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I am submitting herewith a thesis written by John A. Oakberg entitled "An Evaluation of Processing Methods for Static Radioisotope Scan Images." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Computer Science.

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AN EVALUATION OF PROCESSING METHODS FOR STATIC RADIOISOTOPE SCAN IMAGES

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

John A. Oakberg

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ABSTRACT

Radioisotope scanning in the field of nuclear medicine provides a method for the mapping of a radioactive drug in the human body to produce maps (images) which prove useful in detecting abnormalities in vital organs. At best, radioisotope scanning methods produce images with poor counting statistics. One solution to improving the body scan images is using dedicated small computers with appropriate software to process the scan data.

Eleven methods for processing image data are presented. Seven of the methods are applications of a well-known algorithm and four of the methods are applications of an algorithm which heretofore had not been used in processing radioisotope scan data. Two implementations of domains used for processing the data are also presented. It is shown that of the eleven methods developed, six provide desirable processing characteristics and the remaining five either gave strong indications of unacceptable results or were limited by computing facilities. It is also shown that the implementation of a different method for processing the smoothing domains can offer substantial savings in computer processing time.

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CHAPTER I

INTRODUCTION

The reaction of the bodies of humans or animals when absorbing and distributing a radioactive tracer serves as a valuable tool for diagnosing internal diseases. By detecting regions of anomalous uptake of tracer isotopes, diseased areas can be located when other methods, such as X-rays, fail to indicate any abnormality.

Radioisotope scanning is the procedure by which the uptake of a radioisotope by areas of a human or animal body can be observed by detecting gamma rays emitted from that isotope. An isotope is selected based on the area of the body which is of interest. For example, if a scan of the thyroid gland area were desired, a radioactive isotope of iodine would most likely be administered. After allowing sufficient time for uptake, a posterior and anterior scan would be made of the body and the gamma ray counts from the isotope would be recorded.

In nuclear medicine the main objective is to provide the diagnostician a map of the processed data in such a form that qualitative judgments can be made to determine areas of abnormal uptake of the isotope. In some areas (such as the liver), an anomalous uptake is shown by an area of low concentration while an abnormality in another organ may be pinpointed by an area of high absorption.

The main problem in analyzing the scan data is that the current scanning methods produce images of moderate resolution and insufficient counting statistics. The poor quality of the images is attributable to

the low dose levels administered, the short duration of the scan, and the detection method.

To prevent damaging healthy tissues within the patient, the dose of the radioactive agent must be held to a required minimum, resulting in low counting statistics. Improved counting statistics could be obtained by an increase in scanning time. This, however, requires that the patient remain still for an extended period of time which is impossible in many cases. Besides, such an increase in scan time would greatly lower the patient throughput in already overloaded medical facilities. An improvement in sensitivity could be made with little loss of resolution by increasing the size of the detector. However, this proves to be quite expensive, making cost a main limiting factor in medical instrumentation.[1]

An alternative solution to the problem of improving scan quality is that of using computer processing to enhance the body images. It has been shown that a small computer can be used satisfactorily in an imaging environment to collect, process, and display body scan data.[2] Ideally, the processing algorithms should be capable of: (1) reducing the unwanted counts to an acceptable level; (2) keeping changes of the major features of the data to a minimum; and (3) using a minimum amount of computer time and resources.

I. PROBLEM DEFINITION

This investigation was directed toward the development of eleven methods for processing radioisotope scan images to determine their overall effectiveness based on the ideal properties of such processing.

Of additional interest was the evaluation of a different method of processing the smoothing domains. Attention was given to problem areas: (1) the general smoothing and removal of noise; and (2) the consideration of extreme conditions which, due to the nature of isotope imaging, can be spurious counts or valid data. In the specific example shown in Figure 1, curve A represents the closest fit of the data and curve B represents a more realistic interpretation of how the data should appear. This exaggerated example indicates how a single spurious count can cause a false indication of a potential high uptake area. However, should there exist several high counts in the area of the single one shown, then the fit of curve A would perhaps better represent the data.

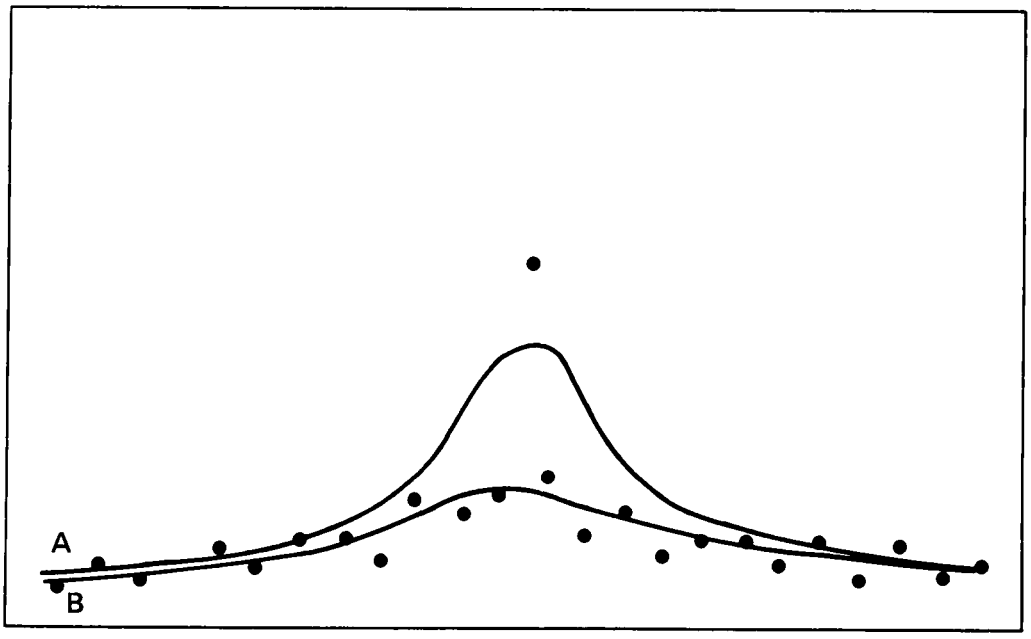


Figure 1

Sample Count Distribution

As can be seen from Figure 1, the best fit of the data is not necessarily the true representation of the image. Most smoothing algorithms in the field of nuclear medicine are second degree functions in order to keep from duplicating the noise in the smoothed result. The use of functions greater than second degree in nuclear medicine for the smoothing of the scan data is uncommon. Direct linear methods are used, but tend to reduce the key features of the data to a point where major inflections in the scan are difficult to detect.

Each processing method was developed on a large computer system for evaluation. The methods which offered the best indication of acceptable smoothing characteristics were then implemented on a small computer using body scan data.

As a result of the development and implementation of the processing methods described herein, satisfactory results were obtained for six of the eleven applications. The degree of image enhancement achieved varied from good fits of the data to images in which key features of the data were obliterated.

II. BACKGROUND

Digital filtering has been applied to biomedical image data at the Jet Propulsion Laboratory. In discussing the several types of linear and non-linear filters which were developed, Selzer[1] mentioned that the most common filtering method used in processing low-contrast pictures containing excessive noise was a low-pass filter. The images presented, which are similar to those used in this study, became blurred when processed by a low-pass filter.

Downward continuation and the calculation of the first and second derivatives of fields can be used to enhance hidden features of magnetic and gravitational data.[3] These methods developed in the field of geophysics have applications in nuclear medicine.

E. D. Grossman[4] used radioisotope scans for locating defective regions in the liver. By limiting an investigation to a single organ and certain isotopes, such a study is more tractable than that of a complete body scan.

The application of a Fredholm integral for singular-value decomposition was presented by Ekstrom.[5] This method is used to compensate for large variations from a known image caused by small errors in a noisy observed image.

A digitized representation of a mountain range in New Mexico was used by James[6] as an image free from statistical error for testing the effectiveness of quadratic and cubic least-squares smoothing. A brain scan was also processed in the investigation to determine the effectiveness of the methods for smoothing whole body gamma ray scans.

The presentation of the image display form is an important area in an imaging system. Data can be displayed using a line printer with the varying intensities of the image depicted by character overstrikes or by use of a cathode-ray tube in which a monochromatic image is presented by varying intensities proportional to the count values.

An interesting extension to the display of images is pseudo color, a method of generating color from black and white images described by Andrews.[7] This added feature gives to the viewing clinical technician a different perspective of the varying intensities of an image.

Considerable work has been accomplished by Bell[8] in the development of the Oak Ridge Imaging System (ORIS-74), an updated version of the Oak Ridge Scanning System (ORSS-72). ORIS-74 is a complete on-line system for the acquisition and processing of images obtained from a rectilinear scanner or camera. The system is capable of processing the entire image, from raw data to an observed picture.

CHAPTER II

DEVELOPMENT OF PROCESSING METHODS

An established method of smoothing two-dimensional scan images involves the replacing of each element of the image with a new data point which is a function of the domain surrounding the element under consideration. This domain is processed by some mathematical smoothing function in order to produce the new data element. A 7 x 7 element domain was chosen in this study since it is compatible with the other smoothing functions of ORIS-74. The 7 x 7 area was implemented in ORIS-74 because it best represents the resolution of the detector, or the minimum separation between unique features of the scan distinguishable by the detector.

The application of the methods to the image data was accomplished by calculating a set of coefficients using the basic algorithms for the smoothing of a 7 x 7 area. The smoothing process was then applied in one of two ways: (1) only a new smoothed center point for each 7 x 7 domain was calculated (referred to herein as a point-by-point method); and (2) each entire 7 x 7 smoothed domain with a one-row, one-column overlap of areas was calculated (referred to herein as an area method).

Eleven methods for processing image data were developed and implemented using two basic algorithms: the well-known polynomial least-squares and a method described by Rempel,[9] referred to hereafter as the Rempel method. Least-square fits of degree one through six were developed as area processors and a second degree least-square fit was developed as a point-by-point processor. Three applications of the

Rempel method are presented as area processors and one is described as a point-by-point processor. The implementation of the area methods are applications which heretofore had not been applied to radioisotope images. Since the point-by-point methods are a more widely accepted implementation, one application of each algorithm using single point processing was developed for comparison purposes.

I. LEAST-SQUARE METHODS

The seven applications of the least-square methods were: (1) a linear least-square fit to an area; (2) a second degree least-square fit, point-by-point; (3) a second degree least-square fit to an area; (4) a third degree least-square fit to an area; (5) a fourth degree least-square fit to an area; (6) a fifth degree least-square fit to an area; and (7) a sixth degree least-square fit to an area. The fit to a single point was developed for comparison to the results of James,[10] and was used in evaluating the effectiveness of each of the processing methods implemented in this study. Each least-square method was utilized also to provide the smoothed images required by the Rempel methods. The fits of the data to an area were implementations of the least-square function which had been previously untried in the field of nuclear medicine.

The development of the least-square fits followed the procedures presented by Hoel[11] in his application of these methods to polynomial regression. A polynomial of degree n can be written as

$$p(x) = \sum_{i=0}^{n} a_i x^i \qquad (1)$$

In the case of the radioisotope images, the objective is to find the equation of a plane which best fits, in a least-square sense, the domain of the points to be smoothed. The general first, second, third, and fourth degree equations for a plane are shown in equations (2), (3), (4), and (5) respectively. Higher degrees can be readily derived by extending equation (5).

First degree: $$Z = a_0 + a_1 x + a_2 y \tag{2}$$

Second degree: $$Z = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2 \tag{3}$$

Third degree: $$Z = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2 + a_6 x^3 + a_7 x^2 y + a_8 x y^2 + a_9 y^3 \tag{4}$$

Fourth degree: $$Z = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2 + a_6 x^3 + a_7 x^2 + a_8 x y^2 + a_9 y^3 + a_{10} x^4 + a_{11} x^3 y + a_{12} x^2 y^2 + a_{13} x y^3 + a_{14} y^4 \tag{5}$$

By applying the well-known method of least-squares to equations (2), (3), (4), and (5), a set of normal equations is obtained for each. These normal equations are then solved for the coefficients a_0, a_1, ... a_n, which are used in applying the least-square polynomials.

II. REMPEL METHODS

A desired feature of a processing method is the ability to remove spurious high and low counts which may be regarded as noise. Such counts, which occur rather frequently in radioisotope imaging, can cause inflections in a smoothed image which are not an actual part of the real data. An algorithm to compensate for such high or low counts, as presented by Rempel,[9] has been used successfully in processing geophysical data.

The smoothing process of the Rempel algorithm is accomplished by calculating the new image based on the difference between the raw image data and a smoothed image of the data. A more detailed breakdown of the smoothing process is presented in three phases: (1) a given domain (the 7 x 7 areas) is smoothed by a least-square method; (2) an array of weights is calculated based on the differences between the 7 x 7 area of raw data and the corresponding 7 x 7 area of smoothed data; (3) the new image is calculated using the weighted array.

The second degree least-square fit was chosen for the initial smoothing to agree with the original statement of the algorithm; however, a second method was implemented using a third degree least-square fit for the first smoothing pass.

Image data smoothed by the Rempel methods are calculated from the coefficients of a set of simultaneous equations which are derived in a manner similar to the polynomial least-squares from the relationship

$$\sum_{k=-n}^{n} \{F(x_{i+k}) - H(x_{i+k})^2\} W_k = \text{minimum} \qquad (6)$$

where

$$W_k = \frac{\max d_k - d_k}{(2n+1) \max d_k - \Sigma d_k}, \tag{7}$$

$$d_k = \{F(x_{i+k}) - G(x_{i+k})\}^2 \tag{8}$$

$2n + 1$ = length of smoothing interval

x_i = point to be smoothed

F = raw image

G = least-square smoothed image of F

H = smoothed result of Rempel algorithm.

The weights (W_k) depend upon the size of the deviation of the points in the raw image, F, from the smoothed result, G.

Since the implementation to the body scan involved two dimensions, the Rempel algorithm was extended to a two-dimensional case by the author from equations (6), (7), and (8). The method is represented for an n x n domain as follows

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \{F(x_{ij}) - H(x_{ij})\}^2 W_{ij} = \text{minimum} \tag{9}$$

where

$$W_{ij} = \frac{\max d_{ij} - d_{ij}}{n^2 \max d_{ij} - \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}} \tag{10}$$

$$d_{ij} = \{F(x_{ij}) - G(x_{ij})\}^2 \tag{11}$$

x_{ij} = area to be smoothed,

$i-3 \geqslant i \leqslant i+3$,

$j-3 \geqslant j \leqslant j+3$

F = raw image

G = least-square smoothed image of F

H = smoothed result of Rempel algorithm

If additional levels of smoothing are required, a third polynomial can be calculated using the same conditions stated above, except that the representations of the image as contained in H and G are used to replace those of G and F, respectively.

A potential problem arises in the calculation of the weights for this two-dimensional application if the area to be smoothed is a constant value. This condition could cause the denominator of equation (10) to become zero. However, in radioisotope imaging, the probability of such an occurrence is very remote. In the event the situation does occur, additional image processing is bypassed for that particular area.

Initially, the Rempel algorithm is stated as a point-by-point method. One application to a point was developed and tested. Three methods utilizing the Rempel algorithm were developed and implemented as area processors. As a result of unacceptable results from one of the Rempel methods, a deviation from the implementation of the smoothing domains as originally stated in the algorithm was made.

The two types of processors for each domain used are hereafter referred to as the Rempel method one and the Rempel method two.

For clarity in explanation of the Rempel methods one and two, let Figure 2 represent an image of seven rows with thirteen elements per row.

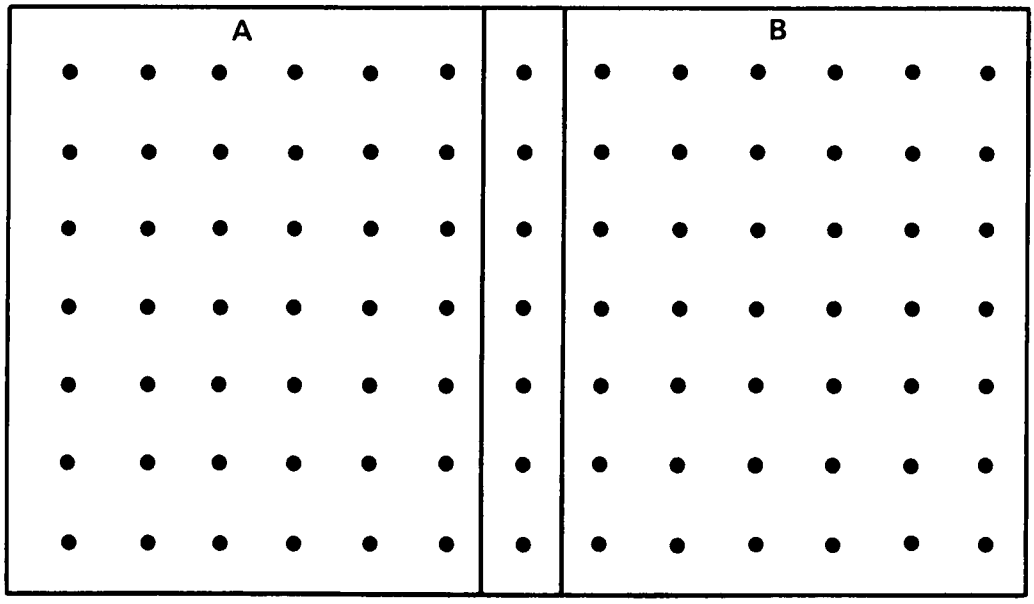


Figure 2

Rempel Method Description

Rempel method one, in processing the 7 x 7 areas A and B, smoothed A and B giving intermediate smoothed results A' and B'. The final results, A" and B" were obtained using the differences between A and A', B and B'. The common column between A" and B" was then averaged to give the final smoothed image.

The second Rempel method performed the initial smoothing on A and B giving A' and B', the smoothed domains. The common column between A' and B' was then averaged and final smoothed results, A" and B" were calculated using A, B, A', and B'.

The difference between the two Rempel methods lies in the one-row one-column averaging of the intersection of the 7 x 7 areas. The first method utilizes a single average between the areas during two smoothing processes while the second method performs an average for each smoothing pass.

CHAPTER III

IMPLEMENTATION OF SMOOTHING METHODS

Two separate computer systems were used in testing and implementing the smoothing methods developed in this investigation. Initially, the programs were written in FORTRAN IV, Level G, and were tested on an IBM 360/370 intercoupled configuration using the Attached Support Processor (ASP) system as shown in Figure 3.[12] After they were successfully tested on the large computer, the programs, which were written in FORTRAN II compatible format, were transferred to a Digital Equipment Corporation PDP/8e small computer having the configuration shown in Figure 4. This hardware, in association with the Oak Ridge Imaging System (ORIS-74), was used in processing body scan images. Sufficient hardware was available to allow the development of several large programs without exceeding the capacity of the computing resources.

For hard-copy display of the image data, two recording methods were used. Image output from the IBM 360/370 system was plotted on an Electronic Associates Incorporated (EAI 430) flatbed electronic plotter using routines[13] which provide perspective views of the data. Pictures of the body scan data were obtained using a Polaroid camera mounted on a Hewlett-Packard 1300A CRT display with P7 phosphor coupled to the PDP/8e system. By utilizing the scaling and intensity functions of ORIS-74, different displays of the same data were readily available. The selectible image exposure feature of ORIS-74 was used to allow proper exposure of the Polaroid film.

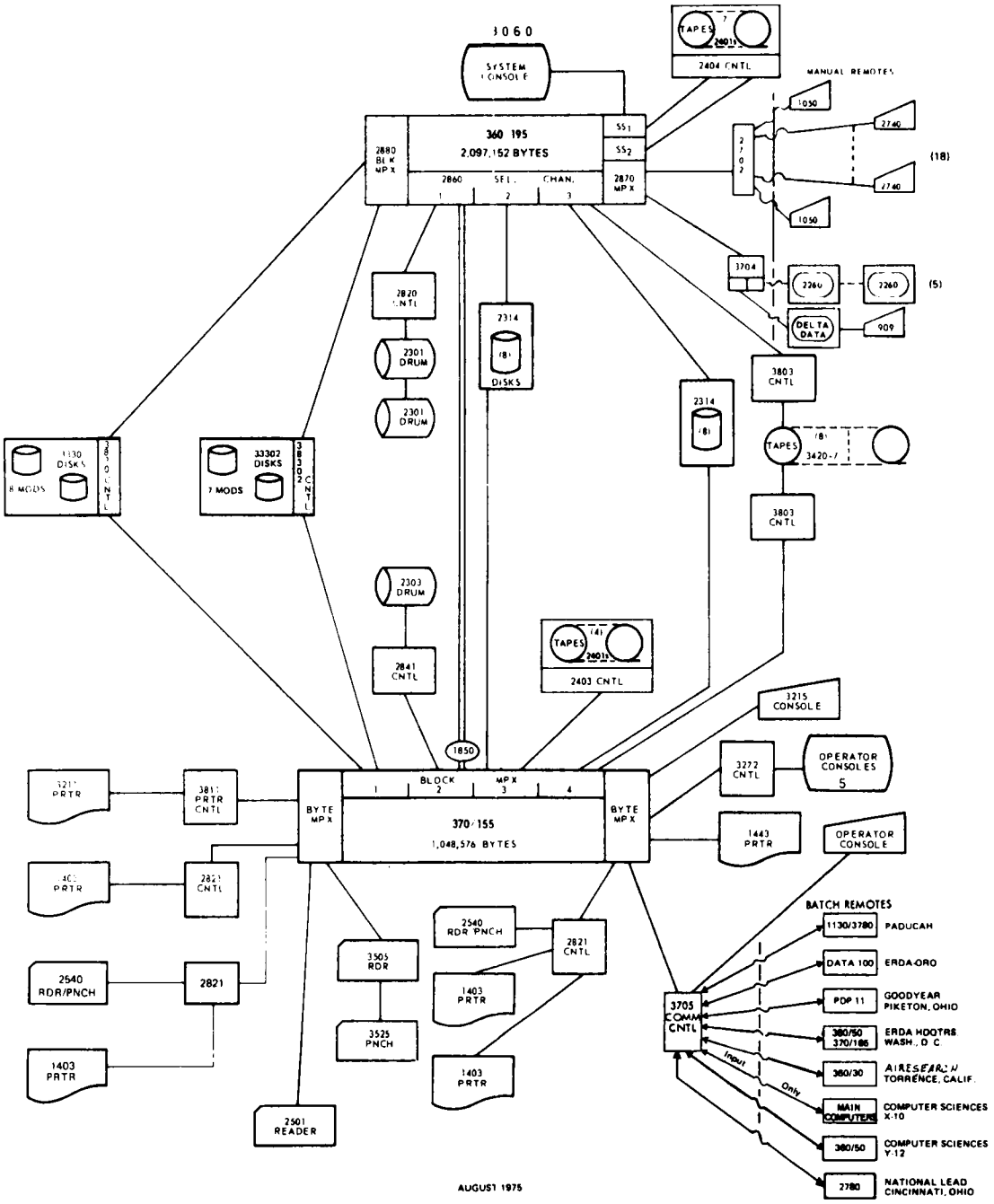


Figure 3

IBM 360/370 Configuration

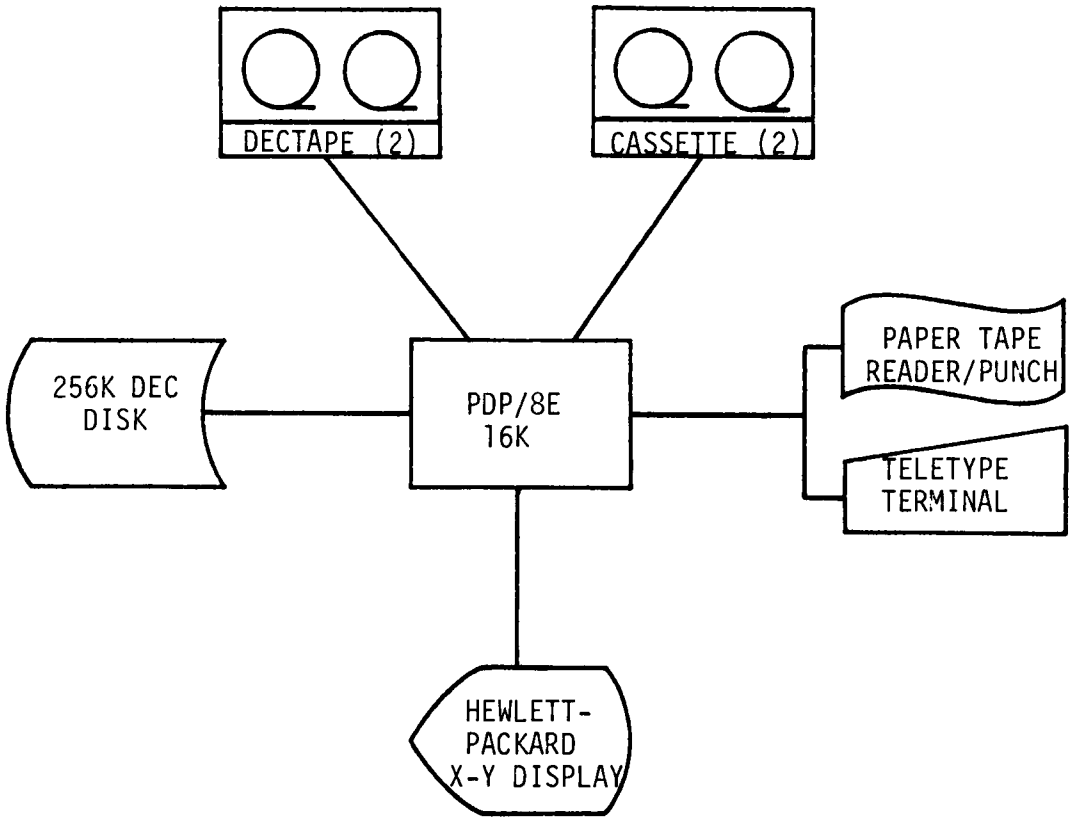


Figure 4

PDP/8e Configuration

I. PROGRAMMING CONSIDERATIONS

In the development of the image processing programs, primary consideration was given to the memory size of the computer to be used in the application of these algorithms since sufficient memory was not available to store the entire image during processing. The scan was read into memory in blocks of seven rows to provide the data for the 7 x 7 smoothing domains. After the 7 x 7 matrix "window" was moved across the seven rows and the new image was calculated, the first six rows of the processed image were written on the new image file, the bottom row was moved to the top of the raw data buffer, and six new rows of raw data were read into rows two through seven. This "scrolling" method, while reducing memory requirements, was also helpful in calculating the averages of the one-row and one-column overlaps of the 7 x 7 areas. As the new image was being created, the smoothed result was added into the output area, and, before recording the data in the new image file, a simple arithmetic average was taken of the overlapped points. A basic flow chart of the smoothing function logic is shown in Figure 5.

The major programming difference between implementing the routines on the IBM 360/370 and the PDP/8e was that of reading and writing the scan data. The input-output routine used on the PDP/8e was capable of reading 128 words from the DECTAPE after which the data were broken into the correct number of rows and columns. A general case program for solving a least-square polynomial of degree n was developed and implemented on the IBM 360/370 system and is shown in Appendix A. Due to computing

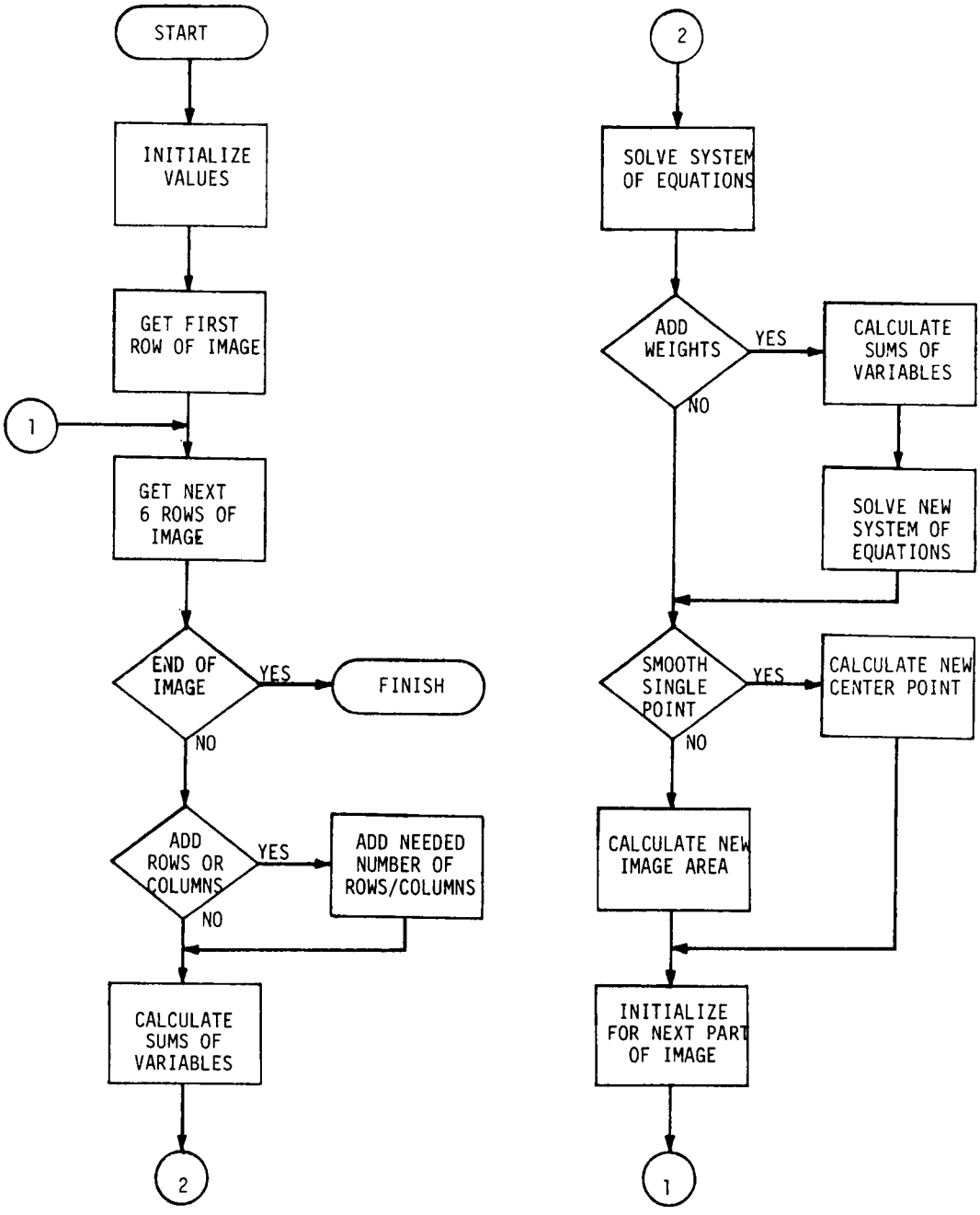


Figure 5

Logic Flow of Smoothing Algorithms

limitations on the PDP/8e, nominal changes to the program were made for implementation of the specific second and third degree cases. The programs on the IBM 360/370 were designed to read only one row of the image at a time instead of reading image blocks. Except for the changes in data transfer and development for the specific rather than the general case, the programs which were implemented were the same for both of the computers used.

Since the ORIS-74 scan display routines output the images in integral grey levels, round-off of the results is a potential area for losing picture detail. To compensate for this possible rounding error, each smoothed data element in floating point was multiplied by a factor of four before the rounded integer value was calculated. For example, instead of a value of 1.20 being rounded down to 1, it was stored as the value 5.

To solve a system of equations, the routine DMATEQ,[14] which employs a Gaussian elimination method, proved acceptable in processing speed. The routine was converted from double precision to single precision because the double precision feature was not available on the PDP/8e system used. Loss of accuracy as a result of using single precision was determined to be negligible.

II. DESCRIPTION OF TEST DATA

The processing methods were initially developed, tested, and evaluated on the IBM 360/370 system using the following five sets of data:

1. A flat (planar) surface;
2. A symmetric image representing a hemisphere;
3. Data representing a topographic map of the Magollon mountain range in New Mexico;
4. A randomly generated image of points with a Poisson distribution about a mean of three;
5. A smoothed image of the mountain data of (3) above with noise added.

The flat surface was the initial test of the routines where the smoothed image would be expected to be equal to the input data. The planar surface was especially useful in checking that portion of the procedures which overlaid the 7 x 7 areas by one row and one column. The symmetry of the second test image gave a more thorough exercising of the ability of the methods to interpolate a simple quadratic while simultaneously offering an easy means of checking the effectiveness of the routines, i.e., hand calculations on one axis applied to the corresponding opposite axis. The first two sets of data were used only in debugging the programs.

Data representing the elevations of a mountain range, as suggested by James,[6] are shown in Appendix B and were used as an image free from statistical error. This image provided data for use in visually evaluating the results of each processing method.

An image comprised of a set of random points of a Poisson distribution with a mean of three was used to offer some indication of noise smoothing. Since the noise encountered in radioisotope imaging follows

a Poisson distribution,[15] it would be expected that the processing methods would reduce the random data to a more uniform image (i.e., a flatter image).

The smoothed mountain image to which noise had been added was used as the most significant test of noise removal. The data were developed for comparison to the results of similar work by James[13] for determining how close the smoothing functions followed the noise instead of the image.

Both sets of noise data were created on the IBM 360/370. The image of Poisson noise was generated based on the following conditions:

$$Z_{ij} = N \tag{12}$$

where Z_{ij} = image of Poisson Distribution

N = k such that $(r_1 \times r_2 \times r_3 \ldots r_k) < e^{-n}$

n = mean of the distribution

$r_1, r_2, \ldots r_k$ are independent uniformly distributed random numbers* in the interval 0 to 1.

The second type of noise was chosen so that a direct comparison to the results of corresponding work by James could be made. The element without noise, Z, was replaced by the value Z1 by the following relationship:

$$Z1 = Z + N \tag{13}$$

---

*All random numbers were generated using a routine from the IBM System 360 Scientific subroutine package, (16) which utilizes a power residue method for the number calculations.

where N is the noise calculated by the equation

$$N = (\cos 2\pi R_1)\ (-2Z \ln R_2)^{1/2}$$

and R_1 and R_2 are independent uniformly distributed random numbers* in the interval 0 to 1.

III. DESCRIPTION OF BODY IMAGE DATA

A single radioisotope scan image was used as input to the methods that were implemented on the PDP/8e small computer. The image is actual data from a patient scan (using Gallium-67 as the radioisotope) taken in 1973 at the Medical Division of the Oak Ridge Associated Universities, Oak Ridge, Tennessee. This specific image was selected for two basic reasons: (1) it was a representative sample of radioisotope body scan data; (2) it was a narrow image (49 elements per row), which reduced computer memory requirements.

The images processed on the PDP/8e system were stored on a magnetic tape formatted by ORIS-74. Each image record contained internal counts of the number of rows and columns that comprised the individual scans. The image used by the methods consisted of 139 rows with 49 elements per row, of which only the upper two-thirds were displayed in the final results. The varying intensities of the gamma ray counts are represented in 32 integral grey levels.

---

*All random numbers were generated using a routine from the IBM System 360 Scientific subroutine package, (16) which utilizes a power residue method for the number calculations.

CHAPTER IV

EVALUATION OF PROCESSING METHODS

Each method was tested on the IBM 360/370 system using data described in Chapter III. After the results of the tests were evaluated, five of the eleven methods were selected for transfer to the PDP/8e computer for processing the body scan data. The criteria for selection were based on four factors: (1) ability to smooth the mountain data; (2) low average chi-square value per element when processing a noise image; (3) relatively fast computer run time; (4) computer memory requirements. The processing of another noise image was found to support the results of the methods but was not a very strong determinant of effectiveness. Areas of prime concern in each set of mountain and body image data are selected and presented as those features which indicate, as best as possible, the effectiveness of the routines. How well a given feature is handled depends upon the data of most interest, i.e., if high count areas are of the most concern, those routines which flatten the peaks the least may be the most desirable.

A quantitative measure of method effectiveness was obtained by processing two noise images and then performing basic statistics on the results. The two noise images were used to test the capabilities of the methods to process noisy data elements similar to those encountered in radioisotope imaging.

The results of the test image processing are presented first, followed by a summary of the effectiveness of each method based on the test images. Justification for selection of the methods to process the body image data is also given.

I. APPLICATION OF METHODS TO MOUNTAIN DATA

To provide a qualitative comparison of the effectiveness of each routine in smoothing the mountain data as well as a comparison of the results of all methods, three distinct subsets (each representing a 7 x 7 area of the unprocessed data in Figure 6) were selected as follows:

1. Area 'A', rows 6-12, columns 9-15, an area of high counts;
2. Area 'B', rows 32-38, columns 2-8, an area with a single peak;
3. Area 'C', rows 30-36, columns 29-35, a fairly uniform area which is a "gently sloping" valley.

Row 1 and column 1 are oriented to the upper left-hand corner of the data.

Isometric plots of the processed mountain data are shown in Figures 6 through 17. The corresponding numerical data are listed in Appendix B. Areas A, B, and C have been marked on the plots and the data to facilitate comparisons of the smoothing effect. In addition to the areas delineated above, the general middle area of the plots can also be used for evaluation of the image data.

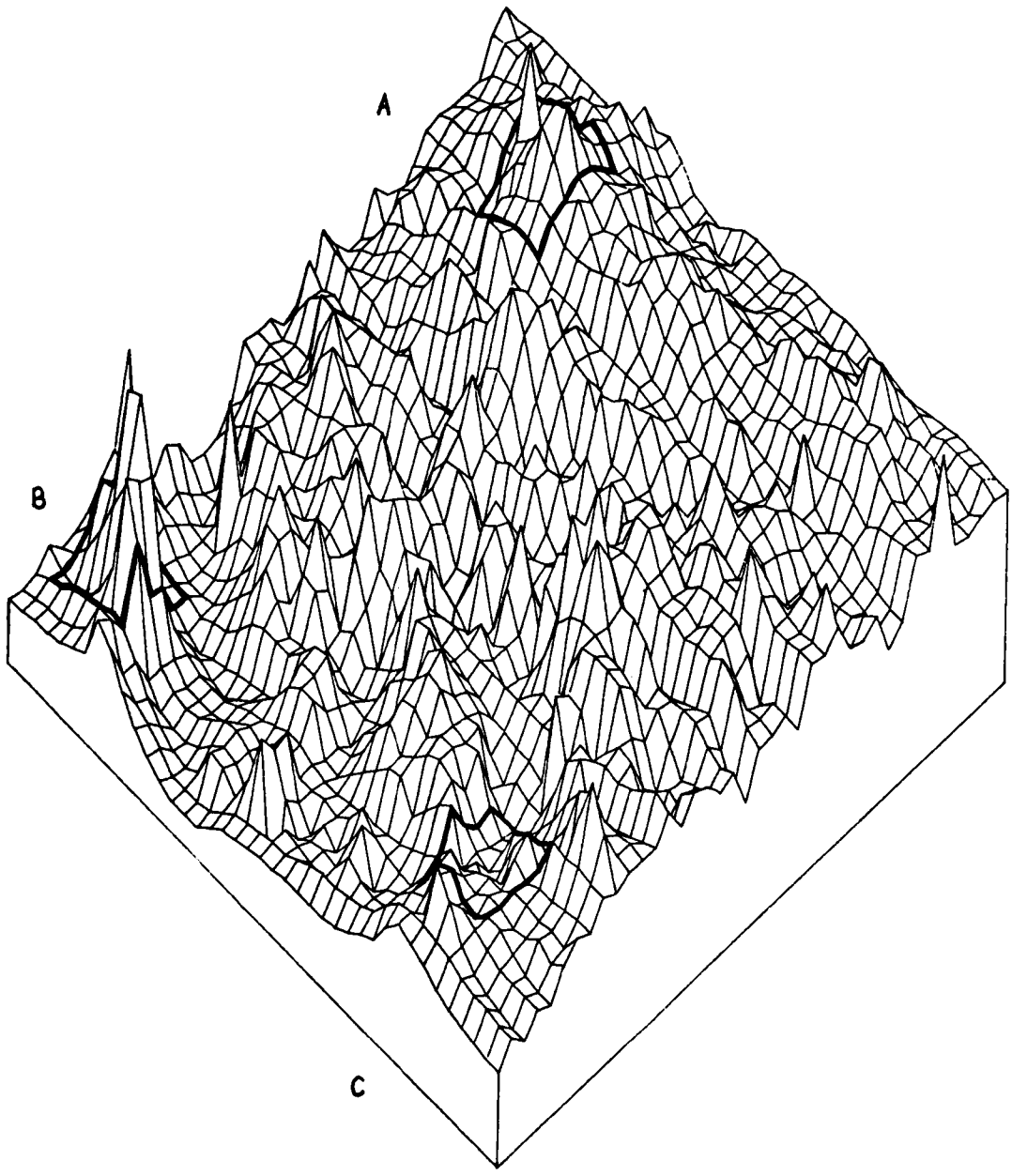


Figure 6

Plot of Mountain Data, Unprocessed

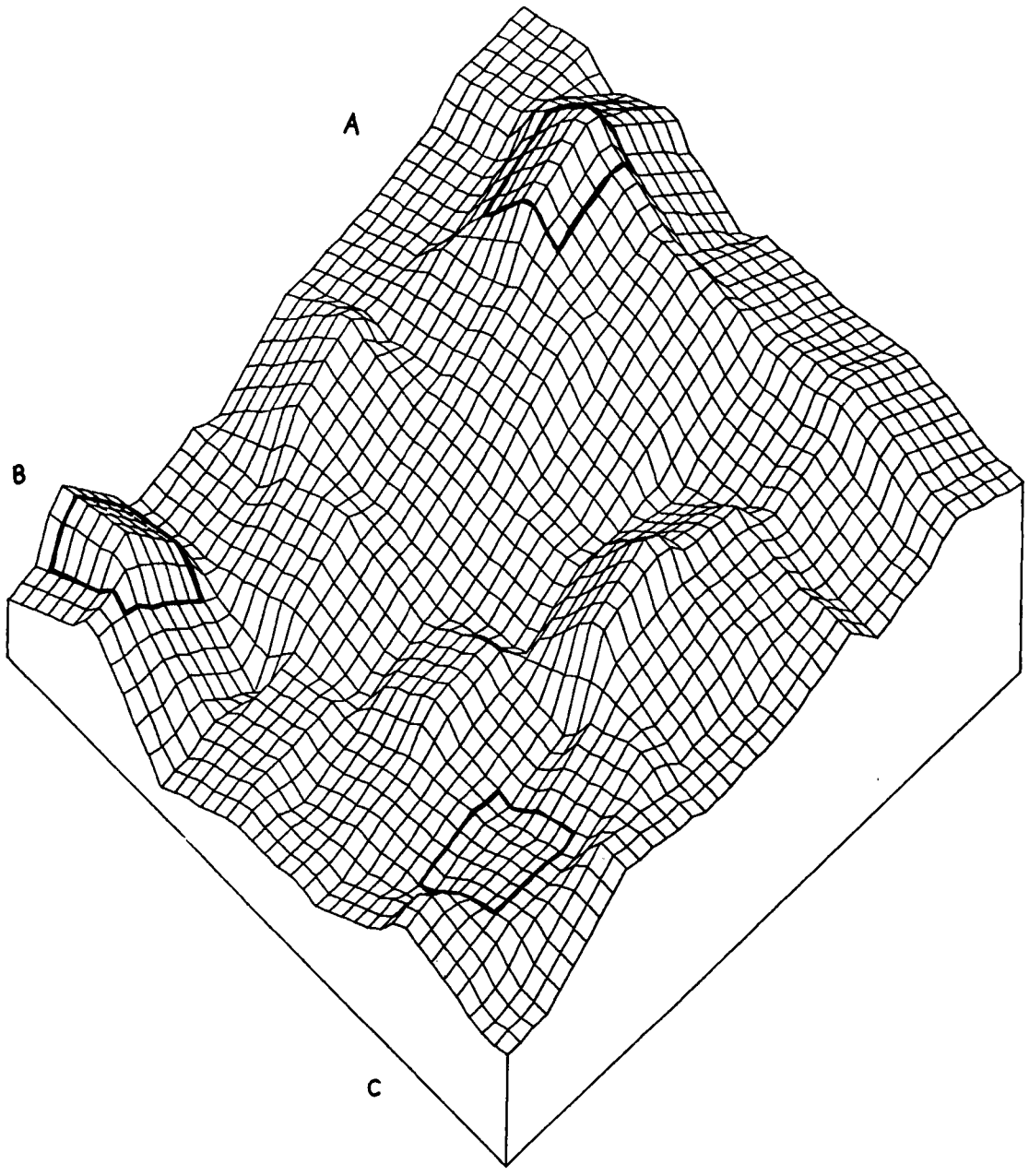


Figure 7

Plot of Mountain Data, First Degree Least-Squares (Area)

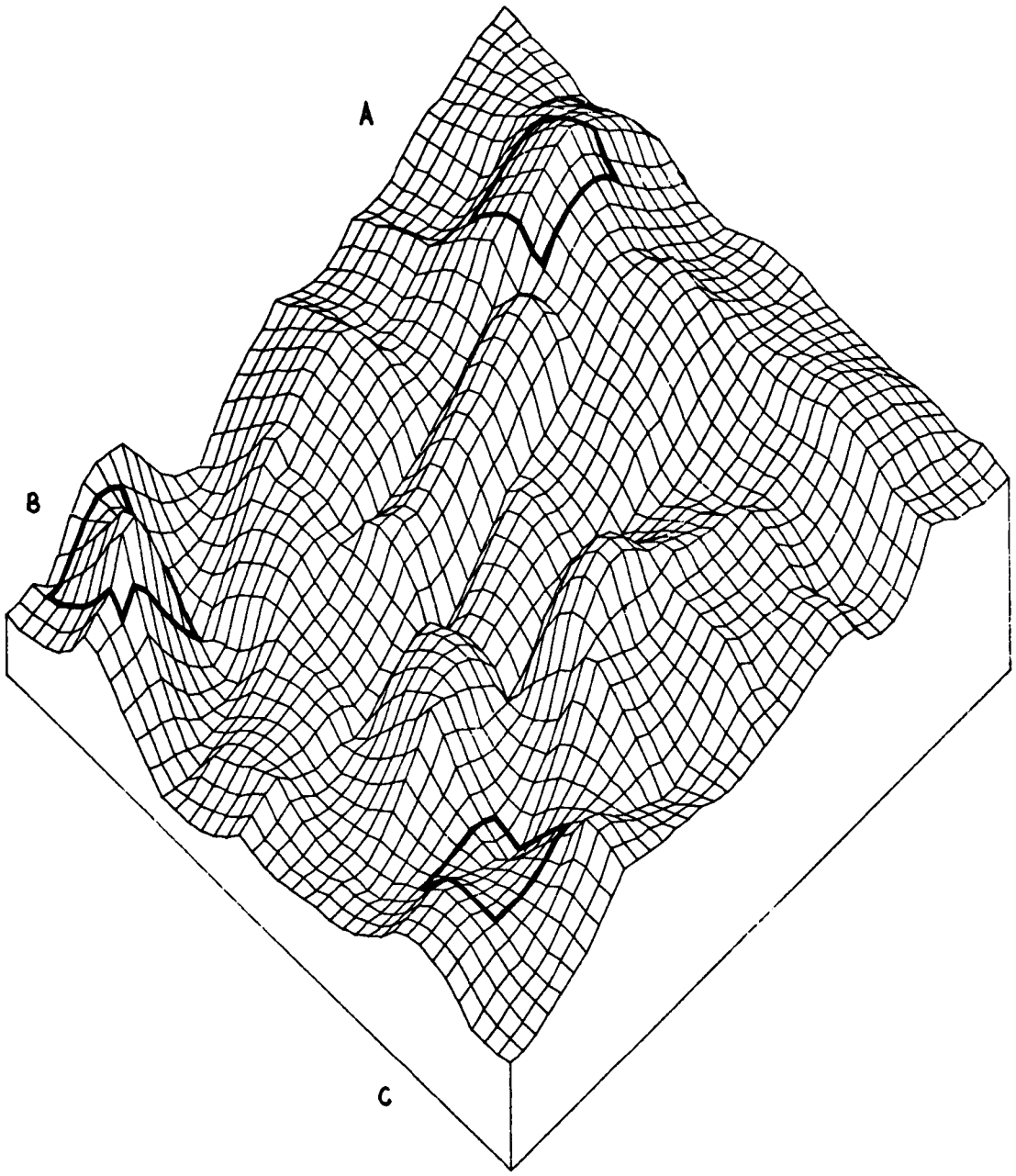


Figure 8

Plot of Mountain Data, Second Degree Least-Squares (Area)

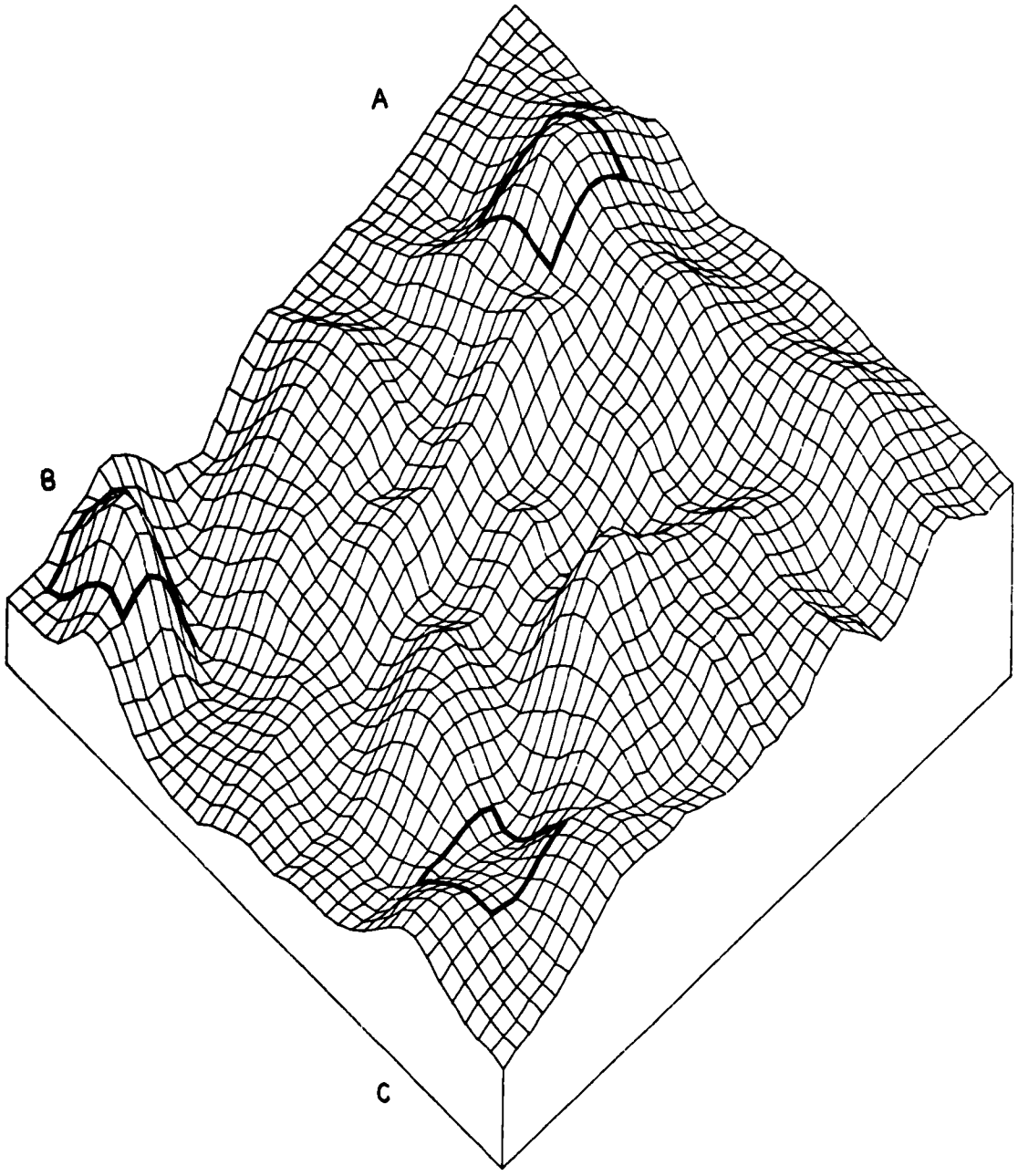


Figure 9

Plot of Mountain Data, Second Degree
Least-Squares (Point-by-Point)

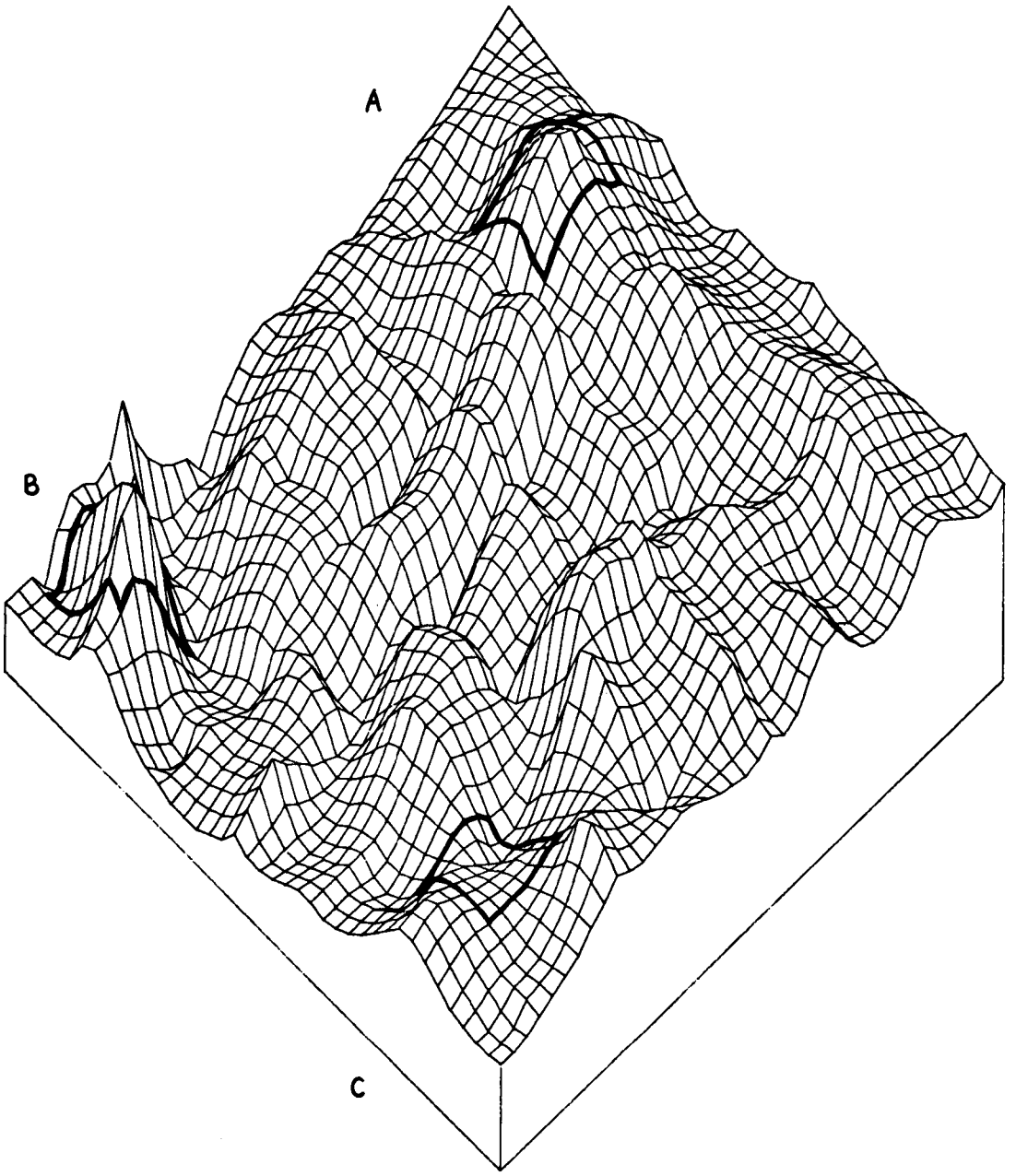


Figure 10

Plot of Mountain Data, Third Degree Least-Squares (Area)

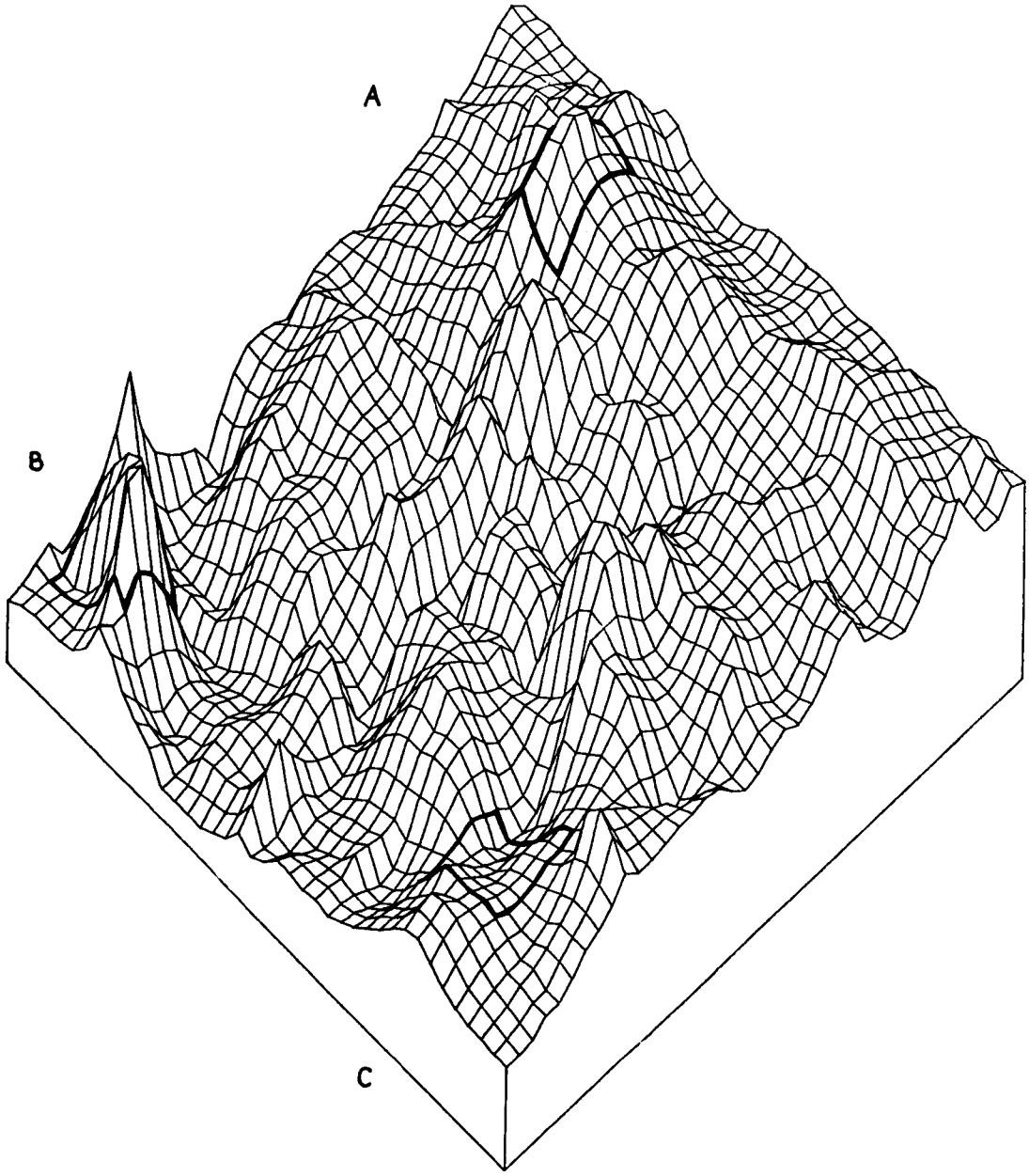


Figure 11

Plot of Mountain Data, Fourth Degree Least-Squares (Area)

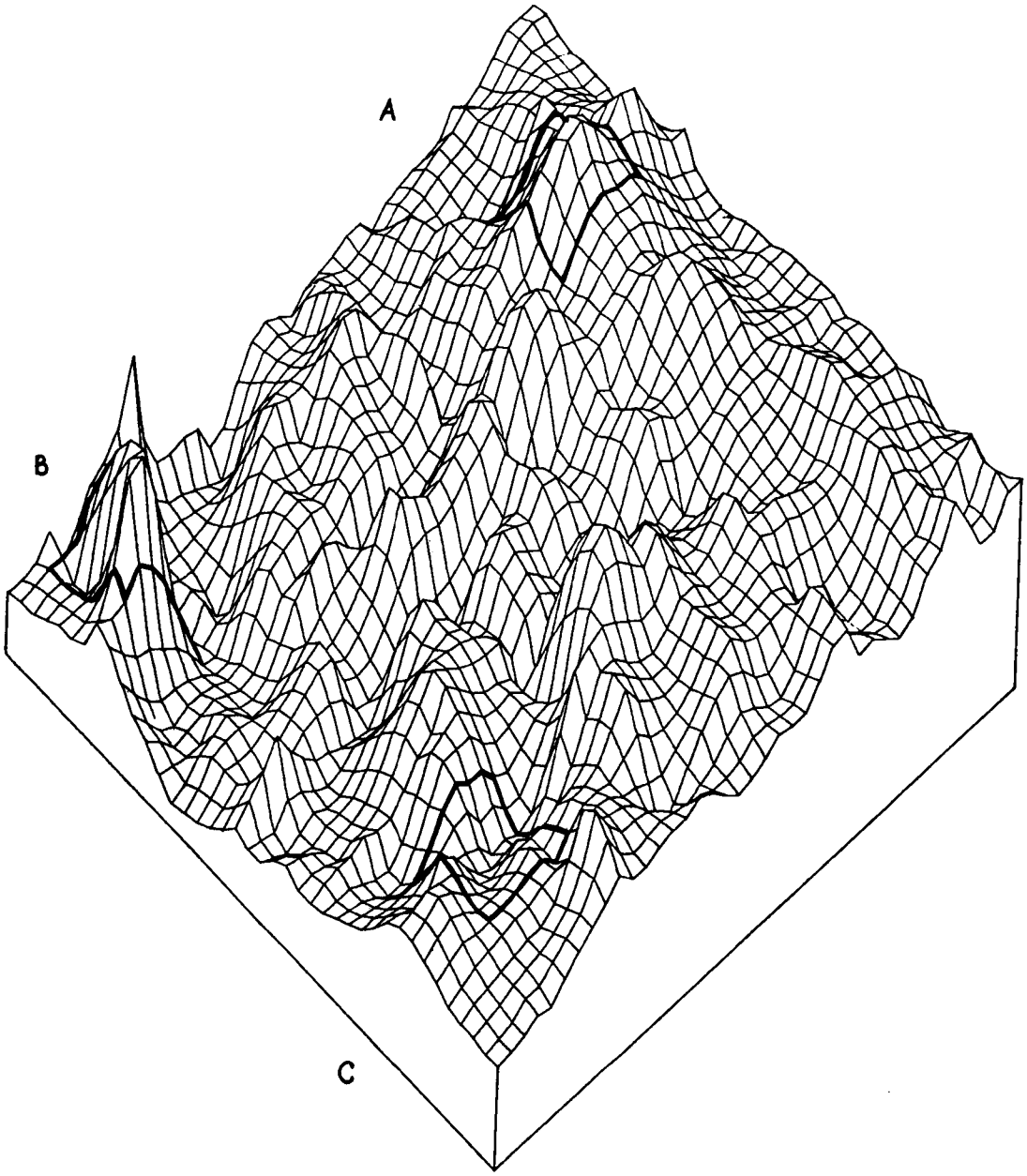


Figure 12

Plot of Mountain Data, Fifth Degree Least-Squares (Area)

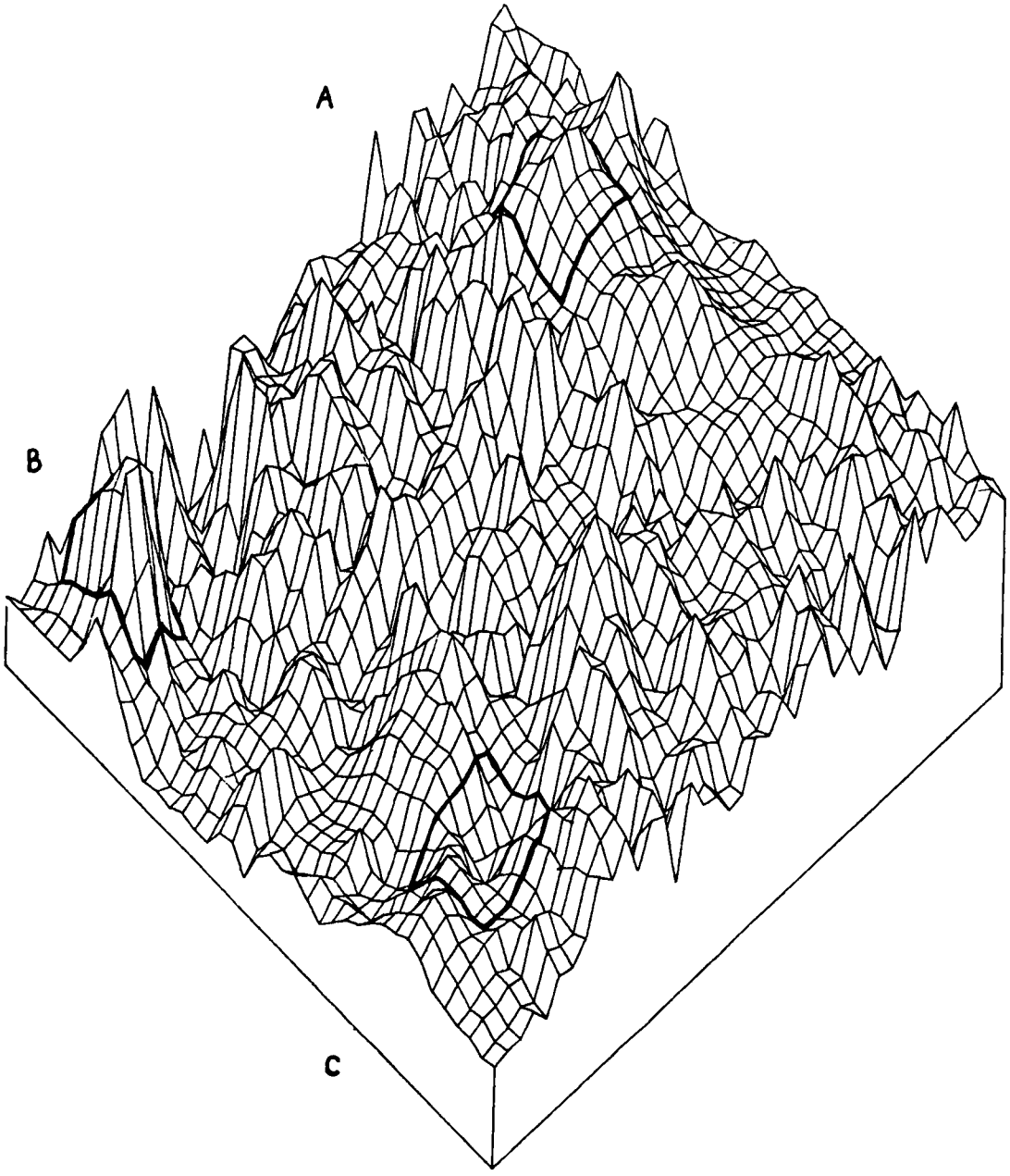


Figure 13

Plot of Mountain Data, Sixth Degree Least-Squares (Area)

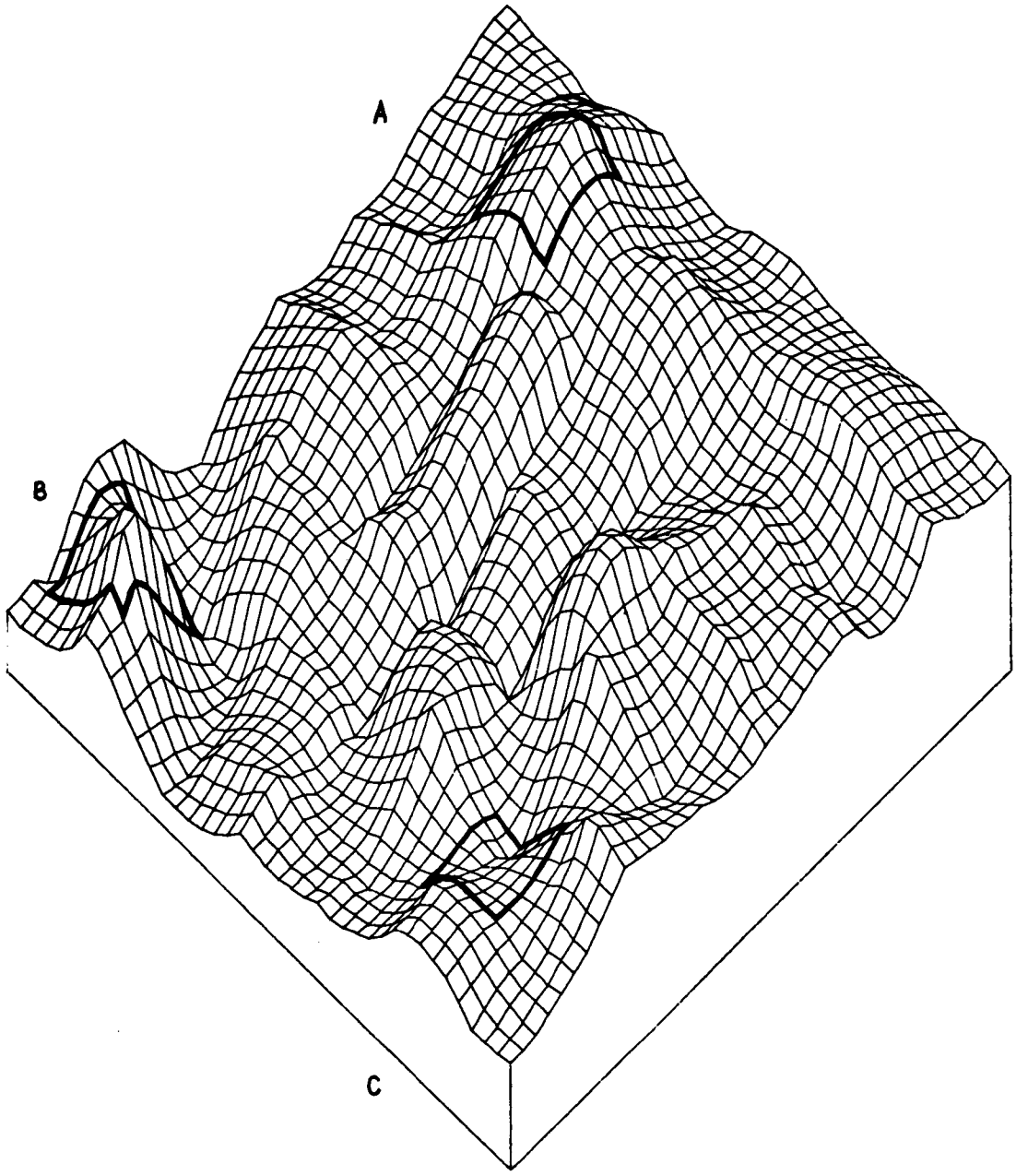


Figure 14

Plot of Mountain Data, Rempel Method One (Area)

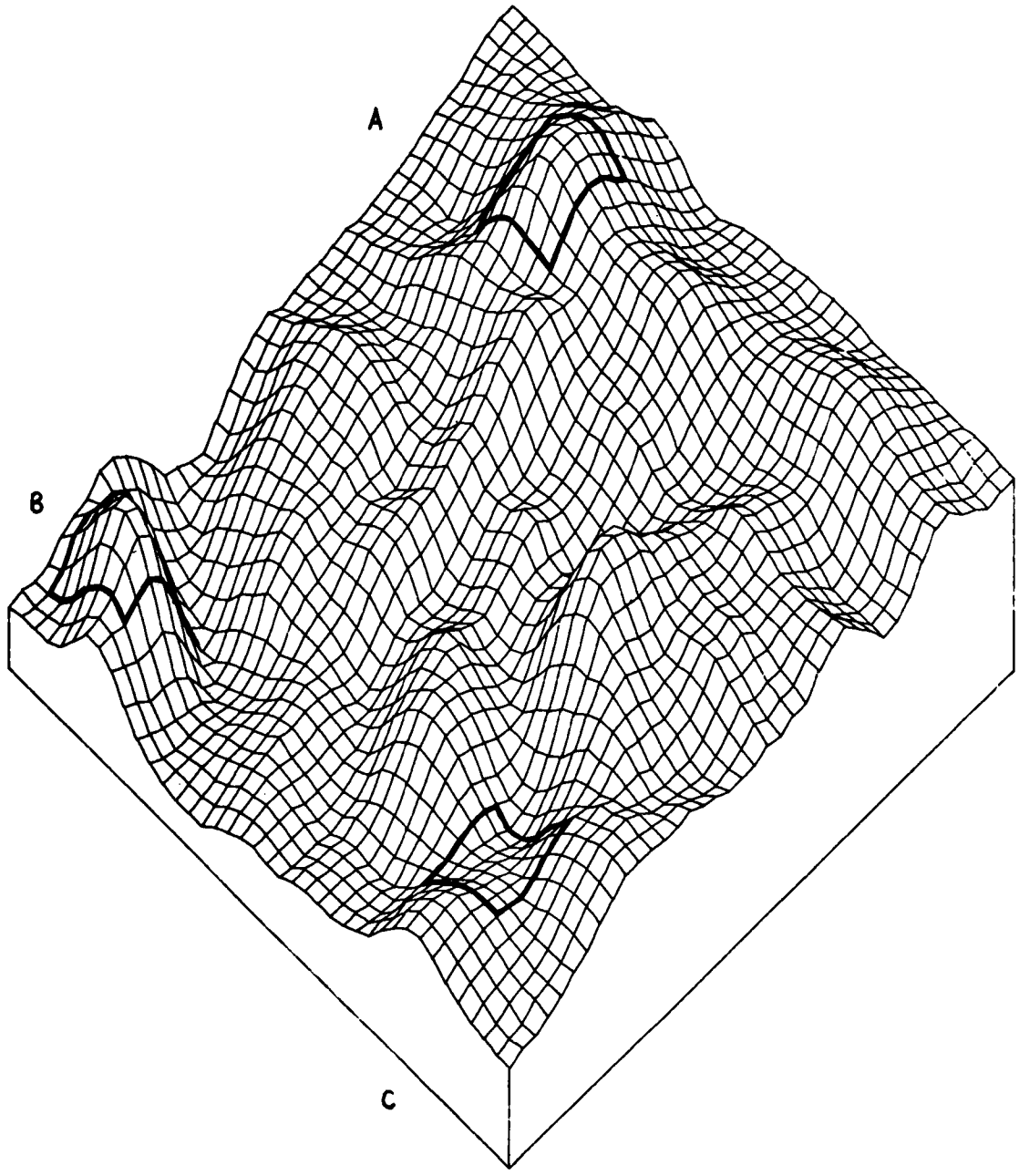


Figure 15

Plot of Mountain Data, Rempel Method One (Point-by-Point)

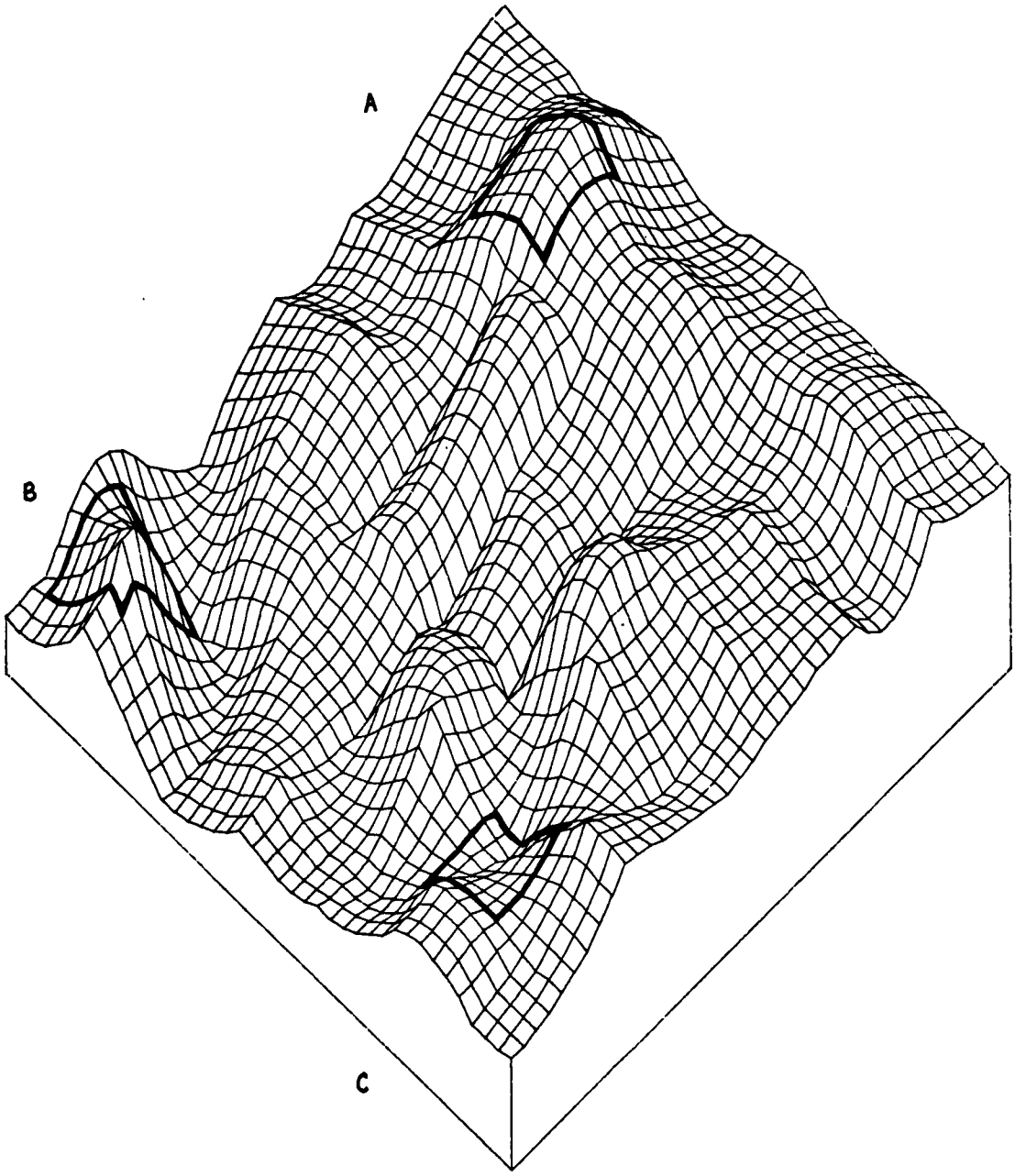


Figure 16

Plot of Mountain Data, Rempel Method Two (Area)
First Pass: Second Degree Least-Squares

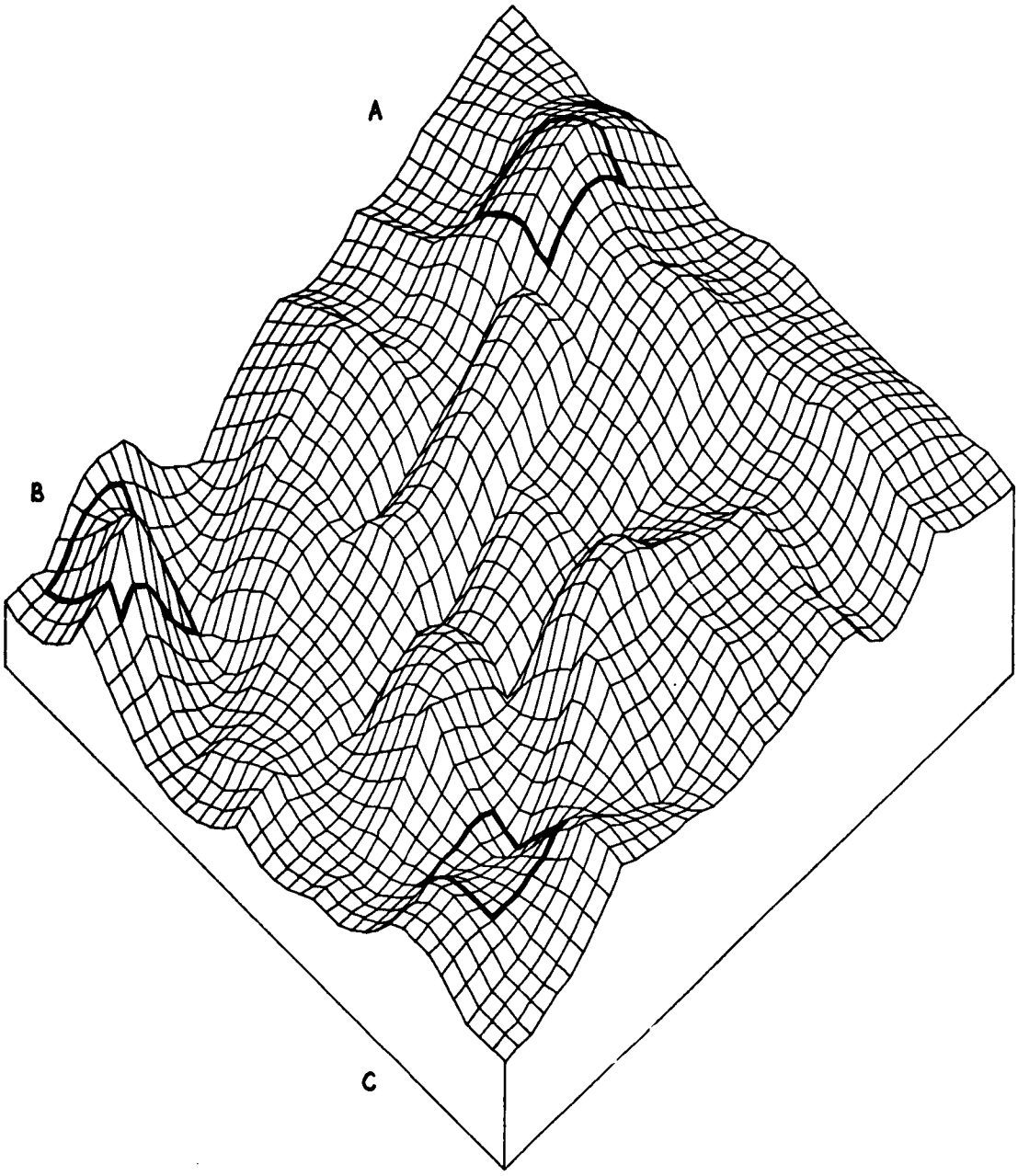


Figure 17

Plot of Mountain Data, Rempel Method Two (Area)
First Pass: Third Degree Least-Squares

II. APPLICATION OF METHODS TO A KNOWN IMAGE WITH NOISE

For testing the ability of each method to remove noise from an image without altering the features of the image, a set of noise data was generated and added to a known image. This noisy data was then processed by each method and the results were compared to the known image using the chi-square test. The chi-square test was chosen for giving a numerical evaluation of the effectiveness of the processing while at the same time providing a method for comparison with the results of James.[13] It would be expected that the method which most closely reproduced the original image would offer the greatest level of noise removal.

The results of the chi-square tests are given in Table 1. A graphic display of the data in Table 1 is presented in Figure 18.

From Figure 18 it is obvious that the removal of the noise is at maximum for second degree methods. Linear methods reduce the overall features of the data while methods of degree greater than two begin to follow the noise instead of the image data. The second degree methods offered the largest factor of improvement over the unprocessed test image. The fourth degree case was approximately half as effective at noise removal as the second degree; the sixth degree actually overcompensated for the noise and came close to reproducing the noise image. It would be expected that fits of degree greater than six would tend to fit the noise even closer.

TABLE 1

CHI-SQUARE TEST VALUES

Method	Chi-square value	Chi-square value per element	Improvement factor*
Unprocessed (raw data)	1321	.826	0
Linear least-squares (area)	353	.221	3.74
Second degree least-squares (area)	198	.124	6.66
Second degree least-squares (point-by-point)	196	.123	6.72
Third degree least-squares (area)	273	.171	4.83
Fourth degree least-squares (area)	398	.249	3.32
Fifth degree least-squares (area)	460	.287	2.88
Sixth degree least-squares (area)	1766	1.103	0.75
Rempel method one (area)	199	.124	6.66
Rempel method one (point-by-point)	197	.123	6.72
Rempel method two (area), first pass: second degree least-squares	195	.122	6.77
Rempel method two (area), first pass: third degree least-squares	196	.122	6.77

*Degree of improvement over the unprocessed data.

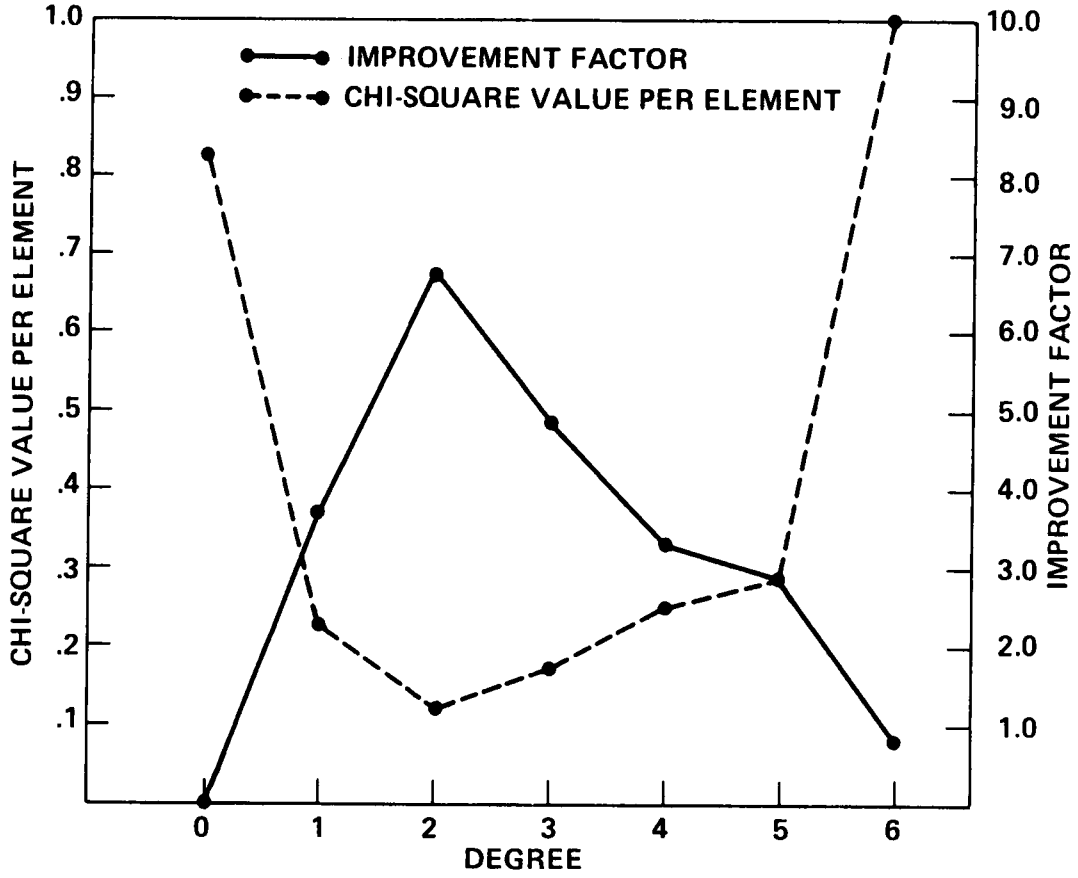


Figure 18

Chi-square Comparison of Results

In testing a second degree least-squares fit, James[13] achieved a reduction of the average chi-square value per element from 3.97 to 0.60, a reduction by a factor of approximately 6.62. The same algorithm implemented in this study reduced the average chi-square value per element from .826 to .123, a reduction by a factor of 6.72. This closeness in reduction factors for the same method and same type of noise supports the effectiveness of the routines. All of the second degree methods produced satisfactory results using the chi-square value per element as a test for goodness of noise removal. The third degree least-square fit lay on a borderline between good noise removal and following the noise too closely.

III. APPLICATION OF METHODS TO POISSON DATA

The image of Poisson noise with a mean of 3 was processed by each routine and the result was evaluated for "flatness." This was done by visual inspection of the data after the smoothing and by calculating the mean of the smoothed image. While this does not guarantee that the best fit possible has been made, it does provide a method of evaluating the effectiveness of a method on this type of image. The means of the smoothed noise images are shown in Table 2 and are plotted in Figure 19. The data in Figure 19 represent the results for the least-square area methods only.

The mean values presented do not indicate a strong preference of one method over another. It is interesting to note that the second degree least-square area method most closely approaches the mean value of three.

TABLE 2

POISSON IMAGE MEAN VALUES

Method	Mean
Unprocessed (raw data)	3.0000
Linear least-squares (area)	2.9810
Second degree least-squares (area)	2.9967
Second degree least-squares (point-by-point)	3.0075
Third degree least-squares (area)	2.9874
Fourth degree least-squares (area)	3.0106
Fifth degree least-squares (area)	2.9893
Sixth degree least-squares (area)	3.0244
Rempel method one (area)	2.9981
Rempel method one (point-by-point)	2.9956
Rempel method two (area), first pass: second degree least-squares	3.0081
Rempel method two (area), first pass: third degree least-squares	2.9794

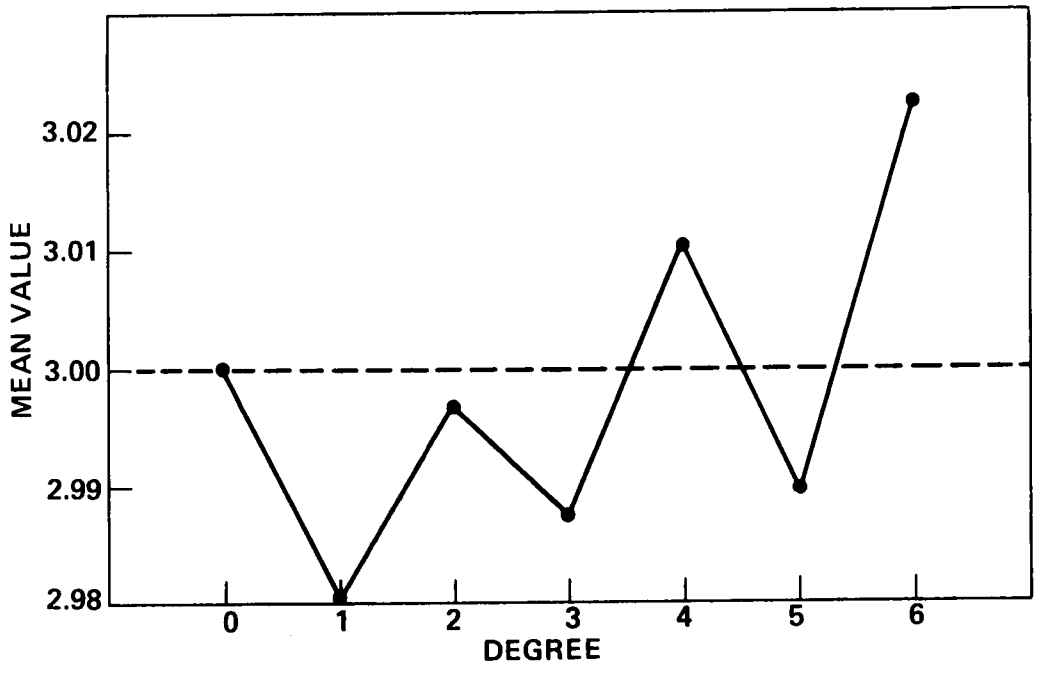


Figure 19

Comparison of Mean Values

IV. SELECTION OF METHODS FOR SCAN PROCESSING

Upon examination of the selected areas in Figures 7-17, pp. 27-37, the corresponding data in Appendix II, the results of the chi-square test and computer processing time, a qualitative evaluation of the methods is made. Of the eleven methods developed in this study, five were selected for implementation on the PDP/8e small computer. These were:

1. A second degree least-squares;
2. A third degree least-squares;
3. The Rempel method one;
4. The Rempel method two, with the first pass smoothed using a second degree least-squares;
5. The Rempel method two, with the first pass smoothed using a third degree least-squares.

The linear least-square fit was not selected due to the oversmoothing of the data shown in Figure 7, page 27, Computer processing time was the only reason that the second degree least-square, point-by-point and the Rempel method one, point-by-point methods were not selected. These two routines offer the most promise for good results once computation time (estimated at 35 and 96 times slower, respectively) can be reduced. The remaining methods were not selected due to computer memory restrictions and poor noise removal as indicated in Figure 18. General smoothing characteristics as shown in Figures 11, 12, and 13, pp. 31-33, are also not promising. One additional factor, that of computer processing time, becomes more important as the degree of polynomial is increased. The three remaining routines which did not meet the necessary criteria for selection were:

1. A fourth degree least-squares;
2. A fifth degree least-squares;
3. A sixth degree least-squares.

V. APPLICATION OF METHODS TO BODY SCAN DATA

In the absence of quantitative method for comparison of the methods when applied to the scan data, one is left with a subjective evaluation of the results. All of the results are shown in photographs made from a cathode-ray tube upon which the data were displayed. Each image is presented in three levels to delineate the smoothing process. The three levels, which comprise the A, B, and C portion of each figure for a given method, are defined as follows:

1. Level one (A), data are displayed at normal levels;
2. Level two (B), data are multiplied by a factor of two before display;
3. Level three (C), data are multiplied by a factor of four before display.

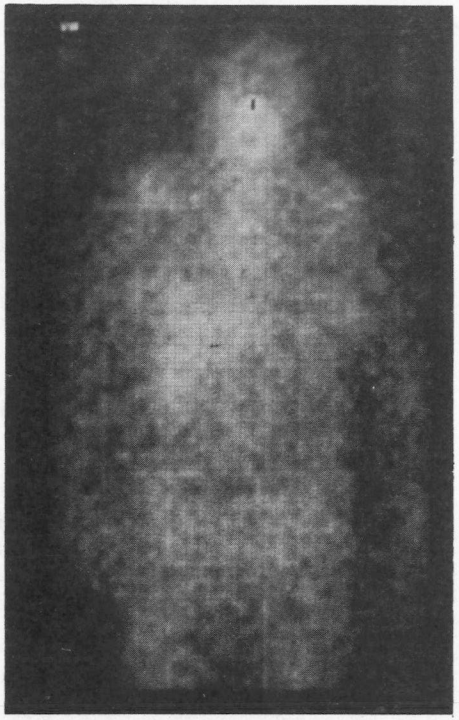
A function of the display routines of ORIS-74 outputs the data to a given threshold and then resets the intensity of the display to zero. For example, if there were 32 intensity levels available on the display screen, the output level is defined as the remainder after dividing the data element by 32.

Thus, at level one, a grey level of 18 would be displayed at a brightness level of 18; at level two, it would be shown at a brightness level of 4; and at level three, it would be shown at a brightness level of 22. This display feature in effect delineates areas of high or low intensity by varying the brightness at the point where the data crosses the threshold of the display. However, it is realized that in rapid

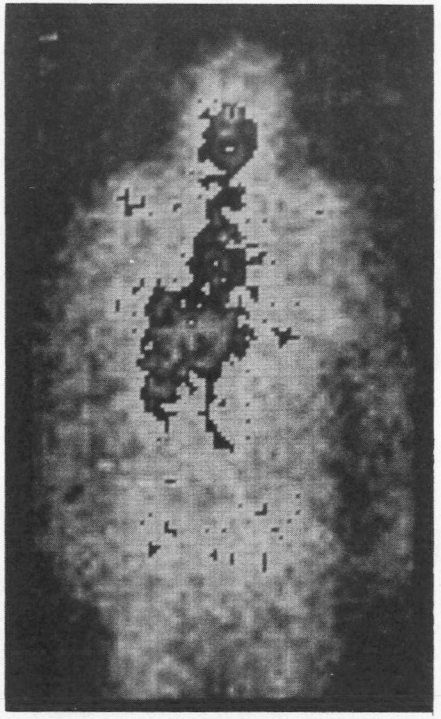
changes of the data, which are infrequent in radioisotope imaging, it may be difficult to determine from the picture alone of the data are increasing or decreasing in intensity.

The unprocessed body scan is shown in Figure 20 and the results as smoothed by the five selected algorithms are shown in Figures 21-25. Of the results shown in Figures 21-25, the Rempel method one in Figure 23 does not offer satisfactory smoothing characteristics. The results shown in Figure 23 are what prompted the development of the Rempel method two in which the order of smoothing was altered. The other methods offer varying degrees of improvement from the raw image in Figure 20. Several general features of the image data are selected and presented in Table 3. Figure 23, due to the unacceptable results, is omitted from the evaluation.

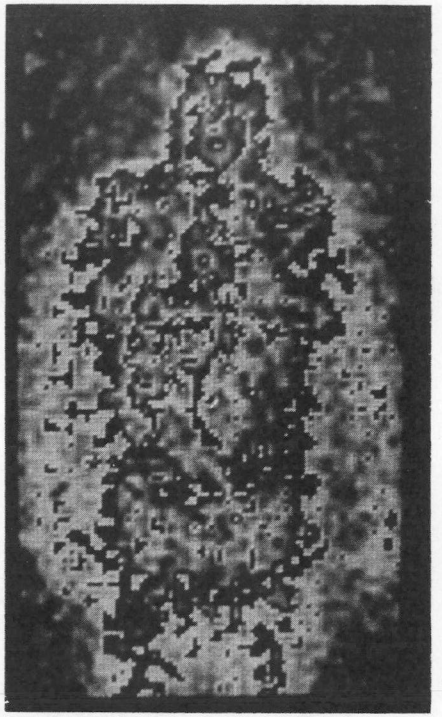
In general, the third degree least-squares (Figure 22) showed peak areas the most vividly while at the same time following the raw data too closely in some areas. The second degree methods (Figures 21 and 24) removed more noise, but also suffered in areas of higher counts in that such areas were not prominently shown. The Rempel method two with the first smoothing pass a third degree least-squares (Figure 25) offered the best compromise between noise level and feature retention.



a. Level One



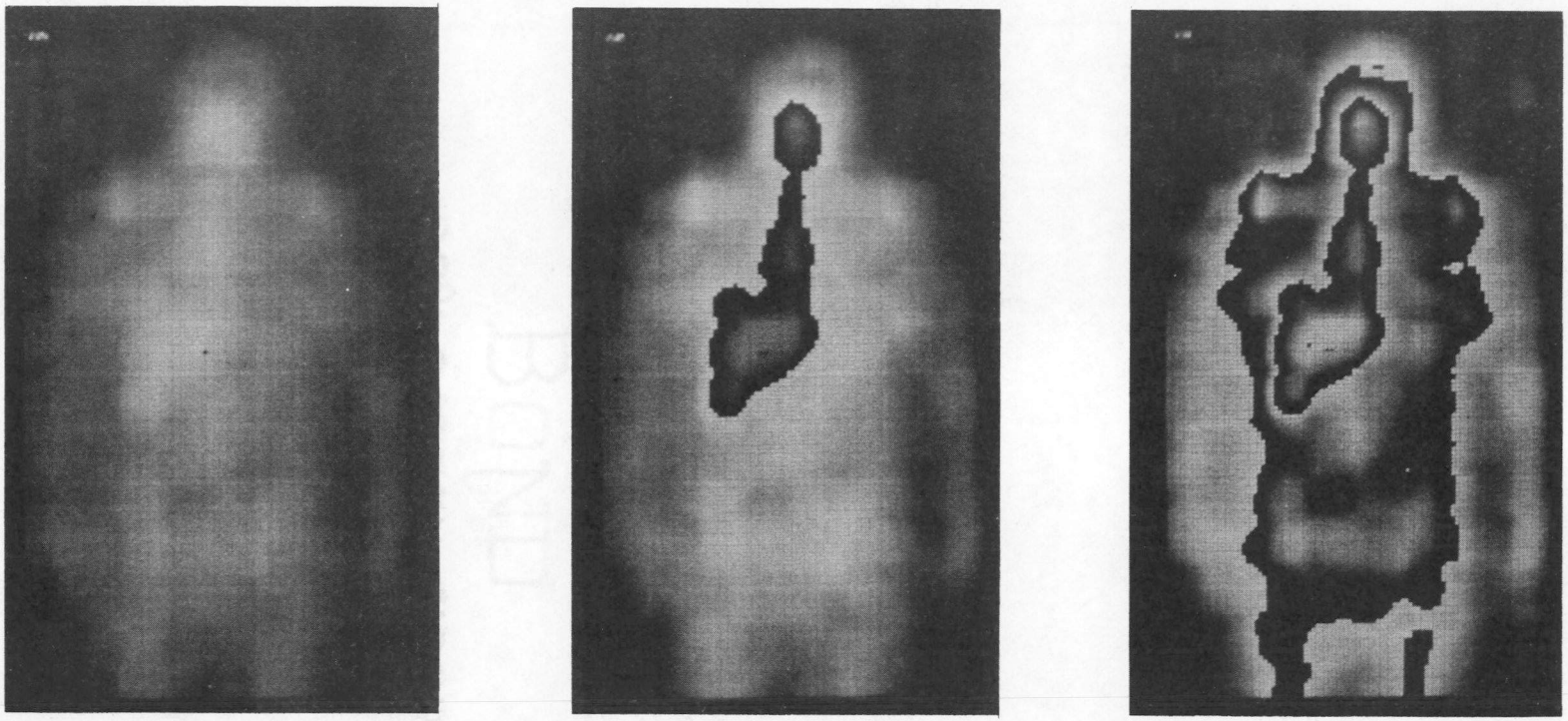
b. Level Two



c. Level Three

Figure 20

Body Image, Unprocessed



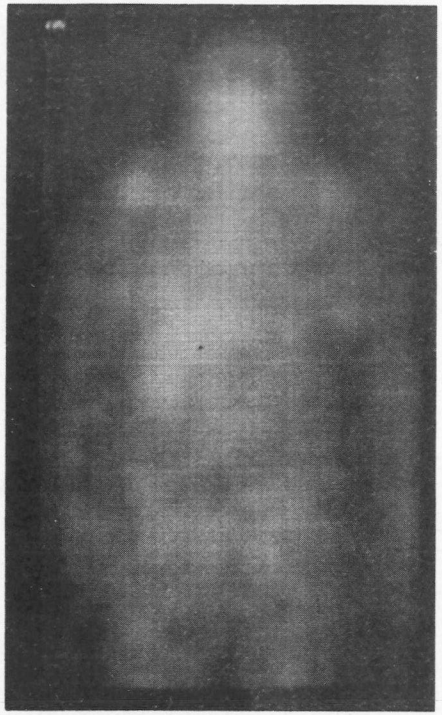
a. Level One

b. Level Two

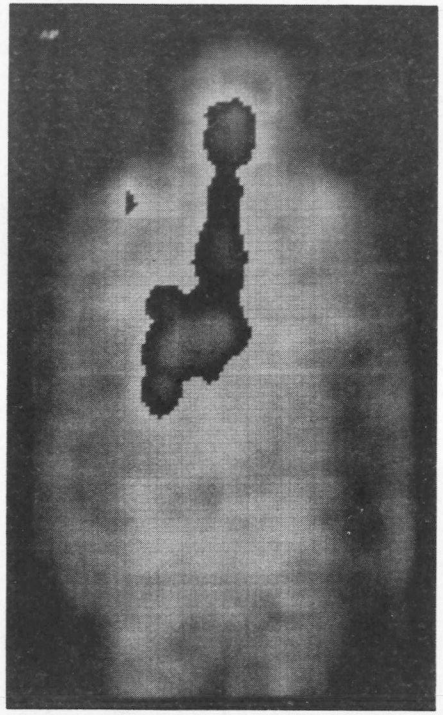
c. Level Three

Figure 21

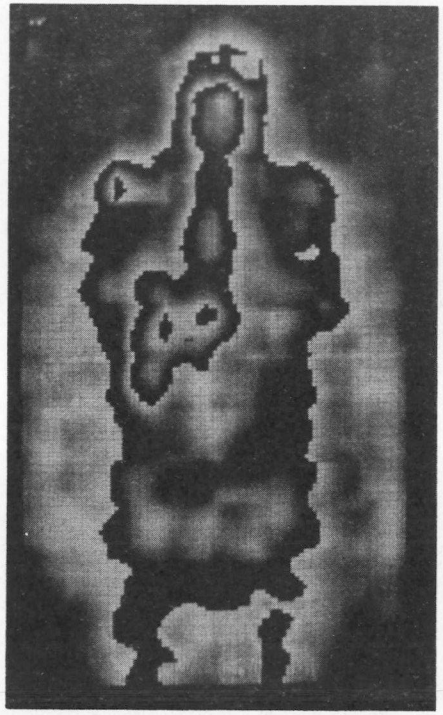
Body Image, Second Degree Least-Squares (Area)



a. Level One



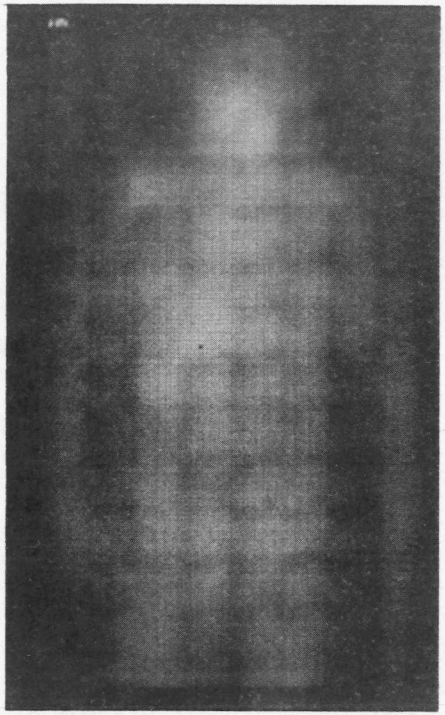
b. Level Two



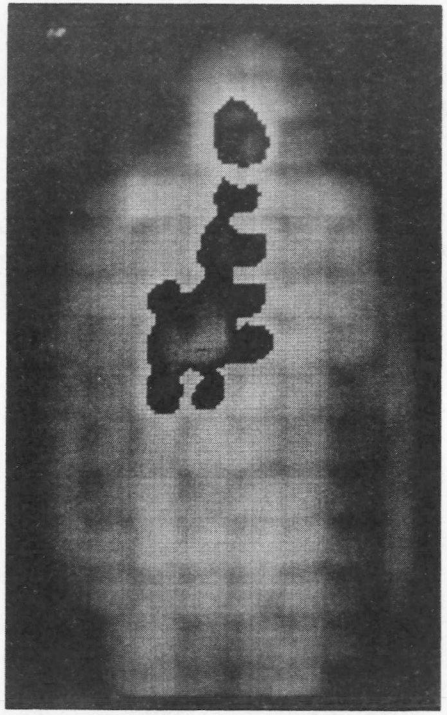
c. Level Three

Figure 22

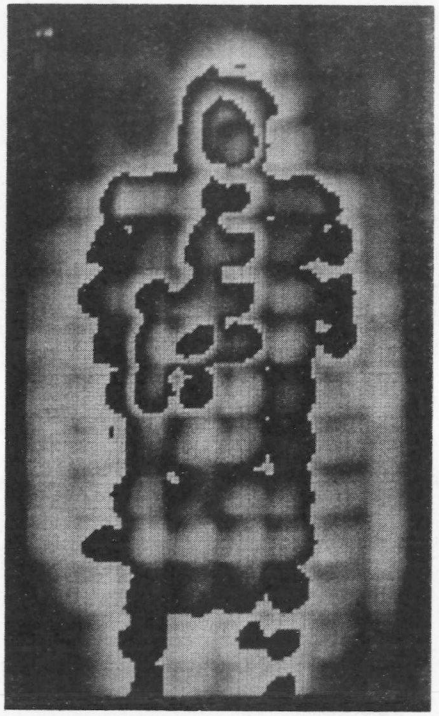
Body Image, Third Degree Least-Squares (Area)



a. Level One



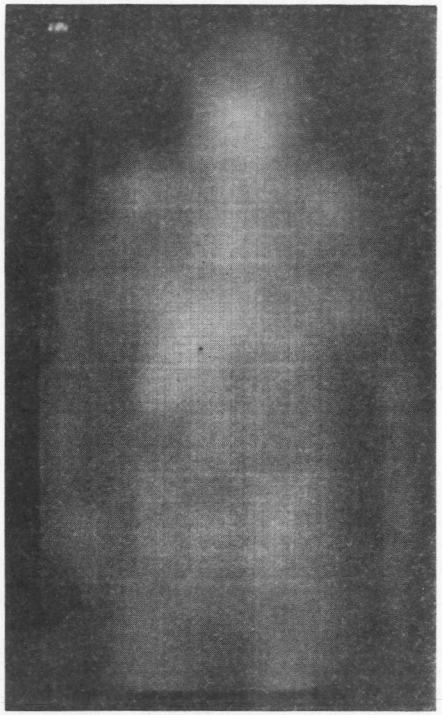
b. Level Two



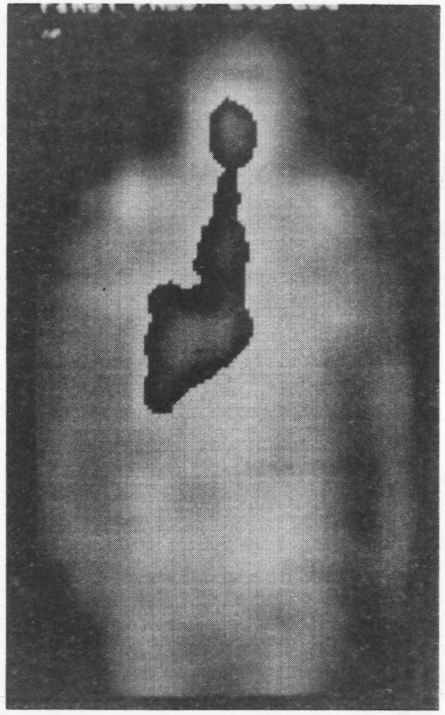
c. Level Three

Figure 23

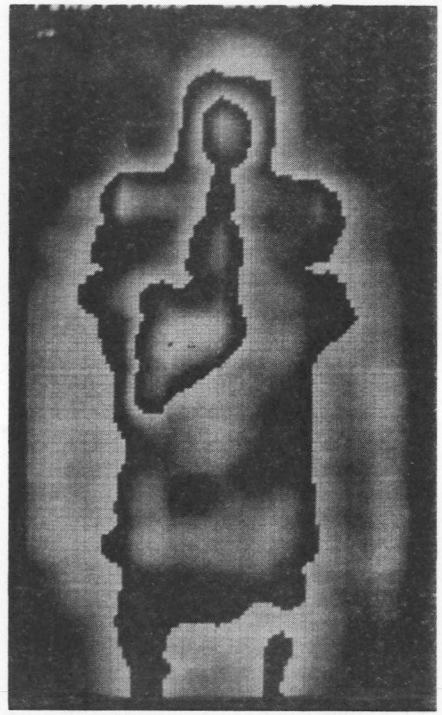
Body Image, Rempel Method One (Area)



a. Level One



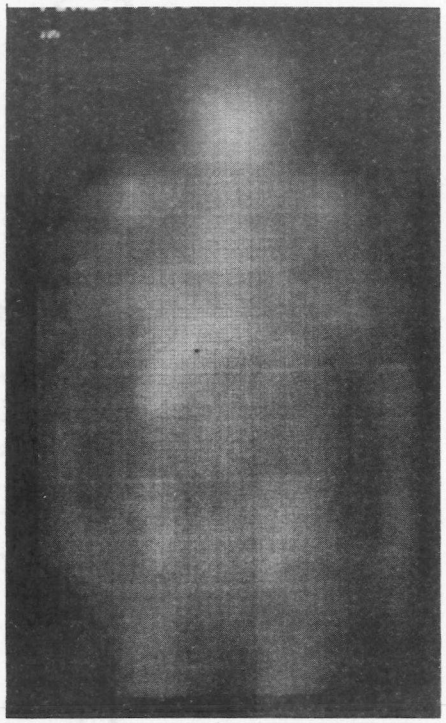
b. Level Two



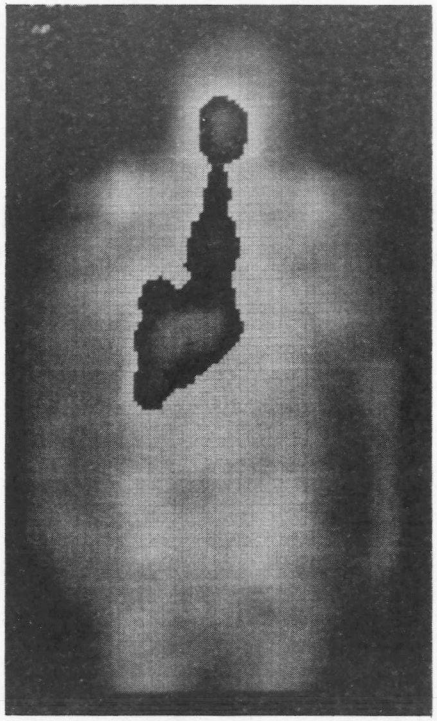
c. Level Three

Figure 24

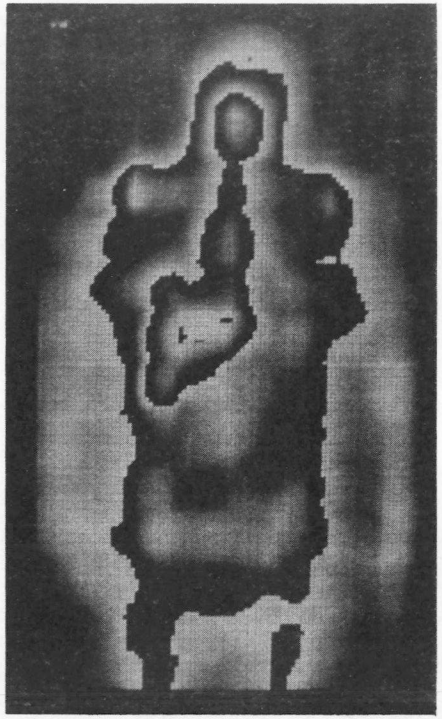
Body Image, Rempel Method Two (Area) First Pass:
Second Degree Least-Squares



a. Level One



b. Level Two



c. Level Three

Figure 25

Body Image, Rempel Method Two (Area) First Pass:
Third Degree Least-Squares

TABLE 3

EVALUATION OF BODY SCAN PROCESSING

Feature	Comments on Processing
General body outline	Best shown in Figures 21 and 24; edge followed the closest in Figure 22. Good compromise between Figures 21, 22, and 24 shown in Figure 25.
High peak area in right shoulder of image	High uptake area visible in all images, shown in Figures 20 and 22 the most prominently, more localized in Figure 22.
High peak areas in liver	Shown most prominently in Figures 22 and 25; most localized in Figure 25. Barely locatible in Figures 21 and 24.
Detail in lower abdomen	Approximately same detail shown in Figures 21, 24, and 25. Figure 22 without features likely to be that of the large intestine.
High counts in facial area	Shown in similar manner in all figures, perhaps most intense in Figure 22.
Outline of internal area in figures displayed at level two	Most closely followed in Figure 22. Figures 21, 24, and 25 very similar.

CHAPTER V

ANALYSIS OF RESULTS

One persistent problem in the evaluation of the smoothed test images, mountain data, and body scans is that of a specific method for determining the relative effectiveness of the routines developed in this study. The rating of the methods is further complicated by the absence of the area methods being previously applied to radioisotope imaging. For this reason one must make a subjective evaluation of the results of this study, bearing in mind that the value of an algorithm depends on the type of data which are of interest. Also, there are times when the smoothest image is not necessarily the best result, depending again on features of the data which are of interest. Each method is presented with an evaluation of its possible use in radioisotope imaging.

I. COMPARISON OF PROCESSING TIMES

One quantitative measure of the performance of the methods is processing time. A time savings is realized by developing the methods to replace the 7 x 7 domains with a one-row, one-column overlap instead of replacing only the center point of each domain. For an image of m rows and n columns, the smoothing operation would be performed (i + 1) x (j + 1) times (where i, j are the integer results of $\frac{m}{7}$ and $\frac{n}{7}$ respectively) for replacing the entire area instead of m x n times when replacing only the center point. Thus, for a 49 x 49 image, the smoothing

operation would be performed 2401 times using a point-by-point processor when an area method would require only 64 smoothing calculations.

A measure of the time required by each routine to process a 49 x 49 image was made for comparison purposes. The times are shown in Table 4.

TABLE 4

PROCESSING TIMES ON PDP/8e

Method	Time Required to Process a 49 x 49 Image
Second degree least-squares (area)	8 minutes
Third degree least-squares	14 minutes
Rempel Method 1	24 minutes
Rempel Method 2, first pass: second degree least-squares	20 minutes*
Rempel Method 2, first pass: third degree least-squares	20 minutes*

*Does not include time required for the first smoothing pass. Extra I/O time is also a factor in the time shown. The additional time would be significantly reduced by using Assembly Language.

II. GENERAL COMPARISON OF PROCESSING METHODS

Depending on the data features of interest, certain methods were more effective in the processing of the image data. An overall comparison of the effectiveness of each method applied to the scan data is given in Table 5.

TABLE 5

GENERAL CONCLUSION OF EFFECTIVENESS

Method	Conclusion
Second degree least-squares (area)	Best for processing low background areas, not effective for high background areas, shows low peaks well, fastest method developed.
Third degree least-squares (area)	Good for processing high background areas, perhaps too good of a fit to low background areas. Best for showing high peaks, shows low peaks well.
Rempel Method 1 (area)	A poor fit of the data, unacceptable due to the bias of the data at the overlapped portions of the 7 x 7 areas.
Rempel Method 2 (area) first pass: Second degree least-squares	Slight advantage over the second degree least-square fit to an area in the valley portions and in high count areas. Perhaps not enough of an advantage to warrant the additional time required for processing.
Rempel Method 2 (area) first pass: Third degree least-squares	Effective for processing high peak and background areas, worth the extra processing using the third degree least-squares fit to an area. A good compromise between the second and third degree least-square methods.

III. CONCLUSION

The Rempel Method 2 using a third degree least-square fit and the direct third degree least-square fit offer the best improvement for high background areas and for feature detection. For handling sharp peak and low background areas, all of the methods except for the Rempel Method 1 are adequate. The best all-purpose methods of those tested would be the second and third degree least-square fits to an area and the Rempel Method 2 using a third degree least-square fit in the first pass. The evaluations of the above routines have been made without consideration to processing time on the computer system. Also, the rating of the algorithms has been made based not necessarily on the inability to properly consider a data feature but rather on the ability of another routine to process the feature in a better manner.

The processing of the data using the replacement of the entire 7 x 7 area instead of replacing the center point offers a good level of smoothing. Effective noise removal of the data is obtained as well as a substantial savings in computer run time.

During the development, implementation, and analysis of the methods, a number of areas were observed where further work could be directed:

1. Using domains other than 7 x 7, such as a 5 x 5 which has been implemented in ORIS-74 or domains in line with other image processing applications;

2. Applying the algorithms to specific areas of the body, i.e., brain scans;
3. Testing the ability of the algorithms to isolate a known "hot spot" within a specific area;
4. Implementing the single point replacement algorithms by using a more efficient programming language and by testing a smaller set of data;
5. Developing an overlapping scheme for the smoothing domains which uses a method other than a simple arithmetic average; which may make the Rempel Method 1, area replacement more feasible;
6. Enhancement using an adjustable weighting array changeable by the using technician or medical doctor.

Two unexpected results of this study were: (1) the location of the high points (peaking or cusping) of the data as compared to the unprocessed data; and (2) the evaluation of the isometric plots proved to be a good indicator of how effectively the routines would process the scan data.

As a result of this investigation, it has been shown that the applications presented herein offer acceptable results that can be of assistance in processing radioisotope scan images. It is also shown that the area replacement method is effective as a means of processing the scan data.

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APPENDICES

APPENDIX A

PROGRAM LISTINGS

I. LEAST-SQUARES SOLUTION, GENERAL CASE

```
      COMMON A,B,IN,OUT,IO
      DIMENSION A(28,28),B(28,28),C(28,28)
      DIMENSION  IN(7,90),OUT(7,90),IO(7,90),IPARM(3)
      DIMENSION ISUB (28,2)
      READ (1,1010) IV, IDEG
 1010 FORMAT (2I1)
      WRITE (6,6030) IDEG
 6030 FORMAT (' LEAST-SQUARE FIT OF DEGREE ',I1)
      READ (1,1020) IPARM
 1020 FORMAT (3I4)
      WRITE(6,6000) IV,IPARM
 6000 FORMAT(' MATRIX SIZE ',I1,' IMAGE # ',I5,' COLUMNS/ROWS ',2I5)
      IRTOT=0
      LENGTH=IPARM(2)
      IH=LENGTH
      IHITE=IPARM(3)*(-1)
      NM=IH/(IV-1)
      IH=(NM*(IV-1)) + 1
      IM=IV-1
      IF(IH-LENGTH) 200,210,210
  200 IH=IH+IM
      NM=NM+1
  210 IHIGH=IV
      LOW=1
      NV=NM
      NN=0
      ID=0
      LOOPCT=IHITE/(IV-1)
      IROWS=(LOOPCT*(IV-1))+1
      IF(IROWS-IHITE) 220,240,240
  220 IROWS=IROWS+IM
      LOOPCT=LOOPCT+1
  240 IE=IH-IM
      WRITE(6,6010) NM,IH,LOOPCT,IM,IE,IV
 6010 FORMAT (6I4)
      II=0
      MDEG=IDEG+1
      DO 300 IX=1,MDEG
      I=IX-1
      DO 300 JX=1,MDEG
      J=JX-1
      IF ((I+J).GT.IDEG) GO TO 300
      II=II+1
      ISUB(II,1)=J
      ISUB(II,2)=I
```

```
 300 CONTINUE
     WRITE (6,6070) II
6070 FORMAT (' NUMBER OF COEFFICIENTS ',I2)
     NN=II
     ID=II
     DO 380 I=1,NN
     WRITE (6,6066) ISUB (I,1), ISUB (I,2)
6066 FORMAT (' ',I2,' ',I2)
 380 CONTINUE
     DO 370 I=1,28
     DO 370 J=1,28
     C(I,J)=0.0
 370 CONTINUE
     DO 320 II=1,NN
     DO 320 JJ=1,NN
     I=ISUB(II,1)+ISUB(JJ,1)
     J=ISUB(II,2)+ISUB(JJ,2)
     Y=0.0
     SUM=0.0
     DO 330 I1=1,IV
     Y=Y+1.0
     X=0.0
     DO 330 J1=1,IV
     X=X+1.0
     SUM=SUM+(X**J)*(Y**I)
 330 CONTINUE
     C(II,JJ)=SUM
 320 CONTINUE
     WRITE (6,6060) C
6060 FORMAT (' ',10F13.0)
     DO 100 J=1,IH
     OUT (1,J)=0.0
 100 CONTINUE
     DO 900 ILOOP=1,LOOPCT
     CALL INCALC (IV,IH,LENGTH,IM,IHIGH,LOW,IROWS,IHITE,ILOOP,
    C    LOOPCT,IN,OUT)
 528 DO 130 I=1,28
     DO 130 J=1,28
     A(I,J)=C(I,J)
     B(I,J)=0.0
 130 CONTINUE
     DO 270 I=LOW,IV
     WRITE (6,600) (IN(I,J),J=1,LENGTH)
 600 FORMAT(' ',40I3)
 270 CONTINUE
     CALL BCALC (NM,IM,IV,IN,B,IDEG,MDEG)
     CALL DMATEQ (A,B,NN,NV,ID)
     CALL CALCB (NM,IM,IV,OUT,B,IDEG,MDEG)
     CALL OUTCAL (IM,IH,LENGTH,IV,OUT,IO,ILOOP,IE,IRTOT,LOOPCT)
 900 CONTINUE
     WRITE (6,6020) IRTOT
6020 FORMAT (' ROWS WRITTEN ',I4)
     CALL EXIT
     END
```

II. MATRIX EQUATION SOLVER

```
      FUNCTION DMATEQ(A,B,III,JJJ,ID)
      DIMENSION A(28,28),B(28,28)
      KK=III
      NV=IABS(JJJ)
      D=1.
      IF(JJJ) 200,210,210
  200 D=0.0
  210 KKM=KK-1
      DO 90I=1,KKM
      S=0.0
      DO 10J=I,KK
      R=ABS(A(J,I))
      IF(R-S) 10,220,220
  220 S=R
      L=J
   10 CONTINUE
      IF (L-I) 230,50,230
  230 DO 20 J=I,KK
      S=A(I,J)
      A(I,J)=A(L,J)
      A(L,J)=S
   20 CONTINUE
      IF(NV) 40,40,240
  240 DO 30 J=1,NV
      S=B(I,J)
      B(I,J)=B(L,J)
      B(L,J)=S
   30 CONTINUE
   40 D=-D
   50 IF(A(I,I)) 250,90,250
  250 IPO=I+1
      DO 80J=IPO,KK
      IF(A(J,I)) 260,80,260
  260 S=A(J,I)/A(I,I)
      A(J,I)=0.0
      DO 60 K=IPO,KK
      A(J,K)=A(J,K)-A(I,K)*S
   60 CONTINUE
      IF(NV) 80,80,270
  270 DO 70 K=1,NV
      B(J,K)=B(J,K)-B(I,K)*S
   70 CONTINUE
   80 CONTINUE
   90 CONTINUE
C     DO 100I=1,KK
C     D=D*A(I,I)
C100 CONTINUE
      IF(NV) 130,130,280
  280 KMO = KK-1
      DO 120K=1,NV
      B(KK,K)=B(KK,K)/A(KK,KK)
      DO 120I=1,KMO
      N=KK-I
      DO 110J=N,KMO
      B(N,K)=B(N,K)-A(N,J+1)*B(J+1,K)
  110 CONTINUE
      B(N,K)=B(N,K)/A(N,N)
  120 CONTINUE
  130 DMATEQ=D
      RETURN
      END
```

III. REMPEL METHOD WEIGHTING ROUTINE

```
      SUBROUTINE WEIGHT (NM,IM,IV,A,B,IN,ID,NV,ISUB,NN,IWCNT)
      DIMENSION  W(10,10),HUT(7,7),A(10,10),B(10,20),IN(7,90),V(15),
     C  BB(10),ISUB(6,6)
      NNV=1
      N=1
      V(15)=1.0
      DO 100 K=1,NM
      M=N
      N=M+IM
      DMAX=0.0
      DSUM=0.0
      DO 130 I=1,10
      BB(I)=0.0
  130 CONTINUE
      DO 230 I=1,14
      V(I)=0.0
  230 CONTINUE
      Y1=0.0
      DO 210 I=1,IV
      X1=0.0
      Y1=Y1+1.0
      Y2=Y1*Y1
      DO 210 J=1,IV
      X1=X1+1.0
      X2=X1*X1
      ZH=B(1,K)*X2+B(2,K)*Y2+B(3,K)*X1*Y1+B(4,K)*X1+B(5,K)*Y1+B(6,K)
      HUT(I,J)=ZH
  210 CONTINUE
      DO 140 I=1,IV
      J1=0
      DO 140 J=M,N
      J1=J1+1
      HLD=FLOAT(IN(I,J))-HUT(I,J1)
      HLD=HLD*HLD
      DSUM=DSUM+HLD
      IF(HLD-DMAX) 140,140,150
  150 DMAX=HLD
  140 CONTINUE
      DMAXN=DMAX*IV*IV
      MAXSUM=DMAXN-DSUM
      IF(MAXSUM) 100,100,160
  160 DO 170 I=1,IV
      J1=0
      DO 170 J=M,N
      J1=J1+1
      DEV=FLOAT(IN(I,J))-HUT(I,J1)
      DEV=DEV*DEV
      W(I,J1)=(DMAX-DEV)/MAXSUM
  170 CONTINUE
      Y1=0.0
      DO 180 I=1,IV
      Y1=Y1+1.0
      X=0.0
      DO 180 J=1,IV
      WVAL=W(I,J)
      X=X+1.0
      X1=X*WVAL
      Y2=Y1*Y1
```

```
      Y3=Y2*Y1
      Y4=Y3*Y1
      X2=X*X1
      X3=X*X2
      X4=X*X3
      V(1) =V(1) +X4
      V(2) =V(2) +X2*Y2
      V(3) =V(3) +X3*Y1
      V(4) =V(4) +X3
      V(5) =V(5) +X2*Y1
      V(6) =V(6) +X2
      V(7) =V(7) +Y4*WVAL
      V(8) =V(8) +X1*Y3
      V(9) =V(9) +X1*Y2
      V(10)=V(10)+Y3*WVAL
      V(11)=V(11)+Y2*WVAL
      V(12)=V(12)+X1*Y1
      V(13)=V(13)+X1
      V(14)=V(14)+Y1*WVAL
      HLD=HUT(I,J)
      BB(1) =BB(1) +X2*HLD
      BB(2) =BB(2) +Y2*HLD*WVAL
      BB(3) =BB(3) +X1*Y1*HLD
      BB(4) =BB(4) +X1*HLD
      BB(5) =BB(5) +Y1*HLD*WVAL
      BB(6) =BB(6) +HLD*WVAL
  180 CONTINUE
      DO 190 I=1,6
      DO 190 J=1,6
      II=ISUB(I,J)
      A(I,J)=V(II)
  190 CONTINUE
      IWCNT=IWCNT+1
      CALL CMATEQ(A,BB,NN,NNV,ID)
      DO 200 I=1,ID
      B(I,K)=BB(I)
  200 CONTINUE
  100 CONTINUE
      RETURN
      END
```

IV. DATA INPUT ROUTINE

```
      SUBROUTINE INCALC (IV,IH,LENGTH,IM,IHIGH,LOW,IROWS,IHITE,
     C    ILOOP,LOOPCT,IN,OUT)
      DIMENSION IN(7,90),OUT(7,90)
  110 DO 120 I=2,IV
      DO 120 J=1,IH
      OUT (I,J)=0.0
  120 CONTINUE
      IF(ILOOP-1) 410,128,410
  410 LOW=2
      DO 122 J=1,IH
      IN(1,J)=IN(IV,J)
  122 CONTINUE
      IF(ILOOP-LOOPCT) 128,420,128
  420 IHIGH=IV-(IROWS-IHITE)
  128 DO 129 I=LOW,IHIGH
      READ(7,700) (IN(I,J),J=1,LENGTH)
  700 FORMAT (20I4)
  129 CONTINUE
      IF (ILOOP-1) 800,800,800
  810 IN(1,1)=1
      IN(1,2)=64
      IN(1,3)=-64
  800 IF(IH-LENGTH) 460,460,450
  450 DO 470 I=LOW,IHIGH
      DO 470 J=LENGTH,IH
      IN(I,J)=IN(I,LENGTH)
  470 CONTINUE
  460 IF(ILOOP-LOOPCT) 515,480,515
  480 IF(IROWS-IHITE) 515,515,490
  490 ICNT=IROWS-IHITE
      ICNT=ICNT+IHIGH
      DO 510 I=IHIGH,ICNT
      DO 510 J=1,LENGTH
      IN(I,J)=IN(IHIGH,J)
  510 CONTINUE
  515 CONTINUE
      RETURN
      END
```

V. DATA OUTPUT ROUTINE

```
      SUBROUTINE OUTCAL (IM,IH,LENGTH,IV,OUT,IO,ILOOP,IE,IRTOT,LOOPCT)
      DIMENSION OUT(7,90),IO(7,90)
      IF(ILOOP-1) 430,200,430
  430 DO 190 J=1,IH
      OUT(1,J)=OUT(1,J)/2.0
  190 CONTINUE
  200 DO 210 I=1,IM
      DO 210 J=IV,IE,IM
      OUT(I,J)=OUT(I,J)/2.0
  210 CONTINUE
C
C   PERFORM ROUNDING OF ARRAY OUT
C
      IF(ILOOP-LOOPCT) 610,600,610
  600 DO 620 J=IV,IE,IM
      OUT(IV,J)=OUT(IV,J)/2.0
  620 CONTINUE
      IM=IV
  610 DO 310 I=1,IM
      DO 310 J=1,IH
      OUTHLD=OUT(I,J)
      INOUT=IFIX(OUTHLD)
      CALC=OUTHLD-FLOAT(INOUT)
      IF(CALC-0.5) 300,440,440
  440 OUTHLD=OUTHLD+0.5
      INOUT=IFIX(OUTHLD)
  300 IO(I,J)=INOUT
  310 CONTINUE
      IF(LENGTH-IH) 530,530,520
  520 DO 530 I=1,IM
      DO 530 J=IH,LENGTH
      IO(I,J)=IO(I,IH)
  530 CONTINUE
      DO 320 I=1,IM
      IRTOT=IRTOT+1
      WRITE (8,800) (IO(I,J),J=1,LENGTH)
  800 FORMAT (' ',40I3)
      WRITE (2,2000) (IO(I,KK),KK=1,LENGTH)
 2000 FORMAT (20I4)
  320 CONTINUE
      DO 220 J=1,IH
      OUT(1,J)=OUT(IV,J)
  220 CONTINUE
      RETURN
      END
```

VI. DATA CALCULATE ROUTINES

```
      SUBROUTINE BCALC (NM,IM,IV,IN,B,IDEG,MDEG)
      DIMENSION B(28,28),IN(7,90)
      N=1
      DO 140 K=1,NM
      M=N
      N=N+IM
      DO 140 I=1,IV
      X1=0.0
      Y1=FLOAT(I)
      DO 140 J=M,N
      X1=X1+1.0
      CALC=FLOAT(IN(I,J))
      KK=0
      DO 100 IY=1,MDEG
      II=IY-1
      DO 100 JX=1,MDEG
      JJ=JX-1
      IF((II+JJ).GT.IDEG) GO TO 100
      KK=KK+1
      B(KK,K)=B(KK,K) + CALC*(X1**JJ)*(Y1**II)
  100 CONTINUE
  140 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CALCB (NM,IM,IV,OUT,B,IDEG,MDEG)
      DIMENSION OUT(7,90),B(28,28)
      N=1
      DO 150 K=1,NM
      M=N
      N=N+IM
      DO 150 I=1,IV
      X1=0.0
      Y1=FLOAT(I)
      DO 150 J=M,N
      X1=X1+1.0
      ZH=0.0
      KK=0
      DO 100 IY=1,MDEG
      II=IY-1
      DO 100 JX=1,MDEG
      JJ=JX-1
      IF((II+JJ).GT.IDEG) GO TO 100
      KK=KK+1
      ZH=ZH+B(KK,K)*(X1**JJ)*(Y)**II)
  100 CONTINUE
      OUT(I,J)=OUT(I,J)+ZH
  150 CONTINUE
      RETURN
      END
```

VII. ROUTINE TO GENERATE POISSON VALUES

```
C    CALCULATE RANDOM POISSCN VALUES WITH A MEAN OF 3
C
       DIMENSICN IMAGE (49,49),NOISE(49,49)
       IX=179
       CALL RANDU(IX,IY,YFL)
       CNT3=0.0
       TOT=0.0
       FL=3.0
       PROB3=EXP(-FL)
       PROB3=PROB3*(3.0**3)
       PROB3=PROB3/6.0
C    PROB3 IS THE CALCULATED PROBABILITY A THREE OCCURS
C    CALCULATE THE IMAGE AND CCUNT THE THREES
       DO 100 I=1,49
       DO 100 J=1,49
       CALL IPOIS(II,FL)
       IMAGE(I,J)=II
       TOT=TOT+FLOAT(II)
       IF(II-3) 100,110,100
  110 CNT3=CNT3+1.0
  100 CONTINUE
       WRITE (6,600)
  600 FORMAT(' POISSON IMAGE ')
       APROB=CNT3/2401.0
       AVG=TOT/2401.0
       WRITE(6,610) CNT3, PROB3, APROB
  610 FORMAT (' NUMBER OF THREES ',F6.0,' CALCULATED PROBABILITY OF A
     C     'THREE ',F8.5,' ACTUAL PROBABILITY ',F8.5)
       WRITE (6,660) AVG
  660 FORMAT (' MEAN VALUE ',F8.5)
       WRITE (6,620)
  620 FORMAT (' ')
       WRITE (6,630) IMAGE
  630 FORMAT (' ',49I2)
       WRITE (7,700) IMAGE
  700 FORMAT (20I4)
       CALL EXIT
       END
```

```
       SUBROUTINE IPOIS(I,FL)
       E1=EXP(-FL)
       I=0
       E2=1.0
   10 CALL RANDU(IX,IY,YFL)
       E2=E2*YFL
       IF (E2.LT.E1) GO TO 20
       I=I+1
       GO TO 10
   20 IPOIS=I
       RETURN
       END
```

VIII. ROUTINE TO GENERATE ROOT-LOG-COSINE NOISE

```
C    CALCULATE NOISE USING ROOT-LOG-COSINE METHOD
C    ADD NOISE TO SMOOTHED MOGOLLON MOUNTAIN DATA
C
      DIMENSION IN(40,40),IOUT(40,40),NOISE(40,40),R(40,40),
     -  MNOISE(40,40)
      READ (5,500) IN
  500 FORMAT (20I4)
      IX1=179
      IX2=1003
      DO 100 I=1,40
      DO 100 J=1,40
      Z=FLOAT(IN(I,J))
      SQ=2.0*SQRT(Z)
      IMAX=IFIX(SQ)
      IMIN=IMAX*(-1)
      MNOISE(I,J)=0
      CALL RAN1 (IX1,IY1,YFL1)
      CALL RAN2 (IX2,IY2,YFL2)
      R(I,J)=YFL2
      FCTR1= COS(6.283185*YFL1)
      SQRD=(-2.0*Z)*ALOG(YFL2)
      FCTR2=SQRT(SQRD)
      CALC=FCTR1*FCTR2
      INOISE=IFIX(CALC)
      IF(INOISE-IMAX) 130,110,120
  120 INOISE=IMAX
      MNOISE(I,J)=1
      GO TO 110
  130 IF(INOISE-IMIN) 140,110,110
  140 INOISE=IMIN
      MNOISE(I,J)=1
  110 NOISE(I,J)=INOISE
      IOUT(I,J)=IN(I,J)+NOISE(I,J)
  100 CONTINUE
      WRITE (7,700) IOUT
  700 FORMAT (20I4)
      WRITE (6,610)
  610 FORMAT('1RAW IMAGE')
      WRITE (6,600) IN
  600 FORMAT (' ',40I3)
      WRITE (6,620)
  620 FORMAT('1NOISE IMAGE')
      WRITE (6,600) NOISE
      WRITE (6,630)
  630 FORMAT('1RAW IMAGE + NOISE')
      WRITE (6,600) IOUT
      WRITE (6,640)
  640 FORMAT ('1RANDUM NUMBERS')
      WRITE (6,650) R
  650 FORMAT (' ',40F3.2)
      WRITE(6,660)
  660 FORMAT ('1MAX NOISE ELEMENTS')
      WRITE (6,600) MNOISE
      CALL EXIT
      END
```

IX. ROUTINE TO COMPUTE CHI-SQUARE VALUES

```
C    COMPUTE CHI SQUARED COMPARISON OF MAGOLLAN MOUNTAINS
C    A1=INPUT IMAGE 1(EXPECTED), UNIT 3
C    A2=INPUT IMAGE 2(OBSERVED), UNIT 4
C    SUM=CALCULATED CHI SQUARE VALUE FROM IMAGES
C
      DIMENSION A1(40,40),A2(40,40)
      READ (3,3000) A1
 3000 FORMAT (20F4.0)
      READ (4,4000) A2
 4000 FORMAT (20F4.0)
      SUM=0.0
      ICNT=40
      IDIV=ICNT**2
      DIV=FLOAT(IDIV)
      DO 100 I=1,ICNT
      DO 100 J=1,ICNT
      HSUM=((A1(I,J)-A2(I,J))**2)/A1(I,J)
      SUM=SUM+HSUM
  100 CONTINUE
      AVG=SUM/DIV
      WRITE (6,6010) SUM
 6010 FORMAT (' CALCULATED CHI-SQUARE FROM IMAGES ',F12.4)
      WRITE (6,6020) AVG
 6020 FORMAT (' AVERAGE CHI SQUARE PER ELEMENT ',F12.6)
      CALL EXIT
      END
```

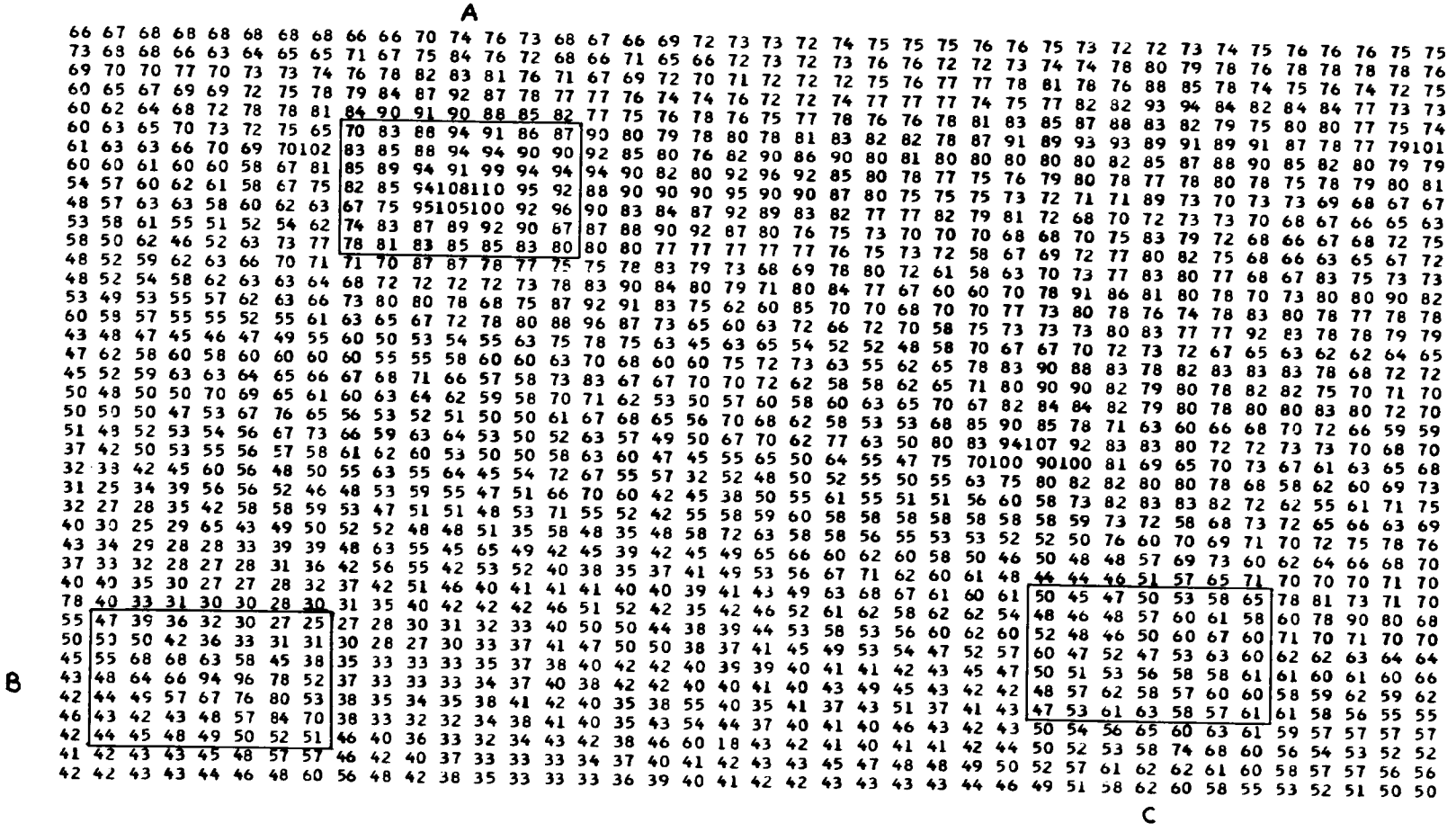
X. ROUTINE TO GENERATE RANDOM NUMBERS

```
      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY) 5,6,6
    5 IY=IY+2147483647+1
    6 YFL=IY
      YFL=YFL*.4656613E-9
      IX=IY
      RETURN
      END
```

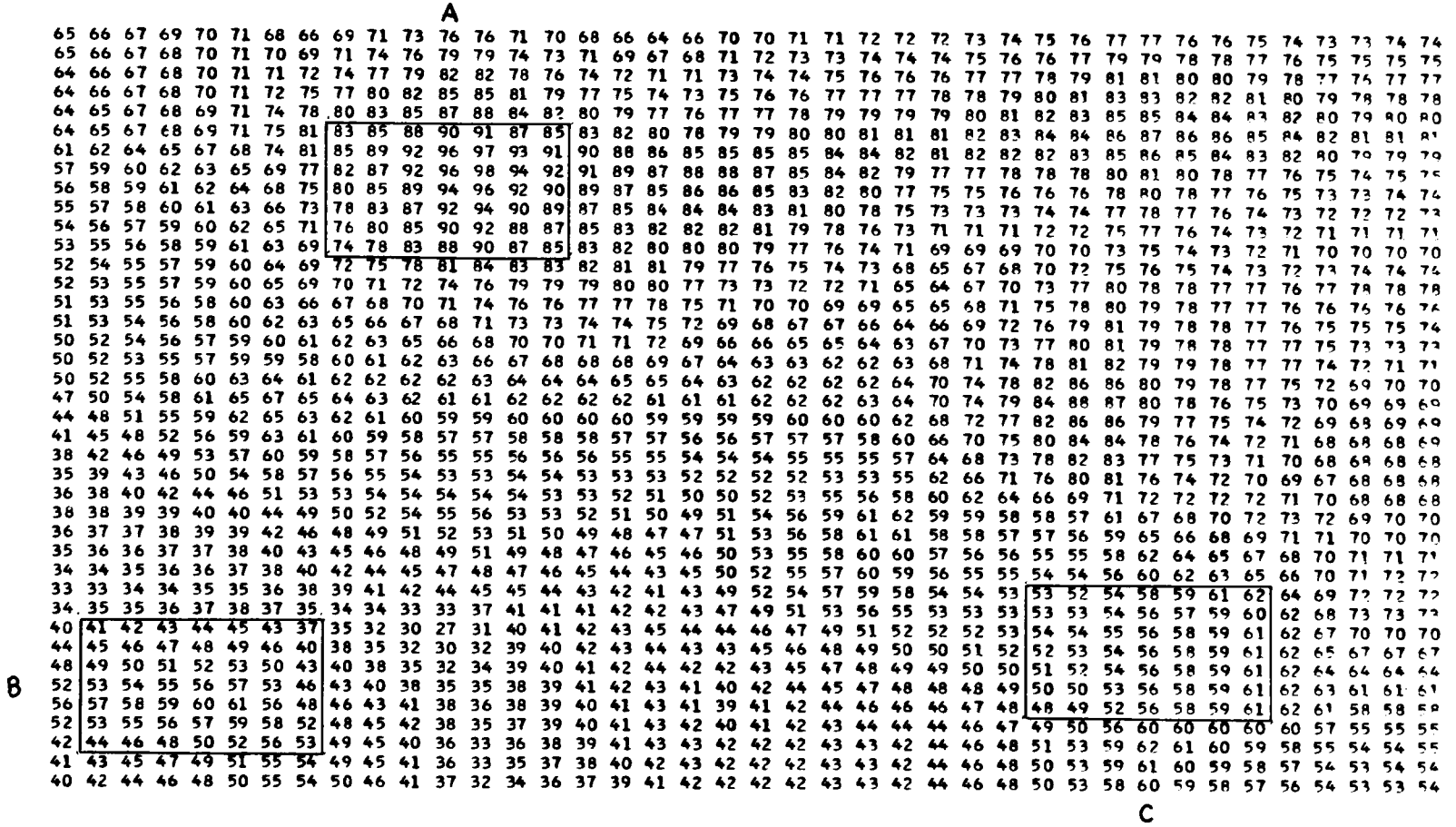
APPENDIX B

MOUNTAIN DATA

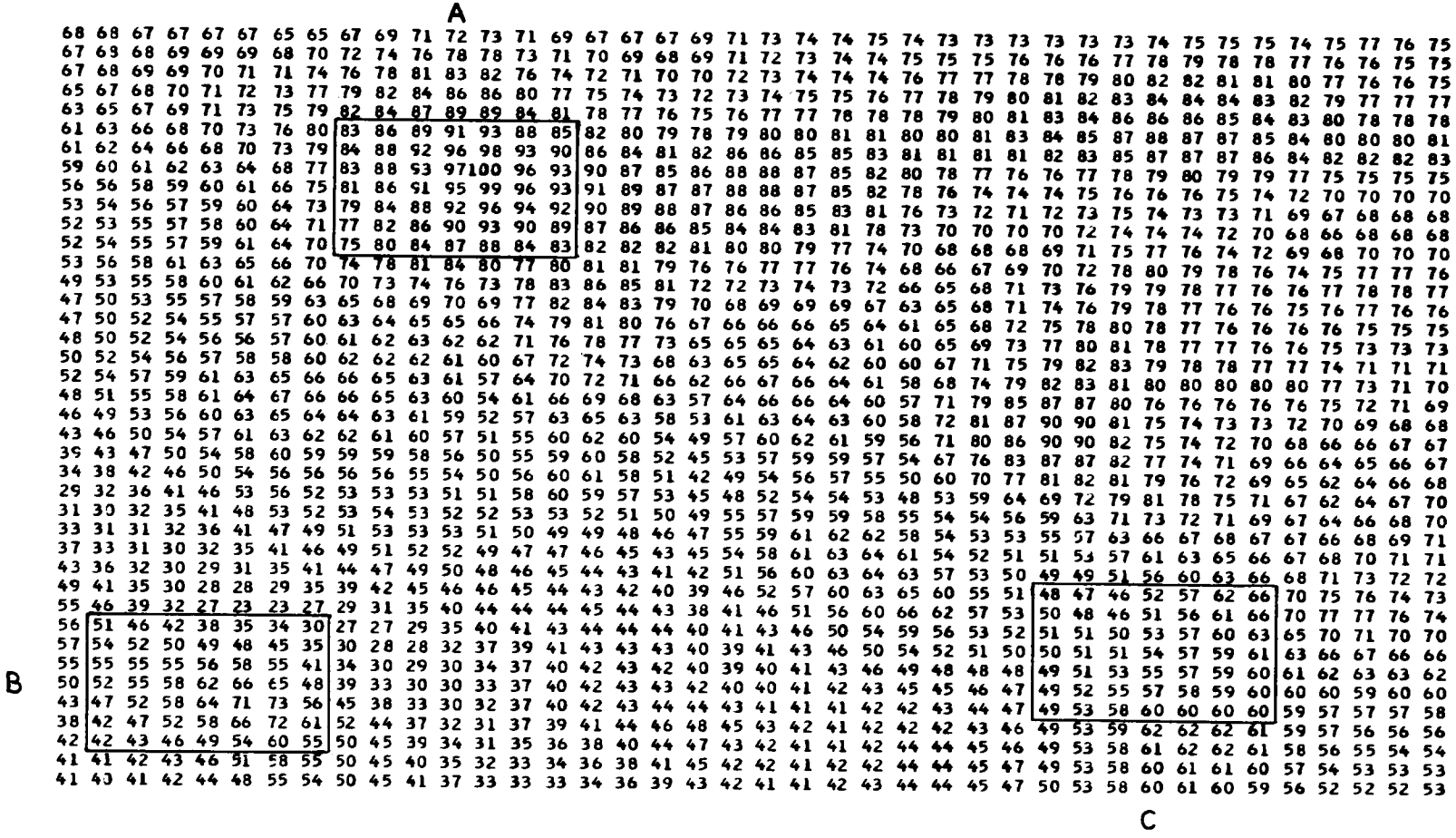
I. MOUNTAIN DATA, UNPROCESSED



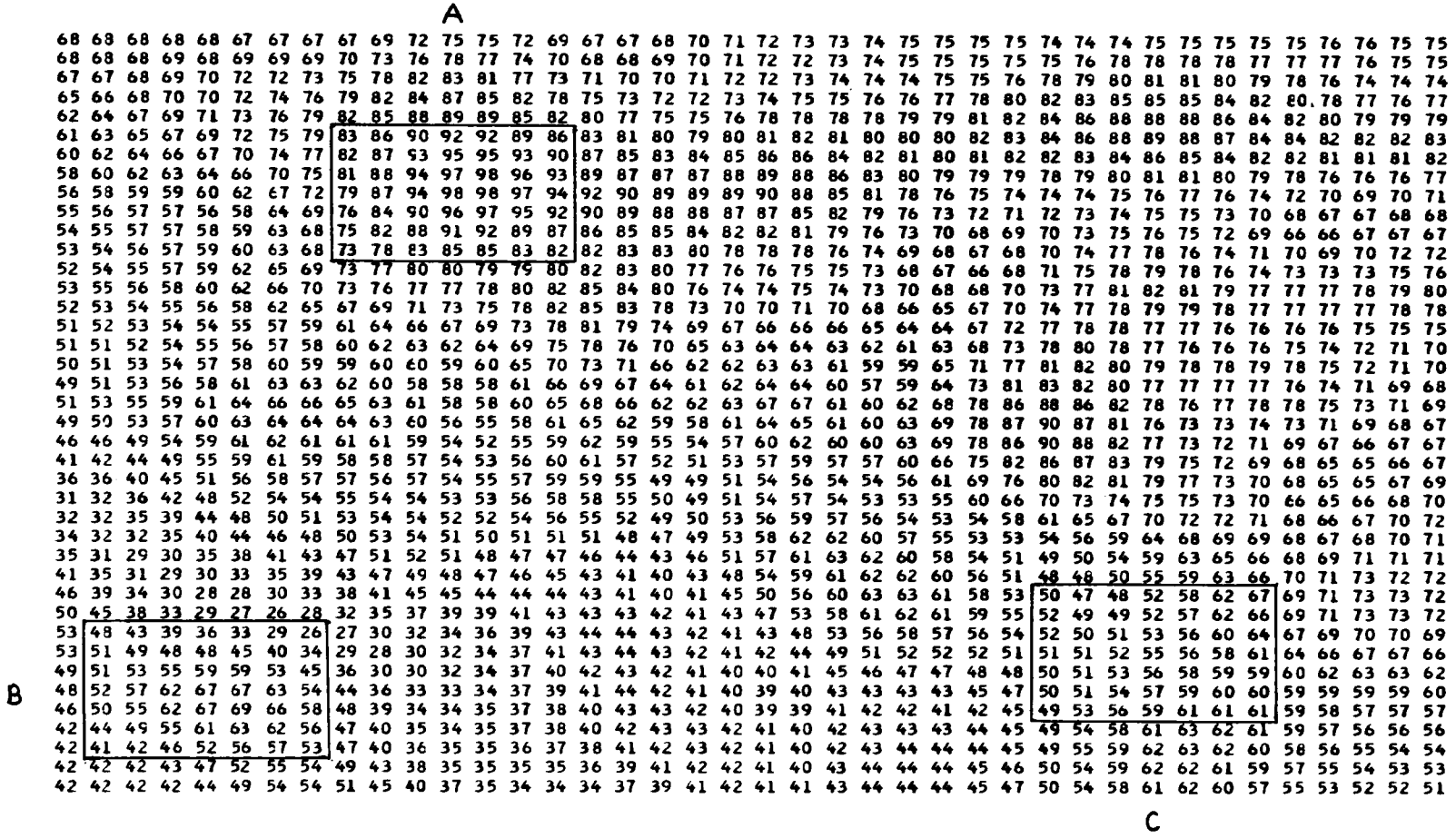
II. MOUNTAIN DATA, FIRST DEGREE LEAST-SQUARES (AREA)



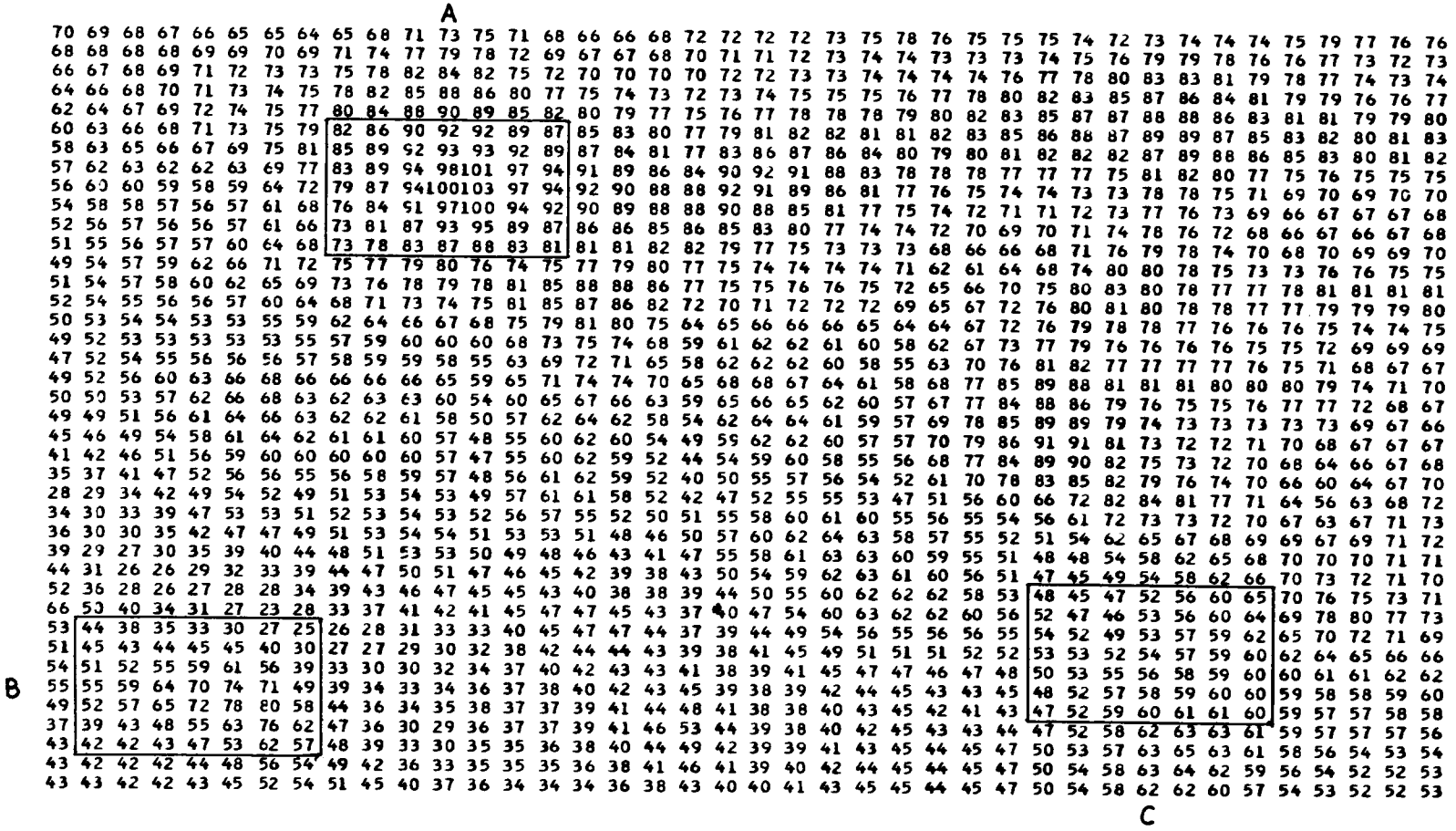
III. MOUNTAIN DATA, SECOND DEGREE LEAST-SQUARES (AREA)



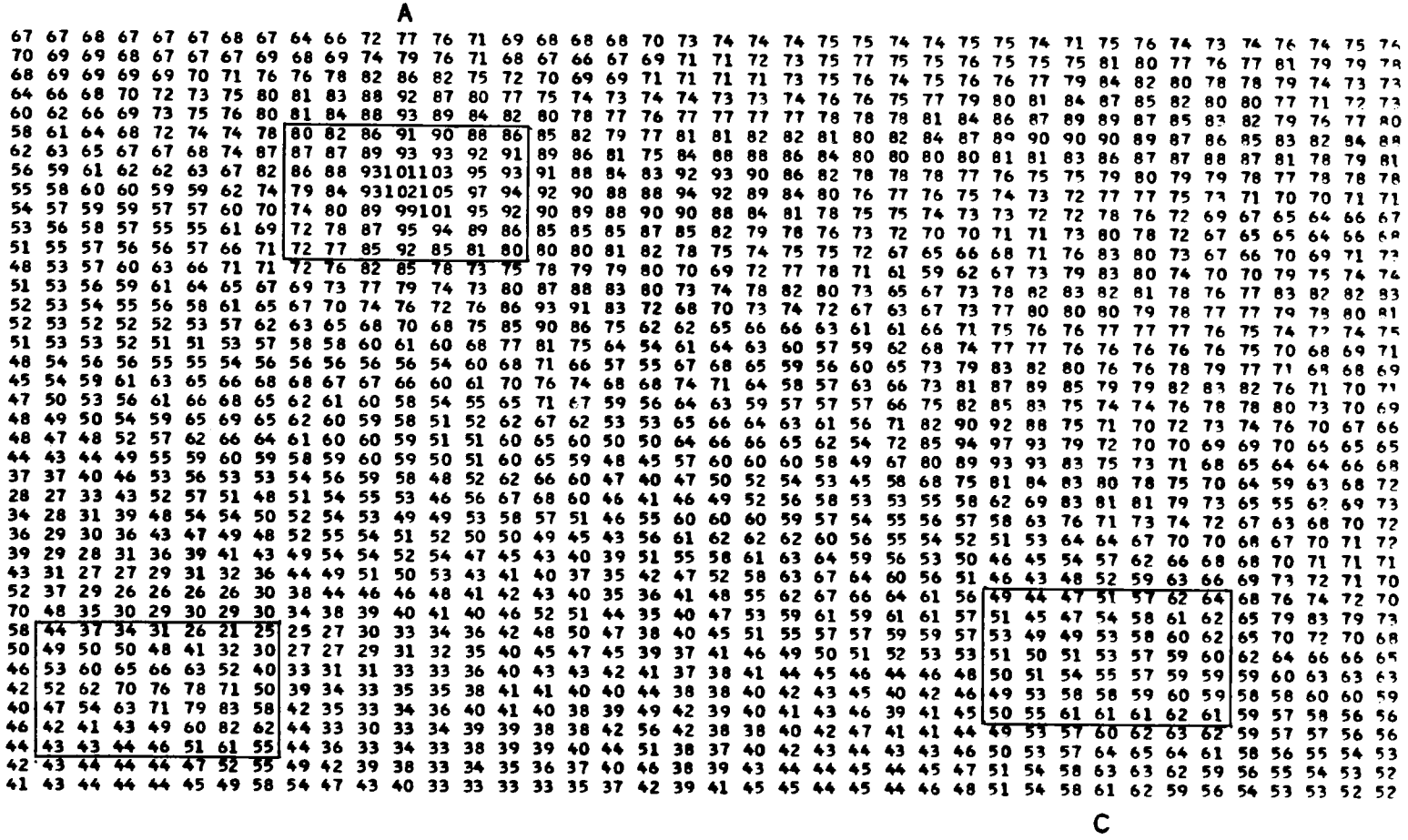
IV. MOUNTAIN DATA, SECOND DEGREE LEAST-SQUARES (POINT-BY-POINT)



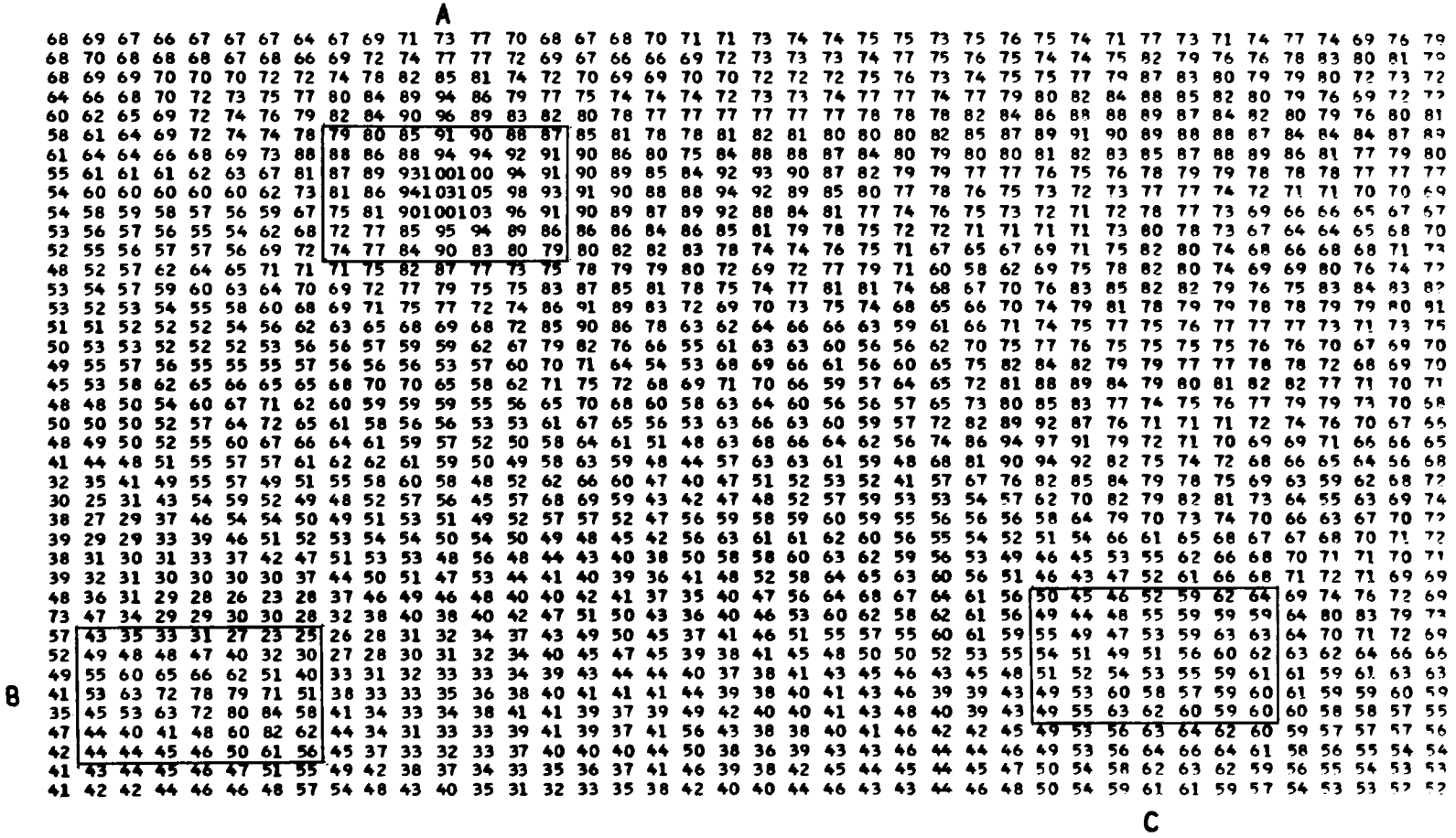
V. MOUNTAIN DATA, THIRD DEGREE LEAST-SQUARES (AREA)



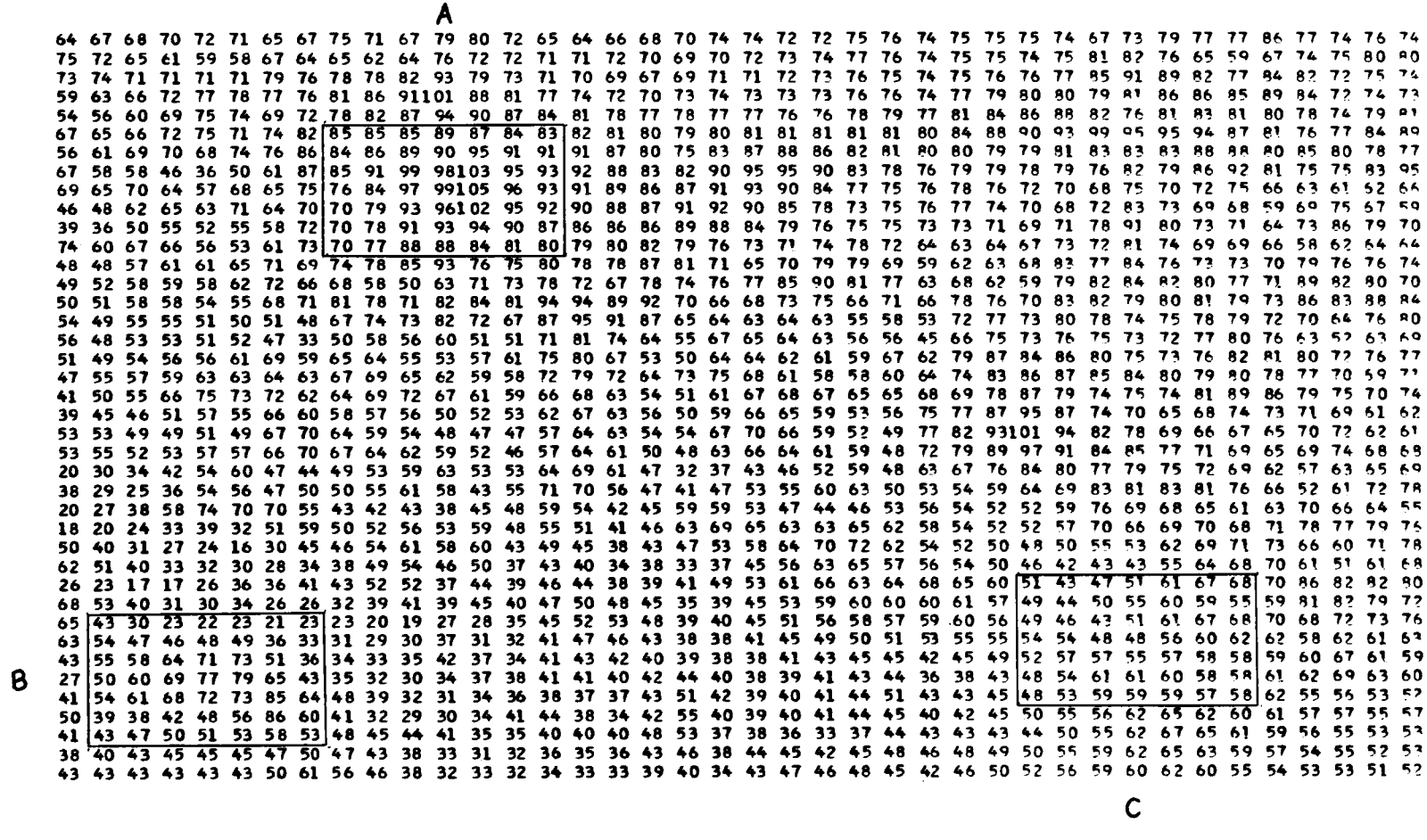
VI. MOUNTAIN DATA, FOURTH DEGREE LEAST-SQUARES (AREA)



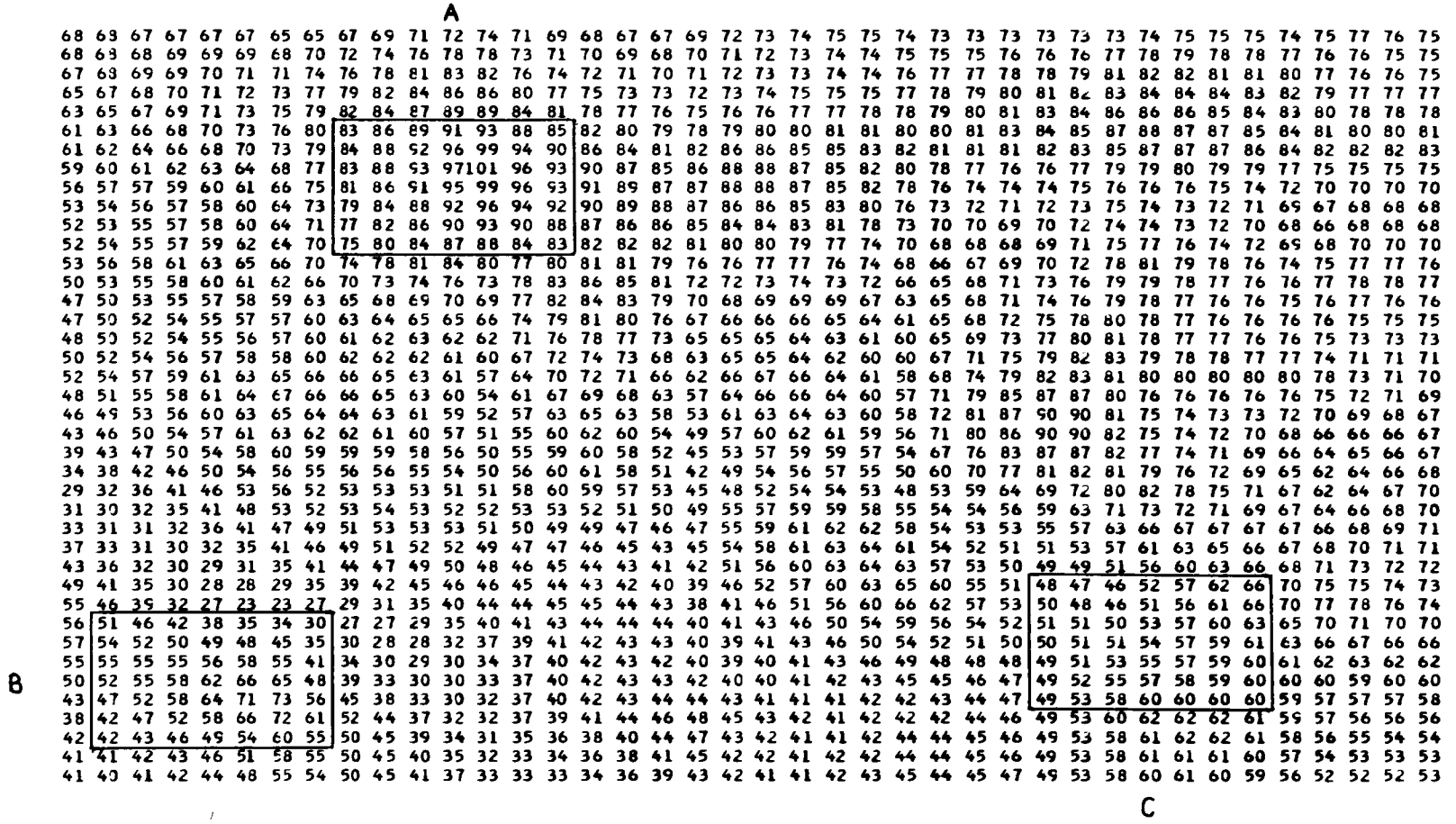
VII. MOUNTAIN DATA, FIFTH DEGREE LEAST-SQUARES (AREA)



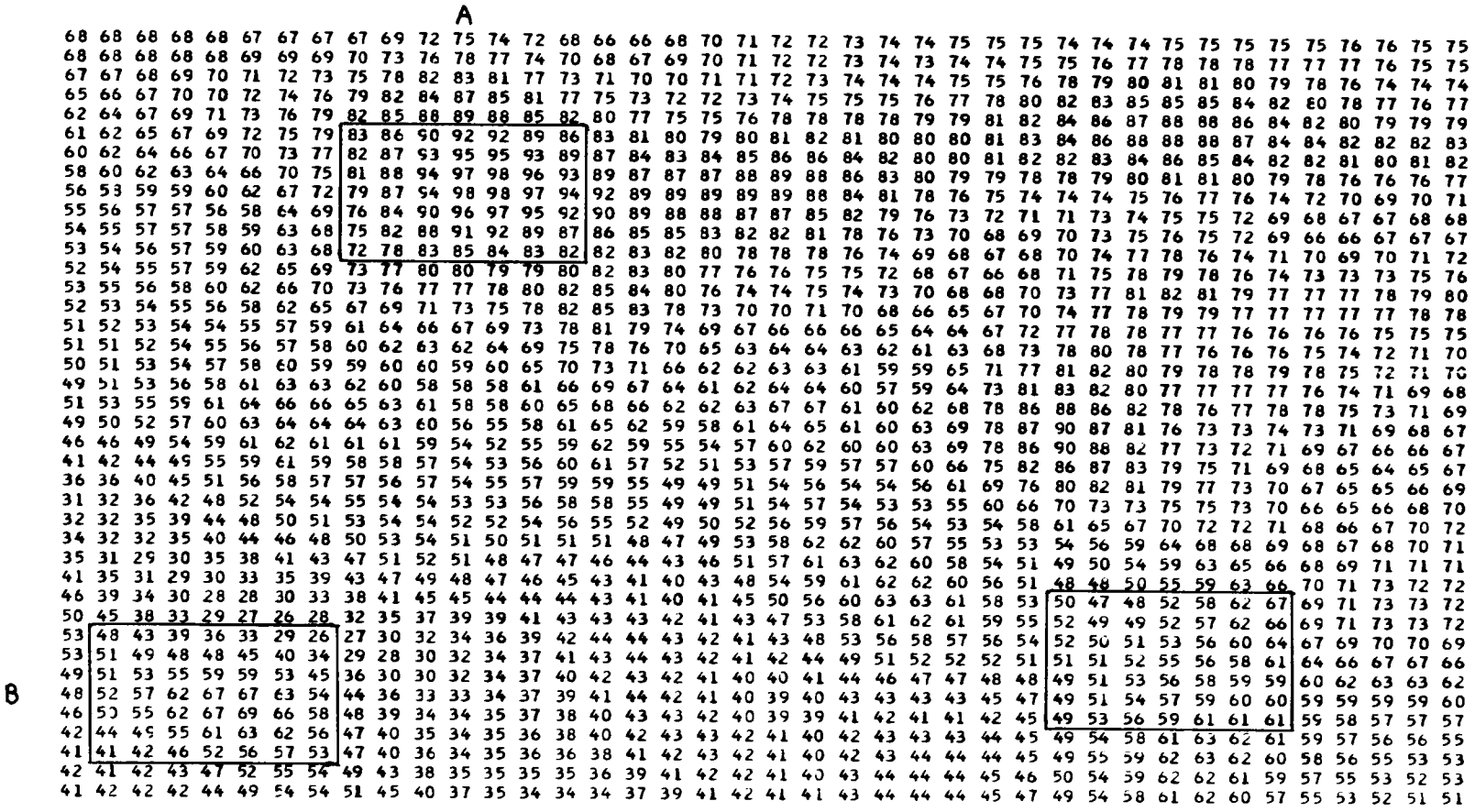
VIII. MOUNTAIN DATA, SIXTH DEGREE LEAST-SQUARES (AREA)



IX. MOUNTAIN DATA, REMPEL METHOD ONE (AREA)



X. MOUNTAIN DATA, REMPEL METHOD ONE (POINT-BY-POINT)



XI. MOUNTAIN DATA, REMPEL METHOD TWO (AREA)
FIRST PASS: SECOND DEGREE LEAST-SQUARES

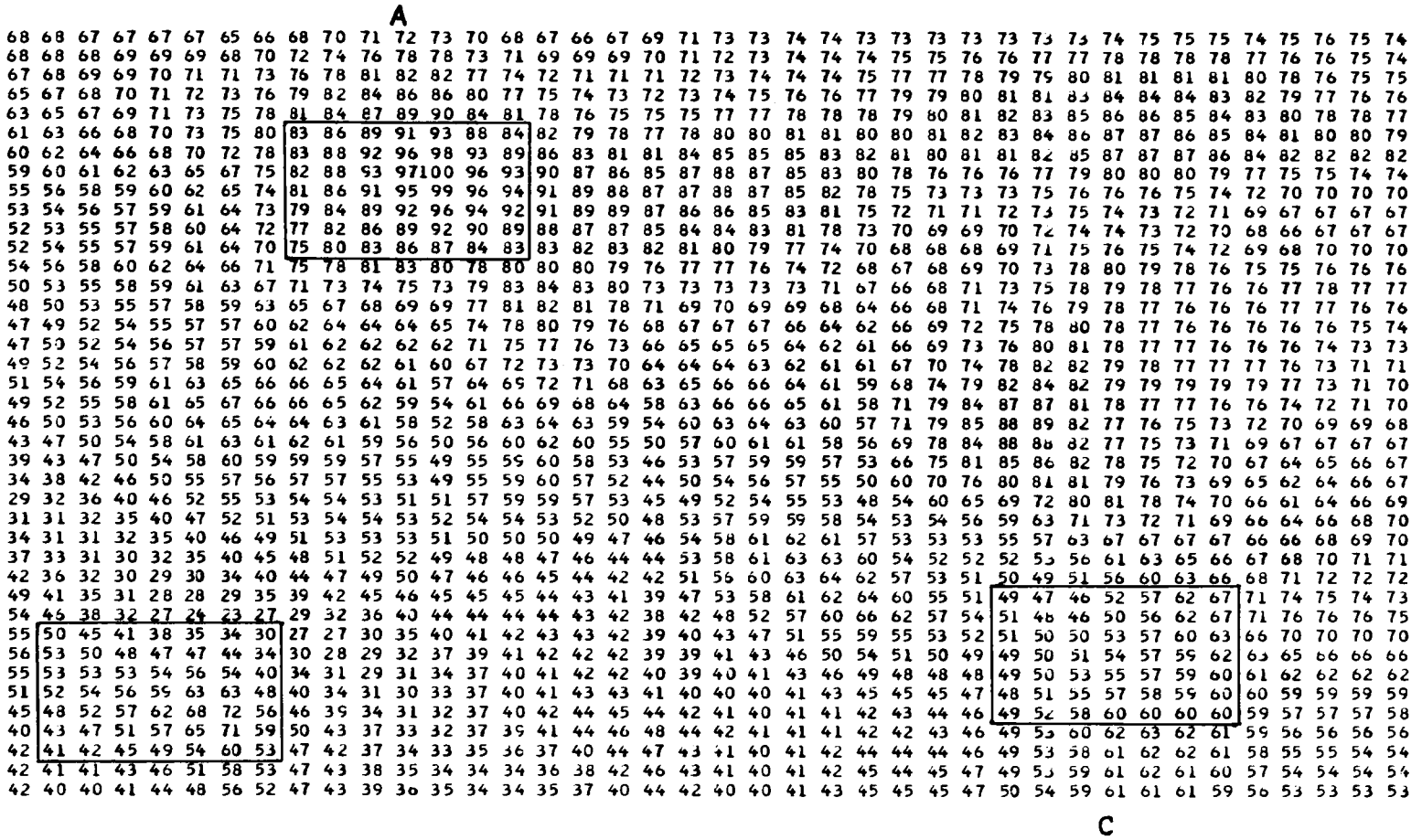
A

B

C

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XII. MOUNTAIN DATA, REMPEL METHOD TWO (AREA)
FIRST PASS: THIRD DEGREE LEAST-SQUARES



VITA

John A. Oakberg was born in East Lancing, Michigan on October 1, 1947. He attended the public school system in Oak Ridge, Tennessee and was graduated in 1965. He then entered Auburn University in Auburn, Alabama where he received a Bachelor of Science degree in mathematics in June, 1969. In July of 1969, he was employed by Union Carbide Corporation, Nuclear Division, and is currently a programmer/analyst. He is married to the former Linda Ruth McKinley of Oak Ridge.