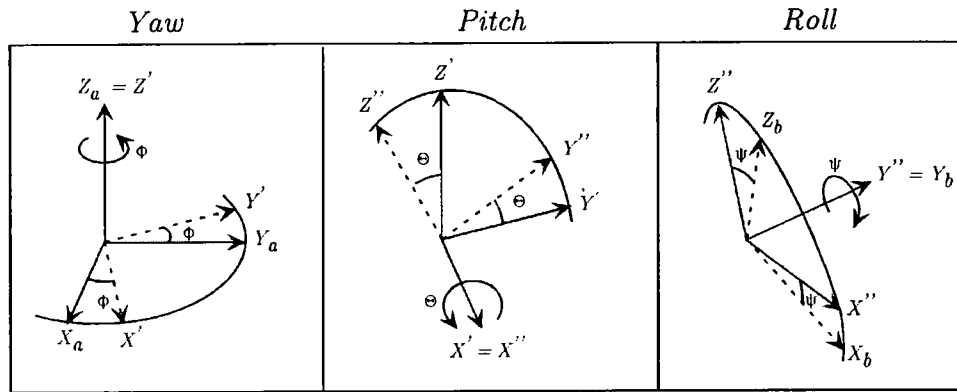


Figure 3.4: Yaw, Pitch, and Roll rotations.

3.2.1 Inertial Stabilization

For inertial stabilization to be achieved, the rotation matrices between an inertial reference frame and the rotating frame of concern must be determined. Once determined, these matrices along with the position vectors from the origin of the inertial frame to the origin of the rotating frame can be used to determine the corresponding homogeneous transformation matrix. Figure 3.4 shows the rotations about each of the axes of a Cartesian coordinate system.

As the vehicle maneuvers over a terrain, the attitude of the vehicle can be determined by gyros and accelerometers. Figure 3.5 illustrates keeping an inertial orientation of the sensor platform reference frame as the vehicle experiences arbitrary orientations. This figure shows arbitrary orientation of the vehicle frame at two times. In this figure it is shown that the camera frame for these two times has retained the same orientation as the inertial frame of reference.

The order of rotation is important since the sensors are to be mounted to a pan/tilt head with only yaw and pitch capability. The order yaw/pitch/roll was used for the rotation matrices. The rotation matrices and the position vectors were determined and are as follows.

The rotation matrix and position vector from the inertial reference frame to the vehicle

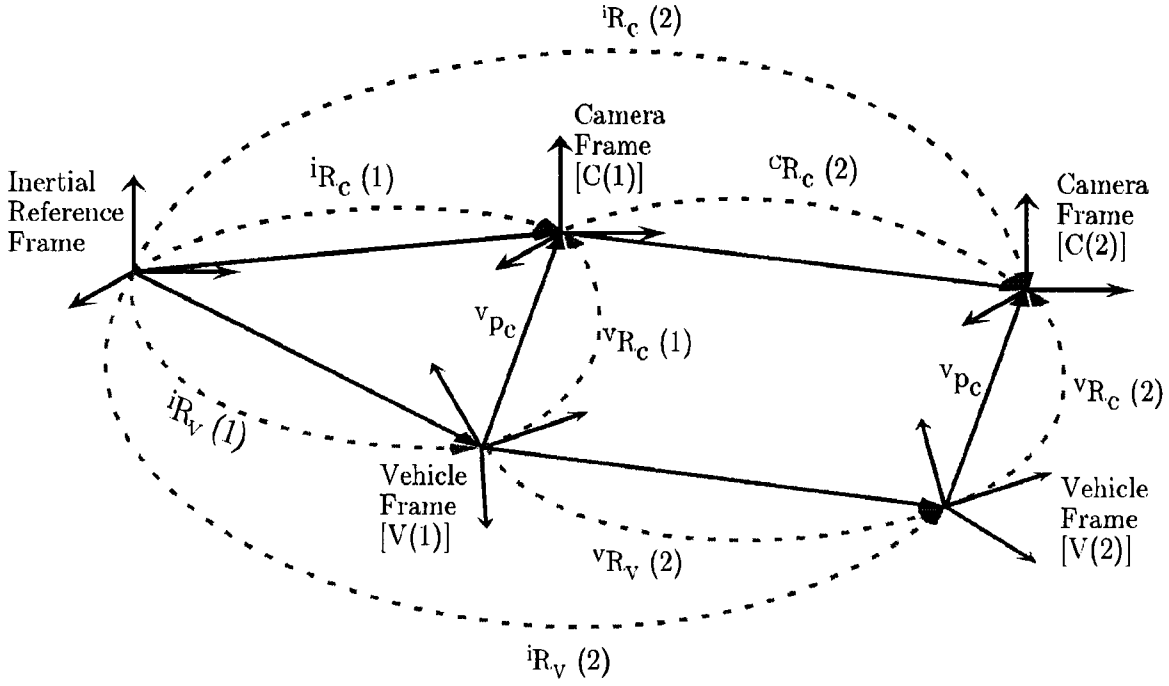


Figure 3.5: Transformations for sensor stabilization.

frame are

$${}^I\mathbf{R}_V = \begin{bmatrix} \cos(yaw) & -\sin(yaw) & 0 \\ \sin(yaw) & \cos(yaw) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(pitch) & -\sin(pitch) \\ 0 & \sin(pitch) & \cos(pitch) \end{bmatrix} \begin{bmatrix} \cos(roll) & 0 & \sin(roll) \\ 0 & 1 & 0 \\ -\sin(roll) & 0 & \cos(roll) \end{bmatrix} \quad (3.7)$$

and

$${}^I\vec{p}_V = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad (3.8)$$

respectively. The rotation matrix and position vector from the inertial reference frame to

the sensor frame are

$${}^I\mathbf{R}_C = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix} \begin{bmatrix} \cos(\gamma) & 0 & \sin(\gamma) \\ 0 & 1 & 0 \\ -\sin(\gamma) & 0 & \cos(\gamma) \end{bmatrix} \quad (3.9)$$

and

$${}^I\vec{p}_C = \begin{bmatrix} x + x_{vc} \\ y + y_{vc} \\ z + z_{vc} \end{bmatrix}, \quad (3.10)$$

respectively, where x_{vc} , y_{vc} , and z_{vc} make up the constant vector from the vehicle to the camera, ${}^V\vec{p}_C$.

The rotation matrix from the vehicle to the sensor frame can then be determined from these rotation matrices to be

$${}^V\mathbf{R}_C = ({}^I\mathbf{R}_V)^T \cdot {}^I\mathbf{R}_C, \quad (3.11)$$

where $(^T)$ indicates the transpose of the matrix. This matrix is given by

$${}^V\mathbf{R}_C = \begin{bmatrix} c(\phi) \cdot c(\psi) - s(\phi) \cdot s(\theta) \cdot s(\psi) & -s(\phi) \cdot c(\theta) & c(\phi) \cdot s(\psi) + s(\phi) \cdot s(\theta) \cdot c(\psi) \\ s(\phi) \cdot c(\psi) + c(\phi) \cdot s(\theta) \cdot s(\psi) & c(\phi) \cdot c(\theta) & s(\phi) \cdot s(\psi) - c(\phi) \cdot s(\theta) \cdot c(\psi) \\ -c(\theta) \cdot s(\psi) & s(\theta) & c(\theta) \cdot c(\psi) \end{bmatrix}, \quad (3.12)$$

where $c(\phi)$, $c(\theta)$ and $c(\psi)$ equals $\cos(\phi)$, $\cos(\theta)$ and $\cos(\psi)$, respectively, and $s(\phi)$, $s(\theta)$ and $s(\psi)$ equals $\sin(\phi)$, $\sin(\theta)$ and $\sin(\psi)$, respectively. It is desired to determine the pitch and yaw angles from this matrix in order to orient the sensor's coordinate frame. In order to orient the pan/tilt head to obtain camera stabilization, the pitch and yaw angles from

equation 3.12 need to be determined. To obtain these angles from ${}^V\mathbf{R}_C$, assume that

$${}^V\mathbf{R}_C = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \quad (3.13)$$

is given. Equating each element of equation 3.12 and equation 3.13 results in the following equations.

$$c(\phi) \cdot c(\psi) - s(\phi) \cdot s(\theta) \cdot s(\psi) = R_{11} \quad (3.14a)$$

$$-s(\phi) \cdot c(\theta) = R_{12} \quad (3.14b)$$

$$c(\phi) \cdot s(\psi) + s(\phi) \cdot s(\theta) \cdot c(\psi) = R_{13} \quad (3.14c)$$

$$s(\phi) \cdot c(\psi) + c(\phi) \cdot s(\theta) \cdot s(\psi) = R_{21} \quad (3.14d)$$

$$c(\phi) \cdot c(\theta) = R_{22} \quad (3.14e)$$

$$s(\phi) \cdot s(\psi) - c(\phi) \cdot s(\theta) \cdot c(\psi) = R_{23} \quad (3.14f)$$

$$-c(\theta) \cdot s(\psi) = R_{31} \quad (3.14g)$$

$$s(\theta) = R_{32} \quad (3.14h)$$

$$c(\theta) \cdot c(\psi) = R_{33} \quad (3.14i)$$

From equations 3.14g and 3.14i

$$\cos(\theta) = \pm \sqrt{R_{31}^2 + R_{33}^2}. \quad (3.15)$$

The angle θ was then determined from equations 3.14h and 3.15 to be

$$\theta = \text{atan2}(\pm R_{32}, \pm \sqrt{R_{31}^2 + R_{33}^2}). \quad (3.16)$$

The angles ϕ and ψ were then determined. From (3.14b) and (3.14e)

$$\phi = \text{atan2}(\mp R_{12}, \pm R_{22}), \quad (3.17)$$

and from (3.14g) and (3.14i)

$$\psi = \text{atan2}(\mp R_{31}, \pm R_{33}). \quad (3.18)$$

These angles can then be used as commands to the pan/tilt head for camera stabilization. The plus or minus that was used in this analysis is shown in the computer code found in appendix A.

3.2.2 Camera Aiming

As the vehicle maneuvers it is desirable not only to obtain inertial stability for the sensor platform but also to orient the camera in order to keep a desired LOS. This is important because it will be desirable to maintain certain fields of view (i.e. keeping a target in the camera's field of view or road following).

Line of Sight (LOS) Positioning

Once a target is detected and the range to target is known by the computer, the target's position can be determined with respect to a position and orientation reference frame. If the UGV then maneuvers, a rotation matrix for the cameras can be established that will orient the LOS of the camera so that it continues to point towards the target as shown in

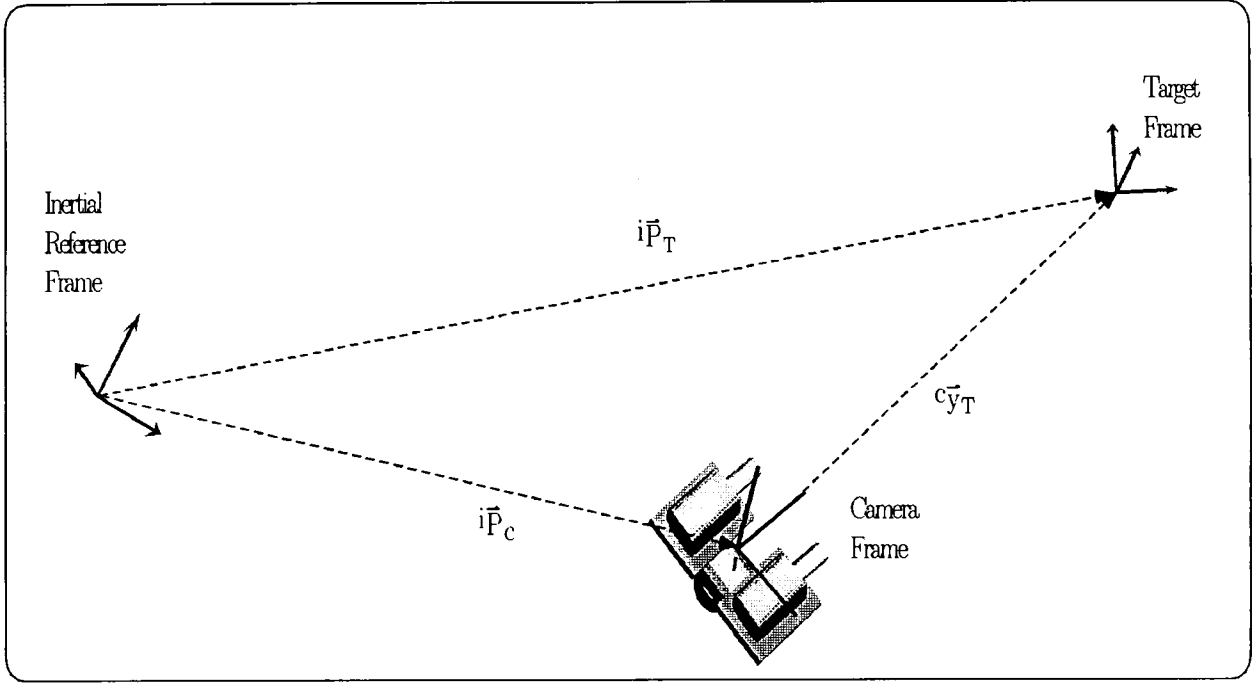


Figure 3.6: Determining angles from range to target.

figure 3.6. The vector from the inertial frame to the target is given by

$${}^I\vec{p}_T = \begin{pmatrix} x + x_c \\ y + y_c \\ z + z_c \end{pmatrix} + \begin{pmatrix} 0 \\ {}^c y_t \\ 0 \end{pmatrix}, \quad (3.19)$$

where ${}^c y_t$ is the known distance along the LOS from the camera to the target. Once this vector is known, the vector from the various sensor position to the target positions can be determined. This vector is given by

$${}^C\vec{p}_T = {}^I\vec{p}_T - {}^I\vec{p}_C. \quad (3.20)$$

The rotation matrix from an inertially stabilized reference frame to camera reference frame that will keep a LOS that points at the target is the same as equation 3.13. The elements

R_{12} , R_{22} , and R_{32} make up the y-coordinate of the desired rotation matrix. Since the vector ${}^C\vec{p}_T$ is along the y-axis of the desired coordinate system, the elements R_{12} , R_{22} , and R_{32} can be determined. These elements were found to be

$$R_{12} = \frac{{}^C p_{T11}}{|{}^C\vec{p}_T|}, \quad (3.21)$$

$$R_{22} = \frac{{}^C p_{T21}}{|{}^C\vec{p}_T|}, \quad (3.22)$$

$$R_{32} = \frac{{}^C p_{T31}}{|{}^C\vec{p}_T|}. \quad (3.23)$$

Equations 3.21, 3.22, and 3.23 are compared to the elements of the second column of the yaw/pitch/roll rotation matrix, equation 3.12, to determine the angles. From equations 3.14b and 3.14e

$$\cos(\Theta) = \pm\sqrt{R_{12}^2 + R_{22}^2}. \quad (3.24)$$

The angle Θ was determined from (3.14h) and (3.2.2) to be

$$\Theta = \text{atan2}(\pm R_{32}, \pm\sqrt{R_{12}^2 + R_{22}^2}). \quad (3.25)$$

The angle Φ was determined from (3.14b) and (3.14e) to be

$$\Phi = \text{atan2}(\mp R_{12}, \pm R_{22}). \quad (3.26)$$

These angles are used to point the camera's LOS towards the identified target. Each measurement of the UGV's position and orientation, from the IMUs, will be used to determine the angles required to maintain the desired LOS.

Following a Known Path

It is also desirable for the cameras to "look ahead" along a known path (i.e. the road that it is traveling). This can be accomplished in a similar fashion as shown in the previous section. As the vehicle travels along the road, the heading of the vehicle can be determined based on the output from the accelerometers. This heading along with the desired heading of the vehicle can be used to determine the desired orientation of the cameras.

The algorithms developed in the previous sections determine the necessary angles for obtaining the mission objectives. Computer code for simulating the "null out" and camera pointing procedures is given in Appendix A. In order for these mission objectives to be accomplished though, it is necessary to estimate the degree of difficulty that is involved in stabilizing the camera. By developing a conceptual sensor platform, stabilization difficulty can be estimated. The conceptual platform can be excited at amplitudes and frequencies corresponding to those that a physical system would experience "in the field". The response characteristics of the conceptual platform can then be used to give an indication of the angular velocities and translational accelerations that an actual sensor platform would experience as the vehicle travels over various terrains.

Chapter 4

Pan/Tilt Platform Pedestal

The motivation behind developing a model of a sensor platform pedestal for UGVs is to determine sensor pointing performance based on excitations experienced by the vehicle's chassis. The UGV/sensor platform system is shown in figure 4.1. The sensor platform, as shown in the figure, was modeled as a simple beam element with a concentrated mass connected to one end. The concentrated mass represents the mass of sensors mounted to a pan/tilt head. The equations of motion derived for this model are based on classical beam theory since the translational and rotational deflections of the concentrated mass are assumed to be small with respect to the base of the platform (i.e. the beam element has high stiffness properties).

4.1 Platform Pedestal Model

A suitable model for the beam support element and concentrated mass has to be determined in order to acquire meaningful data concerning the response characteristics of the concentrated mass. Once the model is determined, the position and orientation of the pan/tilt head can be approximated as functions of time based on excitations transmitted through the vehicle to the base of the platform. As shown in the figure, the positive directions for

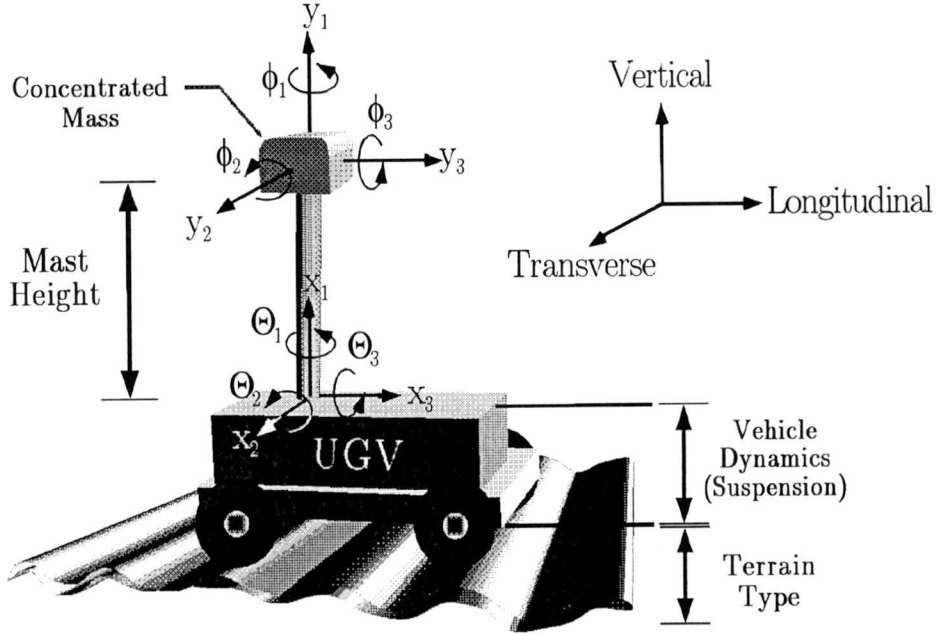


Figure 4.1: UGV with mounted sensor platform pedestal.

concentrated mass deflections are given by $(y_1, y_2, y_2, \phi_1, \phi_2, \phi_3)$ and positive directions for base deflections are given by $(x_1, x_2, x_2, \theta_1, \theta_2, \theta_3)$.

Assumptions and Mast Height Constraint

The assumptions made for this model are as follows:

- The beam element is a straight bar with circular cross section.
- The beam is able to resist axial forces, bending moments about the two principal axes in the plane of its cross section, and twisting moments about its centroidal axis.
- Due to the stiffness of the beam, the translational and rotational deflection of the concentrated mass are small and therefore the effective inertia of the mass is time invariant.

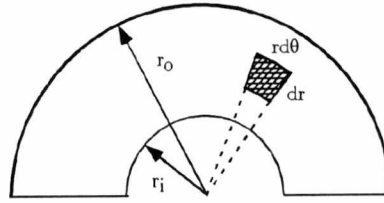
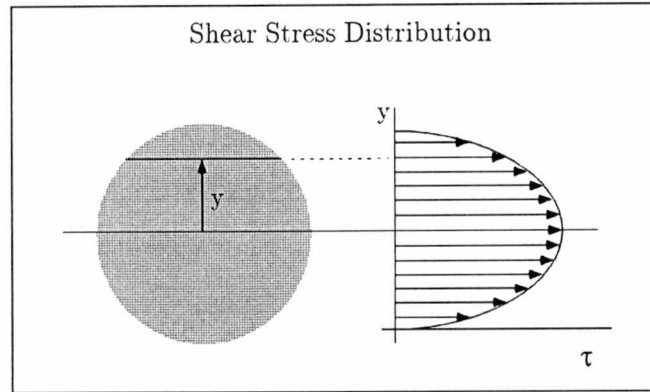
Figure 4.2: Cross sectional area of a pipe with differential area $rd\theta dr$.

Figure 4.3: Shear stress distribution for a uniform, cantilever beam.

- Since the beam is a straight bar of circular cross section, the area moments of inertia are equal in the directions normal to the centroidal axis of the beam.
- Shear deflection is neglected.
- Structural damping and damping due to air are small.

The assumption that the deflection of the beam is due only to bending, so that shear deflection can be neglected, strongly influences equations of motion derived for this system. This assumption constrains the length of the beam to be a function of the beam's inner and outer diameters. This constraint is determined in the following procedure.

Figure 4.2 shows the cross sectional area of half of the beam and the differential element described in cylindrical coordinates. The shear stress distribution for a uniform, cantilever beam of length L , loaded with an end load P , is shown in figure 4.3. The magnitude of the

maximum shear stress is given by the equation

$$\tau_{max} = \frac{PQ}{2I(r_o - r_i)}, \quad (4.1)$$

where I is the cross sectional area moment of inertia, r_o is the outer radius of the beam, and Q is the first area moment. For a beam of circular cross section the average shear stress is two thirds of the maximum shear stress. The average shear stress is given by the equation

$$\tau = \frac{2}{3}\tau_{max}. \quad (4.2)$$

The first area moment for the area above the neutral axis of the beam is given by the equation

$$Q = \int_A y dA, \quad (4.3)$$

where, the integral is taken over half the entire cross sectional area and $y dA$ is area of the differential element. For the uniform beam shown in figure 4.2

$$y = r \sin \theta, \quad (4.4a)$$

$$dA = r dr d\theta. \quad (4.4b)$$

The first area moment for a cross section shown in figure 4.2 is then given as

$$\begin{aligned} Q &= \left[\int_0^\pi \int_0^{r_o} r^2 dr \sin \theta d\theta - \int_0^\pi \int_0^{r_i} r^2 dr \sin \theta d\theta \right] \\ Q &= \frac{2}{3} [r_o^3 - r_i^3]. \end{aligned} \quad (4.5a)$$

The deflection of a beam element due to shearing, as shown in figure 4.4, is given by the equation

$$\delta_\tau = \gamma L \quad (4.6)$$

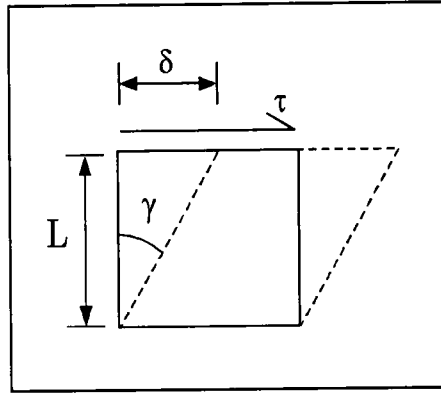


Figure 4.4: Deflection due to shear.

where, γ is the shear strain. The shear strain is related to the shear stress by the equation

$$\gamma = \frac{\tau}{G}, \quad (4.7)$$

where G is the shear modulus. The deflection due to shear is then given by the equation

$$\delta_\tau = \frac{\tau L}{G}. \quad (4.8)$$

It is shown in [1] that for a uniform beam the maximum bending deflection is given by the equation

$$\delta_\sigma = \frac{PL^3}{3EI}. \quad (4.9)$$

It was stated well in [1] when the author said,

“...for beams of any length the shear deflection is completely negligible. Only for beams that are but a little longer than they are high is the shear deflection a sensible fraction of the bending deflection. ...Short beams are not true beams, and when we apply our beam formulae to them, we must expect bad agreement with test. But for $l > 5h$ we can speak of a true beam, and then the shear deflection is less than 4 per cent of the bending deflection. We were therefore completely justified in neglecting it from the beginning.” (p. 108)

The ratio derived from this analysis for a hollow beam $\frac{\delta_\sigma}{\delta_{\tau au}}$ is given by

$$\begin{aligned}
\frac{\delta_\tau}{\delta_\sigma} &= \left[\frac{3EI}{PL^3} \right] \left[\frac{PQL}{2I(r_o - r_i)G} \right] \\
&= \frac{E}{G} \left[\frac{2[r_o^3 - r_i^3]}{2(r_o - r_i)L^2} \right].
\end{aligned} \tag{4.10a}$$

Setting the ratio in 4.10a equal to 4% reveals the following relation for the constraint on the length of the beam

$$L \geq \sqrt{\frac{E}{G} \left[\frac{[r_o^3 - r_i^3]}{0.04(r_o - r_i)} \right]}. \tag{4.11}$$

This constraint must be implemented to ensure that the equations of motion that will be derived for the sensor platform are valid.

4.1.1 Potential and Kinetic Energy Equations

This section details the analysis used to determine the kinetic and potential energy of the sensor platform. Once the kinetic and potential energies are found, an energy method is used to derive the system's equations of motion.

Bending Potential Energy Equations

The displacement of the free end of a beam relative to a fixed end will occur due to a combination of a force and a moment acting on the free end of the beam. Figure 4.5 shows the displacement of the free end of the beam when subjected to an applied force and moment. In this figure it should be noted that Δ and β have components due to both the force and moment applied to the beam. Since the deflection of the beam is assumed to be small, the deflection of the beam in the y_2 direction, as shown in figure 4.1, is assumed to be negligible. The translational and rotational deflection of the free end of the beam are then given by

$$\Delta = \Delta_F + \Delta_M = \frac{L^3}{3EI}F + \frac{L^2}{2EI}M \tag{4.12a}$$

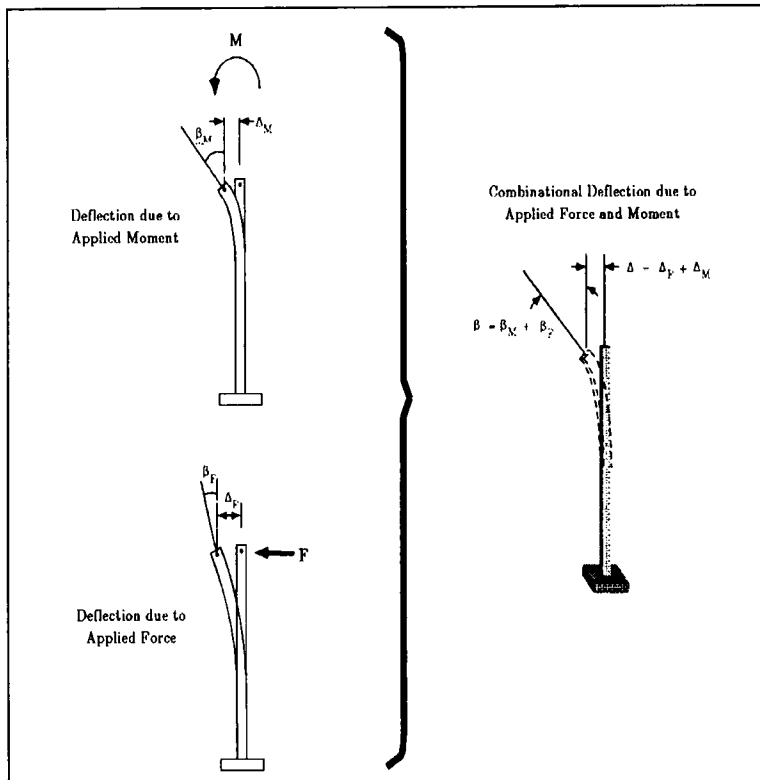


Figure 4.5: Combined translation and rotation due to force and moment applied to free end of beam.

$$\beta = \beta_F + \beta_M = \frac{L^2}{2EI}F + \frac{L}{EI}M. \quad (4.12b)$$

In these equations $\left[\frac{L^3}{3EI}\right]$, a translational stiffness, and $\left[\frac{L^2}{2EI}\right]$, a rotational stiffness, are associated with a force applied to the beam while $\left[\frac{L^2}{2EI}\right]$, a translational stiffness, and $\left[\frac{L}{EI}\right]$, a rotational stiffness, are associated with a moment applied to the beam. The elements of these stiffnesses are

E = elastic Modulus (Young's Modulus),

I = beam cross sectional area moment of inertia,

L = beam length.

The translational potential energy due to a force and moment applied to the free end of the beam is given by

$$U_{Translational} = \frac{1}{2}F\Delta = \frac{1}{2}F[\Delta_F + \Delta_M] = \frac{F^2L^3}{6EI} + \frac{FML^2}{4EI}. \quad (4.14)$$

The rotational potential energy due to a force and moment applied to the free end of the beam is given by

$$U_{Rotational} = \frac{1}{2}M\beta = \frac{1}{2}M[\beta_M + \beta_F] = \frac{M^2L}{2EI} + \frac{FML^2}{4EI}. \quad (4.15)$$

The total potential energy ($U_{Translational} + U_{Rotational}$) for the case of bending is then

$$U_T = \frac{F^2L^3}{6EI} + \frac{FML^2}{2EI} + \frac{M^2L}{2EI}. \quad (4.16)$$

It should be noted that in equation 4.16 that $U = f(F, M)$. In order to determine the system's equations of motion in terms of the desired coordinate displacements, the potential

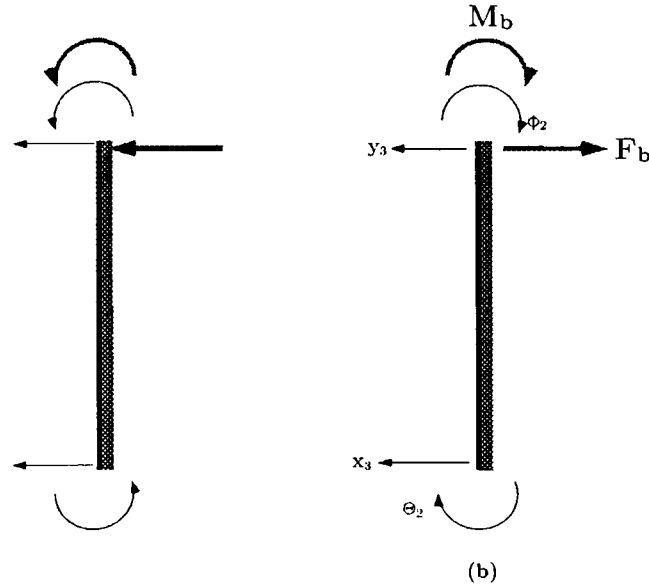


Figure 4.6: Sign convention.

energy for the case of bending needs to be shown as a function of y_2 , y_3 , ϕ_2 , and ϕ_3 . The potential energy for bending was determined due to a force and a moment applied to the free end of a cantilever beam. In order to determine the potential energy for bending as a function of the desired coordinate displacements, the displacement of the free end of the beam is now determined with respect to a base that is free to translate and rotate.

When determining the potential energy for the beam it is important to observe the sign conventions that were chosen for positive displacements shown in figure 4.1. The positive directions for determining the potential energy for the cases of bending, which result from the sign conventions chosen for the platform, are shown in figure 4.6.

Case (a) figure 4.6

When a force and a moment are applied at the (y_2, ϕ_3) end of the beam, in the positive y_2 direction and ϕ_3 direction, respectively, the equations for the translational and rotational deflections were determined to be

$$y_2 = x_2 + \frac{F_a L^3}{3EI} + \frac{M_a L^2}{2EI} + \theta_3 L, \quad (4.17a)$$

$$\phi_3 = \theta_3 + \frac{F_a L^2}{2EI} + \frac{M_a L}{EI}. \quad (4.17b)$$

These equations were then solved simultaneously for F_a and M_a as shown in matrix form in the following equation

$$\begin{Bmatrix} F_a \\ M_a \end{Bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} y_2 - x_2 - \theta_3 L \\ \phi_3 - \theta_3 \end{bmatrix}, \quad (4.18)$$

where

$$a_{1,1} = \frac{L^3}{3EI}, \quad a_{1,2} = \frac{L^2}{2EI}, \quad a_{2,1} = \frac{L^2}{2EI}, \quad a_{2,2} = \frac{L}{EI}. \quad (4.19)$$

The equations for F_a and M_a as functions of y_2 and ϕ_3 are then

$$F_a(y_2, \phi_3) = a_{41} [y_2 - x_2 - \theta_3 L] - a_{42} [\phi_3 - \theta_3] \quad (4.20a)$$

$$M_a(y_2, \phi_3) = -a_{51} [y_2 - x_2 - \theta_3 L] + a_{52} [\phi_3 - \theta_3], \quad (4.20b)$$

respectively, where

$$\begin{aligned} a_{4,1} &= \frac{a_{2,2}}{a_{3,1}}, \quad a_{4,2} = \frac{-a_{1,2}}{a_{3,1}}, \\ a_{5,1} &= \frac{-a_{2,1}}{a_{3,1}}, \quad a_{5,2} = \frac{a_{1,1}}{a_{3,1}}, \\ a_{3,1} &= [a_{1,1} a_{2,2} - a_{1,2}^2]. \end{aligned}$$

Case (b) figure 4.6

When a force and a moment are applied at the (y_3, ϕ_2) end of the beam, in the negative y_3 direction and positive ϕ_2 direction, respectively, the equations for the translational and

rotational deflections were determined to be

$$y_3 = x_3 - \frac{F_b L^3}{3EI} - \frac{M_b L^2}{2EI} - \theta_3 L, \quad (4.22a)$$

$$\phi_2 = \theta_2 + \frac{F_b L^2}{2EI} + \frac{M_b L}{EI}. \quad (4.22b)$$

These equations were solved simultaneously for F_b and M_b as shown in matrix form in the following equation

$$\begin{Bmatrix} F_b \\ M_b \end{Bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} -y_3 + x_3 - \theta_2 L \\ \phi_2 - \theta_2 \end{bmatrix}. \quad (4.23)$$

The equations for F_b and M_b as functions of y_3 and ϕ_2 are then

$$F_b(y_3, \phi_2) = -a_{41} [y_3 - x_3 + \theta_2 L] - a_{42} [\phi_2 - \theta_2], \quad (4.24a)$$

$$M_b(y_3, \phi_2) = a_{51} [y_3 - x_3 + \theta_2 L] + a_{52} [\phi_2 - \theta_2]. \quad (4.24b)$$

The equations for F_a , M_a and F_b , M_b can then be put into equation 4.16 to determine the potential energy for the two cases of bending as a function of y and ϕ .

Case (a)

$$U_a(y_2, \phi_3) = \frac{[F_a(y_2, \phi_3)]^2 L^3}{6EI} + \frac{[F_a(y_2, \phi_3)][M_a(y_2, \phi_3)] L^2}{2EI} + \frac{[M_a(y_2, \phi_3)]^2 L}{2EI}. \quad (4.25)$$

Case (b)

$$U_b(y_3, \phi_2) = \frac{[F_b(y_3, \phi_2)]^2 L^3}{6EI} + \frac{[F_b(y_3, \phi_2)][M_b(y_3, \phi_2)] L^2}{2EI} + \frac{[M_b(y_3, \phi_2)]^2 L}{2EI}. \quad (4.26)$$

Axial and Torsional Potential Energy Equations

The axial potential equation and torsional potential equations are more straight forward than the case of bending. The axial potential equation is given by

$$U_{axial}(y_1) = \frac{1}{2}k_{axial}(y_1 - x_1)^2, \quad (4.27)$$

where

$$k_{axial} = \frac{EA}{L}, \quad (4.28)$$

where A is the cross sectional area of the beam. The torsional potential equation is given by

$$U_{torsional}(\phi_1) = \frac{1}{2}k_{torsional}(\phi_1 - \theta_1)^2, \quad (4.29)$$

where

$$k_{torsional} = \frac{GJ}{L}, \quad (4.30)$$

where G is the shear modulus and J is the area moment of inertia.

Potential Energy for the Sensor Platform Model

The potential energy of the model is found by summing the previously determined potential energy equations for the cases of bending, axial, and torsional loadings. The potential energy is given by

$$U = U_{axial}(y_1) + U_{torsional}(\phi_1) + U_a(y_2, \phi_3) + U_b(y_3, \phi_2). \quad (4.31)$$

Kinetic Energy

The total kinetic energy of the system is determined by the inertia and total velocity of the point of the mass center of the concentrated mass. The kinetic energy of the system has components due to translation and rotation. These components are shown in equation

4.32, where the first summation is the translational component and the second summation is the rotational component.

$$T = \frac{1}{2}m \sum (\dot{y}_i + \dot{x}_i)^2 + \frac{1}{2}I_i \sum (\dot{\phi}_i + \dot{\theta}_i)^2, \quad \text{where } (i = 1, 2, 3). \quad (4.32)$$

The inertia terms shown in the second summation are about the principal axes of the concentrated mass. In equation 4.32, the quantity of mass (m) connected to the free end of the beam remains the same for all (i) and therefore is not given a subscript. Although, the three inertia terms associated with the rotational components are quite different. These terms are determined based on the distribution of mass in the concentrated mass. The geometry chosen for the concentrated mass was a solid rectangular parallelepiped with dimension given as

a = dimension of the mass in the y_2 direction

b = dimension of the mass in the y_1 direction

c = dimension of the mass in the y_3 direction

where the directions were shown in figure 4.1. For the chosen geometry the inertia terms are given by the following equations.

$$I_1 = J = I_2 + I_3$$

Mass rotary moment of inertia about the y_1 axis

$$I_2 = \frac{1}{12}m(b^2 + c^2)$$

Mass moment of inertia about the y_2 direction

$$I_3 = \frac{1}{12}m(a^2 + b^2)$$

Mass moment of inertia about the y_3 direction

4.2 Equations of Motion

Determining the equations of motion for a system with an energy method relies on the concept of virtual work. Virtual work is based on changes in the energy state of the system resulting from virtual displacements experienced by the system. Lagrange's method is based on this concept and is used to determine the platform model's equations of motion. Lagrange's method is based on the equation

$$\frac{d}{dt} \left[\frac{\partial T}{\partial \dot{q}_i} \right] - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = \vec{Q}_i, \quad (4.35)$$

where the q_i 's are the generalized coordinates, or the coordinates required to describe the motion of the system. The generalized coordinates for this system are given by

$$\vec{q} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}. \quad (4.36)$$

$\vec{Q}_i$ represents nonconservative, generalized forces. If the equations of motion are derived based on the deflections of the free end of the beam with respect to the base of the platform and if it is assumed that there is no damping in the system, $\vec{Q}$ is the zero vector. The deflection of the beam was determined with respect to the base of the platform and since damping was included in this system the nonconservative, generalized forces represent energy dissipation terms associated with the damping.

Applying equation 4.35 to the kinetic energy of the system, as given in equation 4.32, resulted in

$$\frac{d}{dt} \left[\frac{\partial T}{\partial \dot{q}_i} \right] = \begin{Bmatrix} m(\ddot{y}_1 + \ddot{x}_1) \\ m(\ddot{y}_2 + \ddot{x}_2) \\ m(\ddot{y}_3 + \ddot{x}_3) \\ J(\ddot{\phi}_1 + \ddot{\theta}_1) \\ I_2(\ddot{\phi}_2 + \ddot{\theta}_2) \\ I_3(\ddot{\phi}_3 + \ddot{\theta}_3) \end{Bmatrix}, \quad (4.37)$$

$$\left[\frac{\partial T}{\partial q_i} \right] = \{0\}. \quad (4.38)$$

For the potential energy, applying equation 4.35 to equation 4.31 resulted in

$$\left[\frac{\partial U}{\partial q_i} \right] = \begin{Bmatrix} k_{axial}(y_1 - x_1), \\ k_1(y_2 - x_2) + k_2(\phi_3 - \theta_3) + k_3\theta_3L \\ k_4(y_3 - x_3) + k_5(\phi_2 - \theta_2) + k_6\theta_2L \\ k_{torsional}(x_1 - y_1) \\ k_7(x_3 - y_3) + k_8(\phi_2 - \theta_2) + k_9\theta_2L \\ k_{10}(y_2 - x_2) + k_{11}(\phi_3 - \theta_3) + k_{12}\theta_3L \end{Bmatrix}, \quad (4.39)$$

where L is the length of the beam and the stiffness coefficients k_j ($j = 1, 2, \dots, 12$) are given as

$$\begin{aligned} k_1 &= [2a_{41}^2a_{61} - 2a_{41}a_{51}a_{62} + 2a_{51}^2a_{71}] \\ k_2 &= [-2a_{41}a_{42}a_{61} + a_{41}a_{52}a_{62} + a_{42}a_{51}a_{62} - 2a_{51}a_{52}a_{71}] \\ k_3 &= [-2a_{41}^2a_{62} + 2a_{41}a_{51}a_{62} - 2a_{51}^2a_{71}] \end{aligned}$$

$$\begin{aligned}
k_4 &= k_1 \\
k_5 &= -k_2 \\
k_6 &= \left[2a_{41}^2 a_{61} - a_{41} a_{51} a_{62} - a_{51} a_{41} a_{62} + 2a_{51}^2 a_{71} \right] \\
k_7 &= -k_2 \\
k_8 &= \left[2a_{42}^2 a_{61} - 2a_{42} a_{52} a_{62} + 2a_{52}^2 a_{71} \right] \\
k_9 &= -k_2 \\
k_{10} &= k_2 \\
k_{11} &= k_8 \\
k_{12} &= -k_2
\end{aligned}$$

where

$$a_{6,1} = \frac{L^3}{6EI}, \quad a_{6,2} = \frac{L^2}{2EI}, \quad a_{7,1} = \frac{L}{2EI}. \quad (4.41)$$

It should be noted that in the equations for ($k_1 \rightarrow k_{12}$) that the negative sign in some of the results is an indication of direction only. The actual stiffness is the absolute value.

The generalized forces, $\vec{Q}_i$, shown in equation 4.35 result from including damping in the system. The damping for this system was assumed to be modal. Since the damping for this system was assumed to be small, it was approximated as if the system was composed of six second order, single degree of freedom equations subjected to harmonic excitations. For this case the damping term is a function of the damping ratio, ζ , given by the equation

$$\frac{c_{eq}}{Inertia} = 2\zeta\omega_n = 2\zeta\sqrt{\frac{k}{Inertia}}, \quad (4.42)$$

where k is the stiffness associated with the material properties and direction of beam deflection, and $Inertia$ is the associated inertia term. The damping coefficients for this system were evaluated as

$$\frac{c_{axial}}{m} = \text{sign}(k_{axial}) 2\zeta \sqrt{\frac{|k_{axial}|}{m}} \quad (4.43a)$$

$$\frac{c_1}{m} = \text{sign}(k_1) 2\zeta \sqrt{\frac{|k_1|}{m}} \quad (4.43b)$$

$$\frac{c_4}{m} = \text{sign}(k_4) 2\zeta \sqrt{\frac{|k_4|}{m}} \quad (4.43c)$$

$$\frac{c_{torsion}}{J} = \text{sign}(k_{torsion}) 2\zeta \sqrt{\frac{|k_{torsion}|}{m}} \quad (4.43d)$$

$$\frac{c_8}{I_2} = \text{sign}(k_8) 2\zeta \sqrt{\frac{|k_8|}{I_2}} \quad (4.43e)$$

$$\frac{c_9}{I_2} = \text{sign}(k_8 - k_9) 2\zeta \sqrt{\frac{|k_8 - k_9|}{I_2}} \quad (4.43f)$$

$$\frac{c_{11}}{I_3} = \text{sign}(k_{11}) 2\zeta \sqrt{\frac{|k_{11}|}{I_3}} \quad (4.43g)$$

$$\frac{c_{12}}{I_3} = \text{sign}(k_{11} - k_{12}) 2\zeta \sqrt{\frac{|k_{11} - k_{12}|}{I_3}} \quad (4.43h)$$

Based on these damping terms, the generalized force vector is given by

$$\vec{Q} = \left\{ \begin{array}{c} -\frac{c_{axial}}{m} \dot{y}_1 + \frac{c_{axial}}{m} \dot{x}_1 \\ -\frac{c_1}{m} \dot{y}_2 + \frac{c_1}{m} \dot{x}_2 \\ -\frac{c_4}{m} \dot{y}_3 + \frac{c_4}{m} \dot{x}_3 \\ -\frac{c_{torsion}}{J} \dot{\phi}_1 + \frac{c_{torsion}}{J} \dot{\theta}_1 \\ -\frac{c_8}{I_2} \dot{\phi}_2 + \frac{c_9}{I_2} \dot{\theta}_2 \\ -\frac{c_{11}}{I_3} \dot{\phi}_3 + \frac{c_{12}}{I_3} \dot{\theta}_3 \end{array} \right\}. \quad (4.44)$$

Dynamic Equations of Motion

The results from equations 4.44, 4.37, 4.38, and 4.39 led to the following equations of motion for the system.

$$\ddot{y}_1 + \frac{c_{axial}}{m} \dot{y}_1 + \frac{k_{axial}}{m} y_1 = \left[\frac{k_{axial}}{m} + \omega_1^2 \right] x_1 + \frac{c_{axial}}{m} \dot{x}_1 \quad (4.45a)$$

$$\ddot{y}_2 + \frac{c_1}{m}\dot{y}_2 + \frac{k_1}{m}y_2 + \frac{k_2}{m}\phi_3 = \left[\frac{k_1}{m} + \omega_2^2 \right] x_2 + \frac{c_1}{m}\dot{x}_2 + \left(\frac{k_2}{m} - \frac{k_3}{m}L \right) \theta_3 \quad (4.45b)$$

$$\ddot{y}_3 + \frac{c_4}{m}\dot{y}_3 + \frac{k_4}{m}y_3 + \frac{k_5}{m}\phi_2 = \left[\frac{k_4}{m} + \omega_3^2 \right] x_3 + \frac{c_4}{m}\dot{x}_3 + \left(\frac{k_5}{m} - \frac{k_6}{m}L \right) \theta_2 \quad (4.45c)$$

$$\ddot{\phi}_1 + \frac{c_{torsion}}{J}\dot{\phi}_1 + \frac{k_{torsion}}{J}\phi_1 = \left[\frac{k_{torsion}}{J} + \omega_4^2 \right] \theta_1 + \frac{c_{torsion}}{J}\dot{\theta}_1 \quad (4.45d)$$

$$\ddot{\phi}_2 + \frac{c_8}{I_2}\dot{\phi}_2 + \frac{k_8}{I_2}\phi_2 + \frac{k_7}{I_2}y_3 = \left[\frac{k_8}{I_2} - \frac{k_9}{I_2}L + \omega_5^2 \right] \theta_2 + \frac{c_9}{I_2}\dot{\theta}_2 + \frac{k_7}{I_2}x_3 \quad (4.45e)$$

$$\ddot{\phi}_3 + \frac{c_{11}}{I_3}\dot{\phi}_3 + \frac{k_{11}}{I_3}\phi_3 + \frac{k_{10}}{I_3}y_2 = \left[\frac{k_{11}}{I_3} - \frac{k_{12}}{I_3}L + \omega_6^2 \right] \theta_3 + \frac{c_{12}}{I_3}\dot{\theta}_3 + \frac{k_{10}}{I_3}x_2 \quad (4.45f)$$

For the purpose of analysis these equations of motion were converted to state space in the form

$$\dot{\vec{z}} = [A]\{z\} + [B]\{u\} \quad (4.46a)$$

$$\vec{y} = [C]\{z\} + [D]\{u\}, \quad (4.46b)$$

where $\vec{z}$ is the state vector which includes the generalized coordinates and their time derivatives and $\vec{u}$ is the input vector. The matrix $\mathbf{A}$ in state form is

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-k_{axial}}{m} & \frac{-c_1}{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-k_1}{m} & \frac{-c_2}{m} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-k_2}{m} & \frac{-c_3}{m} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-k_4}{m} & \frac{-c_4}{m} & 0 & 0 & \frac{-k_5}{m} & \frac{-c_5}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-k_{torsion}}{m} & \frac{-c_6}{m} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-k_7}{m} & \frac{-c_7}{m} & 0 & 0 & \frac{-k_8}{m} & \frac{-c_8}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-k_{10}}{m} & \frac{-c_9}{m} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-k_{11}}{m} & \frac{-c_{10}}{m} \end{bmatrix}^{12 \times 12} \quad (4.47)$$

The matrix $\mathbf{B}$ in state form is

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{2,1} & 0 & 0 & 0 & 0 & 0 & b_{2,7} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{4,2} & 0 & 0 & 0 & b_{4,6} & 0 & b_{4,8} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{6,3} & 0 & b_{6,5} & 0 & 0 & 0 & b_{6,9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{8,4} & 0 & 0 & 0 & 0 & 0 & b_{8,10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{10,3} & 0 & b_{10,5} & 0 & 0 & 0 & 0 & 0 & b_{10,11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{12,2} & 0 & 0 & 0 & b_{12,6} & 0 & 0 & 0 & 0 & 0 & b_{12,12} \end{bmatrix}^{12 \times 12} \quad (4.48)$$

where

$$b_{2,1} = \frac{k_{axial}}{m} + \omega_1^2, \quad b_{2,7} = \frac{c_1}{m} \quad (4.49)$$

$$b_{4,2} = \frac{k_1}{m} + \omega_2^2, \quad b_{4,6} = \frac{k_2}{m} - \frac{k_3}{m}, \quad b_{4,8} = \frac{c_2}{m} \quad (4.50)$$

$$b_{6,3} = \frac{k_4}{m} + \omega_3^2, \quad b_{6,5} = \frac{k_5}{m} - \frac{k_6}{m}, \quad b_{6,9} = \frac{c_4}{m}, \quad (4.51)$$

$$b_{8,4} = \frac{k_{torsional}}{m} + \omega_4^2, \quad b_{8,10} = \frac{c_6}{J} \quad (4.52)$$

$$b_{10,3} = \frac{k_7}{I_2}, \quad b_{10,5} = \frac{k_8}{m} - \frac{k_9}{I_2} + \omega_5^2, \quad b_{10,11} = \frac{c_8}{I_2} \quad (4.53)$$

$$b_{12,2} = \frac{k_{10}}{I_3}, \quad b_{12,6} = \frac{k_{11}}{I_3} - \frac{k_{12}}{I_3} + \omega_1^2, \quad b_{12,12} = \frac{c_{10}}{I_3}. \quad (4.54)$$

The matrix $\mathbf{C}$ in state form is

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^{6 \times 12} \quad (4.55)$$

and the matrix $\mathbf{D}$ is the zero matrix

$$\mathbf{D} = \begin{bmatrix} 0 \end{bmatrix}^{12 \times 12}. \quad (4.56)$$

The harmonic inputs used to excite the system are of the form

$$\delta = |\delta| \sin(\omega t), \quad (4.57)$$

where δ can either indicate the translational deflection (x_1 , x_2 , or x_3) or the rotational

deflection (θ_1 , θ_2 , or θ_3) and the frequency term ω is the frequency driving the system and can be ω_1 , ω_2 , ω_3 , ω_4 , ω_5 , or ω_6 . If damping was not included in the system then the base excitations would be of the form shown in equation 4.57. Since damping was included in the system, additional excitation terms have to be included. These terms are associated with the velocity of the harmonic inputs given as

$$\dot{\delta} = \omega|\delta|\cos(\omega t). \quad (4.58)$$

From equations 4.57 and 4.58 the harmonic inputs can then be shown in vector form as

$$\vec{u} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix} = \begin{Bmatrix} X_1 \cdot \sin(\omega_1 \cdot t) \\ X_2 \cdot \sin(\omega_2 \cdot t) \\ X_3 \cdot \sin(\omega_3 \cdot t) \\ \Theta_1 \sin(\omega_4 \cdot t) \\ \Theta_2 \sin(\omega_5 \cdot t) \\ \Theta_3 \sin(\omega_6 \cdot t) \\ \omega_1 X_1 \cos(\omega_1 \cdot t) \\ \omega_2 X_2 \cos(\omega_2 \cdot t) \\ \omega_3 X_3 \cos(\omega_3 \cdot t) \\ \omega_4 \Theta_1 \cos(\omega_4 \cdot t) \\ \omega_5 \Theta_2 \cos(\omega_5 \cdot t) \\ \omega_6 \Theta_3 \cos(\omega_6 \cdot t) \end{Bmatrix}. \quad (4.59)$$

Chapter 5

Platform Pedestal Analysis

Approach

The purpose for developing the platform pedestal model is to simulate the response characteristics of a physical platform. Reference [3] goes into detail concerning response characteristics for various systems in time and frequency domains. Information in these domains reveals platform model response characteristics. The time and frequency domain analysis is then compared to amplitude spectrum plots constructed from shock and vibration data obtained from test runs conducted on the NGUV to make predictions on camera pointing performance. The computer software that was used in this analysis was MATLAB¹. The correspondence between an actual UGV and sensor platform system and the model analysis is shown in figure 5.1.

The system's plant, or equations of motion, was modeled as a linear time invariant multi-input, multi-output (MIMO) system. When MATLABTM determines response characteristics in the time and frequency domain for a MIMO system, it separates the system into single input, single output (SISO) open loop transfer functions based on the state space

¹TM MATLAB is a registered trademark of MathWorks, Inc.

Physical System / Platform Model Correspondence

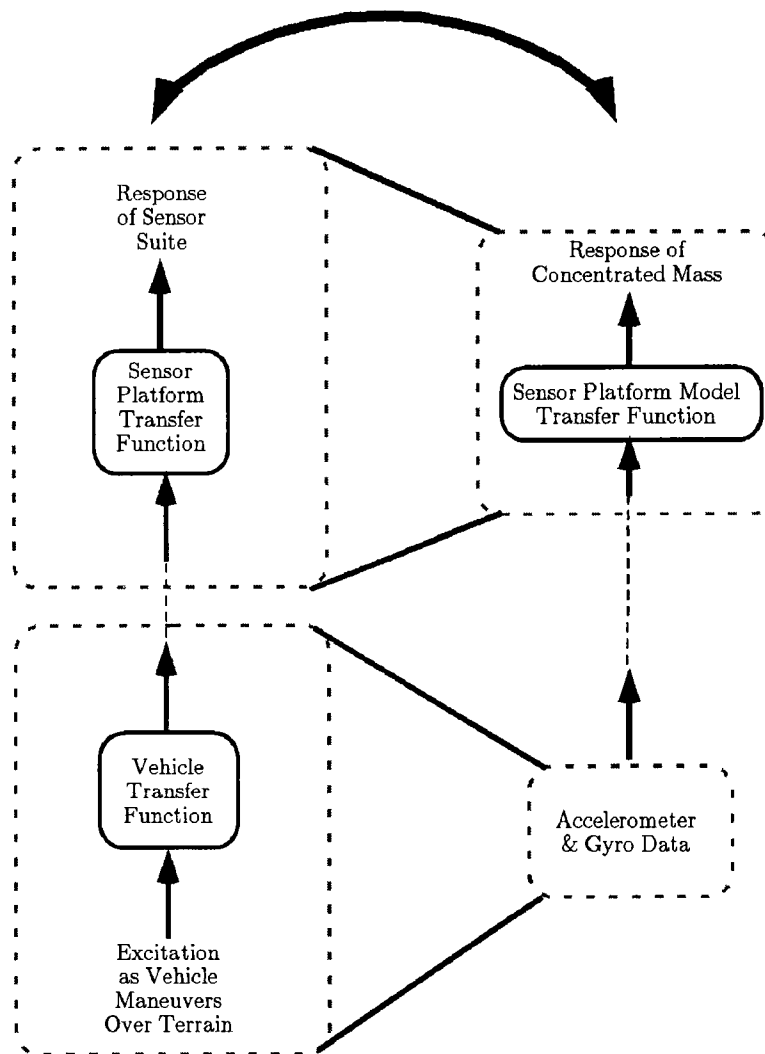


Figure 5.1: Physical UGV and sensor platform / Sensor platform model analysis.

Table 5.1: Single Input, Single Output open loop transfer functions G_i for various input/output relationships.

G_i	Input at Base of Platform	Response of Concentrated Mass
1	Translation in Vertical Direction	Translation in Vertical Direction
2	Translation in Transverse Direction	Translation in Transverse Direction
3	Translation in Longitudinal Direction	Translation in Longitudinal Direction
4	Rotation about Vertical Direction	Rotation about Vertical Direction
5	Rotation about Transverse Direction	Rotation about Transverse Direction
6	Rotation about Longitudinal Direction	Rotation about Longitudinal Direction
7	Translation in Transverse Direction	Rotation about Longitudinal Direction
8	Translation in Longitudinal Direction	Rotation about Transverse Direction
9	Rotation about Longitudinal Direction	Translation in Transverse Direction
10	Rotation about Transverse Direction	Translation in Longitudinal Direction

matrices **A**, **B**, **C**, and **D** derived in section 4.2. It should be noted that since the SISO transfer functions represent input/output open loop transfer functions, the matrices **B** and **D** reduce in size. Columns 7 through 12 of matrix **B** can be discarded since they resulted from assuming that the inputs for the system were of a harmonic nature which introduced the six cosine terms in the input vector shown in equation 4.59. The **D** matrix becomes a 6×6 zero matrix. These SISO transfer functions, G_i ($i = 1, 2, \dots, 10$), correspond to the input/output relationships shown in table 5.1. The direction shown in this table correspond to figure 4.1 and are stated as follows :

- Vertical is positive in the up direction.
- As viewed from the rear of the vehicle, transverse is positive in a direction to the right of the vehicle.
- Longitudinal is positive in the vehicle's forward direction.

It should be noted that there are ten SISO transfer functions of concern since the equations of motion derived for this platform include two (1 DOF) equations and four (2 DOF) equations.

The analysis approach for evaluating the response characteristics of the sensor platform is shown in figure 5.2. As shown in this figure, the platform model is analyzed in the time and frequency domains in order to evaluate sensor platform response characteristics. The frequency domain analysis gives information concerning the natural frequencies of the SISO transfer functions. A simulation was constructed for the time domain analysis that evaluates the response characteristics of the platform when excited by an “impulse” function. This “impulse” function was modeled as a change in translational or rotational position over a finite period of time. This allows information to be obtained concerning the maximum displacement and settling time of the concentrated mass corresponding to the amplitude and duration of the impulse function.

The platform model was then excited with harmonic inputs corresponding to accelerometer and gyro measurement data obtained from test runs conducted on the NGUV. This measurement data was analyzed to determine amplitude and frequency information needed to construct the harmonic inputs. The response characteristics of the sensor platform, when excited with these inputs, were then compared to the frequency and time domain analysis in order to make predictions on sensor pointing performance.

Nominal Plant

A nominal plant was chosen for the platform model. A list of the parameters chosen for the nominal plant are shown in table 5.2. The parameters of the nominal plant were chosen to correspond to parameters that an actual sensor platform might possess. The beam length constraint, as given in equation 4.11, requires that the length of the beam be more than thirty five inches based on the nominal values chosen for the inner and outer diameters of the beam. The damping ratio was chosen to be fairly small based on the assumption that the structural damping of the platform and aerodynamic damping effects are relatively small.

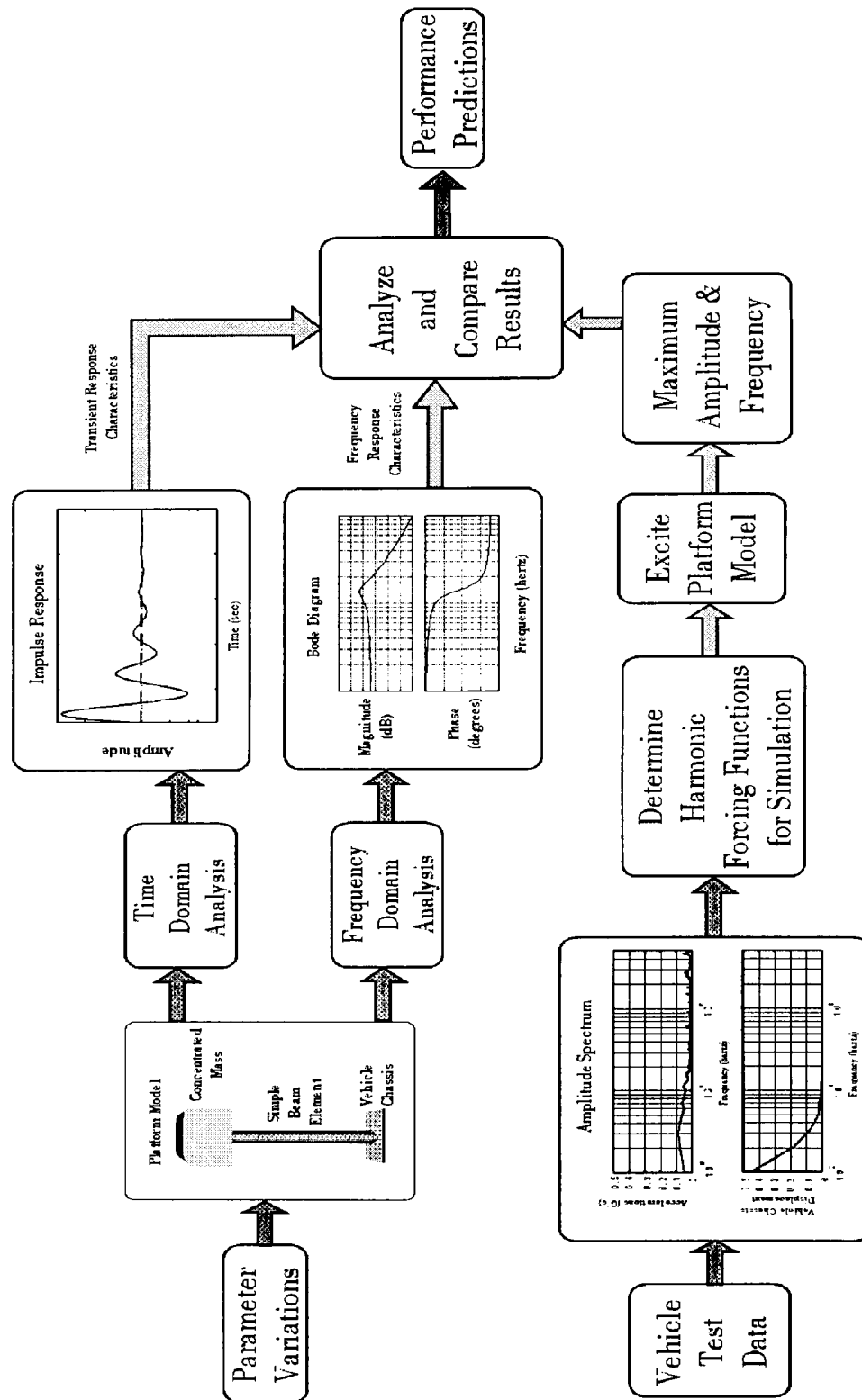


Figure 5.2: Sensor platform analysis concept

Table 5.2: Nominal Plant Setup.

Mast Height	5 feet
Concentrated Mass	100 lbs
Beam Material	Aluminum
Beam Outer Diameter	6.0 inch
Beam Thickness	$\frac{1}{8}$ inch
Damping Ratio (ζ)	0.1

5.1 Frequency Analysis

5.2 Nominal Plant Frequency Domain Response Characteristics

Information can be obtained for the sensor platform model in the frequency domain by looking at the Bode plots of the system's SISO open loop transfer functions. These plots are shown in figures 5.3 through 5.12. The Bode plots for the SISO open loop transfer functions show the natural frequencies of the system. In order to determine how sensitive the model is to various parameters (e.g. mast height, weight of concentrated mass) it is useful to observe the response characteristics of the system in the frequency domain based on parameter variations. Parameter variations will shift the system's natural frequencies. This information quantifies frequency bands that restrict the performance of sensor/camera pointing.

From the Bode plots, the system's natural frequencies were determined for each SISO transfer function as shown in table 5.3. It should be noted that a majority of the bode plots show two peaks in the response curves. This is due to the coupling between translation and rotation since the beam is being modeled as a second-order lumped parameter system. It can be seen in figures 5.4 and 5.5 that the response curves are very similar. The difference in these plots is due solely to the choice of the geometry of the concentrated mass attached

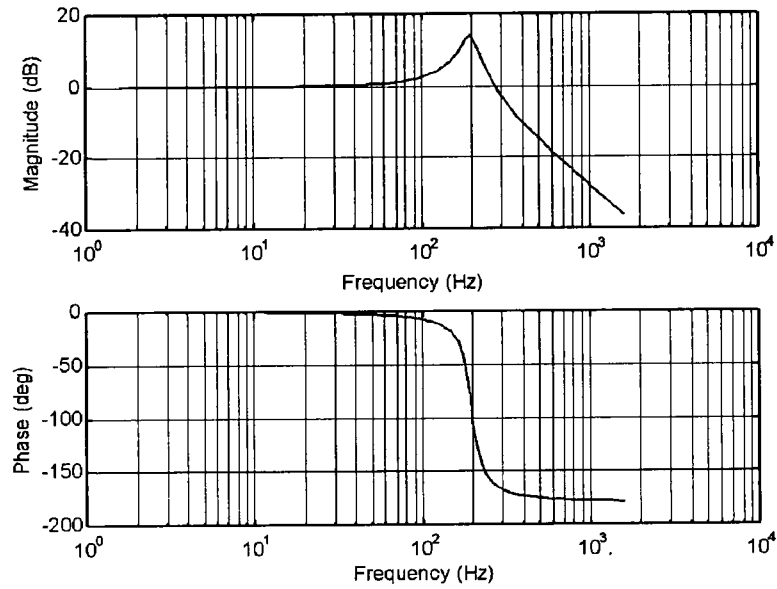


Figure 5.3: Bode plot for SISO open loop transfer function G_1 corresponding to translational input in the vertical direction, translational output in the vertical direction.

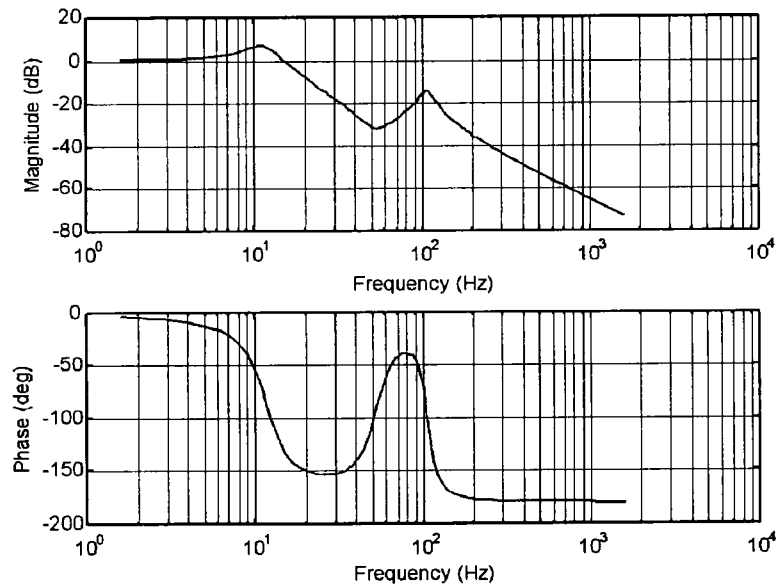


Figure 5.4: Bode plot for SISO open loop transfer function G_2 corresponding to translational input in the transverse direction, translational output in the transverse direction.

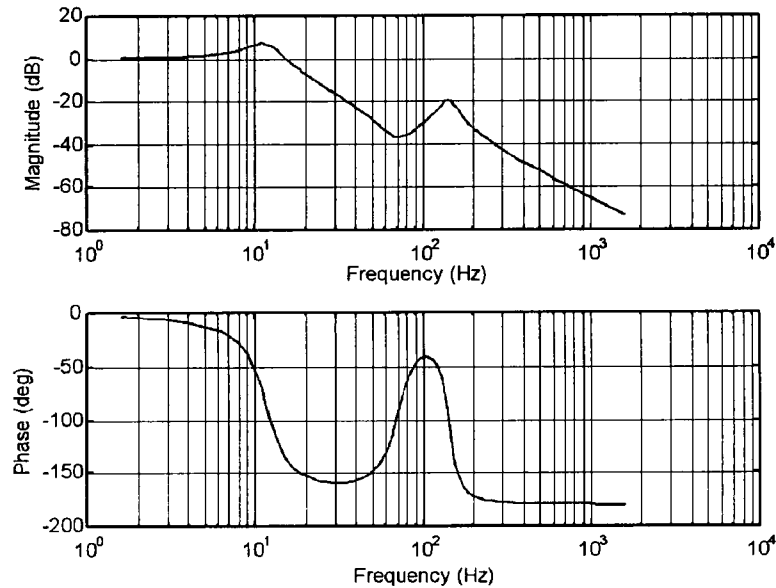


Figure 5.5: Bode plot for SISO open loop transfer function G_3 corresponding to translational input in the longitudinal direction, translational output in the longitudinal direction.

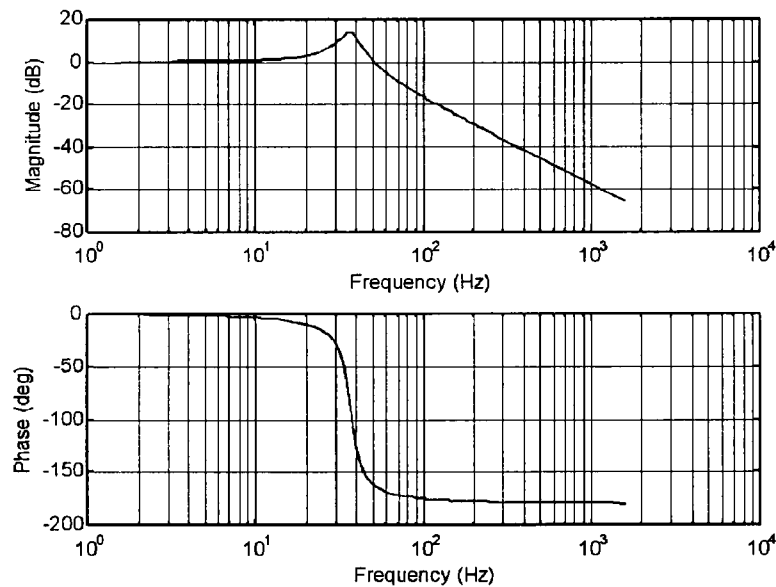


Figure 5.6: Bode plot for SISO open loop transfer function G_4 corresponding to rotational input about the vertical direction, rotational output about the vertical direction.

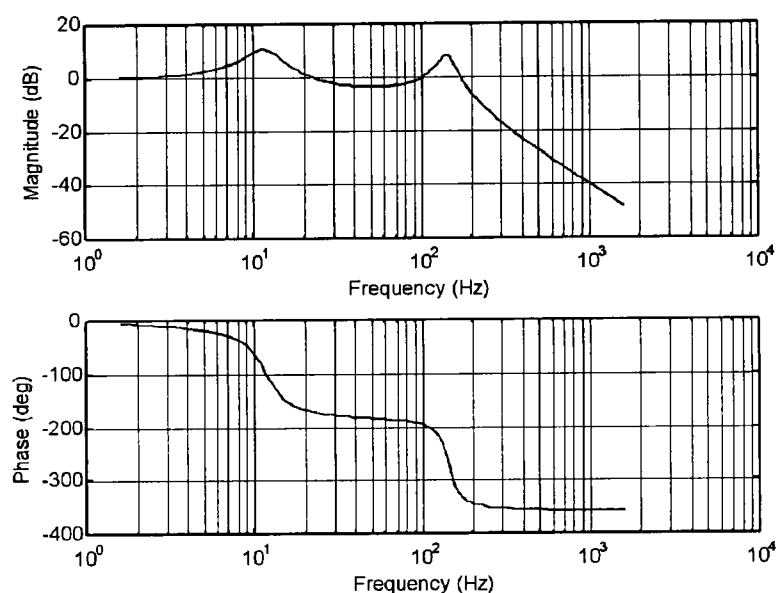


Figure 5.7: Bode plot for SISO open loop transfer function G_5 corresponding to rotational input about the transverse direction, rotational output about the transverse direction.

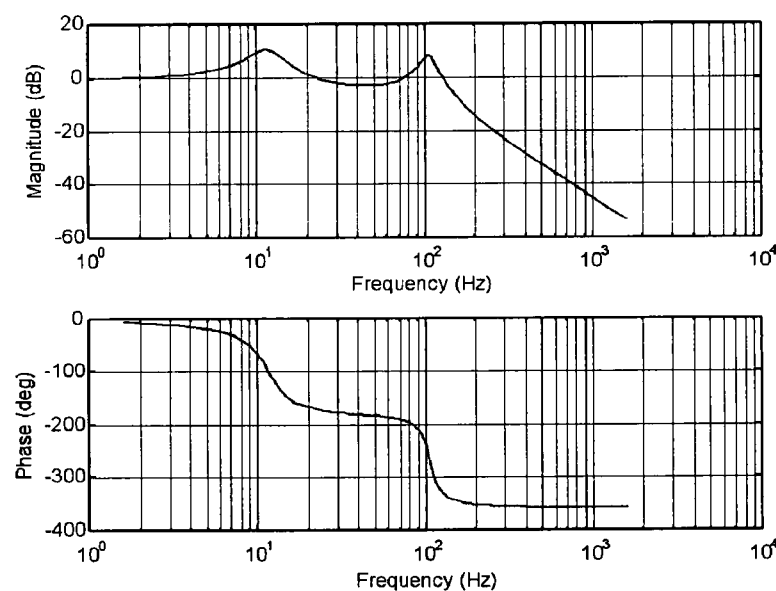


Figure 5.8: Bode plot for SISO open loop transfer function G_6 corresponding to rotational input about the longitudinal direction, rotational output about the longitudinal direction.

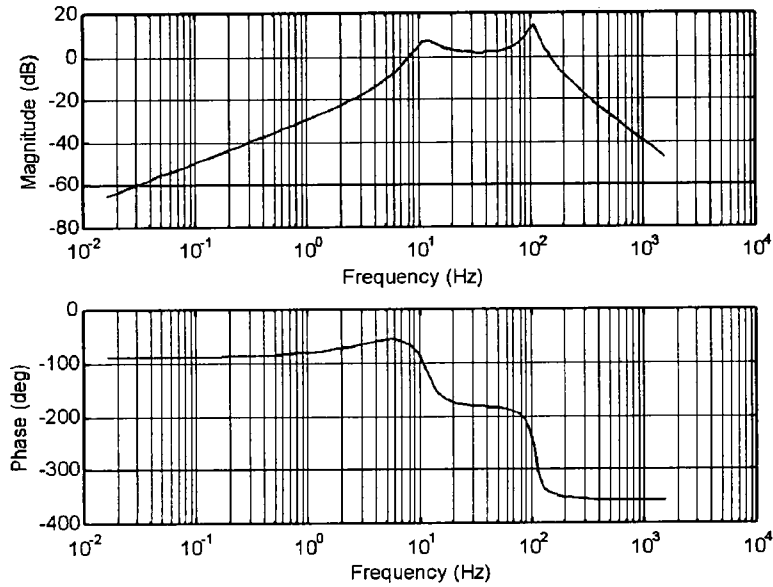


Figure 5.9: Bode plot for SISO open loop transfer function G_7 corresponding to translational input in the transverse direction, rotational output about the longitudinal direction.

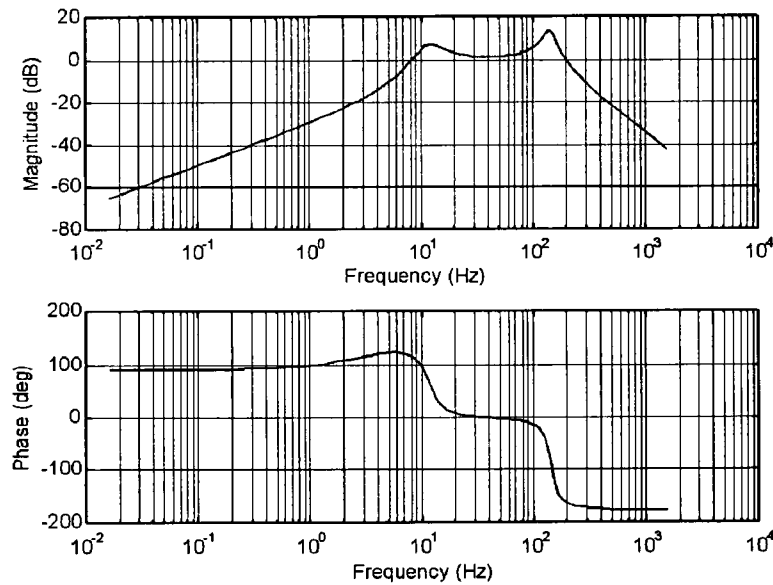


Figure 5.10: Bode plot for SISO open loop transfer function G_8 corresponding to translational input in the longitudinal direction, rotational output about the transverse direction.

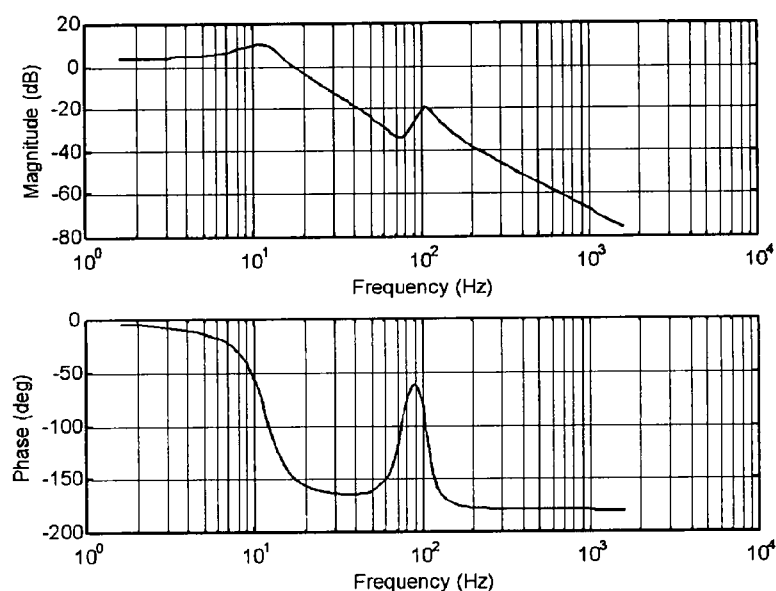


Figure 5.11: Bode plot for SISO open loop transfer function G_9 corresponding to rotational input about the longitudinal direction, translational output in the transverse direction.

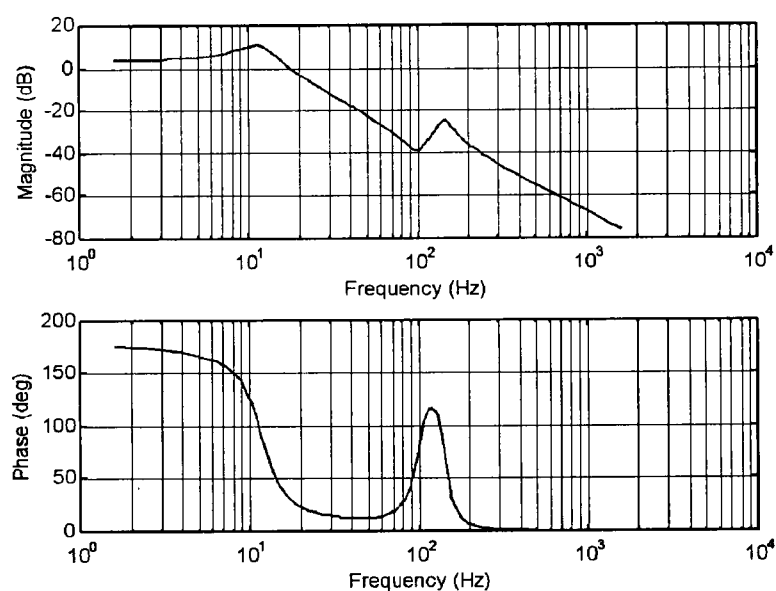


Figure 5.12: Bode plot for SISO open loop transfer function G_{10} corresponding to rotational input about the transverse direction, translational output in the longitudinal direction.

Table 5.3: SISO transfer functions natural frequencies.

SISO Transfer Function (G_i)	ω_{n1} (Hz)	ω_{n2} (Hz)
1	200	N/A
2	12	110
3	12	150
4	37	N/A
5	12	150
6	12	100
7	12	100
8	12	150
9	12	100
10	12	150

to the end of the beam (i.e. the mass moments of inertia I_2 and I_3 are different due to mass geometry). The difference in mass geometry is also evident in the response curves shown in figures 5.7 and 5.8.

The characteristics of the frequency response curves shown in figures 5.9 through 5.12 are not as obvious. The response is attenuated in the low frequency range and increases around resonant frequencies. The peaks in these curves are the natural frequencies of the cross coupling between translation and rotation. The response curves do not originate along the 0 dB axis due to low frequency attenuation, but the natural frequencies for these SISO transfer functions are well defined. It should be noted that the lowest natural frequency present in this nominal plant system is 12 Hertz. This should be considered when evaluating the robustness of the sensor platform and is further discussed in chapter 6.

5.3 Approximate Frequency Response Sensitivity Based on Variations from Nominal Plant Values

Two of the dominant parameters for the sensor platform are the mast height of the beam and the amount of concentrated mass attached to the beam. As either one of these parameters are varied, the characteristics of the response curves for the system will change. These changes in the system's response characteristics can be seen in the frequency domain by observing shifts in the system's natural frequencies. When a parameter is varied, the degree of frequency shift is a measure of the system's sensitivity to that individual parameter. If the frequencies that will be driving the system can be approximated *a priori*, the parameters can be varied so as to maintain a desired range between driving frequencies and system natural frequencies.

Effects of Mast Height Variations

In order to perform the RSTA mission it is desirable for the sensor suite to be at a higher elevation than the vehicle. This will allow the vehicle to conceal itself behind obstacles while performing RSTA. Therefore it is advantageous to determine the model's sensitivity to variations in the mast height. The length of the beam is the dominating parameter of stiffness in the equations of motion since it is usually squared or cubed, see equation 4.18. Figure 5.13 shows the frequency response of the system as the mast height is changed from 3 feet to 5 feet to 8 feet. From this figure it can be seen that the system is quite sensitive to the mast height. As the mast height is increased it can be seen that the system's lowest natural frequencies is reduced from about 25 hertz to about 5.5 hertz.

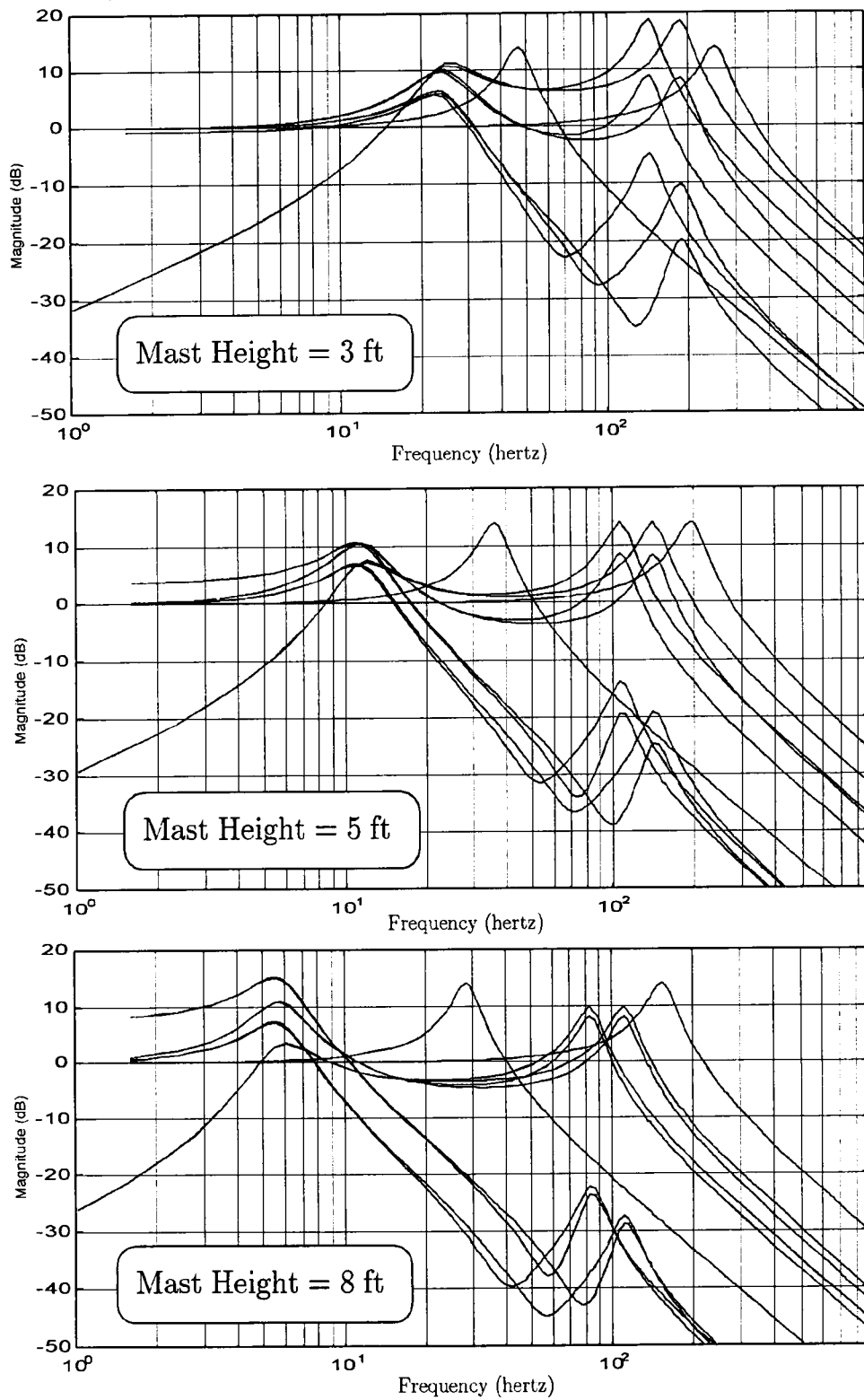


Figure 5.13: Plant Frequency Response for the case where the beam length is 1 ft, 5 ft, and 8 ft.

Effects of Mass Variations

The weight of the pan/tilt head and cameras will also affect the system's frequency response. To capture the frequency response range as a function of mass, the concentrated mass was varied from 50 lbs - 150 lbs. The frequency response plot is shown in figure 5.14. This figure shows that the system's lowest natural frequency was reduced from about 17 hertz to about 9 hertz as the mass was increased.

The natural frequencies from the previous analysis give an indication of the excitation frequencies that will cause the concentrated mass to oscillate with maximum amplitude with respect to the base of the platform. Therefore, if the base of the platform is excited above the lower limits of these natural frequency bands, the actuators used to orient the sensor suite will not only need to "null out" the rotations of the vehicle, they will also need to "null out" rotation of the sensor suite with respect to the base of the platform. When comparing the two figures it can be seen that the system's most sensitive parameter is indeed the mast height.

5.4 Time Domain Analysis of Measurement Data obtained from Test Conducted on the Next Generation Unmanned Ground Vehicle (NGUV)

The equations of motion derived in section 4.2, which include harmonic base excitations as the forcing functions, are used in this time domain analysis. In these equations the input frequencies ω_i ($i = 1, 2, \dots, 6$) and amplitudes X_1 , X_2 , X_3 , Θ_1 , Θ_2 , and Θ_3 were obtained based on the amplitude spectrum plots for the NGUV test run data. This data was in the form of accelerometer and gyro measurements obtained for the NGUV for various terrain conditions. The test runs conducted on the NGUV were over four terrain types at various speeds. A description of the test runs is given in table 5.4. The test runs considered in this

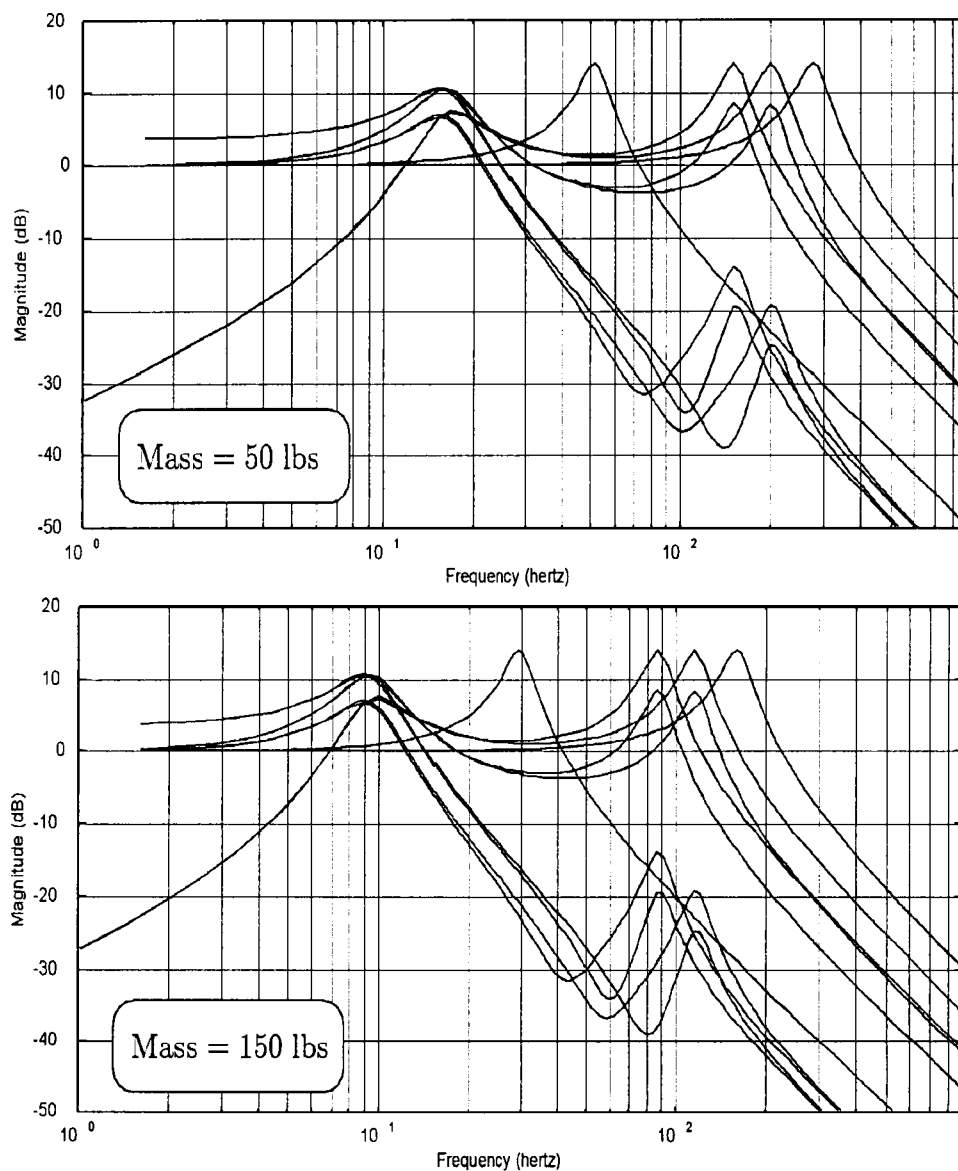


Figure 5.14: Plant Frequency Response for the case where the mass is 50 lbs compared to the case where the mass is 150 lbs.

Table 5.4: Description of test runs.

Terrain Type	Description
3-Inch Spaced Bump	3 inch total height, 3 feet long, 30 ft centers
Belgian Block	2 inch bed of sand and concrete, unevenly laid granite blocks (some straight, some curved)
2-Inch Washboard	Sinusoidal terrain, 2 inch total amplitude, 2 ft wavelength, gentle turns

Table 5.5: Runs considered from the test conducted on the NGUV.

Run No.	Terrain Type	Speed (mph)
4	3-Inch Spaced Bump	12
8	Belgian Block (Turn Section)	12
11	Belgian Block (Straight Section)	13
15	2-Inch Washboard	8

research correspond to cases where the vehicle was run over the terrain at the maximum speed. The test runs considered in this research are shown in table 5.5.

Data from each test run was provided which included data from three perpendicularly oriented accelerometers and three perpendicularly oriented angular rate gyros located on the top center portion of the NGUV. Plots showing the amplitude spectrum for the data obtained from the four test runs are shown in Appendices E, F, G, and H. These amplitude spectrum plots were determined using the Fast Fourier Transform (FFT) method. The FFT method determines a signal's harmonic components, i.e. amplitude and frequency. This technique was appropriate since the data from the test runs was both harmonic and transient, the sampling rate was small ($t_s = 0.00048$ seconds), and the number of measurements taken was on the order of 60,000 for each measurement device. The amplitude spectrum plots for the test data were determined by the following operations.

- Subtract out the DC component of the data

- Section the data into blocks with 2048 elements
- Perform the FFT operation on each sectioned block
- Determine the absolute value of the FFT data
- Determine the average for all sectioned blocks

The computer code for this procedure is found in Appendix D.1.

In each of the amplitude spectrum plots there are two figures. The top figure corresponds to the amplitude spectrum obtained for the raw test data. The raw test data was in units of (g's) for the accelerometers and (deg/second) for the gyro data. Since the FFT method reveals the harmonic contents of a signal, the magnitudes shown in the amplitude spectrum plots could be divided by their corresponding frequencies to obtain velocity and position data from the accelerometer amplitude spectrum and rotational angles from the gyro amplitude spectrum. For example, the amplitude spectrum plot for the vertical accelerometer data, as shown in figure 5.15, was used to determine harmonic inputs that could be used to excite the base of the sensor platform. The upper plot in this figure shows the harmonic components of the accelerometer data. The lower plot was obtained by dividing the amplitudes of the acceleration data by the square of their corresponding frequencies². The lower plot in this figure is then positional information of the vehicle in the vertical direction. Data from the lower plot was then put into the form

$$x_{vertical} = \sum_{i=0}^n X_i \sin(2\pi f_i t), \quad (5.1a)$$

$$\dot{x}_{vertical} = 2\pi \sum_{i=0}^{1024} f_i X_i \cos(2\pi f_i t), \quad (5.1b)$$

where X_i is the magnitude of the data shown in the lower plot and f_i is the frequency

²The frequencies in this figure are in Hertz. The frequencies need to be multiplied by 2π before dividing the data by the frequencies so that the frequency is in units of $\frac{rad}{sec}$.

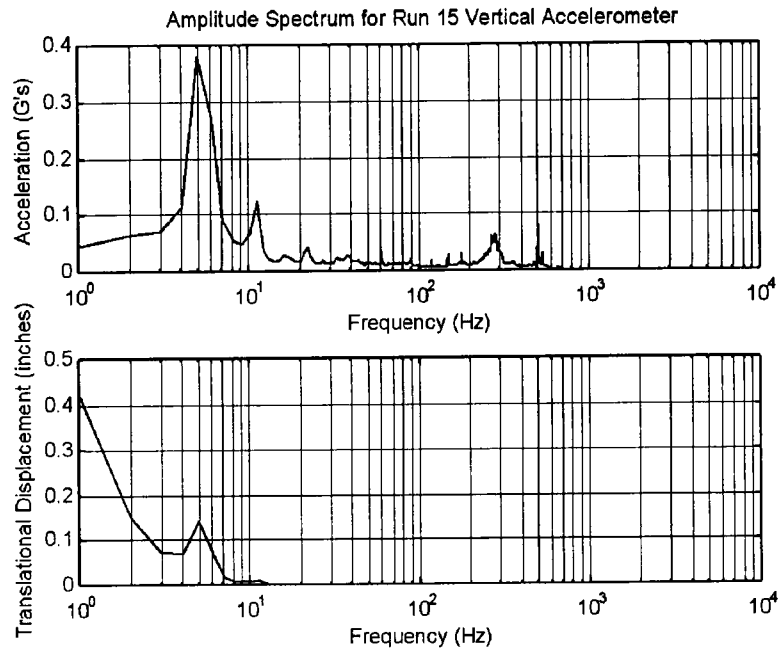


Figure 5.15: Amplitude Plot for Run 15 Vertical Accelerometer.

corresponding to X_i . Equations 5.1a and 5.1b can then be used as the position and velocity harmonic inputs for the simulation, respectively. This procedure was performed on the data for each measurement device. In the case of the gyro amplitude spectrum plots, it was only necessary to divide the amplitudes by their corresponding frequencies, as opposed to the square of the frequencies, since the gyro data was already in the form of rotational rates.

The frequencies that were used to excite the system model, given in equation 4.59, correspond to the frequencies obtained for each measurement device as shown in table 5.6. The data obtained from the amplitude spectrum plots indicates that the dynamics of the vehicle (i.e. the suspension) absorbs a large portion of the excitation experienced by the vehicle when traveling over the various test runs and does not transmit much of the excitation to the base of the platform. The next section shows response curves of the concentrated mass when the system was excited with harmonic inputs obtained from the amplitude spectrum plots for the various test runs. The response curves were obtained by

Table 5.6: Base excitation driving frequency corresponding to measurement device frequency band.

Driving Frequency	Measurement Device
ω_1	Vertical Accelerometer
ω_2	Transverse Accelerometer
ω_3	Longitudinal Accelerometer
ω_4	Yaw Gyro
ω_5	Pitch Gyro
ω_6	Roll Gyro

Table 5.7: Maximum displacement relative to ground obtained from simulation response curves when excited with excitations from test run 4 harmonic inputs.

Direction of Displacement	Maximum Amplitude
Translational Vertical Direction	0.75 inches
Translational Transverse Direction	0.85 inches
Translational Longitudinal Direction	0.25 inches
Rotational Yaw	1.5 degrees
Rotational Pitch	0.5 degrees
Rotational Roll	0.45 degrees

using the command '*lsim*' in MATLABTM.

5.4.1 Beam Deflection based on Test Run Harmonic Excitations

Mass Deflection based on Run 4 Data

Figures 5.16 and 5.17 show the translation and rotational displacements of the concentrated mass when the simulation was excited with harmonic inputs obtained for test run 4. The maximum displacement with respect to ground, as shown in the figures, is given in table 5.7.

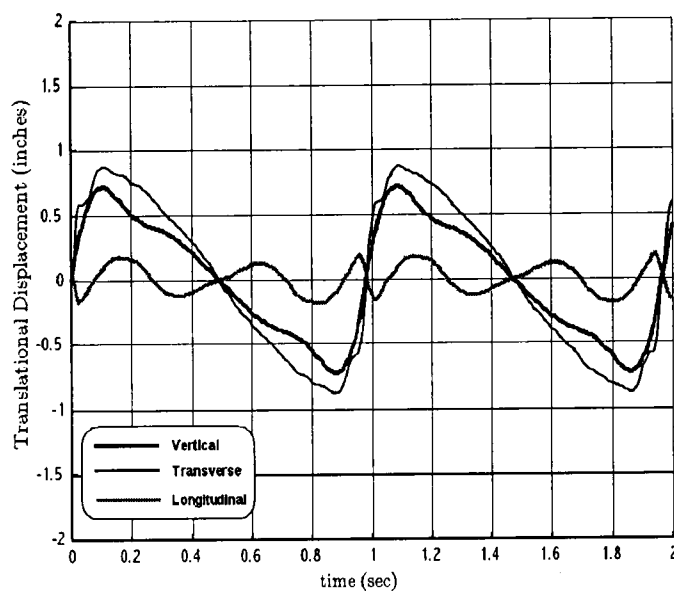


Figure 5.16: Translational displacements of the concentrated mass when excited with harmonic inputs obtained from run 4 data.

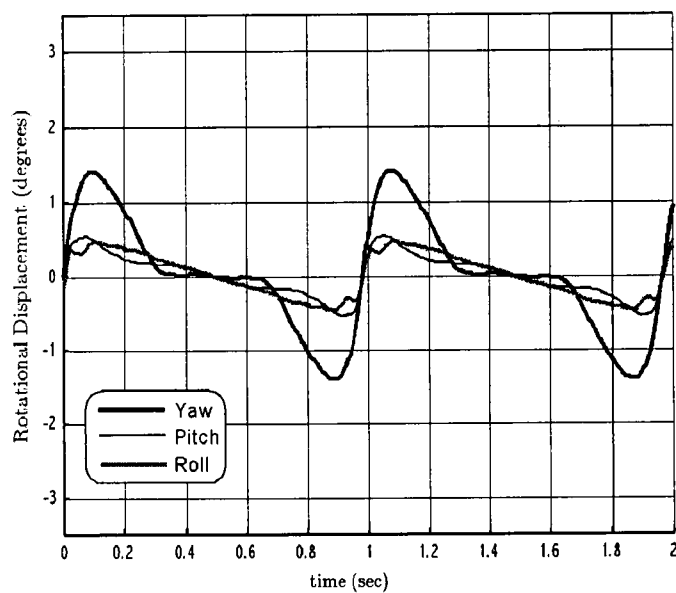


Figure 5.17: Rotational displacements of the concentrated mass when excited with harmonic inputs obtained from run 4 data.

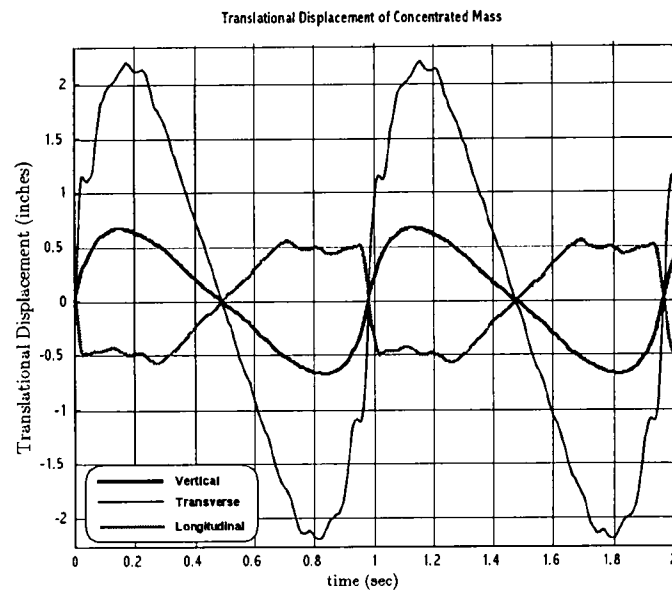


Figure 5.18: Translational displacements of the concentrated mass when excited with harmonic inputs obtained from run 8 data.

Mass Deflection based on Run 8 Data

Figures 5.18 and 5.19 show the translation and rotational displacements of the concentrated mass when the simulation was excited with harmonic inputs obtained for test run 8. The maximum displacement with respect to ground, as shown in the figures, is given in table 5.8.

Mass Deflection based on Run 11 Data

Figures 5.20 and 5.21 show the translation and rotational displacements of the concentrated mass when the simulation was excited with harmonic inputs obtained for test run 11. The maximum displacement with respect to ground, as shown in the figures, is given in table 5.9.

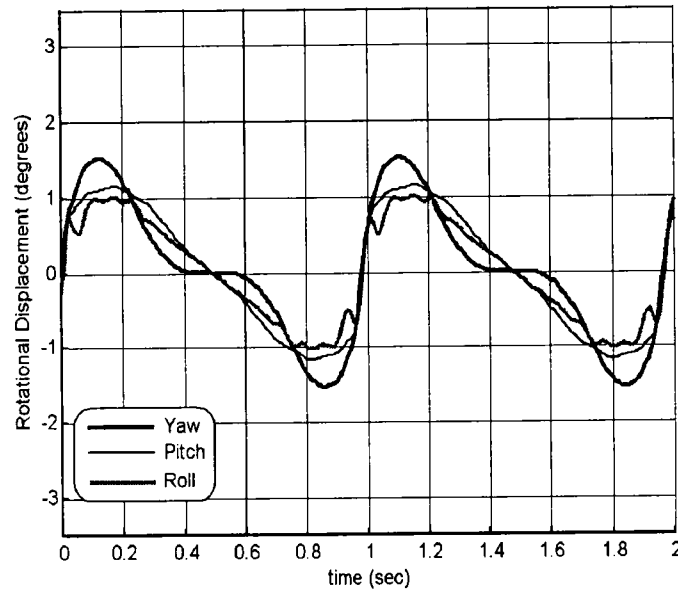


Figure 5.19: Rotational displacements of the concentrated mass when excited with harmonic inputs obtained from run 8 data.

Table 5.8: Maximum displacement relative to ground obtained from simulation response curves when excited with excitations from test run 8 harmonic inputs.

Direction of Displacement	Maximum Amplitude
Translational Vertical Direction	0.7 inches
Translational Transverse Direction	2.2 inches
Translational Longitudinal Direction	0.505 inches
Rotational Yaw	1.5 degrees
Rotational Pitch	1.1 degrees
Rotational Roll	1.0 degrees

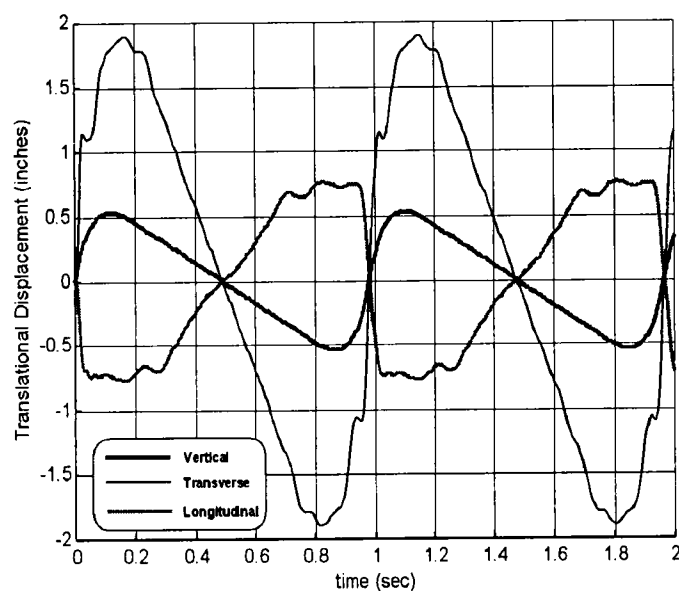


Figure 5.20: Translational displacements of the concentrated mass when excited with harmonic inputs obtained from run 11 data.

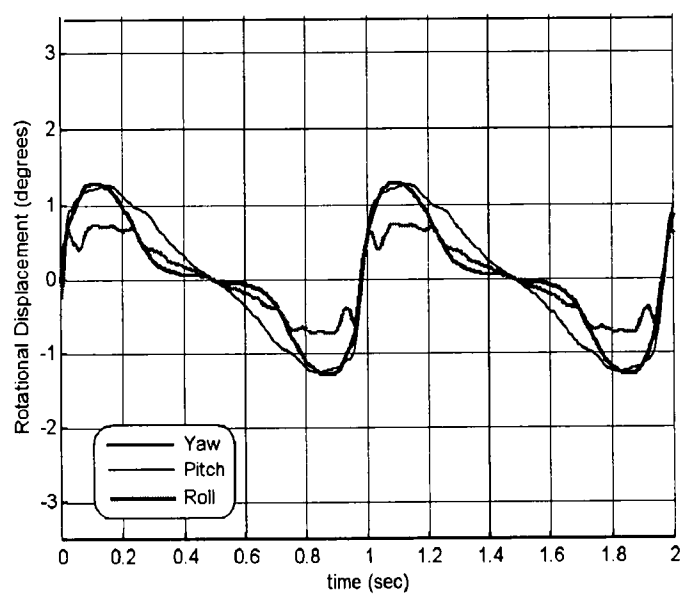


Figure 5.21: Rotational displacements of the concentrated mass when excited with harmonic inputs obtained from run 11 data.

Table 5.9: Maximum displacement relative to ground obtained from simulation response curves when excited with excitations from test run 11 harmonic inputs.

Direction of Displacement	Maximum Amplitude
Translational Vertical Direction	0.505 inches
Translational Transverse Direction	1.85 inches
Translational Longitudinal Direction	0.75 inches
Rotational Yaw	1.2 degrees
Rotational Pitch	1.2 degrees
Rotational Roll	0.75 degrees

Table 5.10: Maximum displacement relative to ground obtained from simulation response curves when excited with excitations from test run 15 harmonic inputs.

Direction of Displacement	Maximum Amplitude
Translational Vertical Direction	0.51 inches
Translational Transverse Direction	1.35 inches
Translational Longitudinal Direction	0.45 inches
Rotational Yaw	1.5 degrees
Rotational Pitch	0.6 degrees
Rotational Roll	0.75 degrees

Mass Deflection based on Run 15 Data

Figures 5.22 and 5.23 show the translation and rotational displacements of the concentrated mass when the simulation was excited with harmonic inputs obtained for test run 15. The maximum displacement with respect to ground, as shown in the figures, is given in table 5.10.

Simulation Result

The figures for the various test runs give information concerning the position and orientation of the concentrated mass as the vehicle maneuvers over the different terrains. In these figures it is important to notice the frequency of oscillation and maximum amplitude in the

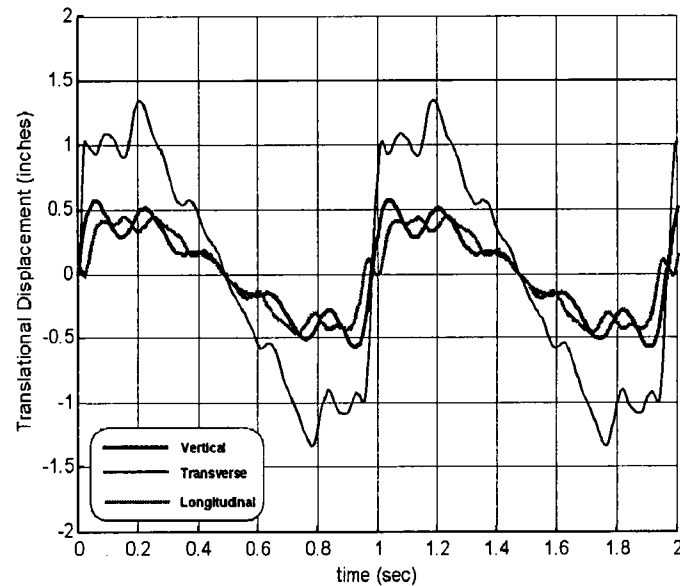


Figure 5.22: Translational displacements of the concentrated mass when excited with harmonic inputs obtained from run 15 data.

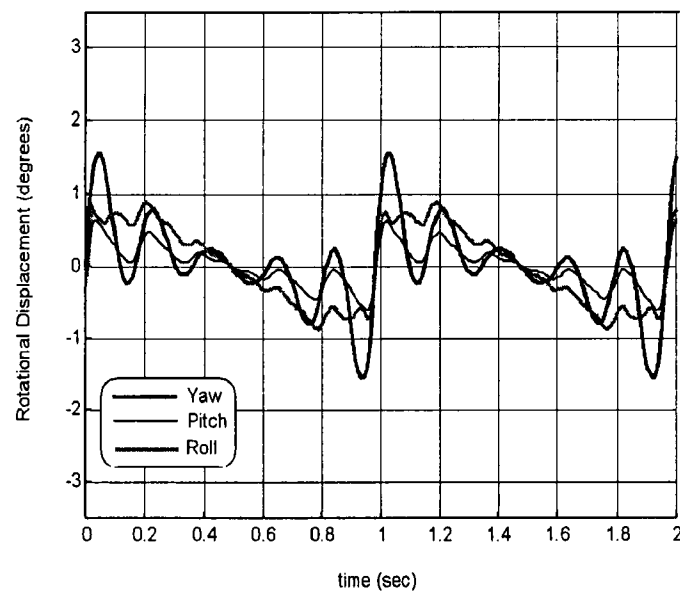


Figure 5.23: Rotational displacements of the concentrated mass when excited with harmonic inputs obtained from run 15 data.

response curves. The frequency content of each response curve can be seen in these figures.

Figures 5.22 and 5.23 show that the frequencies contained in the corresponding harmonic input for this simulation run were higher than those of the other test run harmonic inputs. This simulation proved to be the worst case scenario as far as camera pointing is concerned. The character of the response curves for this simulation run implies that maximum sensor module motion effects will occur when the base of the platform is excited with the harmonic inputs corresponding to the test data obtained from test run 15.

Overall simulation result characteristics are listed as the following.

- Simulation results from test runs 4, 8, and 11 do not possess the high frequency characteristics that test run 15 produced.
- Mass deflection was shown to be the highest for simulation run 8.
- The maximum deflection for any of the simulation runs was not very high in translation or rotation, ≤ 2.2 inches translation and ≤ 1.5 degrees rotation.
- The range of the frequency content that can be seen in any one of the simulation response curves is ≤ 11 Hertz.

These results follow what was expected for the different test runs. The vertical displacement of the sensor pod was higher when the simulation was run with the three inch spaced bumps and Belgian block than it was for the two-inch washboard case. It was also seen that the frequency of oscillation was higher for the two-inch washboard case.

5.5 Simulated Impulse Response

While the UGV is maneuvering over various terrain types it will undergo elevation changes, positive or negative, such as ascending or descending curbs, railroad ties, potholes, etc. . . This change in elevation will occur over a finite period of time depending on the speed of the

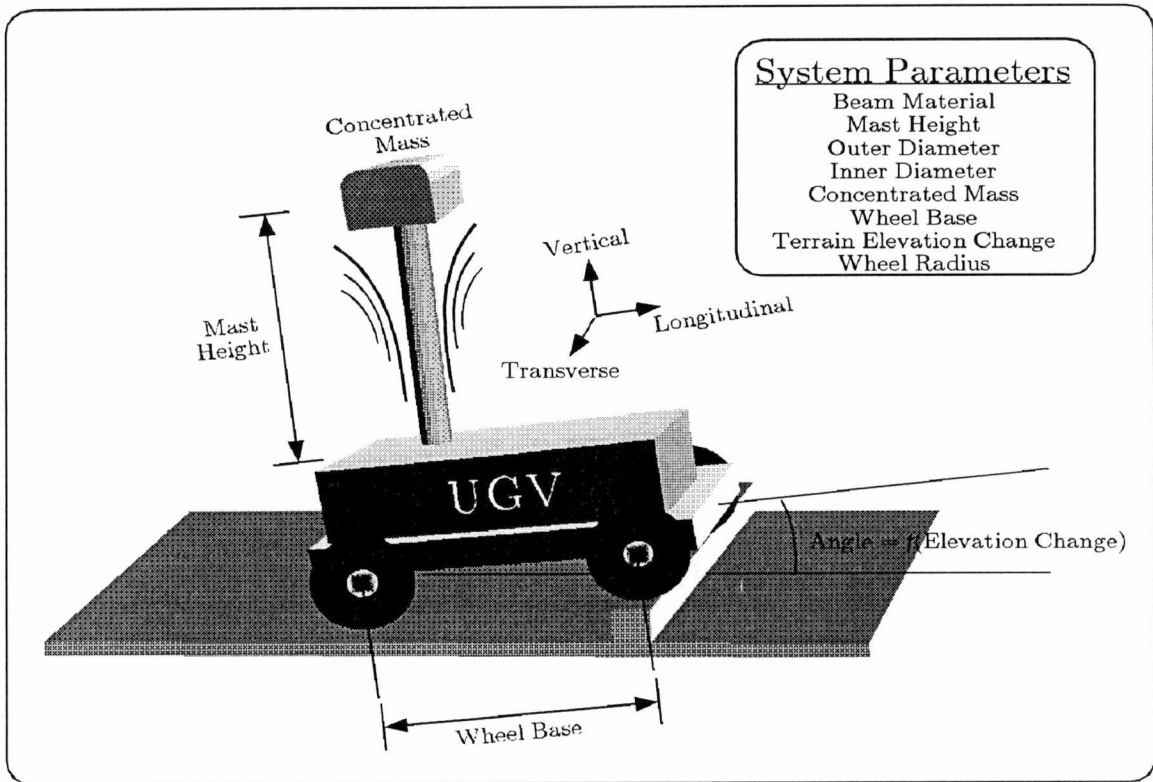


Figure 5.24: Simulated impulse applied to UGV system.

vehicle. As the vehicle maneuvers over terrain of this type, the base of the sensor platform could experience high accelerations over a very small period of time which implies high excitation forcing functions at the base of the platform. Therefore, it is important to determine the response characteristics of the concentrated mass when the base of the platform is excited with this type of force. A simulation was developed in MATLABTM to compute the response of the system due to an “impulse” excitation. Figure 5.24 shows the UGV/sensor platform system excited by a simulated impulse excitation. From this figure it can be seen that when the vehicle “climbs” the obstacle, the coordinate frame that describes the position and orientation of the base of the platform undergoes a translational displacement in the positive vertical and negative longitudinal directions and a rotational displacement about the positive transverse direction. Each one of these displacement changes take place

over a period of time that is a function of the vehicle's velocity. In order to simulate the response of the sensor platform when maneuvering over an obstacle of this type, the model was excited with an amplitude change, over this finite period of time, which was a function of the obstacle's and vehicle's geometry.

To this point, the dynamic characteristics of the vehicle are not taken into account (i.e. the vehicle's transfer function = 1). For this simulation, consider the input to the base of the platform as the excitation that would be transmitted through the transfer function describing the vehicle's dynamics. To date, the simulation only includes straight ahead vehicle motion. A more modular simulation would include vehicle dynamics and turning maneuvers.

The simulation was constructed so that input parameters could be specified by the user. These user parameter variations are as follows:

- Distance from front wheels to back wheels of the vehicle
- Wheel radius
- Beam inner and outer diameter
- Beam mast height (based on height constraint)
- Height and width of positive or negative obstacle
- Forward velocity of the vehicle
- Weight of concentrated mass

The impulse excitation is modeled simply as an elevation change from a zero reference followed by an elevation change back to the zero reference in a finite amount of time. The "impulse obstacle's" height, as defined by the user, indicates the magnitude of the excitation in the vertical direction. The "impulse obstacle's" width, also defined by the user, indicates

Table 5.11: Simulation parameters

Parameter	Impulse
Obstacle Height (in)	1
Obstacle Width (in)	1
Vehicle Velocity (mph)	4
Beam Outer Diameter (in)	6
Beam Inner Diameter (in)	5.75
Mast Height (ft)	5
Concentrated Mass (lbs)	100
Wheel Base (ft)	7
Wheel Radius (ft)	1

the time duration of the vertical impulse based on the velocity of the vehicle. The magnitude of the translational impulse in the longitudinal direction and the rotational impulse about the transverse direction are functions of both the amplitude of the obstacle, and the user defined wheel base.

5.5.1 Simulation Run

The parameters used in the simulation are shown in table 5.11. Figures 5.25 through 5.27 show the translational and rotational displacement response characteristics of the concentrated mass with respect to ground when the impulse simulation was run with these parameters. In these figures it is important to notice the magnitude and time duration of the transient response. As expected, figure 5.25 shows that the displacement of the concentrated mass in the vertical direction exactly followed the impulse forcing function in that direction. The other two figures show that the transient response “dies out” very quickly, but it is evident in the figures that there is oscillation in the response curve which will need to be accounted for as far as camera pointing is concerned. It should be noted that including vehicle dynamics will affect the response characteristics of the simulation. Therefore, when running the simulation it is important to excite the system only with inputs that would be

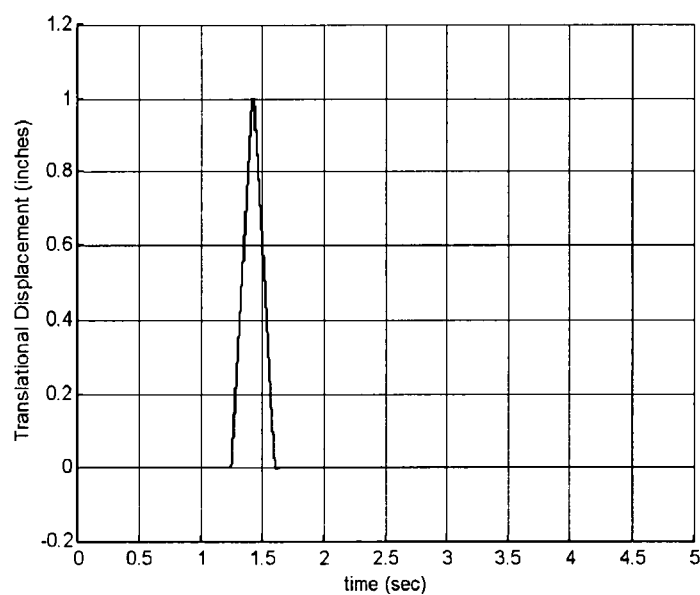


Figure 5.25: Mass translational displacement in vertical direction with respect to ground when excited with an impulse.

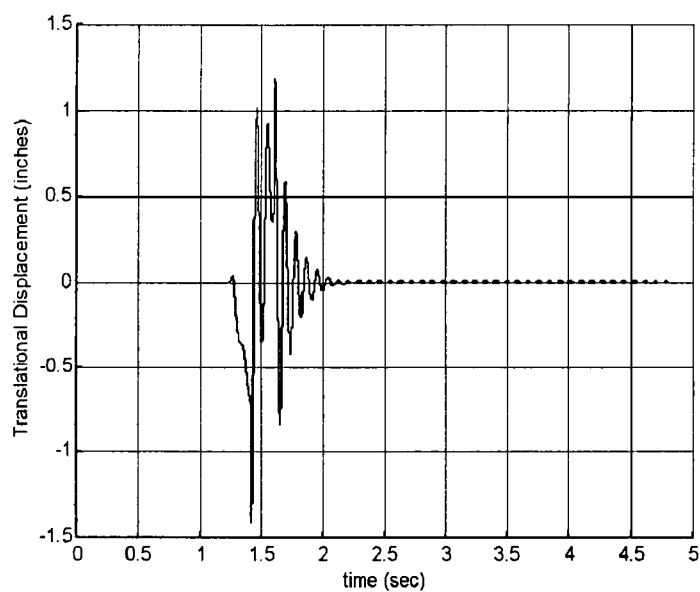


Figure 5.26: Mass translational displacement in longitudinal direction with respect to ground when excited with an impulse .

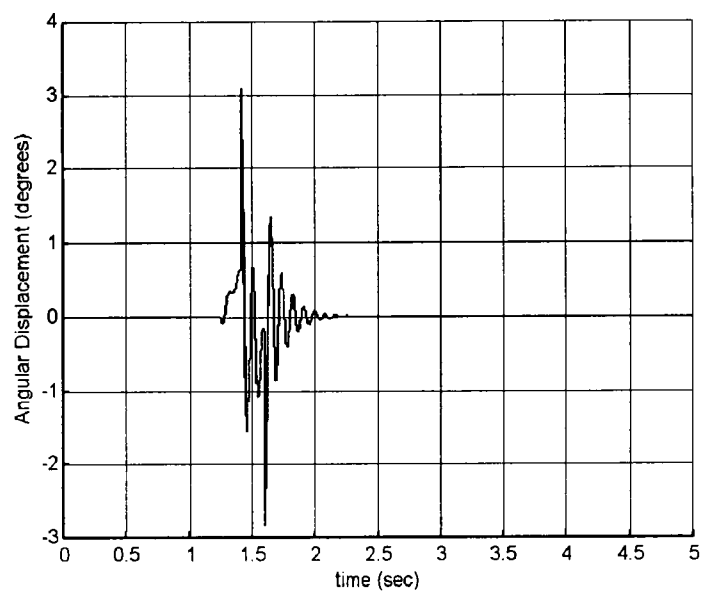


Figure 5.27: Mass rotational displacement about the transverse direction with respect to ground when excited with an impulse.

transmitted through the vehicle's transfer function.

Chapter 6

Conclusions

Overview

Unmanned ground vehicles UGVs have a vital role in military operations. The motivation for the UGV program is to place machines in hostile or dangerous environments so that hazards to soldiers are reduced. One of the key goals of the UGV is to perform operations such as reconnaissance, surveillance, and target acquisition (RSTA). As the vehicle travels over various terrains, the position and orientation of the cameras/sensors in the RSTA sensor suite need to be determined for sensor suite stabilization and/or camera pointing. In order to determine the translational and rotational displacement characteristics of the sensor suite, a sensor platform model was developed and analyzed based on various vehicle/terrain situations. This analysis allows performance predictions to be made concerning sensor pointing while performing RSTA on the move. Performing RSTA on the move is sensitive to the large amplitude and high frequencies of oscillation of the sensor suite when the vehicle is in motion.

The sensor platform was modeled as a simple beam element with a mass connected one end. The beam was assumed to be long, slender, of circular cross section, and has negligible mass compared to the concentrated mass attached to the free end. Once the model was

developed, a simulation was constructed in MATLABTM to evaluate the response characteristics of the concentrated mass when the base of the platform was excited by different forcing functions. These forcing functions include three translational inputs and three rotational inputs located at the base of the beam element (i.e. this model is treated as a base excitation problem). The computed displacement output of concern for the model included three translational displacements and three rotational displacements of the center of gravity of the concentrated mass. Assumptions were made for the platform model parameters corresponding to what would be expected of the configuration of an actual platform. The translational and rotational deflections of the concentrated mass are assumed to be small and are due only to bending. Shear deflection is neglected.

Sensor Platform Model Analysis Results

When amplitude spectrum plots were constructed for data obtained from the test runs conducted on the NGUV it was indicated that chassis rotational and translational amplitudes were evident primarily in the low frequency range (1-11 hertz). Although, it can be seen from the amplitude spectrum plots that the maximum amplitude occurred primarily at 1 hertz. These plots showed that these maximum amplitudes were very small which indicates that the vehicle suspension system attenuates most of the excitation experienced by the vehicle while traveling over the various test run terrains. In order to obtain information concerning the maximum amplitudes and frequencies of oscillation that would be experienced if a sensor platform was mounted to the vehicle, the platform model was excited with harmonic inputs corresponding to these amplitude spectrum plots.

Sensitivity analysis results indicated that the platforms lowest natural frequency decreased from 25 hertz to 12 hertz to 6.5 hertz when the mast height was increased from 3 feet to 5 feet to 8 feet, respectively, with all other system parameters nominal. This shows that the lowest natural frequency of the system, with the nominal mast height value of

5 feet, borders the excitation frequency range (1 - 11 hertz) obtained from the amplitude spectrum plots. Therefore it is concluded that mast heights above 5 feet, with all other parameters nominal, are not desirable. It is recommended that for mast height values above 5 feet, other system parameters should be changed to increase beam stiffness (e.g. change material and/or beam thickness) or to decrease the amount of concentrated mass. Changing these parameters will offset a portion of the frequency shift when the mast height is increased.

Varying the amount of mass connected to the end of the beam showed that the system was not as sensitive to the mass parameter as it was to the mast height of the beam. Although, it was found that the lowest natural frequency decreased from 17 hertz to 9 hertz when the mass was increased from 50 lbs to 150 lbs. This should be considered when determining what the mast height of the beam need to be in order to perform the various task required of the UGV.

Even though the nominal plant had natural frequencies close to the frequencies that were used to excite the system, the harmonic excitation simulation analysis for this model indicated that the sensor platform would be well suited in applications for performing RSTA on the move. The maximum deflection and frequency of oscillation experienced by the sensor platform model shows sensor/camera aiming is servo-controllable while the vehicle is in motion without a high degree of difficulty and/or cost.

Impulse Simulation

A simulation for obtaining impulse response information was developed for the platform model that allowed the user to specify a range of system parameters. This simulation show the response characteristics of the sensor platform with respect to an inertial orientation reference when excited by an impulse function. The results from the simulation show that the transient response "dies out" fairly quickly, after about 1 second, and that the amplitude

of displacement is small, about 1.4 inches translation and 3 degrees rotation.

The next step for the impulse/step simulation is to include vehicle dynamic characteristics (i.e. vehicle suspension system) that will enable the simulation to capture a more realistic response profile of the sensor suite.

Future Work

Future work should include the following.

- Mast height was the dominating sensitivity parameter for the model. Therefore, in order to push the natural frequencies of the system higher as the mast height increases, combinations of other parameter variations should be considered.
- An experimental setup to test the performance of a physical platform pedestal based on the results from the platform model results and cost issues.
- The analysis conducted in this research considered the response characteristics of the platform model when the vehicle was in a dynamic mode of operation. Analysis should be conducted when the vehicle is in a static position. Affects such as wind velocities and/or vehicle static position vibration should be considered.
- This analysis did not consider any vehicle dynamics. The platform model was excited with harmonic inputs constructed from NGUV test run data. This data was obtained from a vehicle that did not possess a sensor platform, which affects the overall weight of the vehicle. Therefore, in order to more accurately determine the response characteristics of the sensor pod, it would be advantageous to obtain data from test conducted on a vehicle that contained a sensor platform.

Bibliography

Bibliography

- [1] J. Den Hartog. *STRENGTH OF MATERIALS*. General Publishing Company, Ltd., 1977.
- [2] F. Fisher. Design and application of a robotic vehicle driving package for the HMMWV, Demo I to project MUSTANG. *SPIE*, 2738:143–150, 1996.
- [3] G. Franklin, J. Powell, and A. Emami-Naeini. *FEEDBACK CONTROL OF DYNAMIC SYSTEMS*. Addison Westley, 1991.
- [4] B. Gothard, R. Spomer, J. Medina, and W. Chun. Design issues with the SSV platform. *SPIE*, 1831:320–331, 1992.
- [5] M. Herman, L. Matthies, J. Albus, and M. Juberts. Mobility white paper for Demo III, April 1997.
- [6] G. Klager, P. David, T. Mills, G. Haas, and P. Burt. Reconnaissance, surveillance and target acquisition. *Military*, Oct 1997.
- [7] A. Lawrence. *MODERN INERTIAL TECHNOLOGY*. Springer-Verlag, 1993.
- [8] A. Meirovitch. *METHODS OF ANALYTICAL DYNAMICS*. Magraw-Hill Book Company, Inc., 1976.
- [9] MilitaryDraft. Next generation unmanned ground vehicle modeling, simulation and experimentation, April 1997. Drafted by Mounted Maneuver Battlespace Laboratory (Fort Knox, Kentucky) and at Army Research Laboratory, (Aberdeen Proving Grounds, Maryland).
- [10] S. Munkeby and J. Spofford. UGV / Demo II program: Status though Demo C and Demo II preparations. *SPIE*, 2738:110–121, 1996.
- [11] D. Philip. Real-Time Target Acquisition for the Army's SCOUT Mission. *SPIE*, 2739:324–331, 1996.

Appendices

Appendix A

Code for Tracking Angles

```
% This program nulls out the rotations of the vehicle so that the SRTS
% can remain inertially orientated for the UGV Army Project.
% Author : David C Lennon

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%load ang.mat
%global ang

%The Identity Matrix

I = [1 0 0; 0 1 0; 0 0 1];
I4 = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1];

%Set up initial conditions
zero = 0;
dummy_variable = 1;
xold = 0;
yold = 0;
zold = 0;

i_p_c_old = [0; 0; 0];
i_p_v1 = [0; 0; 0];
R_tr_old = I;
v_R_c_old = I;
i_R_c_old = I;
i_R_v_old = I;
v_T_c1 = I4;
rot_n = I;
rot_n_old = I;

%The identified target will occupy an array of pixels in the camera's
%pixel array. From this array the distance to target can be
%determined. Once the target is found, the initial range to target
%is given by the variable DIST.

DIST = 8;

% The pixel array gives an indication of the distance to target

% This is the number of gimbal angles for measured for the vehicle
% (IF CHANGED THEN THE ANGLE VECTORS (ROLL/PITCH/YAW) MUST ALSO
% BE CHANGED.)
[r,c] = size(theta);
n = r;

%This makes up the zero matrix with size = size(theta)

zero = zeros(r,c);

%These vectors set up the inertial coordinate frame
xi=[0 1];

x2=[0 0];
yi=[0 0];
y2=[0 1];
zi=[0 0];
```

```

z2=[0 1];

plot3(0,0,0)
%These plots make up the inertial coordinate system.
plot3(x1,y1,z1, '-b')
plot3(x2,y2,z1, '-b')
plot3(x2,y1,z2, '-b')
j = 1:n;

grid
hold on

i = 1;

while i<n+1

%Pan/Tilt Head Rotation Angles
roll(j) = ang(:,1); %[0; 5; 10; -7; 10];
pitch(j) = ang(:,2); %[0; -2; 5; 4; 3];
yaw(j) = ang(:,3); %[0; -3; -4; +3; -10];

roll = (roll(i)*pi/180);
pitch = (pitch(i)*pi/180);
yaw = (yaw(i)*pi/180);

%Tracking Rotation Angles
alpha(j) = zero(:,1); %[0; 0; 0; 0; 0];
beta(j) = zero(:,2); %[0; 0; 0; 0; 0]; %[5; 3; 6; 6; 4];
gamma(j) = zero(:,3); %[0; 0; 0; 0; 0]; %[10; 3; -5; -2; 5];

alpha = (alpha(i)*pi/180); %roll angle
beta = (beta(i)*pi/180); %pitch angle
gamma = (gamma(i)*pi/180); %yaw angle

position = ones(r,c);

x(j) = POSIT(:,1); %[1; 1; 1; 1; 1];
y(j) = POSIT(:,2); %[-5; 1; 1; 1; 1];
z(j) = POSIT(:,3); %[1; 1; 1; 1; 1];

% x_i, y_i, and z_i positions of the vehicle measured by the
% accelerometers from position (i-1) to (i)

x_i = x(i);
y_i = y(i);
z_i = z(i);

% x, y, and z are the positions of the vehicle with respect to the
% inertial reference at time (i)

x = xold + x_i;
y = yold + y_i;
z = zold + z_i;

v_p_v_i = [x_i; y_i; z_i];
i_p_v = i_p_v1 + v_p_v_i;

% x, y, and z positions of the SRTS with respect to the vehicle
xc = +2;
yc = -2;
zc = +3;
v_p_c = [xc; yc; zc];

% position vector from the inertial frame to the SRTS
i_p_c = i_p_v + v_p_c;

% Vectors from the inertial frame to the vehicle and to the SRTS
xveh = [0 x];
yveh = [0 y];
zveh = [0 z];

xSRTS = [0 (x+xc)];
ySRTS = [0 (y+yc)];
zSRTS = [0 (z+zc)];

% The range distance is used to determine the vector from the camera
% to the target. Since the LOS of the camera is along the y-axis
% this vector is given by:

if dummy_variable > 0

```

```

c_p_t = [0; DIST; 0];

% The vector from the inertial reference to the target is give by:
i_p_t = i_p_c + c_p_t;

end
dummy_variable = -1;

c_p_t = i_p_t - i_p_c;

%These are the vectors to the vehicle and the SRTS
%plot3([0 i_p_t(1,1)], [0 i_p_t(2,1)], [0 i_p_t(3,1)], '-r')
%plot3(i_p_t(1,1), i_p_t(2,1), i_p_t(3,1), '*')
plot3([i_p_c(1,1) i_p_c(1,1)+c_p_t(1,1)], [i_p_c(2,1) i_p_c(2,1)+c_p_t(2,1)], [i_p_c(3,1) i_p_c(3,1)+c_p_t(3,1)], 'w:');

mag_cpt = sqrt(c_p_t(1,1)^2 + c_p_t(2,1)^2 + c_p_t(3,1)^2);

R12 = (c_p_t(1,1))/mag_cpt;
R22 = (c_p_t(2,1))/mag_cpt;
R32 = (c_p_t(3,1))/mag_cpt;

yaw_n = atan2(-R12, R22);
pitch_n = atan2(R32, sqrt(R12^2 + R22^2));
roll_n = 0;

y_n = [cos(yaw_n) -sin(yaw_n) 0; sin(yaw_n) cos(yaw_n) 0; 0 0 1];
p_n = [1 0 0; 0 cos(pitch_n) -sin(pitch_n); 0 sin(pitch_n) cos(pitch_n)];
r_n = [cos(roll_n) 0 sin(roll_n); 0 1 0; -sin(roll_n) 0 cos(roll_n)];

rot_n_i = y_n * p_n * r_n;
%rot_n_i = I;

%Rotation matrices for the pan/tilt mechanism from the tracking algorithm :
track_y = [cos(gamma) -sin(gamma) 0; sin(gamma) cos(gamma) 0; 0 0 1];
track_p = [1 0 0; 0 cos(beta) -sin(beta); 0 sin(beta) cos(beta)];
track_r = [cos(alpha) 0 sin(alpha); 0 1 0; -sin(alpha) 0 cos(alpha)];

c_R_c = track_y*track_p*track_r;
R_tr = R_tr_old * c_R_c;
i_R_c = R_tr*rot_n_i;

%Forced Transformation matrix of the SRTS wrt inertial frame
i_T_c = ([[i_R_c; 0 0 0] [i_p_c; 1]]);

%Rotation matrices from the gimbal angles.
gim_y = [cos(yaw) -sin(yaw) 0; sin(yaw) cos(yaw) 0; 0 0 1];
gim_p = [1 0 0; 0 cos(pitch) -sin(pitch); 0 sin(pitch) cos(pitch)];
gim_r = [cos(roll) 0 sin(roll); 0 1 0; -sin(roll) 0 cos(roll)];

v_R_v = (gim_y*gim_p*gim_r);
i_R_v1 = i_R_v_old*v_R_v;

%Transformation matrix of the vehicle wrt inertial frame
i_T_v = ([[i_R_v1; 0 0 0] [i_p_v; 1]]);

%The rotation matrix between the vehicle and the SRTS
v_R_c = i_R_v1'*i_R_c;

%The angles required for the pan/tilt mechanism are then found to be:

theta = atan2(v_R_c(3,2),sqrt(v_R_c(3,1)^2+v_R_c(3,3)^2));
psi = atan2(-v_R_c(1,2),v_R_c(2,2));
%phi = 0;
phi = atan2(-v_R_c(3,1),v_R_c(3,3));

psi1 = psi*180/pi ;
theta1 = theta*180/pi ;
phi1 = phi*180/pi ;

YAW_PITCH_ROLL = [psi1 theta1 phi1];

Y = [cos(psi) -sin(psi) 0; sin(psi) cos(psi) 0; 0 0 1];
P = [1 0 0; 0 cos(theta) -sin(theta); 0 sin(theta) cos(theta)];
R = [cos(phi) 0 sin(phi); 0 1 0; -sin(phi) 0 cos(phi)];

v_R_c_ypr = (Y*P*I);

%Transformation matrix of the SRTS wrt vehicle frame
v_T_ci = ([[v_R_c_ypr; 0 0 0] [xc;yc;z;1]]);
v_T_c = v_T_ci*v_T_ci;

```

```

%Transformation matrix of the SRTS wrt inertial frame
I_T_C = i_T_v*v_T_ci;

%*****

%This point is the translation of the vehicle coordinate system
%plot3(x,y,x,'*')
%text(x, y, z, 'Veh')
%axis('square')

%plot3(x+xc,y+yc,z+zc, '*')
%text(x+xc,y+yc,z+zc, 'RSTA')

%These are the vectors to the vehicle and the SRTS
%plot3(xveh, yveh, zveh, '-y')
%plot3(xSRTS, ySRTS, zSRTS, '-y')

%%This is the roation of the vehicle
%This is the new x coordinate vector after the transformation
x2=[0+x (i_T_v(1,1)+x)];
y2=[0+y (i_T_v(2,1)+y)];
z2=[0+z (i_T_v(3,1)+z)];
plot3(x2, y2, z2, '-r')

%This is the new y coordinate vector after the transformation
x3=[0+x (i_T_v(1,2)+x)];
y3=[0+y (i_T_v(2,2)+y)];
z3=[0+z (i_T_v(3,2)+z)];
plot3(x3, y3, z3, '-b')

%This is the new z coordinate vector after the transformation
x4=[0+x (i_T_v(1,3)+x)];
y4=[0+y (i_T_v(2,3)+y)];
z4=[0+z (i_T_v(3,3)+z)];
plot3(x4, y4, z4, '-g')

%%This is the rotation of the pan/tilt mechanism
%This is the new x coordinate vector after the transformation
%x5=[0+x+xc (I_T_C(1,1)+x+xc)];
%y5=[0+y+yc (I_T_C(2,1)+y+yc)];
%z5=[0+z+zc (I_T_C(3,1)+z+zc)];
%plot3(x5, y5, z5, '-r')

%This is the new y coordinate vector after the transformation
x6=[0+x+xc (I_T_C(1,2)+x+xc)];
y6=[0+y+yc (I_T_C(2,2)+y+yc)];
z6=[0+z+zc (I_T_C(3,2)+z+zc)];
plot3(x6, y6, z6, 'g'); % x6, y6, z6, 'v:')
%text(I_T_C(1,2)+x+xc, (I_T_C(2,2)+y+yc), (I_T_C(3,2)+z+zc))

%This is the new z coordinate vector after the transformation
%x7=[0+x+xc (I_T_C(1,3)+x+xc)];
%y7=[0+y+yc (I_T_C(2,3)+y+yc)];
%z7=[0+z+zc (I_T_C(3,3)+z+zc)];
%plot3(x7, y7, z7, '-g')

%axis([0 8 0 8 0 8])
title('RSTA Inertial Stabilization')
xlabel('pitch (TILT) (x-axis)')
ylabel('roll (y-axis)')
zlabel('yaw (PAN) (z-axis)')

xold = x;
yold = y;
zold = z;
i_p_c_old = i_p_c;
i_p_v1 = i_p_v;
rot_n_old = rot_n_i;
R_tr_old = R_tr;
v_R_c_old = v_R_c;
i_R_c_old = i_R_c;
i_R_v_old = i_R_v1;
v_T_c1 = v_T_c;

i = i+1;

end
hold off

```

Appendix B

Frequency Analysis Code

B.1 Code for the State Space Platform Model

```
% Inertia parameters
Jp = pi*(Dout^4-din^4)/32 ; %m^4
I = (pi/64)*(Dout^4 - din^4); %m^4

%These are the inertia terms for the pan/tilt head

a = 24*0.0254; %meters
b = 12*0.0254; %meters
c = 16*0.0254; %meters

I2 = (1/12)*m*(b^2 + c^2); %kg*m^2 Mass moment of inertial in Transverse direction
I3 = (1/12)*m*(a^2 + b^2); %kg*m^2 Mass moment of inertial in Longitudinal direction
J = I3+I2; %kg*m^2 Rotary mass moment of inertia in vertical direction

% These are the coefficients acquired by determining the energy
% of the system.

a11 = (L)^3/(3*E*I);
a12 = (L)^2/(2*E*I);
a21 = (L)^2/(2*E*I);
a22 = (L)/(E*I);
a31 = (a11*a22 - (a12)^2);
a41 = a22/a31;
a42 = a12/a31;
a51 = a21/a31;
a52 = a11/a31;
a61 = (L^3)/(6*E*I);
a62 = (L^2)/(2*E*I);
a71 = L/(2*E*I);

ka = E*area/(L) ;
ca = sign(ka)*2*zeta*sqrt(abs(ka)*m);
kt = G*Jp/(L) ;
ct = sign(kt)*2*zeta*sqrt(abs(kt)*J);

k1 = (2*(a41^2)*a61 - 2*a51*a41*a62 + 2*a71*a51^2);
c1 = sign(k1)*2*zeta*sqrt(abs(k1)*m);
k2 = (-2*a41*a42*a61 + a62*a52*a41 + a51*a42*a62 - 2*a71*a51*a52);
k3 = L*(-2*(a41^2)*a61 + 2*a51*a41*a62 - 2*a71*a51^2);

k4 = (-2*a41*a42*a61 + a62*a52*a41 + a51*a42*a62 - 2*a71*a51*a52);
k5 = (2*(a42^2)*a61 - 2*a42*a52*a62 + 2*a71*a52^2);
c5 = sign(k5)*2*zeta*sqrt(abs(k5)*I3);
k6 = L*(2*a41*a42*a61 - a62*a52*a41 - a51*a42*a62 + 2*a71*a51*a52);

k7 = (2*a41*a42*a61 - a41*a52*a62 - a42*a51*a62 + 2*a51*a52*a71);
k8 = (2*(a42^2)*a61 - 2*a42*a52*a62 + 2*(a52^2)*a71);
c8 = sign(k8)*2*zeta*sqrt(abs(k8)*I2);
k9 = L*(2*a42*a41*a61 - a52*a41*a62 - a42*a51*a62 + 2*a52*a51*a71);

k10 = (2*(a41^2)*a61 - 2*a41*a51*a62 + 2*(a51^2)*a71);
```

```

c10 = sign(k10)*2*zeta*sqrt(abs(k10)*m);
k11 = (2*a51*a52*a71 - a42*a51*a62 - a41*a52*a62 + 2*a41*a42*a61);
k12 = L*(2*(a51^2)*a71 - 2*a41*a51*a62 + 2*(a41^2)*a61);

A = [ 0      1      0      0      0      0      0      0      0      0      0      0 ;
      -ka/m  -ca/m   0      0      0      0      0      0      0      0      0      0 ;
      0      0      0      1      0      0      0      0      0      0      0      0 ;
      0      0     -k1/m  -c1/m   0      0      0      0      0      0     -k2/m   0 ;
      0      0      0      0      0      0      1      0      0      0      0      0 ;
      0      0      0      0     -k10/m  -c10/m   0      0     -k11/m   0      0      0 ;
      0      0      0      0      0      0      0      1      0      0      0      0 ;
      0      0      0      0      0      0      0     -kt/J   -ct/J   0      0      0 ;
      0      0      0      0      0      0      0      0      0      1      0      0 ;
      0      0      0      0     -k7/I2   0      0      0     -k8/I2  -c8/I2   0      0 ;
      0      0      0      0      0      0      0      0      0      0      0      1 ;
      0      0     -k4/I3   0      0      0      0      0      0     -k6/I3  -c5/I3] ;

```

```
b2_1 = ((ka/m));
```

```
b4_2 = ((k1/m));
```

```
b4_6 = k2/m - k3/m;
```

```
b6_3 = ((k10/m));
```

```
b6_5 = k11/m - k12/m;
```

```
b8_4 = ((kt/J));
```

```
b10_3 = k7/I2;
```

```
b10_5 = ((k8/I2) - (k9/I2));
```

```
b12_2 = k4/I3;
```

```
b12_6 = (k5/I3) - (k6/I3) ;
```

```

B = [ 0      0      0      0      0      0      0      0 ;
      b2_1   0      0      0      0      0      0      0 ;
      0      0      0      0      0      0      0      0 ;
      0      b4_2   0      0      0      0      b4_6   0 ;
      0      0      0      0      0      0      0      0 ;
      0      0      b6_3   0      b6_5   0      0      0 ;
      0      0      0      0      0      0      0      0 ;
      0      0      0      b8_4   0      0      0      0 ;
      0      0      0      0      0      0      0      0 ;
      0      0      b10_3   0      b10_5   0      0      0 ;
      0      0      0      0      0      0      0      0 ;
      0      b12_2   0      0      0      b12_6   0      0 ];

```

```
%This is the matrix [C] 6x12
```

```

C = [ 1      0      0      0      0      0      0      0      0      0      0      0 ;
      0      0      1      0      0      0      0      0      0      0      0      0 ;
      0      0      0      0      1      0      0      0      0      0      0      0 ;
      0      0      0      0      0      0      1      0      0      0      0      0 ;
      0      0      0      0      0      0      0      0      1      0      0      0 ;
      0      0      0      0      0      0      0      0      0      0      1      0 ];

```

```
%This is the matrix [D] 6x12.
```

```

D = [ 0      0      0      0      0      0 ;
      0      0      0      0      0      0 ;
      0      0      0      0      0      0 ;
      0      0      0      0      0      0 ;
      0      0      0      0      0      0 ;
      0      0      0      0      0      0 ];

```

B.2 Code for the bode plot

```
%This program creates the bode plots for the nominal plant values
clear
```

```
warning off
```

```

% Parameters of aluminum pipe

E = 7.10e10; %N/(m^2)
G = 2.67e10; %N/(m^2)

%Inside and outside pipe diameter

Dout = 6*0.0254; %meter
thickness = 0.0254*1/8;
din = Dout - 2*thickness; %meter

%Mast Height
L = 5*12*0.0254; %ft*(in/ft)*(m/in) = m

%Cross sectional area of beam
area = (pi/4)*(Dout^2-din^2); %m^2

%Concentrated mass for the support and pan/tilt head
m = 100*(0.454); %kg

%Nominal value of the damping ratio
zeta = 0.1;

%Call the file system.m that has the
%equations of motion in steady state form
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
system %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

sys = ss(A,B,C,D);

tfsys = tf(sys);
% s2 = tf2ss(tfsys(2,2));

%SISO transfer function for input in 1 output in 1
[m1,p1,w1]=bode(tfsys(1,1));
i1=1:max(size(m1));
m(i1) = m1(:,i1);
n = 20*log10(m);
p(i1) = pi(:,i1);
w = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(v,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(v,p)
grid
xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod11

%SISO transfer function for input in 2 output in 2
[m1,p1,w1]=bode(tfsys(2,2));
i1=1:max(size(m1));
m(i1) = m1(:,i1);
n = 20*log10(m);
p(i1) = pi(:,i1);
w = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(v,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(v,p)
grid
xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod22

%SISO transfer function for input in 3 output in 3
[m1,p1,w1]=bode(tfsys(3,3));
i1=1:max(size(m1));
m(i1) = m1(:,i1);
n = 20*log10(m);

```

```

p(i1) = p1(:,i1);
w = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(v,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(v,p)
grid
xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod33

%SISO transfer function for input in 4 output in 4
[m1,p1,w1]=bode(tfsys(4,4));
i1=1:max(size(m1));
m(i1) = m1(:,i1);
n = m(1:max(size(m1)));
n = 20*log10(n);
p(i1) = p1(:,i1);
p = p(1:max(size(m1)));
w = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(v,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(v,p)
grid
xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod44

%SISO transfer function for input in 5 output in 5
[m1,p1,w1]=bode(tfsys(5,5));
i1=1:max(size(m1));
m(i1) = m1(:,i1);
n = m(1:max(size(m1)));
n = 20*log10(n);
p(i1) = p1(:,i1);
p = p(1:max(size(m1)));
w = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(v,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(v,p)
grid
xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod55

%SISO transfer function for input in 6 output in 6
[m1,p1,w1]=bode(tfsys(6,6));
i1=1:max(size(m1));
m(i1) = m1(:,i1);
n = m(1:max(size(m1)));
n = 20*log10(n);
p(i1) = p1(:,i1);
p = p(1:max(size(m1)));
w = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(v,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(v,p)
grid

```

```

xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod66

%SISO transfer function for input in 2 output in 6
[m1,p1,w1]=bode(tfsys(6,2));
ii=1:max(size(m1));
m(ii) = m1(:,ii);
m = m(1:max(size(m1)));
m = 20*log10(m);
p(ii) = p1(:,ii);
p = p(1:max(size(m1)));
w = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(w,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(w,p)
grid
xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod26

%SISO transfer function for input in 3 output in 6
[m1,p1,w1]=bode(tfsys(6,3));
ii=1:max(size(m1));
m(ii) = m1(:,ii);
m = m(1:max(size(m1)));
m = 20*log10(m);
p(ii) = p1(:,ii);
p = p(1:max(size(m1)));
w = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(w,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(w,p)
grid
xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod35

%SISO transfer function for input in 6 output in 2
[m1,p1,w1]=bode(tfsys(2,6));
ii=1:max(size(m1));
m(ii) = m1(:,ii);
m = m(1:max(size(m1)));
m = 20*log10(m);
p(ii) = p1(:,ii);
p = p(1:max(size(m1)));
w = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(w,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(w,p)
grid
xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod62

%SISO transfer function for input in 5 output in 3
[m1,p1,w1]=bode(tfsys(3,5));
ii=1:max(size(m1));
m(ii) = m1(:,ii);
m = m(1:max(size(m1)));
m = 20*log10(m);
p(ii) = p1(:,ii);
p = p(1:max(size(m1)));

```

```

v = w1/(2*pi);

figure
subplot(2,1,1)
semilogx(w,m)
ylabel('Magnitude (dB)')
grid

subplot(2,1,2)
semilogx(w,p)
grid
xlabel('Frequency (Hz)')
ylabel('Phase (deg)')
print -dmeta bod53

```

B.3 Sensitivity Analysis in the Frequency Domain

```

%This program determines how sensitive the platform model is to
%changes in the mast height of the beam
clear

warning off
% Parameters of aluminum pipe

E = 7.10e10; %N/(m^2)
G = 2.67e10; %N/(m^2)

%Inside and outside pipe diameter

Dout = 6*0.0254; %meter
thickness = 0.0254*1/8;
din = Dout - 2*thickness; %meter

%Cross sectional area of beam
area = (pi/4)*(Dout^2-din^2); %m^2

%Concentrated mass for the support and pan/tilt head
m = 100*(0.454); %kg

%Nominal value of the damping ratio
zeta = 0.1;

%This section evaluates the frequency response curves for the system
%with the different mast heights
%In order to determine the sensitivity of the system to the amount of
%mass connected to the end of the beam do the same process with changes
%in the mass instead of length

L = 3*12*0.0254; %ft*(in/ft)*(m/in) = m
system
sys1 = ss(A,B,C,D);
tfsys = tf(sys1);
bodplots

L = 5*12*0.0254; %ft*(in/ft)*(m/in) = m
system
sys2 = ss(A,B,C,D);
tfsys = tf(sys2);
bodplots

L = 8*12*0.0254; %ft*(in/ft)*(m/in) = m
system
sys3 = ss(A,B,C,D);
tfsys = tf(sys3);
bodplots

```

Appendix C

Amplitude Spectrum Code

C.1 Plant Model Code

```
%The measurement data is found in files dat[run#] where
%dat[run#] = [VerticalAcc TransverseAcc LongAcc PitchGyro RollGyro YawGyro]
%for runs 4, 8, 11, and 15
%The file dat[run#] needs to be loaded before running this program
%(i.e. at the command prompt type 'load dat15' to load the data
%from run 15 into the MATLAB workspace)

Fs=2083.333;           % sampling frequency
Fn=Fs/2;              % Nyquist frequency

%The (a) is the vector considered corresponding to
%the measurement device of question
%a = 1 calls the data for the Vertical Accelerometer
%a = 2 calls the data for the Transverse Accelerometer
%a = 3 calls the data for the Longitudinal Accelerometer
%a = 4 calls the data for the Pitch Gyro
%a = 5 calls the data for the Roll Gyro
%a = 6 calls the data for the Yaw Gyro
for a = 1:1:6
%Define the vector of question with DC bias removed

vec = dat15(:,a)-mean(dat15(:,a));
%Measurement data signal is sectioned into blocks of
NFFT = 2048;
n = NFFT;
%Since the FFT is symmetric only the first half of the
%FFT needs to be considered
NumUniquePts = ceil((NFFT+1)/2);

%final is the number of blocks considered from the signal
final = 29;

%This loop sections the signal data into
%the blocks with 2048 elements
for i = 1:1:final

    dat(:,i) = vec((i-1)*n+1:i*n);

end

%Determine the FFT and PSD of the sectioned measurement data
%f(i) is the FFT of the sectioned data
%p(i) is the PSD of the sectioned data
%FFTX keeps only the first half of the FFT
%MX(i) determines the magnitude of the FFT

f1 = fft(dat(:,1),n);

FFTX=f1(1:NumUniquePts);
MX1=abs(FFTX);
p1 = FFTX.*conj(FFTX)/n;
```

```

f2 = fft(dat(:,2),n);

FFTX=f2(1:NumUniquePts);
MX2=abs(FFTX);
p2 = FFTX.*conj(FFTX)/n;

f3 = fft(dat(:,3),n);

FFTX=f3(1:NumUniquePts);
MX3=abs(FFTX);
p3 = FFTX.*conj(FFTX)/n;

.
.
.

f27 = fft(dat(:,27),n);

FFTX=f27(1:NumUniquePts);
MX27=abs(FFTX);
p27 = FFTX.*conj(FFTX)/n;

f28 = fft(dat(:,28),n);

FFTX=f28(1:NumUniquePts);
MX28=abs(FFTX);
p28 = FFTX.*conj(FFTX)/n;

f29 = fft(dat(:,29),n);

FFTX=f29(1:NumUniquePts);
MX29=abs(FFTX);
p29 = FFTX.*conj(FFTX)/n;

%PX and MX are the average PSD and FFT of the sectioned data, respectively
PX = (p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12+p13+p14+p15+p16+p17+p18 ...
+p19+p20+p21+p22+p23+p24+p25+p26+p27+p28+p29)/final;

MX = (MX1+MX2+MX3+MX4+MX5+MX6+MX7+MX8+MX9+MX10+MX11+MX12+MX13+MX14+MX15+MX16+MX17+MX18 ...
+MX19+MX20+MX21+MX22+MX23+MX24+MX25+MX26+MX27+MX28+MX29)/final;

% Multiply by 2 to take into account the fact that we
% threw out second half of FFTX above
MX=MX*2;
PX=PX*2;

%Define the size of the FFT and PSD
[mxrow mxcolumn] = size(MX);
[pxrow pxcolumn] = size(PX);

% Account for endpoint uniqueness
MX(mxrow)=MX(mxrow)/2;
PX(pxrow)=PX(pxrow)/2;

% Scale the PSD and FFT so that they are not a function of the
% length of vec.
MX=MX/NFFT;
PX=PX/NFFT;
MX1 = MX;
PX1 = PX;

%Determine the frequency range in hertz
f=(0:NumUniquePts-1)*2*Fn/NFFT;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%The units of the amplitude spectrum plots need to be in inches
%for the accelerometer since position is needed and degrees for
%the gyro since rotation angle is needed

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%FOR THE ACCELEROMETERS
%The units of the accelerometer data is in (g's)
%386 takes the data from g's to inches/sec^2
%Since the data is of a harmonic nature
% dividing by (2*pi*f)^2 puts the data in inches
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if a <= 3
    f1 = f.^2;
    MX = MX*386;
    MX = (1/(2*pi)^2)*MX'./f1;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%FOR THE GYROS
%The units of the gyro data is in (deg/sec)
%Since the data is of a harmonic nature
% dividing by (2*pi*f) puts the data in inches
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if a > 3
    MX = (1/(2*pi))*MX'./f;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Account for the first point of the FFT and PSD due to
%bias in the data
MX(1) = 0;
PX(1) = 0;
MX1(1) = 0;
PX1(1) = 0;

%Create the figures of the amplitude and PSD
%for the measurement device of question

omega = f*2*pi;

if a == 1
    AMP = MX*0.0254;
end
if a == 2
    AMP = MX*0.0254;
end
if a == 3
    AMP = MX*0.0254;
end
if a == 4
    AMP = MX*(pi/180);
end
if a == 5
    AMP = MX*(pi/180);
end
if a == 6
    AMP = MX*(pi/180);
end
t1 = 0:0.005:5;

if a == 1
    fp_old = zeros(max(size(t1)),1)';
    fv_old = zeros(max(size(t1)),1)';
    for i = 1:1:20
        force_p_new = AMP(i)*sin(omega(i)*t1);
        force_v_new = omega(i)*AMP(i)*cos(omega(i)*t1);
        %force_pos = [force_p' force_p(i,:)'];
        %force_vel = [force_v' force_v(i,:)'];

        h_vt_pos = fp_old + force_p_new;
        h_vt_vel = fv_old + force_v_new;
        fp_old = h_vt_pos;
        fv_old = h_vt_vel;
    end
    figure
    plot(t1,h_vt_pos/0.0254)
    grid
    xlabel('time (sec)')
    ylabel('Amplitude of Excitation (inches)')
    title('Excitation in Vertical Direction')
end

if a == 2

```

```

    fp_old = zeros(max(size(t1)),1)';
    fv_old = zeros(max(size(t1)),1)';
    for i = 1:1:20
        force_p_new = AMP(i)*sin(omega(i)*t1);
        force_v_new = omega(i)*AMP(i)*cos(omega(i)*t1);
        %force_pos = [force_p' force_p(i,:)'];
        %force_vel = [force_v' force_v(i,:)'];

        h_tt_pos = fp_old + force_p_new;
        h_tt_vel = fv_old + force_v_new;
        fp_old = h_tt_pos;
        fv_old = h_tt_vel;
    end
    figure
    plot(t1,h_tt_pos/0.0254)
    grid
    xlabel('time (sec)')
    ylabel('Amplitude of Excitation (inches)')
    title('Excitation in Transverse Direction')

end

if a ==3
    fp_old = zeros(max(size(t1)),1)';
    fv_old = zeros(max(size(t1)),1)';
    for i = 1:1:20
        force_p_new = AMP(i)*sin(omega(i)*t1);
        force_v_new = omega(i)*AMP(i)*cos(omega(i)*t1);
        %force_pos = [force_p' force_p(i,:)'];
        %force_vel = [force_v' force_v(i,:)'];

        h_lt_pos = fp_old + force_p_new;
        h_lt_vel = fv_old + force_v_new;
        fp_old = h_lt_pos;
        fv_old = h_lt_vel;
    end
    figure
    plot(t1,h_lt_pos/0.0254)
    grid
    xlabel('time (sec)')
    ylabel('Amplitude of Excitation (inches)')
    title('Excitation in Longitudinal Direction')

end

if a ==4
    fp_old = zeros(max(size(t1)),1)';
    fv_old = zeros(max(size(t1)),1)';
    for i = 1:1:20
        force_p_new = AMP(i)*sin(omega(i)*t1);
        force_v_new = omega(i)*AMP(i)*cos(omega(i)*t1);
        %force_pos = [force_p' force_p(i,:)'];
        %force_vel = [force_v' force_v(i,:)'];

        h_vr_pos = fp_old + force_p_new;
        h_vr_vel = fv_old + force_v_new;
        fp_old = h_vr_pos;
        fv_old = h_vr_vel;
    end
    figure
    plot(t1,h_vr_pos*180/pi)
    grid
    xlabel('time (sec)')
    ylabel('Amplitude of Excitation (degrees)')
    title('Excitation about Vertical Direction')

end

if a ==5
    fp_old = zeros(max(size(t1)),1)';
    fv_old = zeros(max(size(t1)),1)';
    for i = 1:1:20
        force_p_new = AMP(i)*sin(omega(i)*t1);
        force_v_new = omega(i)*AMP(i)*cos(omega(i)*t1);
        %force_pos = [force_p' force_p(i,:)'];
        %force_vel = [force_v' force_v(i,:)'];

        h_tr_pos = fp_old + force_p_new;
        h_tr_vel = fv_old + force_v_new;
        fp_old = h_tr_pos;
        fv_old = h_tr_vel;
    end

```

```

end
figure
plot(t1,h_tr_pos*180/pi)
grid
xlabel('time (sec)')
ylabel('Amplitude of Excitation (degrees)')
title('Excitation about Transverse Direction')

end

if a ==6
fp_old = zeros(max(size(t1)),1)';
fv_old = zeros(max(size(t1)),1)';
for i = 1:i:20
force_p_new = AMP(i)*sin(omega(i)*t1);
force_v_new = omega(i)*AMP(i)*cos(omega(i)*t1);
%force_pos = [force_p' force_p(i,:)'];
%force_vel = [force_v' force_v(i,:)'];

h_lr_pos = fp_old + force_p_new;
h_lr_vel = fv_old + force_v_new;
fp_old = h_lr_pos;
fv_old = h_lr_vel;
end
figure
plot(t1,h_lt_pos*180/pi)
grid
xlabel('time (sec)')
ylabel('Amplitude of Excitation (degrees)')
title('Excitation about Longitudinal Direction')

end

end
v1 = 1*2*pi;
v2 = 1*2*pi;
v3 = 1*2*pi;
v4 = 1*2*pi;
v5 = 1*2*pi;
v6 = 1*2*pi;

```

Appendix D

Harmonic Simulation Code

D.1 Plant Model Code

```
%This program simulates the beam/mass system model with inputs
%determined from MDARS vehicle shock and vibration test data
%This program corresponds to run 15
%Author David C. Lennon
warning off
% Parameters of aluminum pipe

E = 7.10e10; %N/(m^2)
G = 2.67e10; %N/(m^2)

%Inside and outside pipe diameter

Dout = 6.625*0.0254; %meter
thickness = 0.0254*1/8; %meter
din = (Dout-2*thickness)*0.0254; %meter

%ft = input('What is the length of the beam (in feet) that you want to simulate?');

L = 8*12*0.0254; %ft*(in/ft)*(m/in) = m

area = (pi/4)*(Dout^2-din^2); %m^2

% Inertial parameters

Jp = pi*(Dout^4-din^4)/32 ; %m^4
I = (pi/64)*(Dout^4 - din^4); %m^4

%This is the equivalent mass for the support and pan/tilt head

m = 100*(0.454); %kg
g = 9.8;

%These are the inertia terms for the pan/tilt head

a = 24*0.0254; %meters
b = 12*0.0254; %meters
c = 16*0.0254; %meters

I2 = (1/12)*m*(b^2 + c^2); %kg*m^2 Mass moment of inertial in 2 direction
I3 = (1/12)*m*(a^2 + b^2); %kg*m^2 Mass moment of inertial in 3 direction
J = I3+I2; %kg*m^2

run15input
% model contains the dynamics of the system in state space form
% The following command calls the file model.m as an input to this program
%*****
system%
%*****

sys = ss(A,B,C,D);

%This is the matrix of inputs [U] 6x1 with frequencies v(i)
```

```

U = [h_vt_pos ;
      h_tt_pos ;
      h_lt_pos ;
      h_vr_pos ;
      h_tr_pos ;
      h_lr_pos ;
      h_vt_vel ;
      h_tt_vel ;
      h_lt_vel ;
      h_vr_vel ;
      h_tr_vel ;
      h_lr_vel] ;

%This is the function that evaluates the dynamic response of the system

[y,x] = lsim(A,B,C,D,U,t1);

%Convert to inches for y1 through y3 degrees for y4 through y6

ya = y(:,1)/0.0254;
yb = y(:,2)/0.0254;
yc = y(:,3)/0.0254;
yd = y(:,4)*(180/pi);
ye = y(:,5)*(180/pi);
yf = y(:,6)*(180/pi);

%These are the figures for the output vectors as a function of time

figure
hold on
title('Sensor Pod Translational Displacement in the Vertical Direction')
xlabel('time (sec)')
ylabel('Translational Displacement (inches)')
plot(t1,ya'+U(1,:)/0.0254,'b')
%plot(t1,ya,'g')
grid
print -dmeta hari5a
hold off

figure
hold on
title('Sensor Pod Translational Displacement in the Transverse Direction')
xlabel('time (sec)')
ylabel('Translational Displacement (inches)')
plot(t1,yb'+U(2,:)/0.0254,'b')
%plot(t1,yb,'g')
grid
print -dmeta hari5b
hold off

figure
hold on
title('Sensor Pod Translational Displacement in the Longitudinal Direction')
xlabel('time (sec)')
ylabel('Translational Displacement (inches)')
plot(t1,yc'+U(3,:)/0.0254,'b')
%plot(t1,yc,'g')
grid
print -dmeta hari5c
hold off

figure
hold on
title('Sensor Pod Rotational Displacement about the Vertical Direction')
xlabel('time (sec)')
ylabel('Rotational Displacement (degrees)')
plot(t1,yd'+U(4,:)*180/pi,'b')
%plot(t1,yd,'g')
grid
print -dmeta hari5d
hold off

figure
hold on
title('Sensor Pod Rotational Displacement about the Transverse Direction')
xlabel('time (sec)')
ylabel('Rotational Displacement (degrees)')
plot(t1,ye'+U(5,:)*180/pi,'b')
%plot(t1,ye,'g')
grid

```

```

print -dmeta har15e
hold off

figure
hold on
title('Sensor Pod Rotational Displacement about the Longitudinal Direction')
xlabel('time (sec)')
ylabel('Rotational Displacement (degrees)')
plot(t1,yf'+U(6,:)*180/pi,'b')
%plot(t1,yf,'g')
grid
print -dmeta har15f
hold off

```

```

figure
hold on
plot(t1,ya,'b');%'+U(1,:)/0.0254,'b')
gtext('Vertical')
plot(t1,yb,'r');%'+U(2,:)/0.0254,'r')
gtext('Transverse')
plot(t1,yc,'g');%'+U(3,:)/0.0254,'g')
gtext('Longitudinal')
xlabel('time (sec)')
ylabel('Translational Displacement (inches)')
title('Translational Displacement of Sensor Pod')
axis([0 2 -2 2])
grid
print -dmeta run15trans
hold off

```

```

figure
hold on
plot(t1,yd,'b');%'+U(4,:)*180/pi,'b')
gtext('Yaw')
plot(t1,ye,'r');%'+U(5,:)*180/pi,'r')
gtext('Pitch')
plot(t1,yf,'g');%'+U(6,:)*180/pi,'g')
gtext('Roll')
xlabel('time (sec)')
ylabel('Rotational Displacement (degrees)')
title('Rotational Displacement of Sensor Pod')
axis([0 2 -3.5 3.5])
grid
print -dmeta run15rot
hold off

```

Appendix E

Amplitude Spectrum for Test Run 4 Data

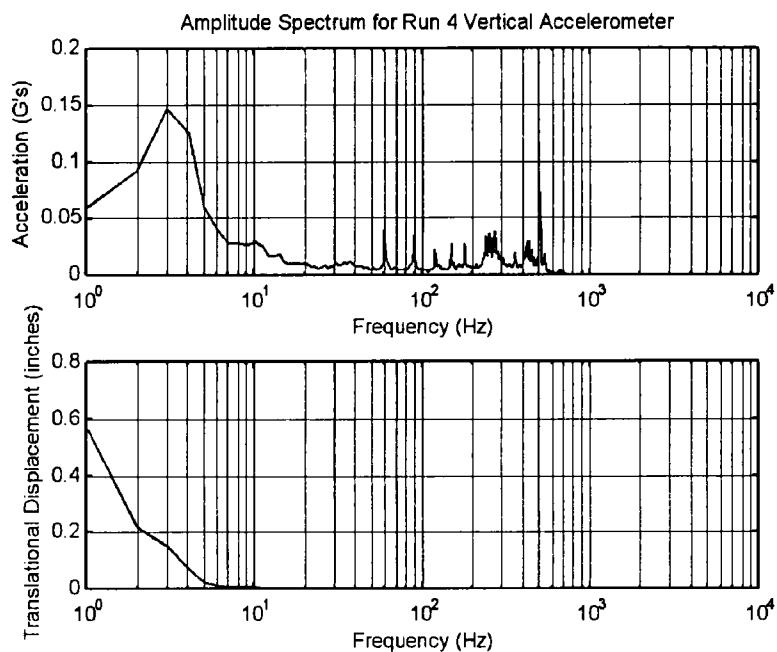


Figure E.1: Amplitude Plot for Run 4 Vertical Accelerometer.

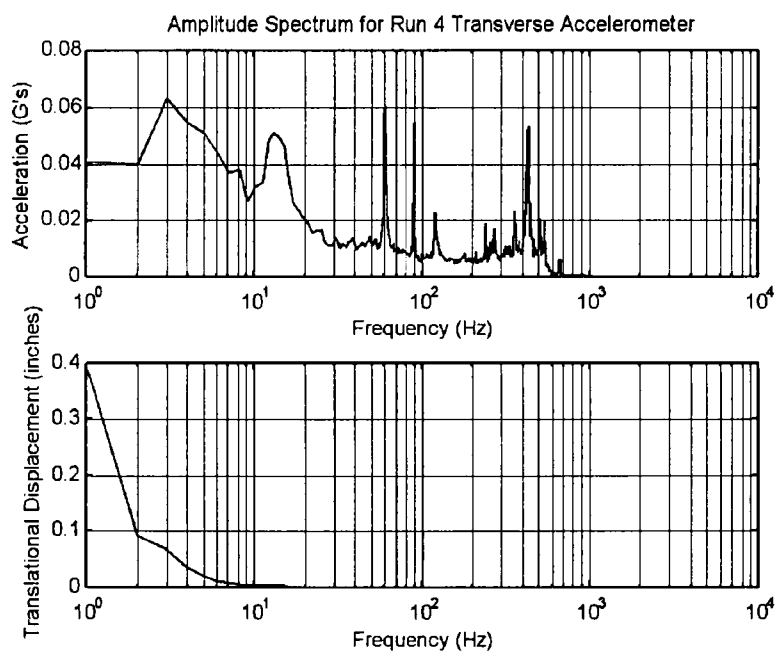


Figure E.2: Amplitude Plot for Run 4 Transverse Accelerometer.

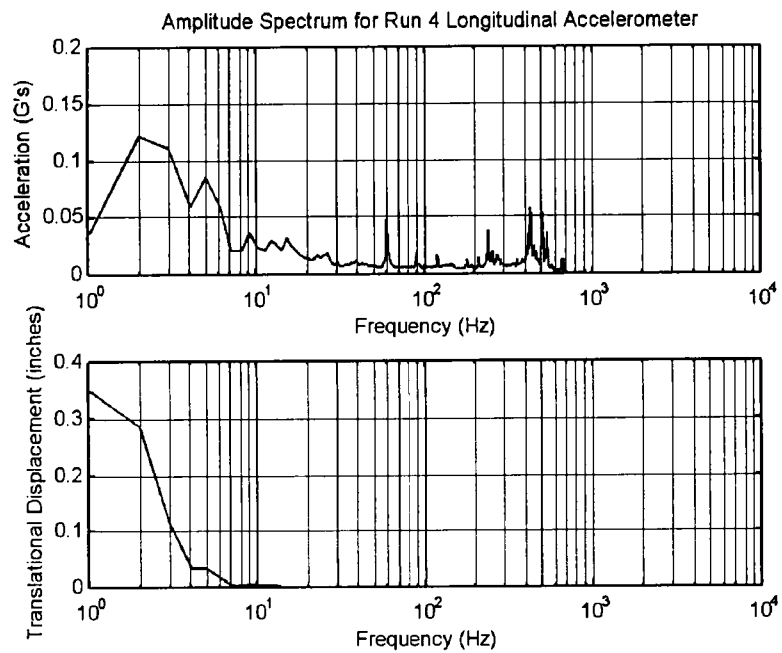


Figure E.3: Amplitude Plot for Run 4 Longitudinal Accelerometer.

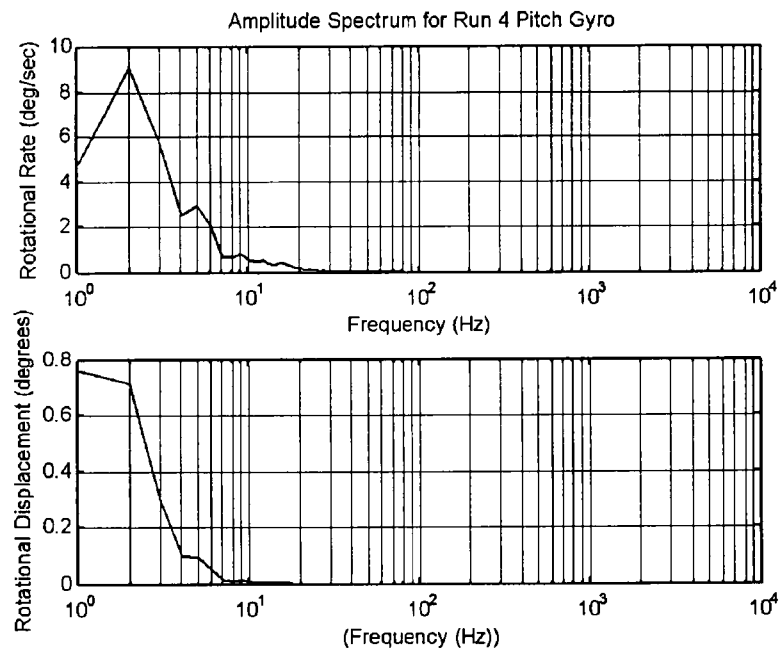


Figure E.4: Amplitude Plot for Run 4 Pitch Gyro.

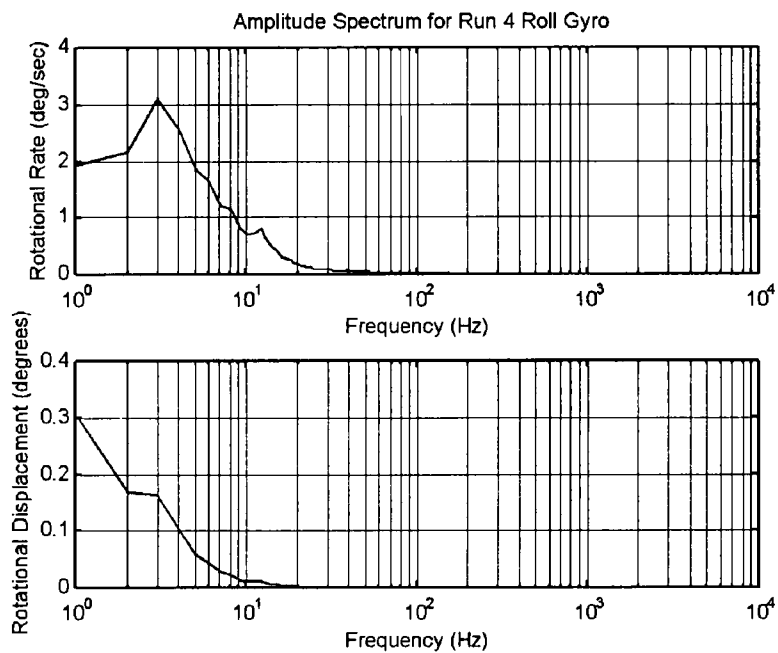


Figure E.5: Amplitude Plot for Run 4 Roll Gyro.

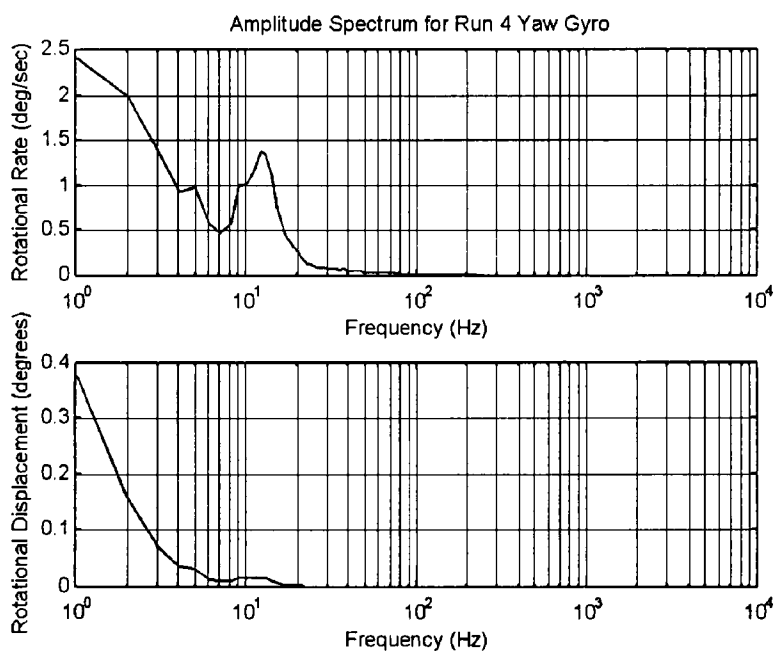


Figure E.6: Amplitude Plot for Run 4 Yaw Gyro.

Appendix F

Amplitude Spectrum for Test Run 8 Data

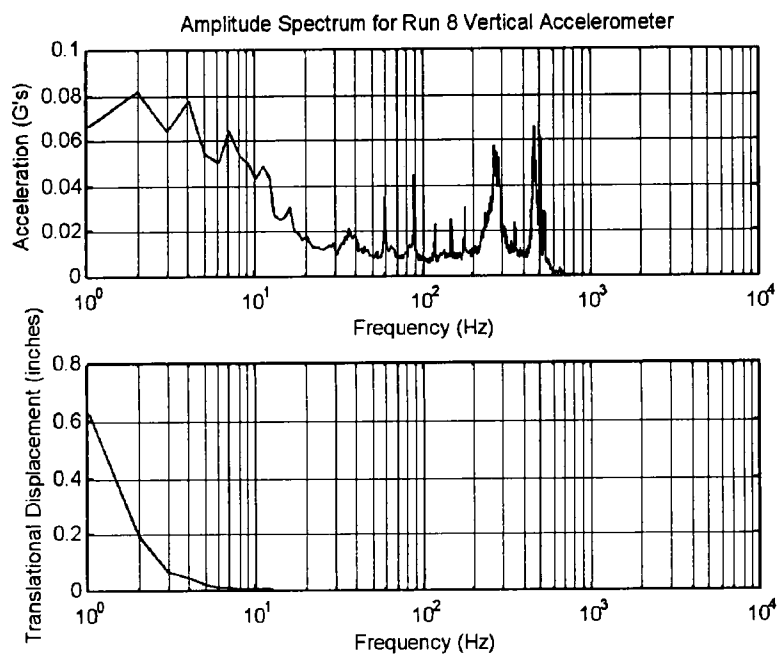


Figure F.1: Amplitude Plot for Run 8 Vertical Accelerometer.

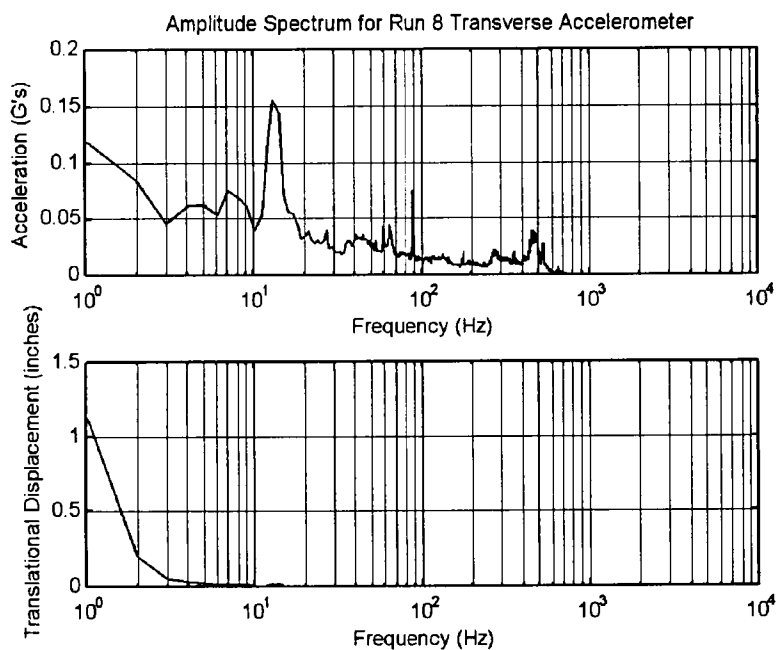


Figure F.2: Amplitude Plot for Run 8 Transverse Accelerometer.

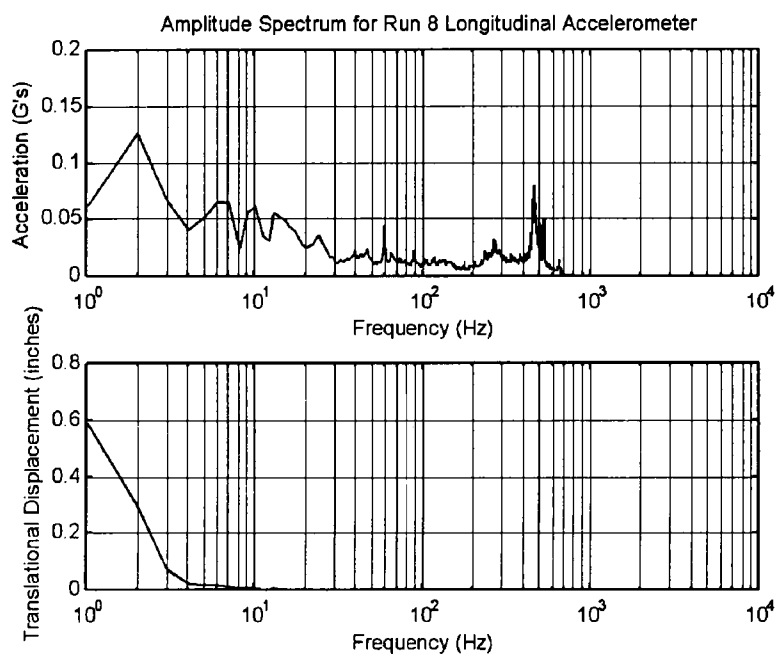


Figure F.3: Amplitude Plot for Run 8 Longitudinal Accelerometer.

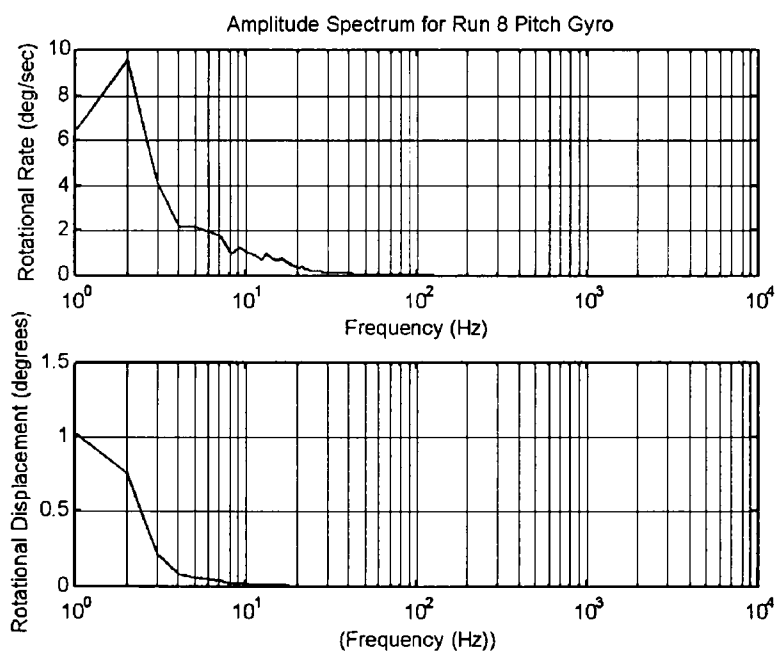


Figure F.4: Amplitude Plot for Run 8 Pitch Gyro.

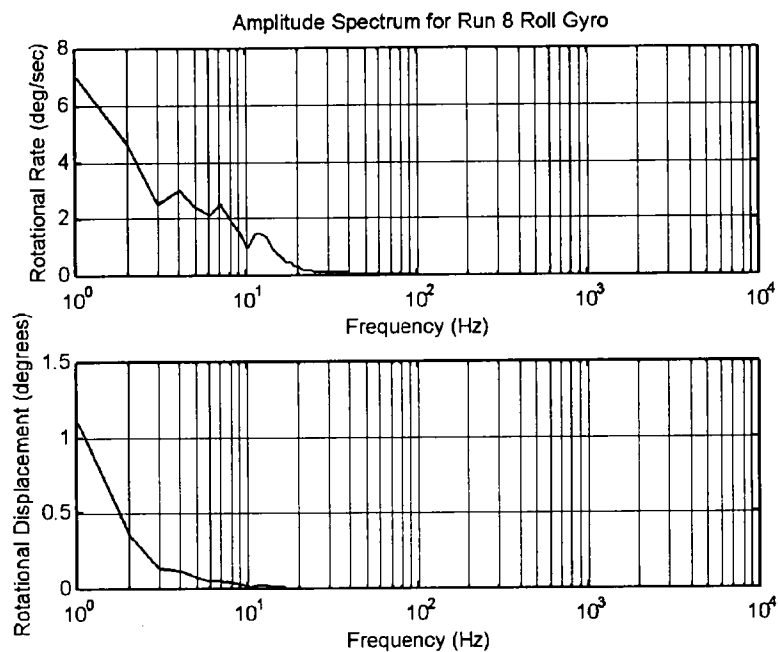


Figure F.5: Amplitude Plot for Run 8 Roll Gyro.

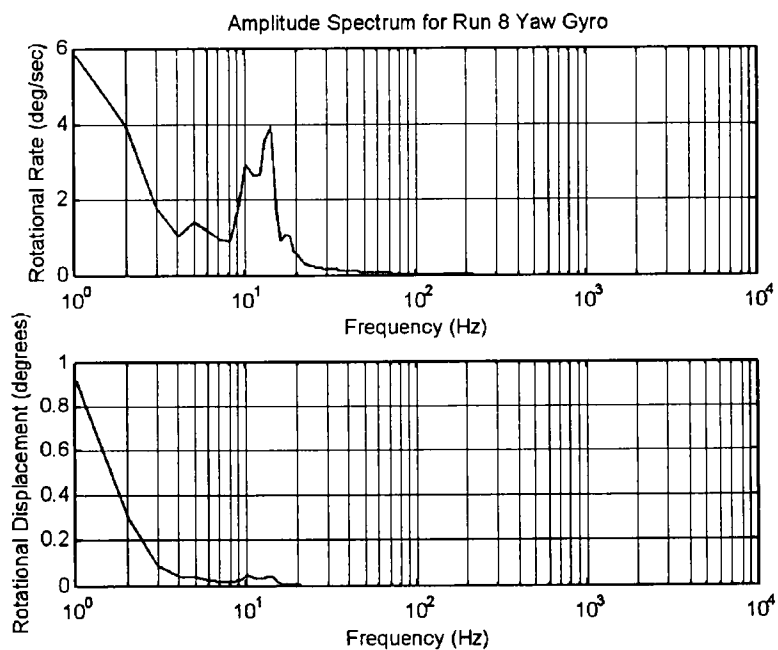


Figure F.6: Amplitude Plot for Run 8 Yaw Gyro.

Appendix G

Amplitude Spectrum for Test Run 11 Data

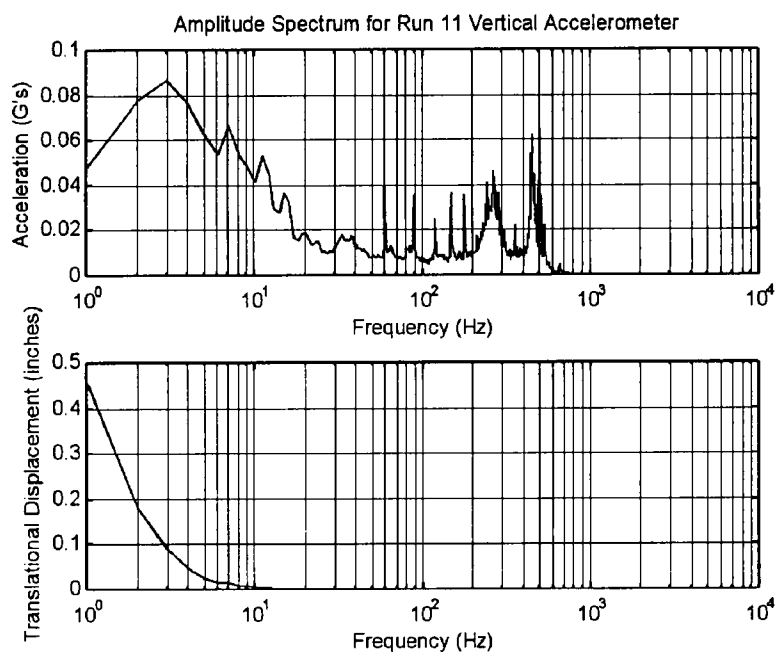


Figure G.1: Amplitude Plot for Run 11 Vertical Accelerometer.

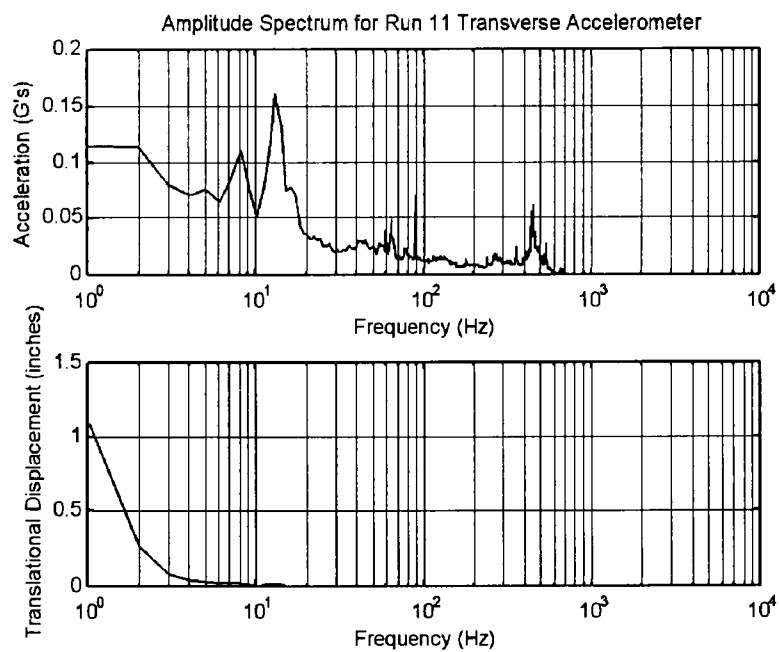


Figure G.2: Amplitude Plot for Run 11 Transverse Accelerometer.

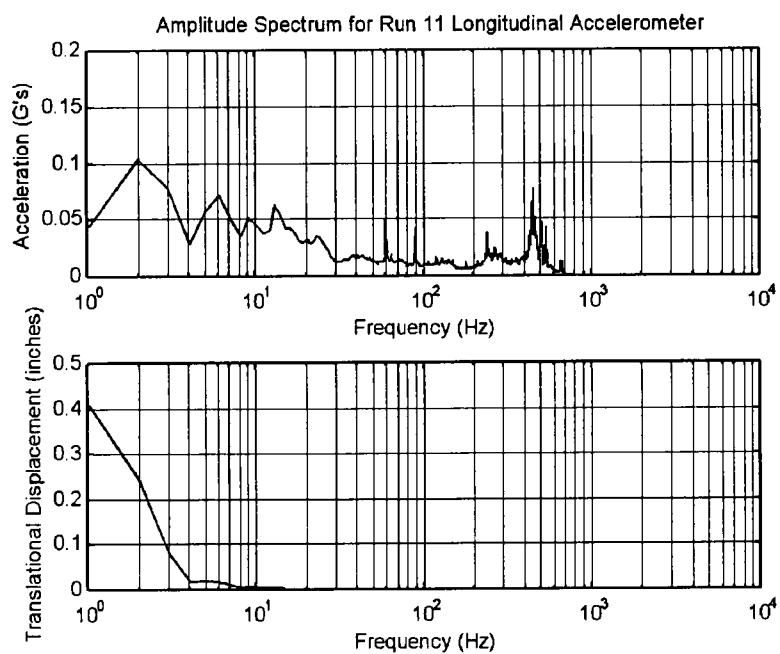


Figure G.3: Amplitude Plot for Run 11 Longitudinal Accelerometer.

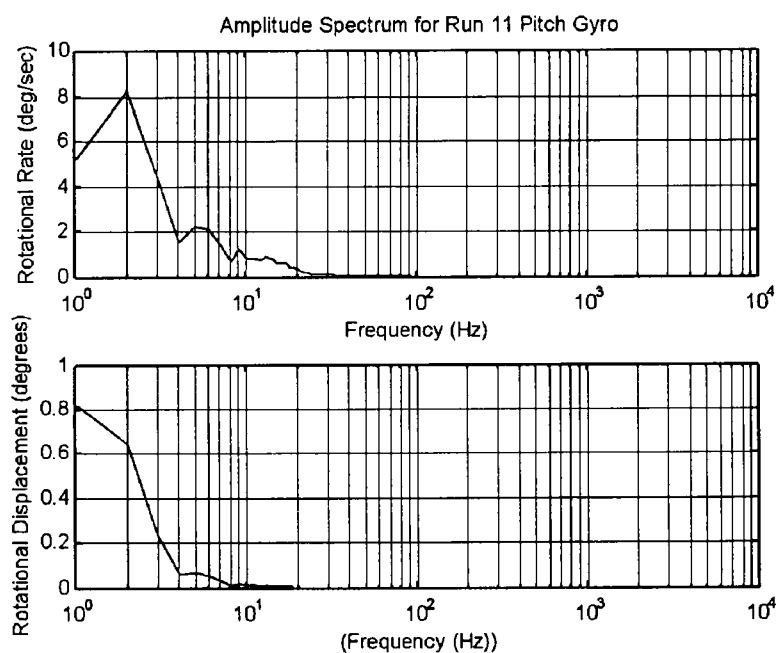


Figure G.4: Amplitude Plot for Run 11 Pitch Gyro.

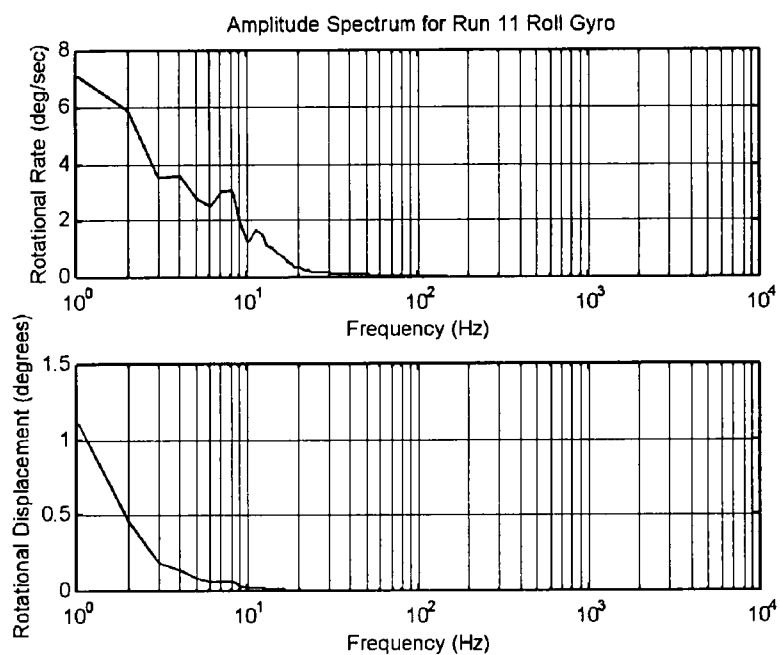


Figure G.5: Amplitude Plot for Run 11 Roll Gyro.

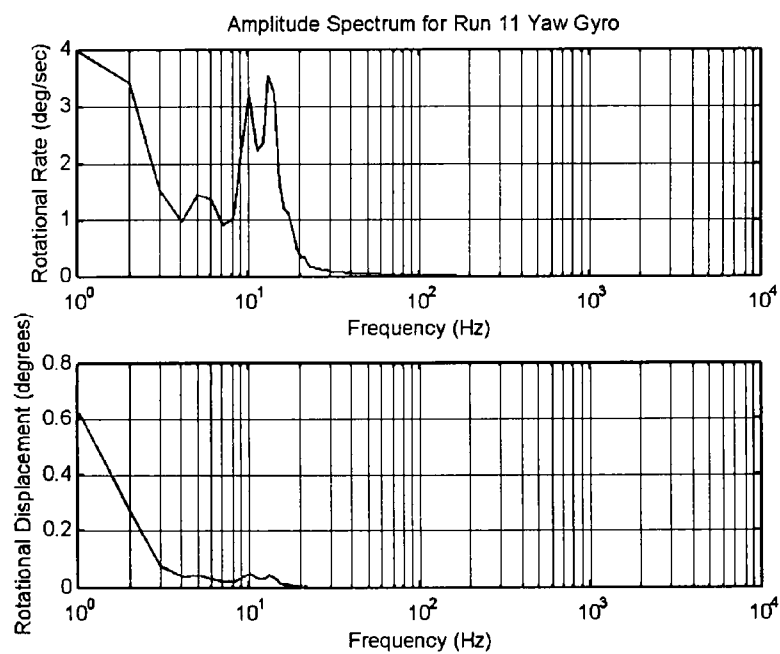


Figure G.6: Amplitude Plot for Run 11 Yaw Gyro.

Appendix H

Amplitude Spectrum for Test Run 15 Data

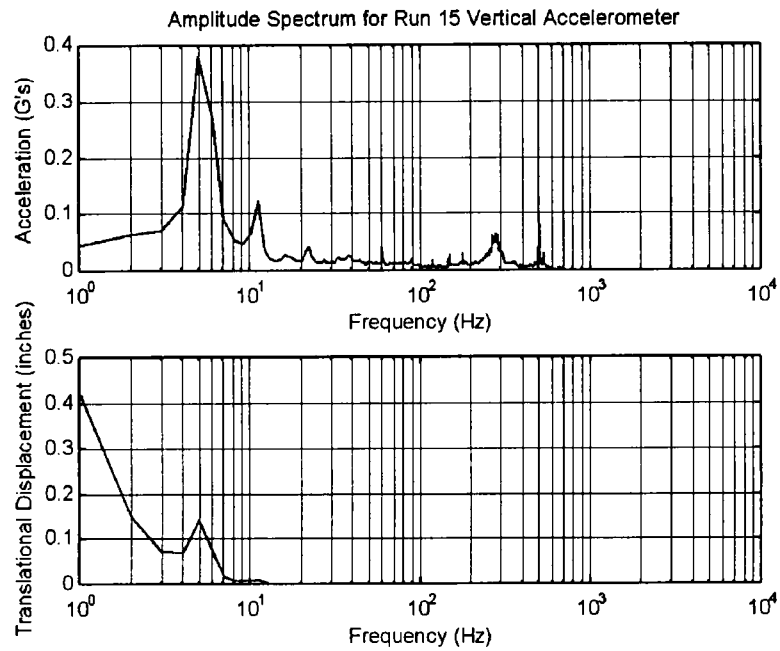


Figure H.1: Amplitude Plot for Run 15 Vertical Accelerometer.

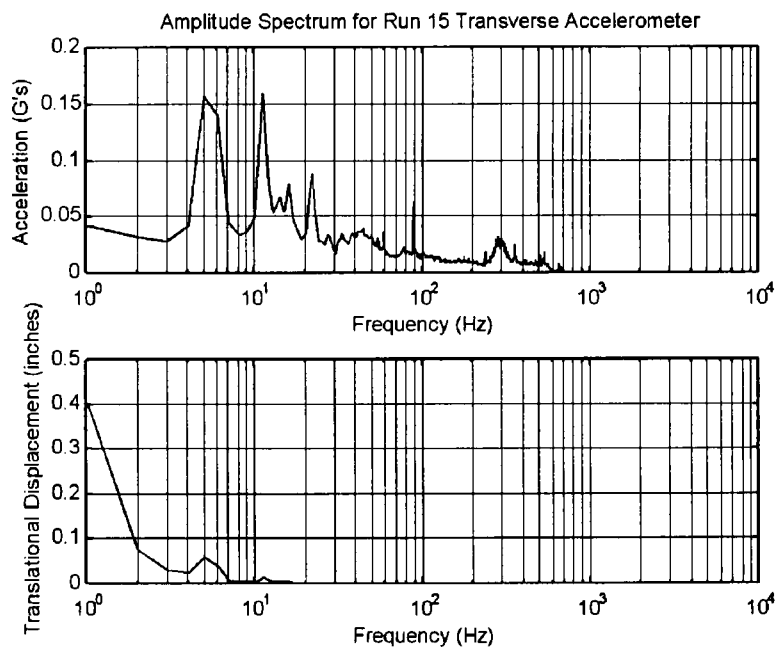


Figure H.2: Amplitude Plot for Run 15 Transverse Accelerometer.

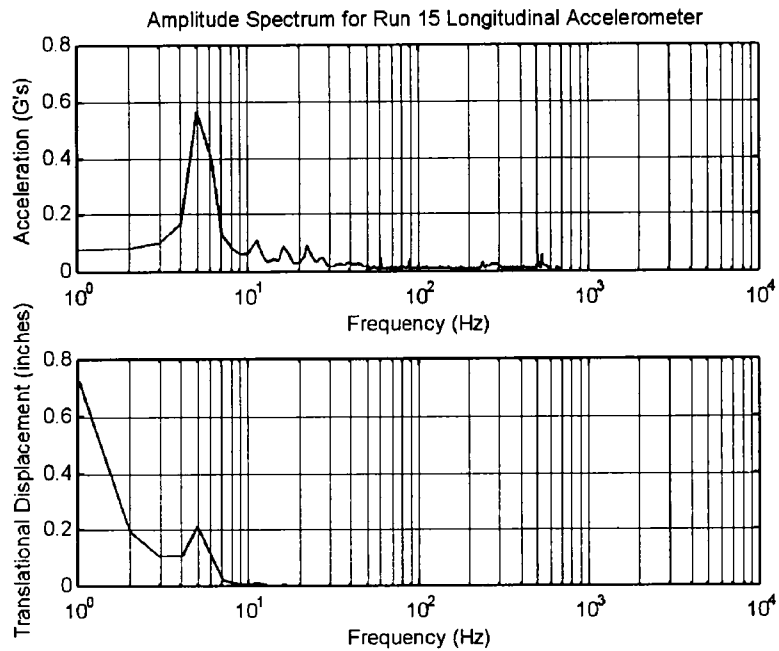


Figure H.3: Amplitude Plot for Run 15 Longitudinal Accelerometer.

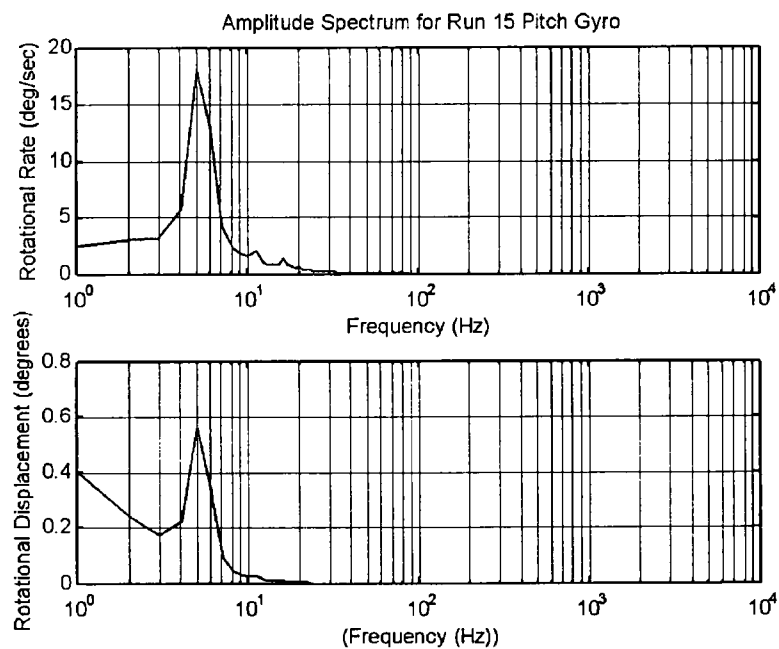


Figure H.4: Amplitude Plot for Run 15 Pitch Gyro.

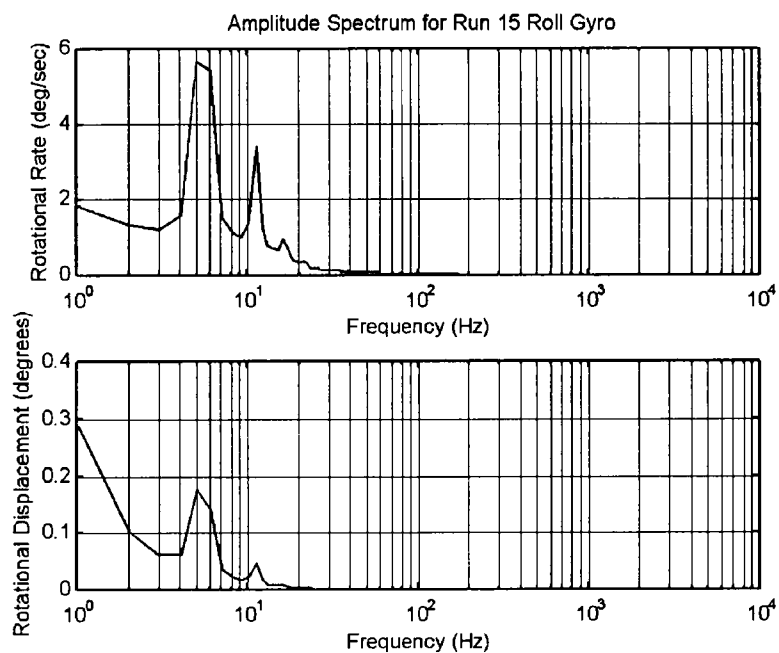


Figure H.5: Amplitude Plot for Run 15 Roll Gyro.

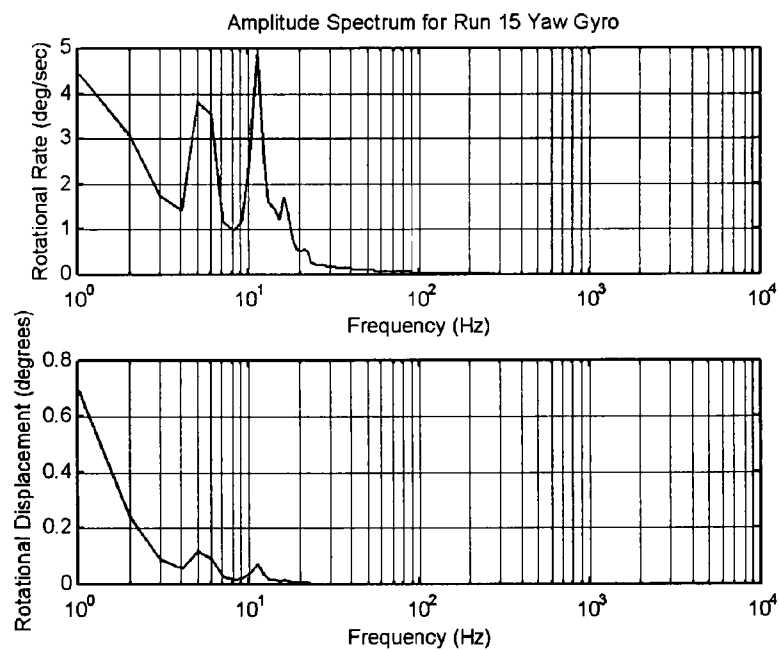


Figure H.6: Amplitude Plot for Run 15 Yaw Gyro.

Vita

David Christopher Lennon was born in Memphis, Tennessee on October 31, 1973. David lived in Memphis for eight years and then moved to Nashville, Tennessee. At a very young age David became involved in sports activities, namely soccer. Outdoor activities were always high on David's priorities. David attended Father Ryan High School in Nashville, Tennessee where he was very active in sports, various clubs, and other extracurricular activities. David was the president of his senior class, president of The Fellowship of Christian Athletes, and attended leadership conferences. After completing high school in 1992 David attended Loyola University in New Orleans where he entered into the pre-engineering program. After his first year of college he transferred to The University of Tennessee, Knoxville where he continued his studies in Mechanical Engineering. David became an active member in $\Phi\Delta\Theta$ fraternity, he was a member of The University of Tennessee's men's soccer team, and he was involved with the Habitat for Humanity program. In 1996 David graduated with a Bachelor of Science with a major in Mechanical Engineering. The summer of 1996 David back-packed across Europe for two months with three close friends. Upon his return from Europe, David entered into the master's program at the University of Tennessee, Knoxville. In 1998 David graduated with a Master of Science with a major in Mechanical Engineering.

David then went to work for International Paper as a project engineer on July, 6 1998. He is currently residing in New Boston, Texas where he works in the Wood Products Division for International Paper.