Extension of zigzag search algorithms for power system multi-objective optimization

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ABSTRACT

The work presented in this thesis focuses on the application and extension the zigzag search algorithms in power systems. The zigzag search method is a multi-objective algorithm which has recently been applied in multiple engineering fields, such as oil well replacement, with fast computational time and accurate results.

Multi-objective optimization algorithms in power systems have been investigated for years. Most of the literatures focus on evolutionary algorithms (EA) such as a non-dominated sorting genetic algorithm (NSGA) or multi-objective particle swarm optimization (MOPSO) for their simplicity and ease of implementation. However, there have been several issues regarding the evolutionary algorithm (EA). For example, the computational time of EA is significant and the parameter configurations are complicated. Other approaches mainly reply on the weight sum method by lumping together different objective functions to form a new single objective function; however, the priority is hard to determine and the characteristic between different objectives may be lost.

In order to improve the performance of power system multi-objective optimization problems, this thesis will first introduce the zigzag search algorithm. Second, by modifying the classic zigzag search algorithm, the zigzag interior point method and zigzag genetic algorithm method will both be proposed to broaden the applications of the classic zigzag search method. Also, in order to provide a systematic method for step-size configuration, a zigzag search method with adaptive step-size will be proposed. Thirdly, all

algorithms will be applied to several practical power system multi-objective problems to demonstrate their practicability and effectiveness.

The case study will be carried out on a modified IEEE 30-bus system and the IEEE 118-bus system. A comparison will be made with classic multi-objective algorithms which have been widely applied in power systems to demonstrate the effectiveness and efficiency of the proposed zigzag search methods.

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CHAPTER ONE
INTRODUCTION AND GENERAL INFORMATION

1.1 General optimization background

The process of minimizing or maximizing objective functions by adjusting the decision variables while satisfying a set of constraints is called optimization [1]. It is a mathematics tool to provide guidelines for decision makers. In almost all real world decision making processes, optimization is an indispensable part.

For example, in a decentralized electricity market, independent system operators (ISO) need to optimize the unit commitment and economic dispatch problems to determine the commitment status of all generation units and optimal outputs for committed units.

In steel making plants, there are six steps: iron making, primary steel making, secondary steel making, continuous casting, primary forming, and manufacturing. The process time for each step is different. Therefore, optimization is utilized to enhance the coordination of each steps.

In the modern stock market, the optimization technique is used to determine the optimal portfolios of different types of stocks.

There are numerous categories of optimization problems that have been proposed. Linear programming means both the objective functions and constraints are linear [2]. Integer programming studies linear systems where some or all the decision variables are in integer value [3]. Quadratic programming allows the objective function to be quadratic but the constraint sets are linear equalities or inequalities [4]. Stochastic programming attempts to include uncertainty behaviors into an optimization problem [5]. From the

viewpoint of the structure of objective functions, optimization problems can be specified as two types: single objective optimizations and multi-objective optimizations.

1.1.1 Single objective optimization

The single objective optimization is to obtain the so called "best" solution which is an objective function's minimum or maximum value [1]. It enables the decision maker to get a view of the nature of the problems. There have been many algorithms developed for single objective optimization. Several algorithms which will be related to the zigzag search methods are briefly reviewed as follows.

▲Steepest descent algorithm

The Steepest Descent Algorithm is a common algorithm for non-constrained optimization problems. It is based on the first order derivative to find the local minimum [6]. In figure 1.1, the blue circle is the contour for the objective function.

It is obvious the fastest way to obtain the optimal value is to follow the red line by equation (1) where $a_n$is the current solution, $a_{n+1}$is the next

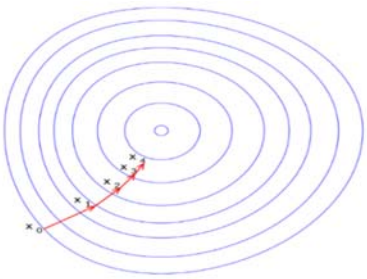

Figure 1.1 steepest descent algorithm

solution, $\partial F(\mathrm{a}_n)$ is the gradient of objective function at current solution, and s is the predetermined step size.

$$a_{n+1} = a_n - s \times \partial F(\mathrm{a}_n) \tag{1}$$

Then by a sequent of iterations, the optimal value is obtained.

▲Interior point method

Interior point method is another common type of method for convex optimization [7]. It aims to iteratively approach the optimal solution from the interior feasible set by forming barrier function. A general form of convex optimization model is shown in Eq. (2)-(5).

$$\min_x f(x) \tag{2}$$

s.t.

$$g_i(x) \geq 0, i = 1, 2, ..., m_1 \tag{3}$$

$$h_j(x) = 0,\ j = 1, 2, ..., m_2 \tag{4}$$

$$x \geq 0 \tag{5}$$

By reformulating it into Eq. (6)-(8), all the iterations will be ensured to remain in the feasible set. Here the barrier function is predetermined as logarithmic term but it can be other type

$$\min_x B(x, \mu) \tag{6}$$

s.t

$$h_j(x) = 0,\ j = 1, 2, ..., m_2 \tag{7}$$

$$B(x, \mu) = f(x) - \mu(\sum_{i=1}^{m_1} \log(\mathrm{g}_i(x)) + \sum_{l=1}^{n} \log(x_l)) \tag{8}$$

Then, by relaxing the equality constraints, the Lagrange function is formed as shown in Eq. (9).

$$f(x) - \mu(\sum_{i=1}^{m_1} \log(g_i(x)) + \sum_{l=1}^{n} \log(x_l)) - \sum_{j=1}^{m_2} \lambda_j h_j(x) \tag{9}$$

$$\nabla_\lambda L(x, \lambda) = 0 \tag{10}$$

$$\nabla_x L(x, \lambda) = 0 \tag{11}$$

The Karush-Kuhn-Tucker (KKT) condition equations are applied to solve the Lagrange function, as shown in Eq. (10)-(11). There have been several modifications on the original interior point method, such as prime-dual interior point or conjugate interior point method.

▲Genetic algorithm

The genetic algorithm (GA) is also a relatively new approach for single objective optimization. Traditional algorithms normally have requirements on either the form of objective functions or the constraints, and when the parameter set is large, the derivative is hard to obtain. The idea of GA is inspired by natural evolution. It can be divided into five parts: encoding; fitness function evaluation; selection; recombination; evolution scheme [8]. A classic encode method uses a bit string scheme, which means by choosing from {0, 1} a series of solutions can be formed. Fitness function evaluation assesses the value of objective functions. Therefore, the quality of each solution is determined. Selection is based on the value of fitness function evaluation. Those solutions that have better objective function values will have higher chances of being selected. A typical selection method is the Roulette Wheel method, which assigns a possibility to each solution by the solution's proportion of the sum of the fitness function value of all the solutions. Recombination recombines the previous population to form the next generation according to the possibility assigned in the selection.

Crossover and mutation are two key factors. Crossover switches some bits in the selected two parent solutions to form a child solution. Then, by generating a random number between 0 and 1, it determines if the crossover operation occurs. The mutation determines whether to flip a bit in the new solution according to a random number between [0, 1]. Therefore, a new population can be formed after mutation and crossover. In the end, evolution tests if the new population satisfied the stop criterion.

1.1.2 Multi-objective optimization

In many real-world decision making processes, multiple goals need to be considered, such as minimizing the risk while maximizing the profit. In this case, single objective optimization is not enough, because there will exist multiple objective functions and the best solution will no longer exist. Multi-objective optimization is able to deal with multiple conflicting objective functions and provide a set of trade-off solutions for decision makers [9]. In the single objective optimization, the comparison between different solutions can be easily determined by objective function values, while if multiple objective functions exist the previous comparison method is no longer useful. Therefore, there are several important definitions that need to be noted.

DEFINITION 1. For feasible solution *x1* and *x2*, *x1* is said to weakly dominate *x2*, denote as $x1 \geq x2$ if equation (12) holds. *F* represent the objective functions. *i* is the index for different objective functions. *m* is the number of the objective functions [10].

$$F_i(x1) \geq F_i(x2) \ \forall i \in 1,2,...m \tag{12}$$

DEFINITION 2. For feasible solution *x1* and *x2*, *x1* is said to strictly dominate *x2*, denote as $x1 > x2$ if equation (13) (14) holds [10]. *F* represent

the objective functions. i is the index for different objective functions. m is the number of the objective functions.

$$F_i(x1) \geq F_i(x2) \ \forall i \in 1,2,...m \tag{13}$$

$$F_i(x1) > F_i(x2) \ \exists i \in 1,2,...m \tag{14}$$

DEFINITION 3. For a set of feasible solutions, if all the solutions in this set is not strictly dominated by another member in this set, then this set is called as non-dominated solution set [9].

DEFINITION 4. The non-dominated solution set over the entire feasible solution space is known as the Pareto optimal solution set [11].

DEFINITION 5. The boundary formed the Pareto optimal solution set is called Pareto optimal front [9].

Roughly speaking, the multi-objective optimization is to obtain the Pareto optimal front solutions [12]. There have been numerous techniques developed especially for multi-objective optimizations. For example, the weighted sum method, ε-constraint method, weighted metric method, Multi-Objective EAs, and a Non-Dominated Sorting GA. Some of those algorithms will be reviewed in detail in Chapter Two.

1.2 Structure

This thesis will be organized as follows:

Chapter 2 will briefly review different techniques of multi-objective optimization solutions which have been widely used in power systems and different types of multi-objective optimization models that have been investigated in power systems.

Chapter Three will present the general approach of the classic zigzag search method, zigzag IP method, zigzag GA method, and zigzag search method with adaptive step-size.

Chapter Four will formulate and analyze the economic emission dispatch problem and economic dispatch considering CVaR when under wind uncertainty.

Chapter Five will show the simulation results, comparing the results from the proposed methods and other algorithms that have been widely applied.

Finally, conclusions and future works will addressed in Chapter Six.

CHAPTER TWO
POWER SYSTEM MULTI-OBJECTIVE OPTIMIZATION

As introduced in Chapter One, single objective optimization is to obtain the best solution for a proposed model, which may not be desirable in a real-world decision making process because it fails to provide trade-offs with respect to concerns from different sides. On the contrary, multi-objective optimization techniques simultaneously deal with two or more conflicting objectives. In many real life applications, the attempt to improve one objective will inevitably lead to the degradation of another [1]. Hence, the multi-objective optimization is able to provide a set of alternative solutions to decision makers. Especially in power systems, much effort has been done on multi-objective optimization. For example, in the decentralized electricity market, the solutions to economic dispatch problems determine the optimal power output for each power plant. Traditional economic dispatch problems only consider fuel cost while satisfying power balance constraint with various security requirements. However, with increasing concerns from environmental protection, economic emission dispatch (EED) starts to lead the direction of research [14] [15] [16]. Similarly, in [17], fuel cost and dynamic security are optimized together; in [18], fuel cost and variability mitigation for the micro grid system are optimized together; in [19], economic aspects and risk impacts are two conflicting objectives when including high wind penetration; or, in [20], investment cost, reliability, and congestion cost are optimized altogether.

Generally, there are two types of methods to solve a multi-objective optimization problem in power systems based on current literatures: the scalarization method and a genetic algorithm.

2.1 Scalarization methods

Scalarization methods will reform the multi-objective optimization problem into single objective optimization. However, it is not desirable in power system application because the scalarization methods will always need parameters that not included in either the objective functions or constraints.

2.1.1 Weight sum method

By assigning the priority to different objective functions, weight sum method is able to reformulate the original multi-objective optimization problem into a single objective optimization problem as shown in equation (15)-(17). For the single objective optimization problem, the methods have been introduced in Chapter One or other classic algorithm can be applied.

$$\min \ F(x) = \sum_{k=1}^{\mathrm{K}} w_k \times F_k(x) \tag{15}$$

$$\textit{subject to } \mathrm{g}_m(\mathrm{x}) \geq 0 \quad \forall \mathrm{m} \in 1,2,....\mathrm{M} \tag{16}$$

$$\mathrm{h}_n(\mathrm{x}) = 0 \quad \forall n \in 1,2,....\mathrm{N} \tag{17}$$

By changing the priority value w_k, different points in the Pareto front can be found. However, it is hard to identify the priority values unless you have extra information for the optimization problem besides the model itself. Furthermore, varying the weights may not result in an accurate Pareto front [21].

In power system application, the weight sum method typically utilizes either traditional methods like lambda iteration, gradient search, bender decomposition [22], and Lagrangian relaxation [23] or population based methods such as a GA [16], the hybrid bacterial foraging Nelder–Mead algorithm [25], gravity search algorithm [26], artificial bee colony, bat algorithm [24] and flower pollination algorithm [27].

2.1.2 ε-constraint method

The ε-constraint method is another way to convert multi-objective optimization into single objective optimization. This method only optimize one objective and reformulates all other objectives into constraints, as shown in equation (18)-(21). ε is an user defined value to confine the other objective functions. By choosing different ε values, the Pareto front can be formed. However, the user defined value ε is hard to justify and the obtained Pareto front may be not evenly distributed [28].

$$\min f_1(\mathrm{x}) \tag{18}$$

$$\textit{subject to}\, f_i(\mathrm{x}) \le \varepsilon \quad \mathrm{i} \in 1,2,....\mathrm{M} \tag{19}$$

$$\mathrm{g}_m(\mathrm{x}) \ge 0 \quad \forall \mathrm{m} \in 1,2,....\mathrm{M} \tag{20}$$

$$\mathrm{h}_n(\mathrm{x}) = 0 \quad \forall n \in 1,2,....\mathrm{N} \tag{21}$$

In [29], the amount of emission cannot exceed the maximum emission amount, and the optimization problems are optimized by a genetic algorithm (GA). In [30], the voltage will be controlled in specified security region.

2.2 Evolutionary method

2.2.1 Non-dominated sorting genetic algorithm-II

The non-dominated sorting genetic algorithm-II (NSGA-II) is a multi-objective optimization tool modified from an NSGA. It is first proposed by [16]. The objective of an NSGA-II algorithm is to perform modification on a set of initial populations until the final solution set is close enough to the true Pareto front. It made two improvements based on NSGA. Firstly, it propose a new fast non-dominant sorting method. Original sorting method in NSGA needed every individual solution to compare with other solutions for each objective function value at each Pareto front level, which made the algorithm slow. The new algorithm needs two entities to be calculated: the domination count n_p and dominated solution numberS_p. Then by reducing n_p from setS_p, each solution is assigned a Pareto domination level. The pseudo code is shown below.

```
def fast_nondominated_sort( P ):
  F = [ ]
  for p in P:
      Sp = [ ]
      np = 0
      for q in P:
          if p > q:
              Sp.append( q )
          else if p < q:
              np += 1
      if np == 0:
```

```
        p_rank = 1
    F1.append( p )
  F.append( F1 )
  i = 0
  while F[i]:
      Q = [ ]
      for p in F[i]:
          for q in Sp:
              nq -= 1
              if nq == 0:
                  q_rank = i+2
                  Q.append( q )
      F.append( Q )
      i += 1
```

Secondly, it made modification on the diversity in order to maintain a good spread of solutions in the obtained set. NSGA utilize sharing function to ensure diversity. However the sharing parameter depend on user experience. The NSGA-II propose a crowded-comparison method to replace the sharing function. The smallest cuboid around current solution is defined as the density estimation.

The overall procedure is shown in figure 2.1.

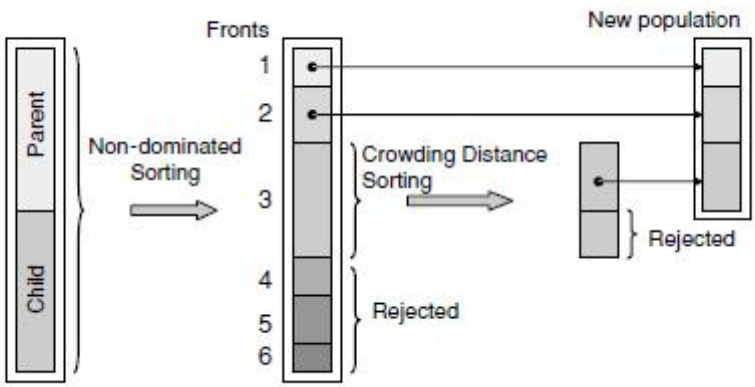

Figure 2.1 main loop of NSGA-II

An NSGA-II has been applied into power system multi-objective optimization for years and successfully achieved satisfactory results. In [31], an NSGA-II is applied to solve the siting and sizing problem of wind farms and FACTS devices. Cost and improvement on voltage profile are both considered. NSGA-II is utilized in [32] to design a power system stabilizer so that the maximum of damped response is obtained for all contingencies. In [33], power loss reduction and reliability are both considered as objectives in order to determine the allocation of reclosers under the load uncertainty.

2.2.2 Multi-Objective particle swarm optimization

For MOPSO, the Parent solutions are generated within the feasible area randomly. For each solution i, a position POS and a velocity VEL are determined. The solutions will update their positions and velocities to move towards the optimal solutions found so far. The current Pareto optimal solutions will be kept in the repository. The procedure of moving towards the optimal solution is shown as follows:

$$\text{VEL(i)} = \chi[VEL(i) + \phi_1 r_1(PBEST(i) - POS(i)) + \phi_2 r_2(REP(h) - POS(i))] \quad (22)$$

$$POS(i) = POS(i) + VEL(i) \quad (23)$$

Here φ_1 and φ_2 are weighting factors which will determine the weight for the local best solution and global best solution; r_1 and r_2 are random numbers within the range [0-1]. χ is calculated as shown in (24):

$$\chi = \begin{cases} \dfrac{2k}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|} & \text{if } \varphi \geq 4 \\ k & \text{if } 0 \prec \varphi \prec 4 \end{cases} \quad (24)$$

where 0< k<1and φ=φ_1+φ_2,with $\varphi_1 = \varphi_2 = 2.05$. *PBEST (i)* is the past optimal position for the particle i; *REP(h)* is a value that is taken from the repository; the Roulette-Wheel selection will decide the index *h*.

The overall steps of MOPSO is presented as follows.

1: Parent solution, velocity, iteration counter are determined.

2: Fitness value calculation.

3: Pareto optimal solution obtained from the non-dominated solution set and set the repository equal to non-dominated solution set.

4: For each solution, the local best solution is first defined as the current position for each particle, form non-dominated set.

5: The local best solution and the global best will be defined for each particle.

6: Update the velocity for each solution.

7: Update each solution's position.

8: Calculate the fitness function for each solution.

9: By use of the non-dominant sorting method, searching for the non-dominated solutions.

10: Expand and update non-dominated global optimal solution set.

11: Expand and update non-dominated local optimal solution set.

12: Update the repository.

13: Determine the local best solution and the global best solution.

14: Check if the maximum iterations is met ?: If it is then stop. Otherwise, go tos 6.

END

MOPSO is also a prevailing multi-objective optimization tool that has been applied in power systems. In [34], MOPSO is used to solve the traditional economic dispatch with maximum generation company profit. Ref [35] made modification on MOSPO to solving the siting and sizing problem of FACTS devices.

CHAPTER THREE
ZIGZAG SEARCH METHOD

3.1 Classic zigzag search method

The zigzag search method was proposed for multi-objective optimization by Dr. Honggang Wang in 2012. It tries to find a set of non-dominant solutions sequentially within single optimization iteration by zigzagging tightly around a Pareto front surface [36].

The routine of the zigzag algorithm consists of three steps: Find the First Pareto optimal (FFPO) search, zig search and zag search.

An FFPO search is based on a line search to find the first minimum solution for f1 while maintaining the smallest value of f2. It consists of two major parts: a regular line search will return an optimal solution for f1 and then take a horizontal search for f2 which means a search along the projection of g2 to the hyperplane of g1:

$$g = g_2(x^0) - < g_1(x^0), g_2(x^0) > * g_1(x^0) \tag{25}$$

where <, > is the vector dot production, g1 (x) is the gradient of f1, and g2 (x) is the gradient of f2.

A zig search is trying to find a solution that relaxes the value of f1 somewhat while keeping f2 the same. It projects the gradient of f1 to the hyperplane of f2 :

$$g = g_1(x) - < g_1(x), g_2(x) > * g_2(x) \tag{26}$$

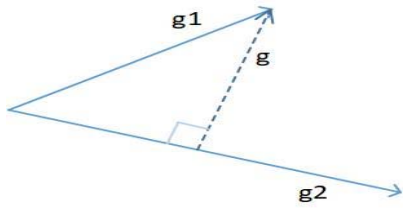

Figure 3.1 projection

Then along this direction, Xn+1=Xn-δ*g will be obtained; δ is step size, as shown in Figure 3.1.

Pseudo code will be shown as follow:

```
if g1 (x0) = 0 then
set g = rand( )
else if  g2 (x0) = 0 then
set  g = g1 (x0)
else
        set α= angle(g1 (x0)，  g2 (x0))
        if α !=pi then
                set g2 (x0) = g2 (x0)/norm(g2 (x0),2)
                set g = g1 (x0) − (g1 (x0)，  g2 (x0)) ×g2 (x0)
{project g1  to the orthogonal plane of g2}
else
        set g = g1 (x0)
end if
end if
set x= x0 + ô×g
set x = project(x) {project x into X}
```

A zag search is similar to a zig search, also searching along the projection of one objective to another. However, it will follow the projection of f2 to f1:

$$g = g_1(x_0) - < g_1(x_0), g_2(x_0) > * g_2(x_0) \tag{27}$$

It is used to find the best solution for f2 while trying to keep f1 the same. Xn+1=Xn-δ*g will be applied. Pseudo code will be shown as follows:

```
set n = 0; xn+1 = xn
while xn+1>=  xn do
        set n = n + 1
        if  g2(xn) = 0 then
                set g2 (xn) = rand( )
        end if
        if  g1 (xn) = 0 then
                set  g= g2 (xn)
        else
        set α= angle(g1 (xn)，  g2 (xn))
        if α !=pi then
                set g1 (xn) = g1 (xn)/norm( g1 (xn),2)
                set g = g2(xn) − <g1 (xn)，  g2 (xn )> ×g1 (xn )
  else
                        set g = g2 (xn)
                end if
        end if
        set  xn+1 = xn - ô ×g; x = project(x)
end while
return  x= xn
```

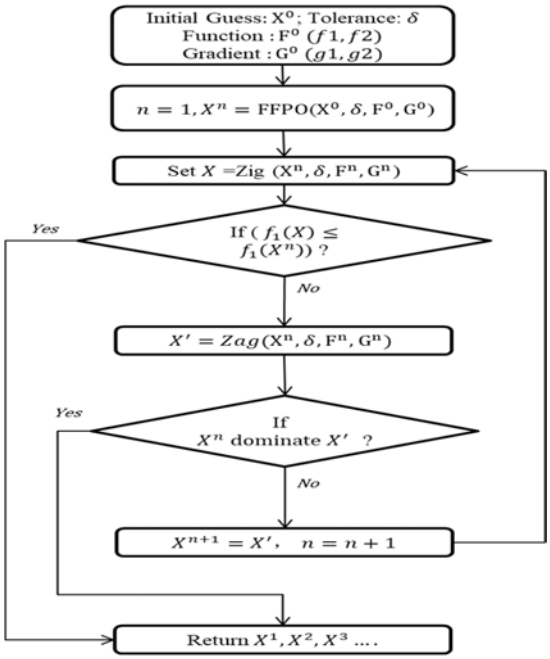

Figure 3.2 zigzag search procedure

Continued zigzagging from the solution obtained from FFPO enables the formation of a whole Pareto front. The simplified procedure of the zigzag method can be found in the above flow chart (Figure 3.2).

3.2 Modification of zigzag search method

A set of new variants of the zigzag search algorithm can be formulated since the zigzag search can also be seen as a framework that can incorporate any desired search method. The classic zigzag search algorithm is based on the line search method and searches from one Pareto optimal to another. Therefore it is desirable if applied in a small system. The classic zigzag will try to decrease its step size in order to keep the solution within the limitations. However, when applied in a large scale power system, there will be many linear or non-linear binding constraints. If the current solution is at the edge

of limitation then any progress on the gradient will violate the constraints. Therefore the zig or zag step may fail or stop early, which will cause the returned solution to be inaccurate and lead to a premature stop for the whole algorithm.

3.2.1 Zigzag interior point method

The zigzag interior point method is proposed for large scale convex problems. Here instead of a line search, the interior point (IP) method is used, in order to improve accuracy and prevent the premature stop issue. The flow chart is shown in Figure 3.3.

The interior point method has been demonstrated as an efficient tool for quadratic convex programming [7]. By relaxing all inequality constraints to form a barrier function, a Newton step is applied to solve the KKT equations. If the Newton step fails, a conjugate gradient method will be applied as a backup option. A zigzag interior point method is therefore proposed implemented with an interior search method. Still zig is a step that relaxes one function somewhat but, instead of following the projection of f1 to f2, at each iteration one objective function will be converted as an equality constraint and the optimization problem can be solved as a single objective optimization problem. It is convenient and effective, which will be tested in the demonstration.

3.2.2 Zigzag genetic algorithm method

Population-based algorithms can also be incorporated into zigzag search algorithms at the researchers' preference. A hybrid version zigzag GA method is also proposed to demonstrate its feasibility and effectiveness. A GA is

chosen for its ease of implementation and speed of convergence among its peers. A GA step is inserted into the zigzag search algorithm. The classic zigzag is fast in getting results but it is sensitive to an initial guess and easily stuck at a local optimal. Population-based methods need randomized parent populations which takes substantial computation time but which will search in a whole solution space to have a certain chance of obtaining the global optimal, which is desirable when encountering non-convex problems. The proposed method aims to combine the advantages of both. If the line search stuck at the local minimum, then the use of the GA may help the solution jump out of it. A GA will be initially used to find the first Pareto solution. After each zigzag step, the angle between the last Pareto solution and the current obtained Pareto solution will be calculated：

$$\alpha=\arcsin\left(\frac{f_1^1-f_1^0}{f_2^0-f_2^1}\right) \tag{28}$$

If the angle is large enough, then the current solution is satisfactory. Otherwise, a GA step can be used. Whenever a zag or zig step fails, a GA step will also be utilized. In this way, the zigzag GA will at least have the same or better results with the classic zigzag and the concern that the randomness of evolutionary algorithms will worsen the situation is eliminated.

The flow chart is given in Figure 3.4. The GA method can also be replaced by any population-based method like particle swarm optimization, flower pollination algorithm, bacterial colony chemotaxis algorithm, or

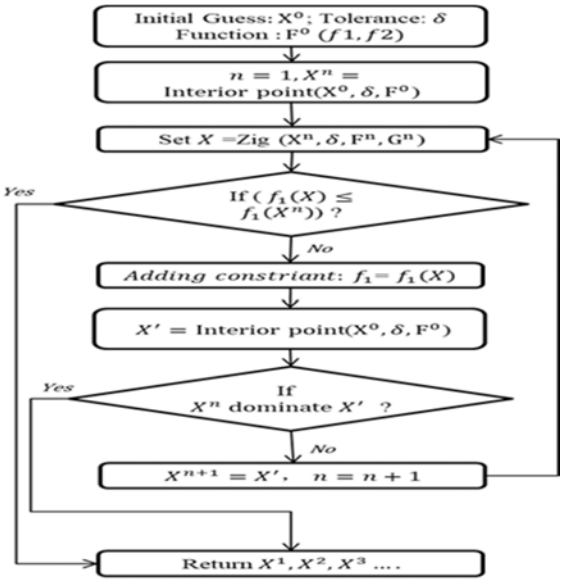

Figure 3.3 zigzag IP procedure

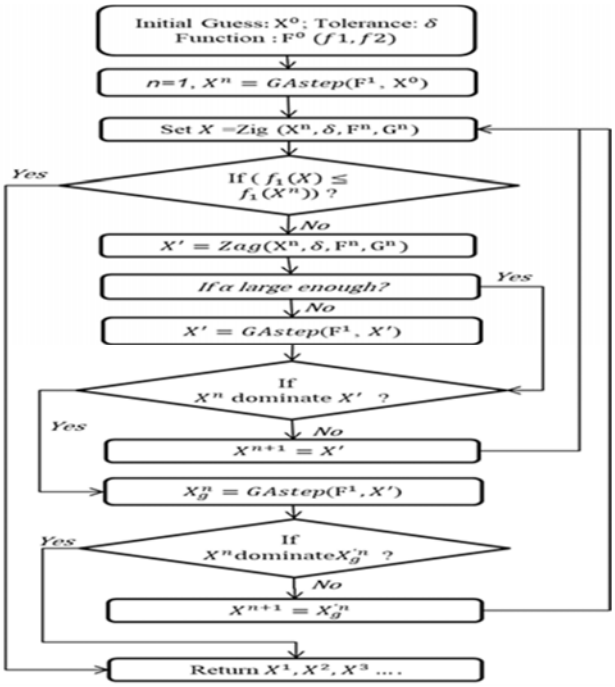

Figure 3.4 zigzag GA procedure

3.2.3 Zigzag search method with adaptive step-size selection

Artificial bee colony algorithm etc. Further researches will focus on how to design a more suitable heuristic algorithm within zigzag framework.

The step-size for a classic zigzag search method is firstly defined by the user. Then in the line search, the current step size will be doubled if better function value is obtained. Otherwise, if the new solution violates any constraints, the step size will be reduced to half until the candidate solution is feasible.

Therefore, if the user defined step size is unsatisfactory, then the quality of the Pareto front is hard to be guaranteed. Especially when the step size is large, the Pareto front solution will be inaccurate. Another way around is to reset the step size to a very small value then it will be doubled until current solution is close enough to the boundary. However, the small step size will make the zig-zag search method bring too much unnecessary solutions.

Inspired by the steepest gradient descent [6], a zigzag search with adaptive step size is proposed to determine the step size automatically. Additionally, instead of using the fixed step size, the desirable range of two adjacent Pareto front solutions can be assigned by users. As shown in equation (29), ε^u and ε^l will give the user the desirable diversity of the Pareto front solution.

$$\varepsilon^s < \left| f_1(\mathrm{X}_{n+1}) - f_1(\mathrm{X}_n) \right| < \varepsilon^u \tag{29}$$

In steepest gradient descent, the step size selection can be attained by applying equation (30). x^k is the current solution; λ is the optimal step size; d^k is the gradient.

$$\min f(x^k + \lambda d^k) \tag{30}$$

$$\min f_2(x^k + \lambda g^k) \tag{31}$$

$$\varepsilon^s < \left| f_1(x^k + \lambda g^k) - f_1(x^k) \right| < \varepsilon^u \tag{32}$$

In the zigzag search method, two objective functions are involved. As equation (25) shows, g^2 will be obtained by the projection of g_1 to g_2.

CHAPTER FOUR
POWER SYSTEM PROBLEMS FORMULATIONS

4.1 Economic emission dispatch

Currently, economic dispatch is the major methodology for ISO to determine the optimal output of each power plant. It is utilized in satisfying the demand with the least cost. However, for traditional units, fossil fuels are the major source of generating electric power. With the increasing demand consumption, environment protection has become a serious problem. The solo cost optimization for economic dispatch no longer satisfied needs. Therefore, economic emission dispatch (EED) serves as an alternation method for ISO to dispatch the units. EED problems are a multi-objective mathematical model which take both costs and pollution into consideration.

4.1.1 Economic emission dispatch formulation

The formulation of objective functions and overall models for EED will be given in this section. Fuel cost and emission will be set as two opposing objective functions while satisfying the power balance and transmission limit constraints.

▲ Objective functions

➢ Minimization of fuel costs

The goal of economic dispatch is to achieve minimization of operation costs through optimal generation dispatch. This function can be defined:

$$F = \sum_{i=1}^{N} F_i(P_{G_i}) \tag{33}$$

where N represents the total number of generators, PGi is the power output of

the generator i and Fi(PGi) is the total generation cost for the generator i. Then F will be the overall operation cost.

Traditional dispatch problem makes an assumption that power output increases quadratically or linearly with power efficiency [37]. Typically, Fi(PGi) is represented by quadratic functions:

$$F_i(P_{G_i}) = \mathrm{a}_i + b_i P_{G_i} + c_i P_{G_i}{}^2 \tag{34}$$

where ai, bi , and ci are fuel cost coefficients of the generator i.

In reality, stream enters the turbine through different set of nozzles. Those nozzles are opened in a sequence to achieve the highest efficiency, which is called valve-point effect. Therefore, there will be a rippled term in fuel cost function, as shown:

$$F_i(P_{G_i}) = \mathrm{a}_i + b_i P_{G_i} + c_i P_{G_i}{}^2 + \left| \mathrm{e}_n \sin(f_n (P_{G,n}{}^{\min} - P_{G,n})) \right| \tag{35}$$

where en and fn are fuel cost coefficients for valve-point effects.

- Minimization of emission

Extreme amount of pollution are generated while power plants provide electricity. For example, Sulfur Oxides, Nitrogen Oxides, Osmium tetroxide, Oxygen difluoride, Perchloryl fluoride, Phosgene, Phosphorus pentafluoride, Selenium hexafluoride or Carbon Dioxide are all detrimental to both the environment and human body.

The relationship between emissions and power output can also be represented as quadratic function [38]:

$$E_i(P_{G_i}) = \alpha_i + \beta_i P_{G_i} + \delta_i P_{G_i}{}^2 \tag{36}$$

where αi, βi , and δi are emission coefficients of the generator i.

▲ **Constraints**

➢ Equality constraint:

$$\sum_{i=1}^{N} P_{G_i} = \sum_{i=1}^{NL} P_{\mathrm{L}_i} \tag{37}$$

where NL is the total number of loads. The equality constraint for an EED problem is power balance. The total amount of generation output must be equal to the sum of demand, in order to achieve secure operation.

➢ Generator output limit

$$P_{G_i}^{\mathrm{Min}} \le P_{G_i} \le P_{G_i}^{\mathrm{Max}} \quad \forall i \tag{38}$$

For secure generator operation, the output of each generator must be within its power loading limits. PMin is the minimum value for generator. PMax is the maximum value for generator. Gi is the index for a specific generator.

➢ Transmission line thermal limit

For the purpose of steady operation of the power system, load flow run at each line should not exceed its thermal limits.

$$\left|P_k\right| \le P_k^{Lim} \qquad \mathrm{k=1,2,3,.....,N} \tag{39}$$

where N is the total number of transmission lines and PLim is the flow limit at a line. K is a specific index for a line. Most researches on EED [24] [26] [27] [39] doesn't consider this constraint since it will derive too many constraints as the test system grows larger. However, transmission limits are extremely important for secure operation. Therefore, in the later case study, we will take the line flow limit into consideration.

4.1.2 Model reformulation

It is clear from the above problem formulations that the economic emission problem is a non-linear multi-objective optimization problem. Decision variables are the outputs for each power plant which will be confined within a certain range. The parameter configuration of the zigzag search algorithm for the EED problem is based on trial and error.

Equality constraint is hard to be dealt with when applying a zigzag search algorithm, which is also an important issue for most optimization algorithms. However, in any economic dispatch related optimization problem there is no escape from power balance constraint. Here, it is dealt with representing output of one power plant with outputs from the others. Then the output of generator j is given by:

$$P_{G_j} = \sum_{i=1}^{NL} P_{L_i} - \sum_{i=1, i\neq j}^{N} P_{G_i}) \tag{40}$$

The optimization problem can be reformulated as shown in the following:

$$\begin{aligned} Min\ F_i(P_{G_i}) &= \sum_{i=1, i\neq j}^{N} a_i + b_i P_{G_i} + c_i P_{G_i}{}^2 + \left| e_i \sin(f_i(P_{G,i}{}^{\min} - P_{G,i})) \right| \\ &+ a_j + b_j(\sum_{i=1}^{NL} P_{L_i} - \sum_{i=1, i\neq j}^{N} P_{G_i}) + c_j(\sum_{i=1}^{NL} P_{L_i} - \sum_{i=1, i\neq j}^{N} P_{G_i})^2 \\ &+ \left| e_j \sin(f_j(P_{G,j}{}^{\min} - (\sum_{i=1}^{NL} P_{L_i} - \sum_{i=1, i\neq j}^{N} P_{G_i}))) \right| \end{aligned} \tag{41}$$

$$\begin{aligned} Min\ E_i(P_{G_i}) &= \sum_{i=1, i\neq j}^{N} \alpha_i + \beta_i P_{G_i} + \delta_i P_{G_i}{}^2 + \alpha_j \\ &+ \beta_j(\sum_{i=1}^{NL} P_{L_i} - \sum_{i=1, i\neq j}^{N} P_{G_i}) + \delta_j(\sum_{i=1}^{NL} P_{L_i} - \sum_{i=1, i\neq j}^{N} P_{G_i})^2 \end{aligned} \tag{42}$$

st:

$$P_{G_i}^{\text{Min}} \le P_{G_i} \le P_{G_i}^{\text{Max}} \ \forall i \in N \ i \ne j \tag{43}$$

$$P_{G_j}^{\text{Min}} \le \sum_{i=1}^{NL} P_{\text{L}_i} - \sum_{i=1, i \ne j}^{N} P_{G_i} \le P_{G_j}^{\text{Max}} \tag{44}$$

$$\left| P_k \right| \le P_k^{Lim} \qquad \text{k=1,2,3,.....,N} \tag{45}$$

In the above optimization formulation, PGj is an unknown variable while other symbols represent parameters. The equality constraint will be enforced as other inequality constraints are satisfied.

4.2 Economic dispatch considering CVaR

Renewable energy like wind power is usually environmental-friendly and cost-efficient, which is beneficial to green and economic operation for the power system [40]. However, because of its intermittent behavior, the randomness and uncertainty it brings will be detrimental to power system secure operation. In decentralized electricity market, economic dispatch is utilized to determine the optimal output for each power plant in terms of fuel cost. With increasing penetration of wind power, a challenge has been posed concerning how to deal with the intermittence of wind power in the economic dispatch problems.

In this section, a multi-objective economic dispatch model under wind generation uncertainty will be proposed with the consideration of both the operation cost and CVaR.

4.2.1 Wind penetration

▲Weibull Distribution

A Weibull probability distribution function (PDF) is a prevailing method to model wind speed distribution [41][42]. The mathematical formulations are

shown from equations (46) - (48).

$$f(\omega)=\frac{r}{c}(\frac{\omega}{c})^{r-1}\exp[-(\frac{\omega}{c})^{r}] \tag{46}$$

$$r=(\frac{\sigma}{\omega_{mean}})^{-1.086} \tag{47}$$

$$c=\frac{\omega_{mean}}{G(1+1/r)} \tag{48}$$

▲Conversion

The wind power output is closely related to the distribution of wind speed. The higher the wind speed, the more wind power will be generated if the wind speed is below the cut-out speed. This paper adopts the conversion method from [43], as shown in equation (49). In this equation, k is a constant number, C_p is the maximum power coefficient, $\rho$is the air density, A is the area for the rotor, V is the velocity of the wind speed, and P_{rated} is the maximum wind power which can be integrated.

$$P_{wind}(\omega)=\begin{cases}0 & 0\le\omega\le\omega_1\\ \frac{1}{2}kC_p\rho AV^3 & \omega_1\le\omega\le\omega_r\\ P_{rated} & \omega_r\le\omega\le\omega_{cut-out}\\ 0 & \omega\ge\omega_{cut-out}\end{cases} \tag{49}$$

4.2.2 Security and risk assessment

The attempt to decrease the cost of generation units in the day-ahead DA market will inevitably increase the scheduled power output of wind energy. However, the real power output for wind is uncertain and assumed to follow the Weibull distribution to be introduced in Section III of this chapter.

Therefore, the over estimation of wind power output will lead to the inability to satisfy the load. As a result, the deficiency will be compensated by buying extra power in the balancing market. The optimal DA market operation cost may correspond to significant financial loss in the RT market.

VaR was proposed by J. P. Morgan in 1996, which is defined by the maximum loss in a portfolio under a certain confidence level [50]. The formulation of VaR can be seen from equation (50) where a is a predetermined confidence level. It normally selected from 0.8, 0.9, 0.95 and 0.99. x is the random variable *and* z is the financial losses.

$$VaR_a(x) = \min\{z \mid f_x(z) \geq a : a \in [0,1]\} \tag{50}$$

CVaR is defined as the expected value of a loss exceeding VaR, as shown in equation (51) and (52) where z is the loss value and F_X is the cumulative probability function.

$$CVaR_a(x) = \int_{-\infty}^{+\infty} z dF_X^a(z) \tag{51}$$

$$F_X^a = \begin{cases} 0 & z < VaR_a(x) \\ \dfrac{F_X(z) - a}{1 - a} & z \geq VaR_a(x) \end{cases} \tag{52}$$

There are several risk assessment techniques that have been widely used in power system application [44] [45] [46]. In this paper, CVaR is employed since it has two advantages: (1) it is a convex optimization; and (2) it is designed to be sensitive to extreme losses. As shown in Figure 4.1, the CVaR value is obtained by calculating the expectation of the shadow area. α is the confidence level.

4.2.3 Model formulation

▲Objective functions

➢ Minimization of fuel costs

The objective of economic dispatch is to determine the optimal output from each unit economically. The operation cost function can be described in as follows

$$F = \sum_{i=1}^{N} F_i(P_{G_i}) \tag{53}$$

where P_{Gi} is the power output for i^{th} generator, F_i is the cost function for each unit, and N is the total number of power plants. Typically, F_i (P_{Gi}) is represented by quadratic functions:

$$F_i(P_{G_i}) = c_i + b_i P_{G_i} + a_i P_{G_i}^{\ 2} \tag{54}$$

where a_i, b_i , and c_i are fuel cost coefficients of the generator i.

➢ Minimization of CVaR

Under wind power uncertainty, the evaluation of CVaR can be formulated in equation (55).

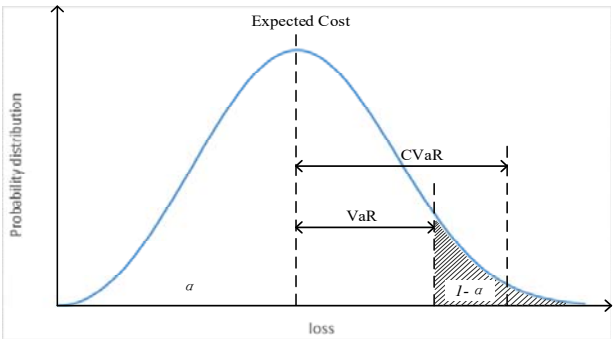

Figure 4.1 illustration of CVaR value

$$F_{\beta}(P_G,\alpha)=\alpha+\frac{1}{q(1-\beta)}\sum_{k=1}^{q}[f(P_G,P_W)-\alpha] \tag{55}$$

where α is the VaR value and β is a predetermined confidence level. k represents a scenario index for wind power output and q is the total scenarios generated. $f(P_G, P_W)$ is the financial loss function in RT market for ISOs, as shown in equation (56).

$$f(P_G,P_W)=\begin{cases}\rho_1\times(P_{W,i}{}^{k}-P_{W,i}{}^{s}) & P_{W,i}{}^{k}-P_{W,i}{}^{s}\geq 0\\ \rho_2\times(P_{W,i}{}^{s}-P_{W,i}{}^{k}) & P_{W,i}{}^{s}-P_{W,i}{}^{k}<0\end{cases} \tag{56}$$

where ρ_1 is the purchase value of extra power at RT market, ρ_2 is the excessive wind power penalty coefficient, $Pk\ w,i$ is the actual wind power and $Ps\ w,i$ is the scheduled wind power.

▲Technical constraints

- Power balance

$$\sum_{i=1}^{N}P_{G_i}+\sum_{i=1}^{NW}P_{W_i}{}^{\mathrm{s}}=\sum_{i=1}^{NL}P_{\mathrm{L}_i} \tag{57}$$

Power balance constraint ensures the secure operation of the power system. The total amount of generation outputs must be able to satisfy the sum of the demand. NL is the total number of loads and NW is the total number of wind plant.

- Generation limits

$$P_{G_i}{}^{\mathrm{Min}}\leq P_{G_i}\leq P_{G_i}{}^{\mathrm{Max}}\quad ,\forall i\in N \tag{58}$$

The output of each generator will be confined in its power loading limits. $PMin\ G_i$and $PMax\ G_i$are the minimum and maximum value for power output respectively.

- Wind power forecast constraint

The forecasted wind power generation is limited by the maximum and minimum wind power capacity.

$$P_{W,i}^{\min} \le P^{s}_{W,i} \le P_{W,i}^{\max}, \forall i \in NW \tag{59}$$

CHAPTER FIVE
SIMULATION RESULTS AND DISCUSSIONS

The zigzag search algorithms are applied to both the IEEE 30-bus system and IEEE 118 bus system. In order to show its effectiveness, the zigzag search algorithms are compared with both the NSGA-II and MOPSO.

All algorithms are implemented in Matlab 2014 and run in a computer with an Intel i7-3720 processor and 8GB RAM.

5.1 Simulation results from economic emission dispatch

5.1.1 Description of test system

▲IEEE 30 bus system

The single line diagram of the IEEE 30-bus system is shown in Figure 5.1 [4]. 6 units are dispatched to fulfill a total 283.4 MW load. Cost and emission coefficients data is shown in Table 1. Load data and branch data can be found at [47].

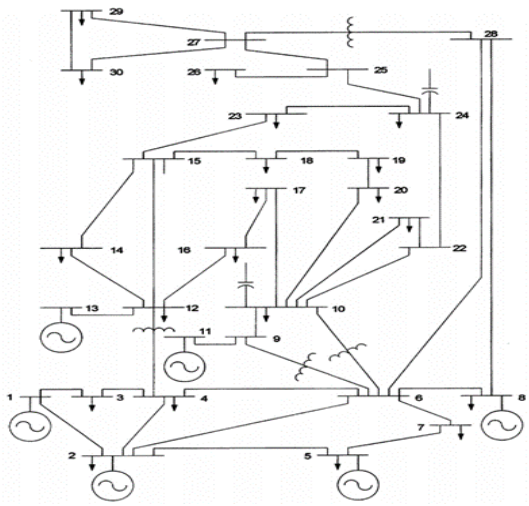

Figure 5.1 one-line diagram of IEEE 30-bus system

Table 1 cost and emission data for 30-bus system

Generator NO	a_1($/ MW)	b_1($/ MW)	c_1($/ MW)	α_1(kg/ MW)	β_1(kg/ MW)	γ_1(kg/ MW)
1	0.00375	2	0	0.0126	-1.1	22.983
2	0.0175	1.75	0	0.02	-0.1	25.313
3	0.0626	1	0	0.027	-0.01	25.505
4	0.00834	3.25	0	0.0291	-0.005	24.9
5	0.025	3	0	0.029	-0.004	24.7
6	0.025	3	0	0.0271	-0.0055	25.3

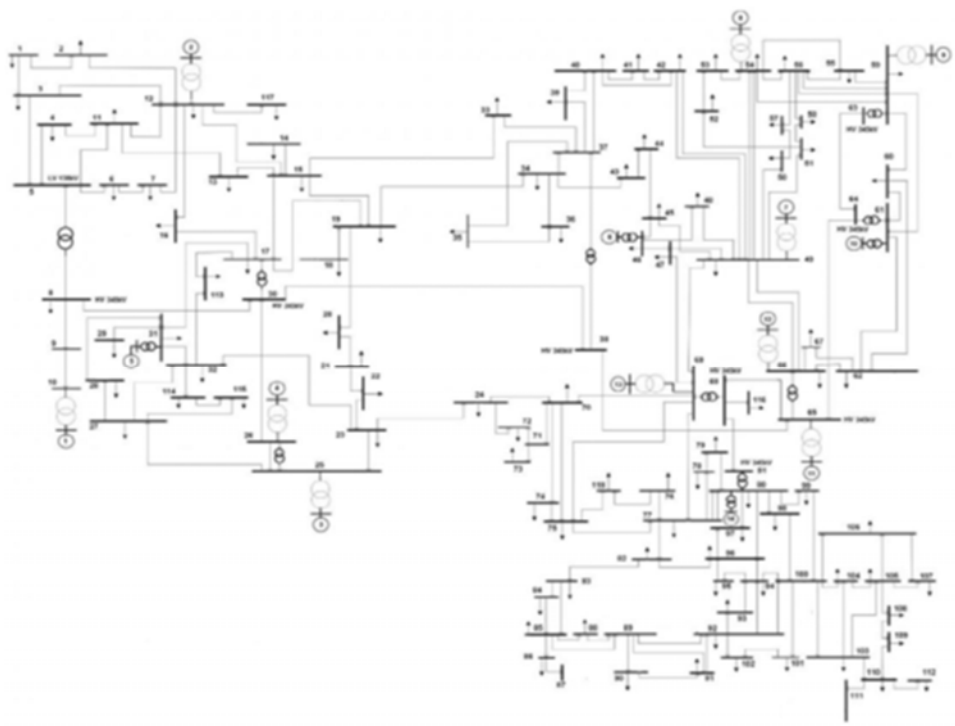

Figure 5.2 one-line diagram of IEEE 118-bus system

Table 2 cost and emission data for 118-bus system

Generator NO	a_1($/MW)	b_1($/MW)	c_1($/MW)	α_1(kg/MW)	β_1(kg/MW)	γ_1(kg/MW)
1	0.022	20	0	0.016	-1.5	23.33
2	0.1176	20	0	0.031	-1.82	21.022
3	0.045	20	0	0.013	-1.249	22.05
4	0.0318	20	0	0.012	-1.355	22.983
5	0.4286	20	0	0.02	-1.9	21.313
6	0.526	20	0	0.007	+0.805	21.9
7	0.049	20	0	0.015	-1.401	23.001
8	0.2083	20	0	0.018	-1.8	24.003
9	0.0645	20	0	0.019	-2	25.121
10	0.0625	20	0	0.012	-1.36	22.99
11	0.0256	20	0	0.033	-2.1	27.01
12	0.0255	20	0	0.018	-1.8	25.101
13	0.0194	20	0	0.018	-1.81	24.313
14	0.021	20	0	0.03	-1.921	27.119

Table 3 computation time and solution number comparison

Algorithm	Run time (s)	Pareto solution
NSGA-II	258.85	120
MOPSO	120.18	200
Zigzag	40.13	897

▲IEEE 118 bus system

A typical IEEE 118-bus system is used to demonstrate the effectiveness of the proposed method. The total load is 2067.5MW which is provided by 14 units. Cost and emission coefficients data can be found in Table 2. The single-line diagram is shown in Figure 5.2. All other related data can be found in [47] [48] [49].

5.1.2 Results from the IEEE 30-bus systems

This test system is a small-size system with six generators. Generator capacity constraints, power balance constraint and transmission limits are all considered. The zigzag search successfully obtain 897 Pareto fronts in 40.1314 seconds, as shown in Table 3. It starts from f1= 767.8439, f2=430.5725 and keeps zigzagging until f1= 827.7445, f2=330.6526 while NSGA-II and MOPSO only obtain 120 solutions and 200 solutions in 258.8536 and 120.1838 respectively.

In order to get enough Pareto fronts, the population size for both MOPSO and NSGA-II is set to a relatively high value. It is obvious that the zigzag search obtains more alternative solutions in a much less computation time than the other two algorithms.

In terms of accuracy, the zigzag search algorithm outperforms NSGA-II and MOPSO, as shown in Figure 5.3 and Figure 5.4.

▲Results from the IEEE 118-bus system

There is hardly any research investigating economic emission problem with IEEE 118-bus systems while considering all the constraints mentioned in problem formulation.

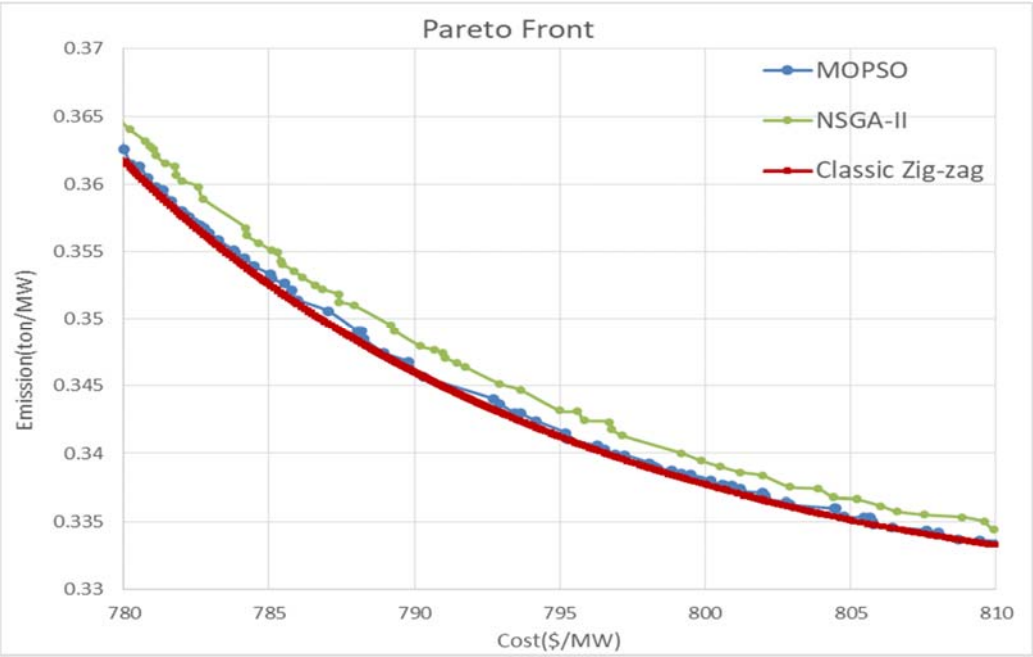

Figure 5.3 detailed Pareto front

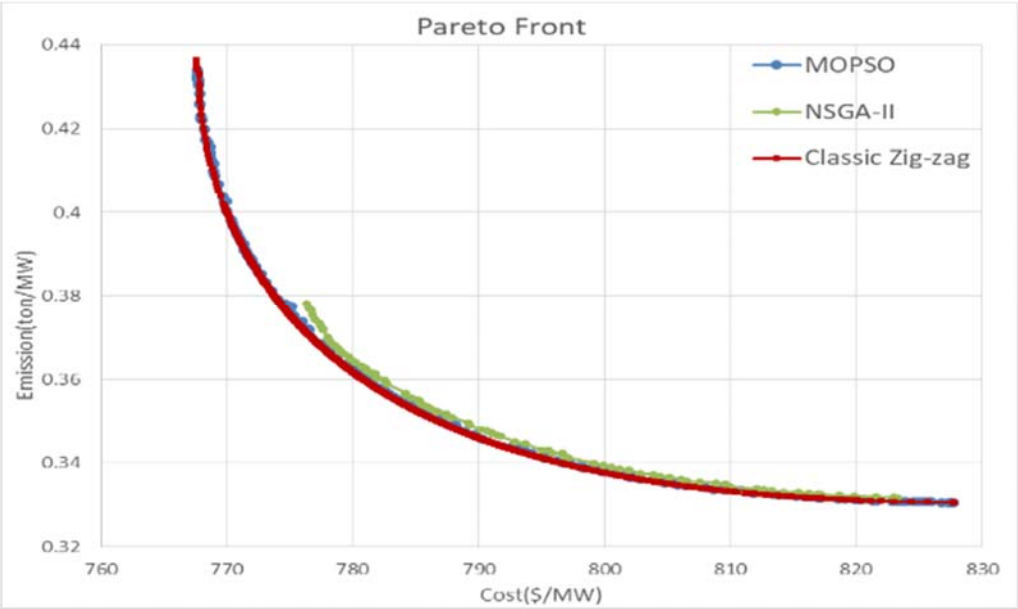

Figure 5.4 comparison by MOPSO, NSGA-ii and classic zigzag search

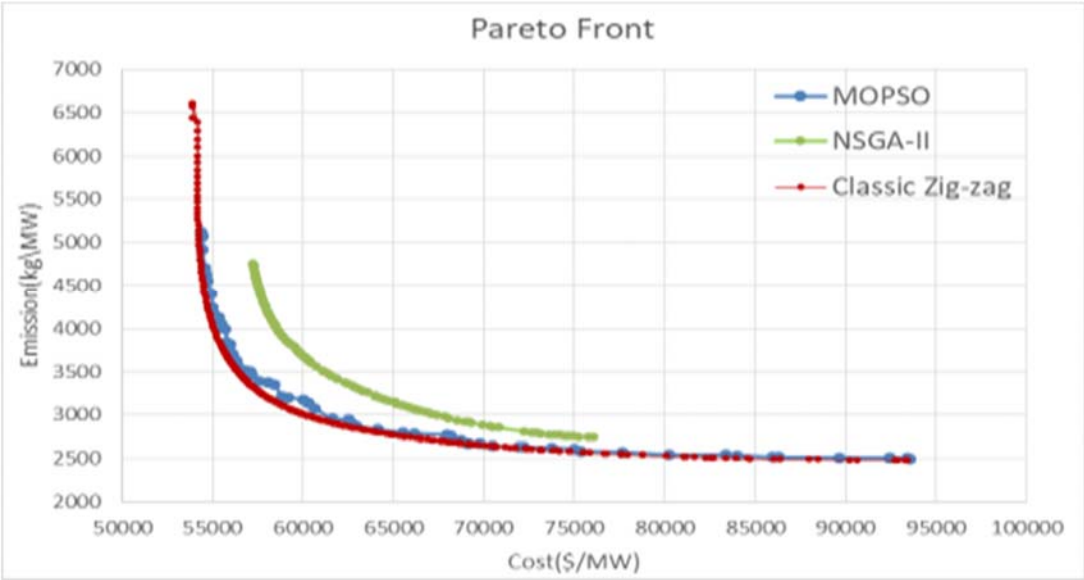

Figure 5.5 comparison for case 1 between MOPSO, NSGA-ii and classic zigzag search

Table 4 computation time and solution number comparison

Algorithm	**Run time (s)**	**Pareto solution**
NSGA-II	780.13	120
MOPSO	664.46	101
Zigzag	314 .61	894

Table 5 best solution comparison

Algorithm	**Best solution(f_1,f_2)**
Zigzag IP	(60080.2, 8121.6)
MOPSO	(60100,8381.3)

Therefore the proposed methods are applied to this system. Comparisons are made and three cases are considered to further reveal the helpfulness of the zigzag method, zigzag IP, and zigzag GA.

- Case 1: ignoring transmission limits

In this case only generation limits and power balance constraints are considered. All the algorithms behave well and consistent Pareto fronts are obtained (Figure 5.5).

The accuracy of NSGA-II starts to decrease while MOPSO still obtains rather accurate results. However, the zigzag search still outperforms these two in terms of accuracy.

The following Table 4 shows that the zigzag search method saves much computational time while obtaining more alternative solutions.

- Case 2: considering transmission limits

When taking transmission limits into consideration, the Pareto front obtained from NSGA-II is so far from the Pareto front for zigzag and MOPSO that it is dropped from the comparison.

The classic zigzag algorithm doesn't form a suitable Pareto front (Figure 5.6). The algorithm stop early if no suitable initial guess is provided, as explained in section III.

In order to overcome this drawback, the zigzag IP is proposed and applied to show its effectiveness. From Figure 5.7, it is obvious that the zig-zag IP outperforms both MOPSO and the classic zigzag algorithm but it is to some extent at the sacrifice of computation time when comparing with the classic zigzag.

If accuracy is the major concern then the zigzag IP is more preferred. If

only a few solutions are needed and computation time is most important, then the classic zigzag can be applied.

Table 5 shows the computation time for each proposed method and Table 6 shows selected best solution for each method.

- Case 3: considering valve-point effect

In this case, an additional sinusoidal term will be applied in the fuel cost function to represent the valve-point effect in addition to those constraints posed on case 2, which will lead the optimization problem to become non-convex. In this situation, although a fair initial guess is provided to the classic zigzag algorithm in order to form the Pareto front, the accuracy is less than satisfactory, as shown in Figure 5.8.

However, the Pareto front is successfully obtained by the zigzag GA (Figure 5.9). The results are also compared with result obtained from MOPSO.

As shown in Table 7, although the zigzag GA consumes more time, it returns more solutions with better accuracy.

5.2.2 Simulation results

▲Wind Penetration

However it is not guaranteed and the result from the zigzag GA may occasionally be the same with the classic zigzag. This is because the Pareto solution returned from the GA is not the same every time but if more computation time is allowed or a better way to initial parent populations, Zigzag GA will have higher chance to return a better result. It is demonstrated that the zigzag frame is suitable for incorporating evolutionary algorithms

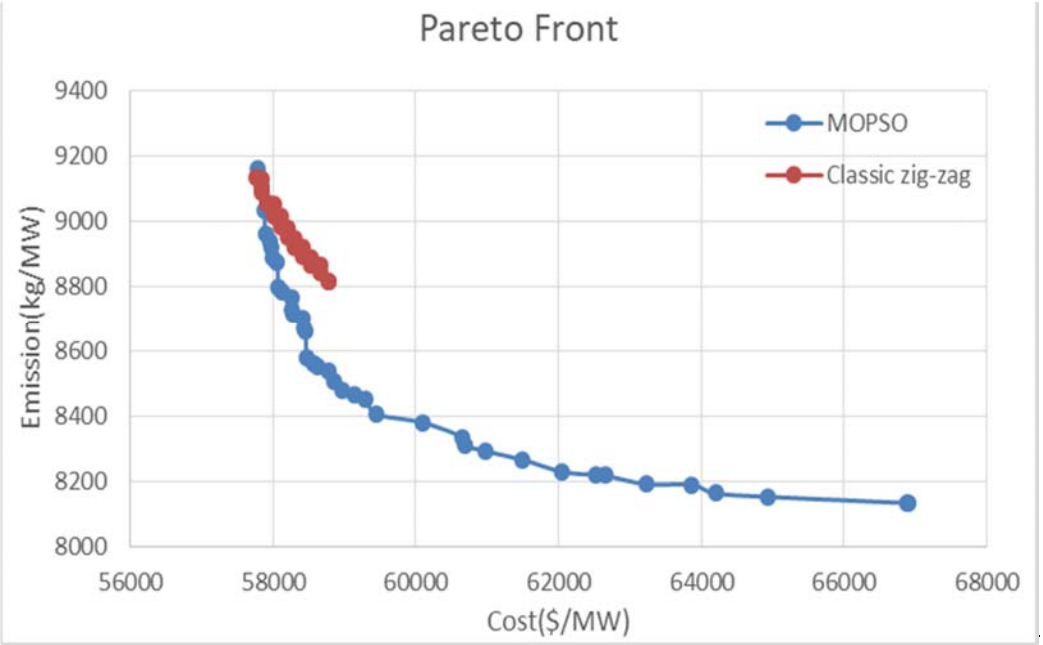

Figure 5.6 comparison for case 2 between MOPSO and zigzag search

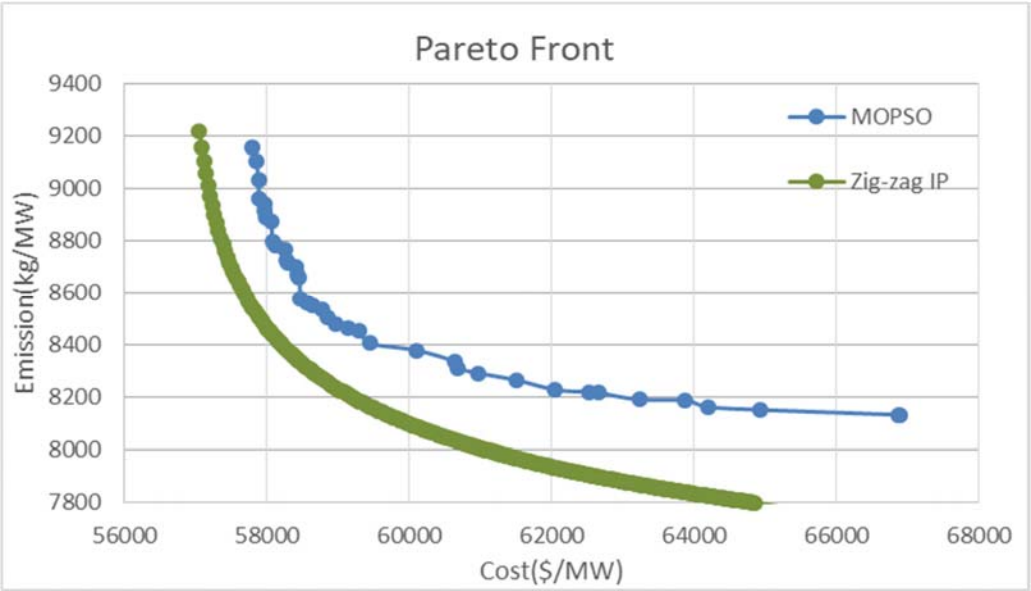

Figure 5.7 comparison for case 2 between MOPSO and zigzag IP

Table 6 computation time comparison

Algorithm	Run time (s)	Pareto solution
Zigzag IP	483.62	651
Classic zigzag	17.64	22
MOPSO	3401.23	39

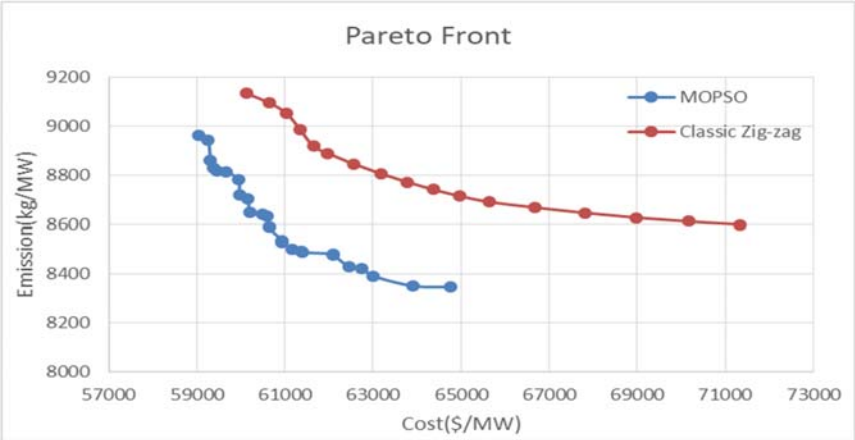

Figure 5.8 comparison for case 3 between MOPSO and classic zigzag

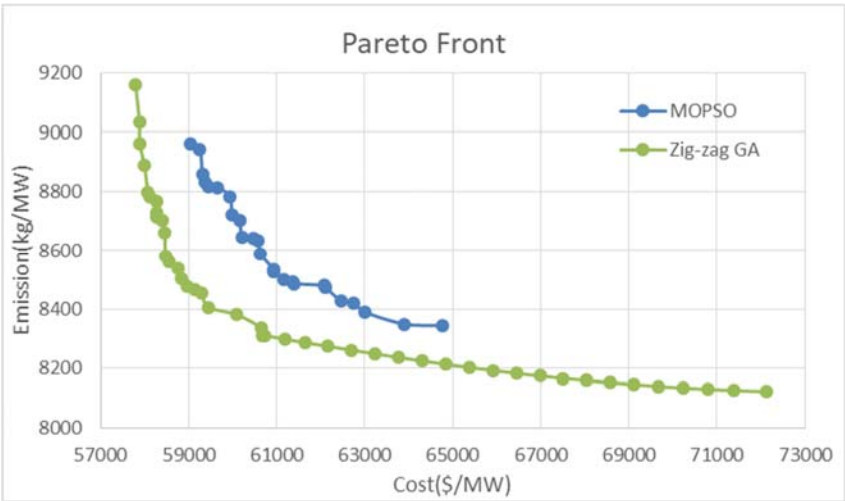

Figure 5.9 comparison for case 3 between MOPSO and zigzag GA

Table 7 computation time and solution number comparison

Algorithm	**Run time (s)**	**Pareto solution**
MOPSO	3628.44	27
Zigzag GA	8876.3	139
Classic Zigzag	134.5	15

Table 8 generation capacity and cost parameters

Generator	P_i^{min}	P_i^{max}	a_i ($/MW)	b_i ($/MW)	c_i ($/MW)
1	5	50	100	200	10
2	5	60	120	150	10
3	5	100	40	180	20
4	5	120	60	100	10
5	5	100	40	180	20
6	5	60	100	150	10

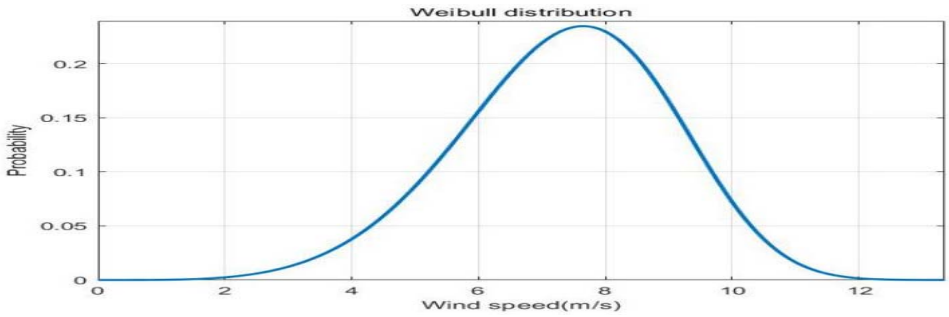

Figure 5.10 wind speed weibull distribution

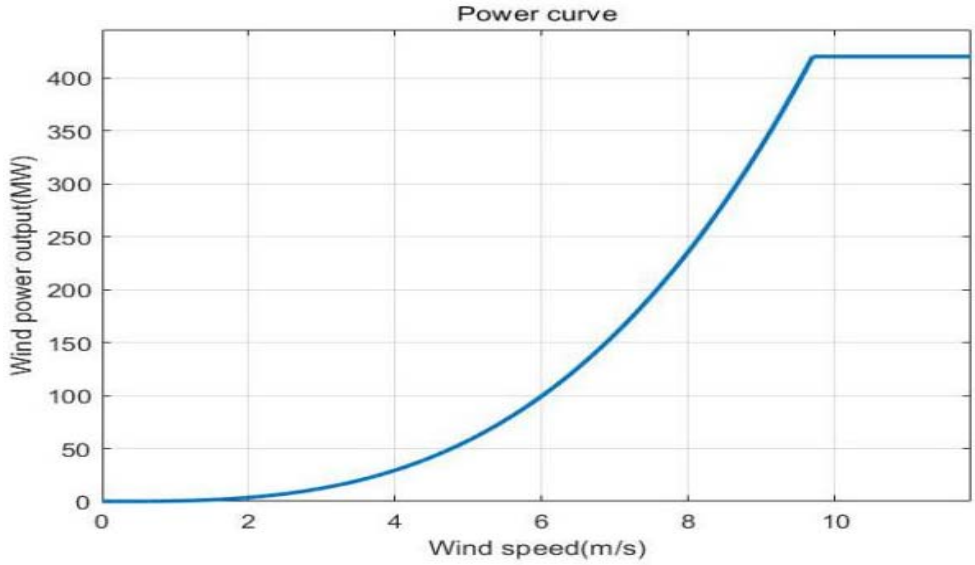

Figure 5.11 wind power output curve

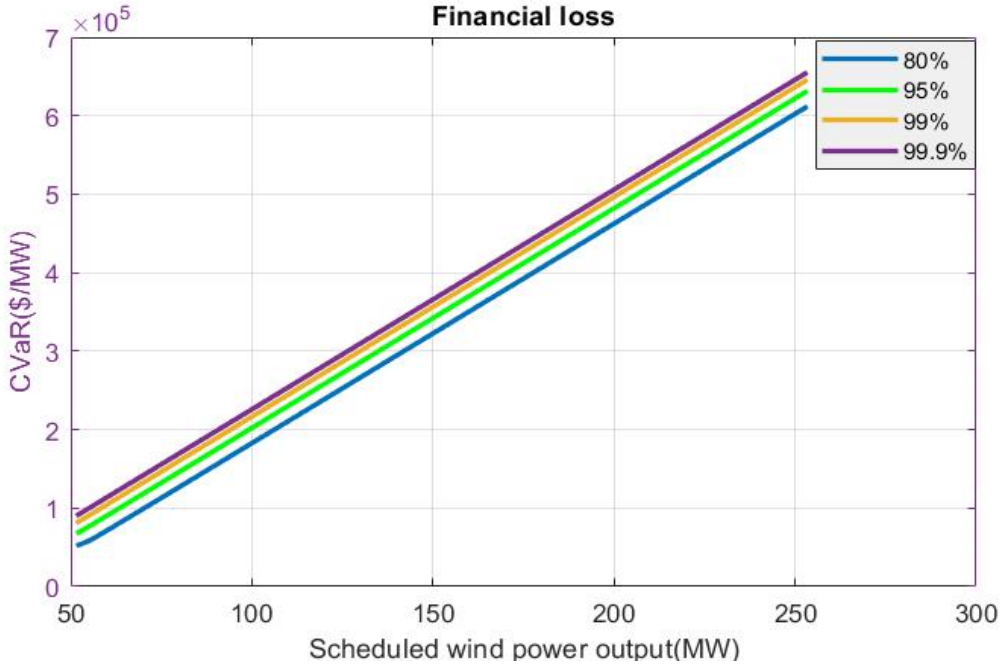

Figure 5.12 CVaR under different wind power integration

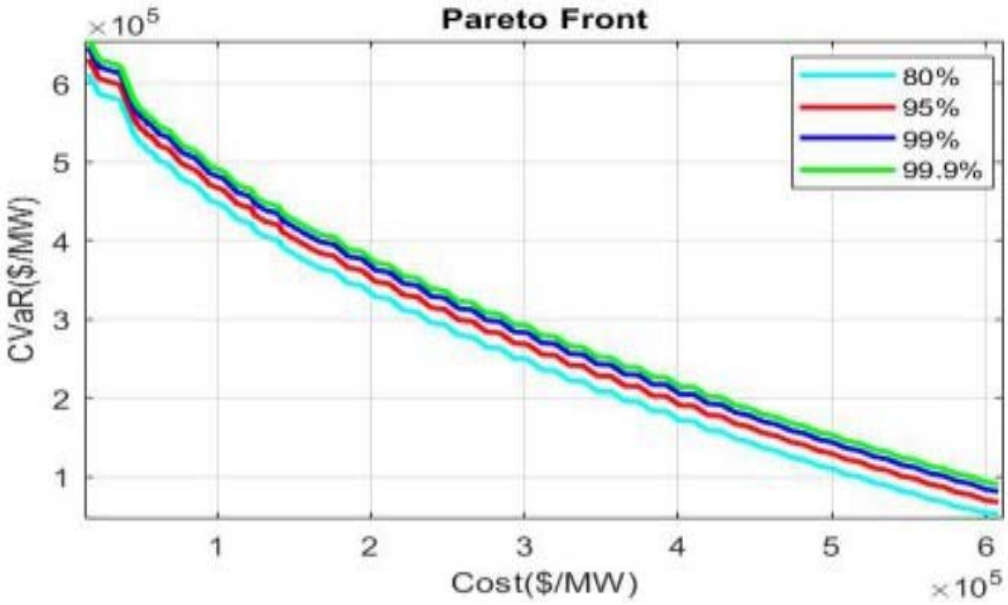

Figure 5.13 pareto front with different confidence level

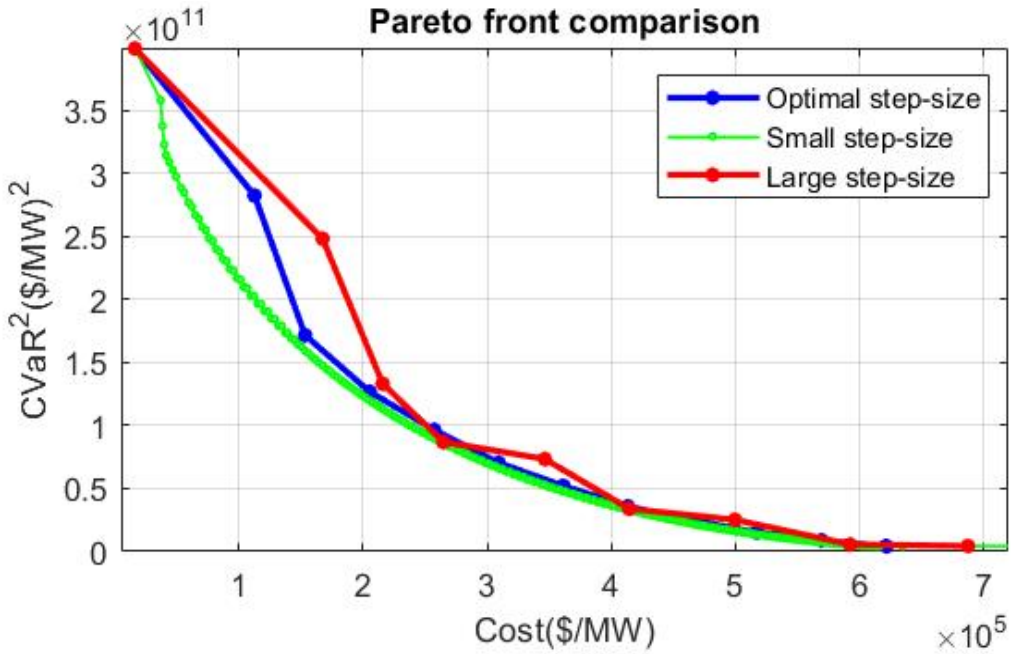

Figure 5.14 pareto front comparison

5.2 Case study for economic dispatch considering CVaR

5.2.1 Description of test system

A modified IEEE 30-bus system is applied and the generation capacity and cost efficient are shown in Table 8. The total electricity demand is 283.4 MW.

Two wind farms are located at the same place with generator 1 and generator 2. Each of the wind farm is composed of 32 wind turbines. The wind speed curve is modeled by Weibull distribution, as shown in Figure 5.10.

The Enercon E-126 EP4 4.2 MW turbine model is selected as a wind turbine model because it has high power output. The r and c are set at 5 and 8, respectively. Power curve is shown in Figure 5.11. According to this distribution, 1000 scenarios are constructed.

▲Financial Risk under wind penetration

Figure 5.12 shows the CVaR values under confidence level 0.8, 0.95, 0.99, 0.999 respectively with different scheduled wind power outputs, from which it can be seen that the CVaR value monotonically increases with scheduled wind power output. When ISO schedules more wind power in the DA market, it will face losing substantial amounts of money in the RT market. In the extreme case when all thermal units are set at the minimum output, wind power scheduled will be 253.4MW which accounts for 89% of total demands, as shown at the endpoint of each line in Fig.2. The CVaR value is more than $600,000. The higher the CVaR value is, the more financial loss may be induced in RT market. Therefore, the financial risk can be reduced if less wind power is scheduled by ISO.

However, increasing wind penetration enables ISO to switch off

traditional thermal units that have high operation costs. The more wind power is scheduled, the more operation cost can be saved from the DA market, as shown in Figure 5.13. At the same extreme case mentioned before, although the CVaR is rather high, the operation costs are reduced to $16,380. With costs decreasing, the CVaR value inevitably increases. Based on the Pareto front, ISO can tradeoff between the possible loss in the RT market and operation cost at the DA market.

▲Comparing Pareto Optimal Solutions

To illustrate the improvements on the algorithm more clearly, instead of using the original CVaR value, the square of CVaR is applied. CVaR under 95% confidence level is selected as the representative case for comparison. In Figure 5.14，the proposed zigzag search method with adaptive step-size is compared with the classic zigzag search method with both the small step-size and large step-size. By applying the small step size, the Pareto front obtained will be accurate but there will be too many Pareto front solutions calculation which may be unnecessary. By applying the large step-size, the accuracy is not guaranteed, the solution keeps jumping away from the true Pareto front and the Pareto solutions may be too sparse. Moreover, if the system gets larger, the accuracy will worsen. The Pareto front obtained by adaptive step-size shows better accuracy than the Pareto front obtained by applying the large step-size and it has a more consistent pattern. After the operation cost is larger than $150,000, the accuracy of Pareto front obtained by the proposed method is almost the same with the small step method. Also, the proposed method avoids calculating unnecessary solutions. The distance of adjacent two Pareto front solutions totally depends on users' preference.

5.3 Simulation result discussion

5.3.1 Zigzag IP search and zigzag GA search

In both the 30-bus system and the 118-bus system, the zigzag search algorithms outperform NSGA-II and MOPSO. It also provides more solutions in less computation time. In the 30 bus system the computation time of the zigzag search is almost 15% of NSGA-II and 33% of MOPSO and Pareto solutions obtained are up to ten times greater. In a 118 bus system, without power flow limits, computation time of the zigzag search is less than half of NSGA-II and MOPSO and the number of Pareto solutions obtained are close to 8 times the solutions obtained from them. When flow limit is posed, the accuracy of the classic zigzag algorithm is not as accurate as before but still a few alternative solutions can be obtained in a short amount of time. The modified zigzag search algorithms are also applied. Depending on different situations, a better result can be achieved by applying either the zigzag IP or the zigzag GA.

5.3.2 Zigzag search with adaptive step-size

A new economic dispatch model with CVaR risk management under wind power output uncertainty is proposed to demonstrate the usefulness of the zigzag search with adaptive step-size. In the IEEE 30-bus sytem, the proposed method outperform the classic zigzag search method. The fixed and user-defined step-size for classic zigzag search method is replaced by an adaptive step-size selection, to guarantee the accuracy and avoid unnecessary calculation.

CHAPTER SIX
CONCLUSIONS AND FUTURE WORKS

6.1 Conclusion

In this thesis, an analytical method, the zigzag search algorithm, for power system multi-objective optimization is proposed, modified and applied to two test studies. The contribution and advantages can be summarized as follows: (1)a zigzag search algorithm to an economic emission dispatch problem and successfully obtaining satisfactory results; (2) modified versions of the zigzag search algorithm are proposed to extend the original zigzag search to a broader application range: zigzag IP for large scale convex problems and zigzag GA for non-convex problems; (3) the step-size selection is improved by applying a steepest descent method to simplify the step size selection procedure and obtain more accurate results; and(4) test cases are carried out to demonstrate the zigzag search algorithms' efficiency and effectiveness for implementation in both small-size and large-size problem instances. Through comparison with other techniques published in literature, the proposed approaches can provide better solution than other algorithms for power system optimization problems.

6.2 Future work

For more practical applications and further improvement of the zigzag algorithm, the following improvement can be investigated.

(1) Obtaining a more precise approach to automatically determine the step size for the zig and zag step will improve the zigzag algorithm proposed in this thesis.

(2) Further modification of the zigzag GA algorithm can be focused on replacing GA with other evolutionary algorithms such as PSO, artificial bee algorithm or ant colony algorithm. By testing different evolutionary algorithm, the most suitable algorithm can be determined to be applied.

(3) Extending the original zigzag search algorithm to a discrete zigzag search algorithm can enable the zigzag search algorithm to be applied into unit commitment problem or PMU location optimization.

(4) By combining the zigzag adaptive step-size algorithm and other zigzag based algorithm, a new variant of zigzag search algorithm can be developed.

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