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Date December 2, 1992

A THEORETICAL SENSITIVITY STUDY OF WET THERMAL ANTI-ICING

A Thesis

Presented for the

Master of Science

Degree

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Charles Scott Bartlett

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DEDICATION

This thesis is dedicated to my parents, Chuck and Helen Bartlett, whose love, encouragment, and support have enabled me to attain this goal in my life. They have continually encouraged my education and supported me with love and understanding. They have always fosterd my desire to understand and study new areas of interest.

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ABSTRACT

A theoretical study has been conducted of the influences of changes in the physical parameters of an icing condition on the heat addition rate required to keep the stagnation line of a cylinder ice-free (anti-ice). The governing equations of energy and mass transport for the stagnation region of a cylinder are presented. A parametric analysis was conducted to determine the relationships between required surface anti-icing heating rate and the physical parameters of the process. A sensitivity analysis was conducted to determine the rate of change in the anti-icing heating rate with respect to the physical parameters. The physical parameters considered were the static air pressure, static air temperature, liquid water content, mass median droplet diameter, and airspeed. The analyses were conducted for three cylinder diameters, 0.25, 2.0, and 6. inches.

The analyses demonstrated the relationship of the required anti-icing heating rate and the sensitivity of the heating rate to the governing parameters. The influences of static pressure changes were found to be relatively insignificant. The static temperature was found to be the most influential governing parameter on required anti-icing heating rate. The influences of the

liquid water content, droplet diameter, and airspeed on anti-icing heating rate were found to depend strongly on the particular icing conditions established.

TABLE OF CONTENTS

SECTION		Page
1.	INTRODUCTION	1
	1.1 BACKGROUND	3
	1.2 NATURAL ICING ENVIRONMENT	5
	1.4 STATEMENT OF PROBLEM	7
2.	LITERATURE SURVEY	10
3.	SENSITIVITY STUDY	15
	3.1 GOVERNING EQUATIONS	15
	3.2 PARAMETRIC STUDY	20
	3.3 SENSITIVITY ANALYSIS	27
4.	SUMMARY AND CONCLUSIONS	32
5.	RECOMMENDATIONS	37
BIBLIOGRAPHY		39
APPENDIXES		43
A.	FIGURES	44
B.	TABLES	62
VITA		66

LIST OF FIGURES

FIGURE		Page
1.	Airfoil ice shape tracing and lift and drag coefficients as a function of angle of attack for a five minute icing encounter at five degrees angle of attack	45
2.	Areas of a typical airplane that may require active ice protection	46
3.	Cloud icing conditions	47
4.	Approximate cloud extreme values of liquid water content and mass median droplet diameter observed in clouds below 10,000 feet above ground level.	49
5.	Comparison of incipient ice predictions using mathematical model to experimental data	50
6.	Anti-icing heating rate as a function of air pressure at 300 ft/sec airspeed.	51
7.	Anti-icing heating rate as a function of air pressure at 100 and 600 ft/sec airspeeds.	52
8.	Anti-icing heating rate as a function of air temperature at 300 ft/sec airspeed.	53
9.	Anti-icing heating rate as a function of air Temperature at 100 and 600 ft/sec airspeeds.	54
10.	Rate of change in the anti-icing heating rate with respect to air temperature as a function of airspeed.	55
11.	Anti-icing heating rate as a function of liquid water content at 300 ft/sec airspeed.	56
12.	Anti-icing heating rate as a function of liquid water content at 100 and 600 ft/sec airspeeds.	57

FIGURE Page

13. Rate of change in the anti-icing heating rate with respect to liquid water content as a function of airspeed. 58

14. Anti-icing heating rate as a function of mass median droplet diameter at 300 ft/sec airspeed. 59

15. Anti-icing heating rate as a function of mass median droplet diameter at 100 and 600 ft/sec airspeeds. 60

16. Anti-icing heating rate as a function of airspeed. 61

LIST OF TABLES

TABLE		Page
I.	Baseline Icing Condition at Three Airspeeds and Three Cylinder Diameters.	63
II.	Influence of One Percent Parameter Change Ratios on the Anti-Icing Heating Rate.	64
III.	Influence of Specified Parameter Change Ratios on the Required Anti-Icing Heating Rate.	65

LIST OF SYMBOLS

C_p	Specific heat of air at constant pressure
C_w	Specific heat of water
^oC	Degrees centigrade
D_m	Mass median droplet diameter
D_o	Cylinder diameter
E	Vapor pressure
F	Calculated parameter
^oF	Degree Fahrenheit
FAA	Federal Aviation Administration
ft	Feet
gm	Grams mass
h_a	Air side convective heat transfer coefficient
H_{fg}	Latent heat of vaporization of water
in	inch
lbm	pounds mass
LWC	Liquid water content
m	Meter
M	Mass flux
MMD	Mass median diameter
NACA	National Advisory Committee for Aeronautics
P	Pressure
Pr	Prandtl Number
psia	pounds force per square inch absolute
Q	Heat transfer rate

Q_a	Anti-icing heat addition rate
sec	second
T	Temperature
U.S.	United States
W	Watts
V	Airspeed
x	Independent parameter
y	Independent parameter
β	Local stagnation line collection efficiency
Δ	Change in a parameter
ρ	Air density

Subscripts

c	Supercooled liquid water
D	Datum
f	Film
fs	Freestream conditions
k	Kinetic
L	Liquid
l	Boundary layer
s	Surface
v	Water vapor
w	Liquid water
∞	Freestream conditions

Superscripts

n	Prandtl number exponent

1 INTRODUCTION

Aircraft routinely fly through clouds composed of super cooled liquid water droplets. In most instances, the liquid water content of the cloud, or the droplet sizes in the cloud, or the aircraft speed preclude the droplets from impinging on the aircraft and freezing there. However, under certain conditions called icing conditions, ice accumulation will occur on aircraft surfaces. Ice usually forms and accumulates on the forward facing surfaces of the aircraft. These surfaces are the most susceptible surfaces for the formation of ice. This formation of ice occurs as the supercooled liquid water droplets impact these components and almost instantly freeze. The ice formation grows through a process known as ice accretion. The rate of ice accretion on a component is dependent on the pressure, temperature, liquid water content, droplet diameter, the geometry of the collection surface, and the airspeed during the icing encounter.

The accreted ice can lead to safety hazards by adversely altering the aircraft operation. This is documented by Bowden, Gensemer, and Skeen [1][1]:

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1 Numbers in brackets refer to similarly numbered references in the bibliography.

"The formation of ice on aircraft surfaces has been a problem since the early days of aviation. Aircraft carrying U.S. mail were often forced down by ice accretion on wings, tail, and propellers. If the aircraft were undamaged, ice was removed from the canvas covered surfaces and the flight was continued."

The consequences of ice accretion can range from undetectable to catastrophic. The aerodynamic characteristics of unprotected aircraft surfaces can be affected by ice accretion. Beyond the weight added by the ice accretion, the performance of both aerodynamic surfaces and the engines of the aircraft can be significantly reduced. Changes in the aerodynamic lift and drag coefficients are among the most notable consequences of ice accretion. Potapczuk and Berkowitz [2] demonstrated the changes in the lift and drag coefficients of various multi-element airfoils due to ice accretion. The change in the lift and drag coefficient of an airfoil as a function of the angle of attack is shown in Figure 1. The lift coefficient of the iced airfoil approaches the stall condition at a lower angle of attack, indicating earlier stall, than the clean airfoil. The drag coefficient is greater over most of the range of angle of attack studied. Changes to other aero-

dynamic surfaces can occur in icing. Propellers, helicopter rotors, turbine engine stators and rotors, air data probes, windshields, and radar radomes are often adversely affected by ice accretion.

To prevent or reduce to acceptable levels the consequence of flight through icing conditions, several methods of active ice protection have been devised. Two major categories of active ice protection devices are anti-icing (continuous prevention of ice formation) and deicing (intermittent removal of accreted ice). Deicing systems allow ice to accrete but are designed to prevent ice from accreting to dangerous levels. The areas of a typical aircraft often receiving active ice protection are shown in Figure 2 [1].

1.1 BACKGROUND

Many anti-icing and deicing ice protection systems rely on applying heat to the surface susceptible to ice formation. Enough heat is applied to prevent ice formation or to melt the layer bonding the accreted ice to the surface. Sogin describes the types of anti-icing systems commonly used for aircraft [3]. Dry anti-icing systems are those where the rate of heating of the surface susceptible to icing is high enough, relative to the rate of

liquid water impinging, so that no water freezes in the impinging region, nor is there water flowing backwards from the area of droplet impingement. Wet anti-icing systems permit water runback from the impinging region in thin rivulets. These wet anti-icing systems are designed to heat the surface to a level greater than 32 °F. Evaporative anti-icing systems are designed to completely vaporize all water along the airfoil so that none freezes or leaves the surface as a liquid. The study reported herein is limited to the study of wet anti-icing.

During the development and demonstration of thermal anti-icing systems, tests are conducted in icing conditions. These tests are conducted to validate design decisions or demonstrate compliance with regulatory criteria. Icing tests are conducted in conditions corresponding as closely as possible to flight through natural icing conditions. Proper testing requires that the tests be conducted with adequate knowledge of the atmospheric or natural icing conditions and the uncertainties of the conditions in which the test are performed.

Icing testing is conducted in actual flight through icing conditions, or in simulated icing conditions provided by ground test facilities or in flight. Descriptions of several test methods and test facilities for

icing testing have been reported [4,5,6]. In either mode of testing, uncertainty exists in the knowledge of the exact conditions under which the tests are conducted. These uncertainties in the test conditions cause uncertainties in the test results.

1.2 NATURAL ICING ENVIRONMENT

Most aircraft icing related studies and test programs are based on early investigations of the icing atmosphere conducted by the National Advisory Committee for Aeronautics (NACA) [7,8]. More recently the Federal Aviation Administration (FAA) has compiled findings of studies conducted to investigate icing conditions under 10,000 feet above ground level [9].

Two categories are generally used to distinguish between the relatively short extent, high liquid water content clouds and longer extent, lower liquid water content clouds [10]. These clouds are classified as intermittent and continuous, respectively. Intermittent clouds correspond to cumuliform, and continuous clouds correspond to stratiform clouds.

Generally envelopes that define the conditions often encountered when icing has occurred on aircraft have been developed [11] and are reproduced in Figure 3, and specif-

ically for clouds below 10,000 feet above ground level [9] in Figure 4. These envelopes do not, in themselves, define icing conditions for test purposes. The conditions that exist at the aircraft components, local icing conditions, must be selected to represent the free stream icing conditions for which the aircraft is to be certified. One notable example of the difficulty in specifying icing conditions for test purposes occurs with the inlet system of modern jet propelled aircraft. The flow entering the inlet is either accelerated or decelerated from free stream conditions hence the free stream icing conditions may not be precisely those that would exist at the engine face. Details of analyses to study the magnitude of these changes have been reported [12,13].

The physical parameters that constitute an icing condition are termed the governing parameters. The governing parameters considered in this report are the air static pressure, air static temperature, cloud liquid water content, cloud droplet diameter, and airspeed of the flow field approaching the icing surface.

The definition of liquid water content and droplet size terms relative to icing conditions are defined below. The liquid water content (LWC) of a cloud is a term that serves to quantify the amount of supercooled liquid water,

expressed in grams, contained in a cubic meter of cloud or air. Typical values of LWC range from 0.1 to 0.8 gm/m^3 for stratiform clouds and from 0.2 to 2.5 gm/m^3 for cumuliform clouds. Droplet diameter is expressed in microns, millionths of a meter. The mass median diameter (MMD) is the single numerical value commonly used to characterize a complete distribution of droplet sizes within a cloud [12]. The MMD is defined as the diameter of those droplets for which one-half of the entire liquid water content is contained in droplets whose diameter is less than the MMD. MMDs generally range from 15 to 40 microns for icing encounters.

Based on many research flights to explore icing conditions, the icing conditions generally occur at altitudes less than twenty two thousand feet above sea level. Icing typically occurs at ambient temperatures between minus 22 and 32 °F. Water droplets do not typically exist in a liquid state below minus 22 °F.

1.3 STATEMENT OF PROBLEM

The influences that uncertainty in, or changes of anti-icing test conditions have on, the test results, for the required heating rate to achieve successful anti-icing are largely unknown. Knowledge of the relative influence

of the test conditions on the required anti-iced surface heating rate identifies those parameters that are most important to accurately control and monitor in conducting the anti-icing tests. Without that knowledge, proper weighing of the efforts to achieve proper simulation and to measure test conditions can not be achieved.

A theoretical study will be presented of the influences of changes in the icing test conditions on the required heat addition rates for anti-icing. The study is reported in three sections. The first section describes a study of the energy and mass transport at the anti-iced surface. The governing equations and assumptions of the analysis are discussed. The mathematical model is validated by comparison to experimental data. The second section is a summary of a parametric study of the effects of changes in the governing parameters on the theoretical wet anti-icing heating rate. The effects on theoretical heating rate for wet anti-icing conditions are calculated for the stagnation line of cylinders of different diameters, for variations in the air static pressure, air static temperature, cloud liquid water content, cloud mass median droplet size, and airspeed. A sensitivity analysis of the relative

influence of perturbations in the governing parameters on the required anti-icing heating rate is reported in section three.

2 LITERATURE SURVEY

The topic of thermal anti-icing has often received treatment as reported in the literature. A large body of knowledge has been established in the theoretical treatment of thermal ice protection systems.

In some of the earliest work, Kantrowitz studied the aerodynamic heating of airfoil surfaces and the deflection of small water drops, associated with atmospheric icing, by aircraft bodies [14]. The study was conducted to provide information to aid investigations of the use of hot engine exhaust to prevent or remove ice formation on aircraft.

An extensive study of the mechanics of droplet impingement and heat transfer processes in an icing environment were reported by Neel, Norman, Jukoff, and Schalff [15]. The process of thermal anti-icing and the theoretical equations for the anti-icing process were verified with experimental data collected in natural icing conditions using NACA 0012 and NACA 65-0216 eight foot chord airfoils. Measurements of the meteorological variables and airfoil thermal data were made simultaneously. Many of the thermal anti-icing reports subsequent to this report use versions of the mathematical model presented in

this report.

Gelder and Lewis conducted studies to compare heat transfer from airfoils in natural and simulated icing conditions [16]. The investigators used the same NACA 65-0216 airfoil used in the flight tests reported in earlier NACA work [15]. The results of the study indicated that the convective heat transfer in natural icing conditions is considerably different from that measured in clean, dry air but only slightly different from the values obtained in the simulated icing conditions of the NACA icing research tunnel.

In 1952, Gray presented simple graphical solutions to the evaluation of the heat transfer and evaporation for surfaces heated to prevent icing [17].

In 1953, Gelder, Lewis, and Koutz investigated methods for predicting the heating requirements and performance penalties associated with turbojet transport aircraft ice protection [18]. In this report it was noted that the heating requirements to maintain aircraft safety could, in some instances, be substantially reduced trough the use of cyclic thermal deicing systems.

Messinger presented an analysis and results for calculating the equilibrium surface temperature of unheated icing surfaces [19]. Messinger demonstrated close comparison between theory and experimental data.

Sogin presented a detailed treatment of double skin thermal anti-icing systems for aircraft usage [2]. An extensive study of mass and heat balances on airfoils in icing conditions was given. Design procedures for performance analysis of the system were discussed.

Carr described a procedure for the design of air heated ice prevention devices [20]. The required heat transfer and pressure loss equations for various hot air anti-icing systems were presented. The methods of selecting meteorological conditions for flight over specified geographical areas and for the calculation of water droplet impingement rates were presented. An electrical analogue was devised to help solve the heat transfer equations.

Willbanks and Schulz investigated anti-icing heating rate requirements during their study of the techniques for the simulation of icing for turbine engines in altitude test cells [11]. They used a mathematical model of the anti-icing surface similar to that presented by Neel, Norman, Jukoff, and Schlaf [15]. Prior to the Willbanks and

Schulz report, most of the literature addressed the physics of anti-icing but did not directly address the influence of the test variables on anti-icing heat addition requirements. Willbanks and Schulz conducted studies to determine the influence of the liquid water content and the diameter of the water droplets of typical icing encounters on the anti-icing heating rate requirements for turbine engines.

Fergus [21] conducted studies to predict incipient icing of airfoils. During his studies, Fergus developed a mathematical model of incipient icing and used the model to predict the contribution of the various heat transfer mechanisms on the incipient icing as functions of primarily vehicle flight Mach number and ambient temperature. Fergus presents experimental data for comparison against predictions of the incipient ice accumulation mathematical model.

An extensive thermal analysis of modern turbine engine inlet anti-icing systems was reported by Kamel, Keith, and DeWitt [22]. A hot air anti-icing system was numerically analyzed. A potential flow model with compressibility corrections was used to calculate the inlet flow field. A trajectory model was used to calculate local droplet

impingement. Energy balances were performed to determine the temperature distribution along the wet anti-iced surface. Analysis of the runback water was included.

Although extensive studies have been reported concerning anti-icing models, test data and model refinements, there is a shortfall in the documentation of studies to address the influence changes in the governing parameters of icing conditions have on test results. The need to understand the parametric relationships between icing conditions and test results exits particularly in the icing test field. Understanding of the influences will enable tolerances in setting, maintaining, or knowing the icing test conditions without significant influence of the test result to be determined. The current study helps fill the literature in this regard.

3 SENSITIVITY STUDY

A sensitivity analysis is presented of the heating rate required to maintain an ice free surface in icing conditions is presented.

3.1 GOVERNING EQUATIONS

The mathematical modeling of the thermal anti-icing process used here is similar to that reported by Neel [15]. The predictions of the mathematical model are compared to experimental data presented by Fergus [21], The rate and area of disposition of the liquid water contained in supercooled droplets in atmospheric icing impinging on an airfoil are critically important to proper modeling. The process of droplet impingement has been reported by several investigators [23,24,25,26]. The water impingement region for typical aircraft icing surfaces at small angles of attack is limited to the leading edge airfoil stagnation region. The water flux to the surface and the heat transfer away from this region combine to require a substantial amount of anti-icing energy be applied to this region. For the study reported herein, only the stagnation line anti-icing process will be modeled. Neither the area aft of the stagnation line nor the runback liquid

film will be modeled. Further, a cylindrical substitution method [20] often used to simplify leading edge-anti icing studies will be used for liquid water droplet impingement and heat transfer estimation.

For the analysis, the surface is specified to maintain a temperature at or above the normal water freezing point and that enough heat is applied to the surface to prevent any ice formation. Radiation from the icing surface and conduction heat transfer are negligible and ignored [15]. Vaporization from the liquid film formed by liquid impingement on the surface into the gas (air) boundary layer will occur. Convective heat transfer and sensible cooling of the surface are modeled. The heating rate required to maintain the leading edge ice free at a prescribed surface temperature is

$$Q_a = h_a(T_S - T_D) + M_v H_{fg} + M_w C_w (T_S - T_w)$$

The terms on the right hand side (RHS) of this equation represent cooling of the surface by convection, evaporation, and sensible heat rejection respectively. The first term on the RHS of the equation describes the heat loss from the liquid film to the air by convection. The surface temperature, T_S, is a prescribed value. The datum temperature, T_D, is given as

$$T_D = T_\infty + \Delta T_k - \Delta T_L$$

The $\Delta T_k - \Delta T_L$ term represents the kinetic temperature of rise of the surface at the point where heat transfer is being computed. In dry conditions the term would represent the recovery temperature. The magnitude of the kinetic temperature rise of an unheated airfoil in icing conditions is reduced due to evaporation from the liquid film of the wetted surface. The kinetic temperature rise is given [15] as

$$\Delta T_k = \frac{V_\infty^2}{2C_p}\left(1 - \left(\frac{V_I}{V_\infty}\right)^2 (1 - Pr^n)\right)$$

where n is equal to 1/2 for laminar airflow and is equal to 1/3 for turbulent flows. A reduction in the kinetic temperature rise due to liquid evaporation is derived by Sogin [3] and concisely expressed [15] as

$$\Delta T_L = 0.622 H_{fg}\left(\frac{E_D - E_I}{C_p P_I}\right)$$

Hence, the datum temperature is calculated from the equation

$$T_D = T_\infty + \frac{V_\infty^2}{2C_p}\left(1 - \left(\frac{V_I}{V_\infty}\right)^2 (1 - Pr^n)\right) - 0.622 H_{fg}\left(\frac{E_D - E_I}{C_p P_I}\right)$$

A trial and error solution to the equation is required since the vapor pressure at the datum temperature depends upon the datum temperature itself.

A method for estimation of the air side convective heat transfer coefficient at the stagnation line, substituting a cylinder for the airfoil leading edge [27], is

$$h_a = 0.194 T_f^{0.49} \left(\frac{V_\infty \rho_\infty}{D_o} \right)^{0.5}$$

The heat loss due to evaporation from the surface is calculated by the product of the evaporated mass rate and the heat of vaporization of the water

$$Q_v = M_v H_{fg}$$

The relation to estimate the rate of mass transfer, liquid evaporation, is

$$M_v = 0.622 h_a \left(\frac{E_S - E_D}{C_p P_l} \right)$$

It is assumed that the liquid film is at a uniform temperature and is equal to the datum temperature, T_D.

The third term on the right hand side of the anti-icing heating rate equation represents a heat loss from the surface to warm the impinging supercooled liquid droplets to the uniform surface liquid film temperature. The droplets are assumed to pass through the air boundary layer and strike the surface with no temperature or mass

change (no evaporation in the boundary layer).

The impinging mass flux of droplets is estimated by

$$M_w = LWCV_\infty\beta$$

The term β represents the collection efficiency for the droplets of various diameters in the stagnation region of a cylinder in a cross flow of air. The local stagnation region collection efficiency is a function of the droplet diameter, the relative velocity between the droplet and the cylinder, the diameter of the cylinder and the viscosity of the air stream. Bowden, Gensemer, and Skeen [1] give an analysis and present values of stagnation region collection efficiency for droplets intercepted by cylinders.

The droplet kinetic temperature, T_w, assuming the droplets are not slowed as they pass through the air boundary layer, is calculated from

$$T_w = T_c + \left(\frac{V_\infty^2}{2C_w}\right)$$

To evaluate the mathematical model, predictions of icing onset were compared to experimental data reported by Fergus [21]. The data were collected for icing onset on airfoils. Icing onset data were experimentally collected

for a range of altitude, liquid water content, flight Mach number, and ambient temperature. The experimental data are presented as a range of flight Mach number and ambient temperature combinations where icing just begins to form on the leading edge of the airfoil. The mathematical model predictions are shown as lines of flight Mach number and ambient temperature where the anti-icing heating rate added to the icing surface to prevent icing is equal to zero, indicating that with no heat addition to the surface, incipient icing is occurring. The comparison of model predictions to experimental data shown in Figure 4.5, indicate reasonable agreement, and the model is considered valid for use in the parametric and sensitivity studies reported herein.

3.2 PARAMETRIC STUDY

An analysis is presented of the influence that variations in the governing icing condition parameters have on the calculated theoretical anti-icing heating rate. Heat energy is applied to maintain the surface at 32 °F. The changes in the wet anti-icing heating rate calculated with the theoretical model are discussed. The governing parameters of air pressure, air temperature, liquid water content, mass median droplet diameter, and airspeed will be

varied over a range of values representative of icing conditions. The study provides an understanding of the relationship of the wet anti-icing heating rate to the governing parameters of icing test conditions.

BASELINE

A baseline icing condition was selected for the parametric study. The governing parameters were varied from this arbitrary baseline condition. The baseline icing condition is:

Air Pressure	10.0 psia
Air Temperature	-4. ^{o}F
Liquid Water Content	1.7 gm/m^3
Droplet Diameter (MMD)	20. microns
Airspeed	300. ft/sec

To illustrate the influence of collection cylinder size on the wet anti-icing process, results of calculations for three different cylinder diameters are presented. The three cylinder diameters are 0.25, 2.0 and 6.0 in. To illustrate the effect of airspeed on the anti-icing process, solutions are presented for airspeeds of 100 and 600 (ft/sec). The nine conditions representing the variations in cylinder diameter and airspeed are

listed in Table I.

AIR PRESSURE EFFECT

Model solutions were obtained at pressures representative of standard altitudes [28] within icing envelopes. The pressures studied ranged from 6.0 to 14.7 psia, representative of altitudes ranging from approximately 22,000 feet above sea level to sea level. The wet anti-icing heating rate as a function of air pressure for the baseline icing conditions at 300 ft/sec airspeed is shown in Figure 5. The influence of altitude is small, being less than one watt/in^2 over the complete range of air pressures for all three cylinder diameters studied. The heating rate requirements tend to decrease as pressure is increased (altitude decreased) up to a point where the trend reverses. The reversal in trend occurs at higher pressures for larger cylinders. The trend does not reverse itself for the 6.0 in. diameter cylinder, with the required heating rate continuing to decrease as the pressure is increased over the range of pressure considered.

Solutions were also obtained for airspeeds of 100 and 600 ft/sec. The results from those solutions are shown in

Figure 6. At the 100 ft/sec airspeed, increases in air pressure tend to increase the heating rate requirement for the 0.25 in. diameter cylinder. At the higher airspeed of 600 ft/sec, increases in air pressure tend to decrease the heating rate requirements for all three cylinder diameters studied. These observations indicate that the point where the trend in the influence of air pressure on the heating rate reverses direction occurs at higher pressures as the airspeed is increased.

AIR TEMPERATURE EFFECT

Model solutions were obtained at air temperatures ranging from minus 22 to 32 °F, representative of the range of normal icing conditions. The wet anti-icing heating rate as a function of air temperature for the baseline icing conditions at 300 ft/sec airspeed is shown in Figure 7. The required anti-icing heating rate decreases with increasing air temperature. The influence of temperature changes is greatest for the smallest cylinder. The influence is approximately linear over the range studied. The rates of change in the anti-icing heating rate with respect to air temperature are approximately -0.54, -0.33, and -0.16 $W/in^2/°F$ for the 0.25, 2.0, and

6.0 in. diameter cylinders respectively.

Solutions were also obtained for air temperature changes at airspeeds of 100 and 600 ft/sec. The results of the calculations are shown in Figure 8. For the airspeeds studied, the influence of changes in air temperature increases as the airspeed is increased and as the cylinder diameter is decreased.

The increasing influence of air temperature on the required anti-icing heating rate, as airspeed is increased for the three cylinders studied is shown in Figure 9. In this figure the rate of change in the heating rate with respect to air temperature is plotted for airspeeds from 100 to 600 ft/sec. The rate of change is decreased as the cylinder diameter is increased.

LIQUID WATER CONTENT EFFECT

Model solutions were obtained for the required anti-icing heating rate at liquid water content values from 0.2 to 3.0 gm/m^3, representative of values typically associated with natural icing conditions. The wet anti-icing heating rate as a function of liquid water content for the baseline icing condition at 300 ft/sec airspeed is shown in Figure 10. The required anti-icing heating rate increases linearly over the complete range of liquid water

contents studied for each of the three cylinders. The rate of change of the heating rate with respect to changes in the liquid water content are 4.4, 3.1, and 1.9 $W/in^2/(gm/m^3)$ for the 0.25, 2.0, and 6.0 in. diameter cylinders. The influence of changes in the liquid water content on the heating rate increases as the cylinder diameter decreases.

The solutions for variations in liquid water content obtained at 100 and 600 ft/sec airspeed indicate that the influence of liquid water content variations increases as the airspeed increases. The results for the three different sized cylinders are shown in Figure 11. The rate of change in the heating rate with respect to liquid water content at airspeeds from 100 to 600 ft/sec is shown for the three cylinder diameters in Figure 12.

DROPLET DIAMETER EFFECT

The droplet diameter used to obtain the required heating rate is the mass median droplet diameter. Solutions were obtained for mass median droplet diameters from 10 to 40 microns, representing typical icing condition values. The wet anti-icing heating rate as a function of droplet diameter for the baseline icing conditions at 300 ft/sec airspeed is shown in Figure 13. The influence of drop

size on heating rate is smallest for the small cylinder and increases as the cylinder diameter is increased. Similar results are illustrated in Figure 14 for the influence of droplet diameter on the heating rate at airspeed of 100 and 600 ft/sec. The influence of the droplet size is greater at higher airspeeds.

AIRSPEED EFFECT

Model solutions were obtained for variations in airspeed from 100 to 800 ft/sec. The wet anti-icing heating rate as a function of airspeed for the baseline icing conditions is shown in Figure 15. The required anti-icing heating rate increases as the airspeed increases up to a point. After this point is reached, further increases in airspeed decrease the required heating rate. The airspeed at which reversal in trend occurs increases as the cylinder diameter increases, occurring at approximately 450, 625, and 700 ft/sec for the 0.25, 2.0, and 6.0 diameter cylinders respectively. These airspeeds are called the critical anti-icing air speeds. The influence of changes in airspeed are greater for the smaller diameter cylinders. The results shown in Figure 15 indicate that, for each cylinder diameter, there is an air speed above

which no input heating rate is required. This indicates that active anti-icing is no longer required at the stagnation line above the critical airspeed.

3.3 SENSITIVITY ANALYSIS

An analysis is presented of the sensitivity of the wet anti-icing heating rate to small variations in the governing parameters. The sensitivity analysis follows accepted methods of error propagation analysis [29]. The sensitivity of the wet anti-icing heating rate to changes in the governing parameters is determined at the baseline icing condition for the three cylinder diameters, 0.25, 2. and 6. in. for three airspeeds, 100, 300, and 600 ft/sec. listed in Table I.

Generally, a process or function, F, of independent governing parameters x and y can be expressed as

$$F = F(x, y)$$

and the influence or change in the calculated solution for small changes in the individual governing parameter is expressed as

$$\delta F = \left(\frac{\partial F}{\partial x}\right)\delta x + \left(\frac{\partial F}{\partial y}\right)\delta y$$

If, for convenience, the terms δx and δy are expressed as ratios, called governing parameter change ratios, the equation for the influence function becomes

$$\frac{\delta F}{F} = \left(\frac{x}{F}\right)\left(\frac{\partial F}{\partial x}\right)\frac{\delta x}{x} + \left(\frac{y}{F}\right)\left(\frac{\partial F}{\partial y}\right)\frac{\delta y}{y}$$

The terms $(x/F)(\partial F/\partial x)$ and $(y/F)(\partial F/\partial y)$ are defined as the influence coefficients of x and y on the calculated solution. A change in x, $\delta x/x$, while δy is set equal to zero, will cause a change, $\delta F/F$, in the calculated solution equal to the product of the influence coefficient and the change. The change in the calculated solution is called the "influence" of x on the solution.

The influence terms are evaluated at specified conditions for a sensitivity analysis. For the sensitivity analysis presented here, the function F represents the wet anti-icing heating rate, and the governing parameters that influence this function are the air static pressure, air static temperature, liquid water content, mass median droplet diameter, and airspeed. The influence function for the wet anti-icing heating rate is obtained by evaluating the effects of small perturbations in one of the governing parameters with all other parameters held constant. This perturbation analysis is conducted for each of the governing parameters at each of the baseline

icing conditions listed in Table I.

The influence terms are evaluated at specified parameter change ratios. If the individual parameter change ratios are specified to be equal, a direct comparison of the influences on the anti-icing heating rate can be made. The influences resulting from a specified parameter change ratio of 0.01, a one percent change in the parameter are listed in Table II for each of the specified icing test conditions.

The results listed in Table II indicate that static air temperature is the strongest influencing parameter on the anti-icing heating rate, and therefore the anti-icing heating rate is most sensitive to the air temperature. The sensitivity to pressure is relatively small for all icing conditions studied. The ranking of sensitivity to changes in the other parameters varies throughout the range of icing conditions investigated. The sensitivity to liquid water content increases as the airspeed increases and as the cylinder diameter increases. The sensitivity to droplet diameter decreases as the airspeed is increased but increases as the cylinder diameter increases. The sensitivity to airspeed generally decreases as the baseline airspeed is increased and increases as the cylinder diameter increases.

If the specified parameter change ratios are selected to be approximately equal to magnitudes of uncertainty in each of the governing parameters that are expected in ground testing, a comparison of these influences on the experimental values of the anti-icing heating rate can be made. The selection of the specified parameter change ratios is based upon the following assumptions of the expected uncertainty in the governing parameters. Air pressures can be measured accurately to approximately one percent, and temperature can be determined to approximately one-half percent, or about 2 °F for the baseline condition. The liquid water content can be measured to approximately fifteen percent accuracy. Droplet diameter can be measured to approximately five microns, representing a twenty five percent uncertainty, and the airspeed can be determined to approximately five percent accuracy. Although these values of accuracy in the governing parameters are not based on published information, the author believes that they are typical and representative of ground test measurement accuracies, and will serve for demonstration purposes.

For the assumed uncertainty in the governing parameters, the specified parameter change ratios are specified to be 0.01 for air pressure, 0.005 for air temperature,

0.15 for liquid water content, 0.25 for droplet diameter, and 0.05 for airspeed. The corresponding influences in the wet anti-icing heating rate for the baseline icing test conditions are listed in Table III.

The results listed in Table III indicate that at these specified parameter change ratios the sensitivity of the wet anti-icing heating rate is closer to being equal for the air temperature, liquid water content, droplet diameter and airspeed than for the uniform one percent parameter change sensitivity analyzed above, where temperature influence dominated. The sensitivity to the temperature changes is generally greatest at lower airspeeds for the smaller diameter cylinders. The sensitivity to the liquid water content change becomes greater at higher airspeeds and for larger diameter cylinders. The sensitivity to the droplet diameter change becomes the greatest of all parameters for the larger cylinder diameters. The sensitivity to the airspeed change is smallest for the smallest cylinder and decreases at higher airspeeds. The influence of air pressure is relatively insignificant.

The sensitivity analysis results indicate that the sensitivity of the wet anti-icing heating rate to the governing parameters vary significantly between different icing conditions.

4 SUMMARY AND CONCLUSIONS

The objective of the investigation into the sensitivity of the calculated anti-icing heating rate to changes in the icing condition governing parameters was satisfied. The problem of aircraft icing was introduced, and some discussion of its consequence and methods of icing protection was given. The use of thermal energy as an ice protection and ice prevention mechanism has been used since the early days of aviation. Numerous studies to characterize the aircraft icing environment have been documented.

Much attention has been given to the modeling of the thermal anti-icing process, primarily to aid in the design of thermal anti-icing systems. The investigation reported herein focused on determining the sensitivity of the wet anti-icing process to changes in the parameters that govern the icing process. The sensitivity study was divided in three sections. The first section presented the equations governing the transport of energy and mass at the stagnation line of a cylinder in icing conditions. The second section presented a parametric study to investigate the trends of the calculated wet anti-icing heating rate over wide ranges of the governing icing condition

parameters. The third section presented a sensitivity study of the wet anti-icing process to specified changes in the governing parameters.

The governing parameters used in the parametric and sensitivity studies were the air pressure, air temperature, liquid water content, droplet diameter, and the airspeed. The effects of changes in the governing parameters on three different cylinder diameters at three different airspeeds were studied.

In the parametric study the effect of air pressure was found to be small over the complete range of pressures from 6. to 14.7 psia, representing standard altitudes from approximately 22,000 feet above sea level down to sea level.

Changes in the air temperature caused changes in the anti-icing heating rate over the complete range from -22 to 32 ^{o}F. As the air temperature is increased, the required heating rate decreases in an approximately linear fashion until no heat addition is required to prevent ice accumulation. The rate of change in the anti-icing heating rate with respect to the air temperature decreased as the cylinder diameter increased and as the airspeed decreased.

Changes in liquid water content caused approximately linear changes in the required heating rate. Increases in the liquid water content require additional heat be added to the surface to maintain an anti-iced condition. The rate of change in the heating rate with respect to liquid water content decreased as the cylinder diameter increased and as the airspeed decreased.

As the droplet mass median diameter varied from 15 to 40 microns, the required anti-icing heating rate increased. The effect was not linear however, as the rate of change in the heating rate with respect to the droplet diameter decreased as the droplet diameter increased.

The effects of airspeed over a range of 100 to 800 feet per second indicated that the required anti-icing heating rate increased as the airspeed increased up to certain airspeed value. Speed increases beyond this value then resulted in decreased heat addition rates required to anti-ice. The point of reversal occurred at higher airspeeds for the larger cylinders. Up to the point of reversal, the rate of change of the required heating rate with respect to airspeed increased as the cylinder diameter decreased.

The sensitivity analysis quantified the relative sensitivity of the required anti-icing heating rate to

changes in each of the governing parameters. The study was conducted by perturbing only one of the governing parameters at a time and determining the influence that governing parameter had on the solution of the required anti-icing heating rate. The perturbation analysis was conducted for specified parameter changes.

The results of the sensitivity analysis when each of the governing parameters was changed by a value of one percent from the baseline value of the parameter showed the air temperature to be the most influential parameter on the heating rate and the air pressure to be the least influential. The sensitivity of the heating rate to changes in the liquid water content , droplet diameter, and airspeed varied substantially throughout the range of icing conditions studied for the three different diameter cylinders.

A second sensitivity analysis was conducted with the parameters not changed in equal percentages but in values roughly approximating uncertainties in knowledge of the parameters. For those specified changes in the governing parameters, the greatest influence in the heating rate was more evenly divided between temperature, liquid water content, droplet size, and airspeed. The influence of pressure was found to be relatively insignificant.

The sensitivity analysis results indicate that the sensitivity of the wet anti-icing heating rate to the governing parameters varies significantly between different icing conditions.

5 RECOMMENDATIONS

The results of the study presented herein are useful in understanding and quantifying the icing test conditions which are most influential in determining the heating rate required to achieve wet thermal anti-icing. The identification of the icing condition parameters in an aircraft mission that will most strongly influence the design of the ice protection equipment will aid the thermal systems designers. The results will benefit test conductors by providing a sound basis for determining the allowable tolerances in setting icing test conditions such that the test results are valid. The result will benefit researchers developing measurement technologies for icing by identifying which icing condition parameters require the most accurate measurement. All of these benefits promote proper investment of resources through identification of areas where proper pay back of investment can be obtained.

Areas for further work can be imagined. The model results should be further validated with experimental data; however as noted in the analysis, the influence of the uncertainties in the test parameters can have significant influence on the test data, and must be accounted for

in the validation process. Models to study the various other anti-icing configurations (dry and evaporative) should be used to conduct similar sensitivity analyses. Sensitivity analyses of other icing related areas such as deicing, ice accretion, and ice accretion effects on aerodynamic characteristics should be considered. Further, the effects of icing on turbine engine and aircraft systems as a whole should be considered. The results of such analyses could yield great insight into the parameters that most strongly influence the processes to be controlled.

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BIBLIOGRAPHY

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APPENDIXES

APPENDIX A

FIGURES

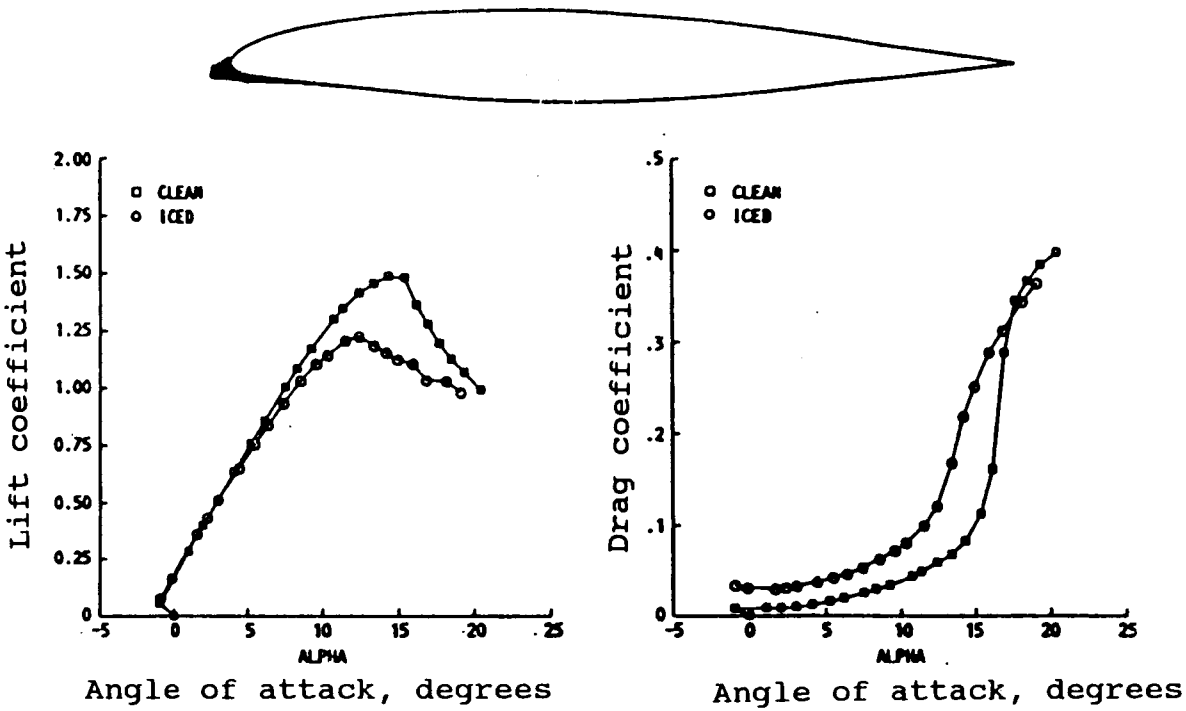


Figure 1. Airfoil ice shape tracing and lift and drag coefficients as a function of angle of attack for a five minute icing encounter at five degrees angle of attack

Adopted From: Potapczuk, M.G., and Berkowitz, B.M., "An Experimental Investigation of Multi-Element Airfoil Ice Accretion and Resulting Performance Degradation." AIAA-89-0752, January 1989.

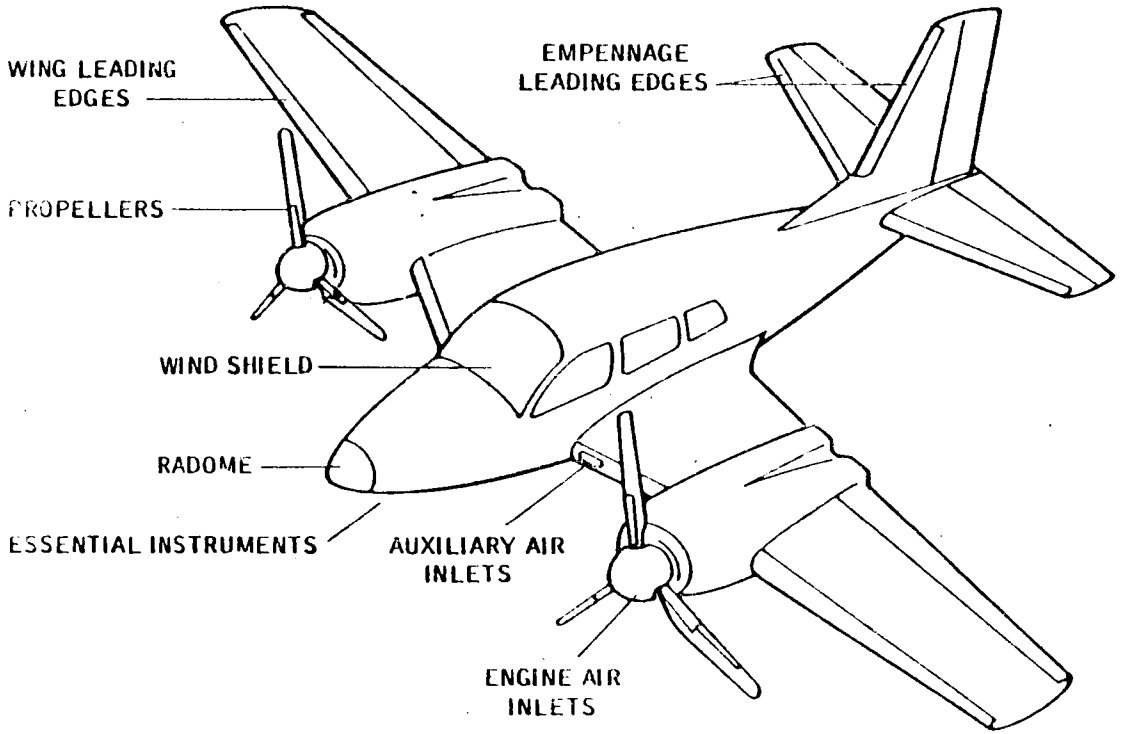
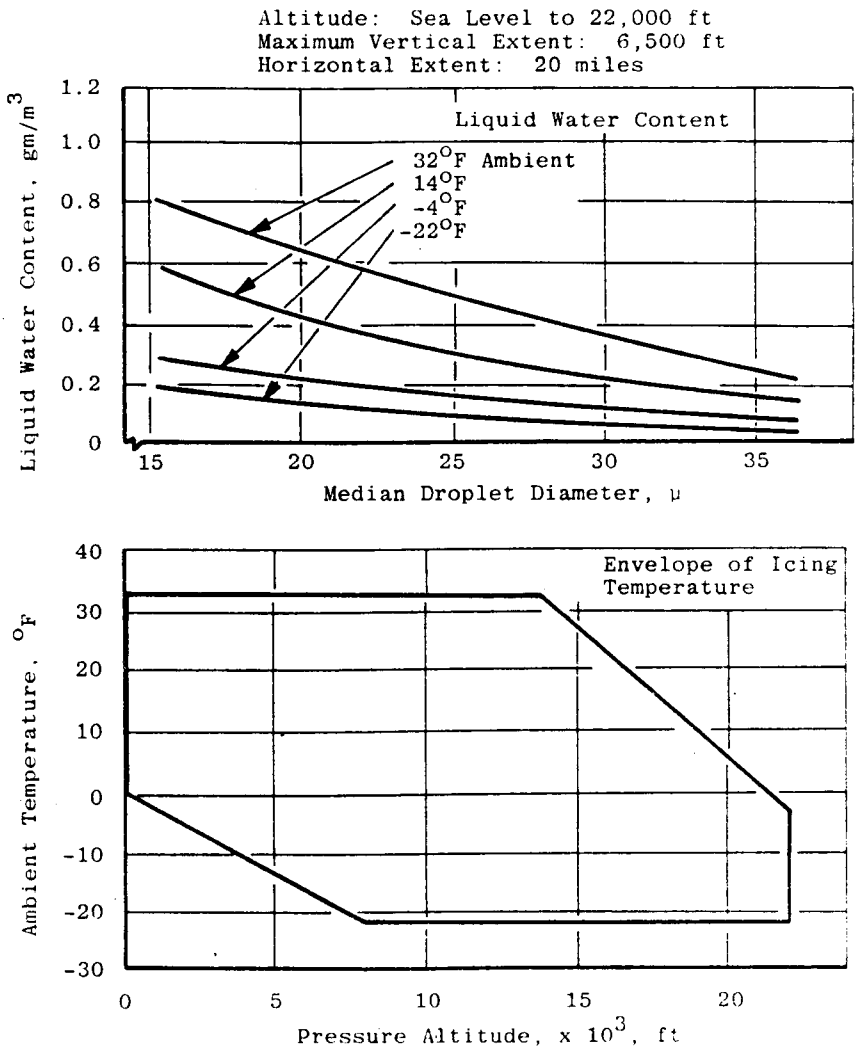


Figure 2. Areas of a typical airplane that may require active ice protection

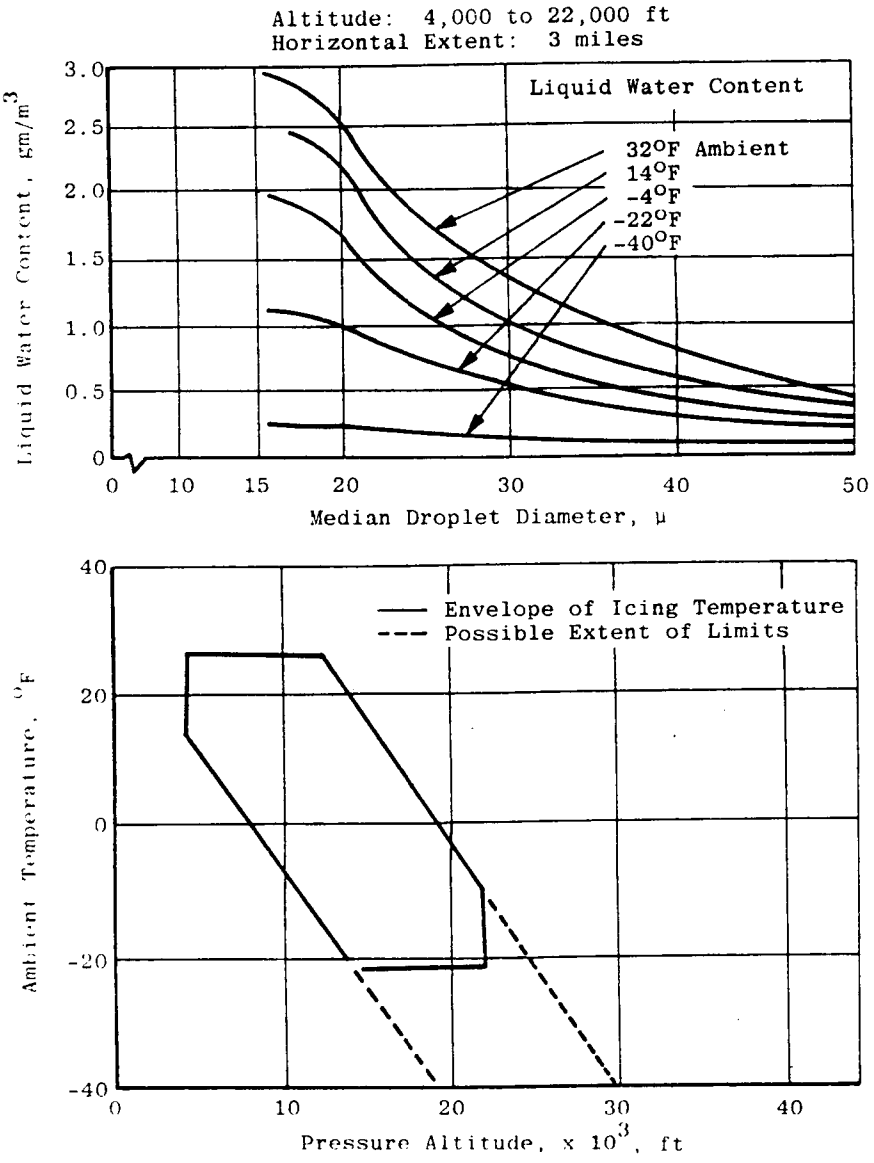
Source: Bowden, D.T., Gensemer, A.E. and Skeen, C.A. "Engineering Summary of Airframe Icing Technical Data." FAA ADS-4, December 1963.



a. Continuous maximum (stratiform clouds)

Figure 3. Cloud icing conditions

Source: "Airworthiness Standards: Transport Category Airplanes; Appendix C." Federal Aviation Standards, Part 25, Federal Aviation Administration.



b. Intermittent maximum (cumuliform clouds)

Figure 3. Concluded.

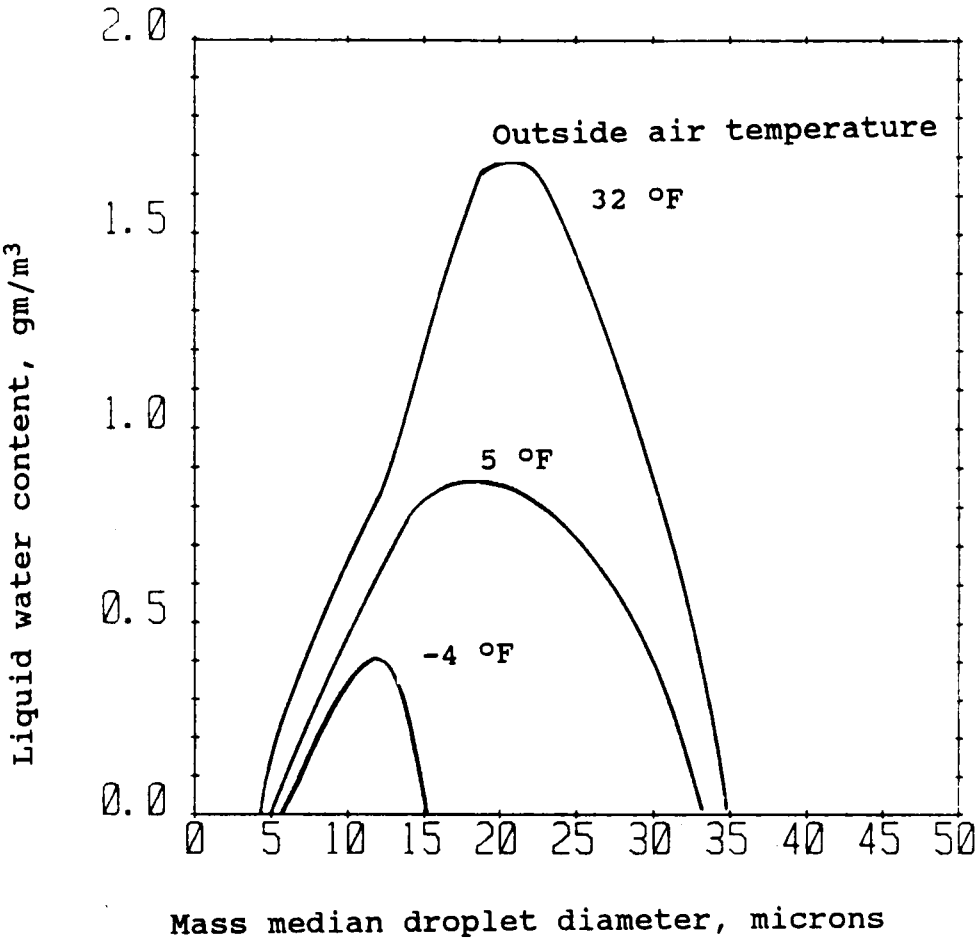


Figure 4. Approximate cloud extreme values of liquid water content and mass median droplet diameter observed in clouds below 10,000 feet above ground level.

Source: Jeck, R.K. "A New Data Base of Supercooled Cloud Variables for Altitudes up to 10,000 Feet AGL and the Implications for Low Altitude Aircraft Icing." DOT/FAA/CT-83-21, August 1983.

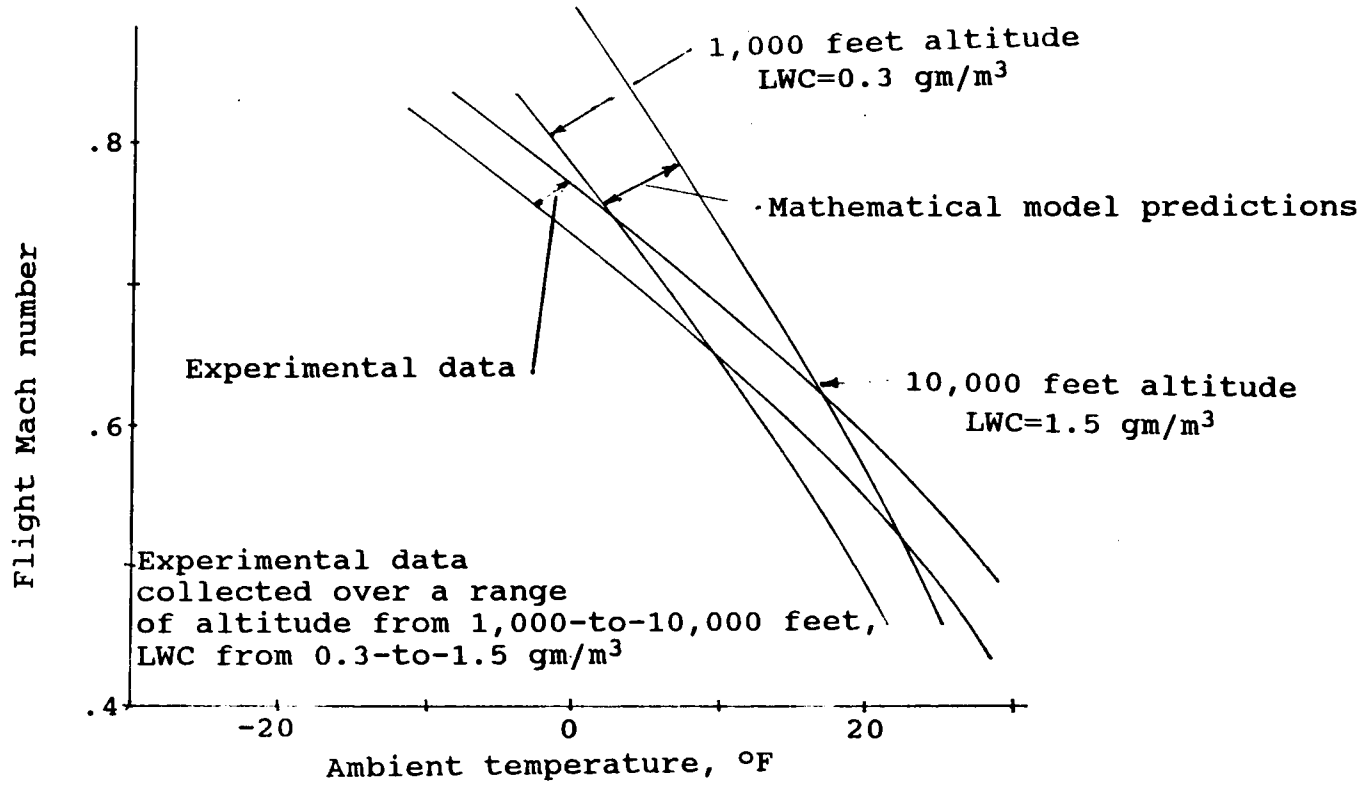


Figure 5. Comparison of incipient ice predictions using mathematical model to experimental data.

Data Adopted From: Fergus, J.L. Jr., "A Steady State Thermal Model for Analysis of Incipient Icing on an Airfoil Leading Edge." AEDC-TR-83-2, July 1983.

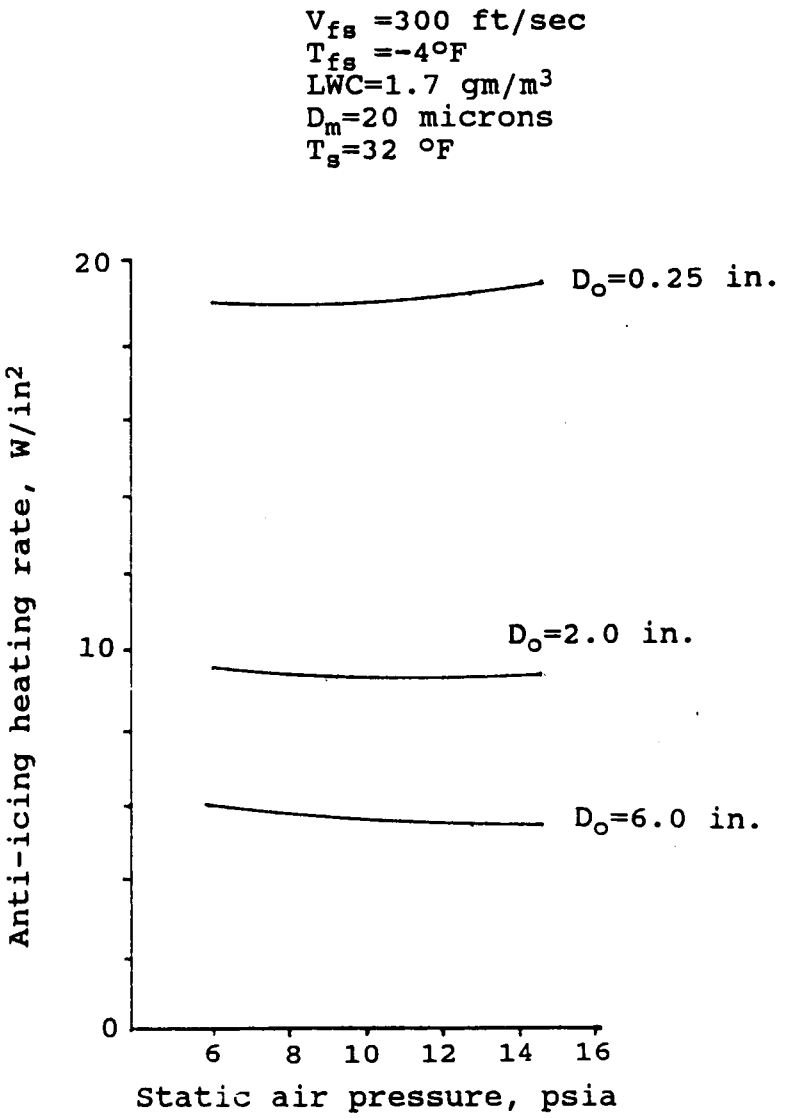


Figure 6. Anti-icing heating rate as a function of static air pressure at 300 ft/sec airspeed.

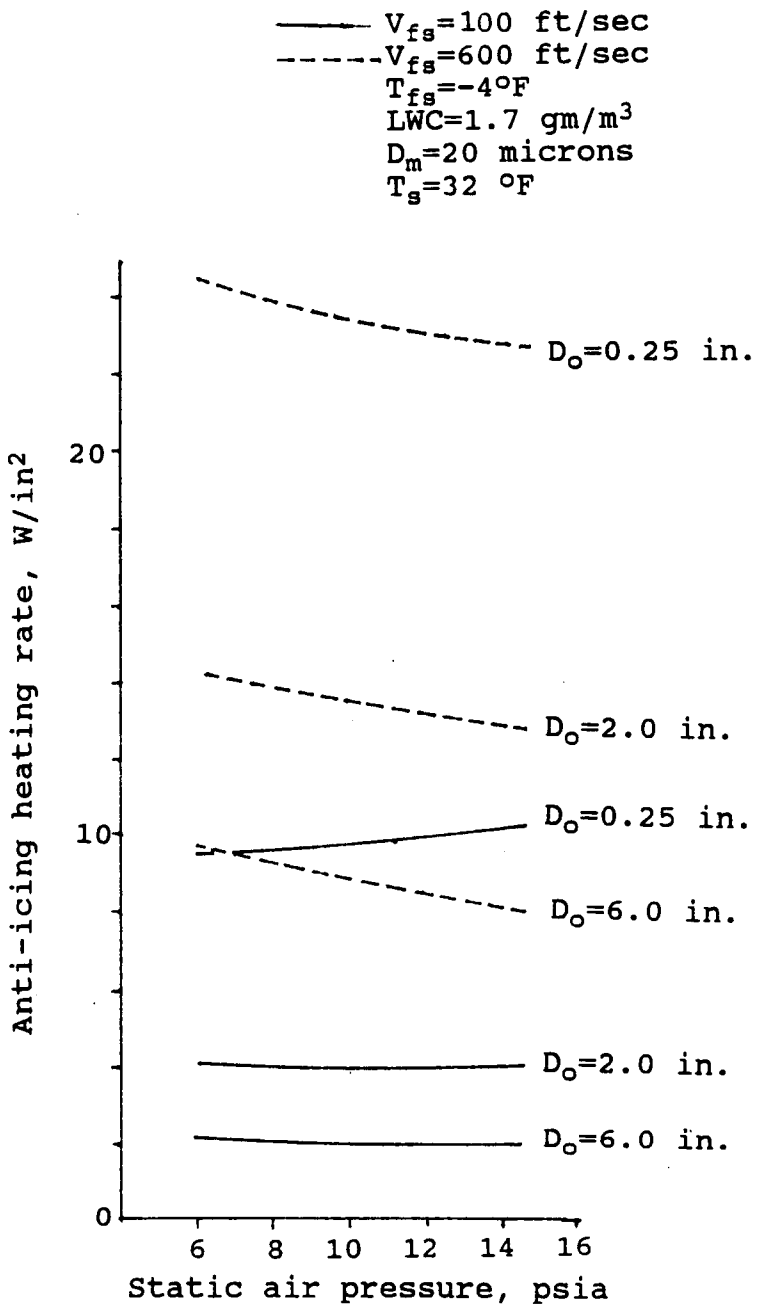


Figure 7. Anti-icing heating rate as a function of static air pressure at 100 and 600 ft/sec airspeeds.

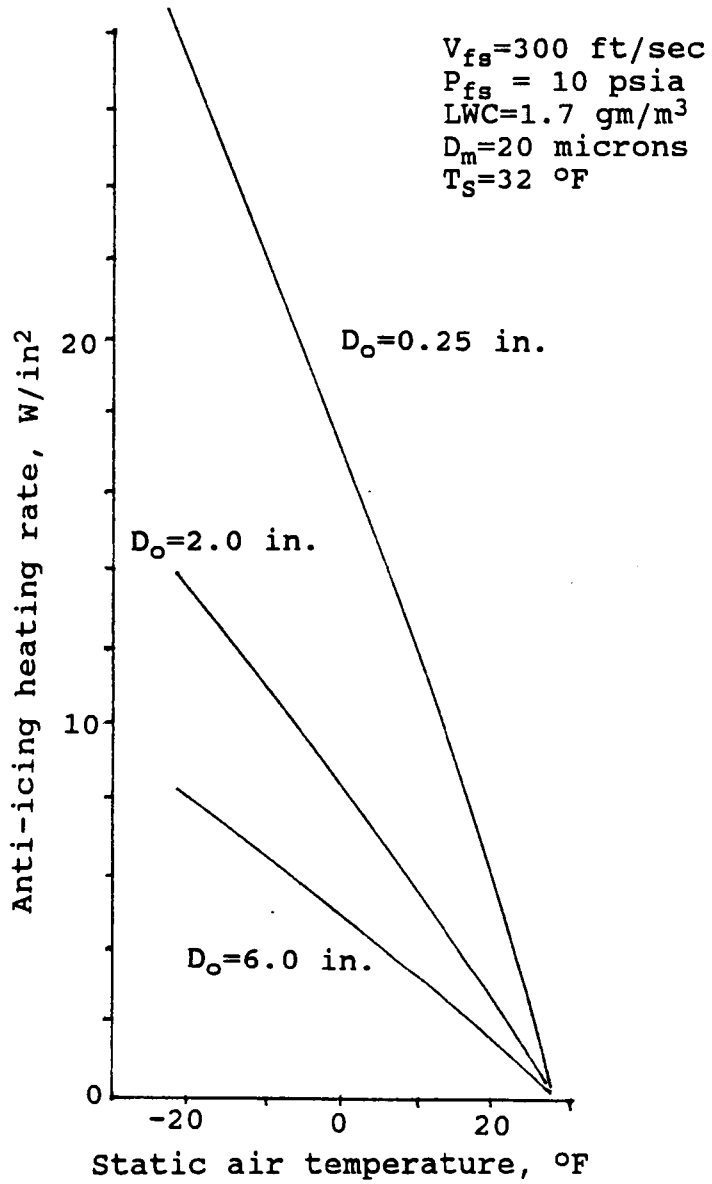


Figure 8. Anti-icing heating rate as a function of static air temperature at 300 ft/sec airspeed.

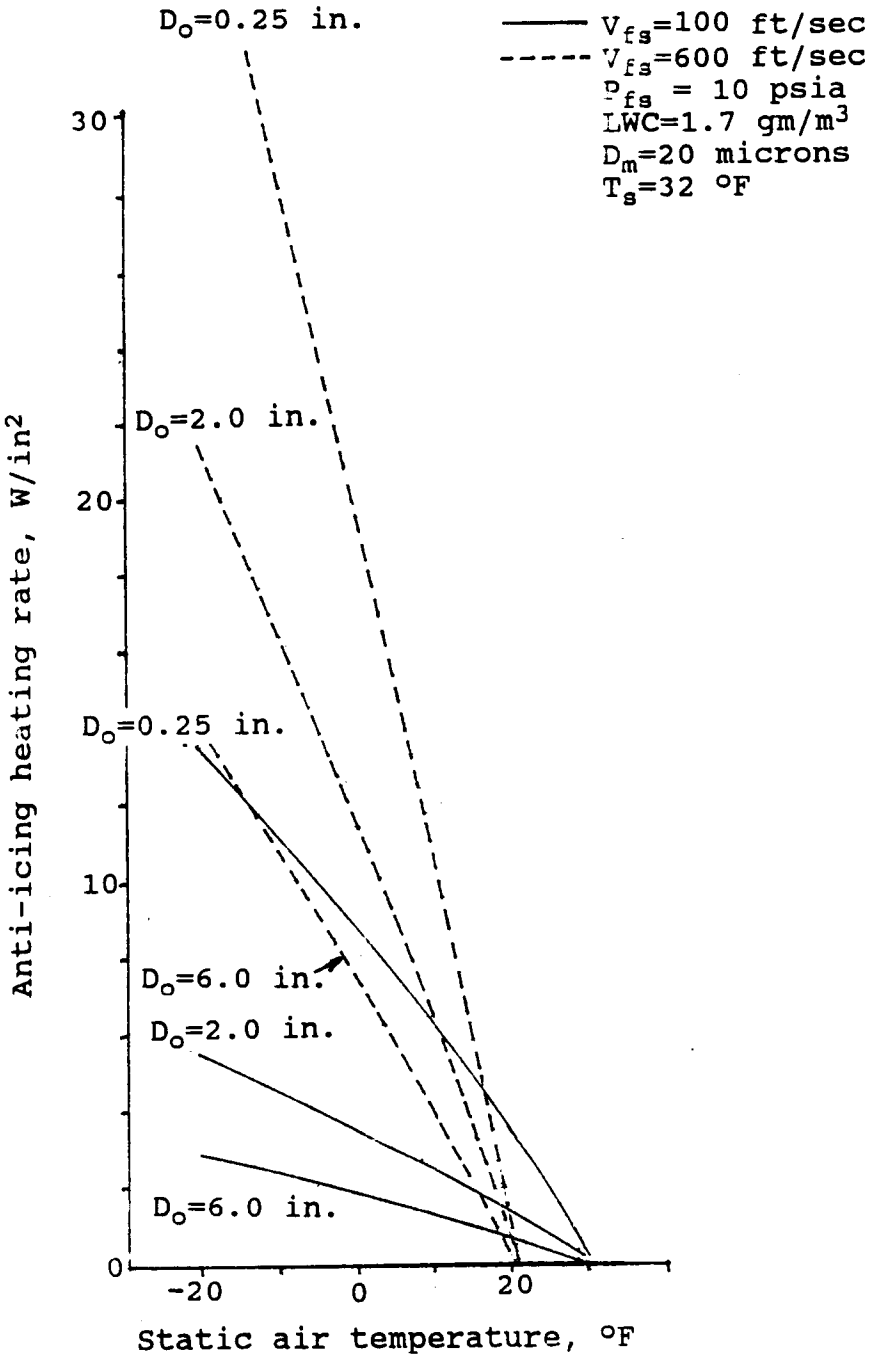


Figure 9. Anti-icing heating rate as a function of static air temperature at 100 and 600 ft/sec airspeeds.

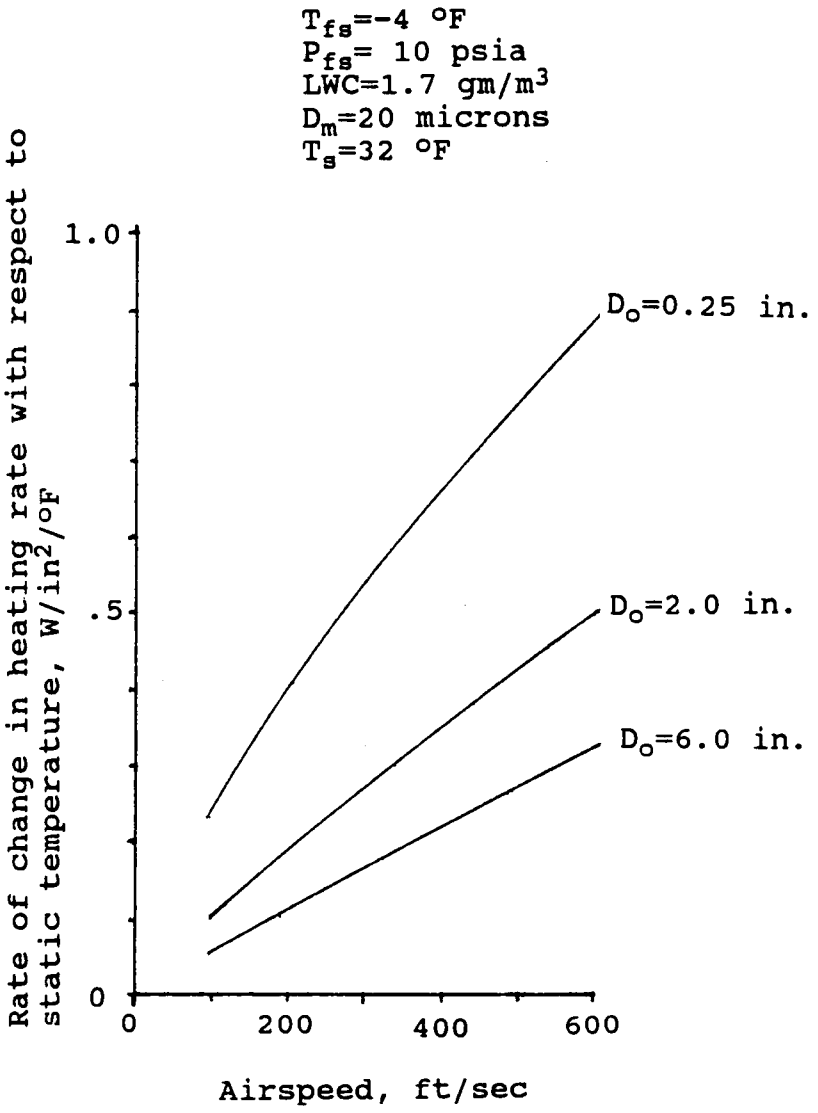


Figure 10. Rate of change in the anti-icing heating rate with respect to static air temperature as a function of airspeed.

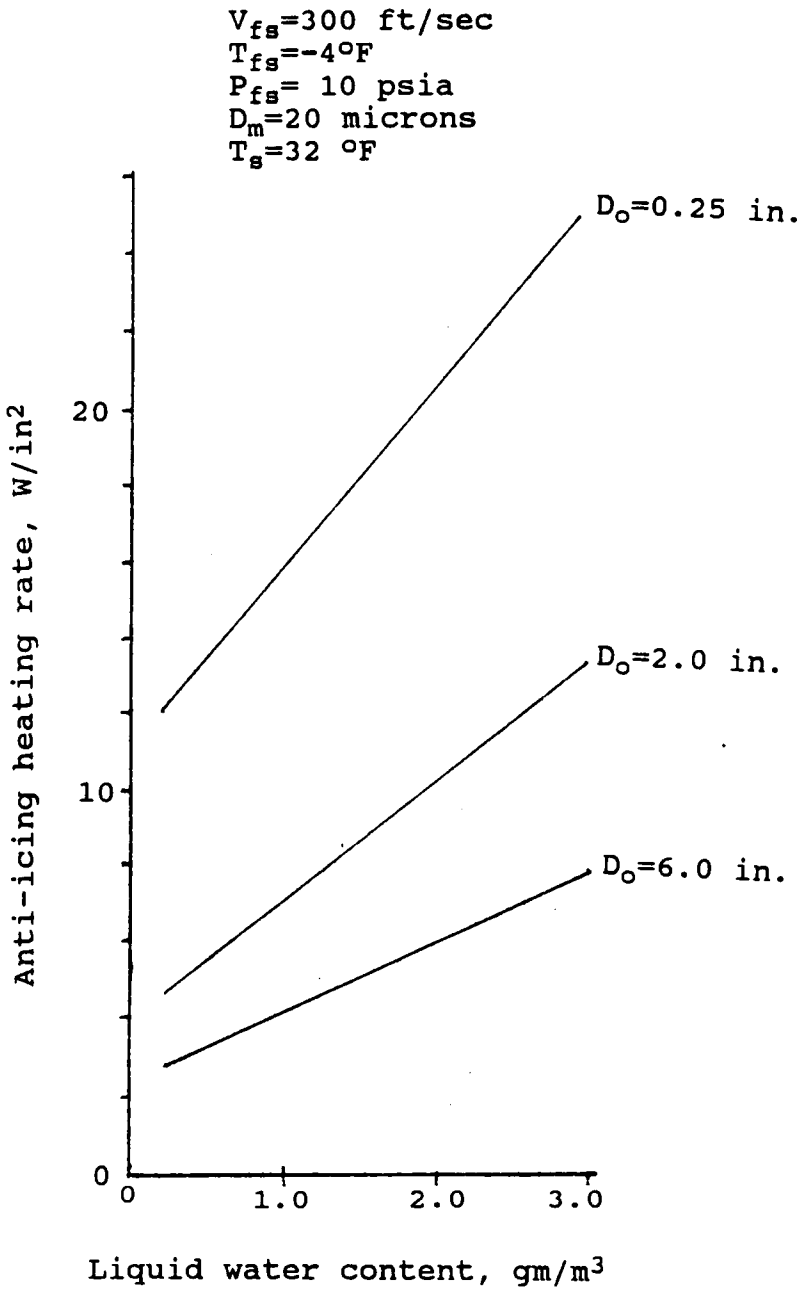


Figure 11. Anti-icing heating rate as a function of liquid water content at 300 ft/sec airspeed.

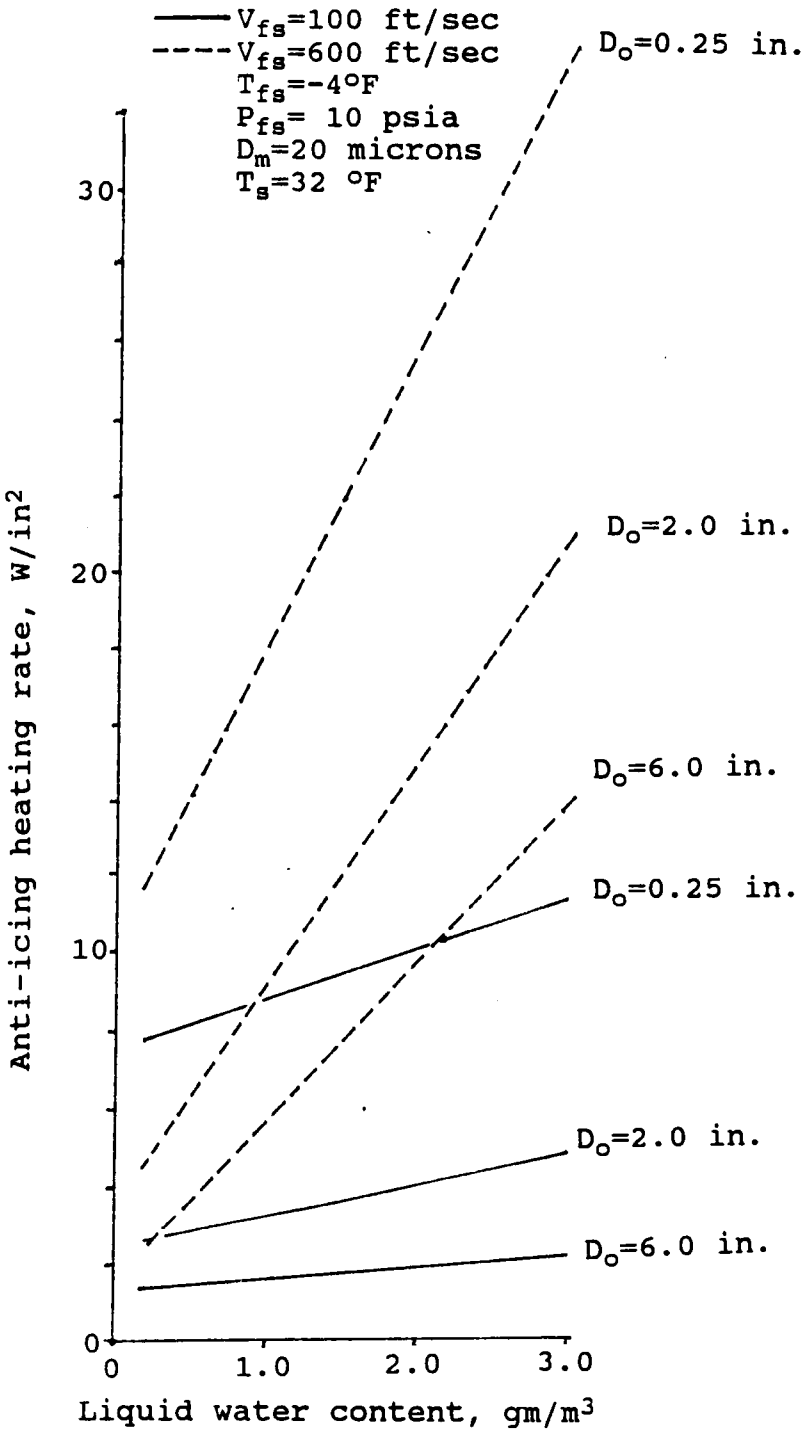


Figure 12. Anti-icing heating rate as a function of liquid water content at 100 and 600 ft/sec airspeeds.

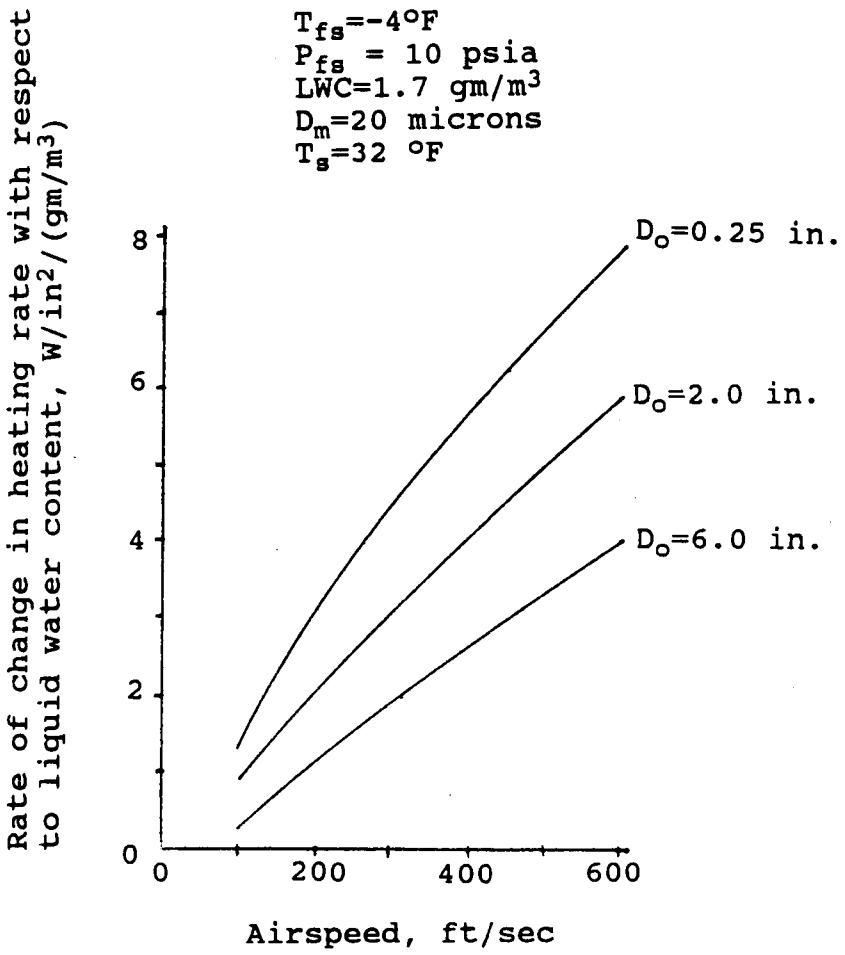


Figure 13. Rate of change in the anti-icing heating rate with respect to liquid water content as a function of airspeed.

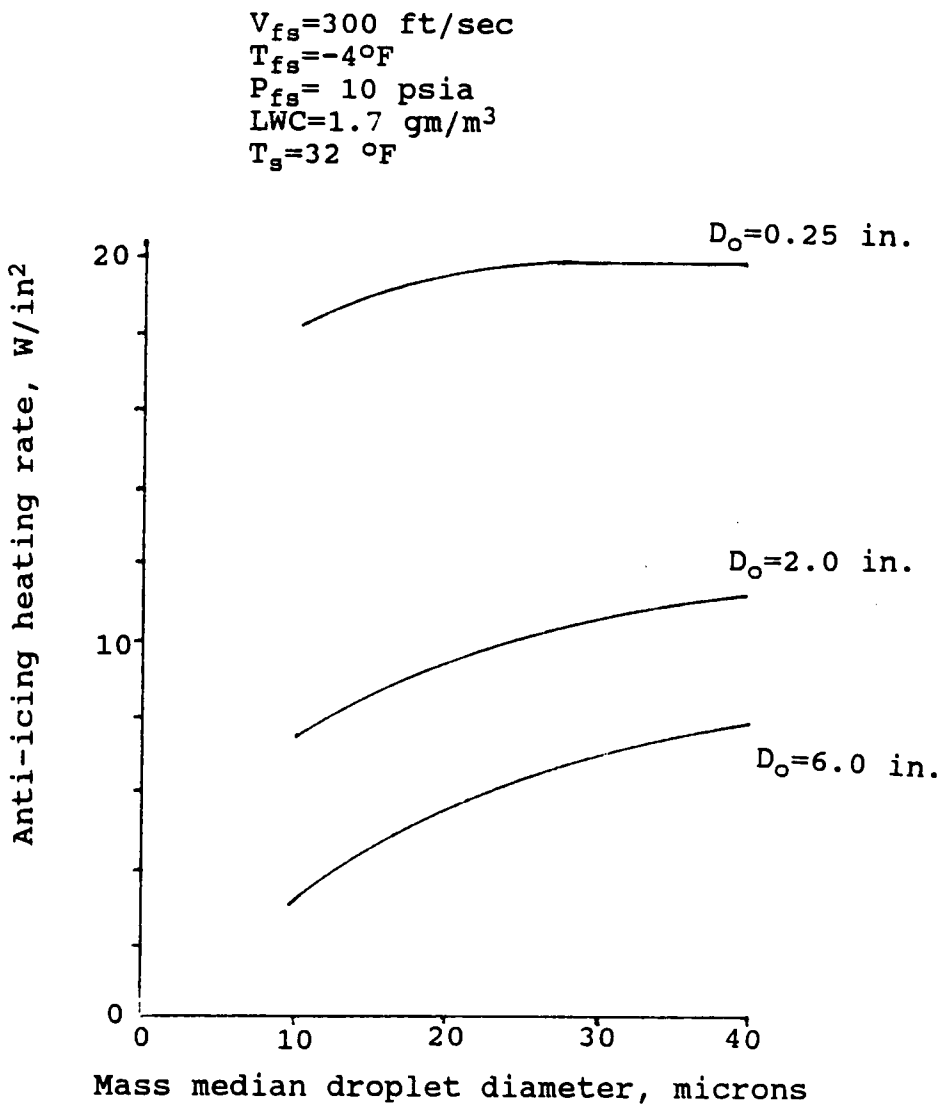


Figure 14. Anti-icing heating rate as a function of mass median droplet diameter at 300 ft/sec airspeed.

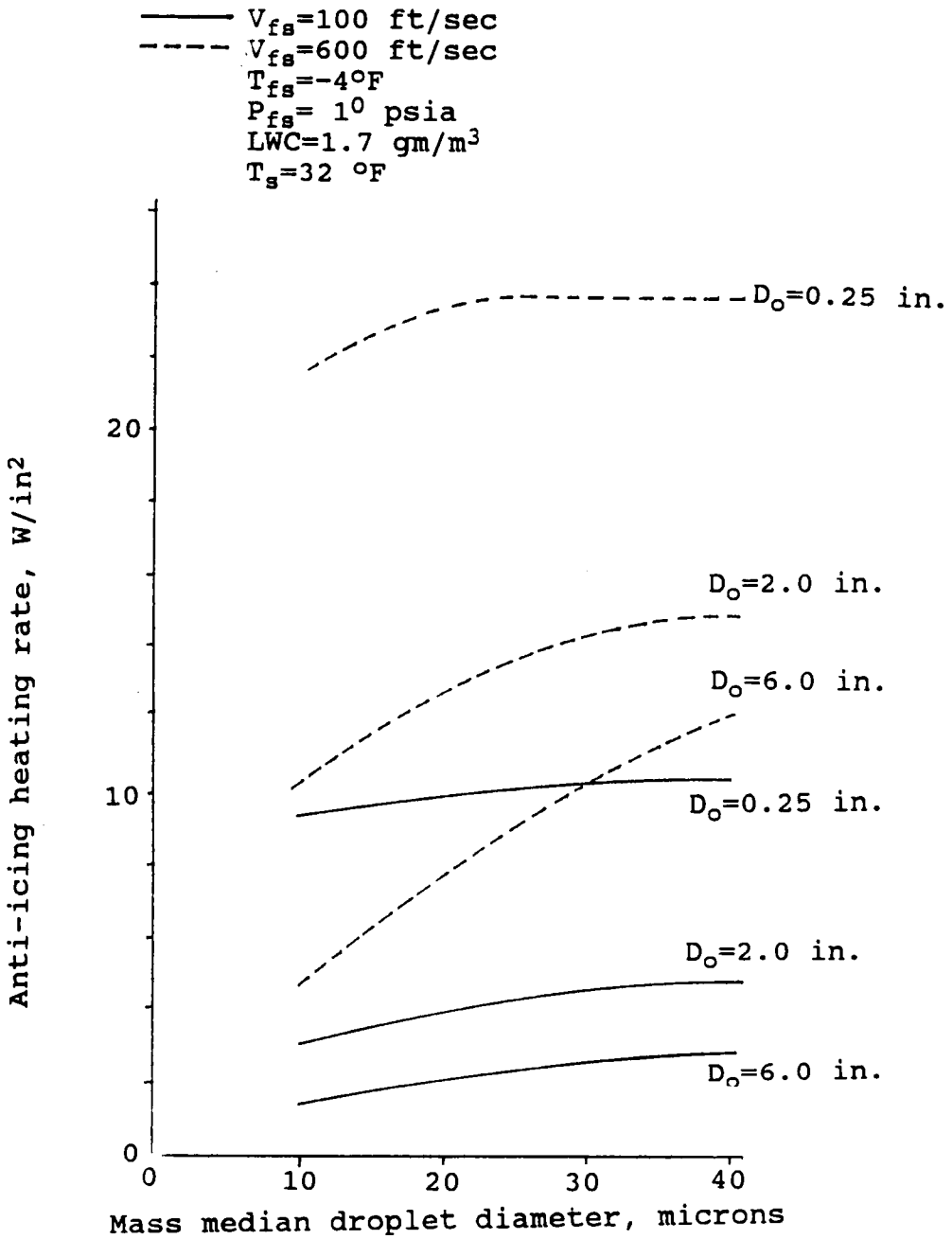


Figure 15. Anti-icing heating rate as a function of mass median droplet diameter at 100 and 600 ft/sec airspeeds.

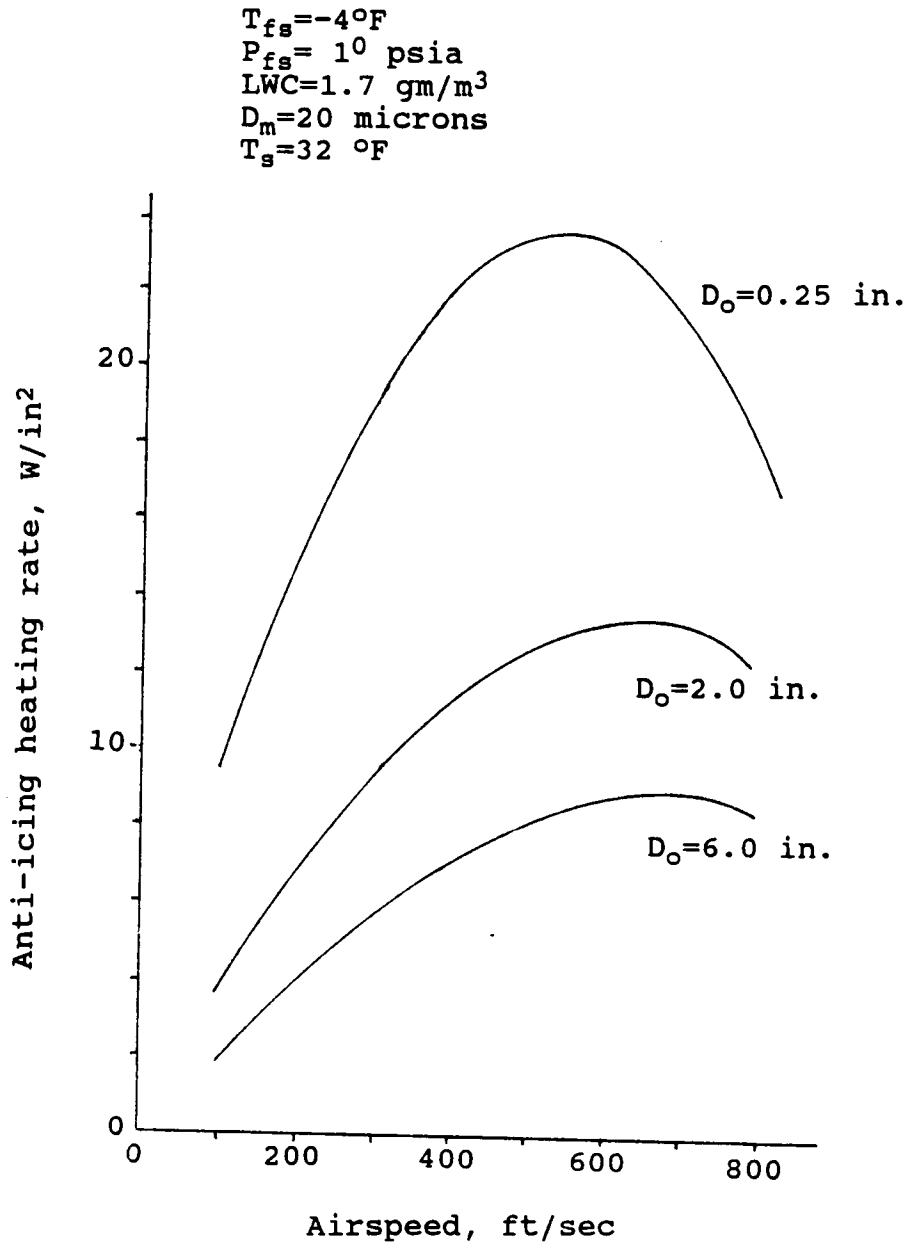


Figure 16. Anti-icing heating rate as a function of airspeed.

APPENDIX B

TABLES

Table I. Baseline icing conditions at three airspeeds and three cylinder diameters.

Condition	P_{fs}, psia	T_{fs}, ^{o}F	LWC, gm/m^3	D_m, microns	V_{fs}, ft/sec	D_o, in	W, watts/in^2
1	10	-4	1.7	20	100	.25	9.8
2	10	-4	1.7	20	300	.25	19.7
3	10	-4	1.7	20	600	.25	23.4
4	10	-4	1.7	20	100	2.0	4.0
5	10	-4	1.7	20	300	2.0	9.3
6	10	-4	1.7	20	600	2.0	13.6
7	10	-4	1.7	20	100	6.0	2.1
8	10	-4	1.7	20	300	6.0	5.5
9	10	-4	1.7	20	600	6.0	8.9

Table II. Influence of one percent parameter change ratio on the anti-icing heating rate.

Condition from Table I	P_{fs}	T_{fs}	LWC	D_m	V_{fs}
1	.0010	-.1123	.0024	.0005	.0062
2	.0004	-.1253	.0039	.0014	.0055
3	-.0006	-.1734	.0057	.0000	-.0024
4	.0005	-.1152	.0034	.0030	.0078
5	-.0001	-.1280	.0056	.0033	.0074
6	-.0013	-.1677	.0074	.0024	.0013
7	.0002	-.1142	.0025	.0062	.0087
8	.0009	-.1294	.0057	.0072	.0091
9	.0022	-.1667	.0077	.0054	.0028

Table III. Influence of specified parameter change ratio on the required anti-icing heating rate.					
Condition from Table I	P_{fs}	T_{fs}	LWC	D_m	V_{fs}
1	.0010	-.0558	.0356	.0127	.0306
2	.0004	-.0623	.0588	.0253	.0271
3	-.0006	-.0863	.0859	.000	-.0144
4	.0005	-.0573	.0513	.0603	.0388
5	.0001	-.0638	.0838	.0764	.0365
6	.0013	-.0836	.1109	.0075	.0042
7	.0002	-.0568	.0371	.1496	.0432
8	.0009	-.0646	.0855	.1463	.0451
9	.0022	-.0832	.1154	.1085	.0118

VITA

Charles Scott Bartlett was born in Orlando, Florida on April 26,1959. He attended Elementary and Junior High Schools in Winchester, Tennessee and graduated from Franklin County High School in May 1977. He entered Tennessee Technological University the fall of 1977, and received a Bachelor of Science degree in Mechanical Engineering in December 1982. Following graduation he accepted a position as an experimental test engineer with Pratt and Whitney Aircraft in West Palm Beach Florida. In June 1984 he accepted a position with Sverdrup Technology, Inc. at the Arnold Engineering Development Center. He began taking graduate courses in September 1984 at the University of Tennessee Space Institute. He graduated with a Master of Science degree in Mechanical Engineering in December, 1992. He is a member of the American Institute of Aeronautics and Astronautics.