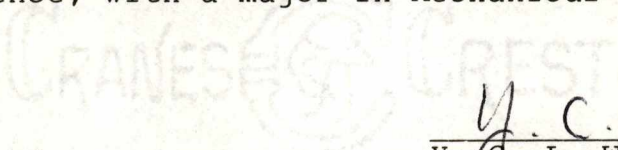


To the Graduate Council:

I am submitting herewith a thesis written by Jyh-Ren Dee entitled "The Application of an Integral Method to a Slagging Magnetohydrodynamic Channel Flow." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.


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THE APPLICATION OF AN INTEGRAL METHOD TO A SLAGGING
MAGNETOHYDRODYNAMIC CHANNEL FLOW

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Jyh-Ren Dee
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ABSTRACT

An integral method analysis for the performance of a magnetohydrodynamic (MHD) channel is developed based on a coupled core flow-boundary layer approach. A model for the thermal behavior of the slag layer coated on the channel internal surfaces of a coal-fired MHD generator, and a model for the current leakage phenomena are incorporated into the analysis.

Comparisons between The University of Tennessee Space Institute (UTSI) Energy Conversion Facility (ECF) generator TP40A-17 test results and the present calculations are discussed. The present method also compares to the multidimensional model calculations with good agreement. With these verifications, the integral boundary layer method is found to be well suited for the entire channel analysis in an engineering development application. On the other hand, the integral boundary layer analysis has greatly relaxed some empiricism from the one-dimensional calculations. The present method is found to be adequate for the weak-to-moderate MHD interactions of MHD generators.

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LIST OF SYMBOLS

A	Cross-sectional area
A_{ce}	Effective core area
a, b, c	Coefficients of temperature profile in slag layer
$\vec{B}$	Magnetic field intensity vector
C_1-C_6	Coefficients
C_f	Skin friction coefficient
$\vec{E}$	Electric field intensity vector
H	Channel height
h	Enthalpy
I_L	Load current
I_g	Generator current
I_ℓ	Leakage current
$\vec{j}$	Current density vector
K_s	Roughness height
$\dot{m}$	Mass flow rate
Pr	Prandtl number
P	Pressure
$\dot{q}$	Heat flux
R_g	Electrode resistance
St	Stanton number
$\dot{S}(x)$	Slag deposition rate
T	Temperature
t	Slag layer thickness

u	Axial velocity
$\vec{V}$	Velocity vector
W	Channel width
x, y, z	Coordinates

Greek Symbols

β_0	Hall parameter
κ	Thermal conductivity
Δ	Dimensionless voltage drop parameter
δ	Boundary layer thickness
δ^*	Displacement thickness
θ	Momentum thickness; field angle
μ	Viscosity coefficient
ρ	Density; pitch of electrode
σ	Electrical conductivity
τ	Shear stress
ϕ	Enthalpy thickness

Subscripts

c	Core value
o	Stagnation value
s	Slag layer value
w	Wall value

CHAPTER I

INTRODUCTION

Coal-fired, open-cycle magnetohydrodynamic (MHD) power generation is important because of its potential for producing significantly higher efficiency and lower pollution at low cost for central-power station applications. The Department of Energy (DOE) has set up a national program for the development of commercial MHD power plants [1].¹ A schematic diagram of such a plant is shown in Fig. 1.1.

In the combustor, pulverized coal is burned in preheated air with optional oxygen enrichment. The combustion product is seeded with an easily ionizable material, such as a potassium compound, to increase its electrical conductivity. A combustion temperature of approximately 2,900^oK is necessary to produce sufficient electrical conductivity for efficient energy conversion. This combustion gas is accelerated through a nozzle and enters the MHD channel which is surrounded by a magnet. As the ionized gas passes through the channel and cuts the magnetic field lines produced by the magnet, an

¹Numbers in brackets refer to similarly numbered references in the Bibliography.

electromotive force is induced. When a proper circuit is set up, current will flow in accordance with the Faraday laws of induction.

There are obvious advantages of this type of power generation over conventional plants; namely, the heat source temperature can be much higher because no moving parts are necessary and, therefore, higher inlet temperature and hence, higher thermal efficiency is attainable. After passing through the MHD power train, the gas is still hot enough to drive a conventional power plant to produce additional electricity.

Since the channel is the principal part of the MHD power train, numerous channel studies and computer codes have been developed to analyze the gasdynamic and the electrical characteristics of the channel flow. These include Doss et al. [2] and Maxwell et al. [3] who developed detailed three-dimensional codes based on the finite-difference scheme. Ishikawa and Wu [4] developed a three-dimensional code based on the finite-element method for the electrohydrodynamic problem with slagging effects; while Liu [5] developed a two-dimensional finite-difference code to calculate the coupled fluid and electrical phenomena for a slag layer coated channel. Multidimensional codes give detailed solutions to the problem, but the required computer time and core storage

are usually expensive. Furthermore, in many engineering applications, such detailed information is not required.

The relatively simple one-dimensional models, such as the one developed at The University of Tennessee Space Institute (UTSI) [6], provide fast calculations, but are generally inadequate for the complete analysis of the MHD channel. The lack of structure in the fluid parameter profiles, such as velocity, temperature and so forth, prevent the calculation of surface losses. Consequently, these losses (skin friction, heat flux and voltage drop) must be obtained either experimentally or empirically.

To alleviate the excessive empiricism of the one-dimensional approach, the integral method has been adopted for MHD channel analysis. Wilson and Williams [7] developed a computer code based on this method which considers the effect of electrical shorting in boundary layers; Gertz et al. [8] proposed special modeling of the velocity and enthalpy profiles which are crucial at higher MHD interactions.

The fundamental advantage of the integral method is that the problem can be modeled with a set of Ordinary Differential Equations (O.D.E.), rather than the Partial Differential Equations (P.D.E.). Therefore, significant

reduction in computer core storage and run time can be achieved in comparison with the multidimensional calculations.

Furthermore, the cross-channel mesh division in the boundary layer can be taken much finer than the multidimensional approaches since it involves only integration. The finer mesh division gives the capability of modeling the high gradient boundary region phenomena, provided that reasonable profiles are assumed, and the integral method can provide a result reasonably close to that of a multidimensional calculation with much less computation.

In view of the foregoing facts, the purpose of the present study is to develop an analytical model based on the integral method which will predict the boundary layer growth and the overall thermal and gasdynamic properties, and to provide a realistic estimation of the channel electrical performance.

In contrast to the previously mentioned earlier integral analysis, the present study considers not only the integral boundary layer solution, but also a slag layer model which includes both the thermal and electrodynamic aspects. In addition, a current leakage model is also incorporated. The voltage drop is computed locally by integrating the electrical field intensity across the channel so that the dimensionless voltage drop parameter

used in the one-dimensional code is no longer a required input.

The slagging channel model is essential for the correct modeling of coal-fired generators. The code presented here will be verified by using the experimental results obtained at UTSI and by comparison with the multi-dimensional calculations of Lineberry et al. [9].

CHAPTER II

THEORETICAL FORMULATION

In this study, the channel flow field, shown schematically in Fig. 2.1, is simulated by the development of a coupled core flow-boundary layer-slag layer model. Adjacent to the internal walls of the channel is a layer of slag, which is treated from the conservation of energy point of view. A thermal balance of the slag layer is discussed in Section 2.3. The gas flow outside the slag layer is treated as two parts: the core flow region and the boundary layer region. The flow in the central core is assumed to be one-dimensional; that is, there are no transverse variations in any of the fluid properties. This allows the conventional quasi-one-dimensional gasdynamic equations to be utilized to predict the behavior of the central core. The development of the boundary layers along the channel walls is calculated by solving the integral momentum and energy equations, with appropriate Lorentz force and Joule heating terms included to account for the magnetohydrodynamic (MHD) effects.

The integral equations for the boundary layer are solved simultaneously with the one-dimensional core equations, along with the necessary corrections to reduce the effective core area by the deduction of the area of

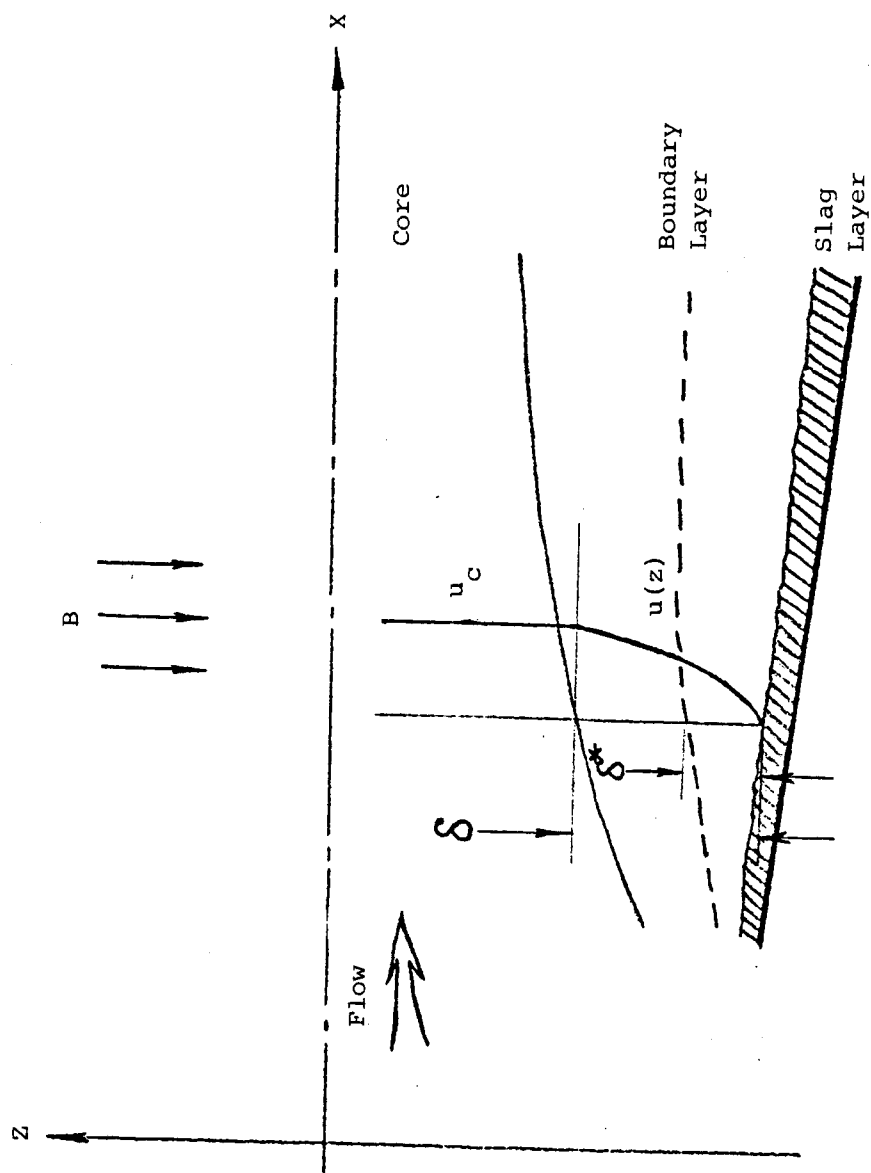


Figure 2.1. MHD channel flow field.

displacement boundary layer thickness and slag layer thickness. Infinite segmentation is assumed in treating the electrical parameters. Equilibrium chemical reactions are considered for the working fluid while real gas and slag properties are used.

2.1. Development of the Core Flow Equations

The core flow can be approximated by a quasi-one-dimensional, steady, inviscid compressible flow. The following equations are employed for the analysis:

Continuity:

$$\rho_c u_c A_{ce} = \dot{m} , \quad (1)$$

Momentum:

$$\rho_c u_c \frac{du_c}{dx} = - \frac{dP_c}{dx} - j_{yc} B_z , \quad (2)$$

Energy:

$$\rho_c u_c \left[\frac{dh_c}{dx} + \frac{d}{dx} \frac{u_c^2}{2} \right] = \vec{j}_c \cdot \vec{E}_c , \quad (3)$$

Equation of State:

$$P_c = \rho_c \frac{R_o}{W_c} T_c , \quad (4)$$

where R_o is the universal gas constant and W_c is the molecular weight.

The subscript, c, in the above equations indicates core values. Dependent variables chosen are: pressure, P_c , and temperature, T_c . All other variables then become explicit functions of pressure and temperature. The thermodynamic properties such as the enthalpy, h , are calculated using the National Aeronautics and Space Administration (NASA) SP-273 chemical equilibrium computer code [10] for a given set of pressure and temperature values to generate a thermal data table. The present computer code reads this thermal table and applies the spline curve-fitting method [11] to perform the thermodynamic calculations. The electrical properties, such as the electrical field intensity, $\vec{E}_c$, and the current density, $\vec{j}_c$, shall be treated in Section 2.6.

By choosing P_c and T_c as the dependent variables, one obtains from Eqs. (1) through (4) the following set of equations:

$$\frac{dP_c}{dx} = \frac{(C4 \cdot C5 - C2 \cdot C6)}{(C1 \cdot C4 - C2 \cdot C3)} , \quad (5)$$

$$\frac{dT_c}{dx} = \frac{(C1 \cdot C6 - C3 \cdot C5)}{(C1 \cdot C4 - C2 \cdot C3)} , \quad (6)$$

where,

$$C1 = \rho_c \left[\frac{u_c^2}{Z_c} \left(\left. \frac{\partial Z}{\partial P} \right|_c - \frac{Z_c}{P_c} \right) \right] + 1 , \quad (7)$$

$$C2 = \rho_c \frac{u_c^2}{Z_c} \left(\left. \frac{\partial Z}{\partial T} \right|_c + \frac{Z_c}{T_c} \right) , \quad (8)$$

$$C3 = \rho_c u_c \left[\frac{u_c^2}{Z_c} \left(\left. \frac{\partial Z}{\partial P} \right|_c - \frac{Z_c}{P_c} \right) + \frac{\partial h}{\partial P} \right] , \quad (9)$$

$$C4 = \rho_c u_c \left[\frac{u_c^2}{Z_c} \left(\left. \frac{\partial Z}{\partial T} \right|_c + \frac{Z_c}{T_c} \right) + \frac{\partial h}{\partial T} \right] , \quad (10)$$

$$C5 = \frac{\rho_c u_c^2}{A_{ce}} \frac{dA_{ce}}{dx} - j_{yc} B_z , \quad (11)$$

$$C6 = \frac{\rho_c u_c^3}{A_{ce}} \frac{dA_{ce}}{dx} + \vec{j}_c \cdot \vec{E}_c , \quad (12)$$

where $Z_c = R_o/W_c$. These equations are well suited for numerical integration.

2.2. Development of the Integral Boundary Layer Equations

The derivation of the integral boundary layer momentum and energy equations is achieved by integrating the differential momentum and energy equations. The basic differential equations for the side wall boundary layers (Z-direction) are:

Continuity:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho w)}{\partial z} = 0 , \quad (13)$$

Momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho w \frac{\partial u}{\partial z} = -\frac{\partial P}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} - j_y B_z , \quad (14)$$

Energy:

$$\begin{aligned} \rho u \frac{\partial h_0}{\partial x} + \rho w \frac{\partial h_0}{\partial z} = & \frac{-\partial \dot{q}}{\partial z} + \frac{\partial}{\partial z} (u \tau_{xz}) \\ & + j_x E_x + j_y E_y , \end{aligned} \quad (15)$$

where $h_0 = h + u^2/2 + w^2/2$ is the total enthalpy. A similar set of equations exists for the electrode wall boundary layers (Y-direction).

2.2.1. Integral Momentum Equations

Integrating Eq. (14) with respect to Z and with limits of $Z = 0$ to $Z = \delta$ yields:

$$\begin{aligned} \int_0^\delta \rho u \frac{\partial u}{\partial x} dz + \int_0^\delta \rho w \frac{\partial u}{\partial z} dz = & - \int_0^\delta \frac{\partial P}{\partial x} dz + \int_0^\delta \frac{\partial \tau_{xz}}{\partial z} dz \\ & - \int_0^\delta j_y B_z dz . \end{aligned} \quad (16)$$

In classical boundary layer theory, pressure, which is a function of axial location (X-direction) only

in the boundary layer, provides the coupling with the core flow. However, with the presence of Lorentz force, this assumption is true only when the transverse body force is small. In the present study, the Lorentz force contribution in the Y-direction is considered to be small and is neglected in the analysis. This assumption is acceptable for a small interaction MHD channel. The boundary layer developments are identical on cathode and anode surfaces due to this assumption. In the Z-direction, since $(\vec{j} \times \vec{B})_z$ is zero, one may employ the core flow momentum equation to obtain the matching condition for the pressure gradient:

$$\frac{-\partial P}{\partial x} = -\frac{dP_c}{dx} = \rho_c u_c \frac{du_c}{dx} + j_{yc} B_z . \quad (17)$$

Thus, Eq. (16) becomes:

$$\begin{aligned} & \int_0^\delta \rho u \frac{\partial u}{\partial x} dz + \int_0^\delta \rho w \frac{\partial}{\partial z} (u - u_c) dz \\ &= \int_0^\delta \rho_c u_c \frac{du_c}{dx} dz + \int_0^\delta \frac{\partial \tau_{xz}}{\partial z} dz + \int_0^\delta (j_{yc} - j_y) B_z dz , \quad (18) \end{aligned}$$

where the following identity is used:

$$\int_0^{\delta} \rho u \frac{\partial u}{\partial x} dz + \int_0^{\delta} \rho w \frac{\partial u}{\partial z} dz = \int_0^{\delta} \rho u \frac{\partial u}{\partial x} dz + \int_0^{\delta} \rho w \frac{\partial}{\partial z} (u - u_c) dz , \quad (19)$$

since u_c is not a function of z .

By employing the continuity equation (Eq. (13)) and by the following definitions:

Displacement Thickness:

$$\delta^* = \int_0^{\delta} \left(1 - \frac{\rho u}{\rho_c u_c} \right) dz , \quad (20)$$

Momentum Thickness:

$$\theta = \int_0^{\delta} \left(1 - \frac{u}{u_c} \right) \frac{\rho u}{\rho_c u_c} dz , \quad (21)$$

Skin Friction Coefficient:

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho_c u_c^2} , \quad (22)$$

Current Density Displacement Thickness:

$$\delta_{JB}^* = \int_0^{\delta} \left(1 - \frac{j_y}{j_{yc}} \right) dz , \quad (23)$$

Eq. (18) can then be reduced to:

$$\frac{d\theta}{dx} + \theta \left[\frac{\frac{\delta^*}{\theta} + 2}{u_c} \frac{du_c}{dx} + \frac{1}{\rho_c} \frac{d\rho_c}{dx} \right] = \frac{C_f}{2} - \frac{j_{yc} B_z}{\rho_c u_c^2} \delta_{JB}^* . \quad (24)$$

This is the final form of the integral momentum equation and is similar to the classical von Karman integral momentum equation [12], except for an additional term on the RHS representing the MHD body force contribution to the momentum deficit thickness. The current density displacement thickness represents the deficit in the current density flux through the boundary layer compared to the current flow that would exist in core flow. As for the electrode wall boundary layers, current continuity gives $j_y = j_{yc}$ throughout the boundary layer, and the integral momentum equation for it simply reduces to:

$$\frac{d\theta}{dx} + \theta \left[\frac{\frac{\delta^*}{\theta} + 2}{u_c} \frac{du_c}{dx} + \frac{1}{\rho_c} \frac{d\rho_c}{dx} \right] = \frac{C_f}{2} . \quad (25)$$

To solve these equations, one needs to know the skin friction (discussed in Section 2.5) for an incompressible flow. In the case of compressible flow, the energy equation must be considered, which is discussed in the next section.

2.2.2. Integral Energy Equations

A formal integration of the energy equation gives:

$$\begin{aligned} & \int_0^\delta \rho u \frac{\partial h_o}{\partial x} dz + \int_0^\delta \rho w \frac{\partial h_o}{\partial z} dz \\ &= \int_0^\delta -\frac{\partial \dot{q}}{\partial z} dz + \int_0^\delta \frac{\partial}{\partial z} (u \tau_{xz}) dz + \int_0^\delta (j_x E_x + j_y E_y) dz . \quad (26) \end{aligned}$$

Applying the following relations:

Core Flow Energy Equation:

$$\rho_c u_c \frac{dh_{co}}{dx} = j_{xc} E_{xc} + j_{yc} E_{yc} , \quad (27)$$

Enthalpy Thickness Parameter:

$$\phi = \int_0^\delta \left[1 - \frac{h_o - h_{so}}{h_{co} - h_{so}} \right] \frac{\rho u}{\rho_c u_c} dz , \quad (28)$$

Stanton Number:

$$St = - \dot{q}_g / \left[\rho_c u_c (h_{co} - h_{so}) \right] , \quad (29)$$

Power Density Displacement Thickness:

$$\delta_{JE}^* = \int_0^\delta \left[1 - \frac{j_x E_x + j_y E_y}{j_{xc} E_{xc} + j_{yc} E_{yc}} \right] dz , \quad (30)$$

and following a similar development to that performed for the momentum equation, the following energy equation results:

$$\frac{d\phi}{dx} + \phi \left[\frac{1}{\rho_c} \frac{d\rho_c}{dx} + \frac{1}{u_c} \frac{du_c}{dx} + \frac{1}{h_{co} - h_{so}} \frac{d(h_{co} - h_{so})}{dx} \right], \quad (30)$$

$$= St + \frac{(j_{xc} E_{xc} + j_{yc} E_{yc})}{\rho_c u_c (h_{co} - h_{so})} (\delta_{JE}^* - \delta^*). \quad (31)$$

The subscript, s, denotes the slag-gas interface values. The relations for the Stanton number shall be discussed in Section 2.5.

2.3. Slag Layer Modeling

In a coal-fired MHD generator, a layer of slag coats the internal walls of the channel. The effects of the slag layer on the performance of the MHD generator have been identified as the following [13]:

1. The presence of the slag layer tends to reduce the heat losses and increase the MHD interaction.
2. The slag-gas interface temperature is the appropriate temperature seen by the gas instead of the wall temperature. This higher temperature decreases the conductivity nonuniformity

associated with the boundary layers, thereby reducing the boundary layer voltage drop.

3. Axial current leakage through the slag layer occurs between electrodes. This tends to lower the Hall voltage and reduces the power extraction.

In summary, the reduction in heat flux alleviates the nonuniformities, so that voltage drops are decreased. On the other hand, current leakage through the slag layer has a degrading effect on power production.

The governing equations for the slag layer (Fig. 2.2) used in this study are given as:

Continuity:

$$\frac{d}{dx} \int_0^t \rho u \, dy = \dot{S}(x) , \quad (32)$$

Momentum:

$$\frac{dP}{dx} = (\vec{j} \times \vec{B})_s + \frac{d}{dy} \left(\mu_s \frac{du}{dy} \right) , \quad (33)$$

Energy:

$$\frac{d}{dy} \left(\kappa \frac{dT}{dy} \right) = \frac{j^2}{\sigma_s} , \quad (34)$$

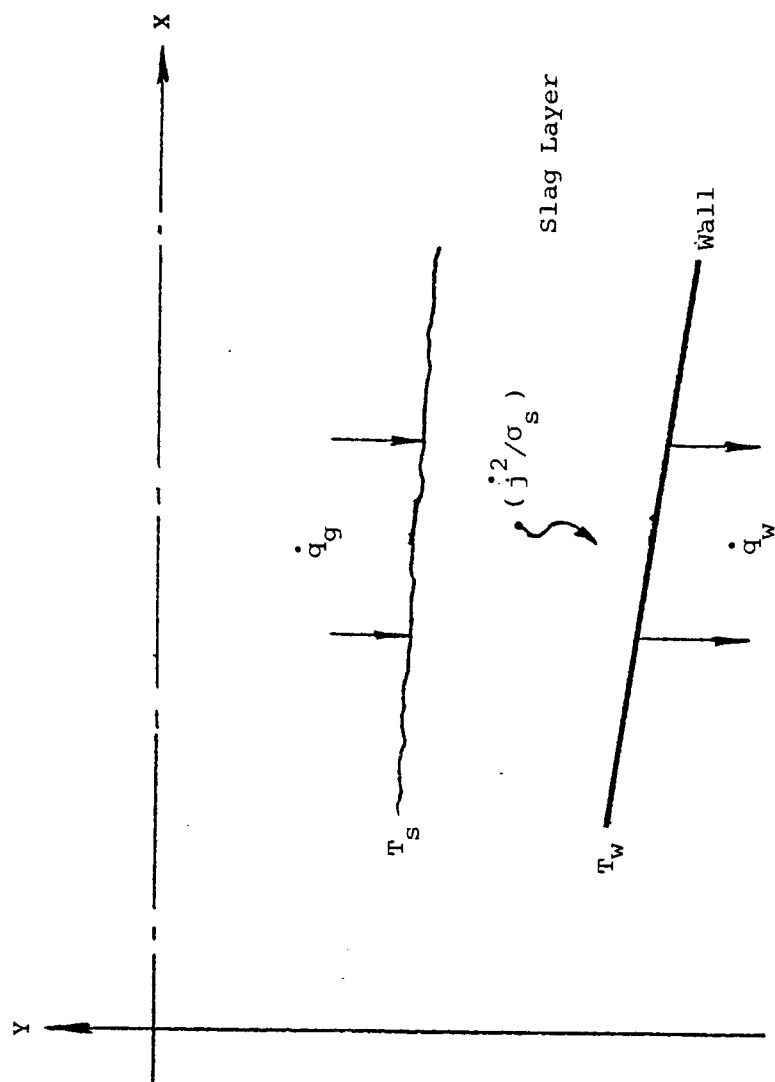


Figure 2.2. Slag layer model.

where

t = the slag layer thickness ,

$\dot{S}(x)$ = the local slag deposition rate ,

μ_s = the slag viscosity coefficient ,

κ = the slag thermal conductivity ,

σ_s = the slag electrical conductivity .

Convective terms are neglected since those terms are small within the slag layer. Furthermore, radiative heat transfer is not considered, since this simplification is reasonable for a small channel such as the Energy Conversion Facility (ECF) channel. It is not true, however, for larger channels.

Several models exist for the slag deposition rate in MHD channels; however, none has been verified by experiment. In the present study, where only overall behavior of the slag layer is of interest, an integral approach can be applied for the slag layer consistent with the plasma analysis. The presence of slag in qualitative terms is accounted for by its effects on the heat flux, which is more meaningful here. In this fashion, a mean slag layer thickness can be determined from thermal balance considerations.

Integrating the energy equation (Eq. (34)), one has:

$$T(y) = \int_0^y \left[\frac{1}{\kappa} \int_0^y \left(\frac{j^2}{\sigma_s} \right) dy \right] dy + C_1 y + C_2 . \quad (35)$$

Let

$$\left\langle \frac{j^2}{\sigma_s} \right\rangle = \frac{1}{t} \int_0^t \frac{j^2}{\sigma_s} dy .$$

The boundary conditions for the energy equation are:

$$T(0) = T_w \quad \text{on the wall ,} \quad (36)$$

$$T(t) = T_s \quad \text{on the slag-gas interface ,} \quad (37)$$

and the compatibility conditions are:

$$-\kappa \left. \frac{dT}{dy} \right|_t = \dot{q}_g \quad \text{on the slag-gas interface ,} \quad (38)$$

$$\begin{aligned} -\kappa \left. \frac{dT}{dy} \right|_0 &= \dot{q}_g + \left\langle \frac{j^2}{\sigma_s} \right\rangle t \\ &= \dot{q}_w \quad \text{on the wall ,} \end{aligned} \quad (39)$$

where T_s is the slag-gas interface temperature.

Since there are two arbitrary constants in Eq. (35), there are only two independent conditions in Eqs. (36) through (39), and the electrical conductivity is

a strong function of temperature, the energy equation is actually nonlinearly coupled. Hence, the integration of Eq. (35) must be performed in an iterative manner. However, in the present approach, the temperature profile is approximated by a second-order polynomial:

$$T(y) = a + by + cy^2, \quad (40)$$

which is assumed to satisfy conditions in Eqs. (36) through (38). The original nonlinear equation (Eq. (34)) is replaced by Eq. (39), which balances heat flux in a global sense. This approximation would be exact if $\langle j^2/\sigma_s \rangle$ were a constant. In this integral approach, nonlinear iteration is avoided.

The coefficients of temperature profile are determined as:

$$a = T_w = T_s - \dot{q}_y \frac{t}{\kappa} - \langle \frac{j^2}{\sigma_s} \rangle \frac{t^2}{2\kappa}, \quad (41)$$

$$b = - \frac{\dot{q}_g}{\kappa} - \langle \frac{j^2}{\sigma_s} \rangle \frac{t}{\kappa}, \quad (42)$$

$$c = \langle \frac{j^2}{\sigma_s} \rangle / 2\kappa. \quad (43)$$

Solving for the slag layer thickness from Eq. (41), one has:

$$t = \left\{ \frac{4\dot{q}_g^2}{\langle \frac{j^2}{\sigma_s} \rangle} + \frac{\delta\kappa}{\langle \frac{j^2}{\sigma_s} \rangle} (T_s - T_w) \right\} - \frac{2\dot{q}_g}{\langle \frac{j^2}{\sigma_s} \rangle} . \quad (44)$$

For the case of $j^2/\sigma_s = 0$, the heat flux entering the slag layer, $\dot{q}_g$, must be equal to that leaving it, $\dot{q}_w$, so that:

$$a = T_w = T_s - \dot{q}_g \frac{t}{\kappa} , \quad (45)$$

and

$$t = (T_s - T_w) \frac{\kappa}{\dot{q}_g} . \quad (46)$$

From Eqs. (44) and (46), one expects that for a given T_s and $\dot{q}_g$, the higher wall temperature yields a lower slag layer thickness. On the other hand, for a given T_s and T_w , t increases with decreasing $\dot{q}_g$. In an MHD channel, $\dot{q}_g$ decreases along the channel; hence, the slag layer thickness increases accordingly.

Equations (44) and (46) could be applied for computing the slag thermal behavior from different points of view: If the slag-gas interface temperature and the slag layer thickness as functions of channel axial location are known, one can compute the wall temperature necessary to satisfy the energy equation. By this method, a solution can be obtained as one which yields wall temperatures

which agree with experimental data. Similarly, if the slag layer thickness and wall temperature are specified at experimental levels, the solution of the energy equation is used to predict the slag-gas interface temperature. The predicted temperatures are reasonable if they are within the range of the slag melting temperatures. One can also specify the wall temperature and the slag-gas interface temperature and then calculate the slag layer thickness.

The latter approach is adopted in this study. The comparison of the calculated result with the experimental slag layer thickness is good. Therefore, one can conclude that the simple slag layer thermal balance can represent adequately the slag thermal behavior.

A final note on the slag layer model is made for the Joule heating term, $\langle j^2/\sigma_s \rangle$, in Eq. (44). Since the Hall parameter and the velocity are zero in the slag layer, the Joule heating generated in the slag is:

$$\langle \frac{j^2}{\sigma_s} \rangle = \langle \sigma_s \rangle E_x^2 + j_y^2 \langle \frac{1}{\sigma_s} \rangle . \quad (47)$$

The slag thermal conductivity is fixed at a constant value of 1.75 w/m-°K, which is consistent with past studies of the coal used and of ECF generator slag properties [14]. The slag electrical conductivity is

quite different from that in the gas region, and the following expressions [14] are adopted:

for $T < 1,341^{\circ}\text{K}$,

$$\sigma_s = 8.44 e^{(-4,419/T)} \quad (48)$$

for $T \geq 1,341^{\circ}\text{K}$,

$$\sigma_s = 7.7 \times 10^5 e^{(-19,736/T)} (\text{ohm-m})^{-1} . \quad (49)$$

2.4. Coupling of the Core Flow, the Boundary Layer and the Slag Layer

The appropriate coupling for mass conservation is achieved by defining an effective core area, A_{ce} , shown in Fig. 2.3:

$$A_{ce} = (W - 2\delta_z^* - 2t) (H - 2\delta_y^* - 2t) , \quad (50)$$

where

W = width of the channel ,

H = height of the channel ,

δ_z^* = displacement thickness on sidewalls ,

δ_y^* = displacement thickness on electrode walls ,

t = slag layer thickness .

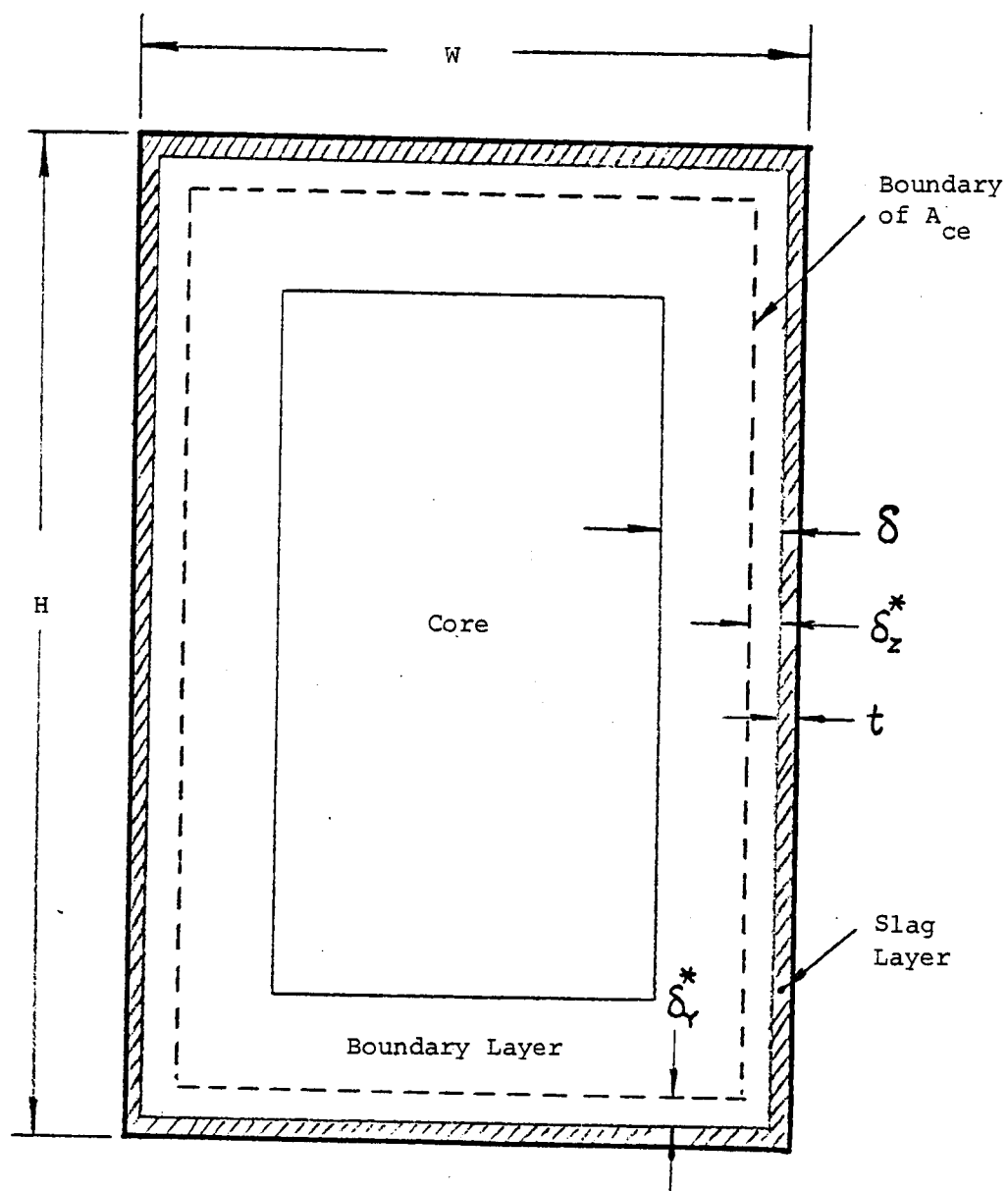


Figure 2.3. Effective core area, A_{ce} .

In this study, the channel area distribution is specified, and an iterative solution for the core flow and the boundary layer equations is required. The iteration process starts by first solving the core flow equations; then the appropriate core flow gradients can be calculated. By integrating the boundary layer equations, one obtains the growth of the boundary layer. Then, by solving the slag energy equation, one has the growth of the slag layer. The displacement thickness, the slag layer thickness and the effective core area can then be computed according to Eq. (50), which allows a reestimation of the core flow solutions based on the new effective core area and dA_{ce}/dx . The process is continued until satisfactory convergence on local pressure as well as mass flow rate are achieved.

2.5. Skin Friction and Heat Transfer Coefficients

Appropriate relations for the skin friction coefficient, C_f ; and heat transfer coefficient, the Stanton number, S_t , are needed to integrate the boundary layer equations (Eqs. (24) and (31)). At the present time, there are no experimental data for these coefficients for a turbulent compressible MHD flow. The integral method generally requires the specification of functional forms for wall heat flux and wall friction. However, it is not only difficult to prescribe empirically the wall heat

flux and skin friction (particularly for rough surfaces and in the presence of pressure gradients) as a function of all the relevant variables that influence them, but also, the heat transfer and skin friction are modified by the MHD effects.

In this study, a formula suggested by Ahluwalia and Doss [15] for the skin friction coefficient is adopted:

$$C_f = 0.0068 \left(\frac{\theta}{K_s} \right)^{-0.25} \quad 0.8 \leq \frac{\theta}{K_s} \leq 8, \quad (51)$$

where θ is the momentum thickness, and K_s is the wall roughness height. In the presence of slag layer, K_s is taken as the slag-gas interface roughness height instead of the wall roughness. The effects of the pressure gradient and MHD terms on skin friction are incorporated in the above relation since the C_f is expressed in terms of the local descriptions of the boundary layer behavior; that is, the momentum thickness.

Following Wilson's approach [7], the Stanton number is expressed by the following:

$$St = \frac{1}{2} C_f / \left[1 + 5 \sqrt{\frac{C_f}{2}} (Pr - 1) + \ln \left(\frac{5 Pr + 1}{6} \right) \right], \quad (52)$$

where Pr is the Prandtl number.

2.6. Electromagnetic Field Relations and Current Leakage

The electrical equations considered here are based on the generalized Ohm's law [16]; that is:

$$\vec{j} = \sigma (\vec{E} + \vec{V} \times \vec{B}) - \frac{\beta_e}{|\vec{B}|} (\vec{j} \times \vec{B}) . \quad (53)$$

Expanding this relation in the flow direction, X, and in the transverse direction, Y, with the magnetic field in the negative Z-direction, that is, $\vec{B} = -B \times \vec{k}$, leads to:

$$E_x = \frac{j_x}{\sigma} - \frac{\beta_e}{\sigma} j_y , \quad (54)$$

$$E_y + uB_z = \frac{\beta_e}{\sigma} j_x + \frac{j_y}{\sigma} . \quad (55)$$

The application of the infinitely fine segmentation approximation implies that the variation of the fields $\vec{E}$ and $\vec{j}$ will be small in the flow direction; that is:

$$\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y} .$$

This approximation is satisfied at small segmentation ratios which characterize MHD generator channels. With this assumption, Maxwell's equations reduce to:

$$\frac{\partial E_x}{\partial y} = 0 \quad \frac{\partial j_y}{\partial y} = 0 .$$

These relations simply relate that E_x and j_y are not functions of Y . Rearranging Eqs. (54) and (55), one has:

$$E_y(Y) = \beta_e E_x + \left(\frac{\beta_e^2 + 1}{\sigma} \right) j_y - u B_z \quad (56)$$

and

$$j_x(Y) = \sigma E_x + \beta_e j_y . \quad (57)$$

The configuration of the channel considered in this study is of the Diagonal Conducting Wall (DCW) type, as illustrated in Fig. 2.4. The condition for this type of generator is:

$$\frac{\langle E_y \rangle}{E_x} = \tan \theta = \phi . \quad (58)$$

where $\langle \rangle$ denotes the averages over the Y -direction. The generator current in the DCW generator is:

$$I_g = (\langle j_x \rangle + \phi j_y) A , \quad (59)$$

where A is the channel cross-sectional area. The average of Eqs. (56) and (57) becomes:

$$\langle E_y \rangle = \langle \beta_e \rangle E_x + \left\langle \frac{\beta_e^2 + 1}{\sigma} \right\rangle j_y - \langle u \rangle B_z$$

$$\langle j_x \rangle = \langle \sigma \rangle E_x + \langle \beta_e \rangle j_y .$$

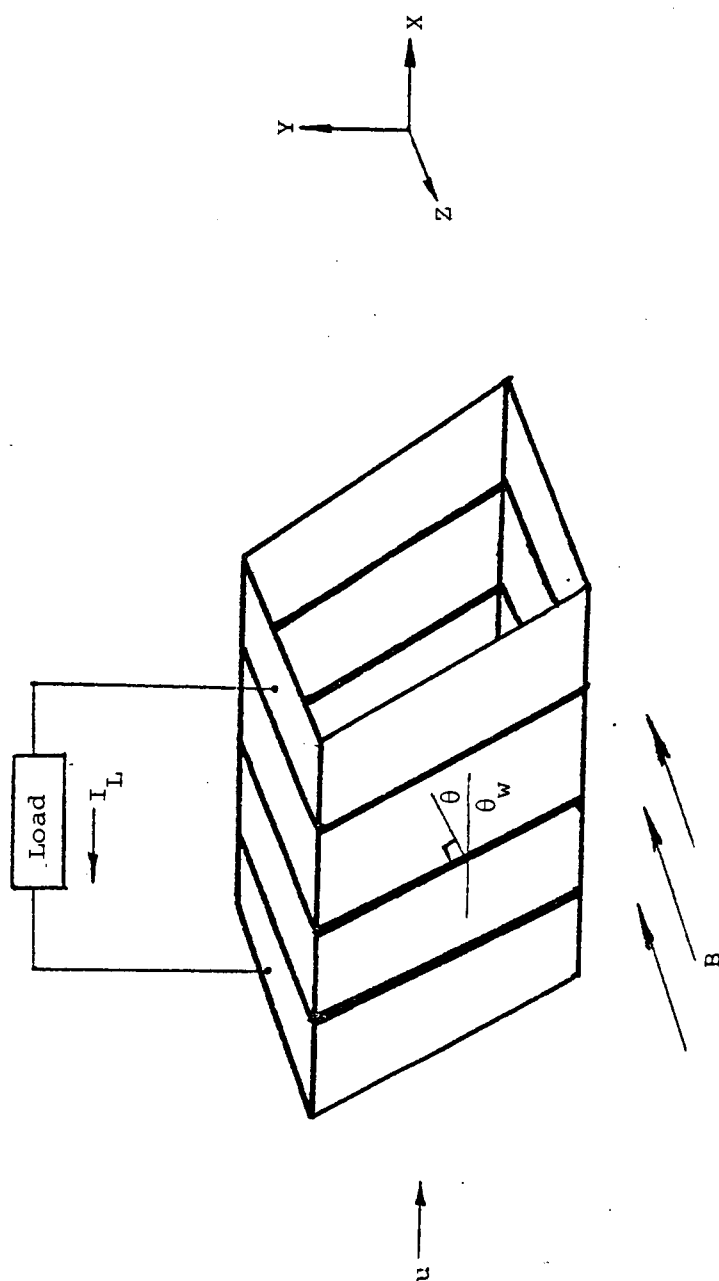


Figure 2.4. Schematic of DCW generator.

Applying Eqs. (58) and (59) to the averages and solving for E_x , j_y , one obtains the following set of equations:

$$E_x = \frac{\left\langle \frac{\beta_e^2 + 1}{\sigma} \right\rangle \frac{I_g}{A} - (\langle \beta_e \rangle + \phi) B_z \langle u \rangle}{\langle \sigma \rangle \left\langle \frac{\beta_e^2 + 1}{\sigma} \right\rangle + \phi^2 - \langle \beta_e \rangle^2}, \quad (60)$$

$$j_y = \frac{(\langle \beta_e \rangle - \phi) E_x - B_z \langle u \rangle}{\left\langle \frac{\beta_e^2 + 1}{\sigma} \right\rangle}, \quad (61)$$

where $E_y(Y)$ and $j_x(Y)$ are determined from Eqs. (56) and (57). In the core region, E_y and j_x are constant. However, in the boundary layer region, velocity, temperature, electrical conductivity and Hall parameter are functions of Y ; thus, E_y and j_x are not constant. In the slag layer region, since velocity and Hall parameter are practically zero, Eqs. (56) and (57) reduce to:

$$E_{ys}(Y) = \frac{1}{\sigma_s} j_y,$$

$$j_{xs}(Y) = \sigma_s E_x.$$

The derivations given above provide the fundamental relations for the electrical parameters in each region.

At low electrode temperature and current densities of interest to MHD generators, the electrical current

transport occurs in an arc mode [17]. The uniform current density induced in the core region is transported through a nonuniform velocity and temperature boundary layer. Because of the decrease in electrical conductivity, it tends to constrict and arc [18]. In the presence of a slag layer, the arc can either start forming in the gaseous boundary layer and punch through the slag layer, or current transport may be uniform until it reaches the slag layer surface and then constrict across the slag towards the electrode. In the multidimensional calculations, a cutoff in the electric field is often used near the cold wall to simulate the voltage drop of arc emission. An alternative approach is to use an artificial cut-off temperature.

In this study, an arcing temperature of $2,000^{\circ}\text{K}$ is assumed and the electrical conductivity is frozen on that cut-off temperature. This temperature corresponds to an electric field intensity on the order of $30,000 \text{ V/m}$, which is a reasonable value for arc emission to occur. This temperature is also used in the multidimensional calculations discussed in Chapter III. It is obvious that the value of this arcing temperature will affect the results. Nevertheless, in the present investigation, a consistent value is more important for the purpose of comparisons.

Another phenomenon of the current transportation investigated in this study is the leakage current through

the channel frame insulators and slag layers. By current conservation, the current in the generator must be equal to the sum of the load current, I_L , and the leakage current, I_ℓ :

$$I_g = I_L + I_\ell . \quad (62)$$

Furthermore, the leakage current can be written as:

$$I_\ell = \frac{-E_x \rho}{R_g} , \quad (63)$$

where R_g is the effective resistance between adjacent electrodes, and ρ is the pitch of an electrode. These two values can be determined by direct measurements.

To calculate the electrical performance, one needs to know the value of generator current, I_g , of Eq. (60); however, only the load current can be measured experimentally. Thus, one needs to rewrite Eq. (60) in terms of I_L .

Applying Eq. (62), one has:

$$E_x = \frac{\langle \frac{\beta_e^2 + 1}{\sigma} \rangle \frac{I_L}{A} - (\langle \beta_e \rangle + \phi) B_z \langle u \rangle}{\langle \sigma \rangle \langle \frac{\beta_e^2 + 1}{\sigma} \rangle + \phi^2 - \langle \beta_e \rangle^2} + \frac{\langle \frac{\beta_e^2 + 1}{\sigma} \rangle \frac{I_\ell}{A}}{\langle \sigma \rangle \langle \frac{\beta_e^2 + 1}{\sigma} \rangle + \phi^2 - \langle \beta_e \rangle^2} , \quad (64)$$

and by Eq. (63), one finally obtains:

$$E_x = \frac{\left\{ \frac{\langle \frac{\beta_e^2+1}{\sigma} \rangle \frac{I_L}{A} - (\langle \beta_e \rangle + \phi) B_z \langle u \rangle}{\langle \sigma \rangle \langle \frac{\beta_e^2+1}{\sigma} \rangle + \phi^2 - \langle \beta_e \rangle^2} \right\}}{\left\{ 1 + \frac{\langle \frac{\beta_e^2+1}{\sigma} \rangle \frac{1}{A} \frac{\rho}{R_g}}{\langle \sigma \rangle \langle \frac{\beta_e^2+1}{\sigma} \rangle + \phi^2 - \langle \beta_e \rangle^2} \right\}} . \quad (65)$$

Equation (65) is the equation for electric field intensity in the X-direction, taking into account current leakage.

The rest of the electrical parameters, that is, E_y , j_x , and j_y , can be found from Eqs. (56), (57) and (61), respectively.

Figure 2.5 illustrates the voltage distribution in the generator channel and the definition of the voltage drop, V_d , which is calculated locally from the distribution of E_y :

$$V_d = E_{yc} \cdot H - \int_0^H E_y dy . \quad (66)$$

The dimensionless voltage drop parameter, Δ [6], for the one-dimensional analysis, is no longer needed in this integral method.

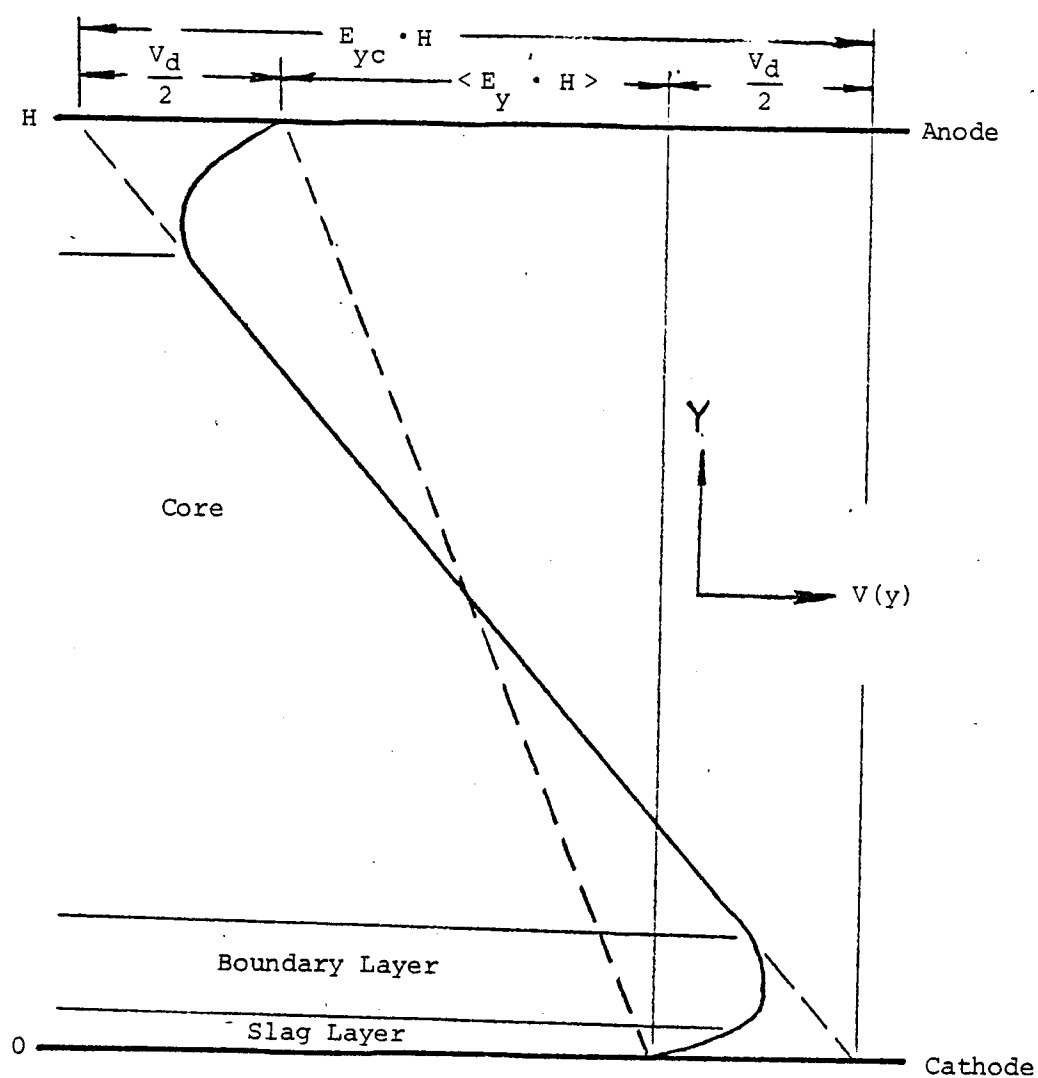


Figure 2.5. Schematic of voltage drop behavior.

CHAPTER III

VERIFICATION OF THE ANALYTICAL MODEL

The accuracy and adequacy of the analytical model are verified by comparing the computed results with experimental data obtained from The University of Tennessee Space Institute (UTSI) Energy Conversion Facility (ECF) generator and with other analytical models.

In this study, two finite-difference computer codes developed by Doss et al. [2, 19] are used for comparison with the integral method. These two codes also include means of approximated modeling of arcing, inter-electrode current leakage and slag layer phenomena. One of the codes is a two-dimensional finite difference scheme [19], while the other is a quasi-three-dimensional modeling [2]. In addition, a comparison is made with the one-dimensional code developed at UTSI which utilizes empirical parameters. All of these codes, as well as the integral method presented here, are compared with the UTSI TP40A-17 experimental data using a set of consistent input conditions.

3.1. Experimental Conditions

The UTSI TP40A-17 test [20], from which the data presented herein are taken, was a comprehensive study of both magnetohydrodynamic (MHD) generator and flow system hardware. The ECF generator was operated under coal-fired conditions, such that total slag carryover occurred through the entire test train. Nominal operating conditions for the combustor and channel are summarized in Table 3.1.

The ECF generator is a supersonic channel with 60-degree Diagonal Conducting Wall (DCW) design. The internal loft of the generator varies from 8 square inches at the channel inlet to 24 square inches at the entrance of the diffuser. The total length of the channel is 42 inches. The rectangular duct has a constant 2 inches width throughout, as dictated by the magnetic bore. Figure 3.1 is provided to show the magnetic field distribution and generator loft along with a sketch of the ECF generator.

The channel geometry has a high aspect ratio, since it is very narrow throughout its entire length. This results in high side wall surface losses and the merger of the side wall boundary layers.

The maximum magnetic field intensity is 2 tesla. This is a rather weak magnetic field, so that only a

Table 3.1. Average Operating Conditions for
ECF Generator TP40A-17 Test

Flow Rate (K_g/sec)	
Total	1.52
Coal ^a + Seed	0.43
Kerosene	0.04
O ₂	0.87
N ₂	0.17
Seed Content, percent weight	2.20
Stoichiometry	0.83
Thermal Input, Mw_t	13.70
Slag Carryover, percent	100.00
Combustor P_0 , atmosphere	4.80

^aThe coal contains 66 percent C (by weight); 5.2 percent H; 1.2 percent N; 4.4 percent S; 11.2 percent O; and 12 percent ash.

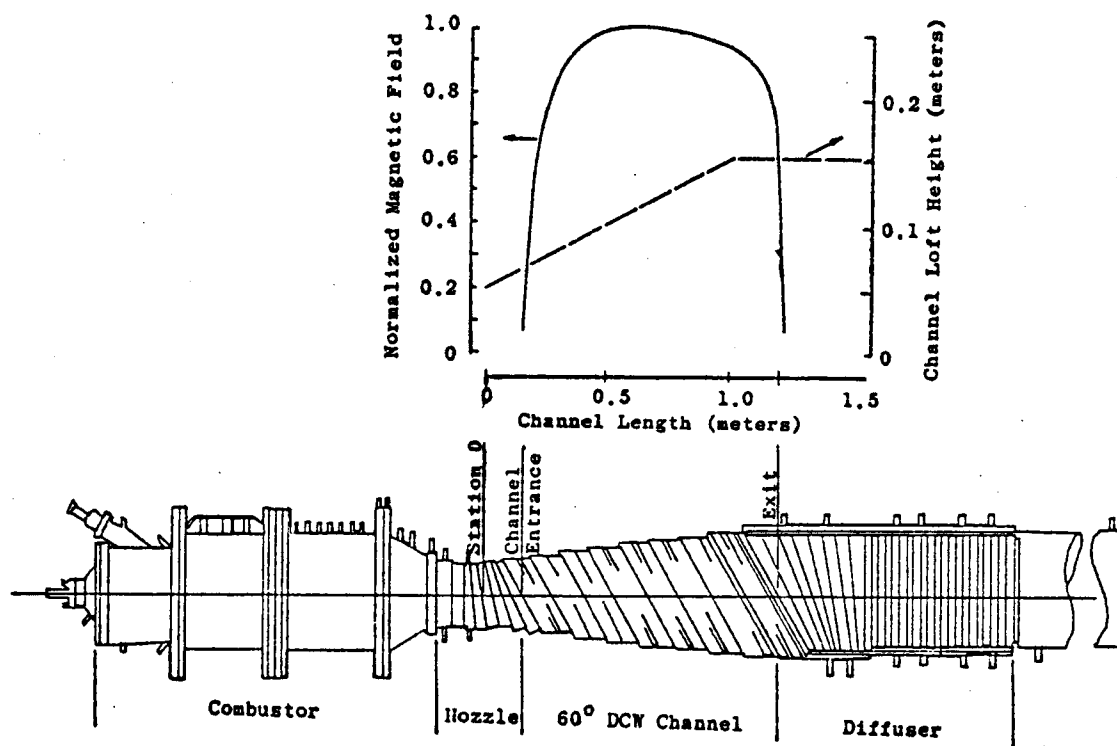


Figure 3.1. Physical characteristics of the ECF test train.

slight degree of MHD interaction is present. In large channels with high MHD interaction, it has been noted that the phenomena of side wall velocity overshoot can be present [21]. In the case of the ECF channel, the low MHD interaction along with the narrowness of the channel leads to aerodynamically well-behaved cross-channel profiles, so that the wall heat flux and the viscous losses are far more dominant effects on the flow field than the Lorentz force or Joule heating. Since the MHD interaction is weak for this channel, power law expressions for the turbulent flow velocity and temperature profiles are considered to be valid; that is:

$$\frac{u}{u_c} = \left(\frac{y}{\delta} \right)^{n_1}, \quad (67)$$

$$\frac{T - T_s}{T_c - T_s} = \left(\frac{y}{\delta} \right)^{n_2}. \quad (68)$$

The values of n_1 and n_2 used in the present model were 1/7 and 1/10, respectively. The "single-layer" concept discussed by Wilson [22] was adopted for the profile model.

The integration for the boundary layer properties, such as the boundary layer thickness, is numerically computed using Simpson's composite rule [11].

Through post-test inspection, a fairly uniform layer of slag, from 1 mm to 3 mm thick, was found to completely cover the lower surface of the channel walls. Slag formation on the side walls can best be described as having a thickness which varies in an elliptical pattern; the thick region occurring around the outer edge of the oval, with the center portion less than 0.5 mm thick. Slag on the upper surface of the channel is thinner and is less than 1 mm.

The leadout current distributions on the Hall voltage distributions of the test data obtained for three different load conditions are summarized in the Appendix.

3.2. Comparisons

In order to obtain a meaningful comparison between the experimental data and the theoretical results, a fixed set of operating conditions for the channel is formulated which is based on preliminary investigations of the channel aerodynamics. These conditions are used in each code for the prediction of generator performance.

An aerodynamic matching of the experimentally measured flow field is undertaken prior to any attempt to compute generator performance. For the weak MHD interaction ECF channel, this is a plausible analysis. The matching allows one to determine several parameters primarily relating to channel surface conditions, such as

the roughness. Matching of the generator heat flux measurements and pressure distribution for the aerodynamic channel at zero magnetic field is achieved using each code. The test data for the heat flux and the pressure distribution are summarized in the Appendix. Figures 3.2 and 3.3 show the numerical results of pressure and heat flux distributions for each code. The roughness height is determined as 1 mm, with good agreement with the experiment.

Using the method presented by Lineberry et al. [9], the channel entrance centerline temperature is determined as $2,660^{\circ}\text{K}$; this value is used as a common input to all codes for the purpose of comparison.

The electrode resistance, R_g , for the current leakage analysis, is set to be 2.1 ohms. Table 3.2 summarizes the common input values for the four computer codes studied here.

The comparison of the distributions of the skin friction coefficient between the present method and the multidimensional calculations is given in Fig. 3.4. The present method yields results which are extremely close to the two-dimensional result. The quasi-three-dimensional calculation gives a jagged distribution. One can conclude that Eq. (51) is suitable for the integral analysis.

As shown in Fig. 3.5, a smooth electrode wall boundary layer growth is obtained by the present method.

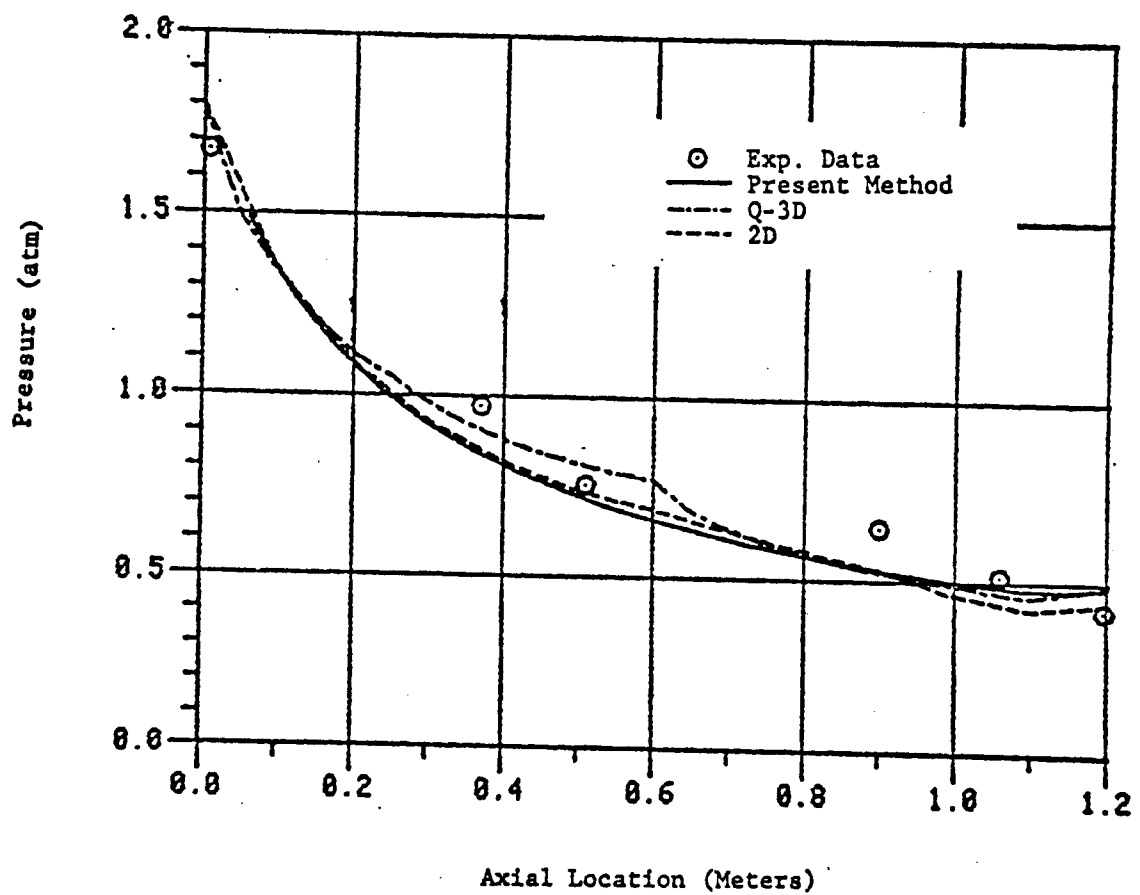


Figure 3.2. Comparison of experimental pressure distribution against numerical calculations.

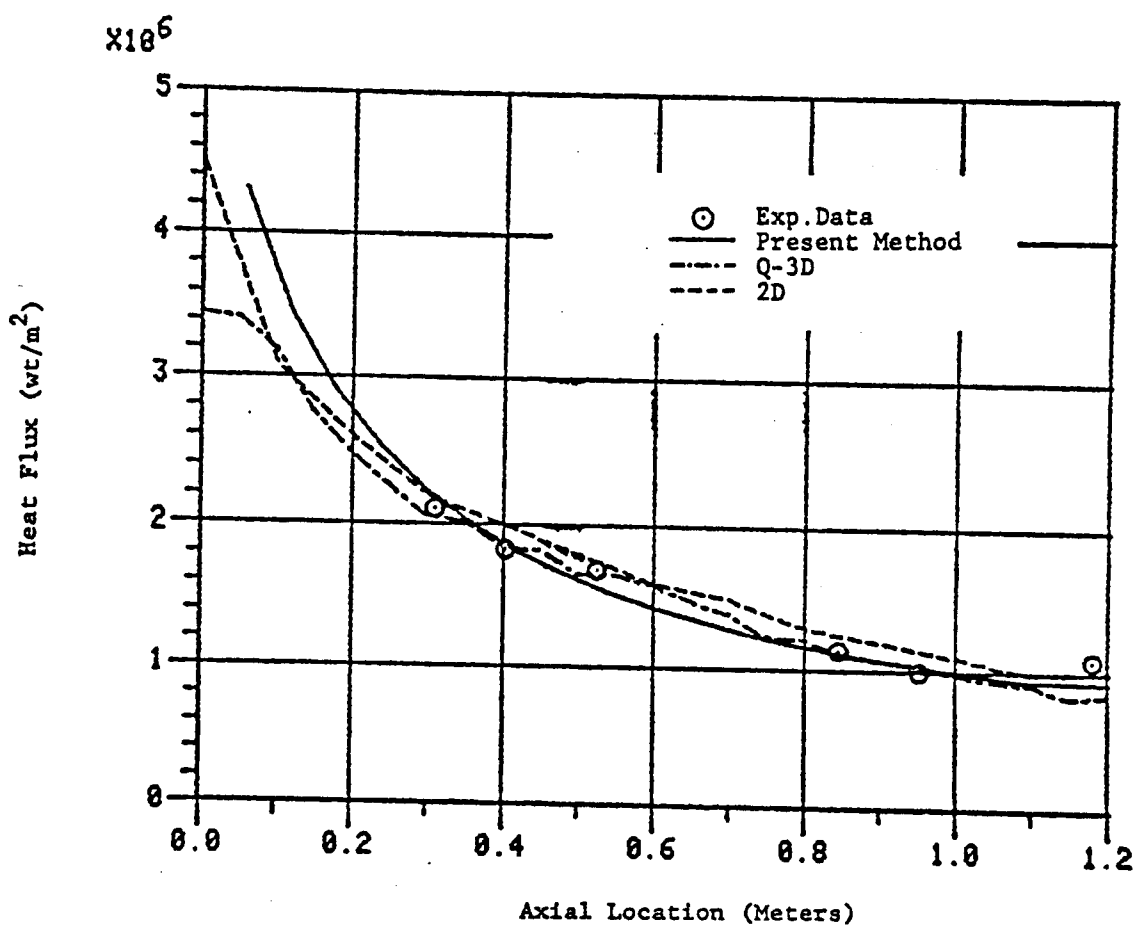


Figure 3.3. Comparison of experimental heat flux data to theoretical predictions.

Table 3.2. Common Input Conditions

Entrance Centerline Temperature, °K	2660.0
Entrance Pressure, atmosphere	1.8
Total Mass Flow Rate, K _g /S	1.52
Arcing Temperature, °K	2000.0
Electrode Resistance, R _g , ohms	2.1
Roughness Height, mm	1.0

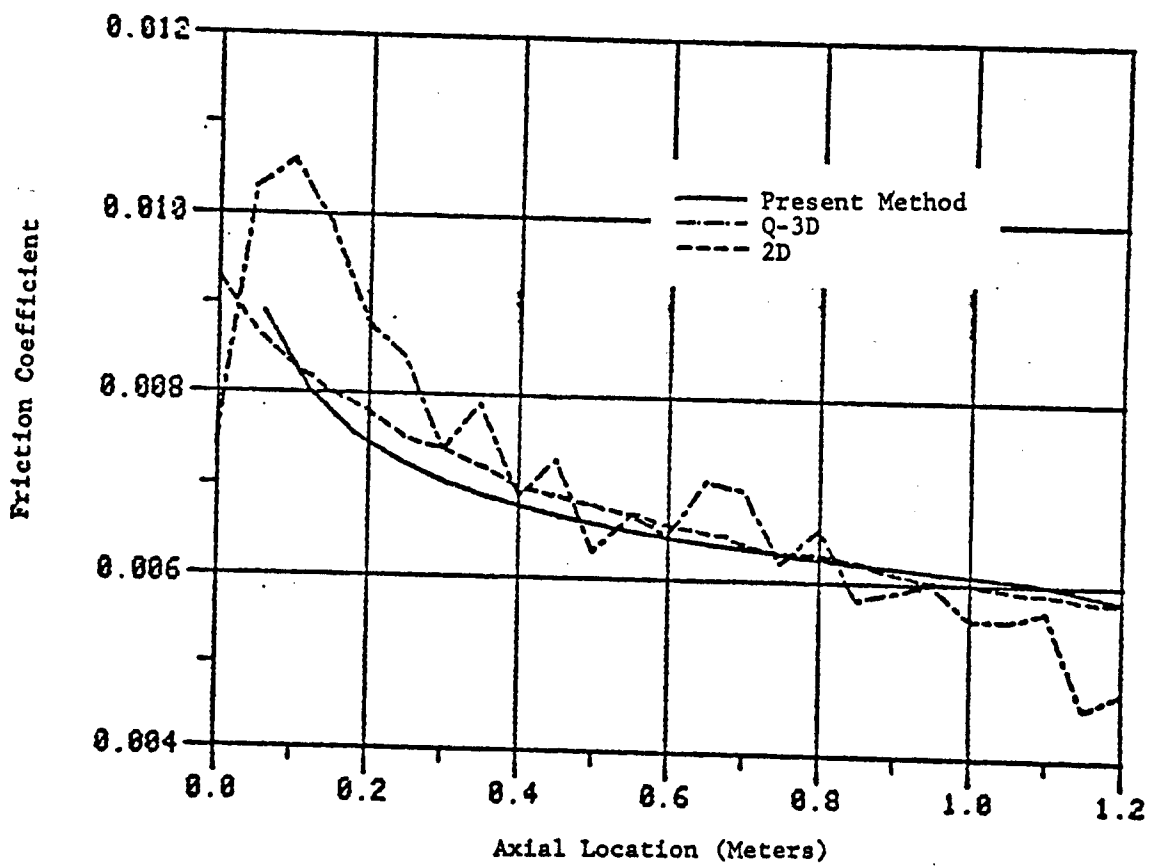


Figure 3.4. Comparison of friction coefficient distributions.

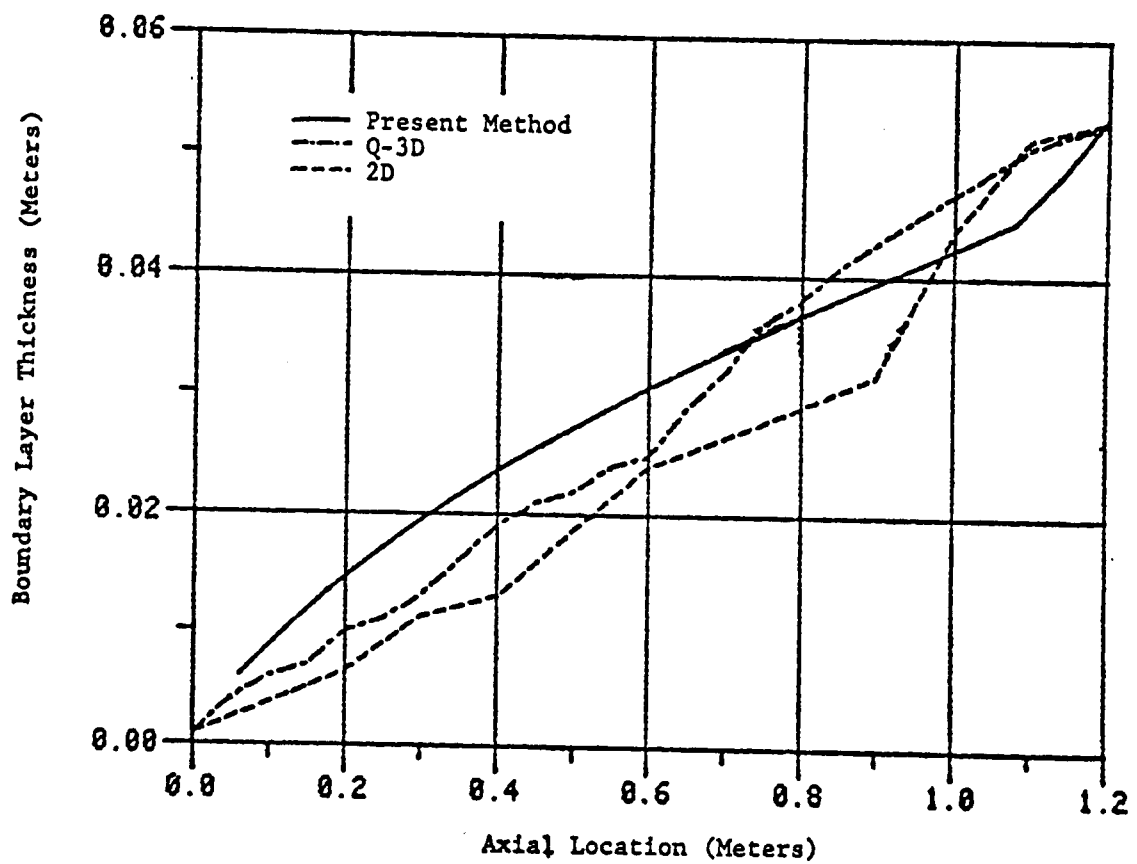


Figure 3.5. Comparison of electrode wall boundary layer developments.

The present method gives a slightly thicker boundary layer in the front part of the channel. Figure 3.6 shows the side wall boundary layer developments. The calculations indicate that the side wall boundary layers merge at mid-channel. From the merging point to the downstream exit, the computations were made by assuming that the side wall boundary layer thickness is equal to the half-width of the gas flow region.

Figures 3.7 and 3.8 give the Mach number and temperature distributions obtained using each code. Close inspection of these figures reveals a slight discontinuity at mid-channel for the multidimensional calculations. This discontinuity occurs at the location where the side wall boundary layers merged. The merging location calculated by the multidimensional codes is very close to the location calculated by the present method, as can be seen in Fig. 3.6. The one-dimensional calculations for the Mach number and temperature distributions are also given in Figs. 3.7 and 3.8. The integral method results are very close to the multidimensional calculations, while the one-dimensional calculation deviates greatly from those calculations. It can be concluded that the present method predicts adequately the overall MHD flows at weak MHD interactions, and that it is superior to the one-dimensional computational model.

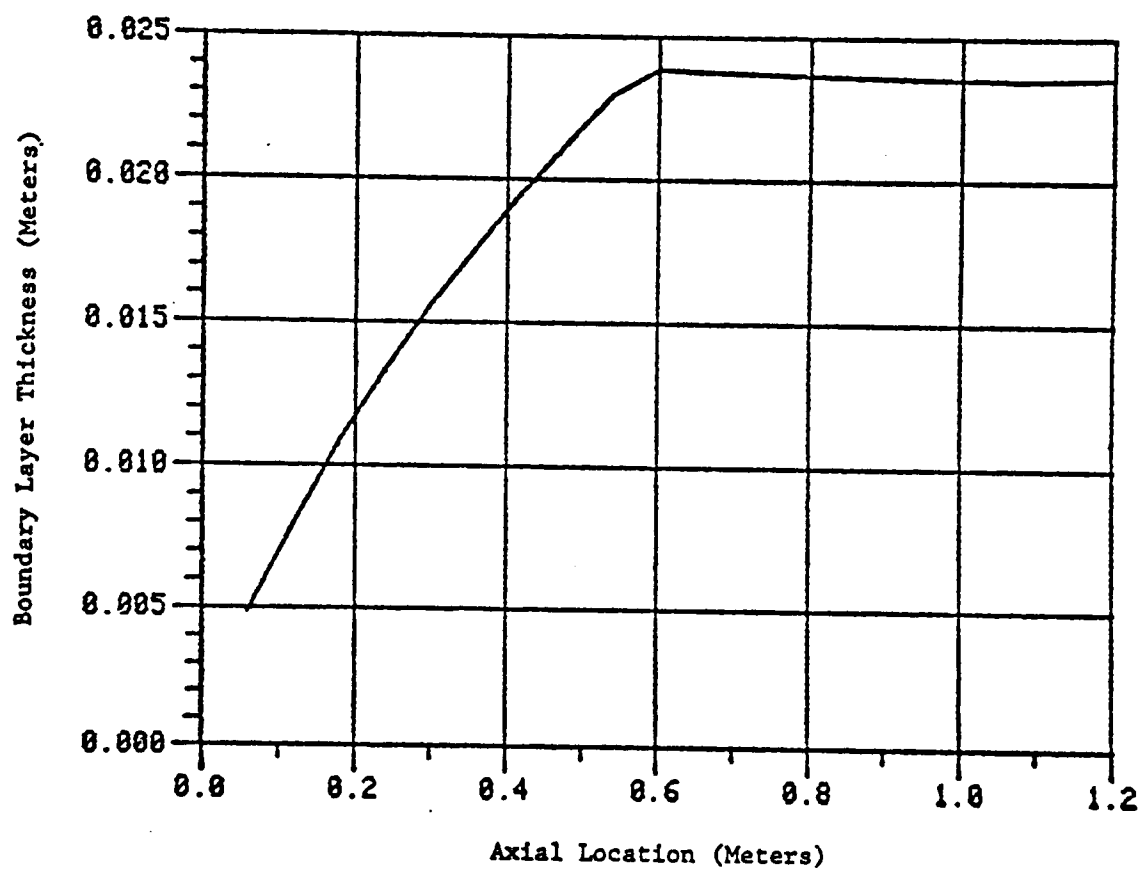


Figure 3.6. Side wall boundary layer development.

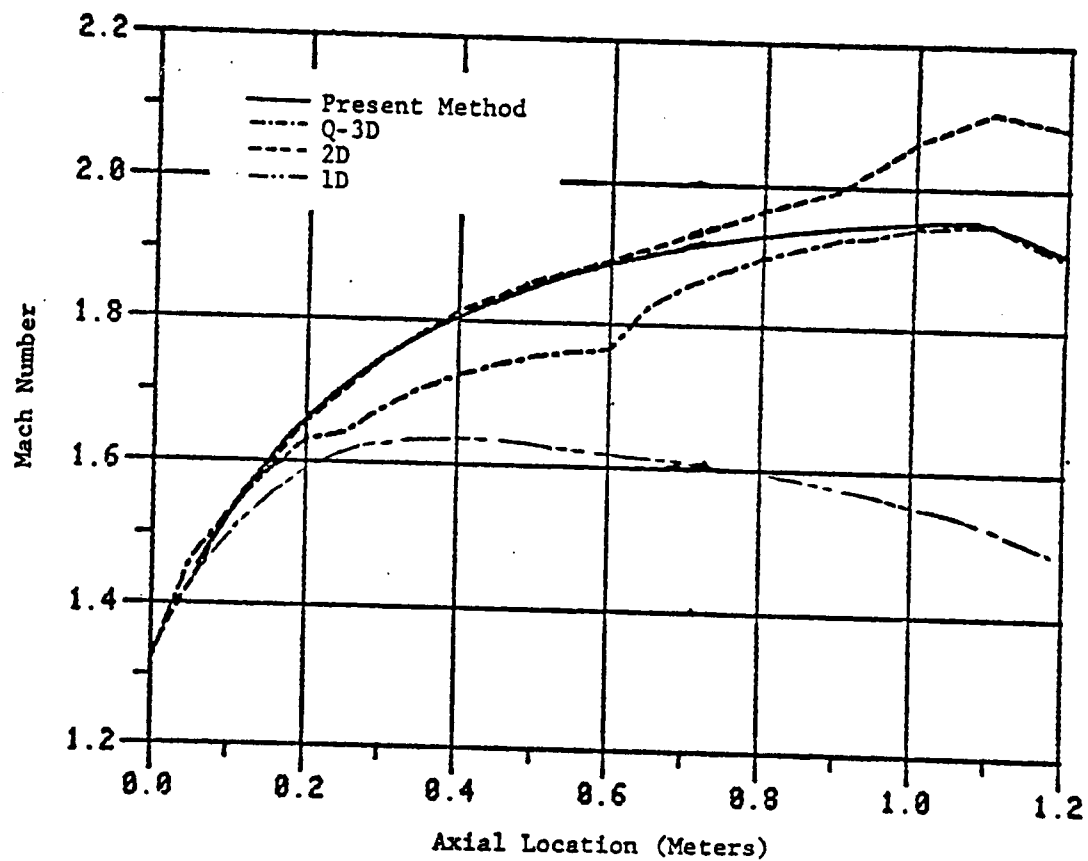


Figure 3.7. Comparison of Mach number distribution.

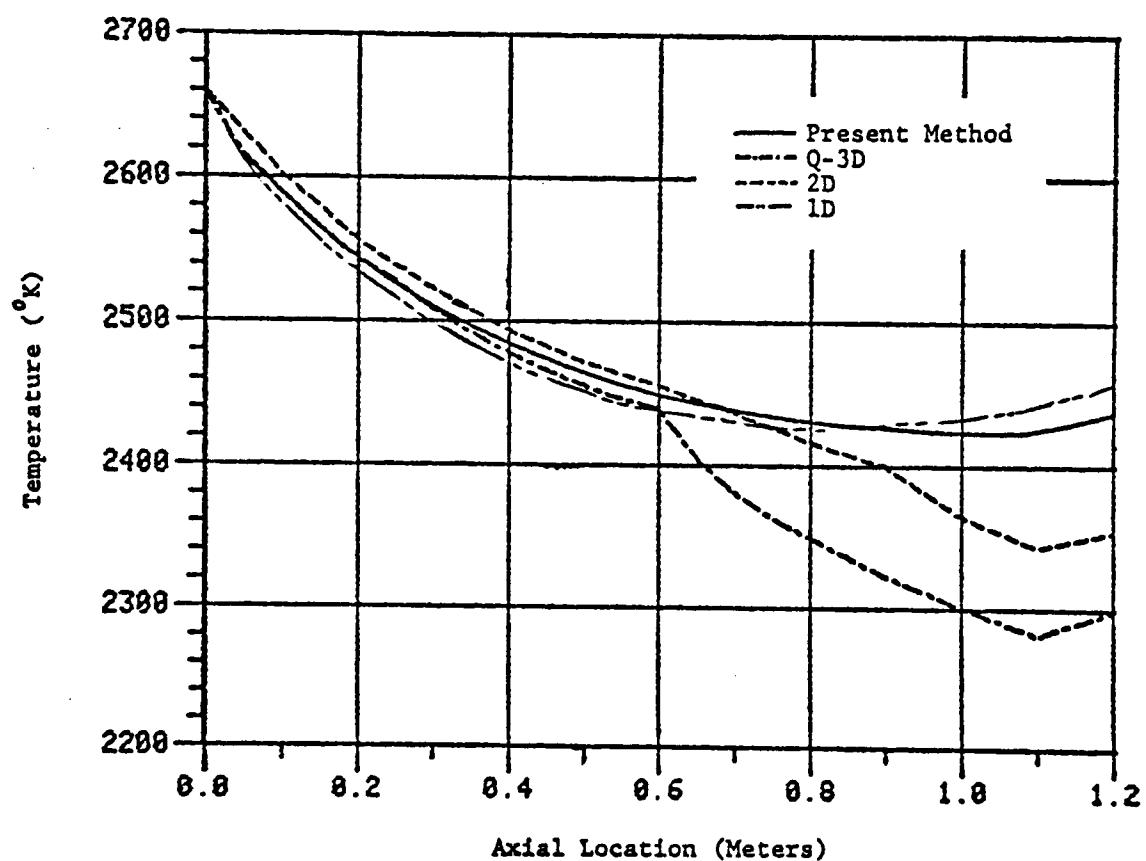


Figure 3.8. Comparison of temperature distribution.

The present code has been used to calculate results for performance of the TP40A-17 test under three different load conditions. The comparisons of the Hall voltage distribution for each load are given in Figs. 3.9, 3.10 and 3.11. The results of the present method closely match the experimental data; this means that the present method predicts the electrical performance very well. The voltage drop (Fig. 3.12) calculated by the present method is compared with the multidimensional calculations. The quasi-three-dimensional calculation gives the lowest voltage drop level; this is probably due to the lack of a slag model. The present method gives a rather large value for voltage drop; however, the shapes of the three voltage drop distributions are similar.

Finally, the predicted slag layer thickness distribution is given in Fig. 3.13. The result compares quite well with the thickness (1 mm to 3 mm) observed during the post-test inspection.

The present method uses about 30 seconds of VAX-11/780 computer CPU time, which is about three times slower than the one-dimensional calculations, but more than 60 times faster than the multidimensional calculations.

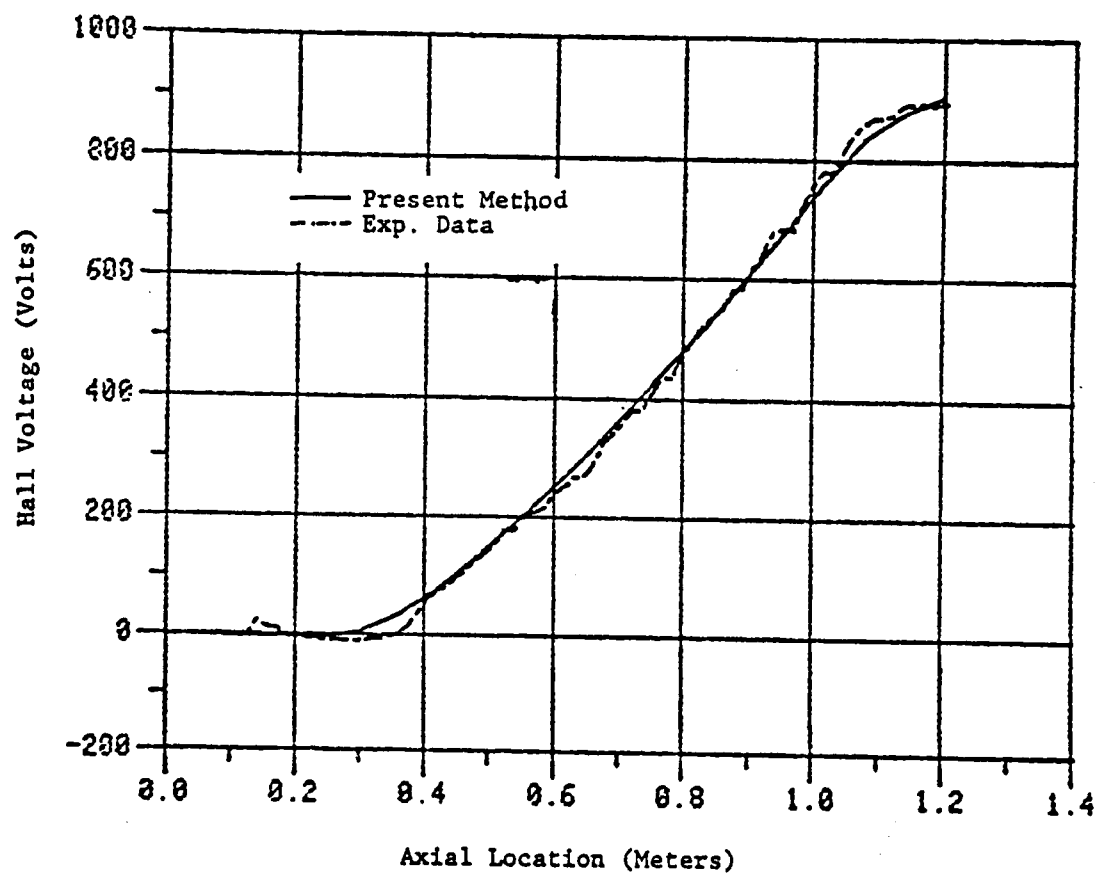


Figure 3.9. Comparison of TP40A-17 16-ohms load experimental Hall voltage distribution to theoretical prediction.

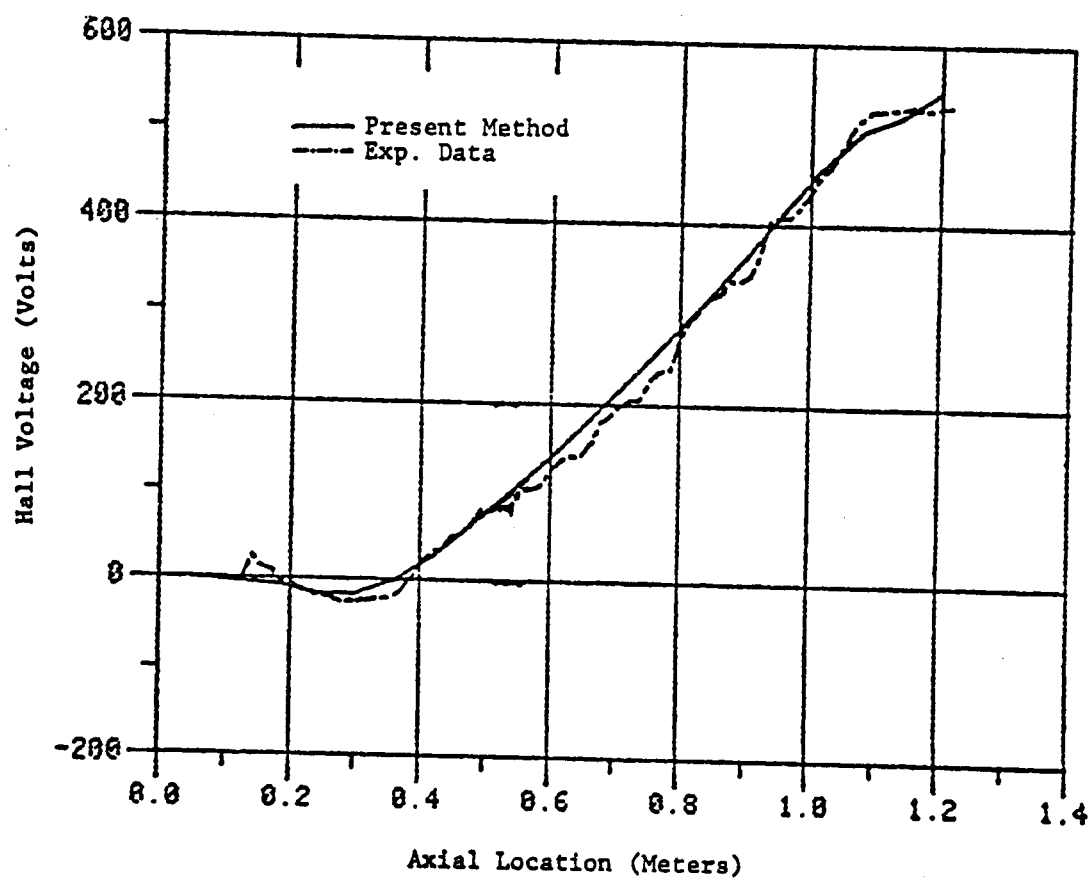


Figure 3.10. Comparison of TP40A-17 8-ohms load experimental Hall voltage distribution to theoretical prediction.

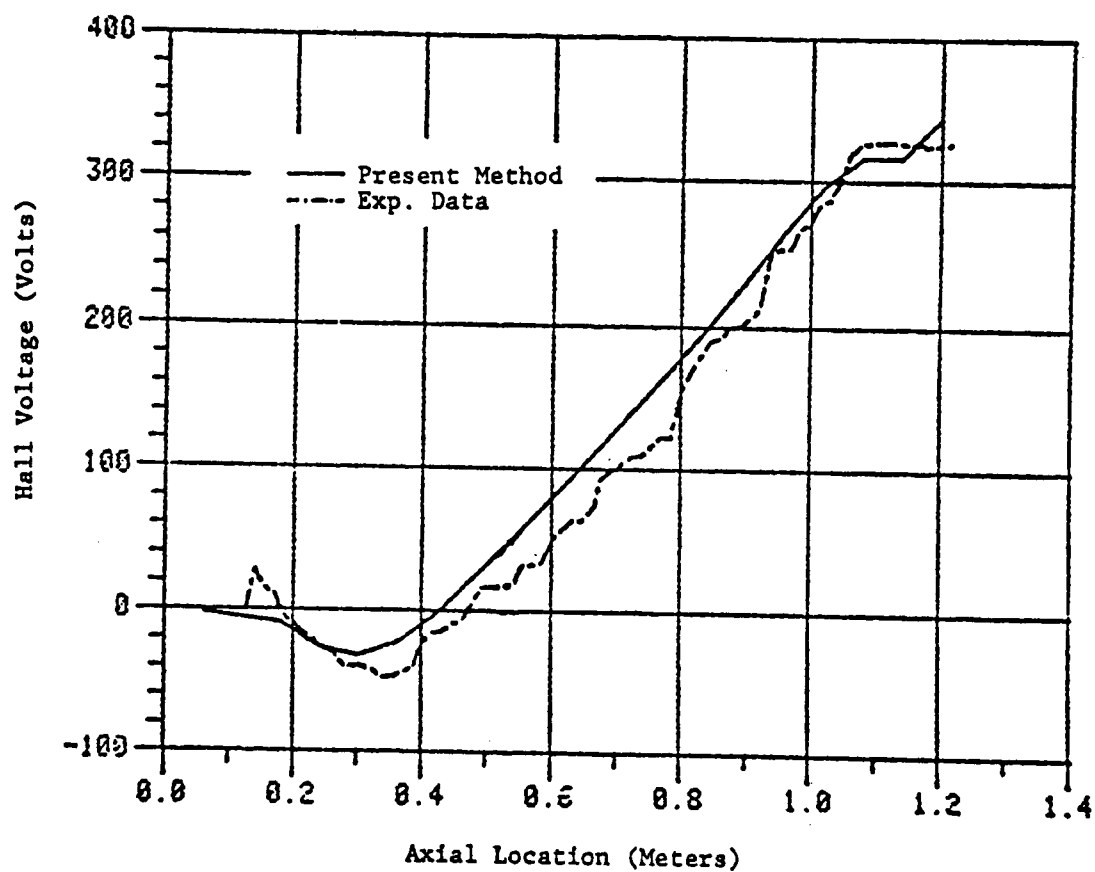


Figure 3.11. Comparison of TP40A-17 4-ohms load experimental Hall voltage distribution to theoretical prediction.

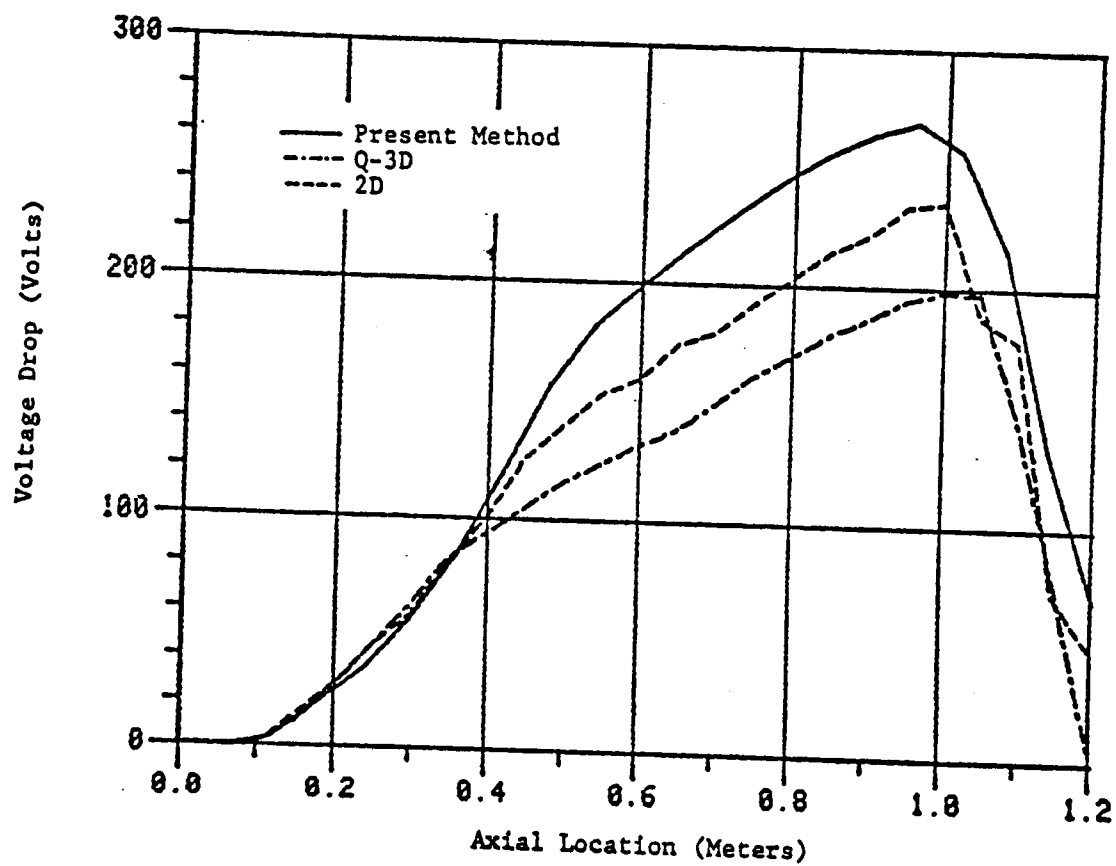


Figure 3.12. Comparison of voltage drop distribution.

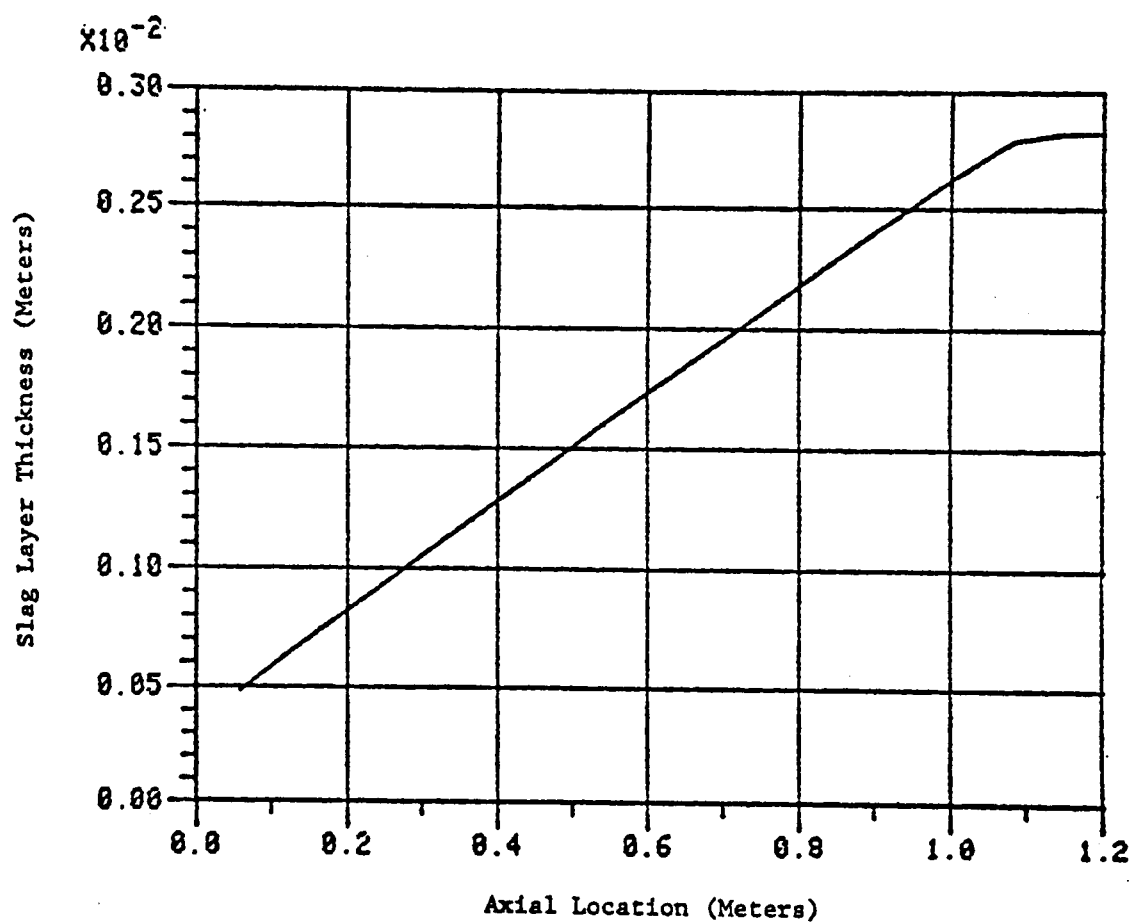


Figure 3.13. Slag layer thickness distribution.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions were reached during the course of this investigation:

1. A coupled core flow-boundary layer-slag layer model for analyzing the performance of magnetohydrodynamic (MHD) channel flows was developed that is capable of predicting the gasdynamic as well as the electrical performance of MHD generators.
2. A slag layer model and a current leakage model were incorporated into the integral analysis.
3. The present method was verified using the experimental results and the multidimensional model calculations. The present method was found to be superior to the one-dimensional model computations. (The computation speed was much faster than the multidimensional codes.)

In summary, the present method can provide a realistic prediction of MHD channel flow behavior.

Several problems still remain that should be investigated further. The following recommendations are offered:

1. Experimental measurements of the boundary layer velocity and temperature profiles in a turbulent compressible MHD channel flow are needed to aid the development of profile models.
2. The effect of MHD interaction needs to be considered on modeling of the boundary layer profiles, so that the velocity overshoot phenomena in a strong MHD interaction flow can be analyzed.
3. Radiative heat transfer should be considered, especially for larger generator channels.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Shanklin, R. V. "National Program for the Development of Commercial MHD." American Institute of Aeronautics and Astronautics Paper No. 79-0188, AIAA Seventeenth Aerospace Sciences Meeting, New Orleans, LA, January 1979.
2. Doss, E. D., and R. K. Ahluwalia. "Quasi-Three-Dimensional Modeling of Flow in MHD Channels," Numerical Heat Transfer, 3:67-87, 1980.
3. Maxwell, C. D., and S. T. Demetriades et al. "STD/MHD Code Descriptions for Computer Programs Utilized under Contract AC-01-79-ET15501," STD (S. T. Demetriades) Research Corporation Report No. STDR-79-3, Arcadia, CA, January 1979.
4. Ishikawa, M., and Y. C. L. Wu. "Three-Dimensional Current Distribution and Slagging Effects in Coal-Fired MHD Channel," Proceedings of the Nineteenth Symposium on Engineering Aspects of MHD, The University of Tennessee Space Institute, Tullahoma, TN, 1981. University, MS: Dr. John Fox, Department of Mechanical Engineering, University of Mississippi, 1981. Pp. 5.2.1-5.2.7.
5. Liu, B. L., and Y. C. L. Wu et al. "Two-Dimensional Coupled Electrical and Fluid Calculations for a Slagging MHD Channel," Proceedings of the Nineteenth Symposium on Engineering Aspects of MHD, The University of Tennessee Space Institute, Tullahoma, TN, 1981. University, MS: Dr. John Fox, Department of Mechanical Engineering, University of Mississippi, 1981. Pp. 5.4.1-5.4.10.
6. Wu, Y. C. L. "Performance Theory of Diagonal Conducting Wall MHD Generators," AIAA Journal, 4(No. 10):1362-1368, October 1976.
7. Wilson, D. R., and T. S. Williams. "Off-Design Performance Analysis of MHD Generator Channels." American Institute of Aeronautics and Astronautics Paper No. AIAA-80-0179, Eighteenth Aerospace Sciences Meeting, Pasadena, CA, January 1980.

8. Gertz, J., et al. "Modeling of MHD Channel Boundary Layers Using an Integral Approach," Proceedings of the Eighteenth Symposium on Engineering Aspects of MHD, Butte, MT, 1979. University, MS: Dr. John Fox, Department of Mechanical Engineering, University of Mississippi, 1979. Pp. B.4.1-B.4.9.
9. Lineberry, J., et al. "A Comparison of Experimental Results from the UTSI Coal-Fired MHD Generator to Theoretical Predictions." American Institute of Aeronautics and Astronautics Paper No. AIAA-81-0031, AIAA Nineteenth Aerospace Sciences Meeting, St. Louis, MO, January 1981.
10. Gordon, S., and B. McBride. "Computer Program for Calculation of Complex Chemical Equilibrium Composition, Rocket Performance and Incident and Reflected Shocks and Chapman-Jouguet Detonations," National Aeronautics and Space Administration SP-273, Lewis Research Center, Cleveland, OH, 1971.
11. Burden, R. L., J. D. Faires, and A. C. Reynolds. Numerical Analysis. Boston: Prindle, Weber and Schmidt, 1978.
12. Schlichting, H. Boundary Layer Theory. New York: McGraw-Hill Book Company, 1968.
13. Martinez-Sanchez, M., and I. Sadovnix. "Effect of Slagging in MHD Generator Ducts." American Society of Mechanical Engineers paper presented at the AIChE-ASME Heat Transfer Conference, Salt Lake City, UT, August 1977.
14. Crawford, L. W., et al. "Investigation of Slag Deposits and Seed Absorption in a Direct-Fired MHD Power Generator," Proceedings of the Sixth International Conference on MHD Electrical Power Generation, 2:51-65, 1965.
15. Ahluwalia, R. K., and E. D. Doss. "Convective Heat Transfer in MHD Channels and Its Influence on Channel Performance," Journal of Energy, 4(No. 3):126-134, May 1980.
16. Rosa, R. J. Magnetohydrodynamic Energy Conversion. New York: McGraw-Hill Book Company, 1968.

17. Sadovnix, I., M. Martinez-Sanchez, and A. Solbes. Proceedings of the Nineteenth Symposium on Aspects of MHD, The University of Tennessee Space Institute, Tullahoma, TN, 1981. University, MS: Dr. John Fox, Department of Mechanical Engineering, University of Mississippi, 1981. Pp. 10.1-10.11.
18. Sadovnix, I. "Electrode Arcing Phenomena in MHD Generators." Ph.D. dissertation, Massachusetts Institute of Technology, 1980.
19. Doss, E., et al. "Two-Dimensional MHD Channel Design." Paper presented at the Symposium on Fluids Engineering, 1978 Winter Annual Meeting, San Francisco, CA, December 1978.
20. Galanga, F. L., et al. "Experimental Results of the UTSI Coal-Fired MHD Generator and Investigation of Various Power Take-Off Schemes." American Institute of Aeronautics and Astronautics Paper No. 81-0030, AIAA Nineteenth Aerospace Sciences Meeting, St. Louis, MO, January 1981.
21. Gerywall, M. S., and J. M. Smith. "The Effect of Velocity Overshoot on the Performance of Magneto-hydrodynamic Subsonic Diffusers." American Institute of Aeronautics and Astronautics Paper No. 80-0251, AIAA Eighteenth Aerospace Sciences Meeting, Pasadena, CA, January 1980.
22. Wilson, D. R. "Analysis of MHD Channel Flows by a Coupled Core Flow-Boundary Layer Model." Ph.D. dissertation, University of Texas at Arlington, 1973.

APPENDIX

Table A.1. Leadout Current for Test TP40A-17

Frame No.	16-ohm Load Current (amps)	8-ohm Load Current (amps)	4-ohm Load Current (amps)
Nozzle	5.989	6.618	7.421
1	5.753	6.230	7.001
2	1.106	0.822	1.053
3	3.350	3.554	3.478
4	3.811	4.047	4.459
5	5.671	6.561	7.439
6	5.964	7.168	8.378
7	6.820	8.773	10.425
8	6.925	9.522	11.046
9	7.635	9.660	11.206
69	10.600	12.998	14.832
70	10.600	12.997	14.832
71	10.600	12.998	14.833
72	9.634	12.997	15.232
Diffuser	11.540	12.998	14.427

Table A.2. Voltage Distribution for Test TP40A-17

X-Location	16-ohm Voltage (volts)	8-ohm Voltage (volts)	4-ohm Voltage (volts)
0.127	0.158	0.095	0.158
0.142	22.804	25.635	28.291
0.157	11.231	12.110	16.345
0.173	9.150	9.823	11.496
0.188	-1.489	-3.276	-4.330
0.203	-2.505	-6.291	-9.697
0.218	-4.702	-11.612	-15.765
0.234	-6.167	-17.083	-21.231
0.249	-11.806	-18.429	-26.188
0.264	-9.631	-21.399	-28.720
0.280	-11.430	-25.378	-38.360
0.295	-11.573	-25.663	-38.541
0.310	-9.252	-24.435	-38.562
0.326	-7.118	-22.037	-42.529
0.341	-4.121	-20.738	-46.703
0.356	0.856	-20.150	-46.666
0.371	9.211	-12.564	-42.427
0.387	30.539	0.809	-39.785
0.402	55.556	18.787	-22.167
0.417	73.010	29.995	-15.139
0.433	80.501	34.290	-14.517
0.448	97.735	47.897	-8.195
0.463	109.158	50.559	-6.510
0.479	123.926	63.833	5.971
0.494	138.669	78.784	16.614
0.509	151.607	79.477	17.073
0.524	177.452	92.024	18.418
0.540	177.871	82.726	19.113
0.555	207.617	104.436	32.578
0.570	207.103	104.118	32.428
0.586	215.494	108.324	35.484
0.601	239.297	125.204	49.131
0.616	254.669	132.010	57.905
0.632	268.661	141.379	63.765
0.647	269.669	141.977	64.111
0.662	300.468	160.187	74.588

Table A.2. (continued)

X-Location	16-ohm Voltage (volts)	8-ohm Voltage (volts)	4-ohm Voltage (volts)
0.677	324.490	179.092	93.249
0.693	344.501	187.354	99.518
0.708	364.915	199.997	103.497
0.723	381.274	204.962	109.560
0.739	382.139	206.003	110.228
0.754	409.598	226.485	116.327
0.769	435.827	237.512	122.911
0.785	437.451	238.375	123.443
0.800	476.988	274.425	154.751
0.815	497.605	293.375	169.111
0.830	520.071	305.406	180.224
0.846	540.768	319.065	192.059
0.861	551.880	323.638	193.223
0.876	584.352	337.505	200.217
0.892	585.677	339.361	201.671
0.907	621.518	344.411	207.725
0.922	646.215	365.357	213.857
0.938	683.393	407.497	253.342
0.953	684.975	408.557	253.964
0.968	683.639	409.269	254.237
0.983	724.877	419.769	268.356
0.999	761.777	436.672	271.473
1.014	782.215	454.747	285.397
1.029	784.618	464.568	287.232
1.045	815.675	481.027	300.148
1.060	843.284	502.594	319.491
1.075	859.075	517.859	325.627
1.091	870.481	527.103	327.147
1.106	870.573	527.366	327.549
1.121	878.155	528.148	327.255
1.136	890.876	529.430	327.627
1.152	895.239	533.101	326.369
1.167	894.408	530.419	326.464
1.182	889.398	527.813	324.752
1.198	894.211	531.556	325.845
1.213	896.371	533.050	326.794

Table A.3. Heat Flux Distribution
for Test TP40A-17

X-Location (meters)	Heat Flux (w/cm ²)
0.310	210.45
0.402	183.16
0.524	168.76
0.647	117.11
0.846	113.98
0.953	99.17
1.182	108.41

Table A.4. Pressure Distribution
for Test TP40A-17

X-Location (meters)	Pressure (atm)
0.007	1.676
0.371	0.971
0.509	0.755
0.800	0.645
1.060	0.514
1.198	0.416

VITA

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