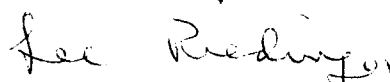


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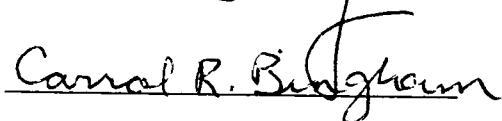
I am submitting herewith a thesis written by Wing Kwong Luk entitled "Coriolis Alignment and Band Crossing of the $h_{g/2}$ Proton Band in $^{175,177,179,181}\text{Re}$." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Physics.



Lee Riedinger, Major Professor

We have read this thesis and
recommend its acceptance:





Accepted for the Council:



Vice Chancellor
Graduate Studies and Research

CORIOLIS ALIGNMENT AND BAND CROSSING OF THE
 $h_{9/2}$ PROTON BAND IN $^{175,177,179,181}\text{Re}$

A Thesis

Presented for the

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Wing Kwong Luk

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ABSTRACT

Preliminary data on a rotational band in ^{175}Re produced by the reaction $^{159}\text{Tb}(^{20}\text{Ne}, 4n)$ at 102 MeV have been acquired. The yrast band built upon the $1/2^- [541]$ orbital was constructed from the experimental data as an extension of the systematics of the $h_{9/2}$ bands known in $^{177, 179, 181}\text{Re}$. The level scheme of the $h_{9/2}$ proton band for this isotopic series was fitted by a Coriolis model. The calculations require an increasing energy separation between the $1/2^- [541]$ and $3/2^- [532]$ Nilsson orbitals for decreasing neutron number (N), which is in qualitative agreement with the trend expected for decreasing hexadecapole (ϵ_4) deformation.

A Cranked Shell Model (CSM) calculation was also performed to fit this $h_{9/2}$ band in this isotopic series. The calculation gives a fairly accurate fit of the level energies in the rotating frame to within ± 70 keV. Our analysis also indicates that the aligned angular momentum of this $h_{9/2}$ band is smoothly increasing from $2.9 \hbar$ for $A = 175$ to $3.6 \hbar$ for $A = 181$. This trend in alignments is nicely reproduced by the CSM calculations. The $i_{13/2}$ neutron crossing frequency, as indicated by the CSM calculation, is shifted upward as a result of larger deformation of the nucleus when excited into the $h_{9/2}$ proton state.

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CHAPTER I

INTRODUCTION

The study of high spin features of nuclei via the mechanism of (heavy ion, xn) reactions provides the opportunity for exploring new aspects of nuclear dynamics. This project involves a comprehensive study of the high spin states of the isotopic series $^{181,179,177,175}\text{Re}$. The nucleus ^{175}Re was produced by the reaction $^{159}\text{Tb}(^{20}\text{Ne},4n)^{175}\text{Re}$ at a beam energy of 102 MeV at the Oak Ridge Isochronous Cyclotron (ORIC). This newly collected data on ^{175}Re along with existing data on $^{181,179,177}\text{Re}$ provide the experimental values to be used for discussion, reasoning and calculation.

This remarkable phenomenon of backbending which manifests itself in apparent compressions of level spacings in rotational bands, has led to extensive study of the high spin states of even-even rare earth nuclei. By studying the neighboring odd-mass nuclei to high spins, we can investigate the interaction of the even-even core with the odd nucleon, thereby observing the importance of selected single particle orbits in relation to the mechanism behind backbending. In the light rare earth region ($A \sim 160$), backbending is a very common effect. Many backbending nuclei are now known in this region, and a few outside it. The first backbend around spin $10-12 \hbar$ is generally believed to be due to the decoupling from the rotating core and the Coriolis alignment of the two low- Ω $i_{13/2}$ neutrons (Ω is the projection of the single particle angular momentum, j , along the symmetry axis). The second backbend is seen in a few nuclei with $N = 90$ (e.g., ^{160}Yb). It is proposed

to result from the alignment of $h_{11/2}$ protons. In the heavy transition region ($A \sim 182$), the first backbend is still observed in some nuclei. However, the exact mechanism behind it is not fully understood. It is believed, according to some people, that the $i_{13/2}$ neutron is still responsible for the first backbend in this region. But another high j , low Ω orbit, the $h_{9/2}$ proton, is located fairly close to the fermi surface in this heavier region. The $h_{9/2}$ protons should align easily at low rotational frequency, and thereby may have a significant effect. To date, studies on odd- A Pt, Ir, Os, Re are incomplete and therefore inconclusive about the exact nature of the cause of the backbend. Little is known, especially about the collective bands built upon the $h_{9/2}$ proton excitation throughout the rare-earth region. Hence, it is our primary motivation to acquire preliminary data on ^{175}Re in order to extend the systematics of the $h_{9/2}$ band known in $^{177,179,181}\text{Re}$. The study of this series of nuclei has the advantage that the proton number, Z , is unchanged. In this way, it eliminates the complications and uncertainties arising from changing both the neutron and proton numbers.

For $^{177,179,181}\text{Re}$, three bands have been observed in experimental work;¹³ namely, the bands that are built on the $1/2^- [541]$, $5/2^+ [402]$, $9/2^- [514]$ Nilsson orbitals. A (HI, xn) reaction favors the population of these bands, because in all cases they provide the maximum angular momentum with the least excitation energy. In the analysis of our data on ^{175}Re , we can only conclude the yrast $1/2^- [541]$ band. The statistics were not good enough to identify the side bands. The energies of the levels in the $1/2^- [541]$ band in $^{177,179,181}\text{Re}$ have been calculated in the particle plus rotor model (Coriolis model) by Leigh et al.¹³ and Singh et al.²¹ The fitted parameters in these calculations were compatible with a

changing hexadecapole deformation, assuming that the quadrupole deformation is fairly constant in this region. We have extended this systematic trend and performed detailed Coriolis calculations on $^{175,177,179,181}\text{Re}$ with the addition of a pairing attenuation on the Coriolis matrix elements. The level scheme of the yrast band ($1/2^- [541]$) can be fitted equally well with slightly different parameters for all four isotopes and the above mentioned trend is reproduced successfully with the addition of one more member (^{175}Re) in this series.

The second part of our calculation involves the use of the Cranked Shell Model. The purpose is dual: first, we attempt to see if the experimental $h_{9/2}$ proton band energies and angular momenta can be fit by this model and secondly, to investigate the occurrence or absence of band crossings in the $h_{9/2}$ band and in adjacent even-even nuclei. Reasonable agreement between the data and the CSM calculations leads to deeper understanding of the physics involved in these nuclei and suggests that the CSM formalism can be used as a valuable guideline for the planning of future experiments.

The puzzle why there is no consistent backbend observed in the $h_{9/2}$ proton bands of these Re isotopes while a strong backbend is found in the ground band of the even-even neighbors, Os and W, is explained by the CSM calculation as a deformation effect. It is suggested, as a result of this feasibility study, that much deeper insight can be gained by pushing the experimental data to higher spins by using heavier projectiles in future experiments.

CHAPTER II

THEORY

In order to interpret the results of experiment or any physical phenomena, a theory or model is utilized. A theory is successful if it is logically consistent and most importantly, is able to reproduce or predict experimental results. Thus, a mathematical model serves not only as a language to explain physical phenomena, but also as a theoretical guideline for future development.

Sometimes, a model may be able to explain certain observed phenomena successfully but fails to account for a similar or related phenomenon, which in turn can be explained by a rather different model. It is from the successes and failures of these theories that the basic trend is acquired. The final goal, of course, is to unify them into a single framework so that it is sophisticated enough to explain all phenomena.

The present situation of theoretical nuclear physics is far from this goal. The macroscopic nuclear models have developed into essentially two distinct paths in their respective attempts to explain nuclear structure, namely: the collective model^{1,2} which stresses the coordinated motion of the nucleons; and at the other extreme, the single particle model^{3,4,5} which emphasizes the effective motion of a single nucleon due to its average interaction with the remaining nucleons. Each model has its own region of greatest success. The single particle shell model has its triumph for nuclei adjacent to the closed shell or

the "magic number" region. The collective model, on the other hand, works in the region midway between the closed shells. However, an interplay of both collective and single particle effects exists in the strongly deformed region. Therefore, to explain the experimental data throughout the deformed region, it is necessary to utilize a combination of both models. A coupling of an odd nucleon to a deformed core explains the low spin features in this region with great success. In light of this, it is important to have a basic understanding of a single particle deformed shell model.

A. THE NILSSON MODEL

The fact that nuclei do possess permanent deformations is manifested by the existence of large quadrupole moments. Therefore, a deformed shell model is a more logical scheme to be used rather than its spherical counterpart. A shell model description based on an axially symmetric static potential was first investigated by Nilsson.⁶ As a starting point, Nilsson assumed a modified oscillator potential as the effective potential generated by the remaining A-1 nucleons

$$V = \frac{1}{2} m(\omega_1^2 x_1^2 + \omega_2^2 x_2^2 + \omega_3^2 x_3^2) + C\vec{\ell} \cdot \vec{s} + D\vec{\ell} \cdot \vec{\ell} \quad (\text{II-1})$$

where

ω_i are the circular frequencies for the harmonic oscillator,

x_i are the Cartesian coordinates,

$C\vec{\ell} \cdot \vec{s}$ is the spin orbit interaction,

and

$D\vec{l}^2$ is a centrifugal term added to round off the harmonic oscillator potential. (The value of D is usually negative. Thus, this term lowers the energy of high angular momentum states.)

If the z-axis is chosen as the symmetry axis of an axially symmetric nucleus, the Nilsson Hamiltonian is distinguished from the spherical shell model via a deformation parameter δ as follows:

$$\omega_1^2 = \omega_2^2 = \omega_0^2(\delta)(1 + \frac{2}{3}\delta) \quad (\text{II-2a})$$

$$\omega_3^2 = \omega_0^2(\delta)(1 - \frac{4}{3}\delta) . \quad (\text{II-2b})$$

The condition of constant volume of the nucleus leads to

$$\omega_1\omega_2\omega_3 = \text{constant} . \quad (\text{II-3})$$

This formula together with (II-2) can be used to express $\omega_0(\delta)$ as a function of δ

$$\omega_0(\delta) = \omega_{00}(1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3)^{-1/6} , \quad (\text{II-4})$$

where

ω_{00} is the value of $\omega_0(\delta)$ if $\delta = 0$. For a first order estimation

$$\hbar \omega_{00} = \hbar \omega_0(\delta=0) = 41A^{-1/3} \text{ MeV} . \quad (\text{II-5})$$

Thus, the Nilsson Hamiltonian is totally defined for a given nucleus if the deformation parameter δ , the spin-orbital interaction coefficient C and the centrifugal coefficient D are chosen properly.

However, the Nilsson Hamiltonian in the form (II-1) gives no clues to the choice of a proper basis set and its relationship to its spherical counterpart. Thus, a change to spherical coordinates is desirable for a more informative form of the Hamiltonian.

The transformation equations are:

$$x_1 = r \sin\theta \cos\phi \quad (\text{II-6a})$$

$$x_2 = r \sin\theta \sin\phi \quad (\text{II-6b})$$

$$x_3 = r \cos\theta \quad (\text{II-6c})$$

Using these equations and (II-2), the Hamiltonian can be expressed into the form

$$H = \left(-\frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} m\omega_0^2 r^2 \right) - m\omega_0^2 r^2 \delta \frac{4}{3} \sqrt{\frac{\pi}{5}} Y_{20}(\theta, \phi) + C \vec{\ell} \cdot \vec{s} + D \vec{\ell}^2 \quad (\text{II-7})$$

Now, it is apparent that the Nilsson Hamiltonian consists basically of four elements. The first term is just the isotropic harmonic oscillator. The second term represents the coupling of the particle to the deformed axis; its magnitude depends on the deformation. The last two terms are the same spin-orbit and centrifugal terms as discussed previously as they are invariant to the transformation given by (II-6).

For the convenience of evaluation of the matrix element, the Hamiltonian can be reduced further by a scale transformation defined by

$$r' = r \sqrt{\frac{m\omega}{\hbar}} . \quad (\text{II-8})$$

After some algebraic manipulation and dropping the prime afterwards, one can put (II-7) into the form

$$H = \frac{\hbar\omega}{2} (-\nabla^2 + r^2) - \frac{4}{3} \hbar\omega \sqrt{\frac{\pi}{5}} \delta r^2 Y_{20} + C \vec{\ell} \cdot \vec{s} + D \vec{\ell} \cdot \vec{\ell} . \quad (\text{II-9})$$

To begin with, we choose the wavefunction generated by the central field Hamiltonian as the basis set.

For the isotropic Hamiltonian, the constants of motion are $\vec{H}_0$, $\vec{\ell}^2$, $\vec{\ell}_3$, $\vec{S}_3$ in a classical sense. The corresponding quantum numbers in a quantum mechanical sense are N , ℓ , Λ , Σ . The eigenfunction corresponding to the eigenvalues is denoted by $|N\ell\Lambda\Sigma\rangle$. Thus, we have

$$\begin{aligned} \vec{H}_0 |N\ell\Lambda\Sigma\rangle &= E_0 |N\ell\Lambda\Sigma\rangle \\ \vec{\ell}^2 |N\ell\Lambda\Sigma\rangle &= \ell(\ell+1) |N\ell\Lambda\Sigma\rangle \\ \vec{\ell}_3 |N\ell\Lambda\Sigma\rangle &= \Lambda |N\ell\Lambda\Sigma\rangle \\ \vec{S}_3 |N\ell\Lambda\Sigma\rangle &= \Sigma |N\ell\Lambda\Sigma\rangle \end{aligned} \quad (\text{II-10})$$

where

N = oscillator shell quantum number,

ℓ = orbital angular momentum,

Λ = z-component of $\vec{\ell}$,

Σ = z-component of $\vec{s}$.

None of these operators commute with the total Nilsson Hamiltonian. However, the projection of the angular momentum on the

symmetry axis operator $\vec{j}_3$ does commute with the total Hamiltonian and is therefore a constant of motion. The corresponding eigenvalue Ω is a good quantum number. In mathematical form, this can be written as

$$\vec{j}_3 = \vec{l}_3 + \vec{s}_3$$

$$\vec{j}_3 |N\ell\Lambda\Sigma\rangle = \Omega |N\ell\Lambda\Sigma\rangle = (\Lambda + \Sigma) |N\ell\Lambda\Sigma\rangle. \quad (\text{II-11})$$

The Nilsson wavefunction for the Nilsson Hamiltonian can now be written as

$$\chi_{N\Omega} = \sum_{\ell\Lambda} a_{\ell\Lambda}^{\Omega} |N\ell\Lambda\Sigma = \Omega - \Lambda\rangle. \quad (\text{II-12})$$

Thus, the problem now reduces to the evaluation of the various matrix elements and diagonalization of the matrix to obtain the energy levels. In this basis, this can be done fairly easily. The isotropic harmonic oscillator term is diagonal and the $\vec{l} \cdot \vec{s}$ and $\vec{l} \cdot \vec{l}$ terms can be expressed in terms of the raising and lowering operators. The Y_{20} term can be reduced to a sum of Clebsch-Gordon coefficients using the Wigner-Eckart theorem and the r^2 matrix is evaluated in terms of the confluent hypergeometric function. However, this basis set is not transparent for practical purposes because it is uncoupled. The expansion over the permitted angular momentum states or the coupled basis is preferred. The wavefunction expressed in this basis can be written as

$$\chi_{N\Omega} = \sum_j C_{j\Omega} |N\ell j\Omega\rangle \quad (\text{II-13})$$

where the $C_{j\Omega}$ coefficients are related to the $a_{\ell\Omega}$ coefficients via the Clebsch-Gordan coefficients

$$C_{j\Omega} = \sum_{\substack{\Lambda\Sigma \\ \Lambda+\Sigma=\Omega}} \langle \ell s \Lambda \Sigma | j \Omega \rangle a_{\ell\Lambda} . \quad (\text{II-14})$$

To label the very many states arising from this deformed oscillator potential, a convention using the good quantum numbers at the limit of very large deformation is employed. For extremely large deformation, the spin-orbit term and the centrifugal term are overshadowed by the deformation term and they can be neglected. In this limit, we go to cylindrical coordinates to solve the axially symmetric problem and get a new set of quantum numbers. The labelling of Nilsson states using this set of quantum numbers can be written as $\Omega^\pi [N n_z \Lambda]$ where n_z is the number of oscillator quanta along the symmetry axis and π is the parity, $(-1)^N$.

The Nilsson model is a valuable tool in predicting which orbital of a given nucleus at a given deformation is near to the Fermi surface, but it fails to give exact correspondence between the single particle excitation energies and the experimental ones. This is because a shell model type of calculation does not take into account the effect of the pairing interaction. Consequently, the calculated single particle energies only serve as a preliminary first order estimation of the true ones. It was pointed out earlier that the coupling of an odd nucleon to a deformed core enjoys great success in explaining the rotational bands of nuclei, because the angular momentum of the odd-A deformed nuclei is due to both the rotational angular momentum of the deformed

core and the angular momentum of the odd nucleon in a deformed potential well. The Nilsson model has shown that for a deformed potential well, each shell model level of angular momentum j breaks up into $j + \frac{1}{2}$ Nilsson levels. Each of these levels may form the ground state or band-head of a rotational band.

Having developed a basic understanding of the Nilsson model, we turn our attention to investigate the coupling of a deformed core plus a single particle (see Figure II-1).

B. THE CORIOLIS MODEL

The total Hamiltonian is given by

$$H = H_{\text{NILSSON}} + \frac{\hbar^2}{2\mathcal{J}} \vec{R}^2 \quad (\text{II-15})$$

The second term is just the kinetic energy due to the rotational motion of the deformed core. Imposing the condition of axial symmetry, i.e., $\vec{R}^2 = R_1^2 + R_2^2$ and using the fact that $\vec{R} = \vec{I} - \vec{j}$, we can rewrite the Hamiltonian as⁷

$$H = H_{\text{NILSSON}} + \frac{\hbar^2}{2\mathcal{J}} (I_1^2 + I_2^2) + \frac{\hbar^2}{2\mathcal{J}} (j_1^2 + j_2^2) - \frac{\hbar^2}{\mathcal{J}} (I_1 j_1 + I_2 j_2) . \quad (\text{II-16})$$

The quantum numbers corresponding to the various operators are easily obtained. The quantization process is expressed by the following:

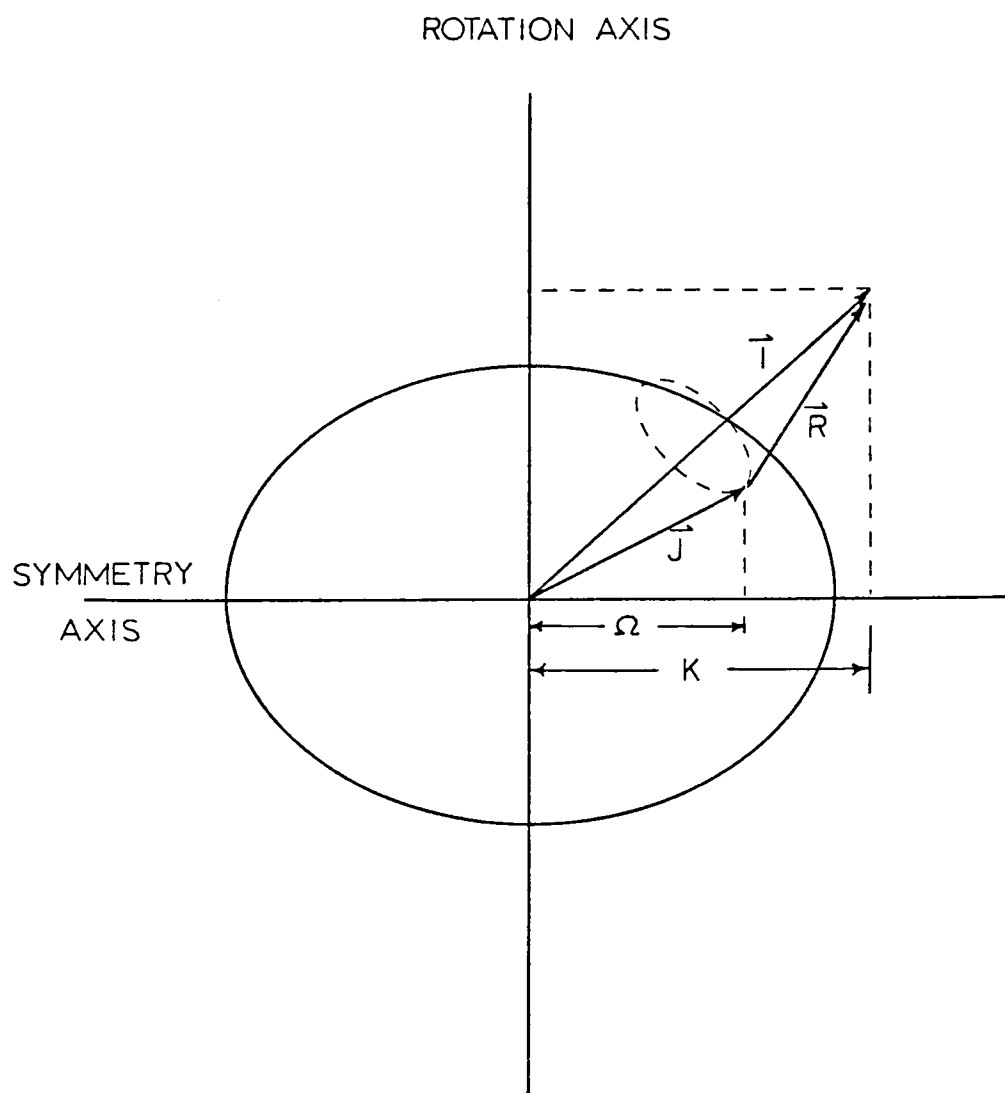


Figure II-1. Coupling scheme of a deformed nucleus.

$$I_1^2 + I_2^2 + I_3^2 \rightarrow I(I + 1) \quad (\text{II-17a})$$

$$I_3^2 \rightarrow K^2 \quad (\text{II-17b})$$

$$j_1^2 + j_2^2 + j_3^2 \rightarrow \langle j^2 \rangle \quad (\text{II-17c})$$

$$j_3^2 \rightarrow \Omega^2 \quad (\text{II-17d})$$

Combining Eqs. (II-16) and (II-17), the total Hamiltonian can be expressed as

$$H = H_{\text{NILSSON}} + \frac{\hbar^2}{2J} [I(I + 1) - K^2] + \frac{\hbar^2}{2J} [\langle j^2 \rangle - \Omega^2] - \frac{\hbar^2}{J} [I_1 j_1 + I_2 j_2] \quad (\text{II-18})$$

This consists basically of four elements. The first term is the potential energy for the particle. The second gives the rotation of the core. The third is the kinetic energy of the particle. The last term is the rotation-particle coupling term or the Coriolis term.

The effect of the Coriolis term on band structure can be visualized more easily by the use of raising and lowering operators. $j_{\pm} = j_1 \pm j_2$, $I_{\pm} = I_1 \pm I_2$. The Coriolis term now becomes:

$$H_C = \frac{\hbar^2}{2J} (I_+ j_- + I_- j_+) \quad (\text{II-19})$$

The matrix elements of H_C can be written as

$$\langle I, \Omega \pm 1 | H_C | I \Omega \rangle = - \frac{\hbar^2}{2J} \sqrt{I(I + 1) - K(K \pm 1)} \langle \Omega \pm 1 | j_{\pm} | \Omega \rangle \quad (\text{II-20})$$

where the matrix elements $\langle \Omega \pm 1 | j_{\pm} | \Omega \rangle$ have to be evaluated from the

Nilsson wavefunction since these operators act on the odd nucleon in the intrinsic system,

$$\langle \Omega \pm 1 | j \pm | \Omega \rangle = \sum_j C_{j\Omega}^* C_{j\Omega} \sqrt{(j \pm \Omega)(j \pm \Omega + 1)} . \quad (\text{II-21})$$

In general, the Coriolis term is nondiagonal, connecting bands that differ in Ω by one unit. These nondiagonal matrix elements can perturb the normal level of a rotational band by a substantial amount.

Having developed the concept that the Coriolis effect is a perturbation on the rotational band via the mechanism of band mixing, we address the significance and limitation of the Coriolis model. The strength of the Coriolis term, as given by the last term of (II-18), can be estimated in a first order approximation as

$$F_{\text{cor}}^{\text{max}} \sim \frac{\hbar^2}{\mathcal{J}} I \cdot j . \quad (\text{II-22})$$

It attains the maximum under the conditions of large I , large j , low Ω and small $\mathcal{J}$. Thus, the coupling has its maximum strength for high j and low Ω Nilsson orbits and for high spin states in a given band. For very high frequency ($I \sim 20$), the Coriolis model can be applied to explain a peculiar behavior called backbending. However, for extremely high spin states, this model fails because as the core rotates faster, the single particle cannot follow the core rotation. At this point, the coupling between the particle and the core breaks down and this model ceases to apply. Hence, in order to account for high spin features of rotational bands, we employ another model that is more compatible with the rapidly rotating physical situation.

C. CRANKED SHELL MODEL

The cranking model of Inglis^{8,9} has been successful in explaining the moments of inertia of deformed nuclei. Such an approach of studying nucleonic behavior in the rotating nuclear potential has been applied to the analysis of quasi-particle bands by Bengtsson and Frauendorf.^{10,11} This cranked shell model has had much success in explaining the properties of observed rotational bands and the crossings which they experience at high frequencies, as discussed, for example, in references 10, 11 and 12.

In the body fixed frame, the shell model Hamiltonian reads

$$H'_{sp} = H_{sp} - \omega j_x \quad (\text{II-23})$$

where H_{sp} is the Nilsson Hamiltonian in the laboratory frame, ω is the collective angular frequency of rotation and j_x is the angular momentum operator along the rotation axis, which is designated as the x-axis.

One can get a more realistic description by taking into account the pairing interaction. Adding the pair field to the potential, one obtains the quasi-particle Hamiltonian. Diagonalization of this with the appropriate quasi-particle transformation gives the quasi-particle excitation energies in the rotating frame as a function of angular frequencies. An example of such a spectrum is shown in Figure II-2.

The parameters necessary for this calculation are ϵ_2 , ϵ_4 , the deformation parameters; Δ , the pairing gap, and λ , the electrochemical potential. To make the problem tractable, the calculation is not performed in a self-consistent manner; that is, these parameters are held

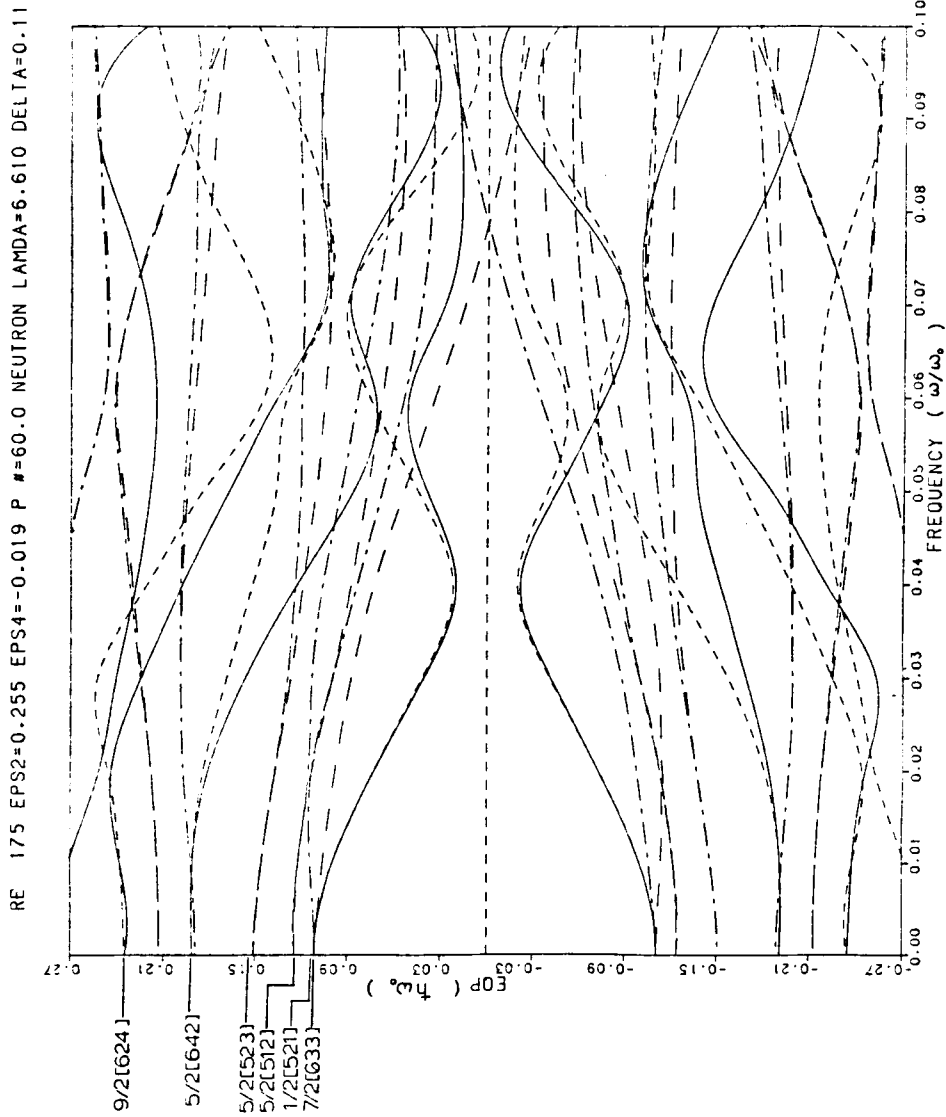


Figure II-2. Cranking model calculation of the proton g.p. energy in the rotating frame. The solid lines correspond to $\alpha\pi = +1/2^+$, dash lines $-1/2^+$, long dashes $+1/2^-$ and dash-dotted lines $-1/2^-$; $\hbar\omega_0 = 41A^{-1/3}$ MeV.

constant with the rotational frequency. The consequences of this approximation have not been studied in detail. However, this approach works up to $\hbar\omega \sim .4$ MeV in view of quantitative agreement between experimental values and theoretical calculations in the light Yb, Er region.

The Cranking model Hamiltonian has only two distinct symmetries. Parity is a good quantum number because of invariance due to space inversion. Also, the Hamiltonian is invariant with respect to rotation by 180° about the axis of rotation. The rotation operator defined by

$$R_x(\pi)\psi = e^{-i\pi\alpha}\psi, \quad \begin{cases} \alpha = 0, 1 & N \text{ even} \\ \alpha = -1/2, +1/2 & N \text{ odd} \end{cases} \quad (\text{II-24})$$

gives rise to the quantum number α , called the signature which essentially distinguishes the two distinct sequences of rotational bands with $\Delta I = 2$, both of which originate from the same Nilsson orbital. This effect of signature splitting due to cranking is the same physical phenomenon as the splitting into favored and unfavored bands.⁷ Thus, the orbits are labelled by parity and signature. For convenience, the diagram also includes the conjugate orbits, with the inverted signature and energy.

It can be seen from (II-23) that the negative slope of the eigenvalues gives the magnitude of the aligned angular momentum,

$$i = \langle j_x \rangle = - \left\langle \frac{\partial H'}{\partial \omega} \right\rangle = - \frac{de'}{d\omega} . \quad (\text{II-25})$$

This is the quasiparticle angular momentum along the rotation axis, i.e., an axis perpendicular to the symmetry axis. Thus, at zero

frequency, there is zero alignment for every orbit except for those orbits with $K = 1/2$, in which case the nonzero Coriolis matrix element $\langle \pm 1/2 | j_{\mp} | \mp 1/2 \rangle$ contributes an intrinsic alignment. As the core rotates, the Coriolis force tends to align the single particle angular momentum of a high j nucleon along the rotation axis. This is counteracted by the pairing force which strives to keep this particle in a time reversed orbit such that its angular momentum projection on the symmetry axis is a good quantum number. Nevertheless, gradual alignment of these high j orbits occurs as a result of this competitive process. This is indicated by the downward sloping of trajectories in the upper half of the energy diagram in Figure II-2.

At zero frequency, one assumes that all negative energy states below the pairing gap are filled to give rise to a quasiparticle vacuum state. This lowest configuration is used as a reference to which all excitation energies are referred. The excitation spectrum is obtained by distributing the quasiparticles over the levels. Each configuration as a function of frequency represents a rotational band. The quasiparticles are distributed over the allowed trajectories according to the rule that if a level is occupied, its conjugate partner must be free. Also, only the excitation energy of one of the conjugate levels is taken into account.

It can be shown that this occupation number representation is equivalent to the Fermi statistics and the usual representation in terms of quasiparticle operators.¹⁶ The states that are classified with respect to α and π form trajectories that exhibit the variation of quasiparticle

motion with frequency. Smooth variations reflect a similarity in the intrinsic structure of adjacent band members.

Another important feature of these quasiparticle energy diagrams is the prediction of band crossings. At a certain critical frequency (e.g., $\omega/\omega_0 = .039$ in Figure II-2), the down-sloping quasiparticle state dips into the vacuum states, crosses another orbit, and becomes the Yrast configuration. This is equivalent to the pair breaking of two quasiparticles and the alignment of their angular momenta along the rotation axis. This phenomenon has been commonly termed "backbending," where the ground band in an even-even nucleus is crossed by an s band composed of two aligned high j particles (i 13/2 neutrons in the light rare-earth region). These cranked shell model calculations demonstrate that these crossings should occur at a certain rotational frequency and that a certain amount of aligned angular momentum should be gained in the crossing.

CHAPTER III

EXPERIMENTAL PROCEDURES AND RESULTS

To investigate the high spin features of rotational bands, a heavy ion reaction is used, as massive angular momentum transfer can be achieved. In this experiment, the reaction $^{159}\text{Tb}(^{20}\text{Ne}, 4n)^{175}\text{Re}$ with a beam energy of 102 MeV was studied. Gamma rays following this reaction have been analyzed using a typical $\gamma\gamma$ time coincidence technique. The basic setup of this experiment is depicted in Figure III-1.

A. ELECTRONICS

The gamma rays that follow from the above reaction interact with the Ge(Li) detector to produce a signal. This electric pulse generated must be further processed by the appropriate electronic circuitry before the data can be collected in useful form. Typically, the pulse has to be shaped, analyzed and registered.

The preamplifier serves the purpose of impedance matching; that is, it transforms the voltage developed across the small capacity at the input of the preamplifier into an approximately equal voltage across the high capacity of the output voltage. The resultant signal is then divided between the main amplifier and the timing filter amplifier. The timing filter amplifier further shapes the pulse to permit optimizing the signal to noise ratio for timing measurements. The signal that feeds into the main amplifier is routed to a stretcher before it is input into the ADC. Thus, the signal that goes to the main amplifier is being

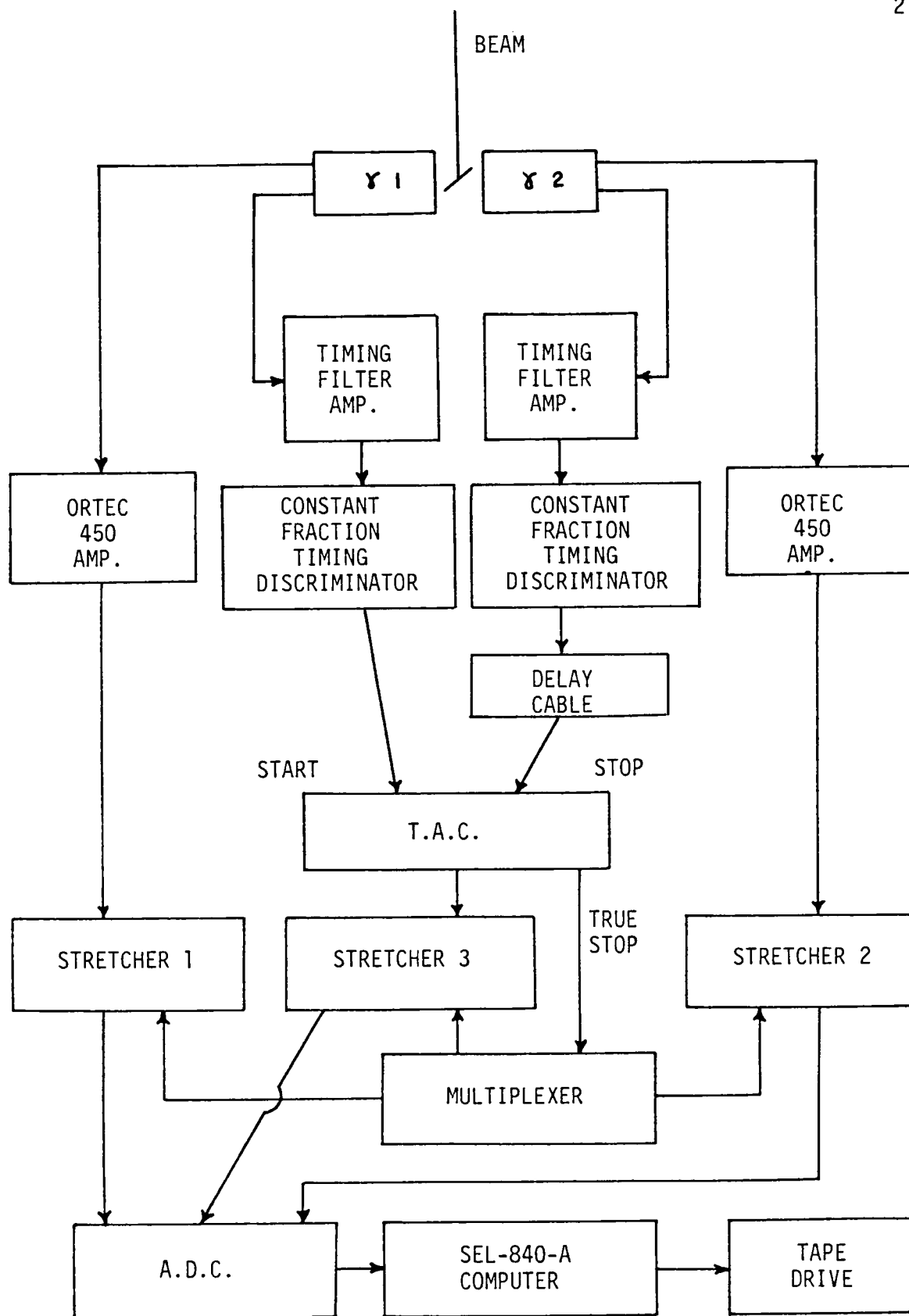


Figure III-1. Coincidence counting apparatus.

processed for energy analysis. The one that goes to the timing filter amplifier is used for timing analysis.

The constant fraction timing discriminator selects only those pulses above a certain level. Therefore, as the name implies, it functions as a discriminator to eliminate the noise. Also, it generates a negative logic signal to act as a START or STOP for the Time-to-Amplitude Converter (TAC). The TAC provides an output pulse with an amplitude that is proportional to the time interval between a start and subsequent stop pulse input. When the output TAC pulses indicate that a coincidence had occurred (within less than 100 nanoseconds), the TAC also puts out a true stop pulse which starts the multiplexer. The multiplexer then records the respective pulse height of the signals in stretchers one, two and three. These peak voltages are then digitized by an analog-to-digital converter (ADC). This device digitizes the pulse height by an auxiliary electronic oscillator. The number of cycles this clock goes through during the linear rise time of a particular pulse is thus proportional to the pulse height. Once the pulse height is digitized, it can be manipulated by various computer techniques. Typically, a memory location in the computer is reserved for each digital pulse height. After each particular pulse is recorded, its corresponding memory location is increased by one. Thus, the cumulative number in each location gives the total number of events or counts for each channel.

The digitized events are also grouped into subsets of three parameters after each interrogation cycle of the multiplexer. These multiparameter events are then stored as three consecutive tracks on a magnetic tape for subsequent analysis.

B. EXPERIMENTAL SETUP

The γ - γ -t coincidence experiment required two Ge(Li) detectors. Each of these was placed opposite to the other at 90° to the beamline. The target which has a thickness of approximately 2 mg/cm^2 was mounted between the two Ge(Li) counters. All of these are housed by a chamber which had been evacuated to a pressure of about 10^{-5} Torr. The distance from the target to each detector was set equal to two inches. This setup was used to subtend a large solid angle so that the coincidence count rate could be maximized.

C. ANALYSIS OF EXPERIMENTAL DATA

It was mentioned in Section III-A that data were stored in three distinct groups: the first and second were the gamma ray energies gathered from the first and second detectors, respectively. The third parameter was the length of time between them. In the subsequent analysis, the total coincidence spectra for both γ_1 and γ_2 were obtained. Using the computer, gates were then set around the peaks of interest. As a rule, we gated on the spectrum with better resolution (γ_1) and built spectra using the other stretcher (γ_2). A 50-nsec window was set around the prompt time peak in the TAC spectrum as various gates were scanned, in order to ensure that mostly true coincidence events were processed. In this manner, spectra of all gammas in coincidence with a certain energy were obtained. Background gates were also set for each peak to account for coincidence events due to Compton scattering. This background count is then offset by subtracting the corresponding background spectra from the peak gates.

During the course of these experiments, singles spectra for both Ge(Li) detectors were also generated for reference in analyzing the data.

D. EXPERIMENTAL RESULTS

Gamma-ray coincidence data were accumulated with two Ge(Li) detectors using the $^{159}\text{Tb}(^{20}\text{Ne},4n)^{175}\text{Re}$ reaction at 102 MeV. A total coincidence spectrum from one counter is shown in Figure III-2. The energies and relative intensities of the observed transitions are listed in Table III-1.

There was no angular distribution measurement taken in this work. Thus, the assignment of spin and parity has to rely totally on analogy to $^{177-181}\text{Re}$. Since the ϵ_2 deformation remains fairly constant in this region,¹³ the use of systematics of nuclei in this region is plausible. The sum of spectra taken in coincidence with the 196 keV, 315 keV and 420 keV γ -rays is shown in Figure III-3. Also, the systematics of the $h_{9/2}$ decoupled bands for the isotopic series is shown in Figure III-4. As can be seen from the diagram, if the assumption that the extrapolation of trends of levels in this region is correct, the correlation of the yrast bands of the four nuclei $^{181,179,177,175}\text{Re}$ seems to be quite reasonable and acceptable.

Only the yrast band can be concluded from the statistics of these preliminary data. We made this assignment from coincidence data. Also, the spin assignments are made strictly from analogy to $^{177,179,181}\text{Re}$.

The yrast band is most strongly populated in ^{177}Re .¹³ It is reasonable to speculate that the lowest state of this band, probably the $5/2^-$ rotational state, is at or near the ground state.¹⁷ The

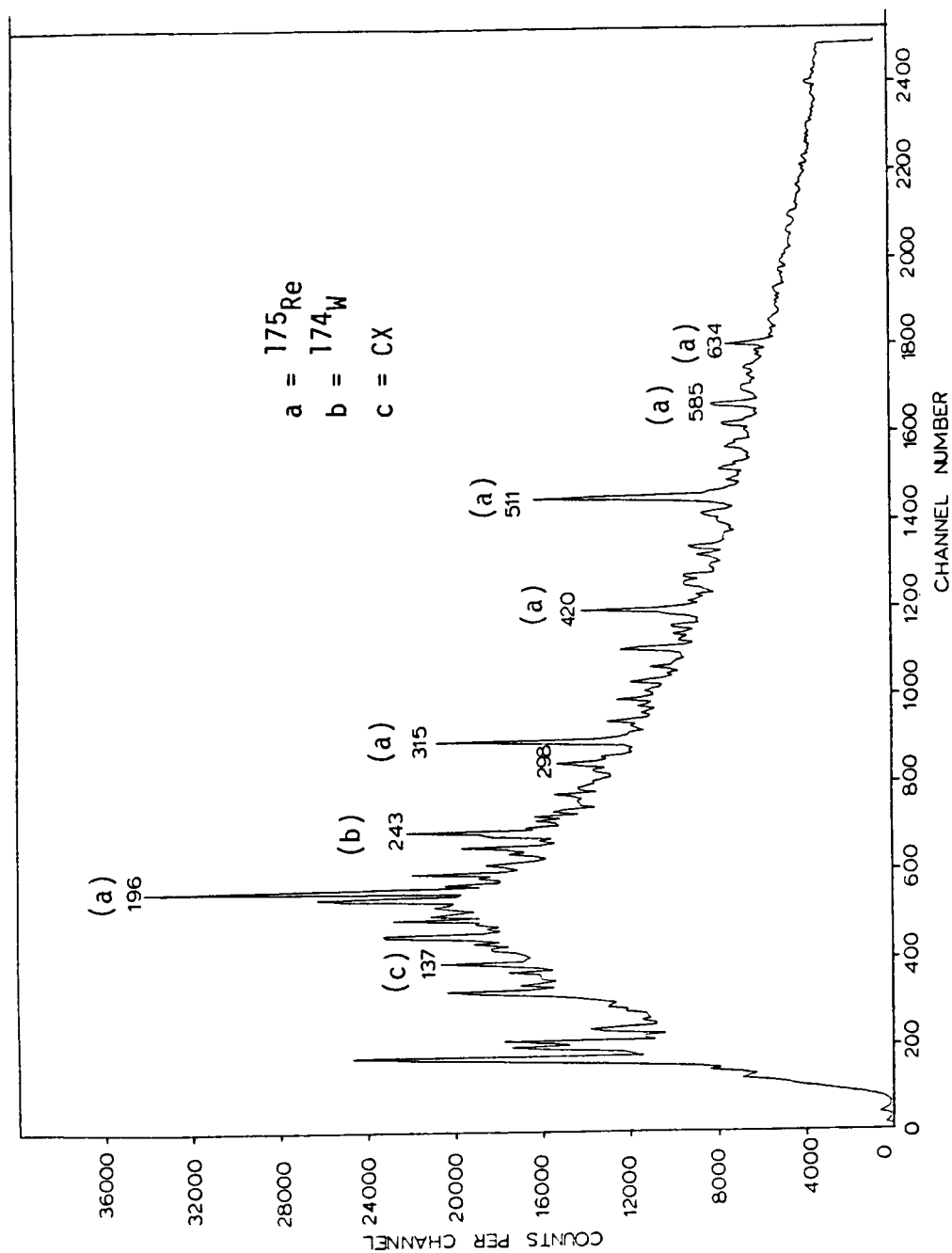


Figure III-2. Y-Ray coincidence spectrum following the $^{159}\text{Tb}(^{20}\text{Ne}, 4n)^{175}\text{Re}$ at 102 MeV reaction. Only transitions definitely assigned to Coulomb excitation, ^{174}W and ^{175}Re are indicated. The 298 keV peak may be an isomeric transition. All peak energies are in keV.

TABLE III-1
GAMMA-RAY TRANSITIONS OBSERVED IN $^{159}\text{Tb}(^{20}\text{Ne},4n)^{175}\text{Re}$ REACTION

E_γ	I_γ (Singles)	I_γ (Tot. Coin.)	Assignment	Coin. Transition*
103.9	252.1	~ 0	CX	
109.8	21.5		$2 \rightarrow 0$, 176_W	
113.1	48.2	14.0	$2 \rightarrow 0$, 174_W	
115.6	23.6	38.8		155?, 190?, 232?, 358?, 493?
121.3	65.5	11.4	CX	103?, 147?, 189?, 358?
125.6	55.8	~ 0		
131.0	24.3	14.6		94?, 240?, 301?
137.4	85.2	22.9	CX	114?, 159W
149.0	23.4	8.6	CX	116W, 196?, 301?, 385?, 401?
154.6	9.5	3.0		
158.7	5.5	9.1	CX	137?, 212?, 257?, 635?
160.8	11.4	14.6	$9/2 \ 11/2^- \rightarrow 9/2 \ 9/2^-$	137?, 190?, 209?, 232?, 267?, 421?
166.5	20.0	2.5		
170.4	5.0	~ 0		
173.7	16.9	20.4		121?, 164?, 182?, 373?, 578?
177.7	11.9	12.9		189?, 301?, 357?, 405?, 564?
184.0	72.8	5.8		121?, 210W, 253W, 362?
190.4	37.5	10.8	CX	No Conclusion
197.0	100.0	100.0	$9/2 \ 13/2^- \rightarrow 9/2 \ 11/2^-$	298S, 315S, 420M, 511M, 585W, 634W
206.5	73.7	11.0	$1/2 \ 13/2^- \rightarrow 1/2 \ 9/2^-$	
210.7	19.8	24.8	$9/2 \ 15/2^- \rightarrow 9/2 \ 13/2^-$	160?, 193W, 256?
217.4	7.5	8.3		113?, 142?
225.4	44.3	3.6	CX	
231.0	13.8	23.7	$9/2 \ 13/2^- \rightarrow 9/2 \ 15/2^-$	116?, 210?
240.2	8.8	15.1	$4 \rightarrow 2$, 176_W	69W, 160?, 191?
243.4	89.1	38.8	$4 \rightarrow 2$, 174_W	113S, 348S
256.0	7.5	11.9		121?, 210?, 225?, 232?
260.6	10.3	32.1		114?, 243?
270.0	35.4	~ 0	CX	

TABLE III-1 (continued)

E_Y	I_Y (Singles)	I_Y (Tot. Coin.)	Assignment	Coin. Transition*
273.6	3.7	10.1		
285.0	6.1	7.2		
298.1	30.2	6.9	494→196	
315.5	52.4	73.1	$1/2\ 17/2^- \rightarrow 1/2\ 13/2^-$	120W, 197S, 421S, 511M, 585W, 634W
332.1	18.7	15.0		
349.1	28.4	14.5	6→4 $174W$	113W, 243S
364.3	32.8	7.4		
374.9	1.9	12.9		
390.9	45.3	6.8		225?, 321?, 357?
408.3	8.6	7.1		No Conclusion
421.2	27.3	48.6		
429.0	25.6	2.3	$1/2\ 21/2^- \rightarrow 1/2\ 17/2^-$	196S, 315S, 511M, 585W, 634W
441.0	5.9	10.4		
448.6	9.1	10.4		
465.2	9.1	7.2		
471.8	4.0	24.8		
511.1	186.6	158.3	$1/2\ 25/2^- \rightarrow 1/2\ 21/2^-$	196M, 315M, 421W, 511S, 585W, 634W
534.0	7.9	18.1		379?
551.7	3.0	9.4		
569.4	3.0	19.5		160?, 260?, 380?
585.6	15.6	52.8	$1/2\ 29/2^- \rightarrow 1/2\ 25/2^-$	196W, 315M, 420W, 511W, 634W
634.4	16.2	36.2	$1/2\ 33/2^- \rightarrow 1/2\ 29/2^-$	196W, 315M, 420M, 511W, 585W, 634W

*S stands for strong coincidence; M stands for medium coincidence; W stands for weak coincidence; ? means it is highly questionable whether it is in coincidence or not.

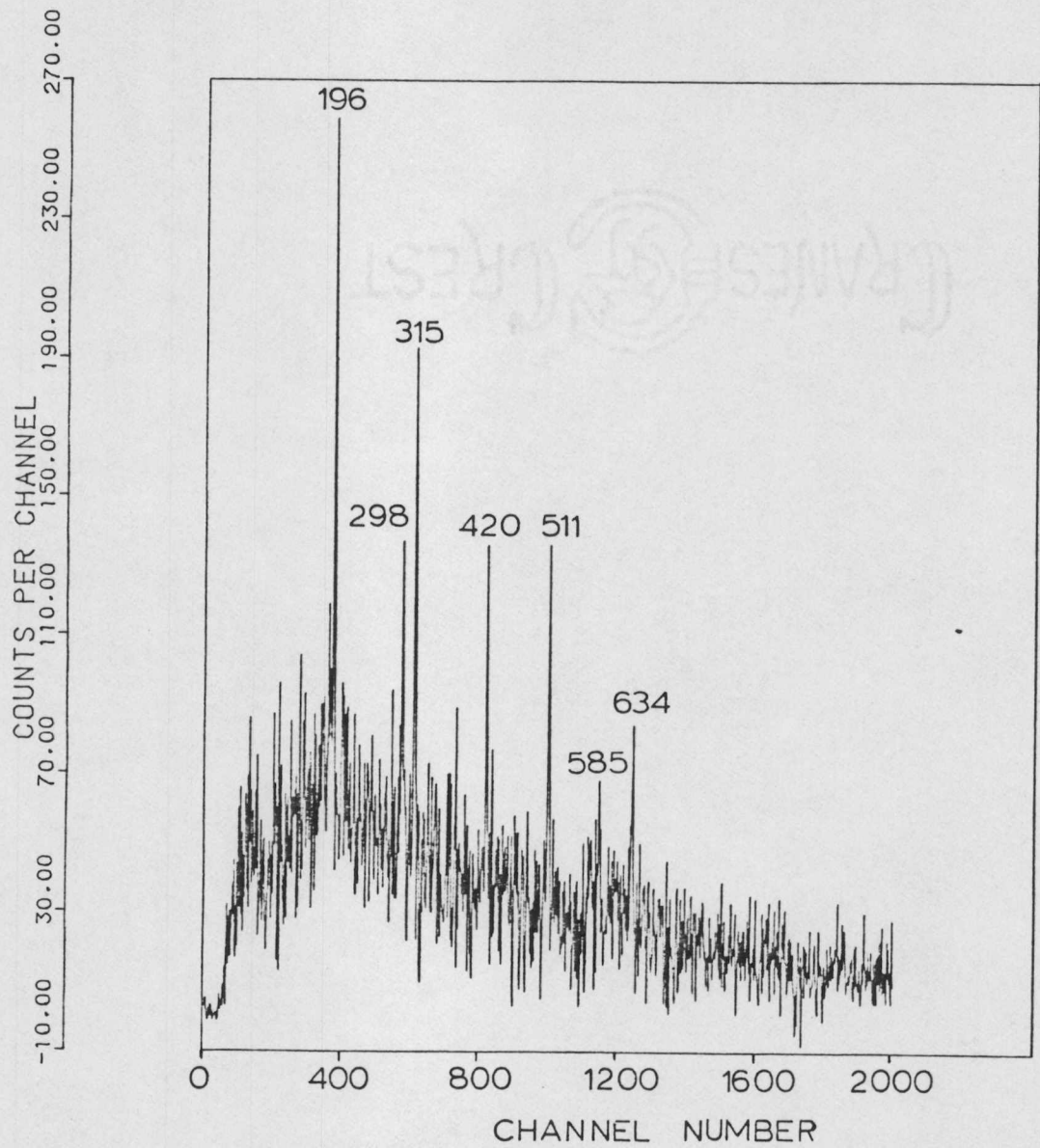


Figure III-3. Sum of 196, 315 and 420 keV coincidence gates in ^{175}Re . These are gates on the stretched E2 transitions in the $h_{9/2}$ decoupled band.

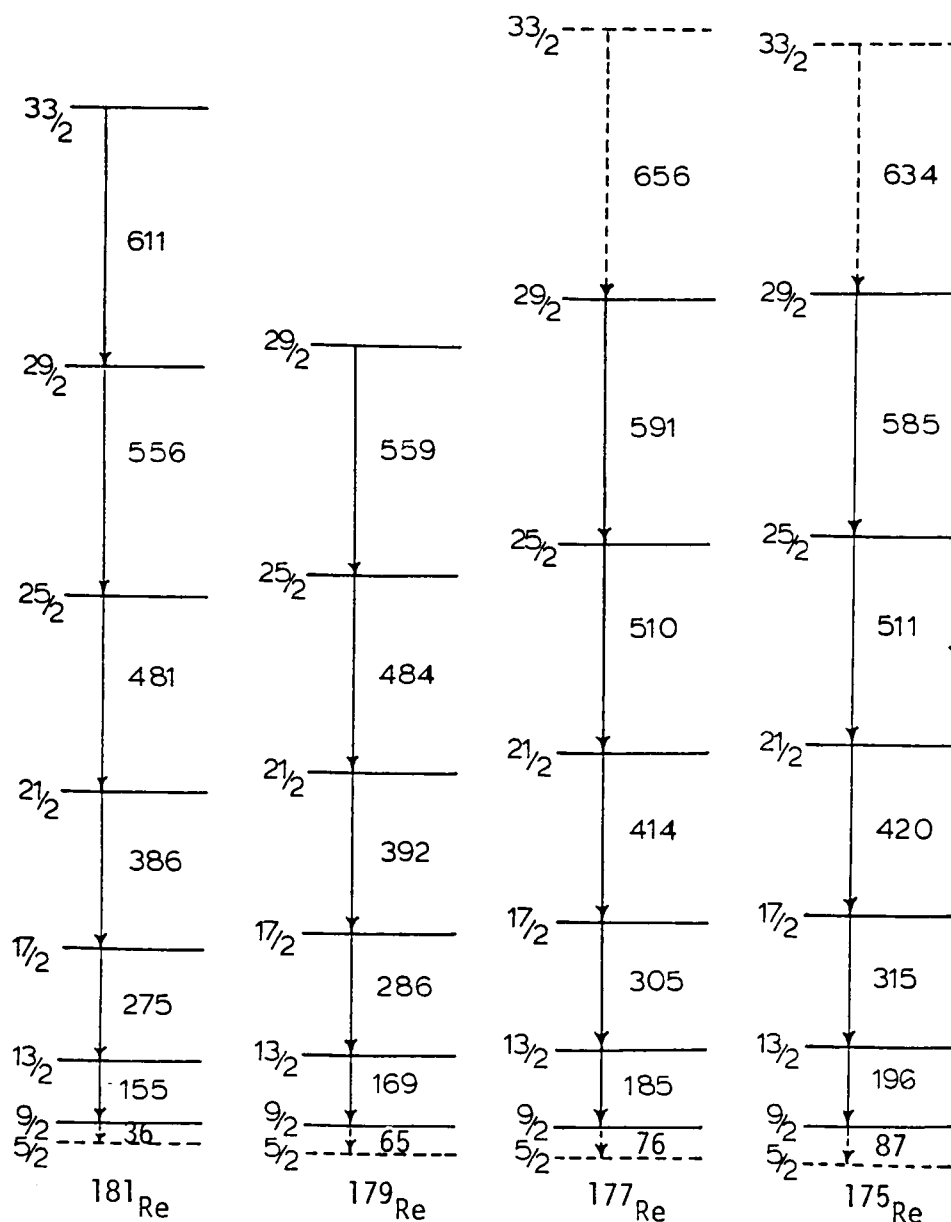


Figure III-4. Systematics of the ground state rotational band for the isotopic series $^{181}, ^{179}, ^{177}, ^{175}\text{Re}$. The $9/2 \rightarrow 5/2$ transition has not been observed in all four isotopes. The energies indicated for the $9/2 \rightarrow 5/2$ transition are as predicted by the Coriolis coupling calculation. All energies are in keV.

$9/2^- \rightarrow 5/2^-$ transition is not observed experimentally in these nuclei. However, this transition was seen in ^{183}Re to be within a 30-40 KeV spacing.¹⁸ Calculation indicates that it should be 37 KeV. Therefore, the transition energies of the $9/2^- \rightarrow 5/2^-$ rotational states in the yrast band for the nuclei $^{181,179,177,175}\text{Re}$ were estimated by Coriolis calculation. The predicted values are shown in dotted arrows in Figure III-4.

CHAPTER IV

CALCULATIONS-COMPARISON TO EXPERIMENT

A. CORIOLIS CALCULATION

For the isotopes $^{181,179,177}\text{Re}$, the level spacings cannot be fitted by the simple rotational formula. However, Coriolis calculations give good fits to the experimental energy spacings.¹³ In the calculations as given by Leigh et al., the pairing attenuation had not been taken into account. However, a smooth variation of parameters does explain many gross properties such as the moment of inertia and hexadecapole deformation parameter of the nuclei in this region. In this project, detailed Coriolis calculations have been performed to fit the experimental data. The intent here is to perform systematic Coriolis calculations on the previous data ($^{177,179,181}\text{Re}$) and on the new data ^{175}Re , following the choice of parameters used by Leigh et al.¹³ The goal is to see if such calculations can reproduce this whole isotopic chain of $h_{9/2}$ bands.

In this calculation, the pairing effect is included.¹⁴ All five bands arising from the $h_{9/2}$ shell have been considered in a pure j calculation. The purpose of fitting the yrast band of this isotopic series is to see if the theoretical trend persists by considering one more additional member in this series.

Before the Coriolis calculation is performed, the Nilsson single particle energies arising from the $h_{9/2}$ orbitals are calculated. The

results are tabulated, as shown in Table IV-1. These values can be used as a first approximation for input into the Coriolis routine.

TABLE IV-1
NILSSON SINGLE PARTICLE ENERGIES FOR $^{181,179,177,175}\text{Re}$,
ASSUMING $\epsilon_2 = 0.25$, $\epsilon_4 = 0.0$

Nilsson Orbitals	^{181}Re	^{179}Re	^{177}Re	^{175}Re
$1/2^- [541]$	0.0	0.0	0.0	0.0
$3/2^- [532]$	0.851	0.857	0.863	0.870
$5/2^- [523]$	2.158	2.173	2.188	2.205
$7/2^- [514]$	3.775	3.798	3.825	3.854
$9/2^- [505]$	5.624	5.663	5.704	5.747

Note: The energy of the $1/2^- [541]$ is set arbitrarily to zero. The energy levels of the other Nilsson states are normalized with respect to this arbitrary zero. All values are in MeV. In this calculation, $\nu_p = 0.493 + 0.649 A/1000$; $\kappa_p = 0.0766 - 0.0779 A/1000$.²⁰

The number of parameters which could be varied in a Coriolis calculation is large. However, some of these can be fixed to some extent by systematics. In our case, the parameters that are allowed to vary are the single particle energy separation between the $1/2^- [541]$ and $3/2^- [532]$ levels, the Fermi surface, the decoupling parameter, and the A and B coefficients in the expansion of the unperturbed rotational energy as a power series in $I(I + 1)$.

In this calculation, the following assumptions are made:

1. The relative single particle energies arising from the $h_{9/2}$ orbitals are as predicted by the Nilsson model, except for the separation the $1/2^- [541]$ and $3/2^- [532]$ orbitals.

2. The quasiparticle energies are given by

$$E_{q.p}^K = \sqrt{(\epsilon_K - \lambda)^2 + \Delta^2} - \Delta \quad (IV-1)$$

where the ϵ_K 's are the Nilsson single particle energies, λ is the energy of the Fermi surface, and Δ is the pairing energy, which is taken to be $12/\sqrt{A}$ MeV.

3. The unperturbed rotational energies of the bands are given by

$$E_K^I = E_{qp}^K + A[I(I+1) + (-1)^{I+1/2} a \delta_{K \frac{1}{2}} + \langle j^2 \rangle - 2K^2] \\ + B[I(I+1) + (-1)^{I+1/2} a \delta_{K \frac{1}{2}} + \langle j^2 \rangle - 2K^2]^2 \quad (IV-2)$$

where a is the decoupling parameter.

4. The Coriolis coupling matrix elements are given by

$$H_{K, K+1}^I = \sqrt{I(I+1) - K(K+1)} \langle K | j_- | K+1 \rangle (u^* u + v^* v) \quad (IV-3)$$

where u and v are the quasiparticle occupation probabilities;

$\langle K | j_- | K+1 \rangle$ can be evaluated from the Nilsson wavefunction.

5. The value of the quadrupole deformation parameter is $\delta = 0.25$, close to the values for doubly even neighbors.

The resulting matrix has been diagonalized using the program CORIOLIS, which allows any of the parameters A , B , a , $\Delta E_{1/2 \rightarrow 3/2}$ and λ to vary to give the best fit to the experimental data.

A variety of combinations of A and a fit the spacing of the yrast band. However, the non-yrast band is quite sensitive to small changes in A and a . Since the energy separation between the $1/2^-$ [541] and $3/2^-$ [532] levels for ^{181}Re is known from radioactive decay study of ^{181}Os , it is used to determine A and a for ^{181}Re . This combination of realistic A and a values serves as a starting value to estimate its corresponding values for other rhenium isotopes. The $\Delta E_{1/2 \rightarrow 3/2}$ and the Fermi surface together somehow determine the energies of the first few transitions. The spacings between the high spin states are not very sensitive to small changes in these parameters. However, the B parameter determines the spacings of these high spin states because of the fact that $[I(I+1)]^2$ term begins to contribute significantly as I becomes large. Thus, the set of parameters that gives the best fit for the entire yrast band is shown in Table IV-2.

The equilibrium calculations of Nilsson based on the Strutinsky method reveal that the quadrupole deformation parameter ϵ_2 is fairly constant in this region.²⁰ Consequently, it is reasonable to assume that the moment of inertia, which is closely related to the A parameter, remains fairly constant also. The decoupling parameter, as given by the Nilsson calculation, is too low to give a satisfactory fit to the energy spacings. In all cases, it is multiplied by a factor of approximately 1.1 in order to produce good fits. Thus, we determine these parameters and allow variation of B , the unknown $1/2^-$ [541] - $3/2^-$ [532] energy separation and the position of the Fermi surface to give the best fit.

TABLE IV-2
INPUT PARAMETER SET FOR CORIOLIS CALCULATION

	^{181}Re	^{179}Re	^{177}Re	^{175}Re
a	4.51	4.42	4.49	4.44
A (MeV)	.0165	.0165	.0165	.0165
B (eV)	- .0026	- .0037	- .0030	- .0037
λ (MeV)	.410	.200	.470	.470
Δ (MeV)	.89	.90	.90	.91
$1/2^- [541]$	0.410	0.200	0.470	0.470
$3/2^- [532]$	0.861	1.137	1.786	2.200
$5/2^- [523]$	2.568	2.373	2.658	2.710
$7/2^- [514]$	4.182	3.998	4.205	4.378
$9/2^- [505]$	6.034	5.863	6.084	6.292
$\Delta E_{1/2-3/2}$ (MeV)	0.451	0.937	1.316	1.730

Note: The energy separation between the $1/2^- [541]$ and $3/2^- [532]$ levels is increasing for decreasing N in order to give good fits to the experimental data. That indicates the decrease of ϵ_4 deformation for decreasing N. In this calculation, ϵ_2 is set to be equal to 0.25 and ϵ_4 is set to be zero for all four isotopes; the energy levels of the single particle Nilsson states are in MeV.

The energy separation between the $1/2^- [541]$ and $3/2^- [532]$ Nilsson state is of crucial significance here. The equilibrium Strutinsky calculations¹³ indicate that the ϵ_2 deformation remains fairly constant as a function of neutron number (N) in these W-Re-Os isotopes, but that the ϵ_4 deformation decreases with decreasing N. Our Nilsson calculations show that decreasing ϵ_4 values result in increasing separation between the $\Omega = 1/2$ and $3/2$ orbitals in the $h_{9/2}$ shell. This trend agrees with our Coriolis coupling calculations, where we found it necessary to increase this separation between orbitals in order to fit the observed $h_{9/2}$ bands.

The result of the calculation is shown in Figure IV-1. It is unreasonable to expect perfect fits to the data from such multi-parameter and complicated calculation. However, the fitting obtained with reasonable variation of parameters is quite good. Moreover, the result of the calculation supports the hypothesis that the ϵ_2 deformation remains fairly constant while the ϵ_4 deformation is decreasing with decreasing mass in this region. This may suggest that the study of level positions of an isotopic series may give a useful indication of changes in the hexadecapole deformation parameter.

B. CRANKING MODEL CALCULATION

As discussed in the theory concerning the cranked shell model, the dominant feature in this scheme is the rotational angular frequency, ω , and the crucial physical quantity that signifies the sudden deviation from the normal rotational mode is the aligned angular momentum, i ,

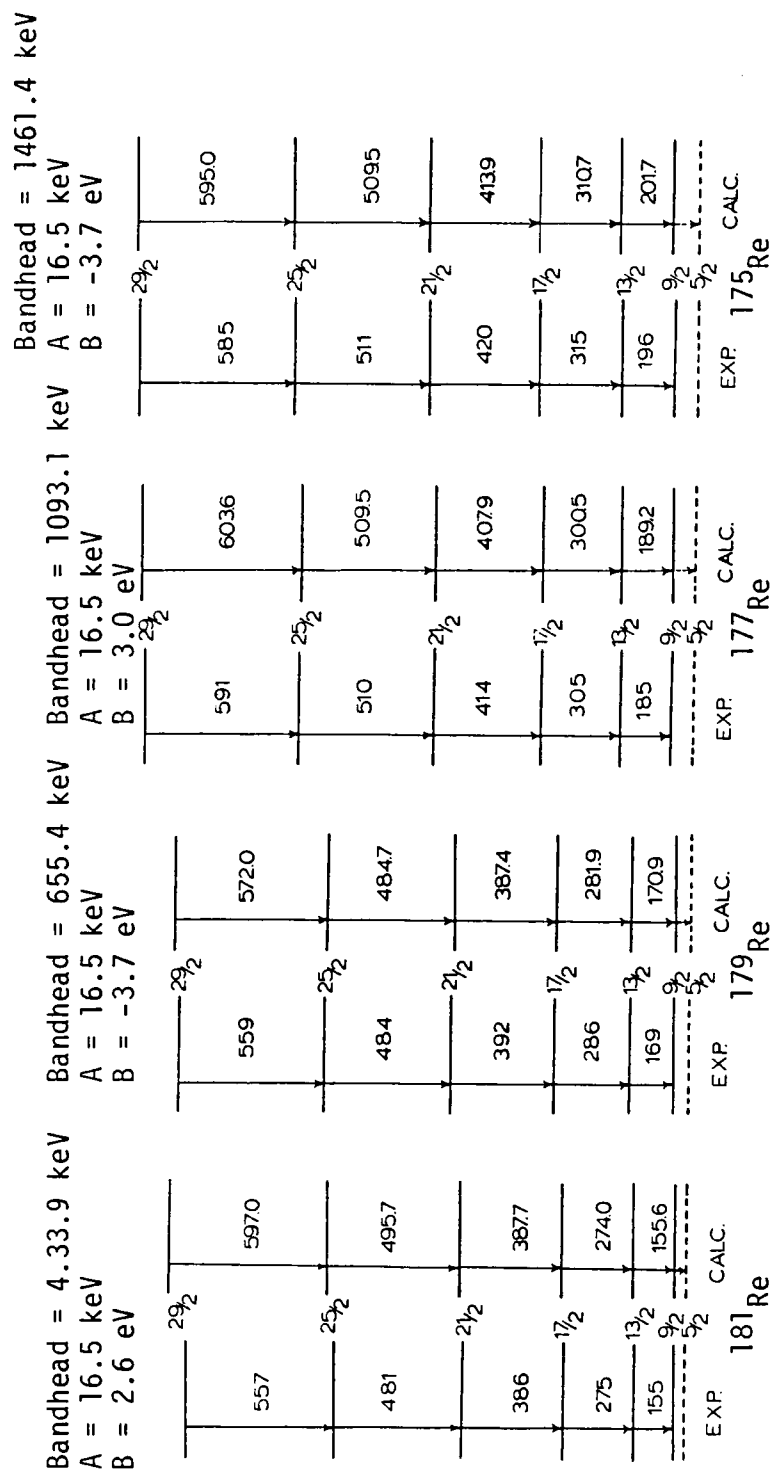


Figure IV-1. Comparison of experimental level scheme and theoretical Coriolis coupling calculation for $^{181}, ^{179}, ^{177}, ^{175}\text{Re}$. The bandhead is defined as the energy separation between the $1/2^- [541]$ and $3/2^- [532]$ Nilsson orbitals. The transitions are in keV. In this calculation, $\epsilon_2 = 0.25$ and $\epsilon_4 = 0$.

which is simply the angular momentum of the excited band relative to the ground configuration at a certain frequency.

$$i = I_x - I_{xg} \quad (\text{IV-4})$$

In this equation, I_x is the angular momentum along the rotational axis of the deformed core and is given by

$$I_x = [(I + \frac{1}{2})^2 - K^2]^{1/2} \quad (\text{IV-5})$$

for a $I + 1 \rightarrow I - 1$ transition in a certain K band.

For the ground band, we use a variable moment of inertia (VMI) description.

$$I_{xg} = (\mathcal{J}_0 + \mathcal{J}_1 \omega^2) \omega \quad (\text{IV-6})$$

where $\mathcal{J}_0$ and $\mathcal{J}_1$ are fitted parameters.

Therefore, to calculate the aligned angular momentum from experimental values, it is crucial to have a correct estimation of $\mathcal{J}_0$ and $\mathcal{J}_1$ to begin with.

Extraction of these $\mathcal{J}_0$ and $\mathcal{J}_1$ values can be done using the experimental values of the rotational band and the formulas:¹⁵

$$\mathcal{J}_b(1) = \frac{I_x}{\omega} = \frac{i + (\mathcal{J}_0 + \mathcal{J}_1 \omega^2) \omega}{\omega} \quad (\text{IV-7a})$$

$$\mathcal{J}_b(2) = \frac{dI_x}{d\omega} = \mathcal{J}_0 + 3\mathcal{J}_1 \omega^2 \quad (\text{IV-7b})$$

where i is assumed to be ω -independent.

These are the two definitions of moment of inertia using the first difference and second difference, respectively. The advantage of $\mathcal{J}_b^{(2)}$ over $\mathcal{J}_b^{(1)}$ is that the confusing effect of the alignment i is eliminated. Thus, if the alignment is not zero (but is constant) as for a one quasiparticle or a two quasiparticle band, $\mathcal{J}_b^{(2)}$ must be used, although there will be fewer data points to fit as a consequence of the second difference. If the alignment is zero as for the ground bands of even-even nuclei, $\mathcal{J}_b^{(1)}$ can be used to give $\mathcal{J}_0$ and $\mathcal{J}_1$ directly.

Thus, if we graph experimental $\frac{I_x}{\omega}$ vs. ω^2 points, the slope and intercept of the best straight line drawn among the data points at low frequencies will give $\mathcal{J}_1$ and $\mathcal{J}_0$, respectively, assuming that $i = 0$. Similarly, from a plot of $\Delta I_x / \Delta \omega$ as a function of ω^2 , we extract the $\mathcal{J}_0$ and $\mathcal{J}_1$ values from the intercept and slope via the same procedure.

For the odd A rhenium nuclei, the method of the second derivative is employed (since $i \neq 0$) whereas for the neighboring even-even isotopes, $^{174,176,178,180}\text{W}$ and $^{176,178,180,182}\text{Os}$, the other alternative is chosen. Both methods yield the same values of $\mathcal{J}_0$ and $\mathcal{J}_1$ for the even-even ground bands, but the first approach is preferred since there are more points to fit. The results are displayed in Figure IV-2 and Table IV-3.

It is readily seen from Table IV-3 that the $\mathcal{J}_0$ values for odd-A nuclei are considerably larger than for neighboring even-even nuclei, as the single nucleon tends to align to the quadrupole field and enhances the deformation. It thus appears that at least the $h_{9/2}$ band in the rhenium isotopes has larger equilibrium deformation than one would expect considering the average values for $Z = 74$ and 76 .

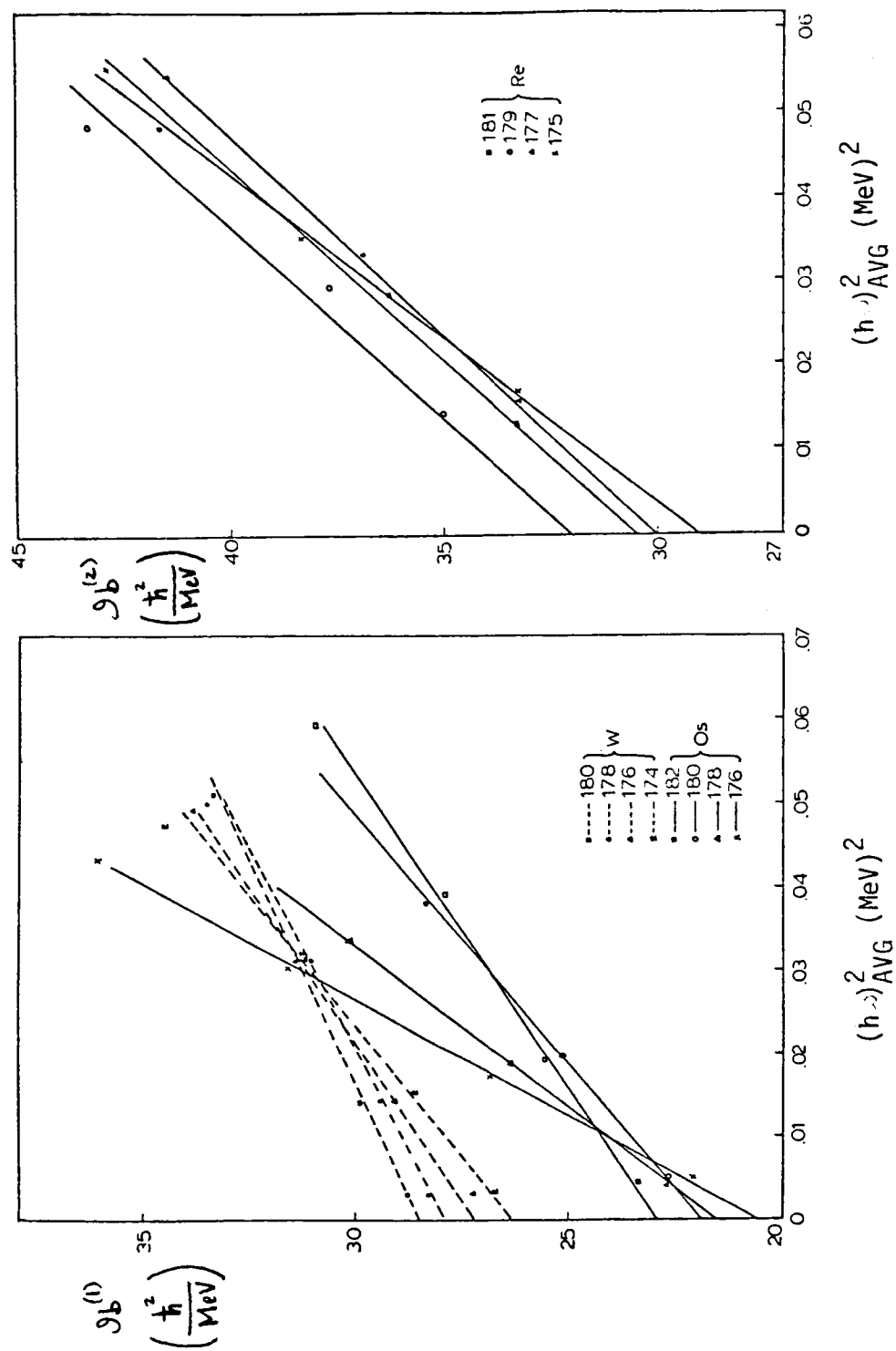


Figure IV-2. Moments of inertia based upon the first and second differences versus the average of the square of rotational frequency for the $\text{hg}/2$ proton band and the ground state even-even isotopic series 174, 176, 178, 180, 182, 184, 186, 188, 190. The second difference is plotted for the odd-A rhenium isotopic series 175, 177, 179, 181, 183, 185, 187, 189, 191.

TABLE IV-3

FITTING PARAMETERS $\mathcal{J}_0$ AND $\mathcal{J}_1$ AS OBTAINED FROM MOMENT OF
INERTIA PLOTS FOR W(z=74), Os(z=76) AND Re(z=75).
 $\mathcal{J}_0$ IS IN $\hbar^2/\text{MeV}$; $\mathcal{J}_1$ IS IN $\hbar^4/\text{MeV}^3$

W	$\mathcal{J}_0$	$\mathcal{J}_1$	Os	$\mathcal{J}_0$	$\mathcal{J}_1$	Re	$\mathcal{J}_0$	$\mathcal{J}_1$
180	28.6	90	182	22.9	135	181	30.4	78
178	28.0	106	180	22.1	165	179	30.0	94
176	27.2	136	178	21.7	258	177	30.0	72
174	26.2	166	176	20.6	360	175	29.0	91

With a knowledge of $\mathcal{J}_0$ and $\mathcal{J}_1$ values, we can deduce the aligned angular momentum of quasiparticle bands using Eqs. IV-4, IV-5, IV-6. The resulting alignments for the rhenium isotopes are graphed in Figure IV-3. The yrast band for all four isotopes have rather constant alignment until the crossing frequency is reached, where an alignment of additional particles occurs.

Only ^{181}Re has experimental values that are high enough to allow the construction of levels up to the crossing frequency. The other isotopes may experience a crossing of the same nature at a similar frequency.

The alignments of the two side bands for ^{181}Re , $5/2^+[402]$ and $9/2^-[514]$, are also computed and plotted on the same graph. The $\mathcal{J}_0$ and $\mathcal{J}_1$ values calculated from an average of the Os and W values in Table IV-3 are used to calculate the alignments because the particles responsible for these bands are not expected to have a significant polarizing effect on the core.

The effect of core rotation on high K orbitals as contrasted to the low K orbitals is fully displayed here. For the yrast band based on the $1/2^-[541]$ Nilsson state, the single particle aligns to the rotation axis quickly and remains fairly constant. For high K bands,

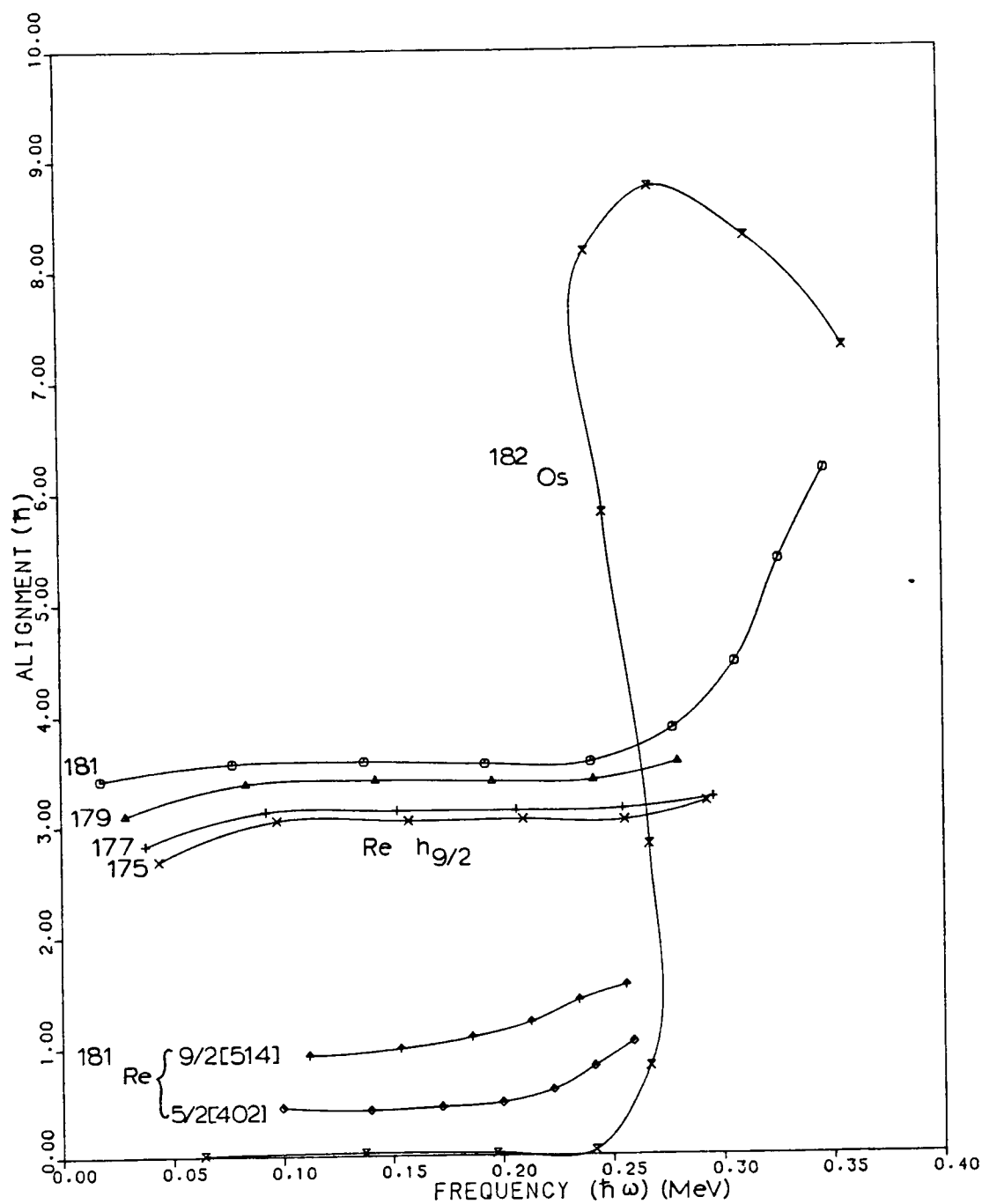


Figure IV-3. Rotational alignment versus rotational frequency. The yrast band of all four rhenium isotopes, the side bands of ^{181}Re originating from the 9/2-[514] and 5/2-[402] Nilsson states and the ground state band of ^{182}Os are plotted for comparison. See text for discussion.

it aligns to the rotation axis gradually with increasing frequency, with the result that little alignment is observed at moderate frequencies.

For the purpose of comparison, the alignment of ^{182}Os as a function of ω is also plotted as shown in Figure IV-3. It illustrates a similar variational pattern as ^{181}Re , with the alignment remaining fairly constant at low frequencies followed by a sharp increase at the crossing frequency. However, the crossing frequency is smaller than that of ^{181}Re . This lowering in crossing frequency may be attributed to a decrease of deformation, even though the alignment of the same pair of $i_{13/2}$ quasi-particles is responsible for the crossing in both cases.

As discussed in II-3, the cranking model calculates quasiparticle energies as a function of frequency in the body fixed frame.

The energy of rotation is ωI_x . Therefore, the energy in the rotating frame is

$$E' = E - \omega I_x \quad (\text{IV-8})$$

where E is the average level energy in the laboratory frame,

$$E(I) = \frac{1}{2} [E(I+1) + E(I-1)] . \quad (\text{IV-9})$$

As discussed previously, the absolute energies are not calculated in the cranked shell model, but only quasiparticle energies in a rotating frame relative to a reference configuration. The reference configuration is defined by

$$E'_g = -\frac{\omega^2}{2} \mathcal{J}_0 - \frac{\omega^4}{4} \mathcal{J}_1 - \frac{1}{8\mathcal{J}_0} . \quad (\text{IV-10})$$

With Eqs. IV-8 and IV-10, we can quote the quasiparticle energies e' in the rotating frame relative to the ground configuration as

$$e' = E' - E'_g . \quad (\text{IV-11})$$

The results of the calculation are tabulated in Table IV-4. The values we obtained here are used to compare with the theoretical calculations later. The odd-even mass difference has been added to the Re level energies in each case, since the reference configuration refers to the even-even core.

The input parameters for the cranking calculation are the deformation parameters, the pairing gap Δ , and the Fermi surface λ , all of which are assumed to be frequency independent in the calculation. The pairing gap is estimated to first order by:²²

$$\Delta n = \frac{1}{4} [B(N-2,Z) - 3B(N-1,Z) + 3B(N,Z) - B(N+1,Z)] \quad (\text{IV-11a})$$

$$\Delta p = \frac{1}{4} [B(N,Z-2) - 3B(N,Z-1) + 3B(N,Z) - B(N,Z+1)] \quad (\text{IV-11b})$$

where Δn and Δp are the pairing gap for neutron and proton, respectively. $B(N,Z)$ is the binding energy of the nucleus with proton number Z and neutron number N .

This approximation seems to give realistic values for protons. For neutrons, it gives values that are unreasonably low. This may be accounted for by the fact that the extrapolation of binding energies of unstable nuclei is not accurate. Thus, the average value of Δp and Δn , which is given by $\Delta = 12A^{-1/2}$ MeV, is used to estimate the odd-even mass

TABLE IV-4
EXPERIMENTAL ROUTHIAN FOR $h_{9/2}$ DECOUPLED BANDS AS A FUNCTION
OF ROTATING FREQUENCY

^{175}Re		^{177}Re		^{179}Re		^{181}Re	
$e^-/\hbar\omega_0$	ω/ω_0	$e^-/\hbar\omega_0$	ω/ω_0	$e^-/\hbar\omega_0$	ω/ω_0	$e^-/\hbar\omega_0$	ω/ω_0
0.132	.006	0.137	.005	0.138	.004	0.165	.007
0.111	.014	0.115	.013	0.113	.012	0.153	.011
0.087	.122	0.088	.021	0.085	.020	0.123	.019
0.066	.029	0.065	.029	0.060	.027	0.096	.027
0.047	.035	0.044	.035	0.038	.033	0.072	.033
0.032	.040	0.026	.041	0.008	.039	0.053	.038
						0.037	.042
						0.023	.045
						0.007	.048

Note: $\hbar\omega_0 = 41 A^{-1/3}$ MeV.

difference for neutrons. Conventionally, 80% of this theoretical value is used for the cranking calculation in order to approximate the inevitable reduced pairing in rotating nuclei.

The chemical potential is obtained by reproducing the correct neutron or proton numbers once ϵ_2 and ϵ_4 are determined. In this calculation, the equilibrium values as calculated in a Strutinsky approach are used to determine ϵ_2 and ϵ_4 . This works correctly for most orbits except the $h_{9/2}$ proton, which seems to have a higher ϵ_2 deformation parameter due to polarization effect. This higher ϵ_2 value is estimated by multiplying the Strutinsky value by a factor given by the square root of the ratio of $\mathcal{J}_0$ for Re divided by the average $\mathcal{J}_0$ of its even-even isotones. The set of deformation parameters that were used is listed in Table IV-5.

TABLE IV-5

QUADRUPOLE DEFORMATION AND HEXADECAPOLE DEFORMATION PARAMETERS
FOR RHENIUM ISOTOPES FOR NORMAL PROTONS AND $h_{9/2}$ PROTONS

Nuclei	Strutinsky Values		$h_{9/2}$ Proton Bands	
	ϵ_2	ϵ_4	ϵ_2	ϵ_4
^{175}Re	.2290	.019	.2547	.019
^{177}Re	.2340	.029	.2595	.029
^{179}Re	.2325	.038	.2549	.038
^{181}Re	.2270	.046	.2450	.046

A cranking calculation for $^{181,179,177,175}\text{Re}$ has been performed using the appropriate input parameters. For the proton calculation, the higher deformations as listed in Table IV-5 were used. Both sets of deformation parameters have been used for neutron calculations. The resulting quasiparticle energies as a function of rotating frequency for ^{175}Re are displayed in Figures IV-4, IV-5, and IV-6. As can be seen in Figure IV-4, the trajectory corresponding to the $1/2^- [541]$ Nilsson state has a nonzero alignment at zero frequency because of the decoupling effect. The single particle gains alignment quickly with increasing frequency until a constant value is reached. This constant slope is measured and compared to the experimental values of the alignment. The theoretical and experimental alignments are calculated and shown in Table IV-6.

The change of alignment as a function of neutron number is compatible in both theory and experiment. This consistent agreement in trend indicates that the general basis of the calculation is correct. The experimental Routhians as calculated and listed in Table IV-4 are overlaid on the proton quasiparticle energy diagram as a comparison between theory and experiment. For a variety of reasons, one cannot expect an exact correspondence between the relative positions of these levels. In our case, this slight disagreement may be due to some uncertainty in the low energy transitions in the odd-A rhenium isotopes and in the odd-even mass parameters Δ for the reference level. To probe the importance of the choice of an appropriate odd-even mass difference, the experimental Routhian is also computed using 80% of the extrapolated odd-even mass differences. The results are overlaid on the theoretical

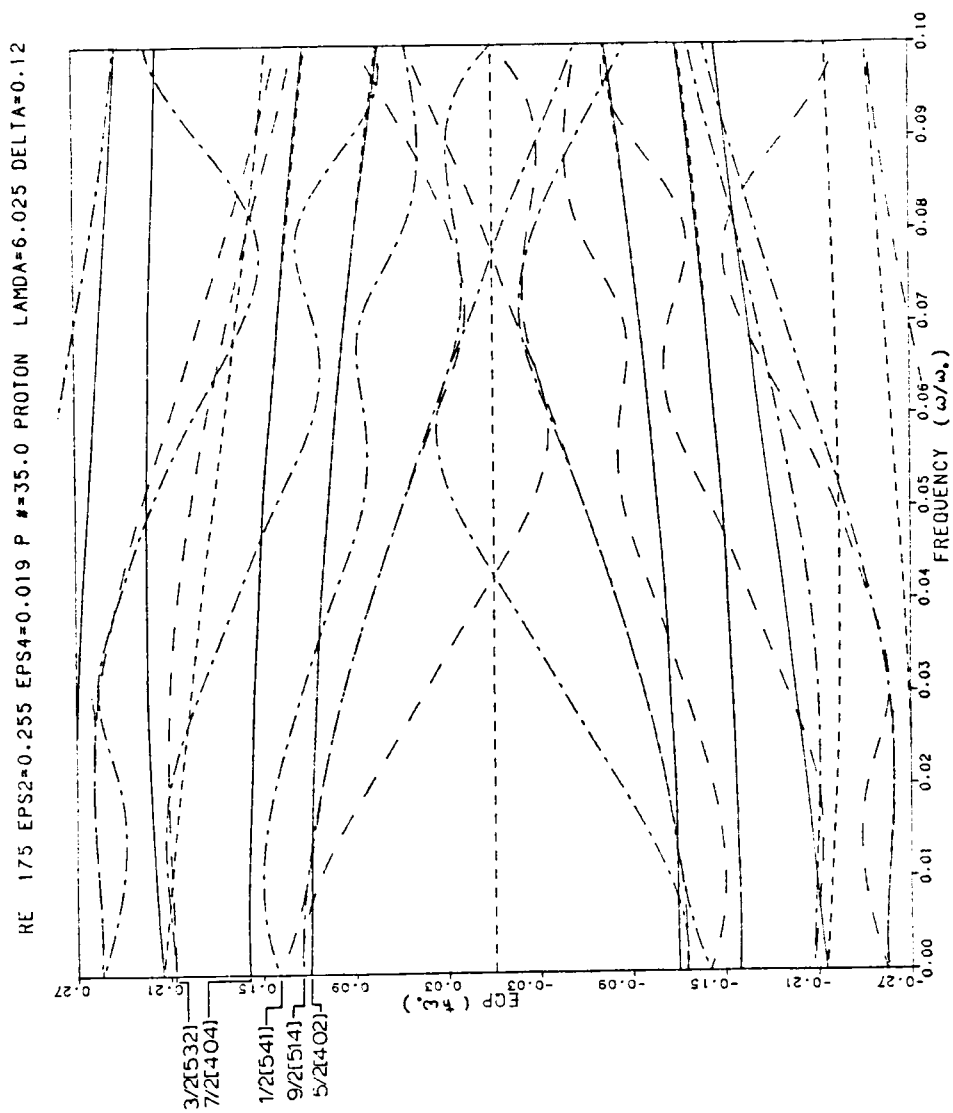


Figure IV-4. Cranking model calculations of proton g.p. energy in rotating frame using the $h_{9/2}$ proton bands ϵ_2 deformation parameter. $\hbar\omega_0 = 41A^{-1/3}$ MeV.

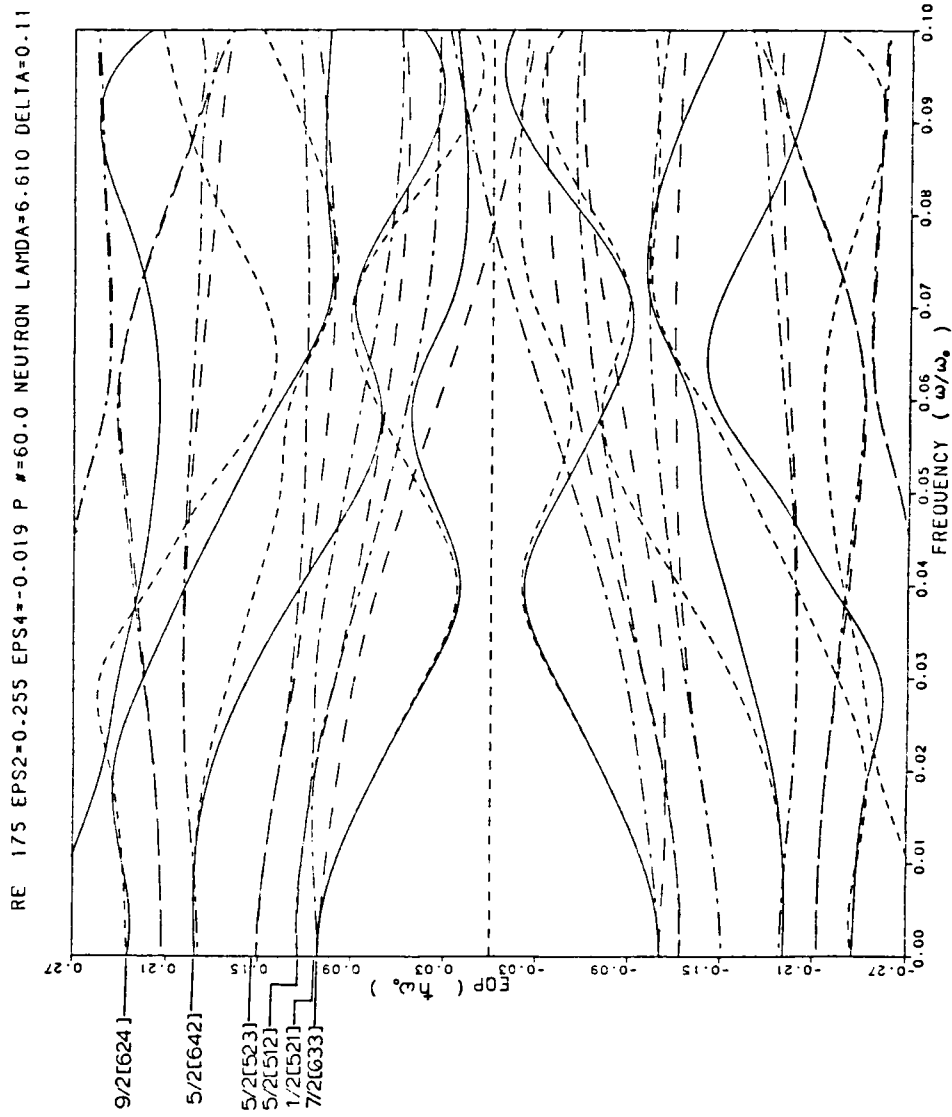


Figure IV-5. Cranking model calculation of neutron g.p. energy in rotating frame using the $h_{9/2}$ proton bands ϵ_2 deformation parameter. $\hbar\omega_0 = 41A^{-1/3}$ MeV.

RE 175 EPS2=0.229 EPS4=0.019 P #=60.4 NEUTRON LAMDA=6.600 DELTA=0.11

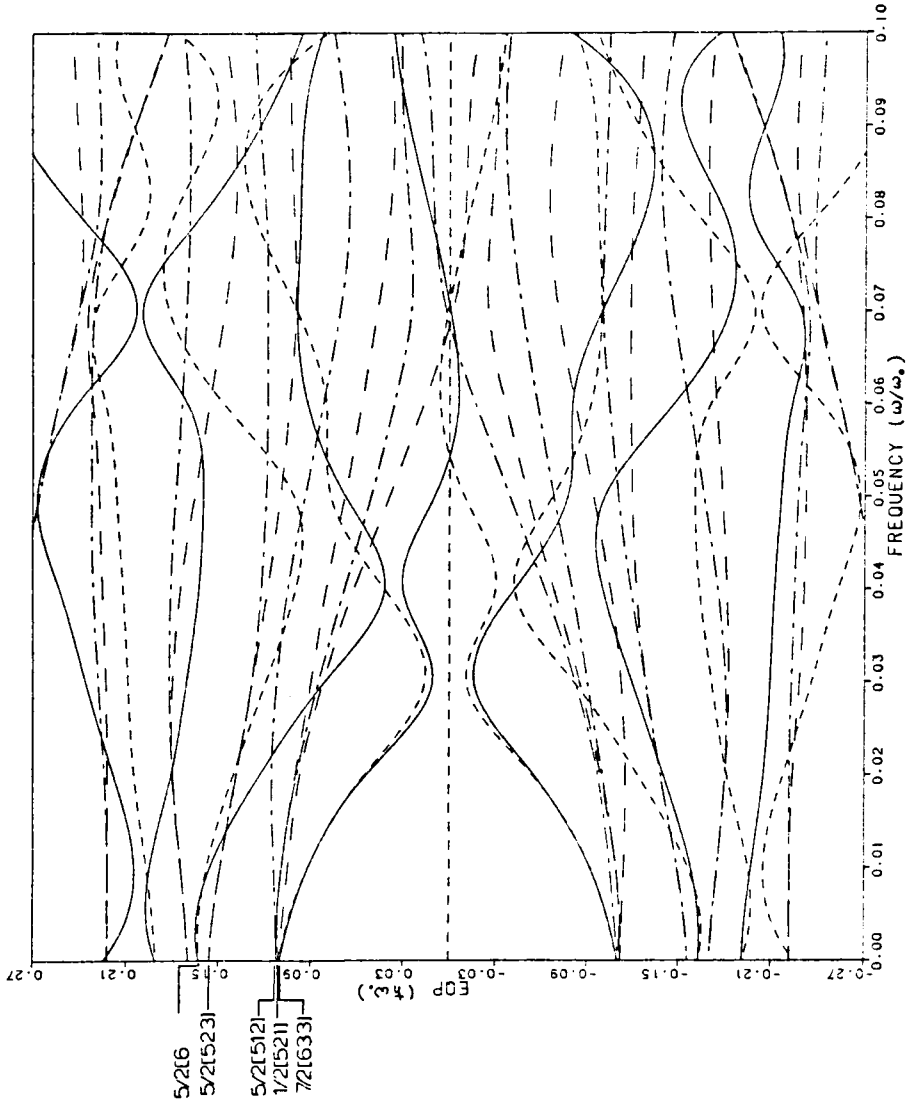


Figure IV-6. Cranking model calculation of neutron g.p. energy in rotating frame using the Strutinsky deformation parameter. $\hbar\omega_0 = 41A^{-1/3}$ MeV.

TABLE IV-6
EXPERIMENTAL AND THEORETICAL VALUES OF ALIGNED ANGULAR
MOMENTUM AS CALCULATED BY THE CRANKED SHELL MODEL
FOR THE $h_{9/2}$ DECOUPLED BAND

A (Re)	$i_{\text{exp}} (\hbar)$	$i_{\text{cal}} (\hbar)$
175	3.05	3.57
177	3.13	3.60
179	3.41	3.97
181	3.57	4.11

quasiparticle diagrams for all four rhemium isotopes (Figures IV-7, IV-8, IV-9, and IV-10). The theoretical trajectory lies in between the 100% and 80% curves within a deviation of ± 70 KeV. Thus, we conclude from this analysis that there is a qualitative agreement between theory and experiment.

Another useful aspect of the cranking calculation is the prediction of the crossing frequency. At the critical frequency, the quasiparticle state crosses a more aligned orbit, which becomes the yrast configuration. This is the frequency of the closest approach between the conjugate trajectories on the quasiparticle energy diagrams. The crossing frequencies for the $i_{13/2}$ neutron in this isotopic series are deduced from the CSM diagrams and tabulated in Table IV-7.

It is evident from these figures that there is a consistent shift in the crossing frequency which is closely related to change of deformation and shift in electrochemical potential. The larger ϵ_2 deformation results in a larger crossing frequency. A comparison with Figure IV-3 (p. 42), which plots the experimental alignment as a

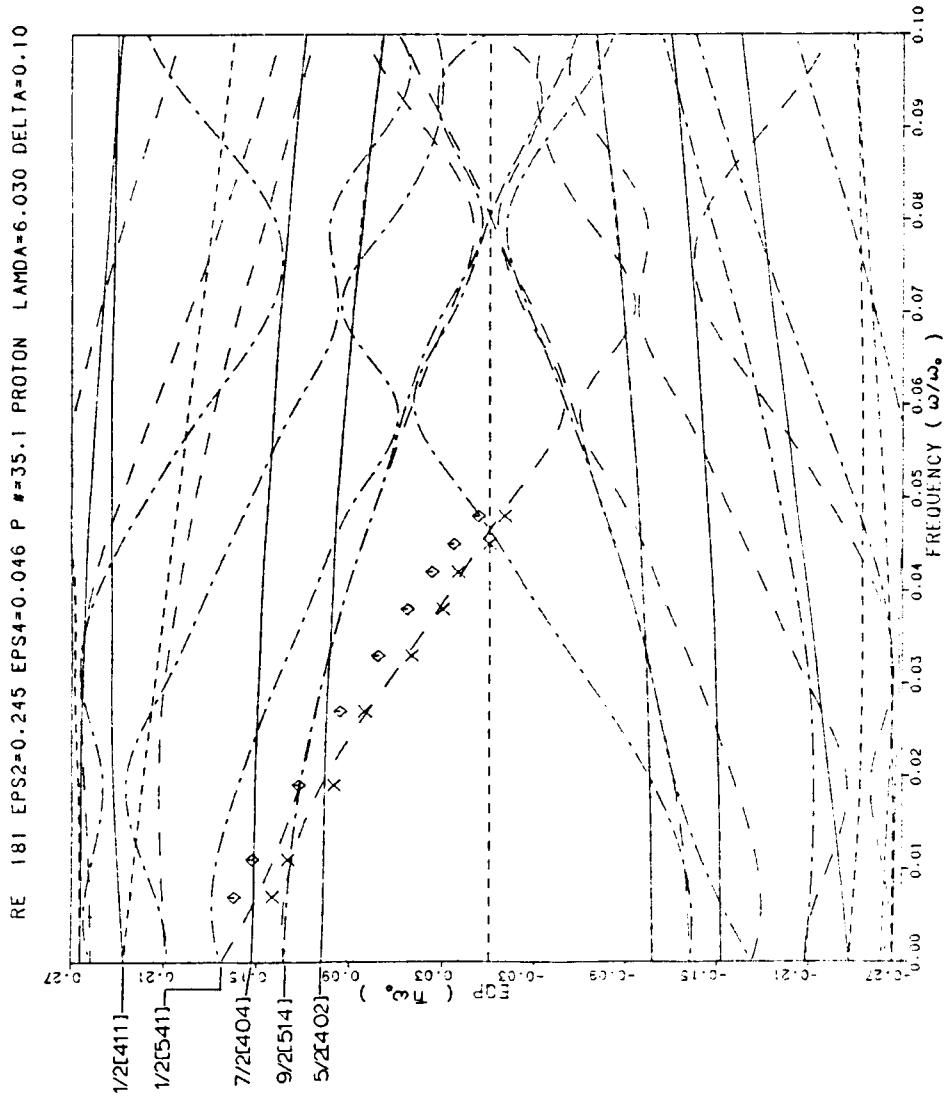


Figure IV-7. Overlay of experimental Routhian on theoretical g.p. trajectories of $h_{9/2}$ $1/2^- [541]$ proton band of ^{181}Re . A diamond corresponds to the computation using 100% odd-even mass difference. A cross corresponds to 80%. $\hbar\omega_0 = 41A^{-1/3}$ MeV.

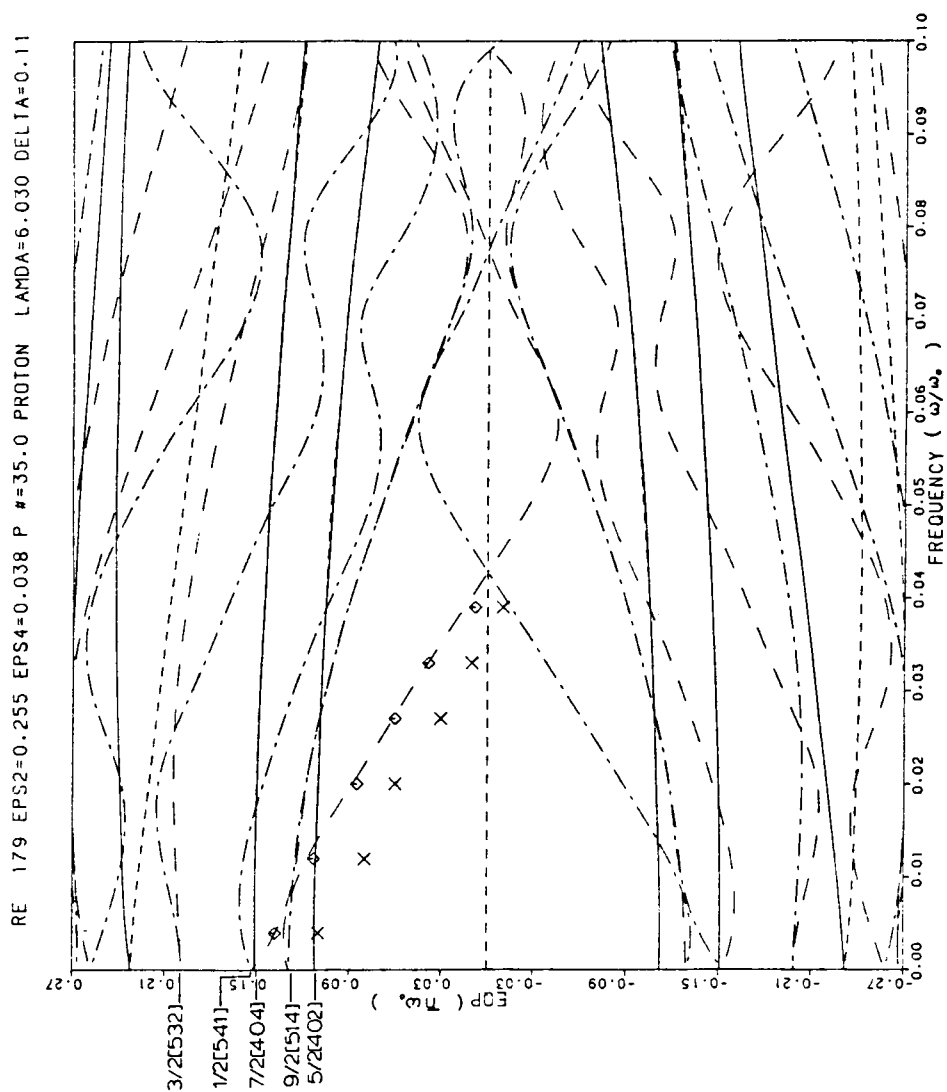


Figure IV-8. Overlay of experimental Routhian on theoretical g.p. trajectories of $h_9/2$ $1/2^- [541]$ proton band of ^{179}Re . A diamond corresponds to the computation using 100% odd-even mass difference. A cross corresponds to 80%. $\hbar\omega_0 = 41A^{-1/3}$ MeV.

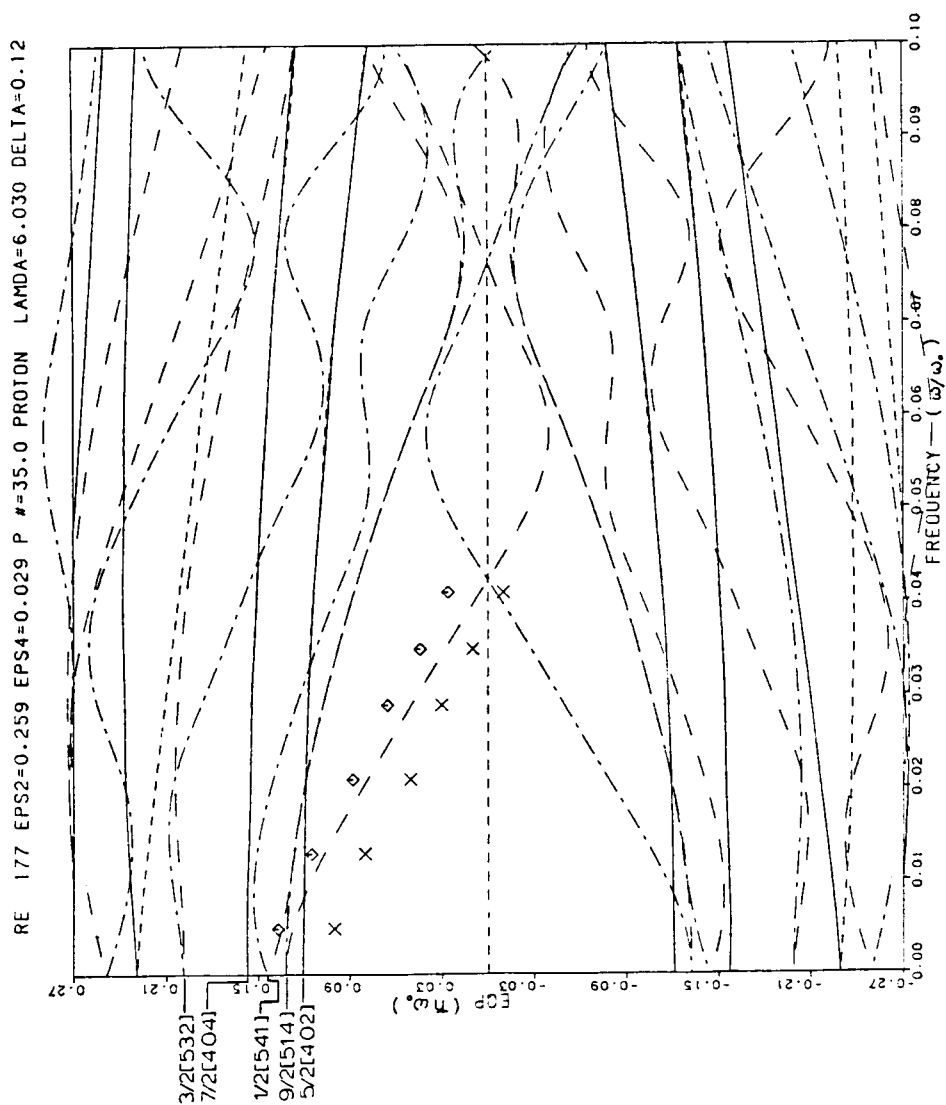


Figure IV-9. Overlay of experimental Routhian on theoretical g.p. trajectories of $h_{9/2}$ $1/2^- [541]$ proton band of 177Re. A diamond corresponds to the computation using 100% odd-even mass difference. A cross corresponds to 80%. $\hbar\omega_0 = 41A^{-1/3}$ MeV.

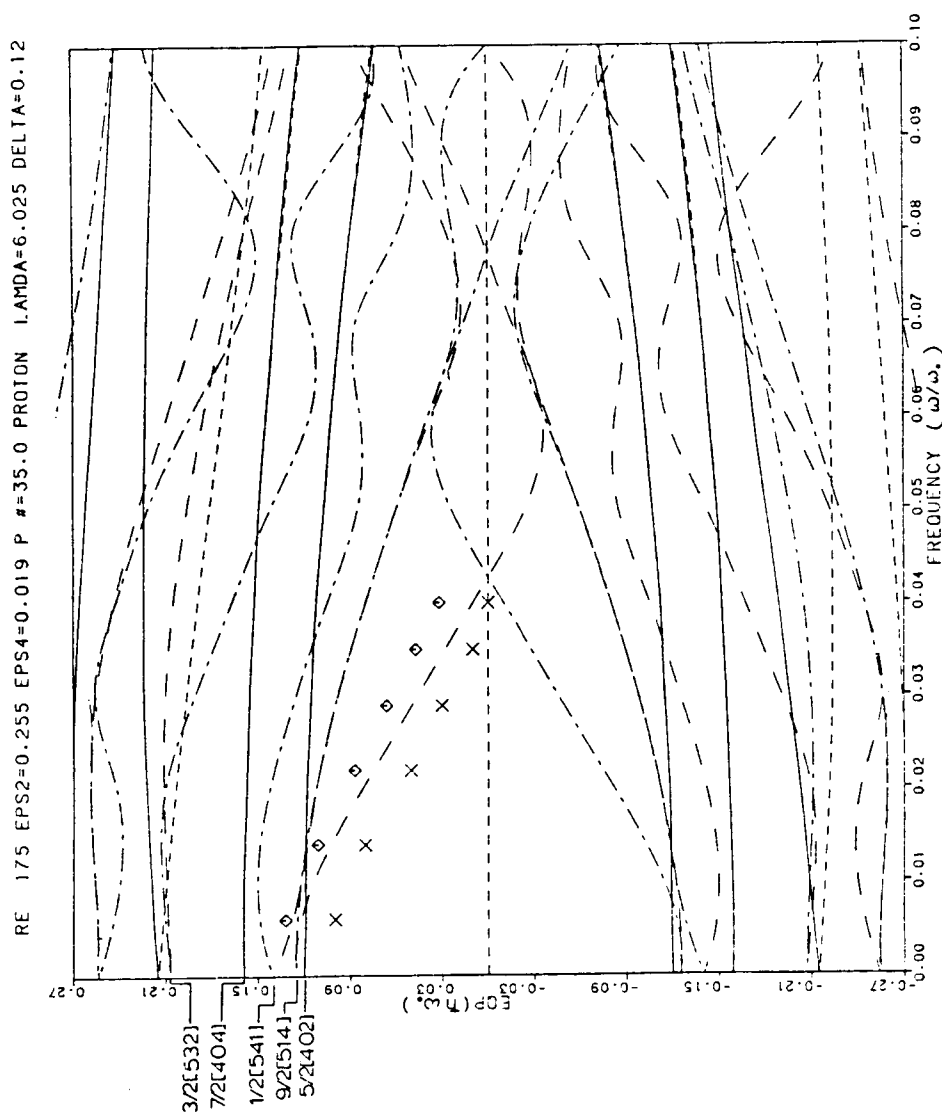


Figure IV-10. Overlay of experimental Routhian on theoretical g.p. trajectories of $h_{9/2}$ $1/2^- [541]$ proton of ^{175}Re . A diamond corresponds to the computation using 100% odd-even mass difference. A cross corresponds to 80%. $\hbar\omega_0 = 41A^{-1/3}$ MeV.

TABLE IV-7

SUMMARY OF DOMINANT FEATURES IN CSM CALCULATIONS OF
 $i_{13/2}$ NEUTRON FOR BOTH HIGH DEFORMATION AND LOW
 DEFORMATION CALCULATIONS

A	$\omega_c (\hbar\omega_0)$	ω_c (MeV)	ϵ_2	ϵ_4	λ (MeV)	$\Delta (\hbar\omega_0)$	V_{int} (MeV)
<u>$i_{13/2}$ ν High ϵ Calculation</u>							
^{175}Re	.035	0.26	.255	.019	6.58	0.11	0.14
^{177}Re	.040	0.29	.259	.029	6.63	0.11	0.09
^{179}Re	.043	0.31	.255	.038	6.67	0.11	0.23
^{181}Re	.044	0.32	.245	.046	6.73	0.11	0.16
<u>$i_{13/2}$ ν Low ϵ Calculation</u>							
^{175}Re	.031	0.23	.229	.019	6.60	0.11	0.10
^{177}Re	.035	0.26	.234	.029	6.63	0.11	0.12
^{179}Re	.038	0.28	.232	.038	6.68	0.11	0.18
^{181}Re	.040	0.29	.227	.046	6.73	0.11	0.12

Note: $\hbar\omega_0 = 41A^{-1/3}$ MeV.

function of frequency, indicates that the theoretical calculation reproduces the correct crossing frequency for ^{181}Re at approximately 0.32 MeV, while the observed ω_c for ^{182}Os is 0.26 MeV, which agrees with the projected trend that the low ϵ calculation gives a substantially lower crossing frequency.

However, for light Re, the calculated ω_c may be too low because there is no experimental sign of an upband for ^{175}Re at $\omega = 0.26$ MeV. One possible explanation for this may be that the ϵ_4 deformation used in the CSM calculation was too large. Thus, calculations were done using lower ϵ_4 values for ^{175}Re to investigate the effect of ϵ_4 deformation on crossing frequency. It does indicate a consistent increase in ω_c for lower ϵ_4 . Very little is known about the effect of the high j particles on the equilibrium ϵ_4 values, so it is difficult to know if a lowering of ϵ_4 values in order to increase the calculated ω_c values is realistic. We can only point out that this calculation gives reasonably close prediction of crossing frequency if appropriate parameters are employed.

Another comparison with experimental data can be made by observing the smoothness of the band crossing. A close examination of Figure IV-3 (p. 42) reveals that ^{177}Re may have a sharp crossing while the others may experience a relatively smoother crossing. In the vicinity of the crossing frequency (~ 0.26 MeV), the alignment plots of $^{181}, ^{179}, ^{175}\text{Re}$ show a slight upbend. On the contrary, the alignment of ^{177}Re remains relatively flat. Thus, if an upbend does occur at higher frequency, the alignment of ^{177}Re would shoot sharply upward whereas the upbend of the alignment of other isotopes will be a more gradual process. This is further confirmed by the fact that the interaction strength for

$h_{9/2}$ proton band, as given by the cranking calculation, is a minimum for ^{177}Re . Small interaction strength signifies a strong or sharp crossing while large interaction strength corresponds to a weak or smooth crossing. Moreover, the interaction minimum shifts to ^{175}Re in the low ϵ or normal proton calculation. The fact that the calculated V_{int} minimum seems to agree with indications from the data for a minimum at $A = 177$ provides some verification of our choice of larger ϵ_2 values for the $h_{9/2}$ bands in the rhenium isotopes.

Based on the above discussion, there is a systematic agreement between theory and experiment. If the general basis of the calculation is correct, it should be able to predict trends in unexplored regions within reasonable accuracy. The gain in neutron alignment due to the $i_{13/2}$ crossing for ^{181}Re , as predicted by the calculation, amounts to approximately $4\hbar$. The $h_{9/2}$ band is not measured high enough in spin to furnish any evidence that this is indeed correct. However, one can set aside this as something to look for in the future. The remarkable feature of backbending, which can be interpreted as a consequence of a sharp crossing between bands, is not observed in the light Re isotopes within the present experimental evidence. However, the projected crossing frequencies by the model can be easily reached experimentally by using slightly heavier projectiles to push further up onto the high spin states. If the future assessment of trends is correct, the cranking model may prove to be a valuable tool in planning future experiments. Also, deeper insight can be gained in the phenomenon of backbending by looking at other odd- Z nuclei to study the crossing trend in $h_{9/2}$ bands.

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