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Nonlinear Behavior of Longitudinal Waves in the Oscillations of a Rijke Tube

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ABSTRACT

The Rijke tube device has been employed since its invention in 1859 in the experimental study of many examples of thermo-acoustic phenomena. The device exhibits generation of acoustic oscillations by heat energy supplied to the flow field in the fashion of a self-excited oscillator. In recent times, the Rijke tube has proved to be a valuable tool in simulation of combustion instability phenomena in rockets and industrial burners. Despite the simplicity of the device, the Rijke tube simulates most important geometrical and physical features that lead to the growth of nonlinear pressure oscillations in combustion chambers. For example it provides a through-flow as in a rocket chamber and is fixed with an energy source that can cause unsteady combustion. The open ends and geometrical simplicity leads to easy accessibility for instrumentation to make measurements that would not be possible in actual combustion chambers. During operation, wave motions are generated by transfer of energy from a heated grid placed at a point within the chamber that can be related to theoretical models for the phenomenon by Rayleigh and other investigators. However, initially, there is exponential growth of these oscillations to high amplitude and transition to a nonlinear limit cycle at a nearly fixed amplitude (usually lasting several seconds) due to natural nonlinearities in the system. The hypothesis advanced in this thesis to explain this nonlinear limiting effect that is the wave steepening occurs in a manner analogous to similar generation of steep wave fronts in rocket motor chambers. The latter proposal is based on: 1) direct observation (using Schlieren techniques) of traveling shock-like waves in axial mode instability, 2) correlation of the observed waves with spectral components similar to that of sawtooth structure, and 3) theoretical calculations showing that the limit amplitude phenomenon is directly related to the cascade of energy from lower frequency standing acoustic modes to higher harmonics leading to characteristic spectrum similar to that of a traveling steep-fronted wave. In prior research, the ‘mechanism’ of initiation of instability in the system has been the main focus. The goal of the research described in this thesis is to measure and to characterize the signal produced during the high-

amplitude (nearly steady state) oscillations at the limit cycle. The intent was to demonstrate in a very simple way that the gas motions produced during the limit cycle in the Rijke tube have the same characteristics observed in many years of rocket testing. The observations again verify the great utility of the Rijke tube in seeking better understanding of the analogous rocket instability.

TABLE OF CONTENTS

1. INTRODUCTION	1
1.1. Combustion instabilities.....	1
1.2. Rijke tube introduction.....	1
1.3. Rijke mechanism explained.....	3
1.3.1. Explanation of this criterion	3
1.4. Applications of Rijke tube.....	5
1.4.1. Oscillations to be discouraged	5
1.4.2. Oscillations to be encouraged.....	8
1.5. Literature survey	8
1.6. Relevance to current topic	9
1.7. Objectives	12
2. THEORY	13
2.1. Shock waves in Rocket engines	13
2.2. Understanding steep-fronted waveforms	13
2.3. Representation of steep-fronted waves	14
3. EXPERIMENTAL SET-UP AND PROCEDURE	16
3.1. Set-up of the experiment	16
3.2. Terminology in Virtual Instruments (VI)	17
3.3. Procedure	18
3.4. Precautions.....	19
4. DATA AND ANALYSIS	22
4.1. Recording of data.....	22
4.1.1. Observations of data for metal screen position $L/8$	23
4.1.2. Readings at $x/L=1/8$	24
4.2. Sound amplitude (dB) vs. frequency (Hz) for $x/L = 3/16$	24

4.2.1. Procedure.....	24
4.2.2. Observations from data for metal screen position $3L/16$	37
4.3. Sound amplitude (dB) vs. frequency (Hz) $x/L = 1/4$	51
4.3.1. Procedure.....	51
4.3.2. Observations from data for metal screen position $L/4$	51
4.4. Interpretation of signal input from microphone	68
4.5. Conclusions.....	74
4.5.1. On data	74
4.5.2. On Instrumentation	74
5. CONCLUSIONS	76
5.1. Conclusions of the experimental data and Theory	76
5.2. Conclusions of this study	76
5.3. Flowchart.....	77
6. RECOMMENDATIONS	79
REFERENCES	81
APPENDIX A.....	85
VITA	95

LIST OF TABLES

Table 1: Specifications of the software	17
Table 2: Format of data input.....	18
Table 3: Higher harmonics and their corresponding frequencies	64

LIST OF FIGURES

Figure 1: Original configuration of the Rijke tube.....	2
Figure 2: Diagram from [21] showing the Rijke tube's fundamental acoustic wave structure.....	4
Figure 3: Behavior of self-excited oscillator effects of nonlinearity [31]	6
Figure 4: Schematic of the thermal system with unsteady heat release	7
Figure 5: Typical pressure measurements in preburner tests.....	10
Figure 6: Schematic of a preburner test device	11
Figure 7: Comparison of measured and theoretical waveforms [14]	13
Figure 8: Experimental and analytical pressure oscillation waveform [17].	15
Figure 9: Experimental setup	20
Figure 10: Detailed construction of metal screen with the thermocouple	21
Figure 11: Spectrum data for 1 st second; metal screen position $L/8$	25
Figure 12: Spectrum data for 2 nd second; metal screen position $L/8$	26
Figure 13: Spectrum data for 3 rd second; metal screen position $L/8$	27
Figure 14: Spectrum data for 4 th second; metal screen position $L/8$	28
Figure 15: Temperature graph for second 1; metal screen position $L/8$	29
Figure 16: Temperature graph for second 2; metal screen position $L/8$	30
Figure 17: Temperature graph for second 3; metal screen position $L/8$	31
Figure 18: Temperature graph for second 4; metal screen position $L/8$	32
Figure 19: Sample data during second 1 for $L/8$ position.....	33
Figure 20: Sample data during second 2 for $L/8$ position.....	34
Figure 21: Sample data during second 3 for $L/8$ position.....	35
Figure 22: Sample data during second 4 for $L/8$ position.....	36
Figure 23: Spectrum data for 1 st second; metal screen position $3L/16$	38
Figure 24: Spectrum data for 2 nd second; metal screen position $3L/16$	39
Figure 25: Spectrum data for 3 rd second; metal screen position $3L/16$	40
Figure 26: Spectrum data for 4 th second; metal screen position $3L/16$	41
Figure 27: Temperature graph for second 1; metal screen position $3L/16$	43

Figure 28: Temperature graph for second 2; metal screen position $3L/16$	44
Figure 29: Temperature graph for second 3; metal screen position $3L/16$	45
Figure 30: Temperature graph for second 4; metal screen position $3L/16$	46
Figure 31: Sample data during second 1 for $3L/16$ position	47
Figure 32: Sample data during second for $3L/16$ position	48
Figure 33: Sample data during second 3 for $3L/16$ position	49
Figure 34: Sample data during second 4 for $3L/16$ position	50
Figure 35: Spectrum data for 1 st second; metal screen position $L/4$	52
Figure 36: Spectrum data for 2 nd second; metal screen position $L/4$	53
Figure 37: Spectrum data for 3 rd second; metal screen position $L/4$	54
Figure 38: Spectrum data for 4 th second; metal screen position $L/4$	55
Figure 39: Temperature graph for second 1; metal screen position $L/4$	56
Figure 40: Temperature graph for second 2; metal screen position $L/4$	57
Figure 41: Temperature graph for second 3; metal screen position $L/4$	58
Figure 42: Temperature graph for second 4; metal screen position $L/4$	59
Figure 43: Sample data during second 1 for $L/4$ position	60
Figure 44: Sample data during second 2 for $L/4$ position	61
Figure 45: Sample data during second 3 for $L/4$ position	62
Figure 46: Sample data during second 4 for $L/4$ position	63
Figure 47: Comparison of sound intensities of fundamental mode in all three positions .65	
Figure 48: Comparison of sound intensities of second acoustic mode in all three positions	66
Figure 49: Comparison of sound intensities of third acoustic mode in all three positions	67
Figure 50: Acoustic oscillation in a Rijke tube	68
Figure 51: Steep waves generated by a piston	69
Figure 52: Velocity plot of steep waves	71
Figure 53: Voltage input from microphone	71
Figure 54: Comparison of time taken for velocity increase and decrease	72
Figure 55: Ratios of time taken for velocity increase and velocity decrease	73
Figure 56: Spectrum plot for the input from the microphone	73

Figure 57: Flowchart for the conclusions chapter	78
Figure 58: Schematic of the experimental setup	86
Figure 59: Spectrum data (upper end)	88
Figure 60: Spectrum data (Lower end)	89
Figure 61: Amplitude in dB at lower end (reference 100 microV)	91
Figure 62: Amplitude in dB at upper end (reference 100 microV)	92
Figure 63: Amplitude in dB at upper end (Reference 10 microV)	93
Figure 64: Amplitude in dB at lower end (Reference 10 microV)	94

1. INTRODUCTION

1.1. Combustion instabilities

One of the technical complexities that has continued to exist ever since the construction of the first rocket engine bears the name combustion instability. Being a very challenging problem, it has intrigued many a great investigators and its eluding nature has called for the dedicated research done by experts in this field. Many decades years after it was first observed. , combustion instability still remains an area where engineers can't help but fumble Combustion instability is perhaps a misnomer because it is not, in general, a fundamental instability in the combustion process though it has not been proved till now. The theory that combustion is not responsible can be proved by the thermo acoustic phenomenon. The study of the actual mechanism behind the oscillations can be conducted conveniently on a device called the Rijke tube.

1.2. Rijke tube introduction

Rijke tube is one of the simplest devices that demonstrate the thermoacoustic phenomenon. Thermo-acoustics is the physics that explains the generation of sound due to the conversion of thermal energy into acoustic energy.

A Rijke tube is a simple hollow, open at both ends, vertically held tube, with a metal screen held inside it. When the metal screen is heated, oscillations are set up in, the tube, which is held vertically, producing sound. This generation of sound due to heat input, depicts the 'thermoacoustic' phenomenon.

The Rijke tube effect was first recorded by Rijke in [1]. He observed and recorded that when the heat source was placed in the lower half of the tube, sound was produced.

Summary [3] of Rijke's observations translated into English:

Rijke discovered that strong oscillations occurred when a heated metal screen was placed in the lower half of an open-ended vertical pipe as shown in Figure 1. The oscillations would stop if the top of the pipe was closed indicating that the convective air current up the pipe was one of the necessary conditions for the phenomenon. Oscillations were the strongest when the heated screen was located one-fourth of the length of the pipe from

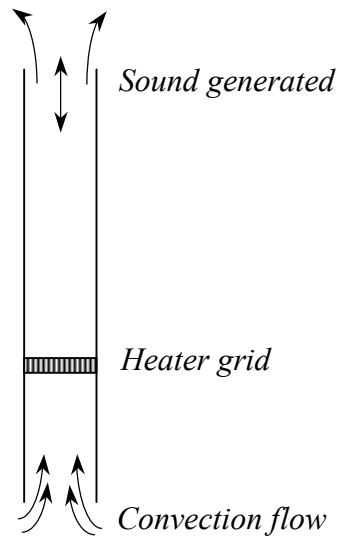


Figure 1: Original configuration of the Rijke tube

the bottom end. For heater positions in the upper half of the pipe, damping instead of driving occurred. Oscillations were observed in pipes from 18" to 10' in length, with diameters between 2" and 5". Rijke explained that the rising convection current expanded in the region of the heated screen and compressed down stream from the heater due to the cooling of the pipe walls. The production of the sound was attributed to these successive expansions and contractions.

His explanation suggests that the fluid elements in the lower half of the tube always experienced expansion while the upper elements experienced compression. But it is well known that when longitudinal oscillations are set up in a tube, every fluid element in it undergoes a variation in pressure. Hence Rijke's theory is limited to the explanation of the excitation of these oscillations which was obviously inadequate to explain the detailed heat exchange mechanism causing the oscillations. His interests were, however, musical and since his tube had no role to play in the musical acoustics he was looking for, there was no motive for further study of this strange occurrence.

1.3. Rijke mechanism explained

Rijke tube later found its explanation in Lord Rayleigh's book, '*The theory of sound*'.

Lord Rayleigh proposed a criterion [2] for heat-driven acoustic oscillations which gave a clear explanation of their mechanism. This criterion in his (Lord Rayleigh's) words is:

“If heat be communicated to and abstracted from, a mass of air vibrating (for example) in a cylinder bounded by a piston, the effect produced will depend upon the phase of the vibration at which the transfer of heat takes place. If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged. On the other hand, if heat be given at the moment of greatest rarefaction or abstracted at the moment of greatest condensation, the vibration is discouraged.”

1.3.1. Explanation of this criterion

In case of the Rijke tube, this criterion refers only to the time-varying component [27] of heat transfer q' where it states that if heat is added ($q' > 0$) during compression half cycle ($p' > 0$) or taken out ($q' < 0$) during an expansion half cycle ($p' < 0$), the acoustic waves would be sustained. In mathematical terms, Rayleigh's criterion can be formulated in terms of Rayleigh's integral, I :

$$I = \frac{1}{T} \oint p' q' dt ,$$

where T is the time period of oscillations, p' is the acoustic pressure, q' is the fluctuation in heat transfer t is the time and $\oint$ denotes integration over one cycle of oscillation.

If $I < 0$, then acoustic oscillations will damp out

If $I > 0$, then acoustic oscillations will grow

If $I = 0$, then oscillations will neither be damped out nor amplified

Another research paper [6], covering both the theory and experimental work clearly indicated that the thermoacoustic oscillations occur only when the heater is located in a region where the acoustic velocity leads the pressure oscillation by a quarter of the period. The representation in Figure 2 gives a clear picture of the nature of the

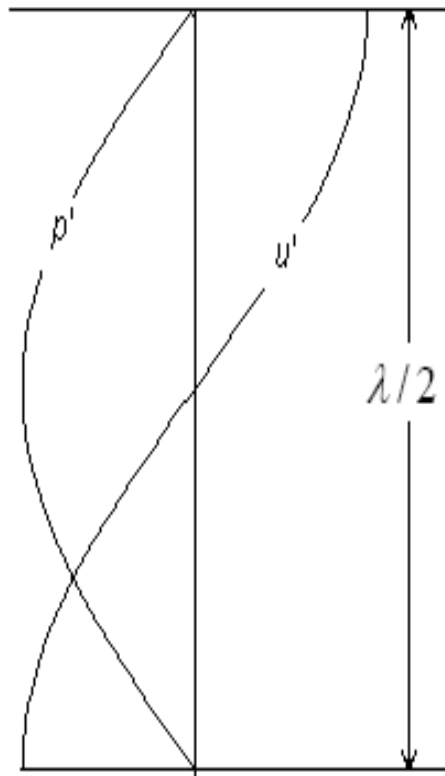


Figure 2: Diagram from [21] showing the Rijke tube's fundamental acoustic wave structure.

oscillations in the tube. Here P' and u' are the pressure and velocity variations respectively and λ is the wavelength

1.4. Applications of Rijke tube

1.4.1. Oscillations to be discouraged

The revival of interest in Rijke tube was largely motivated by problems in jet and rocket engine combustors. In today's era of growing environmental awareness, the aircraft industry is faced with stricter emission norms. One of the most objectionable constituents of jet engine emissions is NO_x . It is a known fact that NO_x emissions in combustion processes are proportional to the temperature [24]. A high ratio of air to fuel in the form of lean, premixed and pre-vaporized (LPP) flame keeps the temperature of the combustion within acceptable limits. However, LPP combustors, beyond a critical fuel-air ratio tend to show low frequency (50-150 Hz) longitudinal acoustic instabilities, known as 'buzz', which can cause serious structural damage.

In the case of rocket engine, which is of prime interest in this work, these longitudinal oscillations with high amplitudes affect the engine components. The similarity in geometry and the principle of heat induced oscillations allows the comparative study of the tube and rocket engines. Construction of experimentally tested baffle configurations on the injector plate is a solution to this problem. The injector plate of Apollo engine was designed with a well tested pattern of baffles. But such an arrangement is effective only in breaking down the transverse oscillations. Longitudinal oscillations continue to exist and can cause irreparable damage to the engine components.

Of crucial importance is that important nonlinear features appear in the vast experimental data set from work of many investigators using the Rijke apparatus. These data are typical of those encountered in the testing of rocket or burner combustion chambers. The analogous features are:

1. Self-excited oscillations *vis-à-vis* externally driven oscillations.

2. Growth of wave motions initiated by random noise in the system transitioning into an exponential growth as predicted by the linear theory of self-excited oscillators
3. Nonlinear transition to a limit cycle mode in which the amplitude of the oscillatory field is at a very nearly constant limit amplitude. This is a distinct feature of the Rijke tube; the oscillations are often maintained for several seconds.

These features are depicted graphically in Figure 3. Figure 4 explains the heat input.

It is important to point out that virtually all analytical work on Rijke tube phenomenon (and to a large extent in the analogous problem of acoustically unstable combustion chambers as well) has focused on the linear aspects of the problem. These analyses are motivated by the fact that frequencies observed are closely predicted by linear acoustic

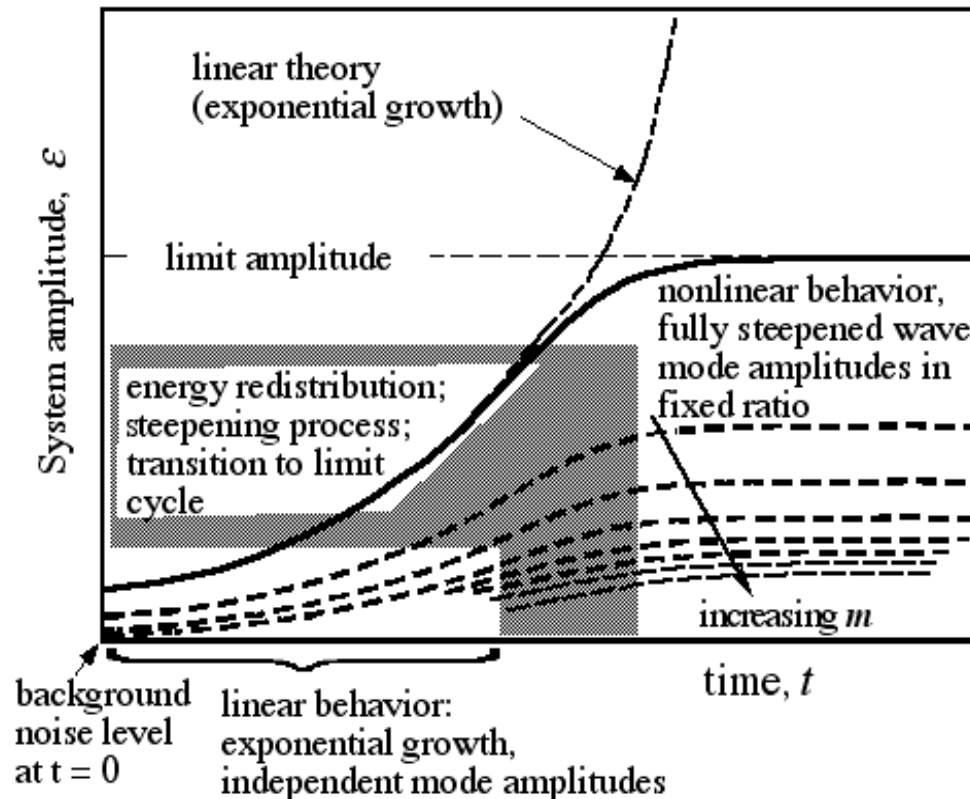


Figure 3: Behavior of self-excited oscillator effects of nonlinearity [31]

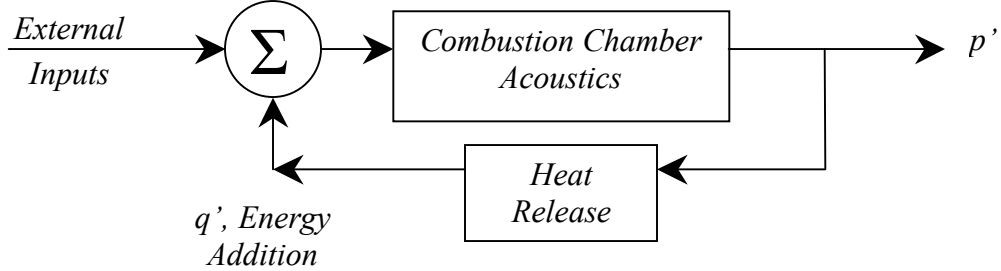


Figure 4: Schematic of the thermal system with unsteady heat release

theory. Thus one attempts to construct models for the driving mechanisms and other aspects of the problem from the standpoint of linear acoustics. These models include the effect of mean flow interactions and a concentrated energy source that is sensitive to local fluctuations mainly those due to the acoustic velocity field. Despite the simplicity of the geometry and the apparently well-understood physical mechanisms, there has not been uniform agreement in the validity of the many proposed analytical models. It must be emphasized that it was not the purpose of the research described herein to further add to the myriad analytical model for the linear part of the problem. One common feature in all of these models is that the gas motions consist of uncoupled acoustic waves that grow exponentially with time. In the case of the Rijke tube these waves conform to the longitudinal modes of an open-open tube as already discussed. As portrayed in Figure 3, the system initially grows. Thus there is practically no information in the literature concerning the transition to the limit cycle. This then is the motivation for the work done in this thesis research. The transition to the limit cycle is clearly a nonlinear effect as is well-known from the vast literature in nonlinear acoustics (Morse and Ingard, [7] (Ch. 14, Section 14.5 pp 874 – 882) and Pierce [11] (Ch. 11))

1.4.2. Oscillations to be encouraged

Another industrial application where the Rijke phenomenon is relevant is the case of pulse combustors and coal bed combustors. Pulse combustors are designed such that the sustained oscillations due to acoustic instability in them becomes a means of improving combustion efficiency by better mixing of fuel and air. Specially designed pulse combustors of a Rijke type use this effect to increase burning rate of heavy fuel oils [19]. In coal bed combustors, almost 70% of the fine ash particles (smaller than 5 microns) can escape through the filtering process. However it is found that intense acoustic energy increases the collision rate between these particles, which can help these particles coalesce and increase their effective size. Such particles can then be efficiently removed by conventional ash removing methods. Research on maximizing the efficiency of these combustors involves innovative methods to encourage the oscillations as opposed to the research in rocket engines where the oscillations have to be discouraged.

1.5. Literature survey

Previous tests have determined the general dependence of the behavior of this device on variations in primary parameters. For instance, in order to make the first acoustic mode unstable, the heater must be located in the upstream half of the tube; the most favorable position is near one quarter of the tube length from the lower end [1]. Study of the mechanics of this tube that involved analytical modeling and experimental validation of the fundamental mode stability boundaries dominated the scene for a long time. In all studies, that sound was produced only when the heat source was located in the lower half of the tube was agreed upon. It was J. J. Bailey [5] who proved by experimental procedures that the second acoustic mode can be sounded in the third quarter of the tube from the lower end. Therefore, to cause second mode excitation, the heater should be located either in the first or third quarter of the tube as measured from the upstream end. Studies also state that the larger the quantity of heat delivered to the airflow, the more prone to instability the system is, with all other parameters kept fixed. Studies of Pulse combustors with Rijke tube [15], [23], phenomenon as the basis have delved into production of the first acoustic mode and its stability boundaries.

In one of the recent works [29], a detailed study has been conducted on the transition of instability of the first acoustic mode by varying the heat input and also the mean flow. This was possible by orienting the Rijke tube horizontally. Proof and mathematical modeling of hysteresis and limit cycle properties were given. Study of nonlinearity due to the presence of higher harmonics in the Rijke oscillations has not been very extensive. An elaborate experimental investigation of stability boundaries and unstable regimes was conducted by [12].

Few works have studied the nonlinearity of Rijke oscillations as applied to the rocket motor combustion instability. The occurrence of higher harmonics in the Rijke tube has not been considered.

1.6. Relevance to current topic

Thermoacoustic instabilities refer to the appearance of pressure oscillation coupled with an unsteady heat release. The process is shown in Figure 4. The combustion chamber, as an example of a thermal device, displays particular acoustic properties. If the heat released in this device is dependent on the pressure and velocity fluctuations, a feedback loop as in the above mentioned figure is formed, that can destabilize the system.

These instabilities occur in real life as longitudinal oscillations in rocket engines causing irreparable damage to the engine components. The oscillations occur with frequencies that are typical of the chamber axial acoustic modes. In order to explain the highly damaging effects of these oscillations, one has to appreciate the presence of steep-fronted wave motions with very high amplitude [30]. This steepening process which begins initially with small amplitude standing acoustic waves grows to result in a shock-like disturbance. In order to bring clarity in the nonlinear problem occurring in these shock-like disturbances, it is considered that the fully steepened traveling wave is a composite of the chamber acoustic modes. This assumption works well in explaining the nonlinearity of the problem. A typical record of a preburner test [30] with low frequency resolution is shown in part (a) of figure 5. Part (b) of Figure 5 shows a typical steep-fronted wave form measured near the injector face. Preburners, the schematic of which is shown in Figure 6, are used to heat and ready propellants for the turbo pumps before

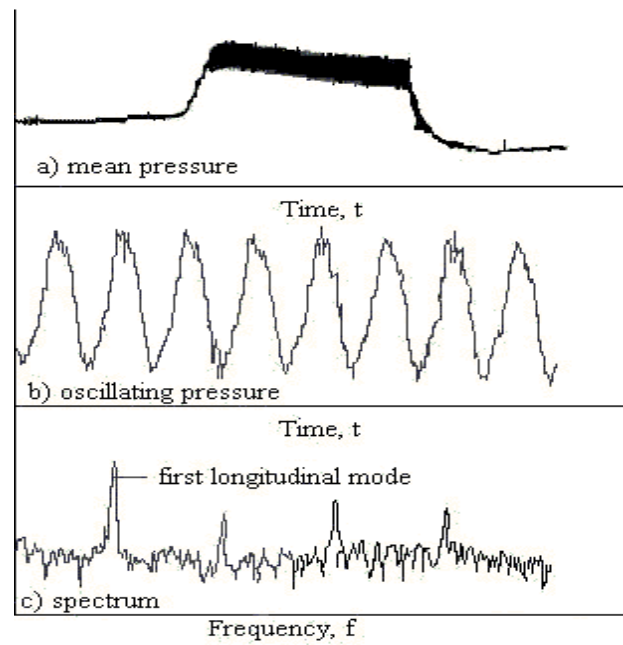


Figure 5: Typical pressure measurements in preburner tests

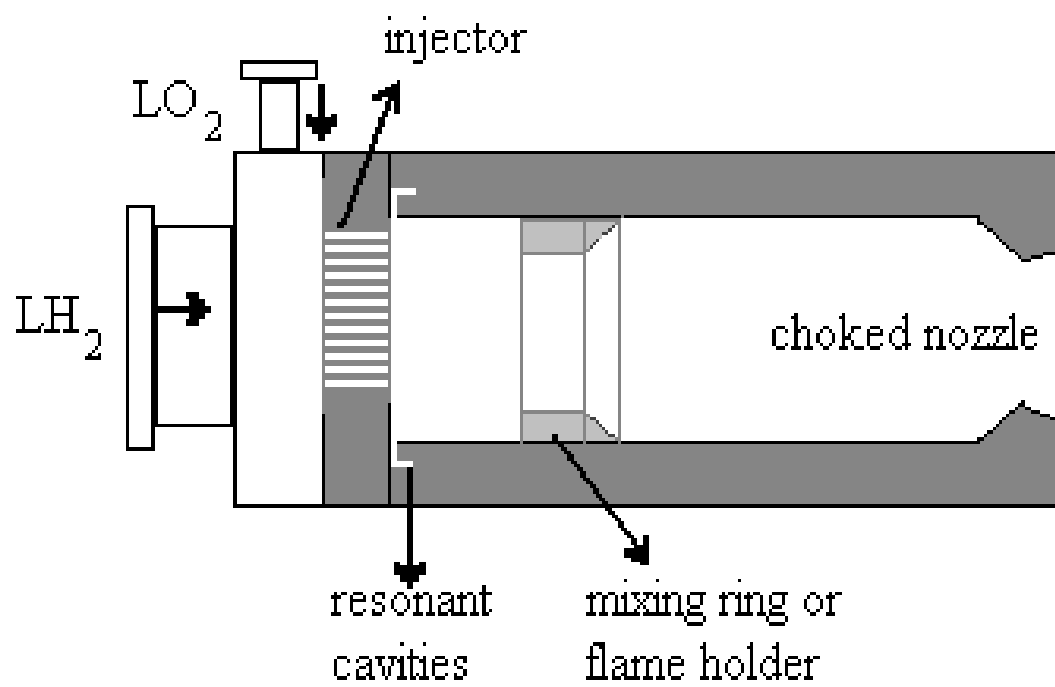


Figure 6: Schematic of a preburner test device

being injected into the main combustion chamber. The frequency of this wave is very close to the fundamental mode of the combustion chamber. Part (c), which plots the frequency spectrums of this pressure data, shows the presence of significant peaks at frequencies which are nearly the same as higher acoustic modes. Study of the presence of higher harmonics in the Rijke tube is analogous to the above mentioned Flandro's theory which has both analytical and experimental validations.

1.7. Objectives

1. To record the thermoacoustic instability occurring in the Rijke tube through a microphone and conduct spectrum analysis on the data to determine the sound amplitudes of frequencies up to 2000Hz.
2. Explain the relation between occurrence of higher harmonics and steep-fronted waves.
3. Examine the analogy between the existences of steep-fronted waves in rocket combustion chambers with the experimentally supported occurrence of higher harmonics in the Rijke tube.
4. Give an explanation of the motivation for this study which encourages further studies of higher harmonics and steep-fronted waves that might justify the high amplitude vibrations recorded as combustion instability.

2. THEORY

2.1. Shock waves in Rocket engines

If shock waves are to be invoked in developing an understanding of high amplitude nonlinear combustion instability, it is necessary to study the conditions under which shock waves would be likely to form in a rocket motor chamber. Many analyses including those intended to address questions of nonlinear behavior start at the acoustic limit. This approach is well founded experimentally, since in many cases the waves grow from initially very weak acoustic motions which usually can be modeled in terms of the standing normal acoustic modes of the chamber.

2.2. Understanding steep-fronted waveforms

The connection between traveling shock waves and the acoustics of the chamber is clearly important since the former obviously results from the development of the latter.

The steep fronted waves can be initiated by pulsing the system or they can grow from initially weak acoustic disturbances. Figure 7 shows a waveform typical of that actually measured in a rocket motor. This waveform closely resembles that based on one-dimensional shock behavior in a duct closed at both ends. The theoretical solution is superimposed for comparison.

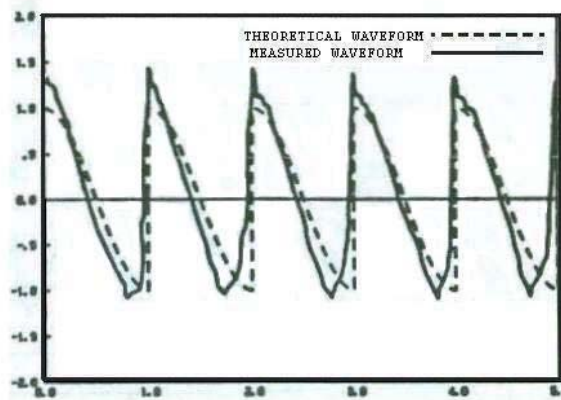


Figure 7: Comparison of measured and theoretical waveforms [14]

Examination of the spectral content in typical data of the latter type suggests steepening comes about as a result of the increasing amplitude of the higher-order acoustic harmonics. This suggests that the steep-fronted traveling waves can be represented by a superposition of standing acoustic Eigen functions.

2.3. Representation of steep-fronted waves

For one-dimensional problems, the Fourier series form of the pressure and velocity are given by

$$\frac{p'}{P_0} = \varepsilon \sum_{n=1}^{\infty} C_n \cos(k_n x) \sin(\omega_n t - \phi_n)$$

$$\frac{u'}{a_0} = \varepsilon \sum_{n=1}^{\infty} C_n \sin(k_n x) \cos(\omega_n t - \phi_n)$$

where C_n is the relative amplitude of mode n and ϕ_n is the relative phase angle. k_n and ω_n are the wave number and frequency for the mode n as given for a tubular chamber of length L by

$$k_n = n\pi / L \text{ and } \omega_n = n\pi a_0 / L$$

A solution that represents traveling waves in a hard-walled chamber with closed ends is

$$C_n = 8n / \pi(4n^2 - 1), \quad \phi_n = 0$$

This exhibits the N-wave traveling shock pressure distribution which is a very simple waveform. In order to get closer to the actual waveforms, we use the solution that is applied/works in tactical rockets.

$$C_n = 2\sqrt{1 + (n\pi)^2} / (n\pi)^2, \quad \phi_n = -\tan^{-1}[1/(n\pi)]$$

The experimental data in the diagram below is that of pressure fluctuations measured in a sonic end-vent burner [10]. The analytical model is the Fourier representation of the pressure fluctuations. There is striking similarity between the experimental data and the analytical model that is plotted using six terms of the Fourier series in Figure 8 below .

Hence the assumption of a steep-fronted wave to be a composite of the chamber normal modes is justified.

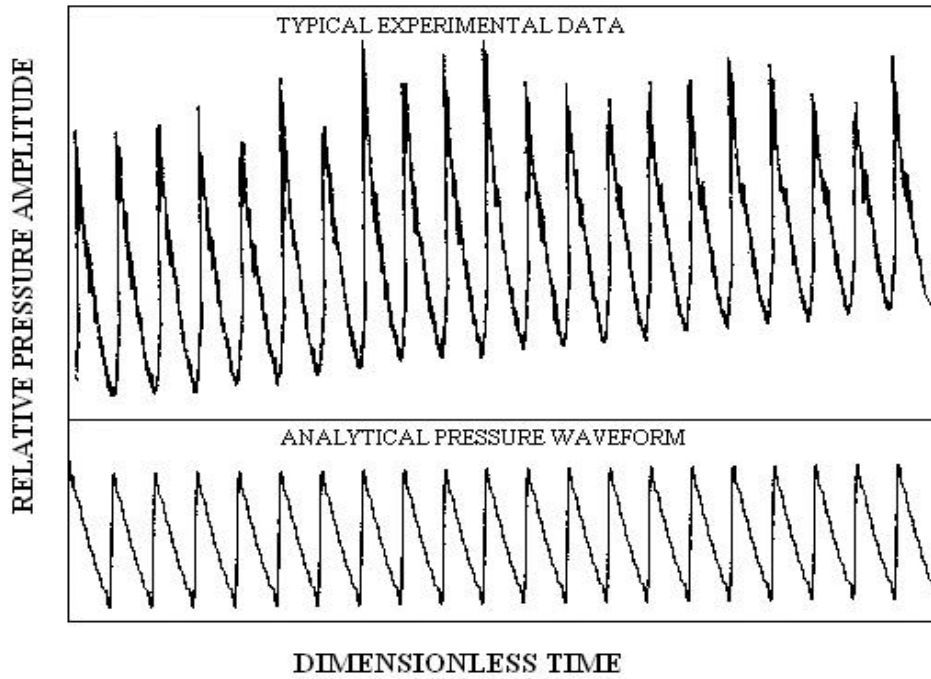


Figure 8: Experimental and analytical pressure oscillation waveform [17].

3. EXPERIMENTAL SET-UP AND PROCEDURE

3.1. Set-up of the experiment

The Rijke tube is a simple apparatus where all the components are easily available off the shelf. The components used were:

1. A steel pipe is used in this experiment as it is easily available with a wide range of dimension specifications. But any tube with L/D ratio such that the one-dimensional assumption of sound waves is valid can be used. This is possible when the radial dimension is smaller than the wavelength. If it happens so, then the motion of the sound wave is effectively confined to the direction in which the tube is aligned. The material is also variable and only needs to withstand the high temperatures. The specifications of the tube used are L (length) = 17.32" and D (diameter) = 1.53". The wavelength for the 1st and 2nd harmonics are greater than the diameter of the tube hence 1-D motion can be assumed.
2. A wire metal screen fixed between two metallic rings in order to give the metal screen stability during alteration of position and also when being heated. Here, the metal screen used was a 16 gauge one.
3. A steel rod with a sturdy flat end in order to vary the position of the metal screen.
4. A stand in order to hold this arrangement in a vertical position with a screw clamp.
5. A thermocouple attached to the screen that measures the temperature of the air less than 1 mm away from the screen. This thermocouple gives an approximate measure of the maximum temperature attained by the air inside the tube. The thermocouple used here is a K-type with maximum working temperature 1800 °C.
6. A microphone placed at a distance of two feet from the tube, in line with the upper end of the tube. The microphone has been calibrated with a standard sound source of 1000Hz with a sound intensity of 114 dB. The software used has recorded this information accurately. So the chances of errors due to microphone inaccuracy have been taken care of.

7. A propane gas torch with variable flow rate of propane in order to heat the metal screen.
8. Analog to digital converter for the input from the microphone and thermocouple.
9. ‘Virtual Instruments (VI)’, a software of National Instruments that conducts the spectrum analysis of the voltage input using the Fast Fourier Transform.

3.2. Terminology in Virtual Instruments (VI)

Scan rate:

It is the requested number of scans per second the VI acquires. The default is 1000 scans/second. A scan is one sample/Channel. Bandwidth is twice the scan rate.

Buffer size:

It is the number of scans (with input operations) or updates (with output operations) you want the circular buffer to hold.

Delta function:

It is the magnitude of the spacing between adjacent tone frequencies. If the start frequency is 100 Hz, and the delta f is ten, and number of tones is three, then the tone frequencies generated are 100Hz, 110Hz, and 120Hz. This value must be an integer multiple of (Sampling Rate)/Samples.

For this experiment, Table 1 shows the specifications of the program run.

Table 1: Specifications of the software

Scan rate	20000/ s
Buffer size	2000000 scans
Number of scans read at a time	5000
Delta f	1 Hz
Time resolution	0.00005 s
Period	20000

The data recorded by the microphone and pressure transducer is analyzed and the output is a table of readings recorded with 20000 time steps in one second. Voltage input from the microphone, temperature from thermocouple and the frequency spectrum are tabulated by the software as shown below.

3.3. Procedure

The apparatus is connected as shown in the diagram. The iron metal screen is heated with the propane torch till the temperature recorded by the thermocouple reaches 1000°C. Once the input from the thermocouple reads a value of 1000°C, information from the microphone is recorded for a period of 4 seconds. Though the choice of the 4 second time period has no particular reason, it is a convenient pick as the temperature remains almost constant, taking into consideration the limitations of the heating device. The input is stored in the format as shown in Table 2.

Table 2: Format of data input

S.no	Time	Voltage (mV)	Frequency (Hz)	Amplitude (dB)	Temperature (°C)
1.					
2.					
3.					
4.					
5.					
6.					

3.4. Precautions

The following precautions need to be taken while conducting the experiment.

1. Since the response of the thermocouple is not very good (when compared to the high speed recording of the software), the flame must be held a little (more than two inches) away from the metal screen in order to have control over the maximum temperature that can be reached.
2. Over a period of time, when the tube has been heated several times, no sound occurs in spite of the temperature being recorded as high as 1300 °C. This can be explained by the fact that the whole tube and the air inside reach a value almost as much as the metal screen temperature (it is to be noted that the temperature of the metal screen used here does not literally refer to the metal screen's temperature. The temperature read by the thermocouple is the temperature of the air around 0.5-1.0 mm above the metal screen surface. This is the same as the temperature of the heat source we mention in this study. The temperature is not of the metal screen. It cannot be possible as the temperature of cherry red steel metal screen will be as high as 2000° C and the thermocouple used here is not constructed to record such a high temperature.). This is when the assumption of a localized heat source breaks down and hence no sound is recorded.

Figure 9 is a schematic of the experimental setup. Figure 10 shows the design of a metal screen whose position can be varied as required.

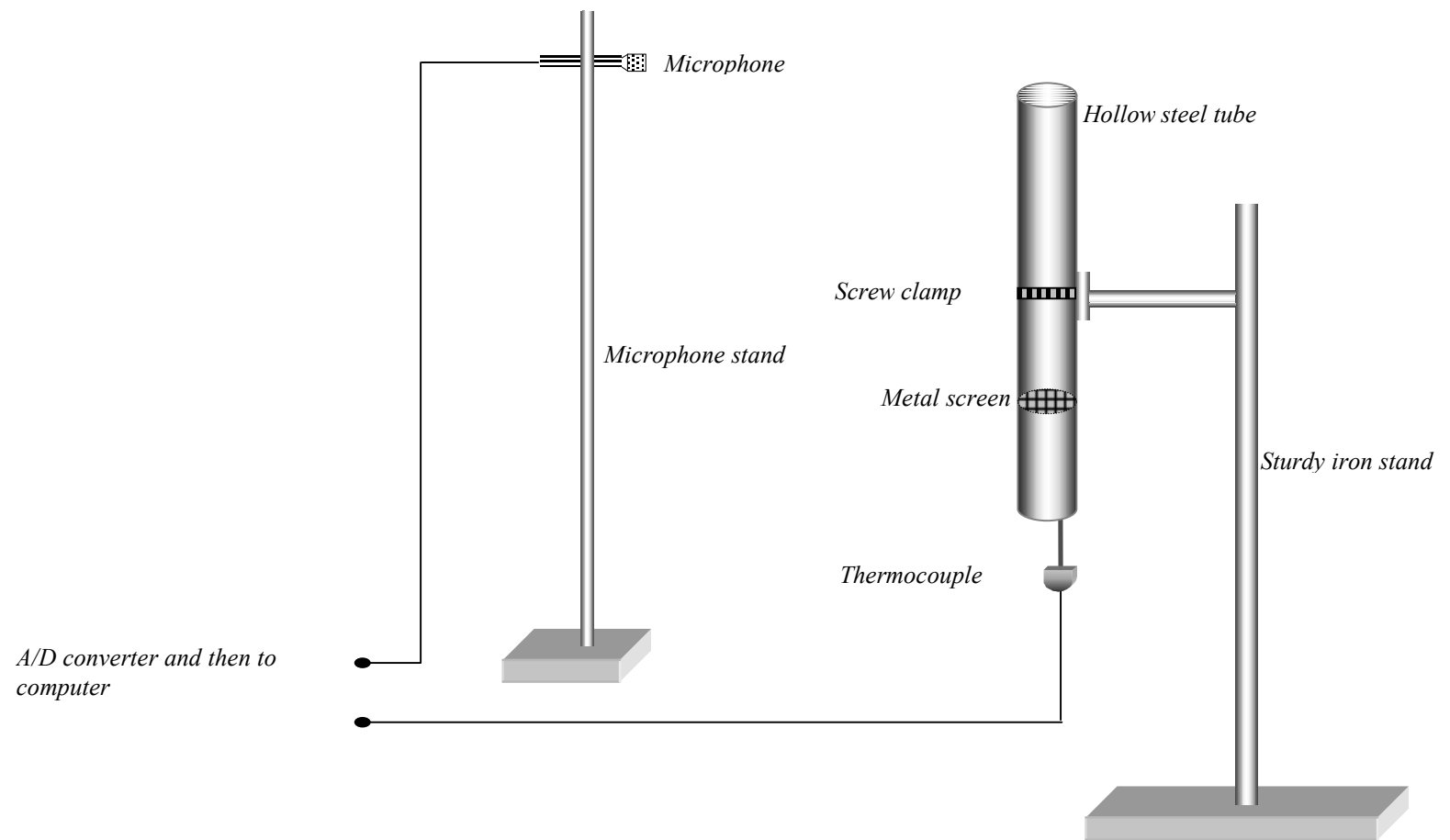


Figure 9: Experimental setup

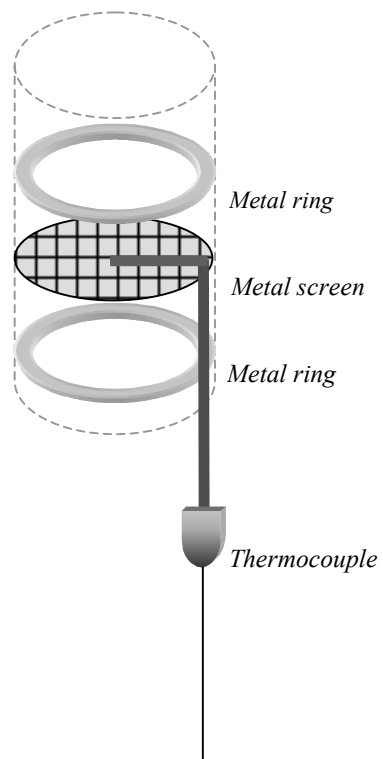


Figure 10: Detailed construction of metal screen with the thermocouple

4. DATA AND ANALYSIS

4.1. Recording of data

In this experiment, readings at positions $L/8$, $3L/16$ and $L/4$ have been recorded. Restriction on the positions of the metal screen is attributed to the simplicity of the experimental setup. Both, the length of the thermocouple and the length of the propane torch (or) the extent to which the propane torch can be inserted into the tube are the reasons.

The spectrum analysis of the voltage output from the microphone is performed using the fast Fourier transform. This plots amplitude against the frequency. In one loop i.e., one second, we have 20,000 data points from which we get 20,000 time domain inputs but only 10,000 frequency domain points. This is because the spectrum analyzer needs two data points in order to generate one reading. Hence we get the intensities of sound produced over a period of one second in the range of 2-10,000 Hz. Inspection of the data revealed that no significant peaks have been noticed after the theoretically calculated fourth acoustic mode of the tube. Hence the graphs have been plotted only till 2000 Hz. The production of higher harmonics in this experiment is limited to three. It can be assertively said that this is a restriction put by the amount of heat supplied to the metal screen. This one of the factors but many other factors could also cause this limitation in the production of harmonics. Previous studies [9] on generation of higher harmonics in Rijke tube made an attempt by placing multiple heat sources (spiral heating coils) across the length of the tube and could excite modes as high as the ninth harmonic. They couldn't excite harmonics beyond the second acoustic mode with the conventional Rijke tube heat source. Absence of sophisticated measurement techniques could be the reason for being unable to notice higher harmonics. It must be mentioned that the software used in this experimental work has many advantages when compared to the conventional spectrum analyzer. It was also stated that production of third and above acoustic modes was 'impossible' in a small Rijke tube. Though the range of dimensions that defined 'small' were not specified, the discussion of tube dimensions affecting the harmonics

generated in the current work is justified to a certain extent by the research work mentioned [9]. Study of higher harmonics didn't seem to have much encouragement hence further work hasn't received much publicity. Possibility of generation of higher harmonics by such a thermo acoustic device might initiate research on the Rijke tube with a more specific application.

4.1.1. Observations of data for metal screen position $L/8$

The data is recorded once the temperature reaches 1000°C . The data for temperature has been taken continuously and has been split into 4 seconds for convenience. In all the four graphs, the first acoustic mode dominates in intensity. With the increase of harmonics, the decibel level decreases, but three harmonics are clearly visible. As the system stabilizes through the four seconds, we can see that the highest peaks correspond to the frequencies that are exactly or with very little tolerance, the theoretically calculated frequencies. As seen in

The fourth acoustic mode that is not clearly visible in the 1st second but begins to show up as the system stabilizes. Its presence can be detected from the 2nd second itself. If the temperature can be maintained at a constant value for a longer time, all the four frequencies can be clearly seen. Also, previous studies have stated that if the heat source can be maintained as a localized one (by saying so we mean to say that the temperature of the rest of the tube is considerably less and of the same order as the ambient temperature), then the sound can be produced infinitely. Since we see the stabilization of the four frequencies by the 4th second, the above mentioned conclusion can be arrived at. This can be inferred from the fact that in the fourth second, there is very little variation in temperature as when compared to the first three seconds. Occurrence of other frequencies with noticeable amplitudes can be attributed to external disturbances. The 6th chapter mentions methods that could possibly avoid such disturbances. As per prior experimental studies, this data is in agreement with the observation that the second acoustic mode can be excited only in the first and third quarters of the tube. Since no readings have been taken in the upper half of the tube, it is not in the capacity of this work to discuss the 3rd quarter of the tube. Strangely, literature survey hasn't revealed much work on the

generation of higher harmonics. Lack of a chance for practical application could be the reason. The study of stability of the first acoustic mode which was more relevant to the then proposed theories in either rocket engines or pulse combustors may have sidelined research in this direction. The information and remarks in Ref [8] and [31] are supportive of this conclusion.

Figures 11, 12, 13 and 14 are plots of the sound intensity over in the frequency domain. Figure 15, 16, 17 and 18, plot the temperature against time. This recorded temperature remains almost constant through the four seconds which facilitates in the stabilization of the system. Figures 19, 20, 21, 22 are plots of the microphone input voltage against time.

4.1.2. Readings at $x/L=1/8$

Sound amplitude (dB) vs. frequency (Hz) $x/L = 1/8$

4.2. Sound amplitude (dB) vs. frequency (Hz) for $x/L = 3/16$

4.2.1. Procedure

The recording of data is stopped and time is taken to let the apparatus cool down. The metal screen inside supplies so much heat to the system that even minutes after the experiment is stopped, hot convective currents can be felt at the upper end of the tube. The temperature of these currents can easily be above 400°C. When the heat input by the torch is stopped, the sound disappears immediately. This means that though the mesh is at a high temperature, no sound is produced since the state of “localized heat source” does not hold good any longer. This heat is carried out by the convective currents. Hence the temperature of these convective flows can be as high as 400°C.

Care must be taken to keep the microphone in the same position. One of the precautions in the setup is to fix the microphone in such a way that it is sturdy and stable enough to not come under the influence of any other physical disturbances.

L/8
Spectrum data 1

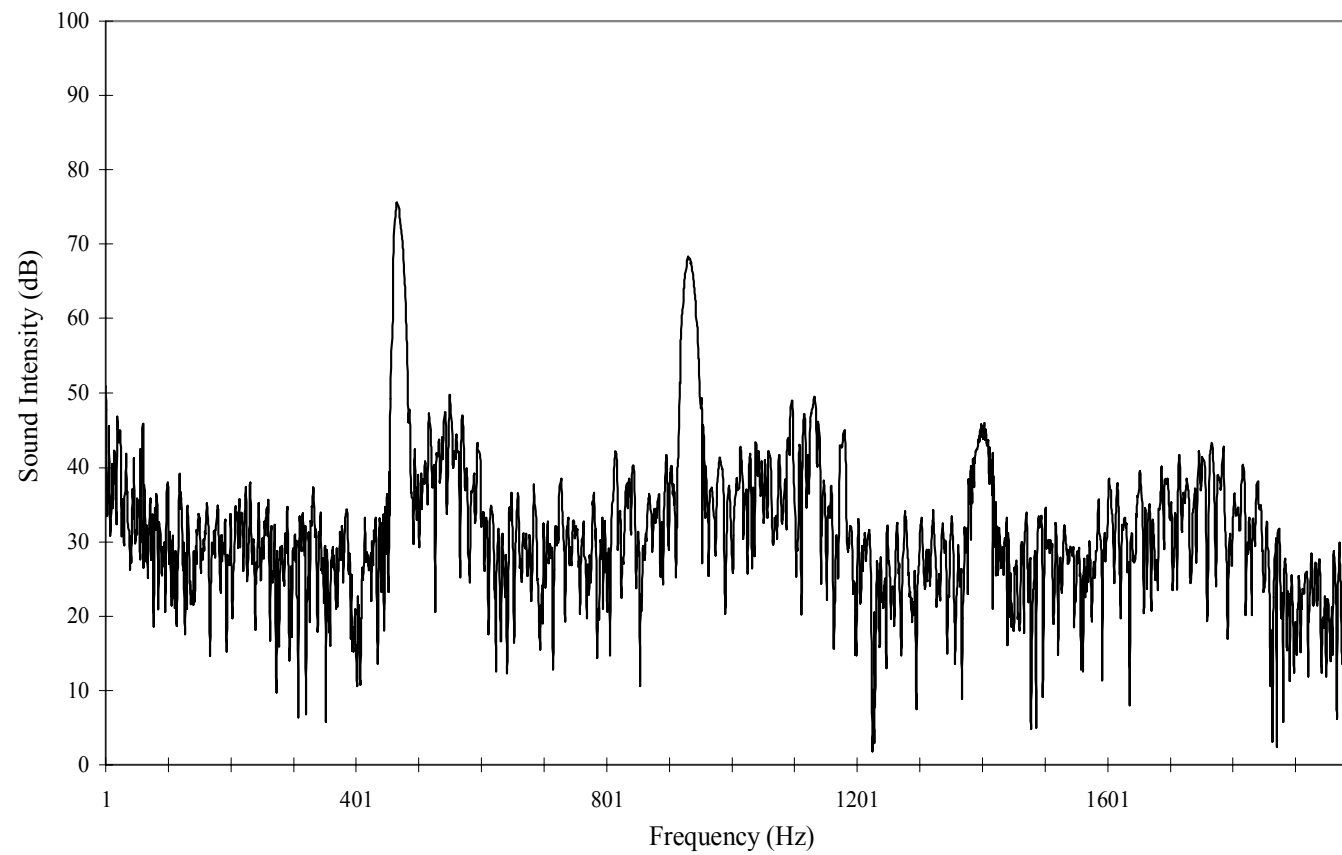


Figure 11: Spectrum data for 1st second; metal screen position L/8

L/8
Spectrum data 2

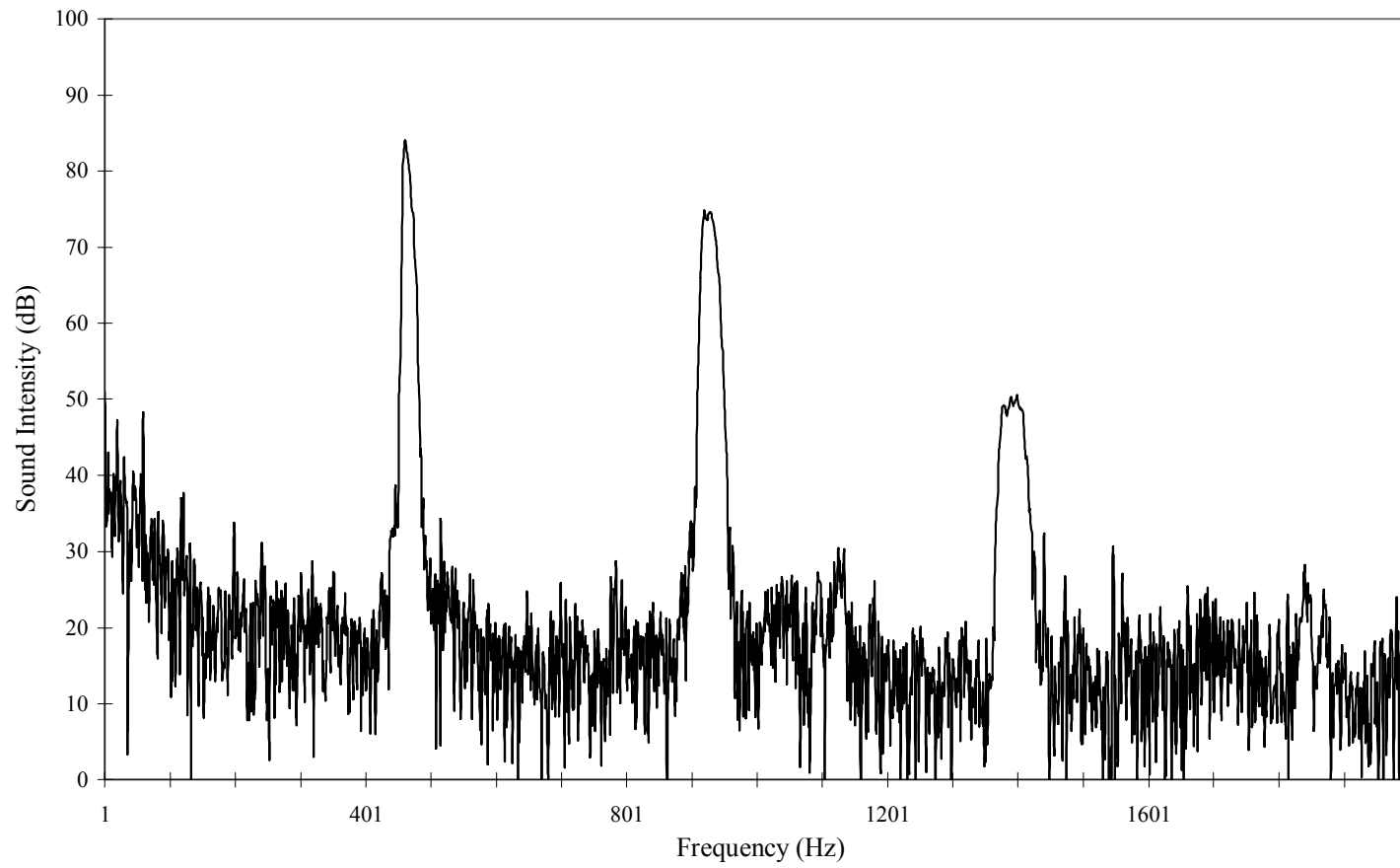


Figure 12: Spectrum data for 2nd second; metal screen position L/8

L/8
Spectrum data 3

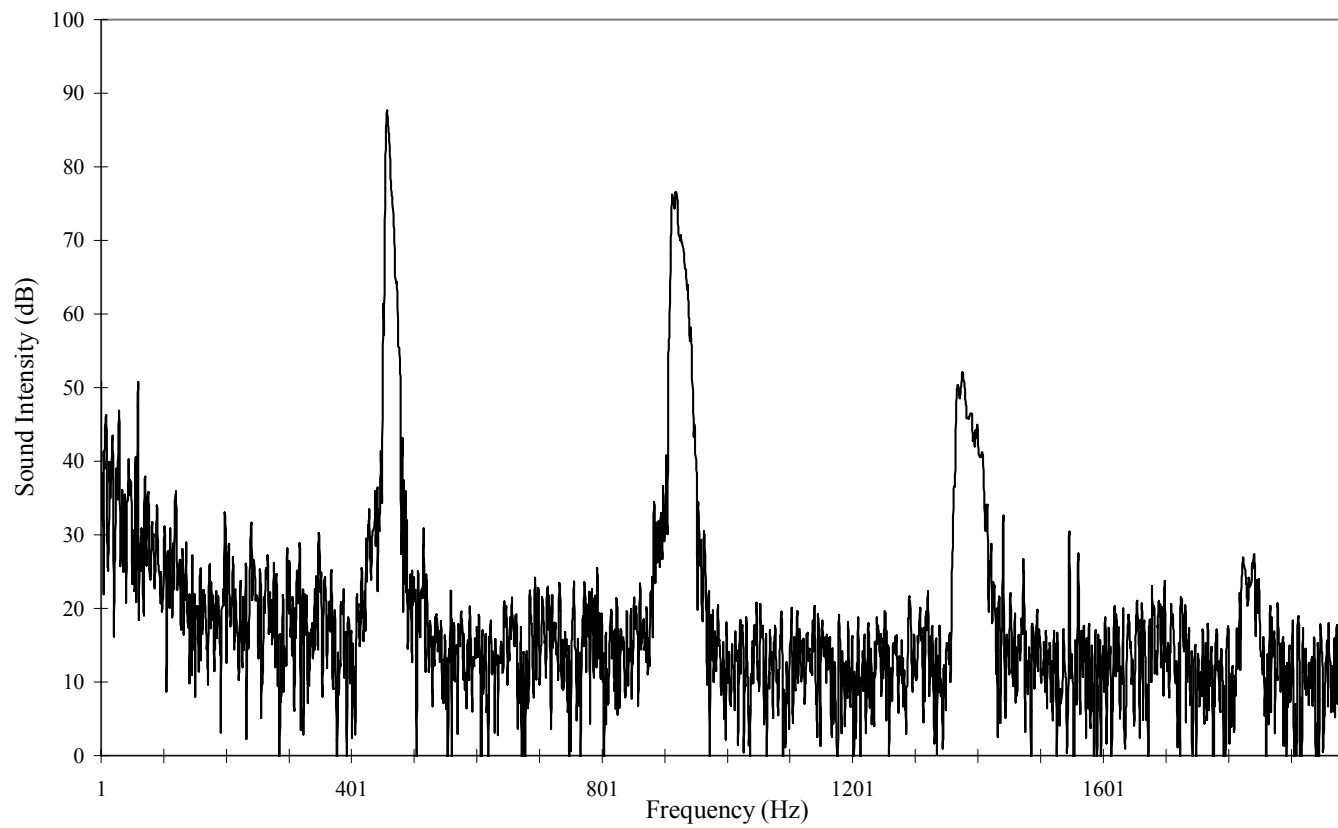


Figure 13: Spectrum data for 3rd second; metal screen position L/8

L/8
Spectrum data 4

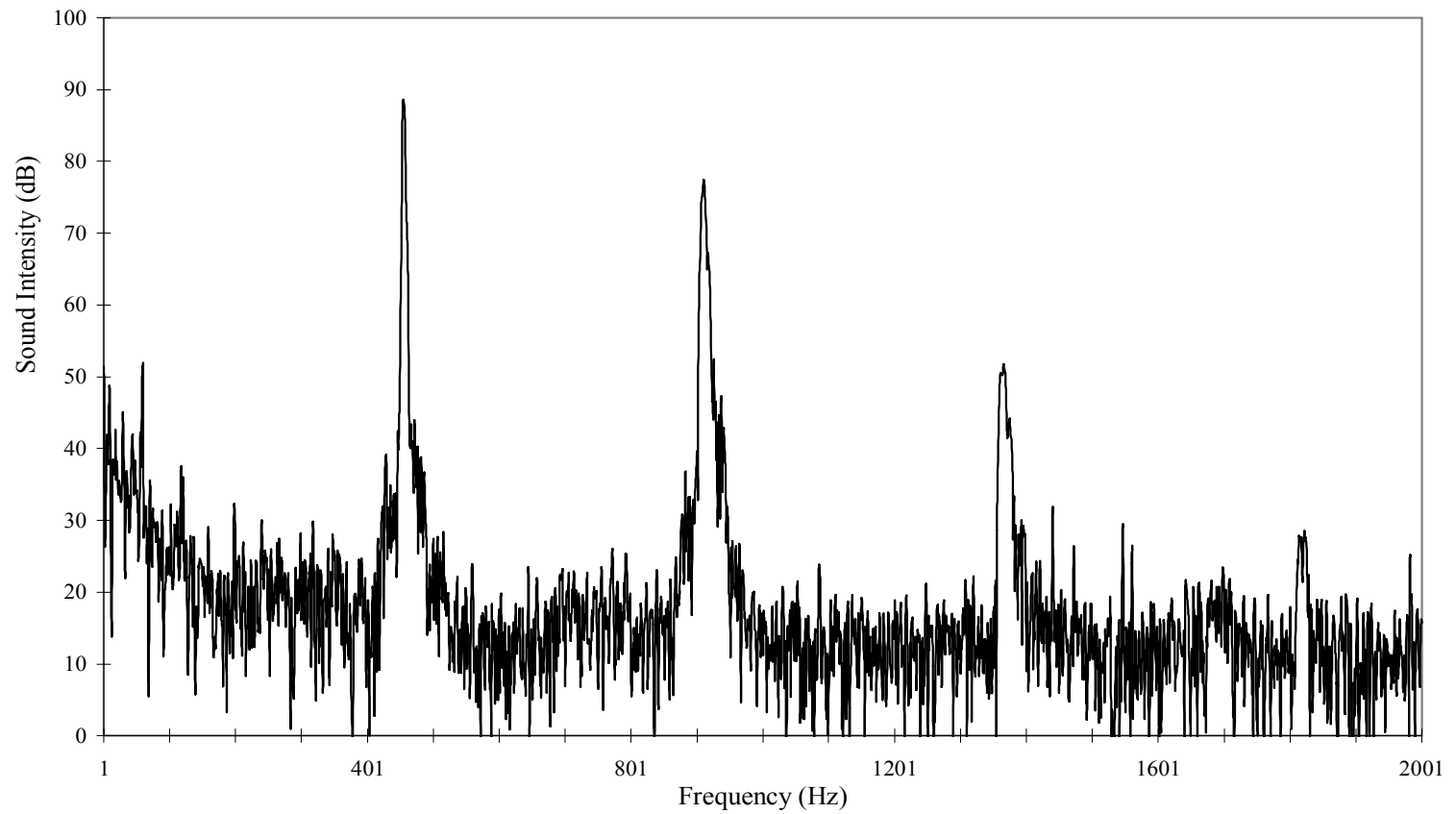


Figure 14: Spectrum data for 4th second; metal screen position L/8

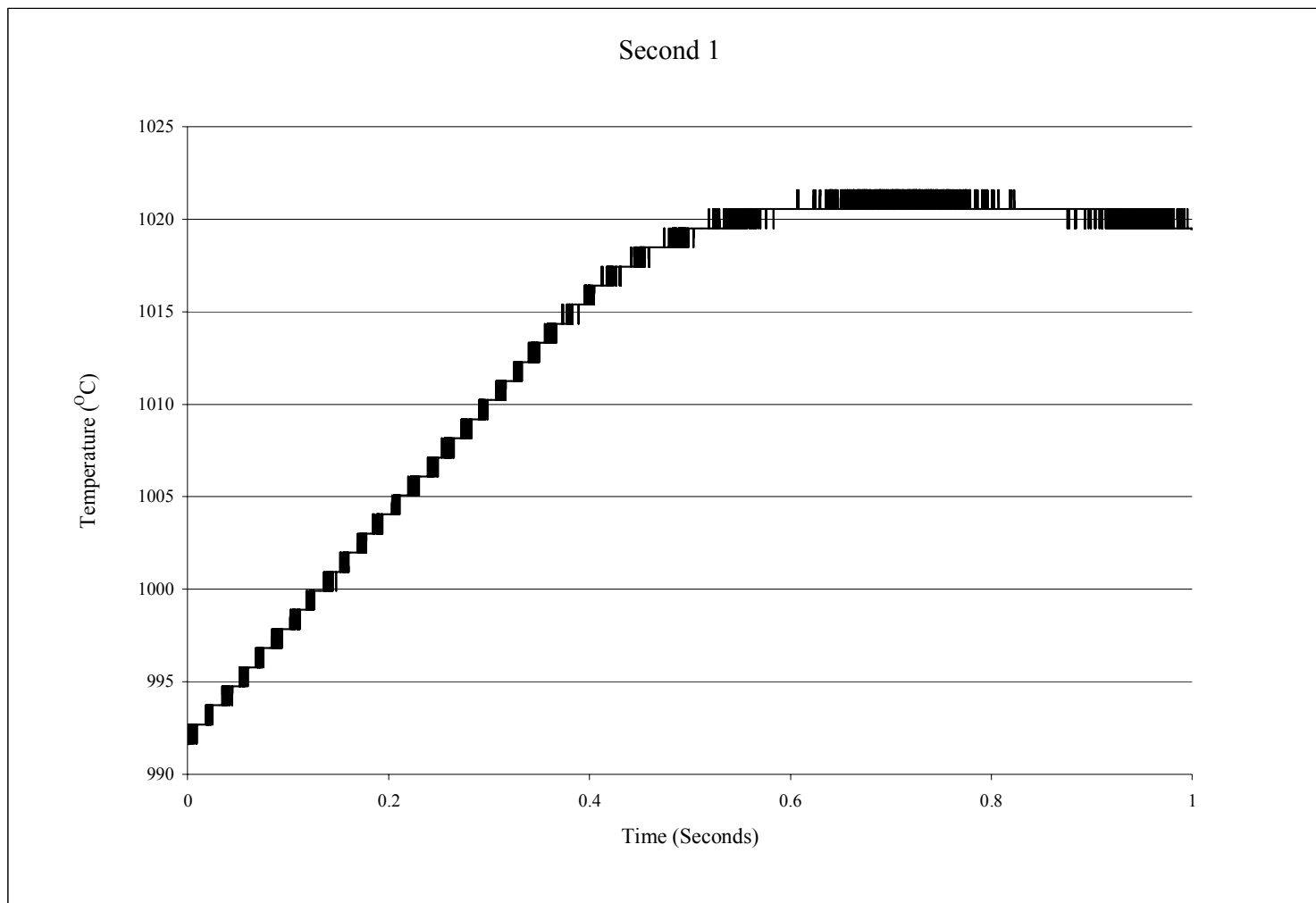


Figure 15: Temperature graph for second 1; metal screen position L/8

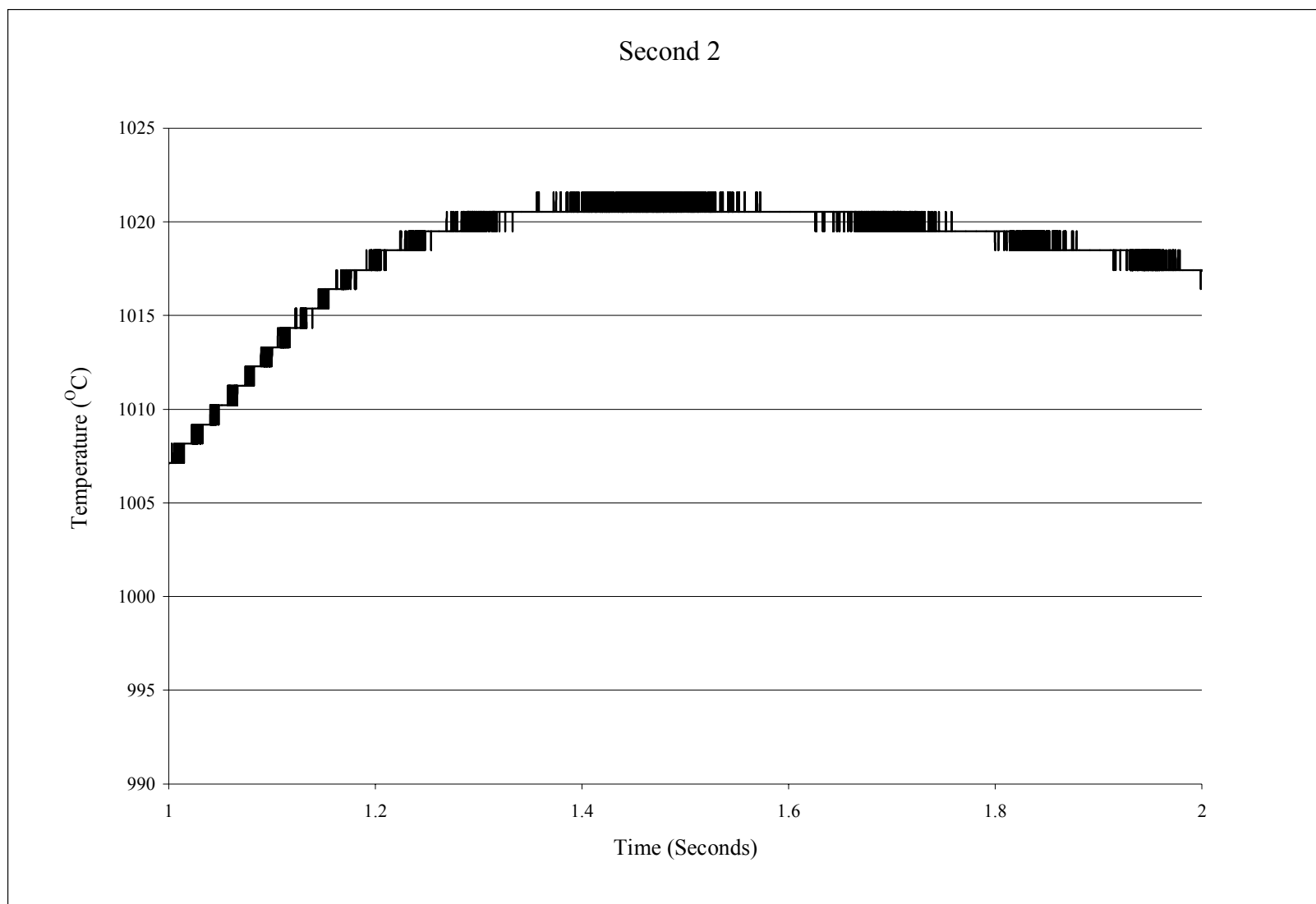


Figure 16: Temperature graph for second 2; metal screen position L/8

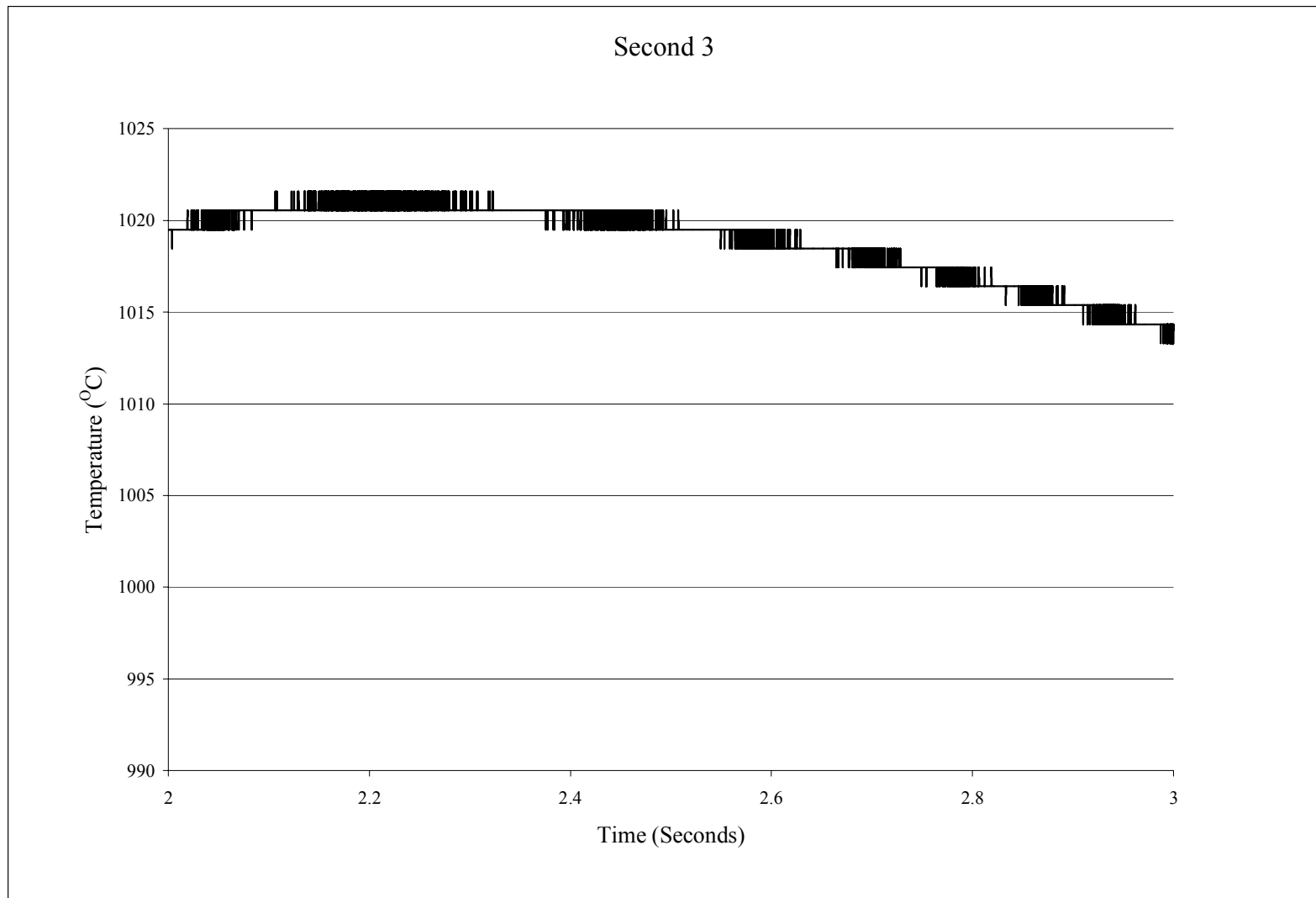


Figure 17: Temperature graph for second 3; metal screen position L/8

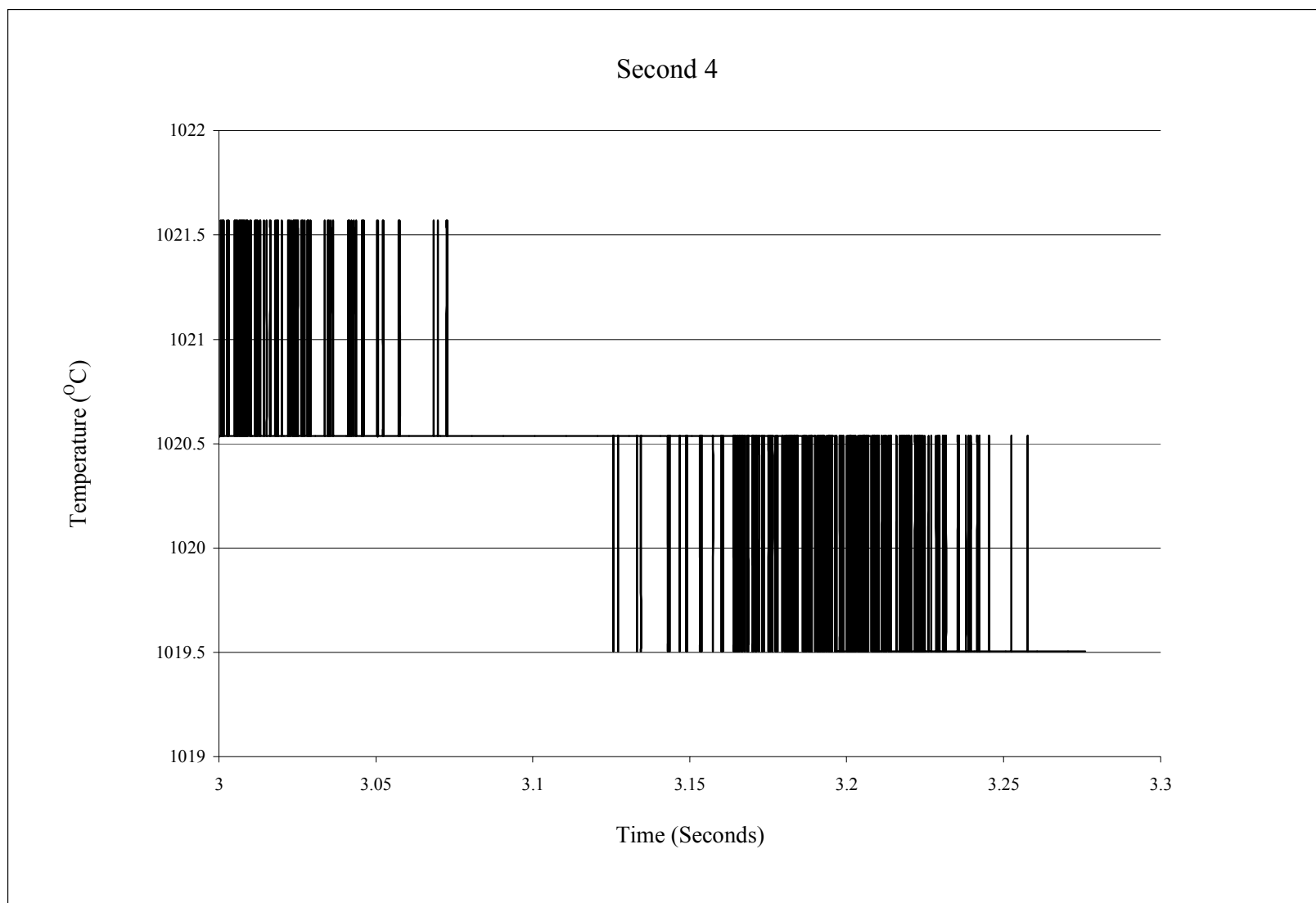


Figure 18: Temperature graph for second 4; metal screen position L/8

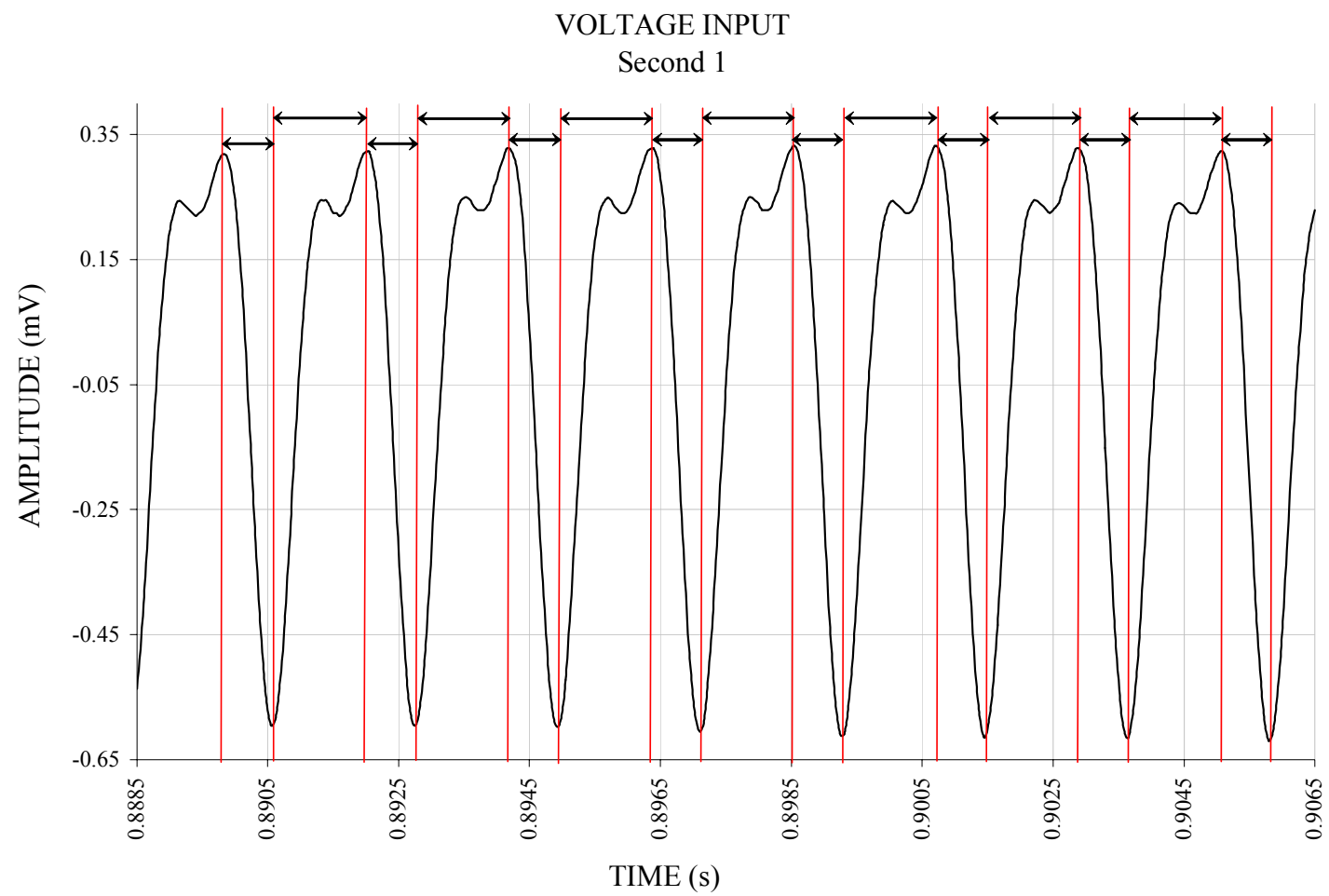


Figure 19: Sample data during second 1 for L/8 position

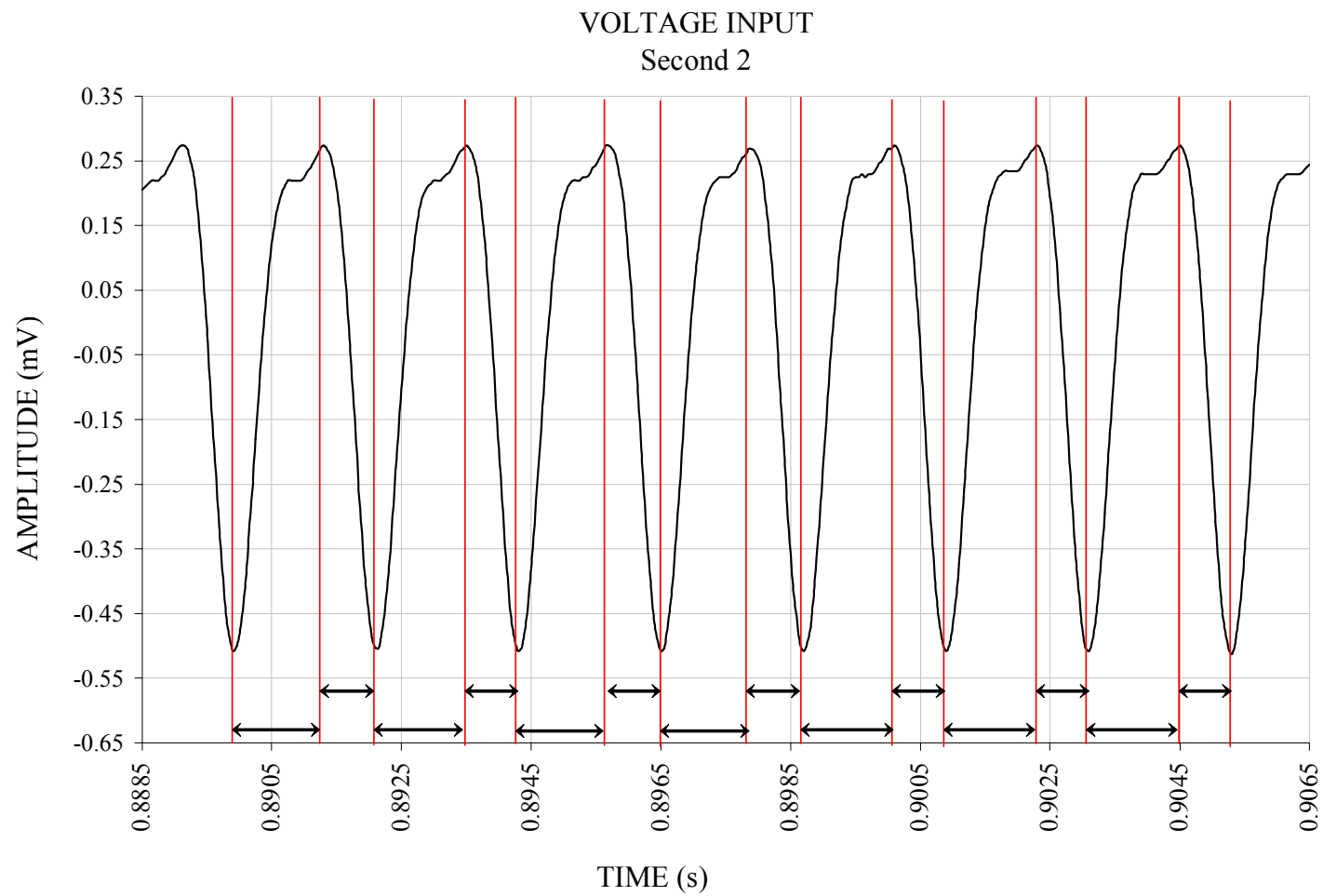


Figure 20: Sample data during second 2 for L/8 position

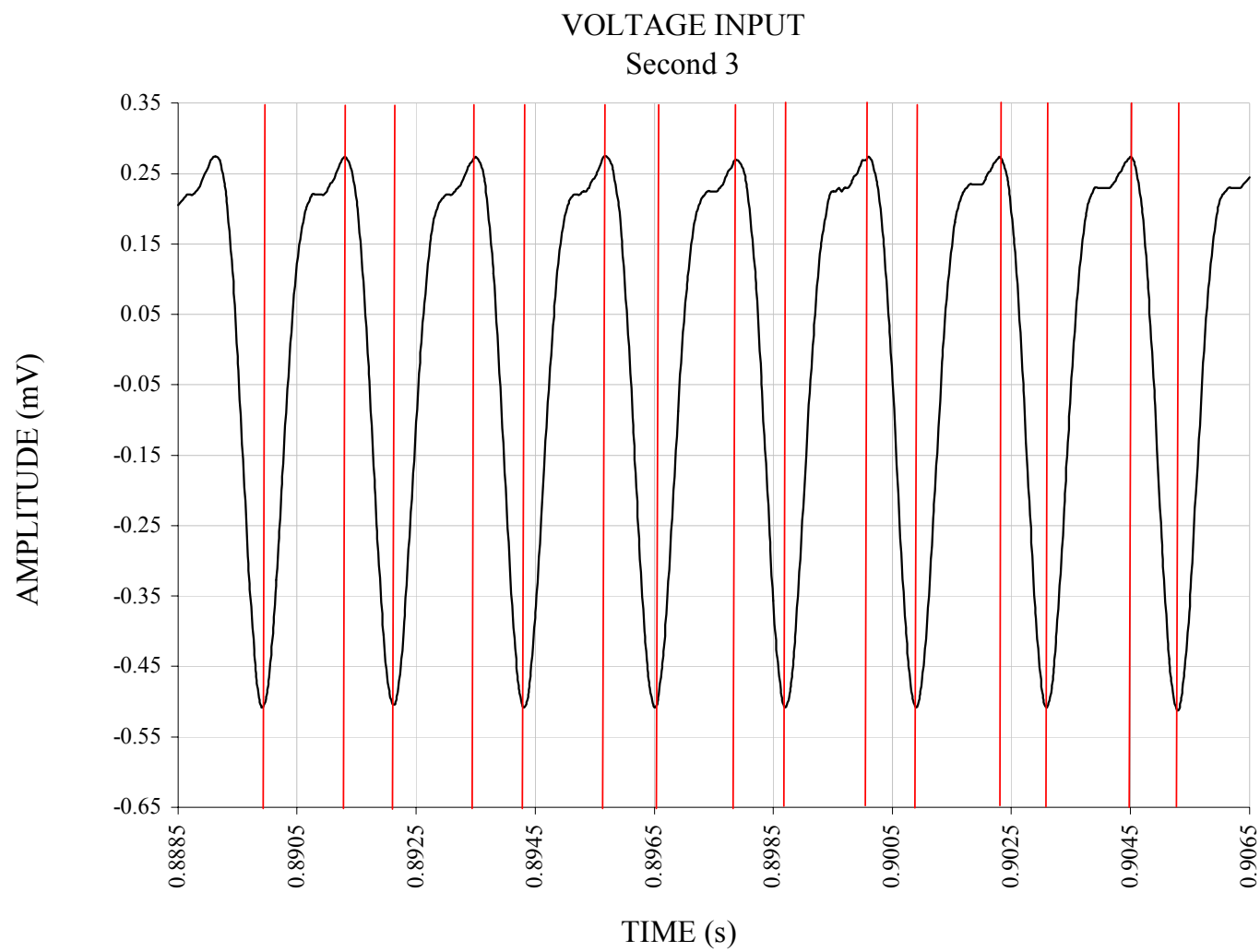


Figure 21: Sample data during second 3 for L/8 position

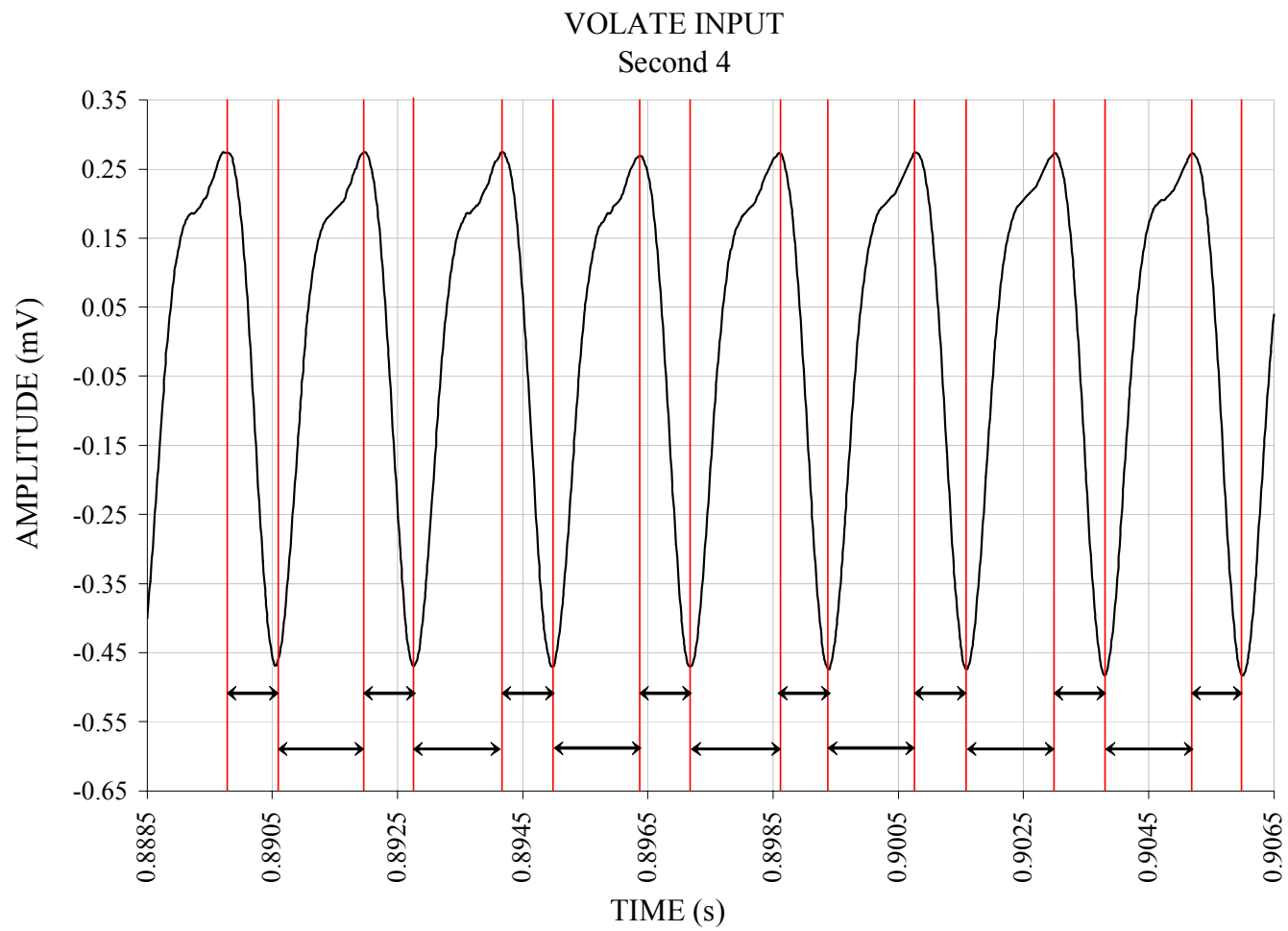


Figure 22: Sample data during second 4 for L/8 position

Once the system has cooled down, the metal screen is moved to the 3L/16 location with the rod that has been constructed for this purpose. Since the rod has gradations marked on it, it is easy to measure the height of the metal screen location inside the tube. Once the metal screen is put in place, the LabVIEW VI program is run.

The temperature of the metal screen can be monitored continuously and is simultaneously displayed graphically on the computer screen. The propane gas torch is switched on and the metal screen is heated till the graph on the screen reaches 1000°C. The torch is then removed. This is when we begin to hear the high intensity sound. At the same moment, the spectrum analyzer is switched on. The voltage input from the microphone can be seen on the screen simultaneously as the information is transferred to the computer. As mentioned previously, four seconds of data is collected and then the spectrum analyzer is turned off. The following are the plots of sound intensity vs. frequency for each of the four seconds.

4.2.2. Observations from data for metal screen position 3L/16

The 1st second in this set of graphs has been recorded earlier than the recordings in other positions. This is the plot for the second even as the propane torch was being removed. There is an evident build up of sound intensities for particular frequencies but nothing can be inferred for sure.

Figure 23 shows a likelihood of the presence of dominating frequencies. The fact that almost all other frequencies are in the same range, i.e., sound intensities of most frequencies lie in a 20 dB range, reduces the usefulness of this plot. But as we proceed to Figure 24, we see the dominant peaks of the acoustic frequencies.

The lower quarter of the Rijke tube is the most favorable for thermoacoustic conversions. The first acoustic mode dominates over the other three modes which also have distinguished intensity peaks. Figure 25 (3rd second) and Figure 26 (4th second) contain data that is of good value as single frequencies in all the three harmonics begin to dominate.

3L/16
Spectrum data 1

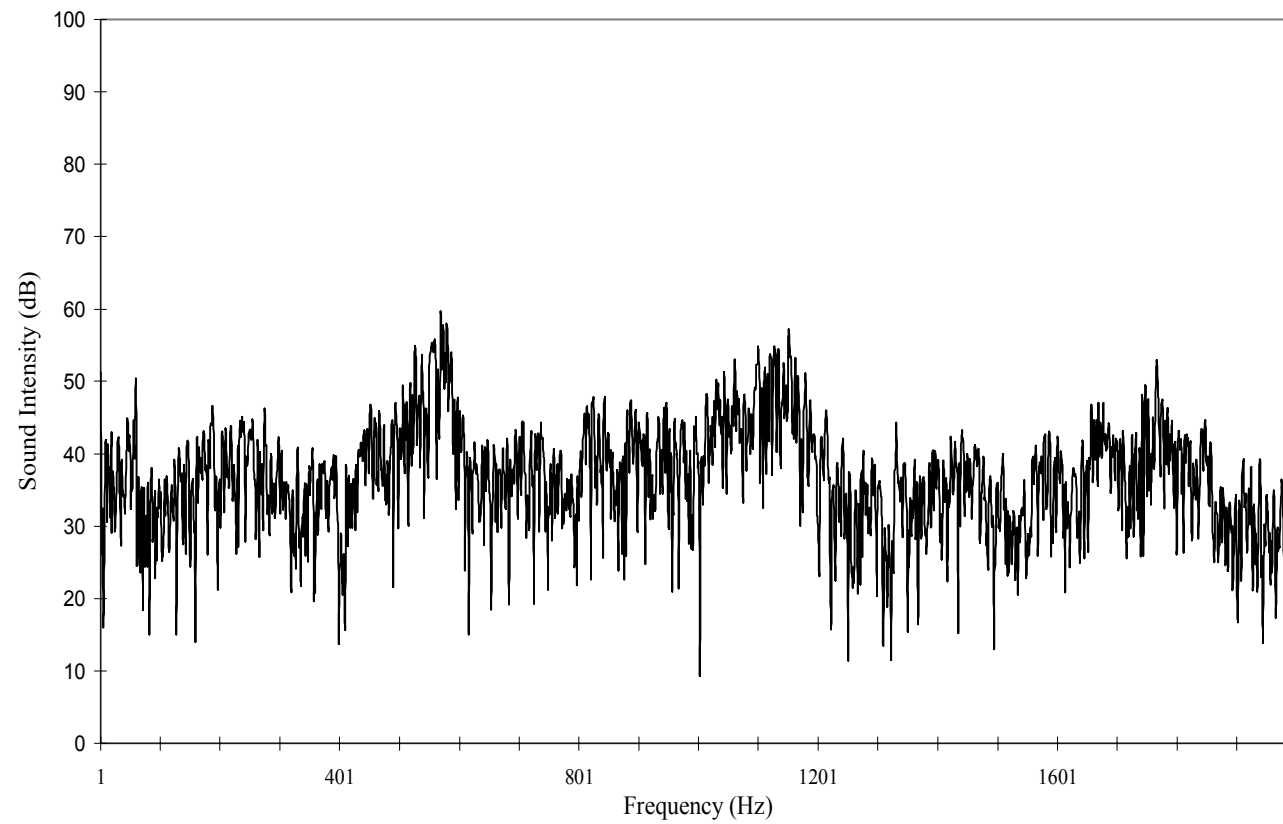


Figure 23: Spectrum data for 1st second; metal screen position 3L/16

3L/16
Spectrum data 2

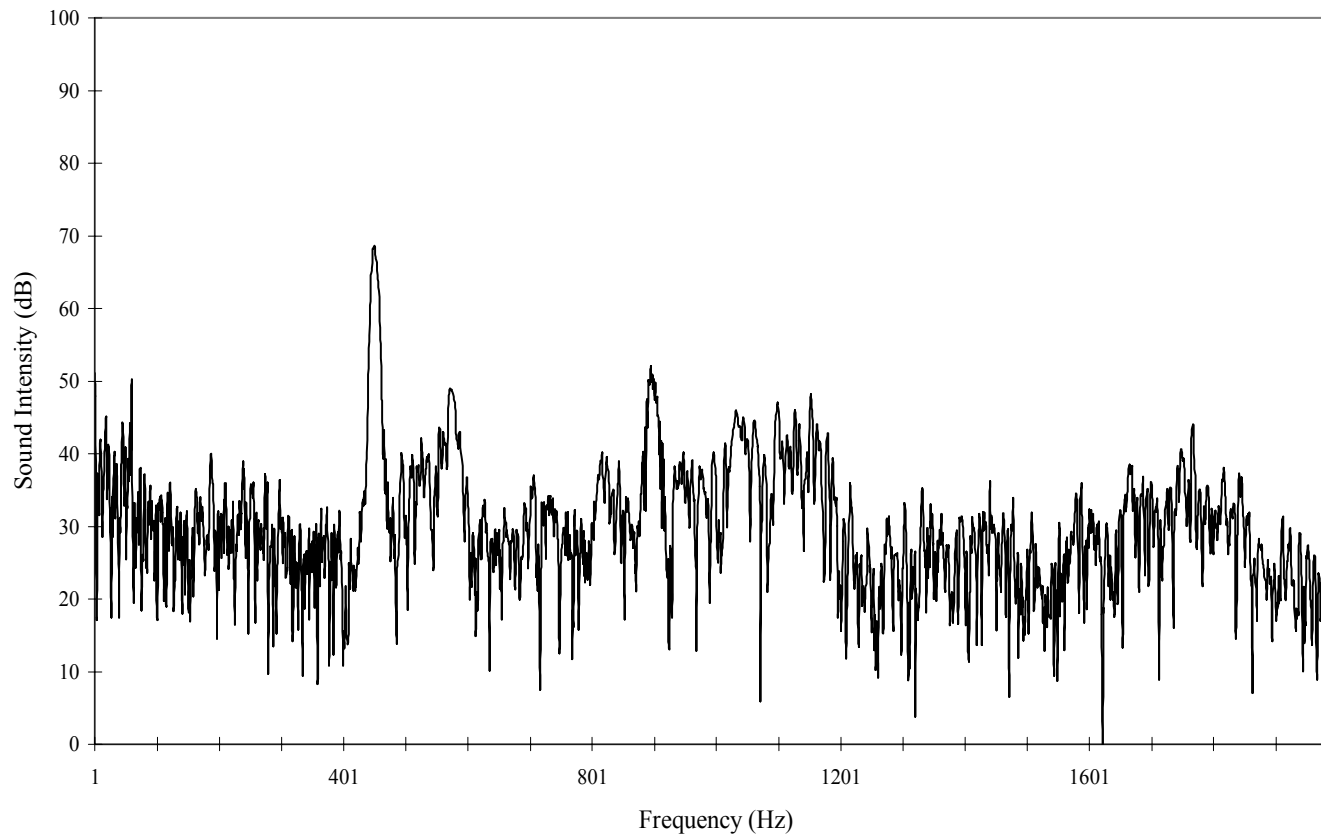


Figure 24: Spectrum data for 2nd second; metal screen position 3L/16

3L/16
Spectrum data 3

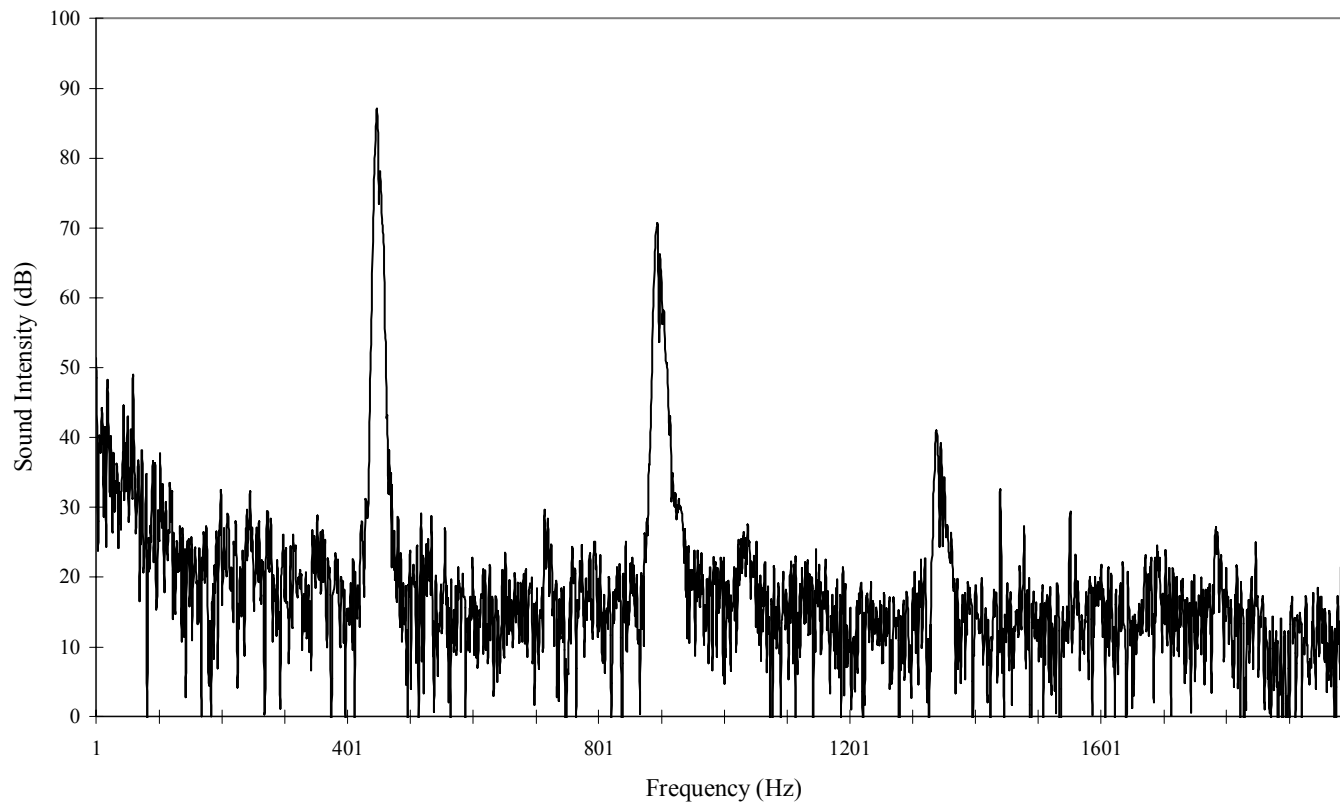


Figure 25: Spectrum data for 3rd second; metal screen position 3L/16

3L/16
Spectrum data 4

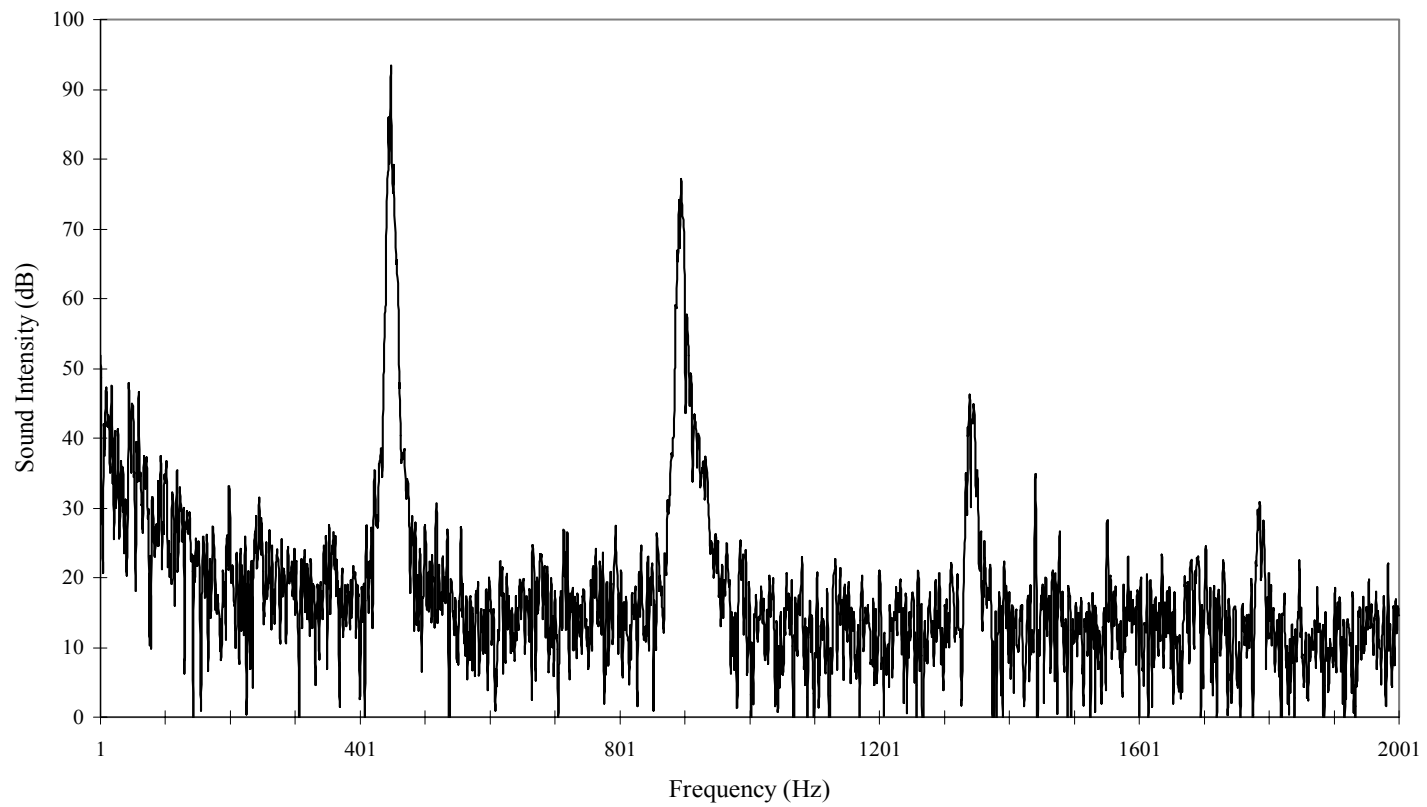


Figure 26: Spectrum data for 4th second; metal screen position 3L/16

These peaks get validation from the calculated values of the frequencies of oscillations. The fourth acoustic mode can be seen only in the 4th second but the behavior over the four seconds is very supportive of its existence if an extrapolation is assumed that this harmonic will also has significant sound intensity. The experiment, as has been mentioned all through, has been conducted only for four seconds. An earnest attempt has been made that all conclusions arrived at are by taking into consideration, this fact.

This is a very unconventional position for the metal screen as it does not exhibit any significant features. Early experiments that were conducted to determine the location of metal screen for highest first acoustic mode intensity might have positioned the metal screen here which of course was just a part of the trial and error method. Normally, experiments are conducted at $L/4$ or $5L/8$ positions as these are the experimentally proven locations that excite the first and second acoustic modes respectively. As mentioned before, interest has been restricted to the first acoustic mode and at the most the second mode, which happens to occur in the upper half of the tube, contrary to Rijke's observations who stated that no sound could be produced in the upper half of the tube.

The need for experimentation in the current work is to record data that exhibits the nonlinearity of the longitudinal waves. Hence, such data points that capture the presence of higher harmonics have been retained. In this position also, the fact that second acoustic mode is encouraged in the first quarter of the tube is demonstrated. Figures 27, 28, 29 and 30 plot the temperature vs. time graph and show that the temperature was almost constant through the four seconds of data recording. The tolerance range is 30°C.

Sample of data obtained in each second from the microphone as voltage input in milliVolts has been plotted in figures 31, 32, 33 and 34.

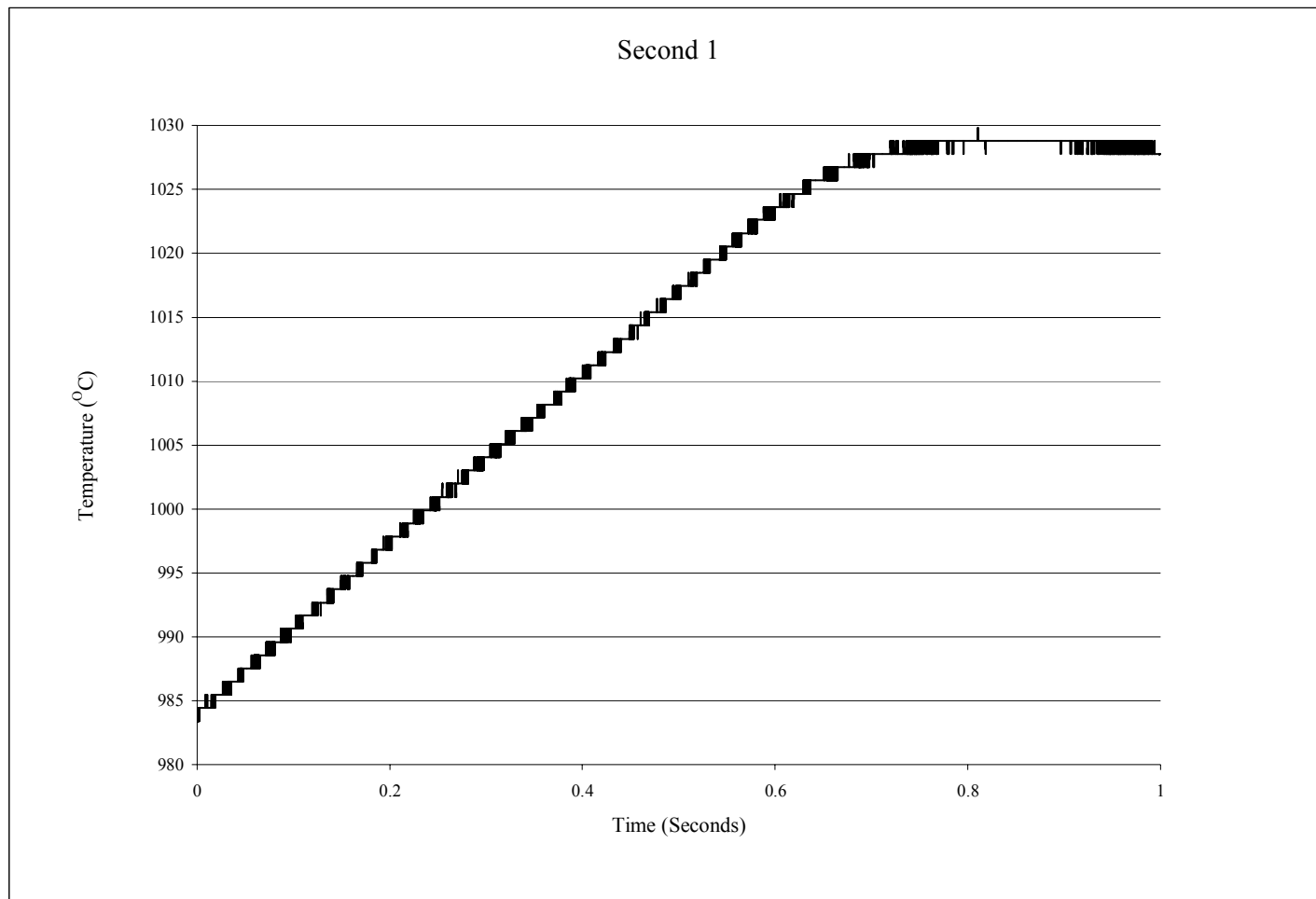


Figure 27: Temperature graph for second 1; metal screen position 3L/16

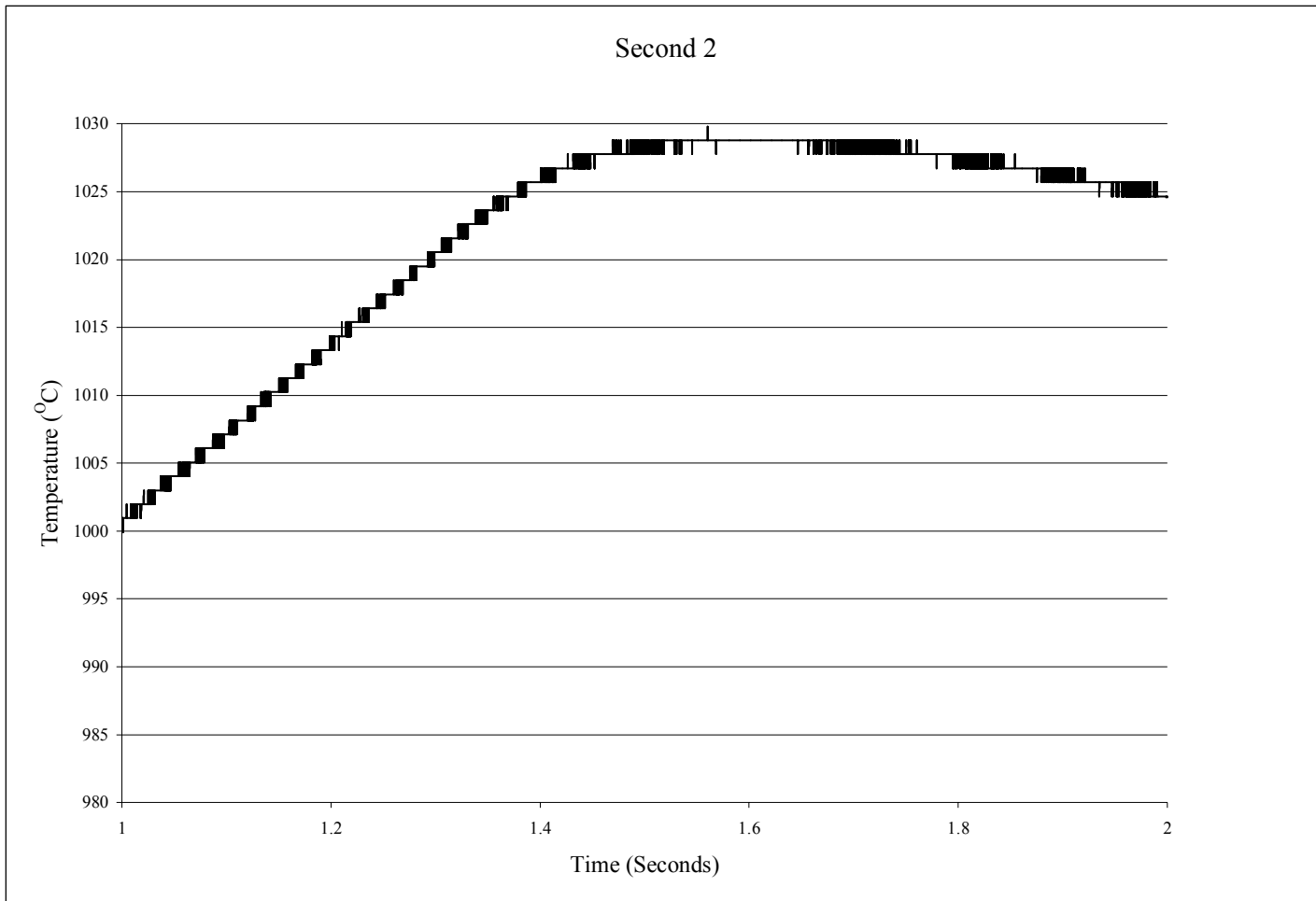


Figure 28: Temperature graph for second 2; metal screen position 3L/16

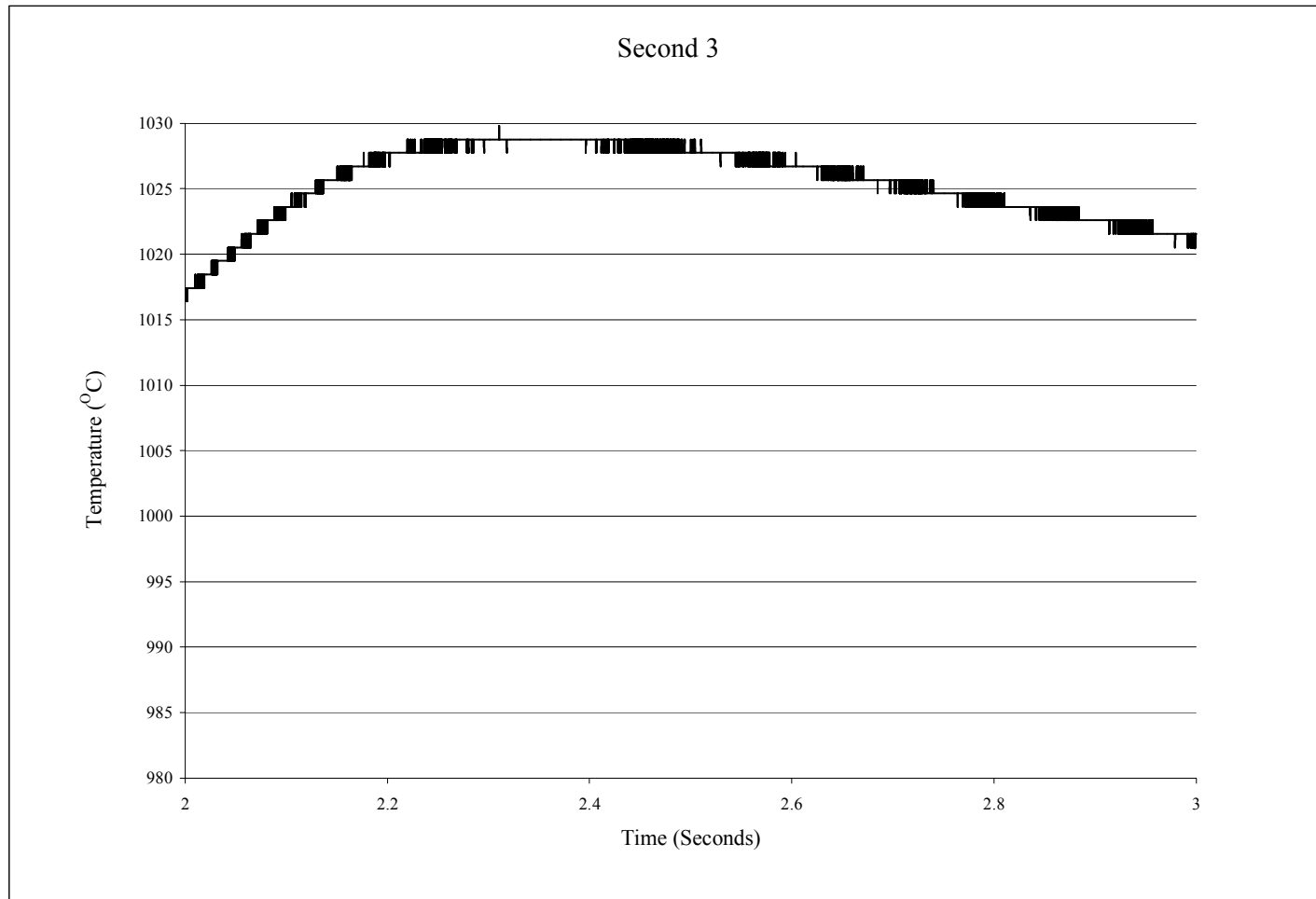


Figure 29: Temperature graph for second 3; metal screen position 3L/16

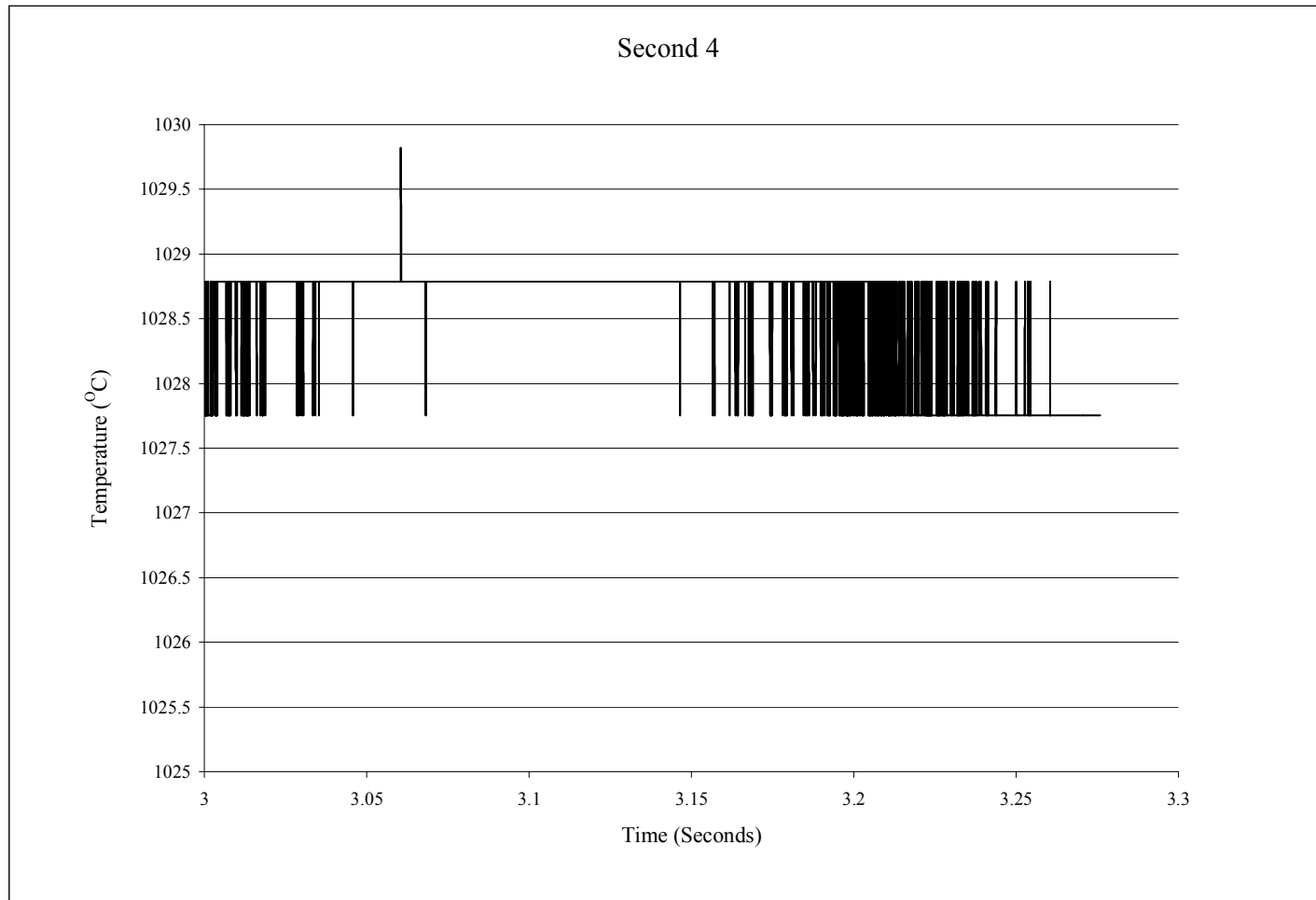


Figure 30: Temperature graph for second 4; metal screen position 3L/16

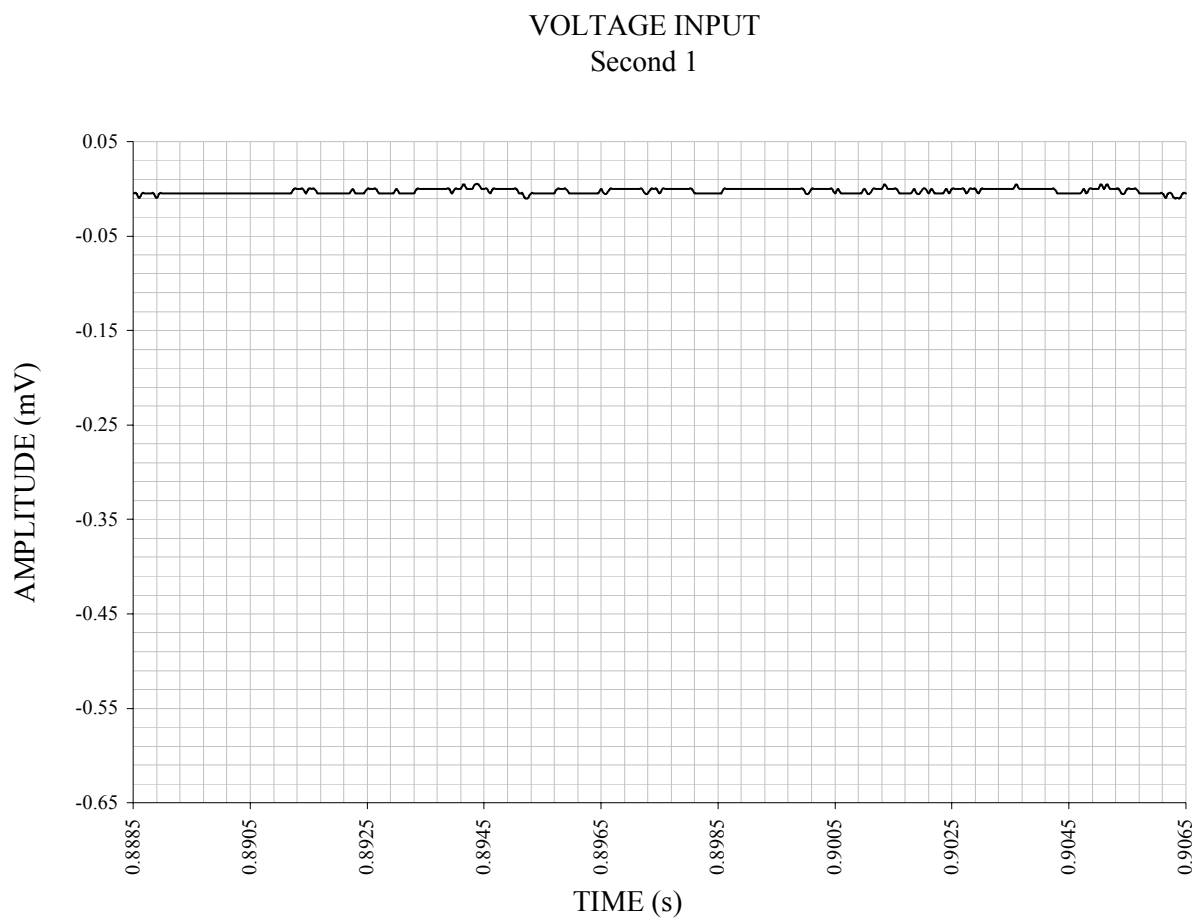


Figure 31: Sample data during second 1 for 3L/16 position

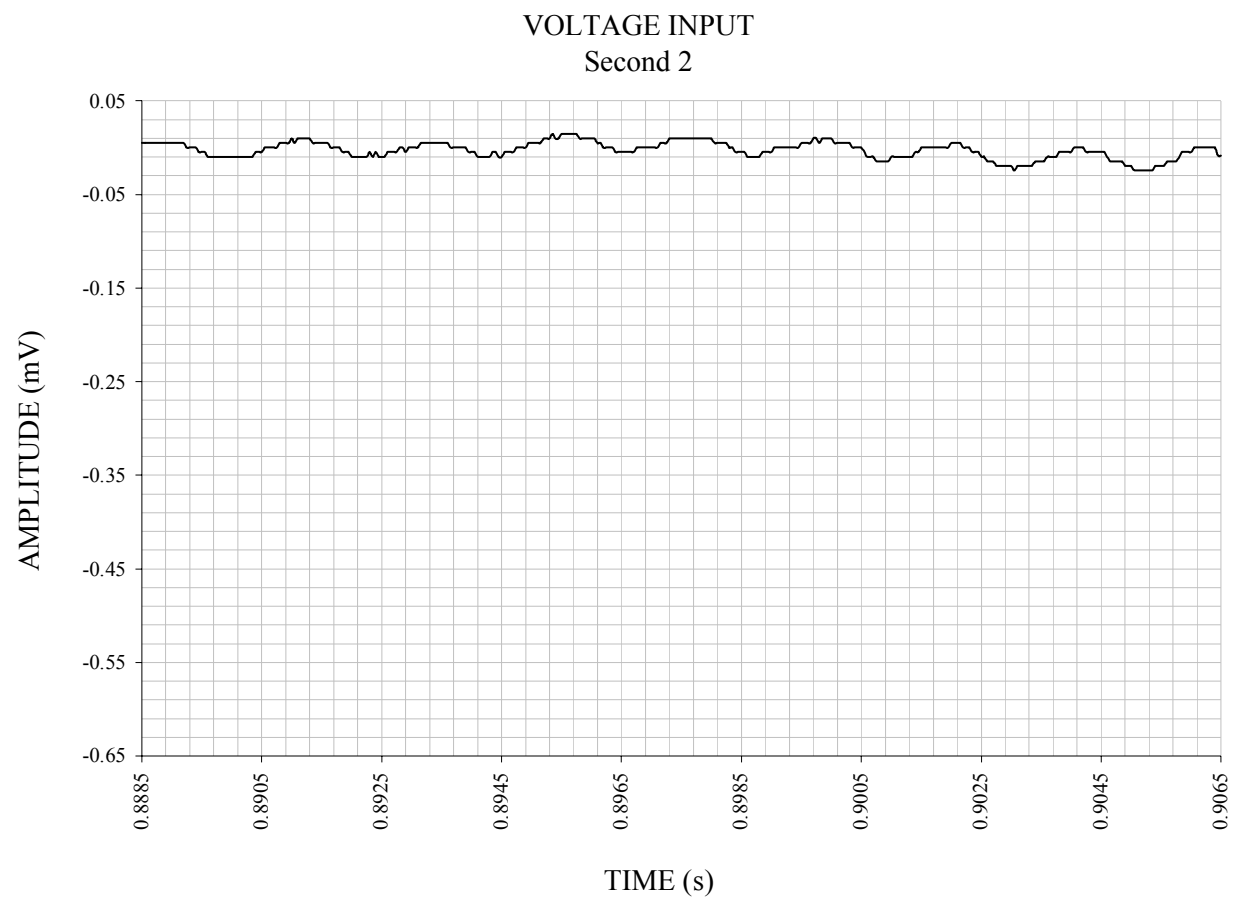


Figure 32: Sample data during second for 3L/16 position

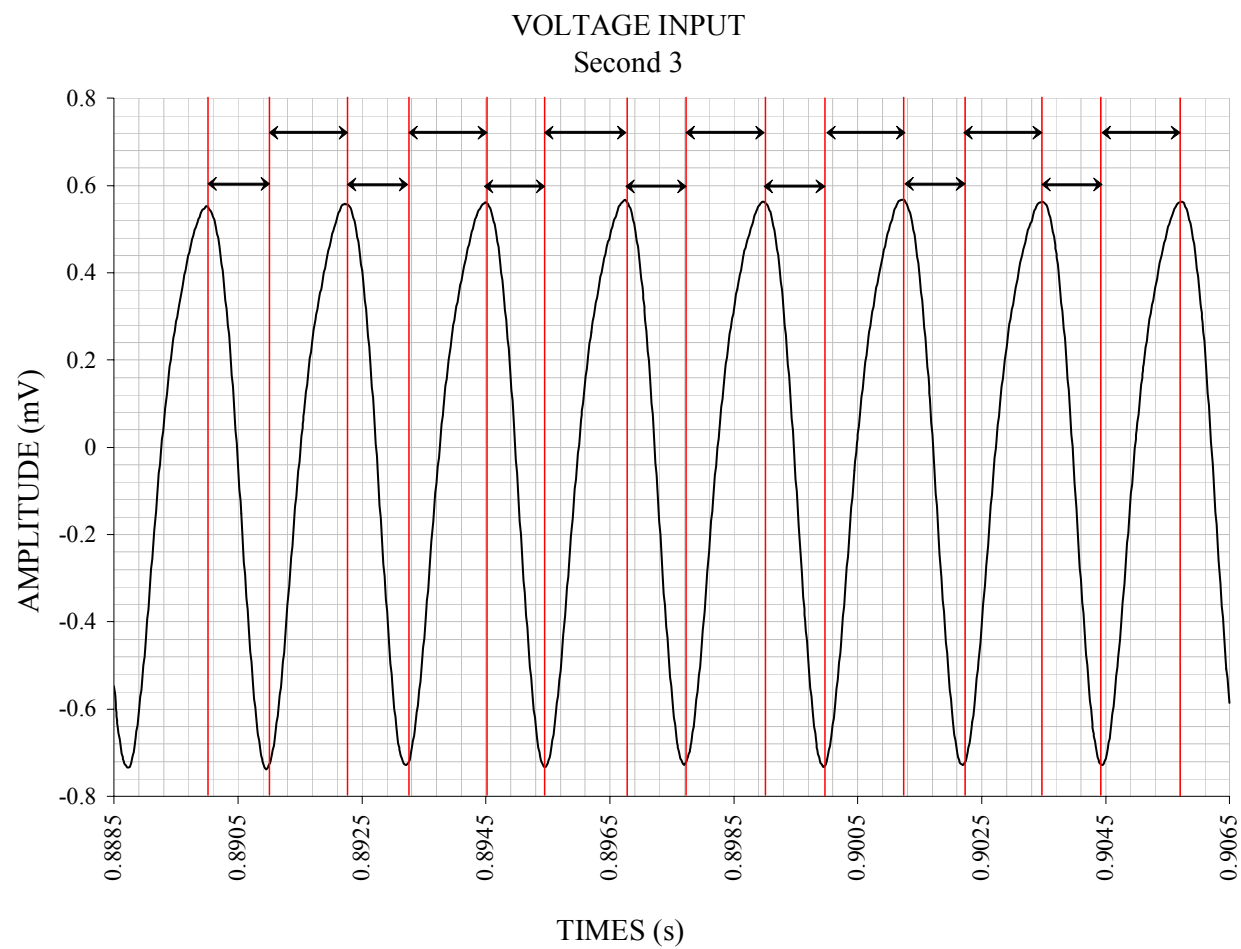


Figure 33: Sample data during second 3 for 3L/16 position

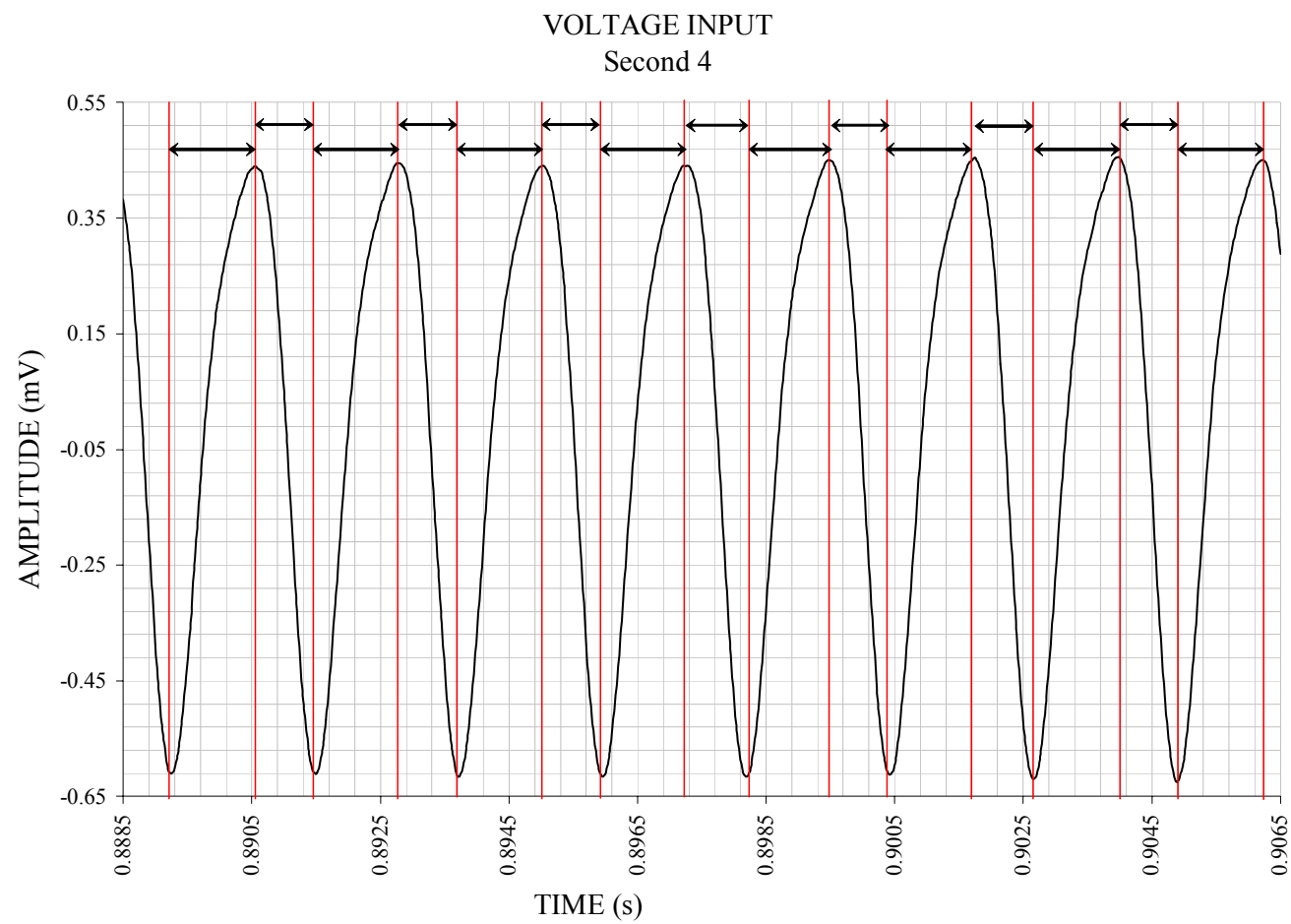


Figure 34: Sample data during second 4 for 3L/16 position

4.3. Sound amplitude (dB) vs. frequency (Hz) $x/L = 1/4$

4.3.1. Procedure

As mentioned for the previous set of readings, the VI is paused. Enough time is given for the tube to cool down completely and now the metal screen is moved to the $L/4$ position. This is the last of the positions for which readings will be taken. This is the maximum depth the thermocouple can reach due to its length. This thermocouple is the largest of the thermocouples that are sold. Thermocouples with specific requirements have to be custom made. Heat the metal screen with the torch becomes inconvenient beyond this position as the nozzle of the torch too has its length limitations.

Four seconds of data is recorded as before. The data are stored in files that can be opened with several Microsoft tools and other plotting software such as Tecplot.

4.3.2. Observations from data for metal screen position $L/4$

Invariably, in all experimental work prior to this, it has been recorded that the fundamental mode is sounded with highest intensity when the metal screen is at this position. The data in this experiment also demonstrates the same behavior. The first fundamental mode is the one with considerable sound intensity as when compared to the sound intensities of other modes. This can be observed in Figures 35, 36, 37, and 38. For example, occurrence of the second acoustic mode is not as strong as it was in the other two positions.

The temperature in this case too has been constant. Figures 39, 40, 41 and 42 are plots of temperatures for each of the four seconds. The variation of temperature was limited to a range of 10°C .

When compared to the data from the other two locations of the screen, the intensity of the first harmonic when compared to the fundamental mode is not that high. Well-focused study in this direction is required in order to be able to explain the decrease of energy input into higher modes. Sample of data obtained in each second from the microphone as voltage input in millivolts has been plotted in figures 43, 44, 45 and 46.

L/4
spectrum data 1

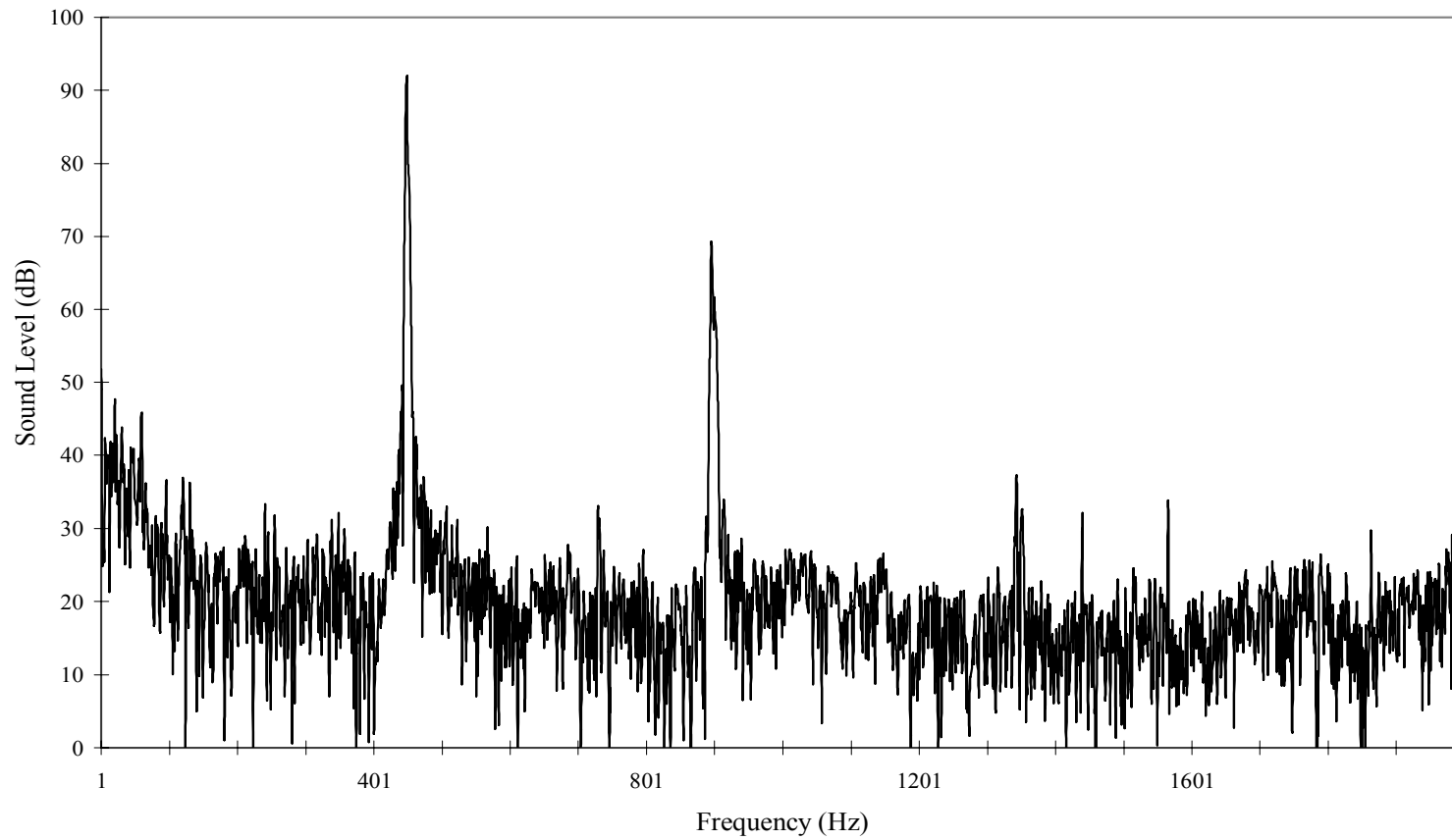


Figure 35: Spectrum data for 1st second; metal screen position L/4

L/4
Spectrum data 2

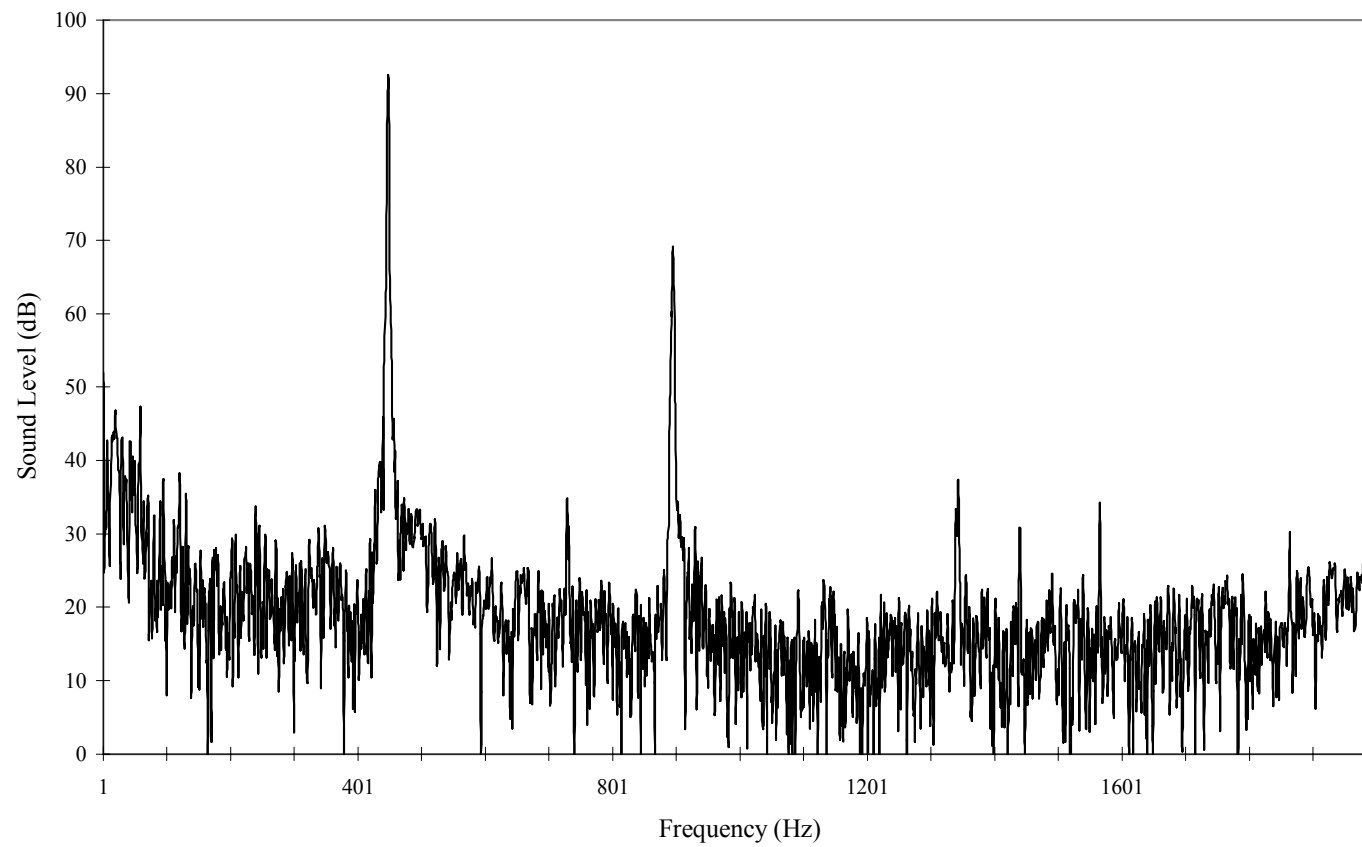


Figure 36: Spectrum data for 2nd second; metal screen position L/4

L/4
Spectrum data 3

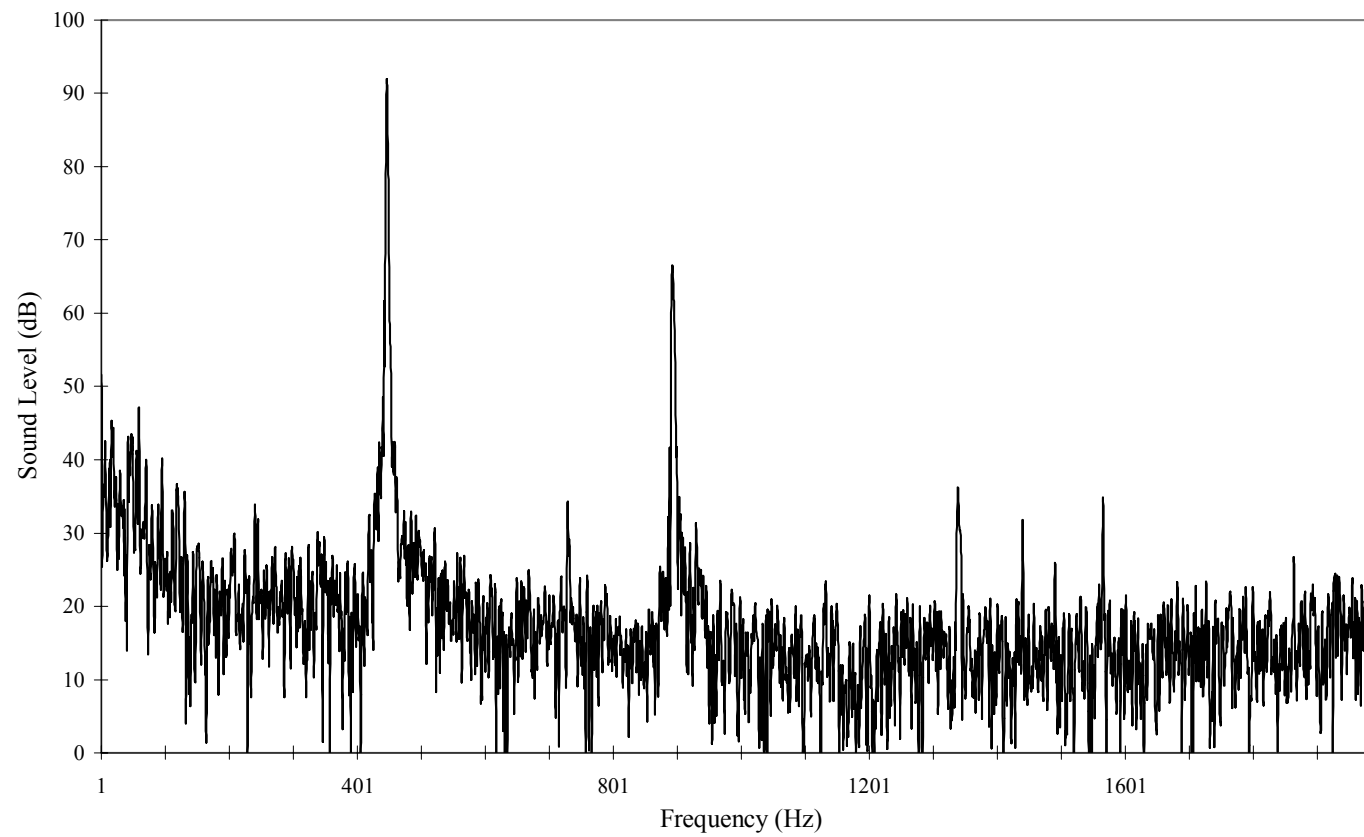


Figure 37: Spectrum data for 3rd second; metal screen position L/4

L/4
Spectrum data 4

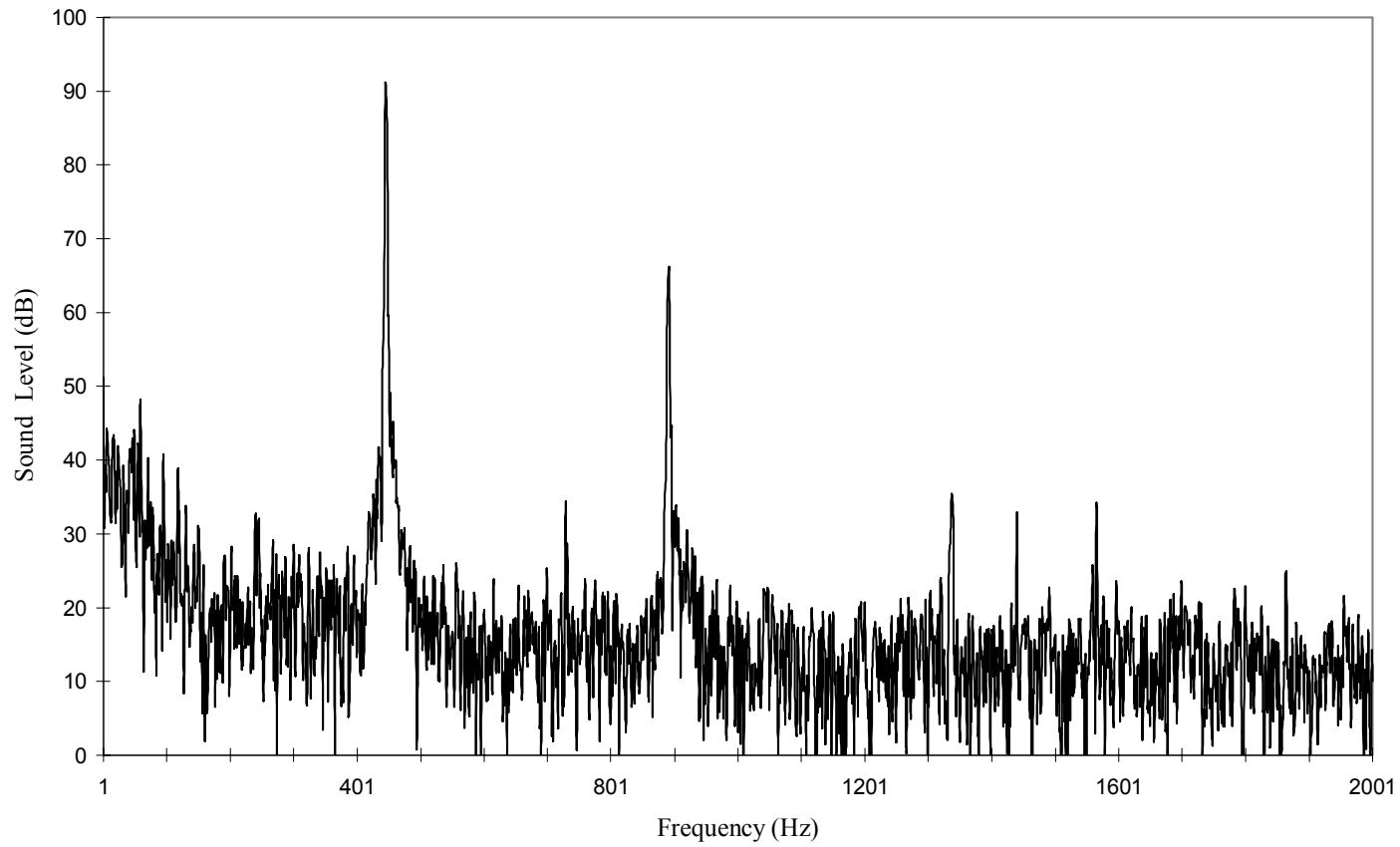


Figure 38: Spectrum data for 4th second; metal screen position L/4

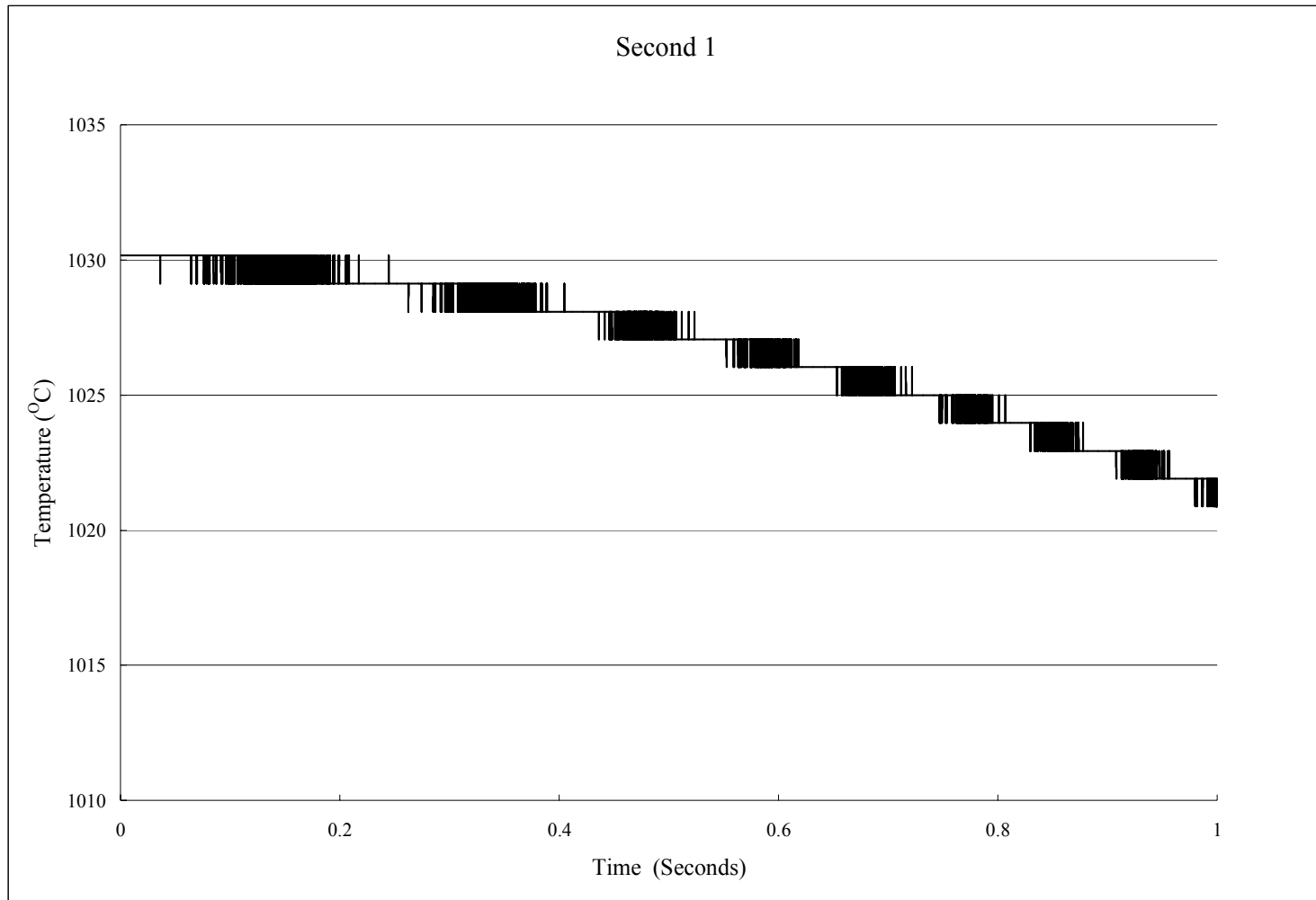


Figure 39: Temperature graph for second 1; metal screen position $L/4$

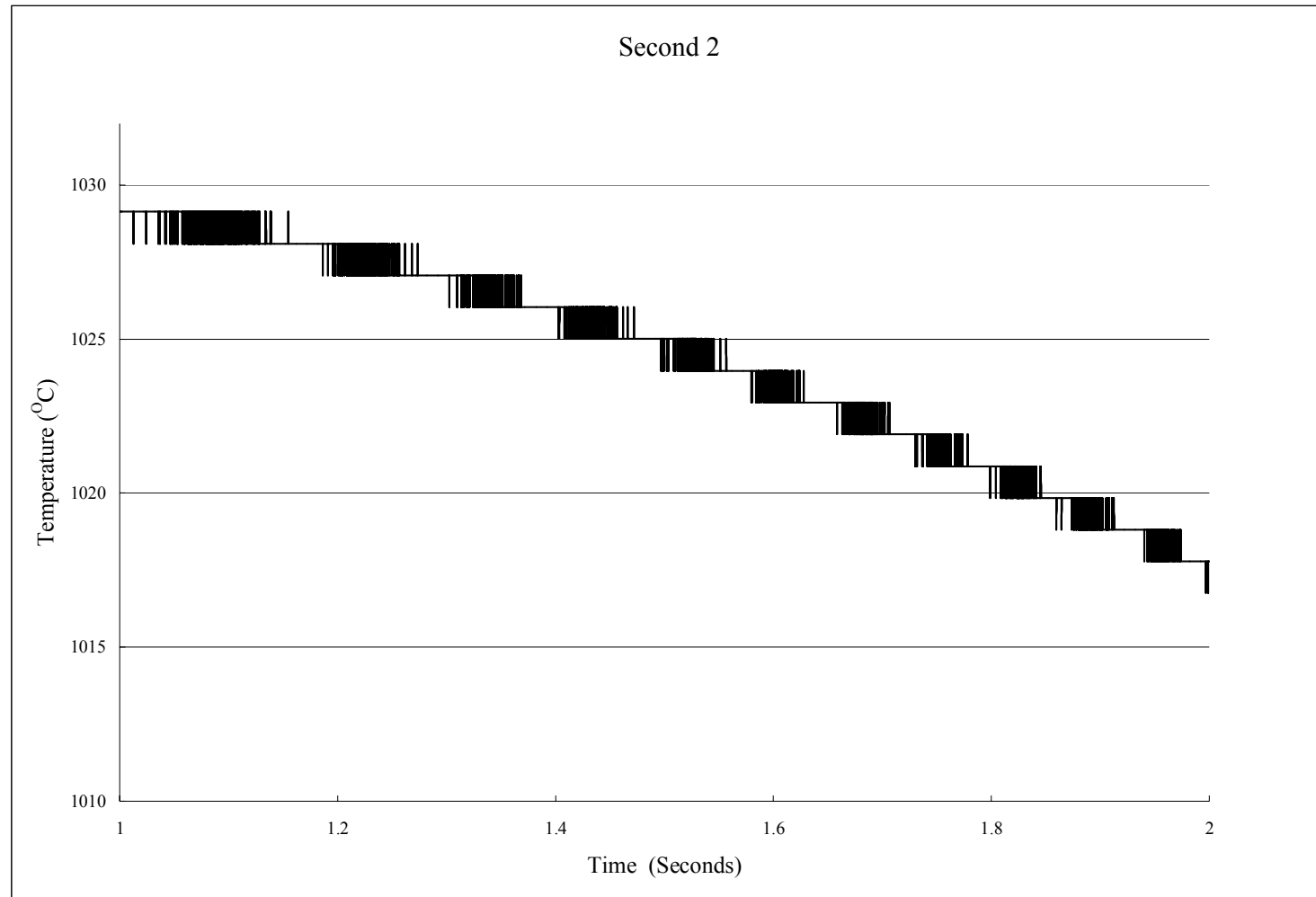


Figure 40: Temperature graph for second 2; metal screen position L/4

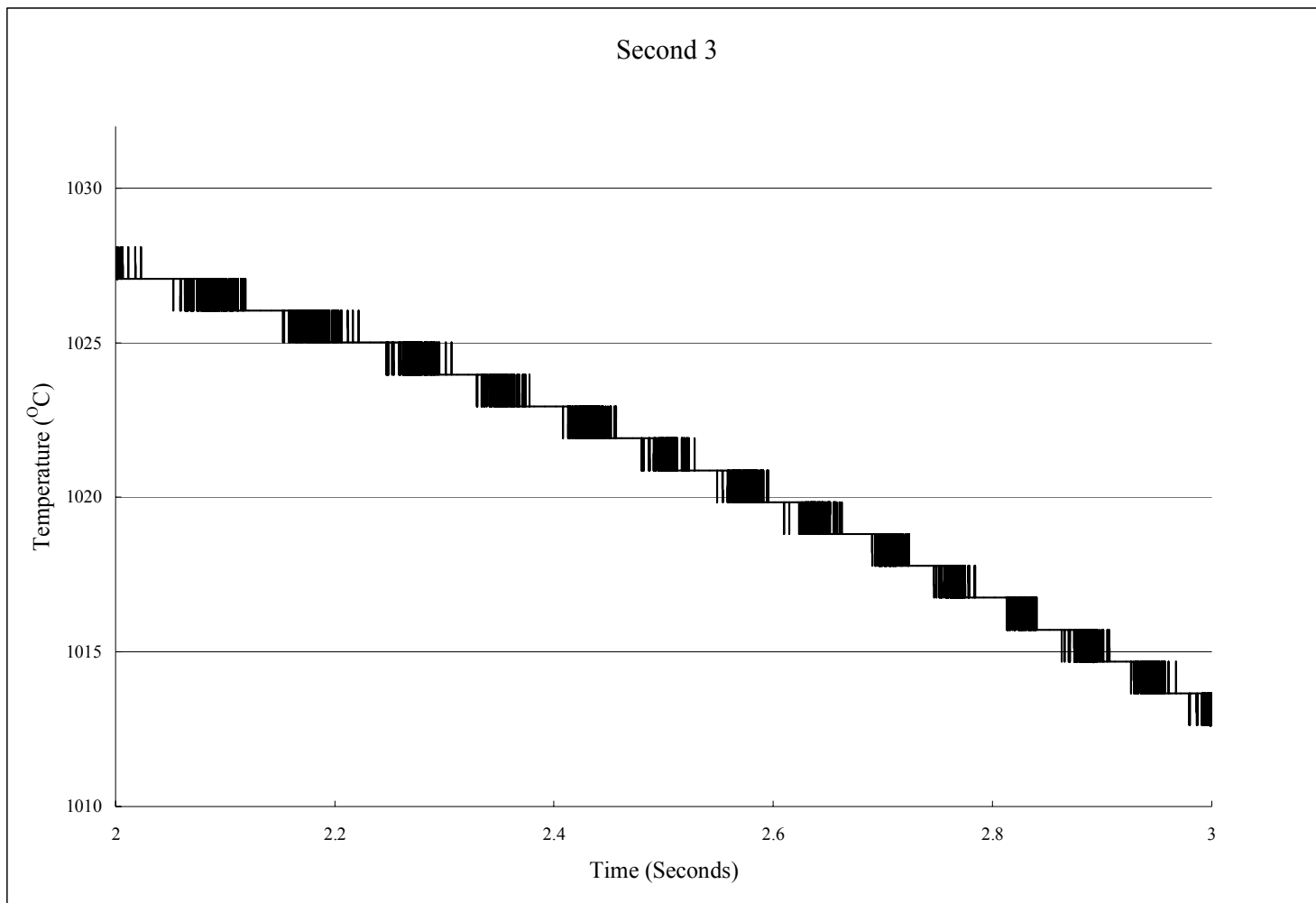


Figure 41: Temperature graph for second 3; metal screen position L/4

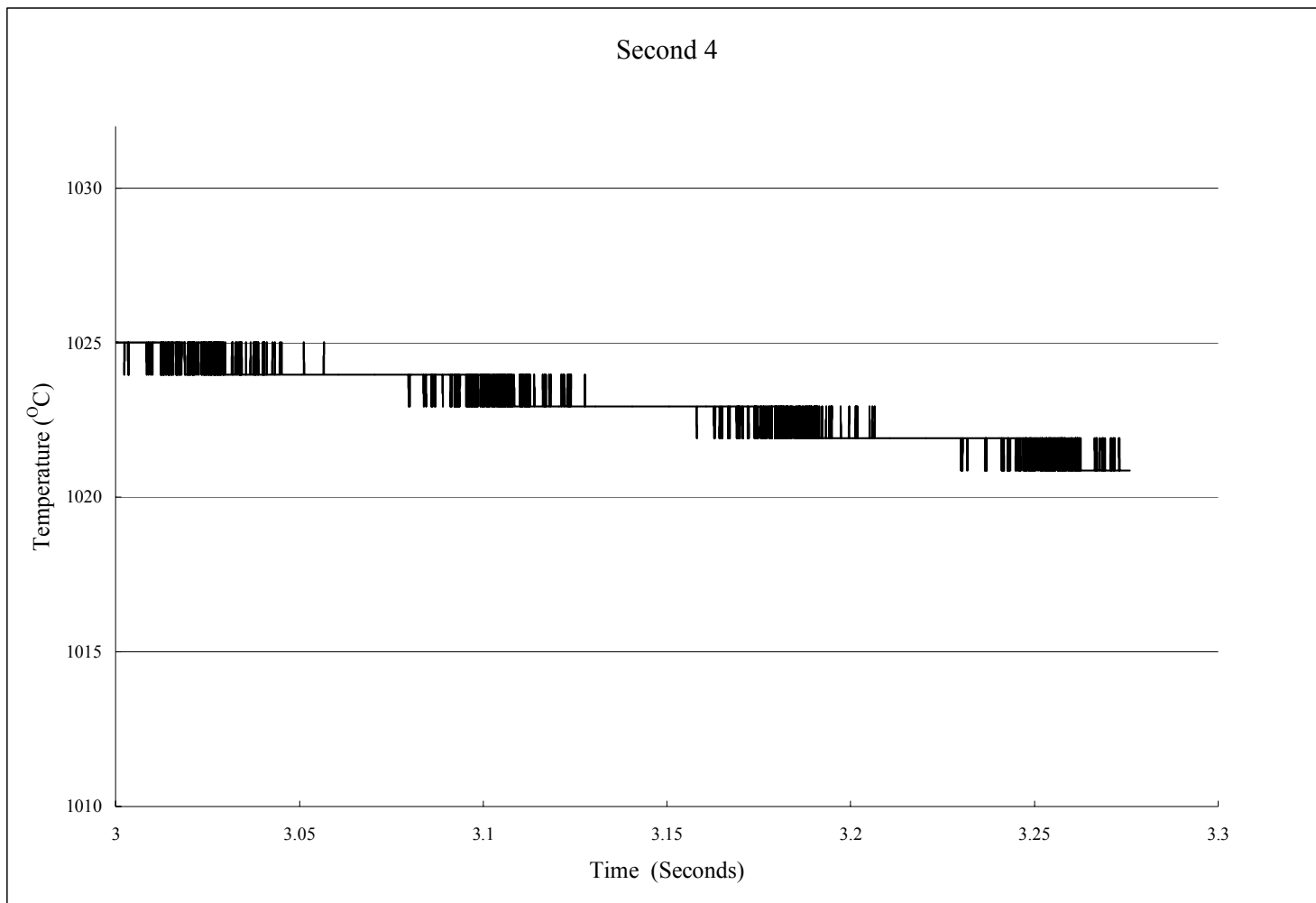


Figure 42: Temperature graph for second 4; metal screen position L/4

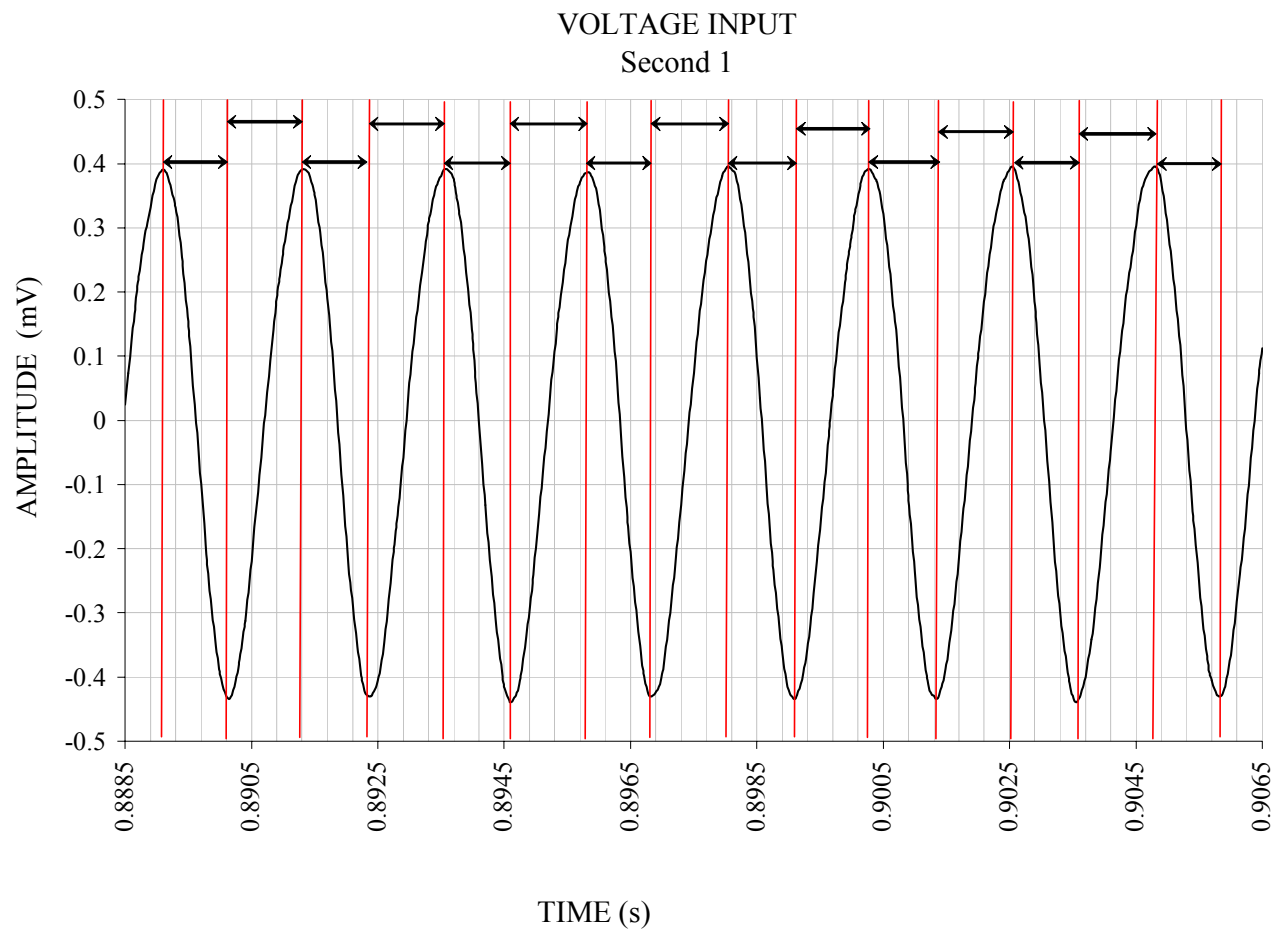


Figure 43: Sample data during second 1 for L/4 position

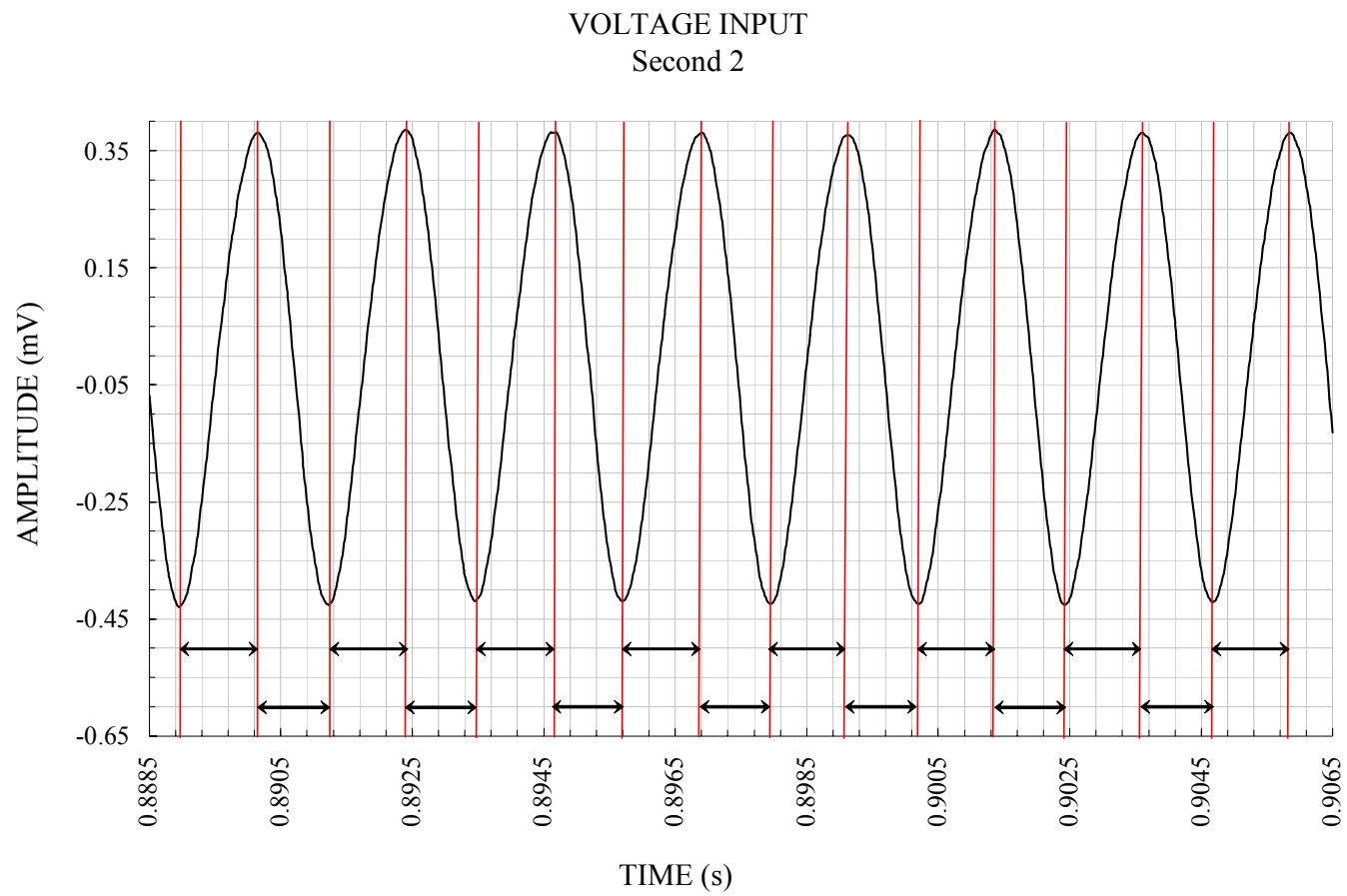


Figure 44: Sample data during second 2 for L/4 position

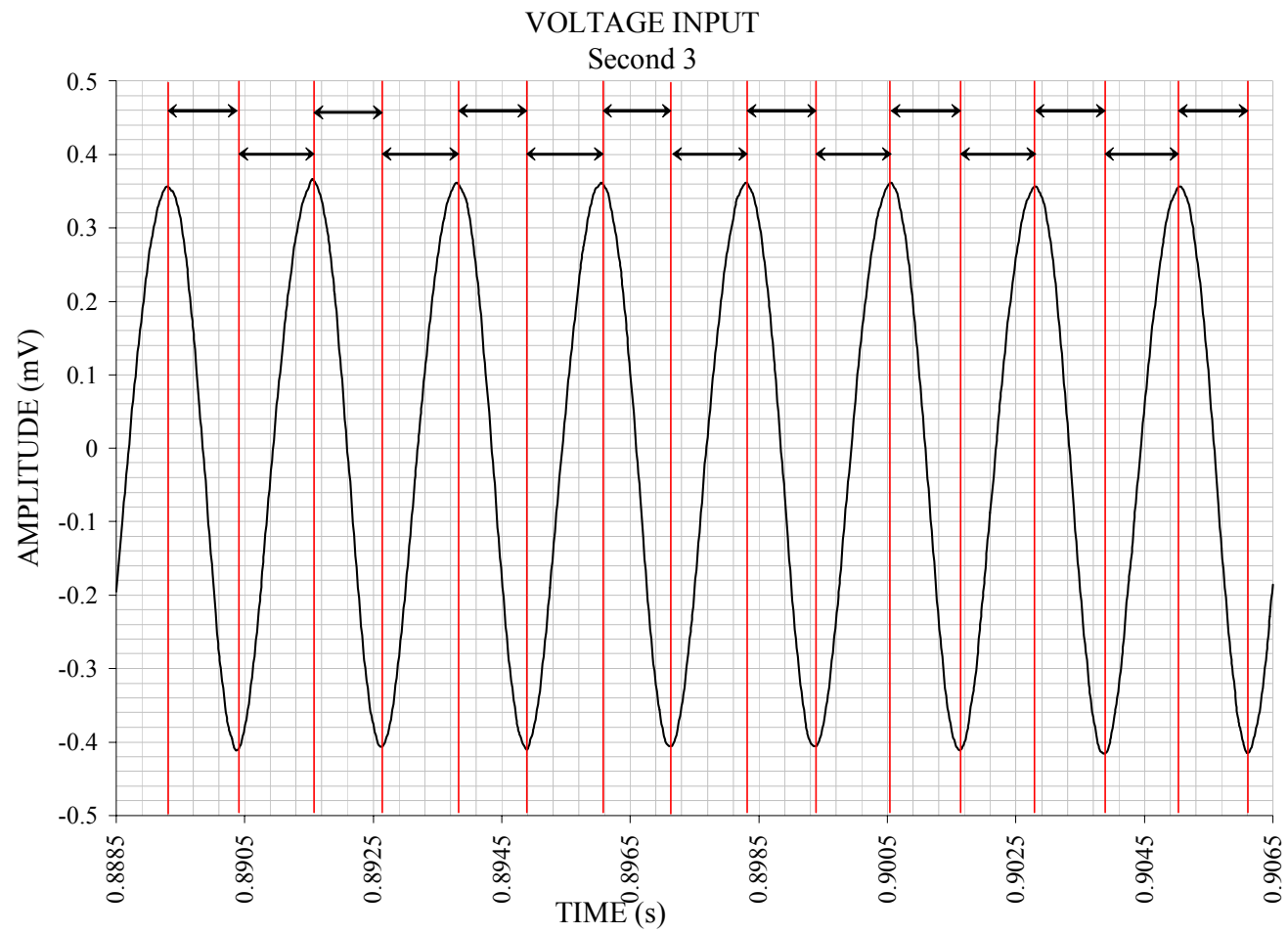


Figure 45: Sample data during second 3 for L/4 position

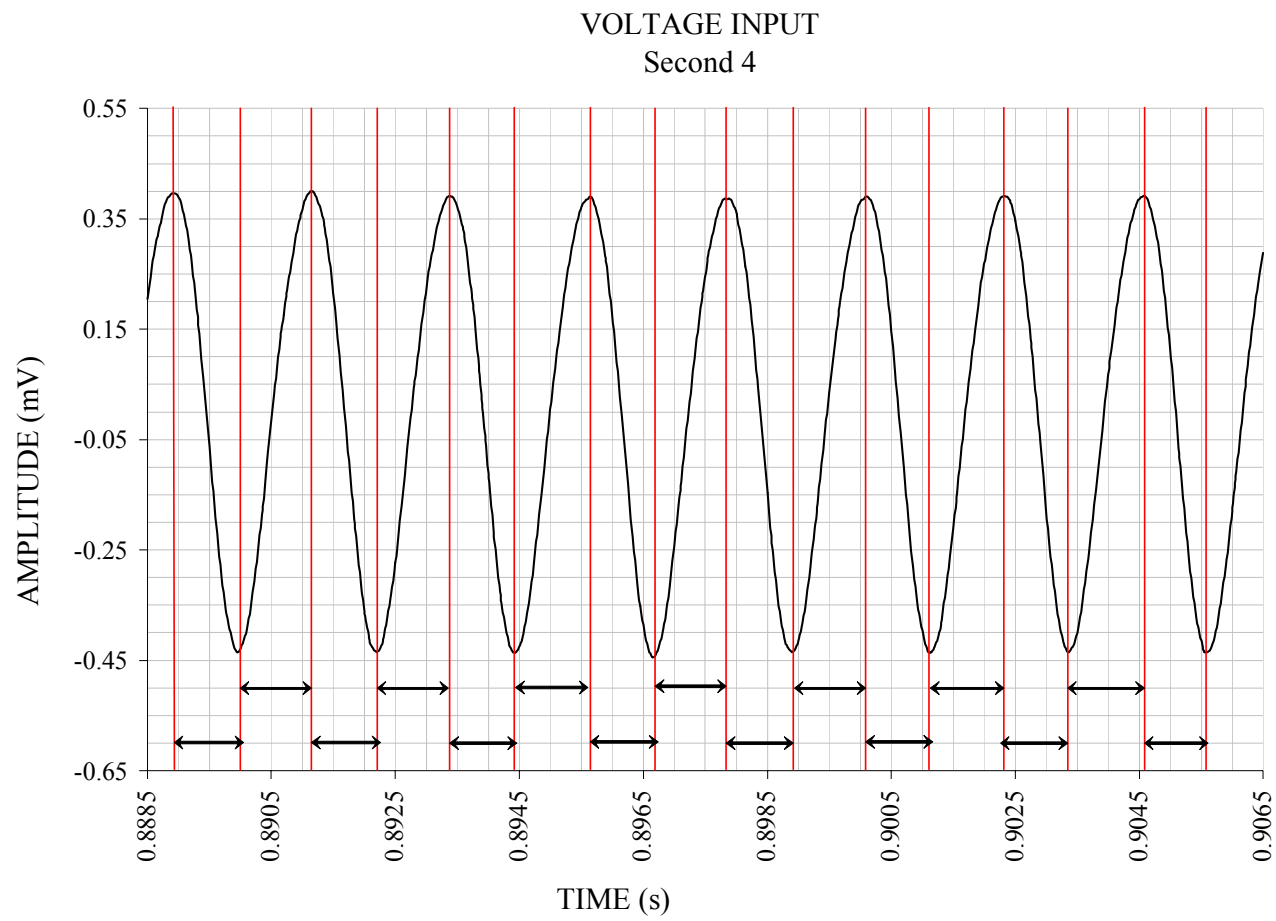


Figure 46: Sample data during second 4 for L/4 position

Table 3: Higher harmonics and their corresponding frequencies

	$\frac{L}{8}$		$\frac{3L}{16}$		$\frac{L}{4}$	
Fundamental frequency	458 Hz	77.62 dB	448 Hz	75.54 dB	448 Hz	88.91 dB
2 nd mode frequency	919 Hz	68.96 dB	895 Hz	62.58 dB	896 Hz	61.89 dB
3 rd mode frequency	1376 Hz	44.75 dB	1346 Hz	38.47dB	1344 Hz	31.3 dB

For the given dimensions of the tube and atmospheric temperature, frequency is calculated to be **400 Hz**. The temperature used in the calculation of speed of sound is the room temperature (93.19 °F) at the particular ambient conditions of the surroundings.. Since the temperature used to calculate the speed of sound must correlate to the temperature of the air inside the tube, the room temperature is very apt, taking into consideration the assumption of the metal screen as a localized heat source. Results from experimental data give the fundamental mode frequency to be around **454 Hz**. The numerical figures have been recorded in table 3.

Graphs of the variation in sound intensity with change in metal screen position are plotted in Figures 47, 48 and 49. As it can be seen in Table 3, the values of the frequencies are not identical. The frequencies here correspond to the data points with maximum sound intensity. As mentioned, the discrepancies in the frequencies of a particular mode for all positions are so small that counting them on a whole as the frequency of that particular mode is agreeable.

The sound intensity of fundamental mode is the highest in the L/4 position which has been proved experimentally and analytically. The intensity at the L/8 position is very low for fundamental frequency but the intensity of the second acoustic mode in the L/8 position is very high. It is same with the third acoustic mode. In this position, we see the domination of higher harmonics.

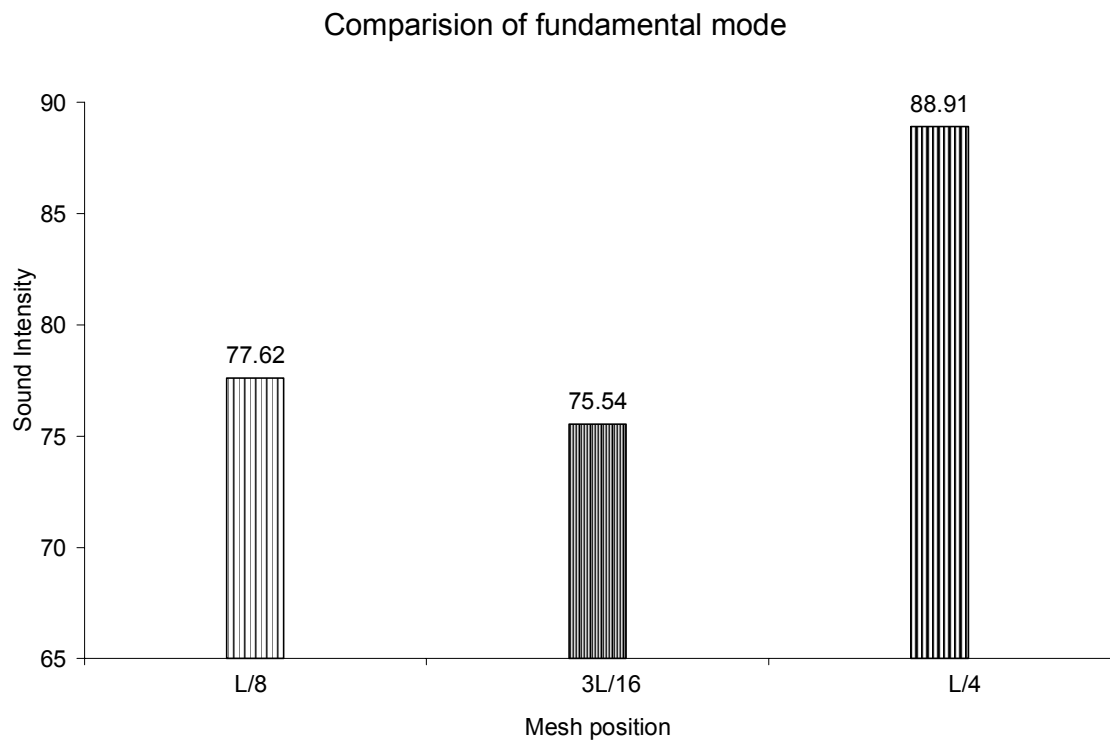


Figure 47: Comparison of sound intensities of fundamental mode in all three positions

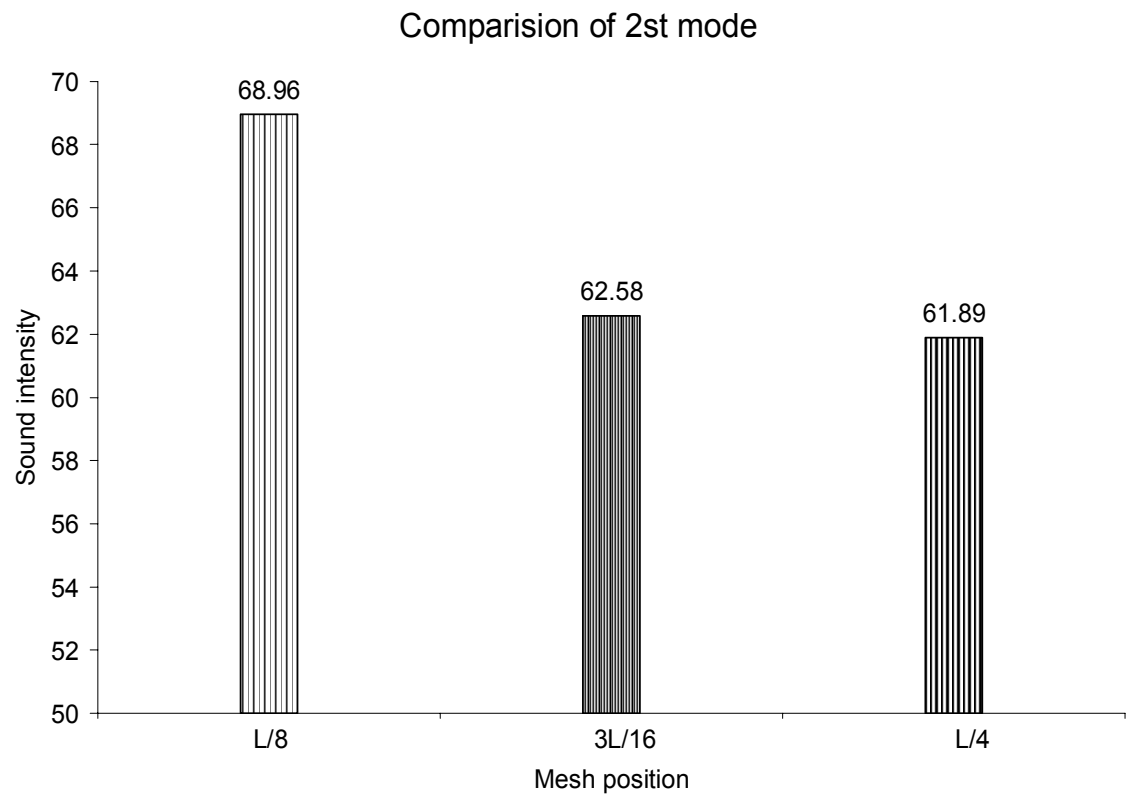


Figure 48: Comparison of sound intensities of second acoustic mode in all three positions

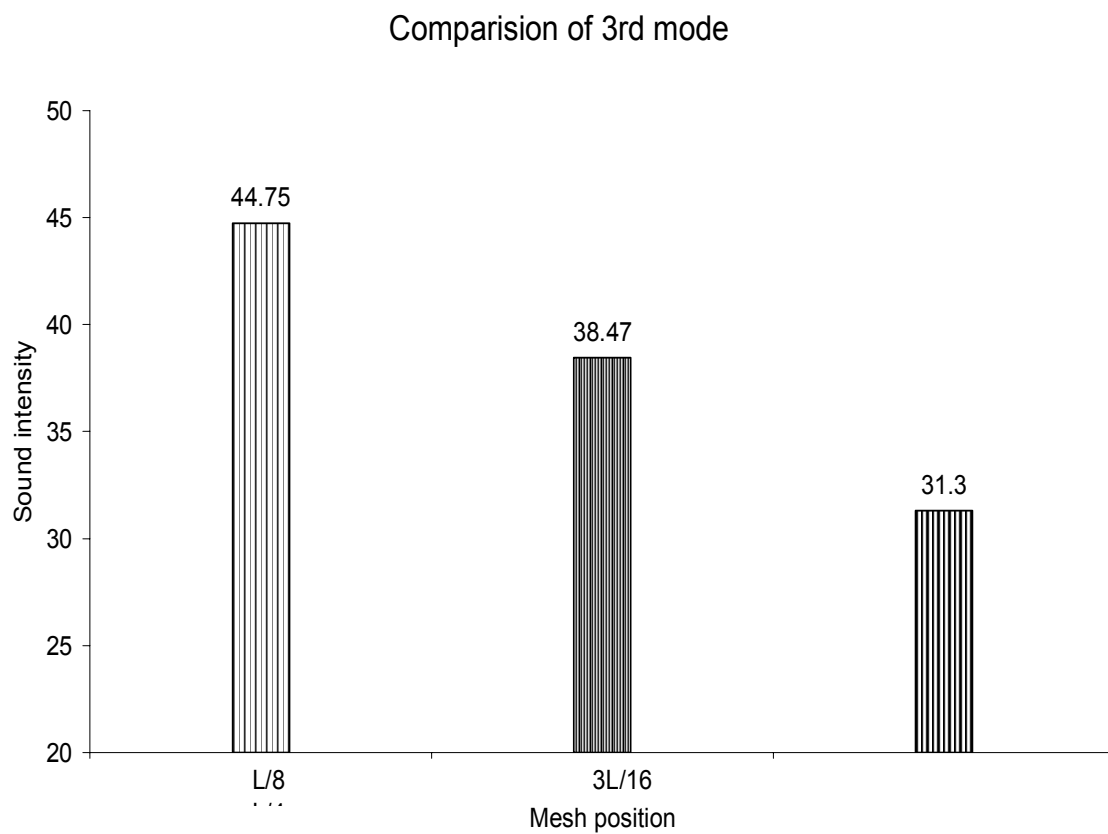


Figure 49: Comparison of sound intensities of third acoustic mode in all three positions

At the $L/4$ position, the fundamental mode dominates very clearly. There is a drastic decrease of the second and third mode intensities once the mesh is at $L/4$ position. But the higher harmonics are never insignificant.

4.4. Interpretation of signal input from microphone

Inside the Rijke tube, the pressure and velocity oscillations are as shown in Figure 50. These oscillations are standing waves. The unsteady pressure component has a node at both open ends of the tube and the velocity component represents an anti-node at these locations. The measurements that were taken in this study are that of the disturbances in the near field radiated from the end of the Rijke tube. The signal recorded contains information from the oscillations within the tube. The disturbance that is propagated into the air outside is clearly due to the velocity fluctuations at the upper end of the tube. This production of an unsteady field is similar to that from a monopole source. The gas motions at the end of the tube can be likened to a piston oscillation in such a way that it transmits the disturbances created within the tube to the surroundings. The signal reflects information created within the tube

When a piston is moved back and forth inside a tube, the system behaves like a monopole source and generates oscillations into the air outside as is shown in Figure 51. The propagation of these oscillations exhibits the nature of the velocity oscillations in the Rijke tube.

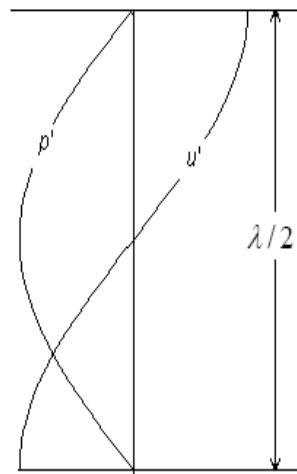


Figure 50: Acoustic oscillation in a Rijke tube

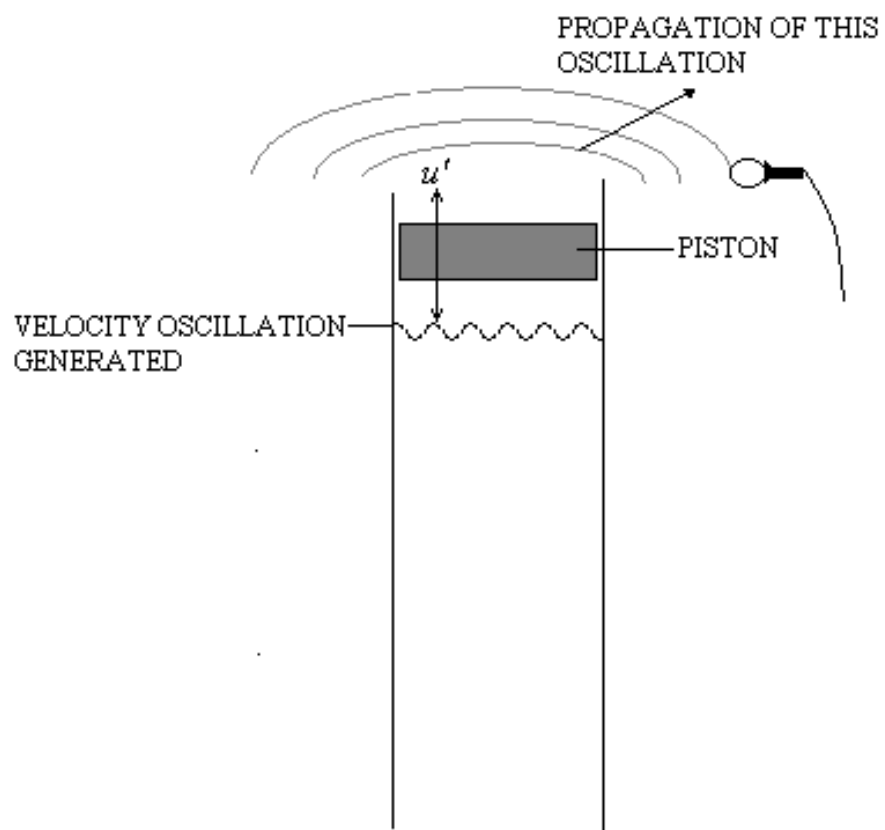


Figure 51: Steep waves generated by a piston

The wave produced is a steep-fronted one as suggested in the velocity-time graph as depicted in Figure 52. The voltage input from the microphone, which is a record of the disturbance, shows evidence of the steepening effects. The input from the microphone is in fact pressure disturbance, which occurs in rocket chambers at higher magnitudes. Figure 53 shows measured amplitude for 200 steps of time (each time step is 5×10^{-5}). This number has been chosen in order to facilitate clarity of the readings. There is no particular reason for choosing this number. This data is when the metal screen is at the 3L/16 position. It must be noted that this position is not one mentioned in the typical research papers on Rijke tubes. Such an observation might be of great importance.

The time difference between time taken for rise and the time taken for fall of velocity is very clear here. These conclusions have been arrived at by studying the amplitude data from the experiment (the input from the pressure transducer mentioned above is what is giving this data).

Figure 54 shows the time difference between times taken for velocity to increase and decrease. The significant difference existing over a considerable period of time is another convincing feature of the experiment that supports the presence of steepening oscillations. Though there is no scope of bringing steepened waves in the the Rijke tube, it is the same phenomenon that is causing this steepening of waves which causes the steepened shocked wave structures in the rocket engines.

Figure 55 is the ratio of these times mentioned above in order to show that the wave is steepening. Also, in Figure 56, the spectrum of this wave shows the presence of higher harmonics. There are many other pressure variations (in the form of sound) that can record higher harmonics but the occurrence of discontinuity which leads to shocked waves is verified by the fact that higher harmonics have dominant frequencies. This is the characteristic of a wave as it grows to become a shock wave. Correlating the presence of higher harmonics in the data and the tendency of steepening waves shows that higher harmonics are the results of steepening of waves.

Therefore, the representation of the steep-fronted waves in the rocket engine oscillations as a Fourier series is justifiable, because as seen in the experimental data above, presence of higher harmonics indicates the presence of steep-fronted waves.

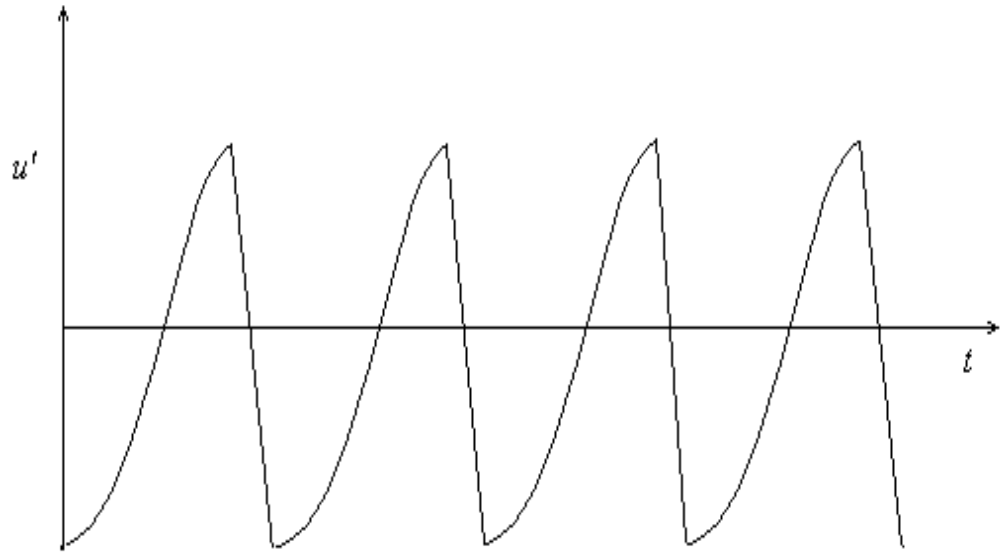


Figure 52: Velocity plot of steep waves

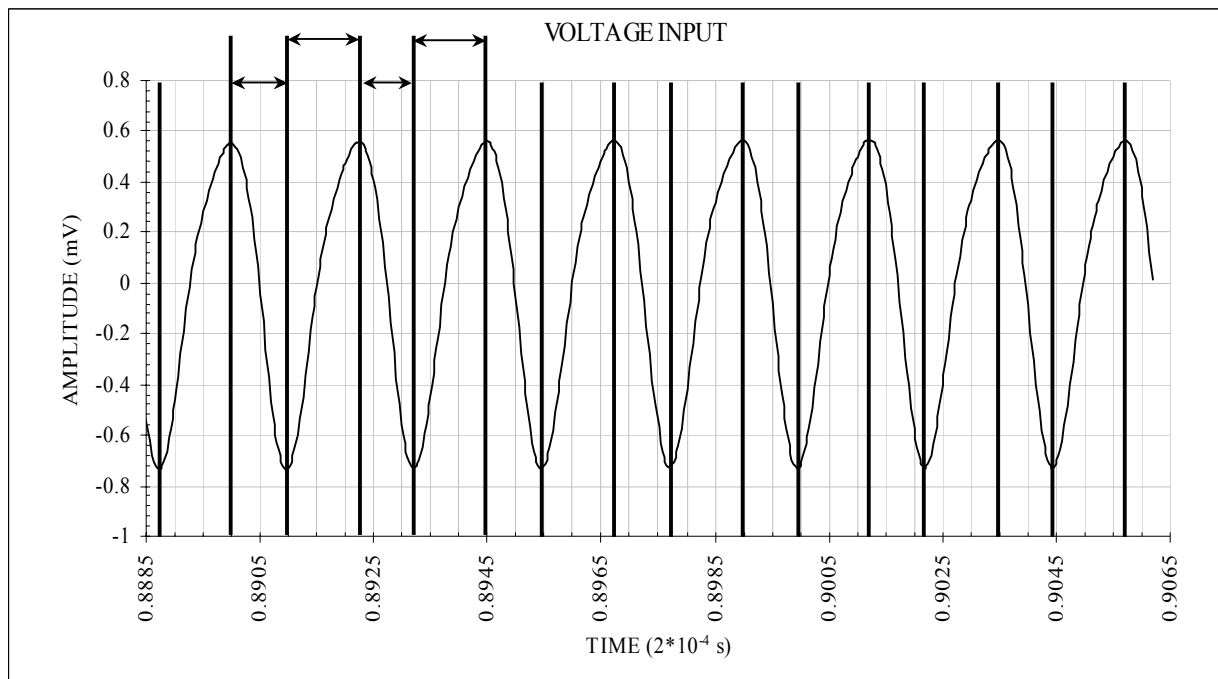


Figure 53: Voltage input from microphone

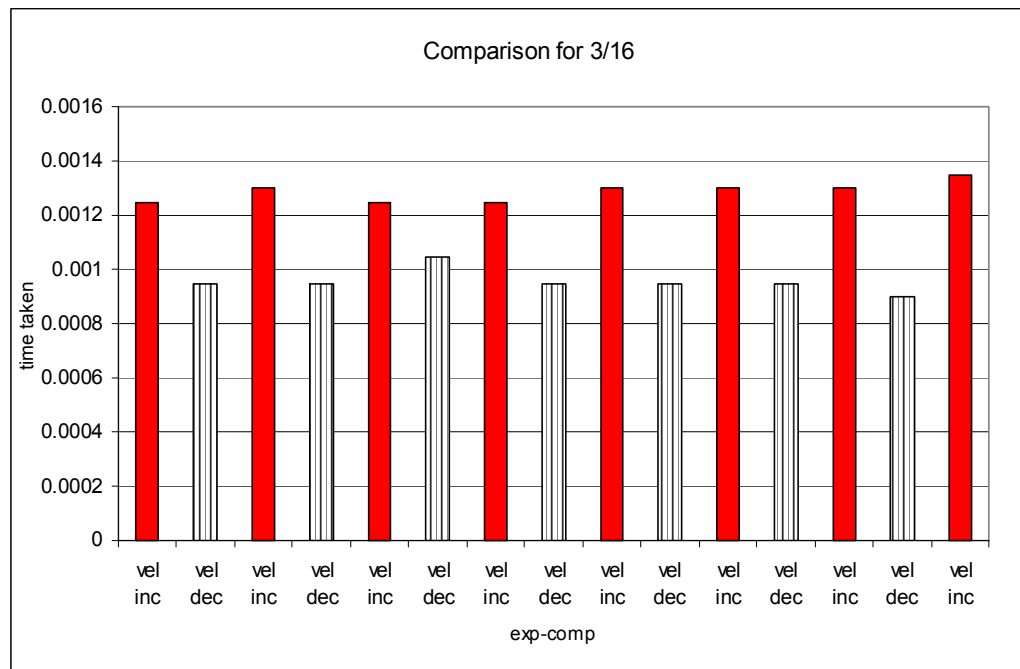


Figure 54: Comparison of time taken for velocity increase and decrease

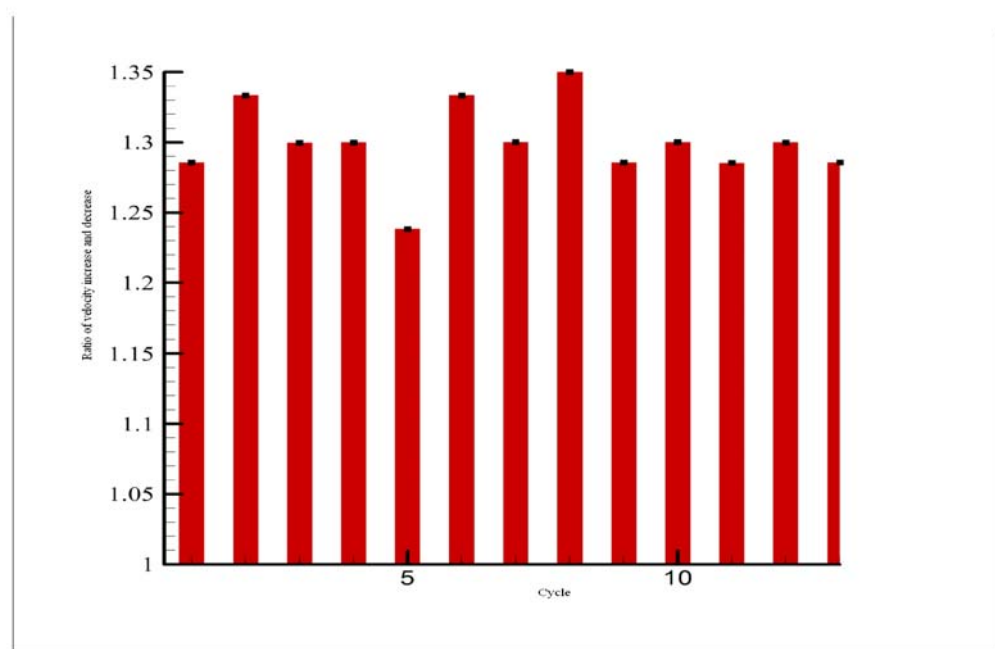


Figure 55: Ratios of time taken for velocity increase and velocity decrease

3L/16

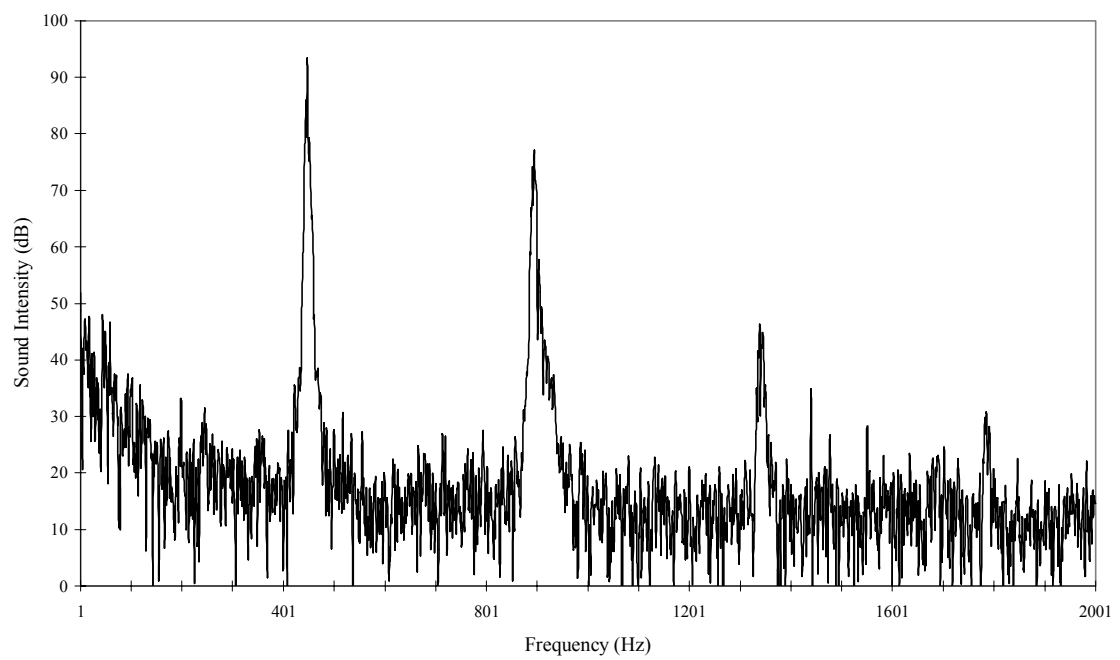


Figure 56: Spectrum plot for the input from the microphone

4.5. Conclusions

There are conclusions and comments on both data obtained and the instrumentation for data acquisition.

4.5.1. On data

1. Readings have been taken at $L/8$, $3L/16$, and $L/4$ positions of the mesh. The fundamental frequency and next three harmonics of the natural frequencies of the tube have been observed at all positions.
2. Maximum sound intensity of the fundamental frequency was recorded at $1/4^{\text{th}}$ of the tube location. At this position, intensity of the fourth acoustic mode was not noticed very clearly.
3. The time duration for increase in velocity is always different from the time duration for decrease in velocity. Normally such a behavior shows a tendency to form steep fronted waves. The extreme condition of such a steepening process results in a shocked acoustic wave.
4. The discrepancy in the theoretical and experimental values of the frequency is due to the assumption of the mean flow temperature to be the room temperature. When actually, $3/4^{\text{th}}$ length of the tube is at a temperature somewhat higher than the room temperature. Many studies, for example [29] have calculated the temperature at various points of the tube and they show profiles where the temperature in the upper $3/4^{\text{th}}$ of the tube is higher than the lower $1/4^{\text{th}}$. But this observation of theirs has not been taken as a basis to assume a random or better put, approximate high temperature.

4.5.2. On Instrumentation

1. The main reason for any kind of deviation of the experimental data from the proposed theory is due to the lack of control over the heat input to the mesh. That the heat source has to be at constant temperature and the temperature of air considered for calculating the frequency that matches the theoretical frequency must have experimental validation cannot be satisfied in this experiment. The use of the propane gas flame does not facilitate proper control over the heating. Also due to the

restriction put by the geometry of the torch nozzle, beyond a certain height inside the tube, heating of the mesh is very ineffective. In this experiment, since readings have been taken only in the lower quarter of the tube, no difficulty has arisen. The data taken at the very start of the oscillations is very critical. These readings are obviously affected by the delay caused in removing the torch in order to allow natural convective flow. The actual behavior of sound can't be accurately predicted as there are sudden fluctuations in the temperature.. In fact, hardly any readings are taken at the 1000 °C limit because the duration of the mesh at that temperature is very momentary. The small variations in the temperature profile for the four seconds are not much when compared to the target 1000°C.. In spite of these changes, there is no change in the particular behavior of oscillations that are of interest here i.e., the occurrence of higher acoustic modes.

2. The use of an electrical heating element could help overcome these inefficiencies. In this case we can control the heat input and hence the temperature of the mesh. This helps us measure the frequency at steady state. Also location of the mesh at any position will not be a problem as long as the connections to the power source are long enough.

5. CONCLUSIONS

From the introduction chapter, it is clear that Rijke tube can be experimented on to study instabilities occurring due to thermoacoustic phenomenon similar to those in rocket engines. The tube is a very simple model that demonstrates much the same behavior as the complex vibrations in the rocket engine and hence experimentation and analyses of the mechanism of the tube simulates that of real engines.

5.1. Conclusions of the experimental data and Theory

From the theory of thermo-acoustics and prior experiments performed, the following results have been obtained which comply with preceding research works.

1. The sound is produced after certain time during which the power transformed from heat to the acoustical motions exceeds the losses. Production of audible acoustics is termed as instability.
2. For various positions from the base till $L/4$ position, sound is heard with increasing amplitude as we move towards the $L/4$ position from the lower end of the tube. This amplitude is the maximum when the mesh is at $1/4^{\text{th}}$ the length of the tube from the base. The fundamental frequency dominates in all positions.
3. The observation that the first harmonic can also be excited in the third quarter (observations tell that the position is $5L/8$) has not been tested due to the limitations of the experimental apparatus.
4. Having known the value of the speed of sound, the fundamental mode frequency can be evaluated and can be compared to the value obtained from the spectral analyzer.

5.2. Conclusions of this study

1. From the data analysis chapter, it is evident that in every of the position of the metal screen that the experiment has been conducted, three modes of oscillations are present. There is no ambiguity in this conclusion. The behavior of the oscillations,

- especially the fundamental and second acoustic modes, is in complete agreement with studies conducted by previous researchers.
2. The presence of higher harmonics is a sign of the existence of steepened wave fronts as observed in combustion chamber of rocket engines. The analogy between Rijke tube oscillations and rocket engine oscillations, which is the basis of this research work allows a conclusion that the same mechanism, that is the thermoacoustic phenomenon is responsible for the occurrence of nonlinearity in the oscillations
 3. Investigating the data, one sees that higher harmonics cannot be neglected as they have been done for quite some time. The study of instability must not be restricted to that of the fundamental mode. Significant amplitudes of higher harmonics call for attention and as mentioned in research papers by Flandro, the Fourier series representation; which includes a number of the chamber's standing acoustic modes becomes a more convincing explanation of the traveling shock waves. This work encourages further research on Rijke tube oscillations that will include experimental studies of the mechanism of wave steepening.

5.3. Flowchart

A schematic representation of the final intentions of this work is shown in Figure 57 as a flowchart.

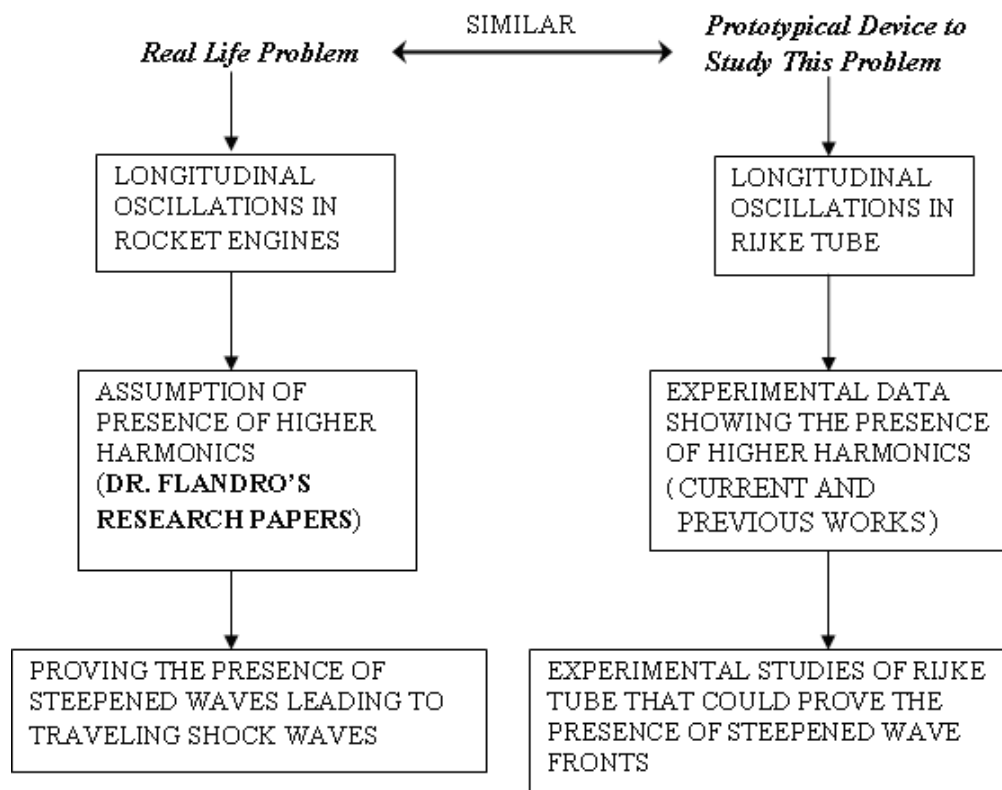


Figure 57: Flowchart for the conclusions chapter

6. RECOMMENDATIONS

For the study of Rijke tube oscillations, two types of heat source are favorable. The first one which is also the classic device is one with metal screen that is heated by a torch or a similar device so that this heated metal screen becomes the supplier of heat to the oscillations for a limited period. This method is attractive because of its simplicity and economic design. Also the data obtained is very reliable. The current study has employed this type of a heat source.

The second and more sophisticated type has a heater element as the metal screen which is connected to an electric supply such that the experimentalist can have control over the heat input to the oscillations.

Recommendations are discussed for the current setup and the electric heat source is mentioned as a recommendation. The reasons and advantages of such a heat source are mentioned in that part of this chapter.

Recommendations for current setup:

1. If the experiments are performed in a closed chamber, other disturbances recorded can be minimized. By doing so higher frequencies such as the 2nd and the 3rd harmonics in this case can be seen well.
2. The thermoacoustic phenomenon that is of interest to us is limited to heat input. Rijke himself, the Rayleigh's criterion and many more experimental works proved that heat input in the lower half of the tube and heat extraction in the upper half of the tube encourage acoustic instability in the tube. Removing the limitations on the position of the metal screen where recording useful data will be possible, will give a lot of information that would be helpful in studying the occurrence of higher harmonics.
3. Apart from using a microphone to gather data, use of pressure transducers to record the pressure fluctuations inside the tube will supply data that can support the presence of steep fronted waves in a more convincing manner.
4. If data can be collected over a larger time interval, the input and hence the system can be stabilized. This helps record isolated, dominating peak for that one frequency which either coincides or is closest to the calculated frequency. This would remove

any remote chance to be doubtful about the correlation with and correctness of the analytical model.

5. Use an electric heating element as the heat source. The uses and advantages of this system are:
 1. Heat input can be made continuous and constant. Control over temperature is possible.
 2. Readings can always be taken when the sound has stabilized. Though there will be no necessity, data over a long duration can be recorded.
 3. The power source can be made variable. This facilitates study of stability boundaries for every frequency. Further study depending upon the advancement in analytical research in this direction can be conducted.
 4. Physical hazards are relatively lower when compared to using a propane flame torch.

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APPENDIX A

Recent studies done at UTSI (Eric Jacob, Dr. Gary Flandro) support the experimental results discussed in this thesis. The data collected in these experiments was analyzed by the conventional spectrum analyzer. Higher harmonics were present in his set of experiments.

The microphone (pressure transducer) was placed inside the tube. The schematic of the set-up is shown in figure 58.

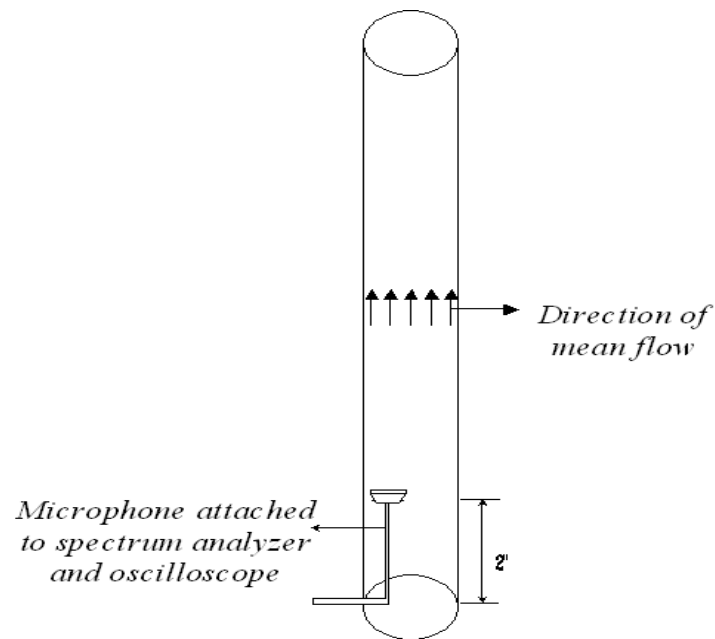


Figure 48 a: Microphone 2" from lower end

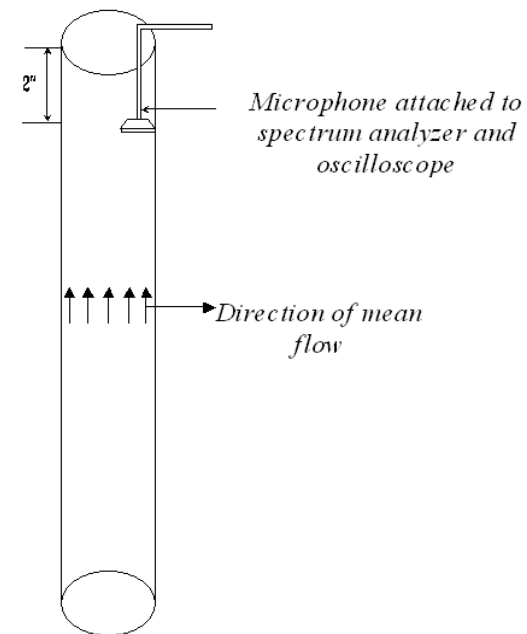


Figure.48 b: Microphone 2" from upper end

Figure 58: Schematic of the experimental setup

The transducers were placed against the direction of the flow in the tube. Data was collected manually from the analyzer and the micro volts of each frequency are plotted. The first two plots are of the amplitude against frequency.

1. In the first run, the transducer was placed two inches above the lower end of the tube.
2. In the second run, the transducer was placed two inches below the upper end of the tube.

Figures 59 and 60 show the amplitude against the frequency in the experiments.

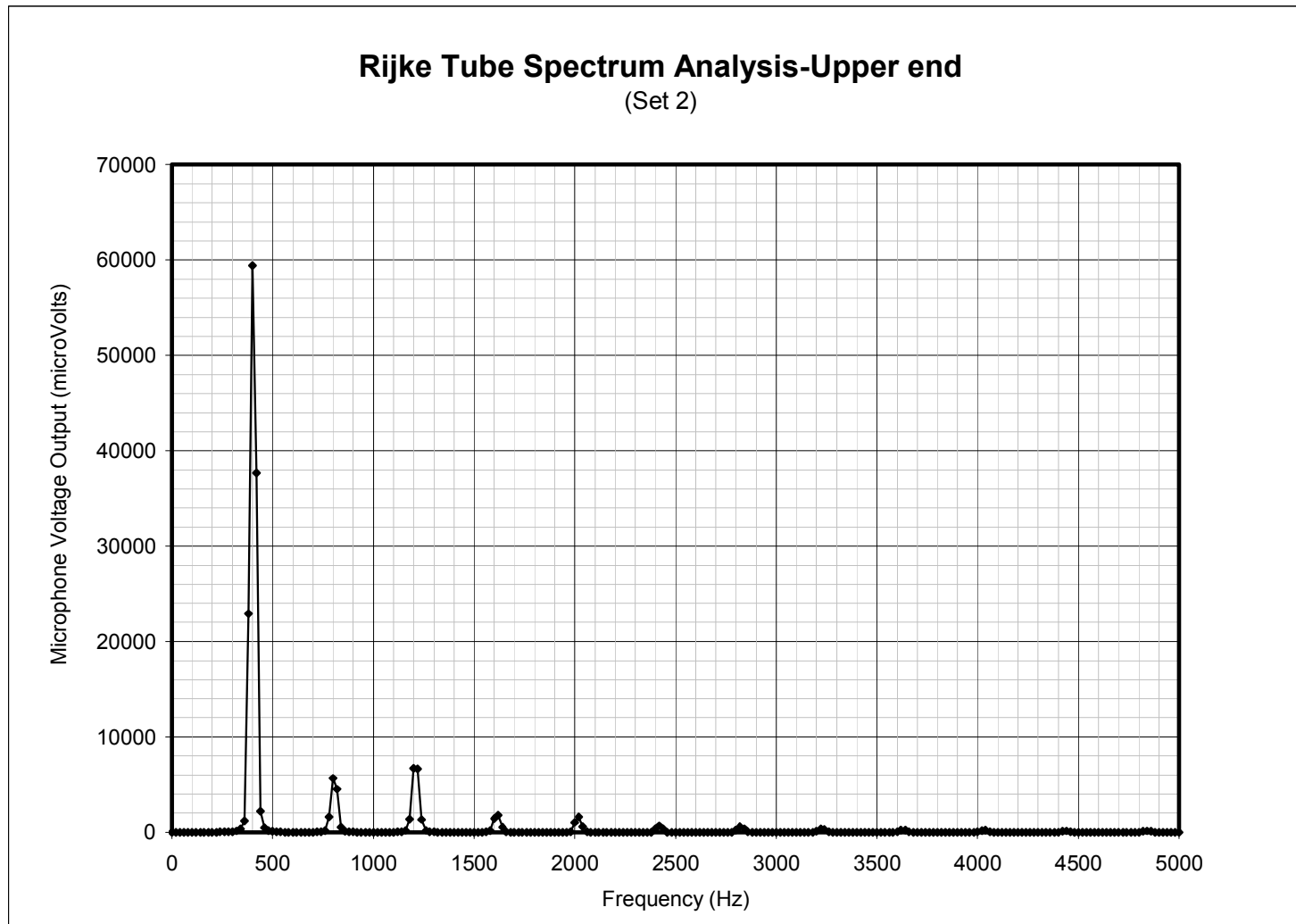


Figure 59: Spectrum data (upper end)

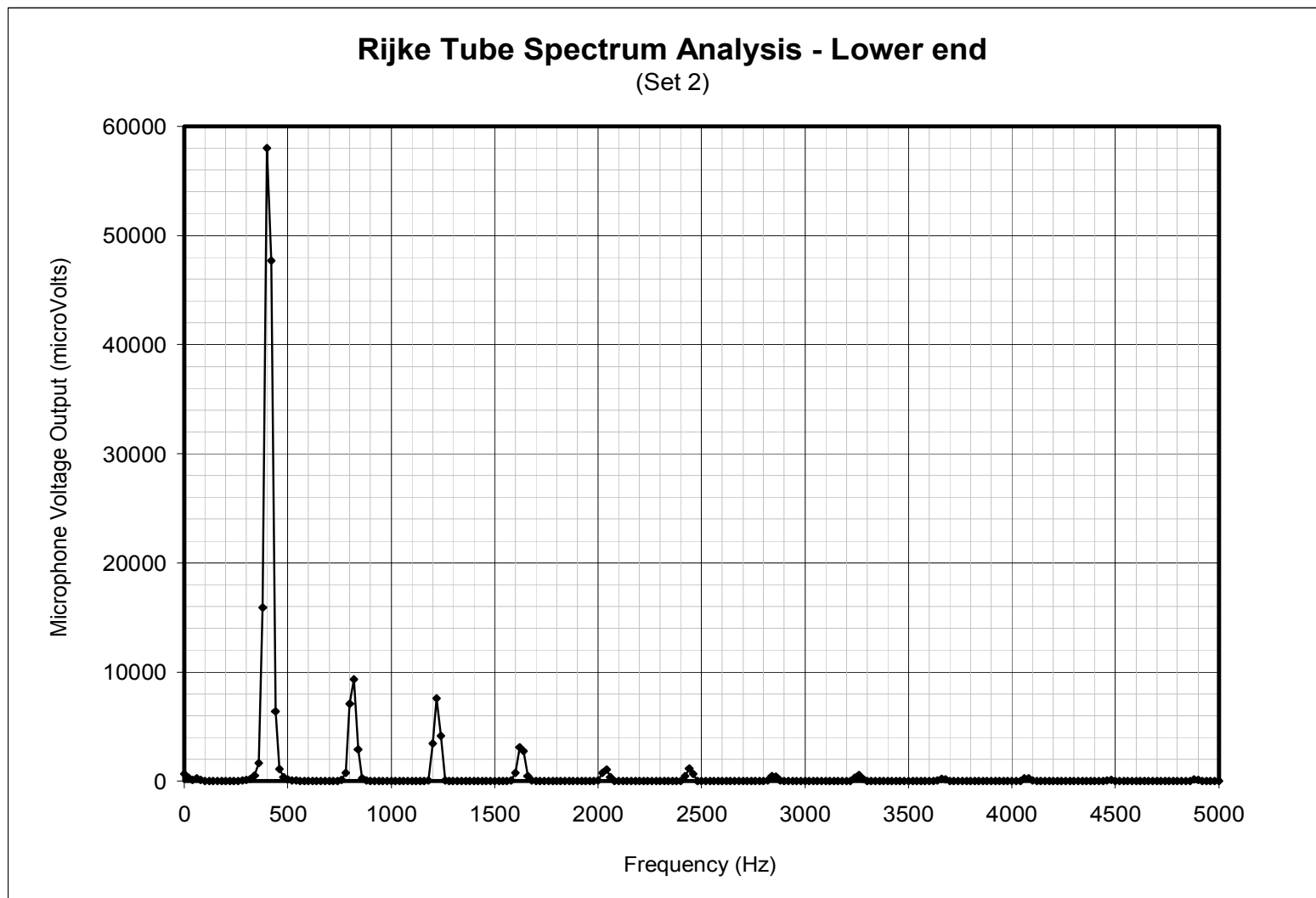


Figure 60: Spectrum data (Lower end)

In order to be able to compare results obtained in this thesis and that of experiments in this chapter, the graphs have been converted to decibel –frequency plots.

Significant amplitudes of higher harmonics can be observed. This is very similar to that of the research in this thesis. There is difference between the decibel levels in both the experiments. This can conveniently be attributed to the discrepancy in the time periods when the data has been collected in each experiment. Since the experiments were not done in connection to each other, the time period over which data was collected can be different in both the works.

In the experiment, the output of the spectrum analyzer is in micro volts. In the set of the original experiments, the output of the LAB View software is in Decibel. In order to compare both these results, the units must be the same.

Voltage can be converted into Decibel using the following formula:

$$\text{dB} = 20 * \text{Log}_{10}(V_1/V_2)$$

Where V_1 is the recorded voltage and V_2 is the reference voltage.

Our interest in these experiments is:

1. Occurrence of higher harmonics
2. Amplitudes of these harmonics.

The reference voltage for these values has been taken as 10 and 100, in order to study the decibel values of various harmonics.

The figures 61, 62, 63, and 64 plot the decibel values of the frequencies occurring in the tube during the experiment.

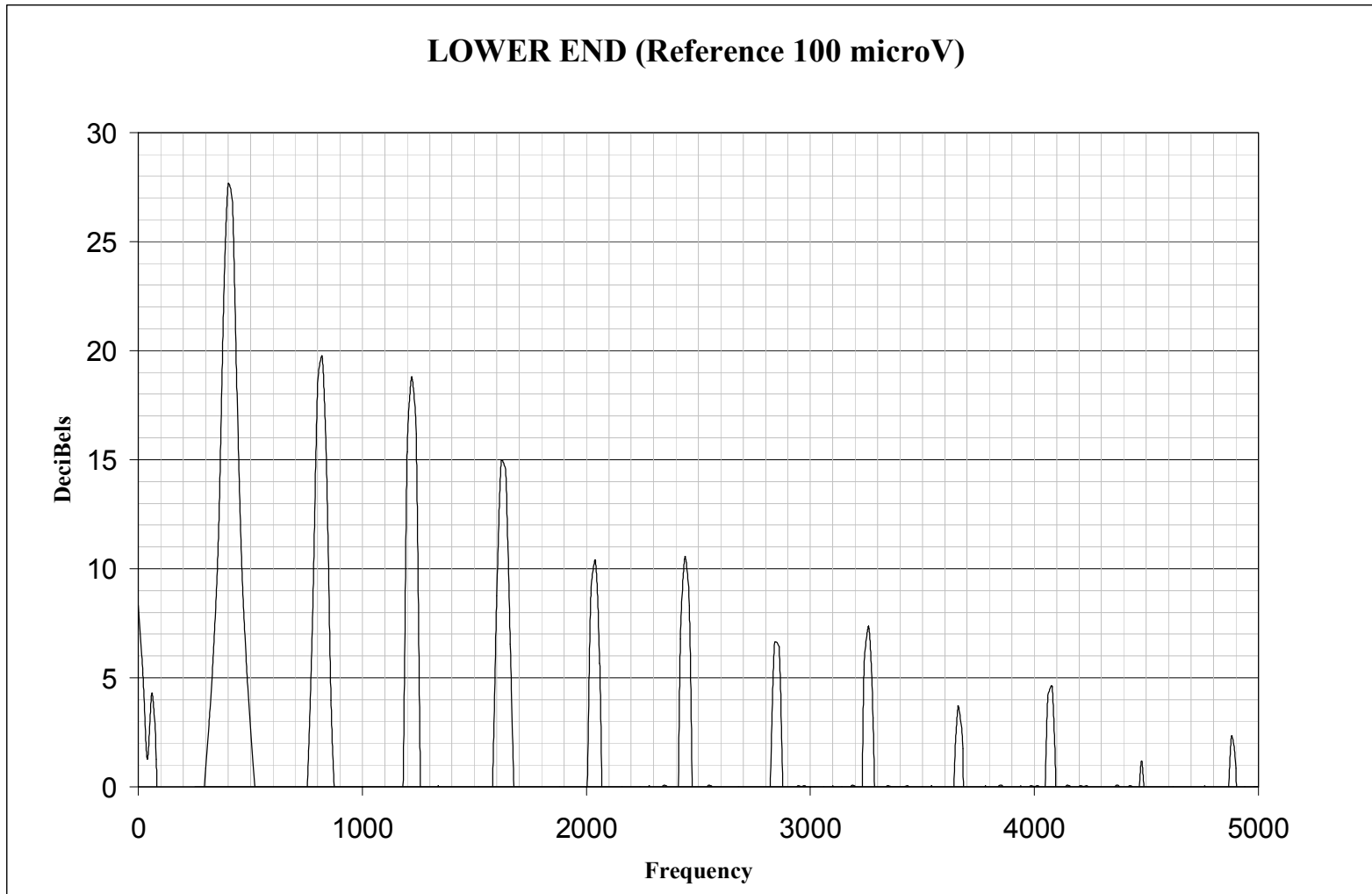


Figure 61: Amplitude in dB at lower end (reference 100 microV)

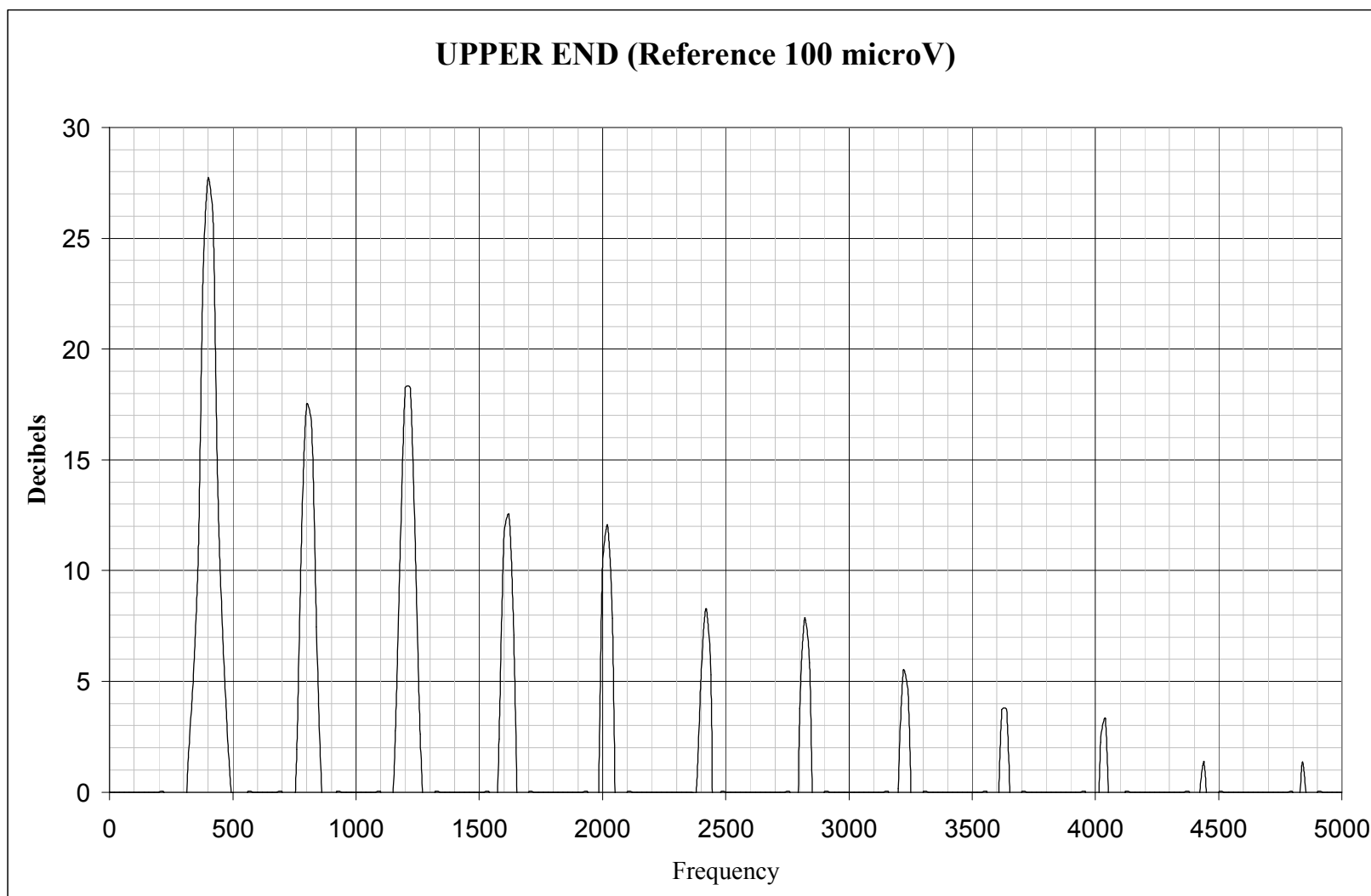


Figure 62: Amplitude in dB at upper end (reference 100 microV)

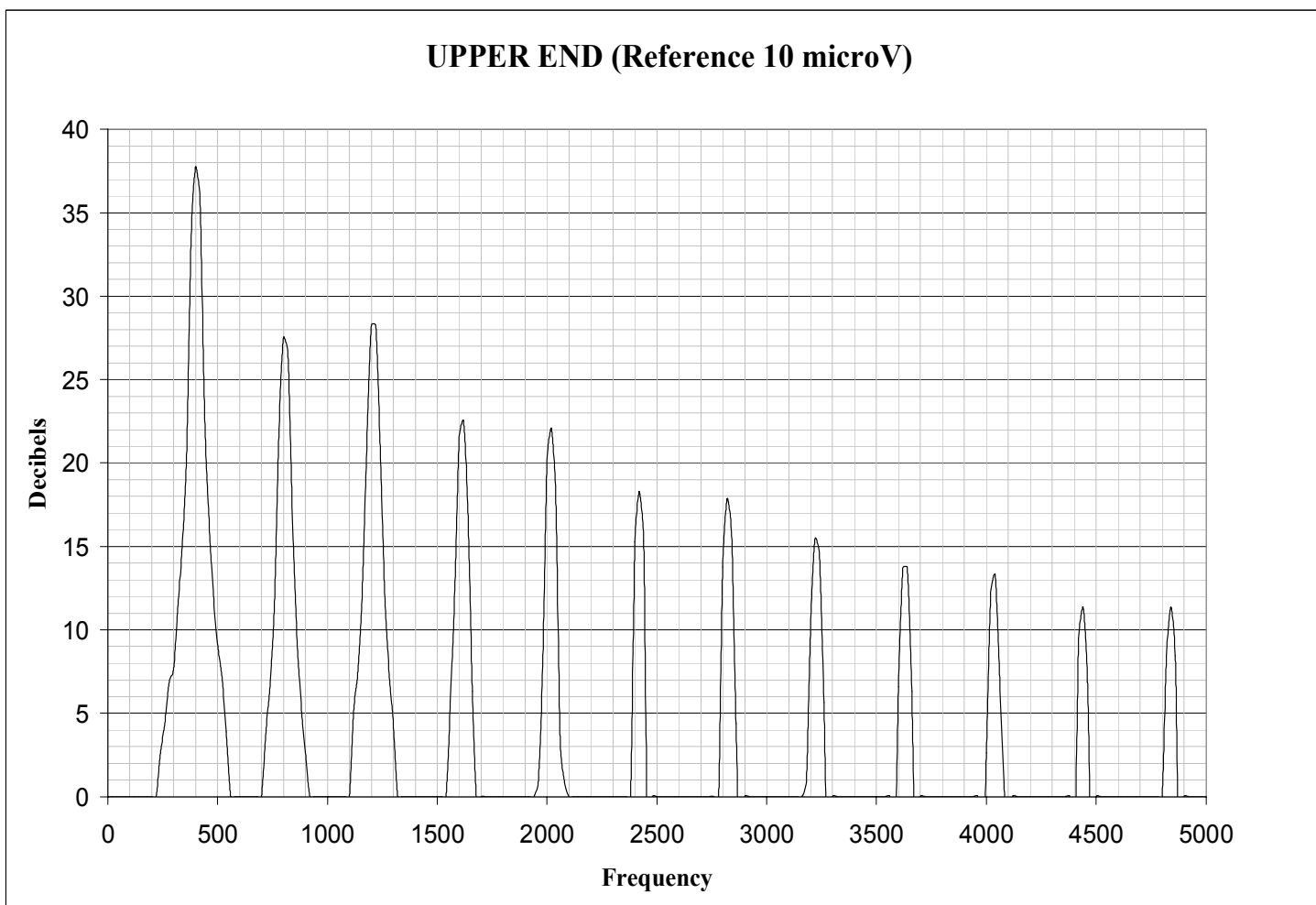


Figure 63: Amplitude in dB at upper end (Reference 10 microV)

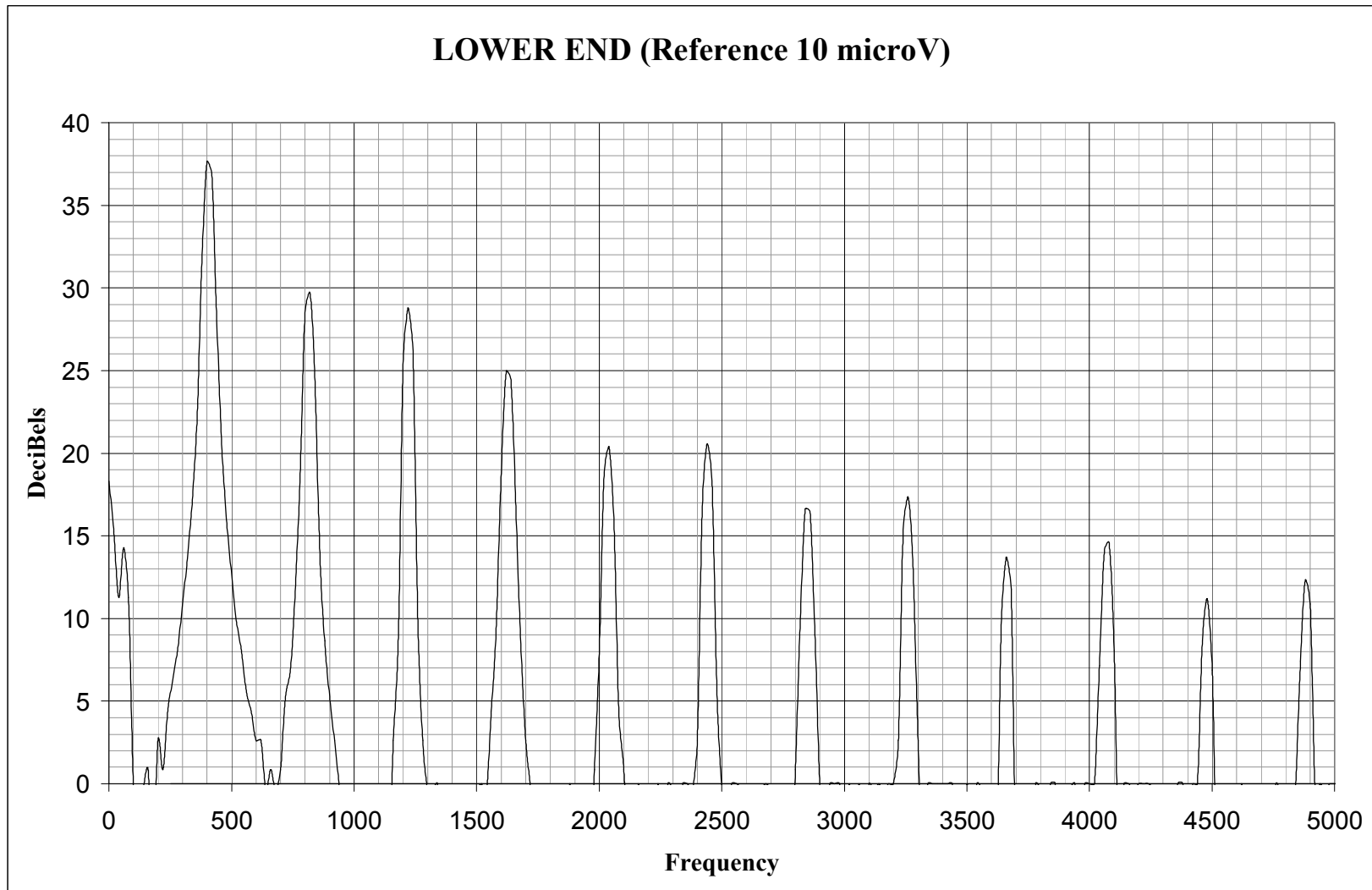


Figure 64: Amplitude in dB at lower end (Reference 10 microV)

VITA

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