

To the Graduate Council:

I am submitting herewith a thesis written by William S. Hamilton Jr. entitled "A Production Function Analysis of Grade A Dairy Farms in Middle and East Tennessee." I recommend that it be accepted in partial fulfillment of the requirements for Master of Science Degree, with a major in Agricultural Economics.

Luther H. Keller
Luther H. Keller,
Major Professor

We have read this thesis
and recommend its acceptance:

Robert M. Ray
Billy J. Shewers

Accepted for the Council:

L. E. van Bock
Vice Chancellor
Graduate Studies and Research

Thesis

78

.H355

cop. 2

A PRODUCTION FUNCTION ANALYSIS OF GRADE A
DAIRY FARMS IN MIDDLE AND EAST
TENNESSEE

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville



William S. Hamilton Jr.

June 1978

1358327

ACKNOWLEDGMENTS

The author wishes to express his appreciation to the people who provided ideas, encouragement, and facilities used in the preparation of this thesis.

I would like to express my gratitude particularly to those who contributed directly to this study, including:

Dr. Billy J. Trevena who contributed the data used in this study and provided invaluable advice about the statistical analysis used in this study.

Dr. Robert M. Ray for his helpful suggestions and encouragement.

Dr. Clark D. Garland for his insights into the data used for this analysis.

Dr. Luther H. Keller, my committee chairman, for his guidance, advice, and encouragement throughout the entire study.

Finally I wish to express my sincere appreciation to my wife, Debbie, and my parents, Mr. and Mrs. W. S. Hamilton Sr. for their inspiration to complete this task.

ABSTRACT

This study was a production function analysis of Grade A dairy farms in Middle and East Tennessee. Records from a select group of Tennessee dairy farms enrolled in the Agricultural Resource Development Program were used for the study. Data were for 1973, 1974, and 1975 and were derived from 241 farm records. At least 50 percent of the gross farm income on each study farm came from the dairy enterprise.

The principal objective was to estimate marginal productivities of resources used on farms in the study. Other objectives were to estimate the returns to scale on Tennessee Grade A dairy farms, to determine the substitution relationship between mechanization and hired labor on Tennessee Grade A dairy farms, to develop a management index for individual milk producers and to compare it with efficiency factors commonly derived from farm record analysis, and to use production function estimates for prediction and to compare predicted output with actual output.

In order to obtain estimates of marginal productivities, production functions were estimated by the least squares technique using the Cobb-Douglas function in logarithmic form. Separate functions were estimated for 1973, 1974, 1975, and 1973-1975 combined. Input categories related to annual gross farm income were investment in dairy cows, investment in buildings and improvements, man equivalents of hired labor per year, man equivalents of unpaid labor available per year, annual machinery and equipment expenses, yearly cash operating expenses, and open acres of land.

Marginal value productivity estimates were derived at the geometric and arithmetic means of each independent variable for each of the time periods. For the year 1973, the input man equivalents of unpaid labor was the only input with a negative marginal value product. In 1974, the marginal value productivities of investment in buildings and improvements and man equivalents of unpaid labor were negative. In 1975 and the 1973-75 period, the marginal value productivities of all inputs were positive. For the average farm in 1973-75, this study indicated that reductions in capital used for buildings and improvements, operating capital, and hired labor combined with increases in investment in dairy cows and machinery and equipment expenses would have increased profitability.

Increasing returns to scale on Grade A dairy farms were exhibited for the 1973-1975 period. This indicated a declining cost per unit of output as all inputs are increased in the same proportion.

Mechanization had a diminishing marginal rate of substitution for hired labor. This analysis indicated that the average farm could have profitably reduced the amount of labor hired and increased machinery and equipment expenses without reducing gross farm income.

The development of a management index from the residuals of the Cobb-Douglas production function estimates indicated that the quality of management on a particular farm is difficult to determine from observable characteristics. Although the management index was found to be correlated with certain efficiency factors, especially those concerning milk production, no single factor was found that could be used as an accurate indicator of managerial ability.

The production function estimated for 1973-1975 was used in conjunction with input data from 82 dairy farm records for 1976 in order to predict 1976 gross farm income. The input output relationship expressed by the production function developed for 1973-1975 data was not found to be significantly different from the input output relationship evidenced by the 1976 data. As a result it was concluded that the general guidelines developed throughout this study would have been appropriate for the 1976 period also.



TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
Objectives of the Study	5
Source of Data	6
Empirical Procedures and Methods of Analysis	7
The production function	7
Specifying the mathematical form of the production function	8
Marginal productivity	10
Returns to scale	11
Substitution of machinery and equipment for hired labor	11
Management or technical efficiency index	12
Prediction of output in 1976 using 1973-75 model	13
Previous Research	13
II. CHARACTERISTICS OF THE FARMS STUDIED	16
III. MARGINAL PRODUCTIVITY OF RESOURCES AND RETURNS TO SCALE . .	26
Empirical Procedures	26
Marginal Productivity Estimates	28
Marginal value productivity of investment in dairy cows	33
Marginal value productivity of investment in buildings and improvements	38
Marginal value productivity of man equivalents of hired labor	41

Marginal value productivity of man equivalents of unpaid labor	44
Marginal value productivity of machinery and equipment expenses	47
Marginal value productivity of operating capital	51
Marginal value productivity of acres of open land . . .	54
Efficiency of Resource Allocation and Opportunities for Improvement.	57
Returns to Scale	59
IV. SUBSTITUTION OF MACHINERY AND EQUIPMENT EXPENSES FOR HIRED LABOR IN PRODUCTION	61
V. DEVELOPMENT OF A MANAGEMENT INDEX	68
Estimation of a Management Index	68
Management Index Analysis	69
Machinery and equipment cost per man equivalent and the management index	70
Number of cows per man equivalent and the management index	73
Pounds of milk per man equivalent and the management index	73
Acres of open land per man equivalent and the management index	73
Investment per man equivalent and the management index .	74
Pounds of milk per cow and the management index	74
Milk sales per cow and the management index	75

CHAPTER

PAGE

Milk sales per acre of open land and the management index	75
Investment per acre of open land and the management index	76
VI. PREDICTION OF 1976 GROSS FARM INCOME	77
Comparison of Data for 1976 and 1973-75	78
Results of Prediction	80
VII. SUMMARY AND CONCLUSIONS	85
Summary	85
Conclusions	89
BIBLIOGRAPHY	93
VITA	95

LIST OF TABLES

TABLE		PAGE
I.	Average Investment on Tennessee Dairy Farms Studied, 1973-75 .	18
II.	Major Expenses on Tennessee Dairy Farms Studied, 1973-75 . . .	19
III.	Income Factors on Tennessee Dairy Farms Studied, 1973-75 . . .	21
IV.	Measures of the Size of Business on Tennessee Dairy Farms Studied, 1973-75	22
V.	A Comparison of Efficiency and Production Measures for Tennessee Dairy Farms Studied, 1973-75	24
VI.	Comparison of Milk and Dairy Feed Prices in the United States, 1973-75	25
VII.	Estimated Resource Productivity on Grade A Dairy Farms Studied, Using the Cobb-Douglas Production Function, by Time Periods	29
VIII.	Estimated Marginal Value Productivities of Seven Resource Inputs Used on Grade A Dairy Farms Studied, by Time Periods	32
IX.	Arithmetic and Geometric Mean Input Levels, by Time Periods	34
X.	Estimated Marginal Value Product of Varying Levels of Investment in Dairy Cows on Grade A Dairy Farms Studied, 1973-75	36
XI.	Estimated Marginal Value Product of Varying Levels of Investment in Buildings and Improvements on Grade A Dairy Farms Studied, 1973-75	39

TABLE

PAGE

XII.	Estimated Marginal Value Product of Varying Levels of Man Equivalents of Hired Labor on Grade A Dairy Farms Studied, 1973-75	42
XIII.	Estimated Marginal Value Product of Varying Levels of Man Equivalents of Unpaid Labor on Grade A Dairy Farms Studied, 1973-75	45
XIV.	Estimated Marginal Value Product of Varying Levels of Machinery and Equipment Expenses on Grade A Dairy Farms Studied, 1973-75	48
XV.	Estimated Marginal Value Product of Varying Levels Of Operating Capital on Grade A Dairy Farms Studied, 1973-75 .	52
XVI.	Estimated Marginal Value Product of Varying Levels of Acres of Open Land on Grade A Dairy Farms Studied, 1973-75	55
XVII.	Estimated Marginal Value Productivity and Cost of Resource Inputs on Grade A Dairy Farms Studied, 1973-75	58
XVIII.	Varying Combinations of Machinery Cost and Hired Labor in Producing a Given Output, 1973-75	63
XIX.	Correlation of Efficiency Measures with the Management Index for 232 Tennessee Grade A Dairy Farms for 1973-75	71
XX.	Relationship Between Management Index and Efficiency Factors on 232 Grade A Dairy Farms in Tennessee, 1973-75	72
XXI.	A Comparison of Input and Output Data on Selected Tennessee Dairy Farms in 1973-75 and 1976	79

LIST OF FIGURES

FIGURE		PAGE
1.	Estimated Marginal Value Productivity of Investment in Dairy Cows Holding Other Inputs Constant at their Geometric Means, Using 1973-75 Estimating Equation (Equation D)	37
2.	Estimated Marginal Value Productivity of Investment in Buildings and Improvements Holding Other Inputs Constant at their Geometric Means, Using 1973-75 Estimating Equation (Equation D)	40
3.	Estimated Marginal Value Productivity of Man Equivalents of Hired Labor Holding Other Inputs Constant at their Geometric Means, Using 1973-75 Estimating Equation (Equation D)	43
4.	Estimated Marginal Value Productivity of Man Equivalents of Unpaid Labor Holding Other Inputs Constant at their Geometric Means, Using 1973-75 Estimating Equation (Equation D)	46
5.	Estimated Marginal Value Productivity of Net Machinery and Equipment Costs Holding Other Inputs Constant at their Geometric Means, Using 1973-75 Estimating Equation (Equation D)	49
6.	Estimated Marginal Value Productivity of Operating Capital Holding Other Inputs Constant at their Geometric Means, Using 1973-75 Estimating Equation (Equation D) . . .	53

FIGURE

PAGE

7. Estimated Marginal Value Productivity of Acres of Open Land Holding Other Inputs Constant at their Geometric Means, Using 1973-75 Estimating Equation (Equation D) . . .	56
8. Combinations of Hired Labor and Mechanization Used in Producing an Annual Gross Farm Income of \$70,905, Developed from 1973-75 Data	65
9. Predicted Versus Actual Gross Farm Income for 82 Tennessee Grade A Dairy Farms in 1976 (Income in Logarithms)	82
10. Predicted Versus Actual Gross Farm Income for 82 Tennessee Grade A Dairy Farms in 1976 (Income in Dollars)	83

CHAPTER I

INTRODUCTION

Farmers in Tennessee marketed \$179,257,000 of dairy products in 1976--nearly 14 percent of total farm cash receipts of the state. In terms of cash receipts, dairying ranked third in importance in Tennessee agriculture trailing only soybeans and cattle.¹

The total milk production in Tennessee comes from two different types of dairy operations--the Grade A dairy and the manufactured milk producer. The produce of the Grade A dairy can be utilized as fluid milk or cream. The manufactured milk producers' product is used in making dairy products such as cheese, butter, nonfat dry milk, and ice cream.

The composition of the Tennessee dairy industry has undergone significant changes in recent years. In 1960, Tennessee had 5,000 Grade A producers and 25,000 manufactured milk producers. In 1976 there were 2,180 Grade A producers and 3,458 manufactured milk producers. Dairy cow numbers have also decreased during this period. Between 1960 and 1976 the number of dairy cows declined from 477,000 to 210,000 and the number of dairy cows per farm more than doubled. This probably indicates that dairymen remaining in business have increased herd size in an attempt to spread increasing fixed production costs over a larger volume of output and to increase efficiency.²

¹Tennessee Agricultural Statistics, U. S. Department of Agriculture and Tennessee Department of Agriculture Annual Bulletin T-14 (Nashville, 1977), p.7.

²D. R. Humberd and Clyde Chappell, "A Brief Resume of the Tennessee Dairy Industry" (unpublished paper for the Tennessee Agricultural Extension Service, 1975), pp. 1-5.

Even though milk production per cow per year increased from 4,600 pounds in 1960 to 9,267 in 1976, total production of milk in Tennessee has declined over 11 percent from 2,194,000,000 to 1,946,000,000 pounds. Grade A producers were responsible for 80 percent of the production in 1976 while manufactured milk producers accounted for only 20 percent.³

This study was concerned only with Tennessee Grade A dairy production. Grade A dairy farms are similar in numerous ways. Most Grade A dairies are multiple enterprise farms and most forage requirements are produced on the same farm. Normally forages used include both hay and silage although some Grade A dairy farms utilize hay as the only forage source. On most Tennessee Grade A dairies pasture is utilized for at least the spring and summer months. A part or all of the grain needs is produced on some Grade A dairy farms; however, purchase of all or most of the grain needs is not uncommon. Although most Grade A dairy farms are multiple enterprise operations, the other enterprises are usually carried on to support the dairy operation. The dairy operation usually provides most of the gross farm income through monthly or bimonthly milk receipts.

A major problem facing Grade A dairy producers is how to efficiently combine their land, labor, and capital resources in production. Inefficient allocation of resources affects not only the income of the individual producer but also reduces the income potential of the region and nation. From an economic standpoint, optimum resource

³Ibid., pp. 1-5.

allocation is attained when a resource is allocated to its highest earning alternative. For example, a unit of farm labor is not being used efficiently if it earns \$2.00 per hour in its present use but could earn \$3.00 per hour in another farm use or in another industry.

The farm operator faces the task of organizing his limited resources into a profitable operating unit. The land, labor, and various forms of capital used on Grade A dairy farms are allocated to various enterprises carried out on the farm. The relative economic returns of the different resources employed will depend on the production practices used in the various enterprises.

The marginal productivity of the different productive resources used on a farm provide the criterion for efficient resource use. The marginal physical productivity of a resource is the "additional" amount added to total output when the usage of that particular resource is increased by one unit, holding other factors constant. The marginal productivity of a resource may also be expressed in value terms and called marginal value product. In this study, output is expressed in dollar terms and, therefore, marginal value product will be the term used.

With unlimited capital a farmer can maximize his returns by using all resources up to a point where the marginal value productivity equals the cost of a unit of the resource. When only limited capital is available, a farmer can maximize returns by allocating each resource to the enterprise where its marginal value productivity is the greatest. For example, a farmer would increase his returns by investing

proportionately more of his limited capital in machinery rather than labor if the returns from an additional dollar spent on machinery was 20 percent while it yields only 10 percent if invested in labor.

Most farmers will not, however, have the empirical data necessary to make precise decisions about such allocations.

Another question facing farmers is what scale of operation is most profitable. Optimum farm size depends on the effect of size on costs of production. Much of the technological change with respect to milk production in recent years has created pressures for expanding scale of operation in order to achieve cost economies. Decisions regarding optimum scale of operation can be indicated by elasticity of production estimates. The elasticity of production is a ratio of the percentage change in output and the percentage change in inputs. This ratio can be used to indicate returns to scale when all inputs are increased by a constant percentage. If the elasticity of production is equal to one, constant returns to scale exist, meaning that a 1 percent increase in all inputs would increase output by 1 percent. If the elasticity of production is less than one, decreasing returns to scale exist, meaning that output will increase by a smaller percentage than the proportional percentage increase in inputs. If the elasticity of production is greater than one, increasing returns to scale exist and output increases by a greater percentage than the proportional percentage increase in inputs. When increasing returns to scale exist, increasing all inputs in proportion will result in lower per unit costs of production. If decreasing returns to scale exist, unit costs will rise as output level is increased. Thus estimates of

elasticity of production can serve as an indicator of optimum scale of operations.

Another important decision confronting every modern farmer is one of finding the least cost combination of labor and mechanization. Increasing levels of mechanization have been used to replace labor in dairying for decades. Efficient reliable labor that is willing to work the long hours necessary on a dairy farm is often difficult to obtain or even unavailable in the area. The dairy farmer is often urged to mechanize to avoid the costs of acquiring and managing hired labor, but at the same time he is often cautioned to guard against "overmechanization." The optimum combination of mechanization and labor depends on the substitution relationship between machinery and labor, and the relative prices of these inputs. The substitution relationship is a measure of how much labor can be replaced by an additional unit of machinery and equipment in order to produce a given output level. The least cost combination of labor and mechanization can be found where the cost of additional machinery and equipment just balances the cost or value in other uses of the labor replaced.

I. OBJECTIVES OF THE STUDY

The overall objective of the study was to estimate the productivity of resources used on Grade A dairy farms in Tennessee. The results of this study should provide useful guidelines about the efficiency of resource allocation at the farm level as well as providing possible empirical evidence for farm policy purposes.

The specific objectives of the study were:

1. To estimate the marginal value productivity of various resources used on Tennessee farms producing Grade A milk. Resource productivity was estimated for building investment, annual machinery and equipment costs, yearly cash operating expenses, acres of open land, investment in dairy cows, man equivalents⁴ of hired labor yearly, and man equivalents of unpaid family labor.
2. To estimate the returns to scale on Tennessee Grade A dairy farms.
3. To determine the substitution relationship between labor and mechanization in farm production on Grade A dairy farms.
4. To develop a management index for individual milk producers and to compare it with efficiency factors commonly derived from farm record analysis.
5. To use production function estimates for prediction and to compare predicted output with actual output.

II. SOURCE OF DATA

Data for the study were obtained from 241 yearly farm records available for a select group of Grade A dairy farms located in the Tennessee Valley portion of Middle and East Tennessee. Records were

⁴A man equivalent of labor is assumed to be 3,000 hours of labor.

available for the years 1973, 1974, and 1975. All the farms, from which records were available, were participants in the Agricultural Resource Development Program which is a joint program between the University of Tennessee Agricultural Extension Service and the Tennessee Valley Authority. The stated purpose of this program is "to demonstrate to Tennessee farmers, by means of a select group of farms, that net farm income can be increased by appropriate resource use adjustments."⁵

The farms selected for this program receive assistance from extension personnel in planning their operations and in keeping accurate records. The farms remain on the program for six years, and as a result, one farm may have as many as three records included in the data used for this study.

III. EMPIRICAL PROCEDURES AND METHODS OF ANALYSIS

Estimates derived in this study rely upon microeconomic theory in general and specifically upon the concept of the production function. This concept is central in deriving the relationships referred to in the objectives.

The Production Function

The term production function refers to the relationship between inputs of resources and an output per unit of time. For example, the relationship between the amount of grain, grass, and water a dairy cow consumes per day and her milk output is called a production

⁵Personal Correspondence, from Clark D. Garland, Associate Professor of Extension Agricultural Economics, Tennessee Agricultural Extension Service, December, 1977.

function. This relationship could be described in general mathematical terms as $Y = f(a, b, c)$ where Y is the output and a , b , and c , are the inputs.

For this study the hypothesized production function included one dependent variable and seven independent variables and could be written in the following general form: $Y = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7)$. Y was the dependent variable and represented yearly gross farm income. The resource inputs were investment in cows (X_1), investment in buildings (X_2), man equivalents of labor hired yearly (X_3), man equivalents of available unpaid family and operator labor yearly (X_4), annual machinery costs (X_5), annual operating capital (X_6), and acres of open land (X_7). Estimates of marginal productivity of the seven different farm resources were made using a combination of cross-sectional and time series observations derived from farm records of the Grade A dairy farms described above.

Specifying the Mathematical Form of the Production Function

Two principal tools are used in deriving production functions from actual empirical data. Economic theory and statistics can be used to obtain a specific expression of the production function. Several types of mathematical equations could be used and the proper form depends to a considerable extent on the kind of use to be made of the estimates.

The specific equation selected should conform to the underlying logic of the relationship to be analyzed. If diminishing marginal returns were hypothesized, then a linear type of equation should not be used; rather an equation such as an exponential equation would be more

appropriate since it is capable of exhibiting diminishing returns. A major statistical criterion often used in selecting the type of equation was the goodness of fit between the equation and the data. Goodness of fit can be indicated by such statistical measures as R^2 , t, and F tests for reliability of estimated coefficients. While goodness of fit is a useful criterion, it must be used in conjunction with other criteria. R^2 can often be increased by adding interaction terms and using higher degree polynomials; however, each term added reduces degrees of freedom and thus decreases the likelihood of identifying significant relationships. In addition, multicollinearity may become a problem as the number of variables is increased and standard errors of estimates may become quite large.

The Cobb-Douglas function was selected as the basis for production function analysis in this study. Since previous studies have frequently used the Cobb-Douglas function, comparisons with the present study were easier to make. Additional reasons for choosing the Cobb-Douglas function included: 1) the ease of obtaining estimates of elasticities of production with respect to the resources used; 2) this form of production function permits diminishing marginal returns without using too many degrees of freedom; and 3) since the Cobb-Douglas equation is linear when transformed into logarithmic form, the method of least squares could be used for estimation.⁶

⁶Gerhard A. Tintner, "A Note on the Derivation of Production Functions from Farm Records," Econometrica, 12:26-34, 1944.

The Cobb-Douglas function is a power function having the following general form:

$$Y = aX_1^{b_1}X_2^{b_2}\dots X_n^{b_n}$$

where:

Y = refers to output

a = refers to a positive constant

X's = refers to inputs

b's = are the estimated coefficients of production and for this function also the elasticity of production of a particular factor.

The logarithmic form of the Cobb-Douglas function is of the following general form:

$$\log Y = \log a + b_1 \log X_1 + b_2 \log X_2 + \dots b_n \log X_n.$$

This was the form of the Cobb-Douglas function used for statistical estimation of the parameters.

Marginal Productivity

If output is expressed in value terms in the Cobb-Douglas function, the marginal value productivity of a resource input can be expressed as:

$$MVP_{X_i} = \left(\frac{b_i}{X_i} \right) \cdot \hat{Y}.$$

MVP_{X_i} refers to the marginal value of product of an additional unit of input X_i holding all other input levels constant. The individual b's indicate the elasticity of production for each input, and the individual X's refer to the level of input use. $\hat{Y}$ is the predicted

output as determined by using the Cobb-Douglas function. The MVP of a given input depends on the level of other inputs. In this study marginal value productivity of particular inputs was estimated for the years 1973, 1974, 1975, and the 1973-75 period combined with other inputs held constant at both the geometric and arithmetic mean levels.

Returns to Scale

Returns to scale were also estimated using the Cobb-Douglas production function. The input coefficients, the b's, indicate the elasticity of production for each input and the sum of these b's indicates the total elasticity of production. If the $\sum b's < 1$, the function exhibits decreasing returns to scale; if the $\sum b's = 1$, the function exhibits constant returns to scale; and if the $\sum b's > 1$, the function exhibits increasing returns to scale. Returns to scale were calculated for Grade A Tennessee dairy farms for 1973, 1974, 1975, and for the period 1973-75.

Substitution of Machinery and Equipment for Hired Labor

The Cobb-Douglas function was also used in estimating the substitution relationship of mechanization for labor. The marginal rate of substitution of machinery and equipment for labor was derived as follows:

$$MRS_{x_2 \text{ for } x_1} = - \frac{b_2}{b_1} \cdot \frac{x_1}{x_2}$$

where:

$MRS_{x_2 \text{ for } x_1}$ = marginal rate of substitution of machinery and equipment for labor

- b_2 = the elasticity of production of the machinery and equipment input
- b_1 = the elasticity of production of the labor input
- x_1 = the level of input use of labor per year
- x_2 = the level of input use of machinery and equipment per year.

Net machinery cost per year was used to measure the level of annual machinery and equipment input. Man equivalents of hired labor were converted into hours of hired labor per year to serve as the labor input. The substitution relationship was estimated using the production function estimated for the 1973-75 period.

Management or Technical Efficiency Index

Management could be considered as an input into the theoretical production function; however, because of the difficulty of quantifying the management input, it was not included in the estimation of the production function. An alternative approach to estimating the relative management ability for each individual farm was developed using the residuals of the estimates of the Cobb-Douglas function for the 1973-75 period. A management or technical efficiency index was developed by dividing actual output by predicted output (found by using the parameters of the estimated statistical function) and multiplying by 100. The resulting index gave a relative indication of management level on each individual farm, and index of 100 meaning that management was average; an index of < 100 meaning that management was below average; and an index of > 100 meaning that

management or technical efficiency was above average on that farm relative to others in the study. Correlations were then calculated between this index and efficiency factors commonly estimated from farm record analysis. The efficiency factors used included those related to labor efficiency, milk production, and investment.

Prediction of Output in 1976 Using 1973-75 Model

The coefficients estimated from fitting the Cobb-Douglas function to 1973-75 data were used with 1976 input data to predict output in 1976. The R^2 derived from this estimation was used as one measure of the accuracy of prediction and indirectly to indicate whether significant changes had occurred in input-output relationships from the 1973-75 period to 1976. An F statistic was also calculated to determine if there was a significant difference in the explanatory power of the model (and thereby the input-output relationship) in the 1973-75 period as opposed to 1976. Predicted and actual outputs were also compared graphically.

IV. PREVIOUS RESEARCH

Numerous production function studies have been made using the Cobb-Douglas equation. A few of these studies were similar to the present study and, therefore, could be used for comparison.

Tintner and Brownlee found diminishing returns to scale in dairying in a Cobb-Douglas production function study. In their 1944 study they found the input category of cash operating expenses to have the highest marginal productivity per dollar of input and the inputs buildings and improvements to have the lowest marginal productivity per dollar of input use. Their results indicated that the

dairy farms in their study were, on the average, overimproved and that additional inputs of liquid and working assets combined with additional expenditures of operating expenses would yield higher returns than additional farm buildings or other improvements.⁷

Heady used the Cobb-Douglas equation in a production function study of Iowa dairy farms in 1946. He found the marginal productivity per dollar of input to be highest for the livestock and feed inputs and lowest for the land input. Diminishing returns to scale were found on the dairy farms he studied.⁸

In 1955, Plaxico observed that bias of the production function coefficients may result from improper aggregation of inputs or products. He suggested that inputs which were perfect complements and, therefore, would be used in fixed proportions, should be treated as one resource. Also, if two inputs were perfect substitutes, having exactly the same effect on production, then they should be weighted and combined into one input category. With regard to product aggregation, he found that the aggregation of products into a single output category could be accomplished without bias only when the set of products were produced in fixed proportions. Because of aggregation problems, Plaxico recommended caution in using Cobb-Douglas production function studies

⁷G. A. Tintner and O. H. Brownlee, "Production Functions Derived from Farm Records," Journal of Farm Economics, 26:566-571, 1944.

⁸Earl O. Heady, "Production Functions from a Random Sample of Farms," Journal of Farm Economics, 28:989-1004, 1946.

as an aid in suggesting intrafarm adjustments. He also questioned the validity of such studies for farm policy recommendations.⁹

Spangler used the Cobb-Douglas equation to study production relationships in Tennessee Grade A dairying in 1968. He found equipment investment to have the highest marginal value product per dollar of input use and livestock investment to have the lowest marginal value of product per dollar of input use. He also found increasing returns to scale on Grade A dairies.¹⁰

⁹J. S. Plaxico, "Problems of Factor Product Aggregation in Cobb-Douglas Value Productivity Analysis," Journal of Farm Economics, 37:664-675, 1955.

¹⁰H. L. Spangler, "Economics Analysis of Commercial Farming in the Elk River Watershed" (unpublished Master's thesis, University of Tennessee, 1968), pp. 49-55.

CHAPTER II

CHARACTERISTICS OF THE FARMS STUDIED

The Grade A dairy farms utilized in this study were located in Middle and East Tennessee. Since the farms were especially selected to participate in the Agricultural Resource Development Program, they would not necessarily be representative. The sample of farms used did, however, represent a wide variation in size of herds, types of enterprises, and resource combinations used. As a result, analysis should provide a rough indicator of resource productivity on Grade A dairy farms in general and be indicative of needed adjustments in resource combinations.

The farms in the study had wide variations in size of operation, number of man equivalents of labor employed, capital investments, and gross farm incomes. The number of dairy cows per farm ranged from a high of 208 cows to a low of 18 cows. The number of open acres per farm ranged from a high of 760 to a low of 23. Man equivalents of labor employed in production per year ranged from one to nine. Total value of investments per farm ranged from a high of \$569,000 to a low of \$29,000. Yearly gross farm income ranged from \$10,000 to \$312,000. The common factor on all the farms in the study was that 50 percent or more of gross farm income came from the dairy enterprise.

In order to provide a more detailed description of the sample of farms used in the study, characteristics of the farms were summarized for three size groups. Among the 241 records, 95 were for

farms with less than 50 cows, 88 were for farms with 50-79 cows, and 58 were for farms with 80 cows or more.

A summary of investments by type and size groups is shown in Table I. Investment in cows ranged from 16.5 to 18.5 percent of total investment among the size groups. Machinery and equipment varied from 14.3 percent to 14.7 percent of total investment while buildings and improvements investments ranged from 10.9 to 17.0 percent. Total investment averaged \$163,158 for all farm--about \$2,550 on a per cow basis or about \$69,500 expressed in terms of per man equivalent. Across size groups, investments increased in a roughly proportional manner.

Investment per cow was slightly lower for the farms having 80 or more cows which may indicate a more efficient use of nondivisible resources. Investment per man equivalent was highest for those farms having 80 or more cows, thus indicating the tendency for the larger dairy farms to substitute more capital for labor.

Major expense items for the farms in the study are summarized by the size groups in Table II. Concentrated feed purchased composed between 29.3 and 32.2 percent of total farm expenses in the various size groups. Fertilizer purchased averaged 7.0 percent of total farm expenses for all farms and varied from 6.7 percent of total expenses on farms with 80 cows or more to 7.7 percent on farms with less than 50 cows. Net machinery cost accounted for 18.9 percent of total farm expenses on all farms and tended to decline as a proportion of all expenses as herd size increased. Hired labor cost

TABLE I
AVERAGE INVESTMENT ON TENNESSEE DAIRY FARMS STUDIED, 1973-75

Item	Investment Per Farm			
	All Farms	Farms with Less Than 50 Cows	Farms with 50-79 Cows	Farms with 80 or More Cows
	-----dollars-----			
Investment in:				
Cows	28,783	15,689	28,343	50,602
Machinery and equipment	23,557	13,926	23,410	39,554
Buildings and improvements	21,520	10,306	27,888	37,965
Other	89,298	54,823	84,476	145,639
Total ^a	163,158	94,744	164,117	273,760
Investment per:				
Cow	2,547	2,570	2,604	2,421
Man equivalent ^b	69,531	62,158	68,902	82,560
Open acre	829	705	933	873

^aTotal investment = value of machinery and equipment + value of dairy cows and calves + value of other livestock + value of feed and supplies on hand + value of land, buildings, and improvements excluding the value of the family dwelling.

^bA man equivalent was assumed to be 3,000 hours of available labor.

TABLE II

MAJOR EXPENSES ON TENNESSEE DAIRY FARMS STUDIED, 1973-75

Expense Item	Expense Per Farm			
	All Farms	Farms with Less Than 50 Cows	Farms with 50-79 Cows	Farms with 80 or More Cows
	-----dollars-----			
Concentrated feed	20,580	10,720	19,343	38,415
Fertilizer	4,637	2,846	4,338	7,958
Net machinery cost ^a	12,471	7,457	12,462	20,696
Hired labor	4,563	1,516	3,978	10,098
Other	23,697	14,063	22,316	42,173
Total	65,948	36,602	62,437	119,340

^aNet machinery cost = depreciation + interest at 5 percent of average investment + gasoline and oil + repairs and maintenance + hauling expenses + machine hire + farm share of auto expenses - receipts from custom work.

accounted for 4.1 percent of total costs on farms with less than 50 cows, but composed 8.5 percent of all expenses on farms with 80 or more cows.

Major income factors for the farms in the study were summarized in Table III. Cash receipts composed between 84 and 86 percent of gross farm income for all size categories. Gross milk sales provided 63 to 70 cents of each dollar of gross farm income for all herd size categories. The farms with 80 or more cows had the highest milk sales per cow, indicating either higher producing cows, higher prices for milk, or a combination of both. Gross farm income averaged about \$83,000 for all farms in the three year period and increased roughly proportional to cow numbers across the size groups (about \$1,250 per milk cow). Both family farm income and operator's labor income increased significantly as herd size increased. Operator's labor income averaged \$7,950 for the farms with less than 50 milk cows and \$19,550 for the farms with 80 or more milk cows.

Size characteristics of the study farms are shown in Table IV. Farms with less than 50 cows had an average of 38 milk cows per farm and used an average of 1.7 man equivalents of labor per year (about 22 cows per man equivalent). Farms with 80 or more cows had an average of 116 milk cows per farm and 3.8 man equivalents of labor (about 30 cows per man equivalent). On the larger dairy farms the capital-labor ratio was somewhat higher. Farms with less than 50 cows had an investment of \$62,158 per man equivalent compared to \$82,560 on farms with 80 or more cows. As herd size increased, an increasing proportion of the total labor supply was hired. On

TABLE III
INCOME FACTORS ON TENNESSEE DAIRY FARMS STUDIED, 1973-75

	Income Per Farm			
	All Farms	Farms with Less Than 50 Cows	Farms with 50-79 Cows	Farms with 80 or More Cows
	-----dollars-----			
Cash receipts	70,691	40,318	67,862	124,734
Milk sales	56,124	30,019	54,618	101,960
Milk sales per cow	840	791	860	892
Milk sales per acre of open land	298	241	336	334
Gross farm income ^a	82,987	47,982	79,807	145,149
Family farm income ^b	17,125	11,667	17,375	25,670
Operator's labor income ^c	11,881	7,950	11,068	19,550

^aGross farm income = all cash receipts + or - change in inventory.

^bFamily farm income = gross farm income - total farm expenses.

^cOperator's labor income = family farm income - interest charged on average investment - value of unpaid family labor.

TABLE IV
MEASURES OF THE SIZE OF BUSINESS ON TENNESSEE DAIRY FARMS STUDIED,
1973-1975

	Averages Per Farm			
	All Farms	Farms with Less Than 50 Cows	Farms with 50-79 Cows	Farms with 80 or More Cows
Man equivalents ^a of total labor	2.6	1.7	2.7	3.8
Man equivalents ^a of hired labor	.8	.3	.7	1.8
Man equivalents ^a of unpaid labor	1.8	1.4	2.0	2.0
Acres of open land	219	146	224	330
Number of cows	66	38	63	116
Hay acreage ^b	43 (89)	32 (88)	40 (95)	67 (81)
Pasture acreage ^b	101 (98)	74 (100)	102 (98)	145 (95)
Corn acreage ^b	36 (53)	25 (48)	30 (57)	60 (53)
Corn silage acreage ^b	44 (78)	27 (72)	42 (80)	68 (84)

^aA man equivalent was assumed to consist of 3,000 hours of available labor.

^bThe numbers in parenthesis indicate the percent of records in each group from which that item was calculated. These average acreages were computed only for those farms reporting the item.

CRANESEST CREST

farms with less than 50 cows about 18 percent of the labor supply was hired as compared to 47 percent on farms with 80 cows or more. However, intensity of land use was greater on the larger farms. Farms with less than 50 cows had an average of about 3.8 acres of open land per cow as compared to 2.8 acres on farms with 80 or more cows. The corn silage acreage per cow varied from 1.4 acres for the smallest herd size category to 1.7 acres for the largest herd size category.

Comparisons of efficiency and production measures for various herd size groups are given in Table V. Farms with 80 or more dairy cows had more cows per man equivalent and produced more milk per cow than those farms having fewer than 50 cows. This was due, at least in part, to higher machinery and equipment costs per man equivalent per year for the larger dairy herds. As shown in Table V, farms with between 50 and 79 cows reported a higher milk production per cow than the farms in the largest herd size category, but milk sales per cow were lower indicating that farms in the intermediate herd size category received an average of \$.32 more per hundred pounds of milk than the farms in the largest herd size category.

Some perspective of the profitability of the Tennessee dairy industry during the study period can be gained by examining milk price and feed relationships shown in Table VI. Based on a comparison of milk and feed prices, 1973 appeared to be the most profitable year for Grade A dairying of the three years in the study period. Milk prices increased over 21 percent from 1973 to 1975, but the prices paid for dairy rations increased over 28 percent. Prices paid for other inputs used on Grade A dairy farms increased in a similar manner during the period.

TABLE V

A COMPARISON OF EFFICIENCY AND PRODUCTION MEASURES FOR TENNESSEE DAIRY FARMS STUDIED, 1973-75

	Averages Per Farm			
	All Farms	Farms with Less Than 50 Cows	Farms with 50-79 Cows	Farms with 80 or More Cows
Open acres PME ^a	97	96	90	107
Machinery and equipment cost PME ^a	5,418	4,862	5,295	6,515
Number of cows PME ^a	28	25	27	35
Pounds of milk PME ^a	276,732	235,455	280,737	338,264
Gross farm income PME ^a	35,135	31,242	33,622	43,805
Pounds of milk per cow	9,807	9,387	10,082	10,078
Milk sales per cow	840	791	860	892
Milk sales per acre of open land	298	241	337	334
Hay (tons/acre) ^b	2.5 (89)	2.3 (88)	2.4 (95)	2.8 (81)
Corn (bushels/acre) ^b	77.6 (53)	71.5 (48)	79.3 (57)	84.1 (53)
Corn silage (tons/acre) ^b	16.5 (77)	16.7 (72)	16.4 (77)	16.1 (84)

^aPME = per man equivalent where a man equivalent was assumed to consist of 3,000 hours of available labor.

^bThe numbers in parenthesis indicate the percent of records in each group from which that item was calculated. These average yields were computed only for those farms reporting the item.

TABLE VI
COMPARISON OF MILK AND DAIRY FEED PRICES IN THE UNITED STATES,
1973-75^a

	Prices Received Per 100 Pounds of Milk ^b	Averages Prices Paid Per 100 Pounds of Dairy Ration ^c	Prices Paid as a Percent of Prices Received
	-----dollars-----		----percent----
1973-75	8.37	5.79	69
1973	7.42	4.88	66
1974	8.66	6.23	72
1975	9.02	6.25	69

^aSource: Dairy Situation, U.S. Department of Agriculture, Economic Research Service, December, 1977, p.6.

^bRefers to milk eligible for fluid consumption.

^cPrice based on 16 percent protein dairy ration.

CRANE & CO. CREST

CHAPTER III

MARGINAL PRODUCTIVITY OF RESOURCES AND RETURNS TO SCALE

A major objective of this study was to estimate the productivity of resources used on a selected group Grade A dairy farms in Tennessee. Four different sets of productivity estimates were derived from farm record data based on the years 1973, 1974, 1975, and 1973-75 combined. The estimates were derived for a total of 241 farm records,¹¹ including 77 farms in 1973, 85 farms in 1974, and 79 farms in 1975.

I. EMPIRICAL PROCEDURES

Most of the later analysis depends upon the reliability of the estimation of a suitable production function for the farms in the study. The general form of the Cobb-Douglas equation used for this estimation was the following:

$$Y = aX_1^{b_1}X_2^{b_2}X_3^{b_3}X_4^{b_4}X_5^{b_5}X_6^{b_6}X_7^{b_7}$$

The variables used were:

Y = total gross farm income per year

X_1 = investment in dairy cows

X_2 = investment in buildings and land improvements

X_3 = man equivalents of labor hired yearly

¹¹Due to the fact that farms participate in the Agricultural Resource Development Program for six years, there were not 241 different farms in the study. There were 241 yearly farm records, however.

X_4 = man equivalents of available unpaid family and operator labor yearly

X_5 = net machinery and equipment cost per year

X_6 = operating capital used per year

X_7 = acres of open land.

The total labor input was divided into paid and unpaid labor categories based on the hypothesis that unpaid family and operator labor might have a higher marginal productivity than hired labor. In addition, varying the two forms of labor represents a separate management decision. Net machinery and equipment cost per year includes depreciation, interest, oil and fuel costs, repairs and maintenance costs, farm share of auto expenses, and machinery work hired less 50 percent of receipts from custom work done. Operating capital contains all farm expenses except net machinery cost and hired labor expense. Acres of open land was used as an input rather than investment in land due to difficulties in obtaining accurate investment data. Acres of open land includes all open land that could be used for pasture or row crops.

This analysis combined cross sectional and time series data. It was hypothesized that a dummy variable should be included to account for differences in data due to different years. To test this hypothesis, a production function was estimated for 1973-1975 using the above variables and yearly dummy variables to account for differences in gross farm income due to time. The dummy variables were highly insignificant in the estimated function as determined by t-tests, and

the coefficients of the other variables were found to vary little whether the dummy variables were included or not. As a result, the effect of time was assumed to have little if any effect on gross farm income and the yearly dummy variables were deleted from the model.

II. MARGINAL PRODUCTIVITY ESTIMATES

The production functions estimated were as follows¹²:

Equation A (1973) -

$$Y = 1.5340X_1^{0.1316}X_2^{0.0168}X_3^{0.0346}X_4^{-0.0120}X_5^{0.3036}X_6^{0.4662}X_7^{0.0891}$$

Equation B (1974) -

$$Y = 2.7806X_1^{0.1940}X_2^{-0.0453}X_3^{0.0636}X_4^{-0.0644}X_5^{0.2477}X_6^{0.3972}X_7^{0.0896}$$

Equation C (1975) -

$$Y = 1.8370X_1^{0.1434}X_2^{0.0373}X_3^{0.0301}X_4^{0.0962}X_5^{0.2252}X_6^{0.4326}X_7^{0.1626}$$

Equation D (1973-75) -

$$Y = 2.0752X_1^{0.1474}X_2^{0.0108}X_3^{0.0313}X_4^{0.0379}X_5^{0.2560}X_6^{0.4312}X_7^{0.1099}$$

Regression coefficients and coefficients of determination for the four equations are compared in Table VII. The negative input coefficients for investment in buildings and unpaid operator and family labor found in Equation A and B are insignificant at the 15 percent level as determined by t-tests. These negative coefficients indicate negative returns to the additional use of these inputs.

¹²The Cobb-Douglas function was converted to logarithmic form to allow estimation by the least-squares method. Results are reported to four decimal places to avoid rounding error problems in later calculations.

TABLE VII

ESTIMATED RESOURCE PRODUCTIVITY ON GRADE A DAIRY FARMS STUDIED, USING THE COBB-DOUGLAS PRODUCTION FUNCTION, BY TIME PERIODS

Resource Input	Equation A (1973)		Equation B (1974)		Equation C (1975)		Equation D (1973-75)	
	Coeffi- cient	Signif- icance Level	Coeffi- cient	Signif- icance Level	Coeffi- cient	Signif- icance Level	Coeffi- cient	Signif- icance Level
Investment in dairy cows	0.1316	.06	0.1940	.01	0.1434	.07	0.1474	.01
Investment in buildings and improvements	0.0168	.68	-0.0453	.19	0.0373	.26	0.0108	.58
Man equivalents of hired labor	0.0346	.12	0.0636	.01	0.0301	.27	0.0313	.01
Man equivalents of unpaid labor	-0.0120	.88	-0.0644	.43	0.0962	.13	0.0379	.19
Machinery and equipment expense	0.3036	.01	0.2477	.01	0.2252	.01	0.2560	.01
Operating capital	0.4662	.01	0.3972	.01	0.4326	.01	0.4312	.01
Open acres of land	0.0891	.12	0.0896	.10	0.1626	.02	0.1099	.01
Elasticity of production	1.0299		0.8824		1.1274		1.0245	
	$R^2 = .90$		$R^2 = .89$		$R^2 = .87$		$R^2 = .88$	

These negative coefficients may have resulted from failures to accurately determine the investment in buildings, and the unpaid operator and family labor actually used in production.

The production functions derived from the farm record data directly indicate the elasticity of production for the different resources. The elasticity of production is the percent increase in output due to a 1 percent increase in the input level of a given resource, holding other input levels constant. The elasticities of production of the various resources are indicated by the coefficients (or b values) of the variables given in the Table VII. For example the elasticity of production for the 1973-75 period of the input X_1 (investment in dairy cows) is 0.1474. This indicates that a 1 percent increase in X_1 would result in a 0.1474 increase in gross farm income.

An increase in gross farm income of less than .1 percent for an increase of 1 percent in a input implies a diminishing marginal product for that input. All of the inputs in each equation exhibited diminishing or negative marginal productivity as indicated by the elasticities of production. The elasticity of production for investment in dairy cows varied from a high of 0.1940 in 1974 to a low of 0.1316 in 1973. The investment in buildings and improvements had an elasticity of production that varied from positive to negative but was positive for Equation D (1973-75) at 0.0108. The elasticity of production of man equivalents of hired labor ranged from 0.0636 to 0.0301 in 1975 and was 0.0313 for the entire 1973-75 period. The elasticity of production for the unpaid labor input ranged from negative to positive values, but was 0.0379 in Equation D (1973-75).

For Equation D (1973-75), the elasticity of production for machinery and equipment was 0.2560 and the elasticity of production for operating capital was 0.4312. The elasticity of production for the input land was 0.1099 for the 1973-75 period but was slightly less in 1973 and 1974. In 1975, the elasticity of production for open acres of land was 0.1626, or nearly twice its value in 1973 and 1974.

Coefficients of determination ranged from .87 for Equation C to .90 for Equation A, indicating a high degree of explanatory power for all of the separate models. All of the coefficients of determination were significant at the .0001 level as determined by F tests.

Input coefficients estimated for the Cobb-Douglas equations were used to compute marginal value productivities for each input at both the geometric and arithmetic mean levels of the variables. The marginal value productivities can be expressed as:

$$MVPX_i = \left(\frac{b_i}{X_i} \right) \cdot \hat{Y}$$

where $MVPX_i$ refers to the marginal value product of an additional unit of input X_i , holding other input levels at some specified level. The b_i 's are the input coefficients reported in Table VII. The X_i are the input levels and Y is the gross farm income as estimated from the Cobb-Douglas equation.

Table VIII compares the marginal value productivities of the inputs calculated: (1) at the geometric mean input levels and (2) at the arithmetic mean input levels. The geometric and arithmetic mean levels of the inputs for each time period (1973, 1974, 1975, and 1973-75) as well as the corresponding gross farm incomes are

TABLE VIII

ESTIMATED MARGINAL VALUE PRODUCTIVITIES OF SEVEN RESOURCE INPUTS USED ON GRADE A DAIRY FARMS STUDIED,
BY TIME PERIODS

Resource Input	At Arithmetic Means				At Geometric Means			
	Equa- tion A (1973)	Equa- tion B (1974)	Equa- tion C (1975)	Equa- tion D (1973- 75)	Equa- tion A (1973)	Equa- tion B (1974)	Equa- tion C (1975)	Equa- tion D (1973- 75)
	-----dollars-----							
Investment in dairy cows	.40	.60	.44	.45	.39	.58	.41	.43
Investment in build- ings and improve- ments	.07	-.21	.13	.04	.08	-.23	.19	.05
Man equivalents of hired labor	3,468	6,805	3,295	3,281	6,655	11,508	6,077	5,976
Man equivalents of unpaid labor	-485	-2,862	6,850	1,854	-414	-2,599	6,599	1,692
Machinery and equip- ment expense	2.40	1.76	1.46	1.80	2.20	1.71	1.36	1.71
Operating capital	.85	.67	.80	.76	.86	.70	.78	.78
Acres of open land	31	39	67	44	30	38	61	42

presented in Table IX. The marginal value productivities computed at the geometric mean input and output levels are generally considered to be more reliable since the Cobb-Douglas function was converted to logarithmic form before estimation by the least squares method. Since the Cobb-Douglas function was linear in logarithmic form, the function was forced through the geometric mean levels of inputs. The marginal value productivities computed at the arithmetic mean input and output levels were included to show the contrast between the marginal productivities computed at the different means.

The marginal value productivities for the inputs may be interpreted as follows: at the geometric mean in Equation A (Table VIII) an additional dollar invested in dairy cows would return \$.39 in additional gross farm income, holding all other inputs constant at their geometric means. The marginal value productivities of the other resources can be interpreted in a similar manner as the increase in gross farm income that would result from a one unit increase in a particular input. All inputs except land and the two types of labor were expressed in dollar terms. Land was expressed as acres of open land and labor as man equivalents.

In the following sections, marginal value productivity will be referred to by the abbreviation MVP. The MVP's were estimated at both the arithmetic and geometric means levels of inputs.

Marginal Value Productivity of Investment in Dairy Cows

When computed at the geometric mean, the estimated MVP of an additional dollar invested in dairy cows ranged from low of \$.39 in

TABLE IX

ARITHMETIC AND GEOMETRIC MEAN INPUT LEVELS, BY TIME PERIODS

Resource Inputs	Arithmetic Means			(1973-75)	Geometric Means			
	(1973)	(1974)	(1975)		(1974)	(1975)	(1973-75)	
Investment in dairy Cows (\$)	26,406	29,872	29,926	28,783	21,796	25,758	25,261	24,264
Investment in build-ings and improve-ments (\$)	18,571	20,131	25,834	21,520	12,842	14,994	14,488	14,110
Man equivalents of labor hired	.81	.86	.83	.83	.33	.43	.36	.37
Man equivalents of unpaid labor	2.01	2.07	1.28	1.79	1.85	1.92	1.12	1.60
Machinery and equip-ment expense (\$)	10,291	12,977	14,052	12,471	8,832	11,182	11,910	10,587
Operating capital (\$)	44,576	54,219	49,467	49,503	34,787	44,015	39,846	39,400
Acres of open land	223	213	221	219	187.	182	191	187
Gross farm income ^a (\$)	81,224	92,082	90,935	87,593	63,887	77,278	72,018	70,905

^a Gross farm income estimates at arithmetic means were calculated by inserting the arithmetic mean input levels into the appropriate estimated Cobb-Douglas function. The gross farm income estimates at the geometric means were calculated in a similar manner using the geometric mean level of inputs.

1973 to a high of \$.58 in 1974 as indicated in Table VIII, page 32. For the 1973-75 period, the estimated MVP of additional dollar invested in dairy cows was \$.43 when computed at the geometric mean of all inputs. The estimated MVP of this input was one to three cents higher for each period when calculated at the arithmetic mean level of inputs.

Table X and Figure 1 illustrate how the MVP of an additional dollar invested in dairy cows would vary as the level of investment in dairy cows was varied, holding other inputs constant at their geometric means. Diminishing marginal productivity was exhibited as the MVP declined from \$.92 with an investment in dairy cows of \$10,000 to \$.13 when the investment was increased to \$100,000.

If the availability of capital is assumed to be unlimited, the efficient allocation of a resource input occurs when the MVP of the input just equals the cost of an additional unit of that input. If an interest rate of 8 percent was charged as the price of an additional dollar invested in dairy cows, the dairy farms in the study could have profitably expanded, on the average, their investment in dairy cows. For the 1973-75 period, at the geometric mean input level (\$24,262), increased investment in dairy cows would have led to a net return of \$.35 per additional dollar invested. The net return would have been \$.37 per additional dollar invested if computed at the arithmetic mean input level (\$28,783) for the same period. As indicated in Table X, the net return would have declined as investment in dairy cows was expanded beyond the mean level.

1973 for which of 2.52 in 1974 as indicated in Table 1. The mean of the 1973-75 period, the estimated MVR of the dairy cows, was 2.42, which is almost at the same level as the 1973-75 period. The estimated MVR of 2.42 was one to three cents higher for each dollar invested in dairy cows than the arithmetic mean level of input. Table 1 and Figure 1 illustrate the MVR of an additional dollar invested in dairy cows would vary as the level of investment in dairy cows was varied, holding other inputs constant at their 1973-75 means. The MVR of an additional dollar invested in dairy cows declined from 2.52 with an investment in dairy cows of \$10,000 to 2.13 when the investment was increased to \$100,000. If the availability of capital is assumed to be unlimited, the efficient allocation of a resource input occurs when the MVR of the input just equals the cost of an additional unit of that input. If an interest rate of 8 percent was charged as the price of an additional dollar invested in dairy cows, the dairy farms in the study could have profitably expanded on the average 1.5% investment in dairy cows. For the 1973-75 period, on the average, the input level of \$135,342, increased investment in dairy cows would have led to a net return of 2.32 per additional dollar invested. The net return would have been 2.32 per additional dollar invested if computed at the arithmetic mean input level, (\$28,752) for the same period. As indicated in Table 1, the net return would have declined as investment in dairy cows was expanded beyond the mean level.

TABLE X

ESTIMATED MARGINAL VALUE PRODUCT OF VARYING LEVELS OF INVESTMENT
IN DAIRY COWS ON GRADE A DAIRY FARMS STUDIED, 1973-75

Investment in Cows (x_1) ^a	Marginal Value Product ^b
-----dollars-----	
10,000	.92
20,000	.51
30,000	.36
40,000	.28
50,000	.23
60,000	.20
70,000	.17
80,000	.16
90,000	.14
100,000	.13

^aThe geometric mean for investment in dairy cows was \$24,264 in 1973-75 while the arithmetic mean was \$28,783 for the same period.

^bOther resources were held fixed at geometric mean levels.

CRANE & CREST

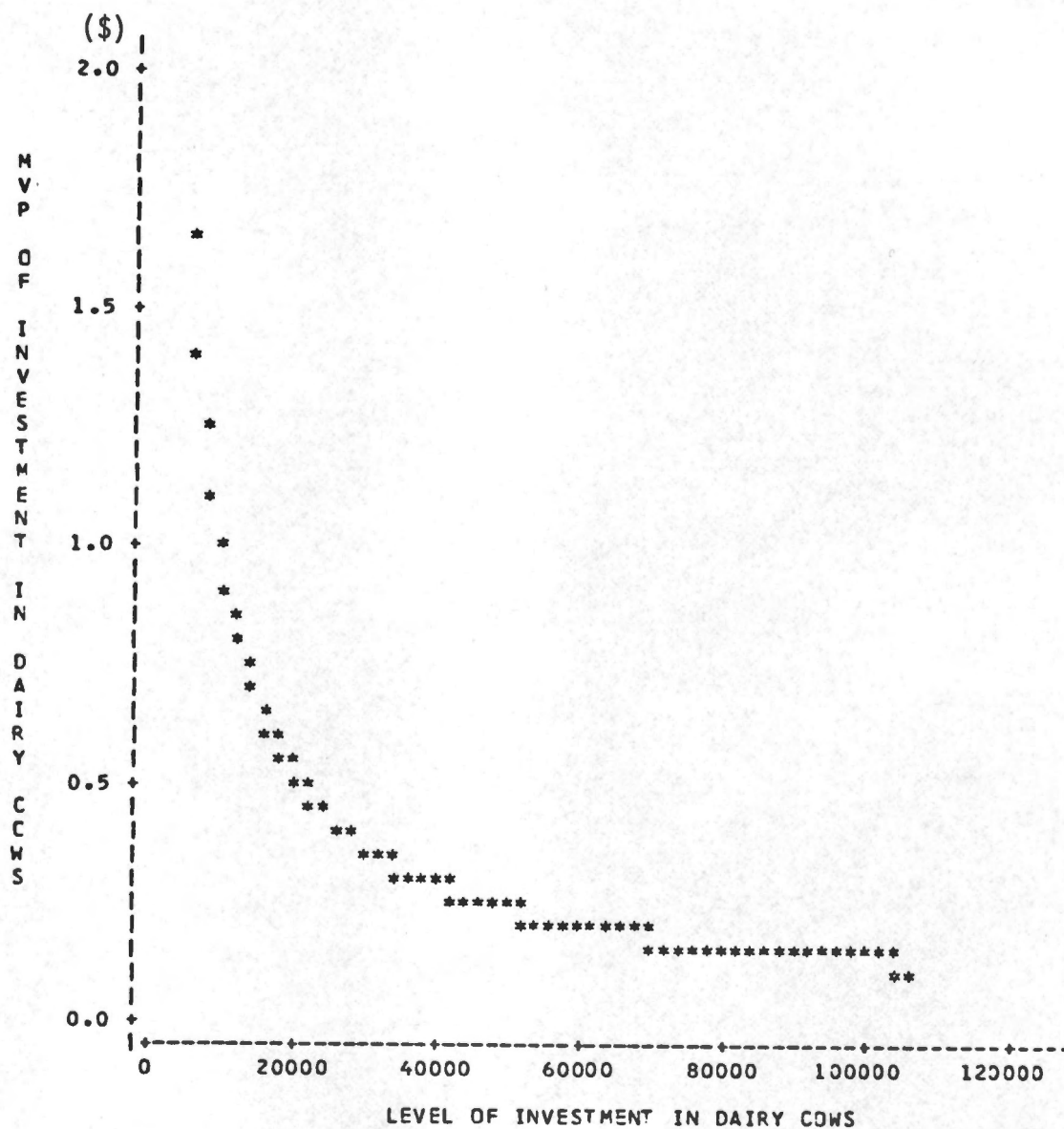


Figure 1. Estimated marginal value productivity of investment in dairy cows holding other inputs constant at their geometric means, using 1973-75 estimating equation (Equation D).

Marginal Value Productivity of Investment in Buildings and Improvements

This input category includes farm buildings, fences, and land improvements. Using geometric mean levels of inputs, the estimated MVP of buildings and improvements ranged from a high of \$.19 in 1975 to a low of \$-.23 in 1974 as indicated in Table VIII (page 32). For the 1973-75 period the estimated MVP of investment in buildings and improvements was \$.05 at the geometric mean level of inputs and \$.04 at the arithmetic mean level of inputs.

The effect of variation of investment in buildings and improvements on MVP, holding other inputs constant at the geometric mean levels, is shown in Table XI and Figure 2, using 1973-75 estimates. Diminishing marginal productivity was exhibited as the MVP of investment in buildings and improvements declined from \$.15 when investment was \$5,000 to \$.02 when investment in buildings and improvements was increased to \$50,000.

Normally, the farm operator would expect to receive a return on building and improvement investment sufficient to cover the cost of the investment. The annual cost of the investment in buildings and improvements would include depreciation, repairs, and a return on capital at least as great as the market rate of interest. If buildings and improvements are assumed to last an average of 20 years, an appropriate rate of depreciation would be 5 percent. Repairs are often expected to average 1 percent of initial total investment per year.¹³

¹³Robert M. Ray and Herbert N. Walch, Farm Planning Manual, (Agricultural Extension Service, University of Tennessee, December, 1975), p. 86.

Measuring the Productivity of Investment in Buildings and Equipment

This study, therefore, includes farm buildings, farms, and land improvements. Using geometric mean levels of inputs, the ratio of MWP of buildings and improvements turned out to be 1.19 in 1950 to 1.00 in 1955 and 1.05 in 1955 to 1.00 in 1960 (see Table VII, page 32). For the 1955-60 period the estimated MWP of investment in buildings and improvements was 1.05, the geometric mean level of inputs and 1.04 of the arithmetic mean level of inputs.

The effect of variation of investment in buildings and improvements on MWP, holding other inputs constant, is shown in Table VIII, page 33. It is shown that MWP of investment in buildings and improvements declined from 1.12 when investment was \$5,000 to 1.02 when investment in buildings and improvements was increased to \$50,000.

Normally, the farm operator would expect to receive a return on building and improvement investment sufficient to cover the cost of the investment. The annual cost of the investment in buildings and improvements would include depreciation, repairs, and a return on capital. Let us treat the market rate of interest as the return on capital and assume to last an average of 20 years; an appropriate rate of depreciation would be 5 percent. Repairs are

often expected to average 1 percent of initial investment per year.

TABLE XI

ESTIMATED MARGINAL VALUE PRODUCT OF VARYING LEVELS OF INVESTMENT
IN BUILDINGS AND IMPROVEMENTS ON GRADE A DAIRY FARMS STUDIED,
1973-75

Investment in Buildings and Improvements (X_2) ^a	Marginal Value Product ^b
-----dollars-----	
5,000	.15
10,000	.08
15,000	.05
20,000	.04
25,000	.03
30,000	.02
35,000	.02
40,000	.02
45,000	.02
50,000	.02

^aThe geometric mean for investment in buildings and improvements was \$14,110 in 1973-75 while the arithmetic mean was \$21,520 for the same period.

^bOther resources were held fixed at geometric mean levels.

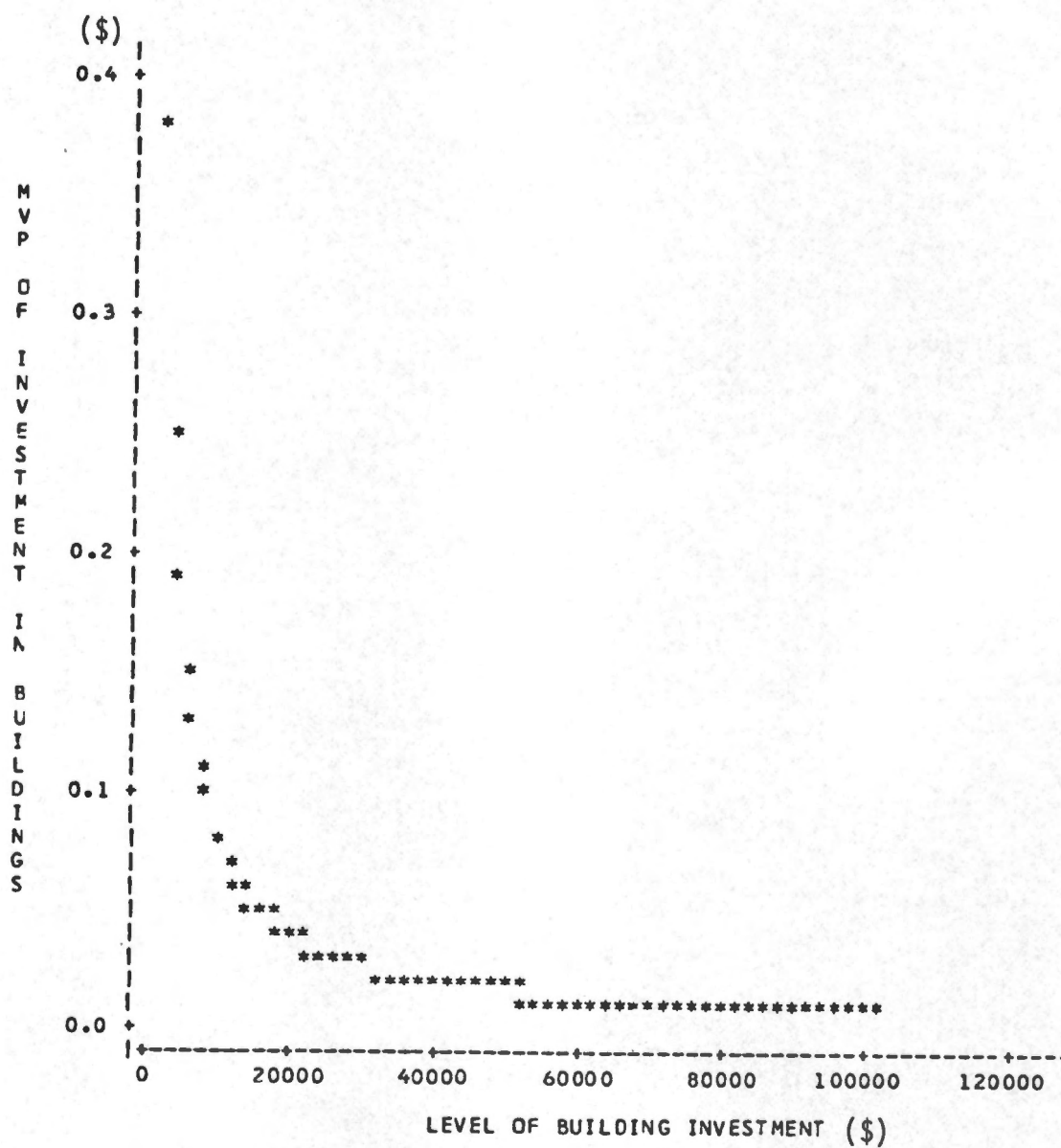


Figure 2. Estimated marginal value productivity of investment in buildings and improvements holding other inputs constant at their geometric means, using 1973-75 estimating equation (Equation D).

CRANESEST CREST

If the market rate of interest could be assumed to be 8 percent per year, the estimated total cost of a dollar investment in buildings and improvements would be \$.14. Given this cost, additional investment in buildings and improvements beyond mean levels would have resulted in a net loss for the 1973-75 period with the MVP of investment in buildings and improvements computed at either the arithmetic or geometric mean input level.

Marginal Value Productivity of Man Equivalents of Hired Labor

A man equivalent of hired labor was defined as 3,000 hours of hired labor. Using the geometric mean levels of inputs, the MVP of an additional man equivalent of hired labor was estimated to range from a low of \$6,655 in 1973 to a high of \$11,508 in 1974 as shown in Table VIII, page 32. The effect of variation in the level of hired labor on the MVP of hired labor, holding other inputs constant at their geometric means, is shown in Table XII, and Figure 3. In Table XII, MVP estimates are shown both in terms of a man equivalent and an hour of labor. Diminishing productivity of hired labor was evident as the MPV of an additional man equivalent of hired labor declined from \$20,296 when .1 man equivalent was employed, to \$532 when 4.5 man equivalents were employed. When man equivalents were converted to hours of labor, the MVP of an hour of labor declined from \$6.76 per hour to \$.18 per hour over the same range of variation.

For most profitable resource allocation, the labor input should be used to the point where the cost of an additional unit of labor just equals its MVP, assuming unlimited capital is available. The

TABLE XII

ESTIMATED MARGINAL VALUE PRODUCT OF VARYING LEVELS OF MAN
EQUIVALENTS OF HIRED LABOR ON GRADE A DAIRY FARMS
STUDIED, 1973-75

Hired Labor (X_3) ^a		Marginal Value Product ^b	
Man Equivalents	Hours	Dollars Per Man Equivalent	Dollars Per Hour
0.1	300	20,296	6.76
0.5	1,500	4,432	1.48
1.0	3,000	2,276	.76
1.5	4,500	1,539	.51
2.0	6,000	1,166	.39
2.5	7,500	939	.31
3.0	9,000	788	.26
3.5	10,500	678	.23
4.0	12,000	596	.20
4.5	13,500	532	.18

^aThe geometric mean for man equivalents of hired labor was .37 for 1973-75 while the arithmetic mean was .83 for the same period.

^bOther resources were held fixed at geometric mean levels.

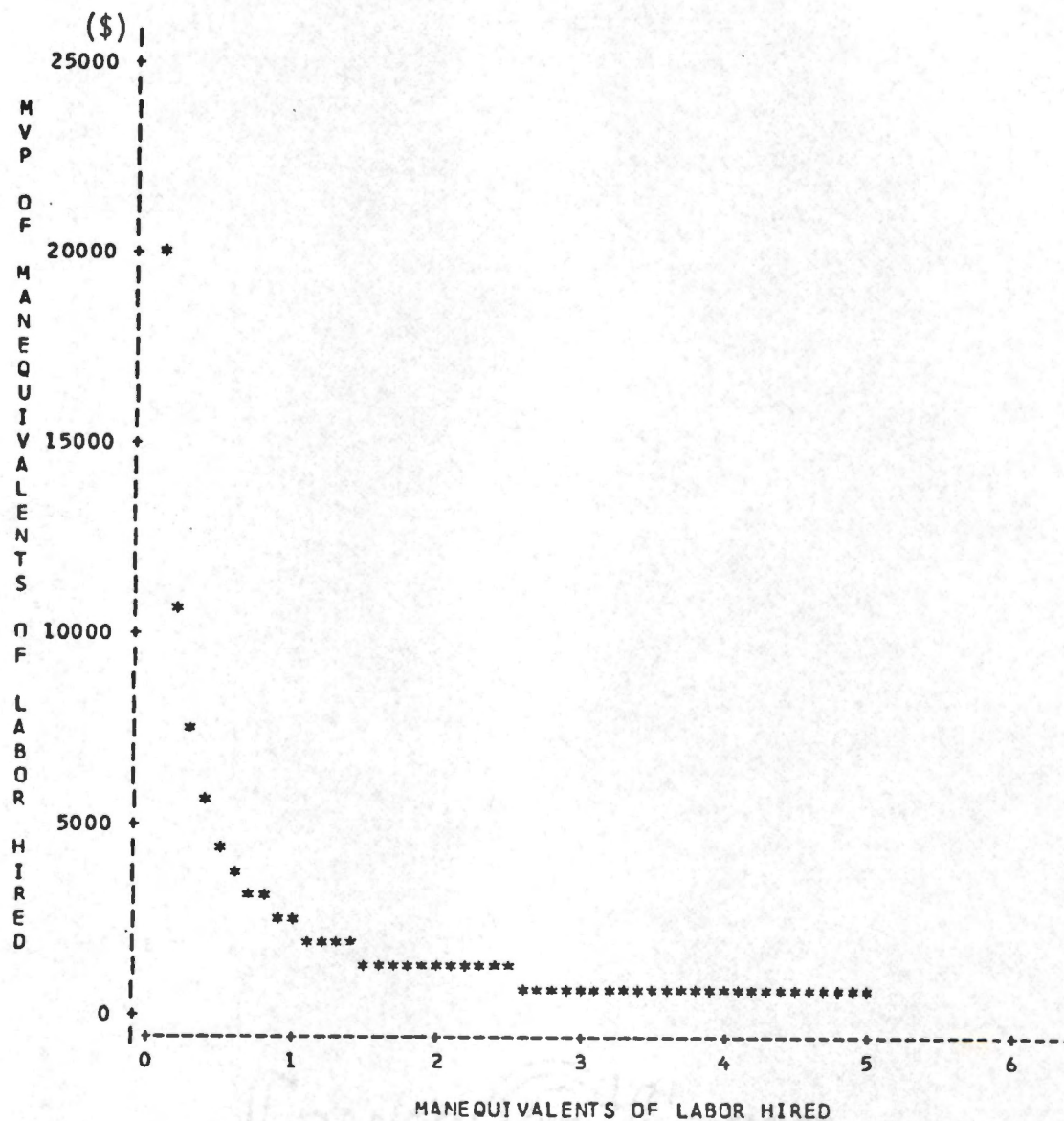


Figure 3. Estimated marginal value productivity of man equivalents of hired labor holding other inputs constant at their geometric means, using 1973-75 estimating equation (Equation D).

average farm wage in Tennessee for the 1973-75 period was \$1.88 per hour or \$5,640 per man equivalent.¹⁴ The MVP of an additional man equivalent in 1973-75 was estimated to be \$5,976 as computed at the geometric mean input levels and \$3,281 at the arithmetic mean input levels. Since the average farm was using hired labor at the arithmetic mean level (.83 man equivalents), it would have been unprofitable for the average farmer to hire more labor given the MVP estimated here and the assumption which was made regarding labor costs.

Marginal Value Productivity of Man Equivalents of Unpaid Labor

A man equivalent of unpaid labor was defined as 3,000 hours of operator and family labor available for use on the farm. When estimated at the geometric mean level of inputs, the MVP of an additional man equivalent of unpaid labor ranged from \$6,599 in 1975 to \$-2,599 in 1974 (Table VIII, page 32). The estimated MVP of this input was \$1,692 when estimated at the geometric mean input levels for the 1973-75 period and \$1,854 when estimated at the arithmetic mean input levels for the same period.

The effect of variations in man equivalents of unpaid labor on MVP of unpaid labor, holding other inputs constant at their geometric means, is shown in Table XIII and Figure 4. The estimated MVP of an additional

¹⁴ Farm Labor Situation, United States Department of Agriculture, Statistical Reporting Service (Washington, D. C., October 1974, and October 1975).

TABLE XIII

ESTIMATED MARGINAL VALUE PRODUCT OF VARYING LEVELS OF MAN
EQUIVALENTS OF UNPAID LABOR ON GRADE A DAIRY FARMS
STUDIED, 1973-75

Unpaid Labor (X_4) ^a		Marginal Value Product ^b	
Man Equivalents	Hours	Dollars Per Man Equivalent	Dollars Per Hour
0.6	1,800	4,321	1.49
0.8	2,400	3,276	1.09
1.0	3,000	2,644	.88
1.2	3,600	2,218	.74
1.4	4,200	1,912	.64
1.6	4,800	1,682	.56
1.8	5,400	1,502	.50
2.0	6,000	1,357	.45
2.2	6,600	1,238	.41
2.4	7,200	1,139	.38
2.6	7,800	1,054	.35

^aThe geometric mean for man equivalents of unpaid labor was 1.60 for 1973-75 while the arithmetic mean was 1.79 for the same period.

^bOther resources were held fixed at geometric mean levels.

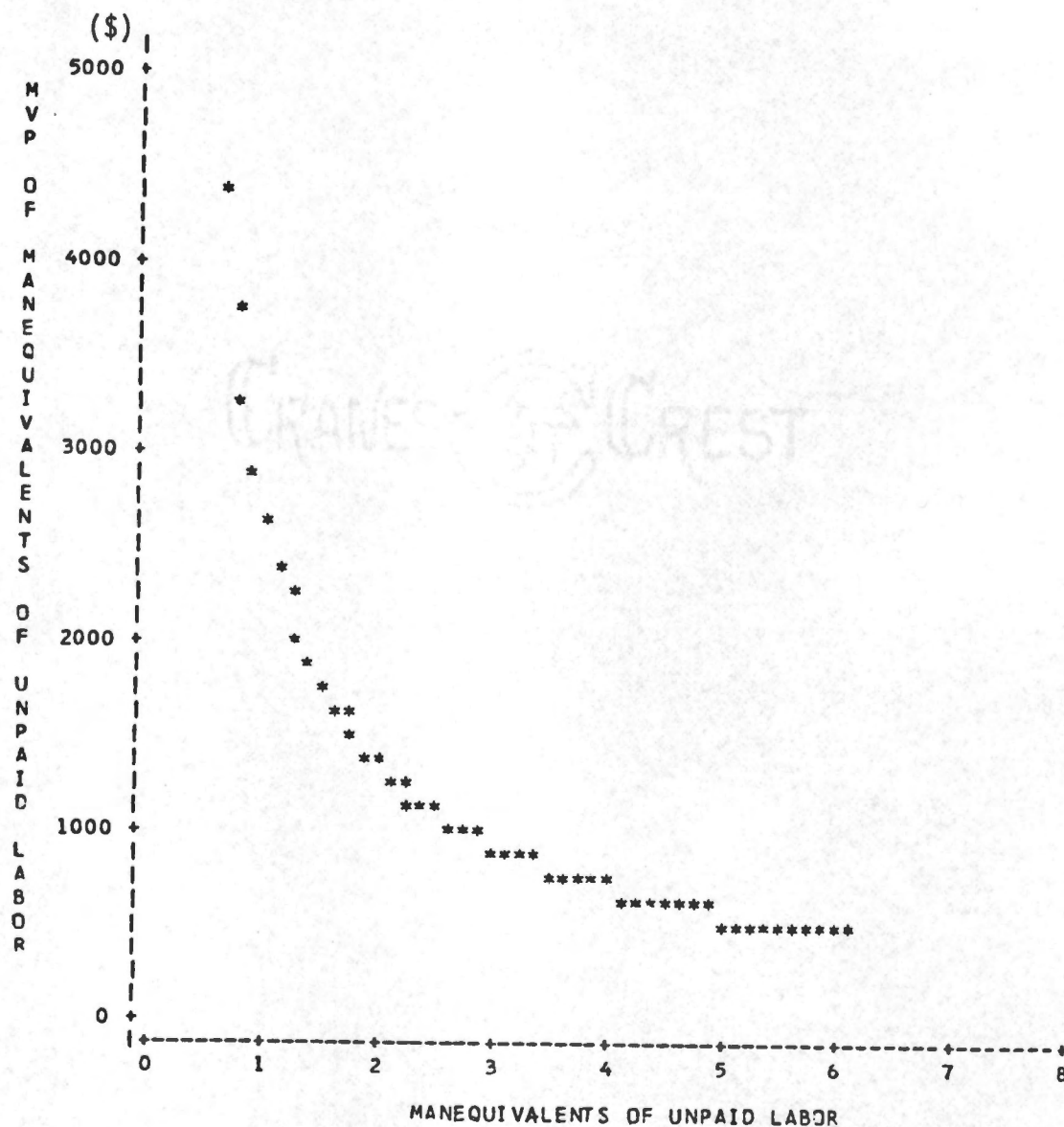


Figure 4. Estimated marginal value productivity of man equivalents of unpaid labor holding other inputs constant at their geometric means, using 1973-75 estimating equation (Equation D).

man equivalent of unpaid labor declined from \$4,321 when only .6 of a man equivalent was available, to \$1,054 when 2.6 man equivalents of unpaid labor were available. Contrary to expectations the estimated MVP of unpaid labor was lower than for hired labor as calculated at either the geometric or arithmetic mean input levels. Differences could be partially explained by differences in methods of measurement. Hired labor was measured in terms of amount used while unpaid labor was measured in terms of availability. Underemployment of unpaid family and operator labor could explain, at least partially, the low marginal productivity estimated for unpaid labor.

Marginal Value Productivity of Machinery and Equipment Expenses:

The input category machinery and equipment expenses includes annual depreciation, interest on investment in machinery and equipment, repairs and maintenance costs, farm share of auto expenses, and machinery work hired less 50 percent of receipts from custom work for both machinery and equipment. At the geometric mean levels of inputs, the estimated MVP of an additional dollar of machinery and equipment expense ranged from a high of \$2.20 in 1973 to a low of \$1.36 in 1975. For the 1973-75 period, the MVP of an additional dollar spent on machinery and equipment expenses was \$1.71 at the geometric mean level of inputs and \$1.80 at the arithmetic mean level of inputs.

The estimated MVP of machinery and equipment expenses at various levels of the input, holding other inputs constant at their geometric means, is given in Table XIV and Figure 5. The estimated MVP of the machinery and equipment input declined from \$4.38 when the input level

TABLE XIV

ESTIMATED MARGINAL VALUE PRODUCT OF VARYING LEVELS OF MACHINERY AND
EQUIPMENT EXPENSES ON GRADE A DAIRY FARMS STUDIED,
1973-75

Machinery and Equipment Expenses (X_5) ^a	Marginal Value Product ^b
-----dollars-----	
3,000	4.38
6,000	2.62
9,000	1.93
12,000	1.56
15,000	1.32
18,000	1.16
21,000	1.03
24,000	.93
27,000	.85
30,000	.79

^aThe geometric mean for machinery and equipment expenses was \$10,587 for 1973-75 while the arithmetic mean was \$12,471 for the same period.

^bOther resources were held fixed at geometric mean levels.

CRANESEST CREST

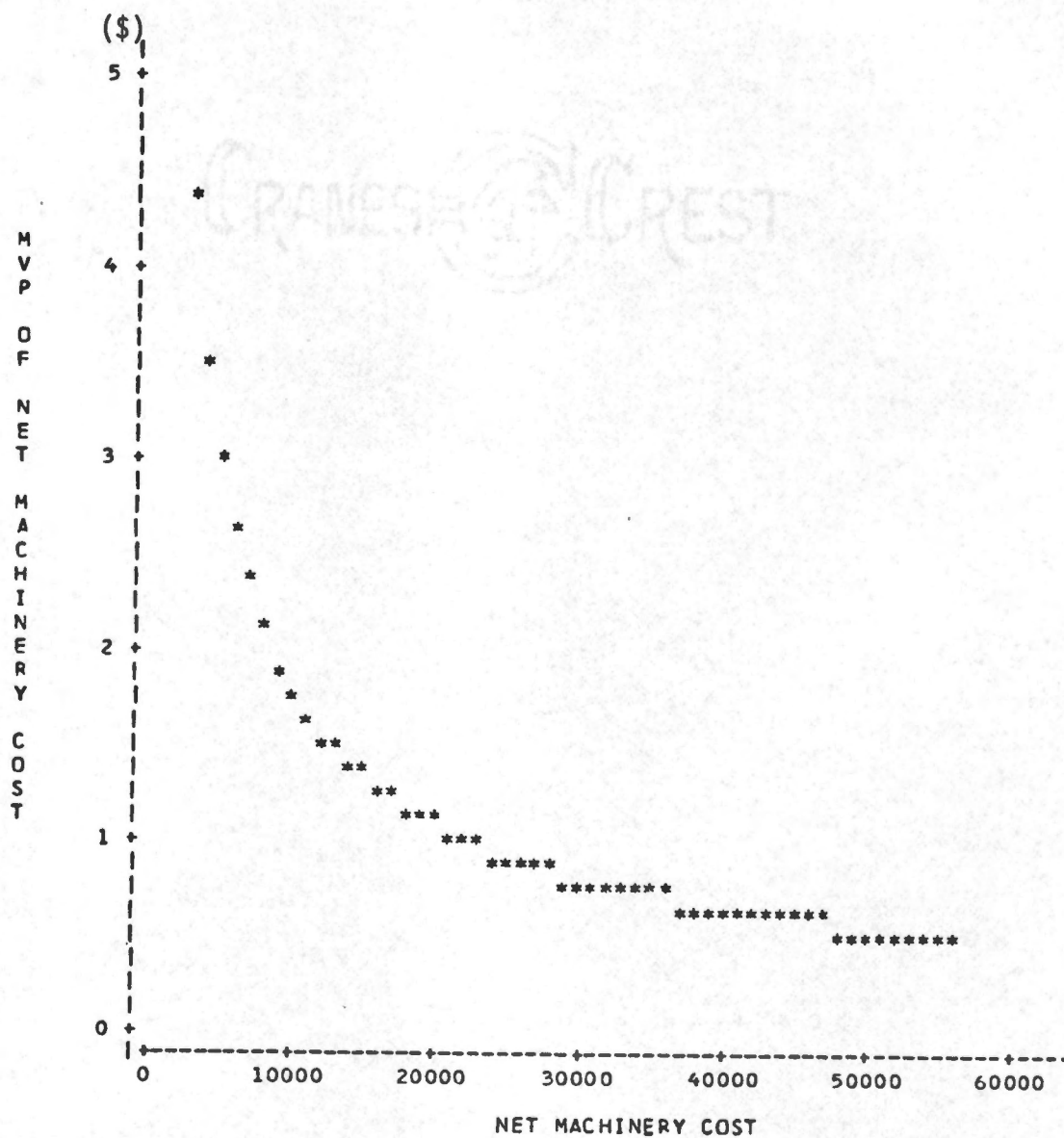


Figure 5. Estimated marginal value productivity of net machinery and equipment costs holding other inputs constant at their geometric means, using 1973-75 estimating equation (Equation D).

was \$3,000 to \$.79 when the input level was \$30,000. The diminishing marginal productivity of machinery and equipment expenses are graphically illustrated in Figure 5.

An interest charge of 5 percent of average investment in machinery and equipment was included in estimating the annual machinery and equipment expense input. In order to determine the appropriate annual cost of a dollar of machinery and equipment expenses, an interest charge had to be placed on the variable part of the machinery and equipment expenses. Approximately 50 percent of the machinery and equipment expense input were annual variable costs. If variable expenses were charged an 8 percent interest cost per year and variable expense capital was assumed to be utilized an average of six months, the appropriate interest cost on the variable expenses would be 4 percent. Thus for determining the efficiency of allocation, the appropriate annual cost of an additional dollar of machinery and equipment expense was \$1.02; a \$1.00 charge for the dollar spent and a \$.02 charge for interest on variable machinery and equipment expenses (which average about 50 percent of total machinery and equipment expenses). The estimated annual cost of \$1.02 for a dollar of annual machinery and equipment expense was considerably below the estimated MVP of an additional dollar spent on machinery and equipment expenses calculated for all four equations at both the geometric and arithmetic mean levels of inputs. Based on these results, farmers in this study could have profitably expanded their expenditures on machinery and equipment expenses.

Marginal Value Productivity of Operating Capital

Operating capital included all farm expenses except annual machinery and equipment expenses and hired labor expenses. As a result, the MVP of operating capital represents the returns to a variety of things such as purchased feed, fertilizer, and insurance. Measured at the geometric mean of all inputs, the estimated MVP of an additional dollar of operating capital ranged from a high of \$.86 in 1973 to a low of \$.70 in 1974. The MVP of this input was \$.78 for 1973-75 as estimated at the geometric mean level of all inputs and \$.76 for the same period when estimated at the arithmetic mean level of inputs.

The estimated MVP of an additional dollar of operating capital at various levels of operating capital, holding other inputs constant at their geometric means, is shown in Table XV and Figure 6. At the \$10,000 level of operating capital, an additional dollar of operating capital had a MVP of \$1.69 as compared to \$.46 at the \$100,000 level of operating capital. The nature of diminishing marginal returns to operating capital is shown in Figure 6.

The cost of a dollar used for operating capital would include the dollar spent plus the associated interest charge for the period the capital was used. If the interest charge is assumed to be 8 percent, and operating capital is assumed to be utilized an average of six months, an appropriate annual cost of using \$1 of operating capital would be \$1.04. Since returns were well below this level for all four equations at both the geometric and the arithmetic mean levels of inputs, the excessive allocation of expenditures of operating capital appeared evident on many farms.

TABLE XV

ESTIMATED MARGINAL VALUE PRODUCT OF VARYING LEVELS OF OPERATING CAPITAL
ON GRADE A DAIRY FARMS STUDIED, 1973-75

Operating Capital (X_6) ^a	Marginal Value Product ^b
-----dollars-----	
10,000	1.69
20,000	1.14
30,000	.91
40,000	.77
50,000	.68
60,000	.61
70,000	.56
80,000	.52
90,000	.48
100,000	.46

^aThe geometric mean of operating capital was \$39,400 for 1973-75 while the arithmetic mean was \$49,503 for the same period.

^bOther resources were held fixed at geometric mean levels.

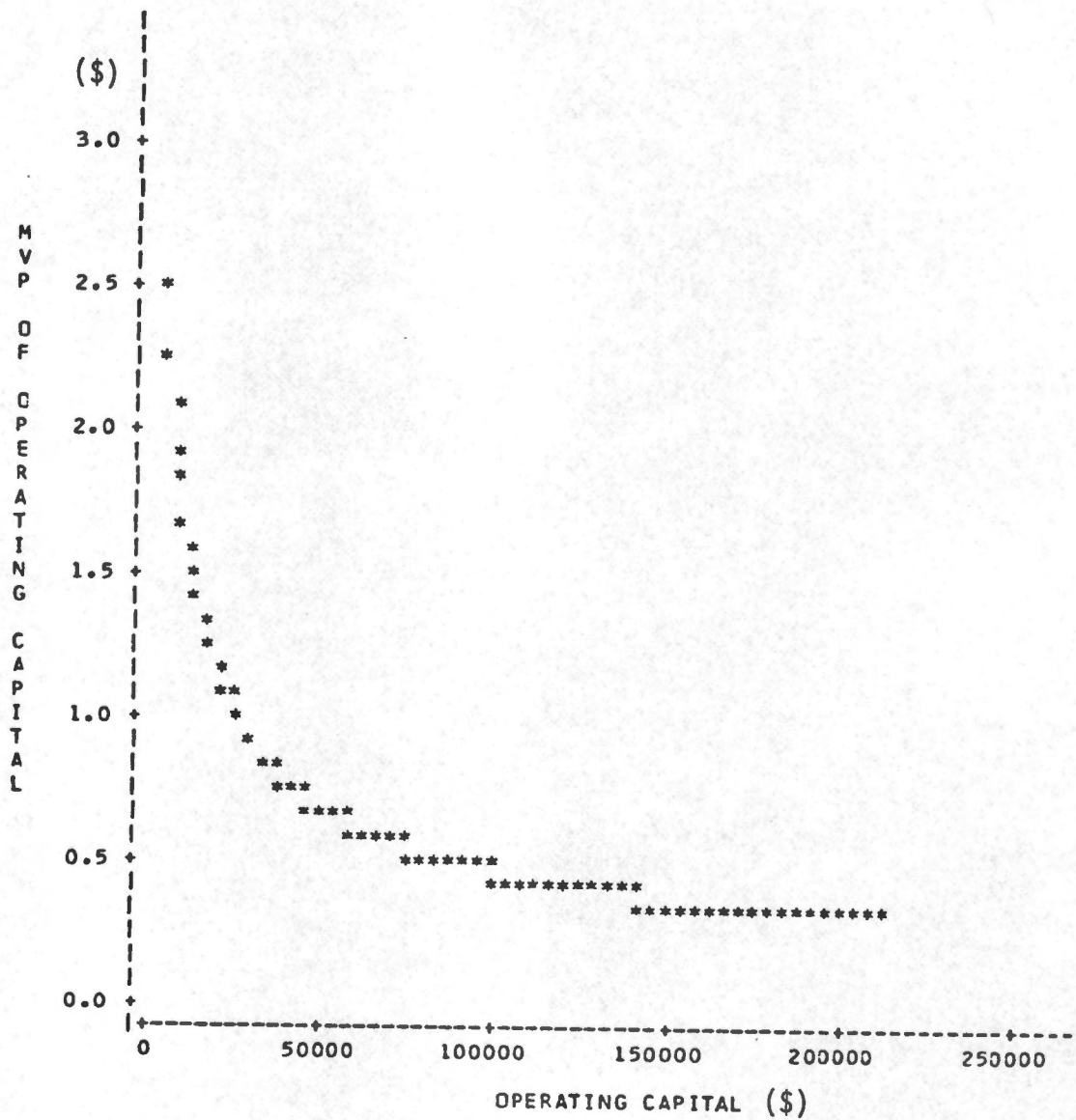


Figure 6. Estimated marginal value productivity of operating capital holding other inputs constant at their geometric means, using 1973-75 estimating equation (Equation D).

Marginal Value Productivity of Acres of Open Land

Land was measured in terms of acres of open land and included both pasture and cropland. At the geometric mean levels of all inputs, the estimated MVP of an acre of open land ranged from \$61 in 1975 to \$30 in 1973.

The estimated MVP of an additional acre of open land is shown in Table XVI and Figure 7 for various levels of acres of open land. With other inputs held constant at their geometric mean levels, at the level of 50 acres of open land, an additional acre had an estimated MVP of \$135. The estimated MVP of an acre of open land declined to \$17 when 500 acres of open land was available. The diminishing marginal productivity of open land as the number of open acres was increased is clearly shown in Figure 7.

The geometric mean level of acres of open land was 187 acres and the arithmetic mean level of open land was 219 acres for 1973-75. Using the 1973-75 estimating equation, the MVP of an additional acre of open land was \$42 when all the inputs were held fixed at their geometric mean level and \$44 when all inputs were fixed at their arithmetic mean levels. Assuming these rates of return would be realized in the future, disregarding risk and uncertainty considerations, and using a capital charge of 8 percent, the capitalized value of an additional acre of land would have been between \$525 and \$550 per acre depending on whether estimates were made at the geometric or arithmetic mean levels of inputs.

The estimated MVP of an acre of open land also gave an indication of the amount dairy farmers could afford to profitably pay in rent for

Productivity of Acres of Open Land

Land was measured in acres of open land and the mean both before and after. As the general mean level of 1950 was estimated to be an acre of open land valued from 1950 to 1955 in 1950.

The estimated MP of an additional acre of open land is shown in Table 1 and Figure 1 for various levels of level of open land. With other things held constant, the estimated MP level of 50 acres of open land, an additional acre had an estimated MP of 1.15. The estimated MP of an acre of open land declined to 1.1 when 50 acres of open land was realized. The declining marginal productivity of open land as the number of open acres was increased is clearly shown in Figure 1.

The general mean level of acres of open land was 100 acres and the estimated mean level of open land was 100 acres for 1950. The 1950-55 estimated MP of an additional acre of open land was 1.15. The input was held fixed at their respective mean level and 100 acres of open land were held at their respective mean levels. Assuming these rates of return would be realized in the future, disregarding risk and uncertainty considerations, and using a capital cost of 5 percent, the capitalized value of an additional acre of land would have been between 1950 and 1955 with the following on which estimates were made at the estimated mean level of open land of 100 acres.

The estimated MP of an acre of open land is shown in Table 1 and Figure 1. The estimated MP of an acre of open land is shown in Table 1 and Figure 1.

TABLE XVI

ESTIMATED MARGINAL VALUE PRODUCT OF VARYING LEVELS OF ACRES OF OPEN LAND
ON GRADE A DAIRY FARMS STUDIED, 1973-75

Open Acres (X_7) ^a	Marginal Value Product ^b
-----acres-----	-----dollars-----
50	134.84
100	72.76
150	50.71
200	39.26
250	32.18
300	27.36
350	23.86
400	21.18
450	19.07
500	17.37

^aThe geometric mean of open acres of land was 187 acres for the 1973-75 period while the arithmetic mean was 219 acres for the same period.

^bOther resources were held fixed at geometric mean levels.

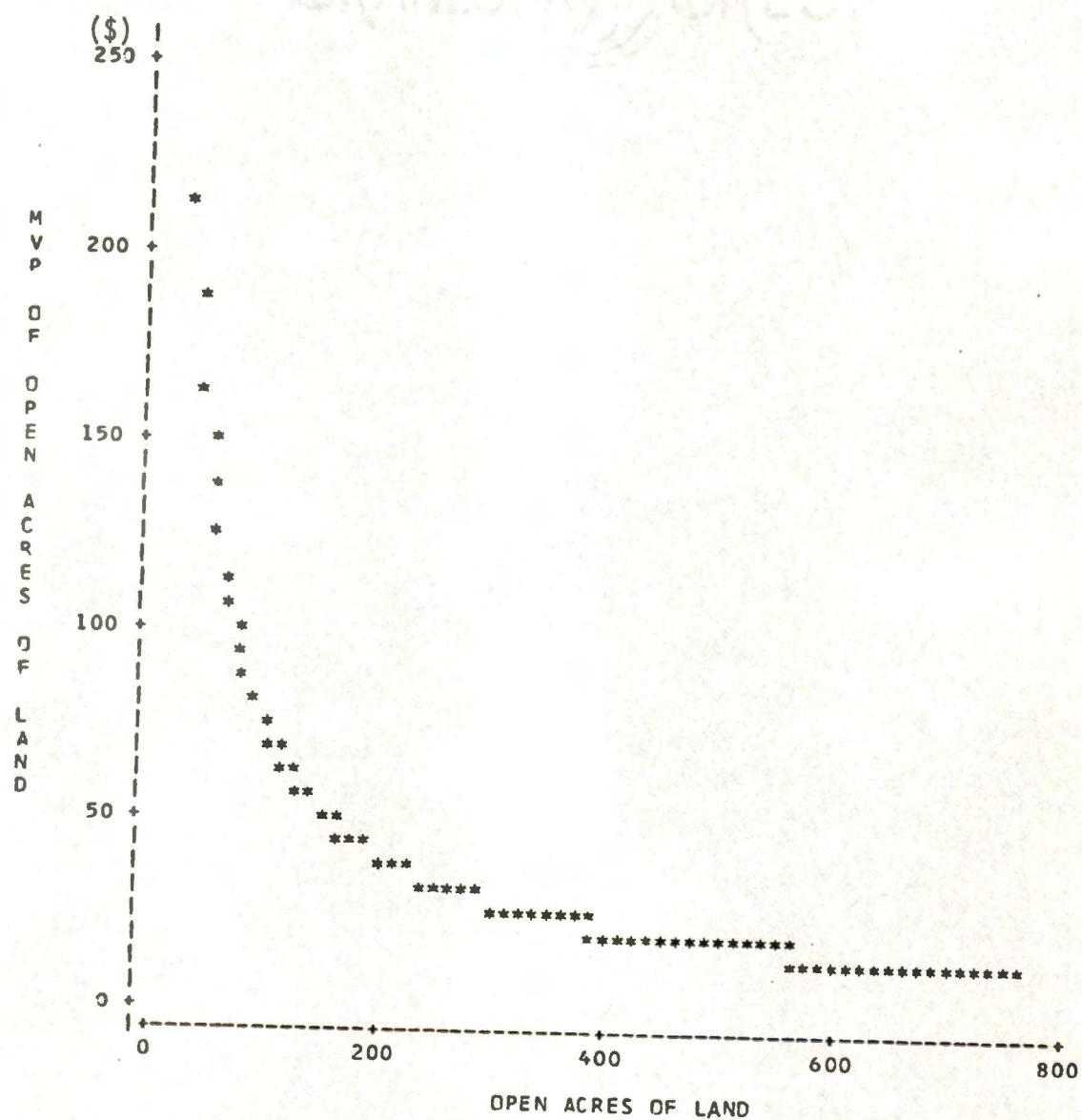


Figure 7. Estimated marginal value productivity of acres of open land holding other inputs constant at their geometric means, using 1973-75 estimating equation (Equation D).

an additional acre of open land. Using the MVP estimated at the arithmetic mean level of inputs for 1973-75, the average dairy farmer in the study could have profitably rented an acre of open land if the rent was no greater than \$44 and was not payable until the end of the calendar year.

III. EFFICIENCY OF RESOURCE ALLOCATION AND OPPORTUNITIES FOR IMPROVEMENT

When considering efficiency of resource allocation, an assumption must be made regarding the capital position of the farm business.

If unlimited capital is assumed to be available, the efficient resource allocation occurs where each resource is used to the point where the marginal value product just equals the cost of an additional unit of that resource. If limited capital is assumed, the efficient resource allocation occurs where the marginal value productivity of the resources are proportional to each of their respective prices.

Neither of the criteria of efficient resource allocation was met by the average of the farms in this study. A comparison of the estimated marginal value productivity of each input category and the estimated cost of a unit of each input is shown in Table XVII. Excessive use of an input would be indicated if the MVP was less than the resource price and under utilization would be indicated if the MVP exceeded the price of the resource. Each individual farmer would need to evaluate the results in terms of his own peculiar circumstances. It should be noted that comparisons are made for arithmetic and geometric mean levels of resource use for 1973-75. The degree of over or under utilization would vary across farms in the study.

TABLE XVII

ESTIMATED MARGINAL VALUE PRODUCTIVITY AND COST OF RESOURCE INPUTS ON
GRADE A DAIRY FARMS STUDIED, 1973-75

Resource Input	Marginal Value Productivity		Price Per Unit of the Resource ^c
	Geometric Means ^a	Arithmetic Means ^b	
	-----dollars-----		
Investment in dairy cows	.43	.45	.08
Investment in buildings and improvements	.05	.04	.14
Man equivalents of hired labor	5,976	3,281	5,640
Man equivalents of unpaid labor	1,692	1,854	--
Machinery and equipment expense	1.71	1.80	1.02
Operating capital	.78	.76	1.04
Acres of open land	42	44	--

^aThese MVP's were estimated at the geometric mean level of each resource, all other inputs held constant at their geometric means.

^bThese MVP's were estimated at the arithmetic mean level of each resource, all other inputs held constant at their arithmetic means.

^cAn interest rate of 8 percent per year was charged to the dairy cow investment. The charge for building and improvements investment included a 5 percent depreciation charge, a 1 percent charge for repairs and maintenance and an 8 percent charge for use of capital. The cost of a man equivalent of hired labor was determined from the average farm wage in Tennessee for 1973-75. The variable expenses portion of machinery and equipment expenses were charged an 8 percent annual interest for an average of six months of utilization which led to the \$1.02 cost of a dollar expense on machinery and equipment. Operating capital was charged an 8 percent interest rate for an assumed average utilization period of six months. Costs were not estimated for man equivalents of unpaid labor or acres of open land due to the extreme variability in the quality of these inputs and thus difficulties in accurately estimating costs.

Investment in dairy cows should in general be increased in order to increase profitability. Using 1973-75 estimates, a net return of \$.37 would have resulted from an additional dollar investment in dairy cows when MVP was estimated at the arithmetic mean level of all inputs. The investment in buildings and improvements and operating capital input categories were being used beyond the point where MVP equals the cost of an additional unit of the resource. At the arithmetic mean level of all inputs, the MVP of an additional man equivalent of hired labor would be less than its cost. Further expenditures on machinery and equipment expenses would have resulted in positive net returns of \$.78 for an additional dollar expenditure when the MVP was computed at geometric mean input levels for the 1973-75 period. Reductions in capital used for buildings and improvements, operating capital, and hired labor combined with increases in investment in dairy cows and machinery and equipment expense would increase allocative efficiency.

Man equivalents of available unpaid labor earned a relatively low return compared to hired labor; and below expected normal returns on labor. The economics of expansion of the land resource depends on the price of an acre of open land and the interest charge associated with it. On the average of farms studied, an acre of open land was returning about \$44 per acre per year. This would be equivalent to a return of 8 percent on \$550 per acre land or 10 percent on \$440 per acre land.

IV. RETURNS TO SCALE

The production functions derived from the farm data directly

indicate not only the elasticity of production for the individual resources but also the total elasticity of production and thus returns to scale. All the resources exhibited diminishing or negative returns since elasticities of production were less than one in every case.

Both elasticities of production for specific resources and total elasticity of production are shown in Table VII, page 29. For equation A (1973) the sum of the elasticities of production was 1.0299. This indicates increasing returns to scale; which means if all the resources were increased by one percent, the resulting gross farm income would have increased by 1.0299 percent. Equation C (1975) and Equation D (1973-75) also exhibited increasing returns to scale. Only Equation B (1974) exhibited decreasing returns to scale. The total elasticity of production from Equation D was 1.0245 indicating increasing returns to scale for the three year period covered by the data. As a result, the dairy farmers in this study could have reduced their costs per unit of output by increasing all inputs proportionally.

CHAPTER IV

SUBSTITUTION OF MACHINERY AND EQUIPMENT EXPENSES FOR HIRED LABOR IN PRODUCTION

Mechanization has been replacing labor in farm production for decades. Milk production has become increasingly mechanized through the use of such equipment as milking machines, bulk milk tanks, and automated grain and forage handling equipment. Mechanization has reduced the labor required in dairy production but at the same time the costs associated with utilizing machinery and equipment have risen. Farmers have often been urged to reduce labor costs but also to guard against over-mechanization.

The optimum combination of mechanization and labor depends on the substitution relationship between mechanization and labor and the relative prices of these inputs. The substitution relationship or the marginal rate of substitution of machinery for labor is a measure of how much labor can be replaced by an additional unit of machinery in order to produce a given output level. In this study, the marginal rate of substitution of machinery and equipment for hired labor was derived as follows:

$$MRS_{x_2 \text{ for } x_1} = - \frac{b_2}{b_1} \cdot \frac{x_1}{x_2}$$

where:

$MRS_{x_2 \text{ for } x_1}$ = marginal rate of substitution of machinery
and equipment for hired labor

- b_2 = the elasticity of production of the machinery and equipment input
 b_1 = the elasticity of production of the hired labor input
 x_1 = the level of input use of hired labor per year
 x_2 = the level of input use of machinery and equipment per year.

The marginal rate of substitution was computed at an output level of \$70,905 of yearly gross farm income (the geometric mean level of gross income in the 1973-75 period). The elasticities of production were taken from estimates found in Cobb-Douglas Equation D (1973-75) reported in Chapter III. The machinery and equipment as well as the hired labor input were measured as in Equation D; as yearly machinery and equipment expenses and as man equivalents of labor hired yearly.

Several combinations of machinery and equipment expenses and hired labor that could be used to produce an annual gross farm income of \$70,905, holding other inputs constant at geometric mean levels, are given in Table XVIII. The marginal rate of substitution of machinery and equipment expenses for hired labor is given for each combination and was expressed as hours of labor. For example, \$8,823 of machinery and equipment expenses and 6,000 hours of hired labor (or 2.0 man equivalents) could have been used to produce an annual gross farm income of \$70,905 for the 1973-75 period. At this particular combination, the marginal rate of substitution of machinery and equipment for hired labor was -5.56 hours. This indicates that at this combination of mechanization and hired labor, an additional dollar of

TABLE XVIII

VARYING COMBINATIONS OF MACHINERY COST AND HIRED LABOR
IN PRODUCING A GIVEN OUTPUT,^a 1973-75

Machinery and Equipment Expenses ^b (dollars)	Hired Labor ^c (man equivalents)	(hours)	Marginal Rate of Substitution ^d (hours)
8,823	2.00	6,000	-5.56
8,937	1.80	5,400	-4.95
9,067	1.60	4,800	-4.33
9,216	1.40	4,200	-3.73
9,391	1.20	3,600	-3.14
9,602	1.00	3,000	-2.56
9,868	0.80	2,400	-1.99
10,221	0.60	1,800	-1.44
10,739	0.40	1,200	-.91
11,374 ^e	0.25	750	-.54
11,688	0.20	600	-.42
12,271	0.10	300	-.20

^aThe given output is the geometric mean value of annual gross farm income of \$70,905. Other inputs were held constant at their geometric mean input levels.

^bThe geometric mean level of annual machinery and equipment expenses was \$10,587 and the arithmetic mean level was \$12,471.

^cThe geometric mean level of hired labor was .37 man equivalents and the arithmetic mean level was .83 man equivalents.

^dThe marginal rate of substitution of machinery and equipment expenses for hired labor indicates the hours of labor that could be replaced by one additional dollar spent on annual machinery and equipment expenses.

^eThis was the least-cost combination assuming labor cost \$1.88 per hour and that a dollar of machinery and equipment expenses cost \$1.02.

machinery and equipment expenses could replace 5.56 hours of hired labor while maintaining an annual gross farm income of \$70,905. Another combination that could have been used to produce the same annual gross farm income was \$12,271 of machinery and equipment expenses and 300 hours (or .1 man equivalent) of hired labor. The marginal rate of substitution for this combination was -.2 hours of labor indicating that an additional dollar spent on machinery and equipment expenses would replace only .2 hours of labor in producing an annual gross farm income of \$70,905.

Data in Table XVIII indicated that as machinery and equipment expenses were increased, less hired labor was required to produce the same gross farm income; however, as machinery and equipment expenses were increased, fewer hours of labor could be replaced by each successive increase. As a result, a diminishing marginal rate of substitution of machinery and equipment for labor was evidenced.

The various combinations of machinery and equipment expenses and hired labor that could be used in producing an annual gross farm income of \$70,905 are illustrated in Figure 8. The curve traced out by the points on the graph illustrates graphically the same relationship shown in Table XVIII. It should be noted that the other resource inputs were held fixed at their geometric means as the machinery and equipment and hired labor inputs were combined in varying proportions. While a large number of combinations of the two inputs could have been used to produce the same annual gross farm income, only one combination will result in the least cost.

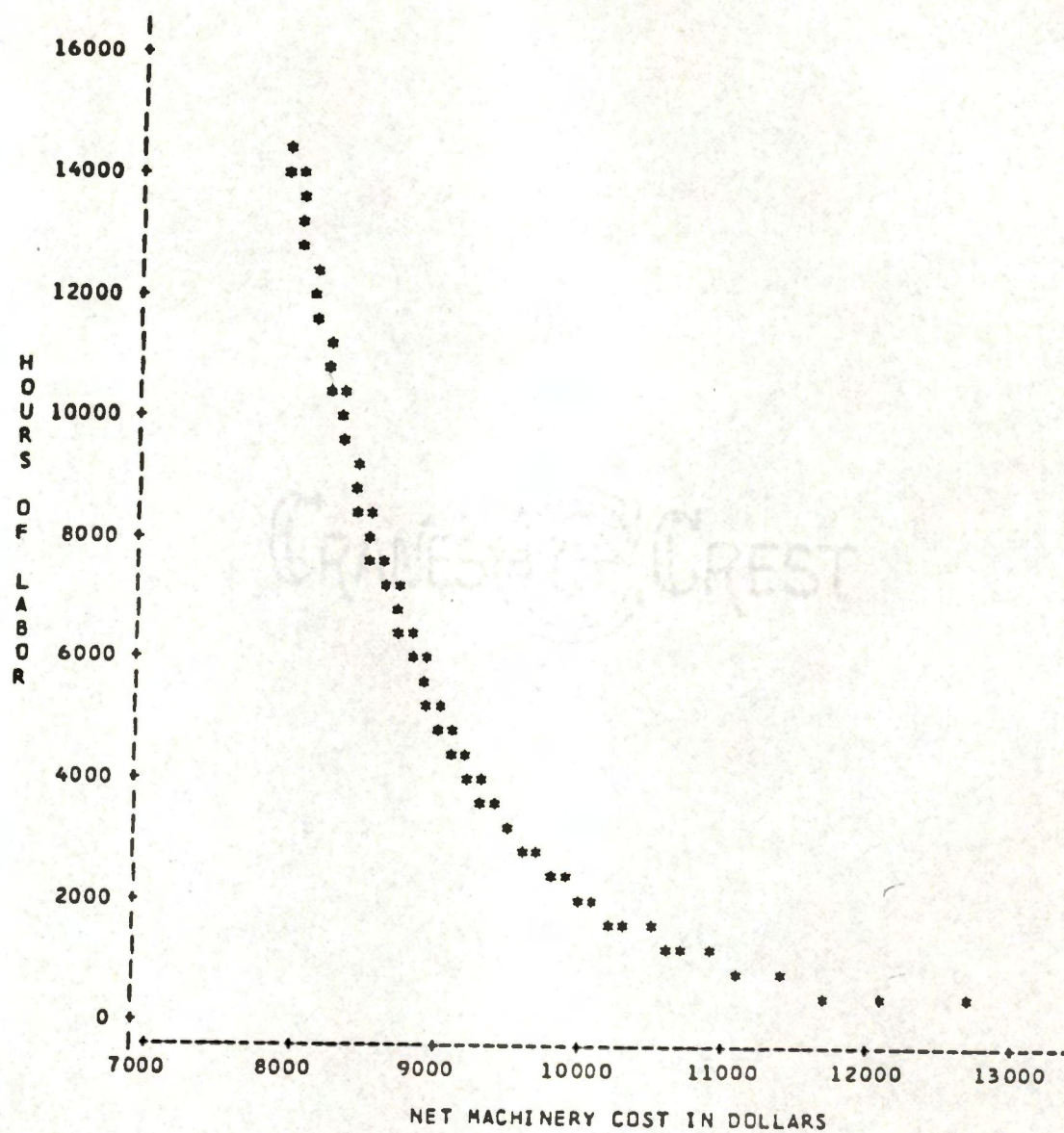


Figure 8. Combinations of hired labor and mechanization used in producing an annual gross farm income of \$70,905, developed from 1973-75 data.

The least-cost combination of mechanization and hired labor can be found where the following equality exists:

$$MRS_{X_2 \text{ for } X_1} = - \frac{P_{X_2}}{P_{X_1}}$$

where:

$MRS_{X_2 \text{ for } X_1}$ = marginal rate of substitution of machinery and equipment expenses for hired labor in producing and annual gross farm income of \$70,905

P_{X_1} = the price of a unit of the hired labor input

P_{X_2} = the price of a unit of the machinery and equipment expense input.

The average cost of an hour of hired farm labor in Tennessee was \$1.88 for the 1973-75 period.¹⁵ The cost of a dollar expenditure on machinery and equipment expenses was estimated to be 1.02.¹⁶ These prices yield a price ratio of .54.

Equating this ratio with the appropriate marginal rate of substitution from Table XVIII yields a least-cost combination of \$11,374 of annual machinery and equipment expenses and 750 hours (.25 man equivalents) of hired labor. If the price of hired labor was increased relative to the price of machinery and equipment expenses, the least-cost combination would include less hired labor and more money spent on machinery costs. For example, if the price of labor

¹⁵ Farm Labor Situation.

¹⁶ For details see Page 50 in Chapter III.

had been \$5.10 per hour and the price of a dollar of machinery and equipment expenses remained at \$1.02, the new price ratio would be .2. The corresponding least-cost combination would then be \$12,271 of machinery and equipment expenses and 300 hours (.1 man equivalent) of unpaid labor.

By contrast, a rise in the cost of a dollar expenditure on machinery and equipment relative to the cost of hired labor would result in less machinery and equipment expenses and more hired labor to have been used in the least-cost combination. A cost of \$1.71 for a dollar of machinery and equipment expenses combined with a cost of \$1.88 per hour of hired labor would yield a price ratio of .91. The new least-cost combination would then have included \$10,739 of machinery and equipment expenses and 1,200 hours (.4 man equivalent) of hired labor.

For farms in this study having an annual gross farm income of approximately \$71,000, the substitution relationships reported would be reasonably accurate for comparisons with the individual farm's level of use of mechanization and hired labor. The results reported could not be meaningfully compared with input use data from farms having annual gross farm incomes significantly different from \$70,905.

CHAPTER V

DEVELOPMENT OF A MANAGEMENT INDEX

The Cobb-Douglas functions used in estimating marginal value productivities of resources were assumed to have included all the important measurable variables that influenced gross farm income. The management resource was the only variable that could not be measured and included in the production function. If it can be assumed that the management factor was responsible for the unexplained variations in gross farm income, a quantitative method can be devised to measure management. Using this assumption, a management index was developed from the residuals between actual and predicted gross farm income using the parameters of the estimated production function for 1973-75.

I. ESTIMATION OF A MANAGEMENT INDEX

A predicted gross farm income was calculated, using Equation D (1973-75), for each yearly farm record used in the study. A management index was then calculated in the following manner:

$$\text{Management Index} = \left(\frac{Y}{\hat{Y}} \right) \cdot 100.$$

Y was actual gross farm income and $\hat{Y}$ was predicted gross farm income.

A management index of greater than 100 indicated that predicted gross farm income was less than actual gross farm income and that the level of management on that particular farm was above average. An index of 100 indicated that the predicted and actual gross farm incomes were equal and that the managerial ability on that farm was average for the group of

farms studied. An index of less than 100 indicated that predicted gross farm income was greater than actual gross farm income and that below average managerial ability was present on that particular farm. The quality of managerial ability in each case was a relative measure as compared to the level of management on the other farms in the study. The average managerial index computed in this manner was approximately 102.¹⁷

The management indices calculated ranged from a high of 211 to a low of 34. Approximately 95 percent of the management indices were between 60 and 140.

If a method could be found to determine the level of management on individual farms by relating it to observable characteristics, it would be possible to include management within a production function. As a result, more accuracy could be obtained in explaining variations in gross farm income. It was hypothesized that one method of accomplishing this would be to determine the relationship between the management index and efficiency factors normally calculated from farm business analysis.

II. MANAGEMENT INDEX ANALYSIS

A simple correlation analysis was conducted relating the management index and the following efficiency measures: (1) machinery and equipment cost per man equivalent per year, (2) number of dairy cows

¹⁷The average managerial index exceeded 100 because predicted gross income was estimated using the log form of the Cobb-Douglas equation. Estimated gross incomes (in log form) were then converted to non-log form before use in calculating the managerial index.

per man equivalent per year, (3) pounds of milk produced per man equivalent per year, (4) acres of open land per man equivalent, (5) investment per man equivalent, (6) pounds of milk per cow per year, (7) milk sales per cow per year (in dollars), (8) milk sales per acre of open land per year (in dollars), (9) investment per acre of open land. The correlations were calculated as Pearson product-moment correlations and are reported in Table XIX. As an aid in understanding these correlations, the farms were divided into three groups, by level of management index, and the average value of each of the nine efficiency factors above are reported in Table XX for each group to provide comparisons. There were 53 farms with a management index of less than 90, 146 farms with an index of 90-120, and 33 farms with an index over 120.

Machinery and Equipment Cost Per Man Equivalent and the Management Index

This efficiency factor gives an indication of the productivity of labor by relating how much machinery and equipment was available for use by the labor employed. This factor was positively correlated with the management index although the correlation was not significant at the 5 percent level. The correlation of .06 indicated that as the management index increased, the machinery and equipment cost per man equivalent increased also. As seen in Table XX, this efficiency factor had the lowest average (\$5,126) for farms with the lowest level of management indices. The farms with the intermediate level of management indices had the highest average machinery and equipment cost per man equivalent of \$5,573.

TABLE XIX

CORRELATION OF EFFICIENCY MEASURES WITH THE MANAGEMENT INDEX FOR 232
TENNESSEE GRADE A DAIRY FARMS FOR 1973-75

Efficiency Measure	Correlation Coefficient (n= 232)
Machinery and equipment cost PME ^a	0.06
Number of cows PME ^a	0.06
Pounds of milk PME ^a	0.20*
Acres of open land PME ^a	0.03
Investment PME ^a	0.13*
Pounds of milk per cow	0.29**
Milk sales per cow	0.31**
Milk sales per acre of open land	0.21**
Investment per acre of open land	0.13**

^aPME = per man equivalent or 3000 hours of labor.

*Significant at the 5 percent level.

**Significant at the 1 percent level.

TABLE XX

RELATIONSHIP BETWEEN MANAGEMENT INDEX AND EFFICIENCY FACTORS ON 232
GRADE A DAIRY FARMS IN TENNESSEE, 1973-75

Efficiency Factors	All Farms (n = 232)	Level of Management Index		
		Less than 90 (n = 53)	90-120 (n = 146)	Greater than 120 (n = 33)
Machinery and equipment cost PME ^a (\$)	5,418	5,126	5,573	5,465
Number of cows PME ^a	28	26	29	28
Pounds of milk PME ^a	276,732	229,239	286,883	316,010
Acres of open land PME ^a	96	83	106	82
Investment PME ^a (\$)	69,531	63,390	70,584	77,313
Pounds of milk per cow	9,807	9,021	9,884	11,230
Milk sales per acre (\$)	840	762	847	976
Milk sales per acre of open land (\$)	298	272	279	425
Investment per acre of open land (\$)	829	893	742	1,099

^aPME = per man equivalent or 3,000 hours of labor.

Number of Cows Per Man Equivalent and the Management Index

This factor gives an indication of the efficiency of labor usage. The factor had a correlation coefficient of .06 with the management index. This positive correlation, although not significant at the 5 percent level, indicated that as the number of cows per man equivalent increased, the management index increased also. The data in Table XX show little variation in the number of cows per man equivalent over the different management level categories. The average number of cows per man equivalent ranged from 26 cows for the farms with the lowest level of management to 29 cows for those farms with the medium level of management indices.

Pounds of Milk Per Man Equivalent and the Management Index

This factor gives an indication not only of labor efficiency, but also of the production level of the dairy herd. This factor was positively correlated with the management index (.20). The correlation coefficient was significant at the 5 percent level. As shown in Table XX, average pounds of milk per man equivalent was 86,000 pounds higher for the farms with the highest management indices than for the farms in the lowest management index group and nearly 30,000 pounds higher than for the farms with the intermediate level of management indices (90-120).

Acres of Open Land Per Man Equivalent and the Management Index

This factor gives an indication of the land-labor ratio. There was a positive correlation of .03 between this factor and the management

index (not significant at the 5 percent level). As shown in Table XX, the farms with the highest and lowest management index groups had approximately the same number of acres of open land (82 and 83 acres respectively). Farms in the intermediate management index group had an average of 106 acres of open land per man equivalent.

Investment Per Man Equivalent and the Management Index

Investment per man equivalent gives an indication of the capital-labor ratio; the amount of capital assets available for use by each man equivalent of labor employed in production. A positive correlation of .13 between this factor and the management index was significant at the 5 percent level. This correlation was consistent with comparisons of the management index groups shown in Table XX. The average investment per man equivalent was nearly \$14,000 greater for the farms in the highest management index group than for the farms in the lowest management index group. The farms with the highest management indices had an average investment per man equivalent of \$77,313 as compared to \$63,390 for those farms with the lowest indices.

Pounds of Milk Per Cow and the Management Index

Pounds of milk per cow is a factor frequently used to describe the level of technical efficiency and as an indicator of the management level on dairy farms. Pounds of milk per cow was positively correlated with the management index at .29 and this correlation was significant at the 1 percent level. As shown in Table XX, average milk production per cow per year was more than 2,000 pounds greater on the group of

farms with the highest level of management indices than on the group of farms with the lowest management indices. Average milk production per cow per year was 11,230 pounds for the group of farms with the highest management indices as compared to 9,021 pounds for the group of farms with the lowest level of indices.

Milk Sales Per Cow and the Management Index

Milk sales per cow would be expected to be highly related to pounds of milk per cow; however, in the analysis of Chapter II it was noted that the herd size group of farms with the highest average pounds of milk per cow did not have the highest milk sales per cow. A positive correlation of .31 was found between milk sales per cow and the management index (significant at the 1 percent level). Average milk sales per cow per year, for the group of farms with the highest management indices, was more than \$200 greater than for the group of farms with the lowest management indices and more than \$100 greater than for those farms with the intermediate level of management indices. Milk sales per cow per year averaged \$976 for the group of farms with the highest management indices.

Milk Sales Per Acre of Open Land and the Management Index

Milk sales per acre of open land indicates the intensity of use of the land resource in terms of the production of milk. There was a positive correlation of .21 between milk sales per acre of open land and the management index and this correlation was significant at the 1 percent level. Milk sales per acre of open land increased from \$272 for those farms with the lowest level of management indices to

\$425 for those farms with the highest level of management indices.

Investment Per Acre of Open Land and the Management Index

Investment per acre of open land is a measure of the capital-land ratio and gives an indication of the capital intensiveness of the farming operation. A positive correlation of .13 was found between the management index and investment per acre of open land (significant at the 5 percent level). Investment per acre of open land averaged \$1,009 for the 33 farms with the highest level of management indices, \$742 for the 146 farms with the intermediate level of management indices, and \$893 for the 53 farms with a management index of less than 90.

The analysis of the relation of efficiency factors and the management index indicated that those factors related to the production of milk were the most highly correlated with the management index. Milk sales per cow was found to be the best single factor in explaining variations in the management index across farms.¹⁸

¹⁸In order to determine if the efficiency factors used in this analysis could collectively explain more variation in the management index than a single factor, the management index was regressed upon the efficiency factors shown in Table XX. Milk production per cow was excluded from the regression to avoid problems of multicollinearity with milk sales per cow. Only about 14 percent of the variation in the management index was explained by the eight efficiency factors collectively. Milk sales per cow per year alone explained approximately 9 percent of the variability in the management index among the 232 observations.

CHAPTER VI

PREDICTION OF 1976 GROSS FARM INCOME

Economic conditions regarding prices received for milk and prices paid for dairy rations were given in Table VI (page 25) for the 1973-75 period. In 1976, dairy farmers in the United States received an average of \$9.93 per 100 pounds for milk eligible for fluid consumption. During the same period dairy farmers paid an average of \$6.30 per 100 pounds for 16 percent dairy ration. These prices yield a prices paid as a percent of prices received ratio of .63. Prices paid for feed as a percent of prices received for milk was .69 for the 1973-75 period. Based on a comparison of milk and feed prices only, 1976 appeared to be a more profitable year for Grade A dairying than the 1973-75 period.¹⁹

In order to determine how well Equation D (page 28), developed for 1973-75 data, would predict 1976 gross farm income, record data for 82 Tennessee Grade A dairies in 1976 was utilized. The 1976 data was available from farms enrolled in the Agricultural Resource Development Program, as were the farm records for the basic study.

Input data for 1976 were used with Equation D and a predicted gross farm income was calculated for each farm. A R^2 statistic was calculated to determine how well Equation D predicted gross farm income. A F statistic was also calculated to test the significance of

¹⁹Dairy Situation, December 1977, p. 6.

the R^2 statistic calculated for 1976. Graphic illustrations were prepared to illustrate the variations between actual and predicted gross farm income.

It was hypothesized that the relationship between inputs and output had not changed significantly from the 1973-75 period to 1976. In order to determine the validity of this hypothesis, an F test was performed to determine if the unexplained variation in gross farm income for 1976, using Equation D, was significantly different from the unexplained variation in 1973-75.

Comparison of Data for 1976 and 1973-75

A comparison of the input and output data for 1973-75 and 1976 is given in Table XXI. Average annual gross farm income was approximately \$33,000 greater for 1976 than for the 1973-75 period, \$116,000 in 1976 as compared to \$83,000 in 1973-75. There was a greater range in annual gross farm income in 1976 than in the 1973-75 period. In 1976 annual gross farm income ranged from \$19,000 to \$504,000 while the range for the 1973-75 period was \$9,500 to \$312,000.

All of the inputs used in the Cobb-Douglas equation had a higher average value for the 1976 data than for the 1973-75 data except unpaid labor as seen in Table XXI. Average investment in cows increased nearly 20 percent from the 1973-75 period to 1976. Average investment in buildings and improvements increased by about 13 percent over the same period. The average number of man equivalents of unpaid labor declined by about 17 percent from 1.79 in 1973-75 to 1.52 in 1976. The average number of man equivalents of labor hired increased

TABLE XXI

A COMPARISON OF INPUT AND OUTPUT DATA ON SELECTED
TENNESSEE DAIRY FARMS IN 1973-75 AND 1976^a

Inputs and Output	Averages	
	1973-75	1976
Investment in dairy cows (\$)	28,783	34,376
Investment in buildings and improvements (\$)	21,520	24,339
Man equivalents of hired labor per year	.83	.98
Man equivalents of unpaid labor per year	1.79	1.52
Annual machinery and equipment costs (\$)	12,471	15,848
Operating capital per year (\$)	49,503	75,007
Acres of open land	219	222
Gross farm income (\$)	82,988	116,085

^aThis comparison was based on data from 241 farm records in 1973-75 and 86 farm records in 1976.

over the same period by nearly 18 percent. Average annual machinery and equipment expenses increased by more than 27 percent from 1973-75 to 1976 and the average amount of operating capital utilized increased by an even greater percentage. The average amount of operating capital increased from \$49,500 in 1973-75 to \$75,000 in 1976, an increase of over 50 percent. The average number of acres of open land per farm increased slightly from 1973-75 to 1976, from 219 to 222 acres.

The average number of dairy cows per farm increased only slightly, from 67 to 70 cows from 1973-75 to 1976. Pounds of milk per cow increased by nearly 950 pounds per cow from 9,800 to 10,750 for the same period.

Average total farm expenses increased over \$30,000 from 1973-75 to 1976. The data indicate that the average farm in 1976 had more gross income than in 1973-75 but that almost 91 percent of this increased gross income was directly offset by increased farm expenses.

Results of Prediction

The R^2 statistic for 1976 was computed using sums of squares in logarithmic form to be consistent with the R^2 's reported for 1973-75. The R^2 found in this manner was .8672 while the R^2 for the 1973-75 period was .8812. This small difference of only .014 means that only 1.4 percent more variation in gross farm income was explained by Equation D on 1973-75 data than on 1976 data. The F value of the R^2 statistic was 73.69 and this value was significant at the 1 percent level. This means that the explanation of variation in 1976 gross farm

income had only a 1 percent chance of occurring due to chance rather than the explanatory powers of Equation D.

Figures 9 and 10 illustrate the variance between actual and predicted gross farm income. The diagonal solid line in each figure represents the points where predicted and actual gross farm income would be equal.

Predicted and actual gross farm income are compared in Figure 9 in terms of natural logarithms of dollars as this was the form used in linear estimation of the Cobb-Douglas function. Comparisons of predicted and actual gross farm income in arithmetic form are made in Figure 10. As can be seen in Figure 10, Equation D tended to underestimate gross farm income at higher income levels. Over all income ranges Equation D underestimated gross farm income for 49 farms while overestimating gross income on 33 farms. This could have been due to the smaller range in gross farm income in the 1973-75 period as compared to 1976. The highest actual gross farm income in 1976 was nearly \$200,000 higher than the highest gross farm income in 1973-75. At lower levels of gross farm income of \$50,000 to \$150,000, Equation D appeared to be most accurate, probably due to the fact that most observations from which Equation D was estimated had gross farm incomes within this range.

The unexplained variations in gross farm income for 1973-75 and 1976 were adjusted for degrees of freedom and combined into a ratio to permit an F test. The calculated F value of 1.22 was less than the critical F value at the 5 percent significance level. This indicated that the unexplained variation in gross farm income in 1976 was not

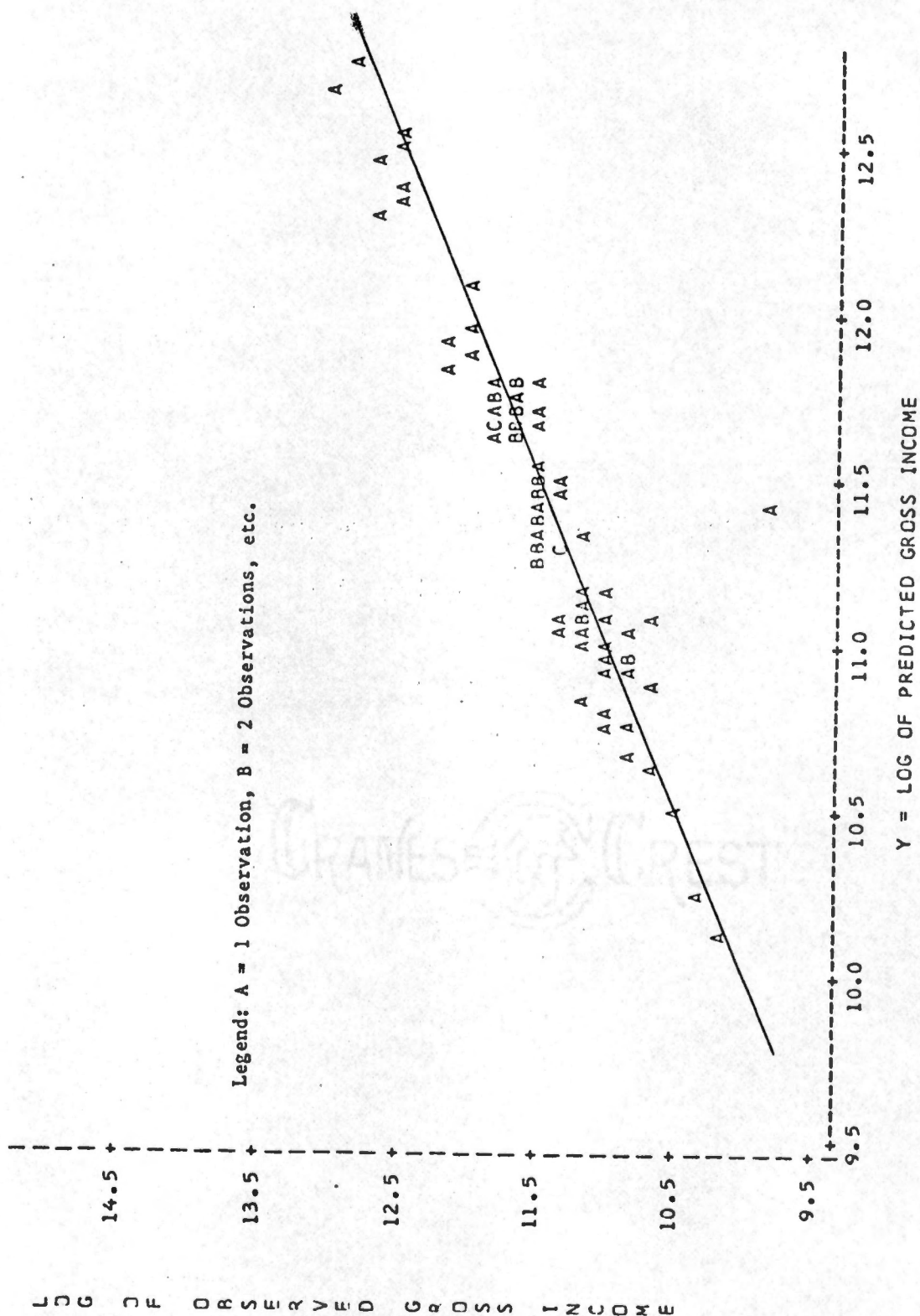


Figure 9. Predicted versus actual gross farm income for 82 Tennessee Grade A dairy farms in 1976 (income in logarithms).

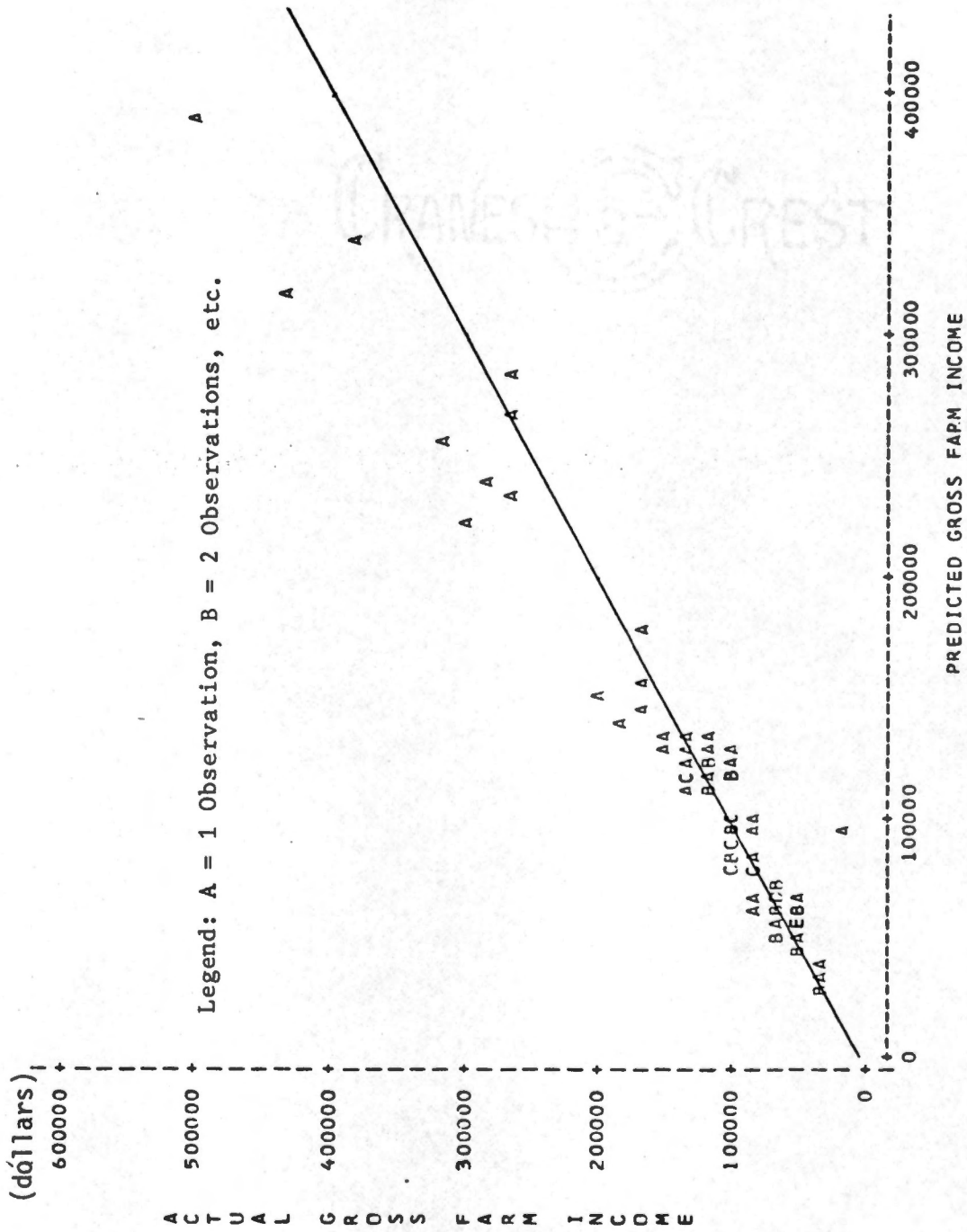


Figure 10. Predicted versus actual gross farm income for 82 Tennessee Grade A dairy farms in 1976 (income in dollars).

significantly different from the unexplained variation in gross farm income in 1973-75 at the 5 percent level. This supported the hypothesis that the input output relationship expressed by Equation D had not changed significantly from 1973-75 to 1976 for the data analyzed.

The value of Equation D as a predictor of gross farm income would likely diminish as the levels of inputs and output rise above the levels which were present in the 1973-75 data from which Equation D was estimated. The 1976 data does not, however, indicate that the input output relationships expressed by Equation D had changed significantly from 1973-75, although the size of operations had increased.

CHAPTER VII

SUMMARY AND CONCLUSIONS

This study was a production function analysis of Grade A dairy farms in Middle and East Tennessee. Records from a select group of Tennessee dairy farms enrolled in the Agricultural Resource Development Program were used for the study. The farms from which the records were taken were similar in that at least 50 percent of the gross farm income on each farm came from the dairy enterprise.

The objectives of the study were:

1. To estimate the marginal productivity of various resources used on Tennessee farms producing Grade A milk.
2. To estimate the returns to scale on Tennessee Grade A dairy farms.
3. To determine the substitution relationship between mechanization and hired labor in farm production on Tennessee Grade A dairy farms.
4. To develop a management index for individual milk producers and to compare it with efficiency factors commonly derived from farm record analysis.
5. To use production function estimates for prediction and to compare predicted output with actual output.

I. SUMMARY

In order to obtain estimates of marginal productivities, production

functions were estimated by the least squares technique using the Cobb-Douglas function in logarithmic form. Separate functions were estimated for 1973, 1974, 1975, and 1973-75 combined. Input categories related to annual gross farm income were investment in dairy cows, investment in buildings and improvements, man equivalents of hired labor per year, man equivalents of unpaid family and operator labor available yearly, annual machinery and equipment costs, yearly cash operating expenses, and acres of open land.

Marginal value productivity estimates were derived at the geometric and arithmetic means of each independent variable for each of the time periods. For the year 1973 man equivalents of unpaid labor was the only input with a negative MVP. In 1974 the MVP's of investment in buildings and improvements and man equivalents of unpaid labor were negative while the MVP's of the other inputs were positive. The 1975 MVP's of all the inputs were positive.

In the 1973-75 period combined, the MVP of investment in dairy cows was \$.43 when estimated at the geometric mean level of inputs and \$.45 when estimated at the arithmetic mean level of inputs. The MVP of investment in buildings and improvements was \$.05 and \$.04 as estimated at the geometric and arithmetic mean level of inputs respectively. The MVP of man equivalents of hired labor was \$5,976 when estimated at the geometric mean and \$3,281 when estimated at the arithmetic mean of all inputs. The MVP of man equivalents of unpaid labor was estimated to be \$1,692 and \$1,854 at the geometric and arithmetic mean input levels respectively. Machinery and equipment.

expenses had a MVP of \$1.71 when estimated at the geometric mean and a MVP of \$1.80 when estimated at the arithmetic mean level of inputs. Operating capital had a MVP of \$.78 when estimated at the geometric mean and \$.76 when estimated at the arithmetic mean level of inputs. The MVP of acres of open land was \$42 at the geometric mean and \$44 at the arithmetic mean level of inputs. Diminishing marginal productivity was exhibited for each of the inputs.

The separate production functions estimated provided direct indications of returns to scale on the farms in the study. Increasing returns to scale were indicated in the periods 1973, 1975, and 1973-75. Only in 1974 did the data indicate decreasing returns to scale for Grade A dairy farming.

The marginal rate of substitution of machinery and equipment for hired labor was estimated for an output level of \$70,905 of gross farm income, the geometric mean level of income for 1973-75. The least-cost combination was found to be \$11,347 of machinery and equipment expenses and 750 hours (.25 man equivalents) of hired labor. The least cost combination was based on a \$1.02 cost per \$1.00 annual machinery and equipment expense and hired labor costs of \$1.88 per hour. A diminishing marginal rate of substitution of machinery and equipment for hired labor was exhibited.

A management index, indicating relative management ability was calculated for each farm in the 1973-75 period as a ratio between actual and predicted gross farm income. This ratio was derived from the residuals of the regression equation used to estimate marginal

value productivities of resources for 1973-75. This procedure was based on the assumption that the unexplained variations in gross farm income resulted from the exclusion of the management variable in the production function.

A correlation analysis related the management index with various labor efficiency, production efficiency, and land efficiency factors. The highest correlations were found between the management index and production efficiency measures, especially those factors relating to milk production per cow. The farms were divided into three groups by level of management index and the average value of each efficiency factor for each group was calculated to provide evidence supporting the results of the correlation analysis.

The production function estimated using 1973-75 data was used to predict gross income for 1976 for a group of 82 dairy farms. A R^2 statistic was used to determine how much variation in gross farm income was explained by the equation. The R^2 calculated was .8672 and was only .014 less than the R^2 for the 1973-75 regression. The predicted gross farm income exceeded actual gross income for 33 farms and was less than actual income for 49 farms. At income levels of \$200,000 or more, the 1973-75 equation tended to underestimate gross farm income, but at lower income levels predictions were more accurate.

The unexplained variation in gross farm income for the 1973-75 and 1976 periods were adjusted for degrees of freedom and combined into a ratio to gain a F statistic. The value of this statistic indicated that the unexplained variation in gross farm income in 1976

was not significantly different from the unexplained variation in the 1973-75 period at the .05 percent level.

II. CONCLUSIONS

The marginal productivity estimates obtained in this study have many limitations and must be interpreted with caution. They do, however, provide some general guidelines as to the level of input use on Tennessee Grade A dairy farms. The following general guidelines were derived from the results of production function analysis for all farms for 1973-75.

The MVP estimated for investment in dairy cows indicated that it would have been profitable for the average farm to invest more in dairy cows. When estimated at the arithmetic mean level of inputs, the MVP of investment in dairy cows was \$.45 for the 1973-75 period, more than sufficient to cover an associated charge for risk and interest on additional investment.

At the arithmetic mean level of inputs, the MVP of investment in buildings and improvements was \$.04 for 1973-75. This rate of return would not have been sufficient to cover interest, depreciation, and maintenance costs on additional investment in buildings and improvements. The average dairy farmer in the study appeared to have over-invested in buildings and improvements.

The MVP of an additional man equivalent of hired labor was estimated as \$3,281 at the arithmetic mean level of inputs for 1973-75. Given the average Tennessee farm wage in this period, the returns

from an additional man equivalent of hired labor would not have covered the costs for the average dairy farm. Applicable wage rates and MVP of additional hired labor would of course, vary from farm to farm.

When calculated at the arithmetic mean level of inputs, the MVP of a man equivalent of unpaid labor was \$1,854 for 1973-75. Thus in the combinations used on the dairy farms in the study, unpaid labor had a lower rate of return than hired labor. This result may be partially a measurement problem. Man equivalents of unpaid labor were measured in terms of available labor while hired labor was measured in terms of amount actually used. Excess or underemployment of family labor would tend to depress the rate of return to unpaid labor.

The MVP of annual machinery and equipment expenses, when estimated at the arithmetic mean level of inputs for 1973-75, was \$1.80. This return would considerably exceed costs associated with a dollar increase in this input, indicating that the average farm in the study could have profitably expanded expenditures on machinery and equipment expenses.

At the arithmetic mean level of inputs, the MVP of an additional dollar of operating capital was \$.76 for 1973-75. Obviously, increasing this input would have yielded a negative return. The study indicated that the average farm in 1973-75 appeared to be using excessive amounts of operating capital.

An additional acre of open land had a MVP of \$44, when estimated at the arithmetic mean level of inputs for the 1973-75 period.

Whether or not the average farm could have profitably expanded acres of open land depends on the price of land in a particular area and the interest rate on capital. If returns were capitalized over time and an interest rate of 8 percent was assumed, the average farm could have profitably purchased land if the price was \$550 per acre or less in the 1973-75 period. At a 10 percent interest rate expansion of acres of open land would have been profitable only if land was priced at \$440 or less. The average farm could have rented acres of open land profitably for costs up to \$44 per acre.

Reductions in capital used for buildings and improvements, operating capital, and hired labor combined with increases in investment in dairy cows and machinery and equipment expenses would have increased profitability on the average farm in 1973-75.

The analysis indicated that for the 1973-75 period increasing returns to scale were evident. This indicates that increasing all inputs in the same proportion would have lowered costs per unit of output and thereby increased profits on the average farm in the study.

The analysis of the substitution relationship between machinery and equipment costs and hired labor indicated that mechanization had a diminishing marginal rate of substitution for hired labor in 1973-75. This analysis combined with the marginal productivity analysis indicated that the average farm could have profitably reduced hired labor and increased machinery and equipment expenses while holding gross income constant.

CRANES CREST

The management index estimated in this study was related to certain observable characteristics of dairy farms but specific conclusions that can be drawn are somewhat limited. The management index was found to be positively correlated with certain efficiency factors. Milk sales per cow was the best indicator of managerial ability found.

The input output relationship expressed by the production function developed for 1973-75 data was not found to be significantly different from the input output relationship evidenced by the 1976 data. As a result, it was concluded that the general guidelines developed throughout this study would have been appropriate for the 1976 period also.

BIBLIOGRAPHY



BIBLIOGRAPHY

- Garland, Clark D., Associate Professor of Extension Agricultural Economics, Tennessee Agricultural Extension Service, personal correspondence, December, 1977.
- Gilson, J. C., and M. H. Yeh. Productivity of Farm Resources in the Carman Area of Manitoba. Technical Bulletin No. 1 (University of Manitoba, Winnipeg), 1959.
- Heady, Earl O. "Production Functions from a Random Sample of Farms," Journal of Farm Economics, 28:989-1004, 1946.
- Heady, Earl O., and John L. Dillon. Agricultural Production Functions (Iowa State University Press, Ames, Iowa), 1961.
- Humberd, D. R., and Clyde Chappell. "A Brief Resume of the Tennessee Dairy Industry" (unpublished paper for Tennessee Agricultural Extension Service, 1975), pp. 1-5.
- Johnson, Glenn L. "Classification and Accounting Problems in Fitting Production Functions to Farm Record and Survey Data," Resource Productivity, Returns to Scale and Farm Size. Edited by E. O. Heady and others (Iowa State College Press, Ames, Iowa), 1956, pp. 90-96.
- Plaxico, J. S. "Problems of Factor Product Aggregation in Cobb-Douglas Value Productivity Analysis," Journal of Farm Economics, 37:664-675, 1955.
- Spangler, H. L. "Economic Analysis of Commercial Farming in the Elk River Watershed" (unpublished Master's thesis, University of Tennessee, 1968), pp. 49-55.
- Tintner, Gerhard A. "A Note on the Derivation of Production Functions from Farm Records," Econometrica, 12:26-34, 1944.
- Tintner, Gerhard A., and O. H. Brownlee. "Production Functions Derived from Farm Records," Journal of Farm Economics, 26:566-577, 1944.
- United States Department of Agriculture, Statistical Reporting Service Farm Labor Situation. (Washington, D. C., October, 1974 and October 1975).
- United States Department of Agriculture and Tennessee Department of Agriculture, Tennessee Agricultural Statistics. Annual Bulletin T-14 (Nashville, 1977), p.7.
- United States Department of Agriculture, Economic Research Service, Dairy Situation. (Washington, D. C., December, 1977), p.6.

VITA

William S. Hamilton, Jr. was born August 9, 1954, in Lincoln County, Tennessee, the son of Mr. and Mrs. W. S. Hamilton, Sr. He was reared on a farm in the Swan Creek Community of Lincoln County and attended Boonshill High School, graduating in May, 1972.

In September, 1972 he entered the University of Tennessee at Knoxville as a student in the College of Agriculture. He majored in Agricultural Economics and was awarded the Bachelor of Science Degree in June, 1976.

During the period of 1976-78, he was employed as a Graduate Research Assistant by the Department of Agricultural Economics and Rural Sociology of the University of Tennessee. He is presently completing the requirements for the Master of Science Degree in Agricultural Economics.