To the Graduate Council:

I am submitting herewith a thesis written by Sunil Kumar Jain entitled "On Some Integral Equation Methods in Two-Dimensional Transonic Flow". I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Aerospace Engineering.

S. N. Chaudhuri, Major Professor

We have read this thesis
and recommend its acceptance:

Accepted for the Council:

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Graduate Studies and Research

A

ON SOME INTEGRAL EQUATION METHODS
IN TWO-DIMENSIONAL
TRANSONIC FLOW

A Thesis
Presented for the
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Degree
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ABSTRACT

The linear flow field horizontal velocities around various airfoils have been obtained analytically and are compared with the velocities calculated using the exponential and inverse square functional relationships between horizontal surface velocity and the vertical coordinate. It is found that the velocities, obtained by using the two different functional relations, match closely with the analytical solution except near the leading edge and trailing edge.

Further, subcritical and supercritical flows over 6 percent biconvex and NACA 0012 airfoils at zero incidence have been obtained using the standard integral equation method but with a better approximate starting solution for the iteration scheme. It is found that this approach makes the solution converge much faster than the existing methods. Gauss Legendre quadrature method has been also used to solve the integral equation for subcritical flows. It is observed that this method is not efficient as far as computing time is concerned.

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LIST OF SYMBOLS

Capital Letters

C_p	Pressure coefficient
E	Euclidean space
F(z,r)	Transverse decay parameter
F'	Function used in Green's theorem, Equation (2.19)
G	Function used in Green's theorem, Equation (2.19)
H_I	Weighting factors in Gaussian Legendre quadrature formula
I	Double integral, Equation (4.2)
I_1	Double integral, Equation (4.14)
K'	Defined in Equation (3.12)
M	Local Mach number
M_∞	Free stream Mach number
N	Total number of streamwise segments used to approximate the velocity distribution over the airfoil
R	Gas constant
T	Temperature
U	Velocity in x direction
U_∞	Free stream velocity
W	Velocity in z direction

Small Letters

a	Local speed of sound
a^*	Critical speed of sound
a_∞	Free stream sonic velocity
c	Airfoil chord
c_p	Specific heat at constant pressure
h	Specific enthalpy
p	Pressure
r	Transverse decay function, Equation (3.19)
r_1	Defined by Equation (2.21)
u	Perturbation velocity in x direction
$\bar{u}$	Reduced perturbation velocity in $\bar{x}$ direction Equation (2.17)
w	Perturbation velocity in z direction
$\bar{w}$	Reduced perturbation velocity in $\bar{z}$ direction
x, z	Cartesian coordinates
$\bar{x}$, $\bar{z}$	Reduced cartesian coordinates, Equation (2.17)
z_1	Airfoil thickness distribution

Greek Letters

β	$(1 - M_\infty{}^2)^{\frac{1}{2}}$
γ	Ratio of Specific heats
ρ	density

ϕ	Perturbation velocity potential
ε_{jk}	Defined in Equation (4.5)
$\bar{\varepsilon}_{jk}$	Defined in Equation (4.8)
$\sigma(x)$	Source strength at point (x,o)
τ	Thickness ratio of the airfoil
∇^2	Laplacian operator, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$

Subscripts

a	Ahead of shock
b	Behind the shock
i	lower side of xz plane
∞	Free stream conditions
l	Linear theory
ns	Non singular solution
s	On the surface of the airfoil
u	upper side of xz plane
x, z	Denotes derivatives with respect to x,z
ξ, ζ	Denotes derivatives with respect to ξ,ζ

Superscipts

*	Sonic condition
n	Condition at nth iterative step

CHAPTER I

INTRODUCTION

Transonic Aerodynamics has been a subject of practical interest and mathematical inquisition since it is known to encompass one of the most vital regimes of flight. The transonic flow is generally characterized by an embedded supersonic flow in the overall subsonic flow terminated abruptly by a shock or smoothly by a sonic line. High subsonic flow, when the flow is always less than sonic, can also be catagorized under transonic flow. Mathematically, the transonic flow is defined by mixed type partial differential equations with nonlinearities to allow discontinuities for the shock waves.

Though some work had been done in transonic flow problems by Chaplygin (4) in the beginning of this century, the real spurt came at the end of 1940's when the basic equations and similarity rules were established by von Karman (13). Then a small number of actual solutions have been obtained by Lighthill (17) and von Mises (20) by transforming the equations to hodograph plane whereby differential equations become linear although still of mixed type. These solutions were exact but were available only for few configurations and therefore had a limited usefulness. The main problem in using hodograph method arose because of difficulties in handling the boundary conditions in the new plane. Also this method can not be applied to three dimensional flow. Inspite of all these limitations, this method has been developed to be of practical use in the design of supercritical shock free airfoils by Nieuwland (24) and more recently by Boerstoel (2) and Garbedian and Korn (7). These methods make use of the similarity between the elementary solutions of the incompressible and compressible hodograph equations by representing the incompressible flow around an ellipse as a series in the elementary incompressible solutions and then replacing this elementary incompressible solution by the compressible elementary solution.

With the development of large computers, numerical techniques for solving the transonic differential equations have gained importance. The first successful general finite difference method was developed by Sells (35). Thereafter Magnus and Yoshihara (19) formulated an unsteady approach, to capture the shock wave, where the steady flow is obtained asymptotically for large times by a proper finite difference marching procedure. While some useful results can be obtained, the time required for the solution to settle down to steady value is enormously high. The other finite difference relaxation technique is introduced by Murman and Cole (22) using the small disturbance equations in the physical plane. Here different schemes are used in the elliptic (subsonic) and hyperbolic (supersonic) regions to conform to the diverse characteristics of the respective subdomains. When the flow changes from subsonic to supersonic at the sonic line, a special "parabolic point" difference operator is used and when the flow decelerates from supersonic to subsonic through a shock wave, the central difference formula is used. Discontinuities are represented by a rapid finite compression.

More recently Garbedian and Korn (8) and Jameson (12) have extended the above procedure by using the exact inviscid equations with more accurate difference schemes. The idea of Jameson is further used by Hafez, South and Murman (11). They have come up with a simpler method to solve the full potential equation by modifying slightly the density (due to the artificial viscosity in the supersonic region) and solving the resulting elliptic like problem iteratively.

However for the present work, the integral equation approach has been used. The integral equation for two dimensional flow was first obtained by Oswatitsch (33). He transformed the nonlinear small perturbation equation to an nonhomogeneous elliptic equation and using Green's theorem, obtained the singular integral equation. This equation contains a line integral over the aerofoil chord and a surface integral over the flow field. To simplify the calculations, he converted the surface integral to line integral by replacing the variation of horizontal velocity component in vertical direction with a function taking into account the boundary conditions at the body and the asymptotic behaviour at infinity. Gullstrand (10) further refined this method and also derived the three dimensional counterpart of the integral equation. Spreiter and Alksne (36) extended this theory to mixed subsonic/ supersonic flows with shock wave. Later various papers have been published by Norstrud (29, 30), Nixon (25, 26, 27), Ogana (32) and Ogana and Spreiter (31) modifying the method of solving the integral equation so as to obtain better results. Niyogi and Mitra (28) obtained an approximate closed form solution of the integral equation for shock free symmetric airfoil at zero incidence while Mitra (21) got the approximate solution for supercritical flow with a shock wave.

As discussed earlier, to convert the surface integral into line integral, various functions, representing the variation of horizontal velocity in vertical direction, have been used in the past. To validate the use of these functions, a closed form solution has been obtained for linear flow field horizontal velocity. The velocities, so obtained, are compared with the velocities obtained using the various decay functions. The calculation and the comparison are given in chapter III. Chapter II contains the detailed derivation of integral equation.

Since nowadays, the main interest is to reduce the computing time while maintaining the same accuracy, it is desirable to have a better solution to start the iteration for solving the integral equation. The approximate closed form solution given by Niyogi and Mitra (28) has been used as the initial solution to start the iteration scheme for the calculation of subcritical flow over NACA 0012 and biconvex aerofoils at zero incidence while for the calculation of supercritical flow with shock, the solution given by Mitra (21) has been used. These results are compared with the present existing schemes in terms of number of iterations taken by the scheme to converge to a given accuracy. Also Gauss Legendre quadrature method is used to solve the surface integral in the integral equation since this method does not require any decay function for the horizontal velocity. Chapter IV consists of description of the methods mentioned above and also the results.

The discussion of results and the conclusions are given in Chapter V.

CHAPTER II

GENERAL ANALYSIS

1. Transonic Small Disturbance Equation

For a steady, two dimensional inviscid flow, the governing equations can be written as:

$$\frac{\partial(\rho\, U)}{\partial x} + \frac{\partial(\rho\, W)}{\partial z} = 0 , \qquad (2.1a)$$

$$U \frac{\partial U}{\partial x} + W \frac{\partial U}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial x} , \qquad (2.1b)$$

$$U \frac{\partial W}{\partial x} + W \frac{\partial W}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial z} , \qquad (2.1c)$$

$$U \frac{\partial h}{\partial x} + W \frac{\partial h}{\partial z} = \frac{1}{\rho} \left(U \frac{\partial p}{\partial x} + W \frac{\partial p}{\partial z} \right) , \qquad (2.1d)$$

where (x, z) is a cartesian coordinate system, and x is in the free stream direction, U and W are velocities in the x and z directions respectively, p is the pressure, ρ is the density and h is the specific enthalpy.

It is assumed that the fluid under consideration is both thermally and calorically perfect and two equations of state required to completely describe the flow can be written as:

$$p = \rho \; R \; T \; , \tag{2.1e}$$

$$h = c_p \; T = \left(\frac{\gamma}{\gamma - 1} \right) \frac{p}{\rho} \quad , \tag{2.1f}$$

where R is the gas constant, T is the temperature, c_p is the specific heat at constant pressure, and γ is the ratio of specific heats.

In transonic flows, where the shock wave that occur are usually almost normal, entropy variations across the shocks are generally of third order, hence the flow can be approximated as isentropic, i.e.,

$$\frac{p}{\rho^{\gamma}} = \text{constant} \; . \tag{2.2a}$$

From Crocco's theorem, therefore, we can assume the flow to be irrotational for adiabatic conditions. This assumption leads to the condition

$$\frac{\partial U}{\partial z} = \frac{\partial W}{\partial x} \quad . \tag{2.2b}$$

Because of addition of two more equations (2.2a, 2.2b) for the description of flow, two of the equations (2.1a-2.1f) can be neglected. Neglecting two momentum equations (2.1b, 2.1c) and combining the rest of the equations, we obtain

$$(a^2 - U^2) \frac{\partial U}{\partial x} + (a^2 - W^2) \frac{\partial W}{\partial z}$$

$$= U\, W \left(\frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} \right) , \qquad (2.3)$$

where,

$$a^2 = \frac{\gamma\, p}{\rho}$$

$$= a_\infty^2 - \left(\frac{\gamma - 1}{2} \right) (U^2 + W^2 - U_\infty^2) . \qquad (2.4)$$

Equation (2.2b) leads to the introduction of a perturbation velocity potential, ϕ, such that

$$u = U - U_\infty = \phi_x , \qquad (2.5a)$$

$$w = W = \phi_z , \qquad (2.5b)$$

where u, w are the perturbation velocities in x and z directions respectively. Using equations (2.4) and (2.5a, 2.5b), equation (2.3) can be reduced to

$$(1 - M_\infty^2) \phi_{xx} + \phi_{zz} =$$

$$M_\infty^2 \left(\frac{\gamma + 1}{U_\infty} \phi_x + \frac{\gamma + 1}{2\, U_\infty^2} \phi_x^2 + \frac{\gamma - 1}{2\, U_\infty^2} \phi_z^2 \right) \phi_{xx} +$$

$$2 \frac{M_\infty^2}{U_\infty^2} \phi_x \phi_z \phi_{xz} + M_\infty^2 \left(\frac{\gamma - 1}{U_\infty} \phi_x + \frac{\gamma + 1}{2 U_\infty^2} \phi_z^2 + \right.$$

$$\left. \frac{\gamma - 1}{2 U_\infty^2} \phi_x^2 \right) \phi_{zz} + 2 \frac{M_\infty^2}{U_\infty^2} \phi_z \phi_{xz} \quad , \qquad (2.6)$$

where M_∞ is the free stream Mach number.

If it is assumed that all perturbation velocities and perturbation velocity gradients are small and that only the first order terms in small quantities need to be retained, equation (2.6) simplifies to well known Prandtl Glaurt equation of linear theory,

$$(1 - M_\infty^2) \phi_{xx} + \phi_{zz} = 0 \quad . \qquad (2.7)$$

But the failure of linear theory in transonic range is evidenced by the calculated values of ϕ_x growing to such magnitude that they can no longer be treated as small quantities with respect to U_∞. Therefore in order to retain the characteristics of transonic flow, it is suggested that the first term on the right hand side of equation (2.6) is significant with respect to other terms on the right hand side. This leads to a simplified version of equation (2.6) which is also known as Transonic Small Disturbance Equation,

$$(1 - M_\infty^2) \phi_{xx} + \phi_{zz} =$$

$$M_\infty^2 \left(\frac{\gamma + 1}{U_\infty} \right) \phi_x \, \phi_{xx} \quad . \tag{2.8}$$

equation (2.8) is valid only in the regions where the necessary derivatives exist and are continuous. Since this equation does not hold where shock waves occur, an additional equation is needed for the transition through the ss shock. The necessary equation is provided by the classical relation for shock polar

$$W_b^2 = (U_a - U_b)^2 \frac{U_a U_b - a^{*2}}{\frac{2}{\gamma + 1} U_a^2 - U_a U_b + a^{*2}} \quad , \tag{2.9}$$

where U and W refer to cartesian velocity components with U parallel to the flow direction ahead of the shock, the subscripts a and b refer to conditions ahead of and behind the shock, and a^* is the critical sound velocity, which can be expressed in terms of U_∞ and M_∞ as

$$\frac{a^*}{U_\infty} = \sqrt{\left[\frac{\gamma - 1}{\gamma + 1} + \frac{2}{M_\infty^2 (\gamma + 1)}\right]} \quad . \tag{2.10}$$

The appropriate simplified version of equation (2.9) is obtained by resolving the velocities into components parallel to the axis of the coordinate system and carrying out a small perturbation analysis analogous to that performed in the derivation of equation (2.8). In this way, the following relation is found between the perturbation velocity components on the two sides of the shock wave.

$$(1 - M_\infty^2) (u_a - u_b)^2 + (w_a - w_b)^2 =$$

$$M_\infty^2 \frac{\gamma + 1}{U_\infty} \left(\frac{u_a + u_b}{2} \right) (u_a - u_b)^2 \quad . \tag{2.11}$$

The boundary conditions for equation (2.9) are:

(a) the tangential condition, i.e.,

$$\frac{w_s}{U} = \frac{dz_1}{dx} \quad , \tag{2.12}$$

where subscript s stands for the surface,

(b) the asymptotic behaviour of perturbation velocities at infinity, i.e.,

$$u\Big|_\infty = w\Big|_\infty = 0 \quad . \tag{2.13}$$

Further the tangential boundary condition (2.12) can be applied on the axis rather than on the surface which is consistent with the small perturbation theory. This leads

$$\frac{w}{U}\Bigg|_{z = 0} = \frac{dz_1}{dx} \tag{2.14}$$

In addition, it is presumed necessary that the direct influence of a disturbance in the supersonic region proceeds only in the downward direction and the Kutta condition applies whenever the flow velocity at the trailing edge is subsonic. Upon solving the above boundary value problem for the perturbation velocity potential ϕ, one may determine the pressure coefficient, c_p by

$$c_p = \frac{p - p_\infty}{\frac{\rho_\infty}{2} U_\infty^2} = \frac{-2 \frac{d\phi}{dx}}{U_\infty} \tag{2.15}$$

2. Transonic Integral Equation

The differenciation of equation (2.8) with respect to x gives

$$(1 - M_\infty^2) u_{xx} + u_{zz} = \frac{\gamma + 1}{U_\infty} M_\infty^2 \left(\frac{u^2}{2} \right)_{xx} . \tag{2.16}$$

A change of variables

$$\bar{x} = x, \quad \bar{z} = \beta z, \text{ and}$$
$$\bar{u} = \frac{\gamma + 1}{U_\infty} \frac{M_\infty^2}{\beta} u , \tag{2.17}$$

transforms the equation (2.16) into

$$\bar{u}_{\bar{x}\bar{x}} + \bar{u}_{\bar{z}\bar{z}} = \nabla^2 \bar{u} = \left(\frac{\bar{u}^2}{2} \right)_{\bar{x}\bar{x}} . \tag{2.18}$$

Now the basic tool in deriving the integral equation is the Green's theorem which relates a surface integral over the surface of domain D to the line integral computed around the complete boundary L. If F'and G are any two functions which, together with their first and second derivatives, are finite and single valued throughout the domain D, Green's theorem states

$$\int_L \left(F' \frac{\partial G}{\partial n} - G \frac{\partial F'}{\partial n} \right) ds = -\iint_D \left(F' \nabla^2 G - G \nabla^2 F' \right) dS \quad , \qquad (2.19)$$

where the directional derivatives on the left side are taken along the normal n, drawn inward, to the boundary L as shown in Figure 1.

It is convenient to let $G = \bar{u}$ and to choose F' as the fundamental solution $\ln(\frac{1}{r_1})$ of the equation $\nabla^2 F' = 0$

$$F' = \ln\left(\frac{1}{r_1}\right) \quad , \qquad (2.20)$$

$$\text{where,} \quad r_1 = (\bar{x} - \bar{\xi})^2 + (\bar{z} - \bar{\zeta})^2 \quad . \qquad (2.21)$$

Using equation (2.18) and (2.20), equation (2.19) becomes

$$\int_L \left[\ln\left(\frac{1}{r_1}\right) \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial}{\partial n} \left(\ln \frac{1}{r_1} \right) \right] ds = -\iint_D \frac{1}{r_1} \nabla^2 \bar{u} \, dS = -\iint_D \frac{1}{r_1} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{\bar{u}^2}{2} \right) dS \quad . \qquad (2.22)$$

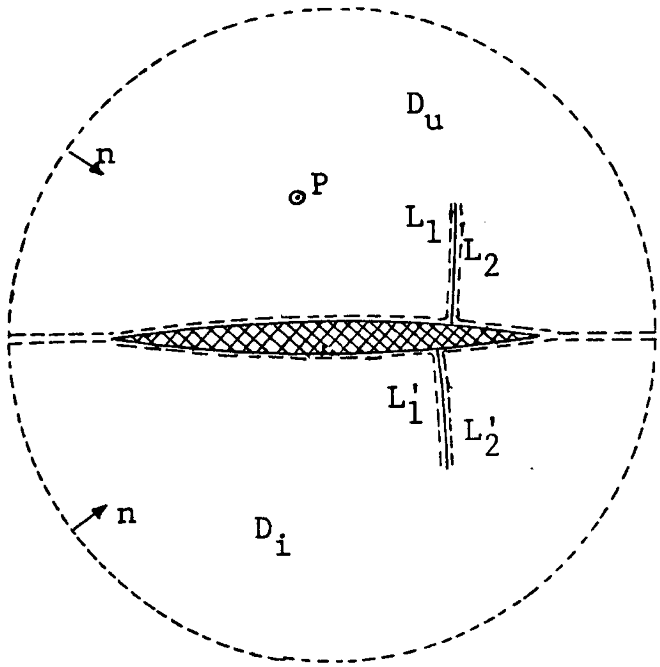

Figure 1. Region of integration

The singularity at $r_1 = 0$, in the above equation, is excluded by enclosing the point P within a small sphere. The discontinuity, because of $\bar{u}$ being discontinous at the shock wave, is taken care by altering the boundary of the surface so that it goes round the shock wave. Thus the equation (2.22) gives

$$\begin{aligned}\bar{u}(\bar{x}, \bar{z}) = \\ &- \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\partial \bar{u}_u}{\partial \bar{\zeta}} \ln \frac{1}{r_1} - \bar{u}_u \frac{\partial}{\partial \bar{\zeta}} \ln \frac{1}{r_1}\right) d\bar{\xi} \\ &- \frac{1}{2\pi}\int_{L_1}\left(\frac{\partial \bar{u}}{\partial n} \ln \frac{1}{r_1} - \bar{u} \frac{\partial}{\partial n} \ln \frac{1}{r_1}\right) ds_{L_1} \\ &- \frac{1}{2\pi}\int_{L_2}\left(\frac{\partial \bar{u}}{\partial n} \ln \frac{1}{r_1} - \bar{u} \frac{\partial}{\partial n} \ln \frac{1}{r_1}\right) ds_{L_2} \\ &- \frac{1}{2\pi}\iint_{D_u} \frac{\partial^2}{\partial \bar{\xi}^2}\left(\frac{\bar{u}^2}{2} \right) \ln \frac{1}{r_1}\, dS \quad , \end{aligned} \tag{2.23}$$

where the subscript u denotes the conditions on the upper side of $\bar{x}\bar{z}$ plane.

Similarly if domain D_i is considered keeping point P still in the upper half plane, equation (2.22) gives

$$\begin{aligned} 0 = &\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\partial \bar{u}_i}{\partial \bar{\zeta}} \ln \frac{1}{r_1} - \bar{u}_1 \frac{\partial}{\partial \bar{\zeta}} \ln \frac{1}{r_1}\right) d\bar{\xi} \\ &- \frac{1}{2\pi}\int_{L_1'}\left(\frac{\partial \bar{u}}{\partial n} \ln \frac{1}{r_1} - \bar{u} \frac{\partial}{\partial n} \ln \frac{1}{r_1}\right) ds_{L_1}' \\ &- \frac{1}{2\pi}\int_{L_2'}\left(\frac{\partial \bar{u}}{\partial n} \ln \frac{1}{r_1} - \bar{u} \frac{\partial}{\partial n} \ln \frac{1}{r_1}\right) ds_{L_2}' \quad - \end{aligned}$$

$$- \frac{1}{2\pi} \iint_{D_{\bar{l}}} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{\bar{u}^2}{2} \right) \ln \left(\frac{1}{r_1} \right) dS \quad , \qquad (2.24)$$

where the subscript $\bar{l}$ denotes the conditions on the lower lower side of the $\bar{x}\bar{z}$ plane.

$$\bar{u}_{\bar{l}}(\bar{\xi}, 0) = \bar{u}_u(\bar{\xi}, 0)$$

and

$$\frac{\partial \bar{u}_{\bar{l}}}{\partial \bar{\zeta}} = \frac{\partial \bar{u}_u}{\partial \bar{\zeta}} \quad , \qquad -\infty \le \bar{\xi} \le -\frac{1}{2}, \ \frac{1}{2} \le \xi \le \infty$$

$$\frac{\partial \bar{u}_{\bar{l}}}{\partial \bar{\zeta}} = - \frac{\partial \bar{u}_u}{\partial \bar{\zeta}} \quad , \qquad -\frac{1}{2} \le \bar{\xi} \le \frac{1}{2}$$

the summation of equations (2.23) and (2.24) give

$$\bar{u}(\bar{x}, \bar{z}) = - \frac{1}{\pi} \int_{-1/2}^{1/2} \left(\frac{\partial \bar{u}_u}{\partial \bar{\zeta}} \ln \frac{1}{r_1} \right)_{\bar{\zeta}=0} d\bar{\xi}$$

$$- \frac{1}{2\pi} \int_{L_1'+L_2'+L_1+L_2} \left(\frac{\partial \bar{u}}{\partial n} \ln \frac{1}{r_1} - \bar{u} \frac{\partial}{\partial n} \ln \frac{1}{r_1} \right) ds_L$$

$$- \frac{1}{2\pi} \iint_{D} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{\bar{u}^2}{2} \right) \ln \frac{1}{r_1} dS \quad . \qquad (2.25)$$

The first integral on the right hand side of above equation is equal to the value $\bar{u}$ given by the linearized theory of subsonic flow about airfoil and can be replaced by $\bar{u}_1$. Further the surface integral is integrated twice by parts with respect to $\bar{\xi}$, and the line integral over the shock wave is decomposed into components parallel to the axes of the coordinate system. By performing these operations, equation (2.25) can be rewritten as

$$\bar{u} = \bar{u}_1 + \frac{\bar{u}^2}{2} - \frac{1}{2\pi}\iint_D \frac{\bar{u}^2}{2} \frac{\partial^2}{\partial \bar{\xi}^2} \ln \frac{1}{r_1}\, d\bar{\xi}\, d\bar{\zeta}$$

$$+ \frac{1}{2\pi}\int_{L_1'+L_1} \{ \ln \frac{1}{r_1} \frac{\partial}{\partial \bar{\xi}} (\bar{u} - \frac{\bar{u}^2}{2})$$

$$- (\bar{u} - \frac{\bar{u}^2}{2}) \frac{\partial}{\partial \bar{\xi}} \ln \frac{1}{r_1} + (\ln \frac{1}{r_1} \frac{\partial \bar{u}}{\partial \bar{\zeta}} - \bar{u} \frac{\partial}{\partial \bar{\zeta}} \ln \frac{1}{r_1})$$

$$\frac{\cos(n,\ \bar{\zeta})}{\cos(n,\ \bar{\xi})} \} d\bar{\zeta} - \frac{1}{2\pi}\int_{L_2'+L_2} \{ \ln \frac{1}{r_1} \frac{\partial}{\partial \bar{\xi}} (\bar{u} - \frac{\bar{u}^2}{2})$$

$$- (\bar{u} - \frac{\bar{u}^2}{2}) \frac{\partial}{\partial \bar{\xi}} \ln \frac{1}{r_1}$$

$$+ (\ln \frac{1}{r_1} \frac{\partial \bar{u}}{\partial \bar{\zeta}} - \bar{u} \frac{\partial}{\partial \bar{\zeta}} \ln \frac{1}{r_1}) \frac{\cos(n,\ \bar{\zeta})}{\cos(n,\ \bar{\xi})} \} d\bar{\zeta} , \quad (2.26)$$

Also normalization of equation (2.11), by introducing the quantities defined in equation (2.17), gives

$$(\bar{u}_a - \bar{u}_b)^2 + (\bar{w}_a - \bar{w}_b)^2 =$$

$$(\frac{\bar{u}_a + \bar{u}_b}{2}) (\bar{u}_a - \bar{u}_b)^2 \quad . \quad (2.27)$$

The above equation can be written into two alternative forms as

$$(1 - \frac{\bar{u}_a + \bar{u}_b}{2}) (\bar{u}_a - \bar{u}_b)^2 + (\bar{w}_a - \bar{w}_b)^2$$

$$= 0 \quad , \qquad (2.28)$$

and

$$(\bar{u}_a - \bar{u}_b) \{ (\bar{u}_a - \frac{\bar{u}_a^2}{2}) - (\bar{u}_a - \frac{\bar{u}_b^2}{2}) \}$$

$$+ (\bar{w}_a - \bar{w}_b)^2 = 0 \quad . \qquad (2.29)$$

To simplify equation (2.26) further, it is assumed that the shock wave is a normal shock wave and the flow is parallel to the x axis, i.e.,

$$\bar{w}_a = \bar{w}_b = 0 \qquad (2.30)$$

$$\cos(n, \bar{\zeta}) = 0 \quad . \qquad (2.31)$$

Substitution of condition (2.30) in equation (2.29) gives

$$(\bar{u} - \frac{\bar{u}^2}{2})_a = (\bar{u} - \frac{\bar{u}^2}{2})_b \quad . \qquad (2.32)$$

With equation (2.31) and (2.32), equation (2.26) can be simplified to

$$\bar{u} = \bar{u}_1 + \frac{\bar{u}^2}{2} - \frac{1}{2\pi} \iint_D \frac{\bar{u}^2}{2} \frac{\partial^2}{\partial \bar{\xi}^2} \ln \frac{1}{r_1} \, d\bar{\xi} \, d\bar{\zeta} \quad , \qquad (2.33)$$

or,

$$\bar{u}(\bar{x}, \bar{z}) = \bar{u}_1(\bar{x}, \bar{z}) + \frac{\bar{u}^2}{2}(\bar{x}, \bar{z})$$

$$- \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\bar{u}^2}{2}(\bar{\xi}, \bar{\zeta}) \frac{\{(\bar{x} - \bar{\xi})^2 - (\bar{z} - \bar{\zeta})^2\}}{\{(\bar{x} - \bar{\xi})^2 + (\bar{z} - \bar{\zeta})^2\}^2} d\bar{\xi}\, d\bar{\zeta} \quad . \qquad \ldots\ldots\ldots(2.34)$$

The above equation is known as Oswatitsch integral equation.

CHAPTER III

LINEAR FLOW FIELD VELOCITY CALCULATION

The main source of difficulty in solving the integral equation (2.31) lies in the double integral. Various functional relationships, between horizontal perturbation velocity in the field and on the surface of the aerofoil for the same x location, have been used in the past to convert the surface integral into line integral. In this chapter, linear flow field perturbation velocity is obtained analytically and the results are compared with the velocities obtained using the various functional relationships.

1. Exact Solution (Linear Theory)

As seen in the previous chapter, the Prandtl Glaurt equation based on linear theory can be written as:

$$(1 - M_\infty^2) \phi_{xx} + \phi_{zz} = 0 \quad . \qquad (3.1)$$

The boundary conditions are

a. $$w = U_\infty \frac{dz_1}{dx} \quad \text{at } z = 0 \; . \qquad (3.2)$$

b. $u = w = 0$ at $z = \infty$. (3.3)

By transforming the variables as

$$\bar{x} = x \quad \text{and} \quad \bar{z} = \beta z ,$$

equation (3.1) can be written as

$$\phi_{\bar{x}\bar{x}} + \phi_{\bar{z}\bar{z}} = 0 . \qquad (3.4)$$

Because of the symmetry of the airfoil, the flow pattern is completely symmetrical with respect to x axis, having no discontinuities in pressure or velocity components across the x axis except for the jump $2 U_\infty dz_1/dx$ in w in the interval $0 \leq x \leq 1$. All these requirements can be met by distributing over the interval (0,1) the two dimensional point source solution of the Laplace equation (3.4). A sheet of these two dimensional source distributed continously over $0 \leq x \leq 1$ and having the strength $\sigma(x')$ per unit length at the point (x', 0), possesses the perturbation potential at any point $(\bar{x}, \bar{z})$:

$$\phi(\bar{x}, \bar{z}) = \frac{1}{2\pi}\int_0^1 \sigma(x') \ln\{(\bar{x}-x')^2 + \bar{z}^2\}^{1/2} dx' . \qquad \text{.....(3.5)}$$

To satisfy the boundary conditions (3.2) and (3.3), we calculate :

$$w(x,0) = \frac{\partial\phi}{\partial z}(x,0) = \beta \frac{\partial\phi}{\partial \bar{z}}(\bar{x},0)$$

$$= \frac{\beta}{2\pi} \lim_{\bar{z}\to 0} \frac{\partial}{\partial \bar{z}} \int_0^1 \sigma(x') \ln\{(\bar{x}-x')^2 + \bar{z}^2\}^{1/2} \, dx'$$

$$= \frac{\beta}{2\pi} \lim_{\bar{z}\to 0} \bar{z} \int_0^1 \frac{\sigma(x')}{(\bar{x}-x')^2 + \bar{z}^2} \, dx' \qquad (3.6)$$

As $\bar{z} \to 0$, the integral vanishes due to the multiplying factor except in the viscinity of the point $x' = x$ and $\bar{z} = 0$, where the integrand tends to infinity. By isolating this region with a small segment of length 2δ where $\delta << 1$, equation (3.6) can be written as :

$$w(x,0) = \frac{\beta}{2\pi} \lim_{\bar{z}\to 0} \bar{z} \int_{\bar{x}-\delta}^{\bar{x}+\delta} \frac{\sigma(x')}{(\bar{x}-x')^2 + \bar{z}^2} \, dx'$$

$$= \frac{\sigma(x)}{2\pi} \beta \lim_{\bar{z}\ 0} \left[\tan^{-1}\left(\frac{\delta}{\bar{z}}\right) - \tan^{-1}\left(\frac{\delta}{\bar{z}}\right) \right]$$

$$= \frac{\sigma(x)}{2} \beta \, . \qquad (3.7)$$

Using equations (3.2) and (3.7), we obtain

$$\sigma(x) = \frac{2 U_\infty}{\beta} \frac{dz_1}{dx} \, . \qquad (3.8)$$

Hence,

$$u(x,z) = \frac{\partial\phi}{\partial x}$$

$$= \frac{1}{2\pi}\int_0^1 \sigma(x') \frac{(\bar{x}-x')}{(\bar{x}-x')^2 + \bar{z}^2} dx'$$

$$= \frac{U_\infty}{\beta\pi}\int_0^1 \frac{dz_1}{dx}(x') \frac{(x-x')}{(x-x')^2 + (\beta z)^2} dx' \quad . \quad (3.9)$$

The Prandtl Glaurt rule can easily be obtained from (3.9) by making z = 0

$$u(x,0) = \frac{1}{\beta}\left[\frac{U_\infty}{\pi}\int_0^1 \frac{dz_1}{dx}(x') \frac{dx'}{(x-x')}\right] . \quad (3.10)$$

Moreover it can be seen from equation (3.9) that Prandtl Glaurt rule is not valid in the flow field because of presence of factor βz instead of z.

The solution to equations (3.9) and (3.10) are given by Chaudhuri (5) and are as follows:

If the equation of the airfoil is defined by

$$z_1 = F_0 \sqrt{x} + \sum_{I=1}^{N} F_I\, x^I , \quad 0< x <1 \quad . \quad (3.11)$$

then the surface perturbation velocity u(x,0) can be written as

$$\frac{u(x,0)}{U_\infty} = K' , \quad (3.12)$$

where

$$\pi K' = F_0\ T_0(x) + \sum_{\nu=1}^{N} F_\nu\ T_\nu(x) \quad , \tag{3.13}$$

$$T_0(x) = \frac{1}{2\sqrt{x}} \ln \left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)$$

$$T_1(x) = \ln \left(\frac{x}{x-1}\right)$$

$$T_\nu(x) = \frac{\nu}{\nu-1} \left(x\ T_{\nu-1} - 1\right), \quad \nu = 2,3 \ldots\ldots N$$

The field perturbation velocity $u(x,z)$ can be written as

$$\frac{u(x,z)}{U_\infty} = \frac{1}{2}\left[x\ S_{-\frac{1}{2}}(x,z) - S_{\frac{1}{2}}(x,z)\right] F_0$$

$$+ \sum_{\nu=1}^{N} \nu\left[x\ S_{\nu-1}(x,z) - S_\nu(x,z)\right] F_\nu \ , \tag{3.14}$$

where

$$S_{-\frac{1}{2}}(x,z) = \frac{1}{\pi z\sqrt{(x^2+z^2)}}\left[C \tan^{-1}\{T(x,z)\} + D \ln \sqrt{L(x,z)}\right]$$

$$S_{\frac{1}{2}}(x,z) = \frac{1}{\pi z}\left[C \tan^{-1} \{T(x,z)\} - D \ln \sqrt{L(x,z)}\right]$$

$$L(x,z) = \frac{(x+1+2\ C)^2 + (z+2\ D)^2}{(x-1)^2 + z^2}$$

$$T(x,z) = \frac{z\ (x+1+2\ C)\ -\ (x-1)(z+2\ D)}{(x-1)(x+1+2\ C)\ +\ z\ (z+2\ D)} > 0.$$

$$C^2 = \frac{1}{2}\ \surd(x^2 + z^2) + \frac{x}{2}$$

$$D^2 = \frac{1}{2}\ \surd(x^2 + z^2) - \frac{x}{2}\ .$$

When $T(x,z) < 0$, $C \tan^{-1}\{T(x,z)\}$ is replaced by $C[\pi - \tan^{-1}\{\ |T(x,z)|\ \}]$.

Also,

$$S_0(x,z) = \frac{1}{\pi z}\ \{\ \tan^{-1}\ (\ \frac{x}{z}\ ) - \tan^{-1}\ (\frac{x-1}{z})\ \}$$

$$S_1(x,z) = x\ S_0(x,z) + \frac{1}{2\pi}\ \ln\{\frac{(x-1)^2 + z^2}{x^2 + z^2}\}$$

$$S_\nu(x,z) = \frac{1}{(\nu-1)\pi} + 2\ x\ S_{\nu-1}(x,z)$$

$$- (x^2 + z^2)\ S_{\nu-2}(x,z)\ ,\quad \nu > 2.$$

2. Approximate Solutions

The variation of $u(x,z)$ with z has been approximated by some simple functions based on Mclaurin's expansion.

The Mclaurin's expansion for $u(x,z)$ for $z > 0$ can be written as:

$$u(x,z) = u(x,0) + z\ u_z(x,0) + \text{------------}\ . \qquad (3.15)$$

Since $u_z(x,0) = w_x(x,0)$ for irrotational flow, equation

(3.15) can be written using tangential boundary condition as

$$u(x,z) = u(x,0) + z\, z_{1_{xx}} + \text{---------} \quad . \qquad (3.16)$$

The various functions used to satisfy approximately equation (3.16) are

$$u(x,z) = u(x,0)\, e^{\frac{-z}{r}} \quad , \qquad (3.17)$$

$$u(x,z) = \frac{u(x,0)}{\left(1 + \frac{z}{2\,r}\right)^2} \quad , \qquad (3.18)$$

where r(x) is calculated using the irrotationality condition and it is written as

$$r(x) = - \frac{u(x,0)}{\left\{ \frac{\partial u}{\partial z} \right\}_{(x,0)}} = - \frac{u(x,0)}{z_{1_{xx}}} \quad . \qquad (3.19)$$

It can be seen that the above equations (3.17) and (3.18) satisfy the boundary condition at infinity.

To compare the results, 6 percent biconvex airfoil and NACA 0012 airfoil have been selected to represent typical sharp and blunt leading edge profiles.

The 6 percent biconvex airfoil geometry is described by

$$z_1 = \pm\, 0.12\ (\ x - x^2\ ) \quad , \quad 0 \leq x \leq 1 \quad . \qquad (3.20)$$

and NACA 0012 airfoil profile is described by

$$z_1 = \pm 0.12 \; (b_0 \sqrt{x} + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4) \quad , \qquad 0 \leq x \leq 1 \quad . \tag{3.21}$$

where

$$b_0 = 1.48450$$
$$b_1 = -0.63000$$
$$b_2 = -1.75800$$
$$b_3 = 1.42150$$
$$b_4 = -0.50750$$

Since the surface velocity solution given by equation (3.12) is singular at the leading edge, the non-singular approximation given by Nixon (27) is used and is as follows

$$u_{ns} = \frac{1 + u_1}{\left\{ 1 + \left(\frac{\tau \, z_{1_x}}{\beta} \right)^2 \right\}^{1/2}} - 1 \quad , \tag{3.22}$$

where u_{ns} stands for nonsingular solution and u_1 is the solution obtained using equation (3.12).

Using the above discussed methods, the field velocities at different x locations for NACA 0012 and 6 percent biconvex airfoils have been calculated at zero incidence for various Mach numbers and the results are given in Figures 2 to 12.

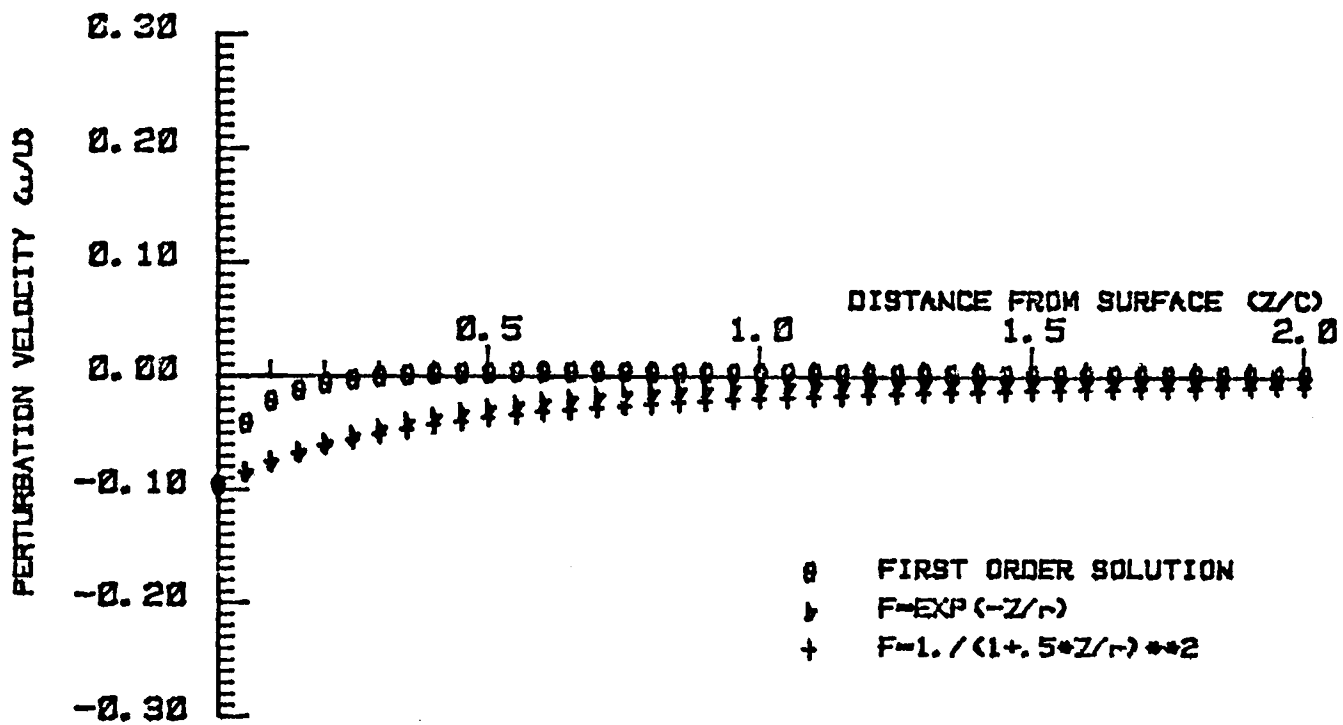

Figure 2. Field velocity distribution over 6% biconvex airfoil at x = 0.01c and $M_\infty = 0.00$

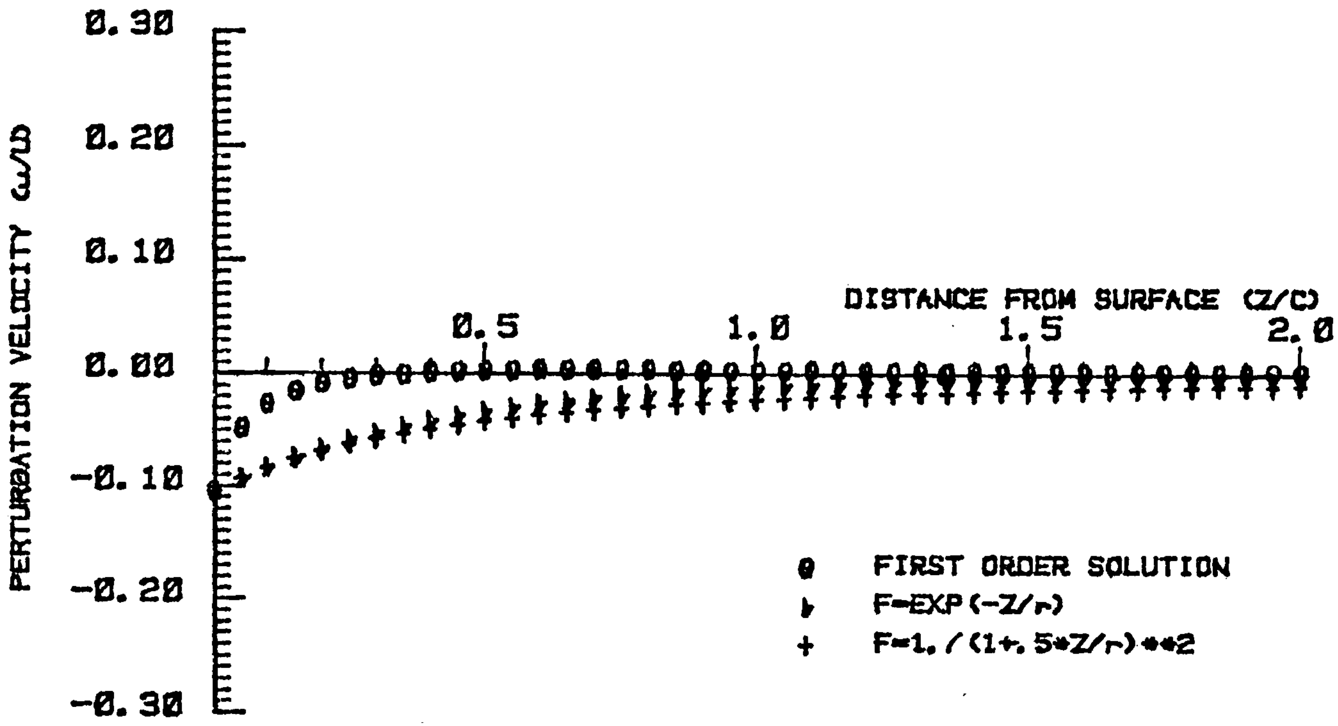

Figure 3. Field velocity distribution over 6% biconvex airfoil at x = 0.01c and M_∞ = 0.30

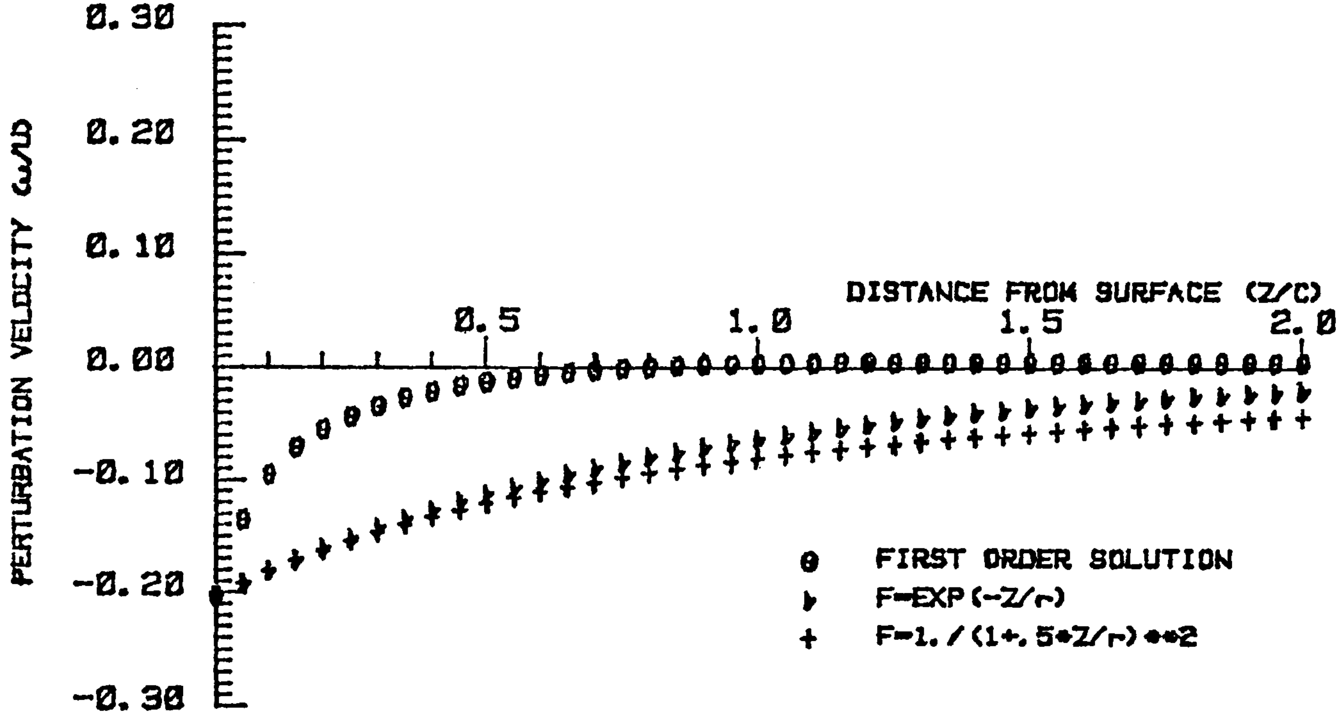

Figure 4. Field velocity distribution over 6% biconvex airfoil at x = 0.01c and $M_\infty = 0.83$.

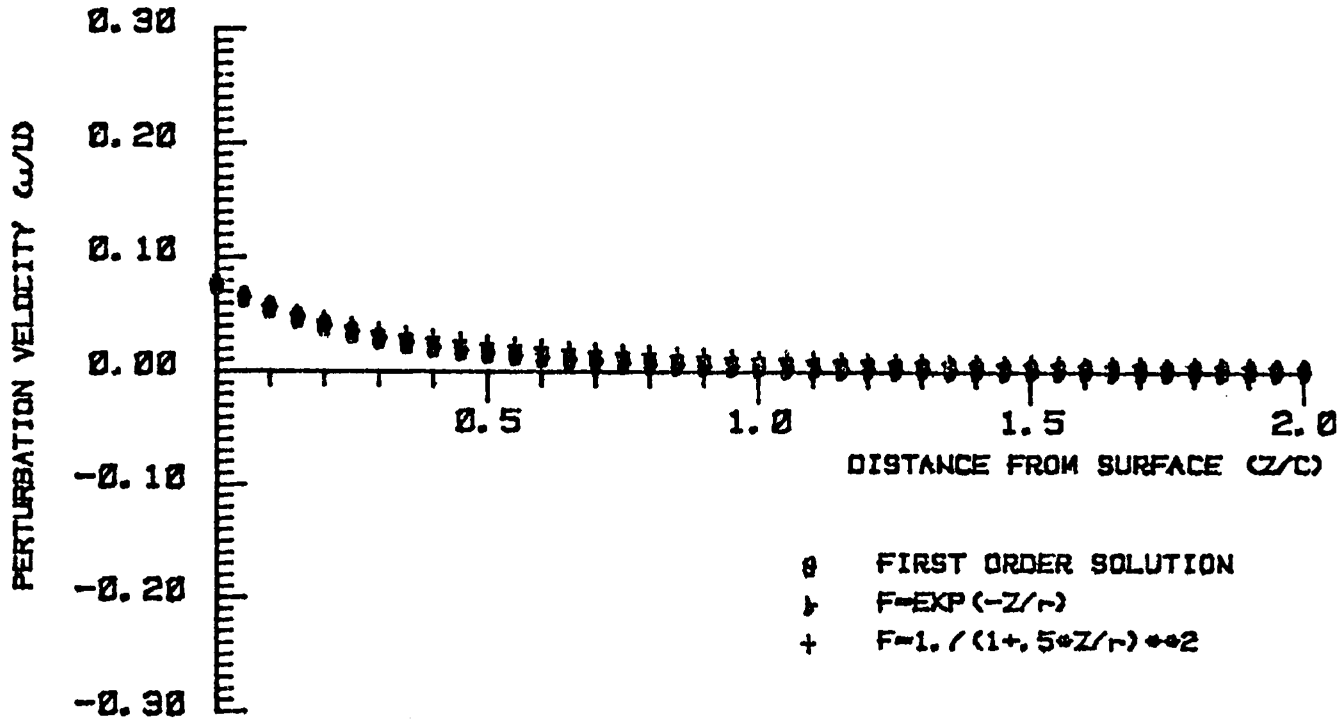

Figure 5. Field velocity distribution over 6% biconvex airfoil at x = 0.50c and $M_\infty = 0.00$.

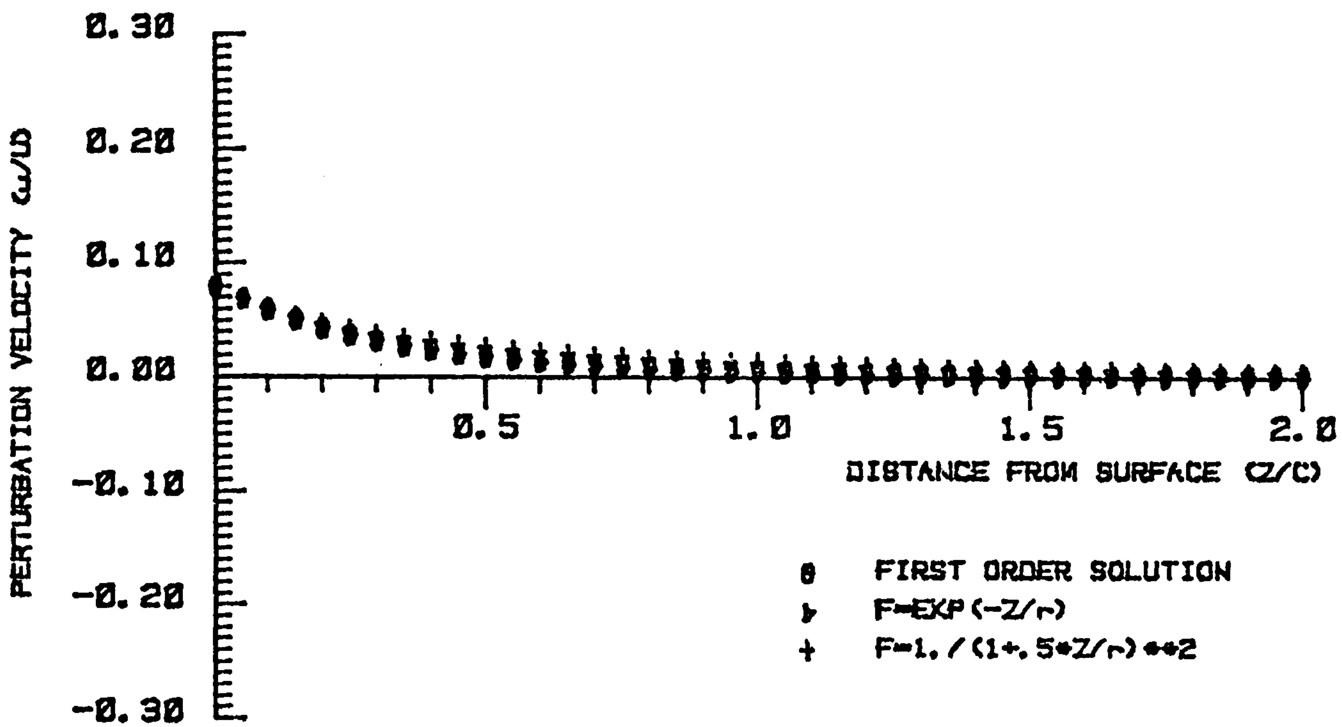

Figure 6. Field velocity distribution over 6% biconvex airfoil at x = 0.50c and M_∞ = 0.30.

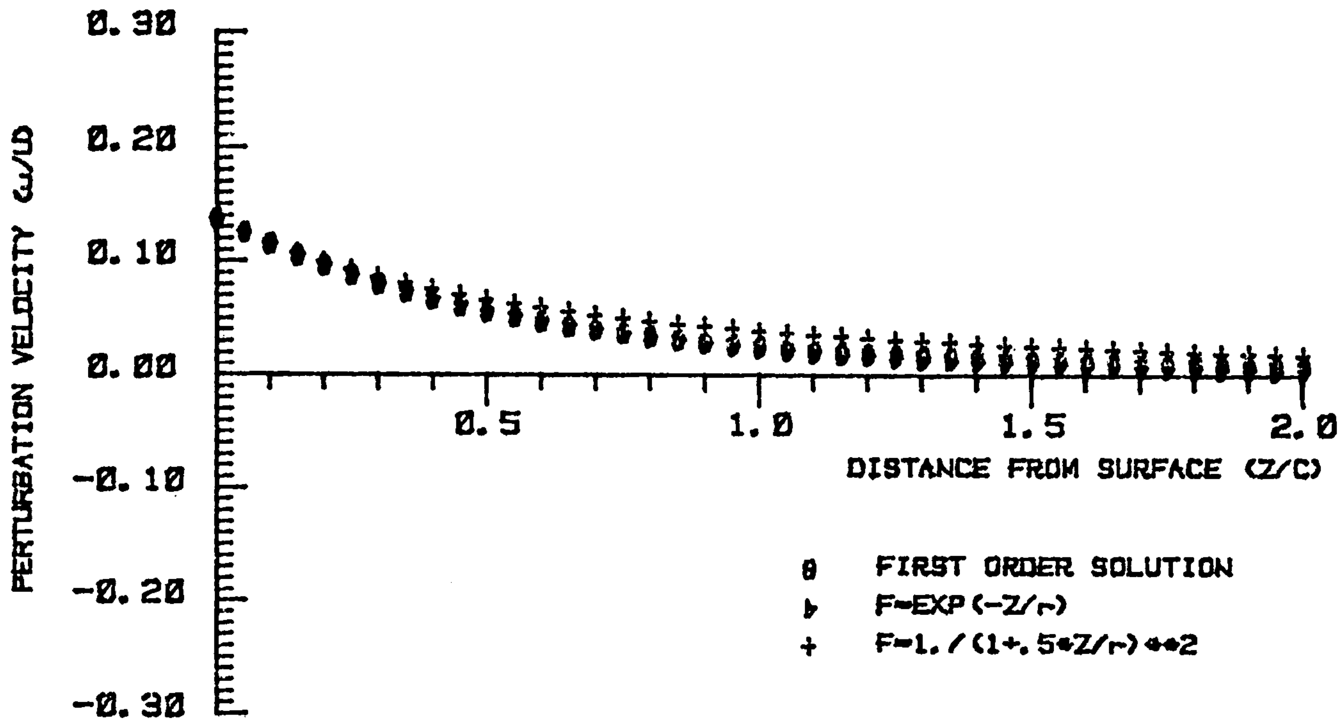

Figure 7. Field velocity distribution over 6% biconvex airfoil at $x = 0.50c$ and $M_\infty = 0.83$

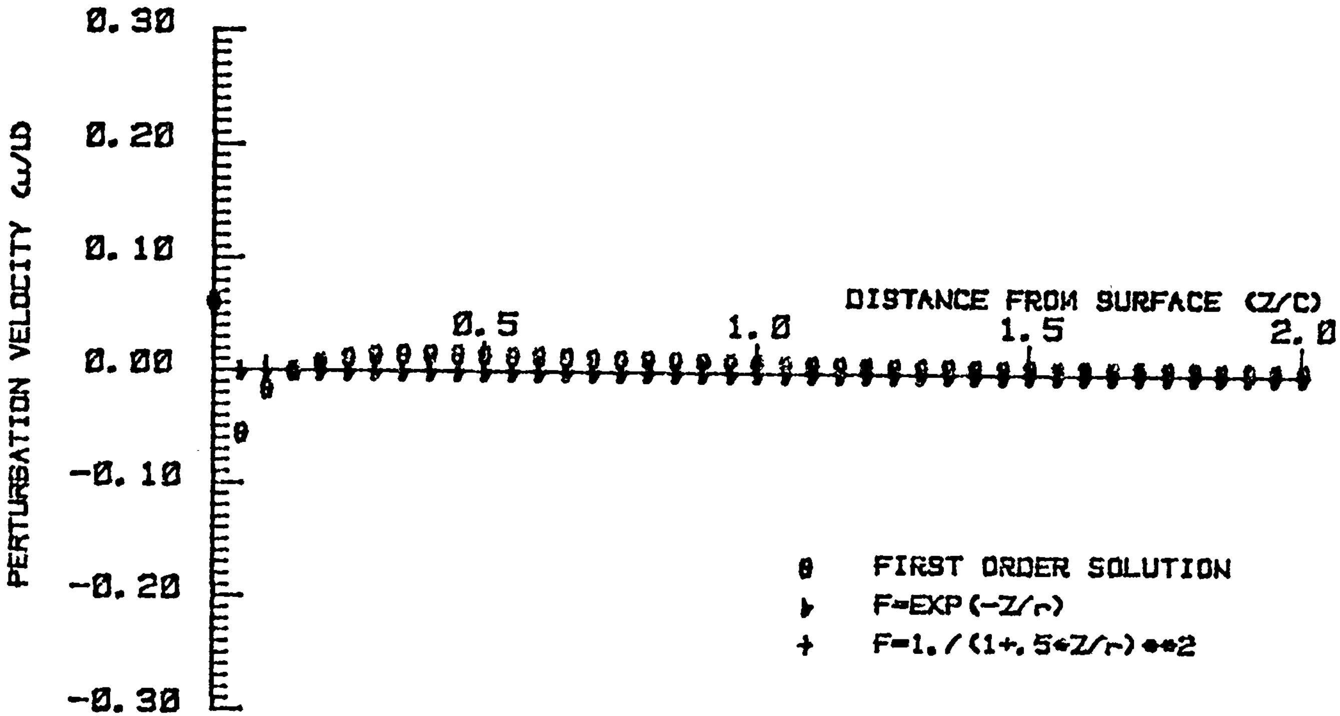

Figure 8. Field velocity distribution over NACA 0012 airfoil at x = 0.01c and M_∞ = 0.30

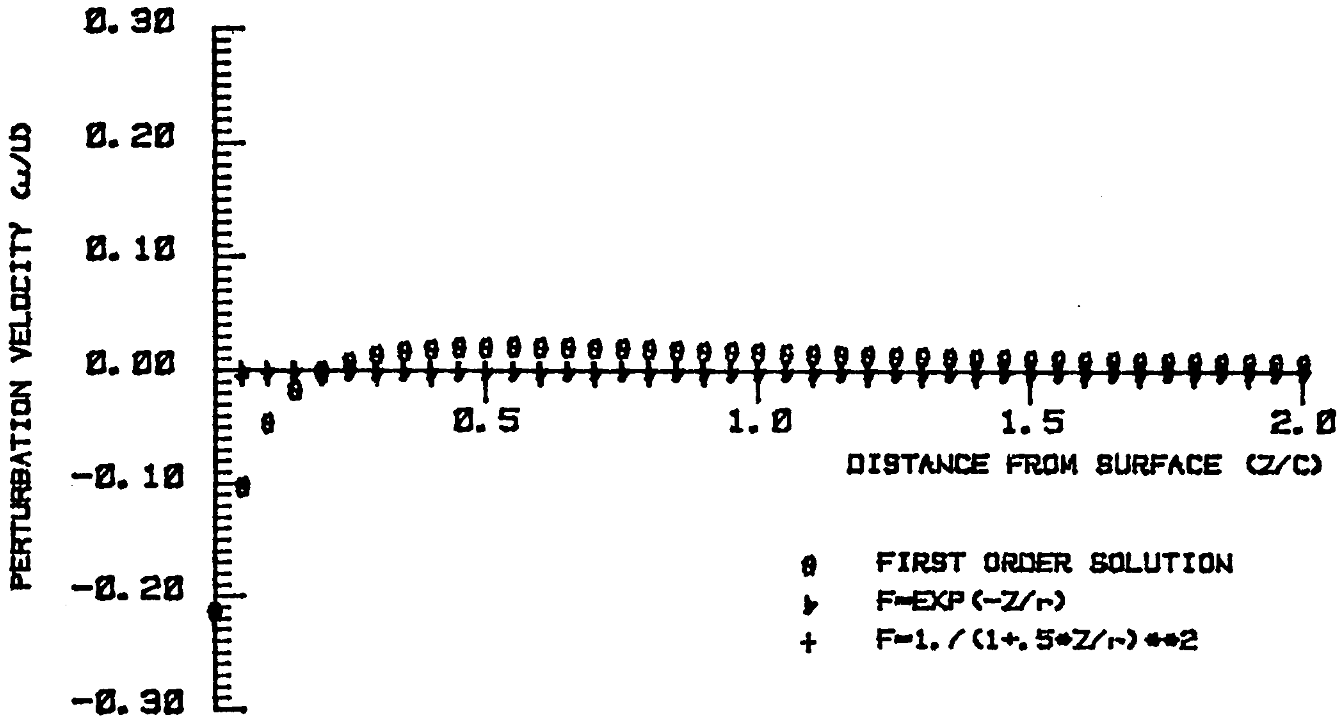

Figure 9. Field velocity distribution over NACA 0012 airfoil at x = 0.01c and M_∞ = 0.72.

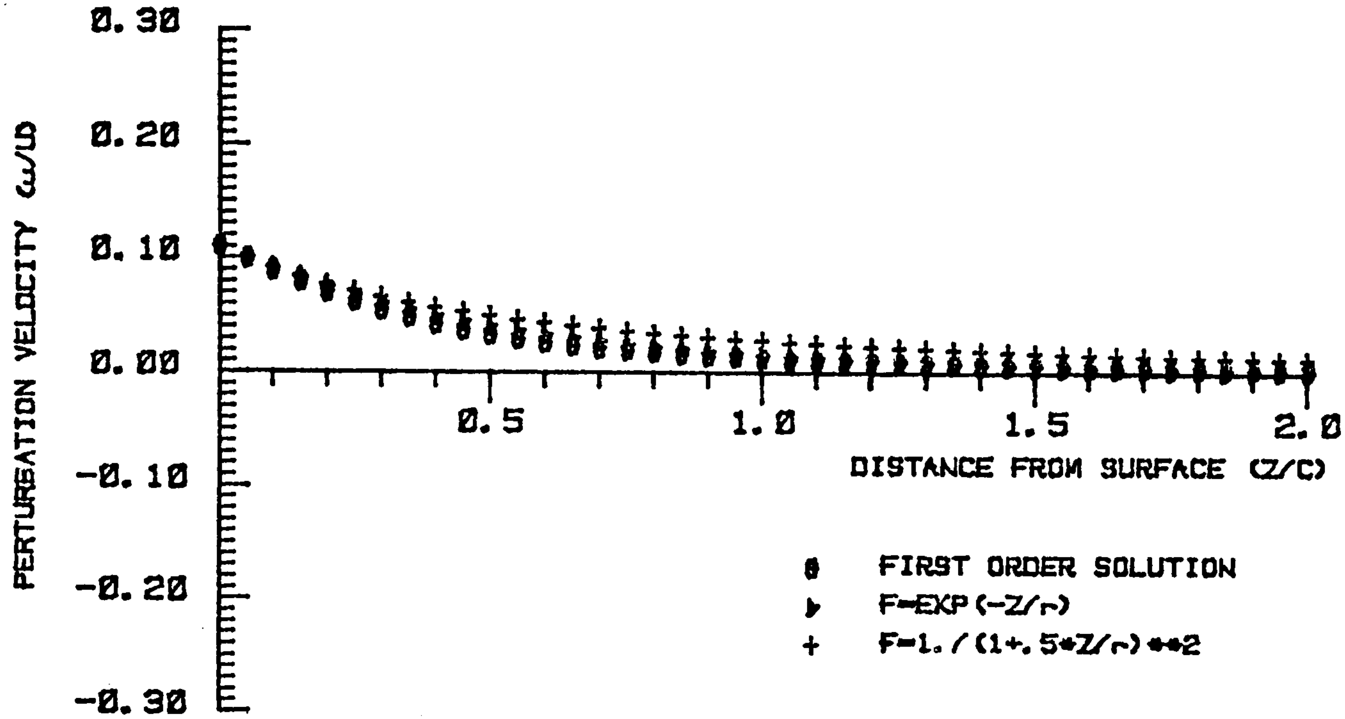

Figure 10. Field velocity distribution over NACA 0012 airfoil at x = 0.50c and M_∞ = 0.30.

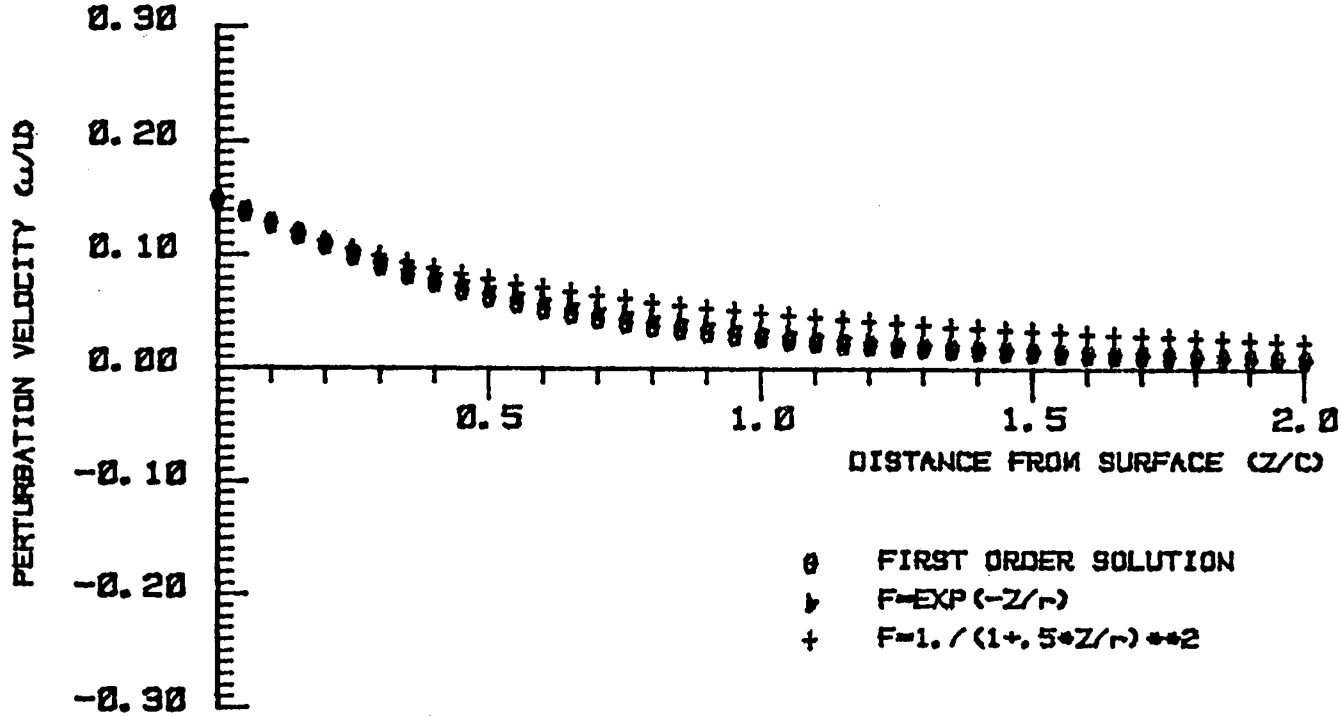

Figure 11. Field velocity distribution over NACA 0012 airfoil at $x = 0.50c$ and $M_\infty = 0.72$.

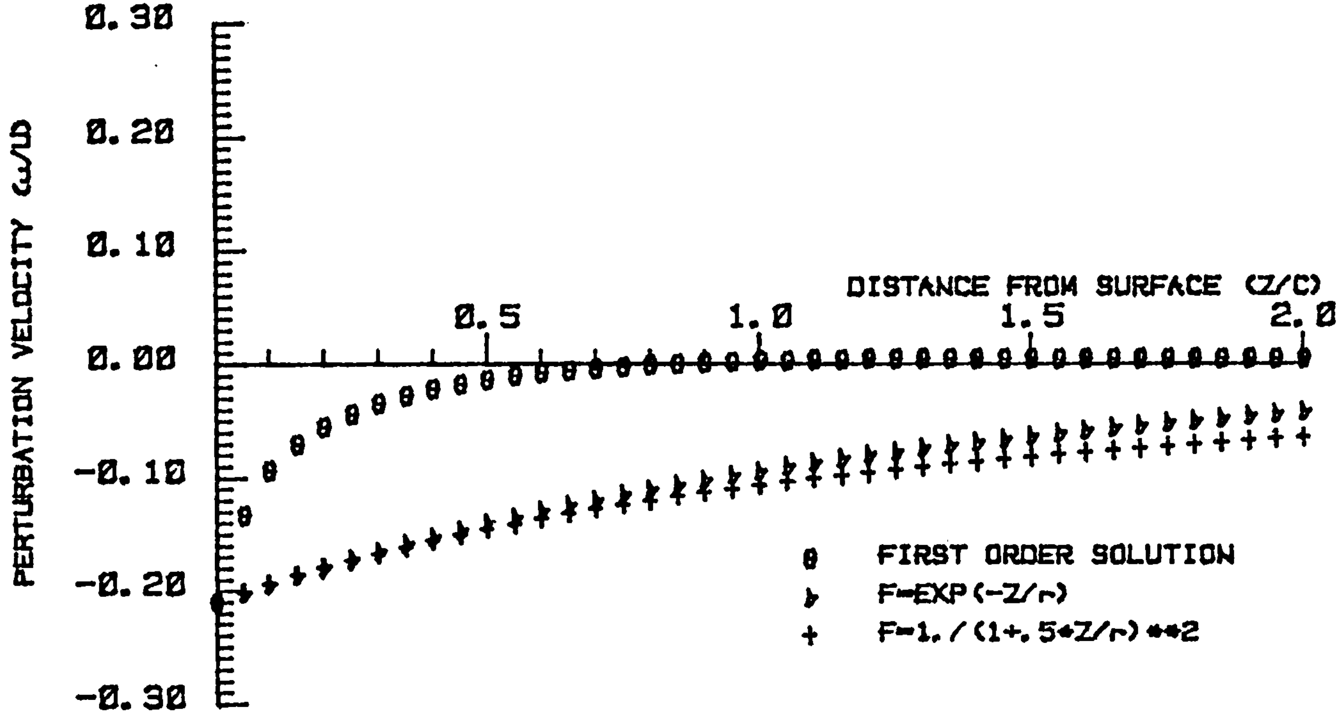

Figure 12. Field velocity distribution over NACA 0012 airfoil at x = 0.99c and M_∞ = 0.72.

CHAPTER IV

SOLUTION OF INTEGRAL EQUATION

The Oswatitsch integral equation (2.34), which is a nonlinear Fredholm integral equation, can be solved numerically using an iteration process. Various techniques can be used for the evaluation of the double integral in the integral equation. One of the techniques, as discussed in Chapter I, is to reduce the double integral into line integral using the approximate functional relationships between horizontal perturbation velocity in the field and on the surface for the same x location. Thereafter the line integral is solved by using quadrature method. Here the airfoil is divided into N elements and by introducing a mean value of decay factor r(x) for each of these elements, the contribution of a single element can be determined by integration over the element. The other technique of solving the double integral numerically can be Gauss Legendre quadrature method where the value of double integral could be obtained without making any assumption regarding the decay of perturbation velocity in vertical direction. These techniques and the iteration schemes used for solving the integral equation are discussed in this chapter.

Equation (2.34) can be rewritten as

$$\bar{u} = \bar{u}_1 + \frac{\bar{u}^2}{2} - I \quad , \tag{4.1}$$

where

$$I = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\bar{u}^2}{2} \frac{\{(\bar{x} - \bar{\xi})^2 - (\bar{z} - \bar{\zeta})^2\}}{\{(\bar{x} - \bar{\xi})^2 + (z - \bar{\zeta})^2\}^2} \, d\bar{\xi} \, d\bar{\zeta}$$

For the symmetric airfoils, I can be further simplified as

$$I = \frac{1}{\pi}\int_{0}^{\infty}\int_{-\infty}^{\infty} \frac{\bar{u}^2}{2} \frac{\{(\bar{x} - \bar{\xi})^2 - \bar{\zeta}^2\}}{\{(\bar{x} - \bar{\xi})^2 + \bar{\zeta}^2\}^2} \, d\bar{\xi} \, d\bar{\zeta}$$

at $\bar{z} = 0$.

Since it can be assumed that u is zero in the region ahead of leading edge and behind the trailing edge of the airfoil, the integration limit in $\bar{\xi}$ direction $(-\infty, \infty)$ can be reduced to (0, 1). Hence

$$I = \frac{1}{\pi}\int_{0}^{\infty}\int_{0}^{1} \frac{\bar{u}^2}{2} \frac{\{(\bar{x} - \bar{\xi})^2 - \bar{\zeta}^2\}}{\{(\bar{x} - \bar{\xi})^2 + \bar{\zeta}^2\}^2} \, d\bar{\xi} \, d\bar{\zeta} \quad , \tag{4.2}$$

on the airfoil surface.

By introducing the relation

$$\bar{u}(\bar{\xi}, \bar{\zeta}) = \bar{u}(\bar{\xi}, 0) \, F(\bar{\zeta}, r) \quad ,$$

I can be written as

$$I(x_j) = \sum_{k=1}^{N} \frac{\bar{u}^2}{2}(\bar{\xi}, 0)\, \varepsilon_{jk} \quad . \qquad (4.3)$$

ε_{jk} has been obtained by Kraft (14) for

$$F(\bar{\zeta}, r) = e^{-\frac{\bar{\zeta}}{r}}$$

where

$$r(x_j) = -\frac{\bar{u}(x_j, 0)\, \beta^3}{(\gamma + 1)\, M_\infty^2\, z_{1_{xx}}(x_j)}$$

is obtained from irrotationality condition.

$$\varepsilon_{jk} = \frac{1}{\pi}\int_{\bar{\xi}_k - \frac{\Delta}{2}}^{\bar{\xi}_k + \frac{\Delta}{2}} \int_0^\infty e^{-\frac{2\bar{\zeta}}{r}} \frac{\{(\bar{x}_j - \bar{\chi})^2 - \bar{\zeta}^2\}}{\{(\bar{x}_j - \bar{\chi})^2 + \bar{\zeta}^2\}^2}\, d\bar{\zeta}\, d\bar{\chi}$$

$$= \frac{1}{\pi}\int_0^\infty \left(\frac{b}{b^2 + \bar{\zeta}^2} - \frac{a}{a^2 + \bar{\zeta}^2} \right) e^{-\frac{2\bar{\zeta}}{r}}\, d\bar{\zeta} \ , \qquad (4.4)$$

where

$$b = \bar{x}_j - \bar{\xi}_k - \frac{\Delta}{2}$$

$$a = \bar{x}_j - \bar{\xi}_k + \frac{\Delta}{2}$$

$$\Delta = \frac{1}{N}$$

Equation (4.4) can be integrated analytically (Reference 9, equation 3.354/ 1) to yield

$$\varepsilon_{jk} = \begin{cases} \frac{1}{\pi} \frac{b}{|b|} H_\infty \{ | \frac{2b}{r(x_j)} | \} - \frac{1}{\pi} \frac{a}{|a|} H_\infty \{ | \frac{2a}{r(x_j)} | \} , & j \neq k \\ 1 - \frac{2}{\pi} H_\infty \{ | \frac{\Delta}{r(x_j)} | \} , & j = k \end{cases} \quad (4.5)$$

where

$$H_\infty(\mu) = ci(\mu) \sin(\mu) - si(\mu) \cos(\mu)$$

where ci and si are the cosine and sine integration fucctions respectively. Thus equation (4.1) can be written as

$$\bar{u}_j = \bar{u}_1 + \frac{\bar{u}_j^2}{2} - \sum_{k=1}^{N} \frac{\bar{u}_k^2}{2} \varepsilon_{jk} \quad , \quad j = 1, \ldots\ldots, N \quad . \quad (4.6)$$

The above equation can be further simplified using the presence of 1 in equation (4.5) for ε_{jj}. Using this, we can write the alternate form of equation (4.6) as

$$\bar{u}_j = \bar{u}_1 - \sum_{k=1}^{N} \frac{\bar{u}_k^2}{2} \bar{\varepsilon}_{jk} \quad , \quad j = 1, \ldots\ldots\ldots, N \quad . \quad (4.7)$$

where

$$\bar{\varepsilon}_{jk} = \frac{1}{\pi} \frac{b}{|b|} H_\infty \{ | \frac{2b}{r(x_j)} | \} - \frac{1}{\pi} \frac{a}{|a|} H_\infty \{ | \frac{2a}{r(x_j)} | \} \quad (4.8)$$

1. Subcritical Flow Iteration Methods

Equation (4.6) or (4.7) can be solved by a simple Picard iteration scheme.

$$\bar{u}_j^{\,n} = \bar{u}_{1_j} + \frac{(\bar{u}_j^{\,n-1})^2}{2} - \sum_{k=1}^{N} \frac{(\bar{u}_k^{\,n-1})^2}{2} \varepsilon_{jk} \,, \qquad (4.9)$$

or

$$\bar{u}_j = \bar{u}_{1_j} - \sum_{k=1}^{N} \frac{(\bar{u}_k^{\,n-1})^2}{2} \bar{\varepsilon}_{jk} \,, \qquad (4.10)$$

where the superscript n indicates the nth iterative step The iteration proceeds until

$$\max_j . \left| \bar{u}_j^{\,n} - \bar{u}_j^{\,n-1} \right|$$

becomes less than some specified tolerance.

As given by Kraft (14), the iteration starts with $\bar{u}_j^{\,0} = \bar{u}_{1_j}$. To demonstrate Kraft's method , solutions have been obtained for NACA 0012 airfoil and 6 percent biconvex airfoil. The results are given in Figures 13, 14 and 15 respectively. In both cases, it takes 5 iterations for the solution to converge to the accuracy of 10^{-3}.

However an approximate closed form solution have been obtained by Niyogi and Mitra (28) which can be used as the starting solution for the Kraft's scheme. In Niyogi and Mitra's method, use is made of an inversion formula for the unique exact solution of a two dimensional

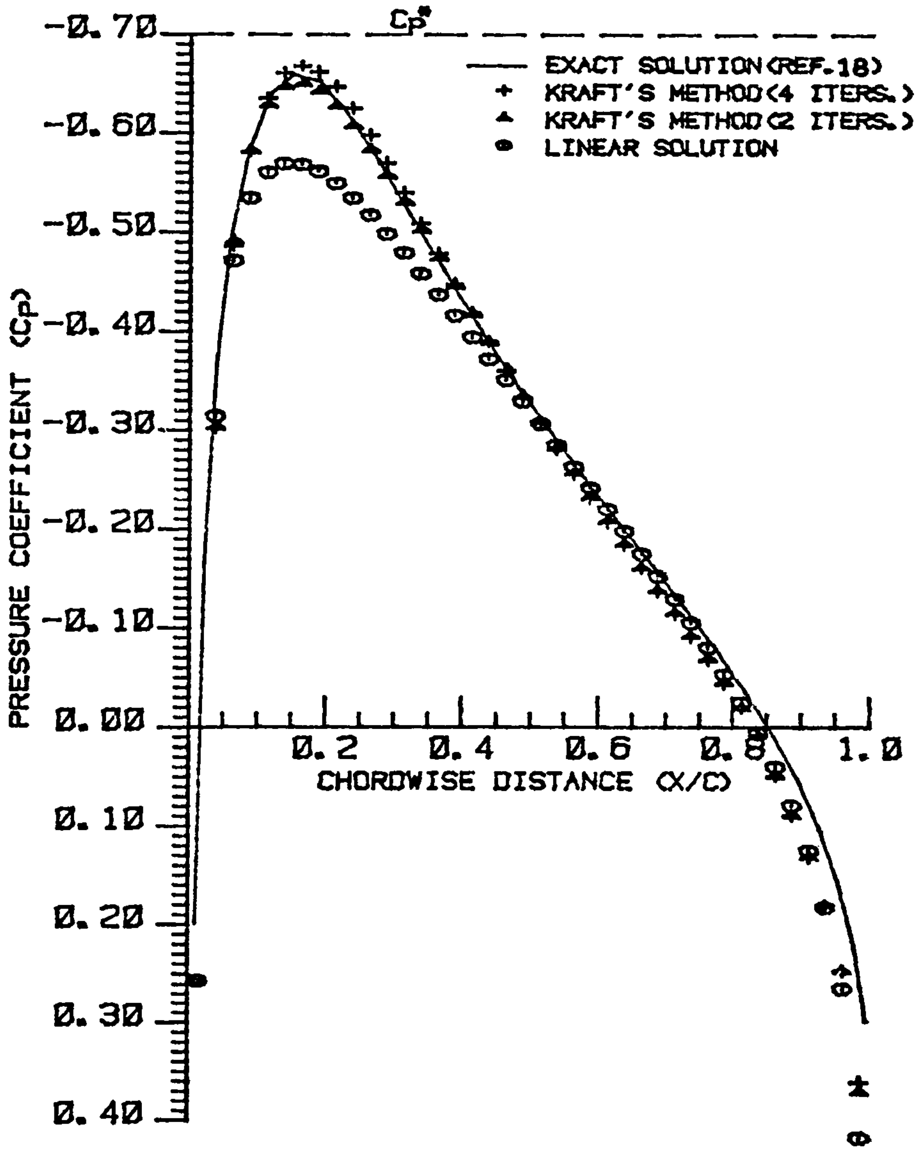

Figure 13. Subcritical pressure distribution over NACA 0012 airfoil at $M_\infty = 0.72$ using Kraft's method.

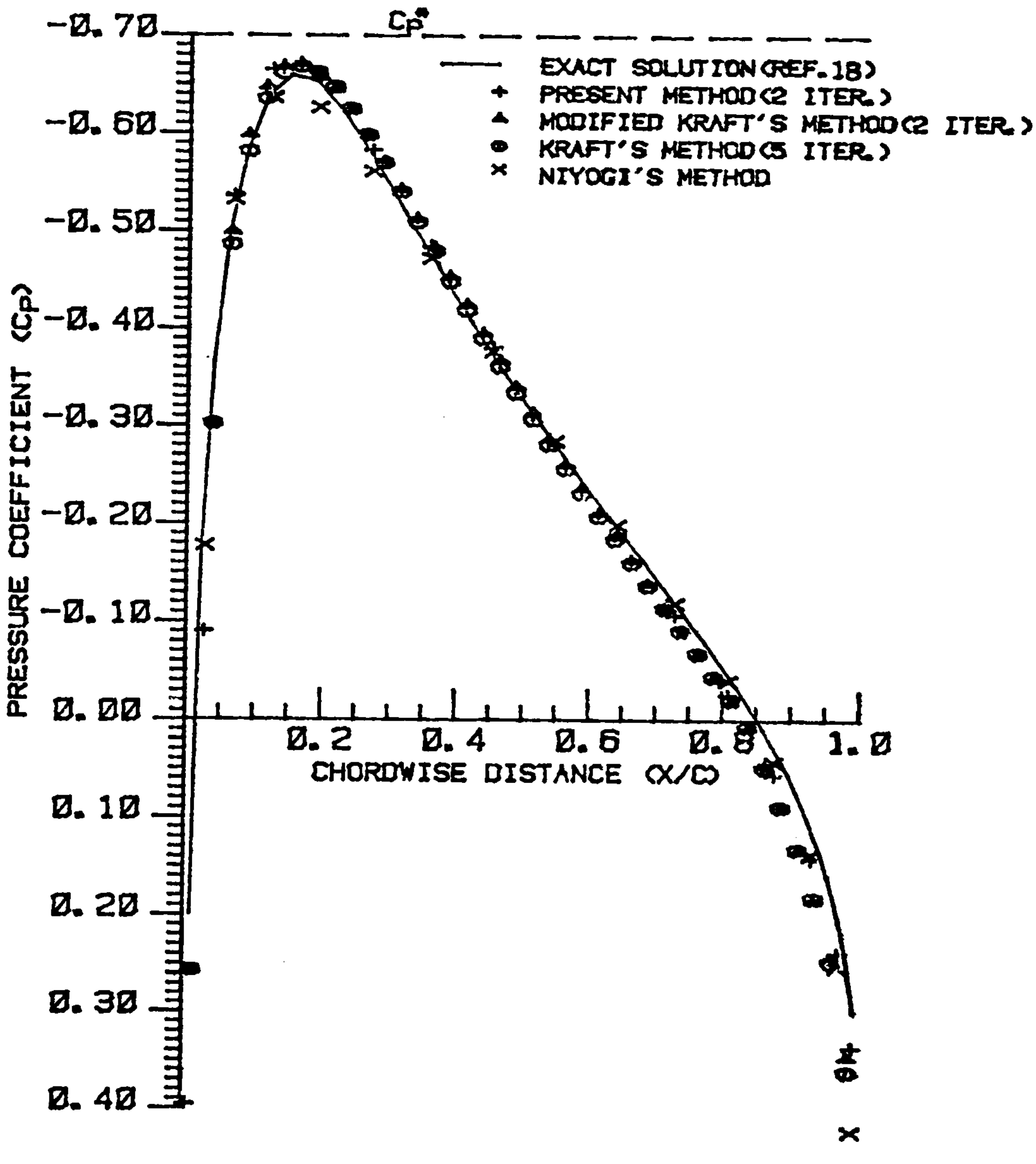

Figure 14. Subcritical pressure distribution over NACA 0012 airfoil at $M_\infty = 0.72$.

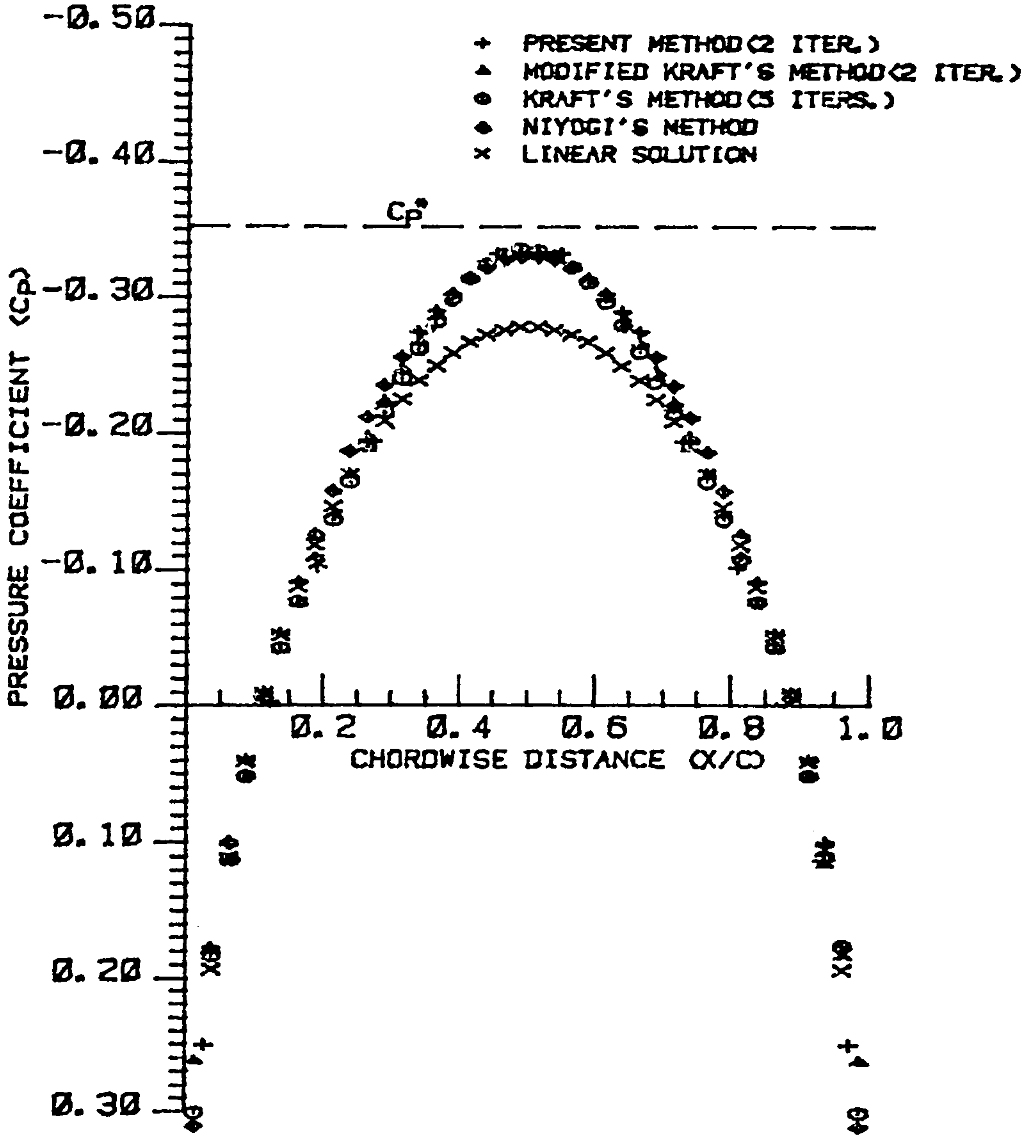

Figure 15. Subcritical pressure distribution over 6% biconvex airfoil at $M_\infty = 0.83$.

linear singular integral equation with constant coefficients, whose kernel is identical with that of the Oswatitsch integral equation. To solve the integral equation (4.1), it is written in the form

$$\bar{a}\,\bar{u}^2(\bar{x},\bar{z}) - \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{u}^2(\bar{\xi},\bar{\zeta})\,\frac{(\bar{\xi}-\bar{x})^2 - (\bar{\zeta}-\bar{z})^2}{\{(\bar{\xi}-\bar{x})^2 + (\bar{\zeta}-\bar{z})^2\}^2}\,d\bar{\xi}\,d\bar{\zeta}$$

$$= 2\,\bar{u}(\bar{x},\bar{z}) - 2\,\bar{u}_1(\bar{x},\bar{z}) + (\bar{a}-\tfrac{1}{2})\,\bar{u}^2(\bar{x},\bar{z}) \quad , \qquad (4.11)$$

where $\bar{a}$ is a parameter greater than half, to be fixed later.

Now the inverse formula, the proof of which is given in reference (28), is: if

$$\bar{a}\,Q(\bar{x},\bar{z}) - \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(\bar{\xi},\bar{\zeta})\,\frac{(\bar{\xi}-\bar{x})^2 - (\bar{\zeta}-\bar{z})^2}{\{(\bar{x}-\bar{\xi})^2 + (\bar{\zeta}-\bar{z})^2\}^2}\,d\bar{\xi}\,d\bar{\zeta}$$

$$= g(\bar{x},\bar{z}) \quad , \qquad (4.12)$$

where $g(\bar{x},\bar{z})$ be a known function square integrable in Euclidean Space E and the parameter a be greater than half, then the unique exact solution in E of equation (4.12) is

$$Q(\bar{x},\bar{z}) = \frac{2}{(4\bar{a}^2-1)^{1/2}}\,g(\bar{x},\bar{z}) +$$

$$\frac{2}{\pi(4\bar{a}^2-1)^{1/2}}\int_{\infty}^{\infty}\int_{\infty}^{\infty} g(\bar{\xi},\bar{\zeta})$$

$$\frac{(2\bar{a}-1)\ (\bar{\xi}-\bar{x})^2 - (2\bar{a}+1)\ (\bar{\zeta}-\bar{z})^2}{(2\bar{a}-1)\ (\bar{\xi}-\bar{x})^2 + (2\bar{a}+1)\ (\bar{\zeta}-\bar{z})^2\ {}^2}\ d\bar{\xi}\ d\bar{\zeta}\ . \qquad \text{.........}\ (4.13)$$

Applying the inverse formula to equation (4.11),we get

$$\bar{u}^2(\bar{x},\bar{z}) = \frac{2}{(4\bar{a}^2-1)^{1/2}}\{2\ \bar{u}(\bar{x},\bar{z}) - 2\ \bar{u}_1(\bar{x},\bar{z}) +$$

$$(\bar{a}-\tfrac{1}{2})\ \bar{u}^2(\bar{x},\bar{z})\ \} + \frac{2}{(4\bar{a}^2-\bar{a})^{1/2}}\ I_1(\bar{x},\bar{z})$$

where

$$I_1(\bar{x},\bar{z}) = \frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(2\bar{a}-1)\ (\bar{\xi}-\bar{x})^2 - (2\bar{a}+1)\ (\bar{\zeta}-\bar{z})^2}{\{(2\bar{a}-1)\ (\bar{\xi}-\bar{x})^2 + (2\bar{a}+1)\ (\bar{\zeta}-\bar{z})^2\}^2}$$

$$\{2\ \bar{u}(\bar{\xi},\bar{\zeta}) - 2\ \bar{u}_1(\bar{\xi},\bar{\zeta}) + (\bar{a}-\tfrac{1}{2})\ \bar{u}^2(\bar{\xi},\bar{\zeta})\}d\bar{\xi}\ d\bar{\zeta} \qquad \text{........}\ (4.14)$$

Solving (4.14) and simplifying, we get

$$\bar{u}(\bar{x},\bar{z}) = K(\bar{a})\quad 1 \pm \{1-\frac{2}{K(\bar{a})}\ (\bar{u}_1(\bar{x},\bar{z}) - \tfrac{1}{2}\ I_1(\bar{x},\bar{z}))\}^{1/2} \qquad \text{..........}\ (4.15)$$

$$K(\bar{a}) = 1 + \frac{(4\bar{a}^2-1)^{1/2}}{(2\bar{a}-1)}$$

It is further shown in reference (28) that for shock free flow, $I_1(\bar{x},0)$ is generally small and therefore

$$\bar{u}(\bar{x},0) = K(\bar{a}) \left[1 - \{ 1 - \frac{2}{K(\bar{a})} \bar{u}_1(\bar{x},0) \}^{1/2} \right] , \quad (4.16)$$

It is also shown that the optimum value of $\bar{a}$, is in general, greater than unity although $\bar{a} = 1$ is a good choice. Using this method, the results are obtained for NACA 0012 and 6 percent biconvex airfoils and these are given in Figures 14 and 15.

This approximate closed form solution is further used as the starting solution for the Kraft's scheme and it is found that the solution converges to the accuracy of 10^{-3} within two iterations. The results can be seen in Figures 14, 15 and 16 under "Modified Kraft's Method."

Since the main interest is to reduce the computing time to obtain a solution of given accuracy, it was thought that Gauss Legendre quadrature method should give a better accuracy with respect to the quadrature method followed in Kraft's scheme for a given number of points, thereby reducing the computing time for a given accuracy. Hence a scheme is developed using Gauss Legendre quadrature method to solve the double integral and is as follows:

The Gauss Legendre quadrature in one dimension can be written as

$$\int_{-1}^{1} f(x)\, dx \simeq \sum_{i=1}^{n} H_i \, f(a_i) , \quad (4.17)$$

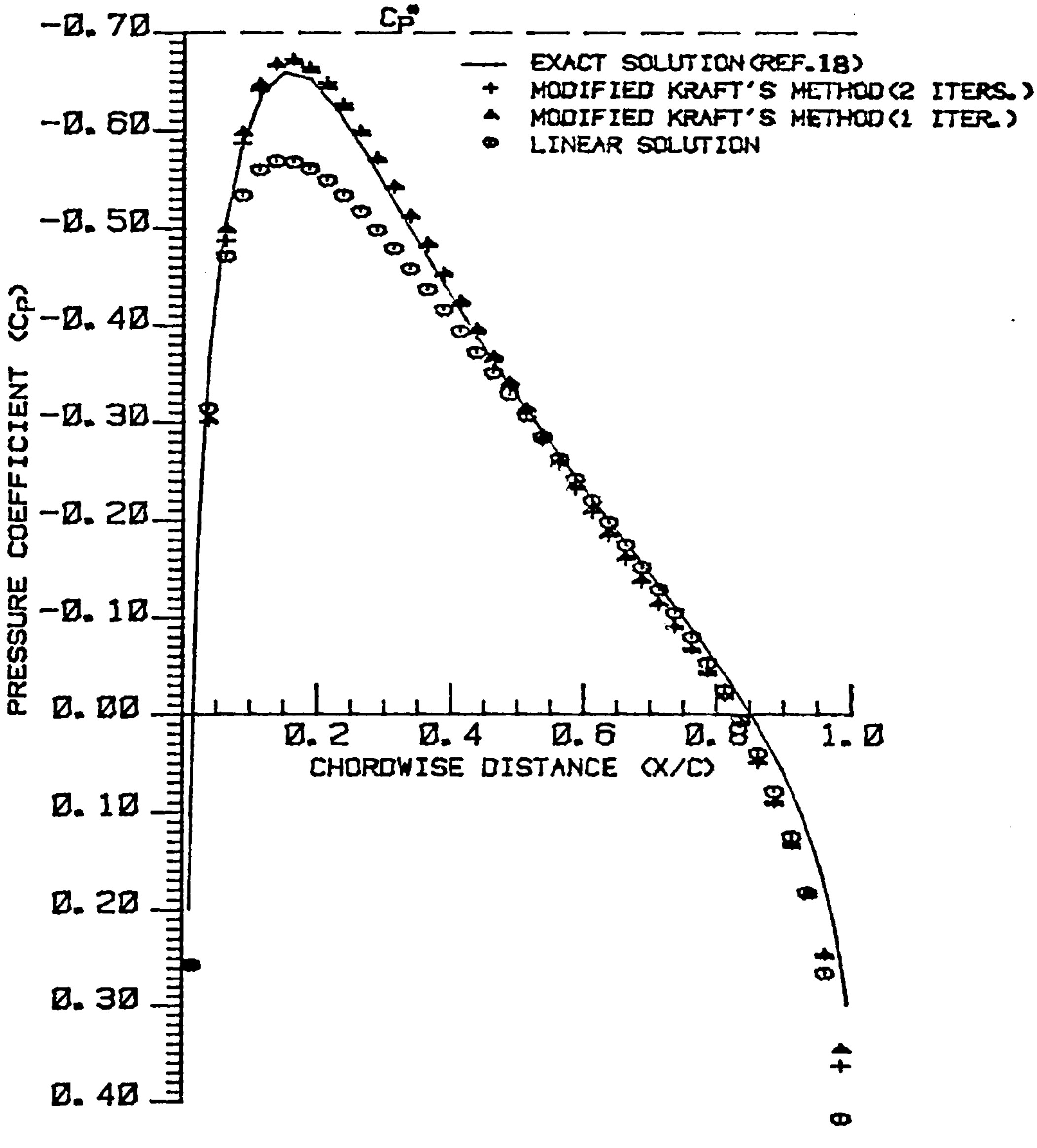

Figure 16. Subcritical pressure distribution over NACA 0012 airfoil at $M_\infty = 0.72$ using modified Kraft's method.

where the ordinates a_i and weights H_i are fixed values depending on n. A typical list is shown in Table 1. It is noted that the finite limit (a', b') can easily be changed to (-1, 1) with a substitution of the form

$$y = \frac{2x - (a' + b')}{b' - a'} \quad . \tag{4.18}$$

Unfortunately the substitution breaks down for infinite limits (-∞, ∞). In case of an integral of the form

$$\int_{-\infty}^{\infty}\int_{0}^{1} f(x,z)\ dx\ dz$$

it is possible to evaluate the integral using Gauss Legendre quadrature method by transforming the limit (0, ∞) by the following substitution

$$z = \tan \frac{\pi}{4}(\ 1+t\ ) \quad , \tag{4.19}$$

to (-1, 1). The limit (0, 1) is changed to (-1, 1) by using the transformation (4.18).

But since the integrand has to be continous, in the region under consideration, for using the Gauss Legendre quadrature method, the above formula can not be straightforward used in case of double integral, in the integral equation, because of presence of singularity.

Gauss Legendre quadrature method is modified for

Table 1. ABSCISSAE AND WEIGHT COEFFICIENTS OF THE GAUSSIAN QUADRATURE FORMULA

n	a_i	H_i	n	a_i	H_i
2	±.57735027	1.00000000	14	±.98628381	.03511946
4	±.86113631	.34785485		±.92843488	.08015809
	±.33998104	.65214515		±.82720132	.12151857
6	±.93246951	.17132449		±.68729290	.15720317
	±.66120939	.36076157		±.51524864	.18553840
	±.23861919	.46791393		±.31911237	.20519846
				±.10805495	.21526385
8	±.96028986	.10122854	16	±.98940093	.02715246
	±.79666648	.22238103		±.94457502	.06225352
	±.52553241	.31370665		±.86563120	.09515851
	±.18343464	.36268378		±.75540441	.12462897
10	±.97390653	.06667134		±.61787624	.14959599
	±.86506337	.14945135		±.45801678	.16915652
	±.67940957	.21908636		±.28160355	.18260342
	±.43339539	.26926672		±.09501251	.18945061
	±.14887434	.29552422			
12	±.98156063	.04717534			
	±.90411725	.10693933			
	±.76990267	.16007833			
	±.58731795	.20316743			
	±.36783150	.23349254			
	±.12533341	.24914705			

n = number of Gaussian points needed to evaluate the integral

f(x) = given function to be integrated

H_i = weighting factors

solving the double integral I, is modified to take care of singularity by using equations (4.3) and (4.4). Picard iteration scheme is used to solve the integral equation at each of the Gaussian pionts. Since the singularity in I occurs at the point where the nonlinear velocity is calculated, the contribution of this Gaussian point to the value of double integral $I(a_j)$ is obtained using equation (4.4) for the interval $(-H_j/2, H_j/2)$ instead of

$$H_j \sum_{k=1}^{n} f(a_j, a_k) H_k$$

Solutions have been obtained using 16 Gaussian points, with Niyogi and Mitra's solution as the starting solution for the iteration scheme, for NACA 0012 and 6 percent biconvex airfoils. The results are given in Figures 17, 14 and 15 under "Present Method."

2. Supercritical Flow Iteration Methods

If, for a fixed geometry, the Mach number is increased such that supersonic flow develops anywhere on the airfoil, the Picard iteration scheme fails to converge. This is because of the existence of an embedded supersonic flow in an overall subsonic flow terminated abruptly by a shock or smoothly by a sonic line. Thus to develop an alternate iteration scheme, several

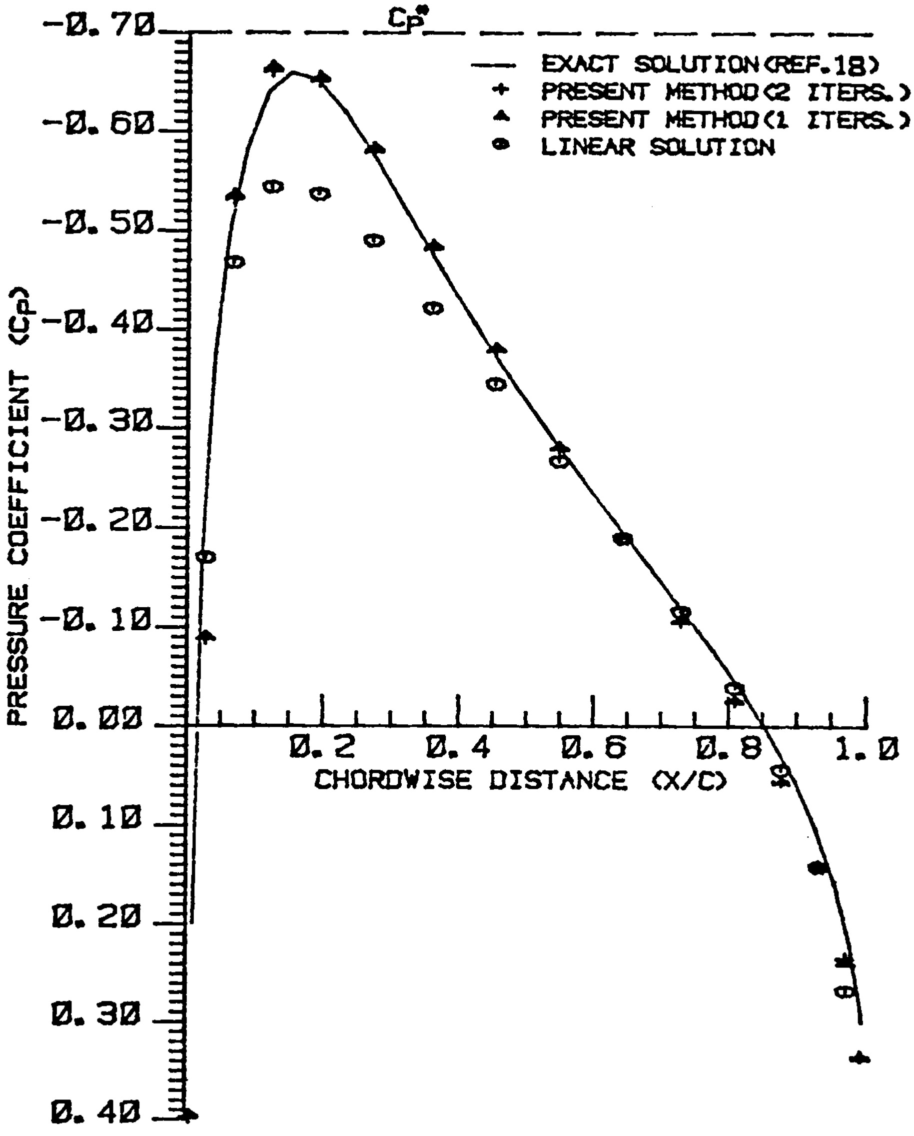

Figure 17. Subcritical pressure distribution over NACA 0012 airfoil at $M_\infty = 0.72$ using present method.

points with regard to equation (4.1) are discussed first. Solution of equation (4.1) for $\bar{u}$ in terms of I and $\bar{u}_1$ can be written as

$$\bar{u} = 1 \pm \surd \{ 2 I - (2 \bar{u}_1 - 1) \} . \tag{4.20}$$

Therefore for real values of $\bar{u}$, the discriminant must always be positive, i.e.,

$$2 I \geq (2 \bar{u}_1 - 1) . \tag{4.21}$$

Furthermore, the choice of plus sign or minus sign in equation (4.20) determines whether the local velocities are subsonic or supersonic. A change in sign where the discriminant is zero corresponds to a smooth transition through the sonic velocity. A change in sign where the discriminant is not zero corresponds to a discontinous jump in velocity. Such discontinuity corresponds to normal shock wave and is permissible when it proceeds from supersonic to subsonic velocity. Discontinuity in the reverse direction is inadmissible since they correspond to expansion shocks, an impossible phenomenon which violates the second law of thermodynamics. These conditions lead to the following relations at the sonic point. First, the flow attains the sonic velocity at the sonic point, or

$$\{ 2 I - (2 \bar{u}_1 - 1) \}_{x^*} = 0, \qquad (4.22)$$

and secondly, to avoid expansion shocks, the flow must pass smoothly through the sonic point, or

$$\frac{\partial}{\partial x}\{ 2 I - (2 \bar{u}_1 - 1) \}_{x^*} = 0. \qquad (4.23)$$

These conditions are used in the fixed shock iterative scheme followed herein. In this scheme, for a fixed airfoil geometry, an initial value of M_∞ and a reasonable distribution of $\bar{u}$ are assumed. This $\bar{u}$ distribution contains a discontinuity, given by equation (2.28) for a normal shock wave, at the specified shock location. This shock location is unaltered throughout the iteration process. Using the initial $\bar{u}$ distribution, the values of discriminant of equation (4.20) are calculated. Thereafter, to check the conditions at the sonic point, the algorithm given by Kraft (14) is used. In this algorithm, the value of local minimum of the N discrete values of discriminant nearest the leading edge is found. Then a quadratic polynomial is fitted through the three values of the discriminant immediately upstream, downstream, and at the minimal value. Using this polynomial, the location of actual minimun of the discriminant (which is the sonic point location if condition (4.22) is satisfied) is determined. The value of Mach number is then adjusted until a distribution of $\bar{u}$ is determined such that the actual minimum of the discriminant is zero. A new set of values can then be calculated from

$$\bar{u}_j = 1 \pm \sqrt{\left[2\ I(\bar{u}_j^{\,n-1}) - \{\ 2\ \bar{u}_1(M_\infty^n) - 1\ \}\right]}\ . \qquad (4.24)$$

The terms of the discriminant in equation (4.24) are written in functional form to emphasize their dependence on $\bar{u}$ and M_∞ for the previous iterative step. Proper care must be taken to change from the minus sign to plus sign at the sonic point and then return to minus sign aft of the prescribed shock location. If the new values of M_∞ and $\bar{u}$ are sufficiently close to the previous values, it is presumed that a converged solution is obtained. However, if such a close correspondence is not obtained, the new values of M_∞ and $\bar{u}$ are taken and the process is repeated. Using this technique, results are obtained for NACA 0012 and 6 percent biconvex airfoil and are shown in Figures 18 and 19 respectively.

However, since a reasonable distribution of $\bar{u}$ is required in this method, it is decided to use the approximate solution obtained for supercritical flow with shock, by Mitra (21) as the starting solution. Mitra (21) started with the solution given in reference (28) for shock free profile flow:

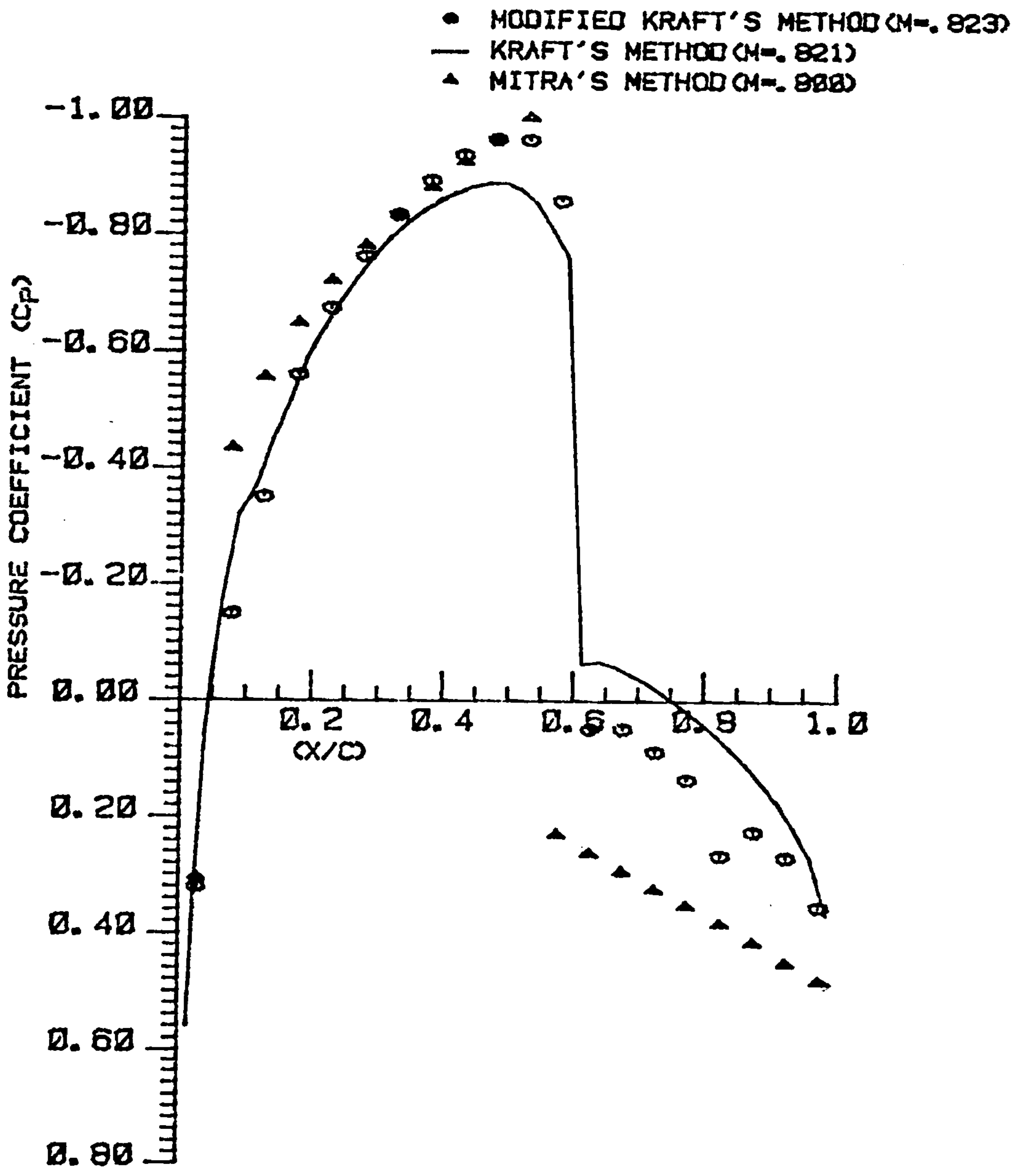

Figure 18. Supercritical pressure distribution over NACA 0012 airfoil.

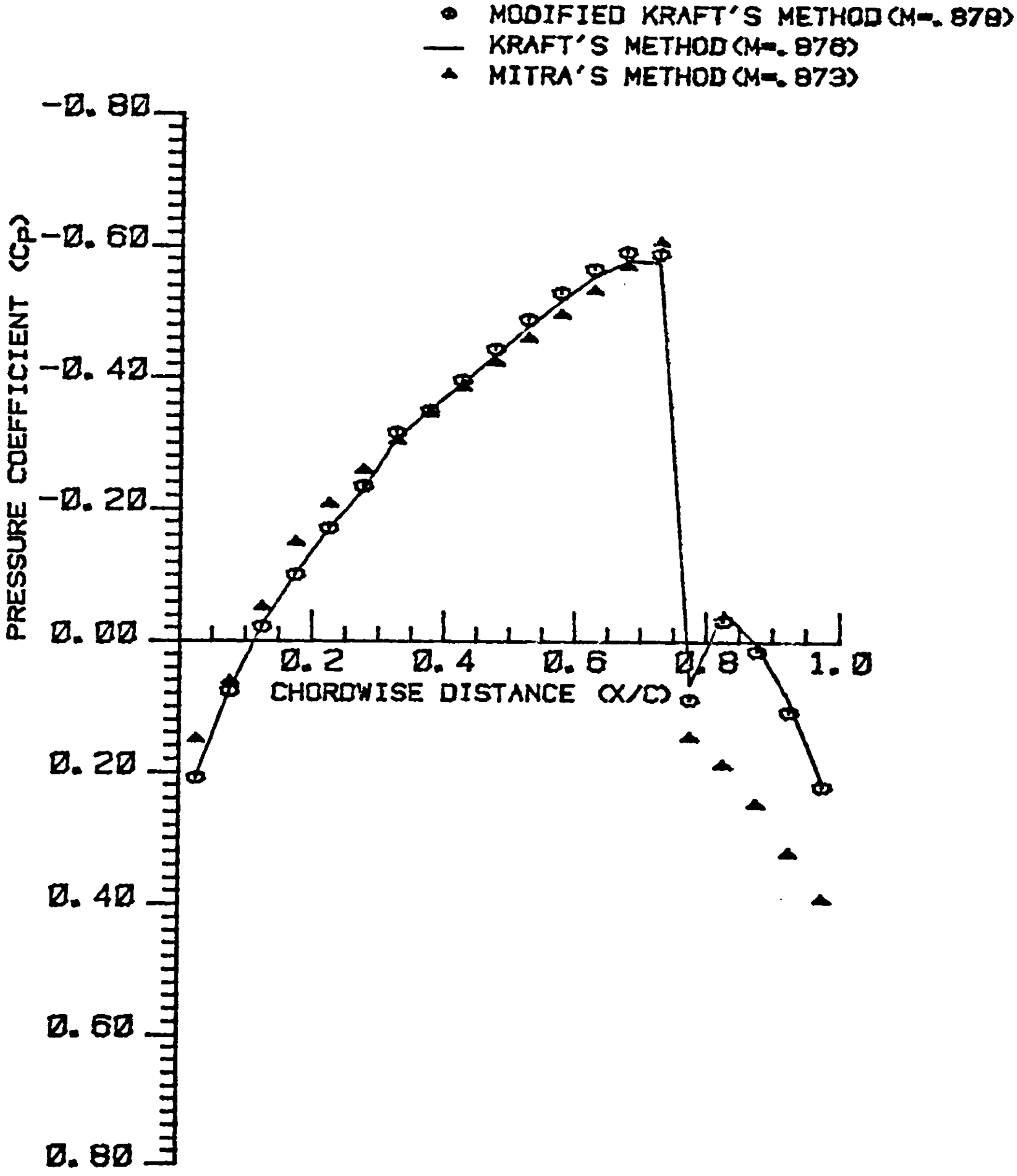

Figure 19. Supercritical pressure distribution over 6% biconvex airfoil.

$$\bar{u}_0(\bar{x},0) = K(\bar{a}) \left[1 - \sqrt{ \left\{ 1 - \frac{2}{K(\bar{a})} \bar{u}_1(\bar{x},0) \right\} } \right] \quad \ldots\ldots\ldots\ldots\ldots \quad (4.25)$$

where

$$K(\bar{a}) = 1 + \frac{\sqrt{(4\bar{a}^2 - 1)}}{(2\bar{a} - 1)} \quad .$$

Then a correction term possessing a jump discontinuity is is determined which leads to an approximate solution for supercritical flow as

$$\bar{u}(\bar{x},\bar{z}) = 1 \pm \Bigg[\{\bar{u}_0(\bar{x},\bar{z}) - 1\}^2 - 2 \left\{ \left(1 - \frac{1}{K(\bar{a})}\right) \frac{\bar{u}_0^2(\bar{x},\bar{z}) - 1}{2} - \left[\bar{u}_1(\bar{x},\bar{z}) - \bar{u}_1(\bar{x}^*,\bar{z})\right] - \int_{\bar{x}^*}^{\bar{x}} \frac{\partial \bar{w}_0}{\partial \bar{z}} \, d\bar{x} \right\} \Bigg]^{1/2} , \quad (4.26)$$

where $\bar{x}^*$ is the sonic point location. Here the upper and lower sign corresponds to $\bar{u}_0 \gtrless 1$. To further simplify the integral on equation (4.26) for $\bar{u}(\bar{x},0)$, the functional relationship between the horizontal perturbation velocity and the vertical direction given by equation (3.17) is made use of. Using the irrotationality condition and boundary conditions, one can obtain

$$\frac{\partial w_0}{\partial \bar{z}} = \left\{ \frac{(\gamma+1)\, M_\infty^2}{U_\infty \, \beta} \right\}^2 \int_{\bar{x}^*}^{\bar{x}} \frac{(d^2 z_1/d\bar{x}^2)}{\bar{u}_0(\bar{x},0)} \, d\bar{x} \quad . \quad (4.27)$$

In this method, it is assumed that the $u_0(\bar{x}^*,0) = 1$ at the sonic point. Thus for a given Mach number and the airfoil, $\bar{u}_0$ can be calculated using equation (4.25) and sonic point location is determined. Next the value of $\partial\bar{w}_0/\partial\bar{z}$ is computed with the help of equation (4.27) by integrating it numerically by Simpson's rule. Then by the same procedure, the integral in equation (4.26) is calculated and finally the corrected value $\bar{u}(\bar{x},0)$ is obtained using equation (4.26).

The results are obtained using this method for NACA 0012 and 6 percent biconvex airfoils and these are given in Figures 18 and 19.

Further, these approximate solutions for NACA 0012 and 6 percent biconvex airfoils have been used as the starting solution for Kraft's algorithm and the results so obtained are again given in Figures 18 and 19.

CHAPTER V

DISCUSSION OF RESULTS AND CONCLUSIONS

Linear flow field horizontal perturbation velocities for NACA 0012 and 6 percent biconvex airfoils have been obtained analytically and also by using the exponential and inverse square functional relationships (equations (3.17) and (3.18) respectively). For biconvex airfoil, the results are obtained at x = 0.01c and 0.50c for M_∞ = 0.00, 0.30 and 0.83. These results are given in figures 2 to 7. As can be seen from these figures, for all Mach numbers, the velocities obtained using both the functional relationships do not agree with the analytical results near the leading edge. Same trend can be seen near the trailing edge of the biconvex airfoil because of its symmetry about x = 0.50c. At x = 0.50c, the results of all the three methods agree closely. The results calculated for NACA 0012 airfoil are given in Figures 8 to 12. For this airfoil also, the comparison is not good near leading and trailing edges. At x = 0.50c, the agreement between different results is good up to $z \simeq 0.5c$ and thereafter the results obtained using exponential decay matches closer with the analytical solution than that obtained using other relationship.

Further, subcritical flow solutions have been computed for NACA 0012 airfoil at $M_\infty = 0.72$ and 6 percent biconvex airfoil at $M_\infty = 0.83$ using various methods described in chapter IV. Figures 13 to 17 consist of these solutions. It is seen in Figure 13 that in case of Kraft's method, it takes 5 iterations for the solution to converge to a given accuracy (10^{-3}) for NACA 0012 airfoil. While with the modified Kraft's method and present method, solution converges to the same accuracy within two iterations. However, the computing time is more in case of present method with respect to other methods. The results for biconvex airfoil (Figure 17) also leads to the same conclusion as obtained for NACA 0012 airfoil.

Finally, the results are obtained for supercritical flow over NACA 0012 and 6 percent biconvex airfoils. These are shown in Figure 18 and 19 respectively. It can be seen from these figures that Mitra's closed form solution represents a good starting solution compared to an arbitrary solution for the Kraft's iterative scheme. Also a kink can be observed in both figures near the sonic point. This may be attributed to the algorithm used to satisfy the conditions at the sonic point.

Thus it is seen that that the exponential and inverse square functional relationships, used to represent the decay of horizontal perturbation velocity with vertical coordinate, are good approximations except near the leading edge and trailing edge. It is further seen that the Kraft's approach for solving the integral equation can be modified to achieve faster convergence, thereby reducing the computing time.

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