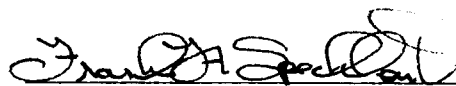


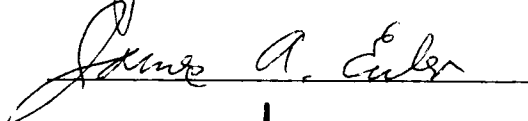
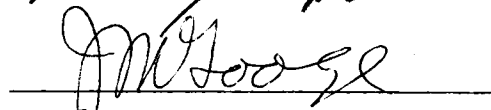
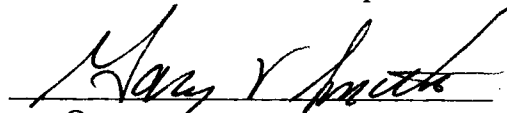
To the Graduate Council:

I am submitting herewith a dissertation written by William Ross Hamel entitled "Analysis of a Cantilever Coriolis Mass Flowmeter Concept." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Mechanical Engineering.



Frank H. Speckhart, Major Professor

We have read this dissertation
and recommend its acceptance:



Accepted for the Council:



Vice Chancellor
Graduate Studies and Research

ANALYSIS OF A CANTILEVER CORIOLIS
MASS FLOWMETER CONCEPT

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

William Ross Hamel

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During my pursuit of a doctorate, I lead the double life of a student and a full-time staff member of the Oak Ridge National Laboratory (ORNL), I am indebted to the management of the Oak Ridge National Laboratory for their support, tolerance, and encouragement during this time. The mutual interest that Mr. C. A. Mossman and Dr. F. R. Mynatt of the Instrumentation and Controls Division and Mr. D. B. Trauger have shared in my academic goals has been essential to my success. I must also acknowledge the help that Mr. Richard F. Spille, a truly superior microprocessor expert, gave me in setting up the experimental data acquisition system. His assistance and counsel greatly reduced the amount of time necessary to make all the hardware and software work correctly.

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Because I was working full-time during all of this, it was necessary for me to divert a great deal of time from my beloved family. My wife, Libby; my son, Luke; and my daughter Alison were all supportive during this time to an extent far beyond reasonableness. Because each of them realized that this was something really important to me, they were all willing to sacrifice to my benefit and to maintain a home environment which actually made my lot easier. I may never know if I made the correct tradeoff with respect to my family, but rest assured that I will attempt to repay each of them in the years to come.

Finally, I believe that all knowledge and wisdom comes from God (Col. 2:3). Consequently, I owe thanks to the Lord for giving me the physical strength, the mental faculties, and opportunity to enjoy this accomplishment and the gratification that it has given me.

ABSTRACT

A new flowmeter concept which simultaneously measures density, mean velocity, and mass flowrate has been conceived, analyzed, and experimentally verified. The concept is based on the dissipative effect of Coriolis inertial forces present in a vibrating hydroelastic cantilever. The concept also demonstrates: (1) the use of dynamic governing equations as a basis of measurement and, (2) the value of microprocessor algorithm implementation.

The cantilever Coriolis flowmeter concept was analyzed using literature associated with hydroelastic system stability as a basis. The governing equation is a fourth-order non-self-adjoint partial differential equation for which no closed-form solution is known. The boundary value problem associated with the cantilever geometry was solved numerically. Approximate single-mode analytical solutions were obtained using perturbation theory and a related simplified technique. In the range of interest, all three solutions agreed very well.

The theoretical analysis showed that the dynamic response of the cantilever flow sensor with static deflection motion excitation is essentially a second-order damped response in which the decay constant and frequency of vibration are dependent on the flowing fluid density and mean velocity. For this form of motion excitation, the cantilever Coriolis flowmeter operates as a sampled-data sensor. The analytical results were used to develop algorithms which transform the recorded transient waveform decay and frequency characteristics into fluid density, mean velocity, and mass flowrate.

A proof of principle experiment was performed to verify the analytical results. Using a microprocessor-based data acquisition and computation system three cantilever flow sensors were evaluated over a range of 0 to 1000 kg/h using a simple water circulation loop. Each cantilever Coriolis flowmeter was compared against an industrial thermal flowmeter. Thin wall tubing sensors worked best inspite of difficulties with multimodal response characteristics. The mass flow measurement tracking between the cantilever Coriolis and the thermal flowmeters was good with relative linearity in the range +5% to +10%. Density measurement results between the Coriolis density calculation and the known water density were +1% to +5%. Follow-on work will include prototypic experimentation and evaluation of several enhancements.

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NOTATION

a_x, a_y	Acceleration in x and y directions.
a_1, a_2, a_3	Cantilever Coriolis flowmeter calibration constants.
A	Internal cross-sectional area of tube.
A_o	Material cross-sectional area of tube.
c_{eq}	Equivalent linear damping coefficient.
c	External damping coefficient per unit length.
$\bar{c}$	Nondimensional external damping coefficient.
d_i, d_o	Inside and outside diameters of a circular tube.
$e(t)$	Deflection sensor output.
E	Modulus of elasticity.
F	Lateral force per unit length between the tube and fluid.
F_j	Perturbation expansion functions, $j = 0, 1, \dots$
I	Moment of inertia of tube.
K_e	Deflection measurement channel gain.
K_ρ	ρ, β relationship constant for circular tubes.
K_m	Mass-flow rate calibration constant.
K_k	Orthonormalization constant.
K_j	Orthogonality constants for $X(x)$, $j = x, x', x''$
L	Length of the tube.
M, m	Masses of fluid and tube, respectively, per unit length.
$\dot{m}$	Fluid mass-flow rate.
$\underline{M}$	Tube bending moment.
P_j	Waveform j-th peak value.

$P(t)$	Damping power.
p	Internal fluid pressure.
q	Shear stress between fluid and tube.
Q	Transverse shear force due to tube bending.
R	Real part.
R_{ω}	Ratio of data sampling frequency to beam fundamental natural frequency.
S	Internal perimeter of tube.
t	Time.
$\bar{t}, t^*$	Stretched time variables.
t_j	Waveform j -th zero-crossing.
T_k	Waveform k -th half-period value.
$\underline{T}$	Kinetic energy.
$\tilde{T}$	Time scale factor.
T	Longitudinal tension in tube.
$T(t)$	Time function in product solution.
U	Mean flow velocity in tube.
u	Nondimensional flow velocity.
$\underline{V}$	Potential energy.
$W_d(t)$	Work done by damping.
x	Longitudinal coordinate.
X	Spatial eigenfunction, or space function in product solution.
y	Lateral deflection of the tube.
y_s	Static deflection of the tube.
α_j	Spatial eigenvalues, $j = 1, \dots, 4$.

γ	Nondimensional external damping.
$\bar{\omega}$	Frequency of oscillation for serendipity solution.
ω	Time eigenvalue.
ω_s	Data sampling frequency.
ψ	Slow time scale expansion variable, $j = 0, 2, \dots$
τ_0, τ	Nondimensional decay time constant.
ρ, ρ_T	Fluid density; tube material density.
$\sigma_{\max}$	Maximum flexure stress.

INTRODUCTION

Flow-rate measurement in realistic plant operating conditions continues as one of the most difficult, yet important, tasks facing process instrumentation technology. Accurate and reliable measurements of flow rate have always been an essential factor in process operations for functions such as material balance calculations and process control.

Two major trends are accelerating the need for effective flow-rate measurements in the process industry. First, rising energy costs have substantially increased the interest in precise and optimum energy management in production operations; more accurate metering of heating/cooling fuels and fluids is an integral part of such goals. Second, the next generation of petrochemical process systems is certain to use more sophisticated chemical component-level control [1]. On-line control of specific chemical components in streams will be accomplished through new advances in analytical instrumentation that permit the concentration of specific components to be measured readily. With this shift to a "component level" perspective in process unit operations, the routine applications and requirements for flow measurements will also increase.

The state of the art in process flow-rate measurements is described concisely by Liptak [2]. Mass flow rates are measured by either of two methods: (1) coupling independent volumetric flow rate and fluid density measurements, or (2) transducer schemes which measure mass flow rate implicitly. The first case is the most prevalent today. In both methods,

applications usually involve undesirable performance compromises in terms of accuracy, measurement range, or other factors. The standard techniques usually include significant dependence upon other process variables such as temperature, pressure, and viscosity [2, pp. 416-417].

One concludes that there is considerable opportunity for flow measurement performance improvement in process industry applications. This basic need represents the fundamental practical motivation for the research reported here. In particular, techniques which use Coriolis inertial accelerations to measure mass flow rate and density are studied. These techniques fall into the general class of measurement concepts involving flowing stream energy addition rather than extraction and as such have greater potential for low Reynolds Number applications.

The basic purpose of this present work was to conceive, analyze, and demonstrate the feasibility of a new method of fluid flow measurement based upon the Coriolis measurement principle.

A secondary purpose was to show the important role that available microprocessor technology can play in the implementation of complex measurement algorithms. To this end, special interest was given to transient techniques that are more difficult in terms of realtime data acquisition and realtime digital signal processing. Coriolis techniques inherently involve the type of algorithms which require high-performance digital implementation.

Another secondary purpose was to demonstrate the value of the dynamics approach in measurements research. Virtually all process measurement concepts are based upon the steady-state characteristics of

physical phenomena (e.g., the relationship of differential head to velocity from Bernoulli's Equation). The simple algebraic relationships describing steady-state transducer characteristics have historically been necessary to facilitate analog electronic implementations. Usually, considerably greater information is contained in a transducer's dynamic response characteristics. With microprocessor technology and advanced signal processing theories, it is now possible to consider dynamic governing equations as the basis of measurements. Through this type of approach, greater sensitivities and ranges may be achievable for many measurements. The Coriolis concept of flow measurement, particularly in the transient motion excitation case, falls into this class of dynamic measurements.

The remainder of this dissertation describes research results for a new type of Coriolis flowmeter, covering background information, basic modeling and analysis of simple beam concepts, mathematical formulations necessary for flow rate and density measurements, presentation of experimental proof of principle results, and a statement of conclusions and recommendations.

CHAPTER 1

BACKGROUND

The History of Coriolis Flow Measurements

The basic principle of Coriolis flow measurements is to subject flowing fluids, usually single or multiphase liquid/solid streams, to an angular velocity field. With the proper piping geometry and orientation with respect to the angular velocity, the fluid particles experience the " $2\omega \times V$ " Coriolis inertial acceleration. In this way, inertial forces that are a function of the fluid mass and velocity act upon the piping system structure to produce some form of a measurable effect, usually deflection. The primary conceptual degrees of freedom for Coriolis flow measurement schemes are the angular velocity field properties and the geometry of the fluid conveying system. Angular velocities can be generated by either pure rotation, or harmonic vibration, of a sense element. Piping geometries with different flow measurement properties are within the domain of individual creativity and innovation.

Li [3] appears to be the first researcher to pursue Coriolis flow measurement in depth. He developed a simple theory for a geometry involving a "T-shaped" sensing element with pure rotation as the excitation. As shown in Fig. 1, the fluid flows out the legs of the "T," where the angular velocity and fluid velocity are perpendicular. Under these conditions, the Coriolis accelerations produce a twisting moment relative to the axis of rotation. This twisting moment is

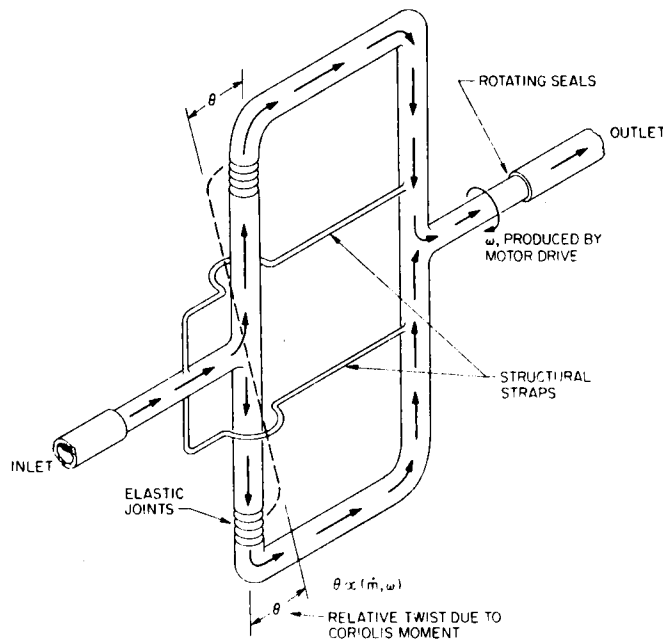


Fig. 1. The rotating "T-shape" configuration.

proportional to mass flow rate and is measured with torsion-sensing strain gages. The practical limitations of this approach are: (1) the necessity of rotating seals between the stationary process piping and the rotating sense element, and (2) the necessity of slip-ring commutation of the low-level strain gage signals.

With exception of a flowmeter based upon angular momentum exchange [4, 5, 6], no other dynamic approaches to fluid flow measurement, particularly Coriolis concepts, are reported in the domestic literature. Perhaps this is related to practical limitations of the Li "T-shape" meter and the generally crude state of the art of electronics in the 1950s.

During this time, however, the Russians were quite active in the development of Coriolis flow measurement techniques as evidenced in the

translation of a book by Katys [7]. Li's work in the rotating "T-shape" was extended considerably. Many different variations of this basic geometry are reported and include the clever use of drive motor schemes that reduce sealing and torque measurement complications. Katys [7] also presents an innovative variation of the rotating "T-shape" in which harmonic vibration is used rather than pure rotation. Bye [8] reports actual experimentation with this type of vibratory design. While these designs eliminate the need for rotating seals and commutators, they require bellows-type elastic couplings to achieve high measurement sensitivities. Very simple electrical circuits are used. Katys [7] is the most complete reference found regarding Coriolis flow measurements.

Since the 1950-1960 period when the earlier work on Coriolis flow measurement techniques was performed, revolutionary advances in microelectronics have been accomplished which have literally rocked every area of modern technology. The impact of electronics advances has been particularly great in engineering and the physical sciences. Microelectronics have in effect rendered the calculational power of the general-purpose digital computer to the lowest levels of application both technically and economically. Microprocessor technology has been significant in the field of instrumentation, where its ability to perform complex calculations in realtime has made many previously impossible measurement concepts possible. The functional capabilities of integrated circuit devices have been a strong inverse function of physical size and component density (e.g., transistors per square millimeter), while device unit costs have decreased or remained constant. As shown in Fig. 2,

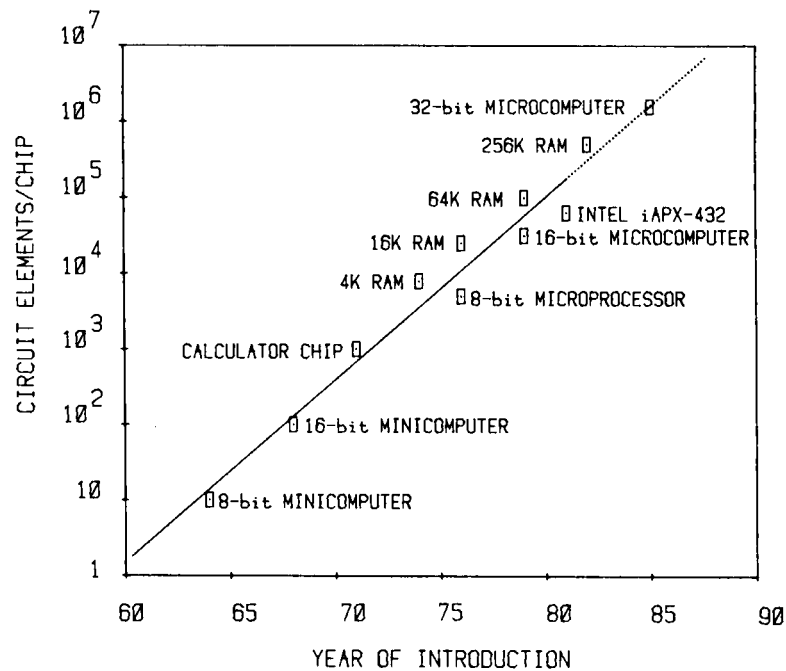


Fig. 2. Trends in microelectronics technology.

component densities continue to increase steadily with the likelihood that very large scale integrated circuit research will provide another quantum jump in density within the next 5 years. It is only reasonable to expect that microelectronics technology will continue to be the most dominant forcing function in instrumentation research and development.

Relative to many of the standard fluid flow measurement techniques, Coriolis concepts involve greater complexity in both the sensing element mechanical design and in the calculational requirements necessary to transform the measured variables into the desired flow variables. A recently introduced industrial flowmeter based on the Coriolis principle demonstrates the dramatic effect that today's microelectronics can play. Micromotion, Inc., introduced their vibratory Coriolis mass flowmeter in 1976, and Plache [9] described its principles of operation. A very

attractive feature of the design is a simple U-tube sensing element configuration which is unobtrusive and does not require any type of moving parts seal.

The unobtrusive feature of this concept is particularly attractive for two-phase flow applications such as that reported by Baucum [10]. As shown in Fig. 3, the plane of the U-tube forms a beam that is vibrated at constant amplitude in its first mode. For nonzero fluid velocity, the Coriolis forces superimpose an alternating twist action on the tube because the fluid velocity directions in the entrance and exit legs are opposing. The twisting moment produces a torsional deflection that is proportional to mass flow rate, and the frequency of oscillation is proportional to fluid density. Consequently, this single meter measures

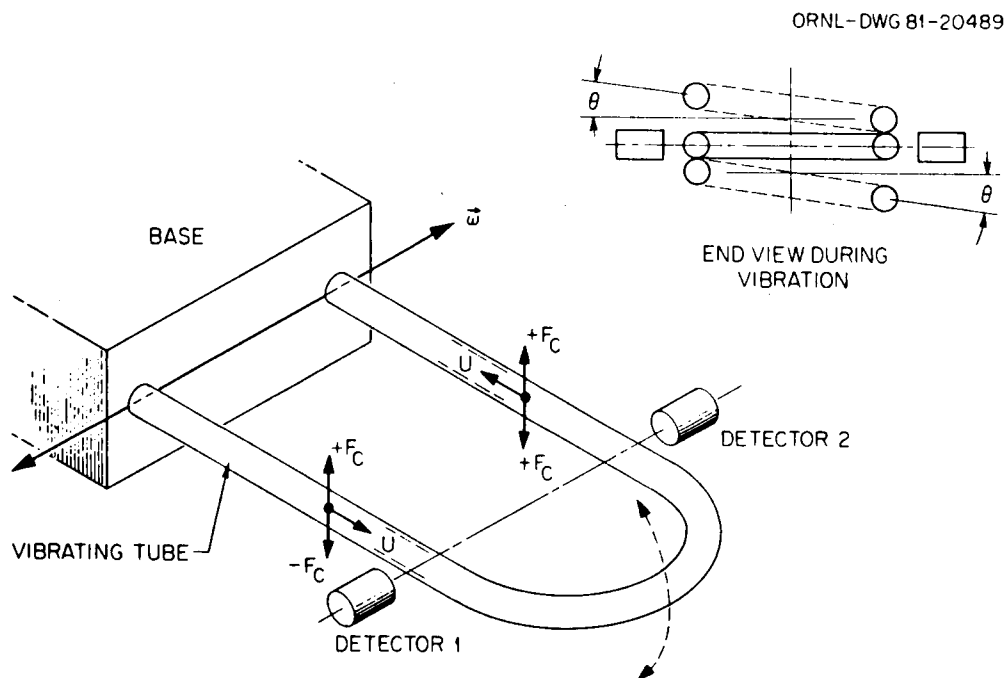


Fig. 3. Micromotion Coriolis flowmeter schematic.

the three flow variables of interest: mass flow rate, density, and mean velocity (or volumetric flow rate). Two aspects of the Micromotion flowmeter were made possible as a result of integrated circuitry. The torsional deflection of the U-tube is measured using optical detectors and digital timing circuits. Also, the beam excitation system is actually a complete servo-control loop with an electromagnetic actuator providing noncontact forcing. Neither of these two basic functions could have been reasonably implemented without microelectronics.

The Micromotion flowmeter certainly represents an important advancement. As is the case with most instrumentation development, it appears to have been developed with a simple theoretical understanding and empirical iteration. Progress has been very slow in the development of Coriolis flow measurements. Clearly, an accurate theoretical analysis of Coriolis concepts and parametric relationships would accelerate the assessment of their potential in real applications.

Related Work in Hydroelastic Systems

Coriolis flow measurements are possible by virtue of the dynamic interaction between the fluid motion with respect to conveying tube and the dynamic motion of the tube itself. The vibration and stability of fluid conveying systems has been an active research topic for the past fifteen years and provides a good foundation for the analytical investigation of the Coriolis flow measurement concept. Hydroelastic systems are of interest because it has been observed, both theoretically and experimentally, that increases in fluid velocity result in natural

frequency changes which can promote structural instability. Centripetal and Coriolis inertia forces are exerted on the fluid containing structure as the fluid is accelerated around the curved path resulting from deflection. Simple beam mathematical models are of academic interest because they constitute non-self-adjoint fourth-order boundary value problems which are quite difficult to solve. Published results of hydroelastic systems research encompass studies of the effects of boundary conditions, tube geometries, fluid velocity and density, and pressure. Much of the work involves numerical solution methodologies for complex geometries.

Earliest researchers, such as Ashley and Haviland [11] and Housner [12], were concerned with vibrational problems in oil pipeline installations. They modeled these systems as straight, fixed-end tubes behaving as beams. Niordson [13] presented an elegant treatment based upon shell theory. For the case of simple supports, he achieved closed-form analytical solutions for two limiting cases of parameter values and numerical results for the region between. Long [14] solved the fourth-order boundary value problem through a power series solution for small fluid velocities, and Handelman [15] used perturbation techniques to approximate frequency and decay properties of the simple hydroelastic beam operator. Benjamin [16] produced a notably thorough Lagrangian approach to the analysis of a system of fluid conveying tubes connected by rotating joints for various end conditions. He extended Lagrange's equation for rigid bodies to properly include the energy of the flowing fluid. This extension of Lagrangian dynamics was used to show that the

hydroelastic vibration of articulated tube systems is conservative for fixed-end conditions and nonconservative for cantilever conditions. Gregory and Paidoussis [17] performed an in-depth analytical and experimental study of the stability characteristics of continuous cantilevers as an extension of Benjamin's work [16]. Good agreement between numerical predictions and experimental results for mode shapes and critical fluid velocities was obtained. Their numerical solution technique involved assumption of a product solution in separated time and spatial variables and solution of the resulting nonlinear complex characteristic and boundary condition determinant equations. This work is recommended as readable and thorough introduction to the hydroelastic systems literature. Hill and Swanson [18] extended Gregory's work and showed that additional discrete masses located along the beam tend to destabilize cantilever fluid conveying tubes.

Wu and Raju [19] considered the case of a single discrete mass. Herrmann [20], Nemat-Nasser [21, 22], and Beal [23] are among authors who have studied this class of systems from a more general mathematical viewpoint under the topic of nonconservative follower-force elastic system stability. Chen [24-29] has published many papers on hydroelastic stability for both straight and curved tubes. In one article [28], he uses a work principle to argue that the cantilever case is generally nonconservative. An analytical solution through the formulation of a companion adjoint operator for the cantilever case is given in [24], but only after the troublesome Coriolis cross-derivative term was discarded for small flow velocities. The longitudinal distribution

of phase and mode shapes for fixed-end conditions, including internal pressure effects, have been considered by Naguleswaran and Williams [30] and Stein [31, 32].

Svetlitskii [33] provides an elegant development of the governing equation for curved rectilinear tubes with solution via Galerkin's Method. Considerable effort has been directed toward techniques for treating planar curved tubes. Springfield [34] analyzed the in-plane and out-of-plane hydroelastic vibration of unpressurized circular arcs with clamped-clamped and clamped-free end conditions. Hopkins [35] studied stability using periodic perturbations. Davis [36] included, in a continuing sequence, the effects of initial in-plane forces.

More recent works continue to increase the analytical sophistication of this area of elastic systems. Singh and Mallik [37] used a parametric excitation approach and Bolotin's Method to consider instability with periodically varying supports. Shayo and Ellen [38] incorporate downstream flow models in their stability analysis.

In summary, a large volume of effort is represented in the papers which have been reviewed [11-38]. The majority of this work has been directed toward stability characterization with respect to fluid flow velocity. The detailed results have not been of direct value in the research being reported here since instabilities generally occur at extremely high fluid velocities far beyond the normal practice in piping design. These works have, however, been very helpful in developing the governing equation for the transverse vibration, defining scaling, and

providing insight into the attributes and possibilities for analytical solution for the Coriolis flow measurement case. These references adequately disclose the inherent theoretical complexity associated with the non-self-adjoint governing equation. The Coriolis inertial acceleration term inexorably couples the spatial and temporal variables in a cross-derivative term. This cross-derivative renders the governing equation and boundary conditions into what is termed a non-self-adjoint boundary value problem even though the differential operator is linear with constant coefficients for the case of steady flow. According to linear differential operator theory [39], the closed and orderly nature of the eigenspace can only be assured when the operator of interest meets the self-adjointness conditions. In the case of non-self-adjoint operators, there is little hope that the classical solution techniques for partial differential equations such as separation of variables or integral transforms will be successful. For application in flow measurement, reasonably simple and accurate expressions in the region of interest which are amenable to implementation as realtime algorithms are essential. These earlier works in hydroelastic stability have served as valuable guideposts in the pursuit of the needed analytical expressions.

CHAPTER 2

THEORETICAL ANALYSIS

The hydroelastic systems literature reviewed in Chapter 1 vividly defines the inherent mathematical complexity of this class of systems. Maximum practical use of idealizations leads to extremely difficult fourth-order boundary value problems. Closed-form solutions are difficult, or impossible, for simple boundary conditions and forcing functions. It is doubtful, under these conditions, that one could obtain compact and useable analytical solutions for any of the complex flowmeter geometries discussed by Katys [7], or such as the Micromotion Coriolis flowmeter [9]. With this as a reference, the prognosis for achieving analytical expressions appropriate for realtime flow measurement algorithms seemed dismal, at best.

In the interest of pressing on, the only recourse was to impose as much simplicity into the problem as possible with the hope that this would result in tractable mathematical models. Accordingly, it was decided to consider the most simple possible geometry — straight fluid conveying tubes with simple end conditions. This restriction assured that only planar vibrations would be introduced and was encouraged by the results of Benjamin [16] and Chen [28] who analytically predicted that cantilever end conditions, at least for the geometries they were studying, result in nonconservative system behavior with respect to the fluid flow. The thought was that significant energy dissipation should produce measurable dynamic response decay properties when transient excitation is used.

This chapter is devoted to various aspects of the theoretical analysis of simple hydroelastic beams from the perspective of utilizing the Coriolis principle to measure fluid flows.

Governing Equations for Simple Beams

This derivation of the governing equation for simple hydroelastic beams is patterned after that given by Gregory and Paidoussis [17] with the addition of external damping in a fashion similar to Nemat-Sasser [21]. Figure 4 is a schematic of a simple cantilever and defines coordinate axes to be used. The x-axis coincides with the centerline for the tube and forms the plane of the beam transverse vibrations with the y-axis.

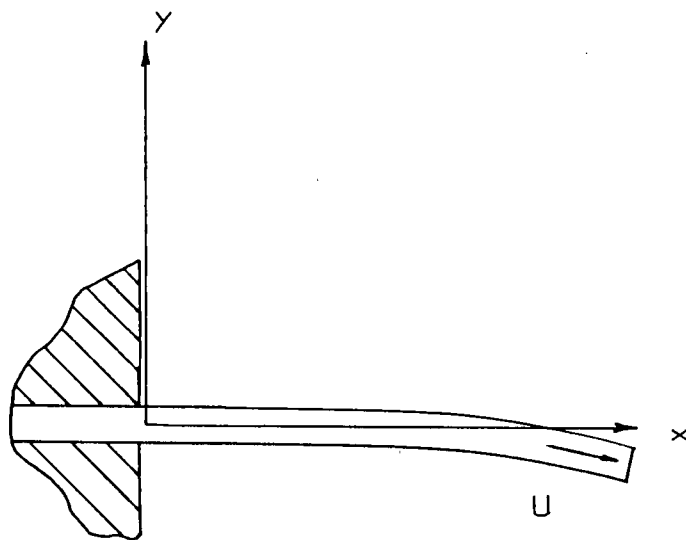


Fig. 4. Schematic diagram of a simple cantilever.

The tubular beam is of length, L , and flexural rigidity, EI , where E is the modulus of elasticity of the tube material and I is the tube cross section area moment of inertia with respect to its neutral axis. The tube, also, has a mass-per-unit length, m . The fluid being conveyed is assumed to be incompressible with a density, ρ , such that its mass-per-unit length of tube is M . The fluid is assumed to be flowing with a steady mean velocity, U . If the beam vibrates in a media of finite density, external damping effects will be present. To describe external damping, the beam is assumed to have a constant damping coefficient per unit length, c . The damping coefficient, c , is assumed to have the proper units for the particular damping model to be used.

A finite elemental volume of the simple hydroelastic beam is actually comprised of two parts: (1) an elemental volume of the fluid contained in the tube section, and (2) an elemental section of the tube itself. Free-body diagrams of these elements are shown in Fig. 5. The internal cross-sectional area, A , available to the fluid flow is assumed to be constant along the length of the beam, and the corresponding internal perimeter is S .

Niordson [13] shows that if the dimensions of the cross section of the tube are small compared with the wavelength of any disturbances to its centerline during vibration, the accelerations of fluid particles along and perpendicular with respect to the x -axis are

$$a_x = 0, \quad (1)$$

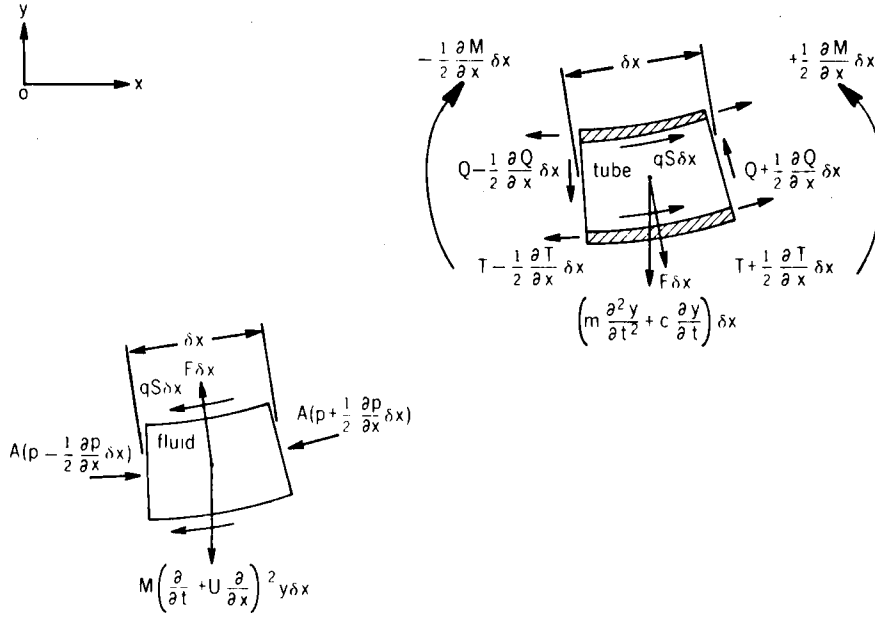


Fig. 5. Free-body diagrams.

and

$$a_y = \left[\left(\frac{\partial}{\partial t} \right) + U \left(\frac{\partial}{\partial x} \right) \right]^2 y \quad (2)$$

to the first order in the small deflections, y . The inertial accelerations of the fluid element are defined using Eqs. (1) and (2). Force equilibrium in the x and y directions of the fluid element requires that

$$\sum F_x = -A \frac{\partial p}{\partial x} - qS = 0, \quad (3)$$

and

$$\sum F_y = F - pA \frac{\partial^2 y}{\partial x^2} - M \left[\left(\frac{\partial}{\partial t} \right) + U \left(\frac{\partial}{\partial x} \right) \right]^2 y = 0, \quad (4)$$

where q denotes the longitudinal shear stress between the fluid and the tube internal surface and F denotes the force per unit length acting between the fluid and the tube. The tacit assumption here is that the

incompressible fluid "slides" along the tube behaving as if it were a solid material in contact with the tube. The normal force acting between the two is accounted for by the reaction force F . The inertial effects of the fluid defined by Eq. (2) are communicated to the tube through F .

The equation of motion of the tube element is defined in the same way:

$$\sum F_x = \frac{\partial T}{\partial x} + qS - Q \frac{\partial^2 y}{\partial x^2} = 0 , \quad (5)$$

and

$$\sum F_y = \frac{\partial Q}{\partial x} + T \frac{\partial^2 y}{\partial x^2} - F - m \frac{\partial^2 y}{\partial t^2} - cf\left(\frac{\partial y}{\partial t}\right) = 0 , \quad (6)$$

where T is the longitudinal tension, and Q is the transverse shear force in the tube resulting from the flexure deflection. Equations (3)-(6) are obtained through a limit analysis of the finite fluid and tube elements shown in Fig. 5 [35]. For negligible rotary inertia, the shear force is a function of the bending moment, $\underline{M}$, acting on the beam according to

$$\frac{\partial \underline{M}}{\partial x} = Q , \quad (7)$$

and $\underline{M}$ is related to the lateral deflection by

$$\underline{M} = EI \frac{\partial^2 y}{\partial x^2} . \quad (8)$$

Combining Eqs. (5), (7) and (8) gives

$$\frac{\partial T}{\partial x} + qS - EI \left(\frac{\partial^3 y}{\partial x^3} \right) \left(\frac{\partial^2 y}{\partial x^2} \right) = 0 . \quad (9)$$

The last term of Eq. (9) is of second order in the small deflection y and is therefore negligible. With this simplification, Eqs. (4), (6), (7), and (8) can be combined to describe the transverse behavior as

$$EI \frac{\partial^4 y}{\partial x^4} + (pA - T) \frac{\partial^2 y}{\partial x^2} + M \left[\left(\frac{\partial}{\partial t} \right) + U \left(\frac{\partial}{\partial x} \right) \right]^2 y + cf \left(\frac{\partial y}{\partial t} \right) + m \frac{\partial^2 y}{\partial t^2} = 0 . \quad (10)$$

With respect to the longitudinal direction, Eqs. (3) and (9) combine to require that

$$\frac{\partial(pA - T)}{\partial x} = 0 , \quad (11)$$

which is to say that $(pA - T)$ is independent of x . For cantilever case, at the free end

$$(pA - T) = 0 . \quad (12)$$

Equations (11) and (12) together necessitate that Eq. (12) applies for all x .

Substituting Eq. (12) into Eq. (10) and simplifying gives the basic governing equation for the transverse vibration of a simple hydroelastic beam as

$$\underbrace{EI y_{xxxx}}_{\text{flexure}} + \underbrace{MU^2 y_{xx}}_{\text{centripetal}} + \underbrace{2MU y_{xt}}_{\text{Coriolis}} + \underbrace{cf(y_t)}_{\text{external damping}} + \underbrace{(M + m)y_{tt}}_{\text{lateral inertia}} = 0 , \quad (13)$$

where the compact notation for partial derivatives has been introduced to simplify the bookkeeping. If the external damping model used is linear,

equation (13) will be linear with constant coefficients (for steady flow), fourth-order in space, and second-order in time. Discussion of the boundary conditions that go with Eq. (13) will be deferred until the next section, where various end conditions will be used to classify its energy characteristics.

The following change is used momentarily to further simplify notation after scaling factors are imposed:

$$y_{\tilde{x}\tilde{x}\tilde{x}\tilde{x}} + \frac{MU^2}{EI} y_{\tilde{x}\tilde{x}} + \frac{2MU}{EI} y_{\tilde{x}\tilde{t}} + \frac{c}{EI} f(y_{\tilde{t}}) + \frac{(M + m)}{EI} y_{\tilde{t}\tilde{t}} = 0 . \quad (14)$$

An important step in the analysis is to properly scale the system variables and parameters into a nondimensional space where in a mathematical sense things are best behaved. First, the variable x is normalized with respect to the beam length by, $x = \tilde{x}/L$, to give

$$y_{xxxx} + \frac{MU^2 L^2}{EI} y_{xx} + \frac{2MUL^3}{EI} y_{x\tilde{t}} + \frac{cL^4}{EI} f(y_{\tilde{t}}) + \frac{(M + m)L^4}{EI} y_{\tilde{t}\tilde{t}} = 0 . \quad (15)$$

Note that for the case of linear external damping Eq. (14) is linear, and nothing is to be gained by scaling y .

For application to flow measurement, it is anticipated that the mass flow rate of the fluid will be relatively small such that the centripetal and Coriolis inertial force terms will be small in comparison to the flexure and transverse inertia. This is to say that the beam will behave essentially as a nonhydroelastic beam with a "small" additive hydroelastic effect. With these assumptions, time scaling which makes the flexure and transverse inertia terms of the same order is desirable.

The correct scale factor is obtained by equating the coefficients of these two terms and the result is

$$\tilde{T} = \sqrt{\frac{(M + m)}{EI}} L^2 , \quad (16)$$

where $\tilde{t} = \tilde{T}t$. Incorporating Eq. (16) into Eq. (15) gives, after some manipulation,

$$y_{xxxx} + \frac{MU^2 L^2}{EI} y_{xx} + 2\sqrt{\frac{M}{M+m}} \sqrt{\frac{ML^2}{EI}} U y_{xt} + \sqrt{\frac{M}{M+m}} \frac{cL^2}{\sqrt{MEI}} y_t + y_{tt} = 0 , \quad (17)$$

where for the moment it has been assumed that the external damping is viscous. The coefficients and variables of Eq. (17) are now nondimensionalized, but the definition of a nondimensional velocity and a mass density ratio of

$$u = \sqrt{\frac{ML^2}{EI}} U \quad \text{and} \quad \beta = \frac{M}{M+m} \quad (18)$$

provides additional compactness. The resulting equation is

$$y_{xxxx} + u^2 y_{xx} + 2\sqrt{\beta} u y_{xt} + \sqrt{\beta} \gamma y_t + y_{tt} = 0 , \quad (19)$$

where the external viscous damping coefficient has also been nondimensionalized as $\gamma = cL/MU$. Equation (19) is the final form of the governing equation as it will be used here and is the same basic form used by most researchers [11-38].

In the literature reviewed in Chapter 1, only linear, or viscous, external damping mechanisms have been considered. Gregory and

Paidoussis [17, pp. 536-540] modified their initial analysis to include a small external viscous damping term by assuming that for small deflections the external flow around the tube would be laminar. They also admitted that there was a strong incentive to avoid the introduction of nonlinearity into Eq. (19). Their analysis showed that the additional damping produced a destabilizing effect for specific β -values ($\beta > 0.3$).

Baker and Allen [40] performed a detailed analysis of the possible linear and nonlinear damping mechanisms of beams vibrating in air. Their results showed that "pressure drag" of the form $f(y_t^2)$, matched experimental data better than laminar "deformation drag" of the form $f(y_t)$. Damping as a combination of internal material friction and "deformation drag" was studied for the case of very small amplitudes. Because of measurement sensitivity limitations, these experiments were inconclusive regarding the relative importance of the two mechanisms.

In the research being reported here, preliminary experiments showed that a significant amount of beam damping was present for zero flow velocity. As a result, there was no alternative but to include some form of external damping in the model. Baker and Allen [40] showed rather conclusively that square-law damping should have the best chance of working well. Square-law damping is difficult analytically because the direction of the damping force must be communicated through an absolute value function of (y_t) . Consequently, to use square-law damping would require much greater complication in an already complex boundary value problem. Following the principle of maximum simplicity established earlier, it was decided to use linear viscous damping as

in Eq. (19) arbitrarily by assuming that it would work well enough for the small vibration amplitudes required in the flow measurements case.

The ultimate performance of specific flow measurement concepts will depend upon the quality of the mathematical model used in developing measurement algorithms. Of course, any system model is only as good as the validity of the assumptions made during its development. To complete this section and in the interest of clarity, the assumptions which are implicit in Eq. (19) are summarized in Table 1.

The most critical assumption is that the secondary flow effects are insignificant compared to the mean flow. The presence of random movements of the flow relative to the principal motion of the tube would be certain to cause measuring errors. Katys [7, p. 50] reports that an experimental investigation of this issue showed that for a given length/diameter ratio there is on the average a mutual statistical compensation of the positive and negative deviations due to turbulence. As a result, the value of the observed Coriolis force was the same, essentially, for both turbulent and laminar conditions. He recommends that length/diameter > 10 will ensure that this assumption is valid.

Classification of Simple Beams

As mentioned earlier, previous investigations [16, 28] have shown that, in the case of negligible external damping, simple hydroelastic beams may be conservative, or nonconservative, depending upon the boundary condition type. In this section, the boundary conditions that

Table 1. Summary of simplifying assumptions

Phenomena	Assumptions
Beam Bending	• thin wall and uniform cross section • simple elastic bending described relative to neutral axis • rotary inertia and transverse shear — negligible • small deflections • all material and geometric parameters — constant
Fluid Flow	• incompressible and steady flow • fully developed flow • tube completely filled • hydrostatic pressure gradients — negligible • for $L \gg A$, small-scale details of flow arising from turbulence — insignificant compared to the mean flow
Beam Vibrations	• no gravitational effects • small deflections • internal material damping — insignificant • viscous external damping (linear) • rotary and longitudinal dynamics — negligible
Fluid/Beam Interaction	• tube vibration effects on fluid pressure and velocity with respect to the tube — insignificant • fluid transmits its inertial force directly to the tube wall through reaction forces at the interface • tube diameter — small in comparison to radius of curvature in the deflected state

are associated with Eq. (19) are introduced and Hamilton's Principle is used to deduce how they stipulate basic energy classification for the case of zero external damping.

Hamilton's Principle states that a conservative and holonomic (e.g., one having only equality constraints) system behaves such that the time integral of the Lagrangian function is minimal. A necessary condition for this function to be stationary is that the first variation of the integral must be zero, or

$$\delta \int_{t_1}^{t_2} (\underline{T} - \underline{V}) dt = 0 , \quad (20)$$

where $\underline{T}$ and $\underline{V}$ are the system kinetic and potential energies respectively. In this application, the equality of Eq. (20) will be used to ascertain whether a particular beam and boundary condition combination is conservative or nonconservative. If the equality is satisfied, the system is conservative; if the equality is not satisfied, the system is nonconservative.

In the case of simple hydroelastic beams, the potential energy and kinetic energy are functions of both space and time such that

$$\begin{aligned} d\underline{T} &= d\underline{T}_{\text{pipe}} + d\underline{T}_{\text{fluid}} \\ &= \frac{1}{2} [m y_t^2 + M(y_t + U y_x)^2 + M U^2] dx , \end{aligned} \quad (21)$$

and

$$d\underline{V} = \frac{EI}{2} y_{xx}^2 dx . \quad (22)$$

For this type of energy analysis, it is more straightforward for the independent variables x and t to be in *unscaled* dimensional space ($t \geq 0$, $0 \leq x \leq L$). Substituting Eqs. (21) and (22) into Eq. (20) gives

$$\delta \int_{t_1}^{t_2} \int_0^L [m y_t^2 + M(y_t + U y_x)^2 + M U^2 - E I y_{xx}^2] dx dt = 0 . \quad (23)$$

The variation operation in Eq. (23) is in the sense of virtual displacements in y with (t_1, t_2) fixed. Moving the variation across the integral operations results in

$$\int_{t_1}^{t_2} \int_0^L [m y_t \delta y_t + M(y_t + U y_x) \delta(y_t + U y_x) - E I y_{xx} \delta y_{xx}] dx dt = 0 . \quad (24)$$

The variations of the derivatives in the integrand can be expressed in terms of the virtual displacement in y as follows,

$$\left. \begin{aligned} \delta y_t &= \delta \left(\frac{\partial y}{\partial t} \right) = \frac{\partial}{\partial t} (\delta y) \\ \delta y_x &= \frac{\partial}{\partial x} (\delta y) \\ \delta y_{xx} &= \frac{\partial^2}{\partial x^2} (\delta y) \\ \delta(y_t + U y_x) &= \delta y_t + U \delta y_x \end{aligned} \right\} . \quad (25)$$

Using Eq. (25), the integrand of Eq. (24) can be expanded and somewhat simplified to give

$$\int_{t_1}^{t_2} \int_0^L [(M + m)y_t \delta y_t + (MUy_t \delta y_x + MU\delta y_t \delta y_x) + MU^2 y_x \delta y_x - EIy_{xx} \delta y_{xx}] dx dt = 0 . \quad (26)$$

Equation (26) can be reduced further using integration by parts for each term. For the first term,

$$\begin{aligned} \int_{t_1}^{t_2} \int_0^L (M + m)y_t \delta y_t dx dt &= \int_0^L \left[\int_{t_1}^{t_2} (M + m)y_t \delta y_t dt \right] dx \\ &= \int_0^L \left[[(M + m)y_t \delta y]_{t_1}^{t_2} - \int_{t_1}^{t_2} (M + m)y_{tt} \delta y dt \right] dx . \end{aligned} \quad (27)$$

Since the virtual displacement is defined such that $\delta y(t_1) = \delta y(t_2) = 0$, Eq. (27) becomes

$$\int_{t_1}^{t_2} \int_0^L (M + m)y_t \delta y_t dx dt = - \int_0^L \int_{t_1}^{t_2} (M + m)y_{tt} \delta y dx dt . \quad (28)$$

Similarly, the second integrand term of Eq. (26) gives

$$\begin{aligned} \int_{t_1}^{t_2} \int_0^L (MUy_t \delta y_x + MU\delta y_t \delta y_x) dx dt \\ = \int_0^L [MUy_x \delta y]_{t_1}^{t_2} dx + \int_{t_1}^{t_2} \left\{ [MUy_t \delta y]_0^L - \int_0^L 2MUy_{xt} \delta y dx \right\} dt \end{aligned} \quad (29)$$

and, again, since the variation is zero at (t_1, t_2) ,

$$\int_{t_1}^{t_2} \int_0^L (MUy_t \delta y_x + MU \delta y_x \delta y_t) dx dt = \int_{t_1}^{t_2} \left\{ [MUy_t \delta y]_0^L - \int_0^L 2MUy_{xt} \delta y dx \right\} dt . \quad (30)$$

Continuing in the same fashion, the third term of the integrand of Eq. (26) produces

$$\int_{t_1}^{t_2} \int_0^L (MU^2 y_x \delta y_x) dx dt = \int_{t_1}^{t_2} \left\{ [MU^2 y_x \delta y]_0^L - \int_0^L (MU^2 y_{xx} \delta y) dx \right\} dt . \quad (31)$$

And, finally, the strain energy term of (26) gives

$$\int_{t_1}^{t_2} \int_0^L (EIy_{xx} \delta y_{xx}) dx dt = \int_{t_1}^{t_2} \left\{ [EIy_{xx}^2 \delta y_x]_0^L - \int_0^L (EIy_{xxx} \delta y_x) dx \right\} dt . \quad (32)$$

Further reduction using integration by parts again is necessary, but on the last term of Eq. (32) only.

$$\int_{t_1}^{t_2} \int_0^L (EIy_{xxx} \delta y_x) dx dt =$$

$$\int_{t_1}^{t_2} \left\{ [EIy_{xx}^2 \delta y_x]_0^L - [EIy_{xxx} \delta y]_0^L + \int_0^L (EIy_{xxxx} \delta y) dx \right\} dt . \quad (33)$$

By combining Eqs. (28), (30), (31), and (33), an equivalent expression for the original equality condition, Eq. (23), is obtained which can be used in deducing the energy classifications for the various types of boundary conditions. The result is

$$\begin{aligned} \delta \int_{t_1}^{t_2} (\underline{T} - \underline{V}) dt = & \int_{t_1}^{t_2} \int_0^L [-EIy_{xxxx} - MU^2 y_{xx} - 2MUy_{xt} \\ & - (M + m)y_{tt}] \delta y dx dt + \int_{t_1}^{t_2} \left\{ [(MUy_t + EIy_{xxx} + MU^2 y_x) \delta y]_0^L \right. \\ & \left. - [EIy_{xx}^2 \delta y_x]_0^L \right\} dt . \end{aligned} \quad (34)$$

Both integrals on the right hand side of Eq. (34) must be zero for the beam to be conservative. Because δy must be arbitrary, the only possibility for the first integral to be zero is if the integrand is zero on the intervals (t_1, t_2) and $(0, L)$. So the first necessary condition for the beam to be conservative is

$$EIy_{xxxx} + MU^2 y_{xx} + 2MUy_{xt} + (M + m)y_{tt} = 0 . \quad (35)$$

This equation is recognized as the basic governing equation, Eq. (13), with the exception of no external damping. The result obtained in

Eq. (35) verifies that the variation of the Lagrangian function leads to Lagrange's equations. Note that there is no loss in generality regarding the relationship between boundary conditions and energy classification due to the omission of external damping. Including viscous damping would have led to external work terms in Eq. (13) which would have deposited the necessary damping term in Eq. (35) without affecting the second integral term of Eq. (34).

An analysis of the second integral of Eq. (34) with respect to the various types of simple-beam end conditions is given in Table 2. As shown, the clamped-free (or cantilever) boundary conditions are not capable of rendering the second integral term to zero as they do for the other end conditions. Consequently, there is no way that the equality condition can be met. Hence, it is concluded that the cantilever case must be generally nonconservative. This conclusion is considered attractive for the case of flow measurement because nonconservative properties suggest a greater dynamic response characteristic that can be used as a basis for measurement.

Solution of the Cantilever Governing Equation

For the most part, the specific Coriolis flowmeter concept chosen in this research resulted from mathematical modeling considerations of the hydroelastic beam system. The theoretical analysis has, in effect, been used to define that which is to be analyzed. The following summary of this somewhat circular process is given to explain how the various decisions were made.

Table 2. Energy classification of simple beams

Support type	Boundary conditions	Energy classification	Conditions associated with equation (34)
Clamped-Clamped	$y(0,t) - y(L,t) = 0$ $y_x(0,t) - y_x(L,t) = 0$	Conservative	$\delta y(0,t) - \delta y(L,t) = 0$ $\delta y_x(0,t) - \delta y_x(L,t) = 0$
Clamped-Pinned	$y(0,t) = y(L,t) = 0$ $y_x(0,t) = y_{xx}(L,t) = 0$	Conservative	$\delta y(0,t) = \delta y(L,t) = 0$ $\delta y_x(0,t) = y_{xx}(L,t) = 0$
Pinned-Pinned	$y(0,t) = y(L,t) = 0$ $y_{xx}(0,t) = y_{xx}(L,t) = 0$	Conservative	$\delta y(0,t) = \delta y(L,t) = 0$ $y_{xx}(0,t) = y_{xx}(L,t) = 0$
Clamped-Free (Cantilever)	$y(0,t) = y_x(0,t) = 0$ $y_{xx}(L,t) = y_{xxx}(L,t) = 0$	Nonconservative	$\delta y(L,t) = 0$ $y_{xx}(L,t) = 0$ $y_x(L,t) = 0$

The hydroelastic configuration to be used has been selectively restricted. The first restriction imposed was that only straight fluid conveying beams with simple end conditions would be considered so that the associated boundary value problem would be as simple as possible. The second restriction, that only cantilever end conditions would be considered, was based on an energy analysis. This analysis showed that cantilever end conditions result in energy dissipation due to the fluid flow while the other simple end conditions do not. These two restrictions together define the flow transducer geometry — a straight cantilever tube — and ensure that (1) the maximum theoretical simplicity is achieved, and (2) the most dominant dynamic response characteristics are available. Admittedly, the free end of the cantilever configuration represents a practical limitation. As will be discussed in Chapter 5, this limitation does not totally compromise the practical application of the concept.

The last conceptual feature to be defined was the way in which the beam vibration would be generated. The motion excitation concept is theoretically associated with either nonhomogeneous forcing terms, or boundary and initial conditions. A number of options pertaining to motion excitation are worthy of discussion. First, motion excitation can be either steady-state harmonic, or transient. It was decided arbitrarily, although consistent with basic research objective of studying transient measurement concepts, to focus on transient vibration excitation. More detail comparing the practical implications of steady-state versus transient excitation is given in Chapter 3. Having selected transient excitation, it was necessary to choose what type of transient

excitation would be the best. Two basic methods were judged readily achievable in terms of repeatability and practicality. First, the beam could be "pulsed" into motion from an initial zero-deflection rest condition. Second, the beam could be excited by the release of stored strain energy created by establishing a static deflection prior to release. For the "pulse" mode, an actuator which could supply a force, or displacement, pulse with controlled strength was judged not difficult to construct. The difficulty with the "pulse" mode concept was found to be theoretical in that it would lead to greater complexity in the initial conditions model. Static deflection excitation, by comparison, is mathematically simpler; but has the disadvantage of requiring static conditions to be established. This means that the overall sampling frequency for the Coriolis flowmeter could be less for static deflection excitation than "pulse" excitation. As with all previous decisions, static deflection excitation was selected for its simpler mathematical form.

After that lengthy, but presumably constructive, synopsis, the complete mathematical specification for the cantilever Coriolis flowmeter can be stated and techniques for solution pursued. Recall that the nondimensionalized governing equation for the transverse vibration of a hydroelastic beam is

$$y_{xxxx} + u^2 y_{xx} + 2\sqrt{\beta} u y_{xt} + 2\sqrt{\beta} u y y_t + y_{tt} = 0 .$$

In the cantilever configuration, the boundary conditions for Eq. (35) are

$$\left. \begin{aligned} y(0,t) &= 0 \\ y_x(0,t) &= 0 \\ y_{xx}(1,t) &= 0 \\ y_{xxx}(1,t) &= 0 \end{aligned} \right\} , \quad (36)$$

where $0 \geq x \geq 1$ and $t \geq 0$. Boundary conditions Eqs. (36) stipulate that the deflection and slope must be zero at the cantilever base while the shear force and bending moment must vanish at the free end.

The static deflection excitation mode can be expressed simply as

$$\left. \begin{aligned} y(x,0) &= X(x) \\ y_t(x,0) &= 0 \end{aligned} \right\} , \quad (37)$$

where $X(x)$ represents the static deflection curve generated by the actuator mechanism. The magnitude of $X(1)$ is arbitrary other than it should not be so large as to violate the small deflections assumption. Equation (35) is fourth-order in space and second-order in time so that along with Eqs. (36) and (37) the boundary value problem is fully defined. Note that simultaneously the entire flowmeter concept is specified as well.

The remainder of this chapter covers various solutions which have been obtained for the flowmeter boundary value problem. A numerical solution is compared with two approximate analytical solutions.

Numerical Product Solution

In keeping with today's propensity for computer analysis, the first solution technique investigated was a numerical analysis of an assumed

product solution. This approach has the same formulation as that apparently used by Gregory and Paidoussis [17, pp. 520-521] and begins with the assumption that the solution is the real part of a product of separated spatial and temporal functions. That is,

$$y(x,t) = R[X(x)T(t)] , \quad (38)$$

where $X(x)$ and $T(t)$ may in general be complex. Since the beam is a damped vibratory system, it is likely that $T(t) = Be^{\omega t}$, where ω will be complex to account for both oscillation and decay. Substituting this solution into Eq. (35) converts the partial differential equation into an ordinary differential equation with complex coefficients,

$$\frac{d^4 X}{dx^4} + u^2 \frac{d^2 X}{dx^2} + (2\sqrt{\beta}u)\omega \frac{dX}{dx} + (2\sqrt{\beta}u\gamma)\omega X + \omega^2 X = 0 . \quad (39)$$

Given the form of Eq. (39), the usual assumption is that $X(x) = Ae^{\alpha x}$, where α must also be complex. This substitution converts Eq. (39) into a complex polynomial with complex coefficients of the form,

$$\alpha^4 + u^2 \alpha^2 + (2\sqrt{\beta}u)(\alpha + \gamma)\omega + \omega^2 = 0 . \quad (40)$$

Equation (40) is the characteristic equation of the beam system and specifies the relationship between the spatial eigenvalues, α , and temporal eigenvalues, ω . Because Eq. (40) is fourth-order, the complete solution of Eq. (35) can be written as

$$y(x,t) = R \left[\sum_{j=1}^4 A_j e^{\alpha_j x} e^{\omega t} \right] , \quad (41)$$

with the usual possible modifications if the roots repeat. Using Eq. (41) in the boundary conditions, Eq. (36), provides four equations in the unknown arbitrary constants, A_j ($j = 1, \dots, 4$),

$$\left. \begin{aligned} A_1 + A_2 + A_3 + A_4 &= 0 \\ \alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3 + \alpha_4 A_4 &= 0 \\ \alpha_1^2 A_1 e^{\alpha_1} + \alpha_2^2 A_2 e^{\alpha_2} + \alpha_3^2 A_3 e^{\alpha_3} + \alpha_4^2 A_4 e^{\alpha_4} &= 0 \\ \alpha_1^3 A_1 e^{\alpha_1} + \alpha_2^3 A_2 e^{\alpha_2} + \alpha_3^3 A_3 e^{\alpha_3} + \alpha_4^3 A_4 e^{\alpha_4} &= 0 \end{aligned} \right\} \quad (42)$$

The system of equations (42) are homogeneous, so that a necessary condition for a nontrivial solution is that the determinant of the coefficients matrix must vanish [41, p. 454]. Thus,

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_1^2 e^{\alpha_1} & \alpha_2^2 e^{\alpha_2} & \alpha_3^2 e^{\alpha_3} & \alpha_4^2 e^{\alpha_4} \\ \alpha_1^3 e^{\alpha_1} & \alpha_2^3 e^{\alpha_2} & \alpha_3^3 e^{\alpha_3} & \alpha_4^3 e^{\alpha_4} \end{vmatrix} = 0$$

$$\begin{aligned} \Delta = & (\alpha_3 \alpha_4)^2 (\alpha_2 - \alpha_1) (\alpha_4 - \alpha_3) e^{\alpha_3 + \alpha_4} \\ & - (\alpha_4 \alpha_2)^2 (\alpha_3 - \alpha_1) (\alpha_4 - \alpha_2) e^{\alpha_4 + \alpha_2} \\ & + (\alpha_4 \alpha_1)^2 (\alpha_3 - \alpha_2) (\alpha_4 - \alpha_1) e^{\alpha_4 + \alpha_1} \\ & + (\alpha_3 \alpha_2)^2 (\alpha_4 - \alpha_1) (\alpha_3 - \alpha_2) e^{\alpha_3 + \alpha_2} \\ & - (\alpha_3 \alpha_1)^2 (\alpha_4 - \alpha_2) (\alpha_3 - \alpha_1) e^{\alpha_3 + \alpha_1} \\ & + (\alpha_2 \alpha_1)^2 (\alpha_4 - \alpha_3) (\alpha_2 - \alpha_1) e^{\alpha_2 + \alpha_1} = 0 \end{aligned} \quad (43)$$

Equations (40) and (43) algebraically define $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and ω for a given set of parameters, u, β and γ . Because Eq. (43) is transcendental, there are an infinite number of roots to the combined solution of Eqs. (40) and (43). Each solution describes one of the vibratory modes of the continuous hydroelastic cantilever.

The eigenvalue problem represented by Eqs. (40) and (43) was solved numerically using the method summarized in Fig. 6. The basic technique was to treat the combination of Eqs. (40) and (43) as an analytic function of ω , the zero of which would be the desired value of ω for a given set of parameters. The subroutine ZANLYT performs an iterative search for the zero using the Muller Method with deflation [42]. Since Eqs. (40) and (43) cannot be manipulated into a single explicit function of ω , Eq. (40) is used inside the iterative search loop of ZANLYT to calculate the values of α corresponding to each ω . The α vector is obtained using the complex polynomial solver CØ2ADF [43]. Each iteration of ZANLYT requires the iterative determination of the complex roots of Eq. (40) using CØ2ADF. Once α and ω have been determined, the complex determinant routine FØ3ADF [43] calculates the left side of Eq. (43) which becomes the analytic function residual used by ZANLYT. These two levels of iteration are executed until the convergence criteria is met (i.e., the residual is very small). After convergence is achieved, the parameters u, β , and γ are incremented as desired and the entire process repeated. The computer programs for this analysis are listed in Appendix A.

The numerically calculated temporal eigenvalues for the first and second vibration modes and various parameter values are shown in

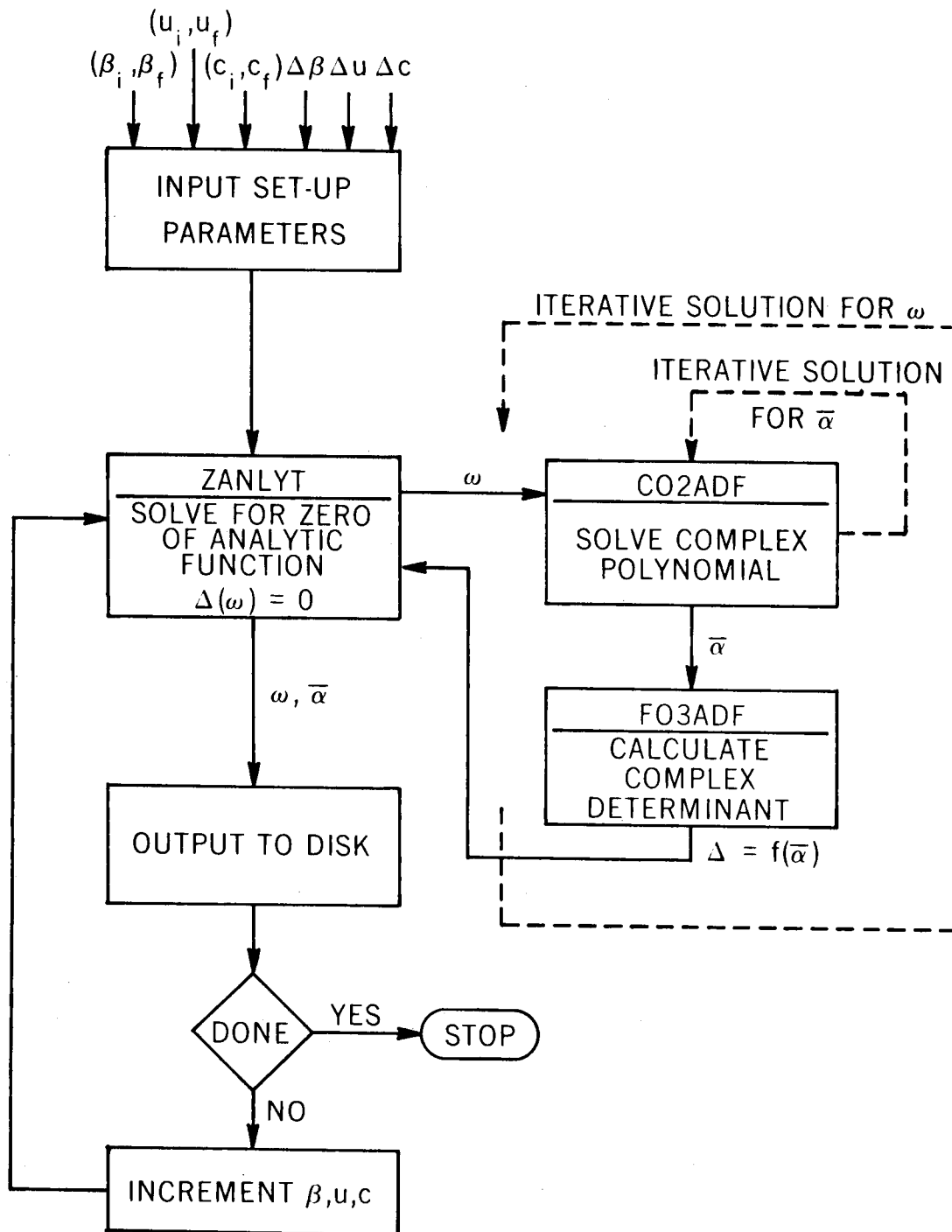


Fig. 6. Numerical product solution technique.

Figs. 7 and 8. These results agree with computational results published by Gregory and Paidoussis [17, p. 525]. Recall that the assumed solutions were defined in Eq. (41) such that the real and imaginary parts of ω describe the decay/growth rate and oscillation frequency respectively. Previous research has been primarily interested in this type of analysis to predict the points of instability where real ω becomes positive. In the flowmeter application of interest here, generally $u \ll 1$ and only the initial region of the curves shown in Figs. 7 and 8 are of interest. The complete root locus diagrams were generated so that these results could be validated through comparison with other published results.

The spatial eigenvalues were calculated along with the temporal eigenvalues inherently and were verified at zero velocity points. The spatial eigenfunctions, or mode shapes, were not calculated since they are not required for flow algorithms which use beam deflection measurement at a single arbitrary location.

Analytical Solutions

As discussed earlier, it was absolutely essential to obtain analytical expressions describing the relationship between the cantilever vibratory response and the fluid flow conditions. Such expressions could be developed through the rather inelegant process of curve fitting functions in the three parameters to the numerical solution results. A physically more meaningful method to develop the needed expressions would be to either solve the boundary value problem in closed form or through an approximate analytical technique. This latter approach was used with both success and failure.

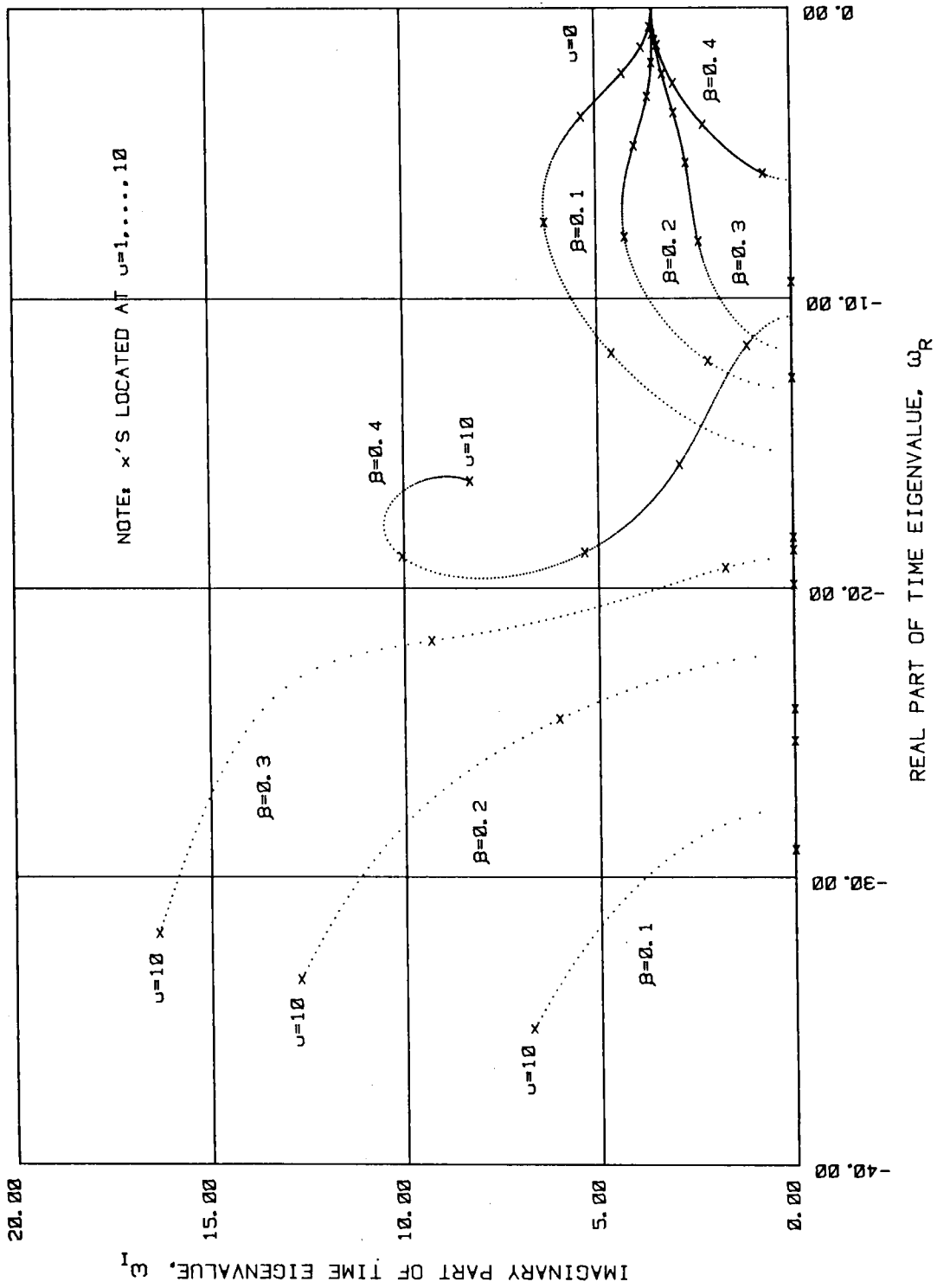


Fig. 7. Time eigenvalues for the fundamental mode.

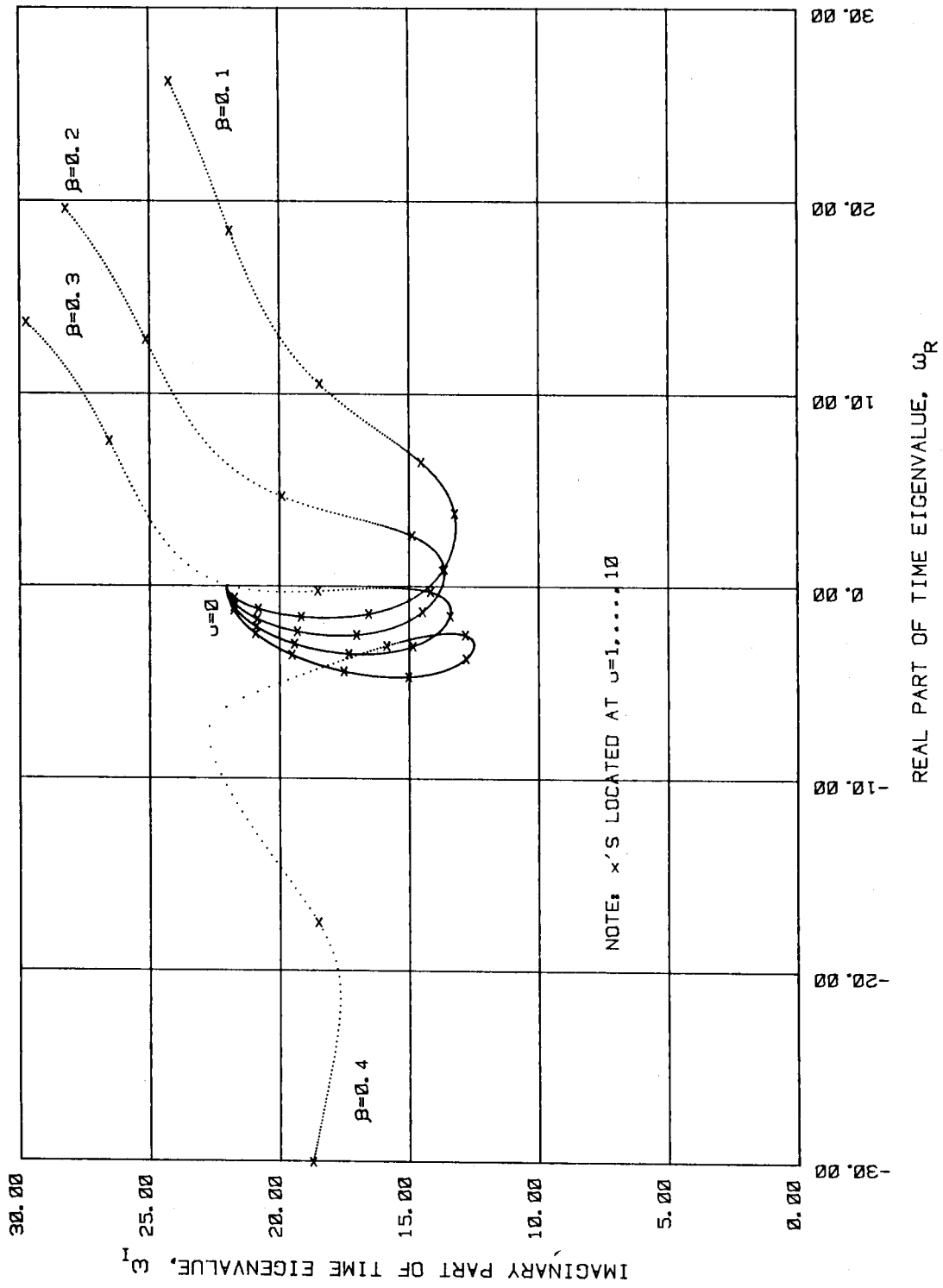


Fig. 8. Time eigenvalues for the second mode.

In spite of the ominous implications of the non-self-adjointness of Eq. (35), a multitude of techniques encompassing integral transforms through combined (x,t) transformations were used in an attempt to achieve a closed-form solution. These efforts, while surely noble and filled with optimism, were totally unsuccessful. After regaining a positive perspective from these failures, attention was diverted to perturbation techniques which have been useful in the analysis of difficult boundary value problems like the highly nonlinear ones associated with fluid mechanics. In the remainder of this chapter, the solution of the boundary value problem is considered using the method of multiple scales and a related, but greatly simplified, technique.

Perturbation solution. Perturbation techniques based upon asymptotic expansions in suitably nondimensionalized parameters have been developed to a high degree of sophistication and have been successfully applied to a wide range of applications, particularly in nonlinear systems [44, 45]. These techniques facilitate the development of approximate analytical solutions in which the quality of the approximation can be systematically controlled. One generally encounters mathematical difficulty in analyzing physical systems that involve wide differences in the size of significant phenomena. The mathematical complexity of the cantilever Coriolis flowmeter can be interpreted as a case of this type. The hydroelastic beam response is comprised of two basic parts resulting from the simultaneous action of large and small forces. The beam oscillates rapidly due to the "large" inertia and flexure forces. Simultaneously,

the beam also experiences exponential decay of the vibratory response due to the "small" fluid damping and the external damping. These damping effects remain small in comparison to the oscillation forces for small nondimensional fluid velocities.

The general class of problems involving large disparaging differences of scale have received considerable attention [44, 45, 46]. The method of multiple scales is the technique that was found to be the most straightforward for application to the cantilever Coriolis flowmeter analysis. In the method of multiple scales, additional independent variables are introduced. These new variables are transformed, or "stretched," to offset the inherent differences of scale. When time is the independent variable of concern, the technique is sometimes called "two-timing." A slow time scale is used for small effects acting for long periods, and a shorter fast time scale is used for large effects acting for shorter periods.

Much work has been done in extending perturbation theory to boundary and initial value problems described with partial differential equations [47, 48]. Davy and Ames [49] used this work as a basis in applying two-timing to an initial value problem which is conceptually very similar to the cantilever Coriolis flowmeter description. The problem they considered was the dynamic response of a nonlinear viscoelastic rod to general displacement, velocity, and acceleration initial conditions. Because of the viscoelasticity of the rod, this system has a large difference in the time scales of significant phenomena. The basic wave transit time associated with the rod elasticity is small compared to the

material relaxation time resulting from its viscous properties. Davy and Ames [49] used an expansion parameter which is the ratio of these two effects to obtain a multimode solution of the governing third-order nonlinear partial differential equation. The solution was carried out through the first-order expansion terms (i.e., the approximation involves truncation not greater than the order of the expansion parameter squared). This work is analogous to the problem being considered here, and has been used as a pattern in the application of two-timing. To obtain simple results useable in flowmeter algorithms, considerable modifications to their approach were required. In the following discussion, two-timing is applied to the problem represented in Eqs. (35)-(37).

The first step is the determination of a suitable expansion parameter. In considering Eq. (35), it is noted that the coefficients of the inertia and flexure terms are of order 1, $O(1)$, while the other coefficients are dependent on the parameters u , β , and γ . Recall that β is the ratio of the fluid density to the sum of the fluid density and the tube material density, and for practical applications can vary over the range $0 < \beta < 1$. The nondimensional external damping, γ , is assuredly small since $\sqrt{ML^2/EI} = O(10^{-3})$ typically and c is only due to air drag. The nondimensional parameter u appears linearly and as a square while the magnitude of the coefficients involving u is proportional to the fluid velocity, U . As a result, the relative size of the systems parameters depends for the most part on the nondimensional parameter u . Typical flowmeter conditions were considered so that the range of u could be estimated. To keep pressure drops reasonable, it is standard

practice to limit maximum pipe velocities to the range $1.25 \text{ m/s} < U < 3.1 \text{ m/s}$ [50]. For this mean velocity range, it was found that typically $u < 0.3$, and in most applications $u \ll 0.3$. These calculations verified what was to be expected and seen in early experimentation – the damping effects are small in comparison to the beam vibration.

Since the interest here is the relationship of beam response to fluid flow effects, the obvious choice of the expansion parameter is u , since γ is constant and β may not necessarily be small. A regular perturbation expansion for the beam deflection as a function of u is

$$y(x,t) \sim F_0(x,t) + uF_1(x,t) + u^2F_2(x,t) + \dots, \quad (44)$$

where F_0 is the solution corresponding to $u = 0$, which is the case of a simple nonhydroelastic cantilever. Equation (44) can be used to reformulate Eqs. (35)-(37) as a power series in u set equal to zero. The expansion parameter, u , must be arbitrary. As a result, the only way Eqs. (35)-(37) can be satisfied in the expanded form is for the coefficients of the powers of u to be zero. In this way, the power series equivalents of Eqs. (35)-(37) reduce to a series of boundary value problems for each F_j where j is the number of terms carried in the expansion and the order of the approximation. For the technique to be successful, the series equivalent boundary value problems must be amenable to closed-form solution.

Secular terms are nonhomogeneous terms which enter into the series of F_j boundary value problems and which result in unbounded solutions.

This is one form of the "singularity" type behavior treated in perturbation theory. If one proceeds with the straightforward regular perturbation expansion given in Eq. (44), secularity occurs in the solution of the cantilever Coriolis flowmeter problem. The first step in avoiding this blow-up behavior is to apply the method of multiple scales in the manner of Davy and Ames [49] by introducing stretched time variables which depend on the small expansion parameter u . The first time variable which is associated with the beam vibratory oscillation is specified by the transformation

$$t^* = \psi(u)t \quad (45)$$

where it is assumed that $\psi(u)$ can be expressed as a power series in the parameter u according to

$$\psi(u) = (\psi_0 + u\psi_1 + u^2\psi_2 + \dots) \quad (46)$$

The second time scale is used to describe the much slower decay response due to damping. To account for this time scale, another stretched time variable,

$$\bar{t} = ut \quad (47)$$

is introduced where the linear stretching is motivated by the appearance of u in the coefficient of the Coriolis term in Eq. (35). For this approach to work, it is necessary for Eqs. (46) and (47) to be independent [44] which requires $\psi_1 = 0$. The remaining coefficients, $\psi_0, \psi_2, \dots$ are left to be defined by conditions which eliminate terms leading to secularity.

With these conditions, the perturbation solution is of the form

$$\left. \begin{aligned} y(x,t) &\sim F_0(x,t^*,\bar{t}) + uF_1(x,t^*,\bar{t}) + u^2F_2(x,t^*,\bar{t}) + \dots \\ \bar{t} &= ut \\ t^* &= (1 + u^2\psi_2 + \dots)t \end{aligned} \right\} . \quad (48)$$

The partial derivatives of Eqs. (35) can be expressed in terms of F_j , x , t^* , and $\bar{t}$ using Eqs. (48) and the chain rule. After considerable manipulations, these expressions can be summarized as follows,

$$\begin{aligned} \frac{d\bar{t}}{dt} &= u \quad \text{and} \quad \frac{dt^*}{dt} = 1 + u^2\psi_2 + \dots, \\ y_t &= \frac{dt^*}{dt} \cdot \frac{\partial y}{\partial t^*} + \frac{d\bar{t}}{dt} \cdot \frac{\partial y}{\partial \bar{t}} = (1 + u^2\psi_2) \frac{\partial y}{\partial t^*} + u \frac{\partial y}{\partial \bar{t}} + O(u^3) \\ &= (1 + u^2\psi_2) \left[\frac{\partial F_0}{\partial t^*} + u \frac{\partial F_1}{\partial t^*} + u^2 \frac{\partial F_2}{\partial t^*} + O(u^3) \right] \\ &\quad + u \left[\frac{\partial F_0}{\partial \bar{t}} + u \frac{\partial F_1}{\partial \bar{t}} + u^2 \frac{\partial F_2}{\partial \bar{t}} + O(u^3) \right] \\ y_t &= \frac{\partial F_0}{\partial t^*} + u \left(\frac{\partial F_0}{\partial \bar{t}} + \frac{\partial F_1}{\partial t^*} \right) + u^2 \left(\frac{\partial F_2}{\partial t^*} + \psi_2 \frac{\partial F_0}{\partial t^*} + \frac{\partial F_1}{\partial \bar{t}} \right) + O(u^3), \end{aligned} \quad (49)$$

and in a similar fashion the other derivatives become

$$\begin{aligned} y_{tt} &= \frac{\partial y_t}{\partial t} = \frac{dt^*}{dt} \cdot \frac{\partial y_t}{\partial t^*} + \frac{d\bar{t}}{dt} \cdot \frac{\partial y_t}{\partial \bar{t}} \\ &= \frac{\partial^2 F_0}{\partial t^{*2}} + u \left[\frac{\partial^2 F_1}{\partial t^{*2}} + 2 \frac{\partial^2 F_0}{\partial \bar{t} \partial t^*} \right] + u^2 \left[\frac{\partial^2 F_2}{\partial t^{*2}} + 2\psi_2 \frac{\partial^2 F_0}{\partial t^{*2}} \right. \\ &\quad \left. + 2 \frac{\partial^2 F_1}{\partial \bar{t} \partial t^*} + \frac{\partial^2 F_0}{\partial \bar{t}^2} \right] + O(u^3), \end{aligned} \quad (50)$$

$$\begin{aligned}
y_{xt} &= \frac{\partial y_t}{\partial x} \\
&= \frac{\partial^2 F_0}{\partial x \partial t^*} + u \left[\frac{\partial^2 F_0}{\partial x \partial \bar{t}} + \frac{\partial^2 F_1}{\partial x \partial t^*} \right] + u^2 \left[\frac{\partial^2 F_2}{\partial x \partial t^*} + \psi_2 \frac{\partial^2 F_0}{\partial x \partial \bar{t}^*} \right. \\
&\quad \left. + \frac{\partial^2 F_1}{\partial x \partial \bar{t}} \right] + O(u^3) , \tag{51}
\end{aligned}$$

$$\left(\frac{\partial^i y}{\partial x^i} \right) = \frac{\partial^i F_0}{\partial x^i} + u \frac{\partial^i F_1}{\partial x^i} + u^2 \frac{\partial^i F_2}{\partial x^i} + (u^3) , \quad i = 1, \dots, 4 . \tag{52}$$

Upon substituting the expansion derivatives, Eqs. (49)-(52), into the governing equation, Eq. (35), and then setting the coefficients of $\{u\}^k$ to zero, the following equations are obtained:

$$u^0 = 0 : \frac{\partial^4 F_0}{\partial x^4} + \frac{\partial^2 F_0}{\partial t^{*2}} = 0 , \tag{53}$$

$$u^1 = 0 : \frac{\partial^4 F_1}{\partial x^4} + \frac{\partial^2 F_1}{\partial t^{*2}} = -2\sqrt{\beta} \frac{\partial^2 F_0}{\partial x \partial t^*} - 2 \frac{\partial^2 F_0}{\partial t^* \partial \bar{t}} - \sqrt{\beta} \gamma \frac{\partial F_0}{\partial t^*} , \tag{54}$$

$$\begin{aligned}
u^2 = 0 : \frac{\partial^4 F_2}{\partial x^4} + \frac{\partial^2 F_2}{\partial t^{*2}} &= - \frac{\partial^2 F_0}{\partial x^2} - 2\sqrt{\beta} \left(\frac{\partial^2 F_1}{\partial t^* \partial x} + \frac{\partial^2 F_0}{\partial \bar{t} \partial x} \right) \\
&- \left(2\psi_2 \frac{\partial^2 F_0}{\partial t^{*2}} + 2 \frac{\partial^2 F_1}{\partial \bar{t} \partial t^*} + \frac{\partial^2 F_0}{\partial \bar{t}^2} \right) - \gamma \sqrt{\beta} \left(\frac{\partial F_1}{\partial t^*} + \frac{\partial F_0}{\partial \bar{t}} \right) . \tag{55}
\end{aligned}$$

The expansion derivatives are also used to transform the initial and boundary conditions to the series form. The results are

$$\left. \begin{aligned}
 y(0,t) = 0 &\Rightarrow F_i(0,t^*,\bar{t}) = 0 \\
 y_x(0,t) = 0 &\Rightarrow \frac{\partial F_i}{\partial x}(0,t^*,\bar{t}) = 0 \\
 y_{xx}(1,t) = 0 &\Rightarrow \frac{\partial^2 F_i}{\partial x^2}(1,t^*,\bar{t}) = 0 \\
 y_{xxx}(1,t) = 0 &\Rightarrow \frac{\partial^3 F_i}{\partial x^3}(1,t^*,\bar{t}) = 0
 \end{aligned} \right\} \quad i = 1, \dots, 4 \quad (56)$$

$$\left. \begin{aligned}
 y(x,0) = X(x) &\Rightarrow F_0(x,0,0) = X(x) \\
 &F_k(x,0,0) = 0, \quad k = 1, 2 \\
 y_t(x,0) = 0 &\Rightarrow \frac{\partial F_0}{\partial t^*}(x,0,0) = 0 \\
 &\frac{\partial F_1}{\partial t^*}(x,0,0) + \frac{\partial F_0}{\partial \bar{t}}(x,0,0) = 0 \\
 &\left[\frac{\partial F_2}{\partial t^*} + \psi_2 \frac{\partial F_0}{\partial t^*} + \frac{\partial F_1}{\partial \bar{t}} \right]_{(x,0,0)} = 0
 \end{aligned} \right\} \quad (57)$$

Equations (53)-(57) represent the series equivalent boundary value problems for the original problem shown in Eqs. (35)-(37). Equations (53)-(55) show the basic nature of perturbation solutions. The original complex governing equation is traded for a set of simpler recurring equations that are coupled through nonhomogeneous forcing terms that increase in complexity with the order of the approximation. As the order (or accuracy) of the desired solution is increased, the difficulty in determining the particular solutions that go with these nonhomogeneous terms increases rapidly.

The recurring operator for the perturbation solution is

$$\frac{\partial^4 F_j}{\partial x^4} + \frac{\partial^2 F_j}{\partial t^{*2}} \triangleq L(F_j) = 0, \quad j = 1, \dots \quad (58)$$

Equation (58) with the boundary conditions given in Eq. (56) is recognized as the governing equation of a simple nonhydroelastic cantilever without damping. The homogeneous solution of this equation can be easily obtained using separation of variables as shown in ref. 41, pp. 323-326. The results of the solution are summarized here for convenience. The homogeneous solution is

$$F_j(x, t^*, \bar{t}) = \sum_{k=1}^{\infty} [A_k(\bar{t}) \cos \alpha_k t^* + B_k(\bar{t}) \sin \alpha_k t^*] X_k(x), \quad (59)$$

where it has been tacitly assumed that the arbitrary constants A_k and B_k may vary slowly with the long time variable to account for the small damping effects known to be present. The spatial eigenfunctions are

$$X_k(x) = K_k [(\sin \sqrt{\alpha_k} + \sinh \sqrt{\alpha_k})(\cos \sqrt{\alpha_k} x - \cosh \sqrt{\alpha_k} x) - (\cos \sqrt{\alpha_k} + \cosh \sqrt{\alpha_k})(\sin \sqrt{\alpha_k} x - \sinh \sqrt{\alpha_k} x)] , \quad (60)$$

$$X_k(1) = 1 \quad \text{and} \quad K_k = \text{normalization constant},$$

and where the spatial eigenvalues are defined by the roots of the transcendental equation

$$\cosh \sqrt{\alpha_k} \cos \sqrt{\alpha_k} + 1 = 0 . \quad (61)$$

With this general form for the homogeneous solution, the total solutions for each of the series equivalent boundary value problems can be pursued. For the zeroth-order solution,

$$\left. \begin{aligned} L(F_0) = 0 : F_0(x, t^*, \bar{t}) &= \sum_{k=1}^{\infty} [A_k(\bar{t}) \cos \alpha_k t^* + B_k(\bar{t}) \sin \alpha_k t^*] X_k(x) \\ F_0(x, 0, 0) = 0 : [A_k(0) \cdot 1 + B_k(0) \cdot 0] X(x) &= X(x) \Rightarrow A_k(0) = 1 \\ \frac{\partial F_0}{\partial t^*}(x, 0, 0) = 0 : [-A_k(0) \alpha_k \cdot 0 + B_k(0) \cdot \alpha_k \cdot 1] X(x) &= 0 \Rightarrow B_k(0) = 0 \end{aligned} \right\} .$$

(62)

Notice that to simplify the $F_0(x, 0, 0) = 0$ initial condition expression, it has been assumed that the static deflection curve coincides with the composite mode shape. The quality of this assumption will be discussed later after other important simplifications have been incorporated.

At this point, only the initial conditions for the unknown constants $A(\bar{t})$ and $B(\bar{t})$ are known. By proceeding to the first-order equation, auxiliary equations specifying these constants will be obtained. The first-order equation is

$$L(F_1) = -2\sqrt{\beta} \frac{\partial^2 F_0}{\partial x \partial t^*} - 2 \frac{\partial^2 F_0}{\partial t^* \partial \bar{t}} - \sqrt{\beta} \gamma \frac{\partial F_0}{\partial t^*} , \quad (63)$$

where $F_0(x, t^*, \bar{t})$ solution given in Eq. (62) can be used to explicitly define the forcing terms in Eq. (63). That is,

$$\left. \begin{aligned}
 \frac{\partial F_0}{\partial t^*} &= \sum_{k=1}^{\infty} [-\alpha_k A_k(\bar{t}) \sin \alpha_k t^* + \alpha_k B_k(\bar{t}) \cos \alpha_k t^*] \chi_k(x) \\
 \frac{\partial^2 F_0}{\partial t^* \partial x} &= \sum_{k=1}^{\infty} [-\alpha_k A_k(\bar{t}) \sin \alpha_k t^* + \alpha_k B_k(\bar{t}) \cos \alpha_k t^*] \frac{\partial \chi_k(x)}{\partial x} \\
 \frac{\partial^2 F_0}{\partial t^* \partial \bar{t}} &= \sum_{k=1}^{\infty} \left[-\alpha_k \frac{dA_k(\bar{t})}{d\bar{t}} \sin \alpha_k t^* + \alpha_k \frac{dB_k(\bar{t})}{d\bar{t}} \cos \alpha_k t^* \right] \chi_k(x)
 \end{aligned} \right\} \cdot (64)$$

With Eq. (64) substituted into Eq. (63), the first-order equation becomes

$$\begin{aligned}
 L(F_1) &= \sum_{k=1}^{\infty} \left\{ 2\alpha_k \sin \alpha_k t^* \left[\frac{dA_k(\bar{t})}{d\bar{t}} \chi_k(x) + \sqrt{\beta} \left(\frac{d\chi_k(x)}{dx} + \frac{\gamma}{2} \chi_k(x) \right) A_k(\bar{t}) \right] \right. \\
 &\quad \left. - 2\alpha_k \cos \alpha_k t^* \left[\frac{dB_k(\bar{t})}{d\bar{t}} \chi_k(x) + \sqrt{\beta} \left(\frac{d\chi_k(x)}{dx} + \frac{\gamma}{2} \chi_k(x) \right) B_k(\bar{t}) \right] \right\} \\
 &\triangleq L_1(A_k, B_k) \cdot
 \end{aligned} \tag{65}$$

The complexity of Eq. (65) is formidable to say the least. Even though the homogeneous solution is easily obtained, the particular solution stems from a forcing term that involves all of the eigenvalues in a complex coupling of space and time. Rather than proceeding in the infinitude of k and dismal complexity of Eq. (65), a digression to the implications and attributes of Eq. (65) will save time and bookkeeping.

Note that the $\sin \alpha_k t^*$ and $\cos \alpha_k t^*$ terms in the forcing term are contained in the homogeneous solutions such that secular terms of the form $t^* \sin \alpha_k t^*$ and $t^* \cos \alpha_k t^*$ are sure to appear in the particular solution. In two-timing, these secular terms are defeated by requiring the expressions in the brackets (involving $A(\bar{t})$ and $B(\bar{t})$) to be zero.

That is

$$\left. \begin{aligned} \frac{dA_k(\bar{t})}{d\bar{t}} X_k(x) + \sqrt{\beta} \left[\frac{dX_k(x)}{dx} + \frac{\gamma}{2} X_k(x) \right] A_k(\bar{t}) &= 0 \\ \frac{dB_k(\bar{t})}{d\bar{t}} X_k(x) + \sqrt{\beta} \left[\frac{dX_k(x)}{dx} + \frac{\gamma}{2} X_k(x) \right] B_k(\bar{t}) &= 0 \end{aligned} \right\} . \quad (66)$$

As can be seen, because of the presence of the spatial terms, the solution of these equations is not straightforward even though the initial conditions have been specified in Eqs. (62). The difficulty caused by these spatial terms is removed by applying an additional condition proposed by Keller and Kogelman [47]. In order that the nonhomogeneous problem possess a solution which is bounded as $t^*, \bar{t} \rightarrow \infty$, the nonhomogeneous forcing terms, both spatial and temporal, must be orthogonal to all of the homogeneous eigenfunctions. This condition is expressed mathematically for the first-order solution as

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \int_0^1 L(A_j, B_j) \begin{bmatrix} \cos \alpha_j t^* \\ \sin \alpha_j t^* \end{bmatrix} X_j(x) dx dt^* = 0 . \quad (67)$$

Condition (66) serves its intended purpose, but as can be seen in Davy and Ames [49] does not significantly reduce the complexity of the nonhomogeneous problems. In the initial application of this method to the cantilever Coriolis flowmeter formulation, the solution was rigorously continued from the point of Eq. (65) only to encounter a level of complexity equal to that experienced by Davy and Ames [49].

By including the simultaneous presence of entire eigenvalue spectrum, expressions like Eq. (66) become coupled making closed-form solutions improbable. Davy and Ames [49] overcame this difficulty by assuming an auxiliary expansion solution for the slowly varying arbitrary constants in another small system parameter. By using this perturbation solution within a perturbation solution and dazzling algebraic skills, they obtained a first-order solution for the entire problem. Their final solution, from the perspective of realtime algorithms, was prohibitively complex.

At this point, things were nearly as dismal as they were in the initial analytical experiences reviewed in the first section of this chapter. While this theoretical work was stalling out, the equipment for the proof of principle experiment was assembled and made operational. Preliminary experiments involved recording the dynamic response of the beam deflection for a 9.25 mm diameter and 50 cm long cantilever. These experiments showed that the harmonic characteristics were surprisingly pure and appeared to be unimodal. A digital signal analyzer (Hewlett-Packard Model 5420) was obtained and used to evaluate the frequency spectrum of the beam deflection response. Typical results of the spectral analysis are shown in Fig. 9. For this particular cantilever, the fundamental mode is at about 30 Hz and the second mode would be near 180 Hz if present. As seen, the spectrum is flat beyond the second mode. These results were most encouraging and were used as justification to seek a single mode solution through the two-timing perturbation technique with the hope that the resulting expressions would be more amenable to realtime algorithm development.

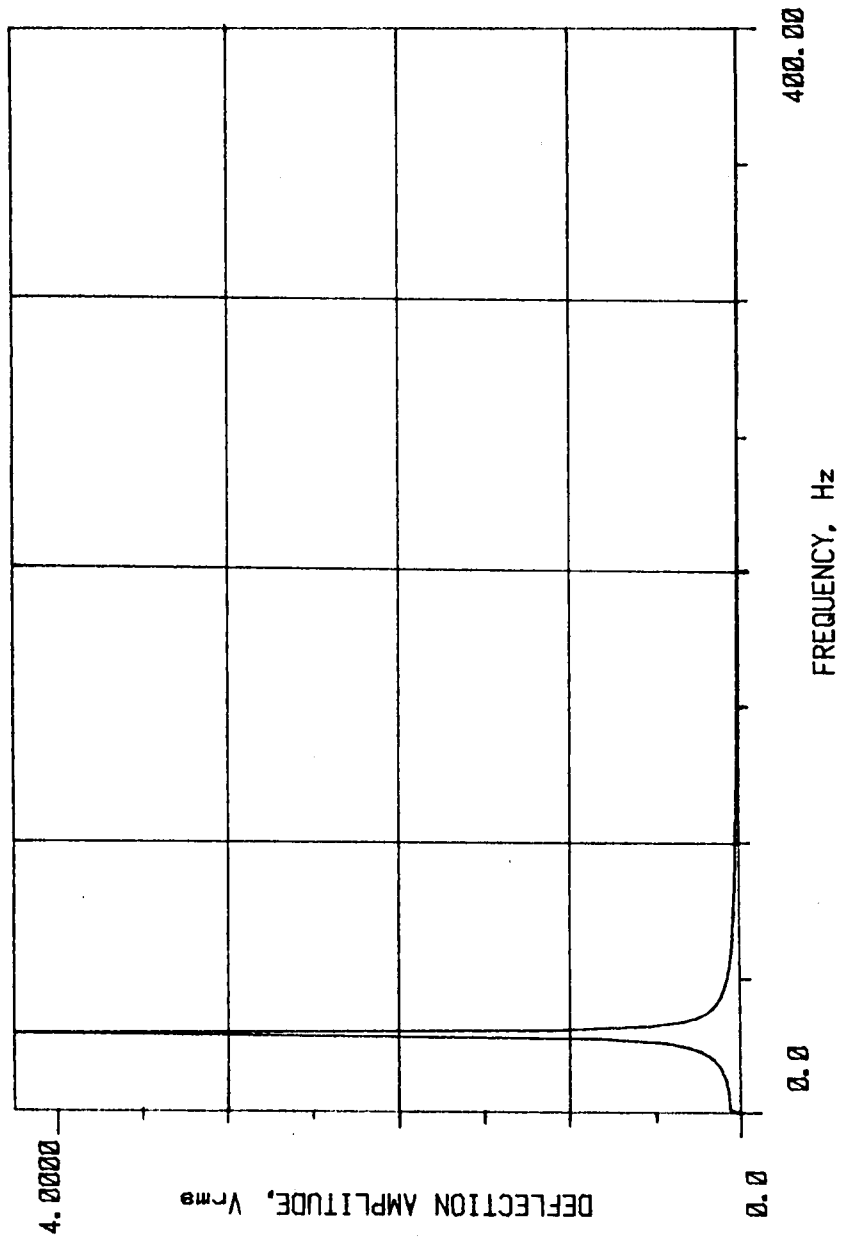


Fig. 9. Typical cantilever beam frequency spectrum.

So with $k = 1$, Eqs. (62) and (65) are simplified to

$$\left. \begin{aligned} F_0(x, t^*, \bar{t}) &= [A(\bar{t}) \cos \alpha t^* + B(\bar{t}) \sin \alpha t^*] X(x) \\ A(0) &= 1 \\ B(0) &= 0 \end{aligned} \right\}, \quad (68)$$

and

$$\begin{aligned} L(F_1) &= 2\alpha \sin \alpha t^* \left[\frac{dA(\bar{t})}{d\bar{t}} X(x) + \sqrt{\beta} \left(\frac{dX(x)}{dx} + \frac{\gamma}{2} X(x) \right) A(\bar{t}) \right] \\ &\quad - 2\alpha \cos \alpha t^* \left[\frac{dB(\bar{t})}{d\bar{t}} X(x) + \sqrt{\beta} \left(\frac{dX(x)}{dx} + \frac{\gamma}{2} X(x) \right) B(\bar{t}) \right] \\ &\triangleq L_1(A, B), \end{aligned} \quad (69)$$

where the eigenvalue subscript has been completely dropped and it is understood that only the fundamental terms are included. To preclude the formation of secular terms, $L_1(A, B)$ must be orthogonal to $L(F_1)$.

The single mode equivalent of Eq. (67) is

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \int_0^1 L_1(A, B) \begin{bmatrix} \cos \alpha t^* \\ \sin \alpha t^* \end{bmatrix} X(x) dx dt^* = 0 \quad (70)$$

and assures that this orthogonality condition is met. Equation (70) actually represents two equations which restrict the functions $A(\bar{t})$ and $B(\bar{t})$ such that secularity is prevented. Because $L(F_1)$ is self-adjoint, its eigenfunctions $X(x)$ are known to be orthogonal [41] such that

$$\left. \begin{aligned} \int_0^1 X^2(x) dx &= K_x \\ \int_0^1 \frac{dX(x)}{dx} X(x) dx &= K_{x'} \\ \int_0^1 \frac{d^2X(x)}{dx^2} X(x) dx &= K_{x''} \end{aligned} \right\} . \quad (71)$$

Substituting $L_1(A, B)$ into Eq. (70) and using Eq. (71) gives

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \left\{ 2\alpha \sin \alpha t^* \left[\frac{dA(\bar{t})}{d\bar{t}} K_x + \sqrt{\beta} \left(K_{x'} + \frac{\gamma}{2} K_x \right) A(\bar{t}) \right] \right. \\ \left. - 2\alpha \cos \alpha t^* \left[\frac{dB(\bar{t})}{d\bar{t}} K_x + \sqrt{\beta} \left(K_{x'} + \frac{\gamma}{2} K_x \right) B(\bar{t}) \right] \right\} \begin{bmatrix} \cos \alpha t^* \\ \sin \alpha t^* \end{bmatrix} dt^* = 0 . \quad (72)$$

Various terms of the integral (72) are

$$\left. \begin{aligned} \int_0^T \sin \alpha t^* \cos \alpha t^* dt^* &= \frac{\sin^2 \alpha T}{2\alpha} \\ \int_0^T \cos^2 \alpha t^* dt^* &= \frac{1}{2\alpha} (\sin \alpha T \cos \alpha T + \alpha T) \\ \int_0^T \sin^2 \alpha t^* dt^* &= \frac{1}{2\alpha} (-\sin \alpha T \cos \alpha T + \alpha T) \end{aligned} \right\} . \quad (73)$$

Applying the limiting process to Eq. (73) gives

$$\left. \begin{aligned} \lim_{T \rightarrow \infty} \left(\frac{\sin^2 \alpha T}{2\alpha T} \right) &= 0 \\ \lim_{T \rightarrow \infty} \left\{ \frac{1}{2\alpha T} (\sin \alpha T \cos \alpha T + \alpha T) \right\} &= \frac{1}{2} \\ \lim_{T \rightarrow \infty} \left\{ \frac{1}{2\alpha T} (-\sin \alpha T \cos \alpha T + \alpha T) \right\} &= \frac{1}{2} \end{aligned} \right\}, \quad (74)$$

which simplifies Eq. (72) to produce the necessary equations in $A(\bar{t})$ and $B(\bar{t})$. That is,

$$\left. \begin{aligned} \frac{dA(\bar{t})}{d\bar{t}} + \sqrt{\beta} \left(\frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right) A(\bar{t}) &= 0 \\ \frac{dB(\bar{t})}{d\bar{t}} + \sqrt{\beta} \left(\frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right) B(\bar{t}) &= 0 \end{aligned} \right\}. \quad (75)$$

With the initial conditions in Eq. (68), the solutions of Eq. (75) can be obtained directly,

$$\left. \begin{aligned} A(\bar{t}) &= e^{-\sqrt{\beta} \left(\frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right) \bar{t}} \\ B(\bar{t}) &= 0 \end{aligned} \right\}. \quad (76)$$

Installing Eqs. (76) into Eqs. (68) gives the complete zeroth-order solution which is

$$F_0(x, t^*, \bar{t}) = e^{-\sqrt{\beta} \left(\frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right) \bar{t}} \cos \alpha t^* X(x). \quad (77)$$

With completion of the zeroth-order solution, it is now time to return to the first-order equation. The choices of $A(\bar{t})$ and $B(\bar{t})$ were based on the condition that $L(F_1)$ and $L_1(A, B)$ be orthogonal so that $L_1(A, B)$ would not introduce unbounded terms into the particular solution. With this condition, the complete solution is, as with Eqs. (62),

$$F_1(x, t^*, \bar{t}) = [C(\bar{t}) \cos \alpha t^* + D(\bar{t}) \sin \alpha t^*] X(x) \quad (78)$$

where the initial conditions for $C(\bar{t})$ and $D(\bar{t})$ are defined by Eqs. (57) as follows

$$\left. \begin{aligned} F_1(x, 0, 0) = 0: [C(0) \cdot 1 + D(0) \cdot 0] X(x) = 0 &\Rightarrow C(0) = 0 \\ \frac{\partial F_1}{\partial t^*}(x, 0, 0) = -\frac{\partial F_0}{\partial \bar{t}}(x, 0, 0): [\alpha C(0) \cdot 0 + \alpha D(0) \cdot 0] X(x) = \\ &+ \sqrt{\beta} \left(\frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right) X(x) \\ &\Rightarrow D(0) = \frac{\sqrt{\beta}}{\alpha} \left(\frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right) \end{aligned} \right\} \quad (79)$$

As previously, the next order equation must be considered to obtain the conditions which specify $C(\bar{t})$ and $D(\bar{t})$. The second-order equation is repeated for convenience,

$$\begin{aligned} L(F_2) = & \left[-\frac{\partial^2 F_0}{\partial x^2} - 2\sqrt{\beta} \left(\frac{\partial^2 F_0}{\partial \bar{t} \partial x} + \frac{\gamma}{2} \frac{\partial F_0}{\partial \bar{t}} \right) - 2\psi_2 \frac{\partial^2 F_0}{\partial t^{*2}} - \frac{\partial^2 F_0}{\partial \bar{t}^2} \right] \\ & - 2\sqrt{\beta} \left(\frac{\partial^2 F_1}{\partial t^* \partial x} + \frac{\gamma}{2} \frac{\partial F_1}{\partial t^*} \right) - 2 \frac{\partial^2 F_1}{\partial \bar{t} \partial t^*} \quad (80) \end{aligned}$$

The orthogonality condition, Eq. (70), will again be applied to prevent secularity from entering through the right-hand side forcing terms.

This condition will not offset the forcing terms in the brackets since they are dependent on $F_0(x, t^*, \bar{t})$. This term is eliminated by exercising the freedom to choose ψ_2 such that

$$\frac{\partial^2 F_0}{\partial x^2} + 2\sqrt{\beta} \left(\frac{\partial^2 F_0}{\partial \bar{t} \partial x} + \frac{\gamma}{2} \frac{\partial F_0}{\partial \bar{t}} \right) - 2\psi_2 \frac{\partial^2 F_0}{\partial t^{*2}} + \frac{\partial^2 F_0}{\partial \bar{t}^2} = 0. \quad (81)$$

Using the definition $\tau_0 \triangleq \left[\sqrt{\beta} \left(\frac{K'_x}{K_x} + \frac{\gamma}{2} \right) \right]^{-1}$ and Eq. (77) in Eq. (81) gives

$$\begin{aligned} e^{-\bar{t}/\tau_0} \cos \alpha t^* \left[\frac{d^2 X(x)}{dx^2} + 2\sqrt{\beta} \left[-\frac{1}{\tau_0} e^{-\bar{t}/\tau_0} \cos \alpha t^* \frac{dX(x)}{dx} \right. \right. \\ \left. \left. - \frac{\gamma}{2} \frac{1}{\tau_0} e^{-\bar{t}/\tau_0} \cos \alpha t^* X(x) \right] - 2\psi_2 \alpha^2 e^{-\bar{t}/\tau_0} \cos \alpha t^* X(x) \right. \\ \left. + \frac{1}{\tau_0^2} e^{-\bar{t}/\tau_0} \cos \alpha t^* X(x) \right] = 0. \end{aligned} \quad (82)$$

The spatial dependence in Eq. (82) can be removed by taking into account the orthogonality of $X(x)$. Upon multiplying by $X(x)$ and then integrating with respect to x over the interval $0 \leq x \leq 1$, Eq. (82) reduces to

$$K_{x''} + 2\sqrt{\beta} \left(-\frac{1}{\tau_0} K_{x'} - \frac{\gamma}{2} \frac{1}{\tau_0} K_x \right) - 2\psi_2 \alpha^2 K_x + \frac{1}{\tau_0^2} K_x = 0 \quad (83)$$

where the notation defined in Eq. (71) has been used. The final expression for ψ_2 is

$$\psi_2 = \left(\frac{1}{2\alpha^2} \right) \left(\frac{K_{x''}}{K_x} - \frac{1}{\tau_0^2} \right) . \quad (84)$$

With this specific choice of ψ_2 , the forcing terms of Eq. (80) are dependent on F_1 only. That is,

$$L(F_2) = -2\sqrt{\beta} \left(\frac{\partial^2 F_1}{\partial t^* \partial x} + \frac{\gamma}{2} \frac{\partial F_1}{\partial t^*} \right) - 2 \left(\frac{\partial^2 F_1}{\partial \bar{t} \partial t^*} \right) . \quad (85)$$

Using Eq. (78), this equation becomes

$$\begin{aligned} L(F_2) = & -2\alpha \sin \alpha t^* \left[\frac{dC(\bar{t})}{d\bar{t}} X(x) + \sqrt{\beta} C(\bar{t}) \left(\frac{dX(x)}{dx} + \frac{\gamma}{2} X(x) \right) \right] \\ & + 2\alpha \cos \alpha t^* \left[\frac{dD(\bar{t})}{d\bar{t}} X(x) + \sqrt{\beta} D(\bar{t}) \left(\frac{dX(x)}{dx} + \frac{\gamma}{2} X(x) \right) \right] . \quad (86) \\ \triangleq & L_2(C, D) \end{aligned}$$

After applying the orthogonality condition, Eq. (70), in the same way that it was done in Eqs. (71)-(74), the restrictions on $C(\bar{t})$ and $D(\bar{t})$ are

$$\left. \begin{aligned} \frac{dC(\bar{t})}{d\bar{t}} + \sqrt{\beta} \left(\frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right) C(\bar{t}) &= 0 \\ \frac{dD(\bar{t})}{d\bar{t}} + \sqrt{\beta} \left(\frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right) D(\bar{t}) &= 0 \end{aligned} \right\} . \quad (87)$$

The solutions to Eqs. (87) are obtained directly using the initial conditions given in Eqs. (79). The result is

$$\left. \begin{aligned} C(\bar{t}) &= 0 \\ D(\bar{t}) &= \frac{1}{\alpha\tau_0} e^{-\bar{t}/\tau_0} \end{aligned} \right\} . \quad (88)$$

So, the complete first-order solution is finally generated and is

$$F_1(x, t^*, \bar{t}) = \frac{1}{\alpha\tau_0} e^{-\bar{t}/\tau_0} \sin \alpha t^* X(x) . \quad (89)$$

An approximate solution for the beam deflection which involves truncation not greater than $O(u^2)$ can now be assembled. The complete solution is

$$\begin{aligned} y(x, t^*, \bar{t}) &\sim F_0(x, t^*, \bar{t}) + uF_1(x, t^*, \bar{t}) + O(u^2) \\ &\sim e^{-\bar{t}/\tau_0} \left(\cos \alpha t^* + \frac{u}{\alpha\tau_0} \sin \alpha t^* \right) X(x) . \end{aligned} \quad (90)$$

The solution, Eq. (90), is more convenient if written in "unstretched" time by using Eqs. (48) and (84) and is summarized as

$$\begin{aligned} \tau_0 &= \left[\sqrt{\beta} \left(\frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right) \right]^{-1} , \\ \alpha' &= \alpha \left[1 + u^2 \left(\frac{1}{2\alpha^2} \right) \left(\frac{K_{x''}}{K_x} - \frac{1}{\tau_0^2} \right) \right] , \\ y(x, t) &= e^{-ut/\tau_0} \left[\cos \alpha' t + \frac{u}{\alpha\tau_0} \sin \alpha' t \right] X(x) , \\ X(x) &= K[(\sin \sqrt{\alpha} + \sinh \sqrt{\alpha})(\cos \sqrt{\alpha} x - \cosh \sqrt{\alpha} x) \\ &\quad - (\cos \sqrt{\alpha} + \cosh \sqrt{\alpha})(\sin \sqrt{\alpha} x - \sinh \sqrt{\alpha} x)] . \end{aligned} \quad (91)$$

The final solution is of the form expected — an exponentially decaying harmonic. The decay constant is comprised of two parts: (1) a part resulting from the nonconservative Coriolis effect that is proportional to u , (2) a part due to the external damping that is independent of u since $\gamma = cL/MU$.

Before proceeding to another analytical solution technique, attention is returned to simplifying assumption for the initial condition model. In Eq. (62) it was assumed that

$$y(x,0) = X(x) \quad (92)$$

where $X(x)$ is the fundamental mode shape of a nonhydroelastic cantilever. The simplest beam excitation system would be an actuator that applies a point load at a specific beam position to achieve the desired initial static deflection. The quality of the assumption implicit in Eq. (92) depends on how close the static deflection curve matches the fundamental mode shape. Large differences might result in undesirable multimode excitation.

The nondimensionalized static deflection curve for a simple cantilever with a unit load applied at the arbitrary location $x = a$, where $L = a + b$, is

$$y_s(x) = \frac{1}{3\left(\frac{a}{L}\right)^2 - \left(\frac{a}{L}\right)^3} \left[\left(x - \frac{b}{L}\right)^3 - 3\left(\frac{a}{L}\right)^2 \left(x - \frac{b}{L}\right) + 2\left(\frac{a}{L}\right)^3 \right] \quad (93)$$

A computer program was written to calculate the fundamental mode shape $X(x)$, the static deflection $y_s(x)$, Eq. (93), and the difference

between the two for various locations of the actuator force. Several iterations were performed until very small differences were observed between the two. The results are shown in Fig. 10. The percent difference is less than 1% over the entire length when the load is located at $3/4 L$. This result was judged to be sufficiently good to justify the assumption.

Solution through serendipity. The two-timing perturbation technique was successful in achieving a not altogether simple solution, Eqs. (91). Recall that in the perturbation solution, the "mixing" of spatial and temporal variables was manipulated by taking advantage of the spatial orthogonality properties of the recurring operator, $L(F_1)$, Eq. (58). Expressions in x , t^* , and $\bar{t}$ were multiplied by $X(x)$ and integrated with respect to x over $0 \geq x \geq 1$. In this way, spatial dependences were replaced by constants by virtue of Eqs. (71).

This type of manipulation motivated thought for a very simple Galerkin-like approach to the solution of the cantilever Coriolis flowmeter problem. If one were to assume at the outset that the spatial eigenfunctions of Eqs. (35)-(37) could be satisfactorily approximated by the eigenfunctions of a simple cantilever in the region of interest, then the orthogonality properties of $X(x)$ could be used to "uncouple" space and time in the original governing equation. As a result, the boundary value problem could then be solved directly through separation of variables. This serendipitous approach was tried on the slight chance that it would provide useful results.

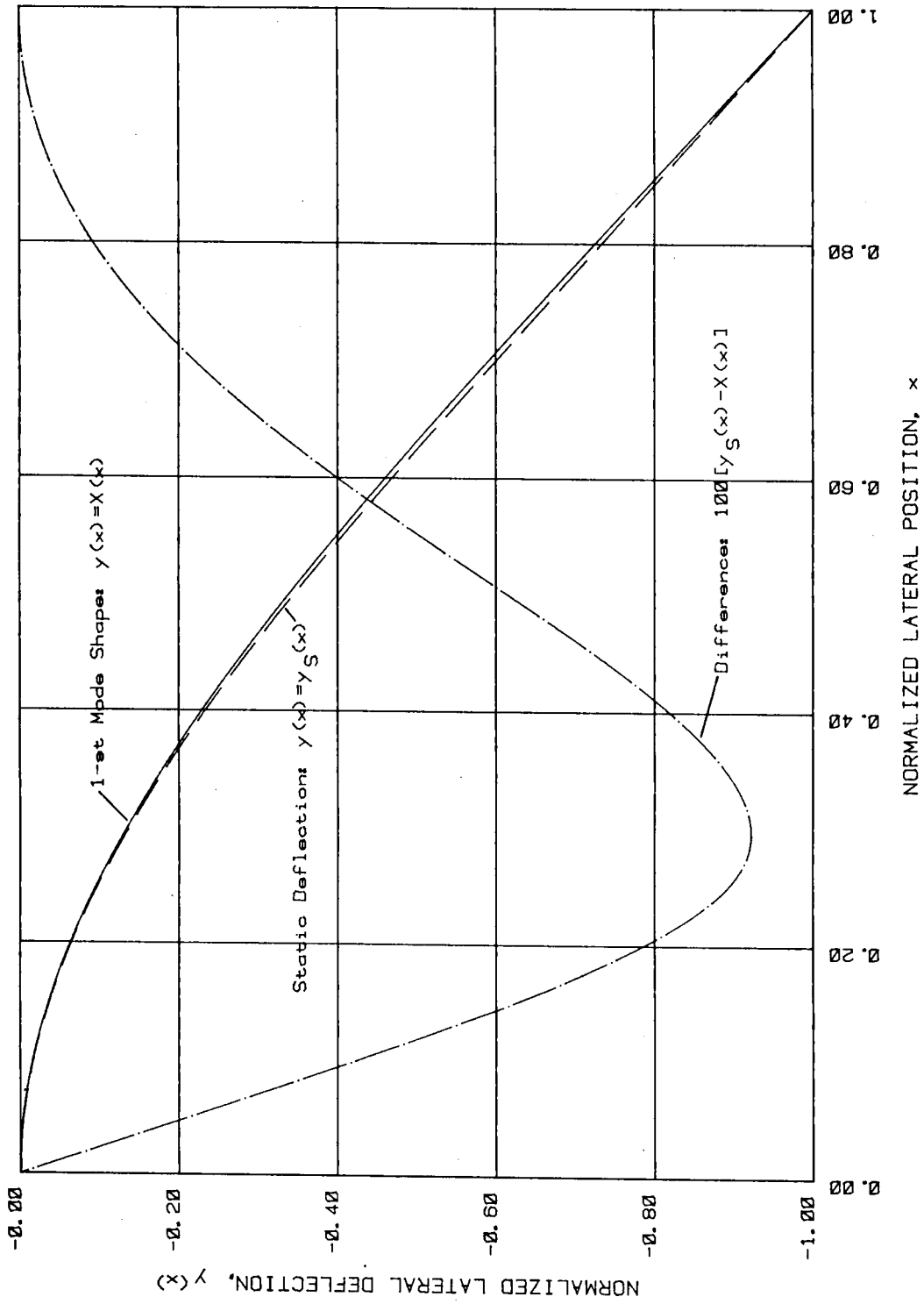


Fig. 10. Comparison of initial static deflection with fundamental mode shape.

The first step was to assume that the solution can be expressed as a product of the form

$$y(x,t) = X(x)T(t) , \quad (94)$$

where $X(x)$ belongs to the operator given in Eq. (58). In this approach there is no need to bury the external damping term within the parameter expansion and a more convenient form of Eq. (35) becomes

$$y_{xxxx} + u^2 y_{xx} + 2\sqrt{\beta} u y_{xt} + \sqrt{1-\beta} \frac{cL^2}{\sqrt{mEI}} y_t + y_{tt} = 0 , \quad (95)$$

where the relationship $M + m = m/(1 - \beta)$ has been used to simplify the external damping term. With Eq. (94), Eq. (95) can be rewritten as

$$\begin{aligned} \frac{d^4 X(x)}{dx^4} T(t) + u^2 \frac{d^2 X(x)}{dx^2} T(t) + 2\sqrt{\beta} u \frac{dX(x)}{dx} \cdot \frac{dT(t)}{dt} \\ + \sqrt{1-\beta} \bar{c} X(x) \frac{dT(t)}{dt} + X(x) \frac{d^2 T(t)}{dt^2} = 0 , \end{aligned} \quad (96)$$

where $\bar{c} = cL^2/\sqrt{mEI}$. Removal of the spatial variables begins with the procedure described above and results in

$$\int_0^1 \left[\frac{d^4 X}{dx^4} T + u^2 \frac{d^2 X}{dx^2} T + 2\sqrt{\beta} u \frac{dX}{dx} \cdot \frac{dT}{dt} + \sqrt{1-\beta} \bar{c} X \frac{dT}{dt} + X \frac{d^2 T}{dt^2} \right] dx = 0 . \quad (97)$$

It can be shown that $d^4 X/dx^4 = \alpha^2 X$, and with Eq. (71), Eq. (97) reduces to the ordinary differential equation

$$\frac{d^2T}{dt^2} + \left(2\sqrt{\beta}u \frac{K_{x'}}{K_x} + \sqrt{1 - \beta} \frac{\bar{c}}{c} \right) \frac{dT}{dt} + \left(u^2 \frac{K_{x''}}{K_x} + \alpha^2 \right) T = 0 . \quad (98)$$

Equation (96) describes the time response of the cantilever and can be solved directly by assuming a solution of the form $T(t) = Ce^{st}$, where s is the familiar Laplace transform variable. The characteristic equation of Eq. (98) then becomes

$$s^2 + \left(2\sqrt{\beta}u \frac{K_{x'}}{K_x} + \sqrt{1 - \beta} \frac{\bar{c}}{c} \right) s + \left(u^2 \frac{K_{x''}}{K_x} + \alpha^2 \right) = 0 \quad (99)$$

for which the two distinct roots are

$$s_{1,2} = \underbrace{\left(\sqrt{\beta}u \frac{K_{x'}}{K_x} + \sqrt{1 - \beta} \frac{\bar{c}}{2c} \right)}_{\text{decay constant, } \tau^{-1}} \pm i \left[\underbrace{\left(u^2 \frac{K_{x''}}{K_x} + \alpha^2 \right) - \left(\sqrt{\beta}u \frac{K_{x'}}{K_x} + \sqrt{1 - \beta} \frac{\bar{c}}{2c} \right)^2}_{\text{frequency, } \bar{\omega}} \right]^{\frac{1}{2}} . \quad (100)$$

This result is the familiar second-order system solution common to the theory of linear vibrations. The solution can be written directly as

$$T(t) = e^{-t/\tau} (A \cos \bar{\omega}t + B \sin \bar{\omega}t) . \quad (101)$$

Applying the initial conditions, Eqs. (37), defines the arbitrary constants such that the solution is

$$T(t) = T(0)e^{-t/\tau} \left(\cos \bar{\omega}t + \frac{1}{\bar{\omega}\tau} \sin \bar{\omega}t \right) . \quad (102)$$

With this result for the time solution, the total solution can be written as

$$y(x,t) = y(x,0)e^{-t/\tau} \left(\cos \bar{\omega}t + \frac{1}{\bar{\omega}\tau} \sin \bar{\omega}t \right) , \quad (103)$$

recalling that $y(x,0) = X(x)$. So with an admittedly unsystematic approximation procedure, a complete solution has been obtained with a small fraction of the effort expended in developing the perturbation solution. In the next section, the numerical solution and the two analytical solutions are compared.

Comparison of Solutions

Three solutions to the cantilever Coriolis flowmeter problem were obtained using different techniques. The numerical product solution was considered to be the closest of the three to an exact solution since it involved the fewest simplifying assumptions and exhibited good convergence properties. The first approach used to compare the solutions was to simply plot the decay constants and frequencies as functions of u and β with $\gamma = 0$. The external damping term contributes a simple offset in the decay constant at $u = 0$ such that a comparison of the total decay constant minus the external damping term provides a complete description. The results of the comparison were both fortifying and surprising. The nondimensional decay constants are shown in Fig. 11 where the real part

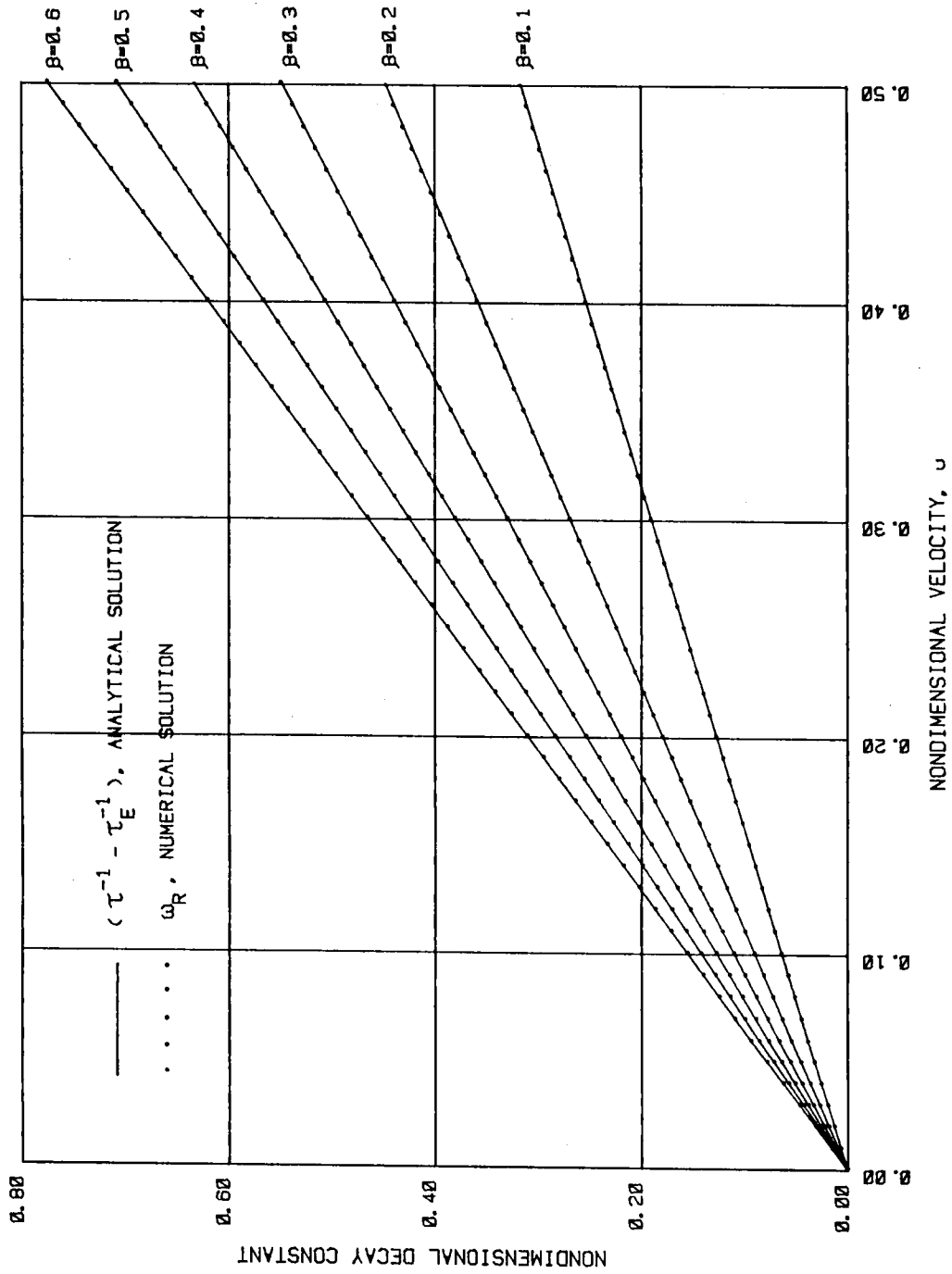


Fig. 11. Comparison of decay constant models.

of the numerically calculated temporal eigenvalues are plotted with the decay constants given in Eqs. (91) and (100). The orthogonality constants, Eq. (71), were calculated using the double-precision numerical integration program listed in Appendix B. The results were $K_x = 0.25000$, $K_x' = 0.50000$, and $K_x'' = 0.21456$. As seen in Fig. 11, the curves superimpose with no perceivable differences at this plotting resolution.

The imaginary part of the numerical eigenvalues are plotted with the frequency expressions given in Eq. (91) and the imaginary part of Eq. (100) in Fig. 12. In this case, the analytical expressions again coincide with no perceivable difference, but both diverge from the numerical calculation as the u increases. This result is compatible with the rigor of the perturbation solution in which the truncation error is known to be $O(u^2)$. As seen in Fig. 12, as u increases the error (i.e., the difference between the analytical expressions and the numerical results) also increases. The difference is small below $u = 0.2$. The frequency shift characteristics are actually quite small as is shown more clearly in percent difference format (with respect to the fundamental eigenvalue) in Fig. 13. One interesting aspect of the frequency shift is that there is a β value between 0.2 and 0.3 for which the frequency is independent of flow velocity, u . This value can be predicted ($\beta \sim 0.248$) using the expression in Eq. (91). No real utility for this property is seen for the flow measurement application.

The excellent correlation between the three independent (at least seemingly) solutions was most encouraging. The unbelievable coincidence of the two analytical solutions suggested that their equivalence might

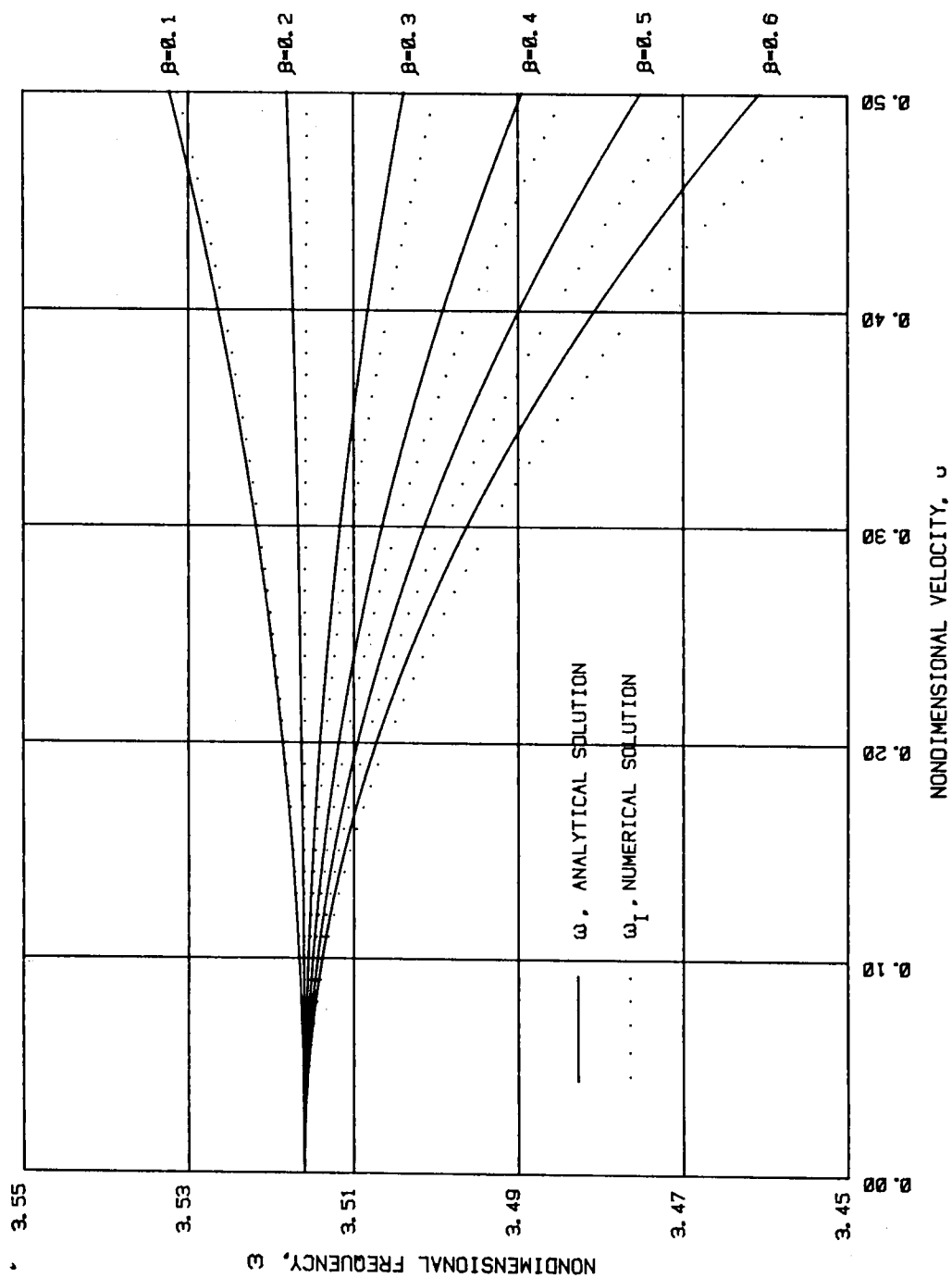


Fig. 12. Comparison of frequency models.

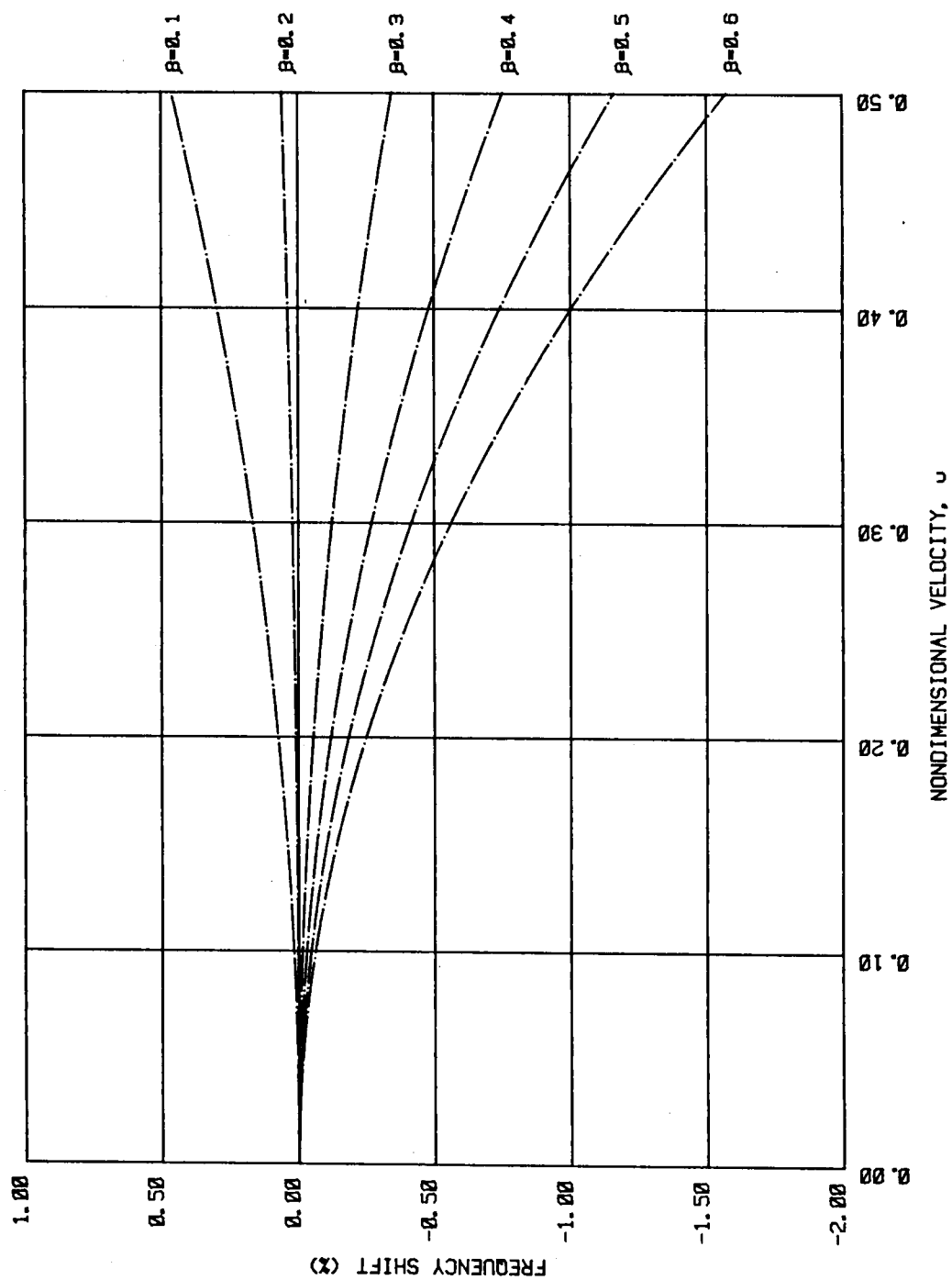


Fig. 13. Frequency shift in percent difference.

be shown algebraically. This equivalence is certainly not obvious by comparing Eqs. (91) and (100) since different expressions for the external damping terms were used in the derivations for convenience. The "perturbation" decay constant in Eq. (91) is

$$u\tau_0^{-1} = \sqrt{\beta} \left(u \frac{K_{x'}}{K_x} + \frac{\gamma}{2} \right). \quad (104)$$

This expression can be altered using the definition given in Eq. (18) for u and $\gamma = cL/MU$ to give

$$u\tau_0^{-1} = \sqrt{\beta} u \frac{K_{x'}}{K_x} + \frac{cL^2}{2\sqrt{MEI}} \sqrt{\beta}. \quad (105)$$

The definition of β given in Eq. (18) can be rewritten as $(1 - \beta)/m = \beta/M$ and used in Eq. (105) to obtain

$$u\tau_0^{-1} = \sqrt{\beta} u \frac{K_{x'}}{K_x} + \frac{\sqrt{1 - \beta}}{2} \frac{cL^2}{\sqrt{mEI}}, \quad (106)$$

and since $\bar{c} = cL^2/\sqrt{mEI}$

$$u\tau_0^{-1} = \sqrt{\beta} u \frac{K_{x'}}{K_x} + \frac{\sqrt{1 - \beta}}{2} \bar{c}. \quad (107)$$

Equation (107) is identical to the real part of Eq. (100) as expected from the plotting results.

To compare the analytical frequency expressions, consider the square of the imaginary part of Eq. (100),

$$\begin{aligned}
\bar{\omega}^2 &= \left[\left(u^2 \frac{K_{X''}}{K_X} + \alpha^2 \right) - \left(\sqrt{\beta} u \frac{K_{X'}}{K_X} + \sqrt{1-\beta} \frac{\bar{c}}{2} \right)^2 \right] \\
&= u^2 \frac{K_{X''}}{K_X} + \alpha^2 - \tau^{-2} .
\end{aligned} \tag{108}$$

Using the equivalence established in Eq. (107), Eq. (108) can be rewritten as

$$\begin{aligned}
\bar{\omega}^2 &= u^2 \frac{K_{X''}}{K_X} + \alpha^2 - u^2 \tau_0^{-2} \\
&= \alpha^2 \left[1 + \frac{u^2}{\alpha^2} \left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right) \right] .
\end{aligned} \tag{109}$$

Now, the α' -expression in Eq. (91) is the "perturbation" frequency term and when squared becomes

$$\begin{aligned}
(\alpha')^2 &= \alpha^2 \left[1 + \frac{u^2}{2\alpha^2} \left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right) \right]^2 \\
&= \alpha^2 \left[1 + \frac{u^2}{\alpha^2} \left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right) + \frac{u^4}{4\alpha^4} \left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right)^2 \right] \\
&= \alpha^2 \left[1 + \frac{u^2}{\alpha^2} \left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right) \right] .
\end{aligned} \tag{110}$$

The last term of Eq. (110) is fourth-order in the small parameter u and is negligible. It can be neglected so that the frequency expressions are also found to be equivalent.

The last basis of comparison involves the amplitudes of the harmonic terms in Eqs. (91) and (100). The cosine terms in both solutions have unity amplitude, but the sine terms appear to be somewhat different. The "serendipity" sine term can be written in the form

$$\begin{aligned} \frac{1}{\omega\tau} &= \frac{u}{\alpha' \tau_0} \\ &= \frac{u}{\alpha\tau_0} \left\{ \frac{1}{\left[1 + \frac{u^2}{2\alpha^2} \left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right) \right]} \right\}, \end{aligned} \quad (111)$$

using Eqs. (110) and (107). Subtracting the "perturbation" sine term amplitude from both sides gives

$$\begin{aligned} \left(\frac{1}{\omega\tau} - \frac{u}{\alpha\tau_0} \right) &= \frac{u}{\tau_0} \left(\frac{1}{\alpha'} - \frac{1}{\alpha} \right) \\ &= \frac{u}{\alpha\tau_0} \left\{ \frac{1 - \left[1 + \frac{u^2}{2\alpha^2} \left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right) \right]}{\left[1 + \frac{u^2}{2\alpha^2} \left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right) \right]} \right\}. \end{aligned} \quad (112)$$

After some manipulation, the amplitude difference can be expressed as

$$\left(\frac{1}{\omega\tau} - \frac{u}{\alpha\tau_0} \right) = \frac{u^3}{2\alpha^3\tau_0} \left\{ \frac{-\left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right)}{\left[1 + \frac{u^2}{2\alpha^2} \left(\frac{K_{X''}}{K_X} - \tau_0^{-2} \right) \right]} \right\}. \quad (113)$$

The denominator of this expression is $O(1)$ such that the entire expression will be $O(u^3)$ within a factor of 10 depending on the actual value of the

external damping. In any event, the amplitude difference between the two solutions is insignificant. The two analytical results are approximately equivalent; with these results it is not possible to indicate which of the two expressions is more correct.

In summary, two analytical expressions for the solution of the cantilever Coriolis flowmeter model have been obtained. Both of these solutions agree well with the numerical product solution, and differences between the results are consistent with the magnitudes of the errors predicted in the approximate perturbation solution. The two analytical solutions were shown to have equivalent decay constants and frequencies. Differences in the amplitudes of harmonic terms were shown to be at least $O(u^2)$ and therefore insignificant.

CHAPTER 3

MATHEMATICAL FORMULATIONS FOR FLOW AND DENSITY MEASUREMENT

In Chapter 2, reasonably good (i.e., having close agreement with an exact numerical solution) analytical expressions for the deflection of a hydroelastic cantilever operating in the parametric range of flow measurement applications were obtained. The good agreement achieved between the three different solution methods provided optimism for the use of these expressions in further cantilever Coriolis flowmeter development.

In this chapter, the results of the previous chapter are focused more directly to flow and density measurements. The analytical expressions are converted from the nondimensional space to realtime and used in the development of measurement and calibration algorithms based on realtime deflection measurements. Next, cantilever Coriolis flowmeter synthesis issues are considered through a simplified parameter sensitivity analysis and a discussion of the attendant design sequence. Finally, the important alternative concept of steady-state harmonic beam excitation is discussed in comparison with the transient excitation being considered in this research.

Realtime Analytical Expressions

In the analysis of the cantilever Coriolis flowmeter boundary value problem, scaling considerations lead to the nondimensionalization of

time. Consequently, the final expressions obtained are not directly useable in the actual flowmeter algorithm development that must be based on realtime measurements (i.e., beam deflection). These nondimensional expressions must be transformed back to realtime variables.

It was decided to use the "serendipity" solution results, Eqs. (98), (100), and (103), in the realtime algorithm development because these expressions are more compact algebraically than the expressions obtained through perturbation analysis, Eqs. (91). All of the solutions were time-scaled according to the scale factor given in Eq. (16). Using this scale factor and the parameter definitions given for u and β in Eq. (18), the "serendipity" expressions can be converted to realtime. The ordinary differential Eq. (98) that governs the time dimension of the beam becomes

$$\begin{aligned} \ddot{\tilde{T}} + \left(2\sqrt{\beta}u \frac{K_{x'}}{K_x} + \sqrt{1 - \beta}c \right) \dot{\tilde{T}} \tilde{T}^{-1} + \left(u^2 \frac{K_{x''}}{K_x} + \alpha^2 \right) \tilde{T} \tilde{T}^{-2} &= 0 , \\ \ddot{\tilde{T}} + \left[2\beta \frac{K_{x'}}{K_x} \frac{U}{L} + (1 - \beta) \frac{c}{m} \right] \dot{\tilde{T}} + \left[\beta \frac{K_{x''}}{K_x} \frac{U^2}{L^2} + (1 - \beta) \frac{\alpha^2 EI}{mL^4} \right] \tilde{T} &= 0 . \end{aligned} \quad (114)$$

In a similar fashion, the solution results can be returned to realtime in the form

$$y(x, \tilde{t}) = e^{-\tilde{t}/(\tau\tilde{T})} \left[\cos(\tilde{\omega}\tilde{T}^{-1})\tilde{t} + \frac{1}{\tilde{\omega}\tau} \sin(\tilde{\omega}\tilde{T}^{-1})\tilde{t} \right] \chi(x) , \quad (115)$$

where

$$(\tau\tilde{T})^{-1} = \beta \frac{K_{x'}}{K_x} \frac{U}{L} + (1 - \beta) \frac{c}{2m} , \quad (116)$$

$$(\tilde{\omega}T^{-1})^2 = (1 - \beta) \frac{\alpha^2 EI}{mL^4} + \beta \frac{K_{x'}}{K_x} \frac{U^2}{L^2} - \left[\beta \frac{K_{x'}}{K_x} \frac{U}{L} + (1 - \beta) \frac{c}{2m} \right]^2. \quad (117)$$

and $X(x)$ is the same as Eq. (60). The decay constant, Eq. (116), and the frequency, Eq. (117), contain terms that are dependent on the empty-tube characteristics only. The external damping coefficient is not a function of flow and is the second term in Eq. (116). The following definition is introduced to simplify notation

$$\tau_e^{-1} \triangleq \frac{c}{2m}. \quad (118)$$

The fundamental natural frequency of the empty tube is the first term in Eq. (117), and it also is independent of flow. This term is the nonhydroelastic eigenvalue given by Wylie [41], and is

$$\omega_e^2 \triangleq \frac{\alpha^2 EI}{mL^4}. \quad (119)$$

Using this definition, the time constant and frequency expressions can be written as combinations of the empty-pipe characteristics, fluid density (implicitly), and fluid mean velocity. That is,

$$\tau^{-1} = \beta \frac{K_{x'}}{K_x} \frac{U}{L} + (1 - \beta) \tau_e^{-1}, \quad (120)$$

$$\bar{\omega}^2 = (1 - \beta) \omega_e^2 + \beta \frac{K_{x''}}{K_x} \frac{U^2}{L^2} - \tau^{-2}. \quad (121)$$

where from this point on, all variables are understood to be in realtime.

A plot of the theoretical response curves in realtime using typical parameter values is shown in Fig. 14.

The parameter β was defined as $\beta = M/(M + m)$, where M is the mass of the fluid per unit length contained in the tube, and m is the mass per unit length of the tube material itself. With this definition, β is a function of the tube material density, the flowing fluid density, and the dimensions and geometry of the tube cross section. Because of the ready availability of a wide range of sizes, beam materials were limited to standard tubular stock with circular cross sections. If it is assumed that the tube has inside and outside diameters of d_i and d_o respectively, then

$$M = \rho A = \rho \frac{\pi d_i^2}{4} \quad (122)$$

and

$$m = \rho_T A_o = \rho_T \frac{\pi}{4} (d_o^2 - d_i^2) . \quad (123)$$

Using Eqs. (122) and (123), β can be expressed as a function of the diameter ratio and the density ratio as follows:

$$\beta = \frac{1}{1 + \frac{\rho_T}{\rho} \left[\left(\frac{d_o}{d_i} \right)^2 - 1 \right]} . \quad (124)$$

Density variations in the flowing fluid affect the hydroelastic beam response through the β parameter. Since the tube material density and diameter ratio should be constant, the β parameter is a function of ρ

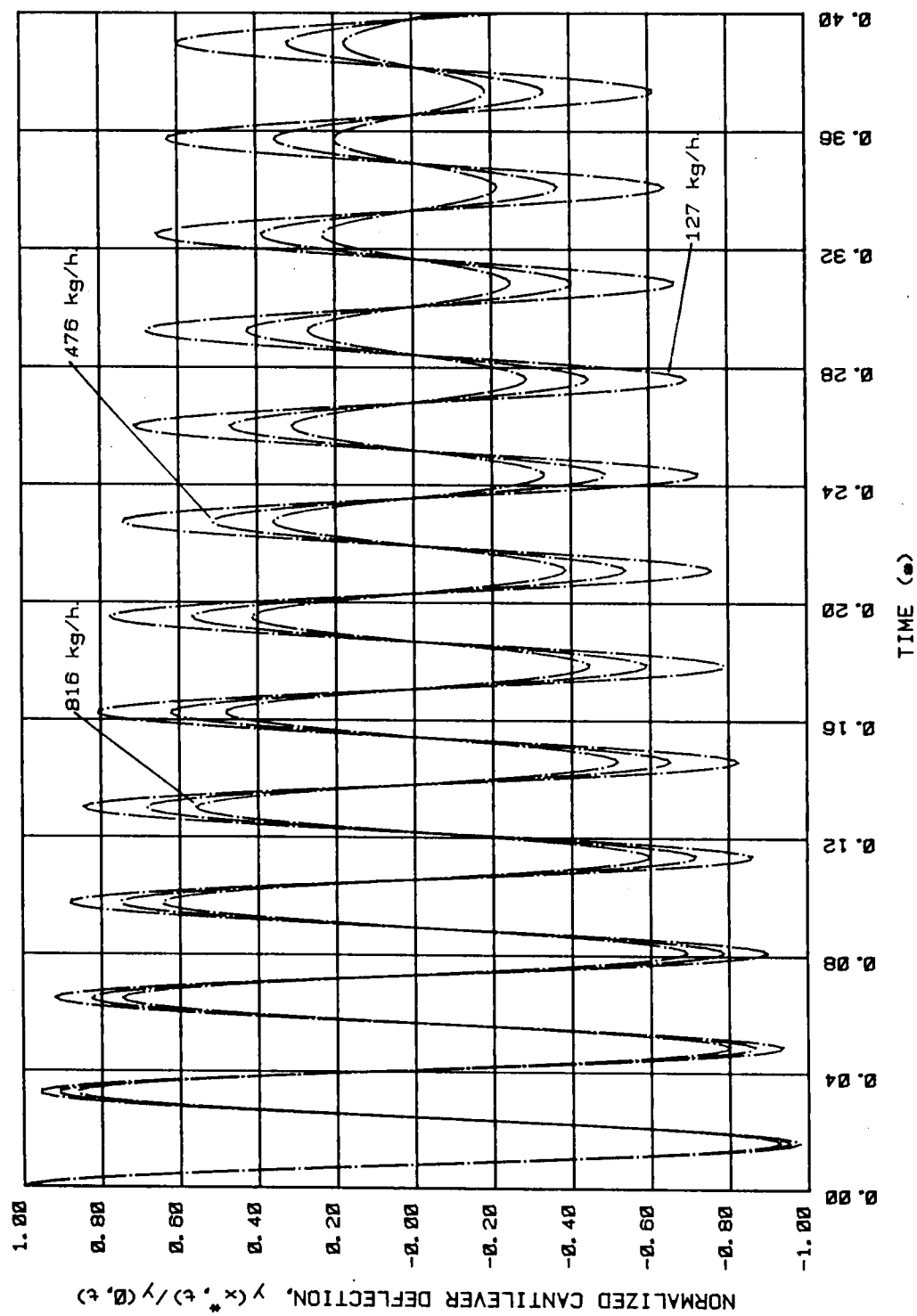


Fig. 14. Theoretical response curves for typical beam parameters.

alone. Note, however, that this is an area where second-order effects such as thermal expansion can introduce measurement error. Curves for β as a function of fluid specific gravity and diameter ratio with a steel-like material density are shown in Fig. 15. Equation (124) can be written as

$$\beta = \frac{\rho}{\rho + K_{\rho}} , \quad (125)$$

where $K_{\rho} \triangleq \rho_T \left[\left(\frac{d_o}{d_i} \right)^2 - 1 \right]$. Solving Eq. (126) for ρ gives

$$\rho = \frac{K_{\rho} \beta}{(1 - \beta)} , \quad (126)$$

which is the companion Eqs. (120) and (121) necessary to define the flowing fluid density.

Flow and Density Measurement Algorithms

Equations (115), (120), (121), and (126) relate the lateral deflection of a hydroelastic cantilever to the mean fluid velocity and density as a function of time and distance along the beam. In addition, the beam dynamic response to an initial static deflection is a function of several constants and empty tube parameters that are assumed not to vary during the short interval of deflection measurement. These equations are used in conjunction with practical transducer concepts

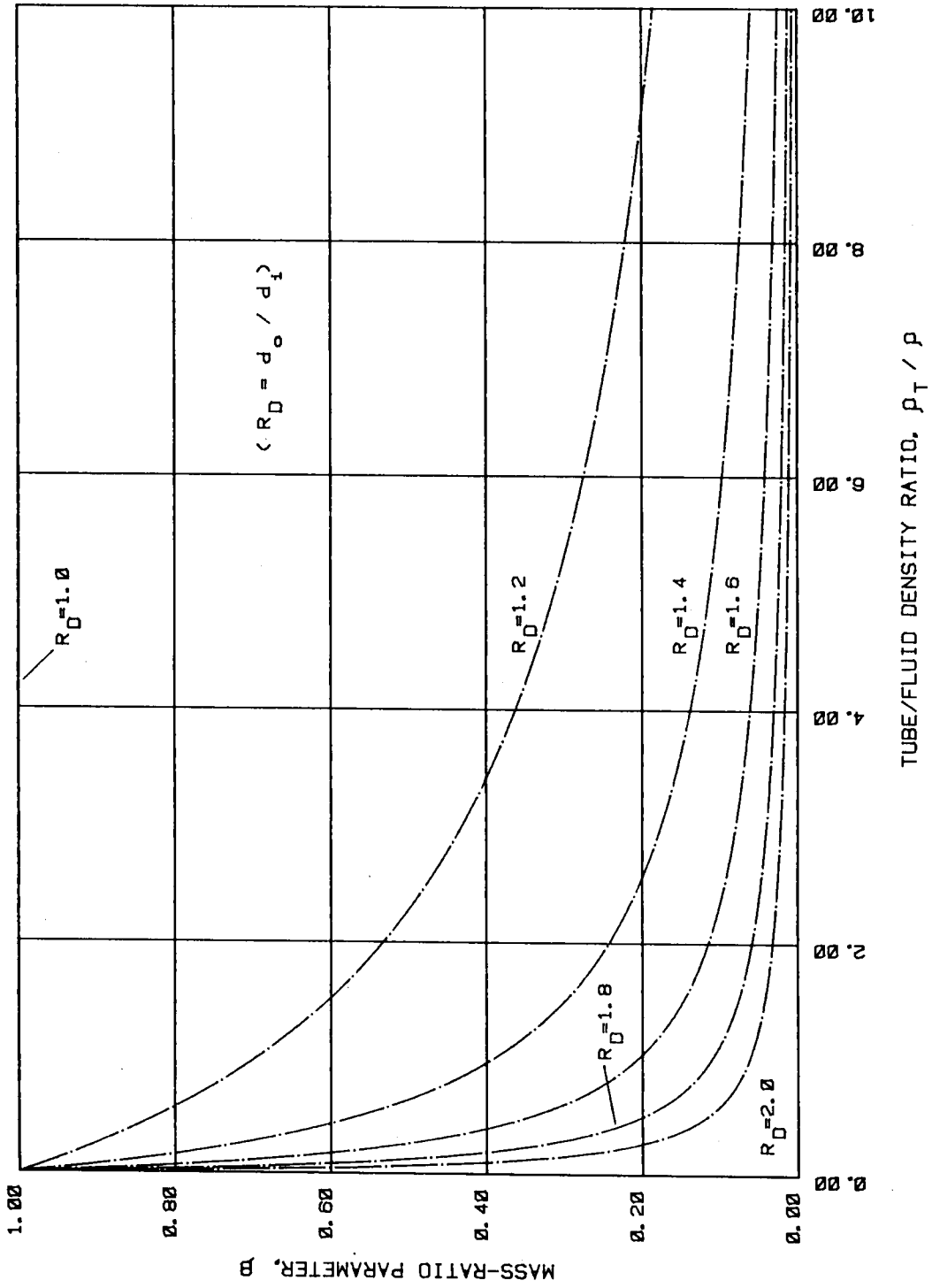


Fig. 15. Mass-ratio parameter characteristics for circular cross sections.

to develop algorithms that can be used to transform physical measurements of the beam response into the mass flow rate, mean velocity, and density.

Direct Deflection Algorithm

Equations (114) and (115) show that the cantilever Coriolis flowmeter dynamic response as viewed from a specific longitudinal position is a damped second-order system response. This is fully characterized by the decay constant, Eq. (120), and the vibration frequency, Eq. (121), with amplitude determined by the magnitude of the initial deflection. Note that if the decay constant and frequency were known Eqs. (120) and (121) could be viewed as two equations in the unknowns U and β (or ρ).

The decay constant and frequency of a second-order linear response can be determined readily if a time record of sufficient length and resolution is available. The direct measurement of the cantilever deflection as a function of time at a specific location can be accomplished easily in a number of different ways using strain measurements or relative deflection measurements. Equation (119) shows that typical flowmeter applications would involve vibration frequencies less than 100 to 200 Hz such that high-speed digital measurements (~ 5000 samples/s) would provide adequate resolution. Obtaining a high-quality "snapshot" of the beam transient response can be achieved with available transducer and digital data acquisition technology. Once the transient "snapshot" is stored in memory, the decay constant and frequency of the waveform can be calculated rather simply. The essential features of the waveform are shown in Fig. 16.

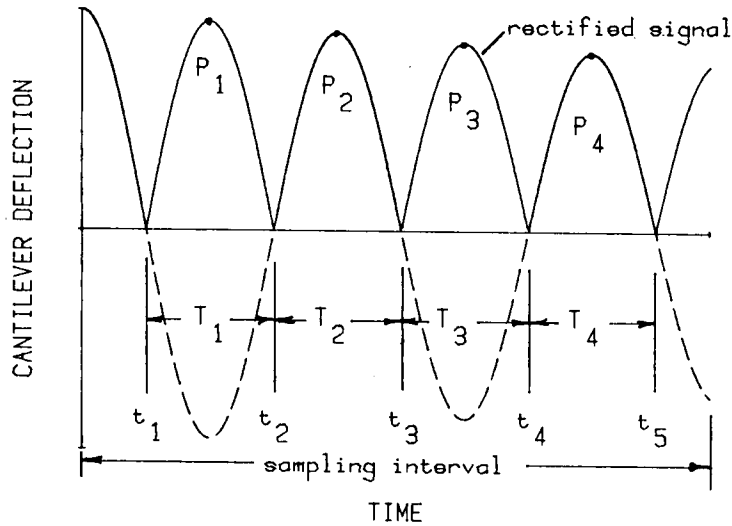


Fig. 16. Waveform analysis.

The frequency can be determined by searching for the zero-crossings of the waveform. The distance between consecutive zero-crossings is one-half of a full period and can be used to calculate frequency as follows:

$$\left. \begin{aligned} T_j &= t_{k+1} - t_k \\ f &= \frac{1}{n} \sum_{j=1}^n \left(\frac{1}{2T_j} \right) \\ \bar{\omega} &= 2\pi f \end{aligned} \right\}, \quad (127)$$

where n is the number of zero crossings. The peaks of the transient waveform can be located in the stored data quickly since they are known to be approximately half-way between the zero-crossings. The decay constant for a simple exponential can be calculated with knowledge

of the waveform peaks and period using logarithmic decrement [51]. That is,

$$\tau^{-1} = \left(\frac{f}{2n_{\text{peak}}} \right) \ln \left(\frac{P_1}{P_{n_{\text{peak}}}} \right) . \quad (128)$$

After first sampling and storing deflection data, Eqs. (127) and (128) and waveform analysis routines which define the period and peak values can be used to attach numerical values to τ^{-1} and $\bar{\omega}$. These actual "readings" for τ^{-1} and $\bar{\omega}^2$ will be called R_1 and R_2 to distinguish them from the variables τ^{-1} and $\bar{\omega}^2$. Since R_1 and R_2 are known through direct measurement, Eqs. (120) and (121) become

$$\left. \begin{aligned} R_1 &= a_2 \beta U + (1 - \beta) \tau_e^{-1} \\ R_2 + R_1^2 &= a_3 \beta U^2 + (1 - \beta) \omega_e^2 \\ \rho &= \frac{a_1 \beta}{(1 - \beta)} \end{aligned} \right\} \quad (129)$$

which are three equations in the three unknowns, β , U , and ρ . The constants a_1 , a_2 , and a_3 are functions of the beam dimensions. The variables τ_e^{-1} and ω_e^{-1} are constants that describe the dynamic characteristics of the cantilever when it is empty. These constants can be determined by either of two ways: (1) they can be estimated using dimensional measurements, material properties data, and the analytical expressions Eqs. (118), (119), (120), (121), and (124); or (2) they can be determined through calibration with respect to a reliable standard that in effect calculates the unknown constants under test conditions

in which U and ρ are measured precisely. The normal practice in instrumentation engineering is the second case because it generally results in higher accuracy and permits meaningful and traceable comparison of measurement results.

Techniques have been developed for the determination of these various calibration constants. The empty tube characteristic constants can be determined by performing the basic measurement sequence (i.e., excitation, data acquisition, and waveform analysis) when the tube is empty. When the tube is empty, $\rho = 0$, $\beta = 0$, and $U = 0$, and Eqs. (129) becomes

$$\left. \begin{aligned} R_1 &\Rightarrow R_{C1} = \tau_e^{-1} \\ R_2 + R_1^2 &\Rightarrow R_{C2} + R_{C1}^2 = \omega_e^2 \\ \rho &= 0 \end{aligned} \right\} . \quad (130)$$

The calibration "readings," R_{C1} and R_{C2} , are saved and used as numerical values of the constants τ_e^{-1} and ω_e^2 . Equations (129) become

$$\left. \begin{aligned} R_1 &= a_2 \beta U + (1 - \beta) R_{C1} \\ R_2 + R_1^2 &= a_3 \beta U^2 + (1 - \beta) R_{C2} \\ \rho &= \frac{a_1 \beta}{(1 - \beta)} \end{aligned} \right\} . \quad (131)$$

The remaining constants are determined by operating (or calibrating) the cantilever Coriolis flowmeter in series with another flowmeter or system that measures the flow variables with known accuracy and precision. This secondary flow measurement defines p and U for given readings of R_1 and

R_2 . In this way, Eqs. (131) can be interpreted as three equations in the unknown calibration constants a_1 , a_2 , and a_3 . The solution becomes

$$\left. \begin{aligned} a_1 &= \frac{\rho(1 - \beta)}{\beta} \\ a_2 &= \frac{R_1 - (1 - \beta)R_{C1}}{\beta U} \\ a_3 &= \frac{(R_2 + R_1^2) - (1 - \beta)C_2}{\beta U^2} \end{aligned} \right\} . \quad (132)$$

The flow conditions (ρ , U) are varied over the flow measurement range so that the statistical properties (means and standard deviations) a_2 and a_3 can be calculated to assess the calibration quality. The calibration constant a_1 quantifies the relationship between fluid density and β as shown in Eq. (125). This is a static relationship and is determined through simple weighing tests using the cantilever tube and fluids of known density. If the standard deviations of these calibration constants are found to be large, it is likely that the assumption that they are constants is invalid. The mean values for the calibration constants are stored for use in the flow measurement algorithm.

For a specific flow reading, the cantilever Coriolis flowmeter model embodied in Eqs. (131) becomes three equations in three unknowns, ρ , β , and U since the constants a_1 , a_2 , a_3 , R_{C1} , and R_{C2} have been determined through calibration. Realtime algorithms that transform the R_1 and R_2 measurements into β , ρ , and U are the final component needed to realize a flowmeter. The algebraic solution of Eqs. (131) provides the necessary expressions. The first two equations can be combined into a quadratic expression in U and the readings R_1 and R_2

as follows:

$$U^2 + C_1 U + C_2 = 0 \quad (133)$$

where

$$\left. \begin{aligned} C_1 &= \frac{a_2(R_{C2} - R_2 - R_1^2)}{a_3(R_1 - R_{C2})} \\ C_2 &= \frac{(R_1^2 + R_2)R_{C1} - R_1 R_2}{a_3(R_1 - R_{C1})} \end{aligned} \right\} . \quad (134)$$

The flow measurement algorithms can be written as

$$\left. \begin{aligned} U &= -\frac{C_1}{2} + \frac{\sqrt{C_1^2 - 4C_2}}{2} \\ \beta &= \frac{R_1 - R_{C1}}{a_2 U - R_{C1}} \\ \rho &= \frac{a_1 \beta}{(1 - \beta)} \end{aligned} \right\} . \quad (135)$$

The positive sign in the quadratic-root pair is selected to ensure that $U \geq 0$. Since damping increases with velocity, $R_1 > R_{C1}$, and since natural frequencies increase with decreasing mass, the fundamental frequency of the cantilever is greater when empty than when it is filled with fluid and $R_2 + R_1^2 < R_{C2}$. All of the calibration constants a_1 , a_2 , and a_3 are nonnegative. These conditions require that $(R_1 - R_{C1}) > 0$ and $[(R_1^2 + R_2)R_{C1} - R_1 R_{C2}] < 0$, or $C_1 > 0$ and $C_2 < 0$. Consequently, the expression in the radical is positive and real. The radical expression is greater than or equal to $C_1/2$ making the positive sign choice

imperative if $U \geq 0$. Reassuringly, physical considerations restrict the mathematical options properly.

Early experimental work revealed some interesting characteristics of the flow measurement algorithm. When the calibration routines were executed, large uncertainties in the a_3 calibration constant were observed. In fact, often the standard deviation of a_3 would be larger than the mean for a 10- to 15-point calibration. This observation suggested that the calibration was insensitive to U term. Taking a closer look at the parameter values being used in the experimental set-up, it was found that $a_3\beta U^2 \sim O(10)$, but $(1 - \beta)\omega_e^2 \sim O(10^4)$, which is to say that the frequency shift due to fluid flow is negligible. In the calibrations, this term is down in the noise level, which causes a_3 to have a very large variance. The algorithm was modified to take into account this important simplification. With $a_3 = 0$, Eqs. (131) become

$$\left. \begin{aligned} R_1 &= a_1\beta U + (1 - \beta)R_{C1} \\ R_2 + R_1^2 &= (1 - \beta)R_{C2} \\ \rho &= \frac{a_1\beta}{(1 - \beta)} \end{aligned} \right\} . \quad (136)$$

The actual flow measurement algorithms used in the proof of principle experiment discussed in Chapter 4 were obtained by solving for specific variables in the right-hand side of Eqs. (136) in terms of the left-hand side variables (the measurements). The result is

$$\left. \begin{aligned}
 (1 - \beta) &= \frac{R_2 + R_1^2}{R_{C2}} \\
 a_2 \beta U &= R_1 - (1 - \beta) R_{C1} \\
 \rho &= \frac{a_1(\beta)}{(1 - \beta)} \\
 \dot{m} &= K_{\dot{m}} \left[\frac{a_2 \beta U}{(1 - \beta)} \right] \\
 U &= \frac{a_2 \beta U}{a_2 \beta}
 \end{aligned} \right\}, \quad (137)$$

where the calibration constants are more effectively calculated through

$$\left. \begin{aligned}
 a_1 &= \frac{\rho(1 - \beta)}{\beta} \\
 a_2 &= \frac{R_1 - (1 - \beta) R_{C1}}{\beta U} \\
 K_{\dot{m}} &= \frac{\dot{m}_{\text{secondary}}(1 - \beta)}{a_2 \beta U}
 \end{aligned} \right\}, \quad (138)$$

using a secondary measurement of ρ , U and $\dot{m}$. Measurement and calibration algorithms given in Eqs. (137) and (138) are the algorithms used in the proof of principle experiments reported in Chapter 4. Computer programs for their realtime implementation are listed in Appendix C in subroutines FLO25T.FOR and EFC25T.FOR.

Deflection Transducer Position

One interesting feature of the measurement algorithms is that they do not require precise longitudinal positioning of the deflection sensor. This is a desirable feature of the cantilever Coriolis flowmeter concept

because it means one less design constraint and possible source of error. To demonstrate that the deflection sensor location is arbitrary, assume that the deflection $y(x^*, t)$ is measured at x^* with a transducer having an output signal $e(t)$, presumably electrical. The cantilever Coriolis flowmeter model, Eq. (103), says that the sensor output will be

$$\begin{aligned} y(x^*, t) &= T(t)X(x^*) \\ &= K_e e(t) . \end{aligned} \quad (139)$$

Solving for $T(t)$ shows that the time varying sensor output is directly proportional to the time response function. That is,

$$\begin{aligned} T(t) &= \frac{K_e}{X(x^*)} e(t) \\ &\propto e(t) . \end{aligned} \quad (140)$$

As long as the deflection sensor has sufficient measurement sensitivity, the actual magnitude of its output due to total channel gain and beam location is not important in the calculation of the τ^{-1} and $\bar{\omega}^2$ represented in $e(t)$.

Flowmeter Synthesis

With the basic form of the measurement and calibration algorithms established, the synthesis problem associated with the cantilever Coriolis flowmeter concept can be considered. The flowmeter design process, in terms of options and constraints, is addressed through a cursory parameter sensitivity analysis and design sequence definition.

Parameter Sensitivity Analysis

The "best" flowmeter is considered to be the one having the greatest sensitivity to flow rate variations over a given range of measurement. Flowmeter parameters should then be chosen to maximize this sensitivity. Flow rate variations in the stream being measured can manifest themselves in either density or mean velocity changes (assuming all other effects are negligible). In the direct deflection measurement algorithm, the flow condition is assessed through the analysis of the decay and frequency characteristics of the vibratory response. Measurement sensitivity may be thought of as the strength of the dependence of these response characteristics to changes in mass flow rate. Mass flow rate is a function of both density and mean velocity for a given tube geometry. In the direct deflection algorithm, density variations alter the vibration frequency, but both density and velocity variations alter the vibration decay rate. One would expect that it would be possible to express the mass flow rate as a function of the system parameters, τ^{-1} and $\bar{\omega}^2$. The mass flow rate is

$$\dot{m} = \rho AU , \quad (141)$$

which with Eq. (131) and the definitions used in Eqs. (120) and (121) can be written as

$$\begin{aligned} \dot{m} &= \left(\frac{a_1 A}{a_2} \right) \left\{ \bar{\omega}^2 e^{\left(\frac{\tau^{-1}}{\bar{\omega}^2 + \tau^{-2}} \right)} - \tau^{-1} e \right\} \\ &= \left[\frac{\alpha^2}{\left(\frac{K_{X'}}{K_X} \right)} \right] \frac{EI}{L^3} \left(\frac{\tau^{-1}}{\bar{\omega}^2 + \tau^{-2}} \right) - \left[\frac{1}{2 \left(\frac{K_{X'}}{K_X} \right)} \right] cL , \end{aligned} \quad (142)$$

where $a_1 = \rho_T(A_0 - A)$ and $a_2 = K_{X'} / K_X L$.

The decay constant and the frequency should be thought of as the flowmeter outputs resulting from the mass flow rate which is the input. In this sense, Eq. (142) is backwards, but nonetheless it can be used to study parameter effects in regard to measurement sensitivity. This "inverted" approach was taken because the derivation of functions for τ^{-1} and $\bar{\omega}^2$ as functions of $\dot{m}$ is much more complex. Consider the first variation of $\dot{m}$ as a function of τ^{-1} and $\bar{\omega}^2$,

$$\delta \dot{m} = \left(\frac{\partial \dot{m}}{\partial \tau^{-1}} \right) \delta(\tau^{-1}) + \left(\frac{\partial \dot{m}}{\partial \bar{\omega}^2} \right) \delta(\bar{\omega}^2) , \quad (143)$$

where

$$\begin{aligned} \left(\frac{\partial \dot{m}}{\partial \tau^{-1}} \right) &= \left[\frac{\alpha^2}{\left(\frac{K_{X'}}{K_X} \right)} \right] \frac{EI}{L^3} \left\{ \frac{\partial}{\partial \tau^{-1}} \left(\frac{\tau^{-1}}{\bar{\omega}^2 + \tau^{-2}} \right) \right\} , \\ \left(\frac{\partial \dot{m}}{\partial \bar{\omega}^2} \right) &= \left[\frac{\alpha^2}{\left(\frac{K_{X'}}{K_X} \right)} \right] \frac{EI}{L^3} \left\{ \frac{\partial}{\partial \bar{\omega}^2} \left(\frac{\tau^{-1}}{\bar{\omega}^2 + \tau^{-2}} \right) \right\} . \end{aligned} \quad (144)$$

The inverses of the sensitivity coefficients given in Eq. (145) relate the gain between changes in τ^{-1} and $\bar{\omega}^2$ to $\dot{m}$, and can fortunately be used to gauge parametric influences on sensitivity. The inverses are

$$\left(\frac{\partial \tau^{-1}}{\partial \dot{m}} \right) = \left[\frac{\left(\frac{K_{X'}}{K_X} \right)}{\alpha^2} \right] \frac{L^3}{EI} \left\{ \frac{\partial}{\partial \tau^{-1}} \left(\frac{\tau^{-1}}{\bar{\omega}^2 + \tau^{-2}} \right) \right\}^{-1} ,$$

$$\left(\frac{\partial \bar{\omega}^2}{\partial \dot{m}}\right) = \left[\frac{\left(\frac{K_x}{K_x}\right)}{\alpha^2} \right] \frac{L^3}{EI} \left\{ \frac{\partial}{\partial \bar{\omega}^2} \left(\frac{\tau^{-1}}{\bar{\omega}^2 + \tau^{-2}} \right) \right\}^{-1} \quad (145)$$

Both functions are multiplied by the same constant expression and to maximize the measurement sensitivity parameters should be selected to make this constant expression as large as possible. This in turn indicates that (1) the cantilever length, L , should be as large as possible; and (2) the beam flexural rigidity, EI , should be as small as possible. Large L and small EI correspond to beams that are long, have thin cross sections, and are made from low modulus of elasticity materials. Beams of this description are what one would intuitively guess to be the most susceptible to fluid flow effects. Equation (142) also shows that the external damping coefficient should be small because it appears as a zero-flow bias term.

Flowmeter Design Sequence

The sensitivity analysis showed that certain trends in the parameters are desirable to maximize flow measurement sensitivity. When these trends are viewed in terms of other system constraints, a particular design sequence is revealed. There are three rigid design constraints applicable to the cantilever Coriolis flowmeter: (1) the maximum pressure drop across the meter must be less than or equal to the maximum allowable pressure drop budgeted for the measurement, (2) the deflection data acquisition rate required must be within the maximum data sampling rate of the equipment, and (3) the bending stress in the cantilever must remain well within the elastic region.

The flowmeter pressure drop allowance, maximum flow rate, fluid density, and viscosity specifications can be used with standard pressure drop formulas [50] to calculate $(L/d_i)_{\max}$ for turbulent flow or $(L/d_i^2)_{\max}$ for laminar flow. The data throughput capabilities of the digital data acquisition system (i.e., computer, analog-to-digital converter, etc.) can be used to specify a maximum deflection data sampling frequency, ω_s . To be able to reconstruct the beam deflection transient from the sampled-data satisfactorily, the sampling frequency should be at least ten times larger than the beam frequency of vibration which will always be less than empty pipe natural frequency [52]. Therefore, the timing constraint can be expressed as a relationship between ω_s and ω_e .

Using Eq. (119) and the data sampling frequency constraint $\omega_s/\omega_e = R_\omega$, the expression

$$\text{Timing Constraint: } \frac{\omega_s^2}{R_\omega^2 \alpha^2 g} = \left(\frac{E}{\rho_T} \right) \left(\frac{d_o^2 + d_i^2}{16L^4} \right) \quad (146)$$

is obtained. For good results R_ω should be 100 or more. The given initial static deflection creates the maximum flexure stress. A simple beam analysis for a given $y(L,0)$ provides the relationship

$$\text{Stress Constraint: } \frac{\sigma_{\max}}{y(L,0)} = \frac{16E}{9} \frac{d_o}{L^2} \quad (147)$$

Pressure drop calculations restrict the length/inside diameter ratio to form the last constraint

$$\text{Pressure Drop Constraint: } \left(\frac{L}{d_i} \right) = \left(\frac{L}{d_i} \right)_{\max}, \quad (148)$$

where the equality has been chosen so that L and I will be as large as possible. Recall also that the $(L/d_i) > 10$ constraint discussed in Chapter 2 due to Katys [7] must also be met.

Equations (146), (147), and (148) represent three equations in the unknowns d_o , d_i , and L when the tube material, initial deflection, maximum stress, and maximum data sampling frequency are specified. The mechanics of the complete design cycle are shown schematically in Fig. 17. One would most likely bracket the values of d_o , d_i , and L in the manner shown. Then the closest values of standard tube or pipe cross sections would be selected and the beam stress, initial deflection, and sampling frequency values would be adjusted to compensate for any differences.

The Case of Steady-State Harmonic Motion Excitation

As mentioned earlier, steady-state harmonic excitation of the cantilever Coriolis flowmeter is a practical alternate to the transient excitation being emphasized. With steady-state harmonic excitation, the cantilever would be forced to vibrate at a known frequency and amplitude. Under these conditions, the power necessary to sustain the beam vibration would be proportional to rate of change of work done by the dissipative damping forces which are dependent on flow rate. Steady-state excitation can be implemented with the same basic idea used in the Micromotion Coriolis flowmeter [9]. In the Micromotion concept, a force from an electromagnetic actuator is applied to the beam. Discrete sensors are

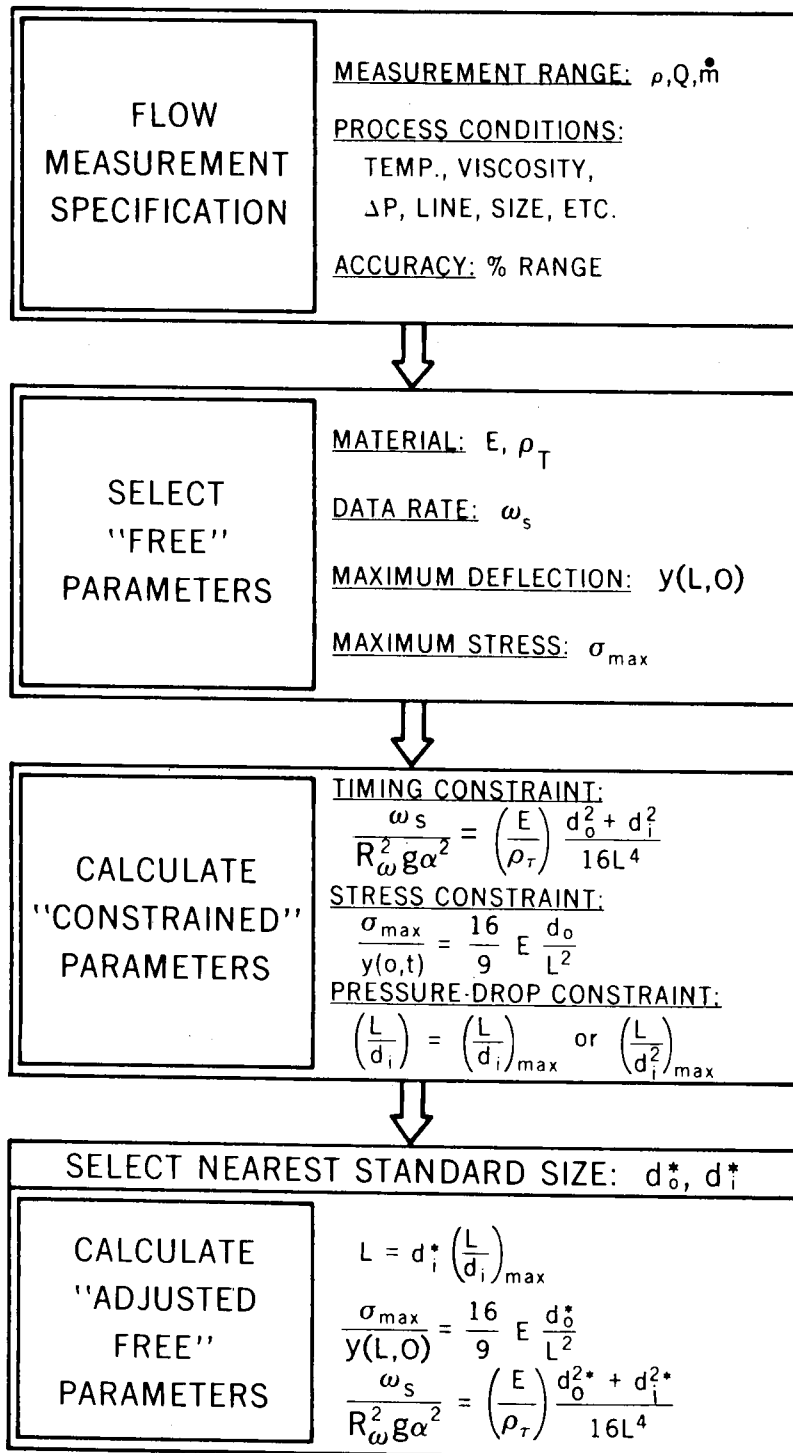


Fig. 17. Synthesis mechanics.

used to detect beam deflection amplitude with respect to the flowmeter chassis (refer to Fig. 3, p. 8). The outputs of these sensors are used to reverse the actuator polarity as the beam intercepts them. This, in effect, creates a vibration of constant amplitude and at the fundamental natural frequency of the beam. It is important to force the beam at its natural frequency since this characteristic is used to define fluid density. It appears that for the cantilever Coriolis flowmeter concept analog deflection measurement will be necessary rather than simple limit detection. This is because as flow rate increases, for example, the electromagnetic actuator force must be increased to maintain a constant deflection amplitude. It is, therefore, necessary to servo-control the excitation actuator power with respect to deflection amplitude. The servo-controller output, an electrical power level, would be related to total beam damping and hence flow rate.

A functional schematic of the steady-state harmonic excitation concept is shown in Fig. 18. This approach would most likely require analog electronic implementation because the beam frequencies will generally be in the range of 20 to 100 Hz. Special high-speed concepts would have to be used to consider microprocessor implementation of the necessary control functions. A cursory analysis of the amount of damping power observed in experiments using the $3/8 \times 0.035$ beam configuration was performed. Damping power at an arbitrary location x was approximated as

$$dP(x,t) = \frac{dW_d(x)}{dt} = \oint C_{eq} \dot{Y}_t^2(x,t) dt dx, \quad (149)$$

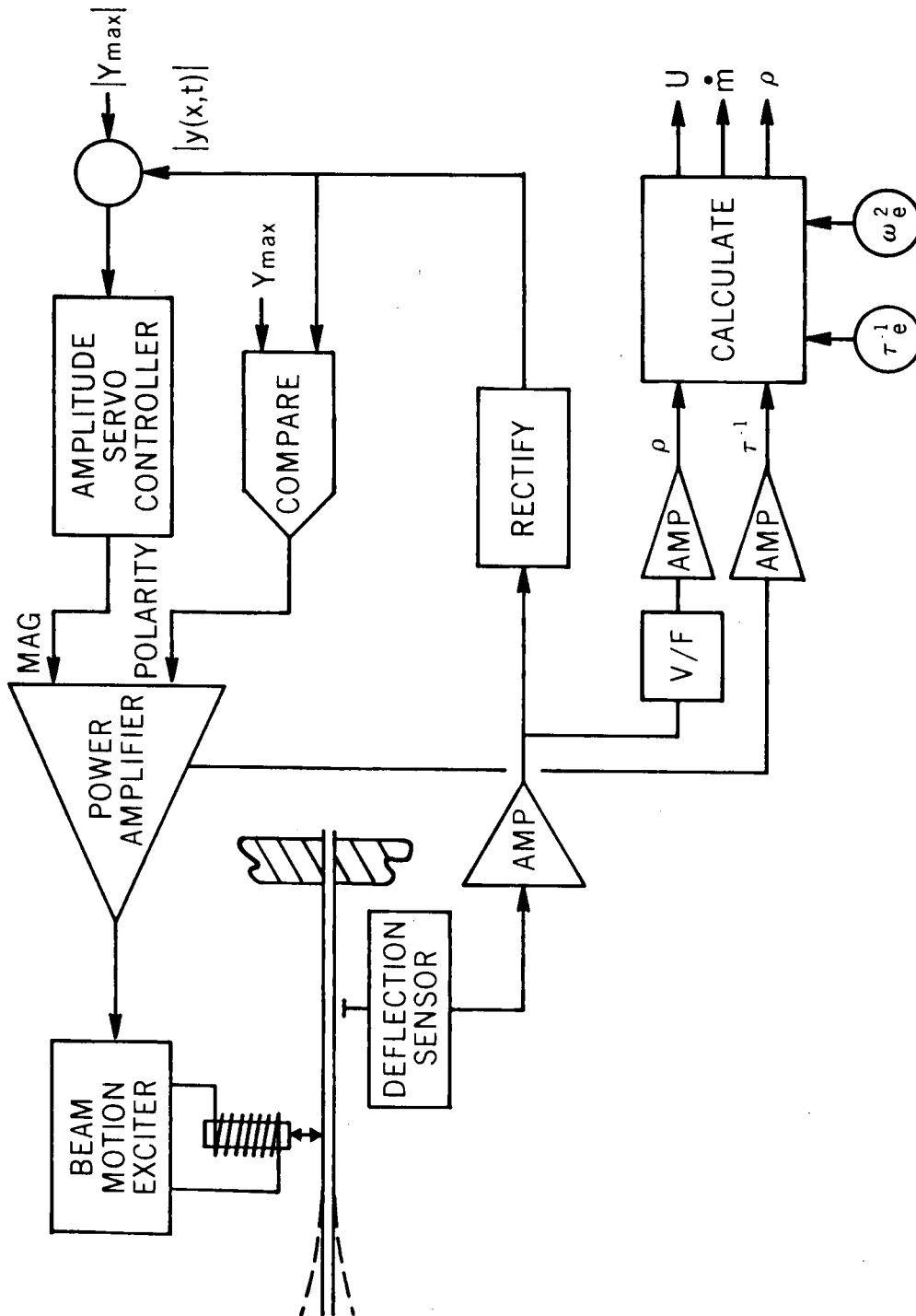


Fig. 18. Steady-state harmonic excitation concept.

where

$$y(x,t) = Y(L,t)X(x) \sin(\bar{\omega}t - \phi)$$

$$c_{eq} \triangleq (m + M) \left[2\beta \frac{K_{x'}}{K_x} \frac{U}{L} + (1 - \beta) \frac{c}{m} \right]. \quad (150)$$

The total beam damping power is found by integrating Eq. (149) over the beam length. That is,

$$|P(t)| = \int_0^L c_{eq} Y^2(L,t) X^2(x) \bar{\omega}^2 dx$$

$$= \frac{c_{eq} Y^2(L,t) \bar{\omega}^2}{4}. \quad (151)$$

At a maximum flow rate of approximately 550 kg/h, the damping power amplitude was found to be on the order of 1 W. For the experiments reported in Chapter 4, the damping power varies from nearly 0 to on the order of 1 W for the different configurations. It is not known if power levels of this size represent serious practical difficulties. The major difficulty with steady-state measurement techniques generally is that uncalibrated or uncompensated systems parameters must be stable over long time intervals (i.e., the elapsed time between calibrations).

At this point, it is not clear which of the two basic approaches for cantilever motion excitation would be the most advantageous. For comparison, a functional schematic of the transient excitation concept is shown in Fig. 19. On this cursory basis, it appears that the

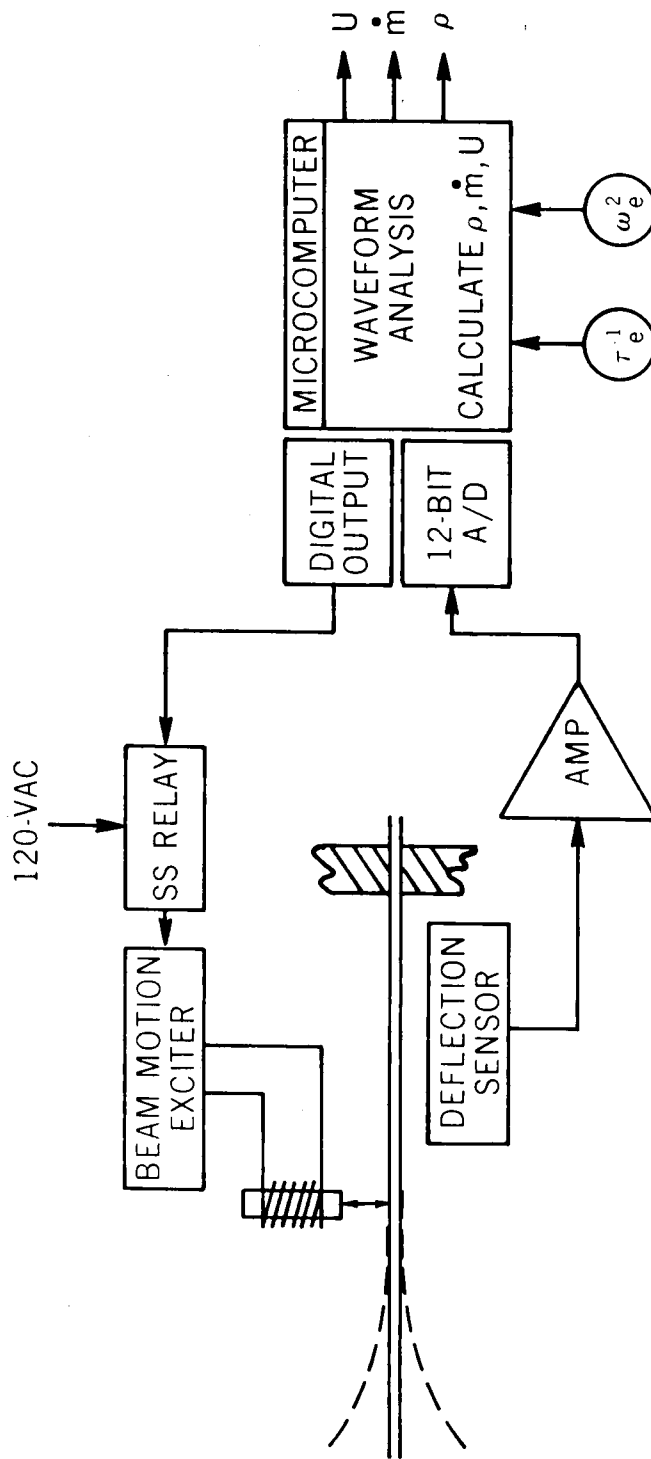


Fig. 19. Transient excitation concept.

transient concept is functionally (and electronically) simpler. Both approaches require the implementation of calculations to transform τ^{-1} and $\bar{\omega}$ to ρ , U , and $\dot{m}$. The transient approach does not require the closed-loop amplitude control or stringent long-term stability in various electronic gains. The transient approach leads to a pulse mode of operation such that the flowmeter output is sampled-data. Depending on the actual overall flow measurement sampling frequency achievable, the transient concept may not be fast enough for flow control applications with narrow stability margins. In contrast, the steady-state concept with its greater use of analog electronics probably can be designed to provide continuous flow measurement output (i.e., from the perspective of typical process flow control dynamics). Clarification of the merits of the two approaches is a topic for further study.

CHAPTER 4

PROOF OF PRINCIPLE EXPERIMENT

An approximate theoretical foundation for the cantilever Coriolis flowmeter concept has been established and measurement algorithms compatible with practical hardware/software capabilities have been derived. The next step was to assess the analytical findings through actual experimentation. In typical instrumentation research and development, analytical work (usually much more simplistic than that presented here) concludes with the beginning of proof of principle experimentation. Experimental evaluation at the proof of principle level is intended to verify the fundamental cause and effect relationships predicted in the analytical phase. As such, proof of principle experiments evaluate the basic trends in these relationships while providing predominantly qualitative rather than quantitative results (e.g., estimates of accuracy and precision). The proof of principle experimental results are normally used as a key checkpoint in instrument development, and satisfactory results are usually imperative for continued development.

A proof of principle experiment has been performed to assess the basic features of the analytical results, and to demonstrate the feasibility of the concept. The equipment used, techniques employed, and results obtained in this experiment are discussed in the following sections.

Basic Approach

The ultimate industrial feasibility of the cantilever Coriolis flowmeter concept is a major interest in this work. As a result, it was decided to perform a proof of principle experiment by comparing a typical industrial-grade flowmeter with a simple cantilever Coriolis flowmeter. To simplify the fluid handling aspects of the experiment, it was decided to perform all experiments with tap water and variable volumetric flow rates. A suitable system of piping, a reservoir, and a pump were used to circulate the water through the industrial and the Coriolis flowmeters connected in series. As discussed in Chapter 3, the cantilever Coriolis flowmeter was calibrated relative to the industrial flowmeter according to Eqs. (138). As the water circulation rate is varied, the two flowmeter outputs were compared to assess the quality of the Coriolis approach.

It is important to define a basis of comparison in this type of relative experimentation. Since the cantilever Coriolis flowmeter was calibrated against the industrial flowmeter, distortions in the zero-intercept and span (slope) of the results may be caused by either of the two meters. The meaningful feature of the results is linearity with respect to straight line and repeatability about that line, or in other words how well the two devices track together as the flow varies. The slope and intercept of the straight line defined by the data relate to simple gain and zero differences between the two devices. Quantitative analysis of these differences would be the subject of subsequent

experimentation based on a traceable calibration standard. For the purposes of this work, good proof of principle results are characterized by "tight" clustering of the data around straight lines when the flowmeter outputs are plotted against each other.

For logistical and economic reasons, it was decided to use an available thermal flowmeter as the industrial standard of comparison. The thermal flowmeter calculates mass flow rate by introducing a measured heat transfer rate between upstream and downstream temperature measurements [2]. As long as the fluid specific heat and other heat transfer parameters remain constant, the ratio of the heating rate to the temperature difference is proportional to mass flow rate. The particular flowmeter used was built by Thermal Instruments Company and was originally calibrated for a range of 0 to 250 kg/h water. This unit was also calibrated with the Low Flow Test Facility of the Oak Ridge National Laboratory Instrumentation and Controls Division. The results of these water calibration tests are shown in Figs. 20 and 21. Note that the flowmeter displays a significant decrease in accuracy below 100 kg/h. The accuracy is generally in the range 1 to 10% depending on flow rate.

Experimental Equipment Description

An experimental system consisting of: (1) a flow circulation system, (2) a microcomputer-based data acquisition and computation system, (3) various Coriolis flow measurement sensors and signal conditioning, and (4) a cantilever motion excitation system was assembled. A photograph of the overall experimental system is shown in Fig. 22.

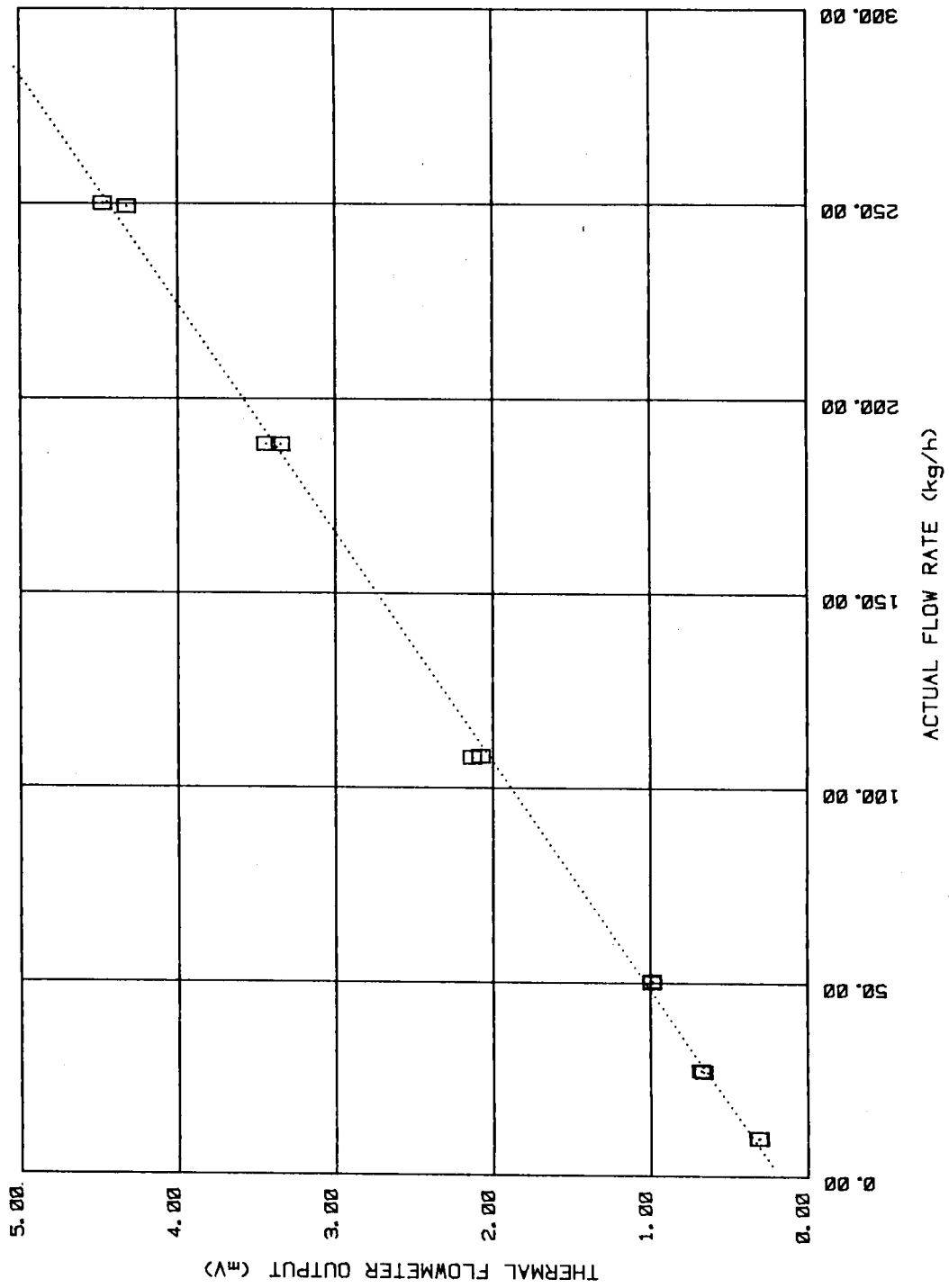


Fig. 20. Thermal flowmeter calibration.

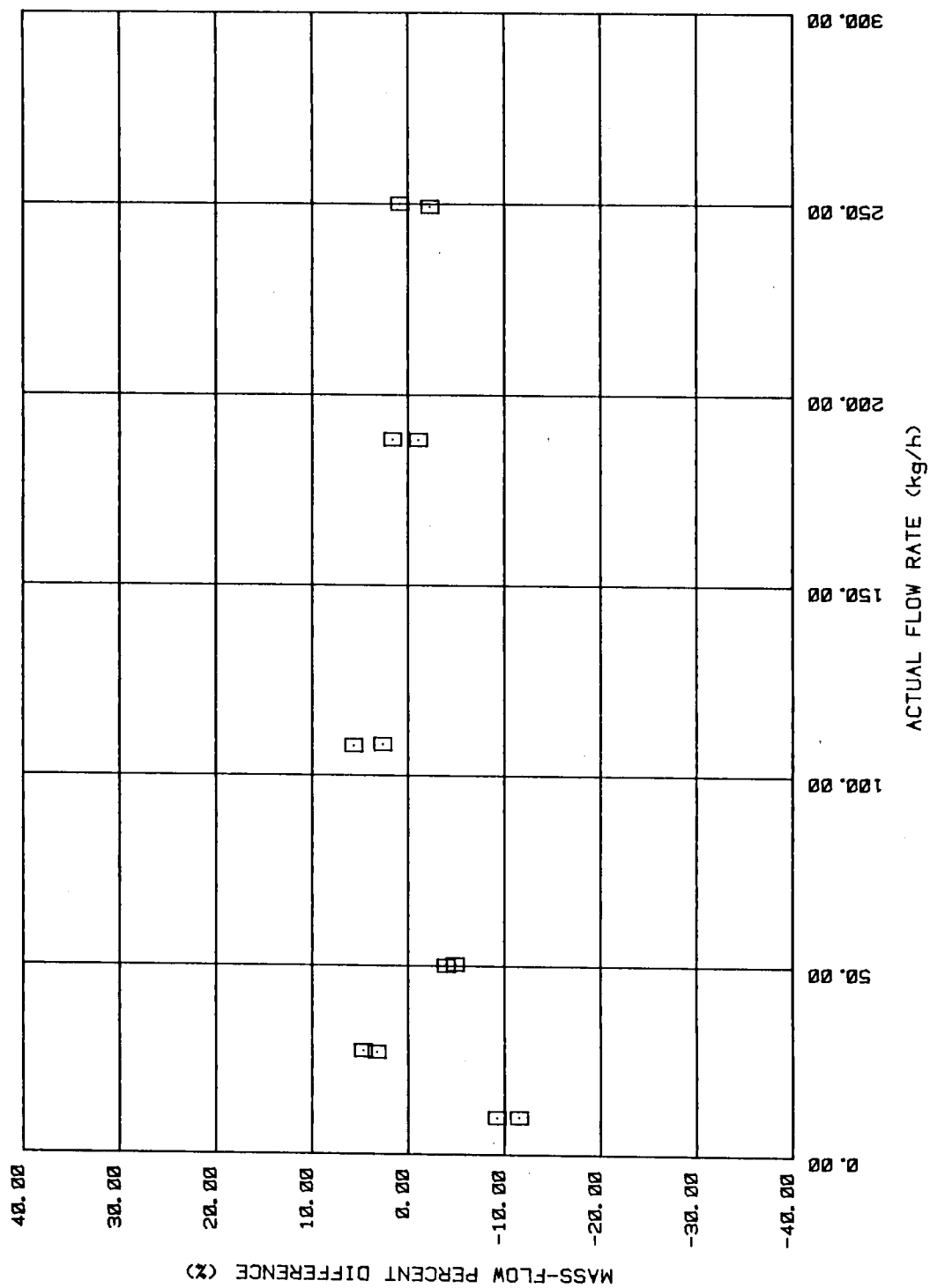


Fig. 21. Thermal flowmeter accuracy.

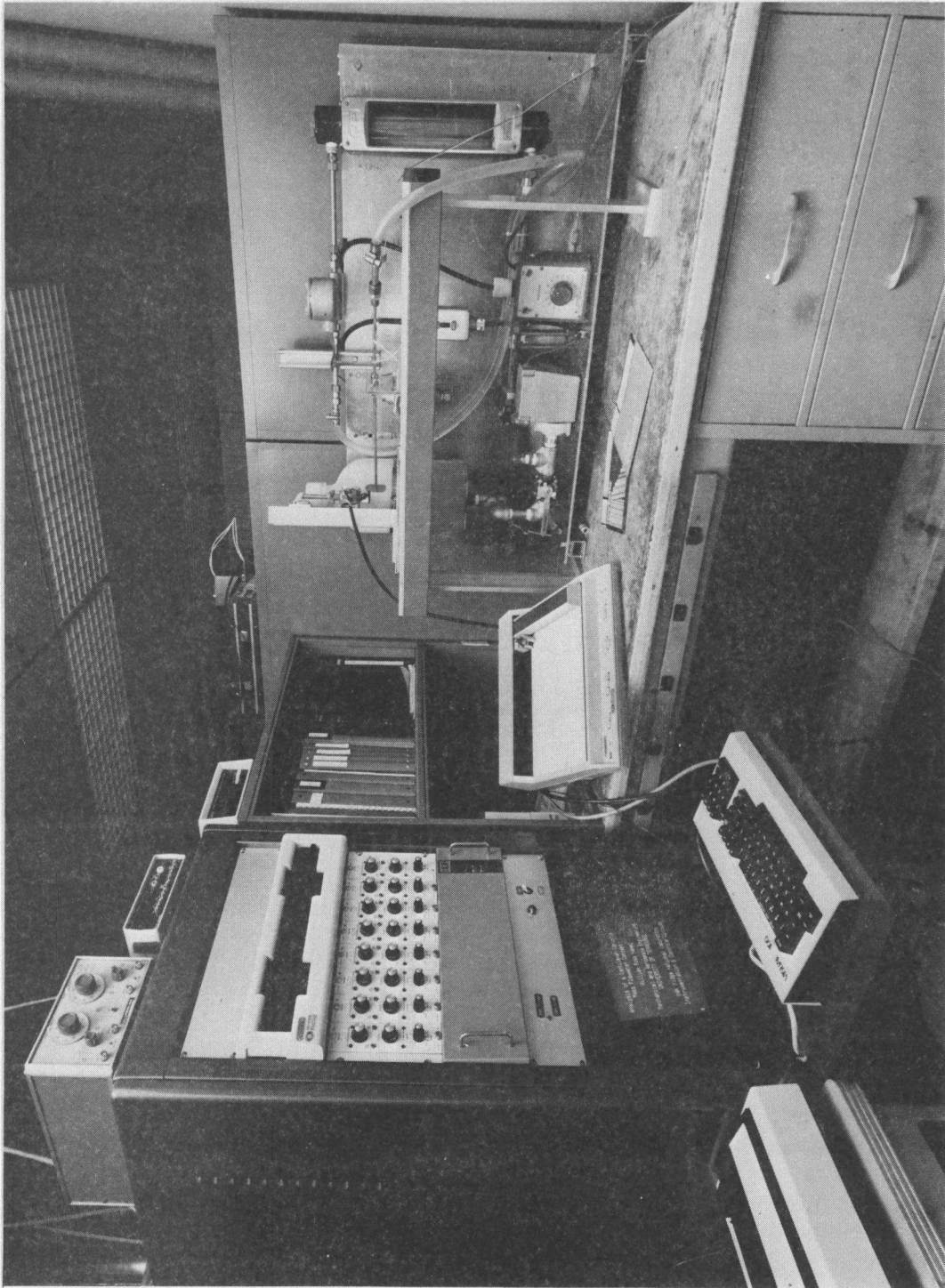


Fig. 22. Overall experimental system.

The component systems making up the overall experimental setup are described in the following sections.

Flow Circulation Loop

A simple flow circulation loop was fabricated using a small centrifugal pump, 12.7-mm tubing and 12.7-mm pipe, necessary isolation valves, and an 8-L reservoir. The arrangement of the flow circulation loop is shown schematically in Fig. 23. The water flow rate in the loop is varied by adjusting the AC supply voltage to the pump drive motor with a manually operated variable transformer (Variac). A variable-area flowmeter (rotameter) was included for quick visual reference of approximate loop flow rate. The outlet of the rotameter is connected in series with the thermal flowmeter and the Coriolis cantilever. The Coriolis cantilever(s) discharge into the reservoir that forms the pump suction. The variable-voltage flow control is actually a form of open-loop control that makes it difficult to set the flow rate with repeatability. Fortunately, for the procedures used in this experiment, this simple flow control was sufficiently repeatable and stable.

Experimental Measurement and Control System

The most complex component of the experimental setup is the realtime microcomputer-based system used to: (1) control the operation of the Coriolis cantilever flowmeter, (2) record and store beam deflection and thermal flowmeter data, and (3) perform necessary flow measurement and calibration algorithm calculations. This system was also used for analyzing and plotting both theoretical and experimental data.

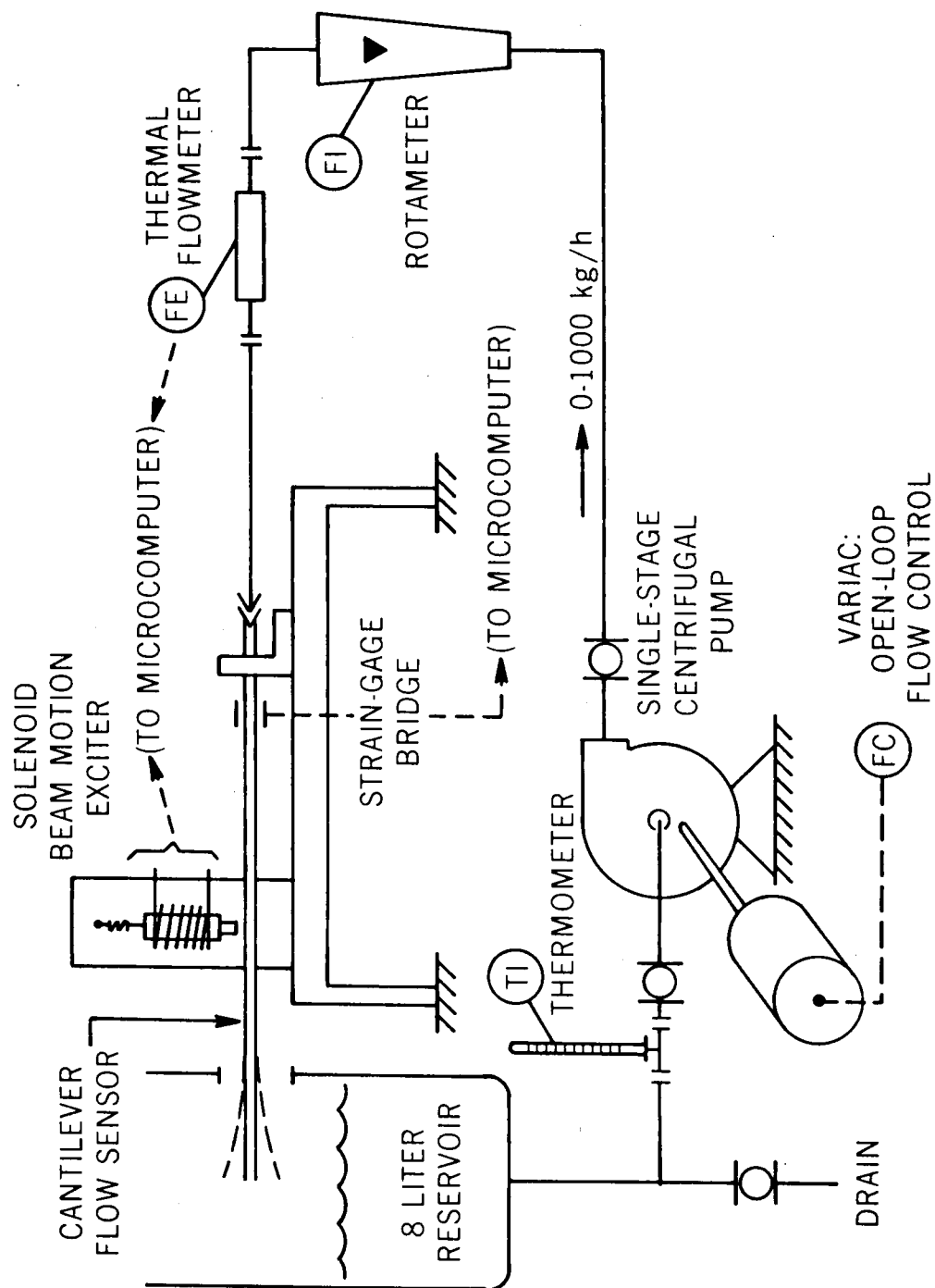


Fig. 23. Flow circulation loop schematic.

A schematic of the experimental measurement and control system is shown in Fig. 24. A Digital Equipment Corporation (DEC) LSI-11/23 16-bit microprocessor is the heart of the system. Two megabytes of bulk data storage is provided by a dual-floppy diskette drive. The system is operated through a black and white video display terminal connected to the LSI-11/23 through a RS-232C serial interface. The serial interface also provides channels for a high-resolution digital plotter (Hewlett-Packard Model 7225A) and a telephone-line connection for the data transmission with the ORNL Central Computer Center.

Data from the cantilever Coriolis flowmeter and the thermal flowmeter were read through an 8-channel 12-bit analog-to-digital (A/D) interface (Data Translation Model DT2782). One output bit of a 16-bit parallel input/output interface (DEC Model DRV11) was used to automatically control the beam excitation actuator. A programmable realtime clock (Data Translation Model DT2769) operating at a 1 MHz clock rate provided precision data sampling intervals.

The entire measurement and control system was operated using the various features of the DEC RT-11 foreground/background realtime operating system. All software programming was written in FORTRAN with the exception of special machine language routines used for controlling the realtime clock, the digital parallel interface, and the A/D interface. Machine language routines from the Data Translation support library, DTLIB, were used for these devices. Listings of the many FORTRAN programs developed for the operation of the experiment and the analysis of data are presented in Appendixes C and D. The experiment control

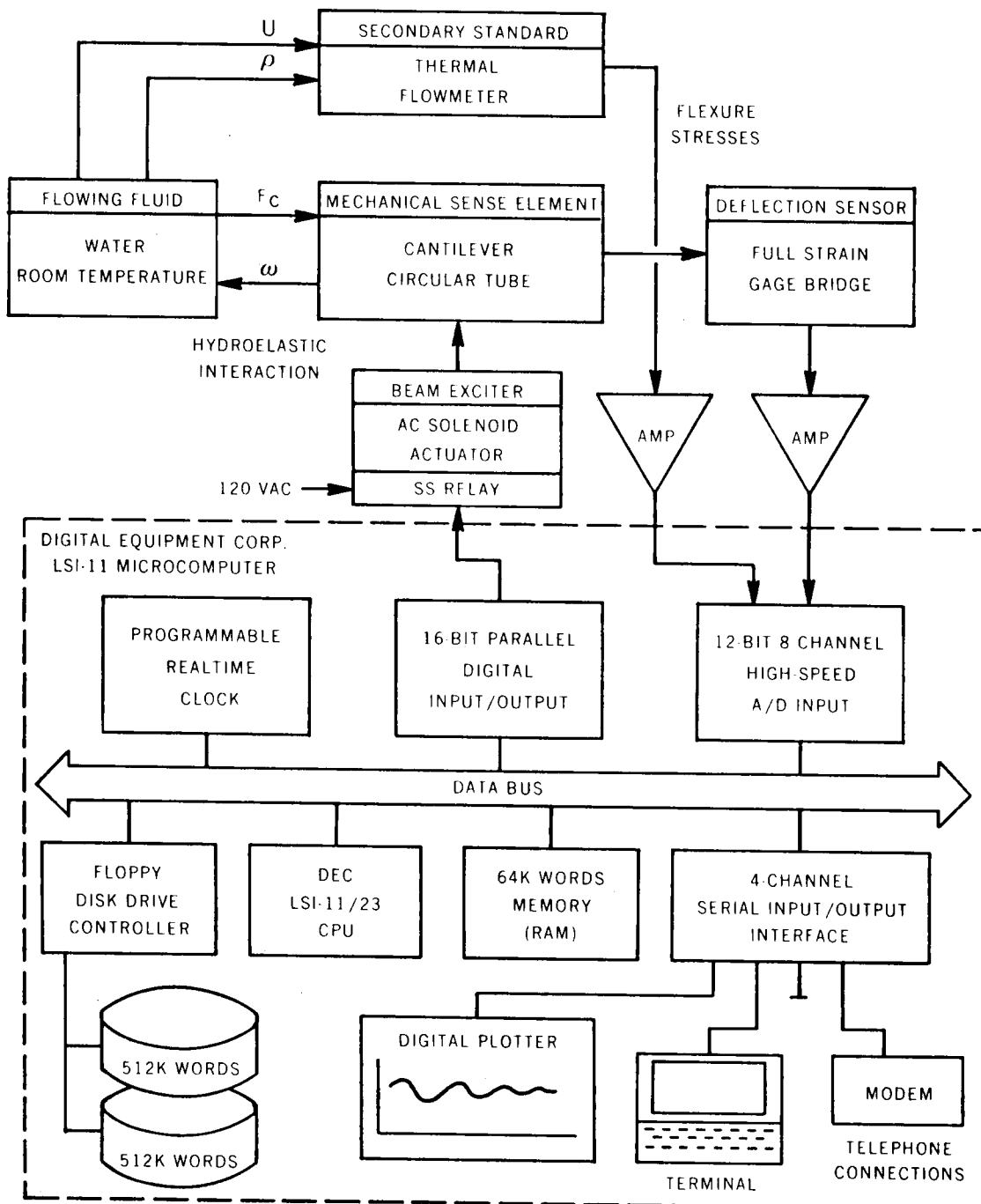


Fig. 24. Experimental measurement and control system functional block diagram.

software routines were written to allow the operator to specify or alter sampling parameters, command measurement execution, and store data results automatically on the floppy diskettes.

The floating point arithmetic capabilities of the LSI-11/23 proved to be more than adequate for this application. Both of the 12-bit analog data channels were sampled at intervals of 0.2 to 0.5 ms for 500 to 1000 sample accumulations. This is on the order of 5,000 channels/s throughput, a moderately high data rate for present technology. The total time interval from when the operator commanded a flow measurement until the algorithm calculations were completed varied from 0.8 to 1.2 s depending on the specific sampling parameters being used.

Flow Sensors and Signal Conditioning

The simplest possible mechanical configuration for the Coriolis cantilever flow sensors was used. As shown in Fig. 25, three lengths and cross-section sizes of conventional stainless-steel tubing were used. These were connected with standard tubing bulkhead fittings to support brackets machined from 7.62-mm aluminum angle stock. The sensor mounting brackets are bolted directly to a "breadboard" table as shown in Fig. 22. No special provisions were incorporated for rigidity or counterbalancing.

Very early experimental work was performed using an "in-line" form of the cantilever concept. The free end of the cantilever was connected back into the flow line using a rubber elastic coupling. This approach was found to be unsatisfactory for two reasons: (1) the high damping effect of the rubber, and (2) a tendency of the beam vibrations to be nonplanar. The damping effect of the rubber coupling was found to be

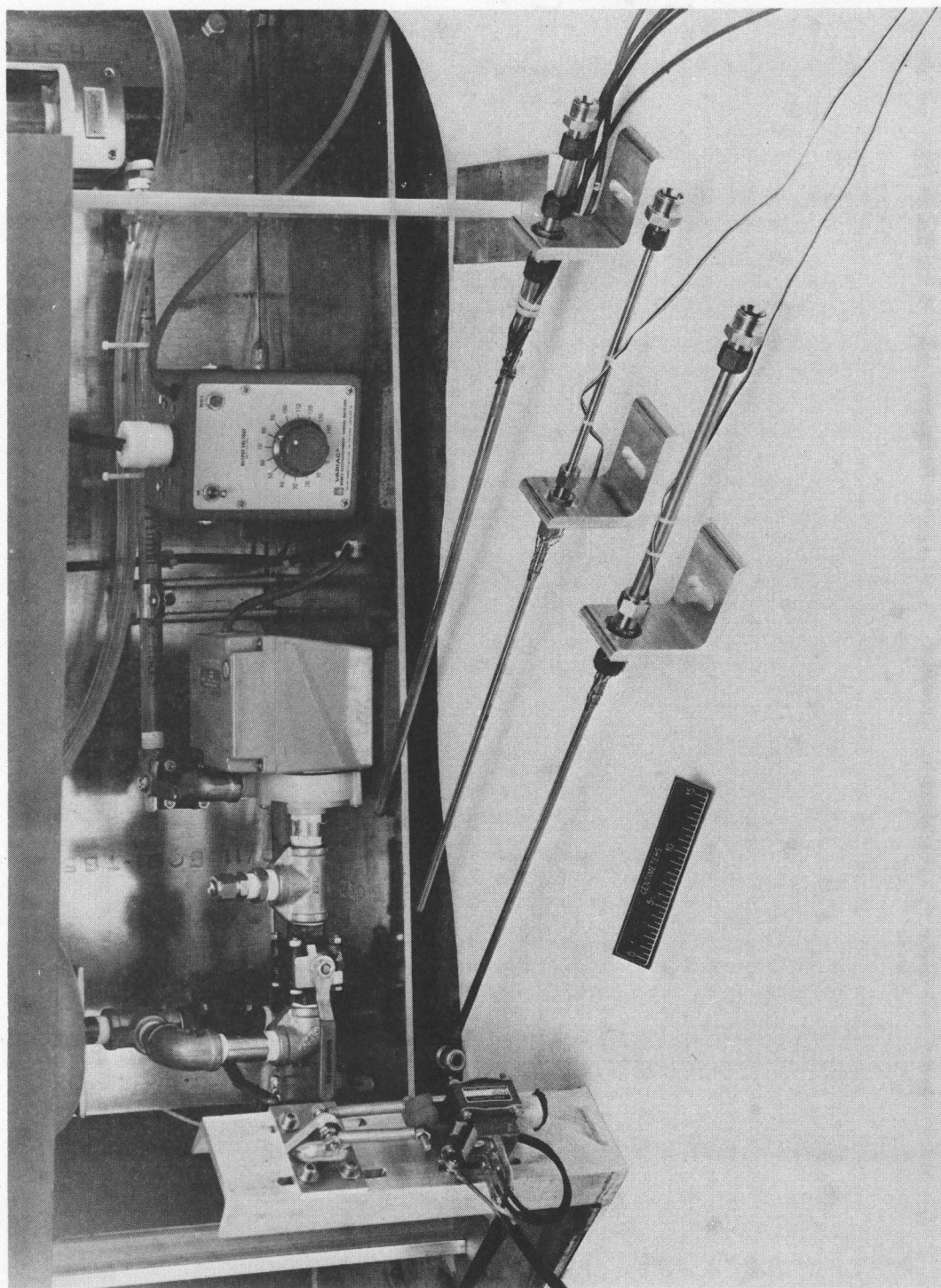


Fig. 25. Cantilever Coriolis flowmeter beams and motion exciter.

3 to 5 times larger than the combined flow and external damping. As a result, the desired signal was a small effect superimposed on a large effect — a very undesirable ratio for measurement purposes.

The dynamic beam deflection was measured using standard metal film strain gages (gage factors ~ 2) mounted near the fixed end where flexure stresses are the greatest. For maximum sensitivity and thermal stability, complete four-arm strain gage bridges were used. The somewhat small 3.175-mm by 0.762-mm gages restricted bridge excitation voltages to 2 to 3 VDC. As a result, about 80 dB of DC amplification was required to match the ± 10 VDC range of the A/D interface.

Motion Excitation System

An off-the-shelf AC solenoid actuator was used to create the static beam deflection necessary to create vibration. As shown in Fig. 25, the 22.2 N solenoid actuator was mounted on a support bracket such that the return-spring force can be adjusted. The magnitude of the initial-static deflection was adjusted by varying the distance between the point of contact and the cantilever support bracket. The maximum initial static deflection was not measured precisely but was estimated to be about 1 cm.

The excitation actuator was automatically controlled by the LSI-11/23 by energizing or de-energizing the solenoid voltage through a solid-state relay controlled by the parallel digital interface. To ensure that a true static deflection was established prior to beam release, the LSI-11/23 was programmed to pause for 0.5 s after energizing the excitation actuator to allow any vibrations to dampen out.

Experimental Parameter Ranges and Limitations

Three cantilever flow sensor sizes were used in an effort to cover a wide range in the nondimensional parameters β and u . In this way, the quality of analytical results (and flow measurement algorithms) would be tested over a broad interval. The ranges of various parameters including β and u are shown in Table 3. Based on Eqs. (137), one might expect the $1/4 \times 0.010$ beam configuration to have a greater flow measurement sensitivity than the other two because of the much higher β . The smaller maximum flow rates achieved with the $1/4 \times 0.035$ beam configuration is due to higher flow resistance of the smaller cross-sectional area.

Note that the nondimensional velocity, u , in the $1/4 \times 0.010$ beam configuration becomes fairly large at maximum flow rates. Recall that the truncation error of the analytical results is $O(u^2)$ such that one might expect poorer flow measurement algorithm performance at high flow rates for this case. The high β and u values of the $1/4 \times 0.010$ beam configuration are indicative of the small wall thickness of this beam. The external damping coefficients increase with both diameter and wall thickness suggesting that material damping plays a role.

The proof of principle experiment was limited in two apparent ways: (1) since water was used as the flowing media, it was not possible to significantly vary the flowing stream density, and (2) the flow test range was limited to about 20:1 because of the simplicity of the flow circulation loop hardware. As a result of the first limitation, the flowmeter response to density variations cannot be assessed. The second

Table 3. Experimental parameter ranges

Parameter	Beam configuration		
	$3/8 \times 0.035$	$1/4 \times 0.035$	$1/4 \times 0.010$
Outside Diameter (mm)	9.25 (0.375 in.)	6.35 (0.250 in.)	6.35 (0.250 in.)
Inside Diameter (mm)	7.75 (0.305 in.)	4.57 (0.180 in.)	5.84 (0.230 in.)
*Length (cm)	50.17 (19.75 in.)	36.83 (14.50 in.)	35.56 (14.00 in.)
*Empty Natural Frequency (Hz)	30	37	39
*External Damping Coefficient $\times 10 [(N-s)/m^2]$	40(9 (1bf-s)/ft ²)	22(5 (1bf-s)/ft ²)	31(7 (1bf-s)/ft ²)
Nondimensional Parameters:			
$\beta = M/(M+m)$	0.2 (*0.16)	0.12 (*0.095)	0.41 (*0.48)
$u = \sqrt{ML^2/EI} U$	0 - 0.07	0 - 0.06	0 - 0.21
Maximum Flow Conditions:			
*Volumetric flow rate (L/h)	900 (4 gpm)	450 (1.5 gpm)	800 (3.5 gpm)
Mean velocity (m/s)	4.9 (16 ft/s)	5.5 (18 ft/s)	7.3 (24 ft/s)
Reynolds Number	41,500	26,400	48,000

*Measured values; other data are nominal values.

limitation of a 20:1 flow range, while respectable in terms of typical process flow measurements, is not sufficient to match the upper range of the $3/8 \times 0.035$ or the lower range of the $1/4 \times 0.010$ beam configurations.

The quality of the deflection measurement channel electronics was thought to be good but by no means optimum for this application. Because the beam deflections were kept quite small to ensure the small deflections assumptions, the strain levels being measured are quite small ($500 \mu\text{m/m}$). Low noise amplification techniques could provide a much higher signal-to-noise ratio in this measurement.

Experimental Results

Experiments intended to bracket the flow measurement quality of the three beam configurations relative to the thermal flowmeter were performed. The same basic experimental procedure was used for each experiment. To ensure that the single-mode analysis assumption was valid, each experiment began with a beam frequency spectrum analysis. Complications were encountered during this stage with the $1/4 \times 0.010$ beam configuration that will be discussed later. With verification that the beam transient was unimodal, the calibration routines given in Appendix C were executed to define the calibration constants, Eqs. (138), by using the thermal flowmeter as a secondary standard and knowing that the density of the circulating water was 997 kg/m^3 . The circulation flow was set at ten different selected values over the range and statistical averages and standard deviations calculated for the constants. Using these calibration constants in Eqs. (137), measurement results from the cantilever Coriolis

flowmeter were obtained for comparison with the thermal flowmeter output. The flow circulation pump was varied over the entire range for the particular beam being tested for a total of 10 to 15 different flows. At each setting, both flowmeter outputs were recorded and stored. Because of the automation of the experiment, these data sets could be collected in a few minutes; in fact, it is believed that in some cases, the dynamic response of the thermal flowmeter was exceeded such that for some data points additional "artificial" error was introduced.

Upon completion of the data acquisition, the two flowmeter outputs were compared by plotting them against each other. A linear regression analysis in the two outputs was then performed to characterize the degree of linearity. The Coriolis density calculation was compared in a similar fashion against the density of water at 20°C (997 kg/m³). In the following sections, the spectrum analysis, calibration results, and measurement results for each beam configuration are discussed.

Results for $3/8 \times 0.035$ Beam Configuration

Recall that the cantilever Coriolis flowmeter concept being considered is based upon motion excitation through creation of an initial static displacement. Some tests were performed on the $3/8 \times 0.035$ beam configuration to verify that the beam excitation software and actuator hardware were producing a "clean static" initial-condition forcing function. The prevue, or pretrigger, function of the digital signal analyzer (Hewlett-Packard Model 5420) was used to record the beam deflection just at release of the actuator solenoid. As shown in Fig. 26

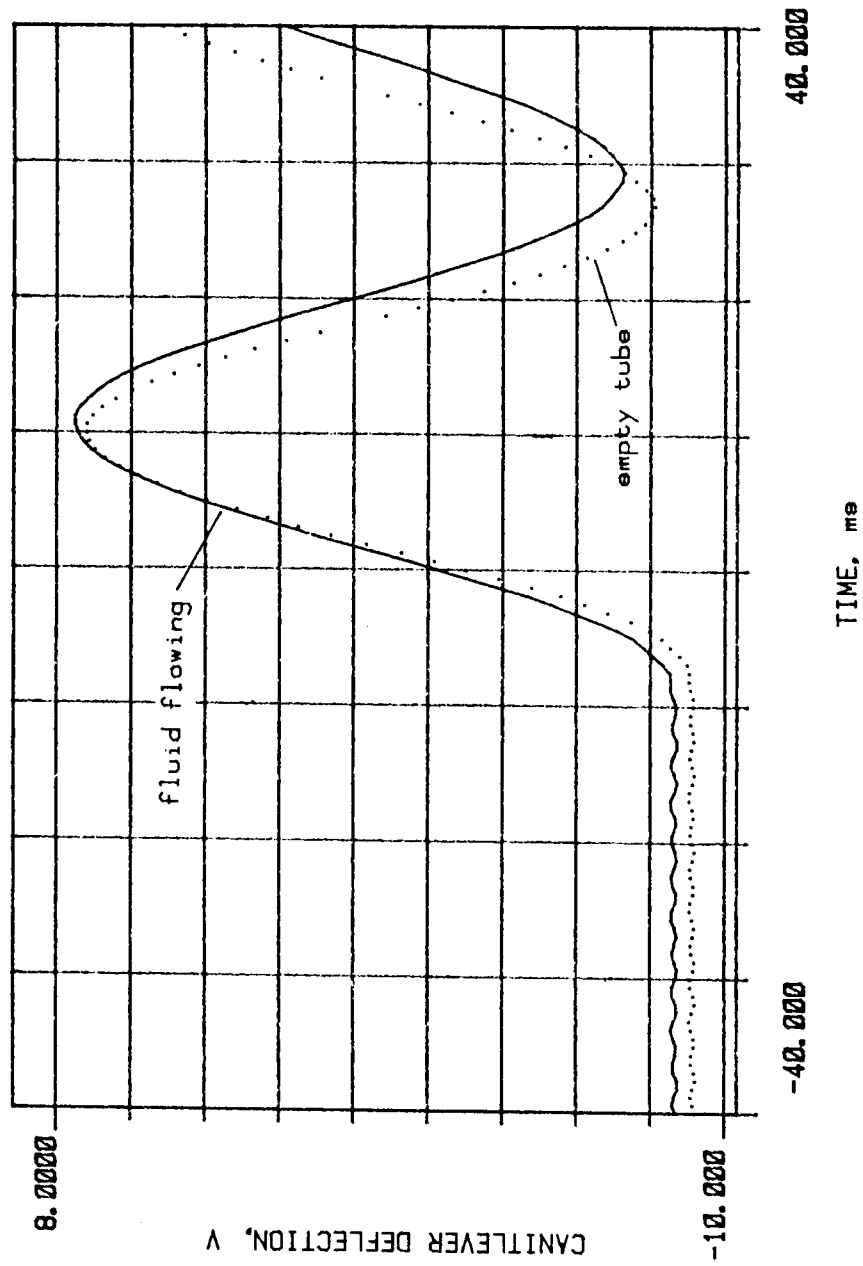


Fig. 26. Beam excitation transient.

the beam transient response agrees very well with the assumption. Response curves for both empty-tube and filled-tube conditions are given and show decreased frequency due to the water. The slight offset is believed to be the result of a small twisting movement of the solenoid core due to the flow-induced reaction force of the beam. The high frequency present when the solenoid is extended and energized is believed to be due to the solenoid "hum."

The frequency spectrum for the $3/8 \times 0.035$ beam configuration is given in Fig. 27, and shows that only the first mode of vibration is present through 250 Hz. Figure 27 indicates that the fundamental frequency is approximately 30 Hz and displays an expected amplitude attenuation and frequency decrease when the fluid flow is introduced.

The actual beam deflection transient as measured by the strain-gage measurement channel is shown for three flow rates in Fig. 28. These curves were generated from the data acquired at 0.4 ms intervals and 1000 samples. These results testify to the good quality of the deflection measurement and the repeatable operation of the excitation sequence. There is, however, a noticeable amount of signal noise present, particularly near the end of the measurement interval when the deflection amplitude has decayed considerably.

Table 4 presents a typical set of calibration results for the $3/8 \times 0.035$ cantilever. The first three parameters are zero-condition biases present on the deflection and thermal flowmeter channels. These biases are subtracted from the channel data at the time of measurement. The filled-tube zero bias defines the static deflection the beam undergoes due to the weight of the water. It is determined by reading the strain-

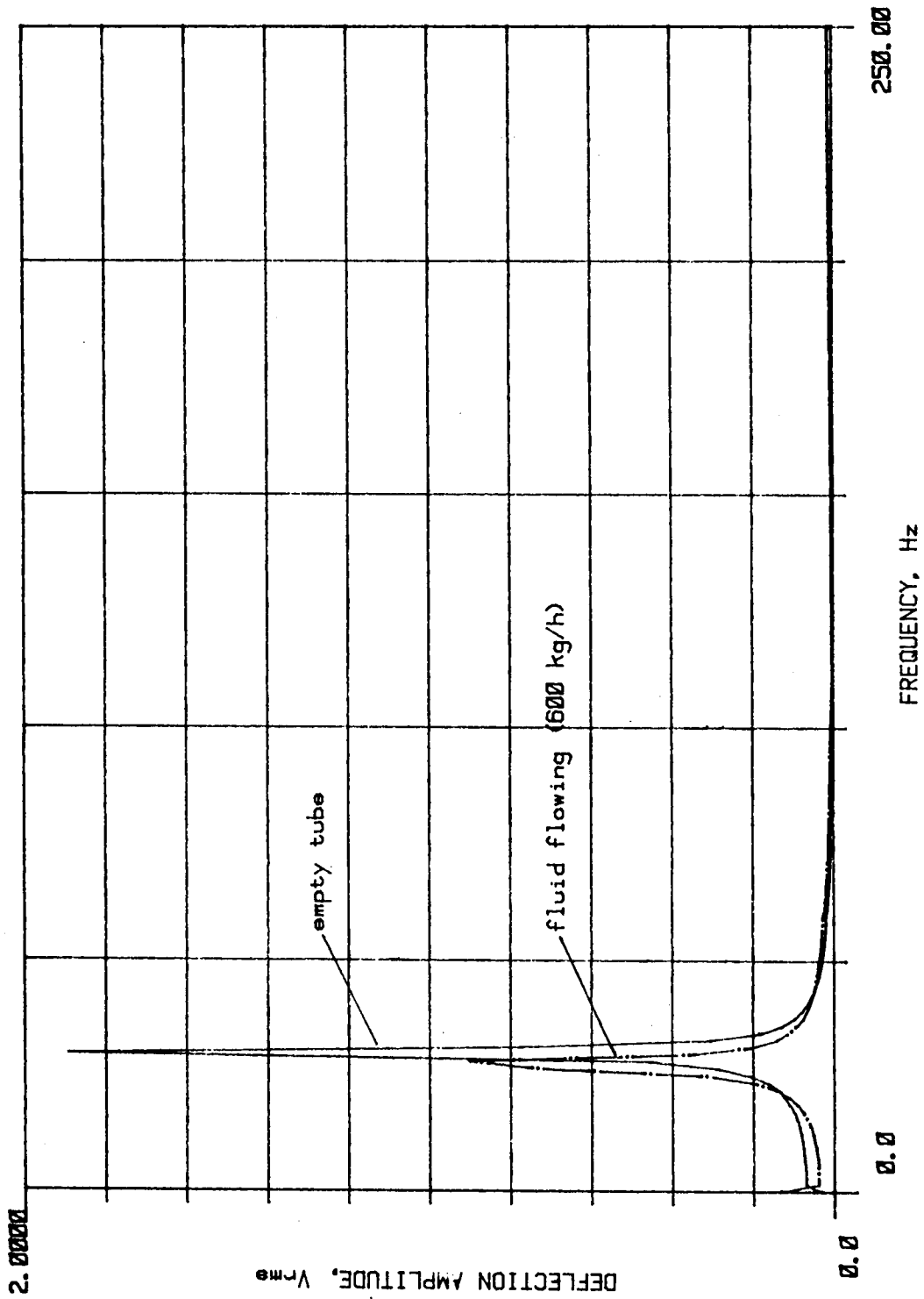


Fig. 27. Frequency spectrum for $3/8 \times 0.035$ beam configuration.

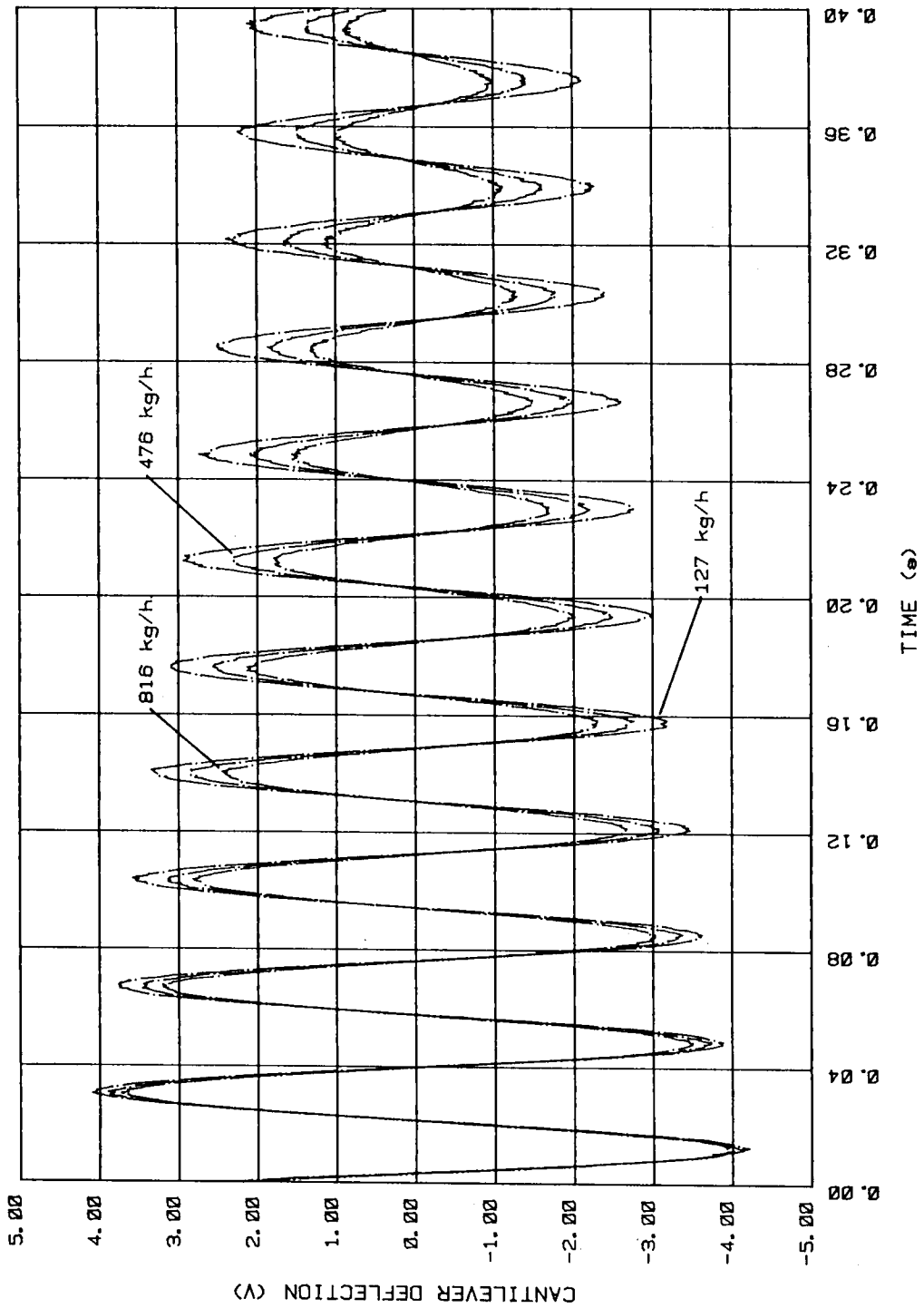


Fig. 28. Dynamic response of $3/8 \times 0.035$ beam configuration.

Table 4. Typical calibration results for
 $3/8 \times 0.035$ beam configuration

Calibration parameter	Mean	Standard deviation (% of mean)
Channel Biases:		
Thermal flowmeter (kg/h)	2.95098	
Cantilever deflection		
-with flow (V)	-0.03180	
-empty (V)	0.12604	
Calibration Constants:		
Empty decay constant, $R_{C1} (s^{-1})$	1.070393	3.98
Empty frequency-squared, $R_{C2} (s^{-2})$	35,495.9	0.28
a_1	318.5714	0.52
a_2	6.851901	3.08

gage channel output with the motion exciter de-energized and the fluid flowing. The standard deviations of the calibration constants are based on a ten point calibration and indicate how close the constants actually approach being constant. The density related parameters R_{C2} and a_2 generally had very small variances for all beam configurations. The parameters R_{C1} and a_1 are mass-flow rate related and were generally an order of magnitude greater in standard deviation. The specific calibration values for the $3/8 \times 0.035$ beam configuration were judged to be good since they were less than $\pm 4\%$ of the mean values for both sets.

Measurement results were displayed on the video display terminal as well as stored on the floppy diskette unit. For convenience, the experiment was performed using English units as shown in the typical display given in Fig. 29. For reporting purposes all units have been converted to the International System of Units (SI).

WAVEFORM STATISTICS: TA = 0.1409338E-01
 SIGTA = 0.1221985E-03

```

*****
***** MEASUREMENT RESULTS *****
*****
Coriolis Flowmeter:
Frequency = 35.5509 Hz
Beta Parameter = 0.0952 {nondimensional}
Density = 62.1742 lb/cu.ft.
Velocity = 9.1367 ft/sec
Mass Flowrate = 6.0229 lb/min
*****
Thermal Flowmeter = 6.7150 lb/min
*****

```

DO YOU WISH TO ALTER THE SET-UP PARAMETERS?[YES OR NO]

Fig. 29. Typical measurement display.

Actual mass-flow rate measurement results are shown in Fig. 30 where the cantilever Coriolis flowmeter is plotted against the thermal flowmeter. The dotted line is a linear regression curve of the cantilever Coriolis flowmeter as a function of the thermal flowmeter and is an indication of the degree of linearity between the two. The regression line indicates a rather large zero-offset for this configuration. It is thought that this offset is a result of the insensitivity of the "hefty" $3/8 \times 0.035$ beam configuration. At the lower end of the range, fluid velocities are low and produce correspondingly small Coriolis forces which in turn are not able to significantly influence the cantilever structural response. One would expect the smaller tube sizes to perform better in this regard. Figure 31 shows the superposition of three separate test scans. All of the data can be collapsed to a single reference by plotting percent differences relative to the regression line for mass-flow rate and the assumed density for density measurement. In Fig. 32, the percent difference between the linear regression line and the cantilever Coriolis flowmeter is shown for the three results given in Fig. 31. The mass-flow percent difference is within $\pm 10\%$ over the full range. Density measurement results are shown in a similar fashion in Fig. 33, where the percent difference between the Coriolis calculation and the known water density (997 kg/m^3) is plotted as a function of the thermal flowmeter output. The density results are much more accurate (as were the density-related calibration parameters) being within $\pm 2\%$ over the full flow measurement range. The density results show a decreasing trend as flow rate increases. It is thought that this trend results primarily from a tendency of the circulation pump to

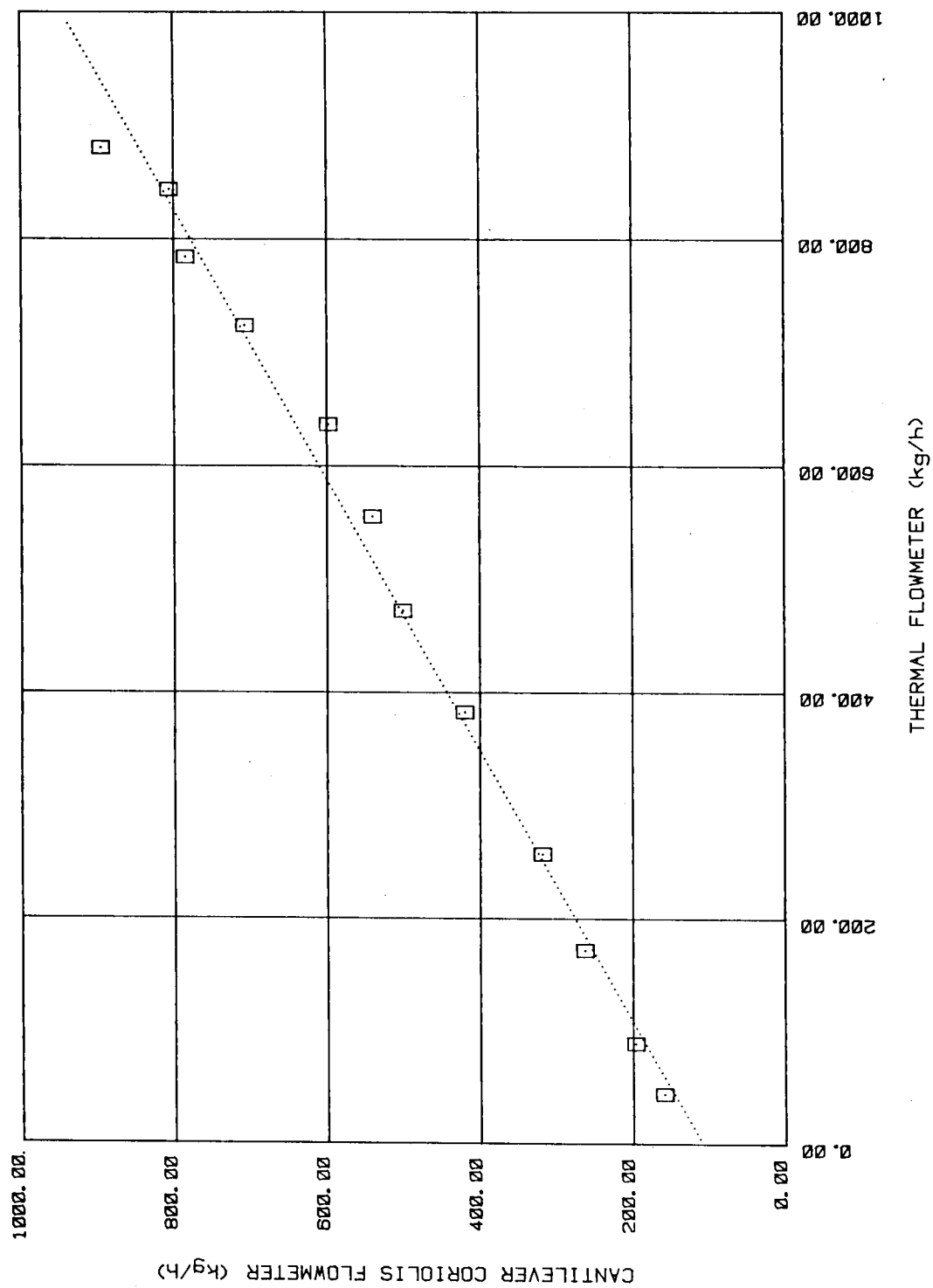


Fig. 30. Typical result and regression for $3/8 \times 0.035$ beam configuration.

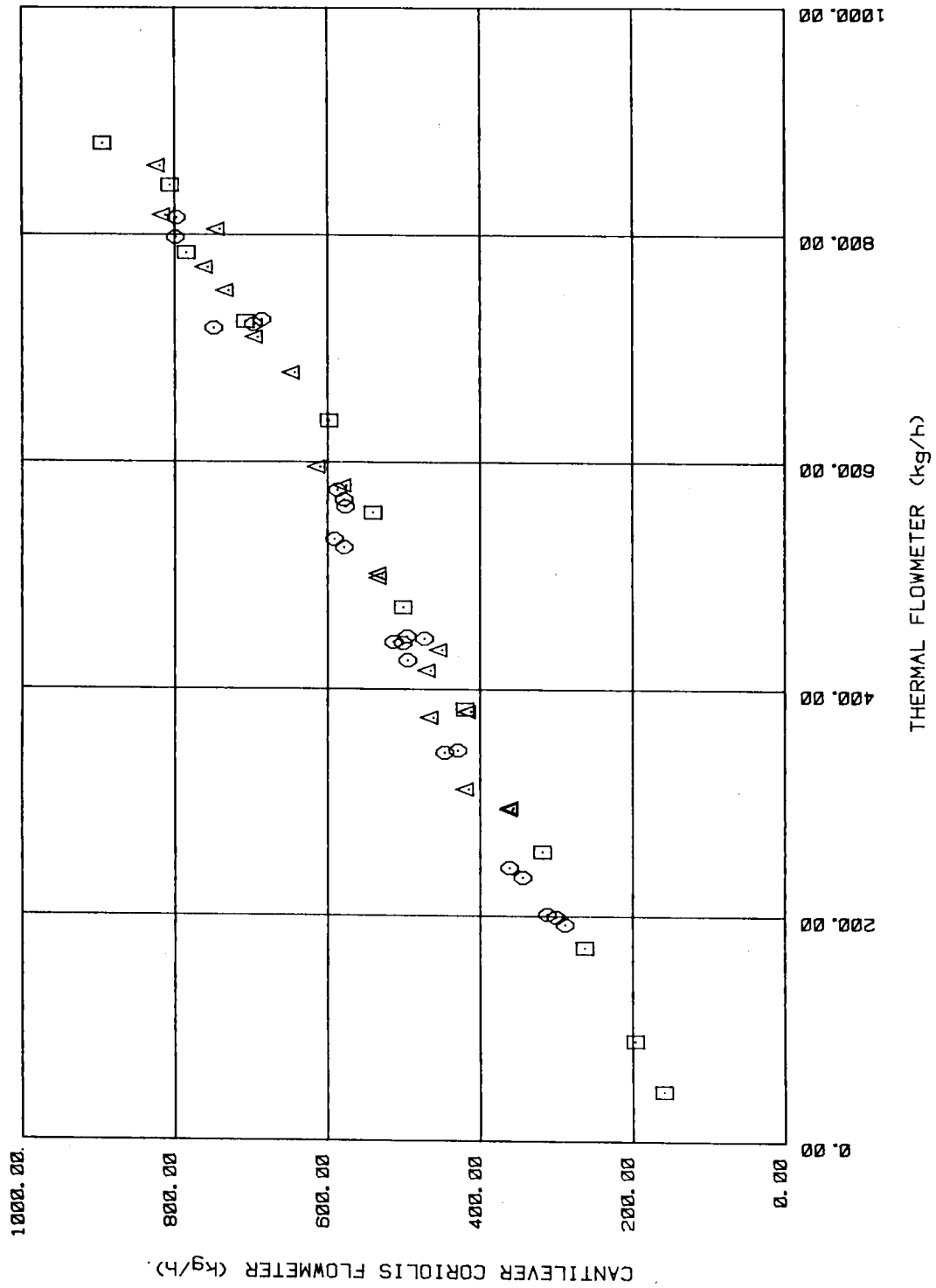


Fig. 31. Multiple results for 3/8 x 0.035 beam configuration.

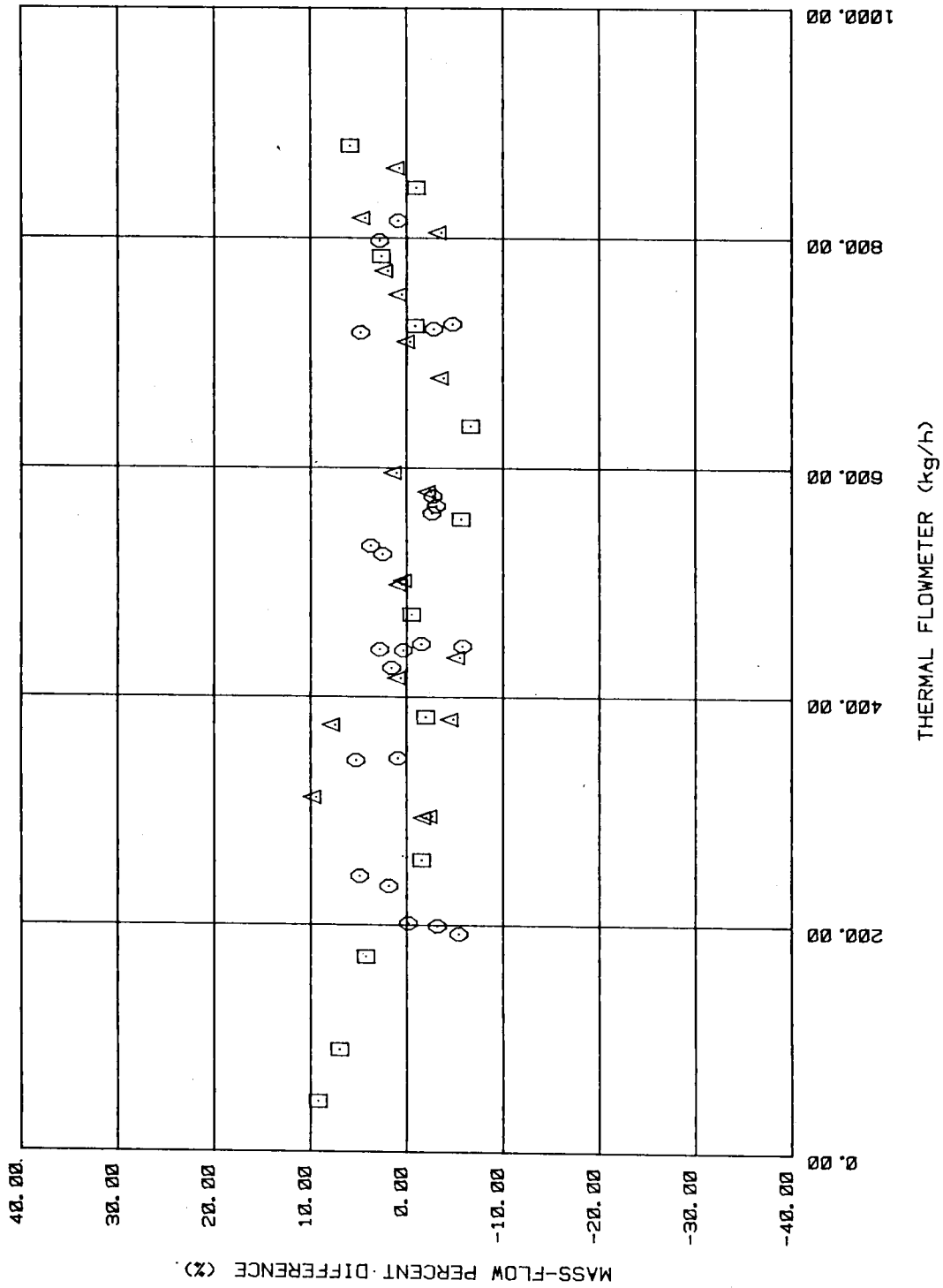


Fig. 32. Mass-flow regression comparison for 3/8 x 0.035 beam configuration.

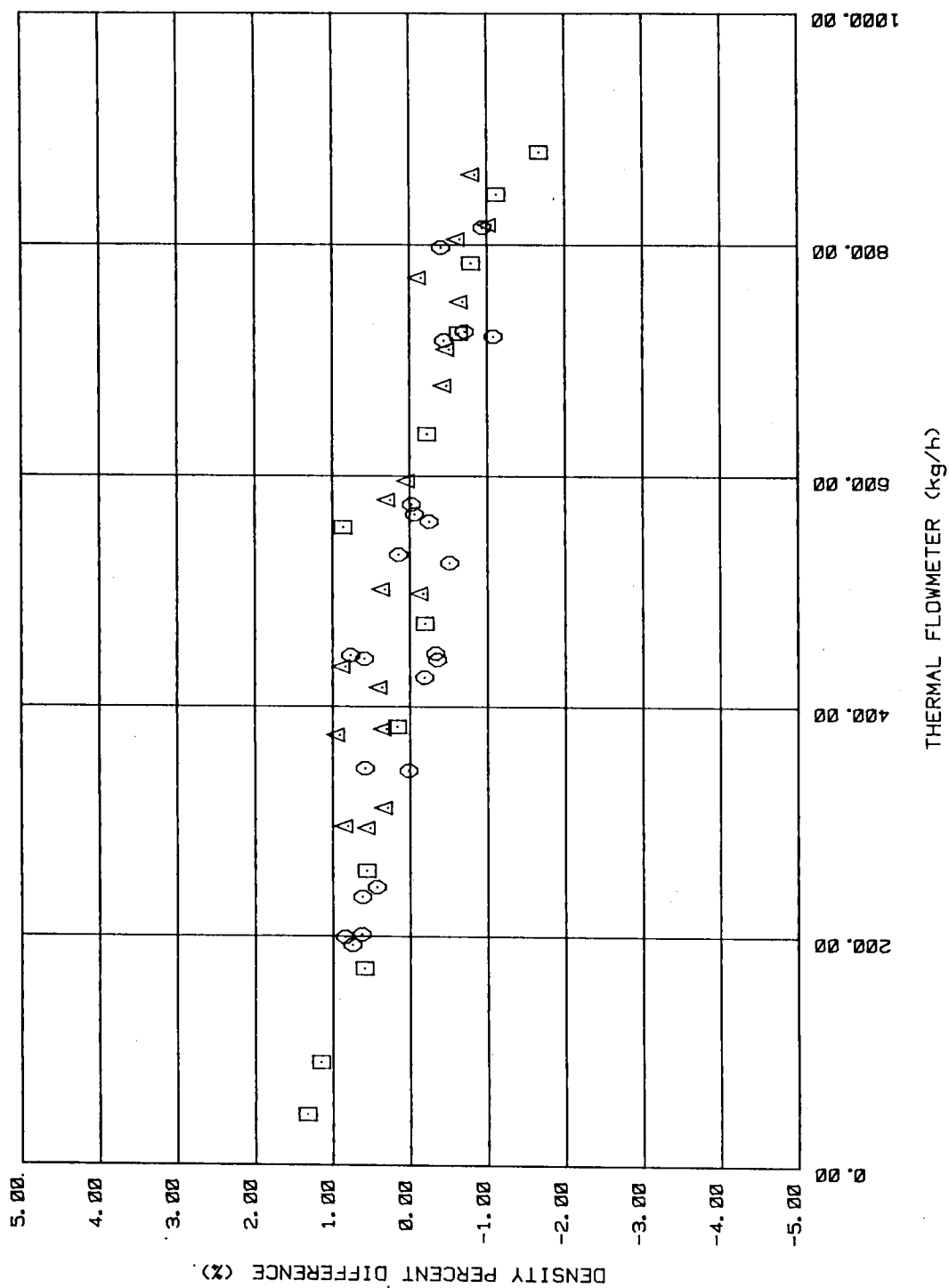


Fig. 33. Density regression comparison for 3/8 x 0.035 beam configuration.

ingest air at the suction by overrunning the reservoir capacity at high flow rates. A small amount of the trend may be attributed to temperature rise of the loop water during the day and during each test run. The temperature rise during a full day of operation would typically be as much as 5 to 10°C, while in early morning operation (i.e., the first runs after overnight cool down) the temperature rise would be 1 to 2°C. Temperature increases of this size will only account for a few tenths of a percent in the total density decrease.

Results for $1/4 \times 0.035$ Beam Configuration

The $1/4 \times 0.035$ cantilever is essentially the same as the $3/8 \times 0.035$ configuration but with a much smaller cross-sectional area, which resulted in much higher mean flow velocities in the lower flow range. Because this beam has the same wall thickness as the $3/8 \times 0.035$ configuration, it is, in a sense, more massive structurally. This is reflected in its lower β value shown in Table 3 (page 118). One would not expect this cantilever to perform as well as the first, because the low β value leads to a lower measurement sensitivity and the smaller cross-sectional area results in a reduced measurement range due to the higher flow resistance of the smaller cross section.

The frequency spectrum analysis of this configuration is shown in Fig. 34, and is consistent with the unimodal approximation used in the algorithm development. The fundamental frequency is slightly higher at 37 Hz. The dynamic response of the beam for three flow rates is shown in Fig. 35. The strain-gage deflection measurement quality appears to be slightly better than the first configuration.

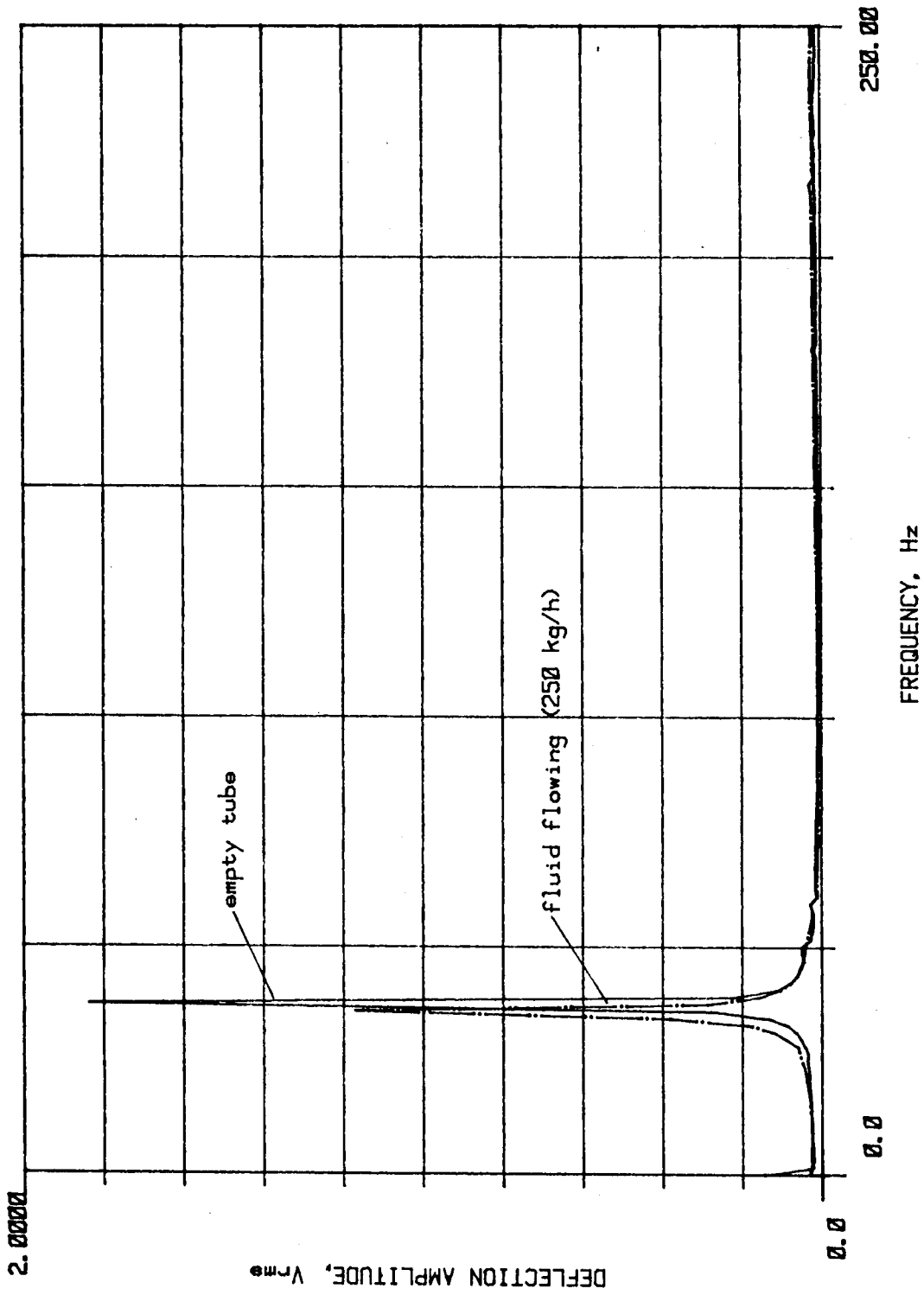


Fig. 34. Frequency spectrum for 1/4 x 0.035 beam configuration

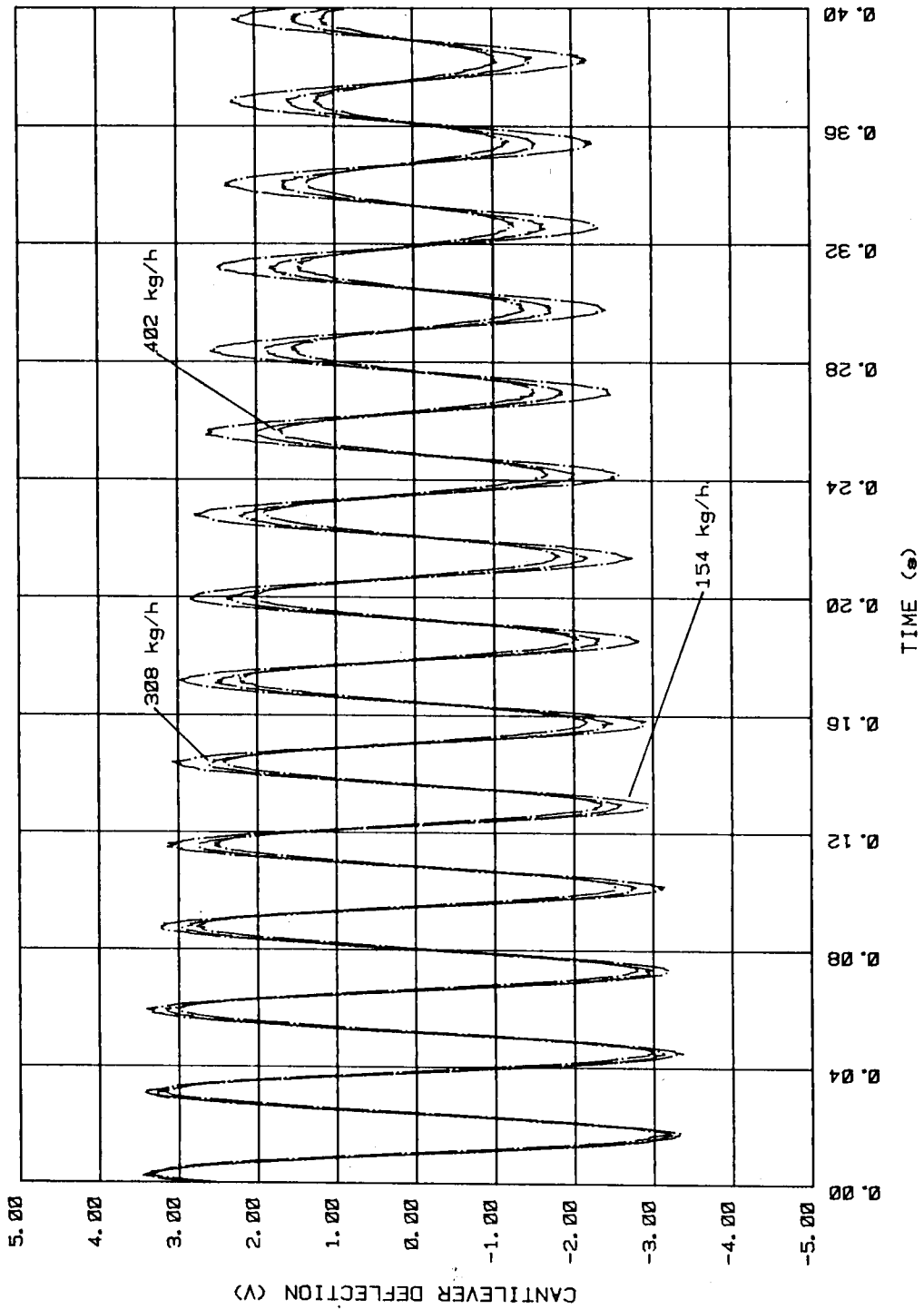


Fig. 35. Dynamic response of $1/4 \times 0.035$ beam configuration.

One set of calibration results for the $1/4 \times 0.035$ cantilever is shown in Table 5. These calibration results are similar to those of the $3/8 \times 0.035$ beam configuration with the exception of the large variance in a_2 . The parameter a_2 is mass-flow related and its high variance would suggest this configuration would have poor measurement performance. In all calibrations for this beam, the a_2 parameter had very large standard deviations typically in the range ± 10 to 25% of the mean. A standard deviation of this size in a presumably invariant parameter is unacceptable.

Table 5. Typical calibration results for
 $1/4 \times 0.035$ beam configuration

Calibration parameter	Mean	Standard deviation (% of mean)
Channel Biases:		
Thermal flowmeter (kg/h)	7.06720	
Cantilever deflection		
-with flow (V)	0.04203	
-empty (V)	-0.05276	
Calibration Constants:		
Empty decay constant, $R_{C1}(s^{-1})$	0.737480	3.07
Empty frequency-squared, $R_{C2}(s^{-2})$	555,142.6	0.05
a_1	590.8546	0.04
a_2	15.19295	36.0

A number of tests were run in spite of the calibration uncertainty. A typical test result and regression analysis is given in Fig. 36 and four different tests are shown in Fig. 37. These results are actually better than might have been anticipated from the calibration results. Notice that the data show a negative-zero bias (or intercept). In

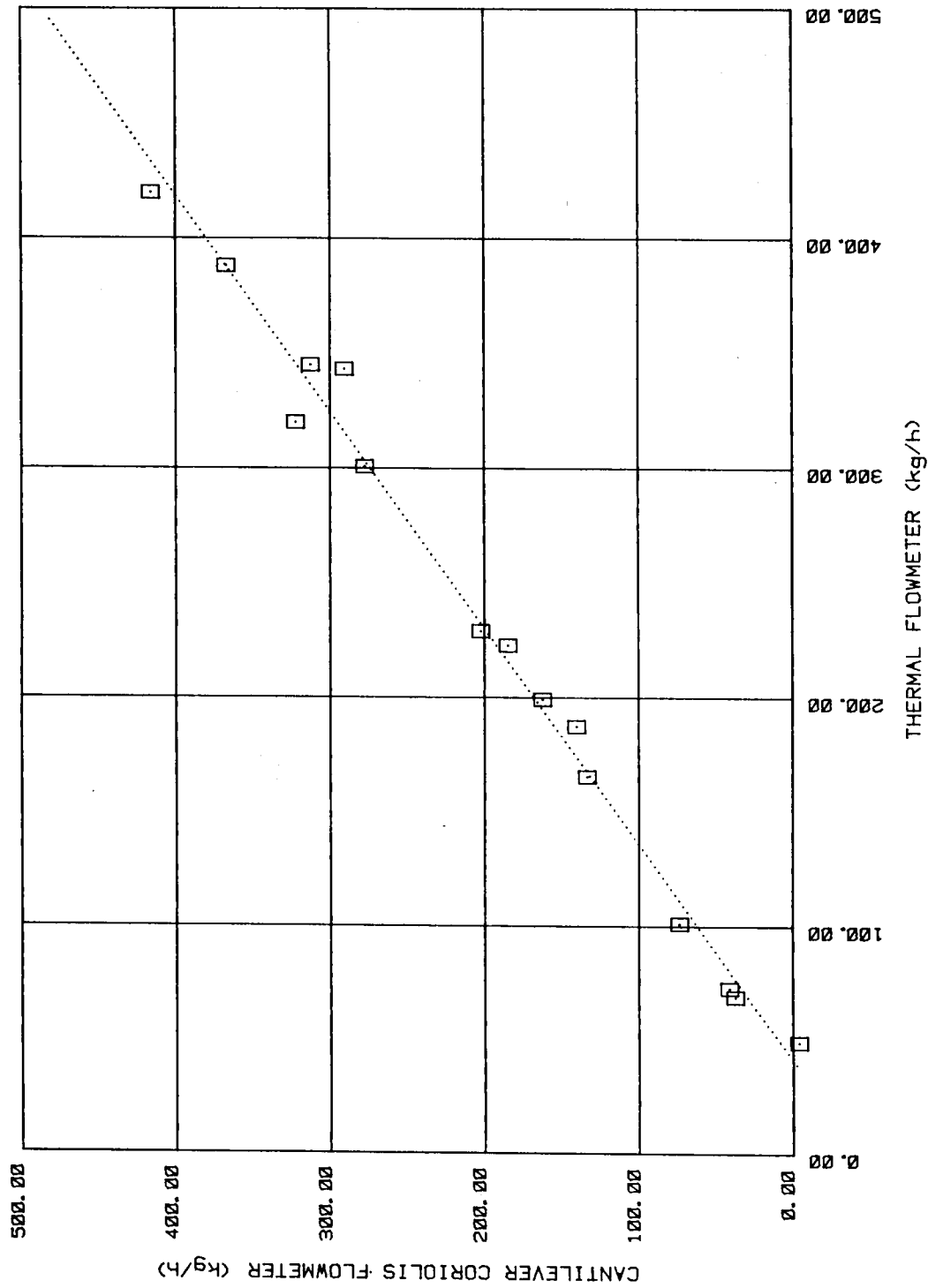


Fig. 36. Typical result and regression for $1/4 \times 0.035$ beam configuration.

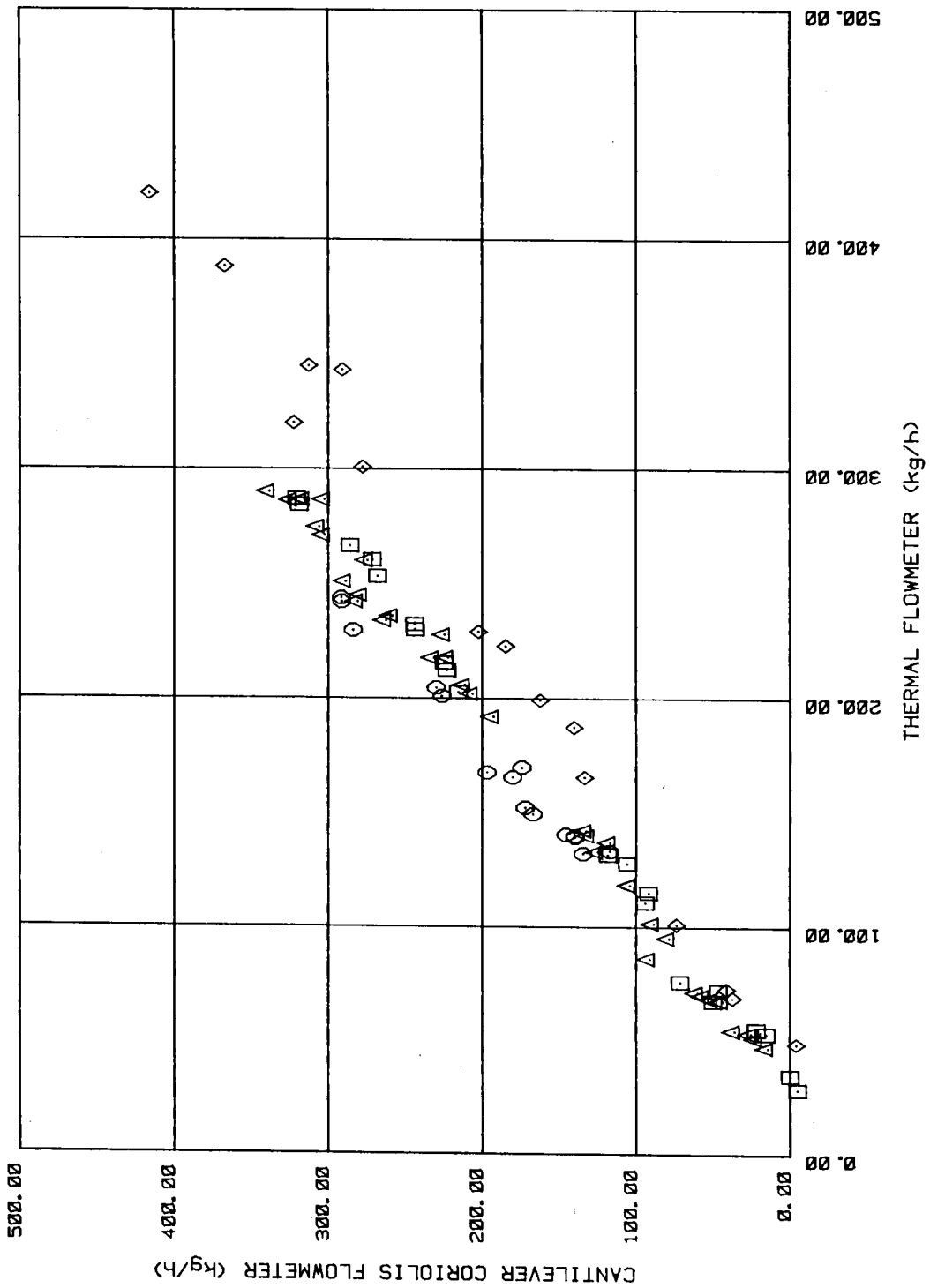


Fig. 37. Multiple results for $1/4 \times 0.035$ beam configuration.

Fig. 37, the test represented with the diamond-data-point symbol was based upon a different calibration than the other three tests shown and accounts for the difference in slope. All four tests have essentially the same negative-zero bias, unlike the positive-offset seen in the $3/8 \times 0.035$ beam configuration results.

A positive offset makes sense when interpreted as a loss of measurement sensitivity at low flow rates. An explanation for the negative bias seen in this case is not apparent. Strange behavior such as this may be related to the thermal flowmeter uncertainty. As seen in Figs. 20 and 21 (pages 107 and 108), the thermal flowmeter has a low-range scatter of about $\pm 10\%$. As shown in Fig. 38 the $1/4 \times 0.035$ Coriolis configuration experiences very large errors ($\pm 40\%$ relative to reading) below 100 kg/h. Above 100 kg/h, the linearity is within $\pm 10\%$. Error divergence at the lower range is not unusual for instrumentation when errors are expressed in terms of reading rather than span. The density measurement performance of the $1/4 \times 0.035$ beam configuration is much better as shown in Fig. 39, and is generally within $\pm 1\%$ of the water value of 997 kg/m^3 . These data do not contain the decreasing trend with increasing flow rate seen with the $3/8 \times 0.035$ beam. This corresponds with the fact that because of the much higher pressure drop a maximum flow rate of only 450 kg/h could be achieved. At this lower flow rate, no problems of ingesting air in the fluid reservoir were experienced. The results for this configuration are essentially as expected in that the data show a greater degree of scatter due to a

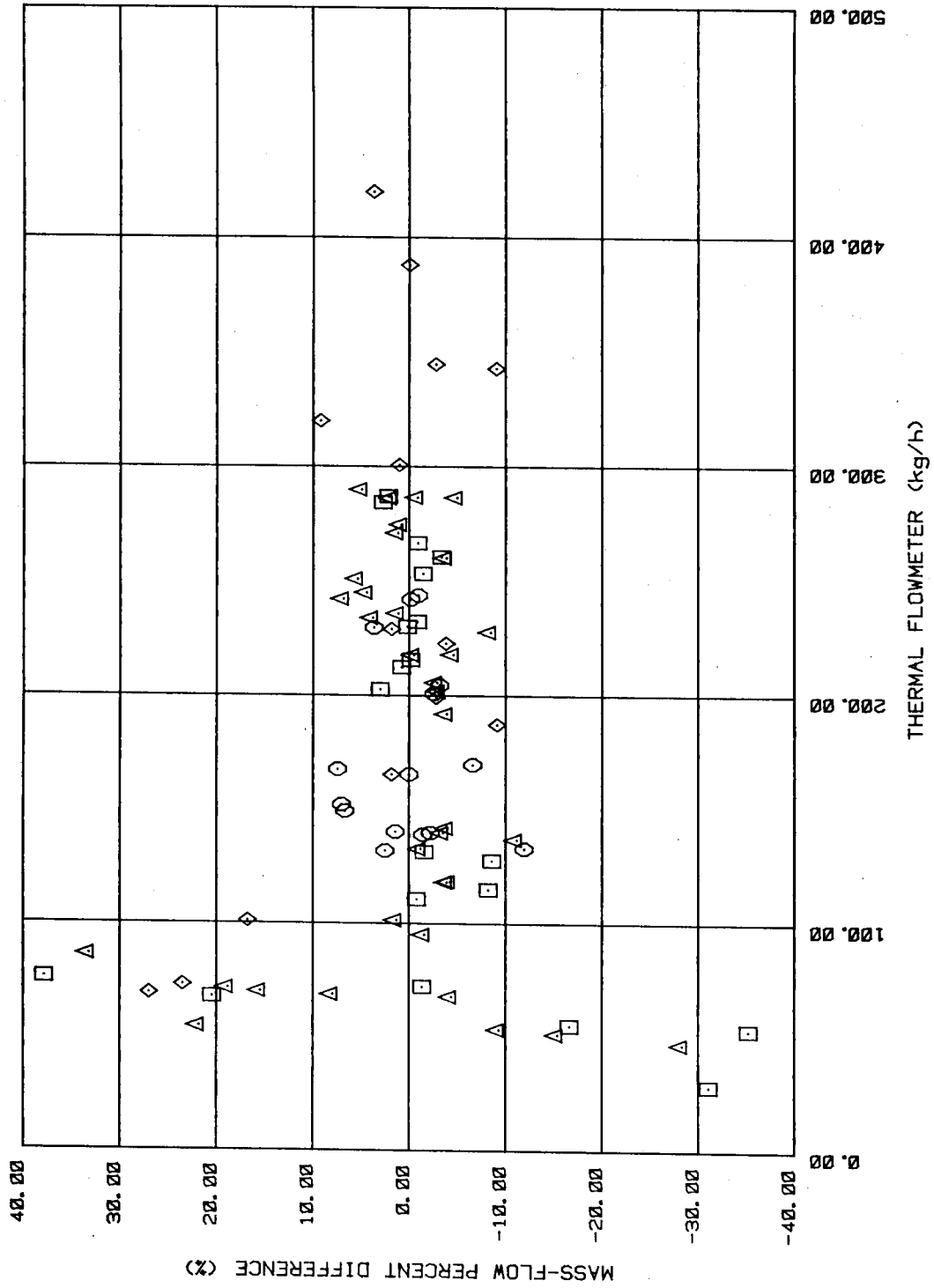


Fig. 38. Mass-flow regression comparison for $1/4 \times 0.035$ beam configuration.

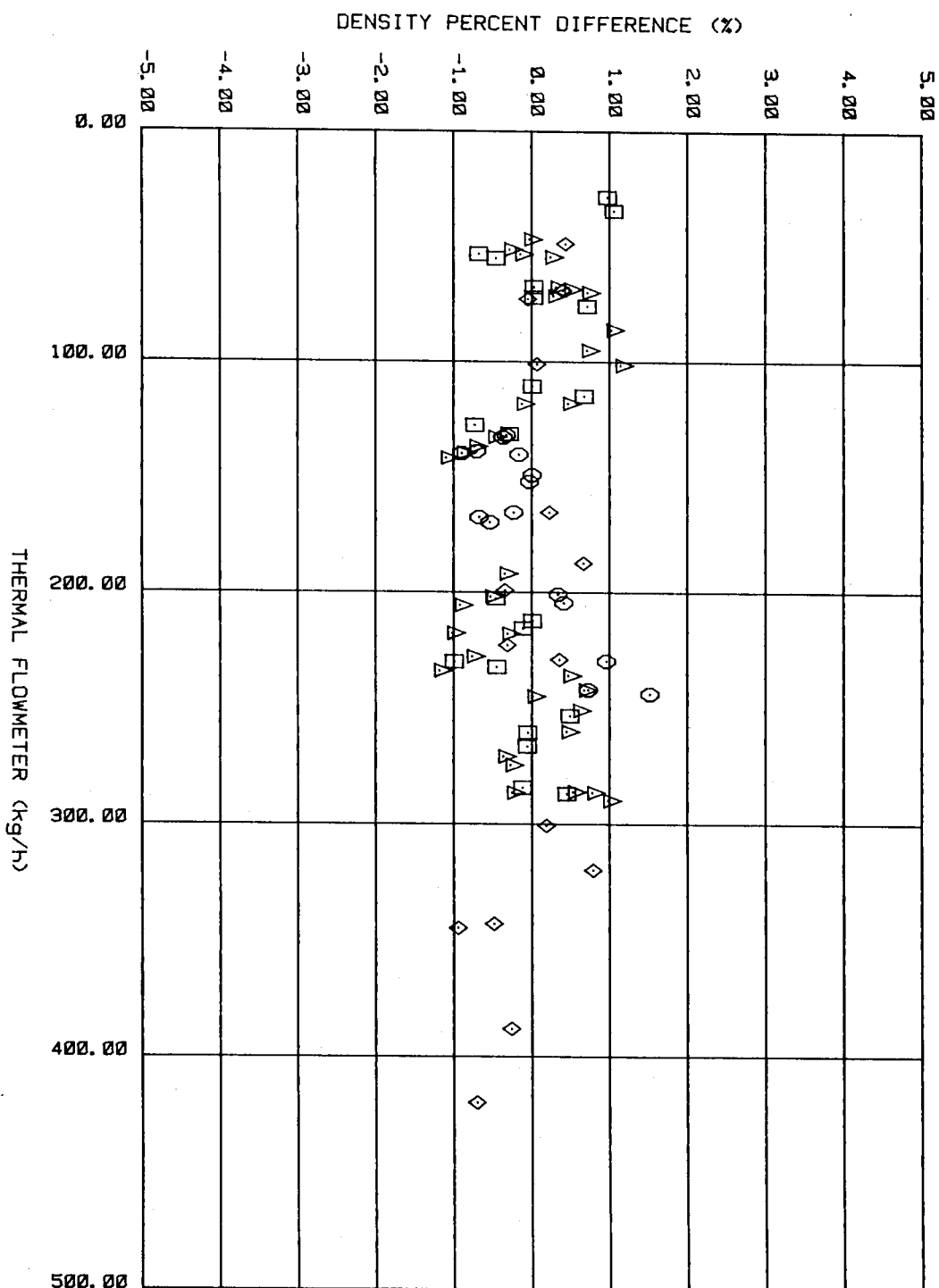


Fig. 39. Density regression comparison for 1/4 x 0.035 beam configuration.

reduction in measurement sensitivity. The source of the negative-zero offset has not been identified.

Results for $1/4 \times 0.010$ Beam Configuration

As discussed in Chapter 3, cantilever Coriolis sensor sensitivity increases with increasing tube thickness and length. For the last test, it was decided to use a sensor made from the thinnest tubing readily available. A sensor was fabricated from 6.35-mm tubing having a 0.254-mm wall thickness. As shown in Table 3 (page 118), this resulted in much higher β value and a potentially high nondimensional velocity, u . Recall that the approximate analytical solution used in the measurement algorithm development is $O(u^2)$, which translates into a truncation error of 4% for $u = 0.2$. Also, recall that a simplification was used in obtaining Eqs. (136) based on the assumption that the frequency shift with respect to u is negligible. As shown in Fig. 13 (page 72), this assumption will introduce an additional error of approximately -0.25% for $\beta = 0.4$ and $u = 0.2$. The $1/4 \times 0.010$ beam configuration stretches the validity of the underlying theoretical analysis far beyond the other two tests. This configuration presents an interesting dichotomy: maximum measurement sensitivity versus potentially significant modeling error.

As with the other two configurations, the first step was to perform a spectral analysis of the beam response. This time the results were both surprising and disappointing. The unadulterated frequency spectrum

for the $1/4 \times 0.010$ beam is given in Fig. 40 and shows the ghastly presence of the second mode of vibration — exactly where it should be at approximately 180 Hz! This configuration is rather fragile such that the method of motion excitation somehow excites the (with disgusting repeatability) second mode.

In this situation, two possible recourses were recognized:

(1) derive a new set of algorithms which permits the presence of both modes, and (2) arbitrarily extract the second-mode signal with simple filtering. It was deemed that derivation of more sophisticated algorithms would be a major task beyond the scope of this work. In expedience, it was decided to try simple blocking of the second-mode frequency using a first-order low-pass filter with a corner frequency at 100 Hz. The fundamental question in this approach is how well the filtered response will match the model prediction. In viewing Fig. 40, the power ($\propto [\text{amplitude}]^2$) in the first mode is much greater than the second mode such that deleting it arbitrarily might not introduce large errors.

The effect of filtering on the transient response is shown in Fig. 41. The simple waveform analysis techniques presented in Chapter 3 will not function properly on the unfiltered waveform with its many local maxima/minima caused by the second mode. The filtered waveform, however, appears to be unimodal with the exception of low-frequency distortion toward the end of the measurement interval. A filtered-frequency spectrum is shown in Fig. 42, and verifies the complete removal of the second mode.

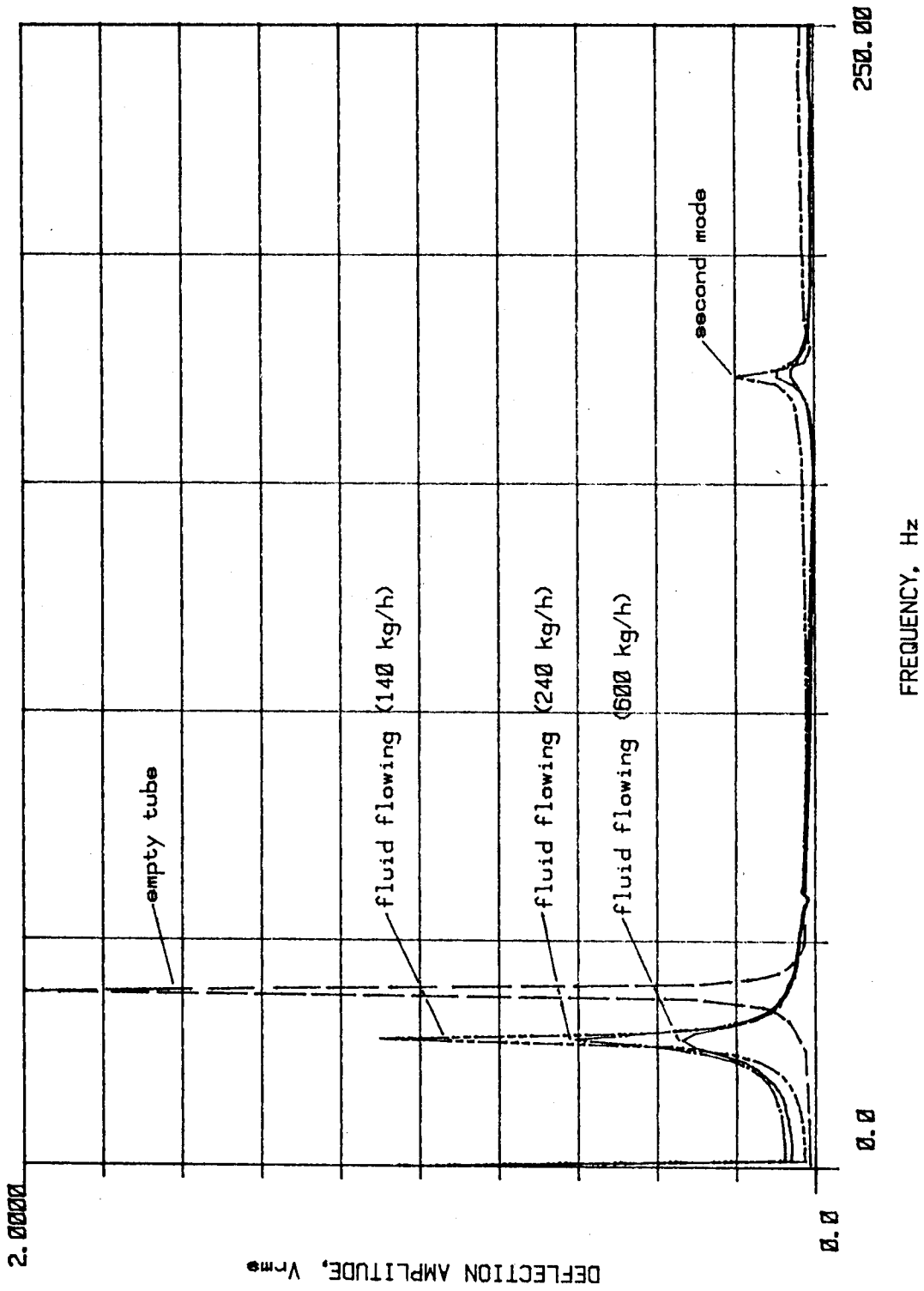


Fig. 40. Frequency spectrum for $1/4 \times 0.010$ beam configuration.

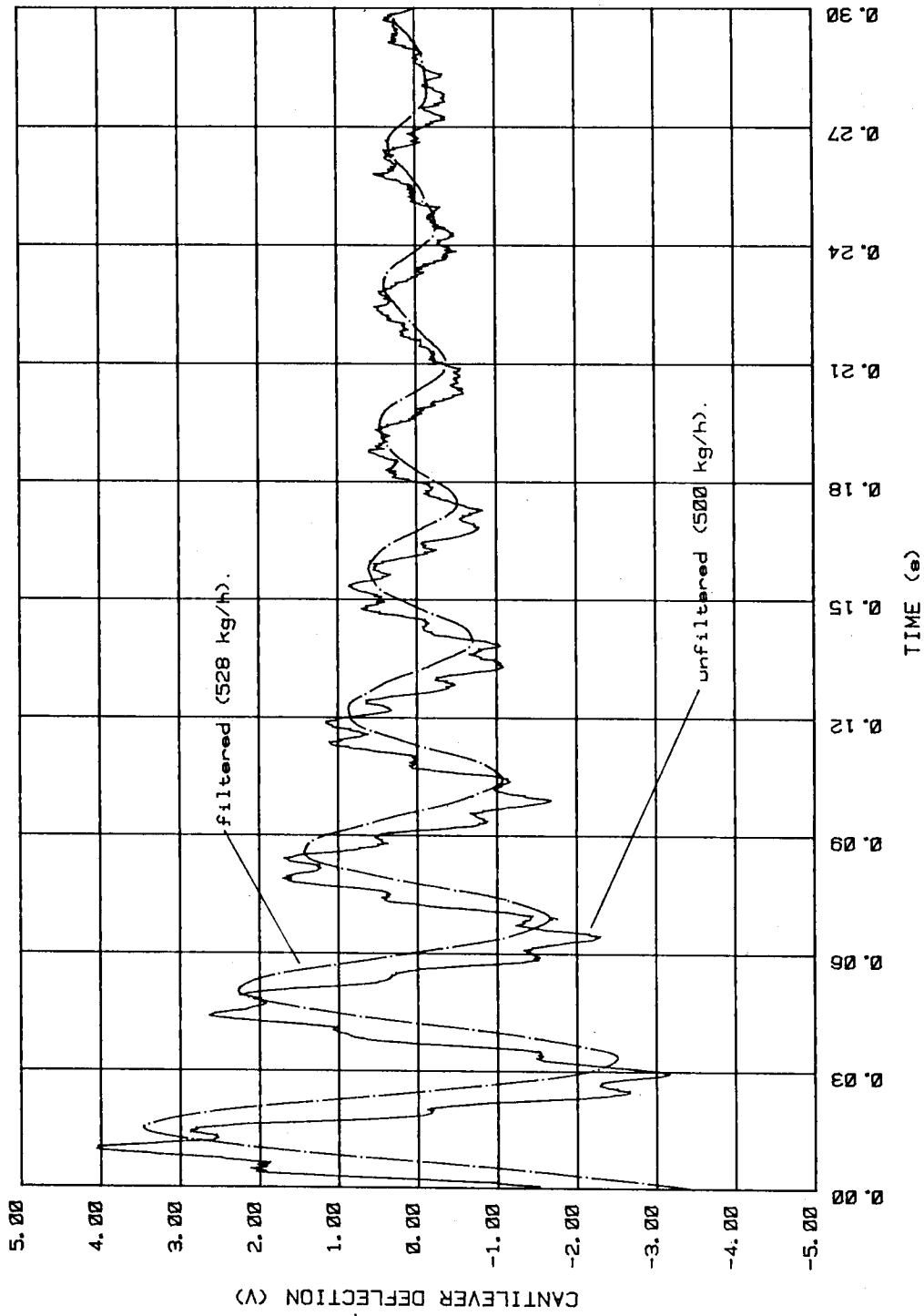


Fig. 41. Filtering effects on the dynamic response of $1/4 \times 0.010$ beam configuration.

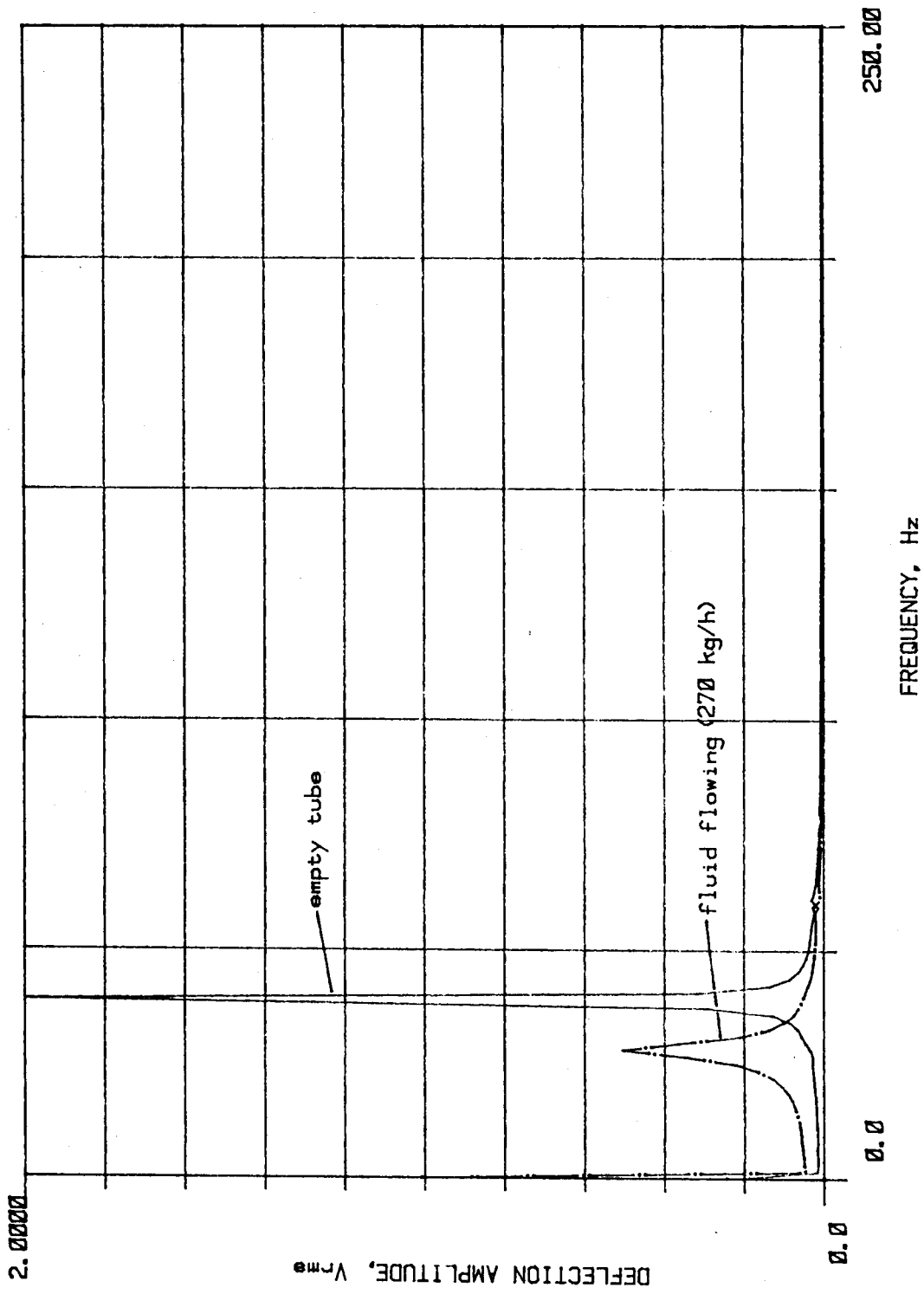


Fig. 42. Filtered frequency spectrum for 1/4 x 0.010 beam configuration.

The $1/4 \times 0.010$ configuration transient response for several fluid flow rates is shown in Fig. 43 and displays the high measurement sensitivity of this beam. At 768 kg/h, the waveform distortion beyond 120 ms is quite large. Typical calibration results for this configuration are shown in Table 6, and are much better than the $1/4 \times 0.035$ beam.

A typical flow measurement result is shown in Fig. 44. These are the best results of the three sensors tested in spite of the artificial extraction of the second mode of vibration. This beam has excellent linearity and rangeability as compared with the thermal flowmeter. In fact, it is believed that the limiting factor in this test was the flow threshold of the thermal flowmeter — not the cantilever Coriolis flowmeter. Figure 45 shows how well four separate test runs "cluster" about the same line, and notice that the zero-intercept bias is positive and the smallest observed. Linearity with respect to regression lines for each test are shown in Fig. 46. The mass-flow rate data are within a band of $\pm 5\%$ and the low range scatter is no greater than that of the thermal flowmeter. The density percent difference shown in Fig. 47 is not as good as the other two configurations because of the waveform distortion. The waveform distortion causes a large variation in the zero-crossing intervals. This, in turn, results in large frequency errors that lead to calculated density errors as large as $\pm 4\%$ compared to about $\pm 1\%$ for the other configurations.

Summary

Three cantilever flow sensors ranging the nondimensional parameters β and u were experimentally evaluated. The experimental trends matched

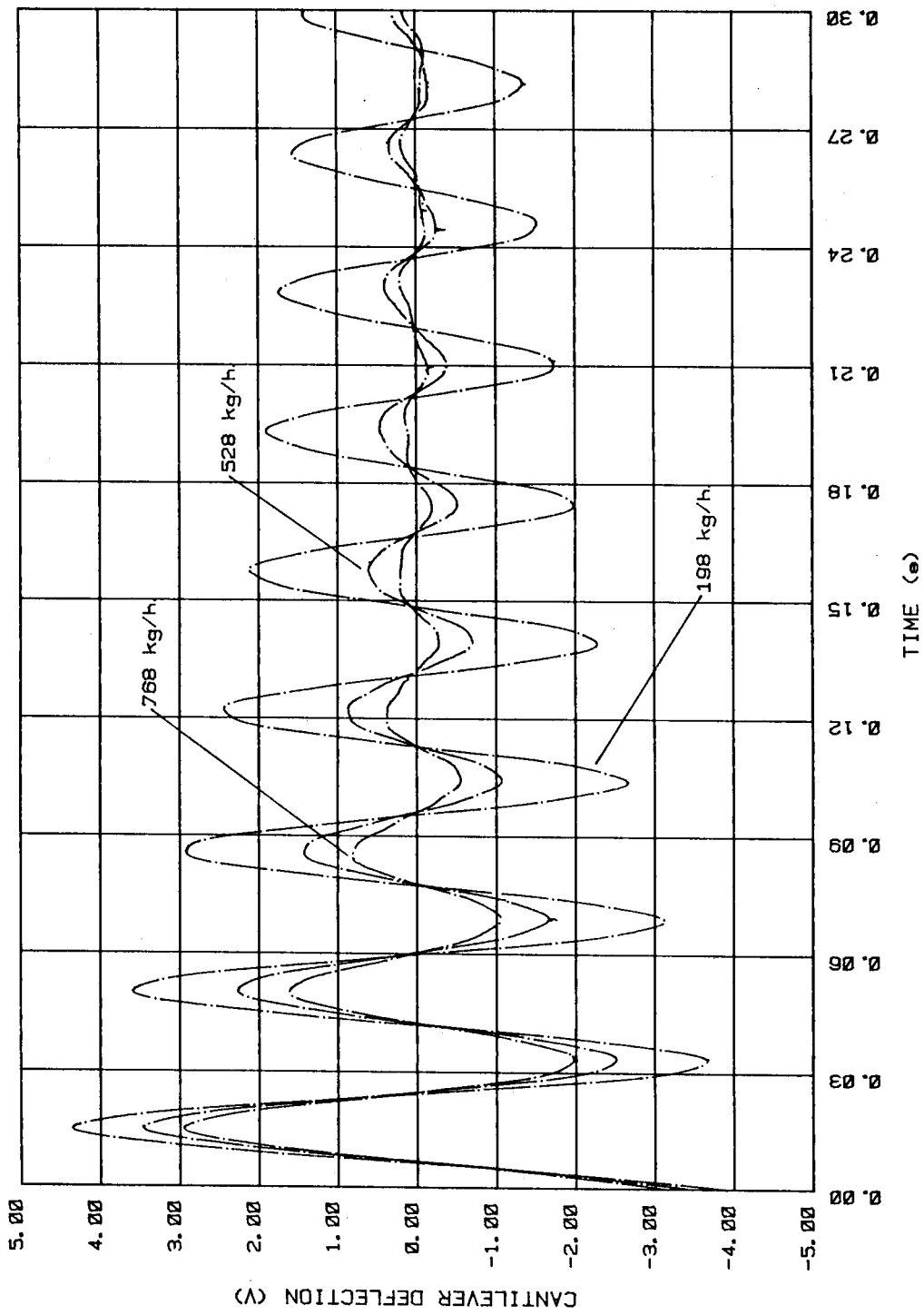


Fig. 43. Filtered dynamic response of $1/4 \times 0.010$ beam configuration.

Table 6. Typical calibration results for
 $1/4 \times 0.010$ beam configuration

Calibration parameter	Mean	Standard deviation (% of mean)
Channel Biases:		
Thermal flowmeter (kg/h)	3.92769	
Cantilever deflection		
-with flow (V)	0.06045	
-empty (V)	-0.68496	
Calibration Constants:		
Empty decay constant, $R_{C1} (s^{-1})$	0.633508	4.10
Empty frequency-squared, $R_{C2} (s^{-2})$	60,211.9	0.09
a_1	67.93414	1.01
a_2	1.210800	2.50

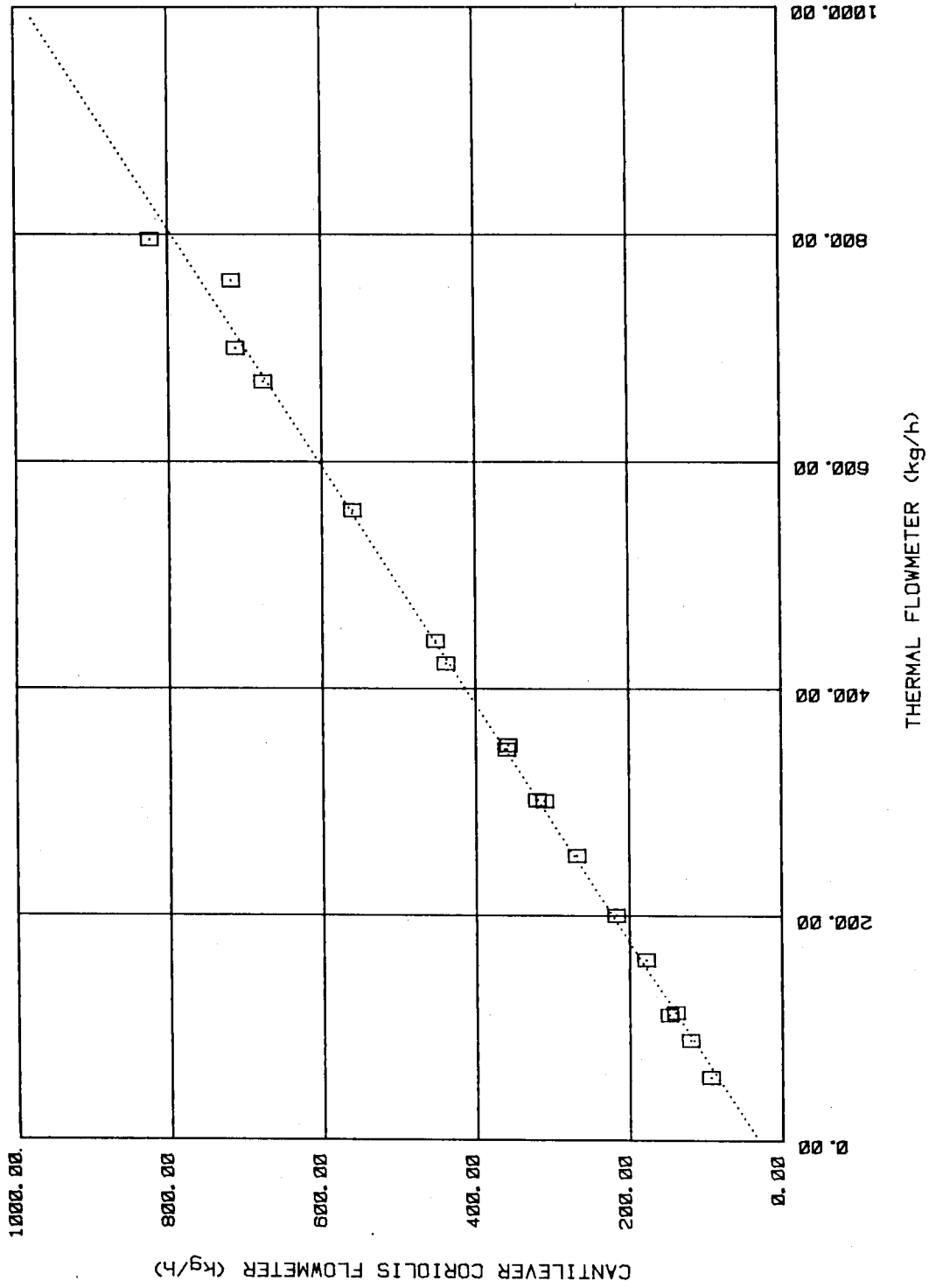


Fig. 44. Typical result and regression for $1/4 \times 0.010$ beam configuration.

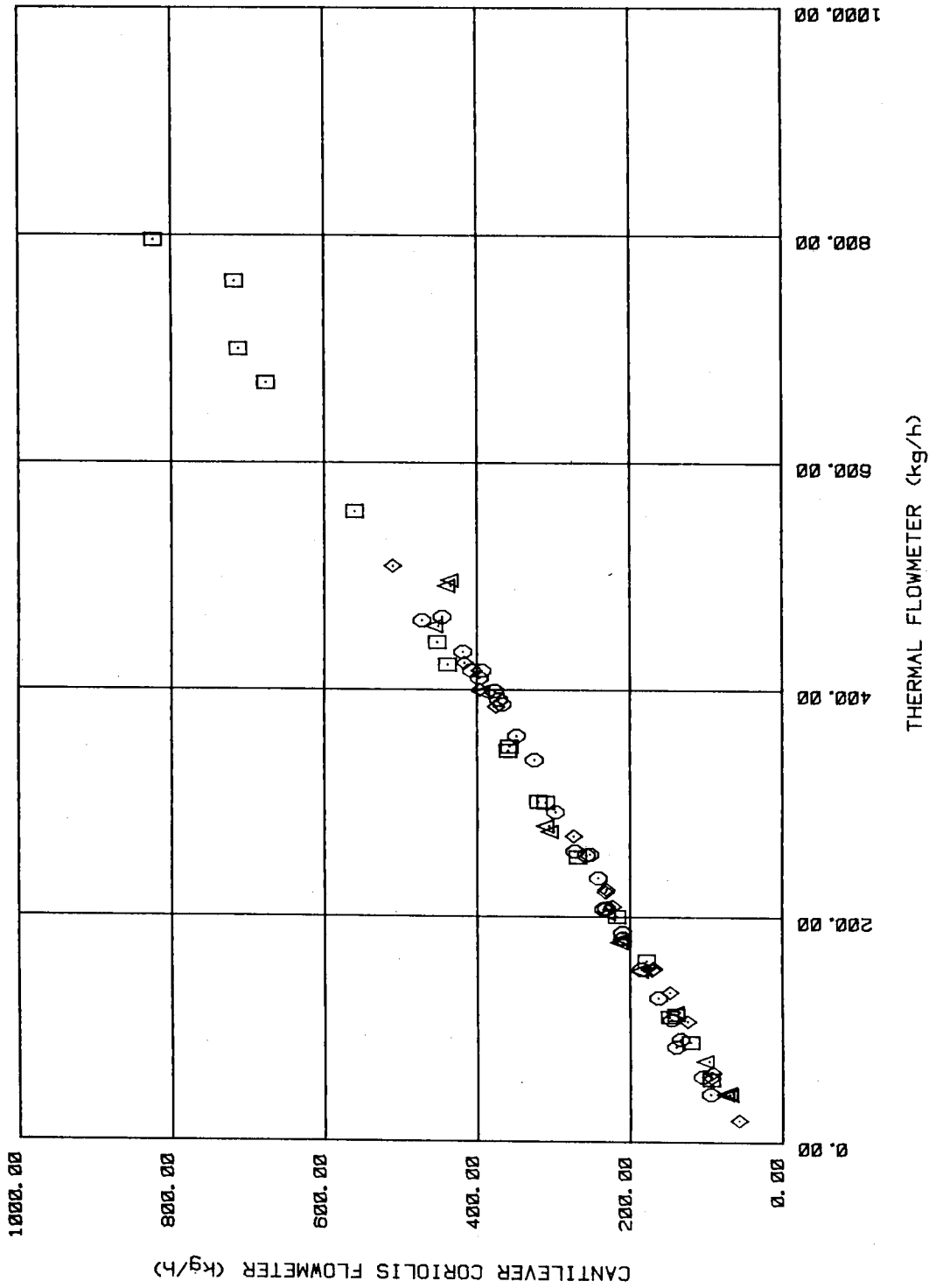


Fig. 45. Multiple results for $1/4 \times 0.010$ beam configuration.

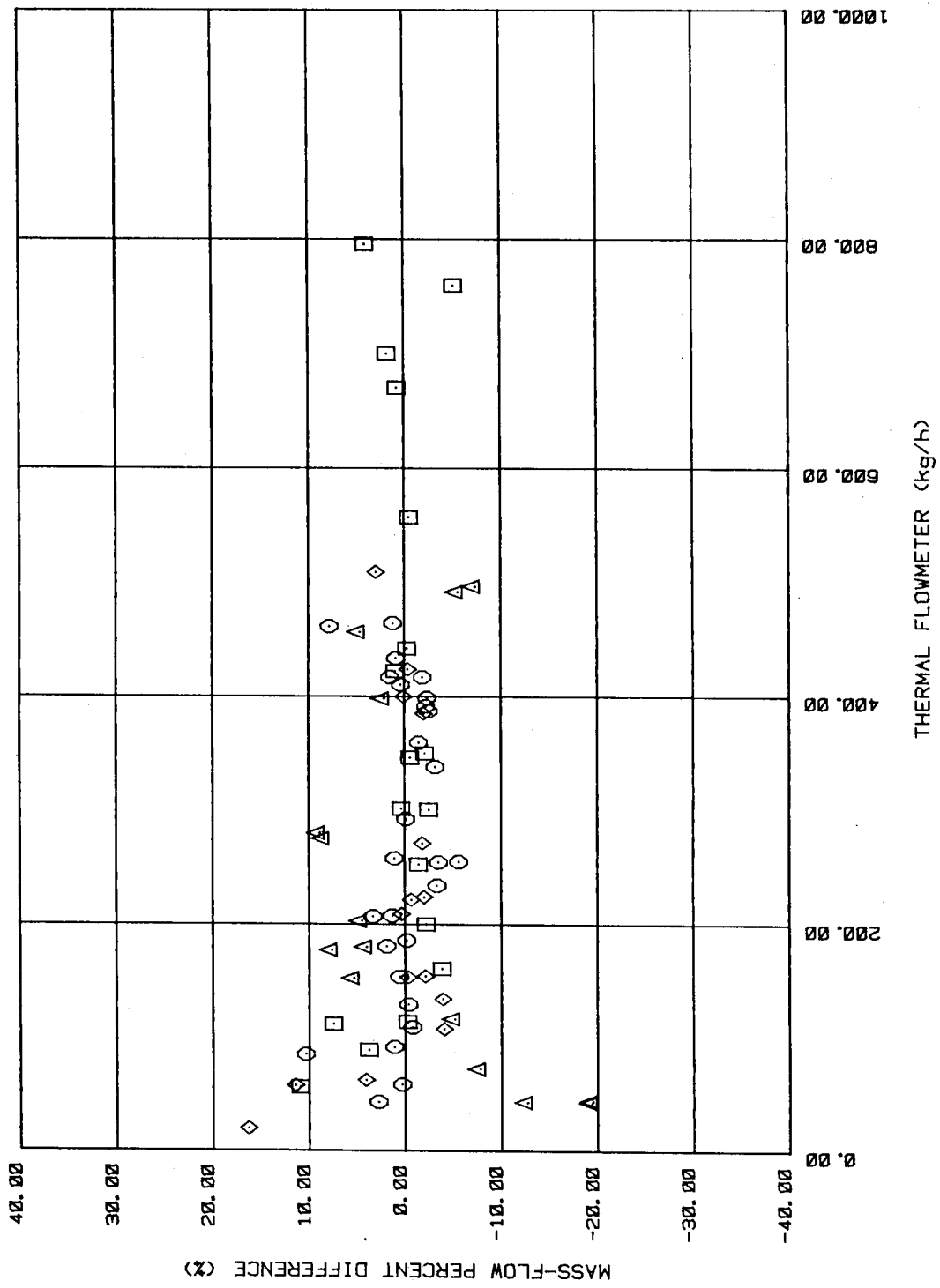


Fig. 46. Mass-flow regression comparison for $1/4 \times 0.010$ beam configuration.

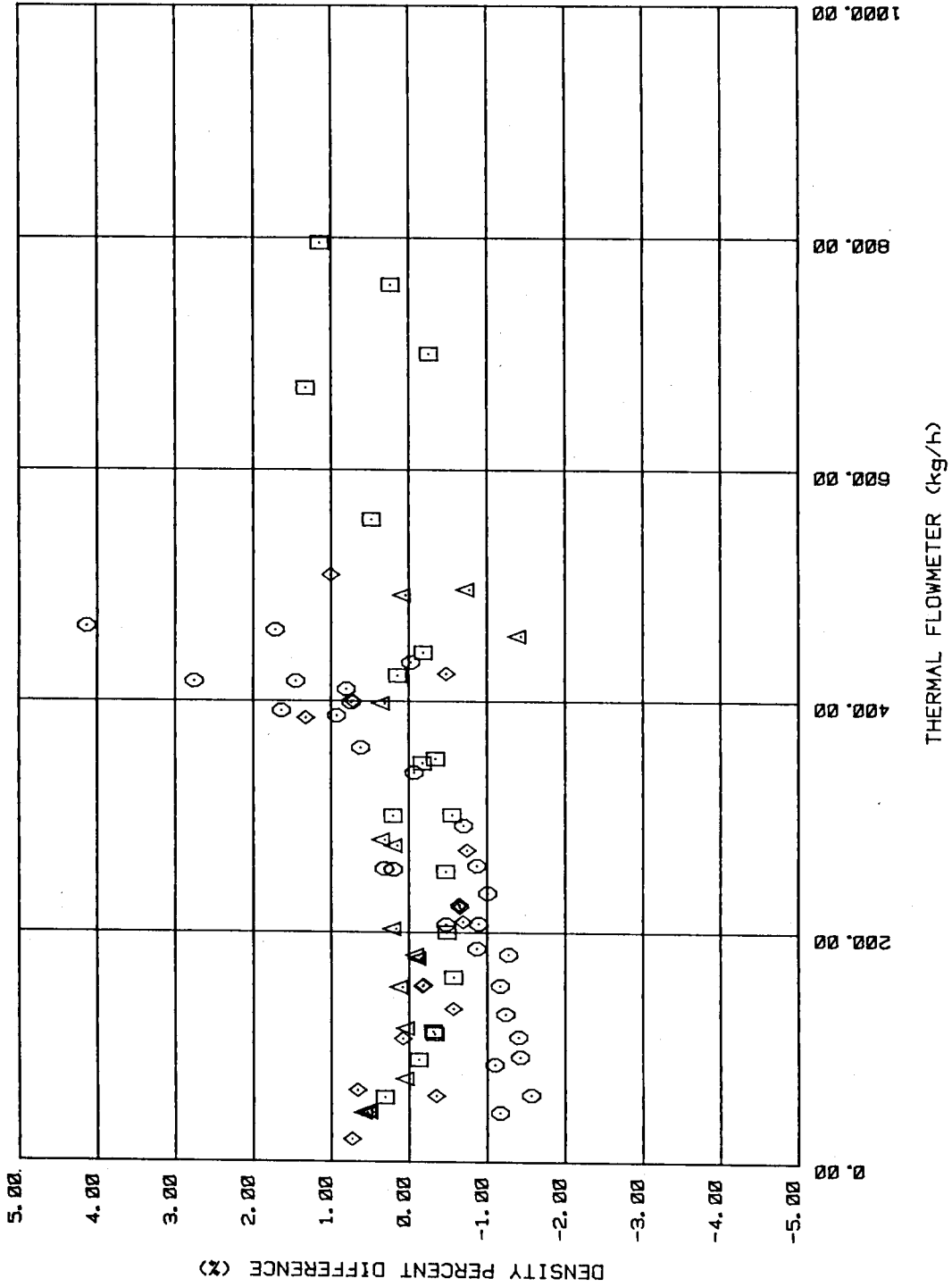


Fig. 47. Density regression comparison for 1/4 x 0.010 beam configuration.

analytical predictions quite well. Very thin beams were found to be the most sensitive to the Coriolis flow effects, but were found to be more susceptible to multimodal excitation from the static deflection forcing technique.

In the case of the $1/4 \times 0.010$ beam configuration, it was found that simple filtering of the undesired second-mode vibrations could be used without destroying the measurement capability. For very simple mechanical flow sensors arrangements, mass-flow rate measurement performance relative to an industrial-grade thermal flowmeter was found to be about $\pm 10\%$ of reading except near zero where errors increased ± 20 to 40% . Density measurement performance was found to be ± 1 to 4% of reading for the known loop water density of 997 kg/m^3 .

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The basic objectives originally established for this research have for the most part been accomplished. A new type of Coriolis flowmeter has been conceived, analyzed in considerable detail, and shown to be feasible through proof of principle experimentation. The new concept has demonstrated the use of dynamic sensor characteristics as a basis of measurement and relied heavily on microprocessor-based data acquisition and realtime computation.

In the remainder of this section, additional specific conclusions that relate to future research in Coriolis flow measurements are discussed.

Theoretical Analysis

The theoretical analysis was a major portion of this work and has verified the inherent complexity of the mathematics associated with hydroelastic systems. It is unlikely that direct solution of the associated boundary value problem will be possible for any flowmeter geometries — even the simplest such as the cantilever configuration studied in this work. Approximate solutions that restrict the number of eigenvalues are possible, and the method of multiple scales in perturbation theory has been shown to work well for this class of systems with small, but significant, damping effects. It was shown

that only clamped-free boundary conditions result in nonconservative behavior for simple hydroelastic beams undergoing planar vibration.

For the case of deflection measurement at a single spatial position, the approximate analytical solutions can be used as a suitable basis for the development of mass-flow rate and density measurement algorithms. The approximate analytical solution, Eqs. (103), when viewed at a single longitudinal position can be expressed as a second-order ordinary differential equation, Eq. (114), or an algebraic equation, Eq. (115). The algebraic expression was successfully used to develop simple calibration and measurement schemes for realizing the transient cantilever Coriolis flowmeter.

Proof of Principle Experiment

The proof of principle experiment showed that the transient concept of a cantilever Coriolis flowmeter that simultaneously measures density, mean velocity, and mass flow rate is feasible. Experimental results for three beam sizes correlated well with the response characteristics and parametric trends predicted by the theory. The experimental results demonstrated that the transient mode of motion excitation could be accomplished. The particular parameter ranges used also showed that the cantilever Coriolis flowmeter will work at very low flow ranges below 100 kg/h. The results are judged to be sufficiently good to justify the next phase of prototypic development.

Practicality Considerations

The most significant practical limitation of the cantilever Coriolis meter is the flow sensor geometry. Traditionally, process flowmeters are

thought of as devices that are relatively small, used in-line, and are installed far away from pipe bends, valves, or other such transitions. In contrast, the cantilever Coriolis flowmeter would be a relatively large end-of-line device that can only be installed such that the cantilever free-end discharges directly into a low-viscosity free-space (e.g., the headspace above the liquid level in process vessels).

Usually, new and different devices must provide significant advantages or performance improvements to be recognized. For cantilever Coriolis flowmeter concepts to achieve applications significance, the utility of the simultaneous measurement of density, velocity, and mass-flow rate must outweigh the "strangeness" of its installation constraints. Its performance (i.e., accuracy and range) will have to be as good, or better, than standard methods. This research has addressed these two issues superficially.

For several reasons, the cantilever flow sensors must be relatively long (e.g., at least 10 pipe diameters). For large line sizes, these flow sensors would approach the size of major equipment components. For large pipe diameters and typical materials, Eq. (146) can be used to define the relationship

$$L \approx \frac{2000 \sqrt{d_o}}{\sqrt{\omega_s}}, \quad (150)$$

where L and d_o are in meters and ω_s is in Hz. A 15 cm sensor-line would be about 2 m in length for a 5 KHz sampling rate. Assuming Sched 40 pipe, the pipe section necessary to fabricate the flow sensor would

have a mass of 50 to 60 kg, and the complete flowmeter assembly would likely be on the order of 100 to 150 kg. And, of course, the size of the motion excitation actuator would increase correspondingly. Such large size requirements would likely restrict the application of this type of flowmeter to smaller flow-range applications.

No other major limitations are envisioned. The deflection measurement and microprocessor-based computation requirements are straightforward.

Recommendations

Like most research, the results of this work suggest a number of possible follow-on activities. These activities are discussed in terms of the following topical areas: (1) prototypic experimentation, (2) mathematical model enhancement, and (3) advanced measurement algorithms.

Prototypic Experimentation

The next level of developmental refinement following the proof of principle experiment would include design of an experiment that demonstrates, for a specific application, as many prototypical mechanical and electronic features as possible. The mechanical design of the cantilever flow sensor would be the major challenge in this phase. The schematic given in Fig. 48 shows a concept depicting additional features including flange mounting provisions for integration into a vessel/piping system, protective shroud, and electromagnetic (noncontact) motion excitation.

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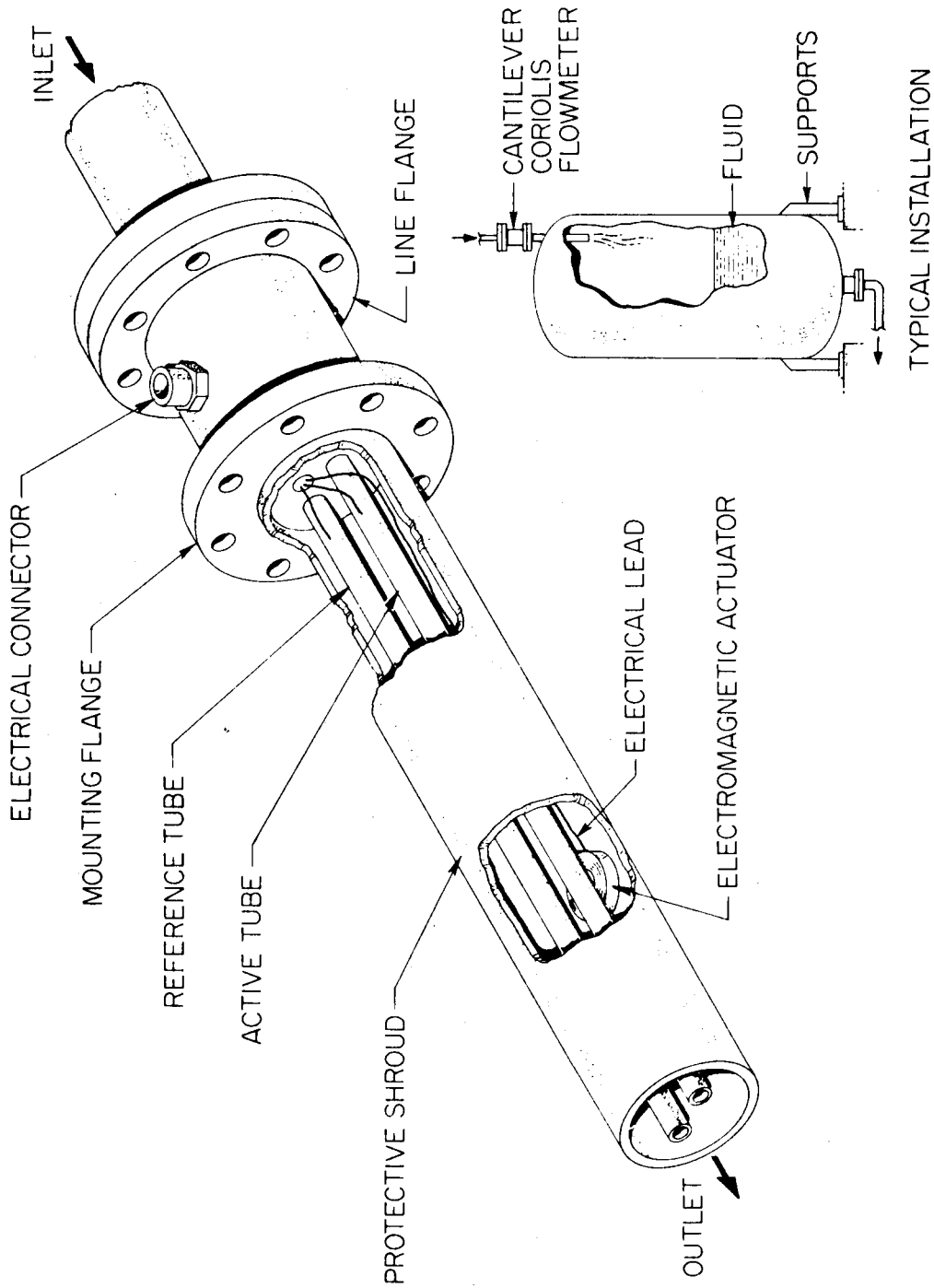


Fig. 48. Prototype cantilever Coriolis flowmeter concept.

Note also the incorporation of a "dry" reference-sensing tube. This tube would be excited in the same fashion as the "wet" sensing tube to provide updated values for the calibration constants, R_{C1} and R_{C2} , at the time of each measurement. This provision would be used to reduce long-term environmental effects on these two calibration constants.

In a prototypical mechanical design, attention would be given to the rigidity of the cantilever mounting and possibly to dynamic counterbalancing of the tube/fluid vibration. These two considerations would tend to minimize unmodeled secondary vibrations.

Alternate optical deflection measurement (e.g., linear diode array cameras) schemes and welded-strain gages for harsh environmental conditions are two areas which should be looked at in more detail.

In the prototypical phase, the experimental verification should include both variable density and variable volumetric flow rate tests which are based on high-quality traceable calibration standards. Test loop conditions should be actively controlled to permit precise repetition of test conditions. Although it would involve a considerable increase in test equipment complexity and calibration, thought might be given to multiphase solid/liquid experiments so that the applications of the cantilever Coriolis flowmeter concept to slurries could be studied.

Mathematical Model Enhancement

The ultimate performance of the cantilever Coriolis flowmeter may be improved by increasing the detail of the mathematical model used in deriving the flow measurement algorithms. The viscous external damping

model used in this work was the simplest permissible form. External damping effects (as considered here) are most likely a nonlinear combination of material damping, pressure drag, $f(y_t^2)$, and viscous damping, $f(y_t)$. Incorporation of a combined damping model while preserving the practical possibility for a useable analytical result should be considered in more depth.

The fluid-flow model might be extended to the unsteady case in which the total fluid acceleration includes du/dt along the tube centerline. If workable, this would lead to a greater measurement bandwidth that could be of value in specific applications.

The unimodal solution technique can theoretically be expanded to include the simultaneous presence of the first two, or three, vibration modes. One would expect this extension to lead to sets of coupled equations corresponding to Eqs. (114) and (115). A multimodal approach should provide greater generality (hence fewer restrictions on input/output response characteristics), but it would require more sophisticated waveform analysis techniques.

Another area in which the mathematical model could be made more elaborate to possibly simplify the flowmeter design is the motion excitation model. Pulse excitation using the proper pulse shape and width should be a means to achieve an optimized and selective response spectrum.

Finally, the system governing equation(s) could be derived to include the much more complex case of three-dimensional motion rather than the planar vibrations case studied here. This extension is

necessary to admit more general flow-sensor geometries. Intuitively, one would not expect to readily obtain useable analytical results because of the inherent complexity of the associated boundary-value problem, which in this case would be represented by coupled torsional and lateral equations.

In considering mathematical model enhancements, the focus should be on the final approximate analytical solutions for the governing equation(s). Analytical expressions, of some form, are essential for the development of measurement algorithms. While the model enhancements suggested are certain to improve the validity of the governing equations, caution should be maintained since they will not necessarily lead to more accurate approximate solutions and measurement algorithms.

Advanced Algorithms

It is regrettable that more time was not available to study different approaches to the measurement algorithms used to transform information from the (y,x,t) -space to the desired $(\rho,U,\dot{m})$ -space. There are several additional areas that should be pursued.

Adaptive waveform analysis is a possible extension to the basic technique used in this work. In this approach, the sampling frequency and sampling interval would be "adapted" automatically to match the response such that the variances in the calculated period and decay constant would be reduced. The need for this capability was apparent in the tests of the $1/4 \times 0.010$ beam configuration as shown in Fig. 43 (page 147). The sampling interval was satisfactory for low flow rates

but was much too long for high flow rates. Better results (particularly in density) could be achieved by increasing the sampling frequency and decreasing the sampling interval as flow increases.

Finally, a number of techniques from general systems theory in the area of parameter identification are applicable to the cantilever Coriolis flowmeter concept [51-55]. The extended-Kalman filter can be applied to the distributed system case as shown by Bhagavan and Nardizzi [56]. The least-squares filter [57, 58] appears to be the most attractive. A recursive least-squares filter following Graupe et al. [58] was actually implemented, but was never satisfactorily operated. The algorithm would not converge properly and required much greater computational times (on the order of 30 s). The cause of the poor performance was not determined, but was most likely the result of software problems. These and other parameter identification algorithms should be evaluated.

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APPENDIXES

APPENDIX A

NUMERICAL PRODUCT SOLUTION PROGRAM

```

C*****
C*
C*      This program calculates the time and spatial      *
C*      eigenvalues of a cantilever Coriolis flowmeter.   *
C*
C*      The boundary condition determinant is satisfied    *
C*      using an analytic function zero solver(IMSL       *
C*      Routine ZANLYT).                                   *
C*
C*      The characteristic polynomial is solved for        *
C*      each iteration of ZANLYT with a complex           *
C*      polynomial solver(NAG Routine C02ADF).             *
C*
C*      The solution can be iterated for various          *
C*      nondimensional parameter ranges:                  *
C*          U      = fluid velocity                        *
C*          BETA   = mass ratio                            *
C*          C      = external damping coefficient          *
C*
C*          William R. Hamel   December,1980              *
C*
C*      Comments within the source explain                 *
C*      the use of the program.                            *
C*
C*****
C
C      EXTERNAL F
C      COMPLEX DUM(2),ALPHA(4),F,OMEGAI
C      COMMON/VAR/C,U,BETA,ALPHA
C      DIMENSION INFER(5)
C
C          Set-up starting values and
C          parameter ranges with the
C          following statements
C
C      DATA EPS,NSIG,KN,NGUESS,N,ITMAX/1.0E-06,8,0,1,1,200/
C      CALL INPUT(OMEGAI,UI,BETAI,CI,DU,DBETA,DC,NU,NBETA,
C      1NC)
C      DUM(1) = OMEGAI
C      U      = UI
C      BETA   = BETAI
C      C      = CI
C      TYPE 10,OMEGAI,UI,BETAI,CI,DU,DBETA,DC,NU,NBETA,NC
10  FORMAT(9X,' INITIAL VALUES: '//,
1     9X,' OMEGA = ',2E15.7/,
2     9X,' U = ',F4.2/,

```

```

3      9X,'      BETA = ',F4.2/,
4      9X,'      C = ',F4.2//,
5      9X,' INCREMENT VALUES: '//,
6      9X,'      DU = ',F4.2/,
7      9X,'      DBETA = ',F4.2/,
8      9X,'      DC = ',F4.2//,
9      9X,' # OF INCREMENTS: '//,
1     9X,'      NU = ',I5/,
2     9X,'      NBETA = ',I5/,
3     9X,'      NC = ',I5//)

```

```

C
C      Iterate on C
C

```

```

C      DO 3 I=1,NC
C

```

```

C      Open disk file for data
C

```

```

C      OPEN(UNIT=20,DEVICE='DSK',ACCESS='SEQOUT',MODE='ASCII',
C      1FILE='EIGEN.DAT',DIALOG)
C

```

```

C      Iterate on BETA
C

```

```

C      DO 1 J=1,NBETA
C

```

```

C      Iterate on U
C

```

```

C      DO 2 K=1,NU
C

```

```

C      Solve for frequency eigenvalue by
C      treating the boundary condition
C      determinant as an analytic function
C      of OMEGA through the characteristic
C      polynomial. Zero found using IMSL
C      Routine ZANLYT.
C

```

```

C      CALL ZANLYT(F,EPS,NSIG,KN,NGUESS,N,DUM,ITMAX,INFER,IER)
C

```

```

C      Print this result on terminal
C

```

```

C      CALL OUTPUT(DUM)
C      U = U+DU
C      CONTINUE
C

```

```

C      Reset OMEGA and U
C      for next BETA value
C

```

```

DUM(1) = OMEGAI
U = UI
BETA = BETA+DBETA
1 CONTINUE
C
C
C           Reset OMEGA,U, and BETA
C           for next C value
C
DUM(1) = OMEGAI
U      = UI
BETA   = BETAI
C = C+DC
3 CONTINUE
CLOSE(UNIT=20,DEVICE='DSK',ACCESS='SEQOUT',MODE='ASCII')
STOP
END

C
C
C           Data(from terminal) Input
C           Routine
C
SUBROUTINE INPUT(OMEGAI,UI,BETAI,CI,DU,DBETA,DC,
1INU,NBETA,NC)
COMPLEX OMEGAI
TYPE 10
10 FORMAT(' TYPE IN THE INITIAL VALUES OF OMEGA,U,BETA, AND C')
ACCEPT *,OMEGAI,UI,BETAI,CI
TYPE 20
20 FORMAT(' TYPE IN INCREMENT VALUES FOR U,BETA, AND C')
ACCEPT *,DU,DBETA,DC
TYPE 30
30 FORMAT(' TYPE IN THE NUMBER OF INCREMENTS FOR U,BETA,AND C')
ACCEPT *,NU,NBETA,NC
RETURN
END

C
C
C           Data Output Routine
C
SUBROUTINE OUTPUT(DUM)
COMMON/VAR/C,U,BETA,ALPHA
COMPLEX DUM(2),ALPHA(4)
C TYPE 100,U,ALPHA(1)
100 FORMAT(9X,' U      = ',F5.2,3X,'ALPHA(1) = ',2E15.7)
C TYPE 200,BETA,ALPHA(2)
200 FORMAT(9X,' BETA = ',F5.2,3X,'ALPHA(2) = ',2E15.7)
C TYPE 300,C,ALPHA(3),ALPHA(4),DUM(1)
300 FORMAT(9X,' C      = ',F5.2,3X,'ALPHA(3) = ',2E15.7/,
124X,' ALPHA(4) = ',2E15.7/,24X,' OMEGA      = ',2E15.7////)

```



```

400 WRITE(20,400)U,BETA,C,ALPHA,DUM(1)
    FORMAT(13F9.4)
    RETURN
    END

C
C           Subprogram for ZANLYT
C

COMPLEX FUNCTION F(DUM)
COMPLEX DUM(2),ALPHA(4),CALPHA
COMMON/VAR/C,U,BETA,ALPHA

C
C           Update polynomial coefficients for
C           current value of OMEGA
C

DIMENSION CR(5),CC(5),ALPHAR(4),ALPHAI(4)
CALL COEFF(DUM,CR,CC)
N=5
IFAIL=0
TOL=0.15E-38

C
C           Solve characteristic polynomial
C           using NAG Routine C02ADF
C

CALL C02ADF(CR,CC,N,ALPHAR,ALPHAI,TOL,IFAIL)
DO 10 K=1,4
10 ALPHA(K)=CMPLX(ALPHAR(K),ALPHAI(K))

C
C           Calculate boundary conditions
C           determinant
C

CALL DET(ALPHA,CALPHA)
F=CALPHA
RETURN
END

C
C           Update Polynomial Coefficients
C           Routine
C

SUBROUTINE COEFF(DUM,CR,CC)
COMMON/VAR/C,U,BETA,ALPHA
COMPLEX DUM(2),C4,C5
DIMENSION CR(5),CC(5)
DO 1 J=1,5
1  CR(J)=0.0
  CC(J)=0.0
  CR(1)=1.0
  CR(3)=U**2

```

```

C4=2,*SQRT(BETA)*U*DUM(1)+SQRT(1,0-BETA)*C
C5=DUM(1)**2
CR(4)=REAL(C4)
CC(4)=AIMAG(C4)
CR(5)=REAL(C5)
CC(5)=AIMAG(C5)
RETURN
END

```

Boundary Conditions Determinant Routine

```
SUBROUTINE DET(ALPHA,CALPHA)
  COMPLEX C1,ALPHA(4),A(4,4),CALPHA
  DIMENSION WKSPCE(4)
```

Form the matrix

```
C1 = (0.0,1.0)
DO 1 J=1,4
A(1,J) = (1.0,0.0)
A(2,J) = ALPHA(J)
A(3,J) = CEXP(A(2,J))*(A(2,J)**2)
A(4,J) = A(2,J)*A(3,J)
```

Now find the determinant

```
IA = 4
N = 4
IFAIL = 0
```

Find complex determinant
using NAG Routine F03ADF

```
CALL F03ADF(A,IA,N,CALPHR,CALPHI,WKSPCE,IFAIL)
CALPHA = CMPLX(CALPHR,CALPHI)
CONTINUE
RETURN
END
```

```

*****
C*
C*      GEBIC.FOR This is the program which actually reads the
C*      the analog interface and stores the beam deflection and
C*      thermal flowmeter data under realtime clock control.
C*
C*      Data acquisition in realtime is implemented using the
C*      DEC RT-11 operating systems and the Data Translation, Inc.
C*      data acquisition and control software library, DTLIB.
C*
C*      William R. Hamel November, 1980
C*
C*****
C
      SUBROUTINE GETDAT
      COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
      1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
      COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
      COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
      1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM

C
C      Define set-up parameters for A/D interface
C

      ANLSBT=20.*.4633/4096.
      ANLSBV=20./4096.
      ISIZ=NREAD
      NBUF=1
      ISTCHN=0
      NCCH=2
      IGAIN=1
      MODE=2
      ICMF=0
      IBEF=0

C
C      Collect data using DTLIB Subroutine RTS
C

      CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,IGAIN,MODE
      1,ICMF,IBEF)
      IVAR=LWAIT(ICMF,0)

C
C      Convert data to engineering units, remove
C      calibration biases, and store
C

      DO 100 J=1,NREAD/NCCH
      TURB(J)=ANLSBT*FLOAT(IBUF(2*J-1))-TBIAS
100  VOLT(J)=ANLSBV*FLOAT(IBUF(2*J))-VBIAS
      RETURN
      END

```

C
C
C
C

This routine calculates measurement biases
during calibration runs

SUBROUTINE BIAS
COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM

C
C
C
C

Set-up A/D interface and realtime clock for
bias measurements

ANLSBT=20.*0.4633/4096.
ANLSBV=20./4096.
ISIZ=NREAD
NBUF=1
ISTCHN=0
NCCH=2
IGAIN=1
MODE=2
ICMF=0
IBEF=0
CALL SETR(-1,,0,0,0)
CALL SETR(1,,9,PRESET,0)

C
C
C

Measure empty pipe biases

CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,IGAIN,MODE
1,ICMF,IBEF)
IVAR=LWAIT(ICMF,0)
TBIAS = 0.0
VBIASE = 0.0
VBIAS = 0.0
DO 100 J=1,NREAD/NCCH
VBIASE=VBIASE+ANLSBV*FLOAT(IBUF(2*J))
TBIAS=TBIAS+ANLSBT*FLOAT(IBUF(2*J-1))

100

C
C
C

Average values

VBIASE=VBIASE/(NREAD/NCCH)
TBIAS=TBIAS/(NREAD/NCCH)

C
C
C

Read voltage bias with flow on

PAUSE ' SET FLOW ON FOR VOLTAGE BIAS'

```
CALL SETR(-1,,0,0,0)
CALL SETR( 1,,9,PRESET,0)
CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,IGAIN,MODE
1,ICMF,IBEF)
IVAR=LWAIT(ICMF,0)
DO 200 J=1,NREAD/NCCH
200  VBIAS=VBIAS+ANLSBV*FLOAT(IBUF(2*J))
    VBIAS=VBIAS/(NREAD/NCCH)
    TYPE 900,TBIAS,VBIAS,VBIASE
900  FORMAT(//
1'    THERMAL FLOWMETER BIAS = ',E15.7,/,
2'    FLOWING VOLTAGE BIAS = ',E15.7,/,
3'    EMPTY VOLTAGE BIAS = ',E15.7//)
RETURN
END
```

APPENDIX B

NUMERICAL INTEGRATION PROGRAM
FOR ORTHOGONALITY RELATIONSHIPS

```

C*****
C*
C*   This program calculates various spatial eigenfunction
C*   integral relationships for a straight cantilever beam
C*   without fluid flow and on the interval X=[0,1]. These
C*   results are used in the approximate analytical solutions
C*   to the cantilever Coriolis flowmeter problem.
C*
C*   Double precision numerical integrations are performed
C*   using the ORNL DEC PDP-10 resident library program
C*   CORLIB QAGS.
C*
C*           William R. Hamel   November, 1980
C*
C*   Comments within the source explain the design of the
C*   program.
C*
C*****
C
C   EXTERNAL X1,X2,XPX,XPPX,XP,XPP
C   COMMON/BLK/KMAX,SALP,S,C
C   DOUBLE PRECISION ABSERR,RELERR,ALPHA,SALP,ONE,ZERO,
C   1           S,C,KMAX,KX,KXP,KXPP,RXPX,RXPPX,ERR
C   2           EST,F1,F2,F3,F4,XD
C
C   OPEN FILE FOR DATA STORAGE ON DISK
C   OPEN(UNIT=20,DEVICE='DSK',ACCESS='SEQOUT',MODE='ASCII'
C   1,FILE='INTGRL.DAT')
C
C           Set-up various constants, with everything
C           in double precision. Function subprograms
C           for double precision hyperbolics are used.
C
C   ABSERR = 1.0D-02
C   RELERR = 1.0D-02
C   ALPHA  = 3.51602I0
C   SALP   = DSQRT(ALPHA)
C   ZERO   = 0.0D0
C   ONE    = 1.0D0
C   S      = DSINH(SALP)+DSIN(SALP)
C   C      = DCOSH(SALP)+DCOS(SALP)
C
C           Calculate normalization constant, X=1
C
C   KMAX = S*(DCOS(SALP)-DCOSH(SALP))-C*(DSIN(SALP)-DSINH(SALP))
C   TYPE 100,KMAX
C   WRITE(20,450)KMAX
C

```

```

C           All integrations are performed using the
C           ORNL FDP-10 CORLIB Routine: QAGS
C
C           Calculate integral [0,1] of  $X^2$ 
CALL QAGS(X2,ZERO,ONE,ABSERR,RELERR,KX,ERREST,IER)
TYPE 200,KX,ERREST,IER
WRITE(20,550)KX,ERREST,IER
C
C           Calculate integral [0,1] OF  $X^3$ 
C
CALL QAGS(XPX,ZERO,ONE,ABSERR,RELERR,KXP,ERREST,IER)
TYPE 300,KXP,ERREST,IER
WRITE(20,550)KXP,ERREST,IER
C
C           Calculate integral [0,1] OF  $X^4$ 
C
CALL QAGS(XPPX,ZERO,ONE,ABSERR,RELERR,KXPP,ERREST,IER)
TYPE 400,KXPP,ERREST,IER
WRITE(20,550)KXPP,ERREST,IER
C
C           Calculate various ratios
C
RXPX = KXP/KX
RXPPX = KXPP/KX
TYPE 500,RXPX,RXPPX
WRITE(20,650)RXPX,RXPPX
C
C           Calculate function values for  $X(X)$ ,
C            $X'(X)$ , and  $X''(X)$ 
C
TYPE 700
XD = 0.0D0
DO 10 J=1,21
F1 = X1(XD)
F2 = XP(XD)
F3 = XFP(XD)
TYPE 600,XD,F1,F2,F3
WRITE(20,750)XD,F1,F2,F3
XD = XD+(0.05)
10  CONTINUE
100  FORMAT(/,5X,' NORMALIZATION CONSTANT = ',D14.7//)
200  FORMAT(5X,' INTEGRAL [0,1] OF  $X^2$  = ',D14.7/,
1      5X,' ESTIMATED ERROR = ',D14.7/,
2      5X,' IER = ',I4//)
300  FORMAT(5X,' INTEGRAL [0,1] OF  $X^3$  = ',D14.7/,
1      5X,' ESTIMATED ERROR = ',D14.7/,
2      5X,' IER = ',I4//)

```



```

400  FORMAT(4X,' INTEGRAL [0,1] OF X*XFP = ',D14.7/,
      1      5X,' ESTIMATED ERROR = ',D14.7/,
      2      5X,' IER = ',I4//)
500  FORMAT(5X,' KXP/KX = ',D14.7/,
      1      5X,' KXPP/KX = ',D14.7///)
600  FORMAT(4D20.7)
700  FORMAT(//,14X,' X',17X,'X[X]',16X,'XP[X]',14X,'XPP[X]',/)
450  FORMAT(D20.7)
550  FORMAT(2D20.7,15)
650  FORMAT(2D20.7)
      CLOSE(UNIT=20,FILE='INTGRL.DAT')
750  FORMAT(4D20.7)
      CONTINUE
      STOP
      END

```

C
C
C

Hyperbolic Cosine

```

DOUBLE PRECISION FUNCTION DCOSH(X)
DOUBLE PRECISION X
DCOSH = (DEXP(X)+DEXP(-X))/2.
RETURN
END

```

C
C
C

Hyperbolic Sine

```

DOUBLE PRECISION FUNCTION DSINH(X)
DOUBLE PRECISION X
DSINH = (DEXP(X)-DEXP(-X))/2.
RETURN
END

```

C
C
C

Function: X**2

```

DOUBLE PRECISION FUNCTION X2(X)
EXTERNAL DSINH,DCOSH
DOUBLE PRECISION X,KMAX,SALP,S,C
COMMON/BLK/KMAX,SALP,S,C
X2 = S*(DCOS(X*SALP)-DCOSH(X*SALP))
1   -C*(DSIN(X*SALP)-DSINH(X*SALP))
X2 = (X2**2)/(KMAX**2)
RETURN
END

```

C
C
C

Function: X*X'

```

DOUBLE PRECISION FUNCTION XPX(X)

```

```

EXTERNAL DSINH,DCOSH
DOUBLE PRECISION X,KMAX,SALP,S,C
COMMON/BLK/KMAX,SALP,S,C
XPX = S*(DCOS(X*SALP)-DCOSH(X*SALP))
1    -C*(DSIN(X*SALP)-DSINH(X*SALP))
XPX = XPX*SALP*(S*(-DSIN(X*SALP)-DSINH(X*SALP))
1    -C*(DCOS(X*SALP)-DCOSH(X*SALP)))
XPX = XPX/(KMAX**2)
RETURN
END

```

C
C
C

Function: X*X''

```

DOUBLE PRECISION FUNCTION XFPX(X)
EXTERNAL DSINH,DCOSH
DOUBLE PRECISION X,KMAX,SALP,S,C
COMMON/BLK/KMAX,SALP,S,C
XFPX = S*(DCOS(X*SALP)-DCOSH(X*SALP))
1    -C*(DSIN(X*SALP)-DSINH(X*SALP))
XFPX = XFPX*(SALP**2)*(S*(-DCOS(X*SALP)
1    -DCOSH(X*SALP))-C*(-DSIN(X*SALP)
2    -DSINH(X*SALP)))
XFPX = XFPX/(KMAX**2)
RETURN
END

```

C
C
C

Function: X[X]

```

DOUBLE PRECISION FUNCTION X1(X)
EXTERNAL DSINH,DCOSH
DOUBLE PRECISION X,KMAX,SALP,S,C
COMMON/BLK/KMAX,SALP,S,C
X1 = S*(DCOS(X*SALP)-DCOSH(X*SALP))
1    -C*(DSIN(X*SALP)-DSINH(X*SALP))
X1 = X1/KMAX
RETURN
END

```

C
C
C

Function: X'[X]

```

DOUBLE PRECISION FUNCTION XP(X)
EXTERNAL DSINH,DCOSH
DOUBLE PRECISION X,KMAX,SALP,S,C
COMMON/BLK/KMAX,SALP,S,C
XP = SALP*(S*(-DSIN(X*SALP)-DSINH(X*SALP))
1    -C*(DCOS(X*SALP)-DCOSH(X*SALP)))
XP = XP/KMAX

```

RETURN
END

C
C
C

Function: $X''[X]$

DOUBLE PRECISION FUNCTION XPF(X)
EXTERNAL DSINH,DCOSH
DOUBLE PRECISION X,KMAX,SALP,S,C
COMMON/BLK/KMAX,SALP,S,C
XPP = (SALP**2)*(-S*(DCOS(X*SALP)+DCOSH(X*SALP))
1 -C*(-DSIN(X*SALP)-DSINH(X*SALP)))
XPP = XPP/KMAX
RETURN
END

APPENDIX C

REALTIME FLOW MEASUREMENT AND CALIBRATION PROGRAMS

```

*****
C*
C*      MOD25T.FOR This is the main fluid flow measurement
C*      program which controls the beam motion excitation, the
C*      beam deflection data acquisition, and calculation of
C*      the fluid flow variables: mass flowrate, density, and
C*      and mean velocity.
C*
C*      MOD25T.FOR is for the 1/4 X 0.035 configuration and is
C*      representative of the programs used for the other configurations.
C*
C*      Data acquisition in realtime is implemented using the
C*      DEC RT-11 operating systems and the Data Translation, Inc.
C*      data acquisition and control software library, DTLIB.
C*
C*      William R. Hamel November, 1980
C*
C*****
C
COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50),
1TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
DIMENSION DIFF(50)
REAL*4 YES,NO,MDOT
INTEGER*2 MASK,IWORD
DATA YES,NO/'YES','NO'/
OPEN(UNIT=8,NAME='25T.CAL',ACCESS='SEQUENTIAL',TYPE='OLD',
1FORM='UNFORMATTED')

C
C      Zero out the digital interface, using the DTLIB
C      routine IDOR
C
C      MASK = '1
C      IWORD = '177777
C      CALL IDOR(0,0,MASK,IWORD)
50 CONTINUE
C
C      Close data file for parameter alterations case
C
C      CLOSE(UNIT=8)
C
C      Turn realtime clock off, using the DTLIB routine
C      SETR
C

```



```

2'          TBIAS = ',E15.7,/,
3'          VBIAS = ',E15.7,/,
4'          VBIASE = ',E15.7,/,
5'          RC1 = ',E15.7,'(',E15.7,')',/,
6'          RC2 = ',E15.7,'(',E15.7,')',/,
7'          A1 = ',E15.7,'(',E15.7,')',/,
8'          A2 = ',E15.7,'(',E15.7,')',/)

C
C          Open file for data storage
C
OPEN(UNIT=8,NAME='M25TXX.DAT',ACCESS='SEQUENTIAL',TYPE='NEW',
1FORM='UNFORMATTED')
660 CONTINUE
C
C          Initialize all data storage arrays
C
DO 8 J=1,2000
8   IBUF(J)=0
DO 88 J=1,1000
   TURB(J)=0.0
88  VOLT(J)=0.0
DO 888 K=1,50
   TIME(K) =0.0
   TZERO(K) =0.0
   JTIME(K) =0
   PEAK(K) =0.0
   TP(K) =0.0
DO 888 J=1,2
888 ZERO(K,J)=0.0
   TA = 0.0
   RADIAN = 0.0
   KPEAK = 0
   KCROSS = 0
   RHO = 0.0
   VEL = 0.0
   MDOT = 0.0
   THERM = 0.0

C
C          Begin measurement sequence by 'FIRING'
C          the motion excitation system
C
CALL FIRE

C
C          Execute the ALGORithms for calculating the
C          period and peaks of the vibratory waveform
C
CALL ALGOR

```

```

C
C           Calculate the statistics of the
C           vibratory waveform analysis
C
DO 19 J=1,KCROSS-2
19  DIFF(J) = TZERO(J+1)-TZERO(J)
    SIGTA = 0.0
    DO 199 J=1,KCROSS-2
      TYPE 197,J,DIFF(J)
197  FORMAT(I3,E15.7)
199  SIGTA = SIGTA+(DIFF(J)-TA)**2
      SIGTA = SQRT(SIGTA/(KCROSS-3))
      TYPE 1999,TA,SIGTA
1999 FORMAT(//
1'  WAVEFORM STATISTICS: TA = ',E15.7,/
2'                      SIGTA = ',E15.7//)

C
C           Do calculations necessary to throw away
C           any bad period data
C
DO 297 J=1,KCROSS-2
  PERCEN = ABS((TA-DIFF(J))/DIFF(J))
  IF(PERCEN.GE.0.03)DIFF(J)=0.0
297  CONTINUE

C
C           Count the number of good periods
C           and the better average
C
SUM = 0.0
K = 1
DO 299 J=1,KCROSS-2
  SUM = SUM + DIFF(J)
  IF(DIFF(J).EQ.0.0)K=K-1
299  K = K+1
    TA = SUM/(K-1)

C
C           Calculate the statistics of the
C           reduced waveform analysis
C
SIGTA = 0.0
DO 399 J=1,KCROSS-2
  TYPE 197,J,DIFF(J)
  IF(DIFF(J).EQ.0.0)GO TO 399
  SIGTA = SIGTA+(DIFF(J)-TA)**2
399  CONTINUE
    SIGTA = SQRT(SIGTA/(K-2))
    TYPE 1999,TA,SIGTA

```



```

C
C          Calculate density and FLOWrate
C
C      CALL FLOW
C
C          Store results on disk file DY1:M25TXX.DAT
C
C      WRITE(8) RHO,VEL,MDOT,THERM
C
C          Check to see if raw data is to be STORed
C          for this measurement
C
700      CONTINUE
        IF(STORE.EQ.YES)GO TO 900
        IF(STORE.EQ.NO) GO TO 1000
        TYPE 800
800      FORMAT(' DO YOU WISH TO STORE DATA? [YES OR NO]')
        ACCEPT 400,STORE
        GO TO 700
900      CALL STOR
1000     CONTINUE
C
C          Check to see if raw data is to printed
C          for this measurement
C
        IF(RITE.EQ.YES)GO TO 1200
        IF(RITE.EQ.NO) GO TO 1300
        TYPE 1100
1100     FORMAT(' DO YOU WISH TO PRINT DATA? [YES OR NO]')
        ACCEPT 600,RITE
        GO TO 1000
1200     CALL RITER
1300     CONTINUE
C
C          Check to see if it is desired to change any
C          parameters for the next measurement
C
        TYPE 1400
1400     FORMAT(' DO YOU WISH TO ALTER THE SET-UP PARAMETERS?[YES
        1 OR NO]')
        ACCEPT 400, ALTER
        IF(ALTER.EQ.YES)GO TO 50
        IF(ALTER.EQ.NO) GO TO 1500
        GO TO 1300
1500     CONTINUE
C
C          Check to see if operator is ready for the

```

```
C          next measurement
C
      TYPE 1600
1600  FORMAT(' ANOTHER MEASUREMENT?[YES OR NO]')
      ACCEPT 400, ANOTHR
      IF(ANOTHR.EQ.YES)GO TO 660
      IF(ANOTHR.EQ. NO)GO TO 1700
      GO TO 1500
1700  CLOSE(UNIT=8)
C
C          Remind operator to rename the data file
C          uniquely
C
      TYPE 1800
1800  FORMAT(' BE SURE TO RENAME M25TXX.DAT...')
      CONTINUE
      END
```

```

C*****
C*
C*      CAL25T.FOR This is the main flow measurement calibration
C*      program which controls the beam motion excitation, the
C*      beam deflection data acquisition, and calculation of
C*      empty pipe and flowing pipe calibration constants. This pro-
C*      gram also calculates, stores and reports the mean and
C*      standard deviations of the various calibrations.
C*
C*      CAL25T.FOR is for the 1/4 X 0.035 configuration and is
C*      representative of the programs used for the other config-
C*      urations.
C*
C*      Data acquisition in realtime is implemented using the
C*      DEC RT-11 operating systems and the Data Translation, Inc.
C*      data acquisition and control software library, DTLIB.
C*
C*      William R. Hamel November, 1980
C*
C*****
C
      COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
      1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
      COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
      COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50),
      1TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
      REAL*4 YES,NO,DIFF(50)
      INTEGER*2 MASK,IWORD
      DATA YES,NO/'YES','NO'/

C
C      Zero out the digital interface
C
      MASK = '1
      IWORD = '177777
      CALL IDOR(0,0,MASK,IWORD)
50  CONTINUE
C
C      Turn realtime clock off
C
      CALL SETR(-1,,0,0,0)
      TYPE 100
100  FORMAT(' TYPE IN THE SAMPLING PERIOD IN MILLISECONDS. ')
      ACCEPT *,PRESET
      PRESET = PRESET*1000.
      TYPE 200
200  FORMAT(' TYPE IN THE NUMBER OF SAMPLES TO BE TAKEN. ')
      ACCEPT *,NREAD

```

```

NCCH = 2
NREAD = NREAD*NCCH

C
C      Set-up realtime clock
C

IRATE = 1
MODE = 9
ICMF1 = 0
CALL SETR(IRATE,,MODE,PRESET,ICMF1)

C
C      Open calibration data file: DY1:CAL25T.DAT
C

OPEN(UNIT=8,NAME='CAL25T.DAT',ACCESS='SEQUENTIAL',TYPE='NEW',
1FORM='UNFORMATTED')

C
C      Measure channel electronic biases
C

CALL BIAS

C
C      Store biases on DY1:CAL25T.DAT
C

WRITE(8)TBIAS,VBIAS,VBIASE

C
C      Calibrate empty pipe parameters
C      take five readings for an average
C      put VBIASE in VBIAS temporarily
C      so GETDAT will be ok
C

TEMP = VBIAS
VBIAS = VBIASE
DO 300 JJ=1,5
CALL FIRE
CALL ALGOR

C
C      Calculate the statistics of the
C      waveform analysis
C

DO 19 J=1,KCROSS-2
19  DIFF(J) = TZERO(J+1)-TZERO(J)
SIGTA = 0.0
DO 199 J=1,KCROSS-2
TYPE 197,J,DIFF(J)
197  FORMAT(I3,E15.7)
199  SIGTA = SIGTA+(DIFF(J)-TA)**2
SIGTA = SQRT(SIGTA/(KCROSS-3))
TYPE 1999,TA,SIGTA
1999 FORMAT(//

```

```

1' WAVEFORM STATISTICS: TA = ',E15.7,/'
2'                               SIGTA = ',E15.7//)

C
C           Do calculations necessary to throw away
C           any bad period data
C

DO 297 J=1,KCROSS-2
PERCEN = ABS((TA-DIFF(J))/DIFF(J))
IF(PERCEN,GE,0.03)DIFF(J)=0.0
297 CONTINUE

C
C           Count the number of good periods
C           and the better average
C

SUM = 0.0
K = 1
DO 299 J=1,KCROSS-2
SUM = SUM + DIFF(J)
IF(DIFF(J),EQ,0.0)K=K-1
299 K = K+1
TA = SUM/(K-1)

C
C           Calculate the statistics of the
C           waveform analysis
C

SIGTA = 0.0
DO 399 J=1,KCROSS-2
TYPE 197,J,DIFF(J)
IF(DIFF(J),EQ,0.0)GO TO 399
SIGTA = SIGTA+(DIFF(J)-TA)**2
399 CONTINUE
SIGTA = SQRT(SIGTA/(K-2))
TYPE 1999,TA,SIGTA
300 CALL ECAL

C
C           Restore vbias
C

VBIAS = TEMP

C
C           Calibrate flowing pipe parameters
C           take ten readings for an average
C           over the entire flow range
C

PAUSE ' SET FLOWRATE FOR THE FIRST CALIBRATION POINT'
DO 700 JJ=1,10
CALL FIRE
CALL ALGOR

```

```

C
C           Calculate the statistics of the
C           waveform analysis
C
DO 119 J=1,KCROSS-2
119  DIFF(J) = TZERO(J+1)-TZERO(J)
    SIGTA = 0.0
    DO 1199 J=1,KCROSS-2
    TYPE 197,J,DIFF(J)
1199  SIGTA = SIGTA+(DIFF(J)-TA)**2
    SIGTA = SQRT(SIGTA/(KCROSS-3))
    TYPE 1999,TA,SIGTA

C
C           Do calculations necessary to throw away
C           any bad period data
C
DO 1297 J=1,KCROSS-2
    PERCEN = ABS((TA-DIFF(J))/DIFF(J))
    IF(PERCEN.GE.0.03)DIFF(J)=0.0
1297  CONTINUE

C
C           Count the number of good periods
C           and the better average
C
    SUM = 0.0
    K = 1
    DO 1299 J=1,KCROSS-2
    SUM = SUM + DIFF(J)
    IF(DIFF(J).EQ.0.0)K=K-1
1299  K = K+1
    TA = SUM/(K-1)

C
C           Calculate the statistics of the
C           waveform analysis
C
    SIGTA = 0.0
    DO 1399 J=1,KCROSS-2
    TYPE 197,J,DIFF(J)
    IF(DIFF(J).EQ.0.0)GO TO 1399
    SIGTA = SIGTA+(DIFF(J)-TA)**2
1399  CONTINUE
    SIGTA = SQRT(SIGTA/(K-2))
    TYPE 1999,TA,SIGTA
    CALL FCAL
    PAUSE ' SET ANOTHER FLOWRATE FOR NEXT CALIBRATION POINT '
700  CONTINUE
C

```

```

C           Close raw calibration data file
C
C       CLOSE(UNIT=8)
C
C           Go calculate the calibration statistics
C
CALL CALAVE
CONTINUE
STOP
END
SUBROUTINE CALAVE
COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50),
1TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
DIMENSION RC1A(5),RC2A(5),A1A(10),A2A(10)
C
C           Put raw calibration data in memory; CAL25T.DAT is
C           still open at this point
C
OPEN(UNIT=8,NAME='CAL25T.DAT',ACCESS='SEQUENTIAL',TYPE='OLD',
1FORM='UNFORMATTED')
READ(8)TBIAS,VBIAS,VBIASE
DO 9 J=1,5
9   READ(8)RC1A(J),RC2A(J)
DO 99 J=1,10
99  READ(8)A1A(J),A2A(J)
CLOSE(UNIT=8)
C
C           Calculate the mean values
C
RC1 = 0.0
RC2 = 0.0
A1 = 0.0
A2 = 0.0
DO 10 J=1,5
10   RC1 = RC1+RC1A(J)
      RC2 = RC2+RC2A(J)
RC1 = RC1/5.
RC2 = RC2/5.
DO 20 K=1,10
20   A1 = A1+A1A(K)
      A2 = A2+A2A(K)
A1 = A1/10.
A2 = A2/10.
C

```

```

C          Now, calculate the variances; note that
C          initially SIGRCI is SIGRCI**2
C
SIGRC1 = 0.0
SIGRC2 = 0.0
SIGA1  = 0.0
SIGA2  = 0.0
DO 30 J=1,5
          SIGRC1 = SIGRC1+(RC1A(J)-RC1)**2
30      SIGRC2 = SIGRC2+(RC2A(J)-RC2)**2
SIGRC1 = SQRT(SIGRC1/4.)
SIGRC2 = SQRT(SIGRC2/4.)
DO 40 K=1,10
          SIGA1 = SIGA1+(A1A(K)-A1)**2
40      SIGA2 = SIGA2+(A2A(K)-A2)**2
SIGA1 = SQRT(SIGA1/9.)
SIGA2 = SQRT(SIGA2/9.)
C
C          Store statistical results on DY1:25T.CAL
C          open new file for calibration statistics
C
OPEN(UNIT=8,NAME='25T.CAL',ACCESS='SEQUENTIAL',TYPE='NEW',
1FORM='UNFORMATTED')
WRITE(8)TBIAS,VBIAS,VBIASE,RC1,RC2,A1,A2,SIGRC1,SIGRC2,SIGA1,SIGA2
CLOSE(UNIT=8)
C
C          Type out results on the system terminal
C
TYPE 50,TBIAS,VBIAS,VBIASE,RC1,SIGRC1,RC2,SIGRC2,A1,SIGA1,A2,SIGA2
50  FORMAT(//
1'  CALIBRATION:                                MEAN(STANDARD DEVIATION)',/
2'          TBIAS = ',E15.7,/
3'          VBIAS = ',E15.7,/
4'          VBIASE = ',E15.7,/
5'          RC1 = ',E15.7,'('',E15.7,'')',/
6'          RC2 = ',E15.7,'('',E15.7,'')',/
7'          A1 = ',E15.7,'('',E15.7,'')',/
8'          A2 = ',E15.7,'('',E15.7,'')',/
RETURN
END

```



```

C*****
C*
C*      GEBIC.FOR This is the program which actually reads the
C*      the analog interface and stores the beam deflection and
C*      thermal flowmeter data under realtime clock control.
C*
C*      Data acquisition in realtime is implemented using the
C*      DEC RT-11 operating systems and the Data Translation, Inc.
C*      data acquisition and control software library, DTLIB.
C*
C*      William R. Hamel November, 1980
C*
C*****
C
      SUBROUTINE GETDAT
      COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
      1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
      COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
      COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
      1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM

C
C      Define set-up parameters for A/D interface
C
      ANLSBT=20.*.4633/4096.
      ANLSBV=20./4096.
      ISIZ=NREAD
      NBUF=1
      ISTCHN=0
      NCCH=2
      IGAIN=1
      MODE=2
      ICMF=0
      IREF=0

C
C      Collect data using DTLIB Subroutine RTS
C
      CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,IGAIN,MODE
      1,ICMF,IREF)
      IVAR=LWAIT(ICMF,0)

C
C      Convert data to engineering units, remove
C      calibration biases, and store
C
      DO 100 J=1,NREAD/NCCH
      TURB(J)=ANLSBT*FLOAT(IBUF(2*J-1))-TBIAS
      100 VOLT(J)=ANLSBV*FLOAT(IBUF(2*J))-VBIAS
      RETURN
      END

```

C
C
C
C

This routine calculates measurement biases
during calibration runs

SUBROUTINE BIAS

COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO

1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2

COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)

COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)

1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM

C
C
C
C

Set-up A/D interface and realtime clock for
bias measurements

ANLSBT=20.*0.4633/4096.

ANLSBV=20./4096.

ISIZ=NREAD

NBUF=1

ISTCHN=0

NCCH=2

IGAIN=1

MODE=2

ICMF=0

IBEF=0

CALL SETR(-1,,0,0,0)

CALL SETR(1,,9,PRESET,0)

C
C
C

Measure empty pipe biases

CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,IGAIN,MODE

1,ICMF,IBEF)

IVAR=LWAIT(ICMF,0)

TBIAS = 0.0

VBIASE = 0.0

VBIAS = 0.0

DO 100 J=1,NREAD/NCCH

VBIASE=VBIASE+ANLSBV*FLOAT(IBUF(2*J))

TBIAS=TBIAS+ANLSBT*FLOAT(IBUF(2*J-1))

100
C
C
C

Average values

VBIASE=VBIASE/(NREAD/NCCH)

TBIAS=TBIAS/(NREAD/NCCH)

C
C
C

Read voltage bias with flow on

PAUSE ' SET FLOW ON FOR VOLTAGE BIAS'

```
CALL SETR(-1,,0,0,0)
CALL SETR( 1,,9,PRESET,0)
CALL RTS(IBUF,ISIZ,NBUF,NREAD,ISTCHN,NCCH,,IGAIN,MODE
1,ICMF,IBEF)
IVAR=LWAIT(ICMF,0)
DO 200 J=1,NREAD/NCCH
200  VBIAS=VBIAS+ANLSBV*FLOAT(IBUF(2*J))
    VBIAS=VBIAS/(NREAD/NCCH)
    TYPE 900,TBIAS,VBIAS,VBIASE
900  FORMAT(//
1'    THERMAL FLOWMETER BIAS = ',E15.7,/,
2'    FLOWING VOLTAGE BIAS = ',E15.7,/,
3'    EMPTY VOLTAGE BIAS = ',E15.7//)
RETURN
END
```

```

C*****
C*
C*      ALGORC.FOR This is the main program which performs
C*      the beam deflection response waveform analysis. ALGORC
C*      calls PERIOD and PEAKER. PERIOD calculates the waveform
C*      frequency by calculating the the period from the zero-
C*      crossings. PEAKER finds the deflection value at each
C*      peak.
C*
C*      Data acquisition in realtime is implemented using the
C*      DEC RT-11 operating systems and the Data Translation, Inc.
C*      data acquisition and control software library, DTLIB.
C*
C*      William R. Hamel November, 1980
C*
C*****
SUBROUTINE ALGOR
COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
REAL*4 YES,NO
DATA YES,NO/'YES','NO'/
K = 1
J = 0

C
C      Check to see if this is the first data point
C
IF(VOLT(1).LT.0.)GO TO 200
100 CONTINUE
C
C      Look for a + to - transition
C
CALL PTOM(J)
C
C      Check to see if this is the end of this data set
C
IF(J.GE.NREAD/NCCH)GO TO 300
C
C      Store values on both sides of this transition
C
CALL TRAP(J,K)
C
C      Look for a - to + transition
C
200 CALL MTOP(J)

```

```

      IF(J.GE.NREAD/NCCH)GO TO 300
C
C      Store values on both sides of this transition
C
      CALL TRAP(J,K)
      GO TO 100
300    K = K-1
C
C      KCROSS = # of zero-crossings minus one
C
      KCROSS = K
C
C      Calculate the waveform frequency
C
      CALL PERIOD
C
C      Find the waveform peaks
C
      CALL PEAKER
C
C      Type out the results
C
      TYPE 400, TA,RADIAN,KPEAK
400    FORMAT(//
      1' WAVEFORM ANALYSIS: PERIOD = ',E15.7,'SEC',/,
      2' FREQUENCY = ',E15.7,'RAD/SEC',/,
      3' NO. OF PEAKS = ',I5,/,
      4' PEAK VALUES:',/)
      TYPE 500,(PEAK(J),J=1,KPEAK)
500    FORMAT(
      1' ',E15.7/)
      CONTINUE
      RETURN
      END
C
C      This routine searches the waveform for a
C      + to - transition
C
      SUBROUTINE PTOM(J)
      COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
      1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
      COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
      COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
      1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
100    J = J+1
      IF(J.GE.NREAD/NCCH)GO TO 200
C

```

C Look for a + to - transition
C

SWITCH = SIGN(VOLT(J),VOLT(J+1))
IF(SWITCH.LT.0.0)GO TO 200
GO TO 100
200 CONTINUE
RETURN
END

C This routine searches the waveform for a
C - to + transition
C

SUBROUTINE MTOP(J)
COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MIDOT,THERM
100 J = J+1
IF(J.GE.NREAD/NCCH)GO TO 200

C Look for a - to + transition
C

SWITCH = SIGN(VOLT(J),VOLT(J+1))
IF(SWITCH.GT.0.0)GO TO 200
GO TO 100
200 CONTINUE
RETURN
END

C This routine stores the deflection data
C on either side of the zero-crossing
C

SUBROUTINE TRAP(J,K)
COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MIDOT,THERM

C Store values on either side of the transition
C

ZERO(K,1) = VOLT(J)
ZERO(K,2) = VOLT(J+1)
TIME(K) = J*PRESET*1.0E-6
JTIME(K) = J
IF(K.LE.1)GO TO 100

```

IF(NREAD/NCCH-J,LE,5)GO TO 200

C
C           This is at least the second zero-crossing,
C           so take "giant" step to the next.
C

T = TIME(K)-TIME(K-1)
J = J+IFIX(T/(PRESET*1.0E-6))-5
100 CONTINUE

C
C           K = # of zero-crossings
C

K = K+1
200 CONTINUE
RETURN
END

C
C           This routine calculates the waveform frequency
C           by averaging the time intervals between zero-
C           crossings
C

SUBROUTINE PERIOD
COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
REAL*4 OMEGA
SUM = 0.0
DO 100 J=1,KCROSS
100 TZERO(J)=TIME(J)+(ABS(ZERO(J,1))*PRESET*1.0E-6)
1/(ABS(ZERO(J,2))+ABS(ZERO(J,1)))
DO 300 J=1,KCROSS-2
300 SUM = SUM+TZERO(J+1)-TZERO(J)
TA = SUM/(KCROSS-2)
OMEGA = 0.5/TA
RADIAN = 6.28318*OMEGA
RETURN
END

C
C           This routine finds and stores the peak values
C           for each half cycle
C

SUBROUTINE PEAKER
COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)

```

```

1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
DIMENSION AMPL(1000)
DO 10 I=1,NREAD/NCCH
10  AMPL(I) = ABS(VOLT(I))
    JPERD = IFIX(TA/(PRESET*1.0E-6))
C
C          # of peaks = 1/2 # of zero-crossings
C          truncation to the nearest number of
C          of peaks if KCROSS is odd.
C
    KPEAK = KCROSS/2
    DO 30 L=1,KPEAK
C
C          Jump data by 2*TA increments
C
    JL = JTIME(2*L-1)+3*JPERD/8
    JR = JTIME(2*L-1)+5*JPERD/8
    PEAK(L) = 0.
    DO 20 I=JL,JR
    20  TP(L) = I*PRESET*1.0E-6
    30  PEAK(L) = AMAX1(PEAK(L),AMPL(I))
    CONTINUE
    RETURN
    END

```



```

C*****
C*
C*      FLO25T.FOR This program is called by MOD25T.FOR and
C*      calculates mass flowrate, density, and velocity from
C*      the measured frequency and decay constant.
C*
C*      FLO25T.FOR is for the 1/4 X 0.035 configuration and is
C*      representative of the programs used for the other configurations.
C*
C*      Data acquisition in realtime is implemented using the
C*      DEC RT-11 operating systems and the Data Translation, Inc.
C*      data acquisition and control software library, DTLIB.
C*
C*      William R. Hamel November, 1980
C*
C*****
C
      SUBROUTINE FLOW
      COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
      1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
      COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
      COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
      1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
      REAL*4 MDOT
      AREA = 5.073577E-4
      R1 = (1./(2.*TA*KPEAK))*ALOG(PEAK(1)/PEAK(KPEAK))
      R2 = RADIAN**2

C
C      BETAM1 = 1.0-BETA
C
      BETAM1 = R2/RC2
      BETA = 1.-BETAM1
      MDOT= A2*(R1-BETAM1*RC1)/BETAM1
      RHO = A1*(BETA/BETAM1)
      VEL = MDOT/(RHO*AREA)

C
C      Calculate average thermal flowmeter output
C      for the last 100 readings
C
      THERM = 0.
      DO 15 L=NREAD/NCCH-100,NREAD/NCCH
      THERM = THERM+TURB(L)
      THERM = (62.4/7.481)*THERM/100
      TYPE 20,RHO,VEL,MDOT,THERM,BETA,RADIAN
      FORMAT(/,
      1' MEASUREMENT RESULTS: RHO = ',E15.7,' LB/CU.FT.',/,

```

```
2'          U = ',E15.7,' FT/SEC',/,  
3'          MDOT = ',E15.7,' LB/MIN',/,  
4'          THERMAL = ',E15.7,' LB/MIN',/,  
5'          BETA = ',E15.7,/,  
6'          RADIAN = ',E15.7,//)  
RETURN  
END
```

```

C*****
C*
C*      EFC25T.FOR This program is called by CAL25T.FOR, and
C*      calculates the empty pipe and flowing pipe calibration
C*      constants from frequency and decay constant measurements
C*      during calibrations.
C*
C*      EFC25T.FOR is for the 1/4 X 0.035 configuration and is
C*      representative of the programs used for the other config-
C*      urations.
C*
C*      Data acquisition in realtime is implemented using the
C*      DEC RT-11 operating systems and the Data Translation, Inc.
C*      data acquisition and control software library, DTLIB.
C*
C*      William R. Hamel November, 1980
C*
C*****
C
      SUBROUTINE ECAL
      COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
      1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
      COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
      COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
      1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
      SM = 0.1264
      RADO = RADIAN
      RC1 = (1./((2.*TA*KPEAK))*ALOG(PEAK(1)/PEAK(KPEAK)))
      R2 = RADO**2
      RC2 = R2+RC1**2
      CEXT = RC1*2.*SM

C
C      Store the raw data from this pass
C      DY1:CAL25T.DAT is currently open
C
      WRITE(8)RC1,RC2

C
C      Type out results for this pass
C
      TYPE 30,RADO,CEXT,RC1,RC2
      FORMAT(/,
      1'      CALIBRATION CONSTANTS:  RADO = ',E15.7,/
      2'                                CEXT = ',E15.7,/
      3'                                RC1 = ',E15.7,/
      4'                                RC2 = ',E15.7//)
      RETURN
      END
30

```

```

SUBROUTINE FCAL
COMMON/PA/PRESET,NREAD,NCCH,STORE,RITE,CALIB,CEXT,RADO
1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50)
1,TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
REAL*4 MDOT,MDOTTH
AREA = 5.073577E-4
RHO = 62.4

```

C
C
C

Calculate the decay constant

```

R1 = (1/(2.*TA*KPEAK))*ALOG(PEAK(1)/PEAK(KPEAK))
R2 = RADIAN**2

```

C
C
C

BETAM1 = 1.0-BETA

```

BETAM1 = R2/RC2
BETA = 1.0-BETAM1

```

C
C
C
C
C

Calculate the average thermal
flowmeter output for the
last 100 readings

```

THERM = 0.
DO 15 L=NREAD/NCCH-100,NREAD/NCCH
THERM = THERM+TURB(L)
MDOTTH = (62.4/7.481)*THERM/100.

```

C
C
C

calculate A1 and A2

```

A1 = BETAM1*RHO/BETA
A2 = BETAM1/(R1-BETAM1*RC1)
A2 = MDOTTH*A2
TYPE 20,A1,A2,RC1,RC2,MDOTTH,BETA,R1,R2

```

20

```

FORMAT(//,
1' CALIBRATION RESULTS: A1 = ',E15.7,/
2' A2 = ',E15.7,/,
3' RC1 = ',E15.7,/,
4' RC2 = ',E15.7,/,
5' MDOTTH = ',E15.7,' LB/MIN',/,
6' BETA = ',E15.7,/,
7' R1 = ',E15.7,/,
8' R2 = ',E15.7,/)

```

C
C
C

Store data for this pass on DY1:CAL25T.DAT

```

WRITE(8)A1,A2
RETURN
END

```

```

*****
C*
C*   FIREC.FOR  This program controls the beam excitation
C*   actuator using RT-11/SYS.F4 timing and a parallel digital
C*   interface controlled by the DTLIB Subroutine IDOR. After
C*   excitation, Subroutine GETDAT is called for data acquis-
C*   tion.
C*
C*   Data acquisition in realtime is implemented using the
C*   DEC RT-11 operating systems and the Data Translation, Inc.
C*   data acquisition and control software library, DTLIB.
C*
C*           William R. Hamel  November, 1980
C*
*****
C
      SUBROUTINE FIRE
      REAL*4 YES,NO
      INTEGER*2 MASK,IWORD
      DATA YES,NO/'YES','NO'/

C
C           Zero out the parallel interface
C
      MASK = '1
      IWORD = '177777

C
C           Issue ready query
C
      TYPE 400
400    FORMAT(' IS THE BEAM EXCITER ARMED?')
      TYPE 100
100    FORMAT(' INITIATE A READING? [YES OR NO]')
50     CONTINUE
      ACCEPT 300,DECIDE
300    FORMAT(A4)
      IF(DECIDE.EQ.YES)GO TO 2
      IF(DECIDE.EQ.NO) GO TO 3
      TYPE 200
200    FORMAT(' PLEASE TYPE <YES> FOR MEASUREMENT, OR <NO> TO
      1TERMINATE')
      GO TO 50
2      CONTINUE

C
C           Command given to deflect beam
C
      CALL IDOR(0,0,MASK,'0)
C

```

```
C           Hold beam down for 1 second
C
C  CALL ISLEEP(0,0,01,0)
C
C           Release beam
C
C  CALL IDOR(0,0,MASK,IWORD)
C
C           Now, so set the data
C
C  CALL GETDAT
3  CONTINUE
   RETURN
   END
```

```

C*****
C*
C*   STORC.FOR  These programs store the data acquired for
C*   the current run on the floppy disk.  Subroutine READIT
C*   allows the operator to scan selected data on the terminal.
C*
C*   Data acquisition in realtime is implemented using the
C*   DEC RT-11 operating systems and the Data Translation, Inc.
C*   data acquisition and control software library, DTLIB.
C*
C*           William R. Hamel  November, 1980
C*
C*****
C
C   SUBROUTINE STOR
C   COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,RADO,CEXT
C   1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
C   COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
C   COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50),
C   1TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
C   LOGICAL*1 FILE(15)
C
C           Get file name from the terminal
C
C   TYPE 5
C   5   FORMAT(' TYPE IN FILE SPECIFICATION...,DY1:NAME,DAT')
C   ACCEPT 10,FILE
C   10  FORMAT(14A1)
C   FILE(15)=0
C
C           Open disk file DY1:NAME.DAT for data storage
C
C   OPEN(UNIT=2,NAME=FILE,ACCESS='SEQUENTIAL',TYPE='NEW',
C   1FORM='UNFORMATTED')
C
C           First, write sampling parameters for this case
C
C   WRITE(2)PRESET,NREAD,NCCH
C
C           Now, write deflection and turbine meter data
C           into the sequential unformatted file
C           DY1:NAME.DAT
C
C   WRITE(2)(VOLT(J),TURB(J),J=1,NREAD/NCCH)
C
C           Close disk file DY1:NAME.DAT

```

```

C      CLOSE(UNIT=2)
C
C      Send data stored message to terminal
C
100    TYPE 100,(FILE(KK),KK=1,15)
      FORMAT(' DATA STORED ON',15A1)
      RETURN
      END
C
C      This routine allows the operator to print
C      out selected data on the terminal
C
      SUBROUTINE RITER
      COMMON/PAR/PRESET,NREAD,NCCH,STORE,RITE,CALIB,RADO,CEXT
      1,TBIAS,VBIAS,VBIASE,A1,A2,RC1,RC2
      COMMON/DAT1/IBUF(2000),VOLT(1000),TURB(1000)
      COMMON/DAT2/ZERO(50,2),TIME(50),TZERO(50),JTIME(50),PEAK(50),
      1TA,TP(50),RADIAN,KPEAK,KCROSS,RHO,VEL,MDOT,THERM
      REAL*4 YES,NO
      DATA YES,NO/'YES','NO'/
10     CONTINUE
      TYPE 100
100    FORMAT(' SPECIFY BEGINNING AND LAST[ARRAY INDICES] TO
      1BE PRINTED.',/,,' FIRST>1 AND LAST<NREAD/NCCH')
C
C      Get first and last data of printout
C
      ACCEPT *,IFIRST,ILAST
      TYPE 200
200    FORMAT(/,/,
      1'   NT      BEAM DEFLECTION(VOLTS)   TURBINE METER(GPM)')
      TYPE 201
201    FORMAT('   ----   -----   -----
      1')
      TYPE 300,(J,VOLT(J),TURB(J),J=IFIRST,ILAST)
300    FORMAT(2X,I4,7X,F9.5,15X,F9.5)
      TYPE 310
310    FORMAT(' IS THAT ALL YOU WISH TO PRINT? [YES OR NO]')
400    CONTINUE
      ACCEPT 500,RIT
500    FORMAT(A4)
      IF(RIT.EQ.YES)GO TO 700
      IF(RIT.EQ.NO) GO TO 10
      TYPE 600
600    FORMAT(' MORE PRINTING? [YES OR NO]')
      GO TO 400
700    CONTINUE
      RETURN
      END

```


APPENDIX D

EXPERIMENTAL AND THEORETICAL DATA ANALYSIS PROGRAMS

```

C*****
C*
C*                               GENPLT.FOR
C*
C* This is a general-purpose driving program for
C* using the Hewlett-Packard Model 7225A digital
C* plotter. The program operates through a self-
C* explanatory user dialogue during execution and
C* uses the subroutines in PLTSUB.FOR
C*
C*                               William R. Hamel
C*                               March 1981
C*
C*****

```

C

```

REAL*4 YES,NO,NPLOT
INTEGER UPX,DOWNX,UPY,DOWNY
DATA YES,NO/'YES','NO'/
INTEGER PATER,NPATER
CALL LDATA(NPLOT,NUMBER,PATER)
NPATER = PATER
CALL GDATA(XMIN,XMAX,YMIN,YMAX,XOFSET,YOFSET,TICINX,
1TICINY,TICLBX,TICLBY,UPX,DOWNX,UPY,DOWNY)
CALL LINTYP(-1,0.)
CALL SELPEN(NUMBER)
CALL SCALE(XMIN,XMAX,YMIN,YMAX)
IF(NPLOT,EQ.NO)GO TO 10
CALL TICLN(UPX,DOWNX)
CALL XAXIS(YOFSET,TICINX,XMIN,XMAX,TICLBX)
CALL TICLN(UPY,DOWNY)
CALL YAXIS(XOFSET,TICINY,YMIN,YMAX,TICLBY)
10 CALL SELPEN(NUMBER)
CALL LINTYP(NPATER,0.)
CALL GRAPH(XMIN,XMAX,YMIN,YMAX)
CALL SCRIBE
CALL SELPEN(0)
STOP
END

```

C

C

C

This subroutine sets-up line parameters

```

SUBROUTINE LDATA(NPLOT,NUMBER,PATER)
INTEGER PATER
REAL*4 YES,NO,NPLOT
DATA YES,NO/'YES','NO'/
TYPE 10
10 FORMAT(' FIRST PLOT FOR THE SET OF AXES WHICH HAS
1BEEN DRAWN?[YES OR NO]')
ACCEPT 11,NPLOT
11 FORMAT(A4)
TYPE 20
20 FORMAT(' WHICH PEN DO YOU WANT TO USE?...[1,2,3,4]')

```

```

ACCEPT *,NUMBER
TYPE 30
30  FORMAT(' WHICH LINE TYPE DO YOU WANT TO USE?...[-1,0,1,2,3,4,5,
      16; SEE MANUAL]')
ACCEPT *,PATTERN
RETURN
END

C
C      This subroutine sets-up graph axes parameters
C

SUBROUTINE GDATA(XMIN,XMAX,YMIN,YMAX,XOFFSET,YOFFSET,TICINX,
1TICINY,TICLBX,TICLBY,UPX,DOWNX,UPY,DOWNY)
REAL*4 YES,NO
DATA YES,NO/'YES','NO'/
TYPE 10
10  FORMAT(' WHAT ARE THE MINIMUM AND MAXIMUM VALUES OF THE HORIZON
      1TAL AXIS?')
ACCEPT *,XMIN,XMAX
TYPE 20
20  FORMAT(' WHAT ARE THE MINIMUM AND MAXIMUM VALUES OF THE VERT
      1ICAL AXIS?')
ACCEPT *,YMIN,YMAX
TYPE 30
30  FORMAT(' WHAT ARE THE HORIZONTAL AND VERTICAL AXIS OFFSETS?')
ACCEPT *,XOFFSET,YOFFSET
TYPE 40
40  FORMAT(' WHAT X-AXIS TIC MARK SPACING DO YOU WANT
      1 [AXIS UNITS PER TIC]?')
ACCEPT *,TICINX
TYPE 41
41  FORMAT(' WHAT Y-AXIS TIC MARK SPACING DO YOU WANT
      1 [AXIS UNITS PER TIC]?')
ACCEPT *,TICINY
TYPE 50
50  FORMAT(' WHAT Y-AXIS LABEL SPACING DO YOU WANT [IN TICS PER
      1LABEL]?')
ACCEPT *,TICLBY
TYPE 51
51  FORMAT(' WHAT X-AXIS LABEL SPACING DO YOU WANT [IN TICS PER
      1LABEL]?')
ACCEPT *,TICLBX
55  CONTINUE
TYPE 60
60  FORMAT(' DO YOU WANT A GRID?[YES OR NO]')
ACCEPT 70,GRID
70  FORMAT(A4)
      IF(GRID,EQ,NO)GO TO 90
      IF(GRID,EQ,YES)GO TO 80
      GO TO 55
80  CONTINUE
TYPE 85
85  FORMAT(' WHAT UPX AND DOWNX TIC SIZES?')

```

```

ACCEPT *,UPX,DOWNX
TYPE 86
86  FORMAT(' WHAT UPY AND DOWNY TIC SIZES?')
    ACCEPT *,UPY,DOWNY
    GO TO 95
90  UPX=0.
    UPY=0.
    DOWNX=0.
    DOWNY=0.
95  CONTINUE
    RETURN
    END

C
C      This subroutine performs labeling
C

SUBROUTINE SCRIBE
LOGICAL*1 STRING(80),DUM(80)
REAL*4 YES,NO,BIG,AGAIN
DATA STRING/80*3/
DATA YES,NO/'YES','NO'/
STATE = 0

C      BE SURE CHARACTER ANGLE IS 0
CALL CHDIR(STATE)
CALL PEN(STATE)
TYPE 10
10  FORMAT(' POSITION PLOTTER PEN WHERE YOU WANT THE HORIZONTAL
    1 LABEL TO START')
    TYPE 20
20  FORMAT(' TYPE IN LABEL...UP TO 80 CHARACTERS')
    ACCEPT 100,LEN,DUM
100  FORMAT(Q,80A1)
    DO 200 J=1,LEN
200  STRING(J) = DUM(J)
    ENCODE(80,300,STRING)
300  FORMAT(80A1)
C      PRINT LABEL
    STATE = 1
    CALL PEN(STATE)
    CALL LABEL(STRING)
    STATE = 0
    CALL PEN(STATE)

C      ROTATE CHARACTERS FOR VERTICAL LABEL
    CALL CHDIR(90)
    DO 25 J=1,80
25  STRING(J)=3
    TYPE 30
30  FORMAT(' POSITION PLOTTER PEN WHERE YOU WANT THE VERTICAL
    1 LABEL TO START')
    TYPE 20
    ACCEPT 100,LEN,DUM
    DO 400 J=1,LEN
400  STRING(J) = DUM(J)

```

```

      ENCODE(80,300,STRING)
C      PRINT LABEL
      CALL PEN(1)
      CALL LABEL(STRING)
      CALL PEN(0)
      CALL CHDIR(0)
34     CONTINUE
      DO 35 J=1,80
35     STRING(J)=3
      TYPE 40
40     FORMAT(' POSITION PLOTTER PEN WHERE YOU WANT A NOTATION
      1 TO START')
      TYPE 20
      ACCEPT 100,LEN,DUM
      DO 45 J=1,LEN
45     STRING(J)=DUM(J)
      ENCODE(80,300,STRING)
52     CONTINUE
      TYPE 41
41     FORMAT(' BIG CHARACTER SIZE?[YES OR NO]')
      ACCEPT 42,BIG
42     FORMAT(A4)
      IF(BIG,EQ,NO)GO TO 46
      IF(BIG,EQ,YES)GO TO 51
      GO TO 52
51     CALL CSIZE(1.5,3.0)
46     CALL PEN(1)
      CALL LABEL(STRING)
      CALL PEN(0)
      CALL CFLT(0.,0.)
      CALL CSIZE(0.75,1.5)
53     CONTINUE
      TYPE 47
47     FORMAT(' ANOTHER NOTATION?[YES OR NO]')
      ACCEPT 42,AGAIN
      IF(AGAIN,EQ,NO)GO TO 50
      IF(AGAIN,EQ,YES)GO TO 34
      GO TO 53
50     CONTINUE
      RETURN
      END

```

```

C*****
C*
C*      NUMPLT.FOR  These are a series of SUBROUTINE
C*      GRAPHS used for plotting numerical and analytical
C*      results on the Hewlett-Packard digital plotter.
C*      SUBROUTINE GRAPH is called from MAIN located in
C*      file GENPLT.FOR
C*
C*      William R. Hamel  March, 1981
C*
C*****
C
C      Subroutine for plotting imaginary Omega
C      versus nondimensional velocity, u
C
      SUBROUTINE GRAPH(XMIN,XMAX)
      COMPLEX ALPHA(4,300),OMEGA(300)
      DIMENSION U(300),BETA(300),C(300)
      REAL OMEGAR,OMEGAI
      LOGICAL*1 FILE(15)
      TYPE 5
5      FORMAT(' TYPE IN FILE SPECIFICATION...DY1:NAME.DAT')
      ACCEPT 10,FILE
10     FORMAT(14A1)
      FILE(15)=0
      I = 1
      OPEN(UNIT=8,TYPE='OLD',ACCESS='SEQUENTIAL',FORM=
100     1'FORMATTED',NAME=FILE)
      READ(8,400,END=700)U(I),BETA(I),C(I),(ALPHA(J,I),J=1,4)
400     1,OMEGA(I)
      FORMAT(13F9.4)
      I = I+1
      GO TO 100
700     I = I-1
      TYPE 501
501     FORMAT(' END OF FILE')
      CLOSE(UNIT=8)
      TYPE 500,(U(J),OMEGA(J),J=1,I)
500     FORMAT(3F9.4)
      CALL PEN(0)
      PAUSE ' CHANGE PEN?'
      DO 600 J=1,I
      OMEGAR = ABS(AIMAG(OMEGA(J)))
      CALL PLOT(U(J),OMEGAR)
      IF(J.EQ.1)CALL PEN(1)
600     CONTINUE
      CALL PEN(0)
      RETURN
      END
C
C      Subroutine for plotting real Omega versus
C      nondimensional velocity, u

```

C

```

SUBROUTINE GRAPH(XMIN,XMAX)
  COMPLEX ALPHA(4,300),OMEGA(300)
  DIMENSION U(300),BETA(300),C(300)
  REAL OMEGAR,OMEGAI
  LOGICAL*1 FILE(15)
  TYPE 5
5  FORMAT(' TYPE IN FILE SPECIFICATION...DY1:NAME.DAT')
  ACCEPT 10,FILE
10  FORMAT(14A1)
  FILE(15)=0
  I = 1
  OPEN(UNIT=8,TYPE='OLD',ACCESS='SEQUENTIAL',FORM=
    1'FORMATTED',NAME=FILE)
100  READ(8,400,END=700)U(I),BETA(I),C(I),(ALPHA(J,I),J=1,4)
    1,OMEGA(I)
400  FORMAT(13F9.4)
    I = I+1
    GO TO 100
700  I = I-1
    TYPE 501
501  FORMAT(' END OF FILE')
    CLOSE(UNIT=8)
    TYPE 500,(U(J),OMEGA(J),J=1,I)
500  FORMAT(3F9.4)
    CALL PEN(0)
    PAUSE ' CHANGE PEN?'
    DO 600 J=1,I
      OMEGAR = ABS(REAL(OMEGA(J)))
      CALL PLOT(U(J),OMEGAR)
      IF(J.EQ.1)CALL PEN(1)
600  CONTINUE
    CALL PEN(0)
    RETURN
  END

```

C

C

C

C

Subroutine for plotting imaginary Omega
versus real Omega

```

SUBROUTINE GRAPH(XMIN,XMAX)
  COMPLEX ALPHA(4,300),OMEGA(300)
  DIMENSION U(300),BETA(300),C(300)
  REAL OMEGAR,OMEGAI
  LOGICAL*1 FILE(15),STRING(80)
  DATA STRING/80*3/
  TYPE 5
5  FORMAT(' TYPE IN FILE SPECIFICATION...DY1:NAME.DAT')
  ACCEPT 10,FILE
10  FORMAT(14A1)
  FILE(15)=0
  I = 1
  OPEN(UNIT=8,TYPE='OLD',ACCESS='SEQUENTIAL',FORM=

```

```

1'FORMATTED',NAME=FILE)
100 READ(8,400,END=700)U(I),BETA(I),C(I),(ALPHA(J,I),J=1,4)
    1,OMEGA(I)
400  FORMAT(13F9.4)
    I = I+1
    GO TO 100
700  I = I-1
    TYPE 501
501  FORMAT(' END OF FILE')
    CLOSE(UNIT=8)
    TYPE 500,(OMEGA(J),J=1,I)
500  FORMAT(2F9.4)
    CALL PEN(0)
    PAUSE ' CHANGE PEN?'
    K = 0
    L = 11
    DO 600 J=1,I
    K = K+1
    OMEGAR = REAL(OMEGA(J))
    OMEGAI = ABS(AIMAG(OMEGA(J)))
    IF(K.EQ.L)GO TO 598
    CALL PLOT(OMEGAR,OMEGAI)
    IF(J.EQ.1)CALL PEN(1)
    GO TO 600
598  CALL PEN(0)
    CALL PLOT(OMEGAR,OMEGAI)
    JJ = J/10
    K = 0
    L = 10
    IF(JJ.GE.10)GO TO 597
    STRING(1)=JJ+48
    GO TO 596
597  STRING(1)='1'
    STRING(2)='0'
596  CALL PEN(1)
    CALL LABEL(STRING)
600  CONTINUE
    CALL PEN(0)
    RETURN
    END

```

C
C
C
C

Subroutine for plotting realtime
analytical solution

```

SUBROUTINE GRAPH(XMIN,XMAX)
  INTEGER STATE
  REAL XMIN,XMAX,L,KRHO,OMEGA,OMEGA0,MDOT

```

C
C
C
C
C

This routine plots the realtime response
of the beam using the "serendipity" analytical
solution.


```

C          Define Various Constants, these particular
c          values are for a 3/8 X 20' X 0.035 tube.
C

KRHO = 250.0
RKXP = 2.0
RKXP2 = 0.8582424
EI = 113.7
L = 20.0/12.0
SM = 0.1254/32.2
AREA = 5.074E-04
ALPHA = 3.5160

C          Get Specific Gravity, Mass Flowrate and External
C          Damping Coefficient
C
TYPE 10
10  FORMAT(' Type in the specific gravity, mass flowrate(lb/min),
      1 and external damping coefficient')
      ACCEPT *,SPGR,MDOT,CBAR

C          Calculate the Parameters
C
RHO = 62.4*SPGR
U   = MDOT/(60.*RHO*AREA)
BETA = RHO/(RHO+KRHO)
C          TAU is actually 1/TAU
TAU = BETA*RKXP*U/L+(1.0-BETA)*CBAR*0.5/SM
C          OMEGA0 is actually [ OMEGA0 ]**2
OMEGA0 = (1.0-BETA)*(ALPHA**2)*EI/(SM*(L**4))
OMEGA = SQRT(OMEGA0+BETA*RKXP2*(U**2)/(L**2)
1-TAU**2)
CALL PEN(0)
DO 100 I=0,200
  T = FLOAT(I)*0.001
  Y = EXP(-T*TAU)*(COS(OMEGA*T)+(TAU/OMEGA)
1*SIN(OMEGA*T))
C          Plot this Point
CALL PLOT(T,Y)
STATE = 1
IF(I.EQ.0)CALL PEN(STATE)
100  CONTINUE
STATE = 0
CALL PEN(STATE)
CALL SELPEN(0)
RETURN
END

```

```

C*****
C*
C*   EXPPLT.FOR  These are a series of SUBROUTINE
C*   GRAPHS used for plotting various experimental
C*   results on the Hewlett-Packard digital plotter.
C*   SUBROUTINE GRAPH is called from MAIN located in
C*   file GENPLT.FOR
C*
C*
C*           William R. Hamel  March, 1981
C*
C*****
C
C           Subroutine for plotting deflection
C           voltage versus time
C
      SUBROUTINE GRAPH(XMIN,XMAX)
      DIMENSION VOLT(1000),TURB(1000)
      CALL READIT(PRESET,NREAD,NCCH,VOLT,TURB)
      T=PRESET*1.E-6
      DO 100 J=1,NREAD/NCCH
      X=T*(J-1)
      Y=VOLT(J)
      CALL PLOT(X,Y)
      IF(J.EQ.1)CALL PEN(1)
100  CONTINUE
      CALL PEN(0)
      RETURN
      END

C
C           Subroutine for plotting Coriolis mass flow
C           rate versus thermal flowmeter mass flow rate
C
      SUBROUTINE GRAPH(XMIN,XMAX)
      DIMENSION X(100),Y(100)
      LOGICAL*1 FILE(15)
      REAL*4 MDOT
      TYPE 4
4     FORMAT(' TYPE IN THE DATA POINT CHARACTER',/,
      1'          SQUARE   = 1.',/,
      2'          TRIANGLE = 2.',/,
      3'          CIRCLE   = 3.',/,
      4'          DIAMOND  = 4.')
      ACCEPT *,TYPE
      TYPE 5
5     FORMAT(' TYPE IN FILE SPECIFICATION...DY1:NAME.DAT')
      ACCEPT 10,FILE
10    FORMAT(14A1)
      FILE(15)=0
      OPEN(UNIT=8,NAME=FILE,ACCESS='SEQUENTIAL',TYPE='OLD'
      1,FORM='UNFORMATTED')
      I=1
50   READ(8,END=100)RHO,VEL,MDOT,THERM

```

```

X(I)=THERM
Y(I)=MDOT
I=I+1
GO TO 50
CLOSE(UNIT=8)
100 J=I-1
    PAUSE ' CHANGE PEN?'
    CALL PEN(0)
    DO 150 K=1,J
        CALL PEN(1)
        CALL PLOT(X(K),Y(K))
        CALL PEN(0)
        CALL USCHAR(TYPE)
150    CONTINUE
        CALL PEN(0)
        CALL LINTYP(3)
        PAUSE ' CHANGE PEN?'
        DO 250 J=1,200
            XX = XMAX*FLOAT(J)/202.
            YY = XX
            CALL PLOT(XX,YY)
            IF(J.EQ.2)CALL PEN(1)
250    CONTINUE
        CALL PEN(0)
        RETURN
    END

C
C      Subroutine for plotting percent difference
C      of a linear regression between the Coriolis
C      and thermal flowmeters versus the thermal
C      flowmeter mass flow rate.
C

SUBROUTINE GRAPH(XMIN,XMAX,YMIN,YMAX)
    DIMENSION X(100),Y(100),RESULT(20)
    LOGICAL*1 FILE(15)
    REAL*4 MDOT
    TYPE 4
4    FORMAT(' TYPE IN THE DATA POINT CHARACTER',/,
1'          SQUARE    = 1.',/,
2'          TRIANGLE  = 2.',/,
3'          CIRCLE    = 3.',/,
4'          DIAMOND   = 4.')
    ACCEPT *,TYPE
    TYPE 5
5    FORMAT(' TYPE IN FILE SPECIFICATION...DY1:NAME.DAT')
    ACCEPT 10,FILE
10   FORMAT(14A1)
    FILE(15)=0
    OPEN(UNIT=8,NAME=FILE,ACCESS='SEQUENTIAL',TYPE='OLD'
1,FORM='UNFORMATTED')
    I=1
50   READ(8,END=100)RHO,VEL,MDOT,THERM

```

```

X(I)=THERM
Y(I)=MDOT
I=I+1
GO TO 50
CLOSE(UNIT=8)
100 J=I-1
C      GO DO REGRESSION ANALYSIS
CALL G02CAF(J,X,Y,RESULT,IFAIL)
TYPE 160,(FILE(I),I=1,15),RESULT(6),RESULT(7)
PAUSE ' IS PRINTER ON?'
PRINT 160,(FILE(I),I=1,15),RESULT(6),RESULT(7)
160 FORMAT(/
1'  DATA FILE:',15A1,
2'  REGRESSION RESULTS:',
3'      MDOT = ('E15.7,')*THERMAL+('E15.7,')')
PAUSE ' CHANGE PEN?'
CALL PEN(0)
DO 150 K=1,J
CALL PEN(1)
REG = (RESULT(6)*X(K)+RESULT(7))
Y(K) = Y(K)-REG
Y(K) = Y(K)*100./REG
CALL PLOT(X(K),Y(K))
CALL PEN(0)
CALL USCHAR(TYPE)
150 CONTINUE
RETURN
END

C
C      Subroutine for plotting the percent difference
C      between the Coriolis density calculation and
C      the calibration density, 62.4 lb/ft**3.
C

SUBROUTINE GRAPH(XMIN,XMAX)
DIMENSION X(100),Y(100)
LOGICAL*1 FILE(15)
REAL*4 MDOT
TYPE 4
4  FORMAT(' TYPE IN THE DATA POINT CHARACTER',/,
1'      SQUARE   = 1.',/,
2'      TRIANGLE = 2.',/,
3'      CIRCLE   = 3.',/,
4'      DIAMOND  = 4.')
```

ACCEPT *,TYPE

```

TYPE 5
5  FORMAT(' TYPE IN FILE SPECIFICATION...DY1:NAME.DAT')
ACCEPT 10,FILE
10  FORMAT(14A1)
FILE(15)=0
OPEN(UNIT=8,NAME=FILE,ACCESS='SEQUENTIAL',TYPE='OLD'
1,FORM='UNFORMATTED')
I=1

```

```
50      READ(8,END=100)RHO,VEL,MDOT,THERM
        X(I)=THERM
        Y(I)=RHO
        I=I+1
        GO TO 50
        CLOSE(UNIT=8)
100     J=I-1
        PAUSE ' CHANGE PEN?'
        CALL PEN(0)
        DO 150 K=1,J
        CALL PEN(1)
        Y(K) = Y(K)-62.4
        Y(K) = Y(K)*100./62.4
        CALL PLOT(X(K),Y(K))
        CALL PEN(0)
        CALL USCHAR(TYPE)
150     CONTINUE
        RETURN
        END
```

```

C*****
C
C          PLTSUB.FOR
C
C      SUBROUTINES TO OPERATE THE HEWLETT-PACKARD 9872A DIGITAL
C      PLOTTER.  SINCE THE PLOTTER HAS AN IEEE-488 INTERFACE, THE
C      ICS MODEL 4885A RS-232 TO IEEE CONTROLLER IS USED.
C
C      WRITTEN BY RICHARD SPILLE
C      LAST REVISED 12-MAY-81
C
C*****
C          SUBROUTINE SCALE(XMIN,XMAX,YMIN,YMAX)
C*****
C      PARAMETERS PASSED ARE TYPE REAL.
C      WITH XMAX GREATER THAN XMIN AND YMAX GREATER THAN YMIN.
C      THE PARAMETERS XMIN AND YMIN DEFINE THE USER-UNIT
C      COORDINATES OF P1; LIKEWISE XMAX AND YMAX DEFINE THE
C      USER-UNIT COORDINATES OF P2.
C
C      REAL XMIN,XMAX,YMIN,YMAX
C      REAL UONEX,UONEY,UTWOX,UTWOY,PONEX,PONEY,PTWOX,PTWOY
C      COMMON /XYZZX/UONEX,UONEY,UTWOX,UTWOY,PONEX,PONEY,PTWOX,PTWOY
C
C      UONEX=XMIN
C      UTWOX=XMAX
C      UONEY=YMIN
C      UTWOY=YMAX
C      TYPE 9000
9000  FORMAT('+', '!WRT05,0P')
C      TYPE 9001
9001  FORMAT('+', '!RED05')
C      ACCEPT 9002,PONEX,PONEY,PTWOX,PTWOY
9002  FORMAT(4F6.0)
C      RETURN
C      END
C
C*****
C          SUBROUTINE PLOT(X,Y)
C*****
C      BOTH PARAMETERS MUST BE SPECIFIED(COORDINATE PAIR).
C      THE X COORDINATE PARAMETER SPECIFIES THE X LOCATION
C      TO WHICH THE PEN WILL MOVE IN USER UNITS.  THE Y
C      COORDINATE PARAMETER SPECIFIES THE Y LOCATION TO
C      WHICH THE PEN WILL MOVE IN USER UNITS.  IF THE SCALE
C      SUBROUTINE HAS NOT BEEN CALLED SINCE THE PLOTTER WAS
C      TURNED ON THEN THE UNITS WILL BE 0 TO 16000 IN X AND
C      0 TO 11400 IN Y.
C
C      REAL X,Y,XSCAL,YSCAL
C      REAL UONEX,UONEY,UTWOX,UTWOY,PONEX,PONEY,PTWOX,PTWOY
C      COMMON /XYZZX/UONEX,UONEY,UTWOX,UTWOY,PONEX,PONEY,PTWOX,PTWOY

```

C

```

XSCAL=((PTWOX-PONEX)/(UTWOX-UONEX))*X+PONEX-UONEX*
* ((PTWOX-PONEX)/(UTWOX-UONEX))
YSCAL=((PTWOY-PONEY)/(UTWOY-UONEY))*Y+PONEY-UONEY*
* ((PTWOY-PONEY)/(UTWOY-UONEY))
IF(XSCAL.LT,-32767.0) XSCAL=-32767.0
IF(XSCAL.GT.32767.0) XSCAL=32767.0
IF(YSCAL.LT,-32767.0) YSCAL=-32767.0
IF(YSCAL.GT.32767.0) YSCAL=32767.0
TYPE 9005,INT(XSCAL),INT(YSCAL)
9005  FORMAT('+', '!WRT05,PA',I6,',',I6)
      CALL DONE
      RETURN
      END

```

C

```

C*****
SUBROUTINE PEN(STATE)
C*****
C      THE PARAMETER PASSED IS TYPE INTEGER AND DETERMINES
C      THE PLOTTER PEN STATE.  A PARAMETER OF ONE WILL
C      CAUSE THE PEN TO GO DOWN ON THE PAPER AND REMAIN
C      THERE UNTIL PEN IS CALLED AGAIN WITH A PARAMETER
C      OF ZERO.
C
C      INTEGER STATE
C
C      IF(STATE.NE.1) TYPE 9010
9010  FORMAT('+', '!WRT05,PU')
      IF(STATE.EQ.1) TYPE 9011
9011  FORMAT('+', '!WRT05,PD')
      CALL DONE
      RETURN
      END

```

C

```

C*****
SUBROUTINE SYMMOD(SYMBOL)
C*****
C      THE PASSED PARAMETER IS OF TYPE INTEGER AND MUST BE
C      BETWEEN 35 AND 122.  A PARAMETER OF ZERO WILL CANCEL
C      THE SYMBOL MODE, CHARACTERS WITH THE FOLLOWING DECIMAL
C      VALUES ARE NOT ALLOWED; %-37 &-38 '-39 (-40 )-41 /-47
C      !-58 ;-59 <-60 =-61 >-62 ?-63 @-64 [-91 \-92 ]-93 ^-94
C      --95 ` -96.  THE CSIZ AND CHDIR CALLS AFFECT THE SYMBOL
C      PLOTTED.  THE SYMBOL IS DRAWN AT THE POSITION OF THE
C      PEN AFTER A PLOT CALL.  CONSULT AN ASCII CHART TO OBTAIN
C      THE DECIMAL VALUES OF THE CHARACTERS.
C
C      INTEGER SYMBOL
C
C      IF(SYMBOL.LT.35.OR.SYMBOL.GT.122) GOTO 10
C      IF(SYMBOL.GT.36.OR.SYMBOL.LT.42) GOTO 10
C      IF(SYMBOL.EQ.47) GOTO 10

```

```

        IF(SYMBOL.GT.57.OR.SYMBOL.LT.65) GOTO 10
        IF(SYMBOL.GT.90.OR.SYMBOL.LT.97) GOTO 10
        TYPE 9010,SYMBOL
9010    FORMAT('+', '!WRT05,SM',A)
        GOTO 20
10      CONTINUE
        TYPE 9020
9020    FORMAT('+', '!WRT05,SM')
20      CONTINUE
        CALL DONE
        RETURN
        END

```

C

C*****

SUBROUTINE SELPEN(NUMBER)

C*****

```

C      THE PASSED PARAMETER IS OF TYPE INTEGER AND MUST BE
C      BETWEEN 0 AND 4. A PARAMETER OF ZERO WILL CAUSE THE
C      PLOTTER TO DROP OFF THE PEN IT HAD AND RETURN WITHOUT
C      PICKING UP ANOTHER ONE. USE ZERO WHEN YOU ARE FINISHED
C      WITH YOUR PLOT. A PARAMETER OF 1 TO 4 CORRESPONDS TO
C      THE PEN NUMBERS STAMPED ON THE PLOTTER AND WILL CAUSE
C      THE PLOTTER TO PICKUP THE NUMBERED PEN AND RETURN TO
C      THE PREVIOUS PEN POSITION.

```

C

INTEGER NUMBER

C

IF(NUMBER.LT.0.OR.NUMBER.GT.4) NUMBER=0

TYPE 9010,NUMBER

9010 FORMAT('+', '!WRT05,SP',I)

CALL DONE

RETURN

END

C

C*****

SUBROUTINE LABEL(STRING)

C*****

```

C      THE PASSED PARAMETER IS A LOGICAL*1 ARRAY WITH UP TO 80
C      ELEMENTS. THE TEXT CONTAINED IN THE ARRAY WILL BE PRINTED
C      AT THE PRESENT PEN POSITION. THE TEXT IN THE ARRAY IS
C      DELIMITED BY A 3(NOT AN ASCII 3) THEREBY ALLOWING VARIABLE
C      LENGTH TEXT TO BE PASSED.

```

C

LOGICAL*1 STRING(80),ETX

C

DATA ETX/3/

C

I=1

TYPE 9005

9005 FORMAT('+', '!WRT05,LB',%)

10 CONTINUE

IF(STRING(I).EQ.ETX) GOTO 20


```

      TYPE 9010,STRING(I)
9010  FORMAT('+',A1,$)
      I=I+1
      GOTO 10
20    CONTINUE
      TYPE 9020,ETX
9020  FORMAT('+',A1)
      CALL DONE
      RETURN
      END

C
C*****
      SUBROUTINE XTIC
C*****
C      CALLING THIS SUBROUTINE WILL CAUSE A VERTICAL TIC MARK
C      TO BE DRAWN AT THE PRESENT PEN POSITION.
C
      TYPE 9010
9010  FORMAT('+','!WRT05,XT')
      CALL DONE
      RETURN
      END

C
C*****
      SUBROUTINE YTIC
C*****
C      CALLING THIS SUBROUTINE WILL CAUSE A HORIZONTAL TIC MARK
C      TO BE DRAWN AT THE PRESENT PEN POSITION.
C
      TYPE 9010
9010  FORMAT('+','!WRT05,YT')
      CALL DONE
      RETURN
      END

C
C*****
      SUBROUTINE TICLN(UP,DOWN)
C*****
C      PASSED PARAMETERS ARE OF TYPE REAL AND MAY BE
C      POSITIVE ONLY.  THE FIRST PARAMETER SETS THE UPWARD
C      PORTION OF A TIC MARK DRAWN ALONG THE X AXIS AND THE
C      RIGHT-SIDE PORTION OF A TICK MARK DRAWN ALONG THE Y
C      AXIS.  THE SECOND PARAMETER DETERMINES THE DOWNWARD
C      PORTION OF A TIC MARK ALONG THE X AXIS AND THE LEFT-
C      SIDE PORTION OF A TIC MARK ALONG THE Y AXIS.
C
C      REAL UP,DOWN
C
      IF(UP.GT.100.) UP=100.
      IF(DOWN.GT.100.) DOWN=100.
      IF(UP.LE.0..AND.DOWN.LE.0.) GOTO 10

```

```

      IF(DOWN.LE.0.) GOTO 20
      TYPE 9005,UP,DOWN
9005  FORMAT('+', ' !WRT05,TL',F6.2,' ',F6.2)
      GOTO 30
10    CONTINUE
      TYPE 9010
9010  FORMAT('+', ' !WRT05,TL.5,.5')
      GO TO 30
20    CONTINUE
      TYPE 9020,UP
9020  FORMAT('+', ' !WRT05,TL',F6.2)
30    CONTINUE
      CALL DONE
      RETURN
      END

```

```

C
C*****
      SUBROUTINE CSIZE(WIDTH,HEIGHT)
C*****
C      PASSED PARAMETERS ARE OF TYPE REAL AND MAY BE +.004
C      TO +127.999.  PARAMETERS OF 0 WILL SET THE DEFAULT
C      VALUES OF .75 , 1.5.  THE FIRST PARAMETER DETERMINES
C      CHARACTER WIDTH AND THE SECOND CHARACTER HEIGHT AS A
C      PERCENTAGE OF THE P1 AND P2 POINTS.

C
      REAL WIDTH,HEIGHT

C
      IF(WIDTH.GE.127.999.OR.HEIGHT.GE.127.999) GOTO 10
C      IF(WIDTH.LE..004.OR.HEIGHT.LE..004) GOTO 10
      TYPE 9005,WIDTH,HEIGHT
9005  FORMAT('+', ' !WRT05,SR',F7.3,' ',F7.3)
      GOTO 20
10    CONTINUE
      TYPE 9010
9010  FORMAT('+', ' !WRT05,SR')
20    CONTINUE
      CALL DONE
      RETURN
      END

```

```

C
C*****
      SUBROUTINE CPLT(SPACES,LINES)
C*****
C      PASSED PARAMETERS ARE OF TYPE REAL AND MAY BE
C      NEGATIVE OR POSITIVE.  THE FIRST PARAMETER CAUSES THE
C      PLOTTER TO FORWARD OR BACKSPACE THE PASSED NUMBER OF
C      TIMES.  THE SECOND PARAMETER CAUSES THE SAME OPERATION

```

C IN THE VERTICAL DIRECTION(PLUS AND MINUS LINEFEED).
 C THESE SPACES AND LINEFEEDS ARE SCALED TO THE PRESENT
 C CHARACTER SIZE. IF SPACES AND LINES BOTH EQUAL
 C ZERO THEN A CARRIAGE RETURN LINE FEED IS SIMULATED.

C

REAL SPACES,LINES

C

IF(SPACES.EQ.0.AND,LINES.EQ.0) GOTO 10
 IF(SPACES.GT.127.999.OR,SPACES.LT,-127.999) GOTO 10
 IF(LINES.GT.127.999.OR,SPACES.LT,-127.999) GOTO 10
 TYPE 9005,SPACES,LINES
 9005 FORMAT('+', 'IWRTO5,CP',F8.3,',',F8.3)
 GOTO 20
 10 CONTINUE
 TYPE 9010
 9010 FORMAT('+', 'IWRTO5,CP')
 20 CONTINUE
 CALL DONE
 RETURN
 END

C

SUBROUTINE LINTYP(PATERN,LENGTH)

C THE PASSED PARAMETERS ARE OF TYPE INTEGER. THE PATERN
 C PARAMETER CAUSES VARIOUS LINE TYPES TO BE USED.

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

-1 - SOLID LINE
 0 - SPECIFIES DOTS ONLY AT THE PLOTTED POINTS.
 1 -
 2 - ----
 3 - ----
 4 - ---- . ---- . ---- .
 5 - ----
 6 - ----
 !-----! ONE PATERN LENGTH

THE LENGTH PARAMETER DEFINES THE LENGTH OF ONE COMPLETE
 SEGMENT OF THE PATTERN AND IS EXPRESSED AS A PERCENTAGE
 OF THE DIAGONAL DISTANCE BETWEEN THE SCALING POINTS, P1
 AND P2. THE DEFAULT LENGTH IS 4% AND VALID PARAMETERS
 ARE +.004 TO +127.999. USEFUL VALUES FOR PATTERN LENGTH WITH
 DEFAULT P1 AND P2 ARE BETWEEN 1 AND 5 AND DEPEND SOMEWHAT
 ON THE PATTERN NUMBER. A PATTERN VALUE OF 0 WILL CAUSE THE
 LAST VALUE RECEIVED TO BE USED. AFTER CALLING THIS ROUTINE,
 ALL SUBSEQUENT PLOT COMMANDS WITH THE PEN DOWN WILL CAUSE THE
 SPECIFIED PATTERN TO BE DRAWN. ANY PORTION OF THE PATTERN
 NOT USED WILL BE CARRIED OVER TO THE NEXT VECTOR.

INTEGER PATERN
REAL LENGTH

```

C      IF(PATERN.LT.0) GO TO 10
      IF(LENGTH.LT..004.OR.LENGTH.GT.127.999) GO TO 20
5      CONTINUE
      TYPE 9005,PATERN,LENGTH
9005   FORMAT('+', '!WRT05,LT',I6,',',I6)
      GO TO 30
10     CONTINUE
      TYPE 9010
9010   FORMAT('+', '!WRT05,LT')
      GO TO 30
20     CONTINUE
      TYPE 9020,PATERN
9020   FORMAT('+', '!WRT05,LT',I6)
30     CONTINUE
      CALL DONE
      RETURN
      END

```

```

C
C*****
      SUBROUTINE XAXIS(YOFSET,TICINT,START,FINISH,TICLBL)
C*****
C      THE PASSED PARAMETERS ARE OF TYPE REAL.  THE VALUES ARE
C      IN USER-UNITS DETERMINED BY THE LAST CALL TO SCALE.
C      YOFSET IS THE POINT YOU WANT THE X-AXIS TO CROSS THE
C      Y-AXIS.  TICINT IS THE TIC-MARK INTERVAL PARAMETER AND
C      DETERMINES WHETHER OR NOT TIC MARKS ARE DRAWN ALONG THE
C      AXIS.  IF TIC MARKS ARE TO BE DRAWN, THE PARAMETER VALUE
C      SPECIFIES THE SPACING, IN SCALE CALL UNITS, BETWEEN TICS.
C      A VALUE OF 0 RESULTS IN NO TIC MARKS.
C      THE START AND FINISH PARAMETERS SPECIFY THE LOCATION
C      OF THE ENDPOINTS OF THE AXIS.  THE AXIS IS ALWAYS
C      DRAWN FROM THE START TO THE FINISH POINT.
C      THE TICLBL(NUMBER-OF-TICS/LABEL) PARAMETER DETERMINES
C      WHETHER OR NOT THE TIC MARKS ON THE AXIS WILL BE LABELED.
C      SPECIFYING A 0 RESULTS IN NO LABELS.  IF LABELS ARE DESIRED,
C      THE PARAMETER SPECIFIES THE NUMBER OF TIC MARKS BETWEEN
C      LABELS.  ALL LABELS ARE LETTERED ACCORDING TO THE CURRENT
C      CHARACTER SIZE CALL AND THE NUMBER FORMAT IS F10.2.
C
      REAL YOFSET,TICINT,START,FINISH,TICLBL,XTEMP
      REAL COUNT
      LOGICAL*1 ETX

C      DATA ETX/3/

C
      IF(TICINT.NE.0.) GOTO 10
      CALL PLOT(START,YOFSET)
      CALL PEN(1)
      CALL PLOT(FINISH,YOFSET)

```

```

      CALL PEN(0)
      GOTO 100
10    CONTINUE
C     DO LOOP WITH REAL NUMBERS
C     DO 20,XTEMP=START,FINISH,TICINT
      COUNT=1.
      XTEMP=START
15    CONTINUE
      CALL PLOT(XTEMP,YOFSET)
      IF(XTEMP,EQ.START) CALL PEN(1)
      CALL XTIC
      XTEMP=START+TICINT*COUNT
      COUNT=COUNT+1.
      IF(XTEMP,LE.FINISH) GOTO 15
20    CONTINUE
      CALL PEN(0)
      IF(TICLBL,EQ.0.) GOTO 100
      CALL CHDIR(90)
C     DO LOOP WITH REAL NUMBERS
C     DO 30,XTEMP=START,FINISH,TICLBL*TICINT
      COUNT=1.
      XTEMP=START
25    CONTINUE
      CALL PLOT(XTEMP,YOFSET)
      CALL CPLT(-11.,0.)
      TYPE 9000,XTEMP,ETX
9000  FORMAT('+', 'IWRTO5, LB', F10.2, A1)
      XTEMP=START+(TICLBL*TICINT*COUNT)
      COUNT=COUNT+1.
      IF(XTEMP,LE.FINISH) GOTO 25
30    CONTINUE
      CALL CHDIR(0)
100   CONTINUE
      CALL DONE
      RETURN
      END

C
C*****
      SUBROUTINE YAXIS(XOFSET,TICINT,START,FINISH,TICLBL)
C*****
C     THE PASSED PARAMETERS ARE OF TYPE REAL.  THE VALUES ARE
C     IN USER-UNITS DETERMINED BY THE LAST CALL TO SCALE.
C     XOFSET IS THE POINT YOU WANT THE Y-AXIS TO CROSS THE
C     X-AXIS.  TICINT IS THE TIC-MARK INTERVAL PARAMETER AND
C     DETERMINES WHETHER OR NOT TIC MARKS ARE DRAWN ALONG THE
C     AXIS.  IF TIC MARKS ARE TO BE DRAWN, THE PARAMETER VALUE
C     SPECIFIES THE SPACING, IN SCALE CALL UNITS, BETWEEN TICS.
C     A VALUE OF 0 RESULTS IN NO TIC MARKS.
C     THE START AND FINISH PARAMETERS SPECIFY THE LOCATION
C     OF THE ENDPPOINTS OF THE AXIS.  THE AXIS IS ALWAYS
C     DRAWN FROM THE START TO THE FINISH POINT.
C     THE TICLBL(NUMBER-OF-TICS/LABEL) PARAMETER DETERMINES

```

```

C      WHETHER OR NOT THE TIC MARKS ON THE AXIS WILL BE LABELED.
C      SPECIFYING A 0 RESULTS IN NO LABELS.  IF LABELS ARE DESIRED,
C      THE PARAMETER SPECIFIES THE NUMBER OF TIC MARKS BETWEEN
C      LABELS.  ALL LABELS ARE LETTERED ACCORDING TO THE CURRENT
C      CHARACTER SIZE CALL AND THE NUMBER FORMAT IS F10.2.
C
      REAL XOFSET,TICINT,START,FINISH,TICLBL,YTEMP
      REAL COUNT
      LOGICAL*1 ETX
C
      DATA ETX/3/
C
      IF(TICINT.NE.0.) GOTO 10
      CALL PLOT(XOFSET,START)
      CALL PEN(1)
      CALL PLOT(XOFSET,FINISH)
      CALL PEN(0)
      GOTO 100
10     CONTINUE
C     REAL NUMBER DO LOOP
C     DO 20,YTEMP=START,FINISH,TICINT
      COUNT=1.
      YTEMP=START
15     CONTINUE
      CALL PLOT(XOFSET,YTEMP)
      IF(YTEMP.EQ.START) CALL PEN(1)
      CALL YTIC
      YTEMP=START+TICINT*COUNT
      COUNT=COUNT+1
      IF(YTEMP.LE.FINISH) GOTO 15
20     CONTINUE
      CALL PEN(0)
      IF(TICLBL.EQ.0.) GOTO 100
C     REAL NUMBER DO LOOP
C     DO 30,I=START,FINISH,TICLBL*TICINT
      COUNT=1.
      YTEMP=START
25     CONTINUE
      CALL PLOT(YOFSET,YTEMP)
      CALL CFLT(-11.,0.)
      TYPE 9000,YTEMP,ETX
9000   FORMAT('+', 'IWR05, LB', F10.2, A1)
      YTEMP=START+(TICLBL*TICINT*COUNT)
      COUNT=COUNT+1.
      IF(YTEMP.LE.FINISH) GOTO 25
30     CONTINUE
100    CONTINUE
      CALL DONE
      RETURN
      END
C
C*****

```

```

SUBROUTINE CHDIR(ANGLE)
C*****
C    THE PASSED PARAMETERS ARE OF TYPE INTEGER.
C    ANGLE PARAMETER SPECIFIES THE ANGLE AT WHICH
C    CHARACTERS WILL BE LABELED USING THE LABEL CALL.
C    0 LABELS HORIZONTALLY AND 90 LABELS VETRICALLY AND
C    180 LABELS UPSIDE DOWN. THE ANGLE REMAINS IN EFFECT
C    UNTIL CHANGED BY ANOTHER CALL TO CHDIR.
C
C    INTEGER ANGLE
C    REAL RISE,RUN,RAD
C
C    RAD=FLOAT(ANGLE)/180.*3.14159
C    RISE=100.*SIN(RAD)
C    RUN=100.*COS(RAD)
C    TYPE 9005,RUN,RISE
9005    FORMAT('+', '!WRT05,DI',F8.3,',',F8.3)
C    CALL DONE
C    RETURN
C    END
C*****
SUBROUTINE USCHAR(TYPE)
C*****
C    THIS ROUTINE DRAWS SPECIAL DATA POINT CHARACTERS
C    AT THE LOCATION OF THE CURSOR. PASSED PARAMETERS
C    ARE OF TYPE REAL AND MAY BE 1.,2.,3.,OR 4.
C    TYPE IS DEFINED AS FOLLOWS:
C        1. = SQUARE
C        2. = TRIANGLE
C        3. = CIRCLE(OR OCTAGON)
C        4. = DIAMOND
C    THE SPECIAL CHARACTERS ARE SIZED RELATIVE TO THE CURRENT
C    PLOTTER CHARACTER SIZED IN USE
C
C
C
C
C    TYPE 9000
9000    FORMAT('+', '!WRT05,CP-.5,-.375')
C    IF(TYPE.EQ.1.)GO TO 10
C    IF(TYPE.EQ.2.)GO TO 20
C    IF(TYPE.EQ.3.)GO TO 30
C    IF(TYPE.EQ.4.)GO TO 40
C    INCORRECT TYPE SENT, RETURN WITHOUT
C    ACTION
C    GO TO 100
C    DRAW A SQUARE
10    TYPE 9001
9001    FORMAT('+', '!WRT05,UC +99,6,0,0,12,-6,0,0,-12')
C    GO TO 100
C    DRAW A TRIANGLE
20    TYPE 9002
9002    FORMAT('+', '!WRT05,UC -99,0,3,+99,6,0,-3,12,-3,-12')

```

```

      GO TO 100
C      DRAW A CIRCLE
30     TYPE 9003
9003   FORMAT('+', '!WRT05,UC -99,2,0,+99,2,0,2,4,0,4,-2,4,
            1-2,0,-2,-4,0,-4,2,-4')
      GO TO 100
C      DRAW A DIAMOND
40     TYPE 9004
9004   FORMAT('+', '!WRT05,UC -99,3,0,99,3,6,-3,6,-3,-6,3,-6')
100    CONTINUE
      TYPE 9005
9005   FORMAT('+', '!WRT05,CP-.5,.375')
      CALL DONE
      RETURN
      END

C
C*****
      SUBROUTINE DONE
C*****
C      THIS ROUTINE IS A HANDSHAKE SIMULATOR FOR THE PLOTTER.
C      THE PLOTTER RUNS AT VARYING SPEEDS DEPENDING ON THE
C      OPERATION BEING PERFORMED AND THE RS-232 TO IEEE-488
C      CONVERTER ONLY HAS A 255 BYTE BUFFER SO SOME WAY OF
C      KNOWING WHEN THE PLOTTER IS FINISHED AN OPERATION IS
C      NEEDED. THIS IS DONE BY REQUESTING THE PLOTTER TO SEND
C      OVER IT'S PRESENT ERROR CODE. WHEN THE ERROR CODE IS
C      RECEIVED THEN WE KNOW THE PLOTTER IS READY TO CONTINUE.
C      THIS METHOD IS INEFFICIENT BUT IS THE ONLY ONE PRESENTLY
C      AVAILABLE.
C
      TYPE 9996
9996   FORMAT('+', '!FMT1,9B,1T')
      TYPE 9997
9997   FORMAT('+', '!WRT05,OE')
      TYPE 9998
9998   FORMAT('+', '!RED05,1')
      ACCEPT 9999,I
9999   FORMAT(I)
      RETURN
      END

```



```

C*****
C*
C*   READIT.FOR This is a general purpose program for reading *
C*   raw experimental data into memory from disk files.      *
C*
C*   Data acquisition in realtime is implemented using the    *
C*   DEC RT-11 operating systems and the Data Translation, Inc. *
C*   data acquisition and control software library, DTLIB.    *
C*
C*   William R. Hamel November, 1980                          *
C*
C*****
C
C   SUBROUTINE READIT(PRESET,NREAD,NCCH,VOLT,TURB)
C   DIMENSION VOLT(1000),TURB(1000)
C   LOGICAL*1 FILE(15)
C
C       Get file name from terminal
C
C   TYPE 5
5   FORMAT(' TYPE IN FILE SPECIFICATION...DY1:NAME.DAT')
C   ACCEPT 10,FILE
10  FORMAT(14A1)
C   FILE(15)=0
C
C       Open disk unformatted sequential file FILE.DAT
C
C   OPEN(UNIT=8,NAME=FILE,ACCESS='SEQUENTIAL',TYPE='OLD',
C   1FORM='UNFORMATTED')
C
C       Read parametric data
C
C   READ(8)PRESET,NREAD,NCCH
C
C       Now, read deflection and turbine meter data
C
C   READ(8)(VOLT(J),TURB(J),J=1,NREAD/NCCH)
C
C       Close disk file
C
C   CLOSE(UNIT=8)
C
C       Send data read message
C
C   TYPE 100
100 FORMAT(' DATA READ INTO ARRAY STORAGE')
C   RETURN
C   END

```

VITA

William Ross Hamel was born in Cleveland, Ohio, on June 11, 1945. His parents returned to their home in the Kanawha Valley of West Virginia when he was six months old where he was raised in Appalachian splendor in the small town named "St. Albans." Being the son of a well-known machine designer, he grew up in an environment of model airplanes, quarter midgets, go-karts, etc., and was essentially destined to an engineering career from the beginning — in fact it was never questioned.

Upon surviving St. Albans High School during the "real" rock-and-roll years, he entered West Virginia University where he received a B.S. degree in mechanical engineering. Realizing how little he really knew and feeling some compulsion to follow Horace Greeley's call to "go west" he decided to pursue graduate work at Oklahoma State University. He was able to convince his college sweetheart, the good looking Elizabeth (Libby) Ann Caldwell, to marry him and to join him in his journey west.

After barely avoiding the intense military draft caused by the Vietnam War and with a great deal of help from Libby, he completed his M.S. (specializing in automatic controls) in 1969 and went to work for TRW Systems. At TRW in Houston, he performed preflight and postflight analysis of the Apollo Lunar Module digital autopilot software performance. He was soon (2 years) to realize that the aerospace was not the solid career opportunity that he wanted so he returned to West Virginia to work for Union Carbide Corporation in the area of process control and simulation. This job also had its frustrations and disappointments,

and through an unimportant sequence of trauma and change encompassing another two years, he ended up at the Oak Ridge National Laboratory. In east Tennessee, life took on a new level of quality and enjoyment. At ORNL, he learned the true meaning of engineering where a desirable mixture of hardware, software, and paperwork is readily available. He has been working in the Instrumentation and Controls Division for 9 years and has had a variety of instrumentation engineering assignments in fossil energy-related and nuclear energy-related research. He has most recently been working in the areas of remote systems technology and robotics.

He and Libby have now assumed a normal middle-aged societal role and are extremely happy. They have a home in suburbia, a four-door sedan, a station wagon, two children, and a dog. Luke is seven and Alison is four and they both provide special blessings every day.

Bill holds memberships in the American Society of Mechanical Engineers, the Institute of Electrical and Electronic Engineers, the IEEE Control System Society, Tau Beta Pi, Phi Kappa Phi, and Sigma Xi.