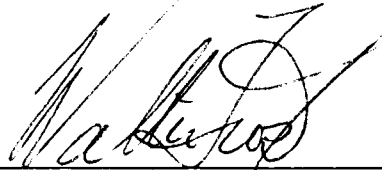


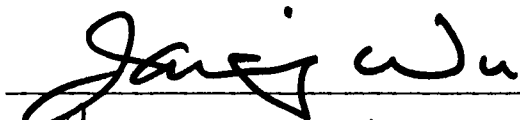
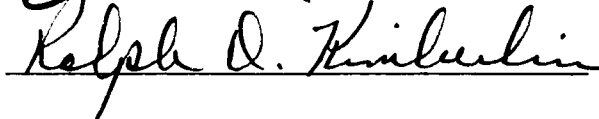
TO THE GRADUATE COUNCIL:

I am submitting herewith a thesis written by Erik Amund Ringnes entitled "Analysis of Spanwise Turbulence Effects on Airplane Performance." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science with a major in Aerospace Engineering.



Dr. Walter Frost, Major Professor

We have read this thesis and
recommend its acceptance:

Accepted for the Council:



The Graduate School

ANALYSIS OF SPANWISE TURBULENCE EFFECTS
ON AIRPLANE PERFORMANCE

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Erik Amund Ringnes
December 1984

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ABSTRACT

The purpose of this study is to determine the influence of spanwise turbulence on airplane dynamic behavior. Calculations are based on data collected from test flights with the NASA B-57 research aircraft. The approach taken is to first compute aerodynamic forces and moments due to a spanwise distribution of angle of attack and relative wind. Secondly, these quantities are incorporated into the equations of motion. Simulation of flights done with the effects of spanwise turbulence included are compared to simulations without any spanwise turbulence. The findings of the study are that the moments developed by turbulence along the span are significant and that more realistic flight simulation can be achieved by including the spanwise turbulence terms.

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LIST OF SYMBOLS

A_n	Fourier series coefficients (dimensionless)
a_∞	Two-dimensional lift curve slope (rad^{-1})
C_D	Drag coefficient (dimensionless)
C_{D_0}	Drag coefficient at zero angle of attack (dimensionless)
C_{D_α}	Drag derivative with respect to angle of attack (rad^{-1})
$C_{D_{\delta_r}}$	Drag derivative with respect to rudder deflection (rad^{-1})
$C_{D_{\delta_a}}$	Drag derivative with respect to aileron deflection (rad^{-1})
C_L	Lift coefficient (dimensionless)
C_{L_0}	Lift coefficient at zero angle of attack (dimensionless)
C_{L_q}	Lift derivative with respect to pitching rate (rad^{-1})
C_{L_α}	Lift derivative with respect to angle of attack (rad^{-1})
$C_{L_{\dot{\alpha}}}$	Lift derivative with respect to rate of change of angle of attack (rad^{-1})
$C_{L_{\delta_e}}$	Lift derivative with respect to elevator deflection (rad^{-1})
$C_{L_{i_t}}$	Lift derivative with respect to tail incidence angle (rad^{-1})
$C_L(y)$	Section lift coefficient
C_{ℓ_p}	Rolling moment derivative with respect to rolling rate (rad^{-1})

C_{ℓ_r}	Rolling moment derivative with respect to yawing rate (rad^{-1})
C_{ℓ_β}	Rolling moment derivative with respect to sideslip angle (rad^{-1})
$C_{\ell_{\delta_a}}$	Rolling moment derivative with respect to aileron deflection (rad^{-1})
$C_{\ell_{\delta_r}}$	Rolling moment derivative with respect to rudder deflection (rad^{-1})
C_m	Pitching moment coefficient (dimensionless)
C_{m_0}	Pitching moment coefficient at zero angle of attack (dimensionless)
C_{m_q}	Pitching moment derivative with respect to pitching rate (rad^{-1})
C_{m_α}	Pitching moment derivative with respect to angle of attack (rad^{-1})
$C_{m_{\dot{\alpha}}}$	Pitching moment derivative with respect to rate of change of angle of attack (rad^{-1})
$C_{m_{\delta_e}}$	Pitching moment derivative with respect to elevator deflection (rad^{-1})
C_{n_p}	Yawing moment derivative with respect to rolling rate (rad^{-1})
C_{n_r}	Yawing moment derivative with respect to yawing rate (rad^{-1})
C_{n_β}	Yawing moment derivative with respect to sideslip angle (rad^{-1})
$C_{n_{\delta_r}}$	Yawing moment derivative with respect to rudder deflection (rad^{-1})

$C_{n_{\delta a}}$	Yawing moment derivative with respect to aileron deflection (rad^{-1})
$C_{y_{\beta}}$	Side force derivative with respect to sideslip angle (rad^{-1})
$C_{y_{\delta r}}$	Side force derivative with respect to rudder deflection (rad^{-1})
$c(y)$	Section chord length (feet)
$\bar{c}$	Mean chord length (feet)
D	Drag force (lbf)
D_i	Induced drag force (lbf)
$d_i(y)$	Induced drag per unit span (lbf/feet)
F_y	Side force (lbf)
h	Normal distance from a vortex (m)
I_{nm}	Matrix with n rows and m columns
L	Lift force (lbf)
L	Rolling moment (ft-lbf)
L/W	Lift-to-weight ratio (dimensionless)
$l(y)$	Lift per unit span (lbf/ft)
L_{SPANWISE}	Lift force in a spanwise distributed wind (lbf)
L_{UNIFORM}	Lift force in a uniform wind field (lbf)
m	Mass of aircraft
M	Pitching moment (ft-lbf)
N	Yawing moment (ft-lbf)
P_{shape}	Shape factor (dimensionless)
P	Rolling rate (rad/sec)

q	Pitching rate (rad/sec)
r	Yawing rate (rad/sec)
s	Half-span length (ft)
t	Time (sec)
V	Relative wind (ft/sec)
V_{avg}	Average airspeeds (ft/sec)
V_c	Center probe airspeed (ft/sec)
V_L	Left probe airspeed (ft/sec)
V_R	Right probe airspeed (ft/sec)
V_R	Resultant relative wind (ft/sec)
$V(y)$	Relative airspeed (ft/sec)
v_i	Induced velocity (ft/sec)
$w(y)$	Downwash velocity (ft/sec)
x	x-axis in body coordinates (feet)
y	y-axis in body coordinates (feet)
z	z-axis in body coordinates (feet)

Greek Symbols

α	Angle of attack at aircraft center of gravity (rad)
$\alpha(y)$	Angle of attack at position y along the span (degrees)
α_{avg}	Average angle of attack (degrees)
α_c	Center probe angle of attack (degrees)
α_L	Left probe angle of attack (degrees)
α_R	Right probe angle of attack (degrees)

$\alpha_o(y)$	Zero lift angle (rad)
$\alpha_{\text{eff}}(y)$	Effective angle of attack (rad)
β	Sideslip angle (rad)
δ_a	Aileron deflection (rad^{-1})
δ_e	Elevator deflection (rad)
δ_r	Rudder deflection (rad)
$\delta\Gamma$	Strength of finitely small vortex filament
ΔD_{ADD}	Drag due to spanwise turbulence (lbf)
ΔL_{ADD}	Lift due to spanwise turbulence (lbf)
$\epsilon(y)$	Downwash angle at a position y along the span (rad)
$\Gamma(y)$	Circulation strength per unit span at a position y along the span
$\mu(y)$	Parameter defined by Equation 2.23 (rad^{-1})
θ	Pitching angle (rad)
ρ	Air density (lbm/ft^3)
ϕ	Rolling angle (rad)
ψ	Yawing angle (rad)

Other

$(\dot{})$	Time derivative
$(\overline{})$	Time-averaged value

CHAPTER I

INTRODUCTION

Atmospheric turbulence is a major hazard to safe flight operation. Past studies of airplane behavior in turbulent air have been constrained to assume homogeneous turbulence and a point mass aircraft (Newman and Foster 1970). The effect of turbulence across the wing span of an aircraft has been ignored, primarily due to a lack of data. However, recent test flights conducted by NASA now facilitate studying spanwise turbulence. The B-57 test aircraft with a wing span of 64 feet is equipped to measure relative wind and angles of attack at three locations, in the nose and on each wing tip (Figure 1.1). The B-57 instrumentation is discussed in detail in Camp et al. (1983), Painter and Camp (1983), and Campbell et al. (1983). In this study, linear distributions of angle of attack and relative wind between the nose and either wing tip are assumed to represent spanwise turbulence. Any turbulence that is not symmetric laterally, produces asymmetric distributions of lift and drag (Figure 1.2). Such uneven lift and drag forces create roll and yaw moments. These moments, due to spanwise turbulence, are neglected by assuming the entire airfoil experiences uniform conditions.

The motive of this study is to utilize the data gathered with the NASA B-57 aircraft to analyze the effects of spanwise turbulence on aircraft performance. The purpose is to provide design criteria and to contribute to the continuous effort of improving state-of-the-art simulator technology. Realistic simulation of flight in critical weather

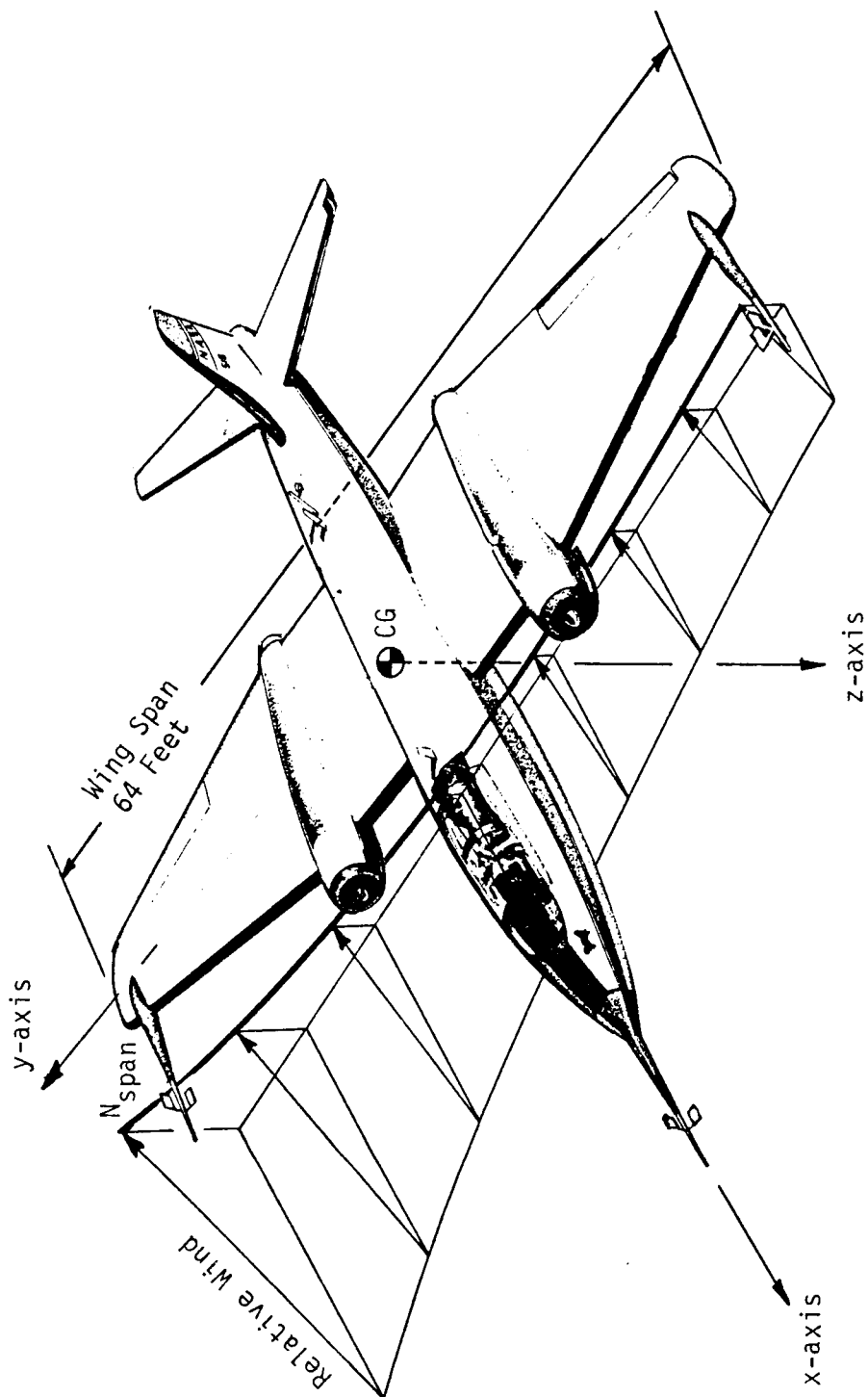


Figure 1.1. Spanwise variation of relative wind and angle of attack.

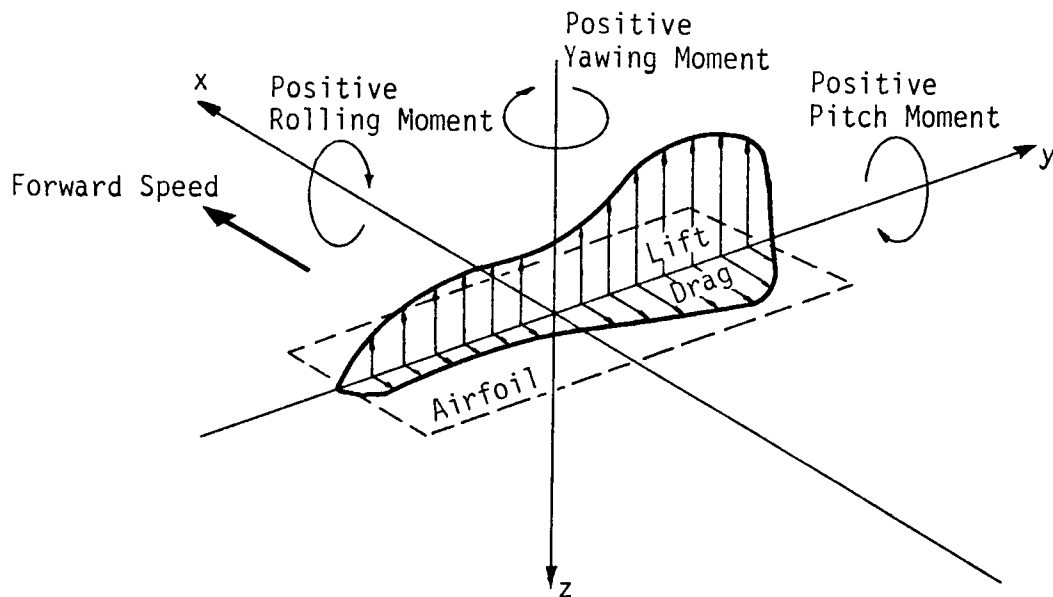


Figure 1.2. Asymmetric distributions of lift and drag.

conditions depends upon the turbulence models and more importantly on the methods of incorporating turbulence effects into aircraft dynamic modeling.

Spanwise turbulence creates only additional aerodynamic roll and yaw moments. The pitch moment is assumed to be unaffected. Total lift and drag forces can be calculated based upon spanwise or uniform distributions of angles of attack and airspeed. The difference between such two calculations are considered the effect of spanwise turbulence on aerodynamic forces. Mathematically the additional roll moment is expressed as:

$$L_{ADD} = \int_{-s}^s \ell(y)ydy \quad (1.1)$$

and similarly the additional yaw moment is written:

$$N_{ADD} = \int_{-s}^s d_i(y) y dy \quad (1.2)$$

where s is the half-span length and y is in body axis coordinate system which is indicated in Figure 1.1. Sectional lift and induced drag, $l(y)$ and $d_i(y)$, are functions of the spanwise position. Incremental changes in lift and induced drag due to spanwise turbulence are given as:

$$\Delta L_{ADD} = L_{SPANWISE} - L_{UNIFORM} \quad (1.3)$$

$$\Delta D_{iADD} = D_{iSPANWISE} - D_{iUNIFORM} \quad (1.4)$$

The details of these calculations are explained in Chapter III. These four quantities L_{ADD} , N_{ADD} , ΔL_{ADD} , and ΔD_{iADD} are then the representation of the effects due to spanwise turbulence.

The first part of this study is devoted to calculating aerodynamic forces and moments due to spanwise turbulence. In Chapter II the necessary theory is derived, and in Chapter III results of applying the theory to representative case studies are presented.

The influence spanwise turbulence has on aircraft dynamic behavior is estimated in the second part of the study. A six-degrees-of-freedom flight simulation program (6-D program) developed at FWG Associates, Inc. is used for this purpose (Frost and Bowles 1984). In Chapter IV the methods used to implement the spanwise turbulence effects into the simulation routine are described. Chapter IV also compares these with the results of computer-simulated flights and flight data. Chapter V covers a final conclusion and recommendations. The computer program used to calculate the effects of spanwise turbulence is documented in

Appendix A. Appendix B shows in detail the calculation of the gust alleviation factor referred to in Chapter III. Appendix C lists the B-57 characteristics used in the 6-D program. The automatic control laws used in the 6-D program are presented in Appendix D.

CHAPTER II

DERIVATION OF STRIP THEORY

Vortex theory is the method most commonly used to calculate the lift and drag distributions of a finite airfoil. The strip theory developed in this chapter is a modified vortex theory which allows for random spanwise variations in angle of attack and relative wind. As the name implies, the airfoil is divided into a finite number of strips. At each strip, lift and drag can be evaluated. By integrating these forces across the wing span, the desired moments are obtained.

1. DERIVATION OF TOTAL QUANTITIES

The lift distribution across an airfoil can be represented by a distribution in the strength of the bound circulation $\Gamma(y)$ as shown in Figure 2.1. At any position in the spanwise direction, the lift per unit span can be expressed by Zhukovsky's theorem as (Kuethe and Chow 1976):

$$L(y) = \rho V(y) \Gamma(y) \quad (2.1)$$

where ρ is the air density and $V(y)$ is the speed of the airfoil along the direction of the x-axis in the wind coordinates. The frame of reference in wind coordinates is moving with the mean wind. The x-direction is chosen to be parallel to the direction of flight. Contrary to traditional analysis, $V(y)$ is considered a variable in the body axis y-direction. Similarly the induced (vortex) drag per unit span at any location again by Zhukovsky's theorem becomes:

$$d_i(y) = \rho W(y) \Gamma(y) \quad (2.2)$$

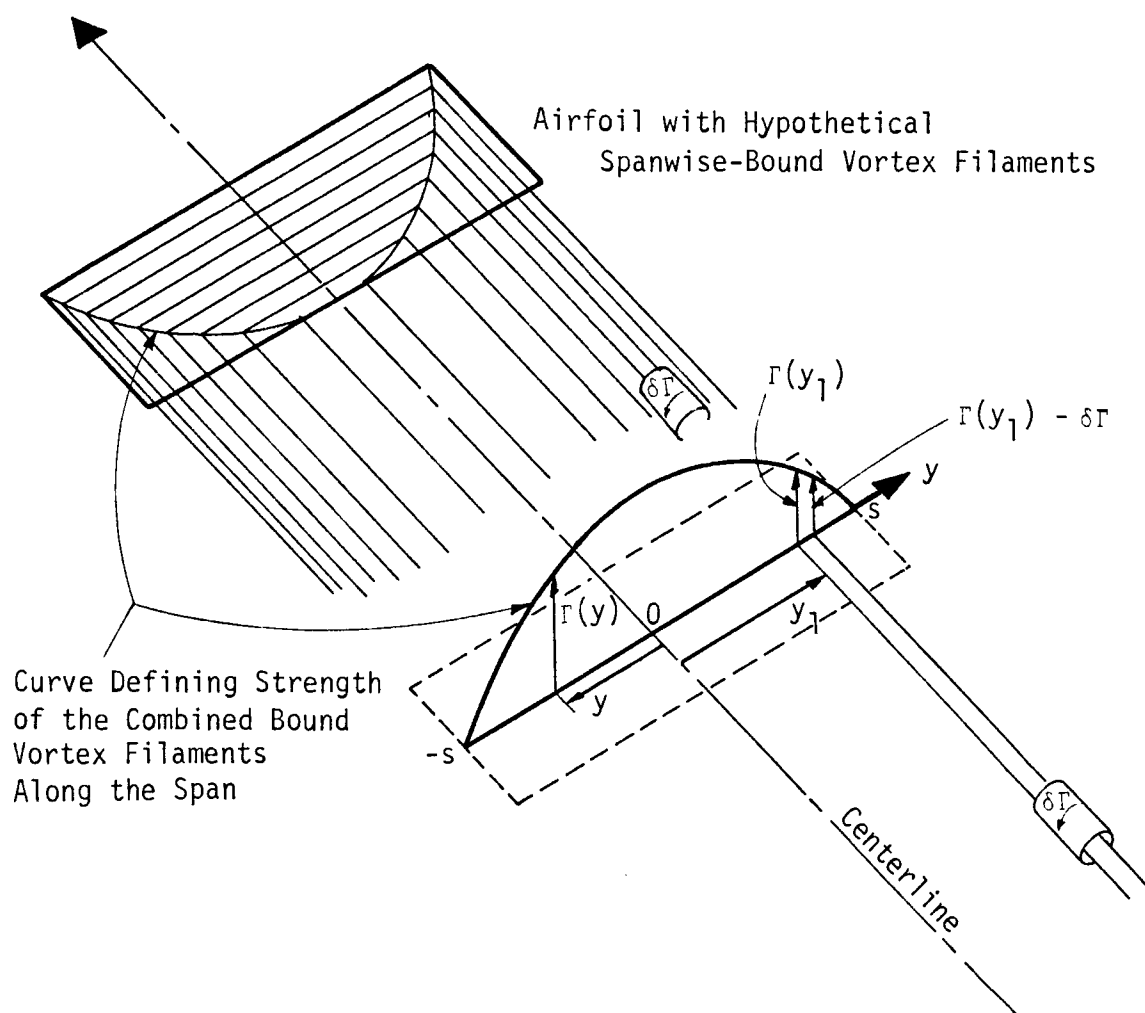


Figure 2.1. The relation between spanwise load variation and trailing vortex strength.

where $w(y)$ is the downwash velocity. The total lift is found by integrating over the span:

$$L = \int_{-s}^s \rho V(y) \Gamma(y) dy \quad (2.3)$$

In the same manner, total induced drag is:

$$D_i = \int_{-s}^s \rho w(y) \Gamma(y) dy \quad (2.4)$$

As a first approximation, the circulation is often assumed to be elliptical in nature, which leads to closed form solutions of the integrals. In this study the integrals are solved numerically since the circulation can be non-elliptical and asymmetric.

The lift per unit span as in Equation 2.1 is also written:

$$l(y) = \frac{1}{2} \rho V^2(y) c(y) C_L(y) \quad (2.5)$$

where $c(y)$ is the chord length and $C_L(y)$ is the sectional lift coefficient. Combining Equations 2.1 and 2.5 and solving for $\Gamma(y)$, the following expression is obtained:

$$\Gamma(y) = \frac{1}{2} V(y) c(y) C_L(y) \quad (2.6)$$

The continuation of the derivation is simplified by designating the variable y in terms of the angle θ as illustrated in Figure 2.2.

Equation 2.6 is now written:

$$\Gamma(\theta) = 4sV(\theta)P_{\text{shape}}(\theta) \quad (2.7)$$

where P_{shape} is a dimensionless shape parameter which includes the variation in both lift coefficient $C_L(y)$ and the chord length $c(y)$. The circulation is made proportional to four times the half-span length rather than the root chord length for convenience.

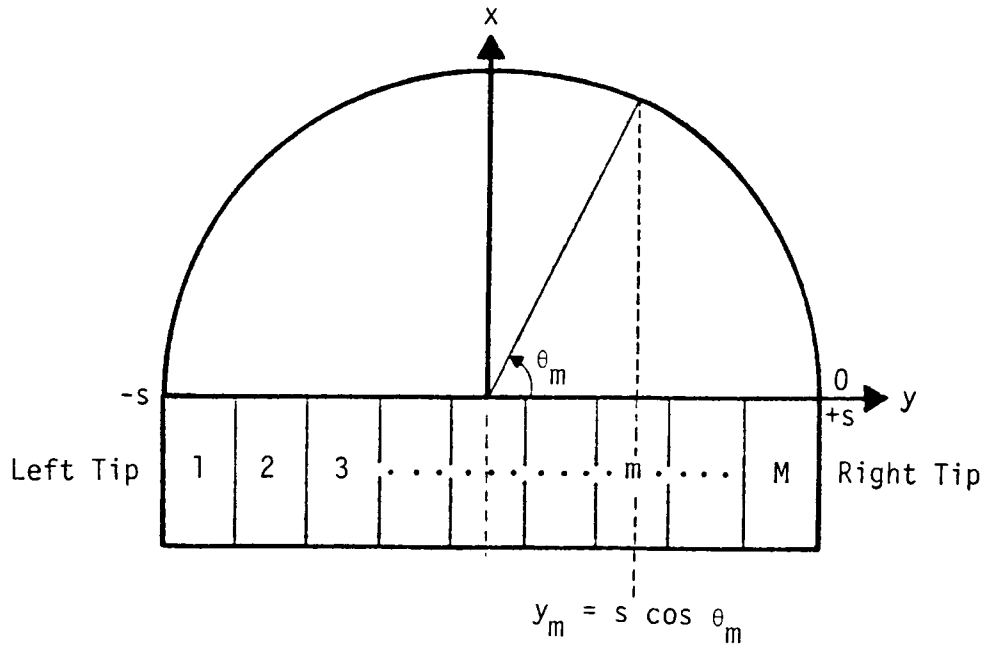


Figure 2.2. Change of variable.

The shape parameter is represented as a Fourier sine series:

$$P_{\text{shape}}(\theta) = \sum_{n=1}^{\infty} A_n \sin n\theta \quad (2.8)$$

The sine series satisfies the end conditions of the curve reducing to zero at the tips where $n\theta$ is zero or π (n is an integer). A_n is the amplitude of each sine curve making up the shape parameter. In Figure 2.3 a sample asymmetrical shape parameter is illustrated.

The total quantities of lift, induced drag, and roll and yaw moments given by Equations 2.3, 2.4, 1.1, and 1.2, respectively, are now written by substitution of Equations 2.7 and 2.8 as:

$$L = \sum_{n=1}^{\infty} A_n \int_0^{\pi} 4s^2 \rho V^2(\theta) (\sin n\theta \sin \theta) d\theta \quad (2.9)$$

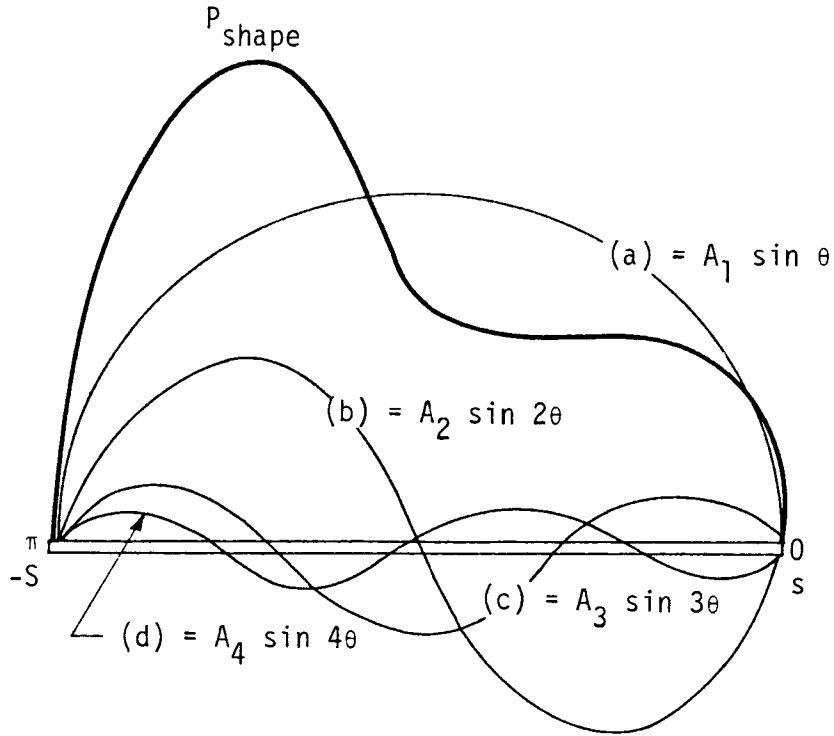


Figure 2.3. Loading as described by a shape parameter.

$$D_i = \sum_{n=1}^{\infty} A_n \int_0^{\pi} 4s^2 \rho V(\theta) (\sin n\theta \sin \theta) w(\theta) d\theta \quad (2.10)$$

$$L_{ADD} = \sum_{n=1}^{\infty} A_n \int_0^{\pi} 4s^3 \rho V^2(\theta) (\sin n\theta \sin \theta \cos \theta) d\theta \quad (2.11)$$

$$N_{ADD} = \sum_{n=1}^{\infty} A_n \int_0^{\pi} 4s^3 \rho V(\theta) (\sin n\theta \sin \theta \cos \theta) w(\theta) d\theta \quad (2.12)$$

The change in variable going from y to θ is presented below:

$$\int_{-S}^S dy = \int_{\pi}^0 \left(\frac{dy}{d\theta} \right) d\theta = \int_{\pi}^0 (-s) \sin \theta d\theta = \int_0^{\pi} s \sin \theta d\theta \quad (2.13)$$

The obvious task then becomes to find solutions for the A_n coefficients as well as to evaluate the integrals in Equations 2.9 through 2.12. As already mentioned, the approach taken is to divide the airfoil into a finite number M sections (Figure 2.4). The integrals are then evaluated numerically using Simpson's rule. As will be explained in the following section, the number of A_n coefficients that can be solved for is the same as the number of strips, M . Thus, the Fourier series representation of P_{shape} is approximated to the desired accuracy by choosing a sufficiently large number M .

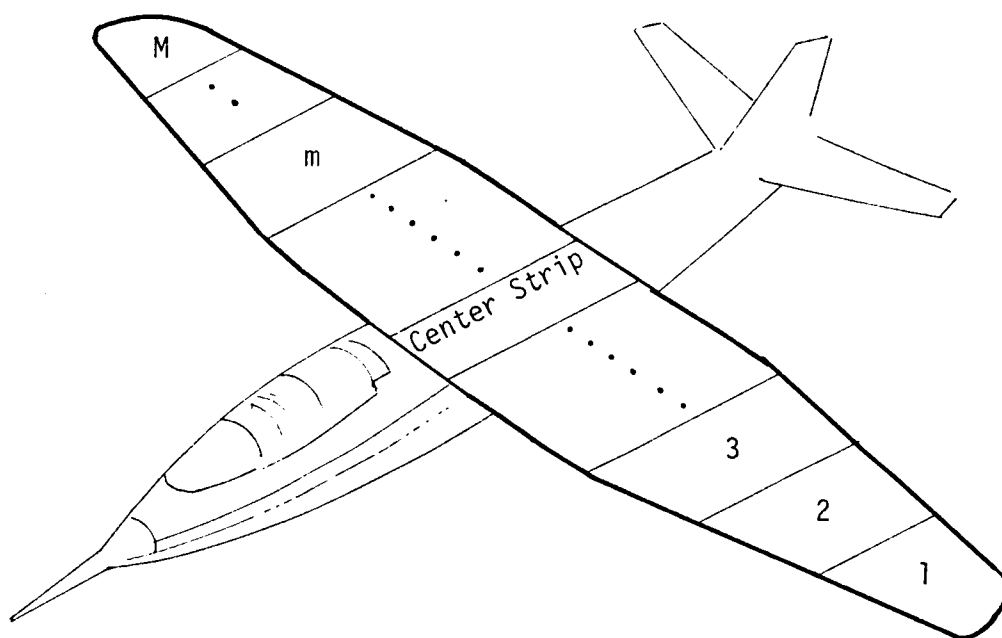


Figure 2.4. Division of airfoil into strips.

2. DETERMINATION OF THE A_n COEFFICIENTS AND DOWNWASH

The lift coefficient at a section y along the span in terms of the lift curve slope is given as:

$$C_L(y) = a_\infty[(\alpha(y) - \alpha_o(y)) - \epsilon(y)] \quad (2.14)$$

where a_∞ is the two-dimensional lift curve slope; $\alpha(y)$ is the angle of attack; $\alpha_o(y)$ is the section zero lift angle; and $\epsilon(y)$ is the downwash angle. Since airfoils commonly are designed with washout or a variable incidence angle, $\alpha_o(y)$ is written as a variable of y . The angle $\alpha_o(y)$ is negative and adds to the measured angle of attack $\alpha(y)$. The downwash angle $\epsilon(y)$ is positive and reduces the effective angle of attack $\alpha_{eff}(y)$. The geometry of Equation 2.14 can be visualized by use of Figure 2.5.

Solving Equation 2.6 in terms of $C_L(y)$ and equating it with Equation 2.14 leads to the following result:

$$V(y)[\alpha(y) - \alpha_o(y)] = \frac{2\Gamma(y)}{c(y)a_\infty} + w(y) \quad (2.15)$$

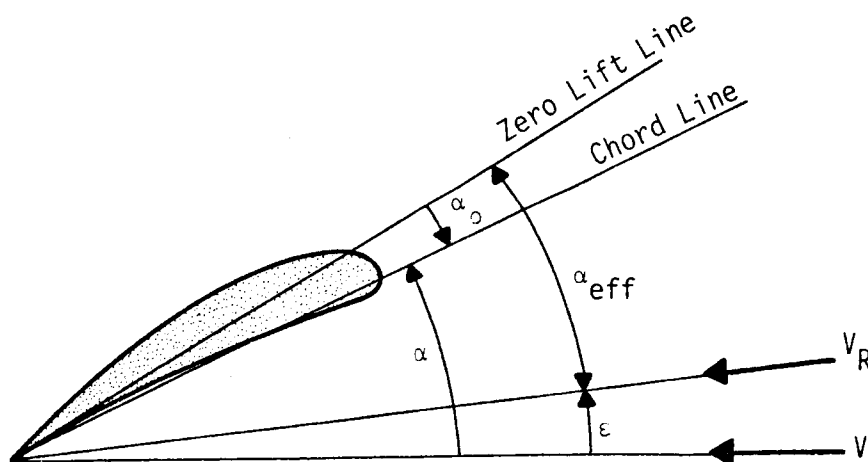


Figure 2.5. Effective angle of attack.

The downwash angle $\epsilon(y)$ is given by:

$$\epsilon(y) = \tan^{-1} \left(\frac{w(y)}{V(y)} \right) \quad (2.16)$$

Using a small angle approximation downwash becomes:

$$w(y) = V(y)\epsilon(y) \quad (2.17)$$

The downwash term in Equation 2.15 must be expressed as a function of circulation before solutions to the A_n coefficients can be found. Using the Biot-Savart law, velocity induced at a point y due to a semi-infinite vortex line is given by:

$$v_i = \frac{\Gamma}{4\pi h} \quad (2.18)$$

where Γ is the circulation strength and h is the perpendicular distance from the vortex line to the point y (Figure 2.6). Consider now the influence on downwash at a position y along the airfoil (Figure 2.1, page 7) due to a trailing vortex filament of strength $\delta\Gamma$ shed at y_1 . Using Equation 2.18 this is expressed:

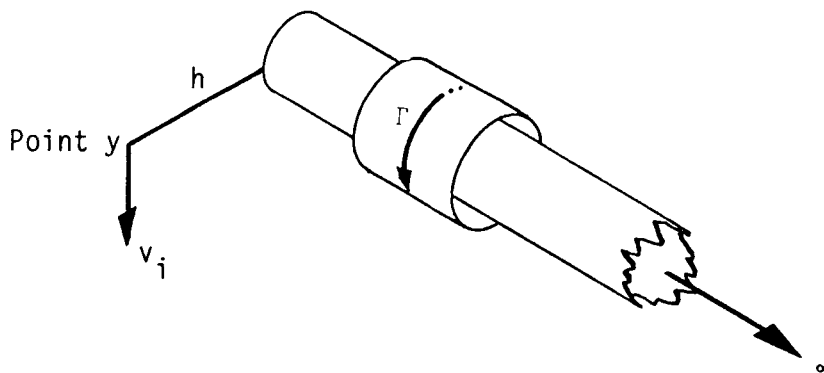


Figure 2.6. The Biot-Savart law.

$$\delta w(y) = \frac{-\frac{d\Gamma(y_1)}{dy_1} dy_1}{4\pi(y_1 - y)} \quad (2.19)$$

Since all trailing vortices influence the downwash velocity at position y , the total downwash is found by integrating Equation 2.19 across the span:

$$w(y) = -\frac{1}{4\pi} \int_{-s}^s \frac{\frac{d\Gamma(y_1)}{dy_1} dy_1}{y_1 - y} \quad (2.20)$$

Changing the variable and integrating Equation 2.20 by parts the downwash becomes:

$$w(\theta) = -\frac{1}{\pi} \int_0^\pi \frac{V(\theta_1) \sum_{n=1}^{\infty} A_n \sin n\theta_1 \sin \theta_1 d\theta_1}{(\cos \theta_1 - \cos \theta)^2} \quad (2.21)$$

Finally, by substituting Equation 2.21 into Equation 2.15 and expressing the circulation in terms of the Fourier series, the following equation is obtained:

$$\mu(\theta)[\alpha(\theta) - \alpha_o(\theta)] = \sum_{n=1}^{\infty} A_n \left[\sin n\theta - \frac{\mu(\theta)}{\pi V(\theta)} \int_0^\pi \frac{V(\theta_1) \sin n\theta_1 \sin \theta_1}{(\cos \theta_1 - \cos \theta)^2} d\theta_1 \right] \quad (2.22)$$

where $\mu(\theta)$ is:

$$\mu(\theta) = \frac{c(\theta)a_\infty}{8s} \quad (2.23)$$

At this point in the derivation the finite number (M) strips are introduced. Assuming that $V(\theta)$, $c(\theta)$, $\alpha(\theta)$, and $\alpha_o(\theta)$ all are known at each

spanwise position m . Equation 2.22 is rewritten as:

$$\mu(\theta_m)[\alpha(\theta_m) - \alpha_o(\theta_m)] = \sum_{n=1}^N A_n \left(\sin n\theta_m - \frac{\mu(\theta_m)}{\pi V(\theta_m)} \int_0^{\pi} \frac{V(\theta_1) \sin \theta_1 \sin n\theta_1}{(\cos \theta_1 - \cos \theta_m)^2} d\theta_1 \right) \quad (2.24)$$

Solution to the A_n coefficients is then given by:

$$A_n = \sum_{M=1}^M \mu(\theta_m)[\alpha(\theta_m) - \alpha_o(\theta_m)] I_{nm}^{-1} \quad (2.25)$$

where I_{nm} is the matrix identifiable in Equation 2.24. Immediately, solutions of Equation 2.25 only exist when N equals M . Furthermore, the singularity inside the integral at $\theta_1 = \theta_m$ must be avoided. For most convenient numeric evaluation, I_{nm} is written:

$$I_{nm} = \sin n\theta_m - \mu(\theta_m) \left(\frac{n \sin n\theta_m}{\sin \theta_m} - \frac{1}{\pi V(\theta_m)} \int_0^{\pi} \frac{(V(\theta_1) - V(\theta_m)) \sin \theta_1 \sin n\theta_1}{(\cos \theta_1 - \cos \theta_m)^2} d\theta_1 \right) \quad (2.26)$$

The downwash given by Equation 2.21 can now be calculated at each of the spanwise sections as follows:

$$w(\theta_m) = \sum_{n=1}^N A_n \frac{V(\theta_m)}{\mu(\theta_m)} [I_{nm} - \sin n\theta_m] \quad (2.27)$$

3. COMPUTATION OF TOTAL QUANTITIES

Equations 2.9 through 2.12 can now be solved for ΔL_{ADD} , $\Delta D_{i_{ADD}}$, L_{ADD} , and N_{ADD} , respectively. The quantities inside the integrals are

all known at M finite positions along the span. The integrals are then estimated numerically and the summations are performed with a finite number N A_n coefficients.

In this study the airfoil is divided into 21 strips. Thus, the computations involved, in addition to estimating integrals numerically, include a 21 by 21 matrix inversion. A computer code written in Fortran has been developed to execute the large number of calculations. The program is documented in Appendix A.

CHAPTER III

STRIP THEORY CALCULATIONS

The reason for developing a strip theory with angles of attack and relative wind as spanwise variables is to utilize the data that has become available from the NASA Gust Gradient Program. Using the theory developed in the previous chapter, meaningful calculations of spanwise turbulence effects can now be performed. In this chapter the computation procedures are described and results presented.

1. DATA AVAILABLE

The NASA B-57 research aircraft is equipped to measure a large number of quantities. Table 3.1 lists all the variables directly recorded or calculated in the format received. Of interest in this part of the study are the angles of attack and relative airspeeds. The B-57 is capable of recording these values at three different positions. This is made possible by installing flow vanes and pitot tubes at each wing tip in addition to the nose (see Figure 3.1).

All variables are sampled at a rate of 200 per second aboard the airplane. But the digitized magnetic data tapes that are produced by NASA Langley Research Center, Hampton, Virginia, contain only 40 data points per second. In this study a 10-point average is calculated from the digital tapes which then gives only four data points per second.

Each test flight consists of a number of runs. An individual run represents a continuous period of the flight at which data was recorded.

TABLE 3.1. Statistical Summary of Parameters Recorded on Flight 60, Run 18.

THE MAX, MIN, MEAN, AND RMS VALUES FOR EACH CHANNEL OF SERIAL NUMBER 3 FOLLOWS:							
CHANNEL	UNITS	HIGH	LOW	MEAN	RMS	STD	POINTS
TIME	SECONDS	61185.608	61035.533	61110.57060	61110.58596	43.33377	6004.
PHI DOT	RAD/SEC	.175	-.290	-.00183	.05280	.05277	6004.
ACCL N CG	G UNITS	2.482	-.194	1.00663	1.03454	.23870	6004.
THETA DOT	RAD/SEC	.073	-.079	-.00269	.01140	.01107	6004.
THETA	RAD	.093	-.015	.04460	.04944	.02135	6004.
PHI	RAD	.179	-.258	.01150	.06534	.06432	6004.
PSI 1	RAD	359.118	-.155	347.43847	350.83069	48.67315	6004.
DEL PSI 1	RAD	2.701	-12.613	-4.31112	5.10750	2.73899	6004.
PSI 2	RAD	359.374	344.560	352.65590	352.66595	2.66272	6004.
DEL PSI 2	RAD	.962	-15.092	-4.07908	5.07909	3.02652	6004.
ACCL N LT	G UNITS	3.094	-1.062	1.09925	1.17582	.41740	6004.
ACCL N RT	G UNITS	3.347	-1.545	1.01530	1.10742	.44223	6004.
ACCL X CG	G UNITS	.187	-.044	.03026	.03708	.02142	6004.
ACCL Y CG	G UNITS	.121	-.155	-.01133	.03533	.03347	6004.
ALPHA CTR	RAD	.058	-.169	-.04786	.05286	.02244	6004.
BETA CTR	RAD	.050	-.174	-.05051	.05867	.02986	6004.
TEMP 1	DEG F	59.947	59.058	59.57441	59.57472	.19304	6004.
TEMP P	DEG F	63.579	63.219	63.35632	63.35639	.09361	6004.
ACCL Z INS	G UNITS	2.448	-.177	1.01149	1.04024	.24286	6004.
ALPHA RT	RAD	.080	-.155	-.03612	.04343	.02411	6004.
BETA RT	RAD	.076	-.125	-.02053	.03323	.02613	6004.
ALPHA LT	RAD	.116	-.127	-.01360	.02792	.02439	6004.
BETA LT	RAD	.059	-.150	-.04308	.05082	.02695	6004.
PSI DOT	RAD/SEC	.101	-.078	-.00129	.02679	.02676	6004.
TEMP TOT	DEG C	20.476	15.957	18.10955	18.14388	1.11579	6004.
QC LT	PSID	1.220	.723	.92961	.93251	.10493	6004.
QC CTR	PSID	1.185	.721	.90358	.90931	.10196	6004.
QC RT	PSID	1.316	.779	.96044	.96654	.10841	6004.
PS	PSIA	11.488	11.379	11.41524	11.41526	.02327	6004.
TEMP IRT	VOLTS	4.609	3.359	4.46645	4.46885	.14621	6004.
HYGROM	DEG C	-20.140	-20.707	-20.40253	20.40276	.09688	6004.
QC2 LT	PSID	.100	-.117	.00019	.02324	.02324	6004.
QC2 CTR	PSID	.098	-.117	.00004	.02123	.02123	6004.
QC2 RT	PSID	.136	-.100	.00009	.02323	.02323	6004.
DAR	DEG	2.963	-2.203	1.20337	1.35712	.62747	6004.
DAL	DEG	2.885	-2.133	-.29814	.63107	.55625	6004.
DELEV	DEG	1.211	-1.629	.14973	.40968	.38136	6004.
DSTAB	DEG	-.184	-.271	-.22094	.22118	.01017	6004.
DRUD	DEG	6.528	-1.706	1.47132	1.86019	1.13831	6004.
DTHRR	PCT MAX	65.918	55.469	59.89818	59.94931	2.47558	6004.
DTHRL	PCT MAX	62.402	52.441	56.94753	56.99267	2.34215	6004.
DFLP	POSITION	1.000	.996	.99643	.99643	.00100	6004.
DSB	POSITION	.002	.000	.00195	.00196	.00003	6004.
D TO G	METERS	8724990.221	8721912.960	*****	*****	916.37648	6004.
B TO D	DEGREES	80.086	80.056	80.07075	80.07075	.00821	6004.
LONG	DEGREES	-105.277	-105.280	-105.27896	105.27896	.00109	6004.
LAT	DEGREES	40.066	39.914	39.99197	39.99199	.04360	6004.
TRK ANG	DEGREES	359.999	.002	130.35229	215.43624	171.53995	6004.
HOG	RADIANS	6.283	.000	6.08410	6.13840	.81474	6004.
VE	M/SEC	11.334	-6.138	1.28519	3.51669	3.27371	6004.
VN	M/SEC	122.446	103.442	111.82108	111.94539	5.27446	6004.
ALTITUDE	KM	2.106	2.030	2.08036	2.08042	.01637	6004.
TEMP C	DEGREES C	12.849	10.350	11.84025	11.84967	.47245	6004.
EW WND SPD	KNOTS	41.319	-9.162	10.56366	14.59437	10.07083	6004.
NS WND SPD	KNOTS	14.651	-19.233	-.12215	4.24683	4.24542	6004.
WIND SPEED	KNOTS	41.340	.044	12.26670	15.19971	8.97623	6004.
WIND DIREC	DEGREES	359.993	.091	245.89200	257.19053	75.39918	6004.
AIRSPEED R	M/SEC	134.803	104.464	115.44075	115.61524	6.35020	6004.
AIRSPEED C	M/SEC	128.139	100.604	112.06540	112.23505	6.16936	6004.
AIRSPEED L	M/SEC	129.915	100.713	113.62457	113.79647	6.25288	6004.
DELTA ALT	METERS	21.705	-54.721	-3.93629	16.83466	16.36936	6004.
INRTL DISP	METERS	16.440	-57.215	-2.56090	17.85698	16.97045	6004.
UG RIGHT	M/SEC	7.001	-10.846	.00000	2.18479	2.18498	6004.
UG CENTER	M/SEC	7.112	-10.385	.00000	2.03625	2.03642	6004.
UG LEFT	M/SEC	7.623	-10.927	.00000	2.08562	2.08579	6004.
VG RIGHT	M/SEC	752.132	-25.212	.34578	98.37485	98.38244	6004.
VG CENTER	M/SEC	733.269	-22.977	.32006	95.41415	95.42156	6004.
VG LEFT	M/SEC	741.958	-24.110	.28268	96.44861	96.45623	6004.
WG RIGHT	M/SEC	12.362	-16.573	-.02489	3.18635	3.18652	6004.
WG CENTER	M/SEC	10.405	-13.607	-.02681	2.80001	2.80011	6004.
WG LEFT	M/SEC	10.856	-13.847	-.03034	3.01159	3.01169	6004.

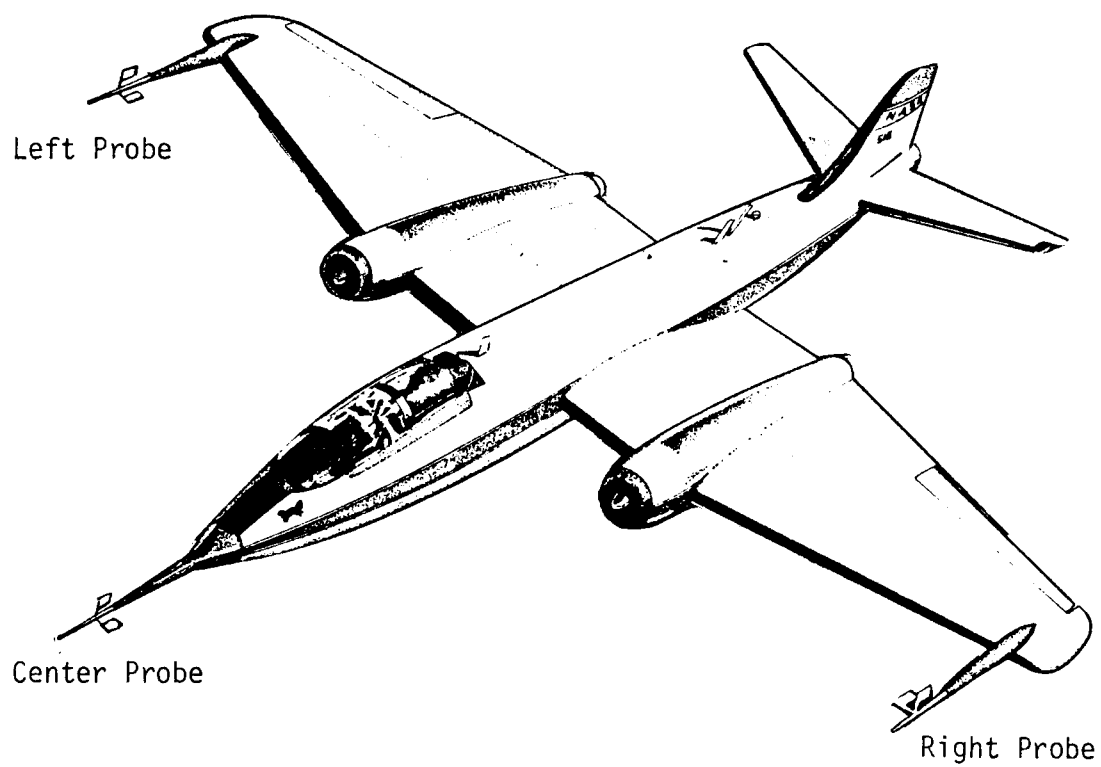


Figure 3.1. Spatial measurement of turbulence with the B-57.

The length of a run may vary between less than half a minute and several minutes.

This first part of the study analyzes two runs originating from two separate flights. They have both been chosen because of expectedly high levels of turbulence. The first is Run 21 of Flight 6 recorded July 14, 1982, at Denver, Colorado, as part of the Joint Airport Weather Studies (JAWS) Project. The other is Run 18 of Flight 60 recorded February 1, 1984, at Boulder, Colorado, recorded during the Orographic Effects Campaign. Additional information on the flights is listed in Appendix E. These two runs represent level flights approximately 1000 feet above the ground. The data tapes from Flight 60 include records of pilot control inputs but Flight 6 does not. For the performance analysis in Chapter IV, this will prove to be of significance.

2. CALCULATION DESCRIPTION

The strip theory developed in Chapter II has been programmed. The computer code is referred to as Program I in Appendix A. Inputs to the program are data files from the B-57 test flights containing time histories of angles of attack and relative wind at the three different spanwise positions. At each time step of the input file the strip theory calculations are performed. No aircraft dynamics enter into Program I; each time step is simply treated uniquely assuming steady-state condition. The output file is a time history of lift, induced drag, and roll and yaw moments equal in length to the input file. The rate at which calculations can be done on the Pixel Supermicro Motorola 68,000 computer is substantially slower than required to keep up with inputs in real

time. Each time step of 0.25 seconds will consume approximately one minute of computer time. This is largely due to the inversion of the sizable 21 by 21 matrix. The results of Program I are stored, and as described in Chapter IV, a dynamic performance analysis based on these results is carried out.

The calculations in this chapter are designed so that the results can be directly utilized in FWG's six-degrees-of-freedom performance analysis computer program. The input file is a time history representation of angles of attack and airspeeds recorded by the B-57 at the three spanwise positions. A unique linear distribution of the input quantities is associated with each time step (Figure 3.2). The calculations applied at a single time step are iterated until the entire input data file is analyzed. Next, in a second iteration through the input file the calculations are based on inputs uniform across the span. Each time step again has a unique value of angle of attack and airspeed, but they represent the average of the three spanwise recordings (Figure 3.3). The two calculations allow the evaluation of additional lift and drag forces, ΔL_{ADD} and $\Delta D_{i_{ADD}}$, due to spanwise turbulence (Equations 1.3 and 1.4). The moment calculated in the first calculations are identical to additional moments, L_{ADD} and M_{ADD} , due to spanwise turbulence. In the second calculations, these moments vanish since by definition (Equations 1.1 and 1.2) they represent only moment due to asymmetry of aerodynamic forces.

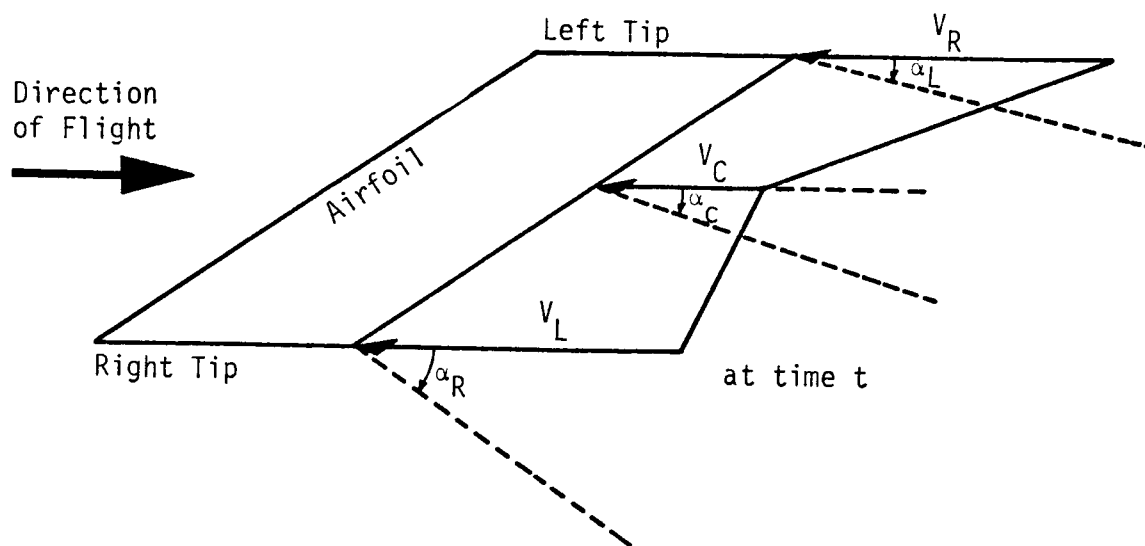


Figure 3.2 Sample of a linear distribution of inputs to Program I.

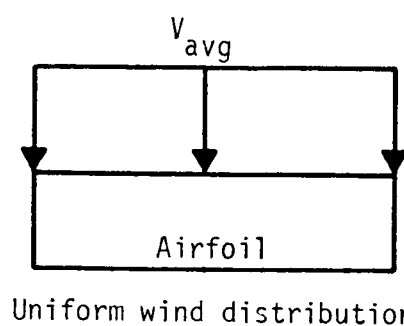
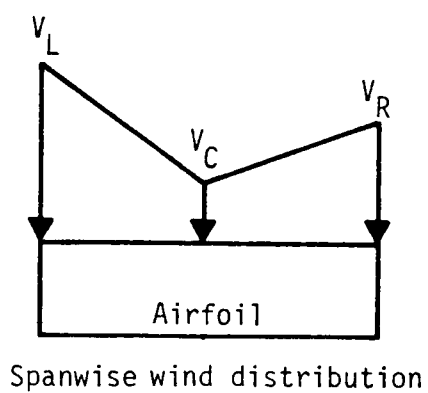


Figure 3.3. Spatial and uniform spanwise distributions of relative wind.

3. ANALYSIS OF STRIP THEORY RESULTS

In the analysis of strip theory results, two problems were uncovered. First, the value of lift calculations appeared to be exaggerated. Secondly, the angles of attack seemed to contain a constant offset. The method used to solve these complications is presented preceding the final results in this section.

At each step of the calculations in Program I assume a steady-state condition. It would then be expected that the instantaneous lift and drag values are overestimated in gusts. A short literature review led to the estimation of a gust alleviation factor K . This factor $K = 0.7$ is applied to the results as described in Appendix B.

A second complication attributed to the inclination angles of the probes on the B-57 introduces offsets in the recorded angles of attack. Suspected errors in the calculations applied to Run 18 of Flight 60 initiated an investigation. It showed that the time-averaged angle of attack for Flight 60, Run 18, differs between the three recording probes. Specifically, the left probe indicates an angle of attack about two degrees higher than the center probe, whereas the right probe lies close to half of a degree below on a time-averaged basis. Moments calculated with Program I are of varying magnitude but suspiciously of only one polarity. This is not in accordance with expectations, namely moments varying in both directions. This problem is avoided by correcting the angles of attack data by a constant value representing the difference of the time-averaged values. Several runs of Flight 60 represent the basis for estimating these correction values. The left probe angle of attack is reduced by 2.02 degrees and the center probe

reduced by 0.67 degrees. Figure 3.4 shows the angles of attack as recorded during a 50-second period of Flight 60, Run 18, and Figure 3.5 shows angles of attack with the corrections included.

The first direct result of strip theory calculations is shown in Figure 3.6. Using the spanwise linear distribution, the lift obtained in the first calculations with Program I is plotted. The lift is normalized with aircraft weight. The aircraft lift-to-weight ratio is computed and is compared with a plot of load factor recorded by the on-board inertial navigation system (INS) in the figure. The theoretical lift-to-weight ratio is computed independently of the INS and is based on data collected by use of pitot tube and angle of attack probes. Any dynamic influence on lift is neglected since the test flight is close to being level and unaccelerated. The dynamic terms are, however, expected to be small. The agreement between the two curves suggest the strip theory model is reliable in estimating lift on an airfoil. Note that the entire 41.5-second Run 21 of Flight 6 is analyzed, whereas only a 50-second period inside the total 140-second Run 18 of Flight 60 is studied. The first 40 seconds of Run 18 of Flight 60 is omitted as is indicated by the time variable of that run in the figures. Also, the mean airspeed of Flight 60, Run 18, is 113 m/s and only 96 m/s in Flight 6, Run 21. With the independent variable as distance (i.e., $x = Vt$), the abscissa would be approximately 15 percent elongated in a plot of Flight 60.

Figure 3.7 represents the ΔL_{ADD} in Equation 1.3, which is computed as the difference of the two calculations illustrated schematically in Figure 3.3. The lift difference is generally small but can reach values

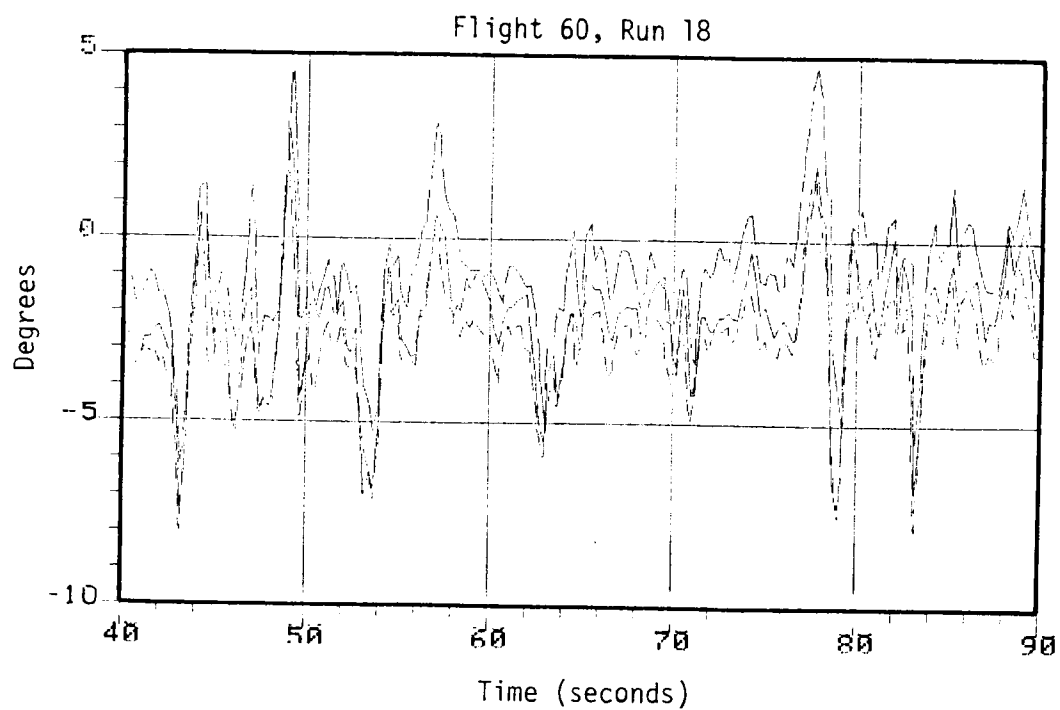


Figure 3.4. Recorded angles of attack.

— Center Probe
 ---- Right Probe
 - · - Left Probe

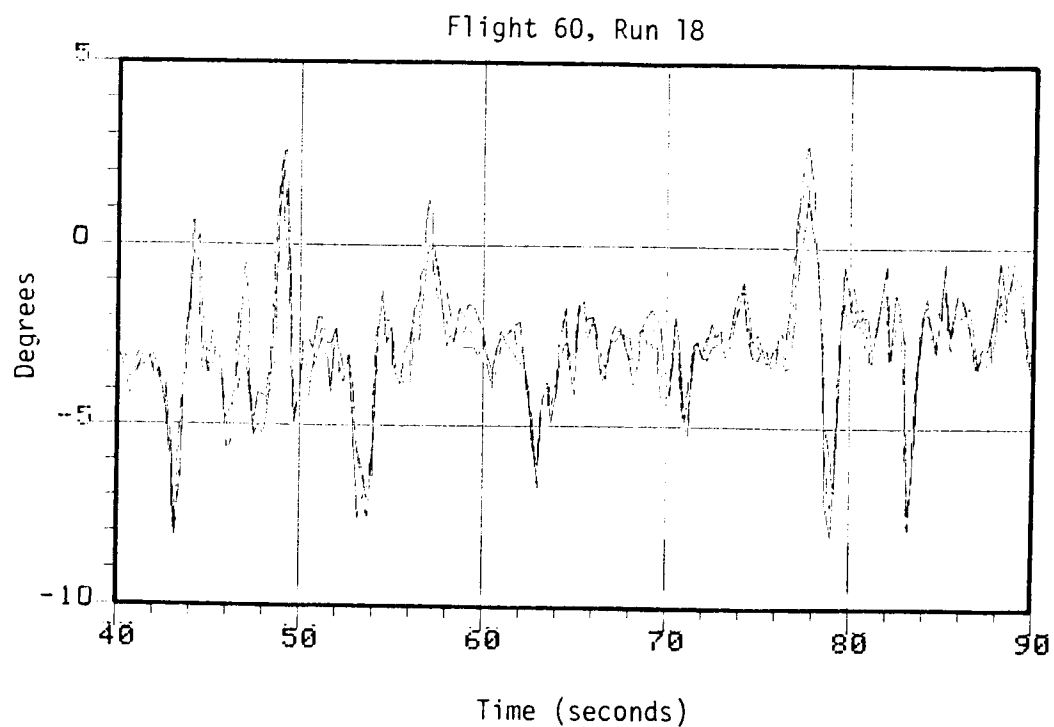


Figure 3.5. Corrected angles of attack.

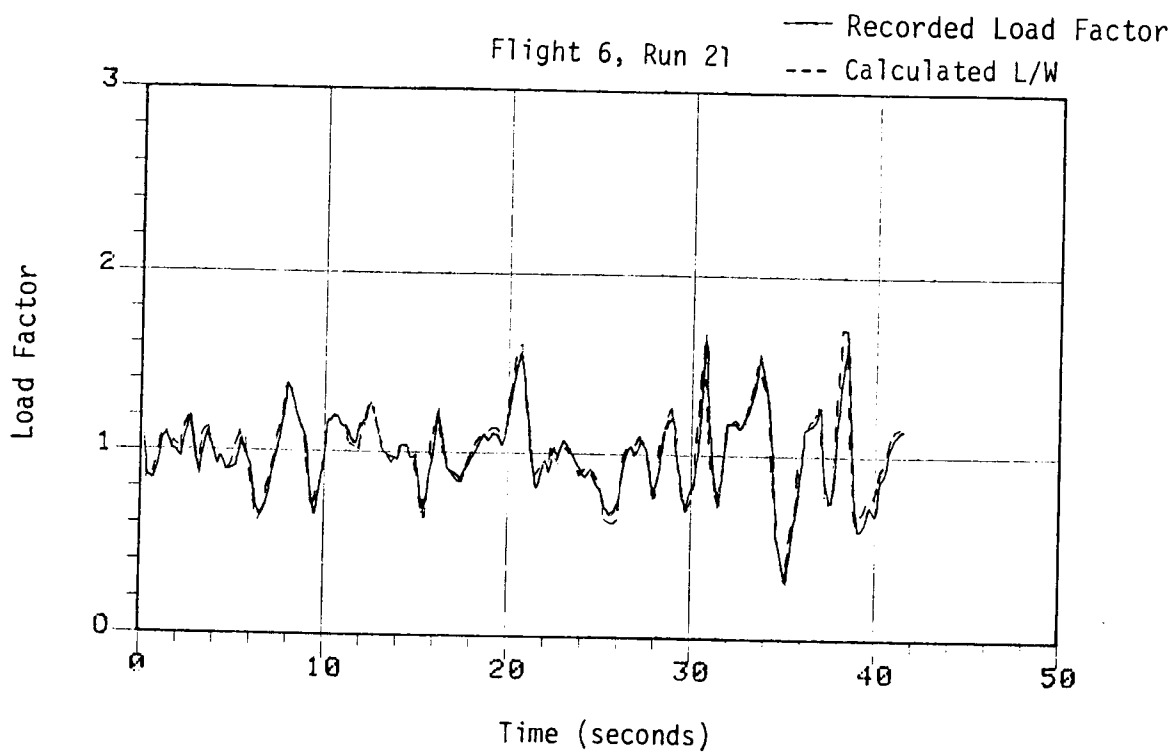
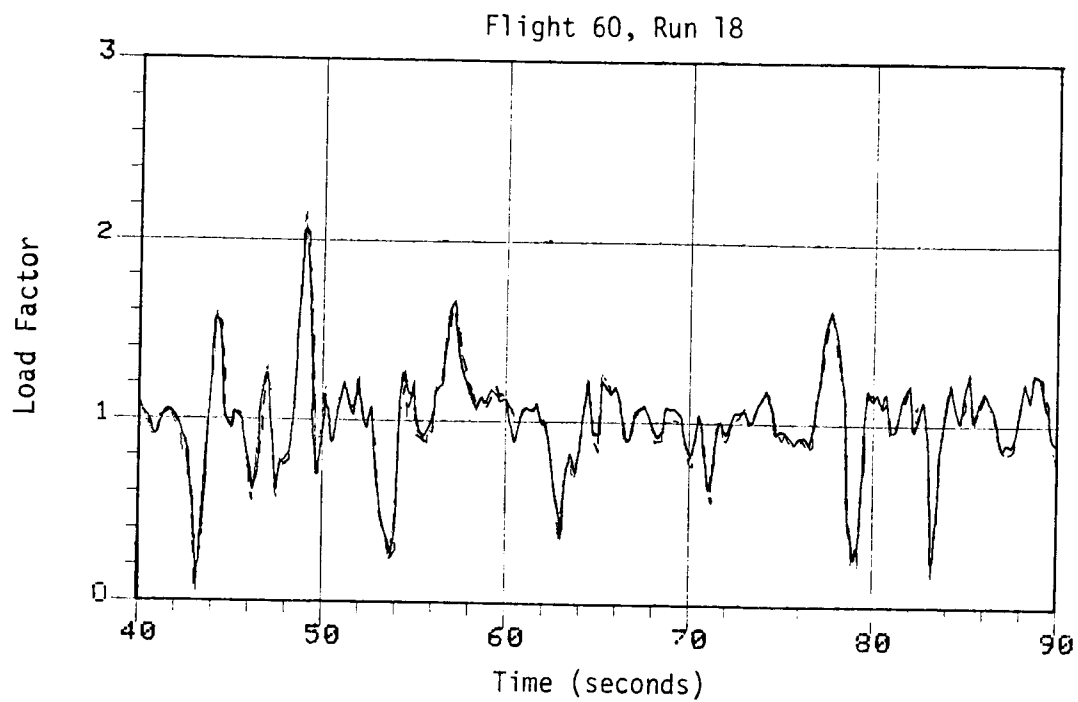


Figure 3.6. Strip theory lift calculations.

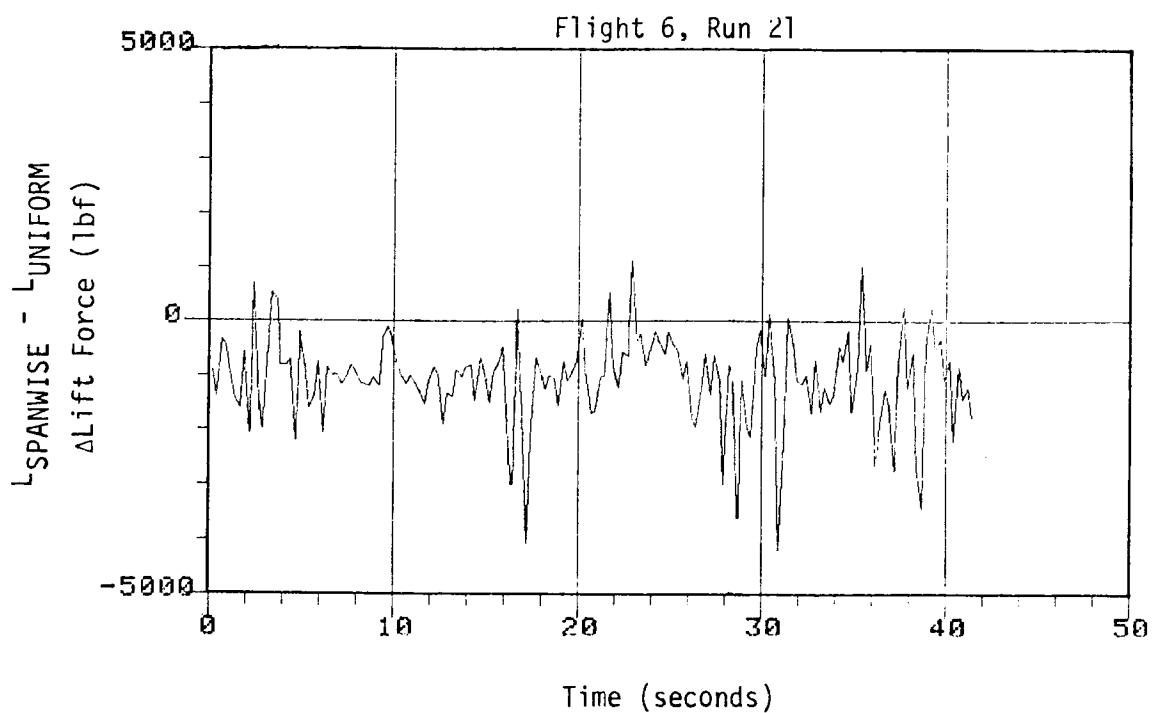
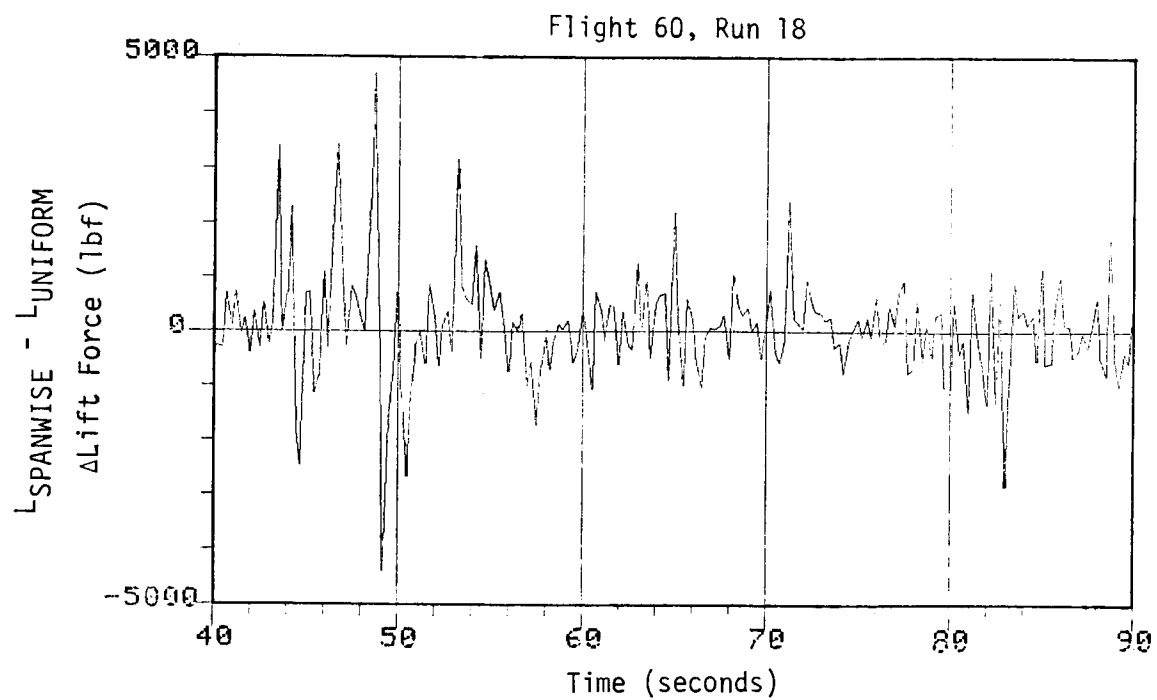


Figure 3.7. Additional lift due to spanwise wind variation.

up to 10 percent of the weight. The problem with offset due to the discrepancy in angle of attack between probes is corrected in Flight 60, Run 18, but not in Flight 6, Run 21. This can explain the apparent negative time-averaged ΔL_{ADD} in Flight 6, Run 21. It is a result of an approximately one half of a degree lower time-averaged angle of attack recorded in the nose of the aircraft, see Figure 3.8. This makes the time-averaged lift with uniform distributions greater than the time-averaged lift with spanwise distributions which results in the negative difference. In this case this difference may simply be caused by upwash at the wing tips and not an inclination induced error in the center probe.

Figure 3.9 is identical to Figure 3.7 except that induced drag not lift is plotted. ΔD_{ADD} is much smaller which is anticipated.

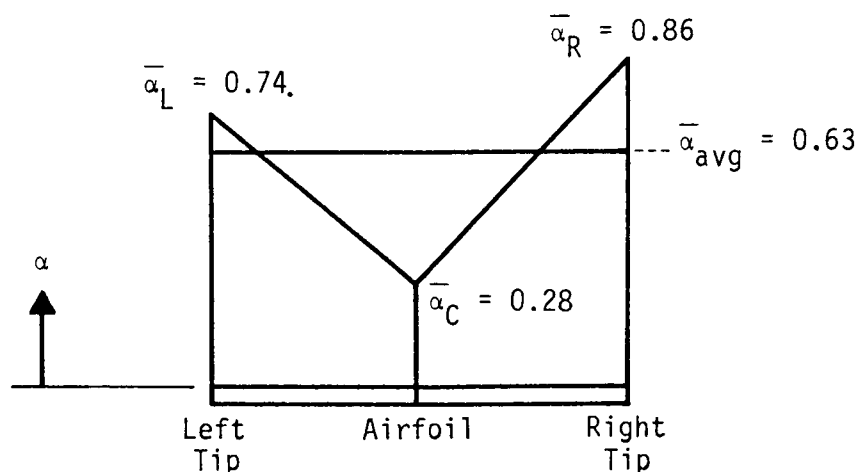


Figure 3.8. Time-averaged angles of attack for Flight 6, Run 21.

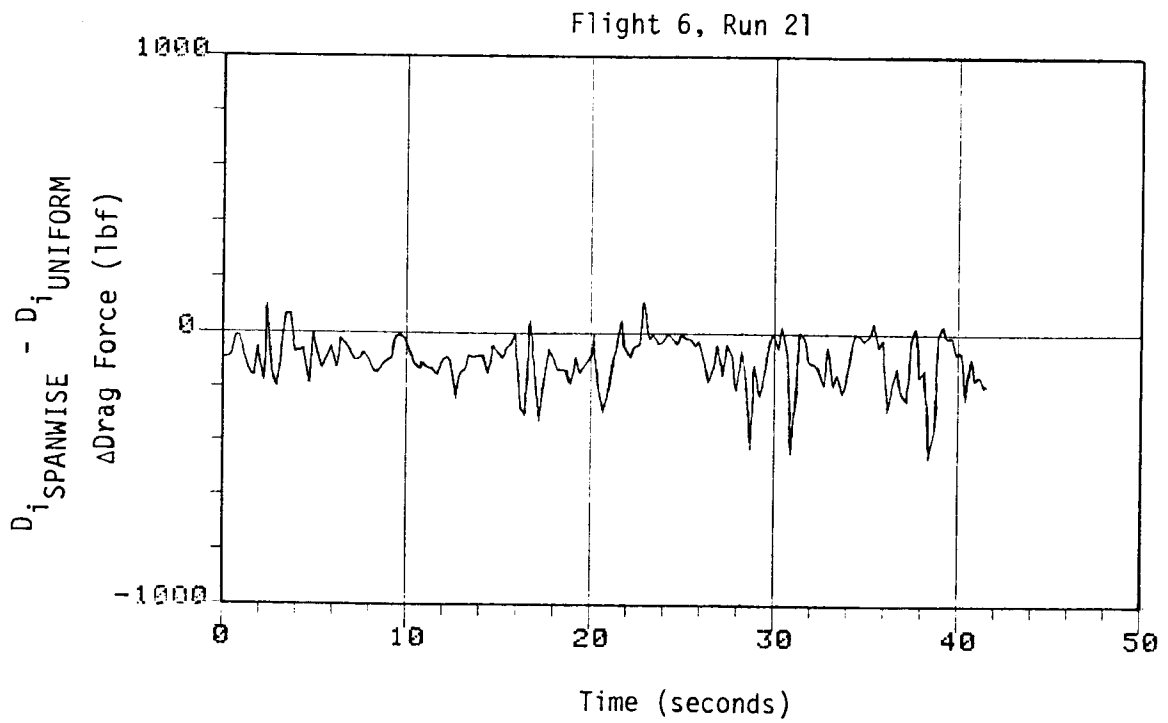
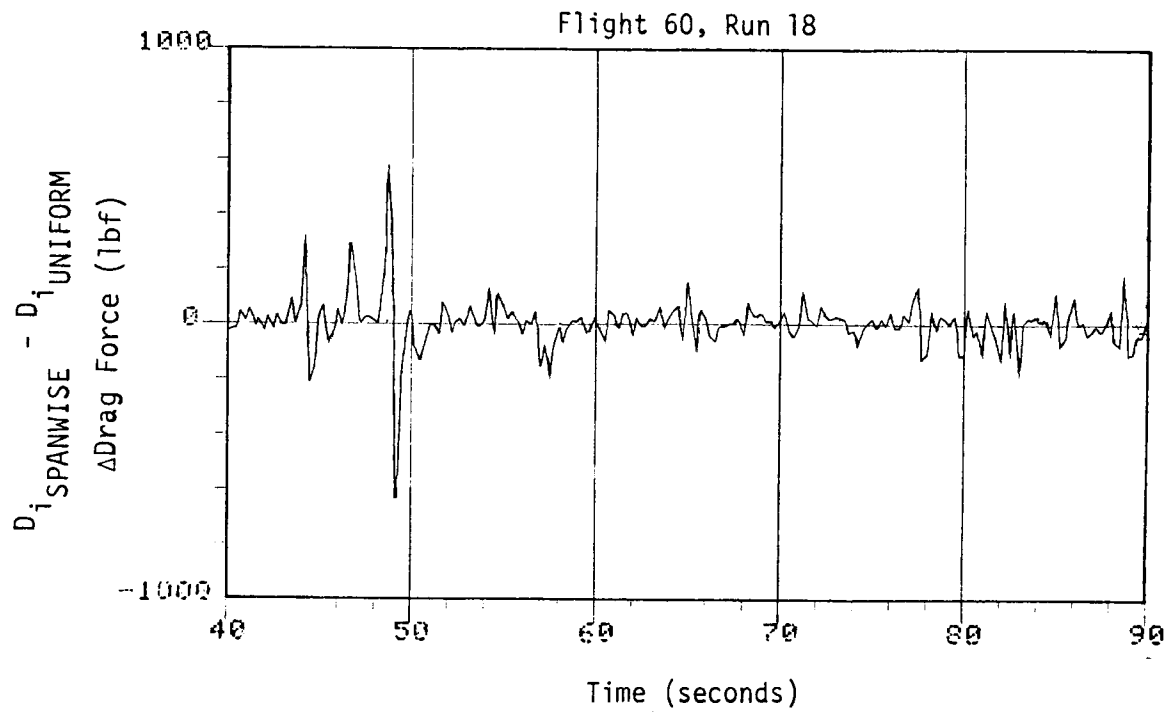


Figure 3.9. Additional induced drag due to a spanwise wind variation.

The roll moment L_{ADD} in Equation 1.1 due to spanwise turbulence is shown in Figure 3.10. The mean value of L_{ADD} in Flight 6, Run 21, is negative because of the only slightly higher time-averaged angle of attack on the right wing tip compared to the left wing tip. As noted, this asymmetry is also displayed in Figure 3.7, and could possibly be real because of the relatively short time period over which the calculation is made. The roll moment of Flight 60, Run 18, in Figure 3.10 is based on the corrected angles of attack as previously mentioned (Figure 3.4, page 25). Similar results for the yaw moment due to spanwise turbulence is presented in Figure 3.11.

The magnitudes of L_{ADD} and N_{ADD} are of little meaning unless they are weighed against known quantities. To provide the reader with a feel for the magnitudes of these values, the roll moment developed as a function of aileron input and yaw moment due to a rudder deflection are plotted in Figure 3.12. It shows the equivalent to the largest sustained roll moment in Figure 3.10 is close to 10 degrees aileron input. Similarly, the yaw moments due to spanwise turbulence can reach magnitudes comparable with a 3 degree rudder deflection, Figure 3.13.

The second part of this study investigates the influence of airplane performance generated by the four quantities ΔL_{ADD} , ΔD_{ADD} , L_{ADD} , and N_{ADD} .

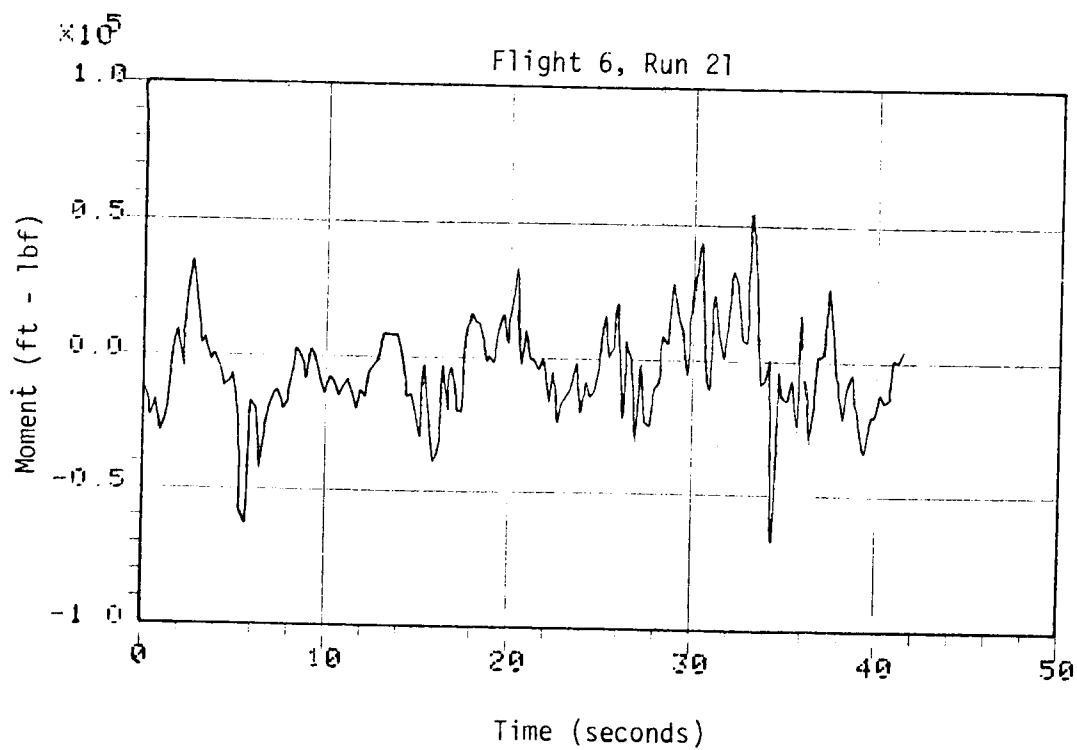
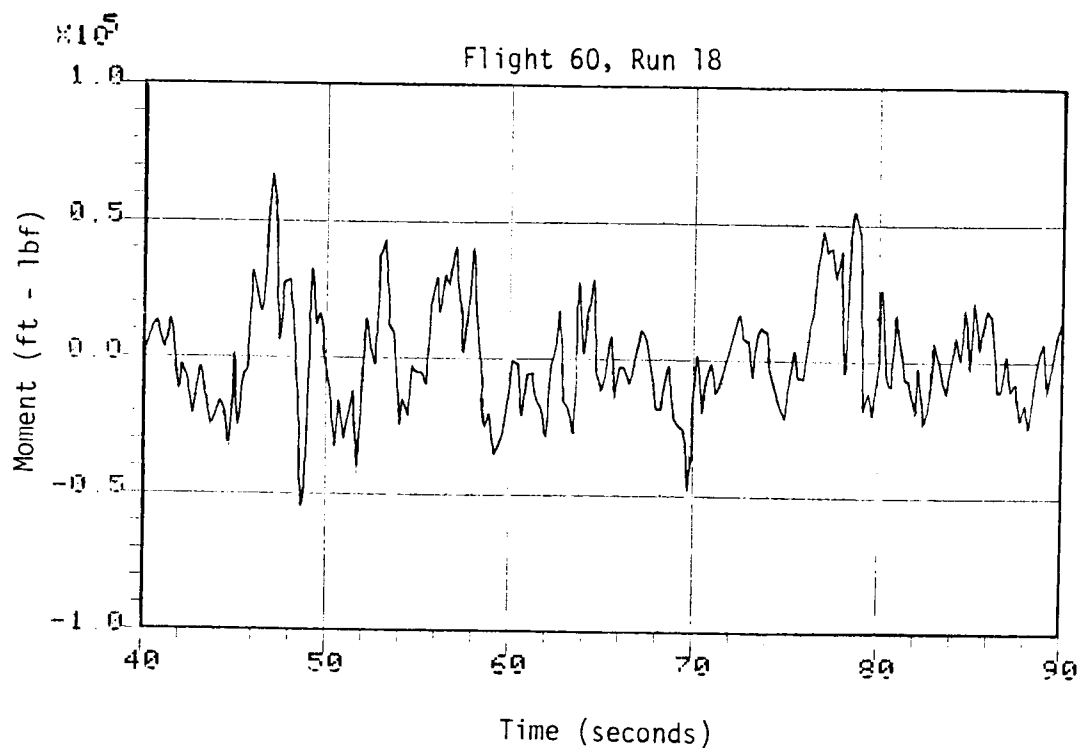


Figure 3.10. Roll moment due to a spanwise wind variation.

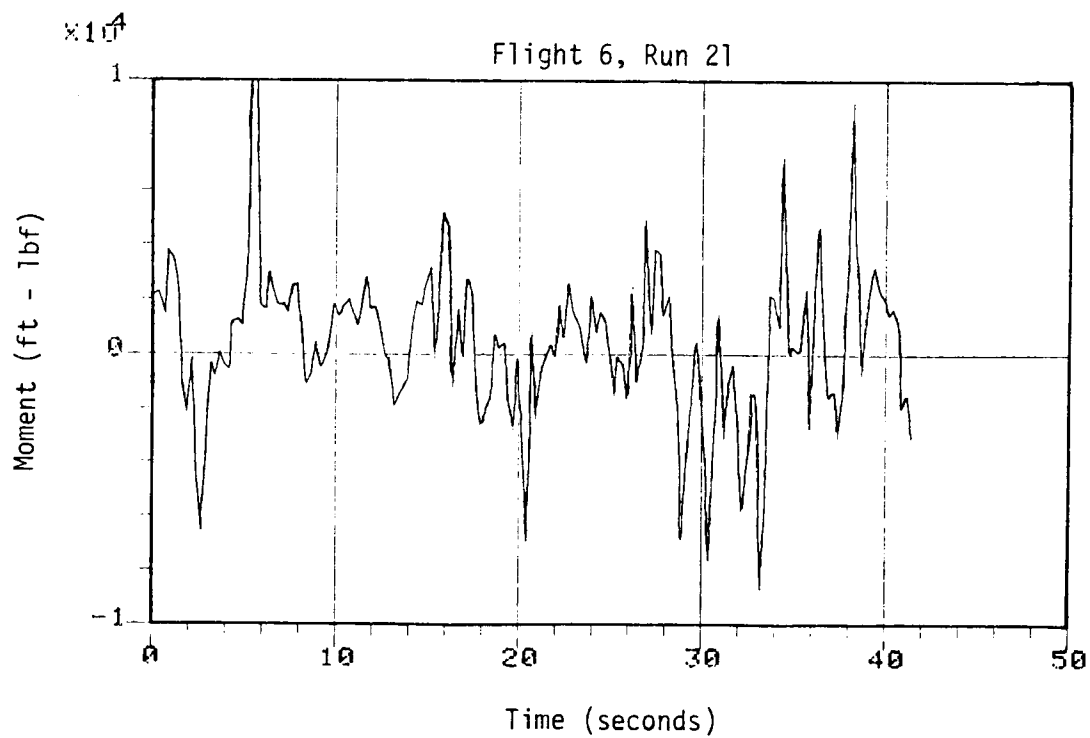
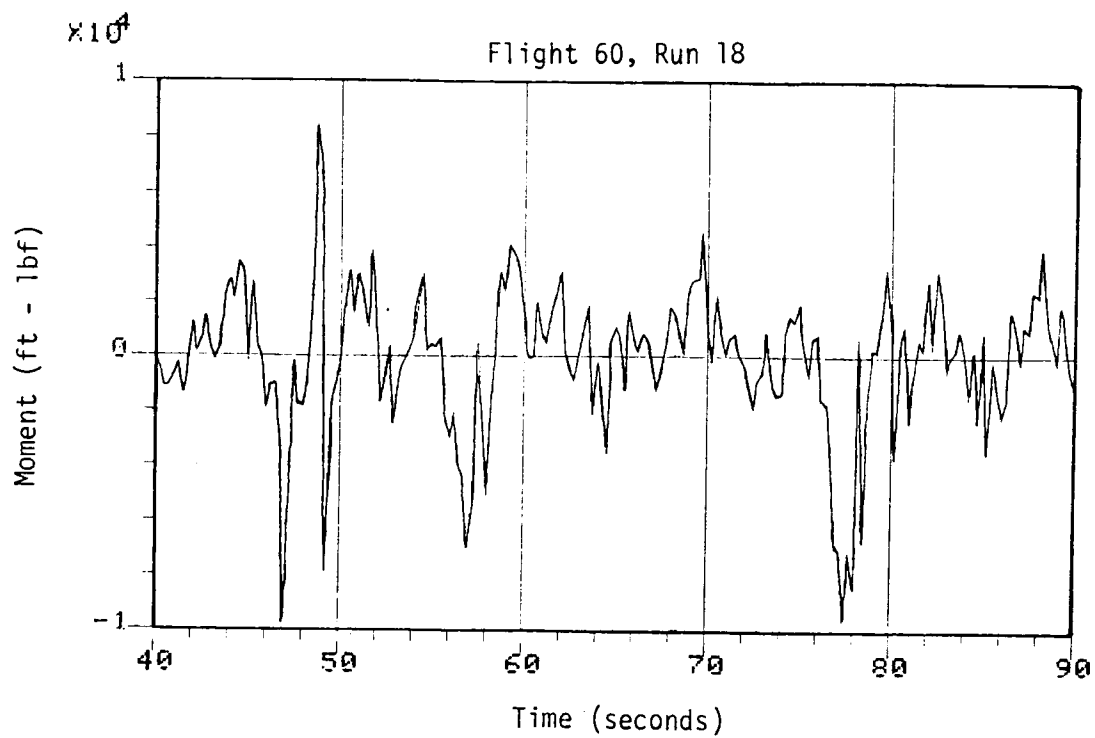


Figure 3.11. Yaw moment due to a spanwise wind variation.

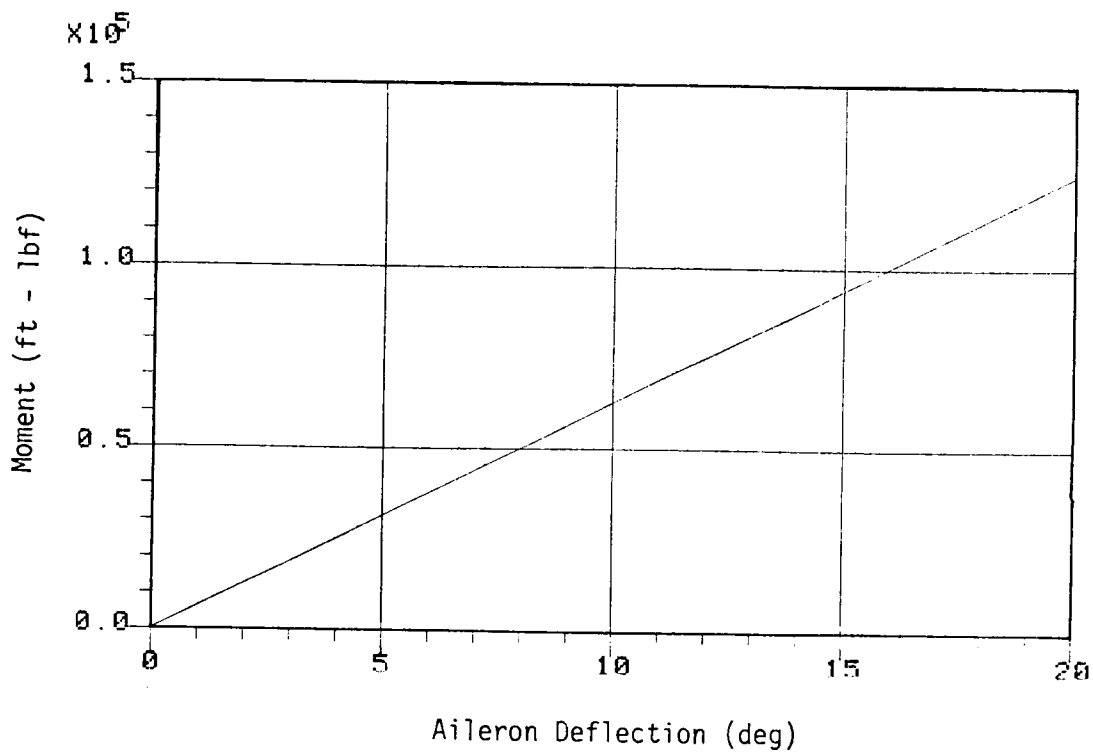


Figure 3.12. Roll moment by use of aileron.

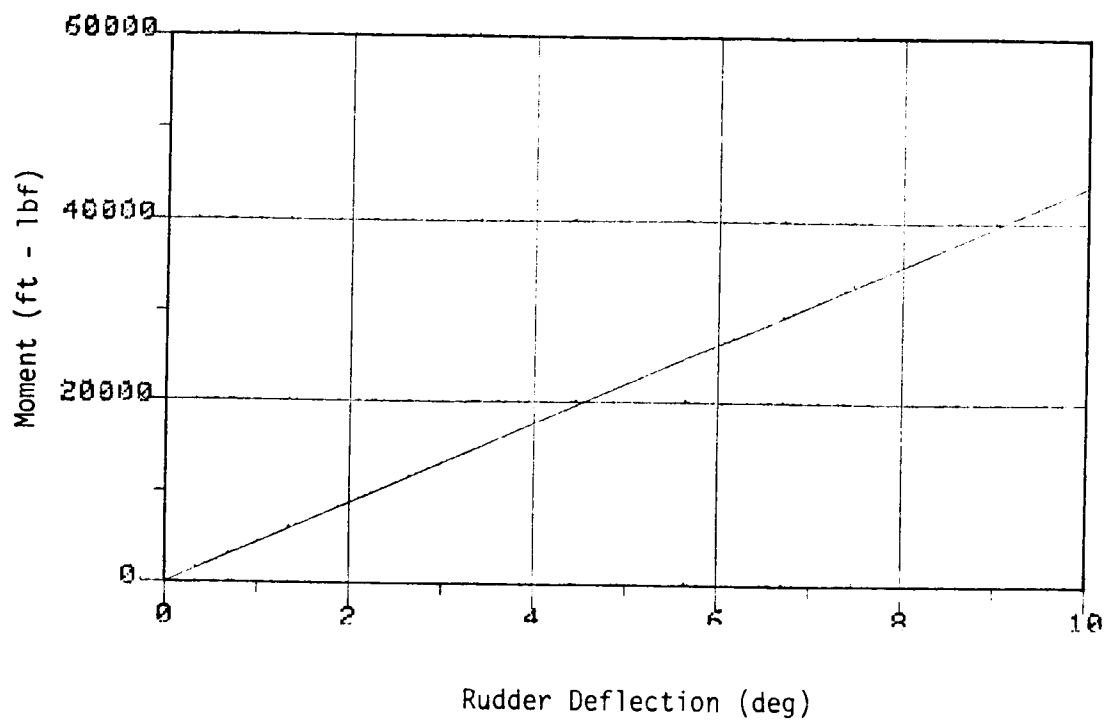


Figure 3.13. Yaw moment by use of rudder.

CHAPTER IV

FLIGHT SIMULATION USED TO EVALUATE PERFORMANCE EFFECTS

The first part of this study has only dealt with assessing the static aerodynamic forces and moments due to spanwise turbulence. The next progression in the study is to estimate the effect spanwise turbulence has on the dynamic behavior of an aircraft. This chapter presents methods used to implement the spanwise turbulence into a flight simulation program. The results of this second part of the study are also shown in this chapter.

1. IMPLEMENTING SPANWISE TURBULENCE

A second computer code is used in the performance analysis. This program is FWG's six-degrees-of-freedom (6-D) flight simulation routine. Two sets of calculations are employed. Initially the spanwise turbulence effects are not implemented, i.e., point mass approximation. Later the program is modified to include the results of the strip theory calculations. In this manner, a comparison of the two sets of results is possible. Finally, the simulation results are checked against the recorded B-57 flight data. Figure 4.1 is a block diagram outline of the procedure for the aircraft performance analysis.

The 6-D program is the same as used in a study by Frost et al. (1984). The original program includes wind gradients in the equations of motion (Frost and Bowles 1984) but for this purpose they are not considered. The only difference lies in the incorporation of spanwise turbulence. Under the assumption that L_{ADD} , N_{ADD} , ΔL_{ADD} , and ΔD_{ADD}

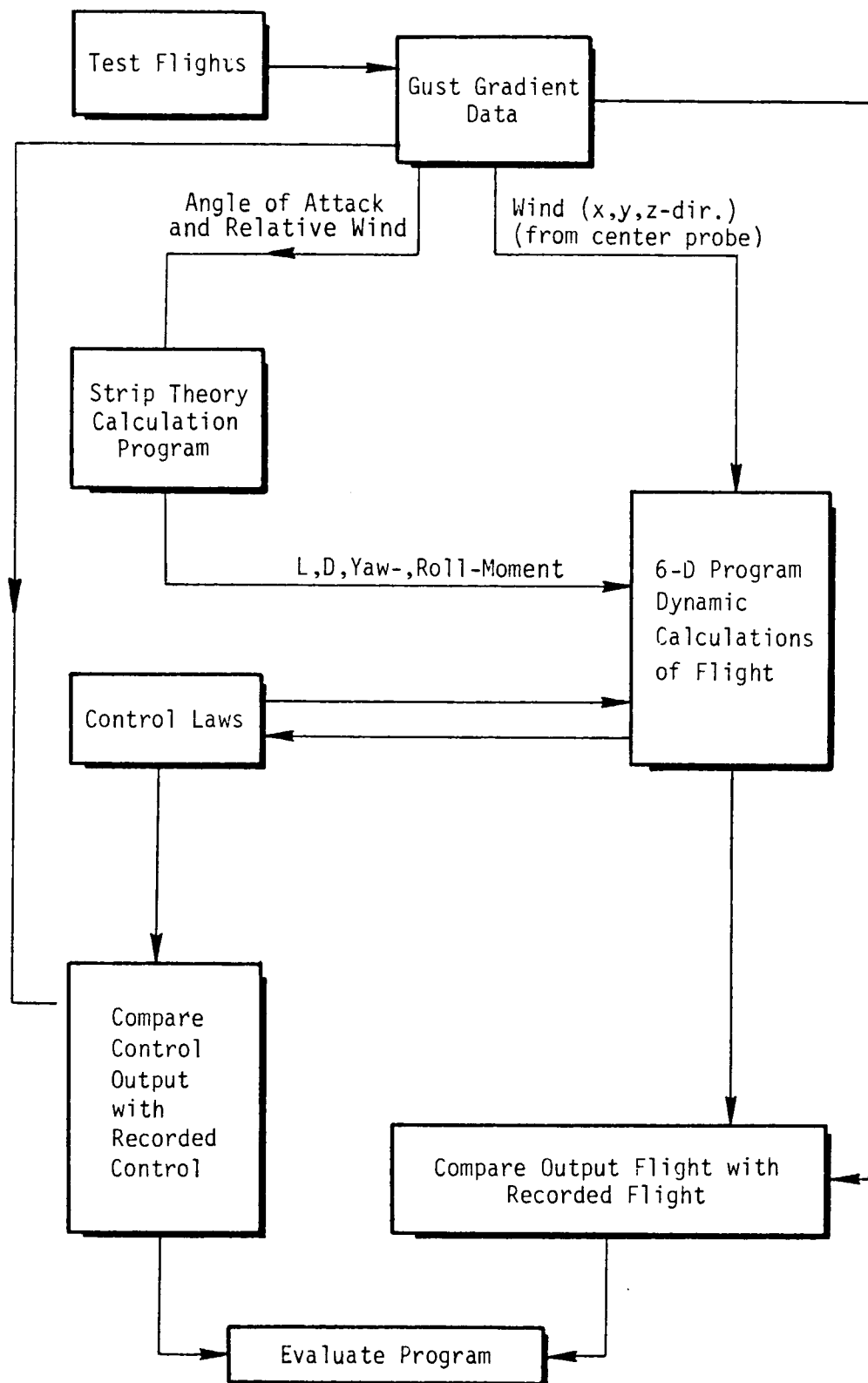


Figure 4.1. Outline of analysis procedure.

contain the effects of spanwise turbulence, these four terms are added into the force and moment equations in the following way:

$$L = \frac{1}{2} \rho V^2 S (C_{L_0} + C_{L_\alpha} \alpha + C_{L_{\delta_e}} \delta_e + C_{L_q} q + C_{L_{\dot{\alpha}}} \dot{\alpha}) + \Delta L_{ADD} \quad (4.1)$$

$$D = \frac{1}{2} \rho V^2 S (C_{D_0} + C_{D_\alpha} \alpha + C_{D_{\delta_r}} \delta_r + C_{D_{\delta_a}} \delta_a) + \Delta D_{ADD} \quad (4.2)$$

$$F_Y = \frac{1}{2} \rho V^2 S (C_{Y_\beta} \beta + C_{Y_{\delta_r}} \delta_r) + L \sin \phi \quad (4.3)$$

$$L = \left(C_{\ell_\beta} \beta + C_{\ell_{\delta_r}} \delta_r + C_{\ell_{\delta_a}} \delta_a + C_{\ell_p} \frac{pb}{2V} + C_{\ell_r} \frac{rb}{2V} \right) \frac{1}{2} \rho V^2 S b + L_{ADD} \quad (4.4)$$

$$M = \left(C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta_e}} \delta_e + C_{m_q} q + C_{m_{\dot{\alpha}}} \dot{\alpha} \right) \frac{1}{2} \rho V^2 S b \quad (4.5)$$

$$N = \left(C_{n_\beta} \beta + C_{n_{\delta_r}} \delta_r + C_{n_{\delta_a}} \delta_a + C_{n_p} \frac{pb}{2V} + C_{n_r} \frac{rb}{2V} \right) \rho V^2 S b + N_{ADD} \quad (4.6)$$

The addition of the precalculated time histories of spanwise turbulence into the 6-D flight simulation program is considered a good first order approximation. Necessarily, the computer simulation and the real flight, from which spanwise turbulence effects are obtained, will show some differences. However, since the recorded flight only has small perturbations from the trim condition, spanwise turbulence is assumed to be independent of dynamic influence and, thus, can be superimposed on the 6-D simulation. Also, the 6-D program regards the aircraft as a perfect rigid body. The corresponding error introduced is expectedly less for the B-57 than for a high aspect ratio wing airplane. The air density and the weight of the B-57 is assumed to be constant. Furthermore, the aerodynamics of the B-57 are represented by aerodynamic

coefficients (Appendix C) that are assumed to be linear over the range they are used. The airplane is controlled through the use of simple linear gain automatic control algorithms. The mathematical control laws are explained in further detail in Appendix D.

2. DATA UTILIZED

The B-57 Gust Gradient Program is responsible for making the B-57 available as a research aircraft. The immense number of flight parameters recorded during a test flight is shown in Table 3.1, page 18. Early flights, however, lacked recordings of pilot control inputs. In the second part of this study such records are useful for comparison of computed and measured results, and consequently Flight 60 was chosen as it became available in the spring of 1984. Flight 6 dates back to 1982 and does not contain pilot control records and is not analyzed any further.

In the strip theory calculations, the three angles of attack and the corresponding airspeeds are utilized. In the 6-D simulation program the atmospheric conditions from Flight 60, Run 18, measured independently of these angles and relative airspeeds are used. Air density is obtained from temperature and static pressure. The wind components measured with the INS system in inertia coordinates are input to reconstruct the wind conditions of the test flight. The roll, pitch, and yaw angles along with pilot control inputs in Flight 60, Run 18, are plotted to provide a comparison with the computer-simulated flight. Recorded airplane altitude variation with time also is used for evaluation of the 6-D program.

3. RESULTS OF 6-D FLIGHT SIMULATIONS

This section presents the results of the 6-D flight computer simulations of Flight 60, Run 18. The two calculations, one incorporating strip theory terms and one without, are both plotted on the same graph to make comparison easy.

The three wind components of Flight 60, Run 18, as measured with the INS during the flight are plotted in Figures 4.2, 4.3, and 4.4, respectively. Figure 4.3 shows the airplane encountered a strong side-wind a few seconds into the time period considered of the test run.* The strong changes in north/south wind occurring around the 50-second

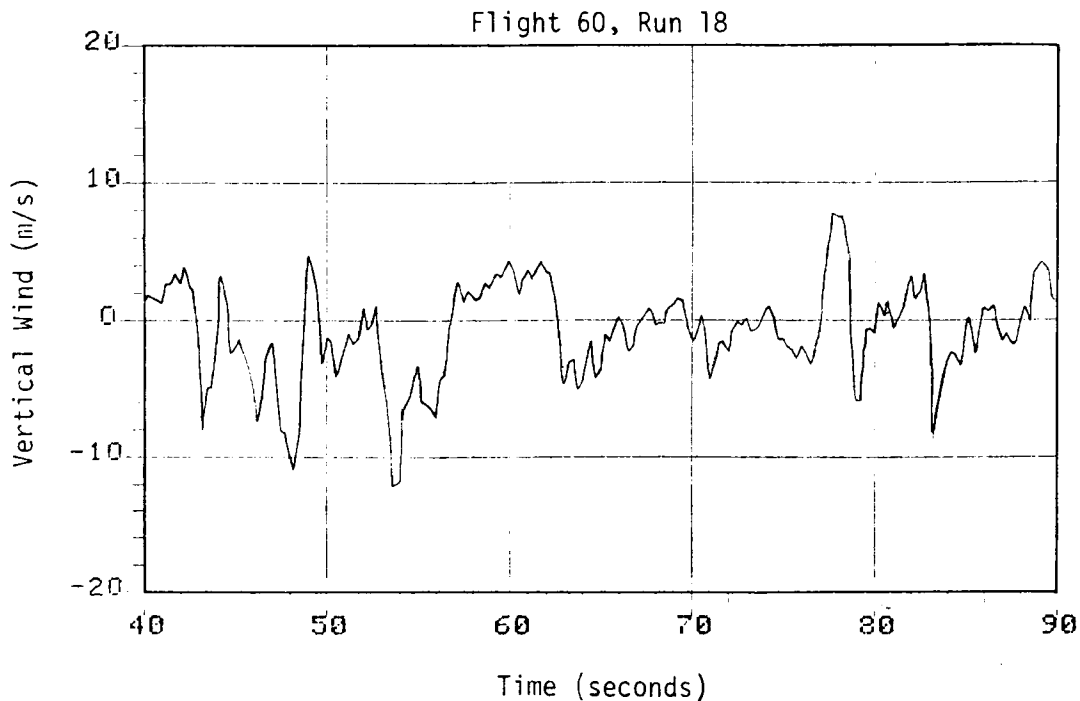


Figure 4.2. Vertical wind component.

*Note that the airplane heading is due north on Flight 60, Run 18.

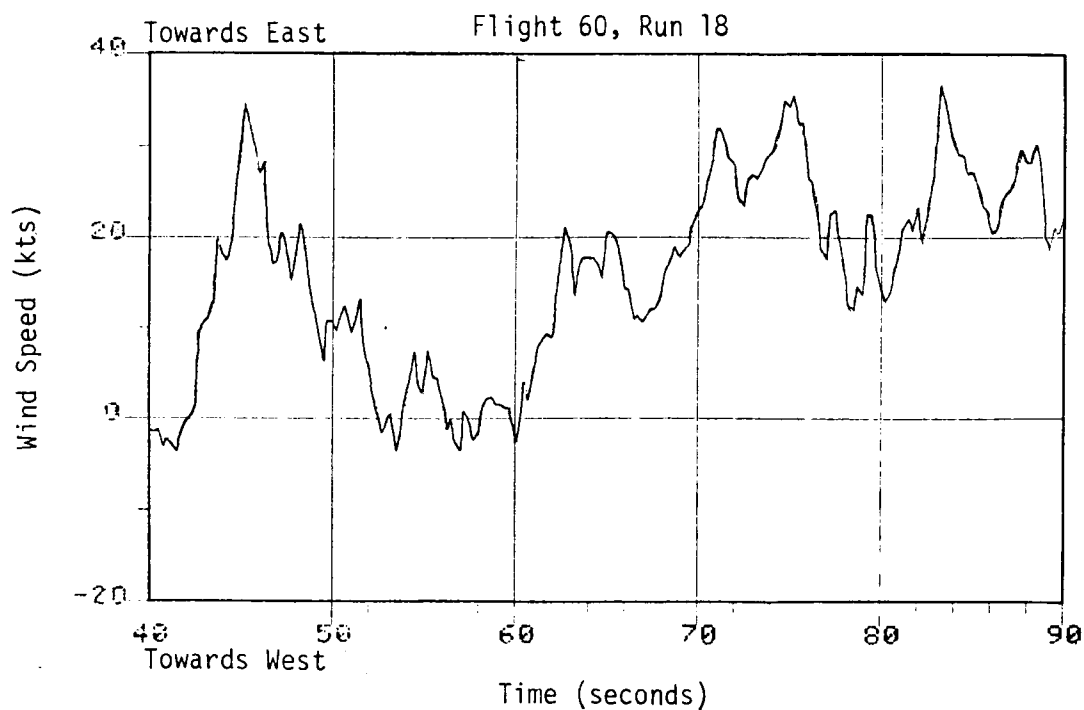


Figure 4.3. Lateral wind component (east/west).

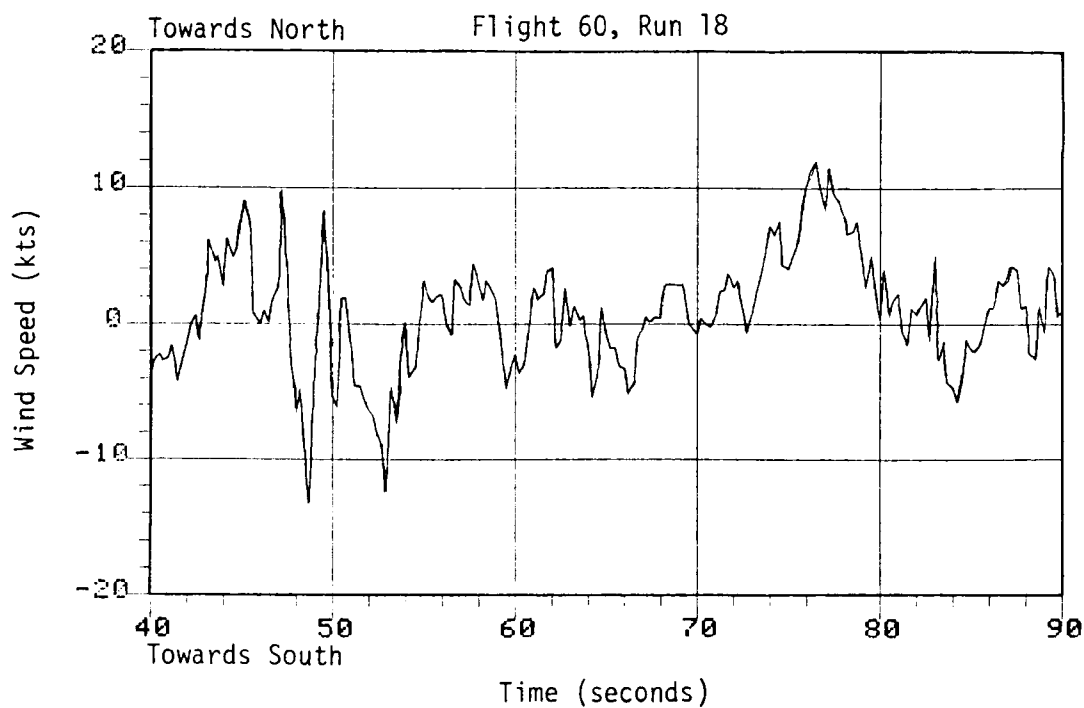


Figure 4.4. Longitudinal wind component (north/south).

mark (Figure 4.4) not surprisingly produce the strongest spanwise turbulence reflected in ΔL_{ADD} in Figure 3.6, page 26.

Figure 4.5, recorded yaw angle, and Figure 4.6, computed yaw angle, show an astonishing similarity between the actual and simulated flight. Notice the negative yaw angle or airplane nose pointing to the left as the sidewind from that direction increases (Figure 4.3). The consequence of including strip theory terms is illustrated in Figure 4.6. The amplitude of the yaw oscillations are increased noticeably by spanwise turbulence and is close to the magnitude of the measured value.

The rudder deflections the pilots used and the automatic rudder control inputs from the 6-D program are shown in Figures 4.7 and 4.8, respectively. The frequency of inputs are not compatible, the pilot only periodically adjusts the rudder control, whereas the automatic control changes the deflection every time step or four times per second. It is further apparent the pilot can control the aircraft with lower deflection angles. It is expected that the gain factors in the control law, the rudder power term $C_{n_{\delta_r}}$, or a combination are to be blamed for the difference. Adjusting the gain factors is done by trial and error and is time consuming. Further fine tuning of the gain factors could not be justified for this study. It is more important to study the relative difference when including strip theory terms.

In Figures 4.9 and 4.10 the pitch angle behavior of the actual flight and of the two computed flight simulations are plotted. The comparison between the computer-simulated and recorded flight data is not as impressive as in the case with the yaw angle. The last 20 seconds of the simulation are much closer than the first 30 seconds,

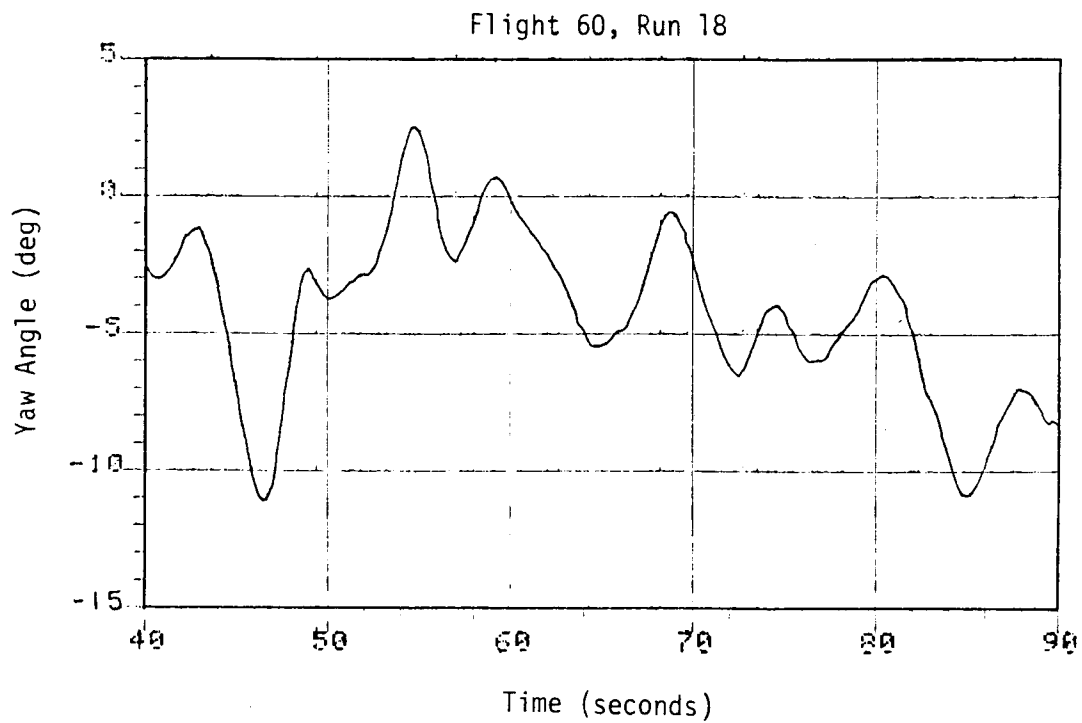


Figure 4.5. Recorded yaw angle.

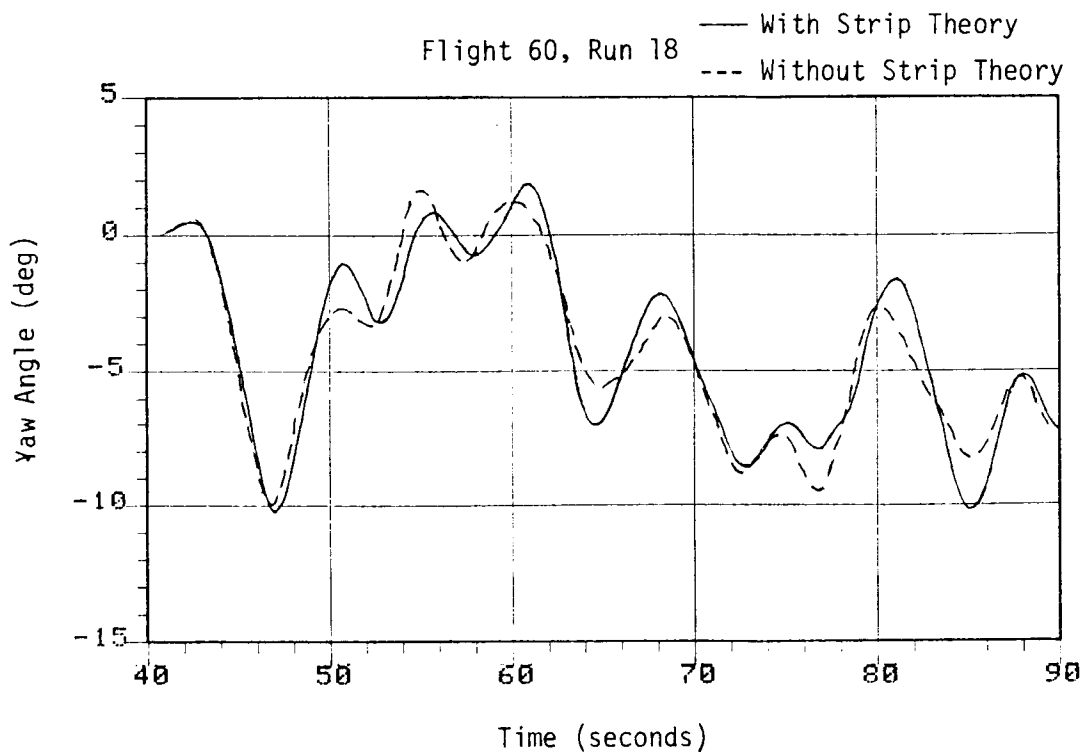


Figure 4.6. Calculated yaw angle.

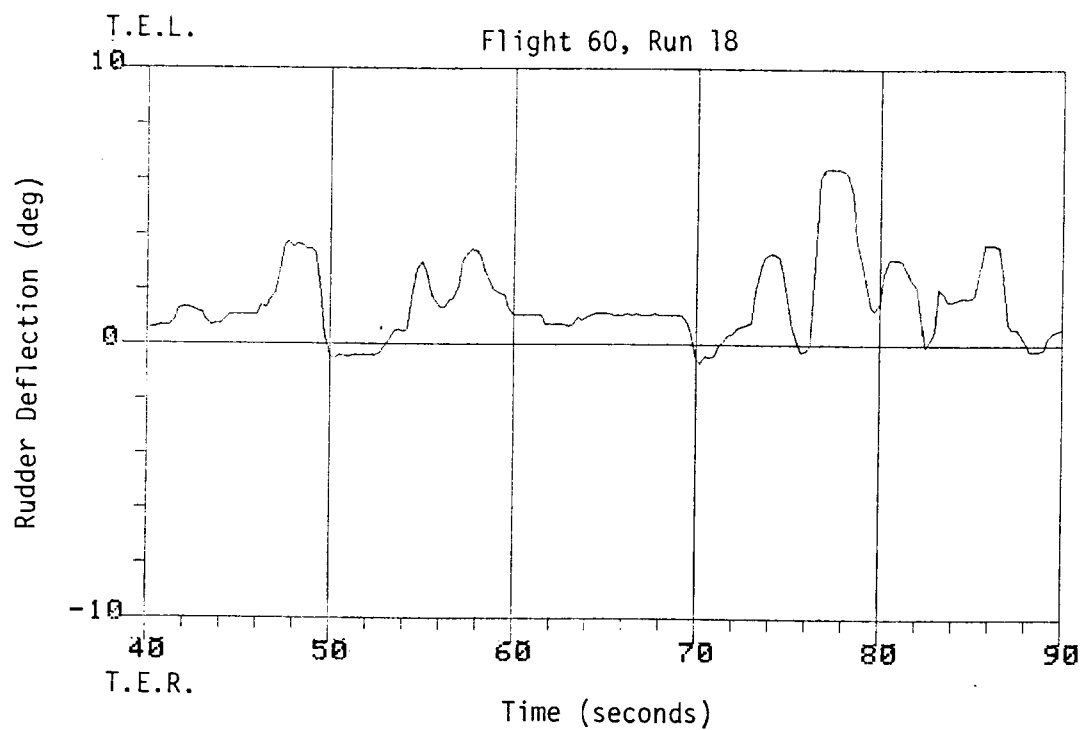


Figure 4.7. Recorded rudder deflection.

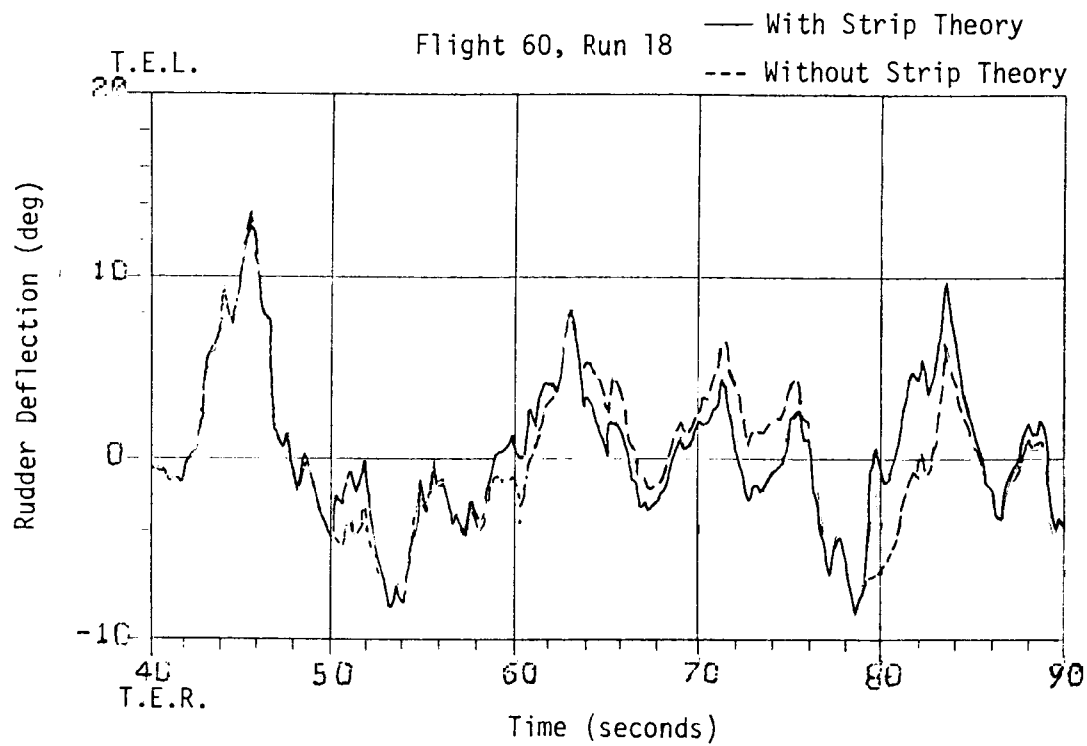


Figure 4.8. Calculated rudder deflection.

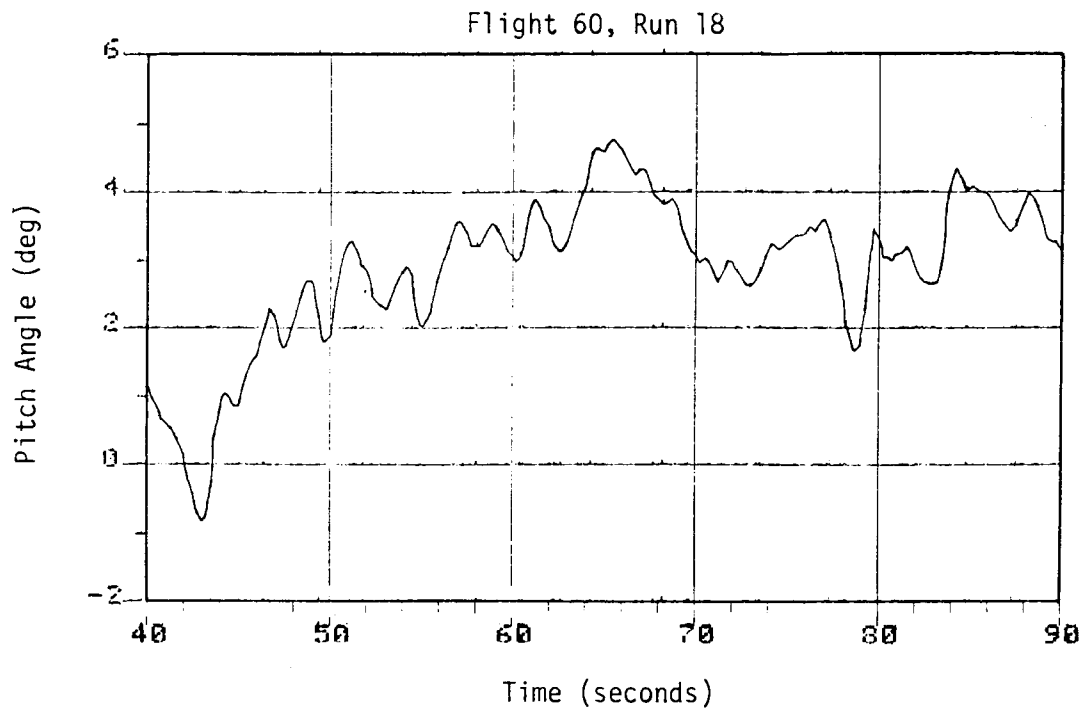


Figure 4.9. Recorded pitch angle.

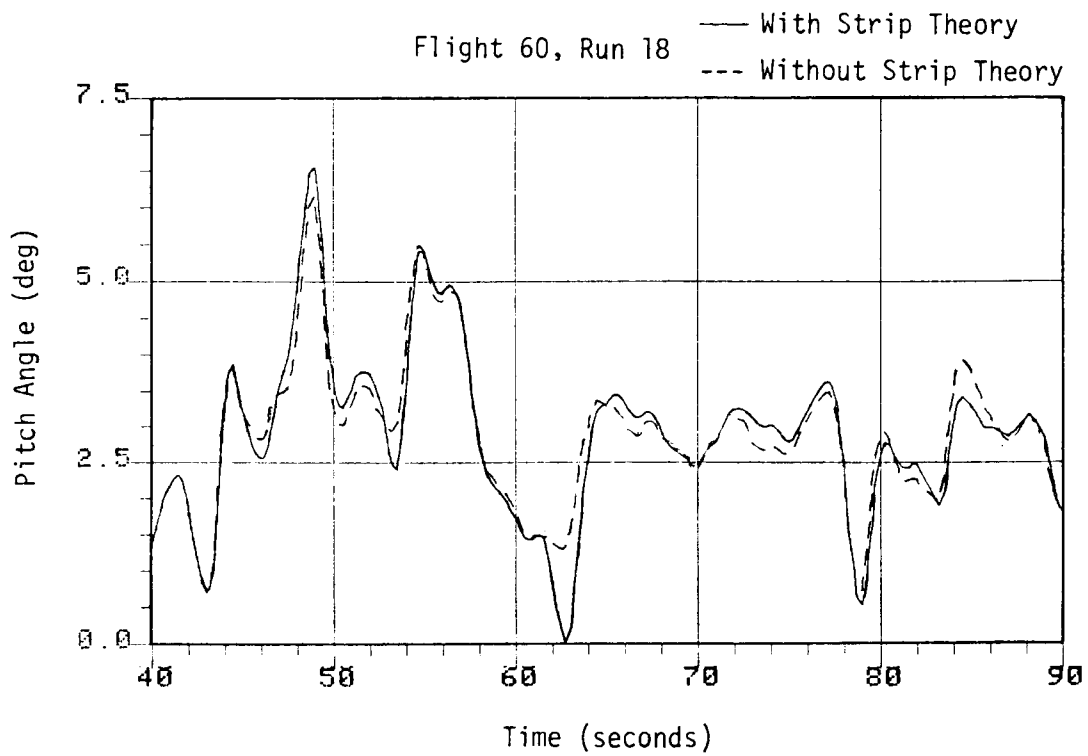


Figure 4.10. Calculated pitch angle.

however. One explanation for this phenomena is that the actual aircraft has already adjusted to the conditions, whereas in the simulated flight the aircraft is instantaneously brought out of trim at time zero corresponding to the 40-second mark and will oscillate until reaching equilibrium with the wind. A study of the frequency response of the pitch control law could help determine such effects. If the simulation of the flight was started at the beginning of the run, the curve may have become similar to the flight-recorded pitch at time equal 40 seconds. The difference induced by strip theory is pronounced but less severe than for yaw angle.

The elevator deflection angles are of the same order of magnitude for the simulated flights as for the real flight (Figures 4.11 and 4.12). The difference with strip theory included is negligible.

Figures 4.13 and 4.14 show the behavior of the recorded roll angles versus calculated roll angles. The 6-D program roll angles resemble the flight-recorded yaw angle. The explanation is probably found in the nature of the control laws. As soon as a left sidewind displaces the aircraft to the right of the intended flight path, a left turn with a negative roll is initiated as Figure 4.14 shows. The pilot sees no such imaginary line and may be more concerned with maintaining the wings level. More meaningful is the relative difference when strip theory terms are used in the simulation. The magnitude of oscillations is increased sharply, and a slight frequency shift is also introduced. The influence of spanwise turbulence, as expected, appears to be most influential on the roll mode.

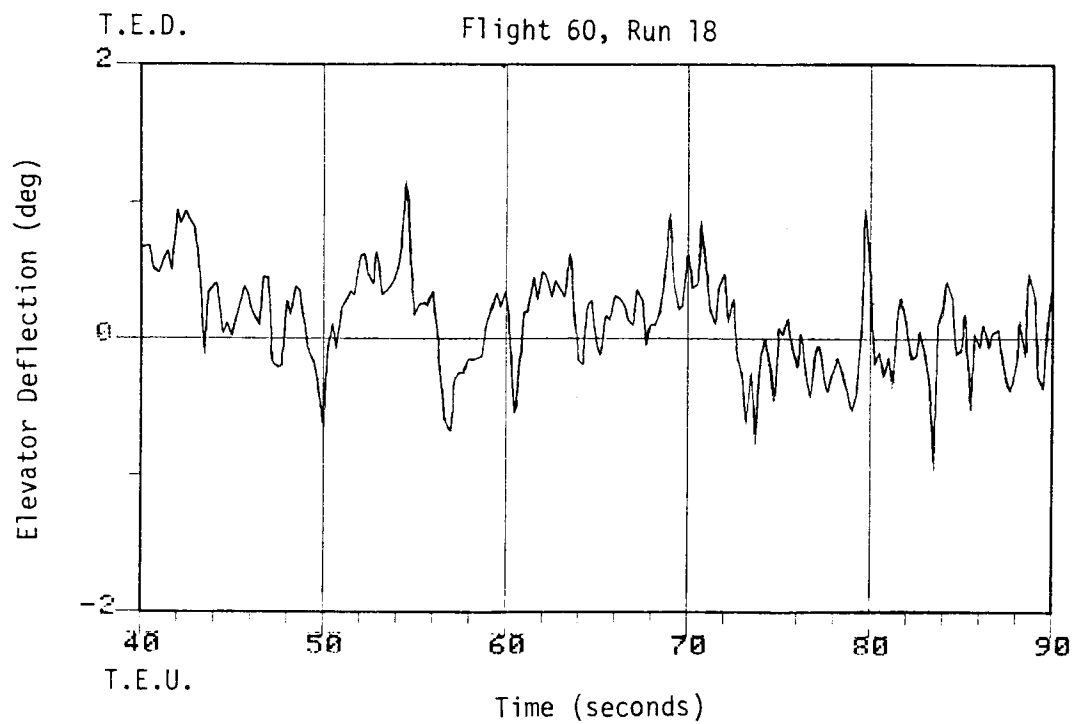


Figure 4.11. Recorded elevator deflection.

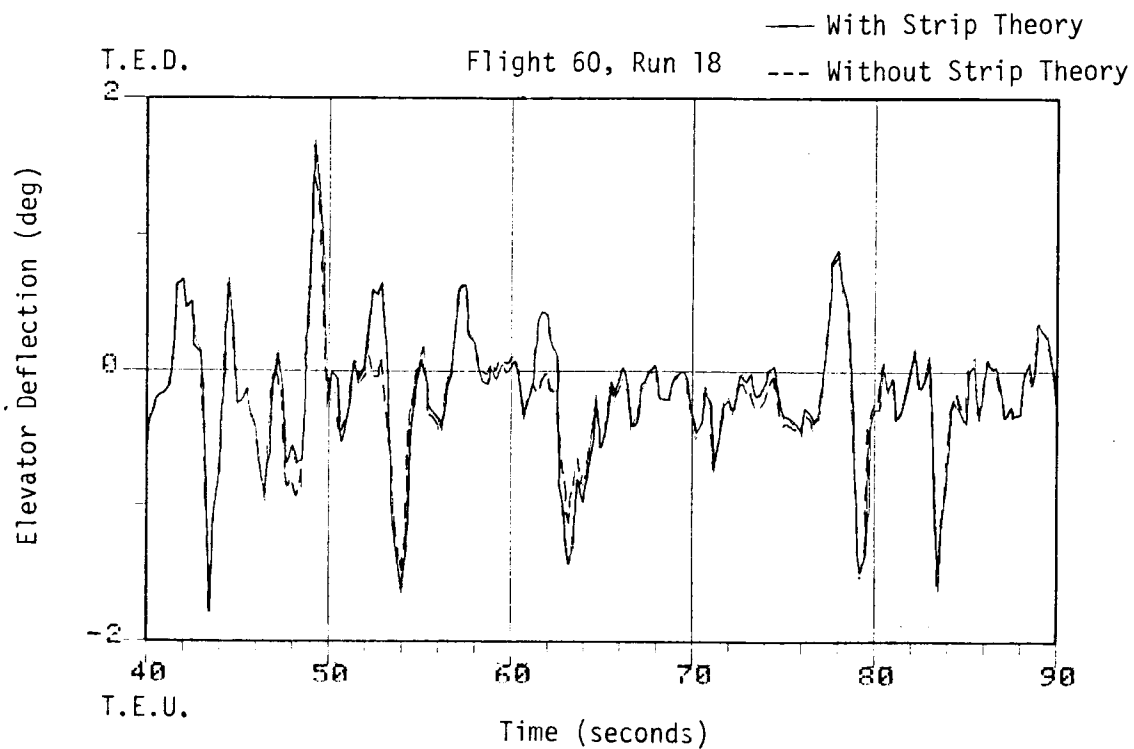


Figure 4.12. Calculated elevator deflection.

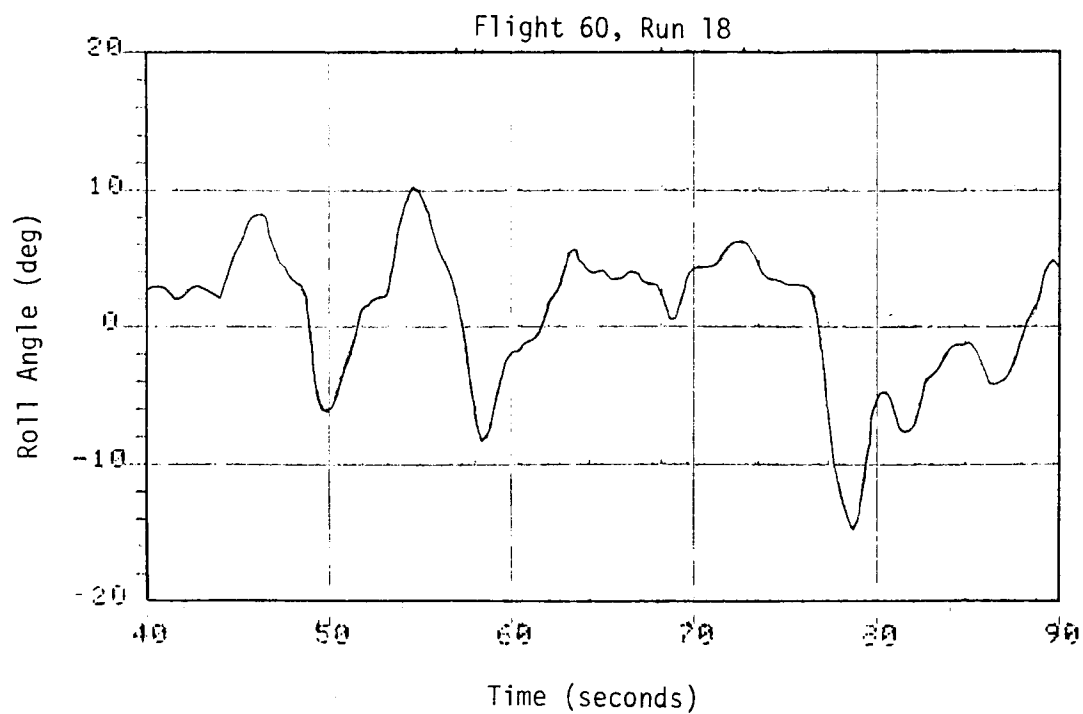


Figure 4.13. Recorded roll angle.

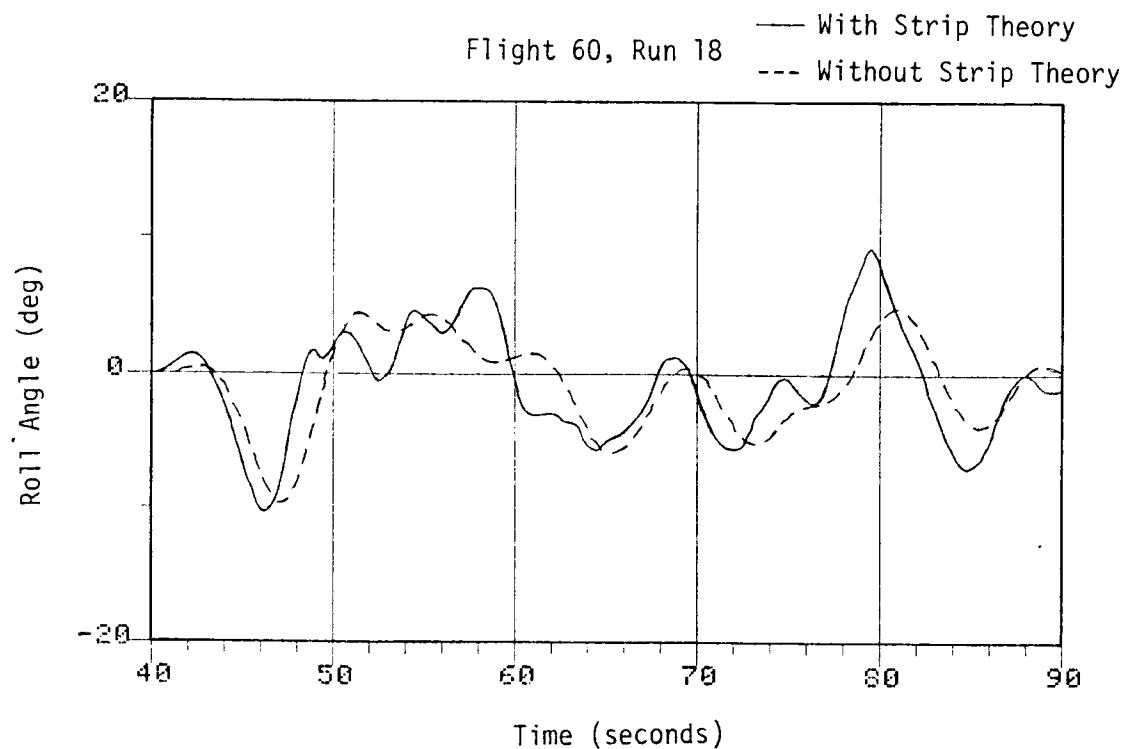


Figure 4.14. Calculated roll angle.

Figures 4.15 and 4.16 again show the relationship between aileron inputs used by the pilot and those calculated in the two flight simulations. It is appropriate at this point to emphasize that cross coupling between the roll and yaw modes cannot be ignored. The rudder deflection also introduces a roll moment and the aileron deflections, even if less severe, affect the yaw behavior. Thus, aileron behavior does not reveal the complete picture of roll disturbances. Figure 4.16 nevertheless illustrates that the incorporation of spanwise turbulence into the simulation will translate into increased aileron workload.

The influence of spanwise turbulence terms becomes more evident in Figure 4.17. The figure represents the total roll moment calculated in the 6-D program. However, much of the high frequency energy content is damped out by the inertia of the aircraft. Figure 4.18 shows similarly the total yaw coefficient calculated in the 6-D program. The influence by including spanwise turbulence terms is relatively smaller on the yaw coefficient compared to the roll coefficient.

In Figure 4.19 the altitude recorded by the B-57 on Flight 60, Run 18, is given. Figure 4.20 shows for comparison the altitude of the calculated flights. The spanwise turbulence terms do not significantly influence calculated airplane trajectory in the vertical plane. Note, however, that the vertical range is greater in Figure 4.19 than in Figure 4.20.

In order to enhance the usefulness of the results presented so far, a statistical analysis is applied to some of the parameters. Calculations of spectra and standard deviations given in the last part of this section are carried out with a fast Fourier transform computer code.

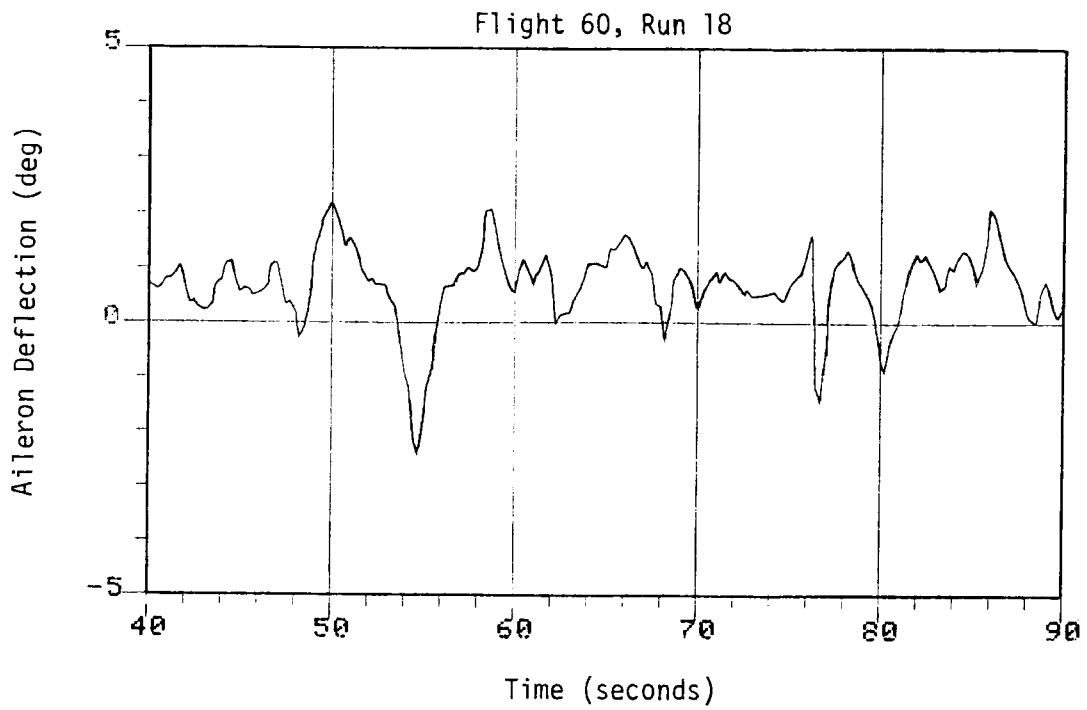


Figure 4.15. Recorded aileron deflection.

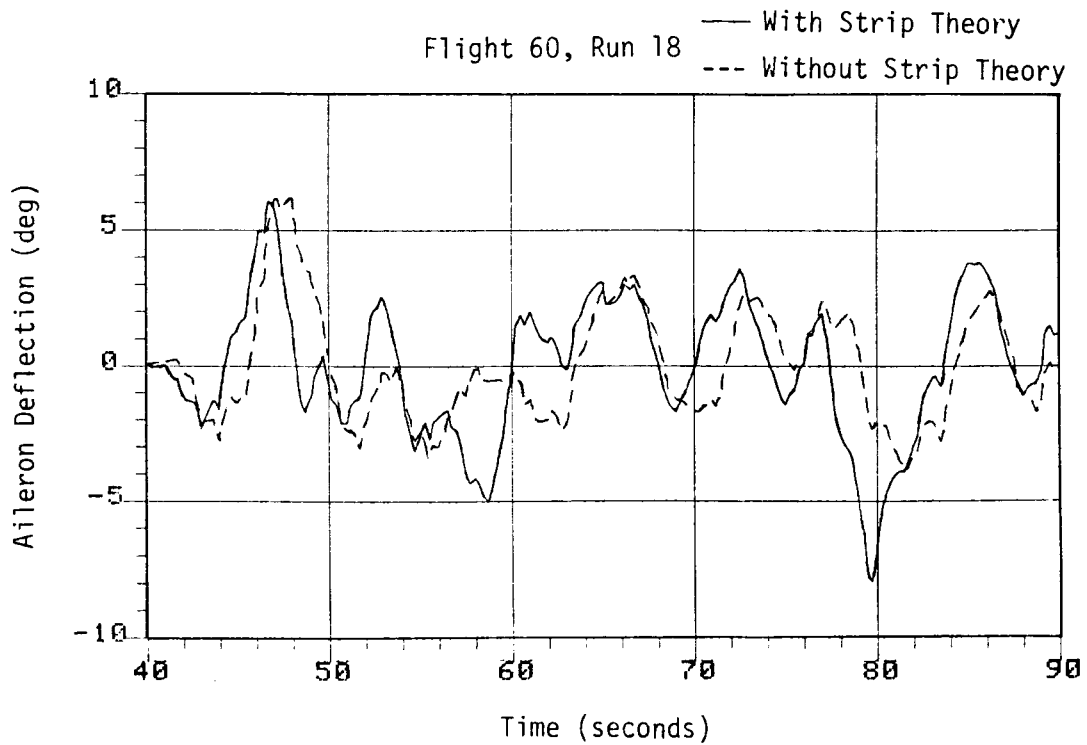


Figure 4.16. Calculated aileron deflection.

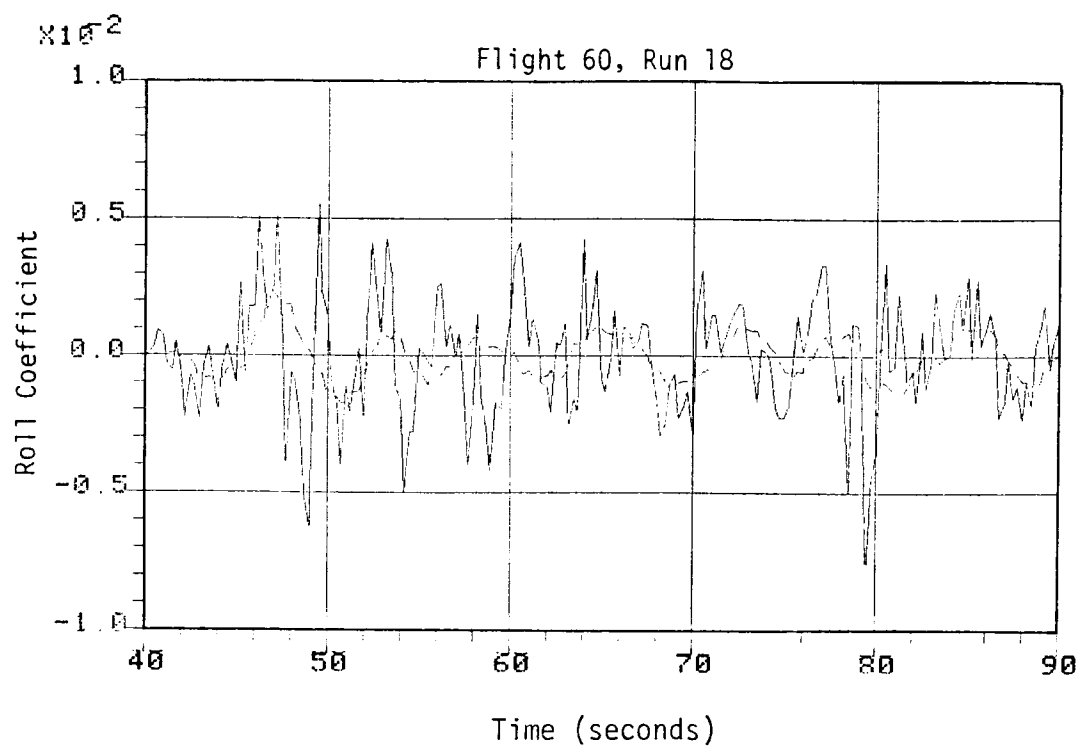


Figure 4.17. Calculated total roll moment coefficient.

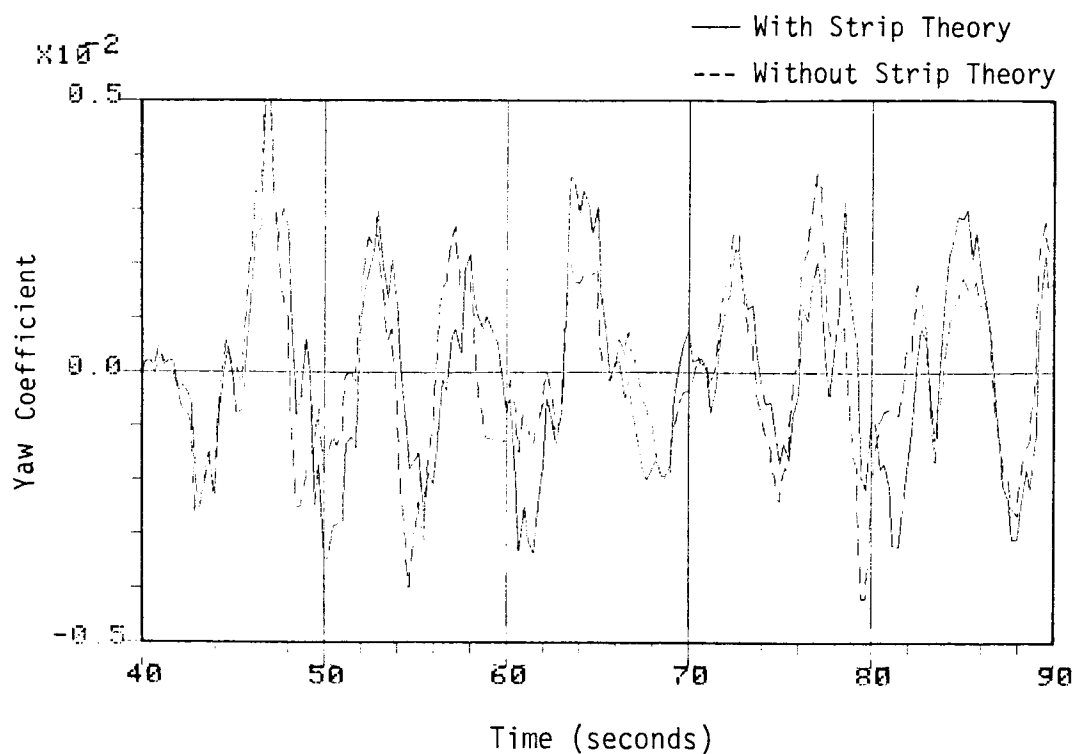


Figure 4.18. Calculated total yaw moment coefficient.

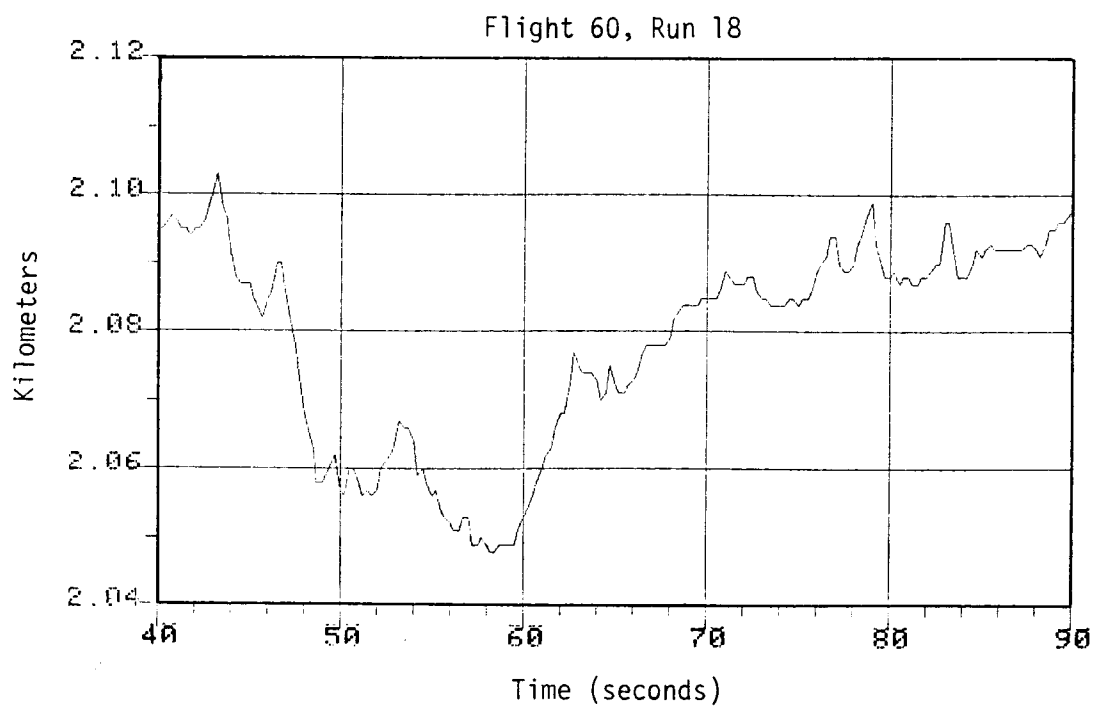


Figure 4.19. Recorded height.

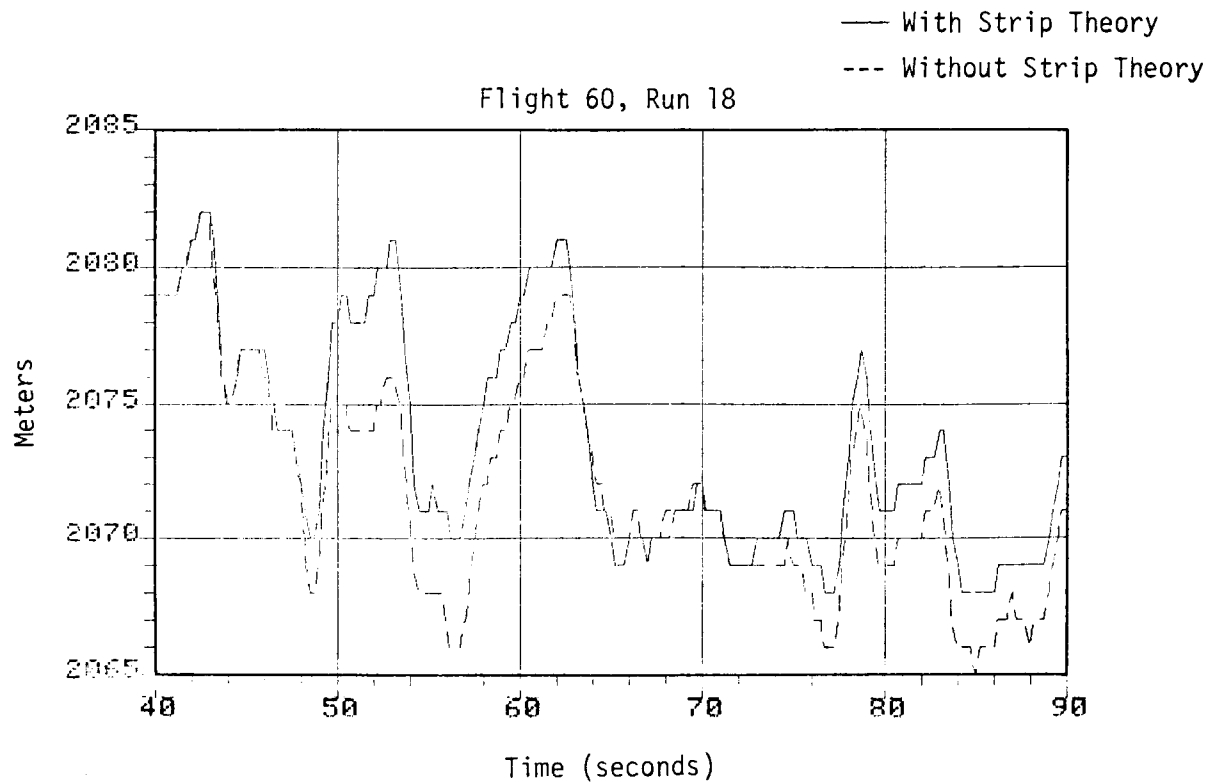


Figure 4.20. Calculated height.

The spectral analysis is performed with a small number of data points. The computation requires an input which is a power of the number 2; thus, only 128 points can be used in the 50-second run containing 201 points.

Figure 4.21 shows the spectra of L_{ADD} , the roll moment due to spanwise turbulence, calculated in Chapter III. The frequencies 0.15 Hz and 0.3 Hz are where the highest energy is contained.

Figure 4.22 gives the spectrum of the roll angle recorded by the B-57 on Flight 60, Run 18. The energy peaks at 0.15 Hz. This is approximately the same frequency at which the L_{ADD} spectrum peaks. Note that the magnitudes of the spectra in Figures 4.21 and 4.22 cannot be compared directly due to the non-compatible inputs, moment and angle.

Figure 4.23 presents the spectra of the 6-D program calculations of roll angle. The relative magnitude of the spectra is in this case compatible with the spectrum in Figure 4.21. Figure 4.22 shows that the strip theory terms L_{ADD} , N_{ADD} , ΔL_{ADD} , and $\Delta D_{i_{ADD}}$ introduce an additional energy peak at 0.15 Hz in the calculated roll angle.

Finally, the dynamic airplane behavior in terms of the roll, pitch, and yaw angles are estimated on the basis of standard deviations. Also, control surface deflections are analyzed similarly. Table 4.1 lists results obtained in the following manner. The standard deviation for all six variables are computed from calculated flight data both with and without the spanwise turbulence included. For each variable the percentage change in standard deviation by implementing spanwise turbulence into the flight simulation is computed. As Table 4.1 shows, the roll mode is most effected.

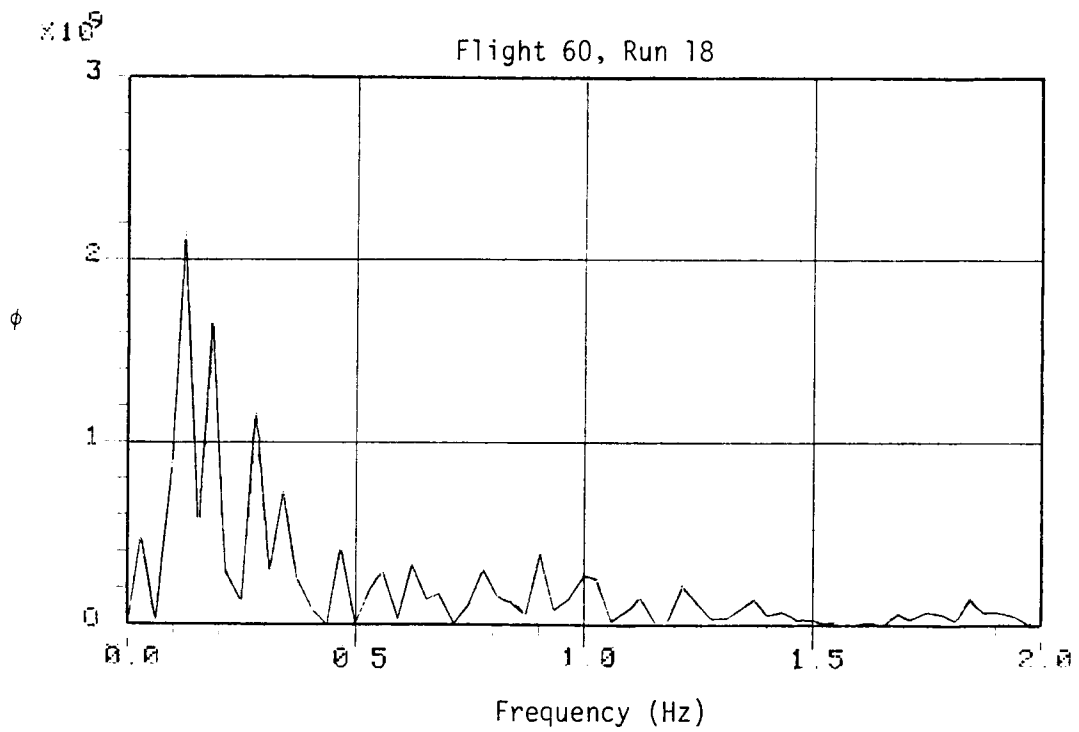


Figure 4.21. Spectrum of roll moment due to spanwise wind variation.

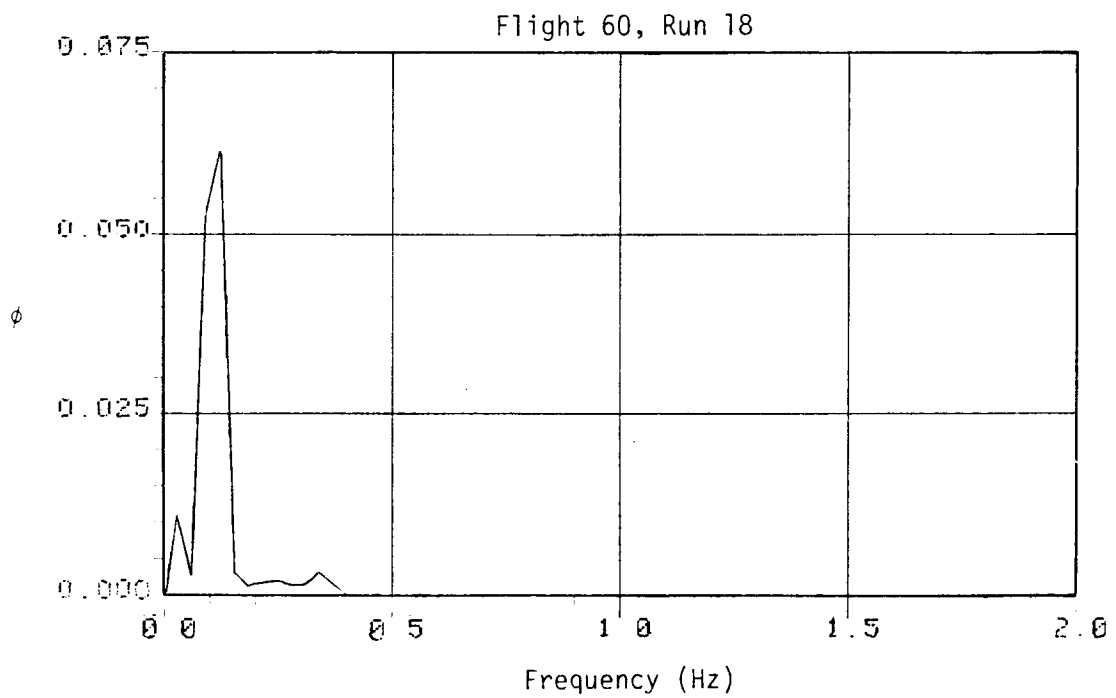
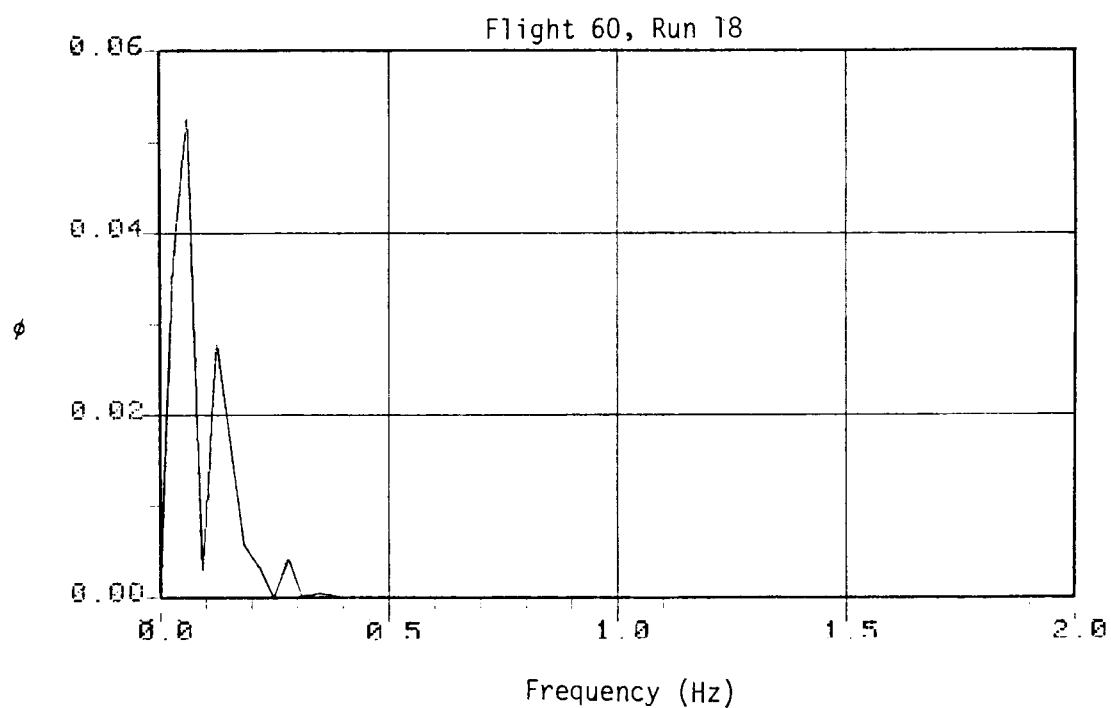
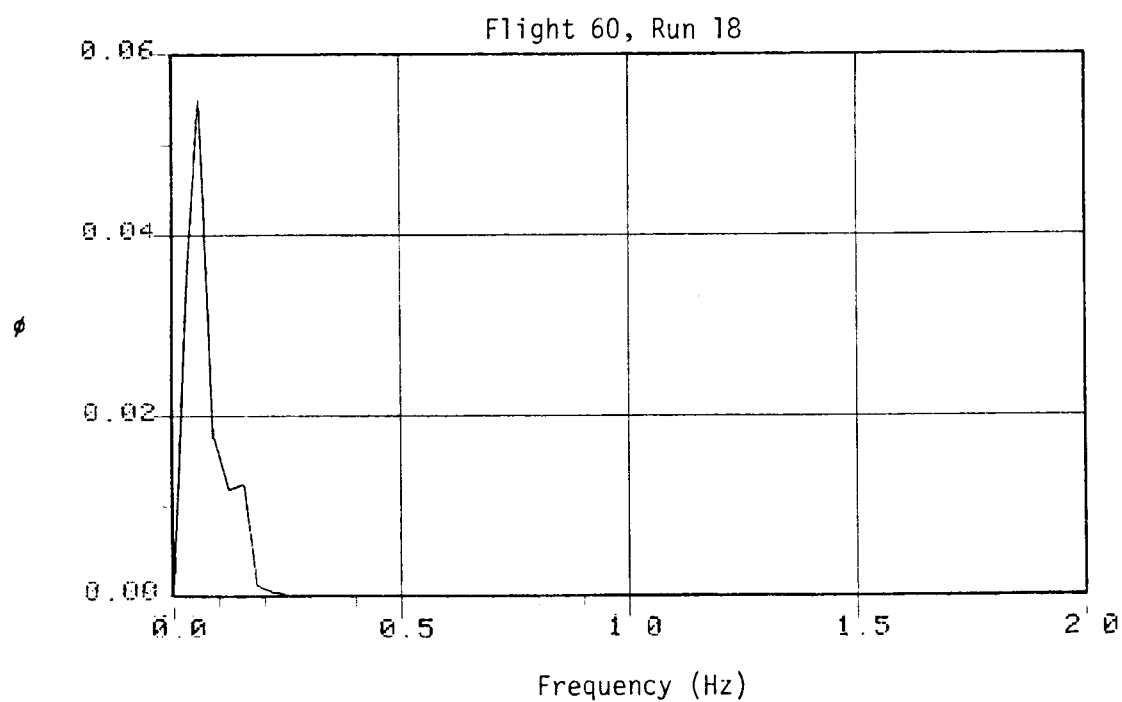


Figure 4.22. Spectrum of recorded roll angle.



(a) With strip theory



(b) Without strip theory

Figure 4.23. Spectra of calculated roll angles.

TABLE 4.1. Change in Statistical Properties of Calculated Dynamic Flight by Including Spanwise Turbulence Effects.

<u>Parameter</u>	<u>Percentage Increase in Standard Deviation</u>
Roll Angle	+19%
Aileron Deflection	+19%
Yaw Angle	+3.5%
Rudder Deflection	-5%
Pitch Angle	+11%
Elevator Deflection	+10%

CHAPTER V

CONCLUSIONS

The results of this study are based on NASA B-57 flight test data. The magnitude of spanwise turbulence effects is estimated and also the influence of spanwise turbulence on airplane dynamic behavior is calculated.

It is evident from the results that the wind variation across the airfoil can indeed introduce large aerodynamic moments on the wings of the B-57. Such roll moments can be equivalent to 10 degrees of aileron input. The frequency of roll moments resulting from spanwise turbulence contain the most energy of 0.15 Hz and 0.3 Hz. This information is of importance in estimating vibrations of a wing design. The influence of spanwise turbulence on the calculated roll mode is significant. The pilot workload is also increased.

The spanwise wind variation does also affect the pitch and yaw modes of the aircraft as is illustrated by the calculations of this study. The relative increase in upsets of the aircraft pitch and yaw angles are less severe than for roll angle. The yaw moments introduced by spanwise turbulence only represent a fraction of similar roll moments. Yaw moments are furthermore of less importance from a design view point as the airfoil stiffness inherently is much greater about the z-axis than the x-axis (body coordinates).

The calculations of aerodynamic forces on the wing showed only small changes when a spanwise wind distribution was replaced by the traditional uniform distribution. The greatest difference in lift

calculated due to spanwise turbulence is 10 percent of the aircraft weight, but generally the difference is closer to 2 percent.

The strip theory lift calculations were expressed as a lift-to-weight ratio and compared with the flight data load factor. With the gust alleviation factor (see Appendix B) applied to the calculated loading, the difference between the two curves is negligible (Figure 3.6, page 26). The contribution to lift from aircraft dynamics is ignored in the theoretical calculations. This comparison suggests the strip theory lift calculations are of high quality.

The six-degrees-of-freedom flight simulation is limited by a number of necessary assumptions. The aerodynamic modeling of the B-57, in particular, is critical. The aerodynamic coefficients and derivatives were estimated without any wind tunnel or test flight information. Nevertheless, spanwise turbulence effects are shown to substantially influence aircraft performance.

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BIBLIOGRAPHY

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APPENDICES

APPENDIX A

DOCUMENTATION OF PROGRAM I

The computer code documented in this appendix is based on the strip theory derived in Chapter II. The version presented calculates aerodynamic forces and moments due to spanwise distributions of angle of attack and relative airspeed. To obtain results with a uniform distribution, simply average angle of attack and airspeed before calling the main part of the program. Observe, that in the latter case the moments vanish.

1. FIRST PART OF PROGRAM

The first part of the program is simply a loop in which the main program is repeated for a desired number of times corresponding to the length of the input file. The inputs are angles of attack and relative airspeeds. Lift, induced drag, and roll and yaw moments due to asymmetric loading are the outputs. Key parameters are:

nrun		input file
NALAIR		output file
ALR	α_L	angle of attack left tip
ALC	α_C	angle of attack center
ALL	α_R	angle of attack right tip
VL	V_L	airspeed left tip
VC	V_C	airspeed center
VR	V_R	airspeed right tip
FORCE(3)	L	lift

FORCE(1)	D_i	induced drag
AOMEN(1)	L_{ADD}	roll moment
AOMEN(3)	N_{ADD}	yaw moment
T	t	time

```

1      REAL T
2      COMMON/A2/CINTG(21),CSPAN(51),SINTG(40,21),SSPAN(40,51),YINTG(21)
3      1  ,YSPAN(51),EULER(3),EULEP(3),AB(3,3),XW(3),WECG(3),UB(3),UBW(3),
4      2  WE(3),WB(3),WBCG(3),V1(21),VM(51),RM(3,3),XECG(3),XEW(3),W(40,51)
5      3  ,AALP(40),UE(3),FORCE(3),AOMEN(3),WDOWN(51),DLTH1,
6      4  DLTHM,PI,AMU(21),SPAN,ALPO
7      COMMON/A5/ALL,ALC,ALR,VL,VC,VR
8      OPEN(7,FILE='nrun',STATUS='NEW')
9      OPEN(2,FILE='NALAIR',STATUS='OLD')
10     DO 1 N=1,201
11
12     READ(2,3)T,ALR,ALC,ALL,VR,VC,VL
13     3  FORMAT(7E12.4)
14
15     VL=3.28084*VL
16     VC=3.28084*VC
17     VR=3.28084*VR
18
19     CALL PROG
20
21     WRITE(7,12)T,FORCE(3),FORCE(1),AOMEN(1),AOMEN(3)
22     12  FORMAT(5E12.4)
23
24     1  CONTINUE
25     STOP
26     END

```

2. MAIN PART, SUBROUTINE PROG

This subroutine is the main program. The option of using a symmetric distribution which gives closed form solutions is included in the program. The control parameters are NOIMT and NOINT. Next, airplane parameters and atmospheric conditions are input. The coordinate transformation from y to the angle θ is performed in subroutine SCINT. In Lines 55 through 77 the spanwise chord length distribution of the B-57 is calculated. The velocities needed for the numerical evaluations

of the integrals are obtained in subroutines VINTG and VSPAN. The values of V_l and V_m differ by a small quantity so that singularity is avoided. In subroutine DOWN the downwash and the matrix I_{nm} is calculated. Subroutine INVERS gives I_{nm}^{-1} , which allows for solution of the A_n coefficients (Line 91). If NOIMT is negative, subroutine AERODY is called. That subroutine gives the final solutions of the four quantities which are printed in the first part of the program. Key parameters are:

C(N)	C	chord length
AINF	a_∞	lift curve slope
ALPO	α_o	zero lift angle
COEF(I,J)	I_{nm}	matrix in Equation 2.24
COEFI(I,J)	I_{nm}^{-1}	inverse of I_{nm}
AK(I)	A_n	A_n coefficients

```

27
28      SUBROUTINE PROG
29
30      DOUBLE PRECISION AK,A,R
31      DIMENSION A(1600),R(40),COEFU(1600)
32      DIMENSION COEF(21,21),COEFI(21,21),C(21)
33      COMMON/A1/ AK(40)
34      COMMON/A2/CINTG(21),CSPAN(51),SINTG(40,21),SSPAN(40,51),YINTG(21)
35      1 ,YSPAN(51),EULER(3),EULEP(3),AB(3,3),XW(3),WECG(3),UB(3),UBW(3),
36      2 WE(3),WB(3),WBCG(3),V1(21),VM(51),RM(3,3),XECG(3),XEW(3),W(40,51)
37      3 ,AALP(40),UE(3),FORCE(3),AOMEN(3),WDOWN(51),DLTHI,
38      4 DLTHM,PI,AMU(21),SPAN,ALPO
39      COMMON /A3/NTerm,NINTG,NSPAN,INTG,ISPAN,NSKIP,MID,NOINT
40      COMMON/A4/ZO,USTAR
41      COMMON/A5/ALL,ALC,ALR,VL,VC,VR
42 C CONTROL OF INTEGRATION PARAMETERS
43 C INTEGRATION WHEN NEGATIVE
44      NOIMT=-1
45      NOINT=-1
46 C SPANWISE DIVISIONS
47      NSPAN=40
48      PI=3.14159265
49      RHO=0.0600812
50 C WINGSPAN IN FEET
51      SPAN=63.95
52 C SLOPE OF THE LIFT CURVE
53      AINF=5.844
54      CALL SCINT
55 C TAPERED WING
56      CC=19.
57      CT=7.666667
58      DO 100 N=1,21
59      IF(N.GT.10) GO TO 11
60      IF(YINTG(N).LT.-12.06) THEN

```

```

61      C(N)=CT+(CC-CT)/19.916*(SPAN/2.+YINTG(N))
62      ENDIF
63      IF(YINTG(N).GT.-12.06)THEN
64      C(N)=CC
65      ENDIF
66      11  IF(N.NE.11) GO TO 12
67      C(N)=CC
68      12  IF(N.LT.12) GO TO 13
69      IF(YINTG(N).LT.12.06)THEN
70      C(N)=CC
71      ENDIF
72      IF(YINTG(N).GT.12.06)THEN
73      C(N)=CC-(CC-CT)/19.916*(YINTG(N)-12.06)
74      ENDIF
75      13  CONTINUE
76      AMU(N)=C(N)*AINF/(8.*SPAN/2.)
77      100  CONTINUE
78  C  ZERO LIFT ANGLE
79      ALPO=-0.12035714
80      DO 40 I=1,3
81      DO 40 K=1,3
82      40  RM(I,K)=0.0
83      CALL VINTG
84      CALL VSPAN
85      CALL DOWN(COEF)
86      CALL INVERS(COEF,COEFI,NTERM)
87      DO 777 I=1,NTERM
88      SUM=0.
89      DO 778 J=1,NTERM
90      SUM=SUM+COEFI(J,I)*AALP(J)
91      777  AK(I)=SUM
92      397  FORMAT(21F10.5)
93      IF(NOIMT)730,732,732
94      732  CONST=(PI*RHO*((VM(MID)*SPAN)**2))/32.2
95      FORCE(3)=CONST*AK(1)/2.
96      SUM=0.0
97      DO 713 I=1,NTERM
98      SUM=SUM+I*(AK(I)**2)
99      713  FORCE(1)=CONST*SUM/2.
100      AOMEN(1)=SPAN*CONST*AK(2)/8.
101      NTERM=NTERM-1
102      SUM=0.
103      DO 714 J=1,NTERM
104      SUM=SUM+(2*J+1)*AK(J)*AK(J+1)
105      AOMEN(3)=SPAN*CONST*SUM/8.
106      AOMEN(2)=0.
107      FORCE(2)=0.
108      GO TO 735
109      730  CALL AERODY
110      735  CONTINUE
111      RETURN
112      END

```

3. SUBROUTINE SCINT

This subroutine calculates the sine and cosine terms needed for the evaluation of the integrals. The angles θ_1 and θ_m are calculated separately so that at a particular section of the airfoil they differ by a small finite value. That way the problem of singularity is avoided.

```

113
114 SUBROUTINE SCINT
115 C CALCULATES SIN&COS TERMS FOR INTEGRATION
116 DOUBLE PRECISION AK
117 COMMON/A1/ AK(40)
118 COMMON/A2/CINTG(21),CSPAN(51),SINTG(40,21),SSPAN(40,51),YINTG(21)
119 1 ,YSPAN(51),EULER(3),EULEP(3),AB(3,3),XW(3),WECG(3),UB(3),UBW(3),
120 2 WE(3),WB(3),WBCG(3),V1(21),VM(51),RM(3,3),XECG(3),XEW(3),W(40,51)
121 3 ,AALP(40),UE(3),FORCE(3),AOMEN(3),WDOWN(51),DLTHI,
122 4 DLTHM,PI,AMU(21),SPAN,ALPO
123 COMMON /A3/NTERM,NINTG,NSPAN,INTG,ISPAN,NSKIP,MID,NOINT
124 DLTHI=PI/20
125 INTG=21
126 MID=11
127 NTERM=21
128 DO 1 I=1,10
129 I2=22-I
130 AUG=(I-1)*DLTHI
131 CINTG(I)=COS(AUG)
132 YINTG(I)=-SPAN*CINTG(I)/2.
133 CINTG(I2)=-CINTG(I)
134 YINTG(I2)=-YINTG(I)
135 DO 1 N=1,21
136 AUGN=N*AUG
137 SINTG(N,I)=SIN(AUGN)
138 SINTG(N,I2)=((-1)**(N+1))*SINTG(N,I)
139 1 CONTINUE
140 LALT=3
141 DO 4 N=1,21
142 IF(N-LALT)10,11,11
143 11 COUNT=-1.
144 LALT=LALT+4
145 GO TO 12
146 10 COUNT=1.
147 12 CONTINUE
148 4 SINTG(N,MID)=(0.5-0.5*((-1)**N))*COUNT
149 CINTG(MID)=0.0
150 YINTG(MID)=0.0
151 MID=21
152 EPSI=0.005
153 DLTHM=(PI-4.*EPSI)/(NSPAN-2)
154 ISPAN=41
155 CSPAN(1)=COS(EPSI)
156 YSPAN(1)=-SPAN*CSPAN(1)/2.
157 DO 2 M=2,20
158 M2=(42-M)
159 AUG=(M-2)*DLTHM+2.*EPSI
160 CSPAN(M)=COS(AUG)
161 CSPAN(M2)=-CSPAN(M)
162 YSPAN(M)=-SPAN*CSPAN(M)/2.
163 YSPAN(M2)=-YSPAN(M)
164 DO 2 N=1,21
165 AUGN=N*AUG
166 SSPAN(N,M)=SIN(AUGN)
167 SSPAN(N,M2)=((-1)**(N+1))*SSPAN(N,M)
168 2 CONTINUE
169 CSPAN(MID)=0.00001
170 YSPAN(MID)=0.
171 LALT=3
172 DO 3 N=1,21
173 IF(N-LALT)16,13,13
174 13 COUNT=-1.
175 LALT=LALT+4
176 GO TO 15
177 16 COUNT=1.
178 15 CONTINUE
179 SSPAN(N,MID)=(0.5-0.5*((-1)**N))*COUNT
180 SSPAN(N,1)=SIN(N*EPSI)
181 3 SSPAN(N,ISPAN)=((-1)**(N+1))*SSPAN(N,1)
182 CSPAN(ISPAN)=-CSPAN(1)
183 YSPAN(ISPAN)=-YSPAN(1)
184 RETURN
185 END

```

4. SUBROUTINE VINTG

This subroutine calculates the V_j terms needed for the evaluation of the integrals.

```

186
187      SUBROUTINE VINTG
188 C   CALCULATION OF V1'S
189      DIMENSION ALPHA(21)
190      DOUBLE PRECISION AK
191      COMMON/A1/ AK(40)
192      COMMON/A2/CINTG(21),CSPAN(51),SINTG(40,21),SSPAN(40,51),YINTG(21)
193      1 .YSPAN(51),EULER(3),EULEP(3),AB(3,3),XW(3),WECG(3),UB(3),UBW(3),
194      2 WE(3),WB(3),WBCG(3),V1(21),VM(51),RM(3,3),XECG(3),XEW(3),W(40,51)
195      3 ,AALP(40),UE(3),FORCE(3),AOMEN(3),WDOWN(51),DLTH1,
196      4 DLTHM,PI,AMU(21),SPAN,ALPO
197      COMMON /A3/NTerm,NINTG,NSPAN,INTG,ISPAN,NSKIP,MID,NOINT
198      COMMON/A5/ALL,ALC,ALR,VL,VC,VR
199      DO 10 N=1,21
200      XW(2)=YINTG(N)
201      IF(N.GT.10)GO TO 1
202      V1(N)=VC+(VC-VL)/(SPAN/2.)*XW(2)
203      ALPHA(N)=ALC+(ALC-ALL)/(SPAN/2.)*XW(2)
204      1 IF(N.NE.11)GO TO 2
205      V1(N)=VC
206      ALPHA(N)=ALC
207      2 IF(N.LT.12)GO TO 3
208      V1(N)=VC-(VC-VR)/(SPAN/2.)*XW(2)
209      ALPHA(N)=ALC-(ALC-ALR)/(SPAN/2.)*XW(2)
210      3 CONTINUE
211      AALP(N)=AMU(N)*(ALPHA(N)-ALPO)
212      10 CONTINUE
213      RETURN
214      END

```

5. SUBROUTINE VSPAN

The V_m terms are calculated in this subroutine.

```

215
216      SUBROUTINE VSPAN
217 C   CALCULATIONS OF VM'S
218      DOUBLE PRECISION AK
219      COMMON/A1/ AK(40)
220      COMMON/A2/CINTG(21),CSPAN(51),SINTG(40,21),SSPAN(40,51),YINTG(21)
221      1 .YSPAN(51),EULER(3),EULEP(3),AB(3,3),XW(3),WECG(3),UB(3),UBW(3),
222      2 WE(3),WB(3),WBCG(3),V1(21),VM(51),RM(3,3),XECG(3),XEW(3),W(40,51)
223      3 ,AALP(40),UE(3),FORCE(3),AOMEN(3),WDOWN(51),DLTH1,
224      4 DLTHM,PI,AMU(21),SPAN,ALPO
225      COMMON /A3/NTerm,NINTG,NSPAN,INTG,ISPAN,NSKIP,MID,NOINT

```

```

226      COMMON/A5/ALL,ALC,ALR,VL,VC,VR
227      NSKIP=2
228      M2=0
229      NADD=1
230      DO 50 M=1,41
231      XW(2)=YSPAN(M)
232      IF(M.GT.20)GO TO 1
233      VM(M)=VC+(VC-VL)/(SPAN/2.)*XW(2)
234      1  IF(M.NE.21)GO TO 2
235      VM(M)=VC
236      2  IF(M.LT.22)GO TO 3
237      VM(M)=VC-(VC-VR)/(SPAN/2.)*XW(2)
238      3  CONTINUE
239      50  CONTINUE
240      RETURN
241      END

```

6. SUBROUTINE DOWN

The matrix I_{nm} needed to solve for the A_n coefficients and the downwash, $w(y)$, is found in this subroutine.

```

242      SUBROUTINE DOWN(COEF)
243      DOUBLE PRECISION AK
244      DIMENSION COEF(21,21)
245      COMMON/A1/ AK(40)
246      COMMON/A2/CINTG(21),CSPAN(51),SINTG(40,21),SSPAN(40,51),YINTG(21)
247      1  ,YSPAN(51),EULER(3),EULEP(3),AB(3,3),XW(3),WECG(3),UB(3),UBW(3),
248      2  WE(3),WB(3),WBCG(3),V1(21),VM(51),RM(3,3),XECG(3),XEW(3),W(40,51)
249      3  ,AALP(40),UE(3),FORCE(3),AOMEN(3),WDOWN(51),DLTHI,
250      4  DLTHM,PI,AMU(21),SPAN,ALPO
251      COMMON /A3/NTERM,NINTG,NSPAN,INTG,ISPAN,NSKIP,MID,NOINT
252      DO 2 N=1,21
253      AN=N
254      NADD=1
255      M2=0
256      DO 2 M=1,41
257      IF(NOINT)12,13,13
258      13  W(N,M)=0.
259      GO TO 14
260      12  W(N,M)=(V1(1)-VM(M))*SINTG(1,1)*SINTG(N,1)/(CINTG(1)-CSPAN(M))
261      DO 1 I=1,17,2
262      1  W(N,M)=W(N,M)+4.0*((V1(I+1)-VM(M))*SINTG(1,I+1)*SINTG(N,I+1)/(CINT
263      1  G(I+1)-CSPAN(M)))+2.0*((V1(I+2)-VM(M))*SINTG(1,I+2)*SINTG(N,I+2)/(
264      2  CINTG(I+2)-CSPAN(M)))
265      J=21
266      W(N,M)=W(N,M)+4.0*((V1(20)-VM(M))*SINTG(1,20)*SINTG(N,20)/(CINT
267      1  G(20)-CSPAN(M)))
268      W(N,M)=-((W(N,M)+(V1(21)-VM(M))*SINTG(1,21)*SINTG(N,21)/(CI
269      1  NTG(21)-CSPAN(M)))*DLTHI/3.)
270      14  FIRST=(AN*SSPAN(N,M)/SSPAN(1,M)+W(N,M)/(PI*VM(M)))
271      NADD=NADD+1
272      IF(M-MID)5,6,5
273      6  NADD=2
274      5  CONTINUE
275      IF(NADD-NSKIP)3,4,4
276      4  M2=M2+1
277      COEF(N,M2)=SSPAN(N,M)+AMU(N)*FIRST
278

```

```

279      NADD=0
280      3  CONTINUE
281      2  W(N,M)=VM(M)*FIRST
282      RETURN
283      END

```

7. SUBROUTINE AERODY

With solutions to I_{nm}^{-1} and the A_n coefficients, this subroutine calculates the total quantities lift, induced drag, and roll and yaw moments.

```

284      SUBROUTINE AERODY
285      DOUBLE PRECISION AK
286      COMMON/A1/ AK(40)
287      COMMON/A2/CINTG(21),CSPAN(51),SINTG(40,21),SSPAN(40,51),YINTG(21)
288      1  ,YSPAN(51),EULER(3),EULEP(3),AB(3,3),XW(3),WECG(3),UB(3),UBW(3),
289      2  WE(3),WB(3),WBCG(3),V1(21),VM(51),RM(3,3),XECG(3),XEW(3),W(40,51)
290      3  ,AALP(40),UE(3),FORCE(3),AOMEN(3),WDOWN(51),DLTHI,
291      4  DLTHM,PI,AMU(21),SPAN,ALPO
292      COMMON /A3/NTERM,NINTG,NSPAN,INTG,ISPAN,NSKIP,MID,NOINT
293      RHO=0.0600812
294      CONST=(SPAN**2)*RHO*DLTHM/(32.2*3.)
295      DO 3 I=1,3
296      FORCE(I)=0.0
297      3  AOMEN(I)=0.0
298      DO 4 M=1,41
299      WDOWN(M)=0.
300      DO 4 N=1,21
301      4  WDOWN(M)=WDOWN(M)+W(N,M)*AK(N)
302      DO 1 I=1,21
303      SUM=VM(I)*SSPAN(I,1)*SSPAN(1,1)
304      SUML=SUM*VM(I)
305      SUMD=SUM*WDOWN(1)
306      SUMLR=SUM*VM(I)*YSPAN(1)
307      SUMN=SUM*WDOWN(1)*YSPAN(1)
308      ANUM=-1.
309      DO 2 K=2,NSPAN
310      ANUM=-1.*ANUM
311      COUNT=4.0-(1.-ANUM)
312      SUM=COUNT*VM(K)*SSPAN(1,K)*SSPAN(1,K)
313      SUMD=SUMD+SUM*WDOWN(K)
314      SUMN=SUMN+SUM*WDOWN(K)*YSPAN(K)
315      SUML=SUML+SUM*VM(K)
316      2  SUMLR=SUMLR+SUM*VM(K)*YSPAN(K)
317      SUM=VM(41)*SSPAN(1,41)*SSPAN(1,41)
318      SUML=(SUML+SUM*VM(41))*CONST
319      SUMD=(SUMD+SUM*WDOWN(41))*CONST
320      SUMLR=(SUMLR+SUM*VM(41)*YSPAN(41))*(-CONST)
321      SUMN=(SUMN+SUM*WDOWN(41)*YSPAN(41))*CONST
322      FORCE(3)=FORCE(3)+SUML*AK(I)
323      FORCE(1)=FORCE(1)+SUMD*AK(I)
324      AOMEN(3)=AOMEN(3)+SUMN*AK(I)
325      1  AOMEN(1)=AOMEN(1)+SUMLR*AK(I)
326      RETURN
327      END

```

8. SUBROUTINE INVERS

In the following subroutine a square matrix A of order N is inverted which yields the solution AI.

```

329
330      SUBROUTINE INVERS(A,AI,N)
331      DIMENSION A(N,N),AI(N,N)
332      DIMENSION Q(21,21),QH(21,21),R(21,21),RI(21,21)
333      CALL DA(A,N,D)
334      IF(ABS(D).NE..0) GO TO 10
335      WRITE(*,7)
336      7 FORMAT(//.2X,'THE DETERMINANT OF A =.0 PROGRAM STOP',/)
337      STOP
338      10 CONTINUE
339      18 FORMAT(10X,E13.6)
340      CALL GS(A,Q,N)
341      DO 20 I=1,N
342      DO 20 J=1,N
343      20 QH(I,J)=Q(J,I)
344      CALL MUL(QH,A,R,N)
345      CALL MI(R,RI,N)
346      CALL MUL(RI,QH,AI,N)
347      RETURN
348      END
349      SUBROUTINE GS(A,Q,N)
350      DIMENSION A(N,N),Q(N,N)
351      DIMENSION SUM1(100),APHA(100),B(100,100)
352      DO 10 J=1,N
353      IF(J.NE.1) GO TO 30
354      DO 20 I=1,N
355      20 B(I,J)=A(I,J)
356      GO TO 70
357      30 CONTINUE
358      DO 45 K=1,J-1
359      SUM2=.0
360      DO 40 I=1,N
361      B(I,J)=A(I,J)
362      40 SUM2=SUM2+B(I,K)*A(I,J)
363      45 APHA(K)=SUM2/SUM1(K)
364      DO 50 K=1,J-1
365      DO 50 I=1,N
366      50 B(I,J)=B(I,J)-APHA(K)*B(I,K)
367      SUM1(J)=.0
368      DO 100 I=1,N
369      100 SUM1(J)=SUM1(J)+B(I,J)*B(I,J)
370      70 CONTINUE
371      SUM1(J)=.0
372      DO 80 I=1,N
373      80 SUM1(J)=SUM1(J)+B(I,J)*B(I,J)
374      DO 90 I=1,N
375      90 Q(I,J)=B(I,J)/SQRT(SUM1(J))
376      10 CONTINUE
377      RETURN
378      END
379      SUBROUTINE MUL(QH,A,R,N)
380      DIMENSION QH(N,N),A(N,N),R(N,N)
381      DO 10 IR=1,N
382      IC=.0
383      DO 10 J=1,N
384      IC=IC+1
385      R(IR,IC)=.0
386      DO 10 I=1,N
387      10 R(IR,IC)=R(IR,IC)+QH(IR,I)*A(I,IC)

```

```

388      RETURN
389      END
390      SUBROUTINE MI(R,RI,N)
391      DIMENSION R(N,N),RI(N,N)
392      DO 20 II=1,N
393      I=N-II+1
394      DO 25 J=1,N
395      25 RI(J,J)=1./R(J,J)
396      DO 20 J=1,N
397      IF(I.EQ.J) GO TO 20
398      IF(I.GT.J) GO TO 30
399      RI(I,J)=0.
400      DO 40 K=I+1,N
401      40 RI(I,J)=RI(I,J)-R(I,K)*RI(K,J)
402      RI(I,J)=RI(I,J)/R(I,I)
403      GO TO 20
404      30 RI(I,J)=0.
405      20 CONTINUE
406      RETURN
407      END
408      SUBROUTINE DA(A,N,D)
409      DIMENSION A(N,N),AD(21,21)
410      DO 5 I=1,N
411      DO 5 J=1,N
412      5 AD(I,J)=A(I,J)
413      NRA=0
414      DO 20 K=1,N
415      IF(ABS(AD(K,K)).NE.0.0) GO TO 10
416      DO 30 I=K,N
417      IF(ABS(AD(I,K)).EQ.0.0) GO TO 40
418      DO 50 J=K,N
419      TR=AD(I,J)
420      AD(K,J)=AD(I,J)
421      50 AD(I,J)=TR
422      GO TO 30
423      40 NRA=NRA+1
424      30 CONTINUE
425      10 DO 20 I=K,N-1
426      AF=AD(I+1,K)/AD(K,K)
427      II=I+1
428      DO 20 J=K,N
429      AD(II,J)=-AF*AD(K,J)+AD(II,J)
430      20 CONTINUE
431      D=1.
432      DO 60 I=1,N
433      60 D=D*AD(I,I)
434      DD=(-1)**NRA
435      D=DD*D
436      RETURN
437      END

```

APPENDIX B

GUST ALLEVIATION FACTOR

The strip theory calculations of lift in Chapter III are based on an instantaneous angle of attack in a steady-state condition. While this is approximately true for a gradually applied disturbance, it overestimates the loads for a sudden disturbance such as that due to a sharp-edged gust. The change of lift is gradual (transient) until steady-state value is reached. The response depends upon the characteristics of the gust as well as the aircraft.

For a gradual upgust as illustrated in Figure B.1, the normal acceleration response will be as shown in Figure B.2 (Babister 1980). The time (t_1) it takes the airplane to fly through the transition (s_1) is related to the airspeed V as follows:

$$s_1 = Vt_1 \quad (B.1)$$

The vertical acceleration increases linearly up to time t_1 and then falls off exponentially. A gust alleviation factor K is then defined as the ratio of the maximum vertical acceleration experienced due to a given gust to the theoretical maximum vertical acceleration in a sharp-edged gust with steady-state conditions. As expected, the gust alleviation factor decreases as the transition region increases. Assuming steady-state conditions, the gust factor becomes unity as s_1 goes to zero. Furthermore, as the transition region becomes large, the unsteady flow effects get smaller. Figure B.3 which represent a plot of the factor K reproduced from Bisplinghoff (1951). Variable H is the normalized gust transition region defined as:

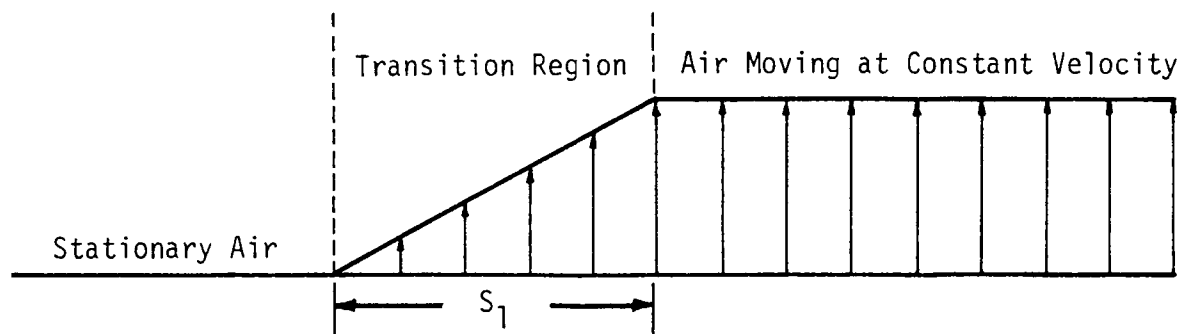


Figure B.1. A gradual gust.

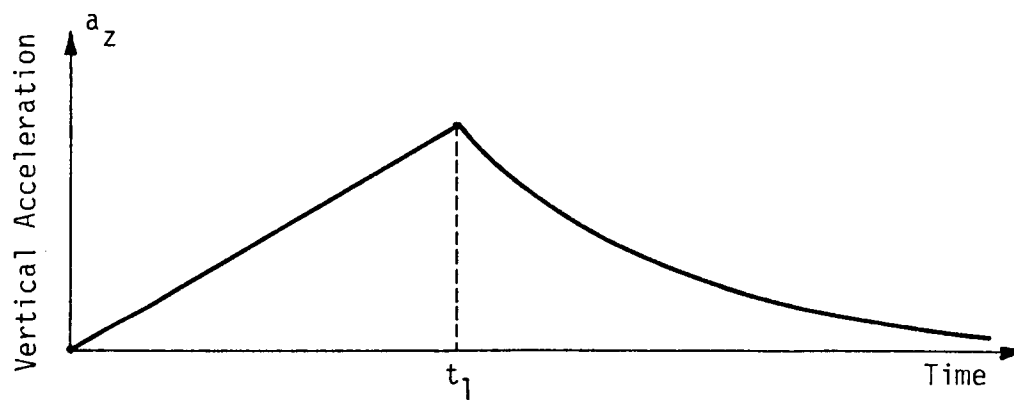


Figure B.2. Vertical acceleration due to a gradual gust.

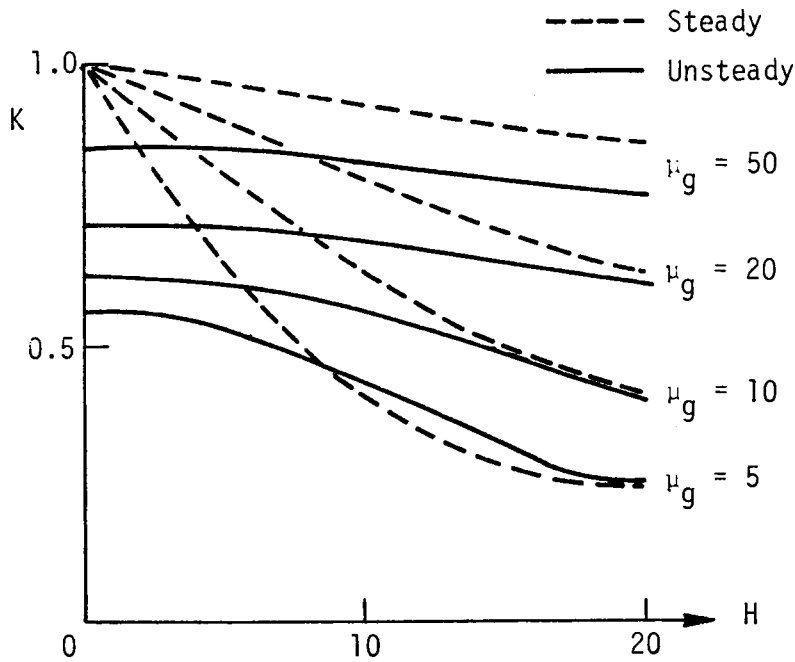


Figure B.3. Gust alleviation factor.

$$H = s_1 / \bar{c} \quad (B.2)$$

and μ_g is a mass parameter given by:

$$\mu_g = \frac{m}{\frac{1}{2} S a_\infty \bar{c}} \quad (B.3)$$

The plot illustrates the importance of the unsteady effects for sharp-edged gusts.

The mass parameter indicates that for an aircraft of low mass, the alleviation factor or acceleration ratio gets small as the transition region increases. For a high mass aircraft, the acceleration ratio is less dependent upon the length of the transition region. However, the maximum acceleration itself is inversely proportional to the mass parameter and is given by:

$$a_{z_{\max}} = \frac{V \omega_z}{\mu_g c} \quad (\text{B.4})$$

where ω_z is the maximum vertical gust velocity.

For the B-57, the following values are obtained under the flight conditions in question:

$$\mu_g = 24.6$$

$$H = 10.6$$

$$K = 0.7$$

This value for the gust gradient coefficient is applied to the lift calculations in the following way. The difference between lift and weight is computed at each time step, and this difference is multiplied with K . That fraction is added back to the weight to give the corrected lift. The procedure is illustrated in Figure B.4 for a lift curve which is artificially constructed.

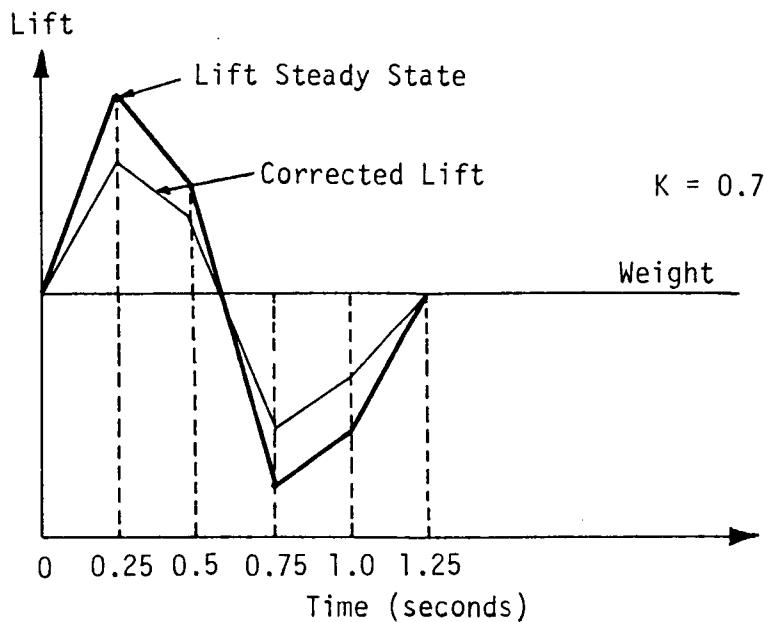


Figure B.4. Correction to lift using the gust alleviation factor K .

APPENDIX C

B-57 CHARACTERISTICS USED IN SIMULATION OF FLIGHT 60, RUN 18

1. TRIM CONDITIONS FOR FLIGHT 60, RUN 18

Trim airspeed	$v = 113.0 \text{ m/s}$
Air density	$\rho = 0.96186 \text{ kg/m}^3$
Tail incidence angle	$i_t = -0.22 \text{ degrees}$
Thrust force	$T = 36,509 \text{ Newtons}$
Aircraft mass	$W = 20,420 \text{ kg}$
Level flight, no flaps, gear up	

2. AIRPLANE PARAMETERS

Wing area	$S = 89.2 \text{ m}^2$
Wing span	$b = 19.5 \text{ m}$
Mean aerodynamic chord	$c_{mac} = 4.57 \text{ m}$
Aspect ratio	$AR = 4.3$
Moments of inertia	$I_{xx} = 325,300 \text{ kg} \cdot \text{m}^2$
	$I_{yy} = 271,100 \text{ kg} \cdot \text{m}^2$
	$I_{zz} = 650,600 \text{ kg} \cdot \text{m}^2$
	$I_{zx} = 11,000 \text{ kg} \cdot \text{m}^2$

3. AERODYNAMIC COEFFICIENTS AND STABILITY DERIVATIVES

$$C_{L_0} = 0.07$$

$$C_{L_\alpha} = 4.98/\text{rad}$$

$$C_{L_{\delta_e}} = 0.419/\text{rad}$$

$$C_{L_{i_t}} = 0.670/\text{rad}$$

$$C_{L_q} = 2.90/\text{rad}$$

$$C_{L_{\dot{\alpha}}} = 1.45/\text{rad}$$

$$C_{D_0} = C_L^2/13.5$$

$$C_{m_0} = 0.064$$

$$C_{m_x} = -1.0/\text{rad}$$

$$C_{m_{\delta_e}} = -0.860/\text{rad}$$

$$C_{m_{i_t}} = -1.72/\text{rad}$$

$$C_{m_q} = -9.65/\text{rad}$$

$$C_{m_{\dot{\alpha}}} = -5.83/\text{rad}$$

$$C_{y_\beta} = -0.286/\text{rad}$$

$$C_{y_{\delta_r}} = -0.071/\text{rad}$$

$$C_{n_{\beta}} = 0.0573/\text{rad}$$

$$C_{n_{\delta_r}} = -0.0321/\text{rad}$$

$$C_{n_{\delta_a}} = -0.00286/\text{rad}$$

$$C_{n_p} = -0.070/\text{rad}$$

$$C_{n_r} = -0.200/\text{rad}$$

$$C_{\ell_{\beta}} = -0.0286/\text{rad}$$

$$C_{\ell_{\delta_r}} = 0.0040/\text{rad}$$

$$C_{\ell_{\delta_a}} = 0.0458/\text{rad}$$

$$C_{\ell_p} = -0.5/\text{rad}$$

$$C_{\ell_r} = 0.081/\text{rad}$$

APPENDIX D

CONTROL LAWS

The six-degrees-of-freedom simulation uses the following automatic control laws:

$$\delta_e = DE1(V_z - V_{e_{ref}}) + DE2(\theta - \theta_{trim}) \quad (D.1)$$

$$\delta_a = DA1(V_y - V_{a_{ref}}) + DA2 \cdot \phi \quad (D.2)$$

$$\delta_r = DR1(V_y - V_{r_{ref}}) + DR2 \cdot \psi \quad (D.3)$$

where:

$$DE1 = -0.0025$$

$$V_{e_{ref}} = 2.0 \text{ m/s}$$

$$DE2 = 0.06$$

$$\theta_{trim} = 1.203 \text{ degrees}$$

and

$$DA1 = -0.005$$

$$V_{a_{ref}} = -0.08 V_y$$

$$DA2 = -0.80$$

and

$$DR1 = 0.015$$

$$V_{r_{ref}} = -0.12 V_y$$

$$DR2 = 0.50$$

APPENDIX E

FLIGHT INFORMATION

1. Flight 6, Run 21

Date: July 14, 1982

Location: Denver, Colorado

Project: Joint Weather Airport Studies (JAWS)

Start time: 14:41:58 MDT

Duration: 41.5 seconds

Altitude above ground: 1000 feet

Mean airspeed: 96 m/s

Flight description: Level flight

2. Flight 60, Run 18

Date: February 1, 1984

Location: Boulder, Colorado

Project: Orographic Effects Campaign

Start time: 16:57:15 MST

Duration: 140 seconds

Altitude above ground: 1500 feet

Mean airspeed: 113 m/s

Flight description: Level flight

VITA

Erik A. Ringnes was born in Oslo, Norway, on August 1, 1959. He attended elementary school in Oslo. In June 1978 he graduated from Norwegian high school, Eikeli Videregående Skole, in the same city. From July 1978 to August 1980 he served in the Royal Norwegian Army. The first year he attended under-officer school and was promoted to sergeant in June 1979. Upon release from active duty he was commissioned second Lieutenant. He entered The University of Tennessee, Knoxville in September 1980 and graduated with a Bachelor of Science degree in Aerospace Engineering in June 1983. He started his graduate studies at The University of Tennessee Space Institute, Tullahoma, in September 1983. He received his Master of Science degree in Aerospace Engineering in December 1984.