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I am submitting herewith a thesis written by Eng Chye Yew entitled "Reliability Analysis of Two-Way Flat Plate Floor Slab Systems". I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Civil Engineering.

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Reliability Analysis
Of
Two-Way Flat Plate
Floor Slab Systems

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Master of Science
Degree
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Eng Chye Yew

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DEDICATION

This thesis is dedicated to my parents

Chin Seng Yew and Siew Looi Tan.

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Abstract

In concrete design, progress has been made in revising the current ACI-318 concrete building code to conform to reliability based design specifications. Among the structural components commonly used in concrete buildings is the two-way floor slab. While rigorous reliability analyses have been performed for structural components such as beams and columns, none has been done for the two-way slab system. In order to provide a comprehensive transition to a reliability based concrete design code in future editions of the ACI code, it is necessary to study the reliability index of two-way slab systems using current design practices. This study will serve as a guide to update future design specifications so that the designs will have consistent reliability based on rational assessment. In this thesis, the reliability index for a range of load values and slab sizes is computed. Comments on the reliability index of both the direct design method and finite element method are offered in the discussion of results and conclusions of this thesis.

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Chapter 1

INTRODUCTION

Reliability based design is an inherent part of today's building codes. Several building codes, such as the Load and Resistance Factor Design (LRFD) of the American Institute of Steel Construction (AISC, 1994) and the Canadian Standard Association building codes (CSA, 1985), are reliability based design codes. Recognizing the existence of uncertainties in the applied loads such as people, equipment, wind, snow, and earthquake, and further uncertainties in the resistance such as concrete strength, steel strength, and member size, the ACI code has recommended the use of load and resistance factors in their designs. However, design by the current code does not result in a consistent level of safety (Israel et al., 1987).

The reliability index is a well-known safety measure in the field of structural engineering. Many results of reliability studies have been used to develop reliability based design codes in steel, aluminium, masonry, and timber. In concrete design, reliability studies have been performed for different structural components such as beams and columns in

order to provide references for future editions of the concrete design code. The focus of this thesis is the reliability analysis of reinforced concrete two-way flat slabs.

ACI recommends two design methods for two-way floor slab systems: the "direct design method" and the "equivalent frame method". The method used in this study was the direct design method. The reliability analysis performed consists of two cases. Case 1 evaluates the reliability index of the flexure design by direct design method with the moments used for designs. Case 2 evaluates the reliability index of the flexure design by direct design method with moments obtained using the finite element method.

Different floor slab sizes and loading ratios were studied to investigate the reliability indices of the current ACI design code. Shorter span lengths of 15 ft and 20 ft were chosen, and span ratios of 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0 were studied to determine the consistency of the reliability index at live load to dead load ratios of 0.5, 1.0, 1.5, 2.0, and 2.5.

Chapter 2 describes the structural design practices based on the current design philosophy.

Chapter 3 describes the first order second moment (FOSM) method used to compute the reliability index. Chapter 4 outlines the current design codes by ACI 318-95 for a two-way floor slab system. Chapter 5 describes the elastic analysis of a two-way floor slab system using the finite element method. Chapter 6 discusses the implementation of the analysis. Chapter 7 discusses the results. Finally, Chapter 8 presents the conclusions and recommendations for future study.

Chapter 2

DESIGN BASIS

The main objective of structural design is to provide sufficient resistance to all kinds of loading. Hence, a structure is said to be "safe" if the resistance of a structure is equal to or greater than the loading applied to it. This can be expressed as:

$$R \geq \sum Q \quad (2.1)$$

Where R = Resistance or strength of the structure.

Q = Load effect due to gravitational or environmental forces.

Hence, in designing a particular structure, it is logical to proportion the members to meet the above criteria.

Failure is mainly defined as the load exceeding the actual resistance. However, failure does not necessarily refer to total collapse; it may mean that

a specific limit state is reached. A limit state is defined as, "a condition beyond which a structure or member becomes unfit for service and is judged either to be no longer useful for its intended function (serviceability limit state) or to be unsafe (strength limit state)" (ASCE Standard, 1995). The serviceability limit state mainly corresponds to limits on the adequacy of stiffness. The adequacy of stiffness influences behavior such as deflections, lateral drift, or vibration of the structures that could adversely affect the intended uses and performances of the buildings or structures.

To account for uncertainties that exist in loads and resistances, safety margins are included in all designs. The American Concrete Institute's ACI 318-95 code recommends that reduction factors be applied to the resistance and that amplification factors be applied to the loads. This can be expressed as:

$$\phi R \geq \sum \gamma Q \quad (2.2)$$

Where ϕ = Resistance factor (usually < 1)

γ = Load factor (usually > 1)

This procedure is to secure against the unexpected lower resistance due to material and dimension variabilities, and the amplification factors take into account the uncertainty of unusually large load magnitudes. Further, ACI recommends different load factors for dead loads and live loads. It also recommends the use of an overload factor for the combination of earthquake and wind loads to account for the extremely unusual event when these two loads occur simultaneously. For example, for dead load and live load combined,

$$\phi R \geq \gamma_d D + \gamma_L L \quad (2.3)$$

Where γ_d and γ_L are the load factors for dead load and live load respectively.

The dead load can have a lower load factor than live load; 1.4 for dead load and 1.7 for live load are specified in ACI 318-95. This is due to the fact that dead load can be predicted more precisely than live

load. The predicted live load has a larger range of variance than dead load.

On the resistance side of the design equation, different values of reduction factors are employed depending on the consequence of failure and the variability of strength capacity. Thus, the reduction factor for bending is higher than the reduction factor for shear and bearing. This is due to bending failure is a slower process, and is usually proceeded by warnings, while shear failure is much quicker and occurs without warning.

Chapter 3

RELIABILITY AND ITS MEASUREMENT

3.1 Definition of Reliability

A system or structure is considered safe as long as the supply is greater than the demand. In terms of structural engineering practices, the resistance needs to be greater than the load effects. Defining the capability of a structure to perform adequately is not an easy task, since both the applied loads and resistance of the structural members are random variables that cannot be determined with absolute certainty, as illustrated in Figure 3.1. Hence, one cannot ensure a structure's absolute safety due to the uncertainty of variables and instead must rely on some probabilistic concept that indicates the likelihood that the available strength will adequately withstand the predicted loads.

From Figure 3.1, one can say that the structure is safe, since the mean of R is greater than the mean of Q . However both values of R and Q are random

variables that contain uncertainties as characterized by the distribution functions shown. Hence, there is still a small probability that the structure can fail. For a specific load effect, q , failure can occur when $R < q$, and the probability of failure is represented by the shaded area in Figure 3.1.

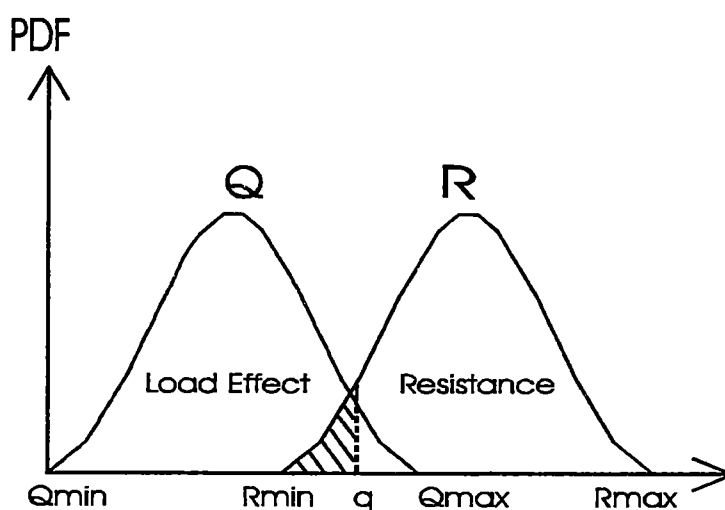


Figure 3.1: Probability density function of load effect, Q , and resistance, R .

One of the commonly used methods to evaluate the safety of a structural member is safety margin, which can be expressed as:

$$SM = R - Q \quad (3.1)$$

Safety margin is simply the random difference between the resistance, R , and load effects, Q , and is greater than zero for a safe structural member. Since both R and Q are random variables, SM is also a random variable that follows its own probability density function (pdf), as shown in Figure 3.2.

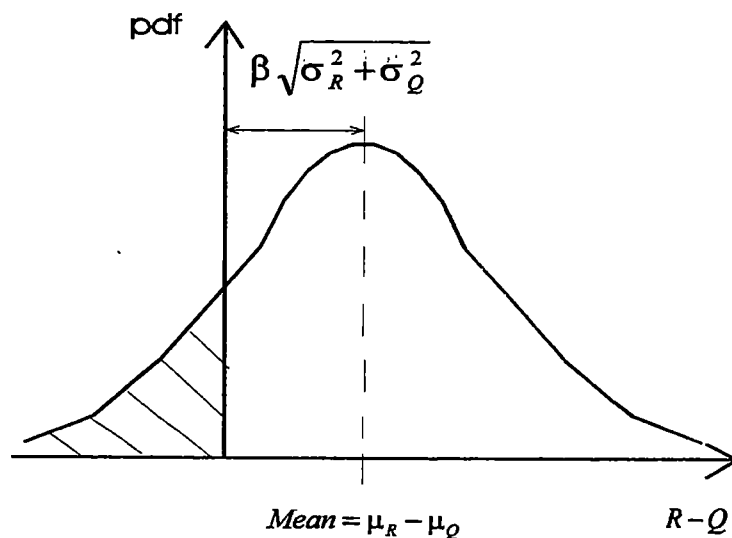


Figure 3.2: Probability distribution function of the safety margin.

In the cases where R and Q are independent of each other, the mean and variance of the safety margin can be defined as follows:

$$\mu_{SM} = \mu_R - \mu_Q \quad \text{and} \quad \sigma_{SM}^2 = \sigma_R^2 + \sigma_Q^2 \quad (3.2)$$

Where μ_{SM} is the mean of the safety margin.

σ_{SM}^2 is the variance of the safety margin.

The probability of failure is defined as the chance of the event where $SM < 0$, which can be mathematically expressed as:

$$p_f = \int_{-\infty}^0 f_{SM}(sm) d(sm) = F_{SM}(0) \quad (3.3)$$

In the cases where SM is a normal distribution as the result of both R and Q are normally distributed, the reliability of the structural member can be derived as follows:

$$r = 1 - F_{SM}(0) = 1 - \Phi\left[\frac{0 - \mu_{SM}}{\sigma_{SM}}\right] = 1 - \left[1 - \Phi\left(\frac{\mu_{SM}}{\sigma_{SM}}\right)\right] = \Phi\left(\frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}\right) \quad (3.4)$$

Where $\Phi(-)$ = cdf (cumulative distribution function) of the standard normal distribution evaluated at -.

The $F_{SM}(0)$ is the probability of failure, which can be represented by the shaded area in Figure 3.2.

Another measure of the adequacy of a structural design is to evaluate its reliability index, which is the ratio between the mean and standard deviation of the safety margin:

$$\beta = \frac{\mu_{SM}}{\sigma_{SM}} = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (3.5)$$

Substituting Eqn.(3.5) into Eqn.(3.4), the reliability becomes:

$$r = \Phi(\beta) \quad (3.6)$$

The further away the mean of the safety margin is from the Y-axis of Figure 3.2, the smaller the probability of failure, p_f . By definition, the

reliability index is also the reciprocal of the coefficient of variation of the safety margin; that is, $\beta = 1/V_{SM}$.

3.2 Measure of Reliability Index

3.2.1 Geometric Formulation of FOSM

The safety margin of a particular structural design is not usually composed of two variables. Instead it may involve multiple variables. In that case Eqn. 3.1 can be expressed as follows:

$$g(\mathbf{X}) = g(X_1, X_2, \dots, X_n) \quad (3.7)$$

In which $g(\mathbf{X})$ is the performance function of the system, and the above equation defines its limit state. Therefore, $g(\mathbf{X}) > 0$ is the safe state, and $g(\mathbf{X}) < 0$ is the failure state of a system. The probability of the failure state can be defined as:

$$P_f = \int \dots \int_{g(x_1, \dots, x_n) < 0} f_x(x_1, x_2, \dots, x_n) dx_1 \dots dx_n \quad (3.8)$$

The mathematical implementation of solving Eqn.3.8 is a difficult procedure. In general, Eqn. (3.8) requires the knowledge of a joint probability distribution function among all random variables. Even when all the random variables are mutually independent, the task of evaluating Eqn. (3.8) is still not simple.

A solution to the problem called the first order second moment method (FOSM) was proposed by Allin Cornell (Cornell, 1969). The FOSM method approximates the performance function as a linear function. Hence, only knowledge of the first and second moments of the random variables are necessary. By the central limit theorem, the performance function can be approximated by the normal distribution. The reliability of a particular system can be expressed in a form similar to safety margin discussed earlier. The reliability measure is then represented by a value called reliability index, β .

For a linear performance function, the exact reliability index can be computed, while for non-linear performance function, geometric formulation

is used to approximate the analysis. The geometric formulation is analytically applicable to both linear and non-linear limit state function.

Consider a performance function with only two statistically independent normal variables, R and Q . The performance function of the limit state can be expressed as:

$$g = R - Q = 0 \quad (3.9)$$

Defining the reduced variables for both R and Q :

$$R' = \frac{R - \mu_R}{\sigma_R} \quad \text{and} \quad Q' = \frac{Q - \mu_Q}{\sigma_Q} \quad (3.10)$$

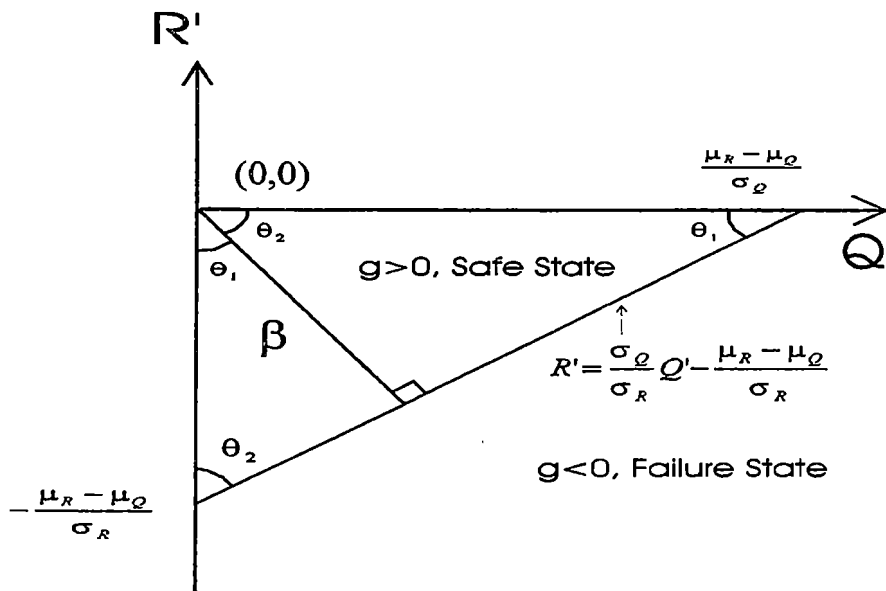
In terms of R' and Q' , Eqn. (3.10) becomes:

$$R = R'\sigma_R + \mu_R \quad \text{and} \quad Q = Q'\sigma_Q + \mu_Q \quad (3.11)$$

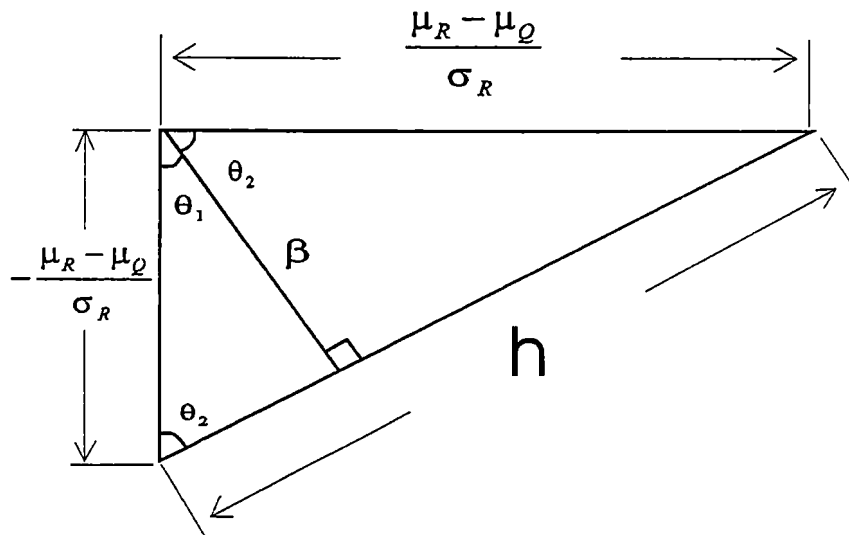
Substituting Eqn. (3.11) into Eqn. (3.9) yields:

$$g(R', Q') = R'\sigma_R - Q'\sigma_Q + \mu_R - \mu_Q = 0 \quad (3.12)$$

Figure 3.3 represents the performance function in the "reduced coordinate system" with the straight line that is generated by the function given by Eqn. (3.12).



a): Geometric formulation of FOSM



b): Geometry of FOSM

Figure 3.3: Geometric formulation of FOSM and geometry of FOSM

The reliability index is represented by the shortest distance from the origin to the failure surface (see Figure 3.3b) and it can be solved geometrically as the following:

- i) Determine the hypotenuse, h of the right triangle, which is

$$h = \frac{\mu_R - \mu_Q}{\sigma_R \sigma_Q} \sqrt{\sigma_R^2 + \sigma_Q^2} \quad (3.13)$$

- ii) Determine the cosine of angles θ_1 and θ_2 geometrically:

$$\cos \theta_1 = \frac{\mu_R - \mu_Q}{\sigma_Q} \frac{\sigma_R \sigma_Q}{(\mu_R - \mu_Q) \sqrt{\sigma_R^2 + \sigma_Q^2}} = \frac{\sigma_R}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (3.14)$$

$$\cos \theta_2 = -\frac{\mu_R - \mu_Q}{\sigma_R} \frac{\sigma_R \sigma_Q}{(\mu_R - \mu_Q) \sqrt{\sigma_R^2 + \sigma_Q^2}} = -\frac{\sigma_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (3.15)$$

- iii) Let

$$\alpha_Q = \cos \theta_2 = \frac{-q^*}{\beta} \Rightarrow q^* = -\beta \alpha_Q \quad (3.16)$$

$$\alpha_R = \cos \theta_1 = \frac{-r^*}{\beta} \Rightarrow r^* = -\beta \alpha_R \quad (3.17)$$

where: α_R and α_Q are the direction cosines for the random variables R and Q respectively.

iv) Substituting r^* and q^* into Eqn.(3.9) yields:

$$g = -\beta\alpha_R\sigma_R + \beta\alpha_Q\sigma_Q + \mu_R - \mu_Q \quad (3.18)$$

v) Solving for β yields:

$$\beta = \frac{\mu_R - \mu_Q}{\alpha_R\sigma_R - \alpha_Q\sigma_Q} \quad (3.19)$$

As expected, Eqn. 3.19 is identical to Eqn. 3.5.

3.2.2 Multiple Variables in Advanced FOSM

In general, the performance function often follows a non-linear function that contains several variables. The performance function or limit state function may be expressed as:

$$Z = g(X_1, X_2, \dots, X_n) \quad (3.20)$$

that defines a critical hypersurface in n -dimensional space, such that $Z > 0$ represents safe state, and $Z < 0$

as failure state. Expanding this function into a Taylor series at the checking point yields:

$$Z^* = g(x_1^*, x_2^*, \dots, x_n^*) + \sum_{i=1}^n (x_i - x_i^*) \left(\frac{\partial g}{\partial x_i} \right) + \sum_{j=1}^n \sum_{i=1}^n \frac{1}{2} (x_i - x_i^*) (x_j - x_j^*) \frac{\partial^2 g}{\partial x_i \partial x_j} + \dots \quad (3.21)$$

The derivatives are evaluated at the checking point $(x_1, x_2, \dots, x_n)$. The standardized variables are:

$$\begin{aligned} X' &= \frac{X_i - \mu_X}{\sigma_X} \Rightarrow X_i = \sigma_X X' + \mu_X \\ x_i^* &= \frac{x_i^* - \mu_X}{\sigma_X} \Rightarrow x_i^* = \sigma_X x_i^{*'} + \mu_X \end{aligned} \quad (3.22)$$

and

$$\frac{\partial g}{\partial X_i} = \frac{\partial g}{\partial X_i'} \left(\frac{\partial X_i'}{\partial X_i} \right) = \frac{1}{\sigma_X} \left(\frac{\partial g}{\partial X_i'} \right) \quad (3.23)$$

The checking point $g(\mathbf{x}^*)$ is on the failure surface in which $g(\mathbf{x}^*)=0$. Since this is a first order approximate analysis, the second and higher orders terms are ignored, and we obtain:

$$g(X_1, X_2, \dots, X_n) = \sum_{i=1}^n (X_i' - X_i^{*'}) \left(\frac{\partial g}{\partial X_i'} \right) + \dots \quad (3.24)$$

If all the X_i 's are mutually independent, the statistical moments of $g(\mathbf{X})$ can be determined as:

$$\begin{aligned}\mu_g &= -\sum_{i=1}^n X_i \cdot \left(\frac{\partial g}{\partial X_i} \right) \\ \sigma_g^2 &= \sum_{i=1}^n \sigma_{X_i}^2 \left(\frac{\partial g}{\partial X_i} \right)^2 = \sum_{i=1}^n \left(\frac{\partial g}{\partial X_i} \right)^2\end{aligned}\quad (3.25)$$

Since the reliability index is the inverse of the coefficient of variation, i.e. μ/σ , the β value can be expressed as the following for uncorrelated random variables:

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{-\sum_{i=1}^n X_i \cdot \left(\frac{\partial g}{\partial X_i} \right)}{\sqrt{\sum_{i=1}^n \left(\frac{\partial g}{\partial X_i} \right)^2}}\quad (3.26)$$

The shortest distance from the origin to the failure surface in the reduced space should be identified as the most probable point in determining the β values. In scalar form, this point of failure can be defined as:

$$x_i^* = -\alpha_i^* \beta \quad i = 1, 2, \dots, n \quad (3.27)$$

The direction cosines can be defined as:

$$\alpha_i^* = \frac{-\sum_{i=1}^n \left(\frac{\partial g}{\partial X_i} \right)_*}{\sqrt{\sum_{i=1}^n \left(\frac{\partial g}{\partial X_i} \right)_*^2}} \quad (3.28)$$

3.3 Non-normal Variates

Section 3.1.2 assumes all the random variables are normally distributed. For the variables that are non-normal, transformation to equivalent normal distributions are required (Ang and Tang, 1986).

The "Rosenblatt Transformation" transforms the non-normal variate at a point to an equivalent point of a standardized normal variate through their cumulative distribution functions. The transformation procedure can be described as follows:

The probability density function of X_i evaluated at the checking point x_i^* can be written in terms of the standardized normal density function, ϕ :

$$f_{X_i}(x_i^*) = \frac{1}{\sigma_{X_i}^N} \phi \left(\frac{x_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N} \right) \quad (3.29)$$

where: $\mu_{X_i}^N, \sigma_{X_i}^N$ = mean and standard deviation of the equivalent normal distribution.

$\phi(-)$ = pdf of the standard normal distribution evaluated at -.

$f_{X_i}(x_i^*)$ = the original pdf of X_i evaluated at x_i^* .

The cumulative probability at x_i^* becomes:

$$F_{X_i}(x_i^*) = \Phi \left(\frac{x_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N} \right) \quad (3.30)$$

where: $F_{X_i}(x_i^*)$ = the original cdf of X_i evaluated at x_i^* .

$\Phi(-)$ = the cdf of the standard normal distribution evaluated at -.

Equation (3.30) can be rearranged to:

$$\mu_{Xi}^N = x_i^* - \sigma_{Xi}^N \Phi^{-1} [F_{Xi}(x_i^*)] \quad (3.31)$$

Equation (3.30) also can be rearranged to the following by substituting Equation (3.29) into its original form:

$$\sigma_{Xi}^N = \frac{\phi [\Phi^{-1}(F_{Xi}(x_i^*))]}{f_{xi}(x_i^*)} \quad (3.32)$$

For non-normal variate, the checking point is:

$$x_i^* = \mu_{xi}^N + x_i^*, \sigma_{Xi}^N = \mu_{Xi}^N - \alpha_i \beta \sigma_{Xi}^N \quad (3.33)$$

Similar to Eqn. 3.28, the direction cosines for a non-normal, mutually independent variates becomes;

$$\alpha_i^* = \frac{\sigma_{Xi}^N \left(\frac{\partial g}{\partial X_i} \right)_*}{\sqrt{\sum_{j=1}^n \left[\left(\frac{\partial g}{\partial X_j} \right)_*^2 (\sigma_{Xj}^N)^2 \right]}} \quad (3.34)$$

The reliability index, β , hence, can be solved with the multiple iterations as discussed in the next section.

3.4 Algorithm for Calculation of Reliability Index

The procedures to calculate the reliability index of a non-linear performance function with non-Gaussian independent variables can be summarized as the following:

- 1) Input the deterministic variables.
- 2) Compute the statistical properties (mean and standard deviation) of the random variable.
- 3) Let the testing point be the given means and standard deviations.
- 4) Convert the non-normal random variables to the equivalent normal random variables.
- 5) Take partial derivatives of the performance function with respect to each random variable.
- 6) Compute the direction cosines, α , with the normalized random variables.

- 7) Calculate $x_i^* = \mu_{Xi}^N - \alpha_i^* \beta \sigma_{Xi}^N$ for each random variables, where β is unknown.
- 8) With the x_i^* computed in step 7), substitute the x_i^* into the performance function, $g(x_1^*, x_2^*, \dots, x_n^*) = 0$, and solve for β .
- 9) Recalculate the new testing point with $x_{i+1}^* = x_i^* - \alpha_i^* \beta \sigma_{Xi}^N$, where β is from step 8).
- 10) Recalculate the new direction cosines, α_{n+1}^* , with the new testing point from step 9).
- 11) With the new testing point and direction cosines, solve for new β .
- 12) Check if the new β is within the acceptable significant floating points, say 0.0001, as the old β .
 - a) If no, go to step 8).
 - b) If yes, end.

The flow chart for the procedure of obtaining the reliability index is presented in Figure 3.4.

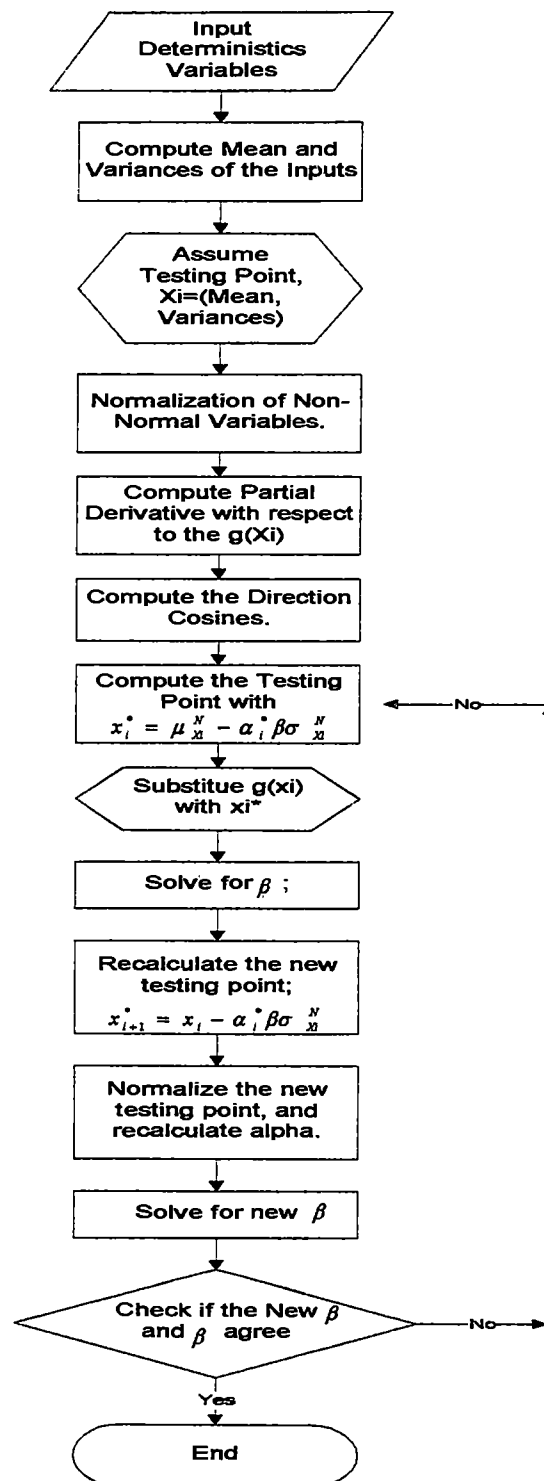


Figure 3.4: Flow chart for reliability analysis

Chapter 4

ACI BUILDING CODE FOR TWO-WAY FLOOR SLAB DESIGN

4.1 Behavior of two-way flat plate slabs

A two-way flat plate, column-supported slab system is a highly indeterminate structural component that requires complex analysis to determine moments and shears. Many theoretical approaches have been derived. A fundamental principle proposed by Nichols is that the entire applied load must be carried in each direction in order to satisfy basic statics (Nichols, 1914). Many practicing engineers at that time did not agree with Nichols' analysis. They insisted on other less conservative ways of analysis, which required field testing before the slab was allowed to be loaded (Eddy & Buel, 1914). Much research involving extensive experiments was conducted in the 1960s; the results led to definition of the design values for moment, shear, and deflection (Sozen & Siess, 1963). The field experiments led to significant changes in the methodology used for slab design which appeared first in the 1971 ACI 318 code.

Figure 4.1 shows a flat plate floor slab supported by columns A, B, C, and D. The total moment in the North-South direction can be plotted as shown in Figure 4.2. In this case, the slab is viewed as a wide flat beam of width l_2 where the bending moment diagram resembles the one for a fixed-fixed beam. Hence, to satisfy static equilibrium, a continuous floor slab system must satisfy the following equation:

$$\frac{1}{2}(M_{AB} + M_{CD}) + M_{EF} = \frac{1}{8}wl_2l_1^2 \quad (4.1)$$

Similarly, for the perpendicular direction, the following static equilibrium requirement must be satisfied:

$$\frac{1}{2}(M_{AC} + M_{BD}) + M_{GH} = \frac{1}{8}wl_1l_2^2 \quad (4.2)$$

in which M_{AB} , M_{CD} , M_{AC} and M_{BD} are the maximum negative moments at the adjacent support;
 M_{EF} and M_{GH} are the maximum positive moments at mid span; and
 w is the load applied (F/L^2 units).
 l_1 and l_2 are the dimensions of the slab.

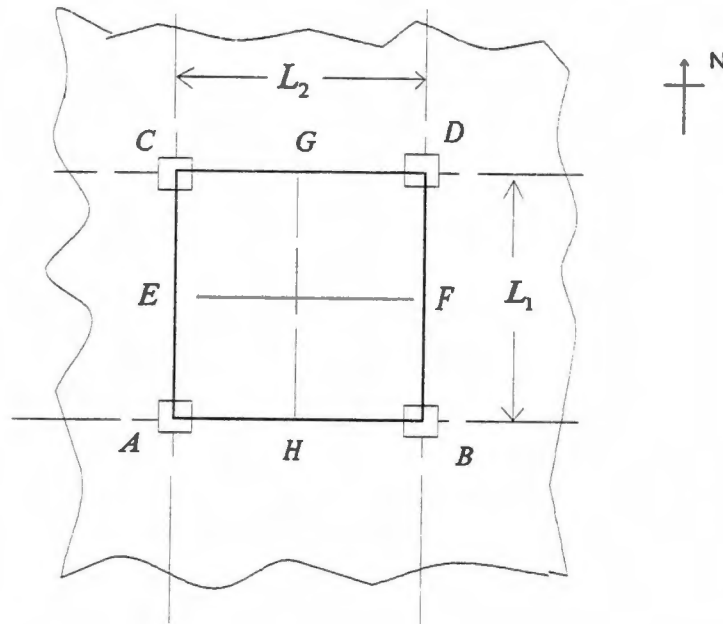


Figure 4.1: Floor plan of view of typical interior flat plate slab.

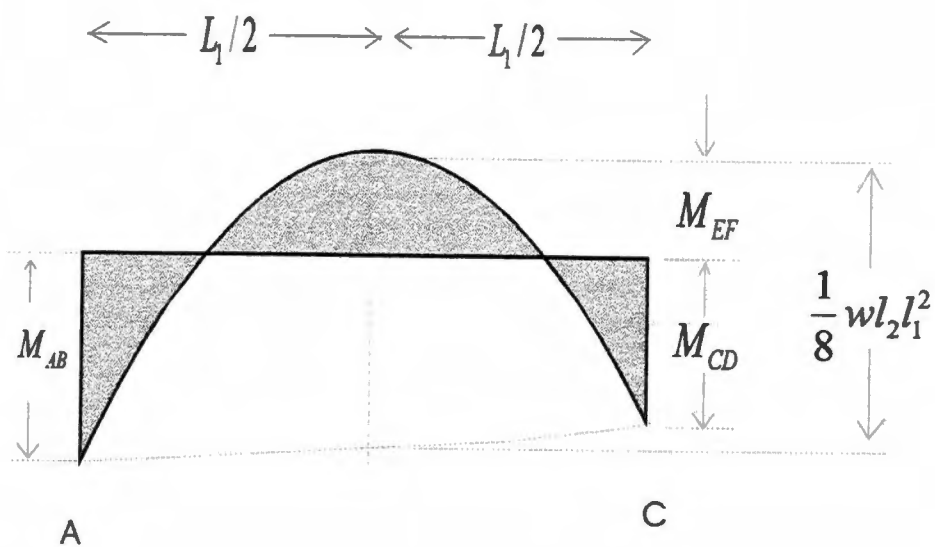


Figure 4.2: Moment variation along a span.

The proportion of the total static moment that exists in the section of interest depends on the relative span length of the adjacent panels, the loading pattern, and the stiffness of the supporting beams and columns. The moments across a particular width of critical section, whether it is positive moment EF, or negative moment AB, are not constant but vary as shown in Figure 4.3.

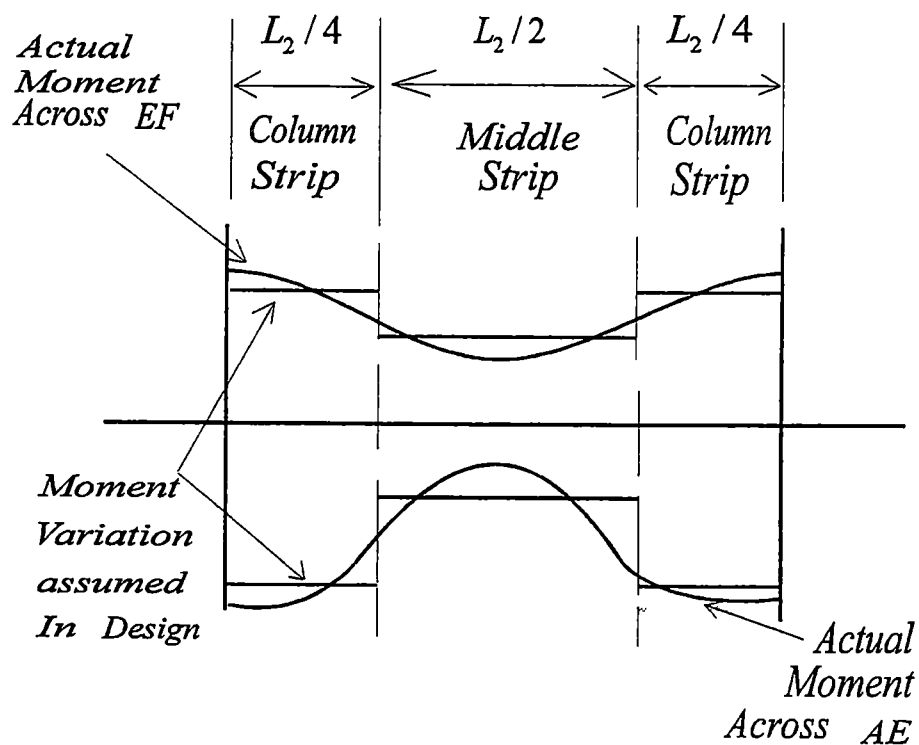


Figure 4.3: Moment variation across the width of critical section.

To facilitate the analysis, a typical panel of the floor slab is divided into column strips and a middle strip as shown in Figure 4.3. A column strip is defined as the subdivided strip having a width of a quarter of the shorter span. A middle strip is the design strip bounded by the column strips on both sides. A typical panel of slab is shown in Figure 4.3 with the moment variation and its column and middle strips indicated. Due to the simplicity of the analysis procedure, a semi-empirical method that follows Nichols' analysis has been found reliable under restricted conditions (Sozen & Siess, 1963) and adopted in the ACI 318 code. It is called the direct design method.

4.2 ACI Flat Plate Floor Slab Design with Direct Design Method

4.2.1 ACI Provision of 13.6 - Limitation of Direct Design Method

ACI 318-1995 gives the following restrictions in the use of the direct design method:

- 1) There shall be a minimum of three continuous spans in each direction.

- 2) The panels shall be rectangular and have an aspect ratio, ratio of longer to shorter span, of 2 or less.
- 3) The adjacent span length in each direction shall not differ by more than 1/3 of the longer span.
- 4) Column may be offset a maximum of 10 percent of the span in the direction of the offset from either axis between centerlines of successive columns.
- 5) Loads shall be due to gravity load only and the ratio of live to dead load shall not be greater than 3.
- 6) If any supporting beams are used on the column lines, the relative stiffness of beams in the two perpendicular directions, given by ratio of

$$\alpha_1 l_2^2 / \alpha_2 l_1^2$$

shall be neither less than 0.2 nor greater than 5.0.

The coefficients of α_1 and α_2 are the γ values in the direction of l_1 and l_2 respectively.

4.2.2 ACI 318-95 Provision in Direct Design Method.

The following are the design provisions as outlined by ACI 318-95 Section R13.6 in addition to other limit states that may control the using of direct design method:

4.2.2.1 Total Static Moment at Factored Load

The total static moment for a particular span is given by

$$M_o = w_u l_n^2 / 8 \quad (4.3)$$

in which l_n is the clear span from face to face of the supporting columns, capitals, brackets or walls and shall not be less than $0.65 l_1$.

4.2.2.2 Determination of Slab Thickness

The slab thickness is generally chosen to prevent excessive deflection that may affect serviceability. Besides, slab thickness is also needed to provide adequate shear strength, especially near the column regions. ACI 9.5.3.3 guidelines for slab thickness to meet serviceability limit-state for a slab without drop panels are shown in Table 4.1. The clear span of l_n in Table 4.1 is in inches.

Table 4.1: Table 9.5c of ACI 318 - 95, minimum thickness of slabs.

Reinforcement Yield Strength f_y , psi	Without Drop Panel		
	Exterior panels		Interior Panels
	Without Edge Beam	With Edge Beam	
40,000	$l_n/33$	$l_n/36$	$l_n/36$
60,000	$l_n/30$	$l_n/33$	$l_n/33$
75,000	$l_n/28$	$l_n/31$	$l_n/31$

4.2.2.3 Compute The Positive and Negative Moment in The Slab.

For an interior span, the total static moment that was computed in section 4.2.2.1 is distributed between the negative and positive bending sections according to the following ratios:

Negative factored moment: $Neg M_u = 0.65 M_o$

Positive factored moment: $Pos M_u = 0.35 M_o$

Figure 4.4 is the graphical illustration of the negative factored moment just next to the column face, and the positive factored moment of a typical interior and exterior panels.

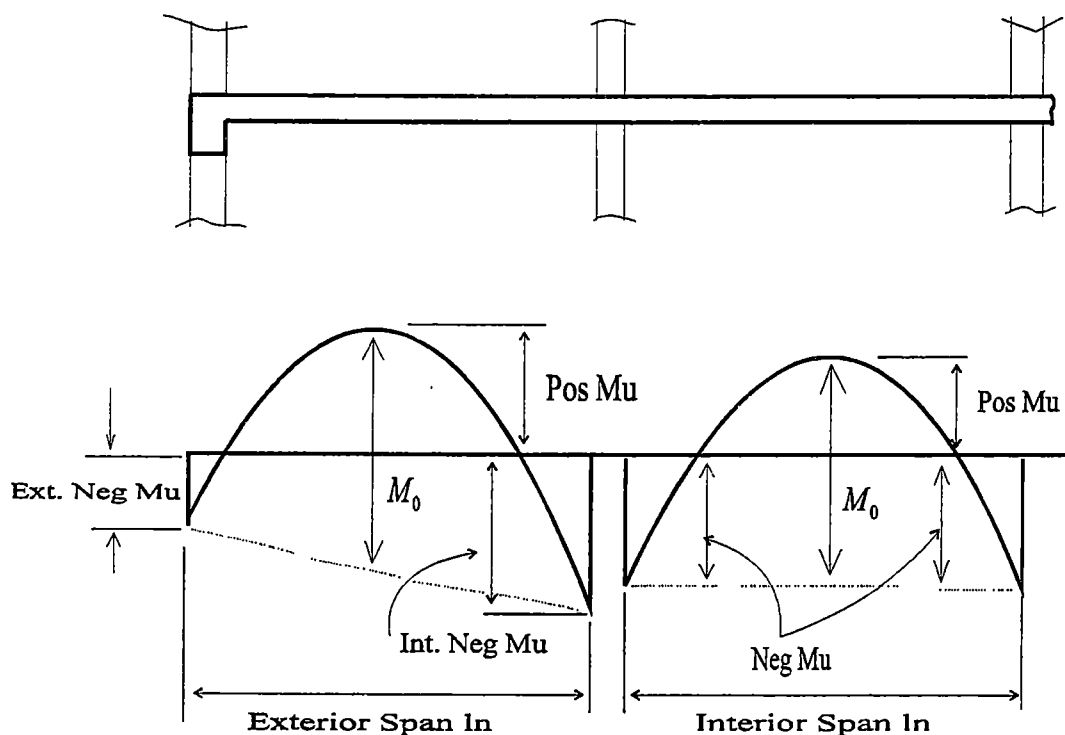


Figure 4.4: Total static moment distribution of a critical section for interior and exterior panels.

For an end span, the total static moment is apportioned differently depending on the flexural restraint of the slab, such as exterior columns, exterior wall, and the supporting beams on the column line. ACI 318-95 section 13.6.3.3 provides the total moment distribution factor for an end span, which is tabulated as shown in Table 4.2:

Table 4.2: Distribution factor for end span moments-
section 13.6.3.3 of ACI 318 - 95

	Exterior edge unrestraint	Slab with beams between all supports	Slab without beams between interior support		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior Neg. Factored Moment	0.75	0.7	0.7	0.7	0.65
Positive Factored Moment	0.63	0.57	0.52	0.5	0.35
Exterior Negative Factored Moment	0	0.16	0.26	0.3	0.65

The distribution factors shown in Table 4.2 were developed based on three-dimensional elastic analysis by Westergaard (Westergaard, 1921) which were modified according to practical experiences and tests (Sozen & Siess, 1963). At interior supports, the negative moments may differ from those framed into the interior panel's supports. In such cases, design should be based on the larger of the two, unless a special analysis is performed to distribute the unbalanced moment.

4.2.2.4 Lateral Distribution of Moments

As the total static moment is distributed to its negative and positive sections as described in section 4.2.2.3, further distribution of negative or positive moments across the width of the critical section is required. To simplify the design procedure, constant moment within strip boundaries is assumed for each critical cross section as shown in Figure 4.3.

The fraction of moment distributed across a critical section depends on the relative stiffness of the beam over slab spanning in either direction, which can be expressed as

$$\gamma = \frac{E_{cb}I_b}{E_{cs}I_s} \quad (4.3)$$

In which E_{cb} and E_{cs} are the moduli of elasticity of the beam and slab respectively. I_b and I_s are the moment of inertia of the beam and slab respectively.

The percentage of total moment in the critical section of column strips depends on the aspect ratio and relative beam to slab stiffness ratio, as tabulated in ACI 318-95 section 13.6.4 (See Table

4.3). For the intermediate aspect ratios, values shall be determined through interpolation.

Table 4.3: Percent of total static moment at critical sections of column strips, ACI 318-95 section 13.6.4.

		L2/L1		
		0.5	1	2
Interior Negative Moment				
$\alpha_1 L_1/L_2 = 0$		75	75	75
$\alpha_1 L_1/L_2 > 1.0$		90	75	45
Exterior Negative Moment $\alpha_1 L_1/L_2 = 0$	$\beta_t = 0$	100	100	100
	$\beta_t > 2.5$	75	75	75
$\alpha_1 L_1/L_2 > 0$	$\beta_t = 0$	100	100	100
	$\beta_t > 2.5$	90	75	45
Positive Moment				
$\alpha_1 L_1/L_2 = 0$		60	60	60
$\alpha_1 L_1/L_2 > 0$		90	75	45

In Table 4.3, the notation α_1 is the γ in the direction of l_1 ; β_t is the ratio of torsional stiffness of edge beam section to flexural stiffness of a width

of slab equal to span length of beam, center to center supports.

The portion of moment that is not resisted by column strip is proportionately assigned to be resisted by the adjacent half-middle strip. Each middle strip is designed to resist the sum of the moments assigned to its two half middle strips. For example, an interior panel of flat slab will have a negative column strip moment of 75%, and the remaining 25% of negative moment will be assigned to the middle strips, with 12.5% on each strip.

4.3 Shear Requirements of A Floor Slab System.

Shear failure is considered a controlling factor for two-way slabs particularly flat-plate slabs. Shear failure is likely to occur near the columns, as shown by most of the tests performed in the 1970s (Simmonds & Alexander, 1987).

One-way shear is rarely a problem, but two-way shear is considered critical in the design of two-way slab. One-way shear is defined as the beam action shear that involves inclined cracks extending across

the entire width of the slab. Two-way shear or punching shear involves the failure mechanism that occurs around a column, forming a truncated cone or pyramid shaped surface. Shear design usually governs the thickness of the slab instead of deflections, discussed in section 4.2.2.2.

4.3.1 Two way Shear Design

Two-way shear without moment transfer should be designed according to the criteria provided by equations 4.4 to 4.7. The basic shear design equation, equation 11-1 of ACI 318-95, can be stated:

$$V_u \leq \phi V_n \quad (4.4)$$

where V_u = Factored shear force due to the loads.
 V_n = Nominal shear strength of the floor slab.

$$\phi = 0.85$$

ACI 318-95 defines the shear capacity of a particular concrete member as:

$$V_n = V_c + V_s \quad (4.5)$$

where V_c and V_s are the shear resistance attributed by concrete and steel respectively.

In practice, V_s is most likely to be taken as zero due to the difficulties of setting steel reinforcement into the floor slab system. The two-way shear capacity of the floor slab contributed by the concrete is defined by the smallest of the following:

$$\begin{aligned} V_c &= \left(2 + \frac{4}{\beta_c}\right) \sqrt{f'_c} b_o d \\ V_c &= \left(\frac{\alpha_s d}{b_o} + 2\right) \sqrt{f'_c} b_o d \\ V_c &= 4 \sqrt{f'_c} b_o d \end{aligned} \quad (4.6)$$

Where β_c = the ratio of long to short side of the column, concentrated load or reaction area.

$\alpha_s = 40$ for interior columns, 30 for edge columns, and 20 for corner columns.

b_o = perimeter of critical section for slabs and footings, in.

4.3.2 One Way Shear Design

The one-way shear strength is also required to be checked. The one way shear strength of a rectangular beam section is (Eqn. 11-28, ACI 318-95),

$$V_c = 2\sqrt{f'_c}b_wd \quad (4.7)$$

Practically, one-way shear is seldom a controlling factor in the design of flat plates or flat slabs.

Chapter 5

ELASTIC ANALYSIS OF SLAB

5.1 Introduction

Slabs can be categorized into the following three types (Macgregor, 1995):

- i) Thick Slab - slab with thickness greater than $1/10$ of the span.
- ii) Thin slab - slab with thickness lesser than $1/50$ of the span.
- iii) Medium-thick slab - slab with thickness between the above two categories.

Behavior of slabs in these categories differs. Thick slabs transmit part of the forces through arch action, which results in larger internal compressive forces than internal tensile forces. Conversely, a thin slab tends to behave like a tensile membrane, where the internal tensile forces are greater than the compressive forces. Most floor slab systems are categorized as medium-thick slabs. The internal forces tend to be in equilibrium, where internal tension equals internal compression. In other words, neither arch action nor membrane action is exhibited.

Figure 5.1 shows an element, which is isolated from a medium-thick two-way slab, with the loads, w , and shear, V . The moments at the edges, denoted m_x and m_y , are parallel to their respective axes, and the twisting moments, denoted m_{xy} and m_{yx} , are perpendicular to their respective axes. Figure 5.1 shows the moment vectors, which are denoted by arrows that follow the right-hand screw rule. Moment that causes compression on the top of the slab is defined as positive moment and vice versa.

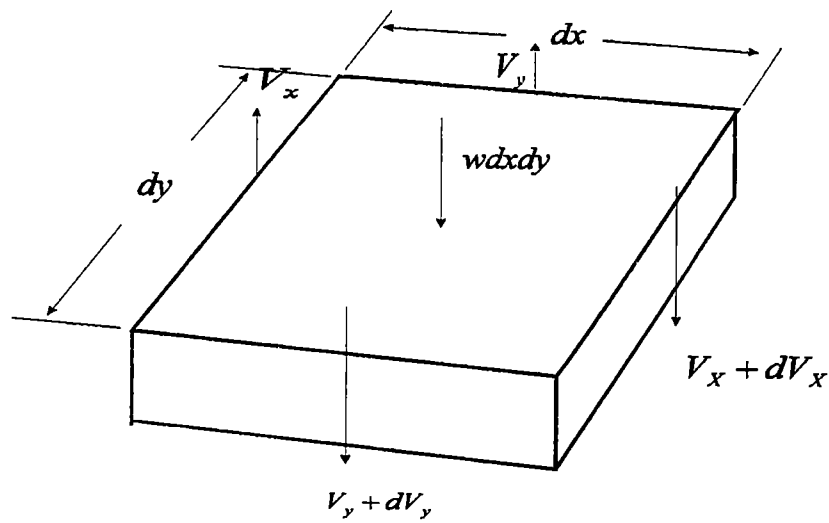
To satisfy force equilibrium,

$$\frac{dV_x}{dx} + \frac{dV_y}{dy} = -w \quad (5.1)$$

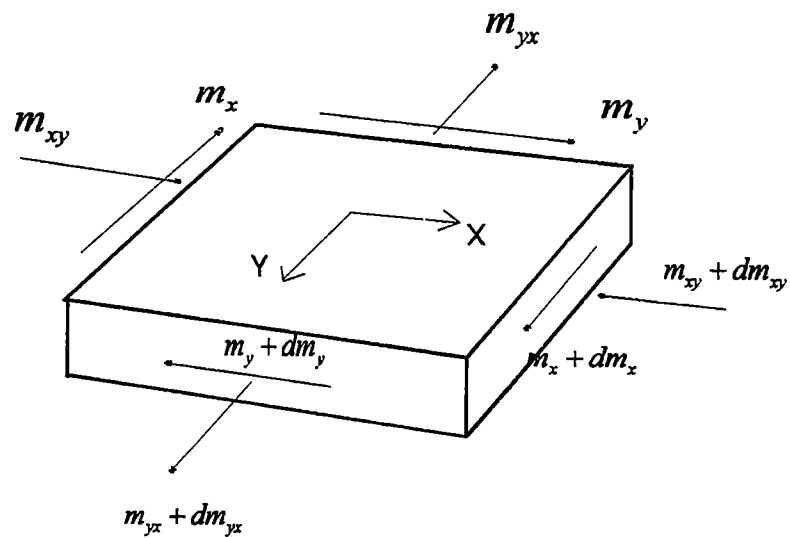
and moment equilibrium,

$$\frac{dm_x}{dx} + \frac{dm_{yx}}{dy} = V_x \quad \text{and} \quad \frac{dm_y}{dy} + \frac{dm_{xy}}{dx} = V_y \quad (5.2)$$

From equilibrium, it can be shown that $m_{xy} = m_{yx}$. Hence, for a small slab element with sides dx and dy ,



a) Shear and loads on a slab element



b) Bending and twisting moments on a slab element

Figure 5.1: Moments and forces in a medium-thick plate

the basic equilibrium equation can be derived by substituting differential Eqn.(5.1) into Eqn.(5.2), which yields:

$$\frac{\partial^2 m_x}{\partial x^2} + \frac{2\partial^2 m_{xy}}{\partial x \partial y} = -w \quad (5.3)$$

For an elastic plate, the deflection, z , due to an applied load of w can be derived from the following fourth order differential equation:

$$\frac{\partial^4 z}{\partial x^4} + 2\frac{\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4} = -\frac{w}{D} \quad (5.4)$$

where the plate rigidity constant, D , is defined as:

$$D = \frac{Et^3}{12(1-\nu^2)} \quad (5.5)$$

where ν = Poisson's ratio of the material.

The deflection, z , can be solved by using Eqn. (5.4). The moments are calculated from

$$m_y = -D \left(\frac{\partial^2 z}{\partial y^2} + \nu \frac{\partial^2 z}{\partial x^2} \right)$$

$$m_{xy} = -D(1-\nu)\frac{\partial^2 z}{\partial x \partial y}$$

$$m_x = -D\left(\frac{\partial^2 z}{\partial x^2} + \nu\frac{\partial^2 z}{\partial y^2}\right) \quad (5.6)$$

where z is positive downward.

5.2 Design Moment from Finite Element Analysis

Solving Eqn.(5.4) requires lengthy computation efforts. Hence, computer aided analysis such as the finite element method has been used to solve Eqn.(5.4). Such an analysis gives the values of m_x , m_y , and m_{xy} for each of the elements comprising the plate. The output of the finite element analysis for m_x , m_y , and m_{xy} are in the units of moment per width of an element.

For a given portion of an element bounded by a diagonal crack as shown in Figure 5.2, the moment about the axis parallel to the crack, m_c can be derived as the following (Macgregor, 1995):

$$m_c \partial s = (m_x \partial y + m_{xy} k \partial y) \cos \theta + (m_y k \partial y + m_{xy} \partial y) \sin \theta$$

or

$$m_c = \left(\frac{\partial y}{\partial x}\right)^2 (m_x + k^2 m_y) \quad (5.7)$$

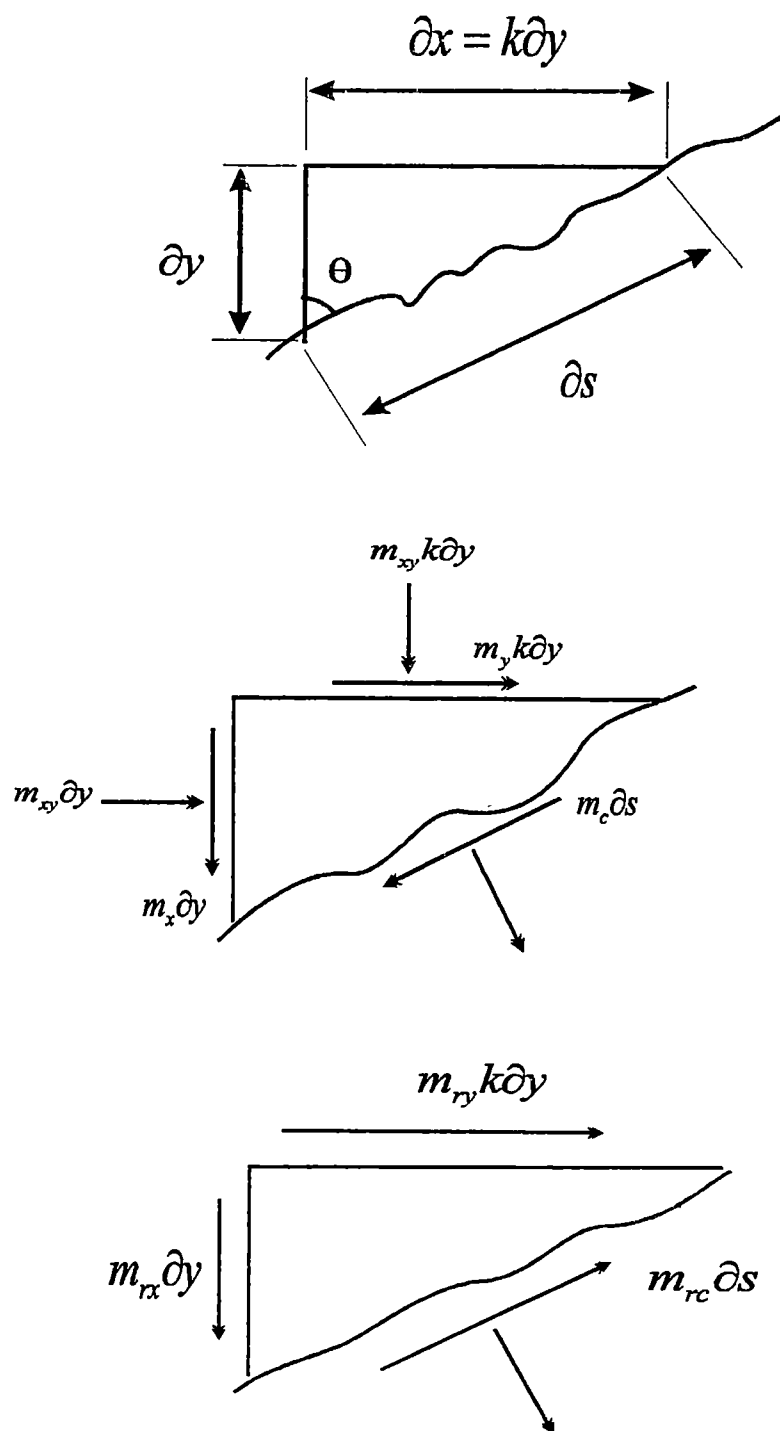


Figure 5.2: Resolution of moments

If the slab is to be reinforced with bars in the direction of x and y with positive moment capacities of m_{rx} and m_{ry} per unit width, then the corresponding moment capacity at the assumed crack is

$$m_{rc} = \left(\frac{\partial y}{\partial x} \right)^2 (m_{rx} + k^2 m_{ry}) \quad (5.8)$$

where k is the aspect ratio of the element.

and m_{rc} must exceed or equal to m_c in order to provide adequate strength. Equating these and solving for the minimum yields:

$$m_{ry} = m_y + \frac{1}{k} m_{xy} \quad (5.9)$$

Since m_{ry} must exceed or equal to m_y , to consider the effect of m_{xy} , the following equation must be true:

$$\frac{1}{k} m_{xy} \geq 0 \quad (5.10)$$

which gives Eqn.(5.9) the following expression:

$$\begin{aligned} m_{ry} &= m_y + \frac{1}{k} |m_{xy}| \\ m_{rx} &= m_x + k |m_{xy}| \end{aligned} \quad (5.11)$$

Note that as k increases, m_{ry} decreases and m_{rx} increases. For the case where the total reinforcement is minimal, $k = 1$ should be adopted for a wide range of moment values (Hillerborg, 1975). Hence, the positive moment reinforcement design in each direction should be provided using the following values:

$$\begin{aligned} m_{ry} &= m_y + |m_{xy}| \\ m_{rx} &= m_x + |m_{xy}| \end{aligned} \quad (5.12)$$

Similarly, the negative reinforcement should be designed to provide the negative moment resistance of:

$$\begin{aligned} m_{ry} &= m_y - |m_{xy}| \\ m_{rx} &= m_x - |m_{xy}| \end{aligned} \quad (5.13)$$

Chapter 6

IMPLEMENTATION OF THE ANALYSIS

6.1 Determination of Performance Function

Recall from Chapter 3, the performance function that consists of 2 random variables:

$$g(X) = R - Q = 0 \quad (3.9)$$

where R is the resistance and Q is the load effect.

R in this case is not a single variable but involves multiple variables. In the case considered here, the moment resistance of a particular strip of the slab behaves exactly like a narrow-width beam. The resistance side of the performance function, R , can be expressed as:

$$R = A_s F_y \left(d - k \frac{A_s F_y}{b F_c} \right) \quad (6.1)$$

Where A_s = area of steel

F_y = yield strength of steel reinforcement

d = depth of slab

$k = 0.59$

b = critical width of the strip section

F_c = compressive strength of concrete

Q represents the moments due to load effects at a particular strip (can be positive or negative moments). Two types of loadings have been considered in the analysis, i.e. live load and dead load.

For the reliability analysis, the design was based on the moments computed using the direct design method. In the first case of the study, the reliability of the slab was assessed based on moment from the direct design method. In the second case of the study, the reliability of the same slab was assessed based on the moment from finite element method. Case 1 has the following variables in the loading side of the performance function:

$$Q = \rho h C_{DL} + M_{LL} \quad (6.2)$$

Where C_{DL} is the moment coefficient computed as moment due to dead load divided by the concrete density and slab thickness.

M_{LL} is the moment due to live load.

Note that the concrete density, ρ , and slab thickness, h , were considered to be individual variables instead of one dead load term in the analysis. Case 2 has only one dead load term, which gives the following loading side of the performance function:

$$Q = M_{DL} + M_{LL} \quad (6.3)$$

Hence, the performance function for both cases can be expressed as the following:

For Case 1:

$$g = A_s F_y \left(d - k \frac{A_s F_y}{b f_c} \right) - C_1 H \rho - M_{LL} \quad (6.4)$$

For Case 2:

$$g = A_s F_y \left(d - k \frac{A_s F_y}{b f_c} \right) - M_{DL} - M_{LL} \quad (6.5)$$

6.2 Probability Functions and Statistical Properties of the Random Variables.

The design variables that were used in this reliability analysis were considered to be random

variables. Their probability functions and statistical properties are summarized in Table 6.1 (Ellingwood et al., 1980; Chou et al., 1983). The subscript "n" for the variables in the column under the heading "Mean" refers to nominal.

6.3 Design Assumption

All the flat-plate floor slabs were designed according to the ACI 318-95 direct design method (Section 13.6). The following assumptions were included in the design process:

- a) The floor slab was assumed to have the shorter span lengths of 15 ft and 20 ft.
- b) The supporting columns were assumed to be 18" X 18" square columns.
- c) No edge beam was provided in either interior or exterior panel of the slab system.
- d) The slab of interest was the center panel of the 5 X 5 panels of floor slab system.
- e) Self-weight of slab was the only dead load considered.

Table 6.1: Statistical properties of the variables

Variables	Mean	Variance	C.O.V	Distribution
As	$1.05As_n$	-	0.035	Normal
Fy	$1.125Fy_n$	-	0.098	Lognormal
D	$D_n - 0.12''$	$0.25''$	-	Normal
K	0.59	-	0.1	Normal
Fc	$0.8056fc_n$	-	0.15	Normal
LL	$1.0LL_n$	-	0.25	Type I Largest
DL	$1.0DL_n$	-	0	Normal
B	$B_n + 0.1''$	$0.15''$	-	Normal
H	$H_n + 0.1''$	$0.47''$	-	Normal
ρ	$1.05\rho_n$	-	0.1	Normal

- f) Neither reduced live load nor pattern loading was considered.
- g) Both live load and dead load were applied uniformly over the 5 X 5 panels simultaneously.
- h) Only gravity loads (live and dead loads) were considered. No lateral load was considered.
- i) Concrete strength used was 4,000 psi. Reinforcing steel yield strength used was 60,000 psi.
- j) In the finite elements analysis, the column supports were modeled as pinned supports, as described in section 6.11.
- k) Elastic behavior was assumed in the finite element analysis.

6.4 Partial Derivatives for Direction Cosines of Variables

In order to calculate the direction cosines, partial derivatives of the performance function with respect to each of the random variables are required. A commercial software called MathCad6.0 has been used to perform the partial derivatives symbolically. The partial derivatives of the performance function with respect to the random variables are:

For Case 1:

$$\frac{\partial g}{\partial A_s} = \frac{F_y (dbf_c - 2kA_s F_y)}{bf_c}$$

$$\frac{\partial g}{\partial F_y} = \frac{A_s (dbf_c - 2kA_s F_y)}{bf_c}$$

$$\frac{\partial g}{\partial D} = A_s F_y$$

$$\frac{\partial g}{\partial k} = -\frac{A_s^2 F_y^2}{bf_c}$$

$$\frac{\partial g}{\partial b} = \frac{A_s^2 F_y^2 k}{b^2 f_c}$$

$$\frac{\partial g}{\partial f_c} = \frac{A_s^2 F_y^2 k}{bf_c^2}$$

$$\frac{\partial g}{\partial H} = -C_1 \rho$$

$$\frac{\partial g}{\partial C_1} = -H \rho$$

$$\frac{\partial g}{\partial LL} = -1$$

For case 2:

$$\frac{\partial g}{\partial A_s} = \frac{F_y (dbf_c - 2kA_s F_y)}{bf_c}$$

$$\frac{\partial g}{\partial F_y} = \frac{A_s(df_c - 2kA_sF_y)}{bf_c}$$

$$\frac{\partial g}{\partial D} = A_sF_y$$

$$\frac{\partial g}{\partial k} = -\frac{A_s^2F_y^2}{bf_c}$$

$$\frac{\partial g}{\partial b} = \frac{A_s^2F_y^2k}{b^2f_c}$$

$$\frac{\partial g}{\partial f_c} = \frac{A_s^2F_y^2k}{bf_c^2}$$

$$\frac{\partial g}{\partial DL} = -1$$

$$\frac{\partial g}{\partial LL} = -1$$

For case 1, the performance function contains ten variables, while for case 2, the $C_1\rho H$ term has been lumped into the M_{DL} term, which gives the performance function only eight terms. Hence, the variables on the resistance side are essentially the same for both cases, except that case 1 has two additional partial derivatives on the load effects sides.

6.5 Normalization of Non-Normal Variables

All the variables in the performance function are normal, except for live load and strength of reinforcing steel, F_y . For the non-normal variables, normalization of the distributions is required.

The procedure of transforming the non-normal variables into normal variables has been discussed in section 3.6.0. The following sections are the discussion of transforming live load (Type 1 Largest) and F_y (Lognormal distribution).

6.5.1 Normalization of Type I Largest

- 1) Compute $\alpha = \frac{\pi}{\sqrt{6} * \mu_x}$
- 2) Compute $u = \mu_x - \frac{0.577}{\alpha}$
- 3) Compute $F_l = e^{-e^{-\alpha(LL-u)}}$
- 4) Compute $f_l = \alpha e^{-\alpha((LL-u)+e)}$
- 5) Substituting F_l into

$$\frac{1}{(2 * (1 + 0.196854x + 0.115194x^2 + 0.000344x^3 + 0.019527x^4))^4} - 1 + F_l = 0$$

And, solve for x .

6) Compute $\phi = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2}$

7) The Normalized Variance: $\sigma_N^2 = \frac{1}{f l \phi}$

8) The Normalized Mean: $\mu_N = \mu_X - \sigma^2 x$

6.5.2 Normalization of Lognormal

1) Compute $\xi = \sqrt{\ln(1+V_R^2)}$

2) Compute $\lambda = \ln(\mu_R) - 0.5\xi^2$

3) The Normalized Variance: $\sigma_N^2 = \mu_R * \xi$

4) The Normalized Mean: $\mu_N = F_y(1 - \ln(F_y) + \lambda)$

6.6 Load Combinations

The nominal dead load used in the analysis was the self-weight of the slabs, which can be determined as:

$$W_{DL} = \rho H \quad (6.6)$$

Where ρ = Unit weight of the normal weight concrete (150 pcf).

H = Thickness of the slab.

The live load used in the analysis was assumed to be 0.5, 1.0, 1.5, 2.0, and 2.5 times of dead load, where the live load can be determined as:

$$W_{LL} = \psi W_{DL} \quad (6.7)$$

Where ψ = Ratio of live load to dead load.

W_{DL} = Dead load as determined from equation (6.6).

6.7 Span Ratio

The span ratios of 1.0, 1.2, 1.4, 1.6, 1.8, and 2.0 were used for the 15 ft span length, while the span ratios of 1.0, 1.4, 1.8, and 2.0 were used for the 20 ft span length. The center panel of the 5 by 5 panel was the panel of interest, and all other panels were assumed to have the same span ratio as the center panel. An example of the layout of the panel of interest is shown in Figure 6.1.

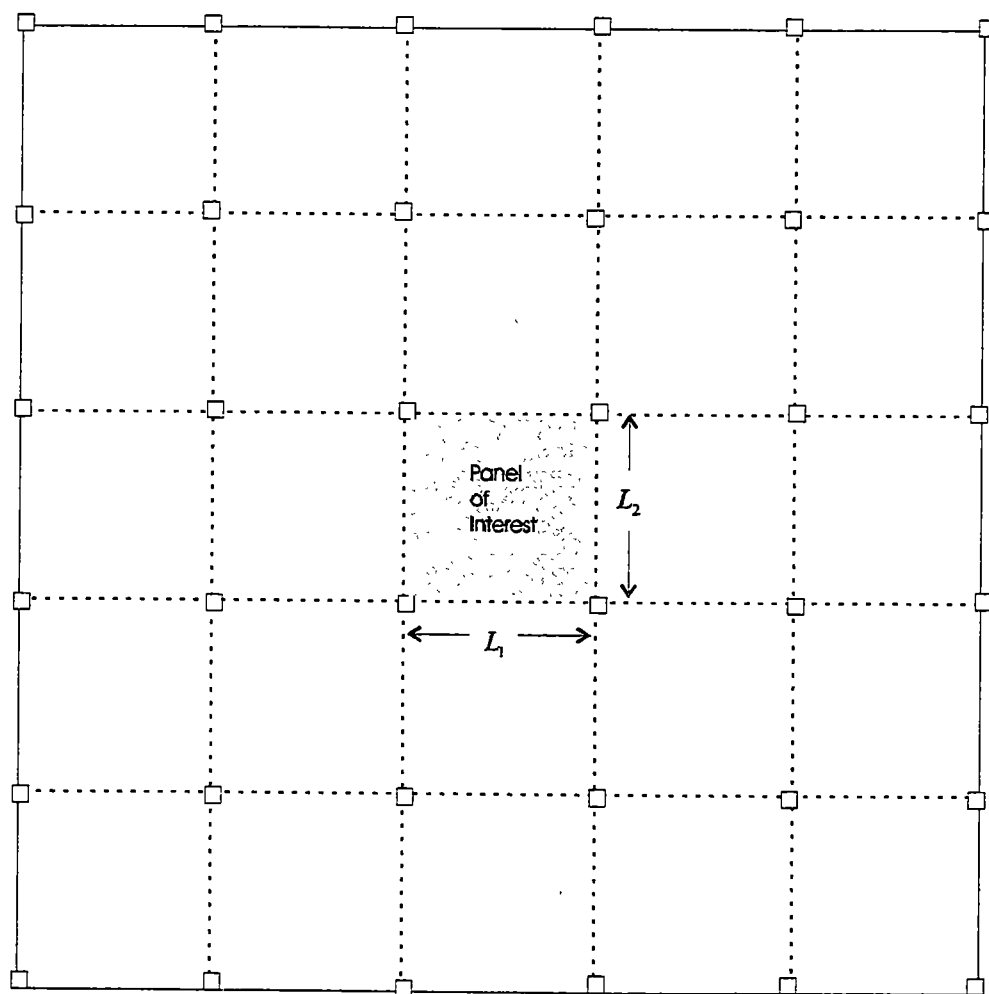


Figure 6.1: Panel of interest

6.8 Cases Studied

One of the main research objectives of this study was to determine the consistency of current design code. The current ACI code was expected to maintain its consistency regardless of span ratio and LL/DL ratio. The reliability indices for different LL/DL ratios were studied for each span ratio described in the previous sections. A total of 50 different LL/DL and span ratio combinations were analyzed. The moment obtained through the direct design method was then reanalyzed using the finite element method.

6.9 Design Procedures

Several design assumptions are outlined in section 6.3. The following design procedures were used for the two-way flat plate floor slab system:

- 1) Input the design parameters: Span ratio, LL/DL ratio, strength of steel, F_y , concrete strength, f_c' .

- 2) Calculate the minimum thickness of the slab, H ; criteria listed in Table 9.5c of ACI 318-95 were applied.
- 3) Calculate the dead and live loads applied as discussed in section 6.6.
- 4) Compute the factored loads (Section 9.2.1 of ACI 318-95): $1.4 \text{ DL} + 1.7 \text{ LL}$ of load factors has been applied for dead and live loads.
- 5) Check if the thickness provided in step 2 satisfies the shear requirement as outlined in section 4.3.
 - a) If yes, proceed to step 6).
 - b) If no, go back to step 2) and resize the slab thickness.
- 6) Calculate the total static moments, M_o :

$$M_o = \frac{W_u L_1 L_n^2}{8}$$

- 7) Calculate the negative and positive moment with the moment coefficients provided in ACI 318-95, section 13.6.3.3.
- 8) Divide the slab into column and middle strips.
- 9) Distribute the moment laterally across the column and middle strips, ACI 318-95 section 13.6.4.

10) Design for flexure with the known strip width, b , and slab thickness, h .

The procedures for designing the two-way flat plate floor slab system can be represented with the flow chart as shown in Figure 6.2.

Step 10 of the design procedure is simply an ordinary flexural beam design procedure. MacGregor (MacGregor, 1995) has made several approximations in the flexure design procedures. For a singly reinforced flexural beam,

$$\phi M_n = \phi \left[A_s F_y \left(d - \frac{a}{2} \right) \right]$$

The moment arm of the internal couple, the $d - \frac{a}{2}$ term, is referred to as jd . Since the F_y term is normally taken as 60 ksi, there are still 2 unknowns in the design equation. MacGregor (MacGregor, 1995) noted in his book that the j (fraction of the reinforcing depth equivalent to the moment arm) for slabs is normally taken as 0.925 for simplicity and to

meet the requirement of minimum reinforcement. With the known j , the trial area of steel can be computed as the following:

$$Trial(A_{REQ'D}) = \frac{Mu}{\phi F_y j d}$$

The depth of the compression zone, a , is then recomputed with the trial steel area computed above and used to compute a more accurate $j d = d - a/2$. This will then be used to compute A_s required at the section of the slab.

Another criteria that needs to be checked while sizing the reinforcement is the spacing requirements recommended by ACI. The requirement simply limits the maximum spacing of reinforcement at the point of maximum positive and negative moment. The maximum spacing is the smaller of two times the slab thickness (ACI Sec. 13.4.2) and 18 inches (ACI Sec. 7.12.3).

Checking of the design to ensure that it meets the minimum steel needed and maximum steel permitted is necessary at the last stage of design.

The entire flexural design procedures can be summarized as shown in Figure 6.2.

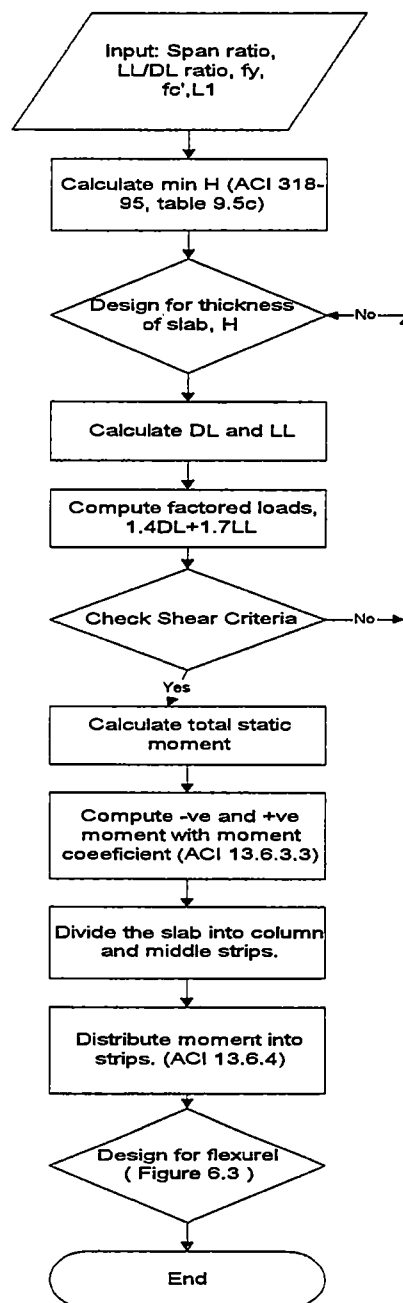


Figure 6.2: Flow chart for two-way floor slab design procedures

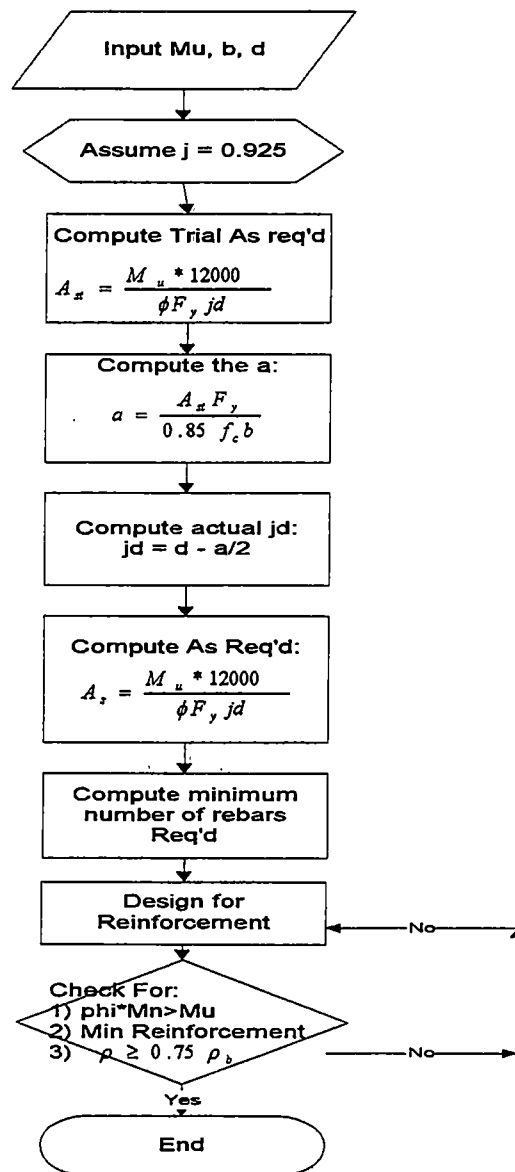


Figure 6.3: Flow chart for flexure design

The design template for the two-way slab system was formulated in *MathCad6.0* to facilitate the multiple design efforts as enclosed in the Appendix B.

6.10 Calculation of Reliability Indices

Once the floor slabs were designed using the ACI 318-95 code, the subsequent step was to input the designs into a program for reliability analysis. The computer program was written in *Matlab4.0*. *Matlab4.0* has the additional capabilities as compared to some other computer languages through their "Toolboxes". The "Optimization Toolbox" enables the users to use the iteration routines to optimize the function of interest. The "Statistical Toolbox" enables the users to transform the probability function.

Both computer programs were written to execute and feed in data from external notepad files that contained statistical data. The output of the programs was designed to track the direction numbers. Reliability indices were computed at 8 different locations of a plate according to the directional numbering shown in Table 6.2:

Table 6.2 Direction numbering represented in the computer programs written

Direction Number	Moment Direction
1	North-South negative moment (Column Strip)
2	North-South negative moment (Middle Strip)
3	North-South positive moment (Column Strip)
4	North-South positive moment (Middle Strip)
5	West-East negative moment (Column Strip)
6	West-East negative moment (Middle Strip)
7	West-East positive moment (Column Strip)
8	West-East positive moment (Middle Strip)

The reliability index of the plate was then given by the lowest reliability index among the 8 locations analyzed. As an illustrative example, the reliability indices for a 15 ft square plate with LL/DL ratio of 0.5 were computed and the results are given in Table 6.3:

Table 6.3: Reliability indices for a square plate with LL/DL ratio of 0.5

DIRECTION NUMBER	RELIABILITY INDEX
1	4.838
2	7.692
3	6.595
4	8.457
5	3.929
6	7.056
7	5.872
8	7.781

Hence, the reliability index for the above case is 3.929.

In some unusual situations where the unstable condition occurs during the iterations, the program will terminate after 15 iterations, and manual interpretation of the results should be implemented. Details of the computer programs can be found in Appendix A.

6.11 Implementation of Finite Element Analysis

The finite element analysis was done using a commercial program called "Visual Analysis 3.1" from Integrated Engineering Software Incorporated. Visual

Analysis 3.1 was chosen because of its graphic capabilities and ease of using the program. The matrix capacities of the software also allow larger mesh of elements to obtain more accurate results.

Visual Analysis 3.1 gives the resulting moments at the centroid and nodes of an element. The centroidal moment was chosen to be the moment used in the design since it is the weighted average of the moment across the element width.

The output moments from Visual Analysis 3.1 are expressed in the units of *kips-ft/ft*. Unit conversion from *kips-ft/ft* to *kips-ft* was needed before the reliability analysis can be performed.

The moment in the units of *kip-ft* is simply the area under the curve between the two nodal moments (*kips-ft/ft*) as shown in Figure 6.4. Expressing the moment in *kips-ft*:

$$M_{(kips-ft)} = \frac{M1+M2}{2}b = M_{Centroid} * b$$

Hence, for a moment value of an element given by the finite element analysis, the moment in the units of

kips-ft can be computed as the centroidal moment, $M_{centroid}$ (*kips-ft/ft*) times the element width, b (*ft*).

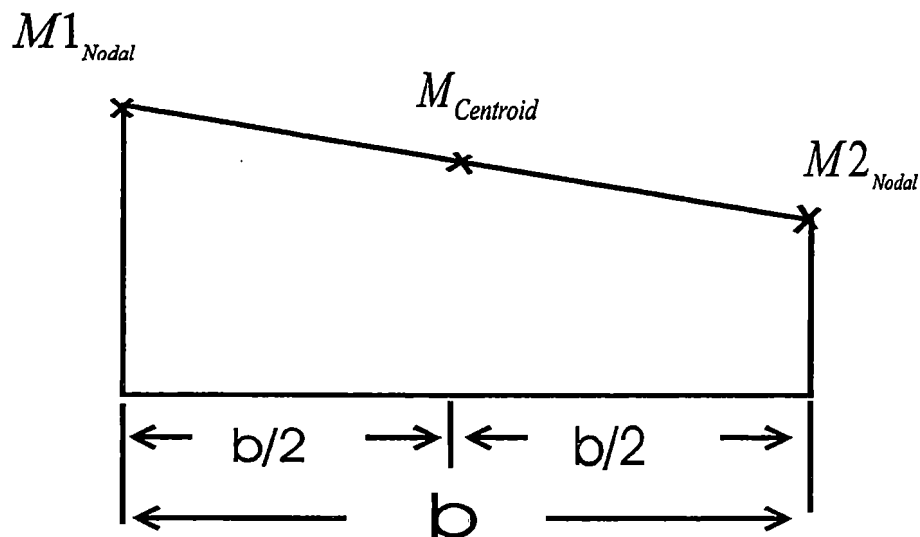


Figure 6.4: Moment at the nodal and centroid.

Recall the total design moments from Eqn. 5.13:

$$\begin{aligned} m_{ry} &= m_y + \frac{1}{k} |m_{xy}| \\ m_{rx} &= m_x + k |m_{xy}| \end{aligned} \quad (5.13)$$

Where k is the aspect ratio of the element (a positive number).

Note that Hillerborg (Hillerborg, 1975) recommended a k value of 1 for a wide range of moments. k value essentially is the aspect ratio of the longer to the shorter span. In these studies the k values varies according to the span ratio that was described in section 6.7.

Due to the varying span ratio, the element aspect ratio also varies accordingly. Hence, all the modeled slabs had the same number of elements regardless of their span ratio; the only difference is the element's aspect ratio.

The accuracy of the analytical result provided by the finite element analysis varies with the appropriateness of the meshing of the model's elements. Hence, verification of the model's results with the known theoretical value is required. Four plate models with simply supported edges have been run to verify the validity of the FEM results. The models are comprised of 2 by 2, 4 by 4, 8 by 8, and 16 by 16 plate elements. The deflection at the center of the plate was computed with the plate theory formula (Szilard, 1974), and comparison of the FEM results of

different n by n element models can be summarized as the following:

Table 6.4: Verification of FEM result

Verification of Finite Element Analysis Thru Visual Analysis

Model Used: Simply Edge Supported Plate

Span Length: 15 ft by 15 ft

Thickness: 5 inches

Load: 0.0625 kips/ft²

n X n Elements	Displacement		% Differences (%)
	Plate theory values (inches)	FEM values (inches)	
2	0.358	0.247	31.01
4	0.358	0.326	8.94
8	0.358	0.348	2.79
16	0.358	0.354	1.12

Note that the deflections obtained through FEM of the plate converged at the 8 by 8 elements with a percent difference of 2.79 %. The plate with 4 by 4 elements was chosen as the mesh used in the analysis due to its simplicity of implementing the total moment computation. The 4 X 4 elements' plate also provided a relatively acceptable percent difference compared to the theoretical value of 8.94 %. Hence, the meshing of

the elements of the plate had been implemented with the 4 by 4 elements on each of the panels.

The meshing of the elements was implemented by generating 200 quadrilateral elements with 36 pinned supports spaced at 4 elements on both directions. A typical slab model is shown in Figure 6.5 :

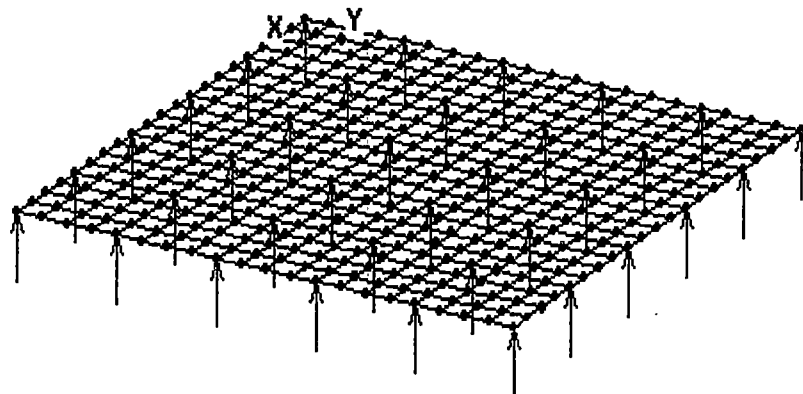


Figure 6.5: Example of finite element model

The supporting columns of the slab were modeled as pinned supports. With the pinned support, the model's support is stiffer, which led to more conservative results.

An example of the calculation of moment at a particular strip is shown in Table 6.5:

Table 6.5: Computation of moment using FEM results

1	2	3	4	5	6	7	8	9
Direction #	Element (#)	M _{xy} (kips-ft/ft)	M _y (kips-ft/ft)	Width (ft)	Aspect Ratio	M _{xy} (kip-ft)	M _y (kip-ft)	Total Moment (kips-ft)
1	169	2.996	-3.957	3.75	2	11.234	-14.839	52.147
2	189	1.793	-3.210	3.75	2	6.725	-12.038	37.526
3	170	0.362	4.817	3.75	2	1.357	18.065	38.844
4	190	0.183	4.769	3.75	2	0.688	17.885	37.145

The M_{xy} and M_y in columns 3 and 4 are the moments obtained from the finite element analysis. The width in column 5 is simply the element width of a particular slab element. Column 6 is the *k* values of the elements. Columns 7 and 8 are the moments in kips-

ft. Column 9 is simply the total design moment computed using Eqn. 5.13.

Chapter 7

RESULTS AND DISCUSSIONS

This section discusses the relationship between span ratio, LL/DL, and the reliability indices. Both case 1 (direct design method) and case 2 (finite element analysis) were plotted on the same chart to determine the differences in β between them. The trends of reliability index versus different span ratios are shown in Figures 7.1 to 7.5 for 15 ft span length on the short side. Figures 7.6 to 7.10 are the reliability index versus span ratio for the 20 ft span length. The relationship between LL/DL ratios for a particular span ratio and the reliability indices are shown in Figures 7.11 to 7.16 to determine the trend of β across different LL/DL ratio for 15 ft span length. Figure 7.17 to 7.20 are the reliability index versus LL/DL for the 20 ft span length.

Figures 7.21 to 7.24 are the 3-dimensional charts that describe the relationship of β versus span ratio and LL/DL ratio for both 15 ft and 20 ft span lengths.

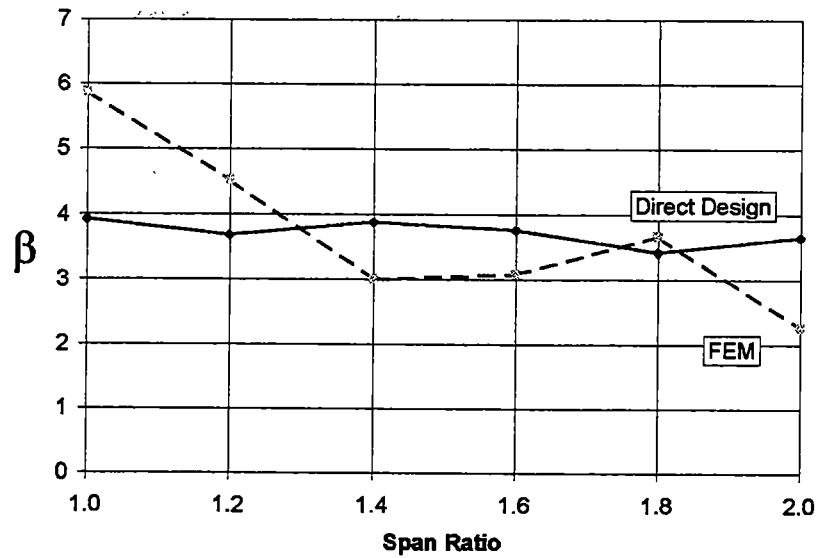


Figure 7.1: β versus span ratio for LL/DL of 0.5
($L1=15$ ft).

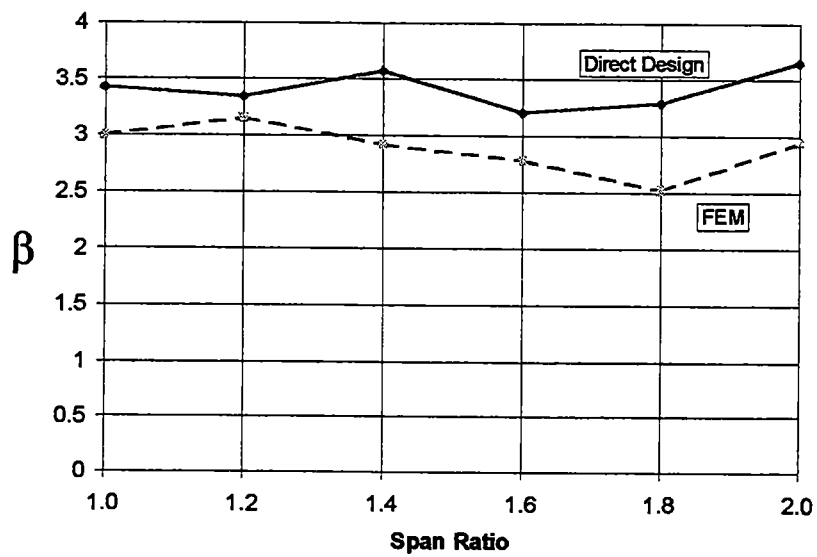


Figure 7.2: β versus span ratio for LL/DL of 1.0
($L1=15$ ft).

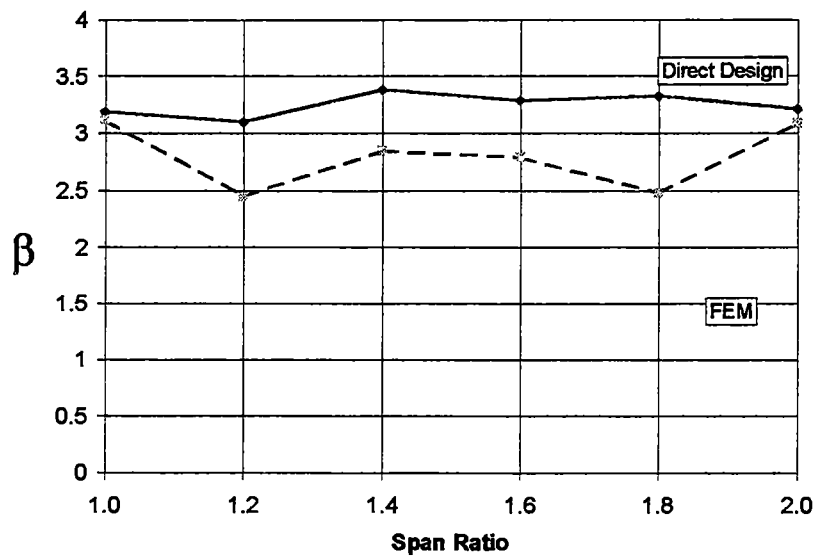


Figure 7.3: β versus span ratio for LL/DL of 1.5 ($L1=15$ ft).

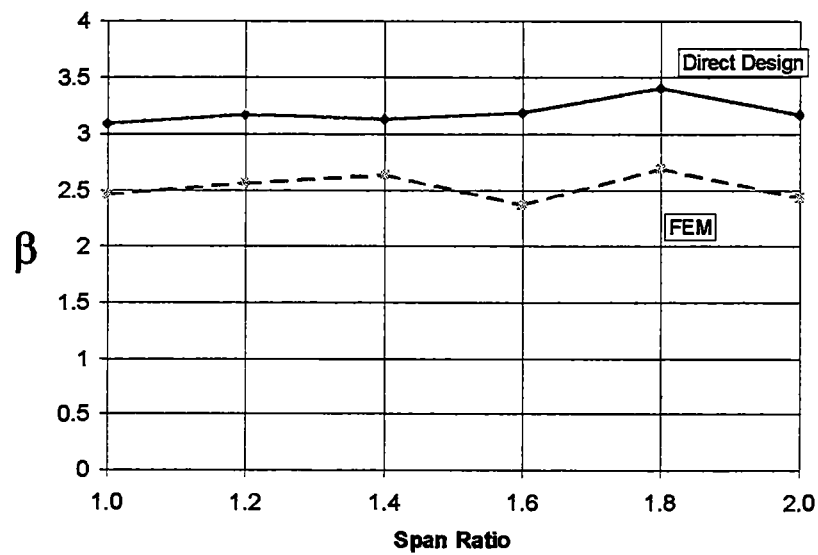


Figure 7.4: β versus span ratio for LL/DL of 2.0 ($L1=15$ ft).

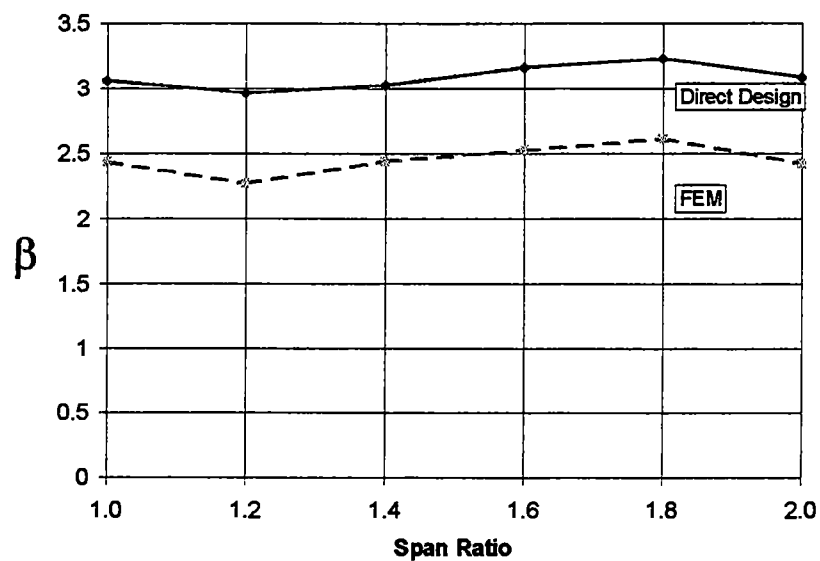


Figure 7.5: β versus span ratio for LL/DL of 2.5
($L1=15$ ft).

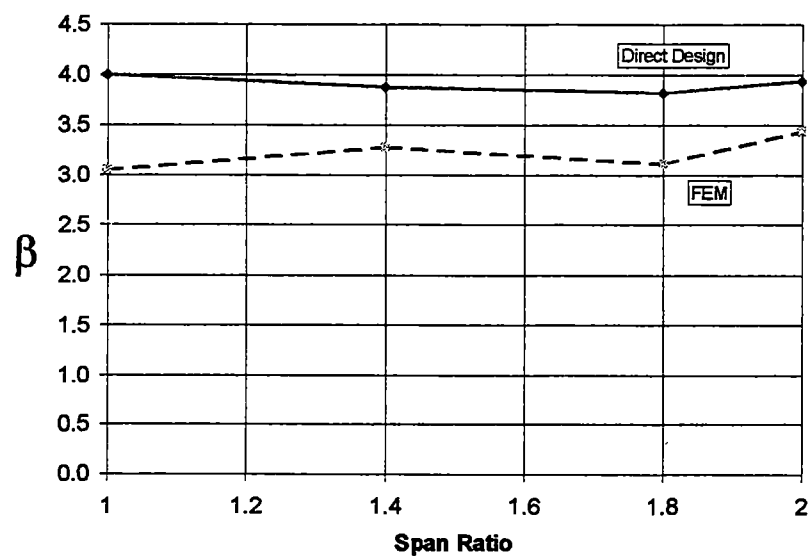


Figure 7.6: β versus span ratio for LL/DL of 0.5
($L1=20$ ft)

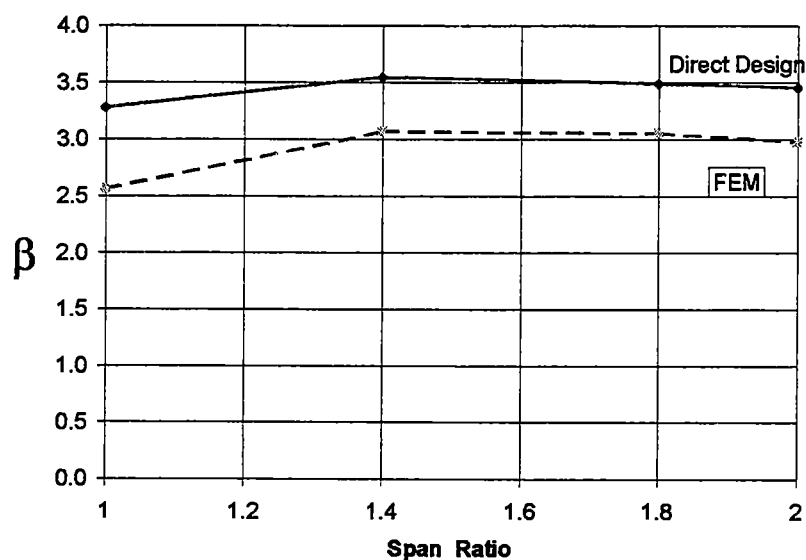


Figure 7.7: β versus span ratio for LL/DL of 1.0 ($L1=20$ ft).

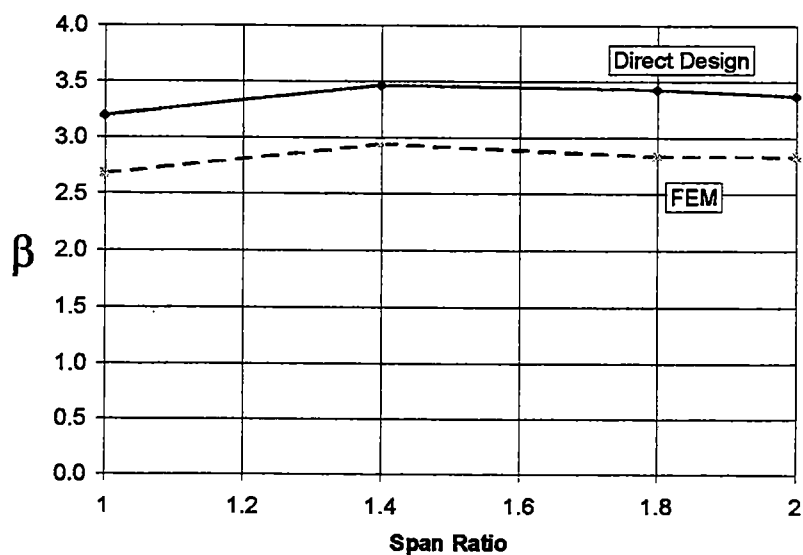


Figure 7.8: β versus span ratio for LL/DL of 1.5 ($L1=20$ ft).

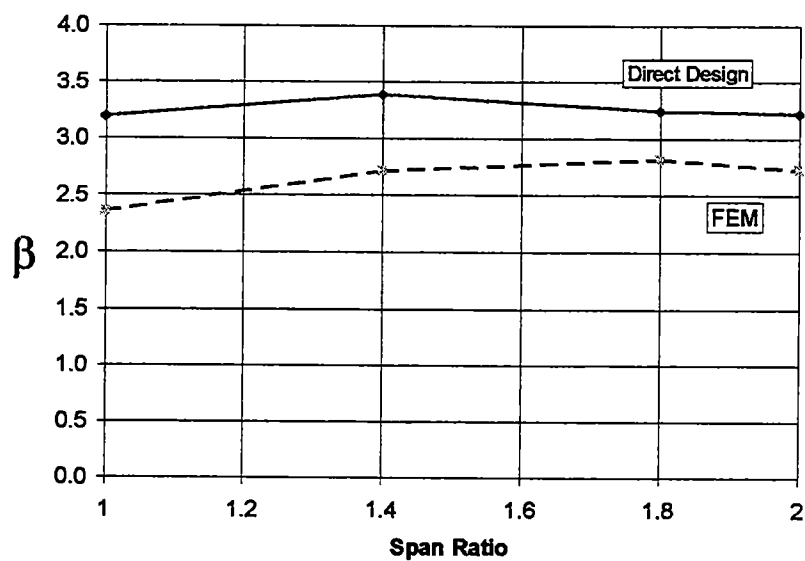


Figure 7.9: β versus span ratio for LL/DL of 2.0 ($L1=20$ ft).

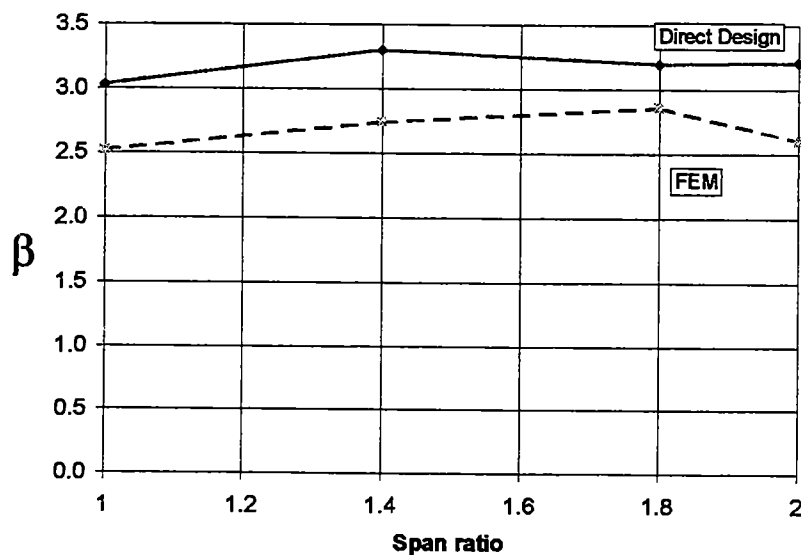


Figure 7.10: β versus span ratio for LL/DL of 2.5 ($L1=20$ ft).

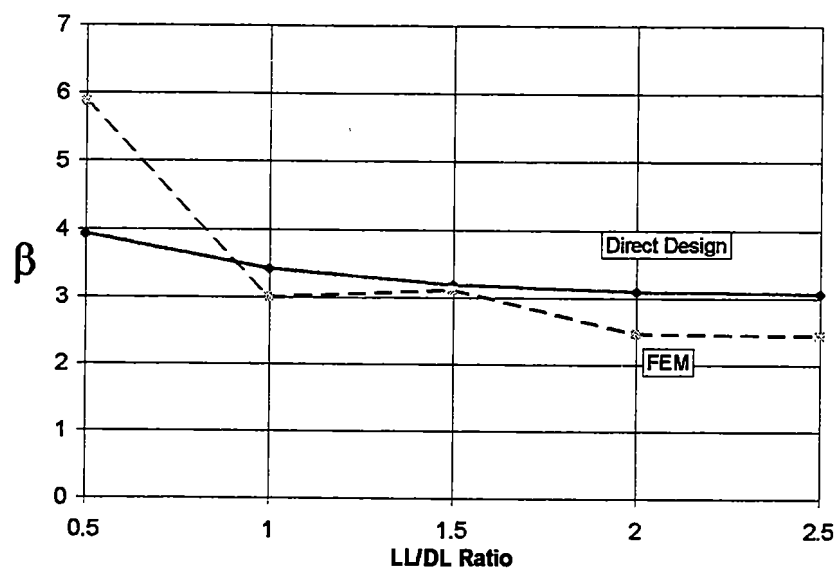


Figure 7.11: β versus LL/DL ratio for span ratio of 1.0 ($L_1=15$ ft)

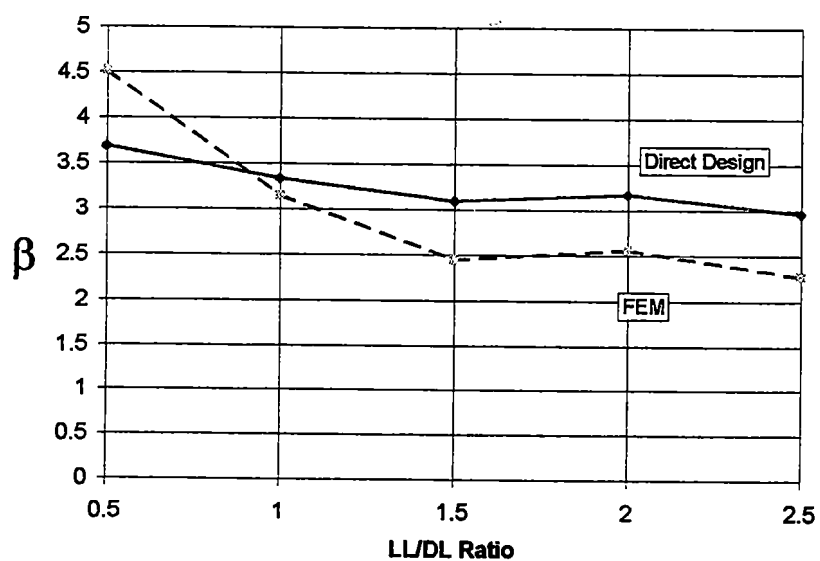


Figure 7.12: β versus LL/DL ratio for span ratio of 1.2 ($L_1=15$ ft)

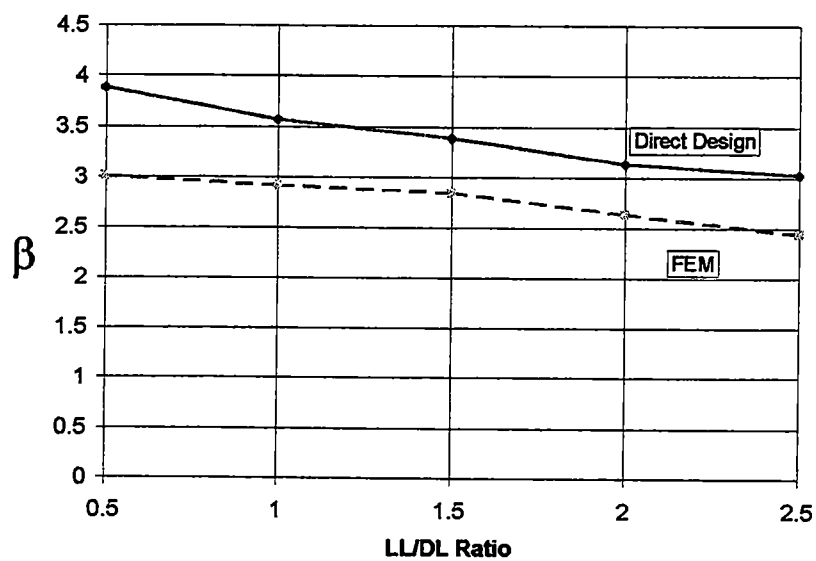


Figure 7.13: β versus LL/DL ratio for span ratio of 1.4 ($L_1=15$ ft).

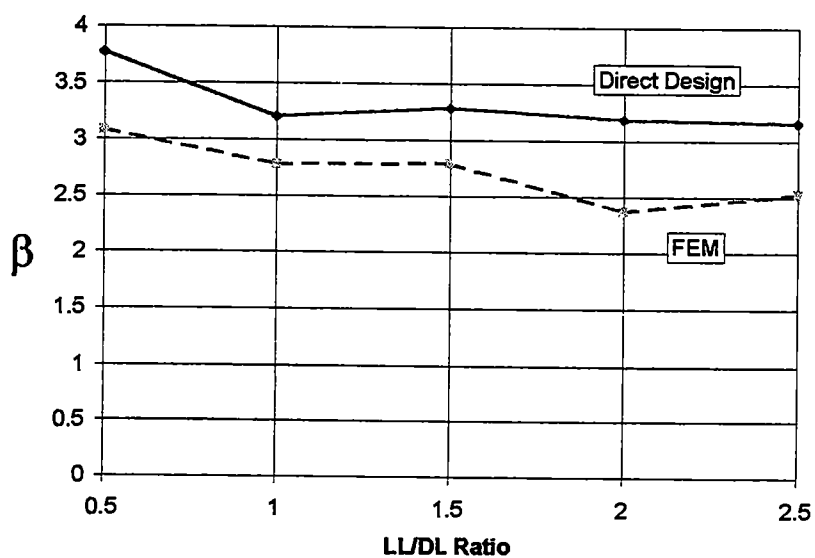


Figure 7.14: β versus LL/DL ratio for span ratio of 1.6 ($L_1=15$ ft).

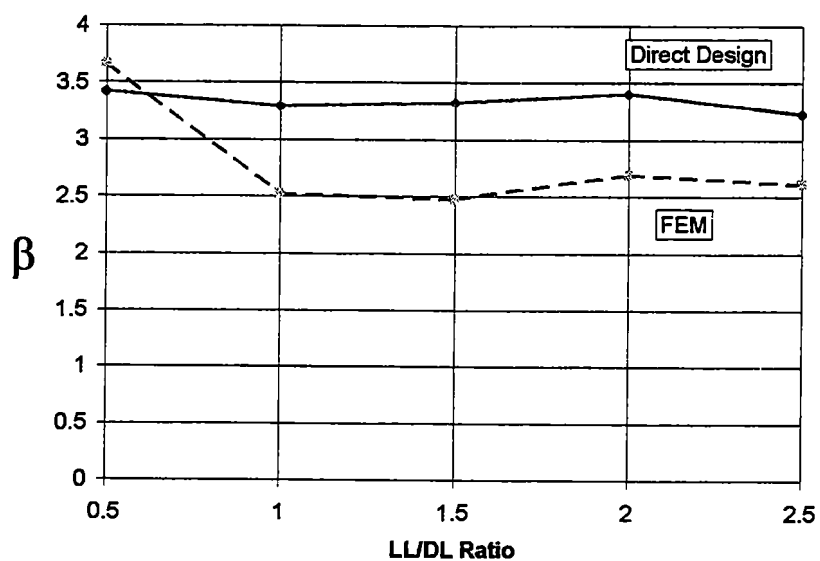


Figure 7.15: β versus LL/DL ratio for span ratio of 1.8 ($L_1=15$ ft).

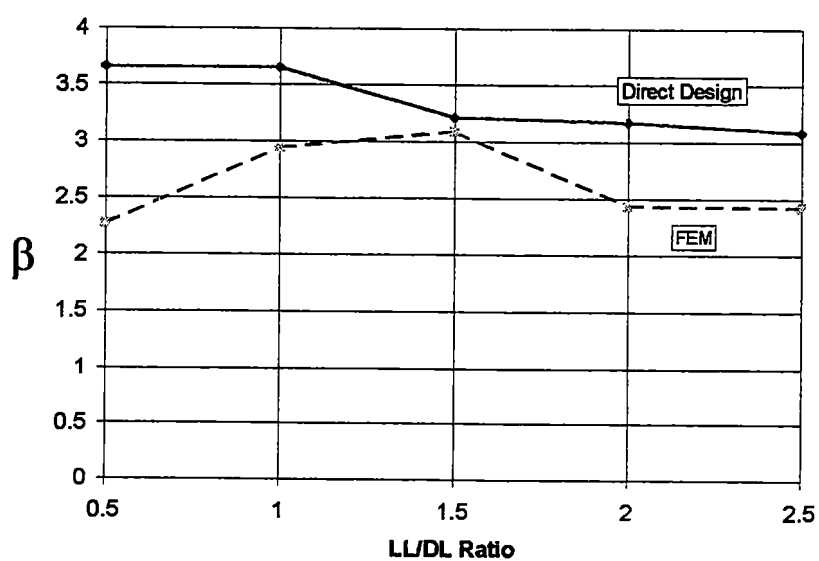


Figure 7.16: β versus LL/DL ratio for span ratio of 2.0 ($L_1=15$ ft).

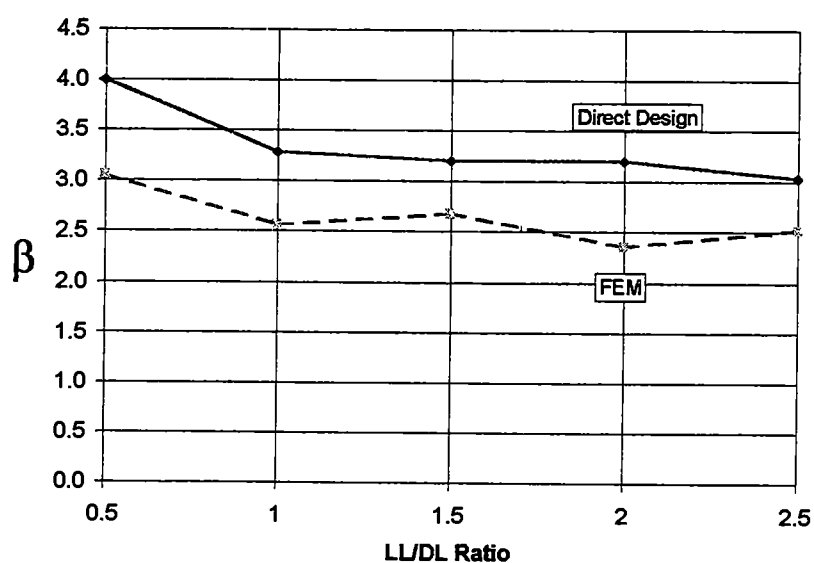


Figure 7.17: β versus LL/DL ratio for span ratio of 1.0 ($L_1=20$ ft).

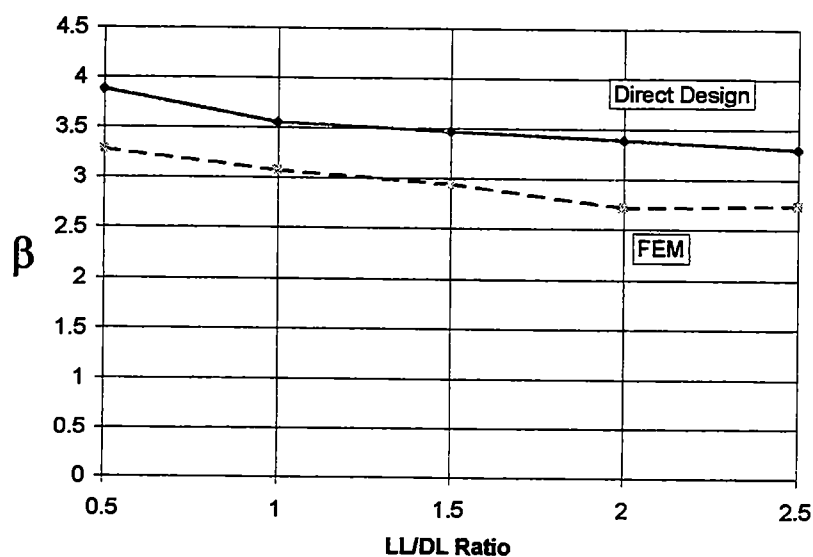


Figure 7.18: β versus LL/DL ratio for span ratio of 1.4 ($L_1=20$ ft).

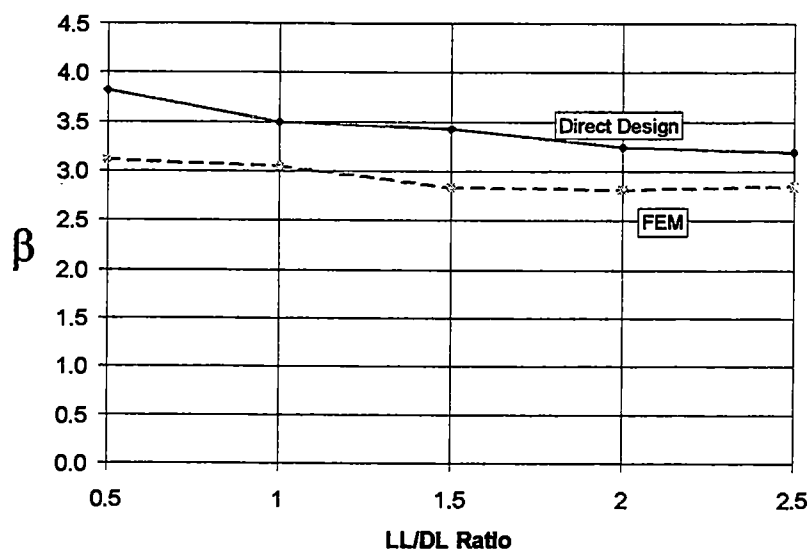


Figure 7.19: β versus LL/DL ratio for span ratio of 1.8 ($L_1=20$ ft).

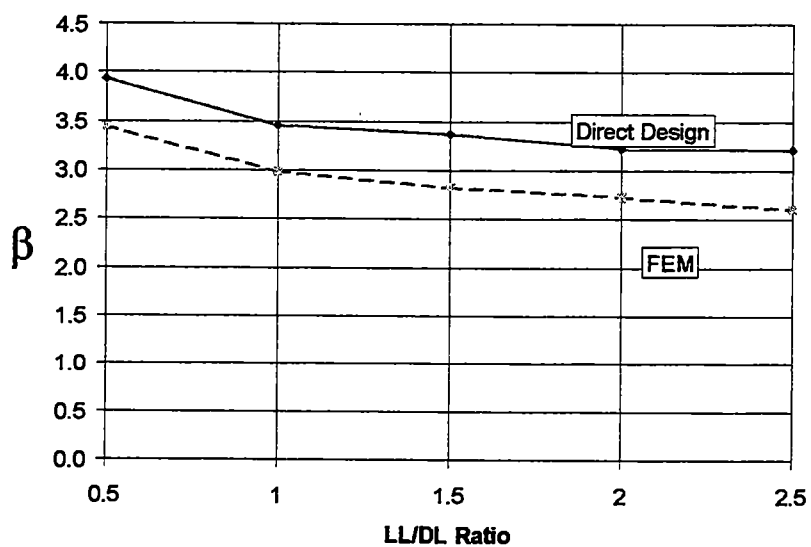


Figure 7.20: β versus LL/DL ratio for span ratio of 2.0 ($L_1=20$ ft).

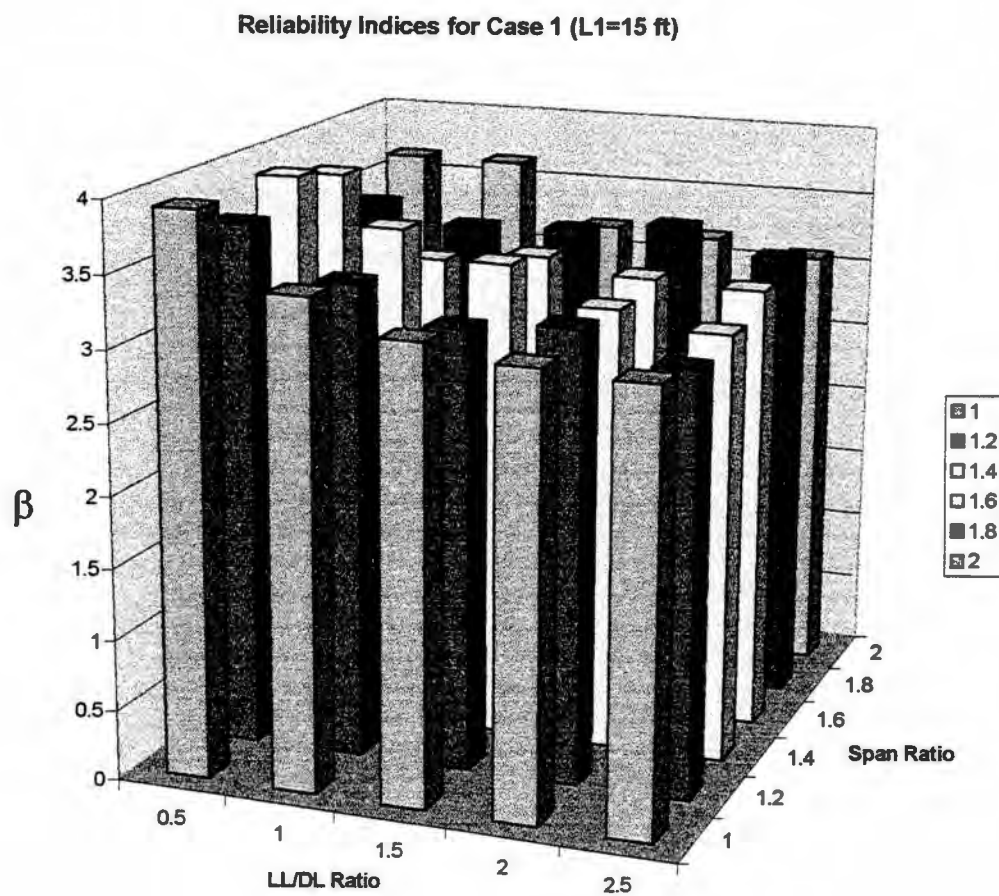


Figure 7.21: Reliability indices for case 1, direct design method analysis (L1=15 ft)

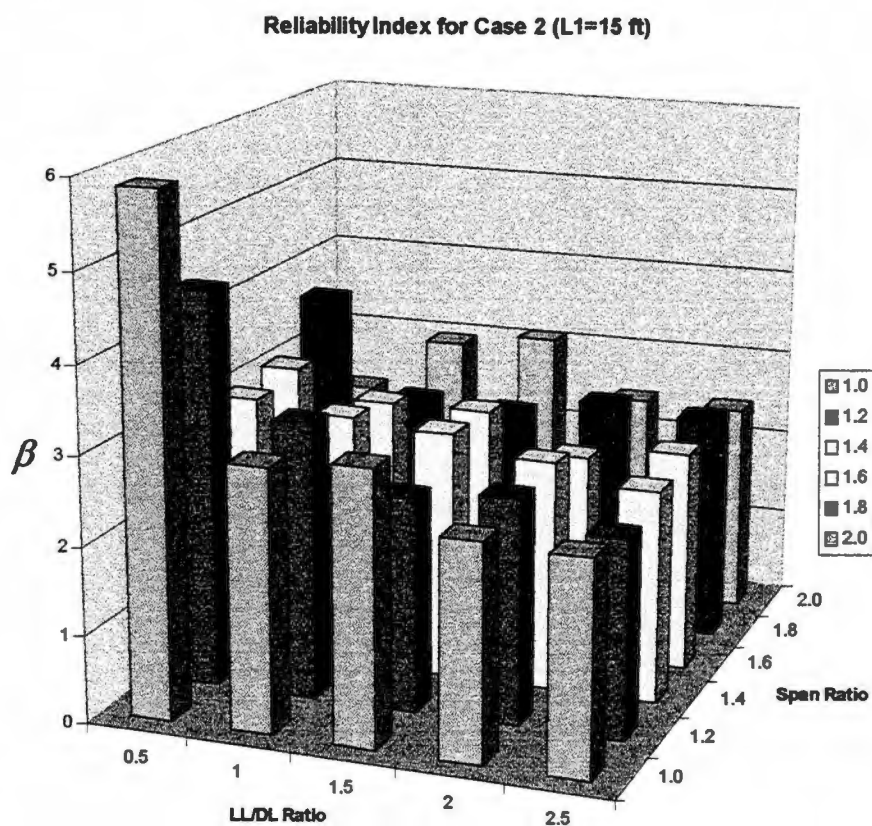


Figure 7.22: Reliability indices for case 2, finite element analysis (L1=15 ft)

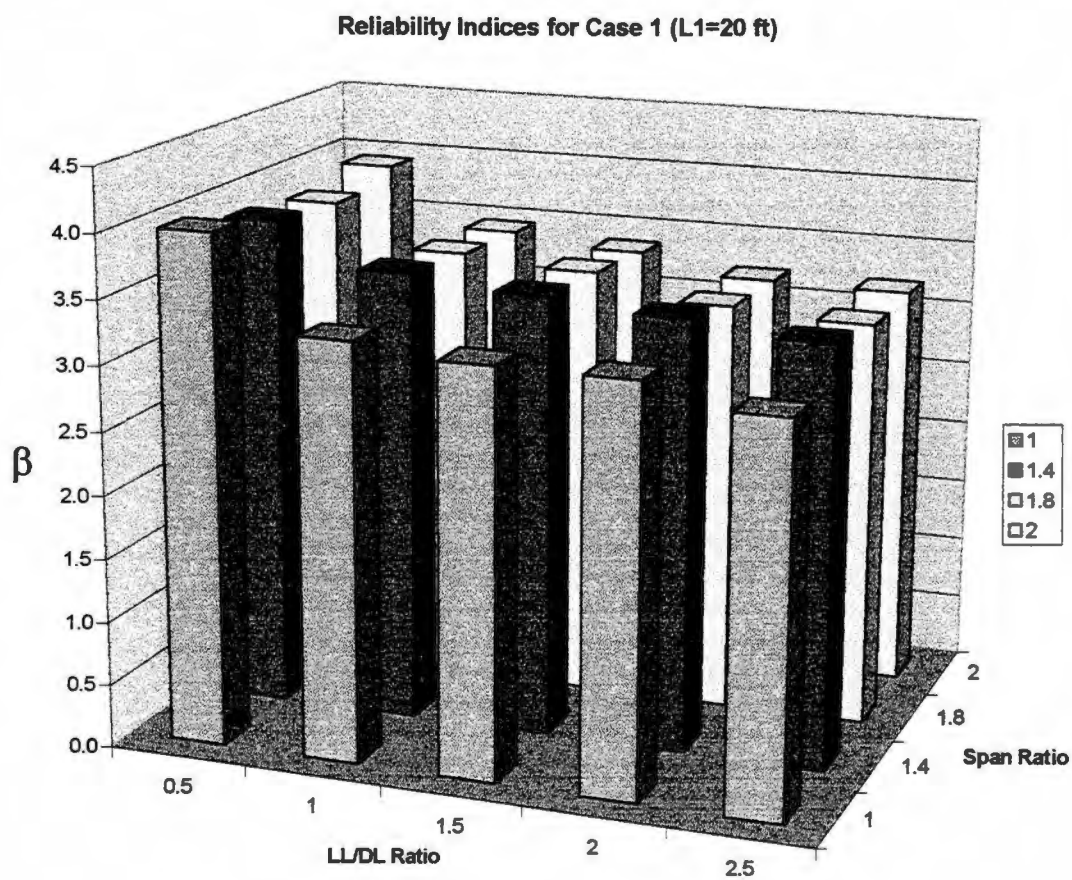


Figure 7.23: Reliability indices for case 1, direct design method analysis (L1=20 ft)

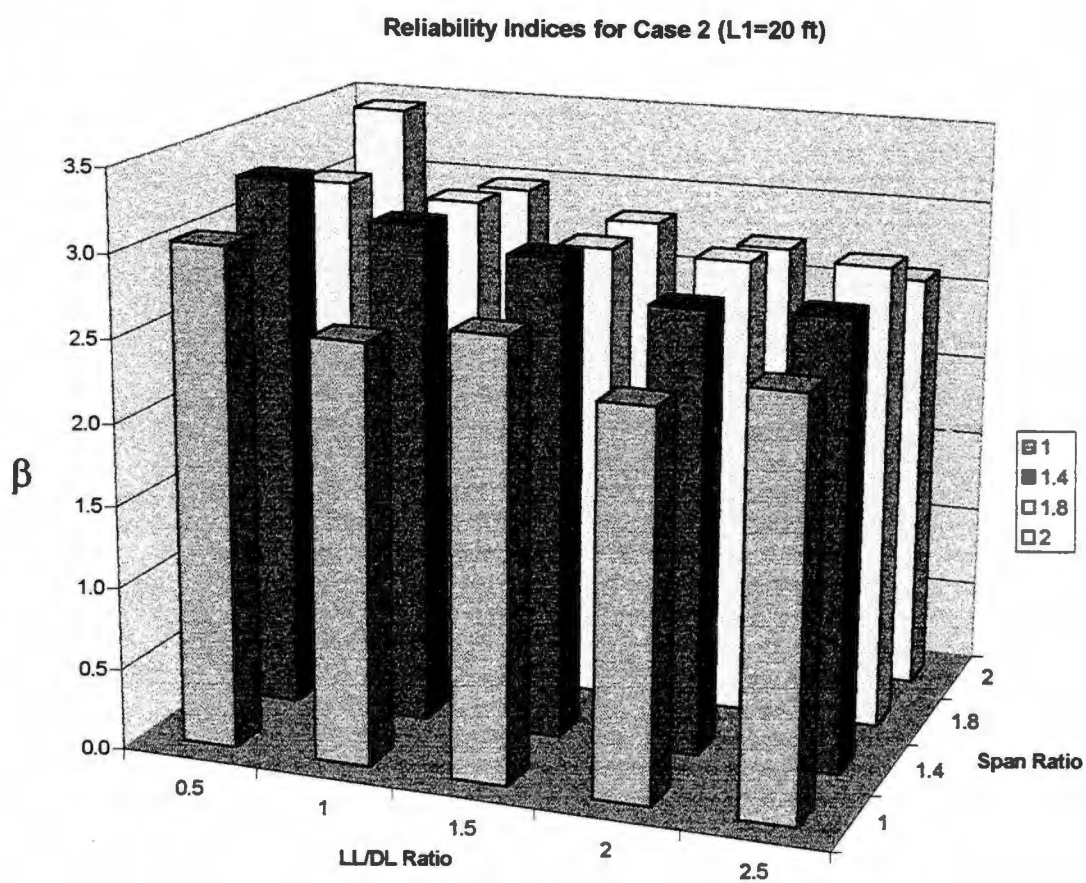


Figure 7.24: Reliability indices for case 2, finite element analysis (L1=20 ft)

The consistency of the reliability indices, β , can be observed for the direct design method (Case 1) between a given LL/DL ratio. The trend of β across the span ratios for all loading ratios displays fairly flat curves as shown in Figures 7.1 to 7.10 for the 15 ft and 20 ft span length. These indicate that ACI 318-95's "Direct Design Method" is a consistent design specification regardless of the span ratio of the two-way slabs assessed at the design moment. The lowest reliability index, 2.964 observed for $L1=15$ ft and 3.029 for $L1 = 20$ ft using direct method indicates that the design has a relatively high level of safety margin as compared to some other structural components like steel beam ($\beta = 3.1$, LL/DL = 2), steel column ($\beta = 3.1$, LL/DL = 1), and steel tension member ($\beta = 2.5$, LL/DL = 2.5) (Ellingwood et al., 1980).

Comparing the results between case 1 and case 2, case 2 has lower reliability indices in all LL/DL ratios except for LL/DL ratio of 0.5 with $L1=15$ ft (Figure 7.1). The reason for this behavior is: The reliability index for case 1 was mostly

controlled by the negative moments, while for case 2 it was controlled by the positive moments except for LL/DL of 0.5 ($L_1 = 15$ ft). To investigate these, the percent differences in moments from finite element analysis were compared with moments from the direct design method. The moments from direct design method and FEM for $L_1=15$ ft, LL/DL ratio of 1.5 and span ratio of 1.2 are given in Table 7.1:

Table 7.1: Comparison of analytical moment for LL/DL of 1.5, $L_1 = 15$ ft and span ratio of 1.2.

	Direction #	FEM (kips-ft)	Direct Design Method (kips-ft)	% Differences
NS Direction	Neg. Moment (Column Strip)	11.97	18.66	-35.89
	Neg. Moment (Middle Strip)	4.52	6.22	-27.39
	Pos. Moment (Column Strip)	7.61	8.04	-5.35
	Pos. Moment (Middle Strip)	6.80	5.36	26.83
WE Direction	Neg. Moment (Column Strip)	11.41	14.99	-23.88
	Neg. Moment (Middle Strip)	1.82	5.00	-63.59
	Pos. Moment (Column Strip)	7.09	6.46	9.72
	Pos. Moment (Middle Strip)	5.35	4.31	24.13

Noticing from Table 7.1 that the negative moments from direct design method are larger than the moments from FEM. On the other hand, the

positive moments from direct design method are smaller as compared to the FEM's moments (percent different in all the controlling positive moments from FEM compared to moments from direct design method are tabulated in Appendix C). Larger positive moment from finite element analysis resulted a reduction in β and eventually became the controlling β in a plate. As expected, the β values computed in case 2 is lower than the β values in case 1, since both of the design were done based on the design moment from the direct design method.

In the case where the β values of case 2 is greater than case 1, like LL/DL ratio of 0.5 with span ratio of 1.0, 1.2, and 1.8 (Figure 7.1), additional steels were provided to satisfy the minimum spacing reinforcement requirement. Table 7.2 is the comparisons of overdiseign ratio with the design moments obtained through direct design method and finite element analysis.

Table 7.2: Comparison of overdesign ratio for case 1 and case 2 for LL/DL of 0.5 and span ratio of 1.0 (L1=15 ft).

Direction #	Moment Capacity (kips-ft)	Direct design (kips-ft)	FEM (kips-ft)	Overdesign Ratio	
				Direct design	FEM
1	37.86	15.62	11.16	2.42	3.39
2	22.32	5.21	2.91	4.29	7.66
3	22.32	6.73	6.89	3.32	3.24
4	22.32	4.49	5.81	4.98	3.84
5	32.50	15.62	11.16	2.08	2.91
6	20.12	5.21	2.91	3.86	6.91
7	20.12	6.73	6.89	2.99	2.92
8	20.12	4.49	5.81	4.49	3.46

The moment capacities computed in Table 7.2 were calculated with the mean of the moment capacity's variables. The controlling location for both direct design method and FEM is direction number 5. The overdesign ratio for direct design method is 2.08 and for FEM is 2.91. As the result of higher overdesign ratio of the controlling location in case 2, higher reliability index of case 2 ($\beta=5.864$) over case 1 ($\beta=3.929$) is observed. Although FEM gave higher positive moments, overdesign due to spacing requirement may result in the higher β values in the positive moment direction compared to negative moment direction. In

the above example, direction numbers 2, 3, 4, 6, 7, and 8 were reinforced with additional steel just to satisfy the minimum spacing requirement instead of flexure requirements. Situations like this rarely occur in the high LL/DL ratio cases, since the rebars were designed to satisfy the flexure requirement instead of spacing requirement and greater FEM's positive moment has controlled among other locations. As a result of higher FEM's positive moment compared to the direct design method's design moment, case 2 appears to have lower reliability index compared to case 1.

Although finite element analysis model was done to simulate the actual floor slab system, idealization of the model may result in greater positive moments. The supporting columns of the slab were modelled as pinned supports in the FEM analysis. This has resulted in a stiffer support compared to column supports in the vertical displacement but lower rotational restraint between column and slab. On the other hand, the clear span in FEM model is longer than the clear span in direct design method (direct design method take

into account the face to face distance between columns). Hence, larger moments in the FEM analysis are expected with this assumption due to greater span length.

One of the findings in the studies was that the thickness of the slab does not influence the design moment in the direct design method. The procedures simply apply coefficients to the total static moment as described in section 4. The thicknesses of the slabs designed by the "Direct Design Method" were tabulated as shown in Table 7.3:

Table 7.3a: Thickness of the slab designed in inches ($L_1=15$ ft)

SPAN RATIO	LL/DL ratio				
	0.5	1	1.5	2	2.5
1	5	5	5	5	6
1.2	6	6	6	6	8
1.4	7	7	7	8	12
1.6	8	8	8	11	15
1.8	9	9	9	14	18
2	10	10	12	16	22

Table 7.3b: Thickness of the slab designed in inches ($L_1=20$ ft)

Span Ratio	LL/DL Ratio				
	0.5	1	1.5	2	2.5
1	7	7	9	14	18
1.4	9	12	17	24	30
1.8	12	18	26	35	42
2	13	21	30	40	48

However, thickness of the slab does influence the analytical moments in the finite element analysis. The gap of the β values between case 1 and case 2 and the fluctuations of the curves for case 2 imply that the slab thickness have impacts on the β values. This is due to the design of both cases are based on the moment obtained through direct design method, while assessed at different analytical moment values. Noticing that for span ratio of 1.0, 1.2, 1.4, 1.6 and 1.8 with LL/DL ratio of 0.5, 1.0, 1.5 having $L_1=15$ ft, the thickness is the same (Table 7.3a). Larger positive moment of FEM gave lower reliability indices, and even lower reliability indices for the slab with

same span ratio and thickness but higher LL/DL. Somehow, for situation where greater thickness (controlled by shear criteria) was involved, gaps between the FEM's moment and design moment has drew closer due to the increase of slab thickness that produced stiffer model in FEM analysis. Stiffer model eventually gave lower positive moments, which drew the gap between the β values of case 1 and case 2 closer. Percent differences of all the controlling positive moment compared to the moment from direct design method have been tabulated in Appendix C.

The thickness of a particular slab was designed based on the minimum slab thickness that bounded by serviceability limits and shear strength criteria. For cases with low LL/DL ratio like 0.5, 1.0 and 1.5, serviceability limits are most likely to control the slab thickness, particularly for the 15 ft span length. Recall that dead load is simply the self-weight of slab, and live load is the LL/DL ratio times the dead load (self-weight of the slab). Hence, for high LL/DL ratio, shear has

become the controlling design criteria and thicker slab is required. Thicker slab means higher dead load and much greater live load (due to high LL/DL ratio) which occasionally required several iterations before the design satisfies the shear criteria. Thicker slab and greater LL/DL ratio have resulted in higher design moment. Although flexure design provides the required reinforcement, the coefficient of variation and mean value in strength that form the performance function is small compared to those variables in the load effects. With the increase in loadings, Q , the variables that form the resistance, R , do not increase proportionally with the increase of load effects. Hence, the decrease of β is observed across the LL/DL ratio of a particular span ratio. As a result of that, case 1 tends to have a down slope trends with the increases of LL/DL ratio for both 15 ft and 20 ft span length as shown in Figure 7.21 and Figure 7.22.

Designs with slab thickness controlled by the shear requirements seem to have relatively flat

curves across the LL/DL ratio and span ratio for case 2. This can be observed through the case with 20 ft span length (Figure 7.6 to 7.10 and Figure 7.17 to 7.20). The differences in reliability indices between case 1 and case 2 are smaller compared to the case with span length of 15 ft. The curves shown relatively similar trends for both case 1 and case 2. The 20 ft span length seems to maintain the consistency of β compared to the 15 ft span. Again, thickness design due to shear requirements may result in the FEM's positive moments that are closer to the design moments.

Chapter 8

CONCLUSIONS AND RECOMMENDATIONS

In this thesis, the reliability of the two-way floor slab system using the direct design method of ACI 318-95 was investigated. Two cases were studied: Case 1, the designed slab system was assessed against the design moments obtained by the direct design method in ACI 318-95; Case 2, the designed slab system was assessed against the analytical moments obtained by the finite element analysis.

The result of the investigation displayed two interesting points. Case 1 showed consistent reliability indices across different span ratios for all LL/DL ratio for both $L1=15$ ft and $L1=20$ ft. Conversely, Case 2 showed inconsistent reliability index across different span ratio for all LL/DL ratio for $L1=15$ ft, but showed consistent reliability indices for $L1 = 20$ ft.

Most of the controlling cases in Case 1 were negative moments, while virtually all the controlling cases in Case 2 were positive moments. The

controlling of positive moment cases is due to the fact that the FEM's analytical positive moment is greater than the design moment from direct design method.

Although the actual moments in a floor slab are difficult to define, the results from Case 1 with the lowest β values of 2.964 ($L_1=15$ ft) and 3.029 ($L_2=20$ ft) indicate that the direct design method of the ACI 318-95 is a fairly reliable method assessing at the design moment.

On the other hand, results from case 2 showed that the β values can be as low as 2.264 ($L_1=15$ ft). Although this may indicate that the ACI building code does not give sufficient safety margin to the design, the idealization of the FEM model may cause higher positive moments as discussed in Chapter 7. While the design moments from the direct design method are independent of the slab thickness, slab thickness affects the finite element method's results. Different analytical moments from FEM have resulted in the fluctuation of β across different span ratio and LL/DL ratio.

These studies indicate that both direct design method and FEM give design moments that are different. The differences in design moments have resulted in the gaps between the β values of case 1 and case 2. Although the design by direct design method has been assessed with the FEM's moment, argument of whether it is a reliable design method still can not be concluded unless the actual moment of the slab is known through experiments.

Several shortcomings of this research are noted below with suggestions for future research:

- 1) The finite element analysis is a linear elastic analysis. In addition, a floor slab system is a highly indeterminate structure; perhaps comparison of the direct design method with the yield lines theories would be more appropriate.
- 2) The analysis was limited only to a flat-plate column supported slab system. Studies also can be made for beam supported floor slab system.

- 3) Reliability analysis also can be implemented on the other design method that is recommended by ACI 318-95, that is, the equivalent frame design method.
- 4) Effect of the lateral loadings together with gravity loads applied simultaneously on the floor slab also can be an interesting subject of studies in the reliability analysis of the floor slab system.
- 5) The reliability analysis also can be implemented on the shear failure in the floor slab system particularly for the punching shear design.

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APPENDICES

APPENDIX A
Computer Program
Written in MATHLAB4.0
For Reliability Index
Computation

```

% Call statistics for Variables from notepad file

fid=fopen('design15.txt','r');
Data=fscanf(fid,'%g',[18,inf]);
status=fclose(fid);
Data=Data';

% Feed in data by routines

for n=1:1:24;

% Define variables as normal variables

N=Data(n,1);
R=Data(n,2);
NAs=Data(n,3);
NVAs=Data(n,4);
NFy=Data(n,5);
NVFy=Data(n,6);
ND=Data(n,7);
NVD=Data(n,8);
NK=Data(n,9);
NVK=Data(n,10);
NFc=Data(n,11);
NVFc=Data(n,12);
NLL=Data(n,13);
NVLL=Data(n,14);
NDL=Data(n,15);
NVDL=Data(n,16);
NB=Data(n,17);
NVB=Data(n,18);

% Tracking the direction number

fprintf('Design Number:%g\n',N)

if R>=1 & R<2
fprintf('North-South Negative Moment (Column
Strip)\n\n')
elseif R>=2 & R<3
fprintf('North-South Negative Moment (Middle
Strip)\n\n')

```

```

elseif R>=3 & R<4
fprintf('North-South Positive Moment (Column
Strip)\n\n')
elseif R>=4 & R<5
fprintf('North-South Positive Moment (Middle
Strip)\n\n')
elseif R>=5 & R<6
fprintf('West-East Negative Moment (Column Strip)\n\n')
elseif R>=6 & R<7
fprintf('West-East Negative Moment (Middle Strip)\n\n')
elseif R>=7 & R<8
fprintf('West-East Negative Moment (Column Strip)\n\n')
else
fprintf('West-East Negative Moment (Middle Strip)\n\n')
end

fprintf('As=%g      VAs=%g\nFy=%g      VFy=%g\nD=%g
      VD=%g\nK=%g      VK=%g\nFc=%g
      VFc=%g\nLL=%g      VLL=%g\nDL=%g
      VDL=%g\nB=%g
      VB=%g\n\n', NAs, NVAs, NFy, NVFy, ND, NVD, NK, NVK, NFc, N
VFc, NLL, NVLL, NDL, NVDL, NB, NVB)
fprintf(' Iteration      Reliability Index\n\n')

% Set initial RI values (dummy value to initial the run
of the program)

RI=0;
NRI=100;
i=0;

% Constraints to exist for the iteration as RI reach its
stable values

while abs(RI-NRI)>=0.001;
i=i+1;
NRI=RI;

if NRI~=0;

% Define variables at its testing point

As=Data(n,3)-a1As*RI*Data(n,4);

```

```

Fy=NFy-alFy*RI*NVFy;
D=Data(n,7)-alD*RI*Data(n,8);
K=Data(n,9)-alK*RI*Data(n,10);
Fc=Data(n,11)-alFc*RI*Data(n,12);
LL=NLL-alLL*RI*NVLL;
DL=Data(n,15)-alDL*RI*Data(n,16);
B=Data(n,17)-alB*RI*Data(n,18);

```

```

else

```

```

LL=NLL;
VLL=NVLL;
Fy=NFy;
VFy=NVFy;
As=NAAs;
VAs=NVAs;
D=ND;
VD=NVD;
K=NK;
VK=NVK;
Fc=NFc;
VFc=NVFc;
DL=NDL;
VDL=NVDL;
B=NB;
VB=NVB;

```

```

end

```

```

% Normalized the Live Load data

```

```

a=pi/(sqrt(6)*Data(n,14));
u=Data(n,13)-0.577/a;
e=-exp(-a*(LL-u));

```

```

Fl=exp(e);
fl=a*exp(-a*(LL-u)+e);
fid=fopen('Fl.txt','w');
fprintf(fid,'%g',Fl);
fclose(fid);
x=[1];
x=fsolve('fun2',x);

```



```

options(14)=100;
options(6)=1;
x=fsolve('fun4',x,options);
RI=x;
fprintf(' %g          %g\n',i,RI)

```

```

% Command to exist the program as iteration exceed 15
runs

```

```

if i>=15;
RI=NRI;
end
end

```

```

fprintf('\n\n\n          ***- End of Reliability Computation
for Design # %g  -***\n',N)
fprintf('          ***-          Direction Assigned No. %g
-***\n\n\n',R)

```

```

end

```

APPENDIX B

Example of The

Design of Two-Way Floor

Slab System

With MathCad6.0

INPUT DATA

CASE: 05-1

RATIO OF LONGER SPAN TO SHORTER SPAN:

$$\xi := 1.0$$

RATIO OF LIVE LOAD TO DEAD LOAD:

$$\gamma := 0.5$$

LENGTH OF THE SHORTER SPAN:

$$L_2 := 15$$

COLUMN WIDTH & LENGTH:

$$C = 1.5$$

$$L_c = 12$$

LEGTH OF THE LONGER SPAN:

$$L_1 = L_2 \cdot \xi$$

$$L_1 = 15$$

STRENGTH OF THE MATERIAL:

$$f_y = 60000$$

$$f_c = 4000$$

COMPUTATION

1) Compute the minimum thickness req'd:

$$L_{\eta 1} = L_1 - C$$

$$L_{\eta 1} = 13.5$$

$$h_{\min} = \frac{L_{\eta 1}}{36} \cdot 12$$

$$h_{\min} = 4.5$$

$$\text{USE } h = 5$$

2) Find Live Load, LL

$$DL = h \cdot \frac{150}{12}$$

$$LL = DL \cdot \gamma$$

$$LL = 31.25$$

3) Compute Factored Wu:

$$W_u = 1.4 \cdot h \cdot \frac{150}{12} + 1.7 \cdot LL$$

$$W_u = 140.625$$

4) Determine the adequacy of minimum thickness is sufficient for shear requirements:

Average depth:

$$d := h - \frac{5}{16} - 0.75$$

a) Wide beam action, assuming 3/4 inches cover and #5 rebar:

$$V_{u1} = \frac{W_u \cdot L_2 \cdot \left(\frac{L_1}{2} - \frac{C}{2} - \frac{d}{12} \right)}{1000}$$

$$V_{u1} = 13.546$$

$$\phi V_c = \frac{0.85 \cdot 2 \cdot \sqrt{f_c} \cdot 12 \cdot L_2 \cdot d}{1000}$$

$$\phi V_c = 76.203$$

Since $\phi V_c > V_u$, OK

b) Two way action.

$$V_u = W_u \cdot \frac{L_1 \cdot L_2 - \left(C + \frac{d}{12} \right) \cdot \left(C + \frac{d}{12} \right)}{1000}$$

$$V_u = 31.171$$

Perimeter of the critical section at $d/2$ around the column:

$$b_0 := 4 \cdot \left(C + \frac{d}{12} \right)$$

$$\frac{b_0}{d} = 1.857$$

$$V_{c1} := 6 \cdot \sqrt{f_c} \cdot b_0 \cdot d \cdot 12$$

$$V_{c2} := \left[\frac{40}{\left[\frac{b_0}{\left(\frac{d}{12} \right)} \right]} + 2 \right] \cdot \sqrt{f_c} \cdot b_0 \cdot d \cdot 12$$

$$V_{c3} := 4 \cdot \sqrt{f_c} \cdot b_0 \cdot d \cdot 12$$

$$V_{ci} := \begin{bmatrix} V_{c1} \\ V_{c2} \\ V_{c3} \end{bmatrix} \cdot \frac{1}{1000}$$

$$V_c := \min(V_{ci})$$

$$V_c = 82.927$$

$$\phi V_n = 0.85 \cdot V_c$$

$$\phi V_n = 70.488$$

$$V_u = 31.171$$

Since $\phi V_n > V_u$, the thickness of the slab is OK. $\Rightarrow$ Use 5 inches.

5) Compute α value, $\alpha=0$

7) Compute Negative and Positive Moment for Slab Strip 1 and 2:

$$L_1 = 15$$

$$L_{\eta 1} = 13.5$$

$$L_2 = 15$$

$$W_u = 140.625$$

$$\text{Total Moment: } M_o = W_u \cdot L_2 \cdot \frac{L_{\eta 1}^2}{8000}$$

$$M_o = 48.054$$

COLUMN

A1

MIDDLE

B1

Moment Coeff

$$MC_A := -0.65$$

$$MC_{M1} := 0.35$$

$$MC_B := -0.65$$

Moment

$$M_A := MC_A \cdot M_o$$

$$M_{M1} := MC_{M1} \cdot M_o$$

$$M_B := MC_B \cdot M_o$$

$$M_A = -31.235$$

$$M_{M1} = 16.819$$

$$M_B = -31.235$$

8) Compute Negative and Positive Moment for Slab Strip A and B:

$$L_1 = 15$$

$$L_2 = 15$$

$$L_{\eta 2} = L_2 - C$$

$$L_{\eta 2} = 13.5$$

$$W_u = 140.625$$

$$\text{Total Moment: } M_o = W_u \cdot L_1 \cdot \frac{L_{\eta 2}^2}{8000}$$

$$M_o = 48.054$$

COLUMN	A1	MIDDLE	A2
Moment Coeff	$MC_1 = -0.65$	$MC_{M2} = 0.35$	$MC_2 = -0.65$
Moment	$M_1 = MC_1 \cdot M_o$ $M_1 = -31.235$	$M_{M2} = MC_{M2} \cdot M_o$ $M_{M2} = 16.819$	$M_2 = MC_2 \cdot M_o$ $M_2 = -31.235$

9) Distribute the Positive and Negative Moment to the Column and Middle Strips:

Strips Width: $L_s = \frac{L_2}{4}$ $L_s = 3.75$

10) Divide the moment to the Column and Middle Strips (Strips 1 And 2) Negative Moment

	Column Strip	Middle Strip	Column Strip
Length	$Cl_s := 2 \cdot L_s$ $Cl_s = 7.5$	$CM1_s := L_2 - L_s - L_s$ $CM1_s = 7.5$	$C2_s := 2 \cdot L_s$ $C2_s = 7.5$
Slab Moment	$M_A = -31.235$		$M_B = -31.235$
Percent Moment	$P1 = 0.75$	$PM = 0.25$	$P2 = 0.75$
Moment	$NMC1 := M_A \cdot P1$ $NMC1 = -23.426$	$NMS1 := M_A \cdot PM$ $NMS1 = -7.809$	$NMC2 := M_B \cdot P2$ $NMC2 = -23.426$
Depth	$d := h - 0.75 - \frac{5}{16}$	$d = 3.938$	
Ast Required	$Ast1 = \frac{NMC1 \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$	$AstM := \frac{NMS1 \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$	$Ast2 = \frac{NMC2 \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$
Compute a	$a1 = \frac{Ast1 \cdot f_y - 1}{0.85 \cdot f_c \cdot Cl_s \cdot 12}$ $a1 = 0.28$	$aM = \frac{AstM \cdot f_y - 1}{0.85 \cdot f_c \cdot CM1_s \cdot 12}$ $aM = 0.093$	$a2 = \frac{Ast2 \cdot f_y - 1}{0.85 \cdot f_c \cdot C2_s \cdot 12}$ $a2 = 0.28$
Compute jd	$jd1 = d - \frac{a1}{2}$ $jd1 = 3.797$	$jdM := d - \frac{aM}{2}$ $jdM = 3.891$	$jd2 = d - \frac{a2}{2}$ $jd2 = 3.797$
As Required	$As1 = \frac{NMC1 \cdot 12000}{0.9 \cdot f_y \cdot jd1}$ $As1 = -1.371$	$AsM = \frac{NMS1 \cdot 12000}{0.9 \cdot f_y \cdot jdM}$ $AsM = -0.446$	$As2 = \frac{NMC2 \cdot 12000}{0.9 \cdot f_y \cdot jd2}$ $As2 = -1.371$

Minimum Number Of Bar:

$$BS := \left(\frac{2 \cdot h}{18} \right)$$

$$n1 := C1s \cdot \frac{12}{\min(BS)}$$

$$n1 = 9$$

$$nM := CM1s \cdot \frac{12}{\min(BS)}$$

$$nM = 9$$

$$n2 := C2s \cdot \frac{12}{\min(BS)}$$

$$n2 = 9$$

Steel Chosen

$$Nb1 = 9$$

$$Db1 = 4$$

$$NbM = 9$$

$$DbM = 3$$

$$Nb2 = 9$$

$$Db2 = 4$$

$$NA1_s = Nb1 \cdot \frac{\left(\frac{Db1}{8} \right)^2}{4} \cdot \pi$$

$$NA1_s = 1.767$$

$$NAM_s := NbM \cdot \frac{\left(\frac{DbM}{8} \right)^2}{4} \cdot \pi$$

$$NAM_s = 0.994$$

$$NA2_s := Nb2 \cdot \frac{\left(\frac{Db2}{8} \right)^2}{4} \cdot \pi$$

$$NA2_s = 1.767$$

Moment Capacity

$$d_{n1} := h - \frac{Db1}{16} - 0.75$$

$$d_{n1} = 4$$

$$d_{nm1} := h - \frac{DbM}{16} - 0.75$$

$$d_{nm1} = 4.063$$

$$d_{n2} := h - \frac{Db2}{16} - 0.75$$

$$d_{n2} = 4$$

$$M_{n1} = NA1_s \cdot \frac{f_y}{12000} \cdot \left(d_{n1} - \frac{NA1_s \cdot f_y}{1.7 \cdot f_c \cdot C1s \cdot 12} \right)$$

$$M_{n1} = 33.812$$

$$NMC1 = -23.426$$

$$M_{nm1} = NAM_s \cdot \frac{f_y}{12000} \cdot \left(d_{nm1} - \frac{NAM_s \cdot f_y}{1.7 \cdot f_c \cdot CM1s \cdot 12} \right)$$

$$M_{nm1} = 19.707$$

$$NMS1 = -7.809$$

$$M_{n2} = NA2_s \cdot \frac{f_y}{12000} \cdot \left(d_{n2} - \frac{NA2_s \cdot f_y}{1.7 \cdot f_c \cdot C2s \cdot 12} \right)$$

$$M_{n2} = 33.812$$

$$NMC2 = -23.426$$

POSITIVE MOMENT

Column
StripMiddle
StripColumn
Strip

Length

$$C1s := 2 \cdot Ls$$

$$C1s = 7.5$$

$$CM1s := L_2 - Ls - Ls$$

$$CM1s = 7.5$$

$$C2s := 2 \cdot Ls$$

$$C2s = 7.5$$

Slab Moment

$$M_{M1} = 16.819$$

$$M_{M1} = 16.819$$

Percent Moment

$$P1 = 0.6$$

$$PM = 0.4$$

$$P2 = 0.6$$

Moment

$$PMC1 = M_{M1} \cdot P1$$

$$PMC1 = 10.091$$

$$PMS1 = M_{M1} \cdot PM$$

$$PMS1 = 6.728$$

$$PMC2 = M_{M1} \cdot P2$$

$$PMC2 = 10.091$$

Depth

$$d = h - 0.75 - \frac{5}{16}$$

$$d = 3.938$$

Ast Required	$Ast1 := \frac{PMC1 \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$	$AstM := \frac{PMS1 \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$	$Ast2 := \frac{PMC2 \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$
Compute a	$a1 := \frac{Ast1 \cdot f_y}{0.85 \cdot f_c \cdot C1s \cdot 12}$	$aM := \frac{AstM \cdot f_y}{0.85 \cdot f_c \cdot CM1s \cdot 12}$	$a2 := \frac{Ast2 \cdot f_y}{0.85 \cdot f_c \cdot C2s \cdot 12}$
	$a1 = 0.121$	$aM = 0.08$	$a2 = 0.121$
Compute jd	$jd1 = d - a1$	$jdM = d - aM$	$jd2 = d - a2$
	$jd1 = 3.817$	$jdM = 3.857$	$jd2 = 3.817$
As Required	$As1 := \frac{PMC1 \cdot 12000}{0.9 \cdot f_y \cdot jd1}$	$AsM := \frac{PMS1 \cdot 12000}{0.9 \cdot f_y \cdot jdM}$	$As2 := \frac{PMC2 \cdot 12000}{0.9 \cdot f_y \cdot jd2}$
	$As1 = 0.588$	$AsM = 0.388$	$As2 = 0.588$
Minimum Number Of Bar:	$BS := \left(\frac{2 \cdot h}{18} \right)$		
	$n1 := C1s \cdot \frac{12}{\min(BS)}$	$nM := CM1s \cdot \frac{12}{\min(BS)}$	$n2 := C2s \cdot \frac{12}{\min(BS)}$
	$n1 = 9$	$nM = 9$	$n2 = 9$
Steel Chosen	$Nb1 = 9$	$PDb1 := 3$	$Nb2 = 9$
	$PDb1 := 3$	$NbM := 9$	$PDbM := 3$
	$PDbM := 3$	$Nb2 := 9$	$PDb2 := 3$
	$PA1_s := Nb1 \cdot \frac{\left(\frac{PDb1}{8} \right)^2}{4} \cdot \pi$	$PAM_s := NbM \cdot \frac{\left(\frac{PDbM}{8} \right)^2}{4} \cdot \pi$	$PA2_s := Nb2 \cdot \frac{\left(\frac{PDb2}{8} \right)^2}{4} \cdot \pi$
	$PA1_s = 0.994$	$PAM_s = 0.994$	$PA2_s = 0.994$
Moment Capacity	$d_{p1} := h - \frac{PDb1}{16} - 0.75$	$d_{pm1} := h - \frac{PDbM}{16} - 0.75$	$d_{p2} := h - \frac{PDb2}{16} - 0.75$
	$d_{p1} = 4.063$	$d_{pm1} = 4.063$	$d_{p2} = 4.063$
	$M_{p1} := PA1_s \cdot \frac{f_y}{12000} \cdot \left(d_{p1} - \frac{PA1_s \cdot f_y}{1.7 \cdot f_c \cdot C1s \cdot 12} \right)$	$M_{p1} = 19.707$	$PMC1 = 10.091$
	$M_{pm1} := PAM_s \cdot \frac{f_y}{12000} \cdot \left(d_{pm1} - \frac{PAM_s \cdot f_y}{1.7 \cdot f_c \cdot CM1s \cdot 12} \right)$	$M_{pm1} = 19.707$	$PMS1 = 6.728$
	$M_{p2} := PA2_s \cdot \frac{f_y}{12000} \cdot \left(d_{p2} - \frac{PA2_s \cdot f_y}{1.7 \cdot f_c \cdot C2s \cdot 12} \right)$	$M_{p2} = 19.707$	$PMC2 = 10.091$

11) Divide the moment to the Column and Middle Strips (Strips A and B) Negative Moment

	Column Strip	Middle Strip	Column Strip
Length	$CA_s := 2 \cdot L_s$ $CA_s = 7.5$	$CM2s := L_1 - L_s - L_s$ $CM2s = 7.5$	$CB_s := 2 \cdot L_s$ $CB_s = 7.5$
Slab Moment	$M_1 = -31.235$		$M_2 = -31.235$
Percent Moment	$PA := 0.75$	$PM := 0.25$	$PB := 0.75$
Moment	$NMCA := M_1 \cdot PA$ $NMCA = -23.426$	$NMS2 := M_1 \cdot PM$ $NMS2 = -7.809$	$NMCB := M_2 \cdot PB$ $NMCB = -23.426$
Depth	$d := h - 0.75 - \frac{5}{16} - \frac{Db1}{8}$	$d = 3.438$	
Ast Required	$AstA := \frac{NMCA \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$	$AstM2 := \frac{NMS2 \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$	$AstB := \frac{NMCB \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$
Compute a	$a1 = \frac{AstA \cdot f_y \cdot 1}{0.85 \cdot f_c \cdot CA_s \cdot 12}$ $a1 = 0.321$	$aM = \frac{AstM2 \cdot f_y \cdot 1}{0.85 \cdot f_c \cdot CM2s \cdot 12}$ $aM = 0.107$	$a2 = \frac{AstB \cdot f_y \cdot 1}{0.85 \cdot f_c \cdot CB_s \cdot 12}$ $a2 = 0.321$
Compute jd	$jd1 = d - a1$ $jd1 = 3.116$	$jdM = d - aM$ $jdM = 3.33$	$jd2 = d - a2$ $jd2 = 3.116$
As Required	$AsA := \frac{NMCA \cdot 12000}{0.9 \cdot f_y \cdot jd1}$ $AsA = -1.67$	$AsM := \frac{NMS2 \cdot 12000}{0.9 \cdot f_y \cdot jdM}$ $AsM = -0.521$	$AsB := \frac{NMCB \cdot 12000}{0.9 \cdot f_y \cdot jd2}$ $AsB = -1.67$
Minimum Number Of Bar:	$BS := \left(\frac{2 \cdot h}{18} \right)$		
	$n1 := CA_s \cdot \frac{12}{\min(BS)}$ $n1 = 9$	$nM := CM2s \cdot \frac{12}{\min(BS)}$ $nM = 9$	$n2 := CB_s \cdot \frac{12}{\min(BS)}$ $n2 = 9$

Steel Chosen NbA := 9 DbA := 4 NbM2 := 9 DbM2 := 3 NbB := 9 DbB := 4

$$NAA_s = NbA \cdot \frac{\left(\frac{DbA}{8}\right)^2}{4} \cdot \pi$$

$$NAA_s = 1.767$$

$$NAM2_s = NbM2 \cdot \frac{\left(\frac{DbM2}{8}\right)^2}{4} \cdot \pi$$

$$NAM2_s = 0.994$$

$$NAB_s = NbB \cdot \frac{\left(\frac{DbB}{8}\right)^2}{4} \cdot \pi$$

$$NAB_s = 1.767$$

Moment Capacity $d_{nA} := h - \frac{Db1}{8} - \frac{DbA}{16} - 0.75$

$$d_{nA} = 3.5$$

$$d_{nm2} := h - \frac{DbM}{8} - \frac{DbM2}{16} - 0.75$$

$$d_{nm2} = 3.688$$

$$d_{nB} := h - \frac{Db2}{8} - \frac{DbB}{16} - 0.75$$

$$d_{nB} = 3.5$$

$$M_{nA} = NAA_s \cdot \frac{f_y}{12000} \cdot \left(d_{nA} - \frac{NAA_s \cdot f_y}{1.7 \cdot f_c \cdot CA_s \cdot 12} \right)$$

$$M_{nA} = 29.394$$

$$NMCA = -23.426$$

$$M_{nm2} = NAM2_s \cdot \frac{f_y}{12000} \cdot \left(d_{nm2} - \frac{NAM2_s \cdot f_y}{1.7 \cdot f_c \cdot CM2s \cdot 12} \right)$$

$$M_{nm2} = 17.843$$

$$NMS2 = -7.809$$

$$M_{nB} = NAB_s \cdot \frac{f_y}{12000} \cdot \left(d_{nB} - \frac{NAB_s \cdot f_y}{1.7 \cdot f_c \cdot CB_s \cdot 12} \right)$$

$$M_{nB} = 29.394$$

$$NMCB = -23.426$$

POSITIVE MOMENT

	Column Strip	Middle Strip	Column Strip
Length	CA _s = 2 · L _s CA _s = 7.5	CM2 _s = L ₁ - L _s - L _s CM2 _s = 7.5	CB _s = 2 · L _s CB _s = 7.5
Slab Moment	M _{M2} = 16.819		M _{M2} = 16.819
Percent Moment	P1 := 0.6	PM := 0.4	P2 := 0.6
Moment	PMCA := M _{M2} · P1 PMCA = 10.091	PMS2 := M _{M2} · PM PMS2 = 6.728	PMCB := M _{M2} · P2 PMCB = 10.091
Depth	d := h - 0.75 - $\frac{5}{16}$ - $\frac{PD_{bM}}{8}$	d = 3.563	
Ast Required	AstA := $\frac{PMCA \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$	AstM2 := $\frac{PMS2 \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$	AstB := $\frac{PMCB \cdot 12000}{0.9 \cdot f_y \cdot 0.925 \cdot d}$

Compute a	$aA := \frac{AstA \cdot f_y}{0.85 \cdot f_c \cdot CAs \cdot 12}$ $aA = 0.133$	$aM2 := \frac{AstM2 \cdot f_y}{0.85 \cdot f_c \cdot CM2s \cdot 12}$ $aM2 = 0.089$	$aB := \frac{AstB \cdot f_y}{0.85 \cdot f_c \cdot CBs \cdot 12}$ $aB = 0.133$
Compute jd	$jdA := d - aA$ $jdA = 3.429$	$jdM2 := d - aM2$ $jdM2 = 3.474$	$jdB := d - aB$ $jdB = 3.429$
As Required	$AsA := \frac{PMCA \cdot 12000}{0.9 \cdot f_y \cdot jdA}$ $AsA = 0.654$	$AsM2 := \frac{PMS2 \cdot 12000}{0.9 \cdot f_y \cdot jdM2}$ $AsM2 = 0.43$	$AsB := \frac{PMCB \cdot 12000}{0.9 \cdot f_y \cdot jdB}$ $AsB = 0.654$
Minimum Number Of Bar:	$BS := \left(\frac{2 \cdot h}{18} \right)$ $nA := CAs \cdot \frac{12}{\min(BS)}$ $nA = 9$	$nM := CM2s \cdot \frac{12}{\min(BS)}$ $nM = 9$	$nB := CBs \cdot \frac{12}{\min(BS)}$ $nB = 9$
Steel Chosen	$NbA := 9$ $PDbA := 3$ $PAA_s = NbA \cdot \frac{\left(\frac{PDbA}{8} \right)^2}{4} \cdot \pi$ $PAA_s = 0.994$	$PNbM2 := 9$ $PDbM2 := 3$ $PAM2_s = PNbM2 \cdot \frac{\left(\frac{PDbM2}{8} \right)^2}{4} \cdot \pi$ $PAM2_s = 0.994$	$NbB := 9$ $PDbB := 3$ $PAB_s = NbB \cdot \frac{\left(\frac{PDbB}{8} \right)^2}{4} \cdot \pi$ $PAB_s = 0.994$
Moment Capacity	$d_{pA} = h - \frac{PDb1}{8} - \frac{PDbA}{16} - 0.75$ $d_{pm2} = h - \frac{PDbM}{8} - \frac{PDbM2}{16} - 0.75$ $d_{pB} = h - \frac{PDb2}{8} - \frac{PDbB}{16} - 0.75$	$d_{pA} = 3.688$ $d_{pm2} = 3.688$ $d_{pB} = 3.688$	
	$M_{pA} := PAA_s \cdot \frac{f_y}{12000} \cdot \left(d_{p1} - \frac{PAA_s \cdot f_y}{1.7 \cdot f_c \cdot CAs \cdot 12} \right)$ $M_{pA} = 19.707$		$PMCA = 10.091$

$$M_{pm1} = PAM_s \cdot \frac{f_y}{12000} \cdot \left(d_{pm1} - \frac{PAM_s \cdot f_y}{1.7 \cdot f_c \cdot CM1s \cdot 12} \right)$$

$$M_{pm1} = 19.707$$

$$PMS1 = 6.728$$

$$M_{pB} = PAB_s \cdot \frac{f_y}{12000} \cdot \left(d_{p2} - \frac{PAB_s \cdot f_y}{1.7 \cdot f_c \cdot CBs \cdot 12} \right)$$

$$M_{pB} = 19.707$$

$$PMCB = 10.091$$

APPENDIX C

Comparison of Positive
Moment from Direct Design Method
Versus Analytical Moment
from Finite Element Analysis

Comparison of the controlling moment for $L1 = 20$ ft:

Case		Moment		% Different
Span Ratio	LL/DL	FEM	Direct Design	
1	0.5	19.3	15.7	-22.6
1	1	25.7	21.0	-22.6
1	1.5	41.3	33.7	-22.6
1	2	77.1	62.9	-22.6
1	2.5	115.7	94.3	-22.6
1.4	0.5	49.3	41.5	-18.8
1.4	1	56.8	50.3	-12.9
1.4	1.5	155.1	130.6	-18.8
1.4	2	262.8	221.2	-18.8
1.4	2.5	383.2	322.6	-18.8
1.8	0.5	84.8	72.8	-16.5
1.8	1	215.3	187.5	-14.9
1.8	1.5	306.0	262.8	-16.4
1.8	2	494.4	424.5	-16.5
1.8	2.5	860.7	765.4	-12.4
2	0.5	99.2	87.6	-13.3
2	1	169.1	146.0	-15.9
2	1.5	264.1	224.6	-17.6
2	2	483.2	417.1	-15.9
2	2.5	591.5	503.2	-17.6

Comparison of the controlling moments for $L1 = 15$ ft:

Case		Moment		% Different
Span Ratio	LL/DL	FEM	Direct Design	
1.0	0.5	-	-	-
1.2	0.5	10.63	9.69	-8.8
1.4	0.5	16.17	13.10	-18.9
1.6	0.5	24.03	19.93	-17.1
1.8	0.5	34.06	28.81	-15.4
2.0	0.5	55.72	39.99	-28.2
1.0	1.0	9.29	5.98	-35.7
1.2	1.0	13.60	10.72	-21.2
1.4	1.0	21.56	17.468	-19.0
1.6	1.0	32.05	26.58	-17.1
1.8	1.0	45.41	38.4	-15.4
2.0	1.0	61.91	53.3	-13.9
1.0	1.5	9.68	7.475	-22.8
1.2	1.5	17.00	13.4	-21.2
1.4	1.5	26.95	21.835	-19.0
1.6	1.5	40.06	33.2	-17.1
1.8	1.5	56.77	48	-15.4
2.0	1.5	92.86	79.955	-13.9
1.0	2.0	11.62	8.98	-22.7
1.2	2.0	16.04	12.93	-19.4
1.4	2.0	35.30	30.14	-14.6
1.6	2.0	57.84	47.36	-18.1
1.8	2.0	83.43	65.77	-21.2
2.0	2.0	103.02	86.11	-16.4
1.0	2.5	16.26	12.56	-22.8
1.2	2.5	24.94	20.12	-19.4
1.4	2.5	64.67	52.42	-18.9
1.6	2.5	63.80	50.23	-21.3
1.8	2.5	125.15	101.71	-18.7
2.0	2.5	114.37	92.17	-19.4

APPENDIX D

Reliability Index

for 8 Locations of all the cases

Under Studies

Comparison of Reliability Index for case 1 and Case 2 (L1 = 15 ft):

LL/DL:	0.5	Span Ratio:	1
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Direction #	β	
	Case 1	Case 2
1	4.838	6.015
2	7.692	11.387
3	6.595	6.363
4	8.457	7.449
5	3.929	5.864
6	7.056	10.065
7	5.872	5.872
8	7.781	6.591

LL/DL:	0.5	Span Ratio:	1.2
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Direction #	β	
	Case 1	Case 2
1	3.686	6.116
2	5.629	7.229
3	4.172	4.613
4	6.307	5.283
5	4.219	5.775
6	8.207	14.773
7	4.909	4.519
8	9.077	7.661

LL/DL:	0.5	Span Ratio:	1.4
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Direction #	β	
	Case 1	Case 2
1	3.884	6.78
2	4.015	5.004
3	4.962	5.586
4	4.11	3.007
5	4.087	5.438
6	8.645	11.142
7	4.289	3.488
8	9.438	8.682

LL/DL:	0.5	Span Ratio:	1.6
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Direction #	β	
	Case 1	Case 2
1	3.769	7.231
2	4.133	4.496
3	4.275	5.222
4	4.087	3.081
5	4.292	5.418
6	7.954	10.487
7	4.476	3.399
8	9.602	7.644

LL/DL:	0.5	Span Ratio:	1.8
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Direction #	β	
	Case 1	Case 2
1	3.419	7.287
2	4.478	4.539
3	4.246	5.367
4	4.583	3.668
5	4.449	5.275
6	7.487	9.526
7	4.939	3.782
8	8.729	7.292

LL/DL:	0.5	Span Ratio:	2
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Direction #	β	
	Case 1	Case 2
1	3.655	7.125
2	3.973	2.893
3	4.301	4.529
4	4.172	2.264
5	4.073	3.966
6	7.858	10.708
7	4.391	2.848
8	8.914	6.931

LL/DL:	1	Span Ratio:	1
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Direction #	β	
	Case 1	Case 2
1	3.422	4.202
2	5.753	7.388
3	4.442	4.191
4	6.128	2.837
5	3.942	4.711
6	5.11	6.677
7	3.893	3.629
8	5.614	2.384

LL/DL:	1	Span Ratio:	1.2
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Direction #	β	
	Case 1	Case 2
1	3.341	5.333
2	4.086	5.442
3	4.97	5.296
4	4.26	3.151
5	3.615	5.031
6	5.487	10.902
7	5.431	5.054
8	6.162	5.459

LL/DL:	1	Span Ratio:	1.4
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Direction #	β	
	Case 1	Case 2
1	3.57	5.684
2	4.169	5.329
3	4.098	4.692
4	3.899	2.918
5	3.627	5.732
6	5.392	5.334
7	4.391	3.945
8	6.275	5.425

LL/DL:	1	Span Ratio:	1.6
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Direction #	β	
	Case 1	Case 2
1	3.203	6.826
2	3.563	3.822
3	3.739	3.225
4	3.655	2.779
5	3.816	4.744
6	5.379	3.995
7	3.881	2.939
8	6.144	5.371

LL/DL:	1	Span Ratio:	1.8
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Direction #	β	
	Case 1	Case 2
1	3.293	7.542
2	3.913	3.922
3	3.837	4.834
4	3.911	3.109
5	3.467	6.522
6	5.167	8.582
7	3.496	2.521
8	5.629	5.069

LL/DL:	1	Span Ratio:	2
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Direction #	β	
	Case 1	Case 2
1	3.335	6.054
2	3.891	3.848
3	3.983	5.108
4	3.654	2.942
5	3.895	4.392
6	5.212	8.463
7	4.443	3.541
8	6.021	5.171

LL/DL:	1.5	Span Ratio:	1
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	β	
Direction #	Case 1	Case 2
1	3.482	5.096
2	4.289	6.785
3	3.59	3.508
4	4.993	3.817
5	3.646	5.025
6	3.774	6.06
7	3.19	3.112
8	4.446	3.336

LL/DL:	1.5	Span Ratio:	1.2
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	β	
Direction #	Case 1	Case 2
1	3.486	5.364
2	3.655	5.061
3	3.555	3.821
4	3.453	2.448
5	3.098	4.326
6	3.879	7.817
7	3.434	3.053
8	3.572	2.661

LL/DL:	1.5	Span Ratio:	1.4
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	β	
Direction #	Case 1	Case 2
1	3.451	5.874
2	3.423	4.137
3	3.563	3.367
4	3.51	3.556
5	3.383	5.679
6	4.182	4.035
7	3.519	2.848
8	4.902	4.06

LL/DL:	1.5	Span Ratio:	1.6
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	β	
Direction #	Case 1	Case 2
1	3.285	6.756
2	3.598	3.834
3	3.895	4.628
4	3.601	2.789
5	3.475	6.831
6	4.054	4.913
7	3.703	2.831
8	4.797	4.013

LL/DL:	1.5	Span Ratio:	1.8
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	β	
Direction #	Case 1	Case 2
1	3.325	7.093
2	3.865	3.868
3	3.539	4.431
4	3.856	3.104
5	3.765	4.404
6	3.674	8.596
7	3.373	2.477
8	4.766	3.909

LL/DL:	1.5	Span Ratio:	2
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	β	
Direction #	Case 1	Case 2
1	3.209	5.556
2	3.392	3.349
3	3.536	4.581
4	4.095	3.395
5	3.893	4.365
6	4.103	7.028
7	3.902	3.089
8	4.856	3.783

LL/DL:	2	Span Ratio:	1
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Direction #	β	
	Case 1	Case 2
1	4.133	5.432
2	3.321	5.701
3	3.718	3.631
4	3.944	2.893
5	3.092	4.275
6	3.997	5.607
7	3.962	3.876
8	3.472	2.459

LL/DL:	2	Span Ratio:	1.2
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Direction #	β	
	Case 1	Case 2
1	3.171	4.992
2	3.703	4.998
3	3.511	3.761
4	4.383	3.338
5	3.371	4.538
6	3.548	6.832
7	3.509	3.137
8	3.429	2.562

LL/DL:	2	Span Ratio:	1.4
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Direction #	β	
	Case 1	Case 2
1	3.136	5.947
2	3.539	4.241
3	3.315	3.781
4	3.255	3.962
5	3.315	6.235
6	3.711	9.663
7	3.281	2.638
8	4.024	3.262

LL/DL:	2	Span Ratio:	1.6
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Direction #	β	
	Case 1	Case 2
1	3.347	6.335
2	3.474	3.693
3	3.866	4.524
4	3.561	3.134
5	3.668	7.322
6	3.731	3.779
7	3.189	2.368
8	3.713	3.034

LL/DL:	2	Span Ratio:	1.8
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Direction #	β	
	Case 1	Case 2
1	3.405	7.611
2	3.497	3.495
3	3.434	4.279
4	3.493	2.788
5	3.573	4.175
6	3.665	6.953
7	3.563	2.691
8	3.693	2.956

LL/DL:	2	Span Ratio:	2
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Direction #	β	
	Case 1	Case 2
1	3.278	6.209
2	3.236	3.193
3	3.326	4.304
4	3.238	2.615
5	3.314	3.721
6	3.535	5.939
7	3.174	2.433
8	4.002	3.053

LL/DL:	2.5	Span Ratio:	1
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Direction #	β	
	Case 1	Case 2
1	3.059	4.507
2	3.257	6.134
3	3.544	3.449
4	3.446	2.435
5	3.177	4.586
6	4.278	7.592
7	3.067	2.978
8	3.514	2.504

LL/DL:	2.5	Span Ratio:	1.2
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Direction #	β	
	Case 1	Case 2
1	2.964	4.839
2	3.273	4.682
3	3.419	3.657
4	3.353	2.408
5	3.203	4.933
6	3.904	6.506
7	3.289	2.922
8	3.113	2.269

LL/DL:	2.5	Span Ratio:	1.4
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Direction #	β	
	Case 1	Case 2
1	3.026	5.489
2	3.422	4.098
3	3.37	3.828
4	3.271	2.442
5	3.242	4.231
6	3.576	4.655
7	3.135	2.508
8	3.616	2.892

LL/DL:	2.5	Span Ratio:	1.6
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Direction #	β	
	Case 1	Case 2
1	3.36	6.389
2	3.373	3.581
3	3.282	3.919
4	3.518	2.753
5	3.375	4.142
6	3.463	4.564
7	3.341	2.537
8	3.16	2.525

LL/DL:	2.5	Span Ratio:	1.8
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Direction #	β	
	Case 1	Case 2
1	3.229	6.817
2	3.399	3.397
3	3.461	4.287
4	3.479	2.795
5	3.418	3.988
6	3.308	8.047
7	3.453	2.611
8	3.667	2.949

LL/DL:	2.5	Span Ratio:	2
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Direction #	β	
	Case 1	Case 2
1	3.085	7.509
2	3.249	3.207
3	3.249	4.188
4	3.244	2.641
5	3.369	3.768
6	3.424	5.588
7	3.438	2.709
8	3.287	2.422

Comparison of Reliability Index for case 1 and Case 2 (L1 = 20 ft):

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LL/DL:	0.5	Span Ratio:	1
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Direction #	β	
	Case 1	Case 2
1	4.001	6.543
2	4.229	8.481
3	4.684	4.991
4	4.496	3.438
5	4.06	6.572
6	4.371	8.614
7	4.129	4.412
8	4.098	3.045

LL/DL:	1	Span Ratio:	1
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Direction #	β	
	Case 1	Case 2
1	3.688	5.458
2	3.913	7.584
3	3.737	3.919
4	3.493	2.558
5	3.737	5.494
6	4.156	8.114
7	3.279	3.458
8	3.568	2.635

LL/DL:	1.5	Span Ratio:	1
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Direction #	β	
	Case 1	Case 2
1	3.197	4.971
2	3.403	7.044
3	3.6566	3.808
4	3.557	2.674
5	3.51	5.244
6	3.926	7.434
7	3.261	3.409
8	5.932	4.842

LL/DL:	2	Span Ratio:	1
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Direction #	β	
	Case 1	Case 2
1	3.619	5.328
2	3.531	6.861
3	3.489	3.627
4	3.365	2.521
5	3.701	5.659
6	3.893	6.035
7	3.268	3.403
8	3.193	2.355

LL/DL:	2.5	Span Ratio:	1
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Direction #	β	
	Case 1	Case 2
1	3.093	4.82
2	3.421	7.019
3	3.268	3.398
4	3.496	2.667
5	3.271	4.941
6	3.759	6.844
7	3.677	3.817
8	3.334	2.515

LL/DL:	0.5	Span Ratio:	1.4
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Direction #	β	
	Case 1	Case 2
1	3.997	7.988
2	4.106	5.506
3	4.034	5.135
4	3.909	3.275
5	3.961	5.883
6	4.102	10.238
7	3.874	3.644
8	4.648	4.176

LL/DL:	1	Span Ratio:	1.4
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Direction #	β	
	Case 1	Case 2
1	3.582	7.292
2	3.607	4.709
3	4.077	3.653
4	3.774	3.268
5	3.837	7.966
6	4.557	4.925
7	3.867	3.455
8	3.549	3.066

LL/DL:	1.5	Span Ratio:	1.4
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Direction #	β	
	Case 1	Case 2
1	3.452	6.893
2	3.582	5.675
3	3.769	4.524
4	3.648	2.941
5	3.467	6.997
6	3.714	5.863
7	3.705	3.294
8	4.091	3.563

LL/DL:	2	Span Ratio:	1.4
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Direction #	β	
	Case 1	Case 2
1	3.387	6.593
2	3.473	5.675
3	3.485	4.196
4	3.388	2.712
5	3.741	6.662
6	3.495	5.993
7	3.595	3.187
8	3.552	3.082

LL/DL:	2.5	Span Ratio:	1.4
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Direction #	β	
	Case 1	Case 2
1	3.298	6.763
2	3.324	6.334
3	3.338	4.007
4	3.396	2.738
5	3.484	5.964
6	3.744	5.334
7	3.361	2.976
8	3.436	2.972

LL/DL:	0.5	Span Ratio:	1.8
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Direction #	β	
	Case 1	Case 2
1	3.821	8.603
2	4.151	4.365
3	4.359	5.414
4	4.266	3.509
5	4.027	5.222
6	4.452	10.295
7	3.945	3.109
8	5.367	4.709

LL/DL:	1	Span Ratio:	1.8
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Direction #	β	
	Case 1	Case 2
1	3.493	8.112
2	3.897	4.049
3	3.889	5.037
4	3.724	3.051
5	3.709	4.715
6	3.686	9.397
7	4.114	3.353
8	3.735	3.157

LL/DL:	1.5	Span Ratio:	1.8
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Direction #	β	
	Case 1	Case 2
1	3.43	7.951
2	3.714	3.848
3	3.719	4.737
4	3.935	3.293
5	3.683	6.965
6	4.281	4.569
7	3.512	2.832
8	3.628	3.089

LL/DL:	2	Span Ratio:	1.8
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Direction #	β	
	Case 1	Case 2
1	3.246	7.096
2	3.498	3.625
3	3.374	4.355
4	3.647	3.054
5	3.642	4.519
6	3.319	6.963
7	3.459	2.816
8	3.987	3.452

LL/DL:	2.5	Span Ratio:	1.8
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Direction #	β	
	Case 1	Case 2
1	3.194	6.329
2	3.208	4.125
3	3.483	4.629
4	3.326	2.852
5	3.562	6.813
6	4.536	6.618
7	3.301	8.245
8	3.218	5.002

LL/DL:	0.5	Span Ratio:	2
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Direction #	β	
	Case 1	Case 2
1	3.933	9.698
2	4.228	4.349
3	4.428	6.001
4	4.512	3.805
5	4.243	5.201
6	4.561	7.563
7	4.136	3.437
8	5.483	4.505

LL/DL:	1	Span Ratio:	2
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Direction #	β	
	Case 1	Case 2
1	3.595	7.971
2	3.458	3.538
3	3.97	5.172
4	3.627	3.027
5	3.918	4.726
6	3.691	3.043
7	3.368	5.402
8	3.766	2.983

LL/DL:	1.5	Span Ratio:	2
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Direction #	β	
	Case 1	Case 2
1	3.368	6.729
2	3.471	3.542
3	3.814	4.935
4	3.628	3.071
5	3.722	4.441
6	4.321	7.375
7	4.114	3.002
8	3.735	2.827

LL/DL:	2	Span Ratio:	2
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Direction #	β	
	Case 1	Case 2
1	3.226	5.263
2	3.579	3.647
3	3.687	4.798
4	3.683	3.146
5	3.358	4.022
6	3.343	2.727
7	3.566	5.116
8	4.028	3.303

LL/DL:	2.5	Span Ratio:	2
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Direction #	β	
	Case 1	Case 2
1	3.232	7.938
2	3.282	3.345
3	3.281	4.349
4	3.473	2.964
5	3.467	4.124
6	3.21	5.654
7	3.28	2.776
8	3.249	2.599

VITA

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