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Edwin G. Burdette

Dr. Edwin G. Burdette, Major Professor

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DESIGN OF REINFORCED CONCRETE COLUMNS SUBJECTED TO BIAXIAL BENDING

A Thesis

Presented for the

Master of Science Degree

The University of Tennessee, Knoxville

Amjad Jamal Khayat

August, 1997

DEDICATION

To my father Dr. Jamal Abdulqader Khayat and my mother Afaf Jawdat Shwakeh. Thank you for bringing me up to be where I am now. There is no way I would know about all the sacrifices that you made for me, but I know it is more than what I can imagine.

ACKNOWLEDGMENT

There are not enough ways to thank Dr. Edwin G. Burdette for all that he did for me. He is the best teacher; he is a professor with a big heart. I learned from his knowledge and he helped me achieve my goals. His extreme generosity is a big asset to The University of Tennessee and the Civil Engineering Department. I'm proud to have his name as my advisor on my committee . Thank you for making this education possible for me.

Dr. J. Harold Deatherage and Dr. David W. Goodpasture: thank you for your sincere kindness, for the education you gave me and for being on my committee. Dr. Richard M. Bennett: thank you for your sincere kindness and the good education that I received in your classes.

My wife Abeer made it come true; it was never possible for me to achieve this without her encouragement, support and extreme patience. My children Afaf and Jamal did not have enough bedtime stories red to them. Thank you for your patience, I hope I will be able to make up the passed times, and I hope our family will have a better future.

Ann L. Lacava: a very special thank you for helping me within my tight schedule.

ABSTRACT

Biaxial bending in reinforced concrete columns in very complex. The Bresler Reciprocal Method (1) is a simplified approximate approach to analyze reinforced concrete columns in biaxial bending. In this thesis a spreadsheet is used to analyze rectangular reinforced concrete columns for biaxial bending and to compare its exact results to the approximate approach using the Bresler reciprocal method.

The results of comparisons confirmed what the ACI Building Code (3) indicated. The Bresler Reciprocal Method is a satisfactory and conservative method to use for biaxial bending and axial load, with a note that the method is "most suitable" when values of P_x and P_y are greater than the balanced axial load for that axis.

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Chapter 1

INTRODUCTION

Columns are, typically, the vertical members in a structure which transmit the loads from other structural members to the footings. They mainly carry axial compression forces combined with bending moments, the relative magnitudes depending on the location of the columns and the beams that they support.

When columns are relatively short, the strength typically depends only on the cross-sectional properties, while for longer columns length effects must be considered. Long columns tend to deflect more than shorter ones, inducing additional or secondary moments due to deflection. This secondary moment effectively reduces the strength of long columns, making it necessary to consider length in the design.

The analysis of columns subjected to an axial compression load is simple. The analysis becomes more involved when moments are present with the axial load. The analysis of rectangular columns subjected to an axial compression load in combination with bending moments about both x and y axis is particularly complicated and time consuming. To simplify column design, charts and empirical approaches are typically used to investigate biaxially loaded columns.

Objective

The objective of this thesis is to write a spreadsheet computer program that analyzes a rectangular reinforced concrete column for biaxial bending and to compare its exact results to the approximate approach using the Bresler reciprocal method.

Scope

As just noted this thesis's objective is to compare the exact results of the output of the spreadsheet to the Bresler reciprocal method (1). The development of the Bresler reciprocal method is discussed and the steps in its evolution are shown. The approach in the spreadsheet is explained, and an example solved by the spreadsheet is presented. The accuracy of the spreadsheet program is documented by comparing an example output to an example worked out previously (2). Four examples are solved by both the Bresler reciprocal method and the spreadsheet program. The results are compared and discussed.

Basic Principles

To simplify the analytical procedure the ACI code (3) allows the use of several assumptions and idealizations. The strain value at failure of the concrete at the extreme compression fiber is set equal to 0.003. The distance between the extreme compression fiber and the neutral axis of the cross section is called "c". The distribution of the compressive stress in the concrete may be taken to be a rectangle with a mean stress of 0.85 f_c' and a depth "a" equal to $\beta_1 c$, where β_1 is equal to 0.85 for f_c' equal to or less than 4000 psi and reduced by 0.05 for each 1000 psi of additional strength with a minimum value of 0.65. The steel stress is equal to the yield stress when the steel strain exceeds the yield strain.

The location of the resultant of the forces acting on a column P_n may be concentric or eccentric. When the load is concentric, no moment is associated with it. But when the load is applied at a distance e_x along the x axis, it applies a moment M_x on the column in addition to the axial load. On the other hand if the load is applied at a distance e_y along the y axis, it applies a moment M_y on the column in addition to the axial load. In some cases the location of the resultant of the forces acting on a column may be applied at a distance e_x and e_y from the center of gravity of the column at the same time, in which case biaxial bending moments are applied on the column in addition to the axial load.

Approximate Approach

Columns fail in either compression or in tension, depending on the strain of the concrete and steel at failure. When the compressive strain of concrete reaches 0.003 before the steel yields, columns fail in compression. When the tensile strain in the steel exceeds the yield strain, the primary failure of the columns is a steel tensile failure. For a given cross section there is an infinite number of failure combinations at which $\mathbf{P_n}$, M_{nx}, and M_{ny} act concurrently. These failure points form a surface which is called a failure envelope.

The failure envelope for a given column is defined by three variables: P_n, M_{nx}, and M_{ny}. M_{nx} and M_{ny} may also be expressed in terms of eccentricities: $e_x=M_{nx}/\mathbf{P_n}$ and $e_y=M_{ny}/\mathbf{P_n}$. Bresler (1) used a reciprocal failure surface (figure 1.1) that was expressed by $1/P_n$, e_x, and e_y. To locate a point on the surface of the failure envelope that he established, he passed

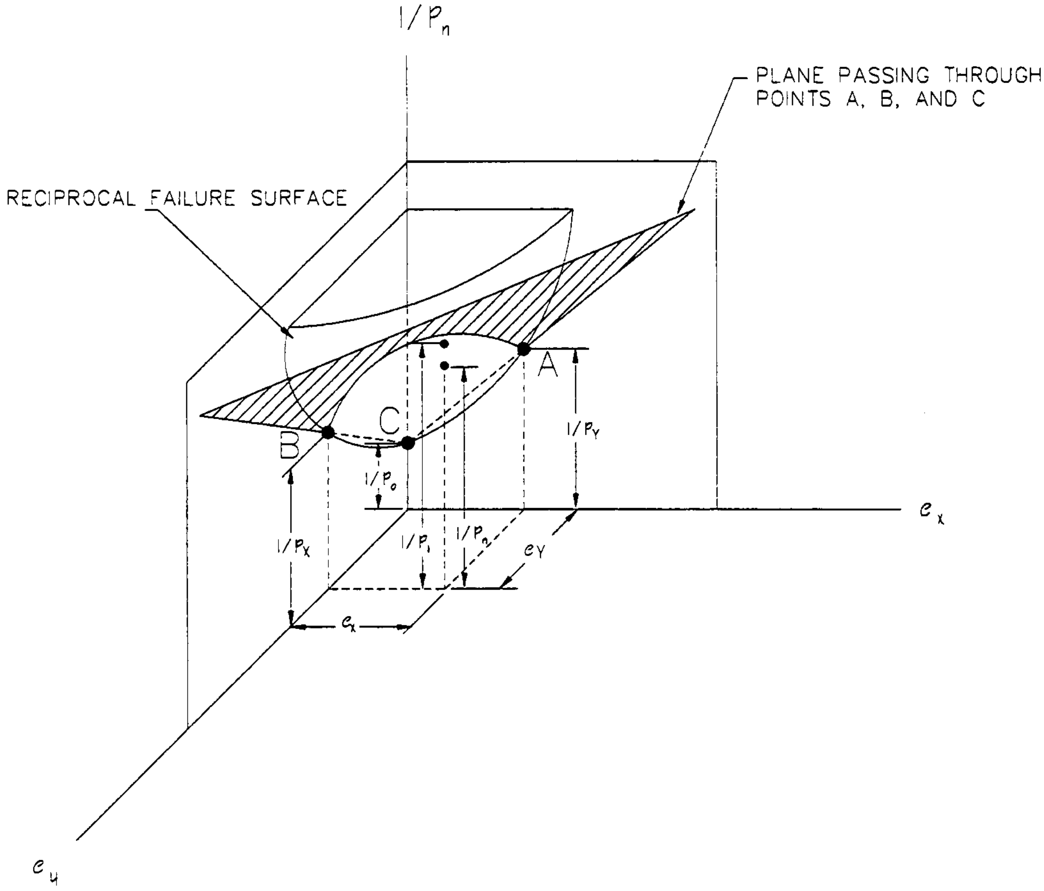

FIGURE 1.1 Assumed Failure surface in Bresler Reciprocal Method

a tangent plane through that point. The failure surface envelope and the plane intersect at only one point.

That point can be defined in terms of ($1/P_n$, e_x , e_y). Each point on the failure surface envelope can be intersected by a different plane and the entire failure surface is defined by an infinite number of planes.

The tangent plane that intersects the point in question can be defined by three points, points A, B, and C. Their coordinates are

point A (e_x, 0, $1/P_y$)

point B (0, e_y, $1/P_x$)

point C (0, 0, $1/P_0$).

P_y is the nominal strength at the uniaxial eccentricity e_x, P_x is the nominal strength at the uniaxial eccentricity e_y, and P_0 is the nominal strength under axial compression alone.

The equation of a plane is

$$A_1 x + A_2 y + A_3 z + A_4 = 0 \quad \text{.......(1)}$$

Substituting the coordinates of the three points into the plane equation gives the following:

$$A_1 e_x + 0 + A_3 1/P_y + A_4 = 0 \quad \text{.......(2)}$$

$$0 + A_2\, e_y + A_3\, 1/P_x + A_4 = 0 \qquad \text{.......(3)}$$

$$0 + 0 + A_3\, 1/P_0 + A_4 = 0 \qquad \text{.......(4)}$$

Solving the above equations in turns of A_4 leads to these expressions:

$$A_1 = (1 / e_x)\,(P_0 / P_y - 1)\, A_4 \qquad \text{.......(5)}$$

$$A_2 = (1 / e_y)\,(P_0 / P_x - 1)\, A_4 \qquad \text{.......(6)}$$

$$A_3 = - P_0\, A_4 \qquad \text{.......(7)}$$

Substituting these equations into the equation of a plane gives

$$A_4 \left[x / e_x\,(P_0 / P_y - 1) + y / e_y\,(P_0 / P_x - 1) - P_0\, z + 1 \right] = 0 \qquad \text{.......(8)}$$

And dividing the above equation by P_0 gives

$$x / e_x\,(1/P_y - 1/P_0) + y / e_y\,(1/P_x - 1/P_0) - z + 1/P_0 = 0 \qquad \text{.......(9)}$$

At the point in question $x = e_x$, $y = e_y$ and $z = 1/P_i$, where P_i is a point on the tangent surface approximating point P_n which is on the failure surface.

$$(1/P_y - 1/P_0) + (1/P_x - 1/P_0) - 1/P_i + 1/P_0 = 0 \qquad \text{.......(10)}$$

Simplifying this equation gives the final expression for the value of P_i:

$$1/P_i = 1/P_x + 1/P_y - 1/P_o \qquad \text{.......(11)}$$

Equation 11 is the fundamental equation of the Bresler reciprocal method referred to in the Commentary of ACI Building Code (3).

Chapter 2

REVIEW OF LITERATURE

Several methods have been used to approximate and to simplify the complexity of the analysis and design of columns subjected to biaxial bending. Some of these approaches are briefly discussed in this chapter.

Superposition

To simplify the complicated process of biaxial design a method of superposition has been used. This method treats each biaxial moment as two separate cases. One uniaxial bending moment, P_u - M_{ux} in one case and P_u - M_{uy} in the second case. Then the reinforcement required for each case separately is found and the two added together to obtain the total reinforcement. This method has no theoretical basis and may lead to errors on the unsafe side because it counts on the concrete strength twice.

Interaction Diagram Approximation

A better method of using a unaxial bending approach is used to draw a simple biaxial interaction diagram. Instead of drawing a curved line, this approach suggests to draw a straight line between M_{ux} and M_{uy} for a given load P_u. This approach is relatively simple and always conservative since the straight line always lies inside the curve (figure2.1).

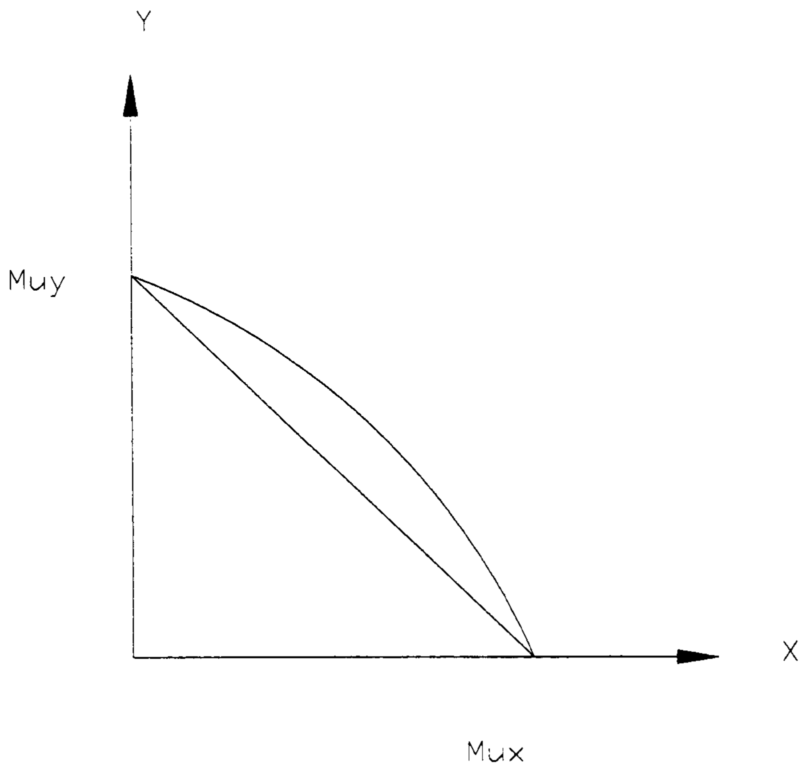

FIGURE 2.1 Simplified straight biaxial interaction diagram

Design Charts

Different design charts have been used for analysis and design of columns loaded in biaxial bending. Weber (4) used conditions of equilibrium and compatibility of strains to draw interaction curves of P_u versus P_u e for square columns with load applied at various eccentricities along various diagonal lines of the section. A rectangular stress block derived for rectangular compressed areas was used to approximate the stress block characteristics of triangular or semi triangular shapes. The charts were developed for square columns with f_c' less than or equal to 4,000 psi and f_y equal to 60,000 psi, and they include the reduction factor ϕ = 0.7 . Even though these charts were developed for square columns, later calculations indicated that they may be used for rectangular columns too.

To use Weber (4) charts, the resultant moment eccentricity e must be calculated with the angle θ between the y axis and the direction of eccentricity e (figure 2.2).

$e_x = P_u / M_{ux}$

$e_y = P_u / M_{uy}$

$e = \sqrt{(e_x^2 + e_y^2)}$

$\theta = \tan (e_x / e_y)$ with $e_x < e_y$

From the charts find the steel required for $P_u / (f_c' h^2)$ and $P_u e / (f_c' h^3)$ acting uniaxially and for $P_u / (f_c' h^2)$ and $P_u e / (f_c' h^3)$ acting diagonally at 45^0. Then by linear interpolation between these two values the designer can find the required steel for the acting angle θ.

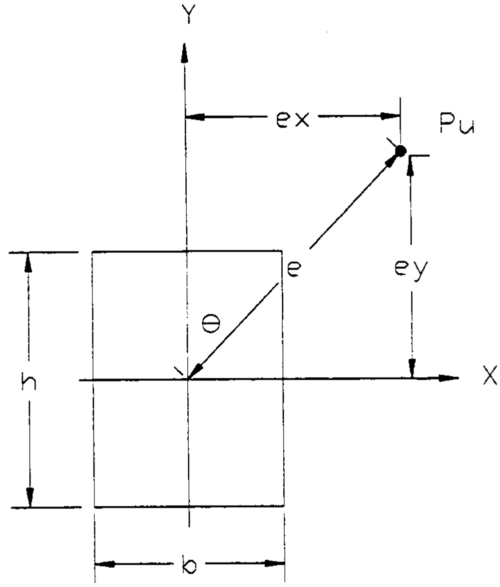

FIGURE 2.2 Load P_u with eccentricity e acting at angle θ with x - axis

The accuracy of this method was checked by Weber (4) by comparing the exact theoretical solution against solution from charts. Maximum error was 5.3% between the two methods.

Row and Pauley (5) improved the accuracy of the linear interpolation by developing charts in which the direction of the eccentricity is inclined at various angles to the major axes, allowing several interpolation points on the constant load interaction line. But still some doubt may be left about the accuracy of the equivalent rectangular stress block that was used by Weber (4) when used with columns with non rectangular compressed areas.

They expressed the eccentricity of the load by a dimensionless parameter K related to the cross section dimensions and the eccentricity of the load (figure 2.2).

$$K = e_x h / e_y b$$

Charts are plotted for different values of K enabling the shape of the interaction surface to be obtained accurately. Row and Pauley also developed their charts for f_c' less than or equal to 4,000 psi and f_y equal to 60,000 psi, but they did not include the reduction factor. The charts plotted the dimensionless value $P_u / (f_c' \, b \, h)$ and $M_{ux} \sqrt{(1 + K^2)} / (f_c' \, bh^2)$, where $M_{ux}=P_u e_x$.

To obtain more accurate results, the charts used the conditions of equilibrium and compatibility of strains and assumed a parabolic stress strain curve for the compressed concrete up to a stress of 0.85 f_c' at a strain of 0.003 at the extreme compression fiber.

To use Row and Pauley (5) charts, first find K:

$K = e_x h / e_y b$

then,

$P_u / (f_c' \, b \, h)$ and $M_{ux} \sqrt{(1 + K^2)} / (f_c' \, bh^2)$

Next determine the angle, $\theta = \tan^{-1} K$, and the steel value for the calculated K value by interpolation.

Row and Pauley charts give more conservative results than Weber because of the concrete compressive stress assumed and the method of interpolation they adopted.

Shape of Interaction Surface

Bresler's Reciprocal Method (1) used the surface of the failure envelope of a cross section to derive an equation to estimate the load of a column P_i under biaxial bending. The failure envelope for a given column is defined by three variables: P_n, M_{nx}, and M_{ny}. M_{nx}, and M_{ny} may also be expressed in terms of eccentricities: $e_x = M_{nx} / \mathbf{P_n}$ and $e_y = M_{ny} / \mathbf{P_n}$. The reciprocal method equation is $1/ P_i = 1/P_x + 1/P_y - 1/ P_0$, in which P_y is the nominal strength at the uniaxial eccentricity e_x, P_x is the nominal strength at the uniaxial eccentricity e_y, and P_0 is the nominal strength under axial compression alone.

Bresler found the ultimate load predicted by his method to be in reasonable agreement with the ultimate load with both theoretical and test results. This method has the disadvantage of being more suitable for analysis than for design. The ACI Building Code (3) indicated that this method is a satisfactory and conservative method to use for biaxial bending and axial load, with a note that the method is "most suitable" when values of P_x and P_y are greater than the balanced axial load for that axis.

Chapter 3

DESCRIPTION OF PROGRAM

The Program

This program plots ϕM_{nx} - ϕM_{ny} for a specified concentrated load ϕP_n for a given rectangular short column.

The column's cross section dimensions and reinforcement are easy to input in the program and to edit. After the required data are entered and the program run, the ϕM_{nx} - ϕM_{ny} plot is displayed on the screen, and the plot can then be printed.

Input

The data that are required to be input into the program are the depth and width of the column, the size and the number of reinforcing steel bars in each face of the column, concrete cover dimensions, the concrete compressive strength, and the steel yield strength. The axial load on the column must also be specified. These are the only inputs needed for the program to run. The program requires a minimum of two bars in each face for a total of four bars in the column with a maximum of ten bars in each face for a total of thirty six bars in the column.

Analysis

The program finds the shape of the concrete compression area and the value of c, the perpendicular distance from the neutral axis to the extreme compression fiber.

There are five possible shapes or cases for the concrete compression area, Case 1 to Case 5 (figure 3.1). For each possible shape, the program tries to find the c value that would result in balanced forces, the sum of the forces of the tension steel and the forces of compression steel and concrete and ϕP_n (figure 3.2) must add to zero.

The area of concrete in compression for any one case has a defined shape, as illustrated in figure 3.1. The length of each side of that shape is related to c and θ_{NA}. The program sets a value for θ_{NA} in order to plot a point on the ϕM_{nx} and ϕM_{ny} curve, and it iterates to find c. For each case and for each iteration it sums the forces in the reinforcing steel bars and the concrete based on the assumed concrete shape, then it picks the shape that gives an equilibrium condition, and the lengths of each side of the compression concrete area is as assumed for that case and the given value of θ_{NA} and the iterated c.

The angle of rotation of the neutral axis θ_{ACTUAL} for the specified column depends on the ratio of ϕM_{nx} and ϕM_{ny}. The program calculates 19 points to plot, from $\theta_{NA}=0^0$ to $\theta_{NA}=90^0$ at 5^0 intervals. For each point it finds the controlling case which determines the

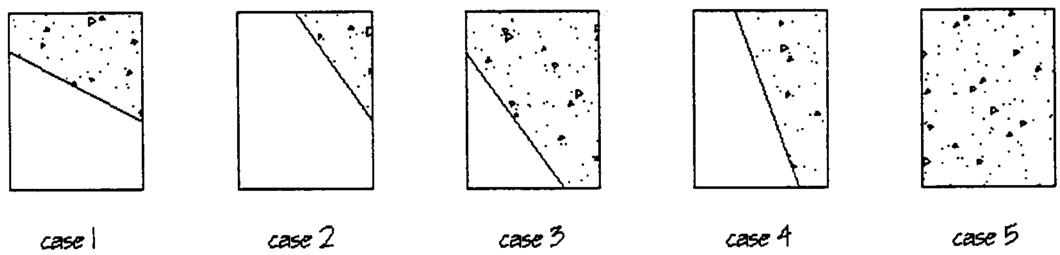

FIGURE 3.1 The five possible shapes for the concrete compression area

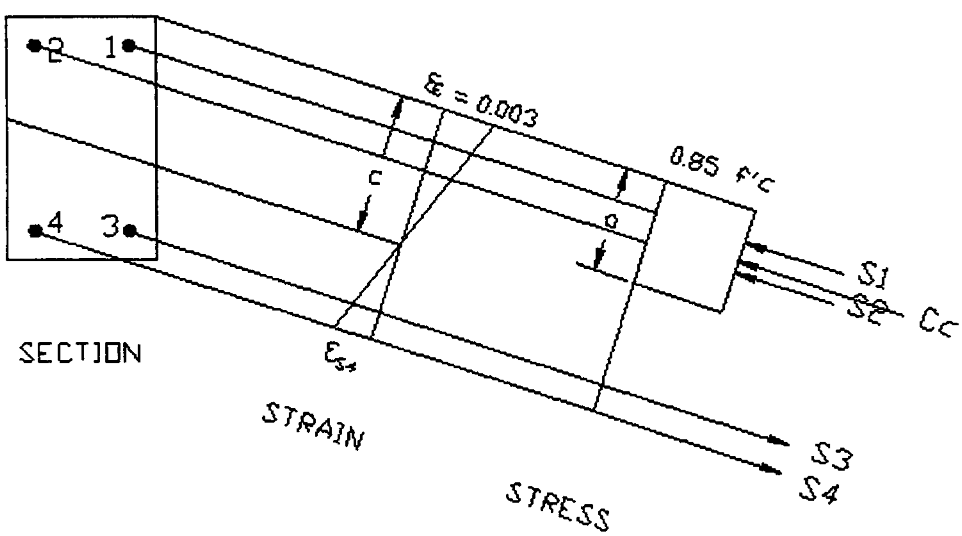

FIGURE 3.2 Stress strain diagram with biaxial bending moment

compression area shape, the c value, the tension and / or compression in the reinforcing bars, the concrete compression forces, and in turn the ϕM_{nx} and ϕM_{ny}.

Results

The output of the program consists of a table for each run showing θ_{NA} and the corresponding ϕM_{nx} - ϕM_{ny}, with the θ_{ACTUAL}, and a plot of the ϕM_{nx} - ϕM_{ny} values.

Assumptions

When the specified column has more than two bars in each face, the program divides the bars into four groups, one in each corner of the column section, of which points 1, 2, 3 and 4 are centroids of each group (figure 3.3).

To illustrate the spreadsheet's output and accuracy, two examples are shown: example 1 of a square column and example 2 of a rectangular column.

Example 1

Given:

- 12"X12" square column with 4#9 bars

- f_c'=4000 psi f_y=60 ksi

- ϕP_n=0

Required:

- Show strain diagram and stress block with equilibrium forces for $\theta_{NA}=10^0$ and 40^0

Solution:

- A summary from the Spreadsheet's output for $\theta_{NA}=10^0$ and 40^0 are shown in table 3.1. The strain diagram and stress block with equilibrium forces for both cases are shown in figure 3.4 and figure 3.5. The values are consistent with hand calculations performed previously (2).

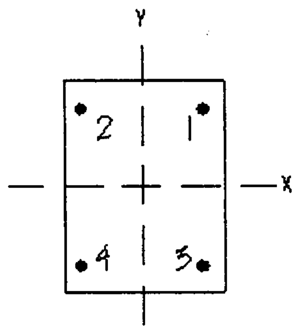

FIGURE 3.3 Assumed locations for reinforcing bars

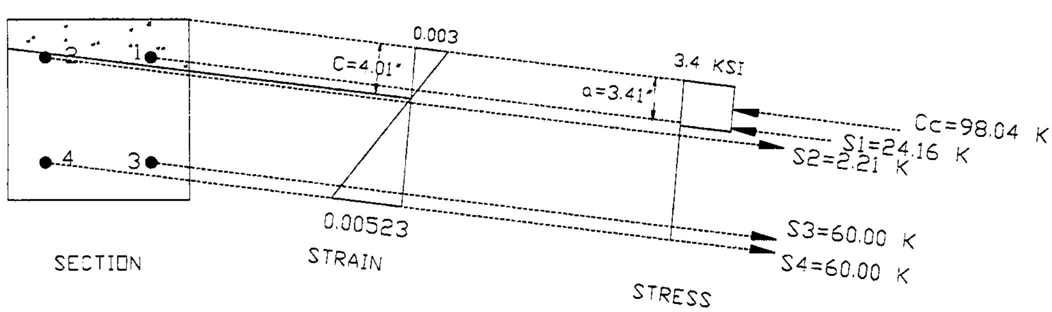

FIGURE 3.4 Stress strain diagram for $\theta_{NA}=10^{0}$

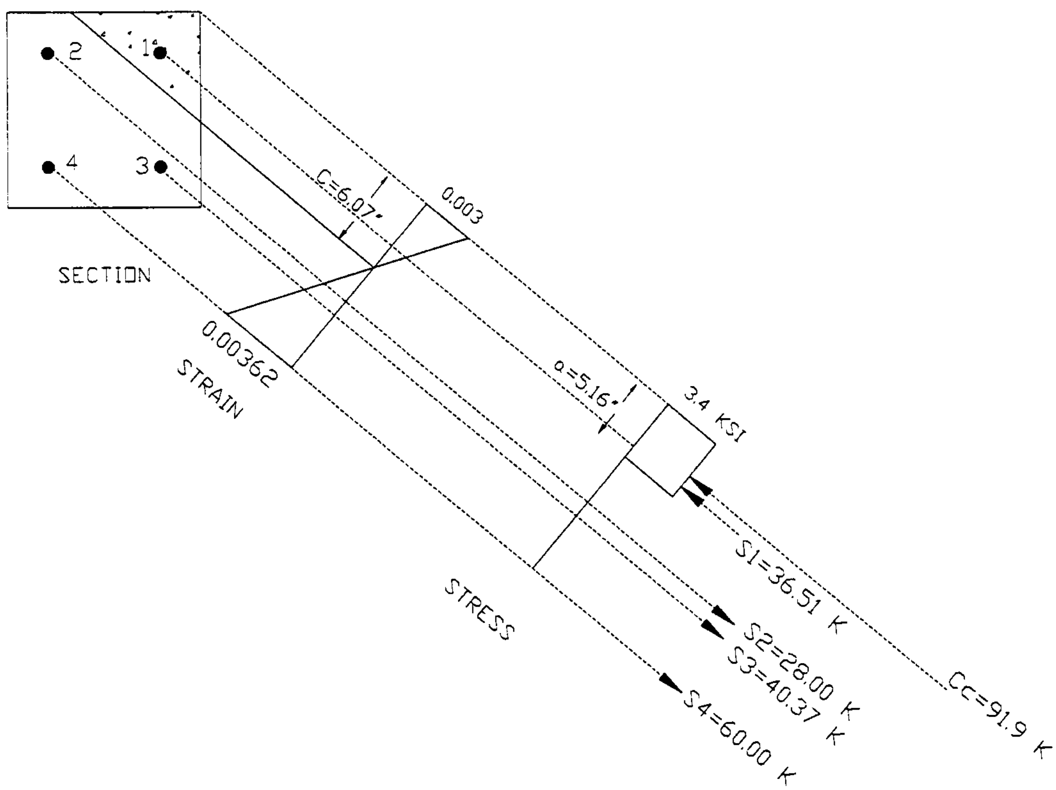

FIGURE 3.5 Stress strain diagram for $\theta_{NA}=40^0$

TABLE 3.1 Summary from the Spreadsheet output

EXAMPLE	θ_{NA}=10	θ_{NA}=40	UNITS	
CASE	1	2		
Ac	28.8	27.0	IN^2	
θ_{NA}	10.0	40.0	DEGREE	
θ_{ACTUAL}	10.5	39.6	DEGREE	
ϕM_{nx}	72.0	39.6	K FT	
ϕM_{ny}	13.4	45.0	K FT	
c	4.0	6.1	IN	
Cc	98.4	91.9	KIP	
ε_{S1}	0.00083	0.00126		
ε_{S2}	0.00008	0.00097		
ε_{S3}	0.00432	0.00139		
ε_{S4}	0.00523	0.00362		
S1	24.2	36.5	KIP	*POSITIVE FOR COMPRESSION*
S2	-2.1	-28.0	KIP	*NEGATIVE FOR TENSION*
S3	-60.0	-40.4	KIP	
S4	-60.0	-60.0	KIP	

Example 2

Given:

- 18"X24" column with 6#14 bars
- f_c'=4000 psi f_y=60 ksi
- ϕP_n=100

Required:

1- Work this example both by hand and by the Spreadsheet

2- Show strain diagram and stress block with equilibrium forces for $\theta_{NA}=10^0$

Solution:

A complete hand solution of this problem is presented in Appendix A.

The strain diagram and stress block are shown with equilibrium forces in figure 3.6 from the hand calculations and in figure 3.7 from the Spreadsheet. A summary from the Spreadsheet's output and the hand calculation for $\theta_{NA}=10^0$ is shown in table 3.3. Note that the Spreadsheet groups the 6 rebars into four groups of 1.5 bar each (figure 3.3). For this reason the forces in the tension bars are:

hand calculation S_{s4} & S_{s5} & S_{s6} = 135 K

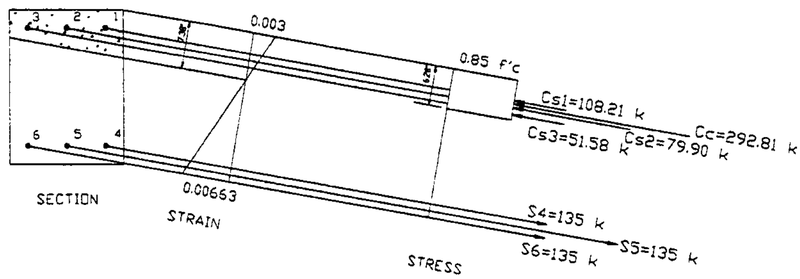

FIGURE 3.6 Stress strain diagram from exact hand calculations for example 2

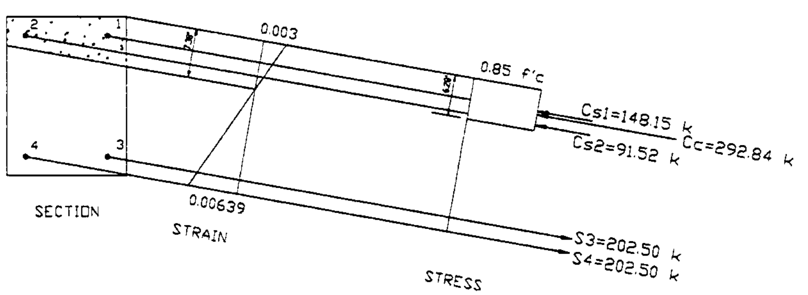

FIGURE 3.7 Stress strain diagram from Spreadsheet for example 2

Spreadsheet S_{s3} & S_{s4} = 1.5 x 135 = 202.5 K

Also

Spreadsheet S_{s1} = hand calculation $S_{s1} + S_{s2} / 2$ = 108.21 + 79.9/2 = 148.16 K

Spreadsheet S_{s2} = hand calculation $S_{s2} / 2 + S_{s3}$ = 79.9/2 + 51.58 = 91.53 K

Correction Factor

When the cross-section has more than 2 reinforcing bars in any one face, the Spreadsheet divides them into four groups. The location of each group depends on the number of bars in the cross-section. The Spreadsheet finds the coordinates of the centroid of each group. This assumption causes the distance for the moment arm from the X and the Y axis of the cross-section to the centroid of any one group to get smaller (figure 3.8), which leads to a smaller calculated moment capacity than the actual moment capacity. To account for this a correction factor is used to adjust the moment capacity. The correction factor is a ratio between the actual moment arm and the assumed moment arm. For example the correction factor for a case with three bars in the x face is (d / d') as shown in figure 3.8. The same thing would apply to the Y face if it had 3 bars. In this thesis a study has been done on groups with no more than three bars.

Accuracy of the Spreadsheet

The full result of example 1 is compared to the class example (2) in table 3.2. Both the spreadsheet output and the class example are the same.

TABLE 3.2 Summary from example 1

θ_{NA}	θ_{ACTUAL}	M_{nx}	M_{ny}
0	0.0	72.6	0.0
5	5.7	72.4	7.3
10	10.5	72.0	13.4
15	14.9	71.2	18.9
20	19.0	70.1	24.2
25	23.0	68.7	29.1
30	28.1	64.7	34.5
35	34.0	59.4	40.0
40	39.6	54.5	45.0
45	45.0	49.7	49.7

TABLE 3.3 Summary of example 2

	Spreadsheet	**Hand Clacs.**	**Units**	
θ_{NA}	10^{o}	10^{o}	DEGREE	
c	7.38	7.38	IN	
Cc	292.84	292.81	KIP	
S1	148.15*	108.21*	KIP	
S2	91.52*	79.90*	KIP	
S3	-202.50*	51.58*	KIP	
S4	-202.50*	-135.0*	KIP	
S5	-	-135.0*	KIP	*POSITIVE FOR COMPRESSION*
S6	-	-135.0*	KIP	*NEGATIVE FOR TENSION*
ϕM_{nx}	567.71	567.72	K FT	
ϕM_{ny}	41.81	41.78	K FT	

* See comments page 26 and figures 3.6 & 3.7 for S_1, S_2, S_3, S_4, S_5, S_6 locations.

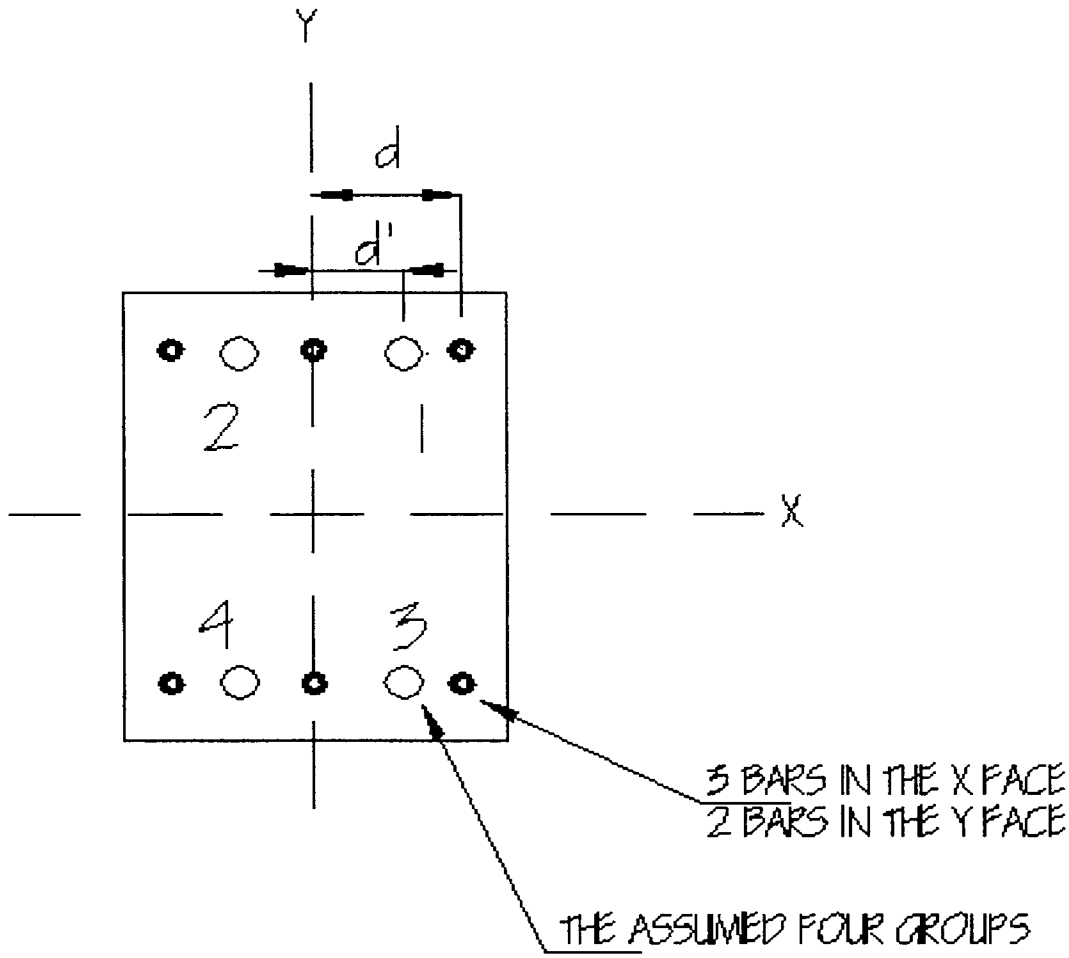

FIGURE 3.8 Locations of the assumed four groups

Chapter 4

ANALYSIS BY RECIPROCAL METHOD

In order to document the accuracy of the Bresler reciprocal method (1), four cross-sections as shown in figure 4.1, were selected to compare with exact solutions obtained from the Spreadsheet. Three of the sections have the same reinforcement ratio ρ. Their cross sectional areas A_g have a ratio of 1: 2.25: 4. The fourth section is rectangular, 18"x24" as shown in example 2. All cross sections have the same concrete strength f_c' of 4,000 psi and steel yield strength f_y of 60,000 psi.

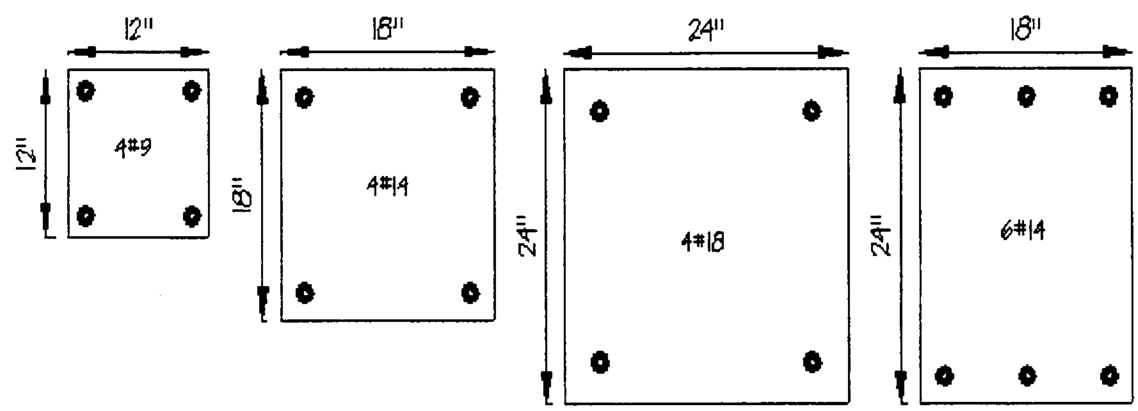

FIGURE 4.1 Four cross sections selected.

Procedure

For each cross section a P-M diagram is drawn. Ten different points on the diagram were chosen to be analyzed by both the Bresler reciprocal method and the spreadsheet. Points chosen for analysis on each P-M diagram were as low as $0.1f_c'A_g$ and up to the maximum design axial load compression strength allowed by the ACI code of $0.8\phi P_n$.

For the cross- sections that are symmetrical, ϕM_{nx} equals ϕM_{ny} for all points, and only one P-e and one P-M interaction diagram are needed for each one. From the ϕM_{nx} - ϕM_{ny} interaction diagram for a given value of ϕP_n, points on the curve are chosen and their corresponding values of ϕM_{nx} and ϕM_{ny} are found.

From the CRSI Design Handbook (6), values of ϕP_n and ϕM_{nx} for each cross section are used to plot P-e and P-M interaction diagrams. Eccentricities e_x and e_y are calculated as follows:

$e_x = \phi P_n / \phi M_{nx}$

$e_y = \phi P_n / \phi M_{ny}$

From the calculated values of e_x and e_y and the P-e interaction diagrams, P_{nx} and P_{ny} are determined. Then the Bresler reciprocal equation is used to find the corresponding P_n and then ϕP_n. The values of ϕP_n from both the Bresler reciprocal method and the spreadsheet can be compared.

To illustrate the procedure that was followed to compare both methods, the steps of the calculations of one point on the 12"X12" column P-M diagram is shown.

Example 3

Column's cross section description:

- 12"X12" square column with 4#9 bars
- f_c'=4000 psi $\quad\quad$ f_y=60 ksi
- ϕP_n=100 Kip

From the P-M diagram, figure 4.2, obtained with the Spreadsheet:

P_{no}=716 Kip

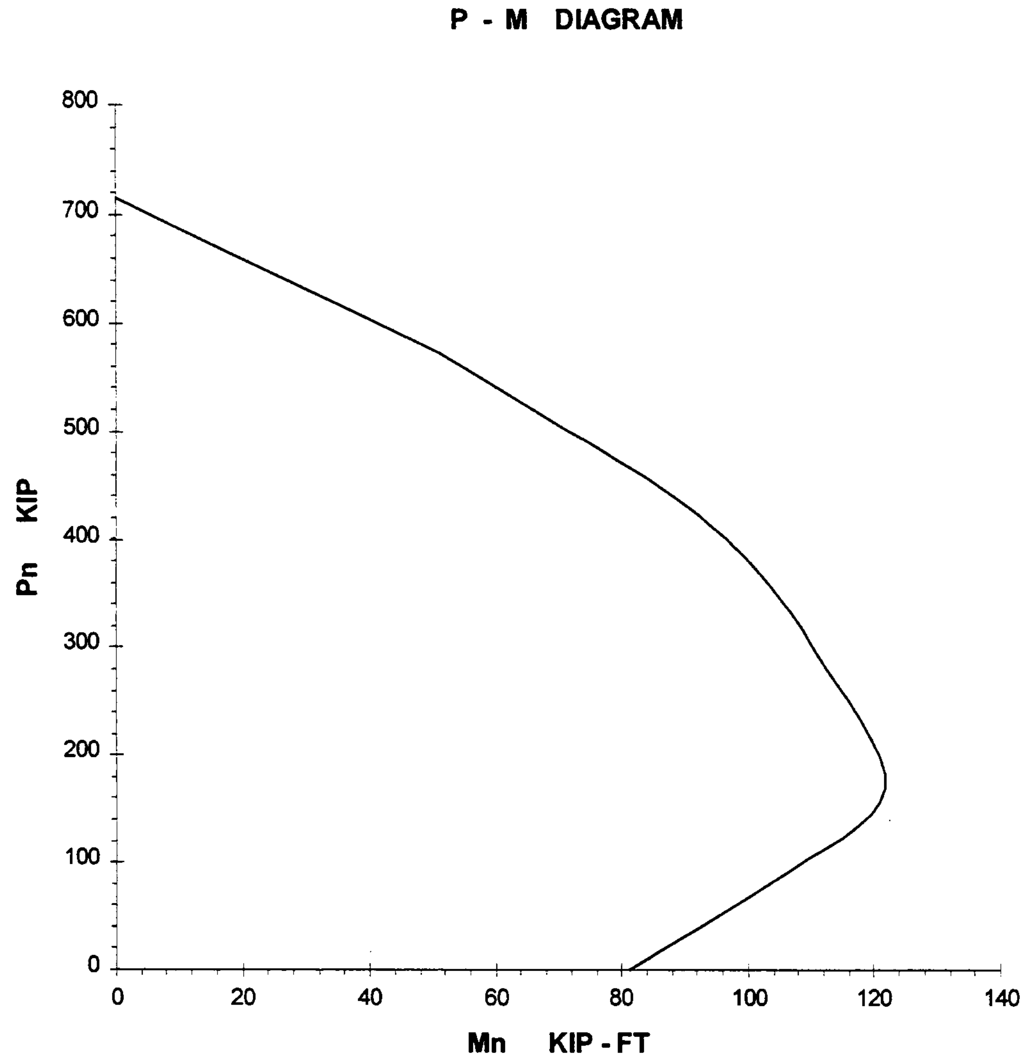

FIGURE 4.2 P - M diagram for 12"x12" column with 4#9

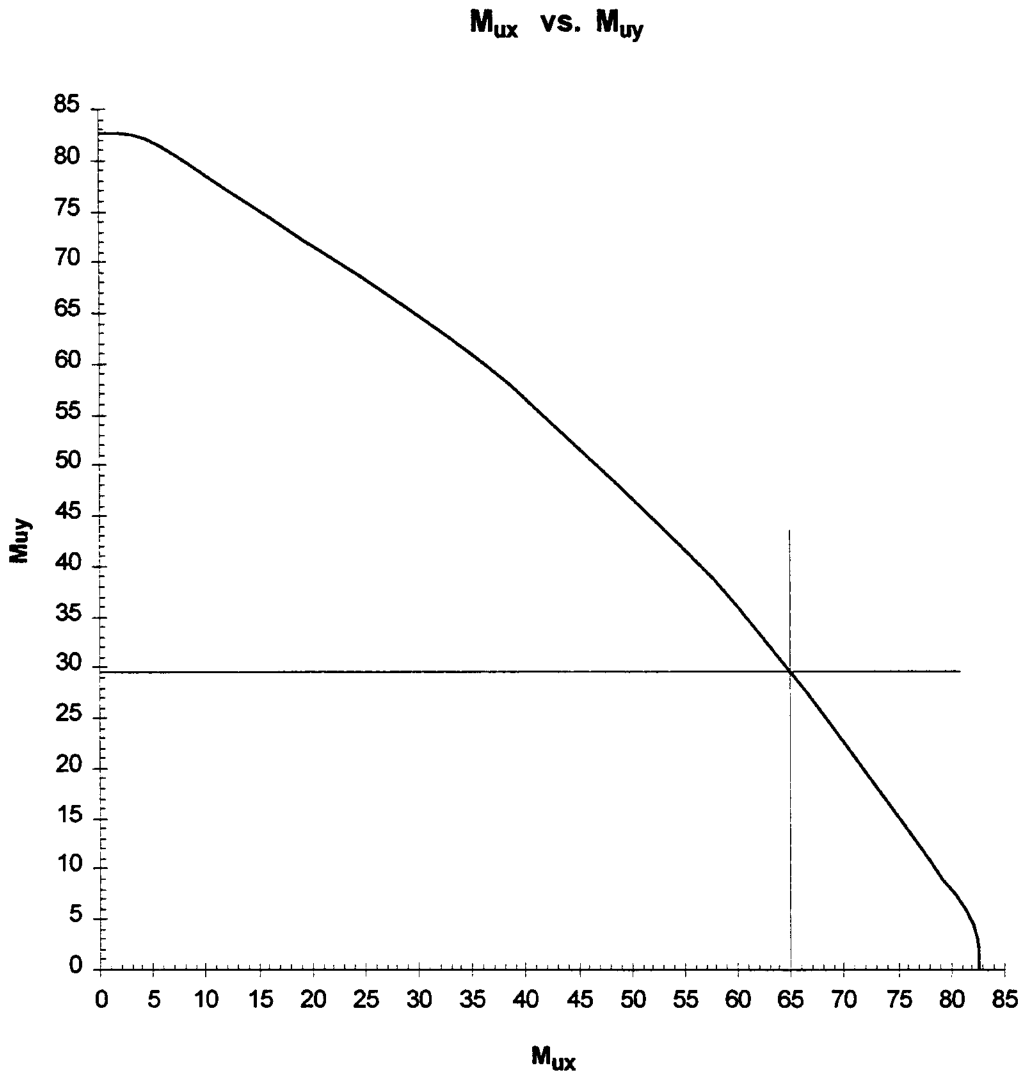

FIGURE 4.3 ϕM_{nx} - ϕM_{ny} diagram for 12"x12" column with 4#9

From the ϕM_{nx} - ϕM_{ny} interaction diagram figure 4.3 obtained with the Spreadsheet:

ϕM_{nx}=65 K FT ϕM_{ny}=29 K FT e_x=65 x 12 / 100

e_x =7.8"

e_y=29 x 12 / 100

e_y =3.5"

From the P-e diagram, figure 4.4 :

P_{nx}=190 Kip P_{ny}=355 Kip

The results from the Bresler reciprocal equation, using appropriate P - values from the Spreadsheet analysis, are as follows :

$1/P_{ni} = 1/P_{nx} + 1/P_{ny} - 1/P_{no}$

$1/P_{ni} = 1/190 + 1/355 - 1/716$

P_{ni} = 149.6 KIP

ϕP_{ni} = 0.7 x 149.6

ϕP_{ni} = 104.7 KIP

Compared to ϕP_n=100 Kip from the Spreadsheet analysis, the reciprocal method leads to

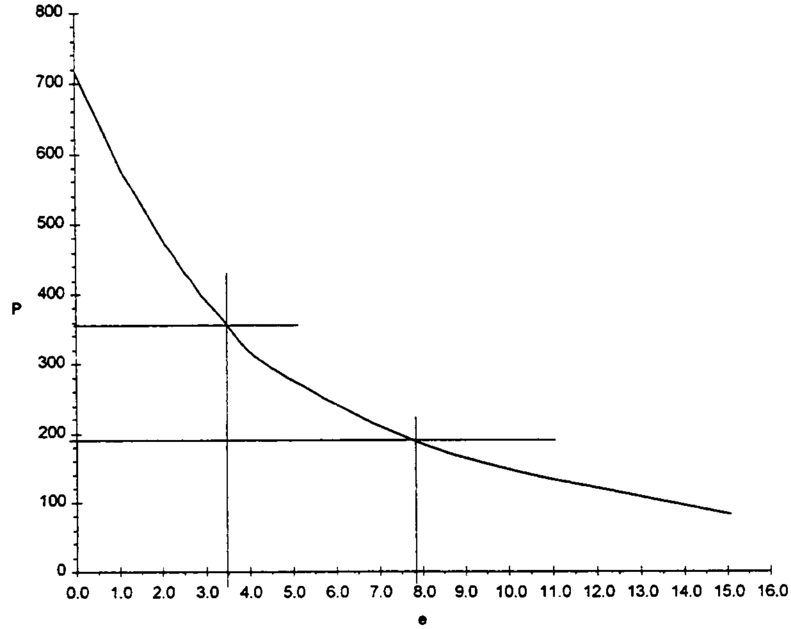

FIGURE 4.4 P - e diagram for 12"x12" column with 4#9

an error of +4.7% on the unconservative side.

To estimate ϕM_{nx} and ϕM_{ny} using Bresler's reciprocal method (1) :

ϕM_{nx} =104.7 x 7.8 / 12

ϕM_{nx} = 68.0 Kip - FT (65.0 Kip - FT from Spreadsheet)

ϕM_{ny} =104.7 x 3.5 / 12

ϕM_{ny} = 30.5 Kip - FT (29.0 Kip - FT from Spreadsheet)

Also ϕM_{nx} has an error of +4.6%, and ϕM_{ny} has an error of +5.2%, both on the unconservative side.

Results

Similar steps were followed for all points for the four cross sections. A summary of the final results is shown in tables 4.1 - 4.4. Figures 4.5 - 4.12 compare the exact ϕM_{nx} - ϕM_{ny} interaction diagrams from the Spreadsheet to the one from Bresler's reciprocal method (1). Ten loads where chosen for each cross-section with some loads below the balance point and the others above it.

As was discussed in example 3, ϕP_n=100 Kip is in the third column in figure 4.1. ϕP_n in the first row, ϕM_{nx}, and ϕM_{ny} are from the Spreadsheet. P_{nx} and P_{ny} are from the CRSI Design Handbook (6) using P-e diagram. . ϕP_n in the sixth row is from the Bresler reciprocal equation. ϕP_n in the first row from the Spreadsheet is compared to ϕP_n in the sixth row from the Bresler reciprocal equation.

Table 4.1 Summary for 12x12 column.

COLUMN SIZE 12X12 REINFORCEMENT : 4#9 $\rho = 2.78$ %

$P_{BALANCE}$ = 168 KIP P_{no} = 716 KIP 0.8 ϕ P_{no} = 401 KIP

ϕP_n	60	80	100	150	168	200	250	300	350	400
ϕM_{nx}	66	62	65	63	66	65	58	54	45	32
ϕM_{ny}	27	31	29	31	26	25	28	25	20	17.5
P_{nx}	118	155	190	275	288	320	405	460	528	580
P_{ny}	263	285	355	430	484	529	548	580	627	647
ϕP_n	64.3	81.7	104.7	153.3	168.9	193.4	241.8	279.8	334.6	373
% off	7.2	2.1	4.7	2.2	0.5	-3.3	-3.3	-6.7	-4.4	-6.6

UNITS IN KIPS & KIP-FT

Table 4.2 Summary for 18x18 column.

COLUMN SIZE 18X18 REINFORCEMENT : 4#14 $\rho = 2.78$ %

$P_{BALANCE}$ = 448 KIP P_{no} = 1611 KIP 0.8 ϕ P_{no} = 902 KIP

ϕP_n	130	150	200	300	400	448	500	700	800	900
ϕM_{nx}	215	240	200	250	200	200	225	185	140	120
ϕM_{ny}	144	110	155	115	158	155	110	90	100	60
P_{nx}	285	300	467	540	767	823	825	1080	1235	131
P_{ny}	425	600	572	910	889	945	1150	1320	1325	145
ϕP_n	133.6	159.8	214.1	300.4	387.2	424	479.1	658.7	741.8	843
% off	2.8	6.5	7.1	0.1	-3.2	-5.4	-4.2	-5.9	-7.3	-6.3

UNITS IN KIPS & KIP-FT

Table 4.3 Summary for 24x24 column.

COLUMN SIZE 24X24 REINFORCEMENT : 4#18 $\rho = 2.78$ %

$P_{BALANCE} = 500$ KIP $P_{no} = 2864$ KIP $0.8\ \phi\ P_{no} = 1604$ KIP

ϕP_n	250	300	400	500	800	830	1000	1200	1400	1600
ϕM_{nx}	590	600	600	610	580	570	535	480	400	300
ϕM_{ny}	305	285	290	300	280	280	260	230	200	150
P_{nx}	510	620	790	940	1330	1380	1620	1850	2100	2300
P_{ny}	935	1125	1330	1500	1790	1980	2165	2320	2450	2580
ϕP_n	261.1	325.2	419.5	506.8	728.1	795	958.9	1124.7	1308	1479
% off	4.4	8.4	4.9	1.4	-9.0	-4.2	-4.1	-6.3	-6.6	-7.6

UNITS IN KIPS & KIP-FT

Table 4.4 Summary for 18x24 column.

COLUMN SIZE 18X24 REINFORCEMENT : 6#14 $\rho = 3.13$ %

$P_{BALANCE} = 642$ KIP $P_{no} = 2233$ KIP $0.8\ \phi\ P_{no} = 1250$ KIP

ϕP_n	175	300	400	500	642	800	900	1000	1100	1250
ϕM_{nx}	371	450	480	420	390	375	350	310	250	190
ϕM_{ny}	182	160	141	168	173	157	147	140	136	96
P_{nx}	461	640	777	975	1167	1365	1458	1595	1723	1870
P_{ny}	516	924	1165	1243	1379	1555	1650	1696	1763	1936
ϕP_n	191.3	318.6	412.4	506.4	617.2	754.	829.3	910.6	1000	1160
% off	9.3	6.2	3.1	1.3	-3.9	-5.7	-7.9	-8.9	-9.1	-7.2

UNITS IN KIPS & KIP-FT

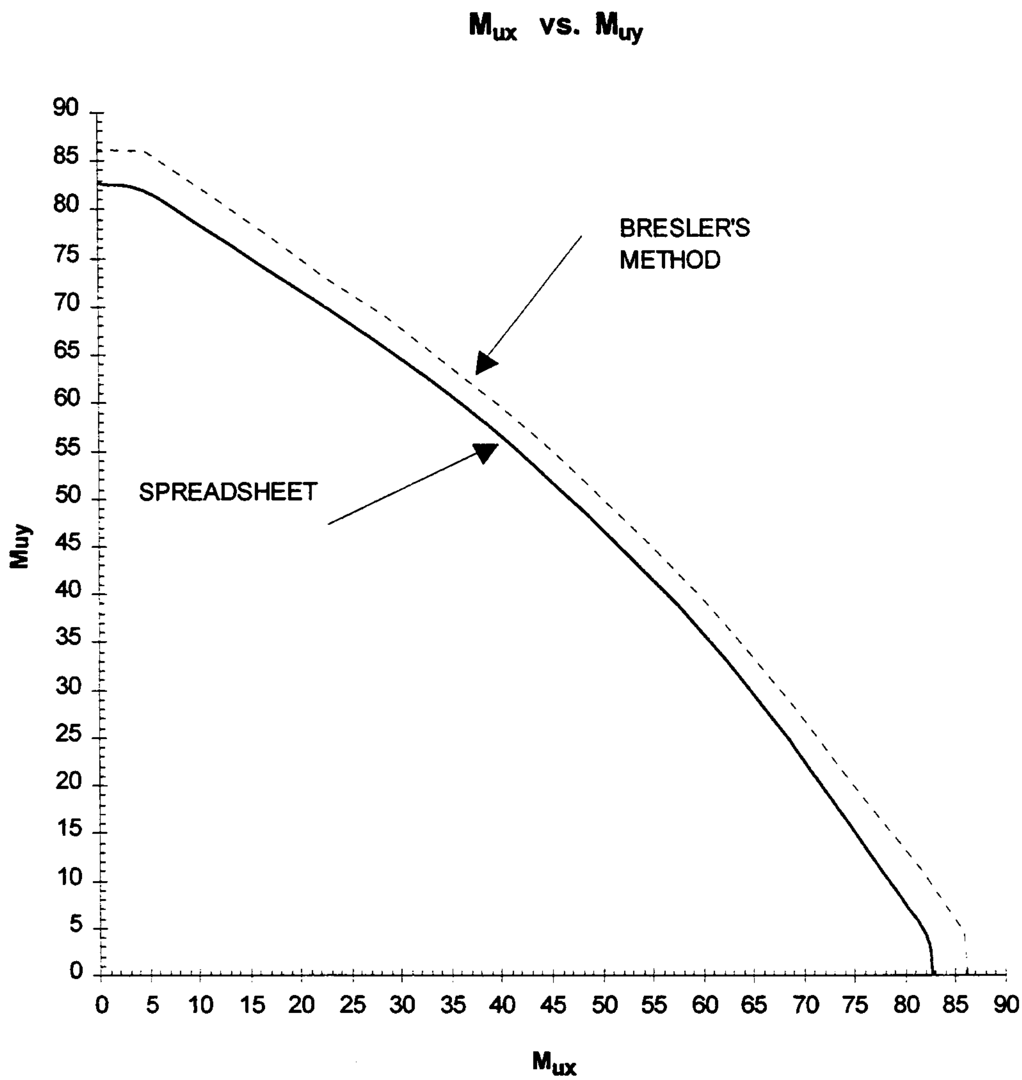

FIGURE 4.5 ϕM_{nx} - ϕM_{ny} interaction diagrams for 12"x12" column. ϕP_n = 100 K

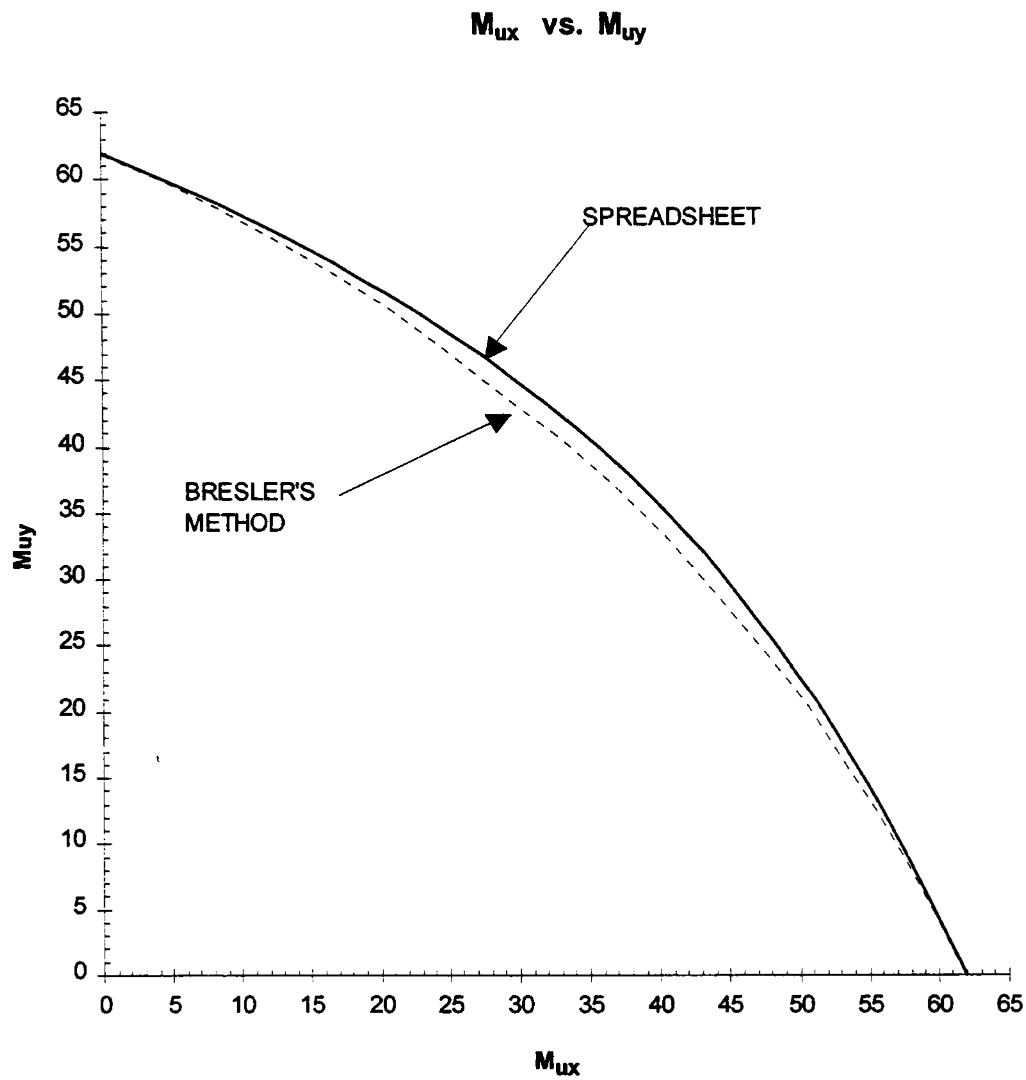

FIGURE 4.6 ϕM_{nx} - ϕM_{ny} interaction diagrams for 12"x12" column. ϕP_n = 300 K

M_{ux} vs. M_{uy}

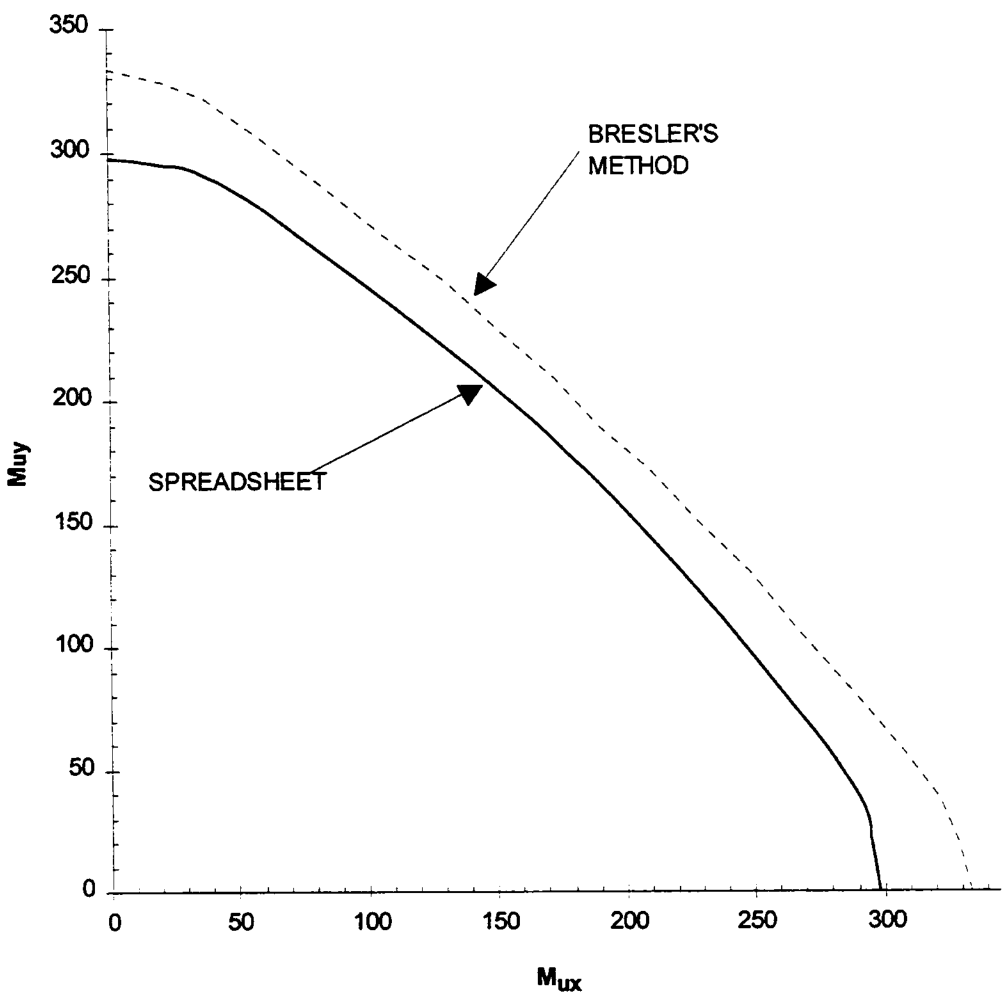

FIGURE 4.7 ϕM_{nx} - ϕM_{ny} interaction diagrams for 18"x18" column. $\phi P_n = 200$ K

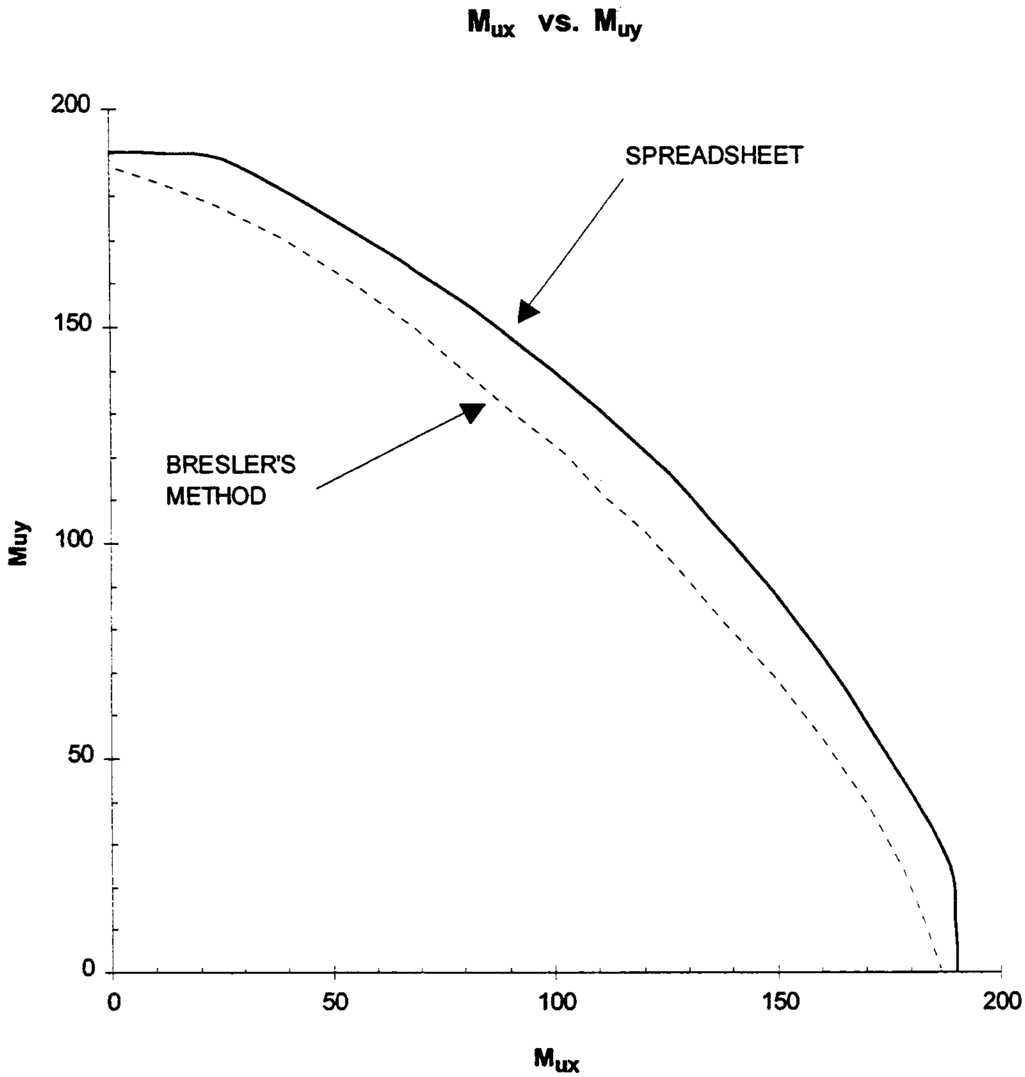

FIGURE 4.8 ϕM_{nx} - ϕM_{ny} interaction diagrams for 18"x18" column. ϕP_n = 800 K

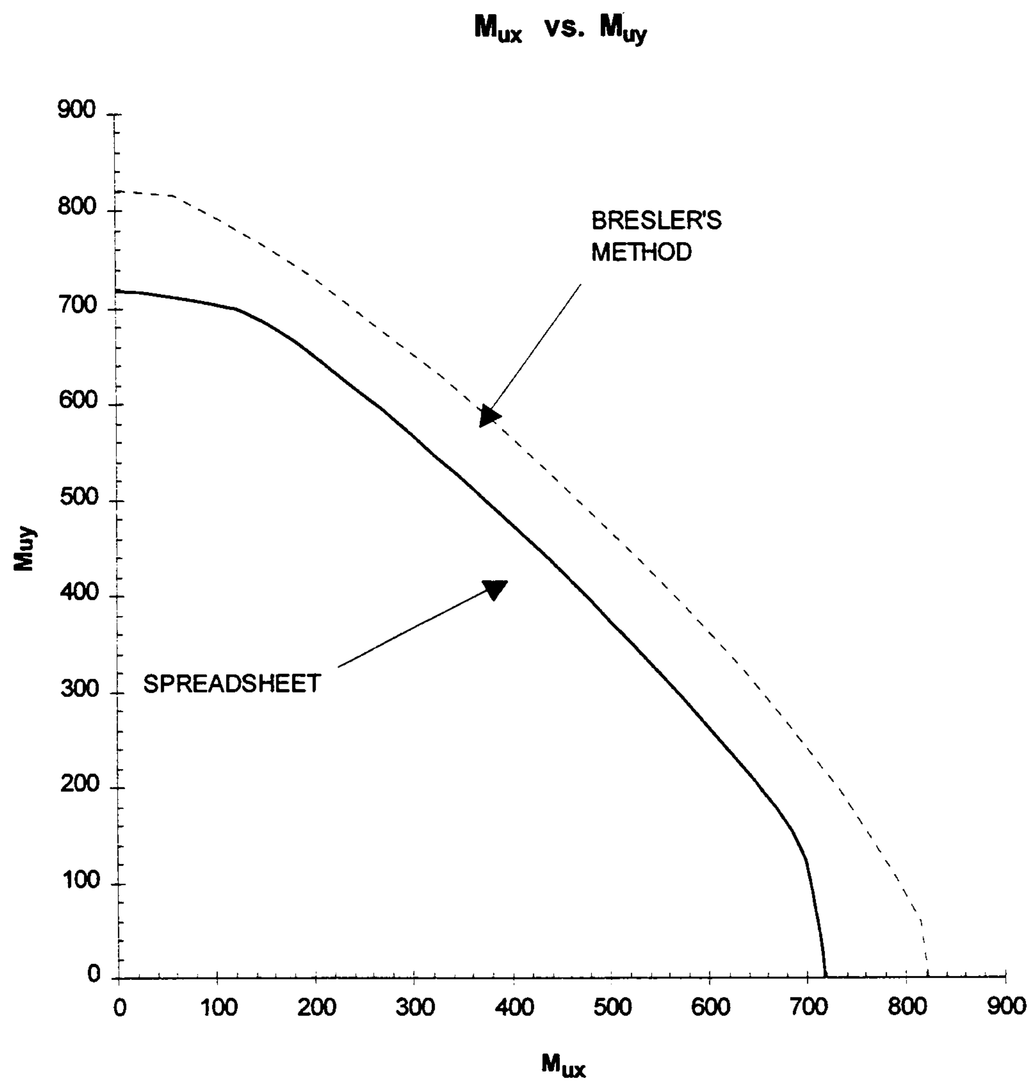

FIGURE 4.9 ϕM_{nx} - ϕM_{ny} interaction diagrams for 24"x24" column. ϕP_n = 300 K

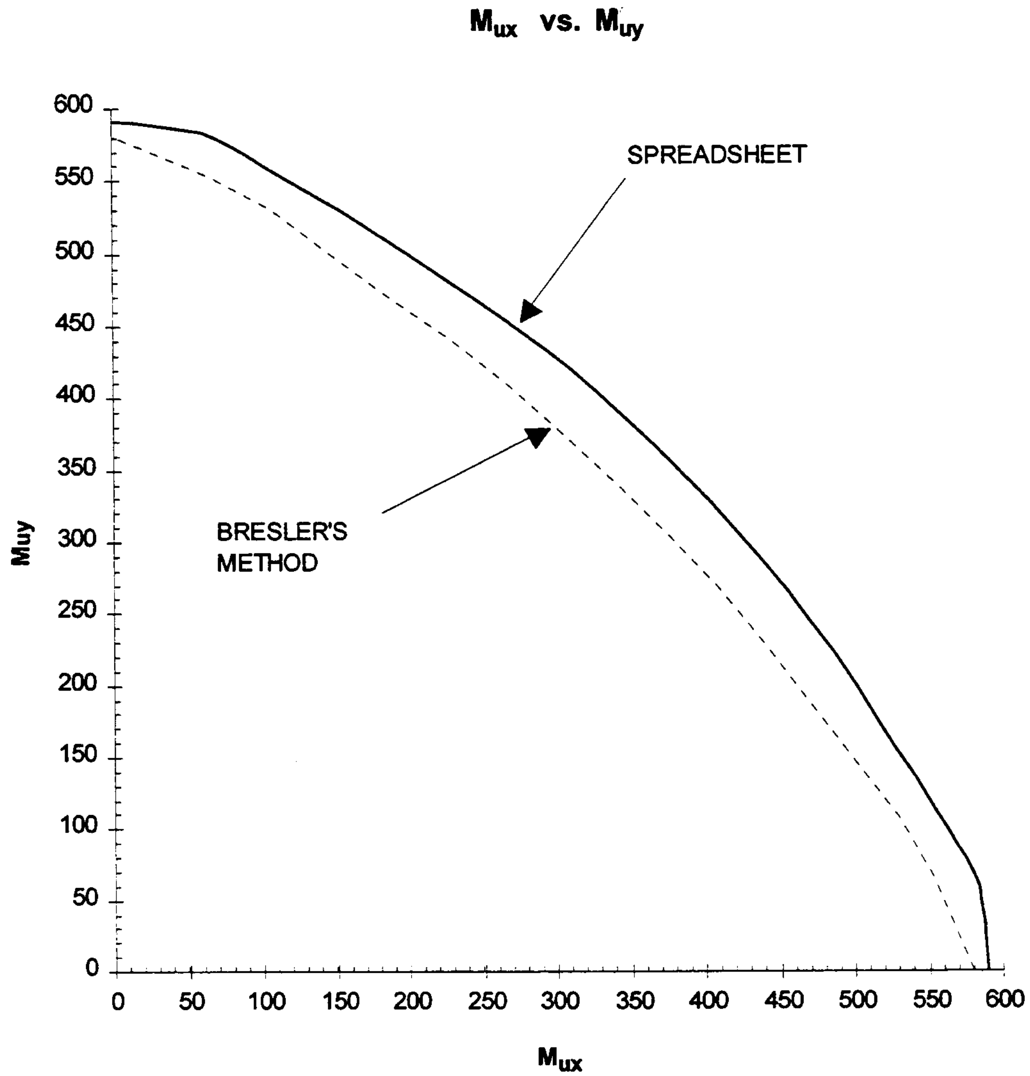

FIGURE 4.10 ϕM_{nx} - ϕM_{ny} interaction diagrams for 24"x24" column. ϕP_n = 1200 K

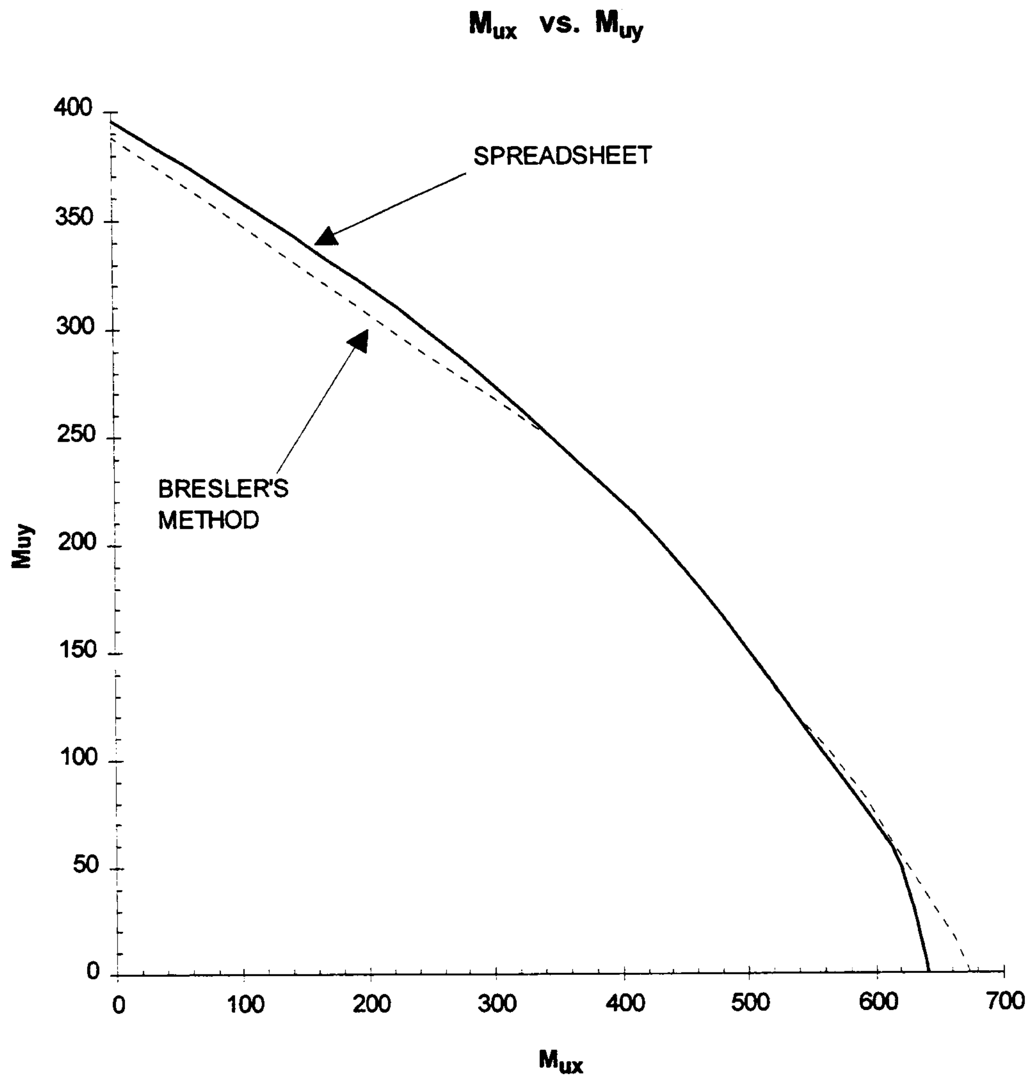

FIGURE 4.11 ϕM_{nx} - ϕM_{ny} interaction diagrams for 18"x24" column. $\phi P_n = 300$ K

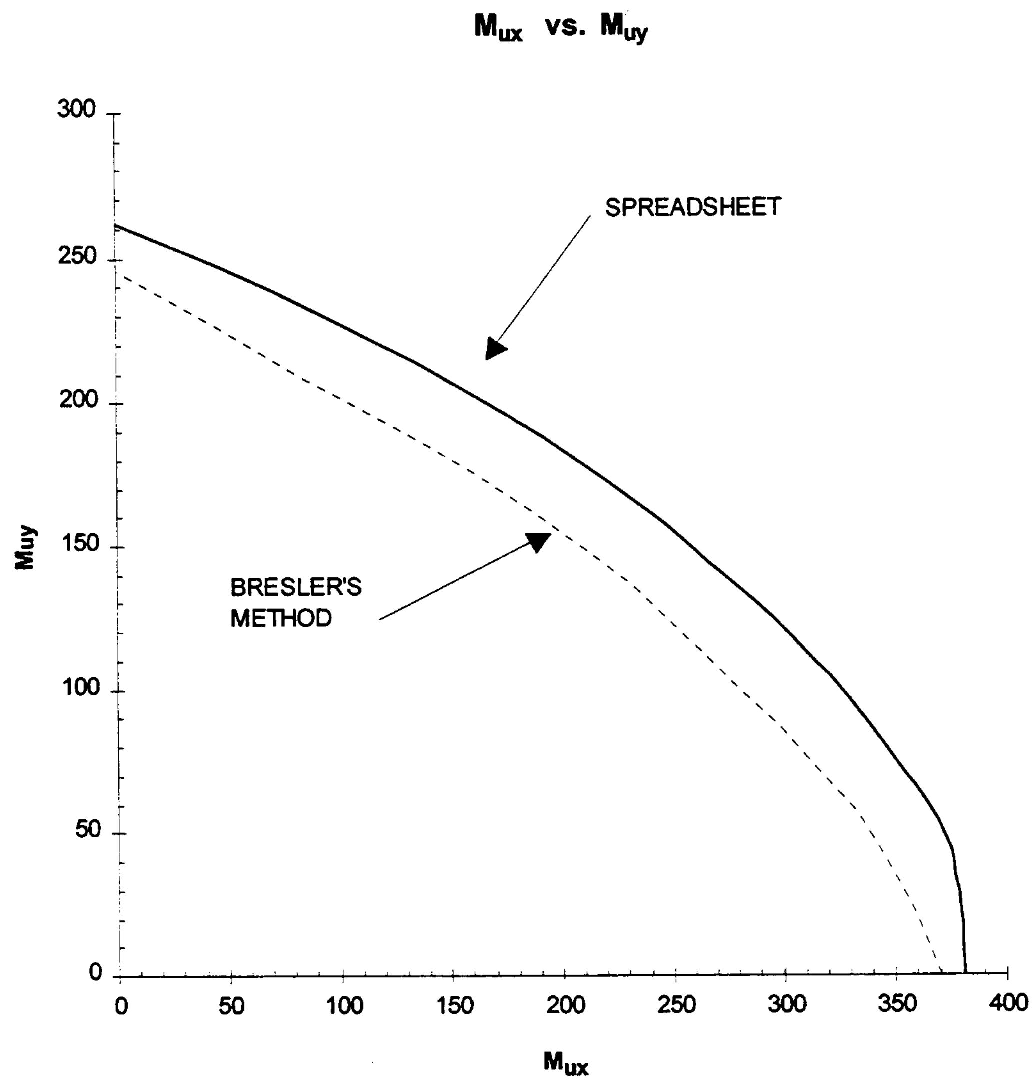

FIGURE 4.12 ϕM_{nx} - ϕM_{ny} interaction diagrams for 18"x24" column. ϕP_n = 1100 K

Chapter 5

CONCLUSION

The results of Bresler's Reciprocal Method (1) were reasonably close to those obtained from the exact analysis of the spreadsheet. From the cross-sections that were compared, the Reciprocal Method (1) tended to be somewhat conservative in the region above the balance point (figure 5.1), with an average of 6.0% on the conservative side with an average deviation of 1.4%. For points below the balance point, the method is reasonable but a bit on the unconservative side with an average error of 4.5% (unconservative) and an average deviation of 2.3%.

The Bresler Reciprocal Method is a reasonable and practical way to approximate the capacity of a given cross section, making it a better tool for analysis than for design. Bresler found his method to be in good agreement with theoretical and test results, with maximum deviations from exact theoretical results of 9.4% and from test results of 16%. These values agree with the exact analysis from the spreadsheet; an overall maximum deviation of 9.3% was obtained with a 5.2% average error and an average deviation of 2.1%.

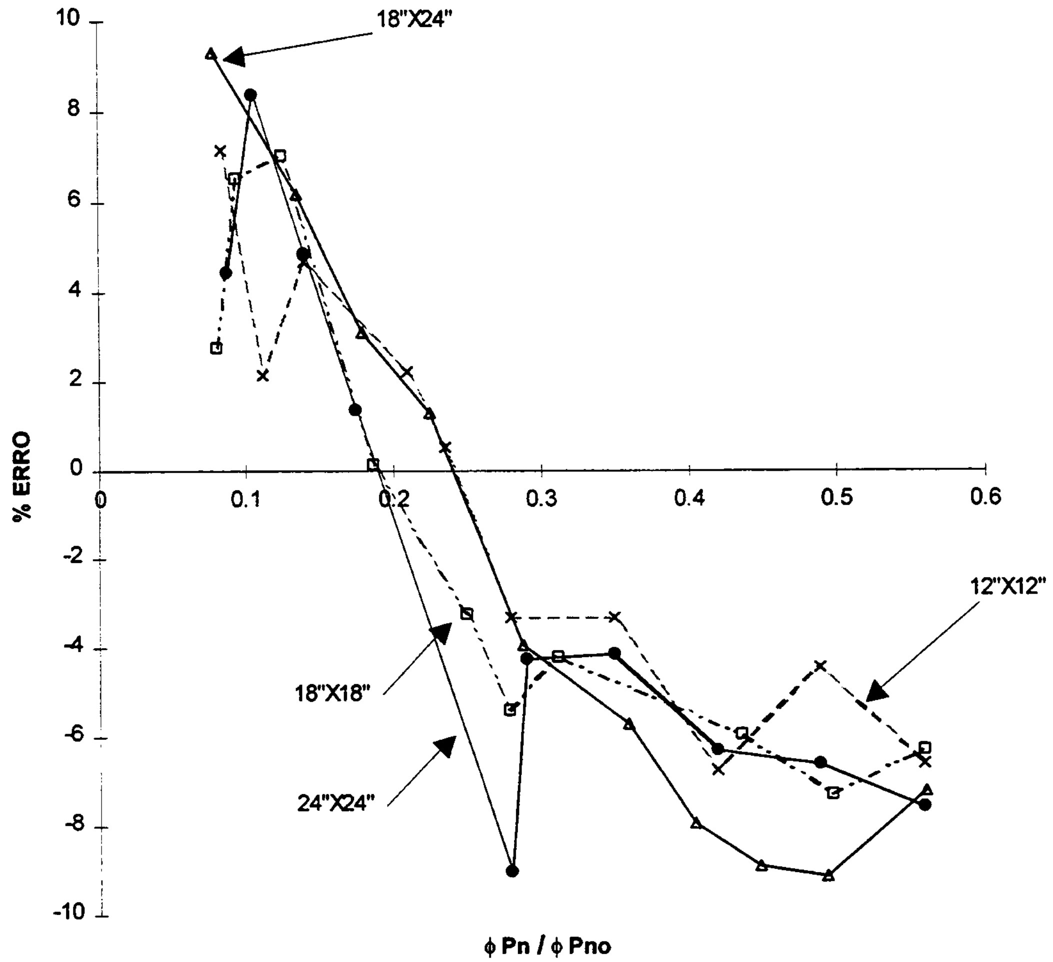

FIGURE 5.1 ϕPn / ϕPno vs. % error diagram

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APPENDICES

Appendix A

EXAMPLE 2 SOLUTION

Assume bars 1, 2, 3 did not yield and are in compression, and bars 4, 5, 6 yield and are in tension (figure 3.6). For transformation of axes coordinates see table A.1.

$\phi = 0.9 - 0.2\,[\,100/(\,0.1 \times 4 \times 18 \times 24\,)\,] = 0.784$

$A_c = 18\,(\,0.863C + 0.863C - 3.174\,)\,/\,2$

$= 15.534C - 28.566$

$C_c = 0.85 \times 4\,(\,15.534C - 28.566\,)$

$= 52.816C - 97.124$

$\varepsilon_{s1} = 0.003\,(C - 3.302\,)\,/\,C$

$\varepsilon_{s2} = 0.003\,(C - 4.370\,)\,/\,C$

$\varepsilon_{s3} = 0.003\,(C - 5.437\,)\,/\,C$

$\varepsilon_{s4} = 0.003\,(21.324 - C\,)\,/\,C$

$C_{s1} = A_s\,E_s\,\varepsilon_{s1}$

$C_{s1} = 2.25 \times 29000 \times 0.003\,(C - 3.302\,)\,/\,C$

$= 195.750\,(C - 3.302\,)\,/\,C$

$C_{s2} = 195.750\,(C - 4.370\,)\,/\,C$

TABLE A.1 Summary of coordinates example 2

18"X24"	$\theta = 10$					As=	2.25	DISTANCE FROM point A
bar	X	Y	r	α	X'	Y'		
A	18.00	24.00	30.00	53.13	13.56	26.76		0.00
1	15.15	21.15	26.02	54.39	11.25	23.46		3.30
2	9.00	21.15	22.99	66.95	5.19	22.39		4.37
3	2.85	21.15	21.34	82.33	-0.87	21.32		5.44
4	15.15	2.85	15.42	10.65	14.42	5.44		21.32
5	9.00	2.85	9.44	17.57	8.37	4.37		22.39
6	2.85	2.85	4.03	45.00	2.31	3.30		23.46

Point A is at the extreme compression fiber.

$C_{s3} = 195.750 (C - 5.437) / C$

$S_{s4} = S_{s5} = S_{s6} = 2.25 \times 60 = 135$ K

$= 135$ K

$\Sigma F = P / \phi$

$C_c + C_{s1} + C_{s2} + C_{s3} + S_{s4} + S_{s5} + S_{s6} = P / \phi$

$52.816C - 97.124 + 195.750 (C - 3.302 + C - 4.370 + C - 5.437) / C - 135 = 100 / 0.874$

$52.816 C^2 - 42.376 C - 2566.087 = 0$

$C = 7.383$ "

$\varepsilon_{s1} = 0.003 (7.383 - 3.302) / 7.383$

$= 0.001658$

$\varepsilon_{s2} = 0.003 (7.383 - 4.370) / 7.383$

$= 0.001207$

$\varepsilon_{s3} = 0.003 (7.383 - 5.437) / 7.383$

$= 0.000769$

$\varepsilon_{s1}, \varepsilon_{s2}, \varepsilon_{s3} < \varepsilon_Y$ OK

$\varepsilon_{s4} = 0.003 (21.324 - 7.383) / 7.383$

$= 0.00575 > \varepsilon_Y$ OK

All assumptions are valid

$C_c = 0.85 \times 4 (15.534 \times 7.383 - 28.566)$

$= 292.81$ K

$C_{s1} = 195.750 (7.383 - 3.302) / 7.383$

$= 108.21$ K

$C_{s2} = 79.90$ K

$C_{s3} = 51.58$ K

ΣF :

$292.81 + 108.21 + 79.90 + 51.58 - 3 \times 135 - 100 / 0.784 = 0$

$\phi M_{nx} = 0.784 \{(108.21 + 79.9 + 51.58) 9.15 + (3 \times 135) 9.15 +$

$(18 \times 3.20 \times 10.4 + 18 \times 3.17 \times 7.74 / 2) 0.85 \times 4.0 \} / 12$

$\phi M_{nx} = 567.72$ K FT

$\phi M_{nY} = 0.784 \{(108.21 - 51.58) 6.15 +$

$[(3.17 \times 9 / 2) 9 / 2 + (3.17 \times 9 / 4) 9 \times 2 / 3 - (3.17 \times 9 / 4) 9 / 3] 0.85 \times 4.0\} / 12$

$\phi M_{nY} = 41.78$ K FT

Appendix B

User's Guide

The program is a Microsoft Excel 5.0 based Spreadsheet. The Spreadsheet file name is Biaxial.xls. It has 3 "sheets"; the first one is called "DATA", the second one is "BIAXIAL MOMENT" and the third one is "M_x vs M_y ".

To start the Excel program double click on the Excel icon. The Microsoft Excel window appears with an empty file called Book1. To open the Biaxial.xls:

From the file menu choose open.

Go to the directory where the Biaxial.xls resides.

Choose the Biaxial.xls and click OK.

After the Biaxial file is opened, the Excel window shows the DATA sheet which is where the column data must be entered. In order to make the data entry easy the data cells are colored yellow.

There are ten cells which need to be edited. They are as follows:

The steel yield strength f_y (in ksi).

The concrete compressive strength fc' (in ksi).

The column width b (in inches).

The column depth h(in inches).

The concrete cover t_x and t_y(in inches).

The number of reinforcing steel bars in one X face.

The number of reinforcing steel bars in one Y face.

The reinforcing steel bars size.

The axial load ϕP_n acting on the column(in kips).

After editing the required cells, click on the start button. The Spreadsheet opens the BIAXIAL MOMENT sheet and starts calculating M_{nx} and M_{ny} for several points to create a table in the BIAXIAL MOMENT sheet and a chart in the M_x vs M_y sheet. After the Spreadsheet finishes the calculation, it displays on the screen the chart in the M_x vs M_y sheet. Any sheet can be reviewed by clicking on its tab in the bottom of the window.

VITA

Amjad Jamal Khayat was born in Nablus, Palestine, in 1961. He received a Bachelor of Science Degree in Civil Engineering with a major in Structures from University of Illinois in 1984. He has over ten years of professional experience in performing structural engineering investigations and design as well as inspection of structures, project management, and civil engineering construction supervision. He worked in Saudi Arabia on many site development and construction projects. In Chicago he worked with the Southwest Transit Group on its new 32 span bridge aerial guideway construction project. After that he worked with Homer L. Chastain & Associates as a structural engineer for the design of several steel plate girder bridges and retaining walls. Then he accepted a position at Alpha Engineering Group in Seattle where he worked on the structural analysis of WSDOT bridges and on routine bridge inspection for ODOT.

In January 1994, he accepted a research assistantship at the University of Tennessee to continue his education toward a Master of Science Degree in Civil Engineering with a major in Structures with a special emphasis on concrete analysis. Currently he is working with The Haskel Company in Jacksonville Florida as a structural engineer.