

Determining the Nature of Superfluidity and Superconductivity in Dense Matter from Neutron Star Observations

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Abstract

I present a quantitative analysis that utilizes observations of neutron stars to constrain the magnitude of the neutron superfluid gap and the proton superconducting gap in dense matter. In the context of the minimal cooling model, the most likely range for the neutron triplet superfluid gap is $2.09^{+4.37}_{-1.41} \times 10^8$ K and the most likely range for the the proton singlet superconducting gap is $7.59^{+2.48}_{-5.81} \times 10^9$ K. In a second analysis, I confirm these basic predictions for the gaps and additionally determine the mass and envelope composition of the neutron stars in our data set. This analysis shows that some neutron stars are likely more massive and thus cool by the direct Urca process. It also shows that the minimal cooling model may not be the best explanation of neutron star cooling observations. Finally, these quantitative results show that further observations of neutron star cooling will continue to provide constraints on both the equation of state and the transport properties of dense matter.

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Chapter 1

A Brief History of Neutron Star Research

The existence of neutron stars was theoretically predicted prior to their discovery in nature. As early as the 1930s, a few theoretical physicists had already suggested the possible existence of neutron stars, several decades prior to the detection of the first neutron star in 1967. The first theoretical proposal for a compact star denser than a white dwarf star is credited to the Soviet physicist Lev Landau, who proposed the idea in 1932 [7]. Landau investigated the stability properties of compact stars supported entirely by degeneracy pressure, which is a good description of the interiors of neutron stars with the exception of their inner cores, wherein the majority of the pressure opposing gravity arises from the repulsive short range behavior of the strong nuclear force. Degeneracy pressure itself arises as a direct consequence of the Pauli exclusion principle, which describes the inability of fermions (e.g. neutrons, protons, electrons) to share the same quantum state.

Landau determined there is a maximum mass above which degeneracy pressure cannot counterbalance gravity, this mass is known as the “Landau limit” and is expressed as:

$$M_L = \frac{3.1}{m^2} \left(\frac{\hbar c}{G} \right)^{\frac{3}{2}}, \quad (1.1)$$

where $\hbar$ is the reduced Planck’s constant, G is the gravitational constant, and c is the speed of light. The only variable in this equation is m , the supplied mass of the fermions repelling

one another as a consequence of the Pauli exclusion principle. When the supplied mass is set equal to twice the mass of a proton $m = 2 \cdot m_p$ (accounting for the degeneracy pressure of both protons and neutrons, whose masses are equivalent to within 0.1% of each other), one obtains a maximum mass limit for compact stars $M_L = 1.8 M_\odot > M_{\text{stable star}}$ ($1 M_\odot$ here being one solar mass, equivalent to 2×10^{33} g), a value close to current predictions for the maximum mass of neutron stars. This inequality states that in order for a compact star supported by degeneracy alone to remain stable over any long period of time, its mass must remain under $1.8 M_\odot$. Consideration of the repulsive nuclear strong force pressure within the inner core raises the allowed maximum mass above this value, but this early prediction yields a good lower limit for the maximum mass of neutron stars. What is perhaps even more remarkable is that Landau made this calculation prior to the discovery of the neutron as an elementary particle.

James Chadwick first experimentally recognized the neutron as being an elementary particle in 1932. One year later, Walter Baade and Fritz Zwicky presented theoretical work which proposed the existence of an astronomical object primarily composed of neutrons. An object predominantly or entirely composed of neutrons, which have no electric charge and thus suffer no repulsive Coulomb force, may contain internal densities exceeding the saturation density of nuclear matter.

The density of uncompressed nuclear matter, $\rho_0 \approx 2.8 \times 10^{14}$ g/cm³ (corresponding to a number density of $\rho_0 \approx 0.16$ fm⁻³), is also referred to as the saturation density. This is due to the peculiar property that, at this special density, the close range repulsive and far range attractive components of the nuclear strong force exactly balance. A consequence of this balance of forces is that at saturation density, additional nucleons may be added to a system of nucleons without any resulting increase in the system's overall density. Nuclear matter at this density is at point of equilibrium, external pressure is necessary to further increase the nucleon density above saturation density.

Baade and Zwicky proposed the means by which a compact astronomical object containing densities exceeding the nuclear saturation density could be created [8]. They noted that some very massive stars, which became particularly bright for a short period of time and subsequently vanished, were very likely undergoing core collapse and subsequently

exploding. These celestial explosions in the night sky appear to the naked eye as new stars, “novus stella” is Latin for ‘new star’, and many transient astronomical events are referred to as novae. The explosion of a supernovae is significantly more energetic and bright than other types of novae, thus giving rise to Baade and Zwicky’s nickname for these phenomenon, supernovae. They proposed that these dramatic explosions could provide enough pressure to create a compact star with a mean density exceeding the nuclear saturation density.

The first experimental detection of a neutron star came in 1967, when a periodic signal was picked up by a radio telescope at Cambridge University. Upon analyzing the signal, Anthony Hewish and Jocelyn Bell deduced that it had originated from a radio source possessing a very short period [9]. Such a short and predictable period, coupled with consistent high frequency emissions could reasonably be explained by the rotation of an extremely massive, stable object with a very strong magnetic field. A neutron star’s strong magnetic field accelerates and collides charged particles at relativistic speeds, making its magnetic poles intense sources of electromagnetic radiation emitted by these collisions. As the star rotates, these poles appear to far away observers here on Earth to blink like lighthouse beacons. This explanation directly accounts for the periodic radio pulses Bell and Hewish detected. This single observation provided convincing evidence that neutron stars do in fact exist.

In the decades following this initial discovery, a fleet of satellites (including Chandra, XMM-Newton, the Fermi satellite, and the Hubble Space Telescope) and ground based x-ray and radio telescopes have discovered thousands of neutron stars. Observations of this increasing population of known neutron stars, as well as theoretical advances in the understanding of dense nuclear matter, has enabled the establishment of a canonical profile for neutron stars. Neutron stars have masses between $1.0 - 3.0 M_{\odot}$ and radii between $9 - 15$ km (see Ref. [10]), and a large collection of neutron stars have masses close to $1.4 M_{\odot}$. The neutron star lower mass limit is obtained from calculated lower mass limits of proto-neutron stars found in current supernovae models (see Ref. [11]), along with the lack of any current observations of neutron stars below this limit [12]. The upper mass limit as detailed in Ref. [13], is informed by the fact that the speed of sound within neutron stars cannot exceed the speed of light. A simple calculation using these canonical values reveals that the mean

mass density of neutron stars is greater than the density of an atomic nucleus (i.e. the saturation density). The exact correlation between individual masses and radii of neutron stars, the mass-radius relation, is determined by the equation of state (EOS). The EOS relates the internal pressures and densities of the neutron star; finding the exact expression of this relation remains an unresolved problem.

Neutron stars are now understood to be one of the possible end products of stellar evolution. They are the dense remnants formed from the core collapse and subsequent supernovae of enormous main-sequence progenitor stars with masses ranging from ~ 8 to $20 M_{\odot}$ [14]. In Ref. [15], it is calculated that a new pulsar is born in our galaxy approximately every century based on the number of observed supernovae events. Over the lifespan of our galaxy, which is estimated to be 13 billion years old, this steady birthrate has produced a galactic population of hundreds of millions of neutron stars. It is then reasonable to ask why we have not yet detected millions of neutron stars. The answer is that most of these stars have cooled beyond the point of being “visible” to observation via telescope.

Fortunately, there are many observable neutron stars currently undergoing detectable cooling [2]. Depending on the phase of cooling the star is detected in, one can use inferred temperature and luminosity observations to make a variety of deductions about the structure, composition, and accompanying cooling mechanisms of the neutron star [16]. Young neutron stars are dominated by neutrino cooling, and thus provide information about the composition and neutrino emitting processes present in the crust and core of the star. Old neutron stars (older than one hundred thousand years old) are dominated by photon cooling, and observations of these stars are useful in constraining the envelope and atmospheric composition near the stars’ outer boundary.

High internal pressures, coupled with relatively low temperatures, allow nucleon superfluids to exist inside neutron stars. This realization was first made by Arkady Migdal in 1959 (see Ref. [17]), but understanding of the nucleon superfluids within neutron stars has greatly increased since then. A superconductor of paired protons and superfluids of paired neutrons are both understood to exist within neutron stars (Ref. [18]), although the critical temperatures at which these phases emerge is presently unknown. The presence of these superfluids has a noticeable effect on neutron star cooling, particularly in the neutrino

cooling phase, therefore it should be possible to determine the critical temperature of nucleon superfluids from surface temperature and luminosity observations of cooling neutron stars [19]. If additional fast cooling of neutron stars is observed, the cooling profiles of these stars should be compared to several current theoretical models which propose the existence of enhanced neutrino emission processes. Various proposed sources of enhanced neutrino cooling range from the activation of the direct Urca process in massive neutron stars, to the presence of exotic particles (hyperons), meson condensates, or quark matter in the neutron star core [20].

Two of the current objectives of neutron star research are to employ observations of neutron stars to obtain a more precise description of the dense matter EOS, and also to study the transport properties of dense matter, especially superfluidity in dense matter. The first goal, obtaining an EOS, describes finding an equation which can accurately predict the observed masses, radii, and composition of neutron stars. The dense matter EOS mutually governs the behavior of nuclei and neutron stars. In nature we see correlations between observations of nuclei and observations of neutron stars. Thus to place precise constraints on the subatomic properties of nuclei is to also constrain astronomical properties of neutron stars and vice versa. This connection is highlighted in experiments such as the determination of the neutron skin thickness in ^{208}Pb (see Ref. [4]), which due to its dependence on the nuclear symmetry energy, is expected to inform more precise constraints on the total neutron star radius. Until this measurement is completed to the desired precision, the radii of neutron stars which best informs the expected value of the neutron skin thickness in ^{208}Pb .

Transport properties determine how momentum and energy are dispersed through dense matter. Ascertaining the allowed temperatures and densities at which nucleon superfluids exist as phases within dense matter plays a key role in determining these transport properties. The role neutron stars play in studying superfluidity and dense matter properties illustrates that neutron stars are not just novel oddities, but powerful instruments for studying the most fundamental properties of spacetime and matter.

1.1 The Composition of Neutron Stars

It is known from exhaustive research of atomic properties at low densities that neutron stars contain at least neutrons, protons, and leptons. Neutrons and protons are the constituents of atomic nuclei, and are collectively referred to as nucleons. Both nucleons are hadrons, particles which participate in nuclear strong force interactions. Protons are positively charged hadrons (they possess an electric charge of equal magnitude and opposite sign to that of electrons), they are the lightest known hadrons (938.272 MeV) and have never been observed to decay in nature. Neutrons are slightly heavier hadrons (939.565 MeV). The masses of the proton and neutron are almost equivalent, with only a 0.1% difference between the two masses. Thus, for strong force interactions, these particles are seen as being degenerate in energy.

Werner Heisenberg was the first to suggest that the neutron and the proton could be viewed as two opposing isospin projections of the same entity, the nucleon. The only difference is that the third isospin component of the proton is considered to be spin up, while the third isospin component of the neutron is considered to be spin down:

$$|p\rangle = |t = 1/2, t_0 = +1/2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |uud\rangle \ , \quad (1.2)$$

$$|n\rangle = |t = 1/2, t_0 = -1/2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |udd\rangle \ . \quad (1.3)$$

The ket on the far right denotes the respective quark composition of protons and neutrons respectively. It is only when one considers Coulomb interactions, many times weaker than the strong force, that this degeneracy is broken. Neutrons are observed to “beta decay” into protons, electrons and neutrinos under terrestrial conditions. The process of beta decay is expressed as:

$$n \rightarrow p + e^- + \bar{\nu}_e \ . \quad (1.4)$$

Note that charge is conserved in this interaction. The process of beta decay in neutron stars is often blocked. This due to the fact that within neutron stars the electrons exist

in a relativistic gas, and the Fermi sea of electrons is filled. The inverse reaction has been famously observed in nuclear reactors:

$$\bar{\nu}_e + p \rightarrow n + e^+ . \quad (1.5)$$

Again, charge is conserved here. Leptons, fermions which do not participate in strong nuclear force interactions, are present in neutron stars in the form of electrons (rest mass of 0.511 MeV), muons (105.7 MeV) and their antiparticles. The most massive leptons, tau leptons (1776.82 MeV), have been generated in collision experiments but are too massive to be generated in neutron stars.

Neutrinos (ν_e, ν_μ, ν_τ), first proposed as theoretical particles in the 1930 by Wolfgang Pauli and subsequently discovered in 1956, are fermions possessing very little mass which have no electric charge. For this reason, they are known to possess very large mean free paths and seldom interact with other particles. This explains the difficulty of detecting these particles. It also explains why neutrinos dominate the early cooling of neutron stars. Photons are a Bose gas interacting with charged particles through Coulomb interactions, taking hundreds of thousands of years to filter through the successive layers of the star. Once the star cools below a certain temperature, neutrinos exit the star near the speed of light.

It is likely that exotic matter exists within the inner core (see section 4.9 on hyperons) of neutron stars. The inner core corresponds to the region of highest density within the star. The extremely high nucleon chemical potentials found at these densities facilitates the generation of exotic matter. Thus, it is possible that the ground state of dense matter at these densities is composed of particles beyond that of nuclear matter as described above. Examples of exotic matter are hyperons, kaon condensates, pion condensates, as well as quark matter. The presence of these phases of exotic matter would have observable effects on neutron star stability and cooling [21].

1.2 The Birth of a Neutron Star

The observed chemical composition of the universe is predominately hydrogen and helium with approximately one helium atom for every three hydrogen atoms, this is a consequence of Big Bang nucleosynthesis. Once the universe cooled below the requisite ambient temperature to produce neutrons from protons, as seen in Eq. (1.7), free neutrons decayed until finding stability in the form of weakly bound deuteron and then tightly bound helium. The stability of helium is an observed consequence of the nuclear shell model (see section 2.8), much like the electron shell model observed in chemical reactions filled nucleon shells have the highest stability. In addition to hydrogen and helium, there are very scarce levels of heavier elements (e.g. carbon, oxygen, calcium). It is the process of stellar nucleosynthesis, the continuous chain of nuclear fusion reactions occurring in main sequence stars such as our sun, which accounts for the observed abundance of all the other elements up to the nuclear mass of iron. Theoretical models of these processes accurately predict the observed relative abundance of elements in the universe [22].

Intermediate mass main sequence stars, such as our sun, sequentially synthesize helium, carbon, oxygen and silicon. In higher mass stars, those above $\sim 10 M_{\odot}$, one finds the nuclear burning and synthesis of elements heavier than silicon and oxygen. The fusion reactions for elements up to the mass of iron are exothermic, producing a net output of thermal energy which provides pressure that prevents the star from undergoing further gravitational collapse. The binding energy, the energy or work necessary to completely disassemble a nucleus into its constituent neutrons and protons, increases with nuclear mass until it peaks at a value of $B.E._{\text{max}} \approx 8.5 \text{ MeV}$ corresponding to the ^{56}Fe nucleus. After the synthesis of the iron-peak elements (examples of these being ^{56}Fe and ^{62}Ni), the high binding energy of these nuclei disallows their use as fuel for further exothermic fusion reactions [23]. Instead, fission reactions wherein mother nuclei radioactively decay into lighter daughter nuclei are increasingly energetically favored for nuclei with masses above ^{56}Fe (see Fig. 1.1). All elements above atomic number 82, corresponding to various isotopes of lead (Pb) which has 82 protons, are unstable to radioactive decay.

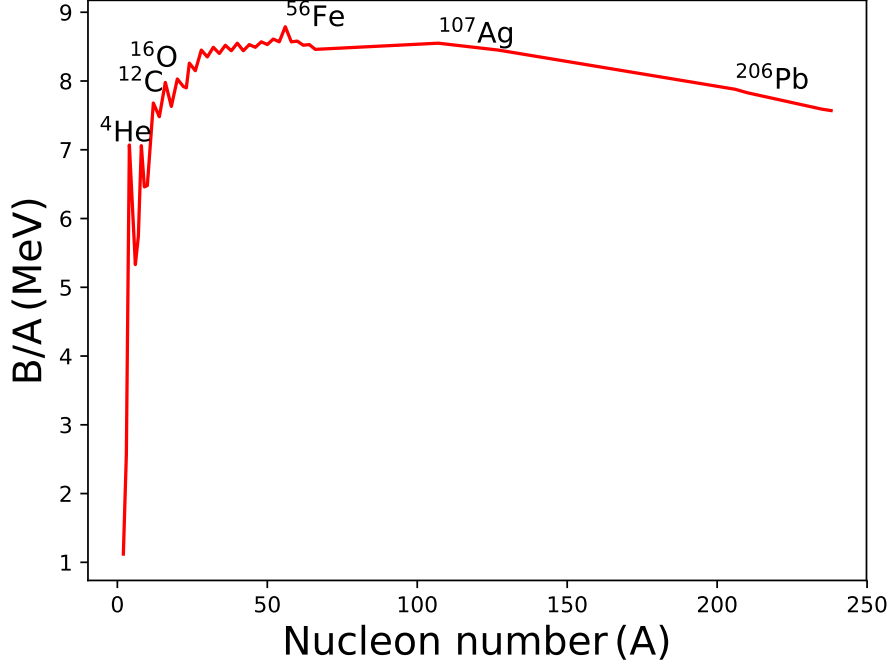


Figure 1.1: Chart of binding energies, note the peak binding energy coinciding with ⁵⁶Fe

The lack of further exothermic nuclear interactions to provide additional thermal pressure ensures the destruction of the star, as now the gravitational force will compress the star radially inward and the star will undergo core collapse. The result of core collapse is a violent explosion, a supernova. It is during this process of collapse that supranuclear densities in the inner core of the remnant are achieved. This remnant will form a proto-neutron star provided it is not massive enough for gravity to overcome all opposing pressure, in which case it will collapse to a black hole. The system achieves minimal energy through neutralizing charge, during the collapse and subsequent explosion there is a mass conversion of protons and free electrons into neutrons:

$$n \rightarrow p + e^{-} + \bar{\nu}_e , \quad (1.6)$$

$$p \rightarrow n + e^{+} + \nu_e . \quad (1.7)$$

Approximately 10^{57} protons are converted into neutrons very rapidly, producing a large population of positrons which quickly undergo pair annihilation with free electrons resulting

in a massive wave of outgoing energy in the form of photons and neutrinos:

$$e^+ + e^- \rightarrow \gamma + \gamma , \quad (1.8)$$

$$e^+ + e^- \rightarrow \nu + \bar{\nu} . \quad (1.9)$$

Additional neutrinos are generated in thermal processes within proto-neutron star. The energy borne away by the neutrinos alone is tremendous, and is approximately 10% of the total binding energy of the neutron star [24]. The binding energy of the star is the difference of the total gravitational and baryon mass of the star:

$$\text{Total Binding Energy} = (M_A - M_G) c^2 . \quad (1.10)$$

The total gravitational and baryon masses of the star are defined respectively as [25]:

$$M_G c^2 = \int_0^R dr 4\pi r^2 \epsilon(r) , \quad (1.11)$$

$$M_A c^2 = m_A \int_0^R dr 4\pi r^2 \frac{n(r)}{\left[1 - \frac{2GM(r)}{c^2 r}\right]^{1/2}} . \quad (1.12)$$

$\epsilon(r)$ and $n(r)$ are the energy density and number density, respectively. Both are functions of the enclosed radius of the star and increase as r goes to zero (the center of the stars).

If each neutrino has 10 MeV of energy, the outgoing torrent of 10^{57} neutrinos carries with it 1.0×10^{58} MeV of energy (comparable to the lifetime energy output of our sun). This massive release of energy was witnessed directly in the case of the supernova 1987A, whose optical detection coincided with the detection of neutrinos using the Earth based detectors Kamiokande and the Sudbury Neutrino detector [26].

The typical proto-neutron star remnant possesses a very large angular momentum [27], and an immense magnetic field. It has been hypothesized that this is due to conservation of angular momentum and magnetic flux from its massive progenitor star. The fastest rotating neutron star observed to date is PSR J1748 – 2446ad, which rotates at 700 Hz. Magnetic fields of exceeding $B \sim 10^{12}$ G have been observed in neutron stars.

1.3 Neutron Star Structure

Starting from the surface of the star and going radially inward, we encounter the atmosphere, envelope, outer crust, inner crust, outer core and finally the inner core sequentially. The pressure increases rapidly toward levels unmatched here on Earth as one descends into the subsequently denser layers of the star. Consequentially, the star’s chemical composition is better understood closer to the surface, while the contents of the inner core remain a mystery.

The outermost layer of the star, the atmosphere, is a gaseous plasma several millimeters thick which absorbs and re-emits electromagnetic energy in the form of photons departing from the neutron star. The atmosphere determines the observed emission spectrum of the neutron star. Comparison of this spectrum to known chemical emission spectra can provide information regarding the chemical composition of the atmosphere. Further analysis of the emission spectrum yields an estimate of the neutron star surface temperature, the mass and radius of the star, and an estimate of the magnitude of the star’s magnetic field [28]. Depending on what theoretical atmosphere is being compared to neutron star observational data, predictions of the radius, emission spectra, and surface temperature can vary considerably. Thus the chemical composition of the atmosphere, from proposed light element atmospheres (containing H, He) to proposed heavy element atmospheres (with emission spectra resembling that of Fe), strongly affects the interpretation of luminosity and effective temperature observations of neutron stars.

Just below the atmosphere is the envelope layer, which thermally insulates the atmosphere from outward cooling through the crust. The thickness and composition of the envelope determines the temperature gradient between the core and atmosphere. The outer crust contains a lattice of nuclei, immersed in a gas of free electrons. The outer crust is characterized as a body-centered cubic lattice of ^{56}Fe [29]. Between the outer crust and inner crust at a density of approximately $\rho \sim 10^{14} \text{ g/cm}^3$, the competition between Coulomb repulsion and the far field attractive behavior of the nuclear strong force give rise to a number of exotic nuclei deformations collectively known as “nuclear pasta” [30].

At a certain radius within the inner crust, a density is achieved known as the “drip density”. It is at this density ($\rho \sim 4 \times 10^{11} \text{ g/cm}^3$) that neutron rich nuclei begin to

shed excess neutrons. The densities lower than the neutron drip density facilitate many nuclear fusion reactions, ensuring that stable nuclei above this density will be neutron rich. The neutron drip process occurs when the nuclei achieve a lower energy state through the unbinding of valence neutrons. The resulting abundance of free, unbound neutrons is described as a neutron fluid. The neutron-rich nature of nuclei at increasing densities ensures the majority of free nucleons are neutrons, but protons can drip out of their parent nuclei as well.

The core of the neutron star is typically characterized as two distinct regions, the outer core and inner core. The outer core contains homogenous nuclear matter consisting of neutron and proton superfluids, and it is this region which plays the most important role in neutrino cooling scenarios allowed within the minimal cooling model paradigm. The inner core of the neutron star contains the highest densities, leading to hypotheses that this region contains exotic matter and enhanced cooling processes.

1.4 Isolated Neutron Stars

In the catastrophic wake of a supernova, the proto-neutron star appears as a dense, hot remnant near the center of a rapidly expanding cloud of ejecta. The proto-neutron star undergoes rapid processes such as gravitational accretion of ejecta and deleptonization. After these short-lived processes, it enters of a long stage of structural stability during which it begins to cool from an initial temperature in the range of 20-50 MeV [31]. Mere minutes from birth, the star cools down a temperature on the order of 1 MeV [32].

If a star exists in a binary orbit with a main sequence star, it will siphon material off the surface of this main sequence star and gain additional mass and heat in a process known as accretion. If the star is not in a binary system, and therefore not undergoing accretion but instead cooling in isolation, it is described as isolated. Isolated neutron stars become isothermal in the first hundred years of their lifetime with the exception of a thin surface shell. These isolated stars then monotonically cool at a rate which is determined by the properties of dense matter. The study of isolated neutron stars therefore plays a critical role in revealing the internal composition and allowed nuclear processes of all neutron stars.

One aspect of neutron star cooling, which we explored in my initial research Ref. [33], is the existence of nucleon superfluidity and its effect on neutron star cooling.

Chapter 2

Research Background

2.1 Spacetime Deviation

In this work, we use units where, for ease of calculating relativistic quantities, we allow:

$$c = \hbar = G = k_B = 1 . \quad (2.1)$$

Utilizing the theory of relativity is absolutely necessary for accurately measuring and modeling neutron stars. A simple test to demonstrate that neutron stars require a relativistic framework is to calculate the spacetime deviation, the first term of our spacetime metric:

$$g_{00} = - (1 - 2M/R) , \quad (2.2)$$

where M and R here are the total mass and radius of the star. Totally flat spacetime (containing absolutely no mass, and thus experiencing no gravity) is characterized by $g_{00} = -1$, and spacetime where gravitational deviations are weak is characterized by $g_{00} \approx -1$. For objects such as a single proton, the Earth and even the sun, spacetime deviations in the vicinity of these objects are weak, as the magnitude of the spacetime deviation remains close to unity.

The powerful gravitational fields of neutron stars generate considerable variation in the calculated spacetime deviation. For neutron stars we see that for a mass of $1.4 M_\odot$ and a

radius of 12 km, well within the canonical limits for neutron stars, for units where $c = G = 1$ calculation shows:

$$\frac{2M}{R} = \frac{2 \cdot 2077 \text{ m}}{12 \times 10^3 \text{ m}} = 0.35 . \quad (2.3)$$

This corresponds to a significant spacetime deviation, as the time component metric of our metric is now $g_{00} = -(1 - 0.35) \approx -0.65$. As this spacetime deviation will result in noticeable time dilation in the vicinity of this object, we can safely say that neutron stars must be described within a relativistic framework.

2.2 Relativistic Effects on Neutron Star Observations

Spacetime deviation is so significant in the proximity of neutron stars that they bend the path of light. This is true of the light departing from the star itself, allowing a portion of the back side of star to be visible (see Eq. (2.7)). The frequency of the emitted light is noticeably decreased by the stars powerful gravitational field. A decrease in the frequency of visible light corresponds to an image becoming more red in appearance, thus the phenomena of gravitational acceleration decreasing the observed frequencies of relativistic objects is often called redshift. Effective temperatures of relativistic bodies, which account for the effect of gravitational redshift, are expressed as [28]:

$$T_{\text{eff}}^{\infty} = \frac{T_{\text{eff}}}{1 + z_g} . \quad (2.4)$$

Here, the redshift parameter z_g is:

$$z_g = \frac{1}{\sqrt{1 - x_g}} - 1 = \frac{\omega}{\omega_{\infty}} - 1 . \quad (2.5)$$

For neutron stars $x_g = \frac{r_s}{R}$ ranges from $\frac{1}{5}$ to $\frac{1}{2}$, where r_s is the Schwarzschild radius, expressed here as $2.95 M/M_{\odot}$ and R is the total measured radius of the star. Here ω is the emitted frequency, while ω_{∞} is the frequency measured by an observer at a location where the effects of gravity are assumed to be non-relativistic (i.e. flat spacetime). More generally the emitted signal frequency and the detected signal frequency can be related by finding the ratio of the

coordinate times in the two coordinates spaces:

$$\frac{\omega_{\text{received}}}{\omega_{\text{emitted}}} = \frac{\sqrt{g_{00,\text{received}}}}{\sqrt{g_{00,\text{emitted}}}} . \quad (2.6)$$

This is sometimes referred to as the relativistic Doppler effect.

The observed radius of the neutron star is also distorted by its strong gravitational potential, and is expressed as:

$$R_{\infty} = R (1 + z_g) = \frac{R}{\sqrt{1 - \frac{2M}{R}}} . \quad (2.7)$$

The “hypotenuse” of the relativistically corrected optical and x-ray radii provides a good overall estimate of the radius of the star:

$$R = \sqrt{R_{\text{opt}}^2 + R_X^2} . \quad (2.8)$$

The apparent luminosity L^{∞} is obtained from the redshift temperature:

$$L^{\infty} = 4\pi (R^{\infty})^2 \sigma_{\text{SB}} (T_{\text{eff}}^{\infty})^4 . \quad (2.9)$$

In practice observations of neutron stars involve counting the number of photons collected from an observed source, the quantity describing the amount of light coming from a certain enclosed area is the flux. Kirchoff’s laws for a black body determine a flux based on temperature. It is this effective flux (F_{∞}) quantity commonly used to derive the apparent luminosity and effective temperature from direct observations and distance measurements to neutron stars (d) :

$$F = \left(\frac{R}{d}\right)^2 \sigma_{\text{SB}} T^4 , \quad (2.10)$$

$$F_{\infty} = \left(\frac{R_{\infty}}{d}\right)^2 \sigma_{\text{SB}} T_{\infty}^4 . \quad (2.11)$$

This highlights the importance of knowing the distance to the source, as uncertainties in distance can often be a limiting factor on obtaining the inferred temperatures, luminosities,

and radii of neutron stars. In a later section of this work, we include neutron stars located in globular clusters for the purpose of mitigating the problem of determining source distances.

2.3 The TOV Equations

To understand how relativistic effects play a role in determining the structural stability of neutron stars, we now consult a special solution of Einstein's field equations. Einstein's field equations describe, in general terms, the interrelation of mass and energy to spacetime curvature. The Schwarzschild solution to Einstein's field equations describes a spherically symmetric, static compact star:

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - \frac{2m(r)}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) . \quad (2.12)$$

Here, ds is the spacetime metric, θ is the polar angle, and ϕ is the azimuthal angle, r is the radial distance. Φ here is the gravitational potential (as from the derived Schwarzschild solution metric) expressed as:

$$e^{2\Phi} = \left(1 - \frac{2M}{r}\right) , \quad (2.13)$$

outside of the star, while inside the star, the gravitational potential depends on enclosed radius and pressure:

$$\frac{d\Phi}{dr} = -\frac{1}{\epsilon} \frac{dP}{dr} \left(1 + \frac{P}{\epsilon}\right) , \quad (2.14)$$

Epsilon here is enclosed energy density. This parameter is important in determining the gravitational lensing effects of the star as described by Eq. (2.7).

This metric motivates the Tolman-Oppenheimer-Volkoff (TOV) equations [34], which relate the star's pressure (P) and enclosed mass (M), along with the enclosed density ρ , as coupled functions of the radial distance from the star's center (r):

$$\frac{dP(r)}{dr} = -\frac{1}{r^2} \frac{[\rho(r) + P(r)][M(r) + 4\pi r^3 P(r)]}{\left[1 - \frac{2M(r)}{r}\right]} , \quad (2.15)$$

$$\frac{dM(r)}{dr} = 4\pi\rho(r)r^2 . \quad (2.16)$$

Two boundary conditions, the first being that the pressure is zero at r_{max} (the surface boundary of the star, this means the star is neither contracting or expanding), and the second being that the central density ρ_{max} at $r = 0$ (the center of the star) is specified and finite, are necessary in order to solve these equations. By specifying a central density, one can also solve for r_{max} and then obtain the total mass of the star for a given density function $\rho(r)$:

$$M = \int_0^{r_{\text{max}}} 4\pi\rho(r)r^2dr . \quad (2.17)$$

Here r_{max} denotes the outermost radius of the neutron star. Solving the TOV equations for a range of possible central densities yields a theoretical mass-radius curve. Each proposed EOS has its own predicted mass-radius curve, but as explained in Ref. [14], a single unique mass-radius curve is expected to describe all neutron stars.

Each point of this mass-radius curve directly corresponds to a pressure-density point on a separate EOS curve. The EOS quantifies the relationship between the internal pressure and density within the star. If provided with a large number of precise simultaneous mass and radius measurements of neutron stars, one could expect to map the mass-radius curve and the dense matter EOS concurrently. Additionally, the EOS predicts a neutron star maximum mass beyond which gravitational collapse is inevitable.

Thus, the discovery of very high mass neutron stars rules out theoretical equations of state whose predicted maximum masses lie below these values. Pressure in neutron stars is principally determined by the number densities of the respective sub-atomic particle species which inhabit the star. Thus, we say that to precisely constrain the EOS is to also constrain the composition of neutron stars [35, 36].

2.4 Causality Limits and Mass Restrictions

The speed at which sound propagates through a material is related to the pressure and energy density of that material. Of course, no matter how high the internal pressure of the material, the speed of sound can never exceed the speed of light. The maximum adiabatic speed of sound can be described by $1 = \frac{dP}{d\epsilon}$. For isolated, rotating stars with well constrained radius measurements, a lower mass limit can be established by noting that a test particle on

the equator of the star cannot exceed the speed of light. This method provides the causality limit for radii as:

$$R > \frac{9}{4} \frac{GM}{c^2} , \quad (2.18)$$

Rearranging, we obtain the upper limit for mass:

$$M < \frac{4}{9} \frac{Rc^2}{G} , \quad (2.19)$$

A calculation of the maximum radius for a $1.4 M_{\odot}$ neutron star is performed in Ref. [35]. For a rigid sphere approximation, the frequency of the neutron star can be expressed as:

$$\nu_k = \frac{1}{2\pi} \sqrt{\frac{M}{R^3}} . \quad (2.20)$$

Any rotational frequency exceeding this would cause the star to become unbound. The highest observed spin rate 641 Hz measured from PSR B1937+21 yields a radius of 15.5 km, assuming a $1.4 M_{\odot}$ star [10].

2.5 Neutron Star Thermodynamics

Several thermodynamic quantities are useful for describing general properties of neutron stars. One of these quantities is pressure, which can be understood in terms of work. Work is pressure integrated over volume, moreover work and energy are the same $Work = W = E = \int_{V_1}^{V_2} P dV$. If the work is being done “on the system” the energy, or work, supplied is negative. In this instance we define the pressure using $n = A/V$ (where A is the number of nucleons and V is the total volume) or $V = A/n$, we say the the pressure $P = -\frac{\partial E}{\partial V} = -\frac{dE}{d(A/n)}$. Taking the derivative of $(-\frac{1}{n}) = \frac{dn}{n^2}$ yield another definition of pressure in terms of energy density ϵ : $P = n^2 \frac{d(\epsilon/n)}{dn} = n \frac{d\epsilon}{dn} - \epsilon = n\mu - \epsilon$. Here $\mu = \frac{d\epsilon}{dn}$ is called the chemical potential. For equilibrium density n_0 there is zero pressure $P(n_0) = 0$ and the chemical potential is $\mu = \frac{\epsilon}{n} = \frac{E}{A}$ [25]. Pressure may also be defined as:

$$P = n^2 \frac{\partial \epsilon}{\partial n} . \quad (2.21)$$

Another approximation for pressure in neutron matter is given in Ref. [37] as:

$$P(n_0) = n^2 \frac{\partial S}{\partial n} . \quad (2.22)$$

Where S is entropy.

The chemical potential is an especially important quantity in neutron star research. For any given species of particle i , the chemical potential is expressed as:

$$\mu_i = \frac{\partial \epsilon_i}{\partial n_i} . \quad (2.23)$$

The quantity ϵ_i here is the energy density contribution for each species of particle i present in the system. In addition to this definition, we can define the chemical potentials in terms of the fermi momenta of the particle species as:

$$\mu(k) = \sqrt{k_i^2 + m_i^2} , \quad (2.24)$$

where m is the mass of the associated particle species (e.g. $i=n,p,e$). For the case where thermal energy is negligible with respect to the total energy of the system we have:

$$\mu_i = \left(\frac{\partial U}{\partial N} \right)_{S,V,N_{i \neq j}} . \quad (2.25)$$

Where U is the system's internal energy and N is the total number of particles. Chemical potentials can be expressed in terms of the above defined energy per nucleon as (Ref. [32]):

$$\mu_e = -\frac{\partial E}{\partial Y_p} = \mu_n - \mu_p , \quad (2.26)$$

Where Y_p here is the proton fraction as defined as quantified in a later section (see Eq. 4.11).

This is very similar to the following definition of chemical potential:

$$\mu_B = \frac{\partial (\epsilon/n)}{\partial Y_B} . \quad (2.27)$$

In general for an arbitrary mixture of baryon and lepton species, a general condition for chemical equilibrium is:

$$\mu_j = B_j \mu_n - q_j \mu_l , \quad (2.28)$$

B is the baryon number of a given particle species j and q_j is the Coulomb charge. The chemical potentials μ_n and μ_l correspond to conservation of baryon number and the conservation of electric charge respectively [25].

The claim that the precise knowledge of the EOS (which relates pressure and energy density) would provide constraints on the particle composition of the star can be supported by the following argument: Beginning with the left hand side of the first TOV equation, we may write this in terms of chemical potential:

$$\frac{dP}{dr} = \frac{dP}{d\mu} \frac{d\mu}{dr} , \quad (2.29)$$

$$\sum_{i=1}^{\text{I set of all species}} N_i d\mu_i = -SdT + VdP , \quad (2.30)$$

and the chemical potential may be expressed as $\mu_i = \left(\frac{\partial U}{\partial N}\right)_{S,V,N}$.

As a special case, consider a single species under isothermal conditions ($dT = 0$). Here $\frac{dP}{d\mu} = \frac{N}{V} = n$, where n is the number density of the particle species. The pressure P may be defined as $P = -\frac{\partial E}{\partial V}$, as seen before an equivalent expression, may be obtained for energy and volume per particle:

$$P = -\frac{\partial \left(\frac{E}{N}\right)}{\partial \left(\frac{V}{N}\right)} = -\frac{\partial}{\partial \frac{1}{n}} \left(\frac{\epsilon}{n}\right) = n\mu - \epsilon . \quad (2.31)$$

Chemical potential here is $\mu = \frac{d\epsilon}{dn}$ and thus the TOV equation for pressure may be expressed in terms of chemical potential as:

$$\frac{d\mu}{dr} = -\frac{\mu (M + 4\pi r^3 P)}{r (r - 2M)} . \quad (2.32)$$

In practice this is more complicated for a neutron star, wherein many species of particles cohabit the same volume and temperatures may change dramatically, but the principle

argument remains valid that the chemical potentials and pressure at a given radius are inextricably coupled [38]. Each species of particle has its own chemical potential and accompanying pressure contribution. Thus observations of neutron star mass and radii can put constraints on what species of particles are possible within the star.

2.6 Superfluid Properties

Superconductivity and the associated phenomena of superfluidity, were discovered experimentally before they were theoretically predicted. Heinke Kamerlingh Onnes, in the year 1911, discovered that below 4.2 K, the electrical resistivity of mercury completely vanishes [39]. He reasoned that the metal had undergone a phase transition, and due to its amazing electrical property of lacking any detectable resistance, he described this phase of matter as a “superconductor”. In 1938, Pyotr Kapista noted the near perfect (that is very high) heat conductivity of ^4He cooled below 2.2 K. He deduced this property resulted from the material having a very low viscosity, and called this phase of matter a “superfluid” [40]. In the same year, John F. Allen and Don Misener discovered that when cooled to 1.15 K, the flow of ^4He through metal tubes was entirely non-laminar, meaning the flow of the fluid was dependent on its inertia alone [41]. These observations were the beginning of the current understanding that superfluids flow without friction, and thus have zero viscosity.

It was not until 1957 that a satisfactory theoretical model was produced to account for these observations. It was then that a microscopic theory of superconductivity was developed by John Bardeen, Leon Cooper, and John Robert Schrieffer. Combining the initials of these theorists last names it is common to refer to this theory as BCS for short. BCS theory accounts for the lack of observed fermion-like behavior at macroscopic levels in superfluids and superconductors (i.e. the observed lack of resistivity or viscosity), by demonstrating that fermions in continuum form pairs (the closed shell model for nucleons, and electron shell model for electrons), and free fermions near the Fermi energy can form composite bosons known as Cooper pairs.

This indistinguishability of Cooper pairs gives rise to the directly correlated phenomena of superconductivity and superfluidity. Cooper pairs in a group are characterized as a superfluid

phase. The defining property of a superfluid is that it has no viscosity (i.e. no resistance to flow), while analogously the defining property of a superconductor is that it has zero electrical resistance. Unlike classical flow, the hydrodynamics of superfluids, called superflow, is characterized by quantized rotating vortices. Superfluids undergoing rotation above a critical angular velocity will generate quantized vortices. The presence of these vortices in nucleon superfluids within neutron stars are hypothesized to account for observed pulsar glitches (sudden increases in the rotational velocity of the star as it slows down) [42].

Once fermions are paired, the pairs no longer exhibit Fermi statistics (they no longer observe the Pauli exclusion principle, the inability to occupy the same quantum state) but instead obey Bose statistics. Thus, below a certain temperature, the valence electrons will form Cooper pairs and, acting as a condensate of composite bosons, occupy a shared ground state energy. It is this sharing of the ground state on the atomic level that accounts for the lack of resistivity at a macroscopic level. As an example, consider the case of electrons in supercooled mercury. The electrons within the material are all occupying the same ground state, any additional current of electrons added into the material will propagate through persistently and perpetually as the electrons in this current are not competing with the electrons in the material for energy states.

The mechanism which generates superconductivity in electrons can be explained as follows: In a lattice, electrons can experience an effective attraction to each-other as a result of phonon coupling. If the electrons are “free”, existing outside a filled Fermi sea of electrons (but possessing energy close to the that of the Fermi energy of the closed Fermi sea), even an arbitrarily weak attraction allows the formation of a bound Cooper pair. BCS theory was the first to explain the experimentally known concept of metallic specific heats.

It is interesting to note that the phonon-electron screening picture, which accounts for the origin of electronic superconductivity in metallic lattices, is not necessarily a good analogy for the origin of nucleonic superfluids of neutrons and proton superconductors. As noted in Ref. [43] however, both superconducting metals and paired nucleons share the common trait of possessing an energy gap between filled Cooper pairs and the filled Fermi sea. The attractive interaction which gives rise to Cooper pairing in the case of nucleons is the far range nuclear strong force, and the repulsive interaction which quenches the interaction for short

distances is the repulsive short range nuclear interaction. It is important to note, however, for both superconducting metals and nucleonic superfluids, it is not the origin of the attractive interactions forming the composite bosons that matters in the formation of a superfluid, but instead the formation of a superfluid is determined entirely by particle statistics [44]. Put another way, it does not matter what the origin of the attractive interaction between the fermions is, as long as they form composite bosons below the critical temperature, they will form a superfluid.

2.7 Nuclear Superfluids

The observed critical temperatures of metallic superconductors formed by Cooper pairs of electrons are quite low (2.17 K for superfluid helium), while calculations of the critical temperatures of nucleon superfluids give results $\sim 10^9$ K. These observations can be reconciled with the understanding that conducting electrons have a very low density, while the extremely high particle number density of the neutron star accounts for its nucleon superfluids having very high critical temperatures. Superfluidity in neutron stars was originally proposed by Arkady Migdal [17] in 1959. His original prediction was that the critical temperature would be of order 1 MeV. Baym and his collaborators [45] would go on to predict a superfluid gap Δ of order 1 MeV in 1969.

Nucleon superfluidity in neutron stars first arises within the inner crust and extends into the inner core of the neutron star, wherein densities exceeding $\rho_d \approx 4 \times 10^{11} \text{g/cm}^3$ compel nucleons to “drip” out of their parent nuclei. This fluid of free neutrons facilitates the existence of the neutron superfluid. At densities in excess of $\rho_{pd} \approx 2 \times 10^{14} \text{g/cm}^3$ [3], protons are observed to drip out of their parent nuclei and form a proton fluid. Mass density increases as one delves deeper into the neutron star, as do the densities of these unbound nucleons. As has been observed with terrestrial unbound electrons, these unbound nucleons occupy the lowest possible energy configuration via the formation of Cooper pairs, wherein two fermions with opposite spins and momenta $(\vec{k}_\uparrow, \vec{k}_\downarrow)$ exhibit weak binding to one another.

Before Cooper pair formation takes place, neutrons and protons are subject to the Pauli exclusion principle, which disallows any two identical fermions to exist in the same quantum

state. This principle is seen to give rise to quantum degeneracy pressure, to which all bound fermions in the star contribute. This degeneracy pressure which partially balances the star's tremendous, centrally-directed gravitational pressure in the outer layers of the star (the inner core is supported by repulsive strong force interactions). In contrast, Cooper pairs exhibit Bose statistics and their constituent fermions attract one another to form a single shared ground state. This allows the composite Cooper pairs, unlike the fermions they are composed of, to share the same quantum state simultaneously.

Once paired, fermions possess lower energy than their free counterparts, which gives rise to an energy gap Δ . This gap is a direct consequence of superconductivity and only occurs once the system has cooled to or below the critical temperature. It is this gap which allows for free fermions to flow in persistent currents with no resistance. A result of the original BCS paper [46] is the following relation for the critical temperature:

$$T_c \sim \omega \cdot \exp \left[-\frac{1}{N(0)V} \right] , \quad (2.33)$$

where $N(0)$ is the density of single spin states per unit energy at the Fermi surface, and ω is the Debye phonon frequency. V is the electron-phonon coupling potential. In the zero temperature limit this relation is expressed as:

$$T_c = 1.14 \cdot \omega \cdot e^{-\frac{1}{N(0)V}} = 1.14 \cdot E_D \cdot e^{-\frac{1}{N(0)V}} , \quad (2.34)$$

where E_D is the Debye cutoff energy. The generation of the Cooper pairs themselves does not depend on the nature of the attractive interaction between the fermions, but the associated critical temperature does. As one nears the critical temperature the gap is expressed by the relation:

$$\Delta(T \rightarrow T_c) = \pi T_c \sqrt{\frac{8}{7\zeta(3)}} \sqrt{1 - \frac{T}{T_c}} \approx 3.07 T_c \sqrt{1 - \left(\frac{T}{T_c}\right)} .$$

Where ζ here represents the Reimann zeta function. In the limit that the system temperature is zero the relation of the gap to the critical temperature is:

$$\Delta(T = 0) = 1.764 \cdot T_c . \quad (2.35)$$

2.8 Nuclear Structure

The Lagrangian we test against nuclear observations is a relativistic mean field theory. The relevant degrees of freedom in any relativistic mean field theory are nucleons, leptons, and mesons. Ideally to model nuclei, we would solve for the equations of motion of a Lagrangian which employs quarks and gluons as the relevant degrees of freedom utilizing the full framework of quantum chromodynamics (QCD). This has never been done in practice due to the inherent non-linearity and complexity of the interactions between quarks and gluons (as well as the interactions amongst gluons), so instead we utilize a mean field approximation to make the problem of modeling nuclear interactions tractable [47].

The intense pressures inside stable neutron stars cause nucleons and leptons alike to travel and collide with relativistic momenta; this condition necessitates the study of Lorentz invariant scalar quantities. Relativistic mean field theories broadly classify approaches which model a system of mesons and nucleons in a static ground state using Lorentz invariant quantities. These quantities are not the nucleon current densities or meson fields themselves, but rather their expectation values in the ground state of the system [25]. For this reason our model is able to reproduce the known properties of nuclear matter at saturation density and always satisfies causality relations, albeit at the cost of ignoring higher order terms in the nucleon-nucleon interaction (see below). The relativistic mean field theory parameters explored in this work are the masses of the omega, rho and sigma mesons, or to be more exact, their ground state expectation values $\langle\sigma\rangle$, $\langle\omega\rangle$ and $\langle\rho\rangle$. We also model the dimensionless couplings b and c , the ρ , σ , and ω meson couplings c_τ , c_s , and c_w , as well as an isoscalar meson interaction term ζ . A higher-order ρ meson self-coupling term ξ is also included. Additional meson interactions are quantified by the parameters $b_1, b_2, b_3, a_1, a_2, a_3, a_4, a_5, a_6$.

The initial guess Lagrangian we used is based on the work found in Ref. [48] (FSUGold EOS), while variations on this Lagrangian and the nomenclature of couplings are based on the work found in Ref. [37]. The Lagrangians in both of these works contain phenomenological parameters which are determined by fitting model predictions to known properties of closed shell nuclei and nuclear matter at and below saturation density. In our notation, the

Lagrangian density from Ref. [48] is expressed as:

$$\begin{aligned}
\mathcal{L}_{\text{int}} = & \bar{\psi} \left[g_{\sigma} \sigma - \left(g_{\omega} \omega_{\mu} + \frac{g_{\rho}}{2} \vec{\tau} \cdot \vec{\rho}_{\mu} + \frac{e}{2} (1 + \tau_3) A_{\mu} \right) \gamma^{\mu} \right] \psi \\
& - \frac{Mb}{3} (g_{\sigma} \sigma)^3 - \frac{c}{4} (g_{\sigma} \sigma)^4 + \frac{\zeta}{4} (g_{\omega}^2 \omega_{\mu} \omega^{\mu})^2 \\
& + \frac{\xi}{24} g_{\rho}^4 \rho^{\mu} \rho_{\mu} + f(\sigma, \omega) \omega^2 (g_{\rho}^2 \rho_{\mu} \rho^{\mu}) .
\end{aligned} \tag{2.36}$$

The couplings are:

$$c_{\sigma} = \frac{g_{\sigma}}{m_{\sigma}}, \quad c_{\omega} = \frac{g_{\omega}}{m_{\omega}}, \quad c_{\rho} = \frac{g_{\rho}}{m_{\rho}} .$$

Couplings are fit to binding energy and saturation density of nuclear matter. Compare these to the couplings from Horowitz and Serot [49]:

$$\begin{aligned}
C_s^2 &= g_s^2 \left(\frac{M^2}{m_s^2} \right) , \\
C_v^2 &= g_v^2 \left(\frac{M^2}{m_v^2} \right) , \\
C_{\rho}^2 &= g_{\rho}^2 \left(\frac{M^2}{m_{\rho}^2} \right) .
\end{aligned}$$

Following the original notation of Ref. [48], the last term denotes the isoscalar-isovector coupling. In the notation we follow, which can be credited to the work of Ref. [50], this term is replaced by a polynomial $f(\sigma, \omega)$:

$$f(\sigma, \omega) = b_1 \omega^2 + b_2 \omega^4 + b_3 \omega^6 + a_1 \sigma + a_2 \sigma^2 + a_3 \sigma^3 + a_4 \sigma^4 + a_5 \sigma^5 + a_6 \sigma^6 .$$

The ρ meson is not active in isospin symmetric matter. The ρ meson couples to the isospin current and coincides with mixing between the ω and ϕ mesons.

Yukawa interactions play a very important role in the previous Lagrangian. Nucleons interact in our model through the Yukawa interaction, wherein mesons act as intermediaries of the strong force between nucleon pairs. The interaction between two fermions (in this case, the two nucleons) is described by Yukawa interactions as follows: The exchanged Yukawa particle has mass μ and coupling constant g , and the potential between the two fermions is $V(r) = -\frac{g^2}{4\pi} \frac{e^{-\mu r}}{r}$. This type of interaction closely resembles the Coulomb interaction. The

difference in the sign of the potential makes the Yukawa attractive for all particles (here nucleons), while the Coulomb interaction is repulsive. The Yukawa interactions appear in the Lagrangian as a coupling between a scalar field ϕ and a Dirac field (the nucleon field) ψ , these nonlinear meson interactions are expressed as:

$$V \approx g\bar{\psi}\phi\psi \text{ (scalar) ,} \quad (2.37)$$

$$V \approx g\bar{\psi}i\gamma^5\phi\psi \text{ (vector) ,} \quad (2.38)$$

where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. For long distances, one-pion exchange is attractive between nucleons, while for short ranges (less than one femtometer) the strong force between the nucleons is repulsive. Phenomenologically, we can describe attractive interactions as being mediated by the scalar meson (the sigma pion in our model), while close range repulsive interactions are mediated by the ω and ρ vector mesons.

Separating this Lagrangian by parts, the code we use solves for each particles individual equation of motion. For the isoscalar-scalar field σ , the equation of motion is:

$$0 = m_\sigma^2\sigma - g_\sigma\rho_{n,s} - g_\sigma\rho_{p,s} + bMg_\sigma^3\sigma^2 + cg_\sigma^4\sigma^3 - g_\rho^2\rho^2\frac{\partial f}{\partial\sigma} .$$

Where the scalar densities for the nucleons are $\rho_{n,s}$ and $\rho_{s,s}$ respectively. The parameters b and c quantify higher order correction to the strength of the scalar meson field σ , in other literature these quantities have been defined as $\kappa = 2Mb$ and $\lambda = 6c$. For the isoscalar meson field ω^μ equation of motion:

$$0 = m_\omega^2\omega - g_\omega\rho_n - g_\omega\rho_p + \frac{\zeta}{6}g_\omega^4\omega^3 + g_\rho^2\rho^2\frac{\partial f}{\partial\omega} .$$

Where the number density for neutrons and protons is ρ_n and ρ_p respectively. and finally for the isovector meson field ρ^μ :

$$0 = m_\rho^2\rho + \frac{1}{2}g_\rho\rho_n - \frac{1}{2}g_\rho\rho_p + 2g_\rho^2\rho f + \frac{\xi}{6}g_\rho^4\rho^3 .$$

The field equation for nucleons is (as expressed in Ref. [51]):

$$\left[\gamma_\mu \left(k^\mu - g_\omega \omega^\mu - \frac{1}{2} g_\rho \tau_3 \rho_3^\mu \right) - (M - g_\sigma \sigma) \right] \psi(k) = 0 . \quad (2.39)$$

These equations are solved for their energy eigenvalues in such a way that beta equilibrium and charge neutrality are satisfied at all times.

The parameter m_s in the Lagrangian is the effective nucleon mass, defined using the standard nucleon mass of $M_{textB} = 939$ MeV as:

$$M_{sB}^* = M_B - g_{\sigma B} \sigma . \quad (2.40)$$

As an example, a M_{sB}^* of 2.6 yields an effective nucleon mass of 512.98 MeV, just over half (0.54) of the bare nucleon mass without the strong presence of the sigma meson field. The vector-meson coupling ζ , is especially important as it is directly constrained by the neutron star maximum mass limit [52, 53, 54]. The current constraints on ζ are $0 < \zeta \leq 0.3$, these constraints arise from an upper limit of a $2 M_\odot$ neutron star. The parameters $b_1, b_2, b_3, a_1, a_2, a_3, a_4, a_5, a_6$ are a summation of terms used to describe the isoscalar-isovector coupling. Their increasing indices denote increasingly higher orders of correction.

Starting above twice nuclear saturation density, we have introduced a high density correction to our EOS. This correction is implemented by enforcing boundary conditions on the polytropic equation of state parameters ΔP_K and ΔP_Γ . We require that the exponent, ΔP_Γ , remains positive but very small. The linear coefficient, ΔP_K , is very large in comparison. The two polytropic EOS parameters, jointly define a polytropic pressure-energy density relation. This ensures that we are probing energy densities twice those of nuclear saturation (saturation density defined here as $n_0 = 0.16 \text{ fm}^{-3}$):

$$\epsilon(n_0) = (M_{\text{nuc}} - E/A - M) \cdot n_0 = (939 \text{ MeV} - 16 \text{ MeV}) \cdot (0.16 \text{ fm}^{-3}) = 147.68 \text{ MeV} \cdot \text{fm}^{-3} , \quad (2.41)$$

$$147.68 \text{ MeV} \cdot \text{fm}^{-3} \approx 0.75 \text{ fm}^{-4} . \quad (2.42)$$

Multiply by a factor of two and we obtain 1.5 fm^{-4} as twice the nuclear saturation energy density:

$$\begin{aligned} P(\epsilon) &= P_{RMF}(\epsilon) + K \left[\epsilon^\Gamma - (1.5 \text{ fm}^{-4})^\Gamma \right] \Theta(\epsilon - 2 \cdot \epsilon_0) \\ &= P_{RMF}(\epsilon) + K \left[\epsilon^\Gamma - (2\epsilon_0)^\Gamma \right] \Theta(\epsilon - 2 \cdot \epsilon_0) . \end{aligned}$$

The interaction between two nucleons, the nucleon-nucleon interaction, is a problem remarked by Hans Bethe in 1957 [55] to have had more man hours devoted to its solution than any previous scientific question in history. The complexity of the nucleon-nucleon interaction emerges from the fact that the interaction potential between the nucleons not only depends on the distance between them, but also the spin and isospin of each nucleon. The only bound two-nucleon system is the deuteron, with a relatively small binding energy of 2.22 MeV. Neutron and proton pairs do not spontaneously form bound pairs at the relatively low densities observed in laboratory experiments, but instead scatter off one another.

At densities near and below nuclear saturation density, the two-body interactions dominate the behavior of nuclear matter. This is not to say that the nucleon many body theory reduces to nucleon pair interactions, merely that pairing interactions are very important to understanding the behavior of nuclei at these densities. At higher densities, three-body interactions along with other higher-order interactions, become increasingly important [56].

The total number of nucleons, the summation of the total number of neutrons and the total number of protons is given as $A = Z + N$. Analogous to the atomic shell model that describes the orbitals of electrons, and thus all the chemical diversity we appreciate in the periodic table, the behavior of nucleons is governed by the nuclear shell model. The nuclear shell model accounts for the greater stability, directly related to the binding energy of the nucleus, enjoyed by some “magic” nuclei $Z, N = 2, 8, 20, 28, 50, 82, 126$. If a nucleus has a magic number of neutrons and a magic number of protons, it is referred to as “double magic”. ^{208}Pb , with $Z = 82$ and $N = 126$ is a heavy nucleus designated as “doubly magic”, it does not decay into daughter nuclei but instead has no measurable half-life and survives indefinitely

as a stable nucleus. There are many stable non-magic nuclei, but it is the particular stability properties of magic nuclei which make them useful for our research.

Exotic nuclei with large numbers of nucleons contain a neutron-rich surface layer, known as the neutron skin. The thickness of the neutron skin, as discussed in a later section of this work, has important implications for the total neutron star radii and the allowance of a powerful cooling process (the direct Urca process). Extrapolation of this trend, that more massive nuclei contain a higher percentage of neutrons, can be extended to a massive nucleus with $\sim 10^{57}$ nuclei. Neutron stars are often compared to giant nuclei.

In all observed nuclei, the density is highest at the center and decreases with radial distance. Extrapolating from a nuclei with finite surfaces to the theoretically considered case of infinite nuclear matter with uniform densities, we can gain an appreciation for the type of nuclear interactions that occur in neutron stars. Pairing of nucleons in infinite matter was explored in Ref. [57], wherein it was shown that nucleon interactions arising from singlet 1S_0 and 3P_2 triplet channels account for the presence of nucleon superfluids in neutron star matter.

To understand what type of physics these channels describe, we can begin with quantum mechanical spectroscopic states formula:

$$^{2S_{\text{spin}}+1}L_J . \quad (2.43)$$

The total angular momentum is the sum of the total spin and total orbital angular momentum:

$$J = S_{\text{spin}} + L . \quad (2.44)$$

The spin anti-aligned ground state ($S_{\text{spin}} = 0$), where the nucleons possess opposite angular momenta $L = 0$ is 1S_0 . Examples of other two nucleon states are $^3P_0, ^3P_1, ^3P_2, ^3F_2$, etc. In the case of the 1S_0 and 3P_2 , calculation of the phase shifts indicate that there is an attractive potential which allows these two channels to form Cooper pairs. The formation of Cooper pairs is understood in terms of the BCS theory in the case of matching nucleons (proton-proton and neutron-neutron), while the proton-neutron Cooper pairs are less understood and are largely omitted from neutron star cooling calculations.

2.9 Neutron Star Cooling

In order to model a neutron star's cooling profile, we must first provide an initial temperature and age estimate. The neutron star's supernova birth imparts it with an initial core temperature of $\sim 10^{11}$ K, and, often, a kick velocity with respect to its origin within the ejecta cloud. Initially, neutron stars are so hot that the neutrino mean free path is too small for neutrinos to exit the star. Neutrinos are trapped inside of dense matter at this early time in the stars evolution and must slowly diffuse through the neutron star matter. Once the neutrino mean free path is large enough, the neutrinos begin freely streaming out of the star without diffusion. This triggers immediate cooling in the core, while the crust remains hot [58]. This vast temperature gradient between the crust and the core triggers a cooling wave, which diffuses radially outward from the core, in a cooling epoch known as thermal relaxation. Once the cooling wave reaches the surface the core and crust will arrive at a temperature equilibrium (apart from a temperature gradient in the envelope as explained in Ref. [59]) and the star is isothermal from this time onward.

Except for a thin shell at the surface, the neutron star becomes isothermal after a few hundred years. In isolated neutron stars without a companion (i.e. stars where accretion is not taking place), the temperature decreases (unless heated by magnetic field dissipation or some dark matter-related process) at a rate determined by the nature of dense matter [1, 60, 61]. In the first 10^5 years, cooling is dominated by the emission of neutrinos from the core, after which photon emission from the surface becomes the most significant source of cooling. The neutrino emission rates strongly depend on the nature of neutron superfluidity and proton superconductivity. Thus, if one obtains temperature and age estimates from a number of cooling isolated neutron stars, the comparison of theoretical models to data results in a constraint on the nature of superfluidity in dense matter.

There are several isolated neutron stars with available age estimates and sufficient x-ray data to produce surface temperature estimates. The two primary methods of obtaining age estimates are the use of kinetic age estimates and spin down age estimates. Surface temperature estimates are generated from redshift astronomical measurements alongside atmospheric modeling of the neutron stars surface. If a neutron star can be associated with

a nearby supernova remnant and its proper motion can be measured, one can determine the kinetic age, t_{kin} . The kinetic trajectory age estimates make use of the fact that some neutron stars are born with a measurable velocity with respect to their supernova remnants. By quantifying the star's relative motion in space as a function of time, we can calculate the star's translational velocity, known as the “apparent kick velocity” since the star received a translational kick from its progenitor supernova. Using the kick velocity and assuming an origin point for the neutron star's birth, often found through observing geometric properties of the supernova remnant, we can derive the kinetic trajectory age t_{kin} since the star's birth.

Alternatively, pulsar ages can be estimated from the spin-down timescale, $t_{\text{sd}} = P/(2\dot{P})$ as seen Eq. (2.47), an age estimate which assumes the neutron star possesses a dipolar magnetic field. The braking of the dipole magnetic field causes the star's period to increase gradually over time. Integrating from an initial to a final period can provide a characteristic spin-down time. We start by re-expressing $PdP = P\dot{P}$ in this integral:

$$\int_{P_i}^{P_f} PdP = \int_0^\tau P\dot{P}dt = P\dot{P} \int_0^\tau dt . \quad (2.45)$$

Because $P\dot{P}$ is constant:

$$\frac{P_f^2 - P_i^2}{2} \equiv P\dot{P}\tau , \quad (2.46)$$

$$\tau = \frac{P}{2\dot{P}} . \quad (2.47)$$

Here, P is the period of the neutron star, $\dot{P}$ is the time derivative of the period, and τ here is equivalent to t_{sd} which is the characteristic spin down age itself. The pulsar is gradually losing angular velocity and slowing down over time, which corresponds to a measurable increase in the period P . By deducing a maximum starting period for the star, we may determine a characteristic age estimate. Spin-down ages can be measured precisely, but they often disagree with kinetic ages by a factor of 3 or more (see Ref. [62]); thus we assume a factor of 3 uncertainty in $\tau = t_{\text{sd}}$. We presume kinetic ages are more accurate than spin-down ages, but this is not certain.

The neutrino driven cooling of high-density matter explains the counterintuitive reality that more massive stars actually cool faster than their smaller counterparts, despite having

greater thermal energy at time of birth. Thus the true importance of the mass is not realized in the bulk thermodynamics of the system but instead in the nuclear processes achievable within the star's range of densities. Neutron star cooling can be approximated (in the case of isothermal stars) by the following heat transport equation [57]:

$$\frac{dE}{dt} = C_v \frac{dT}{dt} = -L_\nu - L_\gamma + H . \quad (2.48)$$

E here represents the total thermal energy, while L_ν and L_γ represent the luminosities of neutrinos and photons respectively. L_ν provides a greater cooling contribution for earlier times, while L_γ dominates cooling for later times. Their negative sign signify that the greater the luminosity, the faster the star will cool. H represents the internal heating energy of star. Although not included in this research, as we assumed no internal heating, possible sources include friction from differential rotation or heating from the decay of a strong magnetic field (ohmic dissipation).

The full neutron star cooling equations, with no approximations, are quantified as two coupled differential equations. The first of these is the energy balance equation:

$$\frac{d(l e^{2\Phi})}{dr} = -\frac{4\pi r^2 e^\Phi}{\sqrt{1-2m/r}} \left[c_v \frac{dT}{dt} + e^\Phi (q_\nu - q_h) \right] , \quad (2.49)$$

which couples to the energy transport equation:

$$\frac{d(T e^\Phi)}{dr} = -\frac{1}{\lambda} \frac{l e^\Phi}{4\pi r^2 \sqrt{1-2m/r^2}} . \quad (2.50)$$

c_v here is the specific heat capacity per volume (at constant volume), and λ is the thermal conductivity. Φ here is the same gravitational potential defined in a previous section (see Eq. (2.13)). l denotes the internal luminosity, this quantity satisfies the boundary condition that $l(r=0) = 0$. The quantities q_ν and q_h , are the neutrino emissivity and neutrino heating rate per unit volume, respectively. This set of equations is capable of describing a neutron star that has not yet come into thermal equilibrium. The approximation of Eq. (2.48) is obtained from these coupled diffusion equations from the approximation that the star is

isothermal (i.e. no radial dependence on temperature), in this limit $\frac{d(Te^\phi)}{dr} = 0$ and we obtain Eq. (2.48) from solving the photon luminosity equation Eq. (2.49).

Minimal cooling excludes the presence of the powerful direct Urca process and also exotic degrees of freedom, such as hyperons, deconfined quarks, Bose condensates, etc. The modified Urca process however, is still permitted and is one of the primary cooling mechanisms in this paradigm. Minimal cooling does not account for any late heating which might occur in stars such as ohmic heating from the dissipation of magnetic fields. In the minimal cooling model, the pair breaking and formation process (PBF) that we study is the most powerful process as ranked by neutrino emissivity when the temperature is near critical temperature T_c , but modified Urca is still the most powerful process over the entire lifetime of the star. The modified Urca can be expressed by the relations:

$$n + n' \rightarrow p + n' + e^- + \bar{\nu}_e , \quad (2.51)$$

$$p + n' + e^- \rightarrow n + n' + \nu_e , \quad (2.52)$$

or the proton branch of modified Urca:

$$n + p' \rightarrow p + p' + e^- + \bar{\nu}_e , \quad (2.53)$$

$$p + p' + e^- \rightarrow n + p' + \nu_e . \quad (2.54)$$

Modified Urca processes utilize spectator nucleons, n' or p' , to provide additional necessary momentum, these interactions are possible for all densities.

The minimal cooling model also includes the nucleon Bremsstrahlung cooling process:

$$n + n \rightarrow n + n + \nu + \bar{\nu} ,$$

$$p + p \rightarrow p + p + \nu + \bar{\nu} ,$$

$$n + p \rightarrow n + p + \nu + \bar{\nu} ,$$

which is of secondary importance in the minimal cooling model due to having only one one-hundredth the neutrino emissivity of the modified Urca process: The last two branches are

suppressed due to the scarcity of available protons [63]. In the neutron star crust, additional neutrino cooling processes exist such as the plasmon decay process:

$$\gamma \rightarrow \nu + \bar{\nu} , \quad (2.55)$$

the pair annihilation process:

$$e^- + e^+ \rightarrow \nu + \bar{\nu} , \quad (2.56)$$

or alternatively:

$$e^- + e^+ \rightarrow \gamma + \gamma , \quad (2.57)$$

and the photo-neutrino process:

$$\gamma + e^- \rightarrow e^- + \nu + \bar{\nu} \quad (2.58)$$

are also available.

Additionally, when Cooper pairs form, they emit outgoing neutrino-antineutrino pairs. The sum of these outgoing neutrinos is a substantial source of neutron star cooling. The PBF describes the formation of Cooper pairs and the subsequent breaking of the pairs via thermal excitation. The pairing first occurs below the critical temperature T_c ; it is at this temperature where superfluidity becomes energetically favorable for the star. It is important to note that each density has its own accompanying critical temperature. However, given that the nucleon superfluids are restricted to the inner crust and outer core regions we concentrate on the critical temperatures of nucleon superfluids at peak densities. After many cycles of breaking and formation, there is not enough heat remaining in the star, below $T_{c,\min}$, to break the pairs. Thus, below $T_{c,\min}$, no further neutrinos will be emitted by the PBF process. It should be noted that $T_{c,\min}$ itself is not a known value, but only serves here as an acknowledgement of the fact that the PBF process takes place within a limited range of fermi momenta, and that same range of fermi momenta limits the possible range of critical temperatures. The

PBF process is as follows:

$$n + n \rightarrow [nn] + \nu + \bar{\nu} , \quad (2.59)$$

$$p + p \rightarrow [pp] + \nu + \bar{\nu} , \quad (2.60)$$

$$[nn] + Q_{\text{heat}} \rightarrow n + n , \quad (2.61)$$

$$[pp] + Q_{\text{heat}} \rightarrow p + p . \quad (2.62)$$

2.10 Enhanced Cooling

The presence or absence of enhanced cooling mechanisms in addition to processes discussed above produces a clear bifurcation in neutron star cooling curves. Both theoretical predictions, minimal cooling and enhanced cooling, appear to match assorted neutron star ages and effective temperatures. This agreement of both cooling models arises from the fact that while many neutron stars are well explained by minimal cooling scenarios, some stars are too cold for their young ages to be explained by models without enhanced cooling.

It would be best to directly obtain enough spectral data from each neutron star to individually model each neutron star's atmospheric composition. In practice however, there is not enough spectral data to do this. An expedient fix for this problem is to model two different families of atmospheres, light element atmospheres and heavy element atmospheres, depending on the observed temperature and age of the neutron star. In Ref. [64], it is demonstrated that both heterogeneous atmospheres of light element atmosphere models and iron envelope models can be used to accurately match neutron star cooling data. Carbon atmospheres, a subset of light element atmospheres, are found to provide especially good explanations for certain neutron star's observed cooling (such as the neutron star in Cassiopeia A) [65].

A clear bifurcation between neutron stars with light envelope compositions and those with heavy element atmospheres exists. In Ref. [66], a technique which treats the light element composition of the neutron star atmosphere as a variable is demonstrated to allow more precise modeling of neutron star atmospheres and envelopes.

Many stars are too hot for their older ages to be effectively explained by either minimal cooling or enhanced cooling models, without some sort of delayed heating mechanism not directly associated with the progenitor supernova. A likely explanation for this observed late heating is the presence of strong magnetic fields cause crustal heating through decay currents or ohmic dissipation over a time scale of $\sim 10^6$ years [67]. The possibility of hot spots generated by magnetic fields leads to ambiguity in interpreting some neutron stars' effective luminosities and temperatures. This is because a star with a large radius and light element atmosphere with hotspots produces very similar observational data to a neutron star with a smaller radius with a blackbody atmosphere.

Chapter 3

Constraining Superfluidity in Dense Matter from the Cooling of Isolated Neutron Stars

3.1 Introduction

Interpreting observations of neutron stars requires an understanding of the transport properties of dense matter. The transition of energy and momentum in dense matter, whether it be related to the properties of shear or bulk viscosity, thermal conductivity, electrical conductivity or neutrino transport, are all strongly affected by the presence of superconductivity and superfluidity [68]. The intensely sought after objective of synthesizing room temperature terrestrial superconductors makes the study of nucleonic superconductivity, which occurs at temperatures many times greater than the internal temperature of our sun, very appealing.

The study of neutron stars is also important as it provides the best opportunity to study dense, strongly interacting matter in the high pressure-low temperature ($\approx 10^9$ K) sector of the quantum chromodynamics (QCD) phase diagram, where the pressure arising from baryon chemical potential is much greater than the thermal pressure. Many particle colliders here on Earth, such as the Relativistic Heavy Ion Collider (RHIC) located in Brookhaven

National Laboratory, study dense matter through ion collisions at relativistic speeds. These collisions occur at high temperatures ($\approx 4 \times 10^{12}$ K), useful for studying the conditions present in the early universe. Unfortunately, these same high temperatures confound the extraction of dense matter transport properties from these experiments. Additionally the matter found in neutron stars is stable, while the quark-gluon plasmas being studied in particle accelerator laboratories are hot, non-equilibrated matter [69]. Hence neutron stars, which contain densities several times larger than the densities at the center of atomic nuclei (See Ref. [14]) and temperatures low enough to extract dense matter transport properties, are often referred to as “unique laboratories”.

Neutron stars have already been employed as unique laboratories for the study of dense and strongly-interacting matter. In previous works, Refs. [70, 71, 10], neutron star mass and radius observations have been used to determine the equations of state (EOS) of dense matter, for densities exceeding nuclear saturation, to within a factor of two. The EOS provides necessary information regarding the relation of density and pressure within the neutron star. Additionally, nuclear theory has recently provided constraints on the energy per baryon for neutron matter at nuclear saturation density to within a confidence interval of several MeV [72]. These EOS constraints alone are nevertheless insufficient to characterize the nature of dense matter, as we still require an understanding of how energy and momentum diffuse through dense matter.

The goal of our research is to put further constraints on the equation of state and transport properties of dense matter. For this purpose, my collaborators and I have studied seventeen neutron stars that have no observed binary partners. Their isolation is significant in one very important regard, namely that accretion is assumed not to occur in these stars. Accreting material forms an accretion disk wherein material siphoned from a binary partner is subsequently deposited on the surface of the accreting neutron star at near relativistic velocities. This violent process adds a significant quantity of energy to the star. The effect accretion has on the observed luminosity and temperature of the accreting neutron star is not fully understood, but is currently being investigated [73]. We initially chose to study neutron stars which cool in isolation in order to avoid the additional variables and confusion that the accretion process poses to our understanding of how these stars gradually cool over

time. The goal of this research was to compare temperature and age observations of isolated neutron stars to theoretical cooling curves based on the minimal cooling paradigm in order to quantitatively constrain the critical temperatures of nucleon superfluids present in neutron stars. My research discussed in the following section was conducted in collaboration with my advisor Andrew Steiner, post-doctoral associate Sophia Han, and Dany Page, who provided the NSCool code and additional input on the project.

3.2 Method

NSCool is a one dimensional relativistic neutron star cooling code that reflects a paradigm known as minimal cooling, which was developed by Dany Page (see Page et al Ref. [1]). Minimal cooling describes four different aspects of neutron star cooling, which are reflected by the four types of input parameters within the code. These four aspects are the mass of the neutron star, its envelope composition, the EOS for dense matter and the superfluid properties of dense matter. In this section, we have fixed the mass input to be $1.4 M_{\odot}$. The EOS input we used is an Akmal-Pandharipande-Ravenhall EOS (APR; [74]) wherein temperature $T = 0$ (corresponding a negligible overall pressure contribution from thermal pressure). This is not to say that temperature is set to zero in this EOS, as finite temperature corrections are performed based on nucleon effective masses. The remaining two inputs, the superfluid parameters and the envelope composition, are treated as variables in our research.

NSCool uses inputs such as the initial temperature and mass of the star to iteratively solve coupled heat and energy transport equations to model neutrino and photon cooling until the star “freezes” (i.e. becomes too cool to be observable). The minimal cooling model assumes that neutron stars are spherically symmetric objects composed entirely of neutrons, protons and leptons. One of the main goals of my research has been to investigate which neutron stars, from a selected group of isolated stars, could be accurately explained by the minimal cooling model using a modified version of NSCool. Stars that cannot be sufficiently described by minimal cooling would require some kind of enhanced cooling or heating mechanism.

Temperature estimates for neutron stars are obtained by matching an atmospheric model with observations of the star’s electromagnetic emission spectra (x-ray and visible) and luminosity. Modeling is necessary because a much larger number of observations would be required in order to directly ascertain each neutron star’s exact atmospheric composition. Our approach in varying the envelope composition parameter is two-fold. First, each of the neutron stars in our data set is assigned an atmospheric model. Ideally, we might determine what their atmospheric composition is directly from x-ray observations of the neutron stars, but, to date there is not enough observational information to complete this task. In lieu of exact composition information, we subdivide our neutron stars into three different models based on their x-ray spectra. Our three different atmosphere models are hydrogen atmosphere models, carbon atmosphere models and black body models. The blackbody atmospheric composition is used because of the fact that the emission spectra of iron (and other heavy elements) very closely resembles the full black body emission spectrum, as it has enough spectral lines to be approximated by a continuous spectrum.

Our modified code uses a series of parameters we refer to as $\eta_{B0833}, \dots$ which describe the light element atmospheric composition of the hydrogen and carbon modeled stars. The black body stars did not require nonzero η parameters because they are not considered to possess any light elements in their atmospheres. With these parameters introduced, we can expand our schema to plot four theoretical cooling curves for the cases of $\eta = 0$, as is the case for black body models, $\eta = 10^{-17}$, for carbon model atmospheres and $\eta = 10^{-12}$ and $\eta = 10^{-7}$, for predominantly light element atmospheres. Each of the η parameters, just like the six cooling parameters (discussed below), are supplied an initial guess value based on the atmospheric composition model of the star which they represent. Then, using a Markov Chain Monte Carlo and Bayesian fitting technique identical to that described later in this section, we determine the maximum likelihood values for each of the light atmosphere composition parameters η_n .

Astronomical data is used to obtain an estimate for the surface temperature and luminosity of the neutron star. We may estimate the radius of the star by combining angle-resolved measurements of the x-ray source along with a derived distance to the source. Once obtained, these apparent radii are fitted with an atmospheric model to produce a best radius

estimate. Hydrogen fitted stars are typically much larger in radii than are their black body fitted counterparts. These assumptions, along with the observation of the neutron star luminosity, allow for the estimation of red-shifted temperature measurements for the neutron star Eq. (2.9). From Eq. (2.7) we obtain R_∞ as the observed radiation radius, which takes into account the gravitational lensing of the star with respect to light emitted from its surface.

Following Ref. [1], we assume that stars younger than 10^5 years can be modeled as having hydrogen atmospheres and stars older than 10^5 years are best described as having a black body spectra model. This assumption emerges from the calculation that young stars when fitted with black body models produce radii too small to be in the acceptable range for neutron stars. For older stars, black body model fits lie within a credible range. Physically, this assumption corresponds to nuclear fusion in the star's crust generating increasingly heavier elements, such as iron, from lighter elements such as hydrogen and helium on the timescale of 10^5 years. Three exceptions to this assumption were made, the first being for PSR J0243+2740, where a magnetized hydrogen model was used due to the lack of availability of a well-constrained black body model (Ref. [75]). The second and third exceptions to this are Cas A and HESS J1731, which are found to be best fitted by a carbon model atmosphere.

To accurately determine the surface temperature, however, requires knowledge of the atmospheric composition of the star. Older neutron stars have undergone many cycles of pycnonuclear reactions yielding atmospheres possibly made of iron-peak elements and these atmospheres are well fit by black body models giving black body radii in the range of 10–13 km expected from theoretical models [76]. The inferred radii from black body fits to younger stars are often much smaller than expected, leading to the idea that younger isolated neutron stars may have light-element atmospheres, and hydrogen (H) atmosphere fits to the data often result in neutron star radii closer to what is expected. For most objects, only black body and H atmosphere fits to the x-ray data are available.

The redshift temperature of the star depends on the composition of the envelope, which is the region between the photosphere and a boundary density near $\rho_b = 10^{10}$ g/cm³. This boundary density is defined so that the luminosity at this boundary is equal to the total

luminosity of the star. If light-elements are present in the atmosphere, they are expected to rise to the top. Their emission spectra will alter the inferred surface temperature. Light-element envelopes are not expected within iron-peak atmospheres described by black body models, as light elements in the envelope will inevitably make their way to the surface. Similar to the procedure used in Ref. [1], we use the temperatures and luminosities implied by H atmosphere fits to the x-ray spectra for younger stars (less than about 10^5 years) in which black body radii are too small to be realistic. In older stars, we use temperatures and luminosities obtained by black body fits to the x-ray spectra. The observational data set is summarized in Table 3.1. In the case of PSR J2043, we use the results from an H atmosphere fit because no black body fit was available.

The true x-ray spectrum of an isolated neutron star is not that of a black body. Modeling heat transport in hydrostatic equilibrium, Ref. [77] found one can obtain a simple relationship between the effective surface temperature which depends on the amount of light elements in the envelope and the temperature at the base of the envelope, $T_b = T(\rho_b)$. This relationship simplifies the calculation considerably, allowing one to connect envelope models on top of a neutron star interior [1]. Younger stars may have light elements which affects the surface temperature, but older stars which have heavy element photospheres are not expected to have light element envelopes (as otherwise the light elements in the envelope would move towards the surface). The work in Ref. [77] was updated in Ref. [78] and our neutron star cooling model uses this work to determine the effective surface temperature and luminosity as a function of the temperature at the base of the envelope. We vary the amount of light elements in the envelope in all neutron stars less than 10^5 years old, which is consistent with the notion that neutron star atmospheres evolve from light elements to iron-peak elements over time through nuclear fusion.

The composition of the envelope is parameterized by a quantity η which takes values from 0 to 10^{-7} , larger values representing a larger contribution from light elements [78]. The cooling code computes three different cooling curves, for $\eta = 0$, $\eta = 10^{-12}$, and $\eta = 10^{-7}$ and results for other values of η are obtained through logarithmic interpolation.

Table 3.1: The data set used in the our initial work is adapted from the earlier work in Refs. [1, 2] and [3].

Star	$\log_{10}(t_{\text{sd}}/\text{yr})$ or $\log_{10}(t_{\text{kin}}/\text{yr})$	B (G)	$\log_{10}(T^\infty/\text{K})$	atmos. model	$\log_{10}(L^\infty)$ erg/s	mass ($M_\odot$)	radius (km)	Ref.
Cas A NS	2.52 (observed) [79]	8×10^{10}	$6.26^{+0.02}_{-0.02}$	C	$33.63^{+0.08}_{-0.08}$	1.4*	12-15	[80]
			$6.447^{+0.012}_{-0.012}$	H		1.4*	4	[80]
			$6.653^{+0.007}_{-0.007}$	BB		1.4*	< 1	[80]
PSR J1119–6127	3.20 (s) [81]	4.1×10^{13}	$6.09^{+0.08}_{-0.02}$	HA	$33.32^{+0.14}_{-0.14}$	1.4*	10*	[82]
			$6.39^{+0.02}_{-0.02}$	BB	$33.39^{+0.11}_{-0.11}$	1.4*	2.7±0.7	[82]
RX J0822–4247	$3.57^{+0.04}_{-0.04}$ (k)	$\sim 10^{12}$	$6.24^{+0.04}_{-0.04}$	HA	$33.93^{+0.08}_{-0.08}$	1.4*	10*	[83]
			$6.65^{+0.04}_{-0.04}$	BB	$33.75^{+0.15}_{-0.15}$	1.4*	≈2	[83]
1E 1207.4–5209 ⁺⁺	$3.85^{+0.48}_{-0.48}$ (k) [84, 85]	3×10^{12}	$6.21^{+0.07}_{-0.07}$	HA	$33.50^{+0.24}_{-0.24}$	1.4*	10*	[86]
			$6.48^{+0.01}_{-0.01}$	BB	$33.29^{+0.59}_{-0.59}$	1.4*	< 1.5	[87, 88]
PSR J1357–6429	3.86 (s)	8×10^{12}	$5.88^{+0.04}_{-0.04}$	HA	$32.63^{+0.59}_{-0.17}$	1.5 – 1.6	10*	[89]
			$6.23^{+0.05}_{-0.05}$	BB	$33.56^{+0.20}_{-0.20}$	1.5 – 1.6	2.5±0.5	[89]
RX J0002+6246 [†]	$3.96^{+0.08}_{-0.08}$ (k)		$6.03^{+0.03}_{-0.03}$	HA	$33.21^{+0.13}_{-0.13}$			[1, 90]
			$6.15^{+0.11}_{-0.11}$	BB	$32.5^{+0.32}_{-0.32}$			[1, 90]
PSR B0833–45	$3.97^{+0.23}_{-0.23}$ (k) [91]	3×10^{12}	$5.83^{+0.02}_{-0.02}$	HA	$32.58^{+0.04}_{-0.04}$	1.4*	13	[92]
			$6.18^{+0.02}_{-0.02}$	BB	$32.16^{+0.12}_{-0.12}$	1.4*	2.1±0.2	[92]
PSR B1706–44	4.24 (s) [93]	3×10^{12}	$5.80^{+0.13}_{-0.13}$	HA	$32.37^{+0.56}_{-0.56}$	1.45 – 1.59	13	[94]
			$6.22^{+0.04}_{-0.04}$	BB	$32.78^{+0.30}_{-0.30}$	1.4*	< 6	[93]
XMMU J1732–344	$4.43^{+0.17}_{-0.43}$ (k) [95]	$\sim 10^{10-11}$	$6.25^{+0.01}_{-0.0045}$	C	$33.99^{+0.04}_{-0.02}$			[96]
PSR J0538+2817	$4.47^{+0.05}_{-0.06}$ (s) [97]	$\sim 10^{12}$	$6.05^{+0.10}_{-0.10}$	HA	$33.10^{+0.50}_{-0.50}$	1.4*	10.5	[98]
			$6.327^{+0.007}_{-0.007}$	BB		1.4*	< 2	[99]
PSR B2334+61	4.61 (s)	$\sim 10^{10-12}$	$5.68^{+0.17}_{-0.17}$	HA	$32.70^{+0.68}_{-0.68}$	1.4*	10-13	[100]
			$6.02^{+0.19}_{-0.19}$	BB		1.4*	< 2	[100]
PSR B0656+14	5.04 (s) [75]	5×10^{12} [101]	$5.71^{+0.03}_{-0.04}$	BB	$32.58^{+0.40}_{-0.40}$	1.4*	12-17	[102]
PSR J1740+1000	5.06 (s) [103]	1.8×10^{12}	$6.04^{+0.01}_{-0.01}$	BB	$32.15^{+0.05}_{-0.05}$			[104] [2]
PSR B0633+1748	5.53 (s)		$5.75^{+0.04}_{-0.05}$	BB	$31.18^{+0.93}_{-0.33}$	1.4*	10*	[105]
RX J1856.4–3754 [‡]	$5.70^{+0.05}_{-0.25}$ (k) [106]	4×10^{12}	$5.75^{+0.15}_{-0.15}$	BB	$31.56^{+0.12}_{-0.12}$		14	[107, 108]
PSR B1055–52 [§]	5.73 (s)	4×10^{12}	$5.88^{+0.08}_{-0.08}$	BB	$32.57^{+0.52}_{-0.52}$	1.4*	13	[109]
PSR J2043+2740 [¶]	6.08 (s)	4×10^{11}	$5.64^{+0.08}_{-0.08}$	HA	$29.62^{+0.52}_{-0.52}$		10*	[75]
PSR J0720.4-3125	6.11 (s) [110]	10^{13} [111]	$5.75^{+0.20}_{-0.20}$	BB	$31.89^{+0.52}_{-0.52}$	1.4*	11-13	[112]

Table 3.1 Caption: As in Ref. [1] we favor kinetic ages over spin-down ages where possible. The letters ‘s’ and ‘k’ in column 2 denote characteristic spin down age and kinetic age, respectively. References are given in column 2 only where our ages differ from the values used in Ref. [1]. We use H atmosphere (HA) fits to stars less than 10^5 years and black body (BB) fits for older stars. In some of the H atmosphere fits, a magnetic field was used (either as a fixed value or as a fit parameter), and this is indicated in the fourth column (mHA). Notes: (*) This value was assumed not derived. (||) For the H atmosphere fit, we use the redshift temperature from Ref. [98], $10^{6.04}$, instead of the value reported as $10^{5.94}$ in Ref. [3]. (‡) As in Ref. [1], we use a range determined by the colder black body component from Ref. [107] and the warmer black body component in Ref. [108]. (§) We have used the updated information from Ref. [109] as in Ref. [3] over the values in Ref. [1]. (¶) We use a H atmosphere fit for this source since a black body fit is not available. (5) As in Ref. [1] we use a range determined by the cold and warm components from the black body model in Ref. [112]. (†) Ref. [113] claims this is not a neutron star. (++) As discussed in Ref. [1], Ref. [114] suggests that this star may be accreting due to its spin-down behavior. As explained in Ref. [80], Cassiopeia A spectral data produces a radius within the canonical range when fitted with a carbon atmosphere.

We also do not include any stars with magnetic fields larger than 10^{14} G in our data set, as the magnetic field has a strong impact on the atmosphere and may cause significant surface temperature gradients which our model cannot accurately describe [28]. In some cases, H atmosphere fits to x-ray spectra imply magnetic fields on the order of 10^{12} G, but we assume that there is no modification to the surface temperature or luminosity from fields of this strength. Instead, we assume a uniform temperature distribution across the neutron star surface.

There are a few objects for which neither H nor black body atmospheres imply a realistic neutron star radius, but where carbon atmospheres fit well. This is the case for the neutron star located in Cassiopeia A and XMMU J1732 located in HESS J1731–347, which we include in our analysis along with the possibility that they also may contain light elements in their envelopes.

As for the superfluid parameters, it should be noted that in the minimal cooling model, the principal unknown quantities in dense matter which impact neutron star cooling are the neutron superfluid and proton superconducting gaps (which determine the emissivity of the PBF process). Superfluidity and superconductivity exponentially suppress the specific heat and modify the neutrino emissivities in dense matter (for a review see Ref. [68]). For the modified Urca active in this model, the suppression factor is $\sim e^{-\frac{2\Delta}{T}}$. If there is no longer enough thermal energy to break Cooper pairs, (i.e. $\Delta \gg T$) neutrino emission is severely reduced. These effects begin when the temperature of the neutron star cools below the critical temperature. In the original BCS theory of superconductivity, the critical temperature and the value of the gap at zero temperature are related by $\Delta(T=0) \simeq 1.8 T_c$. The BCS approximation to superconductivity does not necessarily apply in the strongly-interacting nucleonic fluid, but we retain the standard practice of assuming that the BCS relation is approximately correct.

Two unknown values are required to understand nucleon superfluidity and superconductivity, the critical temperature at which the formation of Cooper pairs becomes favorable and the range of densities over which superfluidity exists. For this research, we focused on two instances of superfluidity found in the outer core of neutron stars, one for the proton singlet gap 1S_0 and the other for the neutron triplet gap 3P_2 . There also exists a third kind of nucleon superfluidity in the inner crust region, arising from the neutron singlet gap 1S_0 , which is omitted from this study because the two former gaps are of greater relevance to the ages of the stars which we survey [115]. The primary reason to neglect the study of neutron singlet gap 1S_0 is that its critical temperature is expected to be too large to be well constrained by data from neutron stars older than several hundred years, which would exclude almost all of the neutron stars in our data set. Hence, by studying the proton singlet gap 1S_0 and the neutron triplet gap 3P_2 , we study both of the most influential gaps in the cooling of star and (fortunately) the gaps for which our data is best capable of providing constraints. In particular, due to the wide range of densities it occupies, the neutron 3P_2 gap is expected to have the most significant effect on neutron star cooling [20].

Greater magnitude of superfluid gaps in general indicates faster cooling, but considering how the superfluid gaps affect the heat capacity of dense matter complicates such a simple

explanation. Superfluid gaps suppress heat capacity for temperatures well below T_c , but increase heat capacity at temperatures just below the critical temperature T_c . This is due to the fact that when the temperature is close to T_c , Cooper pairs are easily broken and reformed while for very low temperatures there is not enough thermal energy to break the pairs and the superfluid exists as a constant phase of matter.

Additionally, the presence of the energy pairing gap suppresses the excited states for fermions within the filled Fermi sea. Each energy gap Δ corresponds to a range of densities and a corresponding range of nucleon momentum k in which superfluidity can occur [116]. Another way that Cooper pairing affects neutron star cooling is once Cooper pairs are formed, the paired nucleons no longer participate in competing nucleon processes such as indirect Urca, or nucleon Bremsstrahlung. The accuracy of our calculation of the PBF process is increased by including the corrections from suppression in the vector channel [117, 118, 119, 120], and also including the axial anomalous contribution to the pair-breaking emissivity from Ref. [121].

A good initial conjecture for the critical temperature relation for nucleons comes from the BCS (Bardeen-Cooper-Schrieffer) theory relating the gap magnitude and critical temperature at absolute zero temperature Eq. (2.35). While it remains unclear to what extent this zero temperature limit can be applied to strongly interacting nucleon fluids, we assume this relation too is valid for nucleon fluids. With appreciation for the multiple ways which superfluid gap formation affects the cooling of neutron stars, we attempt now to quantitatively constrain it.

Despite performing cooling calculations for finite temperatures, we utilize a so called “zero temperature EOS” because the thermal pressure is minuscule in comparison to the pressure arising from fermion degeneracy. $\epsilon_F \gg T$. To express this quantitatively, we study neutron stars several hundred years old whose temperatures have cooled to $\sim 10^6$ K and below. The corresponding thermal energy is approximately 86 eV, less than tenth of a keV. By comparison the chemical potential of nucleons is on the order of MeV, the mass of each individual nucleon alone being 939 MeV. Thus our assumption of zero temperature and entropy is well supported. In contrast to thermonuclear reactions, which rely on thermal motion to overcome Coulomb repulsion and fuse nuclei, there exist nuclear fusion reactions

driven entirely by density. These are so called pyconuclear reactions, and given our zero temperature assumption, this is the mechanism by which nuclei react within the neutron star [56].

We describe each superfluid gap by three superfluid gap parameters, the first being the critical temperatures T_c at which nucleon Cooper pairs appear. The second and third parameters quantify the density dependency of the critical temperatures. These parameters are k_f , the peak Fermi momentum at which pairing occurs proportional to number density (see Eq. (3.1)), and Δk_f , corresponding to the density range over which the pairing occurs. Density increases with proximity to the core and achieves its maximum value at the center of the star. Hence the question of what density, and, correspondingly, at what pressure, Cooper pairing may occur is directly related to the question at what radial distances from the core one can expect to find Cooper pairing and the associated phenomenon of superfluidity. The fermi momenta of nucleons is related to their overall number density by the expression:

$$k_f = (3\pi^2 n)^{1/3} . \quad (3.1)$$

The Equation (3.2) quantifies the critical temperatures dependence on density. In this work, we assume that both the proton singlet and neutron triplet critical temperatures can be described by the Gaussian form:

$$T_c(k_F) = T_{c,\text{peak}} \exp \left[\frac{(k_F - k_{F,\text{peak}})^2}{2\Delta k_F^2} \right] . \quad (3.2)$$

Here $k_{F,\text{peak}}$ is the momentum corresponding to the maximum T_c . The normal distribution physically corresponds to the Cooper pairing being suppressed at extremely low densities as interparticle spacing increases and also being suppressed at extremely high densities as the repulsion between nucleons quenches the attractive interaction. The Gaussian dependence of the critical temperature on density is an informed guess, not a physical fact. It is a hypothesis of the model we test here. From this equation, we can now say that the two superfluids we wish to study, neutron $^3\text{P}_2$ gap and proton $^1\text{S}_0$ gap, can be characterized

by the six superfluid parameters $T_{c,\text{peak},n}$, $k_{F,\text{peak},n}$, $\Delta k_{F,n}$ for the neutron gap and $T_{c,\text{peak},p}$, $k_{F,\text{peak},p}$, $\Delta k_{F,p}$ for the proton gap.

We have attempted to calculate the value of each of these six parameters which correspond to the maximum likelihood determined by a statistical algorithm, the Markov Chain Monte Carlo. This algorithm works by first assuming a probability distribution for each of our six superfluid cooling parameters, here we assumed that each of our parameters has a Gaussian (or normal) probability distribution. In nature, only one true value exists for each cooling parameter, but these values are still undetermined. Hence every value within our distribution is deemed possible, although extreme values have decreased probability.

To ensure our model is physical, and to avoid over-counting models where the gaps vanish, upper and lower boundary conditions are assigned for each of the parameters based on nuclear theory and experiments, as well as astrophysical observations. One obvious boundary is that neither critical temperature can be negative. The critical temperatures' upper bound of $1 \times 10^{10}\text{K}$ is informed by our knowledge of the initial temperature of proto-neutron stars. If the critical temperature were above this maximum boundary temperature, superfluidity would exist for the entire lifetime of the star. This scenario is unlikely. The parameters $k_{F,p,\text{peak}}$ and $k_{F,n,\text{peak}}$ are given a lower bound of $n_{cc} = 0.09 \text{ fm}^{-3}$ which represents the crust-core transition density. For an upper bound of these same parameters, one can determine the central pressure of a $1.4 M_{\odot}$ neutron star to be $n_{\text{central}} = 0.545 \text{ fm}^{-3}$. Using our previous equation (Eq. (3.1)) to convert from density to fermi momenta, the limits may be expressed by the inequalities $0.481 \text{ fm}^{-3} < k_{F,p,\text{peak}} < 1.304 \text{ fm}^{-3}$ and $1.418 \text{ fm}^{-3} < k_{F,n,\text{peak}} < 2.300 \text{ fm}^{-3}$. For the final two parameters, we wish to avoid overcounting scenarios where the gap is nearly independent of density, hence our upper boundaries are expressed as $0 < \Delta k_{F,p} < 1.304 \text{ fm}^{-3}$ and $0 < \Delta k_{F,n} < 2.300 \text{ fm}^{-3}$.

An initial test point that lies within these boundary conditions is used to generate a 'Pairing Control' file in NSCool, which is then executed and produces a theoretical cooling curve dependent on the initial test point. Using a Bayesian fitting procedure discussed below, which compares this generated theoretical cooling curve to known neutron stars and their corresponding age and temperature uncertainties, a likelihood value is calculated for this initial step.

The initial likelihood value is calculated by an analysis of how well our initial theoretical curve fits the age and temperature measurements from our neutron star data set. The technique we used to calculate the likelihood for our theoretical curve takes into account that significant uncertainties exist in both the age and temperature measurements of our data points. We have chosen to use Bayesian inference to calculate the probability that our theoretical model is correct given that our data is correct within its uncertainty (see Appendix M).

It is common in the study of neutron star cooling to give age and temperature values in logarithmic units so as to accommodate their large uncertainties. We have used logarithmic units in our definition of the likelihood. The logarithmic age $\hat{t}$ and temperature $\hat{T}$ are defined as:

$$\begin{aligned}\hat{t} &\equiv \frac{1}{5} \log_{10} \left(\frac{t}{10^2 \text{ yr}} \right) \quad , \text{ and} \\ \hat{T} &\equiv \frac{1}{2} \log_{10} \left(\frac{T}{10^5 \text{ K}} \right) \quad ,\end{aligned}\tag{3.3}$$

which are defined so that typical values are between 0 and 1. There is no physical justification for using logarithmic units; this convention is commonly used without loss of accuracy.

We assume that both variables $\hat{t}$ and $\hat{T}$ have Gaussian distributions with parameters $\hat{t}_j$, $\delta\hat{t}_j$ and $\hat{T}$, $\delta\hat{T}_j$ respectively. The cooling code produces three different cooling curves, one each for hydrogen atmospheres $\hat{T}_H(\hat{t})$, carbon atmospheres $\hat{T}_C(\hat{t})$, and black body atmospheres $\hat{T}_{bb}(\hat{t})$. As an example, for hydrogen atmospheres, the likelihood function is given as:

$$\begin{aligned}\mathcal{L}_H &\propto \prod_j \int d\hat{t} \sqrt{\left\{ \left[\frac{d\hat{T}(\eta_j, \hat{t})}{d\hat{t}} \right]^2 + 1 \right\}} \exp \left\{ \frac{-[\hat{t} - \hat{t}_j]^2}{2(\delta\hat{t}_j)^2} \right\} \\ &\times \exp \left\{ \frac{-[\hat{T}(\eta_j, \hat{t}) - \hat{T}_j]^2}{2(\delta\hat{T}_j)^2} \right\} .\end{aligned}\tag{3.4}$$

Where the product index j denotes all of the stars less than 10^5 years old excluding Cassiopeia A and HESS J1731. No overall normalization is specified, as this is not

required for our results. In practice, this integral is approximated in our code as a product of summations over all the values of our cooling curve array. We let $\hat{T}_H(\hat{t}) \rightarrow (\hat{T}_{H,k}, \hat{t}_k)$ and use finite differencing to obtain the derivative $\left(d\hat{T}/d\hat{t}\right)_k$. Hence, the integral is approximated by the product of summations (see Eq. (3.4)). Specified as a summation array, the integral is re-expressed by allowing the limit $\hat{T}(\eta, \hat{t}) \rightarrow [\hat{T}_k(\eta), \hat{t}_k]$, and finite differencing gives the derivative $[d\hat{T}(\eta)/d\hat{t}]_k$. To a good approximation we can replace the integral by a sum:

$$\begin{aligned} \mathcal{L} \propto \prod_j \sum_k & \sqrt{\left\{ \left[\frac{d\hat{T}(\eta_j)}{d\hat{t}} \right]_k^2 + 1 \right\}} \exp \left\{ \frac{-(\hat{t}_k - \hat{t}_j)^2}{2(\delta\hat{t}_j)^2} \right\} \\ & \times \exp \left\{ \frac{-(\hat{T}_k(\eta_j) - \hat{T}_j)^2}{2(\delta\hat{T}_j)^2} \right\}, \end{aligned} \quad (3.5)$$

over a uniform grid in $\hat{t} \in [0, 1]$. In this context, one can see the purpose for the term under the square root sign: in regions where the cooling curve is nearly vertical, the data covers fewer grid points than in regions where the curve is nearly horizontal. The term under the square root compensates for this, ensuring portions of the cooling curve which are nearly vertical get extra weight. This reweighting is relatively weak in comparison to the data, which exponentially affects the likelihood. We choose a grid of size 100 but increasing the number of grid points does not affect our basic conclusions. When we fit luminosities rather than temperatures, we can just replace $\hat{T}$ with $\hat{L} \equiv (1/4)\log_{10}[L/(10^{30} \text{ erg/s})]$.

The benchmark for what will be considered a good fit can be obtained by incorrectly assuming the temperatures and ages are fully statistically independent. This allows the fitting procedure to be decomposed into two χ^2 procedures, one for the age data and one for temperature data. In this case, a good fit would be a value of $\chi^2 \approx 2N - 17$ where N is the 15 (or 17) neutron stars used (if carbon atmosphere stars are included) and 17 represents the 6 cooling curve parameters plus the 11 eta parameters. Note here that, in the limiting case that one of the two variables has an very small uncertainty, this likelihood function reduces to the more familiar χ^2 procedure.

Note that this likelihood function reduces exactly to the likelihood function for the traditional χ^2 procedure in the limiting cases that one of the two variables has a small uncertainty. A χ^2 fit was used in the case of Cassiopeia A, since it has a very small age uncertainty. The square root operates as a line element, specifying how one defines a distance when integrating the cooling curve along the data. The ambiguity in defining this distance is the exact same as the choice in using different frequentist regression techniques. Our approach makes this ambiguity explicit.

When two-dimensional data points are presented in a plane with comparable uncertainties in both axes, there is no unique “correct” fitting procedure (this conclusion holds in both the frequentist and Bayesian paradigms). In the frequentist picture of statistics, the lack of a unique fitting procedure has led to the use of several methods including orthogonal least squares, orthogonal regression, and reduced major axis regression [122]. These fitting procedures are referred to by several different names in literature. Reduced major axis regression is also referred to as geometric mean regression in Ref. [123] and a linear version obtaining the so-called “impartial line” was first used in Ref. [124]. This ambiguity is part of the reason why more quantitative fits to neutron star cooling data have not yet been performed before this work. When working with the the Bayesian picture of statistics, the non-uniqueness in the fitting procedure described above is manifest in the undetermined prior distribution which one must choose to proceed (see Appendix M).

After this first likelihood is calculated, the MCMC uses the initial guess as a seed value to produce a new guess. A “random-walk” produces a step increment for each value which adds or subtracts from each parameter a randomly determined value (up to half of the seed value of the parameter) and, in doing so, produces a new candidate step. This candidate step is vetted by determining if the likelihood value yielded from identical statistical analysis of its own theoretical cooling curves is greater or less than the likelihood value of the initial step. If the likelihood of the new candidate value is less than or equal to the likelihood of the initial guess, it is rejected, and a new candidate step is generated and tested. If the candidate likelihood is greater than the likelihood of the initial guess then this candidate value is accepted and is treated as the new guess function. This vetting process is a simplified description of the Metropolis algorithm used in our work. This process is then repeated over

many thousands of cycles, using the previously calculated value set as the seed value for generating a new candidate set of values, until a maximum likelihood value which cannot be surpassed over thousands of iterations is calculated.

As is often the case when implementing an MCMC, we discount data from the so called initial transient. The initial transient refers to results generated by our algorithm prior to the mapping of the desired equilibrium distribution. This could mean, for example, that for a 10,000 iteration run, approximately the first 1,000 calculated values will simply be thrown out. This is done to ensure that the system is close to equilibrium. There is nothing special about these example values, the algorithm can be implemented for an arbitrarily large number of iterations, and a larger number of iterations generally corresponds to a better mapping of the target phase space. This step becomes very important if there is a non-equilibrium phase space which might momentarily ‘trap’ our calculation if we make an inaccurate initial guess of our parameters. The good news is that owing to the ergodic nature of our physical system, provided that one’s initial test point is within the boundary conditions given enough iterations, the algorithm is certain to converge upon the maximum likelihood solution (the equilibrium phase space) given a long enough run time.

Our procedure for removing the initial transient isn’t arbitrary, as we use an autocorrelation estimate to observe the range of initial values for which the parameters possess undesired noise associated with the somewhat arbitrary initial guess value. By identifying and removing this initial transient we ensure that the averages and standard deviations of the final abridged data set will reflect the values within the equilibrium state. To perform the autocorrelation estimate we select any integer k less than the integer number of data entries n , and supply μ (the mean) and σ^2 (the variance) as expressed in the following equation:

$$\hat{R}(k) = \frac{1}{n-k} \sum_{t=1}^{n-k} (X_t - \mu)(X_{t+k} - \mu). \quad (3.6)$$

The data was then block averaged to ensure that all the parameter uncertainties were properly estimated. If our assumptions are correct, the values of the equilibrium state approximate the true values that exist in nature [125].

The use of MCMC, similar to method used here, has been successfully applied to constrain the mass-radius curve given neutron star mass and radius observations (the formalism was first developed in Ref. [126] and most recently updated in Ref. [76]). There are two differences in previous works and the work discussed here. First, the term under the square root was ignored. This treatment was appropriate because the radius depends only very weakly on the neutron star mass. Second, the data in that case is not Gaussian in either mass or radius so a more complicated probability distribution was used rather than the product of two Gaussians employed in this work.

3.3 Results

The results of our quantitative analysis of superfluidity and superconductivity in dense matter using observations of isolated neutron stars, assuming the validity of the minimum cooling model paradigm, yielded values for best fit neutron triplet superfluid critical temperature, and the best fit proton singlet superconducting critical temperature, along with their associated statistical uncertainties. We found that the neutron triplet critical temperature is likely $2.09^{+4.37}_{-1.41} \times 10^8$ K and that the proton singlet critical temperature is $7.59^{+2.48}_{-5.81} \times 10^9$ K. However, we also demonstrated that this result only holds if the Vela neutron star was excluded from our data set. When Vela was included, the energy gaps increased significantly to attempt to reproduce Vela's lower temperature given its young age. Further inclusion of neutron stars believed to have carbon atmospheres increases the neutron critical temperature and decreases the proton critical temperature. Our method demonstrated that continued observations of isolated neutron stars can quantitatively constrain the nature of superfluidity in dense matter. Our results indicate that the majority of our neutron star data does not require enhanced cooling processes, except possibly in the case of the Vela pulsar.

For the proton singlet gap, our mean value indicates a gap of magnitude:

$$\Delta(T=0)_p \simeq 1.8 T_c = 1.177 \cdot 10^6 \text{ eV} = 1.177 \text{ MeV} . \quad (3.7)$$

Thus the gap predicted in our work exceeds 1 MeV. For the neutron triplet gap, the mean value indicates a gap of magnitude:

$$\Delta(T=0)_n \simeq 1.8 T_c = 32400 \text{ eV} = 0.0324 \text{ MeV} . \quad (3.8)$$

As pointed out in Ref. [127], this result was consistent with previous results Refs. [128, 19], where the neutron gap was found to be of order $\Delta_{\text{neutron}} \approx 0.1 \text{ MeV}$ and the proton gap was found to be of order $\Delta_{\text{proton}} \approx 1 \text{ MeV}$.

Removing Vela (PSR B0833–45) and the two carbon atmosphere neutron stars in Cas A and XMMU J1732, allowed us to initially limit our research to neutron stars whose cooling profiles could easily be explained within the minimal cooling paradigm. We performed an MCMC simulation as described above, assuming that the minimal cooling model holds, i.e., that the direct Urca process does not operate and that no exotic matter is present. We fit the theoretical effective surface temperatures (accounting for the redshift factor) to the temperatures in Table 3.1 implied by the x-ray spectra. The resulting gap parameters, the envelope compositions, and their uncertainties (which we have assumed symmetric with respect to their central values) are given in the first column of Table 3.2. The posterior cooling curves for $\eta = 0, 10^{-12}$, and 10^{-7} are plotted in the top left panel of Fig. 3.1. We find large uncertainties in the critical temperatures for the singlet proton gap and the triplet neutron gap, and in these large uncertainties our values find agreement with previous results from Ref. [62].

The results from fitting the luminosities rather than the temperatures are presented in the right panel of Fig. 3.2 and the first column of Table 3.3. The results are relatively similar to those obtained by fitting the temperature rather than the luminosity. Representative curves which show the dependence of the superfluid gaps on Fermi momentum are given in Fig. 3.3, showing that the proton superconducting gap is likely largest just near the crust core transition and falls off dramatically at the highest densities in the core. The triplet neutron superfluid critical temperature, on the other hand, may peak at any density, so long as a large enough portion of the core undergoes the superfluid phase transition.

Table 3.2: Posterior parameter values for three fits of the minimal cooling model to data. The first column labels the parameter, the second column gives results obtained without including Vela (B0833–45) or the carbon atmosphere stars, the third column includes Vela, and the fourth column includes all of the stars in the data set. The gap parameters depend most strongly on whether or not Vela is included in the fit. The envelope compositions are relatively insensitive to the data selection, but vary strongly between individual neutron stars.

Quantity	Value and 1- σ uncertainty		
	w/o Vela or carbon	w/o carbon	All
$\log_{10} T_{c,\text{peak},n}$	8.48 ± 0.57	9.11 ± 0.19	9.61 ± 0.22
$k_{F,\text{peak},n} \text{ (fm}^{-1}\text{)}$	1.70 ± 0.12	1.924 ± 0.089	1.750 ± 0.082
$\Delta k_{F,n} \text{ (fm}^{-1}\text{)}$	0.21 ± 0.09	1.980 ± 0.076	1.80 ± 0.10
$\log_{10} T_{c,\text{peak},p}$	8.57 ± 0.60	8.81 ± 0.81	8.72 ± 0.45
$k_{F,\text{peak},p} \text{ (fm}^{-1}\text{)}$	0.67 ± 0.10	0.937 ± 0.086	1.02 ± 0.10
$\Delta k_{F,p} \text{ (fm}^{-1}\text{)}$	0.33 ± 0.12	0.145 ± 0.061	0.219 ± 0.067
$\log_{10} \eta_{\text{Cas A}}$			-9.52 ± 0.87
$\log_{10} \eta_{\text{XMMU J1732}}$			-9.14 ± 0.70
$\log_{10} \eta_{\text{PSR J1119}}$	-14.3 ± 1.4	-16.00 ± 0.77	16.56 ± 0.63
$\log_{10} \eta_{\text{RXJ 0822}}$	-8.80 ± 0.68	-9.80 ± 0.64	-9.97 ± 0.84
$\log_{10} \eta_{\text{1E 1207}}$	-9.57 ± 0.87	-10.8 ± 1.0	-10.22 ± 0.71
$\log_{10} \eta_{\text{PSR J1357}}$	-9.30 ± 0.81	-9.18 ± 0.50	-9.58 ± 0.88
$\log_{10} \eta_{\text{RX J0002}}$	-16.36 ± 0.68	-16.90 ± 0.39	-16.40 ± 0.61
$\log_{10} \eta_{\text{PSR B0833}}$		-8.29 ± 0.49	-8.65 ± 0.67
$\log_{10} \eta_{\text{PSR B1706}}$	-9.35 ± 0.79	-8.9 ± 1.1	-8.30 ± 0.51
$\log_{10} \eta_{\text{PSR J0538}}$	-16.18 ± 0.66	-16.57 ± 0.25	-16.88 ± 0.40
$\log_{10} \eta_{\text{PSR B2334}}$	-8.61 ± 0.79	-8.14 ± 0.32	-8.12 ± 0.47

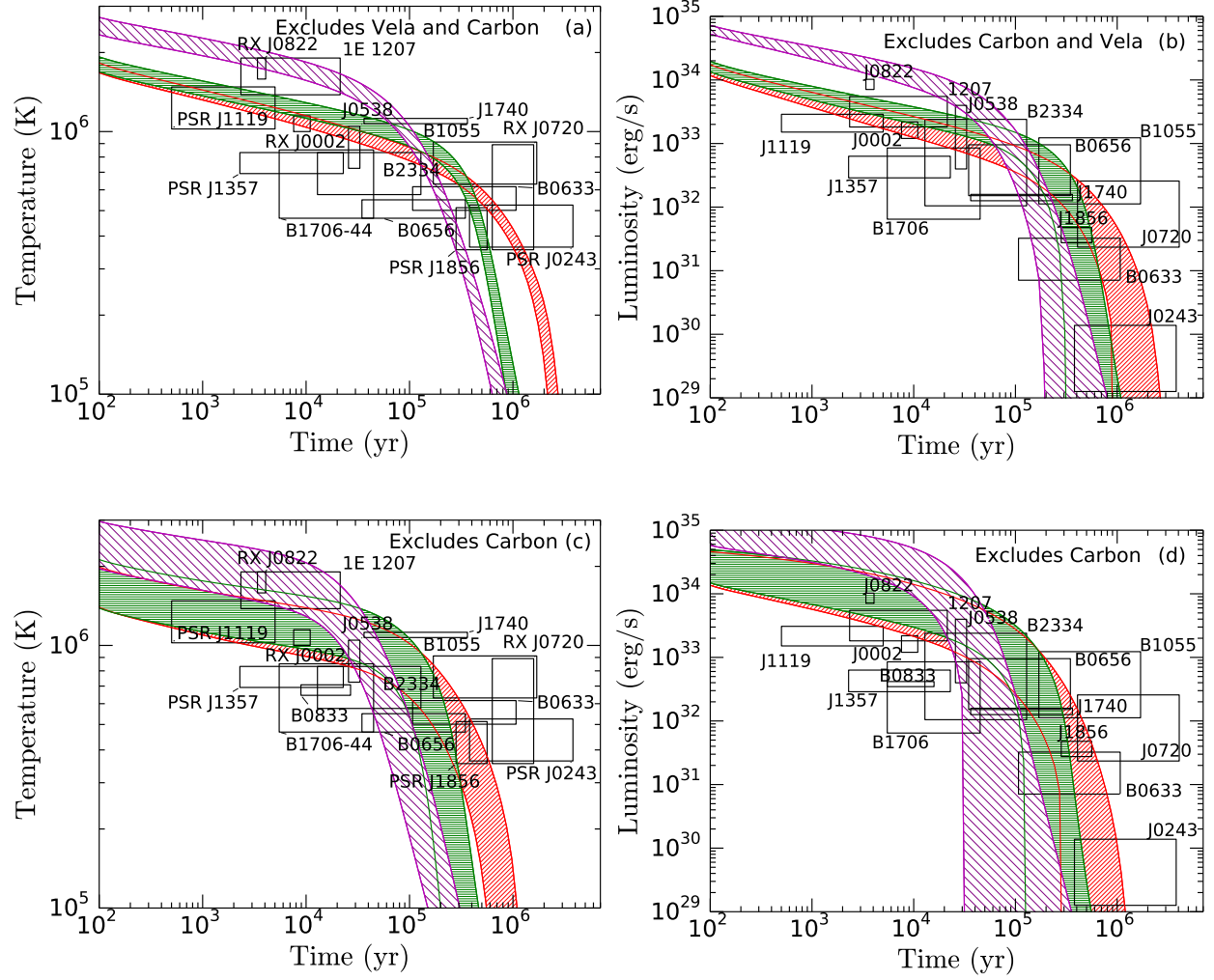


Figure 3.1: Theoretical Cooling Curves Part 1

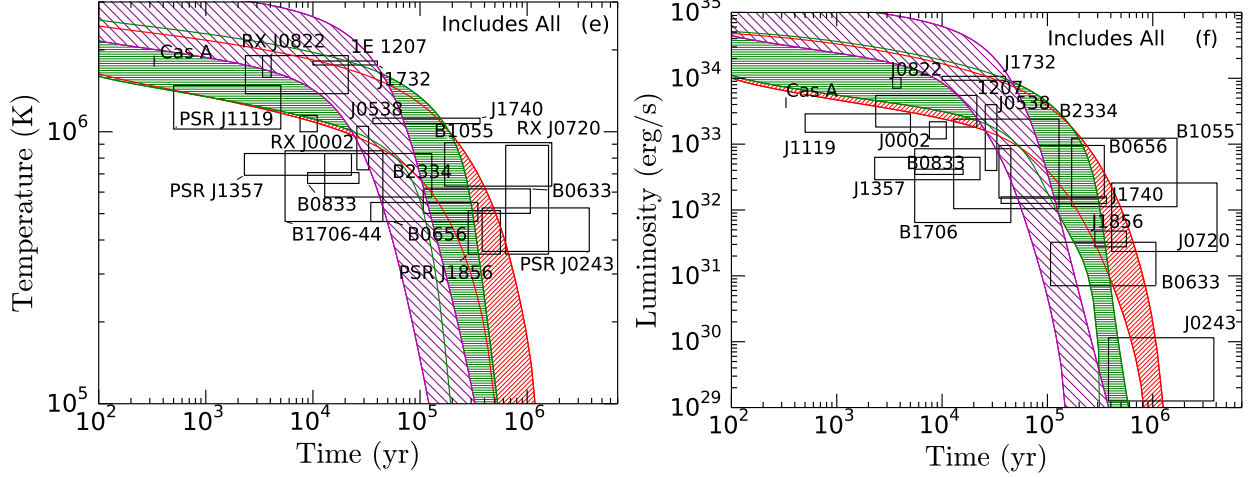


Figure 3.2: Theoretical Cooling Curves Part 2: Theoretical cooling curves illustrating how temperature and luminosity decrease over time. The black boxes represent neutron star cooling data and three colored bands show the $\pm 1\sigma$ uncertainties on the cooling curves (from superfluidity and superconductivity). The three bands represent three different values of η , 10^{-7} (purple, \\\ hatching), 10^{-12} (green, horizontal hatching) and 10^{-17} (red, // hatching). The temperature results [labeled (a), (c), and (e)] correspond to the parameter limits in Table 3.2, while the luminosity results [labeled (b), (d), and (f)] correspond to the parameter limits in Table 3.3

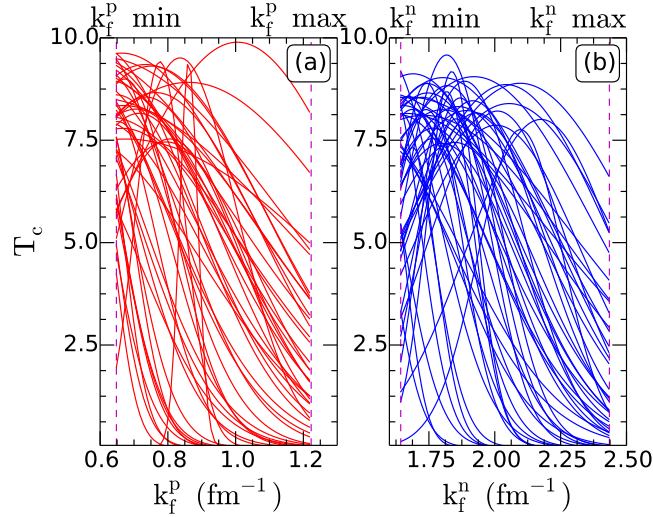


Figure 3.3: Uncorrelated samples from the critical temperatures from the fit to the luminosities as a function of the Fermi momenta for protons [left panel, (a)] and neutrons [right panel, (b)] without Vela or the carbon stars. The left boundary in both panels represents the Fermi momentum at the crust-core transition (denoted “ k_f^p min” and “ k_f^n min”). The right boundary represents the Fermi momentum in the center (denoted “ k_f^p max” and “ k_f^n max”) of a $1.4 M_\odot$ neutron star. The uncertainty in the critical temperatures is large and there is a preference for proton superconductivity to peak at lower Fermi momenta.

Table 3.3: Luminosity fits of the same posterior parameter values.

Quantity	Value and $1-\sigma$ uncertainty		
	w/o Vela or carbon	w/o carbon	all
$\log_{10} T_{c,\text{peak},n}$	8.32 ± 0.49	9.34 ± 0.38	9.55 ± 0.29
$k_{F,\text{peak},n} \text{ (fm}^{-1}\text{)}$	1.78 ± 0.16	1.95 ± 0.12	1.714 ± 0.087
$\Delta k_{F,n} \text{ (fm}^{-1}\text{)}$	0.28 ± 0.12	2.05 ± 0.11	1.744 ± 0.089
$\log_{10} T_{c,\text{peak},p}$	8.88 ± 0.63	9.26 ± 0.46	8.48 ± 0.42
$k_{F,\text{peak},p} \text{ (fm}^{-1}\text{)}$	0.641 ± 0.11	0.928 ± 0.07	0.99 ± 0.09
$\Delta k_{F,p} \text{ (fm}^{-1}\text{)}$	0.276 ± 0.12	0.115 ± 0.047	0.21 ± 0.085
$\log_{10} \eta_{\text{Cas A}}$			-9.51 ± 0.70
$\log_{10} \eta_{\text{XMMU J1732}}$			-9.03 ± 0.81
$\log_{10} \eta_{\text{PSR J1119}}$	-14.39 ± 0.98	-16.41 ± 0.40	16.51 ± 0.61
$\log_{10} \eta_{\text{RXJ 0822}}$	-9.3 ± 1.1	-9.88 ± 0.52	-9.99 ± 0.64
$\log_{10} \eta_{\text{1E 1207}}$	-10.0 ± 1.1	-10.9 ± 0.52	-10.22 ± 0.76
$\log_{10} \eta_{\text{PSR J1357}}$	-9.15 ± 0.94	-9.31 ± 0.68	-9.87 ± 0.88
$\log_{10} \eta_{\text{RX J0002}}$	-16.41 ± 0.80	-16.84 ± 0.39	-16.17 ± 0.55
$\log_{10} \eta_{\text{PSR B0833}}$		-8.27 ± 0.38	-8.44 ± 0.53
$\log_{10} \eta_{\text{PSR B1706}}$	-9.46 ± 1.1	-8.33 ± 0.52	-8.42 ± 0.50
$\log_{10} \eta_{\text{PSR J0538}}$	-16.00 ± 0.10	-16.69 ± 0.57	-16.80 ± 0.46
$\log_{10} \eta_{\text{PSR B2334}}$	-8.46 ± 0.64	-8.03 ± 0.35	-7.88 ± 0.27

The quantitative nature of our fit also allows us to determine the envelope composition for H atmosphere neutron stars. We find PSR J1119-6127, RX J0002+6246 and PSR J0538+2817 most likely have no light elements in their envelopes, in contrast with a small amount of light elements in 1E 1207.4-5209 and a significant contribution from light elements in all of the other H atmosphere stars. Note that neutron stars which lie to the left and below the cooling curves tend to have a large amount of light elements, fitting better to the $\eta = 10^{-7}$ (purple) curve lying to the right of the data point than the $\eta = 10^{-17}$ (red) curve above the data point (because the time uncertainty is larger than the temperature uncertainty).

We then added Vela and redid the temperature fit. The results are summarized in the second column of Table 3.2, and the bottom left panel of Fig. 3.2. This one data point, lying to the left and below the curves, has a strong impact: The critical temperatures implied by the data are much larger than those obtained previously. We find neutron superfluid critical temperatures near 10^9 K are required to explain the data and the width of the Gaussian increases significantly allowing a large part of the core to participate in the Cooper pair neutrino emissivity. The proton superconducting gap also increases slightly and moves to higher densities. The fit to the luminosities shown in the second column of Table 3.3 and the right panel in Fig. 3.2 shows the same trend. Representative curves which show the critical temperature are given in Fig. 3.5. The increase in gaps leads to a larger uncertainty in the cooling curves, as a larger part of the star now participates in the pair-breaking neutrino emissivity and thus the cooling is more sensitive to the gaps. The dramatic effect of Vela is partially because of the age revision of Vela down to $(5 - 16) \times 10^3$ years as obtained in Ref. [91] and discussed in Ref. [62]. The envelope compositions are unchanged (within errors) and the fit prefers a significant amount of light elements in Vela's envelope to become closer to the $\eta = 10^{-7}$ curve lying to the right. The results of including Vela and the carbon stars are displayed in Fig. 3.4.

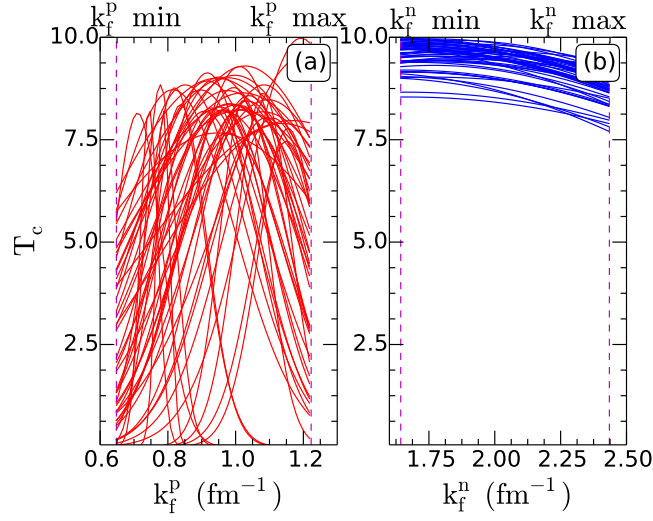


Figure 3.4: Uncorrelated samples from the critical temperatures from the fit to the luminosities as in Figs. 3.3 and 3.5 now having added both Vela (B0833–45) and the carbon-atmosphere stars to the analysis [(a) displays proton critical temperatures; (b) displays neutron critical temperatures]. Comparing to Fig. 3.5, the proton critical temperatures are smaller. The smaller proton critical temperature ensures younger stars are warm enough to match the relatively large luminosities from these two stars.

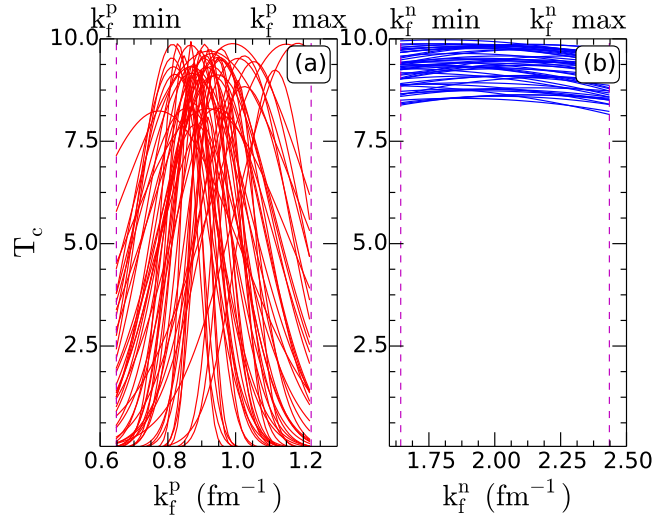


Figure 3.5: Uncorrelated samples from the critical temperatures from the fit to the luminosities as in Fig. 3.3 but now with Vela. Similar to previous works, we find strong proton superconductivity (a) and slightly weaker neutron triplet superfluidity (b). The proton superconductivity moves to higher densities and the density dependence of the neutron superfluid gap broadens to maximize the cooling to match the low luminosity of Vela.

While the absolute normalization of the likelihood function is not meaningful, relative values are physical. A typical data point contributes a factor of 0.5 to the likelihood while Vela’s contribution is 10^{-3} . This is a strong indication that fitting Vela is difficult in the minimal cooling model. The observation of Vela, as it currently stands, provides some evidence for the direct Urca process or the presence of exotic matter in neutron star cores.

3.4 Discussion of Results

Most importantly, our work *quantifies* the extent to which superfluid properties can be constrained from currently available data on the cooling of isolated neutron stars. Most of the previous works on this topic give more qualitative results: These previous results did not employ any particular likelihood function and thus cannot give full posteriors for their parameter values. The extent to which our quantitative approach will be possible without making the assumptions of the minimal cooling model is explored in a later section of this work.

Our analysis has either 14, 15, or 17 parameters corresponding to 15, 16, or 18 data points, respectively. One of the advantages of our Bayesian approach is that our formalism does not require the fitting problem to be strongly over-constrained. Had we not employed the minimal cooling model, we would have required at least four new parameters to describe the EOS and an additional mass parameter for each neutron star (bringing us to a total of 39 parameters for 18 data points). An accurate mass measurement for even a few of the neutron stars in this data set would improve the fitting problem substantially.

One possible extension would be to attempt to explain the surface temperatures of accreting neutron stars as well, as done in Refs. [129] and [130]. It is well known that some of those objects, in particular SAX J1808.4–3658, are too cold to be explained within the minimal cooling model Ref. [131], and thus the direct Urca process is invoked. The approach taken in Refs. [129] and [130] is similar in that they employ a systematic exploration of their parameter space; it is different in that they do not explicitly compute the likelihood of their models as we have done in Eq. (3.4). Extending our method to include the direct Urca process would necessitate also considering the variation in the EOS as well.

Our model does not include variations in temperature across the surface of the star due to magnetic field generated hot spots. Hot spots on the neutron star surface may not create pulsations in the emission if they lie near the axis of rotation. It was argued that fits to the luminosity rather than the effective temperature partially ameliorate this difficulty because uneven temperature distributions impact the shape of the spectrum more strongly than the luminosity [16]. Our results demonstrate that the luminosity and temperature fits obtain qualitatively similar constraints on the superfluid gaps with some quantitative differences (for example, the luminosity fit implies different critical temperatures for proton superconductivity, especially when Vela is included). Nevertheless, fitting to luminosities rather than temperatures may be insufficient to fully explain the data if the temperature variation across the surface is dramatic.

Our model computes an effective surface temperature based on an atmosphere model and the amount of light elements in the envelope (see Ref. [28] for a recent review). The observed x-ray data is analyzed presuming a H atmosphere (sometimes including an estimate of the magnetic field), a carbon atmosphere, or a black body spectrum. Our results are thus limited by these two ingredients insofar as they allow us to correctly determine the temperature at the base of the envelope.

Several authors have examined the cooling of isolated neutron stars outside the minimal model. Sachiko Tsuruta and her collaborators in 2009 [91] examined cooling with hyperons, and finds that superfluidity is required to ensure that the direct Urca process does not make neutron stars too cold. By allowing the direct Urca process, Refs. [132, 133, 134, 135] obtain a strong EOS dependence in their results. These works, along with Refs. [129, 130], find that the data can be explained without exotic matter so long as the direct Urca process operates in some stars. We find (as first found in Ref. [1]), that the isolated neutron stars (with the exception of the Vela pulsar) can be easily explained without having to invoke the direct Urca process, so long as one allows for variations in the envelope composition at early times. Another model found in Ref. [136], has invoked axions in a model which does not include the direct Urca process. While we are performing our work in a model which contains more restrictive assumptions about the nature of dense matter, our statistical analysis allows us

to be more quantitative in our conclusions. Extensions of this work beyond the minimal cooling model are in progress.

For the neutron stars with a carbon atmosphere, Ref. [137] performs a χ^2 fit to the data for the neutron star in Cas A, under the alternative assumption that this neutron star is indeed cooling quickly, as found in Ref. [80]. A χ^2 fit is possible here because there is no uncertainty in the x axis, and thus the likelihood function in Eq. (3.4) gives the same result. We include a larger data set and perform our Monte Carlo over a much larger set of cooling models. In Ref. [138], it is assumed that Cas A is cooling quickly, and the data of this star is explained in terms of the neutrino emissivity of superconducting quarks. The cooling of the carbon atmosphere star XMMU J1732 was addressed in Ref. [139], who also found a large heat blanketing envelope was required to reproduce the data. Ref. [139] also obtained a constraint on the mass and radius of this neutron star because, in their model, the proton superfluid gap is correlated with the mass and radius. In contrast, we treat the EOS and superfluid properties of matter as independent. It was argued in Ref. [140] that the x-ray spectra of Cas A and XMMU J1732 can also be modeled as H atmospheres with hot spots as opposed to uniformly emitting carbon modeled surfaces. This possibility will be considered in future work.

We have presented results with and without Vela, the neutron star in Cas A, and XMMU J1732, but we cannot yet definitively determine whether or not those objects should be included or left out. The decrease in the fit quality may support going beyond the minimal model to explain Vela and an alternative interpretation for XMMU J1732 (such as that in Ref. [140]), but the final answer on this question requires more data or smaller uncertainties.

Chapter 4

Combining Cooling and Radius Observations with Nuclear Structure Data

In the following section, a number of improvements and additions made to the previous analysis are discussed. These additions were made in order to increase the model's accuracy and also incorporate additional physics. These additions include allowing the operation of the direct Urca process for high mass neutron stars, the inclusion of neutron stars under accretion into our data set, incorporating nuclear data into our model, variations on the EOS, and inclusion of hyperons. A crucial advance in this section's research is that, unlike in the previous section, our model no longer treats the EOS and superfluid properties as independent of one another.

4.1 Enhanced Cooling

If it is still found that the minimal cooling paradigm is insufficient to describe the cooling of a neutron star, either some enhanced cooling or late heating mechanism must be present. The following section will discuss the possibility of enhanced cooling processes not permitted in the minimal cooling model, these being the existence of the powerful direct Urca process, and the presence of hyperons in the neutron star core.

The composition of the inner core of neutron stars, in particular NS which lie at the extreme end of the canonical mass range and contain very dense cores, remains unknown. It has been proposed that exotic matter might exist in the core of neutron stars at densities roughly twice nuclear saturation density. Previous works have calculated that the central density of most, if not all, neutron stars is larger than $4 M_\odot$ [76, 141]. Some form of exotica likely exists in stars with large masses. The existence of exotic matter would facilitate additional nuclear processes yielding enhanced cooling [69]. Possible exotica include the Λ hyperon and Σ triplets, K^- meson condensates, Bose condensates or deconfined quarks. In this work, we focus on one possibility, the possible appearance of hyperons in the cores of neutron stars.

4.2 Inclusion of Muons

Proposals to expand on past work have lead to the introduction of muons into the code. Muons appear in the neutron star when the chemical potential of electrons exceeds the muon rest mass $m_\mu = 105.7$ MeV. Muons are introduced via the interactions:

$$e^- \rightarrow \mu^- + \nu_e + \bar{\nu}_\mu , \quad (4.1)$$

$$n \rightarrow p + \mu^- + \bar{\nu}_\mu . \quad (4.2)$$

Their presence also allows for the interaction:

$$p + \mu^- \rightarrow n + \nu_\mu . \quad (4.3)$$

This changes the requisite balance of chemical potentials necessary for beta equilibrium. After the introduction of hyperons, the description of beta equilibrium in terms of chemical potentials is:

$$\mu_n - \mu_p = \mu_e = \mu_\mu , \quad (4.4)$$

and the proton fraction now satisfies charge neutrality under the condition [142]:

$$x_p = x_e + x_\mu . \quad (4.5)$$

Tau leptons τ are not included in this analysis as their mass, 1776.82 MeV, is too high for creation within neutron stars.

4.3 The Direct Urca Process

Previous analysis excluded the direct Urca process, as many of the stars in our data set have old ages but are still observable. The majority of the stars in our previous data set are well explained by cooling curves within the minimal cooling model, while inclusion of the direct Urca process produces fast early cooling, inconsistent with observation of visible old stars. In the minimal cooling model, the fastest cooling processes are the modified Urca process and the PBF process. If direct Urca is present, the star has a cooling mechanism which is one million times faster. The cooling time of neutron stars from direct Urca based on temperature (T), direct Urca emissivity (ϵ_{Durca}) and the heat capacity of the star (c_v) is:

$$\delta t = - \int \frac{c_v}{\epsilon_{\text{Durca}}} dT \equiv - \int \frac{dE}{dT} \left(\frac{dE}{dt} \right)^{-1} dT \rightarrow t_{\text{cooling time}} . \quad (4.6)$$

Once the direct Urca process is activated, the star will cool to the point of being undetectable within several hundred years [25].

The nucleon direct Urca processes is summarized by the following complimentary interactions:

$$n \rightarrow p + \ell + \bar{\nu}_\ell , \quad (4.7)$$

$$p + \ell \rightarrow n + \nu_\ell . \quad (4.8)$$

The direct Urca processes is significantly more powerful than the modified Urca process but, unlike the modified Urca process, can only be satisfied when momentum conservation is

satisfied. Momentum conservation is quantified by the triangle inequality of Fermi momenta:

$$k_{F,n} \geq k_{F,p} + k_{F,\ell} . \quad (4.9)$$

Charge neutrality demands that the following condition is satisfied for protons and electrons:

$$\frac{(k_p)^3}{3\pi^2} = \frac{(k_e)^3}{3\pi^2} . \quad (4.10)$$

It is possible to express this relation entirely in terms of the proton fraction. The proton fraction, the ratio of protons over the total number of nucleons, is expressed as:

$$Y_p = \frac{n_p}{n_b} = \frac{n_p}{n_p + n_n} = \frac{1 - \alpha}{2} . \quad (4.11)$$

The asymmetry parameter is defined as:

$$\alpha = \frac{N - Z}{A} = \frac{n_n - n_p}{n_n + n_p} . \quad (4.12)$$

$\alpha = 0$ represents symmetric nuclear matter containing an equal number of neutrons and protons, while $\alpha = 1$ represents pure neutron matter. A critical proton fraction α is necessary for the direct Urca process to operate [143]. As pointed out in Ref. [32], the momenta of neutrinos and antineutrinos is negligible with respect to that of the considered leptons and nucleons $p_{\nu, \bar{\nu}} \approx T \ll p_n, p_p, p_e, p_\mu$. Thus they are neglected here. The critical proton fraction is:

$$Y_p = \frac{n_p}{n_p + n_n} , \quad (4.13)$$

to satisfy Eq. (4.9) we have:

$$Y_p = \frac{(k_p)^3}{(k_p)^3 + (k_e + k_p)^3} . \quad (4.14)$$

Invoking the condition of charge neutrality $n_e = n_p$ (and thus $k_e = k_p$), the inequality is expressed as:

$$Y_p \geq \frac{(k_p)^3}{(k_p)^3 + (k_p + k_p)^3} \geq \frac{1}{9} . \quad (4.15)$$

Hence the proton fraction for direct Urca without muons is $Y_p = \frac{1}{9} \approx 0.111$. To incorporate muons into this calculation we simply use:

$$\mu_e = \mu_\mu . \quad (4.16)$$

Assuming an equal number of muons and electrons, the critical proton fraction is raised to $Y_p = 0.148$ (Ref. [48]).

Due to the necessary proton fraction of the direct Urca process, observations providing confirmation or disagreement with the predictions of direct Urca cooling help to constrain the proton fraction in neutron stars. The net asymmetry (I), quantified as the difference of neutrons and protons divided by the total number of nucleons is expressed as:

$$I = \frac{N - Z}{N + Z} . \quad (4.17)$$

The neutron-proton asymmetry (δ) is an analogous quantity defined with number densities of nucleons:

$$\delta = \frac{n_n - n_p}{n_n + n_p} . \quad (4.18)$$

Values of I for stable nuclei formed in the laboratory are typically around 0.2, but in the core of the neutron star this value can approach 1 as the nuclei become increasingly neutron rich [37]. In the crust of the star δ is ≈ 0.3 , while nuclei become increasingly neutron rich as density increases. Above the neutron drip density ($4 \times 10^{11} \text{g/cm}^3$ [3]) the asymmetry $\delta \approx 0.7$ (see Ref. [25]), the asymmetry of nucleons has a significant effect on the overall energy of a system of nuclear matter, this is illustrated in the study of the symmetry energy E_{sym} .

By seeing the connection between density and proton fraction, we gain an understanding of why high mass neutron stars are considered to be more likely to possess direct Urca cooling, as they possess higher internal densities. Thus for future work, we will use the most precise available mass observations available for each of the neutron stars in our data set. Each star is assigned a mass value such that EOS and composition constraints can be assigned according to this mass. By investigating the mass of the neutron stars in our data set, we will hopefully find a pattern of cooling for stars within a certain mass range. If a star's mass

is poorly constrained, this research might provide an opportunity of suggesting hypothetical masses for neutron stars based on their cooling patterns.

With regard to modifying previous research, more massive stars would contain greater maximum densities and this would change the allowed range of our variables k_{peak} and Δk . The most significant cooling parameter, which will vary as we assign higher masses to the neutron stars in our data set, more so than bulk cooling or superfluidity parameters, shall be the inclusion of the nucleon direct Urca, and direct Urca processes permitted by the existence of hyperons. One of the results of the previous section is that the Vela (PSR B0833 – 45) is not well explained by minimal cooling predictions. One possible implication of this is that Vela could be a high mass neutron star undergoing direct Urca cooling. This possibility, along with the possibility that accreting neutron stars could have higher proton fractions, is explored in the current sections.

4.4 Accreting Neutron Stars

In the previous section, only isolated neutron stars were considered. Accreting neutron stars in binary systems are included in this section’s analysis. One potential benefit of including these stars is that the higher precision obtained in the measurement of accreting neutron star masses can assist with constraining masses for isolated neutron stars. Conversely, using inferred temperature and luminosity in conjunction with the method of atmospheric modeling used in our study of isolated neutron stars, could yield new insights into the atmospheric composition of neutron stars in binaries.

Neutron stars in close binary systems, wherein the neutron star orbits either main sequence stars or white dwarf stars, commonly exist in either low mass x-ray binaries (LMXBs) or high mass x-ray binaries (HMXBs). In both cases, the neutron stars are known to emit powerful bursts of x-ray radiation powered by the gravitational energy released when accreted material strikes the neutron star surface in thermonuclear bursts. LMXBs draw accretion material from their donor star when the donor fills its Roche lobe; the material which overflows is drawn into the neutron star’s accretion disk. This periodic siphoning of material leads to the neutron star undergoing periodic accretion followed by a period of

quiescence (no accretion). The longer an accreting neutron star’s quiescent period is, the more closely its quiescent cooling resembles that of an isolated neutron star. HMXBs accrete material from a main sequence donor either through a decretion disk surrounding the donor star, or through direct accretion of the stellar wind (see Ref. [144] for a review).

The matter being deposited onto the surface of the star is composed of primarily light elements siphoned from the outer layer of the donor, but the accretion process and subsequent nuclear fusion on the surface of the star quickly converts these light elements into exotic nuclei. Immediately after the initial thermonuclear burst, these exotic nuclei are very proton rich, approaching the proton drip line, but as the material is compressed deeper into the star by additional accreted material, these nuclei will become increasingly neutron rich. Over time, this accretion will replace the crust on a timescale of $10^4 - 10^6$ years [145].

The accretion process is a significant source of heating in the neutron star crust. Analysis of the absorption lines in the x-ray spectrum provides information regarding the chemical composition of the neutron star surface [146]. Determination of the redshift in the thermal emission, alongside a quantification of the apparent radii, allows for constraints on compactness (mass/radius) of the neutron star [147]. In Ref. [148], several LMXBs located within globular clusters (the proximity to these globular clusters helps to constrain the distance to these stars) were used to calculate the neutron star mass-radius curve with uncertainties. For a $1.4 M_{\odot}$ neutron star, this analysis gave radius values in the $10 - 14$ km range, in agreement with canonical values.

Neutron stars have different atmospheres compared to their isolated counterparts, and accretion is a separate heating process not observed in isolated stars. Thus cooling curves of isolated neutron stars and accreting neutron stars will vary noticeably from one another. If the process of accretion is violent enough for a given star, it might obscure evidence of certain cooling processes taking place within the star. Regardless of these caveats, focusing on observable quantities shared by both isolated and accreting stars, we can exploit the cases where accreting neutron stars offer higher precision measurements.

The studies mentioned in this subsection demonstrate that observations of accreting neutron stars are useful for determination of EOS properties, such as the mass-radius curve, and also modeling the composition and nuclear processes taking place near the neutron star

surface. It is also possible that accreting neutron stars could prove useful in studying cooling processes not yet detected in isolated stars. It has been suggested, in Ref. [73], that the cooling of some accreting neutron stars is indicative of the direct Urca process. This example demonstrates further why it is necessary to study both accreting and isolated stars to appreciate the full range of physics found in neutron stars.

4.5 The Equation of State

For any thermodynamic state, there exists a set of parameters which adequately describe all observable properties of the state within a given system. By extension, any system of states has a set of parameters which can adequately describe all of its observables. These parameters are related by the EOS. The primary goal of the previous section's research was to constrain superfluid parameters independent of the EOS. Research in this sections aims to constrain superfluid parameters, while also attempting to constrain the EOS parameters based on mass-radius relations and cooling observations. As discussed in Ref. [149], the EOS and the superfluid parameters could in principle be described by a single nuclear Hamiltonian. However, this Hamiltonian has not yet been fully derived. Thus in the previous section we artificially treat the EOS and superfluid parameters as independent of one another in order to gain the insight necessary to work backwards toward such a master equation.

We utilize the parameters of chemical potential μ , temperature T , density n and entropy S to give a full description of the star. If we choose a limiting case wherein the temperature is set to zero (not to imply that the temperature is zero, but we are neglecting the effects of thermal pressure), and correspondingly the absolute entropy is also zero, we can simplify this down to two remaining parameters; chemical potential and number density. The pressure of the system can be completely described by the chemical potentials of the particle species present:

$$P(\mu, T) \xrightarrow{T \rightarrow 0} P(\mu) . \quad (4.19)$$

The energy of the system may be described entirely by the number densities of the particles, $n_{\text{number}} = \frac{N_i}{V}$, N_i here is the total number of particles i , and V is the volume they are enclosed

in:

$$E(n_{\text{number}}, S) \xrightarrow{S \rightarrow 0} E(n_{\text{number}}) . \quad (4.20)$$

Work in the previous section had assumed a zero thermal pressure APR EOS. This EOS assumes a neutron star is composed of protons, neutrons, and leptons alone. This EOS is also considered an effective field theory, because the degrees of freedom are the hadrons themselves not the quarks which compose them. No quark pairing is assumed in this model and as such we have only considered the quark content of the hadrons to the extent that their constituent quarks allow for weak interactions (e.g. direct Urca). This model EOS makes use of both two-body and three-body interactions derived from nuclear theory and scattering experiments, making it a rather comprehensive EOS. Hyperons are included in this EOS, although they were not included in our previous research.

Assuming zero thermal pressure is a reasonable assumption at zeroth order for the EOS due to the degeneracy pressure being orders of magnitude greater than thermal pressure. The zero temperature limit also implies that the neutrino chemical potentials are zero. Physically this means that the neutrinos will exit the star immediately, and won't disrupt the total thermal equilibrium of the system:

$$\mu_{\nu_e} = \mu_{\bar{\nu}_e} = \mu_{\nu_\mu} = \mu_{\bar{\nu}_\mu} = 0 . \quad (4.21)$$

So for APR EOS (where the pressure due to temperature is negligible), the pressure is entirely determined by the chemical potentials of the particle species within the star (See Eq. (2.25)). Here U denotes the internal energy of the system:

$$U = TS - PV + \sum_i \mu_i N_i . \quad (4.22)$$

The variable i here denotes the different species of particles, e.g. neutron, protons, etc.

In general, there are two broad qualitative descriptions of equations of state. A stiff EOS is one in which an increase in density produces an immense increase in pressure. For a soft EOS an equivalent increase in density corresponds to a much smaller increase in pressure. Soft equations of state are easily compressed while stiff equations of state are more difficult

to compress. Soft equations of state produce neutron stars that have a low maximum mass and smaller radii, whereas stiff equations of state produce neutron stars that have large maximum mass and larger radii. Stars with stiff EOSs have a lower central density when compared to soft EOSs for the same masses.

4.6 Testing the Validity of an EOS

Any valid EOS must reproduce empirical nuclear observations densities below and around saturation density, as well as the high density behavior which yields the mass-radius curve of neutron stars. For the binding energy per nucleon and saturation density we have canonical values centered around these accepted mean values: $E/A - M = -16$ MeV for the binding energy and $n_0 = 0.16 \text{ fm}^{-3}$ [150] for the saturation density. Alternative expressions are provided in Ref. [151]. The saturation density:

$$n = n_p + n_n = \frac{2k_F^3}{3\pi^2} = 0.153 \text{ fm}^{-3} . \quad (4.23)$$

The binding energy may also be expressed as:

$$\frac{\rho}{n} - m_n = -16.3 \text{ MeV} . \quad (4.24)$$

Another quantity tested against our model is the Landau mass (the value here is supplied from Ref. [151]):

$$m_L = \sqrt{\bar{m}_N^2 + k_F^2} = 0.83 m_N . \quad (4.25)$$

A causality test ensures that adiabatic speed of sound is less than the speed of light.

One simplified way to express an EOS is to write a polytropic function. For pure nuclear matter such a function can be expressed as:

$$P = K \rho_0^\Gamma . \quad (4.26)$$

Where K is the polytropic constant. When Γ is below $\frac{5}{3}$, the EOS is generally considered soft and when it is above $\frac{5}{3}$, it is considered stiff. Such a relation can be used to solve the Tolman-Oppenheimer-Volkoff (TOV) equations for non-rotating stars and solve for their maximum mass and mass-radius relations [34, 152]. Simply by supplying a central density and a simplified EOS, we can make a prediction for the mass of the star in question. Conversely, the observed mass of neutron stars can inform our constraints on the central densities of neutron stars and their equations of state.

Research discussed in this section aims to expand the scope of our cooling model by implementing a family of different EOSs, based on the FSUGold EOS and the EOS introduced by the field theoretical model utilized by Steiner, Prakash, Lattimer and Ellis (SPLE EOS) [37]. The EOS we utilize in this work combines both of these and we will refer to it as the FSUGold-SPLE EOS. FSUGold-SPLE utilizes a relativistic mean field theory framework, and we will exploit its ease of incorporating new particles to study hyperons. The FSUGold-SPLE EOS makes use of a minimum of three input parameters; temperature T , proton fraction Y_p , and density n (APR accommodates proton fraction as well, but FSUGold-SPLE has a larger proton fraction which might allow for the existence of the direct Urca process). Again implementing a zero thermal pressure limit, we can see that the pressure of the neutron star will depend on the proton fraction. This in turn will make the pressure and corresponding radii dependent on the symmetry energy of the nuclear matter within the star. These points are further illustrated by the equations:

$$P(\mu_p, \mu_n, T) \xrightarrow{T \rightarrow 0} P(\mu_p, \mu_n) , \quad (4.27)$$

$$E(n_B, Y_p, S) \xrightarrow{S \rightarrow 0} E(n_B, Y_p) . \quad (4.28)$$

The proton fraction primarily affects neutron stars cooling through allowing or disallowing the direct Urca process (Fig 4.1 for density dependence of proton fraction in neutron stars). The proton fraction depends on the symmetry energy, the symmetry energy in turn also correlated to the point-neutron distribution in radii, which in turn determines the total radius of a neutron star. Thus the proton fraction is correlated to the overall radius of the

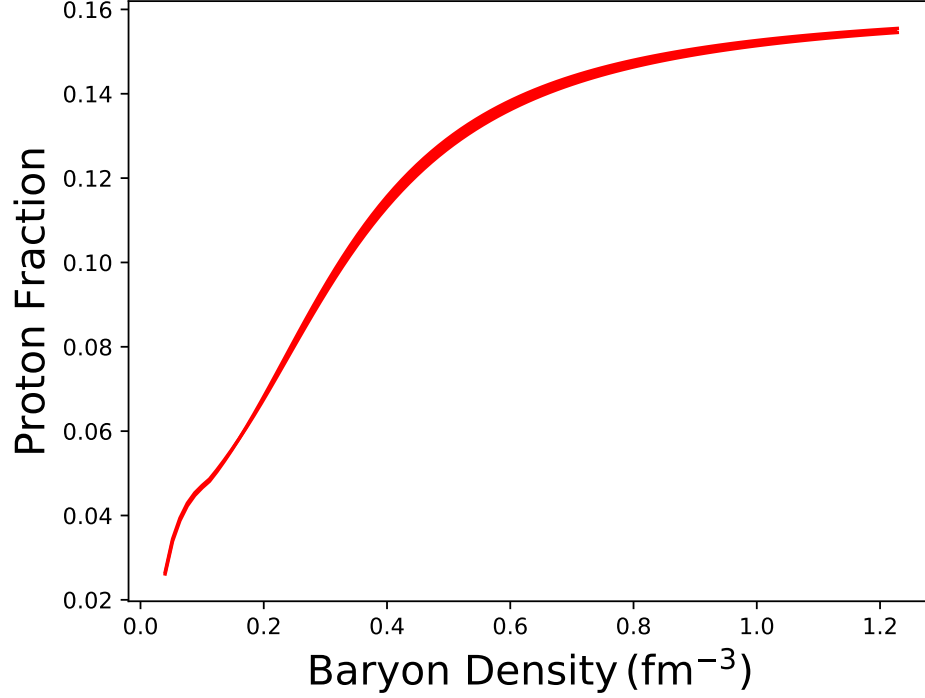


Figure 4.1: Proton fraction as a function of nucleon density.

neutron star and this provides a secondary effect on cooling in addition to the determination of the direct Urca process [153].

The symmetry energy is the lowering of the nuclear binding energy arising from a difference in the number of neutrons and protons. The degree of this asymmetry in nucleon number is described by the proton fraction Y_p [154, 155, 156]. We obtain the nuclear symmetry energy S_E :

$$S_E(n) = \left. \frac{1}{2} \frac{\partial^2 E}{\partial^2 \alpha} \right|_{\alpha=0, n=n_0} . \quad (4.29)$$

The parameter α here is the same as expressed in Eq. (4.12). This derivation ignores Coulomb energy and surface term corrections as in Ref. [157]. Summing the energy per nucleon as in Ref. [158] we find:

$$E(n, \alpha) = E_0(n, \alpha = 0) + S_E \alpha^2 . \quad (4.30)$$

Here, n corresponds to the saturation density, and E is the energy per nucleon. Taylor expansion of $S_E(n)$ around the nuclear saturation density ($n_0 = 0.16 \text{ fm}^{-3}$), at which point

the nuclear symmetric matter achieves its peak binding energy of ≈ 16 MeV, yields the following relation:

$$S_E(n) = S_0 - L\epsilon + 1/2K_{\text{sym}}\epsilon^2 + \mathcal{O}(\epsilon^3) . \quad (4.31)$$

Where S_0 is typically listed as ≈ 32 MeV at saturation density, $\epsilon = \frac{n_0 - n}{3n_0}$, and L and K are the curvature and slope parameters at n_0 . Another name for K is the compressibility, or incompressibility, of the nucleus. This parameter is correlated to the stiffness of the EOS and determines what central densities the neutron star may achieve:

$$K = 9n^2 \frac{\partial^2 E}{\partial n^2} . \quad (4.32)$$

E here is energy per nucleon [159].

The slope of the symmetry energy is a higher order term:

$$L_{\text{slope}} = 3n_0 \left. \frac{dS_E(n)}{dn} \right|_{n_0} = \frac{3}{n_0} P_0 . \quad (4.33)$$

The density dependence of the symmetry energy is still unknown, especially at high densities, which might explain discrepancies in calculating K [48]. The symmetry energy, as determined by the EOS, is directly related to the pressure within the neutron star. For pure neutron matter, which is a good approximation for dense material in the outer core of the neutron star, at nuclear density the pressure can be approximated as in Eq. (2.22). Thus the symmetry energy as determined by the EOS directly corresponds to the internal pressure and the observed radius of the neutron star. The uncertainty in determining the symmetry energy over a range of densities poses one of the greatest obstacles in an accurate theoretical prediction of neutron star radii (Ref. [35]). Conversely, one may work backwards by this same reasoning and use observations of the neutron star radius in order to provide constraints for the EOS of dense matter. A large symmetry energy yields a large star radius, hence larger radii for lower mass stars would hint at a high symmetry energy and perhaps reveal the nucleon ratios present within these stars.

The EOS determines the abundance of each species of particle. The previous calculations determine the magnitude of the symmetry energy as well as what nuclear processes can occur within the star [160]. One process which has been omitted in previous research is the direct Urca, but with a new range of masses and EOS, this process could play a significant role in our future calculations. A large proton fraction would be significant in neutron star cooling as it could allow for the activation of the powerful direct Urca process. This is significant as our previous work assumed, as in the minimal cooling model, the direct Urca process is blocked for a $1.4 M_{\odot}$ star which fails to satisfy the triangle inequality Eq. (4.9) even at its central pressure. Starting with a system of protons, neutrons and electrons in chemical equilibrium:

$$\mu_n = \mu_p + \mu_e , \quad (4.34)$$

using Eq. (2.26) and assuming beta equilibrium, or zero electrostatic charge:

$$k_p = k_e . \quad (4.35)$$

In terms of Eq. (3.1) we obtain:

$$\frac{(k_p)^3}{3\pi^2} = \frac{(k_e)^3}{3\pi^2} , \quad (4.36)$$

in order to satisfy the triangle inequality Eq. (4.9).

In APR EOS models, direct Urca is forbidden except for the most massive neutron stars $1.68 - 1.83 M_{\odot}$ [149]. All of our current neutron stars have masses assumed or observed to be below this limit. Hence, if we wish to study the possible effects of the direct Urca process, we will need to make use of an EOS where either the Urca triangle inequality is satisfied or hyperons exist in the core of the neutron stars.

Additional parameters which can be tuned in EOS measurements are the total energy density and total pressure of the star. A simplified Fermi gas EOS yields:

$$\epsilon_{\text{total}} = \sum_{i=n,p,e} \epsilon_i ; \quad (4.37)$$

where each element of this sum may be expressed in terms of our momentum variable k :

$$\epsilon(k)_i = \int_0^{k_f} (k^2 + m_i)^{1/2} k^2 dk , \quad (4.38)$$

and the total pressure of the star:

$$P_{tot} = \sum_{i=n,p,e} P_i , \quad (4.39)$$

and the pressure elements may be expressed in terms of k as well (Ref. [161]):

$$P(k)_i = \int_0^{k_f} (k^2 + m_i)^{-1/2} k^4 dk . \quad (4.40)$$

Both ϵ_{tot} and P_{tot} are input parameters for APR and FSUGold-SPLE EOSs. These equations of state in turn are used to solve the TOV equations, and thus provide our models theoretical predictions of neutron stars structure (i.e. radii and masses).

As we work backwards toward the derivation of a complete nuclear Hamiltonian based on observation, we can also make use of the latest tentative version of the nuclear Lagrangian. A complete description of the Lagrangian is equivalent to a full description of the Hamiltonian:

$$H = \sum_{i=1}^n \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L . \quad (4.41)$$

The Lagrangian can be written in two parts one being the ground state Lagrangian and the other part being the interacting Lagrangian:

$$L = L_0 + L_{\text{int}} , \quad (4.42)$$

$$L_0 = \sum_j \bar{\psi}_j (i\gamma_\mu \partial^\mu - m_j) \psi_j . \quad (4.43)$$

m_j are the masses of the hadrons (here nucleons) and ψ and $\bar{\psi}$ correspond to the initial and final state of the hadrons. This noninteracting Lagrangian describes nucleons moving as a free fermi gas, with no interactions between one another. This is hardly a realistic picture,

thus we employ the interaction Lagrangian to supply the electromagnetic and strong force interactions necessary to model the group behavior of nucleons. The interacting term is expressed as:

$$\begin{aligned}
L_{\text{int}} = & \sum_j \bar{\psi}_j \left[g_s \phi - \left(g_v V_\mu + \frac{g_\rho}{2} \boldsymbol{\tau} \cdot \mathbf{b}_\mu + \frac{e}{2} (1 + \tau_3) A_\mu \right) \gamma^\mu \right] \psi_j \\
& - \frac{\kappa}{3!} (g_s \phi)^3 - \frac{\lambda}{4!} (g_s \phi)^4 + \frac{\zeta}{4!} (g_v^2 V_\mu V^\mu)^2 \\
& + \Lambda_v (g_\rho^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu) (g_v^2 V_\mu V^\mu) .
\end{aligned} \tag{4.44}$$

This Lagrangian density utilizes Yukawa interactions to describe how the nucleon field couples to assorted meson fields. Here again, ψ and $\bar{\psi}$ correspond to the initial and final state of the nucleon j 's isodoublet nucleon field, ϕ is the isoscalar-scalar meson field. Three vector fields are utilized: A^μ is the photon field, $\mathbf{b}$ is an isovector and V^μ is an isoscalar. The four nonlinear meson interactions, the isoscalar meson self-interactions (through associated terms κ , λ and ζ) ζ in particular reduces K. Λ_v is a mixed isovector-isoscalar coupling which alters the density dependence of the symmetry energy (softening) [50, 48].

4.7 Testing Nuclear Structure Predictions

In this work, we utilized an EOS based on relativistic mean field theories (mentioned in a previous background section), these being the APR and the combined FSUGold and SPLE parameter sets. One of the historically recognized weaknesses of using field-theoretical models is that despite satisfying causality limits and reproducing properties of nuclear matter at saturation, this approach lacks a direct connection to nucleon-nucleon scattering data Ref. [25]. In this work we correct for this weakness by including a fit to such experimental data as established by scattering experiments. Low density EOS predictions are verified in our treatment at densities below nuclear saturation by nuclear scattering and spectroscopy experiments.

Mass spectroscopy and collision experiments have provided the binding energies and charge radii observables for the two nuclei: Zr^{90} and Pb^{208} . Each EOS provides theoretical

predictions for these values, which are compared to their experimental values to produce a likelihood fit. We use physically motivated Gaussians to model probability distributions of the total energies and charge radii of the nuclei like so (note binding energies are negative here to indicate a positive amount of work must be done to deconstruct a bound nuclei) :

$$\begin{aligned}\mathcal{L}_{\text{Zr}} &= \exp \left[\frac{(E - E_{\text{peak}})^2}{2(\delta E)^2} \right] \cdot \exp \left[\frac{(r - r_{\text{peak}})^2}{2(\delta r)^2} \right] \\ &= \exp \left[\frac{(E + 8.71 \text{ MeV})^2}{2(0.16 \text{ MeV})^2} \right] \cdot \exp \left[\frac{(R - 4.27 \text{ fm})^2}{2(0.12 \text{ fm})^2} \right] ,\end{aligned}$$

and for Pb:

$$\begin{aligned}\mathcal{L}_{\text{Pb}} &= \exp \left[\frac{(E - E_{\text{peak}})^2}{2(\delta E)^2} \right] \cdot \exp \left[\frac{(r - r_{\text{peak}})^2}{2(\delta r)^2} \right] \\ &= \exp \left[\frac{(E + 7.87 \text{ MeV})^2}{2(0.16 \text{ MeV})^2} \right] \cdot \exp \left[\frac{(R - 5.50 \text{ fm})^2}{2(0.12 \text{ fm})^2} \right] .\end{aligned}$$

The main source of uncertainty in calculated the binding energy (δE) comes from the calculation of the mass defect, which requires accurate modeling of the nuclear many body potential (a very difficult problem to fully model) [162]. The uncertainty in the nuclear radii (δr) is related to the neutron skin thickness (see Fig 4.2). Both of these radii are very neutron rich, thus knowing the precise neutron skin thickness and associated symmetry energy are important inputs necessary to put further theoretical constraints on these nuclear radii [163]. The total likelihood function for the fits to nuclear experimental data in our model is:

$$\mathcal{L}_{\text{Nuc}} = \mathcal{L}_{\text{Zr}} \cdot \mathcal{L}_{\text{Pb}} . \quad (4.45)$$

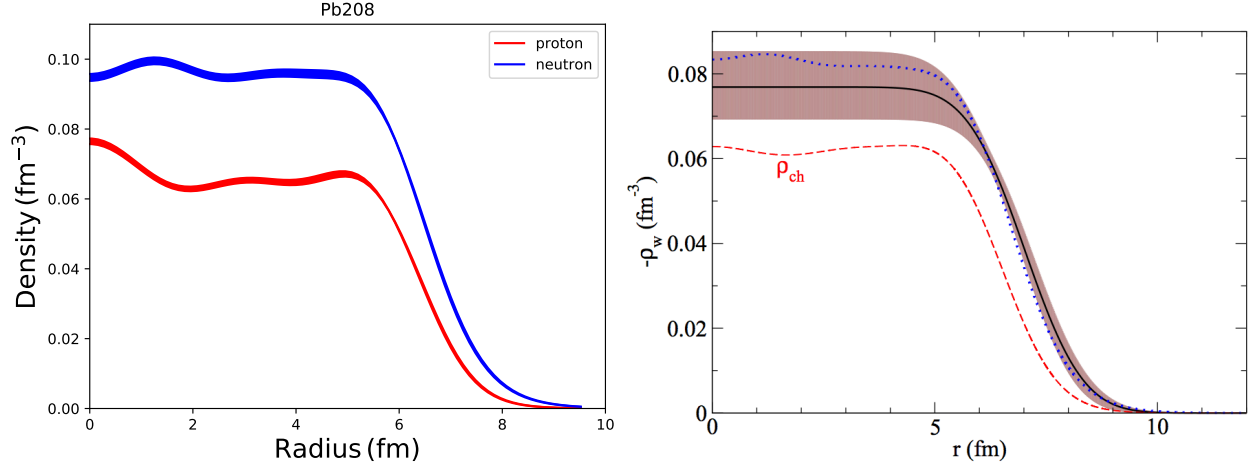


Figure 4.2: Model Predictions of density distribution of neutrons and protons within the ^{208}Pb nucleus. Experimental data (black line) from PREX result from Ref. [4].

4.8 Variation in the Equation of State

A total likelihood function for the candidate EOS is calculated as the product of all of these likelihoods:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{cooling}} \cdot \mathcal{L}_{\text{M-R}} \cdot \mathcal{L}_{\text{Nuc}} . \quad (4.46)$$

Relativistic equations for both nuclear matter as well as neutron matter are crucial for a fuller description of the densities and nuclear processes that exist within neutron stars, wherein there exists an inward gravitational force millions of times greater than that of our sun. This incredible gravitational force is countered by a complex combination of the other three fundamental forces of nature. The compact body is stable over the lifetime of the universe and contains particles in its core that travel at relativistic speeds, thus necessitating a fully relativistic model to understand the nuclear processes operating within the neutron star.

The models considered in this work use hadrons and mesons as the relevant degrees of freedom (quantum hadrodynamics) as opposed to more comprehensive and complicated theories which use quarks and gluons as the relevant degrees of freedom (quantum chromodynamics).

The methods used in our current analysis expand upon previous work, wherein an APR EOS was assumed in order to utilize cooling data from isolated neutron stars to constrain

the critical temperatures and onset densities of two nuclear superfluids, those of the proton 1S_0 and neutron 3P_2 energy gaps [33]. The current work was motivated in part by a result of the previous work, wherein it was found that some stars might undergo enhanced cooling. This result was discovered when it was found that Vela Pulsar (PSR B0833-45), possessing a very low temperature despite its young age, did not match well with the surface temperature and luminosity predictions of minimal cooling. Its low temperature appears to indicate that this neutron star is undergoing, or has undergone, fast cooling facilitated by at least one advanced cooling process.

In order to study the possibility of enhanced cooling, we test variations on the masses of the stars in our dataset, alongside correlated variations on the EOS. Some of the higher masses in the distributions we explore could allow for enhanced cooling processes, such as the direct Urca process.

The current work explores variations on a subset of relativistic mean field theory equations of state similar to the ones described by the FSUGold parameter set (Ref. [48]) and the SPLE EOS (Ref. [37]), in order to find an EOS which simultaneously recreates known neutron star mass and radius combinations, theoretically predicted dense matter properties, and lastly predicts known nuclear binding energies and charge radii ascertained by nuclear experiments. The FSUGold parameter set was generated when comparisons were made between the charge radii and binding energies of magic nuclei as predicted by a Lagrangian density containing a coupling between the omega meson field and rho meson field (see Eq. (2.36)), and the experimentally obtained radii and binding energies of these nuclei. Magic nuclei are those nuclei which have closed nucleonic shells, and these nuclei exhibit a particularly high level of stability compared to their isotope neighbors. The SPLE EOS is similar in form to the FSUGold EOS and is fit to the many of the same observables mentioned above, it contains parameters in addition to the FSUGold EOS which model electromagnetic field strengths and additional higher order couplings in order to provide additional freedom to vary the symmetry energy.

Expanding our previous Markov chain Monte Carlo (MCMC) scheme which utilized a Bayesian framework for which we supplied prior probability distributions of six superfluid parameters and envelope composition parameters for each of the stars ($T_{c,\text{peak},n}$, $k_{F,\text{peak},n}$,

$\Delta k_{F,n}$, $T_{c,\text{peak,p}}$, $k_{F,\text{peak,p}}$, $\Delta k_{F,p}$, $\eta_{\text{PSR J11119-6127,...}}$), additional prior probability distributions now include parameters which represent the meson and nucleon coupling values of our relativistic mean field theory, as well as masses for observed neutron stars. Two polytropic EOS parameters, a coefficient of pressure modification (ΔP_K) and an exponent of pressure modification (ΔP_Γ) are also included as additional terms in the net probability distribution.

In total, the prior probability distribution and each subsequent probability distribution calculated by the MCMC, has fifty-two parameters. These parameters represent unknown values which can be constrained by experimental data, as theoretical predictions produced from the parameters can be directly compared to experimental observables. The three types of experimental inputs we use to constrain our model parameters are: observations of neutron stars cooling (redshift temperature and luminosity values paired with an age estimate), observations of nuclear structure, and observations of neutron star masses and radii.

Likelihood functions quantify how well our candidate EOS accurately reproduces each category of observations. The first likelihood function is a reproduction from our previous work [33]. In this previous work, redshift luminosities and temperatures of neutron stars along with their corresponding pulsar ages were used to constrain proton singlet and neutron triplet superfluid critical temperatures. The second likelihood function is calculated by obtaining the difference between the calculated theoretical values of the masses and radii of neutron stars from their experimentally obtained values and uncertainties. The final likelihood function compares predictions of nuclear observables as predicted by our candidate EOS to their known experimental values, the binding energy of the nuclei and the charge radius of individual nuclei.

A total likelihood function for the candidate EOS is calculated as the product of all of these likelihoods (Eq. (4.46)). Following the technique utilized in the previous work, we proceed via Bayesian inference. New steps are chosen by randomly varying each of the fifty-two parameters. The Metropolis algorithm over many iterations is very likely to adequately map the global maximum probability space. Of course each posterior distribution accepted by the algorithm becomes the prior distribution for the next step. We will use the results of results of this Metropolis algorithm to make deductions about possible improvements to the

dense matter EOS, as well as suggest probable mass and radius values for isolated neutron stars.

Regarding the cooling of neutron stars, we renew our assumption of the validity of the minimal cooling model, disallowing the possibility of exotic matter as well as enhanced cooling processes with the critical exception of the direct Urca process for high mass stars. Following previous work each of the isolated neutron stars are assigned an individual quantity η which quantifies the light element composition of their envelopes. We retain the practice of only conducting this analysis on isolated neutron stars, as extending such an approach to accreting neutron stars would ignore the significant effect accretion has on nucleosynthesis in the neutron star envelope.

In previous work we had merely modeled every neutron star in our data set as having a mass of $1.4 M_{\odot}$. In this work each of these stars is assigned a mass variable parameter, with the hope that the values of these parameters in the global maximum probability space correspond to the most likely mass values for each neutron star. All eighteen isolated neutron stars are assigned mass parameters, and in addition to these the more tightly observationally constrained masses of seven binary neutron stars are included in our analysis.

Experimental mass and radius observations of neutron stars supply both the initial guess values of our relevant mass and radius model parameters as well as their constraints. Unfortunately, as noted in Ref. [164], there are no precise simultaneous mass and radius measurements for neutron stars. At best, there are high precision mass measurements, but the radii fits to these value have large systematic errors. In previous work, the data set was entirely comprised of isolated neutron stars. In this analysis, seven accreting neutron stars in binary systems have been added to this data set. All of these accreting stars have been found in globular clusters whose distances are better determined than the distances to isolated stars, consequentially their radii are more precisely constrained than isolated neutron stars. The names of these globular clusters are ω Cen, NGC 6397, M28, M13, NGC 6304, M30 and X7 [148]. The chaotic process of accretion makes interpretation of these binary neutron stars complicated, but doppler shift measurements of their orbital velocities allow for more precise mass measurements of these stars.

Beyond this, more sophisticated techniques such as gravitational lensing can be used. This technique exploits the fact that the extreme gravity of neutron stars bends light emitted from the surface of the star as well as that of background galaxies. The brightness distortions from the star can be used to deduce the overall mass [165].

Other techniques, such as those in Ref. [107] attempt to use assumptions regarding atmospheric models, gravitational potential and the distance to the neutron star in order to make mass calculations. Unfortunately the accuracy of these calculations is restricted by the uncertainty in distance measurements and atmospheric modeling. Depending on the atmospheric model applied, the assumed distance to the star, and the assumed gravitational red shifting, results can vary greatly. The best that can be done in this regard is to see what masses and gravitational redshift values provide the best fit assuming a certain atmospheric model, much as we assumed atmospheric models in our previous research.

Previous work simply set all of the neutron stars we are studying to have a mass of $1.4 M_{\odot}$ in our theoretical model, in place of accurate mass measurements. This is a reasonable approximation for many stars, but several of the stars in our data set have calculated masses which are greater than $1.4 M_{\odot}$. For the sake of fidelity, we should take these larger masses into account, as higher mass stars contain greater central densities which might allow for enhanced cooling processes and exotic matter such as hyperons. As noted in Ref. [166], $1.4 M_{\odot}$ stars are not expected to contain cores with quark matter admixtures, ruling out at least one kind of enhanced cooling for stars of lower mass.

As explained in Ref. [164], the majority of known neutron stars have observed masses in the $1.3 - 1.4 M_{\odot}$ range. Using this observation we assume a uniform distribution of stars, ranging from 1.0 to $2.0 M_{\odot}$ around a mean value within this range. The assumption of Gaussian distribution does not consider that neutron stars born in core collapse supernovae might have different mass distributions, as compared to neutron stars born in O – Ne – Mg supernovae.

The maximum mass of neutron stars is determined by the internal composition, and therefore ultimately by the EOS. Thus empirical observations of very massive stars may jointly constrain the composition of neutron stars as well as the dense matter EOS. The

observation of low mass black holes may also prove useful in identifying the neutron star maximum mass limit [13].

Each EOS has its own theoretical maximum mass limit, above which the neutron star becomes unstable and collapses to form a black hole. This limit, in tandem with accurate measurements of massive neutron stars, plays an important role in ruling out many equations of state. It is well known that the stiffer the EOS the larger the maximum mass [25]. At these immense densities required to achieve the maximum mass, the inner layers of the star achieve stability as a result of possessing a very high neutron fraction. Thus by making maximum mass one of the parameters of our model, we set our sights on further study of the high density portion of the EOS.

Observations of the most massive known neutron stars inform this upper limit. A famously precise example for a lower limit of a neutron star maximum mass comes from the observation of gravitational wave driven precession of the PSR B1913+16 neutron star binary system. It was confirmed via an observed Shapiro delay of the radiation from this signal that the more massive of the two neutron stars has a mass of $1.44 M_{\odot}$ [167]. In recent years, a Shapiro delay observation revealed the neutron star J1614 – 2230 has a calculated mass very close to $2 M_{\odot}$ (Refs. [53, 168] quoted value of $1.97 \pm 0.04 M_{\odot}$). Our initial assumption is that the maximum mass is nearly the same as that of the FSUGold EOS Ref. [48], $1.976 M_{\odot}$. Our modified EOS has a maximum mass limit of $2.0 M_{\odot}$. Any precise confirmation of a neutron star above this mass would rule out this EOS.

4.9 Inclusion of Hyperons

Quarks and leptons are the fundamental building blocks of all observed matter. Combinations of three quarks make up baryons, of which neutron and protons are examples. Leptons are light, negatively charged particles that do not participate in strong interactions (only electromagnetic and weak interactions). There are three species of leptons in all, but it is electrons and muons which are expected to be present in neutron stars. Mesons are intermediaries of the strong force between nucleons. Mesons are composed of a quark and an antiquark. The flavors of quarks are conserved in strong force interactions, but flavor may

Table 4.1: Chart displaying the masses and Coulomb charges of the fundamental particles.

Fundamental Particles		
Name	Charge Q/e	Mass $m_{\text{particle}}c^2$
up	$2/3$	2.4 MeV
down	$-1/3$	4.8 MeV
strange	$-1/3$	95 MeV
charm	$2/3$	1.275 GeV
top	$2/3$	≈ 172.44 GeV
bottom	$-1/3$	≈ 4.18 GeV

change in weak force interactions (beta decay is an example of this). Baryon number is still conserved in weak interactions.

In all there are six quarks (up, down, strange, charm, top, and bottom), but only the three lightest, the up, down, and strange quarks, are theorized to exist in neutron stars (see Table 4.1). Up and down quarks are certain to exist within neutron stars as it is these quarks which compose neutrons (udd) and protons (uud). The prevalence of strange quarks in neutron stars is less certain, but if present these quarks would exist as constituents of hyperons.

Hyperons, exotic heavy particles which contain at least one strange quark, were first identified within cosmic rays in 1947 by Rochester and Butler [169]. Hyperons are baryons which obey Fermi statistics just like protons and neutrons, composed of three quarks bound together by the strong nuclear force. At least one of the hyperons' constituent quarks is a strange quark, which differentiates them from other baryons and motivates the description of hyperons as being comprised of 'strange matter'. Since their discovery, they have been generated and detected in proton beam experiments, leading to the expectation that these particles would exist in abundance in the cores of neutron stars. Once chemical potentials of neutrons and protons become large enough (that is higher than the rest mass of the Λ^0 hyperon, 1115 MeV, conditions become energetically favorable for those neutrons on the Fermi surface to weak decay into Λ^0 hyperons. The critical chemical potential is expressed here with a dependence on the proton fraction (see Eq. 2.27). Thus, there is a prediction that the only requisite condition for hyperons to exist is a central density of 2 – 3 times saturation density. This requisite central density is well within expected central density

Table 4.2: Chart displaying the masses and Coulomb charges of hyperons likely present in neutron stars.

Hyperons			
Name	Charge	Mass	Composition
Λ^0	0	1115.68 MeV	uds
Σ^+	+e	1189.37 MeV	uus
Σ^0	0	1192.642 MeV	uds
Σ^-	-e	1197.449 MeV	dds
Ξ^0	0	1314.83 MeV	uss
Ξ^-	-e	1321.13 MeV	dss

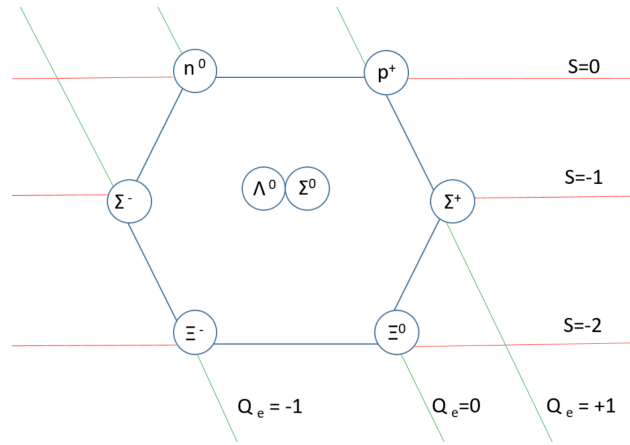


Figure 4.3: The baryon octet

values for massive stars. The existence of some species of hyperons is therefore deemed possible, at least in the case of massive stars [170] (see Table 4.2).

The baryon octet consists of the lightest eight baryons, six of which are strange quark bearing hyperons. We assume in this work that all of the fundamental particles in the octet exist in the core of neutron stars. The octet is composed of Λ^0 (uds) , Σ^+ (uus) , Σ^0 (uds), Σ^- (dds), and the cascade hyperons Ξ^0 (uss) , Ξ^- (dss). All of the hyperons we will study in future research can be arranged by the number of strange quarks they contain, as well as the hadrons overall charge into a geometric diagram known as the baryon octet (see Fig. 4.3).

The existence of hyperons in the core allows for additional cooling processes, as well as possible cooling suppression arising from existence of hyperon superfluids. The issue at the heart of the so called “hyperon puzzle” is whether high mass neutron star observations can

be reconciled with the existence of hyperons in the core as well as cooling observations which might suggest the presence of hyperons.

The majority of the hyperons, by number, expected inside neutron stars are those possessing negative coulomb charge necessary to satisfy the star's beta equilibrium condition. It is worth noting that the possible existence of strangeness-bearing matter would permit the existence metastable proto-neutron stars which would decay into black holes during deleptonization [25]. In an older star (which has survived deleptonization), exotica allow for enhanced cooling processes. The first hyperons to appear are the Σ^- hyperons followed by the Λ hyperons:

$$n + \ell \rightarrow \Sigma^- + \nu_\ell , \quad (4.47)$$

$$p + \ell \rightarrow \Lambda^0 + \bar{\nu}_\ell , \quad (4.48)$$

$$\Lambda^0 \rightarrow p + \ell + \bar{\nu}_\ell , \quad (4.49)$$

$$\Sigma^- \rightarrow n + \ell + \bar{\nu}_\ell . \quad (4.50)$$

This process describes the Direct Urca process for hyperons. ℓ here denotes either an electron or a muon. In the first example, beta equilibrium is satisfied when $\mu_\Lambda = \mu_p + \mu_e$. In general, electric charge and baryon number are conserved quantities throughout the neutron star. The introduction of these initial hyperons gives rise to new beta equilibrium conditions. Until now we have merely assumed that the neutron star satisfies the minimal cooling model condition of beta equilibrium for nuclear matter, that is for every proton there exists an accompanying lepton (e.g. electron, muon) of opposite charge. When this beta equilibrium condition is satisfied, the star is considered to have an overall electric charge of zero:

$$n_e = Z \cdot n_N . \quad (4.51)$$

This beta equilibrium condition can also be described in terms of Fermi-momenta as in Eq. (4.35).

With the introduction of hyperons, whose strange quarks carry a negative charge of $-\frac{2}{3} e$, it becomes energetically advantageous for the system to rid itself of leptons at high densities.

One can think of the negatively charged hyperons as a cost effective alternative to leptons in terms of energy density. Within cores containing hyperons muons will be eliminated and electrons will be reduced from $Y_e = \frac{1}{9}$ down to $Y_e = \frac{1}{100}$ [69]. The expression for beta equilibrium given a system containing all baryons in the octet is given by the equations:

$$\mu_{\Lambda^0} = \mu_n = \mu_p + \mu_\ell , \quad (4.52)$$

$$\mu_p = \mu_n - \mu_\ell , \quad (4.53)$$

$$\mu_{\Sigma^-} = \mu_n + \mu_\ell , \quad (4.54)$$

$$\mu_{\Sigma^+} = \mu_n - \mu_\ell , \quad (4.55)$$

$$\mu_{\Xi^-} = \mu_n + \mu_\ell , \quad (4.56)$$

$$\mu_n = \mu_{\Lambda^0} = \mu_{\Sigma^0} = \mu_{\Xi^0} . \quad (4.57)$$

Using Eq. (3.1) and Eq. (2.24) where electrons and muons are denoted by ℓ we can obtain the electrical neutrality condition in terms of particle densities [171]:

$$n_p + n_{\Sigma^+} = n_{e^-} + n_\mu + n_{\Sigma^-} + n_{\Xi^-} . \quad (4.58)$$

The effective masses of the baryons are:

$$\bar{m}_p = m_p - g_{\sigma N} \bar{\sigma} ,$$

$$\bar{m}_n = m_n - g_{\sigma N} \bar{\sigma} ,$$

$$\bar{m}_{\Lambda^0} = m_{\Lambda^0} - g_{\sigma \Lambda^0} \bar{\sigma} ,$$

$$\bar{m}_{\Sigma^-} = m_{\Sigma^-} - g_{\sigma \Sigma^-} \bar{\sigma} ,$$

$$\bar{m}_{\Xi^-} = m_{\Xi^-} - g_{\sigma \Xi^-} \bar{\sigma} .$$

While the effective potentials are given as (see Ref. [151]):

$$\begin{aligned}
\bar{\mu}_p &= \mu_p - g_{\omega N} \bar{\omega}_0 - \frac{1}{2} g_{\rho N} \bar{\rho}_0^3 - g_{\phi N} \bar{\phi}_0 , \\
\bar{\mu}_n &= \mu_n - g_{\omega N} \bar{\omega}_0 + \frac{1}{2} g_{\rho N} \bar{\rho}_0^3 - g_{\phi N} \bar{\phi}_0 , \\
\bar{\mu}_{\Lambda^0} &= \mu_{\Lambda^0} - g_{\omega \Lambda^0} \bar{\omega}_0 - g_{\phi \Lambda^0} \bar{\phi}_0 , \\
\bar{\mu}_{\Sigma^-} &= \mu_{\Sigma^-} - g_{\omega \Sigma^-} \bar{\omega}_0 + g_{\rho \Sigma^-} \bar{\rho}_0^3 - g_{\phi \Sigma^-} \bar{\phi}_0 , \\
\bar{\mu}_{\Xi^-} &= \mu_{\Xi^-} - g_{\omega \Xi^-} \bar{\omega}_0 + \frac{1}{2} g_{\rho \Xi^-} \bar{\rho}_0^3 - g_{\phi \Xi^-} \bar{\phi}_0 .
\end{aligned}$$

The presence of hyperons would allow for the hyperon direct Urca process to occur. Such a fast and powerful process would have a very large associated neutrino emissivity. The effect on the neutron stars cooling rate from this process would be much more significant than the PBF process which was the focus of our previous research. Examples of enhanced neutrino emission processes which include hyperons are as follows:

$$\Lambda^0 \rightarrow p + \ell + \bar{\nu}_\ell , \quad (4.59)$$

$$p + \ell \rightarrow \Lambda + \nu_\ell , \quad (4.60)$$

$$\Lambda^0 \rightarrow n + \ell + \bar{\nu}_\ell , \quad (4.61)$$

$$n + \ell \rightarrow \Lambda + \nu_\ell . \quad (4.62)$$

A hyperon direct Urca cycle also occurs involving Σ^- hyperons:

$$\Sigma^- \rightarrow n + \ell + \bar{\nu}_\ell , \quad (4.63)$$

$$\ell + n \rightarrow \Sigma^- + \nu_\ell , \quad (4.64)$$

$$\Sigma^- \rightarrow \Lambda^0 + \ell + \bar{\nu}_\ell , \quad (4.65)$$

$$\ell + \Lambda^0 \rightarrow \Sigma^- + \nu_\ell . \quad (4.66)$$

The emissivity of these process is significant, as shown in Refs. [32] for an interaction where the lepton is an electron:

$$\epsilon_{\bar{\nu}} = \frac{2\pi}{\hbar} 2 \sum_i G_F^2 \cos^2(\theta_{\text{Cab}}) (1 + 3g_A^2) f_n (1 - f_p) (1 - f_e) E_{\bar{\nu}} \delta(p_n - p_p - p_e - p_{\bar{\nu}}) . \quad (4.67)$$

Where $E_{\bar{\nu}}$ is the energy of the antineutrino and f denotes a Fermi-Dirac distribution and θ_{Cab} is the Cabibbo angle:

$$\epsilon_{DU} = 4.00 \times 10^{27} \left(\frac{Y_e n}{n_s} \right)^{1/3} \frac{m_{B1} m_{B2}}{m_n^2} \cdot W \cdot T_9^6 \Theta \text{ erg cm}^{-3} \text{ s}^{-1} . \quad (4.68)$$

Here Y_e is the electron fraction, m_{B1} and m_{B2} are the masses of the ingoing and outgoing baryons of the interaction, respectively. m_n is the neutron mass and T_9^6 denotes core temperature in units of 10^9K . W defines a weak interaction matrix factor which is unity for nucleonic direct Urca and 10^{-1} for strangeness conserving direct Urca interactions. Θ can be algebraically expressed as the conditions $\mu_{B1} = \mu_{B2} + \mu_e$ and $p_f(B1) \leq p_F(B2) + p_F(e)$. Inputting the larger baryon masses and much smaller electron fractions present when hyperons appear, we can compare to the the neutrino emissivity of the hyperon direct Urca to the nucleon direct Urca process by calculating antineutrino energy emissivity from neutron decay. The hyperon emissivity is found to be as little as one hundredth of that of the nucleon direct Urca, primarily due the significantly smaller baryon weak decay constants associated with hyperons when compared to analogous constants for nucleon [172].

The appearance of hyperons as a phase of matter softens the EOS [173]. The softening partially arises as a result of the system converting a large quantity of energy into the hyperons which have much greater rest mass than protons and neutrons. The strange quark itself has a bare mass of $90\text{-}100 \text{ MeV}/c^2$ while up and down quarks have bare masses of $1.8\text{-}3.0 \text{ MeV}/c^2$ and $4.5\text{-}5.3 \text{ MeV}/c^2$ respectively. This means that stars which contain hyperon cores, so called hybrid stars, will have a smaller maximum mass compared to stars which do not contain strange matter. Another cause of softening is the removal of relativistic leptons from the core of the neutron star.

We are currently expanding our previously mentioned parameterization of the EOS to include hyperons Eqs. (4.43) and (4.44). Despite a growing volume of experimental data probing nucleon-hyperon interactions as well as hyperon-hyperon interactions, the effect of hyperons on nuclear structure is far from certain [174].

Hyperons under terrestrial conditions rapidly decay through weak processes (mean lifetime of Λ^0 and Σ^- have been measured as $2.6 \cdot 10^{-10}$ s and $1.479 \cdot 10^{-10}$ s) [175]. On Earth, these processes are allowed and we can observe their decay products, but inside the neutron star core nucleon degeneracy forbids the introduction of the nucleonic decay products of these processes. Neutrons and protons already exist in filled Fermi seas, making it energetically very expensive to create new nucleons. Hence these common decay processes are forbidden in neutron stars and the hyperons lifetimes are extended. Because these common processes are blocked, a decay channel which is far less common on Earth becomes the only available process. These allowed processes generate the hyperon direct Urca (see Fig. 4.4 and Fig. 4.5).

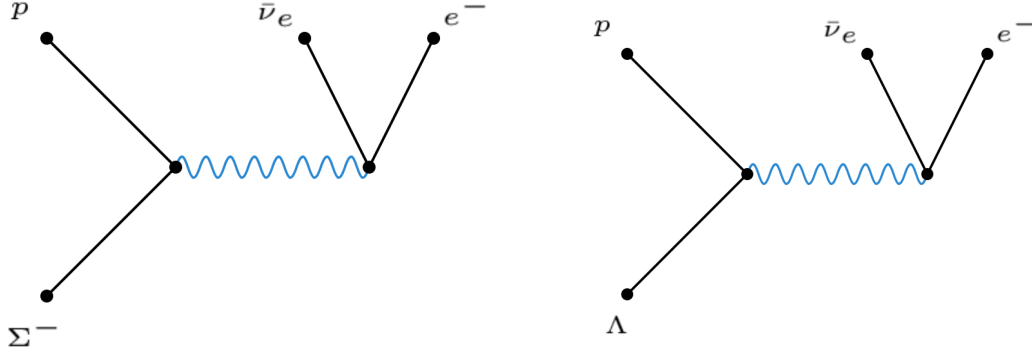


Figure 4.4: Direct Urca with Σ hyperon (left) and Λ Hyperon (right)

The Xi baryons are known as cascade particles, due to their multistep chain of decays [176]:

$$\Xi^0 \rightarrow \Sigma^+ + \ell + \bar{\nu}_\ell, \quad (4.69)$$

$$\Xi^- \rightarrow \Lambda^0 + \ell + \bar{\nu}_\ell, \quad (4.70)$$

$$\Xi^- \rightarrow \Xi^0 + \ell + \bar{\nu}_\ell, \quad (4.71)$$

$$\Xi^- \rightarrow \Sigma^0 + \ell + \bar{\nu}_\ell. \quad (4.72)$$

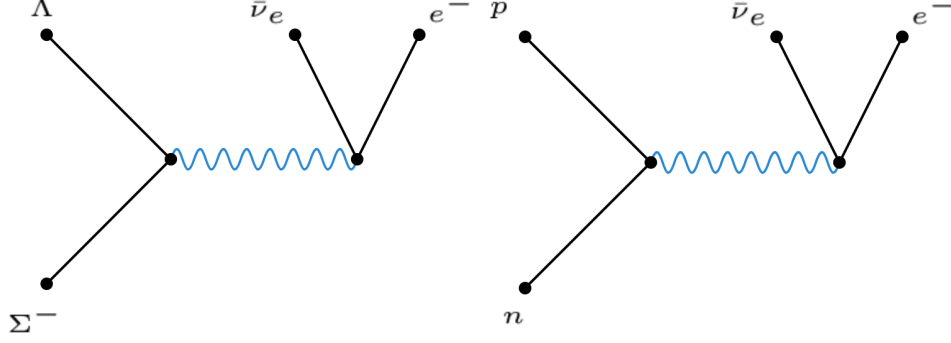


Figure 4.5: Alternative Σ direct Urca with Λ hyperon (left), and nucleon direct Urca (right)

These decays have complimentary processes (in the same order as their respective decay processes) which paired together are considered as additional direct Urca processes [177] :

$$\Sigma^+ + \ell \rightarrow \Xi^0 + \nu_\ell , \quad (4.73)$$

$$\Lambda^0 + \ell \rightarrow \Xi^0 + \nu_\ell , \quad (4.74)$$

$$\Xi^0 + \ell \rightarrow \Xi^- + \nu_\ell , \quad (4.75)$$

$$\Sigma^0 + \ell \rightarrow \Xi^- + \nu_\ell . \quad (4.76)$$

The most plausible decays given the mass range of neutron stars containing hyperons are $\Lambda^0 \rightarrow p + \ell + \bar{\nu}_\ell$, $\Sigma^- \rightarrow n + \ell + \bar{\nu}_\ell$, $\Sigma^- \rightarrow \Lambda^0 + \ell + \bar{\nu}_\ell$, $\Xi^- \rightarrow \Lambda^0 + \ell + \bar{\nu}_\ell$, other processes might require densities which exceed those within the maximum mass of a strange matter star [160]. Due to hyperons softening the EOS for neutron stars, they have lower maximum mass limits than neutron stars which do not contain hyperons. Additionally, the process $\Sigma^- \rightarrow n + \ell + \bar{\nu}_\ell$ is suppressed by neutron pairing.

Strong interactions which exchange mesons allow hyperon conversion are also allowed (see Ref. [178]):

$$\Lambda^0 + p \rightarrow \Sigma^+ + n , \quad (4.77)$$

$$\Lambda^0 + p \rightarrow \Sigma^0 + p . \quad (4.78)$$

If hyperon fractions rise high enough, recent experimental data of hypernuclei reveals that 1S_0 superfluidity of Λ^0 hyperons could be possible at the highest densities where the fraction of Λ^0 exceeds 15 – 20%. Such superfluidity would suppress the before mentioned hyperon direct Urca by a factor proportional to the pairing gap Δ as $e^{-\Delta/k_B T}$. The direct Urca process is suppressed by the formation of Cooper pairs as is the modified Urca processes' rate itself. The overall rate of cooling of modified Urca with PBF is still greater than modified Urca cooling alone. The quenching of these processes ($e^{-2\Delta/k_B T}$) is due to the energy necessary to break nucleon pairs (2Δ), as free nucleons are required to participate both processes. The pairing of Lambda hyperons, results in a PBF process described by the relation:

$$\begin{aligned}\Lambda^0 + \Lambda^0 &\rightarrow [\Lambda^0 \Lambda^0] + \nu + \bar{\nu} , \\ [\Lambda^0 \Lambda^0] + Q_{\text{heat}} &\rightarrow \Lambda^0 + \Lambda^0 .\end{aligned}$$

Once we assume the existence of hyperon superfluidity, there is the possibility of expanding our previous research in order to calculate the T_c , k_{peak} , Δk and an effective mass $m_{\Lambda^0}^*$ in order to constrain Λ^0 1S_0 pairing. It is still unknown over what densities and critical temperatures superfluid pairing of hyperons occurs [57].

The overall beta equilibrium conditions become increasingly complex with the introduction of hyperons. In the innermost core, one approximation would be to consider only the most abundant hadrons; neutrons, protons and Λ^0 hyperons. For an energy density $\epsilon = \epsilon_p + \epsilon_n + \epsilon_{\Lambda^0}$ we can derive the chemical potentials (see Eq. (2.23)). The relevant number densities here are that of protons, neutrons, Λ^0 hyperons [174]. The pressure of the system could then be expressed, in terms of overall density $n = n_n + n_p + n_{\Lambda^0}$ as seen in Eq. (2.21). The presence of Λ^0 hyperons facilitates a greater abundance of neutrons by increasing the neutron drip asymmetry [174].

If we can demonstrate that the introduction of processes permitted by hyperons provides a better explanation of our data and cooling models, we could be one step closer to providing an affirmative answer to the question of whether neutron stars contain strange matter. It is also possible that the tuning of neutron star mass parameter and the EOS parameters alone might provide the most satisfactory theoretical account of cooling, thus obviating the need

for studying hyperon cooling mechanisms. It is also possible that more enhanced cooling processes such as pion condensates and deconfined quarks, or delayed heating processes such as ohmic dissipation are necessary to explain the cooling of neutron stars. We will concentrate for now on the introduction of hyperons, the direct Urca process, as well as EOS and mass variance and leave these other possibilities to future work.

The Lagrangian to be used in this work is (as in Ref. [150]):

$$\begin{aligned} \mathcal{L} = & \sum_B \bar{\psi}_B \left(i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho B} \gamma_\mu \vec{\tau} \cdot \vec{\rho} \right) \psi_B \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu \\ & - \frac{1}{3} b m_n (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4 . \end{aligned} \quad (4.79)$$

This Lagrangian is very similar to our EOS Lagrangian mentioned previously, as it contains terms analogous to those of the previous Lagrangian in Eq. (4.42). The coefficients can be equated as follows: $c = \lambda/6$, $b = \kappa/2m_n$ (where m_n is 939 MeV, the approximate mass of both nucleons). As in previous work, this Lagrangian is solved for the equations of motion of each individual particle, which in turn yields the overall pressure and energy density contribution of each particle species. Summing over all particle species provides the necessary central pressure and energy density inputs necessary to solve the TOV equations. From the TOV solutions, we can determine the structural properties of a neutron star produced by our relativistic mean field EOS.

The neutron star maximum mass limit with the inclusion of hyperons is calculated to be $1.5 - 1.8 M_\odot$ [69]. A lack of any observed neutron stars above this limit would have been strong evidence for the existence of hyperons in neutron stars. Neutron star masses have been accurately measured above this limit however, adding further difficulty to understanding the hyperon puzzle. The study of neutron star cooling, especially the identification of any the above mentioned hyperon driven enhanced cooling processes, plays a key role at present in determining the existence of hyperons in neutron stars.

4.10 Preliminary Results

Our preliminary results of the work just described indicate that the direct Urca process would only be active for very high neutron star masses, around $2.0 M_{\odot}$. Evidence for this conclusion is provided by observations of cooling curves as well as the histogram results for the masses of neutron stars in our data set Fig. 4.6, Fig. 4.7, Fig. 4.8, Fig. 4.9. A calculation of proton fractions over a range of densities within our model reveals that the necessary conditions for direct Urca will only be satisfied within these high mass stars Fig. 4.10. This is a departure from the findings of earlier studies, which concluded that the direct Urca process would operate over a range of higher mass neutron stars $1.4 - 2.0 M_{\odot}$. Some experts have demonstrated it is possible that only very high mass neutron stars could support the high proton fractions necessary for the direct Urca process [179]. There is even evidence that current age and temperature measurements support such claims [1].

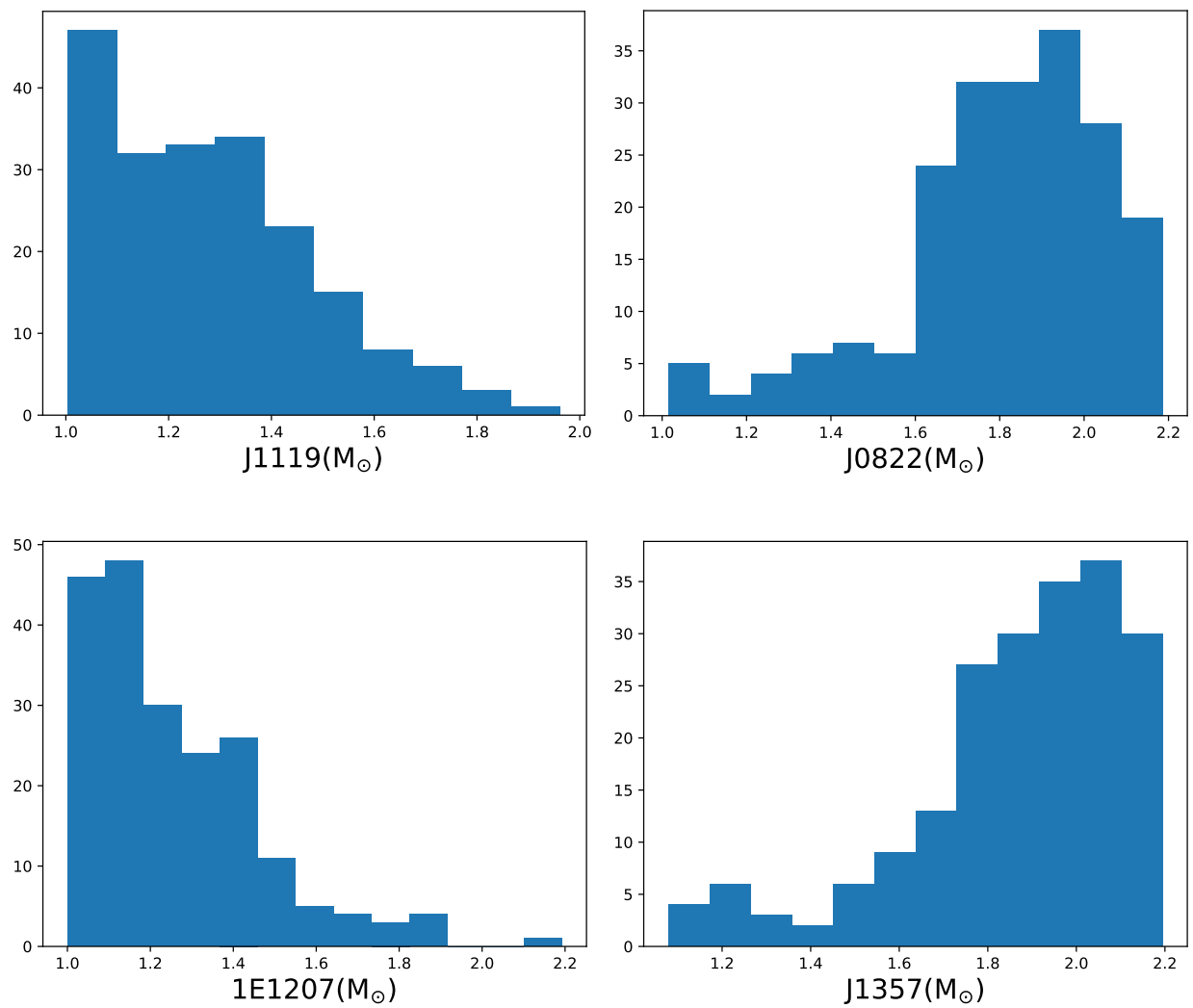


Figure 4.6: Histograms of Masses Part 1

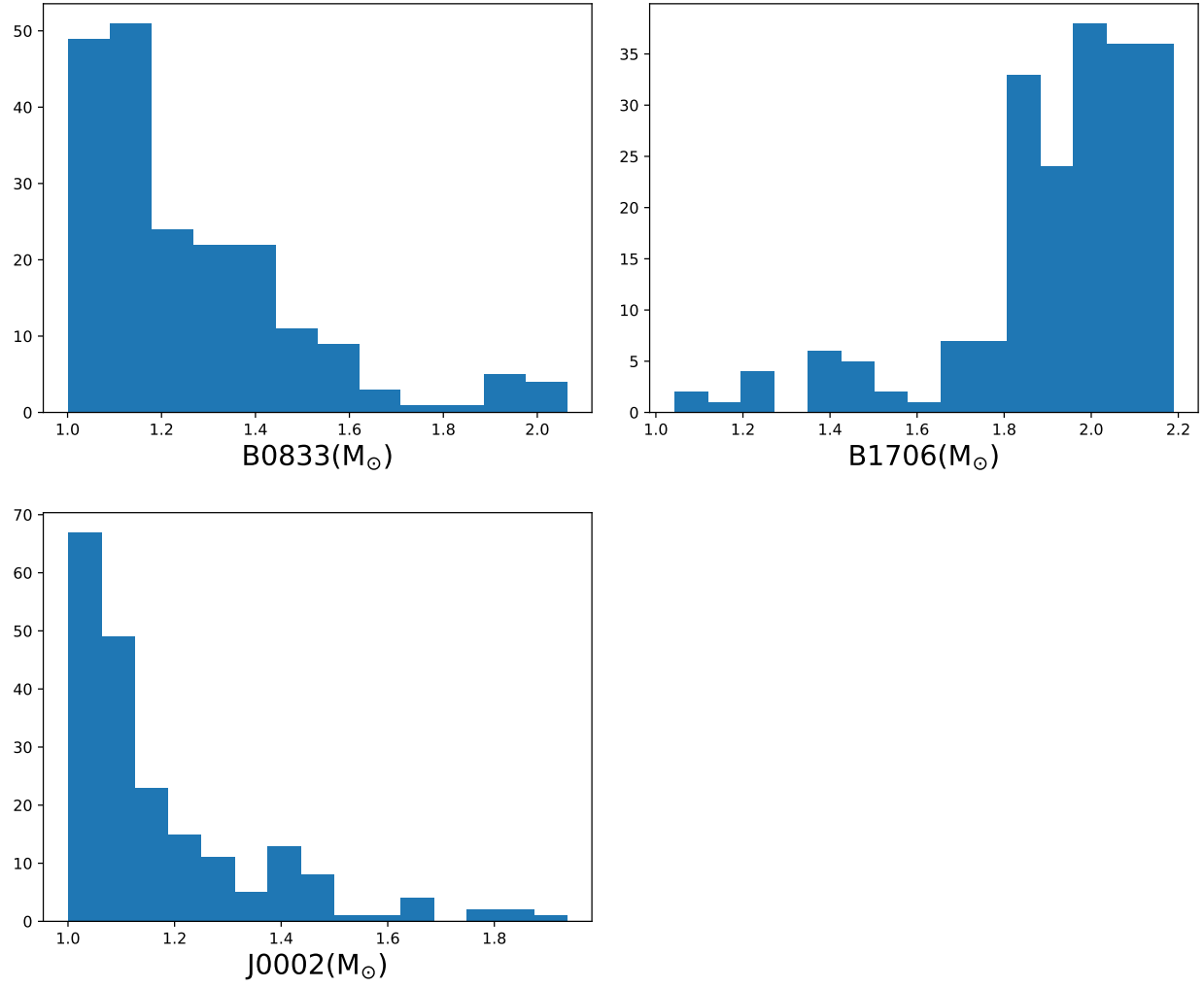


Figure 4.7: Histograms of Masses Part 2

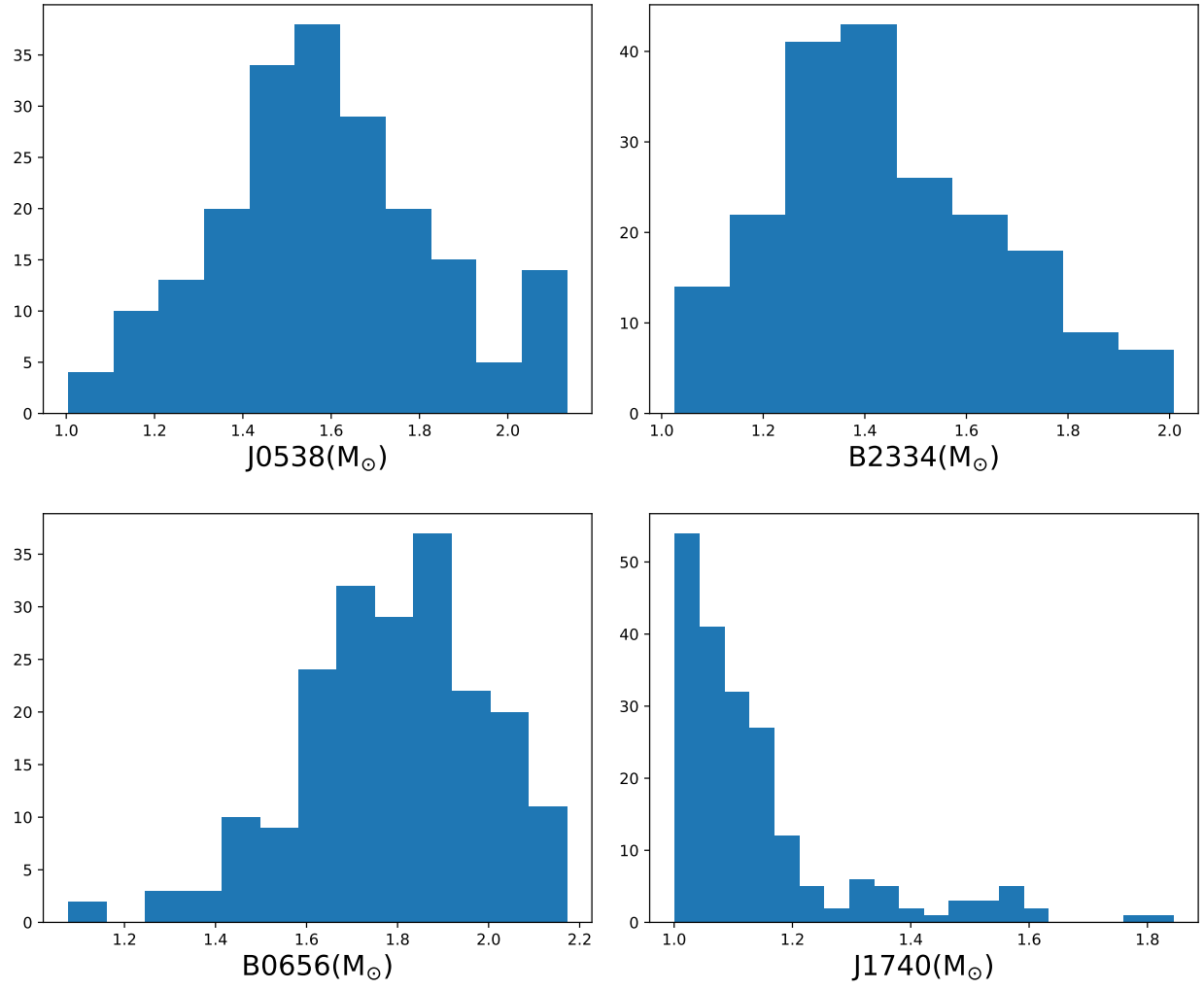


Figure 4.8: Histograms of Masses Part 3

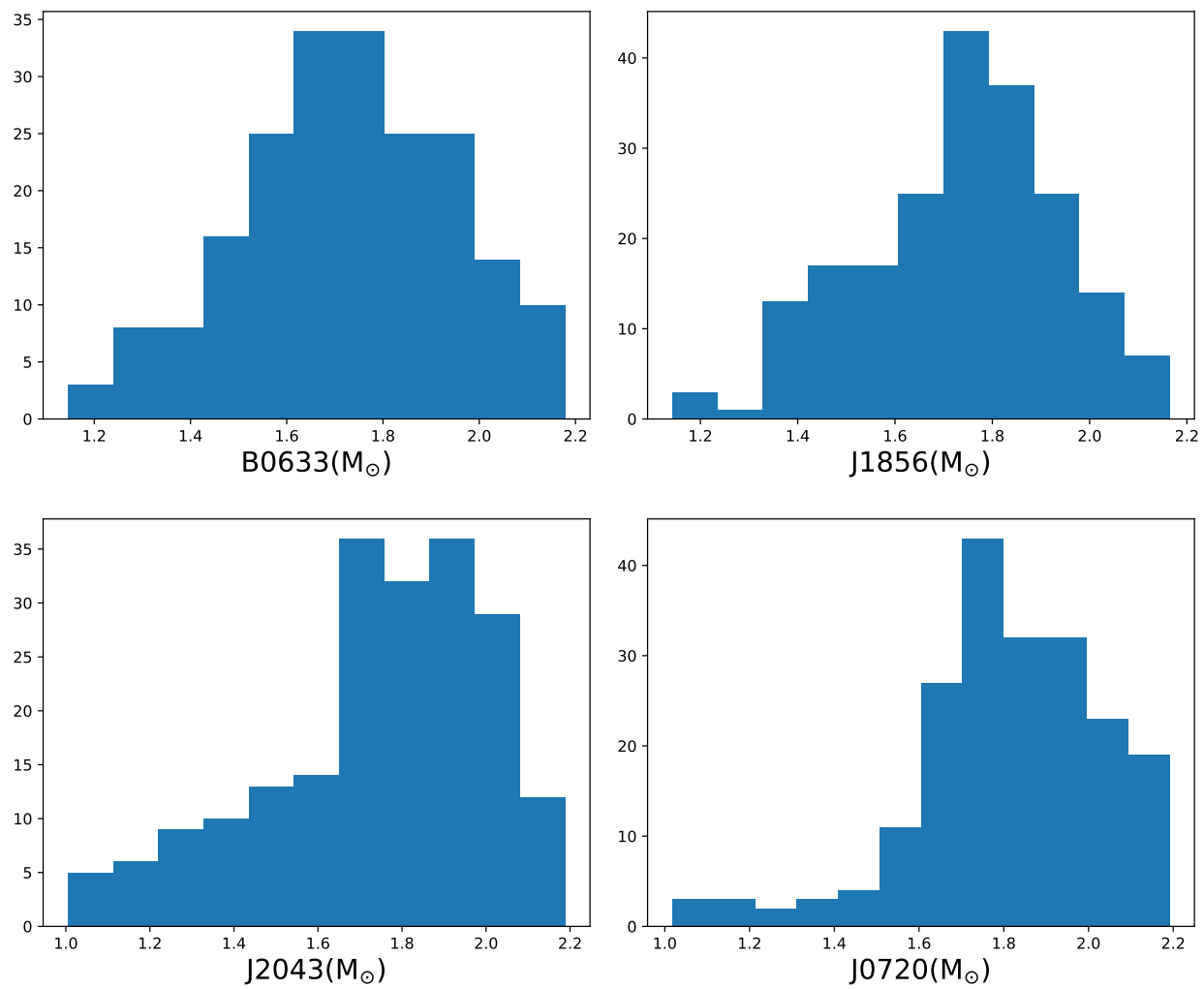


Figure 4.9: Histograms of Masses Part 4

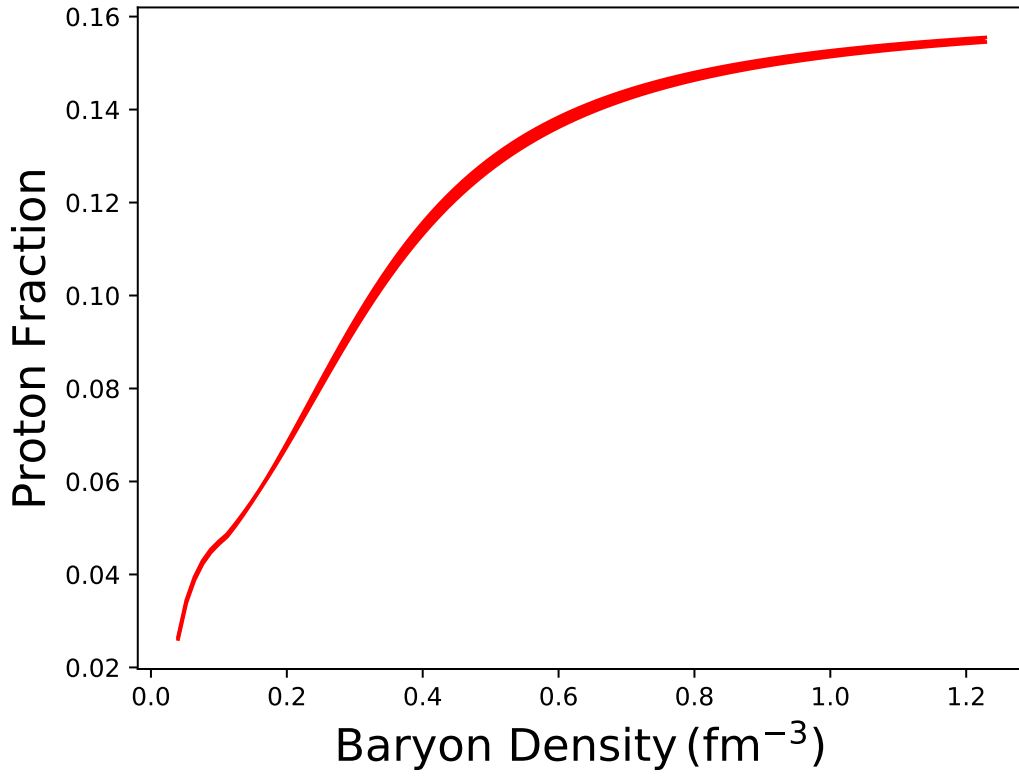


Figure 4.10: Preliminary result demonstrating how the local proton fraction varies with baryon density. This result is consistent with previous calculations. The primary importance of this calculation is determining for which neutron star masses, and their corresponding central densities, the conditions for the direct Urca process are satisfied (see Eq. (4.9)).

In addition to this finding, we observe that the drop in temperature over time associated with the activation of the direct Urca process is much more gradual than the steep drop in temperature found in previous studies of the direct Urca. This could be attributed to a suppression of the direct Urca process due to the presence of nucleonic superfluids. For the direct Urca process, this superfluid suppression factor is $\sim e^{-\Delta/T}$. Looking at the mass histograms below, we realize it is possible that PSR – J1357 and B1706 – 44, given their high calculated masses and low temperatures for their young ages, are undergoing suppressed direct Urca cooling (see Fig. 4.11 and Fig. 4.12).

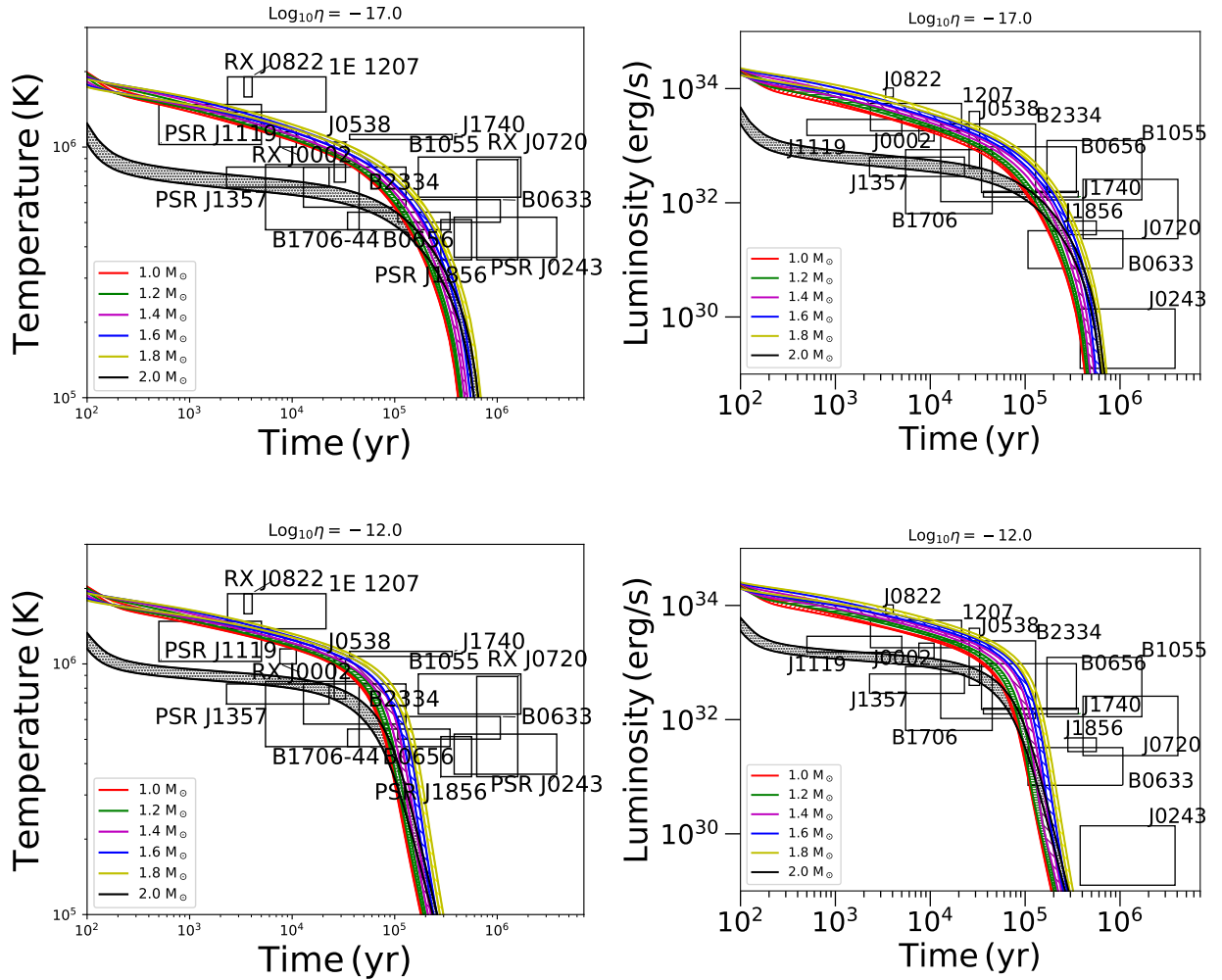


Figure 4.11: Evidence of Direct Urca Part 1

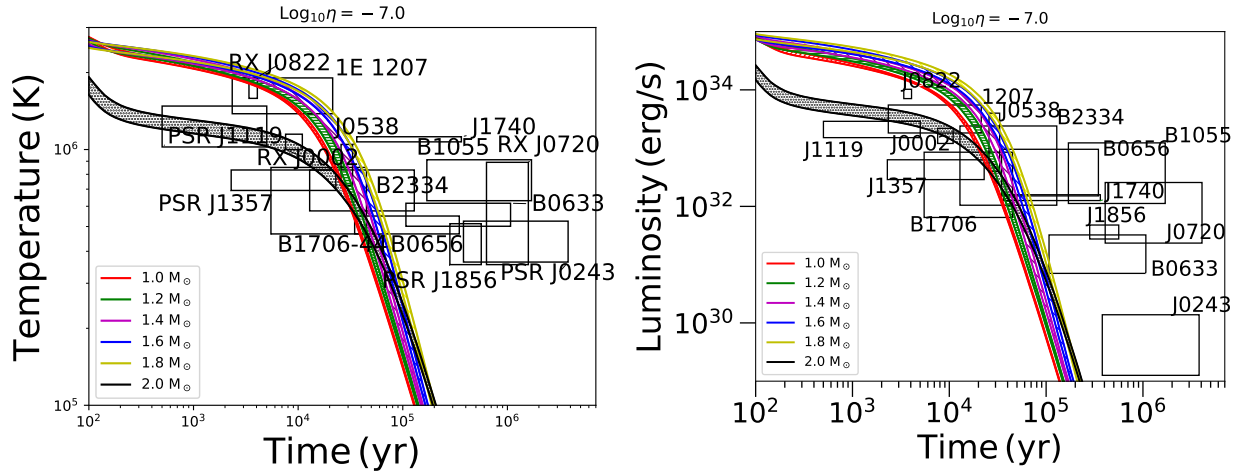


Figure 4.12: Evidence of Direct Urca Part 2: Preliminary results showing temperature (left panels) and luminosity (right panels) cooling curves for various neutron star masses, the heaviest mass stars ($2.0 M_{\odot}$ denoted by the black curves) undergo direct Urca fast cooling. With the exception of stars massive enough to achieve the necessary conditions for the direct Urca process, the variation of neutron star mass has only a secondary effect on neutron star cooling. This can be appreciated in these charts as, for neutron stars less than $2.0 M_{\odot}$, it is clear that there is little difference in the cooling curve profiles arising from bulk cooling alone.

The histograms for the light element envelope compositions (see Fig. 4.13 and Fig. 4.14) collectively reveal that most stars with scarcely any light element atmospheres likely have large masses. In some cases, lower calculated amounts of light elements in young neutron stars and higher amounts of light elements in old stars seems to contradict the assumption that young stars have light elements atmospheres while older stars generally have heavy element atmospheres. It could be that a heterogenous mixture of light elements with heavy elements (as explored in Ref. [66]) is better a better description of young neutron stars atmospheres than the hydrogen model we utilized for some stars.

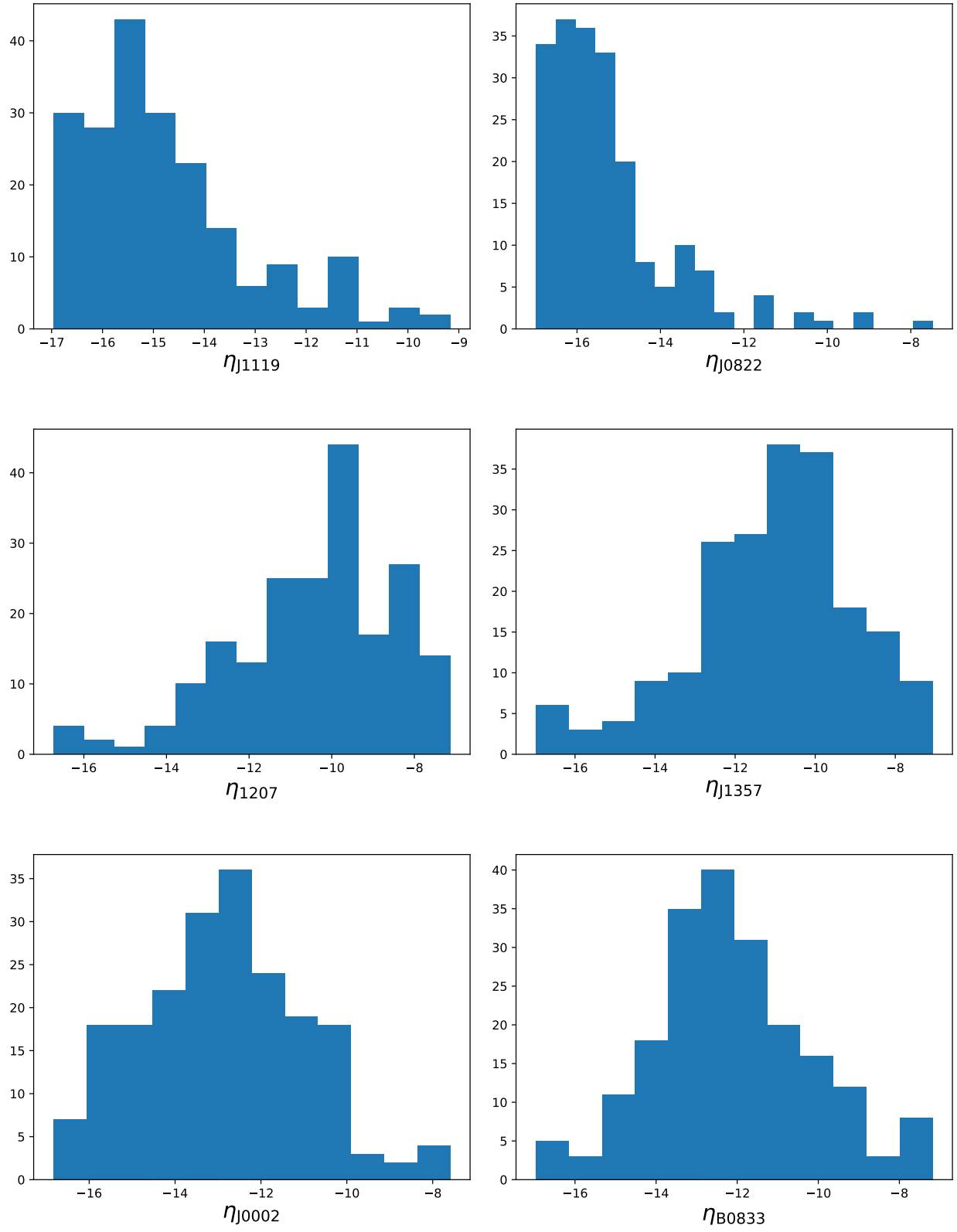


Figure 4.13: Histogram of light envelope compositions

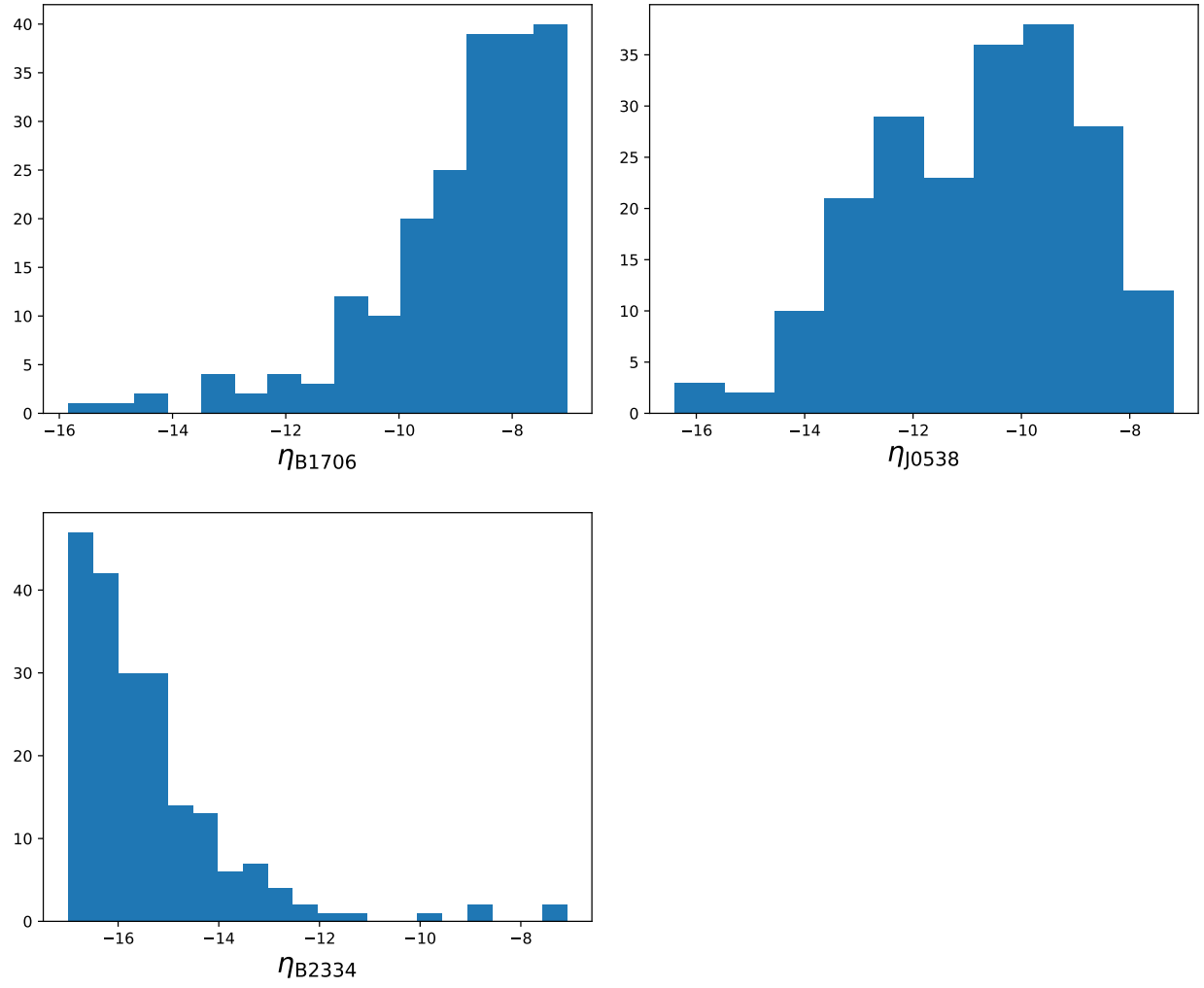


Figure 4.14: Histograms of light envelope composition, continued

An additional result of this work was to demonstrate that some older neutron stars were hotter than predicted given their old ages. As in previous studies, we find that our cooling models cannot account for very old stars with high temperatures. We hypothesized in a previous paper (Ref. [33]) that this is due to the lack of any late cooling mechanisms in our cooling model, such as accretion or heating caused by strong magnetic fields. Given that our new results included accreting stars, a possible explanation of this is ohmic heating caused by the dissipation of strong magnetic fields (greater than 10^{12} G). This possibility was not explored in this research but could prove to be a useful addition to this analysis in future work.

The superfluid critical temperature remains relatively unchanged in the case of the proton singlet temperature as shown in the histograms in Fig. 4.15. However, it is raised significantly in the case of the neutron triplet critical temperature. A higher critical temperature corresponds to greater neutrino emission from the PBF process. There is some ambiguity in interpreting this result, as it could be that the inclusion of accreting neutron stars (which have higher proton fractions) or the variation of masses in isolated neutron stars (higher mass stars also have higher proton fractions) has yielded this departure from our previous result. The calculated peak values for Fermi momenta, and the range of densities associated with each superfluid gap are comparable to the values obtained in the previous analysis.

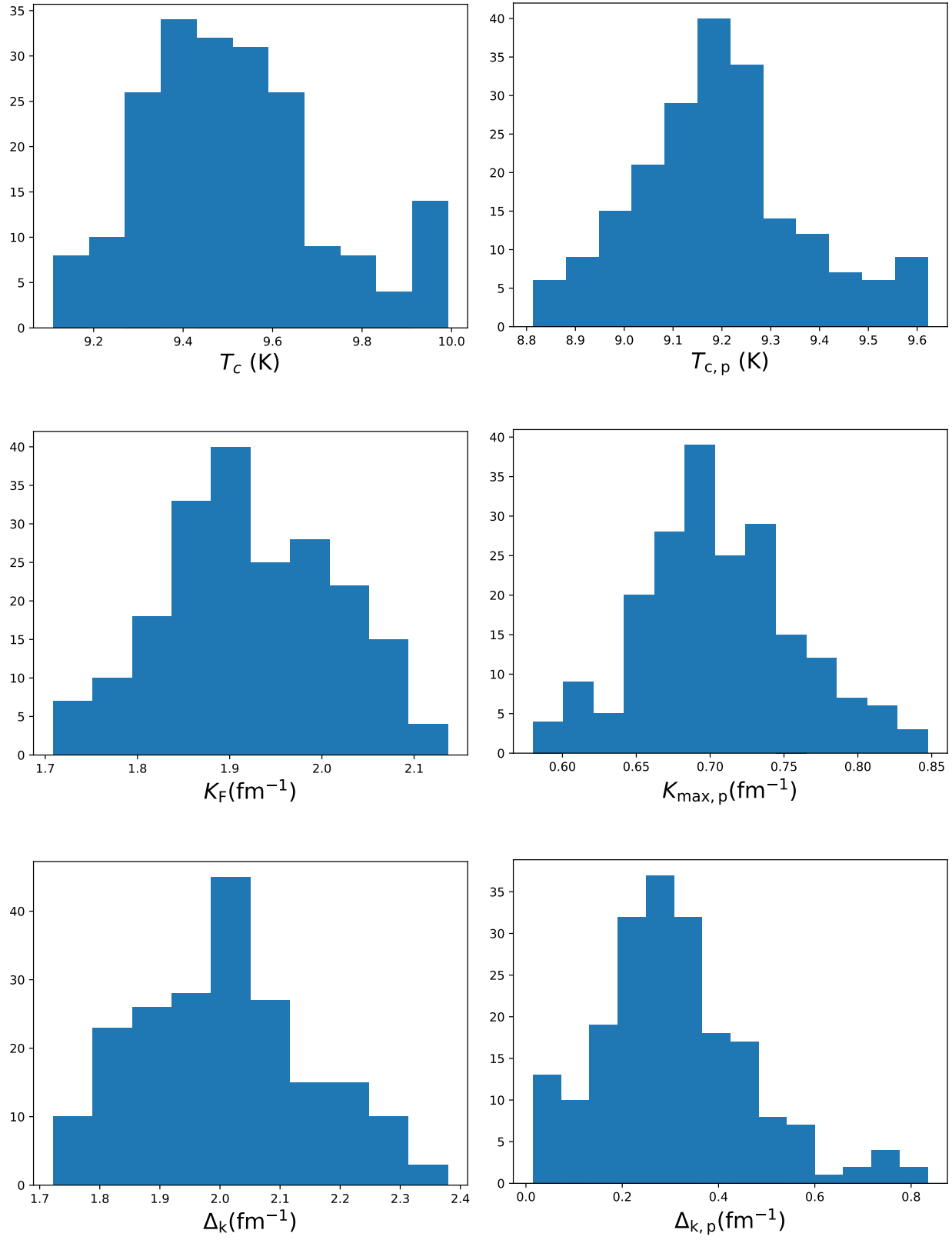


Figure 4.15: Histograms of Cooling Parameters

Our model yields a mass-radius curve, within a range of uncertainty (see Fig. 4.16). The values of this mass radius curve are consistent within current limits provided by astronomical observation and predictions provided by nuclear theory. Of course, if a neutron star with a mass higher than the maximum mass limit of any given EOS is discovered, the predictions of that equation of state are invalid. As for precise observations of neutron stars higher than $2.0 M_{\odot}$, no such observation has been made as of the writing of this work.

4.11 Future Work

We have chosen in this analysis to neglect the structural effects of rotation. We can feel somewhat justified in doing so, as in many cases this will lead a negligible loss of accuracy. The higher the angular velocity, the greater the effect on neutron star structure. For the fastest rotating neutron star observed, PSR J1748 – 2446 possessing a angular frequency of $f = \frac{\Omega}{2\pi}$ it can be shown that the maximum mass will increase as

$$M_{\text{max,rot}} \simeq M_{\text{max,static}} \left[1 + 0.2 \left(\frac{\Omega}{\Omega_0} \right) \right] \quad (4.80)$$

where the angular velocity (Ω_0) arises from neutron star stability limits:

$$\Omega_0 = \sqrt{\frac{M_{\text{max,static}}}{R_{\text{max,static}}^3}} \quad (4.81)$$

Even for the special case of PSR J1748 – 2446, rigid rotation will only increase the maximum mass by about 3% [13].

We also chose to neglect possible effects arising from the presence of exceptionally strong magnetic fields $10^{13} - 10^{14}$ G. If a star possesses a magnetic field of this strength it is possible that additional corrections for hotspots generated by the magnetic field would be necessary when obtaining an implied surface temperature for these stars.

We have also neglected to include heating processes from axions or any other dark matter related cooling process, an alternative hypothesis investigated in Ref. [180]. We also neglected to include the possibility of deconfined quarks, either existing in quark stars or hybrid stars (see Ref. [181], for a review see Ref. [182]).

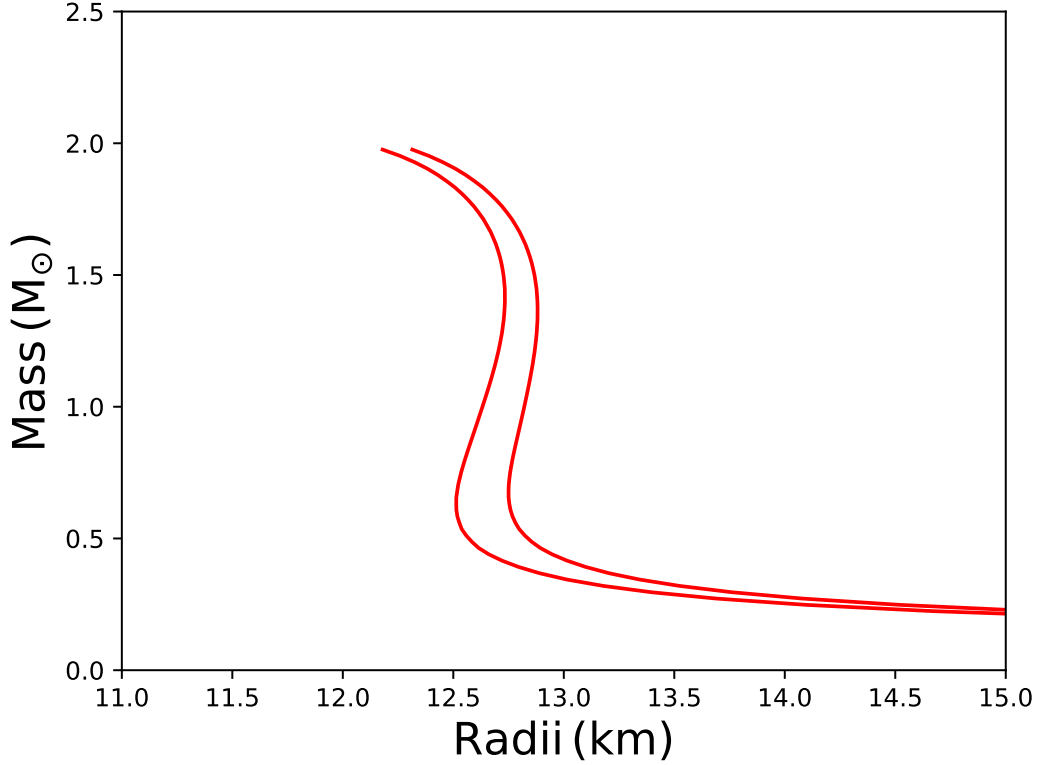


Figure 4.16: Predicted Mass-Radius Curve: The mass-radius curve is calculated using a version of the TOV equations which utilizes the central pressures our EOS predicts for a range of neutron star masses. This curve is consistent with previous mass-radius curves in its distinct shape. Starting on the right, the curve extends off the chart toward a separate mass-radius curve which describes white dwarf stars, which have masses of $0.5 \sim M_{\odot}$ and radii of $\sim 10,000$ km [5]. On the left side of the mass-radius curve assumes a concave shape, reflecting an increase in radius with mass up until a certain point above approximately $1.4 M_{\odot}$, then the radius starts to decrease with increasing mass (reflecting an increase in neutron star density). This continues to the top of the curve, which then terminates at a point of maximum mass, above which the star collapses into a black hole. The width between the two curves reflects the uncertainty of our mass-radius curve. The predicted radius for a neutron star of $1.4 M_{\odot}$ here is ~ 12.75 km, which lies on the higher end of the currently accepted range of radii for a neutron star of this mass [6].

4.12 Outlook for the Field

August 17, 2017 marked the beginning of a new era of neutron star observations, as for the first time ever a neutron star merger was witnessed via the detection of gravitational waves [183]. The accompanying electromagnetic signal gave evidence that neutron stars participate in the r-process, which is responsible for the generation of many of the heavy elements in our galaxy. The relative abundance of elements here on Earth, and more generally our solar system, can be directly attributed to multiple cycles of stellar birth and explosion. The observed merger of these neutron stars gives us one more clue as to the origins of the elemental building blocks of our galaxy, even those necessary for life itself.

With regard to the EOS, the observation of this event (GW170817) has given strong early indications that detection of gravitational waves emitted from binary mergers will provide precise constraints on the EOS. The LIGO and Virgo collaborations have produced inferred masses and radii at the 90% confidence level, with radii of $R_1 = 10^{+2.0}_{-1.7}$ km and $R_2 = 10.7^{+2.1}_{-1.5}$ km (see Ref. [184]) and total mass of $2.73^{+0.04}_{-0.03} M_\odot$ (see Ref. [185]). Additionally, Ref. [185] provides a measurement of the pressure corresponding to twice nuclear saturation density. Further detection and analysis of gravitational waves produced by inspiralling and colliding neutron stars is expected to provide an additional means of obtaining a more precise EOS for dense matter.

Earlier in the same year, June 3, 2017, NASA launched the Neutron Interior Composition Explorer (NICER) and subsequently installed the detection system aboard the International Space Station (ISS). The instrument is designed to observe soft x-rays emitted from the surfaces of neutron stars. The goal of the NICER mission is to provide precise observations of neutron star spectra and radii. It has already obtained x-ray observations of several neutron stars including the accreting neutron star in Serpens X-1 (see Ref [186]), and is scheduled to observe many more (mostly isolated) neutron stars in the near future. The high precision radius measurements NICER obtains could further constrain the dense matter EOS.

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Appendix

M Bayesian Statistics

Bayesian Statistics is named after Reverend Thomas Bayes, who devised a thought experiment wherein a tennis ball is placed on a table which the experimenter cannot see. Subsequent tennis balls are then thrown onto the table and an assistant, without revealing the exact location of the initial ball, informs the thrower whether the ball they threw landed to the right or left, as well as above or below the original ball. The idea is that after throwing many balls randomly and noting their relative position to the initial ball, one could ultimately reveal the absolute location of the initial ball. By appreciating this thought experiment, we may see how increasingly accurate predictions can be made by using new data to update our assumptions [187].

Bayes' theorem is derived from conditional probability: Let event A be "I am full", and let event B be "I ate a lot of food". The joint probability of two events is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (82)$$

The probability that you are full as a result of eating a lot of food ($P(A|B)$ here) is the probability that you are full and ate a lot of food ($P(A \cap B)$) divided by the probability that you ate a lot of food ($P(B)$).

Here, the conditional probability for two completely dependent events $P(A \cap B)$ is equivalent to $P(B|A) \cdot P(A)$ as well as $P(A|B) \cdot P(B)$. This would describe a situation where the only way to get full is by eating a lot of food, and the only result of eating a lot of food is to become full. This along with the realization that $P(A \cap B) = P(B \cap A)$ allows one to use the equations:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ where } P(B) \neq 0 \quad (83)$$

and

$$P(B|A) = \frac{P(B \cap A)}{P(A)}, \text{ where } P(A) \neq 0 \quad (84)$$

yields:

$$P(A \cap B) = P(B \cap A) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A) \quad (85)$$

which in turn yields the expression:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}, \text{ where } P(B) \neq 0 \quad (86)$$

Of course the denominator cannot be zero, one cannot calculate the conditional probability of something which never happened!

It can be shown that this result holds when the events are independent of one another. Perhaps you are full because you ate a bunch of pinecones, or perhaps you ate a lot of food but you are not full. In that case, one has:

$$P(A|B) = \frac{P(A) \cdot P(B)}{P(B)} = P(A) \quad (87)$$

and

$$P(B|A) = \frac{P(B) \cdot P(A)}{P(A)} = P(B) \quad (88)$$

our previous expression Eq. (86) may then be expressed as:

$$P(A|B) = \frac{P(B|A)}{P(B)} P(A) \quad (89)$$

This is Bayes' theorem.

Bayes' theorem in our work is expressed as:

$$P(\mathcal{M}|\mathcal{D}) = \frac{P(\mathcal{D}|\mathcal{M}) P(\mathcal{M})}{P(\mathcal{D})} \quad (90)$$

that is, the probability of reproducing known data $\mathcal{D}$ given a hypothetical model $\mathcal{M}$, is expressed here as $P(\mathcal{D}|\mathcal{M})$. The probability we calculate is the conditional probability of the model given that the data is true:

$$P(M|D) \propto P(D|M)P(M) \quad (91)$$

$P(M)$ is our prior distribution for the our model M , described by our cooling parameters, and $P(D|M)$ is the probability calculated from the neutron star cooling data. $P(M|D)$ itself

is the likelihood we calculate. The non-uniqueness of this fitting procedure is evident in the indeterminate nature of the prior distribution of the model $P(M)$, which we must supply in order to proceed.

Vita

Spencer T. Beloin is the second of Fred and Brenda Beloin's four sons. His older brother Shawn is an attorney, his younger brother Stephen is a CPA, and his youngest brother Samuel has recently started medical school. He was born and raised in Marietta, Ga. Recently, he earned a PhD in nuclear physics, after working as a doctoral student and research assistant in the Physics and Astronomy Department at the University of Tennessee, Knoxville. Spencer previously attended the University of Georgia, where he participated in experimental research on ultrafast optics and earned a bachelor's in physics. Since arriving at Tennessee, he has worked as a teaching assistant and physics tutor for undergraduates. In the Spring of 2015, Spencer joined Professor Andrew Steiner's theoretical research group which focuses on nuclear astrophysics. As a member of this group, he has conducted analysis of neutron star cooling and has published findings in the peer reviewed journal *Physical Review C*.