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I am submitting herewith a thesis written by Catherine Anne Kelly entitled "System Identification and Parameter Estimation of the Piper PA32-301 Saratoga." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Aviation Systems.

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System Identification and Parameter Estimation of the Piper PA32-301 Saratoga

A Thesis
Presented for the
Master of Science Degree
The University of Tennessee, Knoxville

Catherine Anne Kelly
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DEDICATION

This thesis is dedicated to

The Almighty God

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ABSTRACT

The purpose of this project was to estimate the longitudinal stability derivatives of the Piper Saratoga (tail number N22UT), one of the aircraft owned and operated by the University of Tennessee Space Institute (UTSI), using the System Identification Programs for Aircraft, SIDPCAC. After the instrumentation error was reduced by means of a data compatibility analysis, the stability derivatives were estimated using both an output error and an equation error parameter estimation technique.

Following a discussion of the test aircraft and instrumentation and a brief overview of stability derivatives and parameter estimation, the flight test is discussed in detail along with an analysis of the data. The data compatibility analysis showed huge error in the instrumentation measurements. Based on its estimate of the biases and scale factors, the data, particularly the angle of attack and pitch angle, once corrected for the error, had a significant change in scaling when compared to the original measured data. The stability derivative estimates obtained using both parameter estimation methods were close in value for the corresponding parameter and were similar for each test point. The only exception was the Z-force derivative due to the pitch rate which exhibited huge fluctuations. Although the flight test data was truncated to only include the doublet maneuver in the analysis, the output error routines did not reach convergence for a large majority of the test points. A detailed discussion of the results and recommendations are provided.

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NOMENCLATURE

List of Symbols

0 (subscript)	reference value
A, B, C, D	system matrices
a	acceleration (ft/sec ²)
a_x	acceleration in the x direction (ft/sec ²)
a_y	acceleration in the y direction (ft/sec ²)
a_z	acceleration in the z direction (ft/sec ²)
b	wing span (ft)
c.g.	center of gravity
$\bar{c}$	mean aerodynamic chord (ft)
dps	degrees per second
e (subscript)	elevator
ft	feet
g	acceleration due to gravity (32.2 ft/sec ²)
GUI	graphical user interface
i	sample number
I_x	roll inertia (slug-ft ²)
I_y	pitch inertia (slug-ft ²)
I_z	yaw inertia (slug-ft ²)

L	likelihood function
m	mass (slugs)
M	moment (ft-lb)
MAC	mean aerodynamic chord
Ma or M_α	pitching moment due to change in angle-of-attack (s^{-2})
Mde or M_{δ}	pitching moment due to change in elevator deflection (s^{-1})
Mq or M_q	pitching moment due to change in pitch rate (s^{-2})
N	number of sampled data points
p	roll rate (radians/sec)
q	pitch rate (radians/sec)
$\hat{q}$	non dimensional pitch rate
$\dot{q}$	pitch acceleration (radians/sec ²)
$\bar{q}$	dynamic pressure = $\frac{1}{2}\rho V^2$ (lb/ft ²)
R	estimate of error covariance matrix
r	yaw rate (radians/sec)
rad	radians
S	wing area (ft ²)
s or sec	second
T(superscript)	transpose
u	velocity along X body axis (ft/sec)
u(t)	input vector

V	aircraft total velocity (ft/sec)
v	velocity along Y body axis (ft/sec)
w	velocity along Z body axis (ft/sec)
$x(t)$	state vector
$y(i)$	output vector
$z(i)$	output vector
Z_N	sequence of measurements
Z or F_Z	Z-force (lb)
Z_a or Z_α	Z-force due to change in angle-of-attack (ft/sec ²)
$Z_{\delta e}$ or Z_{δ_e}	Z force due to change in elevator deflection (ft/sec)
Z_q or $Z_{\dot{q}}$	Z force due to change in pitch rate (ft/sec ²)
α	angle-of-attack (degrees or radians)
$\dot{\alpha}$	rate of change of angle-of-attack (deg/sec or rad/sec)
β	sideslip angle (degrees or radians)
δ	control deflection (degrees or radians)
θ	pitch angle (degrees or radians); unknown parameters
ρ	air density (slugs/ft ³)
ϕ	roll angle (degrees or radians)
$v(i)$	noise (output error)
C_m	pitching-moment coefficient; $C_m = \frac{M}{qSc}$

C_{m_α} pitching moment coefficient due to change in angle-of-attack; $C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$

$C_{m_{\delta_e}}$ pitching moment coefficient due to change in elevator deflection;

$$C_{m_{\delta_e}} = \frac{\partial C_m}{\partial \delta_e}$$

C_{m_q} pitching moment coefficient due to change in pitch rate; $C_{m_q} = \frac{\partial C_m}{\partial \frac{q\bar{c}}{2V}}$

C_z normal force coefficient; $C_z = \frac{F_z}{qS}$

C_{z_α} normal force coefficient due to change in angle-of-attack; $C_{z_\alpha} = \frac{\partial C_z}{\partial \alpha}$

C_{z_q} normal force coefficient due to change in pitch rate; $C_{z_q} = \frac{\partial C_z}{\partial \frac{q\bar{c}}{2V}}$

$C_{z_{\delta_e}}$ normal force coefficient due to change in elevator deflection; $C_{z_{\delta_e}} = \frac{\partial C_z}{\partial \delta_e}$

CHAPTER 1

INTRODUCTION

Purpose

The purpose of this research was to use a system identification program to estimate six of the longitudinal stability derivatives of UTSI's Piper Saratoga from flight test data, information which had not previously been available. An elevator doublet was used to excite the longitudinal short period response. Using the onboard instrumentation and Daqbook®, a data acquisition unit, fifteen flight parameters were recorded onto a laptop computer while six additional parameters were recorded by hand during the test. These data, in conjunction with other descriptive characteristics of the airplane, were employed by the System Identification Programs for AirCraft, SIDPAC, for a data compatibility analysis (to account for any instrumentation error) and the estimation of the aircraft's stability derivatives.

Details of the Piper Saratoga

OPERATION AT UTSI

The PA-32 series was put into production in 1965. In 1981, the Piper PA32-301 Saratoga (N22UT) was acquired from Piper Aircraft Company by the University of Tennessee Space Institute. N22UT operates as a basic "flying classroom", equipped with flight test instrumentation available at each of the five passenger seats. The Aviation

Systems and Flight Research Department uses the aircraft in all of its short courses, including the Fixed-Wing Stability and Control Flight Testing and Fixed Wing Performance Flight Testing programs. In addition, N22UT has been used in several three month courses for engineers at the Naval Air Station: Patuxent River, Maryland as well as all four Federal Aviation Administration (FAA) Test Pilot/Flight Test Engineer courses in 1991.

AIRCRAFT DESCRIPTION

N22UT is a semi-monocoque, low wing, all metal airplane, accommodating six occupants with two separate one hundred pound capacity baggage compartments. It has a 300 horsepower single Lycoming IO-540-K1G5 engine driving a constant-speed McCauley three- bladed propeller. Two interconnected fuel tanks are located in each wing, holding a total of 107 gallons of fuel, of which 102 gallons are useable.

The engine is equipped with a geared starter, a 60 ampere alternator, dual magnetos, vacuum pump drive, a diaphragm-type fuel pump, and fuel injection. Engine controls consist of a throttle control (adjusts manifold pressure), a propeller control (adjusts propeller speed), and a mixture control lever (adjusts air to fuel ratio).

The 14-volt electrical system includes a 12-volt battery for starting and to back up alternator output. Electrical power is supplied by a 60 ampere alternator. Standard electrical accessories include the starter, the electric fuel pump, the stall warning horn, the ammeter, and the annunciator panel.

The landing gear are in a tri-cycle configuration and are fixed down.

Flight controls consist of a conventional stabilator, rudder, and ailerons. They are dual controlled with a cable system between the controls and surfaces.

The vacuum system operates the air-driven gyro instruments. The system consists of an engine driven vacuum pump, a vacuum regulator, a filter, and the necessary plumbing.

Pitot and static pressure are picked up by the pitot head on the bottom of the left wing. Not standard on the aircraft are a wingtip angle-of-attack (α) vane and a wingtip side slip (β) vane located on the boom off the right wingtip. Airspeed and altitude read on the student consoles in the aircraft are obtained from an aft fuselage static port and a right under wing Keil probe. These are also not standard.

Instrumentation

DATA ACQUISITION EQUIPMENT

The acquisition system was based on the *DaqBook-120*® notebook sized data acquisition unit manufactured by IOTech. This multifunctional unit is capable of A/D converting sixteen analog inputs, D/A converting two analog outputs, and reading or writing to twenty four I/O lines configured as input or output in banks of 8-bits each. The resolution is 12-bits over a $\pm 5V$ input range. Channels are scanned sequentially at a maximum single channel sample rate of 100 kHz.

IOTech manufactures a number of special purpose expansion cards to accept signals having differing characteristics such as low-level thermocouple, strain gage, etc. Although only a total of fifteen analog inputs were required for this application, a *DBK-12* expansion card was needed to incorporate body acceleration and pressure transducer measurements, signals which had to be added for the purpose of this project. The expansion card channels were multiplexed into one of the *Daqbook-120*® channels (ch. 10). The resulting digital data frames were sent to the laptop computer via the parallel port to be recorded on the computer's hard drive.

The *DaqView*® data acquisition software was installed and ran on the laptop computer during the flight recordings. This software enabled the selection of active channels, monitoring of signal activity, selection of file names, selection of data file format, selection of sample rates, and triggering of data acquisition on and off. For this testing, a sample rate of 100 Hz per channel and an ASCII file format were selected. This file format allowed post-processing with *MS Excel*® and *MATLAB*® software packages.

SENSOR SIGNALS

Most of the signals were already available on the aircraft's 25-pin connector wired in parallel with the signals routed to the student consoles. These signals were scaled for a +/- 3V range.

Some signals were not available on the 25-pin connector and therefore additional sensors were installed in the aircraft. This was the case for the differential and absolute

pressure transducers required for dynamic and ambient air pressure measurement, and additionally, separate connections were made to a couple of three-axis accelerometers.

POWER DISTRIBUTION

Power to operate the various items of equipment was derived from a 10A circuit breaker connected to the aircraft's 12V power bus. The 12V power was converted with an inverter to 120V AC at 60 Hz to operate the *Daqbook*® and the computer, and to $\pm 15V$ to power the pressure transducers.

TEST CONDUCT

A configuration file was established to define the data acquisition settings that were to be maintained for each flight. These included the data channel designations, triggering method, sample rate, and file format. It was necessary to establish a unique computer file folder and starting file name prior to each flight. The folder name was coded according to the test date, and the file names were selected sequentially CK1.bin, CK2.bin, CK3.bin, etc. The '.bin' (binary) files were converted to '.txt' following each run. Data acquisition triggering was accomplished by a mouse click on a screen graphic.

After each flight, the data were transferred to a CD and archived for further processing using SIDPAC and other programs.

CHAPTER 2

BACKGROUND

Stability and Control

OVERVIEW

The stability and control characteristics of an airplane are dependent on the purpose of the vehicle. An aircraft, such as a fighter, that is designed to be very maneuverable, will have weak stability characteristics. An aircraft, such as a transport, that lacks high maneuverability will be very stable and not very controllable.

A stability derivative expresses how a moment or force acting on the vehicle changes as the velocity, attitude of the vehicle, or the deflection of a control surface changes. For flight stability, the forces and moments acting on the vehicle must be such as to oppose any changes that may occur. For example, when the elevator is deflected by a control input, the pitch of the aircraft changes in response; but, when the elevator deflection is removed, the aircraft attitude tends to return automatically to the trim condition. The magnitudes of the moment and force changes due to an aircraft attitude change are a function of the stability and control derivatives of the aircraft. $C_{m\alpha}$ is a non-dimensional proportionality factor determining the pitching moment produced by a change in the angle-of-attack (α) of the aircraft, such as with an elevator deflection. The pitching moment acts about the y-axis of the aircraft and tends to restore the aircraft to trim when the deflection is removed. Note that there is a strong cross coupling between the various quantities affecting stability. For that reason, a change in attitude or control

input will affect many flight conditions other than the one directly concerned. These effects, however, are beyond the scope of this study and were not considered during the course of the investigation.

THE DERIVATIVES

Listed below are a summary of the six longitudinal derivatives that were estimated for this project. [9] A dimensional analysis of the units for each derivative is provided in brackets.

The Z-Force derivative with respect to angle-of-attack is:

$$Z_{\alpha} = C_{Z_{\alpha}} \frac{\bar{q}S}{m} \left(\frac{ft}{s^2} \right); \left[\frac{\frac{lb_f}{ft^2} ft^2}{sl} = \frac{lb_f}{sl} = \frac{sl \frac{ft}{s^2}}{sl} = \frac{ft}{s^2} \right]$$

EQUATION 2.1

The Z-Force derivative with respect to pitch rate is:

$$Z_q = C_{Z_q} \left(\frac{\bar{c}}{2u_0} \right) \frac{\bar{q}S}{m} \left(\frac{ft}{s} \right); \left[\frac{\frac{ft \frac{lb_f}{ft^2} ft^2}{\frac{ft}{s} sl}}{\frac{sl}{s}} = \frac{lb_f}{s} = \frac{sl \frac{ft}{s^2}}{s} = \frac{ft}{s} \right]$$

EQUATION 2.2

The Z-Force derivative with respect to elevator deflection is:

$$Z_{\delta_e} = C_{Z_{\dot{\alpha}}} \frac{\bar{q}S}{m} \left(\frac{ft}{s^2} \right); \left[\frac{\frac{lb_f}{ft^2} ft^2}{sl} = \frac{lb_f}{sl} = \frac{sl \frac{ft}{s^2}}{sl} = \frac{ft}{s^2} \right]$$

EQUATION 2.3

The pitching moment derivative with respect to angle-of-attack is:

$$M_{\alpha} = C_{M_{\alpha}} \frac{\bar{q}S\bar{c}}{I_y} (s^{-2}); \left[\frac{\frac{lb_f}{ft^2} ft^2 ft}{sl - ft^2} = \frac{lb_f}{sl - ft} = \frac{sl \frac{ft}{s^2}}{sl - ft} = s^{-2} \right]$$

EQUATION 2.4

The pitching moment derivative with respect to pitch rate is:

$$M_q = C_{M_q} \left(\frac{\bar{c}}{2u_0} \right) \frac{\bar{q}S\bar{c}}{I_y} (s^{-1}); \left[\frac{\frac{lb_f}{ft^2} ft^2 ft}{\frac{ft}{s} sl - ft^2} = \frac{lb_f s}{sl - ft} = \frac{sl \frac{ft}{s^2} s}{sl - ft} = s^{-1} \right]$$

EQUATION 2.5

The pitching moment derivative with respect to elevator deflection is:

$$M_{\delta_e} = C_{M_{\delta_e}} \frac{\bar{q} S \bar{c}}{I_y} (s^{-2}); \left[\frac{\frac{lb_f}{ft^2} ft^2 ft}{sl - ft^2} = \frac{lb_f}{sl - ft} = \frac{sl \frac{ft}{s^2}}{sl - ft} = s^{-2} \right]$$

EQUATION 2.6

Parameter Estimation

OVERVIEW

In parameter estimation, data, often contaminated by measurement error, are used in an estimation routine to optimize the unknown parameters of a mathematical model of a system. In general, these predicted values will produce a model response that should closely match the measured outputs of the system. The process improves upon the initial guess of the parameters that characterize the system so that these predicted parameters, in turn, can provide a model response, that has much less noise than a measured signal. Overall, if the error in the measurement is large, the error in the parameter estimates will be large. The two parameter estimation techniques employed in the analysis for this research paper were both maximum likelihood estimates: the equation error method and the output error method. [2,5]

EQUATION ERROR METHOD [8]

Equation Error assumes that there is process error, but no measurement error. It is based on the principle of least squares, minimizing the error between **computed** output variables (predicted using parameter estimate values) and the **measured** output variables.

To obtain an estimate of the *non dimensional* longitudinal stability and control derivatives using the SIDPAC program, two values of the pitching moment coefficient, C_m , need to be evaluated and compared against each other. The computed value of C_m is

$$C_m = \frac{1}{\bar{q}Sc} [I_y \dot{q} + (I_x - I_z)pr + I_{xz}(p^2 - r^2)]$$

EQUATION 2.7

where all of the quantities on the right hand side of the equation are either known or measured during the flight test. The hypothesized model structure of the pitching moment coefficient is

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_q} \frac{q\bar{c}}{2V} + C_{m_{\delta_e}} \delta_e + C_{m_{\alpha\delta_e}} \alpha\delta_e + v$$

EQUATION 2.8

Each parameter (stability coefficient) on the right hand side of this equation is predicted using equation error parameter estimation to minimize the squared error between the computed values of C_m and the model for C_m . The stability coefficients in equation 2.8 are estimated using the least squares method (lsq.m). This equation is of the form

$$z = y + v$$

EQUATION 2.9

where $z \equiv C_m$ and

$$y = X\theta$$

EQUATION 2.10

In equation 2.10,

$$X \equiv H(x, u) = [1 \quad \alpha \quad \frac{q\bar{c}}{2V} \quad \delta_e \quad \alpha\delta_e] \text{ and}$$

EQUATION 2.11

$$\theta = [C_{m_o} \quad C_{m_\alpha} \quad C_{m_q} \quad C_{m_{\delta_e}} \quad C_{m_{\alpha\delta_e}}]^T$$

EQUATION 2.12

The matrix X is the column of regressors (dependent variables) in the model and the vector θ contains the least squares estimate of the real parameter.

The *dimensional* longitudinal stability and control derivatives were calculated in this same manner. The pitching moment derivatives were estimated using the pitch acceleration. The computed value of $\dot{q}$ was the derivative with respect to time of the pitch rate that was measured during the flight test. The hypothesized model structure of the pitch acceleration was

$$\dot{q} = M_\alpha \alpha + M_q q + M_{\delta_e} \delta_e + M_O$$

EQUATION 2.13

Each parameter (stability derivative) on the right hand side of this equation was predicted using equation error parameter estimation to minimize the squared error between the computed values of $\dot{q}$ and the model for $\dot{q}$.

The stability derivatives in equation 2.13 were estimated using the least squares method (lsq.m). This equation was of the form

$$z = y + v$$

EQUATION 2.14

where $z \equiv \dot{q}$ and

$$y = X\theta$$

EQUATION 2.15

In equation 2.15,

$$X \equiv H(x, u) = [\alpha \quad q \quad \delta_e \quad 1] \text{ and}$$

EQUATION 2.16

$$\theta = [M_\alpha \quad M_q \quad M_{\delta_e} \quad M_o]^T$$

EQUATION 2.17

The matrix X was the column of regressors (dependent variables) in the model and the vector θ contained the least squares estimate of the real parameter.

Likewise, the body-axis Z force *dimensional* derivatives were estimated using $\dot{\alpha}$. The angle-of-attack rate was calculated by taking the derivative of the measured angle-of-attack with respect to time. The measured values in the X matrix were the same as for the pitching moment, and the parameter estimates were

$$\theta = [Z_{\alpha} \quad Z_q \quad Z_{\delta_e} \quad Z_o]^T$$

EQUATION 2.18

OUTPUT ERROR METHOD [5,8]

Output Error assumes measurement noise, but no process noise. It is a batch iterative procedure where all measurements and parameter corrections are computed simultaneously. In this method, two things were estimated- the unknown parameters and the measurement covariance matrix. For this SIDPAC analysis, the unknown parameters consisted of the six longitudinal stability derivatives, Z_{α} , Z_q , Z_{δ_e} , M_{α} , M_q , and M_{δ_e} , as well as the angle-of-attack (α), pitch rate(q), and vertical acceleration (a_z). The dynamic system was described by a first order differential equation for the aircraft state:

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t);$$

EQUATION 2.19

the model output:

$$y(t) = C(\theta)x(t) + D(\theta)u(t);$$

EQUATION 2.20

and the measured output:

$$z(i) = y(i) + v(i) \quad i = 1, 2, 3, \dots, N;$$

EQUATION 2.21

where the noise, v , was normally distributed with 0 mean and R variance.

The Fisher model was the model used in this analysis for the uncertainties in the parameters and measurements. The maximum likelihood parameter estimate for this model was

$$\hat{\theta} = \max_{\theta} L(Z_N | \theta)$$

EQUATION 2.22

where $Z_N = [z(1) \ z(2) \ \dots \ z(N)]$. The likelihood function for this model was

$$L[Z_N | \theta] = (2\pi)^{-Nn_0/2} |R|^{-N/2} \exp \left[\sum_{i=1}^N -\frac{1}{2} v^T(i) R^{-1} v(i) \right]$$

EQUATION 2.23

where

$$v(i) = z(i) - y(i) \quad i = (1, 2, 3, \dots, N)$$

EQUATION 2.24

To make the problem easier to solve, the negative log of the likelihood function was minimized. This technique is the same as maximizing the likelihood function:

$$J = -\ln L(Z_N | \theta) = \frac{1}{2} \sum_{i=1}^N v(i) R^{-1} v^T(i) + \frac{N}{2} \ln |R|$$

EQUATION 2.25

Initial values of the parameter estimates used for this method included the derivatives obtained from the equation error method earlier. The model response was calculated using those initial parameter values and input data from the aircraft. The model response and the measured response were compared and the error was used to calculate the cost function (eq.2.26), which included two unknowns: the parameter estimates, θ , and the noise covariance matrix, R . Equation 2.26 was minimized for a given $R = \hat{R}$:

$$J = \frac{1}{2} \sum_{i=1}^N [z(i) - y(i)]^T \hat{R}^{-1} [z(i) - y(i)]$$

EQUATION 2.26

First R was fixed and θ was estimated; then θ was fixed and R was estimated. The first and second gradients of the cost function were calculated for the alternating iterations and the unknown parameters were updated. The process continued until both θ and R converged.

The maximum likelihood parameter vector estimate was found iteratively from:

$$\hat{\theta} = \theta_0 + \Delta \hat{\theta}$$

EQUATION 2.27

where θ_0 was the estimate of θ and

$$\Delta \hat{\theta} = \left[\sum_{i=1}^N \frac{\partial y^T(i)}{\partial \theta} \hat{R}^{-1} \frac{\partial y(i)}{\partial \theta} \right]_{\theta=\theta_0}^{-1} \left[\sum_{i=1}^N \frac{\partial y^T(i)}{\partial \theta} \hat{R}^{-1} v(i) \right]_{\theta=\theta_0}$$

EQUATION 2.28

For a given θ , R was estimated by setting $\partial J / \partial R = 0$ and solving for R:

$$\hat{R} = \frac{1}{N} \sum_{i=1}^N [z(i) - y(i)][z(i) - y(i)]^T$$

EQUATION 2.29

CHAPTER 3

FLIGHT TEST

Equipment

In addition to the onboard instrumentation, special equipment had to be incorporated. A laptop computer, filter, pressure transducer, data acquisition unit, and power strip were added.

Crew

The flight crew consisted of a pilot, safety pilot, and two flight test engineers.

Test Plan

Because the location of the c.g. affects the magnitude of the pitching moment derivative [11], two flight tests varying the c.g. were conducted over a two day period. The first test was flown with an aft c.g. location and the second with a forward c.g. location. The c.g. was altered by positioning one of the flight test engineers in either the rear (aft) or center (forward) seat. 50 lbs of ballast was also included in the forward baggage compartment during the second flight. All other variables were held constant between the two flight tests. At the beginning of flight 1, the c.g. location was at 92.8 inches aft datum (23.2% MAC) while for flight 2, the c.g. location was at 90.1 inches aft

datum (18.6% MAC). At the end of flight 1, the c.g. location was at 92.7 inches aft datum (23.1% MAC) while for flight 2, the c.g. location was at 90.0 inches aft datum (18.5% MAC). Weight and balance information is located in Appendix D.

The aircraft was flown in nine different configurations each test day. At an altitude of 3000 ft, the Saratoga was trimmed at 70, 90, or 110 kts with a flap setting of 0° (no), 10° (1/4), or 40° (full). The flight test card is located in Appendix F. Once the aircraft was trimmed at a particular setting, the laptop computer was activated to record data. Approximately 5 seconds later, the pilot input an elevator doublet, which, on average, lasted 2-3 seconds. From a neutral position, the pilot sharply pushed the control stick forward, causing the elevator to deflect downward. Next, the control stick was pulled sharply in the opposite direction past its original trim position, causing the elevator to deflect upward. The control stick and elevator were then returned to the original trim position. The laptop continued to record for a few more seconds, logging the aircraft response after the step inputs, before it was deactivated by the flight test engineer. Three to four data points were collected for each of the nine configurations. Both flights, flown in the morning, lasted approximately 1 hour. Both flights also encountered turbulence toward the end of the test. The first flight included a familiarization flight for the crew, so additional time was taken.

CHAPTER 4

METHOD OF ANALYSIS

Overview

The **S**ystem **I**dentification **P**rograms for **A**ir**C**raft (SIDPAC) is a collection of MATLAB® controlled subroutines (called m-files) that work together to determine flight characteristics of the test vehicle from specific flight information collected during the maneuver. The analyses used in this thesis were dependant on the data appearing in a format the SIDPAC routines expected. In order to process the data correctly, each parameter was assembled into a specific column (or channel) in a matrix called *fdata* by SIDPAC (Appendix G).

Assembling the Data Matrix

Sixteen parameters, including time, were recorded using the onboard instrumentation through a laptop computer:

- time
- absolute pressure
- x body axis acceleration
- elevator deflection
- z body axis acceleration

- angle-of-attack
- Euler pitch angle
- pitch rate
- sideslip
- y body axis acceleration
- aileron deflection
- rudder deflection
- Euler roll angle
- roll rate
- yaw rate
- differential pressure

Six parameters were hand recorded during flight:

- manifold pressure
- engine RPM
- outside air temperature
- carburetor air temperature
- heading
- fuel remaining

The following parameters were derived using the above flight information:

- True airspeed
- Air density
- Altitude
- Longitudinal stick deflection
- Lateral stick deflection
- Rudder pedal deflection
- Throttle position
- Thrust

The following was information provided by Piper Aircraft Company:

- roll inertia
- pitch inertia
- yaw inertia
- wing reference area
- wing span
- mean aerodynamic chord

The following were determined by UTSI:

- aircraft mass (empty weight + crew weight + varying weight of the fuel)
- x cg position
- y cg position

- z cg position

From the above information, specific m-files in SIDPAC can calculate the following parameters and input them into the correct column of the fdata matrix:

- roll acceleration
- pitch acceleration
- yaw acceleration
- measured x body axis acceleration
- measured y body axis acceleration
- measured z body axis acceleration
- angle-of-attack rate
- sideslip angle rate
- roll thrust vectoring
- pitch thrust vectoring
- yaw thrust vectoring
- body axis X aerodynamic force coefficient
- body axis Y aerodynamic force coefficient
- body axis Z aerodynamic force coefficient
- aerodynamic rolling moment coefficient
- aerodynamic pitching moment coefficient
- aerodynamic yawing moment coefficient
- aerodynamic drag force coefficient
- wind axis aerodynamic side force coefficient

- aerodynamic lift force coefficient
- thrust force coefficient
- non-dimensional roll rate
- non-dimensional pitch rate
- non-dimensional yaw rate
- x body axis velocity component
- y body axis velocity component
- z body axis velocity component

SIDPAC m-files

The SIDPAC subroutines were initiated in MATLAB® using the **Saratoga_fda.m** file. This file was a modification of **Navion_fda.m**, a flight data analysis script written specifically for the longitudinal flight maneuvers of UTSI's variable stability aircraft. Once the flight data were loaded, converted from voltages to engineering units, and arranged into the standard SIDPAC form, a data compatibility analysis was performed. The data were then processed through an equation error parameter estimation routine and an output error parameter estimation routine to estimate Z_{α} , Z_q , Z_{δ_e} , M_{α} , M_q , and M_{δ_e} . The following were the core SIDPAC subroutines called by the **Saratoga_fda.m** script [5,7]:

eu_convert_Saratoga.m

Loaded the flight test data from an ascii file. Converted measured voltages to engineering units and plotted the longitudinal and lateral response of the aircraft during flight.

fltdatsel_Saratoga.m

Assembled flight test data from the aircraft into the standard SIDPAC data array fdata.

sid_dcmp.m

Created a graphical user interface for data compatibility analysis using data from the standard data matrix fdata.

lesq.m

Computed the least squares estimate of the real parameter.

r_colores.m

Computed the Cramer_Rao bounds for least squares regression parameter estimation (error bound correction). Also computed the least squares estimate of the parameter.

model_disp.m

Displayed the functional form of the model defined (displayed the equation error and output error results).

lon_plot.m

Implemented MATLAB® graphics commands to make standard lines for a longitudinal flight test maneuver.

oe.m;

Computed the output error estimate of the parameter, the Cramer-Rao bound matrix, and the model output using the modified Newton-Raphson optimization.

m_colores.m

Computed the Cramer-Rao bounds for maximum likelihood estimation both conventionally and accounting for the actual frequency content of the residuals.

CHAPTER 5

RESULTS

Data Compatibility Analysis

In the data compatibility analysis, kinematic relationships among the aircraft's 1) translational motion of the c.g., 2) rotational motion about the c.g. and 3) magnitude and orientation of the air-relative velocity were used to check that the measurements were mutually consistent. By using the kinematic equations to check these measurements against one another, this analysis quantified the instrumentation error and the measured data were corrected for the systematic error. [5] Therefore, the parameters being estimated were the biases and scale factors in the instrument measurements.

The data compatibility analysis was initiated with the **sid_dcmp.m** command. A plot of the angle-of-attack versus time (figure 5.1) appears. Through this graphical user interface (GUI), the desired initial and final time of the flight test was chosen, removing any data not wanted for the analysis. For example, this maneuver, flown at a forward c.g. at 90 knots with 10 degrees of flap, has been cut from 9.37 seconds of data to 3.82 seconds (figure 5.2). The cut data included half a second at trim before the maneuver, the excitation, and a bit of the aircraft's response. All of the data points for this project were handled the same way.

Pilot: Jim Wright

Aircraft: Piper Saratoga (N22UT)

Maneuver: Elevator Doublet

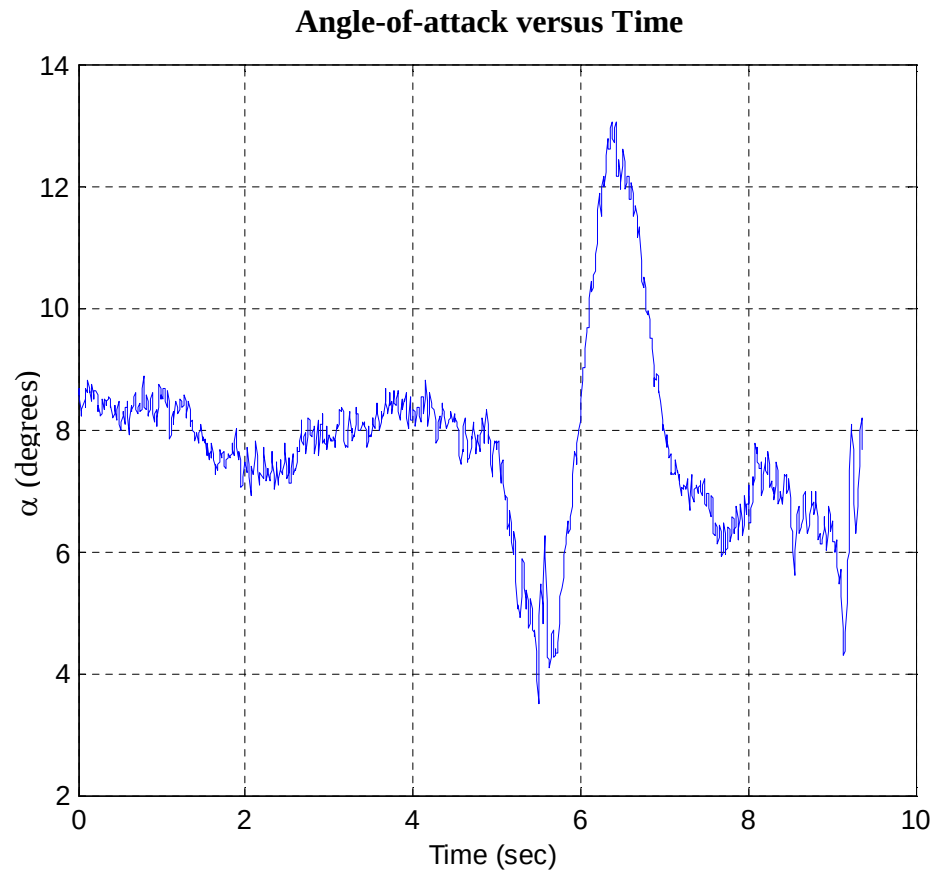
Test Date: 01/25/2006

Gross Weight: 3600 lbs

C.G. Range: 90.1"-90.0" aft datum

Temperature: 25° F

Pressure Altitude: 3000 ft



Created by: Catherine Kelly

Date: 06/03/2006

Program: SIDPAC

FIGURE 5.1: PLOT OF ANGLE-OF-ATTACK BEFORE TEST POINT FWD 90-10 DATA WERE TRUNCATED ¹

¹ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

Pilot: Jim Wright

Aircraft: Piper Saratoga (N22UT)

Maneuver: Elevator Doublet

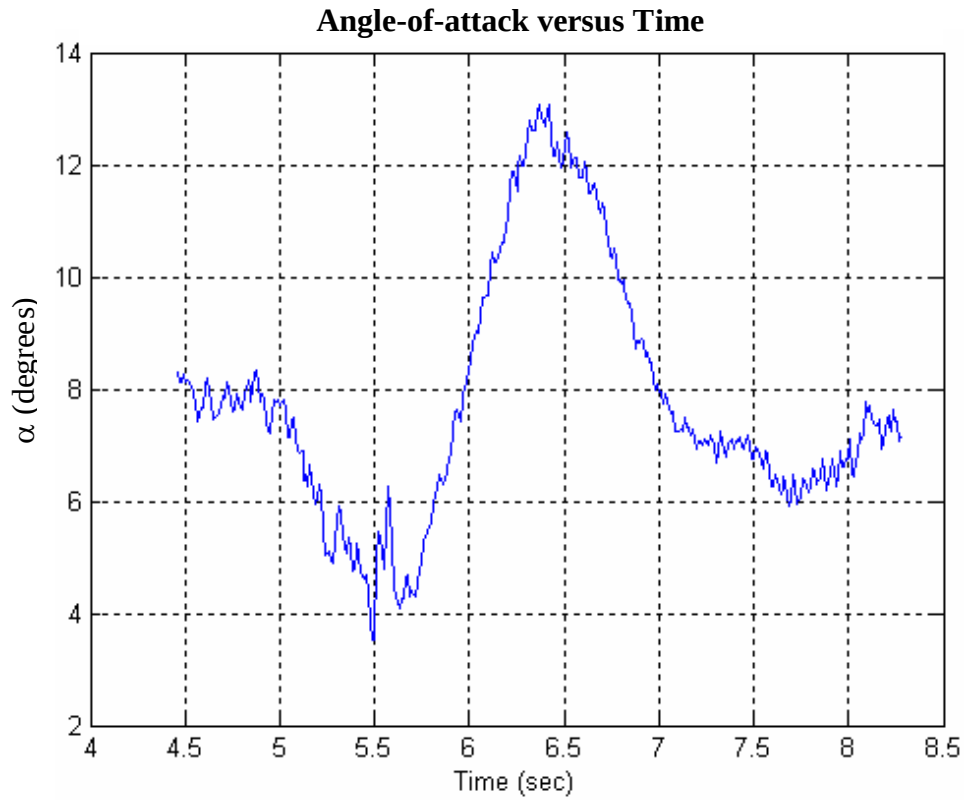
Test Date: 01/25/2006

Gross Weight: 3600 lbs

C.G. Range: 90.1"-90.0" aft datum

Temperature: 25° F

Pressure Altitude: 3000 ft



Created by: Catherine Kelly

Date: 06/03/2006

Program: SIDPAC

**FIGURE 5.2: PLOT OF ANGLE-OF-ATTACK AFTER TEST POINT FWD 90-10² DATA
WERE TRUNCATED**

² Flown at a forward c.g. at 90 knots with 10 degrees of flaps

The next GUI to appear plotted the measured values (labeled “Flight” in the legend) and the calculated values (labeled “Model” in the legend) of roll angle (ϕ), pitch angle (θ), heading angle (ψ), velocity (V), sideslip angle (β), and angle-of-attack (α) (figures 5.3 and 5.4). For this analysis, θ , V , and α were the only parameters calculated using the kinematic equations for the model output:

$$\dot{\theta} = q \cos \phi - r \sin \phi, \quad (\theta = \int \dot{\theta})$$

EQUATION 5.1

$$V = \sqrt{u^2 + v^2 + w^2}$$

EQUATION 5.2

$$\alpha = \tan^{-1} \left(\frac{w}{u} \right)$$

EQUATION 5.3

where:

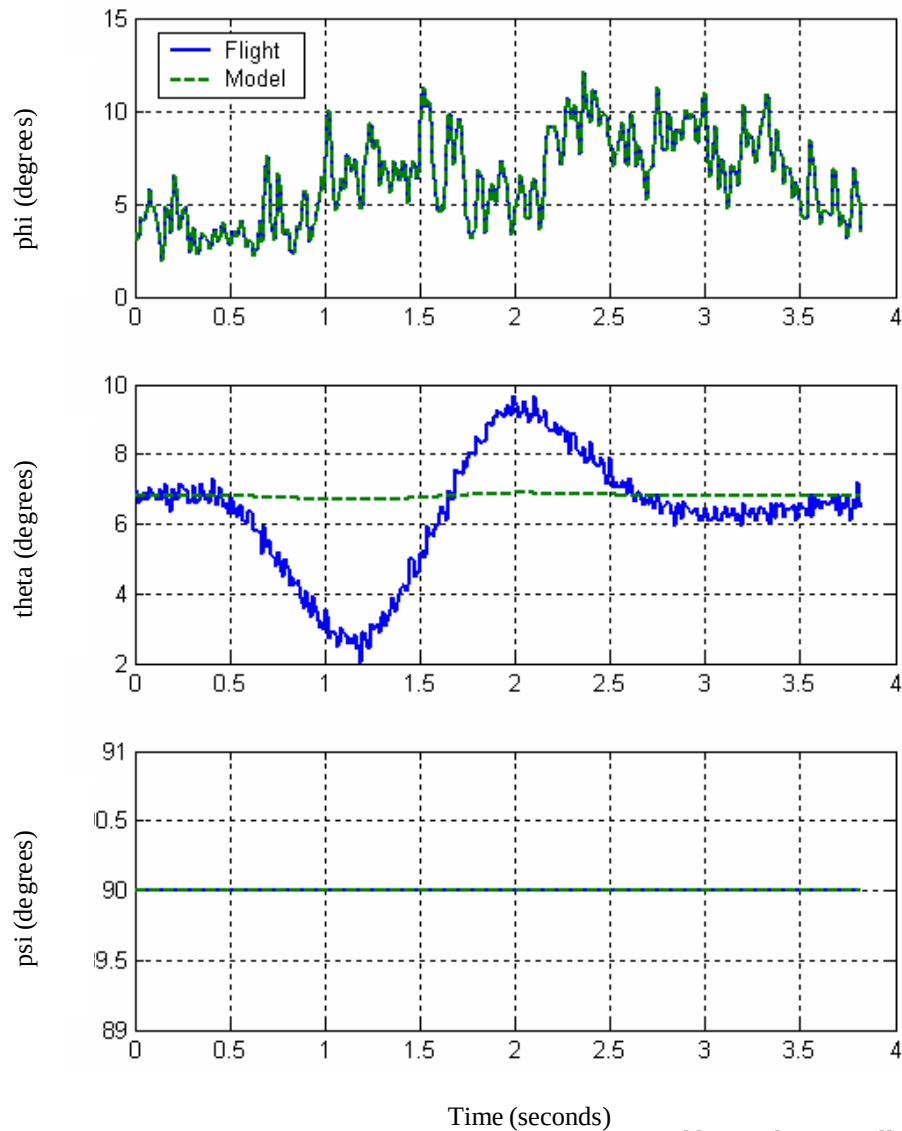
$$\dot{u} = rv - qw - g \sin \theta + a_x, \quad (u = \int \dot{u});$$

EQUATION 5.4

Pilot: Jim Wright
Maneuver: Elevator Doublet
Gross Weight: 3600 lbs
Temperature: 25° F

Aircraft: Piper Saratoga (N22UT)
Test Date: 01/25/2006
C.G. Range: 90.1"-90.0" aft datum
Pressure Altitude: 3000 ft

Data Compatibility Analysis



Created by: Catherine Kelly

Date: 06/03/2006

Program: SIDPAC

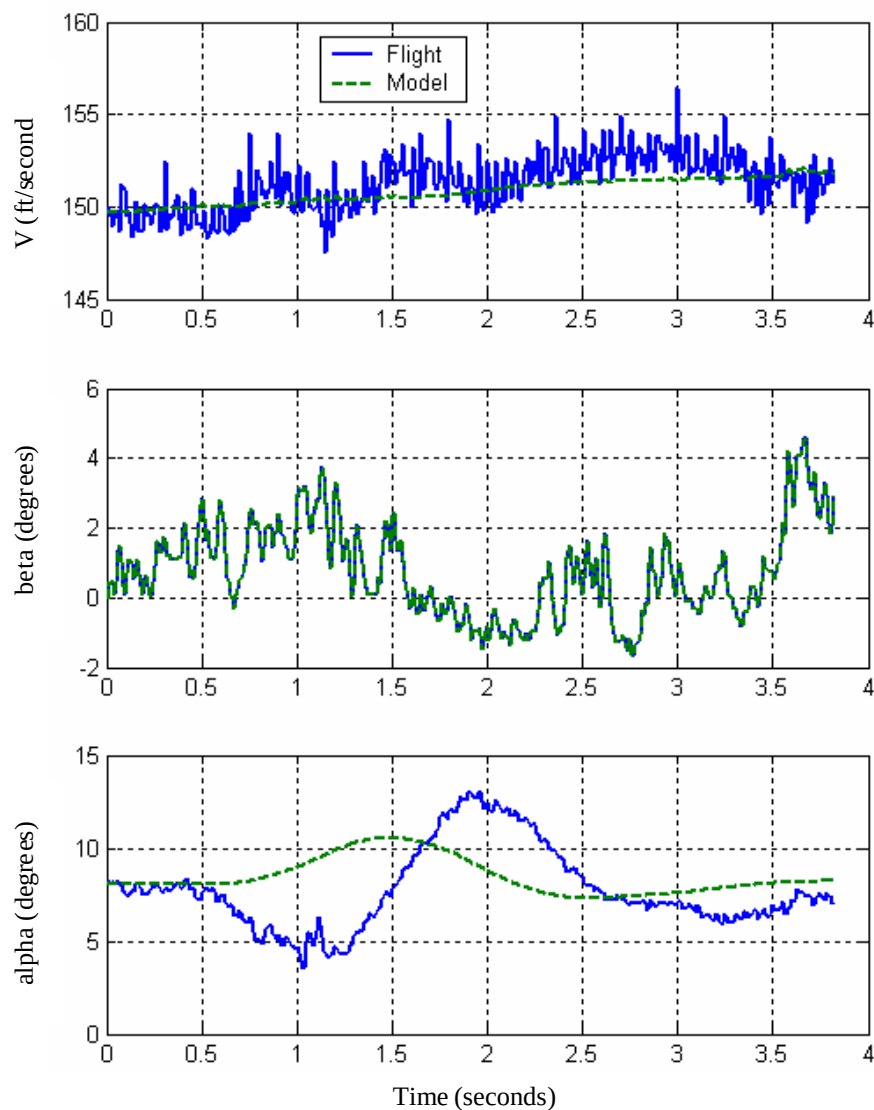
**FIGURE 5.3: DATA COMPATIBILITY ANALYSIS (ROTATIONAL KINEMATICS) OF TEST
POINT FWD 90-10³ BEFORE ESTIMATING ERRORS**

³ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

Pilot: Jim Wright
 Maneuver: Elevator Doublet
 Gross Weight: 3600 lbs
 Temperature: 25° F

Aircraft: Piper Saratoga (N22UT)
 Test Date: 01/25/2006
 C.G. Range: 90.1"-90.0" aft datum
 Pressure Altitude: 3000 ft

Data Compatibility Analysis



Created by: Catherine Kelly

Date: 06/03/2006

Program: SIDPAC

**FIGURE 5.4: DATA COMPATIBILITY ANALYSIS (TRANSLATIONAL KINEMATICS) OF
 TEST POINT FWD 90-10⁴ BEFORE ESTIMATING ERRORS**

⁴ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

$$\dot{v} = pw - ru = g \cos \theta \sin \phi + a_y, \quad (v = \int \dot{v}); \text{ and}$$

EQUATION 5.5

$$\dot{w} = qu - pv + g \cos \theta \cos \phi + a_z, \quad (w = \int \dot{w}).$$

EQUATION 5.6

$\dot{\theta}$, $\dot{u}$, $\dot{v}$, and $\dot{w}$ were calculated using the **dcmp_eqs.m** script within the data compatibility analysis routine. These values were then integrated using **adamb3.m** to obtain the θ , u , v , and w which were used to calculate the θ , V , and α model outputs plotted in figures 5.3 and 5.4. The angular rates, p , q , and r , the accelerations, a_x , a_y , and a_z , and the Euler angles, θ and ϕ , used in equations 5.1, 5.4, 5.5, and 5.6 were the measured output values from flight data. The states, u , v , and w , used in equations 5.4, 5.5, and 5.6 were calculated directly from the measured air data:

$$u = V \cos \alpha \cos \beta$$

EQUATION 5.7

$$v = V \sin \beta$$

EQUATION 5.8

$$w = V \sin \alpha \cos \beta$$

EQUATION 5.9

These values were used to solve equations 5.2 and 5.3.

The measured data were used for the model outputs of ϕ , ψ , and β . After the instrumentation error parameters were estimated in **dcmp_est.m**, the rotation and translation kinematics were plotted again, here with the biases and scale factors applied to the flight data (figures 5.5 and 5.6). The biases and scale factors were estimated using the output error method described earlier but was specifically formulated for the compatibility analysis [5]. The parameters being estimated were the biases in the x and z accelerations (a_x , a_z) and the pitch rate (q), as well as the angle-of-attack (α) and the pitch angle (θ) scale factors. For this test point, the convergence criteria were satisfied after 106 iterations, which took approximately 7 minutes. For this project, the iteration run time for the test points, whether the routine reached convergence or not, ranged from 3 to 12 minutes, with the average test point at 5 minutes. The instrumentation error parameters estimated in this analysis are listed in Table 5.1.

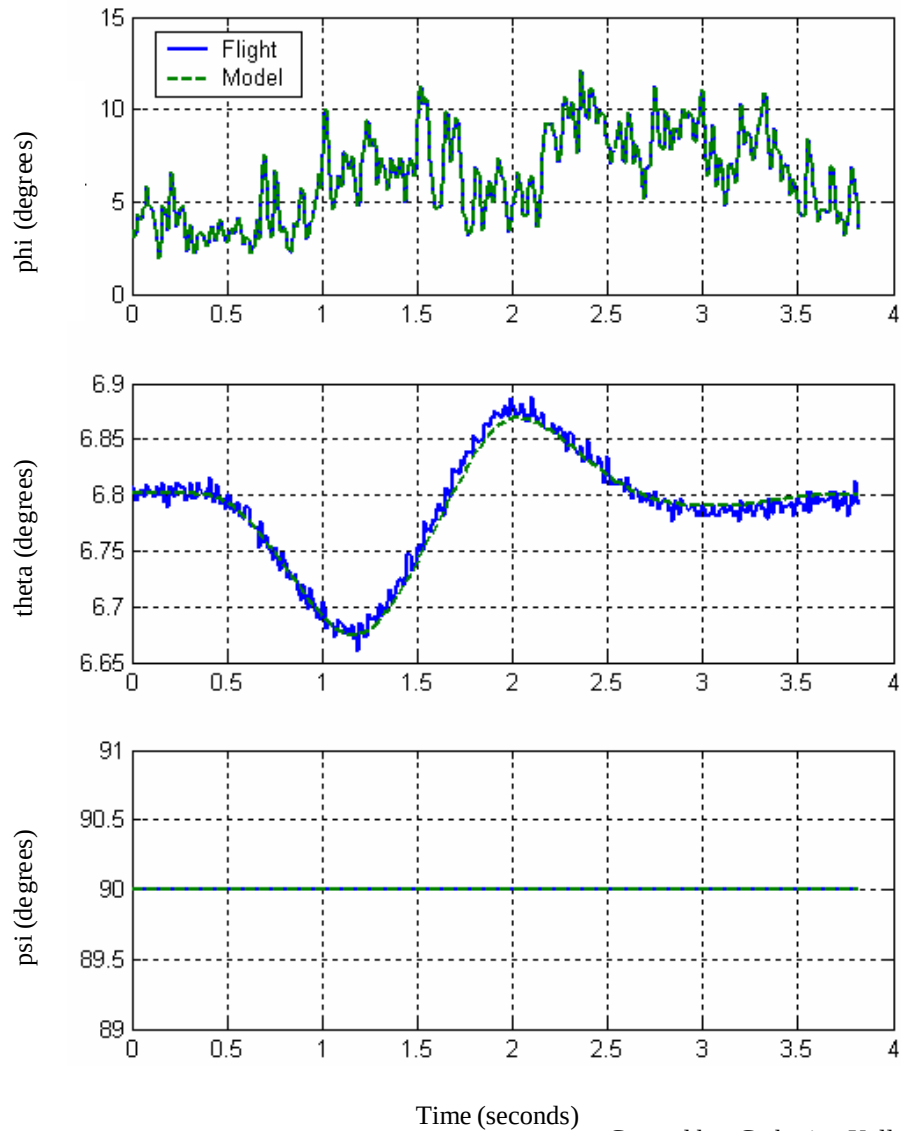
Estimated Stability Derivatives

The six stability derivatives, Z_α , Z_q , Z_{δ_e} , M_α , M_q , and M_{δ_e} , were calculated using the 1) equation error and 2) output error methods. The following results were from the test point flown at a forward c.g. at 90 knots with 10 degrees of flaps.

Pilot: Jim Wright
Maneuver: Elevator Doublet
Gross Weight: 3600 lbs
Temperature: 25° F

Aircraft: Piper Saratoga (N22UT)
Test Date: 01/25/2006
C.G. Range: 90.1"-90.0" aft datum
Pressure Altitude: 3000 ft

Data Compatibility Analysis



Created by: Catherine Kelly

Date: 06/03/2006

Program: SIDPAC

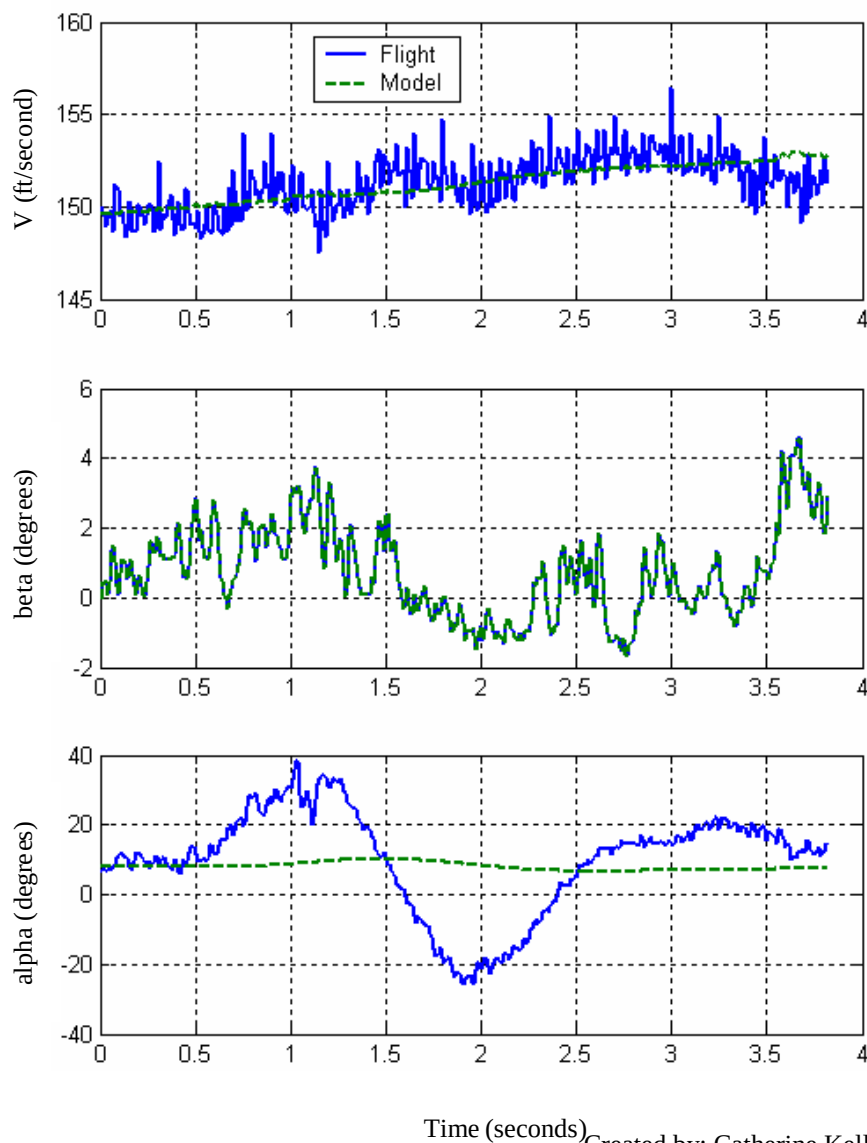
**FIGURE 5.5: DATA COMPATIBILITY ANALYSIS (ROTATIONAL KINEMATICS) OF TEST
POINT FWD 90-10⁵ AFTER ESTIMATING ERRORS AND CORRECTIONS APPLIED**

⁵ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

Pilot: Jim Wright
 Maneuver: Elevator Doublet
 Gross Weight: 3600 lbs
 Temperature: 25° F

Aircraft: Piper Saratoga (N22UT)
 Test Date: 01/25/2006
 C.G. Range: 90.1"-90.0" aft datum
 Pressure Altitude: 3000 ft

Data Compatibility Analysis



Time (seconds) Created by: Catherine Kelly

Date: 06/03/2006

Program: SIDPAC

FIGURE 5.6: DATA COMPATIBILITY ANALYSIS (TRANSLATIONAL KINEMATICS) OF TEST POINT FWD 90-10⁶ AFTER ESTIMATING ERRORS AND CORRECTIONS APPLIED

⁶ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

TABLE 5.1: ESTIMATED INSTRUMENTATION ERROR PARAMETERS OF TEST POINT

FWD 90-10⁷

<i>Parameter</i>	Estimate
$a_x \text{ bias}(\text{ft/s}^2)$	-0.35 $\pm$ 0.26
$a_z \text{ bias}(\text{ft/s}^2)$	-0.57 $\pm$ 0.55
$q \text{ bias}(\text{rad/s})$	-0.00 $\pm$ 0.00
$\alpha \text{ (scale factor)}$	-1.15 $\pm$ 0.03
$\theta \text{ (scale factor)}$	33.58 $\pm$ 0.33

⁷ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

EQUATION ERROR METHOD

The equation error method computed the least squares estimate of the body-axis Z force equation and the pitching moment equation using the **lesq.m** subroutine:

$$y = x * p$$

EQUATION 5.10

where y was based on the model parameter estimate p, the stability derivatives. x, in this case, consisted of the angle-of-attack, α , the pitch rate, q, and the elevator deflection, δ_e . y also matched the measured quantity z (in a least squares sense). For the body-axis Z force equation, z was equal to the rate of change of the angle-of-attack, $\dot{\alpha}$. For the pitching moment equation, z was equal to the pitch acceleration, $\dot{q}$. $\dot{\alpha}$ and $\dot{q}$ were plotted in figures 5.7 and 5.8, respectively. The value of the derivative calculated directly from the measurement was labeled “Data” in the legend. The value calculated using the equation was labeled “Model” in the legend. The residual, the difference between the measured and modeled values, was also included on the plots. The values of the parameter estimates, p(1), p(2), and p(3), which correlated to the p in equation 5.10, were given in table 5.2 under the equation error column along with the corresponding error bound value and were plotted in figure 5.9. The equation used to estimate the body-axis Z force derivatives was of the form

$$y = p(1)*x1 + p(2)*x3 + p(3)$$

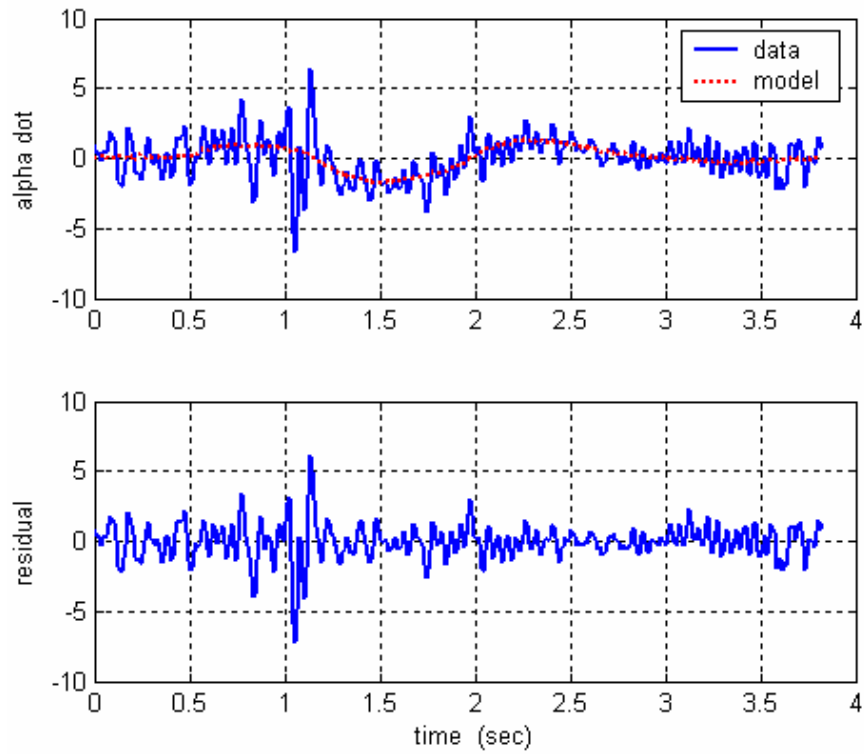
EQUATION 5.11

Pilot: Jim Wright

Aircraft: N22UT

Maneuver: Elevator Doublet

Test Date: 01/25/2006



Created By: Catherine Kelly

Date: 06/03/2006

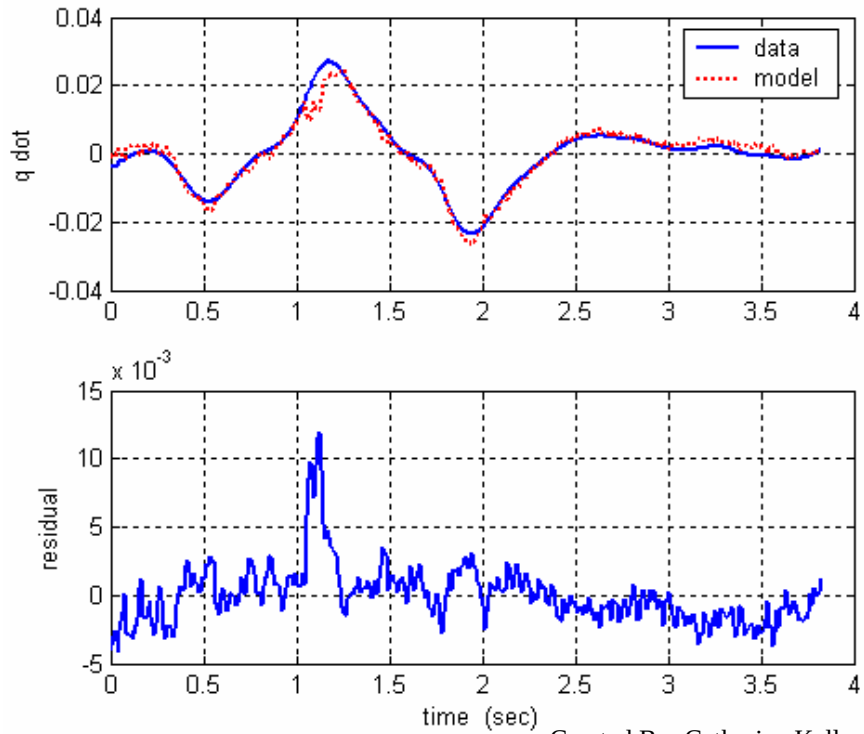
Program: SIDPAC

**FIGURE 5.7: VALUES OF $\dot{\alpha}$ USED TO ESTIMATE THE BODY-AXIS Z FORCE
DERIVATIVES OF TEST POINT FWD 90-10⁸**

⁸ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

Pilot: Jim Wright
Maneuver: Elevator Doublet

Aircraft: N22UT
Test Date: 01/25/2006



Created By: Catherine Kelly

Date: 06/03/2006

Program: SIDPAC

**FIGURE 5.8 VALUES OF $\dot{q}$ USED TO ESTIMATE THE PITCHING MOMENT DERIVATIVES
OF FWD 90-10⁹ TEST POINT**

⁹ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

where $p(1)= Z_{\alpha}$, $p(2)=Z_q$, and $p(3)= Z_{\delta_e}$ and $x1=\alpha$, and $x3=\delta_e$. The equation used to estimate the pitching moment derivatives was of the form

$$y = p(1)*x1 + p(2)*x2 + p(3)*x3 + p(4)$$

EQUATION 5.12

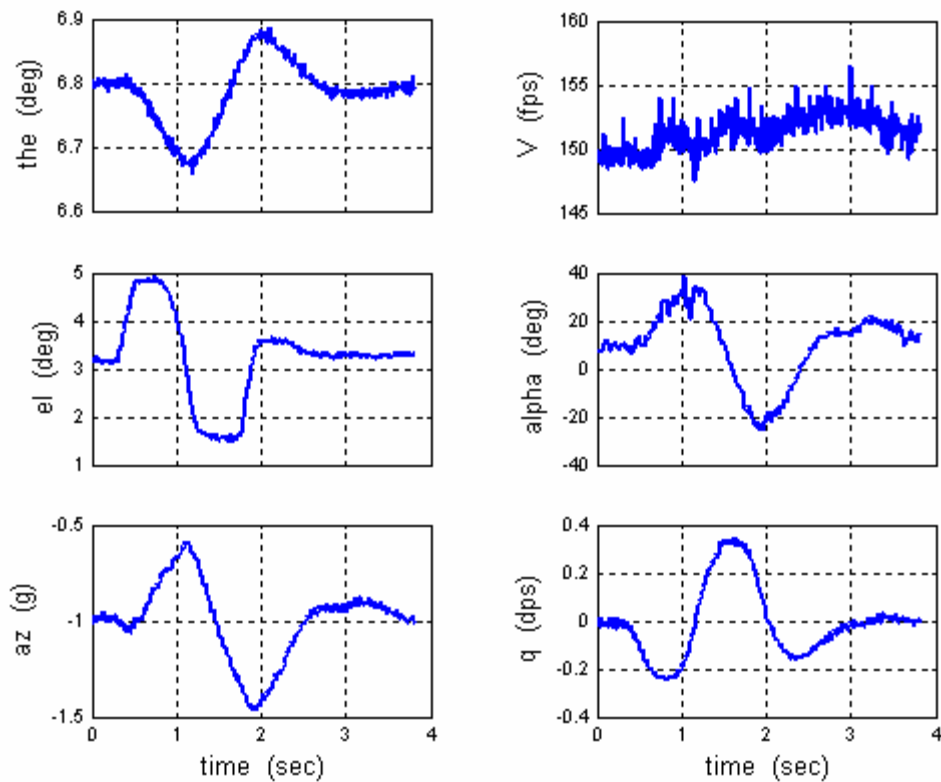
where $p(1)= M_{\alpha}$, $p(2)=M_q$, $p(3)= M_{\delta_e}$, and $p(4)=M_0$ and $x1=\alpha$, $x2= q$, and $x3=\delta_e$. M_0 was not plotted with other estimated parameters.

OUTPUT ERROR METHOD

The output error method computed the estimates of the of the stability derivatives using the **oe.m** subroutine. The measured values of the elevator deflection, δ_e , vertical acceleration, a_z , velocity, V , angle-of-attack, α , and pitch rate, q , plotted in figure 5.9, were used in this routine. The program ran through the iterative process discussed in chapter 2, where, for this analysis, δ_e was used as the control vector, the velocity was used as a constant, and a_z , α , and q were used in the measured output vector. These measured outputs, labeled “Data” in the legend, and the final modeled outputs, labeled “Model” in the legend, were plotted in figure 5.10. The values of the estimates of the stability derivatives from this method are located in table 5.2 under the output error column, along with the corresponding error bound value. The estimates were also plotted in figure 5.11.

Pilot: Jim Wright
Maneuver: Elevator Doublet

Aircraft: N22UT
Test Date: 01/25/2006



Created By: Catherine Kelly
Date: 06/03/2006
Program: SIDPAC

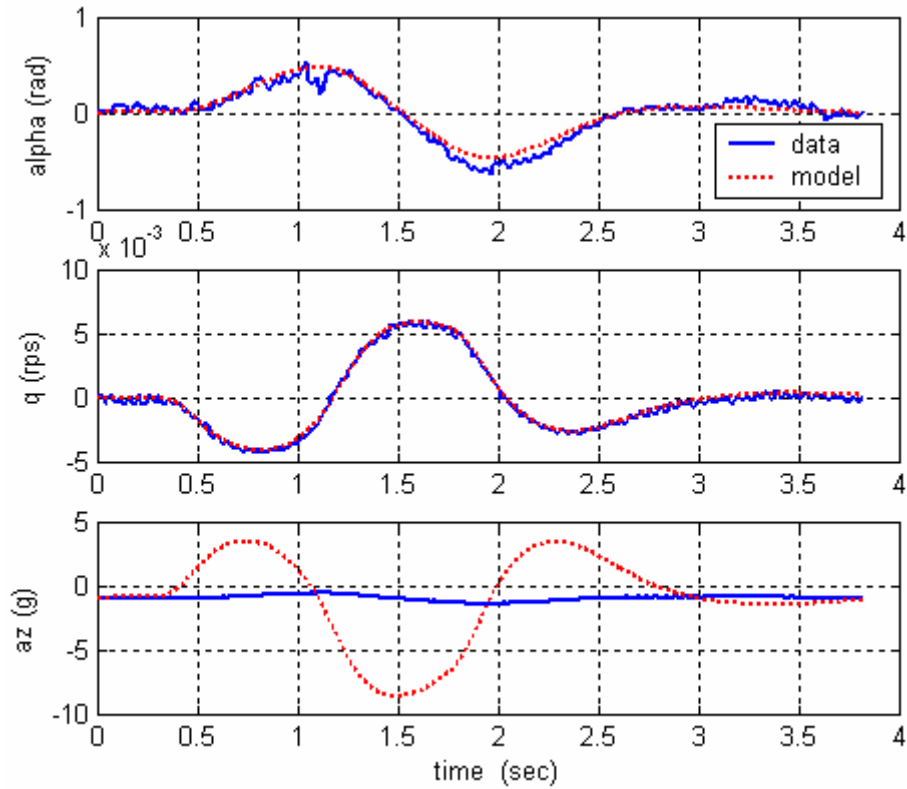
**FIGURE 5.9: MEASUREMENTS USED IN OUTPUT ERROR METHOD TO ESTIMATE BODY-
AXIS Z FORCE DERIVATIVES AND PITCHING MOMENT DERIVATIVES OF TEST POINT**

FWD 90-10¹⁰

¹⁰ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

Pilot: Jim Wright
Maneuver: Elevator Doublet

Aircraft: N22UT
Test Date: 01/25/2006



Created By: Catherine Kelly
Date: 06/03/2006
Program: SIDPAC

FIGURE 5.10: VALUES OF OUTPUT VECTOR USED TO ESTIMATE BODY-AXIS Z FORCE DERIVATIVES AND PITCHING MOMENT DERIVATIVES OF TEST POINT FWD 90-10¹¹

¹¹ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

**TABLE 5.2: EQUATION ERROR AND OUTPUT ERROR ESTIMATES OF BODY- AXIS Z
FORCE DERIVATIVES AND PITCHING MOMENT DERIVATIVES FOR TEST POINT**

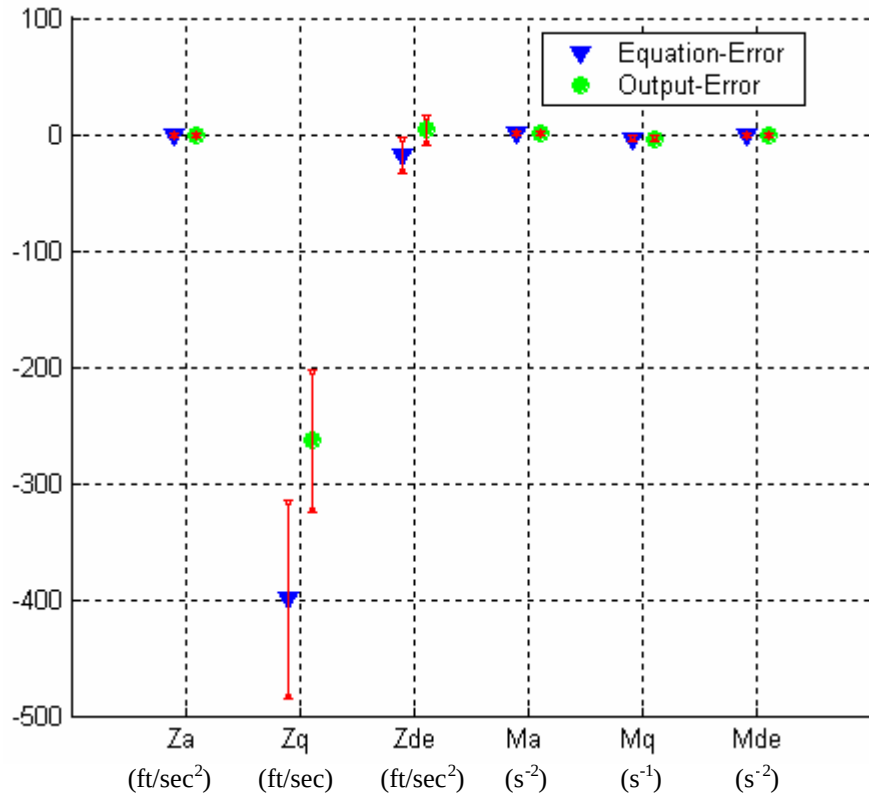
FWD 90-10¹²

<i>Derivative</i>	Equation Error		Output Error	
	Estimates	Error Bounds	Estimates	Error Bounds
<i>Za</i>	-1.06	0.18	-0.93	0.18
<i>Zq</i>	-399.90	41.89	-263.44	29.93
<i>Zde</i>	-18.15	7.08	3.68	5.83
<i>Ma</i>	0.03	0.00	0.03	0.00
<i>Mq</i>	-3.82	0.42	-3.57	0.30
<i>Mde</i>	-0.87	0.07	-0.84	0.03

¹² Flown at a forward c.g. at 90 knots with 10 degrees of flaps

Pilot: Jim Wright
Maneuver: Elevator Doublet

Aircraft: N22UT
Test Date: 01/25/2006



Created By: Catherine Kelly
Date: 06/03/2006
Program: SIDPAC

**FIGURE 5.11: EQUATION ERROR AND OUTPUT ERROR ESTIMATES OF BODY-AXIS Z
FORCE DERIVATIVES AND PITCHING MOMENT DERIVATIVES FOR TEST POINT**

FWD 90-10¹³

¹³ Flown at a forward c.g. at 90 knots with 10 degrees of flaps

Discussion of Results

According to the initial plots of the rotational and translational kinematics (figures 5.3 and 5.4) of the Data Compatibility Analysis, the pitch angle, θ , velocity, V , and angle-of-attack, α , showed instrumentation error when compared to the model values. After the scale factors and biases were estimated, the measured values of θ , V , and α were modified using the parameter estimates. These new values replaced the old ones and were used throughout the remainder of the SIDPAC program. All test points from both flights demonstrated similar trends with θ , V , and α as showcased with the test point given as an example in this chapter. Of the eighteen test points, only six met the convergence criteria, where both the parameter estimates and covariance matrix converged. For the six that converged, the estimated biases for a_x , and a_z showed a notable variation among their respective values, while the estimated biases for q and the estimated scale factors for α and θ were similar among the test points. This trend held for the entire set of flight test points when compared to each other, although the average value for each estimated parameter was usually higher among the points where the data compatibility analysis reached convergence.

The following is a comparison of the two averages for each parameter. The first value is the average among the six test points that converged and the second number is the average for all the test points: for a_x , -2.6 ft/sec² compared to -1.5 ft/sec²; for a_z , 9.5 ft/sec² compared to 4.2 ft/sec²; for q , 0.008 rad/sec compared to 0.002 rad/sec; for α , a scale factor of -1.1 compared to a scale factor of -0.87; and for θ , a scale factor of 32.3 compared to a scale factor of 32.3. Once the biases and scale factors were applied to the

measurements, there was a huge change in the pitch angle and angle-of-attack. As seen in figure 5.9, the pitch angle, θ , was a fraction of the original measurement (before the data compatibility analysis), while the angle-of-attack, α , was two to three times larger. For the most part, this held true for all the test points.

In most cases, the derivatives estimated using the equation error method were similar to those estimated using the output error method. This trend makes sense because the initial values of the derivatives in the output error method were the derivative estimates obtained from the equation error method. For the most part, the derivatives had a tendency to increase in magnitude as the velocity increased. However, there was no set pattern for the effect of the flap setting; the derivatives either increased or decreased. The majority of the derivative estimates were negative. Z_α values were between -1.6 ft/sec and 0 ft/sec; Z_{δ_e} values fluctuated between -25 ft/sec and 200 ft/sec; M_α tended to stay near 0 s^{-2} ; M_q values were between -5 s^{-2} and -1.3 s^{-2} ; and M_{δ_e} values were between -1 s^{-1} and 0 s^{-1} . Z_q estimates had the most extreme fluctuations. The values were often two to four orders of magnitude greater than the other parameter estimates, the pattern holding for each test point. The error bounds were also huge for the estimates of this parameter, showing very little confidence in the value. It was commented in the **Navion_fda.m** script, on which the **Saratoga_fda.m** script was based, that the Z_q term was to be omitted because the maneuver was run at a low trim angle-of-attack, therefore, the same is probably true for this project. Of the eighteen test points, eleven reached the convergence criteria for the output error method, most with the c.g. aft. A table of estimate values, as well as a plot of the parameters, for each test point is located in Appendix C.

CHAPTER 6

CONCLUSIONS

Recap

The purpose of this research project was to estimate the longitudinal stability derivatives from flight data obtained from N22UT using the SIDPAC program. Once the program corrected the data for instrumentation error, it estimated the derivatives using two methods, equation error and output error. According to SIDPAC, the measured angle-of-attack and pitch angle values had significant instrumentation error and the values were updated accounting for any biases or scale factors. Based on these new values, the stability derivatives, Z_α , Z_q , Z_{δ_e} , M_α , M_q , and M_{δ_e} , were estimated and the values compared.

Recommendations

Because of the large instrumentation error SIDPAC encountered, most of the test points did not reach convergence during the data compatibility analysis. It is recommended that the instruments be recalibrated more carefully, especially the angle-of-attack and sideslip wingtip vanes since they were added to the aircraft shortly before the flight test. There is also a concern that the vanes may have had inertia high enough to provide poor results, so a new design may need to be considered such as what is provided

on the Navion, N66UT. Another source of error is the location of the accelerometers with respect to the longitudinal and vertical c.g. The exact location should be measured with care so that SIDPAC can accurately correct the position for the analysis. The vertical gyroscope used in this aircraft is a World War II surplus instrument. The age of the instrument probably affected its performance and it is recommended that it be updated for future tests.

In the interest of time, the data analyzed for this project were truncated to include only the doublet maneuver. It is recommended that in the future the full maneuver be included in the analyses to more accurately model the aircraft. It is also recommended that the output error methods be allowed to run at larger iterations to increase the change of convergence. To aid in uniformity, there should have been a better system in place to allow the pilot to perform the maneuver similarly for each test point, such as marking the yoke. For this flight test, the aircraft was flown with 70 gallons of fuel for both test dates, not giving much of a variation in the c.g. It is recommended that some of the test points be run with less fuel in order to vary the c.g. better. Because this flight test only estimated the longitudinal stability derivatives, future flight tests can be run to test for lateral derivatives and also to take cross coupling into account.

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APPENDICES

APPENDIX A

SIGN CONVENTIONS

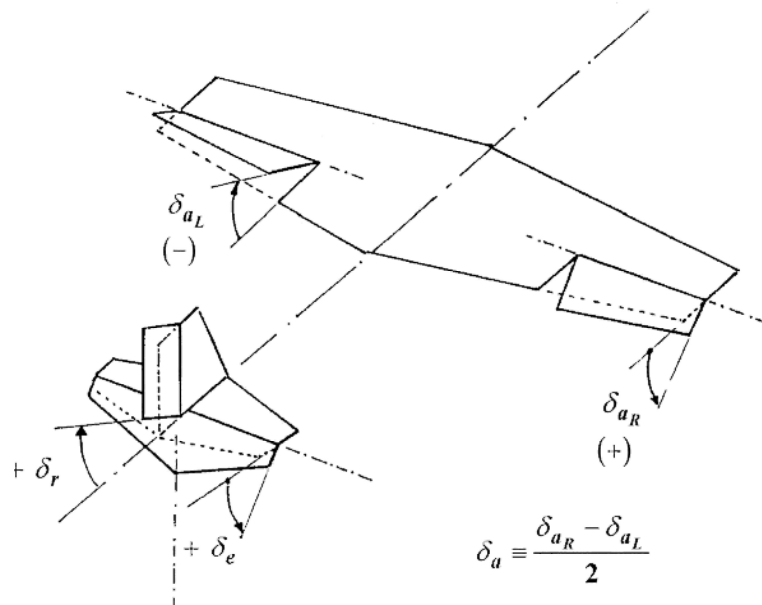
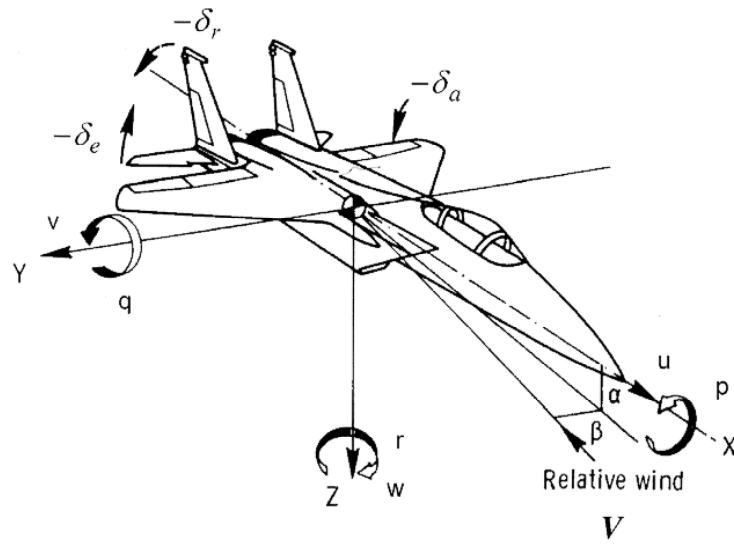


Figure A-1. Sign conventions used in SIDPAC

APPENDIX B

INSTRUMENTATION CALIBRATION

Aircraft Parameter	Conversion Equation	Units
<i>Sensor Voltage to Engineering Units</i>		
Pitch Attitude	$y = 15.377x + 1.5729$	deg
Roll Attitude	$y = 21.107x + 0.4209$	deg
Rudder Position	$y = -0.1104x - 0.7431$	deg
Rt. Aileron Position	$y = -8.0238x - 3.3121$	deg
Elevator Position	$y = -3.2684x + 2.3043$	deg
Absolute Pressure	$y = 6.0797x + 0.6266$	inHg
Differential Pressure	$y = 5.4552x - 0.2594$	inH ₂ O
Pitch Rate	$y = 0.3825x + 0.746$	deg/s
Roll Rate	$y = 0.5487x - 2.1881$	deg/s
Yaw Rate	$y = 0.579x + 0.5139$	deg/s
Longitudinal Acceleration	$y = 0.3989x - 0.0039$	g
Lateral Acceleration	$y = 0.3983x - 0.0123$	g
Normal Acceleration	$y = 0.3989x - 0.0063$	g
AOA Vane	$y = 9.0598x + 0.8312$	deg
Side Slip Vane	$y = 10.96x + 0.118$	deg
<i>Surface Deflection to Control Input</i>		
Pitch Stick Deflection	$y = -0.4033x + 5.9765$	in
Roll Stick Deflection	$y = -0.3032x - 0.421$	in
Rudder Pedal Position	$y = -0.1333x$	in

Figure B-1. Piper Saratoga Conversion Equations

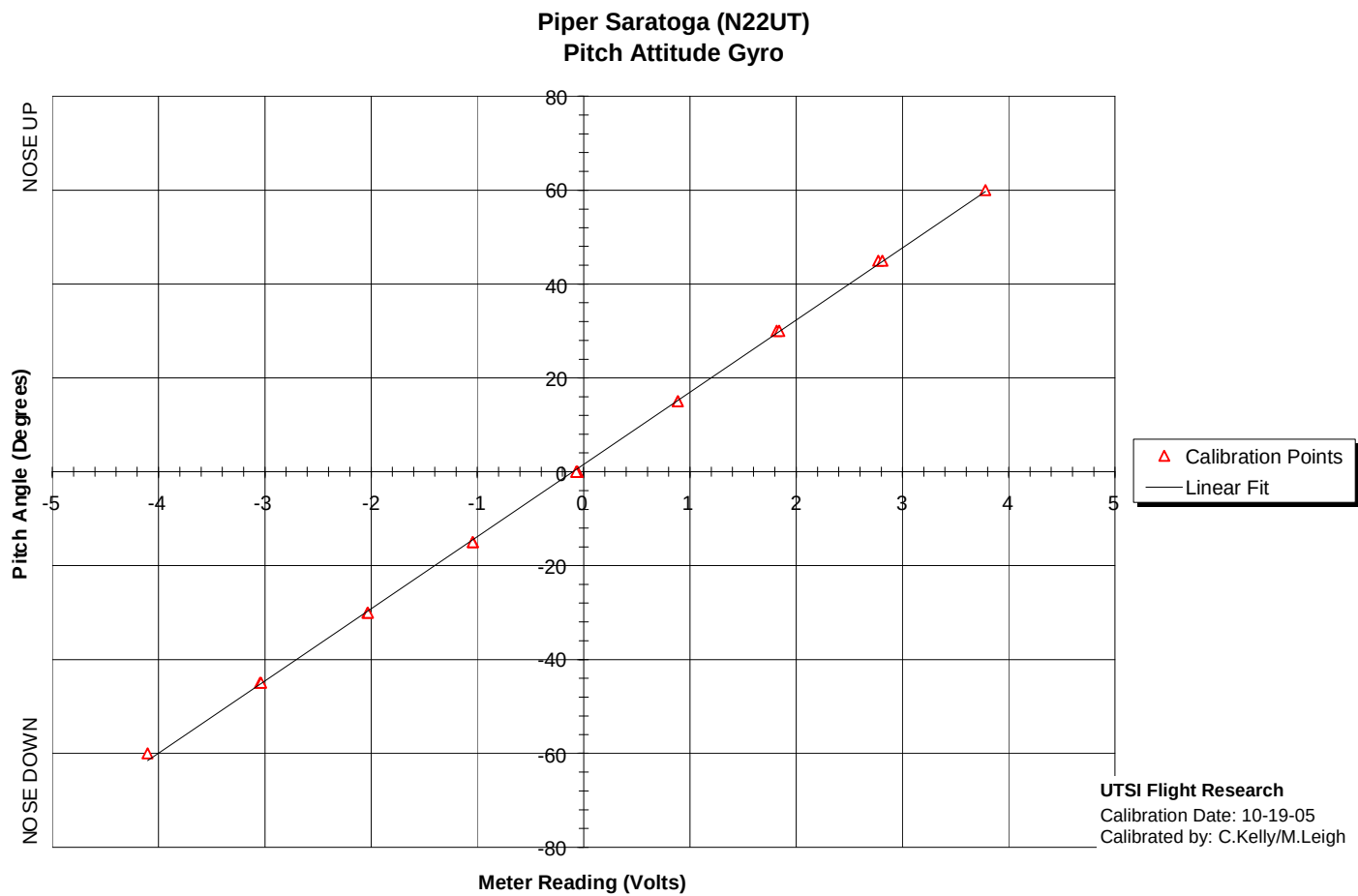


Figure B-2. Pitch Attitude Calibration Chart

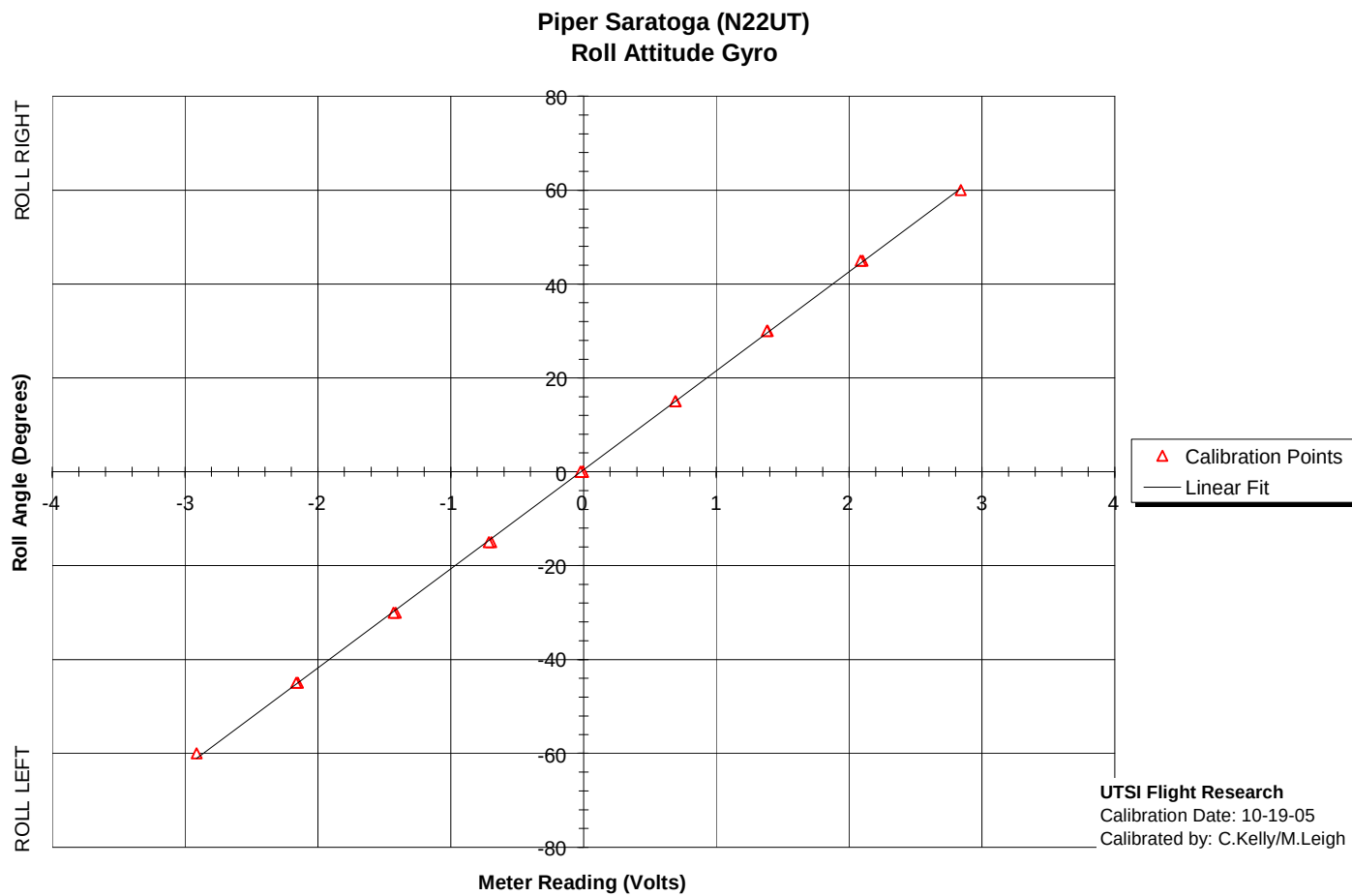


Figure B-3. Roll Attitude Calibration Chart

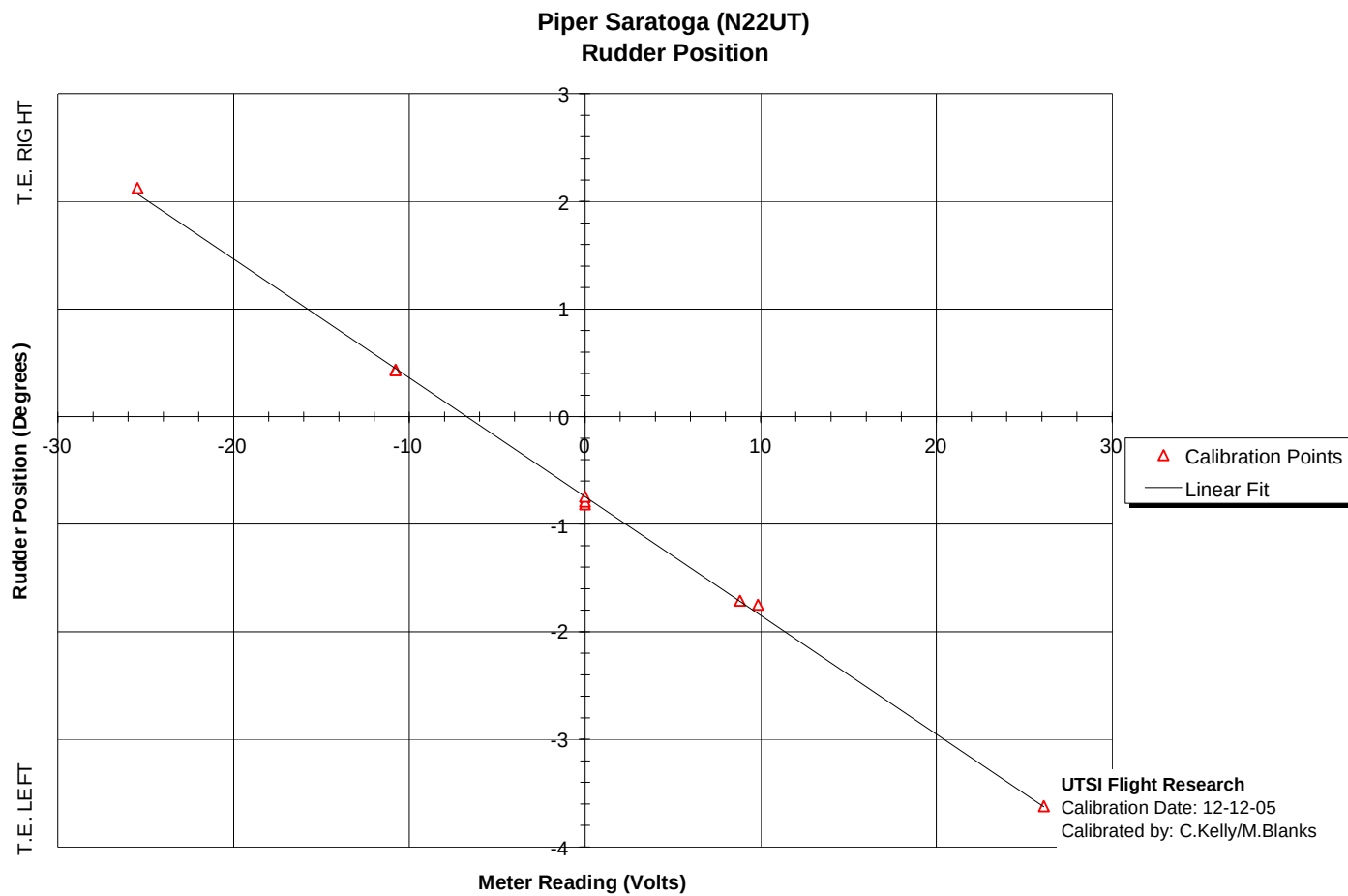


Figure B-4. Rudder Position Calibration Chart

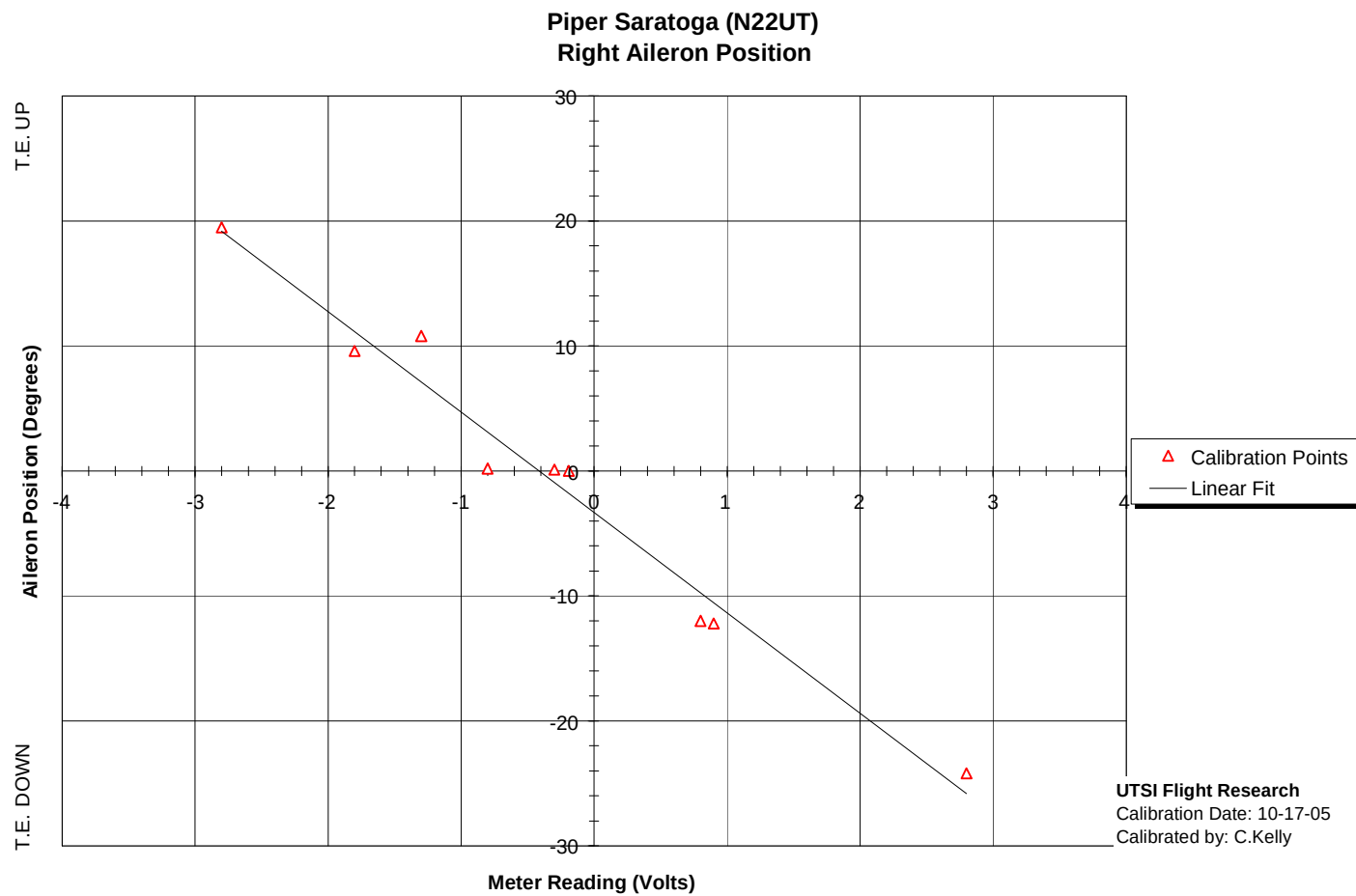


Figure B-5. Right Aileron Position Calibration Chart

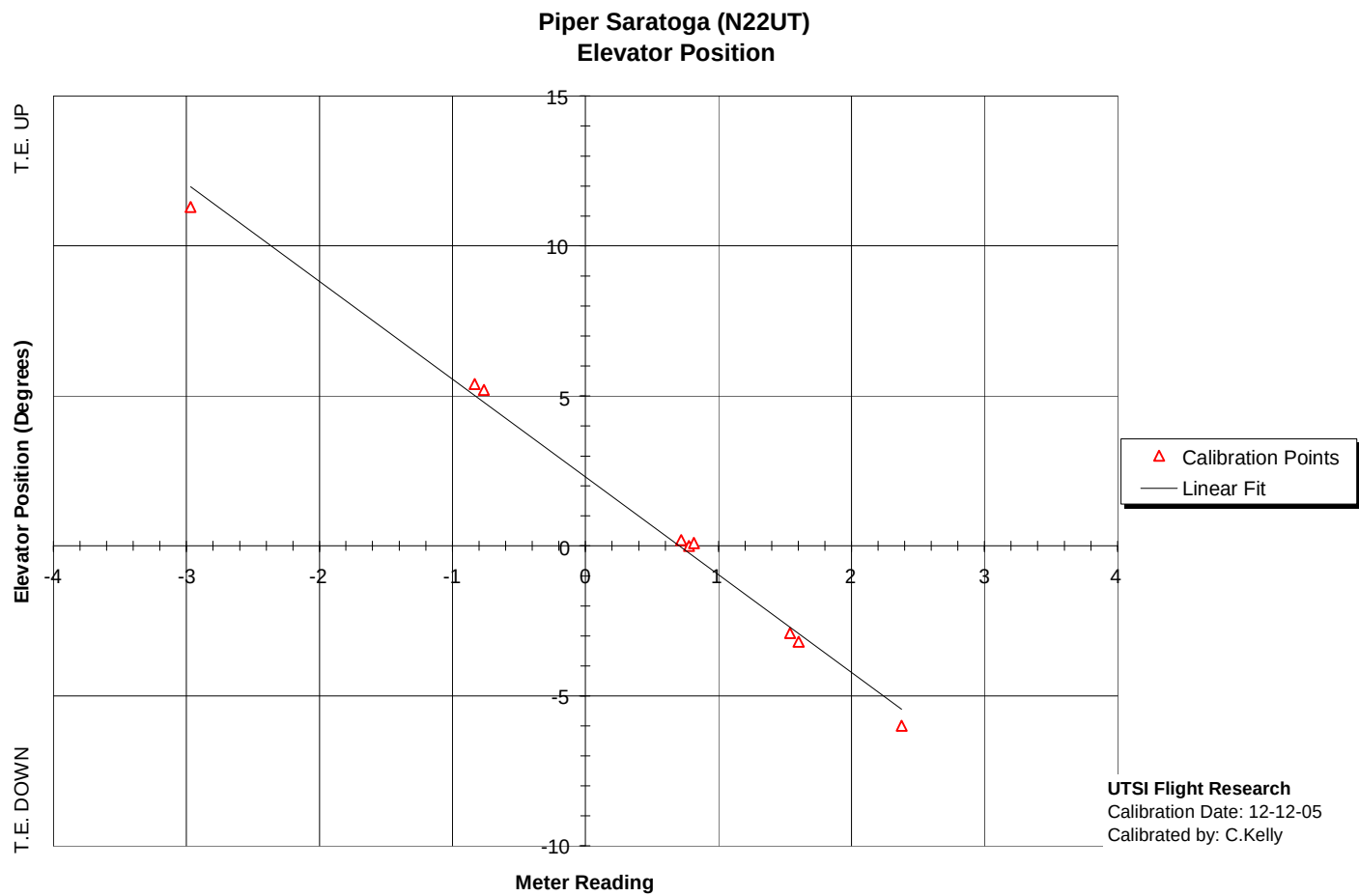


Figure B-6. Elevator Position Calibration Page

**Piper Saratoga (N22UT)
Absolute Pressure Transducer**

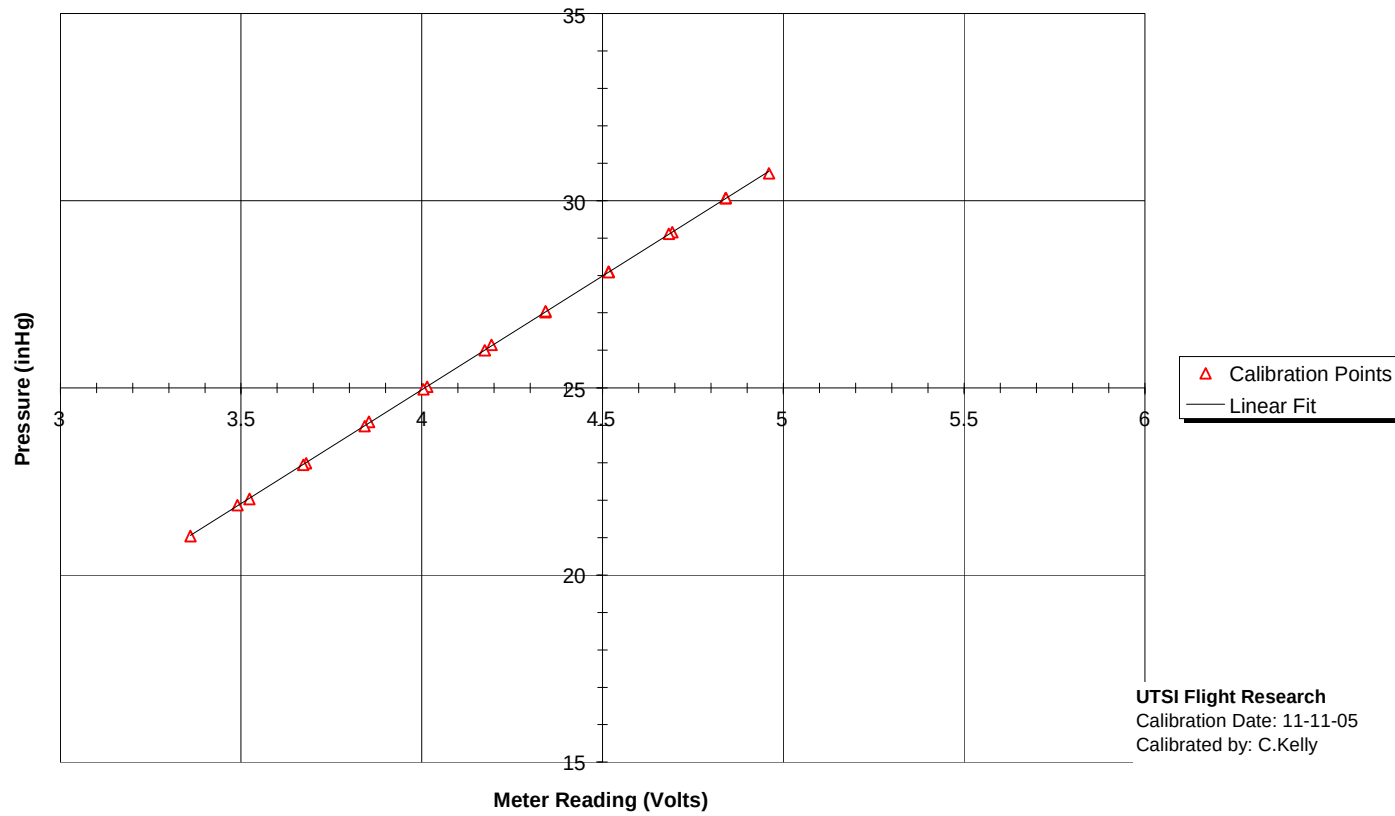


Figure B-7. Absolute Pressure Transducer Calibration Chart

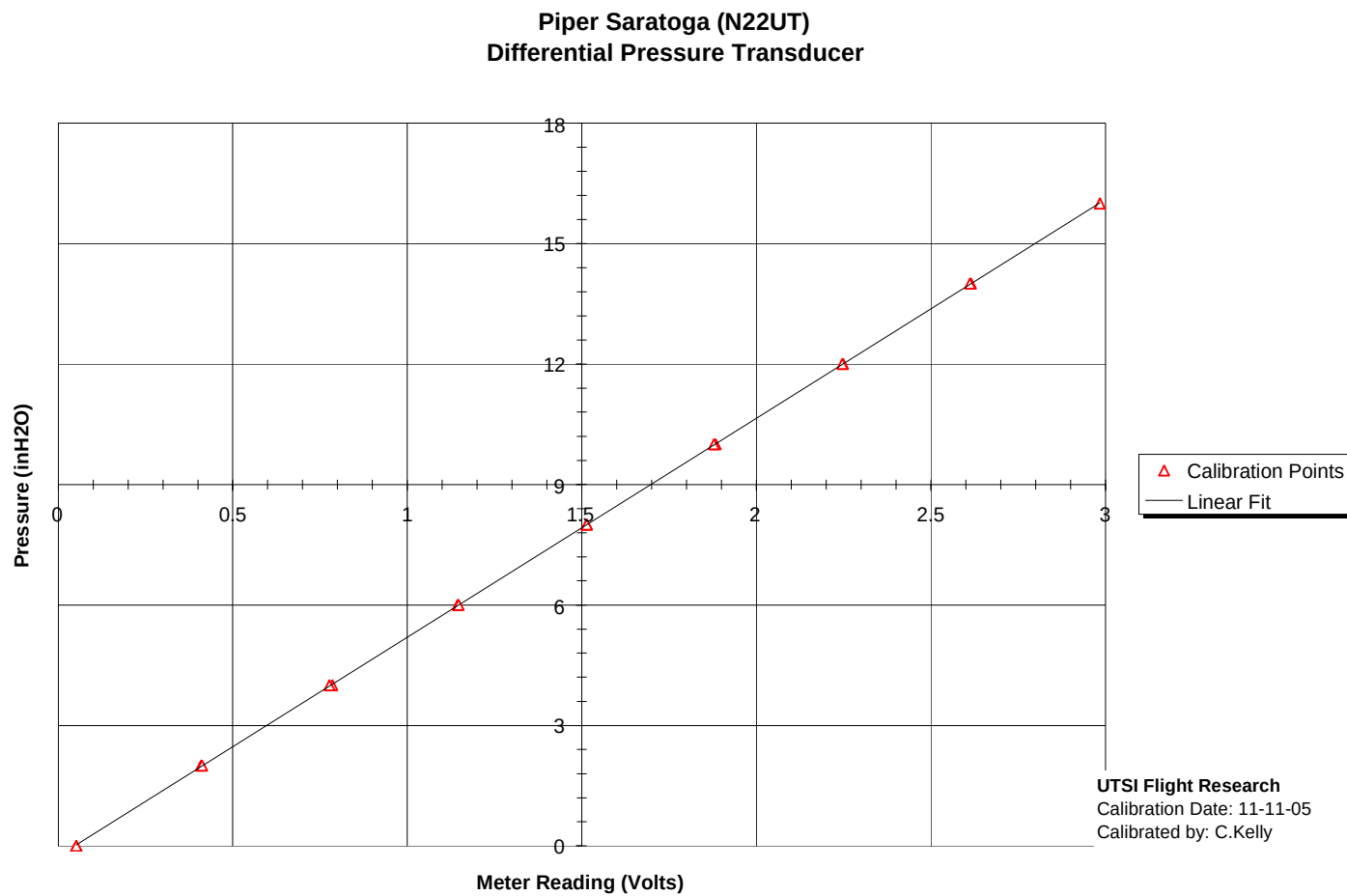


Figure B-8. Differential Pressure Transducer Calibration Chart

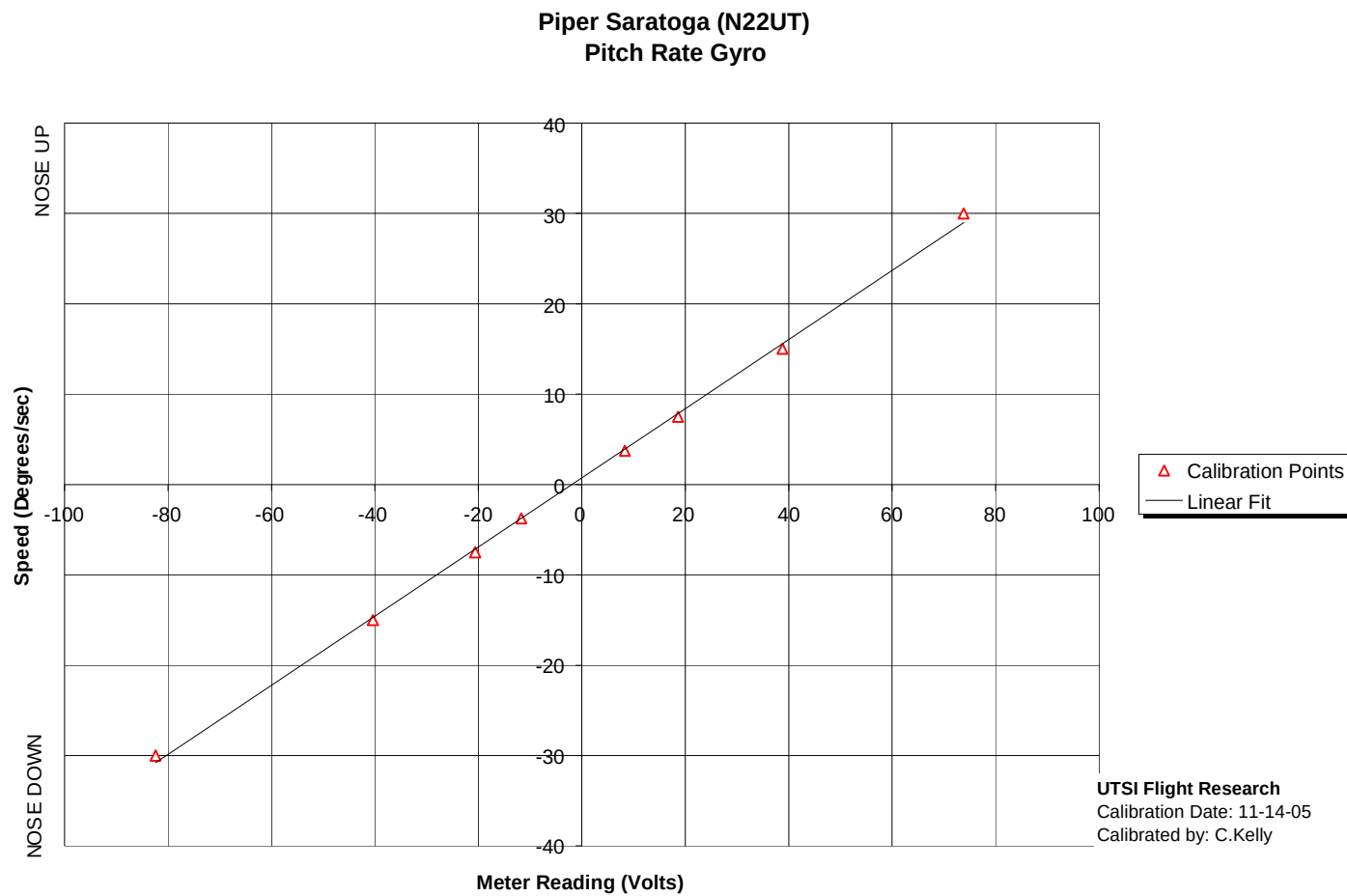


Figure B-9. Pitch Rate Gyro Calibration Chart

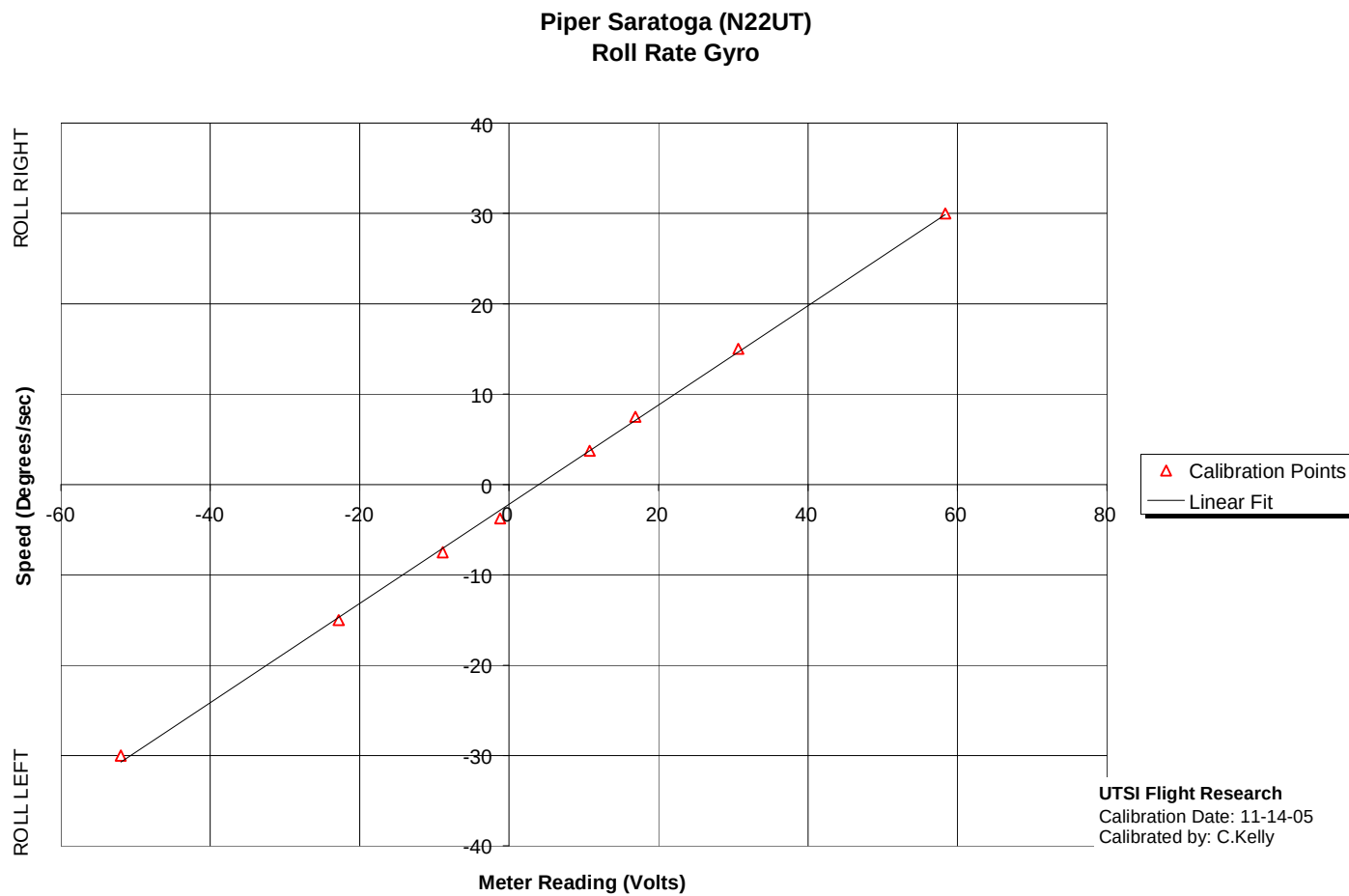


Figure B-10. Roll Rate Gyro Calibration Chart

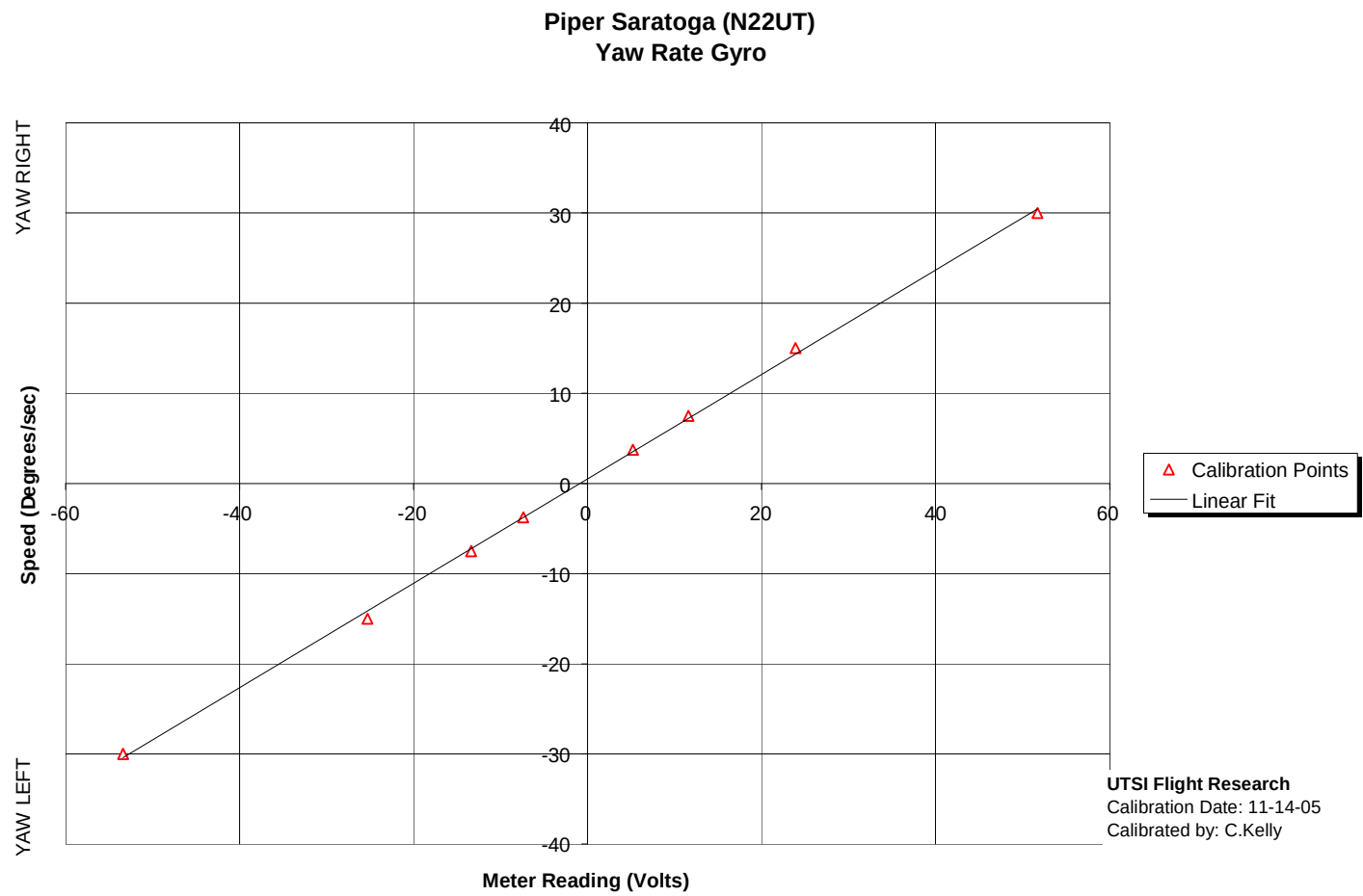


Figure B-11. Yaw Rate Gyro Calibration Chart

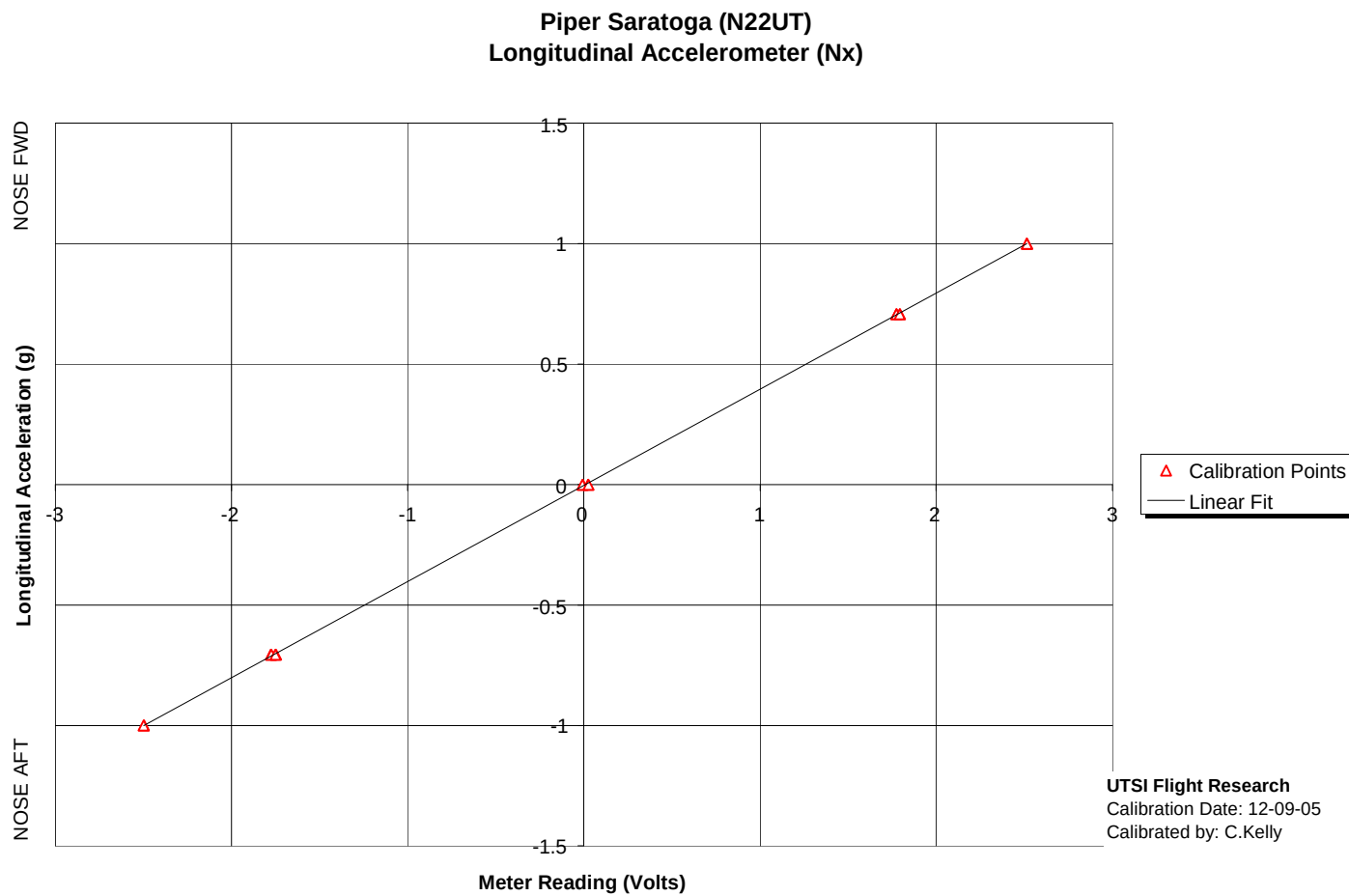


Figure B-12. Longitudinal Accelerometer Calibration Chart

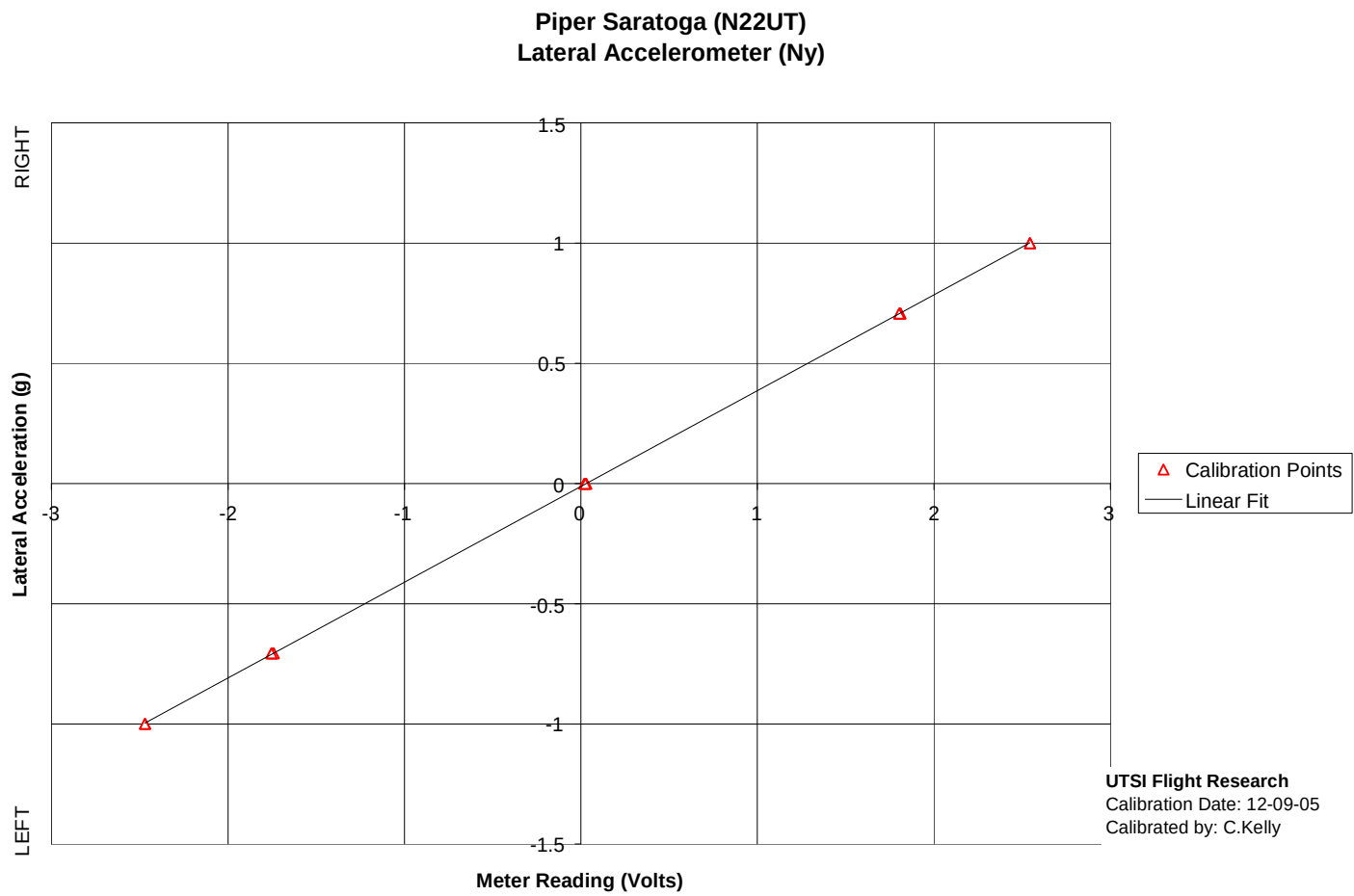


Figure B-13. Lateral Accelerometer Calibration Chart

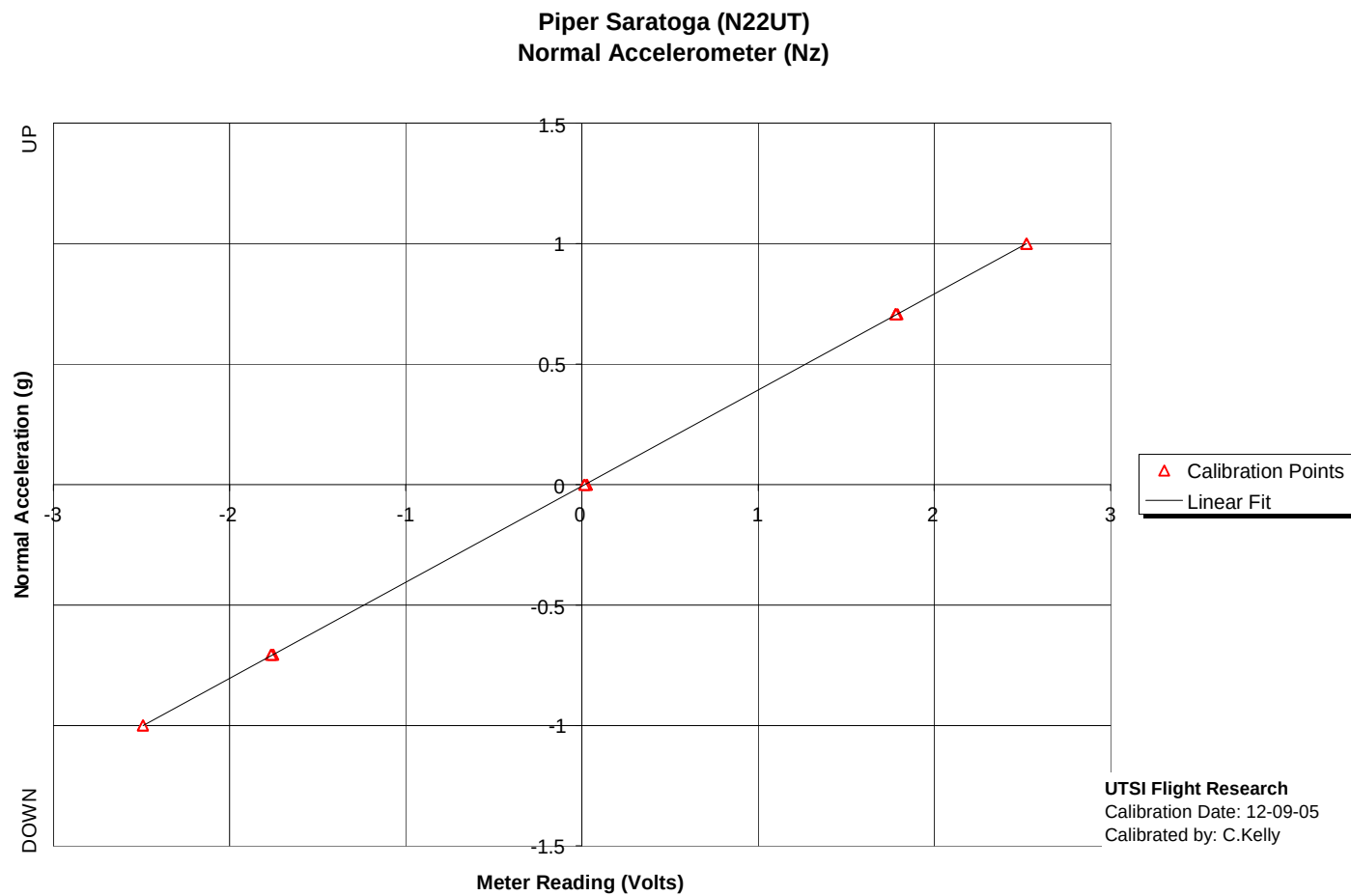


Figure B-14. Normal Accelerometer Calibration Chart

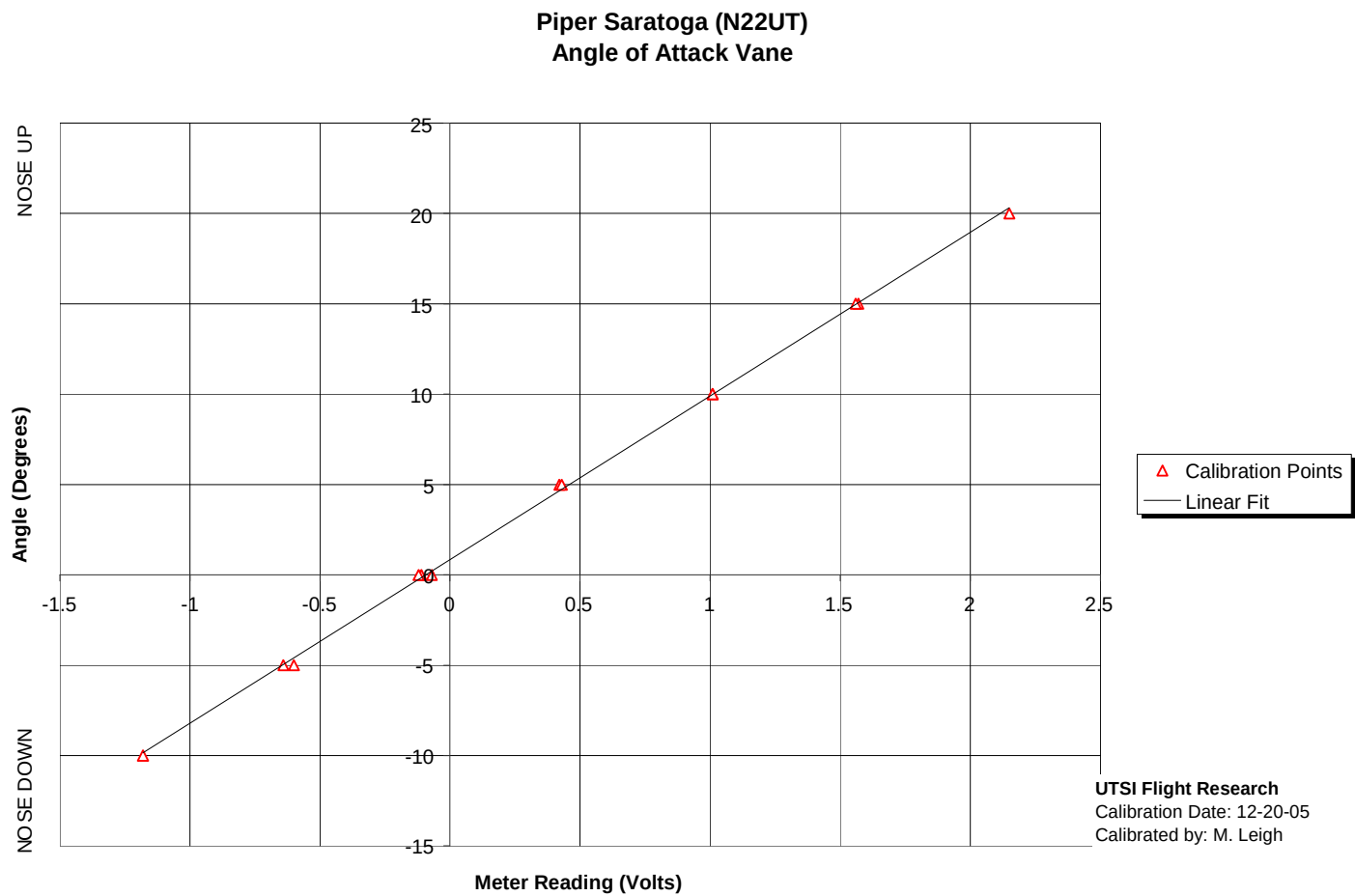


Figure B-15. Angle-of-attack Vane Calibration Chart

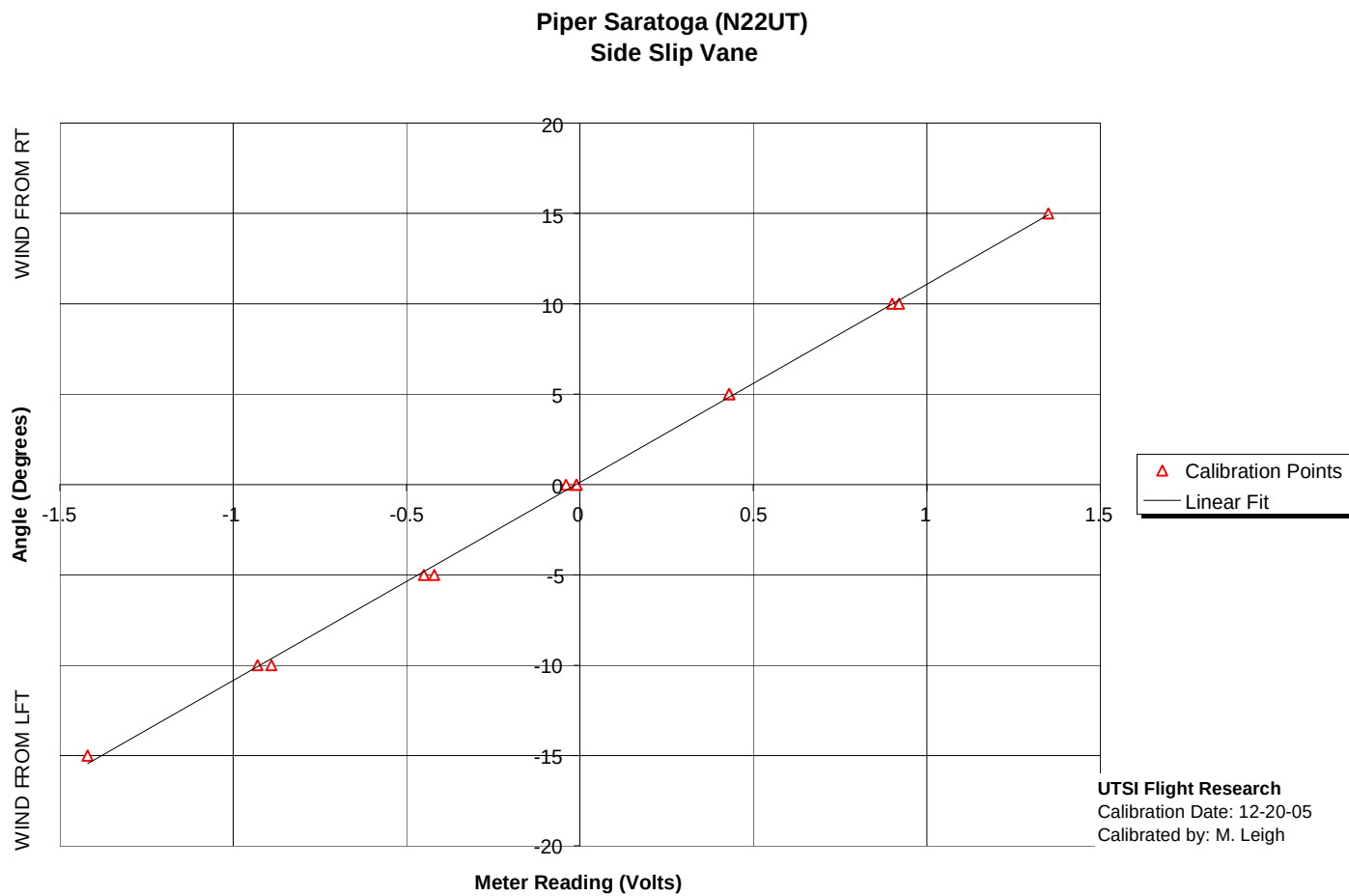


Figure B-16. Side Slip Vane Calibration Chart

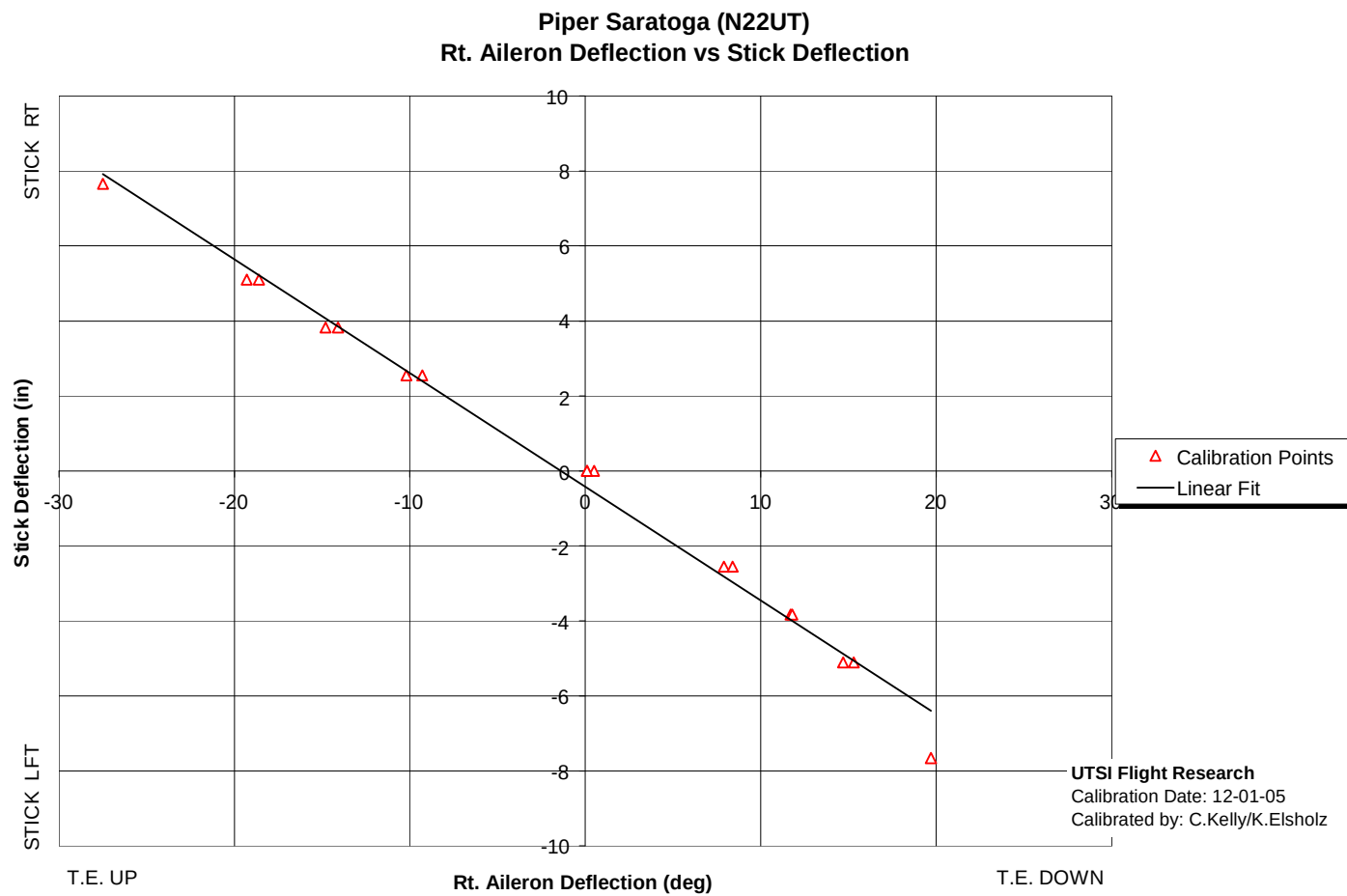


Figure B-17. Pitch Stick Deflection Calibration Chart

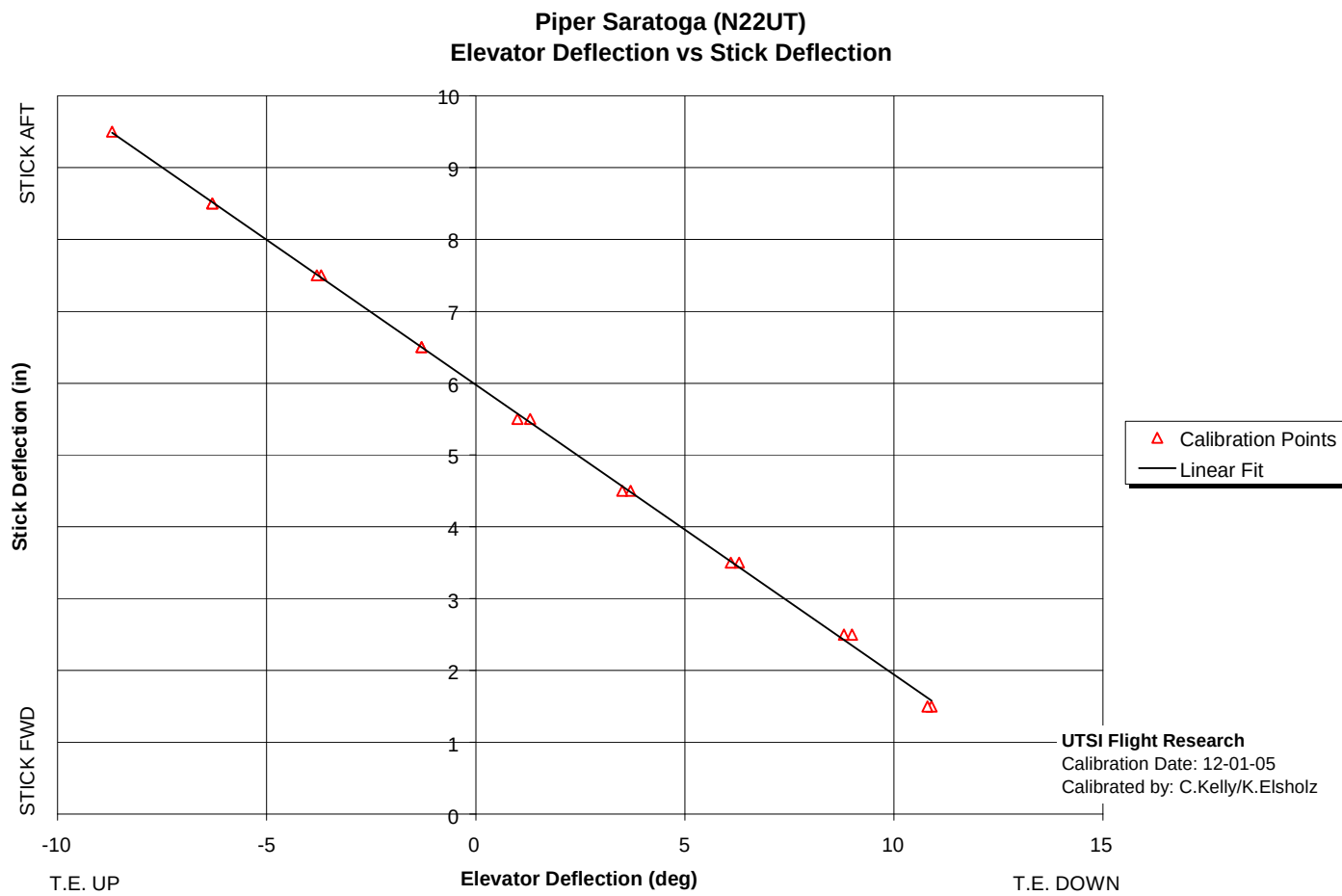


Figure B-18. Roll Stick Deflection Calibration Chart

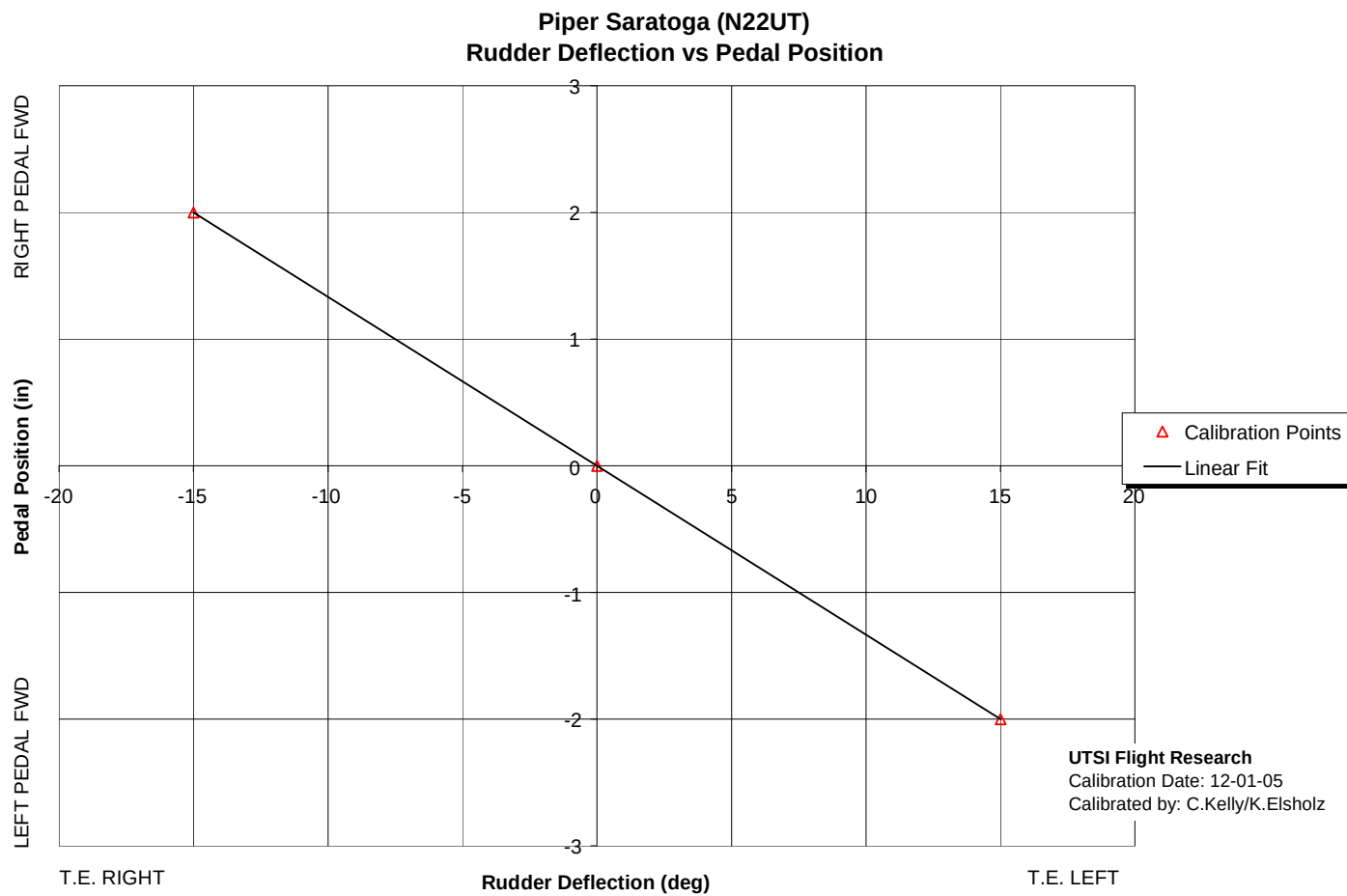


Figure B-19. Rudder Pedal Deflection Calibration Chart

APPENDIX C

STABILITY DERIVATIVES FOR N22UT USING THE EQUATION ERROR AND OUTPUT ERROR METHODS IN THE SIDPAC PROGRAM

Table C-1. Equation Error and Output Error¹⁴ Results for AFT 70-0

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-0.39	0.37	-0.71	0.22
<i>Zq</i>	213.12	25.80	211.91	17.21
<i>Zde</i>	-1.98	3.44	-1.40	1.91
<i>Ma</i>	-0.02	0.00	-0.02	0.00
<i>Mq</i>	-1.49	0.40	-1.62	0.19
<i>Mde</i>	-0.37	0.04	-0.44	0.02

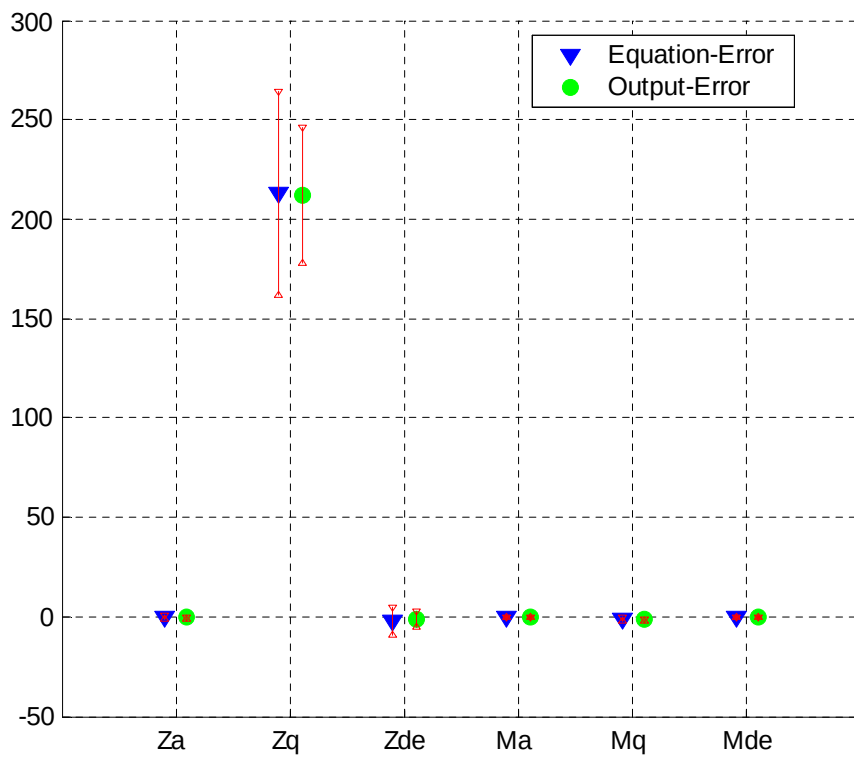


Figure C-1. Plot of Equation Error and Output Error Results for AFT 70-0

Output Error converged after 27 iterations¹⁴

Table C-2. Equation Error and Output Error¹⁵ Results for AFT 70-10

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-0.53	0.31	-1.49	0.58
<i>Zq</i>	395.83	28.10	448.69	47.51
<i>Zde</i>	0.13	4.28	10.51	6.32
<i>Ma</i>	-0.01	0.00	-0.01	0.00
<i>Mq</i>	-1.69	0.19	-1.76	0.22
<i>Mde</i>	-0.37	0.03	-0.42	0.03

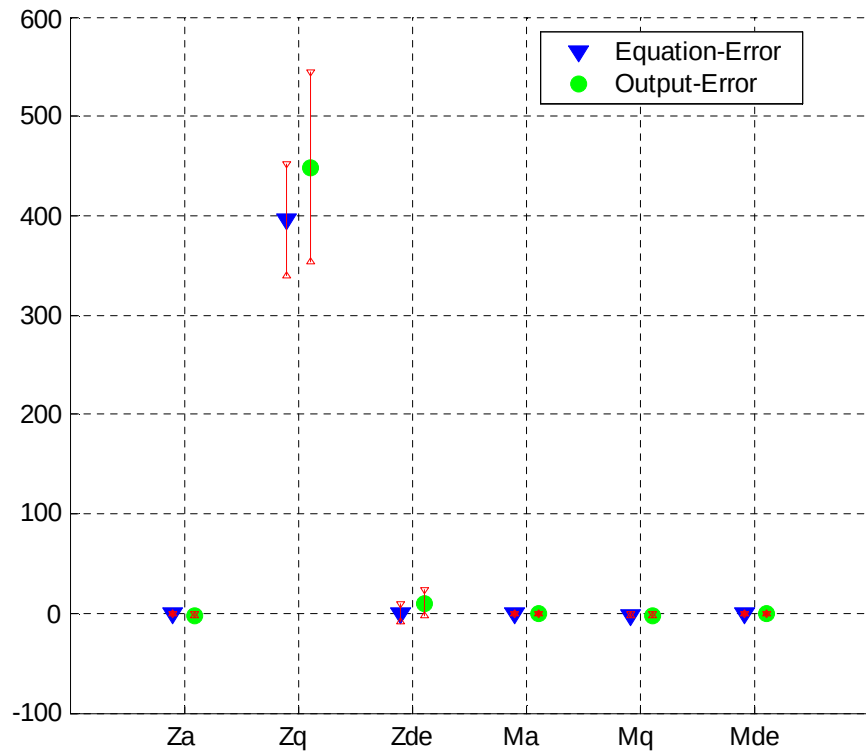


Figure C-2. Plot of Equation Error and Output Error Results for AFT 70-10

Output Error converged after 26 iterations¹⁵

Table C-3. Equation Error and Output Error¹⁶ Results for AFT 70-40

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-0.92	0.17	-3.24	4.11
<i>Zq</i>	99.54	5.84	4.49	8.18
<i>Zde</i>	-0.07	0.68	-1.39	0.44
<i>Ma</i>	-0.03	0.00	-0.85	1.35
<i>Mq</i>	-2.41	0.11	-0.16	4.10
<i>Mde</i>	-0.47	0.01	-0.49	0.01

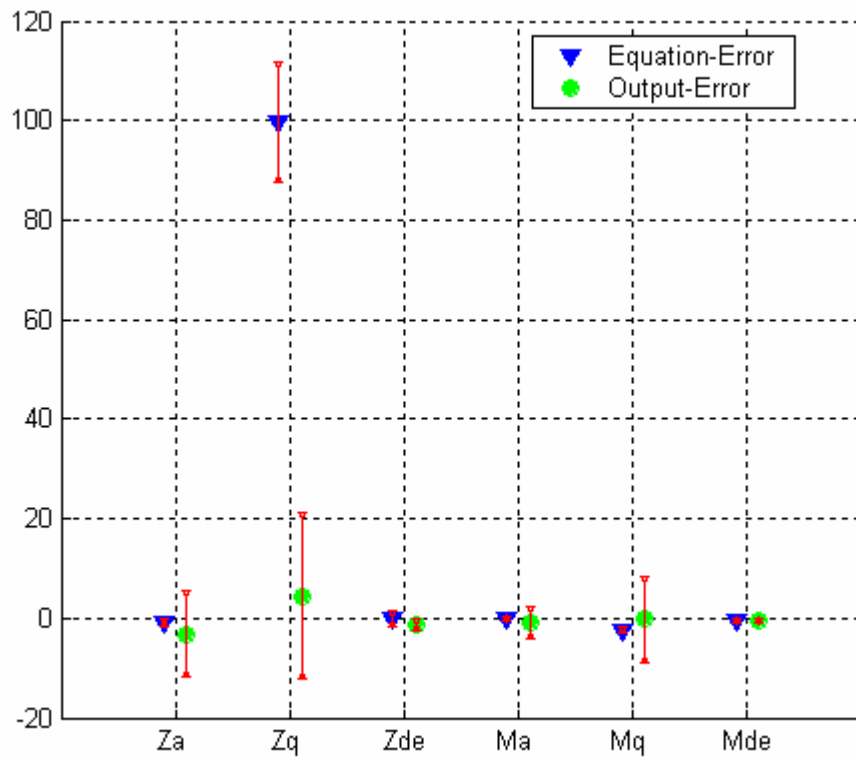


Figure C-3. Plot of Equation Error and Output Error Results for AFT 70-40

Output Error did not converge¹⁶

Table C-4. Equation Error and Output Error¹⁷ Results for AFT 90-0

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.18	0.08	-1.52	0.24
<i>Zq</i>	120.73	3.97	121.02	7.04
<i>Zde</i>	0.48	0.56	0.31	0.92
<i>Ma</i>	-0.05	0.00	-0.06	0.00
<i>Mq</i>	-2.63	0.16	-2.29	0.24
<i>Mde</i>	-0.70	0.02	-0.70	0.03

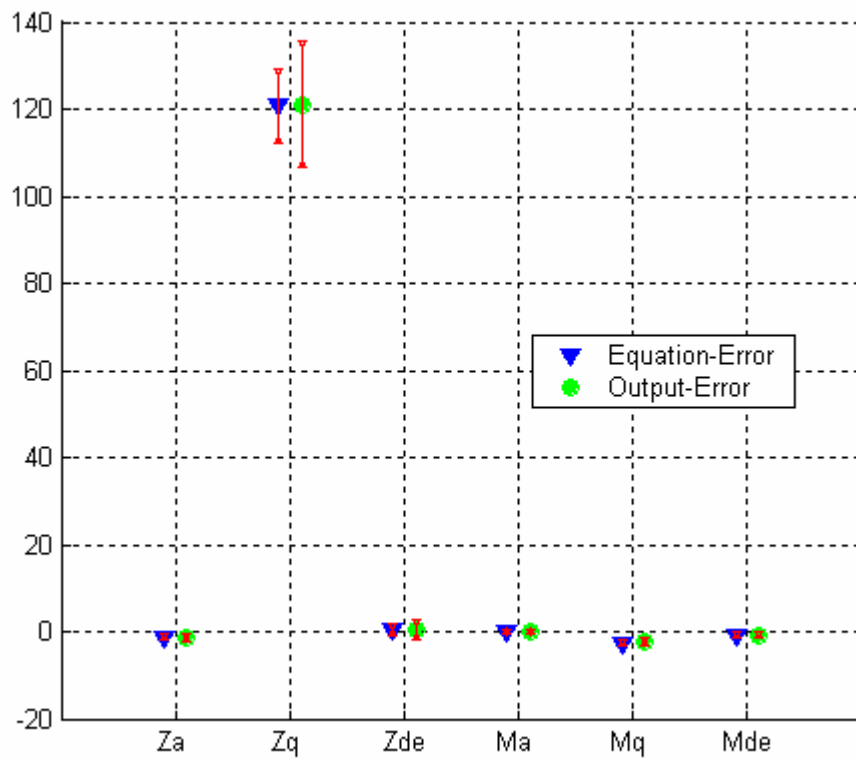


Figure C-4. Plot of Equation Error and Output Error Results for AFT 90-0

Output Error converged after 16 iterations¹⁷

Table C-5. Equation Error and Output Error¹⁸ Results for AFT 90-10

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.33	0.12	-1.51	0.17
<i>Zq</i>	-1695.58	65.50	-1734.86	96.51
<i>Zde</i>	-24.57	10.33	-36.81	13.67
<i>Ma</i>	0.00	0.00	0.00	0.00
<i>Mq</i>	-3.17	0.12	-3.22	0.19
<i>Mde</i>	-0.72	0.02	-0.74	0.03

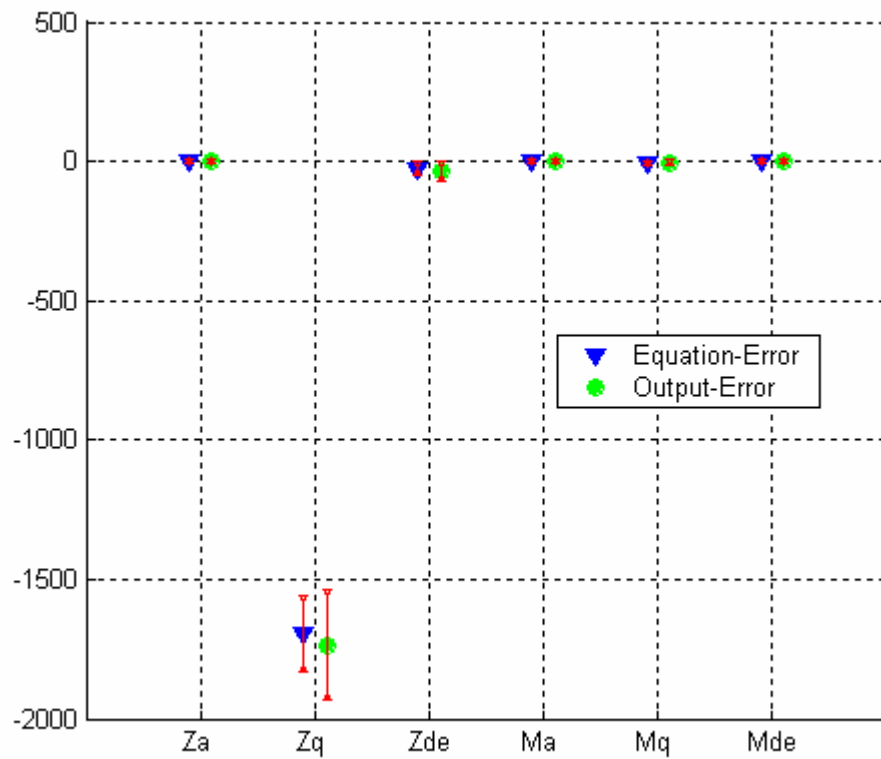


Figure C-5. Plot of Equation Error and Output Error Results for AFT 90-10

Output Error converged after 20 iterations¹⁸

Table C-6. Equation Error and Output Error¹⁹ Results for AFT 90-40

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.20	0.28	-0.99	0.07
<i>Zq</i>	131.04	9.94	117.10	5.39
<i>Zde</i>	0.26	1.40	-1.39	0.76
<i>Ma</i>	-0.03	0.01	-0.04	0.00
<i>Mq</i>	-3.27	0.29	-3.12	0.20
<i>Mde</i>	-0.65	0.04	-0.64	0.02

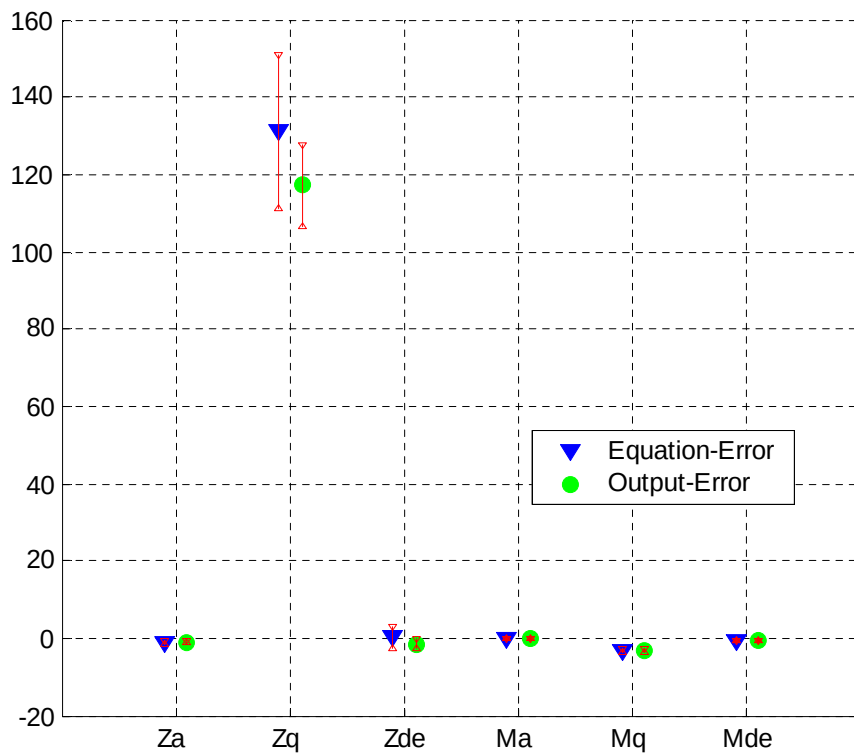


Figure C-6. Plot of Equation Error and Output Error Results for AFT 90-40

Output Error converged after 22 iterations¹⁹

Table C-7. Equation Error and Output Error²⁰ Results for AFT 110-0

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.05	0.44	1.70	0.87
<i>Zq</i>	-467.32	103.71	-19.62	2.51
<i>Zde</i>	10.92	16.58	-2.32	0.29
<i>Ma</i>	0.01	0.00	0.82	0.36
<i>Mq</i>	-3.76	0.39	-6.39	0.79
<i>Mde</i>	-0.92	0.06	-1.00	0.05

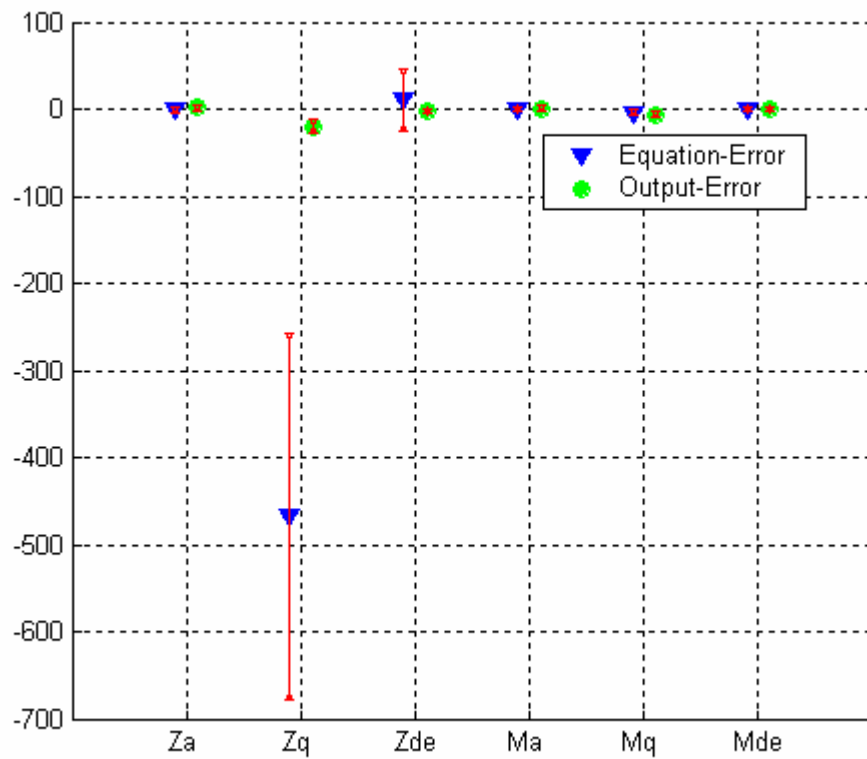


Figure C-7. Plot of Equation Error and Output Error Results for AFT 110-0

Output Error converged after 153 iterations²⁰

Table C-8. Equation Error and Output Error²¹ Results for AFT 110-10

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.54	0.17	-1.29	0.17
<i>Zq</i>	165.38	20.68	95.18	28.51
<i>Zde</i>	1.53	3.36	-7.68	5.03
<i>Ma</i>	-0.04	0.00	-0.03	0.01
<i>Mq</i>	-3.61	0.25	-5.07	0.59
<i>Mde</i>	-0.96	0.04	-1.21	0.09

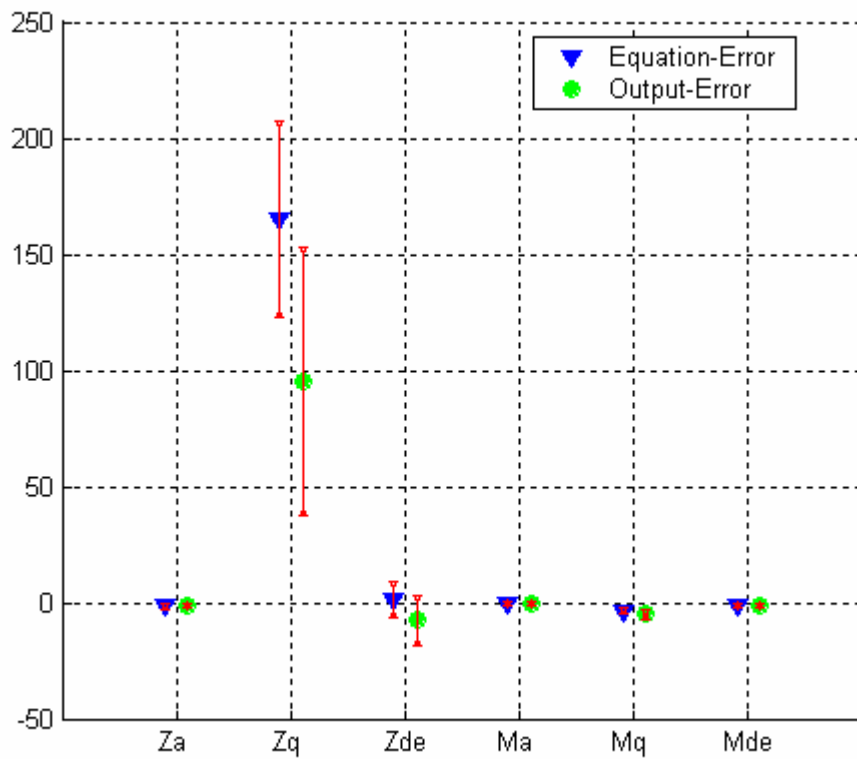


Figure C-8. Plot of Equation Error and Output Error Results for AFT 110-10

Output Error converged after 26 iterations²¹

Table C-9. Equation Error and Output Error²² Results for AFT 110-40

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.62	0.10	-1.40	0.10
<i>Zq</i>	1246.22	82.98	858.21	120.36
<i>Zde</i>	19.38	13.40	-35.74	19.73
<i>Ma</i>	-0.01	0.00	-0.01	0.00
<i>Mq</i>	-4.27	0.61	-4.81	0.49
<i>Mde</i>	-0.98	0.09	-1.09	0.08

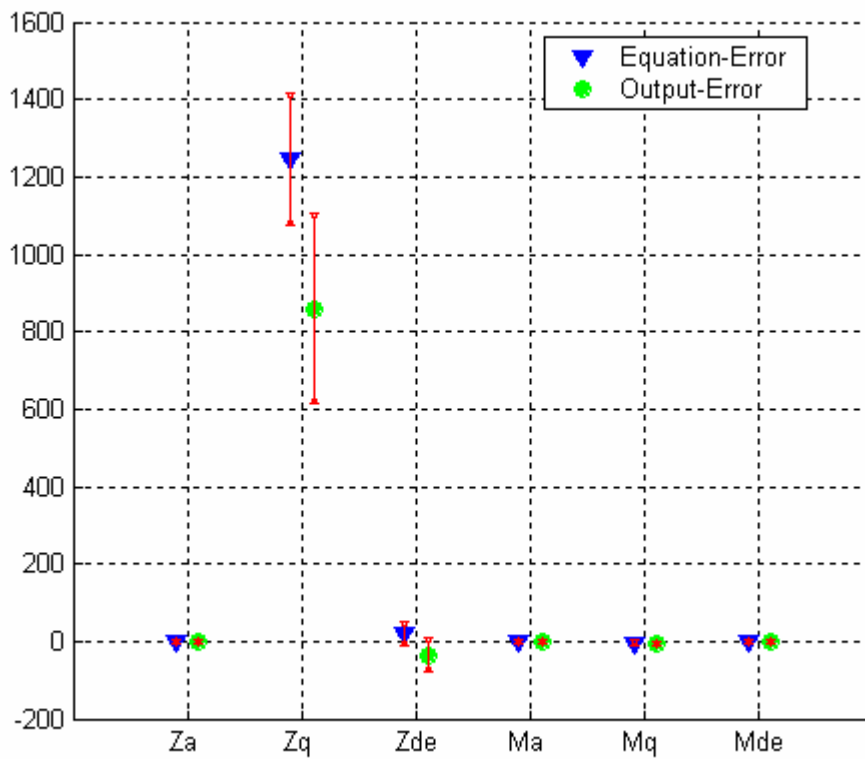


Figure C-9. Plot of Equation Error and Output Error Results for AFT 110-40

Output Error converged after 28 iterations²²

Table C-10. Equation Error and Output Error²³ Results for FWD 70-0

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.52	0.53	-1.21	0.34
<i>Zq</i>	232.74	39.65	159.63	31.18
<i>Zde</i>	4.52	5.62	-2.16	4.07
<i>Ma</i>	-0.04	0.01	-0.06	0.01
<i>Mq</i>	-1.32	0.39	-1.45	0.41
<i>Mde</i>	-0.45	0.05	-0.48	0.02

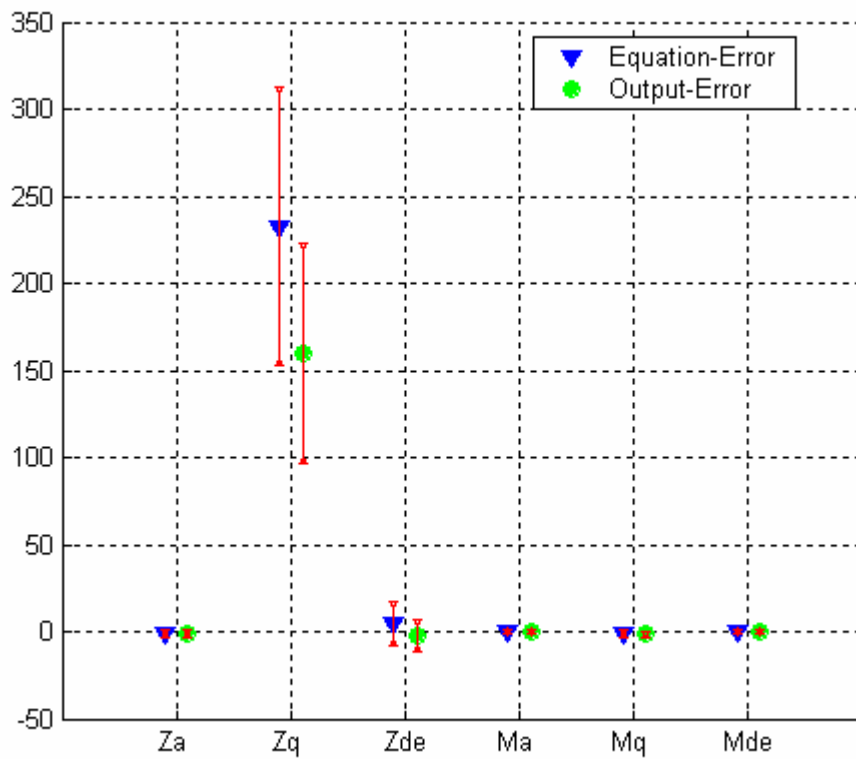


Figure C-10. Plot of Equation Error and Output Error Results for FWD 70-0

Output Error converged after 46 iterations²³

Table C-11. Equation Error and Output Error²⁴ Results for FWD 70-10

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-0.91	0.69	-2.54	1.92
<i>Zq</i>	147.80	33.62	4.99	1.91
<i>Zde</i>	4.57	5.35	-1.08	0.39
<i>Ma</i>	-0.04	0.01	-1.13	0.86
<i>Mq</i>	-1.97	0.44	-0.06	2.01
<i>Mde</i>	-0.50	0.07	-0.54	0.03

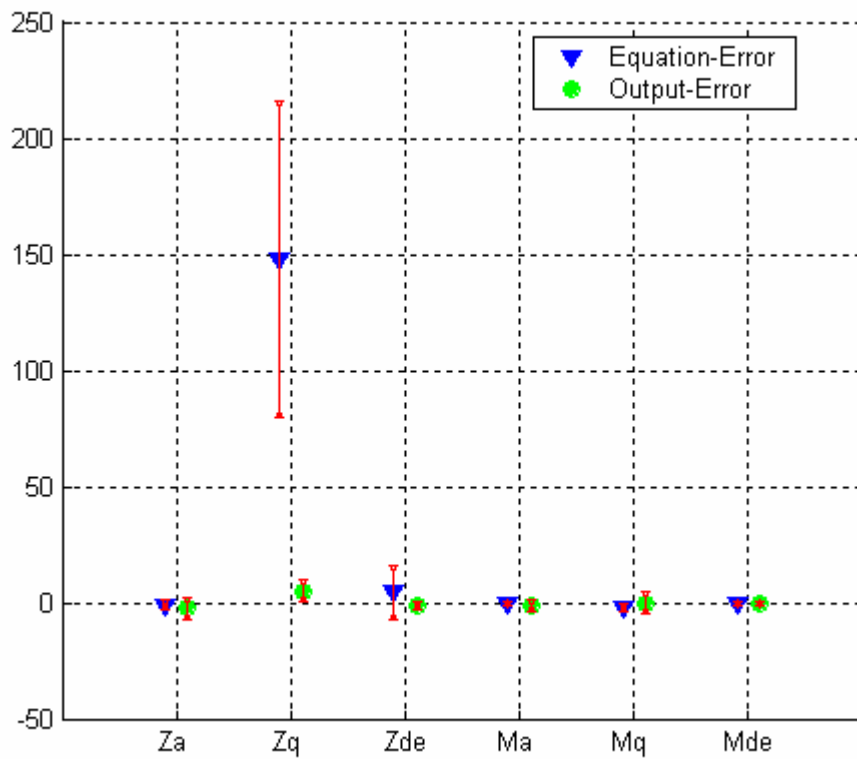


Figure C-11. Plot of Equation Error and Output Error Results for FWD 70-10

Output Error did not converge²⁴

Table C-12. Equation Error and Output Error²⁵ Results for FWD 70-40

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-0.11	1.03	-2.51	5.05
<i>Zq</i>	10.16	61.71	-8.77	10.81
<i>Zde</i>	-8.17	10.11	-2.90	1.05
<i>Ma</i>	-0.04	0.01	-1.25	1.94
<i>Mq</i>	-5.13	0.59	-3.36	5.26
<i>Mde</i>	-0.91	0.10	-1.08	0.09

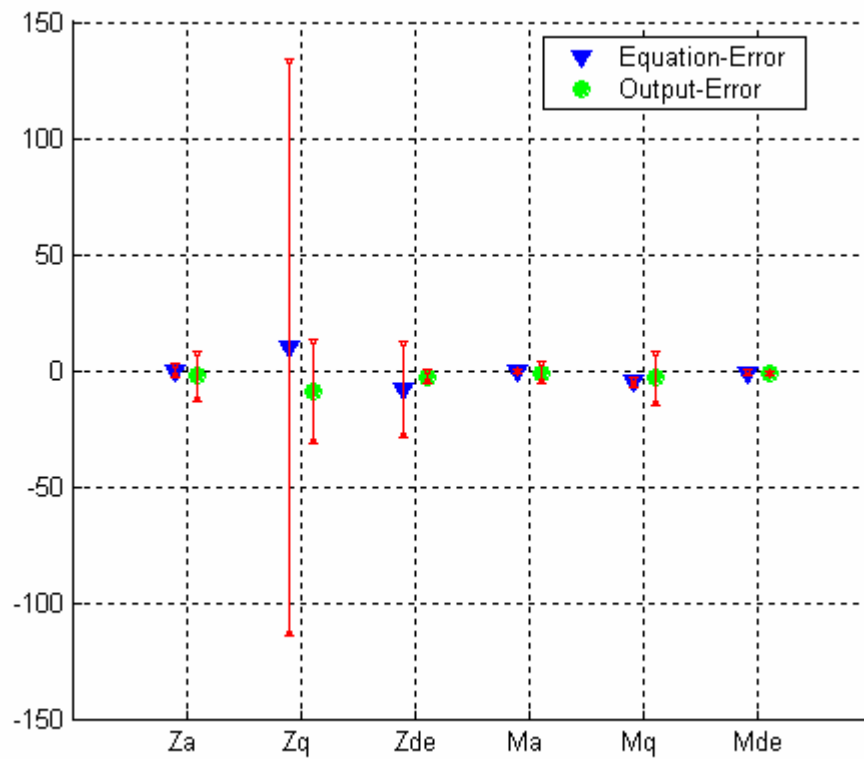


Figure C-12. Plot of Equation Error and Output Error Results for FWD 70-40

Output Error did not converge²⁵

Table C-13. Equation Error and Output Error²⁶ Results for FWD 90-0

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.37	0.42	-1.44	0.45
<i>Zq</i>	3580.58	627.15	2255.69	443.04
<i>Zde</i>	209.67	110.73	-33.22	72.47
<i>Ma</i>	0.00	0.00	0.00	0.00
<i>Mq</i>	-3.18	0.71	-2.85	0.71
<i>Mde</i>	-0.87	0.13	-0.86	0.09

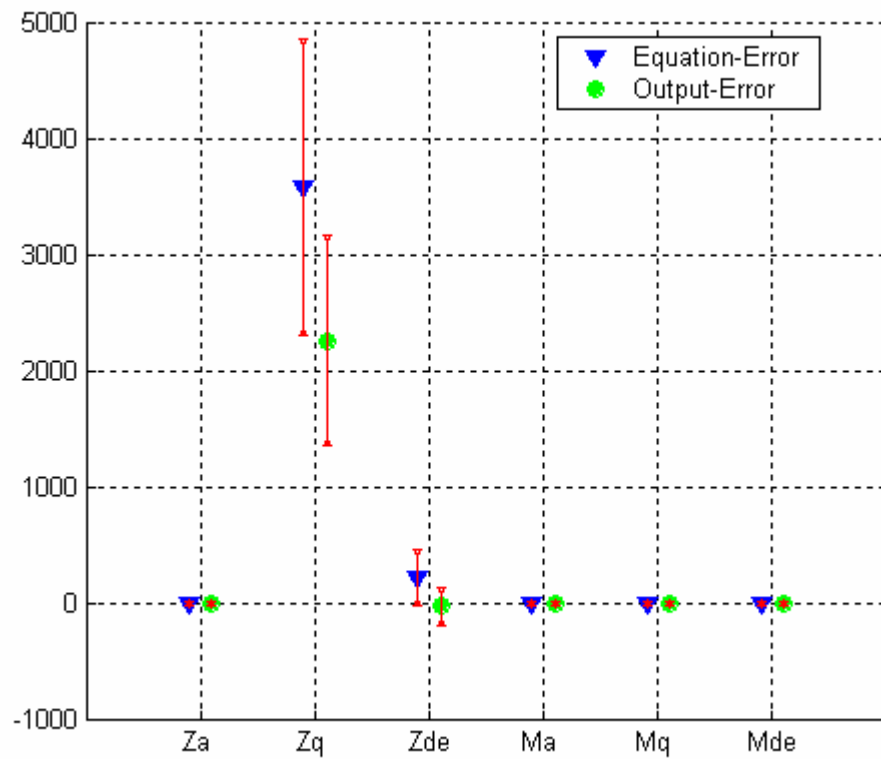


Figure C-13. Plot of Equation Error and Output Error Results for FWD 90-0

Output Error converged after 38 iterations²⁶

Table C-14. Equation Error and Output Error²⁷ Results for FWD 90-40

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-0.74	0.76	-4.54	5.92
<i>Zq</i>	889.10	294.11	5.72	10.76
<i>Zde</i>	47.10	48.92	-2.68	1.56
<i>Ma</i>	-0.01	0.00	-1.34	1.75
<i>Mq</i>	-3.99	0.83	-0.76	6.06
<i>Mde</i>	-0.82	0.11	-0.91	0.03

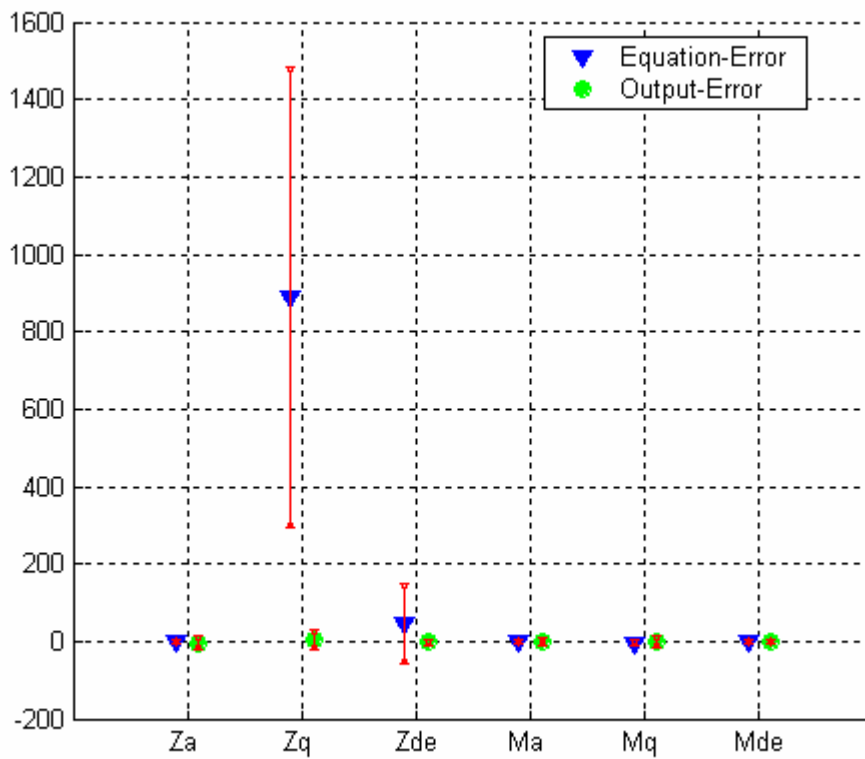


Figure C-14. Plot of Equation Error and Output Error Results for FWD 90-40

Output Error did not converge²⁷

Table C-15. Equation Error and Output Error²⁸ Results for FWD 110-0

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-0.85	0.28	0.00	2.37
<i>Zq</i>	-118.00	16.77	-31.34	27.51
<i>Zde</i>	-3.58	3.65	-1.74	3.94
<i>Ma</i>	0.17	0.01	0.75	0.50
<i>Mq</i>	-2.73	0.70	-6.22	2.32
<i>Mde</i>	-0.83	0.14	-1.31	0.19

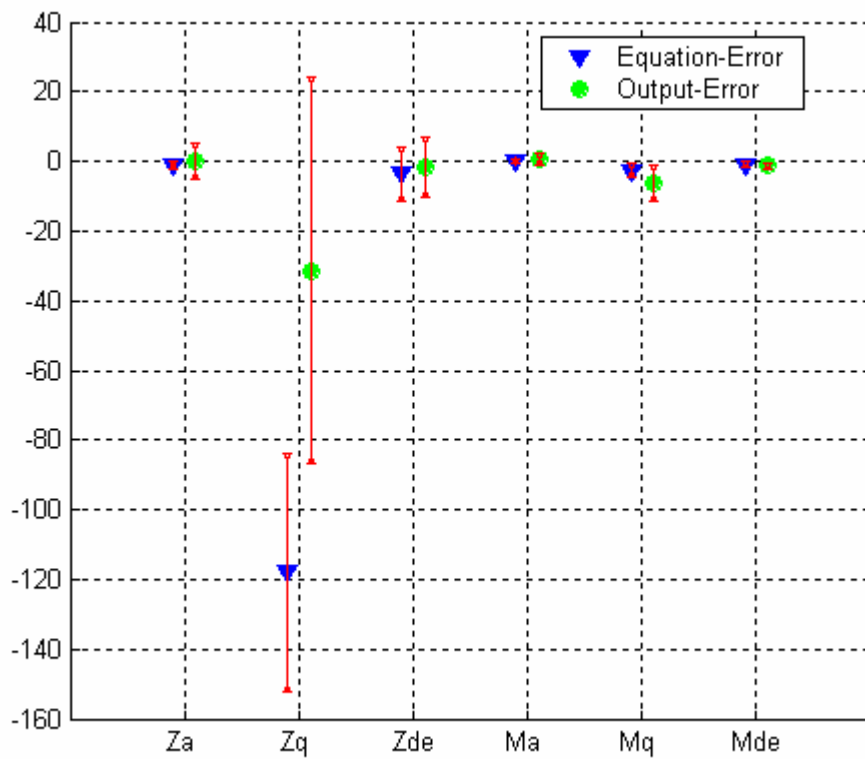


Figure C-15. Plot of Equation Error and Output Error Results for FWD 110-0

Output Error did not converge²⁸

Table C-16. Equation Error and Output Error²⁹ Results for FWD 110-10

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.05	0.25	-3.51	3.10
<i>Zq</i>	1183.02	172.41	8.41	6.69
<i>Zde</i>	89.67	33.42	-3.97	3.41
<i>Ma</i>	-0.02	0.00	-0.84	0.63
<i>Mq</i>	-2.41	1.10	-3.49	3.15
<i>Mde</i>	-0.74	0.20	-1.37	0.08

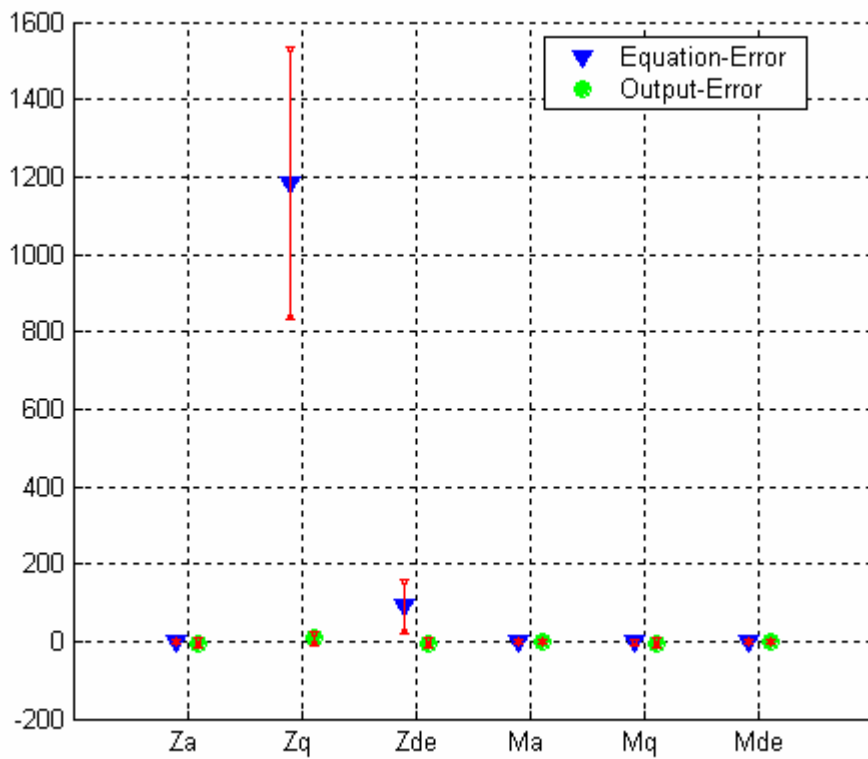


Figure C-16. Plot of Equation Error and Output Error Results for FWD 110-10

Output Error did not converge²⁹

Table C-17. Equation Error and Output Error³⁰ Results for FWD 110-40

<i>Parameter</i>	Equation Error		Output Error	
	Results	Error Bounds	Results	Error Bounds
<i>Za</i>	-1.64	0.28	-1.82	0.45
<i>Zq</i>	168.37	53.57	148.28	34.64
<i>Zde</i>	-4.02	9.92	-4.21	7.30
<i>Ma</i>	-0.07	0.01	-0.09	0.01
<i>Mq</i>	-2.90	0.69	-5.01	1.34
<i>Mde</i>	-0.86	0.12	-1.30	0.20

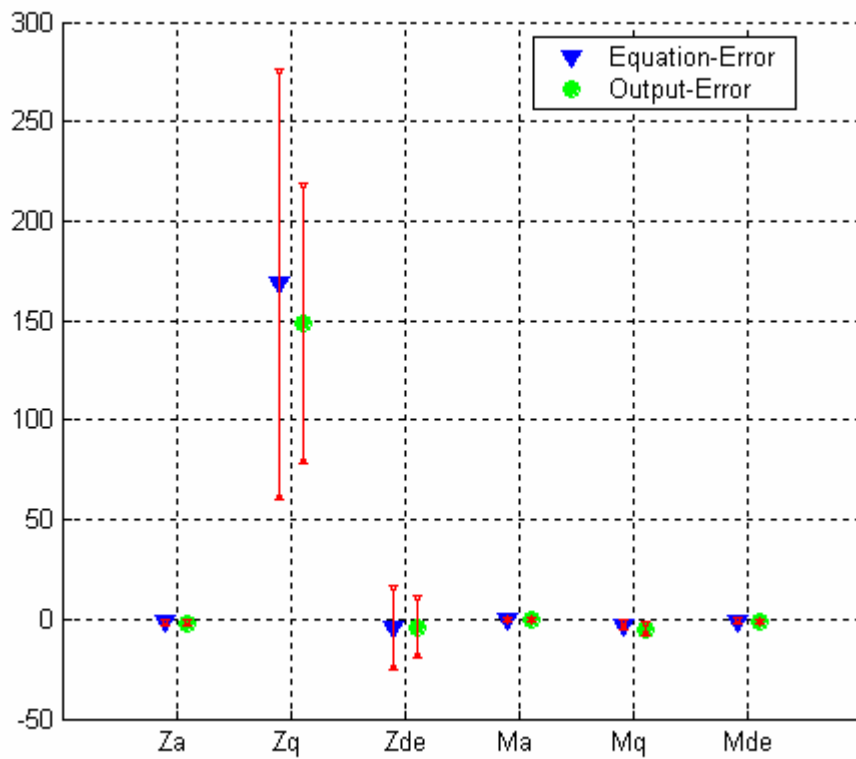


Figure C-17. Plot of Equation Error and Output Error Results for FWD 110-40

Output Error converged after 47 iterations³⁰

APPENDIX D

WEIGHT AND BALANCE

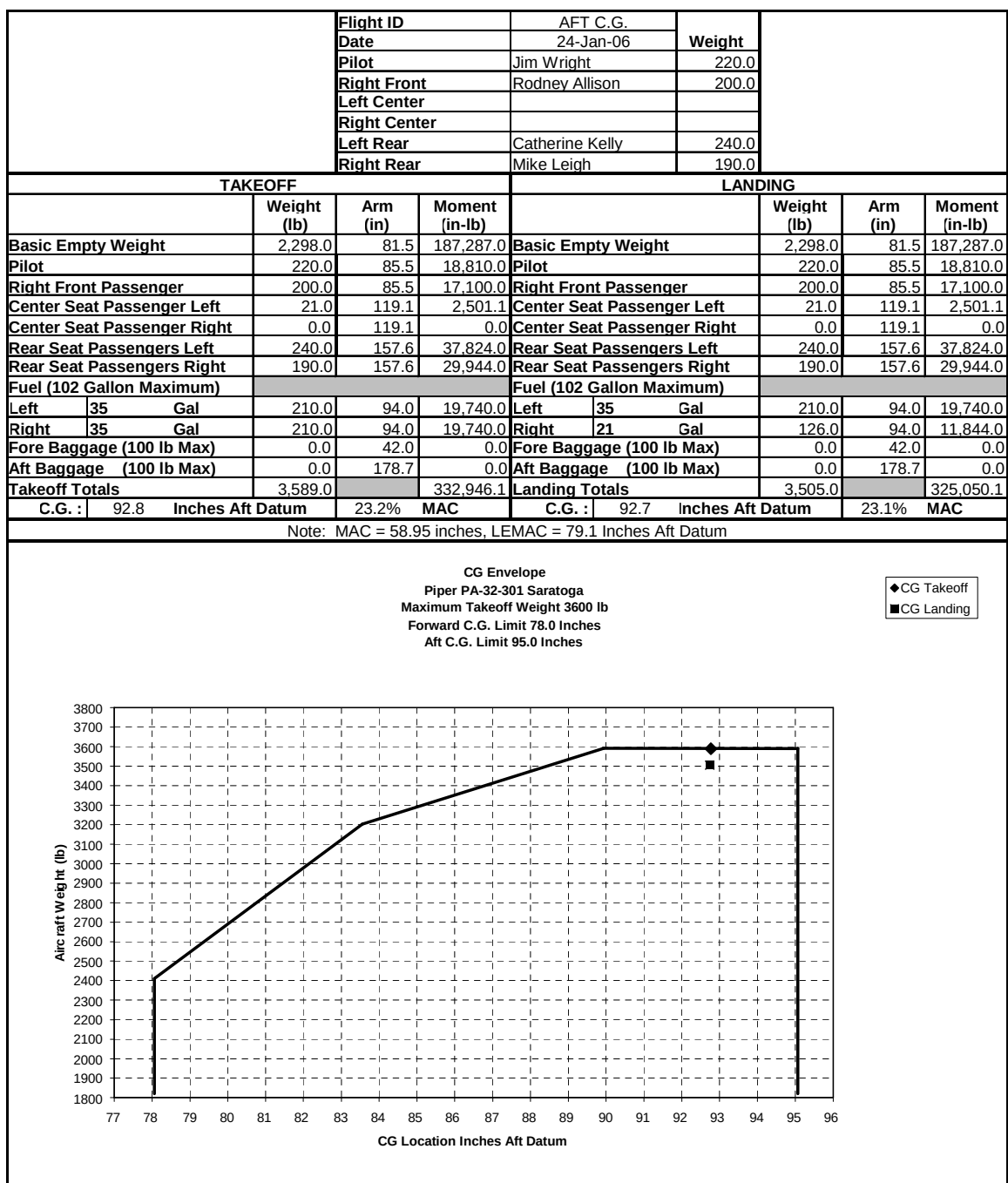


Figure D-1. Weight and Balance of N22UT with aft c.g.

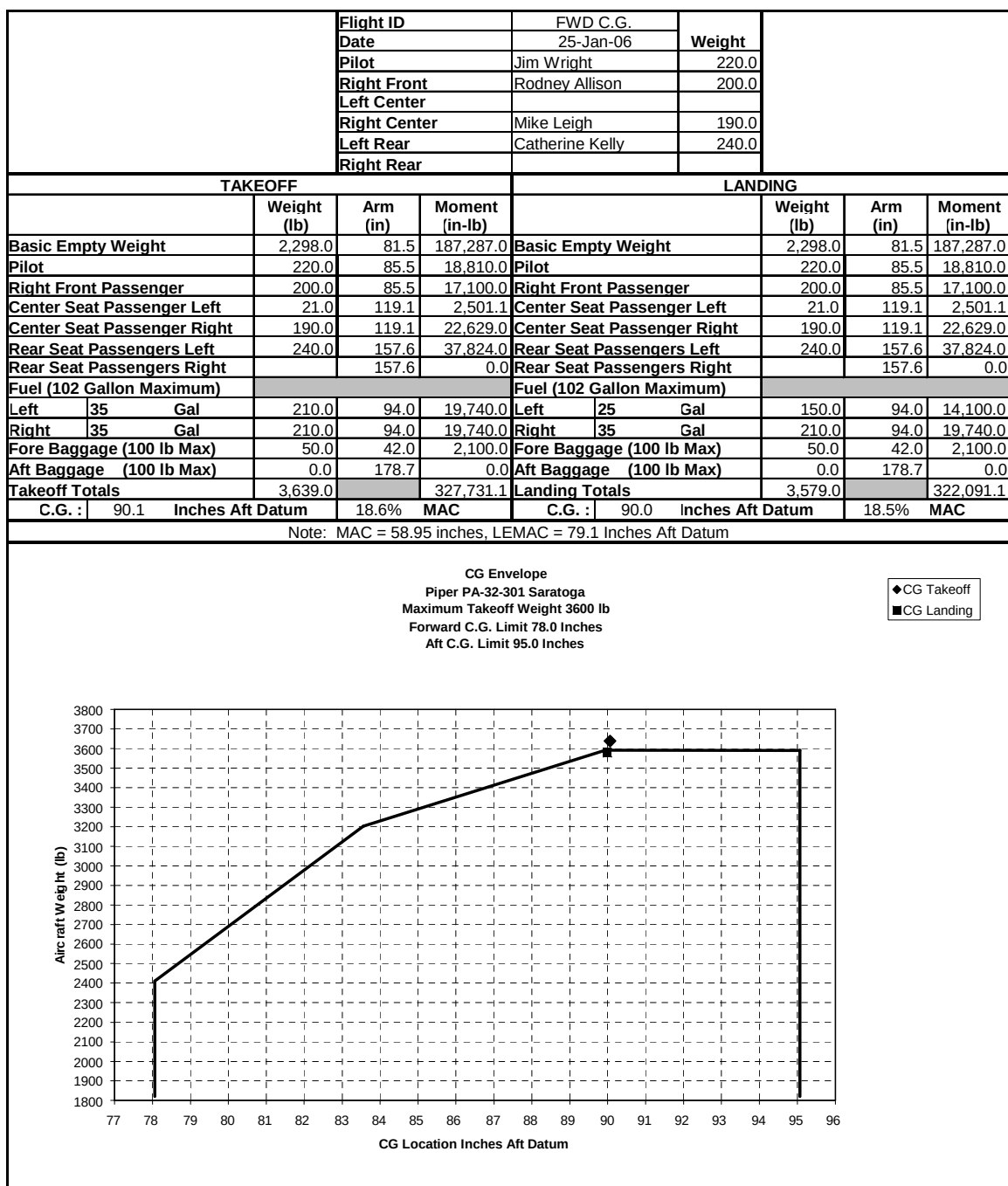


Figure D-2. Weight and Balance of N22UT with forward c.g.

APPENDIX E

N22UT DIMENSIONS

N22UT Dimensions

Surface Areas

Wing.....	178.3 ft ²
Aileron (each).....	5.96 ft ²
Flap (each).....	7.32 ft ²
Fin.....	10.21 ft ²
Rudder.....	4.42 ft ²
Stabilator (including tab).....	32.3 ft ²
Stabilator Tab.....	4.4 ft ²

Wing Data

Airfoil Section.....	NACA 66 ₂ 415 (MOD)
Mean Aerodynamic Chord.....	58.95 in
Aspect Ratio.....	7.4
Dihedral.....	7.0°
Root Chord.....	74.0 in
Chord Constant Section.....	63.0 in
Incident at Root.....	0.0°
Incident at Tip.....	0.0°
Chord at Tip (W.L. 213.54in).....	41.75 in

Aileron

Chord Mean.....13.06 in

Span.....65.73 in

Flap

Constant Chord12.26 in

Span.....85.22 in

General Data

Height.....9.53 ft

Span.....36.17 ft

Length.....27.67 ft

Wheel Base.....7.76 ft

Wheel Tread.....10.54 ft

Turning Radius.....22.9 ft

Power Plant Data

Max Continuous RPM.....2600

Take Off RPM.....2700

Take Off H.P. at Sea Level.....300

Oil Capacity.....12 qts

Model.....10-540-KIG5D

Tail Surfaces Data

Stabilator

Airfoil.....NACA 0012
Chord.....30.0 in
Span.....154.5 in
Aspect Ratio.....5.15
Sweep Back.....0.0°
Dihedral.....0.0°
Hinge Line.....26.9% control surface

Vertical

Root Airfoil.....NACA 63A016
Tip Airfoil.....NACA 0018
Root Chord.....51.6 in
Tip Chord.....27.6 in
Mean Chord.....42.99 in
Span.....60.0 in
Aspect Ratio.....1.52
Sweep Back.....25.5°

Stabilator Tab

Constant Chord.....6.0 in

Span.....106.5 in

The H.L. to Stabilator.....15.93 in

Rudder

Mean Chord.....11.0 in

Span.....50.0 in

Landing Gear Data

Main Gear Tire Size.....6.00x6, 6 ply

Nose Gear Tire Size.....5.00x5, 6 ply

Main Gear Tire Pressure.....55 psi

Nose Gear Tire Pressure.....35 psi

Weights and Loading Data

Gross Weight.....3600 lbs

Max Ramp Weight.....3615 lbs

Std. Empty Weight.....1920 lbs

Useful Load.....1695 lbs

Baggage Capacity

Forward.....100 lbs

Aft.....100 lbs

Total.....200 lbs

Wing Loading (at gross weight).....20.2 lb/ft²

Fuel Capacity (std.).....107 gal

Seating Capacity.....6

APPENDIX F

FLIGHT TEST CARD

Flight Test Card

	Flap Setting		
Airspeed	0°	10°	40°
70 kts			
90 kts			
110 kts			

APPENDIX G

STANDARD SIDPAC DATA CHANNELS

Table G-1. SIDPAC Standard Data Channels³¹

Channel No.	Symbols³²	Description	Units
1	time	time	sec
2	vtrue	true airspeed	ft/sec
3	beta	sideslip angle	deg
4	alpha	angle-of-attack	deg
5	p	roll rate	deg/sec
6	q	pitch rate	deg/sec
7	r	yaw rate	deg/sec
8	phi	Euler roll angle	deg
9	theta	Euler pitch angle	deg
10	psi	Euler heading angle	deg
11	ax	x body axis acceleration	g
12	ay	y body axis acceleration	g
13	az	z body axis acceleration	g
14	de	elevator deflection	deg
15	da	aileron deflection	deg
16	dr	rudder deflection	deg
17	d1	auxiliary control surface 1 deflection	deg
18	d2	auxiliary control surface 2 deflection	deg
19	d3	auxiliary control surface 3 deflection	deg
20	d4	auxiliary control surface 4 deflection	deg
21	d5	auxiliary control surface 5 deflection	deg
22	d6	auxiliary control surface 6 deflection	deg
23	d7	auxiliary control surface 7 deflection	deg
24	d8	auxiliary control surface 8 deflection	deg
25	d9	auxiliary control surface 9 deflection	deg
26	d10	auxiliary control surface 10 deflection	deg
27	qbar	dynamic pressure	lbf/ft ²
28	mach	Mach number	--
29	rho	air density	slug/ft ³

³¹ See reference 5³² Symbols used in program

Table G-1 continued.

Channel No.	Symbols³³	Description	Units
30	h	altitude	ft
31	etae	longitudinal stick deflection	in
32	etaa	lateral stick deflection	in
33	etar	rudder pedal deflection	in
34	thtl1	throttle- engine 1	deg
35	thtl2	throttle- engine 2	deg
36	thtl3	throttle- engine 3	deg
37	thtl4	throttle- engine 4	deg
38	thrust1	thrust- engine 1	lbf
39	thrust2	thrust- engine 2	lbf
40	thrust3	thrust- engine 3	lbf
41	thrust4	thrust- engine 4	lbf
42	pdot	roll acceleration	deg/sec2
43	qdot	pitch acceleration	deg/sec2
44	rdot	yaw acceleration	deg/sec2
45	xcg	x cg position	in
46	ycg	y cg position	in
47	zcg	z cg position	in
48	mass	aircraft mass	slug
49	Ixx	roll inertia	slug-ft2
50	Iyy	pitch inertia	slug-ft2
51	Izz	yaw inertia	slug-ft2
52	Ixz	x-z cross inertia	slug-ft2
53	axm	measured x body axis acceleration	g
54	aym	measured y body axis acceleration	g
55	azm	measured z body axis acceleration	g
56	alpdot	angle-of-attack rate	deg/sec
57	btadot	sideslip angle rate	deg/sec
58	rtv	roll thrust vectoring	deg
59	ptv	pitch thrust vectoring	deg

³³ Symbols used in program

Table G-1 continued.

Channel No.	Symbols³⁴	Description	Units
60	y _{tv}	yaw thrust vectoring	deg
61	C _X	body axis X aerodynamic force coefficient	--
62	C _Y	body axis Y aerodynamic force coefficient	--
63	C _Z	body axis Z aerodynamic force coefficient	--
64	C _l	aerodynamic rolling moment coefficient	--
65	C _m	aerodynamic pitching moment coefficient	--
66	C _n	aerodynamic yawing moment coefficient	--
67	C _D	aerodynamic drag force coefficient	--
68	C _{Yw}	wind axis aerodynamic side force coefficient	--
69	C _L	aerodynamic lift force coefficient	--
70	C _T	thrust force coefficient	--
71	p _{hat}	non-dimensional roll rate	--
72	q _{hat}	non-dimensional pitch rate	--
73	r _{hat}	non-dimensional yaw rate	--
74	u	x body axis velocity component	ft/sec
75	v	y body axis velocity component	ft/sec
76	w	z body axis velocity component	ft/sec
77	s _{area}	wing reference area	ft ²
78	b _{span}	wing span	ft
79	c _{bar}	mean aerodynamic chord	ft
80			
82			
83			
84			
85			
86			
87			
88			
89			
90			

³⁴ Symbols used in program

VITA

Catherine Kelly was born June 3, 1978 in Conway, SC. Her education includes three years in the College of Engineering, Architecture and Physical Sciences at Tuskegee University before obtaining her Bachelor of Science degree in Aerospace Engineering from the University of Maryland, College Park in December 2001 where she was an undergraduate research assistant and a Women in Engineering fellow. A NASA Space Grant recipient, she is a candidate for a Master of Science degree in Aviation Systems at the University of Tennessee Space Institute with a specialization in Research and Development. She is on the board for Col-Kel Education and Training Ministries Inc., a 501(c)(3) non profit corporation. She is also a partner in two real estate holding companies, Kellion-Col Enterprises, LLC and the Kelly Group, LLC. In addition, she is a consultant in the area of science and math education. The oldest daughter of six children, she plans to move to Everett, WA to begin her career as an engineer with Boeing.