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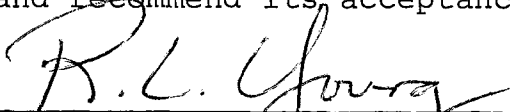
To the Graduate Council:

I am submitting herewith a thesis written by James Kenneth Rochelle entitled "TRAX-A Finite Element Computer Program for Transient Heat Conduction Analysis of Axisymmetric Bodies." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Engineering Mechanics.



Major Professor

We have read this thesis
and recommend its acceptance:



Accepted for the Council:



Vice Chancellor for
Graduate Studies and Research

TRAX-A FINITE ELEMENT COMPUTER PROGRAM FOR
TRANSIENT HEAT CONDUCTION ANALYSIS
OF AXISYMMETRIC BODIES

A Thesis
Presented to
the Graduate Council of
The University of Tennessee

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
James Kenneth Rochelle
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ABSTRACT

The purpose of this thesis is to produce and document TRAX, an improved finite element transient heat conduction program for axisymmetric or plane bodies.

A finite element solution of the transient axisymmetric body heat conduction equation is demonstrated using the Euler Theorem from the calculus of variations. The basic element is chosen to be triangular in cross-section with a linear internal temperature distribution. Anisotropic conductivity at any orientation in the plane of the cross-section is included in the element formulation. Heat generation within the element is considered in the element derivation, but not included in the final formulation. A quadrilateral element is defined to be a combination of two triangular elements. Lumped heat capacity matrices are formulated for both types of element. Element conductivity matrices are formulated as derived for the triangular element. Plane body elements are allowed by assigning a value of one to the axisymmetric element centroid radius. The system matrix equation is derived. It includes boundary conditions for convection and radiation to the body surfaces, and specified temperature at any point in the body.

The transient time period must be divided into a finite number of time steps in any finite element transient heat conduction analysis. In TRAX the transient time period

is first broken into time intervals of variable length. Each time interval is then divided into an even number of time steps.

Changes in boundary conditions, material properties, and the time parameters are allowed in TRAX at every time interval. Variation in the external fluid temperature for convection is allowed at every time step. Steady-state cases are assumed to be solutions for a particular type of time interval.

A Gauss-Cholesky solution routine is used to solve the system matrix equation for the body being analyzed. Computer time requirements are minimized because factoring, the most time consuming part of this solution routine, is only required at the first time step in an interval. Object time dimensioning and overlay techniques are applied to minimize core storage requirements. Various features are included to make data input and output interpretation easy and convenient.

Two example problems are solved using TRAX on an IBM 370-155 computer. The first problem verifies the validity of the solution procedure by comparison with published data. The second problem demonstrates the application of many program features.

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NOMENCLATURE

Temperature

Time

System radial axis

System axial axis

Element axis

Element axis perpendicular to r axis

Angle between system and element axes

Material conductivity in r direction

Material conductivity in z direction

Element conductivity matrix

System conductivity matrix

Specific heat of material

Element heat capacity matrix

System heat capacity matrix

Material density

Heat generated per unit volume of material

Surface conductance

System conductance matrix

Element radius

Temperature vector for current time step

Temperature vector for preceding time step

Radiation heat flux

System heat generation vector

System radiation vector

Element cross-sectional area

External fluid temperature for convection

Distance along body surface

System "stiffness" matrix

System load vector

Time step size

Node numbers for triangular element

CHAPTER I

INTRODUCTION

Computer solutions are necessary for complex transient heat conduction problems. Hand analysis of these problems is not practical without making simplifying assumptions. In many cases only a conservative steady-state solution can be obtained. The cost of systems designed with this approach is typically high and their reliability is low. Some design codes now require transient thermal stress evaluations. The 1971 ASME Boiler and Pressure Vessel Code, Section VIII, Division 2, has this requirement.

In 1966, Wilson and Nickell (1)¹ applied the finite element method of analysis to the transient heat conduction problem. Their publication included a computer program for two-dimensional transient or steady-state heat conduction analysis. This program was combined with a finite element stress program at AEDC² in 1971. The combined program, THERMAS, has been the primary tool at AEDC for thermal stress analysis of axisymmetric bodies such as pressure vessels, pipelines, ducts, and test cells. There are, however, some rather severe limitations in the heat

¹Numbers in parentheses refer to similarly numbered references in the bibliography.

²The Arnold Engineering Development Center, Arnold Air Force Station, Tennessee.

conduction portion of THERMAS. For example, boundary conditions and material properties must be constant for the entire transient period.

Emery and Carson (8) reported similar limitations to finite element heat conduction programs in a 1971 publication. They evaluated then-existing finite element programs in comparison to some standard finite difference analyzers. The finite element programs were found to be superior only in the ease of input and the capacity to increase accuracy by using higher order elements. The finite difference programs were considered vastly superior in ability to handle variable material properties and boundary conditions.

Computer program TRAX is designed to remove those objectionable limitations present in preceding finite element heat conduction programs. It should be competitive with finite difference programs as a general two-dimensional thermal analyzer. In a combined heat conduction and stress analysis finite element program it will provide a definite advantage over THERMAS.

DERIVATION OF ELEMENT AND SYSTEM EQUATIONS

I. THE TRANSIENT HEAT CONDUCTION EQUATION

The transient heat conduction equation will be derived for the general axisymmetric body case. The steady-state, plane body, and isotropic conductivity equations can be obtained by making obvious simplifications of this general equation. Note that the element and system axes are not parallel in the general case. (See Figure 1.)

$$q_1 = -k_r \bar{R} \frac{\partial t}{\partial r} dz$$

$$q_2 = -\left[k_r \bar{R} \frac{\partial t}{\partial z} + \frac{\partial}{\partial r} \left(k_r \bar{R} \frac{\partial t}{\partial r} dr \right) \right] dz$$

$$q_3 = -k_z \bar{R} \frac{\partial t}{\partial z} dr$$

$$q_4 = -\left[k_z \bar{R} \frac{\partial t}{\partial z} + \frac{\partial}{\partial z} \left(k_z \bar{R} \frac{\partial t}{\partial z} dz \right) \right] dr$$

Heat energy generated in the element (e_g) = $\rho \phi \bar{R} dr dz$.

Heat energy stored in the element (e_s) = $\rho c \bar{R} \frac{\partial t}{\partial \tau} dr dz$.

$$(q_2 - q_1) + (q_4 - q_3) = e_g - e_s$$

$$- \frac{\partial}{\partial r} \left(k_r \bar{R} \frac{\partial t}{\partial r} \right) - \frac{\partial}{\partial z} \left(k_z \bar{R} \frac{\partial t}{\partial z} \right) = \rho \phi \bar{R} - \rho c \bar{R} \frac{\partial t}{\partial \tau}$$

$$\frac{\partial}{\partial r} \left(k_r \bar{R} \frac{\partial t}{\partial r} \right) + \frac{\partial}{\partial z} \left(k_z \bar{R} \frac{\partial t}{\partial z} \right) - \rho c \bar{R} \frac{\partial t}{\partial \tau} + \rho \phi \bar{R} = 0$$

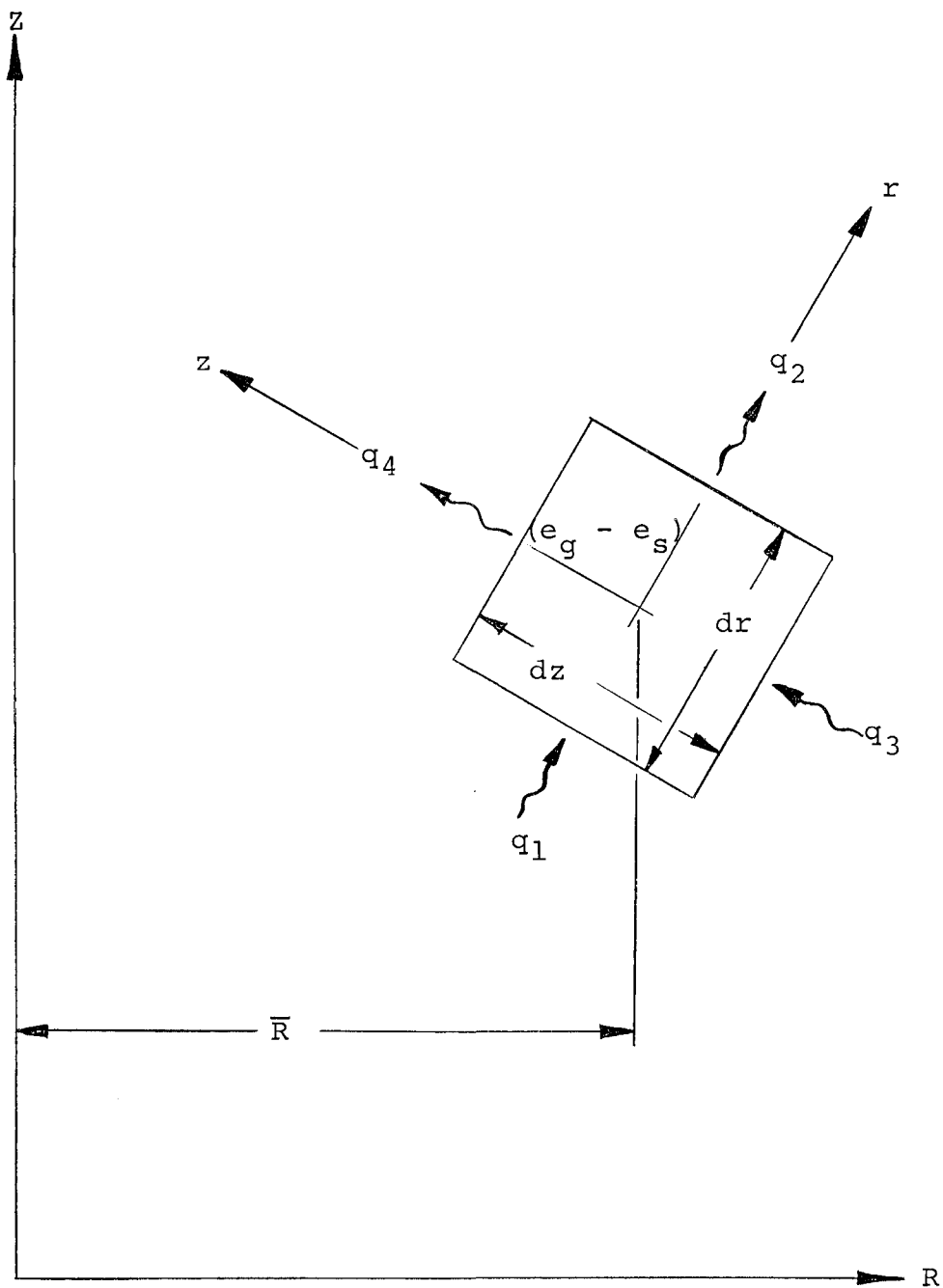


Figure 1. An infinitely small heat conduction element in an axisymmetric body.

II. EULER'S THEOREM APPLIED

From Section I the applicable differential equation is

$$\frac{\partial}{\partial r} \left(k_r \bar{R} \frac{\partial t}{\partial r} \right) + \frac{\partial}{\partial z} \left(k_z \bar{R} \frac{\partial t}{\partial z} \right) - \rho c \bar{R} \frac{\partial t}{\partial \tau} + \rho \phi \bar{R} = 0$$

Euler's Theorem shows that a temperature distribution which minimizes a functional, $x(t)$, is a solution to this equation if,

$$x(t) = \iiint f \left(r, z, t, \frac{\partial t}{\partial r}, \frac{\partial t}{\partial z} \right) dr dz$$

and

$$\frac{\partial}{\partial r} \left(\frac{\partial f}{\partial \left(\frac{\partial t}{\partial r} \right)} \right) + \frac{\partial}{\partial z} \left(\frac{\partial f}{\partial \left(\frac{\partial t}{\partial z} \right)} \right) - \frac{\partial f}{\partial t} = 0, \text{ Reference (3).}$$

x is minimized by setting $\frac{\partial x}{\partial t} = 0$ for the entire body. First

$\frac{\partial x}{\partial t}$ is determined for a typical element within the body.

Define for the element:

$$f^e = \frac{1}{2} k_r \bar{R} \left(\frac{\partial t}{\partial r} \right)^2 + \frac{1}{2} k_z \bar{R} \left(\frac{\partial t}{\partial z} \right)^2 + \rho c \bar{R} \left(\frac{\partial t}{\partial \tau} \right) t - \rho \phi \bar{R} t = 0$$

This definition of f^e satisfies the Euler conditions so long

as $\frac{\partial t}{\partial \tau}$ and $\bar{R}$ are constant. For a typical element the

functional is

$$x^e = \bar{R} \iiint \left(\frac{1}{2} k_r \left(\frac{\partial t}{\partial r} \right)^2 + \frac{1}{2} k_z \left(\frac{\partial t}{\partial z} \right)^2 + \rho c \left(\frac{\partial t}{\partial \tau} \right) t - \rho \phi t \right) dr dz$$

Now an element must be selected and $\frac{\partial x^e}{\partial t}$ determined.

III. THE TRIANGULAR FINITE ELEMENT

Temperature Distribution

A triangular cross-section element with a linear internal temperature distribution is assumed at some location within the continuum (4). (See Figure 2.) The corners of this triangle will be called nodes i, j, and m. Note that the element and system axes are not necessarily parallel.

The temperature at any point within the triangle is expressed by the following equation:

$$t(r,z) = \alpha_1 + \alpha_2 r + \alpha_3 z$$

For nodes i, j, and m the matrix equation is

$$\begin{pmatrix} t_i \\ t_j \\ t_k \end{pmatrix} = \begin{pmatrix} 1 & r_i & z_i \\ 1 & r_j & z_j \\ 1 & r_m & z_m \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

In matrix notation this equation is

$$\{t^e\} = [A^e]\{\alpha^e\}$$

Here e stands for element. Solving for $\{\alpha^e\}$:

$$\{\alpha^e\} = [A^e]^{-1}\{t^e\}$$

By inversion of $[A^e]$:

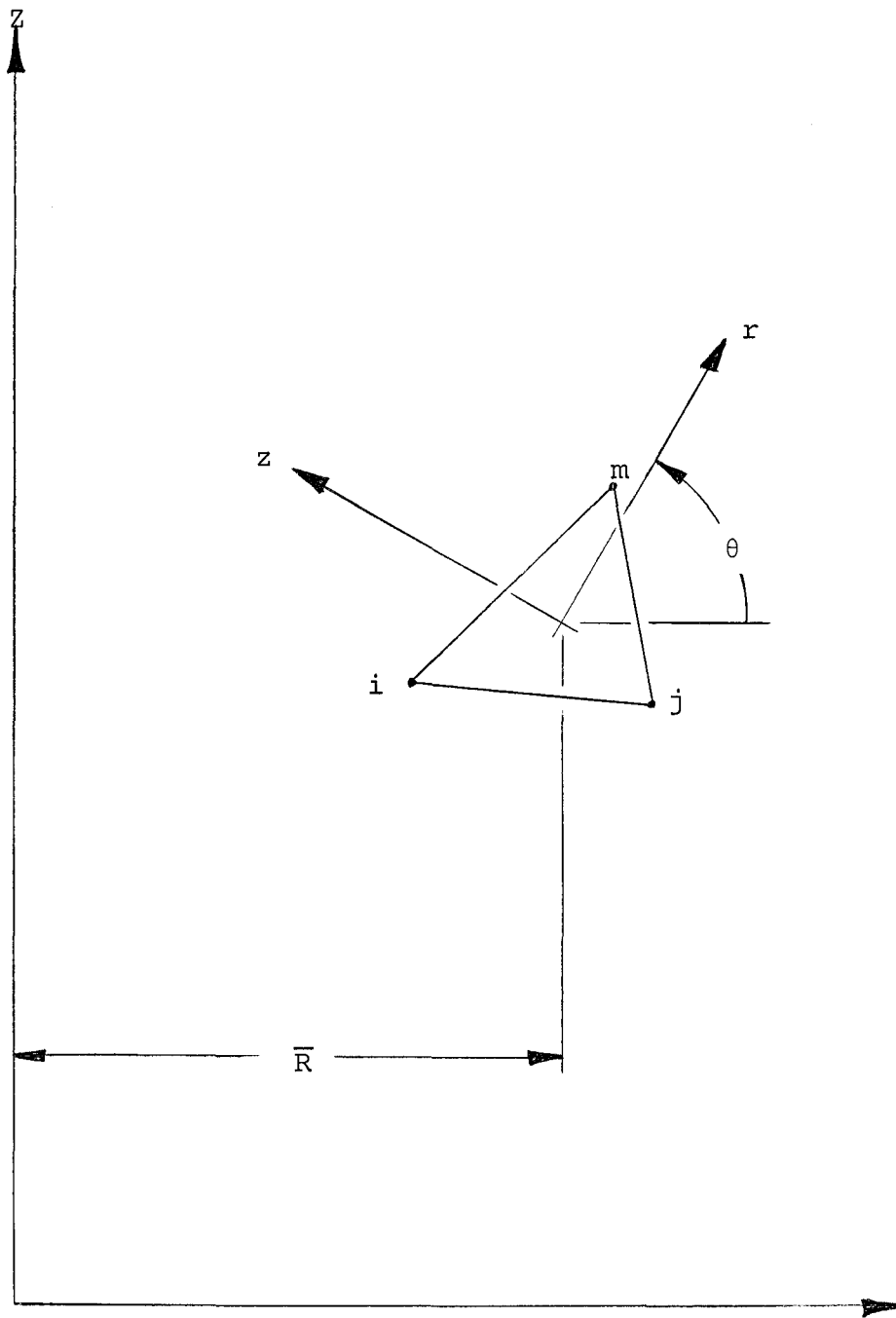


Figure 2. The triangular finite element.

$$[A^e]^{-1} = \frac{1}{2\Delta} \begin{pmatrix} (r_i z_m - z_j r_m) & (r_m z_i - z_m r_i) & (r_i z_j - z_i r_j) \\ (z_j - z_m) & (z_m - z_i) & (z_i - z_j) \\ (r_m - r_j) & (r_i - r_m) & (r_j - r_i) \end{pmatrix}$$

Here, Δ is the cross-sectional area of the triangle and 2Δ is the determinate of $[A^e]$. Thus:

$$2\Delta = r_j z_m + r_i z_j + r_m z_i - r_j z_i - r_m z_j - r_i z_m$$

$$2\Delta = r_i(z_j - z_m) + r_j(z_m - z_i) + r_m(z_i - z_j)$$

Now the matrix equation for $\{\alpha^e\}$ is written

$$\{\alpha^e\} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \frac{1}{2\Delta} \begin{pmatrix} a_i & b_i & c_i \\ a_j & b_j & c_j \\ a_m & b_m & c_m \end{pmatrix} \begin{pmatrix} t_i \\ t_j \\ t_m \end{pmatrix}$$

where, $a_1 = (r_j z_m - z_j r_m)$ for instance. Now the equation for temperature within the triangle can be written in terms of nodal point temperatures as follows:

$$t(r, z) = \frac{1}{2\Delta} [a_i + b_i r + c_i z] t_i + (a_j + b_j r + c_j z) t_j \\ + (a_m + b_m r + c_m z) t_m]$$

In matrix notation this equation is

$$t(r, z) = [N_i, N_j, N_m] \{t^e\}$$

where, $N_i = \frac{1}{2\Delta} (a_i + b_i r + c_i z)$ for instance.

Determination of $(\partial x^e)/(\partial t)$

Recall that the equation for the functional of an element is

$$x^e = \bar{R} \iint \left(\frac{1}{2} k_r \left(\frac{\partial t}{\partial r} \right)^2 + \frac{1}{2} k_z \left(\frac{\partial t}{\partial z} \right)^2 + \rho c \left(\frac{\partial t}{\partial \tau} \right) t - \rho \phi t \right) dr dz$$

For the triangular element:

$$\bar{R} = 1/3(r_i + r_j + r_m)$$

$$\begin{aligned} \frac{\partial x^e}{\partial t} = \bar{R} \iint & \left(\frac{1}{2} k_r \frac{\partial}{\partial t} \left(\left(\frac{\partial t}{\partial r} \right)^2 \right) + \frac{1}{2} k_z \frac{\partial}{\partial t} \left(\left(\frac{\partial t}{\partial z} \right)^2 \right) + \rho c \frac{\partial}{\partial t} \left(\left(\frac{\partial t}{\partial \tau} \right) t \right) \right. \\ & \left. - \rho \phi \frac{\partial}{\partial t} [t] \right) dr dz \end{aligned}$$

Carrying out the differentiations and simplifying:

$$\begin{aligned} \frac{\partial x^e}{\partial t} = \bar{R} \iint & \left(k_r \left(\frac{\partial t}{\partial r} \right) \frac{\partial}{\partial t} \left(\frac{\partial t}{\partial r} \right) + k_z \left(\frac{\partial t}{\partial z} \right) \frac{\partial}{\partial t} \left(\frac{\partial t}{\partial z} \right) + \rho c \left(\frac{\partial t}{\partial \tau} \right) \frac{\partial}{\partial t} (t) \right. \\ & \left. - \rho \phi \frac{\partial}{\partial t} (t) \right) dr dz \end{aligned}$$

Here $\bar{R}$ and $\frac{\partial t}{\partial \tau}$ must be constant. Now the derivatives in this equation must be found.

For $\frac{\partial t}{\partial r}$, $\frac{\partial t}{\partial z}$, and $\frac{\partial t}{\partial \tau}$ the equation for $t(r, z)$ is differentiated.

$$\frac{\partial t}{\partial r} = \frac{\partial}{\partial r} [N_i, N_j, N_m] \{ t^e \} = \frac{1}{2\Delta} [b_i, b_j, b_m] \{ t^e \}$$

$$\frac{\partial t}{\partial z} = \frac{\partial}{\partial z} [N_i, N_j, N_m] \{ t^e \} = \frac{1}{2\Delta} [c_i, c_j, c_m] \{ t^e \}$$

$$\frac{\partial t}{\partial \tau} = \frac{\partial}{\partial \tau} [N_i, N_j, N_m] \{t^e\}$$

Where, $t = t(r, z) = [N_i, N_j, N_m] \{t^e\}$.

Now $\frac{\partial}{\partial t}$ of the applicable terms must be found. $\frac{\partial}{\partial t}$ is defined as

$$\frac{\partial}{\partial t} = \left(\frac{\partial}{\partial t_i}, \frac{\partial}{\partial t_j}, \frac{\partial}{\partial t_m} \right)^T$$

where T stands for transpose. Therefore the following relationships are obtained:

$$\frac{\partial}{\partial t} \left(\frac{\partial t}{\partial r} \right) = \frac{1}{2\Delta} [b_i, b_j, b_m]^T$$

$$\frac{\partial}{\partial t} \left(\frac{\partial t}{\partial r} \right) = \frac{1}{2\Delta} [c_i, c_j, c_m]^T$$

$$\frac{\partial}{\partial t} (t) = [N_i, N_j, N_m]^T$$

The equation for $\frac{\partial x^e}{\partial t}$ can now be written.

$$\begin{aligned} \frac{\partial x^e}{\partial t} = & \bar{R} \iint \left\{ k_r \left(\frac{1}{2\Delta} \right)^2 \begin{bmatrix} b_i \\ b_j \\ b_n \end{bmatrix} [b_i, b_j, b_m] \begin{bmatrix} t_i \\ t_j \\ t_m \end{bmatrix} \right. \\ & + k_z \left(\frac{1}{2\Delta} \right)^2 \begin{bmatrix} c_i \\ c_j \\ c_m \end{bmatrix} [c_i, c_j, c_m] \begin{bmatrix} t_i \\ t_j \\ t_m \end{bmatrix} + \rho c \begin{bmatrix} N_i \\ N_j \\ N_m \end{bmatrix} [N_i, N_j, N_m] \frac{\partial}{\partial \tau} \begin{bmatrix} t_i \\ t_j \\ t_m \end{bmatrix} \\ & \left. - \rho \phi [N_i, N_j, N_m] \right\} dr dz \end{aligned}$$

Recognizing that $\iint dr dz = \Delta$, the cross-sectional area, the first two terms of $\frac{\partial x^e}{\partial t}$ are defined to be $[K^e] \{t^e\}$,

where $[K^e]$ is the element conductivity matrix.

$$[K^e] = \left(\frac{\bar{R}\Delta}{4\Delta^2} \right) \begin{pmatrix} b_i & c_i \\ c_j & c_j \\ b_m & c_m \end{pmatrix} \begin{pmatrix} k_r & 0 \\ 0 & k_z \end{pmatrix} \begin{pmatrix} b_i & b_j & b_m \\ c_i & c_j & c_m \end{pmatrix}$$

The third term of $\frac{\partial x^e}{\partial t}$ is defined to be $[C^e] \frac{\partial}{\partial t} \{t^e\}$, where $[C^e]$ is the element heat capacity matrix.

$$[C^e] = (\bar{R}\rho c) \iint \begin{pmatrix} N_i \\ N_j \\ N_m \end{pmatrix} [N_i, N_j, N_m] drdz$$

For the triangular element this simplifies to

$$[C^e] = \frac{\bar{R}\rho c \Delta}{12} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

This distributed element heat capacity matrix will be discussed later. The last term of $\frac{\partial x^e}{\partial t}$ is defined to be the element heat generation vector, $\{Q^e\}$.

$$\{Q^e\} = (\bar{R}\rho\phi) \iint [N_i, N_j, N_m] drdz$$

For the triangular element this simplifies to

$$\{Q^e\} = \left(\frac{\bar{R}\rho\phi\Delta}{3} \right) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Assembling the matrix terms, the equation for $\frac{\partial x^e}{\partial t}$ is

$$\frac{\partial x^e}{\partial t} = [K^e] \{t^e\} + [C^e] \frac{\partial}{\partial t} \{t^e\} - \{Q^e\}$$

Anisotropic Material

In the preceding derivation, the geometric element properties were assumed to be calculated with reference to the element axes. It is much more convenient to calculate the properties for every element with reference to the system axes. In the equation for $\frac{\partial \mathbf{x}^e}{\partial t}$ only the element conductivity matrix is affected by a conversion from element to system reference axes, because only $[K^e]$ contains directional material properties, k_r and k_z .

Assuming k_r and k_z to be the conductivities along the principle directions for an anisotropic material, the element axes are oriented in these directions. The angle θ is the angle between corresponding system and element axes. (See Figure 2, page 7.)

Recalling from page 9 the definition for $[K^e]\{t^e\}$ can be written:

$$[K^e]\{t^e\} = \bar{R} \iint \left(k_r \left(\frac{\partial t}{\partial r} \right) \frac{\partial}{\partial t} \left(\frac{\partial t}{\partial r} \right) + k_z \left(\frac{\partial t}{\partial z} \right) \frac{\partial}{\partial t} \left(\frac{\partial t}{\partial z} \right) \right) dr dz$$

$\frac{\partial t}{\partial r}$ and $\frac{\partial t}{\partial z}$ are vectors with directional significance. They are formed from geometric properties of the element with reference to the element axes. An equivalent expression for $[K^e]$ must be found with reference to the system axes. In other words expressions for $\frac{\partial t}{\partial R}$ and $\frac{\partial t}{\partial Z}$ must be determined.

$$\begin{bmatrix} \frac{\partial t}{\partial R} \\ \frac{\partial t}{\partial Z} \end{bmatrix} = \begin{bmatrix} (\cos \theta) & (-\sin \theta) \\ (\sin \theta) & (\cos \theta) \end{bmatrix} \begin{bmatrix} \frac{\partial t}{\partial r} \\ \frac{\partial t}{\partial z} \end{bmatrix} = [T] \begin{bmatrix} \frac{\partial t}{\partial r} \\ \frac{\partial t}{\partial z} \end{bmatrix}$$

Here $[T]$ is a transformation matrix. By inspection the determinate of $[T]$ is always one. Thus $[T]$ is a orthogonal matrix and its inverse is equal to its transpose. Therefore:

$$\begin{pmatrix} \frac{\partial t}{\partial r} \\ \frac{\partial t}{\partial z} \end{pmatrix} = [T]^T \begin{pmatrix} \frac{\partial t}{\partial R} \\ \frac{\partial t}{\partial Z} \end{pmatrix} = \begin{pmatrix} (\cos \theta) & (\sin \theta) \\ (-\sin \theta) & (\cos \theta) \end{pmatrix} \begin{pmatrix} \frac{\partial t}{\partial R} \\ \frac{\partial t}{\partial Z} \end{pmatrix}$$

Rearranging terms in $[K^e]\{t^e\}$ the following relationship is obtained:

$$[K^e]\{t^e\} = \bar{R} \iint \left[\left(\frac{\partial}{\partial t} \left(\frac{\partial t}{\partial r} \right) \right) (k_r) \left(\frac{\partial t}{\partial r} \right) + \left(\frac{\partial}{\partial t} \left(\frac{\partial t}{\partial z} \right) \right) (k_z) \left(\frac{\partial t}{\partial z} \right) \right] dr dz$$

In matrix form:

$$[K^e]\{t^e\} = \bar{R} \iint \frac{\partial}{\partial t} \begin{pmatrix} \frac{\partial t}{\partial r} & \frac{\partial t}{\partial z} \end{pmatrix} \begin{pmatrix} k_r & 0 \\ 0 & k_z \end{pmatrix} \begin{pmatrix} \frac{\partial t}{\partial r} \\ \frac{\partial t}{\partial z} \end{pmatrix} dr dz$$

Substituting for $\frac{\partial t}{\partial r}$ and $\frac{\partial t}{\partial z}$:

$$[K^e]\{t^e\} = \bar{R} \iint \frac{\partial}{\partial t} \begin{pmatrix} \frac{\partial t}{\partial R} & \frac{\partial t}{\partial Z} \end{pmatrix} [T] \begin{pmatrix} k_r & 0 \\ 0 & k_z \end{pmatrix} [T]^T \begin{pmatrix} \frac{\partial t}{\partial R} \\ \frac{\partial t}{\partial Z} \end{pmatrix} dr dz$$

Recall from Page 9:

$$\frac{\partial t}{\partial r} = \frac{1}{2\Delta} [b_i, b_j, b_m] \{t^e\}$$

$$\frac{\partial t}{\partial z} = \frac{1}{2\Delta} [c_i, c_j, c_m] \{t^e\}$$

Similarly,

$$\frac{\partial t}{\partial R} = \frac{1}{2\Delta} [b_i, b_j, b_m] \{t^e\}$$

and

$$\frac{\partial t}{\partial Z} = \frac{1}{2\Delta} [c_i, c_j, c_m] \{t^e\}$$

where the geometric factors b , c , and Δ are with reference to the system axes, R and Z . The relationship for

$$\frac{\partial}{\partial t} \left(\frac{\partial t}{\partial R}, \frac{\partial t}{\partial Z} \right) \text{ is:}$$

$$\frac{\partial}{\partial t} \left(\frac{\partial}{\partial R}, \frac{\partial}{\partial Z} \right) = \frac{\partial}{\partial t} \left(\frac{1}{2\Delta} \right) \begin{pmatrix} b_i & c_i \\ b_j & c_j \\ b_m & c_m \end{pmatrix} [t_i, t_j, t_m]$$

$$\frac{\partial}{\partial t} \left(\frac{\partial t}{\partial R}, \frac{\partial t}{\partial Z} \right) = \frac{1}{2\Delta} \begin{pmatrix} b_i & c_i \\ b_j & c_j \\ b_m & c_m \end{pmatrix}$$

The complete equation for $[K^e] \{t^e\}$ is:

$$[K^e] \{t^e\} = \bar{R} \iint \left(\left(\frac{1}{2\Delta} \right) \begin{pmatrix} b_i & c_i \\ b_j & c_j \\ b_m & c_m \end{pmatrix} [T] \begin{pmatrix} k_r & 0 \\ 0 & k_z \end{pmatrix} [T]^T \right. \\ \left. \left(\frac{1}{2\Delta} \right) \begin{pmatrix} b_i & b_j & b_m \\ c_i & c_j & c_m \end{pmatrix} \{t^e\} \right) dr dz$$

$$[K^e] \{t^e\} = \frac{\bar{R}}{4\Delta} [B]^T [T] [k] [T]^T [B] \{t^e\}$$

Define $[T] [k] [T]^T$ as:

$$\begin{aligned}
 \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} &= \begin{pmatrix} (\cos \theta) (-\sin \theta) \\ (\sin \theta) (\cos \theta) \end{pmatrix} \begin{pmatrix} k_r & 0 \\ 0 & k_z \end{pmatrix} \begin{pmatrix} (\cos \theta) (\sin \theta) \\ (-\sin \theta) (\cos \theta) \end{pmatrix} \\
 &= \begin{pmatrix} [(\cos^2 \theta) k_r + (\sin^2 \theta) k_z] & [(\cos \theta) (\sin \theta) (k_r - k_z)] \\ [(\cos \theta) (\sin \theta) (k_r - k_z)] & [(\sin^2 \theta) k_r + (\cos^2 \theta) k_z] \end{pmatrix}
 \end{aligned}$$

The general term in $[K^e]$ is

$$K_{uv}^e = \left(\frac{\bar{R}}{4\Delta} \right) [k_{11} (b_u b_v) + k_{12} (b_u c_v + b_v c_u) + k_{22} (c_u c_v)]$$

IV. CONVECTION AND RADIATION

BOUNDARY CONDITIONS

The functional, x , for the entire body must contain terms for the heat transfer at the surface by convection and radiation. x^s is defined as the functional for heat transferred to an element bordering the surface by convection or radiation. (See Figure 3.) The following definitions of f^s and x^s will satisfy these boundary conditions of $q_n =$

$$h(t_f - t) + q_r = - \left(k_r \frac{\partial t}{\partial r} \cos \psi + k_z \frac{\partial t}{\partial z} \cos \beta \right):$$

$$f^s = -R_s (q_r t + h t_f t - \frac{1}{2} h t^2)$$

$$x^s = -R_s \int (q_r + h t_f) (t) ds + R_s \int \frac{1}{2} h t^2 ds$$

Here q_n is the total heat flux to the surface of the boundary element. q_r is the radiation heat flux and $h(t_f - t)$ is the convection heat flux. This indicates that the basic convection formulation will be included in the

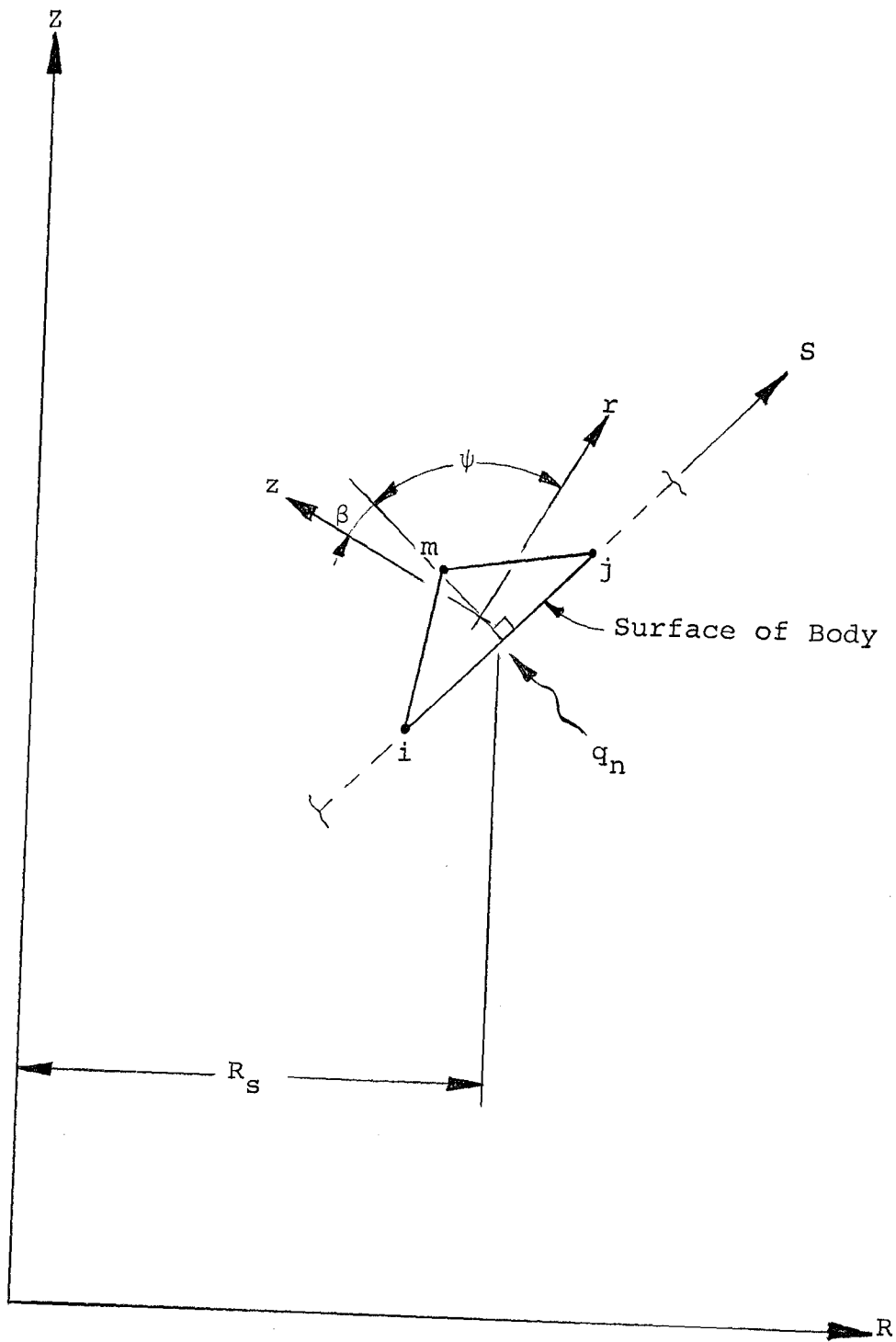


Figure 3. Surface heat transfer.

solution procedure, but radiation will be included on a heat flux basis only. The result of this approach is a set of linear system equations which can be solved directly without iteration.

Now find $\frac{\partial x^s}{\partial t}$. Since the element is assumed to have a linear internal temperature distribution the surface temperature between nodes i and j is

$$t = t_i + \left(\frac{t_j - t_i}{L} \right) s$$

regardless of element or system axis orientation. Now:

$$\frac{\partial x^s}{\partial t} = \left(\frac{\partial}{\partial t_i}, \frac{\partial}{\partial t_j} \right)^T x^s$$

$$\frac{\partial x^s}{\partial t_i} = R_s (q_r + h t_f) \int \left(\frac{s}{L} - 1 \right) ds + R_s (h) \int (t) \frac{\partial}{\partial t_i} (t) ds$$

$$\begin{aligned} \frac{\partial x^s}{\partial t_i} = & R_s (q_r + h t_f) \int \left(\frac{s}{L} - 1 \right) ds \\ & + R_s (h) \int \left(t_i + \left(\frac{t_j - t_i}{L} \right) s \right) \left(1 - \frac{s}{L} \right) ds \end{aligned}$$

$$\begin{aligned} \frac{\partial x^s}{\partial t_i} = & R_s (q_r + h t_f) \left(\frac{-L}{2} \right) + R_s (h t_i) \left(\frac{L}{2} \right) \\ & + R_s \left(\frac{t_j - t_i}{L} \right) \left(\frac{L^2}{2} - \frac{L^3}{3L} \right) \end{aligned}$$

$$\frac{\partial x^s}{\partial t_i} = \frac{-q_r L R_s}{2} - \frac{h t_f L R_s}{2} + h L R_s \left(\frac{t_i}{3} + \frac{t_j}{6} \right)$$

Similarly:

$$\frac{\partial x^s}{\partial t_j} = \frac{-q_r^{LR_s}}{2} - \frac{ht_f^{LR_s}}{2} + hLR_s \left(\frac{t_j}{3} + \frac{t_i}{6} \right)$$

Writing these equations in matrix form:

$$\frac{\partial x^s}{\partial t} = \frac{-q_r^{LR_s}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{ht_f^{LR_s}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{hLR_s}{6} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} t_i \\ t_j \end{pmatrix}$$

The first term represents radiation heat transfer to the surface. The last two terms represent convection heat transfer to the surface. L and R_s are defined as follows:

$$L = \sqrt{(R_i - R_j)^2 + (Z_i - Z_j)^2}, \quad R_s = \frac{R_i + R_j}{2}$$

V. SYSTEM EQUATIONS

The functional for the entire body is $x = \{x^e\} + \{x^s\}$.

To minimize x with respect to t , $\frac{\partial x}{\partial t}$ must be set equal to zero.

$$\frac{\partial x}{\partial t} = \sum \frac{\partial x^e}{\partial t} + \sum \frac{\partial x^s}{\partial t} = 0$$

$$\sum \frac{\partial x^e}{\partial t} = \sum ([K^e]\{t^e\} + [C^e]\frac{\partial}{\partial \tau}\{t^e\} - \{Q^e\})$$

$$= [K]\{t\} + [C]\frac{\partial}{\partial t}\{t\} - \{Q\}$$

$$\sum \frac{\partial x^s}{\partial t} = \sum \left(\frac{-q_r^{LR_s}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{ht_f^{LR_s}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{hLR_s}{6} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \{t\} \right)$$

$$\sum \frac{\partial \mathbf{x}^s}{\partial t} = -\{\mathbf{Q}_r\} - [\mathbf{H}]\{\mathbf{t}_f\} + [\mathbf{H}]\{\mathbf{t}\}$$

$$\frac{\partial \mathbf{x}}{\partial t} = 0 = [\mathbf{K}]\{\mathbf{t}\} + [\mathbf{C}]\frac{\partial}{\partial \tau}\{\mathbf{t}\} - \{\mathbf{Q}\} - \{\mathbf{Q}_r\} - [\mathbf{H}]\{\mathbf{t}_f\} + [\mathbf{H}]\{\mathbf{t}\}$$

This is the system matrix equation. It must be rearranged according to the solution technique used. Recall that $\frac{\partial t}{\partial \tau}$ was assumed constant. For each succeeding solution $\frac{\partial}{\partial \tau}\{\mathbf{t}\}$ can be defined as

$$\frac{\partial}{\partial \tau}\{\mathbf{t}\} = \frac{\{\mathbf{t} - \mathbf{t}_o\}}{\delta \tau} = \frac{\{\mathbf{t}\} - \{\mathbf{t}_o\}}{\delta \tau}$$

where $\{\mathbf{t}_o\}$ is the temperature vector when the time step, $\delta \tau$, begins. The system matrix equation is solved for the temperature vector, $\{\mathbf{t}\}$. This $\{\mathbf{t}\}$ then becomes $\{\mathbf{t}_o\}$ for the next time step. This is referred to as the pure implicit solution technique (12).

The system matrix equation will now be rearranged into its working form.

$$[\mathbf{K}]\{\mathbf{t}\} + [\mathbf{C}]\left(\frac{1}{\delta \tau}\right)\{\mathbf{t}\} - [\mathbf{C}]\left(\frac{1}{\delta \tau}\right)\{\mathbf{t}_o\} - \{\mathbf{Q}\} - \{\mathbf{Q}_r\} - [\mathbf{H}]\{\mathbf{t}_f\} + [\mathbf{H}]\{\mathbf{t}\} = 0$$

$$[\mathbf{K}] + \left[\frac{\mathbf{C}}{\delta \tau}\right] + [\mathbf{H}]\{\mathbf{t}\} = \left[\frac{\mathbf{C}}{\delta \tau}\right]\{\mathbf{t}_o\} + \{\mathbf{Q}\} + \{\mathbf{Q}_r\} + [\mathbf{H}]\{\mathbf{t}_f\}$$

The system "stiffness" matrix, $[\mathbf{K}^*]$, and the system load vector, $\{\mathbf{Q}^*\}$, are defined so that

$$[\mathbf{K}^*]\{\mathbf{t}\} = \{\mathbf{Q}^*\}$$

where,

$$[K^*] = [K] + \left(\frac{C}{\delta \tau} \right) + [H]$$

and

$$\{Q^*\} = \left(\frac{C}{\delta \tau} \right) \{t_o\} + \{Q\} + \{Q_r\} + [H]\{t_f\}$$

VI. SPECIFIED TEMPERATURE BOUNDARY CONDITION

Temperatures at some node points within the body may be known. This necessitates modifying the system matrix equation so that the known temperature is preset prior to solution. A one is placed on the diagonal position of $[K^*]$ which corresponds to the node number where the temperature is known. The remaining terms in that row of $[K^*]$ are given values of zero. The specified temperature is placed in $\{Q^*\}$ at the proper location. Finally the specified temperature value is multiplied by the terms in the appropriate column of $[K^*]$. The results are added to $\{Q^*\}$, and the column terms are set equal to zero.

FORMULATION USED IN TRAX

In this chapter the derived element and system equations are formulated for use in computer program TRAX. Some equations are formulated as derived. Others are modified to achieve desired objectives.

I. THE QUADRILATERAL ELEMENT AND LUMPED HEAT CAPACITY MATRIX

Element data is input easier if quadrilateral elements are used. For TRAX a quadrilateral element, formed from two triangular elements, is defined. (See Figure 4.) The dividing line between the two triangles is along the shortest diagonal of the quadrilateral. The areas and radii of the two triangles are Δ_1 , Δ_2 and $\bar{R}_1$, $\bar{R}_2$. For the quadrilateral the area and radius are defined as follows:

$$\Delta_Q = \Delta_1 + \Delta_2, \quad R_Q = \frac{\Delta_1 \bar{R}_1 + \Delta_2 \bar{R}_2}{\Delta_Q}$$

Ease of input is not the only reason for using a quadrilateral element. Triangular elements with distributed heat capacity matrices cause unacceptable difficulties. These difficulties arise from the linear temperature distribution assumption within the element. In attempting to approximate the actual curve of $\frac{\partial t}{\partial r}$ for instance, this

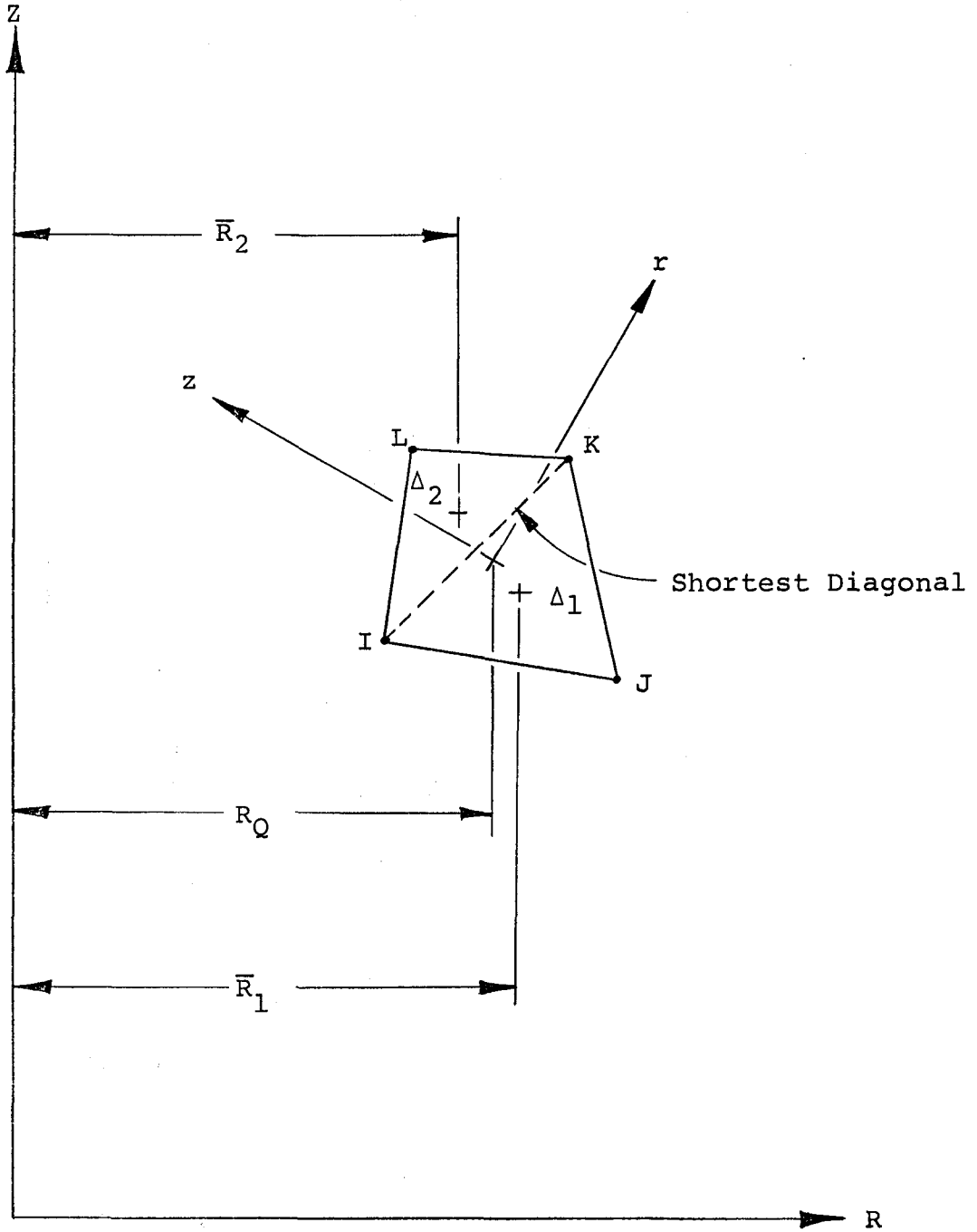


Figure 4. The quadrilateral finite element.

assumption causes a progressive "undershooting" or "overshooting" of node temperatures in the r direction. Also surface temperature fluctuations are calculated where none exist. Two alternatives are using a lumped heat capacity matrix with the linear element or using a distributed heat capacity matrix with a higher order element. Here the lumped heat capacity matrix will be used. For the triangle the terms of the distributed matrix are simply lumped on the diagonal giving:

$$[C^e] = \frac{\bar{R}\rho c\Delta}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This eliminates the undershooting but it does not help the fluctuations. Essentially an even distribution of heat capacity terms is required. The quadrilateral element heat capacity matrix is formulated to facilitate this even distribution. The result is an element heat capacity matrix similar to that used in finite difference programs. For the quadrilateral,

$$[C^e] = \frac{R_Q\rho c\Delta_Q}{4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Quadrilateral elements should be used wherever possible in an analytical model to eliminate surface temperature fluctuations. This quadrilateral element formed from two triangular

elements, is less costly in computing time and storage space than the four triangle element used in References (1), (2) and (7) with little, if any, difference in accuracy. The essential function of either is to form an evenly distributed lumped heat capacity matrix for the system.

For this quadrilateral element the conductivity matrices of its two component triangles are added to the system matrix one at a time. The general term in the conductivity matrix for each triangle is:

$$K_{uv}^e = \left(\frac{R_Q}{4\Delta} \right) [k_{11}(b_u b_v) + k_{12}(b_u c_v + b_v c_u) + k_{22}(c_u c_v)]$$

II. THE TIME PARAMETERS AND CHANGES IN BOUNDARY CONDITIONS AND MATERIAL PROPERTIES

A time step, $\delta\tau$, must be defined for each solution during a transient. The system matrix equation is:

$$[K^*]\{t\} = \{Q^*\}$$

$[K^*]$ and $\{Q^*\}$ are formulated for TRAX as follows:

$$[K^*] = [K] + \left(\frac{C}{\delta\tau} \right) + [H]$$

$$\{Q^*\} = \left(\frac{C}{\delta\tau} \right)\{t_o\} + \{Q_r\} + [H]\{t_f\}$$

Note that the heat generation vector is left out of the definition for $\{Q^*\}$. The reason for this is simply a lack

of demand for the heat generation capability. If a need arises this capability can be readily added to TRAX.

At each time step in the solution $\{t_o\}$, $\{t_f\}$, and $\{Q_r\}$ may be changed without affecting $[K^*]$. $\{t_o\}$ is changed routinely as a part of the solution procedure. At each time step, $\{t_o\}$ is the solution for the preceding time step. The capability to change $\{t_f\}$ at every time step is desirable because of the frequency of primarily convection boundary condition analyses. Radiation heat transfer is a function of the fourth power of absolute temperatures. Most problems will not require adjustments in $\{Q_r\}$ at every time step. Therefore this capability will not be included. Again, it could be readily added if desired.

In TRAX the total time of the transient is broken into time intervals of variable length. Each time interval is then divided into an even number of time steps. At the beginning of each interval all components of $[K^*]$ and $\{Q^*\}$ may change, except geometric properties. Geometric properties are calculated once and stored for further use. Material properties are interpolated from input tabular data according to the temperature of the element at the beginning of the interval. Time parameters and local boundary condition data are input for each interval. (See Chapter V, pages 59-63.) The variation of t_f for the interval is determined by inputting constants A, B and C for the equation:

$$t_f = A + B(N) + C(N)^2$$

Here N is the number of the time step in the interval. The other data remains constant throughout the interval.

A Gauss-Cholesky solution routine is used. The $[K^*]$ matrix is factored at the first time step in an interval. Since $[K^*]$ does not change in an interval, only the forward and backward sweep is required for each succeeding time step after the first one. This procedure is economical in terms of computer time.

III. PLANE BODIES AND STEADY-STATE ANALYSIS

Some minor modifications of the element and system equations are required for analysis of plane bodies and steady-state cases. These modifications are really simplifications of the general case for transient analysis of axisymmetric bodies.

For plane bodies the element and surface radii are all assigned a value of one.

$$\bar{R} = R_Q = R_S = 1 \text{ (for plane bodies)}$$

The steady-state case is assumed to be the solution for a particular type of time interval. For this interval the heat capacity terms are eliminated from the system matrix equation.

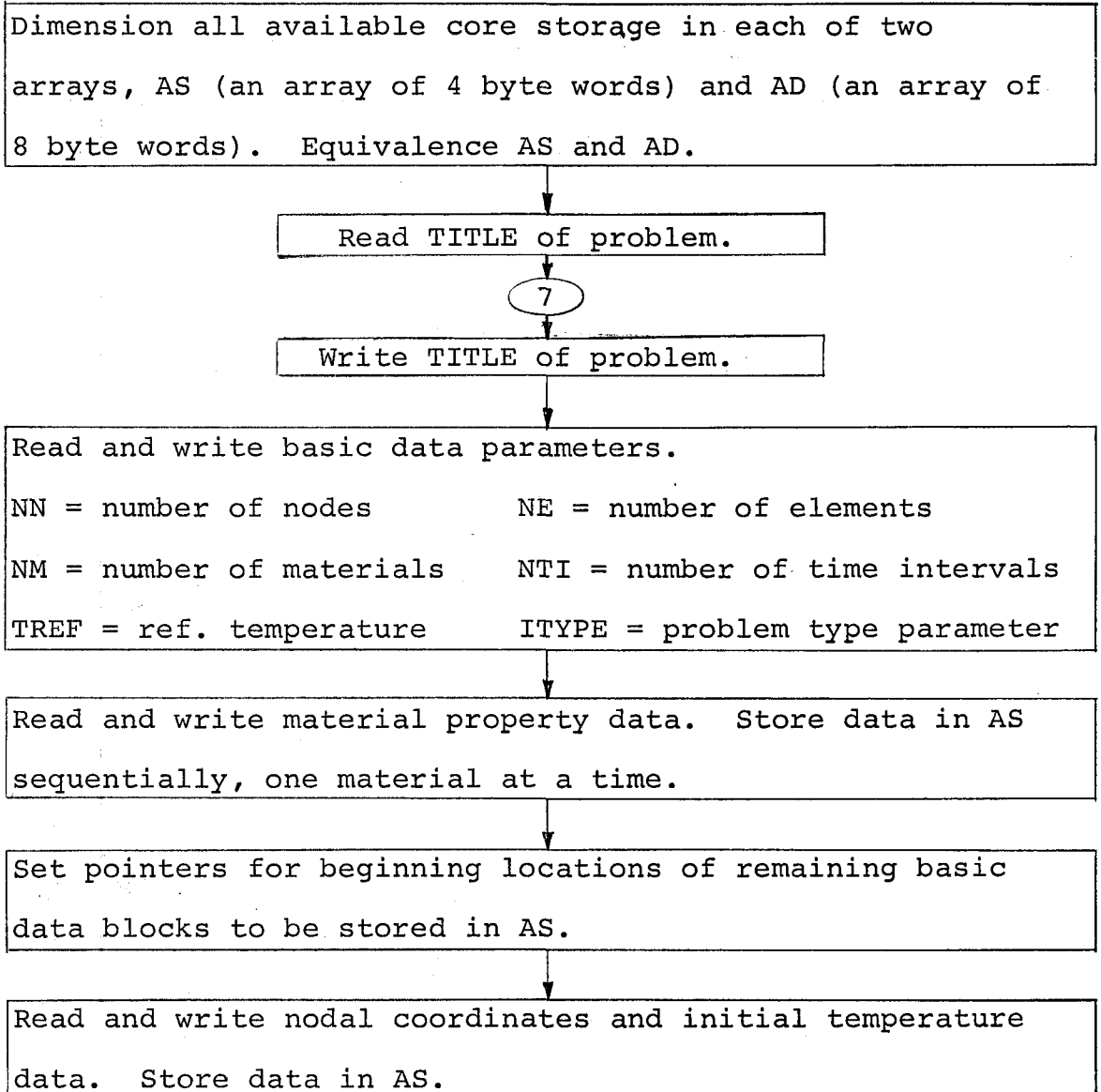
$$[K^*] = [K] + [H]$$

$$\{Q^*\} = \{Q_r\} + [H]\{t_f\} \text{ (for steady-state)}$$

THE PROGRAM

In this chapter a basic outline of program TRAX is presented in block diagram form. Then a complete program listing is given.

I. BLOCK DIAGRAM



Read and write element connectivity, material number,
angle of anisotropic material axes with system axes.
Store data in AS. Calculate element geometric properties.
Store data in AS.

Write all basic data on file 1. Rewind file 1.

Set TIME = 0., LITE = 0

DO 190 I = 1, NTI

Read and write interval-boundary data parameters.
TSS = time step size NTS = number of time steps
INO = print interval NCBC = number of convection cards
NRBC = number of radiation cards
NTBC = number of temperature cards
MPC = material property change indicator

Set pointers for beginning locations of interval-boundary
data blocks and matrices for storage in AS.

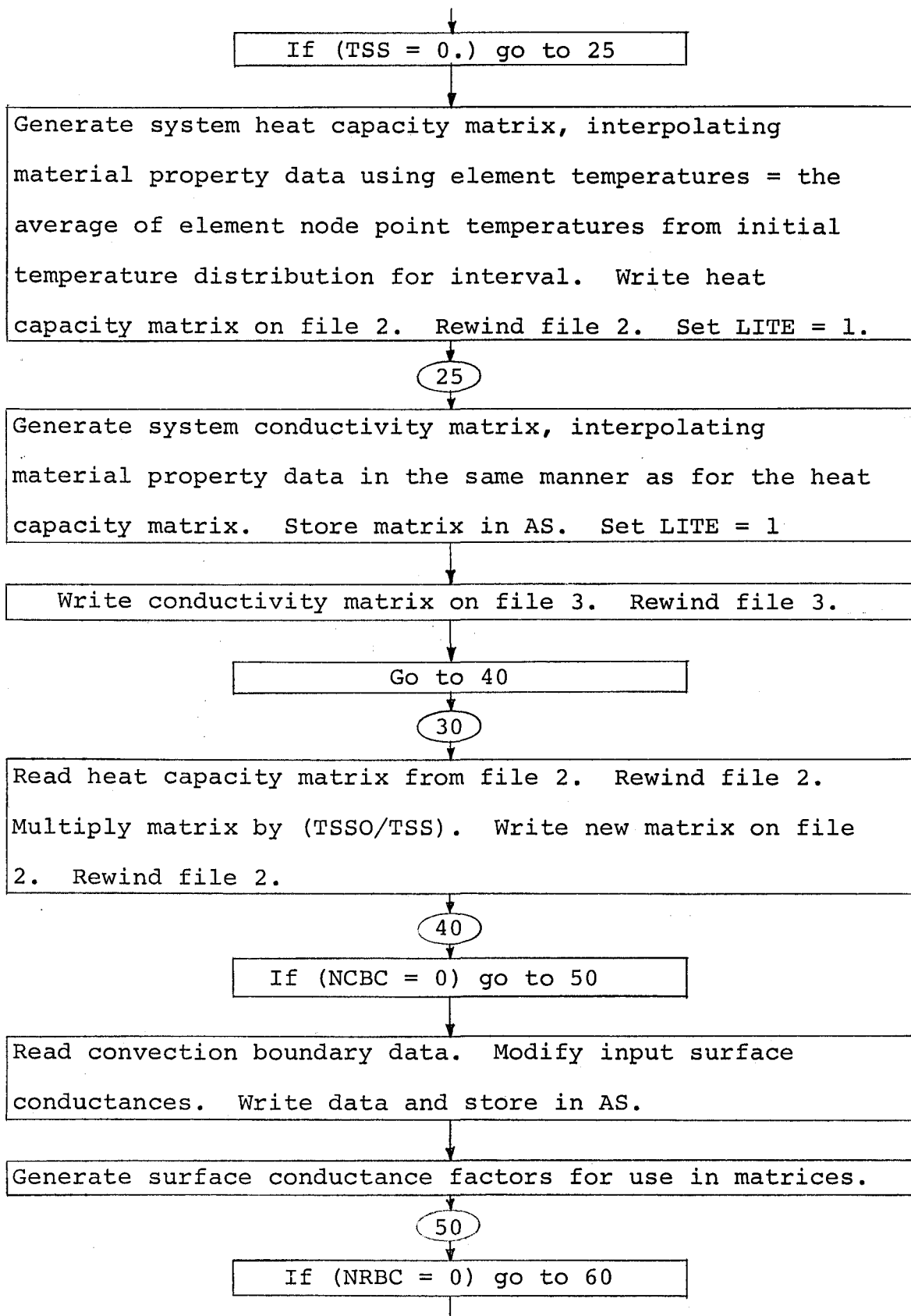
If (LITE = 0) go to 15

Read basic data from file 1 into AS. Rewind file 1.

If (MPC $\neq$ 0) go to 30

Set initial temperature distribution for interval = the
distribution at end of preceding time interval.

15



Read and write radiation boundary data. Generate heat flux factors for use in radiation load vector. Store data in AS.

60

Set pointers for beginning locations of interval-boundary data blocks and matrices for storage in AD.

Take convection boundary data from AS and store it in AD.

If (NRBC = 0) go to 80

Assemble radiation load vector. Store it in AD.

80

If (NTBC = 0) go to 100

Read and write specified temperature boundary data. Store in AD.

100

Read system conductivity matrix from file 3 into AD.
Rewind file 3.

If (NCBC = 0) go to 105

Add surface conductance factors to system matrix in AD.

105

If (TSS = 0.) go to 115

Read heat capacity matrix from file 2. Rewind file 2.
Add heat capacity matrix to system matrix in AD.

115

If (NTBC = 0.) go to 120

Modify system matrix and radiation load vector for specified temperature. (Diagonal matrix terms are not modified. Specified temperatures are not placed in load vector. These are done later.)

120

Normalize system matrix multiplying by BIGD, the inverse of the largest diagonal term. Place a 1 on the diagonal at specified temperature locations.

Factor system matrix. (This is the first part of the Gauss-Cholesky solution procedure.)

DO 170 N = 1, NTS

If (TSS = 0.) go to 140

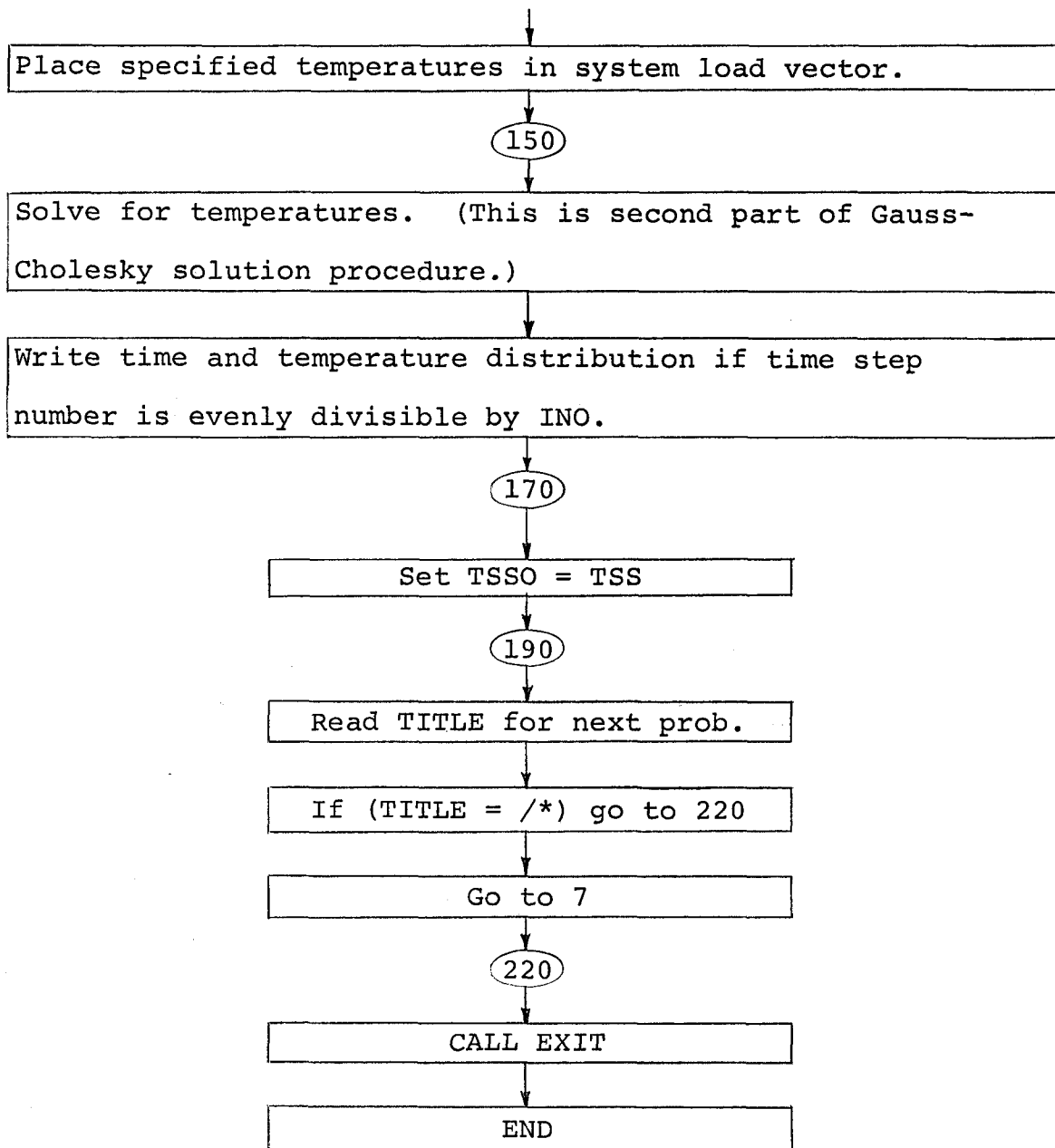
Set TIME = TIME + TSS

Multiply heat capacity matrix by preceding time step temperature distribution thus forming the heat storage load vector. Store it in AD.

140

Form system load vector. Include appropriate terms for each boundary condition. Multiply vector by BIGD. Store vector in AD.

If (NTBC = 0.) go to 150



Comment

All data in AS is stored in 4 byte words, integer or real. The system matrix, heat capacity matrix, load vectors, and temperature distribution are stored in AD in 8 byte real words. Other data in AD is stored in 4 byte words.

II. TRAX LISTING

TRAX - MAIN - 1/24/73
J K ROCHELLE

```

COMMON /PARAM/ NSM(101)
DIMENSION TITLE(20)
REAL*8 AD(8750),BIGD
REAL*4 AS(17500)
EQUIVALENCE (AD(1),AS(1))
KS = 17500
KD = 8750
DO 5 I=1,KS
5 AS(I) = 0.
READ (5,1000) TITLE
7 WRITE (6,1001) TITLE
READ (5,1002) NN,NE,NM,NTI,TREF,ITYPE
WRITE (6,1003) NN,NE,NM,NTI,TREF
IF (ITYPE.NE.0) WRITE(6,1012)
SINGLE PRECISION POINTERS INDICATE BEGINNING OF DATA STORED
NS1 = MATERIAL PROPERTIES
NS2 = NODAL COORDINATES,INITIAL TEMPERATURE
NS3 = ELEMENT CONNECTIVITY,MATERIAL NUMBER,AND ANGLE
NS4 = SYSTEM HEAT CAPACITY,CONDUCTIVITY,AND CONDUCTANCE MATRIX
NS5 = CONVECTION BOUNDARY DATA
NS6 = RADIATION BOUNDARY DATA
NS1 = NN*2 + 1
NSM(1) = NS1
WRITE (6,1004)
DO 10 I=1,NM
READ (5,1005) NTM
NSI = NSM(I)
CALL RPSMP (AS(NSI),NTM,I)
10 NSM(I+1) = NSM(I) + 5*NTM
NS2 = NSM(NM+1)
NE1 = NS2 - 1
NS3 = 3*NN + NS2
NE2 = NS3 - 1
NS3I = 12*NE + NS3
NS4 = 17*NE + NS3
NE3 = NS4 - 1
CALL RPSNC (AS(NS2),AD(1),NN,TREF)
CALL RPSEC (AS(NS2),AS(NS3),AS(NS3I),NN,NE,MBAND,ITYPE)
WRITE (1) (AS(N),N=NS1,NE3)
REWIND 1
LITE = 0
TIME = 0.
DO 190 IN = 1,NTI
READ (5,1006) TSS,NTS,INO,NCBC,NRBC,NTBC,MPC
IF (INO.EQ.0) INO = 1
WRITE (6,1007) IN,TSS,NTS,INO,NCBC,NRBC,NTBC,MPC
NS5 = NN*MBAND + NS4
NE4 = NS5 - 1
NS5I = 2*NCBC + NS5

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```

NS6 = 6*NCBC + NS5
NS6I = 2*NRBC + NS6
NS7 = 3*NRBC + NS6
NE6 = NS7 - 1
IF (NS7.GT.KS) GO TO 200
IF (LITE.EQ.0) GO TO 15
READ (1) (AS(N),N=NS1,NE3)
REWIND 1
IF (MPC.NE.0) GO TO 30
CALL EQT (AD(1),AS(NS2),NN)
15 CONTINUE
DO 20 I=NS4,NE4
20 AS(I) = 0.
NMPR = NS2 - NS1
IF (TSS.EQ.0) GO TO 25
CALL HTCAP2 (AS(NS1),AS(NS2),AS(NS3),AS(NS3I),AS(NS4),NMPR,NN,
1MBAND,NE,LITE,TSS,NS1)
25 CONTINUE
CALL CK (AS(NS1),AS(NS2),AS(NS3),AS(NS3I),AS(NS4),NMPR,NN,MBAND,
1NE,LITE,TSS,NS1)
WRITE (3) (AS(N),N=NS4,NE4)
REWIND 3
GO TO 40
30 CONTINUE
IF (TSS.EQ.0.) GO TO 37
NL = NS4 + NN - 1
READ (2) (AS(N),N=NS4,NL)
REWIND 2
DO 35 N=NS4,NL
35 AS(N) = AS(N) * TSS0 / TSS
WRITE (2) (AS(N),N=NS4,NL)
REWIND 2
37 CONTINUE
READ (3) (AS(N),N=NS4,NE4)
REWIND 3
40 CONTINUE
IF (NCBC.EQ.0) GO TO 50
CALL RPSCBC (AS(NS5),AS(NS5I),AD(1),NCBC,NN)
CALL CKH (AS(NS2),AS(NS5),AS(NS5I),NCBC,NN,ITYPE)
50 CONTINUE
IF (NRBC.EQ.0) GO TO 60
CALL RPSRBC (AS(NS6),AS(NS6I),AS(NS2),NRBC,NN,ITYPE)
DOUBLE PRECISION ARRAY POINTERS INDICATE BEGINNING OF DATA STORED
ND0 = TEMPERATURE VECTOR
ND1 = CONVECTION BOUNDARY DATA
ND2 = RADIATION AND SPECIFIED TEMPERATURE MODIFICATION VECTOR
ND3 = SPECIFIED TEMPERATURE DATA
ND4 = SYSTEM MATRIX
ND5 = FINAL LOAD VECTOR AND HEAT CAPACITY VECTOR
ND6 = HEAT CAPACITY MATRIX
60 ND0 = 1
ND1 = NN + ND0
ND1I = NCBC + ND1
ND2 = 3*NCBC + ND1
ND3 = NN + ND2

```

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      IF (NRBC.NE.0) GO TO 65
      IF (NTBC.EQ.0) ND3 = ND2
65  ND3I = NTBC/2 + ND3 + 1
      ND4 = NTBC + ND3 + 2
      ND5 = NN*MBAND + ND4
      ND6 = NN + ND5
      NL5 = ND6 - 1
      ND7 = NN + ND6
      IF (ND7.GT.KD) GO TO 210
      IF (NCBC.EQ.0) GO TO 70
      CALL SCBD (AD(ND1),AD(ND1I),AS(NS5),AS(NS5I),NCBC)
70  CONTINUE
      IF (NRBC.EQ.0) GO TO 80
      CALL QRV (AD(ND2),AS(NS6),AS(NS6I),NRBC,NN)
80  CONTINUE
      DO 90 N=ND3,ND7
90  AD(N) = 0.
      IF (NTBC.EQ.0) GO TO 100
      CALL RPSTBC (AD(ND3),AD(ND3I),NTBC)
100 CONTINUE
      LSM = ND5 - ND4
      CALL RSM (AD(ND4),LSM,3)
      IF (NCBC.EQ.0) GO TO 105
      CALL ADH (AD(ND4),AD(ND1),AD(ND1I),NCBC,NN,MBAND)
105 CONTINUE
      IF (TSS.EQ.0.) GO TO 115
      LSM = ND7 - ND6
      CALL RSM (AD(ND6),LSM,2)
C    ADD HEAT CAPACITY MATRIX TO SYSTEM K+H MATRIX.
      DO 110 N=1,NN
      NK = ND4 + N - 1
      NC = ND6 + N - 1
110  AD(NK) = AD(NK) + AD(NC)
115 CONTINUE
      IF (NTBC.EQ.0) GO TO 120
      CALL MOD1 (AD(ND4),AD(ND2),AD(ND3),AD(ND3I),NN,MBAND,NTBC,NRBC)
120 CONTINUE
      CALL NORM1 (AD(ND4),AD(ND3),NN,MBAND,BIGD,NTBC)
      CALL FACTOR (NN,MBAND,AD(ND4))
      DO 170 N=1,NTS
      DO 130 II=ND5,NL5
130  AD(II) = 0.
      IF (TSS.EQ.0.) TIME = 0.
      IF (TSS.EQ.0) GO TO 140
      TIME = TIME + TSS
      CALL QOV2 (AD(ND5),AD(ND6),AD(ND0),NN)
140 CONTINUE
      CALL QLV (AD(ND5),AD(ND2),AD(ND1),AD(ND1I),NN,TSS,NCBC,NRBC,NTBC,
1  BIGD,N)
      IF (NTBC.EQ.0) GO TO 150
      CALL MOD2 (AD(ND5),AD(ND3),AD(ND3I),NN,NTBC)
150 CONTINUE
      CALL SOLVE (NN,MBAND,AD(ND4),AD(ND5),AD(ND0))
      AN = N
      AINO = INO

```

```

      BID = N/INO
      AID = AN/AINO
      IF (BID.NE.AID) GO TO 170
      WRITE (6,1010) TIME
160  WRITE (6,1011) (M,AD(M),M=1,NN)
170  CONTINUE
      TSSO = TSS
190  CONTINUE
      READ (5,1000,END=220) TITLE
      GO TO 7
200  WRITE (6,1008) NS6,KS
      GO TO 220
210  WRITE (6,1009) ND7,KD
1000 FORMAT (20A4)
1001 FORMAT ('1',20A4)
1002 FORMAT (4I5,G10.0,I5)
1003 FORMAT('///' '*** BASIC PARAMETERS ***'///' NUMBER OF NODES =',I5/IX,
1'NUMBER OF ELEMENTS =',I5/IX,'NUMBER OF MATERIALS =',I5/IX,
2'NUMBER OF TIME INTERVALS =',I5/IX,'REFERENCE TEMPERATURE = ',
3G12.6)
1004 FORMAT('///' '*** MATERIAL PROPERTIES ***')
1005 FORMAT (I5)
1006 FORMAT (G10.0,6I5)
1007 FORMAT ('1','*** PARAMETERS FOR TIME INTERVAL NO. ',I5,' ***'//
1' TIME STEP SIZE = ',G12.6//IX,'NUMBER OF TIME STEPS =',I5/IX,
2'(PRINT STEP SIZE) / (TIME STEP SIZE) =',I5/IX,
3'NUMBER OF CONVECTION BOUNDARY CARDS =',I5/IX,
4'NUMBER OF RADIATION BOUNDARY CARDS =',I5/IX,
5'NUMBER OF TEMPERATURE BOUNDARY CARDS =',I5/IX,
6'MATERIAL PROPERTY CHANGE INDICATOR =',I5/IX,
7'(A ZERO INDICATES A CHANGE FROM PRECEDING TIME INTERVAL)')
1008 FORMAT (/ ' SINGLE PRECISION STORAGE REQUIREMENT OF ',I5,' WORDS ' /
1 'EXCEEDS ALLOCATED SPACE OF ',I5,' WORDS')
1009 FORMAT (/ ' DOUBLE PRECISION STORAGE REQUIREMENT OF ',I5,' WORDS ' /
1 'EXCEEDS ALLOCATED SPACE OF ',I5,' WORDS')
1010 FORMAT ('1','*** TEMPERATURE DISTRIBUTION ***'//IX,
1'TIME = ',G12.6//2X,'NODE',2X,'TEMPERATURE',3X,'NODE',2X,
2'TEMPERATURE',3X,'NODE',2X,'TEMPERATURE',3X,'NODE',2X,
3'TEMPERATURE'//)
1011 FORMAT (I6,I5,G12.6,2X,I5,I5,G12.6,2X,I5,I5,G12.6,2X,I5,I5,G12.6)
1012 FORMAT (/ ' PLANE SOLID - X=R,Y=Z')
220  CALL EXIT
      END

```

```

SUBROUTINE RPSNC (ANC,TO,NN,TREF)
C   TRAX - RPSNC - 1/16/73
C   J K ROCHELLE
C   DESCRIPTION: READS PRINTS AND STORES NODAL COORDINATES AND INITIAL
C   TEMP IN ARRAY ANC. STORES INITIAL TEMP IN VECTOR TO.
REAL*8 TO(NN)
REAL*4 ANC(NN,3)
WRITE (6,1000)
NCOUNT = 0
10 READ (5,1001) N,NI,(ANC(N,M),M=1,3),RC,ZC,IND
NCOUNT = NCOUNT + 1
IF (ANC(N,3).EQ.0.) ANC(N,3) = TREF
IF (NI.EQ.0) GO TO 25
READ (5,1001) N2,NI2,(ANC(N2,M),M=1,3)
NCOUNT = NCOUNT + 1
IF (ANC(N2,3).EQ.0.) ANC(N2,3) = TREF
IF (NI2.NE.(-NI)) GO TO 100
RT = N2 - N
K = N + NI
NL = N2 - NI
IF (IND.EQ.0) GO TO 11
RA = ANC(N,1)
RB = ANC(N2,1)
ZA = ANC(N,2)
ZB = ANC(N2,2)
R1 = SQRT((RA - RC)**2 + (ZA - ZC)**2)
R2 = SQRT((RB - RC)**2 + (ZB - ZC)**2)
RAVG = (R1 + R2)/2.
RDIF = ABS(R1 - R2)
IF (RDIF.GT.(RAVG/100.)) GO TO 100
CL = SQRT((RA - RB)**2 + (ZA - ZB)**2)
PHI = 2. * ARSIN(CL/(2.*RAVG))
IF (IND.LT.0) PHI = -PHI
ALPHA = ARCOS((RA - RC)/RAVG)
IF (ZC.GT.ZA) ALPHA = -ALPHA
11 CONTINUE
DO 20 J=K,NL,NI
NCOUNT = NCOUNT + 1
AJ = J - N
IF (IND.EQ.0) GO TO 12
THETA = AJ * PHI / RT + ALPHA
ANC(J,1) = RC + RAVG * COS(THETA)
ANC(J,2) = ZC + RAVG * SIN(THETA)
GO TO 20
12 CONTINUE
DO 15 M=1,2
15 ANC(J,M) = (AJ/RT)*(ANC(N2,M) - ANC(N,M)) + ANC(N,M)
20 ANC(J,3) = ANC(N,3)
25 CONTINUE
IF (NCOUNT-NN) 10,30,100
100 WRITE (6,1002)
CALL EXIT
30 CONTINUE
DO 40 N=1,NN

```

```

      TO(N) = 0.
      TO(N) = ANC(N,3)
      WRITE (6,1003) N,(ANC(N,M),M=1,3)
      40 IF (MOD(N,50).EQ.0) WRITE(6,1004)
1000  FORMAT ('1','*** NODAL COORDINATES AND INITIAL TEMPERATURES ***',
      1///6X,'NODE',5X,'R',12X,'Z',9X,'TEMPERATURE'/)
1001  FORMAT (2I5,5G10.0,15)
1002  FORMAT (/ ' ERROR IN NODAL COORDINATE DATA' )
1003  FORMAT (5X,15,3(1X,G12.6))
1004  FORMAT ('1')
      RETURN
      END

```

```

C      SUBROUTINE RPSMP (AMP,NTM,MNO)
C      TRAX - RPSMP - 1/15/73
C      J K ROCHELLE
C      DESCRIPTION: READS PRINTS AND STORES MATERIAL PROPERTIES
C      FOR ONE MATERIAL INTO ARRAY AMP.
C
      REAL*4 AMP(NTM,5)
      WRITE (6,1000) MNO
      DO 10 I=1,NTM
10    READ (5,1001) (AMP(I,J),J=1,5)
      DO 20 I=1,NTM
20    WRITE (6,1002) (AMP(I,J),J=1,5)
1000  FORMAT (//// ' MATERIAL NUMBER',I5//,2X,'TEMPERATURE',4X,
      1'R CONDUCTIVITY',2X,'Z CONDUCTIVITY',2X,'SPECIFIC HEAT',3X,
      2'DENSITY'/)
1001  FORMAT (5G10.0)
1002  FORMAT (1X,G12.6,4(4X,G12.6))
      RETURN
      END

```

```

SUBROUTINE RPSEC (ANC,IEC,AEC,NN,NE,MBAND,ITYPE)
C   TRAX - RPSEC - 1/16/73
C   J K ROCHELLE
C   DESCRIPTION: READS PRINTS AND STORES ELEMENT CONNECTIVITY
C   MATERIAL PROPERTY NUMBER AND ANGLE BETWEEN ELEMENT AND
C   SYSTEM AXES. DETERMINES BANDWIDTH. CALLS GEOM.
REAL*4 AEC(NE,5),ANC(NN,3)
INTEGER*4 IEC(NE,12)
WRITE (6,1000)
LCOUNT = 0
READ (5,1001) L1,(IEC(L1,M),M=1,5),AEC(L1,1),NI
IF (NI.EQ.0) NI = 1
LCOUNT = LCOUNT + 1
10 READ (5,1001) L2,(IEC(L2,M),M=1,5),AEC(L2,1),NI2
IF (NI2.EQ.0) NI2 = 1
LCOUNT = LCOUNT + 1
IF (L2.EQ.(L1+1)) GO TO 50
20 J = L1 + 1
K = L2 - 1
DO 40 I=J,K
LCOUNT = LCOUNT + 1
IP = I - 1
DO 30 M=1,4
30 IEC(I,M) = IEC(IP,M) + NI
IEC(I,5) = IEC(L1,5)
40 AEC(I,1) = AEC(L1,1)
50 L1 = L2
NI = NI2
IF (LCOUNT-NE) 10,60,100
100 WRITE (6,1002)
CALL EXIT
60 MBAND = 0
DO 70 I=1,NE
LARG = 0
LITL = 10000
DO 65 J=1,4
IF (IEC(I,J).GT.LARG) LARG = IEC(I,J)
IF (IEC(I,J).LT.LITL) LITL = IEC(I,J)
IF ((LARG-LITL).GT.MBAND) MBAND = LARG - LITL
65 CONTINUE
CALL GEOM(ANC,IEC,AEC,NN,NE,I,ITYPE)
WRITE (6,1003) I,(IEC(I,M),M=1,5),AEC(I,1)
70 IF (MOD(I,50).EQ.0) WRITE(6,1004)
MBAND = MBAND + 1
1000 FORMAT ('1','*** ELEMENT CONNECTIVITY ***'/1X,
1          '*** MATERIAL PROPERTY NUMBER ***'/1X,
2          '*** ANGLE BETWEEN ELEMENT AND SYSTEM AXES ***'//1X,
3          'ELEMENT',5X,'I',6X,'J',6X,'K',6X,'L',2X,'MATERIAL',2X,'ANGLE'/)
1001 FORMAT (6I5,F10.0,I5)
1002 FORMAT ('/ ERROR IN ELEMENT CONNECTIVITY DATA')
1003 FORMAT (6(2X,I5),2X,F10.5)
1004 FORMAT ('1')
RETURN
END

```

SUBROUTINE GEOM (ANC,IEC,AEC,NN,NE,II,ITYPE)

TRAX - GEOM - 1/17/73

J K ROCHELLE

DESCRIPTION : GENERATES ELEMENT TYPE AND SHORTEST DIAGONAL INDICATORS. CALCULATES RBAR,AREA FOR TRIANGULAR ELEMENTS AND SUB-ELEMENTS. SETS UP INDICATORS FOR SUB-ELEMENT CONNECTIVITY.

REAL*4 ANC(NN,3),AEC(NE,5)

INTEGER*4 IEC(NE,12)

DEFINE NODE NUMBERS

I = IEC(II,1)

J = IEC(II,2)

K = IEC(II,3)

L = IEC(II,4)

DETERMINE ELEMENT TYPE AND SHORTEST DIAGONAL FOR QUAD.

IF (K.EQ.L) GO TO 10

S1 = SQRT((ANC(I,1)-ANC(K,1))**2 + (ANC(I,2)-ANC(K,2))**2)

S2 = SQRT((ANC(J,1)-ANC(L,1))**2 + (ANC(J,2)-ANC(L,2))**2)

IF (S1.LE.S2) IEC(II,6)=1

IF (S1.GT.S2) IEC(II,6)=-1

10 CONTINUE

DO 80 NTR=1,2

IF (IEC(II,6)) 20,40,40

20 CONTINUE

IF (NTR.EQ.2) GO TO 30

IT = I

JT = J

KT = L

GO TO 60

30 IT = J

JT = K

KT = L

GO TO 60

40 CONTINUE

IF (NTR.EQ.2) GO TO 50

IT = I

JT = J

KT = K

GO TO 60

50 IT = I

JT = K

KT = L

60 RI = ANC(IT,1)

RJ = ANC(JT,1)

RK = ANC(KT,1)

ZI = ANC(IT,2)

ZJ = ANC(JT,2)

ZK = ANC(KT,2)

RBAR = (RI + RJ + RK)/3.

IF (ITYPE.NE.0) RBAR = 1.

AREA = (RI*(ZJ-ZK) + RJ*(ZK-ZI) + RK*(ZI-ZJ))/2.

NT1 = NTR + 1

AEC(II,NT1) = RBAR

NT2 = NTR + 3

AEC(II,NT2) = AREA

```

      IF (NTR.EQ.2) GO TO 70
      IEC(II,7) = IT
      IEC(II,8) = JT
      IEC(II,9) = KT
      IF (IEC(II,6).EQ.0) GO TO 90
      GO TO 80
70  IEC(II,10) = IT
      IEC(II,11) = JT
      IEC(II,12) = KT
80  CONTINUE
      AVERAGE RBAR FOR QUAD.
      AEC(II,2) = (AEC(II,2)*AEC(II,4) + AEC(II,3)*AEC(II,5)) /
1(AEC(II,4) + AEC(II,5))
      AEC(II,3) = AEC(II,2)
90  CONTINUE
      RETURN
      END

```

```

SUBROUTINE EQT (TO,ANC,NN)
TRAX - EQT - 1/22/73
J K ROCHELLE
DESCRIPTION: STORES TEMP. VECTOR IN AS IN 4 BYTE WORDS.
REAL*8 TO(NN)
REAL*4 ANC(NN,3)
DO 10 N=1,NN
10  ANC(N,3) = TO(N)
      RETURN
      END

```

```

SUBROUTINE HTCAP2(TMP,ANC,IEC,AEC,AC,NMP,NN,MBAND,NE,LITE,TSS,NS1)
C   TRAX - HTCAP2 - 2/16/73
C   J K ROCHELLE
C   DESCRIPTION: GENERATES SYSTEM HEAT CAPACITY MATRIX AND
C   WRITES IT ON FILE 2.
COMMON /PARAM/ NSM(101)
REAL*4 TMP(NMP),ANC(NN,3),AEC(NE,5),AC(NN,MBAND)
INTEGER*4 IEC(NE,12)
LITE = 1
DO 70 II=1,NE
DO 60 NTR=1,2
CALL TRICON (TMP,ANC,IEC,AEC,IT,JT,KT,RBAR,AREA,ANGL,RCOND,
1ZCOND,SPHT,DENS,II,NMP,NTR,NN,NE,NS1)
IF (IEC(II,6).EQ.0) GO TO 20
IF (NTR.EQ.2) GO TO 10
A1 = AREA
R1 = RBAR
GO TO 60
10 A2 = AREA
R2 = RBAR
CQ = DENS * SPHT * (A1*R1 + A2*R2) / (4. * TSS)
IQ = IEC(II,1)
JQ = IEC(II,2)
KQ = IEC(II,3)
LQ = IEC(II,4)
AC(IQ,1) = AC(IQ,1) + CQ
AC(JQ,1) = AC(JQ,1) + CQ
AC(KQ,1) = AC(KQ,1) + CQ
AC(LQ,1) = AC(LQ,1) + CQ
GO TO 70
20 CT = DENS * SPHT * RBAR * AREA / (3. * TSS)
AC(IT,1) = AC(IT,1) + CT
AC(JT,1) = AC(JT,1) + CT
AC(KT,1) = AC(KT,1) + CT
GO TO 70
60 CONTINUE
70 CONTINUE
WRITE (2) (AC(N,1),N=1,NN)
DO 80 N=1,NN
80 AC(N,1) = 0.
RETURN
END

```

```
SUBROUTINE CK (TMP,ANC,IEC,AEC,ACK,NMP,NN,MBAND,NE,LITE,TSS,NS1)
```

```
TRAX - CK - 1/18/73
```

```
J K ROCHELLE
```

```
DESCRIPTION: GENERATES ELEMENT CONDUCTIVITY MATRICES.
```

```
CALLS SUBROUTINE ASMBL TO ASSEMBLE SYSTEM K MATRIX.
```

```
COMMON /PARAM/ NSM(101)
```

```
REAL*4 B(3),C(3),EK(3,3),K11,K12,K22
```

```
REAL*4 TMP(NMP),ANC(NN,3),AEC(NE,5),ACK(NN,MBAND)
```

```
INTEGER*4 IEC(NE,12)
```

```
LITE = 1
```

```
DO 70 II=1,NE
```

```
DO 60 NTR=1,2
```

```
CALL TRICON (TMP,ANC,IEC,AEC,IT,JT,KT,RBAR,AREA,ANGL,RCOND,  
1ZCOND,SPHT,DENS,II,NMP,NTR,NN,NE,NS1)
```

```
CONST = RBAR / (4.*AREA)
```

```
K11 = ((COS(ANGL))**2)*RCOND + ((SIN(ANGL))**2)*ZCOND
```

```
K12 = (COS(ANGL))*(SIN(ANGL))*(RCOND - ZCOND)
```

```
K22 = ((SIN(ANGL))**2)*RCOND + ((COS(ANGL))**2)*ZCOND
```

```
RI = ANC(IT,1)
```

```
RJ = ANC(JT,1)
```

```
RK = ANC(KT,1)
```

```
ZI = ANC(IT,2)
```

```
ZJ = ANC(JT,2)
```

```
ZK = ANC(KT,2)
```

```
B(1) = ZJ - ZK
```

```
B(2) = ZK - ZI
```

```
B(3) = ZI - ZJ
```

```
C(1) = RK - RJ
```

```
C(2) = RI - RK
```

```
C(3) = RJ - RI
```

```
DO 20 M=1,3
```

```
DO 20 N=1,3
```

```
EK(M,N) = CONST*(K11*B(M)*B(N) + K12*(B(M)*C(N)+B(N)*C(M)) +  
1K22*C(M)*C(N))
```

```
20 CONTINUE
```

```
CALL ASMBL (ACK,EK,IT,JT,KT,NN,MBAND,3)
```

```
IF (IEC(II,6).EQ.0) GO TO 70
```

```
60 CONTINUE
```

```
70 CONTINUE
```

```
RETURN
```

```
END
```

```

SUBROUTINE TRICON (TMP,ANC,IEC,AEC,IT,JT,KT,RBAR,AREA,ANGL,RCOND,
1ZCOND,SPHT,DENS,II,NMP,NTR,NN,NE,NS1)
TRAX -TRICON - 1/19/73
J K ROCHELLE
DESCRIPTION: DETERMINES CONSTANTS FOR ELEMENT CONDUCTIVITY
AND HEAT CAPACITY MATRICES.
COMMON /PARAM/ NSM(101)
REAL*4 TMP(NMP),ANC(NN,3),AEC(NE,5)
INTEGER*4 IEC(NE,12)
IF (NTR.EQ.2) GO TO 20
IT = IEC(II,7)
JT = IEC(II,8)
KT = IEC(II,9)
GO TO 30
20 IT = IEC(II,10)
JT = IEC(II,11)
KT = IEC(II,12)
30 CONTINUE
MNO = IEC(II,5)
NT1 = NTR + 1
RBAR = AEC(II,NT1)
NT2 = NTR + 3
AREA = AEC(II,NT2)
ANGL = AEC(II,1) * (3.14159/180.)
IF (IEC(II,6).EQ.0) GO TO 40
I = IEC(II,1)
J= IEC(II,2)
K = IEC(II,3)
L = IEC(II,4)
TEMP = (ANC(I,3) + ANC(J,3) + ANC(K,3) + ANC(L,3)) / 4.
GO TO 50
40 TEMP = (ANC(IT,3) + ANC(JT,3) + ANC(KT,3))/3.
50 NSI = NSM(MNO)
NSO = NSI - NS1 + 1
NSN = NSM(MNO+1)
NTM = (NSN-NSI)/5
CALL DETMP (TMP(NSO),TEMP,RCOND,ZCOND,SPHT,DENS,NTM)
RETURN
END

```

```

SUBROUTINE DETMP (AMP,TEMP,RCOND,ZCOND,SPHT,DENS,NTM)
C TRAX - DETMP - 1/18/73
C J K ROCHELLE
C DESCRIPTION: DETERMINES MATERIAL PROPERTIES FOR A GIVEN TEMP.
C

```

```

REAL*4 AMP(NTM,5)
IF (TEMP.LE.AMP(1,1)) GO TO 50
IF (TEMP.GE.AMP(NTM,1)) GO TO 60
RATIO = 1.
DO 40 I=2,NTM
IF (TEMP.GT.AMP(I,1)) GO TO 40
J = I - 1
RATIO = (TEMP - AMP(J,1)) / (AMP(I,1) - AMP(J,1))
RCOND = RATIO*(AMP(I,2) - AMP(J,2)) + AMP(J,2)
ZCOND = RATIO*(AMP(I,3) - AMP(J,3)) + AMP(J,3)
SPHT = RATIO*(AMP(I,4) - AMP(J,4)) + AMP(J,4)
DENS = RATIO*(AMP(I,5) - AMP(J,5)) + AMP(J,5)
40 CONTINUE
GO TO 70
50 RCOND = AMP(1,2)
ZCOND = AMP(1,3)
SPHT = AMP(1,4)
DENS = AMP(1,5)
GO TO 70
60 RCOND = AMP(NTM,2)
ZCOND = AMP(NTM,3)
SPHT = AMP(NTM,4)
DENS = AMP(NTM,5)
70 CONTINUE
RETURN
END

```

```

SUBROUTINE RPSCBC (ICB,ACB,TO,NCBC,NN)
C TRAX - RPSCBC - 1/22/73
C J K ROCHELLE
C DESCRIPTION: READS PRINTS AND STORES CONVECTION BOUNDARY
C DATA IN AS. MODIFIES H IF DESIRED.
C

```

```

REAL*4 ACP(NCBC,4)
INTEGER*4 ICB(NCBC,2)
REAL*8 TO(NN)
WRITE (6,1000)
DO 10 N=1,NCBC
READ (5,1001) (ICB(N,M),M=1,2),(ACP(N,M),M=1,4),TF,AN
IF (TF.EQ.0.) GO TO 5
I = ICB(N,1)
J = ICB(N,2)
TW = (TO(I) + TO(J)) / 2.
ACP(N,1) = ACP(N,1)*(TW/TF)**AN
5 CONTINUE
WRITE (6,1002) (ICB(N,M),M=1,2),(ACP(N,M),M=1,4)
10 IF (MOD(N,50).EQ.0) WRITE(6,1004)
1000 FORMAT ('1','*** CONVECTION BOUNDARY CONDITIONS ***'///,4X,'I ',
1'NODE',4X,'J NODE',2X,'SURFACE',8X,'EXTERNAL TEMPERATURE CONSTANT
2S'/23X,'CONDUCTANCE',4X,'A',13X,'B',13X,'C'//)
1001 FORMAT (2I5,6G10.0)
1002 FORMAT (2(5X,I5),4(2X,G12.6))
1004 FORMAT ('1')
RETURN
END

```

SUBROUTINE CKH (ANC,ICB,ACB,NCBC,NN,ITYPE)

TRAX - CKH - 1/19/73

J K ROCHELLE

DESCRIPTION: MODIFIES H VALUES.

REAL*4 ANC(NN,3),ACB(NCBC,4)

INTEGER*4 ICB(NCBC,2)

DO 30 N=1,NCBC

I = ICB(N,1)

J = ICB(N,2)

RI = ANC(I,1)

RJ = ANC(J,1)

ZI = ANC(I,2)

ZJ = ANC(J,2)

H = ACB(N,1)

XL = SQRT((RI - RJ)**2 + (ZI - ZJ)**2)

RBAR = (RI + RJ)/2.

IF (ITYPE.NE.0) RBAR = 1.

CONST = (RBAR * H * XL)/6.

ACB(N,1) = CONST

30 CONTINUE

RETURN

END

SUBROUTINE ASMBL (SS,S,IT,JT,KT,NN,MBAND,NDF)

TRAX - ASMBL - 1/18/73

J K ROCHELLE

DESCRIPTION: ADDS (NDF*NDF) MATRICES INTO UPPER HALFBAND MATRICES

REAL*4 S(NDF,NDF),SS(NN,MBAND)

INTEGER*4 INDEX(3)

INDEX(1) = IT

INDEX(2) = JT

INDEX(3) = KT

DO 1 L=1,NDF

I = INDEX(L)

DO 1 K=1,NDF

J = INDEX(K)

IF (I.GT.J) GO TO 1

JJ = J - I + 1

SS(I,JJ) = SS(I,JJ) + S(L,K)

1 CONTINUE

RETURN

END

```

SUBROUTINE RPSRBC (IRB,ARB,ANC,NRBC,NN,ITYPE)
TRAX - RPSRBC - 1/22/73
J K ROCHELLE
DESCRIPTION: READS PRINTS AND STORES RADIATION BOUNDARY DATA
IN AS. GENERATES HEAT FLUX TERMS WHEN DESIRED.
GENERATES TERMS FOR RADIATION LOAD VECTOR.

```

```

REAL*4 ARB(NRBC),ANC(NN,3)
INTEGER*4 IRB(NRBC,2)
WRITE (6,1000)
DO 10 N=1,NRBC
  READ (5,1001) (IRB(N,M),M=1,2),ARB(N),CW,C1,T1,C2,T2
  I = IRB(N,1)
  J = IRB(N,2)
  TW = (ANC(I,3) + ANC(J,3))/2.
  ARB(N) = ARB(N) + .174*(C1*(T1/100.))**4 + C2*(T2/100.))**4
  * - CW*(TW/100.))**4)
  WRITE (6,1002) (IRB(N,M),M=1,2),ARB(N)
10 IF (MOD(N,50).EQ.0) WRITE(6,1004)
  DO 20 N=1,NRBC
    I = IRB(N,1)
    J = IRB(N,2)
    RI = ANC(I,1)
    RJ = ANC(J,1)
    ZI = ANC(I,2)
    ZJ = ANC(J,2)
    Q = ARB(N)
    XL = SQRT((RI - RJ)**2 + (ZI - ZJ)**2)
    RBAR = (RI + RJ)/2.
    IF (ITYPE.NE.0) RBAR = 1.
  20 ARB(N) = (Q * RBAR * XL)/2.
1000 FORMAT ('1','*** RADIATION BOUNDARY CONDITIONS ***'///,4X,
1'I NODE',4X,'J NODE',4X,'HEAT FLUX'/)
1001 FORMAT (2I5,6G10.0)
1002 FORMAT (2(5X,I5),4X,G12.6)
1004 FORMAT ('1')
RETURN
END

```

```

SUBROUTINE SCBD (ICBD,ACBD,ICB,ACB,NCBC)
TRAX - SCBD - 1/22/73
J K ROCHELLE
DESCRIPTION: STORES CONVECTION BOUNDARY DATA IN AD IN
4 BYTE WORDS.
INTEGER*4 ICB(NCBC,2),ICBD(NCBC,2)
REAL *4 ACBD(NCBC,4),ACB(NCBC,4)
DO 30 N=1,NCBC
  DO 10 M=1,2
10 ICB(N,M) = ICB(N,M)
  DO 20 M=1,4
20 ACBD(N,M) = ACD(N,M)
30 CONTINUE
RETURN
END

```

SUBROUTINE QRV (QR,IRB,ARB,NRBC,NN)

TRAX - QRV - 1/22/73

DESCRIPTION: GENERATES RADIATION LOAD VECTOR. STORES IT IN AD.

REAL*8 QR(NN),Q

REAL*4 ARB(NRBC)

INTEGER*4 IRB(NRBC,2)

DO 10 N=1,NN

10 QR(N) = 0.

DO 20 N=1,NRBC

I = IRB(N,1)

J = IRB(N,2)

Q = 0.

Q = ARB(N)

QR(I) = QR(I) + Q

20 QR(J) = QR(J) + Q

RETURN

END

SUBROUTINE RPSTBC (IT,ST,NTBC)

TRAX - RPSTBC - 1/22/73

J K ROCHELLE

DESCRIPTION: READS PRINTS AND STORES SPECIFIED TEMPERATURE

DATA IN AD IN 4 BYTE WORDS.

INTEGER*4 IT(NTBC)

REAL*4 ST(NTBC)

WRITE (6,1000)

DO 10 N=1,NTBC

READ (5,1001) IT(N),ST(N)

WRITE (6,1002) IT(N),ST(N)

10 IF (MOD(N,50).EQ.0) WRITE(6,1004)

1000 FORMAT ('1','*** SPECIFIED TEMPERATURES ***'///,6X,'NODE',4X,
1'TEMPERATURE'/)

1001 FORMAT (15,G10.0)

1002 FORMAT(5X,15,3X,G12.6)

1004 FORMAT ('1')

RETURN

END

```

SUBROUTINE RSM (A,LSM,I)
TRAX - RSM - 1/25/73
J K ROCHELLE
DESCRIPTION: READS SINGLE PRECISION VECTOR FROM FILE I.
PLACES IT IN DOUBLE PRECISION VECTOR.
REAL*4 A(2,LSM)
READ (I) (A(1,N),N=1,LSM)
REWIND I
RETURN
END

```

```

SUBROUTINE ADH (ACK,ICB,ACB,NCBC,NN,MBAND)
TRAX - ADH - 3/1/73
J K ROCHELLE
DESCRIPTION - ADDS SURFACE CONDUCTANCE VALUES TO SYSTEM MATRIX.
REAL*8 HM(2,2),ACK(NN,MBAND)
REAL*4 ACB(NCBC,4)
INTEGER*4 ICB(NCBC,2)
DO 30 N=1,NCBC
CONST = ACB(N,1)
DO 20 K=1,2
DO 20 L=1,2
IF (K.EQ.L) GO TO 10
HM(K,L) = 0.
HM(K,L) = CONST
GO TO 20
10 HM(K,L) = 0.
HM(K,L) = CONST * 2.
20 CONTINUE
ACB(N,1) = CONST * 3.
DO 1 L=1,2
I = ICB(N,L)
DO 1 K=1,2
J = ICB(N,K)
IF (I.GT.J) GO TO 1
JJ = J - I + 1
ACK(I,JJ) = ACK(I,JJ) + HM(L,K)
1 CONTINUE
30 CONTINUE
RETURN
END

```

SUBROUTINE MOD1 (A,B,IT,TS,NN,MBAND,NTBC,NRBC)

TRAX - MOD1 - 1/23/73

J K ROCHELLE

DESCRIPTION: MODIFIES SYSTEM MATRIX AND RADIATION VECTOR FOR
SPECIFIED TEMPERATURES.

REAL*8 A(NN,MBAND),B(NN),U

REAL*4 TS(NTBC)

INTEGER*4 IT(NTBC)

IF (NRBC.NE.0) GO TO 4

DO 3 II=1,NN

3 B(II) = 0.

4 CONTINUE

DO 10 N=1,NTBC

I = IT(N)

U = 0.

U = TS(N)

IL = I - MBAND + 1

IF (IL.LE.0) IL = 1

DO 1 J=IL,I

JJ = I - J + 1

B(J) = B(J) - A(J,JJ)*U

1 A(J,JJ) = 0.

DO 2 JJ=2,MBAND

J = I + JJ - 1

IF (J.GT.NN) GO TO 10

B(J) = B(J) - A(I,JJ)*U

2 A(I,JJ) = 0.

10 A(I,1) = 0.

B(I) MUST BE SET = TO THE SPECIFIED TEMPERATURE LATER.

RETURN

END

SUBROUTINE NORM1 (A,IT,NR,NC,BIG1,NTBC)

TRAX - NORM1 - 1/22/73

J K ROCHELLE

DESCRIPTION: NORMALIZES SYSTEM MATRIX WITH RESPECT TO
LARGEST DIAGONAL TERM PRIOR TO APPLYING BOUNDARY CONDITIONS.

REAL*8 A(NR,NC),BIG1,TEST,BIG

INTEGER*4 IT(NTBC)

BIG = 0.

DO 1 I=1,NR

TEST = DABS(A(I,1))

IF (TEST.GT.BIG) BIG=TEST

1 CONTINUE

BIG1 = 1./BIG

DO 2 I=1,NR

DO 2 J=1,NC

2 A(I,J) = A(I,J)*BIG1

IF (NTBC.EQ.0) GO TO 10

DO 5 J=1,NTBC

K = IT(J)

5 A(K,1) = 1.

10 CONTINUE

RETURN

END

```

SUBROUTINE FACTOR (NDFREE,IBW,S)
*****
FACTORING OF A BANDED SYMMETRIC MATRIX FOR SOLUTION BY
  A CHOLESKY-GAUSSIAN ELIMINATION PROCEDURE
*****
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION S(NDFREE,IBW)
NDFREE= MAX. DEGREES OF FREEDOM OF SYSTEM
IBW=MAXIMUM HALF BANDWIDTH OF SYSTEM EQUATIONS
S = RECT. MATRIX CONTAINING UPPER HALF BANDWIDTH OF SYS. EQS.
TEMP = 1./S(1,1)
DO 1 J = 2,IBW
1 S(1,J) = S(1,J)*TEMP
DO 6 I = 2,NDFREE
LL = I - 1
NN = NDFREE - LL
2 IF(NN .GT. IBW) NN = IBW
DO 6 J = 1,NN
L = IBW - J
SUM = 0.
IF( L .EQ. 0 ) GO TO 5
IF( LL .LT. L ) L = LL
3 DO 4 K = 1,L
K1 = I - K
K2 = 1 + K
K3 = J + K
4 SUM = SUM + S (K1,K2) * S(K1,K3) * S(K1,1)
5 S(I,J) = S(I,J) - SUM
IF( J .EQ. 1 ) GO TO 6
S(I,J) = S(I,J) / S(I,1)
6 CONTINUE
RETURN
END

```

FACT0010
FACT0020
FACT0030
FACT0040
FACT0050
FACT0060
FACT0070
FACT0080
FACT0090
FACT0100
FACT0110
FACT0120
FACT0130
FACT0140
FACT0150
FACT0160
FACT0170
FACT0180
FACT0190
FACT0200
FACT0210
FACT0220
FACT0230
FACT0240
FACT0250
FACT0260
FACT0270
FACT0280
FACT0290
FACT0300
FACT0310
FACT0320
FACT0330

```

SUBROUTINE QOV2 (QO,CO,TO,NN)
TRAX - QOV2 - 2/16/73
J K ROCHELLE
DESCRIPTION: GENERATES QO VECTOR = HEAT CAPACITY MATRIX *
  TEMPERATURE VECTOR FROM PRECEEDING TIME STEP.
REAL*8 QO(NN),TO(NN),CO(NN)
DO 10 N=1,NN
10 QO(N) = CO(N) * TO(N)
RETURN
END

```

SUBROUTINE QLV (QL,QRM,ICB,ACB,NN,TSS,NCBC,NRBC,NTBC,BIGD,N)

TRAX - QLV - 1/24/73

J K ROCHELLE

DESCRIPTION: GENERATES LOAD VECTOR EXCEPT FOR SPECIFIED
TEMPERATURE VALUES.

REAL*8 QL(NN),QRM(NN),BIGD,QC

REAL*4 ACB(NCBC,4)

INTEGER*4 ICB(NCBC,2)

IF (NRBC.NE.0) GO TO 5

IF (NTBC.EQ.0) GO TO 20

5 CONTINUE

DO 10 I=1,NN

10 QL(I) = QL(I) + QRM(I)

20 CONTINUE

IF (NCBC.EQ.0) GO TO 40

DO 30 L=1,NCBC

I = ICB(L,1)

J = ICB(L,2)

H = ACB(L,1)

A = ACB(L,2)

B = ACB(L,3)

C = ACB(L,4)

SN = N

QC = 0.

QC = H * (A + B*SN + C*SN*SN)

QL(I) = QL(I) + QC

30 QL(J) = QL(J) + QC

40 CONTINUE

DO 50 I=1,NN

50 QL(I) = QL(I) * BIGD

RETURN

END

SUBROUTINE MOD2 (QL,IT,TS,NN,NTBC)

TRAX - MOD2 - 1/24/73

J K ROCHELLE

DESCRIPTION: MODIFIES LOAD VECTOR FOR SPECIFIED TEMPERATURES.

REAL*8 QL(NN),TEMP

REAL*4 TS(NTBC)

INTEGER*4 IT(NTBC)

DO 10 N=1,NTBC

I = IT(N)

TEMP = 0.

TEMP = TS(N)

10 QL(I) = TEMP

RETURN

END

```

SUBROUTINE SOLVE (NDFREE,IBW,S,P,D)
* * * * * SOLV0005
PART TWO OF CHOLESKY-GAUSSIAN SOLUTION SOLV0010
FOLLOWS FACTOR OF SYMMETRIC EQUATIONS SOLV0015
* * * * * SOLV0020
IMPLICIT REAL*8(A-H,O-Z) SOLV0025
DIMENSION S(NDFREE,IBW), P(NDFREE), D(NDFREE) SOLV0030
NDFREE = MAX. DEGREES OF FREEDOM IN SYSTEM SOLV0035
IBW = MAXIMUM HALF BANDWIDTH OF THE SYSTEM SOLV0040
S = RECTANGULAR MATRIX CONTAINING UPPER HALF BANDWIDTH OF SYS. EQS SOLV0045
D = SYSTEM DEGREES OF FREEDOM TO BE DETERMINED SOLV0050
P = SYSTEM COLUMN MATRIX (KNOWN) SOLV0055
D(1) = P(1) / S(1,1) SOLV0060
DO 2 I = 2,NDFREE SOLV0065
  II = I + 1 SOLV0070
  J = II - IBW SOLV0075
  IF(II .LE. IBW ) J = 1 SOLV0080
  IK = I - 1 SOLV0085
  SUM = 0. SOLV0090
  DO 1 K = J,IK SOLV0095
    KK = II - K SOLV0100
    1 SUM = SUM + S(K,KK).* S(K,1) * D(K) SOLV0105
  2 D(I) = ( P(I) - SUM ) / S(I,1) SOLV0110
  DO 4 NN = 2,NDFREE SOLV0115
    I = NDFREE + 1 - NN SOLV0120
    LL = I - 1 SOLV0125
    J = LL + IBW SOLV0130
    IF ( J.GT.NDFREE ) J = NDFREE SOLV0135
    L = I + 1 SOLV0140
    SUM = 0. SOLV0145
    DO 3 K = L,J SOLV0150
      KK = K - LL SOLV0155
      3 SUM = SUM + S(I,KK) * D(K) SOLV0160
    4 D(I) = D(I) - SUM SOLV0165
  RETURN SOLV0170
END SOLV0175

```

CHAPTER V

INSTRUCTIONS FOR TRAX USERS

I. PROGRAM FUNCTION

This finite element program will perform transient or steady-state heat conduction analysis on axisymmetric or plane bodies. The boundary conditions (convection, radiation, or temperature) may be changed at desired time intervals during the transient. The effect of temperature change on material properties may be accounted for at each time interval. Anisotropic conductivities may be specified in directions along a set of element axes. The angle between system and element axes is specified for each element. Input surface conductances, based on constant free stream fluid properties, may be adjusted to account for temperature dependence. Radiation heat flux values may be input directly or calculated from input data. Absolute temperature units are required throughout the program if the special features for surface conductance adjustment or radiation heat flux calculation are used. Otherwise any consistent set of units may be used. Any number of problems may be solved in one computer run.

II. ANALYTICAL MODEL

A problem is modeled in the usual finite element

fashion with quadrilateral or triangular elements. Quadrilateral elements are preferred. For transient analysis element spacing should be closest near thermal boundaries. A minimum of four to five element divisions are recommended through sections where a significant rate of change of heat flux with respect to direction is anticipated. Care should be taken to minimize the bandwidth. Bandwidth is defined to be the maximum difference between node numbers connected to an element, plus one. The size of problem which can be solved is determined as follows:

$$NS = (5 + MBAND)NN + (17)NE + (6)NCBC + (3)NRBC + (5)(NMPC)$$

$$ND = (4 + MBAND)NN + (3)NCBC + NTBC + 2$$

NS must be less than KS, and ND must be less than KD.

Presently, $KS = 17500$ and $KD = 8750$. For each time interval:

MBAND = Maximum Bandwidth

NN = Number of Nodes

NE = Number of Elements

NCBC = Number of Convection Boundary Cards

NRBC = Number of Radiation Boundary Cards

NTBC = Number of Temperature Boundary Cards

NMPC = Number of Material Property Cards

III. INPUT DATA

The data necessary to describe a problem to TRAX is

input by computer cards. A description of each type of input data card follows. The location of data fields on the card for each item of input is given by column numbers. The type of data to be placed in the field is indicated as INTEGER or DECIMAL, except for the identification card data. INTEGER data are fixed point numbers, which must be placed as far to the right as possible in the field. DECIMAL data are floating point numbers, which may be placed anywhere in the field. This assumes that the decimal point is supplied.

Identification Card

Columns 1-80 Job Title, Name, Date, Etc.

Comments. The information on this card should identify the problem being solved.

Basic Parameters Card

Columns	1-5	Number of Nodes	(INTEGER)
	6-10	Number of Elements	(INTEGER)
	11-15	Number of Materials	
		(100 max.)	(INTEGER)
	16-20	Number of Time Intervals	(INTEGER)
	21-30	Reference Temperature	(DECIMAL)
	31-35	Problem Type Indicator	(INTEGER)
		0 = Axisymmetric Body	
		1 = Plane Body	

Comments. This card defines the basic parameters for the problem. Each input item is considered self-explanatory.

Material Properties Cards

First card (one card per material)

Columns 1-5 Number of temperatures for which

```
properties are given      (INTEGER)
```

Following cards (one card per temperature)

Columns 1-10 Temperature (DECIMAL)

11-20 r Conductivity, k_r (DECIMAL)21-30 z Conductivity, k_z (DECIMAL)

31-40 Specific Heat, c (DECIMAL)

41-50 Density, ρ (DECIMAL)

Comments. TRAX calls the first material, for which properties are input, material number one. The second is called material number two, and so on.

Nodal Point Cards

Columns 1-5 Node Number (INTEGER)

6-10 Nodal Increment, NI (INTEGER)

11-20 R Coordinate of Node (DECIMAL)

21-30 Z Coordinate of Node (DECIMAL)

31-40 Initial Temperature (DECIMAL)

41-50 R_G (DECIMAL)

51-60 Z_C (DECIMAL)

61-65 A Directional Indicator (INTEGER)

Comments. Data for a sequence of nodes is input by supplying cards for the first and last nodes in the sequence. On the first card, NI is the numerical difference between

adjacent node numbers in the sequence. On the last card, NI is minus the NI value on the first card. If the sequence includes only one node, NI is left blank.

Nodal coordinates for intermediate nodes in the sequence are program supplied assuming a uniform spacing of nodes along a line connecting the first and last nodes. This line may be either straight or a circular arc. For the circular arc, R_c and Z_c are the coordinates of the center of curvature. A directional indicator of 1 means that to go from the first node to the last node, you proceed in a counterclockwise direction around the center of curvature. An indicator of -1 means you proceed in a clockwise direction. If the directional indicator is left blank, a straight line is assumed.

Initial temperature must be input only if different from the reference temperature on the basic parameters card. For intermediate nodes initial temperature is assumed to be equal to the value for the first node.

Sufficient nodal point cards must be supplied to specify nodal point data for every node.

Element Cards

Columns	1-5	Element Number	(INTEGER)
	6-10	I Node Number	(INTEGER)
	11-15	J Node Number	(INTEGER)
	16-20	K Node Number	(INTEGER)
	21-25	L Node Number	(INTEGER)

26-30	Material Number	(INTEGER)
31-40	Angle, θ	(DECIMAL)
41-45	Node Increment, NI	(INTEGER)

Comments. Data for a sequence of elements is input by supplying a card for the first element in the sequence. Element cards must be input in numerical order assuming data for intermediate elements in a sequence to be program supplied by:

1. Adding one to the preceding element number.
2. Adding NI to the I, J, K and L node numbers of the preceding element. (If NI is left blank, it is assumed to be 1.)
3. Assigning the same material number and angle as for the preceding element.

Triangular elements are designated by repeating the K node number for L.

Node numbers must be ordered counterclockwise around elements.

θ is the angle between system and element axes. (See Figure 2, page 7.) It only has significance for materials with anisotropic conductivities. It is limited to values from 0 to 90 degrees.

Sufficient element cards must be supplied to specify element data for every element.

Interval Parameters Card

Columns	1-10	Time Step Size	(DECIMAL)
---------	------	----------------	-----------

11-15	Number of Time Steps	(INTEGER)
16-20	Print Interval	(INTEGER)
21-25	Number of Convection Cards	(INTEGER)
26-30	Number of Radiation Cards	(INTEGER)
31-35	Number of Temperature Cards	(INTEGER)
36-40	Material Property Change	
	Indicator	(INTEGER)

Comments. This card followed by the applicable boundary cards, must be supplied for each time interval.

The print interval is the ratio of the number of time steps to the number of times the temperature distribution is printed. If print interval is three, for instance, the temperature distribution is printed every third time it is calculated.

If the same material properties are to be used as for the preceding interval, place a one in column 40. Otherwise leave it blank.

For a steady-state solution assign a time step size of zero.

Convection Boundary Cards

Columns	1-5	Node I	(INTEGER)
	6-10	Node J	(INTEGER)
	11-20	Surface Conductance, h	(DECIMAL)
	21-30	Constant A	(DECIMAL)
	31-40	Constant B	(DECIMAL)
	41-50	Constant C	(DECIMAL)

51-60 Free Stream (Static)

Temperature, T_s (DECIMAL)61-70 Exponent, η (DECIMAL)

Comments. I and J are the nodes, on the surface of the body, between which this boundary condition exists.

A, B and C are constants for the equation:

$$t_f = A + B(N) + C(N)^2$$

Here, t_f is the effective fluid (adiabatic wall) temperature and N is the time step number in the current time interval.

The surface conductance, h, for turbulent flow is normally input assuming constant fluid properties at the free stream (static) temperature. T_s and η are used to account for the variation of fluid properties for temperature as follows:

$$q_c = (h) \left(\frac{T_w}{T_s} \right)^\eta (t_f - t), \text{ References (9) and (10)}$$

where q_c is the convection heat flux. t is the current surface temperature. T_w is the average temperature of nodes I and J at the start of the time interval. For gases, Reference (9) recommends an η of $-.5$ for $t_f < t$ and $-.25$ for $t_f > t$.

If T_s is left blank, $\left(\frac{T_w}{T_s} \right)$ is assigned a value of one.

Radiation Boundary Cards

Columns 1-5 Node I (INTEGER)

6-10 Node J (INTEGER)

11-20 Specified Radiation

	Heat Flux	(DECIMAL)
21-30	C_w	(DECIMAL)
31-40	C_1	(DECIMAL)
41-50	T_1	(DECIMAL)
51-60	C_2	(DECIMAL)
61-70	T_2	(DECIMAL)

Comments. I and J are the nodes, on the surface of the body, between which this boundary condition exists.

The radiation heat flux is the amount of radiant heat, per unit area and time, passing through the body surface. Heat input to the body is positive; heat removed is negative. Constants C_w , C_1 , T_1 , C_2 , and T_2 are used in the equation:

$$q_Y = .174 \left(C_1 \left(\frac{T_1}{100} \right)^4 + C_2 \left(\frac{T_2}{100} \right)^4 - C_w \left(\frac{T_w}{100} \right)^4 \right)$$

where, T_1 and T_2 are the temperatures of sources 1 and 2 radiating to the body. T_w is the average temperature of nodes I and J at the start of the time interval. C_1 , C_2 , and C_w are constants corresponding to T_1 , T_2 , and T_w . q_Y is the calculated radiation heat flux.

The total radiation heat flux, q_r , is the sum of the specified and calculated values.

Temperature Boundary Cards

Columns 1-5	Node Number	(INTEGER)
-------------	-------------	-----------

6-15 Specified Temperature (DECIMAL)

Comments. TRAX assumes this node temperature to change from its initial value, at the start of the time interval, to the specified value in the first time step of the interval.

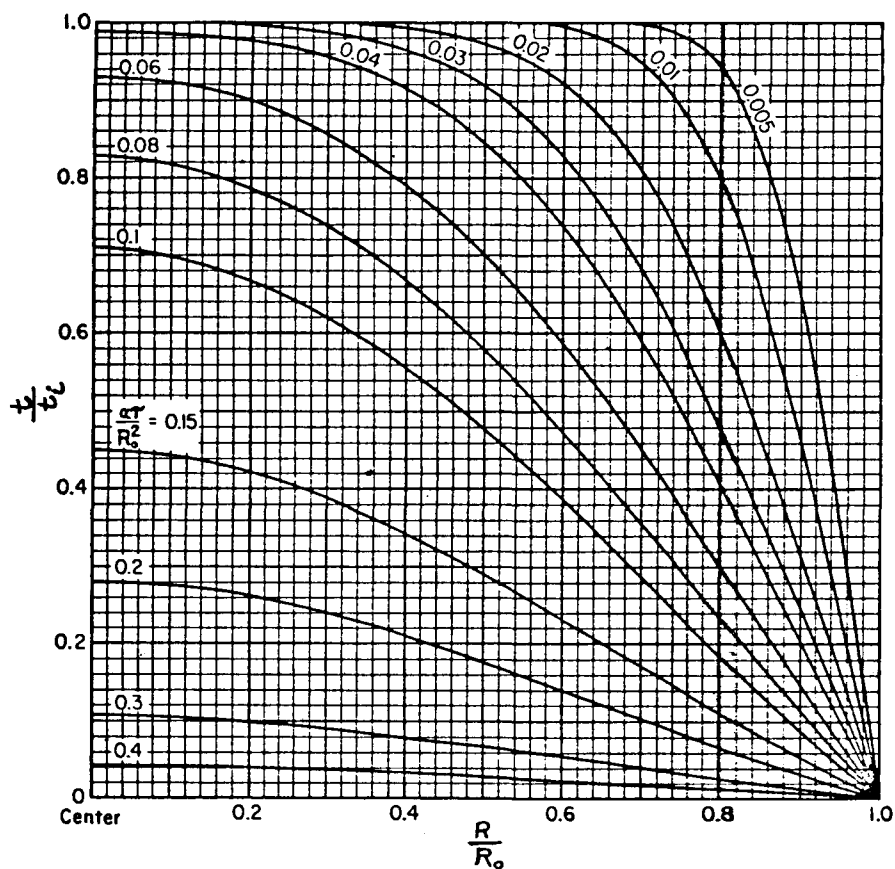
EXAMPLE PROBLEMS

In this chapter, two example problems are solved using TRAX and an IBM 370-155 computer. The first problem demonstrates the validity of a TRAX solution by comparison with a mathematically exact solution of the problem. The second problem is a more typical problem which might be encountered in design or analysis. It demonstrates the application of some program capabilities.

I. EXAMPLE PROBLEM 1--SOLID SPHERE WITH
SPECIFIED SURFACE TEMPERATURE

An exact solution of this general problem is given in Reference (11). The results of that solution are shown in Figure 5 in dimensionless terms. The results obtained from the TRAX solution are comparable to those presented in Figure 5 because for the TRAX solution initial temperature, thermal diffusivity, and outside radius are assumed to be equal to one. A direct comparison of the exact and TRAX solutions is shown in Figure 7.

The analytical model for this problem is shown in Figure 6. The element numbers have circles around them. The other numbers are node numbers. This model has a maximum bandwidth of four. The total computer time required was 4.8 seconds.



t = temperature

R = radius

τ = time

t_i = initial temperature

R_o = outside radius

α = thermal diffusivity

Figure 5. Exact solution results for a solid sphere with specified surface temperature.

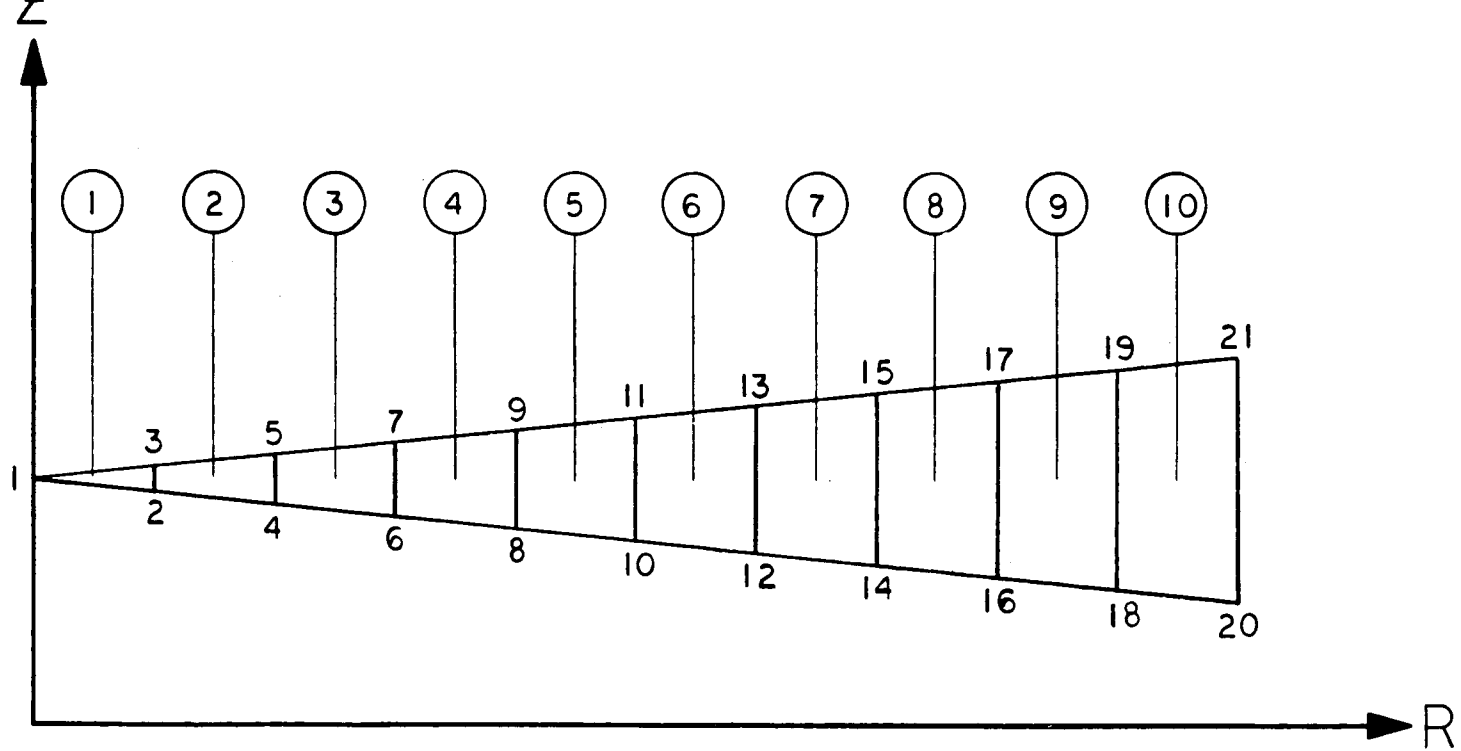
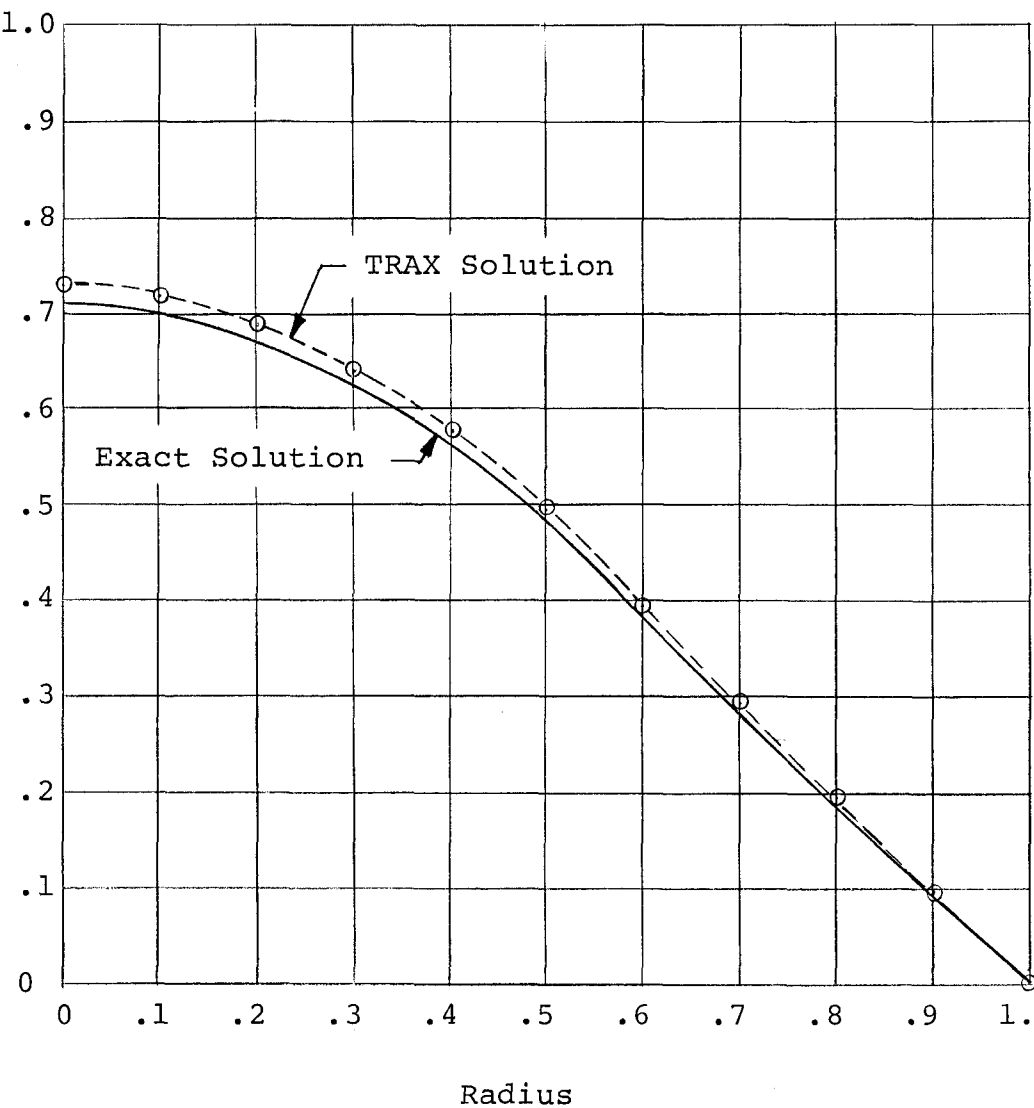


Figure 6. Analytical model for example problem 1.



7. Comparative results for example problem 1 at $e = .1$.

Computer Output--Example Problem 1

EXAMPLE PROBLEM 1 - SOLID SPHERE WITH SPECIFIED SURFACE TEMPERATURE

*** BASIC PARAMETERS ***

NUMBER OF NODES = 21
NUMBER OF ELEMENTS = 10
NUMBER OF MATERIALS = 1
NUMBER OF TIME INTERVALS = 4
REFERENCE TEMPERATURE = 0.0

*** MATERIAL PROPERTIES ***

MATERIAL NUMBER 1

TEMPERATURE	R CONDUCTIVITY	Z CONDUCTIVITY	SPECIFIC HEAT	DENSITY
1.00000	1.00000	1.00000	1.00000	1.00000

*** NODAL COORDINATES AND INITIAL TEMPERATURES ***

NODE	R	Z	TEMPERATURE
1	0.0	0.200000	1.00000
2	0.100000E 00	0.190000	1.00000
3	0.100000E 00	0.210000	1.00000
4	0.200000	0.180000	1.00000
5	0.200000	0.220000	1.00000
6	0.300000	0.170000	1.00000
7	0.300000	0.230000	1.00000
8	0.400000	0.160000	1.00000
9	0.400000	0.240000	1.00000
10	0.500000	0.150000	1.00000
11	0.500000	0.250000	1.00000
12	0.600000	0.140000	1.00000
13	0.600000	0.260000	1.00000
14	0.700000	0.130000	1.00000
15	0.700000	0.270000	1.00000
16	0.800000	0.120000	1.00000
17	0.800000	0.280000	1.00000
18	0.900000	0.110000	1.00000
19	0.900000	0.290000	1.00000
20	1.00000	0.100000E 00	0.0
21	1.00000	0.300000	0.0

*** ELEMENT CONNECTIVITY ***
 *** MATERIAL PROPERTY NUMBER ***
 *** ANGLE BETWEEN ELEMENT AND SYSTEM AXES ***

ELEMENT	I	J	K	L	MATERIAL	ANGLE
1	1	2	3	3	1	0.0
2	2	4	5	3	1	0.0
3	4	6	7	5	1	0.0
4	6	8	9	7	1	0.0
5	8	10	11	9	1	0.0
6	10	12	13	11	1	0.0
7	12	14	15	13	1	0.0
8	14	16	17	15	1	0.0
9	16	18	19	17	1	0.0
10	18	20	21	19	1	0.0

*** PARAMETERS FOR TIME INTERVAL NO. 1 ***

TIME STEP SIZE = 0.100000E-02
 NUMBER OF TIME STEPS = 10
 (PRINT STEP SIZE) / (TIME STEP SIZE) = 5
 NUMBER OF CONVECTION BOUNDARY CARDS = 0
 NUMBER OF RADIATION BOUNDARY CARDS = 0
 NUMBER OF TEMPERATURE BOUNDARY CARDS = 2
 MATERIAL PROPERTY CHANGE INDICATOR = 0
 (A ZERO INDICATES A CHANGE FROM PRECEDING TIME INTERVAL)

*** SPECIFIED TEMPERATURES ***

NODE	TEMPERATURE
------	-------------

20	0.0
21	0.0

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.500000E-02

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	1.00000	2	1.00000	3	1.00000	4	0.999997
5	0.999997	6	0.999982	7	0.999982	8	0.999896
9	0.999896	10	0.999393	11	0.999392	12	0.996549
13	0.996549	14	0.981667	15	0.981667	16	0.912791
17	0.912790	18	0.653902	19	0.653901	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.999999E-02

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.999992	2	0.999979	3	0.999979	4	0.999921
5	0.999921	6	0.999685	7	0.999685	8	0.998714
9	0.998714	10	0.994835	11	0.994835	12	0.980398
13	0.980398	14	0.932180	15	0.932180	16	0.794664
17	0.794663	18	0.484321	19	0.484320	20	0.0
21	0.0						

*** PARAMETERS FOR TIME INTERVAL NO. 2 ***

TIME STEP SIZE = 0.500000E-02

NUMBER OF TIME STEPS = 18

(PRINT STEP SIZE) / (TIME STEP SIZE) = 2

NUMBER OF CONVECTION BOUNDARY CARDS = 0

NUMBER OF RADIATION BOUNDARY CARDS = 0

NUMBER OF TEMPERATURE BOUNDARY CARDS = 2

MATERIAL PROPERTY CHANGE INDICATOR = 1

(A ZERO INDICATES A CHANGE FROM PRECEDING TIME INTERVAL)

*** SPECIFIED TEMPERATURES ***

NODE	TEMPERATURE
------	-------------

20 0.0

21 0.0

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.200000E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.998730	2	0.998087	3	0.998087	4	0.996318
5	0.996318	6	0.992146	7	0.992146	8	0.982275
9	0.982275	10	0.959484	11	0.959484	12	0.909288
13	0.909288	14	0.806906	15	0.806906	16	0.621024
17	0.621024	18	0.336904	19	0.336904	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.300000E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.992925	2	0.990455	3	0.990455	4	0.984417
5	0.984417	6	0.972320	7	0.972320	8	0.948603
9	0.948603	10	0.904003	11	0.904003	12	0.824773
13	0.824773	14	0.694677	15	0.694677	16	0.502798
17	0.502798	18	0.257879	19	0.257879	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.400000E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.979200	2	0.973858	3	0.973858	4	0.961731
5	0.961731	6	0.939840	7	0.939840	8	0.901800
9	0.901800	10	0.839044	11	0.839044	12	0.741659
13	0.741659	14	0.601690	15	0.601690	16	0.419148
17	0.419148	18	0.208535	19	0.208535	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.500000E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.955764	2	0.947121	3	0.947121	4	0.928475
5	0.928475	6	0.897171	7	0.897171	8	0.847188
9	0.847188	10	0.771923	11	0.771923	12	0.665488
13	0.665488	14	0.525576	15	0.525576	16	0.357076
17	0.357075	18	0.174433	19	0.174433	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.600000E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.922762	2	0.911012	3	0.911012	4	0.886567
5	0.886567	6	0.847619	7	0.847618	8	0.789126
9	0.789126	10	0.706665	11	0.706665	12	0.597535
13	0.597535	14	0.462777	15	0.462777	16	0.309065
17	0.309065	18	0.149193	19	0.149193	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.699999E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.881722	2	0.867458	3	0.867458	4	0.838559
5	0.838559	6	0.794242	7	0.794241	8	0.730622
9	0.730622	10	0.645175	11	0.645175	12	0.537438
13	0.537438	14	0.410261	15	0.410261	16	0.270671
17	0.270671	18	0.129591	19	0.129591	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.799999E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.834818	2	0.818789	3	0.818789	4	0.786942
5	0.786942	6	0.739468	7	0.739468	8	0.673566
9	0.673566	10	0.588187	11	0.588187	12	0.484330
13	0.484330	14	0.365729	15	0.365729	16	0.239147
17	0.239146	18	0.113825	19	0.113825	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.899999E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.784287	2	0.767229	3	0.767229	4	0.733822
5	0.733822	6	0.685067	7	0.685067	8	0.619064
9	0.619064	10	0.535836	11	0.535835	12	0.437282
13	0.437282	14	0.327488	15	0.327488	16	0.212717
17	0.212716	18	0.100806	19	0.100806	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.999999E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.732109	2	0.714651	3	0.714651	4	0.680828
5	0.680828	6	0.632242	7	0.632242	8	0.567703
9	0.567703	10	0.487967	11	0.487967	12	0.395449
13	0.395449	14	0.294289	15	0.294289	16	0.190186
17	0.190186	18	0.898341D-01	19	0.898340D-01	20	0.0
21	0.0						

*** PARAMETERS FOR TIME INTERVAL NO. 3 ***

TIME STEP SIZE = 0.100000E-01
 NUMBER OF TIME STEPS = 10
 (PRINT STEP SIZE) / (TIME STEP SIZE) = 5
 NUMBER OF CONVECTION BOUNDARY CARDS = 0
 NUMBER OF RADIATION BOUNDARY CARDS = 0
 NUMBER OF TEMPERATURE BOUNDARY CARDS = 2
 MATERIAL PROPERTY CHANGE INDICATOR = 1
 (A ZERO INDICATES A CHANGE FROM PRECEDING TIME INTERVAL)

*** SPECIFIED TEMPERATURES ***

NODE	TEMPERATURE
20	0.0
21	0.0

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.150000

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.492447	2	0.478061	3	0.478061	4	0.450855
5	0.450855	6	0.413143	7	0.413143	8	0.365154
9	0.365154	10	0.308566	11	0.308566	12	0.245901
13	0.245901	14	0.180270	15	0.180270	16	0.115127
17	0.115127	18	0.539684D-01	19	0.539683D-01	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.200000

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.318055	2	0.308187	3	0.308187	4	0.289657
5	0.289657	6	0.264244	7	0.264243	8	0.232319
9	0.232319	10	0.195209	11	0.195209	12	0.154706
13	0.154706	14	0.112857	15	0.112857	16	0.717952D-01
17	0.717952D-01	18	0.335724D-01	19	0.335724D-01	20	0.0
21	0.0						

*** PARAMETERS FOR TIME INTERVAL NO. 4 ***

TIME STEP SIZE = 0.250000E-01
 NUMBER OF TIME STEPS = 24
 (PRINT STEP SIZE) / (TIME STEP SIZE) = 4
 NUMBER OF CONVECTION BOUNDARY CARDS = 0
 NUMBER OF RADIATION BOUNDARY CARDS = 0
 NUMBER OF TEMPERATURE BOUNDARY CARDS = 2
 MATERIAL PROPERTY CHANGE INDICATOR = 1
 (A ZERO INDICATES A CHANGE FROM PRECEDING TIME INTERVAL)

*** SPECIFIED TEMPERATURES ***

NODE	TEMPERATURE
20	0.0
21	0.0

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.300000

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.135888	2	0.131571	3	0.131571	4	0.123487
5	0.123487	6	0.112446	7	0.112446	8	0.986470D-01
9	0.986470D-01	10	0.826974D-01	11	0.826973D-01	12	0.653899D-01
13	0.653899D-01	14	0.476052D-01	15	0.476052D-01	16	0.302366D-01
17	0.302366D-01	18	0.141247D-01	19	0.141247D-01	20	0.0
21	0.0						

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.400000

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	0.576644D-01	2	0.558251D-01	3	0.558251D-01	4	0.523823D-01
5	0.523823D-01	6	0.476834D-01	7	0.476834D-01	8	0.418160D-01
9	0.418160D-01	10	0.350408D-01	11	0.350407D-01	12	0.276962D-01
13	0.276962D-01	14	0.201562D-01	15	0.201562D-01	16	0.127987D-01
17	0.127987D-01	18	0.597771D-02	19	0.597770D-02	20	0.0
21	0.0						

II. EXAMPLE PROBLEM 2--VENTURI INLET

This venturi inlet has air flowing through it with stagnation temperature and pressure of 700 °F and 70 psi upstream of the venturi. The venturi wall has an initial temperature of 70 °F. The air temperature outside the venturi wall is 700 °F. Forced convection heat transfer is accounted for on the inside surface. Radiation heat transfer is accounted for on the outside surface. The downstream surface of the flange is assumed to be insulated.

The analytical model, Figure 8, has a maximum bandwidth of 12. Special input features for node points and elements were used. Thirty-four nodal point cards were required to specify the data for the 150 nodes. Fifteen element cards were required to specify the data for the 117 elements. The special features for convection and radiation boundary conditions were also used. The same surface conductance values were input for both time intervals. TRAX modified these constant fluid temperature values to account for wall temperature at each time interval. An exponent of $-.25$ was used as recommended in Reference (9). The external temperature for convection was assumed to be the adiabatic wall temperature. For radiation, the heat flux was calculated by TRAX. The constants C_w and C_1 were assumed to be $.005556$. This corresponds to an emissivity of $.8$ multiplied by the proper conversion factor from feet to inches. T_1 was 1160 degrees Rankine. The other units used were inches,

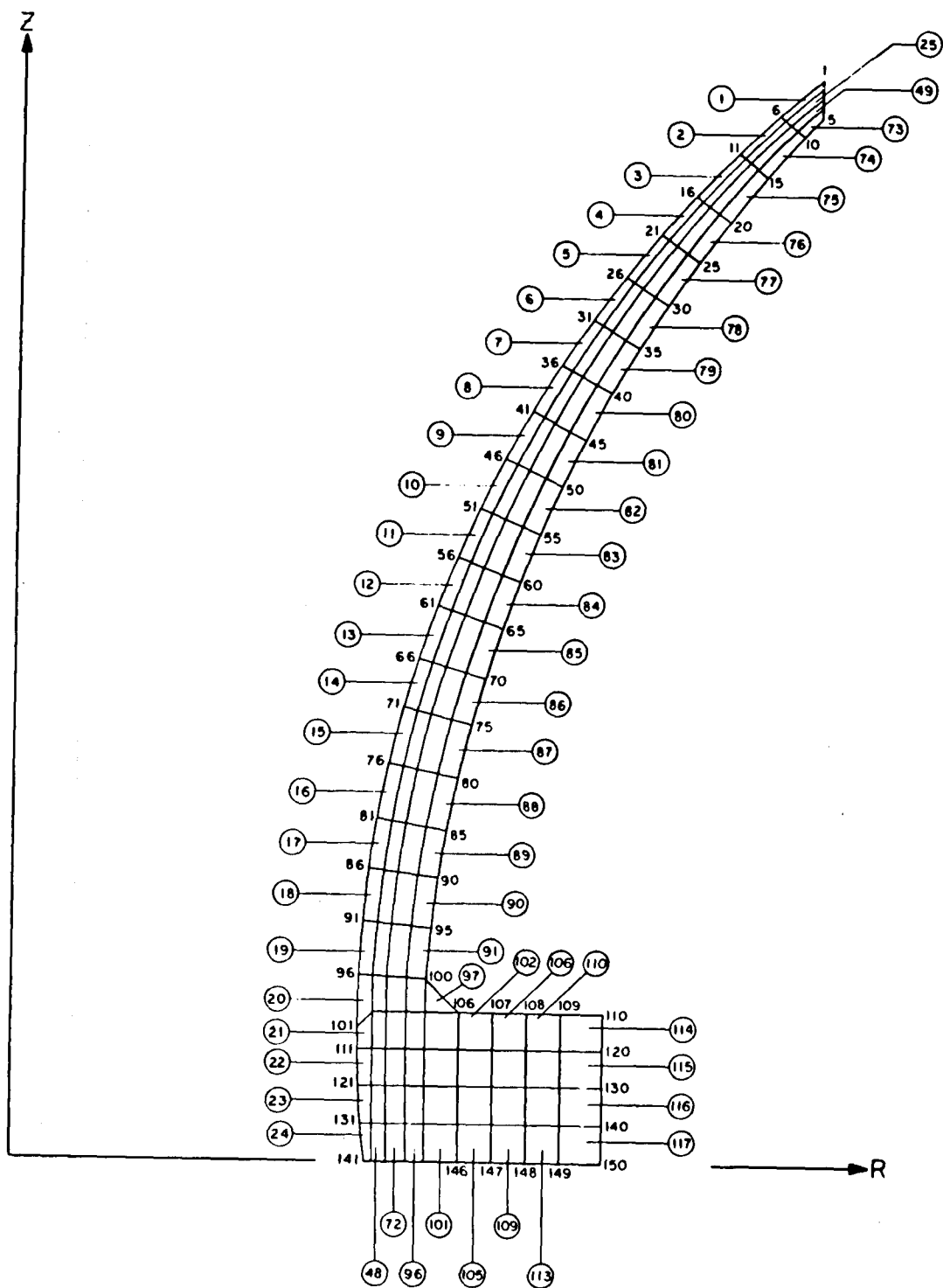


Figure 8. Analytical model for example problem 2.

BTUs, pounds, and hours. The total computer time required was 15.5 seconds.

A review of the results reveals that the convection influence is much greater than the radiation influence. The massive flange remains near the initial temperature longest, as expected. The effect of increasing velocity and hence surface conductance is seen by the increase in wall temperature from the entrance to the throat region. In short, the calculated temperature distributions are logical.

Computer Output--Example Problem 2¹

EXAMPLE PROBLEM 2 - VENTURI INLET WITH CONVECTION AND RADIATION

*** BASIC PARAMETERS ***

NUMBER OF NODES = 150
 NUMBER OF ELEMENTS = 117
 NUMBER OF MATERIALS = 1
 NUMBER OF TIME INTERVALS = 2
 REFERENCE TEMPERATURE = 530.000

*** MATERIAL PROPERTIES ***

MATERIAL NUMBER 1

TEMPERATURE	R CONDUCTIVITY	Z CONDUCTIVITY	SPECIFIC HEAT	DENSITY
492.000	2.04000	2.04000	0.110000	0.284000
672.000	2.04000	2.04000	0.114000	0.284000
852.000	2.02000	2.02000	0.123000	0.284000
1032.00	1.96000	1.96000	0.130000	0.284000
1212.00	1.87000	1.87000	0.142000	0.284000

¹In order to conserve space, only the last temperature distribution from each time interval is included.

*** NODAL COORDINATES AND INITIAL TEMPERATURES ***

NODE	R	Z	TEMPERATURE
1	11.8100	16.3700	530.000
2	11.8100	16.2450	530.000
3	11.8100	16.1200	530.000
4	11.8100	15.9950	530.000
5	11.8100	15.8700	530.000
6	11.1900	15.8000	530.000
7	11.2600	15.7500	530.000
8	11.3300	15.7000	530.000
9	11.4400	15.5700	530.000
10	11.5400	15.4700	530.000
11	10.6004	15.2348	530.000
12	10.6813	15.2580	530.000
13	10.7697	15.1076	530.000
14	10.9035	15.0103	530.000
15	11.0249	14.9119	530.000
16	10.0360	14.6493	530.000
17	10.1272	14.6659	530.000
18	10.2335	14.5105	530.000
19	10.3903	14.4077	530.000
20	10.5321	14.3036	530.000
21	9.49769	14.0397	530.000
22	9.59875	14.0508	530.000
23	9.72241	13.8919	530.000
24	9.90091	13.7855	530.000
25	10.0623	13.6773	530.000
26	8.98668	13.4070	530.000
27	9.09694	13.4138	530.000
28	9.23718	13.2527	530.000
29	9.43625	13.1447	530.000
30	9.61615	13.0341	530.000
31	8.50388	12.7525	530.000
32	8.62266	12.7561	530.000
33	8.77869	12.5941	530.000
34	8.99699	12.4861	530.000
35	9.19423	12.3746	530.000
36	8.05023	12.0775	530.000
37	8.17677	12.0788	530.000
38	8.34767	11.9172	530.000
39	8.58380	11.8110	530.000
40	8.79713	11.6999	530.000
41	7.62660	11.3832	530.000
42	7.76006	11.3831	530.000
43	7.94490	11.2231	530.000
44	8.19731	11.1202	530.000
45	8.42542	11.0109	530.000
46	7.23381	10.6711	530.000
47	7.37329	10.6704	530.000
48	7.57103	10.5130	530.000
49	7.83812	10.4148	530.000
50	8.07959	10.3085	530.000

51	6.87262	9.94241	530.000
52	7.01717	9.94186	530.000
53	7.22672	9.78811	530.000
54	7.50676	9.69588	530.000
55	7.76015	9.59381	530.000
56	6.54372	9.19860	530.000
57	6.69231	9.19886	530.000
58	6.91254	9.04969	530.000
59	7.20377	8.96460	530.000
60	7.46753	8.86767	530.000
61	6.24771	8.44108	530.000
62	6.39934	8.44272	530.000
63	6.62901	8.29894	530.000
64	6.92957	8.22203	530.000
65	7.20213	8.13114	530.000
66	5.98520	7.67132	530.000
67	6.13875	7.67482	530.000
68	6.37663	7.53716	530.000
69	6.68460	7.46932	530.000
70	6.96432	7.38527	530.000
71	5.75670	6.89081	530.000
72	5.91104	6.89653	530.000
73	6.15584	6.76564	530.000
74	6.46924	6.70761	530.000
75	6.75444	6.63104	530.000
76	5.56261	6.10102	530.000
77	5.71661	6.10927	530.000
78	5.96698	5.98570	530.000
79	6.28381	5.93806	530.000
80	6.57277	5.86955	530.000
81	5.40332	5.30347	530.000
82	5.55582	5.31446	530.000
83	5.81039	5.19863	530.000
84	6.12860	5.16186	530.000
85	6.41956	5.10180	530.000
86	5.27916	4.49972	530.000
87	5.42894	4.51354	530.000
88	5.68634	4.40577	530.000
89	6.00385	4.38018	530.000
90	6.29503	4.32888	530.000
91	5.19034	3.69131	530.000
92	5.33623	3.70794	530.000
93	5.59505	3.60851	530.000
94	5.90973	3.59420	530.000
95	6.19934	3.55189	530.000
96	5.13702	2.87977	530.000
97	5.22000	2.85000	530.000
98	5.55000	2.82000	530.000
99	5.83000	2.79000	530.000
100	6.11000	2.75000	530.000

101	5.12300	2.07000	530.000
102	5.33000	2.25000	530.000
103	5.55000	2.25000	530.000
104	5.83000	2.25000	530.000
105	6.11000	2.25000	530.000
106	6.61000	2.25000	530.000
107	7.11000	2.25000	530.000
108	7.61000	2.25000	530.000
109	8.11000	2.25000	530.000
110	8.75000	2.25000	530.000
111	5.12400	1.58750	530.000
112	5.33000	1.68750	530.000
113	5.55000	1.68750	530.000
114	5.83000	1.68750	530.000
115	6.11000	1.68750	530.000
116	6.61000	1.68750	530.000
117	7.11000	1.68750	530.000
118	7.61000	1.68750	530.000
119	8.11000	1.68750	530.000
120	8.75000	1.68750	530.000
121	5.14000	1.02500	530.000
122	5.33000	1.12500	530.000
123	5.55000	1.12500	530.000
124	5.83000	1.12500	530.000
125	6.11000	1.12500	530.000
126	6.61000	1.12500	530.000
127	7.11000	1.12500	530.000
128	7.61000	1.12500	530.000
129	8.11000	1.12500	530.000
130	8.75000	1.12500	530.000
131	5.19000	0.562500	530.000
132	5.33000	0.562500	530.000
133	5.55000	0.562500	530.000
134	5.83000	0.562500	530.000
135	6.11000	0.562500	530.000
136	6.61000	0.562500	530.000
137	7.11000	0.562500	530.000
138	7.61000	0.562500	530.000
139	8.11000	0.562500	530.000
140	8.75000	0.562500	530.000
141	5.23000	0.0	530.000
142	5.33000	0.0	530.000
143	5.55000	0.0	530.000
144	5.83000	0.0	530.000
145	6.11000	0.0	530.000
146	6.61000	0.0	530.000
147	7.11000	0.0	530.000
148	7.61000	0.0	530.000
149	8.11000	0.0	530.000
150	8.75000	0.0	530.000

*** ELEMENT CONNECTIVITY ***

*** MATERIAL PROPERTY NUMBER ***

*** ANGLE BETWEEN ELEMENT AND SYSTEM AXES ***

ELEMENT	I	J	K	L	MATERIAL	ANGLE
1	1	6	7	2	1	0.0
2	6	11	12	7	1	0.0
3	11	16	17	12	1	0.0
4	16	21	22	17	1	0.0
5	21	26	27	22	1	0.0
6	26	31	32	27	1	0.0
7	31	36	37	32	1	0.0
8	36	41	42	37	1	0.0
9	41	46	47	42	1	0.0
10	46	51	52	47	1	0.0
11	51	56	57	52	1	0.0
12	56	61	62	57	1	0.0
13	61	66	67	62	1	0.0
14	66	71	72	67	1	0.0
15	71	76	77	72	1	0.0
16	76	81	82	77	1	0.0
17	81	86	87	82	1	0.0
18	86	91	92	87	1	0.0
19	91	96	97	92	1	0.0
20	96	101	102	97	1	0.0
21	101	111	112	102	1	0.0
22	111	121	122	112	1	0.0
23	121	131	132	122	1	0.0
24	131	141	142	132	1	0.0
25	7	8	3	2	1	0.0
26	12	13	8	7	1	0.0
27	17	18	13	12	1	0.0
28	22	23	18	17	1	0.0
29	27	28	23	22	1	0.0
30	32	33	28	27	1	0.0
31	37	38	33	32	1	0.0
32	42	43	38	37	1	0.0
33	47	48	43	42	1	0.0
34	52	53	48	47	1	0.0
35	57	58	53	52	1	0.0
36	62	63	58	57	1	0.0
37	67	68	63	62	1	0.0
38	72	73	68	67	1	0.0
39	77	78	73	72	1	0.0
40	82	83	78	77	1	0.0
41	87	88	83	82	1	0.0
42	92	93	88	87	1	0.0
43	97	98	93	92	1	0.0
44	102	103	98	97	1	0.0
45	112	113	103	102	1	0.0
46	122	123	113	112	1	0.0
47	132	133	123	122	1	0.0
48	142	143	133	132	1	0.0
49	8	9	4	3	1	0.0
50	13	14	9	8	1	0.0

51	18	19	14	13	1	0.0
52	23	24	19	18	1	0.0
53	28	29	24	23	1	0.0
54	33	34	29	28	1	0.0
55	38	39	34	33	1	0.0
56	43	44	39	38	1	0.0
57	48	49	44	43	1	0.0
58	53	54	49	48	1	0.0
59	58	59	54	53	1	0.0
60	63	64	59	58	1	0.0
61	68	69	64	63	1	0.0
62	73	74	69	68	1	0.0
63	78	79	74	73	1	0.0
64	83	84	79	78	1	0.0
65	88	89	84	83	1	0.0
66	93	94	89	88	1	0.0
67	98	99	94	93	1	0.0
68	103	104	99	98	1	0.0
69	113	114	104	103	1	0.0
70	123	124	114	113	1	0.0
71	133	134	124	123	1	0.0
72	143	144	134	133	1	0.0
73	9	10	5	4	1	0.0
74	14	15	10	9	1	0.0
75	19	20	15	14	1	0.0
76	24	25	20	19	1	0.0
77	29	30	25	24	1	0.0
78	34	35	30	29	1	0.0
79	39	40	35	34	1	0.0
80	44	45	40	39	1	0.0
81	49	50	45	44	1	0.0
82	54	55	50	49	1	0.0
83	59	60	55	54	1	0.0
84	64	65	60	59	1	0.0
85	69	70	65	64	1	0.0
86	74	75	70	69	1	0.0
87	79	80	75	74	1	0.0
88	84	85	80	79	1	0.0
89	89	90	85	84	1	0.0
90	94	95	90	89	1	0.0
91	99	100	95	94	1	0.0
92	104	105	100	99	1	0.0
93	114	115	105	104	1	0.0
94	124	125	115	114	1	0.0
95	134	135	125	124	1	0.0
96	144	145	135	134	1	0.0
97	100	105	106	106	1	0.0
98	105	115	116	106	1	0.0
99	115	125	126	116	1	0.0
100	125	135	136	126	1	0.0

101	135	145	146	136	1	0.0
102	106	116	117	107	1	0.0
103	116	126	127	117	1	0.0
104	126	136	137	127	1	0.0
105	136	146	147	137	1	0.0
106	107	117	118	108	1	0.0
107	117	127	128	118	1	0.0
108	127	137	138	128	1	0.0
109	137	147	148	138	1	0.0
110	108	118	119	109	1	0.0
111	118	128	129	119	1	0.0
112	128	138	139	129	1	0.0
113	138	148	149	139	1	0.0
114	109	119	120	110	1	0.0
115	119	129	130	120	1	0.0
116	129	139	140	130	1	0.0
117	139	149	150	140	1	0.0

*** PARAMETERS FOR TIME INTERVAL NO. 1 ***

TIME STEP SIZE = 0.277800E-03

NUMBER OF TIME STEPS = 10

(PRINT STEP SIZE) / (TIME STEP SIZE) = 2

NUMBER OF CONVECTION BOUNDARY CARDS = 24

NUMBER OF RADIATION BOUNDARY CARDS = 32

NUMBER OF TEMPERATURE BOUNDARY CARDS = 0

MATERIAL PROPERTY CHANGE INDICATOR = 0

(A ZERO INDICATES A CHANGE FROM PRECEDING TIME INTERVAL)

*** CONVECTION BOUNDARY CONDITIONS ***

I NODE	J NODE	SURFACE CONDUCTANCE	EXTERNAL TEMPERATURE CONSTANTS		
			A	B	C
1	6	4.13546	1160.00	0.0	0.0
6	11	4.50036	1160.00	0.0	0.0
11	16	4.86525	1160.00	0.0	0.0
16	21	5.34021	1155.00	0.0	0.0
21	26	5.94705	1155.00	0.0	0.0
26	31	6.43252	1155.00	0.0	0.0
31	36	7.03937	1155.00	0.0	0.0
36	41	7.64621	1155.00	0.0	0.0
41	46	8.35615	1155.00	0.0	0.0
46	51	8.84057	1155.00	0.0	0.0
51	56	9.42530	1155.00	0.0	0.0
56	61	10.1503	1155.00	0.0	0.0
61	66	10.8269	1155.00	0.0	0.0
66	71	11.7627	1155.00	0.0	0.0
71	76	12.4543	1155.00	0.0	0.0
76	81	13.1426	1155.00	0.0	0.0
81	86	13.5570	1150.00	0.0	0.0
86	91	13.8142	1140.00	0.0	0.0
91	96	14.0991	1140.00	0.0	0.0
96	101	14.2626	1135.00	0.0	0.0
101	111	14.2693	1130.00	0.0	0.0
111	121	14.0790	1130.00	0.0	0.0
121	131	13.6592	1130.00	0.0	0.0
131	141	13.0911	1130.00	0.0	0.0

*** RADIATION BOUNDARY CONDITIONS ***

I NODE	J NODE	HEAT FLUX
1	2	16.7414
2	3	16.7414
3	4	16.7414
4	5	16.7414
5	10	16.7414
10	15	16.7414
15	20	16.7414
20	25	16.7414
25	30	16.7414
30	35	16.7414
35	40	16.7414
40	45	16.7414
45	50	16.7414
50	55	16.7414
55	60	16.7414
60	65	16.7414
65	70	16.7414
70	75	16.7414
75	80	16.7414
80	85	16.7414
85	90	16.7414
90	95	16.7414
95	100	16.7414
100	106	16.7414
106	107	16.7414
107	108	16.7414
108	109	16.7414
109	110	16.7414
110	120	16.7414
120	130	16.7414
130	140	16.7414
140	150	16.7414

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.277800E-02

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	1010.07	2	972.016	3	936.437	4	907.197
5	887.534	6	927.237	7	894.275	8	867.209
9	829.133	10	818.594	11	921.355	12	902.225
13	833.623	14	792.805	15	780.838	16	923.226
17	894.088	18	814.245	19	764.917	20	750.346
21	930.300	22	891.253	23	800.701	24	742.445
25	725.340	26	940.028	27	891.355	28	790.548
29	723.998	30	704.591	31	949.619	32	891.978
33	782.105	34	708.354	35	687.024	36	960.343
37	894.084	38	775.829	39	695.650	40	672.684
41	971.362	42	897.133	43	771.137	44	685.349
45	661.032	46	980.473	47	899.325	48	766.827
49	676.564	50	651.255	51	987.601	52	900.559
53	762.950	54	669.218	55	643.214	56	996.238
57	903.620	58	761.048	59	664.025	60	637.374
61	1004.58	62	907.368	63	760.523	64	660.615
65	633.389	66	1013.39	67	912.319	68	761.545
69	658.929	70	631.138	71	1021.51	72	917.613
73	763.539	74	658.594	75	630.280	76	1027.75
77	922.224	78	765.821	79	659.178	80	630.459
81	1030.71	82	925.067	83	767.663	84	660.307
85	631.389	86	1026.26	87	923.277	88	768.153
89	661.659	90	632.884	91	1025.30	92	924.616
93	770.251	94	664.707	95	635.398	96	1023.50
97	963.911	98	766.006	99	663.687	100	620.690
101	1020.02	102	877.510	103	752.859	104	647.779
105	589.719	106	549.784	107	536.584	108	534.283
109	534.504	110	537.125	111	1016.71	112	874.672
113	751.069	114	643.316	115	581.343	116	542.942
117	533.440	118	531.642	119	531.927	120	534.556
121	1017.29	122	883.497	123	757.654	124	646.608
125	582.030	126	542.090	127	532.639	128	530.874
129	531.165	130	533.794	131	1006.08	132	905.956
133	772.929	134	654.814	135	585.721	136	542.767
137	532.626	138	530.727	139	530.995	140	533.620
141	996.010	142	921.841	143	783.008	144	659.991
145	588.025	146	543.263	147	532.699	148	530.717
149	530.969	150	533.593				

*** PARAMETERS FOR TIME INTERVAL NO. 2 ***

TIME STEP SIZE = 0.277800E-02
 NUMBER OF TIME STEPS = 5
 (PRINT STEP SIZE) / (TIME STEP SIZE) = 1
 NUMBER OF CONVECTION BOUNDARY CARDS = 24
 NUMBER OF RADIATION BOUNDARY CARDS = 32
 NUMBER OF TEMPERATURE BOUNDARY CARDS = 0
 MATERIAL PROPERTY CHANGE INDICATOR = 0
 (A ZERO INDICATES A CHANGE FROM PRECEDING TIME INTERVAL)

*** CONVECTION BOUNDARY CONDITIONS ***

I NODE	J NODE	SURFACE CONDUCTANCE	EXTERNAL A	TEMPERATURE B	CONSTANTS C
1	6	3.55673	1160.00	0.0	0.0
6	11	3.91619	1160.00	0.0	0.0
11	16	4.23602	1160.00	0.0	0.0
16	21	4.64392	1155.00	0.0	0.0
21	26	5.15999	1155.00	0.0	0.0
26	31	5.56689	1155.00	0.0	0.0
31	36	6.07580	1155.00	0.0	0.0
36	41	6.58093	1155.00	0.0	0.0
41	46	7.17335	1155.00	0.0	0.0
46	51	7.57349	1155.00	0.0	0.0
51	56	8.05833	1155.00	0.0	0.0
56	61	8.65974	1155.00	0.0	0.0
61	66	9.21726	1155.00	0.0	0.0
66	71	9.99299	1155.00	0.0	0.0
71	76	10.5620	1155.00	0.0	0.0
76	81	11.1332	1155.00	0.0	0.0
81	86	11.4863	1150.00	0.0	0.0
86	91	11.7120	1140.00	0.0	0.0
91	96	11.9576	1140.00	0.0	0.0
96	101	12.1041	1135.00	0.0	0.0
101	111	12.1198	1130.00	0.0	0.0
111	121	11.9622	1130.00	0.0	0.0
121	131	11.6207	1130.00	0.0	0.0
131	141	11.1669	1130.00	0.0	0.0

*** RADIATION BOUNDARY CONDITIONS ***

I NODE	J NODE	HEAT FLUX
1	2	8.17856
2	3	9.48899
3	4	10.5237
4	5	11.2354
5	10	12.3846
10	15	13.5501
15	20	14.1830
20	25	14.6390
25	30	14.9781
30	35	15.2382
35	40	15.4390
40	45	15.5924
45	50	15.7124
50	55	15.8077
55	60	15.8793
60	65	15.9286
65	70	15.9593
70	75	15.9745
75	80	15.9777
80	85	15.9724
85	90	15.9606
90	95	15.9409
95	100	16.0001
100	106	16.3702
106	107	16.6626
107	108	16.7097
108	109	16.7158
109	110	16.7074
110	120	16.7072
120	130	16.7171
130	140	16.7198
140	150	16.7204

*** TEMPERATURE DISTRIBUTION ***

TIME = 0.166680E-01

NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE	NODE	TEMPERATURE
1	1148.50	2	1144.54	3	1140.77	4	1137.56
5	1135.20	6	1134.74	7	1131.47	8	1128.92
9	1125.28	10	1124.77	11	1127.31	12	1125.50
13	1116.51	14	1112.13	15	1111.48	16	1120.16
17	1116.47	18	1104.49	19	1098.20	20	1097.07
21	1114.93	22	1108.94	23	1093.95	24	1085.41
25	1083.72	26	1112.60	27	1103.95	28	1085.50
29	1074.39	30	1072.02	31	1111.12	32	1099.83
33	1078.00	34	1064.31	35	1061.25	36	1110.62
37	1096.64	38	1071.50	39	1055.21	40	1051.46
41	1110.87	42	1094.24	43	1065.87	44	1047.06
45	1042.61	46	1111.16	47	1092.07	48	1060.76
49	1039.63	50	1034.54	51	1111.42	52	1090.14
53	1056.31	54	1033.11	55	1027.46	56	1112.57
57	1089.25	58	1053.17	59	1028.10	60	1021.94
61	1114.09	62	1089.13	63	1051.26	64	1024.64
65	1018.07	66	1116.11	67	1089.86	68	1050.58
69	1022.74	70	1015.84	71	1118.17	72	1091.06
73	1050.76	74	1022.03	75	1014.90	76	1119.93
77	1092.25	78	1051.10	79	1021.79	80	1014.50
81	1119.21	82	1091.50	83	1049.78	84	1020.35
85	1012.97	86	1112.13	87	1085.50	88	1044.19
89	1014.97	90	1007.43	91	1106.55	92	1078.17
93	1032.35	94	1000.37	95	990.958	96	1094.67
97	1072.80	98	998.924	99	952.164	100	921.683
101	1079.54	102	1021.94	103	963.064	104	899.118
105	843.125	106	743.510	107	653.787	108	606.333
109	583.491	110	578.048	111	1072.98	112	1005.75
113	940.662	114	867.772	115	804.777	116	711.701
117	642.580	118	600.504	119	579.171	120	574.095
121	1070.37	122	1003.08	123	933.439	124	855.792
125	789.844	126	697.687	127	634.372	128	595.589
129	575.705	130	571.093	131	1062.60	132	1010.22
133	935.828	134	854.048	135	785.620	136	692.368
137	630.317	138	592.783	139	573.624	140	569.288
141	1055.96	142	1016.77	143	939.130	144	854.955
145	785.203	146	691.144	147	629.181	148	591.914
149	572.950	150	568.696				

CHAPTER VII

FUTURE CONSIDERATIONS

Program TRAX was written to solve the type problems frequently encountered at Arnold Engineering Development Center. For this reason emphasis was placed on convection boundary conditions while less emphasis was placed on radiation boundary conditions. Heat sources within the body were not included in the formulation because no problems requiring this capability have been encountered by the writer.

TRAX can be readily modified to place emphasis in areas where the present version is relatively weak. These additions take computer core space, however. The worth of each new capability must be weighed against the loss in size of problem which can be solved in core. Every effort was made, in programming TRAX, to conserve computer storage by applying object time dimensioning and a system of pointers to locate the beginning of each array of data. In this way no space is wasted. Only that data which is required for current use in the solution procedure is retained in core. Other data required for future use in the program is retained on files. Any modification should apply the same philosophy.

Two areas of study which may lead to future basic improvements in finite element heat conduction programs are

the use of isoparametric elements (5) and special solution procedures which include radiative exchanges implicitly in the formulation in a manner similar to the present method for convective exchanges (6). Reference (12) should provide an excellent source of background information for future developments in the field.

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BIBLIOGRAPHY

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VITA

James Kenneth Rochelle was born on November 19, 1942 in Florence, Alabama to James D. and Katherine M. Rochelle. He attended public schools in Columbia and Gallatin, Tennessee, graduating from Gallatin High School in 1960.

The author attended college at Tennessee Technological University in Cookeville, Tennessee, obtaining a Bachelor of Science degree in Mechanical Engineering in June, 1964. In August, 1963 he married Carolyn Kaufman, the daughter of Dr. J. K. Kaufman of Murfreesboro and Robbie H. Delbridge of Gallatin.

He began work at his present position as an engineer with ARO, Inc. in June, 1964. He became a registered professional engineer in the state of Tennessee in September, 1970.

The Rochelles live at 934 McKeller Drive in Tullahoma. They have two sons--Kenneth Howard and William Kaufman.