

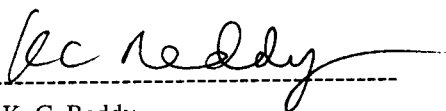
To the Graduate Council:

I am submitting herewith a thesis written by Kyle Troy Snyder entitled "Wavelet Analysis to Characterize Nonlinearities and Predict Limit Cycles of an Aeroelastic System." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirement for the degree of Master of Science with a major in Mathematics.

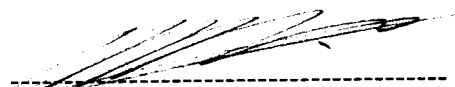


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Wavelet Analysis to Characterize
Nonlinearities and Predict Limit Cycles of an
Aeroelastic System

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May 1998

Abstract

Nonlinearities in the aeroelastic properties of a flight vehicle can result in dangerous limit cycle oscillations during flight; such nonlinear behavior is difficult to analyze by classical methods. This thesis introduces wavelet analysis for processing flight data responses in order to extract information about structural nonlinearities. Features and trends from wavelet transform maps of transient responses are used to detect and characterize a certain class of nonlinear behaviors. These features are also used to formulate functions which may use transient responses at stable flight conditions to predict the onset of limit cycle oscillations.

Preface

The following thesis is presented as the results from continuing a project started in the summer of 1997 in the NASA Academy for Aeronautics at Dryden Flight Research Center in Edwards, CA. The project provided the author, Kyle Snyder, with the opportunity to learn structural dynamics from the universal mathematics involved.

The author spent the summer working with Rick Lind and Marty Brenner, Structural Dynamics Engineers at Dryden. Together they published a paper also titled "Wavelet Analysis to Characterize Nonlinearities and Predict Limit Cycles of an Aeroelastic System," in the journal Mechanical Systems and Signal Processing. The team also has submitted the work to NASA TechBrief titled "Characterizing Nonlinear Dynamics from Wavelet Analysis." Rick Lind presented much of the work at the 1998 AIAA Structures, Structural Dynamics, and Materials Conference, Long Beach CA, in April 1998. That presentation was titled "Investigating Transient and Limit Cycle Behaviors of a Nonlinear Structure by Wavelet Transforms." Both of these reports include the work done to build the simulation, Chapter 2, to initially describe the nonlinear behaviors, Chapter 5, and the work done to develop a prediction function from scale data for the Limit Cycle Oscillation,

Chapter 6. This is the work that was done during the summer at Dryden while Kyle and Rick where working together.

The final Section, "6.2 Energy Analysis," is the work Kyle has done independently while at the University of Tennessee Space Institute. Developing a prediction from the energy information was a goal not achieved during the summer, so it provided additional work. This information is closely related to the rest of the project and should be considered relevant. The new results have been presented to Rick and Marty at Dryden for additional review and use.

Acknowledgments

I want to extend the greatest appreciation to my advisor Dr. Ken Kimble at UTSI. Without his encouragement and belief in wavelets this never would have happened. I have really enjoyed working with you these last two years and have learned more than you can imagine. Thank you!

Thank you Rick Lind at Dryden. Your patience for a math guy with no aerodynamics background is amazing. This was a great project to work on and I want to thank you for sharing so much time on it with me.

I also want to thank Marty Brenner and the rest of the guys in the Structural Dynamics Branch at NASA Dryden Flight Research Center. They showed me that there is so much more to learn outside of school and that it can be fun. Thanks for supporting the NASA Academy program guys, you don't know the influence you had.

I want to send my thanks to the members of my committee at UTSI, Dr. Moulden, Dr. Reddy, and Dr. Whitehead. Thanks for all your help and suggestions.

There is one other person that is responsible for the development of this project, Professor Dominick Andrisani. Taking the summer off from teaching at Purdue University, Dominick was the Dean of the NASA Academy in

Aeronautics at Dryden. So I want to thank Dominick again for inviting me to be a part of the Academy and for showing so much interest in the use of wavelets. Your patience, wisdom, and support are something special.

I also want to thank Thomas Walloschek and Brian Portock for their loyal friendships. We made it!

Despite all the grief I gave him, I want Adel Omrani to know I appreciate all his help too. Thanks for being there.

Finally, I want to thank my parents for encouraging me to chase a dream. This thesis may not make much sense, but it is another step closer to the stars. Thanks for making this all possible, Mom and Dad!

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	(f), $U=10.0\ m/s$ (g), $U=12.0\ m/s$ (h).	100

Nomenclature

a	scale
a_i	dominant scale at time t_i
b	semichord length
c_{l_α}	lift coefficient per angle of attack
c_y	structural damping in force equation for plunge motion
c_α	structural damping in moment equation for pitch motion
e	location of the elastic axis
k_y	stiffness in force equation for plunge motion
k_α	nonlinear stiffness in moment equation for pitch motion
$\hat{k}_\alpha$	linearized function of k_α
m	mass
s	Laplace variable
t	time
t_a	time corresponding to peak derivative of dominant scale
t_o	initial time for simulation
t_i	time value
t_f	final time for simulation
x	state vector

y	plunge displacement
x_α	nondimensional distance between elastic axis and center of mass
A	vector of scales
ψ	wavelet function
$\mathbb{R}^3$	vector space in three-dimensions
C	set of all Complex numbers
F	wavelet map
$\overline{F}_i$	maximum peak magnitude of wavelet map at time t_i
I_α	moment of inertia for pitch motion
L	lift
M	moment
U	airspeed
α	pitch angle
Δ_a	peak magnitude for time derivative of dominant scale
Δ_F	peak magnitude for time derivative of energy
ϕ_S	experimental prediction function from scale
ϕ_F	experimental prediction function from energy

$\hat{\phi}$	linear prediction function
ρ	density of air
ω_y	natural frequency of plunge mode
ω_α	natural frequency of pitch mode
τ	time
ζ_y	modal damping of plunge mode
ζ_α	modal damping of pitch mode

Chapter 1

Introduction

Flight testing aircraft for envelope expansion is a dangerous and costly procedure because the onset of aeroelastic instabilities is often not predictable due to nonlinear behavior [9]. Aeroelastic instabilities include vibrations and structural fatigue that were not anticipated in the original design of the aircraft. One such example is a limit cycle oscillation which is realized as a wing vibration at a constant frequency and amplitude. The vehicle structure is weakened, possibly to the point of failure, as a result of such an oscillation. This **limit cycle oscillation (LCO)** is mathematically stable in the sense of Lyapunov, but is potentially detrimental for flight since it often leads to pilot injury and catastrophic damage to airframe components [5]. The F-16

aircraft, for example, demonstrates LCO behavior with several stores configurations [23]. The F/A-18 also demonstrates an LCO with a particular store configuration [7].

Robust analysis methods have been developed to describe and predict linear flutter instabilities but these are not applicable to nonlinear dynamics [12]. An approach to predict LCO behavior for the F-16 was formulated by considering classical linear analysis utilizing sensitivities and properties of modal damping plots including crossing points and slopes [3]. Another method used a nonlinear model of transonic aerodynamics to predict LCO for fighter aircraft [16]. A data analysis approach based on transient frequencies of forced responses has been shown to predict LCO onset for a particular class of systems [21].

Wavelets have recently been introduced for signal analysis of aeroelastic response data to determine linear dynamical characteristics of aircraft [13]. Wavelet transforms present several advantages for signal analysis as compared to Fourier transforms by relaxing stationarity, linearity and time-invariance assumptions. Also, wavelet processing decomposes the data into localized waveforms rather than into the sums of infinite length sinusoids familiar to Fourier processing. So wavelets naturally characterize features and

patterns of size and time variation.

This thesis considers wavelet analysis for processing response data from nonlinear aeroelastic systems. Wavelet transforms compute time-scale maps which indicate the periodicity of localized components in these responses based on correlations between the signal and sets of wavelets [1]. Two applications are presented that consider decay responses of time-invariant systems with a particular class of nonlinearities.

One application of wavelet analysis is to extract information about the system dynamics from transient response data. Several important features are seen in the time-scale maps generated from wavelet transforms that indicate the influence of nonlinearities. Furthermore, these features and trends are exploited to characterize the nature of the nonlinearities. Such information can potentially be used to develop system identification algorithms for this class of nonlinear systems.

Another application of wavelet analysis is to predict the onset of a limit cycle associated with Hopf bifurcations for this type of nonlinearity. Previous methods have been unsuccessful in producing techniques that can predict the onset of an LCO. Responses from a series of airspeeds ranging up to the bifurcation speed are evaluated by generating wavelet maps. Functions are

developed based on properties of these maps that extrapolate from stable flight conditions to LCO regions.

A nonlinear aeroelastic testbed at Texas A&M University is used to demonstrate these applications [18]. By studying this wind tunnel system and developing analysis techniques from that data, we hope to develop methods that will apply to other configurations, including the F-16. Wavelet maps can detect and characterize different behaviors from a class of structural nonlinearities along with formulating LCO prediction functions. The results presented in this thesis are only directly applicable to this particular system but the concepts may be extended to consider other classes of systems.

Chapter 2

Simulation

An aeroelastic testbed was developed at Texas A&M University for flutter research using a prototypical aeroelastic wing section as shown in Figure 2.1 [18].

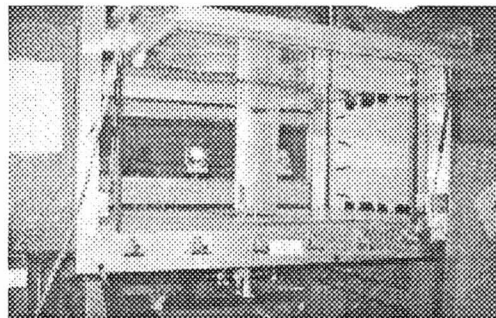


Figure 2.1: Texas A&M University Aeroelastic Testbed

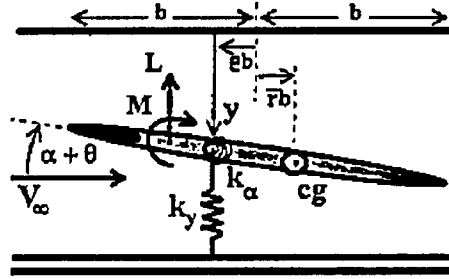


Figure 2.2: Mechanical Interpretation of Texas A&M University Aeroelastic Testbed

Figure 2.2 is a mechanical interpretation of the wind tunnel model. The model is constrained to two degrees of freedom: pitch and plunge motion. **Pitch** is the rotation of the wing about the lateral axis, measured in radians, constrained by the k_α spring. **Plunge** is motion of the wing in the vertical direction, measured in meters, constrained by spring k_y .

Consider the aeroelastic equations of motion for this pitch-plunge system [10].

$$\begin{bmatrix} m & mx_\alpha b \\ mx_\alpha b & I_\alpha \end{bmatrix} \begin{Bmatrix} \ddot{y} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} c_y & 0 \\ 0 & c_\alpha \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \dot{\alpha} \end{Bmatrix} + \begin{bmatrix} k_y & 0 \\ 0 & k_\alpha(\alpha) \end{bmatrix} \begin{Bmatrix} y \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -L \\ M \end{Bmatrix}$$

Table 2.1: Initial Conditions for the system.

$e = -0.6 \text{ m}$	$x_\alpha = 0.2467$	$m = 12.387 \text{ kg}$
$I_\alpha = 0.065 \text{ kg m}^2$	$b = 0.135 \text{ m}$	$c_y = 27.43$
$c_\alpha = 0.036$	$k_y = 2844.4 \text{ N/m}$	$c_{l_\alpha} = 6.28$

This is a coupled nonlinear system on a four-dimensional phase space. The system is, moreover, autonomous; therefore, orbits do not cross in this phase space [22]. The variable y is the plunge position and α is the pitch angle. Other variables include the nondimensional distance between the elastic axis e and the center of mass x_α , the mass of the wing m , the mass moment of inertia I_α , semichord length b , structural damping coefficients in pitch and plunge c_y and c_α , and spring constants k_y and $k_\alpha(\alpha)$. See Table 2.1 for initial values. There are also several constants that are calculated before the simulation is started, see Appendix A.

The lift L and moment M are determined by quasi-steady aerodynamic theory with lift coefficient c_{l_α} .

$$L = \rho U^2 b c_{l_\alpha} \left(\alpha + \left(\frac{1}{2} - e \right) \frac{b}{U} \dot{\alpha} + \frac{1}{U} \dot{y} \right)$$

and

$$M = \left(\frac{1}{2} - e \right) b L$$

Nonlinearities are introduced into the system dynamics through the stiffness associated with pitch movement. This stiffness is described by a nonlinear polynomial function of the pitch angle $k_\alpha(\alpha)$. Such structural nonlinearities occur in physical aeroelastic systems and have been investigated to determine their effect on inducing limit cycle oscillations [4].

Models of the Texas A&M aeroelastic systems are formulated using three types of stiffness functions to investigate a variety of behaviors related to different nonlinearities. The stiffness functions associated with the pitch degree of freedom are chosen to represent a linear spring, a nonlinear hardening spring, and a nonlinear softening spring. Each of these functions is a polynomial function of the pitch angle and affect the system as the $k_\alpha(\alpha)$ variable in the equations of motion.

The linear spring constant is denoted $k_\alpha(\alpha) = k_{lin}$.

$$k_{lin} = 2.82$$

The softening spring function is denoted $k_\alpha(\alpha) = k_{soft}$. A softening spring is a spring that requires less force to keep it extended than a linear or hardening spring.

$$k_{soft} = 2.82 - 200\alpha^2 + 10000\alpha^4$$

The hardening spring function is denoted $k_\alpha(\alpha) = k_{hard}$. The restoring force

for the wind tunnel system is greater for the hardening spring than for the linear and softening springs.

$$k_{hard} = 2.82 - 62.3\alpha + 3709.7\alpha^2 - 24196.0\alpha^3 + 48757\alpha^4$$

The models with each of these springs can be linearized around the equilibrium condition at $\alpha = 0$. The system dynamics of these linear models will be identical because the different spring functions are identical when the terms involving α are eliminated. Thus, the systems at stable flight conditions should behave similarly with nearly identical natural frequencies for responses with small α values. The natural frequencies, ω_y and ω_α , and dampings, ζ_y and ζ_α , associated with the pitch and plunge modes of the linearized models at different airspeeds are given in Table 2.2 [6].

Using the data associated with this testbed, Ko and Kurdila at Texas A&M developed a simulation in Matlab using the fourth and fifth order Runge-Kutta formulas, "ode45," differential equation solver to test different conditions. The Matlab Dynamic System Simulation Tool Simulink was implemented for producing the responses studied in this thesis. Figure 2.3 is the complete system and Figure 2.4 is the subsystem with the same equations of motion that describe the dynamics of the testbed. The simulation begins with the initial conditions that are combined into a vector, the "Mux"

Table 2.2: Predicted Modal Properties of the Linearized Texas A&M University System At Airspeeds U in m/s .

	U=4	U=6	U=8	U=10	U=11.9
ω_y (Hz)	2.73	2.67	2.54	2.36	2.00
ω_α (Hz)	1.09	1.17	1.29	1.48	1.86
ζ_y	.086	.087	.089	.094	.171
ζ_α	.053	.057	.060	.059	-.014

block. The equations of motion are integrated and the continuous responses are separated from the vector for output and analysis, the “Demux” block.

Simulated responses are computed for each of the nonlinear and linear models using a 4th order Runge-Kutta algorithm to integrate the equations of motion with a time step of .001 *sec* for 10 *sec*. The responses simulate free decay behavior of the system to initial conditions given at time $t_o = 0$.

$$\begin{aligned} y(t_o) &= 0.01 \text{ m} & \dot{y}(t_o) &= 0 \text{ m/s} \\ \alpha(t_o) &= 0.1 \text{ rad} & \dot{\alpha}(t_o) &= 0 \text{ rad/s} \end{aligned}$$

These initial conditions are used to initially test the system and for characterizing a set of nonlinear behaviors. Another set of initial conditions is used later for predicting the onset of the LCO.

The responses for the wing motion, produced in the Simulink model, are

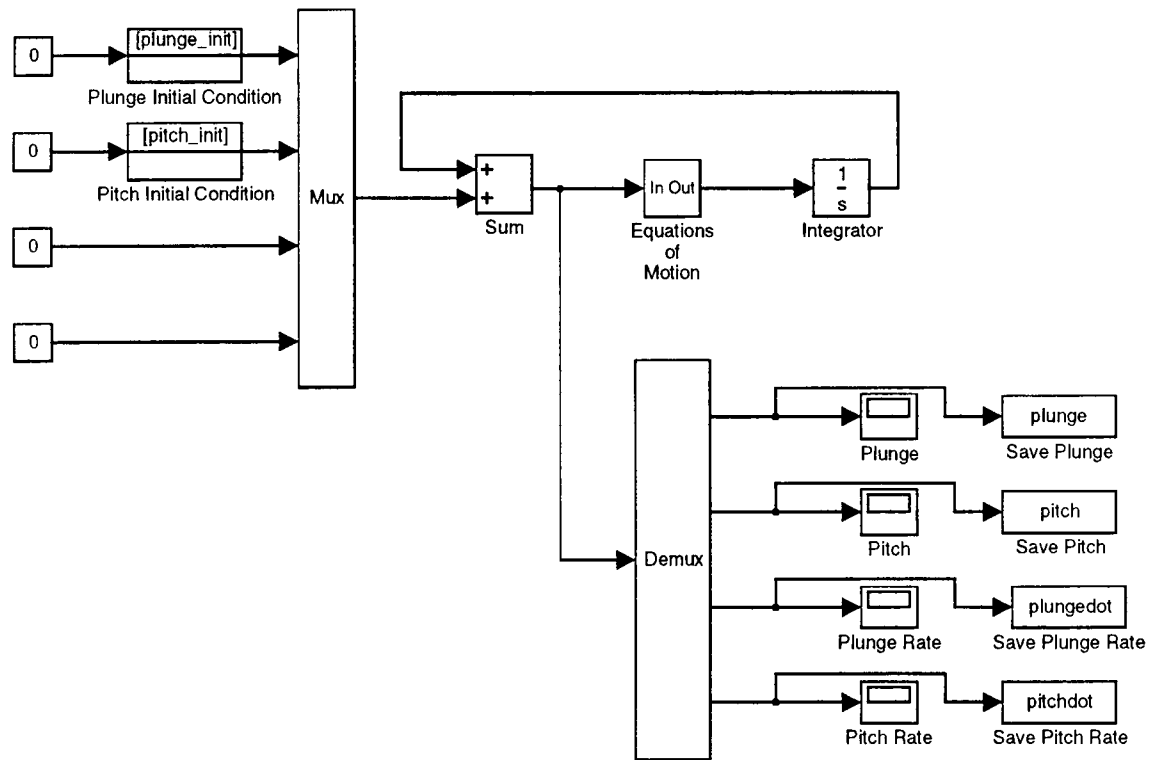


Figure 2.3: Simulink Block Diagram for Texas A&M System

free decay responses, meaning the initial energy in the system provides the motion. The initial energy comes from the original displacement and the airspeed velocity, after that the motion of the wing responds freely to the system. The initial conditions are loaded from a Matlab file before the simulation is started, Appendix A. Figure 2.5 presents the pitch angle responses while Figure 2.6 presents the plunge responses. These show the motion in the

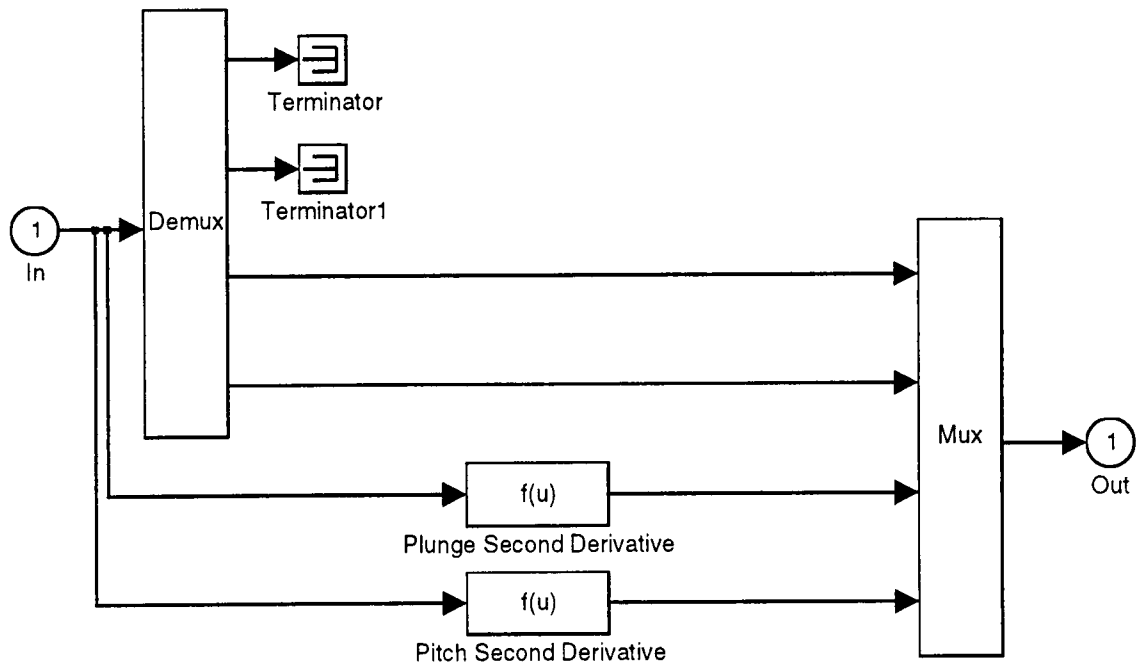


Figure 2.4: Simulink Block Diagram for Texas A&M System

pitch angle degree of freedom and the plunge degree of freedom for the first 10 seconds of the simulation. Each figure shows the responses for a nonlinear hardening spring, a nonlinear softening spring, and a linear spring. These are sample time-histories of what will be analyzed for nonlinear behaviors and for predicting LCOs later. For now, it is worth noticing the smooth decay of the pitch motion. The plunge responses are more affected by coupling with the nonlinear pitch spring and are not as easy to analyze. However, see Appendix J, it is the plunge natural frequency that dominates the coupled

system.

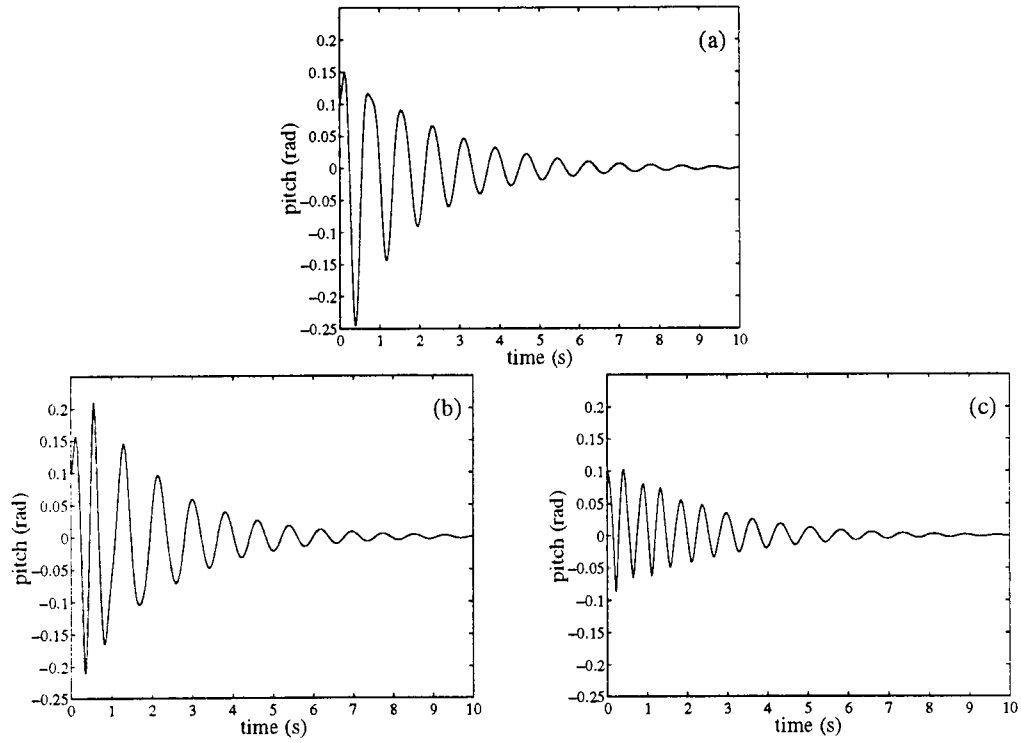


Figure 2.5: Simulated Time Responses of the Pitch Angle of Each Model at Airspeed $U=8 \text{ m/s}$: Linear k_{lin} (a), Nonlinear Softening k_{soft} (b), Nonlinear Hardening k_{hard} (c)

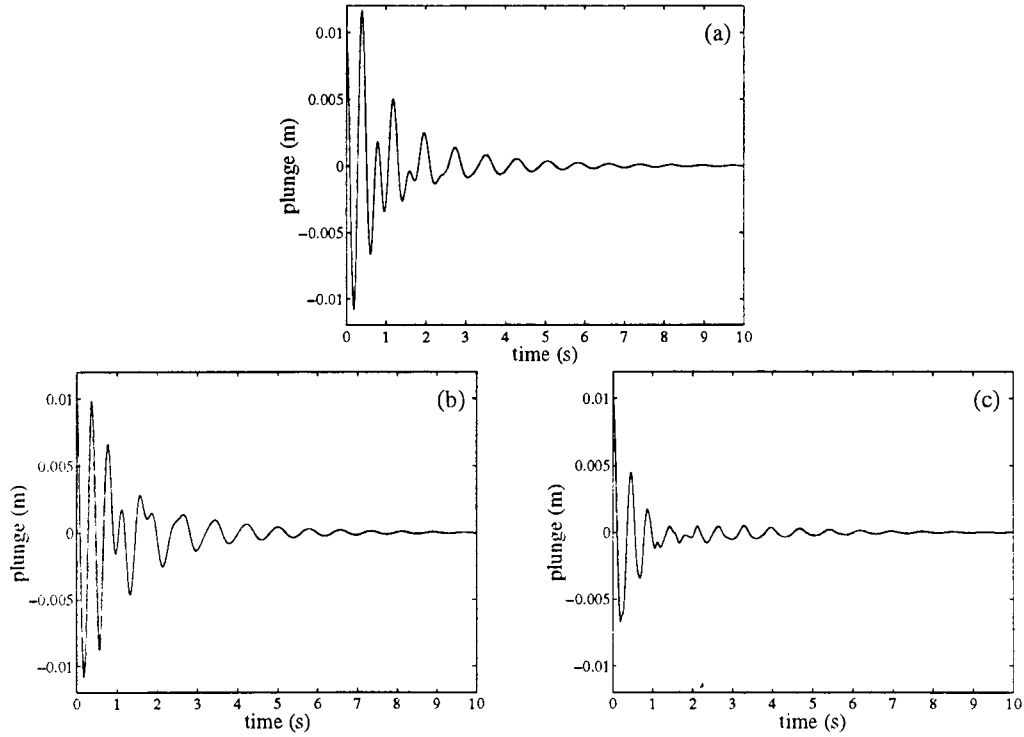


Figure 2.6: Simulated Time Responses of the Plunge Displacement of Each Model at Airspeed $U=8 \text{ m/s}$: Linear k_{lin} (a), Nonlinear Softening k_{soft} (b), Nonlinear Hardening k_{hard} (c)

Chapter 3

Nonlinear Behavior and LCOs

A brief introduction to the characteristics of nonlinear behavior is required to understand the advantages of using wavelet transforms for the data analysis. Stiffness functions are studied for a variety of behaviors related to different nonlinearities of aeroelastic systems. There are two general types of equilibria for the Texas A&M aeroelastic system, point attractors and limit cycles. A two-dimensional parameter space determines the stability regions and bifurcation points of the system for the nonlinear stiffness $k_\alpha(\alpha)$ [22]. The two dependent parameters are the elastic axis location e and the free-stream velocity U . The values of these parameters determine the stability equilibria and consequently determine the response of the wing and the onset of limit

cycle oscillations [20].

For low airspeeds the free decay motion of the wing approaches zero for the Texas A&M model. In a projection of the four-dimensional phase space onto the $(\dot{\alpha}, \alpha)$ subspace, this property is seen as converging to an equilibrium point at the origin. As the trajectories decay, the system approaches the stable origin for airspeeds lower than the bifurcation point. Figure 3.1 shows the free decay of a system that is stable as time increases. Pitch stability is seen in the two-dimensional phase subspace plot as the change in pitch versus pitch rate approaches a point attractor, the origin.

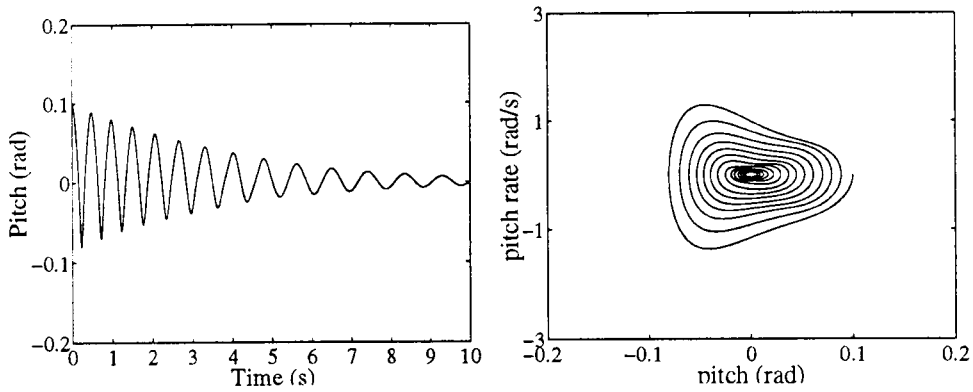


Figure 3.1: Simulated Time Response for the Pitch Angle of a Nonlinear Model at Airspeed $U = 2.0 \text{ m/s}$. Phase Subspace Plot of Pitch Rate (rad/s) versus Pitch (rad) for Simulated Response of the Model at Airspeed $U = 2.0 \text{ m/s}$.

Increasing airspeed forces the system to display a Hopf bifurcation. Hopf bifurcation is the transition from an equilibrium state of a system, a point attractor, to a limit cycle [11]. After the bifurcation has occurred, the attracting set for higher airspeeds changes to a stable periodic orbit, or limit cycle. The previously stable equilibrium point at the origin has now become an unstable equilibrium point in phase space. The Texas A&M system experiences a Hopf bifurcation at the airspeed $U = 9.8 \text{ m/s}$. Figure 3.2 is now a vibration at a constant amplitude and frequency in response to the input energy, thus causing fatigue and structural instabilities. The orbit appears to cross itself before the LCO has formed in the phase subspace plot on the right; this is a result of projecting the orbit onto a two-dimensional cross section of the four-dimensional space.

If the nonlinear conditions are removed from the model, the linear models do not predict any instabilities or limit cycle oscillations for airspeeds less than 11 m/s . All of the simulations show the motion of the wing decaying to zero in time. In phase space, stability is characterized by point attractors for the system with linear conditions at all airspeeds. See Figure 3.3.

Notice several characteristic differences between the linear and nonlinear responses. The time-histories themselves show several examples of nonlinear

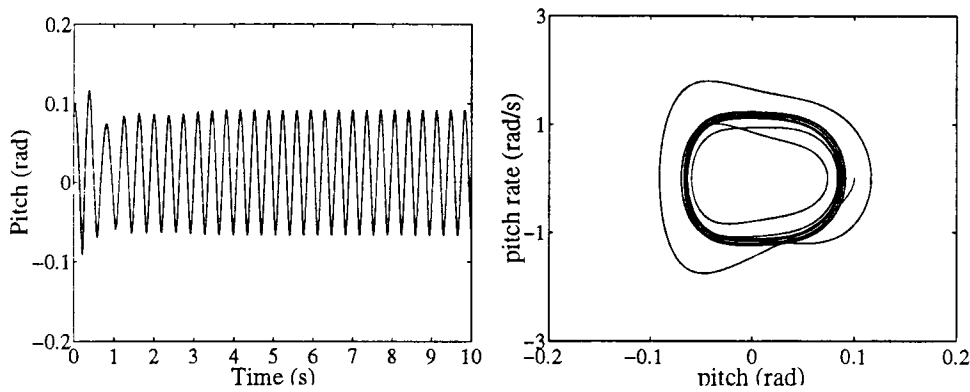


Figure 3.2: Simulated Time Response for the Pitch Angle of a Nonlinear Model at Airspeed $U = 10.0 \text{ m/s}$. Phase Subspace Plot of Pitch Rate (rad/s) versus Pitch (rad) for Simulated Response of the Model at Airspeed $U = 10.0 \text{ m/s}$.

behaviors. The motion for the linearized wing is smooth and predictable with constant frequency in the response. The motion of the nonlinear system does not display a constant frequency or change in amplitude due to the nonlinear stiffness parameter. The nonlinear behaviors will be more obvious later when a different set of initial conditions are used that are further away from the small-amplitude linearized operating range.

Now that the behaviors of nonlinear dynamics have been studied, the basics behind wavelet analysis are needed before the application can be under-

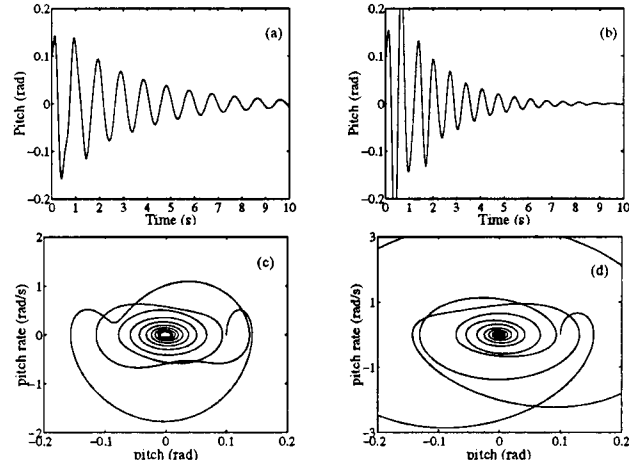


Figure 3.3: Simulated Time Response for the Pitch Angle of Linear Model at Airspeed : $U = 2.0 \text{ m/s}$ (a), $U = 10.0 \text{ m/s}$ (b). Phase Subspace Plots of Pitch Rate (rad/s) versus Pitch (rad) for Simulated Response of the Linear Model: $U = 2.0 \text{ m/s}$ (c), $U = 10.0 \text{ m/s}$ (d).

stood. The phase space analysis will be used again later when the dynamics of the system are covered and the characteristics are needed to explain the wavelet techniques.

Chapter 4

Wavelet Analysis

As wavelet analysis has developed in recent years, one of the most practical applications has been in signal processing. One of the goals of this thesis is to provide new information using wavelet transforms to describe the motion of a prototypical wing. Using the new methods of wavelet transforms to predict the onset of limit cycle oscillations and to describe nonlinear dynamics is a relatively new approach that has not received much attention in the literature. Characterizing such behaviors and predicting fatigue could prevent costly mechanical failures. Although the development of the LCO is evident in the phase space study, the ability to accurately predict the onset of the limit cycle is not as easy. By utilizing the time, energy, and frequency

a : scale, or stretching factor, of the wavelet.

τ : position, or time shift, of the wavelet.

t : time parameter extending to $\pm \infty$.

information that wavelets provide, the goal of predicting limit cycles may be realized. The numerical data provided for analysis are the time-history signals of a wing vibrating at different airspeeds from the Texas A&M simulation.

Begin by defining a wavelet for signal analysis [8].

Definition 4.1 *A wavelet $\psi(\tau, a, t) : \mathbb{R}^3 \rightarrow C$ is a limited duration waveform with an average value of 0 defined for real scalars τ, a, t .*

ψ is a function of time t and scalars τ and a which are interpreted in terms of standard waveform properties.

An example wavelet is shown in Figure 4.1 with $a = 1$ and $\tau = 0$.

The scale acts like a stretching of the wavelet providing wavelets at different frequencies. The Morlet wavelet shown in Figure 4.1 has a dominant sinusoid component with a period of 1.2 s for $a = 1$, but a wavelet defined for $a = 2$ is stretched such that the dominant period is now 2.4 s . Thus, scale is related to periodicity and (loosely) inversely proportional to a sinusoidal

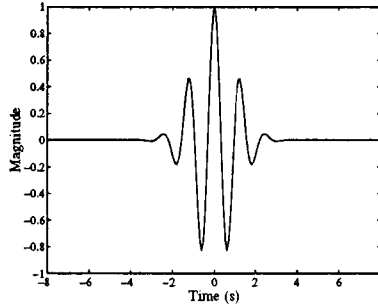


Figure 4.1: Real Morlet Wavelet for $a = 1$ and $\tau = 0$

frequency. Wavelet analysis consists of shifting this example wavelet across different time steps at various scaled values.

The position of the wavelet determines the value of time around which the waveform is centered. Using a value of $\tau = 2$ would change Figure 4.1 to be centered around time $t = 2$. Note a and τ are independent parameters and do not affect each other.

The **continuous wavelet transform (CWT)** is a signal analysis tool utilizing wavelets $\psi(\tau, a, t)$ as basis functions to represent a signal $f(t)$ in the time-scale domain [19].

Definition 4.2

$$F(\tau, a) = \int_{-\infty}^{\infty} f(t)\psi(\tau, a, t)dt$$

This continuous wavelet transform is directly analogous to the commonly

used Fourier transform. The Fourier transform decomposes a signal into a sum of infinite length sinusoids of varying frequencies while the continuous wavelet transform decomposes a signal into a sum of wavelets with varying scales and positions. Although the sinusoids in Fourier analysis are smooth and predictable, wavelets tend to be irregular and asymmetric, which may be more appropriate for describing features of certain signals [17].

The matrix $F(\tau, a)$ represents the magnitude of the signal which correlates with the wavelet $\psi(\tau, a, t)$. The better the wavelet represents the signal at a particular time and scale, the greater the value for F , which is also called a correlation coefficient. This function F can be considered a measure of the energy in the signal at a given scale and time. By having the ability to study time, scale (frequency), and correlation magnitude (energy), wavelets provide a much more comprehensive method than traditional Fourier techniques.

Other continuous wavelets used for comparison are the Haar wavelet and the Daubechies 4 wavelet. One objective before beginning a wavelet analysis, is to determine the wavelet basis that will provide the most information. The best wavelet basis should provide quantitative information not included with other wavelets. For this analysis, a wavelet that represents the signal with correlation coefficients at precise scale values is the best basis. The Haar

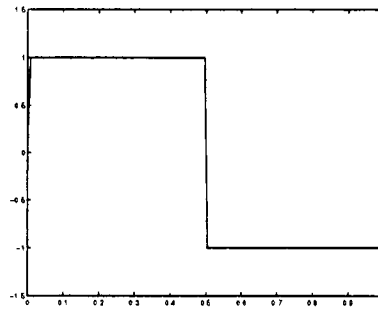


Figure 4.2: Haar Wavelet for $a = 1$ and $\tau = 0$

wavelet, also known as a square wave or “db1”, was first studied in 1910 when Alfred Haar was doing his work with orthogonal systems of functions [19]. The “db4” wavelet is one of the compactly-supported orthonormal wavelets developed by Ingrid Daubechies in her early wavelet research in the 1980s [17]. The Morlet Wavelet is another wavelet that has become standard in wavelet analysis practice [14].

A couple of interesting features of the Haar Wavelet, Figure 4.2, make the wavelet worth considering. The function is obviously not smooth on the time interval $[0,1]$, but it does resemble some sinusoidal features. This Haar Wavelet has a period of 1s for $a = 1$. Because of the jump in the middle of the function, the Haar wavelet is not a valuable tool for analyzing response signals since there is little correlation between the signal and the wavelet. Higher “magnitudes”, or energies, occur when the wavelet and the

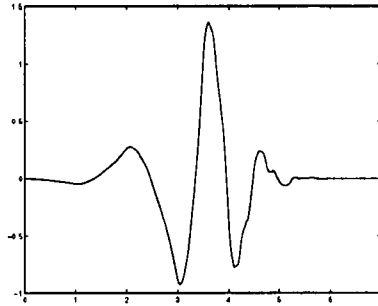


Figure 4.3: Daubachies 4 Wavelet for $a = 1$ and $\tau = 0$

signal have similar qualities. There is essentially no matching between the sinusoidal signal and the step function of the Haar wavelet. So the Haar wavelet is not a reasonable basis for this analysis.

The Daubechies 4 Wavelet, Figure 4.3, also has several unique features. There is roughly a dominant period of $1.5s$ for $a = 1$, but there appears to also be a secondary period of $2.5s$ at the same scale. Although this wavelet does a better job of analyzing the signal than the Haar, the Morlet wavelet Figure 4.1 produced the best results for the system responses studied here.

Once the wavelet coefficients are computed using the continuous wavelet transform, Definition 4.2, they are displayed as wavelet maps for analysis. Understanding what is presented in these maps is the key to understanding how useful wavelet transforms can be. Wavelet maps show the correlation coefficients, F , in the signal across time at a scale. Fourier analysis measures

energies at various frequencies, losing time domain information. The three dimensional data in the wavelet maps provides more information for analysis, therefore a better opportunity for developing a prediction function.

The following wavelet maps analyze the same signals, a nonlinear response at $U = 8.0 \text{ m/s}$ and $U = 10.0 \text{ m/s}$ airspeeds. These are sample maps for comparing how the wavelet functions approximate a signal, not for characterizing nonlinear behavior or predicting LCOs. Each map is a 2-dimensional plot of a 3-dimensional wavelet map. The correlation between the wavelet and signal at each position and scale value is represented by a shade of gray with white implying low correlation and black implying high correlation. This shading approach is not accurate with the Haar and Daubechies wavelets that spread the correlation magnitude across all the scales. For reference, the magnitude of the correlation coefficients is related to energy and the scale values to the frequencies in the signal.

Figure 4.4 shows how difficult a signal can be to understand using the Haar Wavelet in the CWT. The energy (darker area) decays across all scales rather than at a single scale, this is due to the inability of the Haar wavelet to match the signal. The limit cycle seems to be detected in the Haar wavelet map, but it is hard to determine at which scale the wing is actually vibrating.

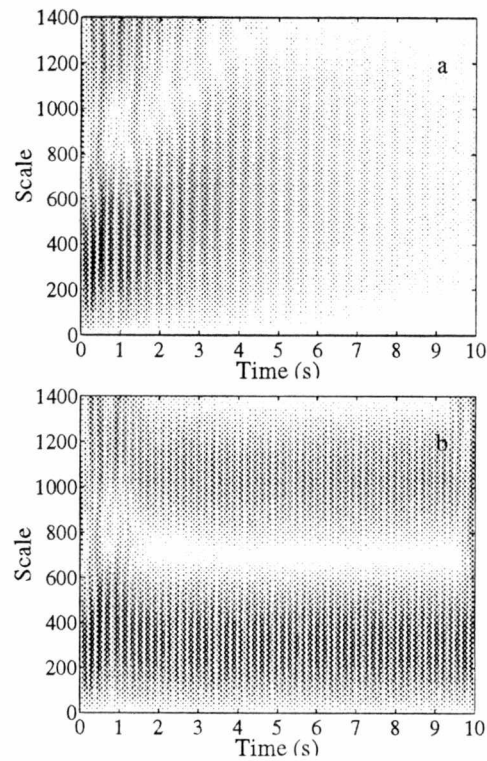


Figure 4.4: Haar Wavelet Maps for Airspeed: $U = 8.0 \text{ m/s}$ (a) and $U = 10.0 \text{ m/s}$ (b).

The step in the wavelet produces a map showing a high frequency and low frequency; this characteristic reveals more about the wavelet function than the signal. Since the wavelet is either a 0 or 1 on all scales, the signal is reduced to two scales also, providing little insight into the features of the signal. The Daubechies maps and finally the Morlet maps will make the signal more manageable.

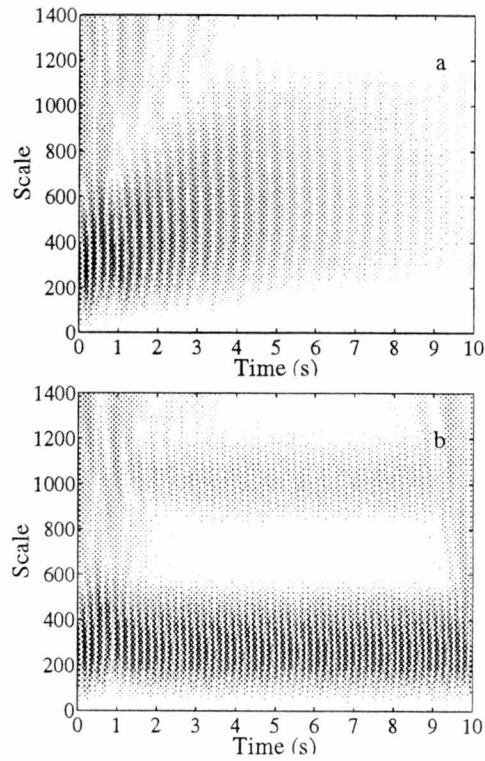


Figure 4.5: Daubechies 4 Wavelet Maps for Airspeeds: $U = 8.0 \text{ m/s}$ (a), $U = 10.0 \text{ m/s}$ (b).

Again the energy appears to be spread over several scales in Figure 4.5 making the information difficult to interpret. Since there are sinusoidal qualities in the Daubechies 4 Wavelet, there is more correlation with the signal than the Haar Wavelet produced, but not enough to study the signal with appropriate accuracy. Early times show a higher concentration of matching coefficients between the wavelet and the signal, these are the darker regions

in the maps. The second map, when $U = 10\text{ m/s}$, is after the bifurcation point and the limit cycle has formed. This map is interesting since it appears to show two regions displaying an LCO; this is due to the fact that the Daubechies 4 wavelet has a dominant and secondary period.

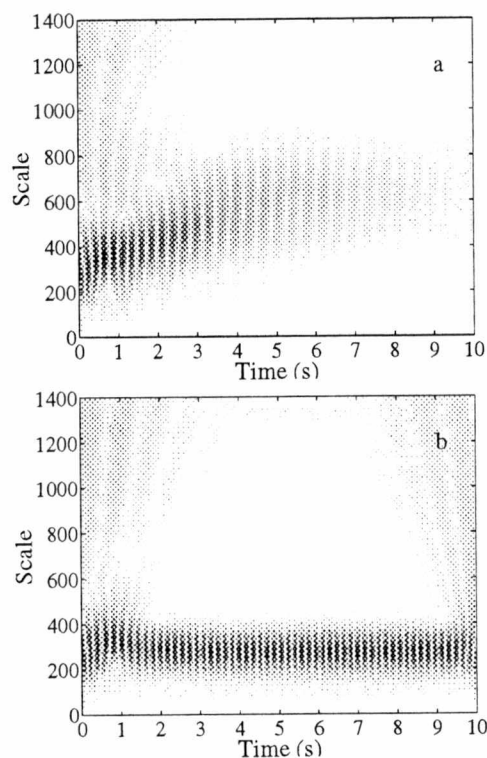


Figure 4.6: Morlet Wavelet Maps for Airspeeds: $U = 8.0\text{ m/s}$ (a), $U = 10.0\text{ m/s}$ (b).

The final examples of wavelet maps, Figure 4.6, are the from the Morlet Wavelet. This is the wavelet used for the analysis in this thesis. Compared

to Figures 4.4 and 4.5, the Morlet Wavelet provides the best approximations for analyzing the system responses studied here. The greatest magnitude correlations are only seen at particular scales and the LCO is seen at one dominant period.

Chapter 5

Time Varying Features in the Wavelet Maps

The analysis of wavelet maps may be eased by extracting information about the dominant features $F(\tau, a)$, which are the wavelet correlation coefficients. Consider a vertical strip $F(t_i, a)$ that represents the magnitude of the correlations between the signal $f(t)$ and wavelets at position $\tau = t_i$ for the vector of scales $a \in A$. Define $\overline{F}_i$ as the maximum peak magnitude correlation for this strip which corresponds to a wavelet with scale defined as $\overline{a}_i \in A$. For this study, it is assumed that the maximum $\overline{F}_i$ only exists once for each scale, $\overline{a}_i \in A$. A threshold factor is usually applied to eliminate portions

of the wavelet map for which no noticeable correlation is obtained between wavelets of any scale $a \in A$ and the response data.

$$\overline{F}_i = F(t_i, \overline{a}_i) = \max_{a \in A} F(t_i, a) = \max_{a \in A} F(\tau, a) \Big|_{\tau=t_i}$$

The parameter $\overline{a}_i$ can be utilized to determine sample length period of the dominant wavelet pattern in the data. This periodicity can be interpreted as a measure of the dominant sinusoidal frequency component in the response data under the approximations that the Morlet wavelet is essentially sinusoidal in nature on local intervals. Figure 5.1 presents the plots of $\overline{a}_i$ corresponding to the peak magnitude wavelets from the maps of the simulated responses for each of the three springs at an airspeed $U = 8$ m/s.

The wavelet maps on the right and the scales $\overline{a}_i$ corresponding to the peaks of those maps in Figure 5.1 show clear differences in the responses from each spring. These plots may not be immediately obvious to interpret; however, a careful examination reveals the wavelet analysis presents information which can be directly compared to properties of the dynamical systems. These wavelets provide a useful framework for data analysis from which properties can be extracted easier than with traditional time-domain or frequency-domain analysis.

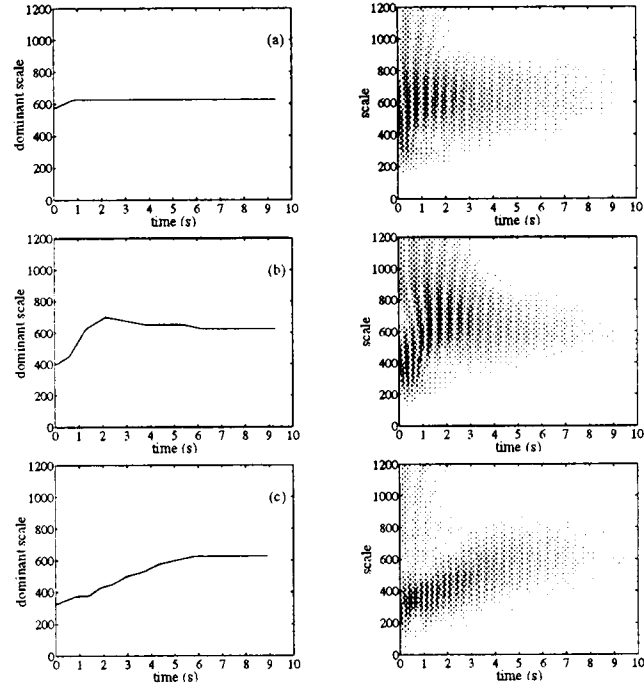


Figure 5.1: Scale $\bar{a}_i$ Corresponding to the Peak Scales of the Wavelet Transform Maps of the Pitch Data Obtained from the Models Simulated at Air-speed $U = 8 \text{ m/s}$: Linear k_{lin} (a), Nonlinear Softening k_{soft} (b), Nonlinear Hardening k_{hard} (c).

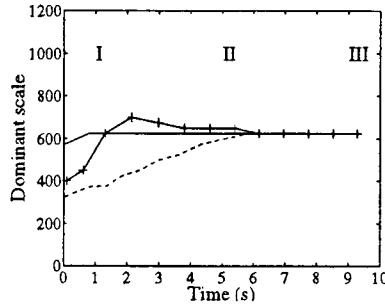


Figure 5.2: Regions for examining time varying features displayed in wavelet maps. Peak coefficients for springs in Figure 5.1 divided into three regions. Linear (—), Nonlinear Softening (+), Nonlinear Hardening (---).

Region I $0 < t < 2 \text{ s}$

Region II $2 < t < 7 \text{ s}$

Region III $7 < t < 10 \text{ s}$

Consider the wavelet information from Region I corresponding to $\tau < 2 \text{ s}$. Each transform map shows the dominant scale factor $\bar{a}_i$ is time-varying with a value increasing until a transition near the end of the Region. This transition occurs at different times for each system with the transition at $\tau = .9 \text{ s}$ for the linear system, $\tau = 1.4 \text{ s}$ for the hardening spring system and $\tau = 2.0 \text{ s}$ for the softening spring system.

The response in Region I, Figure 5.2, is dominated by the dynamics as-

sociated with a single mode as evidenced by the similarity between the time responses and wavelet maps for the pitch data. Also, the dominant scale from the wavelet maps is initially low which corresponds to a higher frequency dynamic such as the plunge mode. The plunge mode is allowed to affect both pitch and plunge measurements through the off-diagonal terms in the mass matrix of the equations of motion. The transition at the end of Region I is caused by the decay of the pitch modal response as clearly demonstrated by the pitch response in Figure 2.5. This decay is also demonstrated in the wavelet maps by a large increase in scale $\bar{a}_i$.

The response in Region II is dominated by the dynamics of the pitch motion with only a small contribution from the coupled plunge motion so this data is useful for analyzing the dynamics of a single degree of freedom pitch system. The scale $\bar{a}_i$ demonstrates significantly different behavior for the wavelet analysis of the three systems as evident from Figure 5.1.

The constant scale $\bar{a}_i$ for the linear system response in Region II is directly indicative of linear system dynamics. The response in this Region results from a linear and time-invariant system with a single mode so the dominant frequency in the response should be constant and thus the dominant wavelet scale should be constant. The Morlet wavelet with the highest correlation to

the linear system response has a scale of $\bar{a}_i = 625$ which can be interpreted as a frequency by dividing into the 1000 *Hz* sampling rate and normalizing by the 1.2 unscaled length of the wavelet. This yields a dominant frequency of 1.333 *Hz* which is similar to the predicted modal natural frequency of the pitch mode for the linear system, see Table 2.2.

$$\bar{a}_i = 625 \quad \Leftrightarrow \quad \omega = 1.33 \text{ Hz}$$

The Region II behavior of the scale $\bar{a}_i$ generated by wavelet analysis of the nonlinear softening and hardening springs is considerably different than the $\bar{a}_i$ behavior associated with the linear spring response. The $\bar{a}_i$ decreases with time for the softening spring response while $\bar{a}_i$ increases with time for the hardening spring response throughout Region II as shown in Figures 5.1b and 5.1c.

These time-varying behaviors of the scale associated with the dominant wavelet agree with expected behaviors of single mode nonlinear dynamical systems. Consider at large pitch angle amplitudes a nonlinear hardening spring will incur a larger restoring force than that incurred by a linear spring. Thus, the decay response of a nonlinear hardening spring system will approach the origin sooner and the response will demonstrate a higher frequency as compared to a linear spring system response. A similar analysis

Table 5.1: Values of $k_\alpha(\alpha)$ at Large and Small Pitch Angles

spring	$\alpha = .06 \text{ rad}$	$\alpha = .01 \text{ rad}$
k_{lin}	2.82	2.82
k_{soft}	2.23	2.80
k_{hard}	7.84	2.54

anticipates the response of a nonlinear softening spring should demonstrate a lower frequency than a linear spring response. The first column of Table 5.1 presents the values of the spring stiffness $k_\alpha(\alpha)$ for a pitch angle that is relevant to Region II to show the difference in restoring force for the different springs.

The difference in response frequency caused by these different spring values can be seen by simulating the model responses with the plunge value constrained to zero. Figure 5.3 presents the pitch responses for this example simulation to clearly demonstrate the different frequencies in the linear and nonlinear system responses.

The Region II behavior of the dominant waveform period for the nonlinear springs matches this simple time domain dynamical analysis. The scales $\bar{a}_i$ are lower for the response of the nonlinear hardening spring than for the

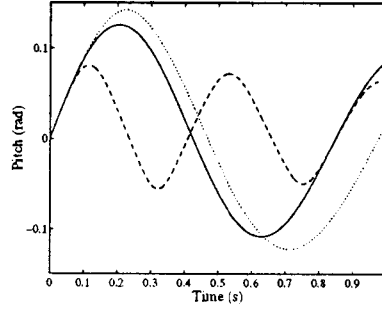


Figure 5.3: Example Simulation of Large Amplitude Time Domain Responses of the Pitch Angle with Plunge Constrained to a Constant : Linear k_{lin} (—), Nonlinear Softening k_{soft} (...), Nonlinear Hardening k_{hard} (- - -)

linear spring, indicating the dominant frequency in the response is higher for the nonlinear system. Conversely, the scales $\bar{a}_i$ are greater for the response of the nonlinear softening spring than from the linear spring, indicating the dominant frequency in the response is lower for the nonlinear system.

Another feature of Region II is the time-varying behavior of the scales $\bar{a}_i$ corresponding to the nonlinear system responses which are smoothly changing until they reach a constant steady-state value in Region III. This behavior, indicative of a nonlinear system, can be explained by further considering the simple example presented in Figure 5.3. The difference in the restoring forces for the different types of springs is greatest for large amplitudes

of the pitch angle. Consequently, as the response of each system decays to the stable pitch angle of 0 rad , the nonlinear system responses will more closely resemble the linear system responses. Region II responses are characterized by initial large amplitude motion with decreasing pitch angles; this is reflected by the initial large differences in $\bar{a}_i$ values between linear and nonlinear responses with a decrease in this difference as time progresses.

The wavelet analysis of Region II responses are clearly indicative of the presence of nonlinearities under the assumptions of free decay responses from single mode time-invariant systems. Furthermore, information about the type of those nonlinearities can be obtained by noting that the time-varying nature of the scales $\bar{a}_i$ indicates the time-varying nature of the dominant local frequency in the responses.

The Region III analysis from Figure 5.2 demonstrates that the responses from each system have the same dominant frequency component, i.e. similar scales $\bar{a}_i$. This similarity is expected since the pitch magnitudes have decayed to small values in this region due to damping so the restoring forces for each spring, as shown in Table 5.1, are nearly identical. Thus, the system responses have similar periodicity and correspondingly the scales $\bar{a}_i$ are similar. This is evidenced by extending the simulation of the small example

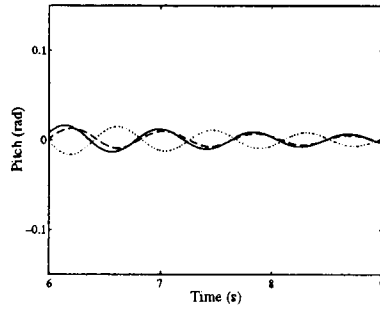


Figure 5.4: Example Simulation of Small Amplitude Time Domain Responses of the Pitch Angle with Plunge Constrained to a Constant : Linear k_{lin} (—), Nonlinear Softening k_{soft} (...), Nonlinear Hardening k_{hard} (---)

from Figure 5.3 for a longer length of time to produce the responses shown in Figure 5.4. These responses may differ in phase but they are close in frequency and magnitude to agree with the wavelet analysis results.

The dominant wavelet period associated with the response of each system is shown to be $\bar{a}_i = 625$ from Figure 5.1, which was noted previously to correspond with a waveform frequency of approximately $\omega = 1.33 \text{ Hz}$. This frequency matches well with the natural frequency of the linearized dynamics to indicate the responses are small amplitudes and the nonlinearities are not strongly excited. Also, the scales $\bar{a}_i$ are constant throughout the Region III responses for each system. This behavior is expected since the system responses, which initially have a small amplitude, are decaying to even smaller

amplitudes. Thus, the nonlinear system responses remain similar to the linear system response and the wavelet analysis notes the dominant frequency component in each response corresponds to the natural frequency of the pitch mode throughout this region.

The analysis in this section demonstrates that wavelet maps and scales $\bar{a}_i$ corresponding to peaks in the wavelet maps may be used to detect nonlinearities from response data for the Texas A&M University aeroelastic system. The time-varying behavior of the $\bar{a}_i$ scales in Region II clearly demonstrates differences between the linear system response and nonlinear system responses. The time-varying nature of the $\bar{a}_i$ scales can even be used to determine if the nonlinearity is similar to a hardening or softening spring effect.

It is important to note there were several restrictive assumptions made in this section that allowed wavelet transforms to be effective in detecting and characterizing nonlinearities. It was always assumed the analysis considered free decay response data from a time-invariant system. The time-varying $\bar{a}_i$ scale for such data could only be caused by a nonlinearity since linear systems theory would preclude such a time-varying response frequency.

This thesis makes no general claims about the ability of wavelets to detect

nonlinearities when analyzing data that violates the assumptions. Example systems could be formulated in which a forced response of a linear time-invariant system could generate wavelet maps similar to those in Figure 5.1. Examples systems could also be generated in which the free decay response of a linear time-varying system could generate these wavelet maps. Therefore, the wavelet analysis presented here is only immediately applicable to systems which are similar in nature to the Texas A&M University system, see also [14].

Chapter 6

Predicting Limit Cycle

Behavior

A new set of simulated responses is now presented for applying wavelet analysis to this system. The focus is again on extracting the dominant features in the wavelet maps; looking for the nonlinear signatures and the limit cycles. The simulated responses presented are for the measurements in the decay of the pitch angle. For this thesis only the pitch responses are considered for the development of the prediction functions. The plunge responses are presented in Appendix J with example responses for an uncoupled system. The results presented here are from the analysis of the pitch decay responses; the

source of the distinguishing features, whether from coupling between pitch and plunge or from the nonlinear spring, is not considered in the analysis. Using the fourth-order polynomial hardening spring $k_\alpha(\alpha)$, and the initial conditions presented at time $t=0$ secs, the following simulated time histories are produced, Figure 6.1.

$$k_{hard} = 2.82 - 62.3\alpha + 3709.7\alpha^2 - 24196.0\alpha^3 + 48757\alpha^4$$

$$y(t_o) = 0.0 \text{ m} \qquad \dot{y}(t_o) = 0 \text{ m/s}$$

$$\alpha(t_o) = 0.3 \text{ rad} \qquad \dot{\alpha}(t_o) = 0 \text{ rad/s}$$

Under these initial conditions, the system starts further away from a linearized equilibrium point. The initial decay rate is very fast, as the wing vibrates at greater angles before falling to zero at low airspeeds and the limit cycle for high airspeeds. The frequency of the system under these initial conditions also needs attention. As time increases, the frequency decreases and converges to the natural frequency of the linearized dynamics (see Table 2.2), so as the motion approaches zero the response begins to reflect a linear system.

The limit cycle is again observed in the final two plots where the airspeed $U \geq 10 \text{ m/s}$. After the bifurcation occurs at $U = 9.8 \text{ m/s}$, the equilibria of the system have changed such that the origin is now unstable and a stable

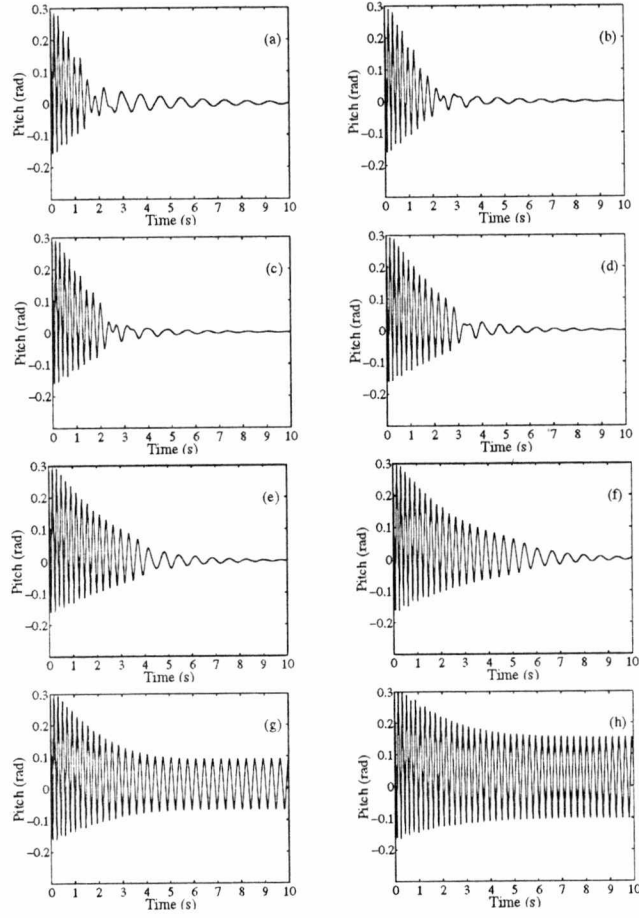


Figure 6.1: Simulated Responses for Pitch (rad/s) with the Nonlinear Hardening Spring k_{hard} : $U=2.0$ m/s (a), $U=4.0$ m/s (b), $U=6.0$ m/s (c), $U=8.0$ m/s (d), $U=9.0$ m/s (e), $U=9.5$ m/s (f), $U=10.0$ m/s (g), $U=12.0$ m/s (h).

periodic attracting set is created. It is expected that the amplitude of the limit cycle at $U = 12 \text{ m/s}$ is greater than the amplitude at $U = 10 \text{ m/s}$, since the behaviors of the system have been changing with airspeed. The radius of the periodic attracting set in phase-plane space increases as U increases, so the magnitude of the LCO increases. The increase in motion at higher airspeeds is caused by a decrease in damping. Since the Lift and Moment equations (L and M in Chapter 2) are functions of $\dot{\alpha}$ and $\dot{y}$, there is airspeed damping. But $\dot{\alpha}$ and $\dot{y}$ are scaled by $1/U$, so as U increases, then airspeed damping decreases.

Returning to the phase subspace plots now that the motion of the wing has been determined, there are several features to notice in Figure 6.2. These plots show the change in pitch against pitch rate for the time domain responses in Figure 6.1. The decay in the motion of the wing can be seen by the decreasing amplitude of the loops in the phase plane plots. The size of the loop is determined by the magnitude of the change between pitch and pitch rate. At higher airspeeds, there is more motion caused by a decrease in damping, so there are more orbits before reaching equilibrium. Another feature that is in the phase subspace plots, which is easily seen in the time domain responses, is the tendency towards positive pitch angles. This fact

is attributed to the asymmetry in the nonlinear stiffness parameter, and describes where the wing is physically vibrating. So the asymmetry in the loops shows that the restoring force is greater for negative pitch angles than for positive pitch angles.

Wavelet transforms are computed for simulated pitch angle responses of the nonlinear models at each airspeed until the onset of a limit cycle oscillation. Figure 6.3 presents the wavelet maps computed with real Morlet wavelets as basis functions. Several of the characteristics that are evident in the time domain and phase plane plots are also evident in the wavelet maps. The time varying trends are evident as the shading of the responses changes across the maps. Darker colors represent higher correlations between data and the wavelets. By considering the inverse of scale as a measure of frequency, the figure shows that the dominant scale is increasing, so the dominant frequency is decreasing. As the color fades to higher scale values for $U < 10$ m/s, this constant scale correlates to the time-invariant frequency seen in Figure 6.1. Again, the responses show linear characteristics for small values of the pitch angle. The LCO is obvious for $U = 10$ m/s and $U = 12$ m/s, since the color is showing a time-invariant scale with constant magnitude after the initial seconds. The wing is vibrating around a periodic

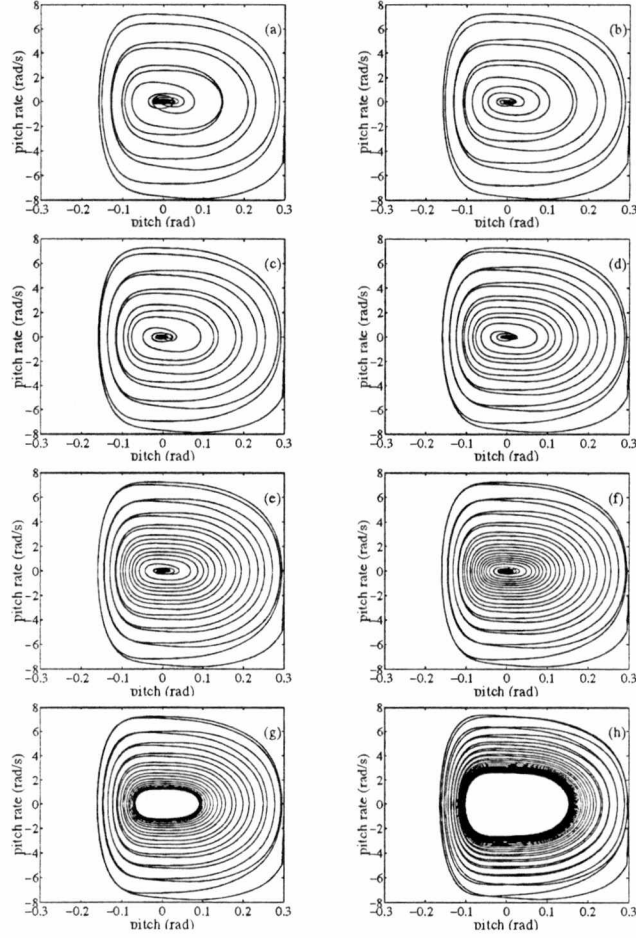


Figure 6.2: Phase Subspace Plots of Pitch Rate (rad/s) versus Pitch (rad) for Simulated Responses of the Model with the Nonlinear Hardening Spring k_{hard} : $U=2.0$ m/s (a), $U=4.0$ m/s (b), $U=6.0$ m/s (c), $U=8.0$ m/s (d), $U=9.0$ m/s (e), $U=9.5$ m/s (f), $U=10.0$ m/s (g), $U=12.0$ m/s (h).

attracting set, in the four dimensional phase space, with constant magnitude and scale. Appendix J shows the limit cycle projected onto the plunge subspace. The color gradient also describes the change in energy in time. The energy that is driving the wing to vibrate at low airspeeds eventually fades in time leaving no evidence. At airspeeds that invoke an LCO, the energy also becomes constant and does not vary after the LCO has begun. Unfortunately, not all the features evident in the time domain are seen in the wavelet maps. The difference in magnitude between $U = 10 \text{ m/s}$ and $U = 12 \text{ m/s}$ limit cycles cannot be visually extracted from the wavelet maps, but the coefficients are calculated and can be used for numerical analysis. Another characteristic not detected is the asymmetry caused by the nonlinearity towards positive pitch angle.

Several features and trends can be noted from a qualitative analysis and visual inspection of Figure 6.3. One immediately obvious feature is the high correlation for small scale wavelets, centered around $a \approx 200$, with the response data for times $t < 2 \text{ s}$. This feature correlates with the Region I response with the same nonlinear hardening spring noted in Figure 5.1; however, the scales in that region were considerably higher. Therefore, this feature appears to be strongly related to the initial condition since the hardening

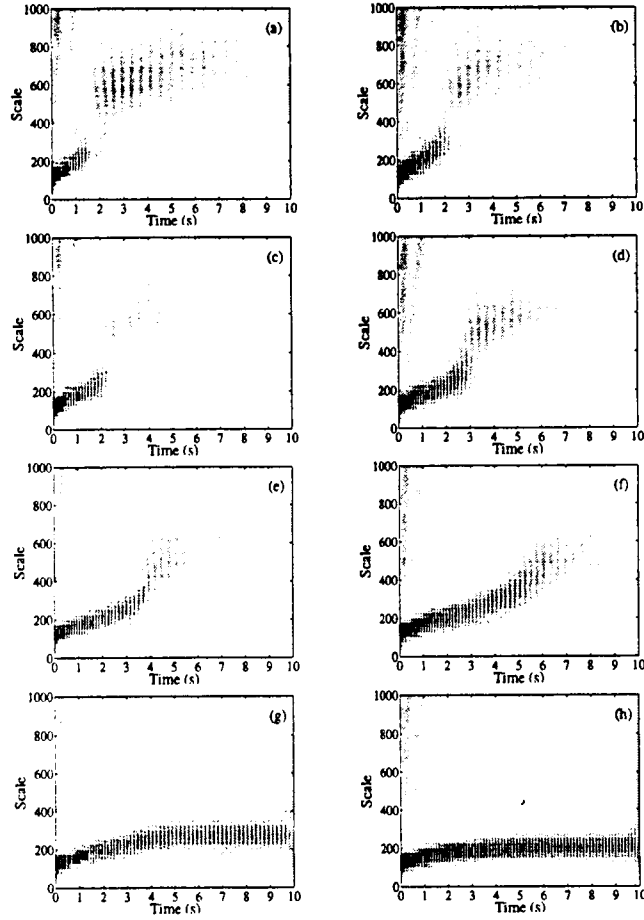


Figure 6.3: Wavelet Transform Maps of the Pitch Data Obtained from the Model Simulations with Nonlinear Hardening Spring k_{hard} for Airspeeds in m/s : $U=2.0 \text{ m/s}$ (a), $U=4.0 \text{ m/s}$ (b), $U=6.0 \text{ m/s}$ (c), $U=8.0 \text{ m/s}$ (d), $U=9.0 \text{ m/s}$ (e), $U=9.5 \text{ m/s}$ (f), $U=10.0 \text{ m/s}$ (g), $U=12.0 \text{ m/s}$ (h).

spring's responses in Figure 5.1 and Figure 6.3 differ only by initial condition. This feature is also related to the type of nonlinearity since the larger initial condition has the effect of more strongly exciting the nonlinearity.

Another immediately obvious feature is the changing behavior of the transient response after time $t = 2$ s for low airspeeds. At the lowest simulated airspeed of $U = 2.0$ m/s, the response data demonstrates a different dominant periodicity between times less than and greater than $t = 2$ s. The scales associated with these motions exhibit a drastic change across this time value with the dominant scale of $a = 200$ for $t < 2$ s and a dominant scale of $a = 700$ for $t > 2$ s. This behavior remains as airspeed increases with the time of the discontinuity increasing and the magnitude of the discontinuity decreasing. As stated in Chapter 5, this discontinuity can be attributed to the interaction of the plunge motion in the early response times. An uncoupled system does not demonstrate such a discontinuous change in frequency, see Figure J.1.

This feature alters at airspeeds $U > 8$ m/s with no discontinuity appearing at any time during the simulation. The scale associated with the dominant periodicity appears to be a smoothly increasing function as the response time continues. This transition is an interesting feature since the

response seems to indicate the impending limit cycle by a delayed transition between the initial low scale periodicity and the final higher scale periodicity as airspeed increases towards the bifurcation.

These features and trends noted by the visual analysis of the wavelet maps can be exploited to develop formal mathematical analysis tools which may indicate the parameters of a bifurcation point and the onset of a limit cycle oscillation. The dominant features of the wavelet maps will be utilized by again computing the highest correlation between the wavelets centered at time $\tau = t_i$ and the response signal. Recall the definition of this highest correlation $\overline{F}_i$ and the scale $\overline{a}_i$ for the wavelet associated with this correlation.

$$\overline{F}_i = F(t_i, \overline{a}_i) = \max_{a \in A} F(t_i, a) = \max_{a \in A} F(\tau, a) \Big|_{\tau=t_i}$$

Using this definition, **prediction functions** are developed by looking at the change in scales, $\overline{a}_i$, across time and the change in energy, $\overline{F}_i$, across time. The Scale Analysis has also been presented in other papers [14].

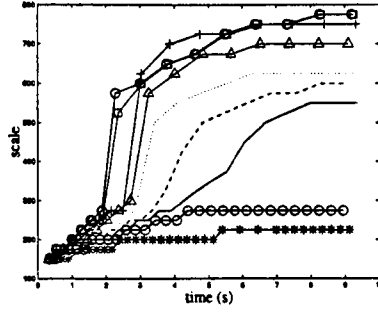


Figure 6.4: Scale $\bar{a}_i$ Corresponding to Peaks of the Wavelet Transform Maps of the Pitch Data Obtained from the Model with Nonlinear Hardening k_{hard} for Airspeeds in m/s : $U=0.5$ ($\square$), $U=2.0$ ($\circ$), $U=4.0$ ($+$), $U=6.0$ ($\triangle$), $U=8.0$ (...), $U=9.0$ (- - -), $U=9.5$ (—), $U=10.0$ ($\circ$), $U=12.0$ (*)

6.1 Scale Analysis

Figure 6.4 presents the $\bar{a}_i$ scale factor as a function of time for each different airspeed. These functions contain only the dominant information of Figure 6.3 and thus are easier to analyze than the wavelet maps, but they still demonstrate the interesting features and trends that were noted by the visual analysis. The $\bar{a}_i$ values for times $t < 2$ s show the similarity for each airspeed. Also, the transition from a low scale dominant periodicity to a higher scale one is easily seen at times $t = 2$ s for low airspeeds with a delayed transition for higher airspeeds.

The transition between dominant scales in the responses presents several characteristics that may be exploited to develop algorithms which may predict bifurcations for this system. The main properties of the transition appear to be the smoothness and magnitude of the change from a low scale to a high scale along with the time position at which this change occurs. This dramatic change can be attributed to the effects of coupling between the pitch and plunge degrees of freedom. For times $t < 2$, Region I, pitch and plunge are both significant factors determining the motion of the wing. In Region II, the plunge motion decays much faster than the pitch motion, explaining the abrupt change in frequency in the pitch responses. Without the coupling, the pitch decay looks almost linear and there is not a drastic change in frequency, see Figure J.1. Finally, in Region III $t > 7$ secs, the pitch responses at different airspeeds show no effects of the plunge motion that has decayed to zero. Thus, the change in scales is also demonstrating the effects of coupling in the system. The derivatives with respect to time of the $\bar{a}_i$ functions are computed to extract some of this information and are presented in Figure 6.5. These derivatives are step function differences between two consecutive scales in time, defined as an Euler derivative. See Appendix F.

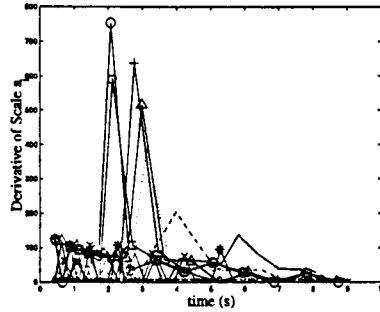


Figure 6.5: Derivatives of Scale $\bar{a}_i$ Corresponding to Peaks of the Wavelet Transform Maps of the Pitch Data Obtained from the Model with Nonlinear Hardening k_{hard} for Airspeeds in m/s : $U=0.5$ ($\square$), $U=2.0$ (o), $U=4.0$ ($+$), $U=6.0$ ($\triangle$), $U=8.0$ ($\dots$), $U=9.0$ ($- - -$), $U=9.5$ ($-$), $U=10.0$ (O), $U=12.0$ ($*$)

The difference derivatives in Figure 6.5 provide a quantitative description of the transition from a dominant low scale waveform to a high scale waveform in the response data. The magnitude of the jump during this transition is clearly seen to be large for the responses at low airspeeds and decreasing as the airspeed increases. Also, the time position at which the largest jump occurs is seen to increase as the airspeed increases. These two features combine to signify the transition from a dominant low scale waveform to a high scale waveform is becoming smoother and delayed as the airspeed increases towards the bifurcation point.

The interesting features of Figure 6.5, namely the magnitude and time of

the peak change of the dominant scale functions, can be extracted to focus on this information. Define parameters Δ_a and t_a such that the difference between $\bar{a}_i$ and a_{i+1}^- achieves its maximum values of Δ_a when evaluated at t_a ; this differencing method is considered a step function derivative. These parameters are functions of airspeed since different Δ_a and t_a correspond to responses from different airspeeds.

$$\Delta_a = \left. \frac{da_i}{dt} \right|_{t=t_a} = \max_{\tau} \left. \frac{da_i}{dt} \right|_{t=\tau}$$

Figure 6.6 presents the values of Δ_a as they vary with airspeed. The curve is computed with eight points corresponding to the eight airspeeds used to compute the derivative functions in Figure 6.5. This plot clearly shows the decrease in magnitude of the peak derivative of the dominant scale as the airspeed is increased. The first point at $U = 0.5 \text{ m/s}$ is removed, because the airspeed is very low and is producing a near pure structural response. Such low airspeeds are nearly linearized and can be neglected, since no valuable information can be obtained.

Figure 6.7 presents the times t_a at which the responses demonstrate the largest change in dominant scale corresponding to the peak derivative Δ_a values from Figure 6.6. This plot clearly shows the time at which the transition from a low to a high dominant scale occurs is delayed as the airspeed is

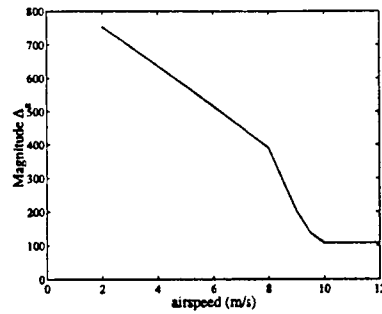


Figure 6.6: Δ_a Variation with Airspeed Corresponding to Wavelet Maps of Figures 6.3,6.4,6.5

increased. After the bifurcation airspeed, a limit cycle will always form, but this figure shows that the LCO takes longer to develop at higher airspeeds.

The information from Figure 6.6 and Figure 6.7 suggests a relationship that may be used to formulate a limit cycle prediction function. There are several desired properties for this function to have, including being smooth and well behaved since any sharp nonlinearities would prevent accurate extrapolation from stable to unstable flight conditions. Another desired function property is linearity since this would allow accurate extrapolation between values at different flight conditions. Also, this function should have a well defined terminus condition that clearly indicates the onset of a limit cycle.

A prediction function, ϕ , can be formulated from the wavelet transform

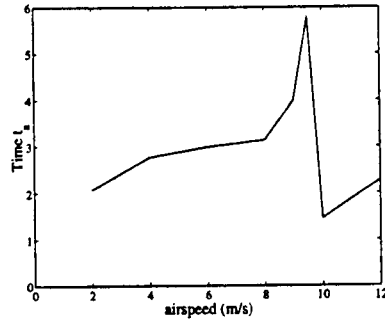


Figure 6.7: t_a Variation with Airspeed Corresponding to Wavelet Maps of Figures 6.3,6.4,6.5

information by dividing the Δ_a values from Figure 6.6 with the t_a values from Figure 6.7.

$$\phi = \frac{\Delta_a}{t_a}$$

This function ϕ should be an acceptable prediction function and have the desired properties. The variation of ϕ with airspeed should be relatively smooth and well behaved since both Δ_a and t_a are smooth and well behaved. ϕ will have some degree of linearity since the rate of change of the decreasing Δ_a values is similar to the rate of change of the increasing t_a values. Also, there is a well defined terminus condition of $\phi = 0$ for the airspeed at which a limit cycle is encountered. This condition arises because any transition between dominant scales for a limit cycle response is smooth and delayed so Δ_a is small while t_a is large causing $\phi \approx 0$. The experimentally computed

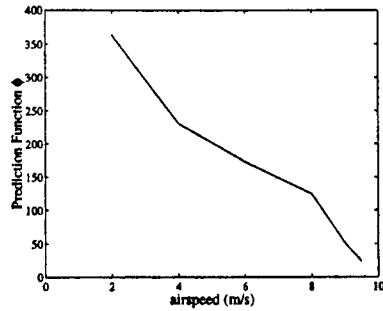


Figure 6.8: Prediction Function ϕ_S

values of ϕ are listed in Table 6.1 and plotted in Figure 6.8.

A linear analytical function $\hat{\phi}$ can be computed from an optimal curve fit to the experimental values of ϕ .

$$\hat{\phi}_S = -44U + 435$$

This function is computed using responses from airspeeds ranging up to the onset of a limit cycle so it logically predicts the correct unstable airspeed of $U = 9.8 \text{ m/s}$; however, such a large amount of information is not necessary. The near linearity of ϕ in Figure 6.8 indicates a function $\hat{\phi}$ could be computed using only responses at airspeeds well below the instability. For example, an optimal linear curve fit of ϕ from airspeeds $U = \{2, 4, 6\} \text{ m/s}$ produces a function $\hat{\phi}$ that predicts the onset of a limit cycle at an airspeed which is within 5% of the true value. Thus, the wavelet transform provides information about this system that can predict a limit cycle from responses

Table 6.1: Wavelet Variables and Prediction Function for Scale Analysis

U	Δ_a	t_a	ϕ
2.0 m/s	753.8	2.07 s	363.6
4.0 m/s	637.5	2.77 s	230.5
6.0 m/s	515.9	2.98 s	173.0
8.0 m/s	389.9	3.13 s	124.6
9.0 m/s	203.6	3.97 s	51.2
9.5 m/s	137.3	5.80 s	23.7

at a limited range of stable flight conditions.

The ability to formulate an accurate predictor function $\hat{\phi}$ is an almost optimal situation for experimental testing; however, the individual properties of Δ_a and t_a functions should also be monitored since their individual behaviors are affected by the flight condition. The Δ_a parameter increases and the t_a parameter decreases near the instability so a threshold test on their values could be used to indicate the proximity to a limit cycle. Also, each parameter is nearly linear until flight regions approach a bifurcation, so another test on the linearity of their behavior could be used to indicate the proximity to a limit cycle. These tests are basically stability indicator tests,

rather than stability predictor tests, but they provide additional information which can augment traditional stability indicators such as damping ratios and any experimental stability predictor functions such as $\hat{\phi}_S$. The combination of these different types of information may decrease errors in stability estimates caused by noise, unobserved dynamics and non-repeatability.

There is more information besides Δ_a and t_a associated with the pitch response data that can be utilized to formulate stability indicators and prediction functions. The peak magnitude $\overline{F}_i$ of the correlation between the signal and the wavelets can be interpreted as a measure of energy in the signal so its decay rate and other trends may indicate properties of the non-linear dynamics, Section 6.2. Also, the same procedures used to develop $\hat{\phi}$ from pitch response data may be used to develop prediction functions from the plunge response data.

The approach adopted to formulate $\hat{\phi}_S$ in Figure 6.8 utilizes local properties of the response data to determine suitable coefficients for the function. This approach is not based on analytical properties of the dynamics and there may not be an obvious physical interpretation of the resulting function; however, this type of approach is often the only useful method. Analytical solutions do not exist for many nonlinear differential equations which de-

scribe dynamical systems so experimental testing must be used to determine characteristics and properties. Indeed, one of the few functions which can accurately predict the onset of a limit cycle for a particular system is a fourth order polynomial which was developed in a manner similar to $\hat{\phi}_S$ [21]. This polynomial is formulated from a curve fit algorithm using localized frequency properties computed from experimental data and is therefore analogous to the localized periodicity properties used to compute $\hat{\phi}_S$.

There are several aspects associated with the response data that have not been considered in the formulation of $\hat{\phi}$ that should be investigated in future research. The main property of the wavelet maps that was used to develop this function is the transition from a dominant low scale periodicity to a dominant high scale periodicity in the response data, disregarding the source of this change. One aspect of the response that may greatly affect this transition is the initial conditions used to begin the free decay motion. Initial conditions with a small magnitude will not strongly excite the nonlinearity so it is unclear how information of a transition can be detected with the wavelet transforms. Another aspect to consider is the response analysis at very low airspeeds because the function $\hat{\phi}_S$ only considered responses for airspeeds $U > 1 \text{ m/s}$. The wavelet maps from the responses at very slow airspeeds

do not seem to have the same features and trends which may indicate some strength of aerodynamic force coupling is required.

6.2 Energy Analysis

Continuing along the same lines, a prediction function from the magnitude data is considered. Figure 6.9 presents the $\bar{F}_i$ magnitude as a function of time for each different airspeed. These functions are measures of how well the wavelet functions match the actual signal. The darker regions in the wavelet maps, Figure 6.3, represent closer correlations between wavelet and signal, which is referred to as energy. The peak magnitudes $\bar{F}_i$ are calculated and plotted for each airspeed in time.

As in the scale trends, there is a transition that may provide insight into how to form a prediction algorithm. The peak correlations appear to dominate the early responses when the wing is initially set in motion. The peak magnitudes for low airspeeds drop as time increases demonstrating less energy and the decay that is evident in all the wavelet maps, Figure 6.3. As expected, the LCO is detected and displays a constant magnitude after $t = 6s$. This characteristic is not surprising considering the near sinusoidal

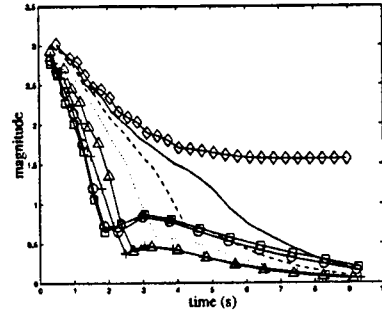


Figure 6.9: Magnitude $\bar{F}_i$ Corresponding to Peaks of the Wavelet Transform Maps of the Pitch Data Obtained from the Model with Nonlinear Hardening k_{hard} for Airspeeds in m/s : $U=0.5$ ($\square$), $U=2.0$ ($\circ$), $U=4.0$ ($+$), $U=6.0$ ($\triangle$), $U=8.0$ (...), $U=9.0$ (- - -), $U=9.5$ (—), $U=10.0$ ($\diamond$)

qualities of the Morlet wavelet and the periodic motion for an LCO. With the wavelet matching the signal, the correlation magnitude is high and constant during the LCO.

The derivative information of energy, $\bar{F}_i$, is provided in Figure 6.10. These derivatives are calculated using the same step function algorithm that calculated the scale derivatives in the previous section. Since the energies for each airspeed are decreasing in time, the difference derivatives are negative values, each point has less energy than the point before it. It appears that it takes longer for the higher airspeed responses to lose energy.

For extracting the dominant features from Figure 6.10, we will use the

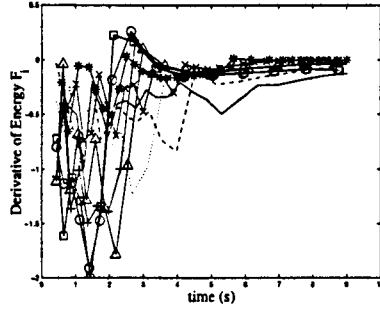


Figure 6.10: Derivative of Magnitude $\bar{F}_i$ Corresponding to Peaks of the Wavelet Transform Maps of the Pitch Data Obtained from the Model with Nonlinear Hardening k_{hard} for Airspeeds in m/s : $U=0.5$ ($\square$), $U=2.0$ ($\circ$), $U=4.0$ ($+$), $U=6.0$ ($\triangle$), $U=8.0$ (...), $U=9.0$ (- - -), $U=9.5$ (—)

same techniques as with the dominant scale qualities. Define parameters Δ_F and t_F such that the derivative of $\bar{F}_i$ achieves its maximum values of Δ_F when evaluated at t_F . These parameters are functions of airspeed since different Δ_F and t_F correspond to responses from different airspeeds.

$$\Delta_F = \left. \frac{d \bar{F}_i}{dt} \right|_{t=t_F} = \max_{\tau} \left. \frac{d \bar{F}_i}{dt} \right|_{t=\tau}$$

Figure 6.11 presents the values of Δ_F as they vary with airspeed. The curve is computed with 7 points corresponding to the 7 airspeeds used to compute the derivative functions in Figure 6.10. This plot clearly shows the decrease in magnitude of the peak differences of the change in energy as the

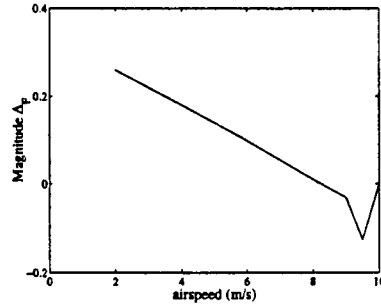


Figure 6.11: Δ_F Variation with Airspeed Corresponding to Wavelet Maps of Figures 6.3,6.9,6.10

airspeed is increased. The sharp spike in the higher airspeeds is due to the sensitivity of the differencing method used to calculate the derivatives. The Δ_F drops just below zero before the limit cycle forms and then stays at zero for the limit cycle.

Figure 6.12 presents the times t_F at which the responses demonstrate the largest change in energy corresponding to the peak derivative Δ_F values from Figure 6.11. This plot clearly shows the time at which the transition from high to low energy occurs is delayed as the airspeed is increased. Since energy remains constant in the LCO, there is no change in time for airspeeds $U > 9.8 \text{ m/s}$.

Finally, Figure 6.13 presents a prediction function for the onset of an LCO from an energy perspective. By studying the change in energy with

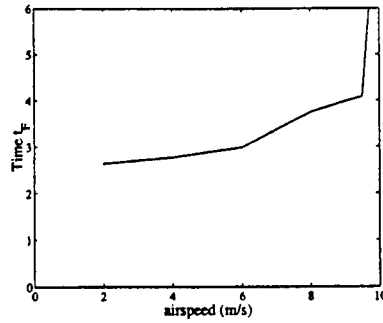


Figure 6.12: t_F Variation with Airspeed Corresponding to Wavelet Maps of Figures 6.3,6.9,6.10

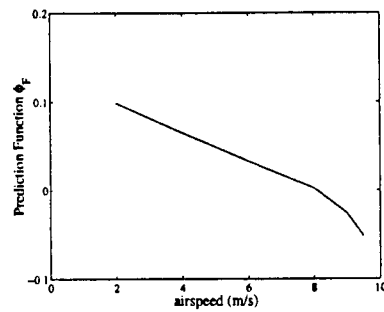


Figure 6.13: Prediction Function ϕ_F

respect to time and airspeed, the ability to predict an impending limit cycle is evident. As in the previous case, the best prediction function is smooth, linear, and has a well-defined terminus. $\phi = \frac{\Delta E}{t_F}$, produces such a function.

Again, a linear analytical function $\hat{\phi}$ can be computed from an optimal curve fit to the experimental values of ϕ_F .

$$\hat{\phi}_F = -0.057U + 0.405$$

Table 6.2: Wavelet Variables and Prediction Function for Energy Analysis

U	Δ_F	t_F	ϕ_F
2.0 m/s	0.26	2.63 s	0.0989
4.0 m/s	0.18	2.77 s	0.0650
6.0 m/s	0.10	2.98 s	0.0336
8.0 m/s	0.01	3.74 s	0.0027
9.0 m/s	-0.10	3.97 s	-0.0252
9.5 m/s	-0.21	4.08 s	-0.0515

This prediction function should be considered accurate since the function ϕ_S developed in the last section predicts the onset of the LCO. Magnitude and scale are related characteristics of a signal, so some degree of agreement is expected when detecting the Limit Cycle Oscillation.

Chapter 7

Conclusions and Recommendations

This thesis develops a methodology to analyze nonlinear dynamical systems by utilizing wavelet transforms. Responses from the Texas A&M University aeroelastic system with a nonlinear stiffness function are used to demonstrate this methodology. The analysis is based on wavelet transform maps that compute time-scale information describing the localized periodicity in experimental data. Properties of these wavelet transform maps from transient response data are demonstrated to detect and characterize the type of nonlinearities affecting the stiffness. These maps are also used to analyze transient

responses from a range of airspeeds to formulate analytical functions that predict the onset of a limit cycle behavior resulting from a Hopf bifurcation. The analysis functions developed in this paper are directly applicable only to the Texas A&M University system but the underlying methodology used in their derivation may be considered for analysis of general nonlinear systems.

This thesis presents results that are directly applicable to a specific system and may not be useful for general nonlinear systems. These results are formulated by exploiting dynamical properties of the Texas A&M University aeroelastic wind tunnel system with a polynomial stiffness function. Formulating similar results for a general class of nonlinear systems would require evaluating dynamical properties specific to that class.

The detection and characterization of nonlinearities is accomplished using wavelet transform maps because of several strong assumptions. These assumptions require response data to be generated from a free decay of time-invariant dynamics which are dominated by a single mode. Additionally, the nonlinearity in the dynamics is assumed to affect the stiffness. Methods of detection and characterization of nonlinearities can probably be developed for nonlinear systems which violate these assumptions but properties other than those presented in this thesis will need to be investigated.

The prediction functions $\hat{\phi}_s$ and $\hat{\phi}_E$ are formulated for the Texas A&M University system with a particular nonlinear polynomial stiffness function and generally should not be accurate for other systems. The order and coefficients of these functions would need to be altered to reflect the properties of a different dynamical system including even the Texas A&M University system with a different stiffness polynomial.

The wavelet based approach presented in this thesis, however, is still applicable to general nonlinear systems since these limitations are associated with the specific analysis functions rather than the methodology used in their derivation. The wavelet based approach can be utilized with a different class of systems to derive new properties for detecting and characterizing nonlinearities and to formulate new functions for predicting the onset of a limit cycle. This approach can be considered for future investigation of many classes of nonlinear systems with behaviors including hysteresis and free play along with different structural and aerodynamic nonlinearities.

With the success of developing the prediction function from the analysis of the pitch data using the scale information, there are several other ideas that can be tested. The simulated data and prediction function can be compared to actual flight data. A control surface, such as a flap, may be added

to the parameters for simulating new responses. Different hardening or softening springs will produce different signals which may provide more insight. Another direction would be to study the plunge data to see what would come from that analysis. Wavelets have opened new doors and are providing new paths for signal analysis, so there is still plenty of work to do here.

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Appendix

Appendix A

Load initial conditions and constants

```
%-----  
% initial conditions, operating parameters and nonlinearity  
%-----  
plunge_init = .01;   % initial condition for plunge  
pitch_init = .1;    % initial condition for pitch  
  
U=4.0;              % parameter condition for airspeed  
a=-0.6;             % parameter condition for elastic axis  
  
ka=2.82*[1.,-22.1,1315.5,-8580.,17289.7];      % polynomial stiffness in pitch  
%-----  
% compute parameters based on the above system  
%-----  
mu=U^2;  
b=0.135;  
rho=1.225;  
span=0.6;  
mw=1.662;  
mt=12.387;  
mm=0.114;  
mf=0.273;  
m=12+mm+mf;
```

```

rcg=0.0873-(b+a*b);
xa=rcg/b;
Ia=0.065;
kh=2844.4;
ch=27.43;
ca=0.036;
muh=0.0221;
mua=0.0113;
%
%
Cla=6.28;
Clb=3.358;
Cma=(0.5+a)*Cla;
Cmb=-0.635;
%
%
d=m*Ia-(m*xa*b)^2;
c1=(Ia*ch+Ia*rho*U*b*Cla+m*xa*rho*U*b^3*Cma)/d;
c2=(Ia*rho*U*b^2*Cla*(1/2-a)-m*xa*b*ca+m*xa*rho*U*b^4*Cma*(1/2-a))/d;
c3=(-m*xa*b*ch-m*xa*rho*U*b^2*Cla-m*rho*U*b^2*Cma)/d;
c4=(m*ca-m*xa*rho*U*b^3*Cla*(1/2-a)-m*rho*U*b^3*Cma*(1/2-a))/d;
%
%
k1=Ia*kh/d;
k2=(Ia*rho*b*Cla+m*xa*(b^3)*rho*Cma)/d;
k3=-m*xa*b*kh/d;
k4=(-m*xa*(b^2)*rho*Cla-m*rho*(b^2)*Cma)/d;
%
%
g3=(-Ia*rho*b*Clb-m*xa*(b^3)*rho*Cmb)/d;
g4=(m*xa*(b^2)*rho*Clb+m*rho*(b^2)*Cmb)/d;
%
%
p=-m*xa*b*ka/d;
q=m*ka/d;

```

Appendix B

Run the simulation

```
%----- hardening nonlinear system response
data_hard;
[t_hard,x,y] = sim('texas');
y_hard = y(:,1);
a_hard = y(:,2);
yy_hard= y(:,3);
aa_hard= y(:,4);

save simdata_hard t_hard y_hard a_hard yy_hard aa_hard U

%----- softening nonlinear system response
data_soft;
[t_soft,x,y] = sim('texas');
y_soft = y(:,1);
a_soft = y(:,2);
yy_soft= y(:,3);
aa_soft= y(:,4);

save simdata_soft t_soft y_soft a_soft yy_soft aa_soft U

%----- linear system response
data_lin;
[t_lin,x,y] = sim('texas');
y_lin = y(:,1);
a_lin = y(:,2);
```

```
yy_lin= y(:,3);  
aa_lin= y(:,4);  
  
save simdata_lin t_lin y_lin a_lin yy_lin aa_lin U
```

Appendix C

Compute wavelet transforms

```
%----- nonlinear system response
load simdata_hard
scales = [25:25:1400];
scales = [25:25:500 550:50:800 900 1000 1200 1400];
cwt_hard = cwt(a_hard,scales,'morl','plot');
[peaks_hard,scale_hard] = max(cwt_hard);

cwt_hard_y = cwt(y_hard,scales,'morl','plot');
[peaks_hard_y,scale_hard_y] = max(cwt_hard_y);

save simdata_hard      t_hard y_hard a_hard yy_hard aa_hard U ...
                        cwt_hard scales peaks_hard scale_hard ...
                        cwt_hard_y

%-----
```

Appendix D

Make the wavelet maps

```
UU = ['20'; '40'; '60'; '80'; '90'; '95'; '10'; '12'];
ss = ['(a)'; '(b)'; '(c)'; '(d)'; '(e)'; '(f)'; '(g)'; '(h)'];

for i=1:8
    string = ['load sim',UU(i,:)];
    eval(string)

    image(t_hard,scales,wcodemat(cwt_hard,128));
    colormap(bone(128));
    map = colormap;
    bone_inv = map(128:-1:1,:);
    colormap(bone_inv);
    axis([0 10 0 1000])
    ylabel('Scale','FontSize',28,'FontName','times')
    xlabel('Time (s)','FontSize',28,'FontName','times')
    set(gca,'XTick',[0:1:10],'XTickLabel',[0:1:10])
    set(gca,'YTick',[200:200:1400],'YTickLabel',[200:200:1400])
    set(gca,'YTick',[0:200:1000],'YTickLabel',[0:200:1000])
    set(gca,'FontSize',24,'FontName','times')
    set(gca,'YDir','normal')
    text(9,900,ss(i,:), 'FontSize',28,'FontName','times')
    string = ['print -deps a_map_',UU(i,:),'.eps'];
    eval(string)
end
```

Appendix E

Compute peak values for energy and scale

```
UU = ['05'; '20'; '40'; '60'; '80'; '90'; '95'; '10'];

for j=1:8

    eval(['load sim',UU(j,:)]);
    p_tmp = peaks_hard;
    t_tmp = t_hard;
    total_index = 0;
    i=1;
    while total_index < max(size(t_hard)-1000)
        if i == 1
            [p(i),index] = max(p_tmp);
        else
            [p(i),index] = max(p_tmp(1:s(i-1)+200));
        end
        t(i) = index+total_index; % time index
        s(i) = scales(scale_hard(t(i))); % scale value
        total_index = total_index + index + floor(s(i)/2); % shift time ahead
        p_tmp = p_tmp(1+index + floor(s(i)/2):max(size(p_tmp)));
        i=i+1;
    end
end
```

```

eval(['t',UU(j,:), '=t_hard(t);']);
eval(['p',UU(j,:), '=p;']);
eval(['s',UU(j,:), '=s;']);

%plot(t_hard,peaks_hard,t_hard(t),p,'o')
%fprintf('U == %s (RETURN)\n', UU(j,:)), pause
clear p s t
end

if 1 == 2
%----- Now do the plot
subplot(211)
plot(t20,p20,'g-.',t40,p40,'o',t40,p40,'-',t60,p60,'-',t60,p60,'+', ...
      t80,p80,'b:',t90,p90,'r--',t95,p95,'y-',t10,p10,'c-', ...
      t05,p05,'*',t05,p05,'-');
ylabel('wavelet correlation')
title('PITCH : 05 (*) 20 (-.) 40 (o) 60 (+) 80 (:) 90 (--) 95 (-)')
subplot(212)
plot(t20,s20,'g-.',t40,s40,'o',t40,s40,'-',t60,s60,'-',t60,s60,'+', ...
      t80,s80,'b:',t90,s90,'r--',t95,s95,'y-',t10,s10,'c-', ...
      t05,s05,'*',t05,s05,'-');
ylabel('scale')
xlabel('time')
print -deps plot_pitch.eps
end

clf
h= plot(t05,s05,'rs',t05,s05,'r-', t20,s20,'go',t20,s20,'g-', ...
        t40,s40,'b+',t40,s40,'b-', t60,s60,'m^',t60,s60,'m-', ...
        t80,s80,'b:',t90,s90,'r--',t95,s95,'y-', ...
        t10,s10,'c-',t10,s10,'cd');
set(h,'MarkerSize',12);
ylabel('scale','FontSize',20,'FontName','times')
xlabel('time (s)','FontSize',20,'FontName','times')
print -deps apeak_pitch.eps

clf
e= plot(t05,p05,'rs',t05,p05,'r-', t20,p20,'go',t20,p20,'g-', ...

```

```

        t40,p40,'b+',t40,p40,'b-',    t60,p60,'m~',t60,p60,'m-', ...
        t80,p80,'b:',t90,p90,'r--',t95,p95,'y-', ...
        t10,p10,'c-',t10,p10,'cd');
set(e,'MarkerSize',12);
ylabel('magnitude','FontSize',20,'FontName','times')
xlabel('time (s)','FontSize',20,'FontName','times')
print -deps apeak_E_pitch.eps

save apeak UU t05 s05 t20 s20 t40 s40 t60 s60 ...
        t80 s80 t90 s90 t95 s95 t10 s10 ...
        p05 p10 p20 p40 p60 p80 p90 p95

```

Appendix F

Calculate derivative information for scale data and Prediction Function

```
load apeak
```

```
order = 8;
for j = 1:9
    eval(['t = t',UU(j,:),',';]);
    eval(['s = s',UU(j,:),',';]);
    poly_val = polyfit(t,s',order);
    poly_fit = polyval(poly_val,t);
    nn = norm(poly_fit-s');
    fprintf('Error in fit == %f\n',nn)
    tt = 0.5:.01:11;
    dog = tt-tt;
    for i=1:order+1
        dog = dog + poly_val(i) * tt.^(order+1-i);
    end

    ddog = tt-tt;
    for i=1:order
        ddog = ddog + (order+1-i)*poly_val(i)* tt.^(order-i);
    end
```

```

eval(['poly_',UU(j,:), '_val = poly_val;']);
eval(['poly_',UU(j,:), '_der = ddog;']);

end

plot(tt,poly_05_der,'r:',tt,poly_20_der,'r:', ...
      tt,poly_40_der,'b--',tt,poly_60_der,'b--', ...
      tt,poly_80_der,'g-.',tt,poly_90_der,'g-.', ...
      tt,poly_95_der,'k',tt,poly_10_der,'y',tt,poly_12_der,'g:')
ylabel('derivative in scale a_i','FontSize',20,'FontName','times')
xlabel('time (s)','FontSize',20,'FontName','times')
axis([0 9 -100 500])
print -deps apeak_derp.eps

      %-----%
      %      Euler derivative      %
      %-----%

clear tt ss
for j=1:9
    eval(['t = t',UU(j,:),',';']);
    eval(['s = s',UU(j,:),',';']);
    for i=1:max(size(t))-1
        tt(i) = (t(i)+t(i+1))/2;
        dt = t(i+1)-t(i);
        ds = s(i+1)-s(i);
        ss(i) = ds/dt;
    end

    [SS,index] = max(ss);
    TT = tt(index);
    sspeak(j) = SS;
    ttpeak(j) = tt(index);
    eval(['tt',UU(j,:),' = tt;']);
    eval(['ss',UU(j,:),' = ss;']);
    eval(['TT',UU(j,:),' = TT;']);
    eval(['SS',UU(j,:),' = SS;']);
end

```

```

h=plot(tt05,ss05,'rs',tt05,ss05,'r-',tt20,ss20,'go',tt20,ss20,'g-', ...
      tt40,ss40,'b+',tt40,ss40,'b-',tt60,ss60,'m^',tt60,ss60,'m-', ...
      tt80,ss80,'b:',tt90,ss90,'r--', ...
      tt95,ss95,'k',tt10,ss10,'yx',tt10,ss10,'y-', ...
      tt12,ss12,'r-',tt12,ss12,'r*');
set(h,'MarkerSize',12)
ylabel('Derivative of Scale a_i','FontSize',20,'FontName','times');
xlabel('time (s)','FontSize',20,'FontName','times')
print -deps apeak_dere.eps

uu = [20 40 60 80 90 95 100 120];
uvec = uu(1:8);
svec = sspeak(2:9);
plot(uvec/10,svec);
axis([0 12 0 800])
ylabel('Magnitude \Delta_a','FontSize',20,'FontName','times')
xlabel('airspeed (m/s)','FontSize',20,'FontName','times')
set(gca,'FontSize',16,'FontName','times')
print -deps apeak_derU.eps

plot(uvec/10,ttpeak(2:9));
axis([0 12 0 6])
ylabel('Time t_a','FontSize',20,'FontName','times')
xlabel('airspeed (m/s)','FontSize',20,'FontName','times')
set(gca,'FontSize',16,'FontName','times')
print -deps apeak_derT.eps

plot(uvec(1:6)/10,sspeak(2:7)./ttpeak(2:7))
axis([0 10 0 400])
ylabel('Prediction Function \phi','FontSize',20,'FontName','times')
xlabel('airspeed (m/s)','FontSize',20,'FontName','times')
set(gca,'FontSize',16,'FontName','times')
print -deps apeak_derF.eps

```

Appendix G

Calculate derivative information for energy data and Prediction Function

```
clear
load apeak

order = 7;
for j = 1:8
    eval(['t = t',UU(j,:),',';]);
    eval(['p = p',UU(j,:),',';]);
    poly_val = polyfit(t,p',order);
    poly_fit = polyval(poly_val,t);
    nn = norm(poly_fit-p');
    fprintf('Error in fit == %f\n',nn)
    tt = 0.5:.01:9;
    dog = tt-tt;
    for i=1:order+1
        dog = dog + poly_val(i) * tt.^(order+1-i);
    end

    ddog = tt-tt;
    for i=1:order
        ddog = ddog + (order+1-i)*poly_val(i)* tt.^(order-i);
```

```

end

eval(['poly_',UU(j,:), '_val = poly_val;']);
eval(['poly_',UU(j,:), '_der = ddog;']);

end

plot(tt,poly_05_der,'r:',tt,poly_20_der,'r:', ...
      tt,poly_40_der,'b--',tt,poly_60_der,'b--', ...
      tt,poly_80_der,'g-.',tt,poly_90_der,'g-.', ...
      tt,poly_95_der,'k',tt,poly_10_der,'w')
ylabel('derivative in energy F_i','FontSize',20,'FontName','times')
xlabel('time (s)','FontSize',20,'FontName','times')
axis([0 9 -100 500])
print -deps apeak_derp_E.eps

```

```

%-----%
%      Euler derivative      %
%-----%

```

```

clear tt ee
for j=1:8
    eval(['t = t',UU(j,:),',';']);
    eval(['p = p',UU(j,:),',';']);
    for i=1:max(size(t))-1
        tt(i) = (t(i)+t(i+1))/2;
        dt = t(i+1)-t(i);
        dp = p(i+1)-p(i);
        pp(i) = dp/dt;
    end

    [PP,index] = max(pp);
    TT = tt(index);
    pppeak(j) = PP;
    ttpeak(j) = tt(index);
    eval(['tt',UU(j,:), ' = tt;']);
    eval(['pp',UU(j,:), ' = pp;']);

```

```

        eval(['TT',UU(j,:), ' = TT;']);
        eval(['PP',UU(j,:), ' = PP;']);
end

h=plot(tt05,pp05,'rs',tt05,pp05,'r-',tt20,pp20,'go',tt20,pp20,'g-', ...
        tt40,pp40,'b+',tt40,pp40,'b-',tt60,pp60,'m~',tt60,pp60,'m-', ...
        tt80,pp80,'b:',tt90,pp90,'r--', ...
        tt95,pp95,'k');
set(h,'MarkerSize',12)
ylabel('Derivative of Energy F_i','FontSize',20,'FontName','times');
xlabel('time (s)','FontSize',20,'FontName','times')
print -deps apeak_dere_E.eps

uu = [05 20 40 60 80 90 95 100];
uvec = uu(1:8);
pvec = pppeak(1:8);
plot(uvec/10,pvec);
axis([ 0 10 -.5 .5])
ylabel('Magnitude \Delta_F','FontSize',20,'FontName','times')
xlabel('airspeed (m/s)','FontSize',20,'FontName','times')
set(gca,'FontSize',16,'FontName','times')
print -deps apeak_derU_E.eps

plot(uvec/10,ttpeak);
axis([0 10 0 6])
ylabel('Time t_F','FontSize',20,'FontName','times')
xlabel('airspeed (m/s)','FontSize',20,'FontName','times')
set(gca,'FontSize',16,'FontName','times')
print -deps apeak_derT_E.eps

plot(uvec(2:7)/10,pppeak(2:7)./ttpeak(2:7))
axis([0 10 -.5 .5])
ylabel('Prediction Function \phi_F','FontSize',20,'FontName','times')
xlabel('airspeed (m/s)','FontSize',20,'FontName','times')
set(gca,'FontSize',16,'FontName','times')
print -deps apeak_derF_E.eps

```

Appendix H

Produce the Phase Plane Plots

```
UU = ['40'; '80'; '90'; '95'; '10'; '12'];

for i = 1:max(size(UU))
    string = ['load sim',UU(i,:)];
    eval(string);
    a_hard(1) = .3;
    plot(a_hard,aa_hard)
    xlabel('pitch (rad)','FontSize',28,'FontName','times')
    ylabel('pitch rate (rad/s)','FontSize',28,'FontName','times')
    axis([-0.3 0.3 -8 8])
    set(gca,'XTick',[-0.3:.1:.3],'XTickLabel',[-0.3:.1:.3])
    set(gca,'YTick',[-8:2:8],'YTickLabel',[-8:2:8])
    set(gca,'FontSize',24,'FontName','times')

    string = ['print -deps phase_plane_',UU(i,:),'.eps'];
    eval(string);
end
```

Appendix I

Produce the Time-history Plots

```
UU = ['05'; '20'; '40'; '60'; '80'; '90'; '95'; '10'];
UU = ['20'; '40'; '60'; '80'; '90'; '95'; '10'; '12'];
ss = ['(a)'; '(b)'; '(c)'; '(d)'; '(e)'; '(f)'; '(g)'; '(h)'];

for i=1:8

    string = ['load sim',UU(i,:)];
    eval(string)

    plot(t_hard,a_hard);
    axis([0 10 -.3 .3])
    ylabel('Pitch (rad)','FontSize',28,'FontName','times')
    xlabel('Time (s)','FontSize',28,'FontName','times')
    set(gca,'XTick',[0:1:10],'XTickLabel',[0:1:10])
    set(gca,'FontSize',24,'FontName','times')

    string = ['print -deps a_time_',UU(i,:)];
    eval(string)

end
```

Appendix J

Plunge Effects

J.1 Effects of Coupling

These responses are calculated using the following initial displacements for pitch and plunge. Without the plunge influence, the uncoupled system takes much longer to decay in the pitch degree of freedom. An initial plunge displacement is used because in an uncoupled system with no initial displacement the linear k_y spring would show no motion in time. Notice the absence of irregular jumps in the uncoupled systems.

$$\begin{aligned}y(t_o) &= 0.03 \text{ m} & \dot{y}(t_o) &= 0 \text{ m/s} \\ \alpha(t_o) &= 0.3 \text{ rad} & \dot{\alpha}(t_o) &= 0 \text{ rad/s}\end{aligned}$$

Note that the uncoupled natural frequency of pitch is significantly higher than that of plunge, but that the coupled system has a frequency closer to the plunge natural frequency.

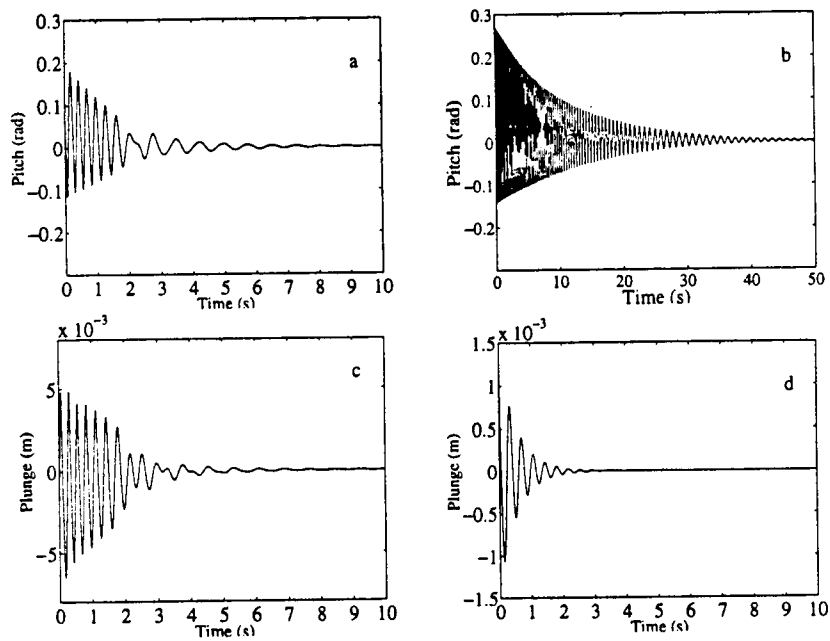


Figure J.1: Simulated Responses for Pitch: (a) coupled with plunge, (b) uncoupled at $U=8.0 \text{ m/s}$. Simulated Responses for Plunge: (c) coupled with pitch, (d) uncoupled at $U=8.0 \text{ m/s}$.

J.2 Plunge Time Histories

These responses are produced using the same initial conditions as those in Figure 6.1 for the coupled system..

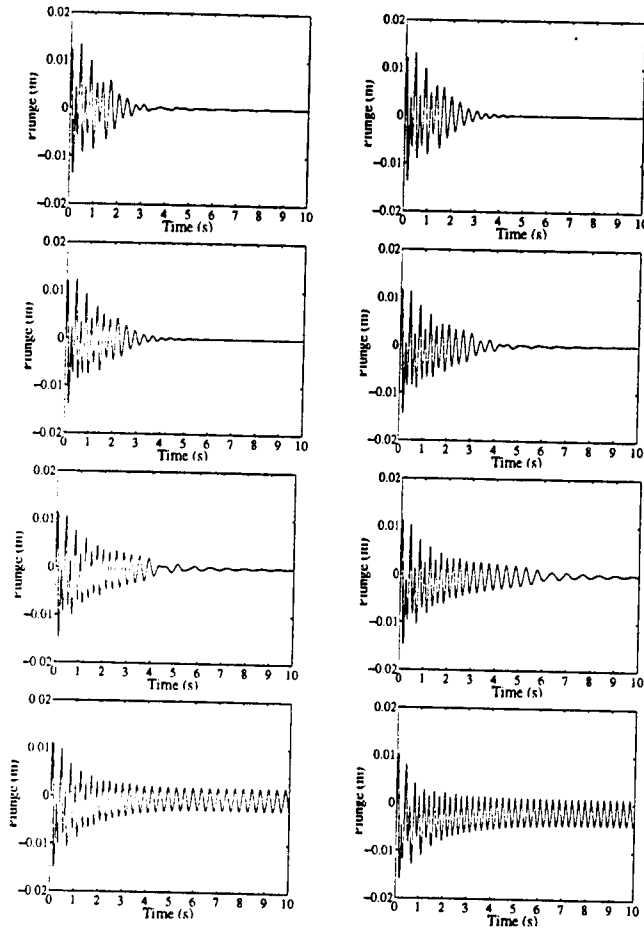


Figure J.2: Simulated Responses for Plunge (rad/s) with the Nonlinear Hardening Spring k_{hard} : $U=2.0 \text{ m/s}$ (a), $U=4.0 \text{ m/s}$ (b), $U=6.0 \text{ m/s}$ (c), $U=8.0 \text{ m/s}$ (d), $U=9.0 \text{ m/s}$ (e), $U=9.5 \text{ m/s}$ (f), $U=10.0 \text{ m/s}$ (g), $U=12.0 \text{ m/s}$ (h).

Vita

Kyle T. Snyder was born June 28, 1974 in High Point, North Carolina. Living in Ocoee, Florida for the early 1980s, Kyle saw the birth of the Space Shuttle program. He graduated from George Washington High School in Danville, VA in 1992. Moving on to Catawba College in Salisbury, NC, Kyle was a founding member of the Dead Athenian Society, played varsity basketball, and majored in Mathematics and Computer Science. He graduated with a double major, a minor in History, and with College Honors in May 1996. For his Master's Degree, Kyle attended the University of Tennessee Space Institute in Tullahoma, TN. He graduated in May 1998 with an M.S. in Applied Mathematics.

This thesis project is an extension of the project Kyle started as a Research Assistant in the first NASA Academy in Aeronautics at Dryden Flight Research Center at Edwards, California in 1997. He worked in the Structural Dynamics Branch at DFRC with Marty Brenner and Rick Lind.