A MATHEMATICAL ANALYSIS OF CURRENTS IN A DISTRIBUTION SYSTEM DURING SHORT CIRCUIT

by

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Contents

I. Introduction.

It is the aim of every electric power distribution system to supply service without interruptions in so far as is possible. Distribution engineers and sub-station operators are daily confronted with the problem concerning conditions that exist in their distribution system during faults. To permit the clearing of these faults before greater damage can be done, it is necessary that adequate protective equipment be provided. The proper setting of this protective equipment is dependent upon a knowledge of conditions existing during such faults.

This paper deals with a mathematical consideration of currents that will flow during short circuit conditions in a ring network of a distribution system.

The study has been attempted at the suggestion of Professor J.G. Tarboux, who became interested in this type of a study as a result of certain peculiarities existing in the distribution system of the city of Knoxville, during short circuit conditions.

II. A General Survey of the Field.

AT present, no satisfactory method, for determining the amount of current that will flow in the various conductors of a complicated ring distribution system during short circuit conditions, by mathematical analysis, is available.

In order that the protective relays in the system may be properly set, it is necessary that the magnitude of the current in each conductor during short circuit, be determined. Actual tests by means of oscillographic pictures are very difficult and expensive to obtain. Hence, the desirability for devising a suitable mathematical process for calculating these short circuit currents.

A problem of this nature occurred within the distribution system of the city of Knoxville which necessitated an expensive oscillographic study by engineers from the home office of the Electric Bond and Share Co. during the latter part of 1931. It is obvious that a problem of this nature may occur in the distribution system of any city.

A mathematical solution of a problem of this nature is exceedingly difficult and requires a relatively large amount of time and work to complete.

III. State of Purpose

It is the purpose of this paper to derive equations enabling one to compute currents in the various conductors of a distribution system during short-circuit conditions. Also it is desired to make an application of these equations to a simple ring distribution system and to determine the currents that will flow during a fault.

Equations derived for this purpose are obtained by an application of the fundamental flux linkage equations to the computation of the mutual inductance between lines, of the system, in terms of the unknown currents. A diagram of the conditions under which this application was made is shown in fig. 5.

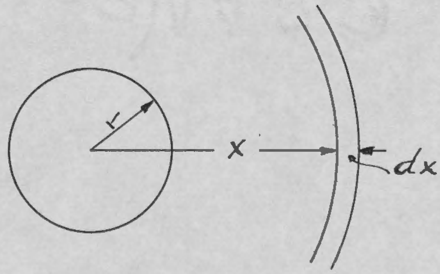
It is to be understood that the application made in this solution does not represent an actual system, but is merely a hypothetical case with dimensions assumed.

IV. General Theory

As has been stated, an application of the fundamental flux linkage equations is involved in deriving the equations used in the calculation of currents under short-circuit conditions in a distribution system, as attempted in this study. For that reason, an understanding of the fundamental flux linkage theory is necessary.

The total flux linking with a current-carrying conductor is the summation of the flux due to the current flowing within the conductor and of the flux due to a current flowing in adjacent conductors. The flux due to the current flowing in the conductor is made up of the flux that is within the conductor and of the flux which is set up around the conductor.

1. Magnetic field around a circular wire carrying current.



"Figure 1"

If a uniform distribution of current be assumed within the wire, then the total ampere turns produced by the wire is numerically equal to the total amperes that are flowing within the wire, since the total number of turns may be considered as one. Let H equal the ampere turns per centimeter of wire. Then the total m.m.f. drop in a circular ring ds thick, at a distance X from the center of the wire will be

$$m.m.f. = 2\pi x H$$

and $H = I/2\pi x$ amp. turns per centimeter of length.

But the density in lines per square centimeter is

$$\beta_a = \mu H = 1.257 \times \frac{I}{2\pi x} = 0.2 \frac{I}{x} \quad \text{lines / sq. cm.} \quad (2)$$

μ for non-magnetic materials = 1.257

2. Magnetic flux within a conductor.

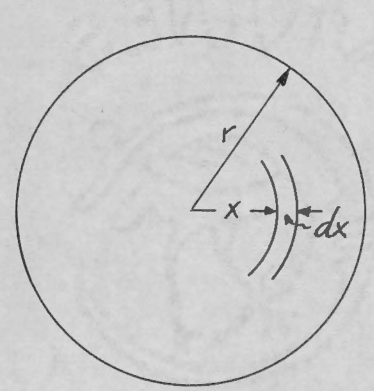


Fig. 2

The current within the radius X of the conductor is proportional to the square of the radii.

That is $I_x = I \times \frac{x^2}{r^2}$

where I = total current.

Therefore the total ampere turns produced by this current will be $I \frac{x^2}{r^2}$ amp. turns

Substituting as before

$$2\pi x H = I \frac{x^2}{r^2}$$

$$H = I \frac{x^2}{r^2} \cdot \frac{1}{2\pi x} = \frac{I x}{2\pi r^2}$$

And $\beta_c = \mu \cdot \frac{Ix}{2\pi r^2} = 0.2 \frac{Ix}{r^2}$ --- lines / sq, cm. (2)

As an explanation of the above equations we may say that equation (1) shows that the flux density outside of the conductor varies inversely as the radius X , while equation (2) shows that the density within the conductor varies directly with X .

When $x = r$ and $\mu = 1.257$

equation (1) becomes $\beta_a = 0.2 \frac{I}{r}$

equation (2) becomes $\beta_c = 0.2 \frac{Ir}{r^2} = 0.2 \frac{I}{r}$

The distribution of the flux density as produced by the current flowing within the wire is shown below in Fig. 3 .

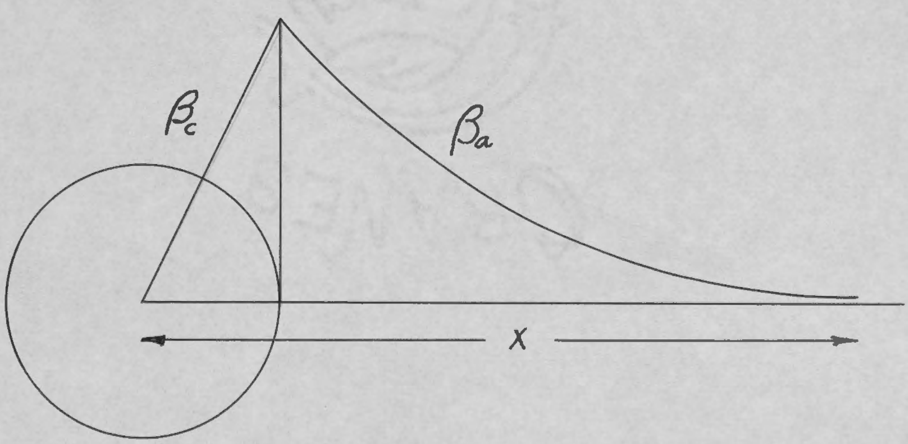


Fig. 3

3. <u>Flux Linkages</u>

The voltage of self inductance within a conductor, due to the flux that links with it may be expressed thus:

$$e = -\frac{d\phi}{dt} \times 10^{-8}$$

or in terms of the inductance

$$e = -L\frac{di}{dt}$$

From the above relations it is seen that

$$L = \frac{d\phi}{di} \times 10^{-8} \qquad \text{henries.}$$

The quanity $\frac{d\phi}{di}$ becomes merely $\frac{\phi}{I}$ in the case of constant current, hence for a particular current

$$L = \frac{\phi}{I} \times 10^{-8} \qquad \text{----------------------------(3)}$$

This equation gives correct results as long as the total flux links the entire current I , but when a portion of the flux does not link with the entire current, then a modification of equation (3) is necessary.

Consider the flux that is outside of the cylindrical conductor. This flux links with the entire current. On the other hand consider the flux within the conductor (see equation 2). This flux does not link with the total current and therefore cannot be divided by the total current to obtain the inductance, L, of the line. Such a proceedure would render an incorrect value of L.

It is required that an equivalent flux be obtained, of such a value that when it is considered outside of the conductor (linking with the total current) it will produce the same voltage of self inductance as the actual distribution of flux when linking with the partial currents. This flux is known as the <u>equivalent flux linkages</u>.

4. Flux Linkages of a Single Phase Line

The total flux per unit length (cm), per conductor, outside of the wire can be obtained from equation (1), by intergrating the expression for flux density between the limits of D and r, where D = the spacing of the conductors in centimeters and r = the radius of the conductors.

$$\phi_a = \int_r^D \beta\, dx = \int_r^D \frac{0.2I}{x} dx = 0.2I \int_r^D \frac{dx}{r}$$

$$\phi_a = 0.2 I \, Ln \frac{D}{r}$$ lines per cm. ------------------(4)

Considering the flux within the wire we now have that the flux within the element dx is

$$d\phi_x = \frac{\mu I x}{2\pi r^2} dx$$ lines per cm.

The equivalent flux linking the current I will be less by the ratio of the areas $\frac{\pi x^2}{\pi r^2}$ or

$$d\phi_{xc} = \frac{\mu I x^3}{2\pi r^4} dx$$ lines / cm.

The total equivalent flux linkages are

$$\phi_{ce} = \int_0^r \frac{\mu I x^3}{2\pi r^4} dx = \frac{\mu I}{2\pi r^4}\left(\frac{x^4}{4}\right)\Big]_0^r$$

$$\phi_{ce} = \frac{\mu I}{2\pi r^4} \times \frac{r^4}{4} = \frac{\mu I}{8\pi}$$ --------------------(5)

The total flux linkages due to one conductor are

$$\phi_T = \phi_a + \phi_{ce} = 0.2 I \, Ln \frac{D}{r} + \frac{\mu I}{8\pi}$$ lines / cm. (6)

5. Flux linkages of a three phase line with unsymetrical spacing.

Given three conductors a, b and c with currents I_a, I_b and I_c. The spacing between the conductors is unsymetrical. The only limitation to the solution is that the vector sum of the three currents shall be equal to zero. Or

$$I_a + I_b + I_c = 0$$

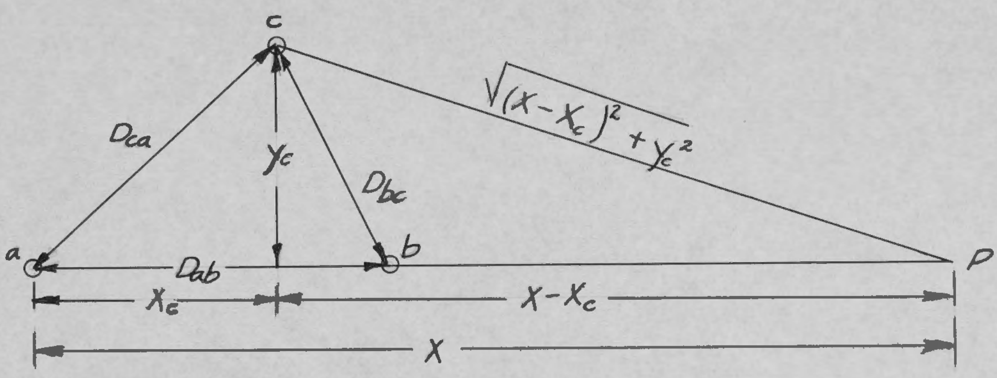


Fig. 4

Let P be some point at a distance X from the conductor "a".

The total flux linkages about the conductor "a" is the summation of the equivalent flux as produced by the current within the conductor itself and also the flux that links with the conductor and is produced by the current flowing within the other conductors. Furthermore the fluxes are not in time phase with each other. But since the flux produced by each conductor is in phase with it's current, the proper results are obtained if each current be considered as a vector quanity instead of a numeric.

The number of linkages set up by conductor "a" which link with "a" and cross the x axis between "a" and point P are as per eq. (5)

$$\phi_{ae} = 0.2 I_a \, Ln \frac{x}{r} + \frac{\mu I_a}{8\pi} \qquad \text{lines / cm.}$$

The number of linkages between conductor "a" and point P which link with conductor "a" and are produced by conductor "b" may be written from equation (4) by using the proper limits.

$$\phi_{ab} = 0.2\, I_b \, Ln \frac{X - D_{ab}}{D_{ab}} \qquad \text{lines / cm.}$$

Similiarly for conductor (c)

$$\phi_{ac} = 0.2\, I_c \, Ln \frac{\sqrt{(X-X_c)^2 + Y_c^2}}{D_{ca}} \qquad \text{lines / cm.}$$

The total flux linkages about (a) are therefore

$$\phi_{TA} = \phi_{ae} + \phi_{ab} + \phi_{ac}$$

$$\phi_{TA} = \left(\frac{\mu}{8\pi} + 0.2 Ln \frac{X}{r}\right) I_a + 0.2 I_b Ln \frac{X - D_{ab}}{D_{ab}} + 0.2 I_c Ln \frac{\sqrt{(X-X_c)^2 + Y_c^2}}{D_{ca}}$$

But

$$I_c = -I_a - I_b$$

Hence,

$$\phi_{Ta} = I_a\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{X D_{ca}}{r\sqrt{(X-X_c)^2 + Y_c^2}}\right) + 0.2 I_b Ln \frac{(X - D_{ab})(D_{ca})}{D_{ab}\sqrt{(X-X_c)^2 + Y_c^2}}$$

Now let the point be moved to infinity so that X equals infinity. But as X approaches infinity, then

$$\frac{X}{\sqrt{(X-X_c)^2 + Y_c^2}} \qquad \text{and} \qquad \frac{X - D_{ab}}{\sqrt{(X-X_c)^2 + Y_c^2}}$$

approaches unity, hence we may now write that

$$\phi_{Ta} = I_a\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{D_{ca}}{r}\right) + 0.2\, I_b \, Ln \frac{D_{ca}}{D_{ab}}$$

which may be rearranged as follows

$$\phi_{Ta} = I_a\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r} + 0.2 Ln D_{ca}\right)$$
$$+ I_b \left(0.2 Ln D_{ca} + 0.2 Ln \frac{1}{D_{ab}}\right)$$

But

$$0.2 I_a Ln D_{ca} - 0.2 I_b Ln D_{ca} = 0.2 I_c Ln \frac{1}{D_{ca}}$$

hence,

$$\phi_{Ta} = I_a\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r}\right) + 0.2 I_b Ln \frac{1}{D_{ab}} + 0.2 I_c Ln \frac{1}{D_{ac}} \qquad \text{lines / cm. --(7)}$$

If equation (7) be examined closely it will be seen that it pocesses a certain peculiar sequence of terms, from examination it is possible to write the total flux linkage expressions for conductors (b) and (c) as follows:

$$\phi_{Tb} = I_b\left(\frac{\mu}{8\pi} + 0.2 Ln\frac{1}{r}\right) + 0.2 I_a Ln\frac{1}{D_{ab}} + 0.2 I_c Ln\frac{1}{D_{bc}} \quad \text{lines / cm.} \text{ --(8)}$$

$$\phi_{Tc} = I_c\left(\frac{\mu}{8\pi} + 0.2 Ln\frac{1}{r}\right) + 0.2 I_b Ln\frac{1}{D_{bc}} + 0.2 I_a Ln\frac{1}{D_{ca}} \quad \text{lines / cm.} \text{ --(9)}$$

Furthermore, from equations 7, 8, and 9 it is possible to write a general equation for flux linkages about one conductor produced by any number of conductors, as follows:

$$\phi_{Ta} = I_a\left(\frac{\mu}{8\pi} + 0.2 Ln\frac{1}{r}\right) + 0.2\left(I_b Ln\frac{1}{D_{ab}} + I_c Ln\frac{1}{D_{ac}} + I_d Ln\frac{1}{D_{ad}} + \text{------} I_n Ln\frac{1}{D_{an}}\right) \quad \text{------------------(10)}$$

8. <u>Reactance drop in the lines.</u>

The complex voltage drop in a circuit of X reactance and carrying I amperes is-

$$jXI$$

But $X = 2\pi f L = \omega L$

Hence, the reactance drop becomes

$$j\omega L$$

but $L = \dfrac{\phi_e}{I} = \dfrac{\text{flux linkages}}{\text{current}}$

therefore,

$$IX = j\omega \frac{\phi_e}{I} \times I = j\omega\phi_e \quad \text{--------------(11)}$$

This result is of extreme importance as it permits the computation of the reactance drops of the lines with either unsymetrical are symetrical spacing of conductors, wh which have unbalanced currents. Equation (11) indicates that the reactance drop of any line equals the flux linkages of that line multiplied by the rotating operator j and by the quanity $\omega = 2\pi f$.

It follows therefore that the impedance drop is

$$IZ = RI + j\omega\phi_e \qquad \text{---------------------(12)}$$

V. Application of Theory to the Solution of an Assumed System.

In this solution a fault is considered as occuring between two phases of a three phase system, that is, the fault is treated as being a line to line short through ground. Figure 5 show a diagram of the assumed problem. Only two conductors are represented, since, the current in the third line is considered to be so small in comparison to the current in the two conductors through which the fault has occured that it may be neglected. Therefore, the assumption that the current flowing in the third conductor is to be negligble, reduces the problem to one of the consideration of a single phase fault.

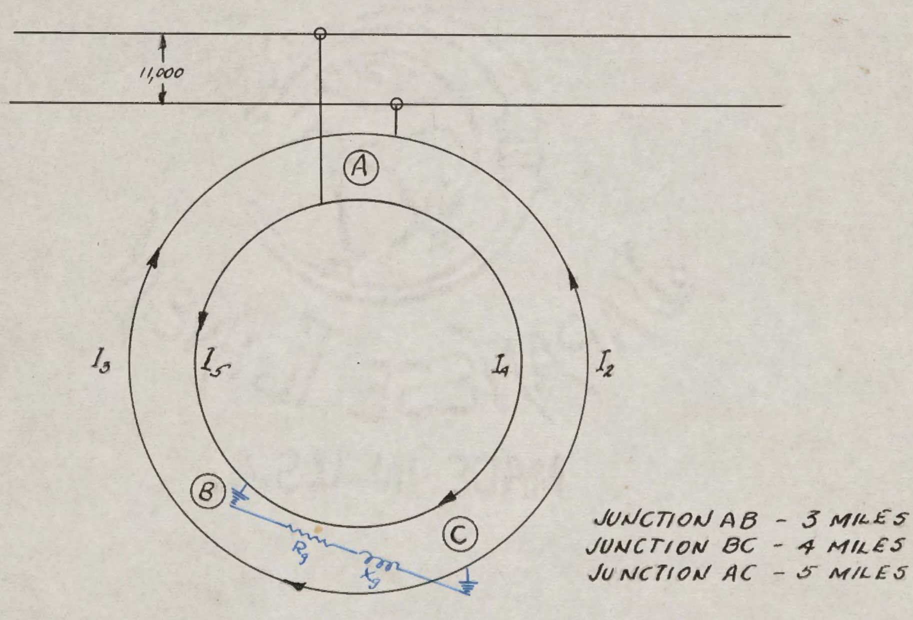


figure 5.

Figure 5 represents a ring distribution system with a fault occuring between phase "a" and "b" of the three phase system.

Setting up flux linkage equations for the several lines as shown, in terms of the unknown currents, we have:

$$\phi_{3AB} = I_3\left(\frac{\mu}{8\pi} + 0.2 \operatorname{Ln}\frac{1}{r_3}\right) - 0.2 I_5 \operatorname{Ln}\frac{1}{D_{35}} \quad (1)$$

$$\phi_{5AB} = I_5\left(\frac{\mu}{8\pi} + 0.2 \operatorname{Ln}\frac{1}{r_5}\right) - 0.2 I_3 \operatorname{Ln}\frac{1}{D_{35}} \quad (2)$$

$$\phi_{2AC} = I_2\left(\frac{\mu}{8\pi} + 0.2 \operatorname{Ln}\frac{1}{r_2}\right) - 0.2 I_4 \operatorname{Ln}\frac{1}{D_{24}} \quad (3)$$

$$\phi_{4AC} = I_4\left(\frac{\mu}{8\pi} + 0.2 \operatorname{Ln}\frac{1}{r_4}\right) - 0.2 I_2 \operatorname{Ln}\frac{1}{D_{24}} \quad (4)$$

$$\phi_{4BC} = I_4\left(\frac{\mu}{8\pi} + 0.2 \operatorname{Ln}\frac{1}{r_4}\right) + 0.2 I_3 \operatorname{Ln}\frac{1}{D_{34}} \quad (5)$$

$$\phi_{3BC} = I_3\left(\frac{\mu}{8\pi} + 0.2 \operatorname{Ln}\frac{1}{r_4}\right) + 0.2 I_4 \operatorname{Ln}\frac{1}{D_{34}} \quad (6)$$

In the computation of the unknown currents as shown by the above equations, wire of the same size is assumed throughout the entire system, symmetrical spacing of the conductors is also assumed. This simplifies the problem to the extent that various constants may now be substituted in the above equations. These constants are as follows:

Let $\left(\frac{\mu}{8\pi} + 0.2 \operatorname{Ln}\frac{1}{r}\right) = A$

and $\left(0.2 \operatorname{Ln}\frac{1}{D}\right) = B$

The equation of inductance as derived in the theory of this thesis is as follows:

Inductance, $L = \frac{\phi}{I} \times 10^{-8}$ henries / cm.

$L = \frac{2.54 \times 12 \times 5280}{10^8} \times \frac{\phi}{I}$ henries / mile

$L = 1.61 \frac{\phi}{I} \times 10^{-3}$ henries / mile

Let $1.61 \times 10^{-3} = k$

Then the reactive voltage drop per mile becomes:

$$IX = \omega k \phi$$

and the actual voltage drop becomes

$$IZ = IR + jIX$$

$$IZ = IR + j\omega k \phi \qquad (7)$$

Constants A and B are substituted in equations 1 to 6, and expressions for voltage drops in the circuit, as shown by figure 5, are written. General equation number 7 is used, with proper substitions being made. R being the resistance per mile of copper wire.

$$I_3 Z_{3AB} = 3R + j\omega 3k(AI_3 - BI_5) \qquad (8)$$

$$I_5 Z_{5AB} = 3R + j\omega 3k(AI_5 - BI_3) \qquad (9)$$

$$I_2 Z_{2AC} = 5R + j\omega 5k(AI_2 - BI_4) \qquad (10)$$

$$I_4 Z_{4AC} = 5R + j\omega 5k(AI_4 - BI_2) \qquad (11)$$

$$I_4 Z_{4BC} = 4R + j\omega 4k(AI_4 + BI_3) \qquad (12)$$

$$I_3 Z_{3BC} = 4R + j\omega 4k(AI_3 + BI_4) \qquad (13)$$

Combining equations 8 with 13, we have

$$I_3 Z_3 = 7I_3 R + j\omega k(7AI_3 + 4BI_4 - 3BI_5) \qquad (14)$$

Rewriting equations 9 and 10, we have

$$I_5 Z_5 = 3I_5 R + j\omega k(3AI_5 - 3BI_3) \qquad (15)$$

$$I_2 Z_2 = 5I_2 R + j\omega k(5AI_2 - 5BI_4) \qquad (16)$$

Combining equations 11 with 12, we have

$$I_4 Z_4 = 9I_4 R + j\omega k(9AI_4 + 4BI_3 - 5BI_2) \tag{17}$$

From an application of Kirchoff's laws of voltage and current, the following equations showing the relations of the several currents to each other and expressions for the several voltage drops within the circuit may be written.

$$I_5 + I_4 = I_T = I_g \tag{18}$$

$$I_3 + I_2 = I_T = I_g \tag{19}$$

$$I_3 + I_2 = I_5 + I_4 \tag{20}$$

$$I_5 Z_5 - I_4 Z_4 = 0 \tag{21}$$

$$I_2 Z_2 - I_3 Z_3 = 0 \tag{22}$$

$$I_4 Z_4 + I_2 Z_2 + I_g Z_g = 11{,}000 \tag{23}$$

Values for IZ drops, equations 14 to 17, are substituted in equations 21 to 23 and the following results are obtained. Substitutions being made in equations 21, 22 and 23 respectively.

$$3I_5 R + j\omega k(3AI_5 - 3BI_3) - 9I_4 R - j\omega k(9AI_4 + 4BI_3 - 5BI_2) = 0$$

From equation 20; $I_3 = I_5 + I_4 - I_2$

$$I_5[3R + j\omega k(3A - 7B)] - I_4[9R + j\omega k(9A + 7B)] + I_2(j\omega k 12B) = 0 \tag{24}$$

$$5I_2 R + j\omega k(5AI_2 - 5BI_4) - 7I_3 R - j\omega k(7AI_3 + 4BI_4 - 3BI_5) = 0$$

$$-I_5[7R + j\omega k(7A - 3B)] - I_4[7R + j\omega k(7A + 9B)] + I_2[12R + j\omega k 12A] = 0 \tag{25}$$

$$9I_4R + j\omega k(9AI_4 + 4BI_3 - 5BI_2) + 5I_2R + j\omega k(5AI_2 - 5BI_4) + (I_5 + I_4)Z_9 = 11{,}000$$

$$I_5[Z_9 + j\omega k(4B)] + I_4[9R + Z_9 + j\omega k(9A - B)] + I_2[5R + j\omega k(5A - 9B)] = 11{,}000 \quad (26)$$

Equations 24,25, and 26 are three simultaneous equations with three unknowns, namely, I_5, I_4, and I_2. As these equations stand, the coefficients are complex quanities. Numerical values for these coefficients can be determined, however, and in order to simplify the solution of these equations, these coefficients are represented by constants as follows:

Let

$$[3R + j\omega k(3A - 7B)] = C$$

$$[9R + j\omega k(9A - 7B)] = D$$

$$[j\omega k(12B)] = E$$

$$[7R + j\omega k(7A - 3B)] = F$$

$$[7R + j\omega k(7A + 9B)] = G$$

$$[12R + j\omega k(12A)] = H$$

$$[Z_9 + j\omega k(4B)] = L$$

$$[9R + Z_9 + j\omega k(9A - B)] = M$$

$$[5R + j\omega k(5A - 9B)] = N$$

Substituting these constants in equations 24, 25 and 26

$$CI_5 - DI_4 + EI_2 = 0 \quad (24)$$

$$-FI_5 - GI_4 + HI_2 = 0 \qquad (25)$$

$$LI_5 + MI_4 + NI_2 = 11{,}000 + j0 \qquad (26)$$

Solving simultaneously, we have

$$-I_4(DF + CG) + I_2(EF + CH) = 0 \qquad (27)$$

$$I_4(FM - GL) + I_2(LH + FN) = 11{,}000F \qquad (28)$$

From equations 27 and 2i

$$I_2 = \frac{11{,}000F(DF + CG)}{[(EF + CH)(FM - GL) + (FN + HL)(DF + CG)]} \qquad (29)$$

Substituting value of I_2 into (28)

$$I_4 = \frac{11{,}000F(EF + CH)}{[(EF + CH)(FM - GL) + (FN + HL)(DF + CG)]} \qquad (30)$$

From equation (24)

$$I_5 = \frac{DI_4 - EI_2}{C} \qquad (31)$$

The following are assumptions made in order to compute values for line conditions.

Height of conductors above ground = 27 feet,

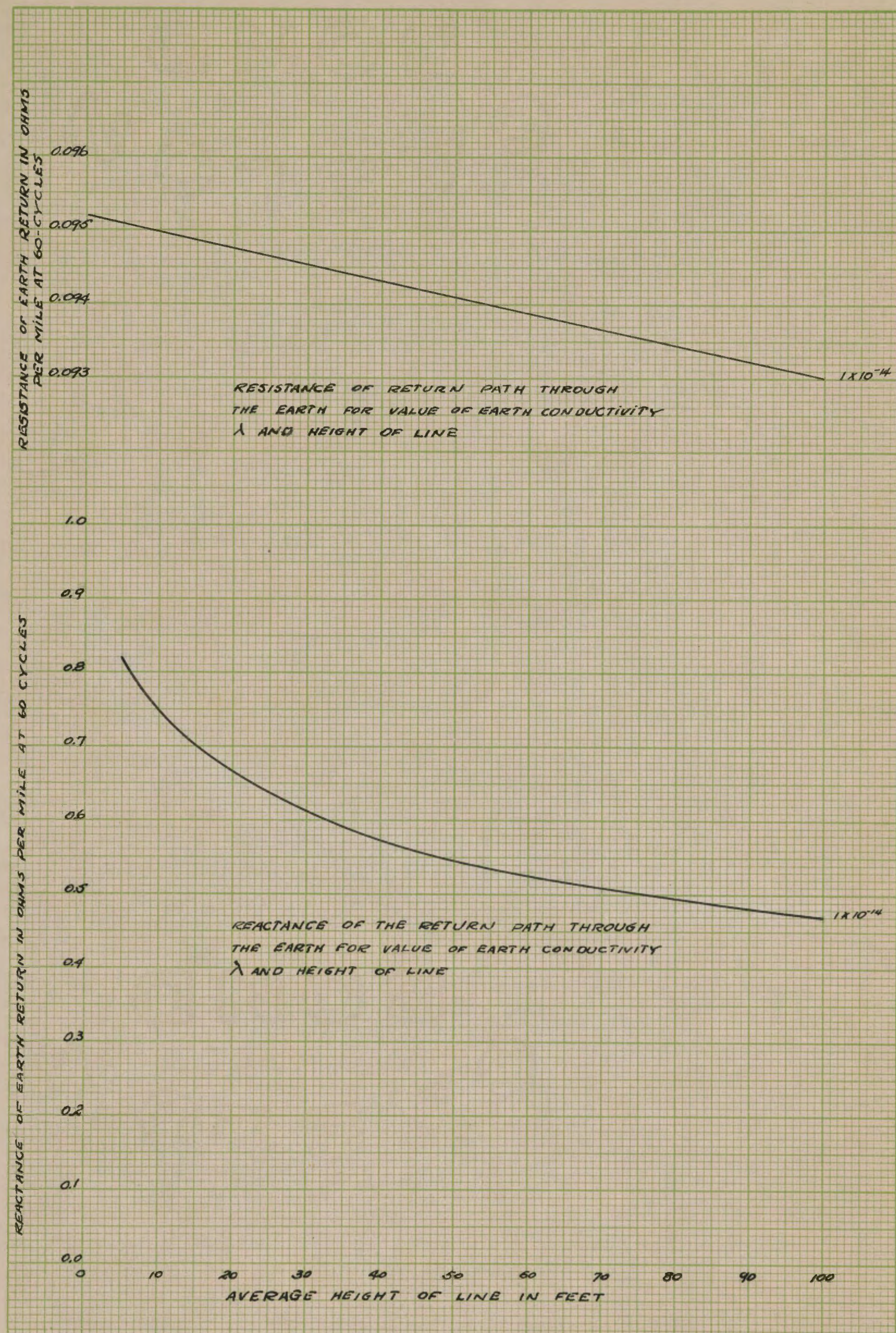
equilateral spacing of conductors = 18 inches, or 45.7 cm.

No. 0, copper wire, radius = $\frac{0.3249}{2}$ inches,

Z_g = 0.095 + j 0.62, this value was obtained from a curve that appeared in an article written by J.E. Clem, of the General Electric Co. This article appeared in the November issue of the "Electrical Engineering", 1931. A reproduction of this curve is given on page 19.

Resistance of conductors per 1000 feet = 0.0964 ohms,

resistance of conductors per mile = 0.509 ohms,



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$A = 0.227$

$B = -\ 0.835$

$wk = 377 \times 1.61 \times 10^{-3} = 0.607$

$C = 3R + jwk(3A - 7B) = 1.53 + j\,3.96 = 4.25\angle 68.8$

$D = 9R + jwk(9A - 7B) = 4.57 - j\,2.30 = 5.10\angle 26.6$

$E = jwk(12B) = 0 - j\,6.07 = 6.07\angle 90$

$F = 7R + j\,wk(\,7R - 3B) = 3.56 + j2.47 = 4.30\angle 35$

$G = 7R + jwk(7A - 9B) = 3.56 - j3.6 = 5.07\angle 45.6$

$H = 12R + jwk(12A) = 6.10 + j\,1.66 = 6.30\angle 15.2$

$L = Z_g + jwk(4B) = 0.1 - j\,1.40 = 1.40\angle 86$

$M = 9R + Z_g - jwk(9A - B) = 4.66 + j\,2.36 = 5.2\angle 27$

$N = 5R + jwk(5A - 9B) = 2.54 + j\,5.25 = 5.83\angle 64.3$

Substituting the above values in equation (29) and solving for I_2.

$$I_2 = \frac{11{,}000F(DF+LG)}{[(EF+CH)(FM-GL)+(FN+HL)(DH+CG)]}$$

$$DF = 21.9\angle 8.4^\circ = 21.6 + j\,3.0$$

$$CG = 21.6\angle 23.2^\circ = \underline{19.9 + j\,8.5}$$

$$(DF+CG) = 41.5 + j11.5 = 43.\angle 15.7^\circ$$

$$11{,}000\times F\times 43\angle 15.7^\circ = 11{,}000\times 4.30\angle 35^\circ\times 43.0\angle 15.7^\circ = 2{,}035{,}000\angle 50.7^\circ$$

$$EF = 26.1\angle -55^\circ = 15.1 - j21.5$$

$$CH = 26.8\angle 84^\circ = \underline{2.9 + j26.7}$$

$$(EF+CH) = 18. + j5.2 = 18.7\angle 16.7^\circ$$

$$FM = 4.3\angle 35^\circ\times 5.2\angle 27^\circ = 22.4\angle 62^\circ = 10.5 + j\,19.8$$

$$GL = 5.07\angle -45.6^\circ\times 1.4\angle -86^\circ = 7.1\angle -131.6^\circ = \underline{-4.75 - j\,5.3}$$

$$(FM-GL) = 15.25 + j25.1$$

$$(FM-GL) = 29.5\angle 59^\circ$$

$$(EF+CH)(FM-GL) = 18.7\angle 16.2^\circ\times 29.5\angle 59^\circ = 552\angle 75.2^\circ = 141 + j535$$

$$FN = 4.3\angle 35^\circ\times 5.83\angle 64.3^\circ = 25.1\angle 99.3^\circ = -4.1 + j\,25.0$$

$$HL = 6.3\angle 15.2^\circ\times 1.4\angle -86^\circ = 8.83\angle -70.8^\circ = \underline{2.9 - j\,8.35}$$

$$(FN+HL) = -1.2 + j\,16.65$$

$$(FN+HL) = 16.6\angle 94^\circ$$

$$(FN+HL)(DF+CG) = 16.6\angle 94^\circ\times 43\angle 15.7^\circ = 714\angle 109.7^\circ = -241 + j672$$

$$(EF+CH)(FM-GL) = 552\angle 75.2^\circ = \underline{141 + j535}$$

$$[(FN+HL)(DF+CG)+(EF+CH)(FM-GL)] = -100 + j1207$$

$$[(FN+HL)(DF+CG)+(EF+CH)(FM-GL)] = 1210\angle 94.5^\circ$$

$$I_2 = \frac{11{,}000\,F(DF+CG)}{[(FN+HL)(DF+CG)+(EF+CH)(FM-GL)]} = \frac{2{,}035{,}000\angle 50.7^\circ}{1210\angle 94.5^\circ} = 1680\angle -43.8^\circ$$

Substituting in equation (30) and solving for I_4,

$$I_4 = \frac{11{,}000\,F(EF+CH)}{[(FN+HL)(DF+CG)+(EF+CH)(FM-GL)]}$$

$$I_4 = \frac{11{,}000 \times 4.3\,\underline{|35^\circ} \times 18.7\,\underline{|16.2^\circ}}{1210\,\underline{|94.5}} = \frac{805000\,\underline{|51.2^\circ}}{1210\,\underline{|94.5^\circ}}$$

$$I_4 = 730\,\overline{|43.3^\circ}$$

$$I_5 = \frac{DI_4 - EI_2}{E} = \frac{(5.10\,\overline{|26.6} \times 730\,\overline{|43.3^\circ}) - (6.07\,\overline{|90^\circ} \times 1680\,\overline{|43.8^\circ})}{4.25\,\underline{|68.8}}$$

$$DI_4 = 5.10\,\overline{|26.6^\circ} \times 730\,\overline{|43.3^\circ} = 3720\,\overline{|69.9^\circ} = 1300 - j3500$$

$$EI_2 = 6.07\,\overline{|90^\circ} \times 1680\,\overline{|43.8^\circ} = 10{,}200\,\overline{|133.8^\circ} = -7500 - j7400$$

$$(DI_4 - EI_2) = 8800 + j3900$$

$$(DI_4 - EI_2) = 9{,}600\,\underline{|23.8^\circ}$$

$$I_5 = \frac{9600\,\underline{|23.8^\circ}}{4.25\,\underline{|68.8}} = 2260\,\overline{|45^\circ}$$

$$I_3 = I_4 + I_5 - I_2$$

$$I_4 = 730\,\overline{|43.3^\circ} = 532 - j501$$

$$I_5 = 2260\,\overline{|45^\circ} = 1600 - j1600$$

$$(I_4 + I_5) = 2132 - j2101$$

$$I_2 = 1680\,\overline{|43.8^\circ} = 1215 - j1185$$

$$I_3 = 917 - j1016$$

$$I_3 = 1370\,\overline{|48^\circ}$$

Following is a tabulated list of the values of current cumputed

$$I_5 = 2260\,\overline{|45^\circ}$$

$$I_4 = 730\,\overline{|43.3^\circ}$$

$$I_3 = 1370\,\overline{|48^\circ}$$

$$I_2 = 1680\,\overline{|43.8^\circ}$$

Conclusions.

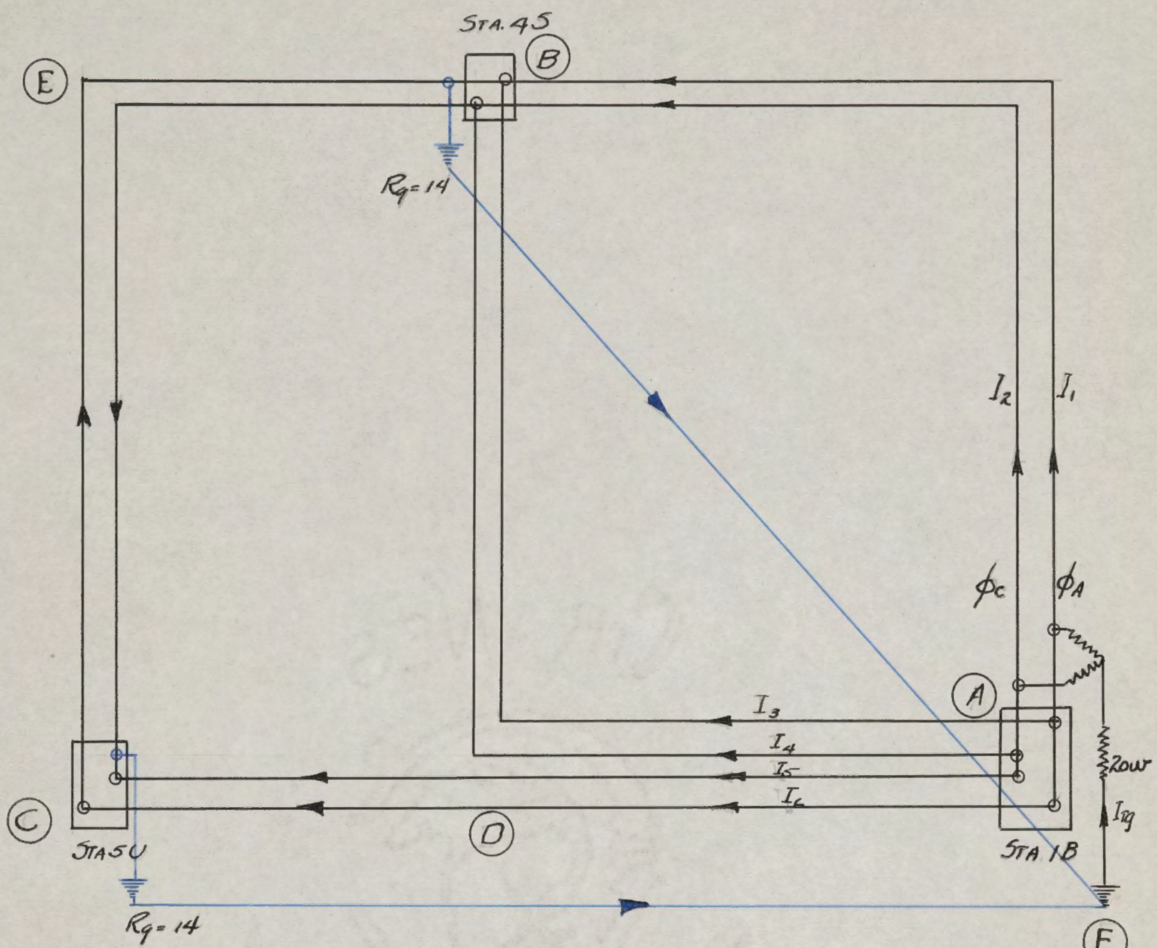
From the foregoing calculations it may be seen that it is possible to calculate the currents that will flow in the various conductors of the system during a fault, by an application of the flux linkage equations. However, it is obvious that this method of determination is by no means a simple problem.

It is the opinion of the authors that many actual problems of this nature, that occur in distribution systems, could be satisfactorily solved by an application of the flux linkage theory as has been followed out in this thesis.

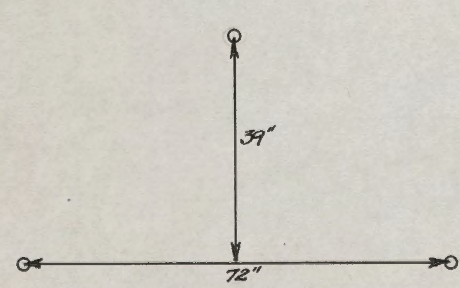
VI Appendix

This paper deals with a computation of a distribution system in which assumed data is used, The main body gives a method of determining conditions, and shows the necessary application of fundamental flux linkage equations, in order to arrive at results. It has been stated that this study is a result of peculiarities existing in the Tennessee Public Service Company's distribution system within the city of Knoxville. The purpose of this appendix is to present the particular section in question, with preliminary flux linkage equations set up and all data with regard to problem given in order that any interested person might continue the study.

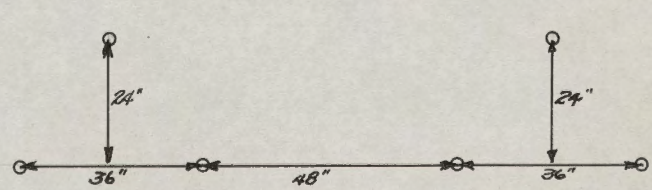
Following is a diagram showing a layout of the section with sub-stations indicated and necessary data regarding conductor spacing, size of conductors, length of lines and etc.



JUNCTION	AD	7500'	DOUBLE	CONSTRUCTION	4/0	STRANDED COPPER
JUNCTION	DC	7500'	SINGLE	CONSTRUCTION	4/0	STRANDED COPPER
JUNCTION	CE	9000'	DOUBLE	CONSTRUCTION	4/0	STRANDED COPPER
JUNCTION	EB	3000'	SINGLE	CONSTRUCTION	4/0	STRANDED COPPER
JUNCTION	BD	2400'	DOUBLE	CONSTRUCTION	4/0	STRANDED COPPER
JUNCTION	BA	11,200'	SINGLE	CONSTRUCTION	2/0	STRANDED COPPER



SPACING - DOUBLE CONSTRUCTION



SPACING - SINGLE CONSTRUCTION

Setting up flux linkage equations for the various parts of the circuit in terms of the unknown currents, we find that there are 14 different flux linkage equations for the circuit. The double subscript notation used in the following equations identifies the part of the circuit to which the equation applies. Hence, a reference to the diagram of the circuit will aid in the understanding of the equations.

$$\phi_{1AB} = I_1\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_1}\right) + 0.2 I_2 Ln \frac{1}{72} \tag{1}$$

$$\phi_{2AB} = I_2\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_1}\right) + 0.2 I_1 Ln \frac{1}{72} \tag{2}$$

$$\phi_{3DB} = I_3\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_2}\right) + 0.2 I_4 Ln \frac{1}{36} \tag{3}$$

$$\phi_{4DB} = I_4\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_2}\right) + 0.2 I_3 Ln \frac{1}{36} \tag{4}$$

$$\phi_{5DC} = I_5\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_2}\right) + 0.2 I_6 Ln \frac{1}{72} \tag{5}$$

$$\phi_{6DC} = I_6\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_2}\right) + 0.2 I_5 Ln \frac{1}{72} \tag{6}$$

$$\phi_{6CE} = I_6\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_2}\right) - 0.2\left(I_2 + I_4\right) Ln \frac{1}{36} \tag{7}$$

$$\phi_{6CE} = I_6\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_2}\right) - 0.2\left(I_2 + I_4\right) Ln \frac{1}{72} \tag{8}$$

$$\phi_{(2+4)BE} = \left(I_2 + I_4\right)\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_2}\right) - 0.2 I_6 Ln \frac{1}{72} \tag{9}$$

$$\phi_{(2+4)EC} = \left(I_2 + I_4\right)\left(\frac{\mu}{8\pi} + 0.2 Ln \frac{1}{r_2}\right) - 0.2 I_6 Ln \frac{1}{36} \tag{10}$$

$$\phi_{3AD} = I_3\left(\frac{\mu}{8\pi} + 0.2 Ln\frac{1}{r_2}\right) + 0.2 I_4 Ln\frac{1}{36} + 0.2 I_5 Ln\frac{1}{84} + 0.2 I_6 Ln\frac{1}{120} \quad (11)$$

$$\phi_{4AD} = I_4\left(\frac{\mu}{8\pi} + 0.2 Ln\frac{1}{r_2}\right) + 0.2 I_3 Ln\frac{1}{36} + 0.2 I_5 Ln\frac{1}{48} + 0.2 I_6 Ln\frac{1}{84} \quad (12)$$

$$\phi_{5AD} = I_5\left(\frac{\mu}{8\pi} + 0.2 Ln\frac{1}{r_2}\right) + 0.2 I_3 Ln\frac{1}{84} + 0.2 I_4 Ln\frac{1}{48} + 0.2 I_6 Ln\frac{1}{36} \quad (13)$$

$$\phi_{6AD} = I_6\left(\frac{\mu}{8\pi} + 0.2 Ln\frac{1}{r_2}\right) + 0.2 I_3 Ln\frac{1}{120} + 0.2 I_4 Ln\frac{1}{84} + 0.2 I_5 Ln\frac{1}{36} \quad (14)$$

To simplify these equations, set up the following constants:

Let $\left(\frac{\mu}{8\pi} + 0.2 Ln\frac{1}{r_1}\right) = A$

Let $\left(\frac{\mu}{8\pi} + 0.2 Ln\frac{1}{r_2}\right) = B$

Let $0.2 Ln\frac{1}{36} = C$

Let $0.2 Ln\frac{1}{72} = D$

Voltage drop per 1000 feet =

$$IX = \omega k \phi$$

$$k = 3.05 \times 10^{-4}$$

$$IZ = IR + jIX$$

Let R_1 = resistance of No. 2/0 conductor, per 1000 feet.

Let R_2 = resistance of No. 4/0 conductor, per 1000 feet.

Substituting in equations (1) through (14), these constants, as obtained above.

$$I_1 Z_{1AB} = 11.2 I_1 R_1 + j\omega k\, 11.2 \left[I_1 A + I_2 D\right] \quad (15)$$

$$I_2 Z_{2AB} = 11.2 I_2 R_1 + j\omega k\, 11.2[I_2 A + I_1 D] \qquad (16)$$

$$I_6 Z_{6CEB} = 12 I_6 R_2 + j\omega k[12 I_6 B - 9(I_2 + I_4) C - 3(I_2 + I_4) D] \qquad (17)$$

$$I_{2+4} Z_{2+4\,BEC} = 12(I_2 + I_4) R_2 + j\omega k[12(I_2 + I_4) B - 3 I_6 D - 9 I_6 C] \qquad (18)$$

Let

$$0.2 \, Ln \frac{1}{48} = E$$
$$0.2 \, Ln \frac{1}{84} = F$$
$$0.2 \, Ln \frac{1}{120} = G$$

$$I_3 Z_{3ADB} = 9.9 I_3 R_2 + j\omega k[9.9 I_3 B + 9.9 I_4 C + 7.5 I_5 F + 7.5 I_6 G] \qquad (19)$$

$$I_4 Z_{4ADB} = 9.9 I_4 R_2 + j\omega k[9.9 I_4 B + 9.9 I_3 C + 7.5 I_5 E + 7.5 I_6 F] \qquad (20)$$

$$I_5 Z_{5ADC} = 15 I_5 R_2 + j\omega k[15 I_6 B + 7.5 I_3 G + 7.5 I_4 F + (7.5C + 7.5D) I_5] \qquad (21)$$

$$I_6 Z_{6ADC} = 15 I_6 R_2 + j\omega k[15 I_6 B + 7.5 I_3 G + 7.5 I_4 F + (7.5C + 7.5D) I_5] \qquad (22)$$

Applying Kirchhoff's laws of voltage and current we may write the following equations.

$$I_1 Z_{1AB} = I_3 Z_{3ADB} = I_6 Z_{6ACEB} \qquad (23)$$

$$I_2 Z_{2AB} = I_4 Z_{4AB} \qquad (24)$$

$$I_2 Z_{2AB} + I_{2+4} Z_{2+4\,BEC} = I_5 Z_{5AC} \qquad (25)$$

$$I_1 Z_{1AB} + (I_{gBF}) 14 = I_5 Z_{5AC} + (I_{gCF}) 14 \qquad (26)$$

$$I_1 Z_{1AB} + (14 I_{gBF}) + I_{Tg}(20) = 13{,}800 \qquad (27)$$

$$I_{Tg} = I_{gBF} + I_{gCF} \qquad (28)$$

$$I_{gBF} = I_1 + I_3 + I_6 \qquad (29)$$

$$I_{gCF} = I_2 + I_4 + I_5 \qquad (30)$$

APPROVAL

The foregoing thesis is hereby approved as a creditable study of an engineering subject, carried out and presented in a manner sufficiently satisfactory to warrant its acceptance as a prerequiste to the degree for which it has been submitted. It is to be understood that by this approval the undersigned does not necessarily endorse or approve any statement made, opinions expressed, or conclusions drawn therein, but approves this thesis only for the purpose for which it is submitted.

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