

**An Experimental Study of Particle Crushing in Granular Material and
Projectile Penetration**

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Doctor of Philosophy
Degree
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DEDICATION

I would like to dedicate this dissertation to my family. It is because of them; I am where I am and who I am.

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ABSTRACT

Laboratory projectile penetration tests have shown extensive particle crushing along the path of the projectile. Careful extraction of projectile tip has shown formation of a false tip, composed of comminuted sand particles, that travels along with the projectile during the penetration process. Particle size analysis of the comminuted fines from the tip and path have shown orders of magnitude in size reduction. Current penetration depth prediction methods (empirical and analytical) do not explicitly account for energy dissipation in the vicinity of the projectile. In the far field, there is very little movement of the bulk mass which can be modelled using a continuum approach. Modeling the penetration process therefore requires a multi-scale approach, able to simulate the far field, the grain-projectile and grain-grain interactions, and granular fracture. A suite of tests was performed on sands with different grain morphology, Ottawa sand (sub-rounded), Q-Rok (angular), and Euroquarz Siligran (sub-angular) to determine the role of particle shape on continuum and micro scale properties. Triaxial tests were performed to investigate the role of particle shape on strength volume relationship. Additionally, digital image correlation technique was employed to better understand the initiation and propagation of localized deformations associated with frictional end triaxial tests. Role of particle shape and size in comminution was explored by performing single grain crushing tests encompassing particles of different shapes and sizes. The tensile strength of single grains decreased with angularity. It was quantitatively shown, using acoustic emission measurements, that crushing smaller particles requires considerably greater energy than crushing larger particles. In high strain rate tests, particle crushing decreased with moisture content and no significant differences in particle crushing were observed for different specimen densities. The role of projectile and target characteristics in the response of projectiles impacting granular material was explored with the help of full flight time histories from onboard data acquisition system. Penetration depth increased with impact velocity. Both projectile and target characteristics produced visible differences in the recorded projectile response. Projectiles with softer tip material went further in the target than those with stiffer tips.

TABLE OF CONTENTS

INTRODUCTION	1
Motivation.....	1
Projectile penetration in granular medium.....	2
CHAPTER I EFFECT OF PARTICLE MORPHOLOGY ON STIFFNESS, STRENGTH AND VOLUMETRIC BEHAVIOR OF ROUNDED AND ANGULAR NATURAL SAND....	6
Abstract	7
Introduction.....	8
Materials	11
Procedure	13
Particle size distribution.....	13
1-D compression	14
Triaxial tests.....	15
Isotropic consolidation.....	15
Results and analysis	15
Particle size distribution.....	15
1-D compression	17
Isotropic consolidation.....	20
Triaxial tests.....	21
Conclusions.....	23
Appendix.....	24
CHAPTER II THE ROLE OF PARTICLE SHAPE ON STRAIN LOCALIZATION IN FRICTIONAL END DRAINED TRIAXIAL COMPRESSION.....	43
Abstract	44
Introduction.....	44
Materials	46
Specimen preparation and mechanical testing.....	47
Digital image correlation (DIC).....	48
Experimental results.....	49
Global results	49
DIC average results.....	49
DIC local results	51
Strain localization	53
Surface strain at failure	54
Conclusions.....	55
Appendix.....	57
CHAPTER III ROLE OF PARTICLE SHAPE IN DETERMINING TENSILE STRENGTH AND ENERGY RELEASE IN DIAMETRICAL COMPRESSION OF NATURAL SINGLE SILICA GRAINS.....	72
Abstract	73
Introduction.....	73
Materials	75

Experimental methods	76
Single particle crushing.....	76
Single particle crushing with acoustic emission (AE) measurement.....	76
Surface imaging	78
High speed imaging	78
Results and discussion	78
Effect of particle morphology on crushing behavior	78
Single particle crushing.....	80
AE emissions	83
Conclusions.....	86
Appendix.....	87
CHAPTER IV PROJECTILE PENETRATION TESTS IN DRY SAND	103
Abstract	104
Introduction.....	104
Materials and methods	107
Results and discussions.....	110
Effect of tip shapes on the response of projectile	110
Effect of grain morphology on the response of projectile	111
Energy dissipation during penetration	111
Projectile and target characteristics on penetration depth.....	112
Poncelet drag coefficient.....	114
Cavity expansion model.....	115
Effect of tip material on penetration depth	116
Conclusions.....	116
Appendix.....	118
CHAPTER V THE ROLE OF PARTICLE SHAPE, DENSITY AND MOISTURE ON PARTICLE CRUSHING UNDER DYNAMIC LOADING	145
Abstract	146
Introduction.....	146
Materials and experimental methods	148
Results and discussions.....	149
Effect of grain shape on particle crushing	149
Effect of moisture on particle crushing.....	149
Effect of density on particle crushing	150
Effect of high strain rate loading in projectile penetration tests	151
Conclusions.....	151
Appendix.....	152
CONCLUSIONS.....	161
LIST OF REFERENCES	163
VITA	174

LIST OF TABLES

Table 1 Void ratio, and some index properties for Ottawa and Q-rok Sand.	24
Table 2 Breakage values computed using volume and number distribution using Lee and Farhoomand (1967) equation and volume distribution using Einav (2007) ultimate fractal distribution method	25
Table 3 Initial states and results of consolidated drained triaxial compression of Ottawa sand and Q-Rok.....	26
Table 4 Failure parameters determined from global and local response	57
Table 5 Shape parameters for different size fractions	87
Table 6 Weibull analysis for fracture stress and flaw size for different clustered groups.	88
Table 7 Median values of characteristic AE parameters at maximum force	89
Table 8 Parameters for Tresca and Mohr-Coulomb failure criteria for predicting penetration using cavity expansion model.	118
Table 9 Summary of the tests results	152

LIST OF FIGURES

Fig. 1 X-ray diffraction powder diffraction of as received (a) Ottawa sand and (b) Q-rok compared with diffraction peaks of (c) α -quartz.	27
Fig. 2 SEM micrographs of (a) Ottawa sand grains and (b) Q-rok grain at 94x magnification; Texture of (c) Ottawa grain and (d) Q-rok grain at 1000x magnification.	28
Fig. 3 Surface images of (a) Ottawa sand, (b) Q-Rok, and (c) Siligran from high resolution confocal laser microscope.	29
Fig. 4 Particle size distribution graph determined by sieve and laser scattering analysis on Ottawa and Q-Rok sand.	30
Fig. 5 Volume and calculated number distribution of Ottawa sand and Q-Rok from laser light scattering technique.	31
Fig. 6 Number distribution from laser light scattering technique and image analysis for Ottawa sand and Q-Rok.	32
Fig. 7 Results of strain controlled 1-D compression tests on loose (L) and dense (D) specimens of Ottawa sand (O) and Q-Rok (Q).	33
Fig. 8 Particle size evolution in dense and loose specimens of (a) Ottawa sand and (b) Q-Rok subjected to 1-D compression loading, shown as volume and number distributions.	34
Fig. 9 Particle shape evolution after 1-D compression loading. (a) Ottawa sand, (b) Q-Rok.	35
Fig. 10 Isotropic consolidation of dense and loose specimens of Ottawa sand and Q-Rok.	36
Fig. 11 Response of Ottawa sand to drained triaxial compression. (a) Stress-strain relationship, (b) Volumetric relationship. O = Ottawa, L = loose, D = dense.	37
Fig. 12 Response of Q-Rok to drained triaxial compression. (a) Stress-strain relationship, (b) Volumetric relationship. Q = Q-Rok.	38
Fig. 13 Response of Euroquartz Siligran to drained triaxial compression. (a) Stress-strain relationship, (b) Volumetric relationship. S = Siligran.	39
Fig. 14 Determination of critical state friction angle.	40
Fig. 15 Effect of D_r on strength and volume parameters in conventional drained triaxial testing.	41
Fig. 16 Effect of particle shape on strength and volume parameters in conventional drained triaxial.	42
Fig. 17 Experimental set-up using a servo-hydraulic loading system, coupled with DIC system for obtaining strain data on the specimen surface.	58
Fig. 18 Global response of the loose and dense specimens of Ottawa sand and Q-Rok.	59
Fig. 19 Typical speckle pattern on specimen membrane for DIC measurements along with regions chosen for extracting strain values.	60
Fig. 20 Response of dense and loose specimens from global displacement measurements and DIC strains.	61
Fig. 21 Typical global and local responses of sand specimens.	62
Fig. 22 Global and local strength and volumetric response Ottawa sand specimens, (a) Global stress strain curve, (b) Volumetric strain determined from average region against global axial strain, (c) – (d) Stress-strain-volumetric response as determined from strains in the active region.	63

Fig. 23 Global and local strength and volumetric response of Q-Rok sand specimens, (a) Global stress strain curve, (b) Volumetric strain determined from average region against global axial strain, (c) – (d) Stress-strain-volumetric response as determined from strains in the active region.....	64
Fig. 24 Major principal strain, ϵ_1 (vertical) from DIC along the height of the Ottawa sand specimens at various stages of global axial strain, ϵ_a : (a) OL69, (b) OL96, (c) OD69, (d) OD 96.....	65
Fig. 25 Major principal strain (ϵ_1) from DIC along the height of the Q-Rok sand specimens at various stages of global axial strain, ϵ_a : (a) QL69, (b) QL96, (c) QD69, (d) QD 96.....	66
Fig. 26 Minor principal strain, ϵ_3 , (lateral) from DIC along the height of the Ottawa sand specimens at various stages of global axial strain, ϵ_a : (a) OL69, (b) OL96, (c) OD69, (d) OD96.....	67
Fig. 27 Minor principal strain (ϵ_3) from DIC along the height of the Q-Rok sand specimens at various stages of global axial strain, ϵ_a : (a) QL69, (b) QL96, (c) QD69, (d) QD96.....	68
Fig. 28 Lateral to axial strain ratios from DIC during triaxial compression (a) Average region Ottawa sand, (b) Average region Q-Rok, (c) Active region Ottawa sand, (d) Active region Q-Rok.....	69
Fig. 29 Surface axial strains, ϵ_1 , approximately at failure for all the specimens. The global axial strain, ϵ_a , is 15% for loose specimens and 5% for dense specimens. The scale is different for each of the specimens as shown in the color bar for better visualization of localized strain. (a) OL69, (b) OL96, (c) OD69, (d) OD96, (e) QL69, (f) QL96, (g) QD69, and (h) QD96.....	70
Fig. 30 Visualization of post peak localization regions in dense specimen at $\epsilon_a = 15\%$, (a) OD69, (b) OD96, (c) QD69, (d) QD96. The color bar has been scaled individually for better visualization of failure planes.	71
Fig. 31 Thin section images of (a) – (b) multigrain Q-Rok, (c) mineral inclusion in Siligran, and (d) bulging recrystallization in Siligran.	90
Fig. 32 Psylotech testing system for particle crushing with AE measurement.....	91
Fig. 33 Explanation of AE parameters.....	92
Fig. 34 High resolution 3D surface images of (a - b) Ottawa sand, (c - d) Q-Rok, (e - f) Siligran.	93
Fig. 35 Example response of the sand grains to diametrical compression, (a) Ottawa sand, (b) Q-Rok, (c) Siligran.....	94
Fig. 36 Weibull plot for the largest size cluster for the three sands.....	95
Fig. 37 Highspeed images of particle fracture in Ottawa sand, Q-Rok and Siligran.....	96
Fig. 38 Size dependence of strength for silica sands and fused silica.	97
Fig. 39 Flaw size scaling with particle size in Ottawa sand, Q-Rok and Siligran.	98
Fig. 40 Example force and energy release time histories for Ottawa sand, Q-Rok and Siligran.	99
Fig. 41 Maximum force required to fail particles of different sizes and energy released in fracture. The color bar scale has been limited to the median energy released in Ottawa sand.	100
Fig. 42 Mass specific energy for particles with different failure stress.	101
Fig. 43 Mass specific energy required to fracture particles of different sizes.	102
Fig. 44 30-mm projectile with interchangeable tips of different shapes.....	119

Fig. 45 Pressurized gas gun with 30 mm smooth bore barrel.	120
Fig. 46 Effect of boundary confinement in high strain rate tests in SHPB.	121
Fig. 47 Material strength parameters for sand from (a) high confining stress triaxial compression tests on Chattahoochee River Sand (Vesic and Clough 1968), (b) high stress and high-strain rate tests on quartz sand (Bragov et al. 2008).	122
Fig. 48 Typical data from laboratory projectile penetration tests for similar target density ($e_0 = 0.75$) and similar impact velocity ($v_0 = 377$ m/s) and varying tip shape.	123
Fig. 49 Response of different tip shapes to impact for similar target densities and impact velocities; $e_0 = 0.75$, $v_0 = 377$ m/s.	124
Fig. 50 The effect of density on the energy release with depth in Siligran 0.125-0.71 target. ...	125
Fig. 51 Response of tip shapes for different particle morphology at similar densities and two different impact velocities.	126
Fig. 52 Comparison of displacement computed by double integration of the acceleration time histories and measured displacements of the projectile in the sand target after careful excavation.	127
Fig. 53 Example of variation of force at different depths of penetration for a hemispherical tip projectile impacting Siligran 0.125-0.71 target at 377 m/s. Energy dissipated was determined by computing the area of the shaded region.	128
Fig. 54 Energy dissipation from integration of force-displacement data recorded by the onboard data recorder (G-Rec) for different impact velocities.	129
Fig. 55 Average energy dissipated along the path of the projectile at different depths, 0-10%, 10-20%, 20-50%, and 50-100% of the penetration depth for different sands and projectile tip shapes. The number in parenthesis is the number of tests where multiple tests with the same sand type and nose shape were performed.	130
Fig. 56 Results of laboratory projectile penetration tests at different impact velocities, target densities and tip shapes. All targets were Siligran 0.125-0.71 except where noted.	131
Fig. 57 Streak of white comminuted particles along the path of the projectile. Ottawa and Q-Rok targets were prepared by placing Ottawa sand Q-Rok along the path of the projectile confined by Siligran.	132
Fig. 58 Maximum deceleration experienced by projectiles in laboratory tests at different.	133
Fig. 59 Relationship between maximum deceleration and penetration depth for different tip shapes and target densities.	134
Fig. 60 The drag coefficient, C_D , for different projectile tips and similar target densities ($e_0 = 0.75$) and impact velocities ($v_0 = 377$ m/s).	135
Fig. 61 Penetration depth predictions from Poncelet's equation (black dashed lines) for average $C_D = 0.80$ compared with that recorded in projectile penetration test for different nose shapes. The mass and diameter of the project were 0.210 kg and 0.03 m respectively, and the target was Siligran 0.125-0.71 sand at $e_0 = 0.75$	136
Fig. 62 The variation of drag coefficient, C_D with impact velocity for different tip shapes, target densities and sand. Most of the targets were Siligran 0.125-0.71. Three targets were of Siligran 1.0-1.6, and two each of Ottawa sand and Q-Rok placed along the path of the projectile.	137
Fig. 63 Schematics showing aspects of cavity expansion model and material model.	138

Fig. 64 Predictions of laboratory projectile penetration depths using cavity expansion model and Tresca failure criterion.	139
Fig. 65 Predictions of laboratory projectile penetration depth using cavity expansion model and Mohr-Coulomb failure criterion.	140
Fig. 66 Maximum frontal stress on the projectile compared with frontal stress computed from cavity expansion model for different nose shapes and failure criteria, Mohr-Coulomb and Tresca failure criteria. MC = Mohr-Coulomb, Hemis = Hemispherical nose shape.	141
Fig. 67 The effect of tip material on projectile penetration depth in Siligran 0.125-0.71 target; $e_0 = 0.63$, $v_0 = 377$ m/s. The target void ratio (e_0) for steel tips was 0.73.....	142
Fig. 68 The response of constant mass (0.210 g) 60° cone projectile with different tip material on Siligran 0.125-0.71 target at similar densities ($e_0 = 0.63$) and two different impact velocities. The e_0 of the target for steel tips was 0.73.....	143
Fig. 69 Instantaneous velocity profiles for constant mass projectiles with different tip materials impacting on Siligran 0.125-0.71 sand targets prepared at similar densities ($e_0 = 0.63$) at two impact velocities.	144
Fig. 70 Particle crushing at different strain rate in 1D compression test of Ottawa sand.....	153
Fig. 71 Particle crushing at different strain rates in 1D compression test of Q-Rok.	154
Fig. 72 Particle crushing at high strain rate in rounded Ottawa sand and angular Q-Rok.....	155
Fig. 73 Particle crushing in high-strain rate 1D compression tests in SHPB in Ottawa sand with different moisture content.	156
Fig. 74 Particle crushing in high-strain rate 1D compression tests in SHPB in Siligran sand with different moisture content.	157
Fig. 75 Particle crushing in high-strain rate 1D compression SHPB tests in Siligran specimens at different densities.....	158
Fig. 76 Images of (a) trail of comminuted particles along the path of the projectile (b) false tip of comminuted particles at the projectile tip, scanning electron microscope images of (c) uncrushed Siligran sand (d) low resolution image of crushed particles from the tip (c-d) high resolution images of crushed particles from the tip.	159
Fig. 77 Particle crushing in high-strain rate 1D compression test in SHPB and along the path and tip of the projectile with impact velocity of 377 m/s.	160

INTRODUCTION

Motivation

Terminal ballistics is the study of the behavior of a projectile as it travels through the target, dissipates its kinetic energy and comes to a rest. Terminal ballistics in geomaterial could involve either a projectile or a meteor impacting on earth's surface. Here, only the case of a projectile impacting granular material is considered. One of the most important parameters in terminal ballistics is the depth of penetration. It determines the damage caused to the target (ground) and the asset (e.g. bunker), if the target is protecting an asset. Therefore, determining the depth of penetration is of paramount importance for both defensive and offensive measures. Laboratory and field tests have shown that projectile characteristics such as impact velocity shape and mass of the projectile along with target characteristics, such as the strength of the target are important considerations in determining the penetration depth.

Laboratory scale projectile penetration tests have shown extensive particle fracture along the path of the projectile (Allen et al. 1957a; Glößner et al. 2017). In addition, a false tip composed of comminuted particles forms around the original tip of the projectile (Allen et al. 1957b; Glößner et al. 2017). The grain size of the crushed particles along the path of the projectile are reduced by as much as three orders of magnitude (Bless et al. 2009). Analyses of the comminuted fines from Glößner et al. (2017) have also revealed two to three orders of size reduction. Penetration depth was reduced in higher density targets. A definitive correlation between penetration depth and tip flatness could not be established in the range of velocities adopted in the test (Glößner et al., 2017), which was attributed to the formation of false tip. However, laboratory tests by Allen et al. (1957a) have shown increasing penetration depth with decreasing cone angle of the projectile tip. Empirical relations developed on a large dataset of field penetration tests have also indicated significant influence of projectile nose shape on penetration depth (Young 1969). These observations indicate that a model to predict penetration depth should endeavor to account for the energy loss in particle fracture in the immediate vicinity of the projectile in addition to the projectile and target characteristics.

In the past, empirical and analytical models have been proposed to determine penetration depth. Empirical methods are attractive due to their ease of use; however, they are case specific

and may require multiple calibration tests for each case. On the other hand, the basic framework of analytical models developed with reasonable assumptions can be used for multiple scenarios. Cavity expansion models are universally used for penetration models and have been shown to successfully predict penetration depths in geomaterials (Forrestal and Luk 1992). A framework to determine projectile penetration depth using continuum approach, and material parameters from single grain crushing tests, and dynamic tests is proposed for the first time in this study.

Projectile penetration in granular medium

Over the past decades a database of projectile penetration into geomaterials have been developed (Young 1969). These large-scale tests have provided valuable information on the penetration depth and resistance to the intruder as it moves in the granular medium. However, such large-scale tests are expensive in terms of time, money, and logistics. Additionally, in the case of projectile penetrating soils, a prohibitively large number of tests would be required to accommodate the effects of soil profile variations and projectile characteristics. Efforts have been made to develop analytical models to predict the penetration depth in granular medium (Backman and Goldsmith 1978; Boguslavskii et al. 1996b; a; Westine 1975). The cavity expansion method is a popular model to predict penetration depth in granular material (Chen and Li 2002; Forrestal and Longcope 1982; Forrestal 1986; Forrestal and Grady 1982; Forrestal and Luk 1992; Shi et al. 2014) . It is simple to use and can be applied equally successfully to rocks (Forrestal 1986), concrete (Forrestal et al. 1994; Forrestal and Tzou 1997; Luk and Forrestal 1987; Warren et al. 2014) and metals (Forrestal et al. 1988, 1995). The stress generated to expand the cavity is used to compute the forces on the projectile nose as it is penetrating the medium. The penetration depth is then obtained by applying Newton's second law. Hence, the mass of the projectile plays an important role in determining the penetration depth. This allows the model to successfully predict penetration depths for both small and large projectiles. The other factors that greatly influence the penetration depth is the strength of the target, and the friction between the target the projectile. Though cavity expansion methods are popular in predicting penetration depths, the continuum approach does not account for the energy loss in comminution of the granular medium during the penetration process. Therefore, the strength of the granular material must be chosen in such a way

that it accounts for the effect of particle fracture on the strength of the medium. The choice of strength value then becomes an important aspect of using the model.

With the advent of cheaper computer hardware, access to high speed computational tools and development of novel numerical frameworks, computer simulations are increasingly used to gain insights into physical processes in many areas of science and engineering. When the physical phenomenon involves length scales ranging in orders of magnitude, these numerical simulations can provide valuable insights into the effects of micro-scale and meso-scale interactions on the observable macro-scale.

Projectile penetrating a granular medium is a complex multi-scale and multi-physics problem. At the micro-scale, is the intra-granular properties such as crystal structure, and surface and volume flaws responsible for the fracture of individual grains. The inter-granular and grain to projectile interactions is the meso-scale phenomena, and at the macro-scale is the overall depth of penetration and the bulk response of the granular mass. Modeling such complex and multi-scale interactions requires advanced numerical frameworks and considerable computational power. Additionally, in order to preserve numerical stability of computer codes operating across multiple scales innovative messaging protocols need to be developed.

One approach to address the multi-scale problem is to couple two existing numerical frameworks developed for the different scales. There are ongoing efforts to couple the discrete granular mechanics with the continuum behavior, coupling the FEM (Finite Element Method)-DEM (Discrete Element Method) frameworks (Guo and Zhao 2014; Oñate and Rojek 2004). Until such codes are fully functional, reliably tested, and routinely available, a framework to determine material parameters from particle crushing tests and dynamic tests results for current empirical or analytical models would be helpful.

As the projectile moves in the medium, particles in the path of the projectile are displaced to the sides (Collins et al. 2011; Omidvar et al. 2016). This region may not extend beyond few projectile diameters. The sand grains in the path of the projectile form short force chains and are displaced axially (Borg et al. 2017). Particles fracture when the force in the chain exceeds the strength of the weakest grain leading to the formation of new chains. The weakest grain among the new chains will fracture, redistributing the load again. Thus, for sands in the near field, directly in front of the projectile tip, the stress state may approach that in 1-D (One-Dimensional)

compression state. The stress state in the material in the far field, which provides confinement, can be approximated by triaxial stress state. Therefore, both granular properties such as single grain crushing strength and material properties for phenomenological continuum models are required for modeling projectile penetration. The granular interactions and crushing can then be modelled using DEM and the far field continuum using FEM.

Penetration in granular medium is accompanied by extensive comminution along the path of the projectile and the formation of false tip. Quantifying comminution in terms of particle size distribution and new surface areas created may help in determining the energy dissipation in the comminution process. Also understanding the comminution of granular material; what are the dominant factors that control the various phases of the fracturing process, is important in field of geomechanics and power industry. This knowledge can also be invaluable to researchers trying to develop realistic micro and meso scale models of particle fracture during the passage a projectile, and in validation of multi-scale models predicting penetration depth.

In the effort to understand and model the various phenomena in the multi-scale problem, Dr. Dayakar Penumadu worked collaboratively with Dr. Nik Petrinic at Oxford University's Impact Laboratory and Dr. Christoph Glößner at Fraunhofer Institute for High-Speed Dynamics, Ernst-Mach-Institut (EMI). Dynamic test on sands of different shapes were performed in Split Hopkins Pressure Bar to understand the effect of strain rate, shape, density, lateral confinement, and moisture on stress-strain behavior at high strain-rates. Some of the results have been reported in De Cola et al. (2018) and results and conclusion will be used in this work. The Oxford group have also made efforts to couple FEM and DEM framework for projectile penetration in granular material simulation. In order to understand the role of tip shape and target characteristics, laboratory projectile penetration tests were performed with instrumented projectiles at EMI. Description of the testing system, G-Rec on-flight data acquisition and recording system, and some preliminary results are available in Glößner et al. (2017). The results of the tests, and the complete flight history data in the form of acceleration, velocity and time histories, were available for independent analyses and are presented in this work. All the data reduction work associated with G-Rec was performed at EMI.

Chapter 1 presents results of conventional drained triaxial compression tests on quartz sands of similar size but different particle morphology. The role of particle shape on the behavior

of granular material was investigated. Particle crushing in 1D (1-Dimensional) compression was quantified and the shape change of particles after crushing is presented.

Chapter 2 presents the results of conventional triaxial compression tests in vacuum consolidated specimens. Since the soil close to the surface is subjected to small vertical stress, triaxial tests were performed under low confining stresses. The confining membrane was speckled patterned, and images captured for spatial resolution of surface strains using 3D (3-Dimensional) digital image correlation. Visualization of surface strains on the surface of vacuum consolidated specimens shows regions of localization in granular material. Investigation of localization in granular materials are important because in laboratory displacements are measured at the top of the specimens and the volume change is measured by the amount of water flowing out of the saturated specimen. Localization immediately invalidates such measurement and hence the subsequent computation of strains and stresses.

Chapter 3 presents results of single grain crushing tests on quartz sands of different grain shapes. The results were modelled using Weibull distribution and the size dependence of strength is presented. The influence of surface features on the single grain crushing strength is discussed with the help of 3D images of grains and high-speed imaging during the test. Acoustic emissions during single grain crushing tests were also measured to understand the differences in the fracture of different shaped grains, and to measure the fracture energy.

Chapter 4 presents results of instrumented projectiles impacting at different velocities. The influence of target density and tip shape was explored. The availability of full flight history provides a unique opportunity to investigate the response of projectile as it travels in the target. The various aspects of target resistance to different nose shapes and tip material are also explored. The penetration depth increases with increasing impact velocity. The maximum deceleration at impact depends on the density of the target and the tip shape, flatter tips and denser targets produce higher maximum decelerations.

Chapter 5 presents results of particle crushing in specimen subjected to high strain rate in Split Hopkins Pressure Bar. Quantitative measure of particle crushing is determined for different particle shape, specimen densities, and moisture content. The effect of density on particle crushing at high strain rate was not very significant. Particle crushing decreased with increasing moisture content.

CHAPTER I
EFFECT OF PARTICLE MORPHOLOGY ON STIFFNESS, STRENGTH
AND VOLUMETRIC BEHAVIOR OF ROUNDED AND ANGULAR
NATURAL SAND

A version of this chapter was submitted by Aashish Sharma, Alexia Lieb, Mohmad M Thakur and Dayakar Penumadu:

Sharma, A., Lieb, A., Thakur, M. M, and Penumadu, D. “Effect of Particle Morphology on Stiffness, Strength and Volumetric Behavior of Rounded and Angular Natural Sand.” *Journal of Geotechnical and Geo-Environmental Engineering*.

Alexia Lieb performed triaxial tests on Ottawa sand and Q-Rok and most of the particle size distribution tests and contributed to the manuscript by writing portions of the introduction. Mohmad M Thakur performed triaxial tests and surface roughness tests on Siligran Euroquarz. Aashish Sharma developed the idea and supervised the experimental tests, performed some experimental tests, analyzed the data and wrote major portions of the paper.

Abstract

The role of particle shape in the response of granular materials to external mechanical loading has been explored to address the geomechanics of projectile penetration problem. Effect of particle shape on deformation behavior has been addressed in the past but the relative contributions of morphology and texture has not been considered. In this study, stress-strain and volume change behavior for clean sands which had distinct particle shape (rounded and angular) with very similar chemical (mineralogical) composition, size and texture, in one-dimensional compression and drained triaxial compression are presented. The effect of particle morphology on crushing behavior in one dimensional loading is explored using laser light scattering technique which is suitable for small volume of crushed particle specimens. Particle size distribution in both volume/mass and number distributions are considered for improved understanding associated with the process of comminution. It was found that rounded sand specimens showed marginally greater crushing than angular sand specimens with higher uniformity coefficient. Densification of angular sand results in a similar improvement in stiffness than replacing it with loose rounded material for example when deciding protection against the impact of a projectile to take advantage of higher friction angle of angular grains. The influence of particle morphology is more evident in loose state than in dense state. The effect of grain shape on critical state friction angle is also quantified.

Introduction

Particle crushing in granular materials is an important phenomenon as it influences the stress strain behavior of granular material subjected to high compressive stresses (Yamamuro and Lade 1996), such as those present at the tip of piles during pile driving, underneath a high dam and near penetrating projectiles. At locations of such high stresses, grain fracture and fragmentation will contribute to the plastic deformation in addition to slippage and reorientation of the particles. Particle fracture will also change the gradation and shape of the particles, which in turn will influence the strength and volumetric response of the crushed mass. It is therefore important to determine the evolution of particle shape and size during particle crushing and subsequently the influence of the evolved particle shape and gradation, on the evolution of strength parameters to determine the response of granular materials subjected to high stresses. Particle crushing and the evolution of particle shape and strength parameters from grain fragmentation is particularly important when a fast-moving projectile impacts a granular mass and travels within the medium. The state of the granular mass and its resistance to the intruder may depend on the magnitude and velocity of the stress waves generated at impact. At high enough impact velocities, the stress waves travelling ahead of the projectile will densify and fracture grains along the way (Van Vooren et al. 2013), leaving a trail of comminuted particles along the path (Allen et al. 1957a; Glößner et al. 2017). There is a high possibility of changed morphology and gradation than the original material. Since resistance to the intruder and the depth of penetration depends on the strength of the resisting mass (Forrestal and Luk 1992), it is therefore important to determine the strength of the medium that the projectile is going to pass rather than the strength before the impact. There is also the possibility of crystallographic phase change, due to increased temperature from friction (Peng and Redfern 2013), at the tip of the projectile as it penetrate the medium.

Comminution is quantified using particle size distribution (PSD) curves, either by comparing the changes in the mass of the material finer than a chosen particle size (Lade et al. 1996; Lee and Farhoomand 1967; Miura et al. 1997) or based on the shift in the particle size distribution curves (Einav 2007; Hardin 1985; Marsal 1967). The PSD curves for soil specimens with particle sizes ranging from clay sized to sand sized is generally determined from sieve analysis for the coarse fraction and from hydrometer analysis for the fine fraction passing No. 200 sieve. The use of sieves with square meshes is inadequate in determining the distribution of fines

smaller than 74 μm . The particle size calculated in the hydrometer analysis is the equivalent diameter of the sphere that settles at the same terminal velocity as the particles. The size in sieve analysis is the smallest dimension that can pass through the square mesh as the grains are bouncing and rolling on the mesh. These two very different definitions of particle size are then integrated in the same particle size distribution curve to present the variation in grain size in the sample. A consistent definition of particle size can be used for a wide range of particle sizes using the laser light scattering technique. It is specially a well-suited method for characterizing comminution (Huang et al. 2014a) as the technique can be used for a wide range of particle sizes, including sub-micrometer (Dishman et al. 1993; Ruckdeschel et al. 2016).

Size distribution in geotechnical engineering is generally determined using sieve analysis even when analyzing comminution (Coop et al. 2004; Hagerty et al. 1993; Hardin 1985; Lade et al. 1996), in which considerable fines smaller than the No. 200 sieve are produced. There are two issues related to characterizing comminution using sieve analysis, one is that it becomes increasingly difficult to characterize fines that are smaller than No. 200 sieve. The other issue is, that it is mass based distribution; a few larger particles can bias the distribution towards larger particle sizes. These issues are more pronounced when the specimen volume is small, as in the case when analyzing comminution in dynamic impact tests in Split Hopkins Pressure Bar tests. Laser light scattering technique is well suited to characterize small volume wide particle size range samples.

The angle of shearing resistance is an important measure of the shear strength of granular materials. The friction angle of coarse grained soils depends on a number of factors, chief among them are: the size of the particles (Koerner 1970), the texture or surface roughness of the grains (Cavarretta et al. 2010; Morris 1960), mineralogy of the grains (Koerner 1970) and shape of the particles (Chen 1948; Cho et al. 2006; Holubec and D'Appolonia 1973; Koerner 1970; Kolbuszewski and Frederic 1963; Miura et al. 1998; Morris 1960; Shin and Santamarina 2013; Taylor 1948; Terzaghi and Peck 1948). Studies have also shown the influence of particle angularity on the steady state friction angle at large strain (Chan and Page 1997; Koerner 1970; Santamarina and Cho 2004). In the past the influence of particle shape on the stress-strain behavior has been evaluated either on datasets of natural and artificial materials (Alshibli and Cil 2018; Miura et al. 1998), on material of same mineralogy but prepared in laboratory by crushing (Koerner 1970), on

materials with different mineralogy (Holubec and D'Appolonia 1973), on materials with very different particle size (Guo and Su 2007), and on material prepared by mixing different fractions of rounded and angular grains (Shin and Santamarina 2013). Attempts have also been made to investigate the role of roughness on peak friction angle and dilatancy angle (Alshibli and Alsaleh 2004), however the effects of roughness and angularity were not decoupled. These studies have generally concluded index densities ($e_{\min}$, $e_{\max}$) increase with increasing angularity and for a given particle size rounded grains packs more densely than angular grains. Also, the peak friction angle increases with increasing grain angularity.

Past studies on the effect of particle morphology on the response of granular materials has addressed specific issues: (a) effect of particle shape on the peak friction angle (Koerner 1970; Miura et al. 1998), (b) effect of particle shape on packing and initial state (Altuhafi et al. 2016; Holubec and D'Appolonia 1973; Koerner 1970; White and Walton 1937), (c) effect of particle shape on dilatancy (Alshibli and Cil 2018; Guo and Su 2007), (d) effect of angularity on the steady state friction angle (Altuhafi et al. 2016; Chan and Page 1997; Koerner 1970; Santamarina and Cho 2004), and (e) effect of roughness on the peak friction angle and dilatancy angle (Alshibli and Alsaleh 2004). Recently, Altuhafi et al. (2016) compiled a database of 25 natural sands and tried to investigate the role of surface roughness, based on quantitative measurements, on packing and critical state parameters. They concluded that roughness increases with angularity and it was not easy to decouple the effect. This paper presents the role of particle shape on the behavior of granular materials via results of experiments performed on two unground sands with the same mineralogy, surface roughness and similar particle size distribution. The mineralogy is verified using X-ray diffraction. Results of tests on a third sand with similar mineralogy, and surface roughness but different particle size distribution is also presented. The effect of particle morphology on the initial state of granular materials is discussed via index properties. One-dimensional (1D) compression tests, isotropic compression tests, and triaxial test results are presented to discuss the effect of angularity on the volumetric response and stress-strain behavior. Also, insights into the effect of particle morphology on grain crushing in granular material subjected to high compressive stresses are provided by analyzing 1D compression test specimens at the end of loading with laser light scattering technique and image analysis method. The evolution

of particle shape due to crushing is presented using some simple shape parameters derived from 2D images of particles.

Materials

Commercially available sands, Ottawa sand and Q-Rok were chosen to study the effect of grain morphology on grain crushing and stiffness in 1D compression stress state, and strength and volumetric response under triaxial stress state. Ottawa sand is 20/40 oil frac unground silica with rounded grains and Q-Rok is unground silica with angular grains. Median roundness values computed from two-dimensional gray scale optical images of the thirty particles retained on US No. 30 sieve were 0.73 and 0.49 for Ottawa sand and Q-Rok. The median sphericity computed as the ratio of the width to length of the grain were 0.84 and 0.76 respectively. These shape factors were determined from computational geometry (Zheng and Hryciw 2016), and are comparable to the shape chart in Krumbein and Sloss (1963). In addition, Euroquarz Siligran 0.175-0.71 mm, a European sand with index densities very similar to the 20/40 Ottawa sand was also chosen for triaxial testing. The median roundness and sphericity values were 0.66 and 0.82 respectively. The mineralogy of Ottawa sand and Q-Rok are very similar with silicon dioxide constituting more than 99.5% of the grains. X-ray powder diffraction patterns for Ottawa sand, and Q-Rok along with indexed peaks for α -quartz are shown in Fig. 1. The specimens for powder diffraction were prepared by pouring the sand into powder sample holder, and gently pressing and smoothing the top for a plane diffraction surface. Only the peaks for those crystallographic planes that were suitably oriented are seen in the diffraction patterns of the two sands. The results indicate that both the sands are mineralogically identical albeit with small differences in orientation of the crystallographic planes when deposited in air. This should have no consequence for the triaxial tests as the tests were performed at low confining stresses and particle crushing is not significant. In 1D compression tests where applied stresses exceeded the fracture strength of sand grains, grain orientation could have, hence unknown effects on the comminution and compressibility.

The Scanning Electron Microscope (SEM) images showing the rounded shape of the Ottawa sand and the angular grains of Q-Rok are presented in Fig. 2 (a) and Fig. 2 (b), respectively and high magnification images of the surface of these sands are shown in Fig. 2(c) and (d) respectively. The defects, depression and holes, on Ottawa sand surface (Fig. 2c) and small

protrusions on Q-Rok surface (Fig. 2d) are few micrometer in size. The angular ridges on the surface of Q-Rok grains are few hundreds of micrometers in size contributing to angularity. Prominent angular ridges are absent in Ottawa sand grains. Not visible in the micrographs are the multi granular nature of larger grains of Q-Rok. Non-contact roughness measurements were made with Keyence VK-X250 confocal laser microscope. Typical surfaces of the three sands area show in Fig. 3. Roughness values are based on 3-D surface profiles of ten sand grain at 50X magnification. Surface roughness were computed from 200 x 200 micrometer area. Roughness is defined as the mean absolute height of the surface points from the average height. The average roughness values were 2.36, 3.83 and 2.68 micrometers respectively for Ottawa sand, Q-Rok and Siligran. The roughness values are very similar for the three sands.

The behavior of coarse-grained material is greatly influenced by its initial state quantitatively referred to as relative density. The minimum (e_{min}) and maximum (e_{max}) void ratios were determined using procedures specified in Standard Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table (D 4253) (ASTM 2014a) and Standard Test Methods for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density (D 4254) respectively. Three tests were completed for each limiting density for Ottawa sand and Q-Rok and the average values are presented in Table 1. In addition to these ASTM methods, a slight modification of Lade et al. (1998) method was also used to determine the extreme limiting void ratios. After each addition the cylinder was lightly tapped with a soft raw-hide mallet six times on four diametrically opposite locations, a total of twenty-four light taps. This process was continued until all the sand was poured in the cylinder. The volume of the sand was determined from the graduated cylinder to the nearest 10 mL. After minimum void ratio, the top of the cylinder was covered with a stopper and the cylinder turned upside down and then slowly placed upright again, in 45 s – 60 s, to determine e_{max} . Three tests were performed to determine the average minimum void ratio, and the average maximum void ratio was determined from ten tests and are presented in Table 1.

The e_{min} and e_{max} using the ASTM method for Ottawa sand were 0.505 and 0.689, and 0.634 and 0.910 respectively for Q-rok. The e_{min} and e_{max} from the cylinder method were 0.51 and 0.75 respectively for Ottawa sand, 0.69 and 1.01 for Q-Rok and 0.52 and 0.78 for Siligran. For the same deposition method and energy, the rounded Ottawa sand packs more densely than the angular

Q-Rok. Also, the difference between the loose and dense state of packing is larger for the angular sand than the rounded sand. The values of limiting void ratios from the cylinder method were used when computing the relative densities (D_r) of the test specimens because similar procedures were adopted for preparing tests specimens. For Ottawa sand, both methods produced similar e_{min} , whereas the ASTM method produces a denser packing for the Q-Rok. The angular and multi granular nature of Q-Rok, especially the larger grains may result in some crushing due to vibration a heavy surcharge in the ASTM method leading to a smaller value. Therefore, the cylinder method maybe more appropriate for angular sands and easily crushable materials. Even though, the sand is deposited from zero height of drop for e_{max} in the ASTM method, the kinetic energy of the flowing sand particles in vertical drop may result in denser state than in the cylinder method where the sand grains gently roll to rest. The ASTM method may not always produce the densest packing; an air pluviation method with drop height of 40 to 50 cm has been shown to produce a denser state (Presti et al. 1992). Siligran is more angular than Ottawa sand, and the similarity of index densities to Ottawa sand is the result of combination of the effects of particle size distribution and angularity on the packing density.

Procedure

Particle size distribution

The particle size distributions were determined using a stack of square meshes as per the procedures specified in the Standard Test Methods for Particle-Size Distribution (Gradation) of Soils Using Sieve Analysis (D 6913) (ASTM 2017). In addition to the sieve analysis, laser light scattering technique using a commercially available instrument, Malvern Mastersizer S, was also used to compute the particle size distribution curves. The instrumental setup consists of a laser source, an array of light sensors at various angle from the direction of the incident light source, and suspended particles flowing in a specimen cell, in between the source and the sensors. The beam passing through the specimen is scattered at various angles, larger particles scattering light at smaller angles and smaller particles scattering light at larger angles. This distribution of scattered intensities is then analyzed using Mie scattering theory (Mie 1908). The Mie theory is a mathematical description of how spherical objects scatter light. The particle size is reported as volume equivalent sphere diameter.

Samples for the laser light scattering were prepared by mixing approximately 2 g of representative mass in 50 ml of water in a glass vial with a cap. The vial was shaken by repeatedly turning it upside down to completely disperse the sand grains in water. A plastic dropper was used to sample from different heights (top, middle and bottom) of the suspension. The steps of shaking the vial and extracting specimen was repeated until enough specimen was gathered for measurement.

Sieve analysis and laser light scattering method do not permit the analysis of particle shape. Both particle size and shape analysis can however be performed using 2D images of the grains captured at suitable magnification. Images of a representative sample of particles placed on a slide were captured using an optical microscope. Particle sizes can vary over a wide range and there arises a necessity to sieve the sample into different particle sizes for appropriate magnification. For proper quantification of the shape and size it is necessary that each grain is represented by adequate number of pixels. The bulk sample were sieved through 250 μm (No. 60) sieve and the fraction retained on 250 μm was imaged at a lower magnification while the fraction passing 250 μm was imaged at higher magnification. Approximately 1-2 g of the two fractions were then placed in separate vials and mixed in 50 ml of water. The vial was repeatedly shaken by turning it upside down and upon standing a drop from the top, middle and bottom was placed on the slide. The steps of shaking and sampling were repeated until there was enough specimen on the glass slide without overloading the slide. The slide was then air dried before imaging. The images were analyzed in the image processing software ImageJ (Schneider et al. 2012). The analysis included conversion to binary image and separation of the contacting grains using the watershed algorithm. Particle size is reported as the projected surface area equivalent diameter.

1-D compression

The 1-D compression specimens were prepared in a steel tube. The internal diameter of the steel tube was 19 mm and the specimen heights were approximately 20 mm. The loose specimens were prepared by placing sand in the tube from zero drop height. The dense specimens were prepared by compacting the sand in three equal layers. After placing a layer, the tube was tapped on the side five times in four diametrically opposite directions. After tapping, the layer was tamped

25 times. A maximum of 22 kN axial load, equivalent to 77 MPa, was applied to the specimens at the rate of 0.1 mm/min in a displacement-controlled testing system.

Triaxial tests

The triaxial tests were performed on cylindrical sand specimens of 71 mm diameter and 178 mm height. Tests were performed on two relative densities (loose and dense) and for three effective confining stresses of 69 kPa, 103 kPa, and 138 kPa. The loose specimens were prepared by pouring from a slowly rising funnel in a circular pattern maintaining a zero height of drop. The dense specimens were prepared by adding 50 grams of sand to the mold and then lightly tapping the diametrically opposite sides six times with a raw-hide mallet, twenty-four taps in total for each layer. Carbon dioxide was used to flush air out of the specimens and were subsequently saturated with deaired water and *B* value check was implemented for ensuring acceptable saturation state of the specimens prior to consolidation.

After saturation, the specimens were consolidated at the desired effective stress for one hour. Volumetric strains were calculated by measuring the volume of pore water flowing out of the specimen by differential pressure transducer (DPT). After consolidation, the specimens were sheared at a rate of 12%/hour to a maximum strain of 25%. Data reduction, and calculations including area correction and membrane correction were performed as stated in Standard Test Method for Consolidated Drained Triaxial Compression Test for Soils, ASTM D7181 (ASTM 2011a).

Isotropic consolidation

Isotropic consolidation test specimens were prepared in the same manner as the triaxial tests specimens. The confining stress was increased in small increments and three cycles of loading and unloading were performed with the maximum effective confining stress of 483 kPa. Each confining stress was held for five minutes before applying the next stress increment.

Results and analysis

Particle size distribution

The grain size distribution curves from the sieve analysis for the two sand samples are shown in Fig. 4. The maximum particle size for Ottawa and Q-Rok was around 850 μm and the

minimum particle size was around 300 μm and 150 μm , respectively. For Siligran the maximum particle size is 710 μm and the minimum particle size is 125 μm . The mean particle size (D_{50}) and coefficient of uniformity (C_u) were 598 μm and 1.43 respectively for Ottawa sand, 470 μm and 1.74 for Q-Rok and 380 μm and 2.21 for Siligran. The classification for the sands is poorly graded sand (SP) as per the Unified Soil Classification System, ASTM-D2487 (ASTM 2011b). The soil classifications and the values for D_{50} , D_{10} and C_u are presented in Table 1.

The particle size range for the light scattering system used in this study was 0.05 μm to 850 μm , thus it was possible to characterize size distribution without having to resort to the dual definition of particle size when analyzing crushed specimens consisting of very fine grains. The particle size distributions in sieve analysis is mass based while those from the light scattering technique are volume distributions. These are equivalent when the specific gravity of the sand particles is assumed to be constant. The volume distribution can be converted to number distribution of equivalent sphere sizes albeit with the introduction of certain error due to the assumption of spherical particle shape. However, such a conversion gives an indication of the number of particles of different sizes present in the granular mass which is important in understanding the crushing behavior of granular materials. The volume distribution and number distribution obtained from the light scattering technique for uncrushed Ottawa sand and Q-Rok are shown in Fig. 5. The D_{50} of the volume distribution for Ottawa sand and Q-Rok is 514 μm and 435 μm respectively and 405 μm and 302 μm respectively in the number distribution. The difference in volume and number distribution is greater in Q-Rok because of the presence of considerably more finer particles.

Image analysis produces a number distribution in which the number of particles for a given size range is counted from 2D images. The number based particle size distribution from the laser light scattering technique and the image analysis for the Ottawa sand and Q-Rok are shown in Fig. 6. The distribution is based on 83 Ottawa sand grains and 331 Q-Rok particles. The size distributions from both the methods are similar. Analysis of 2D images is a simple tool that can be used for particle size characterization of granular materials. A major advantage of image analysis over sieve analysis and laser light scattering technique is that it can be used for shape analysis for determining the evolution of grain shape in particle crushing.

There are limitations of all the three techniques used for size characterization. The sieve analysis is a mass-based technique and susceptible to bias. This is evident in both the distributions shown in Fig. 5, however it is more prominent in the difference between the volume and number distribution for Q-Rok as it contains more fine particles. Even with this limitation, sieve analysis remains the most popular method because of its simplicity, economy and ease of use. The light scattering analysis evaluates particle sizes using an equivalent volumetric sphere method to calculate the diameter. This assumption leads to particle sizes with the smallest possible dimension for a given volume, as the diameter of the sphere is the smallest dimension for a given volume. As a result, it will compute a larger number of particles for a given volume. The other limitation of the laser light scattering technique is the validity of very small specimen volumes representing bulk mass. The gradation curves for sands generated from volume distribution using the laser light scattering technique is also shown in Fig. 4. The similarity between results of sieve analysis and the volume distribution from light scattering suggests that with proper sample preparation technique and specimen extraction method, small specimen volumes may not produce large errors.

The image analysis evaluates particle sizes as equivalent area diameter based on the projected 2D image. The equivalent area diameter is the smallest dimension for the projected area. The image analysis also suffers from small specimen volume. Less than 1 mg of sand was deposited on the glass slides for imaging. However, with proper sampling technique the results from image analysis is not very different from those obtained from the laser light scattering technique for number distribution as shown in Fig. 6.

1-D compression

The results of the 1-D compression tests for the loose and dense specimens of Ottawa sand (OL and OD) and Q-Rok (QL and QD) are presented in Fig. 7. The initial void ratios for OL and OD were 0.76 and 0.53 respectively while those for QL and QD were 1.01 and 0.67. The specimens exhibit bilinear behavior over the range of axial stresses with regions of non-linearity during the initial stage of loading and in the proximity of the threshold stress. The threshold stress is defined as the stress where the slope of the stress-strain curve increases appreciably indicating the initiation of particle crushing. The threshold stresses for OL and OD were 27 MPa and 55 MPa respectively, and 25 MPa for QD. The corresponding void ratios at the threshold stress were 0.65, 0.46 and 0.57

for OL, OD and QD respectively. Densification increases the threshold stress and the threshold stress for QD was comparable to that of OL. The angular Q-Rok specimens, due to the larger initial void ratios exhibit varying degrees of collapse and rearrangement immediately upon the application of the load. The sudden increase in axial strain at the start of loading is approximately 2% and 0.5% for the QL and QD respectively. The axial strain at the maximum axial stress of 77 MPa for the OL and OD are 21% and 11% respectively and those for QL and QD are 29% and 19%. The loose specimens compress significantly more than the dense specimens; 10% more axial strain at the maximum axial stress. The axial compressions in Q-Rok specimens are 8% greater than those for the Ottawa sand specimens for similar packing density. The elastic strain recovery for all specimens was around 3%. Particle morphology and initial density significantly influence volumetric behavior under elevated stresses. Comparing OL and QD, the improvement in stiffness due to compaction is similar to that from changing the morphology of the grain from angular to rounded at similar placement density. With increasing stress there is a tendency for the void ratios to converge to a unique value. The void ratios at the maximum stress were 0.388, 0.367, 0.423 and 0.350 for OL, OD, QL and QD respectively.

The behavior of QL was significantly different from rest of the specimens. QL does not exhibit threshold stress as the other specimens in the range of applied axial stress. This is true even when the void ratio becomes smaller than the void ratio of QD at threshold stress. The void ratio of QL when the slope becomes more gradual at around 43 MPa was 0.564 which is smaller than 0.574 at the threshold stress for QD. The response of QL indicates that particle slippage and readjustment, and asperities and grain fracture occur simultaneously over the range of applied stress.

The particle size evolution after the 1D compression tests in terms of volume and number distributions are shown in Fig. 8. There are very small differences in the volume distribution of uncrushed (as received) sands before and after 1D compression. This is possibly due to small number of larger particles dominating the distribution in the crushed samples after 1D compression. Considerable differences are highlighted by the number distributions. There is significant increase in the number of fines in the micrometer and sub-micrometer size range. Visualizing particle size distribution in both mass/volume and number distribution provides a more complete information in understanding the role of coordination number in comminution. Since the

average force experienced by a grain in a granular mass depends on the number of contacting grains (Turner et al. 2016), the number distribution could provide useful insights into the comminution process. The nature of ultimate crushing is fractal uniquely determined by the maximum particle size before crushing (Coop et al. 2004; Einav 2007; McDowell and Bolton 1998). Crushing begins from the smaller particles as they have the fewer number of contacting neighbors and smaller coordination number. As the smaller grain around the larger grains fracture, either confining the larger particles or filling up the voids and thus increasing the coordination number for the larger particles, it becomes increasingly difficult to fracture these larger grains. Thus, the smaller particles with smaller coordination number continue to crush (Tsoungui et al. 1999). This is evident from the number distribution as the number of small particles have increased significantly, and at the same time the volume distribution still indicates the presence of larger particles. The crushing in 1D compression is quantified using, Einav (2007) relative breakage factor (B_r) defined in Eq 1

$$B_r = \frac{\int_{d_m}^{d_M} (F(d) - F_0(d)) dd}{\int_{d_m}^{d_M} (F_u(d) - F_0(d)) dd} \quad \text{Eq 1}$$

Einav modified Hardin's breakage factor by assuming a fractal nature for the ultimate distribution as given by $F_u(d) = (d/d_M)^{3-\alpha}$, where $F_u(d)$ is the ultimate cumulative PSD, d is the particle size, d_m and d_M are the minimum and maximum particle sizes, α is the fractal dimension taken to be 2.6 (Einav 2007; Sammis et al. 1987), $F(d)$ is the current cumulative PSD and $F_0(d)$ is the initial cumulative PSD. The value of B_r ranges from 0 to 1 for no crushing to complete crushing. The values of B_r for OL and OD were 0.12 and 0.15 and 0.10 and 0.09 for QL and QD respectively, with values for Ottawa sand higher than Q-Rok as shown in Table 2. Also presented are relative crushing values as per Lee and Farhoomand (1967). They defined relative crushing (B_r) as the ratio of D_{15} , particle size at which 15% of the material were finer, before and after crushing (D_{15i}/D_{15a}). The D_{15} of the crushed specimens, in the volume distribution, is translated by 100 μm in Ottawa sand and by 50 μm in Q-Rok irrespective of the density. The relative crushing values are similar for loose and dense packing. The extent of crushing is more evident in the number

distribution in which the D_{15} values are smaller than 1 μm , with 200 – 1000 times more particles than the uncrushed specimen. In general, soils with angular grains show more particle crushing than rounded grains (Lee and Farhoomand 1967), however the small differences in the PSD of Ottawa sand and Q-Rok may have contributed to more crushing in Ottawa sand specimens than in Q-Rok specimens.

The evolutions of particle shape after 1D compression for OD and QD are shown in Fig. 9. There is an increase in the aspect ratio of the particles for OD and decrease in the aspect ratio of the particles for QD. Aspect ratio is defined as the ratio of the major to the minor axis of the ellipse fitted to the 2D image of the particles. The implication is that rounded particles tend to fracture diametrically or cordially thus increasing the aspect ratio of the crushed particles, while asperities breaking in angular particles tend to make angular particles more rounded. Another very important implication from this morphological change is that, after particle crushing, the nature of strength volume response (rounded vs angular) could be significantly different than those determined for the uncrushed specimens. This is particularly important in projectile penetration where an impact wave travelling faster than projectile may change the density and morphology of the medium by the time the intruder passes through the same soil region.

Isotropic consolidation

The results for the isotropic consolidation tests performed on loose and dense specimens of Ottawa sand and Q-Rok are shown in Fig. 10. The volumetric response is non-linear over the range of stress applied. For the Ottawa sand, the volumetric strains at the maximum confining stress, σ'_c , of 483 kPa was 1.16 % for the loose specimen and 0.86% for the dense specimen. For the Q-Rok specimens the volumetric strains were 1.83% and 1.09% for the loose and dense specimens respectively. The elastic rebound upon unloading to 17.2 kPa ranged from 0.92% to 0.76% for loose and dense specimens of Ottawa and 0.96% to 0.89% for loose and dense specimens of Q-Rok specimens. There is significant improvement in volumetric behavior from densification for the angular deposit. The total volumetric strain for the dense Q-Rok is similar to that for the loose Ottawa sand. The volumetric strain during isotropic consolidation was influenced by both relative density and particle shape. However, particle morphology has a greater influence at low relative density. In addition, the total volumetric strain of the loose Ottawa sand specimen

with e_0 of 0.69 and the dense Q-Rok specimen with e_0 of 0.72 are similar. This observation along with the 1D compression results indicate that absolute void ratio could be a bigger influence than particle morphology.

Triaxial tests

The results of the consolidated drained triaxial tests performed on the loose and dense specimens of Ottawa sand, Q-Rok and Siligran are shown in Fig. 11, Fig. 12 and Fig. 13 respectively. The initial specimen states and pertinent test results are presented in Table 3. The shearing resistance increases with increasing effective confining stress and the dense specimens were stiffer and stronger than the loose specimens. The axial strain at failure, ε_p was smaller for the dense specimens than for the loose specimens. There was significant softening after the peak stress in the dense specimens. All specimens continued to dilate even at large strains possibly indicating formation of new shearing bands as compression progresses as reported by (Alshibli et al. 2003; Batiste et al. 2004; Desrues et al. 1996) using computed tomography images. Dilation was considerably greater for the dense specimens than for the loose specimens. Frictional end triaxial tests suffer from strain localization which may start early during the shearing stage depending on the density and confining stress (Alshibli and Sture 2000). Localized volumetric strains in regions of active deformation are different from global volumetric strains which are measured from the volume of pore water flowing in or out of the specimen. These global volume strains are then used to correct axial stresses by assuming uniform deformation. Hence, stress values and by extension critical state friction angle, ϕ'_{cs} , computed based on these global volume strains may not reflect the true value. Bolton (1986) proposed a simple saw blades model in which the peak friction angle, ϕ'_p , is the sum of ϕ'_{cs} and some fraction, k , of the dilation angle, ψ ($\phi'_p = \phi'_{cs} + k\psi$). Bolton proposed $k = 0.8$ for plane shear and approximately 0.5 for triaxial shear. Guo and Su (2007) have reported k values of 0.63 for Ottawa sand and 0.91 for angular crushed limestone. In this series of tests, k for angular grains ranged from 0.55 for Q-Rok, to 0.81 for Siligran. The value for Ottawa sand was 0.62. The value of k appears sensitive to void ratio and angularity. Q-Rok which is more angular but has a lower density than Siligran has a smaller k value. Whereas, Siligran which is more angular than Ottawa sand but has similar density shows higher k value. The dilation angle was computed using the equation proposed by Vermeer and de Borst (1984). Critical state friction

angles determined using Bolton's model is also shown in Fig. 14. The value for Ottawa sand is smaller than that for Q-Rok, displaying the influence of particle shape on ϕ'_{cs} . A similar approach, using rate of dilation, was used by Vaid and Sasitharan (1992) to determine ϕ'_{cs} for Erksak sand with excellent agreement.

There were two major variables in this suite of tests; density and particle morphology. The effect of densification on test parameters are shown in Fig. 15. With increasing D_r , both the ϕ'_p and ψ increases approximately by the same amount suggesting that the additional increase in the ϕ'_p is mainly from dilation. The peak friction angle was defined at maximum deviatoric stress. The axial strain to peak, ϵ_a at ϕ'_p , decreases with increasing D_r . Also, axial strain at the start of dilation decreases with density; the dense specimens begin to dilate almost immediately after the application of load with very little compression. The point of maximum compression was considered as the start of dilation. The effect of particle shape on test parameters are shown in Fig. 16. Particle interlocking in angular sands leads to higher friction angle (Guo and Su 2007) as shown Fig. 16a. Though, Siligran is less angular than Q-Rok, the specimens are relatively denser thus higher ϕ'_p for dense Siligran specimens. Also, shown in Fig. 16a are ϕ'_{cs} from Fig. 14 and the relationship between roundness (R) and ϕ'_{cs} (Cho et al. 2006). The role of particle shape on axial strain at peak friction is shown in Fig. 16b. Angular sands exhibit a larger strain at ϕ'_p , the difference is greater for loose specimens than for dense specimens. Dilation angle (Fig. 16c) is inhibited for Q-Rok, however the difference among the different specimens at a specified density is not very large. As seen in Fig. 16d there is a delay in dilation in Q-Rok as angularity inhibits rolling and sliding.

Relative density has significantly greater influence on the peak friction angle, dilation angle and the axial strain at peak friction angle. The influence of angularity on the ϕ'_p is greater in loose specimens than in dense specimens. Though, rounded sand dilates more than angular sand, the difference is not as significant as that from increasing the density. It also takes larger strain for angular specimens to reach peak strength and to start dilating. The influence of angularity is greater in loose specimens than in dense specimens.

Conclusions

Analysis of comminuted sand after 1D compression reveal that rounded sands become more angular while angular sands become more rounded. The use of laser light scattering presents an opportunity to characterize the grain size distribution for a wide particle size range using consistent definition of particle size. The volume distribution from laser light scattering is similar to the mass distribution from sieve analysis for both rounded and angular particles. Particle size distribution of crushed specimens in terms of both volume and number can be used to better understand the comminution process. Additionally, due to laser light scattering's ability to characterize particle ranging from clay sized to sand sized, a better quantification of comminution is possible in conjunction with fractal nature of ultimate distribution. Two-dimensional image analysis may also be used for both particle size and particle shape characterization. The increase in stiffness in densifying a loose angular deposit is similar to replacing the angular material by loose rounded material. The role of density and particle shape on strength and volume parameters are explored by performing drained triaxial compression tests on natural sands with similar composition, grain size distribution and surface roughness but very different particle shape. Relative density has a greater influence on strength and volumetric parameters than particle shape. The role of particle morphology is greater on loose specimens than on dense specimens.

Appendix

Table 1 Void ratio, and some index properties for Ottawa and Q-rok Sand.

	Ottawa		Q-rok		Siligran
	Cylinder Method	ASTM	Cylinder Method	ASTM	Cylinder Method
e_{min}	0.51	0.507	0.60	0.630	0.52
e_{max}	0.75	0.689	1.01	0.910	0.78
Sieve Analysis					
$D_{50}, \mu m$	595		475		380
$D_{10}, \mu m$	465		300		190
C_u	1.37		1.67		2.21
Classification	SP		SP		SP

Table 2 Breakage values computed using volume and number distribution using Lee and Farhoomand (1967) equation and volume distribution using Einav (2007) ultimate fractal distribution method

Specimen	Lee and Farhoomand (Volume)			Lee and Farhoomand (Number)			Einav (Volume)
	D_{15i}	D_{15a}	B_r	D_{15i}	D_{15a}	B_r	B_r
OL	389.9	288.1	1.35	266.0	0.19	1401	0.12
OD	389.9	283.1	1.38	266.0	0.53	502	0.15
QL	305.6	257.0	1.19	198.0	0.84	235	0.10
QD	305.6	269.0	1.14	198.0	0.20	988	0.09

Table 3 Initial states and results of consolidated drained triaxial compression of Ottawa sand and Q-Rok

	D_r	e_0	e_c	σ'_c	ϕ'_p	ε_p	ψ	Maximum Compression	Strain at Maximum Compression	Maximum Dilation
Specimen	(%)			(kPa)	(°)	(%)	(°)	(%)	(%)	(%)
OL1	32.9	0.669	0.664	69	32.6	9.5	3.4	0.32	1.55	-1.33
OL2	26.9	0.683	0.676	103	31.2	8.2	2.2	0.36	2.67	-0.74
OL3	29.1	0.678	0.669	138	32.2	10.1	2.9	0.33	2.44	-1.15
OD1	97.4	0.518	0.515	69	41.6	2.1	22.2	0.08	0.12	-7.51
OD2	91.5	0.532	0.527	103	40.5	2.8	21.1	0.10	0.20	-7.14
OD3	96.6	0.520	0.514	138	39.1	2.5	23.6	0.12	0.20	-7.57
QL1	29.6	0.917	0.907	69	35.0	13.6	1.5	0.64	4.95	-0.09
QL2	35.8	0.897	0.885	103	34.9	12.8	1.7	0.92	5.80	0.23
QL3	31.8	0.910	0.898	138	34.9	13.5	1.6	0.77	5.09	0.01
QD1	91.7	0.716	0.711	69	42.0	2.6	14.8	0.13	0.28	-6.67
QD2	94.8	0.706	0.700	103	42.7	3.1	15.6	0.15	0.26	-6.68
QD3	94.1	0.708	0.700	138	42.6	3.1	15.1	0.13	0.30	-7.12
SL1	28.5	0.706	0.701	69	33.0	11.5	2.85	0.26	2.15	-1.37
SL2	19.7	0.728	0.721	103	34.2	14.2	2.69	0.51	4.25	-0.50
SL3	25.1	0.714	0.704	138	31.9	14.3	1.86	0.61	4.58	-0.46
SD1	113	0.485	0.482	69	44.4	3.0	16.7	0.19	0.24	-7.16
SD2	109	0.495	0.491	103	43.7	5.1	15.2	0.06	0.23	-7.61

OL = Ottawa loose, OD = Ottawa dense, QL = Q-Rok loose, QD = Q-Rok dense, D_r = Relative density, e_0 = Initial void ratio, e_c = Void ratio after consolidation, σ'_c = Consolidation stress, ϕ'_p = Peak friction angle, ε_p = Axial strain at peak friction angle, ψ = Dilation angle

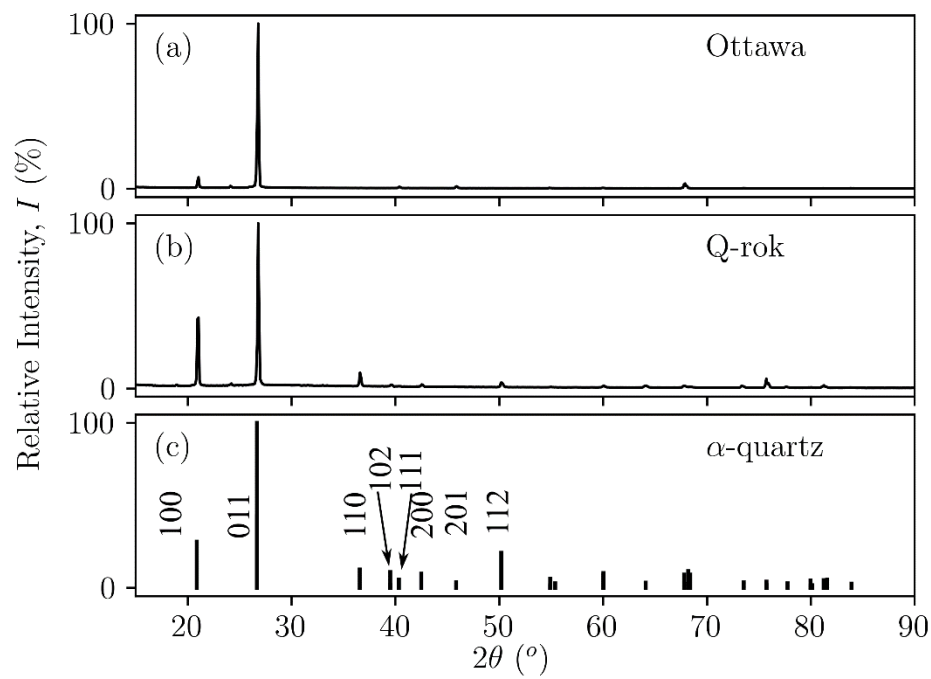


Fig. 1 X-ray diffraction powder diffraction of as received (a) Ottawa sand and (b) Q-rok compared with diffraction peaks of (c) α -quartz.

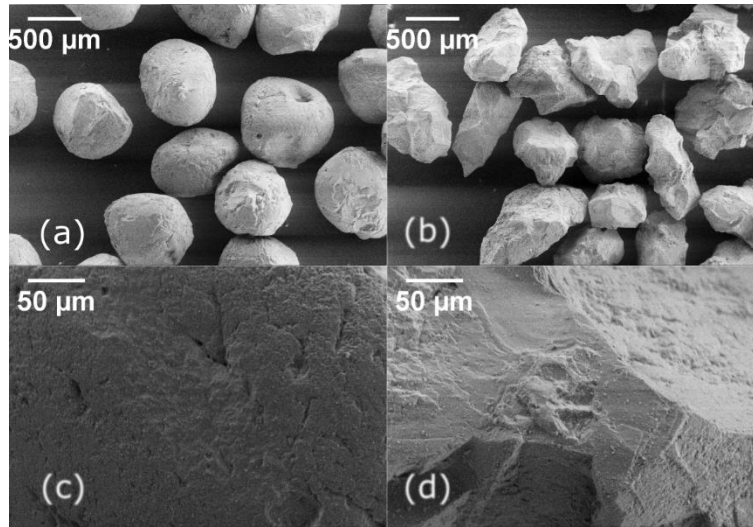


Fig. 2 SEM micrographs of (a) Ottawa sand grains and (b) Q-rok grain at 94x magnification; Texture of (c) Ottawa grain and (d) Q-rok grain at 1000x magnification.

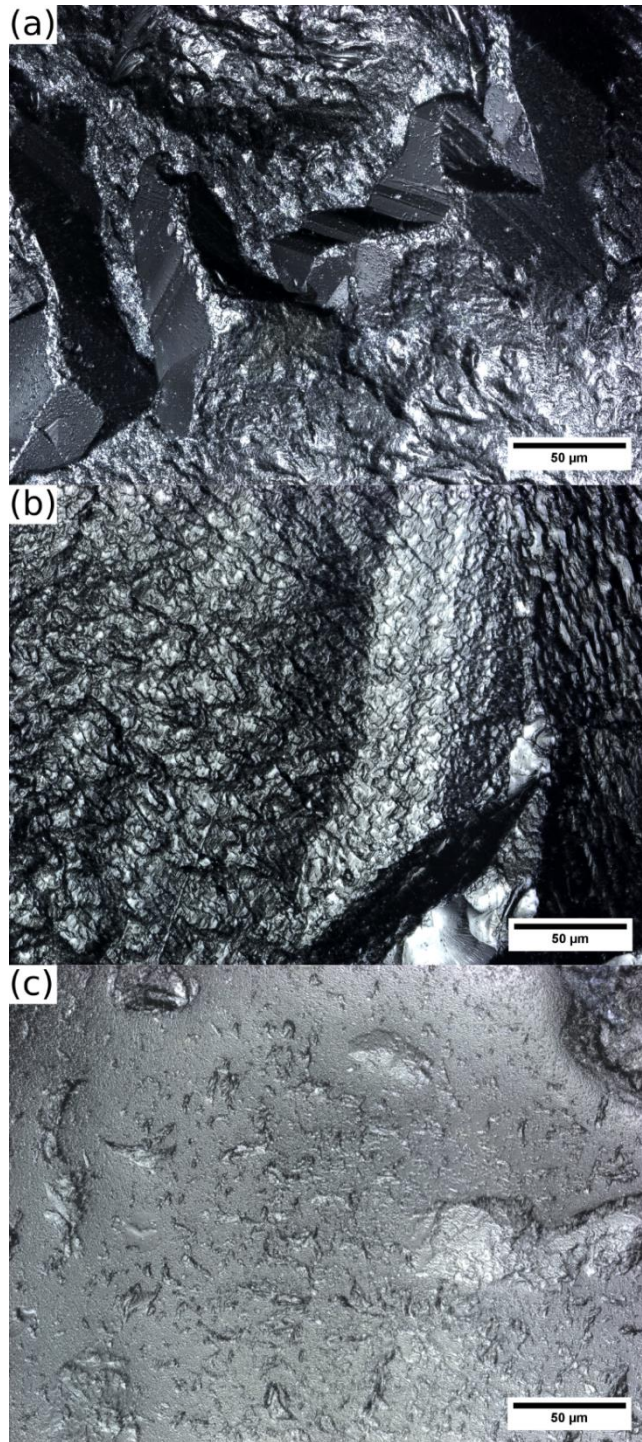


Fig. 3 Surface images of (a) Ottawa sand, (b) Q-Rok, and (c) Siligran from high resolution confocal laser microscope.

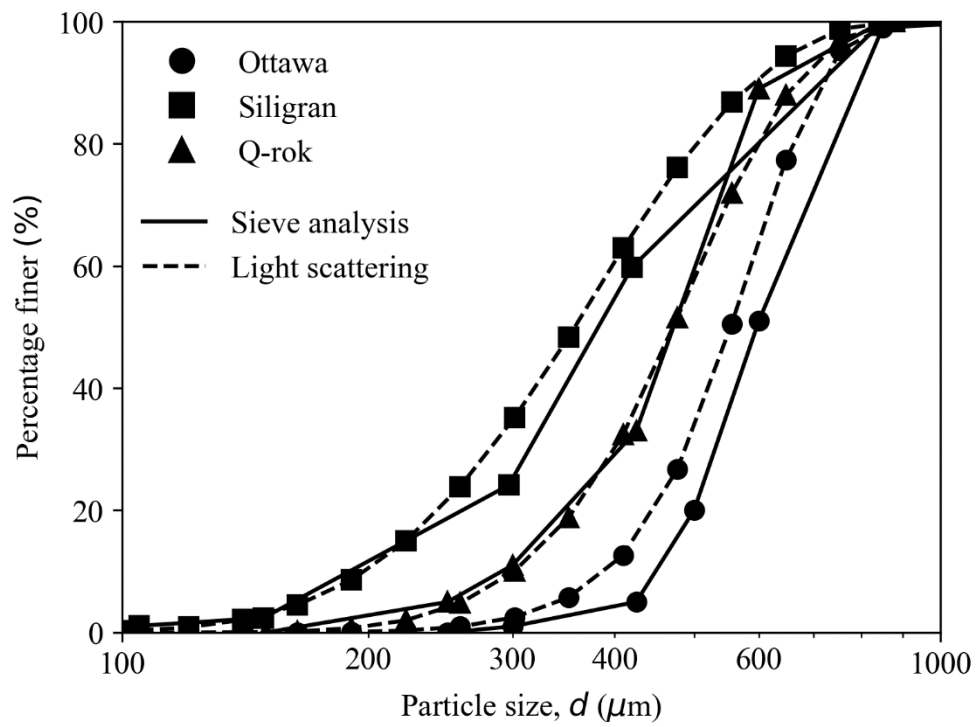


Fig. 4 Particle size distribution graph determined by sieve and laser scattering analysis on Ottawa and Q-Rok sand.

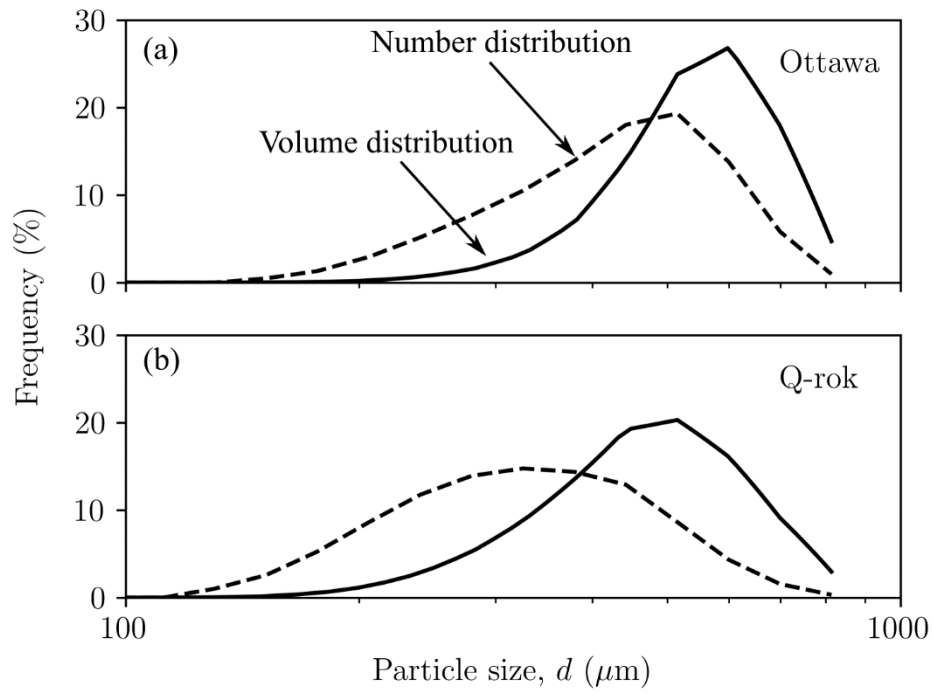


Fig. 5 Volume and calculated number distribution of Ottawa sand and Q-Rok from laser light scattering technique.

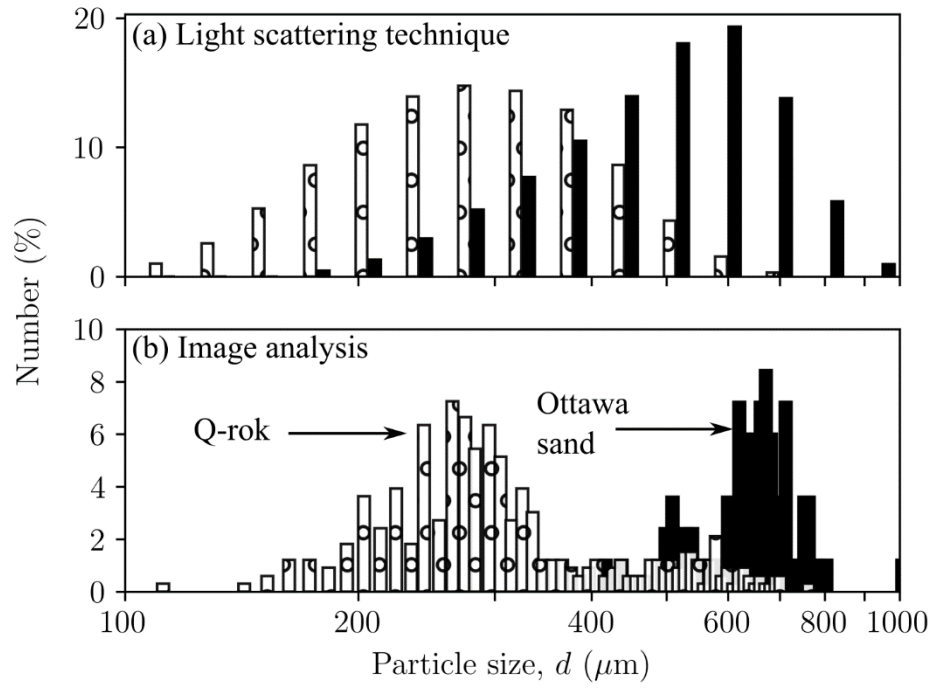


Fig. 6 Number distribution from laser light scattering technique and image analysis for Ottawa sand and Q-Rok.

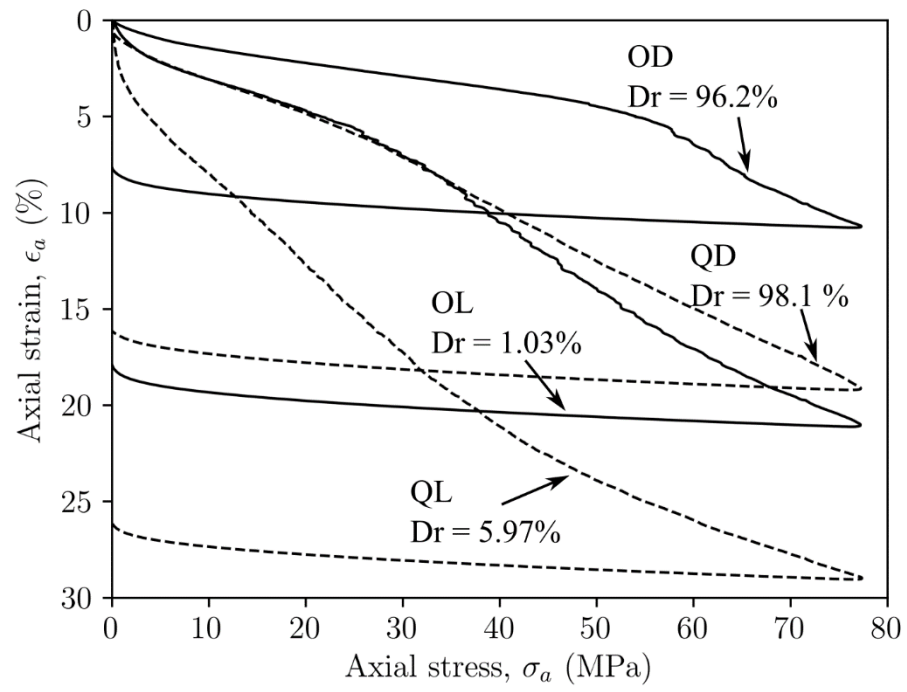


Fig. 7 Results of strain controlled 1-D compression tests on loose (L) and dense (D) specimens of Ottawa sand (O) and Q-Rok (Q).

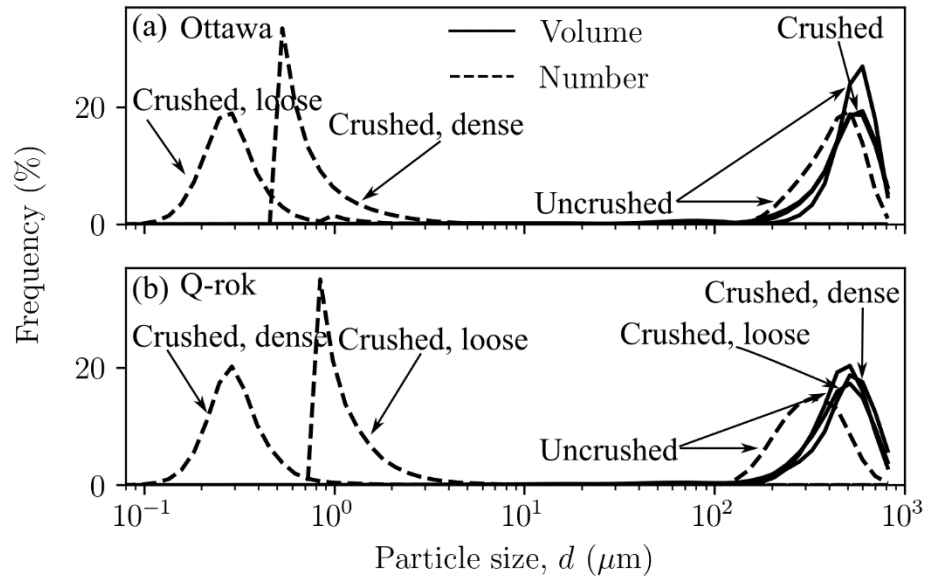


Fig. 8 Particle size evolution in dense and loose specimens of (a) Ottawa sand and (b) Q-Rok subjected to 1-D compression loading, shown as volume and number distributions.

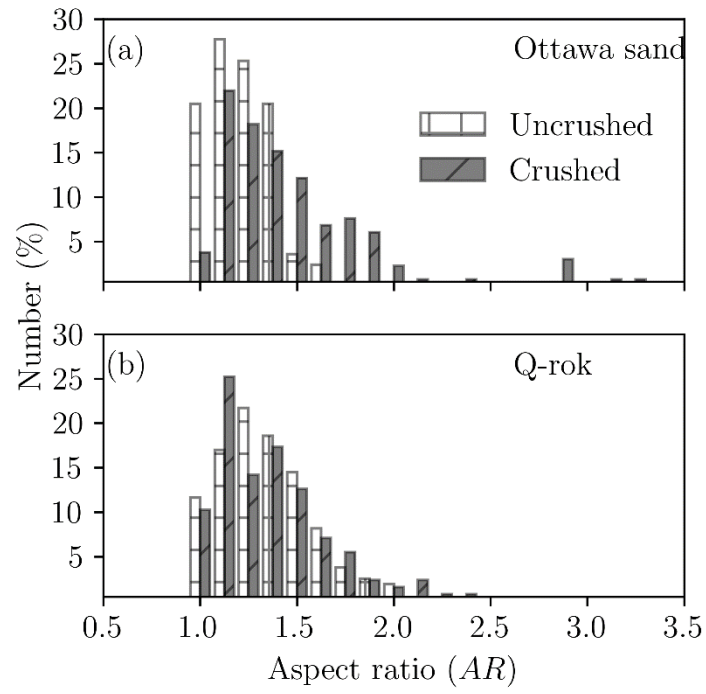


Fig. 9 Particle shape evolution after 1-D compression loading. (a) Ottawa sand, (b) Q-Rok.

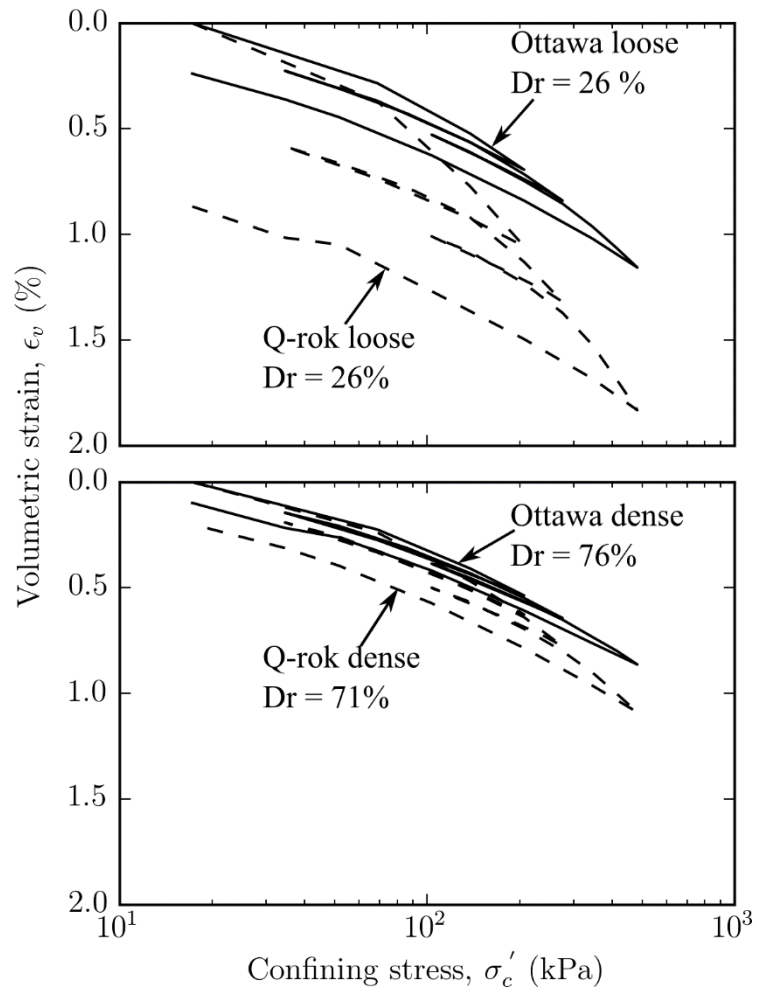


Fig. 10 Isotropic consolidation of dense and loose specimens of Ottawa sand and Q-Rok.

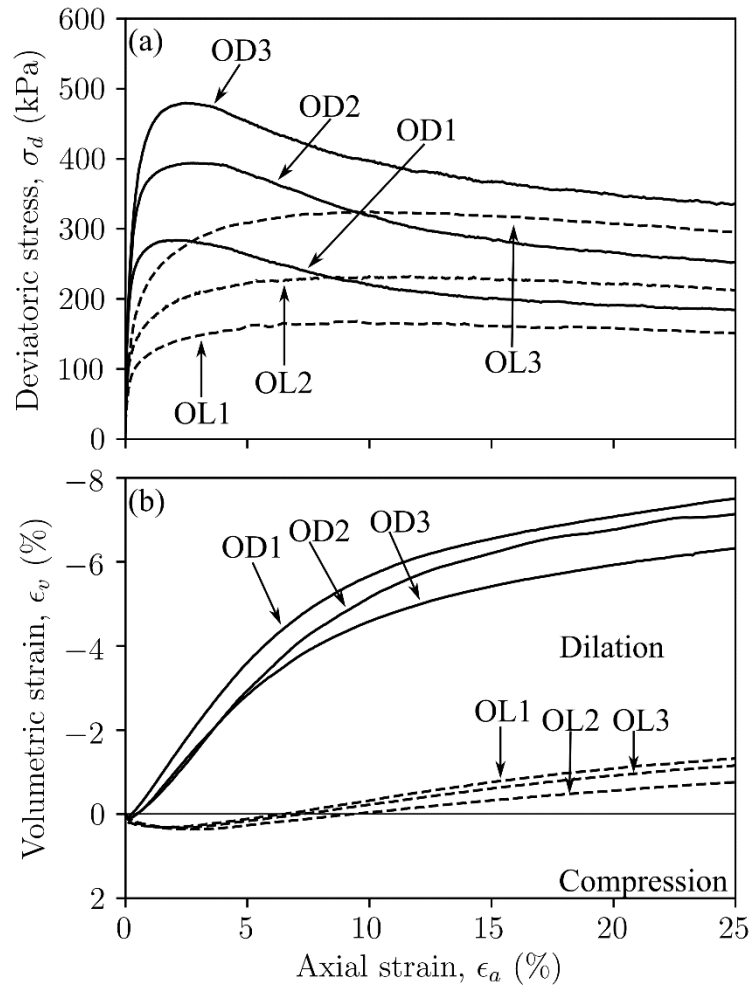


Fig. 11 Response of Ottawa sand to drained triaxial compression. (a) Stress-strain relationship, (b) Volumetric relationship. O = Ottawa, L = loose, D = dense.

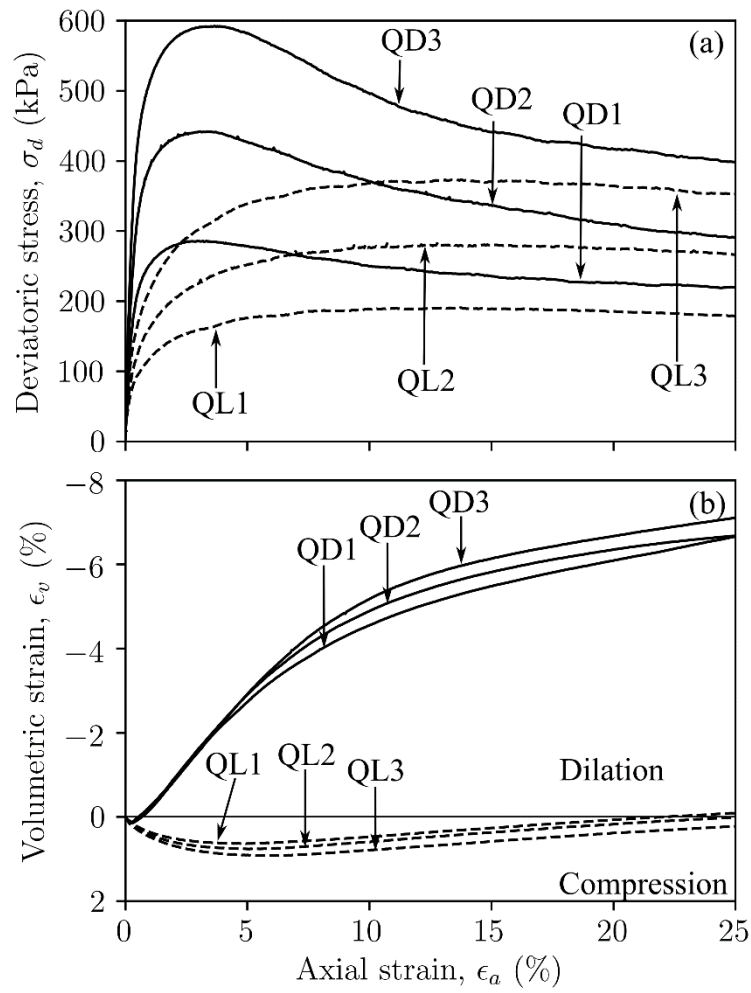


Fig. 12 Response of Q-Rok to drained triaxial compression. (a) Stress-strain relationship, (b) Volumetric relationship. Q = Q-Rok.

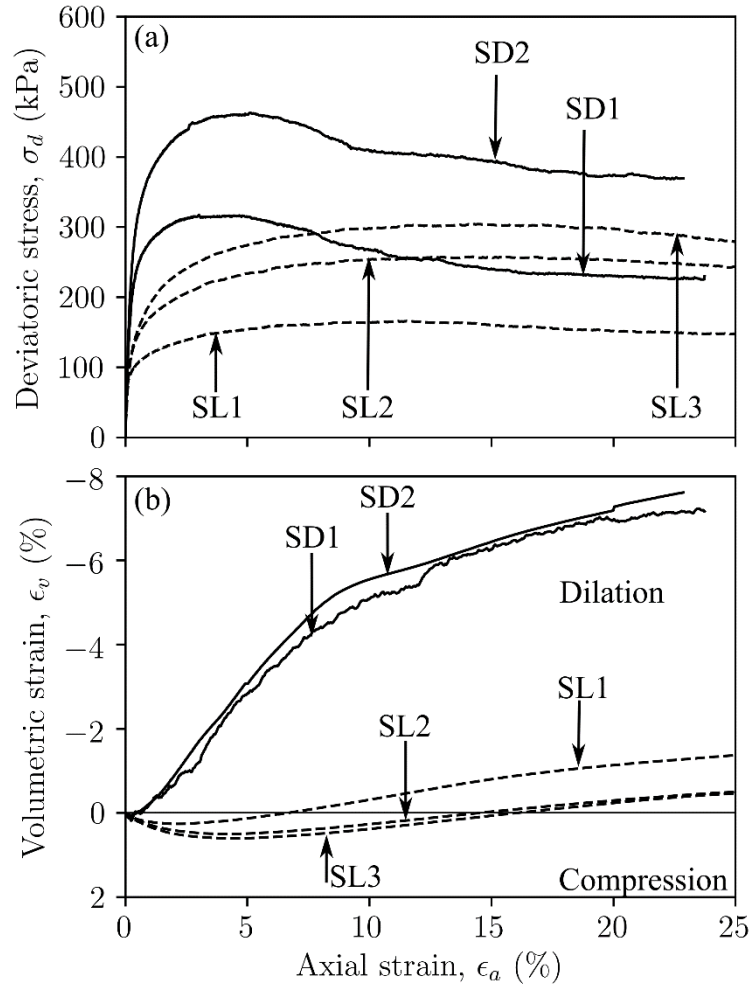


Fig. 13 Response of Euroquarz Siligran to drained triaxial compression. (a) Stress-strain relationship, (b) Volumetric relationship. S = Siligran.

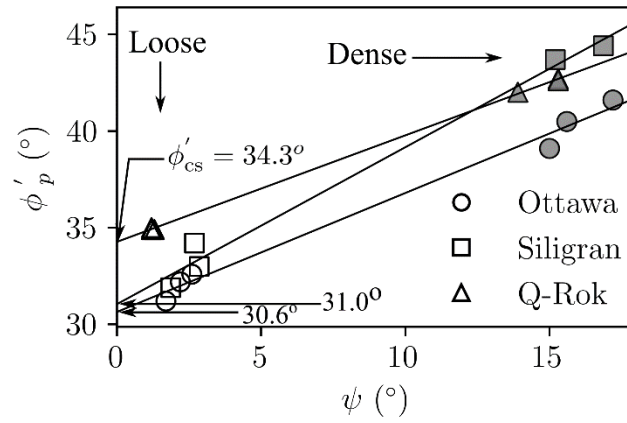


Fig. 14 Determination of critical state friction angle.

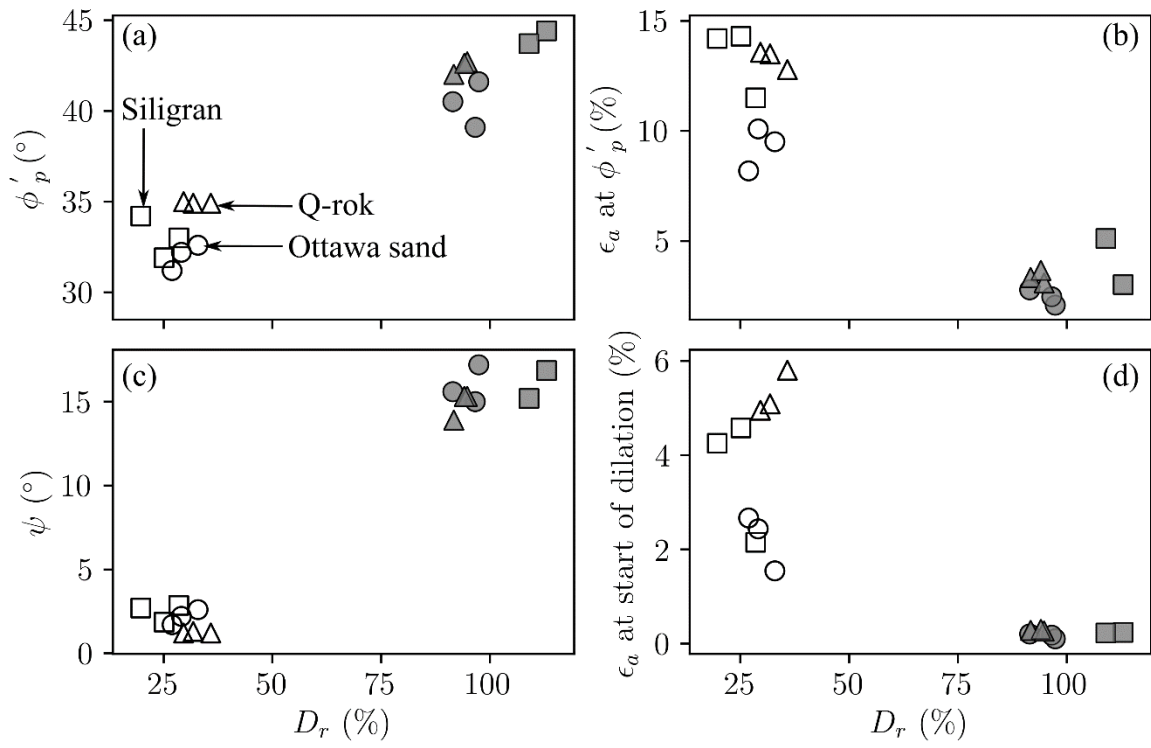


Fig. 15 Effect of D_r on strength and volume parameters in conventional drained triaxial testing.

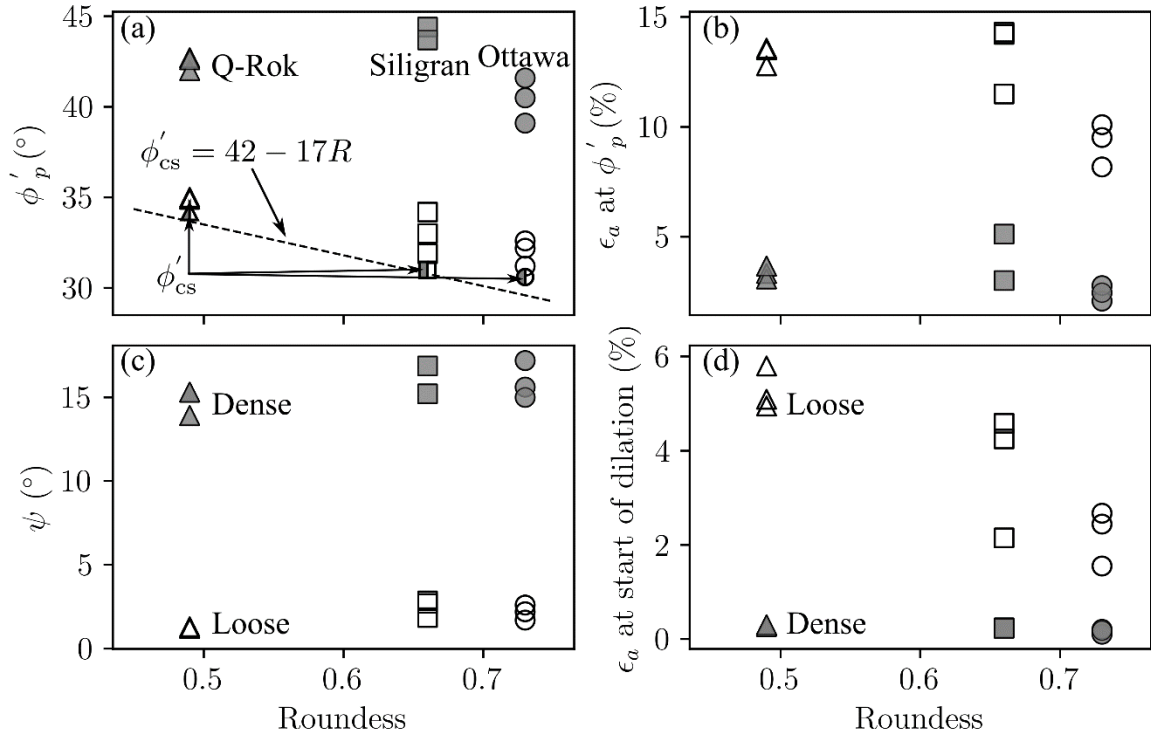


Fig. 16 Effect of particle shape on strength and volume parameters in conventional drained triaxial.

CHAPTER II
THE ROLE OF PARTICLE SHAPE ON STRAIN LOCALIZATION IN
FRICTIONAL END DRAINED TRIAXIAL COMPRESSION

A version of this chapter was submitted by Alexia Lieb, Aashish Sharma and Dayakar Penumadu: Lieb, A., Sharma, A., and Penumadu, D. “The role of particle shape on strain localization in frictional end drained triaxial tests.” *Geotechnical Testing Journal*.

The article will be submitted as it to the journal. Alexia Lieb performed most of the tests and contributed in the preparation of the manuscript by writing the abstract and portions of the introduction, materials, and specimen preparation and mechanical testing sections. Aashish Sharma, supervised the experimental tests, developed the idea, analyzed the data for this chapter and wrote major portions of the paper.

Abstract

The influence of particle morphology has been investigated using triaxial tests to analyze the deformation behavior of sand using three-dimensional digital imaging correlation (3D DIC) based on surface deformation data for a deforming cylindrical specimen of sand. The objective of this research is to investigate how particle morphology, effective stress, and relative density influence deformation during triaxial compression testing under drained conditions. 3D DIC was utilized to obtain a very detailed quantitative surface deformations and strains were interpreted using large strain formulation. The time at which localization occurs as well as the characteristics of localization, whether it be bulging or shear banding, is greatly influenced by the initial relative density and particle morphology for a given effective confining stress. This paper summarizes important experimental observations related to strain localizations which were observed consistently for both sand specimens for varying initial void ratio as a function of particle morphology. Data suggests that angular particles with show significant localizations of deformation under drained shear loading conditions.

Introduction

In the recent past, researchers and engineers have realized the need to deal with more complex issues associated with strain localization and their relation to the constitutive behavior of soil to model realistic boundary value problems. This paper focuses on the localized deformations and shear band type formations within the soil element under triaxial loading/boundary conditions for cohesion-less sand and the role of particle shape and texture for tendency associated with bifurcation phenomenon in granular materials. In this study, techniques are developed for detecting

the onset of strain localization and evaluating the specimen non-uniformity by determining the local deformation profile using an advanced digital imaging technique using two camera system and VIC3D© digital image correlation software.

The use of triaxial testing has been greatly researched and well documented; however, there are inherent flaws associated with basic assumptions used during testing. Triaxial testing relies on a load cell to record force and a vertical measurement device to report strain. These measurements are considered engineering properties, where the values measured are assumed to be the same throughout the specimen. Unfortunately, even at relatively small strains, this assumption is invalid. Strain localization has fully developed before peak stress (Finno and Rechenmacher 2003; Hall et al. 2010; Oda and Iwashita 2000) and can occur as early as 3% global axial strain (Alshibli and Sture 2000; Desrues et al. 1985). Once strain localization begins, it is mostly contained within that small section of the overall sample (Alshibli et al. 2017; Finno and Rechenmacher 2003). Because strain localization begins so early in the compression process, global strain is almost immediately an inaccurate representation of what is occurring throughout the specimen.

Another engineering property easily recorded during a drained triaxial test is the volumetric change of the specimen. A major limitation of measuring the flow of water in and/or out of the specimen with equipment such as a differential pressure transducer (DPT) is that the DPT can only measure the amount of water flow. Therefore, the change in water only exhibits the general change in the entire volume of the specimen due to applied external shear stress but does not reveal precisely where in the specimen that the volume change occurs.

Digital image correlation (DIC) works by tracking the movement of every pixel in every frame and combining the frames to present the overall movement of that point. Mapping the movement of all points on the specimen and combining it with global stresses and strains illustrates behavior of the specimen in compression. Basic triaxial equipment provides an accurate global strain but is ill-suited to provide information as to where strains occur within the sample. Combining a standard triaxial test with digital image correlation has allowed for true strain to be accurately measured throughout a specimen at specific global strains (Rechenmacher and Finno 2004).

This paper describes the investigation of localized deformation by analyzing full field displacement and strain fields on the surface of loose and dense specimens of sand with different

grain morphologies using a non-destructive technique. In particular, major and minor principal strain profiles along the axis of the specimen are compared. Also, global volumetric strains are compared with those occurring in the actively deforming region. Based on these observations, conclusions as to the initiation of localized deformation and the validity of void ratio computations based on global measurements are presented.

Materials

Two commercially available sands, 20/40 Ottawa sand and Q-ROK were used in this study. These sands have very similar mineralogy but very different particle morphologies. The major constituent of both the sands was silica, comprising more than 99.5% of the sand grain. Ottawa sand was unground silica with rounded grains and Q-ROK was unground sand with sub-angular to angular grains. Ottawa sand grains were single crystal whereas larger grains of Q-ROK were composed of multiple crystalline grains. The mean sphericity and roundness values for thirty randomly chosen grains, based on the Krumbein and Sloss (1963), were 0.80 and 0.79 for Ottawa sand and 0.64 and 0.44 for Q-ROK. The $e_{\min}$ and $e_{\max}$ determined using procedures specified in ASTM-D4253 (ASTM 2014a) and ASTM-D4254 (ASTM 2014b) were 0.505 and 0.689 respectively for Ottawa sand and 0.634 and 0.910 respectively for Q-ROK. Limiting void ratios were also determined using a procedure similar to that described by Yamamuro and Lade (1997). For $e_{\min}$, 800 g of sand was poured into 1000 ml graduated glass cylinder in 50 g increments. After each increment, four diametrically opposite locations were tapped lightly six times with a rubber mallet for a total of twenty-four times. The limiting void ratios were 0.51 and 0.75 for Ottawa sand and 0.69 and 1.01 for Q-ROK. The $e_{\min}$ from the cylinder method is similar to that obtained from the ASTM method for Ottawa sand but is higher for Q-ROK. Since the larger grains of Q-ROK were angular and multi-granular and are susceptible to easy breaking, the heavy overburden stress and vibration in the ASTM method could have led to a denser state. For $e_{\max}$, the cylinder with 800 g sand was turned upside down and then slowly (45s – 60s) turned to an upright position. The $e_{\max}$ for both the sands for the cylinder method is higher than those computed from the ASTM procedure. The slow turning of the upside-down cylinder to an upright position imparts very little energy to the sand particles, thus possibly enabling a looser state of deposition. Though the sand is deposited from zero drop height in the ASTM method, nevertheless, the particles have some

kinetic energy while flowing down the funnel which could result in a denser state. The maximum particle size for both the sands was 850 μm , and the minimum particle size was 300 μm for Ottawa sand and 150 μm for Q-ROK. The mean particle size (D_{50}) and coefficient of uniformity (C_u) were 598 μm and 1.43 respectively for Ottawa sand and 470 μm and 1.74 for Q-ROK. The soil classification based on ASTM-D2487 (ASTM 2011b) was poorly graded sand (SP) for both the sands.

Specimen preparation and mechanical testing

Triaxial tests were performed on 71 mm diameter and 178 mm height cylindrical specimens. Loose specimens were prepared by depositing sand from a slowly rising funnel with zero drop height. Dense specimens were prepared by pouring 50 g of sand from the top of the mold and lightly tapping the opposite sides of the mold six times with a raw-hide mallet at the base of the triaxial cell without undercompaction of the lower layers (Ladd 1978); a total of 24 taps for each layer. Porous stones were placed between the sand and the acrylic end platen at the top and the bottom. After removal of the mold, the specimen in the latex membrane was left standing under a small vacuum of 17 kPa. The latex membrane was painted with a thin coating of white paint on which random speckle pattern was created by spraying black paint. To ensure centrally aligned and vertical loading, the triaxial cell was assembled with three tie rods, albeit without the confining acrylic chamber. Three bolts were placed underneath the top cap to hold it in place to guide the piston during load application. The top platen was rigidly attached to the piston to prevent it from rotating during the application of load. The specimen was consolidated by increasing the vacuum to the desired consolidation stress and sheared at a prescribed displacement rate of 12 percent per hour in the MTS 858 biaxial loading system. A separate two inch linearly varying differential transformer (LVDT) integrated with the primary data acquisition system of the loading system was used to measure global axial displacements. Global engineering strains were computed from LVDT displacements and the initial sample height. The axial stress was computed based on the force measured by the loadcell, and the corrected cross-section area. A total of eight tests were performed, dense and loose specimens for each sand confined at two different confining stresses of 68.9 kPa and 96.5 kPa.

Digital image correlation (DIC)

Commercially available hardware, data acquisition software, and analysis code from Correlated Solutions Inc. were used for capturing images at various stages of loading, and for computing displacement and strain fields. The system uses dual-cameras to track the movement of the speckle pattern and to measure three dimensional full-field surface strains. Two obliquely placed cameras are required to provide stereoscopic information of the curved specimen surface. The basic method tracks the gray value pattern in small pixel neighborhoods, called subsets, during deformation. Each camera tracks the movement of the black specks individually and combines the information to compare against the initial state. The movement of the black spots with respect to the initial position allows for the overall sample displacements to be recorded in all three principle directions. To reduce complications resulting from multiple refractions of light rays from the use of water confinement in acrylic cell, tests were performed on sand specimens confined under vacuum. This arrangement simplified the analysis, but no global volume measurements were possible. Digital images were captured more frequently during the early stages of shearing. The analysis code, VIC-3D, tracks the movement of the speckles automatically with advanced image recognition algorithm to render localized strain during the shearing process. A detailed description of same system is provided by (Rechenmacher and Finno 2004; Rechenmacher and Medina-Cetina 2007). The DIC system has been successfully used to study grain scale displacements and strain fields where the different sizes and colors of sand grains in triaxial and plane strain testing were used to provide the random patterns for tracking surface displacements (Rechenmacher 2006; Rechenmacher and Finno 2004; Rechenmacher and Medina-Cetina 2007). Since both Ottawa sand and Q-Rok, used in this study, were either translucent or white, speckled pattern was created on the confining latex membrane. The experimental setup is shown in Fig. 17. Separate computers were used to record global measurements (force and displacement) and images used to compute specimen surface strains. Since both the systems were started within few seconds of one another, the elapsed time from the start was used to determine the force on the specimen corresponding to different images. The major limitation of the DIC method in these series of tests is that surface strains are based on the displacements of the confining latex membrane. With the assumption of no slippage between the moving sand grains and the confining membrane (Abrantes and Yamamuro 2002), the displacements of the membrane are assumed to approximate the

displacements occurring in the specimen. Also, any localized movements that initiate within the specimen are difficult to visualize from surface measurements until those localizations have fully evolved and reached the surface. However, as will be shown later that surface monitoring does enable to unmistakably deduce the initiation of localized deformations.

Experimental results

Global results

The global response, as measured with LVDT are shown in Fig. 18(a) and (b) respectively and some pertinent values are presented in Table 4. Failure is defined as the maximum deviatoric stress within 15% axial strain, or the stresses at 15% axial strain whichever occurs first. Dense specimens of both sands soften after peak strength whereas loose specimens continue to deform with very little softening almost at constant shear stress. The post peak softening is greater in angular specimens than in rounded specimens. The shear strength and friction angles of the dense specimens is higher than those of loose specimens, and the friction angles for angular specimens were higher than those for rounded specimens for similar initial packing state. The failure strain, axial strain at failure, was smaller for dense specimens. The average failure strain was 13.8% for loose specimens, and 3.8% for the dense specimens. Due to the absence of volumetric strain measurements, the cross-section area for computing global deviatoric stress (σ_d) was corrected as shown in Eq 2.

$$A_c = \frac{A_0}{1 - \varepsilon_a} \quad \text{Eq 2}$$

where, A_c is the corrected area, A_0 is the initial cross section area of the specimen, and ε_a is the axial strain based on LVDT readings. This correction assumes no volume change, and as seen later, the assumption is not unreasonable until failure. However, in dense specimens, beyond failure, it over estimates the strength.

DIC average results

The main attraction of DIC is the opportunity to determine spatially resolved strain over large specimen area. Strain values from DIC presented here are principal strains, unless specified.

Principal strains (ϵ_1 and ϵ_3) are more appropriate than vertical and horizontal strains (ϵ_y and ϵ_x), as the specimens were not always perfectly vertical in the images. Vertical and horizontal strains are computed with respect to the image coordinate axes and if the specimen image is not vertical, the vertical strain direction will be at an angle to the specimen axis. A speckle patterned specimen (OL96) is shown in Fig. 19 along with typical area of the specimen surface analyzed. Drawn in the same figure are four rectangular regions chosen to determine average surface strains. In these regions, strains are averaged over the specified rectangular region and one value is computed per image per rectangular region. Contours of vertical displacements along the length of the specimen at 1% axial global strain is also shown in Fig. 19. The LVDT displacement is approximately 1.78 mm. The displacement along the specimen length varies from approximately zero at the bottom to the prescribed displacement at the top of the specimen. Some discrepancy between global displacements and DIC displacements at the top may arise from differences in global data and image acquisition rates; images were not exactly acquired at the desired displacements and the inability to track rigid body movement of the top platen due to uneven surface created by two o-rings. The non-uniform displacement profile seen in the figure, even at this early stage of loading, eventually culminates into higher displacement gradient in the middle (active) region with the progression of loading. Stresses and strains computed using average DIC strains (average region in Fig. 19) are shown in Fig. 20 for OL69 and QD96 along with global responses. These are typical curves for loose and dense specimens. The response computed using DIC strain values are very similar to the global response when the area correction is performed using Eq 2; the axial strain, ϵ_a is replaced with DIC major principal strain, ϵ_1 . Average ϵ_1 is similar to engineering axial strain. However, unless specified all area corrections for DIC responses were performed using Eq 2 (Lade 2016), replacing the radial strain (ϵ_r) with the minor principal strain, ϵ_3 . The deviatoric stresses (σ_d) when associated with DIC strains, were computed by dividing the force by Eq 3, which may give a more realistic value for axial stress after the peak as localized bulging starts to influence the average value of ϵ_3 when considering different regions along the height of the specimen.

$$A_c = A_0(1 - \epsilon_r)^2 \quad \text{Eq 3}$$

DIC local results

The particulate nature of sand, appreciable specimen size, displacement of one end of the specimen during loading, and local non-uniformities are among the chief reasons for non-uniform strain distribution in sand specimens. When frictional end systems are employed, the friction from the porous stone creates additional lateral confinement at the ends which accentuates the non-uniform strain distribution. The nonuniformity of strain leads to nonuniformity of stress in the specimen. DIC permits quantification of these non-uniformities. Three rectangular regions, at the top, middle and, bottom of the specimen, were chosen as shown in Fig. 19. The region in the middle of the specimen was chosen to coincide with actively deforming region and may not always be located at specimen mid-height. The response in these regions are shown in Fig. 21 for QL69 and OD96, which are typical for loose and dense specimens. In general, the top and bottom regions are stiffer and stronger than the active (middle) region and the global response, and the global response is marginally stiffer and stronger than the active region. Frictional ends prevent flow of sand grains to the sides thus rendering a stiffer response in dilatant materials (Vermeer and de Borst 1984). Deviatoric stresses, σ_{df} , failure strains, ε_f , and friction angles, ϕ' , in the average and active regions at global failure strain have also been presented in Table 4. For example, at global failure strain of 13.6%, DIC strains in the average and active regions were 12.7% and 16.4% and the deviatoric stresses computed using Eq 3 were 139 kPa and 128 kPa. Failure parameters based on maximum local shear stress, σ_{dl} , in the active region are also presented as local failure strain ε_{fl} , and local friction angle ϕ'_l . The difference between the global parameters and local parameters for the active region is that global parameters are computed at the instant of global maximum shear stress and local parameters are computed based on local maximum shear stress. Once again, local stresses are derived by dividing the force by Eq 3, using ε_3 of the local region. At global failure, the local shear stress in the actively deforming region is past the peak, hence the strength of the actively deforming region is lower than the global strength in all specimens. The difference is larger in loose specimens than in dense specimens. However, the overestimation of strength from global measurements is less than ten percent and hence may not be of a major concern in geotechnical applications. The top and the bottom regions are stronger, in part due to the confining effect of the frictional ends.

The global and local stress-strain-volumetric responses of dense and loose specimens of Ottawa sand and Q-Rok are presented in Fig. 22 and Fig. 23 respectively. The volumetric strain of the average region is assumed to reflect the global volumetric response and is labelled as global response in the figures. Volumetric strains (ε_v) were computed with small strain assumption, $\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$. The global and the local volume change tendencies are very similar. Loose specimens show small volume contraction in the initial stages of loading before expanding, whereas dense specimens start dilating immediately upon the application of load. All specimens, particularly the dense specimens, continue dilating with large volume changes. None of the specimens reached the critical state condition. None the less, stresses in the active region quickly decrease to those in loose specimens. Investigations of localized behavior in triaxial specimens using computed tomography have shown similar behavior (Alshibli et al. 2003; Batiste et al. 2004; Desrues et al. 1996) of continuing volumetric strains even at large strains. The global and active region volumetric strains are similar at failure in both the dense and loose specimens. However, after failure the active region volumetric strains of the dense specimens are approximately twice the global values, indicating that post failure deformation is mostly confined to the active region. Not disputing theories pertaining to limit void ratio and critical state (Desrues et al. 1996), these significant discrepancies in volume changes in global and local regions bring into question the validity of computing void ratios from global volume change measurements especially for critical state computations.

It was recognized early on (Rowe 1962; Taylor 1948) that the friction angle measured in geotechnical testing is a manifestation of the combined work done in overcoming the frictional resistance between the sand grains and additional resistances to remolding and dilation as the grains displace and roll over one another. Accordingly, the effective peak friction angle (ϕ'_p) is generally modelled as the combination of constant volume change resistance (ϕ'_{cv}), and the dilation angle, ψ , (Hansen 1958) as shown in Eq 4 (Bolton 1986) for plane strain conditions. This concept has been extended to triaxial testing (Vaid and Sasitharan 1992; Vermeer and de Borst 1984). The factor k is 0.8 for plane strain (Bolton 1986) and 0.5 for triaxial testing (Alshibli and Cil 2018).

$$\varphi'_p = \varphi'_{cv} + k \psi \quad \text{Eq 4}$$

The dilation angle was computed using the relationship presented by Vermeer and de Borst (1984) as shown in Eq 5

$$\sin \psi = \frac{(d\varepsilon_v/d\varepsilon_a)}{2 + (d\varepsilon_v/d\varepsilon_a)} \quad \text{Eq 5}$$

where, $(d\varepsilon_v/d\varepsilon_a)$ is the maximum rate of change of volumetric strain with respect to axial strain. The maximum rate of change is assumed to occur at the location of the peak deviatoric stress. Due to uneven strain distribution, the quantitative value of dilation angle varies depending on the choice of axial strain and volume strain region. The average region ψ was computed at the location of peak global deviatoric stress (σ_{df}) and the active region, ψ_l was computed at the location of peak local deviatoric stress (σ_{dl}). Additionally, dilation angle (ψ) was also computed from the slope of the active region volumetric strain with respect to the global axial strain at peak global stress. These three computations reflect three different conditions where the axial and volumetric strains are globally measured, locally measured, and only volumetric strains are locally measured whereas axial strains are global measurements. These values are presented as ψ , ψ_l , and ψ (DIC active) respectively in Table 4. For dense and loose specimens of rounded Ottawa sand, the dilatancy angles computed using global measurements, ψ , are the smallest, and ψ (DIC active) are the largest. As the specimens are actively undergoing localized deformation, the locally computed ψ_l , are generally smaller than the ψ (DIC active) values.

Strain localization

Vertical line profiles of DIC major principal strain, ε_1 , and minor principal strain, ε_3 , along the center line (Fig. 19) of dense and loose specimens of both sands, are shown in Fig. 24 - Fig. 27. The variation in ε_1 , is similar in all the specimens; the maximum value of local strain lies in the actively deforming region of the specimen and it is similar in magnitude for dense and loose specimens. The correlation error in OD69 was slightly larger than the rest of the specimens which could have contributed to the larger strains. With increasing axial global strain, ε_a , the vertical

strain starts to accumulate in the active region of the specimens. This region of strain localization is larger over the length of the specimen for loose specimens than for dense specimens. The initiation of localization is earlier in Ottawa sand however at 10% axial strain Q-Rok shows larger strains in the actively deforming regions. There are, however, some distinct differences in the lateral strain, ε_3 , profiles. Dense specimens exhibit greater bulging, and the region of localized deformation becomes narrower with increasing axial strain. In general, both loose and dense specimens exhibit non-uniform distribution of both ε_1 and ε_3 from very early, as early as 1% global axial strain, stage in loading. The effect of frictional end platens is clearly visible and more severe in dense specimens than in loose specimens. Also, the locations of the maximum strains are not always in the middle third of the specimens, indicating that localization initiates at local inhomogeneities. The difference in the location of maximum strain is more readily visible in dense specimens than in loose specimens. Compacted dense specimens without undercompaction are more likely to display local variations than loose specimens prepared by zero height of fall deposition.

The evolution of the ratio $-\varepsilon_3/\varepsilon_1$, in the average and active region is presented in Fig. 28. There is a significant difference in the $-\varepsilon_3/\varepsilon_1$ ratio in the average and active region. The active region values are larger and increase at a higher rate with the progression of loading than the average region values. In the dense specimens, $-\varepsilon_3/\varepsilon_1$ increases rapidly in the early stages of loading until 1% axial strain in Ottawa sand and 2% axial strain in Q-Rok in both active and average region; after which it increases at a lower constant rate. For angular Q-Rok the ratio exceeds one in the active region at global axial strain of approximately 5%, whereas for Ottawa sand the ratio is less than one for most of the loading process. In the loose specimens, $-\varepsilon_3/\varepsilon_1$ increases at a constant rate through most of the loading and it approaches unity at approximately 15% global axial strain. These indicate that the assumption of constant Poisson's ratio, ν , in numerical modelling may not be valid in general and particularly at early stages of loading.

Surface strain at failure

Spatially resolved surface strains in the post peak region just beyond the global failure are shown for all specimens in Fig. 29. Failure in loose specimens is characterized by diffused bulging with maximum strains distributed over a larger middle region of the specimen. Dense specimens,

especially those under higher confining stress exhibit inclined failure regions. As loading progresses further in the post peak and softening region, rounded and angular sands exhibit different evolution of failure planes. In Ottawa sand, localization region grows larger with increasing loading (ref to Fig. 8) whereas in Q-Rok, post peak behavior is characterized by the development of multiple failure planes as shown in Fig. 30. However, Alshibli et al. (2003), with the help of x-ray computed tomography, have shown the presence of multiple conical shear bands within the volume of Ottawa sand specimens, when the specimen seemed to have deformed uniformly from rectangular patterns drawn on the confining membrane. The observed failure planes are inclined at approximately 45° to the direction of the principal stress. This value is not consistent with either of the Coulomb criterion ($\pi/4 - \phi/2$) or the Roscoe (1970) criterion ($\pi/4 - \psi/2$).

Conclusions

Strain localization poses a problem in determination of stresses and strains in conventional frictional end triaxial testing. The assumption of uniform strain is violated at very early stages of loading; the initiation of non-uniform strain can be visualized from variation of ε_1 and ε_3 along the specimen axis. Since the cross-sectional area for axial stress, is determined from global volume change and axial strain measurements, the error in stress computation becomes larger at higher strains. Location of localized deformation is influenced by local variations which are dependent on method of specimen preparation. Compacted dense specimens without under compaction is more likely to result in inhomogeneous specimens. Also, the strength of the actively deforming region is marginally less than that computed from global measurements and lateral constraints from frictional platens are more severe in dense specimens than in loose specimens.

The volume changes measured globally is significantly different than those occurring in the region of localized deformation. The difference is greater in dense specimens than in loose specimens. In dense specimens, deformation in the actively deforming region dominates the post peak deformation and could significantly affect the computations of void ratios. Thus, void ratios based on global measurements may not be valid especially for determining critical state parameters. The local evolution of the lateral strain to axial strain ratio, $-\varepsilon_3/\varepsilon_1$, is markedly different for dense specimens than for loose specimens. This ratio quickly exceeds unity in dense

specimens where as it is less than unity for most of the loading in loose specimens. The ratio keeps increasing throughout the loading, at a faster rate in the beginning and at a slower rate during the later stages of loading indicating that a constant value of ν might not be appropriate in numerical modelling. Failure at the peak stress is diffused expect for the dense specimens higher confining stress. Inclined failure planes develop well beyond the peak stress and the inclination of the failure planes are not in accordance with either the Coulomb or Roscoe criterion.

Appendix

Table 4 Failure parameters determined from global and local response

SN	D_r [%]	Location	σ'_3 [kPa]	σ_{df} [kPa]	ε_f [%]	ϕ' [°]	ψ [°]	σ_{dl} [kPa]	ε_{fl} [%]	ϕ'_l [°]	ψ_l [°]
OL69	1.1	Global	68.9	142	13.6	30.5	16.0				
		DIC average	68.9	139	12.7	30.2					
		DIC active	68.9	128	16.4	28.8	26.9	130	13.2	29.0	21.7
OL96	4.3	Global	96.5	205	14.2	31.0	18.1				
		DIC average	96.5	200	13.3	30.6					
		DIC active	96.5	183	17.1	29.1	30.0	189	12.9	29.6	19.9
OD69	76	Global	68.9	167	4.0	33.3	18.8				
		DIC average	68.9	162	3.8	32.8					
		DIC active	68.9	152	6.6	31.6	35.1	157	4.2	32.2	23.3
OD96	89	Global	96.5	268	4.2	35.6	14.7				
		DIC average	96.5	265	3.7	35.3					
		DIC active	96.5	254	5.0	34.6	27.0	257	2.4	34.8	19.7
QL69	12	Global	68.5	169	14.6	33.4	17.4				
		DIC average	68.5	165	14.0	33.0					
		DIC active	68.5	149	19.6	31.2	29.1	154	11.3	31.8	13.9
QL96	0.7	Global	96.5	230	12.8	32.9	14.9				
		DIC average	96.5	225	12.3	32.6					
		DIC active	96.5	208	16.2	31.2	25.2	211	10.6	31.5	14.8
QD69	76	Global	68.9	262	4.3	40.9	19.3				
		DIC average	68.9	256	3.9	40.5					
		DIC active	68.9	243	5.5	39.7	34.5	252	2.9	40.3	24.6
QD96	65	Global	96.5	341	3.0	39.7	16.5				
		DIC average	96.5	337	2.7	39.5					
		DIC active	96.5	330	3.5	39.1	27.9	330	2.5	39.1	22.3

σ'_3 effective confining stress; σ_{df} deviatoric stress at global failure; ε_f strain at global failure; ϕ' friction angle at global failure; σ_{dl} local deviatoric stress based on local area correction; ε_{fl} local failure strain; ϕ'_l local friction angle; ψ dilation angle based on global axial strain and volume strain from average region; ψ_l local dilation angle based on active region strains; O is Ottawa; Q is Q-Rok; L is loose; D is dense; Global values determined from LVDT.

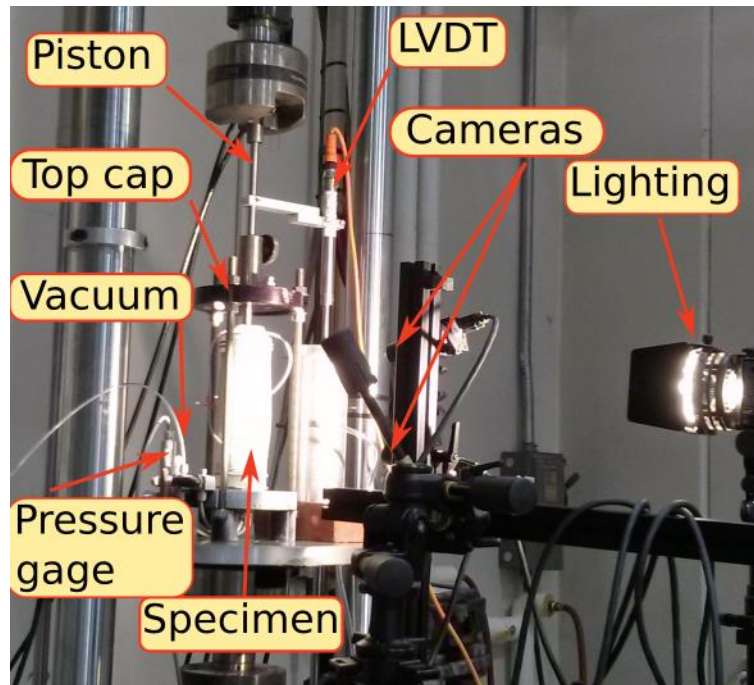


Fig. 17 Experimental set-up using a servo-hydraulic loading system, coupled with DIC system for obtaining strain data on the specimen surface.

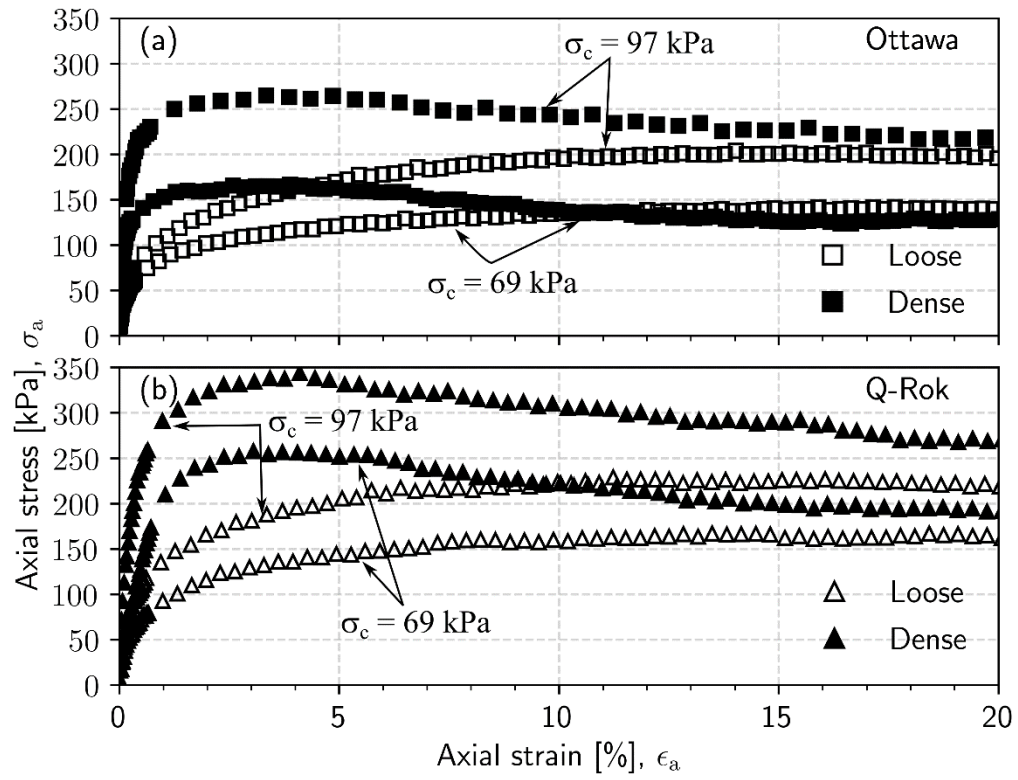


Fig. 18 Global response of the loose and dense specimens of Ottawa sand and Q-Rok.

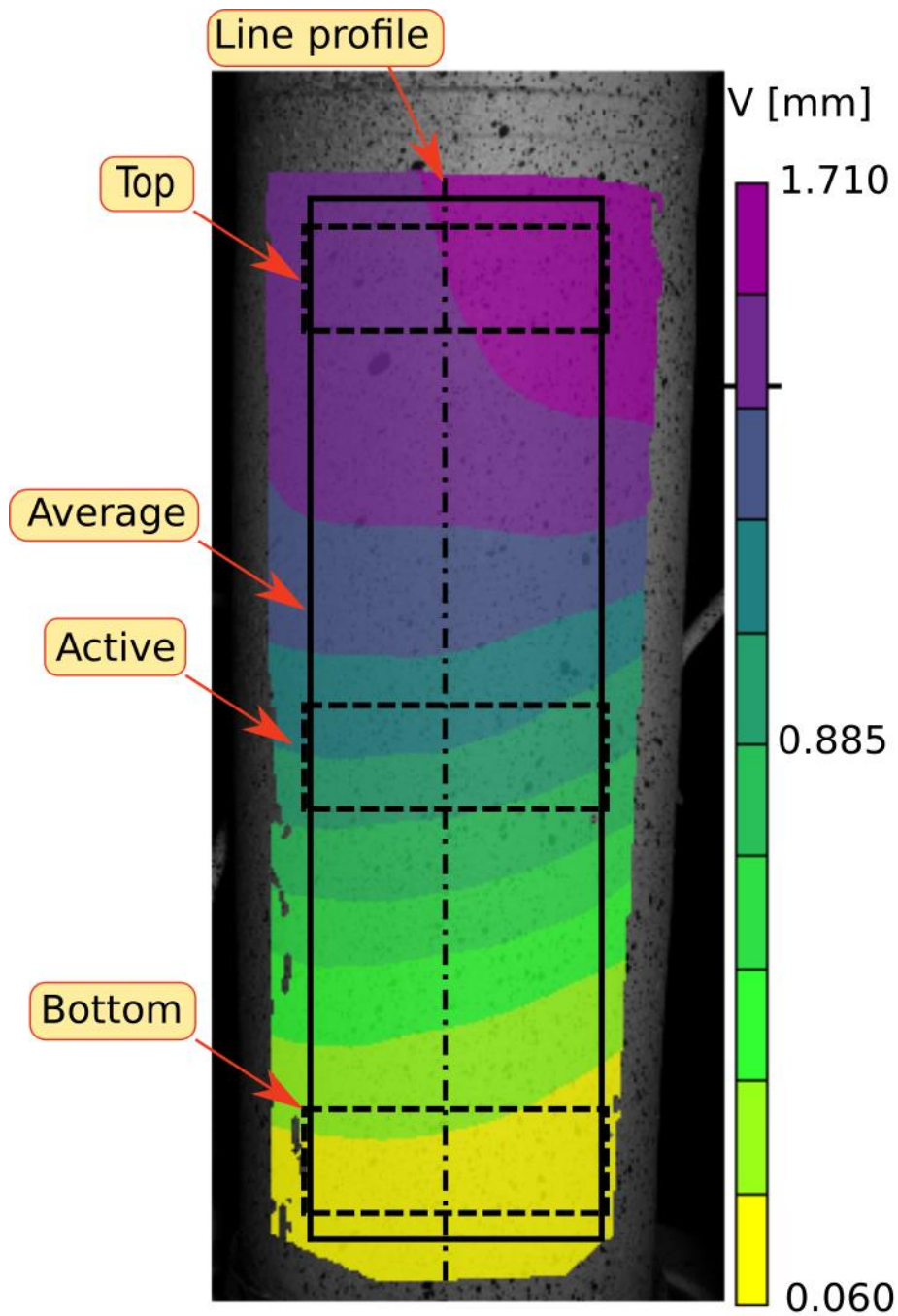


Fig. 19 Typical speckle pattern on specimen membrane for DIC measurements along with regions chosen for extracting strain values.

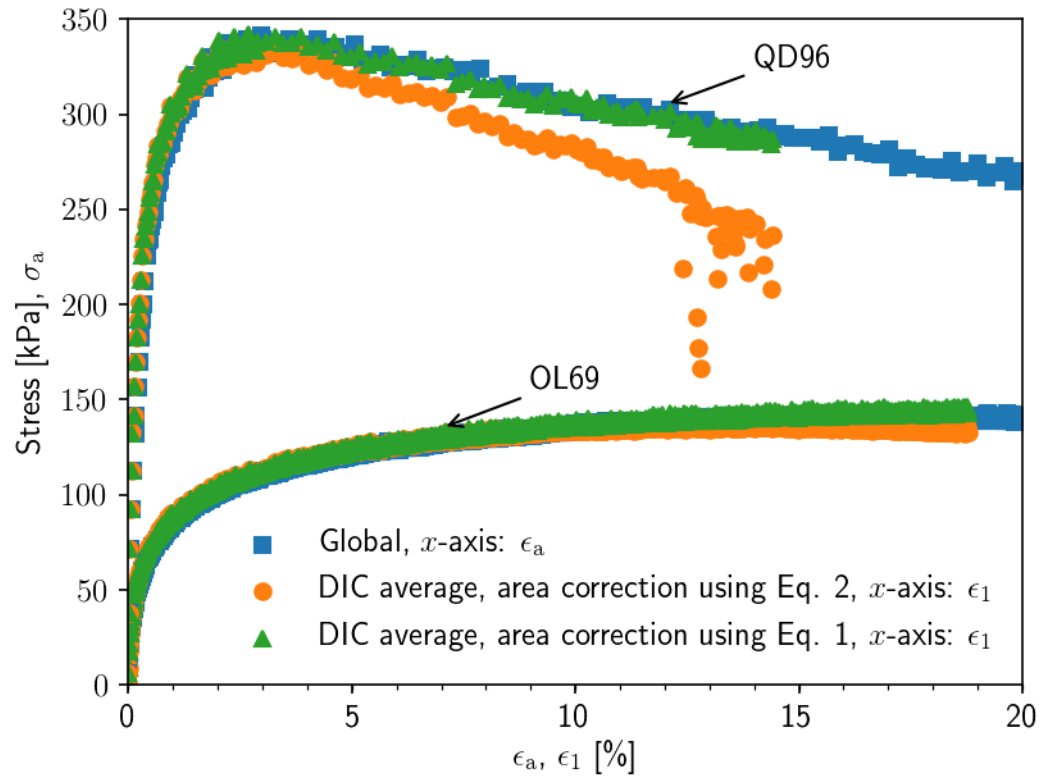


Fig. 20 Response of dense and loose specimens from global displacement measurements and DIC strains.

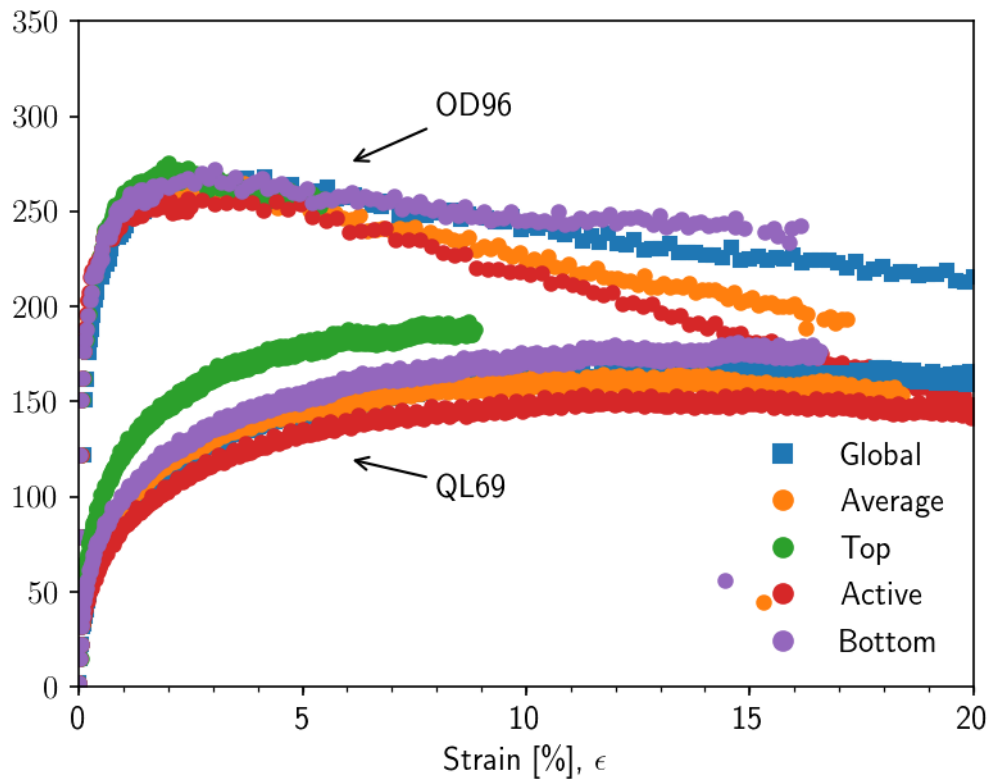


Fig. 21 Typical global and local responses of sand specimens.

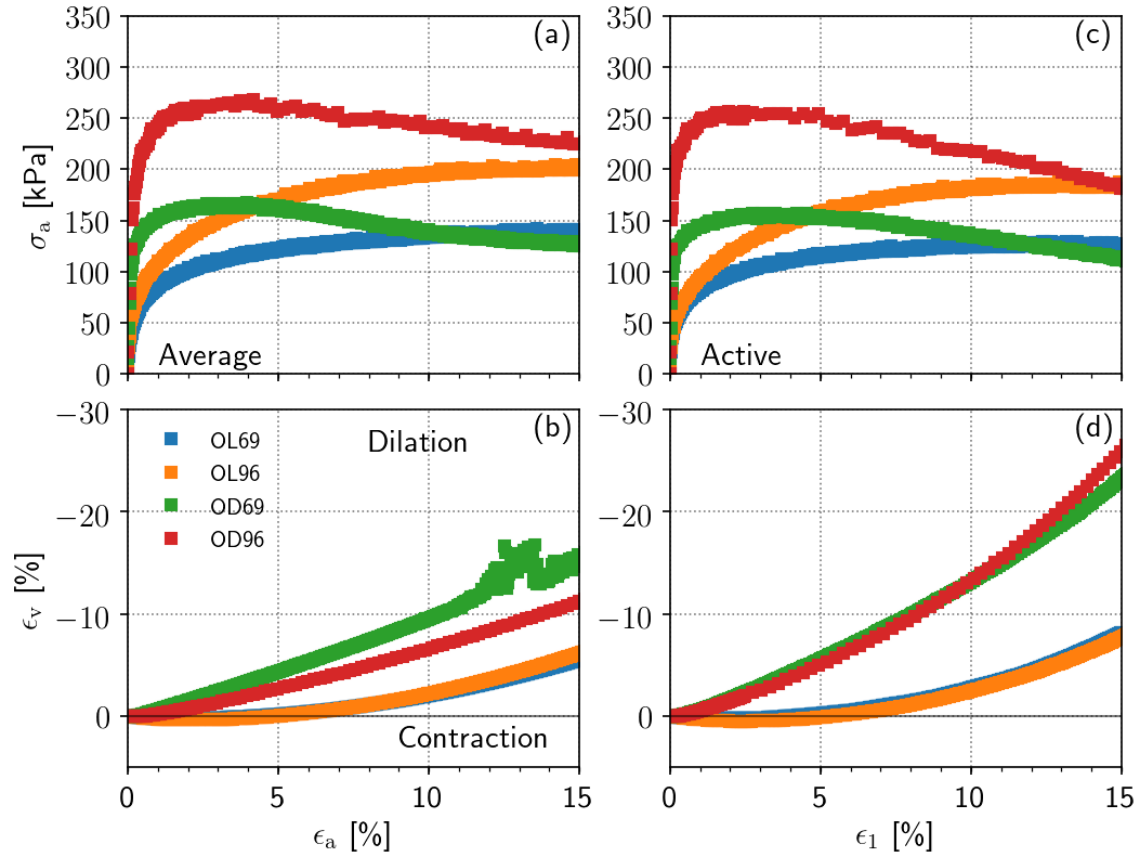


Fig. 22 Global and local strength and volumetric response Ottawa sand specimens, (a) Global stress strain curve, (b) Volumetric strain determined from average region against global axial strain, (c) – (d) Stress-strain-volumetric response as determined from strains in the active region.

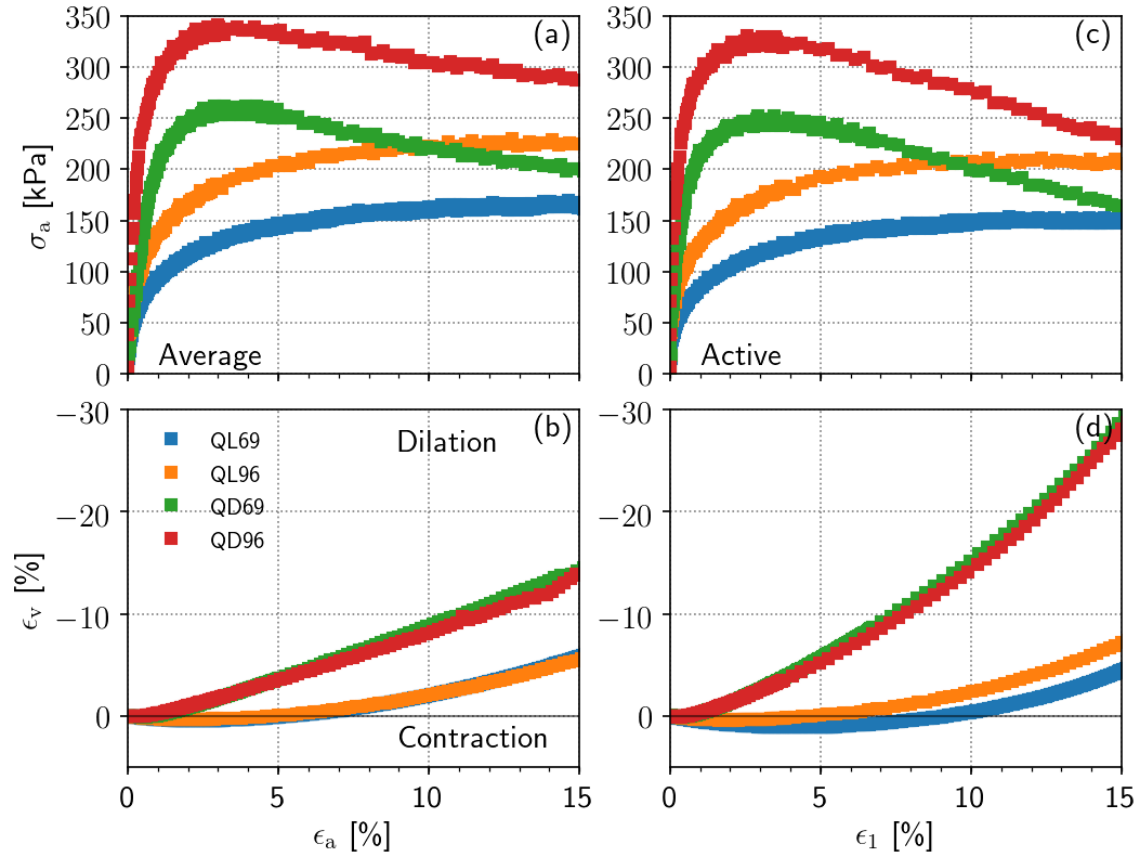


Fig. 23 Global and local strength and volumetric response of Q-Rok sand specimens, (a) Global stress strain curve, (b) Volumetric strain determined from average region against global axial strain, (c) – (d) Stress-strain-volumetric response as determined from strains in the active region.

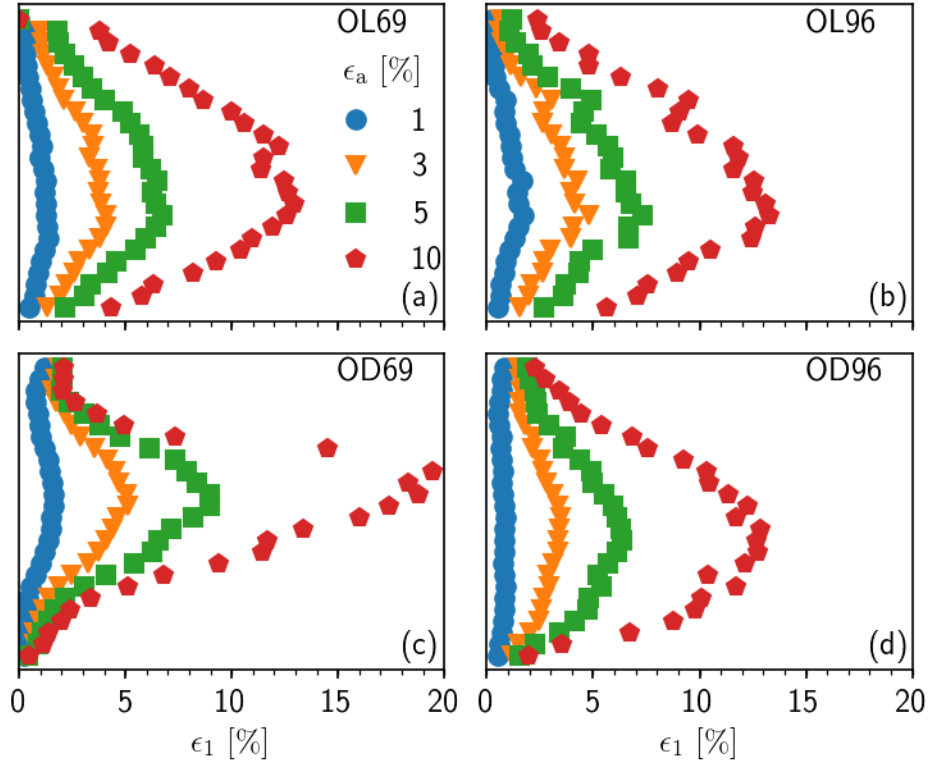


Fig. 24 Major principal strain, ϵ_1 (vertical) from DIC along the height of the Ottawa sand specimens at various stages of global axial strain, ϵ_a : (a) OL69, (b) OL96, (c) OD69, (d) OD 96

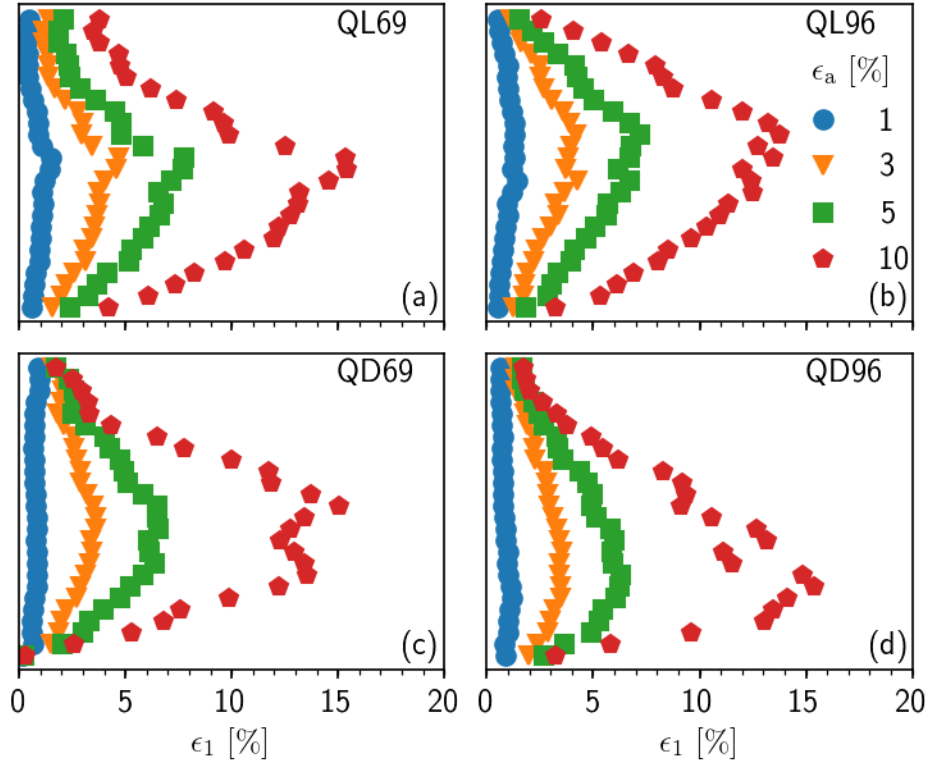


Fig. 25 Major principal strain (ϵ_1) from DIC along the height of the Q-Rok sand specimens at various stages of global axial strain, ϵ_a : (a) QL69, (b) QL96, (c) QD69, (d) QD 96

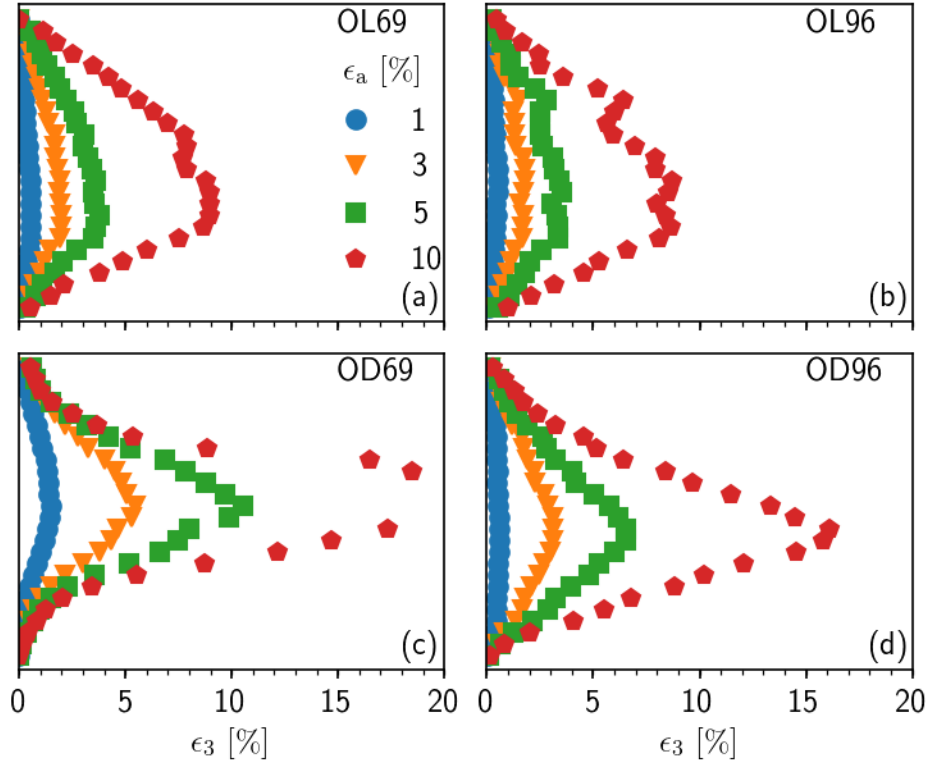


Fig. 26 Minor principal strain, ϵ_3 , (lateral) from DIC along the height of the Ottawa sand specimens at various stages of global axial strain, ϵ_a : (a) OL69, (b) OL96, (c) OD69, (d) OD96

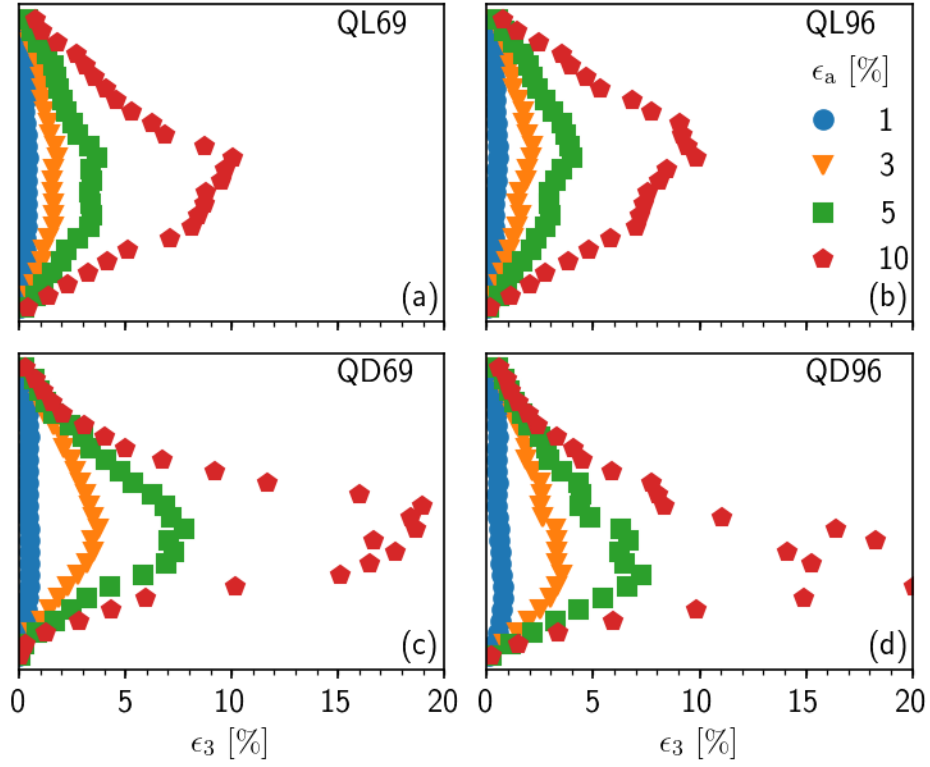


Fig. 27 Minor principal strain (ϵ_3) from DIC along the height of the Q-Rok sand specimens at various stages of global axial strain, ϵ_a : (a) QL69, (b) QL96, (c) QD69, (d) QD96

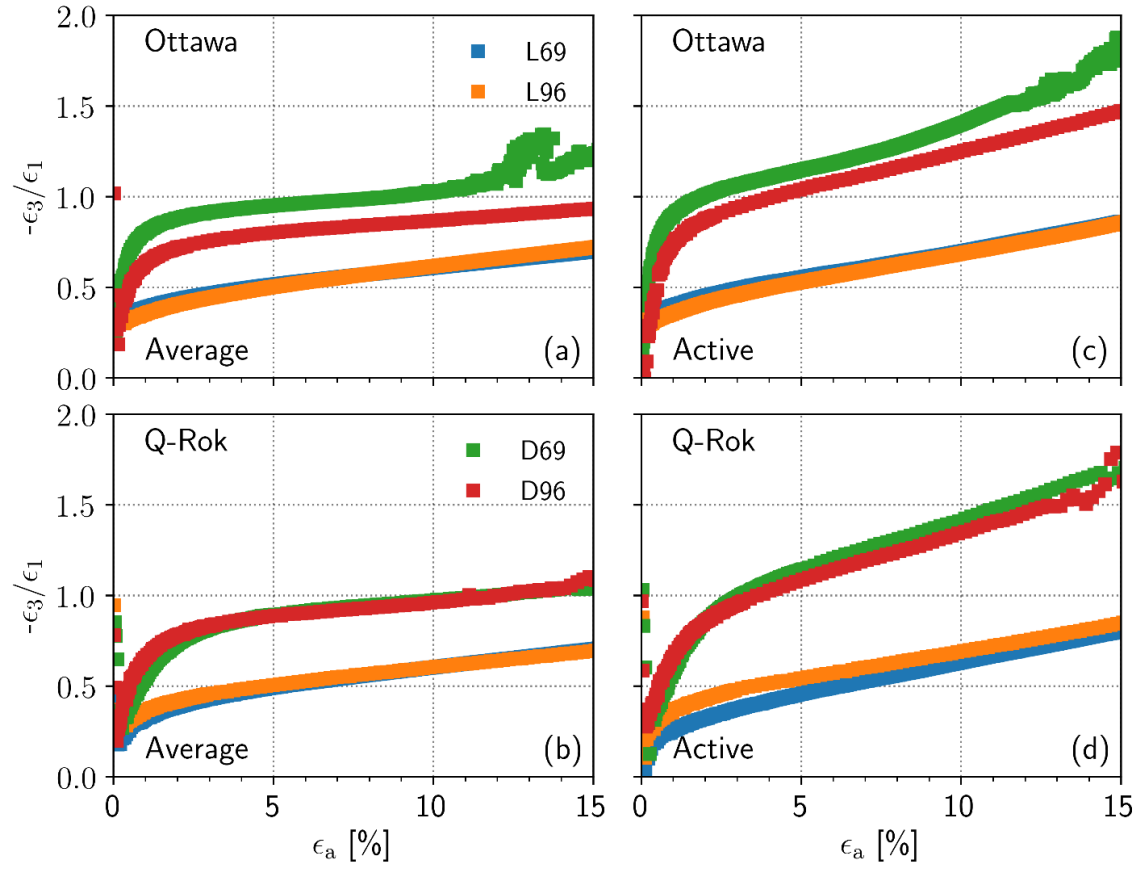


Fig. 28 Lateral to axial strain ratios from DIC during triaxial compression (a) Average region Ottawa sand, (b) Average region Q-Rok, (c) Active region Ottawa sand, (d) Active region Q-Rok.

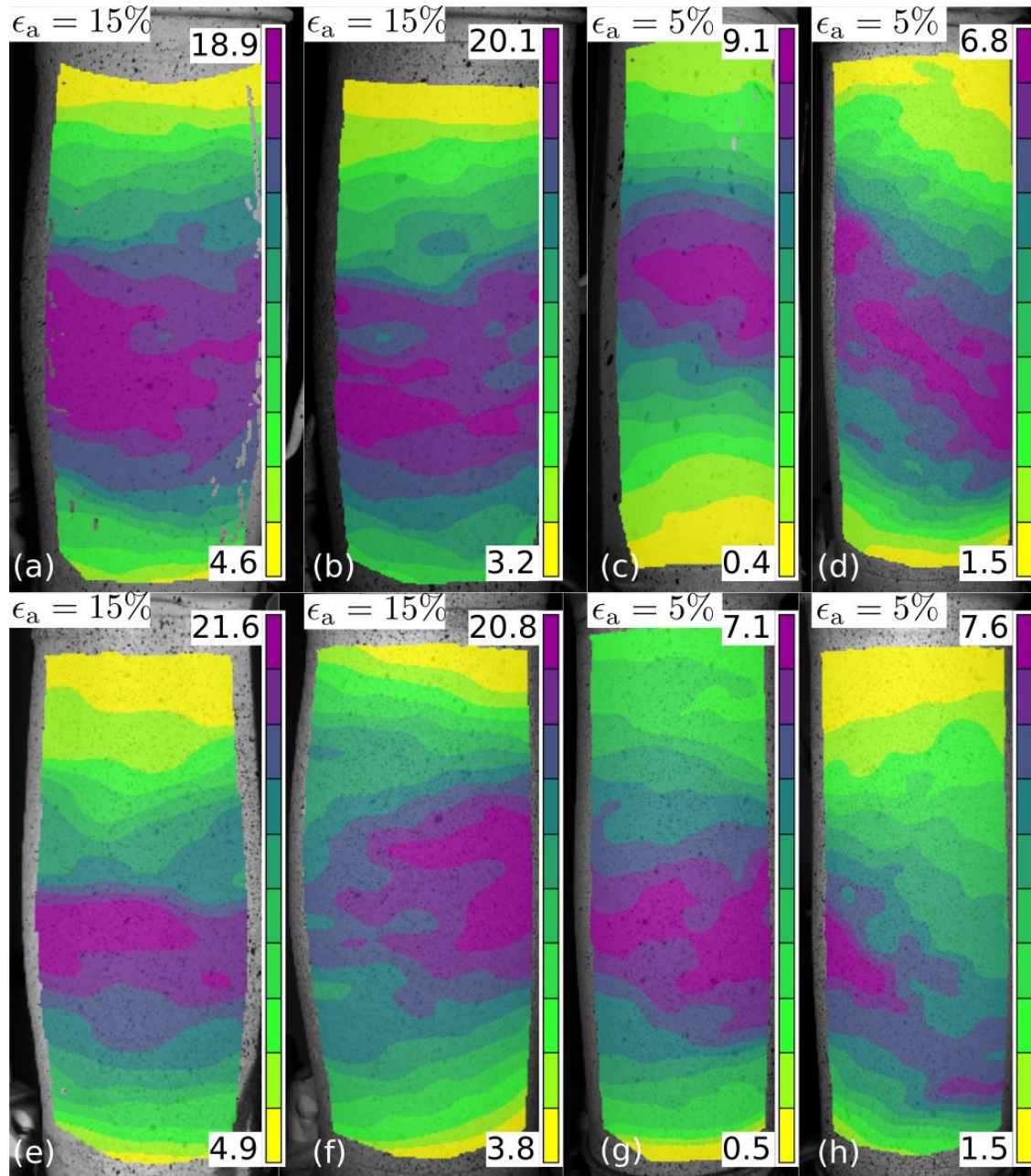


Fig. 29 Surface axial strains, ϵ_1 , approximately at failure for all the specimens. The global axial strain, ϵ_a , is 15% for loose specimens and 5% for dense specimens. The scale is different for each of the specimens as shown in the color bar for better visualization of localized strain. (a) OL69, (b) OL96, (c) OD69, (d) OD96, (e) QL69, (f) QL96, (g) QD69, and (h) QD96.

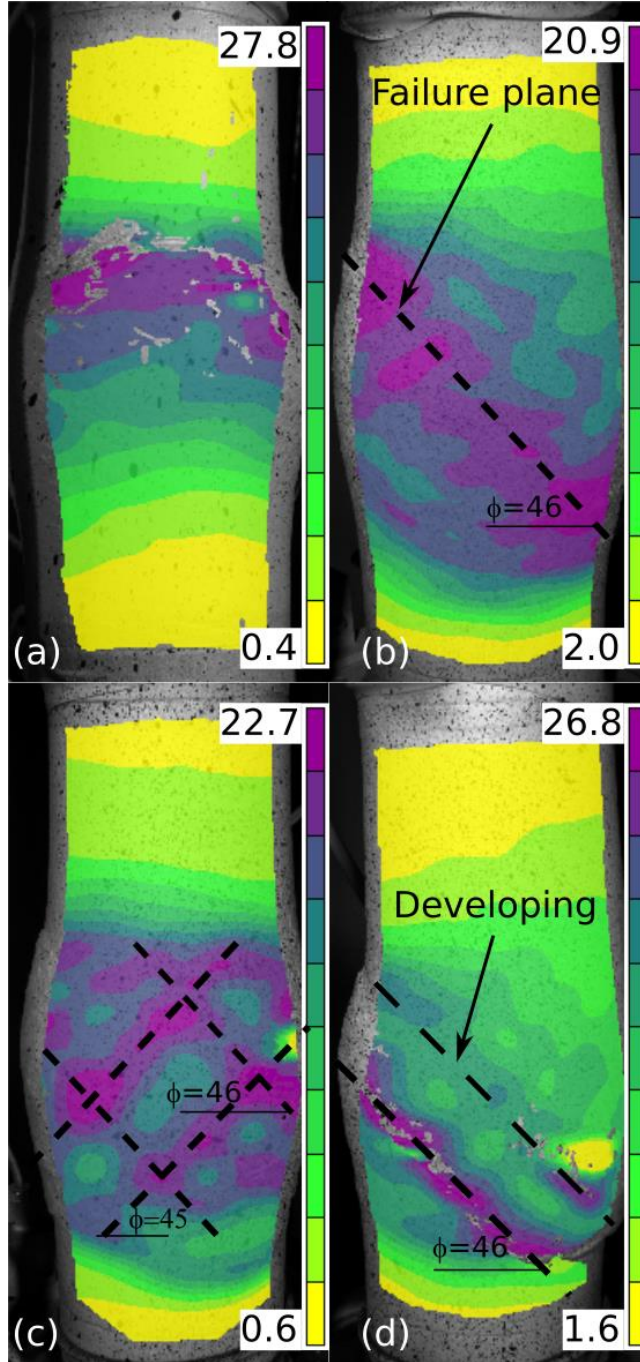


Fig. 30 Visualization of post peak localization regions in dense specimen at $\epsilon_a = 15\%$, (a) OD69, (b) OD96, (c) QD69, (d) QD96. The color bar has been scaled individually for better visualization of failure planes.

CHAPTER III
ROLE OF PARTICLE SHAPE IN DETERMINING TENSILE STRENGTH
AND ENERGY RELEASE IN DIAMETRICAL COMPRESSION OF
NATURAL SINGLE SILICA GRAINS

A version of this chapter was submitted by Aashish Sharma, and Dayakar Penumadu:
Sharma, A., and Penumadu, D. “Role of particle shape in determining tensile strength and energy release in diametrical compression of natural single silica grains” *Soils and Foundations*

Aashish Sharma developed the idea, performed all the experimental tests, data analysis and the writing under the guidance of advisor Dr. Dayakar Penumadu.

Abstract

Extensive particle fracture has been noticed in projectile penetration tests. However current models for predicting projectile penetration depths do not consider particle fracture. In order to understand the role of particle shape on single grain strength and the subsequent comminution process, single grain crushing tests on sand grains of different shapes and sizes were performed. High-resolution, three dimensional images of grain surface, were created using confocal microscope and fracture of grains captured with high-speed imaging. Acoustic emission during single grain crushing was then used to estimate the energy released during grain fracture. It was found that local stress fields influence particle strength and in natural granular material surface flaws maybe the critical flaw causing fracture. The critical flaw size causing fracture were smaller than measured surface roughness. The mass specific fracture energy increases with increasing failure stress and decreasing particle size and is significantly greater than that computed from pure diametrical breaking. In general, mass specific energies for different shapes were similar as angular sands required multiple events to break.

Introduction

Laboratory projectile penetration tests have shown extensive comminution along the path of the projectile and a false tip composed of crushed grains (Allen et al. 1957a; Glößner et al. 2017). Energy is therefore dissipated in fracturing particles in addition to mass movement, friction and as heat. Projectile penetration models based on cavity expansion theory (Forrestal and Luk 1992) are quite successful in estimating the penetration depth, however due to the continuum approach fail to account for energy dissipation in particle fracture. Also simulations employing Discreet Element Model (DEM) generally do not consider particle breakage (Takeda et al. 2018). With increasing sophistication of numerical models and cheaper hardware, multi-scale modeling is being undertaken in science and engineering to understand the effect of micro and meso scale

features on the macro scale behavior. Understanding the meso scale feature of particle fracture with some insights of micro scale granular properties will contribute to the development of models which can exhibit realistic energy dissipation in particle crushing during the passage of the intruder.

When the force on a particle exceeds its tensile strength, grain fractures creating new surfaces and progenies of different sizes. Fracture in brittle material is understood to occur when the stress intensity factor at one of the flaws, either volume or surface, exceeds the critical value. The crack then propagates instantly and uninhibited until it emerges at a boundary. This may lead to the formation of either two or many progenies of smaller sizes and different shapes. Energy is dissipated in the creation of new surfaces, sound, and motion of the progenies. The tensile strength of grains is determined by performing diametrical compression tests on single grains. The results are then statistically modelled using the two parameter Weibull weakest link theory (Weibull 1951). The tensile strength increases with decreasing particle size (McDowell 2001) and angular particles have lower strength than rounded particles (Nakata et al. 2001b).

Fracture of single particles are governed by stress field at the contacts (Zhang et al. 1990). Stress field at the contacts are in turn influenced by the loading geometry and surface features. These geometrical and loading features may manifest as decrease in strength with angularity. The crushing strength then influences the stress strain behavior and the comminution process in high stress isotropic compression, one-dimensional (1D) compression (McDowell 2002; Nakata et al. 2001a), and triaxial loading (Nakata et al. 1999).

There is uncertainty regarding the dominant factor in the comminution process, whether the dominating phenomenon is the coordination number or the size dependence of strength. The consensus thus far is that coordination number influences the fracturing of particles (Einav 2007; McDowell et al. 1996; Tsoungui et al. 1999); smaller particles with fewer neighbors fracture before larger particles with higher coordination number. The dominance of the coordination number leads to a fractal particle size distribution of the crushed particles which has been observed in high stress (Zhong et al. 2018) and high strain tests (Coop et al. 2004). However, Zhang et al. (1990) observed grain crushing in larger particles with smaller particles remaining intact. Also, Kanda et al. (1985), and Tavares and King (1998) have shown increasing energy is required to fracture smaller

particles. Fracture energy of natural sand grains of different shapes and sizes measured with the help of acoustic emission (AE) technique are presented.

Single particle crushing data on rounded and angular sands are presented. In order to better understand the variability of strength in sand grains, thin sections of sand grains were observed for defects and crystallinity of the grains. Further, detailed high-resolution three-dimensional (3D) images of the grain surface were created to gain further insights on the effect of local stress field and grain-platen contact, on single particle strength. In addition, high speed images of particle crushing were captured to observe crack propagation during particle crushing. Energy released during crushing was measured using AE for different particle shapes and sizes. These observations have been synthesized to present a probable framework for comminution process and the observation of fractal nature of comminuted remains.

Materials

Three different sands, Ottawa sand, Q-Rok and Euroquarz Siligran were used for single grain crushing tests. The sands have very similar mineralogy but different particle morphology. Ottawa sand is unground silica with sub-rounded grains, Q-Rok is unground silica with angular grains, and Siligran grains are sub-angular in shape. The maximum grain size of Ottawa sand and Q-Rok was 850 μm and for Siligran the maximum size was 710 μm . Particle shape and size play major roles in determining the fracture strength of single grains. The sands were first washed and then divided into three size fractions using US standard sieves; retained on #30 sieve (R30), passing #30 sieve and retained on #35 sieve (P30R35), and passing #35 and retained on #40 sieve (P35R40). Shape parameters for each fraction were computed from two dimensional (2D) images using computational geometry (Zheng and Hryciw 2016) and are presented in Table 5. The values from the computational geometry algorithm are comparable to the those presented in Krumbein and Sloss (1963). Roundness indicates the degree of angularity; roundness values close to unity indicate smooth rounded grains. Sphericity indicates the closeness of the shape of the 2D image of the grain to a circle and is computed as the ratio of the length and width of the grain. The sphericities of all the sands are similar, and though the roundness values are different for different sands it does not vary significantly for the different size fractions. Thin section images of sands observed under cross polarized light indicated that Ottawa sand grains were single crystal, Q-Rok

grains may contain polycrystalline grains, and Siligran grains may contain other mineral inclusions or bulging recrystallization regions. Thin sections of Q-Rok, showing polycrystalline grain, and inclusion and bulging recrystallization in Siligran are shown in Fig. 31. In addition to the sand grains, four 350 μm diameter and three 800 μm diameter fused silica spheres were also tested.

Experimental methods

Single particle crushing

The Zwick Z2.5 hardness testing machine equipped with hardness measuring head was used to test sand grains in diametrical compression. The hardness measuring head contained 200 N loadcell, a 0.02 μm resolution depth measuring device, a 3 mm diameter flat punch diamond indenter. The transducer that measured displacements was separate from the load cell frame for very precise and zero compliance displacement measurement. The base plate was 5 mm thick and made of Silicon Carbide (SiC). The boundary conditions in single grain crushing can have significant influence on the fracture force (Shipway and Hutchings 1993a) as it affect the contact area with the grain and consequently the stress distribution with the grain. Hard platens were chosen to reduce the contact area between the grain and the platen and determine the smallest critical fracture force. Single grains were placed on the base plate and compressed with the diamond surface of the flat punch indenter at 0.1 mm per minute. A seating load of 0.2N was used to ensure seating of the indenter on the sand grains. The initial height of the specimen was measured as the difference in the displacement values at the base and the top of the grain at the seating load. Approximately 30 specimens were tested for each size fraction of the sands.

Single particle crushing with acoustic emission (AE) measurement

A separate testing system, Psylotech μ -TS was used for single grain crushing test with AE measurement. The system was equipped with 1.6 kN load cell and 9 mm linear variable displacement transformer (LVDT) to measure displacement at full scale. A floating window scale, a proprietary technology, is then able to resolve displacements to sub-micron resolution and force to sub newton resolution enabling very precise displacement control and high-resolution force measurements. This floating window keeps moving as the limits of the window is reached enabling the record of both full-scale measurements and high-resolution windowed measurements throughout the duration of the test. This presents a unique opportunity to measure small variations

in force due to surface features and at the same time the capability to fracture particles of reasonable sizes. The base plate and flat punch diamond indenter used in the Psylotech system were the same as those used with Zwick system. Mistras Group AEWIN was used for recording and analyzing AE signals. A general-purpose sensor, R15a, was attached to the loading head as shown in Fig. 32. The sampling rate was 1MHz, and the threshold amplitude was set at 40 dB to cancel the testing system vibrations, and a 120 kHz high pass filter was applied to cancel audible sound. Approximately 25 specimens were tested from the R30 size fraction and 15 specimens from P35R40 size fraction.

AE measures transient elastic waves that is generated in a medium when energy is suddenly released. In engineering material, generation of elastic waves generally coincides with the formation and propagation of cracks. AE can capture this dynamic phenomenon. The five parameters that are generally used to characterize AE signals are amplitude (*A*), counts (*C*), duration (*D*), rise time (RT) and the measured area under rectified signal envelope (MARSE), which is also known as energy (*E*) with arbitrary units. These parameters are shown in Fig. 33. An event is triggered when the rising signal crosses the threshold and lasts until the signal is smaller than the threshold. Though the sensor was attached to the loading head, energy released at fracture is quickly dissipated via the lower base and free surfaces in the loading head with unknown proportion of the energy released reaching the sensor. Therefore, fused silica spheres were tested in the same configuration in order to scale the measured energy.

AE has diverse applications, from study of damage and damage propagation in brittle materials (Dai and Labuz 1997) to health monitoring of bridges (Yapar et al. 2015). Particularly in the study of material damage, AE has been used to investigate damage in concrete (Ohtsu and Watanabe 2001), relate acoustic energy to fracture energy in concrete beams (Vidya Sagar and Raghu Prasad 2011), monitor failure in fiber composites (Bohse 2000; De Rosa et al. 2009), monitor tests on coarse and fine grained soils (Koerner et al. 1981), and crushing of coarse grained soils (Brzesowsky et al. 2014; Li et al. 2018; Muñoz-Ibáñez et al. 2018). Recently, the technique has also been used to monitor single grain crushing tests (Ibraim et al. 2017; Mao and Towhata 2015).

Surface imaging

High resolution KEYENCE VK-X200 series 3D laser scanning confocal microscope was used for very fine resolution 3D images of few grains at 10X magnification. Confocal microscopes block out of focus-plane light using a spatial pin-hole, allowing light only from the focal-plane. A 3D image of the object is then constructed from 2D images at various heights by moving the focal plane from the bottom to the top of the object. Single sand grains were placed on a dark surface and the start and end focus elevations were set below the base surface and above the top of the grain respectively. Three dimensional images for the three different sands showing typical surface features of the sands are shown in Fig. 34. In addition, ten grains were imaged at higher resolution of 50X to determine the surface roughness of the grains. For each grain the roughness was determined from approximately 200 μm square area as the average sum of the differences of the surface points from the mean surface height. The roughness values of the sands were similar; 2.36, 3.83 and 2.68 μm for Ottawa sand, Q-Rok and Siligran respectively

High speed imaging

High speed images enable visualizing the formation and propagation of cracks in engineering materials. High speed images of particle crushing were captured with Photron Fastcam APX RS high speed camera. A low magnification lens was attached to the camera and the sand grain was illuminated using two focused light beams. Due to explosive nature of fracture, the frame rates for Ottawa sand and Siligran were 3000 frames per second (fps), whereas the frame rate for Q-Rok was 150 fps for longer time duration in order to capture asperities breaking and the final break.

Results and discussion

Effect of particle morphology on crushing behavior

Single grain crushing tests assumes that fracture occurs from tensile stresses (σ) along the axis of loading and is computed using Eq 6 (McDowell 2001; Nakata et al. 2001b).

$$\sigma = \frac{F}{d^2} \quad \text{Eq 6}$$

where, F is the force at failure and d is the height of the particle between the two flat surfaces. Example stress-strain curves for five grains are shown in Fig. 35. The stress is computed using Eq 6 and engineering strain, is computed by dividing the displacement by the original height of the particle. During diametrical crushing between two flat platens, the particles are in contact at their extremity points (highest and lowest) with the platens. For grains devoid of sharp asperities, with few contact points, the stress-strain relationship for Ottawa sand and Siligran are typical as shown in Fig. 35. However, the influence of surface features at the contacts on the strength of the grains are evident. The small contact area due to the pointed top surface of O2 coincides with lower strength than the flatter contact surface of O5. With smaller contact area, maximum tensile stress occurs near the surface, and the volume of the grain under tensile stress along the loading axis increases. This increases the likelihood of encountering a critical flaw leading to failure (Shipway and Hutchings 1993b). There is similar observation with Siligran specimens; the strength of S2 is significantly lower than that of S5 with a large flat contact surface. With highly angular grains there will be multiple contact points of the grains with the bottom surface for stability. During the application of force, the extremity points which may be asperities will break before the particle fractures. Also, on loading the particles may rotate for greater stability (Cavarretta and O'Sullivan 2012). These asperities breaking and readjustments appear as numerous reductions in force as typified by the curves for Q-Rok. The breaking of the small asperities in Q2 (Fig. 34c) and the ultimate failure is seen in the response of Q2 in Fig. 35. Furthermore, if the grains are polycrystalline, the bond between the grains which are weaker, will determine the strength of the grain. The polycrystalline nature of Q4 can be seen in Fig. 34d, which fractured at a very low stress with three large progenies.

High speed images captured during these tests further illustrate the role of particle shape and surface features on particle strength. Ottawa sand and Siligran generally exhibited diametrical fracture. However, surface features dominate, the large surface depression in S2 (Fig. 34e) is responsible for failure as the particle fails by chipping at the flaw. The asperities breaking in Q2, flattens the contact surface during the initial stages of loading, and failure occurs by crack opening at the top. In Q4, the fracture occurs along the grain boundaries of the polycrystalline grain. In most of the tests, probably due to the harder diamond surface at the top, the cracks propagated from the top of the grains.

Single particle crushing

The results of single particle crushing are modelled using the two parameter Weibull distribution with a scale and the shape parameter as shown in Eq 7 (Weibull 1951):

$$P_s(V_o) = \exp \left[- \left(\frac{\sigma}{\sigma_o} \right)^m \right] \quad \text{Eq 7}$$

where, $P_s(V_o)$ is the survival probability of a particle of volume V_o computed based on mean rank survivability (Davidge 1979), σ is the nominal stress as given in Eq 6, σ_o is the characteristic strength at which 37% of the particle survive, also known as the scale parameter, and m is the Weibull modulus or shape parameter, indicating the variability of the test data. The value of m decreases as variability in the data increases.

Size fractions based on sieve analysis results in a very non-uniform particle size. As the grains are bouncing in the mesh, particles with their smallest dimension smaller than the mesh size can pass through the openings. Particles are then chosen randomly based on visual estimation of equal size which may result in particles of varying height in the direction of loading. Therefore, the height of the particles, immediately before the application of the load was used to group the particles into three groups based on k -means clustering algorithm. When the desired number of groups is specified, the algorithm begins by randomly generating the size centers for the desired number of groups and computing the distances to each particle from these centers. The data is then clustered into groups by minimizing within group variance. A new centroid for each group is then computed and the clustering is repeated until the change in the centroid is less than a specified tolerance. The Python module scikit-learn (Pedregosa et al. 2011) was used for clustering single particle crushing data based on grain heights. Weibull plots for the largest size clusters are shown in Fig. 36 and the parameters for all the clusters are presented in Table 6. With increasing angularity both σ_o and m decreases, however the contributing factor is not only angularity but also the surface features and volume flaws present in Q-Rok and Siligran as shown in Fig. 34 and Fig. 31. Despite all three sands consisting of more than 99.5% quartz, there are significant variations in strength due to the formation process. For similar size range silica sand particles, Nakata et al. (2001) have reported values of 72.9 MPa and 2.17 for σ_o and m respectively and for Ottawa sand

Cil and Alshibli (2012) have reported values of 137.9 MPa and 3.26. The value of m for rounded quartz of similar size is in the range of 2.57 (Kanda et al. 1985). For engineering ceramics, the m can be as high as 10. In general, low values of m for Q-Rok suggests high variability of the measured strength. However, even with low m values the results in Table 6 indicate that σ_o increases with decreasing size. As m does not vary significantly over different size clusters, a Weibull analysis may be used to determine size dependent strength for these sands in the size range presented. However, Kschinka et al. (1986), based on tests performed on glass spheres of ten different sizes, cautions that Weibull approach is rather forgiving and can conceal multiple flaw distributions. Additionally, Nakata et al. (2001b) report decreasing m with decreasing particle size and increasing angularity in quartz sand and Kanda et al. (1985) have shown decreasing m with decreasing size in quartz spheres.

The relationship between crushing strength and the size of the grains is shown in Fig. 38 and is a power law of the form $\sigma \propto d^\alpha$ (McDowell 2001; McDowell and Amon 2000; Nakata et al. 2001b). The slope (α) of the regression line in a log-log plot were -0.75, -1.78, -0.92, and -0.63 for Ottawa sand, Q-Rok, Siligran and fused silica. The values for quartz sand may range from -0.79 (Nakata et al. 2001b) to -0.92 (McDowell 2001). The angular nature of Q-Rok coupled with increasing probability of multi-crystallinity in larger grains may have contributed to the shaper reduction of strength with particle size. Assuming volume flaws, the values of m determined as $m = -3/\alpha$ were 4, 1.69, 3.26, and 4.76 respectively for Ottawa sand, Q-Rok, Siligran and fused silica. These values for the sands are larger than those presented in Table 6 for all size fractions. Assuming surface flaws the m values computed as $m = -2/\alpha$ were 2.67, 1.12, and 2.17 which are closer to the values reported in Table 6. Therefore, it is more likely that surface flaws may have dominated the fracture.

The surfaces of natural granular materials contain flaws as seen in Fig. 34. Surface flaws can significantly reduce the strength of grain; Shipway and Hutchings (1993a) report that the fracture force reduced from 167 N for smooth glass spheres to 70 N for abraded glass spheres. The size of Griffith type flaw on grain surface at the edge of the circular Hertzian contact area is computed employing linear elastic fracture mechanics as shown in Eq 8 (Brzesowsky et al. 2011; Zhang et al. 1990).

$$c_f = \frac{K_{IC}^2 4\pi}{Y^2 (1 - 2\nu_g)^2 F^{2/3}} \left(\frac{3r_g}{4E^*} \right)^{4/3} \quad \text{Eq 8}$$

where, c_f is the size of the critical flaw size on the surface and radial in direction, equivalent to edge crack in a plate, which causes failure under diametrical force F , Y is a dimensionless factor equal to 1.12 for single edge crack, K_{IC} is the critical stress intensity factor, r_g is the grain radius and $1/E^* = (1 - \nu_g^2)/E_g + (1 - \nu_p^2)/E_p$ where ν_g and E_g , ν_p and E_p are the Poisson's ratio and elastic modulus of the grain and platen respectively. In compression tests of irregular natural particles, there will be greater number of contact points at the bottom for stability (Cavarretta and O'Sullivan 2012) resulting in lower stresses at the bottom than at the top (Turner et al. 2016). Analysis of high-speed images also showed crack propagating from the top of the specimens in most specimens. Wang and Coop (2016) have reported similar observation from high speed images during fracture for Leighton Buzzard sand. Therefore with the assumption that higher stresses occur at the contact with the diamond surface, c_f was computed using $\nu_g = 0.077$, $E_g = 95.68$ GPa, $\nu_p = 0.20$, $E_p = 1100$ GPa and $K_{IC} = 1 \text{ MPa}\sqrt{\text{m}}$ (Brzesowsky et al. 2011; Ferguson et al. 1987). Assuming surface flaws, and linear scaling of flaw size with particle size ($c_f/r_g = \text{constant}$) Zhang et al. (1990) arrived at excellent agreement between predicted crushing pressure in one dimensional (1D) compression with experimental data. The flaw size scaling with particle size is shown in Fig. 39. The flaw size increases with particle size and if the flaw size scales as a power law ($c_f \propto d^\alpha$), the values of the exponent are 0.5, 1.17, and 0.61 for Ottawa sand, Q-Rok, and Siligran respectively. However, assuming linear scaling (c_f/r_g) as Zhang et al. (1990) the slopes were 6.93×10^{-5} , 1.12×10^{-4} , and 6.66×10^{-5} respectively for Ottawa sand, Q-Rok, and Siligran. The value for Ottawa sand are smaller than those reported by Zhang et al. (1990) for quartz grains with $K_{IC} = 0.3 \text{ MPa}\sqrt{\text{m}}$. The flaw size in Q-Rok, with smaller m , increases at a faster rate with increasing grain size. Characteristic flaw size (c_{fo}) computed assuming a Weibull distribution of the critical surface flaw sizes are shown in Table 6. For Ottawa sand with higher m , the flaw size becomes a larger proportion of the grain with decreasing particle size (McDowell and De Bono 2013). The flaw sizes for Ottawa sand and Siligran are in the range of flaw sizes, $0.004 \mu\text{m}$ to $0.07 \mu\text{m}$, reported by Zhang et al. (1990) for quartz but smaller than $0.115 \mu\text{m}$ reported

by Brzesowsky et al. (2011) for quartz sand in The Netherlands. Flaw sizes in Q-Rok are approximately three times as large as those in Ottawa sand for the largest cluster size and twice as large for the smaller clusters. This difference may have resulted from the polycrystalline nature of the larger grains of Q-Rok. All characteristic flaw sizes, c_{fo} , are smaller than the surface roughness of the sands, however as seen in Fig. 37 surface features may initiate fracture in angular sands.

The values of m for Griffith type surface flaws are higher than those for the Weibull analysis for all sands. The difference is larger for Ottawa sand than angular Q-Rok. The predicted values of c_f for Ottawa sand do not vary as much as those for Q-Rok. In Q-Rok, surface features of the same order of magnitude as the surface roughness may contribute to failure. Zhang et al. (2016) have shown, adopting Hertzian contact and linear fracture mechanics, that the assumed location of critical flaw, either surface or volume, influences the size dependence of strength. The two conditions bound the Weibull analysis, with surface flaws presenting a lower bound and center crack (volume flaw) the upper bound for the m value. However, in the current analysis m values predicted assuming critical surface flaw is larger than that determined from random distribution of flaws in Weibull analysis.

AE emissions

Example force history curves along with the AE energy releases are shown in Fig. 40. Single to few AE events are typical of Ottawa sand and Siligran. In majority of the cases, failure coincides with the maximum force and the maximum energy. Q-Rok on the other hand exhibits significantly greater number of events, also called hits. The median number of events leading to failure were 4, 23, 10 for Ottawa sand, Q-Rok and Siligran respectively. The energy released in each break is also significantly lower in Q-Rok than in Ottawa sand and Siligran. However, in some cases the cumulative energy released at fracture may exceed that released by Ottawa sand. Also, the energy released at the peak force may not always coincide with maximum energy in Q-Rok. The median values of E at maximum force were 1625, 606, 913 and the median value of the cumulative E at break were 1820, 1172, 1342 for Ottawa sand, Q-Rok and Siligran respectively. In tests which did not completely fracture the grain at maximum force, the definition of break is shown in Fig. 40b. The cumulative energy released at the breakpoint is considered as the fracture energy. The relationship between maximum force, particle size and energy, E , is shown in Fig. 41.

Larger force is required to break bigger particles in Ottawa sand and fused silica spheres with release of higher energy. With increasing angularity, Siligran and Q-Rok, there is greater scatter in the data. Q-Rok with very high angularity and presence of polycrystalline grains has the largest scatter. The effect of angularity increases with particle size in Q-Rok; larger grains are more likely to be polycrystalline and fail along the grain boundaries at smaller force. The higher energy release in angular particles generally corresponded to larger number of events leading to the break. The failure force for fused silica spheres are higher than natural sands, one of the 800 μm fused silica sphere failed at a very high load of 317 N and is not shown in the figure.

The relationship between elastic strain energy at fracture and particle strength have been shown to be a power law relationship (Kanda et al. 1985; Yashima et al. 1987). In the above discussion the energy E was in arbitrary units, which is reasonable for comparison purpose. AE systems also compute absolute energy based on measured signal. However, due to the small sensor area compared to the volume of the base platen and loading head, energy dissipation in the audible range, and wave attenuation in elastic medium the energy value computed from the recorded signal is significantly lower than the actual energy released. The AE absolute energy (ABS-E) was hence scaled based on the elastic strain energy at failure of 350 μm spheres. Hertzian contact was assumed between flat platens and the sphere, and the mass specific elastic strain energy (ESE/M) at one contact was computed as shown in Eq 9:

$$\text{ESE/M} = 2.4961 \frac{1}{\pi \rho E^{*2/3}} \left(\frac{F}{d^2} \right)^{5/3} \quad \text{Eq 9}$$

where ρ is the density of the grain (2.65 and 2.22 g/cm^3 for sand and fused silica respectively) and other terms have been previously defined for Eq 8. The values for ν and E for fused silica sphere were 72 GPa and 0.17 and those for silicon carbide base were 410 GPa and 0.16 respectively. The values for sand grains and diamond platen were the same as used in Eq 8. The above equation was applied separately to each contact of the sphere with the top diamond plate and the bottom silicon carbide plate, and the total specific energy was computed as the sum of the two. Similar approach was followed by (Kanda et al. 1985; Yashima et al. 1987) however they computed compressive strength of spheres using $\sigma = 0.9F/d^2$ as proposed by Hiramatsu et al. (1966). Here Eq 7 which

is generally used in geomechanics is consistently used for both sand grains and silica spheres. The average absolute specific energy for two 350 μm spheres from AE measurement was 0.0022 J/kg and from Eq 9 was 20.278 kJ/kg, a scaling factor of 9.221×10^6 . All specific energy values were scaled using this scaling factor and are shown in Fig. 42. The values for mass specific energies are similar to those presented by Kanda et al. (1985) for quartz. Also shown are mass specific energies for fused silica and quartz spheres using Eq 9 at different failure stresses. The relationship between fracture stress and fracture energy appears linear but it is not continuous over the size range of particles tested. At lower stress range, generally corresponding to larger particles, there is a sharper decrease in specific energy.

The mass specific energy required to crush particles increases with decreasing particle size as shown in Fig. 43. Kanda et al. (1985) and Yashima et al. (1987) arrived at similar values for wide range in particle sizes. They computed elastic strain energy assuming Hertzian contact and employed Weibull relationship for size dependence of strength. King and Bourgeois (1993) have reported similar values for 0.5 – 0.7 mm quartz. As particle size decreases greater energy is required for fracture. Also shown in Fig. 43 is the mass specific energy computed assuming diametrical break and surface energy. The average value of specific energy can range from 0.675 J/m² (Brace and Walsh 1962) to 2.678 J/m² (Tromans and Meech 2004), the line shown in the figure was drawn using 2.678 J/m². The mass specific energy of smaller particles is significantly larger than that computed assuming diametrical breaking. With the high stresses required to fracture smaller particles there is a greater possibility of larger number of progenies than that from a diametrical break. With increasing particle size and angularity, the mass specific energy based on surface energy provides a lower limit estimation.

If coordination number is the dominant phenomenon that controls comminution process, then the effect of smaller grains surrounding larger grain is such that it offsets orders of magnitude of higher energy required to crush smaller particles than larger particles. However, this may not be possible until enough fines have been created. Therefore, initially the size dependence of strength maybe the dominant process with a shift to coordination number dominating the comminution as more fines are produced. A stage is then reached that for a given input energy no more crushing is possible. Therefore, for a given starting particle size distribution, relative density and input energy, it is possible to arrive at the same comminuted distribution.

The power law relationship between mass specific energy and particle size ($MSE/M \propto d^\alpha$) is characterized by -4.2, -6.9, -4.2 and -2.4 for the exponent α for Ottawa sand, Q-Rok, Siligran and fused silica respectively. In view of limited number of specimens, though, there were significant difference in the strength of Ottawa sand and Siligran, the similar value for α could be a consequence of similar flaw size and flaw size scaling with particle size. Q-Rok, on the other hand, is highly angular and with increasing grain size there is greater possibility of encountering polycrystalline grains and larger asperities, which may have contributed to rapid decrease in specific energy with increasing grain size. In addition, the compressive strength decreases with increasing size and the particles with higher specific energies are associated with high fracture stress. Hence both strength and specific energy show similar trend with particle size, and King and Bourgeois (1993) have noted that the distribution of mass specific fracture energy is identical to the probability of fracture.

Conclusions

Results of single grain crushing tests have been presented along with some detailed imaging of grain surfaces, high speed imaging of fracture and acoustic emission measurements. Local stress fields at contacts play a major role in fracture strength of single grains. Some variation in fracture strength may be attributed to grain shape and features at contacts. For the size group presented, Weibull distribution is found acceptable for modeling size dependent strength of grains. Weibull modulus is affected by particle shape and decreases with increasing angularity. The computed Weibull moduli are consistent with the assumption of critical surface flaws for all sands. With decreasing grain size, the flaw size is a greater proportion of the grain size in rounded and high Weibull modulus grains, whereas the flaw size increases at a faster rate for angular sands with smaller Weibull modulus. Acoustic emissions reveal that angular sands release lower energy at break and require greater number of events leading up to the failure. The energy released at fracture increases with fracture stress. Low strength angular particles may at times release larger energy due to multiple small asperities breaking leading up to the main fracture. The mass specific energy required to cause fracture increases with decreasing particle size. This coupled with stress reduction in grains with coordination number may explain the fractal distribution of comminuted remains.

Appendix

Table 5 Shape parameters for different size fractions

Fraction	Sand	Median d [μm]	Roundness	Sphericity
R30	Ottawa	644	0.73	0.85
	Q-Rok	524	0.49	0.76
	Siligran	555	0.66	0.82
P30R35	Ottawa	486	0.73	0.82
	Q-Rok	461	0.45	0.74
	Siligran	479	0.55	0.77
P35R40	Ottawa	404	0.68	0.85
	Q-Rok	387	0.45	0.74
	Siligran	411	0.59	0.78

Table 6 Weibull analysis for fracture stress and flaw size for different clustered groups.

Centroid [μm]	Sand	Number of grains	Failure stress		Flaw size, c_f	
			σ_o [MPa]	m	c_{fo} [μm]	m
663	Ottawa	36	144	2.89	0.05	4.10
699	Q-Rok	22	49	1.12	0.14	1.63
737	Siligran	59	93	1.84	0.07	2.66
532	Ottawa	42	157	2.87	0.04	4.36
513	Q-Rok	64	87	1.31	0.09	1.80
526	Siligran	50	96	2.49	0.06	3.71
395	Ottawa	49	210	2.61	0.04	4.02
372	Q-Rok	38	130	1.27	0.07	1.64
389	Siligran	19	138	1.93	0.05	2.81

Table 7 Median values of characteristic AE parameters at maximum force

Sand	Amplitude, A [dB]	Rise time, RT [μs]	Duration, D [μs]	Counts, C	MARSE, E
Ottawa sand	99	41	2080	225	1625
Q-Rok	91.5	43.5	1699	178	606
Siligran	95	41	1926	204	913

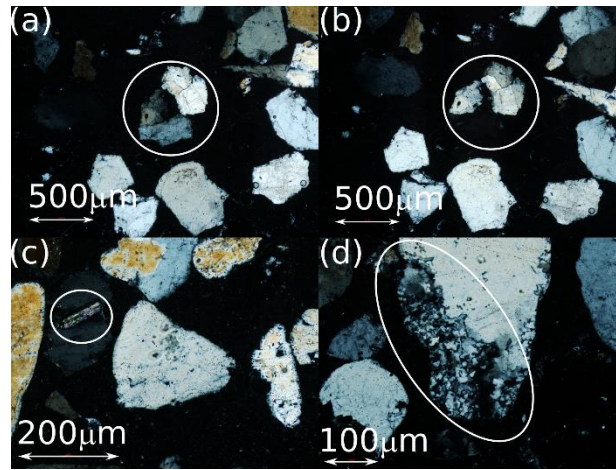


Fig. 31 Thin section images of (a) – (b) multigrain Q-Rok, (c) mineral inclusion in Siligran, and (d) bulging recrystallization in Siligran.

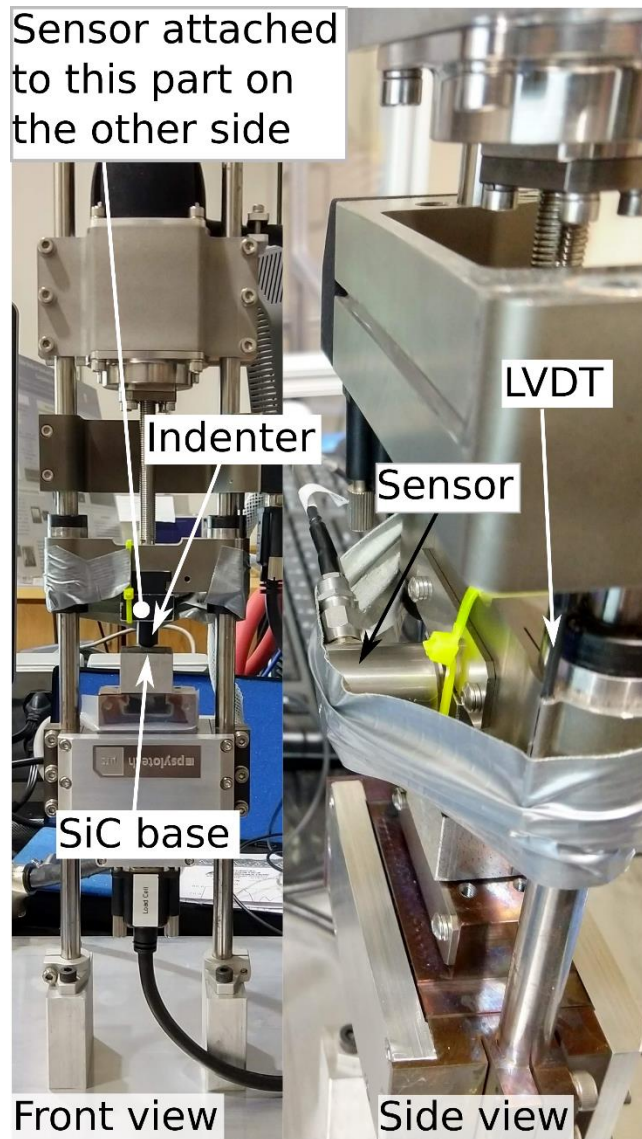


Fig. 32 Psylotech testing system for particle crushing with AE measurement.

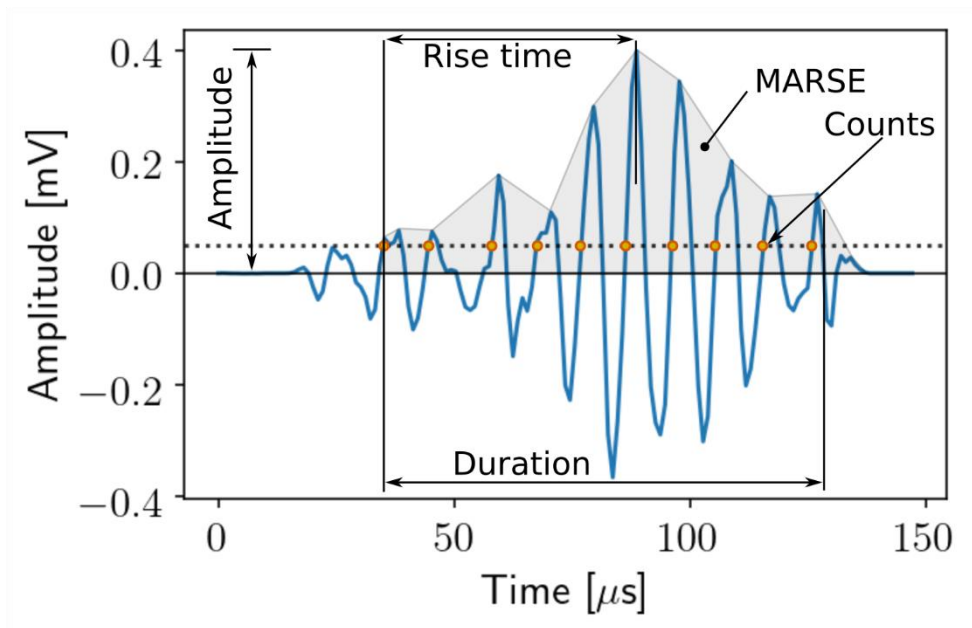


Fig. 33 Explanation of AE parameters.

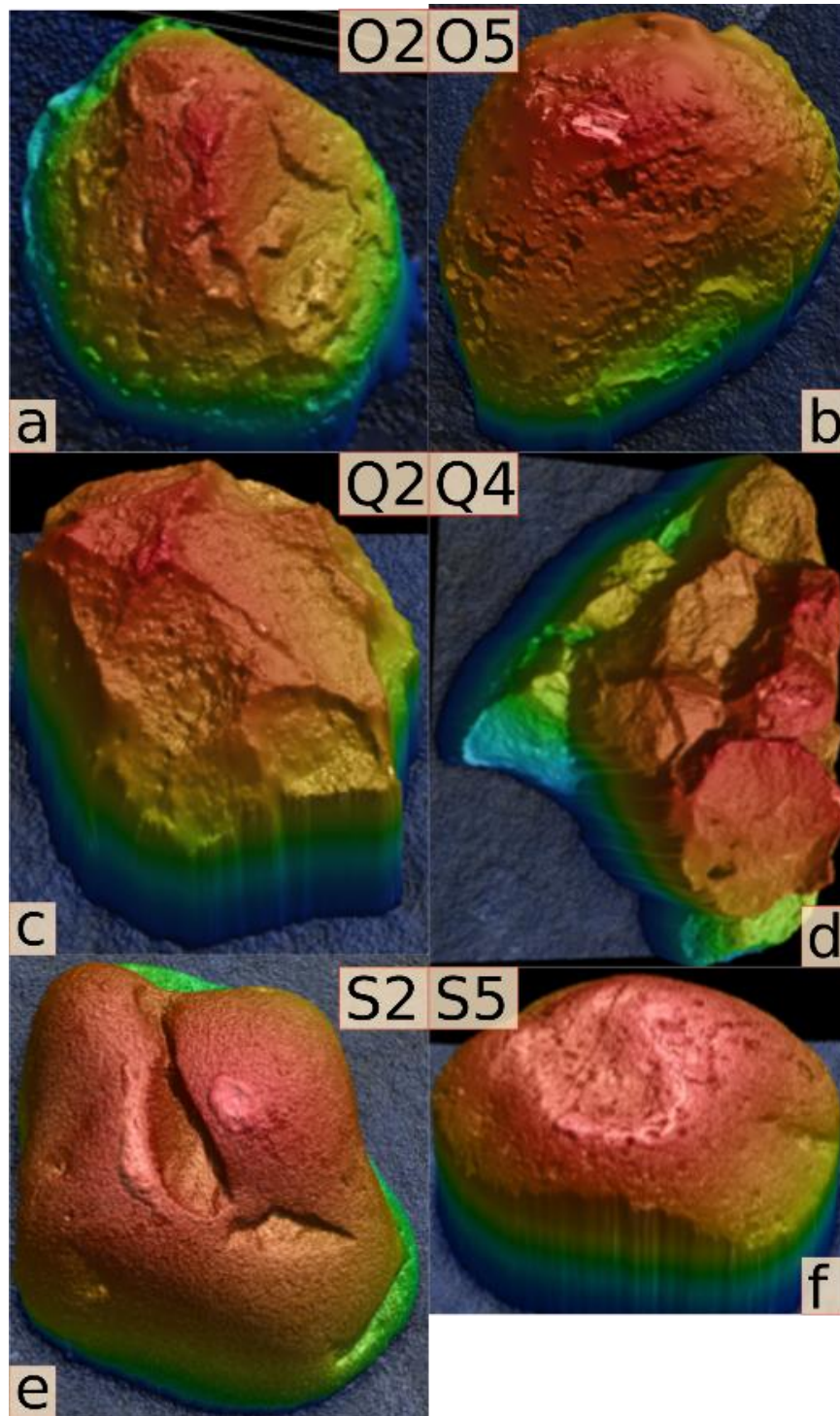


Fig. 34 High resolution 3D surface images of (a - b) Ottawa sand, (c - d) Q-Rok, (e - f) Siligran.

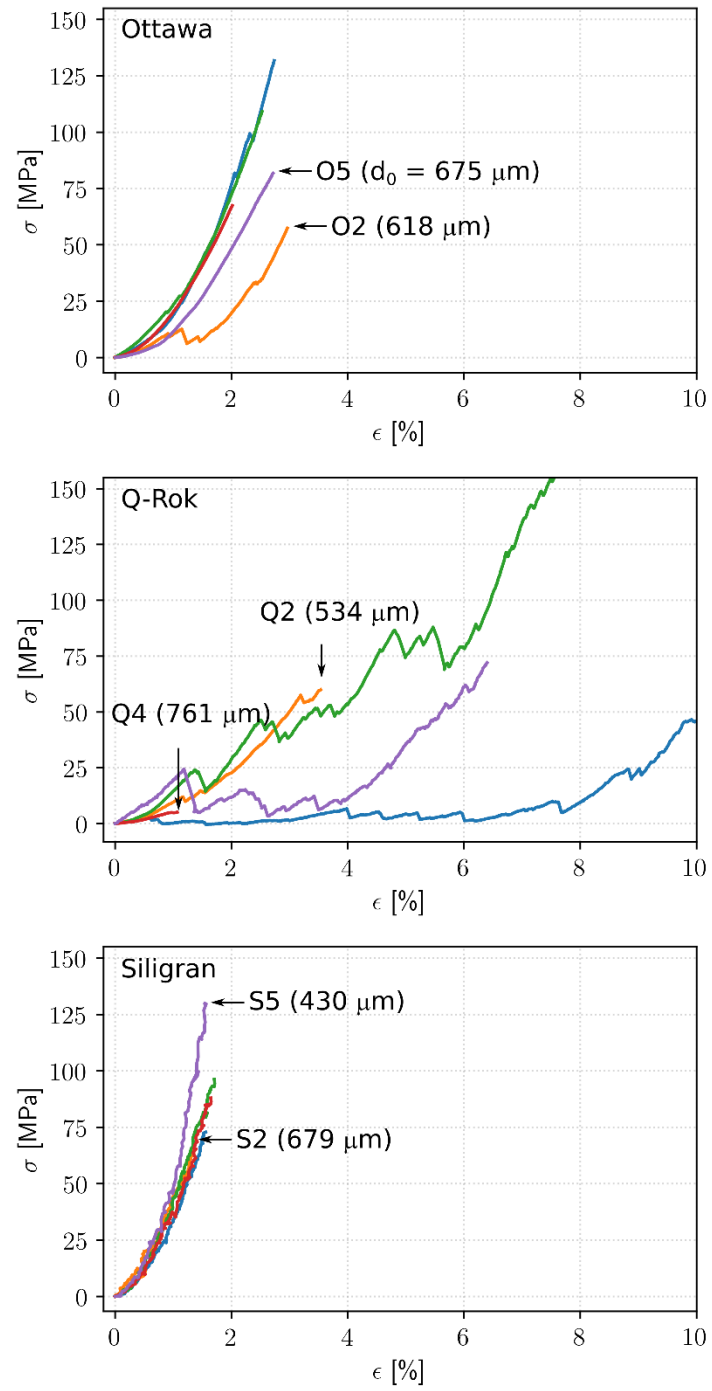


Fig. 35 Example response of the sand grains to diametrical compression, (a) Ottawa sand, (b) Q-Rok, (c) Siligran.

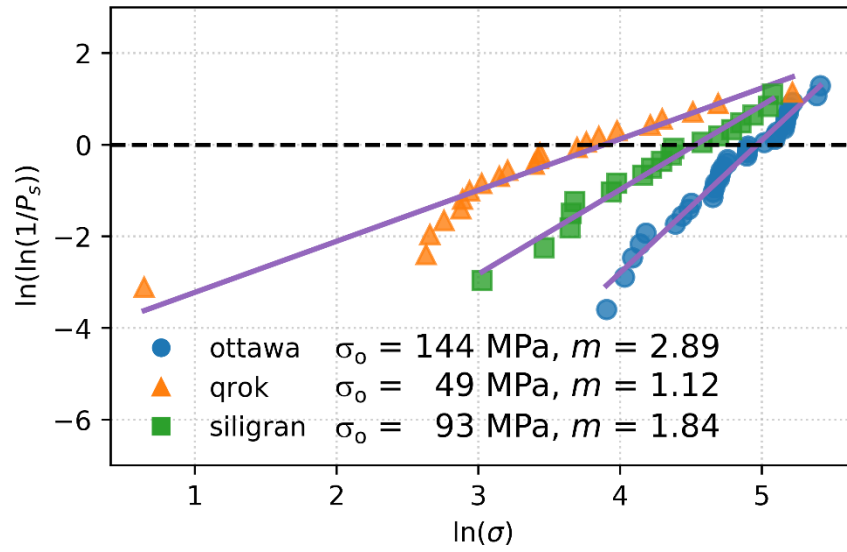


Fig. 36 Weibull plot for the largest size cluster for the three sands.

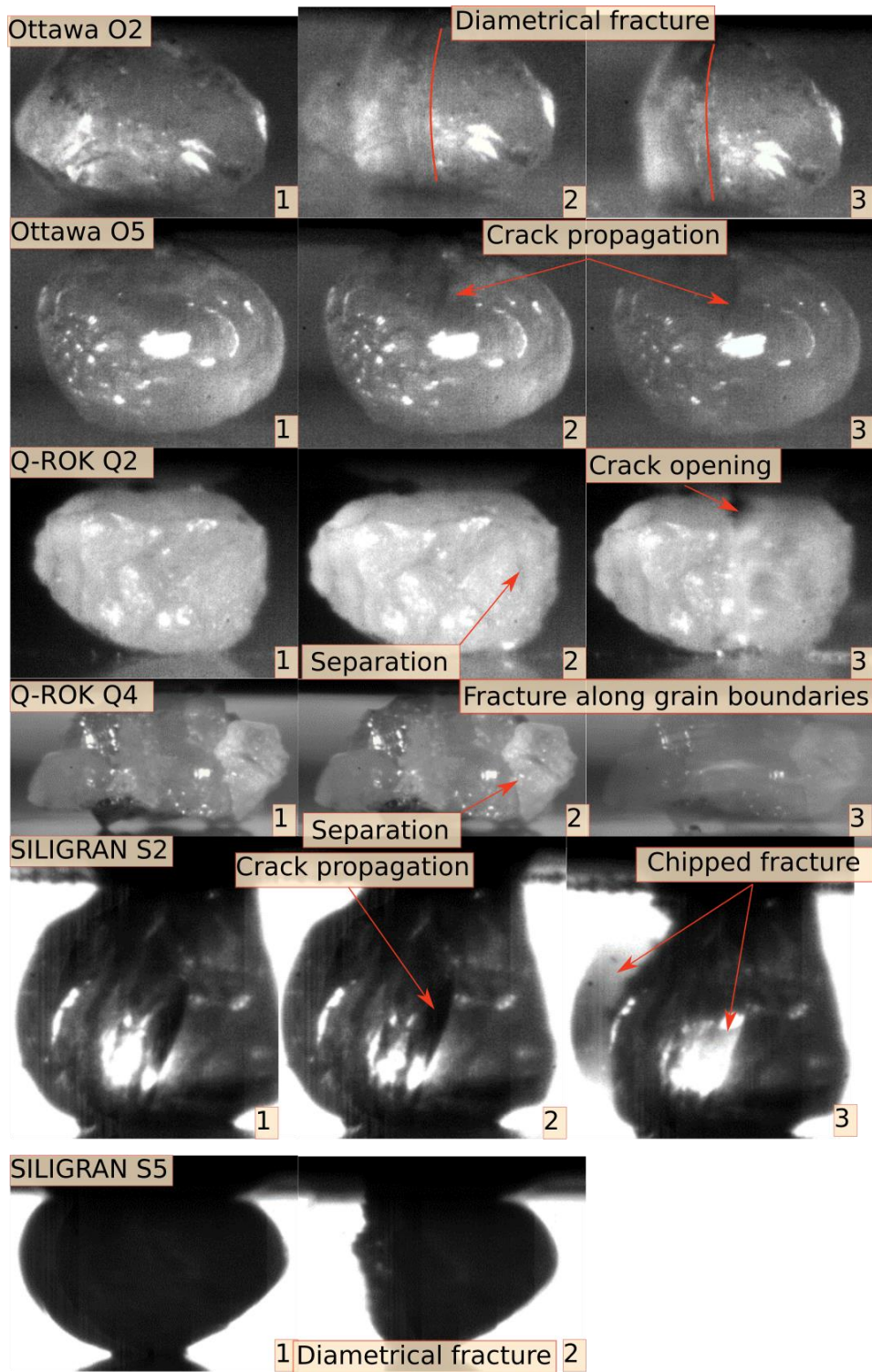


Fig. 37 Highspeed images of particle fracture in Ottawa sand, Q-Rok and Siligran.

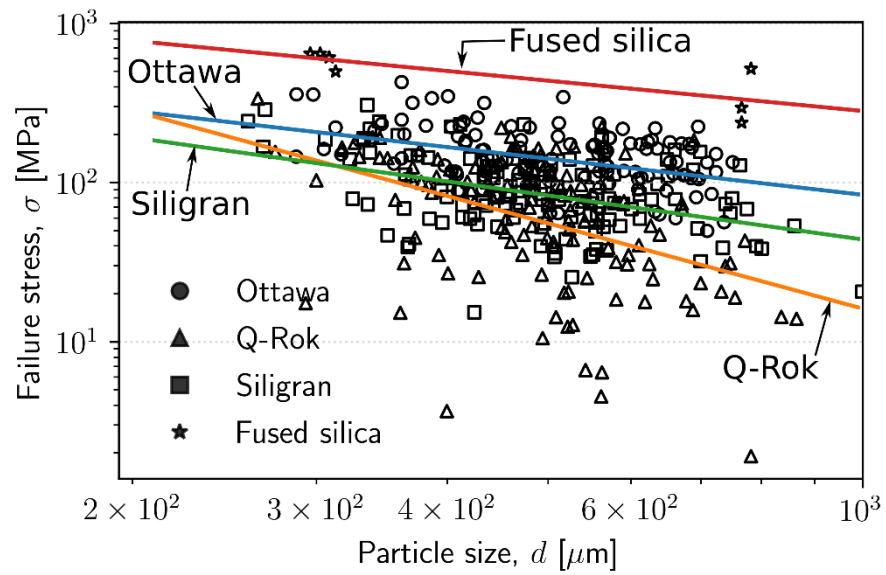


Fig. 38 Size dependence of strength for silica sands and fused silica.

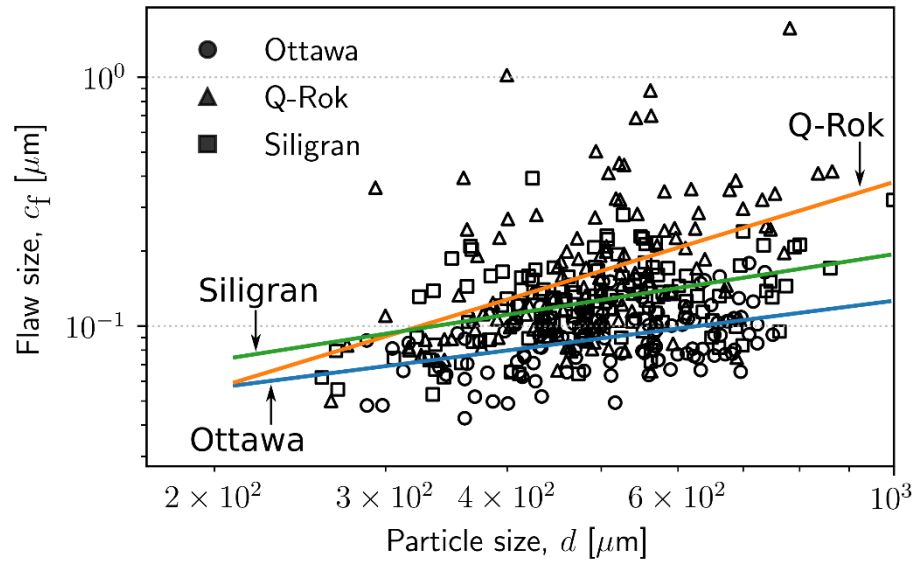


Fig. 39 Flaw size scaling with particle size in Ottawa sand, Q-Rok and Siligran.

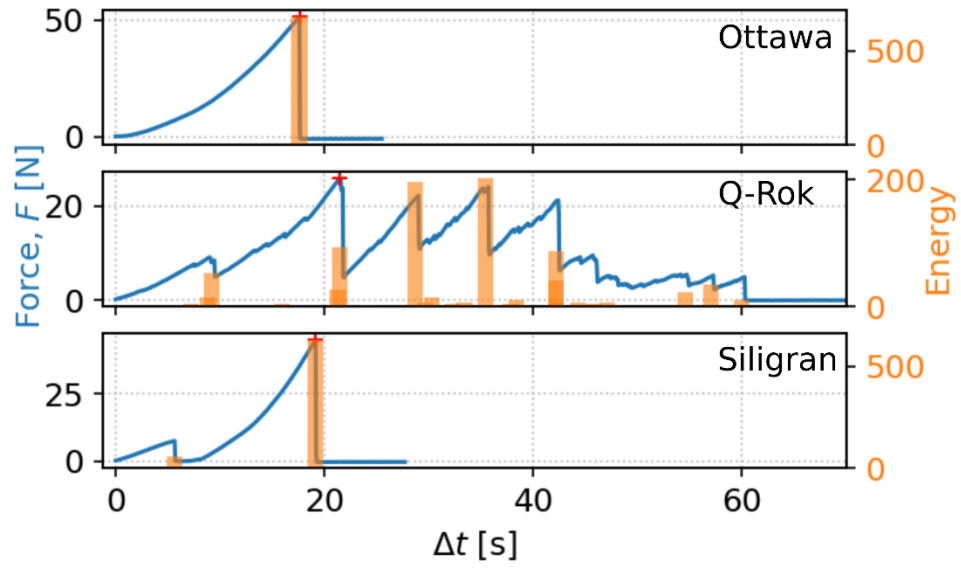


Fig. 40 Example force and energy release time histories for Ottawa sand, Q-Rok and Siligran.

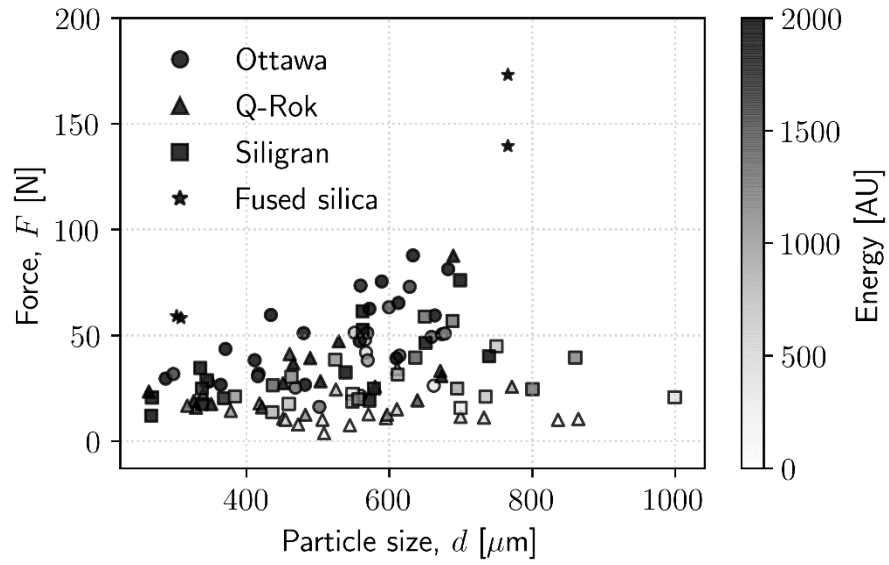


Fig. 41 Maximum force required to fail particles of different sizes and energy released in fracture. The color bar scale has been limited to the median energy released in Ottawa sand.

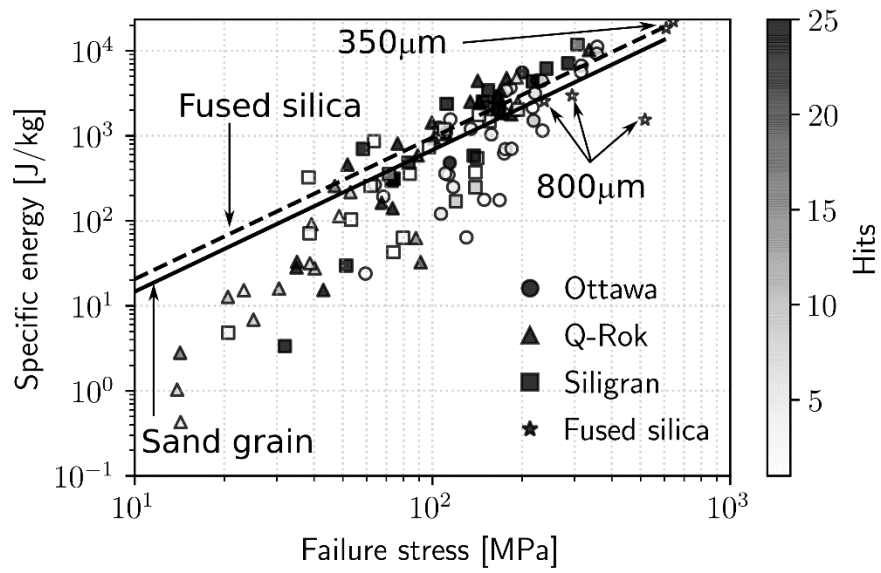


Fig. 42 Mass specific energy for particles with different failure stress.

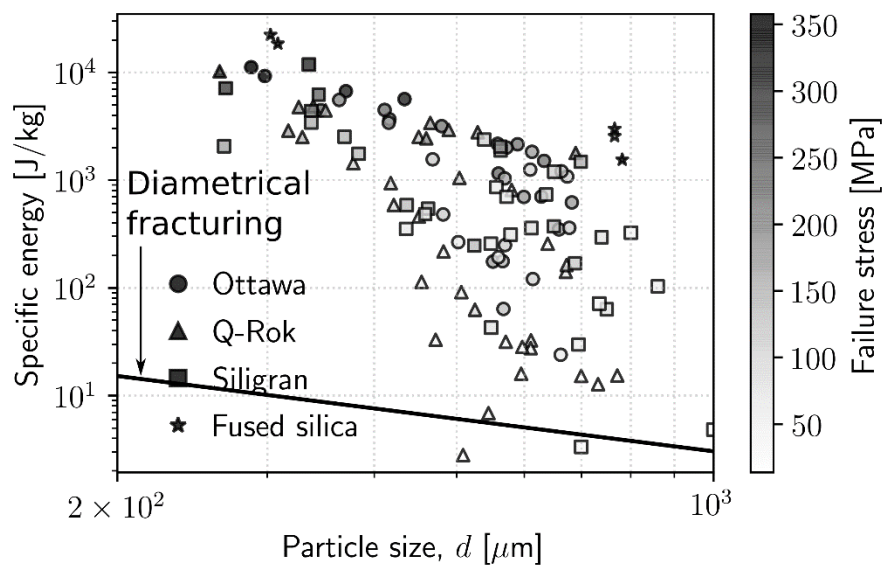


Fig. 43 Mass specific energy required to fracture particles of different sizes.

CHAPTER IV
PROJECTILE PENETRATION TESTS IN DRY SAND

A version of this chapter will be submitted to a journal by Aashish Sharma, Christoph Glößner and Dayakar Penumadu.

Sharma, A., Glößner, C, and Penumadu, D. “Projectile penetration in dry sand.” In preparation, *International Journal of Impact Engineering*.

Christoph Glößner performed all the projectile penetration tests and the data reduction at EMI. Aashish Sharma analyzed the data, developed the ideas presented here along with writing the chapter under the guidance of advisor Dr. Dayakar Penumadu.

Abstract

Laboratory projectile penetration tests for instrumented constant mass projectiles of different tip shapes impacting on sand targets of different densities were analyzed. The response of the projectiles under various target and tip shapes were analyzed using acceleration-time histories of the full flight from the onboard data recorder. The maximum resistive force experienced by projectiles is higher in high density targets and for flatter tipped projectiles. Grain morphology may also influence projectile response, though the data was limited. The depth of penetration increases with increasing impact velocity, though a definitive relationship between penetration depth and projectile tip could not be established. However, in limited the projectile tip material influenced penetration depth, projectiles with softer tips showing greater penetration. Energy dissipation is not uniform along the path, around 50% of the energy is expended for the first 20% of penetration depth and around 80% of the energy is dissipated by 50% of the penetration depth. The maximum deceleration values increase with impact velocity and were in the range of 20,000 – 100,000g. Cavity expansion method may offer a reasonable method to predict penetration depth, however the biggest uncertainty is the soil strength.

Introduction

Penetration depth is the most critical parameter in terminal ballistics. It depends on target and projectile characteristics and can be determined either empirically or analytically. Empirical methods are attractive owing to their simplicity; however, they require full scale tests for calibration. The multitude of influential factors are represented by few regression coefficients and extrapolation of the model beyond the calibration set will generally introduce significant errors. For penetration in soil targets, a prohibitively large number of full-scale tests are required to

account for variations in soil profile and projectile characteristics. Therefore, use of analytical models developed incorporating aspects of constitutive behavior and projectile characteristics provide an opportunity to parameterize the various influencing factors. Though advanced material models require iterative solution, analytical solutions are possible for simple constitutive models.

Young (1969) developed equations for penetration depths in soils based on a large number of full-scale tests, accounting for soil and projectile characteristics. Young's equations were ultimately based on more than 500 field tests (Taylor et al. 1991), and it was found that the projectile mass to area ratio strongly correlated with penetration depth. However, there were uncertainties in the determination of the numerical value of the variable describing the penetrability of the soil. Also, different equations were required for small and large projectiles with an arbitrary separation mass of 27 kg. Taylor (1991) performed penetrations in granular material in centrifuge and developed equations that eliminated the need to separate between small and large projectiles. Again, data indicated a high correlation between penetration depth and projectile mass to area ratio.

Penetration depths are determined by solving Newton's 2nd law of motion as shown in Eq 10.

$$m \frac{dv}{dt} = -F \quad \text{Eq 10}$$

where, m and v are the mass and instantaneous velocity of the projectile and F is the resisting force. Historically, the resistive force is assumed to be a function of v and a general form of F is a polynomial as shown in Eq 11 .

$$F = c_2 v^2 + c_1 v + c_0 \quad \text{Eq 11}$$

where, c_2 , c_1 and c_0 are constants which determine the relative contribution of the quadratic, linear and constant terms respectively. Robins (1742) and Euler (1922) determined P assuming a constant resistance throughout the penetration process, i.e. c_2 and c_1 equal to zero, Poncelet (1839) assumed c_1 to be zero, and Résal (1895) determined P by assuming c_0 equal to zero. The interaction between

the soil and the projectile, and soil and projectile characteristics are all reflected in the coefficients. Here again, a prohibitively large number of experiments are required for the calibration purpose.

The penetration depth in analytical models is also determined using Eq 10. However, the expression for the force is derived solving simplified mechanics of interaction between the projectile and the target. Cavity expansion is a popular analytical model to determine the force on the projectile and the resulting penetration depth (Forrestal and Luk 1992; He et al. 2011; Luk et al. 1991; Shi et al. 2014). Cavity expansion involves computing the stress required to open a cavity in a semi-infinite continuum medium and to grow the cavity at a prescribed rate to the radius of the penetrator. The stress required to grow the cavity is the resistance to the penetrator. Analytical or numerical solutions for the cavity stress can be formulated incorporating constitutive laws and equations of state. Once the force on the projectile nose is determined, Eq 10 can be used to compute the penetration depth.

Volume change is an important aspect of geomaterial behavior and a locked hydrostat behavior is generally assumed (Forrestal and Luk 1992) for simplicity. The behavior of sand in dynamic compression is considerably different than locked hydrostat behavior (Brown et al. 2007). Including dynamic compression volume changes using the $P-\alpha$ model only improves the results for frictionless contact and small friction values between the projectile and particles (Shi et al. 2014). The projectile-sand friction value may not be zero for dry sand. Soil parameters used in cavity expansion methods are determined from pseudo-static triaxial tests (Forrestal and Luk 1992). However, dynamic strengths and particle crushing in high strain rate tests are different than those in pseudo-static tests. Strength generally increases with increasing strain rate (Huang et al. 2014b; Yamamuro et al. 2011) and particle crushing is a time dependent phenomenon with decreased crushing at higher strain rates (Huang et al. 2014b). It would therefore be more pertinent to use dynamic strengths in these computations to accurately reflect the rate effects on the strength of granular material. Under drained conditions rate effects on strength are not significant (Bragov et al. 2008) or marginal (Karimpour and Lade 2010; Omidvar et al. 2012; Song et al. 2009), significant differences could arise in undrained case under partially saturated (Martin et al. 2009) and saturated conditions (Omidvar et al. 2012).

The high strain rate geotechnical tests are not routine. In the absence of dynamic test results, alternative methods maybe employed to determine the strength parameters for analytical

models using the information that drained strength in high strain rate in 1D compression tests are not significantly different from those determined in sudo-static tests. Efforts have also been made to relate single grain crushing tests to characteristic strength, σ_y , in 1D compression tests (McDowell 2002; McDowell and Humphreys 2002; Zhang et al. 1990).

A framework to use single grain crushing test data to determine soil strength which may then be used in cavity expansion model for projectile penetration are presented. The different response of the projectile based on nose shape, and impact velocity is discussed. The role of nose shape, impact velocity and target density on the penetration depth for a constant mass projectile is presented. The strength parameter for cavity expansion model is determined from single grain crushing tests and compared with critical stress in quasi-static 1D compression tests and dynamic Split Hopkins Pressure Bar (SHPB) tests. Cavity expansion model is then used to predict the penetration depths for the laboratory tests.

Materials and methods

Laboratory projectile penetration tests were performed at Fraunhofer Institute for High-Speed Dynamics, Ernst-Mach-Institut (EMI). Instrumented projectiles of 30 mm caliber with aluminum or polycarbonate body and interchangeable steel head as shown in Fig. 44 were fired into 250 mm diameter and 1m long tubular sand targets. The 0.21 kg projectiles were accelerated using 30 mm smooth bore barrel air gun (Fig. 45) to impact velocities ranging between 150 – 850 m/s. The projectiles were instrumented with G-Rec, which is a shock resistant device placed inside the projectile body and is used at EMI for experimental investigation of impact process. The G-Rec is a combination of high-performance accelerometer and an autonomous data recorder developed at Fraunhofer-EMI. This provides a unique opportunity to record the acceleration time history for the entire duration of flight, including when the projectile is travelling inside the target. Other high temporal resolution method such as photon doppler velocimetry (PDV) requires line of sight to operate and will not be able to monitor the projectile flight if the cavity collapses (Omidvar et al. 2015) . Details of experimental method and data reduction are presented in Glößner et al. (2017).

Majority of the targets for the penetration tests were Euroquarz Siligran 0.125 – 0.71. To investigate the role of tip shape and target density on penetration depth, tests were performed with

different tip shapes and placement densities. Additionally, to evaluate the effect of soil type, few targets were prepared by placing Ottawa sand and Q-Rok in the path of the projectile, surrounded by Siligran. Ottawa sand grains are sub-rounded in shape, Q-Rok is an angular sand, and Siligran grains are sub-angular in shape. All three sands have similar mineralogy, more than 99.5% of the grain are composed of quartz and particle size distribution. Siligran contains marginally more fines than Ottawa sand and Q-Rok. Though the morphology and particle size distribution of Ottawa sand and Siligran were different, their index densities were very similar.

Projectile penetration is a dynamic phenomenon and hence dynamic strengths and the influence of various factors on high strain rate (HSR) strength are valuable additions. Such tests may also help in choosing strength parameters for analytical and numerical models. Using SHPB, De Cola et al. (2018) and Song et al. (2009) have shown that the behavior of sand with rigid boundary is different than that in deformable boundary in high strain rate tests. In deformable confinement the behavior may approach elastic-perfectly plastic behavior (Song et al. 2009). The yield stress, σ_y , is lower in deformable boundary than in rigid confinement. The value of σ_y for rigid confinement was between 15 MPa for Q-Rok and 60 MPa for Siligran (De Cola et al. 2018). For deformable boundary, σ_y was around 10 MPa for all three sands (De Cola et al. 2018). The effect of boundary confinement on high strain rate testing is shown in Fig. 46. There were multiple tests at different densities for rigid boundary condition, only intermediate densities for respective sands are shown in Fig. 46. As the penetrator moves in the medium the soil starts to flow to the sides (Collins et al. 2011; Omidvar et al. 2016). However, at high impact velocities compaction waves travel ahead of the projectile tip which may lead to densification of the material in the path of the projectile (Van Vooren et al. 2013). At high strain rates the particles directly in front of the projectile nose may not have enough time to flow laterally. Such grains are displaced in the direction of travel and fracture as the forces on the grains exceed the tensile strength (Van Vooren et al. 2013). Hence, the strength of granular material in the path of the projectile may range between that under rigid and compliant boundary SHPB tests. It may also vary as the projectile travels inside the medium as the confinement stress increases. In the absence of HSR tests, the similarity in σ_y in quasi-static 1D drained compression and in drained HSR tests (Omidvar et al. 2014) may be used.

There have been efforts to correlate grain strength in diametrical compression to σ_y . McDowell and Humphreys (2002) and McDowell (2002) suggest that σ_y is 0.1 – 0.3 times the characteristics strength, σ_o , which is the strength at which 37% of the particles survive in single grain crushing tests. The σ_o for Ottawa sand, Q-Rok, and Siligran were 157 MPa, 87 MPa, and 96 MPa respectively, and therefore, σ_y should be in the range of 16 – 30 MPa, 9 – 26 MPa, and 10 – 30 MPa. Zhang et al. (1990) computed the critical crack length assuming surface flaws (c), Hertzian contact, and linear elastic fracture mechanics from single grain crushing tests. Further, they assumed that c scales with grain radius, R and predicted the value of σ_y , as shown in Eq 12

$$\frac{\sigma_y}{E} = 2.2 \frac{(1 - \nu^2)^2}{(1 - 2\nu)^3} \left(\frac{K_{IC}}{E} \right)^3 (\alpha \psi R)^{-3/2} \quad \text{Eq 12}$$

where, E is the Young's modulus, ν is the Poisson's ratio, K_{IC} is the critical stress intensity factor, $\alpha = c/R$, and ψ is the porosity of the medium. They arrived at excellent agreement with experimental values of σ_y . In the single grain crushing tests performed. the flaw size to particle radius ratio, α , were 6.93×10^{-5} , 1.12×10^{-4} , and 6.66×10^{-5} for Ottawa sand, Q-Rok and Siligran respectively. Using $E = 95.6 \text{ MPa}$, $\nu = 0.077$ and $K_{IC} = 1 \text{ MPa}\sqrt{\text{m}}$ for quartz, σ_y is determined to be 470 MPa – 670 MPa for Ottawa sand, 240 MPa – 350 MPa for Q-Rok and 950 MPa – 1380 MPa for Siligran. These values were computed for median grain sizes and index void ratios. From De Cola et al. (2018), σ_y may vary between 20 MPa and 50 MPa for high strain rate tests with unyielding boundary. For deformable boundary, σ_y is in the range of 10 MPa. The strength drops after the peak stress for compliant boundary before getting stronger again at very large strains as shown in Fig. 46. McDowell and Humphreys (2002) predict a smaller but reasonable estimates of σ_y while not considering the effect of porosity on σ_y . Zhang et al. (1990) predict extremely high and unreasonable values for all three sands. Vesic and Clough (1968) performed triaxial tests under very high confining stresses, ranging from 0.1 MPa to 62 MPa on Chattahoochee River Sand and reported friction angle of around 33°. Their data for loose sand is shown in Fig. 47(a) for strength parameters as defined in Forrestal and Luk (1992). Bragov et al. (2008) performed high strain-rate test in 1D compression on quartz sand at different strain rates. They report lateral stress by measuring the circumferential strain of the specimen holder and

assuming thick-walled cylinder. Their data is shown in Fig. 47(b) along with the determined parameters. Their data shows no strain rate dependence when the 3 mm sand specimen was compressed at striking velocities ranging from 10 – 30 m/s.

The degree of departure from strict 1D compression stress state is unknown in projectile penetration tests and may vary with density of the medium. A value of 10 MPa was chosen for Tresca criterion, and in the presence of limited data Vesic and Clough (1968) test results were chosen for Mohr-Coulomb parameters.

Results and discussions

Effect of tip shapes on the response of projectile

Typical acceleration time histories from G-Rec and subsequent computations of velocities and displacements are shown in Fig. 48. The target was Siligran 0.125-0.71 sand with the average void ratio (e) of 0.75, and the impact velocity was 377 m/s. The impact is characterized by very high decelerations that depends on nose shape; flatter noses experiences larger decelerations. The penetration depth also shows dependence on the nose shape for similar placement density, though these differences may well be within the scatter in the data. In most cases the projectile is brought to a stop within few milli seconds. The force on the projectile as it moves through the medium is shown in Fig. 49. The forces were computed by multiplying deceleration time histories with the projectile mass. The ogive and the cone noses show similar response with smaller peak resistance than hemispherical and flat tips. The hemispherical and flat tips experience higher resisting force. The force in the flat tip rapidly reaches the peak value, whereas the force in the hemispherical tip reaches peak value after some travel in the medium. Though the peak force value of the flat nose is significantly larger than those for ogive and cone, the penetration depths are similar. The flat tip rapidly loses energy in the initial region. The ogive and cone nose experience more gradual loss of energy in the middle region of the travel.

The force at impact also depends on the density of the target. Force profiles for 60° cone tip projectiles for two different target densities are shown in Fig. 50. There is a small difference in impact velocity which may also contribute to the difference in the impact force. The maximum force for the loose target is in the range of 100 kN and 200 kN for the dense target. The maximum resistance in dense target is instant while the maximum force is reached after some travel in the

loose target. The projectile in the dense target rapidly releases its energy upon impact. The impact force decreases to half the maximum value within 0.1 m distance. At similar distance travel the force on the projectile in the loose target is still close to the maximum force.

Effect of grain morphology on the response of projectile

The response of projectile in Ottawa sand and Q-Rok of similar densities are shown for two nose shapes and two impact velocities in Fig. 51. The response is similar for identical nose shapes and impact velocities. Though the relative densities are very different for Ottawa sand and Q-Rok, the response is very similar in terms of penetration depth and energy release profile. This may suggest that density rather than relative density determines the response of the medium to the intruder. The forces at impact are marginally larger in Ottawa sand with smaller relative density than angular Q-Rok with higher relative density. For the hemispherical tips the peak value is reached instantly upon impact, while the build up to peak resistance is slower in the cone tip.

Energy dissipation during penetration

The energy released during the penetration process was computed as the area under the force displacement curve. Displacements were computed by double integration of the deceleration time histories. A comparison of the displacements from the double integration and measured displacements of the projectile after careful excavation is shown in Fig. 52. The penetration depth values computed from the G-Rec data were very similar to the true penetration depth. An example of the variation of the force along the path for a hemispherical tip projectile at 377 m/s is shown in Fig. 53. The area under the force-displacement curve was computed using the trapezoidal method. Energy expended during penetration for different impact velocities are shown in Fig. 54. Also shown is the initial kinetic energy before impact. The maximum deceleration values for the two data points at 30 kJ were approximately 150,000 g, beyond the 100,000 g range of the G-Rec data recorder and may not be reliable. The data reduction did not involve matching kinetic energy with the total energy (Glößner et al. 2017), and the closeness of the dissipated energies to the initial kinetic energy provides a validation for instrumentation and data reduction process. The energy dissipation is not uniform along the path of the projectile. Average energy dissipation along the path is different for different tip shapes as shown in Fig. 55. For majority of the tests, 50% of the energy was dissipated within 20% of the penetration depth and more than 80% of the energy was

dissipated by the time the projectiles travelled 50% of the penetration depth. In general flatter tips are associated with higher impact forces and dissipate most of their energies upon impact and within a short distance of travel.

Projectile and target characteristics on penetration depth

A summary of the penetration tests is shown in Fig. 56. The penetration depth increases with increasing impact velocity and decreases with increasing density. Due to the large scatter in the data, a clear dependence of penetration depth on projectile tip shape is not readily ascertained. Also, a false tip composed of comminuted particles on top of the original tip was seen in many tests which could have further negated the effect of tip shapes. However, hemispherical tips were associated with greater penetration depths than flatter tips (90° cone and flat). A part of the scatter in the data could also be the result of variation in target density along the path of the projectile arising from the difficulty of maintaining homogeneity while depositing sand in a large target. At impact velocities close to 150 m/s the penetration depths are relatively larger than at higher impact velocities. In general, the path of the projectile was characterized by particle crushing for all sands as shown in Fig. 57. If no evidence of extensive crushing was seen along the path in the tests with large penetration depths for 150 m/s, then particle crushing may have a significant role in determining penetration depth in granular materials. Comminuted sand have been noted in projectile speed as low as 100 m/s (Allen et al. 1957a). However, Borg et al. (2013) have reported no visible column of crushed particles along the path of the projectile for velocities up to 212 m/s. Their projectiles were considerably smaller at approximately 0.2 g, and 3.1 mm and 4.0 mm diameter.

There were four tests with Ottawa sand and Q-Rok placed along the path of the projectile in the target as shown in Fig. 57. Due to limited data a definitive conclusion on the role of particle shape on the penetration depth cannot be reached. However, in two tests performed at similar densities and impact velocities, the penetration depth in Q-Rok is either similar or smaller than in Ottawa sand. Angular Q-Rok has higher friction angle than Ottawa sand. The higher strength may have resulted in smaller penetration depths. Angular sands also readily fracture due to asperities breaking and hence energy is readily dissipated in particle fracture which may also result in smaller penetration depths. There were also three tests in which the target was made of larger grain size

Siligran (Siligran 1.0 – 1.6). Here again, due to limited data, and large scatter of the overall data a definitive conclusion regarding the role of particle size on penetration depth cannot be ascertained. The current results indicate, for larger particle size, smaller penetration for cone nose and larger penetration for hemispherical nose. In the size range for Siligran sand used, density increases, and porosity decreases with increasing particle size (Koerner 1969). Also, the strength of single grains, and friction angle decreases with increasing particle size (Koerner 1970; McDowell 2001; Nakata et al. 2001b). Given these opposing influences on the strength of the deposit, the reduction in voids may have a larger impact on the penetration depth. Energy dissipation in the crushing of weaker larger particles may result in more crushing and smaller penetration depth.

The maximum decelerations recorded by the onboard G-Rec data recorder are shown in Fig. 58. The maximum deceleration increases with increasing impact velocity. Deceleration is higher in medium dense and dense targets than in loose targets. For numerous tests performed at 375 m/s the deceleration values range from 50,000 g to 100,000 g. Corresponding to these high decelerations, the forces on the projectile on impact may range from 100 – 200 kN. Even for smaller impact velocities the projectiles are subjected to large forces which may be enough to erode and change the shape of the nose if soft metals are used for the nose. For similar densities, the maximum deceleration values in Ottawa sand is higher than in Q-Rok. Rounded Ottawa sand may form more stable packing than angular Q-Rok thus the higher deceleration may be a consequence of the arrangement of rounded and angular grains at the similar densities. Rounded particles may form more stable networks than angular particles where the contacts between grain fracture through asperities breaking upon impact. Also, in quasi-static 1D compression, the response of dense Q-Rok was similar to the response of loose Ottawa sand. Though, the data is limited, rounded sand may offer higher impact resistance while angular sand may reduce the overall penetration depth. Crushing strength may also influence comminution along the path of travel thus influencing the penetration depth.

The relation between penetration depth and maximum deceleration values is presented in Fig. 59. A concrete relationship between maximum deceleration and penetration depth is not seen because the maximum deceleration depends on target density, projectile nose shape, and the initial impact energy. Until these factors are separated it may not be possible to determine the role of maximum force at impact on the penetration depth.

Poncelet drag coefficient

Many researchers (Allen et al. 1957a; Bless et al. 2018; Chian et al. 2017; Omidvar et al. 2015) have analyzed the resistance to penetration by computing the drag coefficient, C_D . The drag coefficient is obtained as $(ma)/(\rho Av^2)$ where, m is the projectile mass, a is the instantaneous acceleration from G-Rec, ρ is the density of the target, A is the cross-section area of the projectile and v is the instantaneous velocity. This quantity is half of the aerodynamic drag coefficient and is equivalent to Poncelet's coefficient, C , in Eq 13 (Bless et al. 2018).

$$F = m \frac{dv}{dt} = A(C_D \rho v^2 + R) \quad \text{Eq 13}$$

where, R is the bearing strength of the target.

The drag coefficient, C_D for different projectile nose shapes and similar densities and impact velocities are shown in Fig. 60. C_D is constant over most of the flight and does not vary significantly with tip shape for a given target density and impact velocity. The penetration depth predictions from the Poncelet's equation is shown in Fig. 61 for different values of R . Though, the velocity depth profile is significantly different that those recorded by the on-flight data recorder, for $R = 0.5$ MPa, Poncelet's method predicts similar penetration depth value as seen in the experiments. The influence of different target densities, tip shapes and impact velocities on the drag coefficient are shown in Fig. 62. The drag coefficient was computed between 90% of the impact velocity and 80 m/s. When the projectile velocity decreases below a threshold velocity, the drag coefficient increases rapidly as the R term starts to dominate (Bless et al. 2018). This transition velocity is around 80 m/s (Bless et al. 2018). The drag coefficient is generally higher for dense targets and decreases with increasing velocity. Omidvar et al. (2015) reported values of 0.92 – 1.27 for loose sand targets and 1.86 – 2.26 for dense sand targets. The values in this research are closer to 0.54 – 1.45 reported by Chian et al. (2017) for silica sand and projectiles of different tip shapes in target penetrating tests. In the current set of penetration tests, there were limited tests with different sand types. The data from these limited tests suggests that the drag coefficient does not depend on the type of sand used. The C_D for different sands were within the scatter in the data. Similar observation has been reported by Omidvar et al. (2015).

Cavity expansion model

For a constant mass projectile, the major factors that control the penetration depth are, impact velocity, strength of the medium and the nose shape. Particle crushing may influence the penetration depth. Any general model predicting penetration depths must incorporate these factors. Model based on cavity expansion theory includes all the three major factors and the mass of the projectile. Projectile characteristics such as nose shape, mass and impact velocities are determined with high degree of certainty. Soil properties on the other hand are associated with uncertainty resulting from general variability in soil profile, the numerous factors that govern strength, and importantly the rate effects in the case of projectile penetration tests. Forrestal and Luk (1992) developed cavity expansion models to predict penetration depth in soil targets incorporating Tresca and Mohr-Coulomb failure criterion. A schematic of the different aspects of cavity expansion model along with material models is presented in Fig. 63. Forrestal and Luk (1992) modeled the strength of soil specimens collected at different depths at the Sandia Tonopah Test Range using the Tresca failure criterion. They modeled the volume change behavior under dynamic loading as a locked hydrostat. The predictions of penetration depth from the cavity expansion model was in good agreement with the full-scale test data. Using their model, the relation between impact velocities and the penetration depths for various nose shapes are shown for both Tresca and Mohr-Coulomb failure criteria in Fig. 64 and Fig. 65 respectively. The parameters used for these two models are shown in Table 8. The predicted depths from the Tresca failure criterion is smaller than those from the Mohr-Coulomb criterion at lower impact velocities because of the higher τ_0 value. The difference between the two models become smaller at higher velocities. For the strength parameters used, there is good agreement between the test data and the predictions for the 60° cone tip. Given the scatter in the data, and the uncertainty in soil parameters, the range of depth predictions from all the tip shapes may need to be considered as the range in penetration depth at a given impact velocity.

Strength of granular material is pressure dependent and the knowledge of stresses in the vicinity of the projectile will provide valuable information regarding confining stresses required in laboratory tests. The maximum stress exerted by the projectile on the target at impact are shown in Fig. 66. The frontal stress on the projectile was computed by dividing the maximum force at impact by the cross-section area of the projectile. The frontal stress increases with impact velocity

and ranges from 50 MPa to 300 MPa. The range of σ_y at which particle crushing initiates in 1D compression is between 10 MPa and 60 MPa depending on the particle shape and specimen density. Therefore, at high frontal stresses, penetration will be characterized by high degree of particle crushing in the vicinity of the projectile as has been seen by the trail of powdered sand along the path of the projectile. Energy dissipation in particle crushing may then influence the final penetration depth. Also shown in the figure are frontal stresses computed from the cavity expansion model for different nose shapes and failure criteria. The frontal stresses predicted using the Mohr-Coulomb model is in the range of stresses measured in the laboratory tests.

Effect of tip material on penetration depth

Projectile penetration tests with tips made of different materials show depth dependence on the Young's modulus of the tip material. The effect of different tip material on the penetration depth is shown in Fig. 67. At two different velocities the steel tip with the highest Young's modulus has the smallest penetration depth. The tips made of lower moduli materials (Aluminum and Magnesium) have the highest penetration depth and Titanium with intermediate modulus has the penetration depth between steel tip and softer tip projectiles. Rosenberg and Dekel (2016) have explained the penetration depth dependence on projectile material through simulation of aluminum and steel rod in aluminum target in terms of reverberating stress waves and density of the rod. The result of their simulation was that the penetration time for the aluminum rod was shorter and thus it experiences greater deceleration. The lower modulus tips do experience higher impact forces than the steel tips as seen in Fig. 68. The travel time, on the other hand, is greatly reduced in steel nose and the penetration depth is smaller as shown in Fig. 69. This is contradictory to their simulation.

Conclusions

Single grain crushing tests provide reasonable estimates of characteristic stress in 1D compression tests and Split Hopkinson Pressure Bar tests on dry sand. A suite of tests with instrumented projectiles of constant mass in sand targets show that penetration depth is a function of impact velocity, tip shape, and target density. Different tip shapes show different energy release pattern as the projectile travels in the medium and comes to stop. Flatter tips experience instant high resistive force while some travel is required for maximum force to develop in pointed tips.

Most projectiles dissipate around 80% of the initial energy travelling half the penetration depth. The drag coefficient was higher for lower impact velocities and was not dependent on the sand type. Increasing the density of the target increased the maximum resistance and reduces the travel distance to maximum resistance. Though the data was limited, particle morphology may have a major influence on the penetration. The penetration depth in angular sands with higher friction angle was smaller than rounded sands. On the other hand, the stability of the voids may influence the magnitude of maximum resistive forces. Cavity expansion models provide reasonable estimates of impact stresses and penetration depth. Projectile tip material influences the penetration depth with higher penetration depth for lower modulus tips.

Appendix

Table 8 Parameters for Tresca and Mohr-Coulomb failure criteria for predicting penetration using cavity expansion model.

Parameters	Tresca	Mohr-Coulomb
Shear strength, τ_o	10 MPa	0.5 MPa
Friction between projectile and sand	0	0.17
λ	0	1.31
Elastic modulus, E	160 MPa	160 MPa
Initial target density, ρ_o	$1.60 \times 10^3 \text{ kg/m}^3$	$1.60 \times 10^3 \text{ kg/m}^3$
Locked volume strain, η^*	0.22	0.22

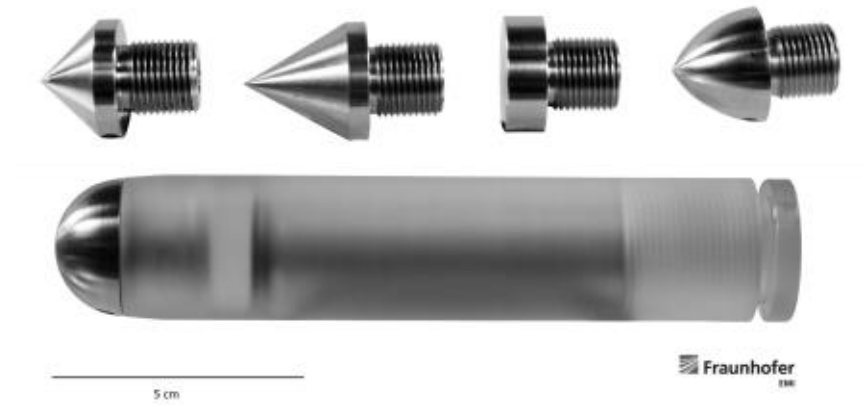


Fig. 44 30-mm projectile with interchangeable tips of different shapes.



Fig. 45 Pressurized gas gun with 30 mm smooth bore barrel.

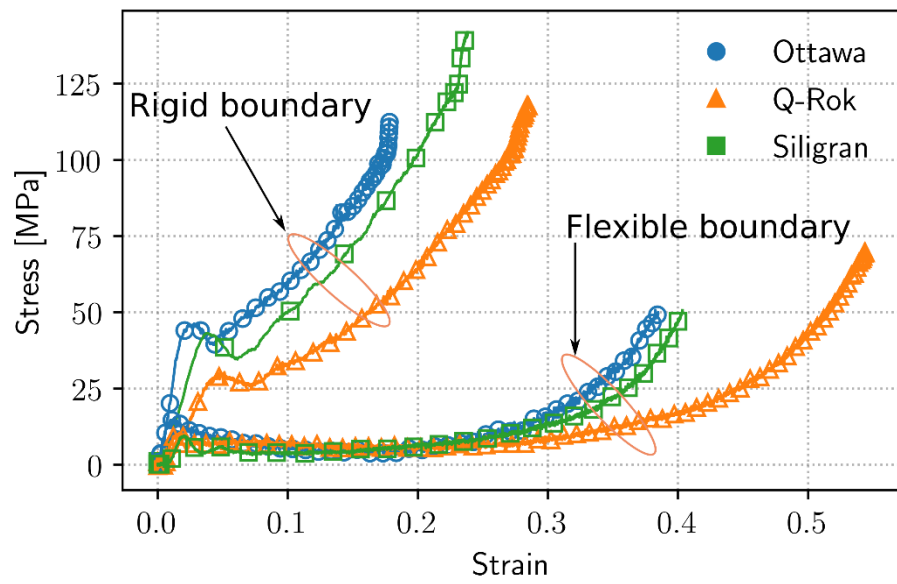


Fig. 46 Effect of boundary confinement in high strain rate tests in SHPB.

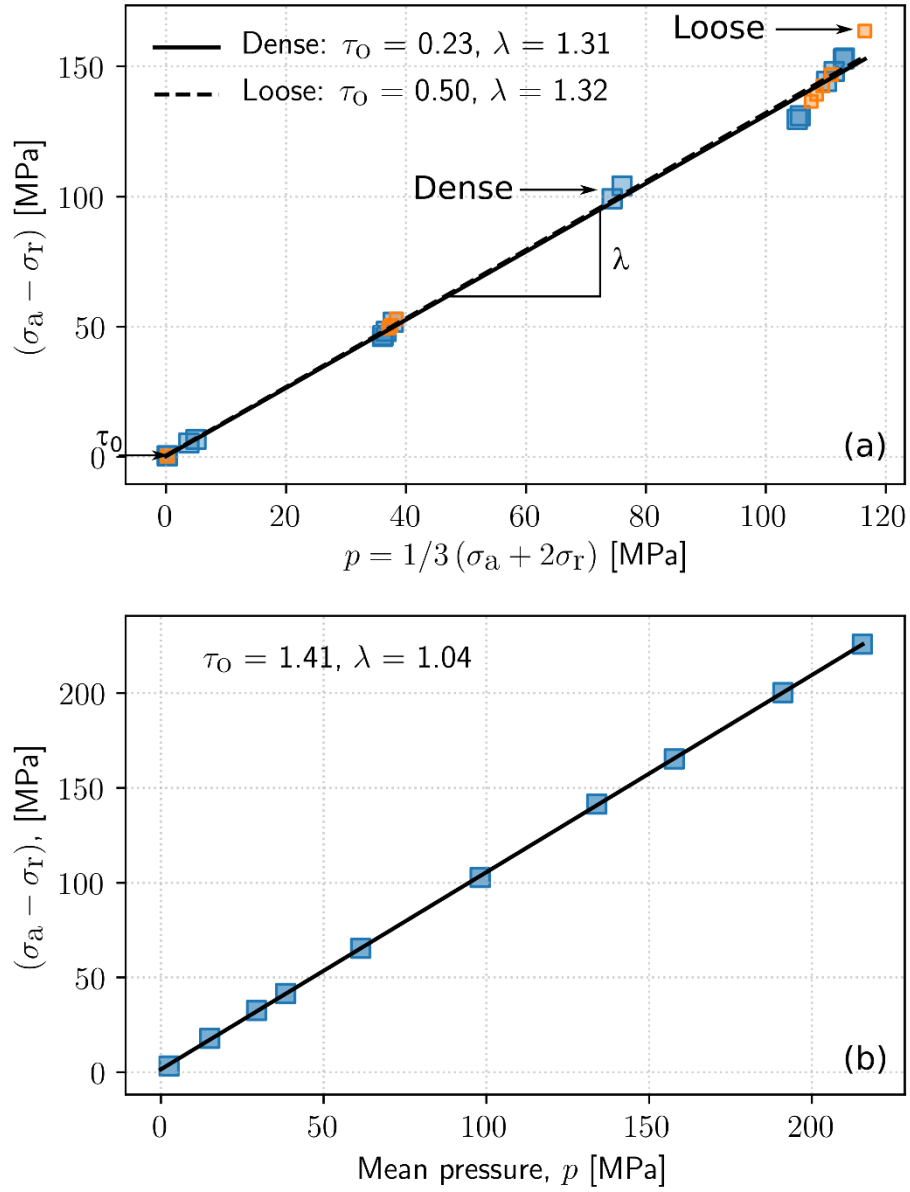


Fig. 47 Material strength parameters for sand from (a) high confining stress triaxial compression tests on Chattahoochee River Sand (Vesic and Clough 1968), (b) high stress and high-strain rate tests on quartz sand (Bragov et al. 2008).

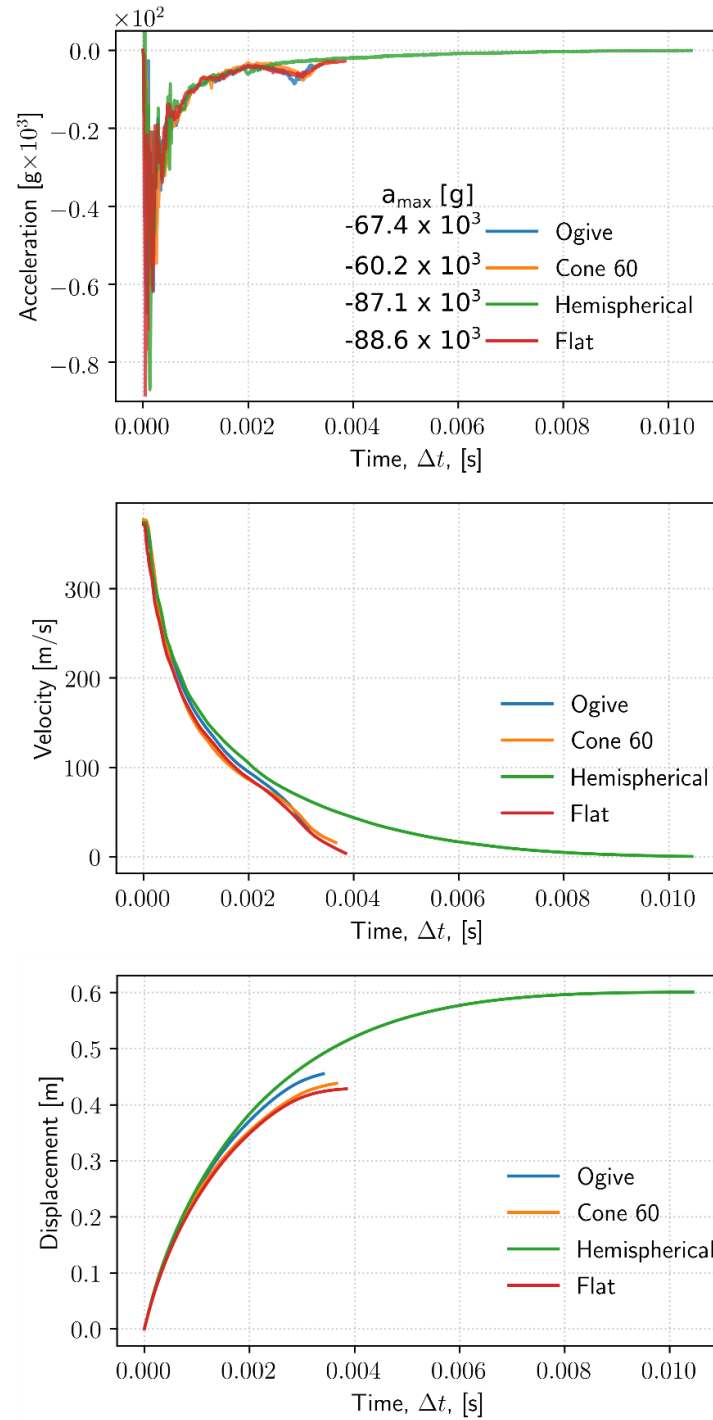


Fig. 48 Typical data from laboratory projectile penetration tests for similar target density ($e_0 = 0.75$) and similar impact velocity ($v_0 = 377$ m/s) and varying tip shape.

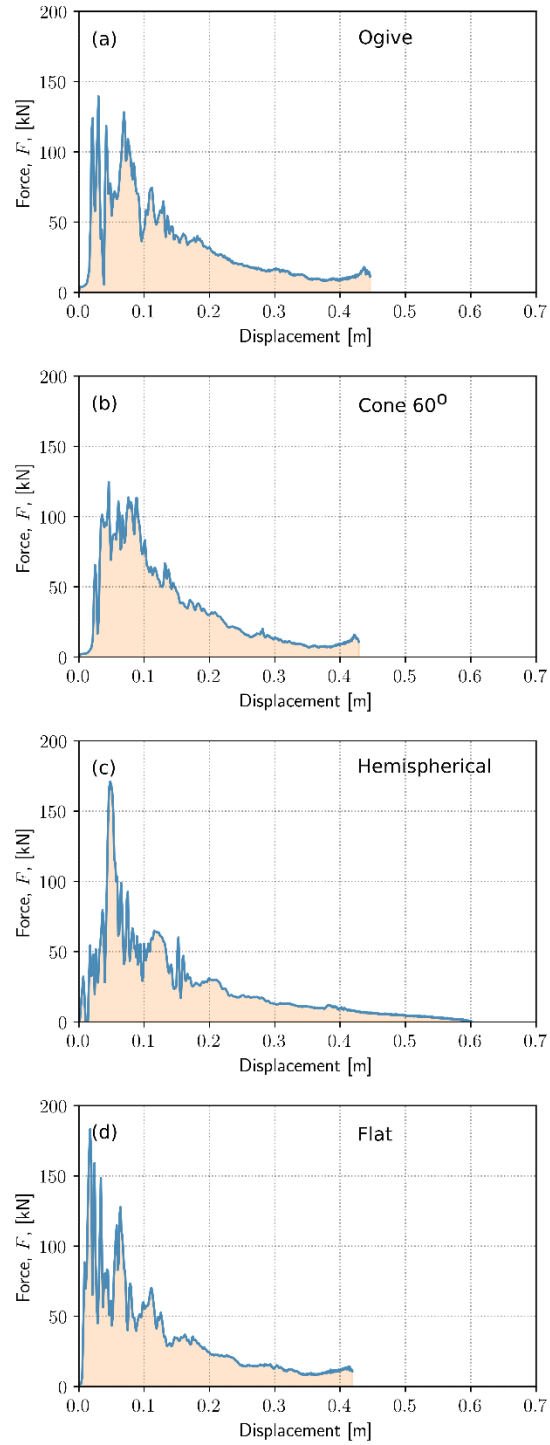


Fig. 49 Response of different tip shapes to impact for similar target densities and impact velocities; $e_0 = 0.75$, $v_0 = 377$ m/s.

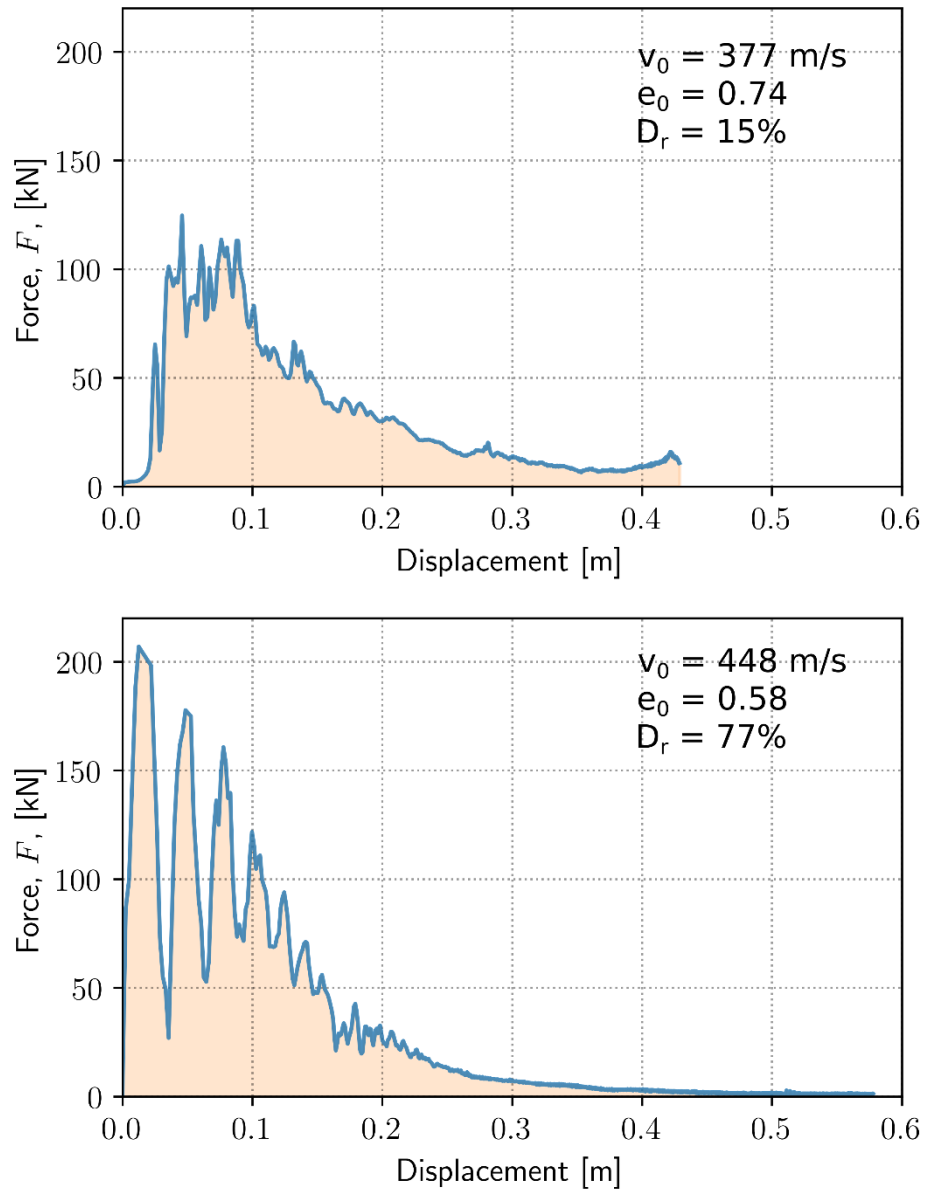


Fig. 50 The effect of density on the energy release with depth in Siligran 0.125-0.71 target.

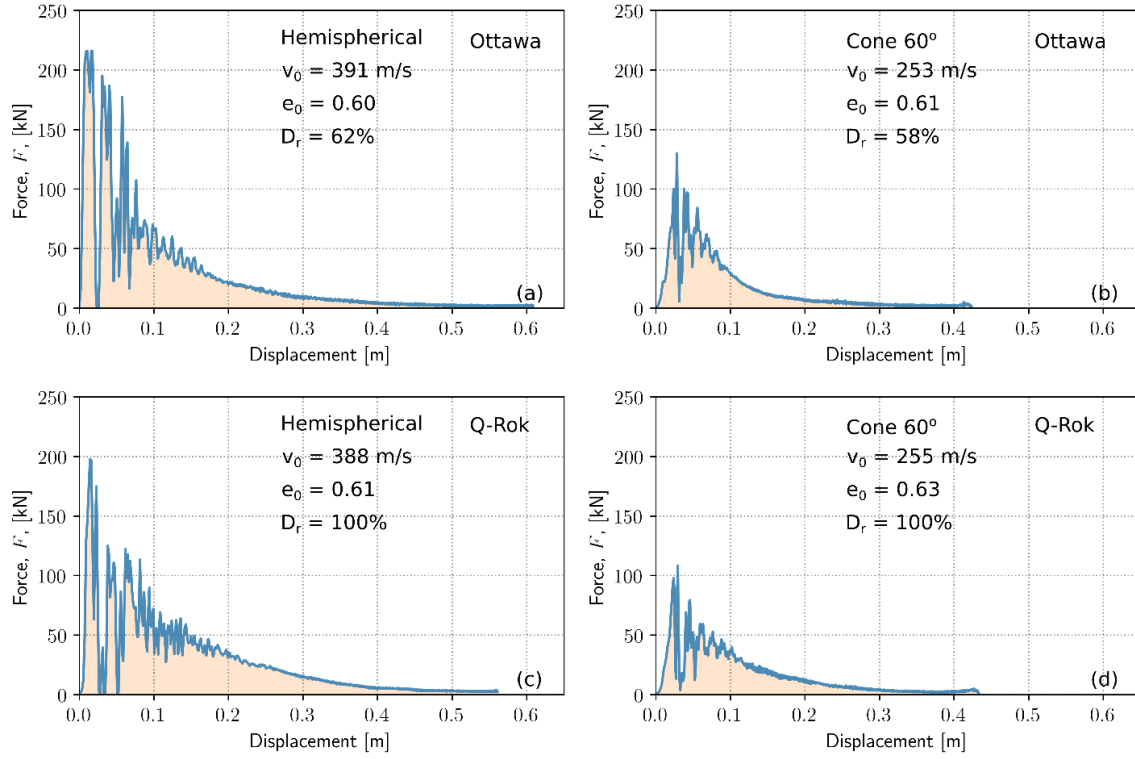


Fig. 51 Response of tip shapes for different particle morphology at similar densities and two different impact velocities.

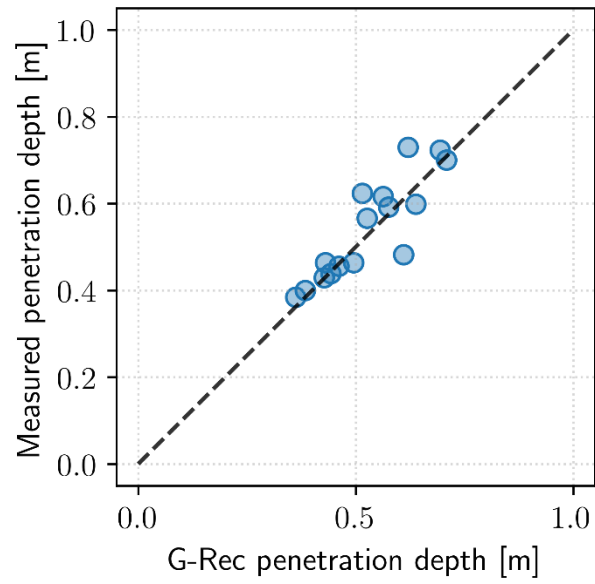


Fig. 52 Comparison of displacement computed by double integration of the acceleration time histories and measured displacements of the projectile in the sand target after careful excavation.

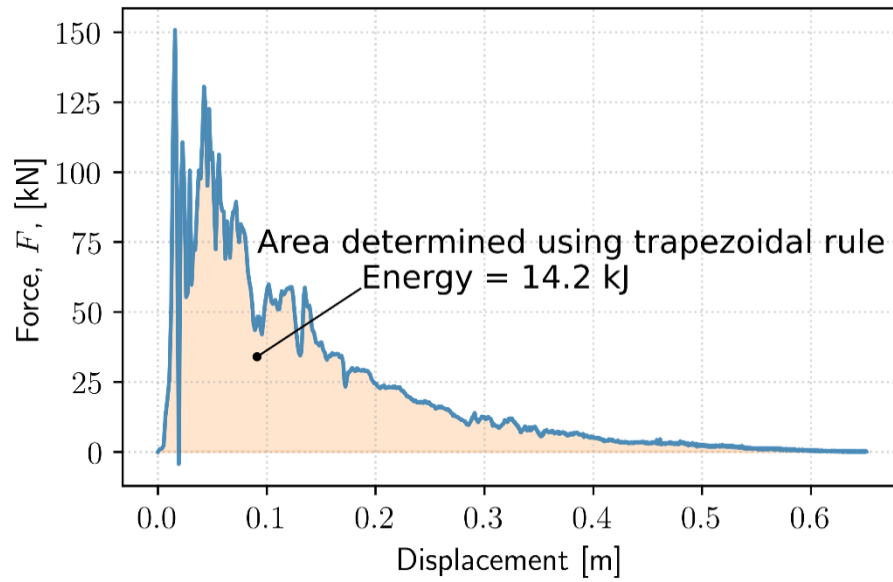


Fig. 53 Example of variation of force at different depths of penetration for a hemispherical tip projectile impacting Siligran 0.125-0.71 target at 377 m/s. Energy dissipated was determined by computing the area of the shaded region.

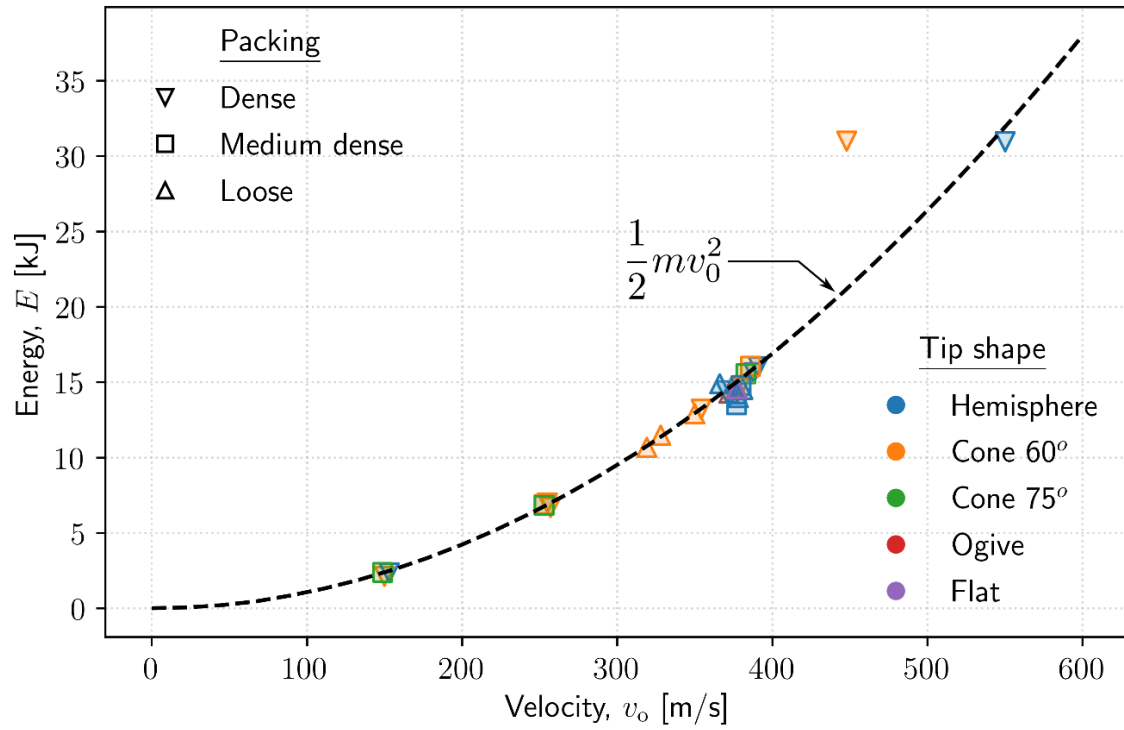


Fig. 54 Energy dissipation from integration of force-displacement data recorded by the onboard data recorder (G-Rec) for different impact velocities.

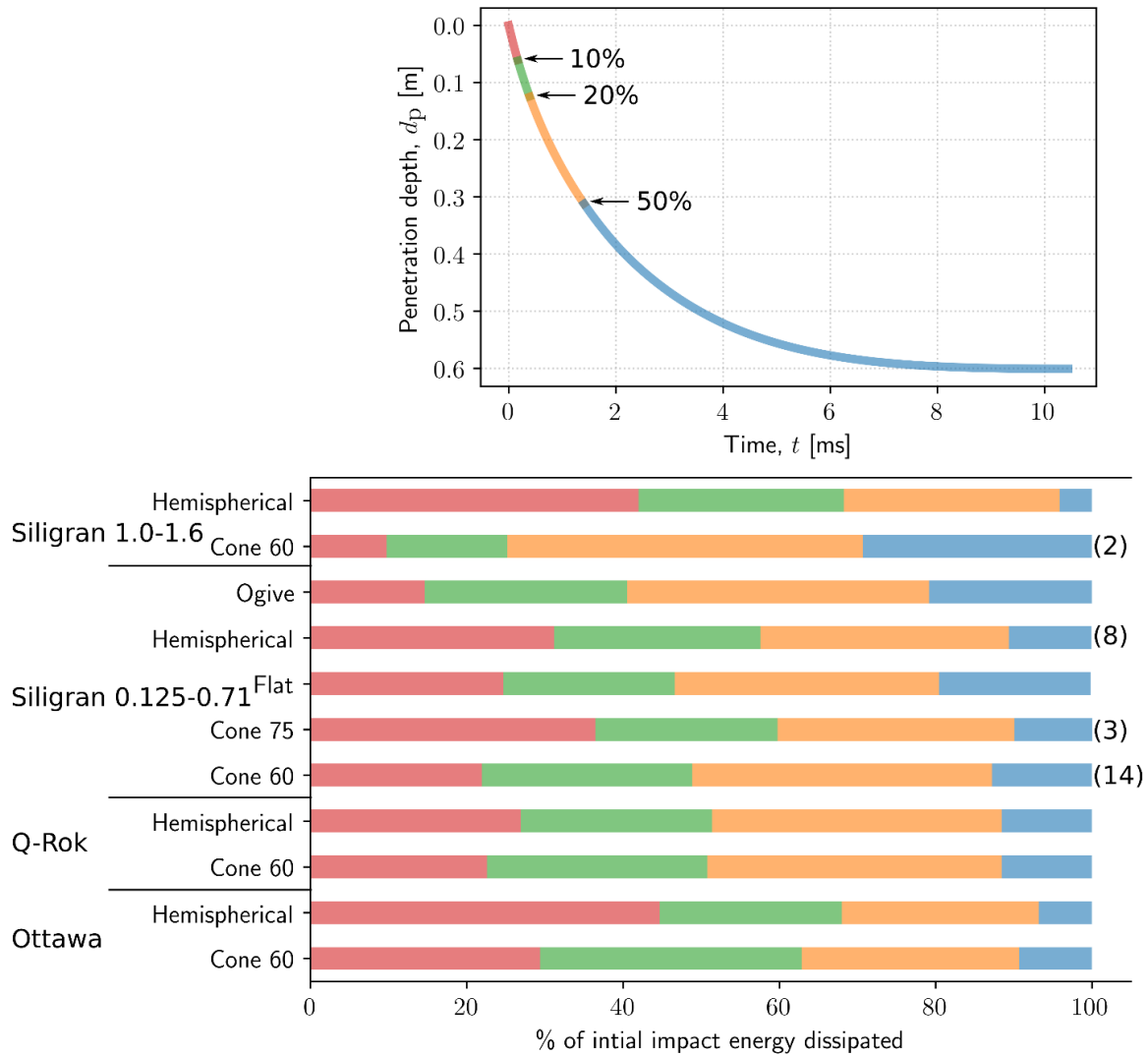


Fig. 55 Average energy dissipated along the path of the projectile at different depths, 0-10%, 10-20%, 20-50%, and 50-100% of the penetration depth for different sands and projectile tip shapes. The number in parenthesis is the number of tests where multiple tests with the same sand type and nose shape were performed.

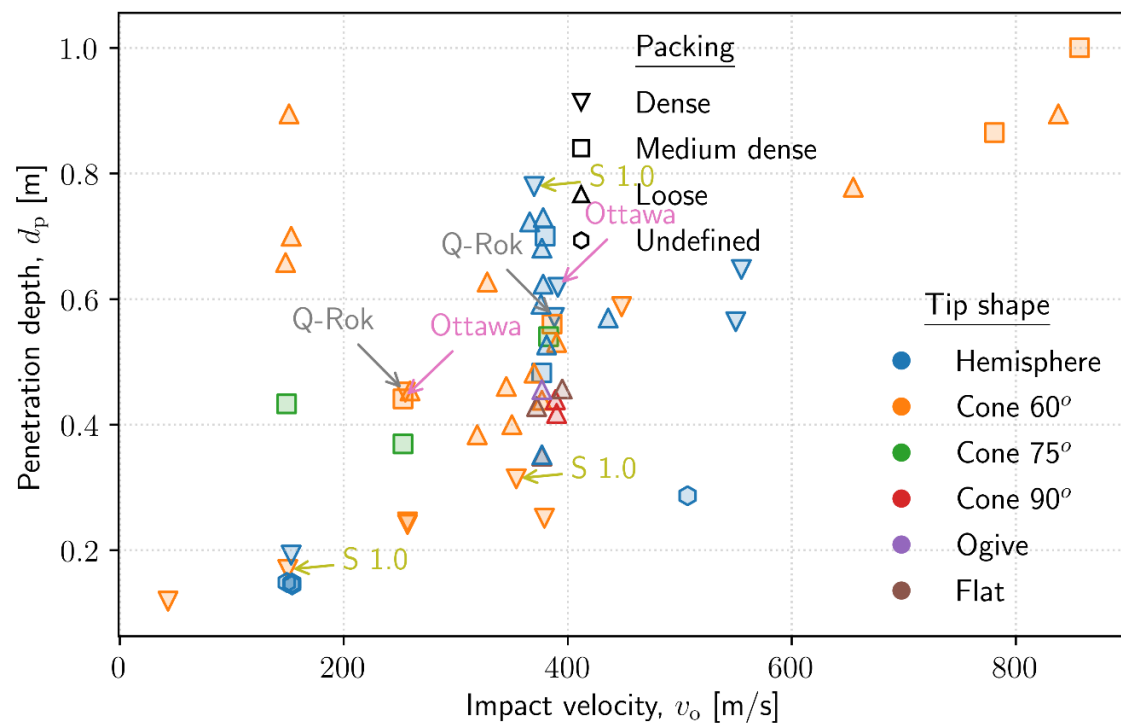


Fig. 56 Results of laboratory projectile penetration tests at different impact velocities, target densities and tip shapes. All targets were Siligran 0.125-0.71 except where noted.

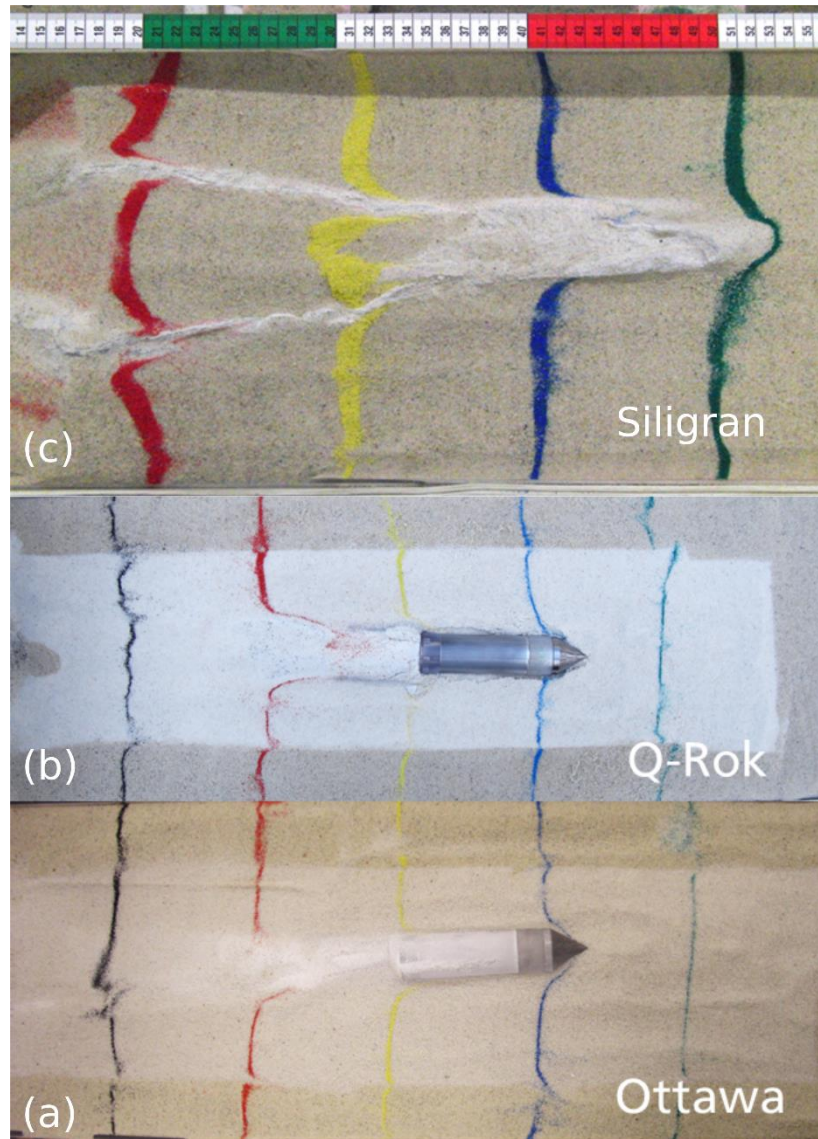


Fig. 57 Streak of white comminuted particles along the path of the projectile. Ottawa and Q-Rok targets were prepared by placing Ottawa sand Q-Rok along the path of the projectile confined by Siligran.

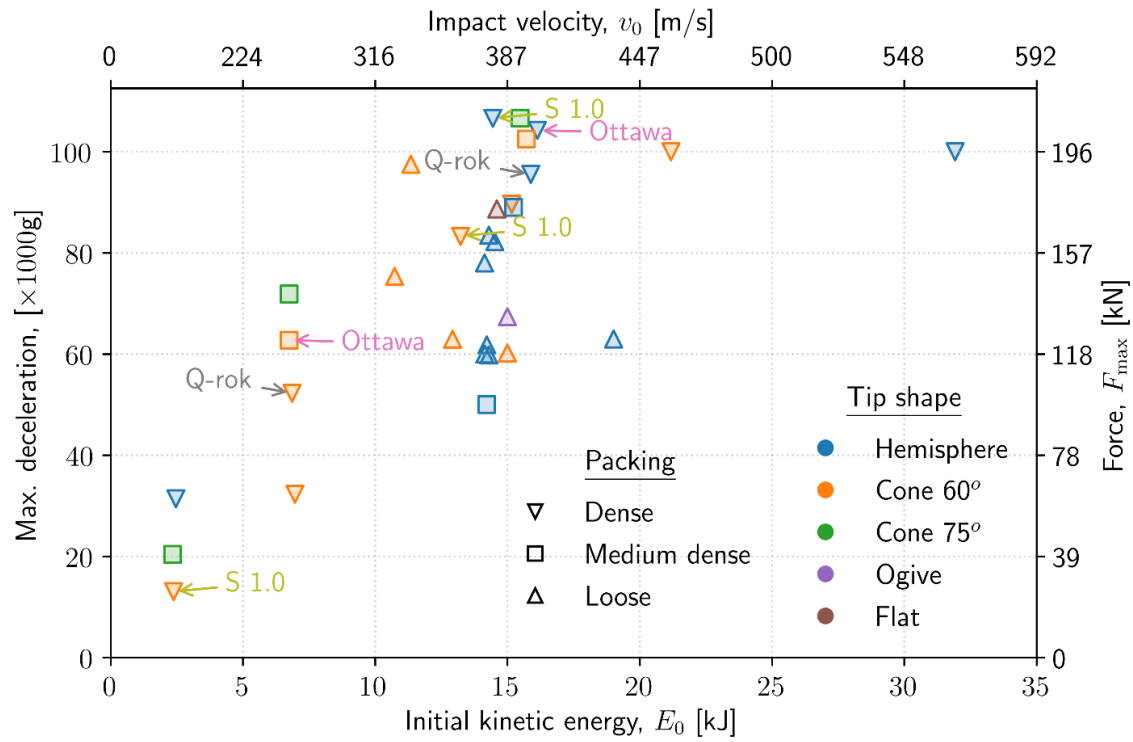


Fig. 58 Maximum deceleration experienced by projectiles in laboratory tests at different.

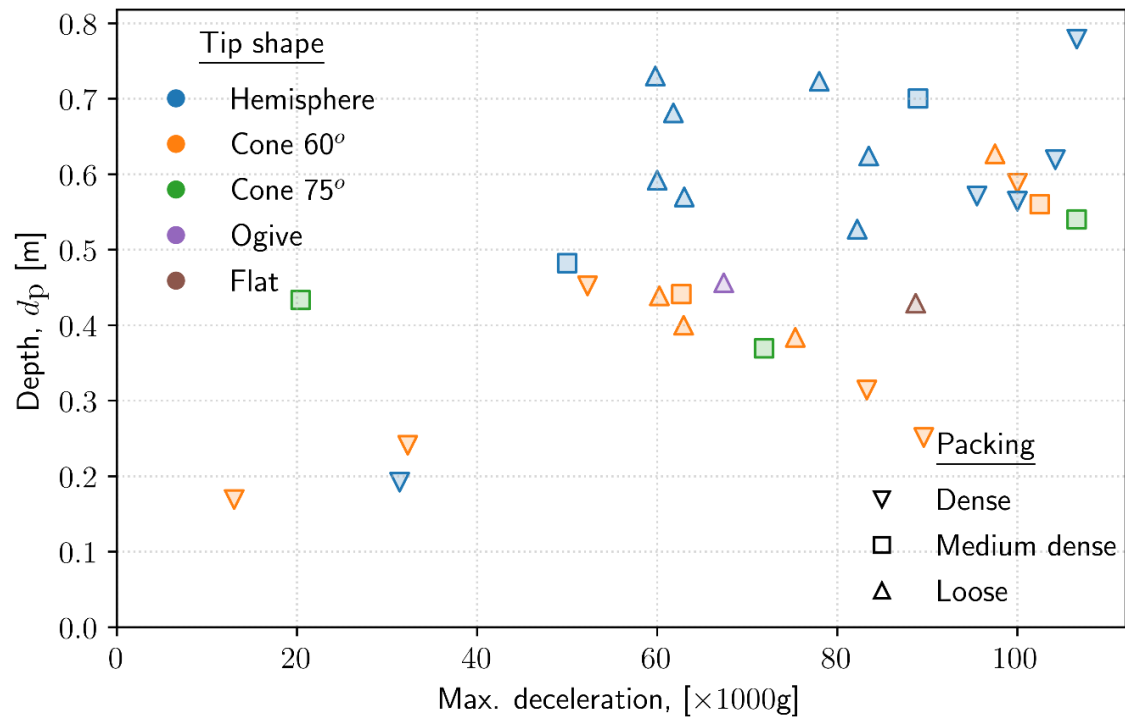


Fig. 59 Relationship between maximum deceleration and penetration depth for different tip shapes and target densities.

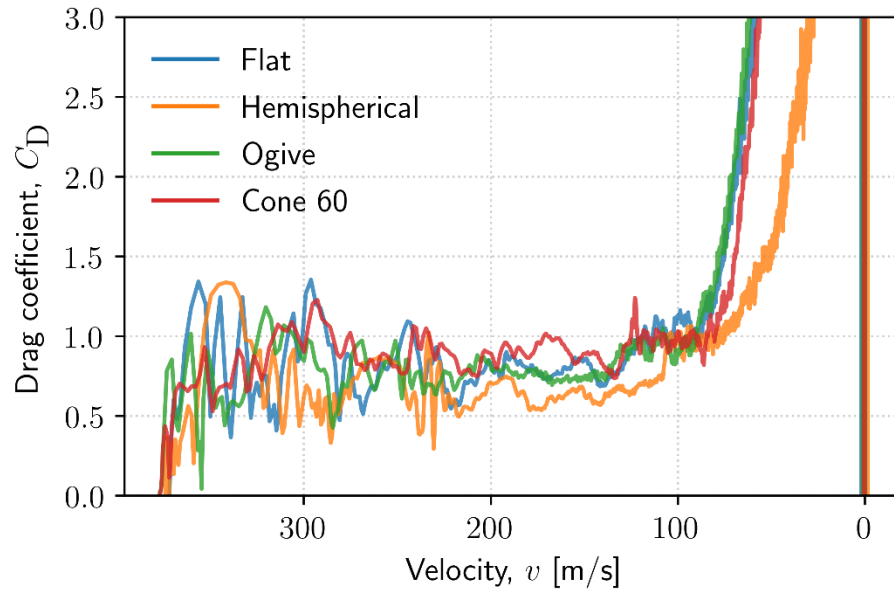


Fig. 60 The drag coefficient, C_D , for different projectile tips and similar target densities ($e_0 = 0.75$) and impact velocities ($v_0 = 377$ m/s).

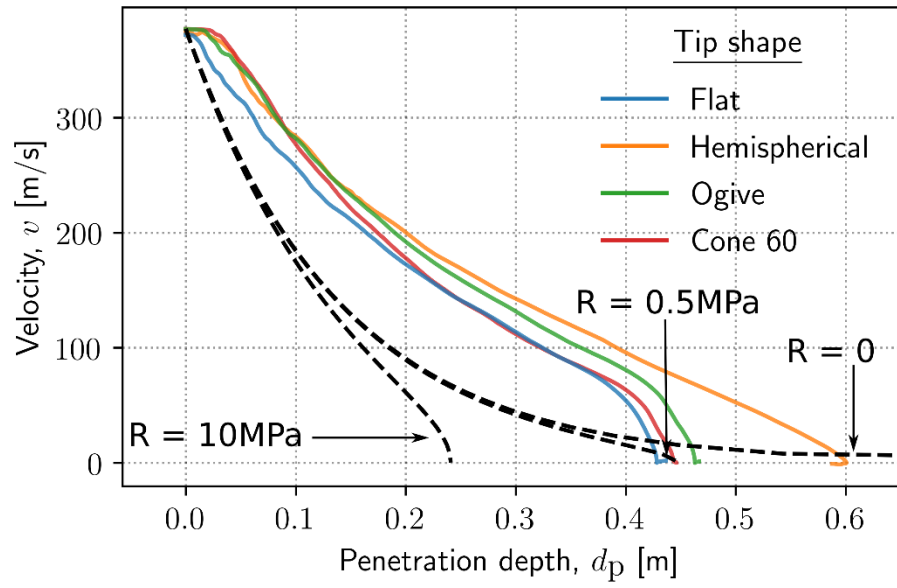


Fig. 61 Penetration depth predictions from Poncelet's equation (black dashed lines) for average $C_D = 0.80$ compared with that recorded in projectile penetration test for different nose shapes. The mass and diameter of the project were 0.210 kg and 0.03 m respectively, and the target was Siligran 0.125-0.71 sand at $e_0 = 0.75$.

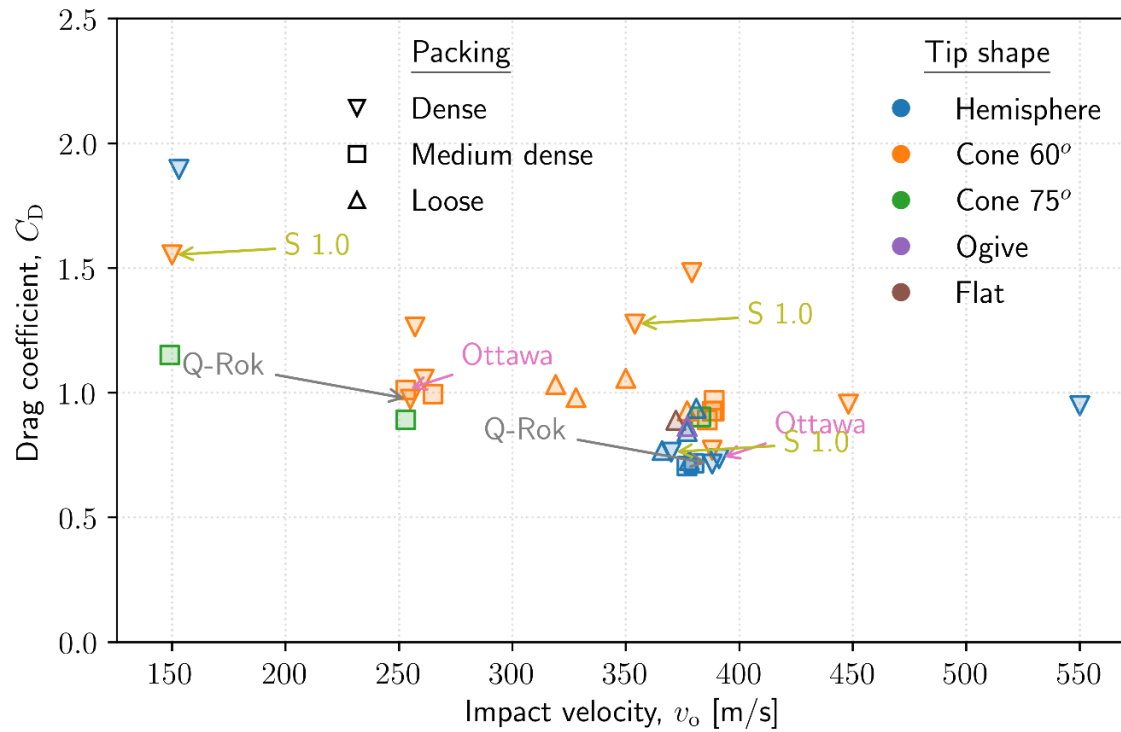


Fig. 62 The variation of drag coefficient, C_D with impact velocity for different tip shapes, target densities and sand. Most of the targets were Siligran 0.125-0.71. Three targets were of Siligran 1.0-1.6, and two each of Ottawa sand and Q-Rok placed along the path of the projectile.

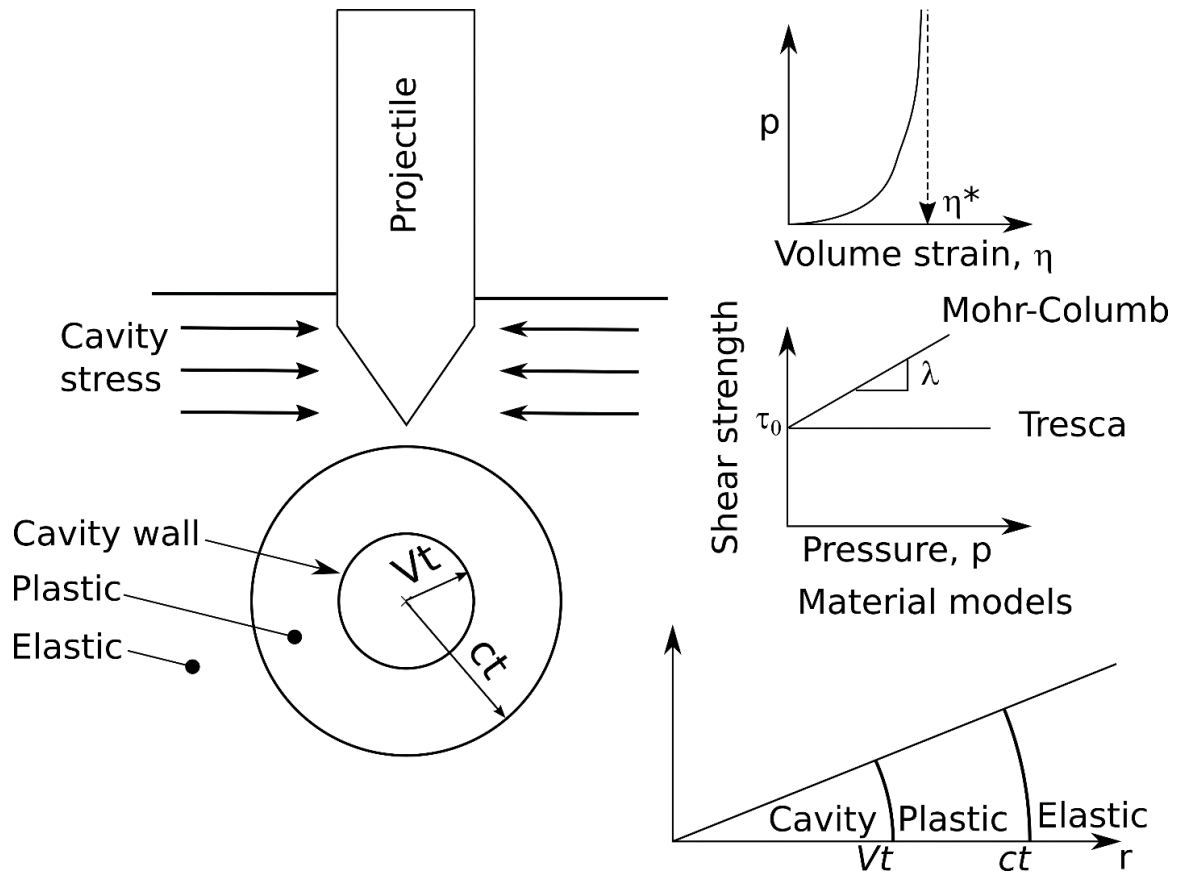


Fig. 63 Schematics showing aspects of cavity expansion model and material model.

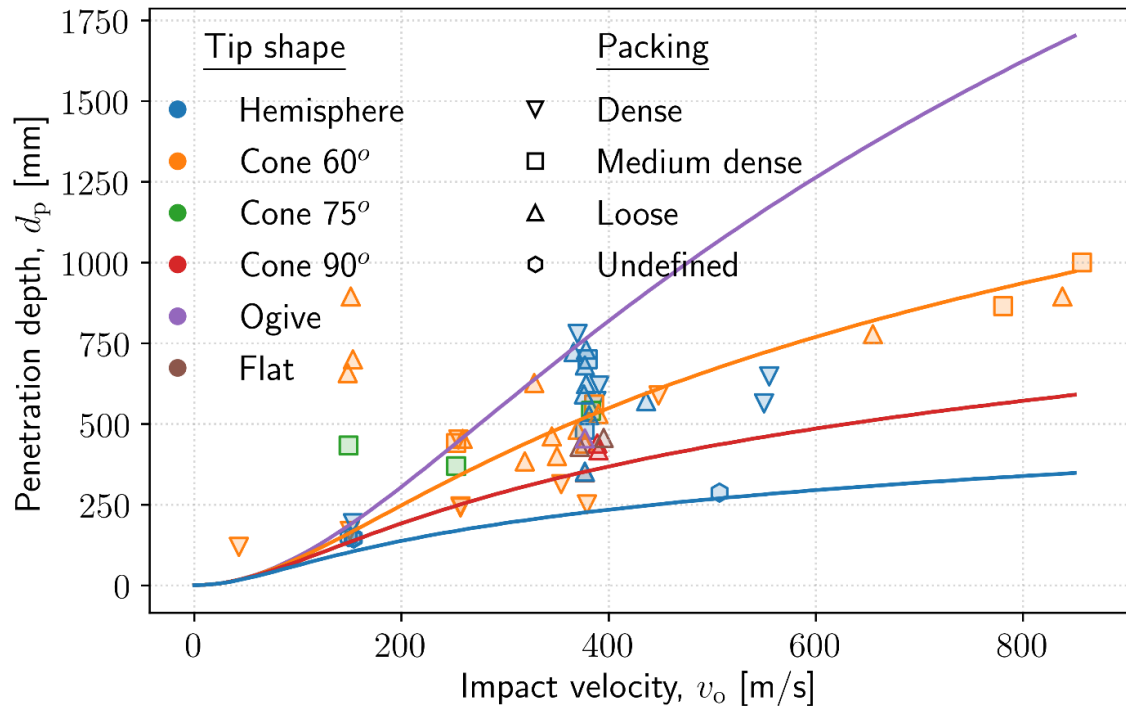


Fig. 64 Predictions of laboratory projectile penetration depths using cavity expansion model and Tresca failure criterion.

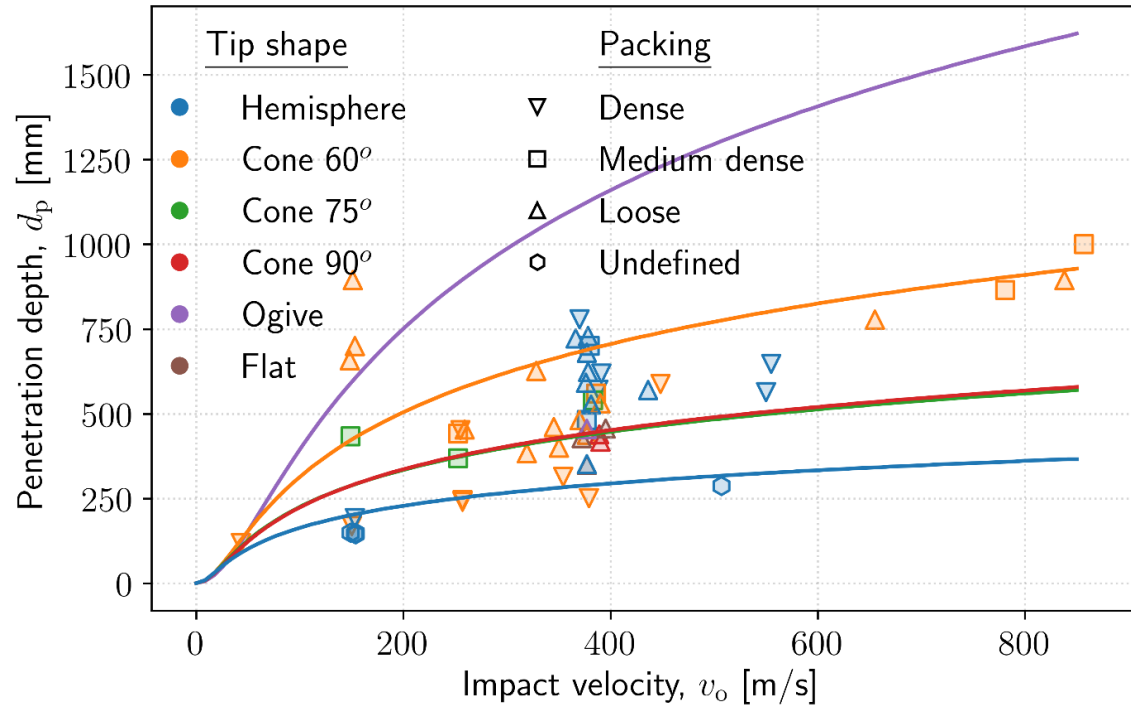


Fig. 65 Predictions of laboratory projectile penetration depth using cavity expansion model and Mohr-Coulomb failure criterion.

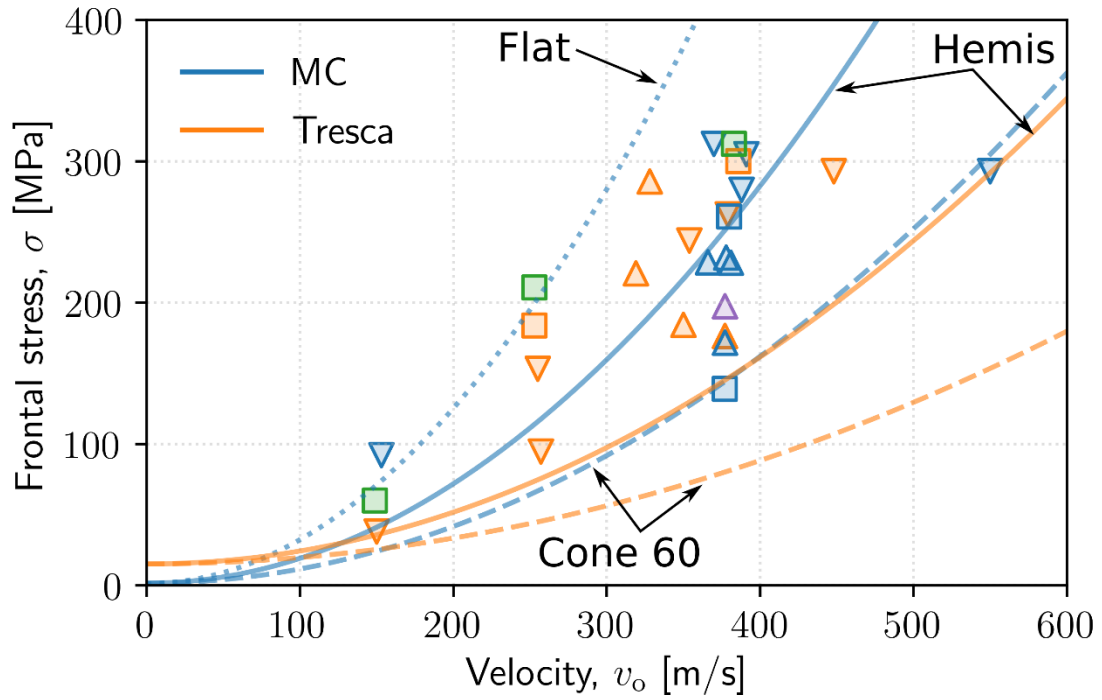


Fig. 66 Maximum frontal stress on the projectile compared with frontal stress computed from cavity expansion model for different nose shapes and failure criteria, Mohr-Coulomb and Tresca failure criteria. MC = Mohr-Coulomb, Hemis = Hemispherical nose shape.

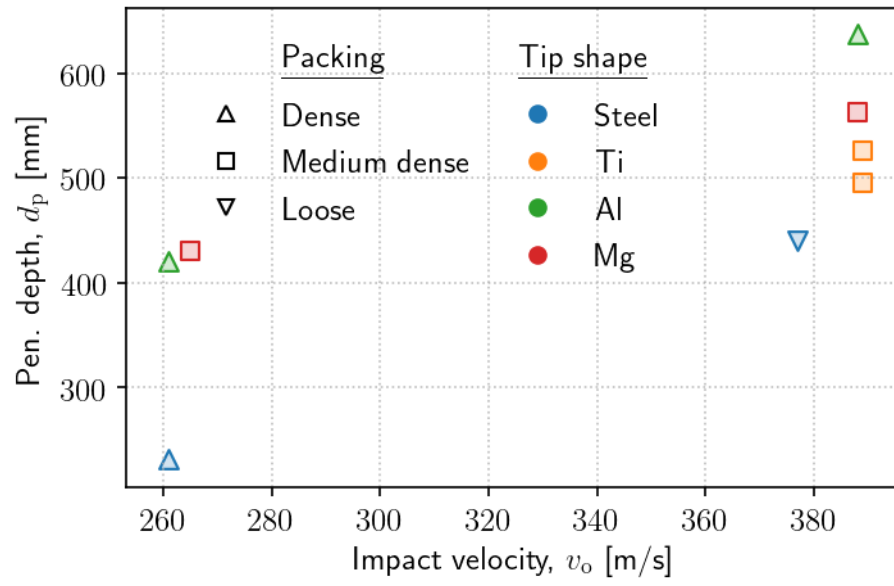


Fig. 67 The effect of tip material on projectile penetration depth in Siligran 0.125-0.71 target; $e_0 = 0.63$, $v_0 = 377$ m/s. The target void ratio (e_0) for steel tips was 0.73.

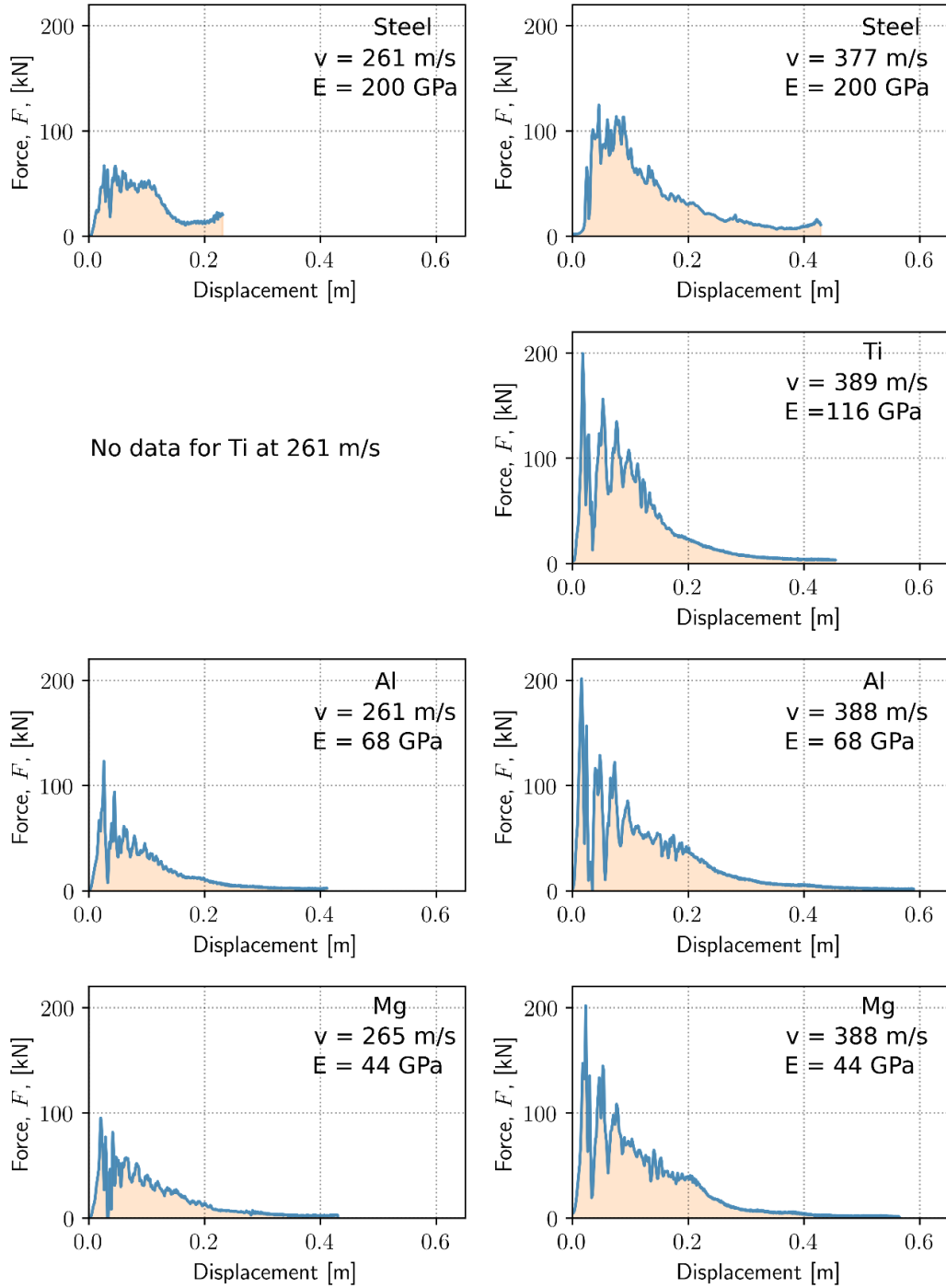


Fig. 68 The response of constant mass (0.210 g) 60° cone projectile with different tip material on Siligran 0.125-0.71 target at similar densities ($e_0 = 0.63$) and two different impact velocities. The e_0 of the target for steel tips was 0.73.

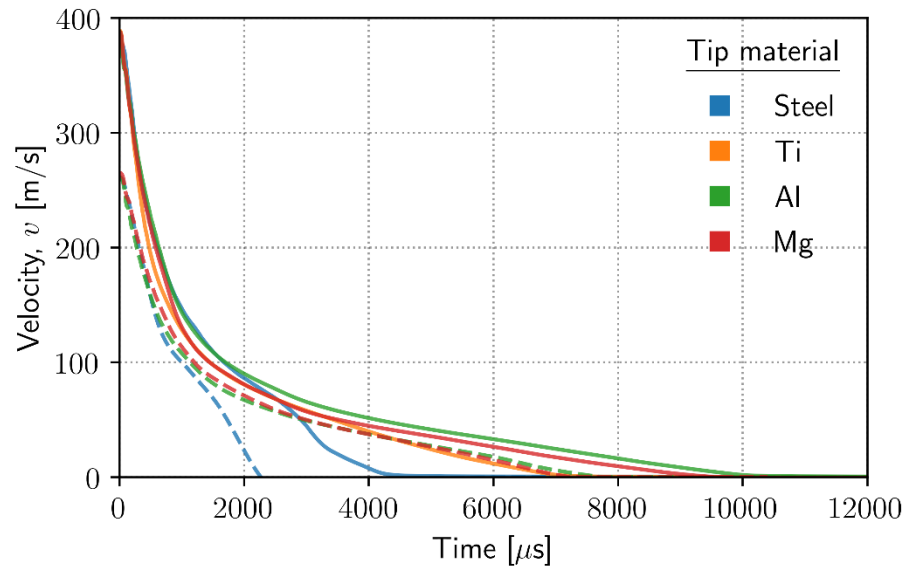


Fig. 69 Instantaneous velocity profiles for constant mass projectiles with different tip materials impacting on Siligran 0.125-0.71 sand targets prepared at similar densities ($e_0 = 0.63$) at two impact velocities.

CHAPTER V
THE ROLE OF PARTICLE SHAPE, DENSITY AND MOISTURE ON
PARTICLE CRUSHING UNDER DYNAMIC LOADING

Abstract

Crushed specimen from high strain-rate Split Hopkins Pressure Bar is analyzed. Laser light scattering technique was used to determine the particle size distribution curves. Quantitative values for crushing are provided. Particle size distribution curves of specimens collected from the path and the tip of the projectile in laboratory projectile penetration tests are compared with Split Hopkins Pressure Bar specimens. Different sands at different moisture contents were analyzed. Crushing decreases with increasing water content. Specimen density did not significantly influence particle crushing. Also, at very high strain-rate, when limit of crushing maybe reached it was found that particle shape did not influence the degree of crushing.

Introduction

Particle crushing in granular material occurs at locations of high stresses such as those found at the tip of piles during pile driving, underneath a high dam, and in the vicinity of penetrating projectile. When particles fracture, energy is expended in creating new surface areas. Comminution also alters the gradation and the density of the granular medium subsequently changing the constitutive behavior. Extensive comminution has been observed in laboratory projectile penetration experiments. Comminuted particles demarcate the path of the projectile and a false tip composed of crushed particles travels along with the projectile (Allen et al. 1957a; Glöckner et al. 2017). However, current methods either empirical (Taylor et al. 1991; Young 1969) or analytical (Forrestal and Luk 1992) used to predict the penetration depth of projectiles in granular medium do not explicitly consider this important aspect. Understanding particle crushing in high strain rate tests would help in the development of methods which would at a minimum account for the energy lost in comminution.

Cavity expansion methods are among the popular analytical methods in predicting the penetration depth of a projectile. The force on the projectile is determined from the stresses required to open a cavity and expand the cavity at a specified rate to the size of the penetrator. Use of simple constitutive behaviors, such as Mohr-Coulomb and Tresca failure enables closed form solutions. Among the most important parameters, which determine the penetration depth, is the strength of the resisting continuum. Strength of the medium is generally determined from quasi-static tests (Forrestal and Luk 1992). A dynamic test would be more appropriate since it would

incorporate the effects of particle crushing during force application and rate effects on the strength of the medium.

High strain rate tests have been performed in granular material, such as sands in the Split Hopkins Pressure Bar (SHPB) to understand the dynamic behavior under varying boundary confinement, specimen density, and moisture content. SHPB have been used to study the rate effects in dry sand (Bragov et al. 2008; De Cola et al. 2018; Song et al. 2009), effect of gradation and size (Huang et al. 2013; Luo et al. 2014), effect of moisture content (Luo et al. 2014; Martin et al. 2009), effect of particle shape (De Cola et al. 2018), and effect of confinement (De Cola et al. 2018; Song et al. 2009). Such tests may prove very useful in studying the comminution in projectile penetration tests due to the possibility of very high strain rates and stresses. Also, flexible confining boundaries may be used to induce stresses that may approximate triaxial stress states.

Particle size distribution of the comminuted particles after SHPB tests have shown that particle crushing increases with increasing stresses (Huang et al. 2013), and decreases with increasing moisture content (Luo et al. 2014). Also, at similar stress levels, particle crushing is more in quasi-static loading than in high strain rate loading (Huang et al. 2014a). A series of high strain rate tests in SHPB were performed by De Cola et al. (2018) to determine the role of particle morphology, confinement and moisture content on high strain rate strength. The results presented here are analyses of the specimens from De Cola et al. (2018). In addition to specimens from SHPB tests, samples were also collected from the path and tip of the projectile in laboratory penetration tests. The laboratory projectile penetration tests were performed at the Fraunhofer Institute for High-Speed Dynamics, Ernst-Mach-Institut (EMI) and are described by Glößner et al. (2017). The circular cylinder targets in the projectile penetration tests were filled with Siligran 0.125-0.71 sand. After the penetration test, the target was carefully excavated to the level of the projectile and comminuted sand collected from the path. It was seen that a false tip of comminuted particles formed around the projectile nose and travelled with the projectile. An intact tip was retrieved to determine the extent of comminution.

Descriptions of the three sands are provided and followed by a summary of the testing procedures. Readers are referred to De Cola et al. (2018) for detailed explanation of the SHPB procedure. Particle size distribution curves of crushed specimens are compared with uncrushed specimens and crushing is quantified using a breakage factor.

Materials and experimental methods

High strain rate tests in SHPB were performed on silica sands, Ottawa sand, Q-Rok and Euroquarz Siligran 0.125-0.71 sand. The major constituent in the three sands was quartz, consisting of more than 99.6%, (De Cola et al. 2018). All three sands classified as poorly graded sand (SP) as per USCS classification and have similar size range. The maximum size of the sands was approximately 850 μm . Siligran has marginally more finer grains than Ottawa sand and Q-Rok. The three sands had very different particle morphologies. Ottawa sand is sub-rounded, Q-Rok is angular and Siligran was sub-angular. The small differences in both particle shape and grain size distribution was such that the index densities of Siligran were very similar to that of Ottawa sand. Thin section images of the sand revealed some polycrystalline Q-Rok grains, and mineral inclusions and recrystallization in some Siligran grains. No such evidence of defects and polycrystalline grains were found in Ottawa sand grains.

High strain rate tests were performed using a Long-SHPB as described by De Cola et al. (2018). Longer striking bar enables a longer load pulse of higher magnitude, consequently achieving higher strains (De Cola et al. 2018). The strain rate of the specimens presented here was 1600 s^{-1} in SHPB. Additional tests were also performed at lower strain rates of 0.001 s^{-1} and 1 s^{-1} on similar sized specimens in Instron testing system.

After the tests the crushed specimens were collected, and particle size distribution determined using laser light scattering. The use of laser light scattering ensures consistent definition of grain size over large particle size range. The long bench Mastersizer S has a range of $0.05\text{ }\mu\text{m}$ to $840\text{ }\mu\text{m}$. For quantitative analysis of comminution Einav's (2007) breakage factor, B_r , which considers the ultimate distribution with the assumption of fractal nature of comminution are presented.

Creation of new surfaces requires energy. When the stress on the particle exceeds the tensile strength of the grain, fracture occurs resulting in two or more pieces and creating new surfaces. The surface area of the specimen from the tip was measured using gas adsorption method in Quantachrome 1-C. The specimen is placed in a glass holder under liquid nitrogen bath and equilibrium pressure is measured after dosing a known volume of gas. Surface area is computed from the number of adsorbed molecules and cross section area of the adsorbed gas molecule.

Generally, nitrogen gas is used as the adsorbate, but Krypton gas has greater sensitivity for very small surface areas and was used when measuring surface of uncrushed and crushed sand.

Results and discussions

Effect of grain shape on particle crushing

Particle crushing is a time dependent phenomenon and thus specimens subjected to different strain rate will undergo different degree of crushing. Particle crushing in Ottawa sand specimens subjected to different strain rates in 1D compression tests, quasi-static and high strain rate tests in SHPB, are shown in Fig. 70. The B_r for the 1s^{-1} strain rate specimens were slightly higher. The B_r for the high strain rate tests in the SHPB approached unity. Though particle crushing has been found to decrease with increasing strain rates for specimens subjected to similar maximum stress, the specimen in the SHPB tests were subjected to a significantly higher stress than in Instron. There is similar observation in angular Q-Rok specimens as shown in Fig. 71.

In Fig. 72, particle crushing in specimens of Ottawa sand, Q-Rok and Siligran compressed at 1600s^{-1} in SHPB is presented. The void ratios for the Ottawa sand and Q-Rok specimens were 0.67 and the Siligran specimen had more voids at 0.72. Particle crushing generally increases with increasing particle angularity and void ratio. All three sands show similar crushing, the B_r was 0.98, 0.93 and 1.0 for Ottawa sand, Q-Rok, and Siligran specimens. The B_r values are close to 1, at extreme crushing close to the ultimate crushing, packing and particle shape may have a limited influence on the degree of crushing.

Effect of moisture on particle crushing

Moisture content, below saturation, increases the strength of the soil by increasing the effective stresses through suction. It may also influence the permeability of the soil specimen. At small moisture contents the specimens may initially have higher strength than dry specimens due to suction. However, increasing moisture content have shown softer specimen response in both quasi-static and high strain-rate tests in uniaxial compression (Martin et al. 2009). This has been attributed to the lubricating effect of water (Martin et al. 2009). With increasing strain, the degree of saturation keeps increasing and at high enough strains the specimens may reach saturation. In high strain rate loading, when the specimen becomes saturated the behavior may approach that in undrained state. The behavior of Ottawa sand specimens at high strain rate loading at different

moisture contents, dry, 3% and 20%, along with the particle size distributions of the specimens after the tests are shown in Fig. 73. The behavior of all the three specimens are similar until the yield stress at around 40 MPa. Beyond the yield stress the dry and the 3% moisture content specimen show similar behavior and reach identical axial stress. The behavior of the 20% moisture content specimen is significantly different after the yield stress. The degree of saturation at 20% moisture content is 94%, therefore the specimen would approach 100% saturation at around 1% strain. At saturation, based on the permeability of the system some of the applied load would be taken up by the water, thus lower stresses in the soil particles leading to less crushing. The B_r value for the 3% water content specimen was 0.82 and it was 0.53 for the 20% water content specimen.

Tests performed in Siligran specimens at different moisture content, and the particle size distribution of the crushed specimens are shown in Fig. 74. At 20% water content the degree of saturation is 73% and the required axial strain to reach 100% saturation is 6.7%. The dry specimen and the 6% water content specimens show similar behavior. The 20% water content specimen behavior is the similar to the 6% water content specimen until 7% axial strain at which point the stress start rising above that lower water content specimen. All the three specimens reach identical axial stress value. The B_r value for 6% water content specimen was 0.64 and for the 20% water content specimen was 0.57. Similar behavior of decreasing particle crushing with increasing moisture content have been reported by Luo et al. (2014). Increasing the water content decreases particle crushing irrespective of grain morphology.

Effect of density on particle crushing

Particle crushing in three specimens of Siligran at three different void ratios is presented in Fig. 75. The yield stress of the dense specimen is 50 MPa and the that of the loose specimen is 25 MPa. All specimens reached similar axial stress at the end of the loading albeit at different strain levels. The B_r value increases with density, 0.78 for loose specimen, 0.86 for medium dense specimen and 0.92 for the dense specimen. Though the B_r values are different for different densities, there is only very minor differences in the particle size distribution curves. Dynamic testing in SHPB tests on Quikrete #1961 sand at different densities indicated almost no difference in particle crushing (Luo et al. 2011).

Effect of high strain rate loading in projectile penetration tests

The trail of comminuted sand along the path of the projectile and the false tip formed in front of the projectile tip in laboratory projectile penetration tests is shown in Fig. 76(a-b). Low and high-resolution scanning electron microscope images of the crushed particles from the tip are shown in Fig. 76(d-f). For comparison purpose uncrushed particles of Siligran is also shown in Fig. 76(c). The particle size has been reduced from approximately 200 μm or larger in the uncrushed images to grain that are less than 1 μm in size in the projectile tip specimen. Along with the fines, there are also few larger grains. The surface area of the tip specimen increased by a factor of four from 0.173 m^2/g to 0.710 m^2/g .

Particle size distribution curves for the false tip and crushed specimen collected from the path are shown in Fig. 77. Also shown are particle size distribution for SHPB tests specimen compressed at 1600s^{-1} . The crushing in SHPB and along the path of the projectile is similar; the B_r is 0.86 for SHPB and 1 for specimen from the path. The tip specimen shows extensive crushing with B_r value exceeding unity. Since volume distributions are easily skewed by few large grains, number distributions provide an alternative visualization of crushing. Number distributions are shown in Fig. 77(b). The extent of crushing can be seen by the abundance of fines created, which reduced the modal particle size from 191 μm to sub 1 μm , the limit of the laser diffraction instrument. The similarity of all three number distributions subjected to different stresses and strains may indicate that particle size limit in crushing may have been reached.

Conclusions

Particle crushing results in dynamic tests are presented. A significant difference in crushing was not observed when the strain rate was varied by three orders of magnitude in both rounded and angular sands. Considerable comminution was seen in very high strain rate tests in SHPB, however the maximum axial stress in these tests were also significantly larger. Particle crushing in sands with different grain morphology was not significantly different. Also, the density of the specimens also did not affect the degree of crushing in high strain rate tests. At very high degree of crushing the grain shape and density may have minimum influence on the crushing. Particle crushing was reduced in the presence of moisture. Particle crushing in laboratory projectile crushing tests was significantly greater than in SHPB tests.

Appendix

Table 9 Summary of the tests results

Test	Sand	Void ratio	Strain rate (s^{-1})	Saturation (%)	Maximum Stress (MPa)	B_r
Strain rate effects						
1	Ottawa	0.78	0.001	0	80	0.74
2	Ottawa	0.78	1	0	80	0.80
4	Ottawa	0.67	1600	0	113	0.98
5	Q-Rok	1.0	0.001	0	80	0.34
6	Q-Rok	1.0	1	0	80	0.44
8	Q-Rok	0.67	1600	0	142	0.93
9	Siligran	0.72	1600	0	142	0.86
10	Siligran Path	-	Penetration test	0	-	1.0
11	Siligran Tip	-	Penetration test	0	-	> 1.0
Saturation effects						
12	Ottawa	0.57	1600	3	114	0.82
13	Ottawa	0.57	1600	20	90.3	0.53
14	Siligran	0.72	1600	6	147	0.64
15	Siligran	0.72	1600	20	152	0.57
Density effects						
16	Siligran	0.80	1600	0	133	0.78
17	Siligran	0.72	1600	0	142	0.86
18	Siligran	0.56	1600	0	138	0.92

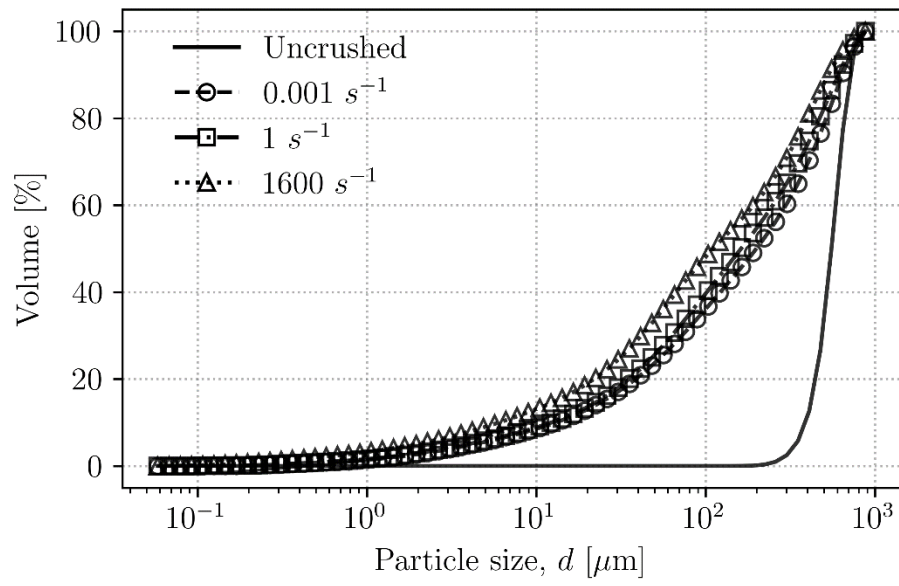


Fig. 70 Particle crushing at different strain rate in 1D compression test of Ottawa sand.

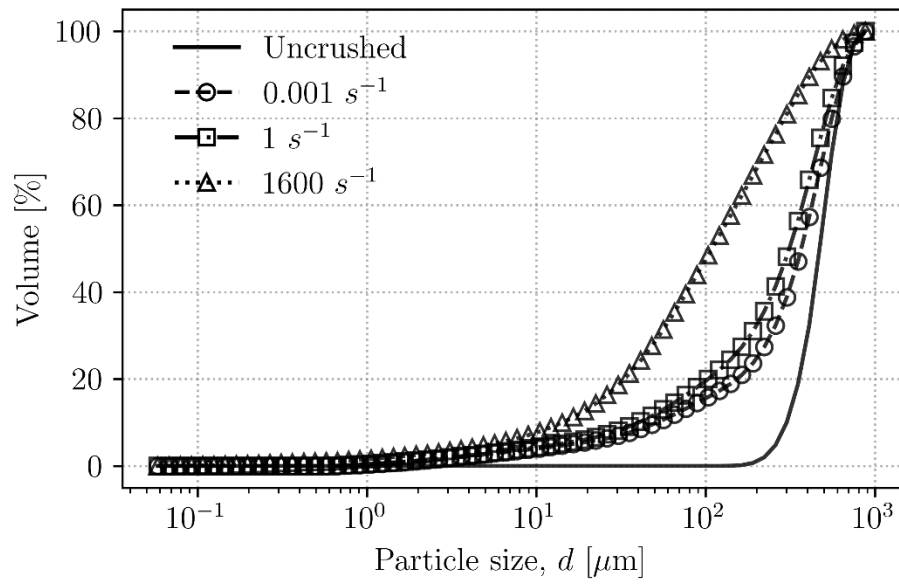


Fig. 71 Particle crushing at different strain rates in 1D compression test of Q-Rok.

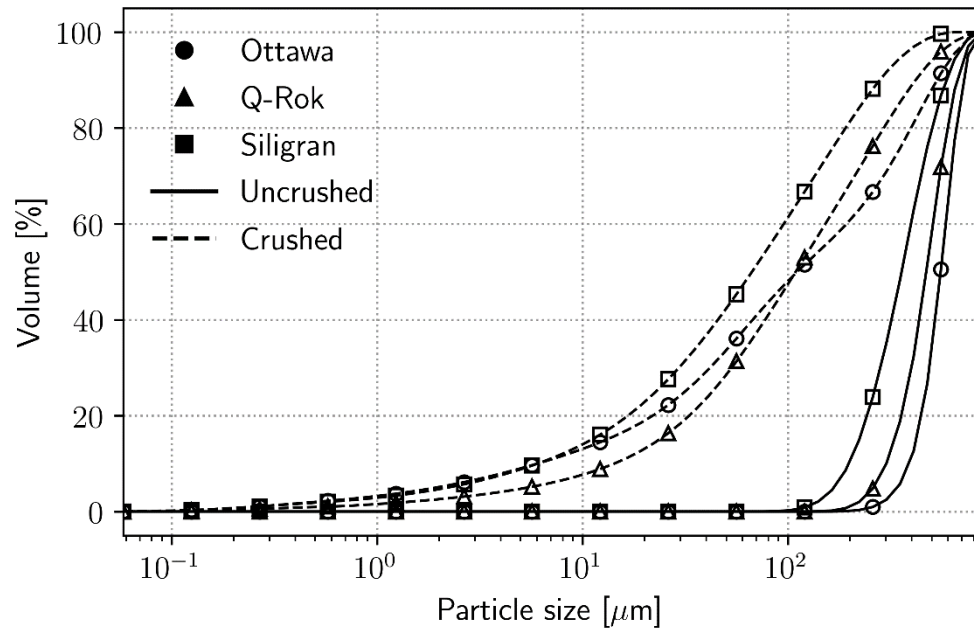


Fig. 72 Particle crushing at high strain rate in rounded Ottawa sand and angular Q-Rok.

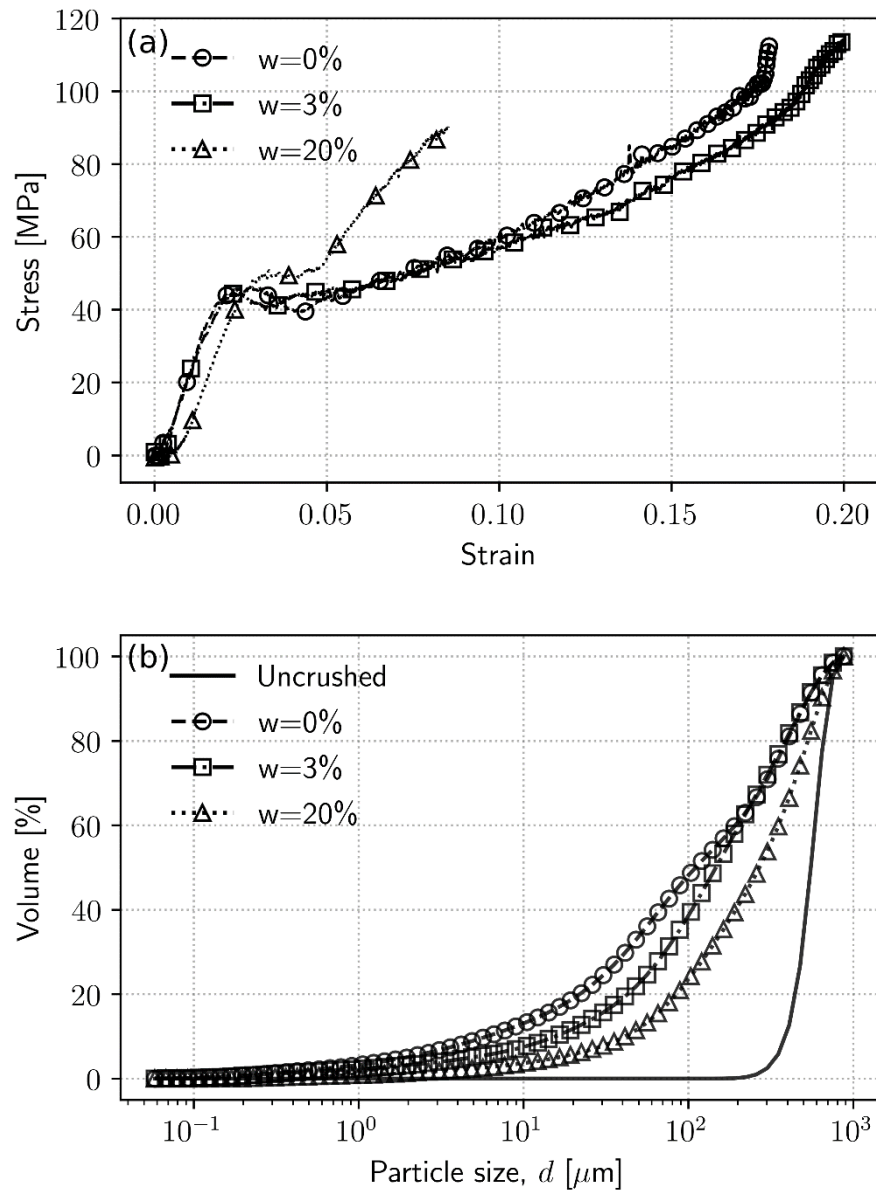


Fig. 73 Particle crushing in high-strain rate 1D compression tests in SHPB in Ottawa sand with different moisture content.

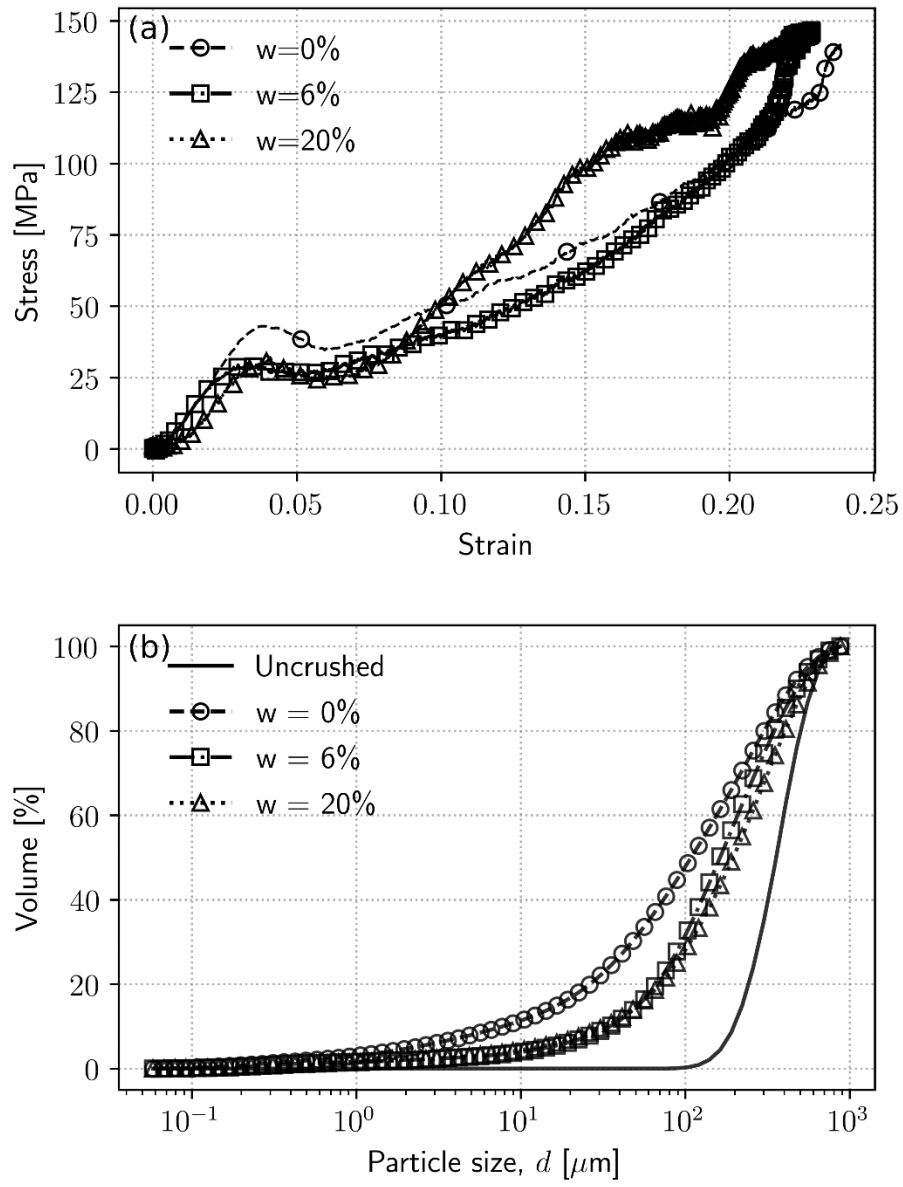


Fig. 74 Particle crushing in high-strain rate 1D compression tests in SHPB in Siligran sand with different moisture content.

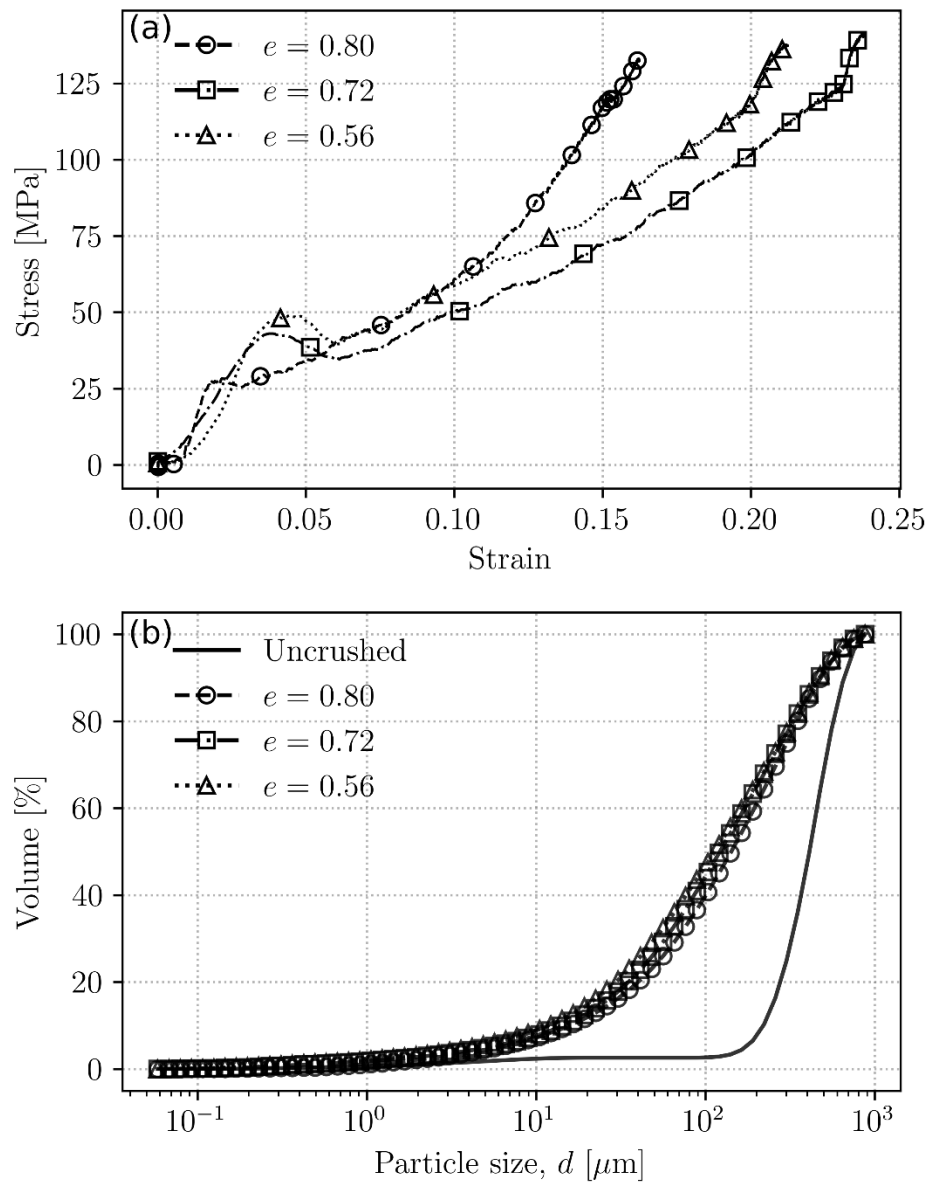


Fig. 75 Particle crushing in high-strain rate 1D compression SHPB tests in Siligran specimens at different densities.

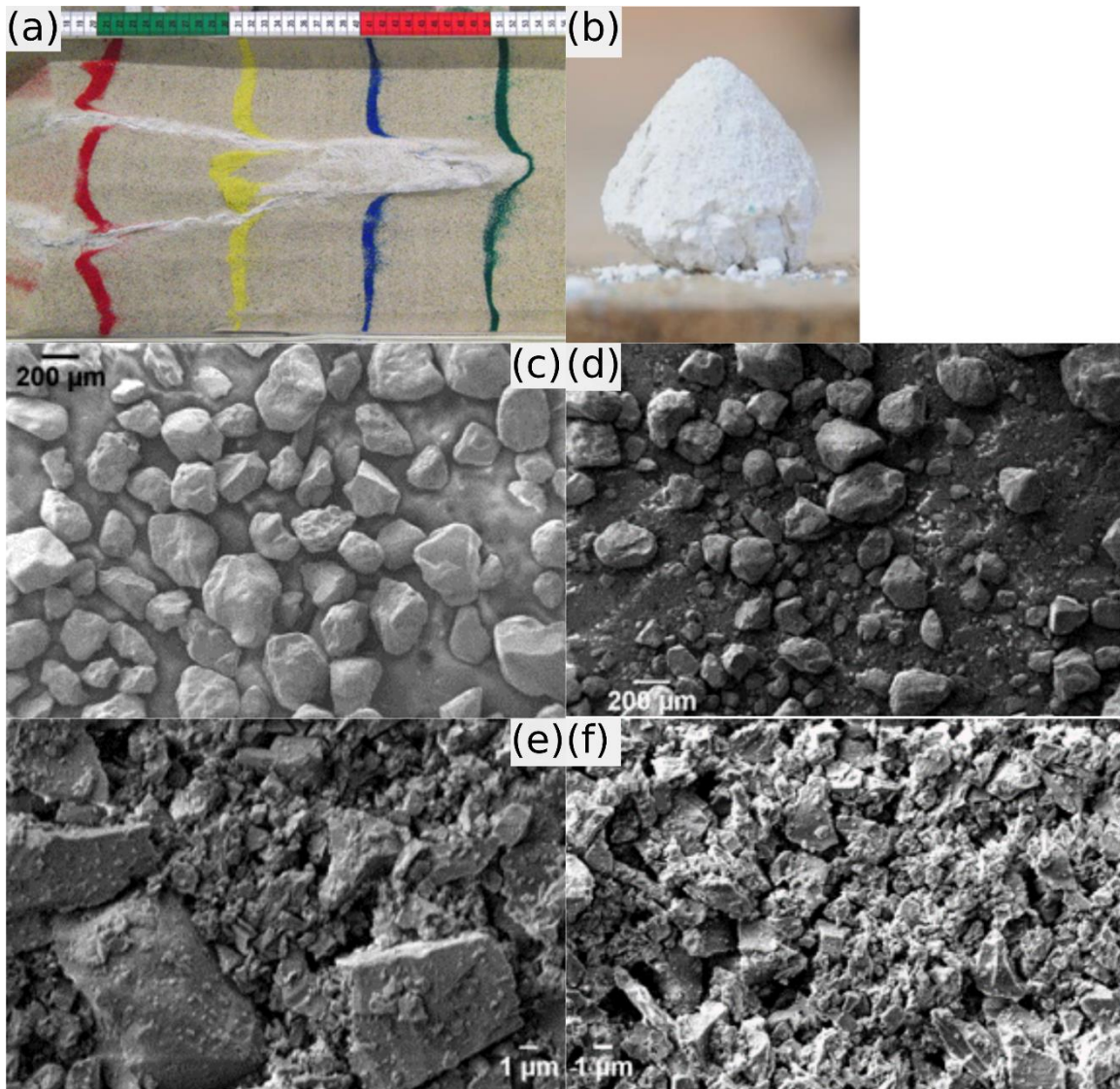


Fig. 76 Images of (a) trail of comminuted particles along the path of the projectile (b) false tip of comminuted particles at the projectile tip, scanning electron microscope images of (c) uncrushed Siligran sand (d) low resolution image of crushed particles from the tip (c-d) high resolution images of crushed particles from the tip.

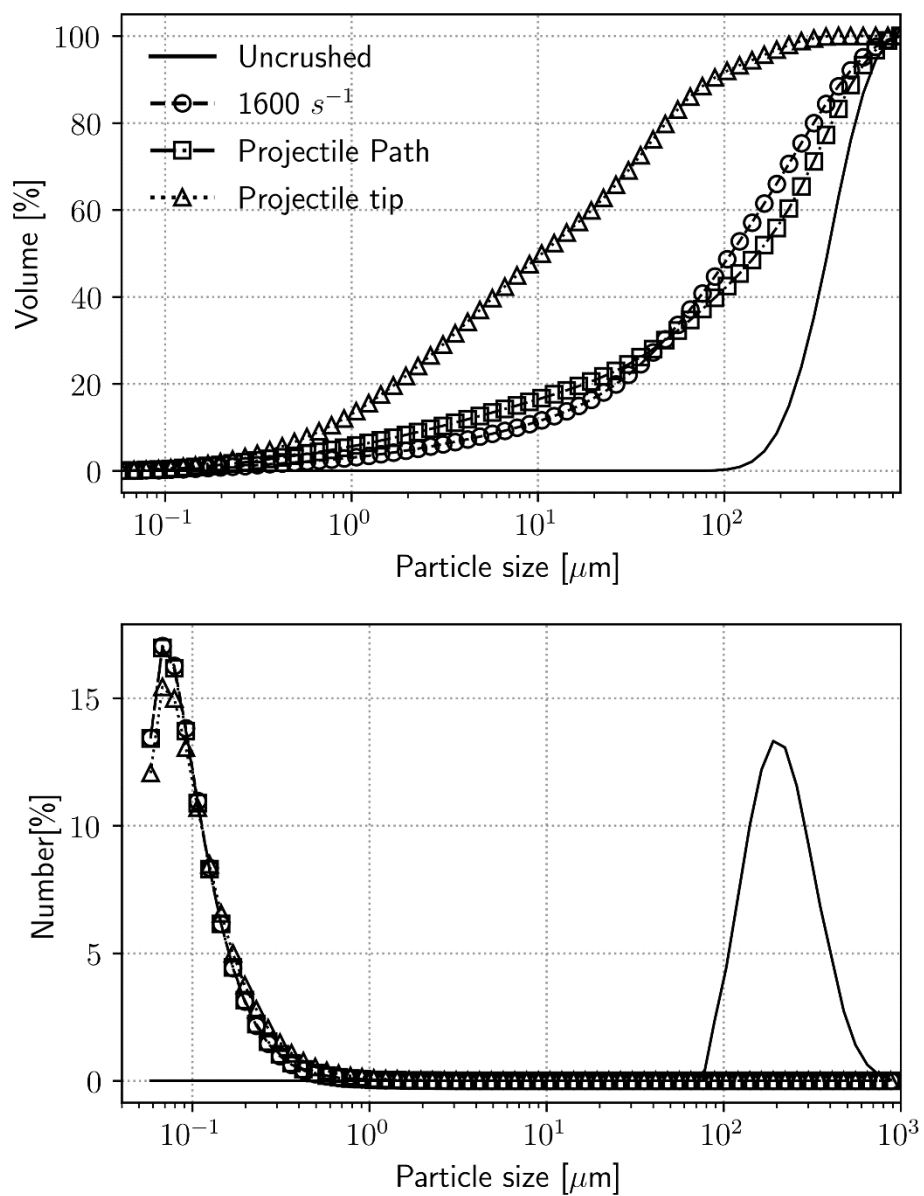


Fig. 77 Particle crushing in high-strain rate 1D compression test in SHPB and along the path and tip of the projectile with impact velocity of 377 m/s.

CONCLUSIONS

Particle crushing is a complex phenomenon that depends on grain shape and size, grain strength, surface features and the rate of loading. In general, angular grains have lower grain strength, and angular specimens have higher void ratio, thus are more susceptible to crushing. Angular grain fracture is accompanied by asperities breaking leading to more rounded grains. Rounded grains fracture diametrically resulting in more angular grains. Hence particle crushing changes the grain morphology in addition to the particle size distribution of the assemblage. The effect of moisture is to reduce crushing and low and high density specimens did not show significant differences in crushing. At high strain-rates which may produce high stresses, grain morphology does not seem to influence particle crushing. Particle crushing in the laboratory projectile penetration tests were significantly greater than in high strain rate tests in Split Hopkins Pressure Bar tests. The stresses at the tip of the projectile were also greater than stresses reached in the high strain rate tests which would result in higher crushing.

Grain morphology influences the behavior of the granular mass. The strength of angular assemblages is generally greater than rounded materials, but angular assemblages are also associated with lower stiffness and can undergo larger displacements. The relative density of the of the medium has a greater influence on the strength and volumetric response than particle shape. The influence of particle shape is greater in loose specimens than in dense specimens.

Specimens are generally tested in triaxial compression for material strength in geotechnical engineering. Localized shearing is a recognized problem, especially in frictional end triaxial testing. DIC provides an excellent opportunity to visualize development of surface strain in triaxial test specimens. With the help of axial and lateral strains the initiation of localized shearing is seen to start at very early stage of loading. In dense specimens this localized deformation dominates the post peak deformation with the development of distinct shear planes. The failure plane is diffused in loose specimens.

Single grain crushing of particles of different shape show rounded grains in general have higher strength than angular grains. The grain strength decreased with increasing particle size and Weibull distribution can be used to model failure strengths of grains, for size range used in this series of tests. With increasing angularity, Weibull modulus decreases indicating greater variability

of strength. Flaw sizes determined with the assumption of critical surface flaws show that for low value of Weibull modulus the flaw size increases at a faster rate with grain size. The fracture energy was measured using acoustic emission technique. Energy required to break particles increases with fracture strength and greater energy is required to break smaller particles than larger particles. Thus, shedding light on the comminution process and the interplay of coordination number and size dependence of particle strength and in the process may explain the process which may lead to fractal distribution of crushed particles.

Laboratory penetration tests have shown extensive crushing along the path of the projectile. A false tip of crushed particles forms around projectile nose. The response of the projectile as it penetrates a granular medium depends on the density, projectile tip shape, impact velocity and the grain shape of the granular medium. In general flatter tips experienced higher resistance on impact, and denser medium provided greater resistance. The Poncelet's drag coefficient did not depend on the sand type and was higher for low impact velocities. It was also seen that projectile tip material influenced the penetration depth.

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