August 11, 1939

To the Committee on Graduate Study:

I am submitting to you a thesis written by Will Otis Leffell entitled "The Design and Application of an Electronic Phase Angle Meter." I recommend that it be accepted for eighteen quarter hours credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

J. G. Tarboux
Major Professor

We have read this thesis
and recommend its acceptance:

Chas. E. Ferris

Edgar D. Eaves

Accepted for the committee

Dean of the Graduate School

THE DESIGN AND APPLICATION OF
AN ELECTRONIC PHASE ANGLE METER

A THESIS

Submitted to
the Committee on Graduate Study
of
The University of Tennessee
in
Partial Fulfillment of the Requirements
for the degree of
Master of Science

by

Will Otis Leffell

August 1939

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CHAPTER I

INTRODUCTION

The purpose of this paper is to consider some problems that are involved in the design and application of an electronic phase angle meter. A phase angle meter is a device that has long been used in power work. However, the meters used take considerable energy from the circuits, and will not respond to very rapid changes in the phase angle. Most of these meters work on the principle of the Tuma phase meter, and commonly indicate the angle between a current and a voltage. In power work, while the energy consumption is not so important, a meter is needed that will read transient angle phenomena, and this is the primary purpose of the type discussed in this paper.

Briefly, the electronic phase angle meter is a device that will produce a D. C. output voltage proportional to the sine, or cosine, function of the angle of displacement, θ, between two sinusoidal A. C. voltages.

Some advantages of such a meter will immediately present themselves. As the two input A. C. voltages are used only as grid excitation on electron tubes of the triode type, the power consumed in making a measurement is very small. This feature is of considerable advantage in communication circuits, where the total available energy is limited. Still another advantage is the rapidity of response to changes in the angle. This permits the observation of transient changes in the angle. A particular application of this type is in the measurement of the angle between the sending and receiving voltages on a transmission line during transient conditions.

When properly connected, the meter can be used as a wattmeter to measure low power levels in communication circuits. It also has possibilities as a harmonic analyzer.

While the circuit used is not original, the theory of its operation, as developed in this paper, is original. Also the application to the measurement of phase angles is, to the best of the writer's knowledge, a new use of the circuit.

The fundamental circuit theory has been briefly treated in a paper written in 1930.[1] However, the authors of this paper are using the circuit primarily as a wattmeter for use in communication circuits. In the paper referred to, matched tubes are assumed, and the plate circuit is loaded by a microammeter. A skew unbalance, produced in the plate voltages, is neglected, and the theory is too simple to be used as a criterion for accurate circuit adjustments.

The writer is deeply indebted to Dr. E. D. Shipley, of the University of Tennessee, for his many helpful suggestions in the preparation of this paper, and to Dr. J. G. Tarboux, who is interested in the development of a meter of this type for use in the study of transmission lines.

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[1] H. M. Turner and F. T. MacNamara, PROC. I. R. E., vol. 18, pp. 1743-1747; October, 1930.

CHAPTER II

THEORY OF OPERATION

In the following pages the theory of operation of the electronic phase angle meter is developed. In this analysis it is assumed that the two tubes used are somewhat identical in operating characteristics. However, as will be shown in this discussion, adjustments can be made to compensate for unlike operating characteristics. Another assumption made is that the two A. C. grid excitation voltages are constant in amplitude and equal in frequency, but differ in phase by the constant angle, θ. The problem of variable amplitude in the grid voltages is taken up in Chapter IV.

It is also assumed that the output voltage of the device is measured with a voltmeter of infinite impedance, so that no current is drawn from the meter output terminals. A means of measuring this voltage is discussed in Chapter III.

The fundamental circuit of the meter is shown in Fig. 1. Frequent reference will be made to this diagram in the following discussion.

The instantaneous plate current of the first tube in Fig. 1 is

$$i_{p1} = A_1\left(e_{g1} + \frac{e_{p1}}{M}\right)^2. \tag{1}$$

This equation holds for all practical purposes as long as the lower, curved, portion of the characteristic curve is considered. It is very necessary in the operation of this meter that the values of plate voltage and grid biasing voltage be chosen so that the operating point of the tube is within the lower, curved, portion of the characteristic curves. In certain types of vacuum tubes, the lower portion follows very closely a parabolic form, and it is essential that the tubes used in this circuit be of one of those types, as the theory of operation is developed on that basis. The constant, A_1, can be found from the characteristic curves. The symbol, M, is the amplification factor of the tube.

Two input signal voltages are assumed, of the form, $e_1 = E_1 \cos \omega t$, and $e_2 = E_2 \cos (\omega t - \theta)$, where $E_1 \neq E_2$. E_1 and E_2 are the maximum values of the voltage waves. The magnitude of E_1 and E_2 should be somewhat less than the value of the D. C. biasing voltage, E_c. They should never be allowed to exceed this value.

From the diagram, it can be seen that the total voltage impressed on the grid of the first tube is

$$e_{g1} = E_{11} \cos \omega t + E_2 \cos (\omega t - \theta) - E_c, \qquad (2)$$

where $E_{11} = 0.5E_1$, as equal grid resistors are assumed.

The plate voltage on the first tube is

$$e_{p1} = E_b - E_{01}, \text{ where } E_{01} = I_{10T} R_{p1}. \qquad (3)$$

I_{10T} is the total D. C. component of i_{p1}. Due to the presence of the condenser, C_1, the A. C. components flow through it and do not appear in the plate resistance R_{p1}, so only I_{10T} flows in R_{p1}, and produces the voltage drop E_{01}.

Following a similar line of reasoning, we can write the following fundamental relations for the second tube,

$$i_{p2} = A_2\left(e_{g2} + \frac{e_{p2}}{M}\right)^2 , \tag{4}$$

and

$$e_{g2} = -E_{12}\cos\omega t + E_2\cos(\omega t - \theta) - E_c, \tag{5}$$

also

$$e_{p2} = E_b - E_{02}, \text{ where, as before, } E_{02} = I_{20T}R_{p2}. \tag{6}$$

The definition of I_{20T} is of the same form as that for I_{10T}. A complete expression for i_{p1} can now be written as follows,

$$i_{p1} = A_1\left[E_{11}\cos\omega t + E_2\cos(\omega t - \theta) + \frac{E_b - ME_c - E_{01}}{M}\right]^2. \tag{7}$$

Expanding, we have,

$$\begin{aligned} i_{p1} = A_1\Big[&E_{11}^2\cos^2\omega t + 2E_{11}E_2\cos\omega t\cos(\omega t - \theta) + \\ &2\,\frac{E_b - ME_c - E_{01}}{M}\,E_{11}\cos\omega t + E_2^2\cos^2(\omega t - \theta) + \\ &2\,\frac{E_b - ME_c - E_{01}}{M}\,E_2\cos(\omega t - \theta) + \\ &\left(\frac{E_b - ME_c - E_{01}}{M}\right)^2\Big] . \end{aligned} \tag{8}$$

The D. C. component of the above current can be obtained by using well known trigonometric identities, and dropping all A. C. terms, which will be by-passed by the condenser, C_1, due to it's low capacitive reactance at these frequencies.

Thus I_{10T} becomes

$$I_{10T} = A_1\left[\frac{E_{11}^2}{2} + E_{11}E_2\cos\theta + \frac{E_2^2}{2} + \left(\frac{E_b - M E_c - E_{01}}{M}\right)^2\right]. \tag{9}$$

As $E_{01} = I_{10T}R_{p1}$, we have

$$E_{01} = A_1R_{p1}\left[\frac{E_{11}^2}{2} + E_{11}E_2\cos\theta + \frac{E_2^2}{2} + \left(\frac{E_b - ME_c - E_{01}}{M}\right)^2\right] . \tag{10}$$

Equation (10) can be rearranged in the form of a quadratic equation, with E_{01} as the variable.

$$\frac{1}{M^2}E_{01}^2 - \left[\frac{1}{A_1R_{p1}} + \frac{2}{M}\left(\frac{E_b}{M} - E_c\right)\right]E_{01} + \frac{E_{11}^2}{2} + E_{11}E_2\cos\theta + \frac{E_2^2}{2} + \left(\frac{E_b}{M} - E_c\right)^2 = 0. \tag{11}$$

By the same procedure, a quadratic equation for the second tube can be written thus:

$$\frac{1}{M^2}E_{02}^2 - \left[\frac{1}{A_2R_{p2}} + \frac{2}{M}\left(\frac{E_b}{M} - E_c\right)\right]E_{02} + \frac{E_{12}^2}{2} - E_{12}E_2\cos\theta + \frac{E_2^2}{2} + \left(\frac{E_b}{M} - E_c\right)^2 = 0. \tag{12}$$

Equations (11) and (12) are now quadratics of the common form

$$ax^2 + bx + c = 0, \tag{13}$$

in which

$$a_1 = a_2 = \frac{1}{M^2}, \tag{14}$$

$$b_1 = -\left[\frac{1}{A_1R_{p1}} + \frac{2}{M}\left(\frac{E_b}{M} - E_c\right)\right], \tag{15}$$

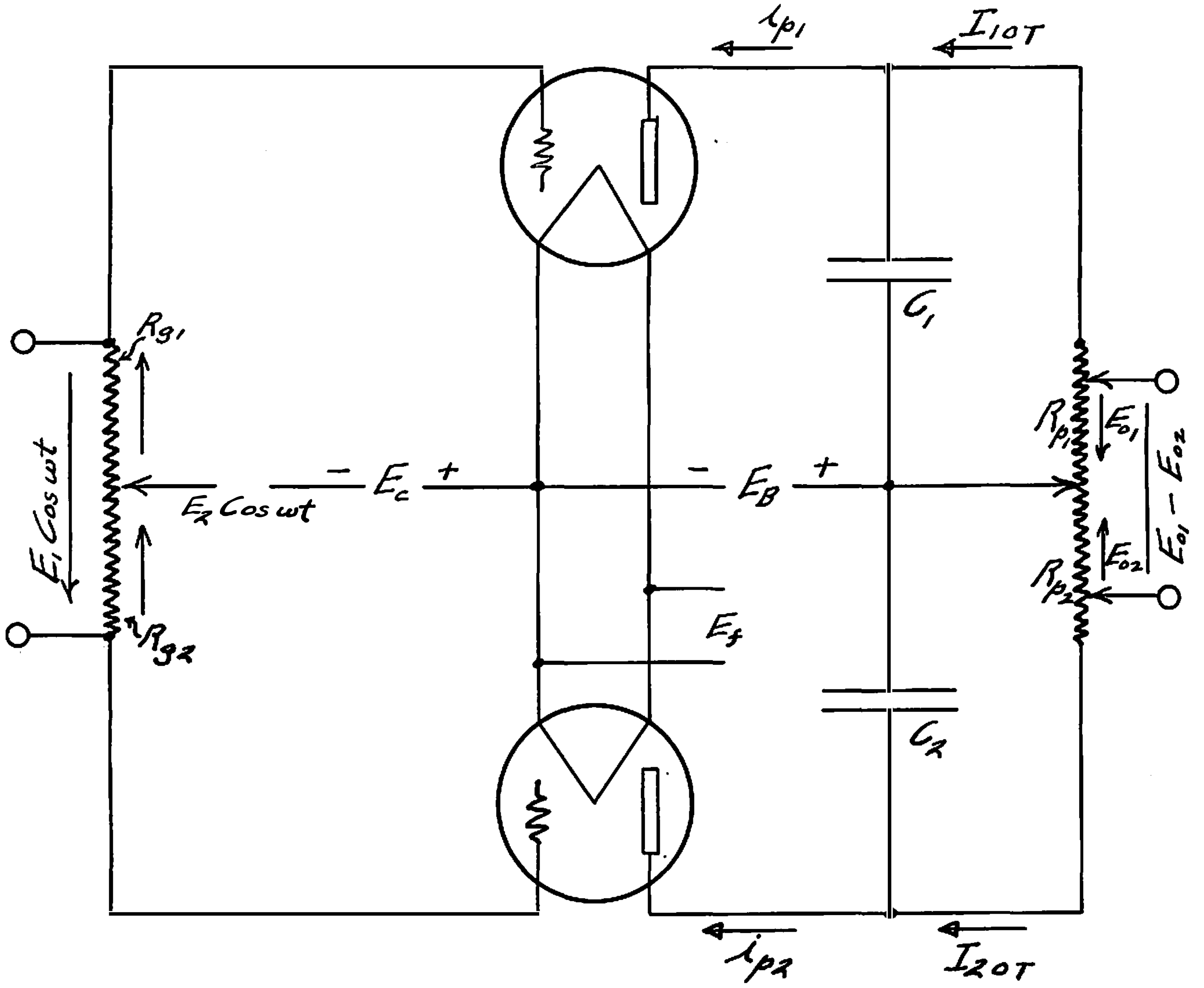

Figure 1

Fundamental Circuit

$$b_2 = -\left[\frac{1}{A_2R_{p2}} + \frac{2}{M}\left(\frac{E_b}{M} - E_c\right)\right], \tag{16}$$

$$c_1 = \frac{E_{11}^2}{2} + E_{11}E_2\cos\theta + \frac{E_2^2}{2} + \left(\frac{E_b}{M} + E_c\right)^2, \tag{17}$$

$$c_2 = \frac{E_{12}^2}{2} - E_{12}E_2\cos\theta + \frac{E_2^2}{2} + \left(\frac{E_b}{M} - E_c\right)^2. \tag{18}$$

An exact solution of equation (13) is, of course,

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \tag{19}$$

An approximation to equation (19) can be obtained by using the Binomial Theorem to expand the radical.

$$x = -\frac{c}{b}\left[1 + \frac{ac}{b^2} + \frac{2a^2c^2}{b^4} + ---\right]. \tag{20}$$

Expressions for both E_{01} and E_{02} can be obtained by applying the above approximation to equations (11) and (12).

$$E_{01} = -\frac{c_1}{b_1}\left[1 + \frac{a_1c_1}{b_1^2} + \frac{2a_1^2c_1^2}{b_1^4} + ---\right], \tag{21}$$

and

$$E_{02} = -\frac{c_2}{b_2}\left[1 + \frac{a_2c_2}{b_2^2} + \frac{2a_2^2c_2^2}{b_2^4} + ---\right]. \tag{22}$$

Before proceeding further, it is well to consider equation (20). Only three terms of the expansion are given, and it will be shown later that the use of only the first two terms will yield sufficiently accurate results.

Subtracting equations (21) and (22), we have,

$$E_{01} - E_{02} = \frac{c_2}{b_2} - \frac{c_1}{b_1} + \frac{a_2 c_2^2}{b_2^3} - \frac{a_1 c_1^2}{b_1^3} . \tag{23}$$

In this expression, $a_1 = a_2 = \frac{1}{M^2}$, if the two tubes used have the same amplification factor. This fact will simplify equation (23), but it can be further simplified if b_1 and b_2 can be made equal by some means. Let us now consider how this can be brought about. In the expression for b_1 and b_2, (equations (15) and (16)), it is obvious that b_1 would be equal to b_2 if $A_1 = A_2$ and $R_{p1} = R_{p2}$, or if $A_1 R_{p1} = A_2 R_{p2}$, and of course if the same amplification factor is assumed. If the two tubes were exactly identical, A_1 would equal A_2, and R_{p1} could be made equal to R_{p2}, and then b_1 would be equal to b_2. However, A_1 may not be equal to A_2, (see fig. 2), but even if this is so, b_1 can be made equal to b_2 by the following circuit adjustment.

Consider the circuit operating with no A. C. grid excitation. Then the plate current of the first tube from equation (1) is all D. C. and is

$$I_{p1} = A_1 \left[-E_c + \frac{E_b - I_{p1} R_{p1}}{M} \right]^2 , \tag{24}$$

also

$$I_{p2} = A_2 \left[-E_c + \frac{E_b - I_{p2} R_{p2}}{M} \right]^2 . \tag{25}$$

Multiplying equations (24) and (25) through by R_{p1} and R_{p2},

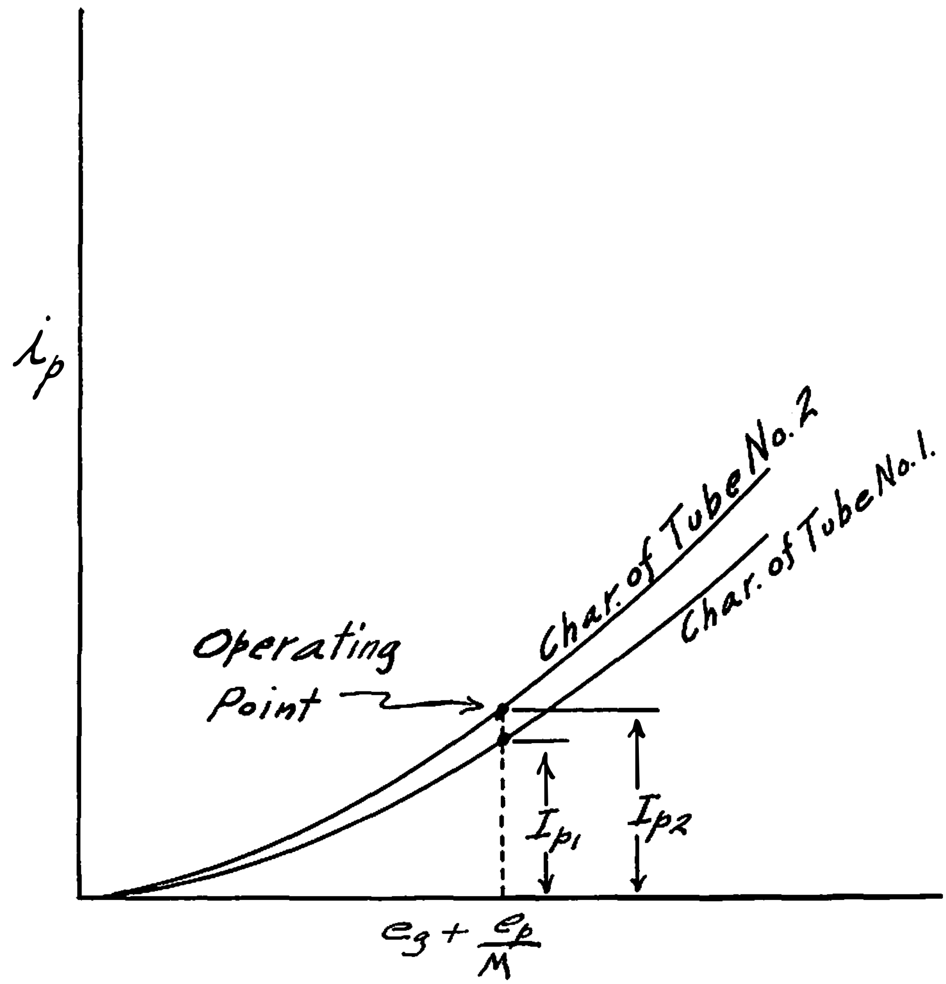

Figure 2

Operating Characteristics

we have,

$$I_{p1}R_{p1} = A_1R_{p1}\left[-E_c + \frac{E_b - I_{p1}R_{p1}}{M}\right]^2, \tag{26}$$

$$I_{p2}R_{p2} = A_2R_{p2}\left[- E_c + \frac{E_b - I_{p2}R_{p2}}{M}\right]^2. \tag{27}$$

A study of equations (26) and (27) will show that if $I_{p1}R_{p1}$ is made equal to $I_{p2}R_{p2}$, then $A_1R_{p1} = A_2R_{p2}$, which is necessary to make $b_1 = b_2$. As the D. C. output of the meter, with no grid excitation, is $I_{p1}R_{p1} - I_{p2}R_{p2}$, the desired condition can be realized by adjusting the tap on the plate circuit resistance until the D. C. output is zero. Then $I_{p1}R_{p1} = I_{p2}R_{p2}$, $A_1R_{p1} = A_2R_{p2}$, and $b_1 = b_2$.

Now, if two A. C. grid excitation voltages are applied that are exactly 90 degrees out of phase, ($\cos\theta = 0$), an examination of equatio ns (17) and (18) will show that $c_1 = c_2$. Under these conditions, the output of the meter from equation (23) should be zero. If it is not zero, it must imply that the two tubes have different amplification factors, so they must be replaced and the above adjustment repeated until two are found that have the same M. It is rare, however, that tubes of the same type have different values of M.

When the above adjustments are completed, equation (23) can be written

$$E_{01} - E_{02} = \frac{c_2 - c_1}{b_1}\left[1 + \frac{a_1}{b_1^2}(c_2 + c_1)\right], \tag{28}$$

and in this, one can see by an examination of equations (17) and (18) that $(c_2 - c_1)$ is proportional to Cos θ, and that $(c_2 + c_1)$ does not involve Cos θ, or

$$(c_2 - c_1) = - E_1E_2 \cos\theta, \tag{29}$$

$$(c_2 + c_1) = \left(\frac{E_1}{2}\right)^2 + E_2^2 + 2\left(\frac{E_b}{M} - E_c\right)^2. \tag{30}$$

This means that equation (28) is of the form

$$E_{01} - E_{02} = K \cos\theta, \tag{31}$$

where

$$K = \frac{-E_1E_2}{b_1}\left[1 + \frac{a_1}{b_1^2}\left\{\frac{E_1^2}{4} + E_2^2 + 2\left(\frac{E_b}{M} - E_c\right)^2\right\}\right]. \tag{32}$$

The value of K can be found by reading the D. C. output voltage when the two grid voltages are applied in phase, so that Cos θ = 1. Then $(E_{01} - E_{02}) = K$, and the meter is calibrated for all values of Cos θ, as long as the magnitude of E_1 and E_2 remain the same.

Now we must consider the error made in the approximation used in writing equation (20), and the fact that only two terms of equation (20) were actually used in writing equation (23).

Using all the terms in equation (20), equation (23) becomes

$$E_{01} - E_{02} = -\frac{c_1}{b} + \frac{c_2}{b} - \frac{ac_1^2}{b^3} + \frac{ac_2^2}{b^3} - \frac{2a^2c_1^3}{b^5} + \frac{2a^2c_2^3}{b^5} \quad , \qquad (33)$$

assuming that $a_1 = a_2$, $b_1 = b_2$, and rearranging the order of the terms. The numerical value of each term will be computed, and a careful consideration of this data, and of equation (33), will show that it is in the form of an alternating series,[1] where each successive term is smaller in numerical value than the one before. There is a mathematical property of a series of this type that is very useful here. In such a series, the difference between the sum of all the terms in the series and the sum of n terms is less than the numerical value of the next, or (n + 1) th term.

Equation (23) takes into account the first four terms of the series, so the correct value of $(E_{01} - E_{02})$, as represented by the sum of all the terms in the series, differs from the sum of the first four by an amount less than the numerical value of the fifth term.

The following data is now submitted, computed on the basis of the operating characteristics of a type 201-A tube; and assuming the following conditions of operation,

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[1] E. B. Wilsonn, Advanced Calculus (New York: Ginn and Company, 1912), pp. 420-1.

after the two adjustments have been made:

$E_b = 60$ volts

$E_c = 4$ volts

$M = 8$

$A_1 = 45 \times 10^{-6} \dfrac{\text{mhos}}{\text{volt}}$

$A_2 = 50 \times 10^{-6} \dfrac{\text{mhos}}{\text{volt}}$

$R_{g1} = R_{g2}$

$R_{p1} = 5260$ ohms

$R_{p2} = 4740$ ohms

$A_1 R_{p1} = A_2 R_{p2} = 0.237$

$E_1 = E_2 = 1$ volt

$\cos\theta = 1.00$

$E_{01} - E_{02} = 0.1993$ volt

$a = 15.625 \times 10^{-3}$

$b = -5.094$

$c_1 = 13.375$

$c_2 = 12.375$

$-\dfrac{c_1}{b} = 2.62563$

$\dfrac{c_2}{b} = -2.42932$

$-\dfrac{ac_1^2}{b^3} = 0.0211$

$\dfrac{ac_2^2}{b^3} = -0.0181$

$\dfrac{-2a^2c_1^3}{b^5} = 0.000343$

$\dfrac{2a^2c_2^3}{b^5} = -0.00026\ 8$

An examination of this data shows that each term is smaller than the one preceding, and the signs alternate, so we have a true alternating series.

The value of $(E_{01} - E_{02})$ obtained by using all the terms in equation (33) is 0.19938 volt. Using four terms, it is 0.19931. Obviously four terms are adequate, as the metering device used to measure (E_{01} - E_{02}) would not measure the value of the voltage to the fifth decimal place. From these figures, the total error made in the approximation is less than $\frac{0.000343}{0.19931} \times 100 = 0.179\%$.

CHAPTER III

THE VACUUM TUBE VOLTMETER CIRCUIT

In Chapter II, the theory of operation of the meter was developed on the basis that the D. C. output voltage was to be measured with a voltmeter of infinite impedance, and that no current was to be drawn from the output terminals. To fulfill these conditions, it is necessary to operate the phase angle meter stage into another stage, which is, essentially, a vacuum tube voltmeter .

The circuit details of this stage are shown in Fig. 3. One tube is used, and the grid voltage of this tube is controlled by the output voltage of the phase angle meter. The voltmeter tube must be operated on the linear portion of the characteristic curve, in order to obtain a linear relationship between the input voltage and the plate current. As a steady plate current flows in the tube at all times, a back-up circuit is provided, to enable the use of a very sensitive meter to detect changes in the plate current produced by any change in $(E_{01} - E_{02})$.

The meter is calibrated by throwing the switch, S_1, to the right and then adjusting the variable resistance, R_1, until the meter reads zero. Then, by throwing the switch to the left, the vacuum tube voltmeter is connected to the phase angle meter stage, and the current read on the meter is proportional to the voltage $(E_{01} - E_{02})$.

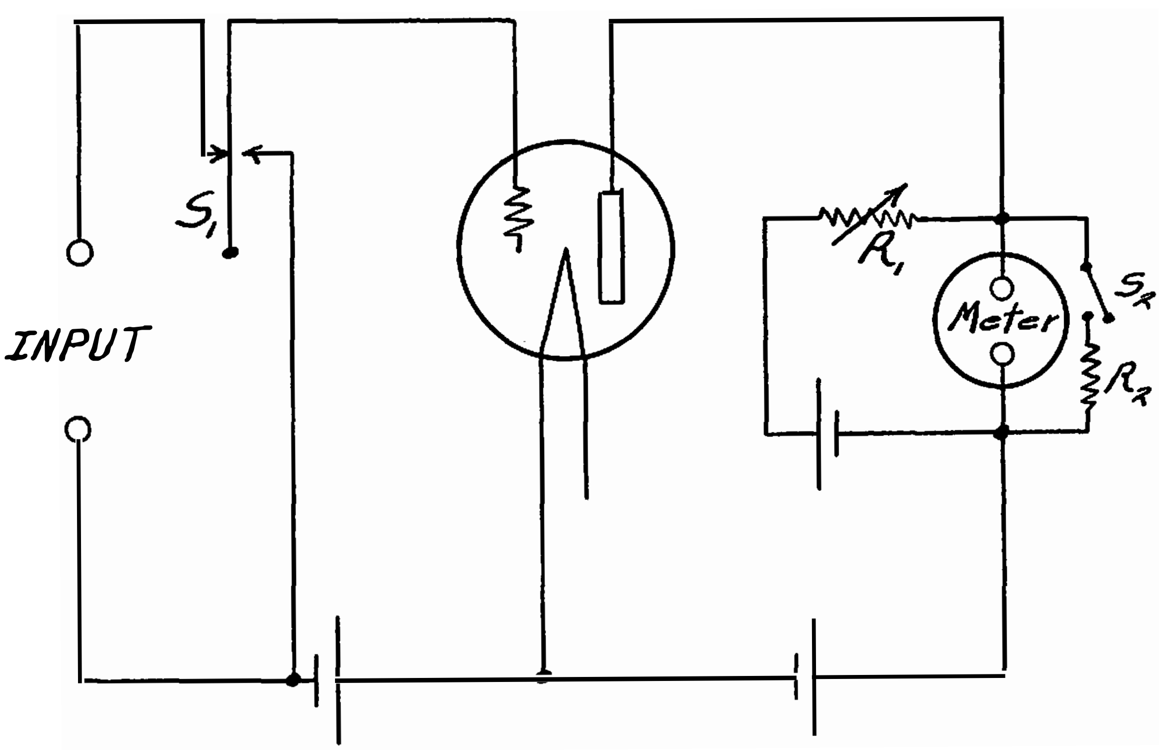

Figure 3

Vacuum Tube Voltmeter Circuit

This circuit, when added to the fundamental circuit of Fig. 1., completes the circuit of the electronic phase angle meter. There is, of course, considerable latitude in choosing the various components for these circuits. However, the tubes used in the phase angle meter should have the parabolic characteristic mentioned previously, as well as a fairly high amplification factor. The voltmeter tube does not require the parabolic characteristic, but should have a reasonable amplification factor.

CHAPTER IV

A SUGGESTED CIRCUIT FOR COMMERCIAL OPERATION

In this chapter, the writer wishes to present a suggested circuit that can be built around the fundamental phase angle meter circuit. This system would have a wide application in the communication and power fields, and could be built up at a reasonable figure.

One obvious defect of the fundamental phase angle meter circuit is the fact that changes in the amplitude of the two A. C. grid voltages will affect the accuracy of the meter reading.

To correct this, a line-up of two phase angle meter stages can be used, as shown in Fig. 4. The output of the first meter is proportional to Cos θ, or

$$E_{dc1} = E_{01} - E_{02} = K \cos \theta.$$

The output of the second meter can be made proportional to Sin θ, or

$$E_{dc2} = E_{03} - E_{04} = K \sin \theta,$$

by shifting the phase of its second grid voltage 90 degrees with a suitable phase shifting network. This can be shown by a review of the theory of Chapter II, substituting for $E_2 \cos(\omega t - \theta)$, $E_2 \sin(\omega t - \theta)$. By proper adjustment of the output taps on each meter, the same value of K could be had

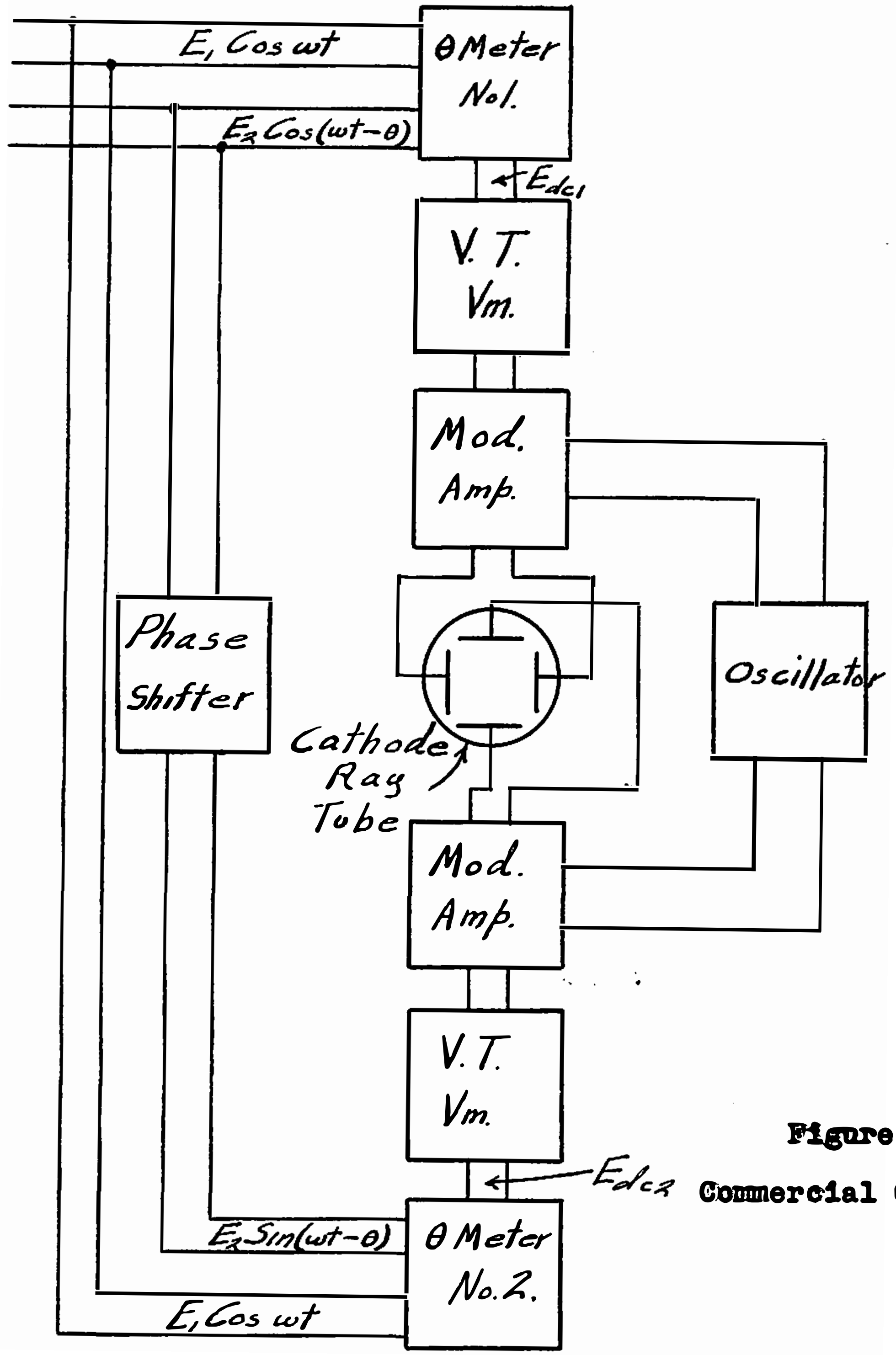

Figure 4

Commercial Circuit

for both meters. As a change in the magnitude of either E_1 or E_2 would change the K for both meters, the value of θ could still be determined by the ratio of the two output voltages, as

$$\frac{K \cos \theta}{K \sin \theta} = \cot \theta,$$

and is independent of the value of K.

A visual picture of the angle θ can be obtained by the use of the other apparatus shown in Fig. 4. It is proposed to use the D. C. output of each meter to modulate the amplitude of two high frequency, in phase, A. C. voltages, that are to be applied to the horizontal and vertical deflecting plates of a cathode ray tube. It can be shown that, after all components are properly adjusted, the resulting pattern on the screen of the cathode ray tube will be a straight line, and the angle of this line with respect to the horizontal axis of the screen is θ.

Such a system could be used to study transient phenomena by photographing the pattern on the screen with a motion picture camera.

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BIBLIOGRAPHY

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