To the Graduate Council:

I am submitting herewith a dissertation written by David Bruce Stewart entitled "An Asset Pricing Model that Corrects for Variance Expectation Bias." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Business Administration.

Phillip R. Daves, Major Professor

We have read this dissertation
and recommend its acceptance:

Accepted for the Council:

Associate Vice Chancellor and
Dean of The Graduate School

An Asset Pricing Model that Corrects for Variance Expectation Bias

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

David Bruce Stewart

August 1997

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DEDICATION

To my wife,

Yvonne Leigh Stewart,

and our son,

Sean David Stewart

ACKNOWLEDGMENTS

I would like to gratefully acknowledge the assistance and guidance of those who have helped me in this academic achievement. First, I would like to thank my chairman, Dr. Phillip R. Daves. His insights have been invaluable throughout this dissertation project, and his donation of large amounts of his time is appreciated. In addition, my other committee members have each helped in countless ways. Dr. James W. Wansley provided many intuitive insights and interpretational tips. Dr. Michael C. Ehrhardt's thorough knowledge of this area of research proved fruitful on several occasions, and Dr. Alan Schlottmann provided helpful advice on presentation of the material. Additionally, I wish to express my sincere thanks to Dr. George C. Philippatos, my finance mentor who gently oversaw my transition into academia.

A very special note of thanks to my parents, Bruce H. Stewart and Marcia A. McCoy, and grandparents is in order. They gave me the foundation and the tools to successfully meet the challenges of a Ph.D. program. Finally, I wish to thank my wife, Yvonne, for her support and patience and my son, Sean, for being a new source of inspiration in my life.

ABSTRACT

Under conditions of predictably-changing variances, traditional estimations of the CAPM beta are biased. This dissertation introduces an estimation procedure that endogenizes both the expected changes in the variance of asset excess returns and the expected changes in the variance of market index excess returns. This dissertation derives, estimates, and tests the resulting predicted betas, then explores the implications for existing market anomalies. The predicted betas are statistically significant in modeling out-of-sample stock excess returns for January and non-January samples in cross-sectional regressions, even when controlling for the level of traditional betas.

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CHAPTER 1

INTRODUCTION & HYPOTHESIS DEVELOPMENT

The purpose of this study is to derive an estimation procedure for a predicted level of systematic risk. The study is motivated by previous findings that the CAPM beta and market premium are nonstationary. The predicted beta is estimated and tested for significance cross-sectionally, and the implications for market anomalies are reported. This chapter provides motivation for this research and includes the derivations of the hypothesized return-generating process, predicted beta, and estimation procedure.

Why Do Researchers Care Whether Betas are Nonstationary?

Financial researchers have long held a goal of modeling the behavior of stock returns. The continuing research reliant upon Markowitz (1952, 1970)[1] and Sharpe (1964) attests that this goal has not been satisfactorily achieved (for examples, see Koutmos (forthcoming), Barber and Lyon (1997), Daniel and Titman (1997), Fama and French (1996a), Fama and French (1996b), and Jagannathan and Wang (1996)).

Markowitz (1970) argues that the behavior of the total stock market can be modeled as reacting to changes in macroeconomic variables. Further, Markowitz (1970) proposes that much of the behavior of individual stock returns can be attributed to reactions to the same changes in macroeconomic variables. However, Markowitz (1970)

[1] It should be noted that the 1970 monograph written by Markowitz was an amalgamation of his previous works.

states that modeling the effects of the macroeconomic variable changes on each individual stock is unreasonable. Markowitz (1970) proposes that researchers make the simplifying assumption that the effects of the macroeconomic variables on the market translate through to the behavior of individual stocks such that researchers can model individual stock returns relative to market returns.

With the Tobin (1958) contribution that investors can choose their risk level by holding a combination of a risky asset with a riskless asset, Sharpe (1964) operationalizes the Markowitz (1970) theory. Sharpe (1964) defines systematic risk and derives the capital asset pricing model (CAPM):

$$E[R_{it}] = R_f + \beta_i (E[R_{Mt}] - R_f),$$

where $E[R_{it}]$ is the expected rate of return on investment *i* during period *t*, R_f is the risk-free rate of return, $E[R_{Mt}]$ is the expected rate of return on the stock market during period *t*, and β_i is the measure of systematic risk for investment *i*. The quantity $(E[R_{Mt}] - R_f)$ is the market return premium during period *t* and represents the additional return earned for bearing the risk of the stock market over that of a riskless investment. In the same manner, the quantity $(E[R_{it}] - R_f)$ is the stock return premium for investment *i* during period *t* and represents the additional return earned for bearing the risk of stock *i* over that of a riskless investment. The risk-free rate of return is assumed to be known and constant. Sharpe (1964) proposes estimating betas using ordinary least squares (OLS) regression and using the resulting estimates for forming predictions. Sharpe (1964) states that if an expectation of the market index's future one-period return can be formed, then the beta of a stock can be used to form an expectation of the stock's future one-period return.

Although Markowitz (1952, 1970) hypothesizes actual betas are nonstationary, Sharpe (1964) assumes that actual betas are constant and that therefore OLS regression estimates of beta are unbiased estimates of out-of-sample-period betas.

If betas are nonstationary, the estimate of beta yielded by an OLS regression on observations from past periods is a biased estimate of the beta for the subsequent period. Using an OLS-estimated CAPM beta as the estimate of the future CAPM beta is relying on an assumption that beta is constant. If betas are nonstationary, OLS-estimated betas from one period do not represent the single-period relationships between stock return premiums and the market return premium during subsequent periods.

The empirical finance research literature has concluded that betas are nonstationary (Koutmos (forthcoming), He, Kan, Ng and Zhang (1996), Ng, Engle and Rothschild (1992), Bera, Bubnys and Park (1988), and Levy (1971)). Levy (1971) notes that relaxing the assumption that beta is constant poses a fundamental question: Can the future "riskiness" of an investment selection be accurately estimated from its past riskiness? The OLS regression estimate of beta is unbiased and consistent if the actual beta is constant over time and if the market risk premium is nonstochastic (Kmenta, 1986). However, OLS estimates of beta are biased if the market risk premium is a dynamic factor, which is a conclusion of Koutmos (forthcoming), Ng, Engle, and Rothschild (1992), and Schwert and Seguin (1990). The estimation of an unbiased systematic risk level for the future investment period requires a time series procedure which predicts an out-of-sample systematic risk level and allows this prediction to differ from the in-sample levels.

Derivation of Hypothesized Return-Generating Process

To reach an unbiased estimate of beta, this dissertation derives and estimates a single-dynamic-factor model which is an alternative to the CAPM. A model with the restriction of a single independent variable to represent systematic risk simplifies the intuitive definition of systematic risk and avoids making additional assumptions about the interactions between multiple independent variables. This dissertation tests the resulting "predicted beta" for statistical significance and explores the predicted beta's influence on existing market anomalies.

The variant of the CAPM hypothesized by this dissertation is:

$$E[R_{i(t+1)}] = R_f + \beta_{\sigma i(t+1)}(E[R_{M(t+1)}] - R_f) \qquad (1)$$

where $E[R_{i(t+1)}]$ is the expected rate of return on investment *i* during period *t+1*, R_f is the risk-free rate of return, $E[R_{M(t+1)}]$ is the expected rate of return on the stock market during period *t+1*, and $\beta_{\sigma i(t+1)}$ is the period *t+1* measure of systematic risk for investment *i* as predicted during period *t* using information from periods *t-n, ... , t-1, t.* Unlike the beta estimated directly using OLS regression, the predicted beta has the subscript σ to signify that this beta is constructed using estimated predictions of the correlation and standard deviations.

To derive a hypothesized return-generating process from equation (1), we rely on an assumption similar to one made by Fama and French (FF, 1992). The assumption relates expected returns to actual returns and is necessitated by the use of actual returns to

estimate an expectations model. An assumption of FF (1992) and of this dissertation is that rational expectations hold such that:

$$R_{i,t} = E[R_{i,t}] + \nu_{it},$$

where $R_{i,t}$ is the period t return of stock i, $E[R_{i,t}]$ is the period t expected return of stock i, and $\nu_{i,t}$ which is distributed $N(0, H_{i,t})$, is a mean-zero error term of period t on the expectation for stock i. However, unlike FF (1992), this dissertation does not assume that the error variance $H_{i,t}$ remains constant over time. Instead, the error variance is assumed to depend on past values, following a generalized autoregressive process:

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} \upsilon^2_{i(t-1)},$$

where $\nu_{i,t}$ is the error term which is distributed $N(0,H_{i,t})$, $H_{i,t}$ is the conditional variance of the error term $\nu_{i,t}$, and where γ_{0i}, γ_{1i} and γ_{2i} are constant coefficients. We follow standard notation by using $H_{i,t}$ to represent the conditional error variance, and using $\sigma^2_{i,t}$ to represent conditional total variance. Likewise, in this dissertation, $H_{M,t}$ will symbolize conditional error variance of the market excess return during period t. The assumption that the error variance follows a generalized autoregressive process is also made by the hypothesized return-generating models of Koutmos (forthcoming), Ghosh (1992), and Bera, Bubnys and Park (1988).

Given the above assumption, a hypothesized return-generating process follows from equation (1):

$$R_{i(t+1)} = \beta_{\sigma i(t+1)} R_{M(t+1)} + u_{i(t+1)}, \qquad (2)$$

$$H_{i(t+1)} = \gamma_{0i} + \gamma_{1i} H_{i,t} + \gamma_{2i} u^2_{i,t}$$

where $u_{i(t+1)}$ is the error term which is distributed $N(0,H_{i,t})$, $R_{i(t+1)}$ is the excess return of stock *i* during period *t+1*, $\beta_{\sigma i(t+1)}$ is the period *t+1* measure of systematic risk for investment *i* as predicted during period *t* using information from periods *t-n, ... , t-1, t*, $R_{M(t+1)}$ is the market index premium during period *t+1*, $H_{i(t+1)}$ is the conditional variance of the error term $u_{i(t+1)}$, and where γ_{0i}, γ_{1i} and γ_{2i} are coefficients to be estimated. Equation (2) specifies that stock excess returns follow a generalized autoregressive conditional heteroskedasticity model with the conditional error variance $H_{i(t+1)}$ depending on one past lag of conditional error variance and one past lag of squared error (GARCH(1, 1)). Note also that "returns" indicates excess returns throughout this dissertation unless otherwise specified.

Equation (2) features an out-of-sample measure of systematic risk, $\beta_{\sigma i(t+1)}$. The construction of $\beta_{\sigma i(t+1)}$, the predicted beta, is discussed in the next section.

Derivation of Predicted Beta from Beta

As stated by Sharpe (1964), the CAPM beta is derived as[2]

$$\beta_i = \frac{\sigma_{Mi}}{\sigma_M^2} = \frac{\rho_{Mi}\sigma_M\sigma_i}{\sigma_M^2} = \frac{\rho_{Mi}\sigma_i}{\sigma_M}.$$

where $\sigma_{Mi} = \rho_{Mi}\sigma_M\sigma_i$, σ_{Mi} is the covariance between the expected market return *M* and the expected risky asset return *i*, ρ_{Mi} is the correlation between the expected market return

[2] Note that variances, covariances, and correlations are unaffected by the addition or subtraction of a constant, so these returns may equivalently be considered to be measured as excess returns if the risk-free rate is assumed constant during the investment period.

and the expected risky asset return, σ_M is the standard deviation of the expected market return, and σ_i is the standard deviation of the expected risky asset return.

Nonstationarity of β_i implies that ρ_{Mi}, σ_M, or σ_i must change over time. If β_i is nonstationary and the correlation ρ_{Mi} is constant then σ_{Mi} and at least one of (σ_M, σ_i) must change.[3] Without loss of generality, assume σ_M and σ_i change by the proportions δ_M and δ_i, respectively, such that

$$\delta_{M(t,t+1)} \equiv \sigma_{M(t+1)} \Big/ \sigma_{M,t}$$

and

$$\delta_{i(t,t+1)} \equiv \sigma_{i(t+1)} \Big/ \sigma_{i,t} .$$

Also define

$$\delta_{(t,t+1)} \equiv \delta_{i(t,t+1)} \Big/ \delta_{M(t,t+1)} .$$

Then the predicted beta, $\beta_{\sigma i(t+1)}$, becomes

$$\beta_{\sigma i(t+1)} \equiv \frac{\rho_{Mi}\sigma_{i(t+1)}}{\sigma_{M(t+1)}} = \frac{\rho_{Mi}\delta_{i(t,t+1)}\sigma_{i,t}}{\delta_{M(t,t+1)}\sigma_{M,t}} = \frac{\delta_{i(t,t+1)}}{\delta_{M(t,t+1)}}\left[\frac{\rho_{Mi}\sigma_{i,t}}{\sigma_{M,t}}\right] = \delta_{(t,t+1)}\beta_{i,t} ,$$

where the subscript σ denotes that this estimate of beta includes the effects of delta, $\delta_{(t,t+1)}$. The predicted beta includes the CAPM measure of systematic risk with $\beta_{i,t}$, and the effects of changing conditional standard deviations of stock returns relative to the conditional standard deviations of market returns with $\delta_{(t,t+1)}$. Allowing the beta to be nonstationary

[3] Kaplanis (1988), during an investigation of international stock market comovements, found that major stock market correlations were stable while the covariances were not stable. The methodology of this dissertation will be applied to international markets in a subsequent investigation.

requires the inclusion of time subscripts: $\beta_{i,t} \equiv \frac{\rho_{Mi}\sigma_{i,t}}{\sigma_{M,t}}$.

Note that the inclusion of time subscripts and relaxation of the assumption of beta stationarity alters neither beta's intuitive interpretation nor beta's graphical representation. With time subscripts, $\beta_{i,t}$ still represents the level of systematic risk represented by stock *i* during period *t*; the difference is that $\beta_{i,t}$ is allowed to differ from $\beta_{i(t-1)}$.

Derivation of an Estimation Procedure

The conditional variance of the market risk premium can be predicted using the time series of conditional error variances. The out-of-sample estimate of the conditional error variance can be estimated using in-sample information in the time series equation for conditional error variance. Then the out-of-sample conditional error variance prediction can be converted to an out-of-sample conditional total variance prediction.[4] The time series equation for the conditional error variances is specified in the second equation of the following model for market returns.

Market index premiums can be modeled with a GARCH(1, 1)-M model as found in Ng, Engle, and Rothschild (1992):[5]

[4] Refer to Appendix B, "Error Variance Process Implies Total Variance Process" for the derivation of the relationship between conditional error variance and conditional total variance.

[5] The model here differs from that found in Ng, Engle, and Rothschild (1992) by including the conditional standard deviation in the mean equation rather than the conditional square root. This difference was necessitated by the use of Version MVS 6.09 of SAS for the estimations. SAS Version MVS 6.12 will allow conditional variance to be included in the mean equation.

$$R_{M,t} = \alpha_0 + \alpha_1 \sqrt{H_{M,t}} + u_{M,t}$$

$$H_{M,t} = \gamma_{0M} + \gamma_{1M} H_{M(t-1)} + \gamma_{2M} u^2_{M(t-1)}$$

where $u_{M,t}$ is the error term which is distributed $N(0,H_{M,t})$, $R_{M,t}$ is the market excess return during period t, $H_{M,t}$ is the conditional variance of the error term $u_{M,t}$, and where α_0, α_1, γ_{0M}, γ_{1M} and γ_{2M} are coefficients to be estimated. This model supplies an estimate of $\sigma_{M(t+1)}$, the period $t+1$ standard deviation of the market risk premium.

The period $t+1$ standard deviation of the stock excess returns, $\sigma_{i(t+1)}$, and the period $t+1$ correlation between the market risk premium and the stock excess return, $\rho_{Mi(t+1)}$, can be estimated simultaneously with a modified GARCH(1, 1)-M model. Perhaps the model can be displayed best by showing its derivation, beginning from a GARCH(1, 1) version of the CAPM:

$$R_{i,t} = \beta_{i,t} R_{M,t} + \varepsilon_{i,t}$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} \varepsilon^2_{i(t-1)}$$

$$\text{where } \beta_{i,t} \equiv \rho_{iM} \frac{\sigma_{i,t}}{\sigma_{M,t}}.$$

$$\therefore R_{i,t} = \left[\rho_{iM} \frac{\sigma_{i,t}}{\sigma_{M,t}} \right] R_{M,t} + \varepsilon_{i,t},$$

and with equation (A1) of Appendix A, this becomes

$$R_{i,t} = \rho_{iM} \left[\frac{\sigma_{\varepsilon,t}}{\sqrt{1 - \rho^2_{iM}}} \right] \frac{R_{M,t}}{\sigma_{M,t}} + \varepsilon_{i,t},$$

$$\therefore R_{i,t} = \left(\frac{\rho_{iM}}{\sqrt{1 - \rho_{iM}^2}} \right) \left[\frac{R_{M,t}}{\sigma_{M,t}} \right] \sigma_{\varepsilon_i t} + \varepsilon_{i,t},$$

$$\therefore R_{i,t} = \lambda_1 \left[\frac{R_{M,t}}{\sigma_{M,t}} \right] \sigma_{\varepsilon_i t} + \varepsilon_{i,t},$$

$$\therefore R_{i,t} = \lambda_1 \left[\frac{R_{M,t}}{\sigma_{M,t}} \right] \sqrt{H_{i,t}} + \varepsilon_{i,t} \qquad (3)$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} \varepsilon_{i(t-1)}^2$$

where $\varepsilon_{i,t}$ is the error term which is distributed $N(0,H_{i,t})$, $R_{i,t}$ is the excess return of stock *i* during period *t*, $R_{M,t}$ is the market excess return during period *t*, $\sigma_{M,t}$ is the conditional total standard deviation of the market premium of period *t*, $H_{i,t}$ is the conditional variance of the error term $\varepsilon_{i,t}$, and where λ_0, λ_1, γ_{0i}, γ_{1i} and γ_{2i} are coefficients to be estimated. This model supplies an estimate of $\sigma_{\varepsilon i(t+1)}$, the period *t+1* error standard deviation of the stock *i* return, which can be converted to obtain an estimate of $\sigma_{i(t+1)}$, the period *t+1* total standard deviation of the stock *i* return. It can easily be shown that an estimate of ρ_{Mi} is $\hat{\rho}_{Mi} = \frac{\lambda_1}{\sqrt{1+\lambda_1^2}}$. Given the assumption that the correlation is constant, the estimation of the correlation from periods $t = 1, \ldots, n$ can be used out-of-sample as the estimate of the correlation for the period $t = n+1$.

Estimating the above stock return model requires knowledge of the market returns and the market standard deviations. The market returns are known and the market standard deviations have been estimated by the market return model. Unfortunately SAS Version MVS 6.09 cannot estimate this form of a GARCH(1, 1)-M model, as SAS does

not allow the multiplication of the conditional error standard deviation in the mean equation to be multiplied by a variable.[6] Therefore, a four-stage estimation procedure also is derived.

Derivation of a Four-Stage Estimation Procedure

An estimate of the predicted beta, $\hat{\beta}_{\sigma i(t+1)}$, can be constructed after estimating $\rho_{Mi(t+1)}$, the period $t+1$ correlation between the market risk premium and the stock return, $\sigma_{i(t+1)}$, the period $t+1$ standard deviation of the stock returns, and $\sigma_{M(t+1)}$, the period $t+1$ standard deviation of the market risk premium:

$$\hat{\beta}_{\sigma i(t+1)} \equiv \frac{\hat{\rho}_{Mi(t+1)}\hat{\sigma}_{i(t+1)}}{\hat{\sigma}_{M(t+1)}}. \qquad (4)$$

We construct predicted betas in four stages. The first stage estimates the market return standard deviation, $\sigma_{M(t+1)}$ using the same market return model as proposed above. The second stage estimates the stock return standard deviation $\sigma_{i(t+1)}$ without including any independent variables in the mean equation. This causes the stock return conditional error variances of the model to be the same as the stock return conditional total variances, so no conversion between the two is necessary. The third stage estimates the constant correlation, ρ_{Mi}, and the fourth stage combines these estimates to form the estimate of the

---

[6] Transforming the mean equation by multiplying by σ_{Mt}/R_{Mt} doesn't solve this estimation problem, as such multiplication also transforms the error term and the variance equation in such a way that the estimation is not supported by SAS Version 6.12.

predicted beta, $\beta_{\sigma i(t+1)}$.

Estimation of Conditional Total Standard Deviations

In order to form predictions of the period $t+1$ standard deviations in equation (4), the first stage assumes the total variances of the market excess returns, $\sigma^2_{M(t+1)}$, follow a time series process. Likewise, the second stage assumes the total variances of the stock excess returns, $\sigma^2_{i(t+1)}$, follow a time series process. Both market and stock total variances are assumed to follow a generalized autoregressive conditional heteroskedastic (GARCH) process with one lag of conditional variance and one lag of squared residual GARCH(1,1)).

The GARCH(1, 1) process for $\sigma^2_{i,t}$, the conditional total variance of the stock excess returns[7], $R_{i,t}$, is assumed to be:

$$\sigma^2_{i(t+1)} = \gamma_{0i} + \gamma_{1i}\sigma^2_{i,t} + \gamma_{2i}u^2_{i,t}, \qquad (5a)$$

where $\sigma^2_{i(t+1)}$ is the period $t+1$ conditional total variance of stock i, $u_{i,t}$ is the transformed period t residual from estimating the return of stock i, and γ_{0i}, γ_{1i}, and γ_{2i} are coefficients to be estimated by the model. Similarly, the GARCH(1, 1) process for $\sigma^2_{M,t}$, the conditional total variance of the market returns, $R_{M,t}$, is assumed to be:

$$\sigma^2_{M(t+1)} = \gamma_{0M} + \gamma_{1M}\sigma^2_{M,t} + \gamma_{2M}u^2_{M,t}, \qquad (5b)$$

[7] The definition of the conditional total variance is fully explained in the Appendix A, "Deriving Conditional Total Variances." Briefly, the conditional total variance $\sigma^2_{i(t+1)}$ is the result of transforming $H_{i,t+1}$, the period $t+1$ conditional error variance obtained from modelling the return of stock i, using the correlation between the independent and dependent variables.

where $\sigma^2_{M(t+1)}$ is the period $t+1$ conditional total variance of the market returns, $u_{M,t}$ is the transformed period t residual from estimating the market return, and γ_{0M}, γ_{1M}, and γ_{2M} are coefficients assumed to be constants.

The conditional total variances of stock excess returns and of market excess returns depend on conditional error variances obtained by estimating stock returns and market returns, respectively. In order to obtain estimates of the parameters to the conditional total variance equation (5a), stock excess returns are modeled with the naïve GARCH(1, 1) model using a sample of t periods:

$$R_{i,t} = \alpha_i + u_{i,t} \qquad (6a)$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} u^2_{i(t-1)},$$

where $u_{i,t}$ is the error term which is distributed $N(0, H_{i,t})$, $R_{i,t}$ is the stock excess return, $H_{i,t}$ is the conditional error variance at time t, and α_i, γ_{0i}, γ_{1i}, and γ_{2i} are coefficients assumed to be constants. The variance equation yields point-in-time estimates of the $t=1$ through $t=61$ conditional error variances of the stock returns. In related research, Bera, Bubnys and Park (1988) assumes $u_{i,t}$ follows an ARCH(1) process, Ghosh (1992) assumes $u_{i,t}$ follows a GARCH(1, 1) process, and Koutmos (forthcoming) assumes $u_{i,t}$ follows an EGARCH(1, 1) process.

With only a constant as the independent variable in the mean equation of equation (6a), the conditional error variance equals the conditional total variance. In this case, the correlation between the dependent and independent variables (as seen in equation (A1) of Appendix A) is the correlation between stock returns and a constant. By definition the

correlation equals zero when one of the variables is a constant. Using the fact that both independent variables of the conditional variance equation are lagged values and the assumption that the estimated coefficients γ_{0i}, γ_{1i}, and γ_{2i} are constants, the model provides an estimate of the conditional total standard deviation for the stock return of period $t+1$ used in constructing the predicted beta.

Market returns are modeled with a univariate GARCH(1, 1)-M model using a sample of t periods:

$$R_{M,t} = \alpha_0 + \alpha_1 \sqrt{H_{M,t}} + u_{M,t} \qquad (6b)$$

$$H_{M,t} = \gamma_{0M} + \gamma_{1M} H_{M(t-1)} + \gamma_{2M} u^2_{M(t-1)},$$

where $u_{M,t}$ is the error term which is distributed $N(0,H_{M,t})$, $R_{M,t}$ is the market return during period t, $H_{M,t}$ is the conditional error variance of $u_{M,t}$, and α_0, α_1, γ_{0M}, γ_{1M} and γ_{2M} are coefficients assumed to be constants. The GARCH(1, 1)-M model was found to be the most appropriate for the two-step procedure of Ng, Engle, and Rothschild (1992). Just as with OLS estimation, an assumption of a GARCH-M model is that the correlation between the independent and dependent variables is constant. This model assumes that the correlation between the market return and conditional error standard deviation, $\rho_{R_M,\sqrt{H_M}}$, is constant. Using this assumption and equation (A1) the model provides estimates of the conditional total market standard deviation, $\hat{\sigma}_{M,t}$, for periods $t=1$ through $t=60$:

$$\hat{\sigma}_{M,t} \equiv \sqrt{\frac{H_{M,t}}{1-\rho^2_{R_M,\sqrt{H_M}}}}.$$

Also using the fact that both independent variables of the conditional variance equation are lagged values and the assumption that the estimated coefficients γ_{0M}, γ_{1M}, and γ_{2M} are constants, the model provides an estimate of the conditional total standard deviation for the market return for period $t+1$ used in constructing the predicted beta:

$$\hat{\sigma}_{M(t+1)} \equiv \sqrt{\frac{H_{M(t+1)}}{1-\rho^{2}_{R_M,\sqrt{H_M}}}}.$$

Estimation of Market Index Return-Stock Return Correlation

The estimation of the correlation between the stock excess returns and market excess returns, which is the third stage, relies on the assumption that the correlation is constant. Following from the in-sample version of (4), $\beta_{\sigma i,t} \equiv \frac{\hat{\rho}_{iM,t}\hat{\sigma}_{i,t}}{\hat{\sigma}_{M,t}}$, the in-sample version of the hypothesized model (2) to be estimated is:

$$R_{i,t} = \frac{\rho_{iM}\hat{\sigma}_{i,t}}{\hat{\sigma}_{M,t}} R_{M,t} + u_{i,t}$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} u^{2}_{i(t-1)},$$

which is estimated as:

$$R_{i,t} = \alpha_i + \rho_{iM}\left[\frac{\hat{\sigma}_{i,t}}{\hat{\sigma}_{M,t}} R_{M,t}\right] + u_{i,t}$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} u^{2}_{i(t-1)},$$

where $u_{i,t}$ is the error term which is distributed $N(0,H_{i,t})$, $R_{i,t}$ is the individual asset excess return during period t, $\hat{\sigma}_{i,t}$ and is the conditional total standard deviation of the stock

excess returns estimated using equations (6a) and (A1), $\hat{\sigma}_{M,t}$ is the conditional total standard deviation of the market returns estimated using equations (6b) and (A1), $R_{M,t}$ is the market return during period *t*, $H_{i,t}$ is the conditional error variance at time *t*, and α_i, ρ_{iM}, γ_{0i}, γ_{1i}, and γ_{2i} are coefficients to be estimated. Note that the mean-equation intercept term, α_i, is hypothesized to equal zero. The assumption of a constant correlation enables the use of this in-sample estimate of the correlation in the construction of the out-of-sample predicted beta. Stage four combines the conditional total variances of the market and stock excess returns and the correlation to construct the predicted beta, as shown in equation (4).

CAPM Assumptions and Applicability to Predicted Beta

Maintenance of Original Beta Definition Intuition

The expected behavior of the predicted beta depends on the assumptions necessary for its derivation. A natural question is whether the predicted beta violates or changes any of the assumptions underlying the traditionally-estimated beta.

The individual-investor assumptions underlying the CAPM are that:

i. utility functions are based on means and variances
ii. investors invest for one period at a time
iii. investors maximize expected end-of-period utility
iv. investors are risk-averse
v. there are no taxes, transactions costs, no margin requirements, no restrictions on short sales, and infinitely divisible assets
vi. there exists a risk-free rate for borrowing and lending
vii. there is a fixed security supply during the investment period

The market assumptions underlying the CAPM are that:

i. investors have homogeneous beliefs
ii. asset returns are jointly normally distributed
iii. unlimited borrowing is available at the risk-free rate

This dissertation's estimation procedure for the predicted beta violates none of the individual-investor CAPM assumptions. In fact, relaxing the assumption that beta is stationary across investment periods permits a model which examines each investment period in turn. Thus allowing beta to be nonstationary seems intuitively consistent with the assumption that investors invest for one period at a time: researchers should allow for the possibility that beta changes as investors add information from the prior period into their expectation for the future period.

Two-fund separation obtains under the three market assumptions. Noting how predicted betas behave under two-fund separation helps define their behavior.

Suppose an investor holds the market portfolio. For the market portfolio the values of delta and the predicted beta are each unity:

$$\delta_{(t,t+1)} \equiv \frac{\delta_{i(t,t+1)}}{\delta_{M(t,t+1)}} = \frac{\delta_{M(t,t+1)}}{\delta_{M(t,t+1)}} = 1;$$

$$\beta_{\sigma\, i(t+1)} \equiv \frac{\rho_{Mi}\sigma_{i(t+1)}}{\sigma_{M(t+1)}}, \quad \therefore \beta_{\sigma\, M(t+1)} \equiv \frac{\rho_{MM}\sigma_{M(t+1)}}{\sigma_{M(t+1)}} = 1.$$

The conditional risky asset return standard deviation, $\sigma_{i(t+1)}$, equals the conditional market return standard deviation, $\sigma_{M(t+1)}$, and the risky asset's change in conditional standard deviation, $\delta_{i(t,t+1)}$, equals the market return's change in conditional standard deviation,

$\delta_{M(t,t+1)}$. In this case, OLS estimation of beta is unnecessary, as the beta will have a value equal to unity by definition.

Suppose an investor chooses to hold a portfolio consisting of weight w_M of the market portfolio and weight $(1-w_M)$ of a risk-free asset. Under two-fund separation, delta, $\delta_{(t,t+1)}$, has a value equal to one, and each the beta, β_{it}, and predicted beta, $\beta_{\sigma i(t+1)}$, equal the percentage invested in the market portfolio, w_M:

$$\sigma_{it} \equiv \sqrt{w_M^2 \sigma_{Mt}^2 + (1 - w_M)^2 \sigma_{R_f t}^2} = w_M \sigma_{Mt},$$

$$\beta_{it} \equiv \frac{\rho_{Mi}\sigma_{it}}{\sigma_{Mt}} = \frac{\rho_{MM} w_M \sigma_{Mt}}{\sigma_{Mt}} = w_M,$$

$$\beta_{\sigma i(t+1)} \equiv \frac{\rho_{Mi}\sigma_{i(t+1)}}{\sigma_{M(t+1)}} = \frac{\rho_{Mi}\delta_{i(t,t+1)}\sigma_{it}}{\delta_{M(t,t+1)}\sigma_{Mt}} = \frac{\rho_{MM}\delta_{M(t,t+1)} w_M \sigma_{Mt}}{\delta_{M(t,t+1)}\sigma_{Mt}} = w_M.$$

For such portfolios, the appropriate measure of systematic risk for the risky portion is equal to the percentage invested in the market portfolio, w_M. For portfolios on the capital market line, the standard result obtains: the portfolio beta equals the weight invested in the market portfolio. The predicted beta differs from the CAPM beta for portfolios which are not on the capital market line and for which delta is not equal to one.[8] If ρ_{Mi} is constant, β_i is also constant if and only if the standard deviation of the individual stock returns changes by the same proportion as does the standard deviation of the stock market returns. If $\delta_{M(t,\,t+1)} = \delta_{i(t,\,t+1)}$ for all $t = 1,\ldots, T$, then $\delta_{(t,\,t+1)} = 1$ and $\beta_{\sigma i(t+1)} = \beta_{it}$; changing standard deviations have no net effect on the predicted beta $\beta_{\sigma i(t+1)}$.

[8] Klein (1973) and Levy (1978, 1980) argue that rational utility-maximizing investors with wealth constraints do not hold the market portfolio given the presence of imperfectly divisible securities or strictly positive transactions costs, respectively. Instead, such investors hold imperfectly diversified portfolios.

In a reply to Kothari, Shanken and Sloan (1995), Fama and French (1996b) mention that the appeal of the CAPM is so strong that many researchers want to assume it holds. The problem is that the CAPM is very difficult (if not impossible) to disprove, while certain estimation procedures have resulted in empirical investigations which clearly do not support the CAPM.?

Potential researcher reactions to the empirical evidence that the CAPM beta does not wholly represent systematic risk (see Fama and French (1993, 1996a), Jagannathan and Wang (1996), and Daniel and Titman (1997)) are to: 1) relinquish the possibility of any single-index model in favor of directly including accounting or macroeconomic factors, or 2) refine the list of CAPM assumptions by relaxing those most at odds with empirical findings and common intuition. For example, even as mutual funds spread investor wealth widely across many stocks, two-fund separation does not hold. Perhaps the most appealing CAPM aspect is the simplicity of modeling risky asset returns using only riskless asset returns and market index returns. This simplicity is especially appealing when the alternative is a multi-factor model based on the intertemporal CAPM of Merton (1973) or on the APT of Ross (1976). The Merton (1973) and Ross (1976) models require intuitive explanations justifying that the additional accounting or economic factors should be orthogonal to market index returns (Daniel and Titman, 1997).

The technique presented in this study uses additional information from the means and variances of the risky asset returns and market index returns than is used by the CAPM. Whether these improvements explain market anomalies is discussed in Chapter 5.

Hypothesis of the Behavior of Predicted Beta

As market risk is the value-weighted average of the systematic risks of securities in the market, the standard deviation of market returns must be positively correlated with the overall changes in the standard deviations of the systematic portion of security risks.

Also, as $\beta_{\sigma i(t+1)} \equiv \frac{\rho_{Mi}\delta_{i(t,t+1)}\sigma_{i,t}}{\delta_{M(t,t+1)}\sigma_{M,t}} \equiv \delta_{(t,t+1)}\frac{\rho_{Mi}\sigma_{i,t}}{\sigma_{M,t}}$, higher levels of delta will result in higher predicted beta estimates compared to beta estimates. Whether stock deltas, $\delta_{i(t,t+1)}$, influence beta estimates strongly enough to be found cross-sectionally statistically significant is the question underlying the hypothesis.

Hypothesis: As predicted betas can be regarded as CAPM betas corrected for nonstationarity, and given the assumed relationship between actual and expectd returns, higher predicted beta deltas uniformly result in higher returns. Higher (lower) deltas signify that the expected level of systematic risk is higher (lower) than that predicted by the CAPM beta. That is, the predicted beta, $\beta_{\sigma i(t+1)}$, is significantly positively cross-sectionally associated with stock returns of period *t+1* after controlling for the level of the traditionally-estimated beta, $\beta_{i,t}$.

Organization of the Research

This dissertation is organized into six chapters. Chapter one provides motivation for this research and includes the derivations of the hypothesized return-generating process,

predicted beta, and estimation procedure. Chapter two details the prior research on beta nonstationarity and on tests of betas estimated using autoregressive conditional heteroskedastic variations on the CAPM. Chapter three presents the methodology, chapter four provides the sample construction and a description of the data, and chapter five recounts the results. Chapter six summarizes the contributions and limitations.

CHAPTER 2

LITERATURE REVIEW

This chapter reviews the related research and compares it to this study. First, literature addressing ARCH-class variations on the CAPM is presented. Next, general literature concerning the CAPM is discussed. Finally, evidence that stock excess returns and the market premium follow ARCH-class processes is summarized.

ARCH-CAPM Research

Research shows the traditional assumption that asset return variance is constant during each investment period does not hold (see Koutmos (forthcoming), Daniel and Titman (1997), He, Kan, Ng, and Zhang (1996), Jagannathan and Wang (1996), Ghosh (1992), Ng, Engle, and Rothschild (1992), Bera, Bubnys and Park (1988), Bollerslev, Engle and Wooldridge (1988), and French, Schwert and Stambough (1987)). Returns of individual assets and of the market index exhibit variances which change over time (Koutmos (forthcoming)). Significant heteroskedasticity is displayed by the error terms from estimating models such as:

$$R_{i,t} = R_f + \beta_i (R_{M,t} - R_f) + u_{i,t},$$

$$R_{i,t} = \alpha + \beta_i R_{M,t} + u'_{i,t} \text{ and}$$

$$R_{M,t} = \mu_0 + u_{M,t},$$

where R_{it} is the return on investment i during period t, R_f is the risk-free rate, β_i is the beta for investment i, R_{Mt} is the market return during period t, and $u_{i,t}$, $u'_{i,t}$, and $u_{M,t}$ are error terms, (Bera, Bubnys and Park (1988), Bollerslev, Engle and Wooldridge (1988), Ghosh (1992), Ng, Engle, and Rothschild (1992), and Koutmos (forthcoming)). Heteroskedasticity of the $u_{i,t}$ causes estimation inefficiency and heteroskedasticity of the $u_{M,t}$ causes bias in the coefficient representing systematic risk in a market model (Kmenta, 1986).

Ordinary least squares regression estimations of the CAPM and of arbitrage pricing theory (APT) models assume the market beta[9] is a constant. Given the mathematical definition of the market beta:

$$\beta_{it} \equiv \frac{\rho_{Mit}\sigma_{it}}{\sigma_{Mt}},$$

it is intuitively improbable that beta is a constant while the correlation ρ_{Mit}, stock return standard deviation σ_{it}, and/or market return standard deviation σ_{Mt} change over time.[10] Further, stock return standard deviations have been found to change over time. Specifically, stock return variances have been found to follow a conditionally autoregressive pattern[11] (Koutmos (forthcoming), Ghosh (1992), Ng, Engle, and Rothschild (1992), Bera, Bubnys and Park (1988), Bollerslev, Engle and Wooldridge (1988), and French, Schwert and Stambough (1987)). As a result a variety of new capital asset pricing models have been proposed.

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[9] The measure of systematic risk is referred to as the "market beta" in APT to distinguish the systematic risk associated with the market risk premium from other hypothetical dimensions of systematic risk.

[10] Refer to the subsection "Condition for Beta to Remain Constant" of Chapter 1 for full details.

[11] A class of statistical models developed to estimate these time series is termed autoregressive conditional heteroskedasticity (ARCH) models.

Bera, Bubnys and Park (BBP, 1988) estimate a modified CAPM where the error variances are assumed to follow an ARCH(1) process:

$$R_{i,t} = \alpha_0 + \alpha_1 R_{M,t} + u_{i,t}$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} u^2_{i(t-1)}$$

where $u_{i,t}$ is the error term which is distributed $N(0,H_{i,t})$, $R_{i,t}$ is the excess return of stock *i* during period *t*, $R_{M,t}$ is the market excess return during period *t*, $H_{i,t}$ is the conditional variance of the error term $u_{i,t}$, and where α_0, α_1, γ_{0i}, and γ_{1i} are coefficients to be estimated. BBP data consist of monthly data from 1976 through 1983 for 126 firms randomly chosen from the CRSP tapes and the with-dividend value-weighted NYSE index as the market portfolio.

The authors find significant ARCH effects, and their comparison of OLS regression model results with ARCH-model results yields the expected conclusion: the point estimates of the betas differ insignificantly, but the standard errors of the beta estimates are smaller for the ARCH model. BBP then form two sets of portfolios for further comparison of OLS regression estimates with ARCH(1) estimates.

BBP alphabetically distribute the 126 firms into ten portfolios for one comparison, then rank the 126 firms by capitalization and distribute them into six size portfolios. The authors find that, again, the point estimates of beta are insignificantly different from those achieved using an OLS regression and the standard errors of the beta estimates are smaller. This result is expected, as heteroskedasticity inflates standard errors of the estimates while not introducing bias (Kmenta, 1986). All 16 portfolios result in significant

ARCH effects as represented by the coefficient γ_{1i}, whereas approximately 75% of the 126 individually-estimated ARCH models result in significant coefficients on the lagged squared error term in the conditional variance equation.

The BBP results are interesting for the finding that the ARCH term was significant for a higher percentage of portfolios (100%) than for individual firms (~50%). However, the BBP methodology allows neither the inclusion of a lagged conditional variance term in the conditional variance equation, $H_{i,t-1}$, (Koutmos (forthcoming), Ghosh (1992)), nor allows nonstationarity of the market returns (Koutmos (forthcoming), Bollerslev, Engle and Wooldridge (1988), French, Schwert and Stambough (1987)). The methodology of this dissertation features the inclusion of a lagged conditional variance term in the conditional variance equation and corrects for nonstationarity of the market returns.

Bollerslev, Engle and Wooldridge (BEW, 1988) employ an MGARCH(1,1)-M model to estimate the conditional covariances of stock, bill, and bond excess returns with a comprehensive market portfolio. The stock returns are proxied by NYSE value-weighted index from CRSP, the bill yields are 6-month Treasury bills from Salomon Brothers, and the bond yields are 20-year Treasury bonds from Salomon Brothers. The stock excess, bill, and bond returns are found by subtracting a proxy for the risk-free rate, the yield on 3-month Treasury bills from Salomon Brothers. Quarterly data from the first quarter of 1959 through the second quarter of 1984 are included. BEW impose diagonality constraints on the conditional variance equation as a result of an assumption that each covariance depends only on its own past behavior, so the estimated model has the form:

$$\begin{bmatrix} R_{S,t} \\ R_{Bi,t} \\ R_{Bo,t} \end{bmatrix} = \begin{bmatrix} \alpha_S \\ \alpha_{Bi} \\ \alpha_{Bo} \end{bmatrix} + \delta \begin{bmatrix} \omega_{S,t-1} & \omega_{Bi,t-1} & \omega_{Bo,t-1} \end{bmatrix} \begin{bmatrix} H_{SS,t} & H_{SBi,t} & H_{SBo,t} \\ H_{BiS,t} & H_{BiBi,t} & H_{BiBo,t} \\ H_{BoS,t} & H_{BoBi,t} & H_{BoBo,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{S,t} \\ \varepsilon_{Bi,t} \\ \varepsilon_{Bo,t} \end{bmatrix}$$

$$\begin{bmatrix} H_{SS,t} \\ H_{SBi,t} \\ H_{BiBi,t} \\ H_{SBo,t} \\ H_{BiBo,t} \\ H_{BoBo,t} \end{bmatrix} = \begin{bmatrix} \gamma_{0,SS} \\ \gamma_{0,SBi} \\ \gamma_{0,BiBi} \\ \gamma_{0,SBo} \\ \gamma_{0,BiBo} \\ \gamma_{0,BoBo} \end{bmatrix} + \begin{bmatrix} \gamma_{1,SS} & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_{1,SBi} & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_{1,BiBi} & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_{1,SBo} & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_{1,BiBo} & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_{1,BoBo} \end{bmatrix} \begin{bmatrix} \varepsilon_{SS,t-1} \\ \varepsilon_{SBi,t-1} \\ \varepsilon_{BiBi,t-1} \\ \varepsilon_{SBo,t-1} \\ \varepsilon_{BiBo,t-1} \\ \varepsilon_{BoBo,t-1} \end{bmatrix}^T + \begin{bmatrix} \gamma_{2,SS} & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_{2,SBi} & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_{2,BiBi} & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_{2,SBo} & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_{2,BiBo} & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_{2,BoBo} \end{bmatrix} \begin{bmatrix} H_{SS,t-1} \\ H_{SBi,t-1} \\ H_{BiBi,t-1} \\ H_{SBo,t-1} \\ H_{BiBo,t-1} \\ H_{BoBo,t-1} \end{bmatrix}^T,$$

where $\begin{bmatrix} \varepsilon_{S,t} \\ \varepsilon_{Bi,t} \\ \varepsilon_{Bo,t} \end{bmatrix}$ is the vector of error terms which is distributed

$$N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} H_{SS,t} & H_{SBi,t} & H_{SBo,t} \\ H_{BiS,t} & H_{BiBi,t} & H_{BiBo,t} \\ H_{BoS,t} & H_{BoBi,t} & H_{BoBo,t} \end{bmatrix} \right),$$ and where $R_{a,t}$ is the excess return on investment *a* during period *t*, *a* = *S* (stock), *Bi* (T-bills), *Bo* (T-bonds), $\omega_{a,t-1}$ is the *t-1* value of investment *a* divided by the *t-1* total value of the three investments, $H_{a1a2,t}$ is period *t* value of the conditional covariance between returns on investments a_1 and a_2, $\varepsilon_{a1a2,t-1}$ denotes the period *t-1* error term associated with investment a_1 multiplied by the period *t-1* error term associated with investment a_2, and α_a, $\gamma_{0,a1a2}$, $\gamma_{1,a1a2}$, and $\gamma_{2,a1a2}$ are parameters to be estimated by the model.

BEW estimate the stock, Treasury bill, and Treasury bond time-varying betas which result from their estimations:

$$\begin{bmatrix} \beta_{S,t} \\ \beta_{Bi,t} \\ \beta_{Bo,t} \end{bmatrix} = \frac{\begin{bmatrix} H_{SS,t} & H_{SBi,t} & H_{SBo,t} \\ H_{BiS,t} & H_{BiBi,t} & H_{BiBo,t} \\ H_{BoS,t} & H_{BoBi,t} & H_{BoBo,t} \end{bmatrix} \begin{bmatrix} \omega_{S,t-1} \\ \omega_{Bi,t-1} \\ \omega_{Bo,t-1} \end{bmatrix}}{\begin{bmatrix} \omega_{S,t-1} & \omega_{Bi,t-1} & \omega_{Bo,t-1} \end{bmatrix} \begin{bmatrix} H_{SS,t} & H_{SBi,t} & H_{SBo,t} \\ H_{BiS,t} & H_{BiBi,t} & H_{BiBo,t} \\ H_{BoS,t} & H_{BoBi,t} & H_{BoBo,t} \end{bmatrix} \begin{bmatrix} \omega_{S,t-1} \\ \omega_{Bi,t-1} \\ \omega_{Bo,t-1} \end{bmatrix}} = \begin{bmatrix} \frac{\hat{\sigma}_{R_S R_M,t}}{\hat{\sigma}^2_M} \\ \frac{\hat{\sigma}_{R_{Bi} R_M,t}}{\hat{\sigma}^2_M} \\ \frac{\hat{\sigma}_{R_{Bo} R_M,t}}{\hat{\sigma}^2_M} \end{bmatrix}.$$

The authors report that the average beta for Treasury bills is less than 0.1, for Treasury bonds is greater than 1.5, and for stocks is "close to one." BEW conclude that the conditional covariance matrix of asset returns is autoregressive and conditional returns are significantly influenced by conditional covariances.

A stock-only, univariate GARCH(1, 1)-M version of the BEW model simplifies to yield a beta which by definition takes a value equal to one. Nonetheless the model is of interest to this dissertation for its substantiated advocacy of the GARCH(1, 1)-M form for market returns.

The BEW multivariate estimation procedure can be used for simultaneous estimation of predicted betas for subportfolios of the market portfolio. The restriction for such use is that the weighted combination of the subportfolios must represent the market portfolio. The BEW procedure can be applied to the case of two portfolios, one being a single stock and the other begin the remainder of the market portfolio.

Applied to subportfolios of the stock market, the procedure of BEW estimates out-of-sample period $t+1$ betas using i) predictions of the period $t+1$ conditional error variances of subportfolio returns and ii) the period t weights calculated as the period t percentages the subportfolio values represent of the market portfolio value. This

dissertation constructs predicted betas using i) predictions of the period $t+1$ conditional total variances of the asset returns and of the market index returns and ii) the assumption of a constant correlation between the asset returns and market index returns. This dissertation's procedure can be employed to estimate predicted betas for stock portfolios without regard to their past values relative to the total market value.

Ghosh (1992) estimates a modified CAPM with an assumption that the errors follow a GARCH(1, 1) process with a *t*-distribution:

$$R_{i,t} = \alpha_0 + \alpha_1 R_{M,t} + u_{i,t}$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} u^2_{i(t-1)},$$

where $u_{i,t}$ is the error term which is distributed $T(0,H_{i,t},d)$, $R_{i,t}$ is the return of stock i during period t, $R_{M,t}$ is the market return during period t, $H_{i,t}$ is the conditional variance of the error term $u_{i,t}$, d is the number of degrees of freedom for the t-distribution, and where α_0, α_1, γ_{0i}, γ_{1i} and γ_{2i} are coefficients to be estimated. Note that Ghosh (1992) does not use a proxy for the risk-free rate of return.

The value-weighted CRSP index is used as the market portfolio, and the model is estimated for each of ten capitalization-based portfolios. Ghosh uses daily data for each year 1985-1989 separately, excluding Octobers of 1987 and 1989. The author reports only the results for the eleven months of 1989, for which the α_0 and α_1 estimates are significantly lower when estimated by the GARCH model than when estimated using OLS regression and the Scholes-Williams nonsynchronous trading correction. Interestingly, Ghosh finds average portfolio betas are smaller for the portfolios of smaller stocks as

predicted by the theory of Levy (1990).

Ng, Engle, and Rothschild (NER, 1992) estimate a model for stock returns in which the estimate of beta changes to allow the market risk premium to follow a GARCH(1, 1)-M process. NER conduct their beta estimation process in two stages. The first stage is the estimation of the GARCH(1, 1)-M model for the market risk premium:

$$R_{M,t} = \alpha_0 + \alpha_1 H_{M,t} + u_{M,t}$$

$$H_{M,t} = \gamma_{0M} + \gamma_{1M} H_{M(t-1)} + \gamma_{2M} u^2_{M(t-1)}$$

where $u_{M,t}$ is the error term which is distributed $N(0,H_{M,t})$, $R_{M,t}$ is the market excess return during period t, $H_{M,t}$ is the conditional variance of the error term $u_{M,t}$, and where α_0, α_1, γ_{0M}, γ_{1M} and γ_{2M} are coefficients to be estimated. For periods $t = 1, ..., t$ this model supplies estimates of $\hat{R}_{M,t}$ and $\sigma^2_{M,t}$, the time series of conditional means and variances of the market risk premium. This model also supplies period $t+1$ predictions of $\hat{R}_{M(t+1)}$ and $\sigma^2_{M(t+1)}$ based on periods $t = 1, ..., t$. Note that this model for market index returns is the same as used by this dissertation.

The series of market excess return conditional means, $\hat{R}_{M,t}$, and the series of market excess return conditional variances, $H_{M,t}$, are used in the second stage to estimate beta:

$$\beta_{i,t} = \beta_{0i} + \beta_{1i} \hat{R}_{M,t} + u_{i,t}$$

$$H_{i,t} = \gamma_{0i} + \beta^2_{1i} H_{M,t}$$

where $u_{i,t}$ is the error term which is distributed $N(0, H_{i,t})$, $\beta_{i,t}$ is the stock portfolio risk measure during period t, $\hat{R}_{M,t}$ is the predicted market excess return during period t, $H_{i,t}$ is

the conditional variance of the error term $u_{i,t}$, and where β_{0i}, β_{1i}, and γ_{0i} are coefficients to be estimated. β_{0i} is the static component of the risk measure of portfolio *i*, whereas β_{1i} is the portfolio *i* risk measure associated with the conditional market excess return. $\beta_{i,t}$ is estimated using ordinary least squares regression:

$$R_{i,t} = \lambda_i + \beta_{i,t} R_{M,t} + u_{i,t}.$$

The NER beta estimation procedure differs from the methodology of this dissertation in two noteworthy ways. First, the NER procedure imposes the restriction that all of the nonstationarity of beta is a result of the ARCH-type process followed by market excess returns. In contrast, the methodology of this dissertation allows both the stock returns and the market returns to follow ARCH-type processes. Second, NER derive conditional total variances from the conditional error variances using beta, whereas the methodology of this dissertation derives the conditional total variances from the conditional error variances using the correlation instead. Refer to Appendix A for a complete description of deriving conditional total variances from conditional error variances.

NER test whether allowing only the market excess returns to follow a GARCH(1, 1)-M process is sufficient for modeling the nonstationarity of betas. NER concludes that beta nonstationarity is insufficiently explained by their model. This dissertation extends the NER procedure by allowing stock returns to follow a GARCH(1, 1) process.

Koutmos (forthcoming) models stock returns using a two-stage procedure featuring ARCH-class models in each stage. The first stage models a constructed market

risk premium, and the second stage models stock excess returns using the first-stage time series of constructed conditional market premia and of the conditional variances of the constructed market premia.

Koutmos' first stage is to estimate a modified exponential GARCH(1, 1)-M model. After defining the market innovation as $\varepsilon_{M,t} \equiv R_{M,t} - R_f - R^p_{M,t}$, where $R_{M,t}$ is the market return proxied by the CRSP with-dividend value-weighted NYSE Index, R_f is the risk-free rate proxied by the rate on 1-month Treasury bills, and $R^p_{M,t}$ is defined to be the market risk premium, it can easily be shown that Koutmos (forthcoming) models 5, 6, and 7 reduce to:

$$R^p_{M,t} = \alpha_0 + \alpha_1 (H_{T,t} - H_{M,t})$$

$$H_{M,t} = e^{\delta_{M0} - \delta_{M3}\left(\frac{2}{\pi}\right)^{0.5}} H^{(\delta_{M1} - \delta_{M2})}_{T,t-1} H^{\delta_{M2}}_{M,t-1} e^{(\delta_{M3}|\varepsilon_{M,t-1}| + \delta_{M4}\varepsilon_{M,t-1}) H^{-0.5}_{M,t-1}},$$

where

$$H_{T,t} = e^{\delta_{M0}} e^{\delta_{M3}\left\{|\varepsilon_{M,t-1}| H^{-0.5}_{M,t-1} - \left(\frac{2}{\pi}\right)^{0.5}\right\}} H^{\delta_{M2}}_{T,t-1},$$

and where $H_{M,t}$ is the conditional variance of the market premium, and $H_{T,t}$ is the conditional variance Koutmos labels as the "time-varying trend" of the conditional variance. α_0, α_1, δ_0, δ_1, δ_2, δ_3, and δ_4 are parameters estimated by the model. (Note that, although the model estimates seven parameters and four variable time series using one known, ($R_{M,t}$ - R_f), and four restrictions, the nonlinearity of the model saves it from overspecification.)

The second estimation stage of Koutmos estimates a modified exponential

GARCH(1, 1) model for stock excess returns. Koutmos models 1-4 combined are:

$$R_{i,t} = \beta_{i0} + \beta_{i1} R^{p}_{M,t} + \varepsilon_{i,t}$$

$$H_{i,t} = \beta^{2}_{i,M} H_{M,t} + e^{\delta_{i0} - \delta_{i1}\left(\frac{2}{\pi}\right)^{0.5}} e^{\delta_{i1}\left(\left|\varepsilon_{i,t-1}\right| + \delta_{i2}\varepsilon_{i,t-1}\right) H^{-0.5}_{i,t-1}} H^{\delta_{i3}}_{\varepsilon_i,t-1},$$

where $H_{\varepsilon_i,t} = H_{i,t} - \beta^{2}_{i,M} H_{M,t}$ and $\varepsilon_{i,t}$ is the error term distributed $N\left(0, H_{\varepsilon_i,t}\right)$, $R_{i,t}$ is the stock excess return, $R^{p}_{M,t}$ is defined to be the market risk premium, $H_{i,t}$ is the conditional total variance of the stock excess return, and $H_{\varepsilon_i,t}$ is the conditional error variance of the stock excess return. β_{0i}, β_{1i}, δ_{i0}, δ_{i1}, δ_{i2}, and δ_{i3} are parameters estimated by the model. Note Koutmos derives the conditional total variance of stock returns by adding the conditional error variance of stock returns to the product of beta squared multiplied by the conditional total variance of market returns. This method of deriving the conditional total variance of stock returns assumes that beta is constant. The methodology of this dissertation is more general, only relying on the assumption that the correlation between the stock returns and market returns is constant.

Koutmos addresses the problem of beta nonstationarity by introducing a new measure of market risk premium and carefully modeling the process of the error terms. The assumptions that i) parameters α_0 and α_1 remain constant and that ii) the next period's market risk premium is best predicted using only the difference of predicted conditional market variances, permit the Koutmos procedure prediction of out-of-sample stock returns. However, the Koutmos methodology results in neither time-varying

estimates of beta nor predictions of beta. Although this dissertation also models the market returns as

being nonstationary, this dissertation offers a prediction of next period's beta value rather than a prediction of next period's market return.

This dissertation proposes a capital asset pricing model whose form follows directly from the original Markowitz (1952) work. As opposed to the systematic risk of the current period as estimated by the traditional estimation procedure for the CAPM, the model proposed here estimates the expected level of systematic risk of the following, out-of-sample period.

General CAPM Research

In their refutation of Kothari, Shanken and Sloan (1995), Fama and French (1996b) argue that the CAPM is obsolete given evidence that beta, as the sole independent variable, does not adequately explain returns. More interesting to this dissertation, however, is the Fama and French (1996b) finding that NYSE portfolio betas are insensitive to the use of monthly or annual investment period assumptions.

Fama and French (1996b) use NYSE data from CRSP and COMPUSTAT covering from 1927 through 1993. For each year the authors form preliminary beta estimates for each NYSE stock by regressing the past five years of monthly returns on their current and the lagged values of the NYSE value-weighted index. For each year these preliminary beta estimates are used to form ten beta portfolios. The resulting equal-

weighted average monthly and annual returns for the ten beta portfolios and for the NYSE value-weighted index are then used to estimate portfolio betas by regressing all post-formation portfolio returns on the current and lagged values of the index. Also, the resulting equal-weighted average monthly and annual returns for the ten beta portfolios and for the NYSE equal-weighted index are then used to estimate portfolio betas by regressing all post-formation portfolio returns on the current and lagged values of the index. Fama and French (1996b) report that the correlations between the portfolio betas estimated using (monthly returns, value-weighted index), (monthly returns, equal-weighted index), (annual returns, value-weighted index), and (annual returns, equal-weighted index) range from 0.98 and 1.00. The authors cite the high correlations as evidence that the choice of monthly or annual investment periods insignificantly affects estimates of beta. The conclusion of Fama and French (1996b) serves as evidence that the assumption of one-month investment periods for research on beta is not overly restrictive when the alternative is using one-year investment periods. The methodology of this dissertation assumes investment periods are one month each.

He, Kan, Ng and Zhang (1996) develop a model which does not allow the estimation of security-specific coefficients, and conclude that their model explains a "small portion" of the variability of stock returns which can be explained by size and the book-to-market ratio. Their model of stock returns allows for time-varying covariances among the stock returns and their explanatory factors, and allows for time-varying reward-to-covariabilities associated with the time-varying covariances:

$$E_t\left[R_{I(t+1)}\right] = \frac{\sigma_{t\left[\left(R_{I(t+1)} - E\left[R_{I(t+1)}\right]\right),\left(R_{M(t+1)} - E\left[R_{M(t+1)}\right]\right)\right]}}{\sigma^2_{t\left(R_{M(t+1)}\right)}} E_t\left[R_{M(t+1)}\right], \quad \text{(H1)}$$

where $E_t[R_{I(t+1)}]$ is the period t expected excess return during period $t+1$ of a portfolio of a "large" number of stocks, the numerator is the period t conditional covariance between i) the $t+1$ portfolio excess return and the expected value of the $t+1$ portfolio excess return and ii) the $t+1$ market premium and the expected value of the $t+1$ market premium, the denominator is the period t conditional variance of the $t+1$ market premium, and $E_t[R_{M(t+1)}]$ is the period t expected market premium.

He, Kan, Ng and Zhang (1996) present equation (H1) in matrix form and adds restrictions. Given z_t, a vector of marketwide information available at period t, the authors add the restrictions that:

$$\text{vector}\left\{\frac{E_t\left[R_{M(t+1)}\right]}{\sigma^2_{t\left(R_{M(t+1)}\right)}}\right\} = Az_t,$$

and

$$\text{vector}\left\{E_t\left[R_{M(t+1)}\right]\right\} = Cz_t.$$

That is, the ratio of the period t expectation of the period $t+1$ market premium to the period t variance of the market premium is allowed to change over time. The ratio multiplies informational factors with constant weights denoted by the constant matrix A, but the ratio changes as the informational factors of vector z_t change over time. Also, the period t expectation of the period $t+1$ market premium is allowed to change over time. The expected market premium is a constant-weight linear combination of the elements of informational vector, z_t. For the elements of the information vector He, Kan, Ng and

Zhang (1996) use the i) S&P 500 Composite Index, ii) 3-month Treasury bill returns minus 1-month Treasury bill returns, iii) average Baa-rated bond yield minus the Aaa-rated bond yield, iv) dividend yield of the S&P 500 Composite Index, and v) the 1-month Treasury bill return.

The authors test this model using the generalized method of moments, by testing the moment conditions:

$$E_t\left[R_{I(t+1)} - R_{I(t+1)}\left(R_{M(t+1)} - Cz_t\right)' Az_t\right] = 0,$$

and

$$E_t\left[R_{M(t+1)} - Cz_t\right] = 0.$$

Note that while this parameterization allows broad generalizations through the z_t vector, a shortcoming of the He, et. al. methodology is that the vector of portfolios, $R_{I(t+1)}$, is required to consist of all the elements of the market portfolio. This shortcoming is also encountered when applying the methodology of BEW to stock portfolios.

The procedure of He et. al. (1996) also does not permit the incorporation of some variable estimates which this dissertation's methodology requires for constructing predicted betas. Variables central to the estimation procedure of this dissertation are past values of the market return conditional variance and the portfolio return conditional variance. While the past values of the market return variance may be included in the z_t vector, portfolio return conditional variances are specifically excluded from the z_t vector which may only contain marketwide informational factors. Indeed, the final conclusion of He, et. al. is the call for further searching for significant additional z_t factors. As past conditional variances have been found significant in modeling stock returns, using a

methodology which necessarily excludes past conditional variances does not seem appropriate.

With monthly returns data from July 1973 through December 1994, Barber and Lyon (1997) employ the procedure of Fama and French (1992) using a sample previously purposely excluded by Fama and French (1992): financial firms. Another significant difference between the procedures of Fama and French (1992) and Barber and Lyon (1996) is that the latter forms size deciles, but not preliminary beta deciles. Not requiring preliminary beta estimates allows Barber and Lyon to require zero years of preliminary data; portfolio returns are estimated for the same years for which the portfolios are formed. Kothari, Shanken and Sloan (1995) argue that the requirement of five previous years of COMPUSTAT data in estimating the Fama and French (1992) procedure induces survivorship bias, as only the successful (surviving) firms are included in the sample. Barber and Lyon's use of size-based portfolios enables the authors to explore the effects of survivorship bias by estimating return relationships without any prior COMPUSTAT data requirement, and again with a required five years of prior COMPUSTAT data, and comparing the results.

Barber and Lyon (1996) find that capitalization size and book-to-market are significant explanatory variables for the cross-sectional variation in stock returns. Of particular importance to this dissertation are the Barber and Lyon findings that: i) the size and book-to-market relationships to returns are significantly different between the 7/73-6/84 and the 7/84-12/94 periods, and ii) there is no evidence of survivorship bias induced by the requirement of five years of COMPUSTAT data prior to the estimation year. The

latter conclusion serves as reassurance that the methodology of this dissertation, which requires five years of COMPUSTAT data prior to portfolio estimations, does not suffer from significant COMPUSTAT-induced survivorship bias.

Knez and Ready (forthcoming) introduce a "least trimmed squares" (LTS) estimation procedure. LTS is a two-pass OLS estimation procedure: the most influential $a\%$ of observations from an OLS regression are removed from the sample, and another OLS regression is estimated on the remaining observations. The LTS specifications are the percentage, a, and the measure of "influence." The authors state that the measure of influence used by the LTS procedure is the size of the squared residual, (the square of the distance from the observation to the estimated line).[12] Knez and Ready (forthcoming) state that by removing the most influential observations, the LTS estimation technique is useful not just for removing outliers, but also for testing the robustness of OLS regression results.

The authors use the LTS procedure for revisiting Fama and French (1992) and comparing OLS and LTS results. Unlike FF (1992), the sample period of Knez and Ready spans from July 1963 through December 1990. Knez and Ready find that when as little as 1% of the sample is trimmed, the size effect changes from significantly negative to significantly positive, although the book-to-market effect remains insignificantly changed. The size effect result indicates that the higher returns associated with smaller capitalizations are due to a small number of influential observations, and the more general

[12] Although not central to this dissertation, it is interesting to note that this is not a standard measure of influence. The distinction is that the actually most "influential" observation may not display a large residual. Standard measures of influence measure each observation's influence by measuring how far the regression line changes with the removal of the observation.

relationship is that larger firms have higher returns during July 1963 through December 1990. Knez and Ready (forthcoming) also examine this size effect result separately for January and non-January months and find a significant negative relationship for January no matter the trimmed percentage. The non-January relationship is insignificant for the untrimmed sample, but is significantly positive when as few as 1% of the observations are excluded. The Knez and Ready results motivate the investigation of January versus non-January months in this dissertation. However, time-series estimation procedures do not allow the removal of select periods for a subsample of the sampled stocks. Time-series estimation assumes period *t* returns rely on period *t-k* information: removing the period *t-k* information for stock *i* also affects the period *t* information for stock *i*. Researchers cannot simultaneously examine the issue of relationship robustness using LTS and the issue of time series relationships. This dissertation examines time series conditional variance patterns in order to construct a predicted beta for an out-of-sample period.

Fama and French (1993) propose three factors for modeling returns. Daniel and Titman (DT, 1997) test whether the three proposed factors uniformly covary with returns or whether another set of firm attributes drives the Fama and French (1993) results. The examination is of a "chicken-and-egg" problem: Do the three Fama and French (1993) factors covary with returns as orthogonal representatives of systematic risk, or is there an underlying measure of systematic risk with which the three factors and returns each covary? DT conclude the latter after comparing empirical results of OLS models based on three alternative return-generating processes. Daniel and Titman state that their first

return-generating hypothesis, which is their null hypothesis, is compatible with the assumptions of Fama and French (1993):

$$R_{i,t} = E_{t-1}\left[R_{i,t}\right] + \sum_{j=1}^{J} \beta_{i,j}\hat{f}_{j,t} + \beta_{Di(t-1)}\hat{f}_{D,t} + \varepsilon_{i,t},$$

where

$$E_{t-1}\left[R_{i,t}\right] = R_{f,t} + \sum_{j=1}^{J} \beta_{i,j}\delta_{j} + \beta_{Di(t-1)}\delta_{D},$$

or

$$R_{i,t} = R_{f,t} + \sum_{j=1}^{J} \beta_{i,j}\left(\delta_{j} + \hat{f}_{j,t}\right) + \beta_{Di(t-1)}\left(\delta_{D} + \hat{f}_{D,t}\right) + \varepsilon_{i,t}, \quad \text{(DT1)}$$

$$\varepsilon_{i,t} \sim N\left(0, \sigma^{2}_{\varepsilon_{i}}\right), \text{ and } f_{j,t} \sim N(0,1),$$

and where $R_{i,t}$ is the period t return of stock i, $E_{t-1}[R_{i,t}]$ is the period $t-1$ expectation of the period t return of stock i, $\beta_{i,j}$ denotes sensitivity j of stock i to dimension j of systematic riskiness, $\hat{f}_{j,t}$ denotes the measure of systematic risk factor j at period t, $\beta_{Di(t-1)}$ denotes stock i's period $t-1$ observed sensitivity to the dimension of systematic risk DT refers to as financial "distress," $\hat{f}_{D,t}$ denotes the *marketwide* period t level of financial distress, $R_{f,t}$ is the period t risk-free return, δ_j is the expected measure of systematic risk factor j, and δ_D is the expected measure of marketwide financial distress.

The first hypothesized return-generating process of Daniel and Titman (1997) assumes the non-distress β`s, the measures of sensitivity to dimensions of systematic risk other than market distress, are constant. Further, DT offer no justification for making the assumption that betas are identical for the expected (δ) and unexpected $\left(\hat{f}_{t}\right)$ components of systematic risk.

The return-generating process assumed by this dissertation, (equations (1) and (2)), differs from (DT1) in four noteworthy ways. As opposed to (DT1), the set of assumptions underlying (1) and (2): i) limits all systematic risk to the market risk, ii) allows the construction of a predicted beta rather than the use of constant and/or lagged dimensions of sensitivity to systematic risk, iii) allows GARCH modeling, and iv) assumes changes in expected returns include changes in the expected market return.

DT's second hypothesized return-generating process, (DT2), differs from (DT1) by holding the distress-related beta constant, but allowing expected returns to include time-varying naïvely-expected values of the marketwide risk premia:

$$R_{i,t} = E_{t-1}\left[R_{i,t}\right] + \sum_{j=1}^{J} \beta_{i,j} \hat{f}_{j,t} + \varepsilon_{i,t}, \qquad \text{(DT2)}$$

where $$E_{t-1}\left[R_{i,t}\right] = R_{f,t} + \sum_{j=1}^{J} \beta_{i,j} \delta_{j(t-1)},$$

$$\varepsilon_{i,t} \sim N\left(0, \sigma_{\varepsilon_i}^2\right), \text{ and } f_{j,t} \sim N(0,1),$$

and where $R_{i,t}$ is the period t return of stock i, $E_{t-1}[R_{i,t}]$ is the period t-1 expectation of the period t return of stock i, $\beta_{i,j}$ denotes sensitivity j of stock i to dimension j of systematic riskiness, $\hat{f}_{j,t}$ denotes the measure of systematic risk factor j at period t, $R_{f,t}$ is the period t risk-free return, and $\delta_{j(t-1)}$ is the measured level of risk premium j during period t-1.

The return-generating process assumed by this dissertation, (equations (1) and (2)), differs from the return-generating process assumed by (DT2) in four noteworthy ways similar to the differences between this dissertation and (DT1). Unlike the assumptions underlying (DT2), the set of assumptions underlying (1) and (2): i) limits the

dimensions of systematic risk to one, the market risk, ii) allows the construction of a predicted beta rather than the use of constant betas, iii) allows GARCH modeling, and iv) allows the expected values at period *t-1* of the period *t* market return to differ from the period *t-1* observed market return: (1) and (2) place no restrictions on the expectation of the period *t* market return, whereas (DT2) specifies the period *t* stock return depends on $\delta_{j(t-1)}$, the measured level of risk premium *j* during period *t-1*.

DT's third hypothesized return-generating process, (DT3), differs substantially from (DT2) and (DT1). The process assumed by (DT3) differs from (DT2) only by the expected return equation, but the implications of the differences are interesting. Note that although "$\beta_{Di(t-1)}$" represents the beta of stock *i* for presumably-well-defined marketwide financial distress in (DT1), $\beta_{Di(t-1)}$ in (DT3) is a factor measured during period *t-1* and representing the financial distress of stock *i* for period *t*:

$$R_{i,t} = E_{t-1}\left[R_{i,t}\right] + \sum_{j=1}^{J} \beta_{i,j} \hat{f}_{j,t} + \varepsilon_{i,t}, \qquad \text{(DT3)}$$

where $$E_{t-1}\left[R_{i,t}\right] = \alpha_0 + \alpha_1 \beta_{Di(t-1)},$$

$$\varepsilon_{i,t} \sim N\left(0, \sigma^2_{\varepsilon_i}\right), \text{ and } f_{j,t} \sim N(0,1),$$

and where $R_{i,t}$ is the period *t* return of stock *i*, $E_{t-1}[R_{i,t}]$ is the period *t-1* expectation of the period *t* return of stock *i*, $\beta_{i,j}$ denotes sensitivity *j* of stock *i* to dimension *j* of systematic riskiness, $\hat{f}_{j,t}$ denotes the measure of systematic risk factor *j* at period *t*, α_0 and α_1 are estimable parameters assumed to be constants, and $\beta_{Di(t-1)}$ is a factor "not directly related to the loadings on the distressed factors."

There are noteworthy similarities between the assumptions underlying (DT3) and the assumptions underlying equations (1) and (2) of this dissertation. For ease of comparison, substitute (DT3)'s expected return equation into the equation for actual returns and compare the result with the lagged (10′):

$$R_{i,t} = \alpha_0 + \alpha_1 \beta_{Di(t-1)} + \sum_{j=1}^{J} \beta_{i,j} \hat{f}_{j,t} + \varepsilon_{i,t} \qquad (DT3')$$

where $\varepsilon_{i,t} \sim N\left(0, \sigma^2_{\varepsilon_i}\right)$, $f_{j,t} \sim N(0,1)$, $R_{i,t}$ is the period t return of stock i, $\beta_{Di(t-1)}$ is measured during period t-1 and represents the financial distress of stock i for period t, $\beta_{i,j}$ denotes sensitivity j of stock i to dimension j of systematic riskiness, $\hat{f}_{j,t}$ denotes the measure of systematic risk factor j at period t, and α_0 and α_1 are coefficients assumed to be constants. Whereas equation (10′) which is equation (9) without the non-market independent variables, is:

$$R_{i,t} = \alpha_0 + \alpha_{1,t} \beta_{\sigma i,t} + u_{i,t} \qquad (10')$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} u^2_{i(t-1)},$$

where $u_{i,t}$ is the error term which is distributed $N(0,H_{i,t})$, $R_{i,t}$ is the period t return of stock i, $\beta_{\sigma i,t}$ is constructed during period t-1 and represents the systematic risk of stock i for period t, $H_{i,t}$ is the period t conditional error variance of $u_{i,t}$; and γ_{0i}, γ_{1i}, γ_{2i}, α_0, and α_1 are coefficients to be estimated by the model.

To transform the model of (DT3′) into that of (10′) requires three changes: i) restrict $\beta_{i,j}=0$ for j = 1, 2, ..., J, ii) include a prediction of the out-of-sample "distress risk" variable in place of the historical measure, and iii) allow the conditional error

variances to follow a GARCH(1, 1) process. Of course the alternative of transforming (10′) into (DT3′) also requires three changes: i) allow additional factors in the mean equation, ii) replace the predicted beta with an in-sample estimate of beta, and iii) restrict $\gamma_{ji} = 0$ for j = 0, 1, 2.

During estimation of models (DT1) and (DT2), Daniel and Titman (1997) restrict the intercept to be the risk-free rate. Citing the three Fama and French (1993) factors as the return premia for small capitalization stocks, high book-to-market ratio stocks, and market risk, the authors restrict $\beta_{Di(t-1)}$ to be the book-to-market ratio during the estimation of (DT1). For choosing among the models, the estimation results are unclear, although DT state that (DT3) is supported by the data. (For one example, faced with generally insignificant intercept terms in Table VII, DT examine the number of positive intercept occurrences versus the number of negative intercept occurrences.)

A result of DT's portfolio formation based on attributes other than preliminary market betas is the generally high levels of statistical significance achieved by market betas in the cross-sectional regressions. As book-to-market data are available beginning in 1962 and some mid-1960's sample size problems are encountered, the authors estimate the models using months from July 1973 through December 1993. The high levels of market beta significance compared to the results of Fama and French (1992) may result not only from not pre-sorting on preliminary betas, but also from using a modern sampling period.

DT also make an argument for using sample periods beginning in the late 1970's or early 1980's. In the conclusion, Daniel and Titman hypothesize that before the end of the 1970's, the relationships between size and returns and between book-to-market ratios

and returns may have been caused by a self-fulfilling prophecy of investor perception. DT further hypothesize that the increasing "ability to access and manipulate the returns/accounting data" would cause a noticeable change in the observed patterns.

Fama and French (1996a) demonstrate that the three-factor APT model advocated by Fama and French (1993) explains many popular asset-pricing anomalies. The three factors are: i) the market excess return, ii) the difference in returns between a small-capitalization portfolio and a large-capitalization portfolio, and iii) the difference in returns between a high book equity-to-market equity portfolio and a low book-equity-to-market equity portfolio. Featuring size-beta portfolios and a rolling sample, the methodology follows that of FF (1992) with the exception that *excess* returns are used: the one-month Treasury bill rate from CRSP serves as their risk-free rate. Fama and French (1996a) note that the second factor, "small minus big," is related to the covariance of returns of small stock issues examined by Huberman and Kandel (1987).[13] Similarly, Fama and French (1996a) note that the third factor, "high minus low," is related to the covariance of returns examined by Chan and Chen (1991).[14] Fama and French (1996a) conclude that OLS estimation of the three-factor model eliminates the significance of the long-term returns reversal found by DeBondt and Thaler (1985), and reduces to insignificance other previously-anomalous factors such as size (market value of issue), book equity-to-market equity, earnings/price, cash flow/price, and past sales growth. Interestingly, the Fama and

[13] Huberman and Kandel (1987) found covariation in the returns of small stock issues that is significantly priced whether or not beta is included in the model.

[14] Chan and Chen (1991) found covariation in returns that is significantly priced whether or not beta is included in the model.

French (1996a) three-factor model is unable to explain the short-term returns autocorrelation found by Jegadeesh and Titman (1993).

Fama and French (FF, 1992) use a rolling-sample estimation procedure. They rank CRSP-tape NYSE firms by size (as measured by the natural logarithm of market equity) and divide this list into decile portfolios each June. The authors further divide each size decile into deciles based on pre-ranking, security-specific β's. These pre-ranking betas are found by summing the slopes of regressing individual security returns on the current and prior values of the NYSE value-weighted return, using 60 periods of monthly data. Fama and French then calculate the equally-weighted monthly returns for the next twelve months for each of these 100 size-beta portfolios and estimate β's for each of the 100 portfolios based on the twelve monthly returns. The authors allocate the resulting portfolio β's to each stock in its respective portfolio. Fama and French mention that this beta estimation technique rids attenuation bias at the price of sacrificing some estimation accuracy. As a test of the CAPM, Fama and French use a cross-sectional regression (at the security level) to predict returns. The authors find that the natural logarithm of book equity-to-market equity and the natural logarithm of market equity serve as statistically significant independent variables in regressions where β and leverage measures are statistically insignificant.

Jagannathan and Wang (1996) simultaneously relax two assumptions of the CAPM: that betas are constant across time periods and that the return on aggregate wealth is sufficiently proxied by the return on the value-weighted portfolio of U.S. stocks. The authors state that practitioners use the CAPM beta for deriving the appropriate

discount rate during project valuation despite a lack of empirical support for the CAPM. JW further posits that a problem with the CAPM may lie with the common estimation assumption that beta remains constant regardless of business cycle activity, technology shocks or taste shocks.

Relaxing the assumption of a constant beta, JW derives a conditional CAPM in which expected returns (conditioned on past information) are a linear combination of the conditional expected return on a "zero-beta" portfolio, $\gamma_{0,t-1}$, and the product of the conditional market risk premium with the conditional beta of stock *i*, $\gamma_{1,t-1}\beta_{i,t-1}$:

$$E[R_{i,t}|I_{t-1}] = \gamma_{0,t-1} + \gamma_{1,t-1}\beta_{i,t-1},$$

$$\text{where } \beta_{i,t-1} = \frac{\text{Cov}(R_{i,t}, R_{M,t}|I_{t-1})}{\text{Var}(R_{M,t}|I_{t-1})}.$$

Note that this conditional CAPM is the same as equation (1). JW derive a model for the unconditional expected returns by taking the unconditional expectation of their conditional CAPM equation. The result is that expected returns are a linear combination of a "constant" average zero-beta portfolio return, γ_0, a "constant" market risk premium, γ_1, multiplied by a "constant" expected beta, $\bar{\beta}_i$, and the covariance between the expected market risk premium with a conditional beta, $\text{Cov}(\gamma_{1,t-1}, \beta_{i,t-1})$:

$$E[R_{i,t}] = \gamma_0 + \gamma_1\bar{\beta}_i + \text{Cov}(\gamma_{1,t-1}, \beta_{i,t-1}).$$

Note that JW derives these "constant" measures as being their expected values from the vantage point of knowing the information revealed during period *t-1*:

$$\gamma_0 = E[\gamma_{0,t-1}], \qquad \gamma_1 = E[\gamma_{1,t-1}], \qquad \bar{\beta}_i = E[\beta_{i,t-1}].$$

JW states that the constant expected beta is defined as the expected value of the current-period beta given the information revealed during prior periods up to and including *t-1*. The constant expected beta is only assumed to be constant from the revelation of information during the current period through the subsequent one period (when, presumably, additional information will be revealed).

Jagannathan and Wang define the "beta-prem sensitivity" as the covariance between the expected market risk premium with a conditional beta divided by the variance of the conditional market risk premium:

$$\vartheta_i \equiv \mathrm{Cov}(\beta_{i,t-1}, \gamma_{1,t-1}) \Big/ \mathrm{Var}(\gamma_{1,t-1}),$$

which simplifies to $\vartheta_i \equiv \rho_{\beta_{i,t-1},\gamma_{1,t-1}} \sigma_{\beta_{i,t-1}}$. The beta-prem sensitivity measures the size of the standard deviation of the conditional beta linearly associated with the conditional market risk premium. JW decompose the conditional beta, $\beta_{i,t-1}$ into three parts: a constant expected beta $\bar{\beta}_i$, the beta-prem sensitivity multiplied by the distance of the *t-1* market risk premium from its expected value given information of time *t-1*, $\vartheta_i(\gamma_{1,t-1} - \gamma_1)$, and a residual term, $\eta_{i,t-1}$. The residual $\eta_{i,t-1} \equiv \beta_{i,t-1} - \bar{\beta}_i - \vartheta_i(\gamma_{1,t-1} - \gamma_1)$, so that

$$\beta_{i,t-1} \equiv \bar{\beta}_i + \vartheta_i(\gamma_{1,t-1} - \gamma_1) + \eta_{i,t-1}.$$

However, JW state an appropriate estimation procedure for the expected beta and for the beta-prem sensitivity would be to regress the time series of conditional betas on the market risk premium. Such a procedure would hold the expected beta constant over the entire sampling period. JW's expected beta would be constant in expectation only under

an additional restriction[15], and the imposition of assuming this restriction would shift the explanatory power of the varying part of the "constant" expected beta onto the beta-prem sensitivity measure.

In practice, Jagannathan and Wang estimate two betas: a "market beta" based on the market return and a "premium beta" on the conditional market risk premium. The market beta is defined as $\beta_i \equiv \mathrm{Cov}(R_{it}, R_{Mt}) / \mathrm{Var}(R_{Mt})$ to measure market risk, and the premium beta is defined as $\beta_i^{\gamma} \equiv \mathrm{Cov}(R_{it}, \gamma_{1,t-1}) / \mathrm{Var}(\gamma_{1,t-1})$ to measure the risk associated with the instability of beta. JW's Appendix A shows that the market beta and premium beta are linear functions of the expected beta and the beta-prem sensitivity under the restriction that the expected beta is assumed to be constant over the entire estimation period. This assumption of a constant expected beta, which is assumed to be constant over many periods, contradicts the condition of JW's first derivation that the expected beta for the subsequent period equals the expected value of beta of the current period. Do JW expect that investors fail to update their expectations of beta given information arriving during subsequent periods?

Jagannathan and Wang state that their estimation procedure is necessary because they "cannot estimate [the beta-prem sensitivity] and [the expected beta]." Such an estimation easily follows from having a time series of estimates of total conditional betas. Estimation of the total conditional betas is a contribution of this dissertation.

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[15] The additional restriction is that the expected value of the product of JW's beta-prem sensitivity with the conditional market risk premium is zero for all assets.

Further, the total conditional betas can be decomposed by cross-sectionally orthogonalizing the conditional betas on the ordinary least squares-estimated betas. The OLS-estimated betas are analogous to JW's expected betas in that the OLS betas represent the level of market risk, and the orthogonalized conditional betas are analogous to JW's beta-prem sensitivity in that the residual betas represent the risk associated with the changes in the betas. However, the total conditional betas estimated by this dissertation are not restricted to be constant over time, as the expected beta and beta-prem sensitivity are restricted in JW. Allowing betas to change over time is consistent with modeling beta nonstationarity.

Levy (1978) notes that investors violate a fundamental property implied by the CAPM: investors do not hold the market portfolio. Levy (1978) observes that investors neither hold all of the securities of the market nor do investors hold all securities in the same proportions as the securities represent in the market.[16] Levy (1978) derives the CAPM under the restriction that a limited number of securities is held in the portfolio, and finds that own-variance should be significant. Levy (1978) conducts empirical tests using two-pass OLS regressions and monthly, semiannual, and annual returns for a random sample of 101 NYSE stocks. During the first-pass regression, T-bill rates "as well as [rates] on government bonds" were used as the risk-free rate. During the second-pass regression Levy (1978) finds that beta, total return variance, and residual variance are each statistically significant when tested independently. For monthly data, none are significant when two of the independent variables are included. For semiannual and annual

[16] Even with the rising use of mutual funds and index funds, I doubt that a significant proportion of investors holds a reasonable proxy of the entire the market portfolio.

data, each beta and residual variance are significant when included in the same regression, but only total variance is significant when included in the same regression with beta. This evidence that investors do not hold the market portfolio in practice is important to this dissertation. Stock betas do not matter to investors who opt to hold the market portfolio.

Fama and MacBeth (1973) estimate: $R_{i,t} = \alpha_0 + \beta_i R_{M,t} + u_{i,t}$ for all NYSE stocks and rank the stocks by their beta estimates. This list is divided into 20 portfolios, each containing a consecutive 5% of the firms in the ranking. Each firm is assigned the average beta of its portfolio. Also estimated for each portfolio are the average squared beta and the average standard deviation of the residuals. Note that the risk-free rate of return for this CAPM estimation was assumed to be uncorrelated with the risky asset returns and the market returns, so was only modeled by the inclusion of an intercept term. Having collected the necessary data, Fama and MacBeth (1973) estimate a market model with the portfolio return as the dependent variable, and an intercept term, the portfolio beta, the portfolio average squared beta, and the portfolio average residual standard deviation as the independent variables. Fama and MacBeth (1973) conclude that beta is generally positive and most often significant, the portfolio average squared beta is generally negative and usually insignificant, and the portfolio average residual standard deviation is consistently insignificant. Using the yield on one-month T-bills as the risk-free rate, Fama and MacBeth (1973) also conclude that the intercept term is significantly different from the risk-free rate.

Although some of the previous literature addresses the nonstationarity of market returns, none of the previous literature constructs an expected value of the subsequent period's CAPM beta. This dissertation derives, estimates, and tests *predicted* betas.

Previous GARCH Model Findings

GARCH(1, 1) Model for Individual Stocks

Individual stock returns have been found to follow a generalized autoregressive conditional heteroskedasticity model with one past lag of conditional variance and one past lag of squared error (GARCH(1, 1)). Bera, Bubnys and Park (1988) use an ARCH(1) specification for modeling the individual and portfolio returns of 126 randomly-chosen firms, and find statistically significant ARCH term coefficients for all portfolios and for 93 of the 126 individual firms. Ghosh (1992) models returns of ten size portfolios using a GARCH(1, 1) model and finds the ARCH and GARCH term coefficients to be statistically significant for all ten portfolios. More recently, Koutmos (forthcoming) uses an EGARCH(1, 1) model for stock returns, but does not report the statistical significance of the coefficients estimated exclusively in the variance equation.

GARCH(1, 1)-M for the Stock Market

Market returns have also been found to follow a generalized autoregressive conditional heteroskedasticity model with one past lag of conditional variance, one past

lag of squared error, and with the conditional standard deviation or conditional variance in the mean equation (GARCH(1, 1)-M). French, Schwert, and Stambough (1987) estimates market returns using two GARCH(1, 1)-M models throughout the paper, one featuring the conditional standard deviation and the other featuring the conditional variance included in the mean equation. The authors show either specification achieves virtually identical explanatory power, and that either measure of conditional volatility in the mean equation is statistically significant. Bollerslev, Engle and Wooldridge (1988) modeled the stock market as one of three types of returns using an MGARCH(1, 1)-M model. If presented in univariate form for modeling only stock market returns, the BEW model reduces to a GARCH(1, 1)-M model. More recently, Ng, Engle and Rothschild (1992) employ a GARCH(1, 1)-M model for market excess returns in the first stage of their estimation of a nonstationary beta.

In summary, stock excess returns and the market premium have been found to follow ARCH-class processes. Nonstationarity of the market premium causes estimation bias when estimating beta using ordinary least squares regression (Kmenta,1986). This study tests the hypothesis that the predicted beta is significant in cross-sectionally modeling returns while controlling for the level of the CAPM beta. Next, the methodology for deriving and testing the predicted beta is presented.

CHAPTER 3

METHODOLOGY

This chapter discusses the methodological approach for this study. Given that the two-stage estimation procedure using equation (3) cannot currently be conveniently estimated, the four-stage procedure is implemented. The four-stage procedure is detailed, then a procedure for checking whether multiple iterations helps the estimation process converge is described. Next, tests of the hypothesis and tests of market anomalies are provided.

A Four-Stage Estimation Procedure

In four stages we construct predicted betas as seen in equation four:

$$\hat{\beta}_{\sigma i(t+1)} \equiv \frac{\hat{\rho}_{Mi(t+1)}\hat{\sigma}_{i(t+1)}}{\hat{\sigma}_{M(t+1)}} \qquad (4)$$

The first stage estimates the market premium standard deviation, $\sigma_{M(t+1)}$. The second stage estimates the stock excess return standard deviation $\sigma_{i(t+1)}$. The third stage estimates the constant correlation, ρ_{Mi}, and the fourth stage combines these estimates to form the estimate of the predicted beta, $\beta_{\sigma i(t+1)}$.

Stage 1

The first stage of the estimation is to estimate a univariate GARCH (1,1)-M model for the market portfolio with a sixty-month sample on a monthly-rolling-sample basis:

$$R_{M,t} = \alpha_0 + \alpha_1 \sqrt{H_{M,t}} + u_{M,t}$$

$$H_{M,t} = \gamma_{0M} + \gamma_{1M} H_{M(t-1)} + \gamma_{2M} u^2_{M(t-1)}$$

where $u_{M,t}$ is the error term which is distributed $N(0,H_{M,t})$, $R_{M,t}$ is the market excess return during period t, $H_{M,t}$ is the conditional error variance of $u_{M,t}$, $(u_{M,t-1})^2$ is the lagged squared error, and where α_0, α_1, γ_{0M}, γ_{1M} and γ_{2M} are coefficients to be estimated. This model style was found to be the most appropriate for the two-step procedure of Ng, Engle, and Rothschild (1992). SAS software uses a quasi-dual Newton method to find the maximum likelihood estimations. Just as with OLS estimation, a GARCH-M assumption is that the correlation between the independent and dependent variables is constant. This model assumes that the correlation between the market premium and conditional error standard deviation, $\rho_{R_M,\sqrt{H_M}}$, is constant. Using this assumption the time series of market premia and the time series of conditional standard deviations is used to calculate the correlation. Given the correlation, the model provides estimates of the conditional total market standard deviation, $\hat{\sigma}_{M,t}$, for periods t=1 through t=60: $\hat{\sigma}_{M,t} \equiv \sqrt{\frac{H_{M,t}}{1 - \rho^2_{R_M,\sqrt{H_M}}}}$. Also using the fact that both independent variables of the conditional variance equation are past information, the model provides an estimate of the conditional total market premium standard deviation for period $t+1$ used in constructing the predicted beta:

$$\hat{\sigma}_{M(t+1)} \equiv \sqrt{\frac{H_{M(t+1)}}{1-\rho^{2}_{R_M,\sqrt{H_M}}}}.$$

Stage 2

For each stock, estimate a GARCH(1, 1) model with a sixty-month sample on a monthly-rolling-sample basis:

$$R_{i,t} = \alpha_i + u_{i,t} \qquad (7)$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} u^2_{i(t-1)}$$

where $u_{i,t}$ is the error term which is distributed $N(0, H_{i,t})$, $R_{i,t}$ is the stock excess return, $H_{i,t}$ is the conditional variance of the residual at time t, and α_i, γ_{0i}, γ_{1i}, and γ_{2i} are coefficients to be estimated. The variance equation yields point-in-time estimates of the $t = 1$ through $t = 61$ conditional total variances (which in this case are equal to the conditional error variances) of the size-beta portfolio returns. The square root of the conditional total variance estimate for period $t = 61$ is the estimate of the conditional total standard deviation of stock excess returns used in constructing the predicted beta.

By excluding nonconstant independent variables from the mean equation of model (7), the conditional total variance of stock excess returns is estimated directly. Including nonconstant independent variables results in estimation of the conditional error variance of stock excess returns, and necessitates using equation (A1) of Appendix A to derive the conditional total variance of stock excess returns. Note that model (7) does not correspond to the hypothesized return-generating process of equation (2): nonconstant independent variables are excluded to directly estimate the conditional total variance of

stock excess returns. The conditional total variance of stock excess returns is unaffected by the inclusion or exclusion of nonconstant independent variables. However, note the market excess return is excluded from being an independent variable in the mean equation of model (7): including the market excess return as an independent variable would cause reliance on the assumption that beta is constant during the estimation sample period.

Stage 3

For each stock, the constant correlation between the stock excess return and the market excess return can be estimated. Given the in-sample version of (4),

$\beta_{\sigma i,t} \equiv \frac{\hat{\rho}_{iM,t}\hat{\sigma}_{i,t}}{\hat{\sigma}_{M,t}}$, the in-sample version of the hypothesized model (2) to be estimated is:

$$R_{i,t} = \alpha_i + \frac{\hat{\rho}_{iM}\hat{\sigma}_{i,t}}{\hat{\sigma}_{M,t}} R_{M,t} + u_{i,t} \qquad (2')$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} u^2_{i(t-1)},$$

where $u_{i,t}$ is the error term which is distributed $N(0,H_{i,t})$, $R_{i,t}$ is the stock excess return during period t, $\hat{\sigma}_{i,t}$ is the conditional total standard deviation of the stock excess returns estimated in Stage 2, $\hat{\sigma}_{M,t}$ is the conditional total standard deviation of the market excess returns estimated in Stage 1, $R_{M,t}$ is the market excess return during period t, $H_{i,t}$ is the conditional variance of the residual at time t, and α_i, ρ_{iM}, γ_{0i}, γ_{1i}, and γ_{2i} are coefficients to be estimated. Equation (2′) can be estimated using the 60 in-sample conditional total market standard deviations from Stage 1, the 60 in-sample conditional total individual standard deviations from Stage 2, and market excess return data to construct the

independent variable. Then for each of the monthly-rolling samples of Stages 1 and 2 estimate the constant correlation between the stock excess returns and the market excess returns with the model:

$$R_{i,t} = \alpha_i + \rho_{iM}\left[\frac{\hat{\sigma}_{i,t}}{\hat{\sigma}_{M,t}} R_{M,t}\right] + u_{i,t}$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i(t-1)} + \gamma_{2i} u^{2}_{i(t-1)},$$

where $u_{i,t}$ is the error term which is distributed $N(0,H_{i,t})$, $R_{i,t}$ is the stock excess return during period t, $\hat{\sigma}_{i,t}$ is the conditional total standard deviation of the stock excess returns estimated in Stage 2, $\hat{\sigma}_{M,t}$ is the conditional total standard deviation of the market excess returns estimated in Stage 1, $R_{M,t}$ is the market excess return during period t, $H_{i,t}$ is the conditional variance of the residual at time t, and α_i, ρ_{iM}, γ_{0i}, γ_{1i}, and γ_{2i} are coefficients to be estimated. The assumption of a constant correlation enables the use of this in-sample estimate of the correlation in the construction of the out-of-sample predicted beta.

Stage 4

This stage derives the predicted beta of equation (4): $\beta_{\sigma i(t+1)} \equiv \dfrac{\hat{\rho}_{Mi(t+1)}\hat{\sigma}_{i(t+1)}}{\hat{\sigma}_{M(t+1)}}$.

Stages 1-3 are estimated for each 60-month period on a monthly-rolling-sample basis, and each period yields an estimate of the predicted betas for the 61st period. The resulting predicted betas are monthly estimates for the months July 1983 through June 1995.[17]

[17] Estimates for July 1995 are also available, but were excluded in order to keep an equal number of estimates representing each month.

Checking for Iteration Necessity

The methodology of this dissertation employs a four-stage estimation procedure rather than the two-stage model using equation (3). The resulting estimations may be improved through the use of multiple iterations. To check whether conducting multiple iterations has a substantial effect on the estimates, Stages 1-4 are estimated, then Stages 3 and 4 are repeatedly reestimated for a random sample of stocks.

The results of initial Stages 1-4 are time series of the estimated conditional total variance of market excess returns, $\hat{\sigma}^2_{M,t}$, the conditional error variance of stock excess returns, $\hat{\sigma}^2_{\varepsilon_i,t}$, the conditional total variance of stock excess returns, $\hat{\sigma}^2_{i,t}$, the constant correlations, $\hat{\rho}_{Mi}$, and the predicted betas, $\hat{\beta}_{\sigma i,t}$. The estimates of predicted beta, conditional total variance of market excess returns, and conditional error variance of stock excess returns can be used to calculate a second-iteration estimate of the conditional total variance of stock excess returns:

$$\hat{\hat{\sigma}}^2_{i,t} = \hat{\beta}^2_{\sigma i,t}\hat{\sigma}^2_{M,t} + \hat{\sigma}^2_{\varepsilon_i,t}.$$

This second-estimate of the series of conditional total variance of stock excess returns can be compared to the initial series, and can also be used in a reestimation of the correlation in Stage 3:

$$R_{i,t} = \alpha_i + \rho_{iM}\left[\frac{\hat{\hat{\sigma}}_{i,t}}{\hat{\sigma}_{M,t}}R_{M,t}\right] + u_{i,t}$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i}H_{i(t-1)} + \gamma_{2i}u^2_{i(t-1)},$$

where $u_{i,t}$ is the error term which is distributed $N(0,H_{i,t})$. These reestimation iterations can

be repeated until the estimates converge. These results are discussed in the Section "Iteration Trial Results" of Chapter 4.

Predicted Beta Performance Tests

To test whether the predicted beta is an improvement over the OLS beta, we test whether the predicted beta is significant in situations where the OLS beta is insignificant, and we test whether the predicted beta explains current market anomalies. The OLS beta is not as consistently significant as expected (for example, Fama and French (1992)). As the predicted beta is not estimated under the restriction that beta is constant, the predicted beta may be more consistently significant. Further, unlike the OLS beta, the predicted beta may explain current market anomalies such as the size effect of the January effect.

First, the predicted beta is tested for signficant explanatory power in addition to that of the OLS beta.[18] Second, the OLS beta and the predicted beta are each used in a test designed by Fama and French (1992) and their results are compared. Third, the relationship between size and the predicted beta is compared to the relationship between size and the OLS beta, and finally, the predicted beta is tested for difference in explanatory power between January and non-January samples. If the predicted beta measures systematic risk and if there is only one dimension of systematic risk, then inclusion of the predicted beta should leave size and January insignificant in explaining stock excess

[18] If the independent variables $\beta_{i,t}$ and $\beta_{\sigma(i,t)}$ are found to be highly correlated, then they will be orthogonalized prior to estimation of the regression. Specifically, the predicted betas will be replaced by their residuals from an OLS regression of $\beta_{\sigma(i,t)} = \alpha_{0i} + \alpha_{1i}*\beta_{i,t}$. See the Section "Testing Predicted Betas while Controlling for Traditional Betas" of Chapter 5 for the results.

returns.

Hypothesis Test

While controlling for the traditional beta, we test whether $\hat{\beta}_{\sigma i,t}$ is statistically significant by the statistical significance of its coefficient, ξ_{2i}, in model (8):

$$R_{i,t} = \xi_0 + \xi_{1i} * \hat{\beta}_{i,t} + \xi_{2i}\hat{\beta}_{\sigma i,t} + u_{i,t} \qquad (8)$$

$u_{i,t}$ is the error term which is distributed N(0, ϕ)

where $R_{i,t}$ is the excess return on portfolio i during period *t*, $\beta_{i,t}$ is the traditional beta and $\beta_{\sigma(i,t)}$ is the predicted beta which is adjusted for the relative expected standard deviation during period *t*.

Do the Predicted Betas Explain Current Market Anomalies?

Anomaly Test 1: Fama and French (1992) Revisited

The F&F (1992) test of beta uses cross-sectional regressions corresponding to the approach of Fama and MacBeth (1973). For each month, the estimates of $\beta_{i,t}$ from an OLS regression and $\beta_{\sigma(i,t)}$ from Stage 4 each are used as the beta values in the regression:

$$R_{i,t} = \xi_0 + \xi_{1i}*\beta_{[i,t \text{ or } \sigma(i,t)]} + \xi_{2i}*\ln(ME) + \xi_{3i}*\ln(BE/ME) + u_{i,t}, \qquad (9)$$

corresponding to the test conducted by FF (1992) which concluded that the traditional beta was statistically insignificant.

Anomaly Test 2: Market Size Examined

Knez and Ready (forthcoming) claim a substantial victory in the debate concerning the stock market value size anomaly. In fact, the authors claim to have solved the anomaly by using a "least trimmed squares" (LTS) estimation procedure. Knez and Ready revisit FF (1992) using the Fama and MacBeth (1973) procedure but estimating each OLS and LTS models. Knez and Ready find that the small firm effect disappears when as little as 1% of the sample is trimmed before model estimation, although the book-to-market and January effects remain. It should be noted, however, that their sampling period is July 1963 through December 1990, despite the fact that Nasdaq data is only available on both CRSP and COMPUSTAT dating back to 1973. This study will provide another test of the firm size affect, but using a sampling period of 1973 through 1995.

Anomaly Test 3: January Effect Examined

One of the three statistically significant dynamic factors included in the full Ng, Engle, and Rothschild (1992) model is a factor related to an indicator variable denoting the month of January. However, as the correlation between the factor and the January variable is less than 0.25 and as the factor also is significantly related to bond spread, the examination of the January effect in this research will follow from (8) and (9). Specifically, to test for January differences, models (8) and (9) will also be estimated separately for January and non-January months.

In the next chapter, the details of sample construction and empirical descriptions of the sample are provided. Discussion of the test results is presented in chapter five.

CHAPTER 4

SAMPLE CONSTRUCTION AND DESCRIPTION OF DATA

The first section of this chapter details the sample construction procedure which follows that of Fama and French (1992). The second section details the results of constructing the predicted beta using multiple iterations, and the final section features descriptive statistics tables using the same presentation format as Fama and French (1992).

Sample Construction

For all stocks listed by CRSP, we collect 1977-1993 year-end market equity values (ME's)[19]. This sample is match-merged by date and CUSIP with book equity values (BE's)[20] gathered from COMPUSTAT. Only nonfinancial firms with December fiscal year-ends are included in the sample.[21]

To estimate the preliminary beta for each stock we use ordinary least squares regressions. The preliminary betas are the beta estimates used for dividing each size decile into beta deciles. The regression model matches that used by Fama and French (1992):

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[19] Fama and French (1992) used year-end market equity values for constructing the book-to-market accounting ratios, and the market equity values of the following June to construct the size-beta portfolios. In this dissertation, year-end market equity values are used for both purposes. It is expected the substitution of year-end market equity values for the market equity values of the following June will have little effect on the size-beta portfolio membership classifications.

[20] In accordance with Fama and French (1992), book equity was calculated as the sum of book value of common equity with balance-sheet deferred taxes, item numbers 60 and 35.

[21] Financial firms excluded were those firms with Standard Industry Classification (SIC) codes beginning with 6 but not with 65. That is, real estate firms were included in the sample as nonfinancials.

$$R_{i,t} = \alpha_1 + \alpha_2 R_{m(t-1)} + \alpha_3 R_{m,t} + \zeta_{i,t},$$

where $\xi_{i,t}$ is the error term which is distributed i.i.d. N(0, $\sigma_{i,t}^2$). The preliminary beta is the sum of the coefficients α_2 and α_3. As in FF (1992), the CRSP value-weighted index is used as a proxy for the market portfolio. A sample size of 60 months is used per stock. For example, for the sample using 1977 year-end market equity and book-to-market data, the preliminary betas for are estimated using the monthly returns beginning July 1973 and ending June 1978. Likewise, the regressions for the preliminary betas of the 1993 sample use the monthly returns beginning July 1989 and ending June 1994. Using a sample which begins in July and ends in the June following the year of the market equity data collection maintains consistency with the methodology of FF (1992). As the preliminary beta regressions for the 1977 sample use data from as early as June 1973, and as 1973 is the first year for which Nasdaq data are available on the CRSP tapes, 1977 is the first year for which market equities and book equities are collected.

Within each sample of firms with the same year of market equity, the firms are sorted by market value of equity into deciles, then sorted within each year-size decile by preliminary beta into further deciles. The result is 100 size-beta portfolios whose member firms change once per year to correspond to changes in firm market equity ranks and preliminary beta ranks. To preserve similarity to the procedure of FF (1992) without editting the data, at this stage the 0.5% of the individual securities with the smallest and largest book-to-market ratios are excluded from the sample to avoid giving these firms

undue influence in the subsequent estimations.[22]

For each size-beta portfolio, the equal-weighted monthly excess returns are calculated for the 12 months following portfolio formation. For example, the portfolios based on year-end 1977 market equities provide monthly portfolio excess returns for July 1978 through June 1979, and the portfolios based on year-end 1993 market equities provide monthly portfolio excess returns for July 1994 through June 1995. The market equities and book equities are collected ending with 1993 because 1995 is then the corresponding latest year for which monthly returns are required, and 1995 is the latest year for which CRSP data are available. The proxy for the risk-free rate is the average of the bid and ask yields on 1-month Treasury bills as provided by CRSP as the risk-free rates from the Fama term structure files.

The result is a sample of equal-weighted monthly returns for 100 size-beta portfolios which spans from July 1978 through June 1995. This data enables monthly-rolling-sample estimation of OLS betas and predicted betas on the portfolio level using 60 months per estimation. The model estimated to gather the traditional betas is:

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \eta_{i,t},$$

where $\eta_{i,t}$ is the error term which is distributed i.i.d. $N(0, \sigma_{i,t}^2)$ and α_i and β_i are estimated by the model. $R_{i,t}$ and $R_{M,t}$ are the stock and market index excess returns, respectively. The resulting OLS β_i estimate for each 60-month period is used as the traditionally-estimated beta for the subsequent, out-of-sample, 61st month. The series of these out-of-

[22] Fama and French (1992) set the smallest and largest 0.5% book-to-market ratio observations equal to those of the 0.995 and 0.005 fractals, respectively, and include these observations in the sample in Table III, page 439. Rather than edit the values of these data, I will exclude these observations as outliers.

sample OLS betas are used to test their significance in modeling returns cross-sectionally at the portfolio and firm levels of specificity. The predicted beta estimation procedure is discussed in Chapter Three.

Iteration Trial Results

The estimations resulting from using the four-stage estimation procedure rather than the two-stage model may be improved through the use of multiple iterations. To check whether multiple iterations improve the estimates, the conditional error variances of stock excess returns and correlations between stock excess returns and market excess returns from multiple iterations are compared. The stocks used for the iterative estimation procedure are a random sample of individual firms. Individual firm excess returns are used as a conservative test, as portfolio excess returns display less noise (Fama and French (1992)).

To form an individual-stock sample, first every 50^{th} CUSIP was chosen from the CRSP tapes, resulting in a sample of 64 firms. Second, the 64 firms were narrowed to a sample of 28 firms which had at least 200 months of continuously non-missing returns. From these 28 firms the 1^{st}, 4^{th}, 7^{th}, etc. (plus one of the remaining two) was selected. The resulting sample of ten individual stocks comprise the sample used for the iteration trial.

Methodology Stages 1-4 are conducted for the total 200-month sample. Stages 3 and 4 are then reestimated four times for each of the ten sample firms. Table 6 summarizes these results.

The mean squared errors (MSE's) between the first- and second-pass time series of conditional error variances of stock returns provides one measure of the effects of reestimation. Given their small size, the MSE's of Table 6 are shown multiplied by 1,000. Although stock 6 (CUSIP 302542) has a much larger MSE than the other nine stocks, its conditional error variance mean only changes by 1.8% between the first and second estimates, and by less than 1.5% between the first and fifth estimates. Rate of convergence judgments are somewhat subjective, but convergence seems to occur quickly.

Perhaps a more tractable measure of the effects of changing series of conditional error variances is the resulting effects on the constant correlation estimates. For the sample of ten firms the average change in correlation estimates between the first and second estimates is 0.337%. Of course the mean percentage change in correlation does not provide information on the dispersion of the percentage changes across the ten firms. Therefore, the mean absolute percentage change is calculated as a measure sensitive to the change dispersion across firms. The mean absolute percentage change of the correlation estimate between the first and second estimates is 2.09%. This number is small enough that we feel comfortable relying on the first-pass estimates.

Data Description

For convenience of comparison, the tables which describe the data are presented in the same manner and order as those of Fama and French (1992). Table 1 displays the time-series means of the size-beta portfolio stock excess return, OLS beta, predicted beta

and of the natural logarithm of market equity (ln(ME)) within each size decile, beta decile, and size-beta portfolio. Table 2 displays the individual firm time-series means of the stock excess return, OLS beta, predicted beta, ln(ME), natural logarithm of book-to-market ratio (ln(BE/ME)), and number of firms by size and beta deciles. Table 3 displays the results of the preliminary regressions. Table 4 displays the individual firm time-series means of the stock excess return, OLS beta, predicted beta, ln(ME), ln(BE/ME) by book-to-market decile. Finally, Table 5 displays the time-series means of the size-beta portfolio mean excess return within each size decile, book-to-market decile, and size-(BE/ME) portfolio. The descriptive statistics and preliminary regression tables are located in Appendix C.

Table 1 - Means within Size-Beta Portfolios

Each panel of Table 1 displays the time-series means of a size-beta portfolio variable within each size decile, beta decile, and size-beta portfolio. For each size-beta portfolio mean the sample size is 144 months, and for each decile mean the sample size is 1,440 (144 months by 10 size-beta portfolios). Panel A displays the size-beta portfolio excess return data. Panel B1 displays the size-beta portfolio OLS beta data, Panel B2 displays the size-beta portfolio predicted beta data, and Panel C displays the size-beta portfolio ln(ME) data.

Panel A - Monthly Excess Return Means

Table 1, Panel A displays the time-series means of the size-beta portfolio excess returns within each size decile, beta decile, and size-beta portfolio. The data show that for the period July 1983 through June 1995, large firms had larger monthly average returns than did medium and small firms. The sole exception is the decile of smallest stock issues which provided the largest average monthly returns. The high returns of the smallest-size decile may be caused by a risk premium associated with size, or by a risk premium associated with stocks traded on the Nasdaq. As Table I, Panel A of FF (1992) spans July 1963 through December 1990, the difference in returns pattern is caused by the difference between pre-July 1983 and post-1990 subperiods.

Interestingly, the highest-pre-beta decile experienced the lowest average returns, followed by the third-largest- and second-largest-pre-beta deciles, respectively. This not only suggests that the mid-1983 to mid-1995 period rewarded low-beta stocks with higher returns, but also that the smallest size decile is not dominated by high-pre-beta stocks. The relationship between size and pre-betas is further illustrated by Table 1, Panel C and Table 2, Panel B.

Panel B1 - OLS Beta Means

Table 1, Panel B1 displays the time-series means of the size-beta portfolio OLS betas within each size decile, beta decile, and size-beta portfolio. The formation of portfolios on size and pre-betas increased the spread of post-ranking OLS betas from a size-grouped-only range of 0.79 to 1.07 to a size-beta-grouped range of 0.48 to 1.52. Use

of the pre-ranking beta groupings enabled a closer approximation to individually-estimated betas than would be obtained using size decile portfolios, without introducing the estimation error bias incurred by estimating pre-ranking betas on individual stocks.

Similar to the finding in FF (1992), the ordering of the post-ranking OLS betas is the same as that of the pre-ranking betas. This serves as an assurance that, although individual beta levels may change over time, beta levels don't change systematically as a function of their pre-ranking beta levels.

Panel B2 - Predicted Beta Means

Table 1, Panel B2 displays the time-series means of the size-beta portfolio predicted betas within each size decile, beta decile, and size-beta portfolio. The formation of portfolios on size and pre-betas increased the spread of post-ranking predicted betas from a size-grouped-only range of 0.80 to 1.05 to a size-beta-grouped range of 0.48 to 1.50. Note that both ranges for predicted betas are smaller than for the OLS betas of Panel B1, which may indicate that predicted betas more accurately model the mean-reversion process of beta evolution.

The rank order of the predicted beta means mirrors that of the pre-ranking beta means. This serves as an assurance that a predicted beta can be regarded as an adjusted beta estimate rather than as an entirely new variable requiring a different intuitive interpretation.

FF (1992) find that average post-ranking betas are "almost perfectly" negatively correlated to size. For the more modern sample period including Nasdaq stocks

throughout, Panels B1 and B2 find weak but positive correlations between average size and average post-ranking beta. The correlation between average OLS beta and average size is 0.71, and the correlation between average predicted beta and average size is 0.60. Neither of these correlations are statistically significant at the 10% level of confidence. The implications are that mid-1963 to mid-1983 is an anomalous period, January 1991 to mid-1995 is an anomalous period, or that the relationship between size and beta has fundamentally changed over time.

Panel C - Size (ln(ME)) Means

Table 1, Panel C displays the time-series means of the size-beta portfolio mean natural logarithm of market equity (ln(ME)) within each size decile, beta decile, and size-beta portfolio. Similar to the finding in FF (1992), the smallest average market equities are found for the highest and lowest pre-ranking beta deciles. However, the distribution of average size across pre-ranking beta deciles is much flatter for the current research than is the case in FF (1992).

The flatter distribution of average sizes across pre-ranking beta deciles may be related to the inclusion of Nasdaq stocks throughout the sampling period of this dissertation. However, note that the range of size deciles within pre-ranking beta deciles is uniformly larger than for the previous research. The smaller average market equities for the smallest size decile may be caused by the inclusion of AMEX stocks during size decile formation, and the larger average market equities for the largest four size deciles may be caused by inflation or by an increase in the size of the largest firms.

Table 2 - Means within Size & Beta Deciles

Each of the two panels of Table 2 displays the time-series means of the individual firm excess returns, OLS betas, predicted betas, ln(ME)'s, natural logarithms of book-to-market ratio (ln(BE/ME)), and numbers of firms for each decile of a variable. Table 2, Panel A displays the means of these individual firm variables across size deciles, and Panel B displays the means if these individual firm variables across beta deciles. For each table, the first and last deciles were divided into halves to more explicitly show the behavior within these extreme deciles.[23]

Panel A - Means within Size Deciles

Table 2, Panel A displays the time-series means of the individual firm excess returns, OLS betas, predicted betas, ln(ME)'s, natural logarithms of book-to-market ratio (ln(BE/ME)), and numbers of firms by size decile. The stark differences between the first column of Panel A here versus FF (1992) are caused by FF's inclusion of Nasdaq-excluding years, during which the smallest size decile is filled by AMEX and NYSE firms. Nonetheless the average returns, ln(ME), and ln(BE/ME) for the first columns are similarly different from their values in the second columns.

Size does not appear to be strongly correlated with returns, nor with either measure of beta. However, size appears to be negatively correlated with average book-to-market equity, as found by FF (1992). The negative correlation between size and book-

[23] The deciles are split into equal-sample-size halves. Note that FF (1992) split these deciles using the decile median instead. The results should not be greatly effected by this difference, which only affects Tables 2 and 4.

to-market appears to indicate that, averaged across size groupings, book values of equity have less dispersion than do market values of equity. Note that such a conclusion is unwarranted if the negative correlation is constructed as a by-product of the size groupings. Table 4 offers further evidence that the correlation between size and book-to-market is negative.

Panel B - Means within Beta Deciles

Table 2, Panel B displays the time-series means of the individual firm excess returns, OLS betas, predicted betas, ln(ME)'s, natural logarithms of book-to-market ratio (ln(BE/ME)), and numbers of firms by beta decile. The correlations between the pre-betas and OLS betas and between the pre-betas and predicted betas are each strongly positive. The differences among the OLS beta and predicted beta averages are striking: although the correlation between the OLS beta and predicted beta averages is 0.913, the OLS beta averages range from -0.11 to 3.18 while the predicted beta averages range from 0.51 to 1.34. Whether the OLS and predicted betas differ in modeling returns is shown in Tables 3 and 6.

Table 3 - Preliminary Regressions

Both panels of Table 3 display the averages across months of the coefficients and R^2's from regressions estimated cross-sectionally across individual firms. Each firm was assigned the OLS beta and predicted beta associated with the firm's portfolio during any particular month. Note that while the size-beta portfolio to which a firm is assigned may

only change annually, the betas associated with that size-beta portfolio are estimated monthly, so may change monthly. Panel A displays the results of estimating the full and reduced-form regressions using all 144 months. Panel B displays the results of estimating the full regressions for the 132 non-January months and for the 12 January months separately.

Panel A - All Months

Table 3, Panel A shows the averages across months of the coefficients and R^2's from regressions estimated cross-sectionally across individual firms. For the results of all months combined, the coefficients on the OLS betas and predicted betas are each negative and significant at the 10% level when beta is the only explanatory variable. In the presence of size and/or book-to-market, the slope of each measure of beta is insignificantly negative. For the July 1963 through December 1990 period, FF (1992) found the slope of the OLS beta to be insignificantly positive when beta is the only explanatory variable, and insignificantly negative when size is also included. The FF (1992) Table AIV examines a sample period of 1981-1990 which is more comparable to the current research. FF (1992) find the slope on beta to be negative and significant at the 5% level for regressions where beta is the sole explanatory variable and where beta and ln(ME) are the two explanatory variables. The finding of a negative coefficient on beta runs counter to the theory supported by the CAPM: the market risk premium is expected to be positive, as higher returns are expected to be associated with the risk of investing in

the market as opposed to the lower returns expected to be associated with the safety of riskless investments.

Unlike the FF (1992) Table III results, the coefficient on size is consistently insignificant. Size may well act differently for this sample because of the consistent inclusion of Nasdaq stocks. The risk premium associated with being traded on the Nasdaq is confounded with the risk premium associated with being a smaller stock issue. The behavior of FF's smallest size decile can be expected to change as Nasdaq stocks become included, and the non-Nasdaq months may be driving the significance of the slope on size. Indeed, the 1981-1990 sample of FF's Table AIV shows ln(ME) to be statistically insignificant irrespective of the inclusion of beta as an explanatory variable.

Panel B - January Separation

Table 3, Panel B shows the averages for non-January and January months of the coefficients and R^2's from regressions estimated cross-sectionally across individual firms. As the January effect remains a largely unsolved anomaly, the regressions including a beta estimate, size and book-to-market were estimated separately for non-January and January samples. For the sample of non-January months, the coefficients on the OLS betas and predicted betas are each negative and significant at the 10% level while controlling for the size and book-to-market effects. The slopes on ln(ME) are significantly positive at the 10% level, while the slopes on ln(BE/ME) remain significantly positive at the 1% level.

The January-only sample results contrast strongly with the previous results. The coefficients on the OLS beta and predicted beta each retain significance at the 10% level,

but switch to being positive. The coefficients on ln(ME) switch to negative and are significant at the 1% level. The coefficients on ln(BE/ME) are statistically insignificant at the 25% level. January remains an anomalous month for modeling stock returns. Unfortunately, FF (1992) does not address the January effect.

Table 4 - Stocks Sorted on Book-to-Market Equity (BE/ME)

Table 4 displays the time-series means of the individual firm excess returns, OLS betas, predicted betas, ln(ME)'s, and ln(BE/ME) by book-to-market decile. For each table, the first and last size deciles were divided into halves (using the decile book-to-market medians) to more explicitly show the behavior within these extreme deciles.

The use of the natural logarithm of the book-to-market ratio does not exist for those firms which have a negative value for book equity. Such observations are excluded from the analysis, consistent with FF (1992).

The positive correlation between returns and BE/ME noted by FF (1992) is found here, despite the fact that positive returns lower the BE/ME ratio. Also interesting is the beta "smile" across BE/ME groups. OLS and predicted beta estimates decrease from the smallest BE/ME half-decile through the BE/ME eighth decile, then increase through the highest BE/ME half-decile. The same pattern is shown in FF's Table IV.

Table 5 - Returns on Portfolios by ME & BE/ME

Table 5 displays the time-series means of the size-beta portfolio mean excess return within each size decile, book-to-market decile, and size-(BE/ME) portfolio. For

each size-(BE/ME) portfolio mean the sample size is 144 months, and for each decile mean the sample size is 1,440 (144 months by 10 size-(BE/ME) portfolios).

Table 5 confirms the generally-positive relationship between returns and book-to-market found by FF (1992), but fails to find the negative relationship between returns and size. Perhaps the increasing business globalization of the recent past has afforded the most lucrative opportunities to the largest firms, or perhaps the largest firms best withstood the increased competition associated with continuing globalization and decreased barriers to trade.

In summary, the descriptive statistics of the sample are very similar to those of Fama and French (1992). The weak relationship between size and stock excess returns found by this study (see Table 1, Panels B1 and B2, and Table 3) also were found by FF (1992) for a 1981-1990 subsample. Also, the four-stage procedure for constructing predicted betas does not seem to greatly benefit greatly from multiple iterations (see Table 6). Finally, in common with findings of FF (1992), the preliminary regressions of this study display a statistically significant negative relationship between beta and returns when beta is the sole explanatory variable.

CHAPTER 5

RESULTS

The first section of this chapter describes the statistical significance of the predicted beta, and the second section discusses the affects of the predicted beta on market anomalies. The tables corresponding to these results are located in Appendix C.

Testing the Predicted Betas while Controlling for Traditional Betas

With the total sample of 100 portfolios and 144 months, pooled, cross-sectional OLS regressions are estimated with either the OLS or predicted beta as the sole explanatory variable: $R_{i(t+1)} = \zeta_0 + \zeta_1 * \beta_{\{i(t+1) \text{ or } \sigma[i(t+1)]\}} + u_{i(t+1)}$,

where $u_{i(t+1)}$ is the error term which is distributed $N(0, \phi)$, $R_{i(t+1)}$ is the excess return on portfolio i during period $t+1$, $\beta_{i(t+1)}$ is the traditional beta, $\beta_{\sigma[i(t+1)]}$ is the predicted beta, and ζ_0, ζ_1 are coefficients estimated by the model. See Table 7, Panel A. As these regressions are run cross-sectionally across portfolios, size and book-to-market variables are excluded. For the sample of all months and for the non-January sample, each estimate of beta has a negative coefficient significant at the 1% level. For the January sample, the coefficient of the predicted beta is statistically significantly negative at the 1% level while the OLS beta is insignificant. The difference in statistical significance between the predicted beta and the OLS beta for the January sample indicates that the two estimates of beta, while highly correlated with each other, do not explain returns in the same manner.

The possibility of this explanatory difference between the predicted beta and the OLS beta is suggested by Table 2, Panel B, where the predicted beta and OLS beta averages differ widely. Panel B examines the explanatory power the predicted has in addition to the OLS beta.

For Table 7, Panel B, the first regression of each category is the result of the regression of OLS betas modeling the predicted betas:

$$\beta_{\sigma(i,t)} = \theta_0 + \theta_1 * \beta_{i,t} + \upsilon_{i,t},$$

where $\upsilon_{i,t}$ is the error term which is distributed $N(0, \psi)$, $\beta_{i,t}$ is the traditional beta, $\beta_{\sigma(i,t)}$ is the predicted beta, and θ_0, θ_1 are coefficients estimated by the model. With the total sample of 100 portfolios and 144 months, a pooled, cross-sectional OLS regression featuring the OLS beta as the independent variable and the predicted beta as the dependent variable resulted in an adjusted R^2 of 0.724. See Table 7, Panel B. This implies a total-sample correlation between the traditional and predicted beta estimates of 0.85 and that the two beta estimates should not be included in the same regression model together. Instead the residuals from the regression will be included in model (8) as the values of "Orth. Beta," or as the values of the predicted beta after orthogonalizing the predicted beta on the OLS beta. Panel B of Table 7 shows that the OLS betas are statistically significant and negative at the 1% level and that the orthogonalized predicted betas are statistically insignificant for the total sample; for the total sample, the predicted betas do not contribute significant explanatory power in addition to the explanatory power of the OLS betas.

Interestingly, the intercept is higher than expected at nearly 2% per month and the coefficient on the OLS beta, which represents the market risk premium, is lower than expected at negative 0.9% per month. This indicates that the sample period is one of generally high returns, with a penalty for taking on market risk. Similarly, Fama and French (1992), for their 1981-1990 sample of their Table AIV, find a 2.35% intercept and a negative 1.01% excess return per month.

For the sample of 132 pooled non-January months, the OLS beta is significantly negative at the 1% level while the additional information provided by the predicted beta is significantly positive at the 5% level. This demonstrates that for non-January months, the returns are penalized for the level of the OLS beta, but rewarded for the level of the orthogonalized predicted beta level.

For the sample of 12 pooled Januarys, the OLS beta is statistically insignificant while the additional information provided by the predicted beta is significantly negative at the 1% level. This demonstrates that, for January, returns are lower when featuring higher levels of orthogonalized predicted beta.

Do the Predicted Betas Explain Current Market Anomalies?

Anomaly Test 1: Fama and French (1992) Revisited

Table 3 summarizes the regression results. As the slopes of the predicted and OLS betas consistently have the same sign and significance level, the predicted beta does not outperform the OLS beta by the strict FF (1992) measure. The negative and significant

slope on either beta estimate when used as the sole coefficient merely confirms FF's 1981-1990 period result of Table AIV. This implies that the 1991 to mid-1995 period also included in this dissertation does not differ by enough from the mid-1985 to December 1990 period to change the beta-return relationship conclusions.

However, FF's Table AIV shows the beta's coefficient to be negative and significant across all months in the presence of ln(ME). The parallel beta estimates in this dissertation have insignificant coefficients.

Anomaly Test 2: Market Size Examined

As found in FF's Table AIV, ln(ME) is statistically insignificant in modeling returns. This is most likely a result of the difference in return-size relationship for January versus the rest of the year: the blending of these two disparate relationships results in an insignificant slope.

Anomaly Test 3: January Effect Examined

Table 3 displays interesting results which, unfortunately, concern a topic which FF (1992) leave unaddressed: the January effect. The book-to-market ratio is insignificant during January. As expected, the relationship between size and return is negative during January, but it was unexpected that the January sample is the only one for which return premiums are earned for incurring additional systematic risk.

The pooled, cross-sectional OLS regression (the second model in each sample) featuring the portfolio returns as the dependent variable and the OLS beta and

orthogonalized predicted beta as the independent variables is estimated separately for January and non-January samples. See Table 7, Panel B. The correlations between the OLS beta and predicted beta for January and non-January samples are very similar: for January months the correlation is 0.969 and for non-January months the correlation is 0.960. However, the results of the model (8) estimation for these subsamples are not similar. For January, the OLS beta is statistically insignificant while the orthogonalized predicted beta is significantly negative at the 1% level. For non-January, the OLS beta is significantly negative at the 1% level and the orthogonalized predicted beta is significantly positive at the 5% level.

These results indicate that January remains an anomalous month. For the January sample, the intercept is a 4.20% return per month and the coefficient on the orthogonalized predicted beta indicates a large negative penalty for firms with market risk. For the non-January sample, the intercept is a 1.73% return per month with a small penalty for market risk as measured by OLS beta and a small bonus for market risk as measured by the orthogonalized predicted beta.

In summary, the OLS beta and the predicted beta are statistically significant and negative in explaining cross-sectional size-beta portfolio excess returns for samples of all months and of non-January months, but only the predicted beta is significant for the January sample (see Table 7, Panel A). After orthogonalizing the predicted beta on the OLS beta, the explanatory power of the predicted beta, in addition to that of the OLS beta, is insignificant for the sample of all months, but is significant for the non-January and

January samples (see Table 7, Panel B). For the non-January sample, the OLS beta is significantly negative and the additional effect of the predicted beta is significantly positive. For the January sample, the OLS beta is insignificant and the orthogonalized predicted beta is significantly negative.

The predicted beta does not explain Current market anomalies. As seen in Table 3, Panels A and B, the predicted beta coefficients are very similar to the OLS beta coefficients in the cross-sectional regression on stock excess returns.

CHAPTER 6

CONTRIBUTIONS AND LIMITATIONS

The first section of this chapter discusses the contributions of this dissertation, such as the unbiasedness or the superior explanatory power of the predicted betas. The second section describes the limitations associated with the predicted beta, such as the assumption of the optimal conditional variance equation and the trade-off between sample size and assumed length of investment horizon.

Contributions of the Research

Predicted Betas as Unbiased Beta Estimates

During portfolio formation or rebalancing with beta as a criteria, weights are assigned to the assets of the portfolio to achieve a desired target portfolio beta. However, the bias of OLS betas limits the accuracy with which the portfolio beta can be set. As the predicted beta is constructed with the estimate of the conditional total variance of the market premium, the predicted beta does not contain the nonstationarity bias present in the OLS beta.

Predicted Betas as Better Betas

In the results of this study, the predicted beta is as statistically significant as the OLS beta in the regressions of Table 3 and in the total and non-January samples of Table

7, Panel A. The predicted beta offers significant explanatory power in addition to the OLS beta in Table 7, Panel B for the non-January and January samples, and the predicted beta is statistically significant in the January sample where the OLS beta is insignificant in Table 7, Panel A.

Overall, the predicted beta is weakly more effective in explaining the cross-section of stock excess returns. As variables required for estimating the OLS and predicted betas are the same, researchers should opt for the more effective measure of systematic risk.

Limitations of the Research

Optimal Conditional Variance Equation Form

Only the most basic conditional mean and conditional variance equations are used in Stages 1 and 2 of the methodology. In practice, the optimal form for each should be estimated and employed. Koutmos (forthcoming) estimates an exponential GARCH for the conditional variance equation. Using an exponential GARCH form would not impair the intuitive interpretation of the findings.

Investment Horizon Vs. Estimation Sample Size

The methodology estimated here has higher sample size requirements than does an OLS estimation. This can pose a limitation, as the estimation procedure assumes that the relationships are stationary during the entire sampling period. Ideally, for this one-period model, each past investment period corresponds to one sample observation period. If the

sampling period units are of shorter length than the investment period units, then bias is introduced by arithmetic averaging of geometric returns.

For example, the ideal sampling period unit is annually for investors who rebalance annually. However, assuming that the return relationships have been stationary over the past five years is of tenuous merit. Survivorship bias becomes a concern as well.

Note that a sample size problem is encountered in this dissertation when checking for the January effect. The small number of Januarys included in the cross-sectional regressions causes the tests of the January effect to have low power. Finding a significant January effect under these conditions is a strong confirmation of January's influence on the risk-return relationship.

In summary, the predicted beta demonstrates significant explanatory power in cross-sectional regressions. No regressions are found in which the OLS beta is significant and the predicted beta is insignificant, although only the predicted beta is significant in the cross-sectional regression on size-beta portfolio returns during January. Although the predicted beta fails to explain the size effect or the book-to-market significance, the additional information of the predicted beta seems to be related to the January effect.

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APPENDICES

Appendix A: Deriving Conditional Total Variances

Conditional total variances can be derived given conditional error variances and the correlation coefficient between the independent and dependent variables. Note that Koutmos (forthcoming) and Ng, Engle, and Rothschild (1992) derive conditional total variances from conditional error variances using beta in the equation:

$$\sigma_{it}^{2(total)} = \beta_{it}^2 \sigma_{Mt}^{2(total)} + \sigma_{\varepsilon_i t}^2 .$$

Using the correlation instead of the beta allows relaxation of the assumption that the dependent variable standard deviation is a constant:

$$\sigma_{it}^{2(total)} = \beta_{it}^2 \sigma_{Mt}^{2(total)} + \sigma_{\varepsilon_i t}^2 = \rho_{iM}^2 \frac{\sigma_{it}^{2(total)}}{\sigma_{Mt}^{2(total)}} \sigma_{Mt}^{2(total)} + \sigma_{\varepsilon_i t}^2 \quad \text{(Greene (1990))},$$

$$\therefore \sigma_{it}^{2(total)} (1 - \rho_{iM}^2) = \sigma_{\varepsilon_i t}^2$$

$$\therefore \sigma_{it}^{2(total)} = \frac{\sigma_{\varepsilon_i t}^2}{(1 - \rho_{iM}^2)}$$

$$\therefore \sigma_{it}^{(total)} = \frac{\sigma_{\varepsilon_i t}}{\sqrt{1 - \rho_{iM}^2}} \quad . \qquad \text{(A1)}$$

Appendix B: Deriving Total Variance Time Series Process

If the conditional error variances follow a GARCH(1, 1) process, then the conditional total variances also follow a GARCH(1, 1) process. The hypothesized model for stock returns is the GARCH(1, 1) of the form:

$$R_{i,t} = \beta_{it} R_{Mt} + \varepsilon_{i,t}$$

$$H_{i,t} = \gamma_{0i} + \gamma_{1i} H_{i,t-1} + \gamma_{2i} \varepsilon_{i,t-1}^2 .$$

The conditional variance equation can be rewritten as an ARMA(1, 1) process:

$$\begin{aligned} \varepsilon_{i,t}^2 &= \gamma_{0i} + \gamma_{1i} H_{i,t-1} + \gamma_{2i} \varepsilon_{i,t-1}^2 + \varepsilon_{i,t}^2 - H_{i,t} \\ &= \gamma_{0i} + (\gamma_{1i} + \gamma_{2i}) \varepsilon_{i,t-1}^2 - \gamma_{1i} (\varepsilon_{i,t-1}^2 - H_{i,t-1}) + (\varepsilon_{i,t}^2 - H_{i,t}) \\ &= \gamma_{0i} + (\gamma_{1i} + \gamma_{2i}) \varepsilon_{i,t-1}^2 - \gamma_{1i} \eta_{i,t-1} + \eta_{i,t} , \end{aligned}$$

where $\eta_{i,t} \equiv \varepsilon_{i,t}^2 - H_{i,t}$. Given that $\varepsilon_t^2 = \hat{\sigma}_{\varepsilon_t}^2$ (Johnston, (1984)), and that

$\hat{\sigma}_{it}^{2(total)} = \dfrac{\hat{\sigma}_{\varepsilon_i t}^2}{(1 - \rho_{iM}^2)}$ from the previous result,[24] it must hold that $\hat{\sigma}_{it}^{2(total)} = \dfrac{\varepsilon_{it}^2}{(1 - \rho_{iM}^2)}$.

Therefore the above ARMA(1, 1) process can be written in terms of $\hat{\sigma}_{it}^{2(total)}$ or $\hat{\sigma}_{it}^2$:

$$\varepsilon_{i,t}^2 = \gamma_{0i} + (\gamma_{1i} + \gamma_{2i}) \varepsilon_{i,t-1}^2 - \gamma_{1i} \eta_{i,t-1} + \eta_{i,t} ,$$

$$\therefore \hat{\sigma}_{it}^2 (1 - \rho_{iM}^2) = \gamma_{0i} + (\gamma_{1i} + \gamma_{2i}) \hat{\sigma}_{it-1}^2 (1 - \rho_{iM}^2) - \gamma_{1i} \eta_{i,t-1} + \eta_{i,t}$$

$$\therefore \hat{\sigma}_{it}^2 = \frac{\gamma_{0i}}{(1 - \rho_{iM}^2)} + (\gamma_{1i} + \gamma_{2i}) \hat{\sigma}_{it-1}^2 - \frac{\gamma_{1i} \eta_{i,t-1}}{(1 - \rho_{iM}^2)} + \frac{\eta_{i,t}}{(1 - \rho_{iM}^2)}$$

$$\therefore \hat{\sigma}_{it}^2 = \gamma'_{0i} + (\gamma_{1i} + \gamma_{2i}) \hat{\sigma}_{it-1}^2 - \gamma_{1i} \eta'_{i,t-1} + \eta'_{i,t} ,$$

[24] E[aX] = aE[X] and if $X_2 \equiv aX_1$, $\hat{X}_2 \equiv a\hat{X}_1$ (Neter, Wasserman, and Kutner (1985)).

where $\eta'_{i,t} \equiv \frac{\eta_{i,t}}{(1-\rho^2_{iM})} = \frac{\varepsilon_{i,t}^{\ 2} - H_{i,t}}{(1-\rho^2_{iM})} = \hat{\sigma}^2_{i,t} - \frac{H_{i,t}}{(1-\rho^2_{iM})} = \hat{\sigma}^2_{i,t} - H'_{i,t}$, and where

$H'_{i,t} = \frac{H_{i,t}}{(1-\rho^2_{iM})}$. In turn, the ARMA(1, 1) written in terms of $\hat{\sigma}^2_{it}$ can be rewritten to show that the GARCH(1, 1) conditional error variance equation implies a GARCH(1, 1) process in terms of conditional total variance:

$$\frac{H_{i,t}}{(1-\rho^2_{iM})} = \frac{\gamma_{0i}}{(1-\rho^2_{iM})} + \gamma_{1i}\frac{H_{i,t-1}}{(1-\rho^2_{iM})} + \gamma_{2i}\frac{\varepsilon^2_{i,t}}{(1-\rho^2_{iM})},$$

$$\therefore H'_{i,t} = \gamma'_{0i} + \gamma_{1i}H'_{i,t-1} + \gamma_{2i}\sigma^2_{i,t-1}.$$

Appendix C: Tables

Table 1 - Means within Size-Beta Portfolios

Panel A - Monthly Excess Return Means

Table 1, Panel A displays the time-series means of the size-beta portfolio excess returns within each size decile, beta decile, and size-beta portfolio. For each size-beta portfolio mean the sample size is 144 months (July 1983 through June 1995), and for each decile mean the sample size is 1,440 (144 months by 10 size-beta portfolios).

Returns	All	Low-Beta	Beta-2	Beta-3	Beta-4	Beta-5	Beta-6	Beta-7	Beta-8	Beta-9	High-Beta
All	1.06	1.13	1.19	1.21	1.22	1.17	1.13	1.09	0.81	0.97	0.68
Small-ME	1.50	1.32	1.68	1.47	1.61	1.57	1.45	1.51	1.38	1.45	1.61
ME-2	0.91	1.05	1.42	1.37	1.02	0.85	0.73	1.15	0.18	0.91	0.45
ME-3	0.85	1.03	1.04	1.07	1.15	1.05	0.84	1.01	0.71	0.64	-0.09
ME-4	0.81	0.97	0.92	0.82	0.87	1.08	0.84	0.81	0.64	0.66	0.49
ME-5	0.90	0.88	1.08	1.19	1.01	0.75	1.29	0.85	0.70	0.89	0.33
ME-6	1.03	1.03	1.14	1.00	1.11	1.40	1.12	1.06	1.04	0.86	0.55
ME-7	1.06	1.16	1.21	1.29	1.37	1.05	1.25	1.19	0.67	0.98	0.45
ME-8	1.12	1.32	1.12	1.08	1.22	1.28	1.47	0.92	0.87	1.06	0.89
ME-9	1.20	1.25	1.09	1.38	1.52	1.32	1.06	1.23	0.99	1.01	1.11
Large-ME	1.21	1.29	1.20	1.38	1.27	1.31	1.29	1.17	0.94	1.25	1.00

Table 1 - Means within Size-Beta Portfolios

Panel B1 - OLS Beta Means

Table 1, Panel B1 displays the time-series means of the size-beta portfolio OLS betas within each size decile, beta decile, and size-beta portfolio. For each size-beta portfolio mean the sample size is 144 months (July 1983 through June 1995), and for each decile mean the sample size is 1,440 (144 months by 10 size-beta portfolios).

Old Beta	All	Low-Beta	Beta-2	Beta-3	Beta-4	Beta-5	Beta-6	Beta-7	Beta-8	Beta-9	High-Beta
All	1.00	0.53	0.71	0.84	0.95	1.00	1.07	1.11	1.18	1.27	1.38
Small-ME	0.78	0.53	0.59	0.59	0.74	0.79	0.81	0.90	0.90	1.00	0.98
ME-2	0.92	0.55	0.68	0.81	0.93	0.87	0.91	1.01	1.12	1.13	1.19
ME-3	0.98	0.54	0.66	0.79	0.91	0.95	1.09	1.01	1.15	1.36	1.35
ME-4	1.03	0.56	0.67	0.84	0.94	1.04	1.04	1.13	1.22	1.33	1.52
ME-5	1.06	0.55	0.73	0.89	0.94	1.09	1.14	1.14	1.25	1.37	1.52
ME-6	1.07	0.56	0.71	0.89	0.97	1.03	1.15	1.23	1.27	1.42	1.50
ME-7	1.07	0.52	0.81	0.94	1.00	0.99	1.06	1.19	1.30	1.39	1.50
ME-8	1.05	0.51	0.81	0.88	0.97	1.06	1.15	1.18	1.26	1.20	1.48
ME-9	1.07	0.48	0.71	0.97	1.06	1.10	1.21	1.16	1.22	1.30	1.46
Large-ME	1.00	0.52	0.70	0.85	0.99	1.07	1.11	1.13	1.15	1.22	1.31

Table 1 - Means within Size-Beta Portfolios

Panel B2 - Predicted Beta Means

Table 1, Panel B2 displays the time-series means of the size-beta portfolio predicted betas within each size decile, beta decile, and size-beta portfolio. For each size-beta portfolio mean the sample size is 144 months (July 1983 through June 1995), and for each decile mean the sample size is 1,440 (144 months by 10 size-beta portfolios).

Pred. Beta	All	Low-Beta	Beta-2	Beta-3	Beta-4	Beta-5	Beta-6	Beta-7	Beta-8	Beta-9	High-Beta
All	0.98	0.52	0.68	0.83	0.93	0.99	1.04	1.08	1.15	1.23	1.33
Small-ME	0.80	0.53	0.60	0.61	0.78	0.81	0.81	0.92	0.91	1.02	1.03
ME-2	0.92	0.56	0.64	0.82	0.94	0.88	0.92	1.02	1.06	1.15	1.20
ME-3	0.97	0.54	0.65	0.78	0.91	0.94	1.06	0.98	1.16	1.35	1.33
ME-4	0.99	0.48	0.63	0.85	0.91	0.97	1.01	1.13	1.18	1.19	1.50
ME-5	1.03	0.54	0.70	0.88	0.95	1.03	1.07	1.13	1.22	1.31	1.48
ME-6	1.05	0.57	0.71	0.77	0.96	1.02	1.13	1.19	1.22	1.40	1.49
ME-7	1.03	0.51	0.81	0.87	0.97	1.00	1.04	1.17	1.23	1.36	1.34
ME-8	1.01	0.49	0.75	0.88	0.86	1.04	1.14	1.16	1.17	1.12	1.45
ME-9	1.02	0.47	0.71	0.96	1.06	1.09	1.13	1.08	1.17	1.28	1.29
Large-ME	0.95	0.51	0.63	0.83	0.98	1.07	1.11	0.98	1.13	1.14	1.13

Table 1 - Means within Size-Beta Portfolios

Panel C - Size (ln(ME)) Means

Table 1, Panel C displays the time-series means of the size-beta portfolio ln(ME)'s within each size decile, beta decile, and size-beta portfolio. For each size-beta portfolio mean the sample size is 144 months (July 1983 through June 1995), and for each decile mean the sample size is 1,440 (144 months by 10 size-beta portfolios).

Size	All	Low-Beta	Beta-2	Beta-3	Beta-4	Beta-5	Beta-6	Beta-7	Beta-8	Beta-9	High-Beta
All	5.05	5.02	5.07	5.06	5.07	5.07	5.08	5.06	5.06	5.04	5.02
Small-ME	1.51	1.27	1.43	1.47	1.49	1.58	1.60	1.62	1.58	1.59	1.50
ME-2	2.88	2.87	2.87	2.87	2.90	2.87	2.91	2.87	2.88	2.90	2.89
ME-3	3.62	3.64	3.62	3.61	3.61	3.63	3.62	3.62	3.62	3.62	3.61
ME-4	4.22	4.22	4.23	4.23	4.22	4.23	4.22	4.21	4.21	4.22	4.22
ME-5	4.75	4.74	4.77	4.75	4.76	4.75	4.76	4.74	4.76	4.74	4.74
ME-6	5.30	5.30	5.30	5.30	5.31	5.31	5.31	5.31	5.29	5.31	5.28
ME-7	5.89	5.91	5.90	5.88	5.90	5.90	5.89	5.90	5.90	5.89	5.85
ME-8	6.53	6.52	6.52	6.49	6.52	6.54	6.54	6.54	6.54	6.51	6.53
ME-9	7.27	7.29	7.28	7.27	7.28	7.24	7.26	7.26	7.26	7.27	7.24
Large-ME	8.56	8.44	8.75	8.69	8.67	8.65	8.67	8.49	8.52	8.37	8.31

Table 2 - Means within Size & Beta Deciles

Panel A - Means within Size Deciles

Table 2, Panel A displays the time-series means of the individual firm excess returns, OLS betas, predicted betas, ln(ME)'s, ln(BE/ME)'s, and numbers of firms by size decile. The first and last size deciles were divided into halves (by median decile size) to more explicitly show the behavior within these extreme deciles. "Firms" is the average number of stocks in the portfolio each month.

Size	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Return	1.98	1.03	0.96	0.89	0.83	0.90	1.03	1.06	1.11	1.20	1.28	1.13
OLS Beta	0.78	0.78	0.91	0.98	1.03	1.06	1.07	1.07	1.05	1.06	1.02	0.99
Pred. Beta	0.80	0.80	0.91	0.97	0.98	1.03	1.04	1.03	1.00	1.02	0.97	0.93
ln(ME)	0.90	2.12	2.89	3.63	4.25	4.77	5.31	5.88	6.53	7.27	7.94	9.18
ln(BE/ME)	0.09	-0.24	-0.30	-0.37	-0.41	-0.45	-0.50	-0.54	-0.49	-0.52	-0.46	-0.78
Firms	314	314	427	360	303	275	260	237	224	214	105	105

Table 2 - Means within Size & Beta Deciles

Panel B - Means within Beta Deciles

Table 2, Panel B displays the time-series means of the individual firm excess returns, OLS betas, predicted betas, ln(ME)'s, ln(BE/ME)'s, and numbers of firms by beta decile. The first and last beta deciles were divided into halves (by median decile beta) to more explicitly show the behavior within these extreme deciles. "Firms" is the average number of stocks in the portfolio each month.

Pre-Beta	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Return	1.10	1.12	1.22	1.23	1.24	1.21	1.15	1.16	0.88	1.05	0.76	0.73
OLS Beta	-0.11	0.34	0.53	0.77	0.95	1.12	1.29	1.46	1.67	1.96	2.07	3.18
Pred. Beta	0.51	0.53	0.66	0.79	0.91	0.95	1.00	1.05	1.11	1.21	1.34	1.24
ln(ME)	3.85	4.85	4.36	4.33	4.35	4.37	4.38	4.38	4.36	4.37	5.19	3.50
ln(BE/ME)	-0.25	-0.14	-0.24	-0.29	-0.30	-0.32	-0.34	-0.36	-0.39	-0.51	-0.66	-0.85
Firms	157	157	310	315	317	316	317	317	315	314	152	152

Table 3 - Preliminary Regressions

Panel A - All Months

Table 3, Panel A shows the averages across months of the coefficients and R^2's from regressions estimated cross-sectionally across individual firms. Each firm was assigned the OLS beta and predicted beta associated with the firm's portfolio during any particular month. Note that while the size-beta portfolio to which a firm is assigned may only change annually, the betas associated with that size-beta portfolio are estimated monthly, so may change monthly. This panel displays the results of estimating the full and reduced-form regressions on stock excess returns using all 144 months: $R_{i,t} = \delta_0 + \delta_1 X_1 + ... + \delta_n X_n + u_{i,t}$.

P-values are listed below estimated coefficients.

	Constant	OLS Beta	Pred. Beta	ln(ME)	ln(BE/ME)	R-Squared
Coefficient	1.952	-0.844				0.010
p-value	0.0001	0.0713				0.0001
Coefficient	1.743		-0.662			0.008
p-value	0.0001		0.0788			0.0001
Coefficient	1.268			-0.038		0.008
p-value	0.0146			0.5381		0.0001
Coefficient	1.233				0.439	0.004
p-value	0.0013				0.0001	0.0001
Coefficient	1.932	-0.623		-0.039		0.017
p-value	0.0001	0.1947		0.5507		0.0001
Coefficient	1.792		-0.557	-0.027		0.016
p-value	0.0001		0.1588	0.6793		0.0001
Coefficient	1.794	-0.543			0.385	0.012
p-value	0.0001	0.2250			0.0001	0.0001
Coefficient	1.653		-0.420		0.382	0.010
p-value	0.0001		0.2378		0.0001	0.0001
Coefficient	1.237			-0.002	0.413	0.012
p-value	0.0147			0.9794	0.0001	0.0001
Coefficient	1.721	-0.413		-0.010	0.363	0.020
p-value	0.0001	0.3694		0.8739	0.0001	0.0001
Coefficient	1.624		-0.385	0.002	0.360	0.019
p-value	0.0001		0.3050	0.9696	0.0001	0.0001

Table 3 - Preliminary Regressions

Panel B - January Separation

Table 3, Panel B shows the averages for non-January and January months of the coefficients and R^2's from regressions estimated cross-sectionally across individual firms. Each firm was assigned the OLS beta and predicted beta associated with the firm's portfolio during any particular month. Note that while the size-beta portfolio to which a firm is assigned may only change annually, the betas associated with that size-beta portfolio are estimated monthly, so may change monthly. This panel displays the results of estimating the full regressions This panel displays the results of estimating the full regressions on stock excess returns for the samples of 132 non-January months and for the 12 January months separately:

$$R_{i,t} = \delta_0 + \delta_1 X_1 + ... + \delta_n X_n + u_{i,t}.$$

P-values are listed below estimated coefficients.

Non-January Months

	Constant	OLS Beta	Pred. Beta	ln(ME)	ln(BE/ME)	R-Squared
Coefficient	1.228	-0.800		0.095	0.367	0.020
p-value	0.0005	0.0771		0.0863	0.0001	0.0001
Coefficient	1.045		-0.707	0.106	0.368	0.018
p-value	0.0034		0.0569	0.0550	0.0001	0.0001

January Months

	Constant	OLS Beta	Pred. Beta	ln(ME)	ln(BE/ME)	R-Squared
Coefficient	7.145	3.852		-1.165	0.317	0.028
p-value	0.0029	0.0980		0.0062	0.2596	0.0001
Coefficient	7.993		3.161	-1.141	0.275	0.026
p-value	0.0008		0.0861	0.0068	0.3209	0.0002

Table 4 - Stocks Sorted on Book-to-Market Equity (BE/ME)

Table 4 displays the time-series means of the individual firm excess returns, OLS betas, predicted betas, ln(ME)'s, and ln(BE/ME) by book-to-market decile. For each table, the first and last size deciles were divided into halves (using the decile book-to-market medians) to show the behavior within these extreme deciles.
The use of the natural logarithm of the book-to-market ratio does not exist for those firms which have a negative value for book equity. Such observations are excluded from the analysis, consistent with FF (1992).

BE/ME	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Return	0.10	0.42	0.78	0.91	1.07	1.13	1.09	1.14	1.11	1.37	1.13	1.08
OLS Beta	1.11	1.10	1.08	1.05	1.04	1.02	0.98	0.94	0.92	0.94	0.97	0.99
Pred. Beta	1.08	1.07	1.05	1.02	1.01	0.99	0.96	0.92	0.90	0.91	0.94	0.97
ln(ME)	5.12	5.09	5.08	5.07	5.06	5.07	5.06	5.05	5.04	5.02	4.96	4.44
ln(BE/ME)	-2.42	-1.57	-1.15	-0.82	-0.58	-0.37	-0.19	-0.02	0.16	0.37	0.64	1.06

Table 5 - Returns on Portfolios by ME & BE/ME

Table 5 displays the time-series means of the size-beta portfolio mean excess return within each size decile, book-to-market decile, and size-(BE/ME) portfolio. For each size-(BE/ME) portfolio mean the sample size is 144 months, and for each decile mean the sample size is 1,440 (144 months by 10 size-(BE/ME) portfolios).

Returns	All	Low	BE/ME-2	BE/ME-3	BE/ME-4	BE/ME-5	BE/ME-6	BE/ME-7	BE/ME-8	BE/ME-9	High
All	1.01	0.31	0.78	0.91	1.07	1.13	1.09	1.14	1.11	1.37	1.17
Small-ME	1.40	0.52	1.51	1.37	1.50	1.26	1.32	1.36	1.48	1.87	1.78
ME-2	0.84	-0.15	0.46	0.51	0.86	0.97	0.87	0.91	1.49	1.09	1.45
ME-3	0.78	-0.20	0.54	0.68	0.90	0.93	1.08	1.11	1.06	1.21	0.52
ME-4	0.74	-0.41	-0.10	0.34	0.98	1.36	0.99	1.00	0.91	1.26	1.10
ME-5	0.78	-0.24	0.73	0.51	1.00	1.05	1.00	0.82	1.09	1.29	0.54
ME-6	0.94	0.32	0.72	1.13	0.85	1.01	1.05	1.13	0.96	1.20	1.00
ME-7	0.99	0.51	0.81	1.03	0.87	1.22	1.06	1.34	1.12	1.38	0.55
ME-8	1.15	0.91	0.88	1.18	1.19	1.03	1.28	1.13	1.02	1.72	1.16
ME-9	1.19	0.70	1.20	1.11	1.20	1.09	1.03	1.36	0.98	1.45	1.84
Large-ME	1.27	1.15	1.11	1.28	1.30	1.33	1.24	1.29	0.99	1.26	1.76

Table 6 - Iteration Results

The sample was formed by first choosing every 50th CUSIP from the CRSP tapes. This resulted in a sample of 64 firms. Second this sample was narrowed to the 28 firms which had at least 200 months of continuously non-missing returns. From these 28 firms the 1st, 4th, 7th, etc. (plus one of the last two) was selected to obtain the final sample of ten individual stocks.

Methodology Stages 1-4 are conducted for the total 200-month sample. Stages 3 and 4 are then reestimated four times for each of the ten sample firms.

"Corr" is the first-pass estimate of the correlation between stock excess returns and market excess returns, "corr2" is the second-pass estimate, etc. "Sig. mse2" is the mean squared error (MSE) between the first- and second-pass time series of conditional error standard deviations of stock excess returns, "sig. mse3" is the MSE between the second- and third-pass estimates, etc. All MSE's have been multiplied by 1,000. "Sig. mean" is the time-series average of the conditional error standard deviation of stock excess returns, and "sig. sig." is the standard deviation displayed by the conditional error standard deviations.

The "% change" is the percentage change in the estimate of the correlation between the first and second estimations, and "abs. % chg." is the absolute value of the percentages of change.

counter	1	2	3	4	5	6	7	8	9	10
cusip	893782	004446	067383	155123	244199	302542	452704	635182	743315	809163
corr	0.196	0.337	0.554	0.693	0.566	0.078	0.549	0.161	0.359	0.481
corr2	0.191	0.335	0.539	0.692	0.574	0.081	0.529	0.166	0.351	0.486
corr3	0.190	0.335	0.535	0.692	0.575	0.080	0.525	0.167	0.349	0.483
corr4	0.190	0.335	0.534	0.692	0.576	0.080	0.525	0.167	0.349	0.482
corr5	0.190	0.335	0.533	0.692	0.576	0.080	0.525	0.167	0.349	0.482
sig. mse2	0.076	0.000	0.004	0.005	0.010	2.782	0.109	0.028	0.046	0.037
sig. mse3	0.000	0.000	0.001	0.001	0.001	0.138	0.003	0.000	0.048	0.002
sig. mse4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
sig. mse5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
sig. mean	0.164	0.066	0.091	0.054	0.082	0.337	0.075	0.112	0.089	0.061
sig. mean2	0.163	0.066	0.092	0.055	0.082	0.331	0.075	0.113	0.089	0.061
sig. mean3	0.163	0.066	0.091	0.055	0.082	0.332	0.075	0.113	0.089	0.062
sig. mean4	0.163	0.066	0.091	0.055	0.083	0.332	0.075	0.113	0.089	0.062
sig. mean5	0.163	0.066	0.091	0.055	0.083	0.332	0.075	0.113	0.089	0.062
sig. sig.	0.019	0.000	0.000	0.006	0.005	0.590	0.004	0.033	0.008	0.036
sig. sig2	0.027	0.000	0.002	0.005	0.002	0.580	0.013	0.033	0.001	0.032
sig. sig3	0.028	0.000	0.003	0.005	0.001	0.585	0.014	0.033	0.008	0.031
sig. sig4	0.028	0.000	0.003	0.005	0.000	0.585	0.014	0.033	0.009	0.031
sig. sig5	0.163	0.000	0.004	0.005	0.000	0.585	0.014	0.033	0.009	0.031
% change	2.726	0.727	2.582	0.200	-1.388	-2.901	3.661	-3.476	2.245	-1.007
average	0.337									
abs. % chg.	2.726	0.727	2.582	0.200	1.388	2.901	3.661	3.476	2.245	1.007
average	2.091									

Table 7 - Pooled Regressions

With the total sample of 100 portfolios and 144 months, pooled, cross-sectional OLS regressions are estimated with either the OLS or predicted beta as the sole explanatory variable:

$$R_{i(t+1)} = \zeta_0 + \zeta_1 * \beta_{\{i(t+1) \text{ or } \sigma[i(t+1)]\}} + u_{i(t+1)},$$

where $u_{i(t+1)}$ is the error term which is distributed $N(0, \phi)$, $R_{i(t+1)}$ is the excess return on portfolio i during period $t+1$, $\beta_{i(t+1)}$ is the traditional beta, $\beta_{\sigma[i(t+1)]}$ is the predicted beta, and ζ_0, ζ_1 are coefficients estimated by the model. P-values are listed below estimated coefficients.

Panel A - OLS Betas & Predicted Betas

All Months

	Constant	OLS Beta	Pred. Beta	R-Squared
Coefficient	1.944	-0.881		0.002
p-value	0.0001	0.0001		
Coefficient	1.804		-0.762	0.002
p-value	0.0001		0.0001	

Non-January Months

	Constant	OLS Beta	Pred. Beta	R-Squared
Coefficient	1.732	-1.038		0.003
p-value	0.0001	0.0001		
Coefficient	1.293		-0.618	0.001
p-value	0.0001		0.0001	

January Months

	Constant	OLS Beta	Pred. Beta	R-Squared
Coefficient	4.197	0.922		0.001
p-value	0.0001	0.2065		
Coefficient	8.061		-2.968	0.016
p-value	0.0001		0.0001	

Table 7 - Pooled Regressions

All Panel B regressions are pooled, cross-sectional OLS regressions with the sample of 100 size-beta portfolios. The "All Months" sample has 144 months, the non-January sample has 132 months, and the January-only sample has 12 months of data. The first regression of each pair is the result of the regression of OLS betas modeling the predicted betas:

$$\beta_{\sigma(i,t)} = \theta_0 + \theta_1 * \beta_{i,t} + \upsilon_{i,t} \qquad \text{(Model A)}$$

where $\upsilon_{i,t}$ is the error term which is distributed $N(0, \psi)$, $\beta_{i,t}$ is the traditional beta, $\beta_{\sigma(i,t)}$ is the predicted beta, and θ_0, θ_1 are coefficients estimated by the model. The second regression of each pair is:

$$R_{i,t} = \xi_0 + \xi_{1i} * \beta_{i,t} + \xi_{2i} * \upsilon_{i,t} + u_{i,t} \qquad \text{(Model B)}$$

where $u_{i,t}$ is the error term which is distributed $N(0, \phi)$, $R_{i,t}$ is the excess return on portfolio i during period t, $\beta_{i,t}$ is the traditional beta OLS-estimated on past observations, and $\upsilon_{i,t}$ is the orthogonalized predicted beta. P-values are listed below estimated coefficients.

Panel B - OLS Betas & Orthogonalized Predicted Betas

All Months

		Constant	OLS Beta	Orth. Beta	R-Squared
Model A	Coefficient	0.051	0.922		0.718
	p-value	0.0001	0.0001		
Model B	Coefficient	1.944	-0.881	-0.269	0.002
	p-value	0.0001	0.0001	0.3377	

Non-January Months

		Constant	OLS Beta	Orth. Beta	R-Squared
Model A	Coefficient	0.051	0.922		0.718
	p-value	0.0001	0.0001		
Model B	Coefficient	1.732	-1.038	0.673	0.004
	p-value	0.0001	0.0001	0.0145	

January Months

		Constant	OLS Beta	Orth. Beta	R-Squared
Model A	Coefficient	0.062	0.927		0.728
	p-value	0.0003	0.0001		
Model B	Coefficient	4.197	0.922	-13.543	0.094
	p-value	0.0001	0.1850	0.0001	

Appendix D: Derivation and Testing of Delta

Traditional CAPM Derivation: Constant Standard Deviations

Minimize the LaGrangian multiplier equation which features the risky asset variance subject to the condition of achieving the desired expected risky asset rate of return, $E[R_{mt}]$.

$$L = \sum_{i=1}^{n} w_i^2 \sigma_{it}^2 + 2\sum_{j>i}^{n} w_i w_j \sigma_{ijt} + 2\lambda \{E[R_{mt}] - \sum_{i=1}^{n} w_i E[R_{it}] - (1 - \sum_{i=1}^{n} w_i) R_f\}$$

$$\frac{\partial L}{\partial w_i}: \quad w_1 \sigma_{1t}^2 + \sum_{j>1}^{n} w_j \sigma_{1jt} = \lambda (E[R_{1t}] - R_f) \qquad (A1)$$

$$w_2 \sigma_{2t}^2 + \sum_{j\neq 2}^{n} w_j \sigma_{2jt} = \lambda (E[R_{2t}] - R_f)$$

$$\vdots$$

$$w_n \sigma_{nt}^2 + \sum_{j=1}^{n-1} w_j \sigma_{njt} = \lambda (E[R_{nt}] - R_f)$$

$$\frac{\partial L}{\partial \lambda}: \quad E[R_{mt}] = \sum_{i=1}^{n} w_i E[R_{it}] + (1 - \sum_{i=1}^{n} w_i) R_f$$

Multiply the first equation of (A1) by w_1, the second by w_2, etc., and sum the products:

$$\sigma_{mt}^2 = \lambda (\sum_{i=1}^{n} w_i E[R_{it}] - \sum_{i=1}^{n} w_i R_f)$$

$$= \lambda \{\sum_{i=1}^{n} w_i E[R_{it}] + (1 - \sum_{i=1}^{n} w_i) R_f - R_f\}$$

$$= \lambda (E[R_{mt}] - R_f) \qquad (A2)$$

(A1) and (A2) yield: $\text{cov}_t(R_i, R_m) = \dfrac{\sigma^2_{mt}(E[R_{it}] - R_f)}{E[R_{mt}] - R_f}$

$$\therefore\ E[R_{it}] = R_f + \frac{\text{cov}_t(R_i, R_m)}{\sigma^2_{mt}}(E[R_{mt}] - R_f)$$

$$\text{Define } \beta_{it} = \frac{\text{cov}_t(R_i, R_m)}{\sigma^2_{mt}}$$

$$\therefore\ E[R_{it}] = R_f + \beta_{it}(E[R_{mt}] - R_f)$$

CAPM Derivation under: $E[\sigma_{it}] = \delta_t\sigma_{i\,(t-1)}$, $E[\sigma_{ijt}] = \delta_i\delta_j\sigma_{ij\,(t-1)}$

Again, minimize the LaGrangian equation which features the expected risky asset variance subject to achieving the desired expected risky asset rate of return, $E[R_{mt}]$.

$$L = \sum_{i=1}^{n} w_i^2 E[\sigma_{it}^2] + 2\sum_{j>i}^{n} w_i w_j E[\sigma_{ijt}] + 2\lambda\{E[R_{mt}] - \sum_{i=1}^{n} w_i E[R_{it}] - (1 - \sum_{i=1}^{n} w_i)R_f\}$$

$$= \sum_{i=1}^{n} w_i^2 \delta_i^2 \sigma_{i(t-1)}^2 + 2\sum_{j>i}^{n} w_i w_j \delta_i \delta_j \sigma_{ij(t-1)} + 2\lambda\{E[R_{mt}] - \sum_{i=1}^{n} w_i E[R_{it}] - (1 - \sum_{i=1}^{n} w_i)R_f\}$$

$$\frac{\partial L}{\partial w_i}:\ w_1\delta_1^2\sigma_{1(t-1)}^2 + \sum_{j>1}^{n} w_j\delta_1\delta_j\sigma_{1j(t-1)} = \lambda\,(E[R_{1t}] - R_f) \qquad \text{(A3)}$$

$$w_2\delta_2^2\sigma_{2(t-1)}^2 + \sum_{j\neq 1}^{n} w_j\delta_2\delta_j\sigma_{2j(t-1)} = \lambda\,(E[R_{2t}] - R_f)$$

$$\vdots$$

$$w_n \delta_n^2 \sigma_{n(t-1)}^2 + \sum_{j=1}^{n-1} w_j \delta_n \delta_j \sigma_{nj(t-1)} = \lambda \, (E[R_{nt}] - R_f)$$

$$\frac{\partial L}{\partial \lambda}: \; E[R_{mt}] = \sum_{i=1}^{n} w_i E[R_{it}] + (1 - \sum_{i=1}^{n} w_i) R_f$$

Multiply the first equation of (A3) by w_1, the second by w_2, etc., and sum the products:

$$E[\sigma_{mt}^2] = \lambda \, (\sum_{i=1}^{n} w_i E[R_{it}] - \sum_{i=1}^{n} w_i R_f)$$

$$= \lambda \, \{\sum_{i=1}^{n} w_i E[R_{it}] + (1 - \sum_{i=1}^{n} w_i) R_f - R_f\}$$

$$= \lambda \, (E[R_{mt}] - R_f) \qquad \text{(A4)}$$

(A3) and (A4) yield: $E[\text{cov}_t(R_i, R_m)] = \dfrac{E[\sigma_{mt}^2](E[R_{it}] - R_f)}{E[R_{mt}] - R_f}$

$$\therefore \; E[R_{it}] = R_f + \frac{E[\text{cov}_t(R_i, R_m)]}{E[\sigma_{mt}^2]} (E[R_{mt}] - R_f)$$

$$\text{Define } \beta_{it}' = \frac{E[\text{cov}_t(R_i, R_m)]}{E[\sigma_{mt}^2]}$$

$$\therefore \; E[R_{it}] = R_f + \beta_{it}' (E[R_{mt}] - R_f)$$

Next we examine the relation between β_{it}' and β_{it}.

An investor owns a risky asset which is actually a portfolio of risky assets. The weights of the market portfolio can be adjusted so that the i^{th} risky asset is replaced by the

investor's risky asset (that includes the i^{th} individual risky asset) without loss of generality. The market weights on the i securities are w_i. Suppose that $\sigma_{i\,t} = \delta_i \sigma_{i\,(t-1)}$ for all $i = 1, \ldots, n$ market securities. Then the i^{th} risky asset systematic risk measure:[25]

$$\beta_{it}' = \frac{E[\sigma_{mit}]}{E[\sigma_{mt}^2]} = \frac{\rho_{mi} E[\sigma_{mt}] E[\sigma_{it}]}{E[\sigma_{mt}^2]},$$

$$= \frac{w_i^2 E[\sigma_{it}^2] + \sum_{j \neq i}^{n} w_i w_j \rho_{ij} E[\sigma_{it}] E[\sigma_{jt}]}{E[\sigma_{mt}^2]}$$

$$= \frac{w_i^2 \delta_i^2 \sigma_{i(t-1)}^2 + \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \delta_i \sigma_{i(t-1)} \delta_j \sigma_{j(t-1)}}{\sum_{j=1}^{n} w_j^2 \delta_j^2 \sigma_{j(t-1)}^2 + 2 \sum_{i=1}^{n} \sum_{j>i}^{n} w_i w_j \rho_{ij} \delta_i \sigma_{i(t-1)} \delta_j \sigma_{j(t-1)}}$$

Case 1: $\delta_i = \delta$ for all $i = 1, \ldots, n$

That is, all expected stock standard deviations change by the same proportion from period *t-1* to period *t*. Therefore

$$\beta_{it}' = \frac{w_i^2 \delta^2 \sigma_{i(t-1)}^2 + \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \delta \sigma_{i(t-1)} \delta \sigma_{j(t-1)}}{\sum_{j=1}^{n} w_j^2 \delta^2 \sigma_{j(t-1)}^2 + 2 \sum_{i=1}^{n} \sum_{j>i}^{n} w_i w_j \rho_{ij} \delta \sigma_{i(t-1)} \delta \sigma_{j(t-1)}}$$

[25] Note that if the APT assumption of a well-diversified portfolio is relaxed, that is if w_i is not small, then the measure of risk depends on a stock's own variance. This is the hypothesis and empirical finding of Levy (1978,1980).

$$= \frac{w_i^2 \sigma_{i(t-1)}^2 + \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \sigma_{i(t-1)} \sigma_{j(t-1)}}{\sum_{j=1}^{n} w_j^2 \sigma_{j(t-1)}^2 + 2 \sum_{i=1}^{n} \sum_{j>i}^{n} w_i w_j \rho_{ij} \sigma_{i(t-1)} \sigma_{j(t-1)}} = \beta_{it}$$

which yields the conclusion (see page 33).

Case 2: $\sigma_{i,t} = (\delta_i \delta_o)\sigma_{i(t-1)}$, $\sigma_{j,t} = \delta_o \sigma_{j(t-1)}$ for all $j = 1, \ldots, n \ni j \neq i$

That is, the i^{th} individual stock has standard deviations which change proportionally more (by δ_i) than do the standard deviations of all other stocks. Therefore

$$\beta_{it}' = \frac{w_i^2 \delta_o^2 \delta_i^2 \sigma_{i(t-1)}^2 + \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \delta_o \delta_i \sigma_{i(t-1)} \delta_o \sigma_{j(t-1)}}{\sum_{j=1, j \neq i}^{n} w_j^2 \delta_o^2 \sigma_{j(t-1)}^2 + w_i^2 \delta_o^2 \delta_i^2 \sigma_{i(t-1)}^2 + 2 \sum_{k \neq i}^{n} \sum_{j>k, j \neq i}^{n} w_k w_j \rho_{kj} \delta_o \sigma_{k(t-1)} \delta_o \sigma_{j(t-1)} + 2 \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \delta_o \delta_i \sigma_{i(t-1)} \delta_o \sigma_{j(t-1)}}$$

$$= \frac{w_i^2 \delta_i^2 \sigma_{i(t-1)}^2 + \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \delta_i \sigma_{i(t-1)} \sigma_{j(t-1)}}{\sum_{j=1, j \neq i}^{n} w_j^2 \sigma_{j(t-1)}^2 + w_i^2 \delta_i^2 \sigma_{i(t-1)}^2 + 2 \sum_{k \neq i}^{n} \sum_{j>k, j \neq i}^{n} w_k w_j \rho_{kj} \sigma_{k(t-1)} \sigma_{j(t-1)} + 2 \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \delta_i \sigma_{i(t-1)} \sigma_{j(t-1)}}$$

$$= \frac{\delta_i \{ w_i^2 \delta_i \sigma_{i(t-1)}^2 + \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \sigma_{i(t-1)} \sigma_{j(t-1)} \}}{\sum_{j=1, j \neq i}^{n} w_j^2 \sigma_{j(t-1)}^2 + w_i^2 \delta_i^2 \sigma_{i(t-1)}^2 + 2 \sum_{k \neq i}^{n} \sum_{j>k, j \neq i}^{n} w_k w_j \rho_{kj} \sigma_{k(t-1)} \sigma_{j(t-1)} + 2 \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \delta_i \sigma_{i(t-1)} \sigma_{j(t-1)}}$$

$$\cong \frac{\delta_i \{ w_i^2 \sigma_{i(t-1)}^2 + \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \sigma_{i(t-1)} \sigma_{j(t-1)} \}}{\sum_{j=1, j \neq i}^{n} w_j^2 \sigma_{j(t-1)}^2 + w_i^2 \sigma_{i(t-1)}^2 + 2 \sum_{k \neq i}^{n} \sum_{j>k, j \neq i}^{n} w_k w_j \rho_{kj} \sigma_{k(t-1)} \sigma_{j(t-1)} + 2 \sum_{j \neq i}^{n} w_i w_j \rho_{ij} \sigma_{i(t-1)} \sigma_{j(t-1)}}$$

$= \delta_i \beta_{it}$.

Under the assumptions that $w_i^2 \sigma_{i(t-1)}^2$ is small compared to $\sum_{j=1, j\neq i}^{n} w_j^2 \sigma_{j(t-1)}^2$ and that

$2\sum_{j\neq i}^{n} w_i w_j \rho_{ij}\ \sigma_{i(t-1)}\ \sigma_{j(t-1)}$ is small compared to $2\sum_{k\neq i}^{n}\ \sum_{j>k,\, j\neq i}^{n} w_k w_j \rho_{kj}\ \sigma_{k(t-1)}\ \sigma_{j(t-1)}$, i.e., that

the i^{th} stock doesn't have a large influence on the risky asset portfolio, it follows that

$\beta'_{i\,t} \cong \delta_i \beta_{i\,t}$.

This case indicates that a more appropriate version of the CAPM is:

$$E[R_{i,t}] = R_f + \beta'_{i\,t} * (E[R_{M,t}] - R_f)$$

where $E[R_{i,t}]$ is the expected return on portfolio i during period t, $\beta'_{i\,t}$ is the predicted beta (adjusted for the relative expected standard deviation during period t), $E[R_{M,t}]$ is the market return during period t, and R_f is the one-period risk-free rate.

Then a more appropriate test of the predicted beta is:

$$R_{i,t} = \xi_0 + \xi_{1i} * \beta_{i,t} + \xi_{2i} * \beta_{\sigma(i,t)} + u_{i,t} \qquad (8)$$

$u_{i,t}$ is the error term which is distributed $N(0, \phi)$,

where $R_{i,t}$ is the excess return on portfolio i during period t, $\beta_{i\,t}$ is the traditional beta OLS-estimated on past observations, and $\beta'_{i\,t}$ is the predicted beta under the proposed methodology. The traditional beta is included as well as the predicted beta to allow for testing whether the difference in estimation techniques significantly improves the model performance after controlling for the traditional beta.

Appendix E: Relationship between R-squared and Delta

The traditional computation of R^2 assumes constant variances and a constant correlation between the dependent and independent variables. R^2 is the market variance divided by the total variance (Sharpe, 1972):

$$R^2 = \frac{\beta^2 \sigma_x^2}{\beta^2 \sigma_x^2 + \sigma_e^2},$$

where e is the residual error from the regression of x on y. As the total variance is the variance of y, R^2 can also be written as:

$$R^2 = \frac{\beta^2 \sigma_x^2}{\beta^2 \sigma_x^2 + \sigma_e^2} = \frac{\beta^2 \sigma_x^2}{\sigma_y^2}.$$

Now substituting for the traditional value of beta, we arrive at the familiar conclusion:

$$R^2 = \frac{\beta^2 \sigma_x^2}{\beta^2 \sigma_x^2 + \sigma_e^2} = \frac{\beta^2 \sigma_x^2}{\sigma_y^2} = \frac{\rho_{xy}^2 \sigma_y^2 \sigma_x^2}{\sigma_x^2 \sigma_y^2} = \rho_{xy}^2.$$

Note that no time subscript is necessary, as the correlation is assumed to be constant, and given ρ_{xy}, R^2 does not depend on σ_x or σ_y.

The computation of $\beta_{\sigma t}$ assumes changing dependent and independent variable variances and a constant correlation between the dependent and independent variable levels. Define delta to be the proportional change of the expected beta between periods *t-1* and *t*:

$$\delta_{i,t} = \frac{E[\beta_{\sigma t}]}{E[\beta_{\sigma(t-1)}]} \equiv \frac{\dfrac{\rho_{MR} E[\sigma_{Rt}]}{E[\sigma_{Mt}]}}{\dfrac{\rho_{MR} E[\sigma_{R(t-1)}]}{E[\sigma_{M(t-1)}]}} = \frac{\dfrac{E[\sigma_{Rt}]}{E[\sigma_{R(t-1)}]}}{\dfrac{E[\sigma_{Mt}]}{E[\sigma_{M(t-1)}]}} = \frac{1+\%\Delta_{(t-1,t)} E[\sigma_R]}{1+\%\Delta_{(t-1,t)} E[\sigma_M]}.$$

We arrive at the conclusion that $\delta_{i,t}$ is analogous to the elasticity of the expected standard deviation of the risky asset returns with respect to the expected standard deviation of the market returns. Autoregressive behavior of this elasticity measure would explain the empirical findings of autoregressive beta behavior by Ohlson and Rosenberg (1982) and D'Souza, Brooks and Oberhelman (1989).

Using the definition for covariance and the assumption that ρ_{xy} is constant, we also note:

$$\delta_{i,t} = \frac{1+\%\Delta_{(t-1,t)} E[\sigma_{MR}]}{1+\%\Delta_{(t-1,t)} E[\sigma_M^2]}.$$

That is, $\delta_{i,t}$ is analogous to the square root of the elasticity of the risky asset's expected market covariance with respect to the expected market variance, and delta is independent of the purely historical measure, R^2. However, note that R^2 and β_σ are dependent.

VITA

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In August of 1993, David enrolled in the Graduate School at The University of Tennessee, Knoxville, to pursue a Ph.D. degree in Finance. He is a member of Financial Management Association, the Multinational Finance Society, the Eastern Finance Association, and the Southeastern Finance Association. David is married to the former Yvonne Leigh Hinson of Charlotte, North Carolina.