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I am submitting herewith a dissertation written by Chandrasekaran Balasubramanyan entitled "Prediction of Aerodynamic Loads on Aircrafts With External Stores at Transonic Speeds." I recommend that it be accepted in partial fulfillment of the requirements for the Doctor of Philosophy degree, with a major in Aerospace Engineering.

S.N. Chaudhuri
S. N. Chaudhuri, Major Professor

We have read this dissertation
and recommend its acceptance:

J. J. Fuller

Walter K. Kuntz

J. R. Mann

H. R. Kumbh

Accepted for the Council:

Levan G. G. G.
Vice Chancellor
Graduate Studies and Research

PREDICTION OF AERODYNAMIC LOADS ON
AIRCRAFTS WITH EXTERNAL STORES
AT TRANSONIC SPEEDS

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ABSTRACT

In the present work, the Integral equation method has been applied to calculate the pressure distribution over an arbitrary, three dimensional lifting wing and a three dimensional wing with an external store, kept beneath the wing. The present study gives a method to calculate the interference effects imposed by an external store on the wing and vice-versa. The wing alone transonic solution is analyzed and compared with the experimental results and as well as a wing with an external store. A theory for the flow field nonlinear velocity prediction is developed, programmed, and applied to a wing with an external store configuration and the flow field velocities are compared with the experimental results. The numerical methods for solving the triple and double integrals with limits ranging from $-\infty$ to $+\infty$ are studied and the application of one such method to the flow field prediction problem is briefly indicated. Future extension of this work is indicated with the limitations and advantages.

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LIST OF SYMBOLS

Capital Letters

C_A	=	Axial force on the store
C_{M_x}	=	Rolling moment of the store
C_{M_y}	=	Pitching moment of the store
C_{M_z}	=	Yawing moment of the store
C_i	=	Cosine integral
C_D	=	Drag
C_L	=	Lift
C_y	=	Side force
E	=	Influence function
I	=	Triple integral in Eq. 3-48
L	=	$2\bar{u}_L - 1$
M	=	Local-Mach number
M_∞	=	Free stream Mach Number
MER	=	Multiple Ejector Rack
N	=	Number of panels on each surface of the wing
R	=	Region of Integration, described in Chapter III
R_B	=	Store body radius
Si	=	Sine integral

- $\bar{U}_T$ = Velocity component of the total velocity in the x-direction, given by the Subsonic Linear Theory, used in Chapter V
- H_v = Sturve function of Integer v
- N_v = Neumann function of Integer v
- TER = Triple Ejector Rack
- U_∞ = Free stream velocity
- V = Total velocity
- V_x = Component of the total velocity in the x-direction
- $V_{x_{at}}$ = Component of the total velocity in the transformed incompressible plane from the compressible plane by Goethert's transformation in the x-direction
- $V_{x_{at\infty}}$ = Component of the free stream velocity at an angle of $\beta\alpha$, in the x-direction, in the transformed incompressible plane by Goethert's transformation
- V_y = Component of the total velocity in the y-direction
- $V_{y_{at}}$ = Component of the total velocity in the transformed incompressible plane from the compressible plane by Goethert's transformation in the y-direction

- $V_{y_{at\infty}}$ = Component of the free stream velocity at an angle of $\beta\alpha$, in the y-direction, in the transformed incompressible plane by Goethert's transformation
- V_z = Component of the total velocity in the z-direction
- $V_{z_{at}}$ = Component of the total velocity, in the transformed incompressible plane from the compressible plane by Goethert's transformation in the z-direction
- X_M = Axial coordinate of the Moment reference point given in eq. 4-6
- Y = Reduced cartesian coordinates as defined in equation 5-59
- Z = Reduced cartesian coordinates as defined in equation 5-59

Small Letters

- a = Local speed of sound
- a^* = Critical speed of sound
- a_∞ = Free stream sonic velocity
- b = Semispan of the wing
- c = Chord of the wing
- c_l = Local lift
- c_p = Pressure coefficient

$$k = \frac{M_\infty^2(\gamma+1)}{U_\infty}$$

$m(x,y)$ = Parameter for the local circulation change defined in eq.(3-68)

$$\left. \begin{matrix} n_1 \\ n_2 \\ n_3 \end{matrix} \right\} = \begin{matrix} \text{Direction cosine of the normal to the shock} \\ \text{surface} \end{matrix}$$

q = Velocity vector, defined in eq. 4-11

$r(x)$ = Function for the velocity decay in eq. 3-57

$$r_2 = [(\bar{x} - \bar{\xi})^2 + (\bar{z} - \bar{\zeta})^2]^{\frac{1}{2}}$$

$$r_3 = [(\bar{x} - \bar{\xi})^2 + (\bar{y} - \bar{\eta})^2 + (\bar{z} - \bar{\zeta})^2]^{\frac{1}{2}}$$

s = Semispan of wing

u = Perturbation velocity in the x-direction

$\bar{u}$ = Reduced perturbation velocity, defined as in eq. 3-20

v = Perturbation velocity in the y-direction

$\bar{v}$ = Reduced perturbation velocity defined as in eq. 3-20

w = Perturbation velocity in the z-direction

$\bar{w}$ = Reduced perturbation velocity, defined as in eq. 3-20

x,y,z = Cartesian Coordinates

$\bar{x}$ = Reduced x coordinate as defined in eq. 3-20

$\bar{y}$ = Reduced y coordinate as defined in eq. 3-20

$\bar{z}$ = Reduced z coordinate, as defined in eq. 3-20

Greek Letters

α	=	Angle of attack
β	=	$\sqrt{1-M_\infty^2}$
γ	=	Ratio of the specific heat
Σ	=	Surface considered for deriving the integral equation
ϵ_1	=	Radius of the small sphere, surrounding the point P
σ	=	Function used in deriving the integral equation in Chapter III, standard deviation in Chapter II
Ω	=	Function used in deriving the integral equation
Λ	=	Angle of sweep
ρ	=	Density of the air; $2 y-\bar{\eta} /r$ in equation 3-70
Φ	=	Total velocity potential
ϕ	=	Disturbance potential
$\bar{\phi}$	=	Reduced velocity potential
ϕ'	=	Correction potential for the lift
ϵ_2	=	Thickness of the shock wave
θ	=	Azimuthal coordinate
v	=	Parameter defining sine and cosine integrals
τ	=	Maximum thickness of the body
ψ	=	$\left(\frac{1}{r_3}\right) \frac{\partial^2}{\partial \xi^2} \left(\frac{\bar{u}^2}{2}\right)$ in equation 3-30
ϵ_{ij}	=	Influence of the panel j at i
χ	=	$2 x-\bar{\xi} /r$ in equation 3-71

Subscripts

A	=	Corresponds to Aircraft Coordinate System
a	=	Ahead of the shock
at	=	Corresponds to the incompressible transformed plane by Goethert's Rule
b	=	Behind the shock
B	=	On the body surface
L	=	Linear theory
∞	=	Free stream conditions
sur	=	On the surface of the store
S	=	On the shock surface
s	=	Pertaining to store coordinate system
w	=	On the wing
1	=	Ahead of the shock
2	=	Behind the shock
x	{	= Denotes the derivative with respect to the independent variables x,y,z
y		
z		
ξ	{	= Denotes the derivative with respect to the independent variables ξ, η, ζ
η		
ζ		

Other Symbols

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\Delta = \text{Incremental operator}$$

CHAPTER I

INTRODUCTION AND TECHNICAL BACKGROUND

The external store problem has been receiving considerable attention during the past decade, since there is a need for developing analytical and numerical methods that are capable of predicting aerodynamic characteristics about arbitrary aircraft - store configurations as accurately as possible. Theoretical studies are important in order to substantiate the experimental results and to set a guideline to understand some basic features of the complicated transonic flow nature present in the multibody problem. Various investigators have been working for a long time on the means of achieving safe store separation. The first step for this is the prediction of the aerodynamic forces involved. There are several methods for calculating the non-uniform flow field around the external stores in subsonic flow as well as some for supersonic flow, based on linear potential theory. So far, there are no computational techniques available for the transonic case, which happens to be the area of main interest since store separation normally occurs in this régime. Subsonic methods are inaccurate above the critical flow conditions, while the supersonic theories, based on linear theories, are limited to slender wing-body configurations. Another inaccuracy that, so far, has not

been corrected is the ignoring of the viscous flow effects on the wing and the store. A simple potential flow model taking the boundary layer effect into account must be constructed for better accuracy.

Until 1974, wind tunnel testing and free flight testing were the only means of analyzing the external store problem, and there were no adequate theoretical methods to support the experimental results. Free flight testing is more costly than wind tunnel tests, which, in turn, are more costly than the theoretical techniques, so the development of the theoretical methods was drawn to the attention of scientists to reduce the cost involved in wind tunnel and free flight testing. If a theoretical technique were to be developed with adequate accuracy to predict the trajectory, along with an idea of the aerodynamic forces involved, this might possibly reduce the tunnel testing programs and free flight testing. Therefore, enormous benefits could be derived by the development of the theoretical prediction techniques. Limitations of the current theoretical methods developed so far are:

1. Subsonic theories are not accurate above critical flow conditions, and there are no techniques to calculate the transonic flows except correction to the linear value to account for the nonlinear effects.

2. The supersonic methods are limited to slender, thin bodies.

3. Boundary layer effect has not been taken into account due to the fact that this requires larger computer storage and a longer time to run.

In the present work, an approximate theory is developed to predict pressure distribution on the wing, fuselage, and pylon with the store, taking into account the nonlinearity imposed by the transonic nature of the flow. The method for the calculation of the nonlinear velocities in the flow field of a wing-body configuration is developed and compared with the experimental results.

This chapter will comment on some of the basic problems inherent in the store separation problems, highlights the technical background, describes the aerodynamic characteristics of the external stores and reviews the available theoretical methods that are applicable in subsonic and supersonic flow regions together with the detailed formulations in subsonic theory for the interference problem.

Chapter II describes the experimental apparatus and procedures used in the wind tunnel testing, together with a brief description of the model mounting and the associated instrumentation for data reduction.

Chapter III describes the formulation and solution of the two- and three-dimensional transonic flow equation, using the integral equation method, and solutions of the integral equation method are indicated by using the

algebraic approximation and by the exponential decay function for the axial component of the velocity in the flow field. For the lifting wing configuration, Norstrud's method of solving the integral equation is outlined and applied to a wing, and the results are compared with experiments. The application of the exponential decay approximation, in the case of two-dimensional airfoils, for the calculation of the pressure distribution on them, is studied and results have been quoted for the subcritical, supercritical and discontinuous flows and compared with the other existing theories. The results of the finite difference method for the wing alone calculation is compared with the experiments. The merits of the integral equation method are briefly outlined.

Chapter IV describes the methods for calculating the interference problem in the case of subsonic and transonic flows. For transonic flow, an approximate method is formulated and applied to a wing-store interference problem, and the results are compared with experiments. Chapter V describes the nonlinear flow field velocity calculation method and the associated numerical work. It also gives the comparison of the flow field velocity data for a wing and store configuration with the experiments.

Chapter VI discusses certain significant conclusions and throws light on problem areas which need further study.

TECHNICAL BACKGROUND

Effect Of The External Stores On The Parent Aircraft's Handling Qualities

The interactions between the parent aircraft and the external stores can be distinguished as store-induced effect on the aircraft and aircraft-induced effects on the store [1] . The first category is primarily concerned with the aircraft's performance losses due to store drag, the changes to aircraft stability caused by external stores, and the resulting effect on handling qualities. The consequences of high aerodynamic drag of external stores are quite obvious and have been aptly discussed in many publications. Perhaps more serious, and generally more common, is the necessity to place even more restrictive operating limitations on certain aircraft/store combinations because of dangerous store release or jettison characteristics. These problems can be avoided, or significantly reduced, by proper aircraft and store design. The ability to properly establish weapons system design criteria depends upon the development and use of accurate and reliable methods of predicting store trajectories. Experience has shown that the proper use of accurate prediction methods will not only result in increased delivery capability and enhance the safety of weapon

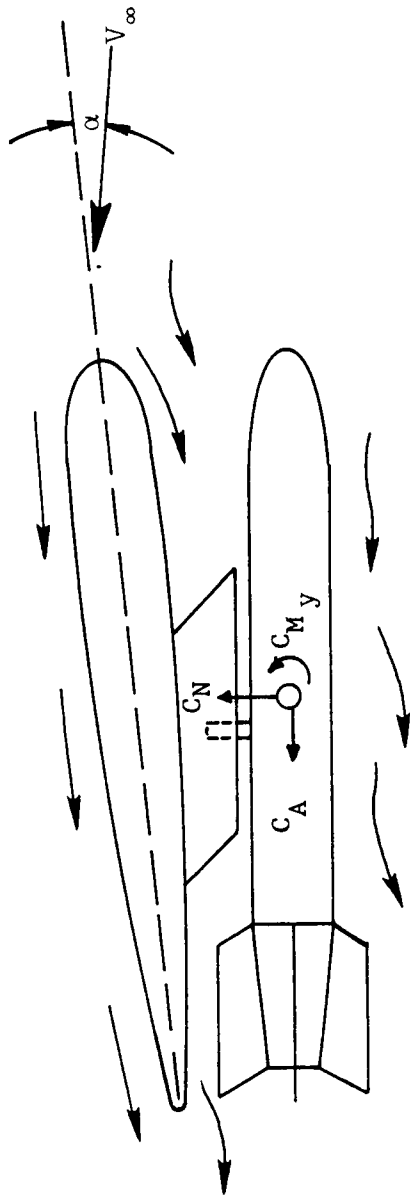
delivery and jettison during flight testing or operational employment, but considerable savings of time and money can also be realized. Aircraft and store designs can be thoroughly investigated using analytical and wind tunnel models and readily changed as necessary prior to production or the fabrication of costly full-scale prototypes.

A. Factors in Store Separation

Store separation categories. The store separation problem can be basically divided into three categories, depending upon the type of the store. The first category will be referred to as the *jettison problem*. This involves the class of stores such as fuel tanks, gun pods, multiple bomb racks complete with weapons, etc., which are released from an aircraft during emergencies or normal operations, with the primary motive being safe separation. There is no requirement for accurate delivery. This mode is considered by far the most dangerous, as many of the items are low density and marginally stable. The second category is the *delivery problem*, which involves stores such as unguided general-purpose bombs and dispenser munitions released in a package for the purpose of hitting a target. This problem not only requires that the item separate safely from the aircraft but also requires a relatively unperturbed release for good delivery accuracy. The third

category is the *launch transient problem*, which is far more difficult to analyze. This involves items such as direct attack electro-optical guided bombs which are locked onto a target before launch. Typically, these weapons have controls which are active during the release to avoid target breaklock due to release perturbations. Normally, these weapons do not pose a danger to the launch aircraft, either in a jettison mode (controls fixed) or during launch with active controls. The danger of these stores comes from control failure and subsequent abnormal deflection soon after release, which could cause collision with the delivery aircraft. As a result, flight control system reliability is one of the major concerns for this type of store. The criteria for satisfactory stores separation vary for each category. A tumbling, empty fuel tank causes no concern provided there is no danger of aircraft collision within the desired jettison envelope.

Store separation parameters. There are many parameters that affect store separation. The more significant ones are depicted in Figure 1.1. Store separation is very complex. The above parameters, shown in Figure 1.1, are highly coupled and react with each other in a most complicated manner. As was pointed out in the previous paragraph, different criteria exist for each category of the store separation problem, and the above parameters play roles of varying importance in each of these categories.



Aerodynamic Parameters

Store Shape
Store Stability
Aircraft Velocity
Aircraft Attitude
Aircraft Mach Number
Aircraft Load Factor
Aircraft Pitch Angle
Aircraft Yaw Angle
Aircraft Roll Angle
Aircraft Configuration
Flow Field Angularity
Flow Field Variation With Displacement

Physical Parameters

Store Mass
Store Inertia
Store Center Of Gravity
Bomb Rack
Ejector Location
Ejector Impulse

Figure 1.1. Store Separation Parameters [2].

In general, stores having high density, high inertia, and high aerodynamic stability, will exhibit the most desirable separation characteristics. It is not always practical to design stores with these characteristics as mission requirements can have a significant impact on external shape and mass properties. For example, a high density, empty fuel tank would impose an impossible weight penalty on the aircraft with the fuel tank in the full configuration or it would be extremely costly, and impractical, to design a rocket launcher (both full and empty) to the specifications of high density, high inertia, and high aerodynamic stability. It is emphasized that store separation parameters are highly coupled and the variation of each plays a different role in each case. In general, the reports and other evidence available in the parametric studies support the following conclusions:

a. The most significant parameters are store stability, the bomb ejector rack induced moments, the aircraft flow field (function of Mach number and aircraft attitude), and the aircraft acceleration at release.

b. The effects of bomb rack ejection force, store weight and dynamic pressure are secondary and approximately the same for aerodynamically stable stores. These effects become more important as store stability decreases.

c. Store moments of inertia are relatively unimportant for aerodynamically stable stores but become more important as store stability decreases.

d. Flight path angle effects are normally small except for longitudinal store displacement; however, at the higher dive angles the store will remain in the disturbed aircraft flow field longer--thus the flow field effects are magnified.

e. Multiple ripple releases are more critical than single releases because of mutual interference effects of the store immediately after release, reduction of ejection force due to multiple rack flexibility, and the increased probability of store-to-store collision.

f. Aerodynamic damping effects are relatively unimportant; however, in some rather special cases for large or very heavy stores, the change in the store aerodynamic damping that can be induced by the motion (oscillation) of the parent aircraft may be large enough to cause significant effects on the flight behavior of the store.

B. Aircraft Flow Field

The interference flow field about a store-laden aircraft may be the most important parameter governing store trajectory. It is normally quite curvilinear in form and varies with aircraft, store, store position, adjacent

stores, flight conditions, and aircraft attitude. Figure 1.2 shows the velocity components in various lateral planes measured about 3 M-117 bombs on a Triple Ejector Rack on the inboard wing station of an F-4 aircraft at 0.85 Mach number [2]. Attention is directed to the large downwash at the nose of the weapon on the bottom station (approximately 100 ft/sec or 30 m/sec). This typical flow characteristic produces an extremely large nose-down aerodynamic pitching moment on large diameter 16-inch (41 cm) stores at higher Mach numbers and results in unsatisfactory trajectories for many weapons. Figure 1.3 shows the same results from a wind tunnel oil flow test conducted at the United States Air Force Academy.

C. Store Separation Prediction Methods

There are basically two classes of methods commonly used to predict store separation techniques. These are the purely theoretical techniques and various techniques based on the use of wind-tunnel data. Each method has its own utility, and the decision as to which method or methods to use for a given problem must be made on the basis of the problem itself.

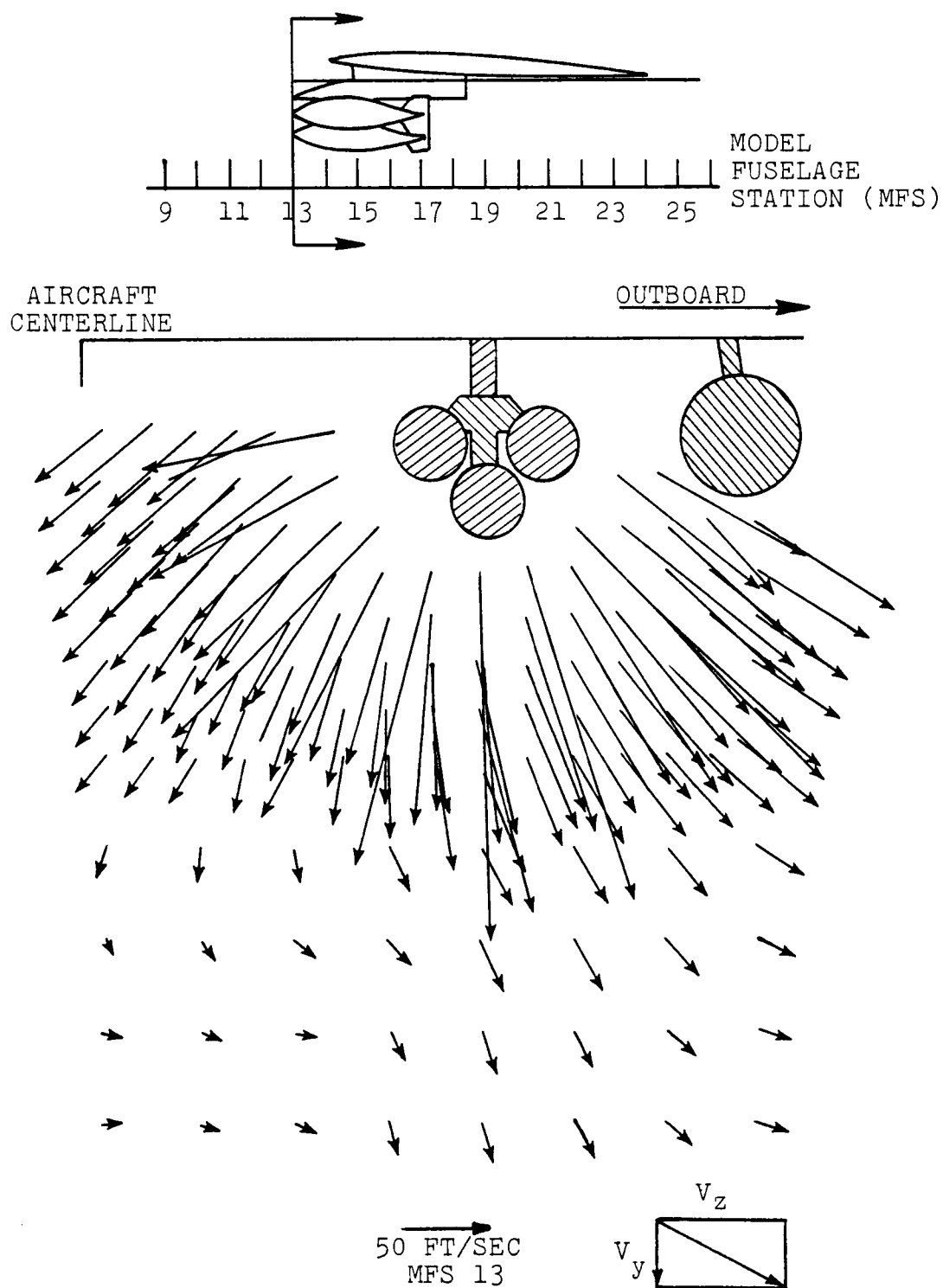


Figure 1.2. F-4 Interference Flow Field at 0.85 Mach Number (Asymmetric Configuration) at Fuselage Station 13. [2]

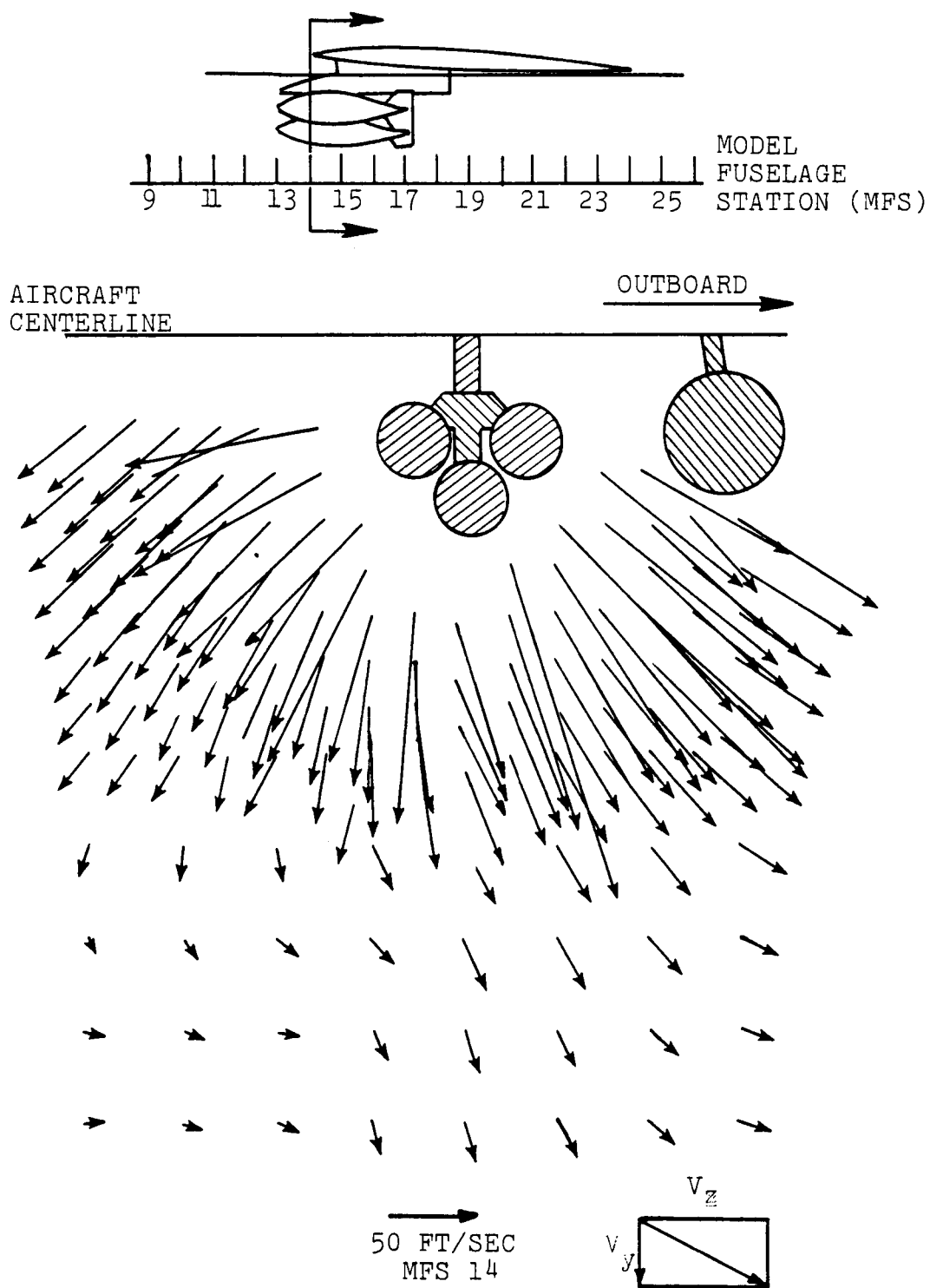


Figure 1.2. (Continued).

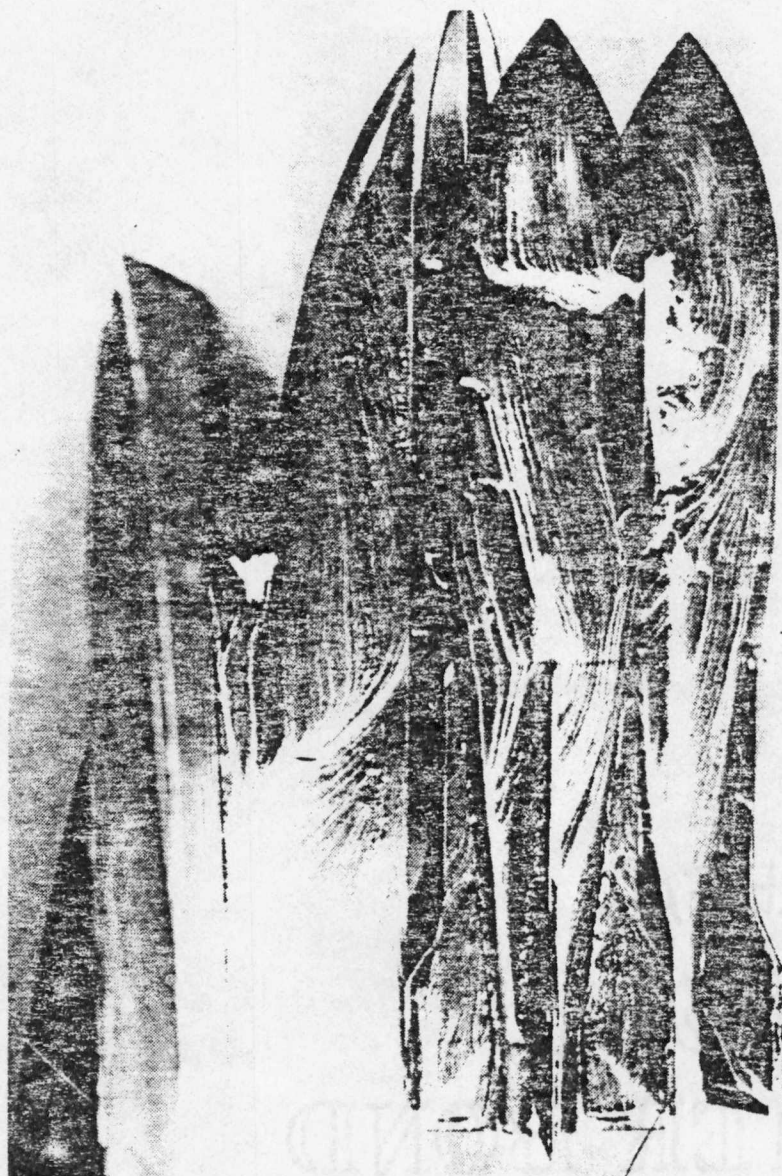


Figure 1.3. Oil Flow Test of 3 M-117 Bombs on an F-4 Inboard Wing Station Triple Ejector Rack. [2]

Present Available Theoretical Methods For The
Analysis Of The External Store Problem
In Subsonic/Supersonic Flow

The theoretical techniques are difficult to apply and limited in application because of the lack of adequate transonic theory. They may, however, be used within their realm of applicability for studies of non-critical problems to explain certain phenomena and, in conjunction with wind tunnel test programs, to reduce the scope of experimental effort. Reference [3] is an excellent report which compares theoretical techniques and experimental wind tunnel techniques to full-scale flight test data. At this time, however, the wind tunnel is the only reliable store separation prediction technique that can be used with confidence throughout the flight regime of our present-day tactical aircraft.

For the subsonic flows below the critical speed range, Nielsen and others [4], Fernandes of General Dynamics [4], and Martin of Auburn University [5] developed panel methods. All these theoretical techniques so far developed in the store separation problems are based on potential flow theory and do not include viscosity. None of the techniques can handle supersonic flow except that of Fernandes, which is applicable to single carriage only.

The Nielsen and Martin methods were developed to handle multiple carriage configurations where mutual aerodynamic interference effects prevail. These methods are excellently described in [4], and a review of the above 3 methods can be obtained from [5], which are summarized in the following sections.

Fernandes' method. The theoretical analysis by Fernandes [6] of General Dynamics under a NASA Contract for the store separation analysis, predicted the disturbance flow field produced by an aircraft using linear theory of source, doublet and vortex distribution.

The lifting problem was represented by a spanwise and chordwise lattice of swept horseshoe vortices distributed to increase the number of vortices near the leading edge of the wing and near the wing-pylon juncture. The strength of each vortex was determined by satisfying the boundary condition that there is no flow through the wing at each control point. Once the strength of each vortex was known, the Biot Savart Law was used to determine the wing lift induced velocity perturbation. Similarly, wing thickness effects were represented by a spanwise and chordwise distribution of elementary source strips whose strengths were functions of the slope of the wing thickness enveloped in the spanwise direction. The interference between the pylon and the sidewash from

the flow around the wing were compensated by representing the pylon with horseshoe vortices to match the boundary conditions of flow tangency at the pylon surface. The pylon thickness effects are modelled by using small source strips similar to those described for modelling the thickness effects. The secondary components of mutual interference between wing and pylon are accounted by mirroring the pylon above the plane of the wing. Similarly, this method has the capability of treating the jet inlet. The aircraft fuselage and any additional stores or racks were represented as bodies of revolution at angle of attack and were modelled mathematically with source and doublet singularities placed along their body axis of symmetry. The strength of each source and doublet was determined by matching the boundary conditions of no flow normal to the body surface at several control points. This method also takes into account for the non-slenderness of the store configuration and it treats stores with wings, canards and/or tails of any aspect ratio. This method predicts the complete set of interference coefficients on the store (normal and side force, pitch and yaw moment, and roll moment coefficients).

To compute the interference loading, the store is divided axially into small sections. The load on each section is computed from the local crossflow by use of a set of loading coefficients which describe the section

aerodynamic properties. These loading coefficients are input to the computer program for the prediction of store loading in nonuniform flow. These coefficients may be obtained from test data in uniform flow or from theory.

For the supersonic flow, Fernandes predicted the flow field about the aircraft by using linear theory, with corrections for shockwave effects. The method calculates the shock location and calculates the singularity strengths to match the boundary conditions of a general jet fighter-bomber aircraft configuration in supersonic flow. The disturbance velocities are calculated by the program over the length of the store for each position of interest, which gives the interference angle of attack and static pressure as seen by the store. The interference coefficients are given by the integrated effect of the variable flow field over the length of the store. These interference coefficients are to be added linearly to the store free-air coefficients to obtain the total store loading at angle of attack during separation.

The aircraft wing, pylon, and jet inlet ramps are represented by source distribution according to linear theory. These aircraft surfaces are divided into small segments of constant source strength. The source strengths to represent wing and pylon crossflow are determined by the "Mach Box" numerical step procedure [7]. There is, however, one important feature of the store

loading in supersonic flow which requires special attention. That is the loading on the store body produced by the variable static pressure in the interference field, termed the buoyancy loading. To improve the precision of the buoyancy force prediction, the supersonic prediction program was coded to calculate the buoyancy loading due to pressure decay along the Mach lines. The results of the calculations using this method showed that the buoyancy loading prediction was negligible. However, this calculation neglects the moment on the store produced by the fact that the Mach lines cross the store at an oblique angle. It should be possible to improve the basic accuracy of the supersonic prediction method by:

1. Accounting for the strengths of flow-field disturbances behind finite shockwaves as they may differ from that given by linear theory.

2. Accounting for the store buoyancy loading produced by a pressure field which is relatively constant along lines which cross the store at an oblique angle. Further studies to improve and evaluate the method should also include data comparisons between theory and experiments for the interference loadings. The Fernandes method of analyzing store separation allows for no interaction between stores, or between the stores and the lifting surfaces. All interference effects except the pylon sidewash interference and the axial nonuniform flow along the body are assumed negligible.

Nielsen's method. The method due to Nielsen [8] accounts more accurately for the mutual interference effects. The calculations for the wing and pylon were similar to those used in the Fernandes method, with a vortex lattice used to account for the lifting wing problem, and source strips to model the thickness. However, the Nielsen method allows the lifting surface (wings and pylons) to be influenced by the thickness effects of the bodies (fuselage, rack and stores), but the source distributions to model the bodies were not influenced by the lifting surfaces. The volume effects of the fuselage, rack, and stores were calculated independently of the lifting surfaces. They were calculated for the body in a uniform flow field.

The main advantage of Nielsen's method is that it also considers the effect of a noncircular fuselage in the canopies and engine air inlets. The shape of the fuselage cross section is arbitrary and may vary from one axial station to the next along the fuselage. In this manner a fuselage of general cross-sectional shape can be modelled including camber, canopy, and engine inlets. The modelling of the engine inlets allows the inlet to free stream ratio to be one or less.

This method essentially consists of vortex lattice for the wing portion and sources for the wing thickness, and sources and doublets for the body portion.

At each time for a captive store trajectory, this program calculates the velocity flow field at those points occupied by the store longitudinal axis at that instant, together with the slender-body theory and viscous cross-flow theory for the stores in the prediction of store airloads.

The trajectory calculation utilizes the full six degree-of-freedom equations. It is not essential that the principal axis of inertia coincide with the axes of symmetry of the store. At each step in the trajectory calculation the flow field in the vicinity of the store, and the resulting forces and moments acting on the store are determined.

The two disadvantages of Nielsen's technique are:

1. The variations in angle of attack and Mach number of the flow around the store are not included in determining the source distribution representing the store.

2. The theory treats the multiple store problem as though the only interference present was between the wing flow field and the stores. Actually, due to their proximity, there are considerable body-body mutual interference effects during MER/TER carriage and release of stores, which has not been taken into account.

Martin's method. Fred W. Martin of Auburn University [9] devised a method which consists of modelling the wing with a vortex lattice and body with a three-dimensional point source distribution. The resulting nonuniform flow field of the wing is used to determine the vortex image system in the body which preserves the body shape boundary condition. The necessary vortex image system is located by applying the Milne-Thompson circle theorem to a pair of counter-rotating planar vortices in the vicinity of a circle. The strengths of the image-vortices are determined by satisfying the three-dimensional body boundary condition. The isolated wing solution may then be obtained by considering the influence at the wing of the body and its image systems. If this iteration results in a significant change in the vortex-strength distribution on the wing, a new series of body images must be calculated.

For the body-body interference, the volume of each body is modelled by a set of three-dimensional point sources distributed along the centerline. The strength of the sources are found to insure no flow occurs across the store boundary at the given Mach number (normal velocity boundary condition). Next, a set of doublets is found which, combined with the single body sources, represents warped body effects and the freestream cross flow due to angle of attack. The third step is to use the

Milne-Thompson circle theorem to find a set of image source-sink pairs for each single body source. Finally, sets of image doublets are found for each unique pair of bodies. The body boundary conditions used in this analysis take into account the flow field generated by all previously determined sources, image pairs, and basic doublets. The major limitation of Martin's technique is that to date the vortex lattice system used to model the wing planform has not included sufficient chordwise elements to accurately model the flow around fighter aircraft wings with complex configurations of dihedral twist, camber and sweep. The memory requirements for running the program are enormous and do not provide a method for readily determining the strength and location of the single body source distribution as does Nielsen's technique which provides a separate source solution program. The counterpart of this method in supersonic flow is described in Reference [10].

So far, we have reviewed the methods which are not "global," i.e., the parent aircraft's effect on the store is considered only as if the store is in the freestream comprised of the disturbed flowfield due to the parent aircraft. If a "global" panel method calculation is performed, in which the complete configuration including the store is simultaneously represented, then all interference terms are properly included within the context of potential flow. The disadvantages of such "global" calculations are the computational time and cost involved.

References [11] and [12] describe one such panel method, which is based on the method for two-dimensional flows and nonlifting three-dimensional flows indicated in References [13] and [14]. The latter is described in somewhat greater detail in [15]. The three-dimensional nonlifting method forms the basis on which is built the lifting method [11] as well as other lifting methods [16, 17, 18, 19]. All these methods of all dimensionality are "exact" in the sense that no approximations are made in the basic formulation, as is done in small-perturbation of lifting-surface theories. The basis of these methods is a distribution of source density over the surface of the body about which flow is to be computed. Application of the normal-velocity boundary condition on the surface of the body yields an integral equation for the distribution function of the source density, where the domain of integration is the body surface. Once this equation is solved for the source distribution, flow velocities both on and off the body surface can be calculated. In lifting two-dimensional cases the required circulation is produced by a surface vorticity distribution whose total strength is determined from the Kutta condition [14].

Discussion of Some Specific Features of the Hess and Smith Method

A very complete discussion of all the formulas and logic of the present method of calculating three-dimensional lifting potential flow is contained in [11], and the computer program itself is documented in [20]. In the interest of brevity the mass of details of the method are not repeated here. Instead certain features are discussed to show what the method involves and some alternatives are mentioned.

Induced velocity formulas. The body surface is approximated by a large number of plane surface elements. Specific formulas have been derived for the velocities induced at the control points of the elements by source and dipole distributions on the elements. A constant source density is used on all elements of both lifting and nonlifting portions of the configuration (Figure 1.4). The dipole distribution on each element of a lifting portion must be a two-variable polynomial of second degree to handle the various options. To save computing time, approximate simplified induced-velocity formulas are used to compute the effect of an element at far-away points.

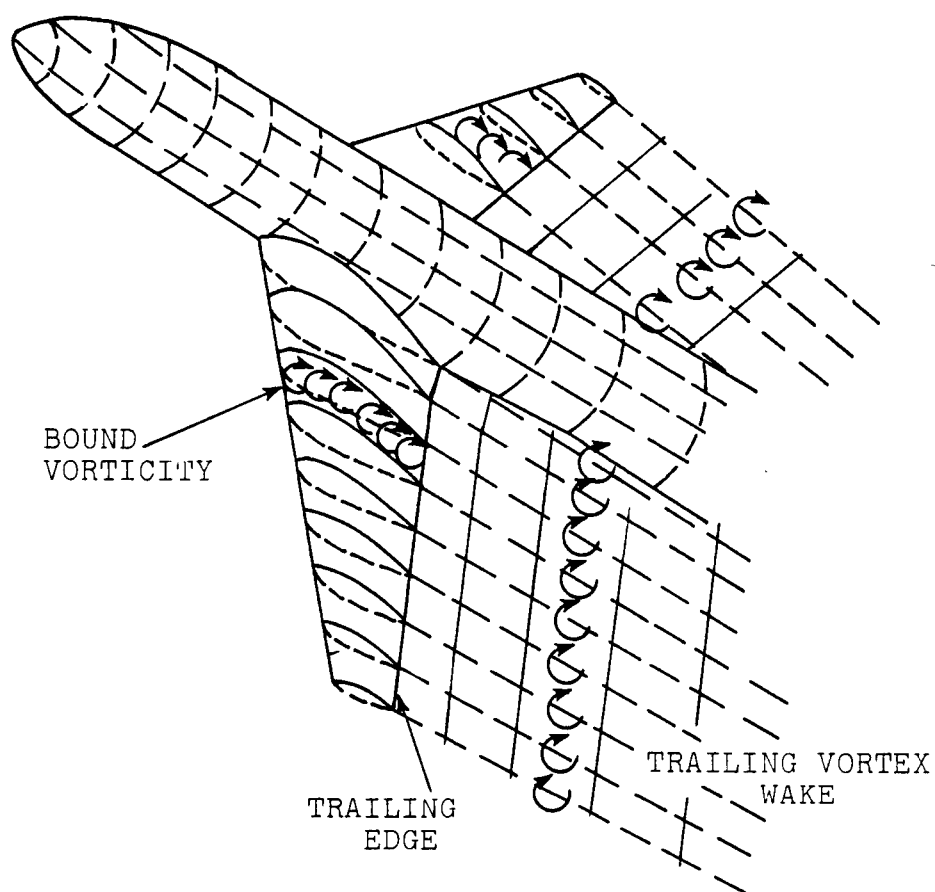


Figure 1.4. Typical Lifting Configuration.

Use Of Hess And Smith Method In The Case Of An External-store Interference Study

To illustrate the use of the Hess and Smith method in an interference study, a simplified shape has been selected but one that has all the essential features of an actual design. The basic geometry is a rectangular wing mounted as a midwing on a round fuselage. Two external-store configurations are considered. The first consists of a tip tank. The geometry and element distribution for this case are shown in Figure 1.5. The second configuration consists of the same external store mounted beneath the wing on a short pylon centered at 60 percent semispan. The geometry and element distribution for this case are shown in Figure 1.5. There will be a "hub vortex" trailing downstream from the tip tank. However, this does not appear to cause any numerical problem, and calculated surface velocities near the downstream end of the tip tank seem entirely reasonable.

Calculations were performed at 6 degrees angle of attack for the 3 configurations: the clean wing, the wing with tip tank, and the wing with pylon-mounted external store. The round fuselage was present in all cases. Calculated results on the wing are compared in Figure 1.6. Figure 1.6a compares spanwise distributions of section lift coefficient. The addition of the tip tank to the wing prevents the lift from falling to zero at the tip, so that

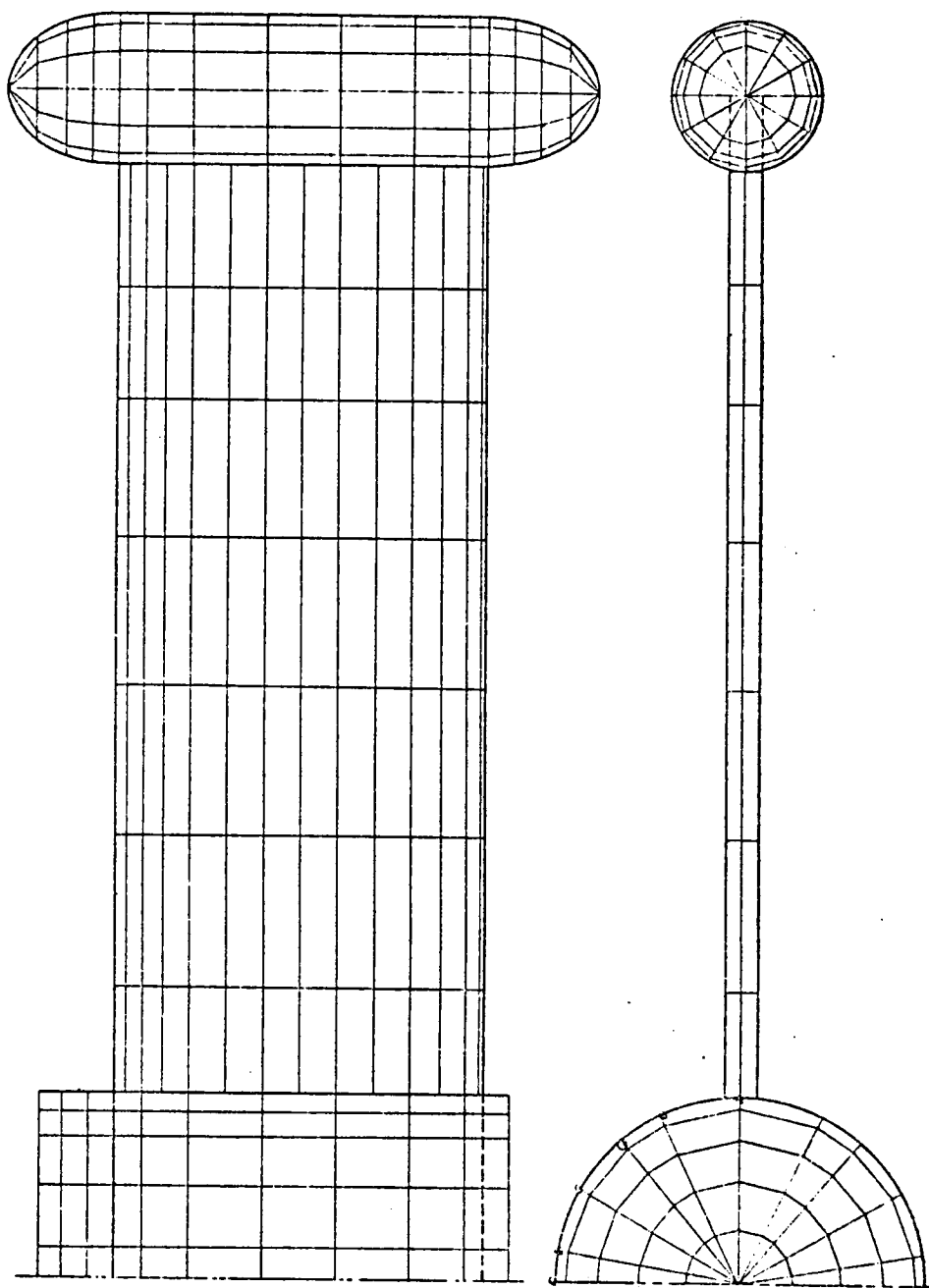
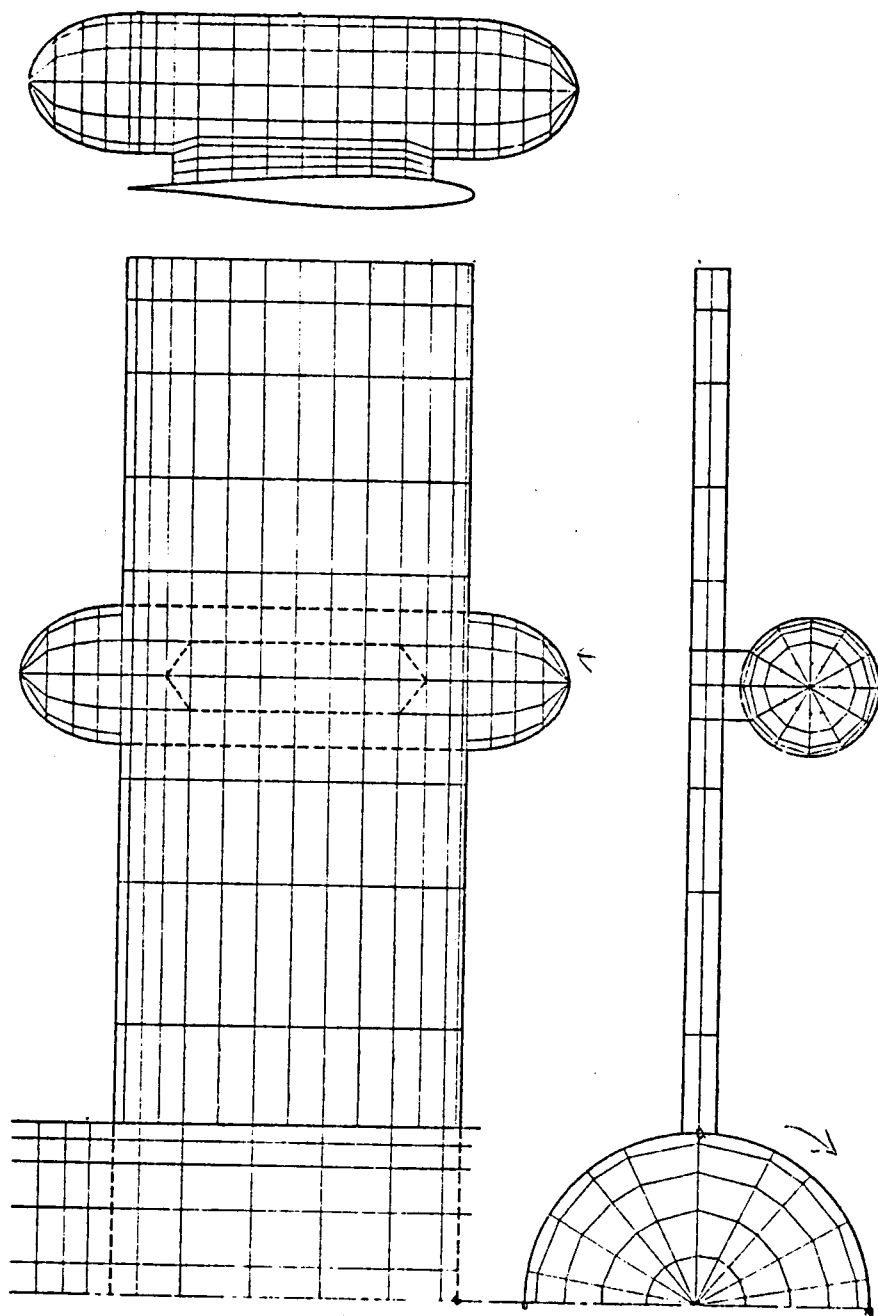


Figure 1.5. The First of Two External-Store Configurations--a Tip Tank.
Source [11]



The Second of Two External-Store Configurations
--a Pylon-Mounted External-Store. Source [11]

Figure 1.5 (continued)

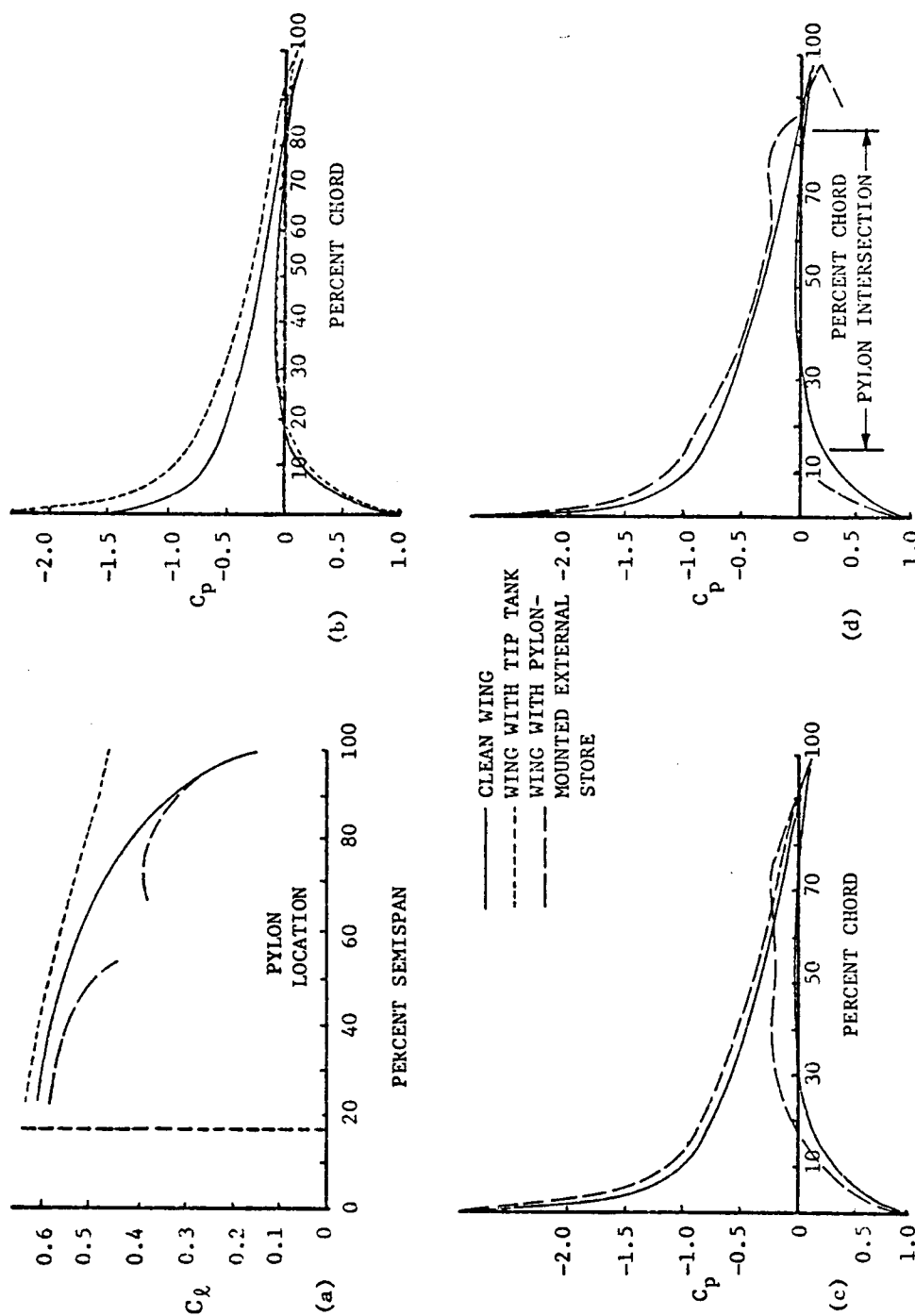


Figure 1.6. Comparison of Calculated Results on a Clean Wing, a Wing with Tip Tank, and a Wing with Pylon-Mounted External Store, All in the Presence of a Round Fuselage at 6 Degrees Angle of Attack. (a) Spanwise Distributions of Section Lift Coefficient. Chordwise Pressure Distributions at: (b) 92.4 Percent Semispan, (c) 53.3 Percent Semispan, (d) 58.3 Percent Semispan. [11]

there is a large increase in lift coefficient in this region. Moreover, as predicted by various theories, the addition of a tip tank increases the effective aspect ratio of the wing and thus increases the section lift coefficient all the way to the fuselage. The effect of the pylon-mounted external store falls to zero at the wing tip, but at the fuselage this effect is about the same size as that of the tip tank but in the opposite direction. The major effect of the pylon is to reduce lift in its vicinity by increasing lower-surface velocities and thus reducing lower-surface pressures (Figure 1.6c). Notice that lift on the wing cannot be meaningfully computed at the spanwise location of the pylon because the lower surface is not exposed to the flow. Of course, there is a force on the external store, but it cannot be meaningfully associated with a particular location on the wing. The bound vorticity distribution is continuous across the span. The general form of this function is quite similar to that of the section lift coefficient. Indeed, it looks as if someone had faired a plausible joining curve between the disjointed portions of the curve of Figure 1.6c.

Chordwise pressure distributions for the clean wing and for the wing with the tank are compared in Figure 1.6b for a spanwise location close to the tip tank. The increase in lift due to the tip tank is seen to be primarily due to

increased velocity on the upper surface of the wing. Figures 1.6c and 1.6d compare chordwise pressure distributions for the clean wing and for the wing with pylon-mounted external store. The spanwise location of Figure 1.6c represents the strip of elements immediately adjacent to the pylon location. The considerable reduction in lower surface pressures due to the presence of the pylon and store is evident. Upper surface pressures are scarcely affected. Figure 1.6d compares pressure distributions on the upper surface of the wing corresponding to a strip of elements that lies directly above the location of the pylon. The pressure distribution computed for the wing with pylon-mounted external store is quite reasonable except for a "hump" between 65 percent chord and 90 percent chord. Examination of the side view of Figure 1.5 shows that in this region the surface elements on the pylon and wing have dimensions that are considerably larger than the local thickness of the wing. Thus, the presence of the pylon is sensed through the wing on the upper surface. An increase in element number could remove this pressure "hump".

Therefore, Hess and Smith's formulation for the potential flow analysis past any arbitrary lifting three-dimensional body can handle interference problems. Due to its exact nature of the formulation and the way in which the interference effects are modelled, in the case

of disjoint surfaces. Even though Hess and Smith's method predicts only for the incompressible flow, compressibility corrections may be made by inputting a value of Mach number and choosing the Goethert transformation for the correction. In this option, the body coordinates are transformed by a value $\beta = \sqrt{1 - M_\infty^2}$, where M_∞ = the freestream Mach number, such that

$$x_{at} = x, \quad y_{at} = \beta y, \quad z_{at} = \beta z, \quad \alpha_{at} = \beta \alpha$$

where the subscript a denotes the transformation. The velocity components at the control points of the transformed geometry are now calculated by the Hess and Smith method as an incompressible calculation and the perturbation velocity components are computed as

$$u_{at} = V_{x_{at}} - V_{x_{at_\infty}},$$

$$v_{at} = V_{y_a} - V_{y_{at_\infty}},$$

$$w_{at} = V_{z_a} - V_{z_{at_\infty}},$$

where $V_{x_{at_\infty}}$, $V_{y_{at_\infty}}$, $V_{z_{at_\infty}}$ are the components of the uniform onset flow at an angle of $\beta \alpha$, in the transformed planes. The Goethert option then computes the final velocity components as

$$V_x = V_{x_\infty} + \frac{1}{\beta^2} u_{at} ,$$

$$V_y = V_{y_\infty} + \frac{1}{\beta} v_{at} ,$$

$$V_z = V_{z_\infty} + \frac{1}{\beta} w_{at} ,$$

at each control point. Where V_{x_∞} , V_{y_∞} , V_{z_∞} are the components of the uniform on set flow in the physical plane the total velocity is $V^2 = V_x^2 + V_y^2 + V_z^2$ and the pressure coefficient c_p can be calculated from

$$c_p = \frac{1}{0.7 M_\infty^2} \left\{ \left[1 + \frac{M_\infty^2}{5} (1 - V^2) \right]^{7/2} - 1 \right\} .$$

CHAPTER II

EXPERIMENTAL APPARATUS AND PROCEDURES USED IN THE
WIND TUNNEL TESTING FOR THE PRESENT WORK

The lack of accurate experimental data has been a major restriction to the development of analytical techniques for computing wing/store interference flow field [21]. Shock waves embedded in the flow fields, often accompanied by regions of flow instability, make measurement by conventional means unreliable. Mechanical probe interference sometimes severely alters the quantities being measured. The laser velocimeter (LV) is capable of obtaining nonperturbed aerodynamic data of the type required. The experimental investigation on wing/store flow field measurement at transonic speeds, using a laser velocimeter, has been carried out at Arnold Engineering Development Center, by Eglin Air Force Base in Florida, and the theoretical studies described in the following chapters substantiate these experiments.

EXPERIMENTAL APPARATUSA. Test Facility

The Arnold Engineering Development Center (AEDC) 1T tunnel is a continuous-flow, open-circuit wind tunnel that can be operated over a Mach number range of 0.20 to 1.50.

The tunnel operates at a constant stagnation pressure of approximately 2850 psfa with the capability of varying the stagnation temperature from 80 to 120°F above ambient temperature. A complete description of the facility is included in Reference [22].

The four test section walls were perforated, with the exception of the window areas. A flat-ground and polished laser velocimeter-quality window was installed in the sidewall adjacent to the laser velocimeter from Station 14.6 to Station 26.6. The wing mount and a lower quality plexiglass port were located between the same tunnel stations along the opposite walls. The installation is shown in Figure 2.1.

B. Test Articles

Test hardware included 2 pressure instrumented 1/20-scale store models, 1 of the MK-83 and 1 of the M-117, in addition to 2 solid models of each store for the multiple-store configuration. The wing was swept 45° with a constant 6-inch chord NACA 0005-34 airfoil cross section. Dimensions of the models are presented in Figure 2.2. The configuration identification, coordinate system and the test configuration are given in Figure 2.3.

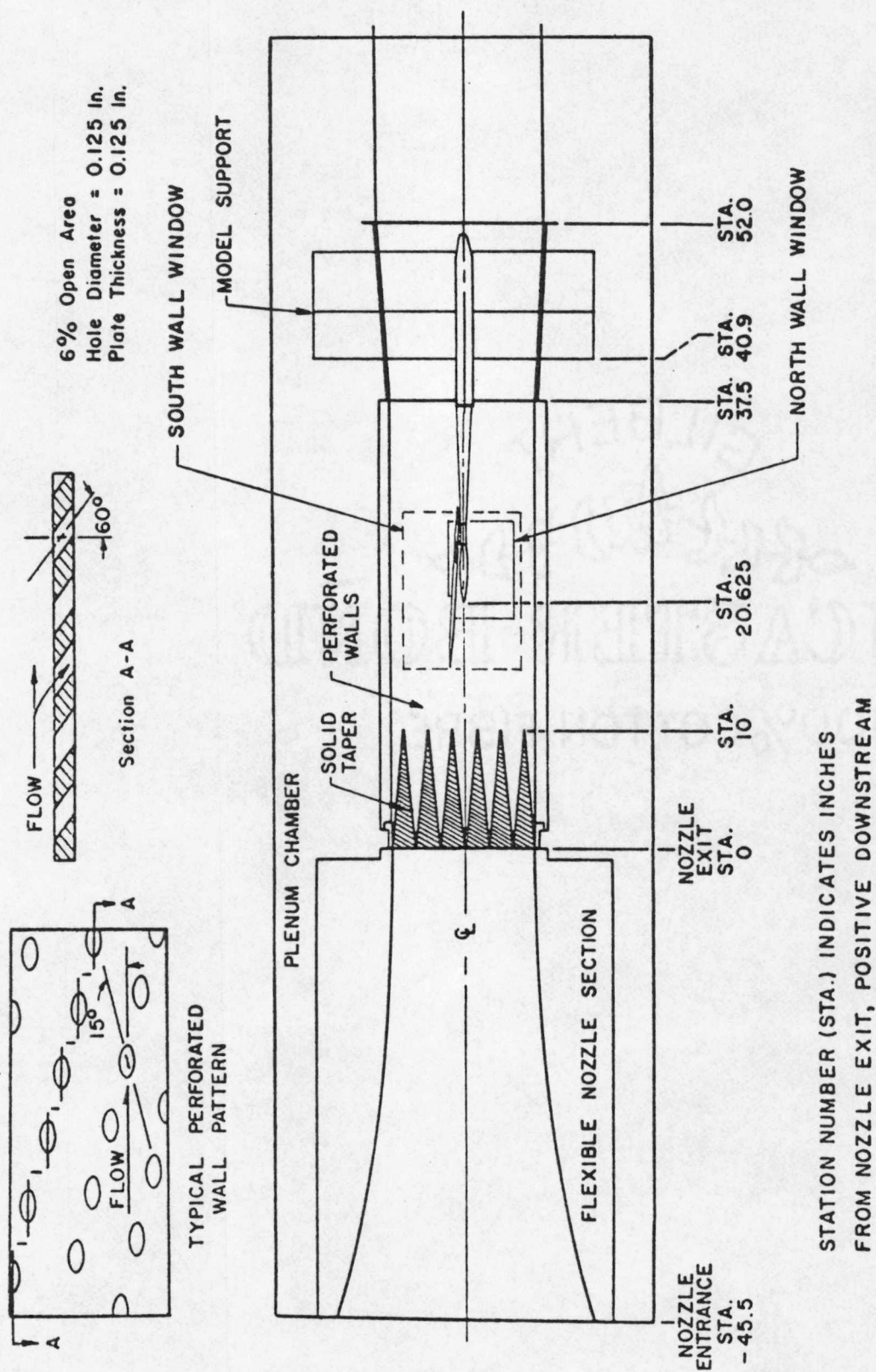
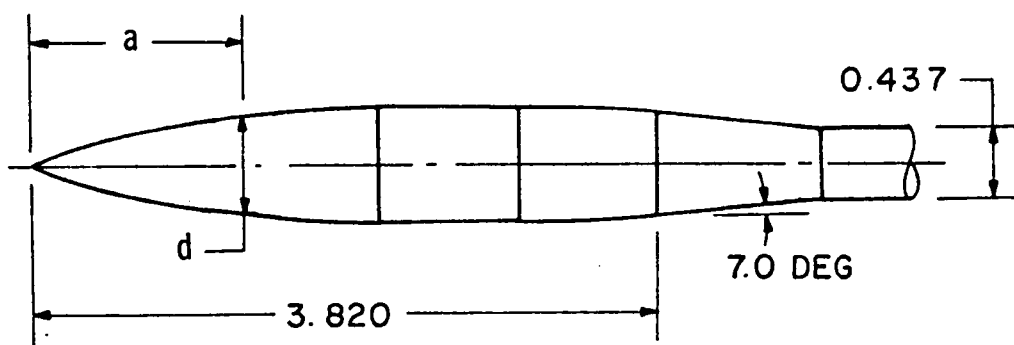


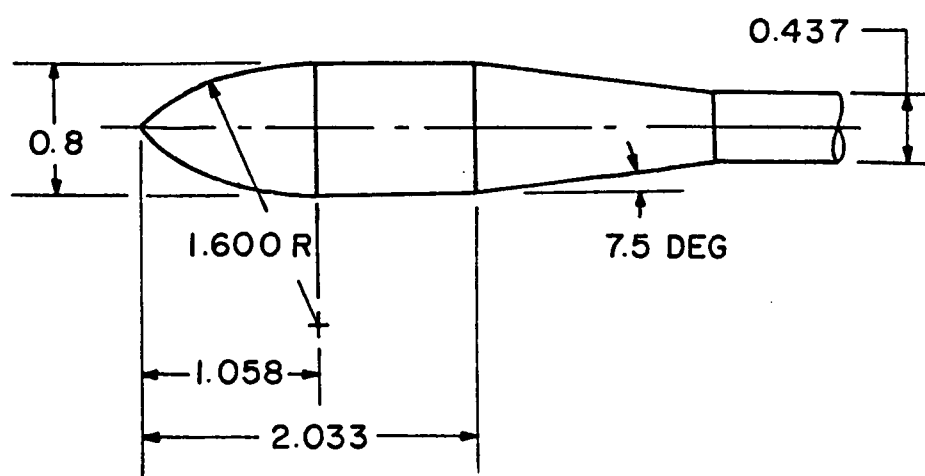
Figure 2.1. Schematic of Tunnel Test Section Showing Model Location [21].



a	d	a	d	a	d
0.0000	0.0000	1.1834	0.5835	2.9833	0.7000
0.2666	0.2313	1.3167	0.6093	3.1166	0.6983
0.3834	0.3073	1.4500	0.6326	3.2500	0.6934
0.5167	0.3777	1.5833	0.6534	3.3833	0.6855
0.6500	0.4349	1.7166	0.6715	3.5166	0.6745
0.7834	0.4819	1.8500	0.6862	3.6500	0.6608
0.9167	0.5211	1.9833	0.6961	3.7833	0.6403
1.0500	0.5545	2.1166	0.7000	3.8200	0.6392

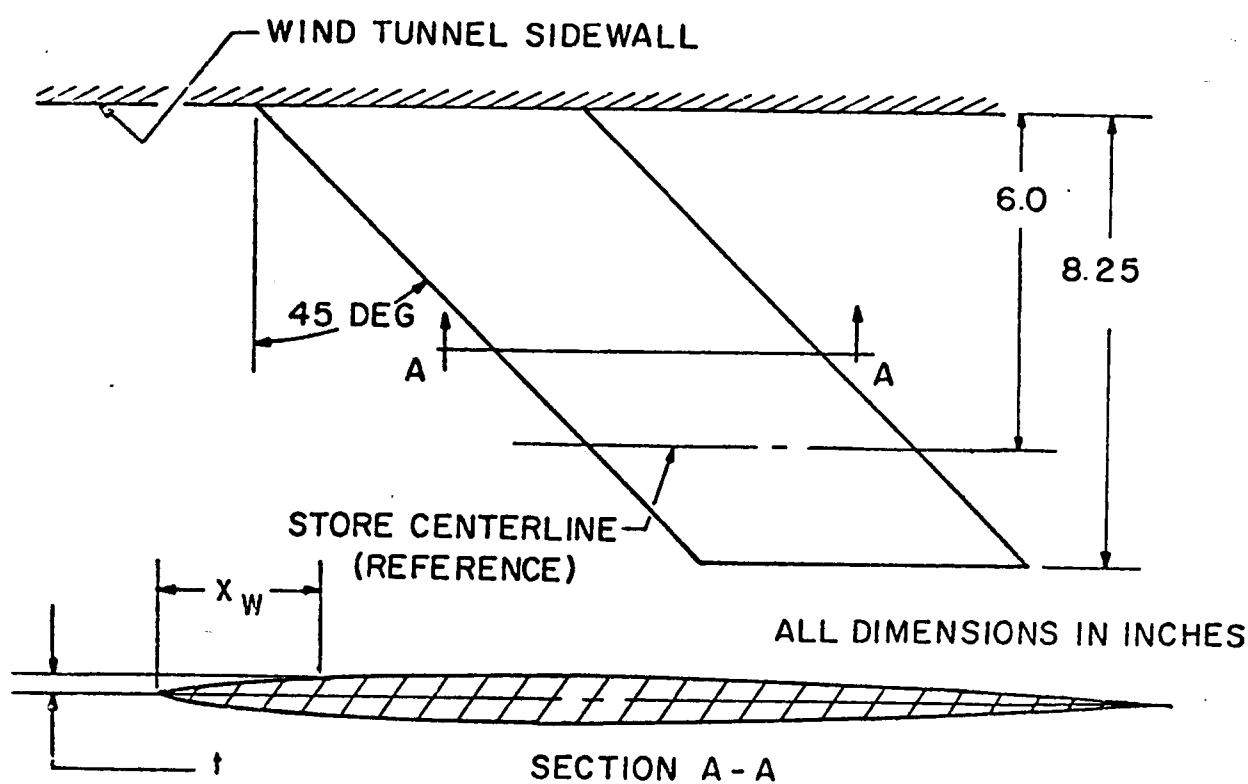
a. MK-83

ALL DIMENSIONS IN INCHES



b. M-117

Figure 2.2. Model Dimensions [21].

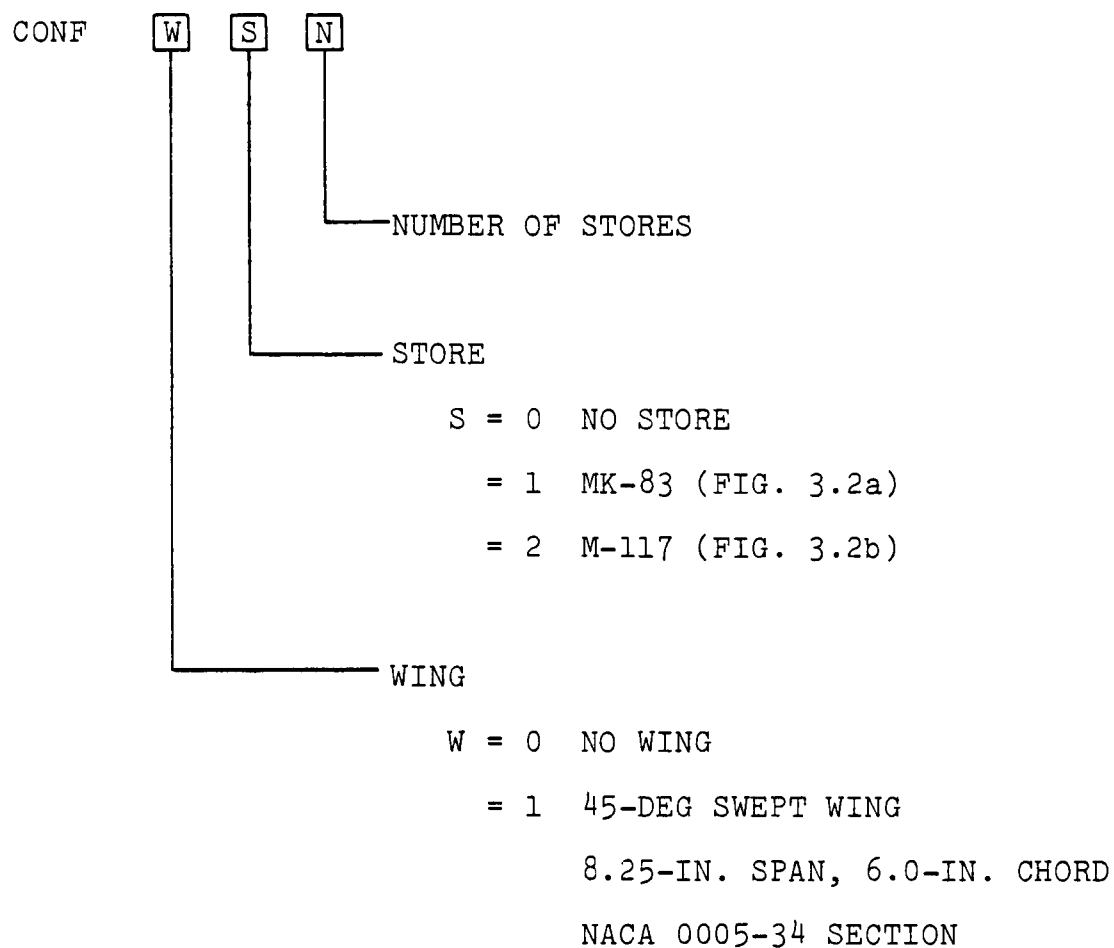


x_w	t
0.0000	0.0000
0.0065	0.0095
0.0750	0.0284
0.1500	0.0420
0.3000	0.0623
0.4500	0.0783
0.6000	0.0914
0.9000	0.1124
1.2000	0.1274

x_w	t
1.8000	0.1450
2.4000	0.1500
3.0000	0.1456
3.6000	0.1330
4.2000	0.1120
4.8000	0.0830
5.4000	0.0466
5.7000	0.0256
6.0000	0.0000

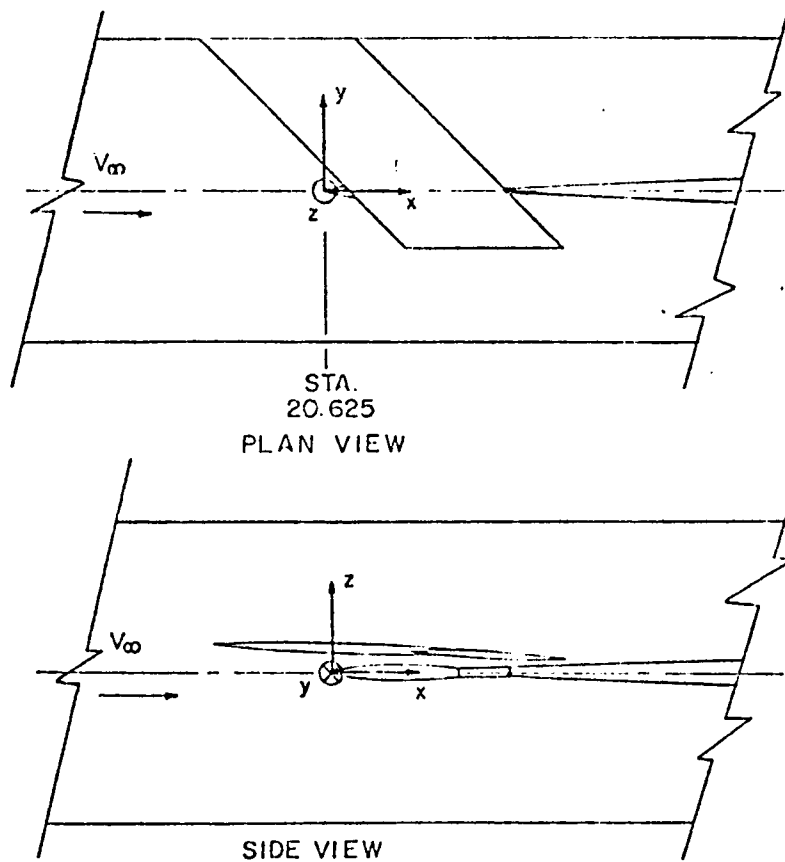
c. Wing

Figure 2.2. (Continued)

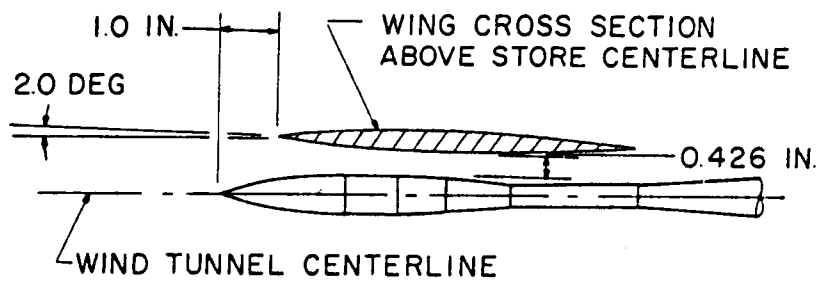


(a). Configuration Identification

Figure 2.3. Configuration Identification and Coordinate System Definition [21].



b. Coordinate System Definition



c. Coordinate System Definition

Figure 2.3. (Continued)

C. Instrumentation

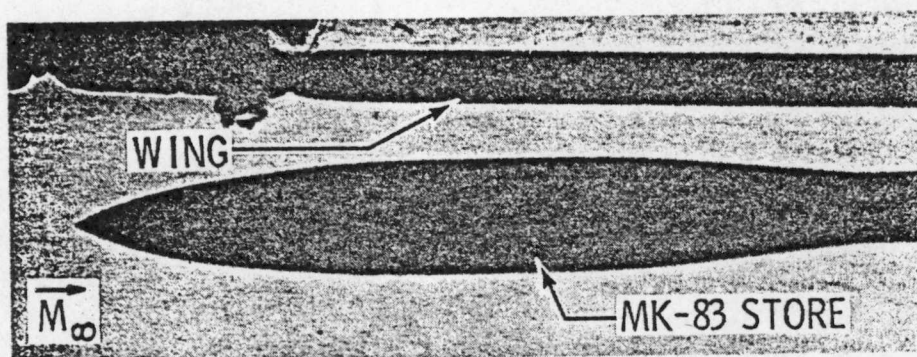
The standard tunnel system was used to record the test conditions and model pressures. A shadowgraph system was used to determine the shock locations for selected model configurations and Mach numbers. High-speed motion pictures were obtained for configuration 111 to assess its vibration characteristics and to determine the relative motion between the store and the wing. Since no significant vibration was observed, no other configurations were photographed.

A 2-component laser velocimeter (References [23] through [28]) was used to provide flow-field measurements.

PROCEDURE

A. Test Conditions

The experimental program was conducted in three phases. Phase I, involving model configurations 111, 100, and 011, was designed to investigate the mutual interference between the selected wing and a single MK-83 store at a transonic Mach number. The particular tunnel condition, $M_{\infty} = 0.92$, was determined experimentally as the lowest Mach number for which a shock was firmly established between the store and the wing lower surface (configuration 111) (Figure 2.4). Configurations 100 and 011 were tested at the same tunnel condition so that zero interference data could be recorded for both the wing and the store.



a. Wind off

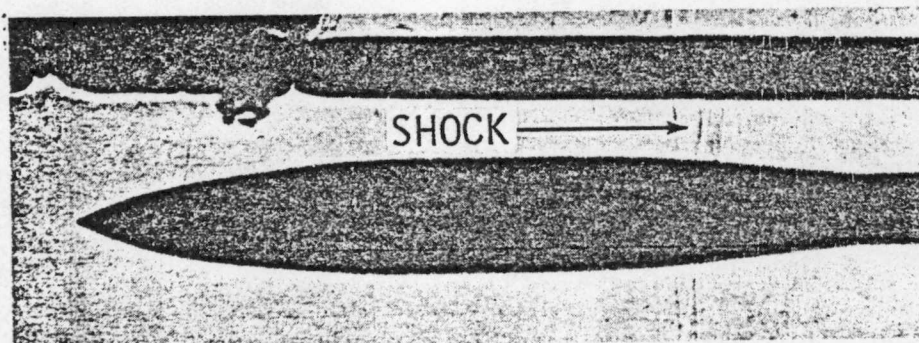
b. $M_\infty = 0.92$

Figure 2.4. Shadowgraphs Showing Shock Between Wing and Store [21].

Phase II of the test program included the MK-83 single and multiple configurations (011, 012 and 013). The models were tested at selected Mach numbers from 0.60 to 1.30.

In Phase III, the 2 M-117 multiple-store configurations (022 and 023) were investigated at test conditions similar to those in Phase II.

A complete summary of test conditions is contained in Chapters III and IV.

B. Data Acquisition

Flow-field surveys were made using the LV system previously described. Nominally, 1000 individual particle velocities were recorded for use in the determination of the flow properties at each selected point in the flow field. In most instances, two minutes were required to complete an acquisition cycle, including the time required to reposition the traversing system. Measurement within the wake or the boundary layer required more time.

The focal volume position relative to the test model was recorded for each velocity data set. Periodic corrections were found to be necessary, however, to minimize spatial error caused by temperature effects. Since thermal expansion caused significant axial movement of the wind tunnel, the linear displacement transducer was used to monitor the test section position. The output

of the device was automatically combined with the traverse axial position indicator signal by the computer to provide an accurate relative measurement.

C. Data Reduction

Analysis of the data was conducted using the digital computer software described in Reference [24]. The statistical quantities computed for each data set include mean and mode velocities, standard deviation, skewness, flatness factor (kurtosis), and Reynolds stress.

D. Precision of Measurements

Tunnel conditions. Care was taken to allow the tunnel to stabilize before the relatively long data-recording period. The resulting variation in mean free-stream Mach number and velocity were as follows:

<u>Parameter</u>	<u>$M_{\infty} = 0.60$</u>	<u>$M_{\infty} = 0.80$</u>	<u>$M_{\infty} = 1.0$</u>	<u>$M_{\infty} = 1.30$</u>
ΔM_{∞}	± 0.001	± 0.002	± 0.003	± 0.001
ΔV_{∞} (ft/sec)	± 2.0	± 2.0	± 6.0	± 5.0

The sidewalls with windows were an additional source of Mach number uncertainty. According to the Tunnel 1T calibration, Reference [29], use of the windows can result in significant variation in Mach number along the tunnel centerline. A statistical analysis of the centerline Mach number resulted in the following values of 2σ (where σ is

the standard deviation) from the mean free-stream Mach number:

$M_\infty =$	<u>0.60</u>	<u>0.80</u>	<u>1.00</u>	<u>1.10</u>	<u>1.20</u>	<u>1.30</u>
$\Delta M =$	± 0.005	± 0.005	± 0.010	± 0.035	± 0.045	± 0.055

Model incidence angle uncertainty was less than $\pm 0.05^\circ$.

The comparison of the experimental pressure distribution on the body with that of the theory is deferred to the subsequent chapters.

CHAPTER III

THEORY OF THE INTEGRAL-EQUATION FORMULATION IN TRANSONIC
FLOW FOR THE WING-BODY COMBINATION WITH
SHOCKS AND ITS APPLICATIONSA. Introduction

One of the most difficult problems yet to be solved to a certain degree of completion is the three-dimensional transonic interference problem. At transonic Mach numbers, three-dimensional effects due to the presence of a fuselage or wing tip can influence the flow field for large distances in the spanwise direction. For low and moderate aspect-ratio wings combined with a body, the flow fields are rarely similar to two-dimensional ones anywhere on the wing except, perhaps, on certain localized regions. Therefore, the treatment of the three-dimensional transonic flow problems, Reference [30], involves more than a trivial extension of existing two-dimensional methods, which are fairly well developed by now.

Some of the major complications are:

1. The circulation changes with the spanwise coordinates and has to be determined as part of the solution.
2. The swept and tapered planform shapes complicate application of wing boundary conditions using finite-difference methods.

3. The shocks, which are nearly normal to the free-stream direction in two dimensions, can be oblique in the lateral direction for finite-swept wings, thereby making them difficult to capture sharply because of poor alignment with the coordinate system.

4. Finally, difficulties in the stability of the results may arise, especially in supersonic regions, in the finite-difference method.

B. Brief Description of the Flow-Development About Swept Wing-Body Combination

Figure 3.1 shows the flow development about a simple swept wing-body combination with increasing free-stream Mach number.

At moderate subsonic speeds, if the lift is not too large, an important effect of the sweep is to move the loading forward at the tip. As a result, the first appearance of supersonic flow occurs in the tip region. Since the isobars near the wing tip lose much of their sweep, the supersonic flow generally gives rise to a shock wave--the initial tip shock which is comparatively weak and lies almost normal to the free stream (Figure 3.1a). This tip shock, though limited in the spanwise extent, extends to considerable distances above and below the wing.

As the free-stream M_∞ is increased, the initial tip shock moves rearwards over the wing surface, but its

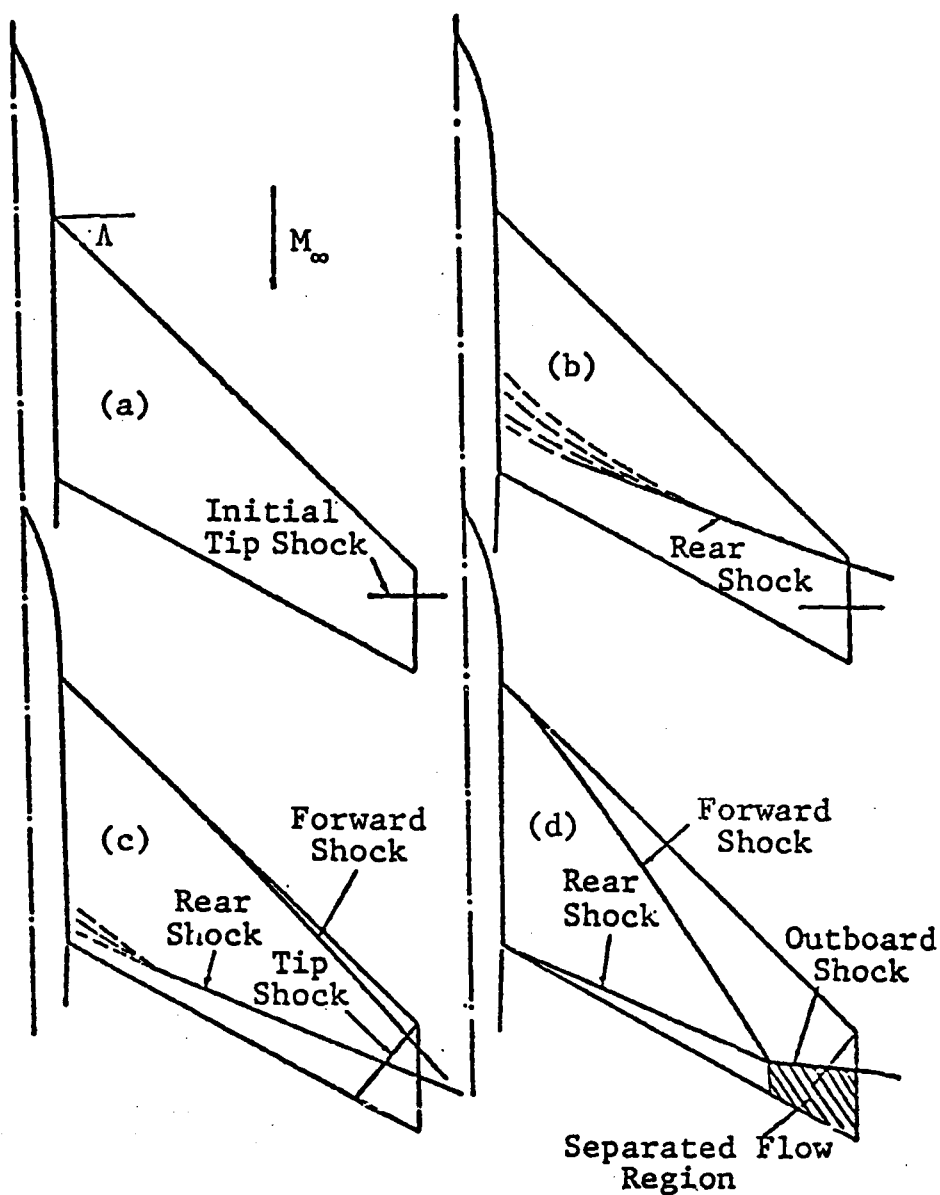


Figure 3.1. Shock Formations on Swept Wing-Body Combination with Increasing M_∞ . [30]

influence on the wing is limited by the appearance of a second shock. This rear shock rapidly develops to affect a large part of the wing span, particularly the flow ahead of the initial tip shock (Figure 3.1b). The rear shock may be regarded as associated primarily with conditions at the wing root. It consists of a compression system propagating outward from the root which coalesces on the outer part of the wing to form a shock wave (Figure 3.1b).

With increasing M_∞ , the rear shock moves aft more rapidly than the initial tip shock which is overtaken and disappears. At a sufficiently high M_∞ the rear shock reaches the trailing edge. The high local flow velocities close to the leading edge over the outer part of the wing lead to flow separation at the leading edge. This starts near the tip and spreads inboard with increasing α . For leading-edge sweeps greater than about 30° , the separated flow rolls up to form a partspan vortex lying obliquely across the wing. Above a certain stream M_∞ the flow changes in type; the leading-edge separation is suppressed, the flow passes smoothly around the leading edge through a forward shock wave which appears to originate close to the leading edge at some spanwise position (Figure 3.1c). The forward shock is, in fact, the boundary of disturbances from the inboard part of the wing leading edge which, because of the higher local supersonic Mach number, propagate over the outer wing in a direction more highly swept than the

leading edge. The Mach number component normal to the leading edge ($M_\infty \cos \Lambda$) for which the flow attachment occurs varies considerably, depending on the sweep angle Λ , and more particularly on the leading edge profile. In general, $M_\infty \cos \Lambda$ has a value between 0.55 and 0.90.

As the free-stream Mach number, M_∞ , is increased beyond that necessary for flow attachment, the forward shock moves inboard and rearward and at some stage intersects the rear shock. Outboard of this intersection a strong shock forms (the outboard shock) which frequently induces a severe flow separation even though the flow is attached behind both the forward and rear shock (Figure 3.1d). When separation is present behind either of these shocks, a vortex tends to form and the associated outflow modifies the flow structure outboard and becomes an important factor in the ultimate breakdown in the attached type of leading-edge flow over the wing. Separation then takes place along almost the entire leading edge, and the shock waves no longer have a direct influence upon the surface pressures which are dominated by a large port-span vortex as for lower subsonic M_∞ .

Many of the features described above are shown in the surface-film pattern of Figure 3.2, obtained at $M_\infty = 0.95$ with $\alpha = 10^\circ$.

Complex flow patterns similar to Figure 3.2 are largely due to the three-dimensional nature of the wing flow

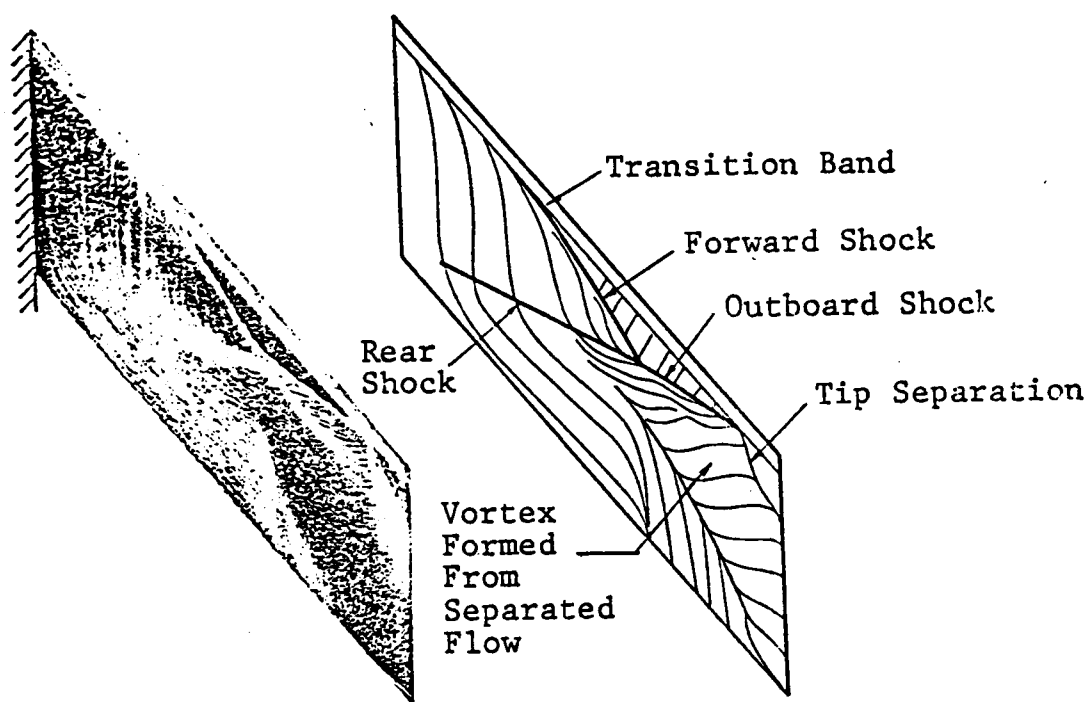


Figure 3.2. Flow Pattern on 50° Swept Wing
at $M_\infty = 0.95$ and $\alpha = 10^\circ$. [30]

and hence to the dominance of the root and tip influence at transonic speeds. The root affects the flow strongly behind the forward shock, which therefore indicates the limit of the root influence and the flow in this region is partly conical in character. The tip influence at transonic speeds is delineated by small disturbance from near the tip leading edge (the tip shock: Figure 3.1c). Ahead of the forward, outboard and tip shocks the flow is almost two-dimensional in character. The extent of this zone depends on the shock positions which, in turn, are dependent on wing planform angle of attack, α , and Mach number, M_∞ . By making the aspect ratio large, or by deliberately attempting to reduce the root and tip influence, flow can be obtained over a large portion of the wing. This flow closely resembles that postulated in the simple sweep-back theory which is so highly desirable.

C. Basic Equations

The basic equations necessary for the discussion of inviscid transonic flow consist of a set of partial differential equations relating the velocity components and their gradients at every point, together with the auxiliary relation giving the velocity jump through a shock wave. For thin airfoils inclined at zero or small angle of attack, the differential equation can be simplified by assuming that the shock waves are sufficiently weak that the flow is irrotational and isentropic, thereby permitting

the introduction of a velocity potential ϕ . The quasilinear partial differential equation satisfied by ϕ can be expressed in the form

$$\begin{aligned} (a^2 - \phi_x^2)\phi_{xx} + (a^2 - \phi_y^2)\phi_{yy} + (a^2 - \phi_z^2)\phi_{zz} \\ - 2\phi_x\phi_y\phi_{xy} - 2\phi_y\phi_z\phi_{yz} - 2\phi_z\phi_x\phi_{xz} = 0, \end{aligned} \quad (3-1)$$

where the subscript notation is used to indicate differentiation and a is the local speed of sound given by the relation

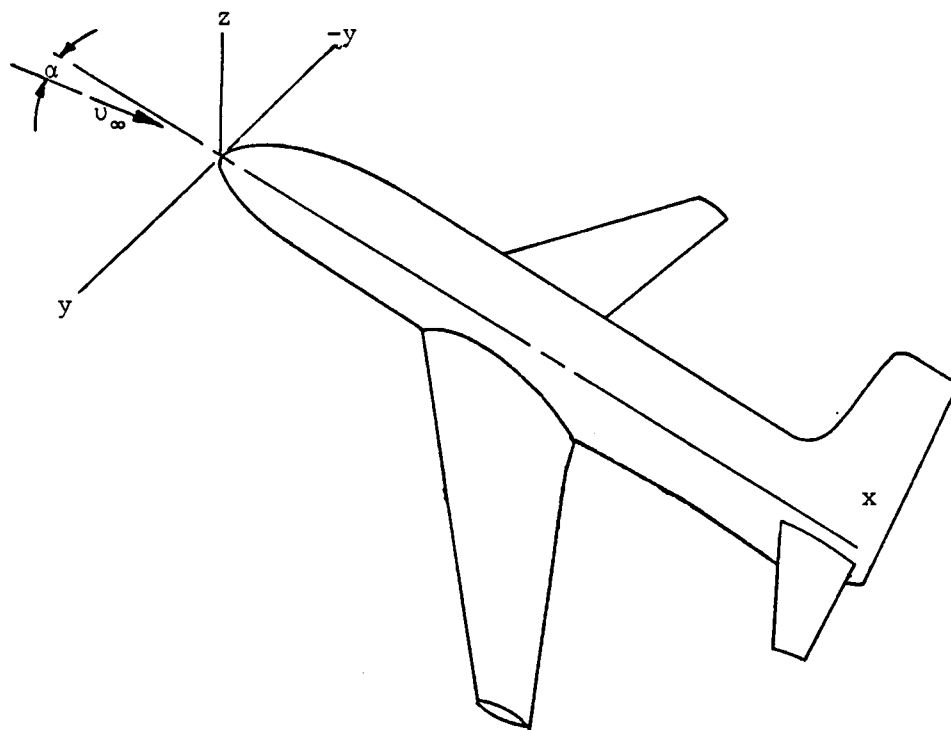
$$a^2 = a_\infty^2 - \frac{\gamma-1}{2} (\phi_x^2 + \phi_y^2 + \phi_z^2 - U_\infty^2), \quad (3-2)$$

where U_∞ and a_∞ are, respectively, the free-stream velocity and the speed of sound in the free-stream, and γ is the ratio of specific heats (for air $\gamma = 1.4$).

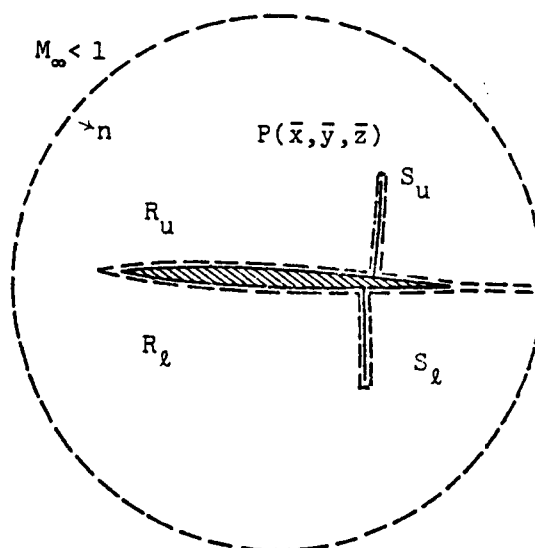
It is convenient to introduce the perturbation velocity potential ϕ , where

$$\phi = -U_\infty x + \phi. \quad (3-3)$$

If it is assumed that all perturbation velocities and perturbation velocity gradients (represented by first and second derivatives, respectively, of ϕ) are small and that only the first-order terms in small quantities need be retained, Equation (3-1) simplifies to the well-known Prandtl-Glauert equation of linear theory,



(a) Coordinate System of the Wing-body Configuration.



(b) Illustration of the Application of the Green's Theorem Over the Wing-body Configuration.

Figure 3.3. Wing-Body Specification Used in the Integral Equation Method.

$$(1 - M_\infty^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad , \quad (3-4)$$

where the free-stream velocity is directed along the positive x-axis, as shown in Figure 3.3a, and where $M_\infty = U_\infty/a_\infty$ is the Mach number of the free-stream. It is well known that Equation (3-4) leads to useful results in the study of subsonic and supersonic flow about thin wings and slender bodies, but that it is incapable, in general, of treating transonic flows, Reference [31]. The failure of linear theory in transonic range is evidenced by the calculated values of ϕ_x growing to such magnitude that they can no longer be regarded as small quantities when compared with U_∞ .

Second-order theory for thin wings would involve solution of the equation

$$(1 - M_\infty^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = M_\infty^2 \left[\frac{\gamma + 1}{U_\infty} \phi_x \phi_{xx} + \frac{\gamma - 1}{U_\infty} \phi_x (\phi_{yy} + \phi_{zz}) + \frac{2}{U_\infty} (\phi_y \phi_{xy} + \phi_z \phi_{xz}) \right] \quad , \quad (3-5)$$

(Reference [32], p. 140). Actually, it is of interest in retaining higher-order terms only to the extent that is necessary to allow the study of transonic flow. Examination of the known properties of transonic flow fields indicates that the first term on the right can often become of importance and should be retained. It is assumed in the small disturbance theory of

transonic flow, however, that the remainder of the terms on the right can be safely disregarded.

The simplified equation is:

$$\begin{aligned} (1 - M_\infty^2) \phi_{xx} + \phi_{yy} + \phi_{zz} &= M_\infty^2 \frac{\gamma + 1}{U_\infty} \phi_x \phi_{xx} \\ &= k \phi_x \phi_{xx} \quad , \end{aligned} \quad (3-6)$$

where

$$k = M_\infty^2 \frac{(\gamma + 1)}{U_\infty} \quad . \quad (3-7)$$

Equation (3-6) is valid only in regions where the necessary derivatives exist and are continuous. Since these conditions do not hold where shock waves occur, an additional equation is needed for the transition through the shock. The necessary equation is provided by the classical relation for the shock polar (Reference [32], p. 108),

$$\bar{v}_b^2 + \bar{w}_b^2 = (\bar{u}_a - \bar{u}_b)^2 \cdot \frac{\bar{u}_a \bar{u}_b - a^{*2}}{\frac{2}{\gamma + 1} \bar{u}_a^2 - \bar{u}_a \bar{u}_b + a^{*2}} \quad , \quad (3-8)$$

where $\bar{u}$, $\bar{v}$, and $\bar{w}$ refer to the cartesian velocity, components with $\bar{u}$ being parallel to the flow direction ahead of the shock, the subscripts a and b refer to conditions ahead of and behind the shock, and a^* is the critical sound velocity, which can be expressed in terms of U_∞ and M_∞ as follows:

$$\frac{a^*}{U_\infty} = \sqrt{\frac{\gamma-1}{\gamma+1} + \frac{2}{M_\infty^2(\gamma+1)}} \quad (3-9)$$

The appropriate simplified equation is obtained from Equation (3-8) by resolving the velocities into components parallel to the axes of the coordinate system and carrying out a small perturbation analysis analogous to that performed in the derivation of Equation (3-6). In this way, the following relation is found between the perturbation velocity components on the two sides of the shock wave:

$$\begin{aligned} (1 - M_\infty^2)(u_a - u_b)^2 + (v_a - v_b)^2 + (w_a - w_b)^2 \\ = M_\infty^2 \frac{\gamma+1}{U_\infty} \left(\frac{u_a + u_b}{2} \right) (u_a - u_b)^2 \\ = k \left(\frac{u_a + u_b}{2} \right) (u_a - u_b)^2, \end{aligned} \quad (3-10)$$

where u , v , w are the perturbation velocity components parallel to the x , y , and z axes. This equation corresponds to the shock polar curve for shock waves of small strength inclined at any angle between that of normal shock waves and that of the Mach lines. On either side of the shock wave the perturbation velocity components are related to the perturbation velocity potential in the usual manner:

$$u = \frac{\partial \phi}{\partial x} \quad , \quad v = \frac{\partial \phi}{\partial y} \quad , \quad w = \frac{\partial \phi}{\partial z} \quad . \quad (3-11)$$

In addition to satisfying the differential equation and the shock-wave equation, the perturbation potential must provide flows compatible with the following physical requirements:

(a) The perturbation velocities must vanish far ahead of the wing, and

(b) the flow must be tangential to the wing surface.

Therefore, the following boundary conditions are to be specified for the perturbation potential:

At $x = -\infty$,

$$\left(\frac{\partial \phi}{\partial x} \right)_{\infty} = \left(\frac{\partial \phi}{\partial y} \right)_{\infty} = \left(\frac{\partial \phi}{\partial z} \right)_{\infty} = 0 \quad . \quad (3-12)$$

At the wing surface, W ,

$$\frac{1}{U_{\infty}} \left(\frac{\partial \phi}{\partial z} \right)_W = \frac{\partial z}{\partial x} \quad , \quad (3-13)$$

where $\partial z / \partial x$ is the local slope of the wing surface in the x direction. Furthermore, it is consistent with the assumption of small disturbances to satisfy the second boundary condition on the two sides of the xy plane rather than on the actual wing surface.

Equation (3-13) becomes:

$$\frac{1}{U_{\infty}} \left(\frac{\partial \phi}{\partial z} \right)_w \approx \frac{1}{U_{\infty}} \left(\frac{\partial \phi}{\partial z} \right)_{z=0} = \frac{\partial z}{\partial x} =$$

$$\tau \cdot \frac{\partial}{\partial \left(\frac{x}{c} \right)} f \left(\frac{x}{c} \cdot \frac{y}{s} \right) , \quad (3-14)$$

where the shape of the wing profile is given by

$$\frac{z}{c} = \tau f \left(\frac{x}{c} , \frac{y}{s} \right) . \quad (3-15)$$

Where c is the chord of the airfoil section and s is the semispan of the wing. In addition, it is presumed necessary to prescribe that the direct influence of a disturbance in the supersonic region proceeds only in the downstream direction and that the Kutta condition applies whenever the flow velocity at the trailing edge is subsonic. Upon solving the above boundary-value problem for the potential, one may determine the pressure coefficient for planar systems by means of the formula:

$$c_p = \frac{p - p_{\infty}}{\frac{\rho_{\infty}}{2} u_{\infty}^2} = - \frac{2}{U_{\infty}} \cdot \frac{\partial \phi}{\partial x} . \quad (3-16)$$

D. Derivation of Transonic Integral Equation for Transonic Flow

Since the principal object of the following analysis is to determine the perturbation velocity components at any point, it is convenient to work with equivalent equations for u , v and w obtained

by differentiating Equation (3-6):

$$(1 - M_{\infty}^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = k \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x^2} ,$$

with respect to x , y and z , respectively:

$$(1 - M_{\infty}^2) \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = k \frac{\partial^2}{\partial x^2} \left(\frac{u^2}{2} \right) , \quad (3-17)$$

$$(1 - M_{\infty}^2) \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = k \frac{\partial^2}{\partial y \partial x} \left(\frac{u^2}{2} \right) , \quad (3-18)$$

$$(1 - M_{\infty}^2) \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = k \frac{\partial^2}{\partial z \partial x} \left(\frac{u^2}{2} \right) , \quad (3-19)$$

since

$$\frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x^2} = u \frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left(\frac{u^2}{2} \right) .$$

We will assume that $M_{\infty} < 1$, i.e., the free stream is a high subsonic velocity though the local speeds on the body exceed unity in some areas.

Then normalizing Equations (3-17), (3-18) and (3-19) by letting

$$\bar{x} = x , \quad \bar{y} = \beta y , \quad \bar{z} = \beta z , \quad \bar{\phi} = \frac{k}{\beta^2} \phi ,$$

$$\bar{u} = \frac{\partial \bar{\phi}}{\partial \bar{x}} = \frac{k}{\beta^2} u , \quad \bar{v} = \frac{\partial \bar{\phi}}{\partial \bar{y}} = \frac{k}{\beta^3} v , \quad \bar{w} = \frac{\partial \bar{\phi}}{\partial \bar{z}} = \frac{k}{\beta^3} w .$$

(3-20)

where

$$\beta = \sqrt{1 - M_{\infty}^2} \quad .$$

Equations (3-6) and (3-17), (3-18) and (3-19) reduce to the following:

$$\frac{\partial^2 \bar{\phi}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{\phi}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{\phi}}{\partial \bar{z}^2} \equiv \nabla^2 \bar{\phi} = \frac{\partial \bar{\phi}}{\partial \bar{x}} \frac{\partial^2 \bar{\phi}}{\partial \bar{x}^2} \quad , \quad (3-21)$$

$$\nabla^2 \bar{u} = \frac{\partial^2}{\partial \bar{x}^2} \left(\frac{\bar{u}^2}{2} \right) \quad , \quad (3-22)$$

$$\nabla^2 \bar{v} = \frac{\partial^2}{\partial \bar{y} \partial \bar{x}} \left(\frac{\bar{u}^2}{2} \right) \quad , \quad (3-23)$$

$$\nabla^2 \bar{w} = \frac{\partial^2}{\partial \bar{z} \partial \bar{x}} \left(\frac{\bar{u}^2}{2} \right) \quad , \quad (3-24)$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2} + \frac{\partial^2}{\partial \bar{z}^2}$$

is the Laplacian operator in the normalized variables $\bar{x}$, $\bar{y}$ and $\bar{z}$.

Before proceeding, it should be noted that the introduction of the reduced perturbation velocity component $\bar{u}$ permits the ready recognition of regions of subsonic and supersonic velocities and emphasizes

the points at which sonic velocity occurs. This becomes apparent from the approximate relation between the local mach number, M , and ϕ_x :

$$1 - M^2 = 1 - M_\infty^2 - k \phi_x \quad (3-25)$$

or

$$\frac{1 - M^2}{1 - M_\infty^2} = 1 - \frac{k}{1 - M_\infty^2} u = 1 - \bar{u} \quad ,$$

from which it is clear, for flows having subsonic free-stream $M_\infty (M_\infty < 1)$, that $\bar{u} < 1$ when the local velocity is subsonic, $\bar{u} = 1$ when it is sonic, and $\bar{u} > 1$ when it is supersonic.

We will now apply Green's theorem to Equation (3-22). The results for Equations (3-23) and (3-24) will follow immediately. If σ and Ω are any two functions which, together with their first and second derivatives, are finite and single valued throughout a region R enclosed by a surface Σ , Green's theorem, which follows from Gauss' divergence theorem, states:

$$\iint_{\Sigma} \left(\sigma \frac{\partial \Omega}{\partial n} - \Omega \frac{\partial \sigma}{\partial n} \right) d\Sigma = \iiint_R (\Omega \nabla^2 \sigma - \sigma \nabla^2 \Omega) dR \quad , \quad (3-26)$$

where the directional derivatives on the left-hand

side are taken along the normal n , drawn inward, to the surface.

Now in Equation (3-26) let $\Omega = \bar{u}$ and choose σ as the fundamental solution $1/r_3$ of the Laplace equation $\nabla^2 \sigma = 0$, i.e.,

$$\sigma = \frac{1}{r_3} = \frac{1}{[(\bar{x} - \bar{\xi})^2 + (\bar{y} - \bar{\eta})^2 + (\bar{z} - \bar{\zeta})^2]^{1/2}} . \quad (3-27)$$

Then we have from Equation (3-26)

$$\begin{aligned} \iint_{\Sigma} \left[\frac{1}{r_3} \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial}{\partial n} \left(\frac{1}{r_3} \right) \right] d\Sigma &= - \iiint_R \frac{1}{r_3} \nabla^2 \bar{u} dR \\ &= - \iiint_R \frac{1}{r_3} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{\bar{u}^2}{2} \right) dR , \end{aligned} \quad (3-28)$$

since from Equation (3-22)

$$\nabla^2 \bar{u} = \frac{\partial^2}{\partial \bar{x}^2} \left(\frac{\bar{u}^2}{2} \right) .$$

The variables of integration in Equation (3-28) are $\bar{\xi}$, $\bar{\eta}$, $\bar{\zeta}$, while $\bar{x}$, $\bar{y}$, $\bar{z}$ are the coordinates of a point P . It must be observed that $1/r_3$ is singular at $r_3 = 0$ and $\bar{u}$ is discontinuous at the shock wave. The point P and the shock must, therefore, be excluded from the region R . A schematic indication of the body (wing or fuselage) and the region of

integration has been shown in Figure (3.3b). The complete three-dimensional extent of the body has not been pictured. It suffices, however, to state that the surface Σ (shown dashed) is composed of a sphere of large radius which forms the external boundary of R , an infinitesimal sphere surrounding P , and a final surface enveloping the wing and body, the wake (for the lifting cases), and the shock waves.

Let us now apply Equation (3-28) to the region R_u bounded by the $\overline{xy}$ -plane and a hemispherical dome of infinite radius lying above this plane, exclusive of the subregions surrounding $P(\bar{x}, \bar{y}, \bar{z})$ and the shock waves (Figure 3.3b). Since, furthermore, $\bar{u}$ may be assumed to diminish sufficiently rapidly with distance, the contributions of the integrals over the hemisphere vanish and the contribution of the surface integral over the small sphere surrounding P may easily be shown to be $4\pi \bar{u}(\bar{x}, \bar{y}, \bar{z})$ when the radius $\epsilon_1 \rightarrow 0$. We have, therefore, from Equation (3.28) the following result:

$$\begin{aligned}
\bar{u}(\bar{x}, \bar{y}, \bar{z}) = & -\frac{1}{4\pi} \iint_{-\infty}^{+\infty} \left[\frac{1}{r_3} \frac{\partial \bar{u}_u}{\partial \bar{\zeta}} - \bar{u}_u \frac{\partial}{\partial \bar{\zeta}} \left(\frac{1}{r_3} \right) \right] d\bar{\xi} d\bar{\eta} \\
& - \frac{1}{4\pi} \iint_{S_u} \left[\left\{ \frac{1}{r_3} \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial}{\partial n} \left(\frac{1}{r_3} \right) \right\}_1 + \left\{ \frac{1}{r_3} \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial}{\partial n} \left(\frac{1}{r_3} \right) \right\}_2 \right] dS \\
& - \frac{1}{4\pi} \iiint_{R_u} \frac{1}{r_3} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{\bar{u}^2}{2} \right) dR, \tag{3-29}
\end{aligned}$$

where the subscript u denotes conditions on the upper side of the $\bar{xy}$ -plane, the subscripts 1 and 2 denote values immediately ahead of and behind the shock wave, and S is the surface of the shock wave. The volume integral is defined as follows when P is ahead of S :

$$\begin{aligned}
\iiint_{R_u} \frac{1}{r_3} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{\bar{u}^2}{2} \right) dR \equiv & \iiint_{R_u} \psi dR = \\
\lim_{\substack{\epsilon_1 \rightarrow 0 \\ \epsilon_2 \rightarrow 0}} & \int_{-\infty}^{\infty} d\bar{\eta} \int_0^{+\infty} d\bar{\zeta} \left[\int_{-\infty}^{\bar{x}_P - \epsilon_1} \psi d\bar{\xi} + \int_{\bar{x}_P + \epsilon_1}^{\bar{x}_S - \epsilon_2} \psi d\bar{\xi} + \int_{\bar{x}_S + \epsilon_2}^{+\infty} \psi d\bar{\xi} \right], \tag{3-30}
\end{aligned}$$

where we have written $\psi \equiv (1/r_3) \cdot \partial^2 / \partial \bar{\xi}^2 (\bar{u}^2/2)$ and ϵ_1, ϵ_2 are the radius of the small sphere surrounding $P(\bar{x}, \bar{y}, \bar{z})$ and the thickness of the surface surrounding the shock wave surface S_u , respectively. Similarly,

when P is behind S , the volume integral is

$$\iiint_{R_u} \psi dR = \lim_{\substack{\epsilon_1 \rightarrow 0 \\ \epsilon_2 \rightarrow 0}} \int_{-\infty}^{+\infty} d\bar{\eta} \int_0^{\infty} d\bar{\zeta} \left(\int_{-\infty}^{\bar{x}_S - \epsilon_2} \psi d\bar{\xi} + \int_{\bar{x}_S - \epsilon_2}^{\bar{x}_P - \epsilon_1} \psi d\bar{\xi} + \int_{\bar{x}_P + \epsilon_1}^{\infty} \psi d\bar{\xi} \right) . \quad (3-31)$$

If P is kept fixed in the upper half space and the region R bounded by the $\overline{xy}$ -plane and a hemispherical dome of infinite radius lying below this plane is considered, it follows in a similar manner that

$$\begin{aligned} 0 = \frac{1}{4\pi} \iint_{-\infty}^{+\infty} \left[\frac{1}{r_3} \frac{\partial \bar{u}_\ell}{\partial \bar{\zeta}} - \bar{u}_\ell \frac{\partial}{\partial \bar{\zeta}} \left(\frac{1}{r_3} \right) \right] d\bar{\xi} d\bar{\eta} - \frac{1}{4\pi} \iint_{S_\ell} \left[\left\{ \frac{1}{r_3} \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial}{\partial n} \left(\frac{1}{r_3} \right) \right\}_1 \right. \\ \left. + \left\{ \frac{1}{r_3} \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial}{\partial n} \left(\frac{1}{r_3} \right) \right\}_2 \right] dS - \frac{1}{4\pi} \iiint_{R_\ell} \frac{1}{r_3} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{\bar{u}^2}{2} \right) dR , \quad (3-32) \end{aligned}$$

where the subscript ℓ denotes conditions on the lower side of the $\overline{xy}$ -plane and the volume integral is defined as

$$\iiint_{R_\ell} \frac{1}{r_3} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{\bar{u}^2}{2} \right) dR \equiv \iiint_{R_\ell} \psi dR = \lim_{\epsilon_2 \rightarrow 0} \int_{-\infty}^{+\infty} d\bar{\eta} \int_{-\infty}^0 d\bar{\zeta} \left\{ \int_{-\infty}^{\bar{x}_S - \epsilon_2} \psi d\bar{\xi} + \int_{\bar{x}_S + \epsilon_2}^{+\infty} \psi d\bar{\xi} \right\} . \quad (3-33)$$

Introducing the notation

$$\Delta \bar{u} = \bar{u}_u - \bar{u}_\ell \quad , \quad \Delta \frac{\partial \bar{u}}{\partial \bar{\zeta}} = \frac{\partial \bar{u}_u}{\partial \bar{\zeta}} - \frac{\partial \bar{u}_\ell}{\partial \bar{\zeta}} \quad , \quad (3-34)$$

and adding Equations (3-34) and (3-32), we have

$$\begin{aligned} \bar{u}(\bar{x}, \bar{y}, \bar{z}) = & -\frac{1}{4\pi} \iint_B \left[\frac{1}{r_3} \Delta \frac{\partial \bar{u}}{\partial \bar{\zeta}} - \Delta \bar{u} \frac{\partial}{\partial \bar{\zeta}} \left(\frac{1}{r_3} \right) \right] d\bar{\xi} d\bar{\eta} \\ & - \frac{1}{4\pi} \iint_S \left[\left\{ \frac{1}{r_3} \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial}{\partial n} \left(\frac{1}{r_3} \right) \right\}_1 + \left\{ \frac{1}{r_3} \frac{\partial \bar{u}}{\partial n} - \bar{u} \frac{\partial}{\partial n} \left(\frac{1}{r_3} \right) \right\}_2 \right] dS \\ & - \frac{1}{4\pi} \iiint \frac{1}{r_3} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{\bar{u}^2}{2} \right) dR \quad , \end{aligned} \quad (3-35)$$

where B represents the wing-body or wing alone and S represents the sum of the surfaces S_u and S_ℓ ; and $R = R_u + R_\ell$. The first integral on the right-hand side of Equation (3-35) will be recognized as the value of $\bar{u}$ given by the linearized theory of subsonic flow past wing or wing-body combination. Denoting this linear value by $\bar{u}_L$, we have

$$\bar{u}_L = -\frac{1}{4\pi} \iint_B \left[\frac{1}{r_3} \Delta \frac{\partial \bar{u}_L}{\partial \bar{\zeta}} - \Delta \bar{u}_L \frac{\partial}{\partial \bar{\zeta}} \left(\frac{1}{r_3} \right) \right] d\bar{\xi} d\bar{\eta} \quad . \quad (3-36)$$

Equation (3-35) may be regarded as the final integral equation for $\bar{u}$ for the nonlifting case. But for numerical computations, it is advantageous to write it in another form by integrating the volume integral twice by parts with respect to $\bar{\xi}$, taking proper cognizance of the definitions given in Equations (3-30), (3-31) and (3-33), and to decompose the surface integral over the shock wave into components parallel to the axes of the coordinate system. If then n_1 , n_2 and n_3 are the direction cosines of the normal to the shock surface drawn inward, as shown in Figure 3.3b, the following equation is obtained from Equation (3-35):

$$\begin{aligned}
 \bar{u} = & \bar{u}_L + \frac{\bar{u}^2}{2} - \frac{1}{4\pi} \iiint_R \frac{\bar{u}^2}{2} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{1}{r_3} \right) d\bar{\xi} d\bar{\eta} d\bar{\zeta} \\
 & + \frac{1}{4\pi} \iint_S \left[\left\{ \frac{1}{r_3} \frac{\partial}{\partial \bar{\xi}} \left(\bar{u} - \frac{\bar{u}^2}{2} \right) - \left(\bar{u} - \frac{\bar{u}^2}{2} \right) \frac{\partial}{\partial \bar{\xi}} \left(\frac{1}{r_3} \right) \right\}_1 \right. \\
 & \quad \left. - \left\{ \frac{1}{r_3} \frac{\partial}{\partial \bar{\xi}} \left(\bar{u} - \frac{\bar{u}^2}{2} \right) - \left(\bar{u} - \frac{\bar{u}^2}{2} \right) \frac{\partial}{\partial \bar{\xi}} \left(\frac{1}{r_3} \right) \right\}_2 \right. \\
 & \quad \left. + \left\{ \left(\frac{1}{r_3} \frac{\partial \bar{u}}{\partial \bar{\eta}} - \bar{u} \frac{\partial}{\partial \bar{\eta}} \left(\frac{1}{r_3} \right) \right)_1 - \left(\frac{1}{r_3} \frac{\partial \bar{u}}{\partial \bar{\eta}} - \bar{u} \frac{\partial}{\partial \bar{\eta}} \left(\frac{1}{r_3} \right) \right)_2 \right\} \left(\frac{n_2}{n_1} \right)_2 \right. \\
 & \quad \left. + \left\{ \left(\frac{1}{r_3} \frac{\partial \bar{u}}{\partial \bar{\zeta}} - \bar{u} \frac{\partial}{\partial \bar{\zeta}} \left(\frac{1}{r_3} \right) \right)_1 - \left(\frac{1}{r_3} \frac{\partial \bar{u}}{\partial \bar{\zeta}} - \bar{u} \frac{\partial}{\partial \bar{\zeta}} \left(\frac{1}{r_3} \right) \right)_2 \right\} \cdot \left[\frac{n_3}{n_1} \right]_2 \right] d\bar{\eta} d\bar{\zeta} \quad , \\
 & \hspace{25em} (3-37)
 \end{aligned}$$

where $\bar{u}_L$ is given by Equation (3-36).

Although the integral equation obtained above appears to be more complicated than Equation (3-35), it is superior from the point of view of approximate solutions for the following reasons:

a. The triple integral of Equation (3-35) shows a very strong influence of the velocities in the neighborhood of P since they are multiplied by $1/r_3$. This influence is largely nullified in the triple integral in Equation (3-37) because part of the region has a negative influence and part has positive influence. The predominant influence in Equation (3-37) is furnished by the term $\bar{u}^2/2$ standing outside the integral.

b. The contribution of distant regions is also diminished in importance in the triple integral of Equation (3-37) since their influence now varies as $1/r_3^3$ rather than $1/r_3$ as in Equation (3-35).

c. The biggest advantage in using Equation (3-37) is that the value of the triple integral in this equation is continuous through a shock wave (due to the presence of the term $\bar{u}^2/2$ outside the second derivative) rather than discontinuous as in Equation (3-35). A point of great importance in the approximate solution arises from the fact that the integration by parts provides extra terms (those containing $\bar{u}^2/2$) in the integrals along the shock surface S which combine with those already present in such a way that

the contribution of these integrals becomes very small when the shocks are nearly normal waves, as is usually the case for high subsonic speeds.

In addition to satisfying the integral equation for $\bar{u}$ given in Equation (3-37), the velocity components on opposite sides of shock waves must be in accord with the simplified relation for the shock polar given in Equation (3-10), which can be rewritten in normalized form, using Equation (3-20), as follows:

$$(\bar{u}_1 - \bar{u}_2)^2 + (\bar{v}_1 - \bar{v}_2)^2 + (\bar{w}_1 - \bar{w}_2)^2 = \left[\frac{\bar{u}_1 + \bar{u}_2}{2} \right] (\bar{u}_1 - \bar{u}_2)^2, \quad (3-38)$$

or with slight manipulations it can be written in the following two alternative forms:

$$\left[1 - \frac{\bar{u}_1 + \bar{u}_2}{2} \right] (\bar{u}_1 - \bar{u}_2)^2 + (\bar{v}_1 - \bar{v}_2)^2 + (\bar{w}_1 - \bar{w}_2)^2 = 0 \quad (3-39)$$

and

$$(\bar{u}_1 - \bar{u}_2) \left[\left[u_1 - \frac{u_1^2}{2} \right] - \left[u_2 - \frac{u_2^2}{2} \right] \right] + (\bar{v}_1 - \bar{v}_2)^2 + (\bar{w}_1 - \bar{w}_2)^2 = 0. \quad (3-40)$$

If the shock wave is a normal wave and the flow is parallel to the $\bar{x}$ -axis (i.e., $\bar{v}_1 = \bar{v}_2 = \bar{w}_1 = \bar{w}_2 = 0$, but $\bar{u}_1 \neq \bar{u}_2$), it can be seen from Equation (3-39) that

$\bar{u}$ jumps from $1 + \Delta$ immediately ahead of the shock to $1 - \Delta$ immediately behind the shock, where $\Delta = \bar{u}_1 - \bar{u}_2/2$. On the other hand, Equation (3-40) shows that the quantity $\bar{u} - \bar{u}^2/2$ is equal on the two sides of the shock. This is consistent with the fact that the latter quantity corresponds, in the transonic approximation, to the mass flow, which is continuous through a normal shock.

The general problem of the three-dimensional transonic flow about nonlifting bodies requires the solution of Equation (3-37) while taking account of the shock relations given in Equations (3-38), (3-39) and (3-40). This is a formidable task well beyond the reach of present analysis. We therefore introduce some further approximations before solutions can be obtained. They are: (a) all shock waves are assumed to lie in a plane perpendicular to the x-axis, and (b) the shock waves are assumed to be normal to the local flow direction. The first assumption corresponds to setting $n_2 = n_3 = 0$, and the second leads to the relations:

$$\left(\bar{u} - \frac{\bar{u}^2}{2}\right)_1 = \left(\bar{u} - \frac{\bar{u}^2}{2}\right)_2 \quad ; \quad \frac{\partial}{\partial \xi} \left(\bar{u} - \frac{\bar{u}^2}{2}\right)_1 = \frac{\partial}{\partial \xi} \left(\bar{u} - \frac{\bar{u}^2}{2}\right)_2 = 0 \quad .$$

(3-41)

Hence Equation (3-37) simplifies to

$$\bar{u} = \bar{u}_L + \frac{\bar{u}^2}{2} - \frac{1}{4\pi} \iiint_R \frac{\bar{u}^2}{2} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{1}{r_3} \right) d\bar{\xi} d\bar{\eta} d\bar{\zeta} . \quad (3-42)$$

A similar, but slightly more involved analysis, given in Chapter V, shows that from Equations(3-23) and (3-24), the equations to determine v and w are:

$$\bar{v} = \bar{v}_L - \frac{1}{4\pi} \iiint_R \frac{\bar{u}^2}{2} \frac{\partial^2}{\partial \bar{\xi} \partial \bar{\eta}} \left(\frac{1}{r_3} \right) d\bar{\xi} d\bar{\eta} d\bar{\zeta} , \quad (3-43)$$

$$\bar{w} = \bar{w}_L - \frac{1}{4\pi} \iiint_R \frac{\bar{u}^2}{2} \frac{\partial^2}{\partial \bar{\xi} \partial \bar{\zeta}} \left(\frac{1}{r_3} \right) d\bar{\xi} d\bar{\eta} d\bar{\zeta} . \quad (3-44)$$

Before proceeding further, it should be observed that the solutions of Equations (3-42), (3-43) and (3-44) must approach those of the linear theory when $M_\infty \ll 1$, since $\bar{u} \ll 1$ and therefore the terms involving $\bar{u}^2$ become negligible, thereby leaving only

$$(\bar{u}, \bar{v}, \bar{w})_{M_\infty \ll 1} \approx (\bar{u}_L, \bar{v}_L, \bar{w}_L) . \quad (3-45)$$

Let us now write Equation (3-42) as follows:

$$\bar{u} = \bar{u}_L + \frac{\bar{u}^2}{2} - \frac{I}{2} , \quad (3-46)$$

where

$$I = 2 \left[\frac{1}{4\pi} \iiint_R \frac{\bar{u}^2}{2} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{1}{r_3} \right) d\bar{\xi} d\bar{\eta} d\bar{\zeta} \right] . \quad (3-47)$$

Although I is a function of $\bar{u}$ and is therefore unknown, it is informative to rewrite Equation (3-46) by solving for $\bar{u}$ in terms of I and $\bar{u}_L$, thus

$$\bar{u} = 1 \pm \sqrt{I - (2\bar{u}_L - 1)} = 1 \pm \sqrt{I - L} , \quad (3-48)$$

where

$$L = 2\bar{u}_L - 1 .$$

Several interesting observations can be made from Equation (3-48). First of all, for real values of $\bar{u}$,

$$I \geq L . \quad (3-49)$$

Furthermore, the choice of the plus or minus sign determines whether the local velocity is subsonic or supersonic. A change in sign at a point where the radical is zero corresponds to a discontinuous jump in velocity. As pointed out following Equation (3-40), such discontinuities correspond to normal shock waves and are permissible when they proceed from supersonic to subsonic velocities or from plus

to minus sign in Equation (3-48), when progressing in the flow direction. Discontinuities in the reverse direction are inadmissible since they correspond to expansion shocks, a phenomenon which violates the second law of thermodynamics.

The values of $\bar{u}_L$, and hence L , can be calculated for any given body and are generally characterized by certain regions in which $\bar{u}_L > 0$ and other regions in which $\bar{u}_L < 0$. The absolute values of $\bar{u}$ increase continuously with increasing M_∞ and the maximum positive values may considerably exceed unity as sonic velocity is approached in the free-stream. Not very much can be stated at this point about the values of I , except that they depend on the distribution as well as magnitude of $\bar{u}$ and that the above inequality must be satisfied.

The starting point for the solution of the full integral equation derived for the three-dimensional cases would be to examine the methods needed for two-dimensional flows given by a number of authors: Oswatitsch, Gullstrand, Spreiter and Alksne, Nixon, and Norstrud.[30]

The analysis of the two-dimensional case is based on the following equation:

$$\bar{u}(\bar{x}, \bar{z}) = \bar{u}_L(\bar{x}, \bar{z}) + \frac{\bar{u}^2(\bar{x}, \bar{z})}{2} - \frac{1}{2\pi} \iint_{-\infty}^{+\infty} \frac{\bar{u}^2(\bar{\xi}, \bar{\zeta})}{2} \frac{[(\bar{x}-\bar{\xi})^2 - (\bar{z}-\bar{\zeta})^2]}{[(\bar{x}-\bar{\xi})^2 + (\bar{z}-\bar{\zeta})^2]^2} d\bar{\xi} d\bar{\zeta} .$$

(3-50)

Approximate solutions of this equation could conceivably be worked out numerically by starting with a two-dimensional grid of suitably selected values of $\bar{u}$ and iterating until convergence is obtained. If the first approximation for $\bar{u}$ is taken to be the results given by incompressible or linearized subsonic flow theory, convergence will be obtained only when M is sufficiently small and the flow is subsonic everywhere. Oswatitsch suggested that mixed flow fields containing shock waves can be obtained if the starting $\bar{u}$ -distribution contains shock waves. The idea then is to start with a reasonable guess for $\bar{u}$, being sure to include a proper discontinuity complying with the shock relations of Equation (3-41), then proceed to the solution. It is not necessary to be highly accurate in the initial guess for $\bar{u}$.

A further simplifying assumption is made by Oswatitsch and others regarding the variation of $\bar{u}$ in the z -direction, which has the effect of reducing the double integral in Equation (3-50) to a single integral. Oswatitsch suggested the following relation:

$$\bar{u}(\bar{x}, \bar{z}) = \frac{\bar{u}_B(\bar{x}, 0)}{\left[1 + \frac{\bar{z}}{b}\right]^2}, \quad (3-51)$$

where $\bar{u}_B(\bar{x}, 0)$ is the value of $\bar{u}$ on the body and b is a function of $\bar{x}$ so chosen that the irrotationality condition $\partial \bar{u} / \partial \bar{z} = \partial \bar{w} / \partial \bar{x}$ is fulfilled at $\bar{z} = 0$. Thus, differentiation of Equation (3-51) with respect to $\bar{z}$ and insertion of the definition of Equation (3-20) and the boundary condition, Equation (3-14), yield the following:

$$\begin{aligned} b &= - \frac{2\bar{u}_w}{\left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)_w} = - \frac{2\bar{u}_w}{\left(\frac{\partial \bar{w}}{\partial \bar{x}}\right)_w} = - \frac{2\beta^3}{k} \frac{\bar{u}_w}{\left(\frac{\partial w}{\partial x}\right)_w} \\ &= - \frac{2\beta^3}{k U_\infty} \frac{\bar{u}_w}{\left(\frac{\partial^2 z}{\partial x^2}\right)} = - \frac{2u_w}{\left(\frac{\partial^2 \bar{z}}{\partial \bar{x}^2}\right)}. \end{aligned} \quad (3-52)$$

where $\bar{z}$ represents the reduced ordinates of the airfoil, related to the actual ordinates by

$$\bar{z}(\bar{x}) = \frac{k U_\infty}{\beta^3} z(x). \quad (3-53)$$

Substitution of Equation (3-51) into the double integral of Equation (3-50) permits integration with respect to $\bar{z}$.

Thus, by performing this integration and setting $\bar{z} = 0$, the following approximate integral equation is obtained for $\bar{u}_w$:

$$\bar{u}_w = \bar{u}_{L_w} + \frac{\bar{u}_w^2}{2} - \int_0^c \frac{\bar{u}_w^2}{2b} E\left(\frac{\bar{\xi} - \bar{x}}{b}\right) d\bar{\xi}, \quad (3-54)$$

where the function E is

$$E\left(\frac{\bar{\xi} - \bar{x}}{b}\right) = E(x) = \frac{4}{\pi(1+x^2)^5} \left[\frac{\pi}{2} |x| (5 - 10x^2 + x^4) - (1 - 10x^2 + 5x^4) \ln |x| - \frac{1}{12} (1+x^2) (25 - 71x^2 - x^4 - x^6) \right] \quad (3-55)$$

Equation (3-54) is solved by iteration procedure by assuming $\bar{u}_w$ value initially and calculating "b" from Equation (3-52), substituting back in Equation (3-54) and calculating new $\bar{u}_w$ from Equation (3-54) until convergence is obtained.

The solution of Equation (3-50) is discussed by various authors by using various techniques. Since Equation (3-50) is a nonlinear Fredholm integral equation which could conceivably be solved numerically by an iteration process. However, the double integral in Equation (3-50) is a source of difficulty in the solution

of Equation (3-50) by an iterative process. If it could be reduced to a single integral by introducing a suitable approximation, the entire problem would be greatly simplified. For this purpose, a functional relationship is sought between the $\bar{u}$ -distribution in the field and the value of $\bar{u}$ at the airfoil surface for the same $\bar{x}$ -location. Assuming

$$\bar{u}(\bar{x}, \bar{z}) = \bar{u}(\bar{x}, 0) F(\bar{z}, r) \quad (3-56)$$

where $r(x)$ is a function so chosen that the irrotationality condition is satisfied on the airfoil. In particular, the parameter $r(x)$ is defined according to

$$r(x) = - \frac{\bar{u}(x, +0)}{\left[\frac{\partial \bar{u}}{\partial \bar{z}} \right]_{x, +0}} \quad (3-57)$$

Various functions $(F(\bar{z}, r))$ have been treated by different authors. Norstrud [33] discusses the nature of four functions commonly used. He indicates that there are almost no differences in these functions once they have been integrated in the z -direction. This fact can be utilized by employing the most simple analytical function $F(\bar{z}, r) = e^{-\bar{z}/r}$ to yield the functional relationship [34]

$$\bar{u}(\bar{x}, \bar{z}) = \bar{u}(\bar{x}, 0) e^{-\frac{\bar{z}}{r}} \quad (3-58)$$

A significant aspect of this approximation is that $\bar{u}$ is zero in the region ahead of the leading edge and behind the trailing edge of the airfoil where r is infinite. The errors in the velocity on the airfoil resulting from this approximation should be small because of the attenuating influence of distance. The approximation also has a profound effect on the region of integration of Equation (3-50), allowing certain mathematical simplifications.

So far, the methods of solving the integral equation in two dimensions have been dealt with. For three-dimensional flows, the triple integral in Equation (3-42) should be simplified to a single integral for solving the Equation (3-42). But the variation of u with respect to x, y, z direction is not known generally and the approximation cited for $u(x, z)$ in the case of two-dimensional flow would not be appropriate generally for three-dimensional flow. The inverse square dependence on $\bar{z}$ at large $\bar{z}$ is consistent with the behavior of a two-dimensional dipole--but not of a three-dimensional dipole nor of a two- or three-dimensional vortex. In general, therefore, some other approximation must be introduced at this point.

Norstrud [34] solved the integral equation method for the three-dimensional lifting wing of arbitrary planform.

E. The Formulation and Solution of the Three Dimensional Integral Equation for Lifting Arbitrary Wing for the Present Application.

The formal integral solution to the transonic small-perturbation equation with associated boundary conditions of tangential flow on the wing surface and constant potential at the far field can be regarded as more amenable to numerical analysis than the differential formulation. This is especially true for subcritical flows. For supercritical flows, however, some additional considerations must be given to guarantee the uniqueness of the flow solution. In either case, the governing integral equation is first replaced by a system of nonlinear algebraic equations and then the method of parametric differentiation is applied to its solution.

Analytical method. Isentropic and irrotational steady flow of an ideal compressible gas around a finite wing is kinematically described by a velocity vector $q = \text{grad } \phi$ with components u, v, w , defined in a body-fixed cartesian coordinate system x, y, z . With further assumption of small disturbance in the plane normal to the direction of oncoming flow (i.e., the yz -plane) the governing partial differential equation for the velocity potential $\phi = \phi(x, y, z)$ can be written as

$$(1 - M_\infty^2) \phi_{xx} + \phi_{yy} + \phi_{zz} = \left(\frac{\gamma+1}{U_\infty} \right)^2 M_\infty^2 \phi_x \phi_{xx} \quad (3-6)$$

where $M_\infty = (U_\infty/C_\infty)$ is the freestream Mach number and γ designates the ratio of specific heats. Equation (3-6) is a particular form of the set of equations which can be reduced from the transonic small perturbation theory, and it can be transformed to

$$\bar{\phi}_{xx} + \bar{\phi}_{yy} + \bar{\phi}_{zz} = \bar{\phi}_x \bar{\phi}_{xx} \quad (3-21)$$

defining a perturbation velocity potential

$$\bar{\phi} = \frac{k}{\beta^2} \phi$$

The employed coordinate transformation $Y, Z = (1 - M_\infty^2)^{1/2} y, z$ corresponds to a Prandtl-Glauert transformation, and the freestream Mach number is thus restricted to subsonic flows, i.e., $M_\infty < 1$. Equation (3-21) resembles a Poisson differential equation for which the potential theory admits the solution

$$\begin{aligned} \bar{\phi}(x, Y, Z) = & \bar{\phi}_L(x, Y, Z) + \bar{\phi}'(x, Y, Z) \\ & - \frac{1}{4\pi} \iiint_{-\infty}^{\infty} [\bar{\phi}_\xi \bar{\phi}_{\xi\xi}]_{\xi, \bar{\eta}, \bar{\zeta}} \frac{1}{r_3} d\xi d\bar{\eta} d\bar{\zeta} \quad , \quad (3-59) \end{aligned}$$

where $r_3 = [(x-\xi)^2 + (Y-\bar{\eta})^2 + (Z-\bar{\zeta})^2]^{1/2}$ designates the inverse of the singular fundamental solution for the domain of integration and $\bar{\phi}_L = \bar{\phi}_L(x, Y, Z)$ represents

the harmonic solution associated with Equation (3-21) for the same boundary conditions which shall apply for $\bar{\phi}$ (i.e., tangential flow at the wing surface and finite velocities at the trailing edge). If now a perturbation velocity vector in the transformed space x, Y, Z is defined as $(\bar{u}, \bar{v}, \bar{w}) = \text{grad } \bar{\phi}$ and if we restrict ourselves to seeking a solution for $\bar{u}(X, Y, 0)$ in the planform plane $Z \equiv 0$, then a differentiation of Equation (3-59) with respect to x will yield, after some manipulation,

$$\begin{aligned} \bar{u}(x, Y, 0) - \frac{1}{2} \bar{u}^2(x, Y, 0) &= \bar{u}_L(x, Y, 0) + \bar{\phi}'_x(x, Y, 0) \\ &+ \frac{1}{4\pi} \iiint_{-\infty}^{\infty} \frac{\bar{u}^2(\bar{\xi}, \bar{\eta}, \bar{\zeta})}{2} \frac{\partial^2}{\partial \bar{\xi}^2} \left(\frac{1}{r_3} \right) d\bar{\xi} d\bar{\eta} d\bar{\zeta} \quad , \quad (3-60) \end{aligned}$$

where $\bar{u}(x, Y, 0)$ is the disturbance velocity component in the axial direction, obtained from the subsonic linear solution.

To facilitate the solution of this nonlinear integral equation, an exponential decay in the Z -direction will be assumed for the perturbation velocity component $\bar{u} = \bar{u}(x, Y, Z)$ as

$$\bar{u}(x, Y, Z) = \bar{u}(x, Y, \pm 0) \exp[\mp Z/r(x, Y, \pm 0)] \quad , \quad (3-61)$$

where the curvature parameter $r^{\pm} = r(x, Y, 0)$ is approximately evaluated from the irrotationality

condition on the wing surface coordinate

$Z_w^\pm = Z_w(x, Y, \pm 0)$ as

$$r^\pm = \left| \left\{ (1-M_\infty^2)^{1/2} \left[\frac{\bar{U}_T(x, Y, \pm 0) - u_\infty}{\bar{U}_\infty} \right] \right/ \frac{d^2}{dx^2} (Z_w^\pm) \right\} \right| \quad (3-62)$$

where $\bar{U}_T$ is the total velocity component in the axial direction, obtained from the subsonic linear theory.

Equation (3-62) substituted in the integral term of Equation (3-60) will then yield the reduced equation

$$\begin{aligned} \bar{u}(x, Y, \pm 0) - \frac{1}{2} \bar{u}^2(x, Y, \pm 0) = & \bar{u}_L(x, Y, \pm 0) + \bar{\phi}'_X(x, Y, \pm 0) \\ & + \left\{ \frac{1}{4\pi} \iint_{-\infty}^{\infty} \bar{u}^2(\bar{\xi}, \bar{\eta}, 0) E(|x-\bar{\xi}|, |Y-\bar{\eta}|; r^+) d\bar{\xi} d\bar{\eta} \right. \\ & \left. + \frac{1}{4\pi} \iint_{-\infty}^{\infty} \bar{u}^2(\bar{\xi}, \bar{\eta}, -0) E(|x-\bar{\xi}|, |Y-\bar{\eta}|; r^-) d\bar{\xi} d\bar{\eta} \right\}, \quad (3-63) \end{aligned}$$

where the kernel $E = E(|x-\bar{\xi}|, |Y-\bar{\eta}|; r)$ takes the form

$$\begin{aligned} E = \frac{-2\pi}{r^2\sigma} \left\{ \left[\frac{4}{\sigma^2} \left(\frac{x-\bar{\xi}}{r} \right)^2 - 1 \right] \left[\frac{2}{\pi} - H_1(\sigma) + N_1(\sigma) \right] + \right. \\ \left. \frac{4}{\sigma} \left(\frac{x-\bar{\xi}}{r} \right)^2 \left[H_0(\sigma) - \frac{1}{\sigma} [H_1(\sigma) + N_1(\sigma)] + N_2(\sigma) \right] \right\}. \quad (3-64) \end{aligned}$$

Here H_v and N_v designate the Sturtevant and the Newmann functions respectively of integer order v with the argument

$$\sigma = 2 \left\{ \left[\frac{(x-\bar{\xi})}{r} \right]^2 + \left[\frac{(Y-\bar{\eta})}{r} \right]^2 \right\}^{1/2} .$$

In the case of lifting flows, the integral term of Equation (3-60) will give rise to a downwash in the plane of the wing planform. This can be compensated by adding an antisymmetric (with respect to $Z = 0$) correction potential $\bar{\phi}' = \bar{\phi}'(x, Y, Z)$ to the harmonic solution $\bar{\phi}_L$, i.e.,

$$\bar{\phi}'(x, Y, Z) = - \frac{1}{\pi} \int_{-\infty}^{+\infty} \gamma'(\bar{\xi}, Y) \tan^{-1} \frac{Z}{x-\bar{\xi}} d\bar{\xi} , \quad (3-65)$$

where the vortex strength $\gamma'(x, Y)$ is necessarily defined as

$$\begin{aligned} \gamma'(x, Y) = \frac{1}{2} [& \bar{u}(x, Y, +0) - \bar{u}_L(x, Y, +0) \\ & - \bar{u}(x, Y, -0) + \bar{u}_L(x, Y, -0)] . \end{aligned} \quad (3-66)$$

Then a differentiation of Equation (3-65) with respect to x together with a limiting process for $\bar{\phi}'_x(x, Y, Z)$ as $Z \rightarrow 0$ will yield the identity

$$\bar{\phi}'_x(x, Y, 0) = \gamma'(x, Y) .$$

Since a unique circulation is already implemented in defining the vortex strength $\gamma'(\bar{\xi}, Y)$, no further considerations have to be made with regard to the Kutta condition at the trailing edge.

Correction for lifting, discontinuous flows. With shock surface discontinuities in the flow field a modification of Equation (3-66) is necessary in order to assure the harmonic nature of $\bar{\phi}'(x,Y,0)$ and, hence, enforce a smooth continuous derivative $\bar{\phi}'_x$. However, instead of defining the function $\bar{\phi}'$ we define its gradient in the x-direction as

$$\bar{\phi}'_x(x,Y,0) = \frac{m(x,Y)}{4} [\bar{u}_L^2(x,Y,+0) - \bar{u}_L^2(x,Y,-0)] \quad (3-67)$$

and let $m(x,Y)$ represent a spanwise-varying scaling parameter which shall depend on the local circulation around the wing in order to satisfy the Kutta condition. Hence, one can write

$$m(x,Y) = 2 \int_{\xi=-\infty}^{\infty} [\bar{u}(\bar{\xi},Y,+0) - \bar{u}_L(\bar{\xi},Y,+0) - \bar{u}(\bar{\xi},Y,-0) + \bar{u}_L(\bar{\xi},Y,-0)] [\bar{u}_L^2(\bar{\xi},Y,+0) - \bar{u}_L^2(\bar{\xi},Y,-0)]^{-1} d\bar{\xi} \quad (3-68)$$

It should be emphasized that Equation (3-67) is not unique and that a better downwash representation at the leading edge could be achieved with, say, an adoption of the Riegel's factor $[1 + (\partial z_w / \partial x)^2]^{-1/2}$.

From the elementary relation

$$\begin{aligned} \bar{u}_L^2(x,Y,+0) - \bar{u}_L^2(x,Y,-0) &= [\bar{u}_L(x,Y,+0) - \bar{u}_L(x,Y,-0)] \\ &\quad [\bar{u}_L(x,Y,+0) + \bar{u}_L(x,Y,-0)] , \end{aligned}$$

where the first and second factor on the right-hand side represents the linearized perturbation velocity due to lift and thickness respectively. One sees from Equation (3-67) that the distribution in chordwise direction of the x-component of the correction potential is defined to be of a similar form as the disturbance due to thickness. This distribution, however, is modified by an amount proportional to the local chordwise lift.

The solution to the nonlinear integral Equation (3.60) cannot be obtained analytically for wing configuration of arbitrary planform and thickness distribution. Hence, a numerical treatment of the general problem is mandatory and the first step in this direction is to discretize the area of integration in accord with the division of the wing planform as used for the numerical evaluation of the harmonic solution. The range of integration shall be confined to the wing planform and the nonlinear compressibility sources outside the cylinder which encloses the planform and which have the generatrix parallel to the Z-direction are therefore not being considered.

Let the upper and lower surface of the left (or right) half of the wing planform each be represented by N number of trapezoidal panels and let the unknown function $\bar{u} = \bar{u}(x, Y, \pm 0)$ assume a constant value within these specified panel boundaries; see Figure 3.4. Then Equation (3.63)

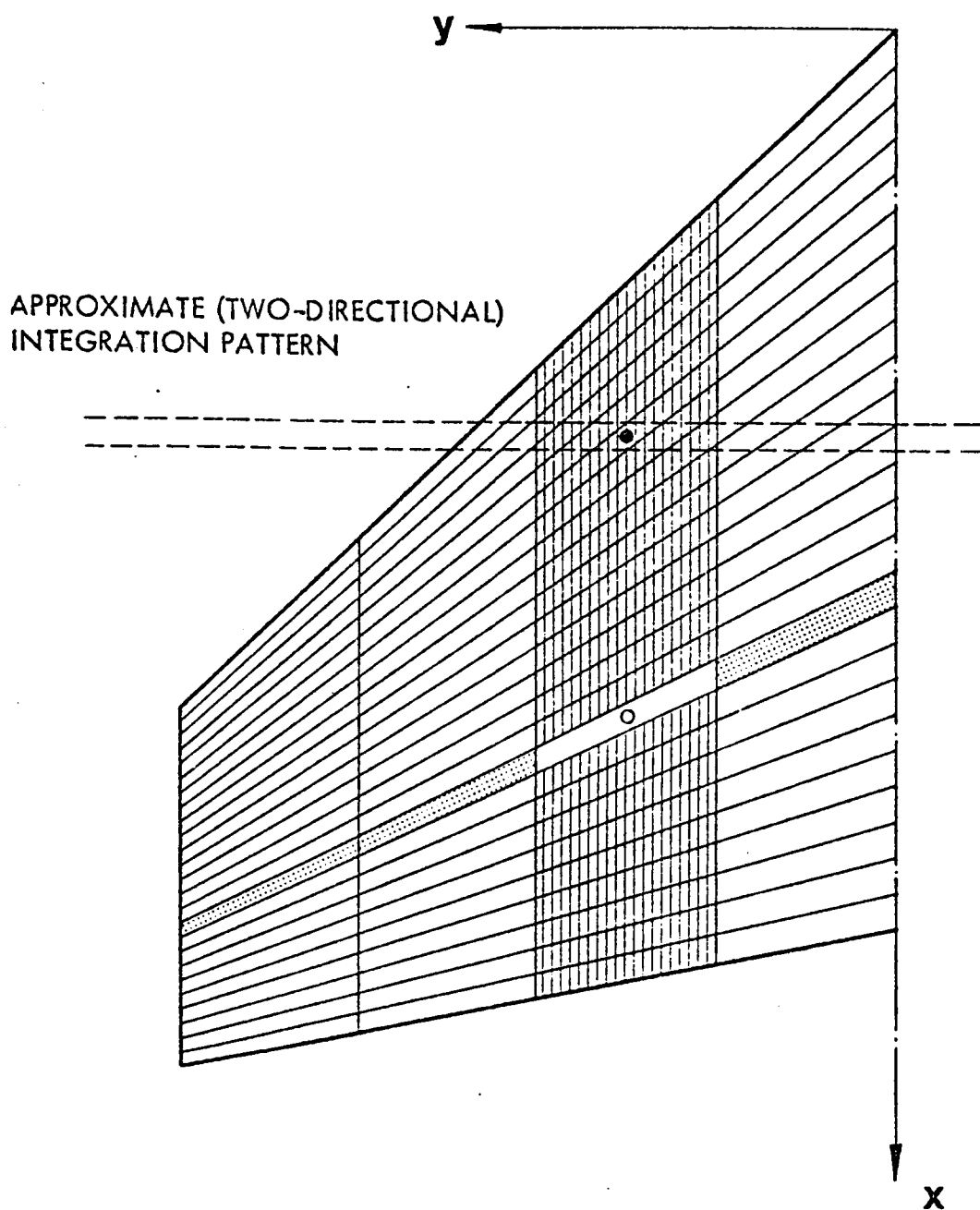


Figure 3.4. Geometric Division of an Arbitrary Planform into a Number of Trapezoidal Panels.

can be replaced by a system of $2N$ algebraic equations which are nonlinear (i.e., quadratic) with respect to

$\bar{u} = \bar{u}(x, Y, \pm 0)$, i.e.,

$$F_i(\bar{u}_1, \bar{u}_2, \dots, \bar{u}_{2N}) = \bar{u}_i - \frac{1}{2} \bar{u}_i^2 - \bar{u}_{L_i} + C_i + \sum_{j=1}^{2N} \epsilon_{ij} \bar{u}_j^2 = 0 \quad (3-69)$$

$$i = 1, 2, \dots, 2N$$

where the index $i \leq N$ shall identify a value on the upper surface of the wing (or panel). Consequently, the lower surface values are associated with the range $N < i \leq 2N$.

Due to the lateral symmetry assumed for the wing planform and its position relative to the oncoming flow (i.e., $v_\infty = 0$) only one-half of the wing need to be considered in the algebraic system (3-69) since the influence from two panels which are positioned symmetrical with respect to the x -axis can be lumped together as $\epsilon_{ij} = \epsilon_{ij_1} + \epsilon_{ij_2}$. (Note that $\bar{u}_{L_1} = \bar{u}_{L_2}$ where the indices 1 and 2 refers to each half of the wing.)

The influence coefficients ϵ_{ij} are integral representations of the influence function $E = E(|x - \bar{\xi}|, |Y - \bar{\eta}|, r)$ as evaluated over a surface element ΔS , i.e. of the form

$$\frac{1}{4\pi} \iint_{\Delta S} E(|x - \bar{\xi}|, |Y - \bar{\eta}|, r) d\bar{\xi} d\bar{\eta} \rightarrow \epsilon_{ij}$$

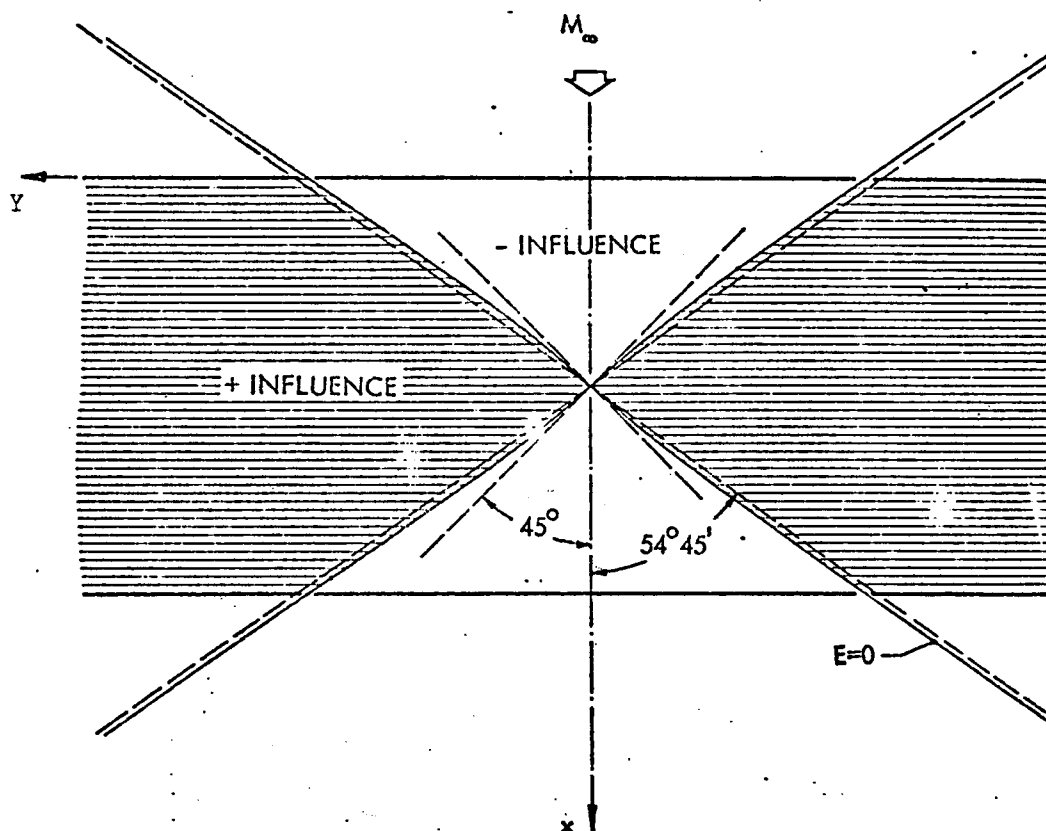


Figure 3.5. Influence Pattern with Reference to the Mid-point of a Rectangular Wing. [34]

for $|x-\bar{\xi}| \neq 0$ and $|Y-\bar{\eta}| \neq 0$. Due to the particular and perhaps unfortunate influence pattern of positive and negative contributions from the integral terms of Equation (3-63), care must be exercised in the numerical evaluation of the coefficients ϵ_{ij} ; see Figure 3.5.

This is further illustrated by the series representations

$$E^X(\rho) = -4 \left[\frac{1}{\rho^2} - \frac{2}{3}\rho + \frac{4}{45}\rho^2 - \dots \right] \quad |\rho| \rightarrow 0 \quad (3-70)$$

$$E^Y(\chi) = 4 \left[\frac{1}{\chi^2} - \frac{1}{\chi} + \frac{1}{3}\chi - \frac{1}{45}\chi^3 + \dots \right] \quad |\chi| \rightarrow 0 \quad (3-71)$$

of the influence function E in chordwise and approximate spanwise directions respectively where $\rho = 2|Y-\bar{\eta}|/r$ and $\chi = 2|x-\bar{\xi}|/r$. Since both equations (3-70) and (3-71) show similar behavior at the singular points $\rho = \chi = 0$, but of opposite sign, we let $\epsilon_{ii} = 0$. Furthermore, we define the influence coefficients ϵ_{ij} as

$$\epsilon_{ij} = \frac{\Delta S}{4} [E^{(1)} + E^{(2)} + E^{(3)} + E^{(4)}] \quad (3-72)$$

Equation (3-72) is derived from a simple application of the mean value theorem of integral calculus by relating the influence strength ϵ_{ij} at the centeroid of each panel to the influence measure at four points $\ell = 1, 2, 3, 4$ located on the four panel boundary lines (see Figure 3.6). A better approximation of ϵ_{ij} would be achieved, however, if a surface spline function is utilized in connection with the four values $E^{(\ell)}$ and the local value of E at the centeroid; see, e.g., Reference [35]. Note that for symmetrical flow problems it is sufficient to consider only a system of N equations with the minor modification of doubling the value of the influence coefficients ϵ_{ij} ($i, j = 1, 2, \dots, N$). However, the following analysis will be presented for a system of $2N$ equations.

The function C_i ($i = 1, 2, \dots, 2N$) appearing in Equations (3-69) is a velocity correction function associated with circulatory flows and is easily obtained from Equation (3-66) as

$$C_i = \frac{1}{2} A_i \quad i = 1, 2, \dots, 2N \quad (3-73)$$

for continuous flow. For circulatory flows with a shock discontinuity, Equation (3-67) approximately defines the required correction as

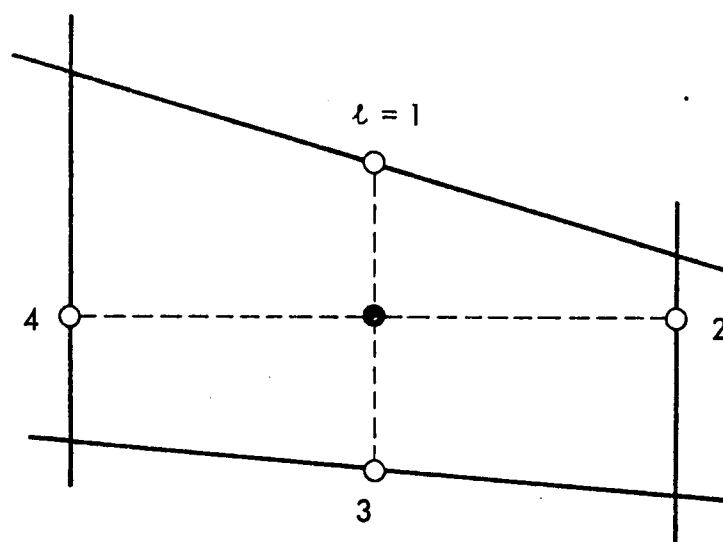


Figure 3.6. Integration Points for the Definition of the Influence Measure from an Elementary Panel.

$$C_i = \frac{1}{2} B_i \sum_{j=1}^N (1 - \delta_{ij}) \frac{A_j}{B_j} \quad i = 1, 2, \dots, 2N \quad (3-74)$$

where δ_{ij} represents the Kronecker delta and where

$$A_i = \begin{cases} \bar{u}_i - \bar{u}_{L_i} - \bar{u}_{i+N} + \bar{u}_{L_{i+N}} & i \leq N \\ \bar{u}_i - \bar{u}_{L_i} - \bar{u}_{i-N} + \bar{u}_{L_{i-N}} & i > N \end{cases}$$

$$B_i = \begin{cases} \bar{u}_{L_i}^2 - \bar{u}_{L_{i+N}}^2 & i \leq N \\ \bar{u}_i^2 - \bar{u}_{L_{i-N}}^2 & i > N \end{cases}$$

In the case of non-circulatory flows, Equations (3-73) and (3-74) will yield the expected corrections of $C_i \equiv 0$. System (3-69) is now completely determined and the task of finding a vector U_i ($i = 1, 2, \dots, 2N$) which satisfies Equations (3-69) for given ϵ_{ij} and U_i will be the subject of the following section.

Methods of solution. Solutions to the nonlinear system (3-69) could be obtained by various iteration or relaxation schemes and each method has its own merits and limitation. The decisive factor in selecting a solution technique, however, is the type of flow under consideration; see Figure 3.7. Since we are also interested in obtaining discontinuous solutions which shall represent flows with compression shocks, we will distinguish between three different types of flow problems,

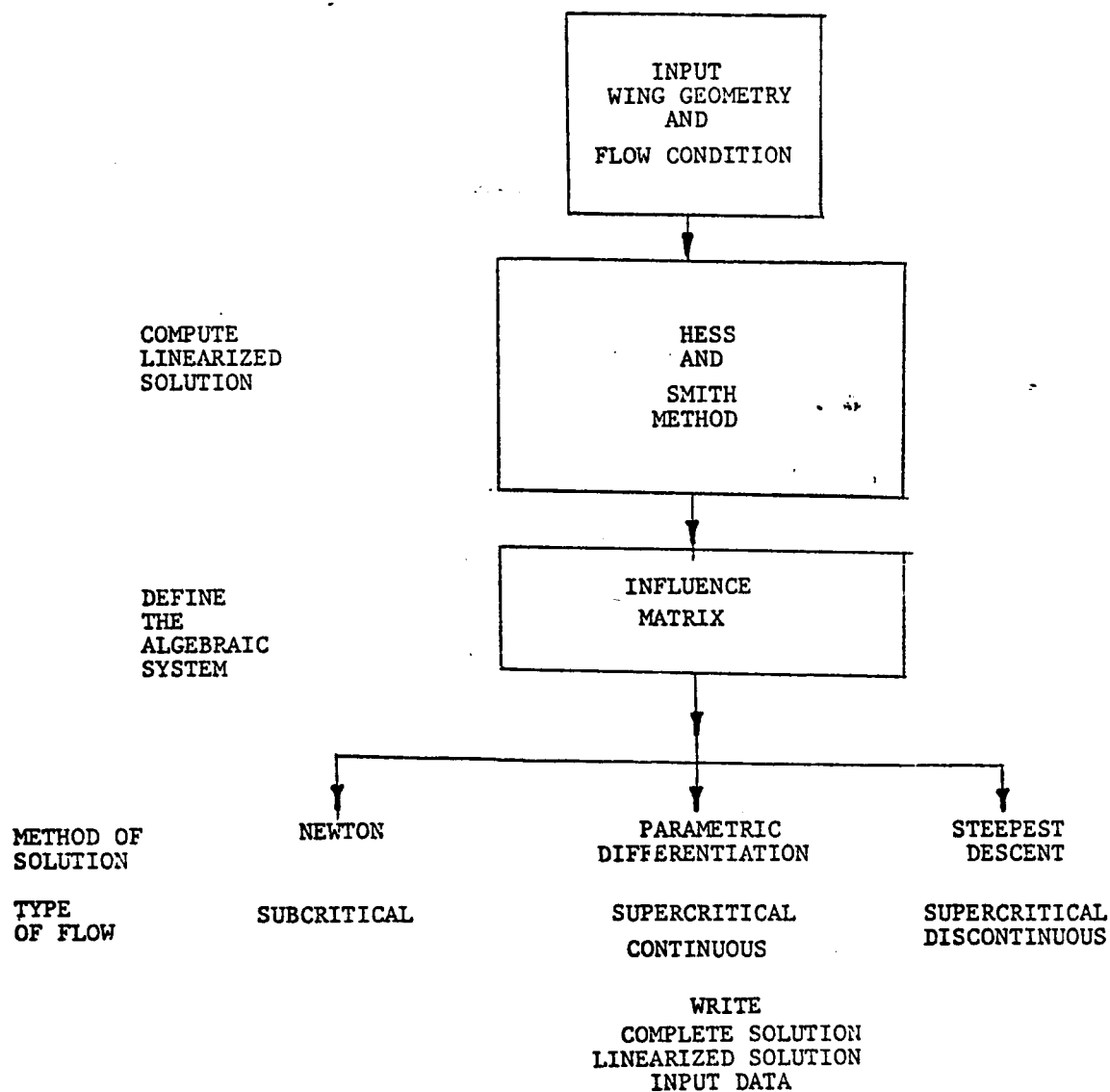


Figure 3.7. Block Diagram of the Computer Program.

namely

- Subcritical flow
- Supercritical flow without discontinuities
- Supercritical flow with discontinuities

and Ref. [34] describes some numerical approaches for their unique solution. In essence, each one of the numerical methods to be introduced should yield the same solution to the quadratic Equations (3-69) for identical given flow configurations.

F. Two-Dimensional Analysis by Exponential Decay Function

For the two-dimensional flow analysis using the exponential decay function for the perturbation "u" velocity, the formulation is as follows:

For wings of infinite aspect ratio the analysis becomes two-dimensional and available results for this case can be utilized in the calculation of flows about finite wings, Reference [36]. The appropriate integral equation [as compared to Equation (3-59)] will then read

$$\bar{\phi}(x, Z) = \bar{\phi}(x, Z) + \bar{\phi}'(x, Z) - \frac{1}{2\pi} \iint_{-\infty}^{\infty} [\bar{\phi}_{\xi} \bar{\phi}_{\bar{\xi} \bar{\xi}}]_{\xi, \bar{\xi}} \ln \frac{1}{r_2} d\bar{\xi} d\bar{\xi} \quad (3-75)$$

where $r_2 = [(x - \bar{\xi})^2 + (Z - \bar{\xi})^2]^{1/2}$. A similar assumption for the decay of the perturbation velocities in the z-direction as made in the foregoing three-dimensional

analysis will reduce Equation (3-75) to an equation involving the unknown quantity $\bar{u} = \bar{u}(x, \pm Z)$ of the form

$$\begin{aligned} \bar{u}(x, 0) - \frac{1}{2} \bar{u}^2(x, 0) = & \bar{u}_L(x, 0) + \bar{\phi}'_x(x, 0) \\ & + \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{u}^2(\xi, \pm 0) E_2(|x - \bar{\xi}|; r^{\pm}) d\bar{\xi} \end{aligned}$$

where the two-dimensional influence function

$E_2 = E_2(|x - \bar{\xi}|; r)$ is given as

$$\begin{aligned} E_2(\chi) &= E_2\left(\frac{2|\bar{\xi} - x|}{r}\right) \\ &= \frac{4}{\pi} \left\{ \sin(\chi) \left[\frac{\pi}{2} - \text{Si}(\chi) \right] - \cos(\chi) \text{Ci}(\chi) \right\} \end{aligned} \quad (3-76)$$

Here Si and Ci are the sine and cosine integrals respectively.

An analogous numerical integration of Equation (3-75) would involve the determination of influence coefficients ϵ_{ij} and this leads to a definition of

$$\epsilon_{ii} = \frac{1}{2r} \int_0^{\Delta x/2} E_2 d\bar{\xi}$$

which yields after a substitution of Equation (3-76) in the integrand of Equation (3-75) the following value

$$\begin{aligned}\varepsilon_{11}(\chi_0) &= \varepsilon_{11}\left(\frac{\Delta x}{r}\right) \\ &= \frac{1}{\pi} \left\{ \cos(\chi_0) \left[\text{Si}(\chi_0) - \frac{\pi}{2} \right] - \sin(\chi_0) \text{Ci}(\chi_0) + \frac{\pi}{2} \right\}\end{aligned}$$

The values of the coefficients ε_{ij} (where $j \neq i$), however, is determined direct from the influence function E_2 as $\varepsilon_{ij} = \Delta x E_2(|x - \xi|; r) / (4r)$. These influence coefficients are used in the approximate integration of the nonlinear influence measure over the wing; see, e.g., Figure 3.4. The above formulation is based on the exponential decay for the u component velocity in the field. The various other available formulations for the two dimensional transonic Integral equation method are as follows:

Integral equation method by Oswatitsch [30]

Integral equation method by Gullstrand [30]

Integral equation method by Spreiter [30]

Integral equation method by Nixon and Hancock [30]

The various finite difference formulations are discussed in Ref. [30] for the two dimensional and for the three dimensional transonic flows. The approximate theories, based on the local linearisation techniques, parabolic methods for the two dimensional flows are described in Ref. [30].

G. Results and Discussions Based on the Integral Equation Method

Two-Dimensional Flows. Figures 3.8 and 3.9 show the pressure coefficient on the surface of the airfoil for subcritical flows calculated by assuming the exponential decay function for the u velocity in the flow field [34] and by solving the integral equation by Newton's iteration method, and for comparison purposes, the exact values are also plotted. The prediction by the integral equation method compares well with the exact values [34, 37]. Figures 3.10 and 3.11 show the pressure coefficient on the surface of the airfoil for supercritical flows, calculated by assuming the exponential decay function for the u velocity component in the flow field and by solving the integral equation by the method of parametric differentiation, and it is compared with the method of finite difference algorithms [38]. The comparison between the two theories is fairly good. Figure 3.12 shows the discontinuous flow past a parabolic arc airfoil calculated by the integral equation method by assuming the exponential decay for the u component of velocity and by applying the method of steepest descent for solving the integral equation. The pressure distribution thus obtained is compared with the numerical solution of Reference [39] and Reference [40], and the comparison is fairly good. Figure 3.13 shows the discontinuous flow past NACA 0012

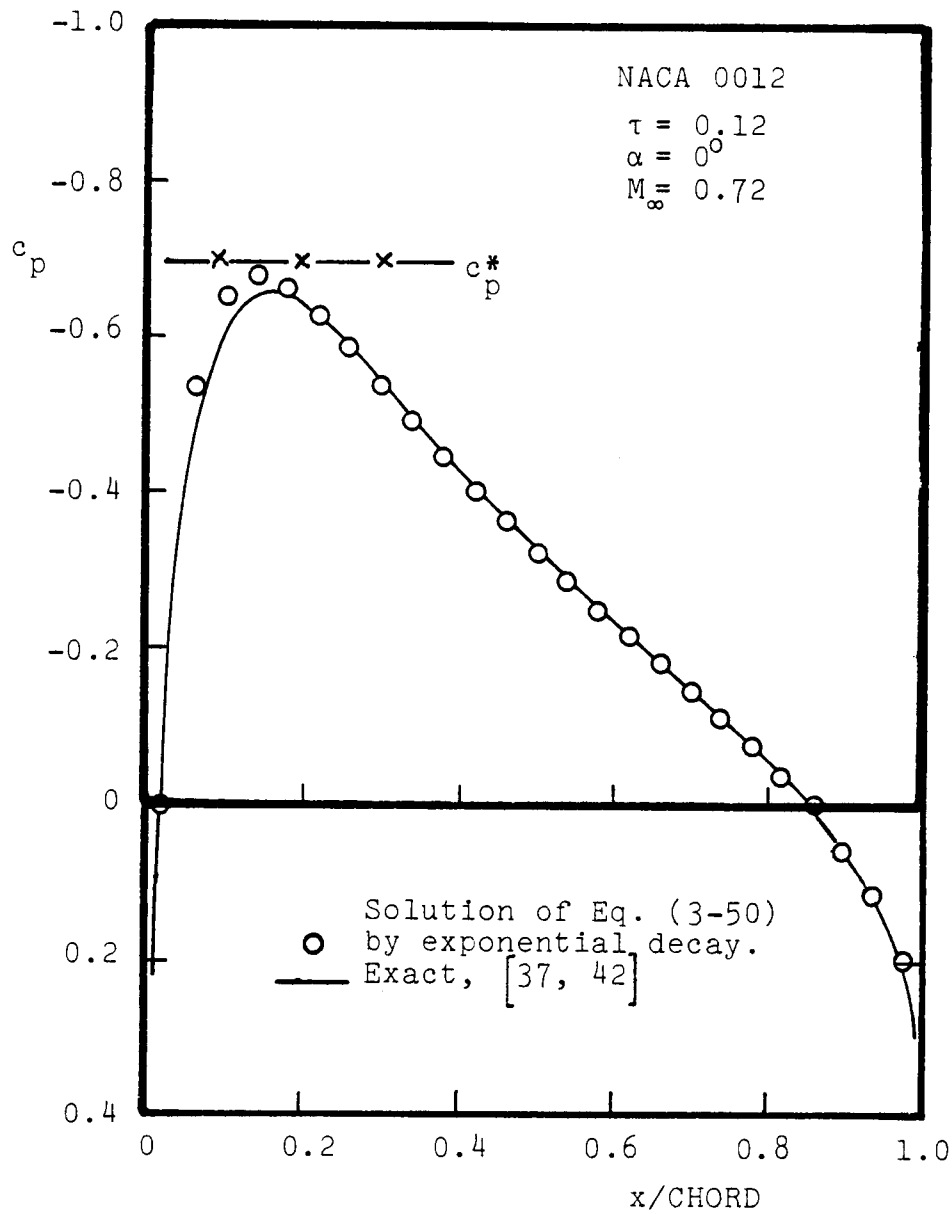


Figure 3.8. Subcritical Flow Past a Non-Lifting Airfoil, by Newton's Iteration Scheme, Source [34].

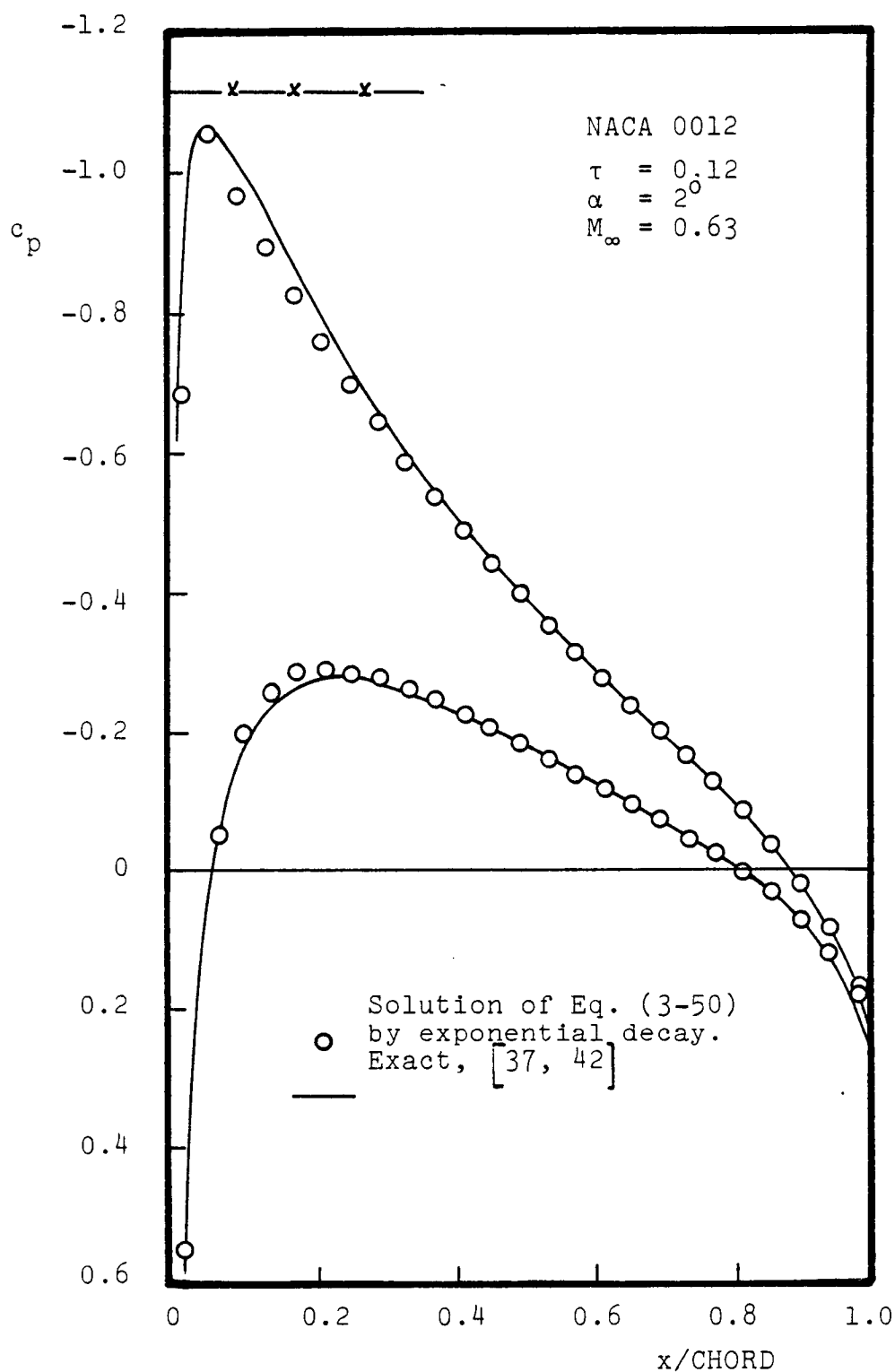


Figure 3.9. Subcritical Flow Past a Lifting Airfoil, by Newton's Iteration Scheme, [34].

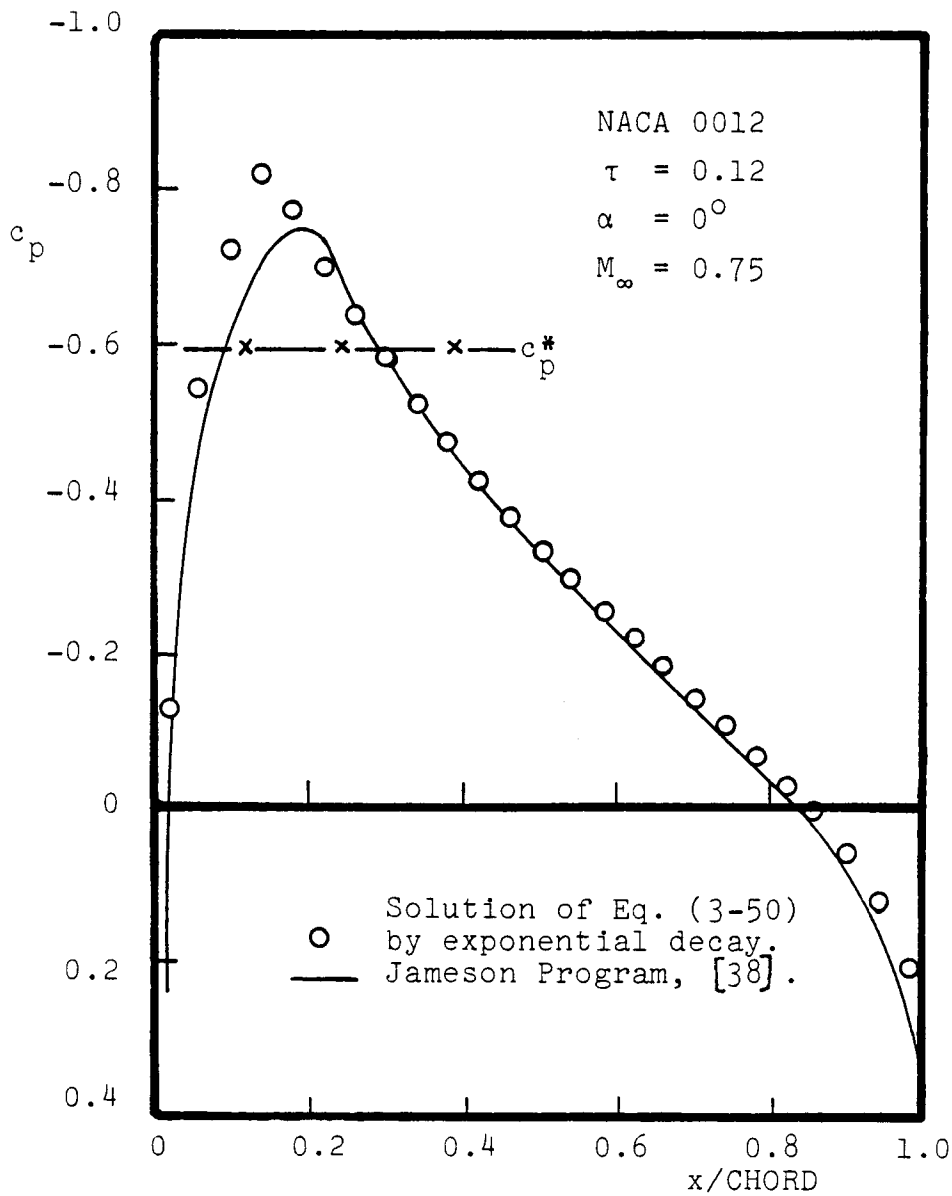


Figure 3.10. Supercritical Flow Past a Non-Lifting Airfoil, by Parametric Differentiation, Source [34].

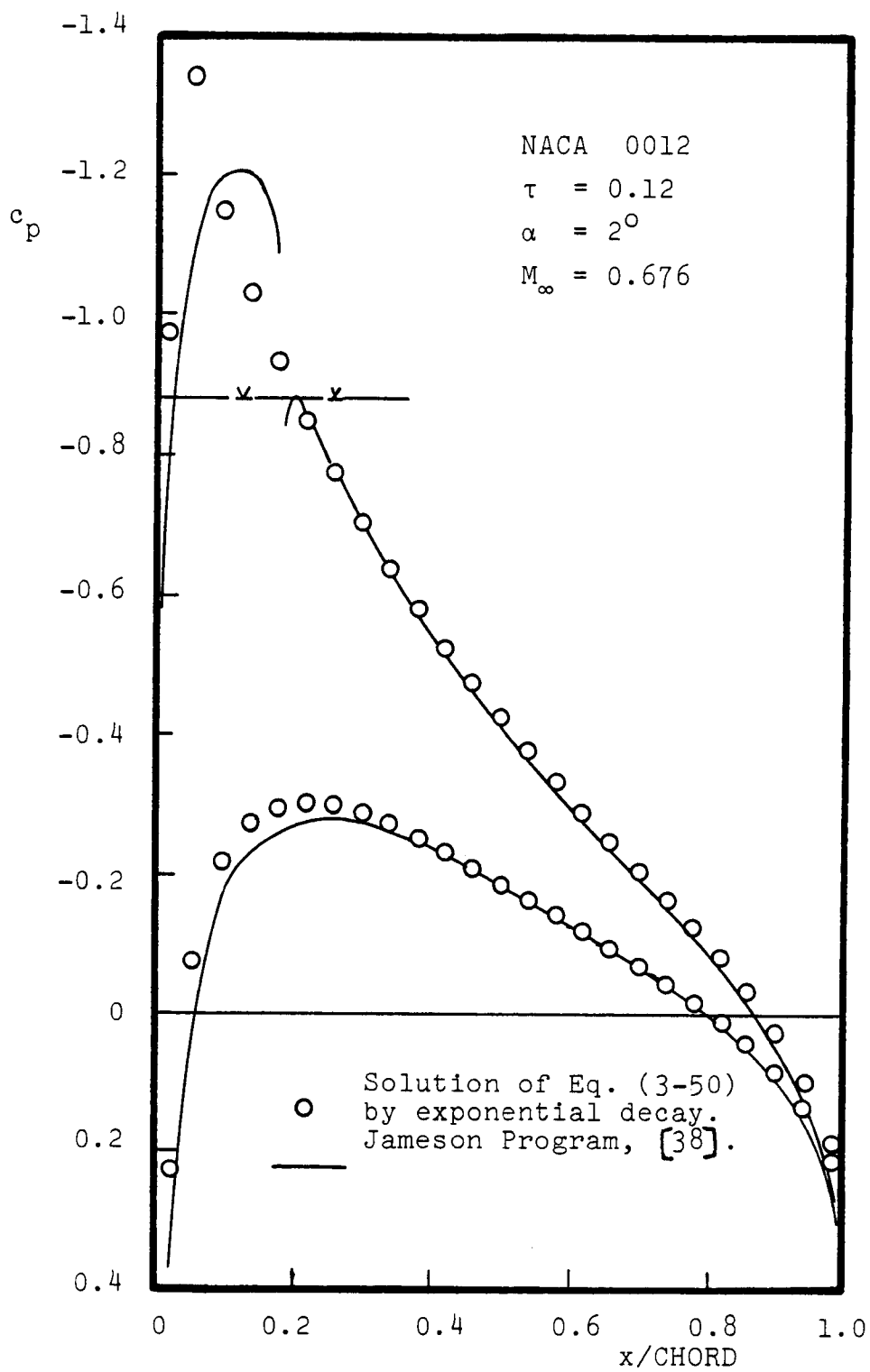


Figure 3.11. Supercritical Flow Past a Lifting Airfoil, by Parametric Differentiation, Source [34].

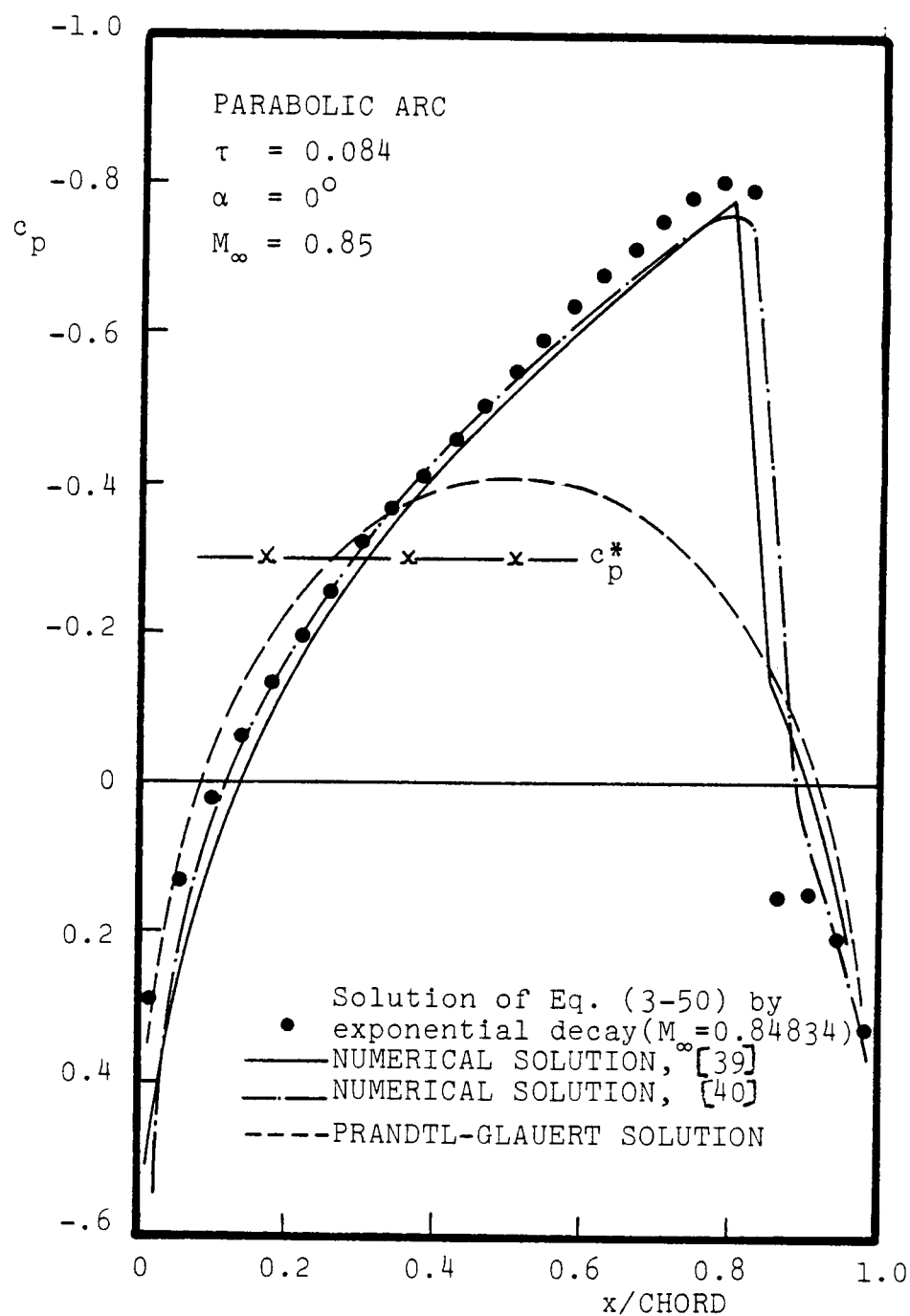


Figure 3.12. Discontinuous Flow Past a Non-Lifting Airfoil by Method of Steepest Descent, Source [34].

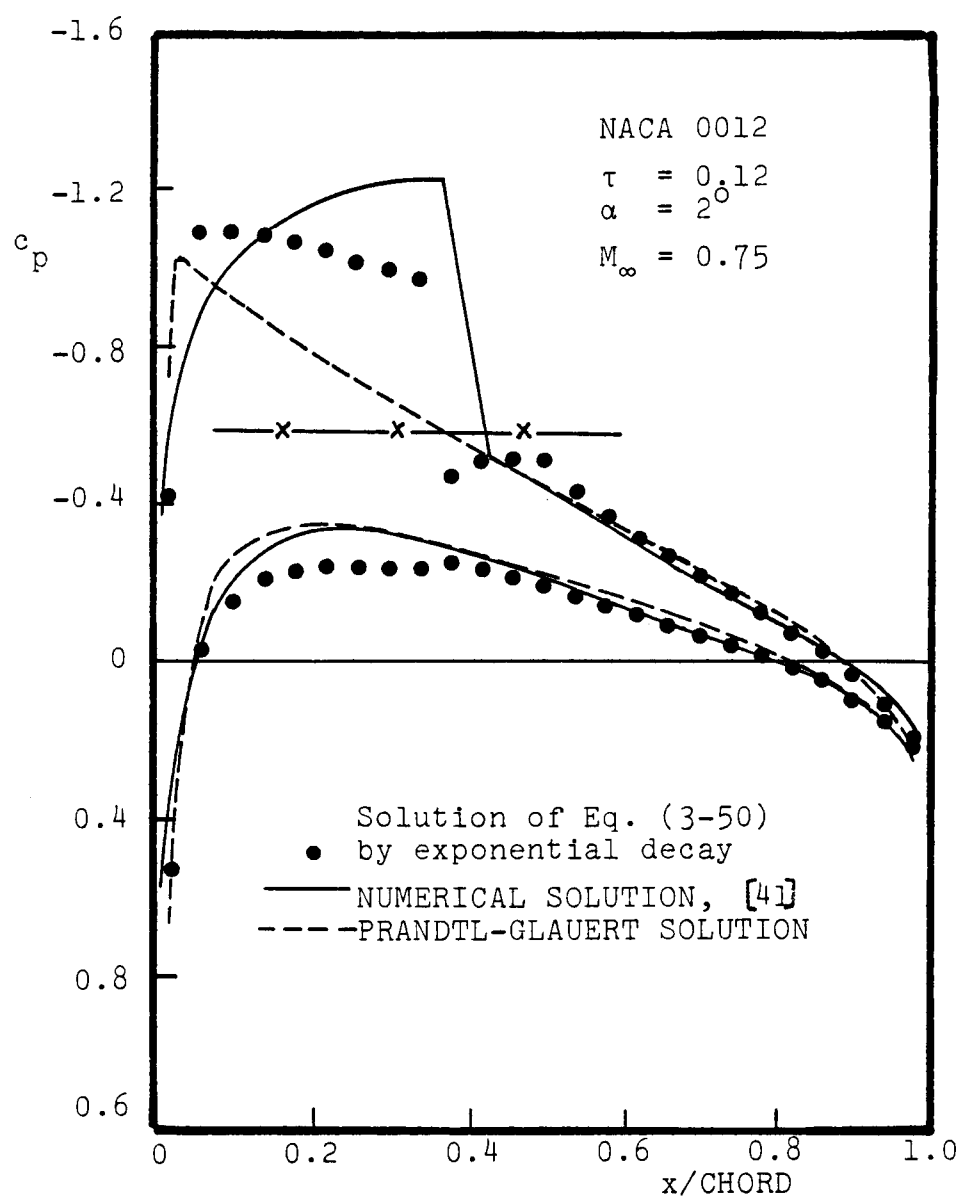


Figure 3.13. Discontinuous Flow Past a Lifting Airfoil by Method of Steepest Descent, Source [34].

airfoil obtained by solving the integral equation by assuming the exponential decay for the u component of velocity and solving the integral equation numerically by the method of steepest descent. The pressure distribution thus obtained is compared with the method indicated in Reference [41] and the comparison is fairly good.

Three-Dimensional Flows. A lifting wing at 2 degrees incidence, as shown in Figure 2.2c, was selected for the application of the three-dimensional integral equation method described earlier.

The linear velocity distribution for the wing, shown in Figure 2.2c, was calculated by the method indicated in Reference [12] for the free-stream Mach number 0.92 and 2 degree angle-of-attack. Goethert's transformation is made use of in running the Hess and Smith program for the calculation of the compressible subsonic linear velocity distribution. The compressible subsonic linear velocity distribution together with the geometry of the wing and free-stream conditions form the input to the nonlinear correction program which solves the transonic small perturbation equation by the integral equation method described earlier in this chapter, Reference [34], and the results obtained from the program are plotted and compared with experimental results and are shown in Figure 3.14. The comparison is fairly good. Since the experimental values are available only for the lower surface of the wing, only the c_p values on the lower surface are plotted and compared.

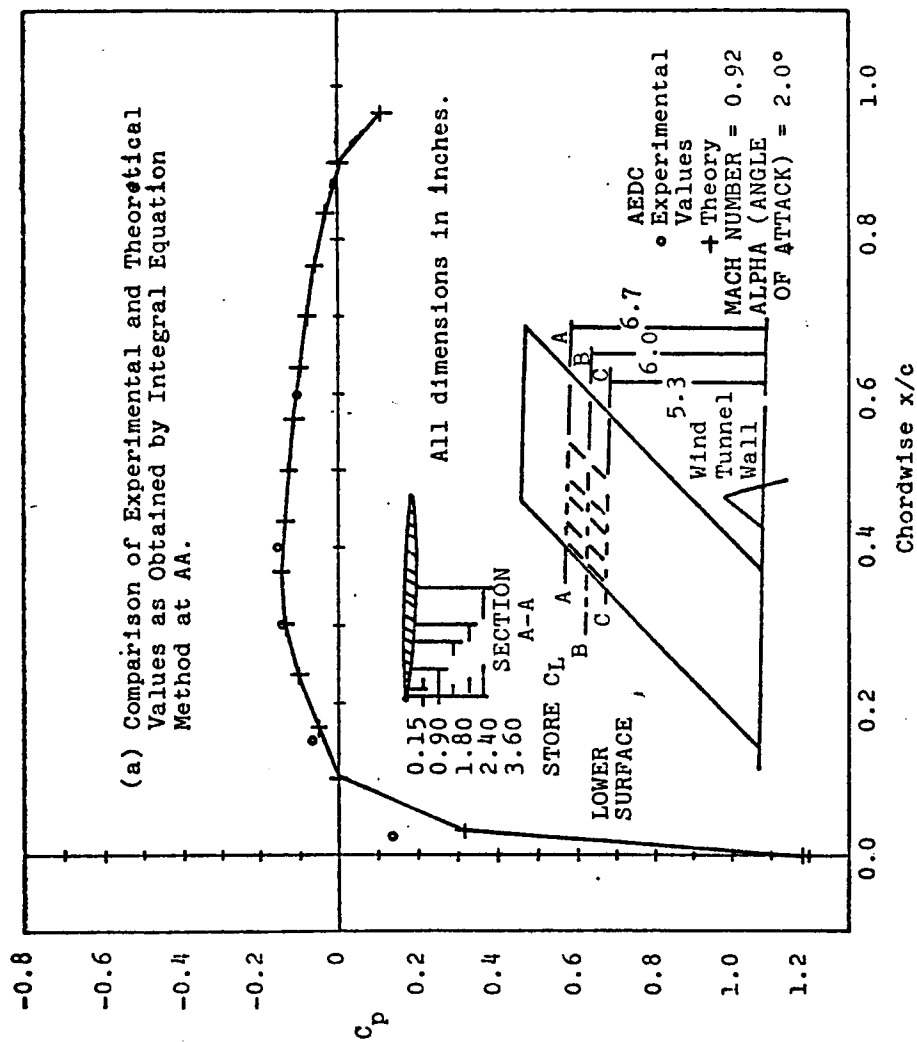


Figure 3.14. Comparison of Theory with Experiments for NACA 0005-34 Wing Alone.

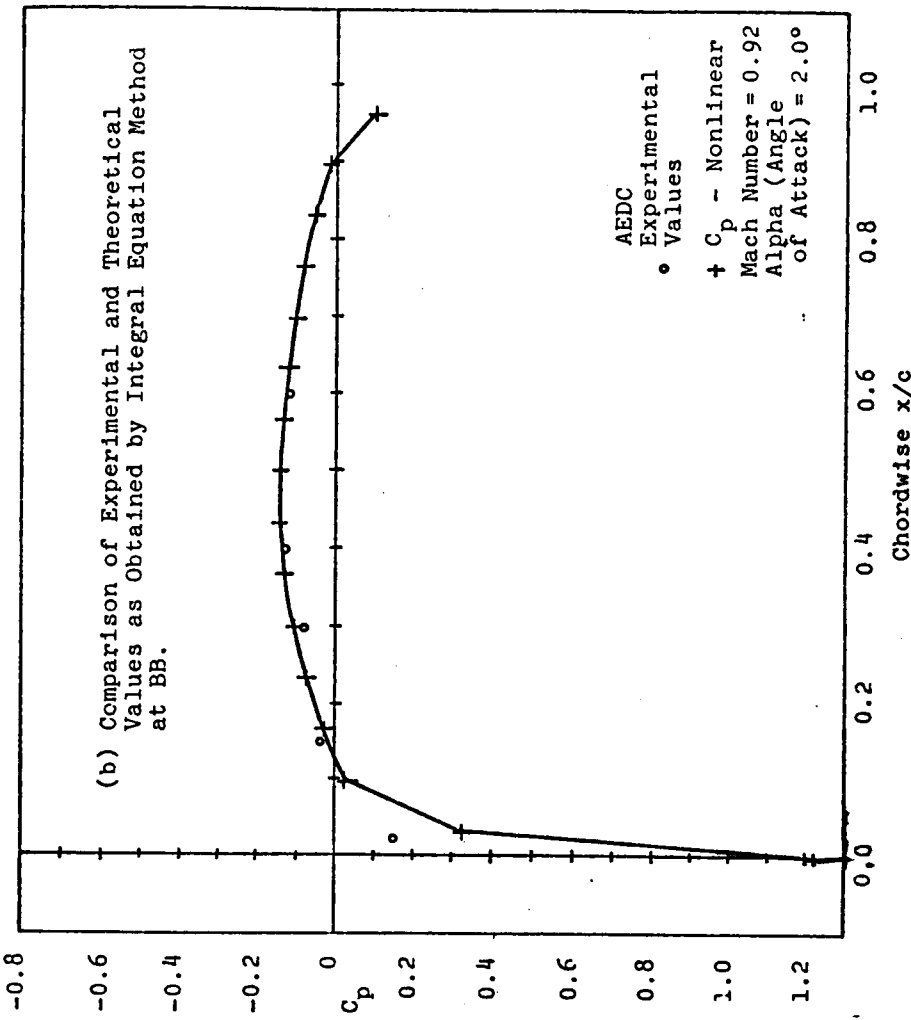


Figure 3.14. (Continued)

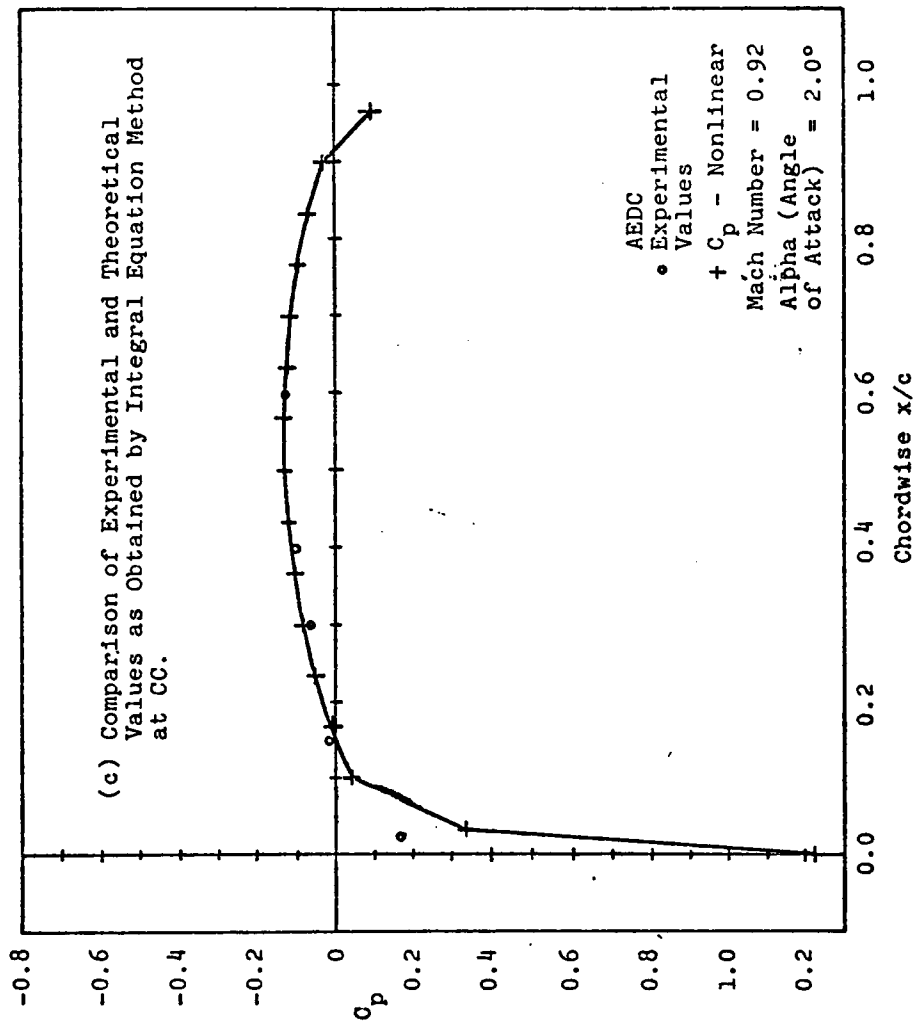


Figure 3.14. (Continued)

The same wing as shown in Figure 2.2c was analyzed by the finite difference algorithm [21] and the pressure distribution thus obtained is compared with experimental results and are shown in Figure 3.15.

The use of an integral equation rather than a differential equation has some advantages in the numerical analysis. Since integration is essentially a smoothing operation, the effect of small errors in evaluating an integral is slight compared to the effect of small errors in a finite difference representation of a differential equation, thus giving greater numerical stability [43,44]. In practice, this means that integral equations converge more rapidly than differential equations. There are, therefore, certain advantages of the integral equation method over finite difference methods. The boundary conditions can be easily and explicitly treated and true, discontinuous solutions are easily obtained in contrast to the finite difference representation of discontinuities in the finite difference methods. It is indicated in the integral equation that only the flow close to airfoil surface needs to be calculated accurately. In addition, the numerical procedures are, in principle, more stable than those for finite difference methods, which leads to more rapid convergence. Because of these advantages, it is anticipated that the computing time for the integral equation method will be lower than that for the existing

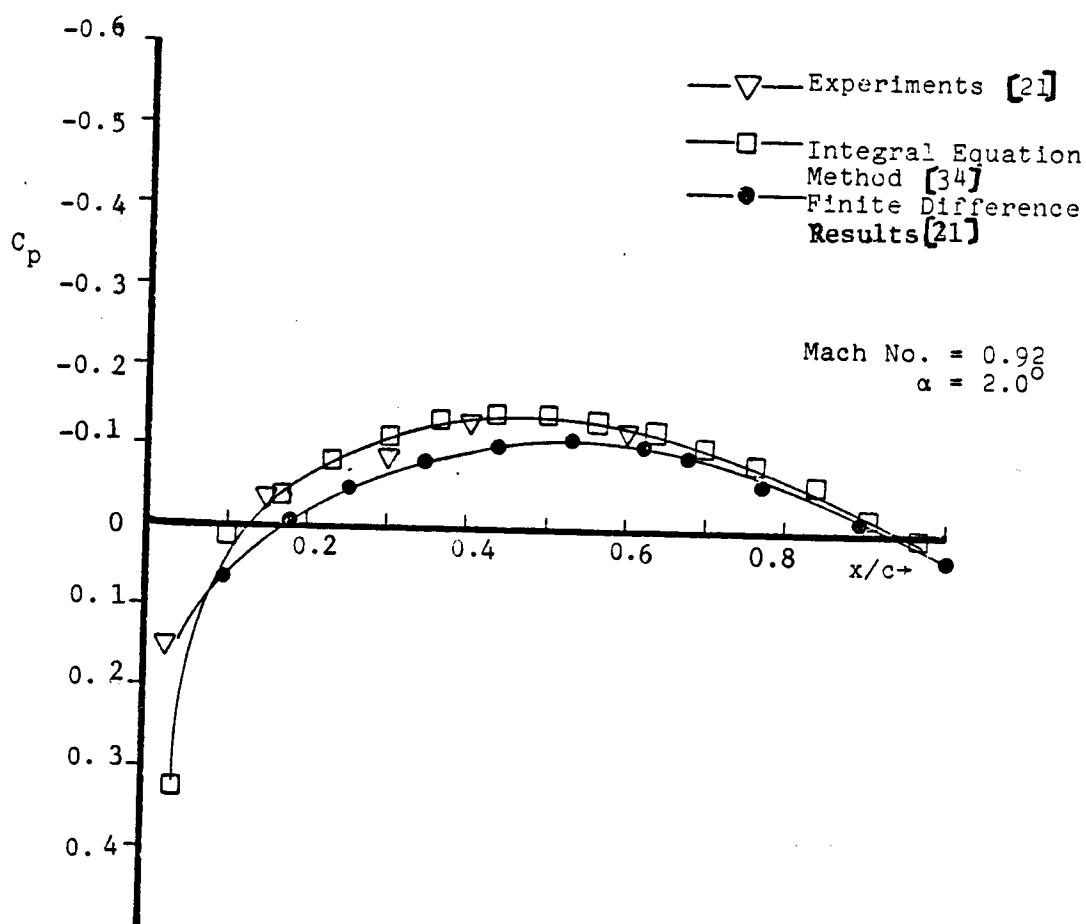


Figure 3.15. Finite Difference Calculation for the NACA 0005-34 Wing at the Center Section. (Refer to Figure 2.2c)

finite difference methods. Table 3.1 has been prepared to highlight these advantages and disadvantages of the integral equation method and the finite difference method.

TABLE 3.1

TRANSONIC FLOW PAST WINGS AND BODIES

THE INTEGRAL EQUATION APPROACH	FINITE DIFFERENCE EQUATION APPROACH
1. The governing equation considered is nonlinear poisson type, derived from Green's theorem. 2. Approximate method. 3. Calculation in the physical plane. 4. The linear velocity distribution on the wing surface is the input, along with the curvature distribution and geometry of the wing, with free stream flow conditions. 5. Viscous effect can be included in the linear velocity distribution, by semi-empirical means. 6. Offers quick and accurate results, sufficient for engineering calculation.	1. The governing equation is the small-disturbance equation. 2. Approximate method, in the sense of the governing equation, is satisfied only at discrete points due to mesh size limitation. 3. Calculation in the physical plane. 4. The potential ϕ in the flow field is the input, along with the geometry of the wing, with free stream flow conditions. 5. No unique way to include viscous effect, i.e., Navier-Stokes equations also have to be solved. 6. The guarantee of the final solution convergence depends on the stability problems in the numerical procedure. Whether the solution will converge or not depends on the free stream Mach number, the thickness-to-chord ratio of the wing section, and mainly on the assumption of the small disturbance approach and on the mesh size.

TABLE 3.1 (Continued)

THE INTEGRAL EQUATION APPROACH	FINITE DIFFERENCE EQUATION APPROACH
7. No coordinate transformation is needed in formulating the problem.	7. Coordinate transformation in differencing the governing equation, and in the case of an interference problem, the geometric representation of the interfering bodies pose great difficulties.
8. Takes less computer time and computer memory storage.	8. Three-dimensional finite difference calculations of transonic flow fields are not economically possible within the present state-of-the-art due to excessive amount of storage and execution time.
9. Can handle shock, by the iterative technique.	9. Can handle shock.
10. Can be extended for the flow field calculations without much effort.	10. Can be extended for the flow field calculation with little more computer storage in order to accommodate larger number of grid points.

CHAPTER IV

CALCULATION OF INTERFERENCE EFFECTS
BY APPROXIMATE METHODSCalculation Of Approximate Types
Of Superposition

In the exact solution of interference problems, the flow equations and the boundary conditions on all of the individual bodies should be satisfied simultaneously. This is a formidable problem for three-dimensional flows, especially with lift, even in the case of an incompressible flow. The method outlined in Reference [12] has this capability of calculating the incompressible flow past multibodies, satisfying the boundary conditions on all of the body surfaces simultaneously. There are some approximate methods available to calculate the interference effects in the case of multibody problem, and in this context the term superposition is used to denote the approximate calculation of the flow about two or more bodies by combining the individual flows about the isolated bodies [45]. The combination may be done in a number of ways. The different ways in which solutions may be combined to account for the effects of the bodies on each other are to different orders of accuracy. In interference applications, the problem may be

stated as that of calculating the flow about a body in the presence of another body. The several types of superposition are all illustrated in Figure 4.1.

Figure 4.1a shows a typical problem. For definiteness, we assume that it is required to calculate the tangential velocity on the surface of the smaller body in the presence of the bigger body. The two bodies may be two-dimensional, axisymmetric, or three-dimensional. For lifting cases the two overall circulations are shown. If the calculation is performed with both bodies present, the result is the exact surface velocity $\vec{V}_T(\text{exact})$.

Figure 4.1b shows the individual flow field of the bigger body at points on the smaller body (without its presence). These are indicated $\vec{V}_s$. The velocity $\vec{V}_0$ is the value at the centroid of the smaller body.

Figure 4.1c shows the velocity distribution on the smaller body isolated from the bigger body when it is immersed in the flow $\vec{V}_\infty$ at angle of attack α . The tangential velocity has been labeled $\vec{V}_T(\text{isolated})$. The difference between $\vec{V}_T(\text{exact})$ and $\vec{V}_T(\text{isolated})$ is the interference effect (i.e., $\vec{V}_{\text{interference}} = \vec{V}_{\text{exact}} - \vec{V}_{\text{isolated}}$). The effectiveness of a method of calculating interference is to be judged by how well it predicts this difference. Thus, the velocity distribution $\vec{V}_T(\text{isolated})$ is a sort of zeroth order solution.

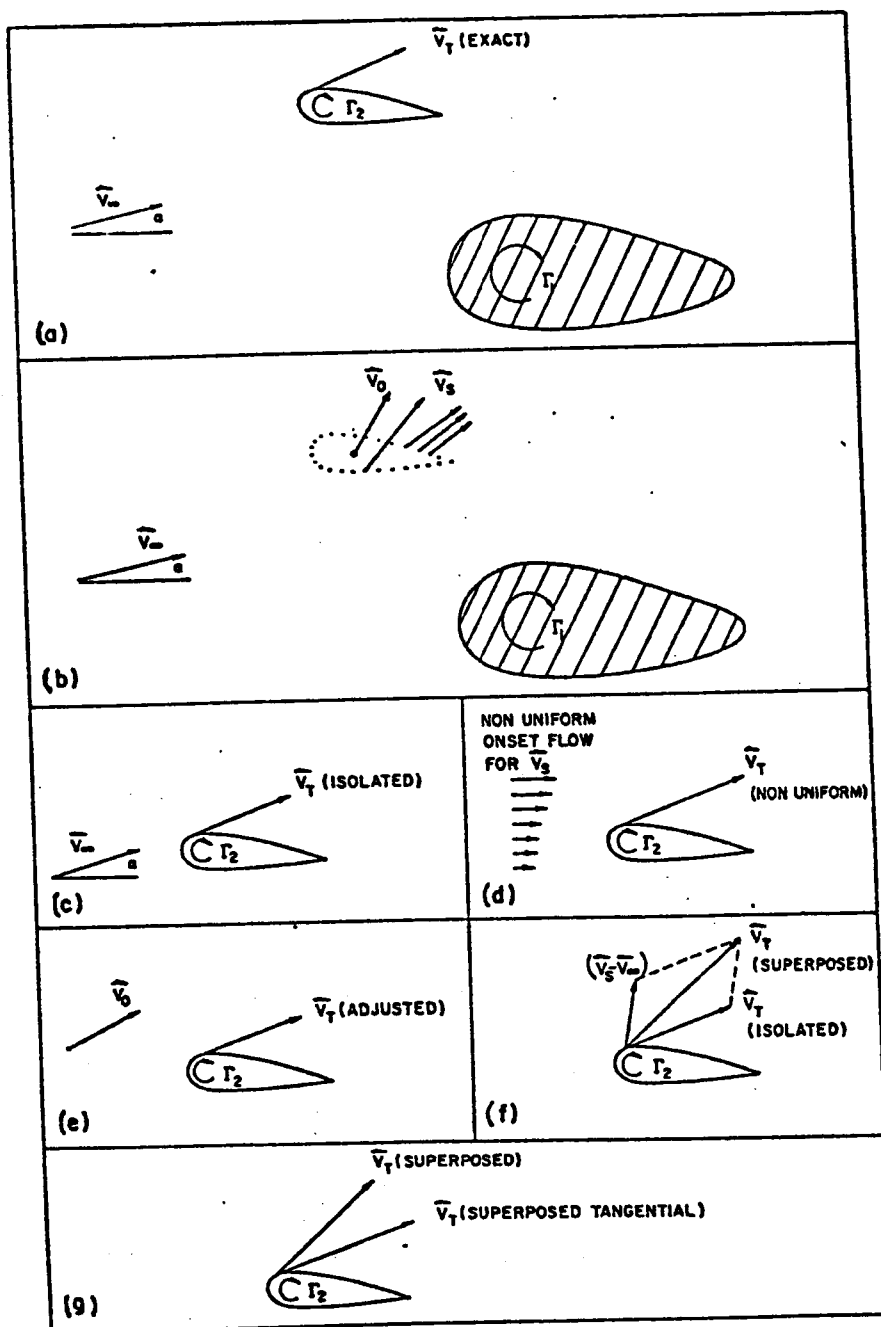


Figure 4.1. Types of Superposition.

Figure 4.1d illustrates a rather accurate type of superposition consisting of taking the velocity distribution $\vec{V}_s$ of Figure 4.1b, which is nonuniform, in place of the uniform initial free-stream velocity $\vec{V}_\infty$. This procedure, of course, neglects the effect of the smaller body on the larger, but this effect enters only as it in turn affects the flow field of the larger body in the vicinity of the smaller body and is thus rather weak. The surface velocity distribution is denoted as $\vec{V}_{T(\text{nonuniform})}$. The calculation of $\vec{V}_{T(\text{nonuniform})}$ in three-dimensional cases can be quite involved if $\vec{V}_s$ is left completely arbitrary. The computing time by numerical methods can be very large.

Figure 4.1e shows a very simple type of superposition. Instead of the initial uniform free-stream velocity $\vec{V}_\infty$, the smaller body in a uniform flow given by $\vec{V}_0$ (Figure 4.1b) is considered. Thus, the magnitude and angle of attack on the isolated smaller body is now different from Figure 4.1a. Typically, the surface velocities in this case are obtained by "scaling-up" those due to a unit free stream; thus denoting this result by $\vec{V}_T(\text{adjusted})$.

In the simplified superposition scheme of Figure 4.1f the mutual interference of the two bodies is neglected and one simply adds velocities due to the individual bodies. Adding vectorally the perturbation velocity $(\vec{V}_s - \vec{V}_\infty)$ due to the larger body and $\vec{V}_T(\text{isolated})$ results in the velocity

distribution $\vec{V}_T$ (superimposed), which, in general, will not be tangential to the smaller body.

Figure 4.1g illustrates that to improve the accuracy of the case shown in Figure 4.1f, one takes the tangential component of $\vec{V}_T$ (superimposed, thus obtaining $\vec{V}_T$ (superposed tangential).

Thus there are four types of superposition to compare with the exact solution of Figure 4.1a and the "zero interference" solution of Figure 4.1e.

Comparison Of Various Types Of Superposition For The Case About Two Circular Cylinders

The radius of the "larger" cylinder is taken as unity. Parameters of the problem are as follows:

1. Radius of the "smaller" cylinder on which the surface velocities are acquired.
2. Separation distance of the two cylinders.
3. Angle of attack α .
4. Two circulations about the cylinders.

Exact and superposition solutions have been obtained for a large number of values of these parameters. The conclusions, based on this example, are as follows:

1. The "superposed" solution of Figure 4.1f is highly unsatisfactory and certainly inferior to the "superposed tangential" solution of

Figure 4.1g. Since the latter is as easy to calculate as the former, the "superposed" solution is rejected.

2. The "adjusted" solution of Figure 4.1e is always more accurate than the superposed tangential" solution of Figure 4.1g.
3. While the accuracy of superposition solutions increases with increasing distance between the cylinders, and with decreasing size of the "smaller" cylinder, the distance effect is much stronger than the size effect. This is clearly indicated in Figure 4.2.
4. The accuracy of the "adjusted" type of superposition is satisfactory if the surfaces of the cylinders are separated by at least one diameter of the larger cylinder.
5. The "nonuniform" type of superposition, Figure 4.1d, is extremely accurate, even when the two bodies are very close together. This is illustrated in Figure 4.3 for the two cylinders.

Superposition Techniques For Wing-Body Combination

The most interesting applications of superposition techniques are three-dimensional, because it is for such cases that computing times and accuracies are most critical.

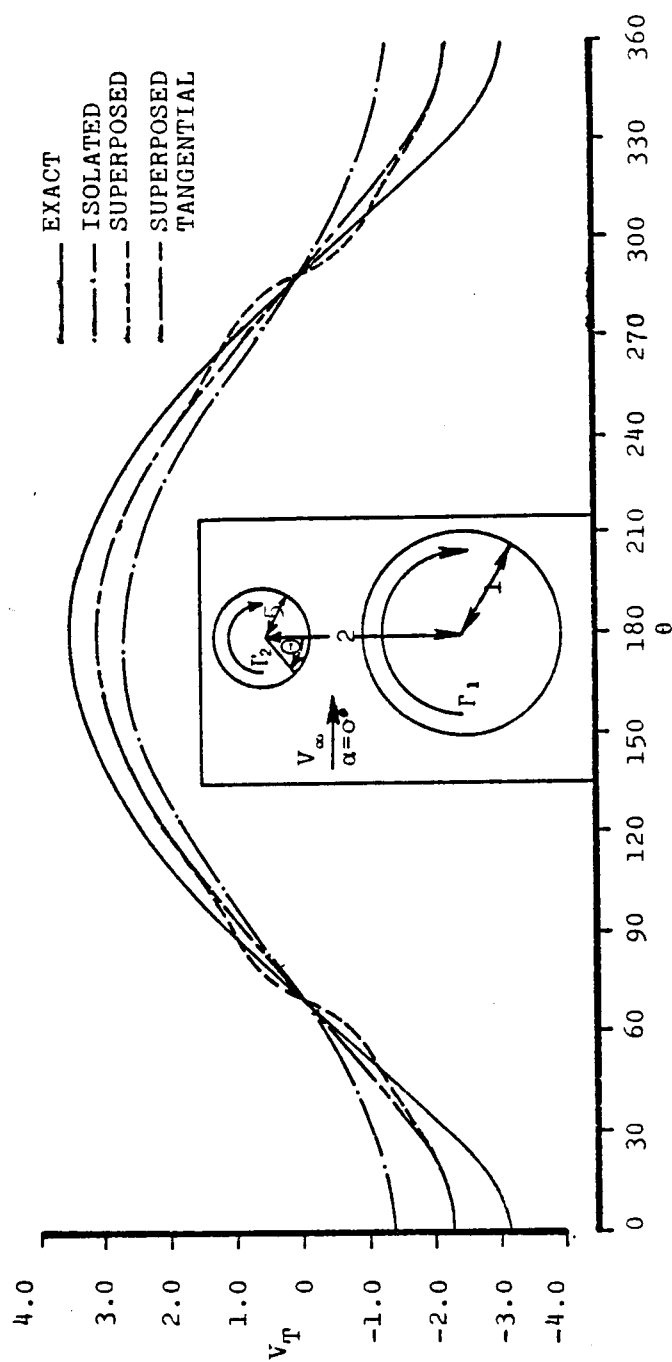


Figure 4.2. Comparison of "Superposed" and "Superposed Tangential" Types of Superposition for a Case of Two Circular Cylinders.

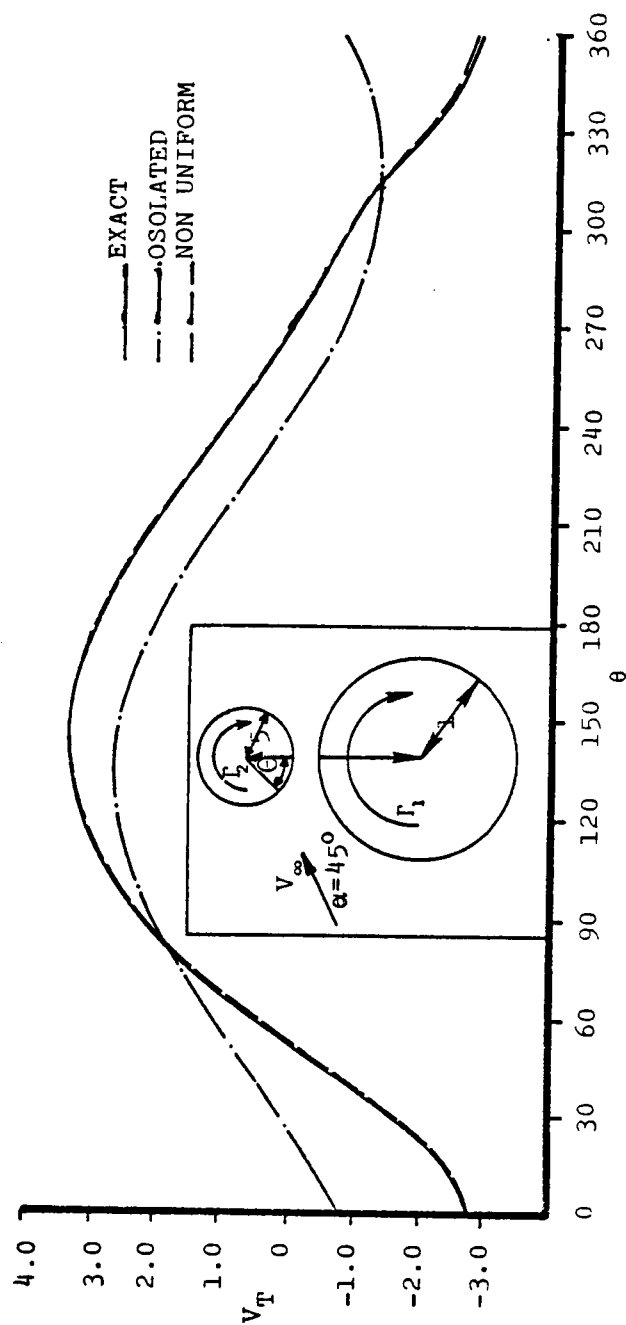


Figure 4.3. "Nonuniform" Superposition for a Case of Two Circular Cylinders.

Often the flow about a complicated configuration can be approximated by superposing flows about two or more simple, three-dimensional bodies. The result is considerable savings in computing time. However, the greatest savings result when a three-dimensional flow can be approximated by superposing two-dimensional and axisymmetric flows. For example, the flow on the wing in the presence of a fuselage can be approximated by superposing the axisymmetric flow about the fuselage and the two-dimensional flow about the airfoil section from which the wing is derived. The curves on the wing surface along which superposition must be carried out are obtained by cutting the wing by planes perpendicular to its sweep-line, and not by planes parallel to the free-stream direction.

The general conclusions are that superposition in three-dimensional cases require care, because there is often more than one way to perform the superposition. It is, however, felt that superposition should be valid over greater ranges in three dimensions than it is in two dimensions, because, in general, disturbance velocities decrease faster with distance from the disturbing body.

General Method Of Predicting The Forces And Moments
On The Store Mounted On Aircraft
In Subsonic Flow By Superposition

The design of external stores for aircraft requires a knowledge of the aerodynamic forces acting on the store. This information is usually obtained in wind tunnel tests. However, when a number of combinations of stores are considered, it is unfeasible to obtain this type of design data by testing. The availability of the high-speed computers has opened the possibility of applying aerodynamic theory to the calculation of forces on stores under interference conditions. If it can be shown that the predictions of the theory are fairly accurate, then a computer program will be a useful tool for preliminary design and for the evaluation of design changes [46].

The flow field data, assumed previously calculated, and hence known, are in the form of perturbation velocity components (u_A, v_A, w_A) in an airplane coordinate system (x_A, y_A, z_A) which is fixed to the airplane and pitches with it as the angle of attack is varied (Figure 4.4). The data may be calculated either at a set of points along a specified store axis, or at the points of a three-dimensional grid covering the space of interest, to be interpolated for the perturbation velocities at any desired point. In subsonic and supersonic flow regimes where linear theory can be

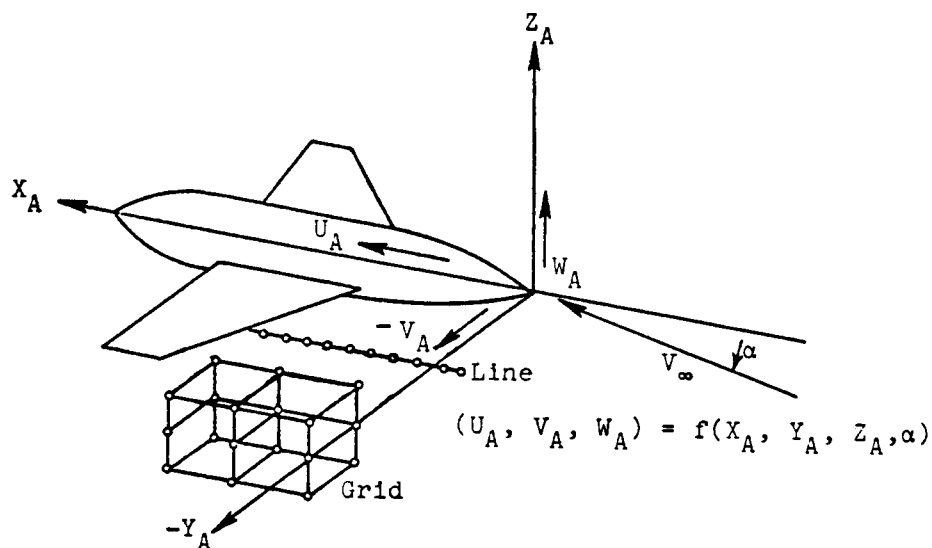


Figure 4.4a. Flow Field Coordinate System.

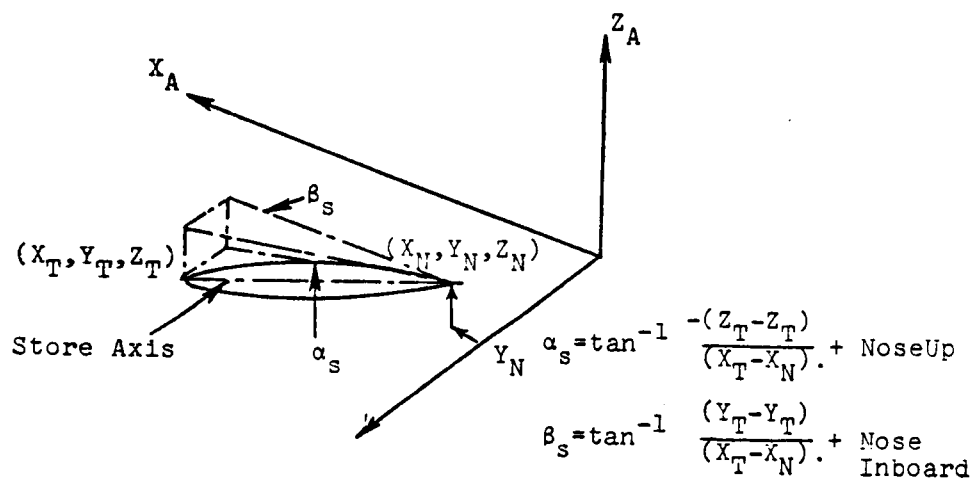


Figure 4.4b. Store Location in Flow Field.

Figure 4.4. Coordinates and Location for the Aircraft and Store Configuration.

used, it is sufficient to calculate (u_A, v_A, w_A) for two angles of attack and interpolate linearly for other angles of attack. In the transonic regime it will be necessary to have at least four sets of data. The store is assumed to be a symmetrical body of revolution and, no provision is made for fins, pylons, or racks. The store is oriented at a nose coordinate location (x_N, y_N, z_N) in the airplane coordinate system and at pitch angle α_s . There is no yaw angle. The store axis system is (x, y, z) .

The store angle of attack is

$$\alpha' = \alpha + \alpha_s, \quad (4-1)$$

The store is located in the nonuniform flow field produced by the aircraft. At each point along the store axis, the perturbation velocities u_A, v_A, w_A are known. The three components of the perturbation velocities along the store axis are given by

$$u_s = u_A \cos \alpha_s - w_A \sin \alpha_s \quad (4-2a)$$

$$v_s = v_A \quad (4-2b)$$

$$w_s = u_A \sin \alpha_s + w_A \cos \alpha_s \quad (4-2c)$$

The free-stream velocity vector resolved into the store axes is given by

$$\vec{V} = U_{\infty} [\cos \alpha' \hat{i} + \sin \alpha' \hat{k}] , \quad (4-3)$$

and the velocity potential is given by

$$\Phi(x,y,z) = U_{\infty} [x \cos \alpha' + z \sin \alpha' + \phi(x,y,z)] , \quad (4-4)$$

where $\phi(x,y,z)$ is the perturbation velocity potential for the body of revolution (store) at the given angle of attack, strictly in the nonuniform flow, but in practice the uniform flow solution is taken.

Introducing the cylindrical coordinates for convenience: x, r, θ , where

$$r = \sqrt{y^2 + z^2} ; \quad \theta = \tan^{-1} \frac{y}{z}$$

$$\Phi(x,r,\theta) = U_{\infty} [x \cos \alpha' + r \sin \alpha' \cos \theta + \phi(x,r,\theta)] . \quad (4-5)$$

The axial, radial and tangential velocity components are given by equations (4-6a), (4-6b), (4-6c).

$$\left. \begin{aligned} u &= \frac{\phi_x}{U_\infty} = \cos\alpha' + \phi_x & (4-6a) \\ v_r &= \frac{\phi_r}{U_\infty} = \sin\alpha' \cos\theta + \phi_r & (4-6b) \\ v_\theta &= \frac{\phi_\theta}{rU_\infty} = -\sin\alpha' \sin\theta + \frac{\phi_\theta}{r} & (4-6c) \end{aligned} \right\}.$$

The tangential flow boundary condition on the store is

$$v_r + v_s \sin\theta + w_s \cos\theta = \frac{dR_B(x, \theta)}{dx} (u + u_s) . \quad (4-7)$$

Here the perturbation velocity components in the field u_s, v_s, w_s have been added to the radial and axial components obtained from potential flow solution.

The radius distribution of the store is given by

$$R_B(x, \theta) = R(x) \quad ; \quad \frac{dR_B}{dx} = \frac{dR}{dx} .$$

Substituting Equations (4-6a), (4-6b), and (4-6c) into Equation (4-7), one obtains an expression for the exact boundary condition:

$$\begin{aligned} \phi_r + \sin\alpha' \cos\theta + v_s \sin\theta + w_s \cos\theta \\ = \frac{dR}{dx} (\cos\alpha' + u_s + \phi_s) . \end{aligned} \quad (4-8)$$

Calculation of the Surface Velocities and Pressures

Adding the external field to Equation (4.6), one obtains an expression for the surface velocity components:

$$u_{\text{sur}} = u_s + \cos\alpha' + \phi_x \quad (4-9a)$$

$$\begin{aligned} v_{\text{sur}} &= v_s + v_r \sin\theta + v_\theta \cos\theta \\ &= v_s + \phi_r \sin\theta + \frac{\phi_\theta}{r} \cos\theta \end{aligned} \quad (4-9b)$$

$$\begin{aligned} w_{\text{sur}} &= w_s + v_r \cos\theta - v_\theta \sin\theta \\ &= w_s + \sin\alpha' + \phi_r \cos\theta - \frac{\phi_\theta}{r} \sin\theta \end{aligned} \quad (4-9c)$$

The pressure coefficient is calculated using the exact isentropic formula,

$$c_p = \frac{2}{\gamma M_\infty^2} \left[1 + \frac{\gamma-1}{2} M_\infty^2 (1 - \bar{q}^2) \right]^{\frac{\gamma}{\gamma-1}} - 1, \quad (4-10)$$

where,

$$\bar{q}^2 = u_{\text{sur}}^2 + v_{\text{sur}}^2 + w_{\text{sur}}^2. \quad (4-11)$$

Calculation of Forces and Moments

The force and moment coefficients are obtained by integrating the C_p over the surface of the body. In the body axis system,

$$C_x = \frac{1}{S} \int_0^{\ell} R(x) \frac{dR(x)}{dx} C(x) dx , \quad (4-12)$$

$$C_y = - \frac{1}{S} \int_0^{\ell} R(x) B(x) dx , \quad (4-13)$$

$$C_z = - \frac{1}{S} \int_0^{\ell} R(x) A(x) dx$$

$$C_{M_x} = 0 , \quad (4-14)$$

$$C_{M_y} = \frac{1}{S\ell} \int_0^{\ell} R(x) \left[\left| x - x_M + R(x) \frac{dR(x)}{dx} \right| A(x) \right] dx , \quad (4-15)$$

$$C_{M_z} = - \frac{1}{S\ell} \int_0^{\ell} R(x) \left[\left| x - x_M + R(x) \frac{dR(x)}{dx} \right| B(x) dx \right] , \quad (4-16)$$

where,

S = reference area

ℓ = store length

x_M = axial coordinate of the moment reference point,

and,

$$\left. \begin{aligned} A(x) &= \int_0^{2\pi} C_p(x, \theta) \cos \theta \, d\theta \\ B(x) &= \int_0^{2\pi} C_p(x, \theta) \sin \theta \, d\theta \\ C(x) &= \int_0^{2\pi} C_p(x, \theta) \, d\theta \end{aligned} \right\} . \quad (4-17)$$

Resolving the forces into the wind axes system, one obtains

$$\left. \begin{aligned} C_L &= C_Z \cos \alpha' - C_X \sin \alpha' \\ C_Y &= C_Y \\ C_D &= C_X \cos \alpha' + C_Z \sin \alpha' \end{aligned} \right\} . \quad (4-18)$$

In the case of transonic flow, the difficulties of determining the nonlinear flow field velocities past the aircraft configuration and solving the governing nonlinear equation for a three-dimensional body with the boundary condition of Equation (4-8) restrict the use of the above method. The interference problem in transonic flow has been tackled by the method to be discussed below.

The Multi-body Problem In Transonic Flow

By The Present Method

The multibody problem in transonic flow is an important task since store separation from high performance fighter aircraft normally occurs in this regime. The problems inherent in this area are enormous, such as the mutual interference between the stores and the interference between the stores and the parent aircraft, the shockwave formation and its reflection between the adjacent stores and with the parent aircraft, its effect on the local aerodynamic characteristics, and the shockwave-boundary layer interactions.

In subsonic flow, the theoretical methods so far developed [6,8,9] treat this problem by superposition theory, which is valid up to the critical Mach number. However, in transonic flow due to the effect of nonlinearity, the solutions can no longer be superposed as in subsonic/supersonic theories. The only way to treat the nonlinear problem is by either finite difference solution [47] or recourse to approximate solutions such as local linearization method [48], parabolic method [50], or nonlinear correction theories [51]. The finite difference solutions are not economically feasible within the present state-of-the-art. The only way to tackle the nonlinear solution is the correction method upon the linear theory values.

The formulation for the store force predictions in transonic flow within the scope of this work is as follows:

1. First, the subsonic linear three-dimensional solution for wing/body/stores configuration by considering them from the outset as a complete entity, is determined from the theory outlined in Reference [12] and this gives the velocity distribution on the wing surface, and on the external stores surface at selected points, and let this value at a particular point on the store be called "A."
2. Then calculate the subsonic linear three-dimensional solution for the same store alone in the same free-stream conditions by the method of Reference [12] and let this be called "B" at the same point for which the velocity "A" is calculated.
3. Finally, calculate the nonlinear three-dimensional solution for the same store alone in the same free-stream conditions, either by finite difference techniques, Reference [52], or by the approximate nonlinear correction theories, Reference [51], at the same selected point, and let this be "C."

Since the solutions A, B, and C individually satisfy the tangential boundary conditions at the same point on the store surface, the nonlinear solution for the configuration within the present state of this formulation is

$$A - B + C ,$$

where A is the subsonic linear solution at the point P on the store surface by considering the wing body and store configuration as a whole entity and satisfying the Prandtl-Glauert equation, and the tangential flow boundary condition on all the bodies simultaneously; B is the store-alone linear solution, which also satisfies the tangential boundary condition at the point P on the same store surface; and C is the store-alone nonlinear solution, which also satisfies the tangential boundary condition at the point P on the same store. The above formulation may be summarized as follows:

1. The linear theory solution for the wing, which includes all of the interference effects due to the body and stores, and which has to be corrected for the nonlinearity by the integral equation method, as discussed in Chapter III.
2. The store solution is already a nonlinear solution. Symbolically, this may be expressed as follows:

$$V_{3-D_{\text{nonlinear}}} = V_{3-D_{\text{linear}}} - V_{3-D_{\text{linear}}} + V_{3-D_{\text{nonlinear}}}$$

wing-body, store	store alone solution	store alone solution
↓		
wing solution corrected for nonlinearity by integral equation method		

In the same outline, the problem of multiple stores, pylon, and fuselage can be treated. The limitations of this formulation are:

1. All of the interference effects (mutual interference between each store, store and wing, wing and pylon, and pylon and stores) are accounted for only by the linear theory, which, in most cases, appears to be adequate.
2. Reflections of the shock wave between the adjacent stores and between the store and wing cannot be accounted for.
3. Finally, viscosity is not considered.

A similar approach using the transonic equivalence rule has been suggested by Stahara [49].

Results And Discussion

The above-mentioned method has been applied for a NACA 005-34 wing with an MK83 store configuration (shown in Figure 2.3c) for a free-stream Mach number of 0.92,

a wing angle of attack of 2 deg, and a store pitch angle of 0 deg [53]. From the store-alone nonlinear solution it is inferred that the local Mach number does not reach unity at all anywhere on the surface of the store. So, for store nonlinear solution, the method of Maeder and Thommen [50] has been used. Figure 4.5 shows the comparison of three theories, namely, References [50], [38], and [12]. Figure 4.6 shows the final nonlinear solution on the store with the presence of the wing, obtained by the above formulation, and it has been compared with the experimental results. The comparison seems to be moderately good. Figure 4.7 shows the the final nonlinear solution on the wing with the presence of the store obtained from the nonlinear correction upon the linear theory values from the integral equation method. Even though the above-mentioned formulation of solving the multibody problem is very approximate, it compares fairly well with the experimental results.

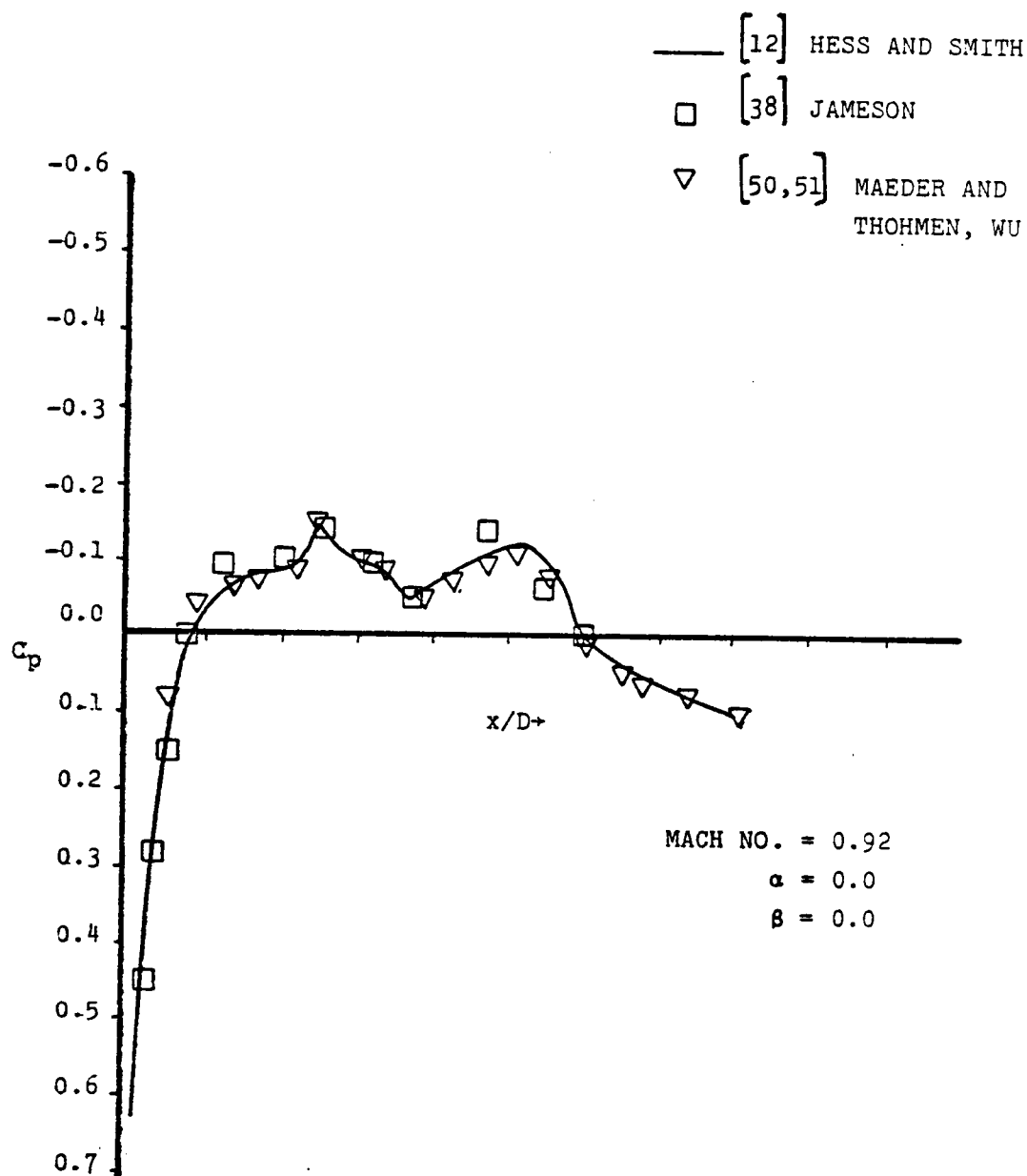
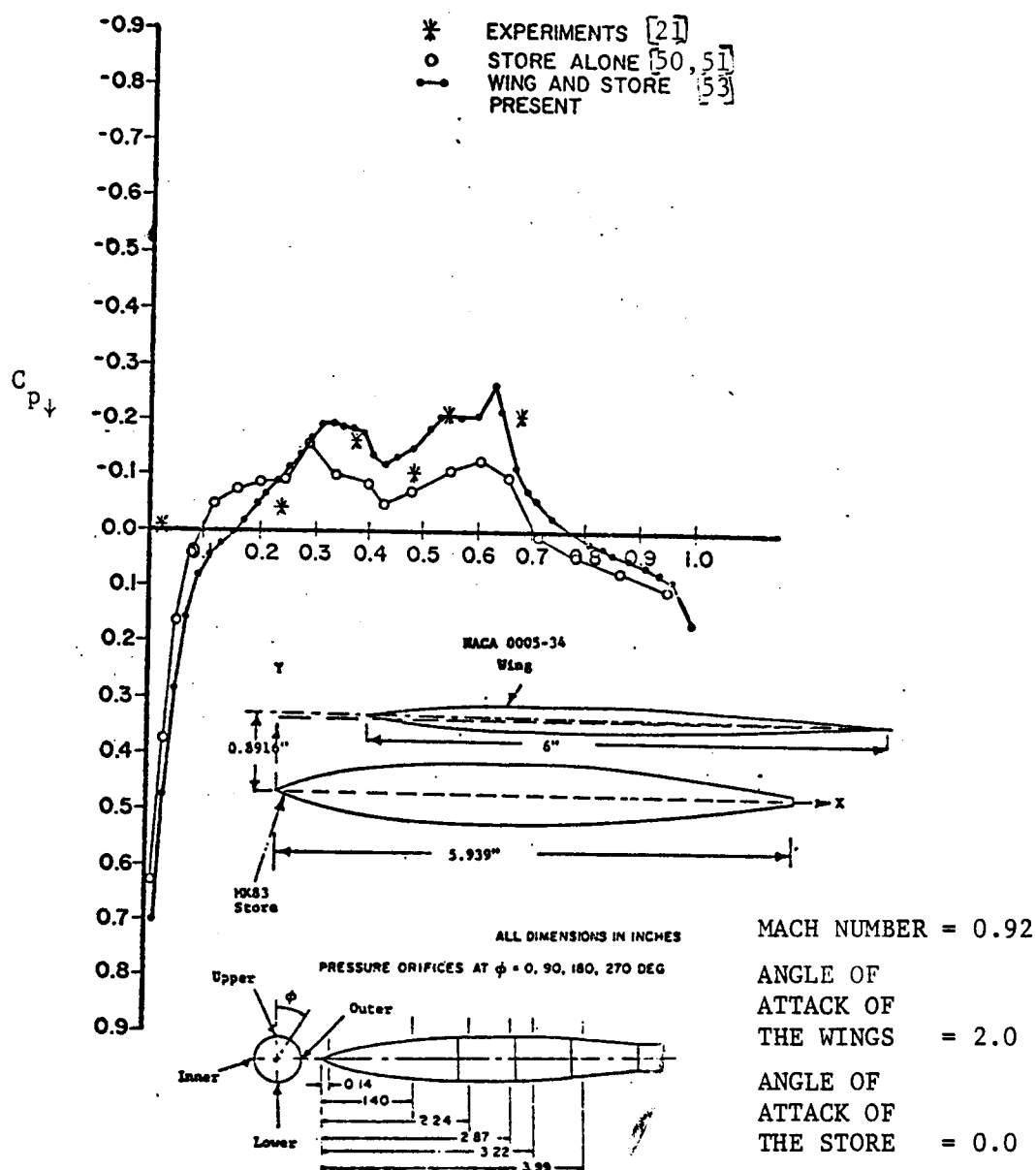
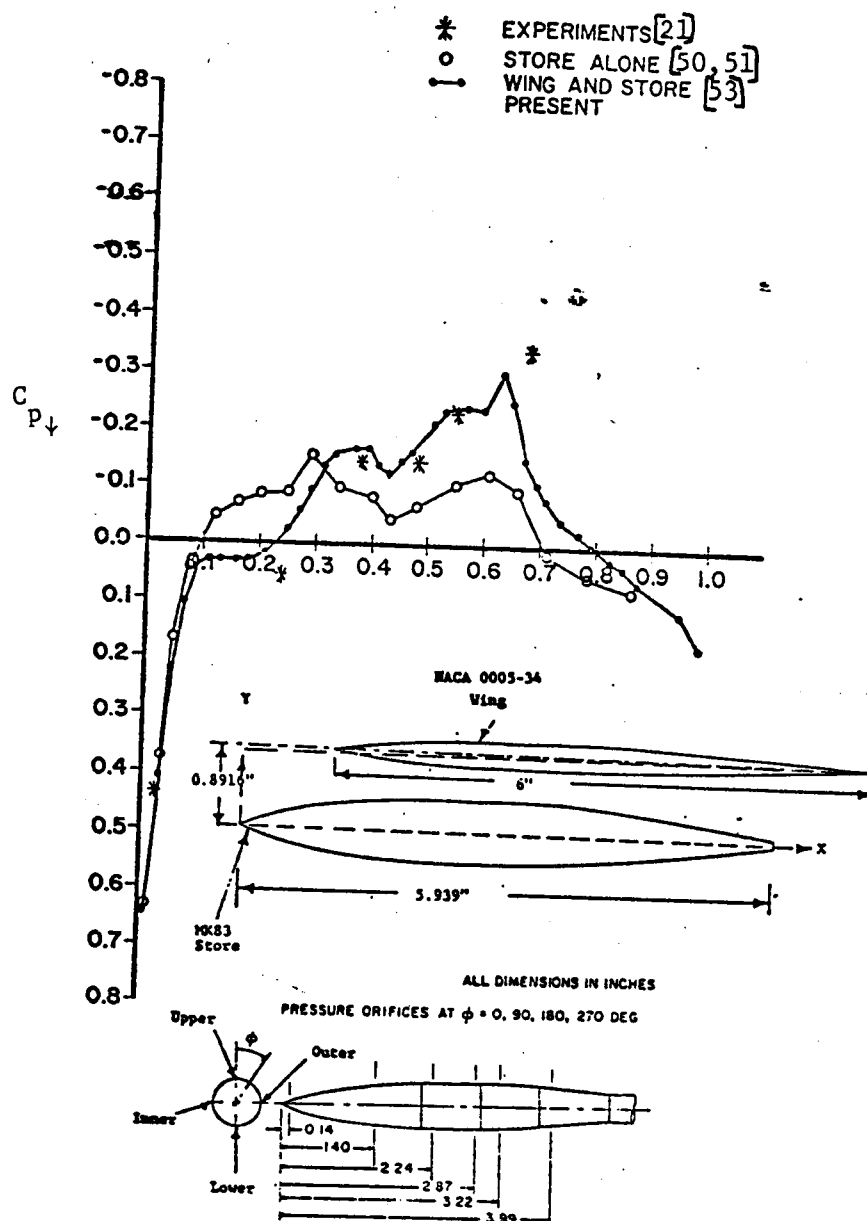


Figure 4.5. Comparison of Various Theories for the Flow Past MK83 Store Alone Configuration. Refer to Figure 2.2 for Store Geometry.



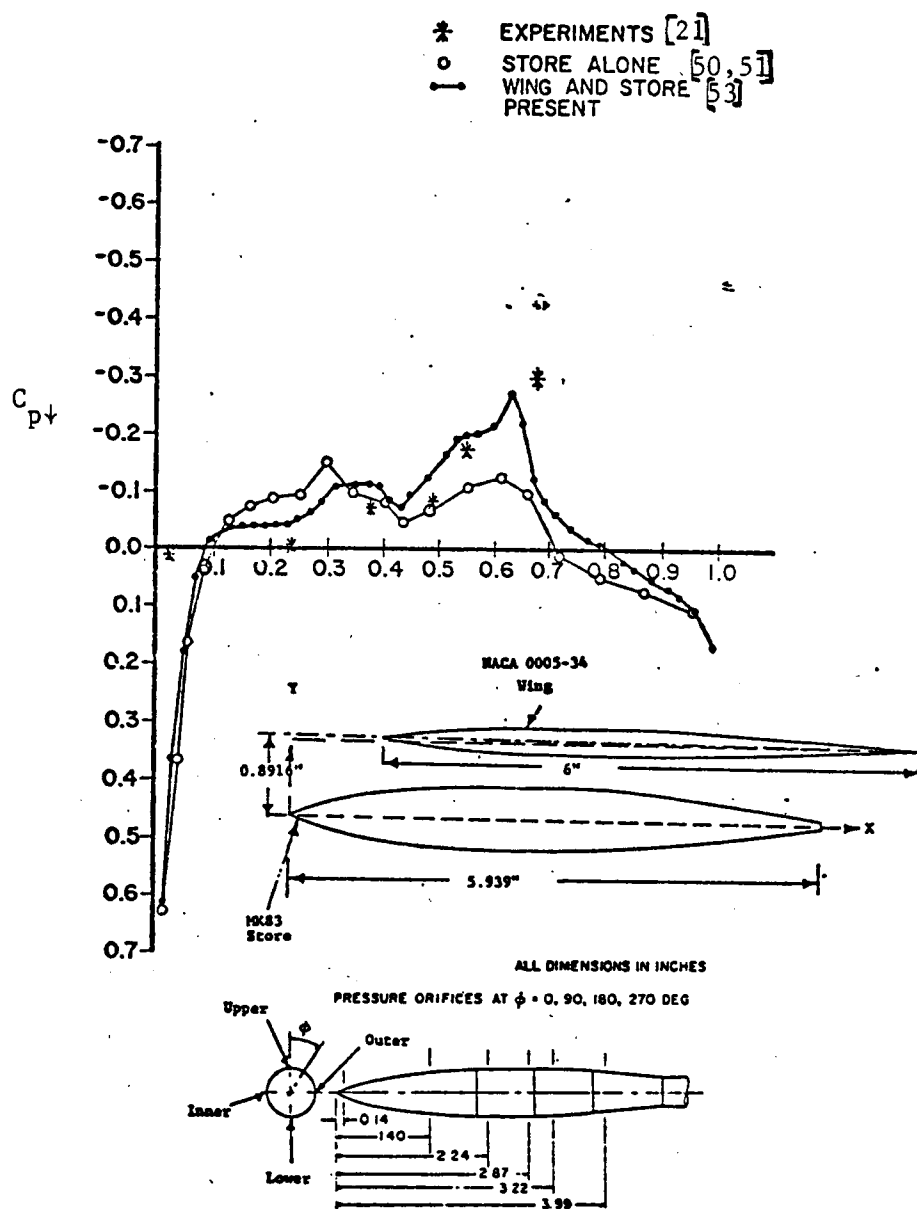
(a) INNER

Figure 4.6. Comparison of the Experimental and the Theoretical Pressure Distribution on the Store Surface with the Presence of the Wing.



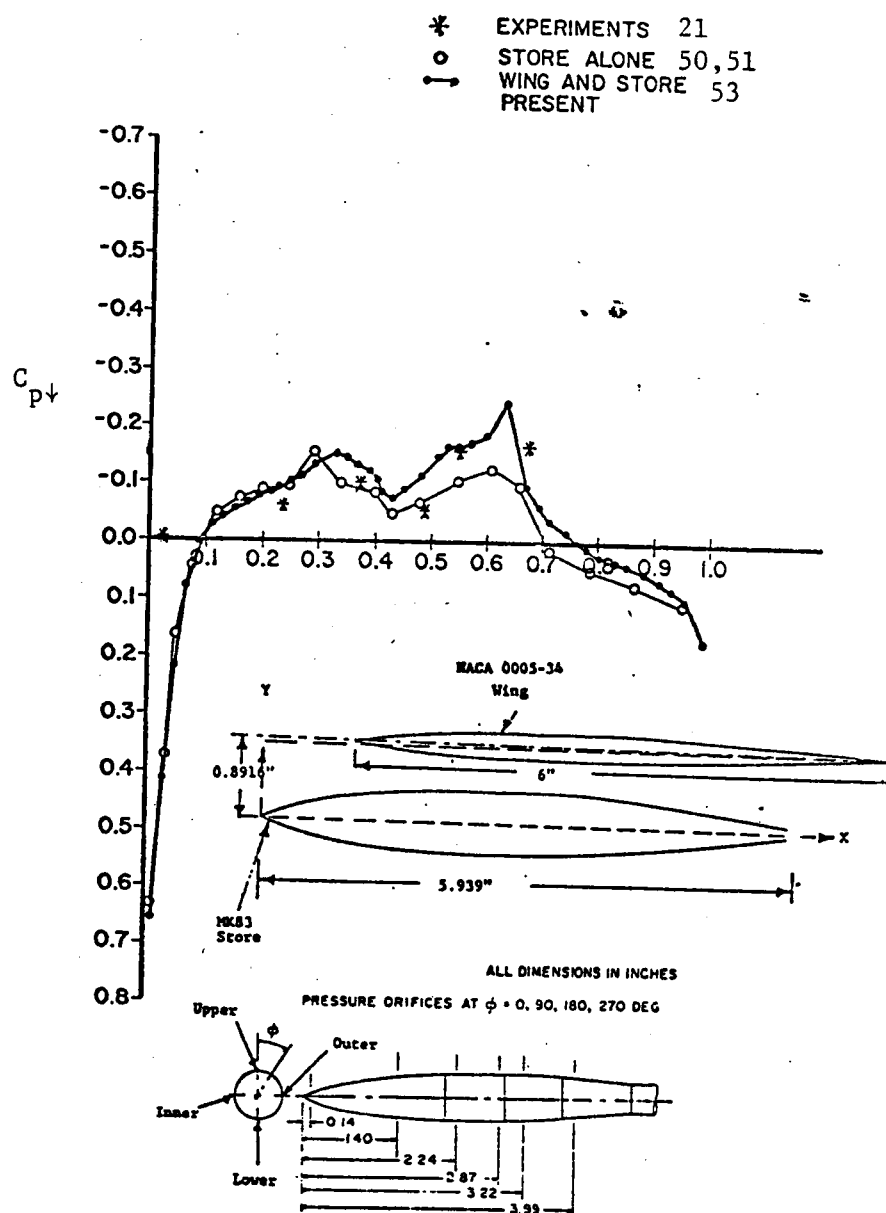
(b) UPPER

Figure 4.6. (Continued)



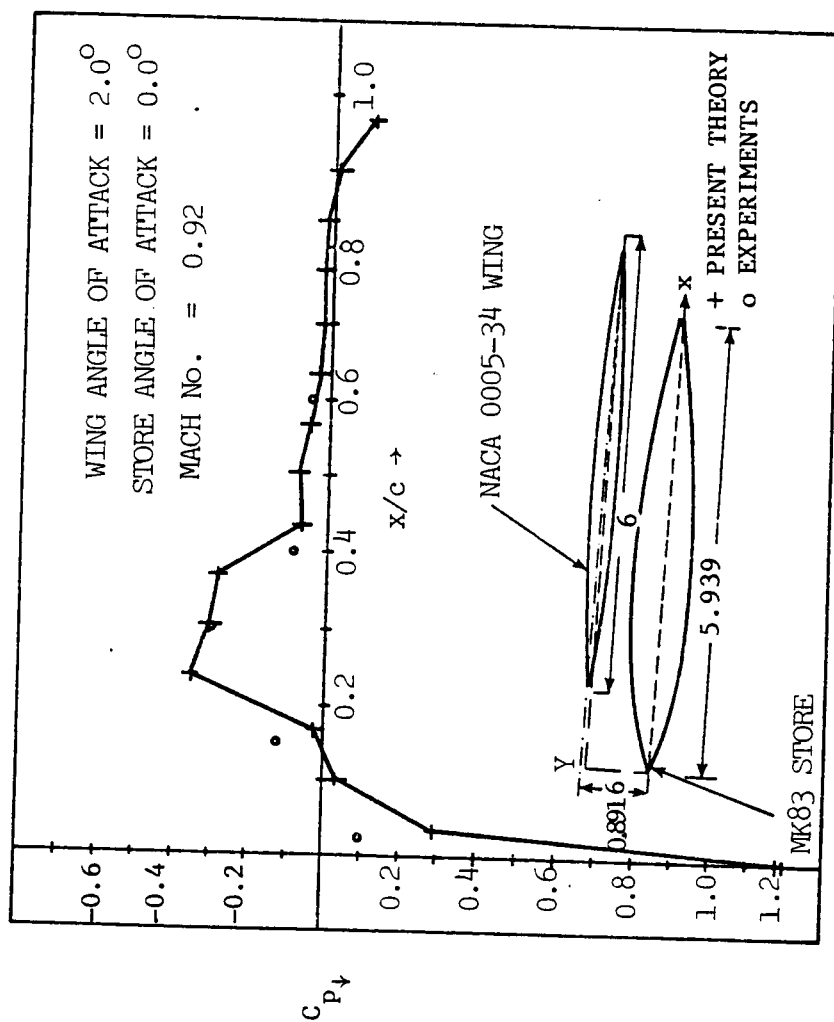
(c) OUTER

Figure 4.6. (Continued)



(d) LOWER

Figure 4.6. (Continued)



(a) At Section AA (Refer Fig. 2.2c)

Figure 4.7. Comparison of the Pressure Distribution on the Wing Lower Surface When Store Is Present.

CHAPTER V

TRANSONIC FLOW FIELD VELOCITY CALCULATION FOR AN AIRCRAFT
WITH EXTERNAL STORES BY THE PRESENT METHOD

The flow field velocity calculation, using the non-linear correction method upon the linear velocity, is discussed in this chapter.

The basic problem reduces to the calculation of the magnitude and direction of downwash and sidewash at positions under the wing or fuselage. This flow field, normally curvilinear in subsonic and transonic flights, will vary with aircraft configuration, the shape and locations of the stores, and flight conditions. There are several methods of calculating the nonuniform flow field around external stores in subsonic flow, as well as some for supersonic flow based on linear potential flow theory. So far, there are no computational techniques available for the transonic case, which happens to be the main area of interest since store separation normally occurs in this regime. Subsonic methods are inaccurate above the critical flow conditions, while the supersonic theories, based on linear theories, are limited to slender-wing-body configurations. Here the flow field calculation problem is treated as a non-linear correction upon the linear subsonic values.

The linear subsonic values at the field points for which the flow field velocity calculations have to be

carried out are determined from the Hess and Smith program [12]. For the determination of the linear perturbation velocity components at these field points, the wing coordinates and angle of attack are transformed from the compressible plane to the incompressible plane by the following Goethert transformations, as explained in Chapter I.

Derivation Of The Flow Field Velocity Components

By The Present Method, For The Three-Dimensional
Lifting Wing-Body Combination

We start with the nonlinear partial differential equation (3-6):

$$(1-M_\infty^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = k\phi_x\phi_{xx} \quad (5-1)$$

where

$$k = M_\infty^2 \frac{\gamma+1}{U_\infty} \quad (5-2)$$

The change of type of the equation is an essential feature of transonic flow and the regions where this occurs are recognized by the sign of the coefficient of ϕ_{xx} :

$$\left[1 - M_\infty^2 - M_\infty^2(\gamma+1) \frac{\phi_x}{U_\infty} \right] \begin{array}{l} > 0 \\ = 0 \\ < 0 \end{array} \begin{array}{l} \text{elliptic (subsonic)} \\ \text{parabolic (sonic)} \\ \text{hyperbolic (supersonic)} \end{array} \quad (5-3)$$

Equation (5-1) is, of course, valid only in regions where the necessary derivatives exist and are continuous. Across a shock surface the normal component of velocity is discontinuous while the tangential component, and therefore ϕ , is continuous. The necessary relation follows from the classical expression for the shock polar given in Chapter III. This approximates in small disturbance transonic theory, to (see equation 3-10)

$$\begin{aligned} (1-M_\infty^2)(\phi_{x_1}-\phi_{x_2})^2 + (\phi_{y_1}-\phi_{y_2})^2 + (\phi_{z_1}-\phi_{z_2})^2 \\ = k \frac{\phi_{x_1}+\phi_{x_2}}{2}(\phi_{x_1}-\phi_{x_2})^2 \end{aligned} \quad (5-4)$$

where the subscripts 1 and 2 refer to conditions ahead of and behind the shock.

Equations (5-1) and (5-4) are usually applicable to the case where the x-axis is parallel to the freestream at infinity; but they also apply where the coordinate system is rotated slightly. Choosing the body-axis system, where x-axis is aligned in the longitudinal axis of the wing and body, the relation between the total velocity potential $\Phi(x,y,z)$ and the perturbation velocity potential $\phi(x,y,z)$ is approximated by:

$$\Phi(x,y,z) = U_\infty(x+\alpha z) + \phi(x,y,z) \quad (5-5)$$

where α is the angle of attack.

The expression for the pressure coefficient C_p is not invariant with respect to small rotations of the coordinate system. In body axes, the proper expression is

$$C_p = -\frac{2}{U_\infty} (\phi_x - \alpha \phi_z) - \frac{1}{U_\infty^2} (\phi_y^2 + \phi_z^2) \quad (5-6)$$

The Boundary Conditions. The condition at infinity yields

$$\phi(\infty) = U_\infty(x + \alpha z)$$

or that

$$\phi(\infty) = 0 \quad . \quad (5-7)$$

An exception to this statement occurs in the vicinity of the wake at great distances behind the wing, but no complications arise due to the relative smallness of this region. The conditions at the airplane surface result in the relation

$$\frac{\partial \phi}{\partial \tilde{n}} \equiv U_\infty(n_1 + \alpha n_3) + n_1 \frac{\partial \phi}{\partial x} + n_2 \frac{\partial \phi}{\partial y} + n_3 \frac{\partial \phi}{\partial z} = 0 \quad (5-8)$$

where n_1 , n_2 , and n_3 are the direction cosines of a normal to the airplane surface with respect to the x , y , and z axes, respectively. This relation is too general. For our needs $n_1 \ll 1$ or $(n_2^3 + n_3^2)^{\frac{1}{2}}$, i.e., slender bodies and wings are considered. Then (5-8) reduces to

$$U_{\infty}(n_1 + \alpha n_3) + n_2 \frac{\partial \phi}{\partial y} + n_3 \frac{\partial \phi}{\partial z} = 0 \quad (5-9)$$

or

$$U_{\infty}(n_1 + \alpha n_3) + \frac{\partial \phi}{\partial n} = 0$$

where n is the normal to the curve bounding a cross-section in a plane normal to the x -axis.

We will now apply Green's theorem for $M_{\infty} \leq 1$.

Let

$$\beta = (|1 - M_{\infty}^2|)^{\frac{1}{2}}; \quad k = M_{\infty}^2 \frac{\gamma + 1}{U_{\infty}} \quad (5-10)$$

Subsonic free stream flow: $M_{\infty} \leq 1$.

Equation (5-1) can be rewritten as

$$\beta^2 \phi_{xx} + \phi_{yy} + \phi_{zz} = k \phi_x \phi_{xx} = k \frac{\partial}{\partial x} \left(\frac{\phi_x^2}{2} \right) \quad (5-11)$$

and the corresponding expression of Green's theorem is

$$\iiint_V [\psi L(\Omega) - \Omega L(\psi)] dV = - \iint_{\Sigma} \left(\psi \frac{\partial \Omega}{\partial v} - \Omega \frac{\partial \psi}{\partial v} \right) d\Sigma \quad (5-12)$$

where Ω and ψ are arbitrary functions and L is the Prandtl-Glauert operator:

$$L \equiv \beta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (5-13)$$

and $\frac{\partial}{\partial v}$ is the derivative along the conormal defined as

$$\frac{\partial}{\partial v} = \beta^2 n_1 \frac{\partial}{\partial x} + n_2 \frac{\partial}{\partial y} + n_3 \frac{\partial}{\partial z} \quad (5-14)$$

where n_1 , n_2 , and n_3 are the direction cosines of the normal to the surface drawn into the region V .

Now choose $\psi = \frac{1}{\sigma}$, where

$$\sigma = \left[(x-\xi)^2 + \beta^2 \{ (y-\eta)^2 + (z-\zeta)^2 \} \right]^{\frac{1}{2}} = r_3 \quad (5-15)$$

so that $L(\psi) = 0$ and let $\Omega \equiv \phi$, the perturbation velocity potential, in equation (5-12). Then from (5-11) and (5-12) we obtain

$$\iint_{\Sigma} \left[\frac{1}{\sigma} \frac{\partial \phi}{\partial v} - \phi \frac{\partial}{\partial v} \left(\frac{1}{\sigma} \right) \right] d\Sigma = - \iiint_V \frac{k}{\sigma} \frac{\partial}{\partial \xi} \left(\frac{\phi \xi^2}{2} \right) dV \quad (5-16)$$

In these equations the running coordinates in the integrations are ξ, η, ζ and ϕ are to be calculated at point $P(x, y, z)$.

If Equation (5-16) is applied to the infinite region V consisting of a sphere of large radius which forms the external boundary V, a sphere of infinitesimal radius surrounding the field point P(x,y,z), and a final surface enveloping the wing-body, its wake, and its shock waves, we obtain the following expression:

$$\begin{aligned} \phi(x,y,z) = & -\frac{1}{4\pi} \iint_{B+W} \left(\frac{1}{\sigma} \frac{\partial \phi}{\partial v} - \phi \frac{\partial}{\partial v} \left(\frac{1}{\sigma} \right) \right) d\Sigma - \frac{1}{4\pi} \iint_{\lambda_1} \left(\frac{1}{\sigma} \frac{\partial \phi}{\partial v} - \phi \frac{\partial}{\partial v} \left(\frac{1}{\sigma} \right) \right) d\Sigma \\ & - \frac{1}{4\pi} \iint_{\lambda_2} \left(\frac{1}{\sigma} \frac{\partial \phi}{\partial v} - \phi \frac{\partial}{\partial v} \left(\frac{1}{\sigma} \right) \right) d\Sigma - \frac{k}{4\pi} \iiint_V \frac{1}{\sigma} \frac{\partial}{\partial \xi} \left(\frac{\phi \xi^2}{2} \right) dv \end{aligned} \quad (5-17)$$

where B+W denotes the wing-body surface and its wake and λ_1 and λ_2 are the shocks, upstream and downstream shock surfaces, respectively. On the shock surface λ the conormals are directly opposed on the upstream and downstream faces λ_1 and λ_2 . On the body itself and the wake, as mentioned earlier, we may approximate

$$\frac{\partial}{\partial v} \approx n_2 \frac{\partial}{\partial y} + n_3 \frac{\partial}{\partial z} = \frac{\partial}{\partial n} .$$

If now the triple integral in (5-17) is integrated by parts twice with respect to x, the resultant form of (5-17) is

$$\begin{aligned}
\phi(x,y,z) = & - \frac{1}{4\pi} \iint_{B+W} \left[\frac{1}{\sigma} \left(\frac{\partial \phi}{\partial n} - \frac{k}{2} \phi_{\xi}^2 n_1 \right) - \phi \frac{\partial}{\partial n} \frac{1}{\sigma} \right] d\Sigma \\
& - \frac{1}{4\pi} \iint_{\lambda_1} \left[\frac{1}{\sigma} \left(\frac{\partial \phi}{\partial n} - \frac{k}{2} \phi_{\xi}^2 n_1 \right) - \phi \frac{\partial}{\partial n} \frac{1}{\sigma} \right] d\Sigma \\
& - \frac{1}{4\pi} \iint_{\lambda_2} \left[\frac{1}{\sigma} \left(\frac{\partial \phi}{\partial n} - \frac{k}{2} \phi_{\xi}^2 n_1 \right) - \phi \frac{\partial}{\partial n} \frac{1}{\sigma} \right] d\Sigma \\
& + \frac{k}{4\pi} \iiint_V \frac{1}{2} \phi_{\xi}^2 \left(\frac{\partial}{\partial \xi} \frac{1}{\sigma} \right) dV
\end{aligned} \tag{5-18}$$

Equation (5-18) is of particular interest because the integrals over the shock surfaces may be shown to vanish as in the nonlifting case. Finally, if we neglect the term $n_1 \phi_{\xi}^2$ in comparison with $\partial \phi / \partial n$, the simplified integral equation is similar to the nonlifting case:

$$\phi(x,y,z) = - \frac{1}{4\pi} \iint_{B+W} \left(\frac{1}{\sigma} \frac{\partial \phi}{\partial n} - \phi \frac{\partial}{\partial n} \frac{1}{\sigma} \right) d\Sigma + \frac{k}{4\pi} \iiint_V \frac{1}{2} \phi_{\xi}^2 \left(\frac{\partial}{\partial \xi} \frac{1}{\sigma} \right) dV \tag{5-19}$$

The first integral on the right-hand side of (5-19) is the expression for $\phi(x,y,z)$ in linearized theory and the spatial integral is a contribution brought about by the nonlinear term of the basic differential equation (5-1). It is of interest to remark that a straight-

forward derivation ignoring the existence of the shock waves leads to the same equation as (5-19). For the majority of cases of practical interest, it can be shown that compensating terms arise and the discontinuity surfaces are taken care of by a formal development that ignores the existence of these surfaces. However, if it is not possible to ignore so completely the existence of the discontinuity surface, equation (5-17) will be preferable to equation (5-19).

Since

$$\frac{\partial}{\partial \xi} \frac{1}{\sigma} = - \frac{\partial}{\partial x} \frac{1}{\sigma}$$

equation (5-19) may be written in the final forms as

$$\phi(x,y,z) = \phi_L(x,y,z) + \frac{k}{4\pi} \iiint_V \frac{1}{2} \phi_\xi^2 \left(\frac{\partial}{\partial \xi} \frac{1}{\sigma} \right) dV \quad (5-20a)$$

or

$$\phi(x,y,z) = \phi_L(x,y,z) - \frac{k}{4\pi} \frac{\partial}{\partial x} \iiint_V \frac{1}{2} \phi_\xi^2 \frac{1}{\sigma} dV \quad (5-20b)$$

Introducing the notation

$$\Delta \left(\frac{\partial \phi}{\partial v} \right) = \left(\frac{\partial \phi}{\partial v} \right)_{\lambda_1} + \left(\frac{\partial \phi}{\partial v} \right)_{\lambda_2}$$

and the variable

$$\omega = \sinh^{-1} \frac{x-\xi}{\beta [(y-\eta)^2 + (z-\zeta)^2]^{\frac{1}{2}}}$$

$$= \frac{(x-\xi)}{|x-\xi|} \ln \frac{|x-\xi| + [(x-\xi)^2 + \beta^2(y-\eta)^2 + \beta^2(z-\zeta)^2]^{\frac{1}{2}}}{\beta [(y-\eta)^2 + (z-\zeta)^2]^{\frac{1}{2}}}$$

and remembering that ϕ is continuous at the shock surface the more exact equation (5-17) may be written in the following two alternative forms:

$$\phi(x,y,z) = \phi_L(x,y,z) - \frac{1}{4\pi} \int_{\lambda} \Delta \left(\frac{\partial \phi}{\partial v} \right) \frac{1}{\sigma} d\Sigma - \frac{k}{4\pi} \iiint_V \left(\frac{\partial}{\partial \xi} \frac{\phi_\xi^2}{2} \right) \frac{1}{\sigma} dV \quad (5-21a)$$

or

$$\phi(x,y,z) = \phi_L(x,y,z) - \frac{1}{4\pi} \frac{\partial}{\partial x} \int_{\lambda} \Delta \left(\frac{\partial \phi}{\partial v} \right) \omega d\Sigma - \frac{k}{4\pi} \frac{\partial}{\partial \xi} \iiint_V \left(\frac{\partial}{\partial \xi} \frac{\phi_\xi^2}{2} \right) \omega dV \quad (5-21b)$$

The perturbation velocity components, u , v and w , are obtained by differentiating ϕ with respect to x , y , z , respectively from any of the equations (5-20) or (5-21).

For our use consider (5-20a) for the x -derivative. After isolating the singularity at the field point by introducing the limits $\xi = x \pm \epsilon$ and using the rule of

differentiating under the sign of integration with variable limits:

$$\begin{aligned} \frac{d}{ds} \int_{g_1(s)}^{g_2(s)} F(x;s) dx &= \int_{g_1(s)}^{g_2(s)} \frac{\partial F}{\partial s} dx + F\{g_1(s);s\} \frac{dg_1}{ds} \\ &\quad - F\{g_2(s);s\} \frac{dg_2}{ds} \end{aligned}$$

we have

$$\begin{aligned} u(x,y,z) &= u_L(x,y,z) + \lim_{\epsilon \rightarrow 0} \frac{k}{4\pi} \frac{\partial}{\partial x} \left[\iint d\eta d\xi \int_{-\infty}^{x-\epsilon} \frac{1}{2} \phi_{\xi}^2 \left(\frac{\partial}{\partial \xi} \frac{1}{\sigma} \right) d\xi \right. \\ &\quad \left. + \int_{x+\epsilon}^{\infty} \frac{1}{2} \phi_{\xi}^2 \left(\frac{\partial}{\partial \xi} \frac{1}{\sigma} \right) d\xi \right] \\ &= u_L(x,y,z) + \lim_{\epsilon \rightarrow 0} \frac{k}{2\pi} \iint \frac{1}{2} \phi_{\xi}^2(x;\eta,\zeta) \frac{\epsilon d\eta d\zeta}{[\epsilon^2 + \beta(y-\eta)^2 + \beta^2(z-\zeta)^2]^{3/2}} \\ &\quad - \frac{k}{4\pi} \iiint_V \frac{1}{2} \phi_{\xi}^2 \left(\frac{\partial^2}{\partial \xi^2} \frac{1}{\sigma} \right) dV . \end{aligned}$$

In the limit as $\epsilon \rightarrow 0$ the influence function in the integrand of the double integral is a two-dimensional Dirac delta function at the point $\eta = y$, $\zeta = z$ and of strength $2\pi/\beta^2$. The expression for u then becomes

$$u(x,y,z) = u_L(x,y,z) + \frac{k}{\beta^2} \frac{u^2(x,y,z)}{2} - \frac{k}{4\pi} \iiint_V \frac{u^2}{2} \left(\frac{\partial^2}{\partial \xi^2} \frac{1}{\sigma} \right) dv \quad (5-22a)$$

Again starting with equation (5-20a) and differentiating with respect to y , we have

$$\begin{aligned} \frac{\partial \phi(x,y,z)}{\partial y} = \frac{\partial}{\partial y} \phi_L(x,y,z) + \frac{k}{4\pi} \lim_{\epsilon \rightarrow 0} \frac{\partial}{\partial y} & \left[\iint d\xi d\zeta \int_{-\infty}^{y-\epsilon} \frac{1}{2} \phi_\xi^2 \left(\frac{\partial}{\partial \xi} \frac{1}{\sigma} \right) d\eta \right. \\ & \left. + \int_{y+\epsilon}^{+\infty} \frac{1}{2} \phi_\xi^2 \left(\frac{\partial}{\partial \xi} \frac{1}{\sigma} \right) d\eta \right] \end{aligned}$$

or

$$\begin{aligned} v(x,y,z) = v_L(x,y,z) - \frac{k}{4\pi} \iiint_V \frac{1}{2} \phi_\xi^2 \left(\frac{\partial^2}{\partial \xi \partial \eta} \frac{1}{\sigma} \right) dv \\ + \lim_{\epsilon \rightarrow 0} \left[\frac{k}{4\pi} \iint \frac{1}{2} \phi_\xi^2(\xi, y; \zeta) d\xi d\zeta \left\{ \frac{x-\epsilon}{[(x-\xi)^2 + \beta^2 \epsilon^2 + \beta^2 (z-\zeta)^2]^{3/2}} \right. \right. \\ \left. \left. - \frac{x-\epsilon}{[(x-\xi)^2 + \beta^2 \epsilon^2 + \beta^2 (z-\zeta)^2]^{3/2}} \right\} \right] \end{aligned}$$

or finally

$$v(x,y,z) = v_L(x,y,z) - \frac{k}{4\pi} \iiint_V \frac{u^2}{2} \left(\frac{\partial^2}{\partial \xi \partial \eta} \right) \frac{1}{\sigma} dv \quad (5-22b)$$

where we have made use of the fact that

$$\frac{\partial}{\partial y} \frac{1}{\sigma} = - \frac{\partial}{\partial \eta} \frac{1}{\sigma}$$

and the double integral tends to zero in the limit for all values of ξ and ζ .

Similarly it is easily seen, by differentiating (5-20a) with respect to z , that

$$w(x,y,z) = w_L(x,y,z) - \frac{k}{4\pi} \iiint_V \frac{u^2}{2} \left(\frac{\partial^2}{\partial \xi \partial \zeta} \frac{1}{\sigma} \right) dV \quad (5-22c)$$

By nondimensionalizing the above equations 6-22a, 6-22b, 6-22c, the following equations are obtained:

$$\bar{u}(\bar{x}, \bar{y}, \bar{z}) = \bar{u}_L + \frac{\bar{u}^2}{2} - \frac{1}{4\pi} \iiint_{-\infty}^{+\infty} \frac{\bar{u}^2}{2}(\xi, \eta, \zeta) \frac{d^2}{d\xi^2} \left[\frac{1}{r_3} \right] d\xi d\eta d\zeta, \quad (5-23)$$

$$\bar{v}(\bar{x}, \bar{y}, \bar{z}) = \bar{v}_L - \frac{1}{4\pi} \iiint_{-\infty}^{+\infty} \frac{\bar{u}^2}{2}(\xi, \eta, \zeta) \frac{\partial^2}{\partial \xi \partial \eta} \left[\frac{1}{r_3} \right] d\xi d\eta d\zeta, \quad (5-24)$$

$$\bar{w}(\bar{x}, \bar{y}, \bar{z}) = \bar{w}_L - \frac{1}{4\pi} \iiint_{-\infty}^{+\infty} \frac{\bar{u}^2}{2}(\xi, \eta, \zeta) \frac{\partial^2}{\partial \xi \partial \zeta} \left[\frac{1}{r_3} \right] d\xi d\eta d\zeta, \quad (5-25)$$

where,

$$\bar{x} = x, \quad \bar{y} = \sqrt{1 - M_\infty^2} \cdot y, \quad \bar{z} = \sqrt{1 - M_\infty^2} \cdot z,$$

$$\bar{u} = \frac{k \cdot u}{1 - M_\infty^2},$$

$$\bar{v} = \frac{k \cdot v}{(1 - M_\infty^2)^{3/2}},$$

$$\bar{w} = \frac{k \cdot w}{(1 - M_\infty^2)^{3/2}},$$

$$r_3 = \frac{1}{[(\bar{x} - \xi)^2 + (\bar{y} - \eta)^2 + (\bar{z} - \zeta)^2]^{1/2}},$$

with

$$k = M_\infty^2 \frac{(\gamma + 1)}{U},$$

and $\bar{u}_L$, and $\bar{v}_L$, and $\bar{w}_L$ are the values of the velocity components at $(\bar{x}, \bar{y}, \bar{z})$ obtained from the usual subsonic linear theory using the Hess and Smith program [12]. In Equations (5-23), (5-24), and (5-25), the evaluation of the integral is facilitated by transforming the limits, $+\infty$, $-\infty$, to $+1$ and -1 , respectively. In the formulation of the integral equation method by Oswatitsch [54] and Spreiter [31], an algebraic decay is assumed for the u -component of velocity in the z -direction given by:

$$\bar{u}(\bar{x}, \bar{y}, \bar{z}) = \frac{\bar{u}_w(\bar{x}, \bar{y}, 0)}{[1 + (z/b)]^2},$$

where $\bar{u}_w(\bar{x}, \bar{y}, 0)$ is the axial component of the velocity in the x-direction on the wing, and b is a function of $\bar{x}$, so chosen such that the irrotationality condition is fulfilled at $z = 0$. Norstrud's [34] formulation considers an exponential decay for the velocity component u, in the z-direction, given by

$$\bar{u}(\bar{x}, \bar{y}, \pm \bar{z}) = \bar{u}(\bar{x}, \bar{y}, \pm 0) \exp \left\{ \frac{\mp \bar{z}}{r(\bar{x}, \bar{y}, \pm 0)} \right\},$$

where $r(\bar{x}, \bar{y}, \pm 0)$ is given by

$$r^\pm(\bar{x}, \bar{y}) = \left\{ \left[(1 - M_\infty^2) \right]^{1/2} \left[\frac{\bar{v}_T \text{ linear}(x, y, 0) - u_\infty}{\frac{d^2}{dx^2}(z_w^\pm)} \right] \right\}$$

In the present work, the triple integral from $+\infty$ to $-\infty$ is evaluated with no assumptions being considered for the u-component velocity decay in the z-direction. The integral limits are converted to +1 to -1 by the following transformation:

$$\begin{aligned} \xi &= \tan \left[\frac{\pi}{2} t_1 \right] \\ \eta &= \tan \left[\frac{\pi}{2} t_2 \right], \\ \zeta &= \tan \left[\frac{\pi}{2} t_3 \right] \end{aligned} \quad (5-26)$$

when,

$$\xi \rightarrow +\infty, \quad t_1 \rightarrow 1$$

$$\xi \rightarrow -\infty, \quad t_1 \rightarrow -1.$$

The terms η and ζ behave similarly.

For the numerical computation of the transformed integral from +1 to -1 limits, Appendix A suggest a method by which the integration is performed by a quadrature. The integrand is evaluated at selected points (which is duly transformed to ξ , η , and ζ , at which u , v , and w linear velocity components caused by the configuration are obtained by the Hess and Smith program). The details are given below. From Equation (5-23), one obtains:

$$\begin{aligned} \frac{\partial^2}{\partial \xi^2} \left(\frac{1}{[(\bar{x}-\xi)^2 + (\bar{y}-\eta)^2 + (\bar{z}-\zeta)^2]^{1/2}} \right) d\xi \\ = \frac{[2(\bar{x}-\xi)^2 - (\bar{y}-\eta)^2 - (\bar{z}-\zeta)^2] d\xi}{[(\bar{x}-\xi)^2 + (\bar{y}-\eta)^2 + (\bar{z}-\zeta)^2]^{5/2}}, \end{aligned}$$

where $\bar{x}, \bar{y}, \bar{z}$ are the coordinates of the point p , the area of interest in the flow field calculation. The terms (ξ, η, ζ) are the dummy field points which are used for the triple integral between -1 to +1. Now, substituting for ξ , η , and ζ from Equation (5-26), one obtains

$$\begin{aligned}
& \cdot \frac{2[\bar{x}-\tan(\frac{\pi}{2}t_1)]^2 - [\bar{y}-\tan(\frac{\pi}{2}t_2)]^2 - [\bar{z}-\tan(\frac{\pi}{2}t_3)]^2}{[[x-\tan(\frac{\pi}{2}t_1)]^2 + [\bar{y}-\tan(\frac{\pi}{2}t_2)]^2 + [\bar{z}-\tan(\frac{\pi}{2}t_3)]^2]^{5/2}} \cdot \\
& \cdot \sec^2(\frac{\pi}{2}t_1) \cdot \sec^2(\frac{\pi}{2}t_2) \cdot \sec^2(\frac{\pi}{2}t_3) dt_1 dt_2 dt_3 .
\end{aligned}$$

Similarly, for v and w components,

$$\begin{aligned}
\frac{\partial^2}{\partial \xi \partial \eta} \left[\frac{1}{r_3} \right] &= \frac{3 (\bar{x}-\xi) (\bar{y}-\eta)}{[(\bar{x}-\xi)^2 + (\bar{y}-\eta)^2 + (\bar{z}-\zeta)^2]^{5/2}} \\
\frac{\partial^2}{\partial \xi \partial \eta} \left[\frac{1}{r_3} \right] & \quad \frac{3 (\bar{x}-\xi) (\bar{z}-\zeta)}{[(\bar{x}-\xi)^2 + (\bar{y}-\eta)^2 + (\bar{z}-\zeta)^2]^{5/2}}
\end{aligned}$$

The values of t_1 , t_2 , and t_3 are given by the numerical method, given in Appendix A and the corresponding values of ξ , η , and ζ are determined from the transformation

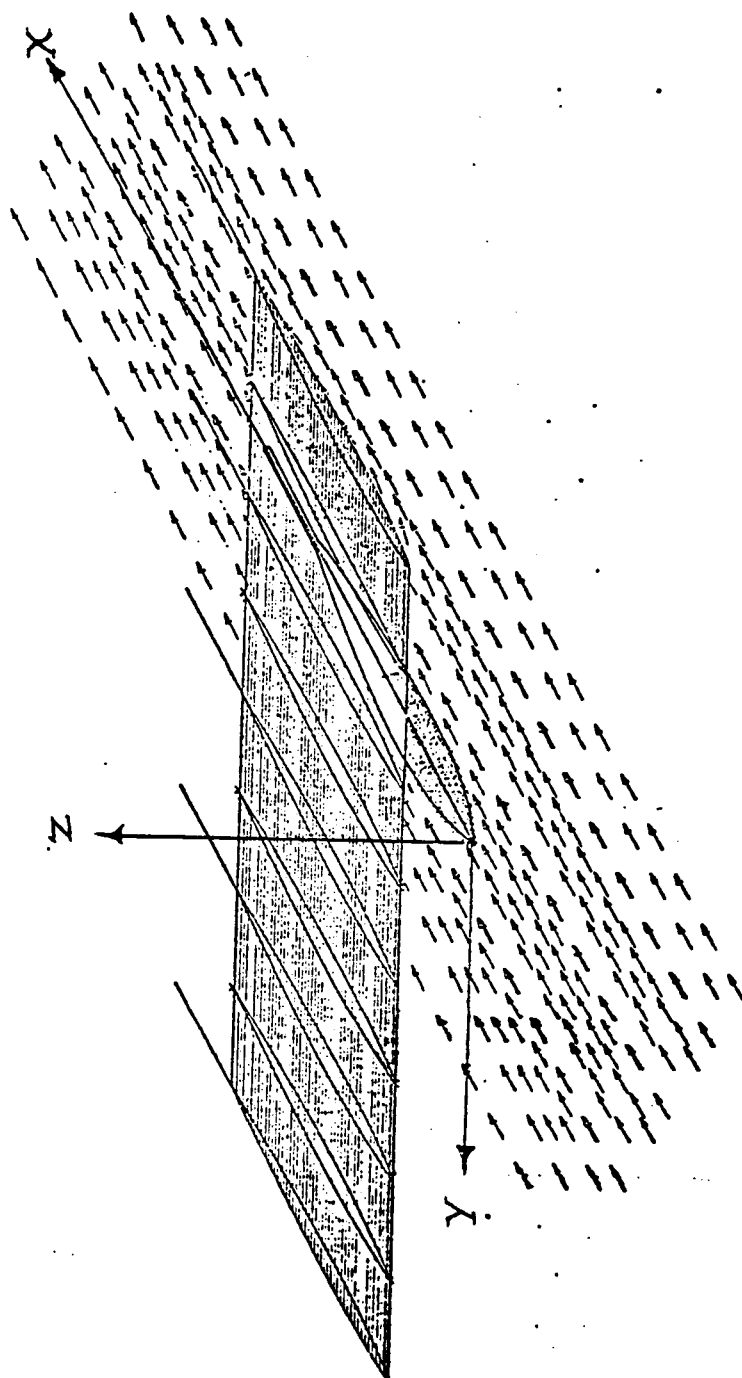
$$\xi = \tan \left(\frac{\pi}{2} t_1 \right)$$

$$\eta = \tan \left(\frac{\pi}{2} t_2 \right)$$

$$\zeta = \tan \left(\frac{\pi}{2} t_3 \right) .$$

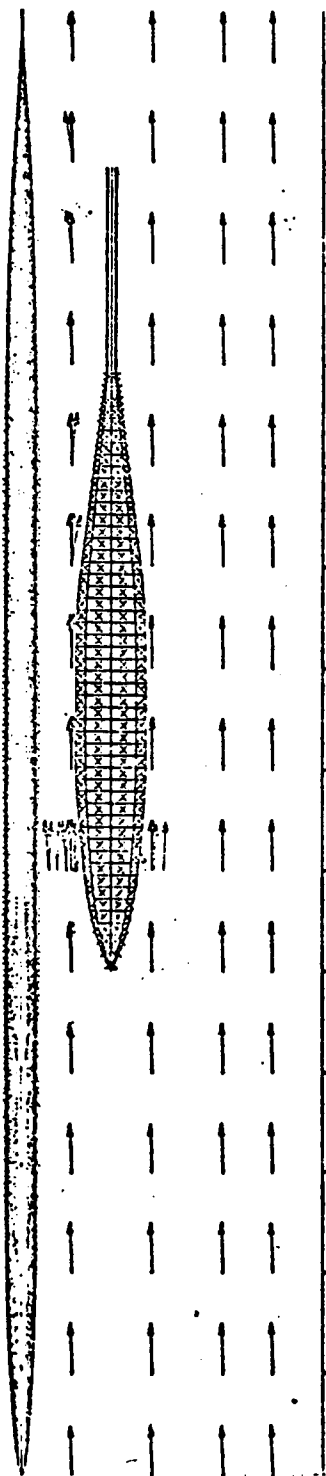
The linear perturbation velocities u , v , and w are found at these (ξ, η, ζ) points with the given configuration, and these values are substituted inside the integral term. The integral is evaluated by the quadrature, as explained in Appendix A. Then it is substituted in equations (5-23), (5-24), and (5-25). At present, the term $u^2/2$ in equation (5-23) is substituted by $u_{\text{linear}}^2/2$. The configuration is shown in Figure 5.1. The overall comparison of the velocity components seems to agree well with experimental results, except at some points, because there is no accounting for the viscosity and for the shock waves, if there are any, and their reflections and interactions, present in the flow. Moreover, since the field velocity theory discussed above is based on the linear theory values, it cannot handle shocks. The integral equation theory for the prediction of the shocks holds good only on the surface of the body and not in the flow field.

The experimental comparisons are shown in Figure 5.2 and they seem to be good except near the nose plane of the stores, where the flow is highly complicated, and the error may be due to the omission of viscosity in the calculation and the singular behavior of the flow properties at the nose stagnation point.



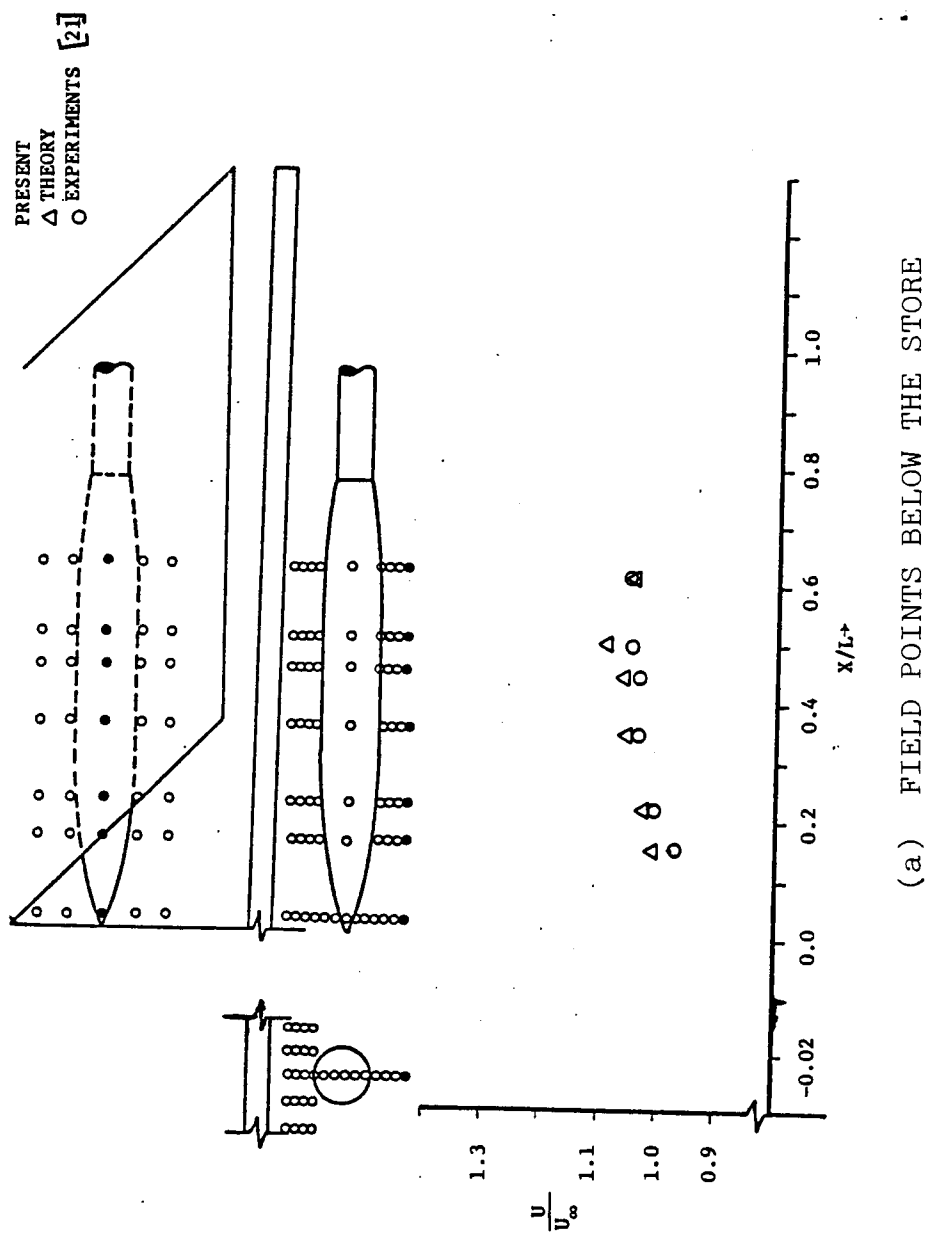
(a) THREE DIMENSIONAL VIEW

Figure 5.1. Configuration of NACA 0005-34 Wing and MK83 Store Without the Pylon, Used for Flow Field Calculation. [Mach Number = 0.92, Wing Angle of Attack = 2° in Pitch, Store Angle of Attack = $0-0^\circ$]



(b) SIDEVIEW

Figure 5.1. (Continued)



(a) FIELD POINTS BELOW THE STORE

Figure 5.2 Comparison of the Present Theory With Experiments.

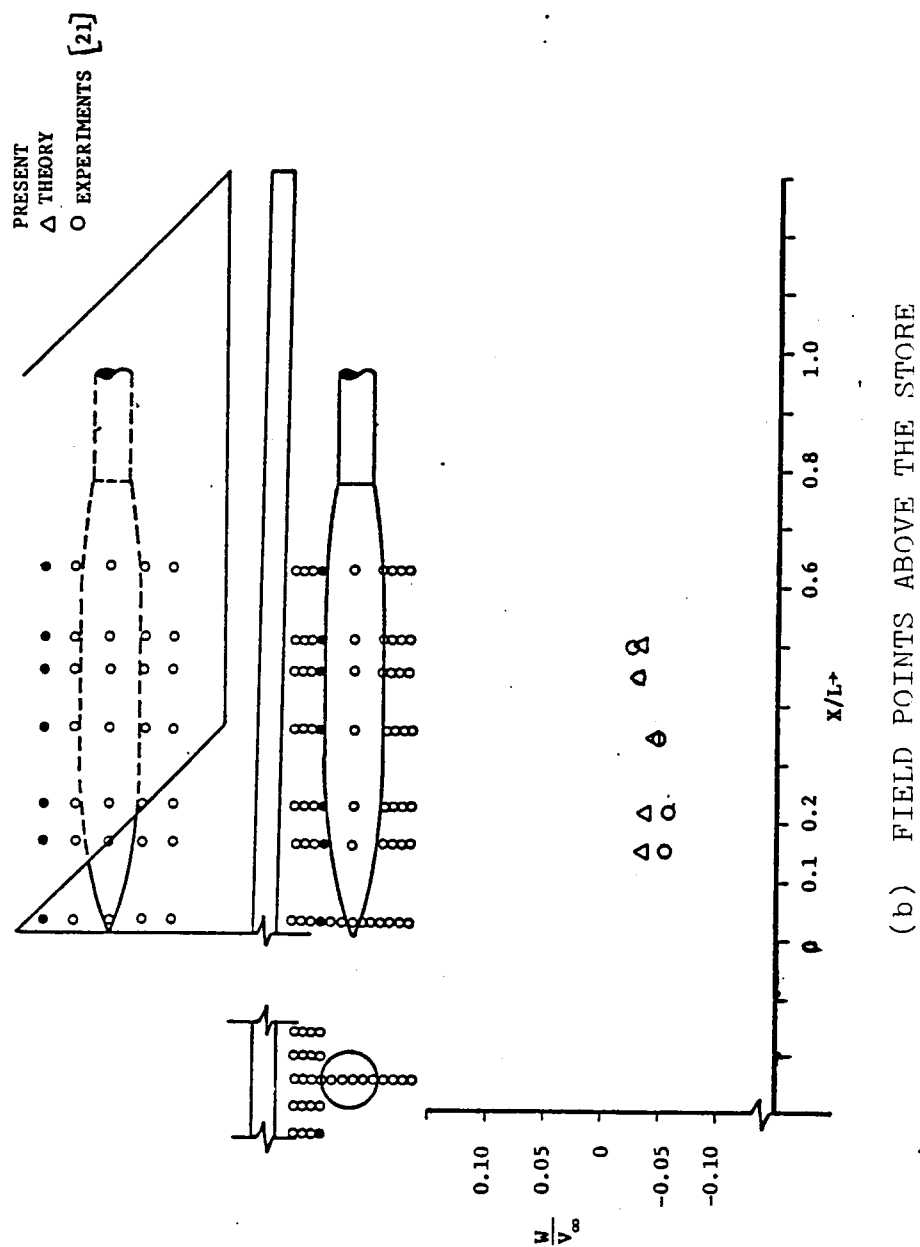
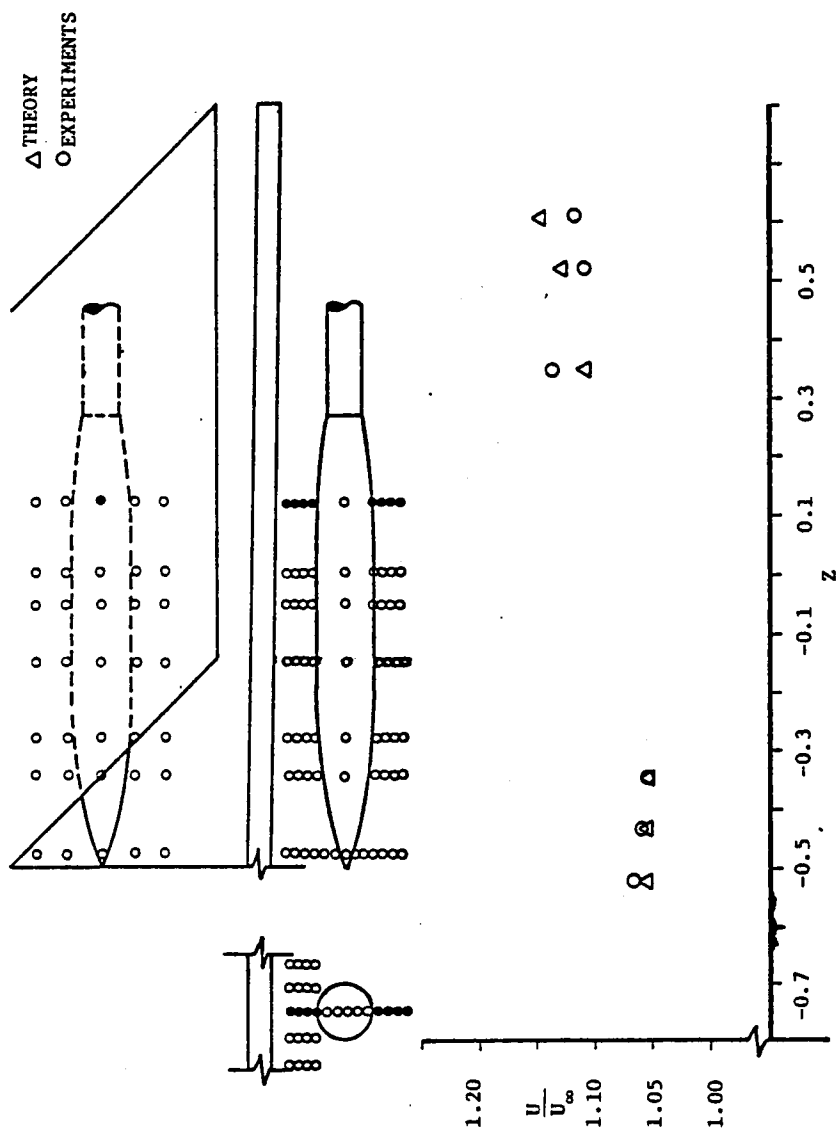


Figure 5.2. (Continued)



(c) FIELD POINTS BOTH ABOVE AND BELOW THE STORE.

Figure 5.2. (Continued)

CHAPTER VI

CONCLUSIONS AND DISCUSSIONS

From the aerodynamic analysis of the external stores problem, discussed in Chapters III, IV, and V, the following conclusions can be drawn:

(a) Even though the interference effects are taken into consideration only by the linear theory, Reference [12], the comparison of the final results obtained by the present formulation discussed in Chapter IV, with the experimental values (Figure 4.6), is good on the surface of the stores. This assures that the interference effects can be adequately handled by the linear theory even though the Mach number is high and the nonlinearity enters only through the store alone solution.

(b) Even though the integral equation method applied on a body requires that the integration limit be from $-\infty$ to $+\infty$ in the z -direction for the wing alone case, its application to a wing with an external store kept underneath has been found to be effective (Figure 4.7). The linear solution on the wing takes into account the effect of the external stores and in correcting the above obtained linear values on the wing for the nonlinearity, the presence of the store is neglected. The comparison of the above obtained nonlinear values on the wing with experimental

values are in fair agreement, even though the presence of the store is neglected in the nonlinear calculations (Figure 4.7).

(c) The store shapes are usually slender and the nonlinear solution does not differ much from the linear solution for the store alone calculation (Figure 4.5). So, for the transonic flow calculations, it is sufficient to consider the linear solution for the store instead of carrying out the nonlinear calculations in the formulation used in Chapter VI.

(d) The entire calculations over the wing store, including all the interference effects and for the flow field velocity calculations take about 15 minutes in VAX 11/780 computer and the memory requirements are of the order of 384 kilobytes, while the finite difference calculations either for a wing alone or for the store body alone, takes about 30 minutes in VAX 11/780 computer and IBM 370/3031.

(e) The integral equation method and its application discussed in Chapter III is a potential method and the results obtained from this theory applied to specific wings, as shown in Figure 3.14, gives confidence and hope that this method will be comparable to finite difference methods in accuracy and much simpler to apply. [Refer Fig. 3.15]. Besides, the time taken and computer memory space are much less than that required for finite difference calculation. The advantages and disadvantages of this integral equation method have already been outlined in Table 3.1.

(f) This method can be easily extended to wings with pylons supporting the multiple store configuration and any complicated geometry can be analyzed.

Future Scope of This Work

Even though the method of Chapter IV has been applied and verified for one particular case, it can be applied to any geometry with pylon and multistore configuration. Since the analysis of the method incorporates various theories at various stages of the prediction program, any change in any part of the method can be easily done without redoing the entire method. Even though the method and computer program are developed for the wing and store interference, wing alone and store alone linear and nonlinear solutions can be obtained along with the interference problem.

The relevant improvements that can be incorporated for future research work are listed below:

1. Include the viscosity model in the linear theory [55]. Many theories and computer programs are available for incorporating this in the subsonic linear theory.

2. Instead of the present formulation described in Chapter IV, the formulation of Reference [46] can be modified to incorporate solution of the nonlinear governing equation for transonic flow, with the boundary conditions, obtained from the nonlinear flow field velocities (equation 4-8) given by the method outlined in Chapter IV.

3. Inclusion of a trajectory program by solving the equations of motion and incorporating this in the present developed program for a captive trajectory store analysis.

4. The flow field velocity prediction outlined in Chapter V has to be modified to include the shocks in the flow field.

5. The reflection of shocks between the wing and the store at higher transonic Mach numbers have to be accounted for and its effect on the boundary layer existing on the store has to be studied.

6. Another important problem is to look into the effects of a fuselage, mounted with a pylon and multiple store configurations. The positioning of the store on the fuselage will have certain interference effects on the flow distribution on the wing and on the fuselage, and study the resulting pressure distributions. This could be used in fighter-aircraft designs.

7. Various wing configurations like swept-forward, kinked and saw-tooth planforms, should be tried by this present method, outlined in Chapter III.

8. To evaluate the present technique for the interference calculations, it needs to be checked with other existing theoretical formulations for a multibody problem in transonic flow, Reference [47].

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APPENDIX

APPENDIX A

NUMERICAL INTEGRATION

The advancement of the digital computer has revolutionized the strategies of numerical integration. For a given integrand, the effort is mostly directed towards the achievement of a stable solution with minimum error, with minimum regard to the number of digits being handled. In most cases, choice can be made for handling large numbers of operations with precise ordinate points and high precision formulae at a minimal increase in computing time. In this Appendix, choice has been made for Gauss-Legendre quadrature, Reference [56], for evaluating the triple integral with limits ranging from $-\infty$ to $+\infty$. The advantages are as follows:

a. For a digital computer, the evaluation of the functional value can be extremely accurate for any ordinate since the integrand is known exactly.

b. The weights of the functional values are all positive so that the error in the result cannot exceed (but can equal) twice the maximum of the errors in the ordinates, Reference [57].

c. The orthogonal properties of the polynomials are fully used in this quadrature to minimize the required number of ordinates.

The Gaussian-Legendre quadrature in one dimension can be written as

$$\int_a^b f(x) dx \approx \sum_{i=1}^n w_i f(x_i) \quad (\text{A-1})$$

where the ordinates x_i and weights w_i are fixed values depending on n . A typical list is shown in Table A-1. It is noted that the finite unit (a,b) can easily be changed to $(-1, +1)$ with a substitution of the form

$$y = \frac{2x - (a+b)}{b-a} \quad (\text{A-2})$$

Unfortunately, the substitution breaks down for infinite limits. $(-\infty, +\infty)$.

In the case of an integral, in the form of

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y,z) dx dy dz$$

it is possible to evaluate this type of integral by transforming the $(-\infty$ to $+\infty)$ limits by the following substitution

TABLE A-1. ABSCISSAE AND WEIGHT COEFFICIENTS OF THE
GAUSSIAN QUADRATURE FORMULA

n	a_i	H_i	n	a_i	H_i
2	$\pm.57735027$	1.00000000	14	$\pm.98628381$	.03511946
4	$\pm.86113631$	.34785485		$\pm.92843488$	.08015809
	$\pm.33998104$	.65214515		$\pm.82720132$	.12151857
6	$\pm.93246951$	.17132449		$\pm.68729290$	.15720317
	$\pm.66120939$	.36076157		$\pm.51524864$	.18553840
	$\pm.23861919$	.46791393		$\pm.31911237$	.20519846
8	$\pm.96028986$	.10122854		$\pm.10805495$	.21526385
	$\pm.79666648$	.22238103	16	$\pm.98940093$	.02715246
	$\pm.52553241$	.31370665		$\pm.94457502$	.06225352
	$\pm.18343464$	.36268378		$\pm.86563120$	.09515851
10	$\pm.97390653$	.06667134		$\pm.75540441$	.12462897
	$\pm.86506337$	.14945135		$\pm.61787624$	.14959599
	$\pm.67940957$	.21908636		$\pm.45801678$	.16915652
	$\pm.43339539$	.26926672		$\pm.28160355$	.18260342
	$\pm.14887434$	.29552422		$\pm.09501251$	.18945061
12	$\pm.98156063$	.04717534			
	$\pm.90411725$	.10693933			
	$\pm.76990267$	.16007833			
	$\pm.58731795$	.20316743			
	$\pm.36783150$	.23349254			
	$\pm.12533341$	.24914705			

n = number of Gaussian points needed to
evaluate the integral

F(x) = given function to be integrated

H_i = weighting factors

$$\int_{-1}^1 f(x) dx = \sum_{j=1}^n H_j f(a_j)$$

$$\begin{aligned}
 x &= \tan \frac{\pi}{2} t_1 \\
 y &= \tan \frac{\pi}{2} t_2 \\
 z &= \tan \frac{\pi}{2} t_3
 \end{aligned}
 \tag{A-3}$$

to (-1 to +1) limits.

The interesting feature of this substitution outlined in (A-3) includes:

a. The infinite limits change to the finite (-1 to +1) limits, for which the standard integration formulae are available.

b. The transformed function is smooth over the whole region, including the origin.

This type of analysis can be suitably applied to either single or multiple infinite integrals. For the evaluation of the triple integral in the finite limit (-1 to +1) the product formula can be applied. With the transformation outlined in A-3, the infinite region is transformed into a unit cube, viz:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y,z) \, dx dy dz
 \tag{A-4}$$

transforms to

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 g(t_1, t_2, t_3) dt_1 dt_2 dt_3 \quad (A-5)$$

The corresponding points (t_1, t_2, t_3) in the finite region for the x, y, z points in the infinite region, can be located using the transformations outlined in A-3 and the integration can be carried out. It might be noted that the disadvantage of this product formula is the rapid increase in ordinates in the multiple integral. If four points are needed for the evaluation of equation (A-1), then $64 = 4^3$ points are needed for the evaluation of equation (A-5). Recently, Irons (Reference 58) has suggested a quadrature rule with 27 points which is comparable with 64 points Gaussian quadrature formula.

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 f(x, y, z) dx dy dz = A_1 f(0, 0, 0) \quad (A-6)$$

$$\begin{aligned} &+B_6 \{f(-b, 0, 0) + f(b, 0, 0) + f(0, -b, 0) + \dots 6 \text{ terms}\} \\ &+C_8 \{f(-c, -c, -c) + f(c, -c, -c) + \dots 8 \text{ terms}\} \\ &+D_{12} \{f(-d, -d, 0) + \dots + f(d, 0, -d) + \dots 12 \text{ terms}\} \end{aligned}$$

where $A_1 = 0.788073483$
 $B_6 = 0.499369002$
 $C_8 = 0.478508449$
 $D_{12} = 0.032303742$
 $b = 0.848418011$
 $c = 0.652816472$
 $d = 1.106412899$

The 64 point product formula has the form:

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 f(x,y,z) \, dx dy dz \approx \sum_{k=1}^{n=4} \sum_{j=1}^{n=4} \sum_{i=1}^{n=4} w_i w_j w_k f(x_i, y_j, z_k)$$

$$w_i w_j w_k f(x_i, y_j, z_k) \quad (A-7)$$

The corresponding values of weights and ordinates are given in Table A-1. Recall that:

It is noted that if the integrand is explicitly defined, Gaussian-Legendre quadrature for single integrals with M points has $(2M-1)$ degree of exactness. For an n -cube region, the product formula will usually contain M^n points and have the corresponding degree d . Recall that equation (A-7) has degree (or degree of exactness) d if it is exact for all polynomials in x, y, z or degree less than, equal to d , and there is at least one polynomial of degree $d + 1$ for which it is not exact.

In order to compare the two formulae (A-6) and (A-7) and finally fix a guideline to choose an integration formula for the specific integration of the field integral, two examples from Reference 59 are chosen, viz:

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \text{Exp} (\text{Sin} x \text{ Sin} y \text{ Sin} z) \, dx dy dz \approx 8.081734973 \quad (\text{A-8})$$

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 (4 + x + y + z)^{-1} \, dx dy dz \approx 2.152142833 \quad (\text{A-9})$$

TABLE A-2. COMPARISON OF THE INTEGRALS OBTAINED FROM FORMULAS A-6 and A-7.

		INTEGRAL A-6	INTEGRAL A-7	SOURCE
Formula with n = 4	A-7	8.081738326	2.152135874	Present
Formula	A-6	8.0965445	2.1517344	Ref. [59]

In all the calculations, double precision in IBM 360 (16 digits) is used. It is interesting to note that equation (A-7) gives fairly accurate values even with as few as 27 points compared to 64 Gaussian points. However, in this specific application, the number of evaluations of the triple integral is not very large.

Another example of the above mentioned technique, as applied to the integral is shown below:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dx dy dz}{(x^2 + y^2 + 1)^{5/2}} \cdot \frac{1}{(1 + z^2)^{3/2}} = 4\pi \quad (\text{A-10})$$

using equations (A-3), (A-10)

$$\frac{\pi^3}{8} \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \frac{\sec^2 \frac{\pi}{2} t_1 \sec^2 \frac{\pi}{2} t_2 \cos \frac{\pi}{2} t_3}{[\tan^2 \frac{\pi}{2} t_1 + \tan^2 \frac{\pi}{2} t_2 + 1]^{3/2}} dt_1 dt_2 dt_3 = 4\pi \quad (\text{A-11})$$

so that

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \frac{\sec^2 \frac{\pi}{2} t_1 \sec^2 \frac{\pi}{2} t_2 \cos \frac{\pi}{2} t_3}{[\tan^2 \frac{\pi}{2} t_1 + \tan^2 \frac{\pi}{2} t_2 + 1]^{3/2}} dt_1 dt_2 dt_3 = \frac{32}{\pi} = 3.24228 \quad (\text{A-12})$$

Table A-3 gives the results of equation (A-11) using different techniques.

TABLE A-3. COMPARISON OF THE INTEGRAL OBTAINED FROM
EQUATIONS (A-6) AND (A-7).

Technique	Result
Equation (A-6)	3.44890704
Equation (A-7) with $n = 4$	3.483001
Equation (A-7) with $n = 6$	3.3600975

It is interesting that equation (A-6) gives a better result this time compared to 64 Gaussian. However, it is obviously inferior to 216 Gaussian point. As stated earlier, the accuracy is much more important, since the time taken in all the cases remains almost the same. Hence, the 216 point Gaussian point integration is chosen for the field integral occurring in the transonic integral equation.

VITA

The author, Chandrasekaran Balasubramanyan, was born in the State of Tamilnadu, India, on March 25, 1952. His college career began at Regional Engineering College, Trichy, India, and he earned his Bachelor of Engineering degree in Mechanical Engineering in June 1973. He then went to graduate school at the Indian Institute of Technology, Madras, India, in July 1973, supported by a graduate assistantship, and received his Master of Technology degree in Aeronautical Engineering in September 1975. He then served in the National Aeronautical Laboratory, Bangalore, India, as a senior research fellow, in the Gas Turbine Research Establishment, Bangalore, India, as a senior scientific officer, and in the National Aeronautical Laboratory, Bangalore, India, as a scientist. He entered the graduate school at The University of Tennessee Space Institute, supported by a research assistantship, in September 1977.

The author is a student member of the American Institute of Aeronautics and Astronautics.