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REPRESENTATION OF THE NON-RIGIDITY BETWEEN A CAM AND A ROLLER FOLLOWER BY A SPRING WITH A DETERMINED CONSTANT

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

James C. Conwell

December 1986

DEDICATION

To Angela, for her love and support through some difficult times. Only she knows how important she is to me.

ACKNOWLEDGMENT

In the course of my research for this thesis, I am indebted to many people who have assisted me. In particular, I would like to acknowledge Professors Charles Brown, James Euler and Frank Speckhart for their help and encouragement.

I would especially like to acknowledge Professor Robert Maxwell, who served as the chairman of my committee. His belief in, and his dedication to the teaching profession have inspired me far beyond what I thought was possible. He made the whole process enjoyable.

ABSTRACT

The amount of non-rigidity that exists between a cam and a follower was investigated. A stress condition for the cam and the follower in contact was determined from the Smith-Liu equations. Then, by using the theory of elasticity the deflections caused by the interactions between the cam and the follower were determined. This amount of non-rigidity in the system was then represented by a spring with a determined spring constant.

The end result of this investigation was an expression for the spring constant representing the non-rigidity in the system. Several additional avenues for study are suggested to build upon the results achieved in this study.

TABLE OF CONTENTS

CHAPTER PAGE

LIST OF FIGURES

FIGURE PAGE

TABLE OF NOMENCLATURE

a	The half width of the contact area, defined in equation (3.28)
C_c	The center of curvature of the cam
C_f	The center of curvature of the follower
d	Diameter of the follower stem
e	The offset of the follower from the center-line of the camshaft
E_c	The modulus of elasticity of the cam
E_f	The modulus of elasticity of the follower
f	Coefficient of rolling friction between the cam and the follower
$F^N_{c/f}$	The normal force on the cam exerted by the follower
$F^N_{f/c}$	Normal force on the follower exerted by the cam
F_{PL}	Preloaded force in the return spring
K_R	Spring constant for the return spring

K_E	Spring constant for the spring representing the non-rigidity in the cam and follower system
K_1, K_2	Parameters used in the Smith-Liu equations
L	Lift of the follower measured from the bottom of its stroke
$\dot{L}$	Velocity of the follower, inches per second
$\ddot{L}$	Acceleration of the follower, inches per second squared
L'	Velocity of the follower, inches per radian
M_f	Mass of the follower
N_B, N_c	Normal forces exerted on follower stem by the frame
P	Normal load per unit length applied to the cam
R_c	Radius of curvature of the cam
R_f	Radius of curvature of the follower
R_{PC}	Radius of the pitch circle

w_c	Deflection of points on the cam in the Z_c direction
w_f	Deflection of points on the follower in the Z_c direction
W_f	Weight of the follower
x_c, y_c, z_c	Position coordinates for points on the cam
x_f, y_f, z_f	Position coordinates for points on the follower
X_c, Y_c, Z_c	Cartesian axes for the cam
X_f, Y_f, Z_f	Cartesian axes for the follower

Symbols

α_{PC}	Deflection of a specific point on the cam, in the Z_c direction
α_{Pf}	Deflection of a specific point on the cam in the Z_f direction
δ,ε	Dimensions used in defining the pressure angle
β	Normal load per unit length applied to the cam

ε_{zc}	Strain of the cam in the Z_c direction
ε_{zf}	Strain of the follower in the Z_f direction
Δ, Ψ, $\overline{\Psi}$	Parameters used in the Smith-Liu equations
ω	Angular velocity of the cam
ϕ	Pressure angle
ℓ_1, ℓ_2	Dimensions used in deriving normal force
μ	Coefficient of sliding friction between the follower stem and its guides
γ_c	Poisson's ratio for the cam
γ_f	Poisson's ratio for the follower
σ_{xN}, σ_{yN}, σ_{zN}	Normal stresses caused by a Hertzian distributed normal load
σ_{xT}, σ_{yT}, σ_{zT}	Normal stresses caused by a Hertzian distributed tangential load

CHAPTER I

INTRODUCTION

Summary

In brief, an investigation was made of the non-rigidity between a disc cam and a roller follower. The non-rigidity was represented by a spring, with a determined spring constant. The expression for the spring constant was derived from known contact stress equations using the theory of elasticity.

This work can be viewed as a first step at modeling an actual cam-follower system. As such, this work could be used to study the behavior of a cam-follower system, as well as perhaps suggest failure criteria for a cam-follower system.

Background Discussion

In the area of machine design, a large volume of literature is devoted to the discussion of the behavior of cams and followers. (For the purpose of this study, the term "cam" will be used to describe a rotating disc cam, while the term "follower" will be used to describe a roller follower.) The majority of this literature is occupied with showing how the displacement, the velocity and the acceleration of the follower may be obtained by choosing a particular cam profile. Typically, one assumption made while generating this information is that the cam and the follower behave as absolutely rigid bodies.

For the purpose of this study, the cam and the follower are assumed to be elastic bodies. Due to the forces that exist between the cam and the follower, small deformations in each body occur. The cam and the follower are elastic in the sense that both members will return to their original shapes if these forces are removed.

This thesis investigates the amount of non-rigidity that exists between a cam and a follower under typical operating conditions. The investigation was completed using a three-part approach, using basic theories of elasticity. The major sections of the investigation are as follows:

1. The determination of the forces that exist between a cam and a follower.

   The force exerted on the follower by the cam is determined using a free-body analysis of the follower. Newton's laws are then applied to determine the force exerted on the cam by the follower.

2. The determination of the stresses on the cam and on the follower.

The first section of the investigation yielded the forces on the cam and on the follower due to the interaction(s) between these two bodies. The stresses caused by these interactive forces are then determined using a published method, the Smith-Liu equations.

3. The determination of the deformation of the cam and the follower bodies.

Once the stresses on each body became known, the deformations of each body were determined by the theory of elasticity.

By knowing the forces on the cam and the follower, the spring constant for a spring representing the non-rigidity between these two members can be determined, as discussed in the first section of this chapter.

Discussion of Previous Work

In recent years there has been much work done in determining the stresses that are caused by the interactions between two cylinders. (For the purpose of this study, the cam and the follower will be treated as two cylinders with parallel axes.) Much of this work has been completed using numerical methods (Ahmadi, Doi, Harnett and Manakuchi), although some of the work more closely related to this has been done experimentally (Midlin).

An extensive computer search failed to locate any relatively recent work that could be directly applied to this thesis. The Smith-Liu equations, published in 1953, still remain the most applicable (at least to this study) expressions for stress available.

CHAPTER II

DEFINING THE PROBLEM IN CONSIDERATION

Introduction to a Cam and Follower System

In general terms, a cam is a machine element designed to have rotational motion and a specially formed surface.[1] The cam transmits motion to another machine element known as the follower. The most basic cam mechanism consists of a cam and follower combination. By rotating, the cam drives the follower. The cam mechanism is used to transform the rotational motion of the cam into intermittent rectilinear motion of the follower, which is the output member.[2]

Cam and follower mechanisms are relatively inexpensive, are not difficult to design or manufacture, and occupy a small amount of space.[3]

For these reasons cam and follower mechanisms are widely used in modern equipment. To meet these many needs there are a variety of types of cams and followers.

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[1] H. A. Rothbart, Cams: Design, Dynamics and Accuracy. (New York: John Wiley and Sons, 1956), p. 26.

[2] F. D. Furmon, Cams: Elementary and Advanced. (New York, John Wiley and Sons, 1921), p. 32.

[3] J. E. Shigley and J. J. Uicker, Theory of Machines and Mechanisms. (New York: McGraw Hill Book Company, 1980), p. 193.

One of the more popular cam and follower mechanisms employs the disc cam in conjunction with the roller follower. For the duration of this thesis, the term "cam" will be used to refer to a disc cam exclusively while the term "follower" should be taken to indicate only a roller follower. A typical disc cam and roller follower are shown in Figure 2.1.

The cam profile is typically designed to produce a particular motion of the follower. This motion is usually dictated by the requirements of the application. Once the cam profile has been chosen, the displacement, velocity and acceleration of the follower are also chosen. The dynamic performance of the follower is a function of the cam profile. This can be visually represented by Figure 2.2.

Throughout this investigation, there will be references to terms describing physical characteristics of the cam. These terms are graphically defined in Figure 2.3.

Description of the Coordinate System Used Throughout the Investigation

The coordinate system used throughout the investigation will be a right-handed orthogonal system. The coordinates used to refer to the cam will be subscripted with a "c" while those used to refer to the follower will be subscripted with an "f".

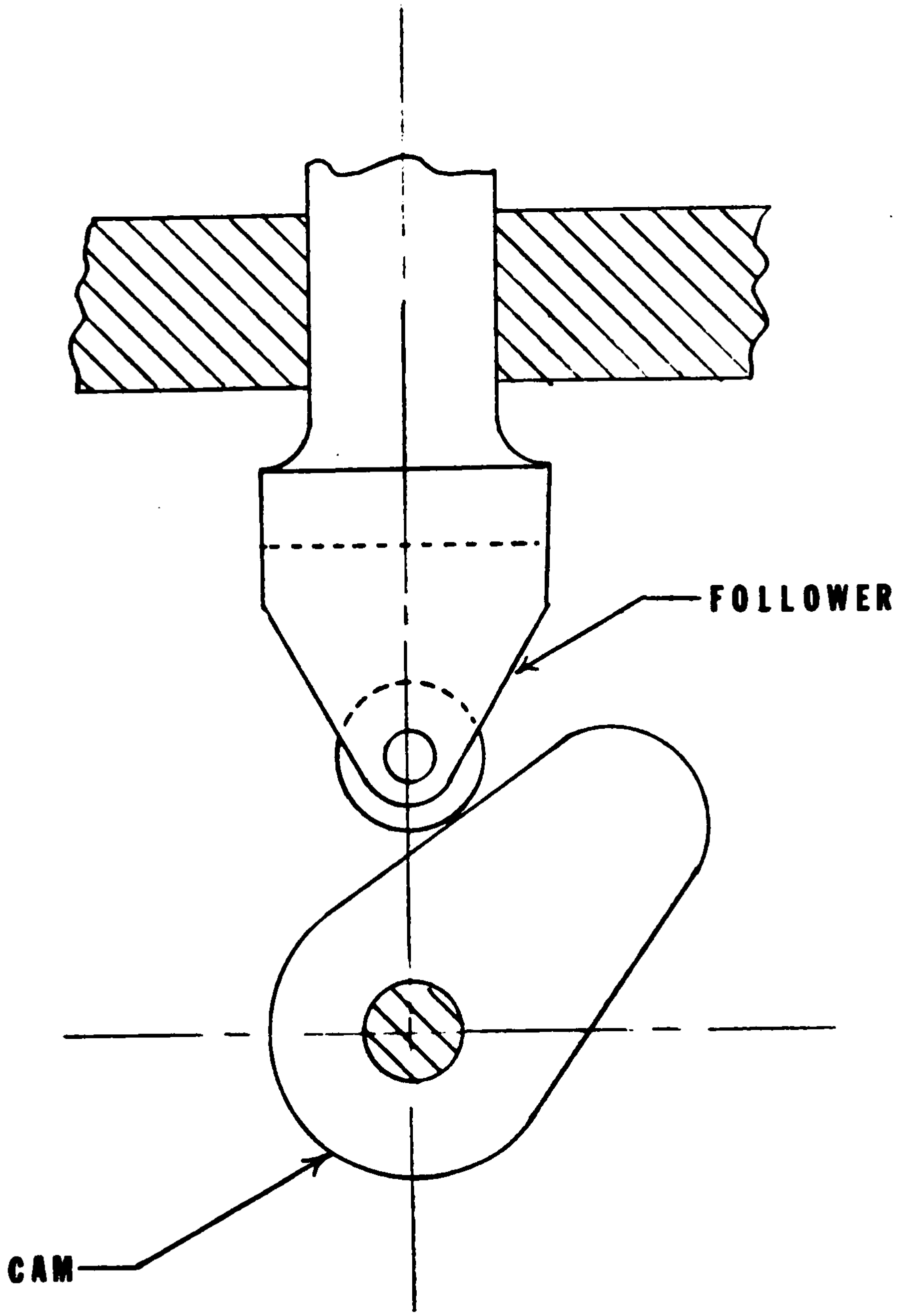

Figure 2.1. A Simplified Representation of a Cam and Follower.

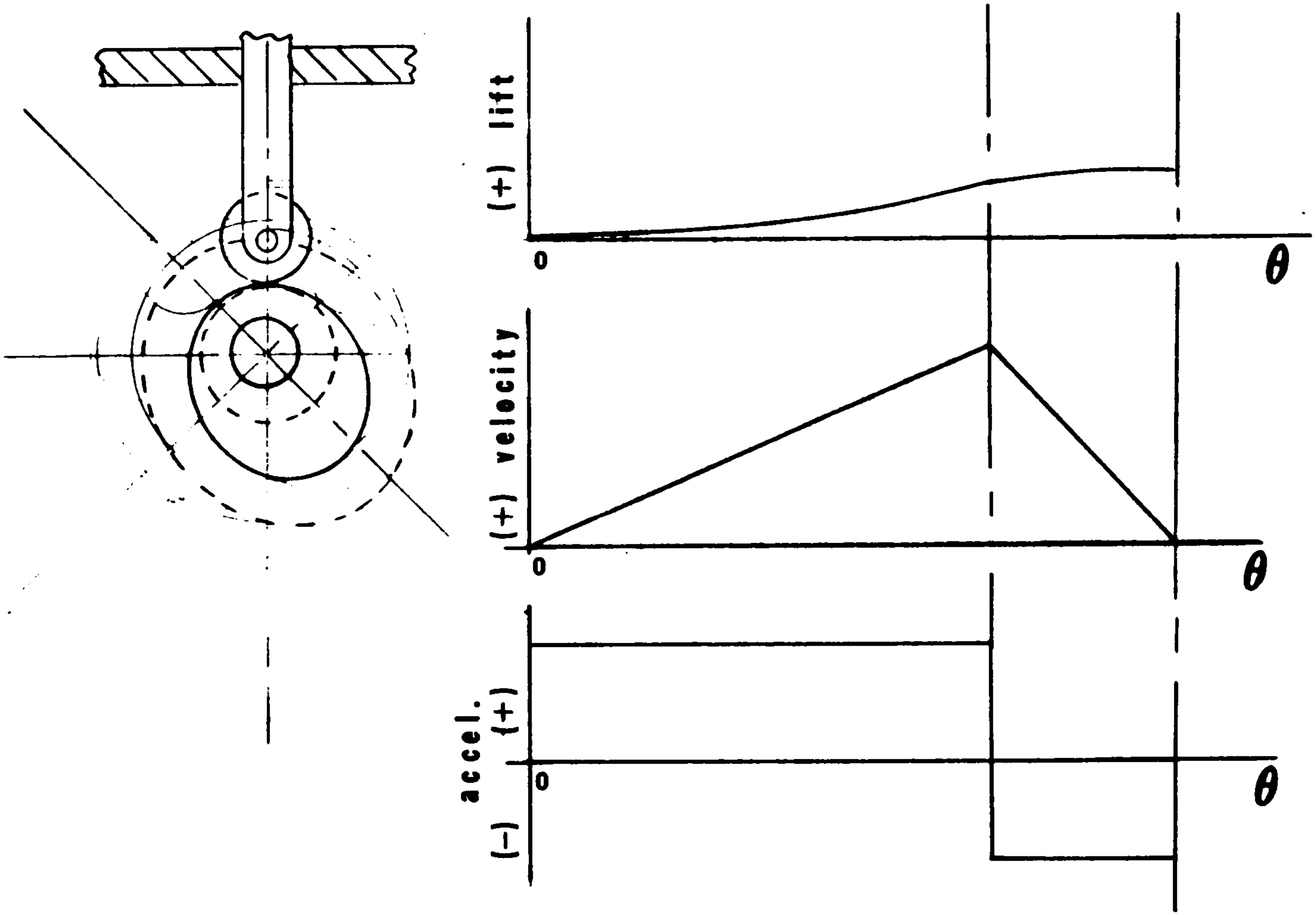

Figure 2.2. Lifter Velocity and Acceleration as a Function of the Cam Profile.

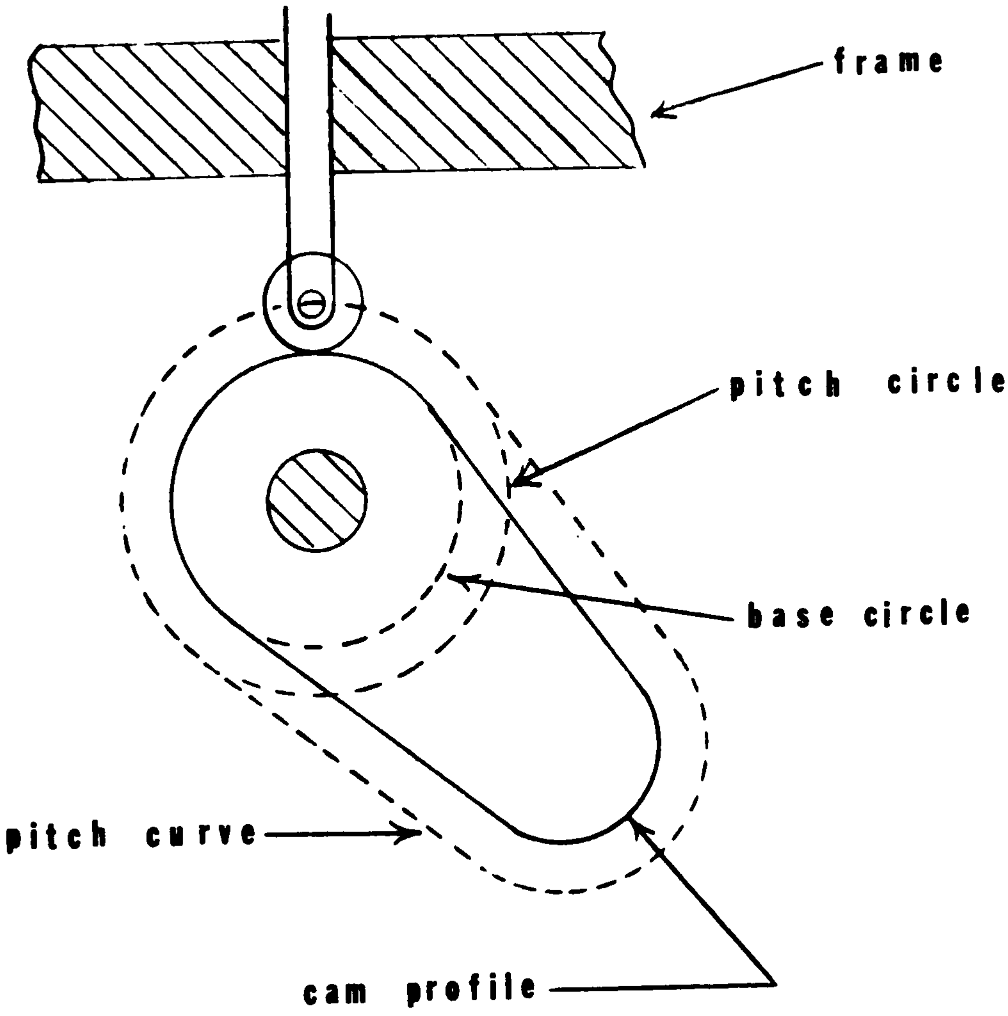

Figure 2.3. Graphical Definition of Cam Terms.

The cam and the follower, along the line of contact, can be considered to be two cylinders with parallel axes. If there is no force pressing these two bodies together, there is only a line of contact between them. The Y_f - Y_c axis will be colinear with this line and will be parallel to the axes of the cam and the follower. The X_c - X_f axis will be tangent to both bodies and will be drawn through this line of contact. The X_c - X_f axis lies in a plane that is perpendicular to the line of contact. The coordinate system is shown in Figure 2.4.

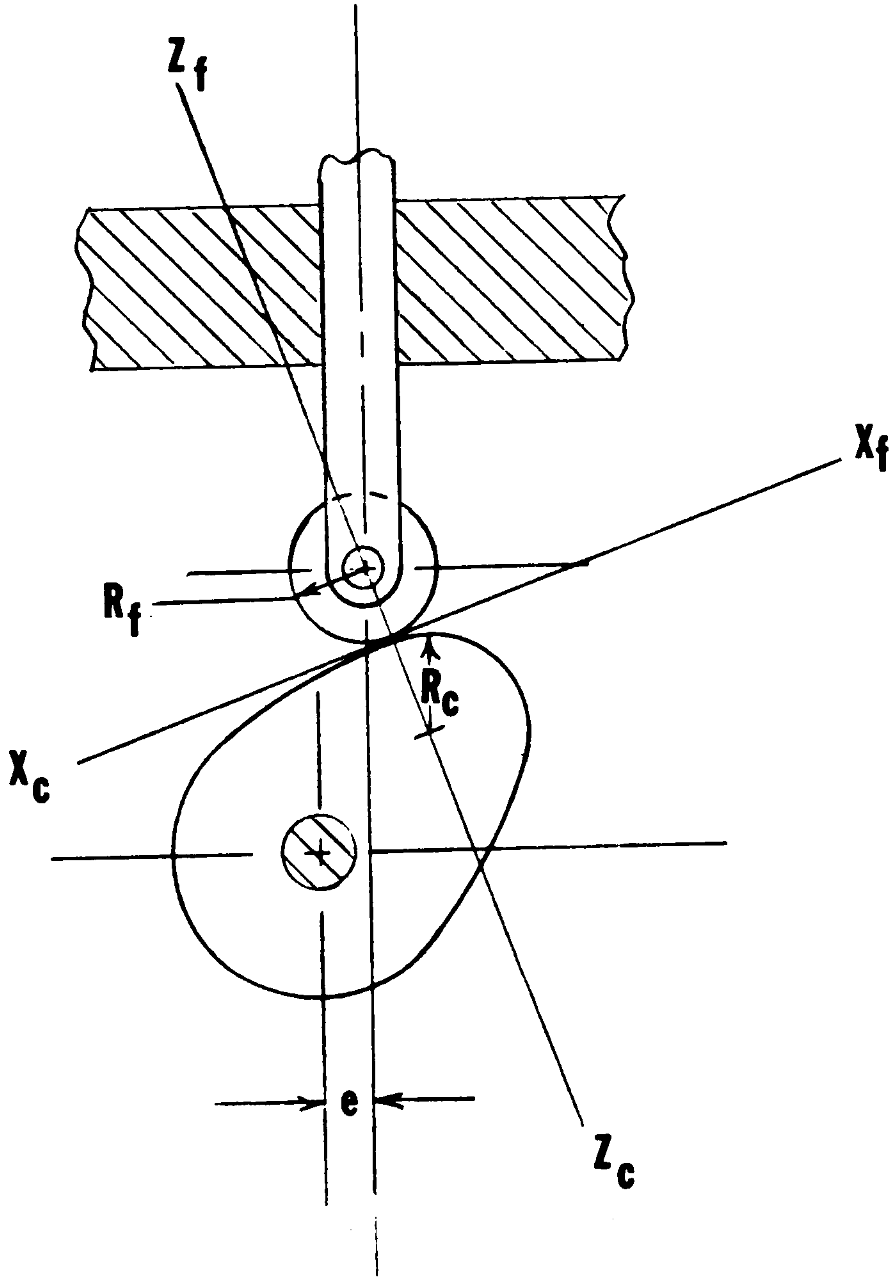

Figure 2.4. The Coordinate System

CHAPTER III

INVESTIGATION OF THE NON-RIGIDITY BETWEEN A CAM AND A FOLLOWER

Assumptions and Limitations to the Study

As discussed in Chapter I, the determination of the non-rigidity between the cam and the follower is the end result of this investigation. Before the first steps can be taken towards achieving this result, it is essential that the assumptions made in this study be discussed in detail. These assumptions are enumerated below. The limitations to the study as a result of these assumptions are handled in the appropriate location(s) throughout the solution.

1. The cam and the follower are each constructed of isotropic materials.

2. The axis of rotation of the cam and the axis of rotation of the follower are parallel.

3. The cam and the follower are each constructed of perfectly elastic bodies for the loads in consideration. The forces on the cam (by the follower) will cause changes in the shape of the cam. By being defined as perfectly elastic, the cam will return to its original shape upon removal

of the loads. A similar discussion can be applied to the follower.

4. The cam and the follower are the only non-rigid members in the cam system. A typical cam system may consist of the cam and the follower, a lift rod and a rocker arm. These other members for the purpose of this investigation are to be considered to be absolutely rigid bodies. The only non-rigidity in the system is located at the contact area between the cam and the follower.

5. The effects of the cam on the follower and the effects on the follower on the cam are localized. The deformations of each body caused by the interaction of that body with the other are localized, and thus each body can be treated as a semi-infinite body.

6. The width of the cam is the same as the width of the follower. To use the coordinate system previously developed, the cam and the follower have the same dimension in the Y-direction.

7. The cam and the follower are in a condition of plane strain. The dimensions in the Y-direction of the cam and follower are very large with respect to the width of the area over which the

cam and the follower are loaded. Additionally, the loading on the cam and the follower will not vary in the Y-direction. It is assumed that there is no axial displacement at any cross section of the cam or the follower.

The Force on the Follower Exerted by the Cam

To evaluate the forces on the follower due to the interaction of the cam with the follower, consider first Figure 3.1. This figure is a simplified representation of a cam follower system. The distance "e" between the centerline of the follower and the centerline of the camshaft is defined as the eccentricity.

The follower is constrained by the frame to move only in a vertical direction. The coordinate L is used to measure the motion of the follower center, with $L = 0$ being arbitrarily chosen when the follower is at the bottom of its stroke. W_f represents not only the weight of the follower, but also the weight of the other members (for example, the push rod) that may also be affecting the forces on the cam.

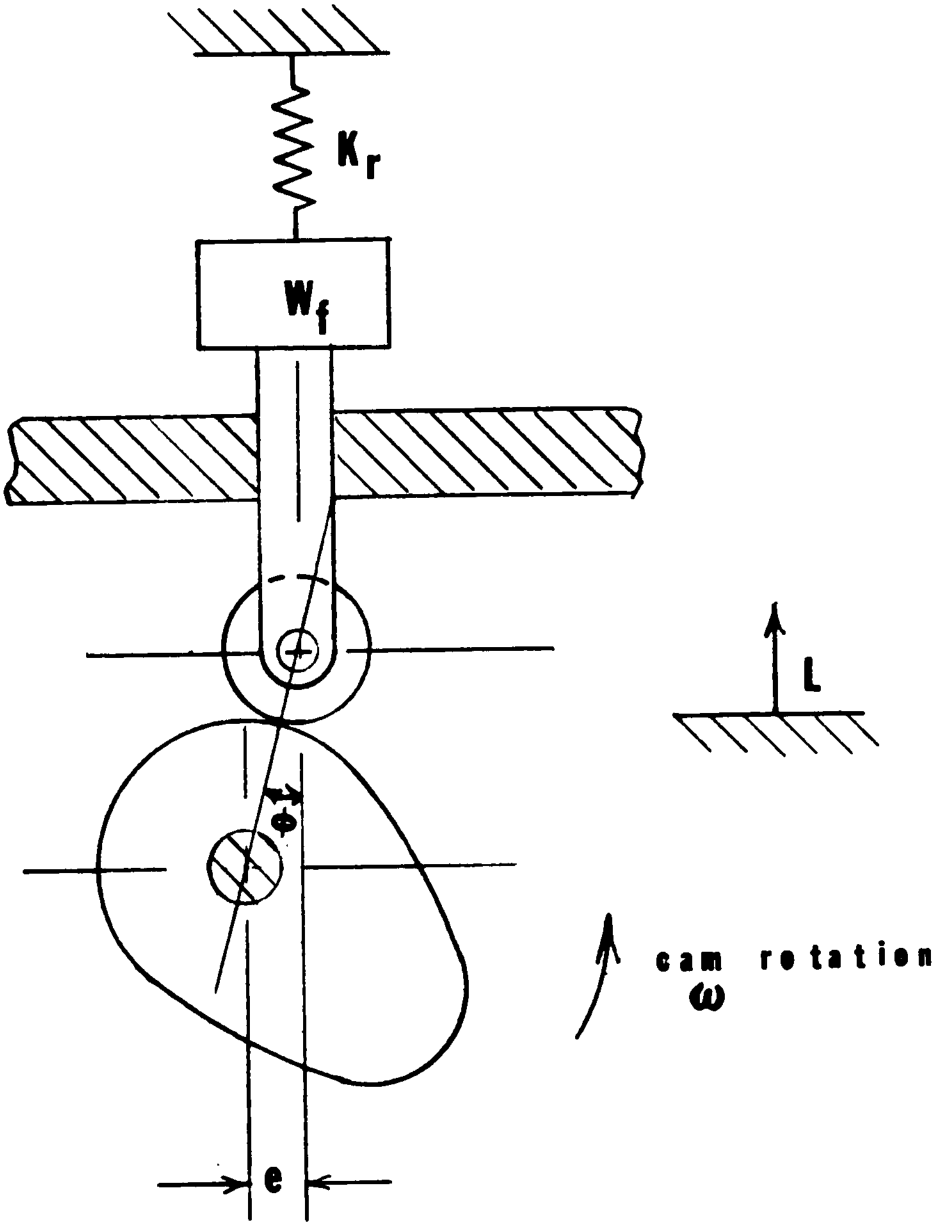

Figure 3.1. A Representation of a Cam System.

The return spring is designed to keep the follower in contact with the cam. The return spring has a known spring constant, K_R, which by definition is the amount of force necessary to compress or extend the spring a unit length. As the spring _must_ keep the follower in contact with the cam, it must be exerting a force even when the follower is at the bottom of its stroke. This force is called the preload force and designated F_{pl}. By definition it is the force exerted by the spring when $L = 0$.

The force exerted by the cam on the follower is directed normal to the cam surface which is typically not the direction of the follower motion. The angle between the force exerted on the follower by the cam and a line drawn parallel to the motion of the follower is defined as the pressure angle, denoted by ϕ.

When the pressure angle is greater than zero, the surface of the cam pushes sideways against the follower. This in turn causes additional resistance to the follower motion thereby increasing the force required to move the follower. As the pressure angle is important to the discussion that follows, a useful means of determining the pressure angle is shown below.

Figure 3.2 shows a cam with a reciprocating follower. The follower is constrained to move only in a vertical direction. The cam is rotating at an angular velocity, ω_{cam}, where,

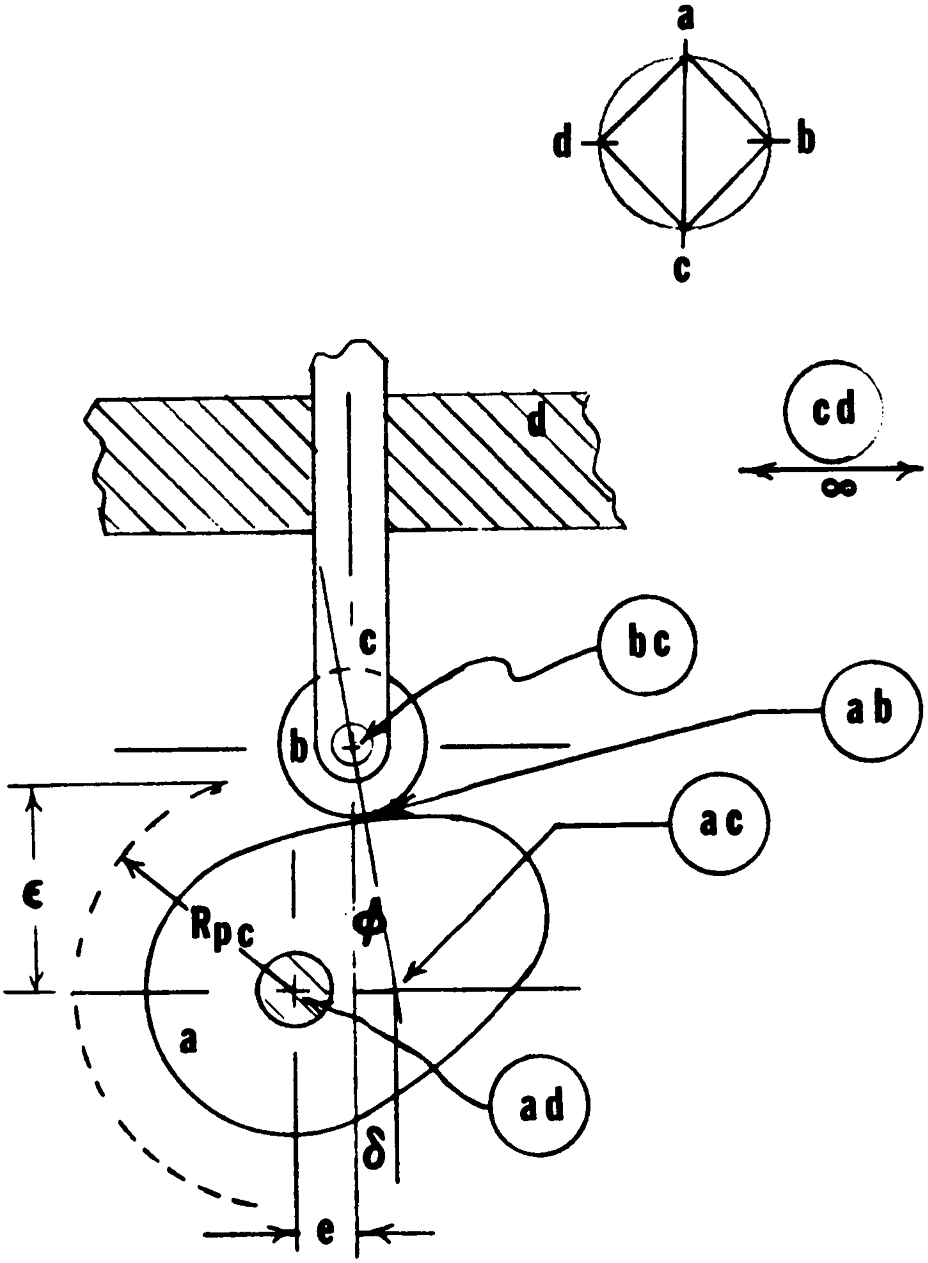

Figure 3.2. Location of the Instant Centers of Velocity.

$$\omega_{cam} = \frac{d\Theta}{dt} \qquad (3.1)$$

Measured in seconds, dt is the time required for the cam to rotate through the angle $d\Theta$. The other parameters shown all retain the same definition that was previously assigned to them.

An instant center of velocity is defined as the location of a pair of concurrent points on two different bodies for which the absolute velocities of the two points are equal. The necessary instant centers to derive an expression for the pressure angle are shown in Figure 3.2. The instant centers were found by application of Kennedy's theorem and with the use of a circle diagram.

For the purpose of this discussion, the cam has been designated as member a, the roller as member b, the follower stem as member c, and the frame, or the fixed body, as member d. The instant centers are indicated with a circle enclosing the two letters designating what instant center is located at the point indicated.

Since the follower is constrained to move only in a vertical direction, the velocity of instant center ac is the velocity of all points of member c, the follower. But, ac is also a point on member a, the cam. As such, the velocity of point ac is a function of the angular velocity of the cam and the distance from ac to ad, the point about which the cam is rotating with respect to the fixed body.

$$V_{ac} = (\omega_{cam})\ \overline{(ac - ad)} \qquad (3.2)$$

Where V_{ac} is the velocity of instant center ac and $\overline{(ac - ad)}$ is the distance from ac to ad. All units must be consistent.

Also, the velocity of ac is the velocity of the follower.

$$V_{ac} = \frac{dL}{dt} \qquad (3.3)$$

Where $\frac{dL}{dt}$ is the instantaneous velocity of the follower.

Rewriting equation (3.2), using the dimensions "δ" and "e" as shown in Figure 3.2,

$$\frac{dL}{dt} = (\omega_{cam})\ (e + \delta) \qquad (3.4)$$

But,

$$\frac{dL}{dt} = \frac{dL}{d\Theta}\,\frac{d\Theta}{dt} \qquad (3.5)$$

By definition,

$$\frac{d\Theta}{dt} = \omega_{cam} \qquad (3.6)$$

Also, by definition,

$$L' = \frac{dL}{d\Theta} = \frac{L}{\omega} \qquad (3.7)$$

Where L' has units of inches/rad.

By substituting equation (3.7) into equation (3.5) the following result is achieved:

$$L' = (e + \delta) \qquad (3.8)$$

By strictly geometrical considerations,

$$\delta = \mathrm{Tan}\,\phi\,(\varepsilon + L) \qquad (3.9)$$

Where ε is the vertical distance from the camshaft to the pitch circle and

$$\varepsilon = \sqrt{R_{pc}^{\,2} - e^2} \qquad (3.10)$$

By substituting equation (3.10) into equation (3.9) and then substituting that expression for δ into equation (3.8), the final expression for the pressure angle can be arrived at.

$$\phi = \mathrm{Tan}^{-1}\,\frac{L' - e}{\sqrt{R_{pc}^{\,2} - e^2} + L} \qquad (3.11)$$

It is extremely important to note that although equation (3.11) is valid for all cam positions, L and L' are functions of the cam profile and are constantly changing as the cam rotates. The pressure angle therefore is also constantly changing as the cam rotates.

Figure 3.3 is a representation of a general case of a cam and follower. The force exerted on the follower by the cam is directed in a direction that is normal to the surface of the cam at the point of contact between the cam and the follower.

In determining the magnitude of this normal force, the following parameters are utilized:

W_f = weight of the follower

F_e = external forces by other members exerted on the follower

L, L, $\ddot{L}$ = the lift, velocity and acceleration of the follower, respectively

F_{Pl} = the preload force in the return spring

K_R = the spring constant of the return spring

$F^N_{f/c}$ = the normal force exerted on the follower by the cam

ℓ_1, ℓ_2 = dimensions of the cam system as shown in Figure 3.3

N_B, N_C = forces normal to the follower stem, exerted

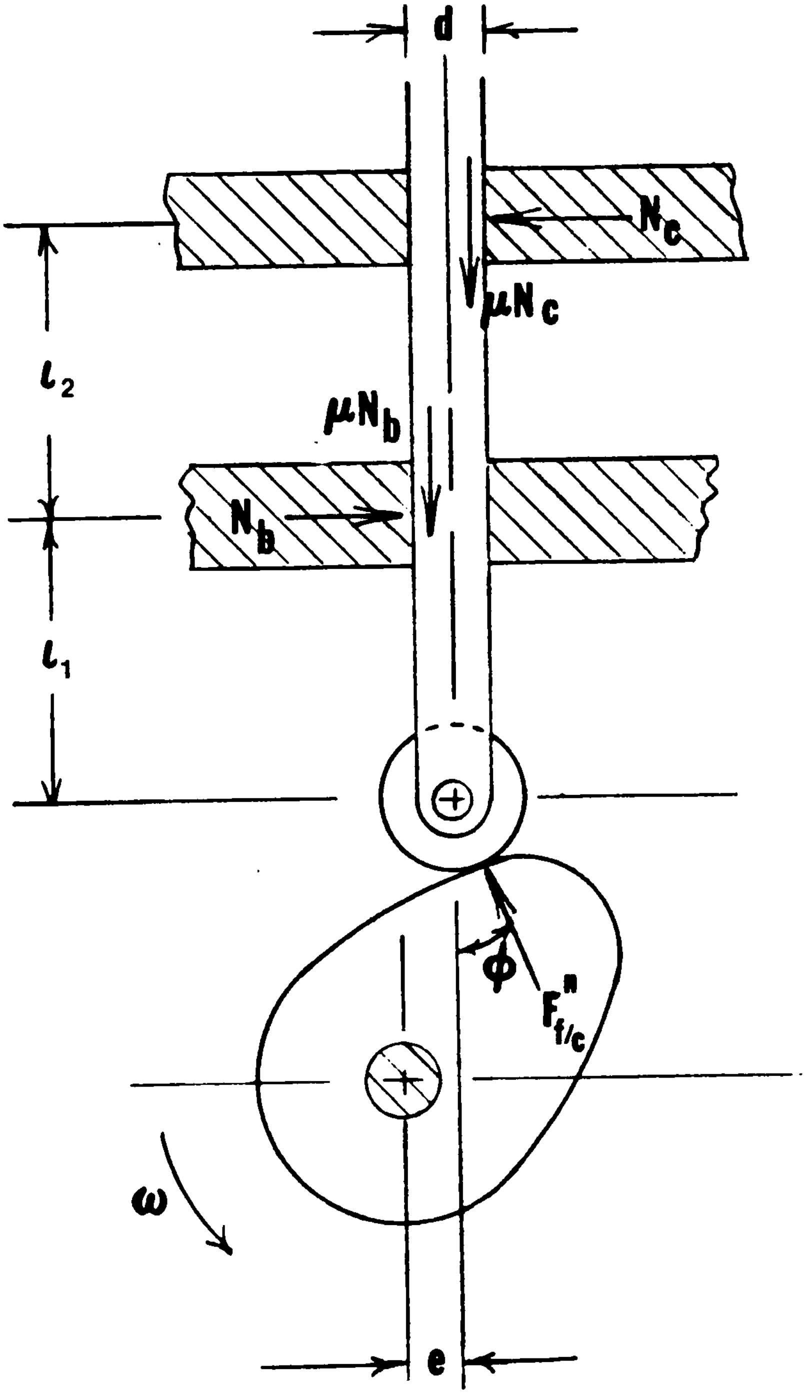

Figure 3.3. Determining the Normal Force.

on the stem at the locations shown in Figure 3.3

μ = coefficient of friction between the follower stem and its guide

ϕ = pressure angle

d = diameter of the follower stem

To determine the direction of the forces due to friction between the follower stem and its guides a sign function is defined.

$$\text{sign}\ \dot{L} = \frac{\dot{L}}{|\dot{L}|} \tag{3.12}$$

As the expression for the magnitude of this normal force is determined in several sources,[1] it is reprinted here without derivation.

$$F_{f/c}^{N} = \frac{F_T \ell_2}{\ell_2 \cos\phi - (2\mu\ \text{sign}\ \dot{L}\ell_1 + \mu\ \text{sign}\ \dot{L}\ell_2 - \mu^2 d)\sin\phi} \tag{3.13}$$

F_T is the sum of the inertial forces, the weight of the follower and the external forces.

---

[1] Robert L. Maxwell, Kinematics and Dynamics of Machinery. (Englewood Cliffs, N.J.: Prentice-Hall, 1960), p. 395.

This force is directed normal to the cam's surface at the point of contact. By Newton's Third Law, the force exerted on the cam by the follower will also be given by equation (3.13).

Discussion on the Expressions for the Force Exerted on the Follower by the Cam and Vice-versa

Several comments are in order concerning the expressions for the forces $F^N_{f/c}$ and $F^N_{c/f}$ derived in the previous section.

The force exerted on the follower by the cam is in a direction normal to the cam's surface at the point of contact. Recalling how the coordinate system defined in Chapter II was oriented, the force $F^N_{f/c}$ is directed in a positive Z_f direction. The force $F^N_{c/f}$ is directed in a positive Z_c direction. Defining and orienting the coordinate system in the manner described in Chapter II was to simplify the descriptions of the directions of the forces $F^N_{f/c}$ and $F^N_{c/f}$. Figure 3.4 shows the application of these forces to the follower and cam respectively, with the coordinate system superimposed.

It is important to note that the force exerted on the follower by the cam (and vice versa) will always be positive, or at the very minimum, zero. If the $F^N_{f/c}$ is less than zero (by algebrically summing the forces on the follower) then the follower has "jumped" off the cam and the analysis no longer holds.

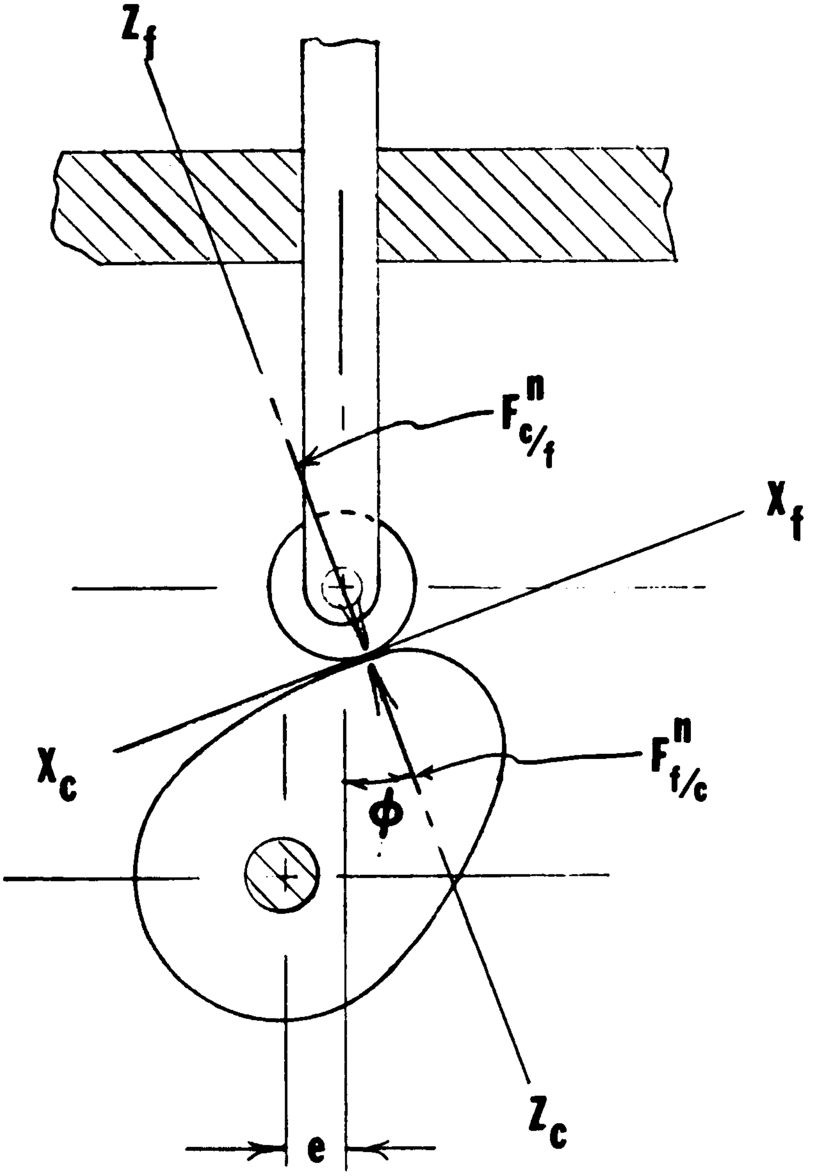

Figure 3.4. The Normal Forces with the Coordinate System Superimposed.

It may also be noticed that the effects of rolling friction between the follower and the cam were ignored in the analysis to determine the forces on the cam and on the follower. This was done for two reasons:

1. The magnitude of the rolling friction will be very small in comparison to the sliding friction between the follower and its guides.

2. The rolling contact friction that may be present will not contribute to the further results obtained in this investigation. This will be shown in a later section.

Expressions for the Stress on the Cam and on the Follower

The first investigation of the contact of elastic bodies was done by H. Hertz in 1881. He computed and verified with experiment the load distribution over the contact area.[1] The case under investigation concerns the contact of two cylinders with parallel axes.

The surface of contact between two cylinders with parallel axes is a narrow rectangle. A Hertzian distribution of the load pressing these two bodies together is assumed. For this assumption to be true, the normal

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[1] Stephen Timoshenko, History of the Strength of Materials. (New York: Dover Publications, 1983), p. 347.

loads (in this case $F_{f/c}^{N}$ and $F_{c/f}^{N}$) are distributed in a semi-elliptical fashion along the width of the surface of contact. The width of the surface of contact is the dimension of the narrow rectangle in the X-Z plane. The integral of the pressures over the contact area is equal to the force pressing these bodies together. Figure 3.5 shows the two bodies being pressed together by the normal force $F_{c/f}^{N}$, while Figure 3.6 represents the elliptical distribution of the normal load.

If in addition to the normal load a tangential load is also considered, then the situation exists which is represented in Figure 3.7. As any presence of a tangential load must be associated with that of a normal load.

$$F_{f/c}^{T} = f\ F_{f/c}^{N} \tag{3.14}$$

Where f denotes the coefficient of friction of the surfaces of contact. It will be shown that for the purposes of this investigation a tangential load will not be of interest. Until that is shown, such a load will be considered to be present and also to be elliptically distributed over the region of contact as shown in Figure 3.8.

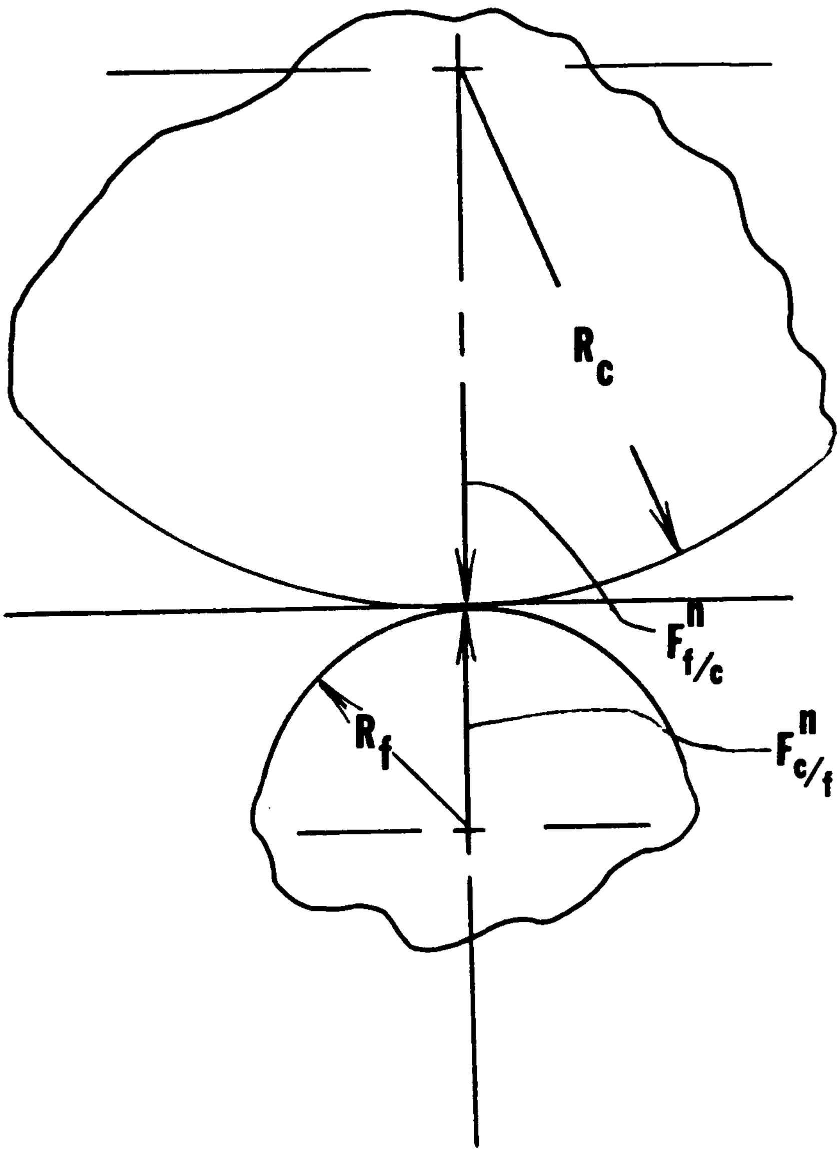

Figure 3.5. The Two Bodies Being Pressed Together Under the Action of the Normal Forces.

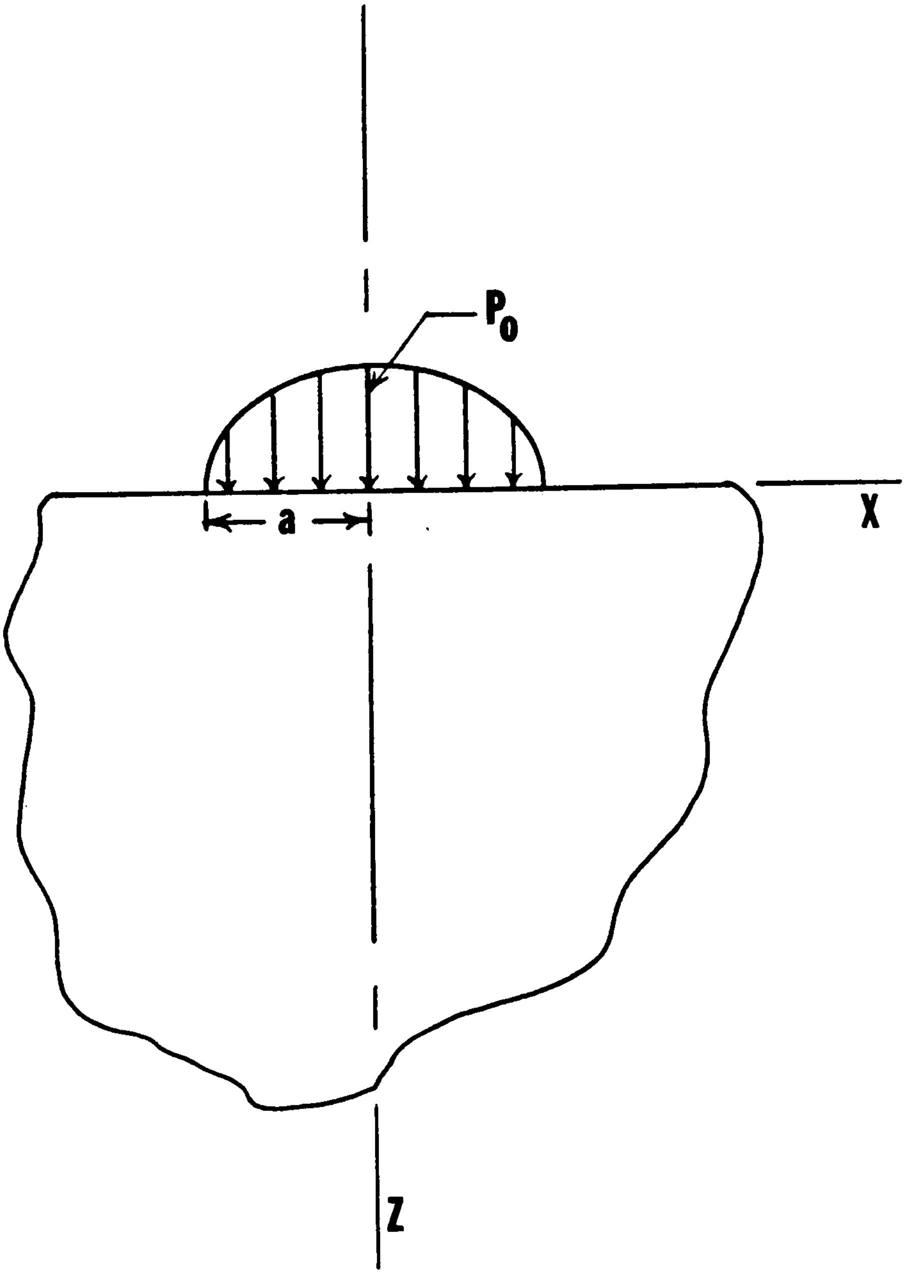

Figure 3.6. A Hertzian Distributed Normal Load.

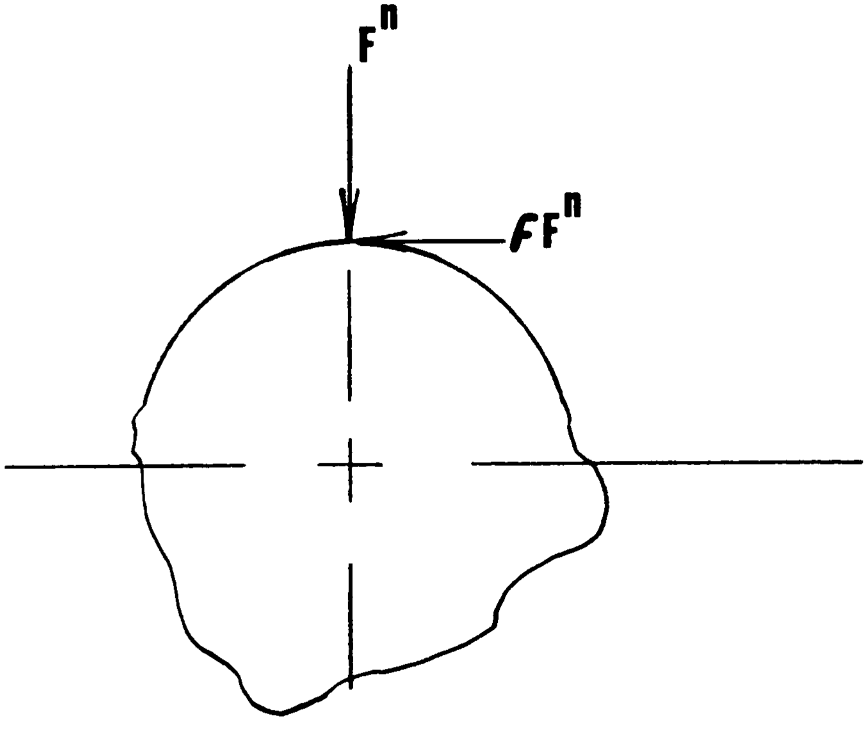

Figure 3.7. A Normal Load and a Tangential Load Acting on a Body.

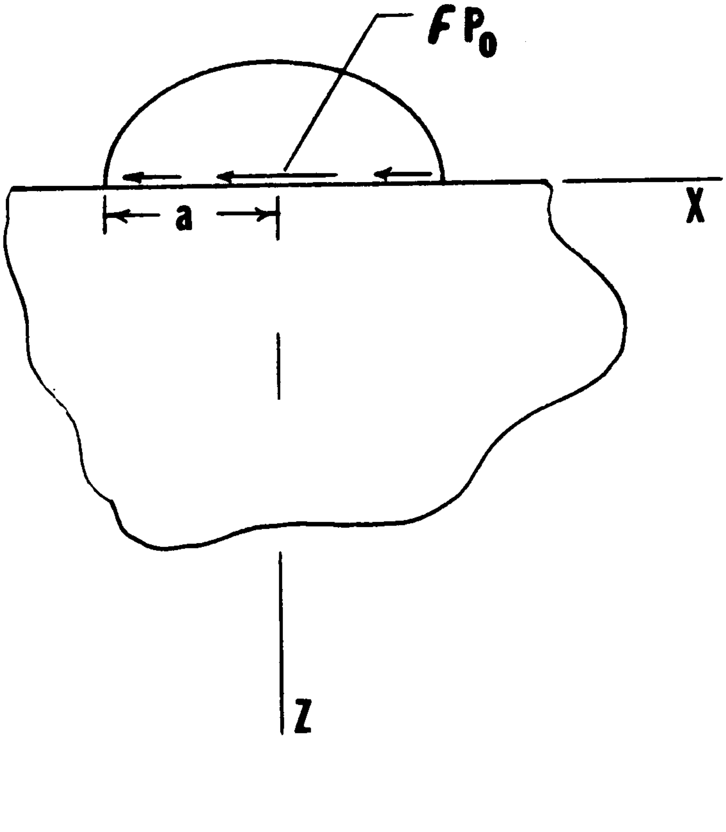

Figure 3.8. A Hertzian Distribution of a Tangential Load.

The Smith-Liu[1] equations are expressions for the stress on a body due to normal and tangential loads distributed in Hertzian (elliptical) fashion over a region on the boundary of the body. These stresses are functions of the material of the body, the geometry of the problem, and the normal load. The tangential load is considered to be a function of the normal load.

In the development of the Smith-Liu equations, several assumptions were made. As these equations will be used throughout the rest of this investigation, it is important to list these.

1. Both the tangential and normal loads are considered to be distributed in elliptical fashion over the area of contact, as shown previously.
2. The magnitude of the intensity of the tangential load is considered to be proportional to that of the normal load.
3. The boundary of the surface of contact is an ellipse. (For the case in consideration, it is a very narrow ellipse and can be approximated by a narrow rectangle.

[1] J. O. Smith and C. K. Liu, "Stresses Due to Tangential and Normal Loads on an Elastic Solid with Application to Some Contact Stress Problems," Journal of Applied Mechanics, June 1952.

4. The surface of contact is not assumed to be a flat surface.

The stresses given by the Smith-Liu equations on either the cam or the follower due to the presence of an elliptical distribution of a normal force over the area of contact are:

$$\sigma_{xN} = -\frac{P_o z}{\pi}\left[\frac{a^2 + 2x^2 + 2z^2}{a}\bar{\Psi} - \frac{2\pi}{a} - 3x\Psi\right] \qquad (3.15)$$

$$\sigma_{zN} = -\frac{P_o z}{\pi}\left[a\bar{\Psi} - x\Psi\right] \qquad (3.16)$$

Where "N" refers to the stresses caused by the distributed normal force.

If the cam and the follower are subject to a tangential force that is distributed along a portion of their boundaries, the stresses caused by this force are:

$$\sigma_{xT} = -\frac{fP_o}{\pi}\left[(2x^2 - 2a^2 - 3z^2)\Psi + 2\pi\frac{x}{a}\right.$$

$$\left. + 2(a^2 - x^2 - z^2)\frac{x}{a}\bar{\Psi}\right] \qquad (3.17)$$

$$\sigma_{zT} = -\frac{fP_o}{\pi} z^2 \Psi \qquad (3.18)$$

The subscript "T" refers to the tangential force causing the stress. The coordinate system defined previously is still in use.

If the normal load per unit length in the y-direction applied on the boundary is defined as

$$\beta = \frac{F^{N}_{f/c}}{w} \tag{3.19}$$

where w is the dimension of the cam in the $Y_c - Z_c$ plane, then the half width of the contact area is given by

$$a = \sqrt{\frac{2\beta\Delta}{\pi}} \tag{3.20}$$

This half width of the contact area is shown in Figure 3.6, and has units of inches. Equation (3.20) was determined by the conditions of compliance between the two bodies.

Δ is defined as

$$\Delta = \frac{1}{\frac{1}{2}\left(\frac{1}{R_c} + \frac{1}{R_f}\right)}\left(\frac{1-\gamma_f^2}{E_f} + \frac{1-\gamma_c^2}{E_c}\right) \tag{3.21}$$

Δ has units of inches cubed per pound.

The symbols R_f, E_f and γ_f represent the radius, the modulus of elasticity and Poisson's ratio, respectively, for the follower and R_c, E_c and γ_c represent the radius at the point of contact with the follower, the modulus of elasticity and Poisson's ratio, respectively, for the cam.

To complete the presentation of the Smith-Liu equations, the parameters Ψ, $\overline{\Psi}$ and P_o need definition.

$$\Psi = \frac{\pi}{K_1} \frac{1 - \sqrt{\frac{K_2}{K_1}}}{\sqrt{\frac{K_2}{K_1}}\sqrt{2\sqrt{\frac{K_2}{K_1}} + \frac{K_1 + K_2 - 4a^2}{K_1}}} \qquad (3.22)$$

and

$$\overline{\Psi} = \frac{\pi}{K_1} \frac{1 + \sqrt{\frac{K_2}{K_1}}}{\sqrt{\frac{K_2}{K_1}}\sqrt{2\sqrt{\frac{K_2}{K_1}} + \frac{K_1 + K_2 - 4a^2}{K_1}}} \qquad (3.23)$$

The expressions K_1 and K_2 are defined as

$$K_1 = (a + x)^2 + z^2 \qquad (3.24)$$

$$K_2 = (a - x)^2 + z^2 \qquad (3.25)$$

As a result of the assumption being made that the load is distributed in elliptical fashion over the contact area, it can be written

$$P_o = \frac{2\beta}{\pi a} \qquad (3.26)$$

Simplifying the Expressions for the Stresses

The Smith-Liu equations are fairly complex and difficult to use in their present form. By recalling that the main objective of this investigation is to represent the non-rigidity between the cam and follower by a spring with a determined constant several simplifications to these equations can be made.

To determine a spring constant for the above purpose, the deflections that occur as the cam and the follower are pressed together and the magnitude of this pressing force are required.

To determine the deflections caused by these bodies being pressed together, the Smith-Liu equations will be used. The specific deflections that need to be determined are those of the points that lie in the $Z - Y$ plane. Additionally, no integrations will be made with respect to X so it can be treated as a parameter rather than a variable.

Therefore, to simplify the Smith-Liu equations the X-terms in the equations can be set equal to zero. This is a simplification unique only to this particular problem and not a general means of using the Smith-Liu equations.

By implementing this simplification,

$$K_1 = a^2 + z^2 = K_2 \tag{3.27}$$

As $K_1 = K_2$ they will be referenced in the future without subscripts. That is,

$$K_1 = K_2 = K \tag{3.28}$$

Substituting equation (3.28) into equation (3.22) the expression for Ψ becomes

$$\Psi = \frac{\pi}{K} \frac{1 - \sqrt{\frac{K}{K}}}{\sqrt{\frac{K}{K}}\sqrt{2\sqrt{\frac{K}{K}} + \frac{K + K - 4a^2}{K}}} \tag{3.29}$$

which readily yields the result

$$\Psi = 0 \tag{3.30}$$

Likewise, by substituting equation (3.28) into equation (3.23) the expression for $\overline{\Psi}$ becomes

$$\overline{\Psi} = \frac{\pi}{K} \frac{1 + \sqrt{\frac{K}{K}}}{\sqrt{\frac{K}{K}}\sqrt{2\sqrt{\frac{K}{K}} + \frac{K + K - 4a^2}{K}}} \tag{3.31}$$

which reduces to the expression

$$\overline{\Psi} = \frac{\pi}{z\sqrt{z^2 + a^2}} \tag{3.32}$$

Using the results from equations (3.30) and (3.32) and again, recalling that the plane of interest is the Y-Z plane, the stress equations can be reduced to

$$\sigma_{xN} = -\frac{P_o(a^2 + 2z^2)}{a\sqrt{a^2 + z^2}} + \frac{2P_o z}{a} \tag{3.33}$$

$$\sigma_{zN} = -\frac{P_o a}{\sqrt{a^2 + z^2}} \tag{3.34}$$

$$\sigma_{xT} = 0 \tag{3.35}$$

$$\sigma_{zT} = 0 \tag{3.36}$$

By recalling that an assumption previously made (Chapter Two) was that the cam and the follower are in a condition of plane strain, it can be written that

$$\varepsilon_y = \frac{1}{E}(\sigma y - \gamma(\sigma_x + \sigma_z)) = 0 \tag{3.37}$$

or

$$\sigma_y = \gamma(\sigma_x + \sigma_z) \tag{3.38}$$

Substituting equations (3.33) and (3.34) into equation (3.38) yields

$$\sigma_{yN} = -\frac{P_o\gamma(a^2 + 2z^2)}{a\sqrt{a^2 + z^2}} + \frac{2\gamma P_o z}{a} - \frac{P_o\gamma a}{\sqrt{a^2 + z^2}} \qquad (3.39)$$

In a previous section, the force exerted on the follower by the cam was determined. In that analysis a tantions (3.35) and (3.36) show that the assumption made in not considering this force was justified. The tangential force will not contribute to the further results of this study. As the stresses created by a tangential distributed load do not contribute to the overall stress distribution, the subscripts will be dropped and therefore $\sigma_x = \sigma_{xN}$, $\sigma_y = \sigma_{yN}$ and $\sigma_z = \sigma_{zN}$. These expressions will be used in the next two sections to determine the deflections of interest.

The Deflection(s) of Points on the Follower

The deflection(s), of points on the follower, in the Z_f direction are the deflections of interest. If w_f is defined as these deflections, Hooke's Law can be written for the strain in the Z-direction of the follower.

$$\varepsilon_{zf} = \frac{1}{E_f}\left[\sigma_{zf} - \gamma_f\left(\sigma_{xf} + \sigma_{yf}\right)\right] \qquad (3.40)$$

and from the theory of elasticity,

$$\varepsilon_{zf} = \frac{\partial w_f}{\partial z_f} \tag{3.41}$$

The parameters are subscripted with an "f" to distinguish them as parameters of the follower.

By substituting equations (3.33), (3.34) and (3.39) into equation (3.40), the expression for the strain becomes

$$\varepsilon_{zf} = -\frac{P_o a}{E_f\sqrt{z_f^2 + a^2}} + \frac{P_o \gamma_f a}{E_f\sqrt{a^2 + z_f^2}} + \frac{2z_f^2 P_o \gamma_f}{aE_f\sqrt{z_f^2 + a^2}}$$

$$- \frac{2\gamma_f P_o z_f}{E_f a} + \frac{P_o \gamma_f^2 a}{E_f\sqrt{a^2 + z_f^2}} + \frac{2P_o \gamma_f^2 z_f^2}{E_f a\sqrt{a^2 + z_f^2}} - \frac{2\gamma_f^2 P_o z_f}{E_f a}$$

$$+ \frac{P_o \gamma_f^2 a}{E_f\sqrt{a^2 + z_f^2}} \tag{3.42}$$

Recalling from equation (3.41) that $\varepsilon_{zf} = \frac{\partial w_f}{\partial z_f}$, an expression for the deflections of points on the follower in terms of (x, z) (where, in this case x = O) can be found by integration of (3.49). The integration of this expression is not difficult, especially as the stresses in the Y-Z plane are only functions of z.

Integrating equation (3.42) yields

$$w_f = -\frac{P_o a}{E_f}\sinh^{-1}\left(\frac{z_f}{a}\right) + \frac{P_o \gamma_f a}{E_f}\sinh^{-1}\left(\frac{z_f}{a}\right)$$

$$+ \frac{2P_o\gamma_f}{aE_f}\left[\frac{z_f\sqrt{z_f^2 + a^2}}{2} - \frac{a^2}{2}\sinh^{-1}\left(\frac{z_f}{a}\right)\right] - \frac{\gamma_f P_o z_f^2}{E_f a}$$

$$+ \frac{P_o\gamma_f^2 a}{E_f}\sinh^{-1}\left(\frac{z_f}{a}\right) + \frac{2P_o\gamma_f^2}{E_f a}\left[\frac{z_f\sqrt{z_f^2 + a^2}}{2}\right.$$

$$\left. - \frac{a^2}{2}\sinh^{-1}\left(\frac{z_f}{a}\right)\right] - \frac{P_o\gamma_f z_f^2}{E_f a} + \frac{P_o\gamma_f^2 a}{E_f}\sinh^{-1}\left(\frac{z_f}{a}\right) + F \tag{3.43}$$

A dimensional check of equation (3.43) yields the units of length for each term--the appropriate units for deflections.

To determine the constant of integration in equation (3.43), it is necessary to consider the boundary conditions of the system. The follower rotates about an axis that passes through the geometric center of the cross section of the follower. In a real system the shaft that the follower rotates about will be designed so that it will not deflect under the action of the load pressing the follower to the cam. Additionally the axis z_f will pass through the center of curvature of the follower. As a boundary condition to determine the value of this constant, and by denoting the radius of the follower as R_f,

$$w_f(z_f = R_f) = 0 \tag{3.44}$$

Rewriting the complete expression for w_f as

$$w_f = f(z_f) + F \tag{3.45}$$

where $f(Z)$ is the expression preceding "F" in equation (3.43). Substituting equation (3.44) in equation (3.45)

$$w_f(z_f) = f(z_f) + F = 0 \tag{3.46}$$

which yields

$$F = - f(z_f) \tag{3.47}$$

To solve for $f(X_f)$ it is only necessary to substitute $z_f = R_f$ in the expression for $F(z_f)$.

Making the substitution and solving for F,

$$F = \frac{P_o a}{E_f} \sinh^{-1}\left(\frac{R_f}{a}\right) - \frac{P_o \gamma_f a}{E_f} \sinh^{-1}\left(\frac{R_f}{a}\right)$$

$$- \frac{2P_o\gamma_f}{aE_f}\left[\frac{R_f\sqrt{R_f^{\,2} + a^2}}{2} - \frac{a^2}{2} \sinh^{-1}\left(\frac{R_f}{a}\right)\right] + \frac{\gamma_f P_o R_f^{\,2}}{E_f a}$$

$$- \frac{P_o \gamma_f^{\,2} a}{E_f} \sinh^{-1}\left(\frac{R_f}{a}\right) - \frac{2P_o \gamma_f^{\,2}}{E_f a}\left[\frac{R_f\sqrt{R_f^{\,2} + a^2}}{2}\right.$$

$$\left. - \frac{a^2}{2} \sinh^{-1}\left(\frac{R_f}{a}\right)\right] + \frac{P_o \gamma_f^2 R_f^{\,2}}{E_f a} - \frac{P_o \gamma_f^{\,2} a}{E_f} \sinh^{-1}\left(\frac{R_f}{a}\right)$$

(3.48)

Again, the complete expression for w_f can be written

$$w_f = f(z_f) + F \tag{3.49}$$

The Deflection(s) of Points on the Cam

By using the results obtained in the previous section, particularly equation (3.43), the general expression for the deflection of points on the cam in the z_c direction can be written

$$w_c = - \frac{P_o a}{E_c} \sinh^{-1}\left(\frac{z_c}{a}\right) + \frac{P_o \gamma_c a}{E_c} \sinh^{-1}\left(\frac{z_c}{a}\right)$$

$$+ \frac{2P_o \gamma_c}{aE_c}\left[\frac{z_c\sqrt{z_c^{\,2} + a^2}}{2} - \frac{a^2}{2} \sinh^{-1}\left(\frac{z_c}{a}\right)\right] - \frac{\gamma_c P_o z_c^{\,2}}{E_c a}$$

$$+ \frac{P_o \gamma_c^{\,2} a}{E_c} \sinh^{-1}\left(\frac{z_c}{a}\right) + \frac{2P_o \gamma_c^{\,2}}{E_c a}\left[\frac{z_c\sqrt{z_c^{\,2} + a^2}}{2}\right.$$

$$- \frac{a^2}{2} \sinh^{-1}\left(\frac{z_c}{a}\right)\Bigg] - \frac{P_o \gamma_c{}^2 z_c{}^2}{E_c a} + \frac{P_o \gamma_c{}^2 a}{E_c} \sinh^{-1}\left(\frac{z_c}{a}\right) + C$$

(3.50)

In keeping with the convention previously used, the subscript "c" refers to parameters of the cam.

To evaluate the constant of integration in equation (3.50) requires an examination of the system. For any real cam system, the radius of curvature of the cam at the point of contact with the follower will be very large with respect to one-half of the dimension of the contact area in the X-Z plane. The proof of this can be found in Appendix C. As a result, w_c will be a local deformation and the same principle applied to the follower can be applied to the cam. To evaluate C, it can be written

$$w_c(z_c = R_c) = 0 \tag{3.51}$$

where R_c is the radius of curvature of the cam at the point of contact with the follower.

Applying equation (3.51) to equation (3.50) in a manner similar to that of the previous section

$$C = \frac{P_o a}{E_c} \sinh^{-1}\left(\frac{R_c}{a}\right) - \frac{P_o \gamma_c a}{E_c} \sinh^{-1}\left(\frac{R_c}{a}\right)$$

$$- \frac{2P_o\gamma_c}{aE_c}\left[\frac{R_c\sqrt{R_c^2 + a^2}}{2} - \frac{a^2}{2}\sinh^{-1}\left(\frac{R_c}{a}\right)\right] + \frac{\gamma_c P_o R_c^2}{E_c a}$$

$$- \frac{P_o\gamma_c^2 a}{E_c}\sinh^{-1}\left(\frac{R_c}{a}\right) - \frac{2P_o\gamma_c^2}{E_c a}\left[\frac{R_c\sqrt{R_c^2 + a^2}}{2}\right.$$

$$\left. - \frac{a^2}{2}\sinh^{-1}\left(\frac{R_c}{a}\right)\right] + \frac{P_o\gamma_c^2 R_c^2}{E_c a} - \frac{P_o\gamma_c^2 a}{E_c}\sinh^{-1}\left(\frac{R_c}{a}\right) \qquad (3.52)$$

Using the notation previously developed

$$w_c = f(z_c) + C \qquad (3.53)$$

where $f(z_c)$ is defined in equation (3.50).

It can be readily seen that the form for the deformation of the cam is identical to that of the follower. Using equations (3.52) and (3.50) would yield the deflection of any point that lies in the $Y_c - Z_c$ plane in the z_c direction. The deflections are not a function of y.

The expressions for w_f and w_c will be utilized in the next chapter to determine a spring constant for a spring representing the non-rigidity between the cam and the follower.

CHAPTER IV

REPRESENTING THE NON-RIGIDITY BETWEEN A CAM AND A FOLLOWER BY A SPRING

Introduction

Figure 4.1 represents a cross section of the cam and the follower in contact. If there is no pressure between the bodies, there is contact only along the line drawn perpendicular to the X-Z plane and passing through the points P_c and P_f. P_c is a point on the cam that lies on this line of contact and has coordinates (x_c, y_c, z_c) of (o, y_c, o). P_f is a point on the follower that also lies on this line and has coordinates (x_f, y_f, z_f) of (o, y_f, o).

However, the two bodies are being pressed together by the normal forces, directed as shown in Figure 4.1. Due to the action of these forces there is local deformation at the point of contact and rather than a line of contact there is contact between the two bodies over a narrow rectangle.

C_f and C_c are the centers of curvature of the follower and the cam, respectively. Before the two bodies were pressed together the distance from C_f to P_f would be the radius of the follower, designated R_f in the previous chapter. Likewise, the distance R_c, the radius

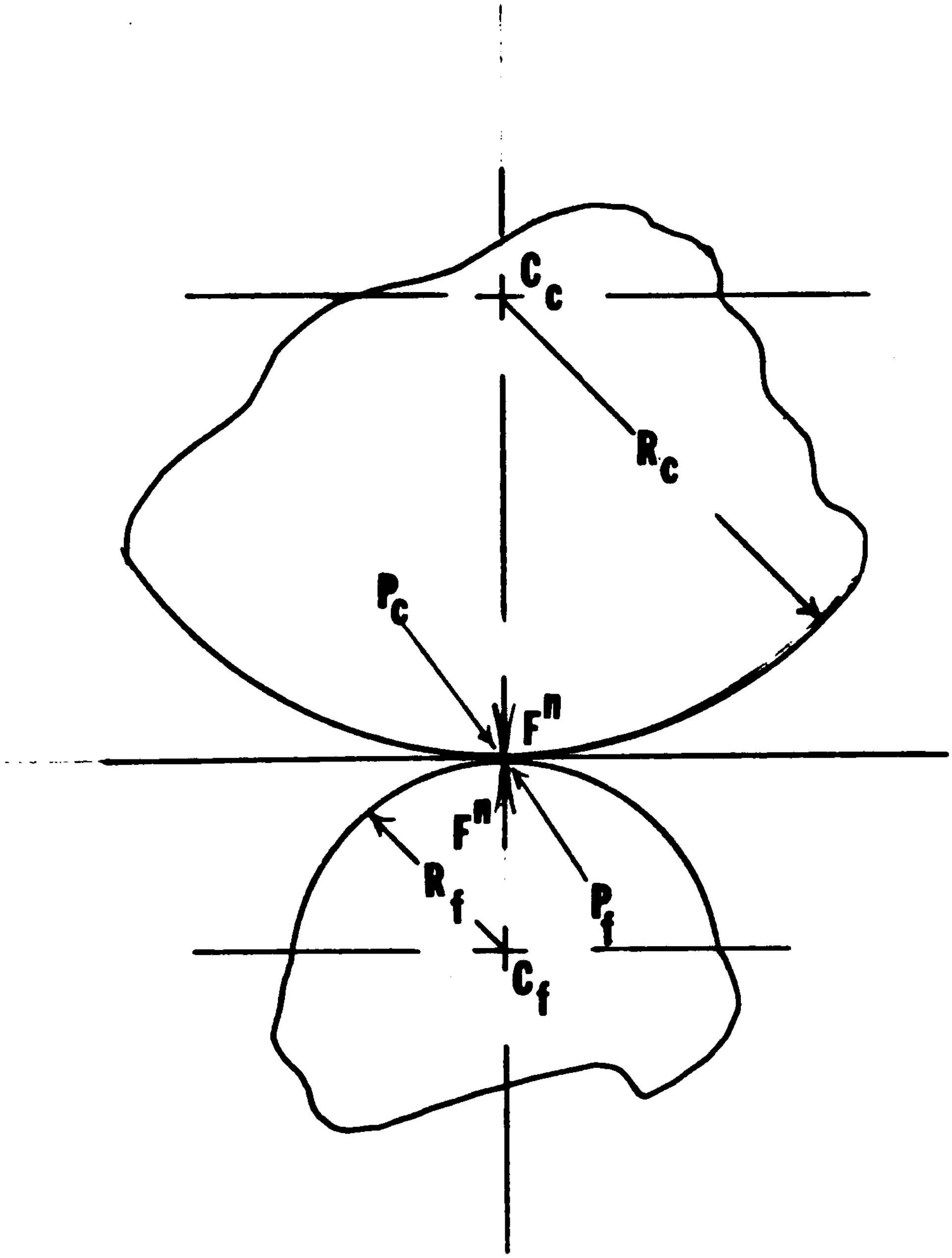

Figure 4.1. The Two Bodies Being Pressed Together.

of curvature of the cam, would be the distance from C_c to the point P_c before the two bodies were pressed together.

As the two bodies are pressed together, there is localized deformation in the vicinity of points P_c and P_f. This local deformation will cause points C_c and C_f to approach each other by a certain amount. This amount will be the sum of the deflection of point P_c in the direction of Z_c and the displacement of point P_f in the direction of Z_f.

Displacement of Point P_c in the the Z_c Direction

The displacement of point P_c in the Z_c direction is not difficult to find, given the fact that equation (3.50) and equation (3.52) in conjunction with each other yield the displacement of any point on the line with coordinates of (x_c, y_c, z_c) of (o, y_c, z_c), in the Z_c direction.

By recognizing that P_c lies on this line and applying equations (3.50) and (3.52), the displacement of point P_c in the Z_c direction, $_{P_c}$, can be written as

$$\alpha_{P_c} = \frac{P_o a}{E_c} \sinh^{-1}\left(\frac{R_c}{a}\right) - \frac{P_o \gamma_c a}{E_c} \sinh^{-1}\left(\frac{R_c}{a}\right)$$

$$- \frac{2P_o\gamma_c}{aE_c}\left[\frac{R_c\sqrt{R_c^2 + a^2}}{2} - \frac{a^2}{2}\sinh^{-1}\left(\frac{R_c}{a}\right)\right]$$

$$+ \frac{\gamma_c P_o R_c^2}{E_c a} - \frac{P_o\gamma_c^2 a}{E_c}\sinh^{-1}\left(\frac{R_c}{a}\right)$$

$$- \frac{2P_o\gamma_c^2}{E_c a}\left[\frac{R_c\sqrt{R_c^2 + a^2}}{2} - \frac{a^2}{2}\sinh^{-1}\left(\frac{R_c}{a}\right)\right]$$

$$+ \frac{P_o\gamma_c^2 R_c^2}{E_c a} - \frac{P_c\gamma_c^2 a}{E_c}\sinh^{-1}\left(\frac{R_c}{a}\right) \qquad (4.1)$$

Displacement of Point P_f in the Z_f Direction

The displacement of point P_f in the Z_f direction is also not very difficult to find. Using the same procedure outlined in the previous section, it can be shown that

$$\alpha_{P_f} = \frac{P_o a}{E_f}\sinh^{-1}\left(\frac{R_f}{a}\right) - \frac{P_o\gamma_f a}{E_f}\sinh^{-1}\left(\frac{R_f}{a}\right)$$

$$- \frac{2P_o\gamma_f}{aE_f}\left[\frac{R_f\sqrt{R_f^2 + a^2}}{2} - \frac{a^2}{2}\sinh^{-1}\left(\frac{R_f}{a}\right)\right] + \frac{\gamma_f P_o R_f^2}{E_f a}$$

$$- \frac{P_o\gamma_f^2 a}{E_f}\sinh^{-1}\left(\frac{R_f}{a}\right) - \frac{2P_o\gamma_f^2}{E_f a}\left[\frac{R_f\sqrt{R_f^2 + a^2}}{2}\right.$$

$$- \frac{a^2}{2} \sinh^{-1}\left(\frac{R_f}{a}\right)\Bigg] + \frac{P_o \gamma_f^2 R_f^2}{E_f a} - \frac{P_o \gamma_f^2 a}{E_f} \sinh^{-1}\left(\frac{R_f}{a}\right) \tag{4.2}$$

Determining the Spring Constant

By definition, a spring is a machine element designed to give an elastic deflection under a given load. Stated inversely, a spring is an element which exerts a force when deformed. This can be shown algebraically as

$$F_s = K_s \delta_s \tag{4.3}$$

Where F_s is the force being exerted by the spring, K_s is the spring constant, and δ_s is the deflection of the spring from its free, unstretched position.

For the problem at hand, to replace the elasticity of the cam and the follower by a spring requires a bit of analysis.

In order for this spring to accurately reflect the elastic properties of the cam and the follower, the deflection of the spring under the applied normal load pressing together the cam and the follower must be identical to the actual deflections of points on the cam and the follower calculated in the previous section.

If such a spring is found, and located in such a manner as to accomplish the above, then the cam and the follower will be assumed to be rigid and the only non-rigidity in the cam system will be the spring itself.

It is extremely important to note that if a spring constant is calculated for a spring representing the non-rigidity in the cam system, it is only valid for the instant of consideration. The spring constant will be a function of the geometry of the problem, the forces on the cam and the overall parameters of the problem such as the modulus of elasticity of the two bodies.

As the cam rotates the spring constant for the spring representing the elasticity in the cam system will change. The spring that does represent the non-rigidity in the cam mechanism will have a constant that is a function of time.

As previously determined in the first two sections of this chapter the deflection of the point P_c on the cam in the direction of Z_c will be α_{P_c} and the deflection of the point P_f on the follower in the direction of Z_f will be α_{P_f}. These deflections are caused by the forces pressing the two bodies together.

Due to these forces C_f and C_c will approach each other by the amount δ_T, where

$$\delta_T = \alpha_{P_c} + \alpha_{P_f} \tag{4.4}$$

The force pressing these two bodies together is the normal force $F_{f/c}^{N}$ (= $F_{c/f}^{N}$). By definition, a spring constant relates the deflection of the spring to the force required to cause that deflection. Utilizing equation (4.3)

$$K_E = \frac{F_{f/c}^{N}}{\delta_T} \tag{4.5}$$

Figure 4.2 shows how the spring representing the non-rigidity of the cam system will be placed. The spring will have the value of the spring constant calculated in equation (4.5).

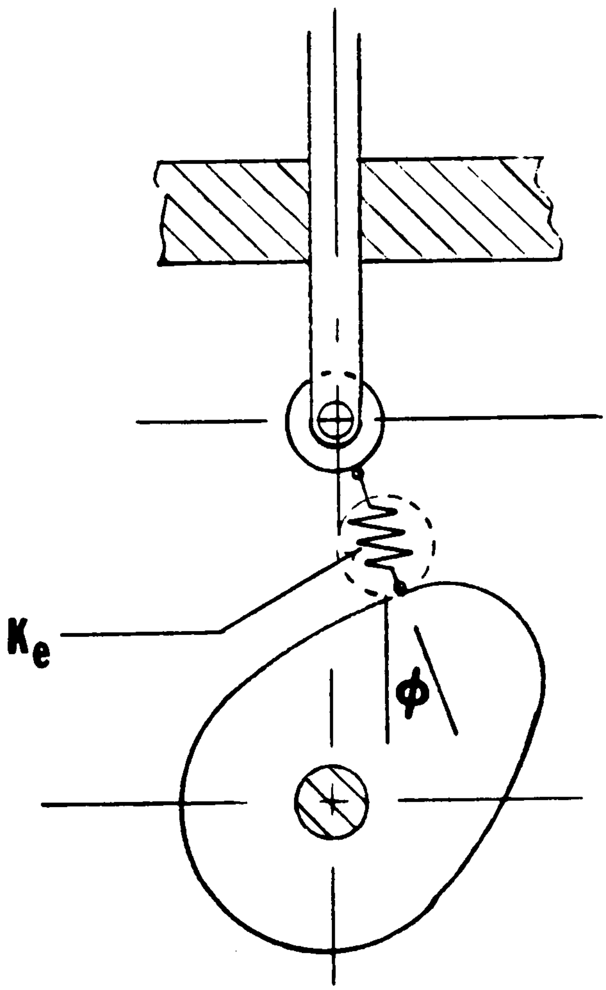

Figure 4.2. A Spring Replacing the Non-Rigidity in the Cam System.

CHAPTER V

DISCUSSION ON THE RESULTS OF THE STUDY

Introduction

The previous four chapters were devoted to determining the amount of non-rigidity in the cam and follower system and representing that non-rigidity by a spring.

This chapter will deal with the results obtained in the previous chapters. It is broken up into sections, each designed to provide a clean analysis of one aspect of the results.

Limitations to the Study

There are some specific warnings that must be made regarding the results of this study. By using the methods developed throughout the previous chapters a spring constant will be determined for a spring that represents the non-rigidity in the cam system. The results are valid only for the instant of consideration. The normal force pressing the cam and the follower together, the radius of curvature of the cam and the parameters associated with the Smith-Liu equations will be constantly changing as the cam rotates. The spring representing this non-rigidity will be a function of time and its constant must be determined at every cam position.

Design of a Spring with a Desired Spring Constant

To determine the characteristics of a spring that may be used to replace the non-rigidity in a cam system for a given cam position, the following parameters are considered.

$R_c = 2$ inches

$R_f = \frac{1}{2}$ inch

Width of the cam and the follower = $\frac{1}{2}$ inch

$E_c = E_f = 30 \times 10^6$ psi

$\gamma_c = \gamma_f = 0.3$

Normal force pressing the cam and the follower together = 1000 lbs.

With the calculations included in Appendix A,

$$K_E = 2.436 \times 10^6 \text{ lb/in}$$

A helical spring with as large a spring constant as the above value would be very difficult to design. Any elastic body would be suitable for use as a spring. To illustrate this principle a longitudinal beam will be placed into service as a spring.

The spring constant for a beam used in such a manner is given by

$$K_B = \frac{EA}{L} \tag{5.1}$$

Where E is the modulus of elasticity of the beam material, A is the cross-sectional area of the beam and L is the length of the beam. If the beam is specified to be constructed of steel with $E = 30 \times 10^6$ psi and the cross-sectional area is arbitrarily chosen to be 0.897 square inches, then the length of the beam is 11 inches.

The physical characteristics of the spring that is chosen could certainly be analyzed to gain insight into the dynamics of a real cam system. Without drawing any hard conclusions this approach suggests a possible way to use the results of this study.

Contact Stress Applications

When discussing contact stresses many authors implicitly assume that the area of contact between the two bodies in consideration is a flat plane.

It is interesting to analyze the results of this study while keeping in mind the above statement. In the calculations in Appendix A, $\alpha_{P_c} = 0.000232$ inches and $\alpha_{P_f} = 0.000178$ inches. If the area of contact between the cam and the follower was a flat plane, the deflection of the point on the surface of the body with the smaller radius of curvature would be significantly greater than a corresponding point on the other body as shown in Figure 5.1.

In determining the deflections of P_c and P_f there were no assumptions made concerning the shape of the contact area. The only assumption made was that at the center of the radius of curvature of both the cam and the follower the deflections were zero. This assumption was made to solve the boundary conditions.

As can be seen from the deflections calculated in Appendix A the follower "digs" into the cam surface and the surface of contact is not a flat plane. The projection of the surface onto the X-Y plane is still a narrow rectangle.

This study does not conclude that the surface of contact is not a flat plane. Rather it proposes that additional study be made in that area to determine what the shape of that surface is.

Deflections

The deflections of the points P_c and P_f caused by the action of the normal force pressing the cam and the follower together are fairly minimal. The dimensions and parameters used in these calculations are for a typical cam and follower system as might be found in an automobile.

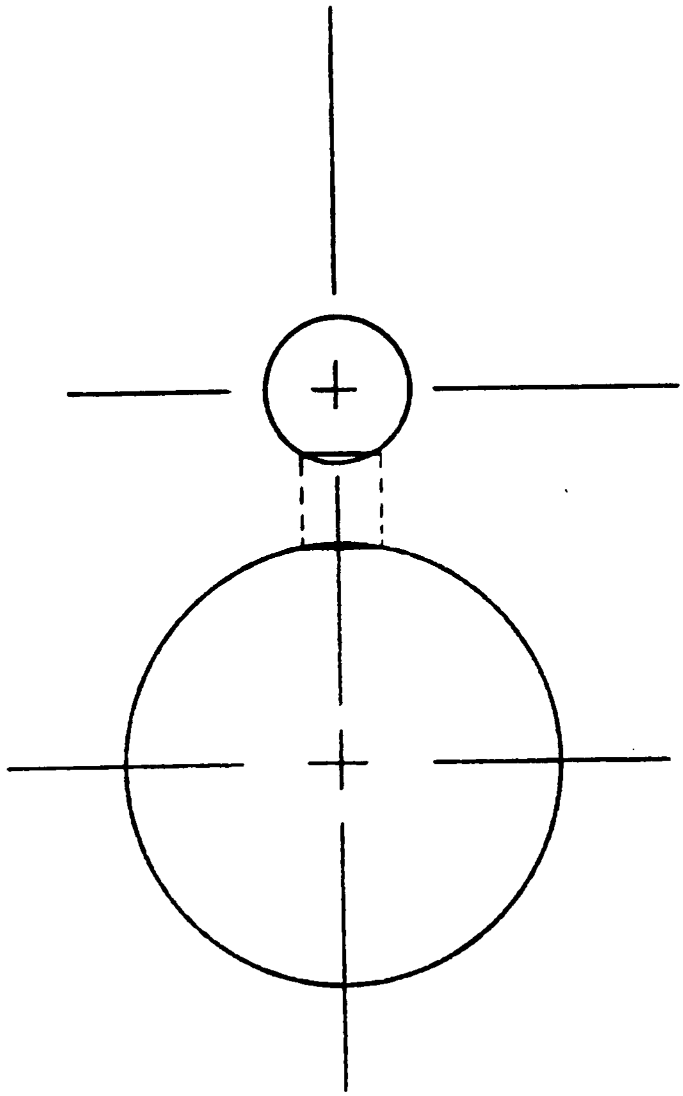

Figure 5.1. Relative Deflection of Bodies if Contact Is Made over a Flat Plane.

With these results it may not be unrealistic in designing a cam system to assume that the cam and the follower are rigid bodies. Additional investigation as suggested by earlier sections in this chapter would be required before any conclusions can be made.

Importance of σ_x and σ_y

Throughout the study a point of discussion was how the stress components σ_x and σ_y affected the deflections of points P_c and P_f. This matter is considered in Appendix B. If σ_x and σ_y are not considered in determining the values α_{P_c} and α_{P_f}, the results are significantly different than if those stresses were considered.

Comparison of Results

There is a published solution to the case of a cylinder with a finite radius of curvature being pressed together with a cylinder with as infinite radius of curvature.[1] The results obtained in this investigation agree very well with these results, within two to three percent.

As might be expected the deflections of points P_c and P_f are identical for a cam and a follower

[1] M. F. Spotts, Design of Machine Elements (New York: Prentice-Hall Book Company, 1971), p. 369.

constructed of the same material with identical radii of curvatures at the point of contact.

Importance of the Tangential Force

A rather important conclusion of this investigation was that the tangential force did not contribute to the deflections of points P_f and P_c. Along the plane of interest, the Y - Z plane, the tangential force did not affect the expressions for the stress.

Plots of Results

Plots of results from this can be found in Appendix D.

LIST OF REFERENCES

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APPENDICES

APPENDIX A

CALCULATIONS TO DETERMINE A SPRING CONSTANT

The following parameters are known:

$$R_c = 2 \text{ inches}$$

$$R_f = 0.5 \text{ inches}$$

$$W = 0.5 \text{ inches}$$

$$E_c = E_f = 30 \times 10^6 \text{ psi}$$

$$\gamma_c = \quad _f = 03$$

$$F^N{}_{F/C} = 1000 \text{ lb.}$$

$$\Delta = \frac{1}{\frac{1}{2}\ \frac{1}{R_c} + \frac{1}{R_f}} \quad \frac{1-\gamma_c{}^2}{E_c} + \frac{1-\gamma_f{}^2}{E_f}$$

$$\Delta = \frac{1}{\frac{1}{2}\ \frac{1}{2} + \frac{1}{0.5}} \quad \frac{1-(0.2)^2}{30 \times 10^6} \quad \frac{1-(0.3)^2}{30 \times 10^6}$$

$$\Delta = 4.8533 \times 10^{-8} \ \frac{IN^3}{LB}$$

Since P = Normal load per unit length,

$$P = \frac{F^N{}_{F/c}}{W} = \frac{1000}{0.5} = 2000 \ \frac{LB}{IN}$$

$$a = \frac{2P\Delta}{\pi} = \frac{(2)(2000)(4.85333 \times 10^{-8})}{\pi}$$

$$a = 0.00786 \text{ inches}$$

$$P_o = \frac{2P}{\pi a} = \frac{(2)\ (2000)}{\pi(0.00786)}$$

$$P_o = 161970 \ \frac{LB}{IN^2}$$

$$\alpha_{P_c} = \frac{P_o a}{E_c} \sinh^{-1} \frac{R_c}{a} - \frac{P_o \gamma_c a}{E_c} \sinh^{-1} \frac{R_c}{a}$$

$$\frac{-2P_c \gamma_c}{aE_c} \; \frac{R_c \; R_c^2 + a^2}{2} - \frac{a^2}{2} \sinh^{-1} \frac{R_c}{a} + \frac{\gamma_c P_o R_c^2}{E_c a}$$

$$- \frac{P_o \gamma_c^2 a}{E_c} \sinh^{-1} \frac{R_c}{a} - \frac{2P_o \gamma_c^2}{E_c a} \; \frac{R_c \; R_c + a^2}{2} - \frac{a^2}{2} \sinh^{-1} \frac{R_c}{a}$$

$$+ \frac{P_o \gamma^2 R_c^2}{E_c a} - \frac{P_o \gamma_c^2 a}{E_c} \sinh^{-1} \frac{R_c}{a}$$

To help reach a solution,

$$\sinh^{-1} \frac{R_c}{a} = \sinh^{-1} \frac{2}{0.00786}$$

$$\sinh^{-1} \frac{R_c}{a} = 6.2322$$

Substituting in numbers,

$$\alpha_{P_c} = \frac{(161970)(0.00786)}{30 \times 10^6} (6.2322)$$

$$- \frac{(161970)(0.3)(0.00786)(6.2322)}{30 \times 10^6}$$

$$- \frac{(2)(161970)(0.3)}{(0.00786((30 \times 10^6)} \; \frac{2(2)^2 + (0.00786)^2}{2}$$

$$- \frac{(0.00786)^2}{2}(6.2322) + \frac{(0.3)(161970)(2)^2}{(30 \times 10^6)(0.00786)}$$

$$- \frac{(161970)(0.3)^2(0.00786)}{30 \times 10^6}(6.2322)$$

$$- \frac{(2)(161970)(0.3)^2}{(30 \times 10^6)\ (0.00786)} \quad \frac{2\ (2)^2 + (0.00786)^2}{2} - \frac{(0.00786)^2}{2}(6.2322)$$

$$+ \frac{(161970)(0.3)^2(2)^2}{(30 \times 10^6)(0.00786)} - \frac{(161970)(0.3)^2(0.00786)}{(30 \times 10^6)}(6.2322)$$

$$\alpha_{P_c} = 0.000194 \text{ in.}$$

Likewise, to calculate α_{P_f},

$$\alpha_{P_f} = \frac{P_o a}{E_f} \sinh^{-1} \frac{R_f}{a} - \frac{P_o \gamma_f a}{E_f} \sinh^{-1} \frac{R_f}{a}$$

$$- \frac{2P_o \gamma_f}{aE_f} \quad \frac{R_f\ R_f^{\ 2} + a^2}{2} - \frac{a^2}{2} \sinh^{-1} \frac{R_f}{a} + \frac{\gamma_f P_o F_f^{\ 2}}{E_f a}$$

$$- \frac{P_o \gamma_f^{\ 2} a}{E_f} \sinh^{-1} \frac{R_f}{a} - \frac{2P_o \gamma_f^{\ 2}}{E_f a} \quad \frac{R_f\ R_f^{\ 2} + a^2}{2} - \frac{a^2}{2} \sinh^{-1} \frac{R_f}{a}$$

$$+ \frac{P_o \gamma_f^{\ 2} R_f^{\ 2}}{E_f a} - \frac{P_o \gamma_f^{\ 2} a}{E_f} \sinh^{-1} \frac{R_f}{a}$$

To help solve for α_{P_f},

$$\sinh^{-1}\frac{R_f}{a} = \sinh^{-1}\frac{0.5}{0.00786}$$

$$\sinh^{-1}\frac{R_f}{a} = 4.8460$$

Substituting,

$$\alpha_{P_f} = \frac{(161970)(0.00786)}{30 \times 10^6}(4.8460)$$

$$- \frac{(161970)(0.00786)(0.3)(4.8460)}{30 \times 10^6}$$

$$- \frac{(2)(161970)(0.3)}{(0.00786)(30 \times 10^6)} \frac{0.5(0.5)^2 + (0.00786)^2}{2}$$

$$- \frac{(0.00786((4.8460)}{2} + \frac{(0.3)(161970)(0.5)^2}{(30 \times 10^6)(0.00786)}$$

$$- \frac{(161970)(0.3)^2(0.00786)}{30 \times 10^6}(4.8460)$$

$$- \frac{(2)(161970)(0.3)^2}{(30 \times 10^6)(0.00786)} \frac{0.5(0.5)^2 + (0.00786)^2}{2}$$

$$- \frac{(0.00786)^2}{2}(4.8460) + \frac{(161970)(0.3)^2(0.5)^2}{(30 \times 10^6)(0.00786)}$$

$$- \frac{(161970)(0.3)^2(0.00786)(4.8460)}{(30 \times 10^6)}$$

$$\alpha_{P_f} = 0.000178 \text{ IN}$$

Since a force of 1000 lbs. is pressing these bodies together,

$$K = \frac{F^N}{\delta_T} = \frac{F^N}{\alpha_{P_c} + \alpha_{P_f}} = \frac{1000}{0.000178 + 0.000194}$$

$$K = 2.436 \times 10^6 \ \frac{\text{LB}}{\text{IN}}$$

APPENDIX B

IMPORTANCE OF THE STRESS COMPONENTS σ_x AND σ_y

The generalized equation for a deflection of a point with coordinates (X, Y, Z) of (O, Y, O) is

$$\alpha = \frac{P_o a}{E} \sinh^{-1} \frac{R}{a} - \frac{P_o \gamma a}{E} \sinh^{-1} \frac{R}{a}$$

$$\frac{-2P_o\gamma}{ae}\ \frac{R\ R^2\ +\ a^2}{Z\qquad Z} - \underline{a}^2 \sinh^{-1} \frac{R}{a} + \frac{\gamma P_o R^2}{Ea}$$

$$\frac{-P_o\gamma^2 a}{E} \sinh^{-1} \frac{R}{a} - \frac{2P_o\gamma^2}{Ea}\ \frac{R\ R^2\ +\ a^2}{2} - \frac{a^2}{2} \sinh^{-1} \frac{R}{a}$$

$$+ \frac{P_o\gamma^2 R^2}{Ea} - \frac{P_o\gamma^2 a}{E} \sinh^{-1} \frac{R}{a}$$

Since in a real system (manufactured with steel, $E = 30 \times 10^6$ psi, and $\gamma = 0.3$) $a \ll 1$, then $a^2 \ll 1$. Assuming then, that $a^2 = 0$, then

$$\alpha = \frac{P_o a}{E} \sinh^{-1} \frac{R}{a} - \frac{P_o \gamma a}{E} \sinh^{-1} \frac{R}{a}$$

$$- \frac{P_o\gamma^2 a}{E} \sinh^{-1} \frac{R}{a} - \frac{P_o\gamma^2 a}{E} \sinh^{-1} \frac{R}{a}$$

If the only stress component considered in the analysis were σ_E, then the deflection α would be completely described by the first term only.

If this were true, then,

$$\alpha = \frac{P_o a}{E} \sinh^{-1} \frac{R}{a}$$

But, the other terms were considered. Substituting $\gamma = 0.3$ yields

$$\alpha = \frac{P_o a}{E} \sinh^{-1} \frac{R}{a} - \frac{0.48a}{\quad} \sinh^{-1} \frac{R}{a}$$

or

$$\alpha = \frac{0.52 P_o a}{E}$$

This suggests that it is important to include the σ_x and σ_y terms when calculating the deflections, as this deflection is much different than obtained when using just the σ_z term.

Obviously, the a^2 terms will contribute to the calculation of the deflections. However, this analysis has indicated the importance of using all the stress components in determining the deflections.

APPENDIX C

RELATIVE VALUE OF a COMPARED TO R_c

From the Smith-Liu equations,

$$\Delta = \frac{1}{\frac{1}{2}\ \frac{1}{R_c} + \frac{1}{R_f}}\ \frac{1 - \gamma_c{}^2}{E_c} + \frac{1 - \gamma_f{}^2}{E_f}$$

It can also be written

$$\Delta = \frac{2R_cR_f}{R_c + R_f}\ \frac{1 - \gamma_c{}^2}{E_c} + \frac{1 - \gamma_f{}^2}{E_f}$$

or

$$\Delta = \frac{R_f(2R_c)}{R_f(\ ^{R_c} + 1)}\ \frac{1 - \gamma_c{}^2}{E_c} + \frac{1 - \gamma_f{}^2}{E_f}$$

As R_c becomes small with respect to R_f, this can be written

$$\Delta = 2R_c\ \frac{1 - \gamma_c{}^2}{E_c} + \frac{1 - \gamma_f{}^2}{E_f}$$

Again, by the Smith-Liu equations, the half width of the contact area is given by

$$a = \frac{2P\Delta}{\pi}$$

can be approximated by the following if both members are constructed by steel:

$$\Delta = \frac{1.82\ R_c}{E}$$

Putting this into the expression for a, and inserting numbers,

$$a = 0.000196\ PR_c$$

where P is the normal load per unit length in the Y-direction.

This shows that a will be much smaller than R_c for any real system.

APPENDIX D

PLOTS OF RESULTS

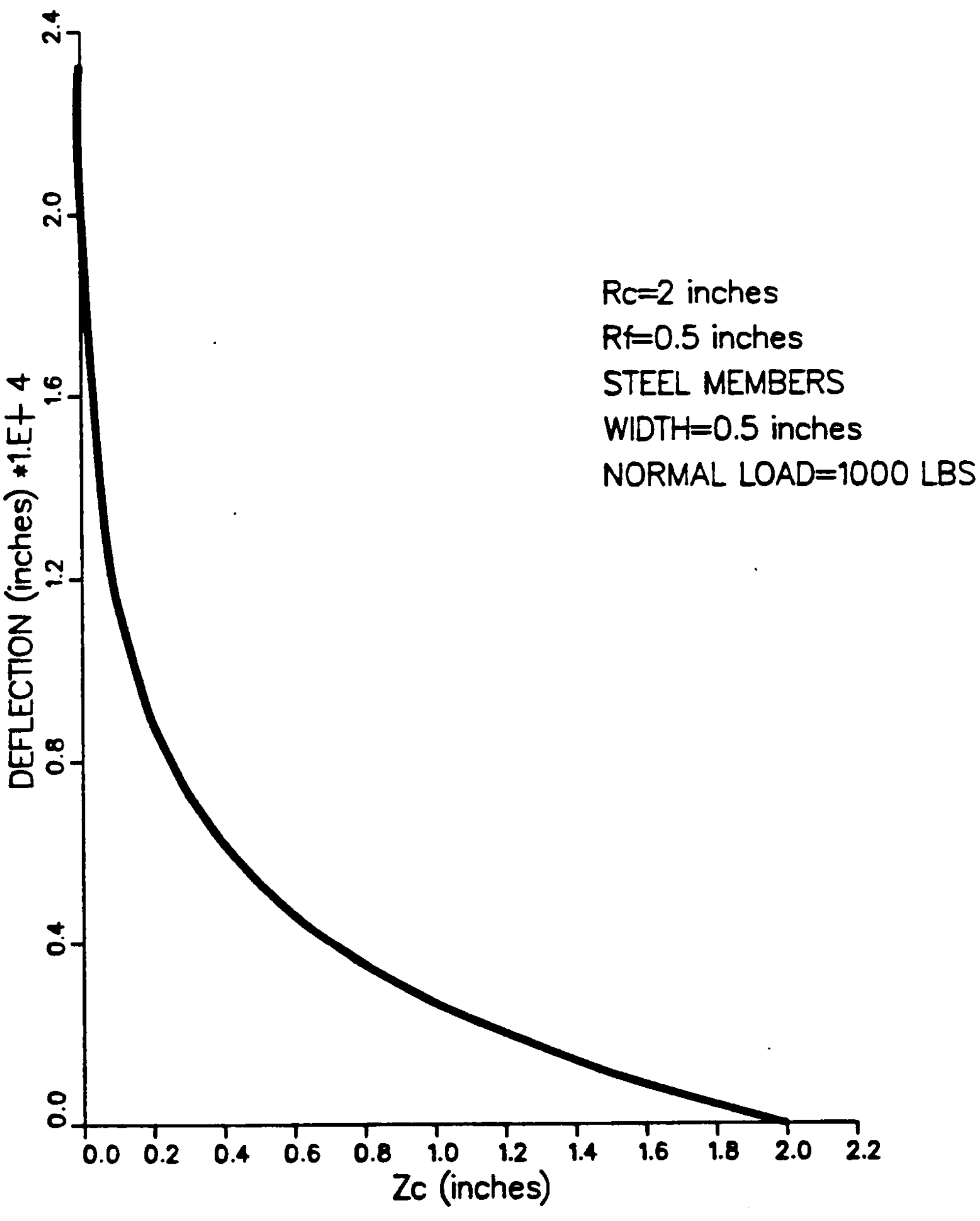

Figure D.1. Deflection of Points on the Cam that Lie on the Y-Z Plane as a Function of Z.

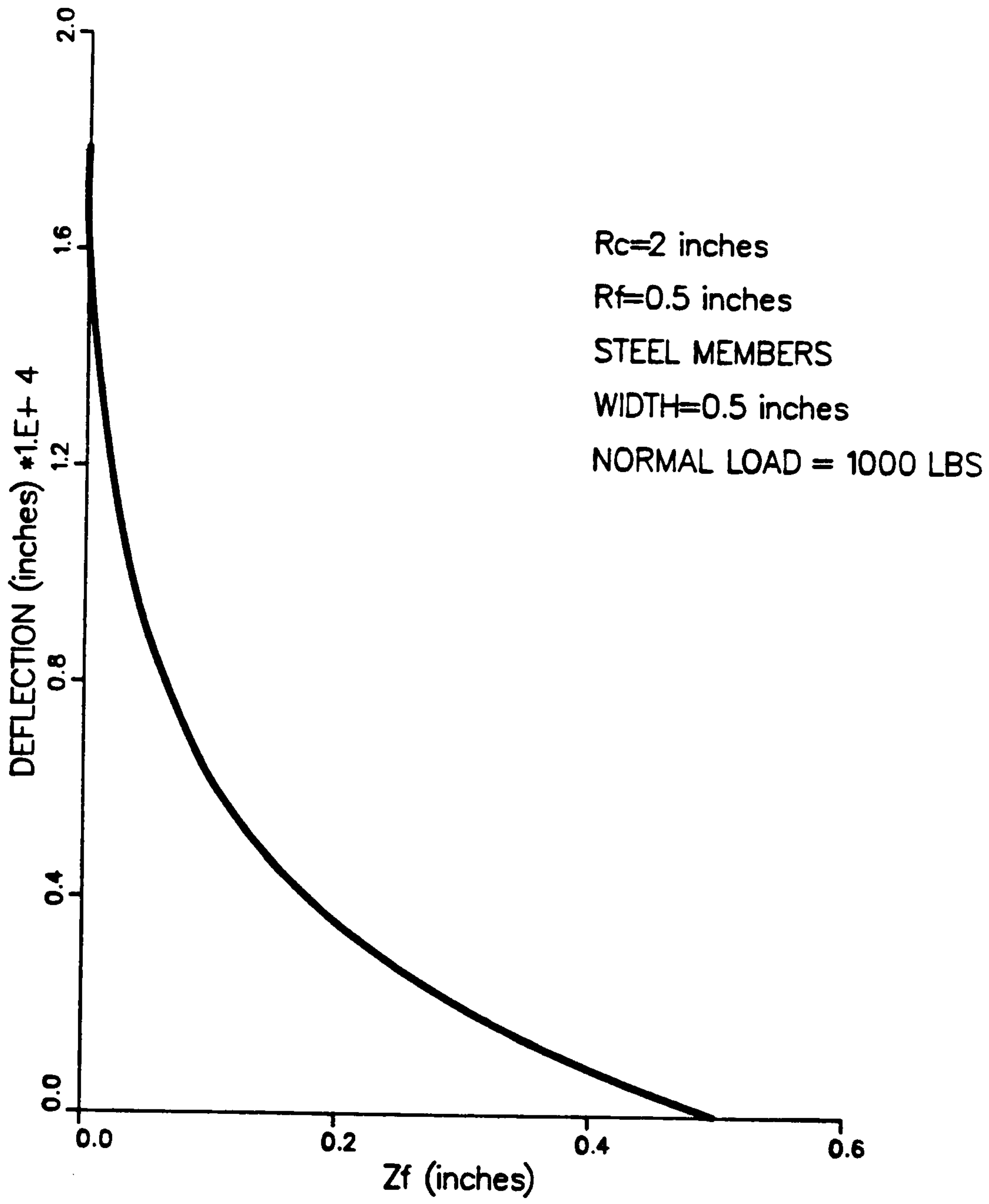

Figure D.2. Deflection of Points on the Follower that Lie on the Y-Z Plane as a Function of Z.

VITA

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