
HEAT TRANSFER ANALYSIS VIA RATE BASED SENSORS

A Thesis Presented for the Master of Science Degree

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DEDICATION

This work is dedicated to my parents, Stan and Rita Plewa, who have given so much of their life for me. I hope to one day repay a fraction of the love they have always given me.

It is also dedicated to my grandparents, Frank and Carmella Plewa, Peter and Josephine Travalin, who have been my constant source of encouragement. I love them and feel sorrow for the ones that couldn't witness the result of their praise and confidence. They told me I could reach so I did.

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ABSTRACT

This work presents an integrated rate-based sensor and method for resolving the surface heat flux of a fundamental inverse heat conduction problem without numerical regularization or differentiation in a semi-infinite geometry that additionally accounts for thermocouple delay due to its intrinsic time constant. The sensor uses well-designed analog filters to directly regularize raw voltage data to eliminate the need for numerical regularization methods. The sensor can also be used in a series of well-designed voltage-rate interfaces that directly measure the voltages from in-depth thermocouples, which are used in conjunction with the thermocouple calibration curve to provide higher-time derivatives of the thermocouple's temperature while minimizing noise caused by the system and differentiation. Using a lumped energy balance about the thermocouple's bead, a first-order model is used for relating the thermocouple temperature to the positional temperature. The required thermocouple's time constant is estimated with the aid of a one dimensional finite difference method to solve the direct heat conduction problem and obtain the required positional temperature. Higher-time derivatives of the in-depth heat flux are produced using time integral relationships between the positional temperature and local heat flux. Finally, the surface heat flux and temperature are estimated using the finite difference based Global Time Method. To verify this concept and acquire real data, an experiment was performed using a well-designed heater sandwiched between two identical plates for producing a symmetric temperature distribution in each plate with an accurately known heat input. Encouraging results are presented from this in depth study indicating the merits of the sensor and methodology. Additionally it is demonstrated that the rate sensor data obtained from an array of indepth thermocouples may be used to determine the transition location in hypersonic flights.

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NOMENCLATURE

A = Magnitude of Signal Input
 A_H = Heated Area of the Block, m^2
 b = Distance from Probe to the Surface, mm
 C_1 = First Capacitive Element, nF
 C_2 = Second Capacitive Element, nF
 E_{in} = Signal Input to Derivative Circuit, V
 E_{out} = Signal Output of Derivative Circuit, V
 F = Linear Operator for Global Time Inverse Method
 F_{apx} = Approximated Filter Gain
 f = Frequency of Signal, Hz
 f_c = Cutoff Frequency of Filters, Hz
 G = Gain
 i = Node Index for Global Time Inverse Method
 k = Thermal Conductivity, W/m K
 L = Thickness of the Sample, mm
 N = Surface Node for Global Time Inverse Method
 n = Index for Order of Differentiation
 Q'' = Total Energy Input Per Unit Area, J/cm^2
 q'' = Heat Flux, W/m^2
 q''_{EXP} = Measured Experimental Heat Flux, W/cm^2
 q''_{FD} = Finite Difference Model Heat Flux, W/cm^2
 q''_H = Heated Area Heat Flux, W/m^2
 q''_{IN} = Inverse Method Predicted Heat Flux, W/cm^2
 R = Resistance of the Heater, Ω
 R_1 = First Resistive Element, $k\Omega$
 R_2 = Second Resistive Element, $k\Omega$
 R_3 = Third Resistive Element, $k\Omega$
 r = Residual, W/m^2
 t = Time, s
 T = Positional Temperature, $^{\circ}C$
 T_i = Temperature at Node i , $^{\circ}C$
 T_o = Initial Temperature, $^{\circ}C$
 T_{tc} = Thermocouple Temperature, $^{\circ}C$
 u = Dummy Time Variable, s
 V = RMS Voltage Applied to Heater, V
 y = Spatial Variable in the y-Direction, m
 x = Spatial Variable in the x-Direction, m

 Greek
 α = Thermal Diffusivity, m^2/s
 Δt_{lag} = Time Lag, s
 Δx = Distance Between Global Time Nodes, m
 μ_r = Mean of Residual, W/cm^2
 σ_r = Standard Deviation of Residual, W/cm^2
 τ = Thermocouple Time Constant, s
 τ_d = Derivative Circuit Time Constant, ΩF
 ϕ = Phase Angle, radians

CHAPTER 1: INTRODUCTION

Inverse heat conduction involves using in-depth temperature measurements for predicting the surface heat flux and temperature. In some cases surface instrumentation for measuring heat flux or temperature is neither practical nor reliable owing to highly hostile aerospace heating environments associated with external hypersonic flows or internal hypersonic combustors. The inverse heat conduction problem (IHCP) is ill-posed, that is, small perturbations in the measured in-depth data produce large fluctuations in the predicted surface heat flux [1]. The main objective in IHCP research involves developing and implementing numerical techniques containing a regularization parameter that effectively stabilizes the prediction. The optimal regularization parameter is often difficult to acquire and the resulting prediction can remain sensitive to its choice. Many regularization methods have been proposed for inverse heat conduction [1]. Back propagation of data from the in-depth probe to the surface inherently involves time differentiation of the measured signal which is both noisy and discrete. This is an inherently ill-posed numerical process that if uncontrolled will subsequently decrease the quality of the projection.

1.1 LITERATURE REVIEW

As mentioned many techniques exist to regularize inverse heat conduction data. Beck uses a method of regularization for conduction that utilizes a number of “future” temperatures to stabilize the calculation of surface flux [1,2]. However the method is highly dependent on the time step and number of future temperatures used in the calculation. Too many temperatures and the prediction will be lagged; too few and the prediction will become unstable. It works well when applied correctly, but it requires skill to select the optimum regularization parameters.

Another method proposed by Beck was a filter solution utilizing function specification and Tikhonov regularization [3]. A simple moving summation containing filter coefficients is inserted in to the Tikhonov regularization. Through some difficulty a series of filter coefficients are solved for and when running under the same materials at similar conditions may be used over again. It is a useful method for a process that is repeated using the same materials or setup.

Also available for regularization is Frankel’s derived Gaussian low pass filter [4]. It requires a cutoff frequency based on physics from the problem to be filtered. In this case it is determined in Hertz, and dependent on the thermal characteristics of the system. It utilizes all time data making it a global method. As shown by Elkins [5] it can also be utilized to obtain the derivative of input data while still retaining the ability to regularize.

Using a novel combination of operational amplifiers and passive electrical components it is possible to develop a “practical differentiator” circuit that produces the time rate of voltage change from a voltage input [6]. It has also been shown that by attaching a thermocouple to such a practical differentiator the circuit can be set up to calculate time rate of change of input temperature [7], in other words the time derivative of temperature. This can be used as an alternative to computer processing.

An important concept to surface projection from in-situ probes is penetration time. When heating a surface it takes time for a “heat front” to propagate its way internally and cause an in-situ location to experience a change in temperature or heat flux [8]. Monte has found through analyzing results that a relationship can be established between the penetration time, depth of the sensor, and thermal diffusivity of material. This will be important to establish the semi-infinite nature of a setup.

In this experiment thermocouples will be used to measure temperature. When used in surface prediction of transient conditions a thermocouple will have large effect on the results. Specifically the reported temperature is not the true temperature at that position. This is due to the contact resistance of the sensor bead, the difference in properties between the material and the sensor, the heat losses through the sensor, and other possible difference between the sensor and an undisturbed piece of material. This difference can drastically change the inverse prediction. Woodbury investigated a first order time constant for a thermocouple used in inverse heat conduction [9]. A larger time constant means the positional temperature is higher and earlier than the measured temperature. He also found that a small time constant could have a drastic negative effect on inverse prediction accuracy. Therefore, it is pertinent to characterize the time constant of a thermocouple in any inverse heat conduction experiment.

It is important to examine experimental setups to find the best method to perform tests. One interesting setup from Ji and Jang utilized two identical copper plates, which were placed on top and bottom of a kapton heater [10]. In order to ensure low contact resistance between all surfaces high thermal conductivity paste was applied in between the heater and copper plates. To reduce losses through the sides of the copper plates insulation was placed around the outside of the setup. A heat flux gage was affixed to the rear of one copper plate and a type K thermocouple to the rear of the other. The setup is designed to measure heat flux and temperature simultaneously in order to verify predictions of the surface conditions of the plate. With some improvements as mentioned by Elkins [5], this experiment could be readily adapted to serve this specific inverse experiment.

1.2 SCOPE

As an alternative to numerical differentiation of discrete data, which represents the most fundamental inverse problem, we propose to directly measure the time derivatives of the measured temperature through carefully designed analog circuitry used in conjunction with the thermocouple calibration curve. The circuitry will contain the proper filtering to lessen the effect of error growth and thus avoid unstable surface prediction [7,11]. Voltage generated from an in-depth thermocouple is used as input to a voltage-rate interface to produce a series of time derivatives of voltage. The time derivatives of temperature are recovered using these measured voltage rates and the thermocouple calibration function. With chain rule calculus, we can retrieve the first-, second-, and third-time derivatives of the thermocouple temperature [11]. It should be noted that due to inherent conditions associated with contact conductance and conductive lead losses, the measured signals do not correspond to actual positional values at the probe site.

This novel rate sensor can be useful to inverse heat conduction involving numerical methods that suffer from implicit or explicit numerical differentiation of noisy data. The recently developed Global Time Method [12] is explicitly derived in terms of the time derivatives of the positional temperature and heat flux. Since inverse methods require actual positional values, the collected data must be adjusted to account for the thermocouple time-delay using a first-order approximation [5]. This adjustment translates the measured data to the positional values required by energy considerations (i.e., the heat equation). In this study data collected from the proposed rate sensor is used in the newly devised numerical method [12]. It should be added that applications such as transition detection in hypersonic flows, detection of icing on airplane wings, and thermal control systems can benefit from availability of multiple analog time derivatives of temperature data.

CHAPTER 2: GLOBAL TIME METHOD FOR INVERSE HEAT CONDUCTION

2.1 GLOBAL TIME METHOD INTRODUCTION

The Global Time Method is a novel inverse heat transfer technique that proves to have many benefits in current experimental applications. As opposed to normal inverse techniques that become unstable with increasing data sampling rate, the Global Time Method increases in accuracy with higher data density [12]. Also most techniques use some form of finite difference method to calculate the time derivative of temperature. This is one cause of instability in inverse methods, as small perturbations in the input result in large fluctuations in the output. In the Global Time Method the time derivatives of temperature are used as data input, this along with the use of ingenuitive data sampling can minimize noise generated by derivatives. Namely the rate sensor introduced in this chapter will measure the time derivatives of temperature directly and avoid the instability in finite differencing and the errors that may result from regularization.

2.2 RATE SENSOR INTRODUCTION

The Rate Sensor is made from a chain of circuits that act together to measure multiple time derivatives of temperature. The Rate sensor consists of an amplifier, a filter, and three practical differentiators [6, 7] in series. By probing the circuit at various points with a data acquisition board, we measure the various time derivatives of voltage and through a simple conversion process the time derivatives of temperature are obtained. The schematic of stages of a rate sensor capable of measuring up to three temporal derivatives is displayed in Figure 2.1. Also displayed in Figure 2.1 is the path for digital derivative comparison, a check of the analog derivatives.

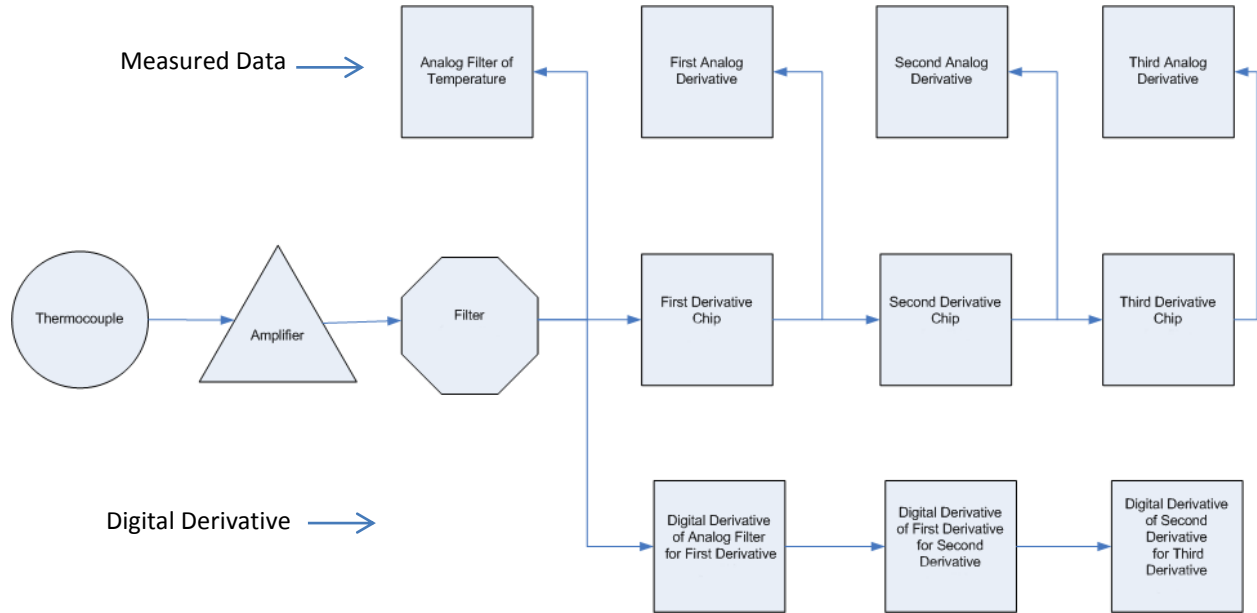


Figure 2.1: Schematic of the Rate Sensor, Sampling Points and Logic Flow for Digital Derivative Comparison.

2.3 SOLUTION PROCEDURE

The Global Time Method begins with the one-dimensional heat equation with constant properties as

$$\frac{\partial T}{\partial t}(x, t) = \alpha \frac{\partial^2 T}{\partial x^2}(x, t), \quad 0 \leq x \leq L, \quad t \geq 0 \quad (2.1)$$

subject to the initial condition of

$$T(x, 0) = T_o, \quad 0 \leq x \leq L \quad (2.2)$$

The objective is to solve for the unknown boundary condition at $x=0$, i.e., surface heat flux $q''(0,t)$. It should be noted that the temperature sensor is located at $x=b$ such that $b \ll L$ and the duration of the heating time is short relative to the thermal penetration time of the slab, i.e., elapsed time for the heat front to reach the back face of the slab at $x=L$. Figure 2.2 displays a Nodal Arrangement of the Global Time Solution for clarification.

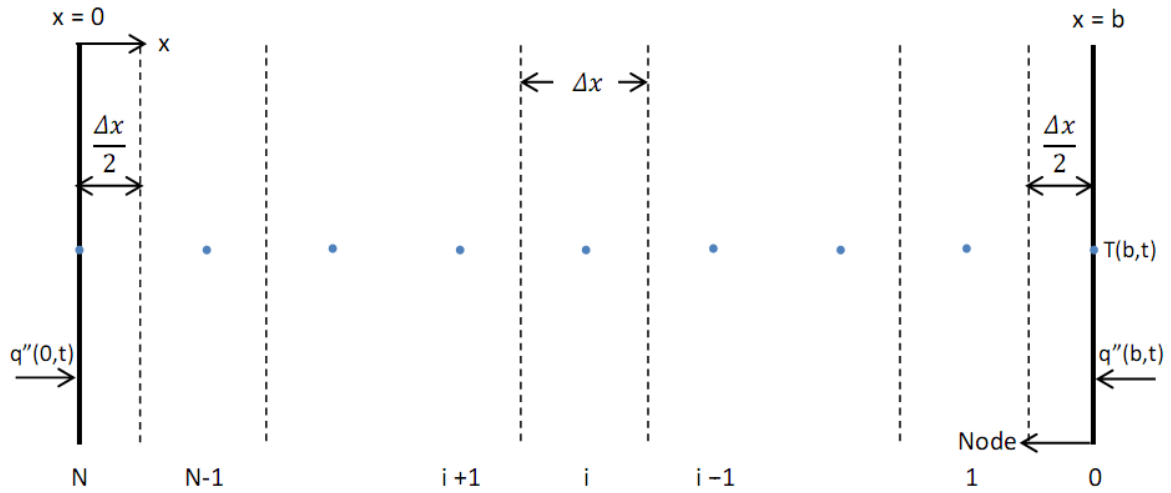


Figure 2.2: Global Time Method Nodal Arrangement for N Control Volumes.

The Global Time Method begins with a simple central difference approximation of the second derivative of temperature with respect to x ,

$$\frac{\Delta x^2}{\alpha} \frac{\partial T_i}{\partial t} = T_{i+1} + T_{i-1} - 2T_i, \quad i = 1, 2, \dots, N-1 \quad (2.3)$$

We define $i=0$ to be our in-depth probe location, which is a distance of b from the surface ($x=b$). This leaves $i=N$ to be the surface ($x=0$) node, so we will divide the distance from the probe site to the surface into N control volumes. For later simplification we now define the F operator,

$$F = \frac{\Delta x^2}{\alpha} \frac{\partial}{\partial t} + 2 \quad (2.4)$$

Equation (2.3) is now rewritten in terms of T_{i+1} :

$$T_{i+1} = FT_i - T_{i-1}, \quad i = 1, 2, \dots, N-1 \quad (2.5)$$

By performing an energy balance at the boundary $x=b$ we can obtain an equation in terms of the probe location or $i=0$,

$$T_1 = \frac{F}{2} T(b, t) + \frac{\Delta x}{k} q''(b, t) \quad (2.6)$$

where $T(b, t)$ and $q''(b, t)$ are the temperature and heat flux at the probe site. In this example we will use $N=3$. Using the control volume energy balance and boundary condition for node $i=0$ and multiple substitutions we can solve for the surface ($i=3$) temperature,

$$T(0, t)_{N=3} = \left(\frac{1}{2} F^3 - \frac{3}{2} F \right) T(b, t) + \frac{\Delta x}{k} (F^2 - 1) q''(b, t) \quad (2.7)$$

Performing another energy balance on the surface leads to:

$$q''(0, t)_{N=3} = \frac{k}{\Delta x} \left(\frac{F}{2} T(0, t)_{N=3} - T_2 \right) \quad (2.8)$$

This can be solved in terms of known temperature and heat flux at the sensor site to yield,

$$q''(0, t)_{N=3} = \frac{k}{\Delta x} \left(\frac{1}{4} F^4 - \frac{5}{4} F^2 + 1 \right) T(b, t) + \left(\frac{1}{2} F^3 - \frac{3}{2} F \right) q''(b, t) \quad (2.9)$$

An inspection of Eq. (2.9) shows that inverse solution of the surface heat flux for $N=3$ will require the fourth temporal derivative of temperature and the third temporal derivative of the heat flux at the probe site. Using an integral relationship with an input of temporal derivatives of the temperature at the probe site, we arrive at the desired temporal derivatives of local flux [4],

$$\frac{\partial^n q''}{\partial t^n}(b, t) = \sqrt{\frac{k^2}{\alpha \pi}} \int_{u=0}^t \frac{\partial^{n+1} T}{\partial u^{n+1}}(b, u) \frac{du}{\sqrt{t-u}} \quad (2.10)$$

It is noted that Eq. (2.10) is applicable to half-space heat conduction ($b \ll L$), where the heating time is short relative to the thermal penetration time of the finite slab of thickness L . It is important to note that all temperatures used in the above equations are positional temperature, not what is measured. The conversion from measured to positional is discussed in later sections.

CHAPTER 3: EXPERIMENTAL SETUP AND PROCEDURE

3.1 DESIGN

Figure 3.1 shows a diagram of an electrical heating experiment with embedded thermocouple sensors. The test samples were two identical Stainless Steel plates with a heater assembly sandwiched between. They were coated with a thin layer of Omega Thermal Paste on their heated-side faces. Since the purpose of the paste was to reduce contact resistance, the thinnest layer possible was used. A thin layer of muscovite mica (0.08mm thick) was laid between the plates. The steel/paste/mica layers were then used to sandwich a 0.125 mm thick custom nichrome heater element. Figure A.1 shows a conceptual sketch of the custom nichrome heater designed by Dr. Majid Keyhani. This created a line of symmetry across the centerline of the heater. Thermophysical and electrical properties of these materials can be seen in Table A.1 of the appendix, while material thicknesses are summarized in Table A.2 also in the appendix.

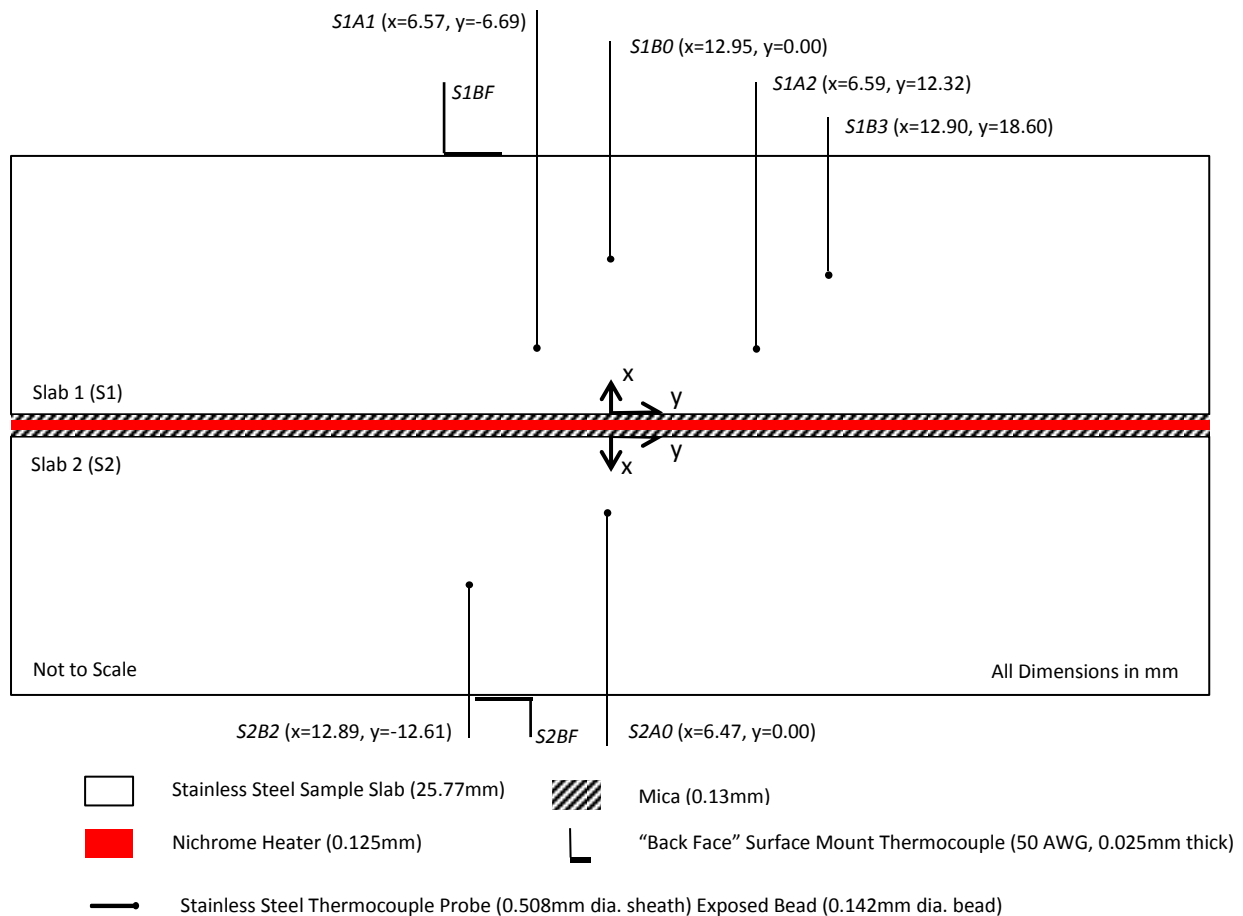


Figure 3.1: Electrical Heating Experimental Setup, a Line of Symmetry Exists along the Centerline of the Heater. Not to Scale.

Multiple 0.047 in. diameter holes were drilled into both of the stainless steel plates from the back surface (perpendicular to the heated surface). These holes were drilled to two different relative depths, the depth closer

to the heated surface known as the “A” depth all of which are approximately 6.5mm from the heated surface. The second depth, “B” depth, had holes all about 12.9mm from the heated surface. The depth of each hole was rigorously measured using a Micro Val coordinate measuring machine; these depths are presented in Table A.3 of the appendix. Each hole follows a naming system that starts with which slab it is imbedded in, S1 for Slab 1 and S2 for Slab 2. Next the depth to the surface is indicated by A or B. And finally the horizontal distance from the slab centerline is indicated by the last number; the higher that number the farther the hole will be from the center of the slab. Therefore S2A0 is on the centerline of slab 2 and about 6.5mm from the heated surface. The horizontal distances from centerline are also displayed in Table A.3.

Type T thermocouple probes (Omega TMTSS transition junction style, 38 AWG thermocouple wire, with exposed bead) with a sheath diameter of 0.020 in. were potted in each hole using Cotronics 989F (alumina paste). Confirmation that the thermocouple bead was in contact with the stainless steel was achieved by measuring the electrical resistance between the thermocouple’s copper lead wire and the stainless steel slab. The resistances are given in Table A.3, which verified contact between the thermocouple bead and the stainless steel slab since the potting compound was an electrical insulator. Fine gage (50 AWG wire) surface mount thermocouples (Omega SA1XL-T) were affixed to the back (unheated) surface of each stainless steel plate.

The thermocouple’s emf outputs were sampled at 200 Hz with a gain of 32 via a DT9824 data acquisition board (DAQ) – a 24 bit, $\pm 10V$ Range, low noise, fully isolated DAQ with simultaneous channel measurement. The measured voltages were converted to temperature via the NIST polynomial calibration curve for type T thermocouples [13].

Unregulated alternating current (60 Hz) was supplied to the heater from a 120VRMS, 50A line. This voltage supply was wired to a Variac to allow variation of the input voltage. The output of the Variac was wired to an 80A solid state relay which was software triggered. The output of the relay was wired to the nichrome heater. The voltage across the heater was also connected to a 1/28.67 voltage divider which was sampled at 4800 Hz via another DT9824 data acquisition board. For a detailed list of equipment and material used in the sandwich experiment see [14]. Finally, the input heat flux (q_H'' , referred to as measured heat flux) that enters each block at the surface is calculated by

$$q_H'' = \frac{V^2}{2 R A_H} \quad (3.1)$$

where V is the measured RMS voltage in the heater, R is the resistance of the heater and A_H is the heated area.

3.2 RATE SENSOR

The entire rate sensor is constructed from resistors, capacitors, and commercially available integrated circuit chips. Figure 3.2 presents an overview schematic of the entire rate sensor. Detailed schematics of each part will be presented and discussed in subsequent figures. The rate sensor is broken up into five parts: the instrumentation amplifier, the pre-filter, and three practical differentiators. The circuit is powered via two 9V batteries that provide a positive and negative power pole as all the chips in the circuit are dual supply. The Rate Sensor is currently assembled on a prototyping “bread board”.

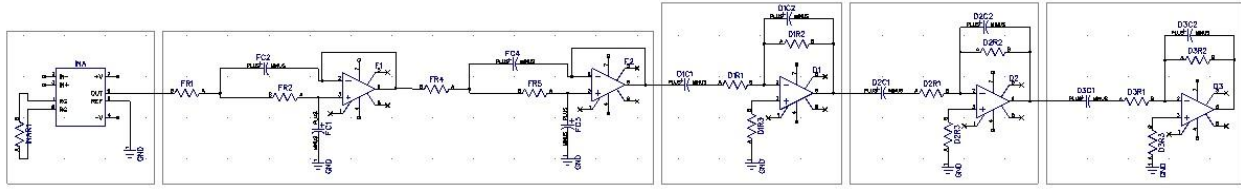


Figure 3.2: Circuit Schematic of Rate Sensor. Each part is boxed in series: INA, Filter, Derivatives.

INSTRUMENTATION AMPLIFIER

The instrumentation amplifier serves three purposes in the circuit. First, it isolates the thermocouple from the rate sensor which reduces interference. Second, it has a high common mode rejection ratio which is important in eliminating electrical noise. Finally it amplifies the thermocouple signal to the volt range allowing more accurate measurement and transformation. In order to minimize space usage, we used a premade INA integrated circuit; it is a Burr-Brown INA114. The schematic below, in Figure 3.3, displays the INA chip and its pin connections. Both ends of the thermocouple are used for input while a resistor labeled INAR1 controls the gain of the chip. The gain of the INA has been verified at 550:1. This gain is suitable for the current application, though higher heating rate experiments may necessitate a change of the gain.

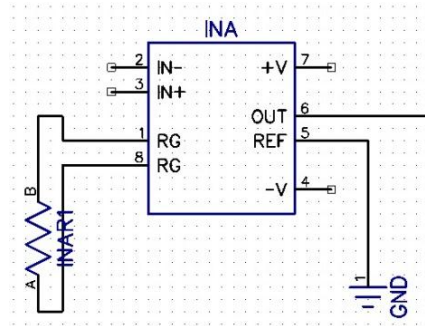


Figure 3.3: Instrumentation Amplifier (INA) Schematic.

PRE-FILTER

The Pre-Filter is a 4th order Butterworth Filter, pictured in Figure 3.4, built on a Sallen-Key Architecture utilizing OP-27s. It serves to clean the incoming signal of unwanted noise present due to background signals. The choice cutoff frequency, f_c , must be selected from analysis of the DFT response of raw thermocouple data [4]. The optimal f_c will remove most incoming noise with minimal effect to the true signal [11].

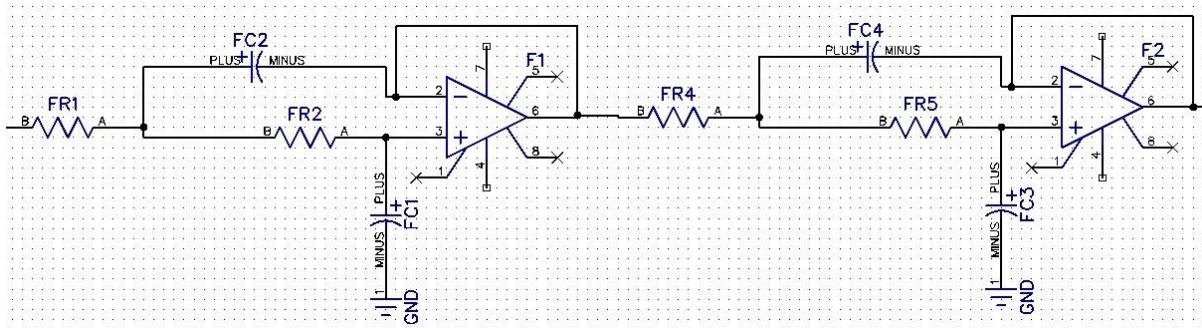


Figure 3.4: Pre-Filter Schematic.

DERIVATIVE CIRCUITS

Figure 3.5 displays the schematic for a single Practical Differentiator Circuit [7,11]. All three circuits follow the same layout and are linked in series so that each takes a further time-derivative of its input, resulting in higher order time-derivatives. Each circuit also includes a built-in, first-order filter to compensate for the noise that can develop and amplify when obtaining derivatives. The op-amp used in all the derivative circuits is an OP-27 chosen for its low noise and low voltage characteristics. Though each differentiator follows the same layout, the individual component values may slightly differ from differentiator to differentiator.

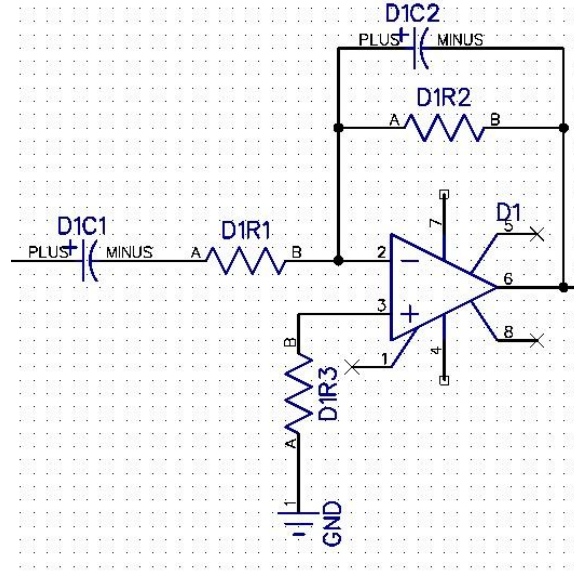


Figure 3.5: Practical Differentiator Schematic.

RATE SENSOR THEORY

The governing equation for each derivative circuit, derived from impedance circuit theory is as follows,

$$\tau_D^2 \frac{d^2 E_{out}}{dt^2} + 2\tau_D \frac{dE_{out}}{dt} + E_{out} = -G \frac{dE_{in}}{dt} \quad (3.2)$$

$$\tau_D = R_1 C_1 = R_2 C_2 \quad (3.3)$$

where R_1 , R_2 , C_1 , and C_2 are the respective resistances and capacitances that make up the derivative circuit, and τ_d is the effective time constant of the derivative circuit. The gain of the derivative circuit, represented by G , from input to output is determined by

$$G = R_2 C_1 \quad (3.4)$$

The component values of the circuits are determined by two user selected inputs, the gain G and the cutoff frequency f_c . The cutoff frequency is defined as

$$f_c = \frac{1}{2\pi\tau_D} \quad (3.5)$$

The cutoff frequency of the circuit determines what frequencies that pass through the circuit get attenuated and which pass through unmolested. In Eq. (3.2) the first two terms in the left hand side can be considered the

filtering effect of the practical differentiator. When the frequency of signal passing through the circuit, f , is much less than the cutoff frequency those terms become very small and can be neglected. Therefore, the output can be approximated as

$$E_{out} = -G \frac{dE_{in}}{dt}, \quad f \ll f_c \quad (3.6)$$

However this will not always be the case and to properly model the circuit Eq. (3.2) must be solved explicitly for E_{out} . In the solution E_{in} is modeled as a sine wave input with a given frequency.

$$E_{in} = A \sin(2\pi f t) \quad (3.7)$$

where t is time and A is the magnitude of the input. Solving for E_{out} results in

$$E_{out} = -\frac{1}{(1 + 4f^2\pi^2\tau_D^2)^2} [-2\pi G A f (-1 + 4f^2\pi^2\tau_D^2) \cos(2\pi f t) + 8AGf^2\pi^2\tau_D \sin(2\pi f t)] \quad (3.8)$$

Equation (3.8) allows direct simulation of the output of the derivative chips. While the solution of E_{out} looks like a complicated figure, graphically it will resemble

$$E_{out,apx} = -F_{apx} G 2\pi f A \cos(2\pi f t + \phi) \quad (3.9)$$

where F_{apx} is the attenuation caused by filtering and ϕ is the phase angle of the circuit. Phase angle is related to the time delay of the circuit via

$$\Delta t_{lag} = \frac{\phi}{2\pi f} \quad (3.10)$$

Time delay, Δt_{lag} , is the amount of time a signal is shifted due to passing through the circuit. Therefore to correctly compare the input and output of the derivative circuit one must account for the shift in time that is the time lag.

RATE SENSOR DESIGN AND TESTING

In our design, after selecting the values for f_c and G , we set $C1$ to a measured value and determine all other component values using Eqs. (3.3)-(3.5). However, since capacitance and resistor values are only available at set discrete intervals, the actual values used in circuit may differ slightly from the calculated values.

One important part of constructing this circuit is verifying that it is working properly. This is a process known as characterization. A precise voltage source, the Fluke 5500A calibrator, was used to put a known input into the circuit then the outputs were measured. The tests showed each circuit to give the correct derivative of a known input, e.g., a cosine output of a sine input.

With this relevant circuit theory it is possible to use the component values in each circuit to generate its characteristics. The measured values of components of each circuit along with the governing characteristics derived from those values are listed in Table A.4 for Pre-Filter and Table A.5 for derivative circuits. It is important to note that the value of τ or time constant is used for time lag. In experimental verification of these characteristics, it was found that τ was an acceptable assumption for time lag.

The characteristics of the rate sensor were decided from a combination of systematic changes based on the test setup and trial and error. The gain of the INA is always determined by the inputs of the test. The goal is to raise the thermocouple signal to in order to maximize the signal to noise ratio. The limitation is the measuring

capability of the DAQ system; the best case will have a signal with a large signal to noise ratio and remain within the bounds that the DAQ can measure.

The cutoff frequency of the filter and derivatives are determined by the frequency content of the signal at each stage in the circuit. The pre-filter cutoff frequency can be chosen via analysis of a discrete Fourier transform (DFT) of the input thermocouple signal [4]. By determining which frequencies are noise and which are actual signal a value for cutoff frequency will become clear. The derivative cutoff frequencies are determined the same way by analyzing the signal input to each circuit. It is vital to ensure that only noise is being attenuated by the filter; otherwise there will be distortions in the real signal. The ideal cutoff frequencies will remove the most noise without disrupting the actual signal. A method of trial and error with different inputs was done to ensure these cutoff frequencies.

3.3 TEST PROCEDURE

The procedure for conducting a run took the form of the following steps. All instruments were allowed their proper warm-up times, the longest being 1hr. First the resistance of the heater was recorded for later use in determining exact heat flux input and to ensure that its value had not changed because two separate segments of the heater were touching. Next the resistance between the sample and the heater was measured. This tests the current condition of the insulating mica, any reading besides overload indicated that the mica has been damaged and the heater is electrically connected to the sample. Next all connections between heater, power source, and the voltage bridge were made and double checked. Connections between the rate sensor and the desired thermocouple were made and recorded; then all rate sensor channels were hooked up to the data acquisition system. The final steps take place in the Matlab program, first all proper settings were input, and then a test run with no input power ensured their accuracy and tested lab conditions for base noise level. Unacceptable base noise, quantified by the standard deviation of the measurements during this 'dry' run, would result in repeating this step or waiting until noise conditions were optimal. This run also had the function of confirming constant temperature conditions throughout the sample. With all pre-tests complete the wall power source is turned on and the final run is made. During the run external voltage and amperage meters are used to verify the DAQ system readings.

3.4 THERMOCOUPLE TIME CONSTANT PROCEDURE

All thermocouple data collected must be corrected for the characteristics of the thermocouple used in the test. In this experiment there is a specific method of determining the characteristics of a thermocouple and correcting all data. To accomplish this first conduction heat transfer inside the sample was modeled with a three layer (half-heater, mica and stainless steel), one-dimensional, constant properties finite difference (FD) code [5]. The code required the specification of the boundary conditions at each surface, namely the heater surface flux and the temperature at the back face. The half-heater surface was specified to be adiabatic. The heater power was modeled as volumetric rate of heat generation, the effect of the thermal paste was neglected, and perfect contact was assumed between each layer. The actual transient heater power measurements, from Eq. (3.1), were converted to volumetric rate of heat generation for input to the FD model. The back face temperature boundary condition was specified to be the filtered measured back face temperature. This resulted in a positional temperature and heat flux at any desired position in the sample. The results of the FD model will be used as a check for all of our prediction methods.

As noted earlier, the inverse projection requires positional data; therefore, the measured values must first be corrected for the thermocouple probe delay (time constant). The thermocouple response is modeled as a first order system and the measured rate sensor data is corrected to positional values by

$$\frac{\partial^n T}{\partial t^n}(b, t) = \tau \frac{\partial^{n+1} T_{tc}}{\partial t^{n+1}}(b, t) + \frac{\partial^n T_{tc}}{\partial t^n}(b, t), \quad t \geq 0 \quad (3.11)$$

where $T(b, t)$ is the positional temperature, b is the depth of the probe from the surface, t is time, τ is the time constant that accounts for contact resistance [5], and T_{tc} is the temperature reading of the thermocouple. For temperature correction $n=0$ is used and by using $n = 1, 2$, and 3 in Eq. (3.11) one can correct the first, second, and third time derivatives of the data to the positional values, respectively. It is noted that the positional temporal derivatives of temperature obtained from Eq. (3.11) are used in Eq. (2.10) to obtain the needed positional temporal derivatives of the heat flux at $x=b$.

With the temperature from the FD model we have a decent approximation of the positional temperature, and the rate sensor provides the measured temperature and measured first time derivative of temperature. Studying Eq. (3.11) all inputs are now known except for τ ; it can now be solved for τ . Since the temperature data is over a range of time, τ was derived using a least squares method on the data over the period of interest. Interestingly the time constant was fairly insensitive to this time range. This results in a unique τ for each data run. With τ in hand, all rate sensor data was converted to positional data using Eq. (3.11) and $n=0:3$. However due to limitations on the rate sensor a digital derivative of the fourth time derivative of temperature had to be used to acquire the third positional time derivative of temperature.

CHAPTER 4: RESULTS

All of the experimental rate sensor runs used the same rate sensor setup, nicknamed Board W. It had an $f_c = 0.5\text{Hz}$ Pre-Filter, $f_c = 1.0\text{Hz}$ first and second derivatives, and $f_c = 2.0\text{Hz}$ third derivative. All data was also sampled with the aforementioned DT9824 at 200Hz and internal gain of 32 except the filtered temperature which required an internal gain of 16; changing this gain decreased the range of the DAQ from the normal $\pm 10\text{V}$. Each experiment set also used the same thermocouple for rate sensor input named S1A1, as displayed in Figure 3.1, which was 6.57mm from the heated surface of the stainless steel slab. All heating curves were applied by physically manipulating a variable voltage transformer. In the Rate Sensor Experiment there are three different test runs named by the type of power input used. Test case 1 is a step run where the power was suddenly turned on and suddenly turned off, test case 2 was a low ramp run where the heating was slowly turned on and slowly turned off to replicate a shape similar to a triangle, and test case 3 was a high ramp run which was a similar shape to case 2 but the max heat flux was much higher.

In order to verify and compare results a few forms of measurement are needed. These metrics will be used to examine the predicted results. First Q'' is defined as the total input energy per unit area, and calculated as

$$Q'' = \int_{u=0}^{u=t_{\max}} q''(0, u) du \quad (4.1)$$

Where $t_{\max}$ is the final time of the experiment and $q''(0, u)$ is the input surface heat flux. The percent difference between calculated total energies of two data sets will be important for comparison of accuracy between predictions and references. This percent difference will be calculated in two different ways; in the first half of the results section a measured experimental heat flux will be compared to a finite difference model to verify the finite difference model. In this case the percent difference will be the FD total energy minus the experimental total energy divided by the experimental value. For the second half the FD model predicted heat flux will become the reference for surface predictions. In this case the percent difference in total energy will be calculated as the inverse surface prediction minus the FD reference divided by FD reference. Another important value is the residual of the two flux data sets; it is defined as

$$r_E(t) = q''_{FD}(t) - q''_{EXP}(t) \quad (4.2a)$$

$$r_I(t) = q''_{IN}(t) - q''_{FD}(t) \quad (4.2b)$$

Where (4.2a) applies to the first half of the results section, comparing FD data to experimental and (4.2b) applies to the second half, the inverse surface predictions vs. the FD model. To easily compare sets of data the mean of the residual, denoted μ_r , and the standard deviation of the residual, denoted σ_r , will be calculated. These metrics are used to ascertain the accuracy of the predicted heat flux.

4.1 THERMOCOUPLE TIME CONSTANT DETERMINATION

As mentioned earlier the FD model was used to determine the time constant for the thermocouple of interest, S1A1. The inputs to the FD model are the power output of the heater and the back face thermocouple temperature. The filtered back face temperature is what was actually used in the FD code, as the level of noise present in the measurement can disrupt the FD code's accuracy. For each test case the two inputs are displayed in Figure 4.1 - Figure 4.3. The most noticeable feature of Figure 4.1a is the increase in noise between 15s and 20s. This noise is caused by turning on power to the heater. In the experiment the electrical heater causes a fair bit of interference with measurement. Many steps were taken to minimize this noise, but all traces of it could not be removed. This is one of the reasons the filtered back face, shown on the graph, is what is actually used in the

Finite Difference Code. The filtered version can be seen to give an excellent approximation of the noisy back face temperature. The digital cutoff frequency of 1.5Hz was chosen by trial and error. It may be noted that the correction of the back face thermocouple data to positional values will have a negligible effect on the FD model results during the half space period of the tests. The input power is calculated from the transient measured voltage used by the heater and the measured resistance of the heater via Eq. (3.1). It appears to have much less noise than the BFTC graph; however, this is due to how this data is obtained. Namely the fact that the data is RMS voltage which means it is averaged over a set of peak to peak measurements. In addition this curve is a high powered step input; this also improves the signal to noise ratio.

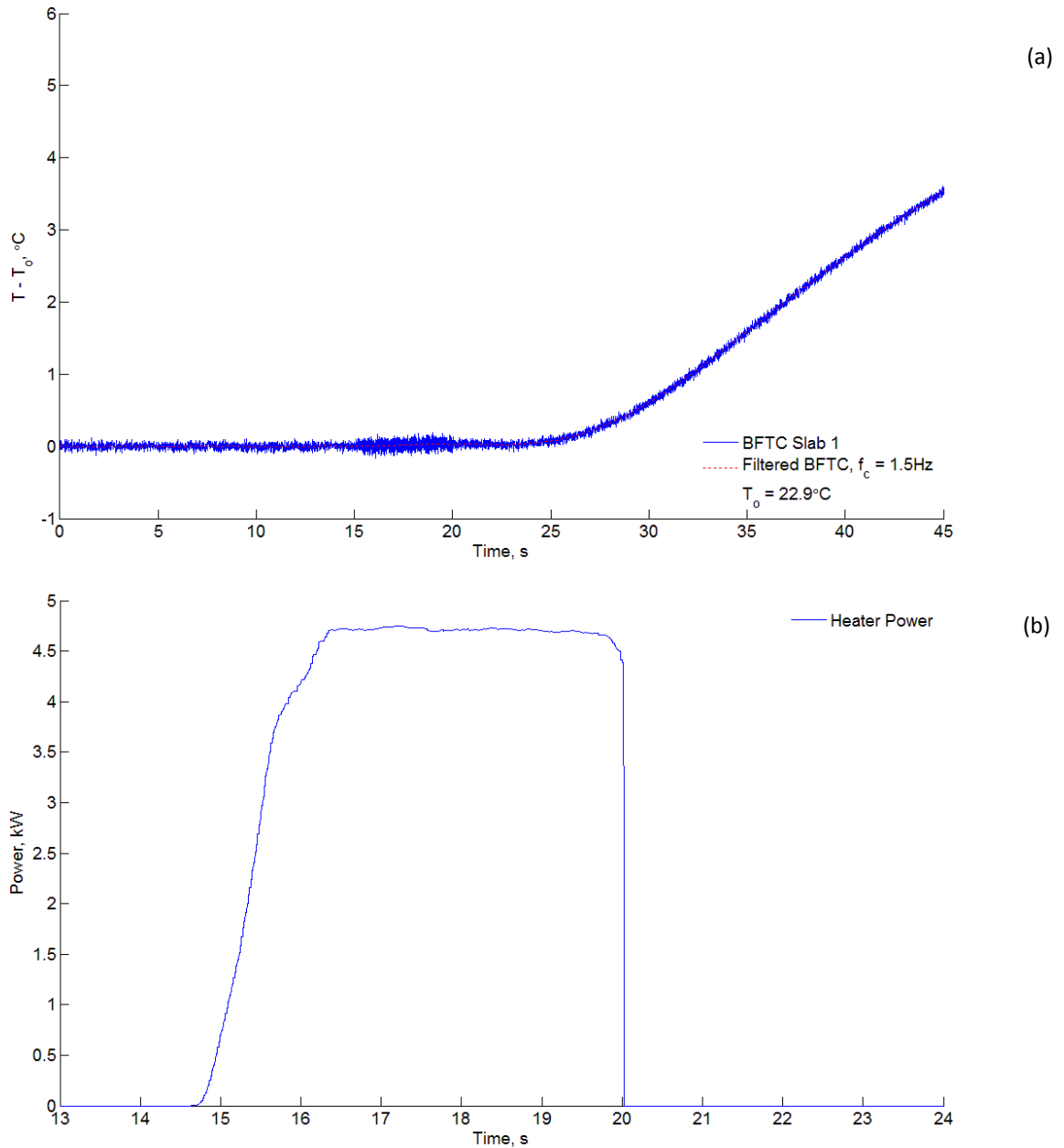


Figure 4.1: Test Case 1 Inputs to Finite Difference Direct Model Code. (a) Measured Temperature of Slab Rear Surface, 27.9mm from surface and (b) Measured Power usage of Heater. Heat Starts at 14.64s and is 5.39s in Duration.

Investigating Figure 4.2 it seems that there has been a large increase in noise. This is true to an extent, but if one looks at the maximum value for both graphs it is apparent that the signal level is much decreased. This is due to the low power input to the heater. This will lower the signal to noise ratio making the data appear to be much noisier. Once again there is an increase in noise during the heating period. These cumulative noises make it even more crucial for the use of the filtered back face temperature. The low power ramp that causes signal to noise drop can be seen in Figure 4.2b; it is a short heating period with gradual rise and fall of power input.

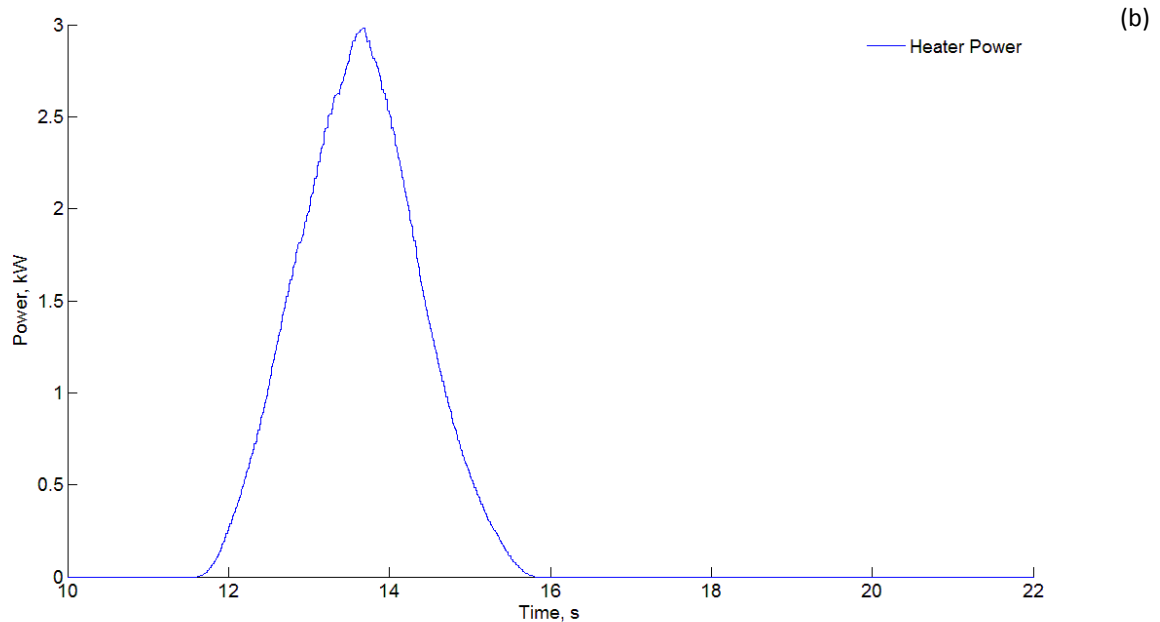
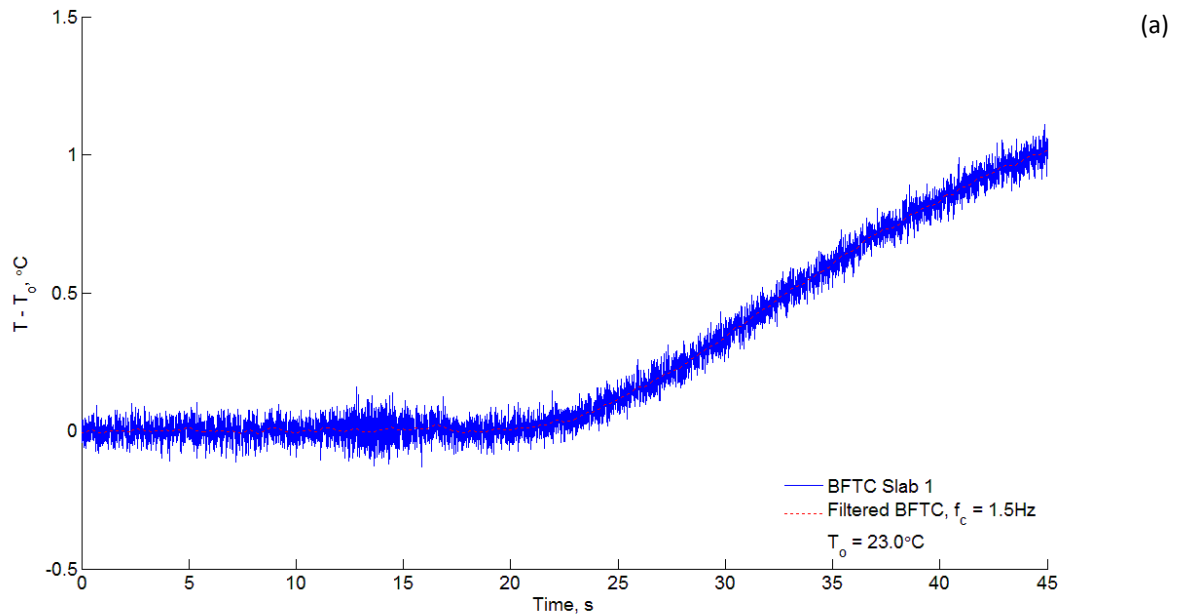


Figure 4.2: Test Case 2 Inputs to Finite Difference Direct Model Code. (a) Measured Temperature of Slab Rear Surface, 27.9mm from surface and (b) Measured Power usage of Heater. Heat Starts at 11.61s and is 4.20s in Duration.

The final test case, in Figure 4.3, had the highest and longest heating of all the other tests. Therefore it also has a very good signal to noise ratio. This greatly improved the results in post-processing. As in the other tests there is an increase in noise during the heating period.

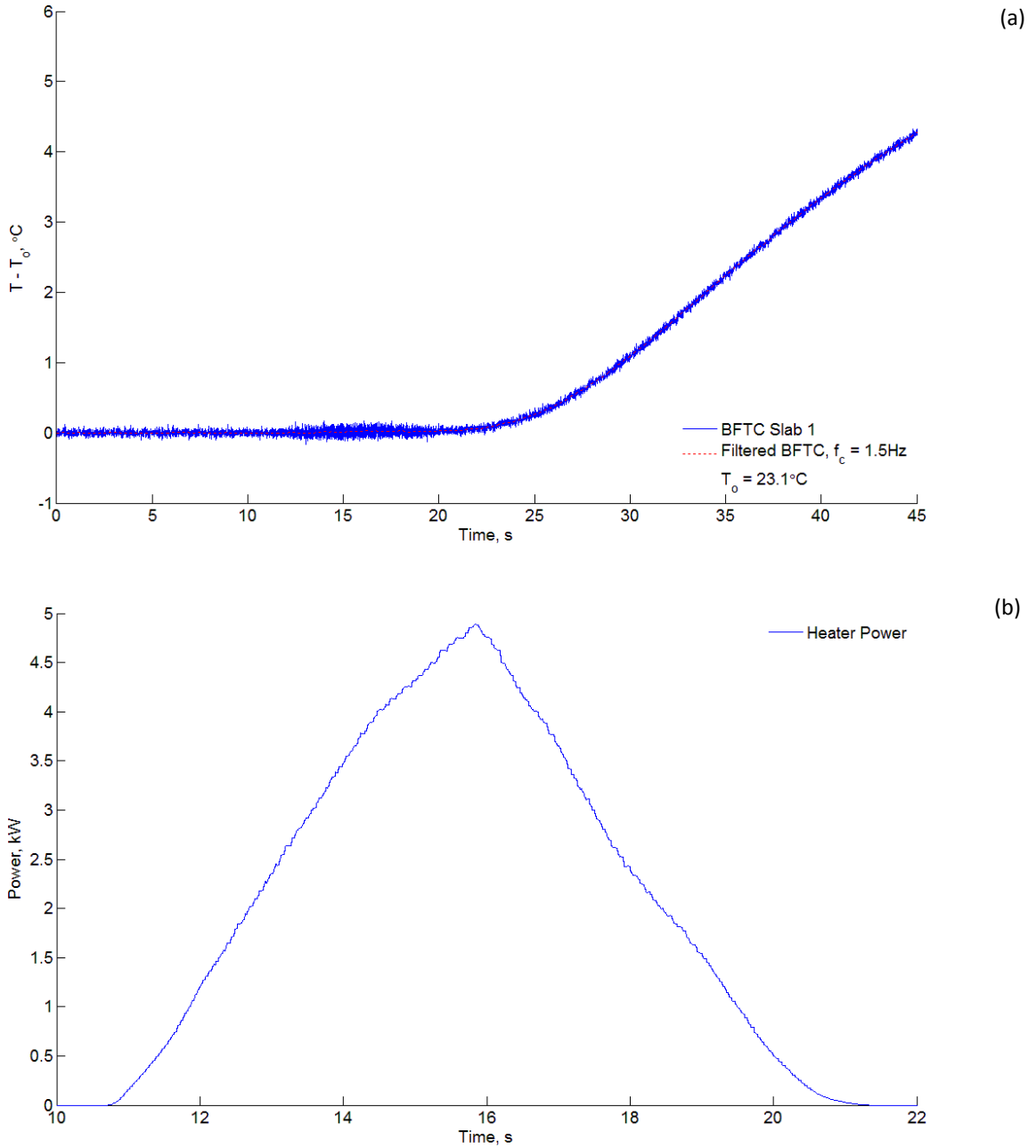
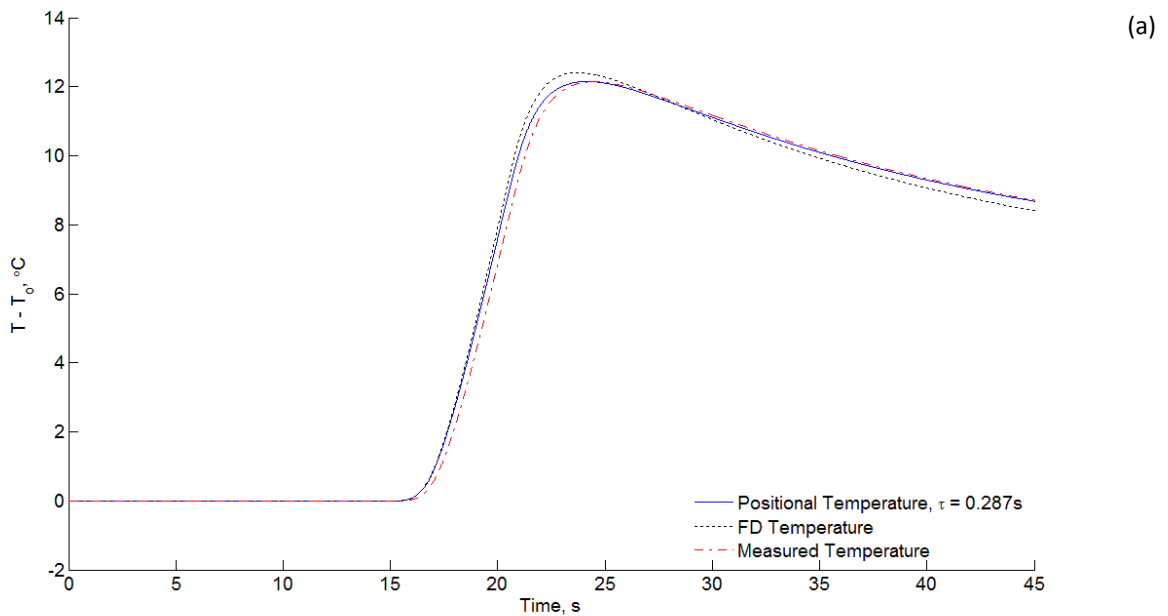


Figure 4.3: Test Case 3 Inputs to Finite Difference Direct Model Code. (a) Measured Temperature of Slab Rear Surface, 27.9mm from surface and (b) Measured Power usage of Heater. Heat Starts at 10.72s and is 10.62s in Duration.

The back face thermocouple will only experience a change in temperature once the heat front from the heater reaches it on the back face. This is called the penetration time of the sample. It is important because before this time elapses the sample may be approximated as an infinite half space. Once this heating of the back face occurs the sample is exposed to a convection/radiation boundary condition. As seen in Figure 4.3a the penetration time is about 12s.

As mentioned before, to calculate τ we compare the measured temperature to the FD temperature result. The FD method is also used to obtain a comparison of heat flux incident on the sample's heated surface. It will be a better match for our prediction than the heat flux calculated from measuring voltage input to the heater because it is located at the surface of the sample instead of the centerline of the heater. Figures 4.4-4.6 detail the pertinent outputs of the FD code for each case, namely the temperature at the thermocouple location and the flux at the surface. Due to small variations in each run the calculated τ 's will differ slightly. How this is dealt with is discussed later.

Figure 4.4a displays the measured temperature at $b=6.57\text{mm}$, the positional temperature obtained from the FD model and the experimental positional temperature. The time constant ($\tau=0.287\text{s}$) listed on the graph is calculated using the FD solution for the positional temperature, $T(b,t)$, and the measured thermocouple temperature, $T_{tc}(b,t)$ in Eq. (3.11) with $n=0$. Differences in the peaks of the finite difference temperature and the corrected positional could be due to a number of factors. First there is some error in the least squares method used to solve for τ ; it can depend on what data is given and the noise that exists in such data. The second reason for the discrepancy could be caused by measurement error. The devices used to measure temperature are never perfect and thus can vary depending on the specific equipment. Finally the FD solution used for comparison could also have errors in it due to many of the same reasons as well as uncertainty in the input thermophysical property values.



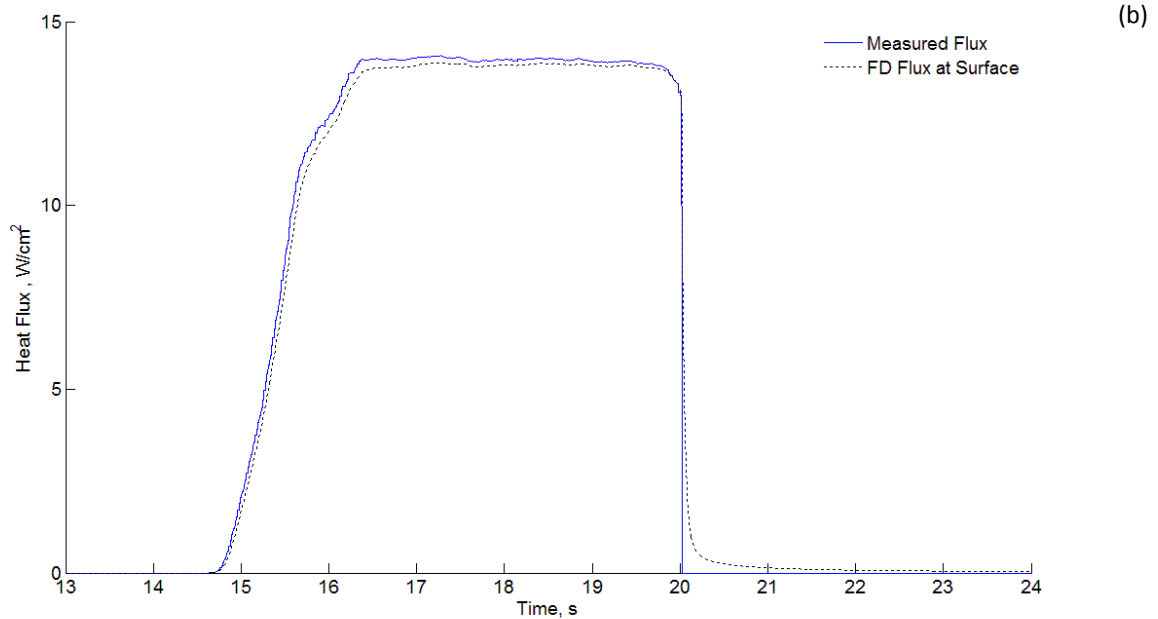


Figure 4.4: Test Case 1 Outputs of Finite Difference Solver. (a) Measured Thermocouple, Finite Difference, and Positional Corrected Temperatures at 6.57mm; and (b) Measured from Heater and Finite Difference Flux at Surface.

Figure 4.4b compares the FD model solution for stainless steel surface heat flux histories with the measured flux, calculated via Eq. (3.1), which neglects the energy storage in the heater and mica. The percent difference between the energy inputs (J/cm^2) of the two heat flux results in Figure 4.4b is -0.26%. The mean of the residual of the difference between the two heat fluxes is $\mu_r = -0.0028 \text{ W}/\text{cm}^2$, and the standard deviation to $\sigma_r = 0.212 \text{ W}/\text{cm}^2$. This indicates the two were extremely close on average and their mean oscillation did not grow very large. In this case the largest residual occurred at the points of discontinuity, the power on and power off times. The calculated incident heat flux to the surface of the stainless steel plate by the FD model was used for comparison with the inverse surface heat flux prediction. The FD model is a better rule of measure because it conveys the actual slab heat flux, unlike the measured data which is a representation of flux at the centerline of the heater.

Figure 4.5a is once again the temperature graph used for determining τ , this time for test case 2. Again using Eq. (3.11), the FD temperature is used as positional temperature and with the measured temperature, τ can be calculated. With $\tau = 0.291 \text{ s}$ in hand and again with Eq. (3.11), applying it to the measured temperature reveals the data listed as positional temperature.

Again like the last set of plots Figure 4.5b displays the measured and FD model flux. Since the FD flux will be used for inverse comparison it must be verified independently for accuracy. The percent difference between energy in the measured and FD resolves to an insignificant -0.25%. The mean residual at $\mu_r = -0.007 \text{ W}/\text{cm}^2$ with a standard deviation of $\sigma_r = 0.055 \text{ W}/\text{cm}^2$ indicate that both values are very close and stay that way over the entire time period. These results provide confidence in using the FD model for surface comparison.

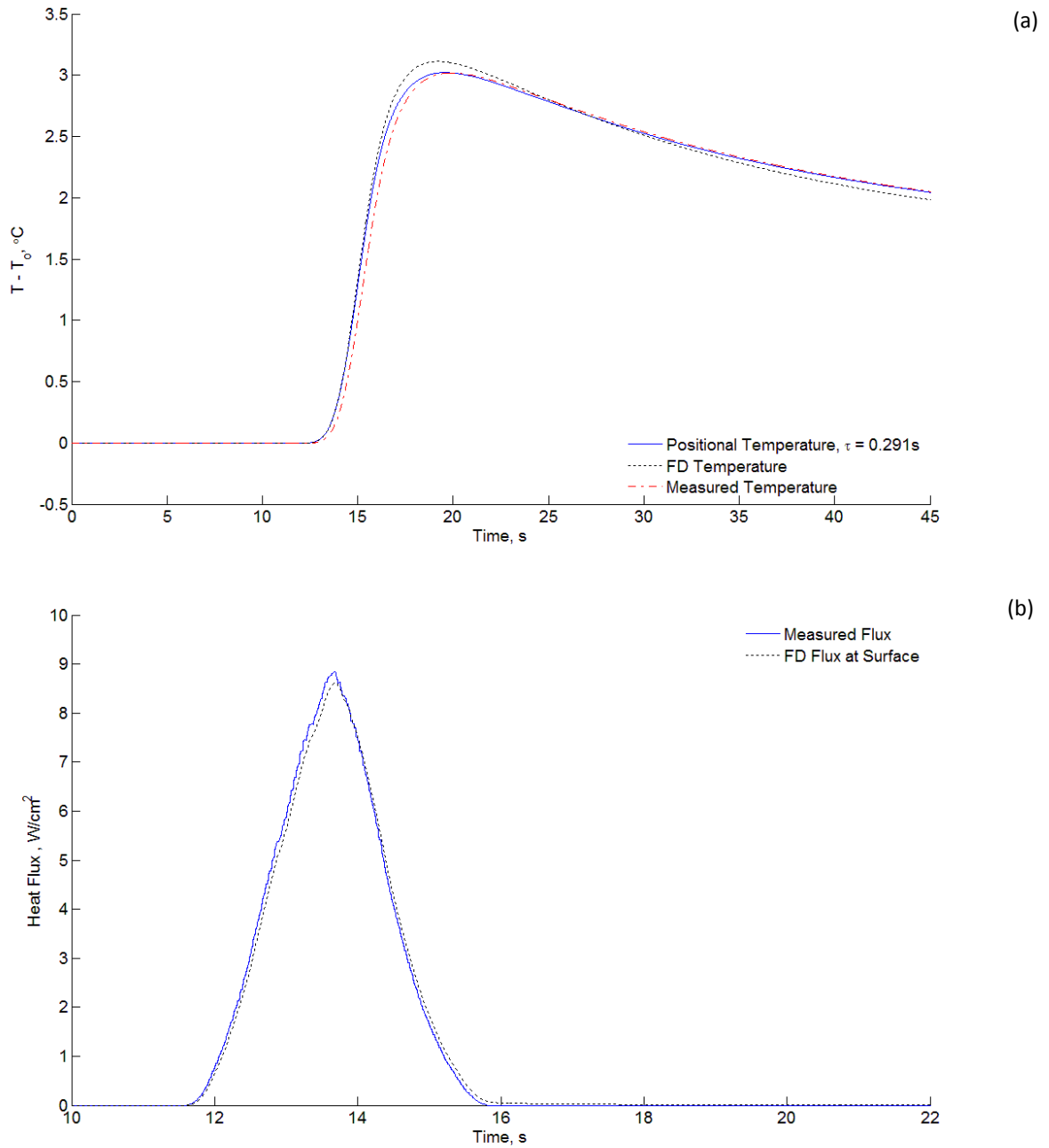


Figure 4.5: Test Case 2 Outputs of Finite Difference Solver. (a) Measured Thermocouple, Finite Difference, and Positional Corrected Temperatures at 6.57mm; and (b) Measured from Heater and Finite Difference Flux at Surface.

Figure 4.6 displays the time constant determination and heat flux examination of the last test case. With the FD data used as the positional, the time constant came to be $\tau = 0.296\text{s}$. Figure 4.6b shows the measured and FD flux as nearly identical with total energy percent difference of -0.26% . They also had a low μ_r of $-0.0031 \text{ W}/\text{cm}^2$ and low σ_r of $0.075 \text{ W}/\text{cm}^2$. The small differences could easily be attributed to the fact that the measured flux is located at the centerline of the heater, while the FD model is located at the surface of the slab.

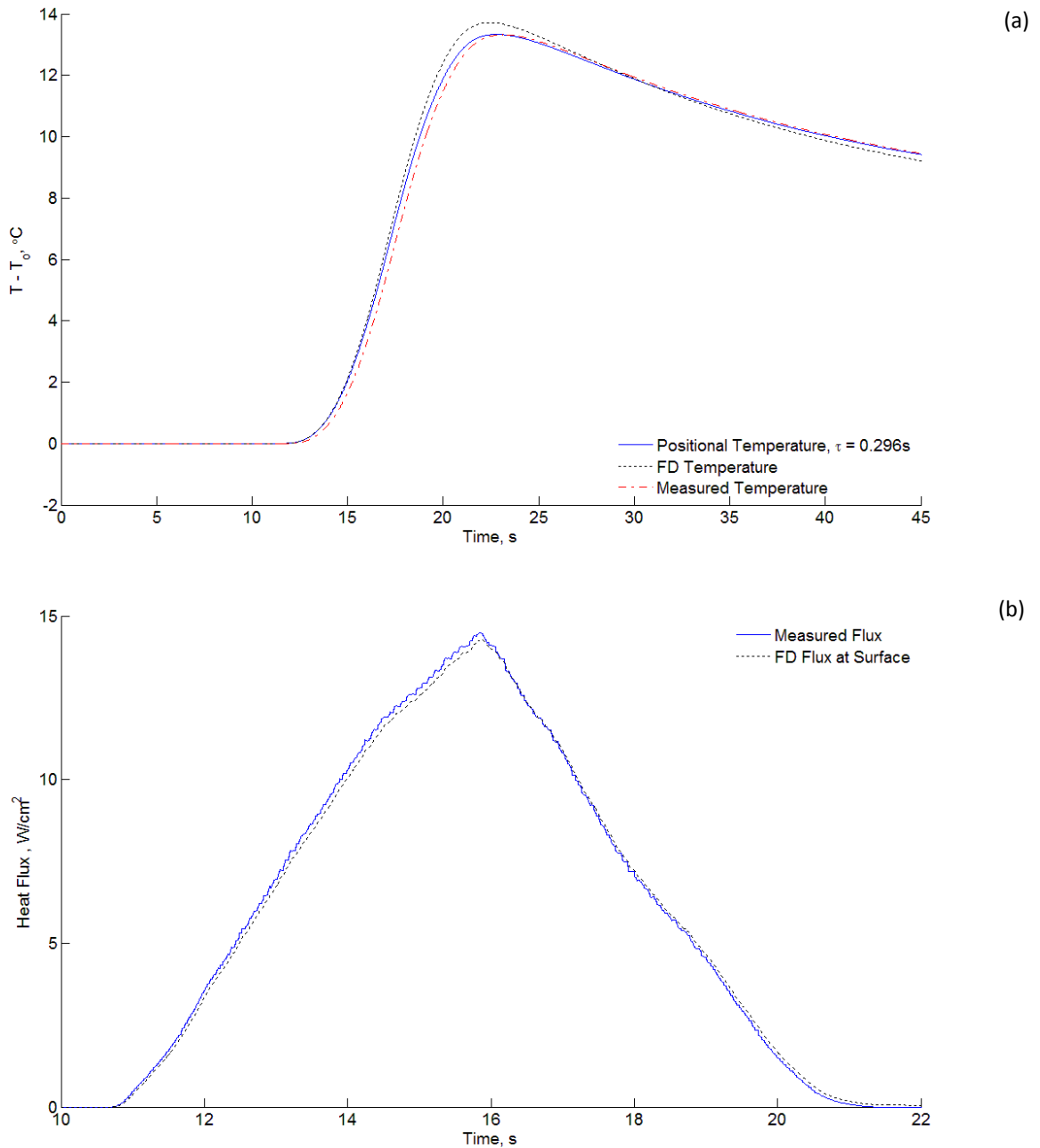


Figure 4.6: Test Case 3 Outputs of Finite Difference Solver. (a) Measured Thermocouple, Finite Difference, and Positional Corrected Temperatures at 6.57mm; and (b) Measured from Heater and Finite Difference Flux at Surface.

Each case has been evaluated individually and produced a separate time constant. This cannot be correct as since each case was performed on the same system, the system must have only one time constant. There are many conditions that may have caused these three different time constants. The most likely culprit is ignoring the axial conduction along the thermocouple leads and data uncertainty. Nonetheless the three calculated time constant values (0.288s, 0.291s, and 0.296s) are very close to their mean value of $\tau = 0.291\text{s}$. In order to rectify the discrepancy the easiest method is to average the three results. There is no information to indicate that any one

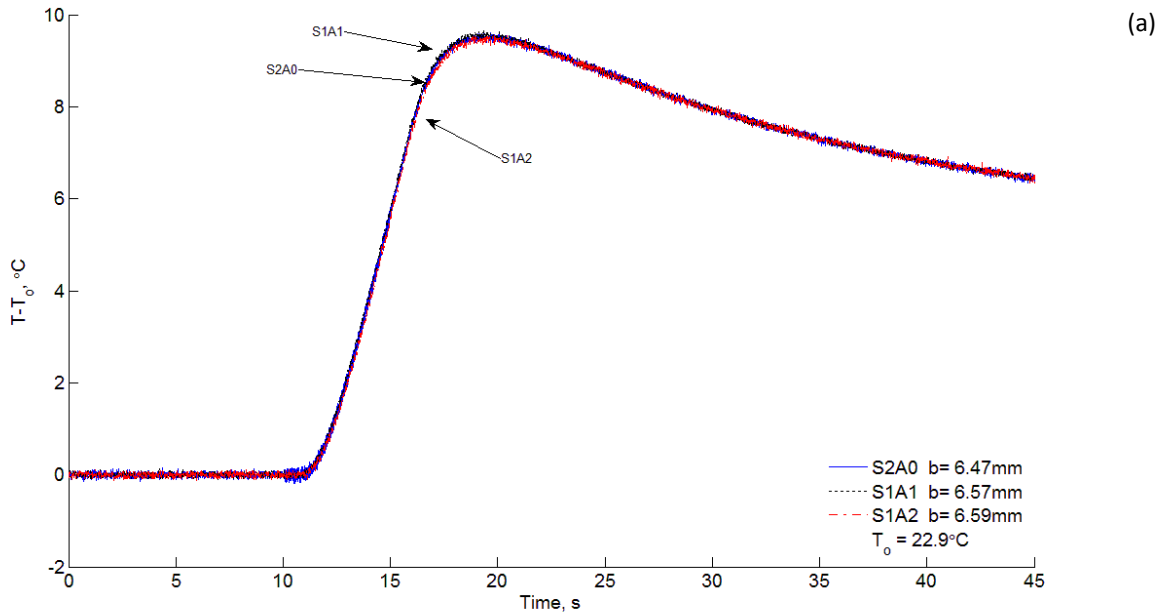
test should have more weight than another. The comparisons of fluxes show confidence in using the FD surface flux as the comparison for the surface predictions later on. The low average residuals increase confidence that the values are similar, but the low residual standard deviations prove they don't oscillate around each other. For final examination all the flux comparisons are presented in Table 4.1. The "control case data" metrics are also listed in Table 4.1.

Table 4.1 Finite Difference Flux Comparison Metrics Measured vs. Finite Difference

Metric	Control Case	Test Case 1	Test Case 2	Test Case 3
Q'' % diff	-0.25%	-0.26%	-0.25%	-0.26%
μ_r (W/cm ²)	-0.0021	-0.0028	-0.0007	-0.0031
σ_r (W/cm ²)	0.217	0.212	0.055	0.075

4.2 CONTROL CASE DATA

In order to show the impact of the rate sensor and the process used to obtain surface conditions, a control case is required. The analog filtering techniques in the rate sensor temperature signal alone are enough to severely impact inverse predictions. Therefore the control case must be a run taken using the same setup, but measuring raw temperature with no rate sensor or analog augmentation. This case will use digital filtering and digital derivate methods to obtain the required data and use it in the Global Time Method to obtain surface conditions. This run was done using the same equipment with all the same settings as all other test case runs. The only difference was that temperatures were recorded directly. No data was sent into the analog rate sensor, neither for filtering nor for derivatives. Figure 4.7 begins with the measured thermocouple temperatures and the heater power measured during the run. Figure 3.1 of the setup indicates positions for the thermocouple temperatures listed in Figure 4.7a and 4.7b.



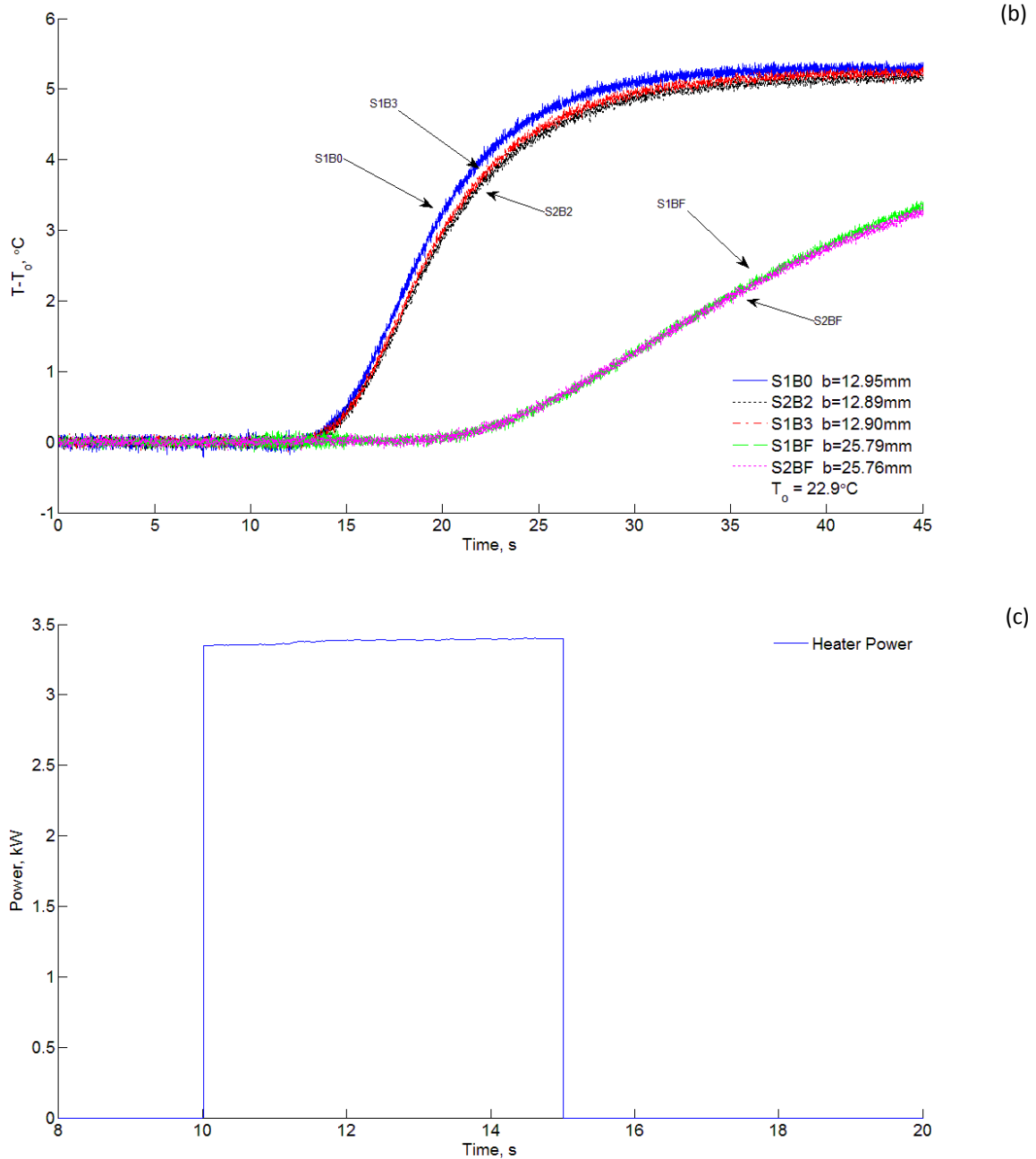


Figure 4.7: Control Case Inputs to Finite Difference Direct Model Code. (a) Measured Temperatures at A Depth Positions, (b) Measured Temperatures at the B Depth Positions and the Back Face Positions, and (c) Measured Power usage of Heater. Heat Starts at 10s and is 5s in Duration.

Figure 4.7a and 4.7b show the measured temperature of each probe in the setup. The back face thermocouples in Figure 4.7b clearly show the thermal symmetry of the two slabs. Normally this is not recorded because of channel limitations of the equipment. These temperatures can be compared to the positional temperatures that will be displayed in later tests as transferring these thermocouple temperatures to positional temperature will not reduce noise. For further analysis thermocouple S1A1 will be used just as it is the input thermocouple to the analog

sensor in all test cases. Figure 4.7c displays the power signal used in the control test. This is also used as the input, along with S1BF, to the FD model for further processing.

Figure 4.8 is the control case output of the FD model. It is the same as Figures 4.4, 4.5, and 4.6. Comparing Figure 4.8 to the test cases, the noise difference is pronounced. This is because there is no longer any analog filtering of the signal to produce such a clean result. This noise will affect the accuracy of prediction.

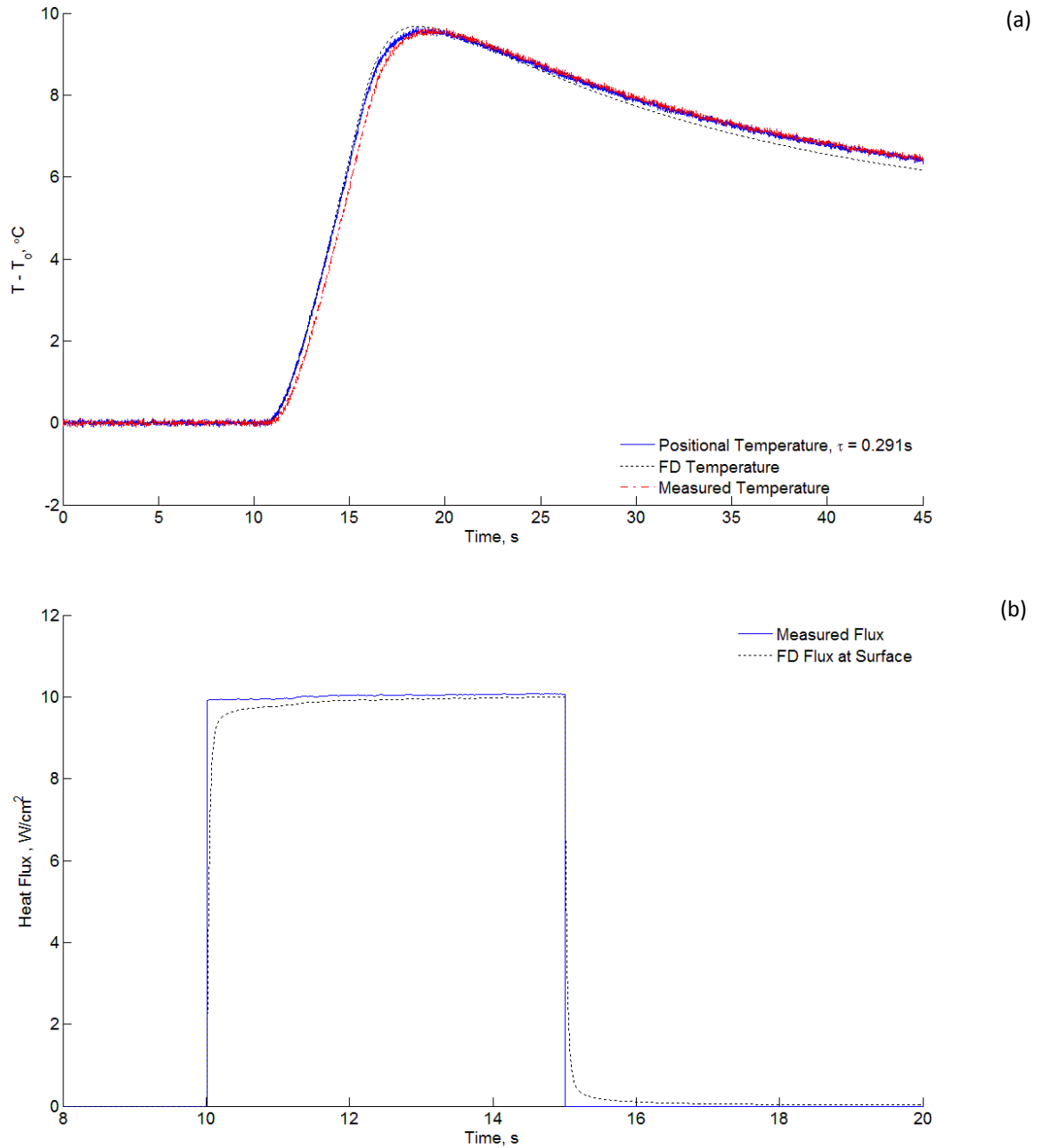


Figure 4.8: Control Case Outputs of Finite Difference Solver. (a) Measured Thermocouple, Finite Difference, and Positional Corrected Temperatures at 6.57mm; and (b) Measured from Heater and Finite Difference Flux at Surface.

The positional temperature of Figure 4.8a is now used with the Gauss filtering function to obtain sufficient time derivatives of temperature. These time derivatives of temperature are used in Eq. (2.10) to resolve the positional heat flux and its time derivatives. Figure 4.8b checks the measured flux with the FD flux result, again because it will be used as the comparator in the surface predictions. The percent total energy difference computes to -0.25% about the same as the analog cases. The μ_r is -0.0021 W/cm² again around the same as the results for test cases 1 to 3; while the σ_r was 0.217 W/cm². It has similar metrics to the test cases 1 to 3; because no data up to this point in all cases has been affected by the rate sensor input. The only data used has been the back face thermocouple, filtered digitally in all cases, and the measured heat flux, which is collected and used the same way throughout all the tests.

With the FD results the gauss filtering function is used to obtain suitable time derivatives of temperature from the positional temperature. These temperature and derivatives are used in Eq. (2.10) to obtain the flux and its time derivatives. Finally all that data is used in the Global Time method to obtain surface predictions. In order to better analyze the digital results, they were computed at two different cutoff frequencies for the Gauss Derivative function.

Figure 4.9 displays the results for the first cutoff frequency. For this test the cutoff frequency was chosen as the highest value that would not result in instability of the inverse prediction. With too much noise the surface prediction will destabilize causing continuous oscillation and increase with no concern for what actually happened during the run. This maximum cutoff frequency was $f_c = 0.7\text{Hz}$, a very low value. Even at a 1Hz cutoff frequency the inverse predictions were useless.

Figure 4.9a presents the surface temperature prediction. The digital projections do have a fair amount of noise in them but remain consistent to the FD model. Normally it is the case that the temperature prediction will be more stable than the flux. Interestingly there seems to be little discernible difference between the N=3, three control volume Global Time method, and the N=7, seven control volume Global Time method. This is due to significant noise in the higher derivatives. The N=7 prediction should be better because it is of higher order; however, if the higher order derivatives are mostly noise there will be little discernible difference between it and a lower order method.

Figure 4.9b presents the predicted surface flux. There is much oscillation in the predicted heat flux when it should report a constant signal. Also it misses the power on and power off of the heater; turning it from a defined discontinuity into a more trapezoid shape. For the N=3 prediction the percent difference total energy was 8.08%, with a $\mu_r = 0.0672\text{ W/cm}^2$ and $\sigma_r = 1.01\text{ W/cm}^2$. The N=7 prediction had a percent difference in total energy of 7.67%, and $\mu_r = 0.0638\text{ W/cm}^2$ and $\sigma_r = 0.792\text{ W/cm}^2$. Looking at that it seems like the mean of the residual is close to zero, but the rather high standard deviation indicates like the figure shows that the digital solution oscillates around the FD model. This is not good because it means the result is nearing instability. This will greatly decrease quality of the results. A new cutoff frequency must be attempted to obtain better results,

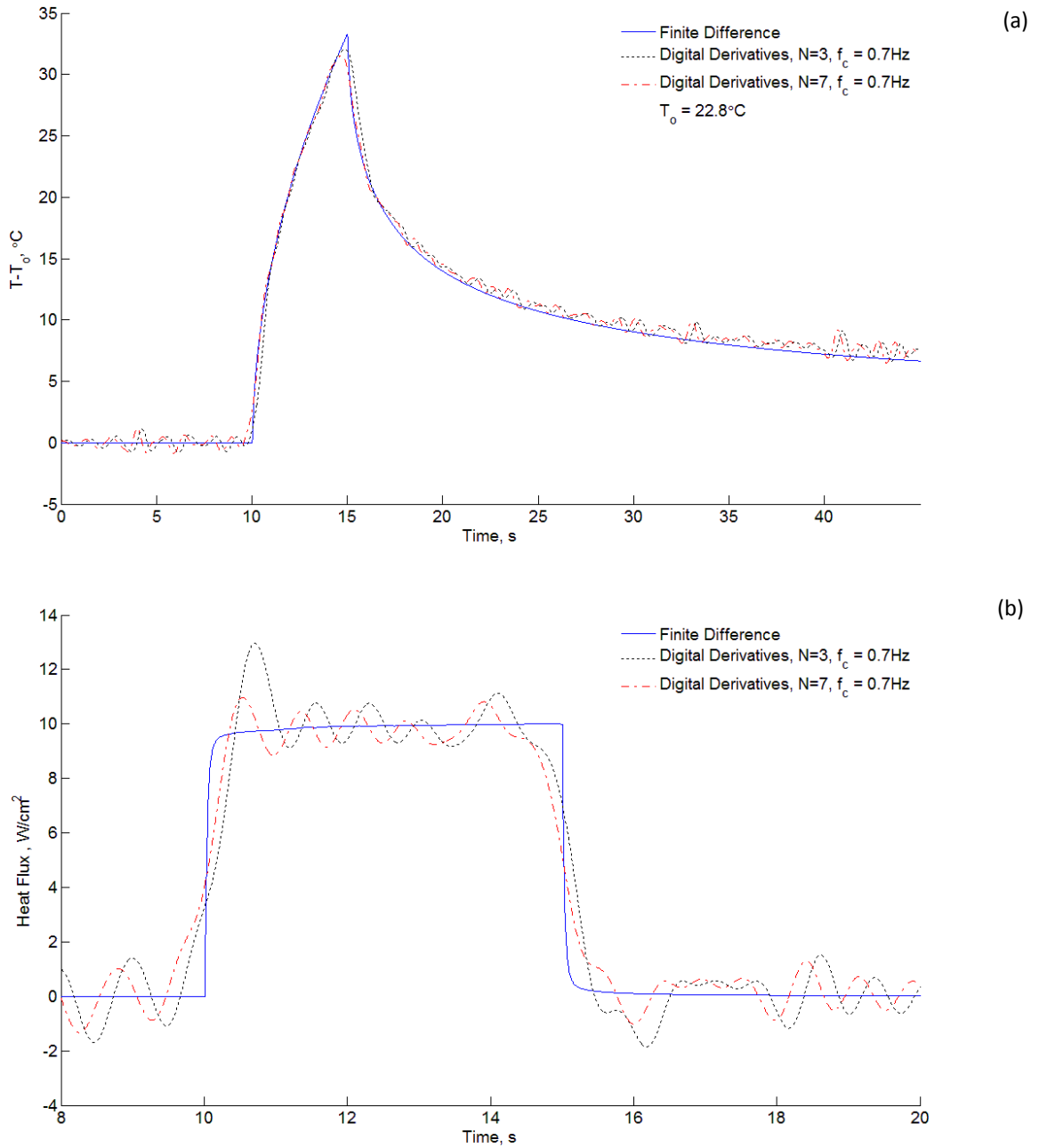


Figure 4.9: Control Case Surface Prediction via Global Time Method with $f_c = 0.7\text{Hz}$. (a) Prediction of Surface Temperature from Digital Derivatives and Finite Difference Solution, and (b) Predictions of Surface Heat Flux from Digital Derivatives and Finite Difference Solution. Heat Starts at 10.00s and is 5.01s in Duration.

For the next digital projection, experimentation revealed that a cutoff frequency of $f_c = 0.5\text{Hz}$ provided signals that were free from a majority of noise and avoided over regularization of the results. The surface temperature prediction displayed in Figure 4.10a is much cleaner than Figure 4.9a. It still misses at the peak, and the $N=3$ prediction is closer than the $N=7$ at the peak as opposed to what is expected when comparing the two methods.

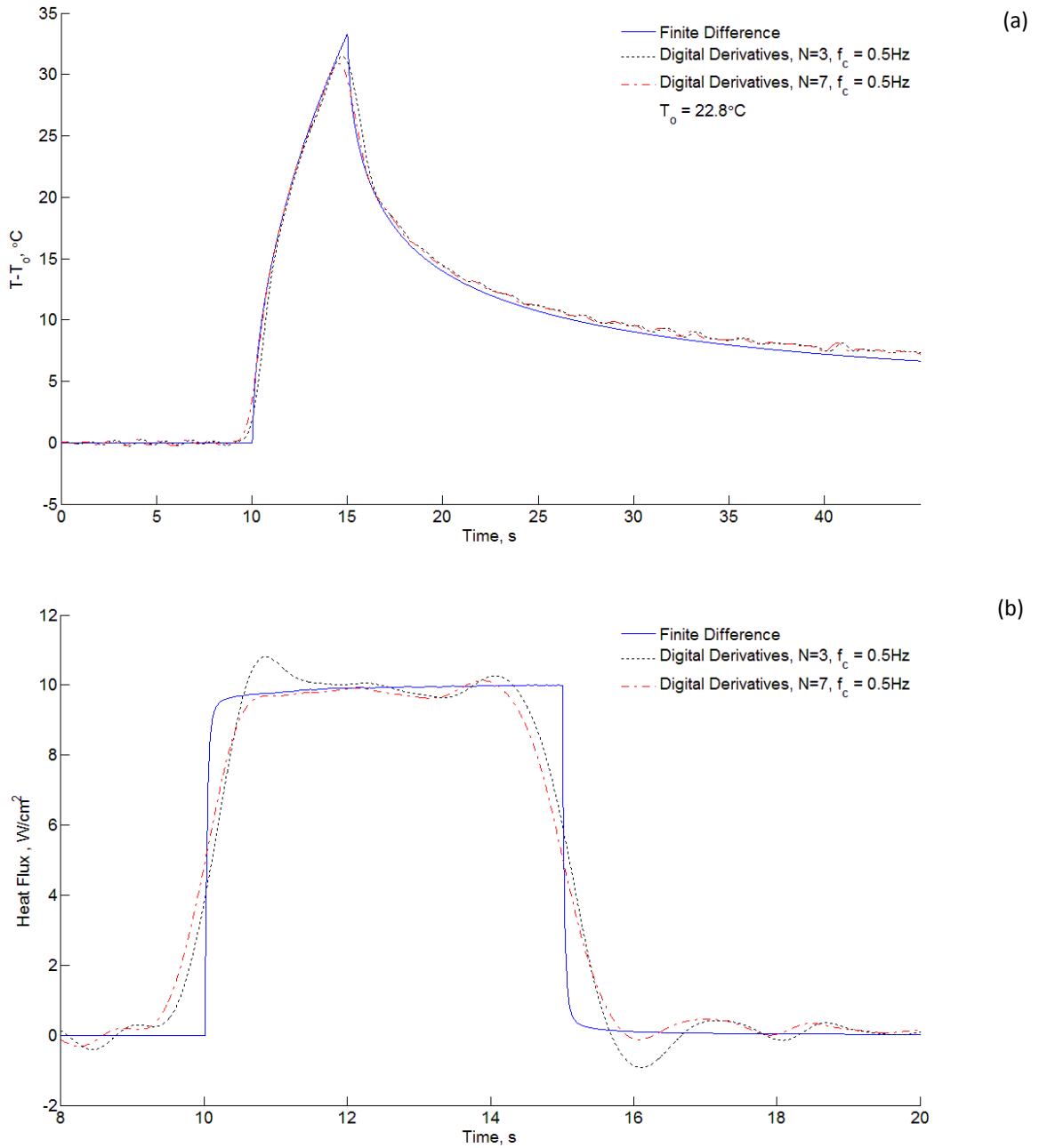


Figure 4.10: Control Case Surface Prediction via Global Time Method with $f_c = 0.5\text{Hz}$. (a) Prediction of Surface Temperature from Digital Derivatives and Finite Difference Solution, and (b) Predictions of Surface Heat Flux from Digital Derivatives and Finite Difference Solution. Heat Starts at 10.00s and is 5.01s in Duration.

The surface heat flux prediction is likewise much better. Figure 4.10b does not have the large oscillation during the periods of constant flux. In the $N=3$ prediction, the Q'' percent difference is 8.58%, with a $\mu_r = 0.0713 \text{ W}/\text{cm}^2$ and $\sigma_r = 0.53 \text{ W}/\text{cm}^2$. The $N=7$ prediction had a Q'' percent difference is 8.60%, and $\mu_r = 0.0715 \text{ W}/\text{cm}^2$ and $\sigma_r = 0.53 \text{ W}/\text{cm}^2$. The lower cutoff frequency for filtering the data has indeed helped the oscillation, but worsened the total accuracy. Unfortunately the lower cutoff frequency did not improve the prediction's ability to capture the rise and fall of the flux; it is still distorting the step into a more trapezoidal rise. These figures now establish the

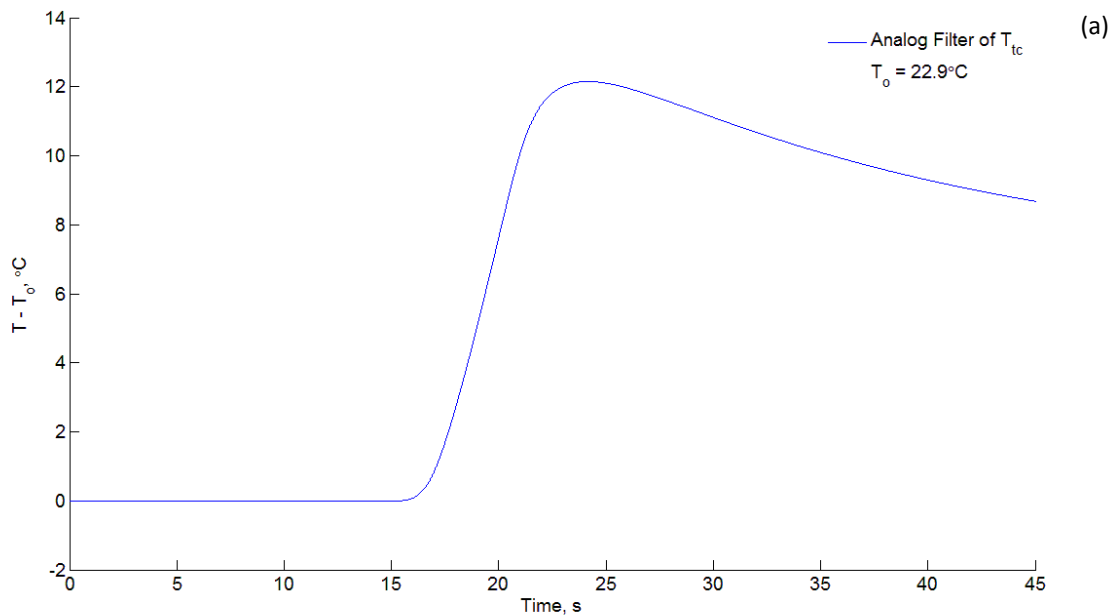
baseline accuracy of the global time method itself. More information about the effect of digital filtering on accuracy can be found in [12]. All further data will be improved by processing via the analog rate sensor. Occasionally looking back to Figure 4.9 and Figure 4.10 will allow a better understanding of the benefits of the analog rate sensor.

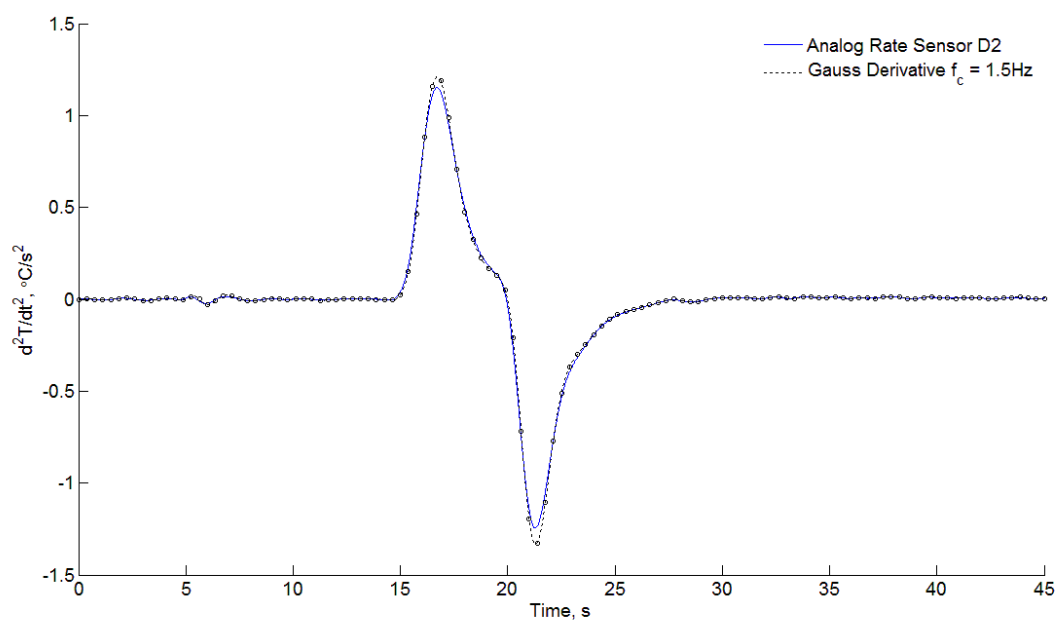
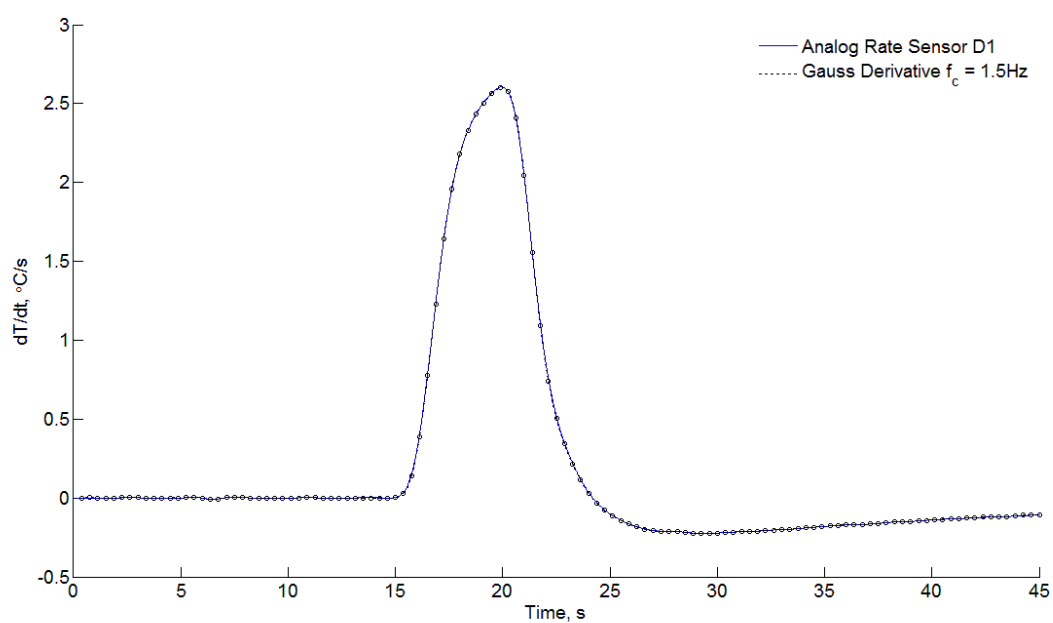
4.3 TEST CASE 1: STEP INPUT

The first test case involved a step input. It attempts to mimic discontinuities at the on and off period, a mathematical difficulty for inverse prediction. It will make any faults in the method or rate sensor self-evident. The final positional temperature results at the probe location are displayed in Figure 4.11. Figure 4.11a presents the positional temperature measured at the probe site after passing through the rate sensor amplifier and filter. The excellent signal to noise ratio exemplifies the quality of the rate sensor filter. It is a key factor in the quality of the results of this inverse method.

Figure 4.11b presents the first time derivative of temperature at the probe location. For comparison, using $f_c=1.5\text{Hz}$, the digital version of the first time derivative of temperature obtained via a method called the Gauss filtering function [5] is shown as a dotted line accentuated by open circles due to the similar nature of the two curves. The similarities between the two curves doesn't just prove the worth of the digital method, it also shows the value of the Butterworth filter to eliminate the noise that would normally create problems for such a digital method.

In Figure 4.11c the comparison of both versions of the second time derivative of temperature at the probe locations are shown. The signal still has a consistent low noise; the only problem is the development of a defect near 5s in the pre-heating data. It is caused by an outside disturbance of the test before the heating begins, but it is very informative to show the development of noise in the process of derivatives. Looking back at Figure 4.11b an insignificant disturbance can be seen around the same time, as the derivatives get higher the effect will be magnified.





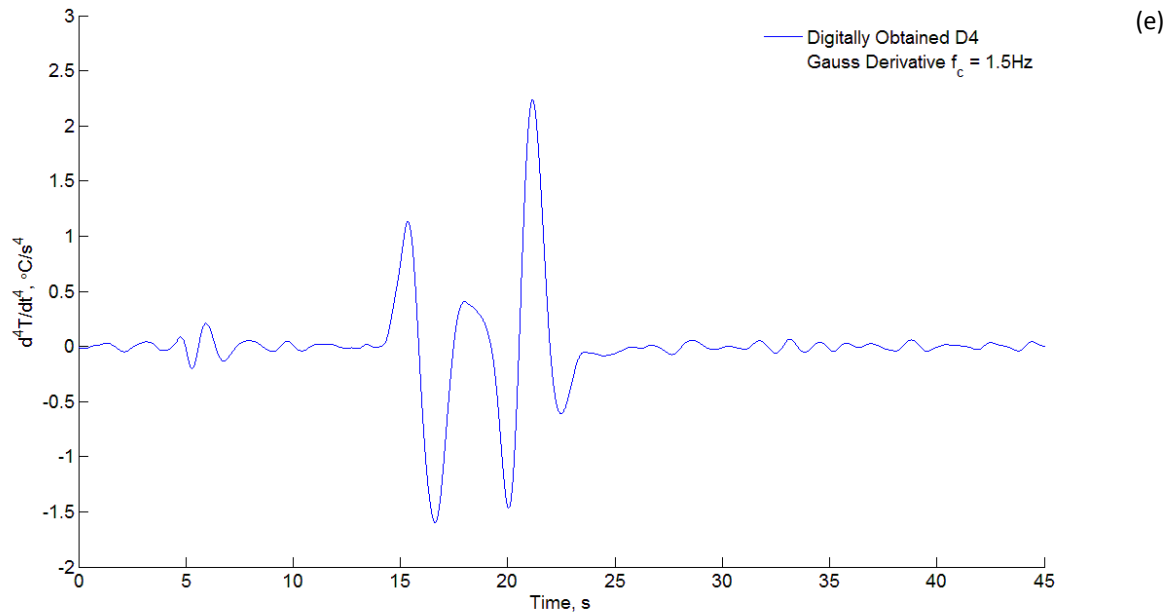
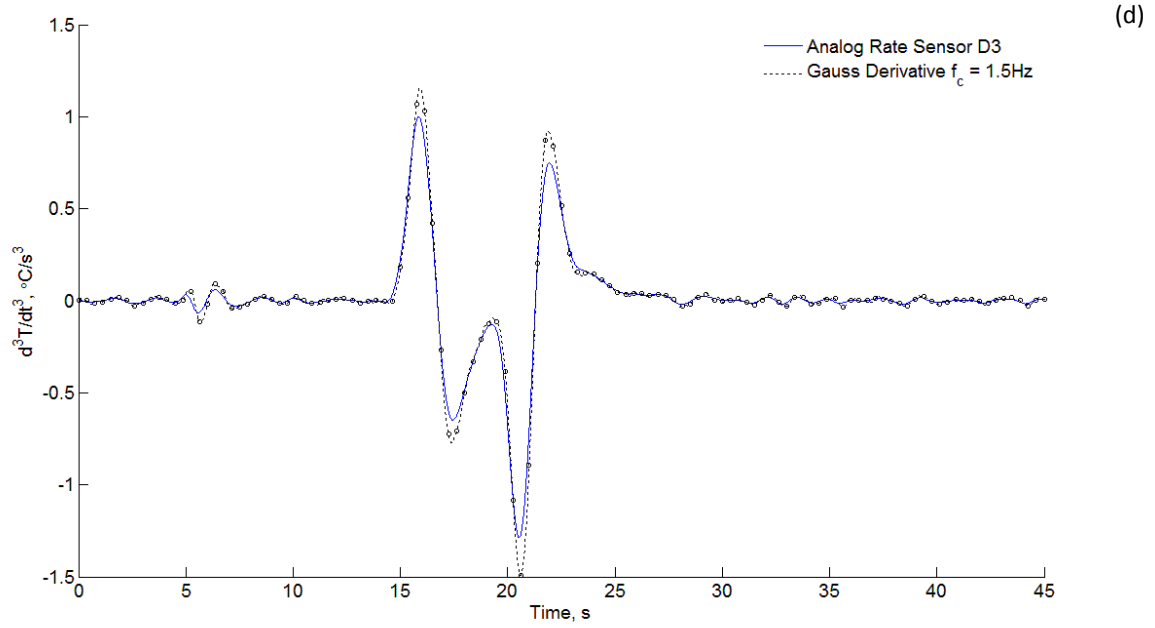


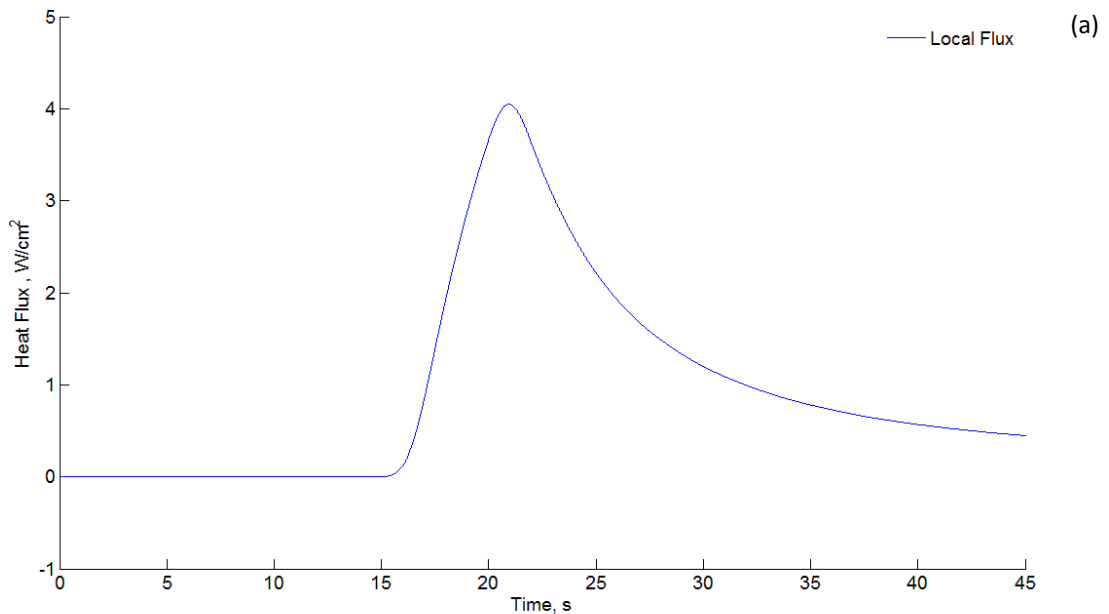
Figure 4.11: Test Case 1 Positional Corrected Rate Sensor Output. (a) Filtered Temperature at depth of 6.57mm, (b) First Time Derivative of Temperature and Digital Comparison, (c) Second Time Derivative of Temperature and Digital Comparison, (d) Third Time Derivative of Temperature and Digital Comparison, (e) Fourth Time Derivative of Temperature, Digitally Obtained through Gauss Function.

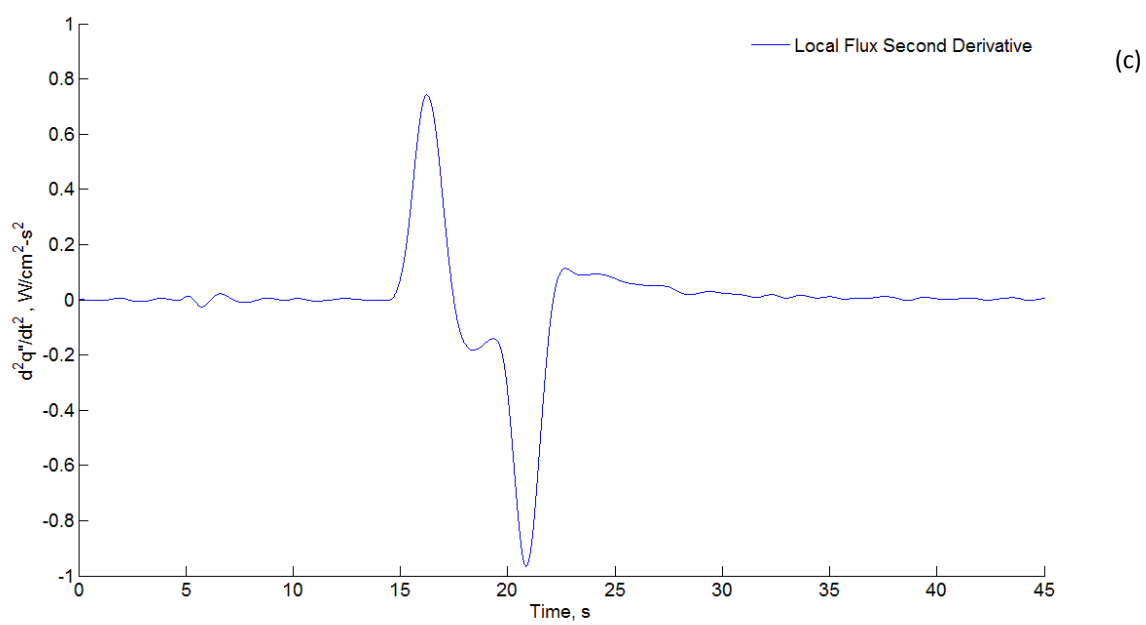
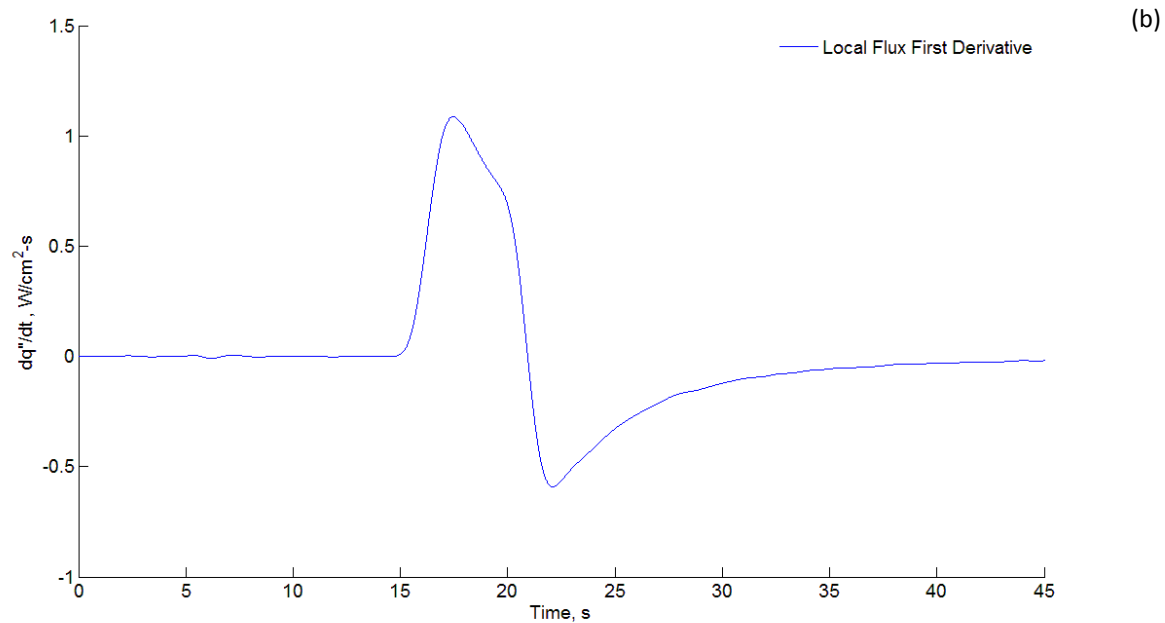
In Figure 4.11d the noise in the test finally becomes apparent, mostly in the pre and post heating data. However it seems the signal to noise ratio is still sufficient to resolve the response during the test run. Interestingly the gauss derivative has begun to fail at the peaks of the derivative. This is caused by its inability to handle the increasing noise from increasing derivatives. It is important to remember that the digital derivative

signal's source is only the actual filtered temperature signal. None of the higher analog derivatives are used in its derivation. Once again the defect at 5s is seen to grow larger still.

Figure 4.11e is different from the other graphs of Figure 4.11. This derivative has been obtained through the same mathematical formula used earlier for comparison to the analog derivatives. As mentioned section 3 due to time constraints the analog sensor was only built to resolve three analog derivatives; however, the Global time method as derived for three control volumes requires four time derivatives of temperature. Therefore the fourth derivative must be obtained digitally. In the comparisons to analog, the gauss derivative method has shown to work well for the first and second derivative of source data. So to ensure the best quality of the fourth derivative the third analog time derivative of temperature was used in the gauss derivative function to obtain the fourth digital time derivative of temperature. The 1.5Hz cutoff frequency for the filter in the gauss derivative was determined by trying various cutoff frequencies in the comparison to the analog derivatives. Once again there is an increase in noise.

Figure 4.12 is the result of the Rate Sensor from Figure 4.11 combined with Eq. (2.10) to arrive at the positional flux and its temporal derivatives. Because Eq. (2.10) involves integrating the input there is no instability and the noise inherent in the input will be improved. This explains the wonderfully smooth signal in Figure 4.12a. Moving on to Figure 4.12b the signal is nearly absent of noise. These time derivatives are obtained by increasing n in Eq. (2.10) and using a higher order temperature derivative to obtain the flux derivatives.





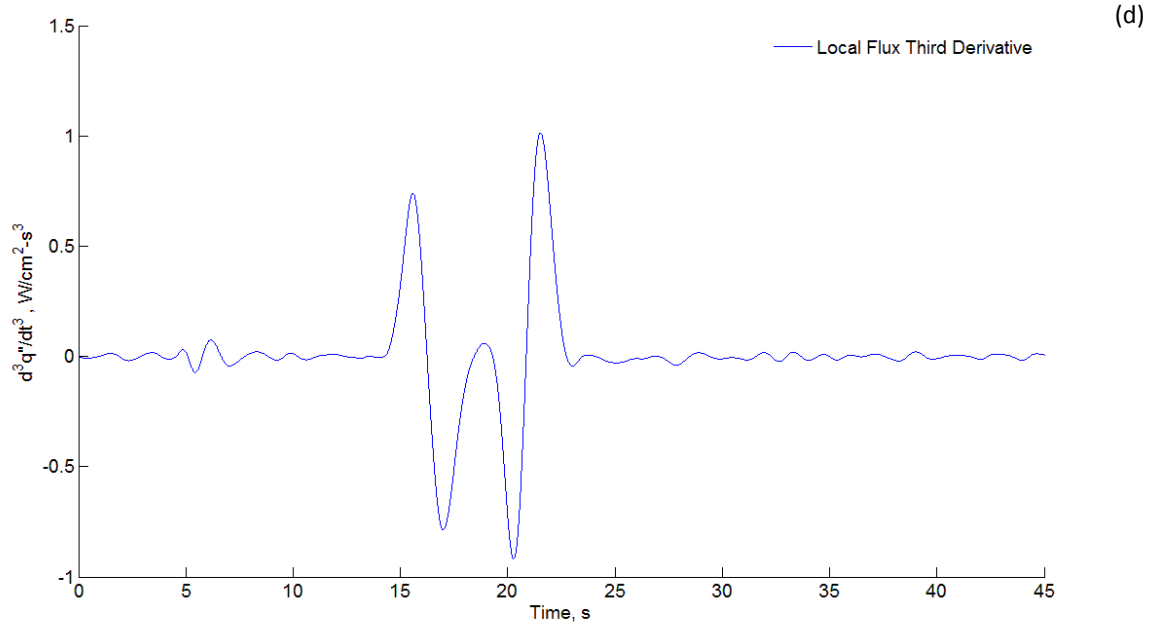


Figure 4.12: Test Case 1 Positional Flux and its Time Derivatives at 6.57mm from Surface Derived from Temperature Derivatives. (a) Local Flux, (b) First Time Derivative of Flux, (c) Second Time Derivative of Flux, and (d) Third Time Derivative of Flux.

Finally in Figure 4.12c the noise of the temperature derivatives begins to show; though it is still well under control by the integration of Eq. (2.10). Figure 4.12c uses $n=2$ in Eq. (2.10), which means that the third temperature derivative from Figure 4.11d is the input for this graph. Looking at Figure 4.12d and earlier graphs the same 5s defect of the temperature graphs transfers over to the flux. These positional fluxes are used in the global time method, both for surface flux and surface temperature prediction. As shown earlier the Global Time Method requires the positional flux and temperature and their temporal derivatives. Figure 4.12a clearly shows that a single probe can be used to calculate the local heat flux during the half space period.

Finally we arrive at the surface predictions. Two comparison cases are shown to the analog surface results. Figure 4.13 contains the analog surface results with a digital comparison obtained with gauss derivatives of cutoff frequency 0.5 Hz. Many different cutoff frequencies were evaluated to find the optimal value. In each graph of this figure there are four sets of data displayed. The first is the results all the previous figures have been building towards; the surface prediction via Global Time Method utilizing analog rate sensor data. The next set of data is the results via the previously mentioned FD model. This is shown to check all of the inverse methods. It was derived via a direct method with inputs from both boundaries; it is therefore the best assumption available for the actual surface conditions.

The next two sets of data are surface predictions made with the Global Time Method but all input to the method was derived digitally from the filtered temperature. This is shown to display the exact benefits of using analog derivatives. The first digitally determined surface condition uses a three control volume global time method, the same equations used by the analog surface predictions. The second digitally determined surface condition uses a seven control volume Global Time Method. This method requires many more derivatives and is much more computationally expensive, but it should produce a better result [12]. It is important to note that the digitally obtained surface conditions are displayed as discreet dots at regular time intervals. They are calculated at

many more time intervals than is displayed, but in order to maintain clarity on the figure they are not displayed at every time step.

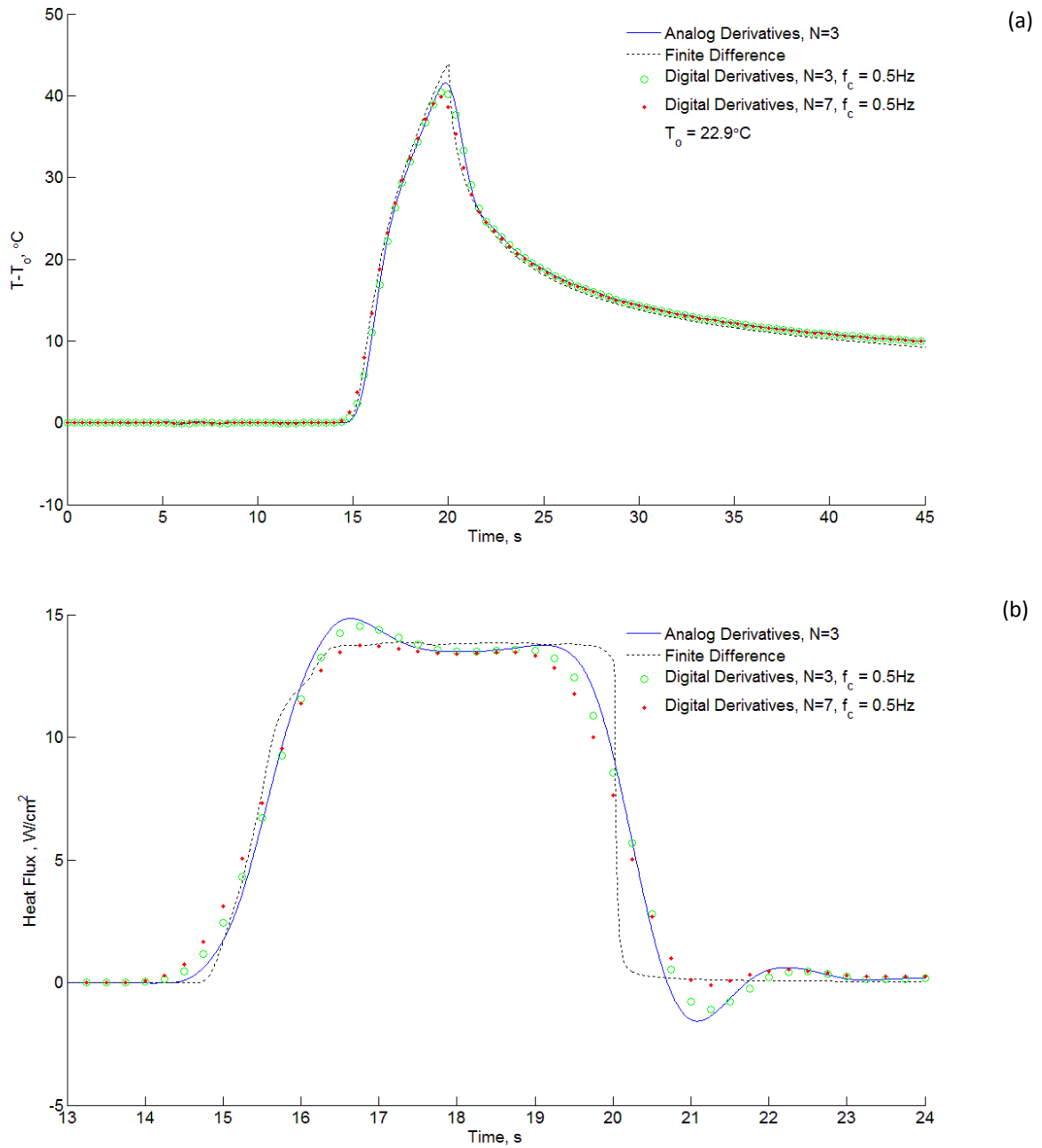


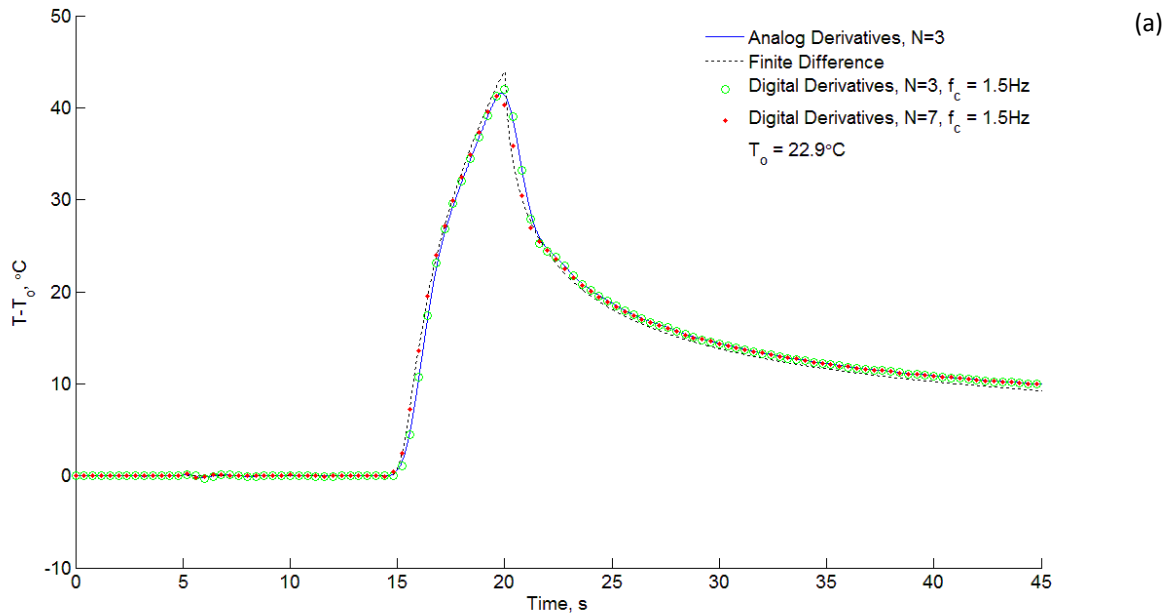
Figure 4.13: Test Case 1 Prediction of Surface Conditions via Global Time Method with Finite Difference Comparison. (a) Predictions of Surface Temperature from Analog Rate Sensor Derivatives and Derivatives Obtained Digitally from Temperature and (b) Predictions of Surface Heat Flux from Analog Rate Sensor Derivatives and Derivatives Obtained Digitally from Temperature. Heat Starts at 14.64s and is 5.39s in Duration. The Digital Predictions are Obtained at $f_c = 0.5\text{Hz}$ and are Not Represented at Each Time Step.

Figure 4.13a presents the predicted surface temperature of the stainless steel slab. It shows a sharp rise after heating start and a long cooling period. The inverse prediction using analog data shows a very stable signal that

follows the shape of the FD results very closely. The only flaws are the blunted peak and lag of the analog result behind the FD result. The peak in the analog is most likely the result of the Global Time Method and analog sensor being unable to accommodate the sharp change at the peak. It is also caused by some over smoothing of derivatives at this point, the derivative becomes extremely large and changes at a very rapid rate; this may be interpreted by the derivative circuit as noise and filtered. The lag is the result of an imperfect procedure for converting measured data to positional data which ignores axial conduction along the thermocouple leads. Increasing the time constant to a higher order correction would complicate the calculations, but produce much better results.

Figure 4.13b displays the surface heat flux predictions. The FD model prediction has a fast rise during heating and sharp off time intended to replicate a discontinuity, this is a comparable setup to the control case. The predicted heat flux using analog data ends up with a percent difference in total energy of 7.54%, which means it over predicted the FD model. Still this is closer to the FD model than any of the control cases. The run has a mean residual of 0.0802 W/cm^2 and a residual standard deviation of 0.542 W/cm^2 . The low standard deviation indicates that the residual is not very oscillatory and the low mean value shows how close the two data sets were. It should be noted that the inability of the Global Time Method to replicate the step change in heat flux is a major contributor to the magnitude of the standard deviation. In subsequent runs without a sudden change in the heat flux the prediction will do a much better job. Another possibility for the inaccuracy is the time constant issue noted earlier. Still the analog data leads to an excellent prediction of the value at the top of the ramp.

Figure 4.14 contains the same analog prediction and FD result as Figure 4.13 and the only change is in the digital projections. Comparisons of Figure 4.13 ($f_c=0.5\text{Hz}$) and Figure 4.14 ($f_c=1.5\text{Hz}$) clearly show the influence of the cutoff frequency on the digital projected results.



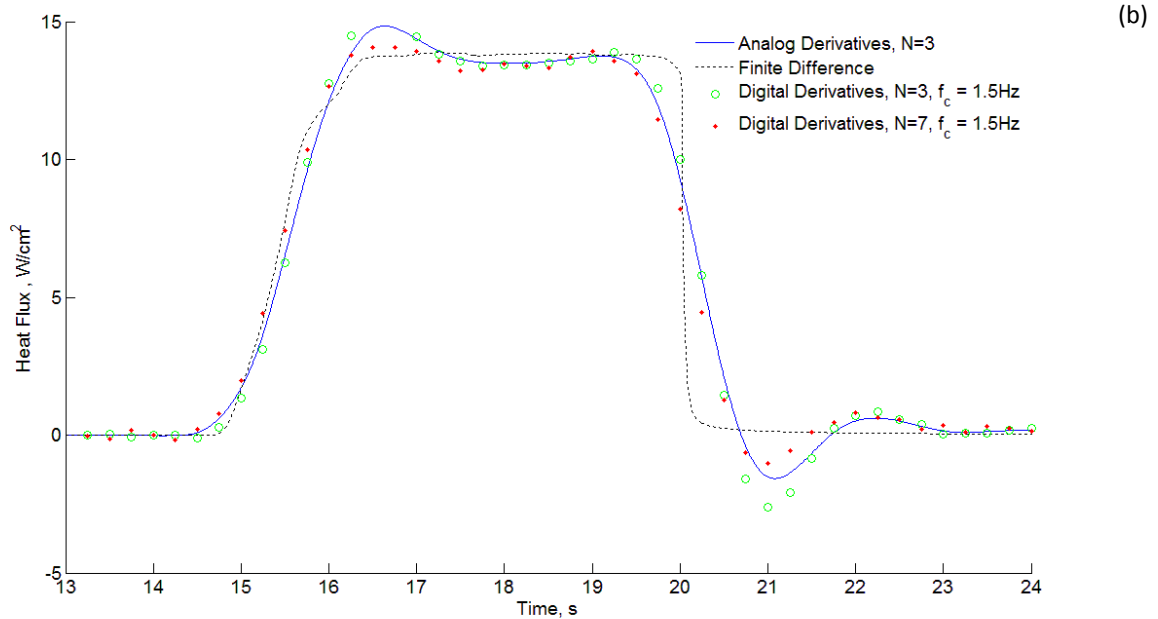


Figure 4.14: Test Case 1 Prediction of Surface Conditions via Global Time Method with Digital Derivative Comparison. (a) Predictions of Surface Temperature from Analog Rate Sensor Derivatives and Derivatives Obtained Digitally from Temperature and (b) Predictions of Surface Heat Flux from Analog Rate Sensor Derivatives and Derivatives Obtained Digitally from Temperature. The Digital Predictions are Obtained at $f_c = 1.5\text{Hz}$ and are Not Represented at Each Time Step.

Originally, the digital projections were calculated using the same cutoff frequency as determined in the control case. However, looking back to Figure 4.13, the digital predictions seem over smoothed. The $N=3$ prediction appears to do better, but it only appears that way due to the discontinuity bump. After the bump the digital prediction is much under the FD data. Both sets of data also rise before actual heating start and fall after heating end. This sluggish behavior around changes in signal is a good indicator of over filtering. The best digital prediction will come from the highest cutoff frequency that doesn't let the solution become unstable. Increasing the cutoff frequency in Figure 4.14 resulted in great improvements. The smoothing of the corner at heat on and heat off is much reduced, showing the signal to follow the shape of the FD data much better. Also the values at the peaks, especially the $N=7$ data, matched the FD result much better. These cutoff frequencies and many other were investigated to find the best fit for the digital projection method. Use of cutoff frequencies higher than 1.5Hz caused significant instability and noise in the prediction, to such a degree as to make the result worthless. Therefore 1.5Hz was chosen for all subsequent digital projections.

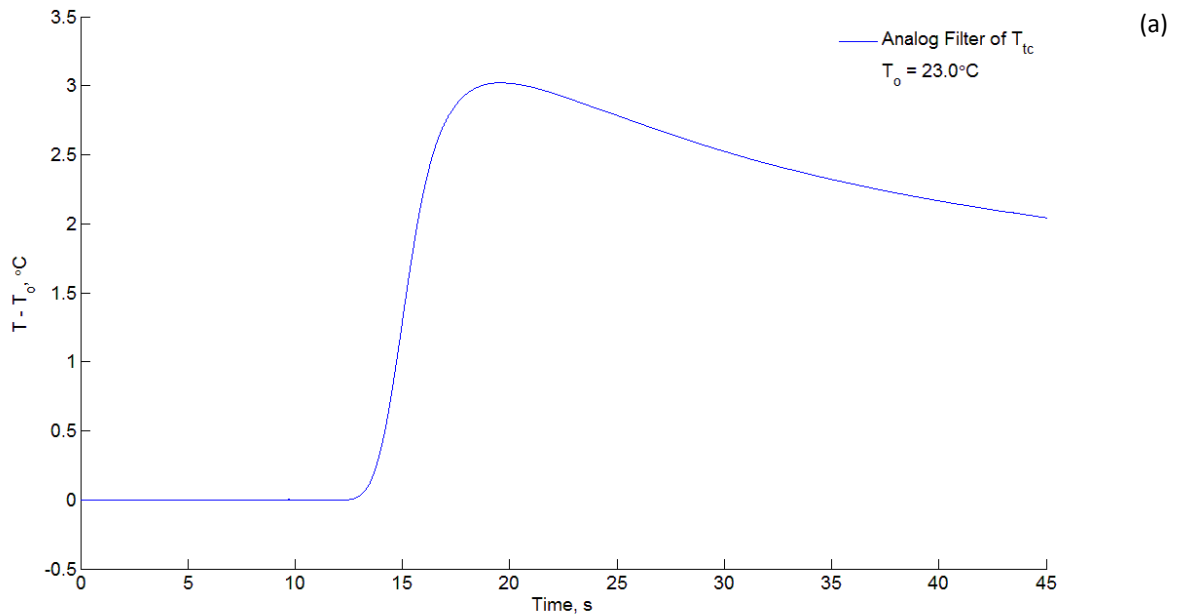
4.4 TEST CASE 2: LOW POWER RAMP INPUT

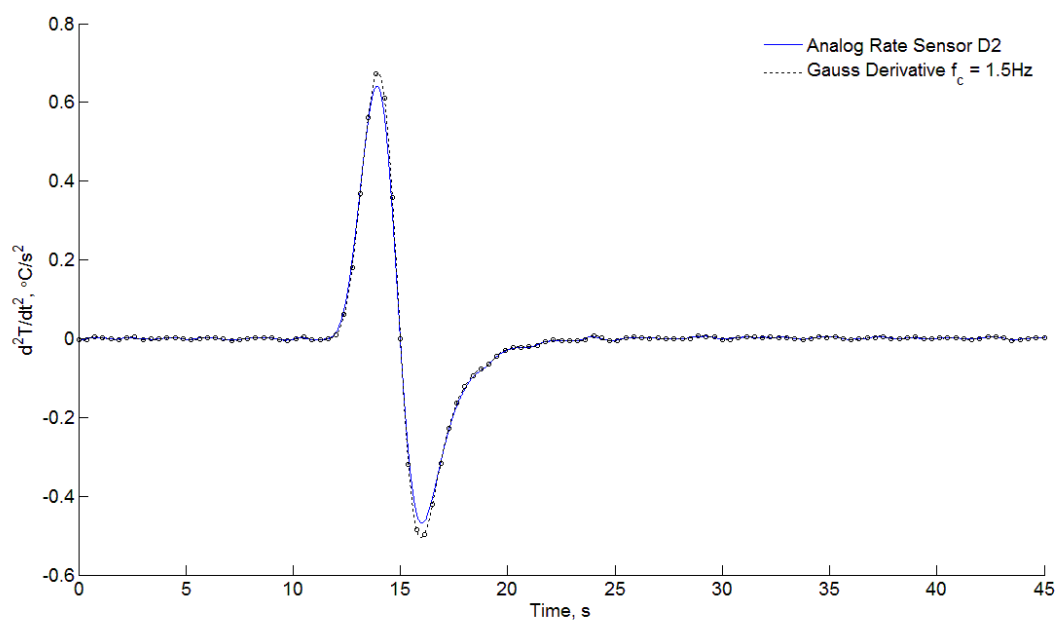
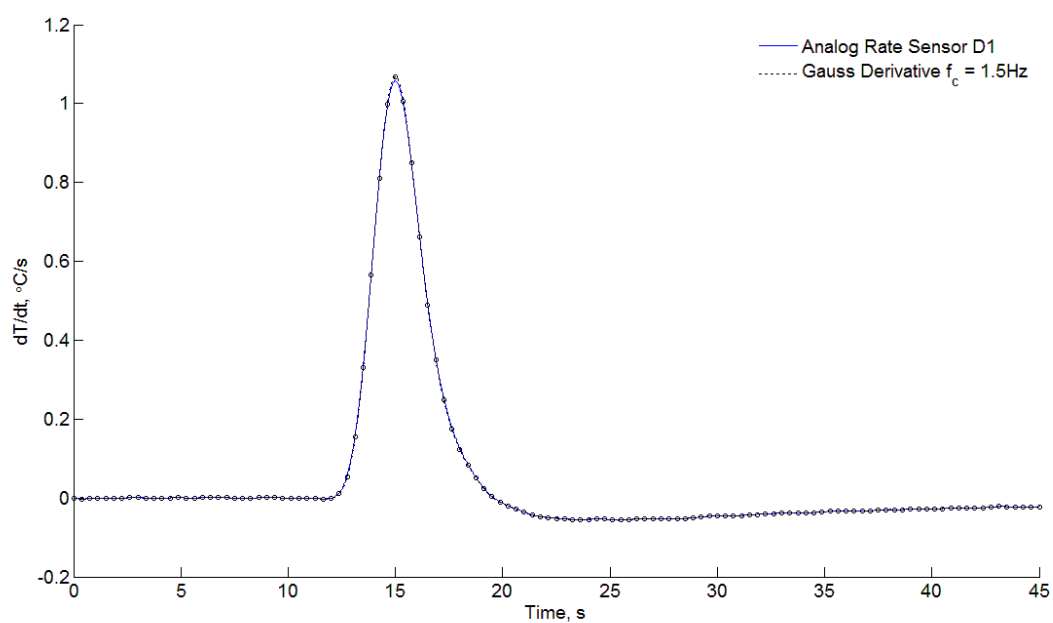
The next test case, the low power ramp, is meant to demonstrate the ability of the rate sensor when the input has a smaller signal to noise ratio. The lower power input to the heater will greatly intensify previous noise problems. This case will be a much greater test of the noise abilities of the rate sensor. Figure 4.15, similar to Figure 4.11, presents the temperature response at the probe location and its subsequent time derivatives.

Figure 4.15a is the temperature response of the sample at the probe location. It has been converted to positional temperature as discussed in section 3. This data has been filtered by the Butterworth filter of the rate sensor. In a cursory glance of Figure 4.15a, it seems much the same as the results from the last test. Again this speaks highly of the Butterworth Filter. Comparing these results to Figure 4.11a more closely shows that

maximum temperature rise above the ambient for test case two was about one quarter of that of test case 1. This means the absolute magnitude of this signal will be one fourth the last test. Because the setup is the same the noise level will be the same or comparable, meaning it will be larger with reference to the actual signal.

Figure 4.15b shows that this increased noise effect is noticeable in the first time derivative of the temperature. In the pre heating data there is a definite oscillation of what should be a constant signal of zero magnitude. The sensor does a good job of handling the noise during the actual run. The Gauss function also resolves it quite well. In the next derivative presented in Figure 4.15c the noise is intensifying but still managed by the rate sensor. However the noise level causes the gauss function peaks to deviate more from the analog peaks. Figure 4.15d shows that the effect of noise in the third derivative becomes even more evident in the data. It is interesting to note here that the differences caused by the change in input profile, a ramp versus a step input, become more apparent in the shape of the higher time derivatives of temperature. This shows the impressive amount of information contained in these higher derivatives and why they are so important to inverse analysis. In Figure 4.15e the digitally obtained fourth time derivative of temperature maintains stability and keeps a favorable signal to noise ratio. Examining its value compared to the previous plot there may be some amplification of the peaks due to noise, but not enough to ruin the surface prediction. Once again it should be noted that this fourth time derivative of temperature is obtained digitally through the Gauss Derivative Function from the third analog time derivative of temperature due to physical constraints in the construction of the rate sensor.





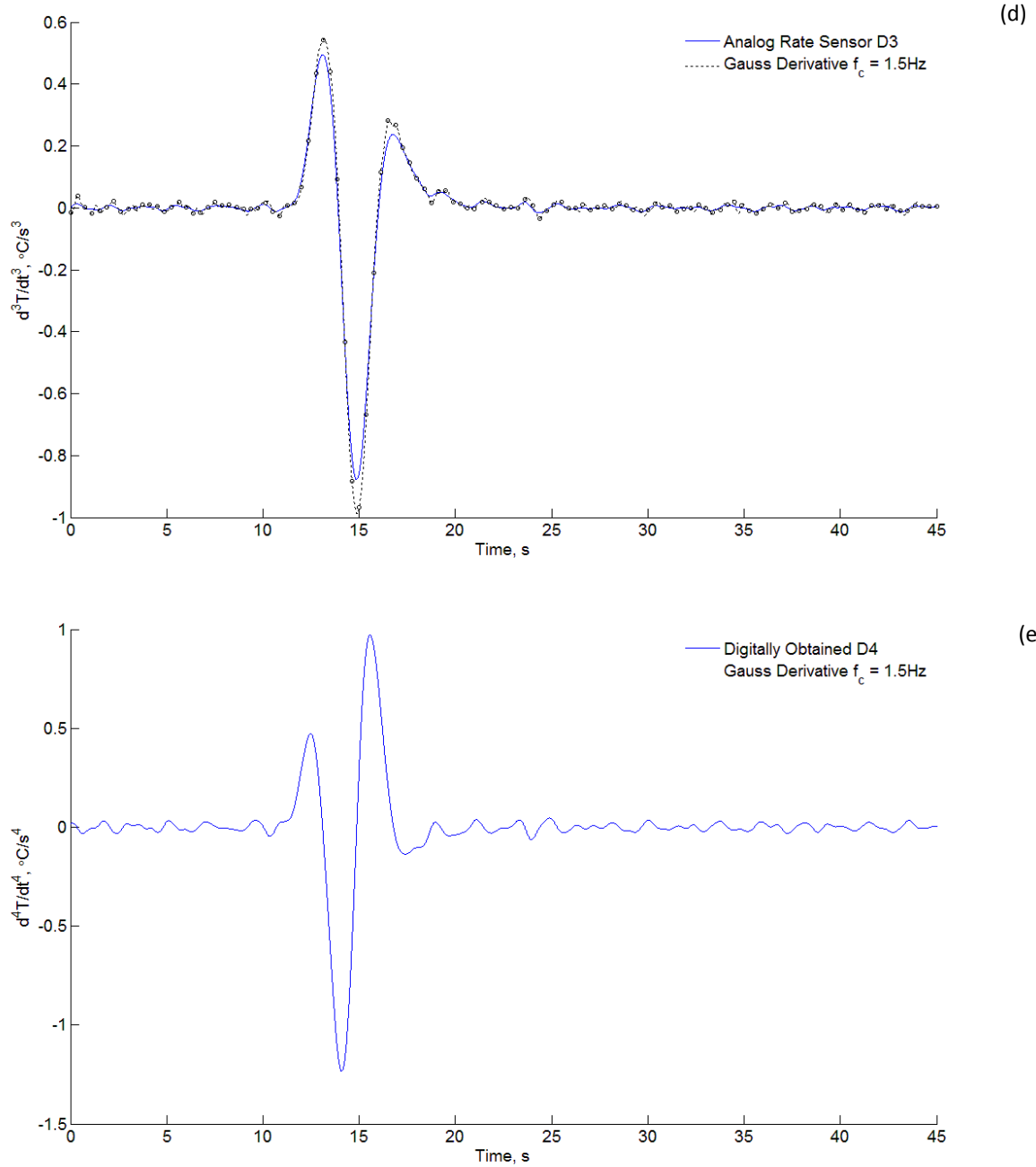
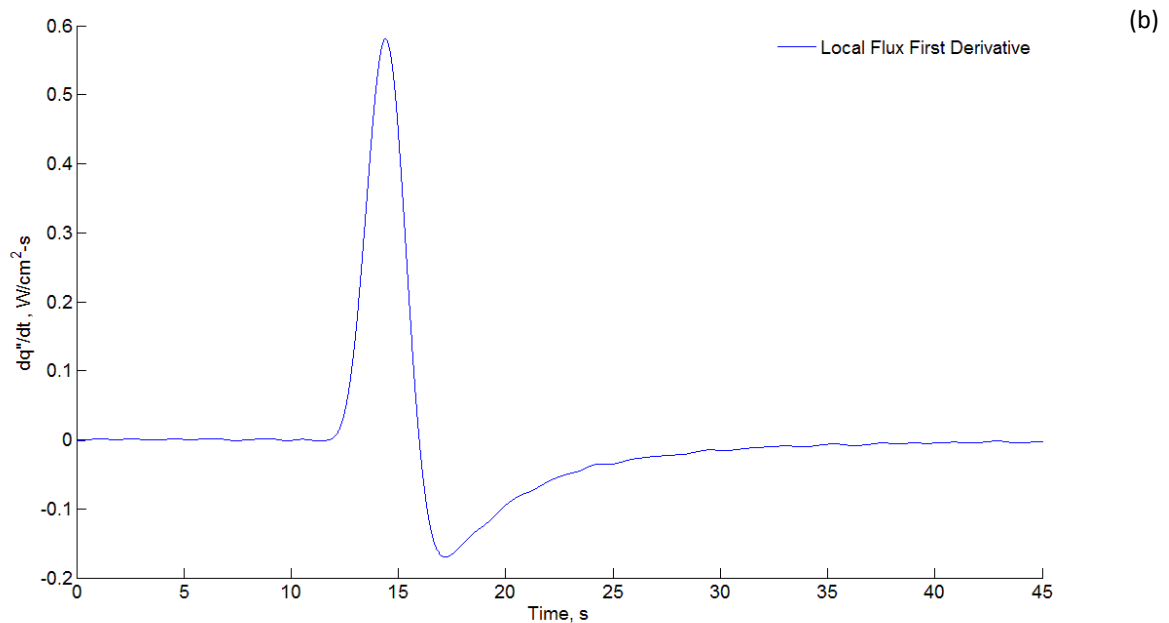
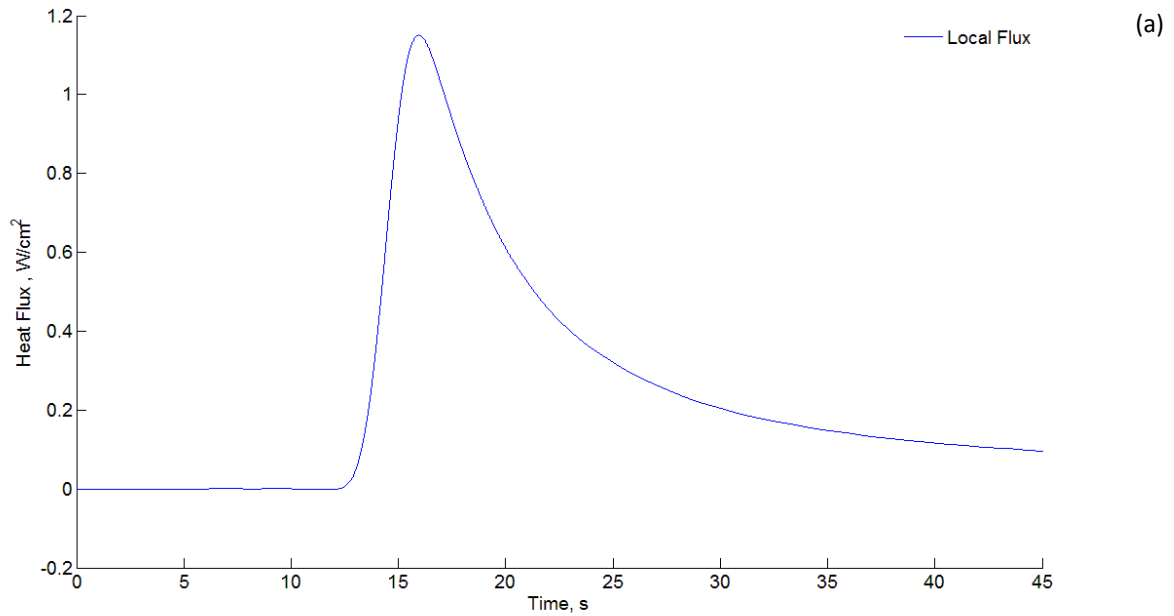


Figure 4.15: Test Case 2 Positional Corrected Rate Sensor Output. (a) Filtered Temperature at depth of 6.57mm, (b) First Time Derivative of Temperature and Digital Comparison, (c) Second Time Derivative of Temperature and Digital Comparison, (d) Third Time Derivative of Temperature and Digital Comparison, (e) Fourth Time Derivative of Temperature, Digitally Obtained through Gauss Function.

The next step in the prediction process is to obtain the positional flux and its time derivatives via Eq. (2.10). In Figure 4.16 once again the positional flux and its time derivatives are shown as determined from Eq. (2.10). From the lower magnitude of the input temperature derivatives it is expected that the flux graphs will have more noise than the previous test. However, the smooth local flux displayed in Figure 4.16a confirms that the analog first derivative of the temperature data is good and confirms the smoothing effects of Eq. (2.10) to improve the results.

Figure 4.16b shows that the noise in the first time derivative of the local heat flux is not noticeable. Figure 4.16c shows that the second time derivative of the local heat flux displays noticeable noise. Finally in Figure 4.16d the noise in the third time derivative of the local heat flux is more pronounced but still held in check; it is amazing that analog techniques allow for excellent retention of these higher order effects. It should be noted that between tests there is no difference in the process or cutoff frequencies used, it is only the inputs that change.



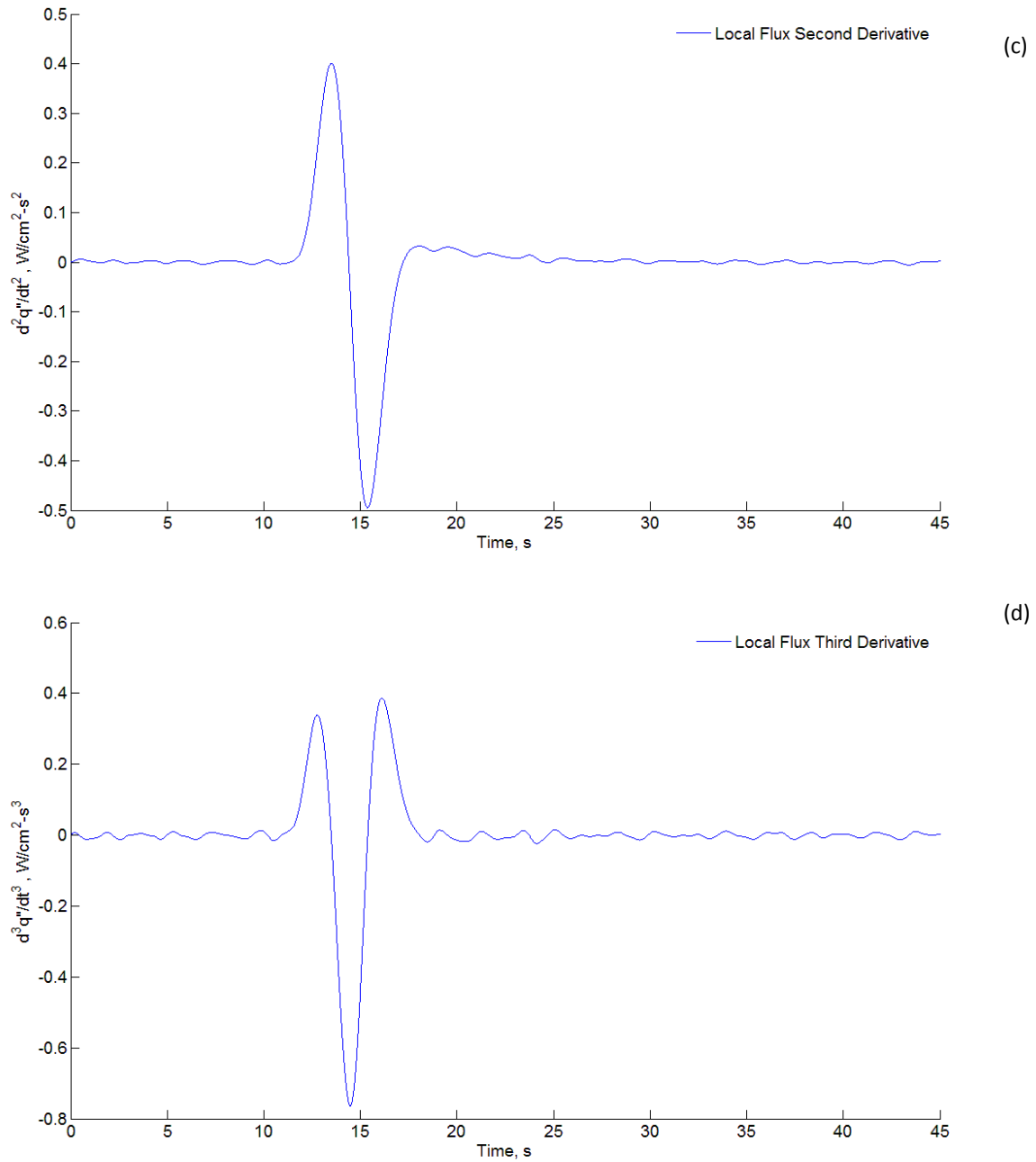


Figure 4.16: Test Case 2 Positional Flux and its Time Derivatives at 6.57mm from Surface Derived from Temperature Derivatives. (a) Local Flux, (b) First Time Derivative of Flux, (c) Second Time Derivative of Flux, and (d) Third Time Derivative of Flux.

Figure 4.17a presents the surface temperature predictions for test case two. The prediction based on the analog data is smooth, stable, and very close to the Finite Difference result. The only defect in the prediction is once again the slight lag in time. In this case the effect of spatial mesh refinement ($N=3$ versus $N=7$) and therefore the need to use higher order derivatives on the prediction via digital method is more apparent. The $N=3$ prediction which requires up to the third time derivative considerably overshoots the prediction, while the $N=7$ prediction

which requires up to the seventh time derivative stays much closer to the FD model. Interestingly the N=3 prediction using the analog derivative data is much better than the prediction using digital derivatives.

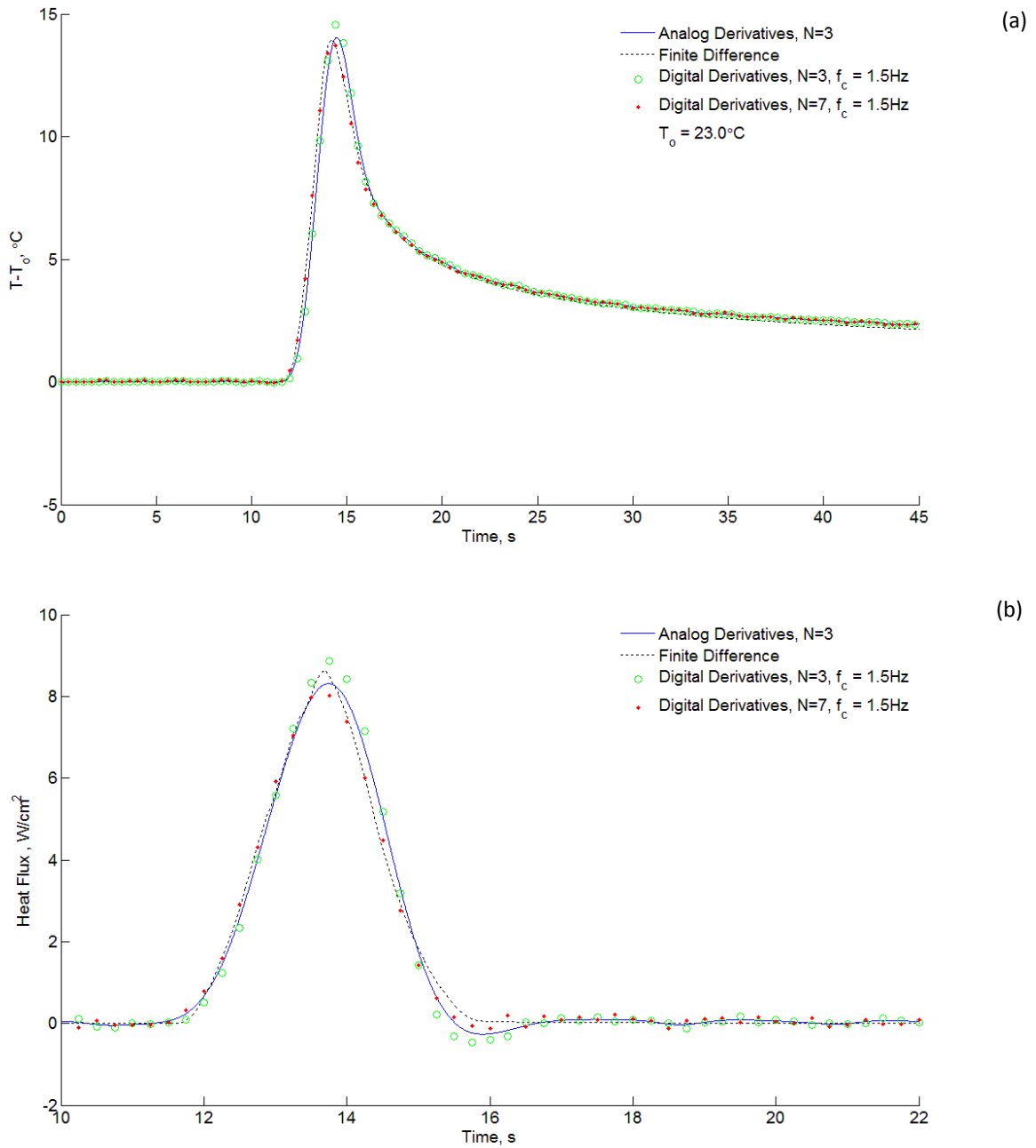


Figure 4.17: Test Case 2 Prediction of Surface Conditions via Global Time Method with Finite Difference Comparison. (a) Predictions of Surface Temperature from Analog Rate Sensor Derivatives and Derivatives Obtained Digitally from Temperature and (b) Predictions of Surface Heat Flux from Analog Rate Sensor Derivatives and Derivatives Obtained Digitally from Temperature. Heat Starts at 11.61s and is 4.20s in Duration. The Digital Predictions are Obtained at $f_c = 1.5\text{Hz}$ and are Not Represented at Each Time Step.

The surface heat flux predictions for test case two are presented in Figure 4.17b. The analog result is very smooth and stable, but lags behind the FD result. One might blame the inconsistencies in the FD model and the analog result on the half space requirement of Eq. (2.10); however, looking back to Figure 4.2a the back of the sample remains at a constant temperature until about 21s into the test. More likely the difference in the peaks of the two methods is caused by losses out of the sides of the slab not accounted for in the one dimensional FD model, uncertainties in the thermophysical properties and data, and also as mentioned before the first order time constant model which ignores the lead losses.

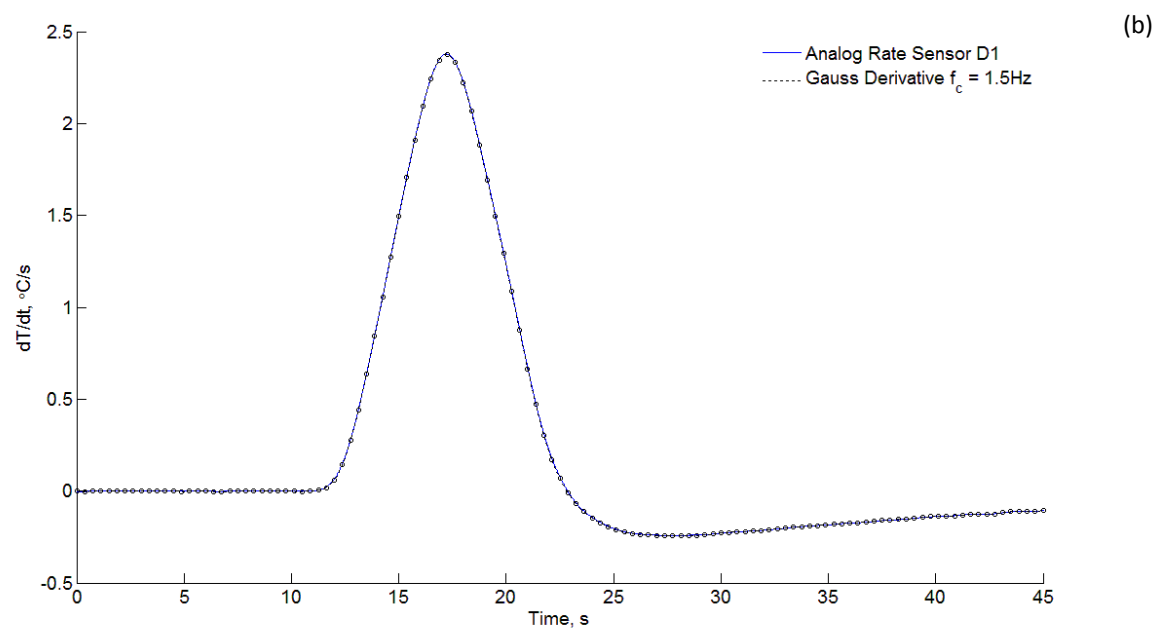
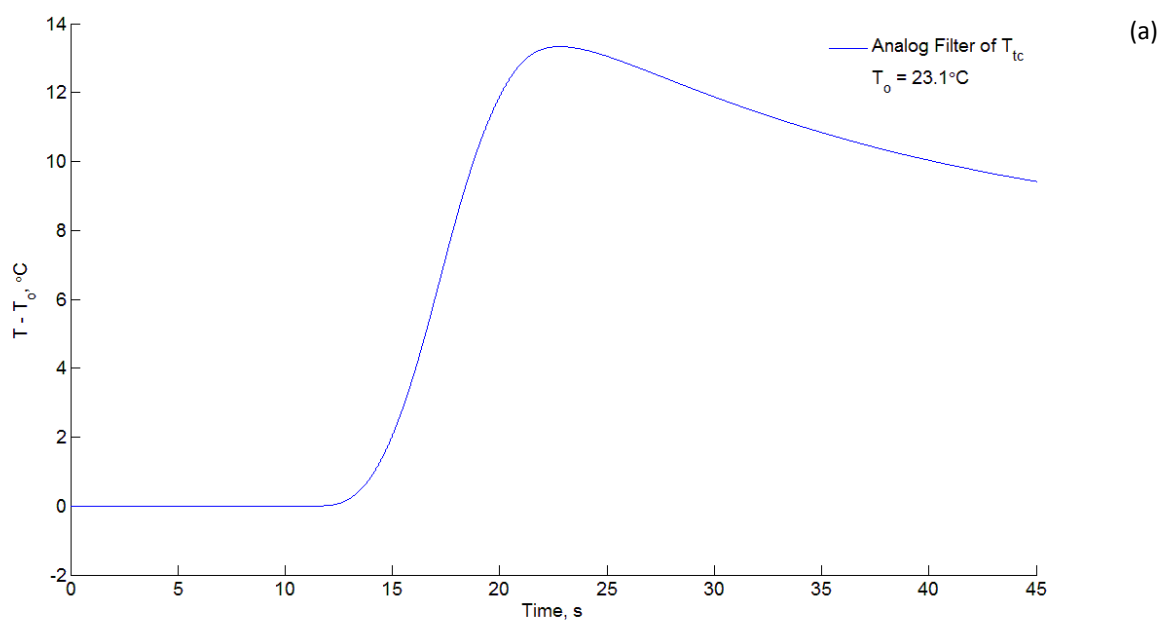
Still the measurement results are very promising. The total energy percent difference is very low at 8.10%, this is better than most other predictions. More impressive is the low mean residual at 0.0214 W/cm^2 . This is better than any test so far; it means that both data sets remain around the same value. The positive indicates that the prediction is generally greater than the FD model. Also the standard deviation of the residual is astounding at 0.104 W/cm^2 . This indicates there are very few outliers in the residual. The two heat fluxes are near the same value for most of the time period. This is also less than one fourth of the other tests; unfortunately not all of that can be attributed to the rate sensor as the input shape is easier to process. Normally the discontinuities cause a very large residual at just those points. It is still interesting that this is much better than test case one or control case.

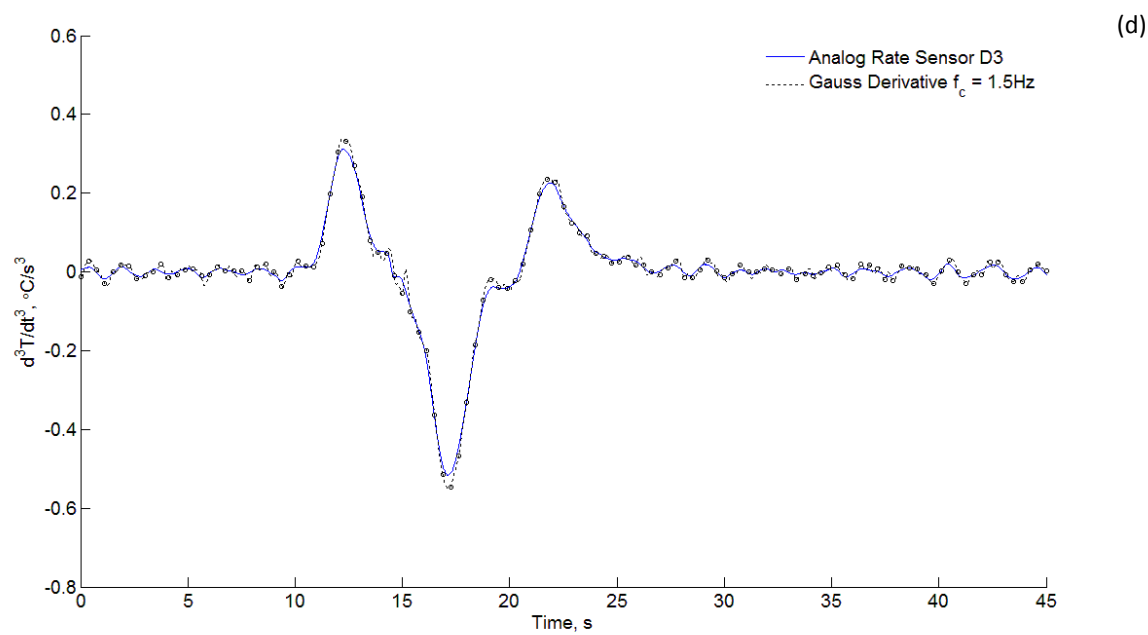
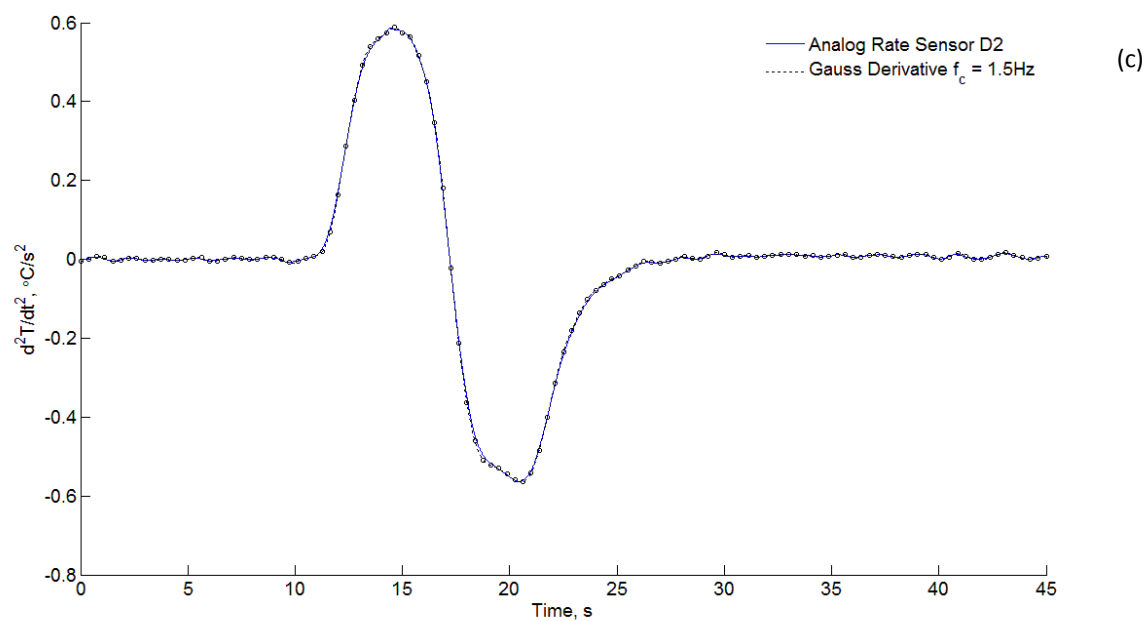
Once again the difference in analog and digital derivatives predictions, even though both are based on analog pre-filtered temperature data, is readily apparent from the $N=3$ digital graph over shooting the FD model while the analog prediction lies just below. The $N=7$ graph drastically undershoots the FD graph most likely due to increased noise in the higher derivatives drowning out the relevant signal and causing it to be over filtered.

4.5 TEST CASE 3: HIGH POWER RAMP INPUT

The final test case has the same general profile as the test case 2, but the run was made utilizing the full power of the system. This run has the highest energy input for all the tests. It lasts longer and reaches a higher power; therefore we expect it to have the best signal to noise ratio of all the runs and return the best results. This will showcase the rate sensor's ability in best case conditions.

Figure 4.18 presents the positional temperature and its time derivatives. Figure 4.18 shows that the maximum temperature rise at the in depth probe location is 13.5°C , higher than any of the previous due to the increased power. At this point because of the Butterworth filter it is hard to see any differences in input noise. Figure 4.18b shows the first time derivative of the temperature with no evidence of noise amplification from the temperature data. The gauss derivative also matches the analog data at every step. Figure 4.18c displays a small amount of increased noise in the second analog derivative before the heating. Unlike in previous tests the gauss matches the analog result very well even at the peaks. The gauss derivative has less trouble with the amount of noise present in the analog filtered temperature data in this case.





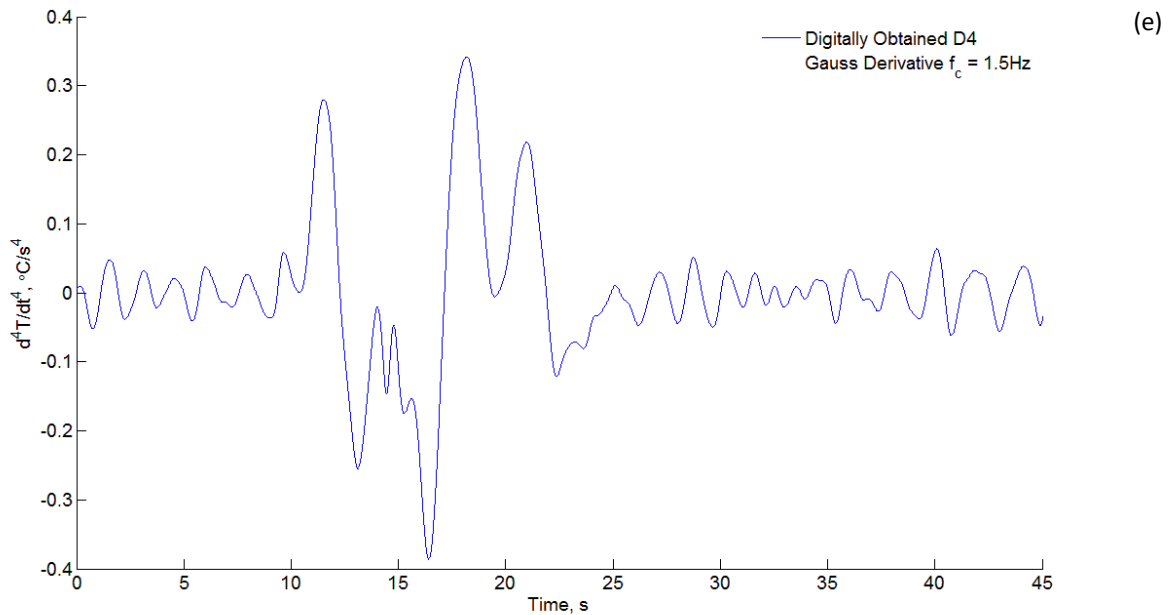
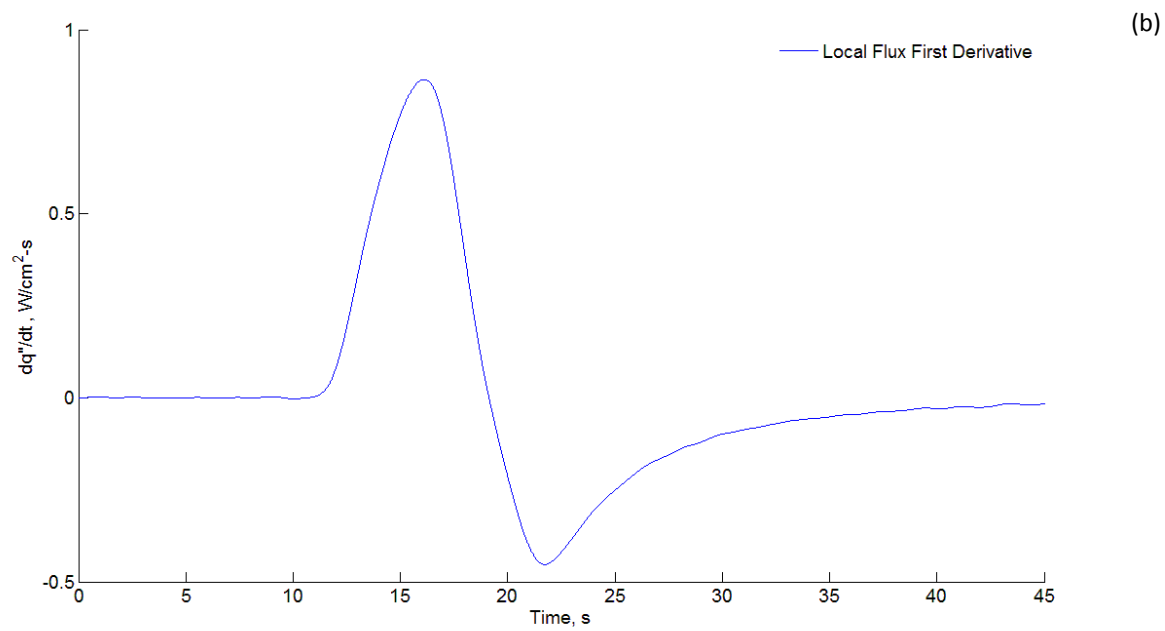
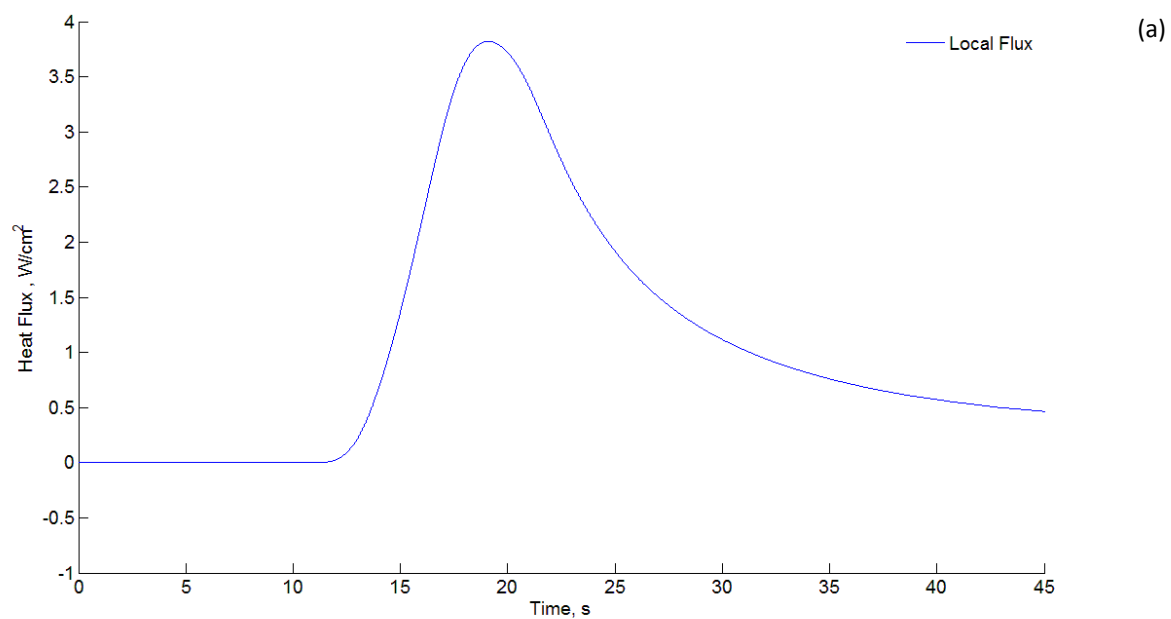


Figure 4.18: Test Case 3 Positional Corrected Rate Sensor Output. (a) Filtered Temperature at depth of 6.57mm, (b) First Time Derivative of Temperature and Digital Comparison, (c) Second Time Derivative of Temperature and Digital Comparison, (d) Third Time Derivative of Temperature and Digital Comparison, (e) Fourth Time Derivative of Temperature, Digitally Obtained through Gauss Function.

For the first time in this test case the gauss third derivate can be seen to deviate noticeably from the analog third derivate in Figure 4.18d. Noise also is seen to creep into the signal as the signal to noise ratio degrades. Moving on to the next graph, the gauss forth derivate exhibits significant noise in Figure 4.18e. It is strange that this test has a worse signal to noise ratio than all previous tests. From what is expected this test should have the lowest noise fourth derivative of any of the other tests due to the higher input levels. One possibility is that the larger input created a higher and more rapidly changing fourth derivative. This rapidly changing derivative was picked up by the filter as noise and attenuated. Another possibility is that the change in the input changed the behavior of the higher derivatives. The final surface prediction is necessary to make any further judgments.

The local flux and its time derivatives are expected to act the same as the previous tests and judging from the temperature and its analog derivatives graphs to exhibit very low noise except for the case of the last derivative. Figure 4.19a shows a familiar smooth, stable local flux. There appears to be no noise in the signal at all. Figure 4.19b also shows nearly no noise in the first time derivative of the local heat flux. It comes from already excellent data and has the benefit of smoothing via the Eq. (2.10) integration process. Moving on to Figure 4.19c, some noise begins to occur in the second time derivative of the local heat flux. Figure 4.19d presents the third time derivative of the local heat flux determined from the unusually noisy gauss forth derivate of the temperature (Figure 4.18e). Due to the filtering effects of integrating data, the third local flux derivative is much cleaner than it's input data (Figure 4.18e). Still the signal has a bit of noise present. The high quality of this data along with the clean temperature data seems to indicate a fairly good prediction.



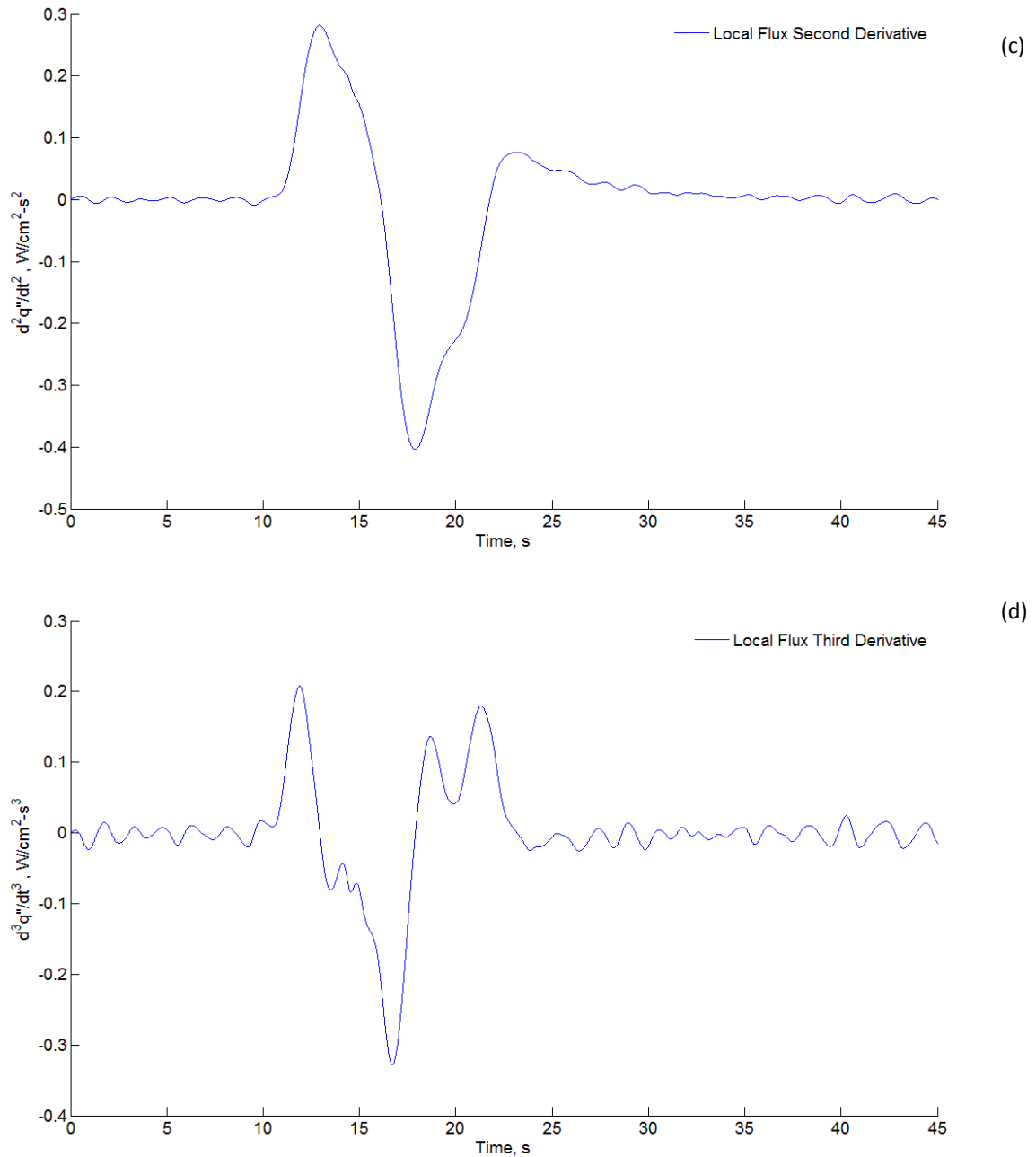


Figure 4.19: Test Case 3 Positional Flux and its Time Derivatives at 6.57mm from Surface Derived from Temperature Derivatives. (a) Local Flux, (b) First Time Derivative of Flux, (c) Second Time Derivative of Flux, and (d) Third Time Derivative of Flux.

The surface predictions for the final test are shown in Figure 4.20. The surface temperature shown in Figure 4.20a is the best of any of the tests. The analog based prediction still lags behind the FD model but the differences are small. The peaks are at a near identical value. Interestingly the N=3 digital derivative prediction is better than the N=7 digital prediction. This indicates that the higher order derivatives contained a large amount of noise that adversely effected the prediction. The N=3 projection without the extra derivatives was better able to match the FD model. This is supported by the unusually noisy fourth time derivative of temperature from Figure 4.18e.

Though that data was digitally produced it stemmed from an analog third derivative. The N=7 projection must use digital derivatives derived from the filtered temperature which unlike the analog derivatives, the noise in the digital process can become unbounded. Therefore the higher order derivatives used in the N=7 case which requires up to seventh time derivative of the temperature may not be meaningful, they are only noise. So the N=3 projection is better because it is unhampered by the meaningless derivatives.

The surface heat flux predictions also show an excellent agreement as shown in Figure 4.20b. The analog result is smooth and stable, matching the FD on the rise and the declining side. At some points like around 11s and 19s the analog prediction lags just behind the FD model like previous tests. Another interesting difference between the FD and analog signal occurs at the peak. The analog prediction has a rounded peak while the FD model has a sharper peak at the maximum point of heating. The factors contributing to the differences between the FD model and the inverse prediction were discussed before. Similar to the surface temperature predictions, the N=3 digital derivative prediction was closer to the FD result than the N=7 projection. Again this is caused by substantial noise in the required higher order derivatives (up to 8th time derivative of the temperature). In this case the useless, noisy higher order derivatives may even have caused the instabilities seen by the oscillatory nature of the N=7 data, especially noticeable around 15s.

The calculated metrics for checking the accuracy of the inverse predictions show strong support of the accuracy of the rate sensor and Global Time Method. The total energy percent difference between FD and the analog prediction was 5.92% that is the lowest value of any test yet. The mean residual was a similar value to the other tests at 0.0708 W/cm². This is still a low value, but like all the other analog predictions it over shoots the FD model. The standard deviation of the residual was 0.114 W/cm². Small standard deviation of the residual along with a close to zero mean of residual suggests that there are few outliers. These metrics lend great support to the analog rate sensor's accuracy.

This entire prediction took place during half space conditions. The half space time for these tests is the time required for the heat front to reach the back face of the plate and reflect back to the probe site. Looking back to Figure 4.3a the rear of the stainless steel slab doesn't experience any incoming flux until 21s into the run. This is about 10s after heating begins and only a small portion of relevant data in Figure 4.20b crosses that time period which does not account for the additional time required for the heat front to reflect back to the probe site.

All these predictions lend great confidence to the Global Time method and the quality of data presented to it by the Rate Sensor. Table 4.2 compiles all the previously mentioned metrics of the surface predictions for easy comparison. It clearly shows that the analog rate sensor maintains better accuracy.

Table 4.2 Surface Prediction Flux Comparison Metrics Surface Prediction vs. Finite Difference

Metric	Control Case, fc=0.7 Hz, N=3	Control Case, fc=0.7 Hz, N=7	Control Case, fc=0.5 Hz, N=3	Control Case, fc=0.5 Hz, N=7	Test Case 1	Test Case 2	Test Case 3
Q'' % diff	8.08%	7.67%	8.58%	8.60%	7.54%	8.10%	5.92%
μ_r (W/cm ²)	0.0672	0.0638	0.0713	0.0715	0.0802	0.0214	0.0708
σ_r (W/cm ²)	1.01	0.792	0.530	0.530	0.542	0.104	0.114

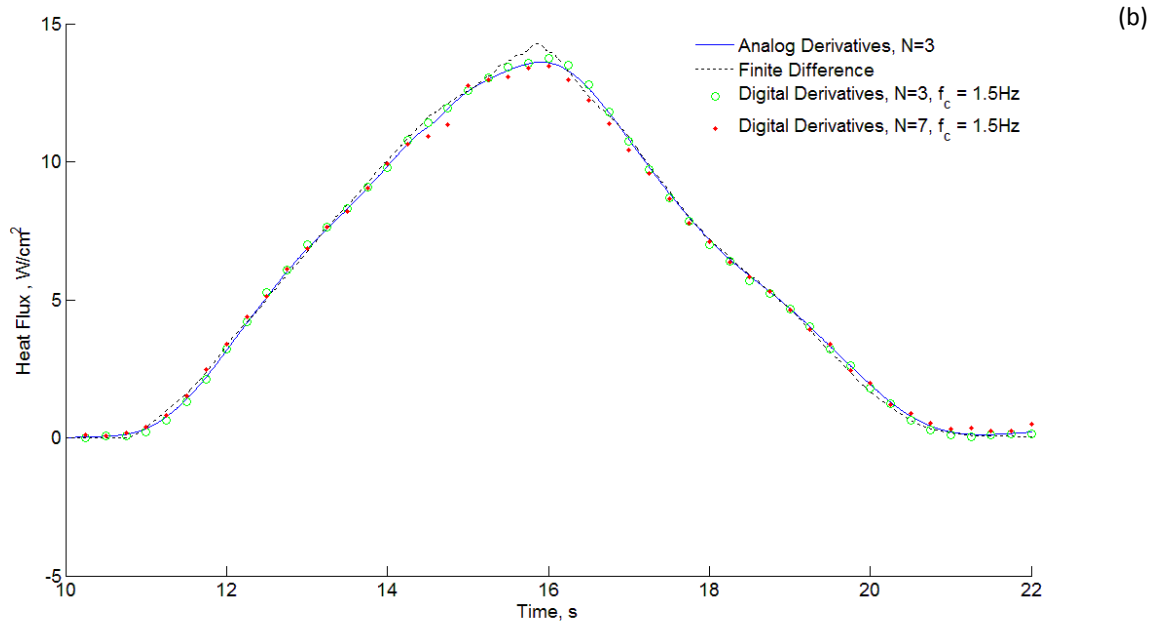
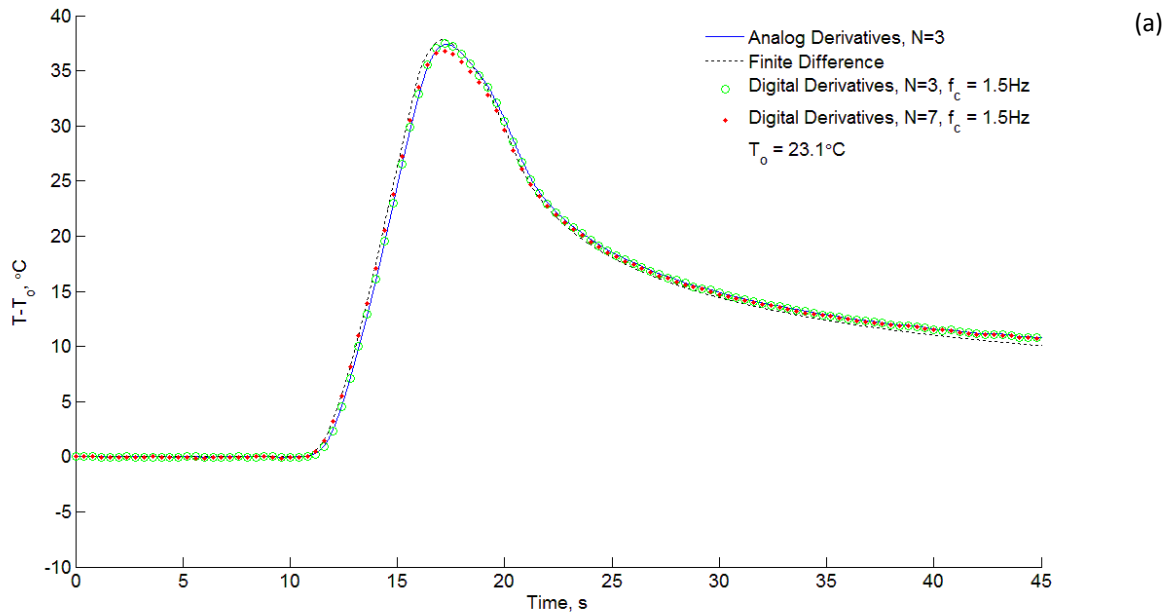


Figure 4.20: Test Case 3 Prediction of Surface Conditions via Global Time Method with Finite Difference Comparison. (a) Predictions of Surface Temperature from Analog Rate Sensor Derivatives and Derivatives Obtained Digitally from Temperature and (b) Predictions of Surface Heat Flux from Analog Rate Sensor Derivatives and Derivatives Obtained Digitally from Temperature. Heat Starts at 10.72s and is 10.62s in Duration. The Digital Predictions are Obtained at $f_c = 1.5\text{Hz}$ and are Not Represented at Each Time Step.

With all the Rate sensor results presented, an interesting trend is seen to develop. Examining the time derivatives of temperature it is possible to estimate the important temporal features of the surface heat flux without any attempt on prediction of the surface boundary condition. In Figure 4.18b the peak of the first derivative is observed at around $t=16s$; comparing this to Figure 4.20b the maximum heat flux occurs around $t=16s$ as well. An examination of the second time derivative of temperature (Figure 4.18c) reveals that departure and return to zero occurs at approximate times of $t=11s$ and $t=26s$. The experimental surface heating (Figure 4.20b) began at $t=10.7s$ and ended at $t=21.3s$; the similarities of these numbers show interesting correlations between the in depth time derivatives of temperature and the surface heat flux. Finally the third time derivative of temperature (Figure 4.18d) has three significant magnitudes occurring at around $t=12s$, $t=17s$, and $t=22s$. The noted times show, respectively, the onset of surface heating ($t=12s$), the time at which the surface heat flux is maximum ($t=17s$), and the end of heating period ($t=22s$). This shows that the rate sensor data obtained from an array of indepth thermocouples may be used to determine the transition location in hypersonic flights.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

The design of an integrated rate-based sensor consisting of a series of well-designed voltage-rate interfaces that directly generate higher-time derivatives of an in-depth thermocouple's temperature is presented and tested in an IHCP experiment. The collected sensor site data is corrected to its positional values using a first-order model for the thermocouple bead. The required thermocouple's time constant is estimated with the aid of a direct finite difference one-dimensional, constant properties heat conduction model. Positional heat flux at the sensor site and its higher-time derivatives are produced using time integral relationships relating the time derivative of the positional temperature to the local heat flux. Finally, the surface heat flux and temperature are calculated using the finite difference based Global Time Method. The results show the following: (a) the analog filter-rate sensor system generates very accurate noiseless temperature and its higher temporal derivatives; (b) the rate sensor is an accurate in-depth local heat flux gauge while half-space conditions prevail; and (c) use of the rate sensor data in an IHCP with only three spatial control volumes produces an accurate prediction of the surface heat flux with an accuracy of total energy input of 5.92% at best. Additionally it is demonstrated that the rate sensor data obtained from an array of indepth thermocouples may be used to determine the transition location in hypersonic flights.

The next step to improve the rate sensor is to increase the number derivative circuits in it. With more derivatives a more complicated method of projection could be used to achieve better accuracy. The physical task of adding the extra circuits to the sensor is relatively easy. The difficulty lies in tuning the characteristics of the chips to still achieve a useable signal from all the derivative circuits. One thing that may make this easier is to change the circuit from being built on an experimental breadboard to a printed circuit board. This will cut down noise, but there is no opportunity to change the configuration of the sensor once it is built. Another possible improvement to the sensor could be made by increasing the order of the Butterworth pre-filter. This would better filter noise with less effect on actual signal. Also if the rate sensor could be reconfigured to allow the circuit characteristics to change easily, it would move the sensor towards being applicable to a wider range of systems. This would allow fast switching when measuring conditions change. It would be difficult to achieve, but creative use of variable resistors or resistor bridges may be sufficient to achieve this. Finally the ultimate goal should be to program the sensor into an integrated circuit and allow all characteristics to be determined by a few external components. This would allow the sensor to be easily used in a wide variety of applications, possibly even being installed into existing measurement devices. With these advancements this sensor could become a staple measurement device used in a wide variety of fields.

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APPENDIX

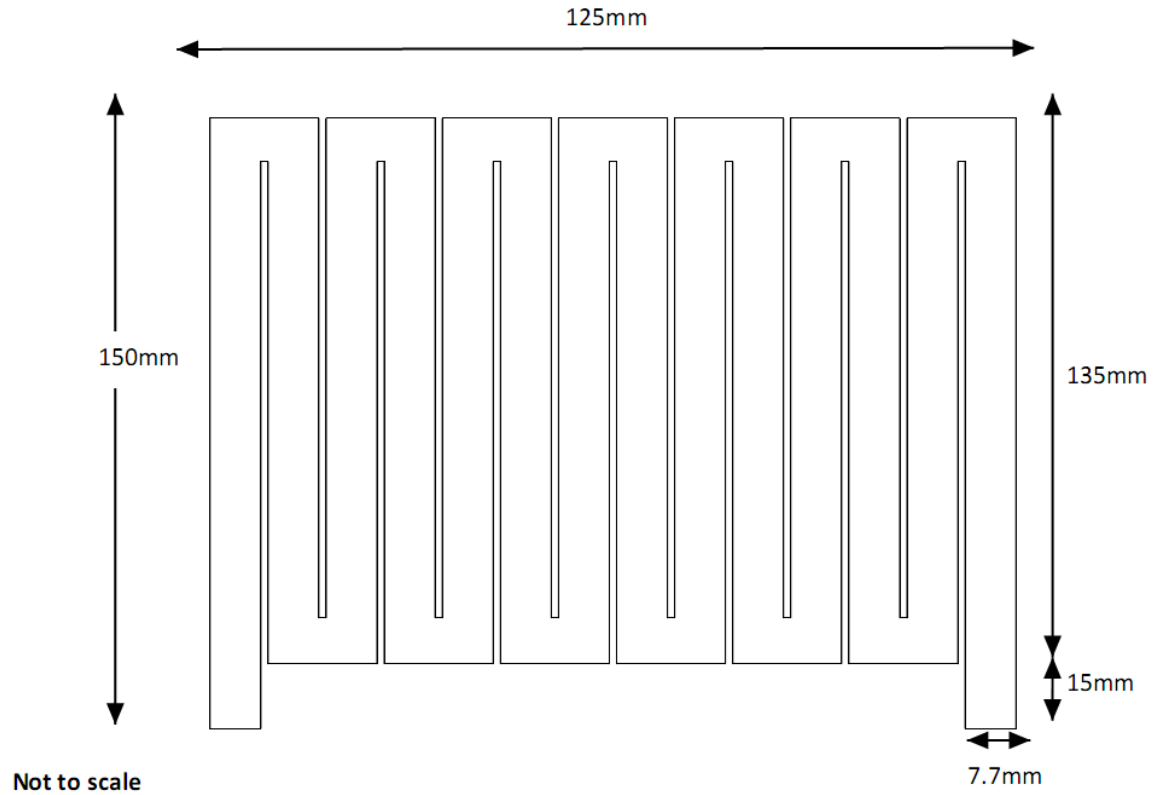


Figure A.1: Conceptual Drawing of the Custom Nichrome Heater By Dr. Majid Keyhani.

Table A.1: Thermophysical Properties of Materials Used.

Property	Value
Stainless Steel	
Thermal diffusivity, α	$3.95 \times 10^{-6} \text{ m}^2/\text{s}$
Thermal Conductivity, k	$14.9 \text{ W}/(\text{m K})$
Mica	
Thermal diffusivity	$4.73 \times 10^{-3} \text{ m}^2/\text{s}$
Density	$300 \text{ kg}/\text{m}^3$
Specific heat	$0.5 \text{ J}/(\text{kg K})$
Heater (nichrome)	
Thermal diffusivity	$7.75 \times 10^{-5} \text{ m}^2/\text{s}$
Density	$1420 \text{ kg}/\text{m}^3$
Specific heat	$1.09 \text{ J}/(\text{kg K})$
Heater resistance	2.185Ω
Potting compound (Cotronics 989F)	
Thermal conductivity	$1.7 \text{ W}/(\text{m K})$

Table A.2: Measured Distances for the Sandwich Experiment.

Parameter	Value
$L_{\text{Stainless Steel}}$	27.9 mm
L_{Mica}	0.076 mm
L_{Paste}	$\lesssim 0.03$ mm
L_{Heater}	0.125 mm
b	6.57 mm

Table A.3: Depths and Characteristics of Thermocouple Holes.

Sample	Slab 1				Slab 2	
Hole	S1B0	S1A1	S1A2	S1B3	S2A0	S2B2
Average Depth (mm)	12.951	6.568	6.586	12.899	6.474	12.893
Distance From Centerline (mm)	0.000	-6.689	12.322	18.600	0.000	-12.608
Resistance to Block (Ω)	2.3	5.9	1.1	1.3M	1.6	0.9

Table A.4: Components and Characteristics of Pre-Filter.

Filter: Component	Stage 1 Measured Value	Stage 2 Measured Value
C1 (μF)	2.45	1.09
C2 (μF)	27.4	9.21
R1 ($\text{k}\Omega$)	6.49	61.9
R2 ($\text{k}\Omega$)	235.9	158.4
Characteristic	Calculated Value	Calculated Value
Gain	1.0	1.0
f_c (Hz)	0.496	0.506
τ (s)	0.594	0.241
Total Lag (s)		0.835

Table A.5: Components and Characteristics of Derivative Circuits.

Chip: Component	D1 Measured Value	D2 Measured Value	D3 Measured Value
C1 (nF)	6900	19950	10580
C2 (nF)	1094	3190	837
R1 (k Ω)	23.26	7.88	7.56
R2 (k Ω)	145.5	49.66	95.06
R3 (k Ω)	19.98	6.87	7.04
Characteristic	Calculated Value	Calculated Value	Calculated Value
f _c (Hz)	0.996	1.009	1.995
τ_{d1} (s)	0.160	0.157	0.080
τ_{d2} (s)	0.159	0.158	0.080
Gain	1.004	0.991	1.006
lag (s)	0.320	0.316	0.160

VITA

Jake Plewa earned a Bachelor's of Science in Mechanical Engineering at the University of Tennessee, Knoxville while working as an Electrician for Amteck and a research and production engineer for Eaton Electrical Assemblies Division. Afterward he went on to earn a Master's of Science in Mechanical Engineering at the University of Tennessee, Knoxville. While completing this degree he worked under the direction of Dr. Majid Keyhani and Dr. Jay Frankel in the field of inverse heat conduction, focusing on its application with analog sensors. His research interests include new experimental and numerical methods of inverse heat transfer especially those utilizing analog circuitry, the use of custom circuits to improve inverse heat transfer measurements and research, and the application of this research to aerospace engineering.