To the Graduate Council:

I am submitting herewith a thesis written by Gwendolyn H. Walton entitled "Upper Semi-Continuous Decompositions on Strongly Developable Spaces." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mathematics.

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UPPER SEMI-CONTINUOUS DECOMPOSITIONS ON STRONGLY DEVELOPABLE SPACES

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

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March 1977

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ABSTRACT

The purpose of this paper is to prove that there exist some conditions under which a strongly developable, locally compact, T_2 space is homeomorphic to its decomposition space. There has been much work done in this area for the case when the space is metric, but not much research has been done for non-metric spaces. This paper presents proofs of two theorems that give sufficient conditions for strongly developable spaces to be homeomorphic to their decomposition spaces.

Theorem 1 uses a special "strongly shrinkable" upper semi-continuous decomposition G in a proof that if X is a strongly developable, locally compact, T_2 space and H_G^* is the countable union of compact sets, then X is homeomorphic to X/G.

Theorem 2 provides an alternative to the "strongly shrinkable" condition by proving the existence of an equivalent condition on G for use when H_G is countable.

There are obvious corollaries to both theorems. Corollaries to Theorem 1 state the theorem holds if H_G is countable or if H_G^* is compact. The corollary to Theorem 2 is a restatement of Theorem 1 for the case where H_G is countable using the equivalent condition for "strongly shrinkable."

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CHAPTER I

INTRODUCTION

X is a locally compact, T_2 space. G is an upper semi-continuous decomposition of X. Under what conditions on G and for what type of spaces X is the decomposition space X/G homeomorphic to X?

This question has been answered for many cases when X is a metric space. R. L. Moore [4] pioneered much of the research into this question for metric spaces with his proof in 1924 that if G is an u.s.c. decomposition of E^2 into compact continua such that no element of G separates E^2, then E^2/G is homeomorphic to E^2. In 1932 Moore [5] also worked with upper semi-continuous decompositions of some non metric spaces. In 1935 G. T. Whyburn [8] gave conditions on an u.s.c. decomposition of E^3 to insure that E^3/G was homeomorphic to E^3. In 1955 Smythe [7] published a theorem stating conditions on G under which X, a complete, locally compact, separable metric space, is homeomorphic to its decomposition space X/G, where G is an u.s.c. decomposition such that every element of G is homeomorphic to the complement of a point. In 1956 Bing [2] gave several conditions on G for $X = E^3$ to be homeomorphic to X/G. Armentrout [1] outlines this and much of the other research for the case when X is E^3. In 1961 Louis McAuley [3] proved that there exists an u.s.c. decomposition G of a complete metric space X such that H_G is countable and a G_δ collection with special "shrinking" conditions on each element of H_G, and X/G is homeomorphic

to X. In 1966 Price [6] gave a proof for $X = E^n$, where n is any positive integer.

However, there has not been much research into the question when X is not a metric space. This paper will use the following definitions to present two theorems that give conditions on G and a strongly developable space X so that X/G is homeomorphic to X.

DEFINITION 1. A space S is called "strongly developable" if there exists a sequence $\{G_i\}_{i=1}^{\infty}$ such that

(1) for every n, G_n is a collection of open sets covering S,

(2) for every n, G_{n+1} is a subcollection of G_n,

(3) for every $x \in S$ and neighborhood N of x, there exists $i > 0$ such that $St(x,G_i) \subset N$,

(4) if H and K are disjoint closed sets, and H is compact, there exists a positive integer m such that no element of G_m intersects both H and K, and

(5) if $\{M_i\}_{i=1}^{\infty}$ is a sequence of closed sets such that for every n, $M_n \supset M_{n+1}$ and for every n, there exists $g_n \in G_n$ such that $M_n \subset \overline{g}_n$, then $\bigcap_{i=1}^{\infty} M_i \neq \emptyset$.

DEFINITION 2. G, an upper semi-continuous decomposition of a space X, is called "strongly shrinkable" if for every G saturated open cover $\mathcal{U}$ and for every open cover $\mathcal{V}$ and for any open set N containing H_G^*, there exists a homeomorphism h of X onto itself such that $h|(X-N) = 1$, and for every $g \in H_G$ there exists $V \in \mathcal{V}$ such that $h(g) \subset V$ and there exists $U \in \mathcal{U}$ such that $h(g) \bigcup g \subset U$.

Theorem 1 of this paper proves that if X is a strongly developable, locally compact, T_2 space and G is a strongly shrinkable upper semi-continuous decomposition of X such that H_G^* is a countable union of compact sets, then X is homeomorphic to X/G. Obvious corollaries state that the theorem also holds if H_G is countable, or if H_G^* is compact. This theorem would also hold if X were metric rather than strongly developable.

Theorem 2 of this paper shows an equivalent condition (*) for G is strongly shrinkable for use in the case when H_G is countable. (*) states that for every $g_0 \in H_G$, any neighborhood N of g_0, and any open cover $\mathcal{V}$ of X, there exists a homeomorphism h of X onto X such that $h|(X-N) = 1$, $h(g_0) \in$ some $V \in \mathcal{V}$, and for every $g \in G$, either $h(g) \subset$ some $V \in \mathcal{V}$ or $h(g) \subset St(g,\mathcal{V})$. As stated by the corollary to this theorem, the theorem gives an alternate method of proving X/G homeomorphic to X when H_G is countable.

CHAPTER II

NOTATION AND TERMINOLOGY

$(X,\mathcal{F})$ is a topological space. G is a decomposition of X means that C is a collection of compact subsets of X such that each point of X belongs to one and only one set of G. G is an upper semi-continuous decomposition of X means that G is a decomposition of X such that if g is any element of G and U is any open set in X containing g, then there exists an open set V in X such that $g \subset V \subset U$, and V is a union of elements of G. (We say that V is G-saturated.)

B = {all subsets W of G such that the union of all the elements of W is an open subset of X} is a base for a topology S for the set G, and (G,S) is the decomposition space associated with G. (G,S) will hereafter be denoted as X/G, and the projection map from X onto X/G will be denoted by π. Because G is upper semi-continuous, π is a closed continuous function.

H_G denotes the set of non-degenerate elements of G, and H_G^* denotes the union of elements of H_G. If U is any open subset of X, the set $U^* = \bigcup \{\pi^{-1}(y) \mid \pi^{-1}(y) \subset U\}$ is open in X.

Let $\mathcal{U} = \{U_\alpha\}$, any collection of sets and A = any set. Then $St(A,\mathcal{U}) = \{U_\alpha \mid U_\alpha \in \mathcal{U}, \text{ and } U_\alpha \cap A \neq \emptyset\}$. $St^2(A,\mathcal{U}) = St(St(A,\mathcal{U}),\mathcal{U})$ $= \{U_\alpha \mid U_\alpha \in \mathcal{U}, \text{ and } U_\alpha \cap St(A,\mathcal{U}) \neq \emptyset\}$. Therefore $St^k(A,\mathcal{U}) = St(St(\ldots(St(A,\mathcal{U}),\mathcal{U}, \ldots),\mathcal{U}),\mathcal{U})$, for every integer k.

CHAPTER III

A DECOMPOSITION THEOREM

THEOREM 1. If X is a strongly developable, locally compact, T_2 space and G is a strongly shrinkable upper semi-continuous decomposition of X such that $H_G^* = \bigcup_{i=1}^{\infty} C_i$, where each C_i is compact. Then X is homeomorphic to X/G.

In 1932 R. L. Moore [5] discussed upper semi-continuous decompositions on strongly developable spaces. He proved that if the space X is strongly developable, then the decomposition space X/G is also strongly developable. This fact and the following lemmas are used to prove Theorem 1.

LEMMA 1.1. Let X be the space described in the theorem, and let h be a homeomorphism of X onto itself. Then for every h(G)-saturated open cover $\mathcal{U}$ of h(X) and every open cover $\mathcal{V}$ of h(X), and for any open set N containing $h(H_G^*)$, there exists a homeomorphism f of h(X) onto itself such that $f|[h(X)-N] = 1$ and for every $h(g) \in h(H_G)$, there exists $V \in \mathcal{V}$ such that $f[h(g)] \subset V$, and there exists $U \in \mathcal{U}$ such that $f[h(g)] \cup h(g) \subset U$.

PROOF. Let $\mathcal{U}' = \{h^{-1}(U)\}_{U \in \mathcal{U}}$ be a G-saturated open cover of X, and $\mathcal{V}' = \{h^{-1}(V)\}_{V \in \mathcal{V}}$ be an open cover of X. $N' = h^{-1}(N)$ is an open set covering H_G^*.

Because, by hypothesis, G is strongly shrinkable, there exists a homeomorphism p of X onto itself such that $p|(X-N') = 1$, and for every $g \in H_G$ there exists $V' \in \mathcal{V}'$ such that $p(g) \subset V'$ and there exists $U' \in \mathcal{U}'$ such that $p(g) \cup g \subset U'$.

I claim that $f = hph^{-1}$ is the desired homeomorphism. Certainly f is a homeomorphism from h(X) onto itself. $f|[h(X)-N] = hph^{-1}|[h(X)-N]$ $= hp|(X-N')$, since h^{-1} is a homeomorphism. Therefore $f|[h(X)-N]$ $= h|[X-N'] = 1|[h(X)-N]$.

Choose $h(g) \in h(H_G)$. By definition of p, there exists $V' \in \mathcal{V}'$ such that $ph^{-1}[h(g)] \subset V'$ and there exists $U' \in \mathcal{U}'$ such that $ph^{-1}[h(g)] \cup h^{-1}[h(g)] \subset U'$. Thus $h[ph^{-1}[h(g)]] \subset h(V') = V \in \mathcal{V}$, and $h[ph^{-1}[h(g)]] \cup h[h^{-1}[h(g)]] \subset h(U') = U \in \mathcal{U}$. Therefore $f[h(g)] \cup h(g) \subset U \in \mathcal{U}$.

LEMMA 1.2. Let X be the space described in the theorem, $\{\mathcal{U}_i\}_{i=1}^{\infty}$ be a development of X/G, and h_1 be a homeomorphism of X onto itself. Let $\{h_i\}_{i=2}^{j}$ be a sequence of homeomorphisms such that h_i is a homeomorphism of $h_{i-1}(X)$ onto itself, and K be a G-saturated compact subset of X.

Then for any given $i \leq j$ and open cover $\mathcal{R}_i$ of $h_i \ldots h_1(X)$ such that for every $g^i \in h_i \ldots h_1(H_G)$, $g^i \subset$ some $R_i \in \mathcal{R}_i$, there exists an integer n so that for every $g^i \subset h_i \ldots h_1(K)$ and for every $m \geq n$, $St^4(g^i, \pi^{-1}\mathcal{U}_m)$ is contained in a compact subset of some $R_i \in \mathcal{R}_i$.

PROOF. Choose any $g^i \in h_i \ldots h_1(K)$, and choose $R_i \in \mathcal{R}_i$ such that $g^i \subset R_i$. There exists $\{N_k\}_{k=1}^{5}$, $h_i \ldots h_1(G)$-saturated open sets such that

$$g^i \subset N_1 \subset \overline{N}_1 \subset N_2 \subset \overline{N}_2 \subset \ldots \subset N_5 \subset \overline{N}_5 \subset R_i,$$

and $\overline{N}_k$ is compact for all $k = 1, \ldots, 5$.

$\{U_i\}_{i=1}^{\infty}$ is a development of X/G. Therefore there exists an integer n_g^i such that for $m \geq n_g^i$, no element of $\pi^{-1}U_m$ intersects both $X - N_{i+1}$ and $\overline{N}_i$ $(i = 1, \ldots, 4)$. Let n_g = maximum of $\{n_g^i\}_{i=1}^4$.

Since K is compact there exists finitely many such N_1^r such that $\bigcup_{r=1}^{p} N_1^r \supset h_i \ldots h_1(K)$. ($N_1^r$ chosen above, so that there exists $g_r \subset K$ where $g_r^i \subset N_1^r$.) Let n = maximum of $\left\{ n_{g_r} \right\}_{r=1}^{p}$.

Claim: for $m \geq n$, for every $g^i \subset h_i \ldots h_1(K)$, $St^4(g^i, \pi^{-1}U_m)$ $\subset$ some $N_5^r \subset \overline{N}_5^r \subset R_i \in \mathcal{R}_i$.

Proof: Choose any $g^i \subset h_i \ldots h_1(K)$. There exists r so that $g^i \subset N_1^r$. If there exists $U_m \in \mathcal{U}_m$ such that $\pi^{-1}U_m \cap g^i \neq \emptyset$, then $\pi^{-1}U_m \cap N_1^r \neq \emptyset$. Therefore $\pi^{-1}U_m \subset N_2^r$. If there exists $U_m \in \mathcal{U}_m$ such that $\pi^{-1}U_m \cap St(g^i, \pi^{-1}U_m) \neq \emptyset$, then $\pi^{-1}U_m \cap N_2^r \neq \emptyset$. Therefore, $\pi^{-1}U_m \subset N_3^r$. If there exists $U_m \in \mathcal{U}_m$ such that $\pi^{-1}U_m \cap St^2(g^i, \pi^{-1}U_m)$ $\neq \emptyset$, then $\pi^{-1}U_m \cap N_3^r \neq \emptyset$. Therefore $\pi^{-1}U_m \subset N_4^r$. If there exists $U_m \in \mathcal{U}_m$ such that $\pi^{-1}U_m \cap St^3(g^i, \pi^{-1}U_m) \neq \emptyset$, then $\pi^{-1}U_m \cap N_4^r \neq \emptyset$. Therefore $\pi^{-1}U_m \subset N_5^r$. Thus, $St^4(g^i, \pi^{-1}U_m) \subset N_5^r$.

LEMMA 1.3. Let X be the space described in the theorem, $\mathcal{U}_1'$ be a G-saturated open cover of X, and h_1 be a homeomorphism of X onto itself so that for every $g \in H_G$, $g^1 \cup g \subset$ some $U_1 \in \mathcal{U}_1'$, where $g^1 = h_1(g)$. Let $\{\mathcal{U}_i'\}_{i=2}^{j}$ be a sequence of $h_{i-1} \ldots h_1(G)$-saturated open covers of $h_{i-1} \ldots h_1(X)$ such that for every i, $\mathcal{U}_i'$ is a subcollection of $\mathcal{U}_{i-1}'$ and

$\{h_i\}_{i=2}^{\infty}$ be a sequence of homeomorphisms such that h_i is a homeomorphism $h_{i-1} \circ \cdots \circ h_1(X)$ onto itself, and for every $g^{i-1} \in h_{i-1} \cdots h_1(H_G)$, $g^i \cup g^{i-1} \subset$ some $U_i' \in \mathcal{U}_i'$. $(g^i = h_i(g^{i-1}))$. Let K be a G-saturated compact subset of X such that for every $g^k \subset h_k \cdots h_1(K)$, $St^2(g^k, \mathcal{U}_{k+1}') \subset$ some $U_k \in \mathcal{U}_k'$.

Then for a given integer $i \leq j$, for every $g \subset K$, and for every $m \geq i$, $g^m \subset St[St(g^i, \mathcal{U}_{i+1}'), \mathcal{U}_{i+1}']$.

PROOF. Fix $g \subset K$ and integer i. Let $m = i + 1$. $g^{i+1} \cup g^i \subset U_{i+1}^{i+1} \in \mathcal{U}_{i+1}'$. Thus $U_{i+1}^{i+1} \subset St(g^i, \mathcal{U}_{i+1}') \subset St^2(g^i, \mathcal{U}_{i+1}')$.

Let $m = i + n$. $g^{i+n} \cup g^{i+n-1} \subset U_{i+n}^{i+n} \in \mathcal{U}_{i+n}'$. Certainly

$U_{i+n}^{i+n} \subset$ some $U_{i+n-1}^{i+n} \in \mathcal{U}_{i+n-1}'$.

$g^{i+n-1} \subset U_{i+n-1}^{i+n} \cap U_{i+n-1}^{i+n-1}$, so $U_{i+n-1}^{i+n} \subset$

$St^2(g^{i+n-2}, \mathcal{U}_{i+n-1}') \subset$ some $U_{i+n-2}^{i+n} \in \mathcal{U}_{i+n-2}'$.

$\vdots$

$g^{i+n-k} \subset U_{i+n-k}^{i+n} \cap U_{i+n-k}^{i+n-k}$, so $U_{i+n-k}^{i+n} \subset$

$St^2(g^{i+n-k-1}, \mathcal{U}_{i+n-k}') \subset$ some $U_{i+n-k-1}^{i+n} \in \mathcal{U}_{i+n-k-1}'$.

$\vdots$

$g^{i+1} \subset U_{i+1}^{i+n} \cap U_{i+1}^{i+1}$, so $U_{i+1}^{i+n} \subset St^2(g^i, \mathcal{U}_{i+1}')$.

Thus $g^{i+n} \subset St^2(g^i, \mathcal{U}_{i+1}')$.

LEMMA 1.4. Let X be the space described in the theorem and

(1) $\{V_i\}_{i=1}^{\infty}$ be a sequence of open covers of X;

(2) Let U_1' be a G-saturated open cover of X and h_1 be a homeomorphism of X onto itself so that for every $g \in H_G$ there exists $U_1 \in U_1'$ so that $g^1 \cup g \subset U_1$;

(3) Let $\{U_i'\}_{i=2}^{\infty}$ be a sequence of h_{i-1} (G)-saturated open covers of $h_{i-1}(X)$ so that for every i, U_i' is a subcover of U_{i-1}' and h_i' a homeomorphism of $h_{i-1}(X)$ onto itself such that for every $g^{i-1} = h_{i-1}(g) \in h_{i-1}(H_G)$ there exists $U_i' \in U_i'$ so that $g^i \cup g^{i-1} \subset U_i'$. Let $h_i = h_i' h_{i-1}$;

(4) Let $\overline{B}$ be a G-saturated compact subset of X such that for a given k and for every $p \in \overline{B}$, $St[St(h_k^{-1}(p), h_k^{-1}U_{k+1}'), h_{k+1}^{-1}U_{k+1}']$ $\subset$ some $h_k^{-1} U_k \in h_k^{-1} U_k'$;

(5) Also, for every $p \in \overline{B}$ and for every k, $h_k^{-1}(p) \cup h_{k+1}^{-1}(p)$ $\subset$ some $h_k^{-1}(U_{k+1}) \in h_k^{-1}U_{k+1}'$.

Then for a given n and $p \in \overline{B}$, for every $m \geq n$, $h_m^{-1}(p) \subset St[St(h_n^{-1}(p), h_n^{-1} U_{n+1}'), h_{n+1}^{-1} U_{n+1}']$.

PROOF. Choose any $p \in X$ such that $p \in \overline{B}$. Let $q_i = h_i^{-1}(p)$ for every i. Thus, by condition (5) above, $q_{k+1} \cup q_k \subset$ some $h_k^{-1} U_{k+1}^{k+1} \in h_k^{-1} U_{k+1}'$ for every k.

Let $m = n + 1$. $q_{n+1} \cup q_n \subset h_n^{-1}(U_{n+1}^{n+1}) \in h_n^{-1} U_{n+1}'$. Thus $q_{n+1} \subset h_n^{-1}(U_{n+1}^{n+1}) \subset St(q_n, h_n^{-1} U_{n+1}') \subset St[St(q_n, h_n^{-1} U_{n+1}'), h_{n+1}^{-1} U_{n+1}']$.

Let $m = n + i$. $q_{n+i} \cup q_{n+i-1} \subset h_{n+i-1}^{-1} U_{n+i}^{n+i} \in h_{n+i-1}^{-1} U_m'$. Certainly $U_{n+i}^{n+i} \subset$ some $U_{n+i-1}^{n+i} \in U_{n+i-1}'$.

$q_{n+i-1} \subset h^{-1}_{n+i-1}\, U^{n+i}_{n+i-1} \cap h^{-1}_{n+i-2}\, U^{n+i-1}_{n+i-1}$ by above statement

and condition (5). Thus

$$h^{-1}_{n+i-1}\, U^{n+i}_{n+i-1} \subset St[St(q_{n+i-2}, h^{-1}_{n+i-2} U'_{n+i-1}),\ h^{-1}_{n+i-1} U'_{n+i-1}]$$

$$\subset \text{some } h^{-1}_{n+i-2}\, U^{n+i}_{n+i-2} \in h^{-1}_{n+i-2}\, U'_{n+i-2}.$$

$\vdots$

$q_{n+i-k} \subset h^{-1}_{n+i-k}\, U^{n+i}_{n+i-k} \cap h^{-1}_{n+i-k-1}\, U^{n+i-k}_{n+i-k}$. Thus

$$h^{-1}_{n+i-k}\, U^{n+i}_{n+i-k} \subset St[St(q_{n+i-k-1}, h^{-1}_{n+i-k-1}\, U'_{n+i-k}),\ h^{-1}_{n+i-k}\, U'_{n+i-k}]$$

$$\subset \text{some } h^{-1}_{n+i-k-1}\, U^{n+i}_{n+i-k-1} \in h^{-1}_{n+i-k-1}\, U'_{n+i-k-1}$$

$\vdots$

$q_{n+1} \subset h^{-1}_{n+1}\, U^{n+i}_{n+1} \cap h^{-1}_{n}\, U^{n+1}_{n+1}$. Thus

$$q_m \subset h^{-1}_{n+1}\, U^{n+i}_{n+1} \subset St[St(q_n, h^{-1}_n\, U'_{n+1}),\ h^{-1}_{n+1}\, U'_{n+1}].$$

PROOF OF THEOREM 1. Let $\{V_i\}_{i=1}^{\infty}$ be a development of X, and $\{U_i\}_{i=1}^{\infty}$ be a development of X/G. Let $\{B_k\}_{k=1}^{\infty}$ be a collection of open sets in X/G such that, for every k, $\pi C_k \subset B_k \subset \overline{B}_k$, and $\overline{B}_k$ is compact. (Thus, $\pi^{-1}\overline{B}_k$ is compact.)

Stage 1. Let $N_1 = \bigcup_{k=1}^{\infty} \pi^{-1}B_k$, $U_1' = \pi^{-1}(U_{n_1})$, where $n_1 = 1$, and $V_1' = V_1$. By hypothesis there exists a homeomorphism h_1 of X onto itself such that $h_1 | (X-N_1) = 1$, and for every $g \in H_G$ there exists $V_1^g \in V_1'$ such that $g^1 = h_1(g) \subset V_1^g$, and there exists $U_1^g \in U_1'$ such that $g^1 \cup g \subset U_1^g$.

Stage 2. Let $N_2 = \bigcup_{k=1}^{\infty} h_1\pi^{-1}B_k = N_1$. By Lemma 1.2 there exists an integer n_2'' such that for every $g \subset \pi^{-1}\overline{B}_1$ and for every integer $m \geq n_2''$, $St^2(g^1, \pi^{-1}U_m) \subset$ some $U_1' \subset U_1'$, and there exists an integer n_2' such that for every $g \in \pi^{-1}\overline{B}_1$ and for every $m \geq n_2'$, $St^4(g^1, \pi^{-1}U_m)$ $\subset$ compact subset of some $V_1 \in V_1'$. Therefore, let $U_2' = \pi^{-1}(U_{n_2})$, where n_2 = maximum of $\{n_2'', n_2', (n_1+1)\}$.

For every $g \in G$, choose $V_1^g \in V_1'$ so that $g^1 \subset V_1^g$. Let $O_1^g = h_1[h_1^{-1}(V_1^g) \cap \pi^{-1}U_1^g]$, where $g \subset \pi^{-1}U_1^g \in \pi^{-1}U_1$. Let $O_1 = \{O_1^g\}_{g \in G}$, and let $V_2' = \{V_2 \cap O_1\}_{V_2 \in V_2, O_1 \in O_1}$.

By Lemma 1.1 there exists a homeomorphism h_2' of $h_1(X)$ onto itself so that $h_2' | [h_1(X) - N_2] = 1$, and for every $g^1 \in h_1(H_G)$ there exists $V_2^g \in V_2'$ so that $g^2 = h_2'(g^1) \subset V_2^g$, and there exists $U_2^g \in U_2'$ so that $g^2 \cup g^1 \subset U_2^g$. Let $h_2 = h_2'h_2$.

Stage k. Let $N_k = \bigcup_{i=1}^{\infty} h_{k-1}\pi^{-1}B_k = h_{k-1}N_{k-1} = N_1$. By Lemmas 1.2 and 1.13 there exists an integer n_k'' such that for every $g \in \bigcup_{i=1}^{k-1} \pi^{-1}\overline{B}_i$ and for every integer $m \geq n_k''$, $St^4(g^{k-1}, \pi^{-1}U_m) \subset$ some $U_{k-1}' \in U_{k-1}'$, and there exists an integer n_k' such that for every $g \in \bigcup_{i=1}^{k-1} \pi^{-1}\overline{B}_i$ and for every integer $m \geq n_k'$, $St^4(g^{k-1}, \pi^{-1}U_m) \subset$ compact subset of some $V_{k-1}' \in V_{k-1}'$. Therefore, let $U_k' = \pi^{-1}(U_{n_k})$, where n_k = maximum of $\{n_k'', n_k', (n_{k-1} + 1)\}$.

For every $g^{k-1} \in h_{k-1}(G)$, choose $V^g_{k-1} \in \mathcal{V}'_{k-1}$ such that $g^{k-1} \subset V^g_{k-1}$. Let $O^g_{k-1} = h_{k-1}[h^{-1}_{k-1}(V^g_{k-1}) \cap \pi^{-1}(U^g_{k-1})]$ such that $g \subset \pi^{-1}U^g_{k-1} \in \pi^{-1}\mathcal{U}_{k-1}$. Let $\mathcal{O}_{k-1} = \{O^g_{k-1}\}_{g\in G}$, and let $\mathcal{V}'_k = \{V_k \cap O_{k-1}\}_{V_k\in\mathcal{V}_k, O_{k-1}\in\mathcal{O}_{k-1}}$.

By Lemma 1.1 there exists a homeomorphism h'_k of $h_{k-1}(X)$ onto itself such that $h'_k | [h_{k-1}(X)-N_1] = 1$, and for every $g^{k-1} \in h_{k-1}(H_G)$ there exists $V^g_k \in \mathcal{V}'_k$ so that $g^k = h'_k(g^{k-1}) \subset V^g_k$, and there exists $U^g_k \in \mathcal{U}'_k$ so that $g^k \cup g^{k-1} \subset U^g_k$. Let $h_k = h'_k h_{k-1}$.

Let $h = \lim_{i\to\infty} \{h_i\}$. We need to show that h is the desired homeomorphism.

h is well defined: choose any $x \in X$.

Case 1: $x \notin N_1$. Thus $h(x) = x$.

Case 2: $x \in N_1$. Therefore $x \in \pi^{-1}B_k$, some k. Thus $x \in$ some $g \subset B_k$, so by Lemma 1.3 and construction of $\{\mathcal{U}'_{i+1}\}$, there exists an integer i such that for all $m \geq i$, $h_m(x) \in g^m \subset St^2(g^i, \mathcal{U}'_{i+1})$. Therefore, by Lemma 1.2, $h_m(x)$ is contained in a compact subset of some $V'_i \in \mathcal{V}'_i$. Thus $h(x)$ exists as a non empty set and is contained in V'_i.

To see that $h(x)$ is a single point, suppose there exists a and b, points of $h(x)$ so that $a \neq b$. a and b are closed, compact sets. Therefore there exists an integer j so that no $V_j \in \mathcal{V}_j$ contains both a and b. But $a \cup b \subset h(x) \subset V'_j \subset$ some $V_j \in \mathcal{V}_j$. This contradicts the statement that $a \neq b$.

h is continuous: choose any $x \in X$. We need to show that for any neighborhood P of $h(x)$ there exists a neighborhood N of x such

that $h(N) \subset P$.

Choose M and M', neighborhoods of $h(x)$ such that $h(x) \subset M \subset \overline{M} \subset M' \subset \overline{M}' \subset P$, and $\overline{M}$ and $\overline{M}'$ are compact. Since $\{V_i\}_{i=1}^{\infty}$ is a development, there exists an integer n' so that $St(h(x), V_{n'}) \subset M$. Also, since $\overline{M}$ is compact and $\overline{M} \cap (X - M') = \emptyset$, there exists an integer n'' so that no $V_{n''} \in V_{n''}$ intersects both $\overline{M}$ and $X - M'$. Let n = larger of n' and n''.

Let $N = h_n^{-1}[St(h(x), V_n)]$.

Claim 1: N is a neighborhood of x. This is obvious if $x \in X - N_1$, since $x = h(x)$. If $x \in N_1$, there exists an integer k such that $x \in \pi^{-1}B_k$. We showed above that for every $m \geq n$, $h_m(x) \subset$ compact subset of some $V_n' \in V_n'$. Thus $h_n(x) \cup h(x) \subset$ some $V_n' \subset V_n \in V_n$. Therefore $h_n(N)$ is a neighborhood of $h_n(x)$, so N must be a neighborhood of x.

Claim 2: $h(N) \subset P$. Choose any $y \in N$. $h_n(y) \in St(h(x), V_n)$. If $y \in X - N_1$, $h(y) = h_n(y) \in St(h(x), V_n) \subset M \subset P$. If $y \in N_1$, for every $m \geq n$, $h_m(y) \subset$ compact subset of some $V_n' \in V_n'$. Thus $h_n(y) \cup h(y) \subset$ some $V_n \in V_n$, so $h(y) \subset S^2(h(x), V_n) \subset St[M, V_n] \subset M' \subset P$. Therefore $h(N) \subset P$.

h(g) = a point: suppose there exists $g \in H_G$ and distinct points x and y so that $x \cup y \subset h(g)$. Since $\{V_i\}_{i=1}^{\infty}$ is a development of X there exists an integer n so that no element of V_n contains both x and y. But for $m \geq n$, by Lemma 1.3, $g^m \subset St^2(g^n, U_{n+1}')$, which is contained in a compact subset of some $V_n' \in V_n'$, by construction of U_{n+1}'. Therefore $h(g) \subset V_n'$; so $x \cup y \subset h(g)$ implies $x \cup y \subset$ some

$V_n \in \mathcal{V}_n$, by construction of $\mathcal{V}'_n$. This contradicts the statement $x \neq y$.

h is onto: choose any $p \in X$. We need to show $p \in h(X)$.

Case 1: $p \notin N_1$. Therefore $p = h(p)$.

Case 2: $p \in N_1$. Therefore $p \in B_n$, some n. We know from Lemmas 1.2 and 1.3 and by construction of $\mathcal{U}'_{k+1}$ that for given integers k and n, for every $q_k \in \bigcup_{i=1}^{n} B_i$ and for every integer $m \geq k$, $h_m(q_k) \subset St^2(h_k(q_k), \mathcal{U}'_{k+1}) \subset$ compact subset of some $V_k \in \mathcal{V}_k$. Therefore $h(q_k) \cup h_k(q_k) \subset V_k$. Let $q_k = h_k^{-1}(p)$. Then $h[h_k^{-1}(p)] \cup p \subset V_k$.

Claim: p is a limit point of $\{h(q_k)\}_{k=1}^{\infty}$.

Proof: Let N = any neighborhood of p. There exists k such that $St(p, \mathcal{V}_k) \subset N$. Also there exists $V_k \in \mathcal{V}_k$ so that $h(q_k) \cup p \subset V_k \subset N$ by proof above and since $\{\mathcal{V}_i\}_{i=1}^{\infty}$ is a development of X.

Therefore, to show $p = h(q)$, some $q \in X$, we only need to show there exists q=limit point of $\{q_k\}_{k=1}^{\infty}$. Fix k. For every $h_k(x) \in h_k(X)$, $h_k(x) \cup h_{k+1}(x) \subset$ some $U'_{k+1} \in \mathcal{U}'_{k+1}$, by construction of h_{k+1}. Thus $h_{k+1}(q_{k+1}) \cup h_k(q_{k+1}) \subset$ some U'_{k+1}. Therefore $h_k^{-1}(p) \cup q_{k+1} \subset h_k^{-1}(U'_{k+1})$. That is, $q_k \cup q_{k+1} \subset h_k^{-1}(U'_{k+1})$, so condition (5) of Lemma 1.4 is satisfied.

We need to show condition (4) of Lemma 1.4 is also satisfied, i.e., given k and d for every $x \in X$ such that $h_k^{-1}(x) \subset \bigcup_{i=1}^{d} \overline{B}_i$, $St[St(h_k^{-1}(x), h_k^{-1}\mathcal{U}'_{k+1}), h_{k+1}^{-1}\mathcal{U}'_{k+1}] \subset$ some $h_k^{-1}U_k \in h_k^{-1}\mathcal{U}'_k$. This statement is true if and only if $h_k\, St[St(h_k^{-1}(x), h_k^{-1}\mathcal{U}'_{k+1}), h_{k+1}^{-1}\mathcal{U}'_{k+1}] \subset h_k h_k^{-1}(U_k)$, i.e., $St[h_k[St(h_k^{-1}(x), h_k^{-1}\mathcal{U}'_{k+1})], h_k h_{k+1}^{-1}\mathcal{U}'_{k+1}] \subset U_k$, i.e., $St[St(x, \mathcal{U}'_{k+1}), h_k h_{k+1}^{-1}\mathcal{U}'_{k+1}] \subset U_k$.

Claim: for every $h_k h_{k+1}^{-1}(U_{k+1}) \in h_k h_{k+1}^{-1} U'_{k+1}$, $h_k h_{k+1}^{-1}(U_{k+1}) \subset St(U_{k+1}, U'_{k+1})$.

Proof: choose $h_k h_{k+1}^{-1}(y) \in h'_k h_{k+1}^{-1}(U_{k+1})$. By construction of h_k and h_{k+1}, $h_{k+1}[h_{k+1}^{-1}(y)] \cup h_k[h_{k+1}^{-1}(y)] \subset$ some $U_{k+1}^{y} \in U'_{k+1}$, i.e., $y \cup h_k h_{k+1}^{-1}(y) \subset U_{k+1}^{y}$. Thus $h_k h_{k+1}^{-1}(y) \subset St(y, U'_{k+1})$; so $h_k h_{k+1}^{-1}(U_{k+1}) \subset St[U_{k+1}, U'_{k+1}]$.

Choose $h_k h_{k+1}^{-1}(U_{k+1})$ such that $h_k h_{k+1}^{-1}(U_{k+1}) \cap St(x, U'_{k+1}) \neq \emptyset$. Thus $St(U_{k+1}, U'_{k+1}) \cap St(x, U'_{k+1}) \neq \emptyset$. Therefore there exists $\tilde{U}_{k+1} \in U'_{k+1}$ so that $\tilde{U}_{k+1} \cap U_{k+1} \neq \emptyset$, and $\tilde{U}_{k+1} \cap St(x, U'_{k+1}) \neq \emptyset$. This implies that $\tilde{U}_{k+1} \subset St^2(x, U'_{k+1})$, and $U_{k+1} \subset St^3(x, U'_{k+1})$. Thus, $St(U_{k+1}, U'_{k+1}) \subset St^4(x, U'_{k+1})$, i.e., $h_k h_{k+1}^{-1}(U_{k+1}) \subset St^4(x, U'_{k+1})$ which is contained in a compact subset of some $U_k \in U_k$, by Lemma 1.2. Therefore, condition (4) of Lemma 1.4 is satisfied.

By Lemma 1.4, for a given integer n, for every $m \geq n$, $q_m \subset St[St(q_n, h_n^{-1} U'_{n+1}), h_{n+1}^{-1} U'_{n+1}]$, which was shown above to be contained in a compact subset P of some $h_n^{-1} U_k$. Therefore $q = \lim_{k \to \infty} q_k$ exists and is contained in P. We proved earlier that p is a limit point of $\{h(q_k)\}_{k=1}^{\infty}$. Thus, $h(q) = h(\lim q_k) = \lim h(q_k) = p$.

$h\pi^{-1}$ is 1-1: suppose there exists f and g $\in$ G so that $h(f) = h(g)$; $f \neq g$. Since $\{U_i\}_{i=1}^{\infty}$ is a development of X/G, there exists an integer n such that $St(g, U_n) \cap St(f, U_n) = \emptyset$. We consider only the most difficult case where $f \cup g \subset \bigcup_{i=1}^{k} \pi^{-1}(B_i)$, for some integer k. Thus, by construction of $\{U'_k\}_{k=1}^{\infty}$ and by Lemma 1.3, for every integer $m \geq n + 1$, $g^m \subset$

$St^2(g^{n+1},\mathcal{U}'_{n+2})$, and $f^m \subset St^2(f^{n+1},\mathcal{U}'_{n+2})$. But, by construction of $\{\mathcal{U}'_k\}_{k=1}^{\infty}$, $St^2(g^{n+1},\mathcal{U}'_{n+2}) \subset$ compact subset of some $V^g_{n+1} \in \mathcal{V}'_{n+1}$ and $St^2(f^{n+1},\mathcal{U}'_{n+2}) \subset$ compact subset of some $V^f_{n+1} \in \mathcal{V}'_{n+1}$. Thus $h(g) \subset V^g_{n+1}$ and $h(f) \subset V^f_{n+1}$. $h(g) = h(f)$, so there exists at least one point $x \in V^g_{n+1} \cap V^f_{n+1}$. $V^f_{n+1} \in \mathcal{V}'_{n+1}$ implies $V^f_{n+1} \subset O^f_n$ such that $f^n \subset O^f_n$, and $V^g_{n+1} \in \mathcal{V}'_{n+1}$ implies $V^g_{n+1} \subset O^g_n$ such that $g^n \subset O^g_n$. Thus $x \in O^f_n \cap O^g_n$, and $h_n^{-1}(x) \subset \pi^{-1}(U^g_n) \cap \pi^{-1}(U^f_n)$, where $f \subset \pi^{-1}U^f_n \in \pi^{-1}\mathcal{U}_n$ and $g \subset \pi^{-1}U^g_n \in \pi^{-1}\mathcal{U}_n$. This is a contradiction, since $St(g,\mathcal{U}_n) \cap St(f,\mathcal{U}_n) = \emptyset$.

<u>πh^{-1} is continuous:</u> choose $x \in X$. We need to show that for any neighborhood P of $\pi h^{-1}(x)$, there exists a neighborhood N of x so that $\pi h^{-1}(N) \subset P$. $h^{-1}(x)$ is compact, so there exists M and M', neighborhoods of $h^{-1}(x)$, such that $h^{-1}(x) \subset M \subset \overline{M} \subset M' \subset \overline{M}' \subset \pi^{-1}(P)$, where $\overline{M}$ and $\overline{M}'$ are compact. Since $\{\mathcal{V}_i\}_{i=1}^{\infty}$ is a development of X there exists an integer n' such that $St(h^{-1}(x),\mathcal{V}_{n'}) \subset M$. Also, since $\overline{M}$ is compact and $\overline{M} \cap (X-M') = \emptyset$, there exists an integer n'' such that no element of $\mathcal{V}_{n''}$ intersects both $\overline{M}$ and X-M'. Let n = larger of n' and n''. Let $N = h_n[St(h^{-1}(x),\mathcal{V}_n]$.

<u>Claim 1</u>: N is a neighborhood of x. This is obvious if $x \in X - N_1$, since $h^{-1}(x) = x = h_n(x)$. If $x \in N_1$ there exists an integer k such that $x \in \pi^{-1}B_k$. We showed before that for every integer $m \geq n$, $h_m^{-1}(x) \subset$ compact subset of some $V'_n \in \mathcal{V}'_n$. Thus there exists $y \in h^{-1}(x)$ such that $h_n^{-1}(x) \cup y \subset V'_n \subset$ some $V_n \in \mathcal{V}_n$. Thus $h_n^{-1}(x) \in St(y,\mathcal{V}_n) \subset St[h^{-1}(x),\mathcal{V}_n] = h_n^{-1}(N)$, i.e., N is a neighborhood of x.

Claim 2: $\pi h^{-1}(N) \subset P$. Choose any $y \in N$. $h_n^{-1}(y)$ $\in St[h^{-1}(x), \mathcal{V}_n] \subset M$. There exists $V_n' \in \mathcal{V}_n'$ such that for every integer $m \geq n$, $h_m^{-1}(y)$ $\in$ compact subset of V_n'. Thus there exists $b \in h^{-1}(y)$ such that $h_n^{-1}(y) \cup b \subset V_n' \subset$ some $V_n \in \mathcal{V}_n$. Therefore $b \in St^2(h^{-1}(x), \mathcal{V}_n) \subset St(M, \mathcal{V}_n) \subset \pi^{-1}(P)$. We proved earlier that πh^{-1} is 1-1. Thus, $\pi h^{-1}(y) = \pi(b) \subset P$. This implies that $\pi h^{-1}(N) \subset P$, so πh^{-1} is continuous.

Let $F = h\pi^{-1}$, a closed, 1-1, onto function of X/G onto X. h is continuous. Therefore F is a homeomorphism.

COROLLARY 1.1. If X is a strongly developable, locally compact, T_2 space, and G is a strongly shrinkable u.s.c. decomposition of X such that H_G is countable, then X is homeomorphic to X/G.

COROLLARY 1.2. If X is a strongly developable, locally compact, T_2 space, and G is a strongly shrinkable u.s.c. decomposition of X such that H_G^* is compact, then X is homeomorphic to X/G.

COROLLARY 1.3. If X is a locally compact, metric space, and G is a strongly shrinkable u.s.c. decomposition of X such that H_G^* is a countable union of compact sets, then X is homeomorphic to X/G.

CHAPTER IV

A DECOMPOSITION THEOREM FOR COUNTABLE H_G

THEOREM 2. Suppose X is a strongly developable, locally compact, T_2 space, and G is a countable u.s.c. decomposition of X. Then G is strongly shrinkable if and only if (*) for every $g_0 \in H_G$, any neighborhood N of g_0, and any open cover $\mathcal{V}$ of X, there exists a homeomorphism h of X onto X such that $h|(X-N) = 1$, $h(g_0) \in$ some $V \in \mathcal{V}$, and for every $g \in G$, either $h(g) \subset$ some $V \in \mathcal{V}$ or $h(g) \subset St(g,\mathcal{V})$.

LEMMA 2.1. Let X be the space described in the theorem. For every $g \in G$ and every neighborhood U of g there exists a neighborhood N such that (the boundary of N) $\cap H_G^* = \emptyset$, and $N \subset U$.

PROOF. Let g_0 = any element of H_G, U any neighborhood of g_0, N_0 = G-saturated open set such that $g_0 \subset N_0 \subset \overline{N}_0 \subset U$, and $\overline{N}_0$ is compact. Let N_0' = G-saturated open set such that $g_0 \subset N_0' \subset \overline{N}_0' \subset N_0$. Enumerate the elements of H_G as $H_G = \{g_i\}_{i=0}^{\infty}$.

Consider $g_1 \in H_G$.

Case 1: $g_1 \subset X - \overline{N}_0$. Then there exists O_1, open set such that $g_1 \subset O_1 \subset \overline{O}_1 \subset X - \overline{N}_0$. Thus $N_1 = N_0$, $N_1' = N_0'$.

Case 2: $g_1 \subset N_0$. Then there exists O_1, open set such that $g_1 \subset O_1 \subset \overline{O}_1 \subset N_0$. $N_1 = N_0$, $N_1' = N_0' \cup O_1^*$.

Case 3: $g_1 \cap$ bdry $N_0 \neq \emptyset$. Then there exist open sets O_1 and M_1 such that $g_0 \subset M_1$, $g_1 \subset O_1$, $\overline{N}_0' \subset M_1 \subset N_0$, $\overline{O}_1 \cap \overline{M}_1 = \emptyset$. Let

$N_1 = (N_0 - \overline{O}_1)^*$ and $N_1' = N_0' \cup M_1^*$.

Consider $g_n \in H_G$.

Case 1: $g_n \subset X - \overline{N}_{n-1}$. Then there exists O_n, open set such that $g_n \subset O_n \subset \overline{O}_n \subset X - \overline{N}_{n-1}$. Let $N_n = N_{n-1}$ and $N_n' = N_{n-1}'$.

Case 2: $g_n \subset N_{n-1}$. Then there exists O_n, open set such that $g_n \subset O_n \subset \overline{O}_n \subset N_{n-1}$. Let $N_n = N_{n-1}$ and $N_n' = N_{n-1}' \cup O_n^*$.

Case 3: $g_n \cap \text{bdry } N_{n-1} \neq \emptyset$. Then there exists open sets O_n and M_n such that $g_0 \subset M_n$, $g_n \subset O_n$, $\overline{N}'_{n-1} \subset M_n \subset N_{n-1}$, $\overline{O}_n \cap \overline{M}_n = \emptyset$. Let $N_n = (N_{n-1} - \overline{O}_n)^*$ and $N_n' = N_{n-1}' \cup M_n^*$.

Claim: $N = \bigcup_{i=1}^{\infty} N_i'$ = open set containing g_0 such that bdry $N \cap H_G = \emptyset$.

Proof: N must be open since it is the union of open sets. For every i, $N_0' \subset N_i'$, thus $g_0 \subset N_i'$, and $g_0 \subset N$. Suppose there exists $g_k \in H_G$ such that $g_k \cap \text{bdry } N \neq \emptyset$.

Case 1: $g_k \subset X - \overline{N}_{k-1}$. Therefore $g_k \subset O_k \subset \overline{O}_k \subset X - \overline{N}_{k-1}$. But, for every i, $N_i' \subset N_{k-1}$, so $O_k \cap N_i' = \emptyset$.

Case 2: $g_k \subset N_{k-1}$. Therefore $g_k \subset O_k \subset \overline{O}_k \subset N_{k-1}$. $N_k' = N_{k-1}' \cup O_k^*$, so $g_k \subset N$ means $g_k \cap \text{bdry } N = \emptyset$.

Case 3: $g_k \cap \text{bdry } N_{k-1} \neq \emptyset$. Therefore, for every neighborhood M of g_k, $M \cap N \neq \emptyset$. Let $M = O_k$ as described before. For $i = 1, \ldots, k-1$, $O_k \cap \overline{N}_i' = \emptyset$ since $\overline{N}_i' \subset \overline{N}_{k-1}' \subset M_k$ and $\overline{O}_k \cap \overline{M}_k = \emptyset$. $\overline{N}_k' = \overline{N}_{k-1}' \cup \overline{M}_k^*$. Thus $\overline{O}_k \cap \overline{N}_k' = \emptyset$. For $i > k$, $\overline{N}_i' \subset \overline{N}_i \subset \overline{N}_k$, and since $\overline{N}_k = (N_{k-1} - \overline{O}_k)^*$, $O_k \cap \overline{N}_k = \emptyset$. Therefore $g_k \cap \text{bdry } N = \emptyset$.

Thus, for every $g_k \in H_G$, $g_k \cap \text{bdry } N = \emptyset$.

LEMMA 2.2. Let X be the space described in the theorem, $\{U_i\}_{i=1}^{\infty}$ be a development of X/G, h_1 be a homeomorphism of X onto itself, $\{h_i\}_{i=2}^{j}$ be a sequence of homeomorphisms such that $h_i = h_i' h_{i-1}$ where h_i' is a homeomorphism of $h_{i-1}(X)$ onto itself, and $\overline{N}_n$ is a compact set. Then for a given $\mathcal{R}_i$ = open cover of $h_i(X)$, and a given integer $i \leq j$, there exists an integer n such that for every integer $m \geq n$, and for every $h_i(x) \in \overline{N}_n$, $St^4(h_i(x), \pi^{-1}U_m) \subset$ compact subset of some $R_i \in \mathcal{R}_i$.

The proof is the same as in Lemma 1.2, and we shall not repeat the details.

LEMMA 2.3. Let X be the space described in the theorem and G a countable u.s.c. decomposition such that (*) is satisfied. Let h be a homeomorphism of X onto itself. Then for every $h(g_0) \in h(H_G)$, any neighborhood N of $h(g_0)$, and any open cover $\mathcal{V}$ of h(X), there exists a homeomorphism f of h(X) onto itself such that $f|[h(X)-N] = 1$, $f(h(g_0)) \subset$ some element of $\mathcal{V}$, and for every $h(g) \in h(G)$, either $f(h(g)) \subset$ some $V \in \mathcal{V}$, or $f(h(g)) \subset St(h(g), \mathcal{V})$.

The proof is like Lemma 1.1.

LEMMA 2.4. Let X denote the space described in the theorem. Suppose $\{U_i'\}_{i=1}^{\infty}$ is a collection of open covers of X, and $\{h_i\}_{i=0}^{j}$ is a sequence of homeomorphisms of X onto itself such that $h_0 = 1$, U_i' is $h_{i-1}(G)$-saturated and for every $g \in G$ and for every $i \geq 1$, $i \leq j$

there exists $U_i \in \mathcal{U}_i$ such that $h_i(g) \cup h_{i-1}(g) \subset U_i$, and suppose $\overline{N}_n$ is an $h_{i(n-1)}(G)$-saturated compact set, where $i(n-1)$ an integer such that for every $h_{i(n-1)}(X) \in \overline{N}_n$, for $j > i\,(n-1)$, $St^2(h_j(x), \mathcal{U}'_{j+1}) \subset$ some $U_j \in \mathcal{U}'_j$, and for $j \leq i\,(n-1)$, $h_j(x) = x$. Then for a given $k > i\,(n-1)$, for every $x \in \overline{N}_n$, and for every $m \geq k$, $h_m(x) \subset St^2(h_k(x), \mathcal{U}'_{k+1})$.

The proof is like Lemma 1.3.

PROOF OF THEOREM 2. ($\Longrightarrow$) G is strongly shrinkable. We will show for any $g_0 \in H_G$, N = any neighborhood of g_0, and $\mathcal{V}$ = any open cover of X, there exists a homeomorphism h of X onto itself such that $h|(X-N) = 1$, $h(g_0) \subset$ some $V \in \mathcal{V}$, and for every $g \in G$, either $h(g) \subset$ some $V \in \mathcal{V}$ or $h(g) \subset St(g, \mathcal{V})$.

Let $\mathcal{U}$ = any G-saturated open cover of X. By Lemma 2.1, there exists N_0 = G-saturated open set containing g_0 such that $N_0 \subset \overline{N}_0 \subset N$, and for every $f \in G$ such that $f \cap \text{bdry } N_0 \neq \emptyset$, f is a point. There exist open sets $\{N_i\}_{i=1}^{\infty}$ such that $\bigcup_{i=1}^{\infty} N_i \supset H_G^* - g_0$, and $\overline{N}_i \cap \overline{N}_0 = \emptyset$ for every $i > 0$. Let $O = \bigcup_{j=0}^{\infty} N_j \supset H_G^*$.

By hypothesis there exists a homeomorphism h' of X onto itself such that $h'|X-O = 1$, and for every $g \in G$, $h'(g) \cup g \subset$ some $U \in \mathcal{U}$ and $h'(g) \subset$ some $V \in \mathcal{V}$. Therefore, let h be defined so that

$$h(x) = \begin{cases} x, \text{ if } x \in X - N_0 \\ h'(x) \text{ if } x \in \overline{N}_0. \end{cases}$$

Claim: h is the desired homeomorphism. Certainly h is 1-1 and onto. (This statement is true when X is locally connected.)

h is a function; $X = \overline{N}_0 \cup X - N_0$; $h|\overline{N}_0$ and $h|(X-N_0)$ are certainly continuous. Therefore h is continuous, and the same reasoning holds to show h^{-1} is continuous. Therefore h is a homeomorphism.

Certainly $h(g_0) \subset$ some $V \in \mathcal{V}$ for every $g \in G$, if $g \subset N_0$, $h(g) \subset$ some $V \in \mathcal{V}$, while if $g \subset X - N_0$, then $h(g) = g \subset St(g,\mathcal{V})$. Thus h is the desired homeomorphism.

($\Longleftarrow$) Let G be a countable u.s.c. decomposition such that (*) holds. To show G is strongly shrinkable, enumerate the elements of H_G as $\{g_i\}_{i=1}^{\infty}$. Let $\{V_k\}_{k=0}^{\infty}$ = development of X, $\mathcal{V}$ = open cover of X, $\{U_i\}_{i=0}^{\infty}$ = development of X/G, $\mathcal{U}$ = any G-saturated open cover of X, and N = any open set containing H_G^*. We will construct a sequence of compact sets $\{\overline{N}_i\}_{i=1}^{\infty}$ and a sequence of homeomorphisms $\{h_i\}_{i=1}^{\infty}$ such that $h = \lim h_i$ satisfies the conditions for G to be strongly shrinkable. To construct $\overline{N}_1$, let $g_{1,1} = g_1 \in H_G$. By Lemma 2.1 there exists a G-saturated open set N_1 such that $g_1 \subset N_1 \subset \overline{N}_1 \subset U \cap N \cap U_1$, where $\overline{N}_1$ is compact, U = some element of $\mathcal{U}$ such that $g_{1,1} \subset U$, U_1 = some element of $\mathcal{U}_1'$ containing $g_{1,1}$ where $\mathcal{U}_1' = \pi^{-1}\mathcal{U}_1$, and bdry $N_1 \cap H_G^* = \emptyset$.

Let i(0) = 0 and $h_0 = 1$. We shall construct a sequence of homeomorphisms $\{h'_k\}_{k=1}^{\infty}$ of $h_{k-1}(X)$ onto itself such that $h'_k|[h_{k-1}(X) - N_1] = 1$, and $h_k = h'_k h_{k-1}$. We shall show that there exists an integer i(1) such that for every $g \in G$ such that $g \subset \overline{N}_1$, $h_{i(1)}(g) \subset$ some $V \in \mathcal{V}$. Then we shall define $\mathcal{U}'_{i(1)+1}$ and construct N_2.

For $n = 2, 3, 4, \ldots$, choose $g_{n,1}$ = 1st element of H_G such that $g_{n,1} \subset X - \bigcup_{i=1}^{n-1} N_i$. By Lemma 2.1 there exists $h_{i(n-1)}(G)$-saturated open set N_n such that $g_{n,1} \subset N_n \subset \overline{N}_n \subset U \bigcap N \bigcap U_{i(n-1)+1}$, where $\overline{N}_n$ is compact, U = some element of $\mathcal{U}$ such that $g_{n,1} \subset U$, $U_{i(n-1)+1}$ = some element of $\mathcal{U}'_{i(n-1)+1}$ containing $g_{n,1}$, and bdry $N_n \bigcap H_G^* = \emptyset$.

We shall construct a sequence of homeomorphisms $\{h_{i(n-1)+k}\}_{k=1}^{\infty}$ of $h_{i(n-1)+k-1}(X)$ onto itself such that $h'_{i(n-1)+k} | [h_{i(n-1)+k-1}(X) - N_n] = 1$, and $h_{i(n-1)+k} = h'_{i(n-1)+k} h_{i(n-1)+k-1}$. We shall show that there exists an integer i(n) such that for every $g \in G$ such that $g \subset \overline{N}_n$, $h_{i(n)}(g) \subset$ some $V \in \mathcal{V}$. Then we shall define $\mathcal{U}'_{i(n)+1}$ and construct N_{n+1}.

Therefore, for $n = 1, 2, 3, \ldots$, we shall work with each compact set $\overline{N}_n$ to construct the sequence of homeomorphisms $\{h'_{i(n-1)+k}\}_{k=1}^{\infty}$ and show the existence of the integer i(n). We begin with the <u>n^{th} compact set, stage 1</u>.

For every $x \in \overline{N}_n$ there exists $V^x \in \mathcal{V}$ such that $x \in V^x$. Also there exists P_0^x and O_0^x such that $x \in P_0^x \subset \overline{P}_0^x \subset O_0^x \subset \overline{O}_0^x \subset V^x$, $\overline{P}_0^x$ and $\overline{O}_0^x$ are compact. $\overline{N}_n$ is compact, thus there exist finitely many P_0^i such that $\bigcup_{i=1}^{d} P_0^i \supset \overline{N}_n$. $\{\mathcal{V}_k\}_{k=0}^{\infty}$ is a development, so for every i there exists n_i such that for all $m \geq n_i$, no element of $\mathcal{V}_m$ intersects both $\overline{P}_0^i$ and $X - O_0^i$. Let $n'_1 = \max \{n_i\}_{i=1}^{d}$. Let $\mathcal{V}'_{i(n-1)+1} = \{\mathcal{V}_{n(1)} \bigcap \mathcal{V}\}$ such that n(1) = larger of n'_1 and $(n_{i(n-1)} + 1)$, where $n_0 = 0$.

Therefore, by hypothesis for n = 1 and by Lemma 2.3 for n > 1, there exists a homeomorphism $h'_{i(n-1)+1}$ of $h_{i(n-1)}(X)$ onto itself such that $h'_{i(n-1)+1} | [h_{i(n-1)}(X) - N_n] = 1$, $h'_{i(n-1)+1}(h_{i(n-1)}(g_{n,1})) \subset$ some $V_{i(n-1)+1} \in \mathcal{V}'_{i(n-1)+1}$, and for every $h_{i(n-1)}(g) \in h_{i(n-1)}(G)$, either

$h'_{i(n-1)+1}(h_{i(n-1)}(g)) \subset$ some $V_{i(n-1)+1} \in \mathcal{V}'_{i(n-1)+1}$, or $h'_{i(n-1)+1}(h_{i(n-1)}(g)) \subset St[h_{i(n-1)}(g), \mathcal{V}'_{i(n-1)+1}]$. Let $h_{i(n-1)+1} = h'_{i(n-1)+1} h_{i(n-1)}$.

Since $h_{i(n-1)+1}(g_{n,1}) \subset$ some $V_{i(n-1)+1} \in \mathcal{V}'_{i(n-1)+1}$, then $h_{i(n-1)+1}(g_{n,1}) \subset$ some $V \in \mathcal{V}$. Let $O_{n,1} = h_{i(n-1)+1}(G)$-saturated open set containing $h_{i(n-1)+1}(g_{n,1})$ such that $O_{n,1} \subset \overline{O}_{n,1} \subset h_{i(n-1)+1}(N_n) \cap V_{i(n-1)+1}$, $\overline{O}_{n,1}$ is compact, bdry $O_{n,1} \cap h_{i(n-1)+1}(H_G^*) = \emptyset$. For every $h_{i(n-1)+1}(g) \in h_{i(n-1)+1}(G)$ such that $h_{i(n-1)+1}(g) \subset O_{n,1}$, $h_{i(n-1)+1}(g) \cup g \subset N_n \subset$ some $U \in \mathcal{U}$, and $h_{i(n-1)+1}(g) \subset$ some $V \in \mathcal{V}$. Therefore, $h_{i(n-1)+1} | O_{n,1}$ satisfies the conditions needed for the homeomorphism in the theorem, so each of the following homeomorphisms will leave $O_{n,1}$ fixed.

<u>n^{th} compact set, stage k</u>: ($k > 1$) Let $j(1,1) = 1$. By Lemma 2.2, there exists $j'(n,k)$ such that for every $h_{i(n-1)+k-1}(x) \in \overline{N}_n$ for every $m \geq j'(n,k)$, $St^4(h_{i(n-1)+k-1}(x), \pi^{-1}U_m) \subset$ compact subset of $V_{i(n-1)+k-1} \cap U_{i(n-1)+k-1}$ such that $V_{i(n-1)+k-1} \in \mathcal{V}'_{i(n-1)+k-1}$ and $U_{i(n-1)+k-1} \in \mathcal{U}'_{i(n-1)+k-1}$. Therefore, let $\mathcal{U}'_{i(n-1)+k} = \pi^{-1}(\mathcal{U}_{j(n,k)})$ where $j(n,k) =$ larger of $j'(n,k)$ and $[j(n,k-1)+1]$.

Let $g_{n,k} =$ first element of H_G such that $g_{n,k} \subset N_n$, and $h_{i(n-1)+k-1}(g_{n,k}) \not\subset \bigcup_{i=1}^{k-1} O_{n,i}$. By Lemma 2.1 there exists $N_{n,k}$, an $h_{i(n-1)+k-1}(G)$-saturated open set such that $N_{n,k} \subset \overline{N}_{n,k} \subset N_n \cap U_{i(n-1)+k}$, where $\overline{N}_{n,k}$ is compact, $\overline{N}_{n,k} \cap (\bigcup_{i=1}^{k-1} \overline{O}_{n,i}) = \emptyset$, and bdry $N_{n,k} \cap h_{i(n-1)+k-1}(H_G^*) = \emptyset$, $\overline{N}_{n,k} \subset St[h_{i(n-1)+k-1}(g_{n,k}), \mathcal{V}'_{i(n-1)+k-1}]$, and $U_{i(n-1)+k} \in \mathcal{U}'_{i(n-1)+k}$.

$\{O^i_{k-2}\}^d_{i=1}$ covers $\overline{N}_n$, $h_{i(n-1)+k-1}(\overline{N}_n) = \overline{N}_n$. Thus $\{O^i_{k-2}\}^d_{i=1}$ covers $\overline{N}_{n,k}$. For every i, $\overline{O}^i_{k-2}$ is a compact subset of some $V \in \mathcal{V}$. Thus there exists O^i_{k-1}, open, such that $\overline{O}^i_{k-2} \subset O^i_{k-1} \subset \overline{O}^i_{k-1} \subset V \in \mathcal{V}$, and $\overline{O}^i_{k-1}$ is compact. Since $\{\mathcal{V}_j\}^\infty_{j=1}$ is a development, for every i, there exists n_i such that for every $m \geq n_i$, no element of $\mathcal{V}_m$ intersects both $\overline{O}^i_{k-2}$ and $h_{i(n-1)+k-1}(X) - O^i_{k-1}$. Let $n'(k) = \max\{n_i\}^d_{i=1}$. Let $\mathcal{V}'_{i(n-1)+k} = \{V_{n(k)} \cap V\}$, where n(k) = larger of n'(k) and n(k-1) + 1.

Thus, by Lemma 2.3, there exists a homeomorphism $h'_{i(n-1)+k}$ of $h_{i(n-1)+k-1}(X)$ onto itself such that $h'_{i(n-1)+k}|[h_{i(n-1)+k-1}(X)-N_{n,k}] = 1$, $h'_{i(n-1)+k}(h_{i(n-1)+k-1}(g_{n,k})) \subset$ some $V_{i(n-1)+k} \in \mathcal{V}'_{i(n-1)+k}$, and for every $h_{i(n-1)+k-1}(g) \in h_{i(n-1)+k-1}(G)$, either $h'_{i(n-1)+k}(g) \subset$ some $V_{i(n-1)+k} \in \mathcal{V}'_{i(n-1)+k}$, or $h'_{i(n-1)+k}(g) \subset St[h_{i(n-1)+k-1}(g), \mathcal{V}'_{i(n-1)+k}]$. Let $h_{i(n-1)+k} = h'_{i(n-1)+k} h_{i(n-1)+k-1}$.

$h_{i(n-1)+k}(g_{n,k}) \subset$ some $V_{i(n-1)+k} \in \mathcal{V}'_{i(n-1)+k}$, so $h_{i(n-1)+k}(g_{n,k})$ $\subset$ some $V \in \mathcal{V}$. Let $O_{n,k} = h_{i(n-1)+k}(G)$-saturated open set containing $h_{i(n-1)+k}(g_{n,k})$ such that $O_{n,k} \subset \overline{O}_{n,k} \subset h_{i(n-1)+k}(N_{n,k}) \cap V_{i(n-1)+k}$, $\overline{O}_{n,k}$ is compact, and bdry $O_{n,k} \cap h_{i(n-1)+k}(H^*_G) = \emptyset$. For every $h_{i(n-1)+k}(g) \in h_{i(n-1)+k}(G)$ such that $h_{i(n-1)+k}(g) \subset O_{n,k}$, $h_{i(n-1)+k}(g) \cup g \subset N_n \subset$ some $U \in \mathcal{U}$, and $h_{i(n-1)+k}(g) \subset$ some $V \in \mathcal{U}$. Therefore, $h_{i(n-1)+k}|O_{n,k}$ satisfies the conditions needed for the homeomorphism in the theorem, so each of the following homeomorphisms will leave $O_{n,1}$ fixed.

Claim: There exists an integer i(n) such that for every $g \in G$ such that $g \subset \overline{N}_n$, $h_{i(n)}(g) \subset$ some $V \in \mathcal{V}$. Let $Y = \bigcup\{g \in G | g \subset \overline{N}_n$, and no P^i_0 contains $g\}$.

Choose any $g \subset \overline{N}_n$. For every $p \geq 1$, if no $V \in \mathcal{V}$ contains $h_p(g)$, then $g \subset Y$.

Proof. Suppose $g \not\subset Y$. Then there exists P_0^i such that $g \subset P_0^i$.

p = 1: By definition of h_1, either $h_1(g) \subset$ some $V_1' \subset V$, a contradiction, or $h_1(g) \subset St(g,\mathcal{V}_1') \subset O_0' \subset$ some $V \in \mathcal{V}$, by construction of $\mathcal{V}_1'$.

p = 2: By definition of h_2, either $h_2(g) \subset$ some $V_2' \subset V \in \mathcal{V}$, a contradiction, or $h_2(g) \subset St(h_1(g),\mathcal{V}_2') \subset St[St(g,\mathcal{V}_1'),\mathcal{V}_2']$ $\subset St(O_0',\mathcal{V}_2') \subset O_0' \subset$ some $V \in \mathcal{V}$, by construction of $\mathcal{V}_2'$.

p = k: By definition of h_k, either $h_k(g) \subset$ some $V_k' \subset V \in \mathcal{V}$, a contradiction, or $h_k(g) \subset St[h_{k-1}(g),\mathcal{V}_k'] = St[St(h_{k-2}(g),\mathcal{V}_{k-2}'),\mathcal{V}_k']$ $\subset St[O_{k-2}^i,\mathcal{V}_k'] \subset O_{k-1}^i \subset$ some $V \in \mathcal{V}$, by construction of $\mathcal{V}_k'$. Therefore, if no $V \in \mathcal{V}$ contains $h_{k(n)}(g)$, then $g \subset Y$.

Suppose for every j there exists $g_{k(j)} \subset \overline{N}_n$ such that $h_j(g_{k(j)})$ is not contained in any $V \in \mathcal{V}$. Then by the above argument, $g_{k(j)} \subset Y$, for every j. Y is a closed subset of $\overline{N}_n$; $\overline{N}_n$ is compact. Therefore Y is compact. Thus $\{g_{k(j)}\}_{j=1}^{\infty}$ has a limit point $x \in Y$, $x \in$ some $g_{k(0)} \in G$ such that $g_{k(0)} \subset \overline{N}_n$. But $h_{k(0)}(g_{k(0)}) \subset$ some $V \in \mathcal{V}$; so there exists some neighborhood N of $g_{k(0)}$ such that $h_{k(0)}(N) \subset$ some $V \in \mathcal{V}$. N is a neighborhood of x; therefore there exists p such that for infinitely many $m \geq p$, $g_{k(m)} \subset N$. $h_j(N) = h_{k(0)}(N)$ for every $j \geq k_0$. Therefore let i(n) = larger of p and k(0). Then for every $m \geq i(n)$, $h_m(g_{k(m)}) = h_{i(n)}(g_{k(m)}) \subset$ some $V \in \mathcal{V}$, a contradiction.

Therefore there exists i(n) such that $h_{i(n)+k}(\overline{N}_n) = h_{i(n)}(\overline{N}_n)$

for every k, and $h_{i(n)}(g) \subset$ some $V \in \mathcal{V}$, and $h_{i(n)}(g) \bigcup g \subset \overline{N}_n$ some $U \in \mathcal{U}$, for every $g \in \overline{N}_n$. Let $\mathcal{U}'_{i(n)+1} = \pi^{-1}\mathcal{U}_j$ where $j = i(n) + 1$.

Now we shall continue constructing the sequence of homeomorphisms as we work with the n + 1 compact set $\overline{N}_{n+1}$ and go through the same procedure to find the integer i(n+1).

Let $h = \lim_i h_i$.

Claim: h is the desired homeomorphism.

Proof. h is well defined--consider any $x \in X$.

Case 1: $x \in X - \bigcup_{i=1}^{\infty} N_i$. Thus $h_i(x) = x$ for every i, and $h(x) = x$.

Case 2: $x \in N_k$, some integer k. Then there exists i(k) such that for every $n \geq i(k)$, $h_n(x) = h_{i(k)}(x)$, and $h(x) = h_{i(k)}(x)$.

h is continuous--choose any $x \in X$. To show h is continuous at x, we need to show that for every neighborhood O of h(x) there exists a neighborhood N of x such that $h(N) \subset O$. Choose M and M', open sets, such that $h(x) \subset M \subset \overline{M} \subset M' \subset \overline{M}' \subset O$, and $\overline{M}$ and $\overline{M}'$ are compact. Since $\{\mathcal{V}_i\}_{i=1}^{\infty}$ is a development, there exists n' such that for every $m \geq n'$, no element of $\mathcal{V}_m$ intersects both $\overline{M}$ and X-M'. Also there exists n'' such that for every $m \geq n''$, no element of $\mathcal{V}_m$ intersects both M' and X-O. Let n = larger of n' and n'', and let $N = h_n^{-1}[St(h(x),\mathcal{V}_n)]$.

Claim 1: N is a neighborhood of x. From Lemma 2.4 and choice of $\mathcal{U}'_{n+1}$, for every $m \geq n$, $h_m(x) \subset St^2(h_n(x),\mathcal{U}'_{n+1}) \subset$ compact subset of some $V'_n \in \mathcal{V}'_n$. Thus $h_n(x) \bigcup h(x) \subset V'_n \subset$ some $V_n \in \mathcal{V}_n$, and $x \in h_n^{-1}[St(h(x),\mathcal{V}_n)]$.

Claim 2: $h(N) \subset O$. Choose any $y \in N$. We need to show $h(y) \subset O$. $y \in N$ implies $h_n(y) \in St(h(x), \mathcal{V}_n)$. But from Lemma 2.4 and the choice of $\mathcal{U}'_{n+1}$, for every $m \geq n$, $h_m(y) \subset St^2(h_n(y), \mathcal{U}'_{n+1}) \subset$ compact subset of some $V'_n \in \mathcal{V}'_n$. Thus $h_n(y) \cup h(y) \subset V'_n \subset$ some $V_n \in \mathcal{V}_n$, and $h(y) \subset St^2(h(x), \mathcal{V}_n) \subset St[M', \mathcal{V}_n] \subset O$.

h is 1-1. Suppose there exist x and $y \in X$ such that $h(x) = h(y)$. By definition of h there exists $n(x)$ such that $h_m(x) = h_{n(x)}(x)$ for every $m \geq n(x)$, and there exists $n(y)$ such that $h_m(y) = h_{n(y)}(y)$ for every $m \geq n(y)$. Let n = larger of $n(x)$ and $n(y)$. Then, for every $m \geq n$, $h_m(x) = h_n(x)$, and $h_m(y) = h_n(y)$. Thus $h(x) = h_n(x)$, and $h(y) = h_n(y)$. This is a contradiction since h_n is a homeomorphism.

h is onto. Choose any $x \in X$. We need to show $x \in h(X)$.

Case 1: $x \in X - \bigcup_{i=1}^{\infty} N_i$. $x = h(x)$.

Case 2: $x \in N_k$, some integer k. $N_k = h(N_k) = h_{i(k)}(N_k)$, for some integer $i(k)$. $h_{i(k)}$ is a homeomorphism implies there exists $y \in N_k$ such that $h_{i(k)}^{-1}(x) = y$. Thus $x = h_{i(k)}(y) = h(y)$.

h^{-1} is continuous. Using Lemma 1.4 and construction of $\{\mathcal{U}'_i\}_{i=1}^{\infty}$, the proof works as in Theorem 1. Therefore h is the desired homeomorphism; so G is strongly shrinkable.

COROLLARY 2.1. If X is a strongly developable, locally compact, T_2 space, and G is a countable u.s.c. decomposition such that G satisfies (*), then X/G is homeomorphic to X.

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